http://oeis.org/A005994 - F(1,4,n)

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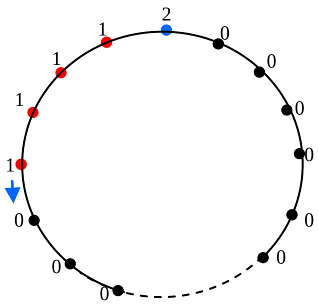
Explanation: Number of bracelets made with 1 blue, 4 identical red and n identical black beads.

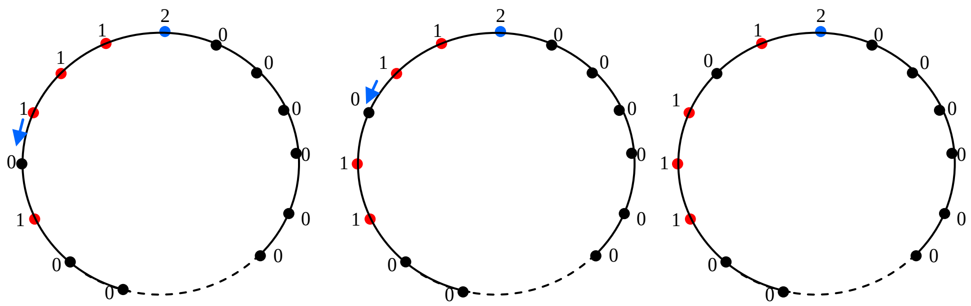
Usage: Chemistry: Paraffin numbers, Maths: Circular permutations of identical objects

Teorem 7 : 1 tane özdeş mavi, 4 tane özdeş kırmızı ve n tane özdeş siyah boncuklar ile yapılacak bilekliklerin sayısı $F(1,4,n)$ ise

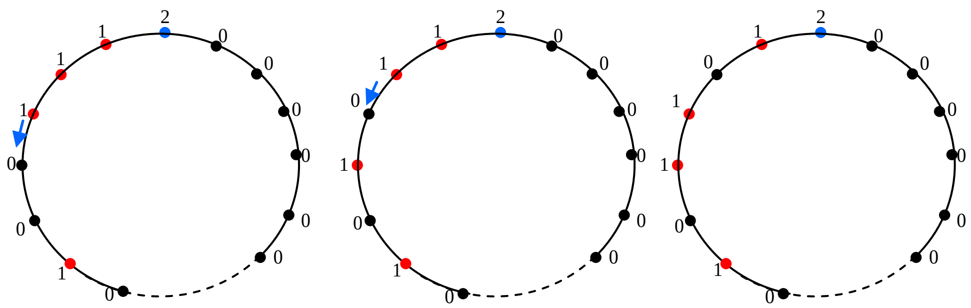
$$F(1,4,n)=\frac{n.(n+1).(n+2)}{6}+F(1,2,n)+F(1,4,n-2)\text{ dir.}$$

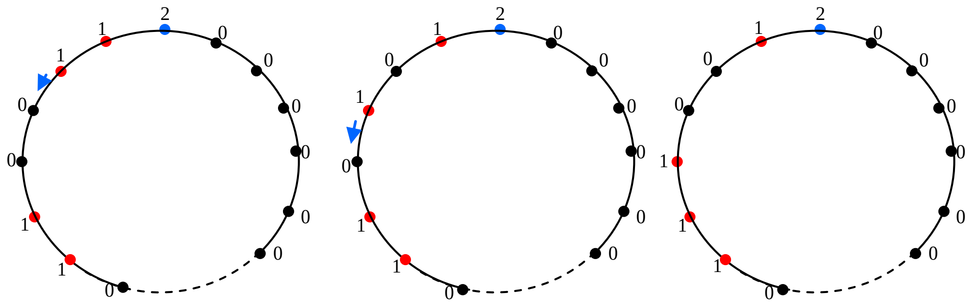


1 durum. Başlangıçtaki 1'i hareket ettirip arkadan gelen 1'leri yanına çekerek genel durumu oluşturmaya çalışalım.



