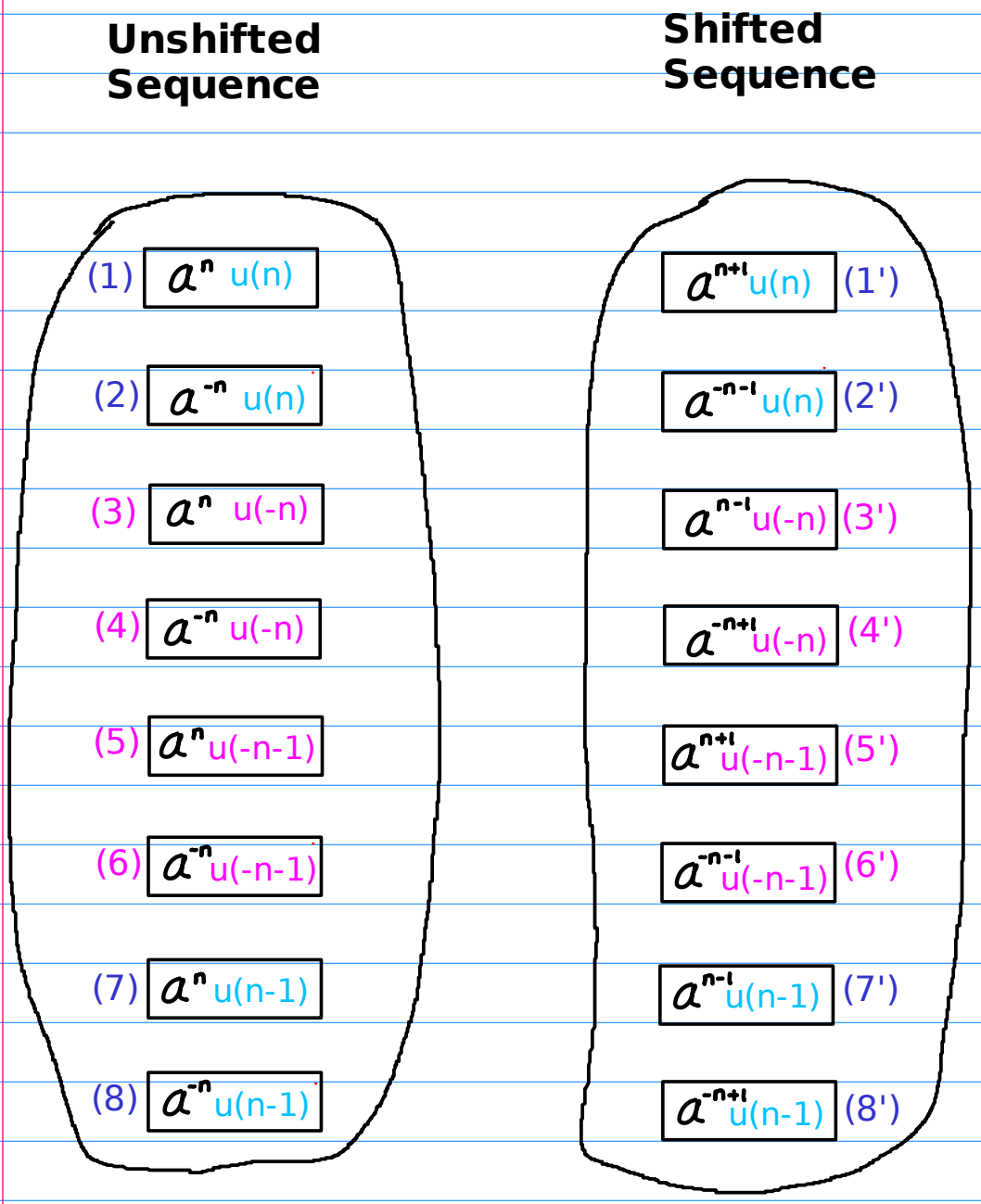
Laurent Series and z-Transform - Geometric Series Permutations B

20240430 Tue



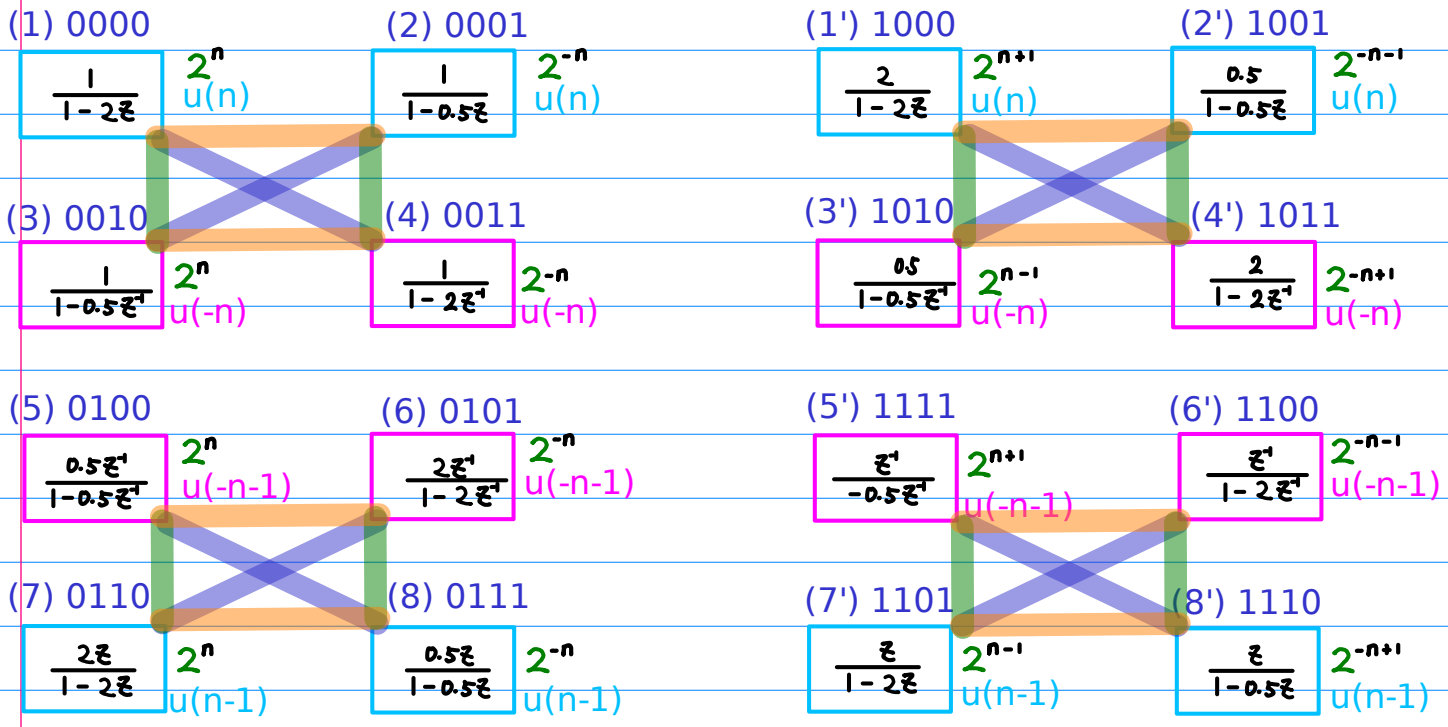
Inter-permutations over unshifted sequnce and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

