http://oeis.org/A018210 - F(1,6,n)

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15.03.2012

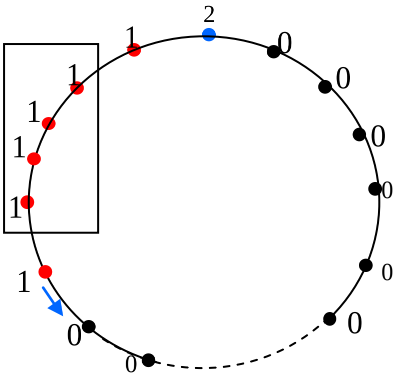
<u>Explanation</u>: Number of bracelets made with 1 blue, 6 identical red and n identical black beads.

<u>Usage</u>: Chemistry: Paraffin numbers, Maths: Circular permutations of identical objects

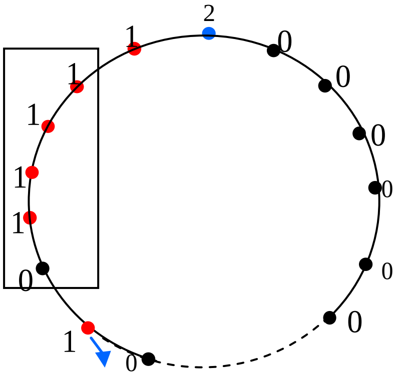
Teorem : If F(1,6,n) is the number of bracelets made with 1 blue, 6 identical red and n identical black beads,

$$F(1,6,n)=\frac{(n+4)(n+3).(n+2).(n+1).n}{120}+F(1,4,n)+F(1,6,n-2)$$

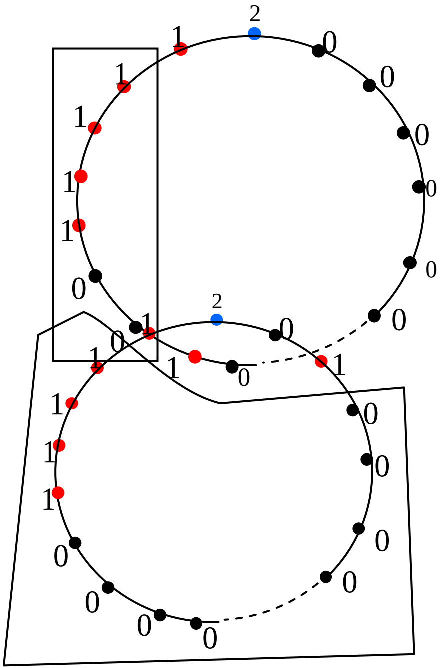
Proof :



For 1111 there are $\binom{4}{4}$ combinatorial states



For 11110 there are $\binom{5}{4}$ combinatorial states



For 111100 kendi there are $\binom{6}{4}$ combinatorial states.

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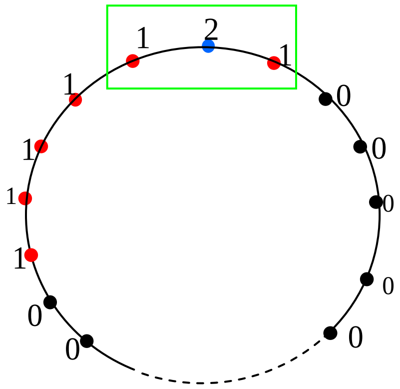
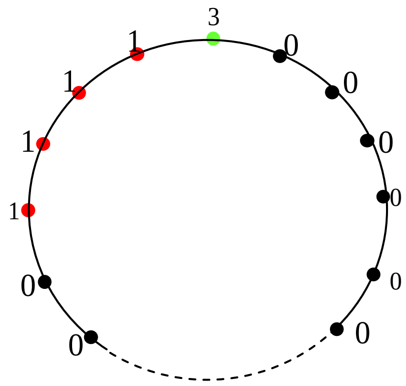
If we continue similarly, for $111\underbrace{00...0}_{n-1}$ there are $\binom{n+3}{4}$ states.