$1+\{1+1\}=3$ tane durum vardır.

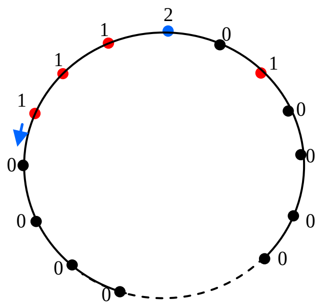




$1+\{1+1\}+\{1+2\}=6$

Tane durum vardır.

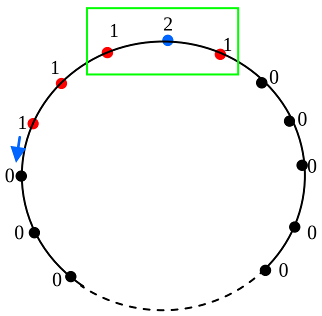
Benzer olarak 1'in 3 birimlik hareketine karşı $1+\{1+1\}+\{1+2\}+\{1+3\}=10$ tane durum vardır.
Benzer olarak devam edilirse;



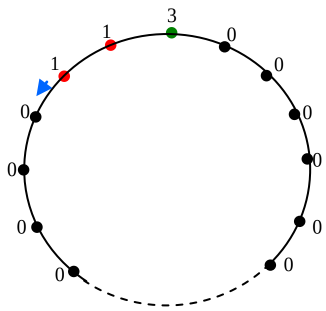
$1+\{1+1\}+\{1+2\}+\{1+3\}+..+\{1+(n-1)\}=\dfrac{n.(n+1)}{2}$ tane durum vardır.

Elde ettiğimiz bütün durumların toplamını:

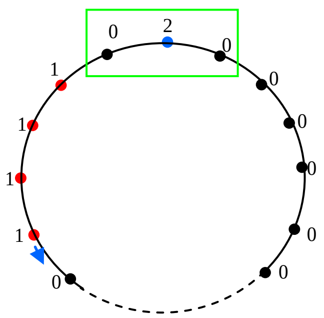
$$\sum_{k=1}^{n}\frac{k.(k+1)}{2}=\frac{1}{2}\sum_{k=1}^{n}(k^2+k)=\frac{1}{2}\left(\frac{n.(n+1)(2n+1)}{6}+\frac{n.(n+1)}{2}\right)=\frac{n.(n+1).(n+2)}{6}$$



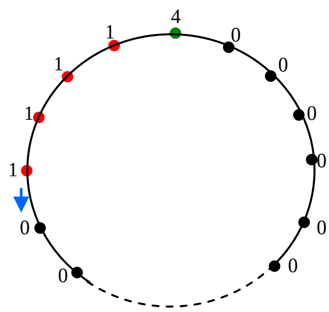
$(121)\rightarrow 3$ ile gösterirsek



$F(1,2,n)$ tane durum vardır.



$(020)\rightarrow 4$ ile gösterirsek



$F(1,4,n-2)$ durum oluşur.

Oluşacak toplam durum sayısı:

$F(1,4,n) = \frac{n.(n+1).(n+2)}{6} + F(1,2,n) + F(1,4,n-2)$ ile ifade edebiliriz.

$$F(1,4,3) = \frac{3.4.5}{6} + F(1,2,3) + F(1,4,1) = 10 + 6 + 3 = 19$$

$$F(1,4,4) = \frac{4.5.6}{6} + F(1,2,4) + F(1,4,2) = 20 + 9 + 9 = 38$$

$$F(1,4,5) = \frac{5.6.7}{6} + F(1,2,5) + F(1,4,3) = 35 + 12 + 19 = 66$$

$$F(1,4,6) = \frac{6.7.8}{6} + F(1,2,6) + F(1,4,4) = 56 + 16 + 38 = 110$$

$$F(1,4,7) = \frac{7.8.9}{6} + F(1,2,7) + F(1,4,5) = 84 + 20 + 66 = 170$$