Inter-permutation (x) --> (x')

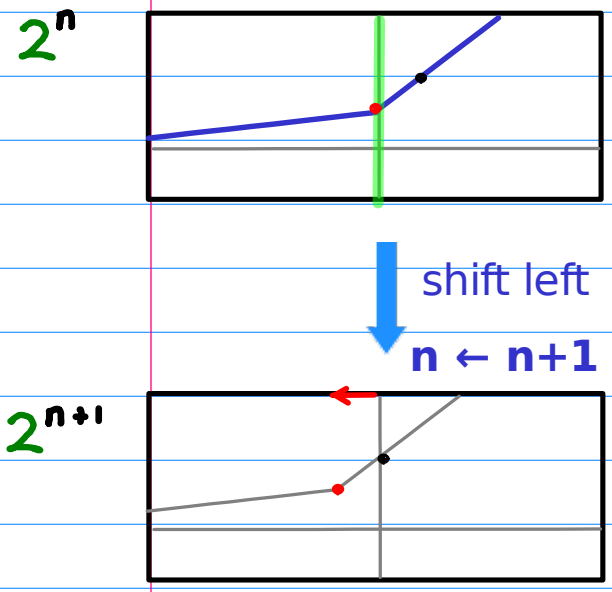
(1)~(8) --> (1')~(8')



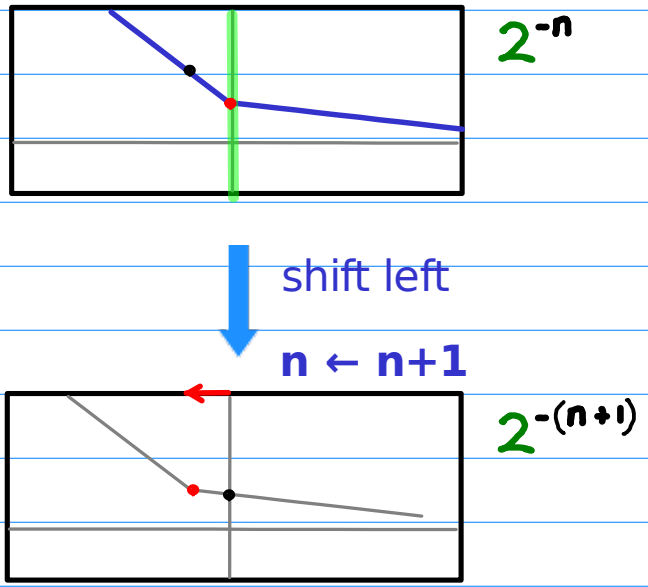
(1)	0	0	0	0	→	1	0	0	0	(1′)	n	→	$n+1$	**shl(n)**
(2)	0	0	0	1	→	1	0	0	1	(2′)	n	→	$n+1$	**shl(n)**
(3)	0	0	1	0	→	1	0	1	0	(3′)	n	→	$n-1$	**shr(n)**
(4)	0	0	1	1	→	1	0	1	1	(4′)	n	→	$n-1$	**shr(n)**
(5)	0	1	0	0	→	1	1	0	0	(5′)	n	→	$n+1$	**shl(n)**
(6)	0	1	0	1	→	1	1	0	1	(6′)	n	→	$n+1$	**shl(n)**
(7)	0	1	1	0	→	1	1	1	0	(7′)	n	→	$n-1$	**shr(n)**
(8)	0	1	1	1	→	1	1	1	1	(7′)	n	→	$n-1$	**shr(n)**

shl(n) = n + 1
shr(n) = n - 1

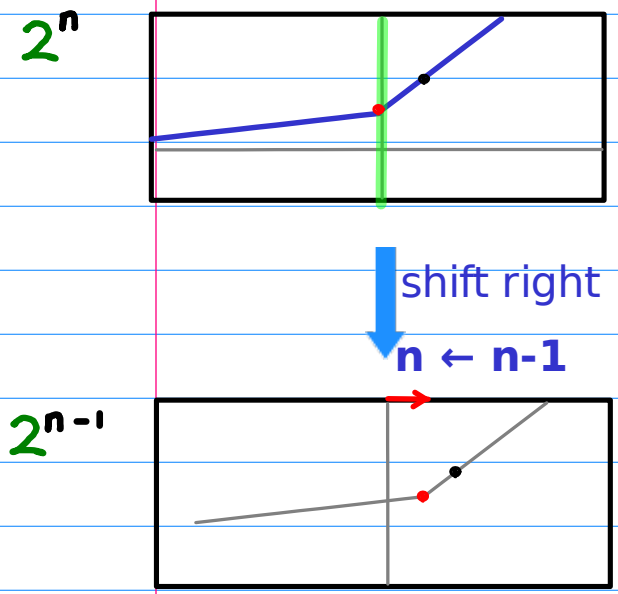
(1) -- (1')
(5) -- (5')



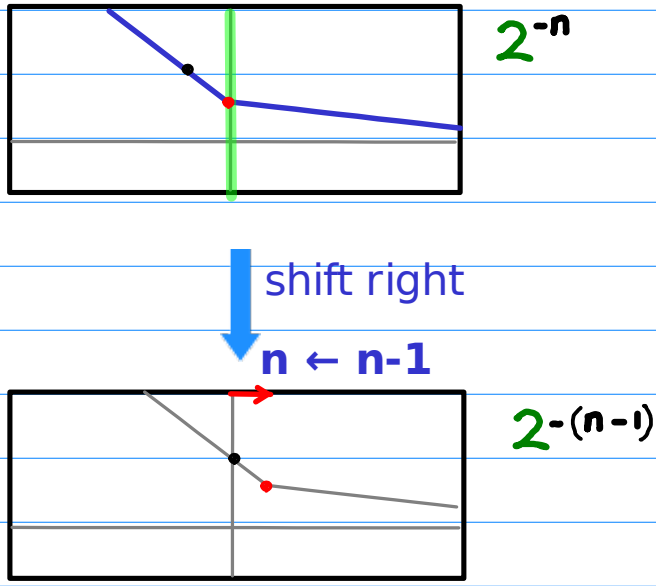
(2) -- (2')
(6) -- (6')



(3) -- (3')
(7) -- (7')



(4) -- (4')
(8) -- (8')