If we sum up all the states,

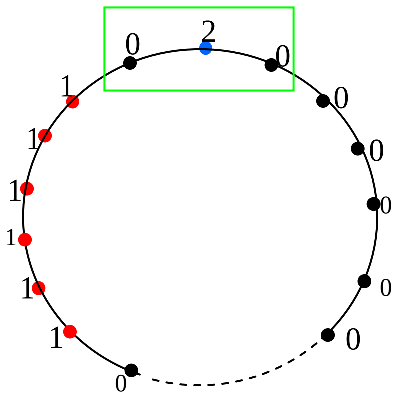
$$\binom{4}{4}+\binom{5}{4}+\binom{6}{4}+...+\binom{n+3}{4}=\binom{n+4}{5}=\frac{(n+4)(n+3).(n+2).(n+1).n}{120}$$



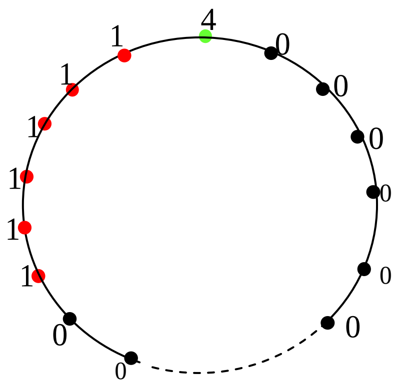
$(121)\rightarrow 3$



There are $F(1,4,n)$ states



$(121)\rightarrow 4$



There are $F(1,6,n-2)$ states.

Total number of possible states:

$$F(1,6,n)=\frac{(n+4)(n+3).(n+2).(n+1).n}{120}+F(1,4,n)+F(1,6,n-2)$$

Using $F(1,4,1)=3$, $F(1,4,2)=9$

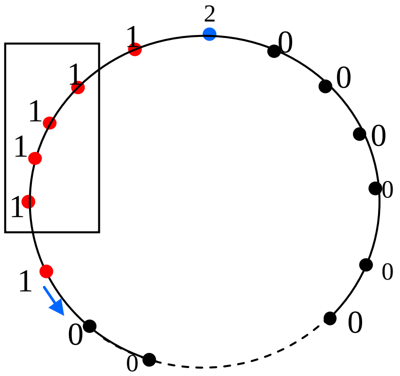
$F(1,6,1)=4$, $F(1,6,2)=16$
$F(1,6,3)=44$, $F(1,6,4)=110$
$F(1,6,5)=236$, $F(1,6,6)=472$

Turkish:

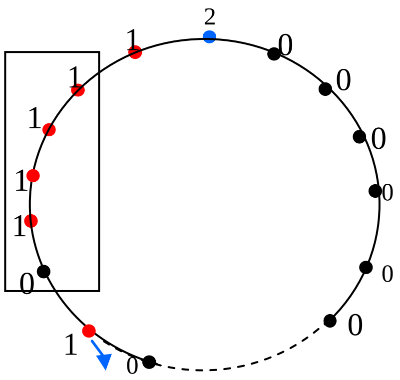
Teorem4 : 1 tane özdeş mavi, 6 tane özdeş kırmızı ve n tane özdeş siyah boncuklar ile yapılacak bilekliklerin sayısı F(1,6,n) ise

$$F(1,6,n)=\frac{(n+4)(n+3).(n+2).(n+1).n}{120}+F(1,4,n)+F(1,6,n-2)$$

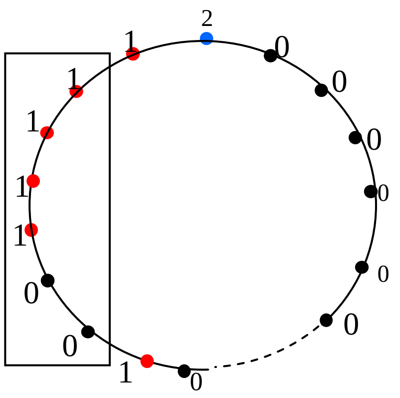
İspat :