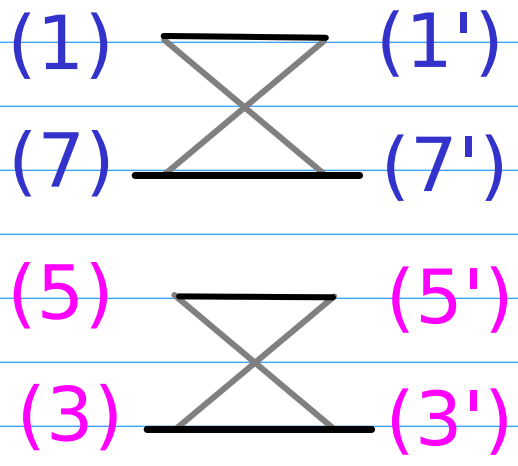


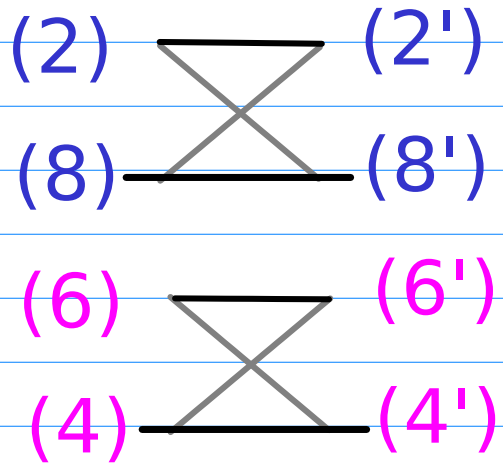
(1) - (1')
(7) - (7')
(5) - (5')
(3) - (3')

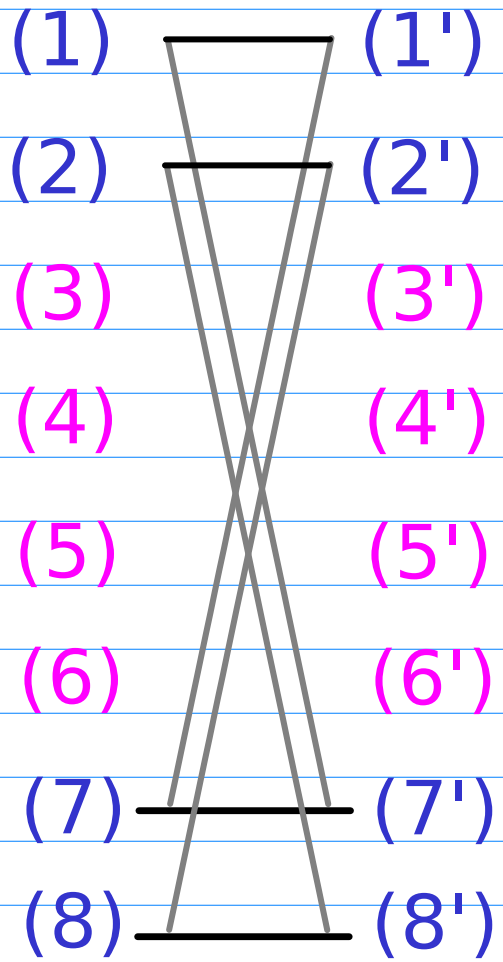
(2) - (2')
(8) - (8')
(6) - (6')
(4) - (4')

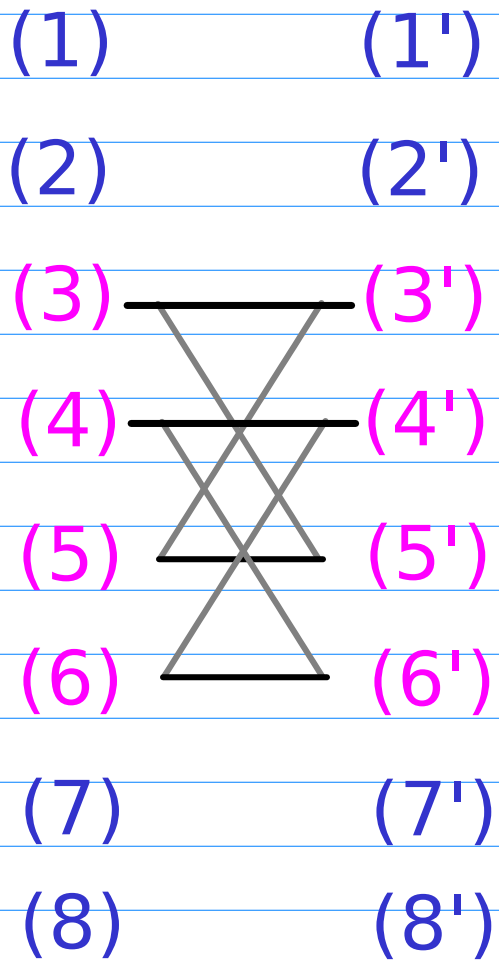
(1) - (1')
(2) - (2')
(3) - (3')
(4) - (4')

(5) - (5')
(6) - (6')
(7) - (7')
(8) - (8')

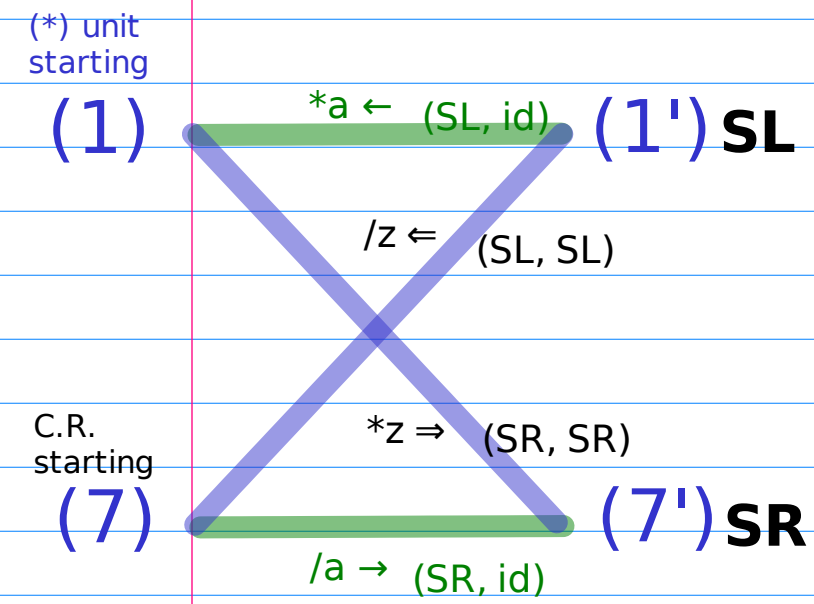


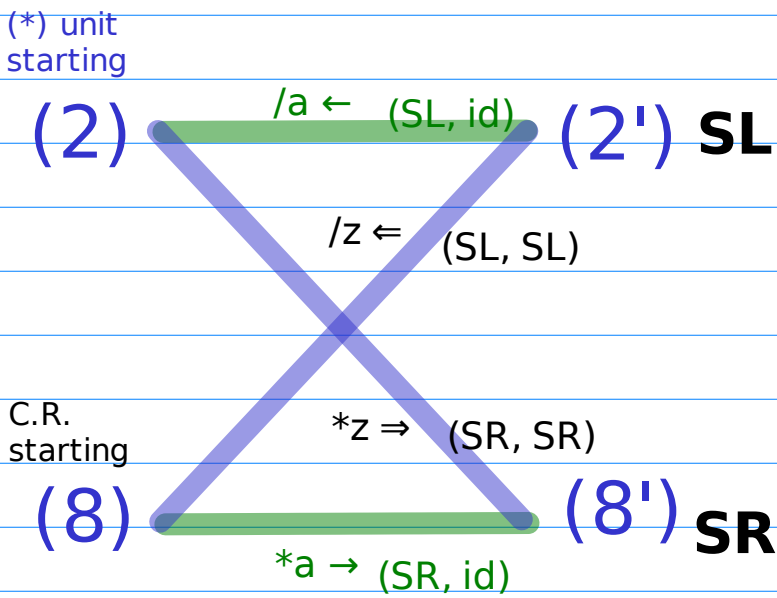


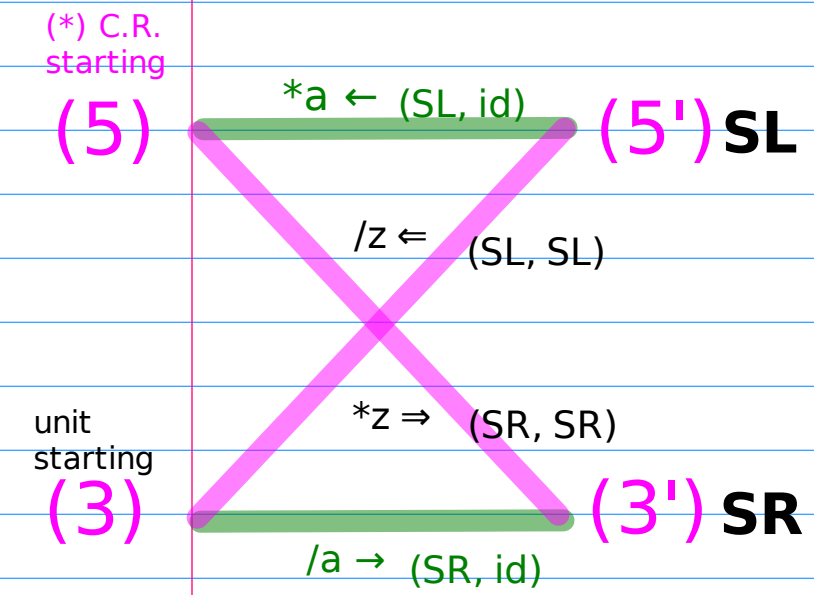


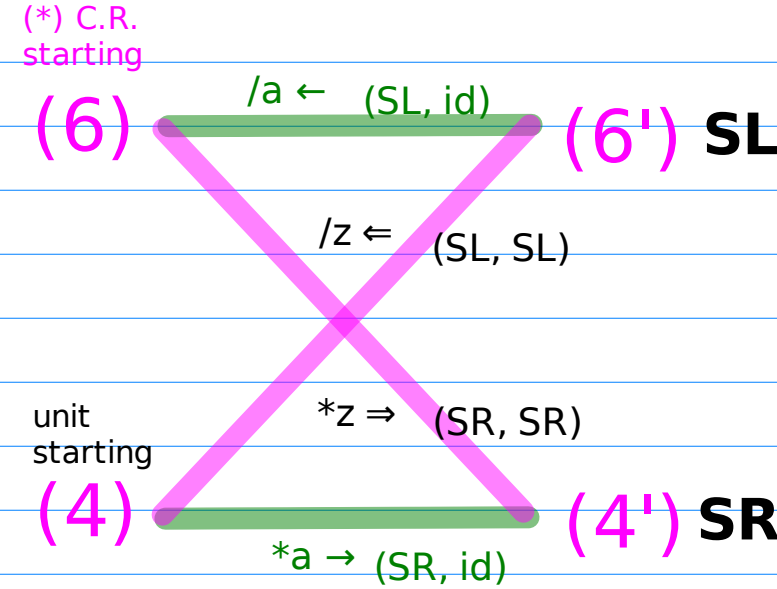