1111 kendi arasındaki sıralaması $\binom{4}{4}$, tane durum vardır



11110 kendi arasındaki sıralaması $\binom{5}{4}$ tane durum vardır



111100 kendi arasındaki sıralaması

$\binom{6}{4}$ tane durum vardır

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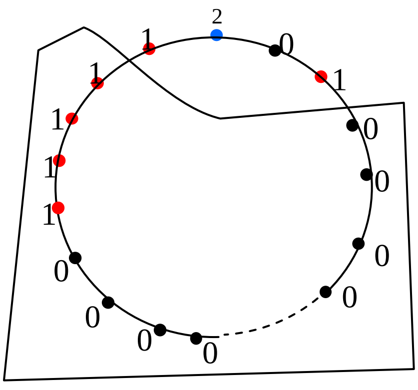
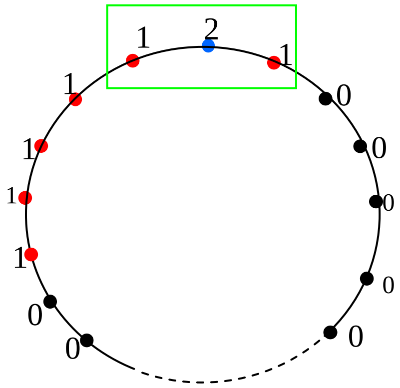


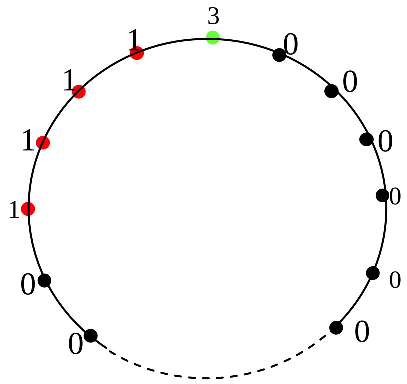
Benzer olarak devam edersek . $111\underbrace{00...0}_{n-1}$ kendi arasındaki sıralaması $\binom{n+3}{4}$ dir.

Elde ettiğimiz bütün durumları toplarsak

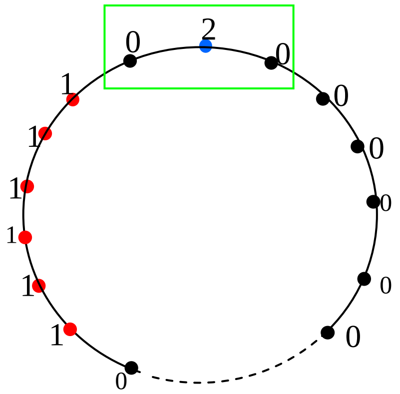
$$\binom{4}{4}+\binom{5}{4}+\binom{6}{4}+\ldots+\binom{n+3}{4}=\binom{n+4}{5}=\frac{(n+4)(n+3).(n+2).(n+1).n}{120}$$



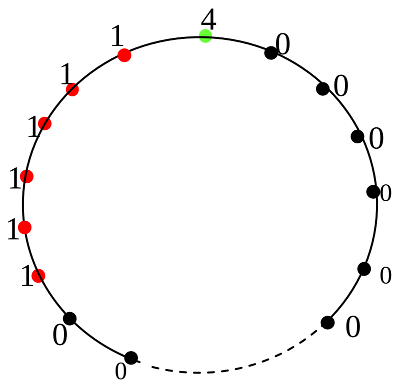
$(121)\rightarrow 3$



$F(1,4,n)$ Tane durum vardır



$(121)\rightarrow 4$



$F(1,6,n-2)$ Tane durum vardır .

Oluşacak toplam durum sayısı:

$$F(1,6,n)=\frac{(n+4)(n+3).(n+2).(n+1).n}{120}+F(1,4,n)+F(1,6,n-2)$$

$F(1,4,1)=3$, $F(1,4,2)=9$

$F(1,6,1)=4$
$F(1,6,2)=16$
$F(1,6,3)=44$
$F(1,6,4)=110$
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