Butterfly Relations



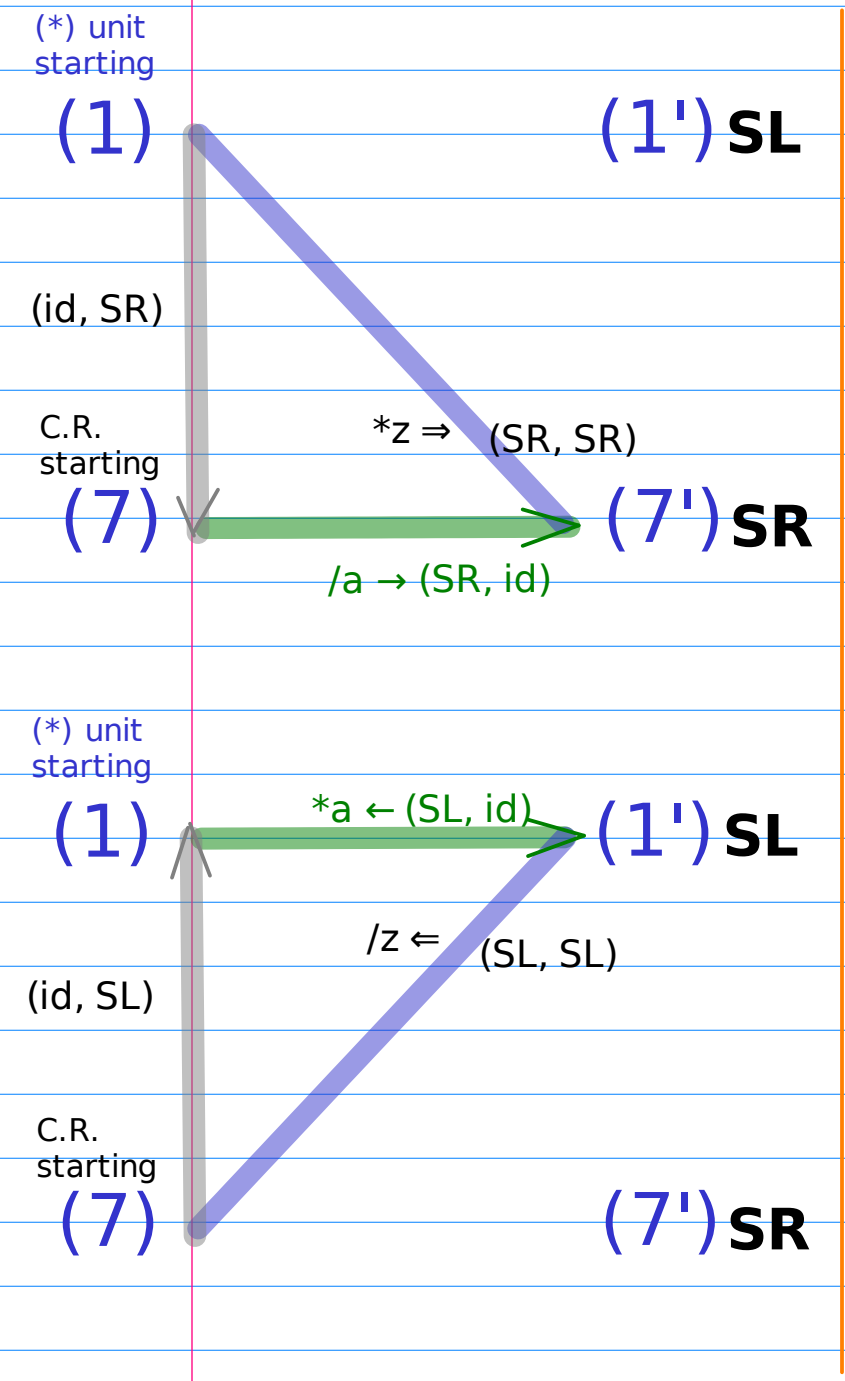


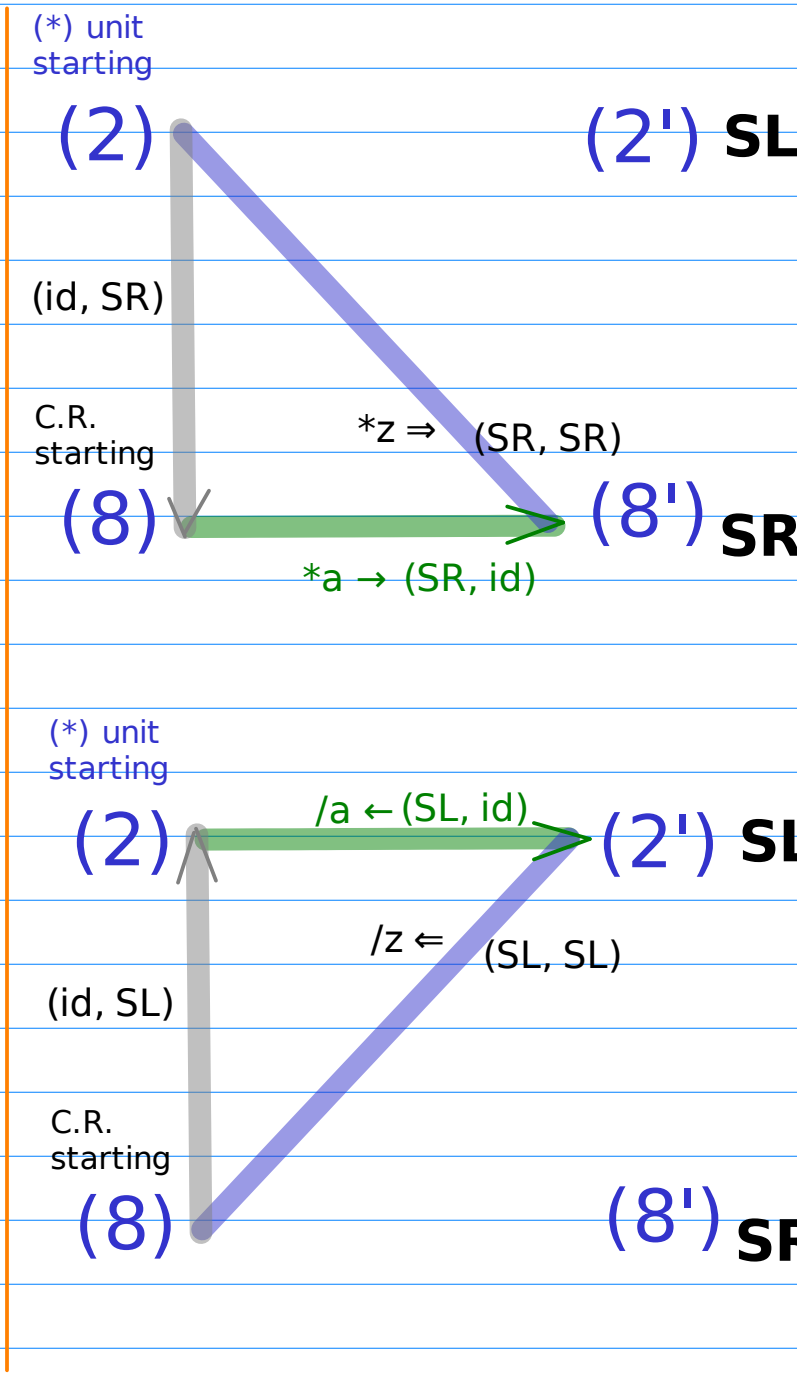




(Exp Shift, Range Shift)

Decomposition of Exp and Rng Shifts

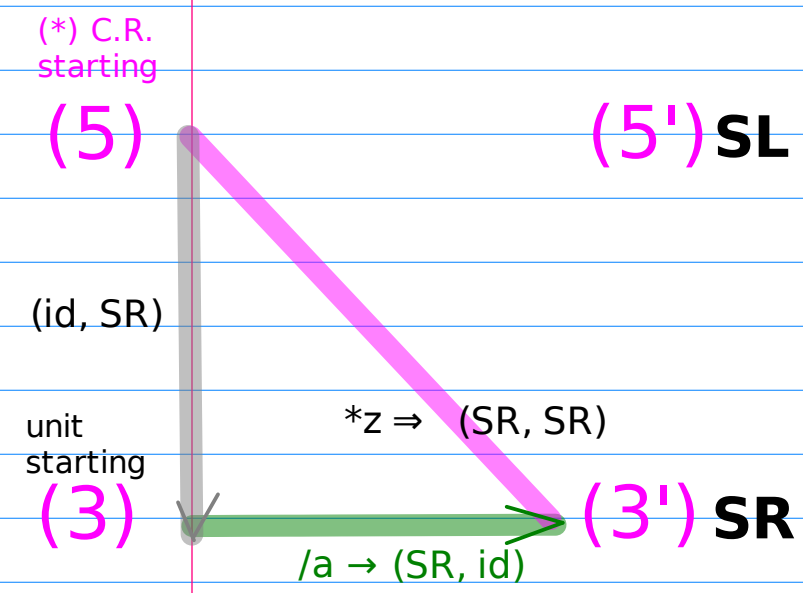


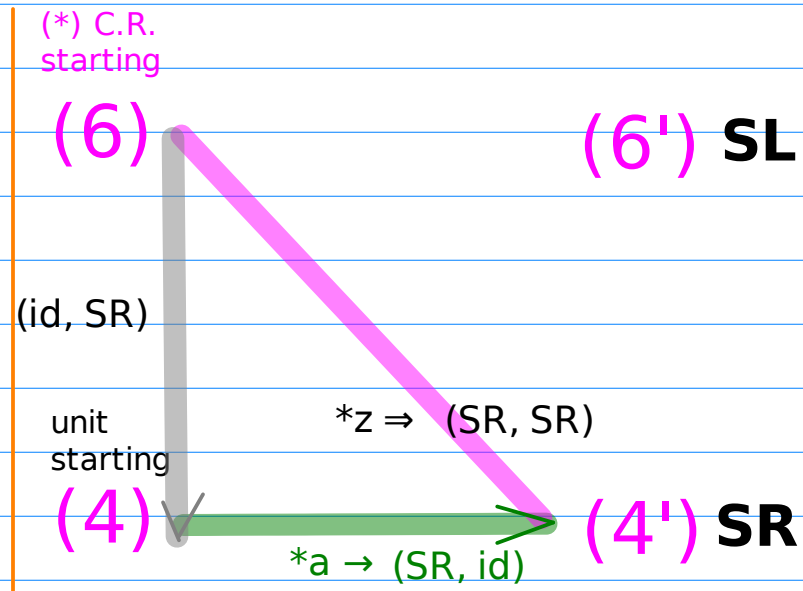


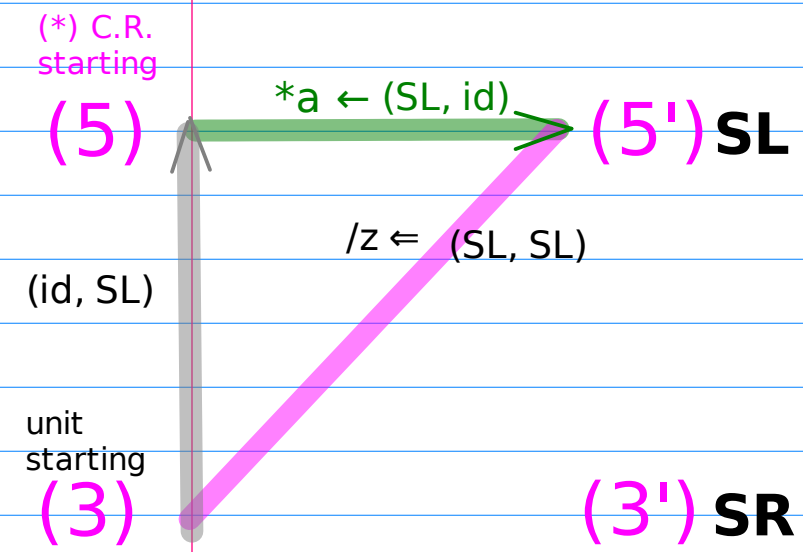
(id, SR) + (SR, id) = (SR, SR)
(id, SL) + (SL, id) = (SR, SL)

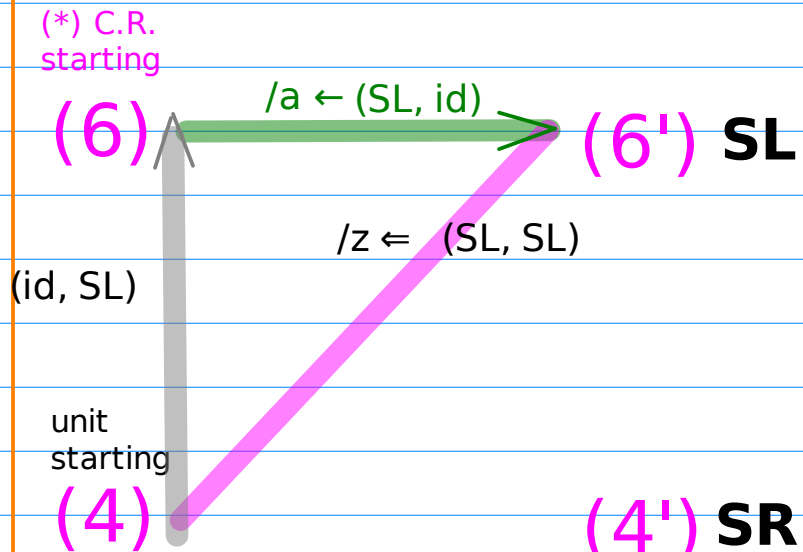
(id, SR) + (SR, id) = (SR, SR)
(id, SL) + (SL, id) = (SR, SL)

Decomposition of Exp and Rng Shifts





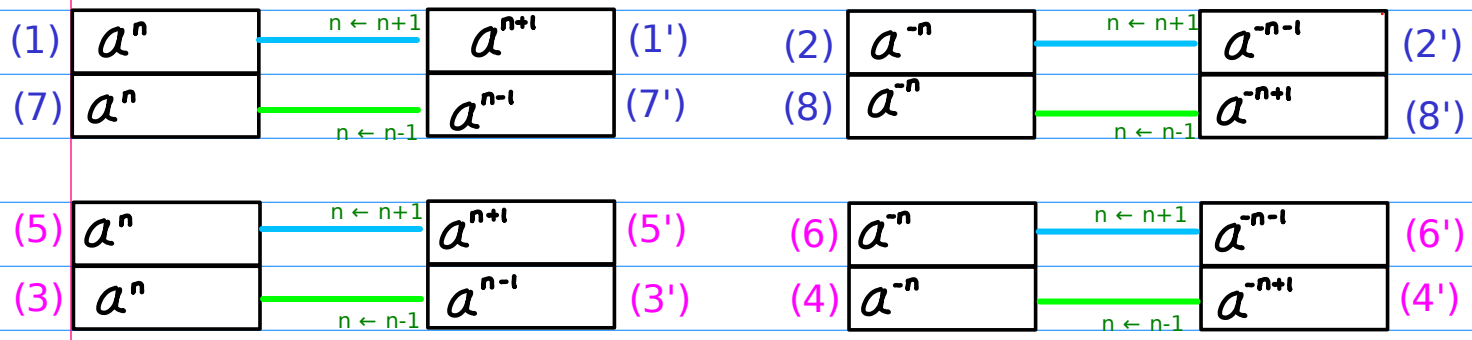




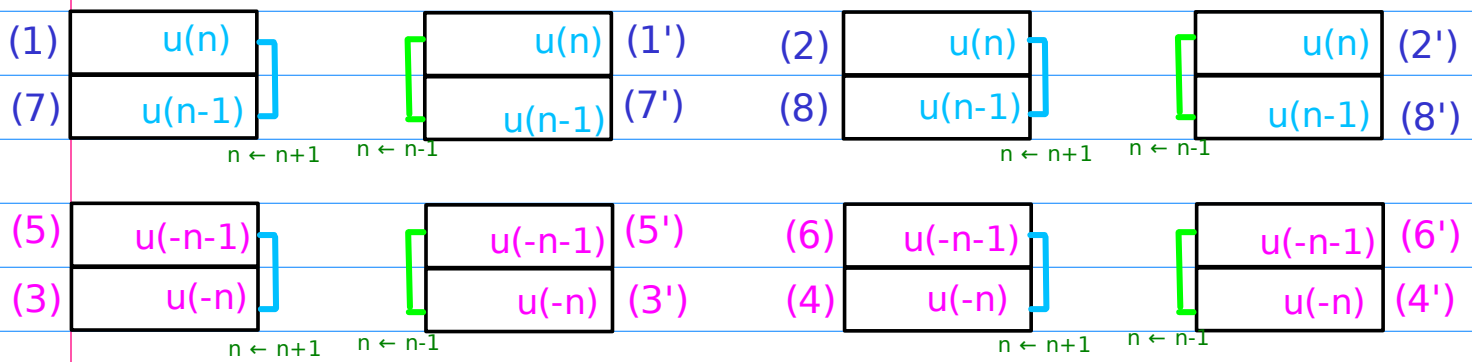
(id, SR) + (SR, id) = (SR, SR)
(id, SL) + (SL, id) = (SR, SL)

(id, SR) + (SR, id) = (SR, SR)
(id, SL) + (SL, id) = (SR, SL)

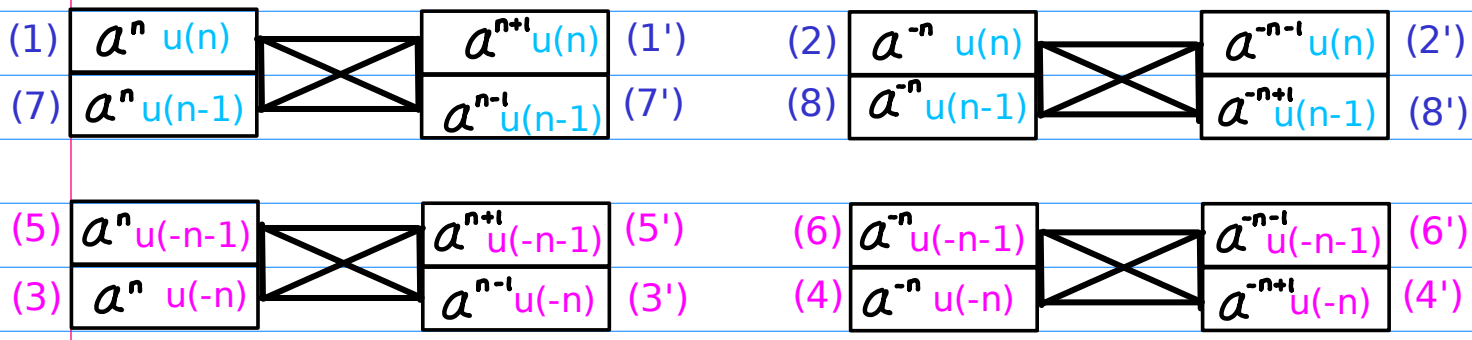
Exponent Shifts



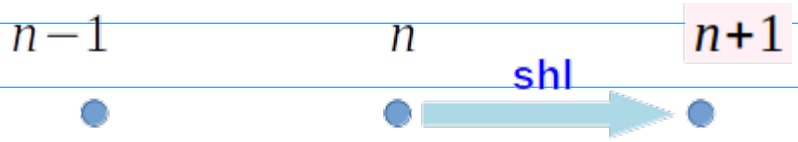
Range Shifts

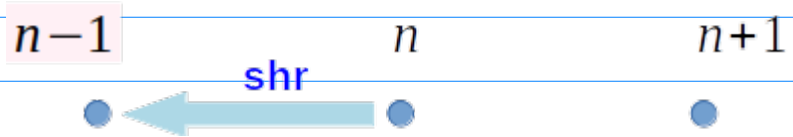


Exponent & Range Permutations

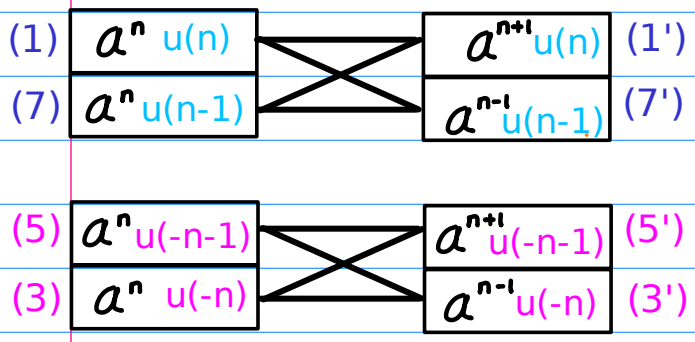


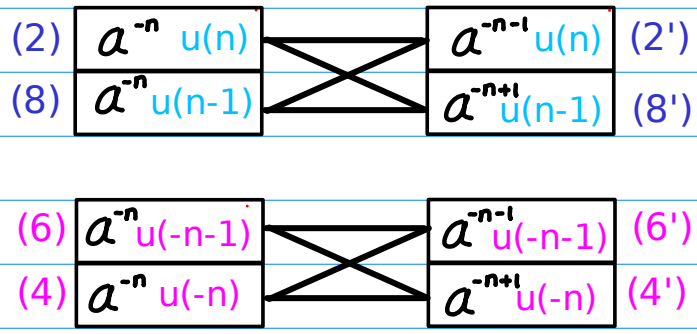
$R(n)$			$SHR(n)$
$u(n)$	$n \leftarrow n-1$	shr	$u(n-1)$
$u(n-1)$	$n \leftarrow n+1$	shl	$u(n)$
$u(-n)$	$n \leftarrow n+1$	shl	$u(-n-1)$
$u(-n-1)$	$n \leftarrow n-1$	shr	$u(-n)$

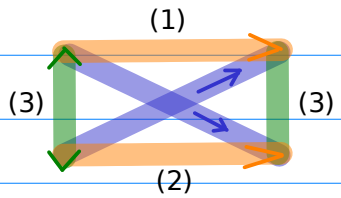


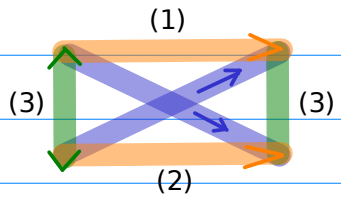


Exponent Permutations









(1) shift left exponent

(2) shift right exponent

(3) shift right/left range

(1) shift left exponent

(2) shift right exponent

(3) shift right/left range

(SR, id) shift right exponent

(id, SR) shift right range

(SR, SR)

(SR, id) shift right exponent

(id, SR) shift right range

(SR, SR)

(SL, id) shift left exponent

(id, SL) shift left range

(SL, SL)

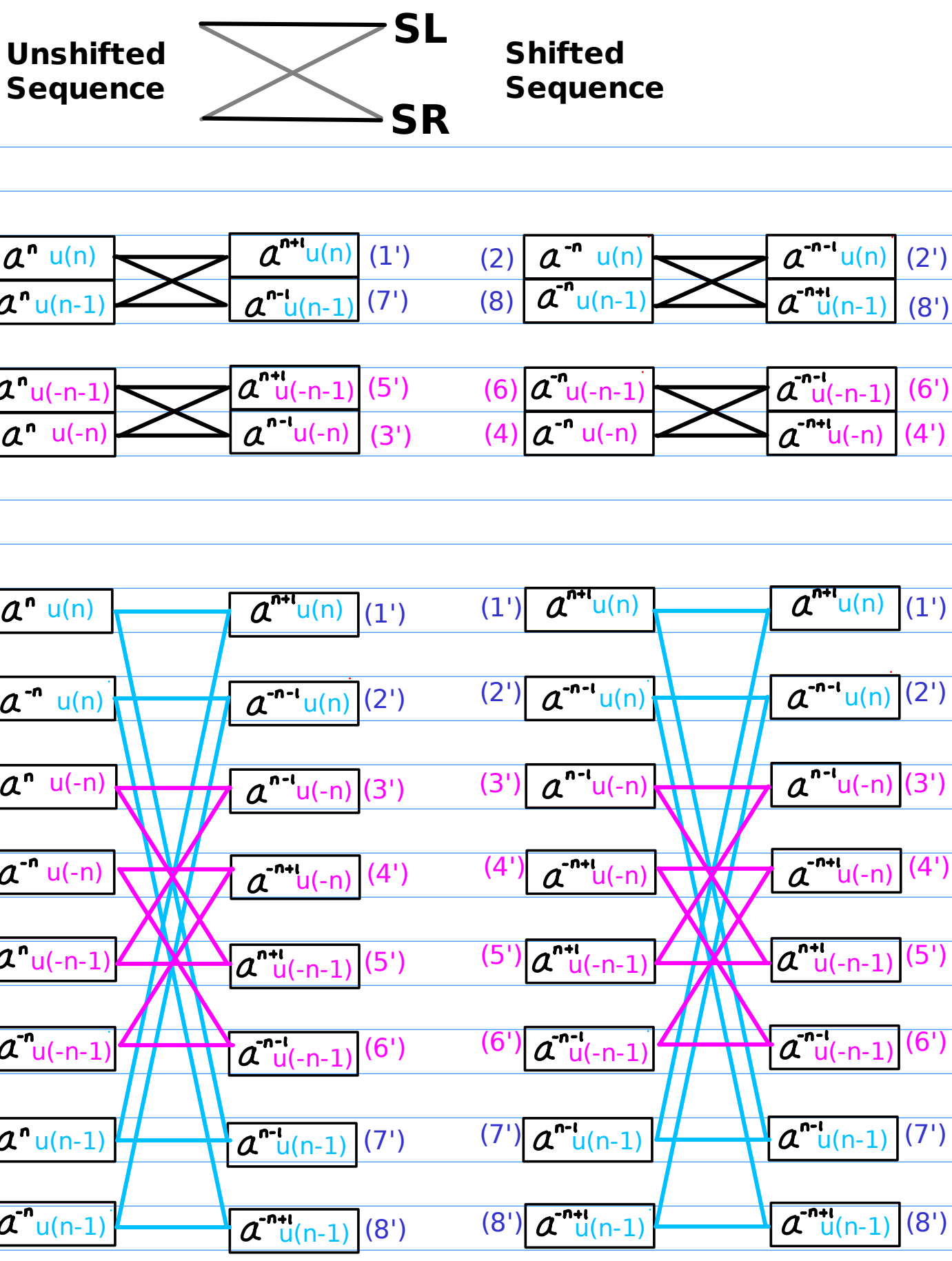
(SL, id) shift left exponent

(id, SL) shift left range

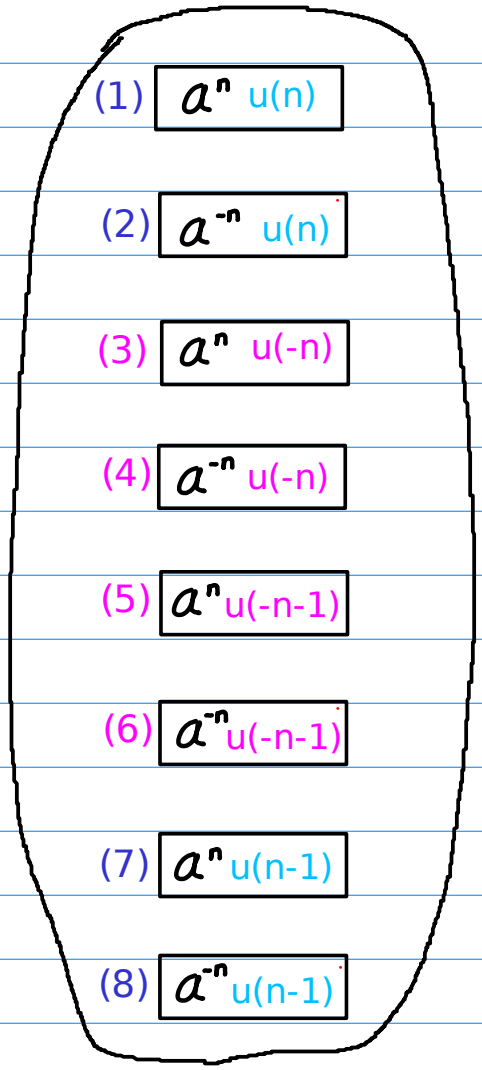
(SL, SL)

(id, SR) + (SR, id) = (SR, SR)
(id, SL) + (SL, id) = (SR, SL)

(id, SR) + (SR, id) = (SR, SR)
(id, SL) + (SL, id) = (SR, SL)


Unshifted Sequence
SL
SR
Shifted Sequence
(1) a^{n} u(n)
(7) a^{n} u(n-1)
a^{n+l}u(n) (1')
a^{n-l}u(n-1) (7')
(5) a^{n}u(-n-1)
(3) a^{n} u(-n)
a^{n+l}u(-n-1) (5')
a^{n-l}u(-n) (3')
(2) a^{-n} u(n)
(8) a^{-n}u(n-1)
a^{-n-l}u(n) (2')
a^{-n+l}u(n-1) (8')
(6) a^{-n}u(-n-1)
(4) a^{-n} u(-n)
a^{-n-l}u(-n-1) (6')
a^{-n+l}u(-n) (4')
(1) a^{n} u(n)
(2) a^{-n} u(n)
(3) a^{n} u(-n)
(4) a^{-n} u(-n)
(5) a^{n}u(-n-1)
(6) a^{-n}u(-n-1)
(7) a^{n}u(n-1)
(8) a^{-n}u(n-1)
a^{n+l}u(n) (1')
a^{-n-l}u(n) (2')
a^{n-l}u(-n) (3')
a^{-n+l}u(-n) (4')
a^{n+l}u(-n-1) (5')
a^{-n-l}u(-n-1) (6')
a^{n-l}u(n-1) (7')
a^{-n+l}u(n-1) (8')
(1') a^{n+l}u(n)
(2') a^{-n-l}u(n)
(3') a^{n-l}u(-n)
(4') a^{-n+l}u(-n)
(5') a^{n+l}u(-n-1)
(6') a^{-n-l}u(-n-1)
(7') a^{n-l}u(n-1)
(8') a^{-n+l}u(n-1)
a^{n+l}u(n) (1')
a^{-n-l}u(n) (2')
a^{n-l}u(-n) (3')
a^{-n+l}u(-n) (4')
a^{n+l}u(-n-1) (5')
a^{-n-l}u(-n-1) (6')
a^{n-l}u(n-1) (7')
a^{-n+l}u(n-1) (8')

Intra-Permuations over (1) ~ (8)



a^{n+1}u(n) (1')

a^{-n-1}u(n) (2')

a^{n-1}u(-n) (3')

a^{-n+1}u(-n) (4')

a^{n+1}u(-n-1) (5')

a^{-n-1}u(-n-1) (6')

a^{n-1}u(n-1) (7')

a^{-n+1}u(n-1) (8')

A. Flipping
- **Base Inverting**
- **Range Flipping**

B. Range Shifting = Range Flipping + Range Complementing
- **Range Flipping**
- **Range Complementing**

C. Complementary Inverting
- **Base Inverting**
- **Range Complementing**

a^n × $R(n)$

a^n	a^{-n}

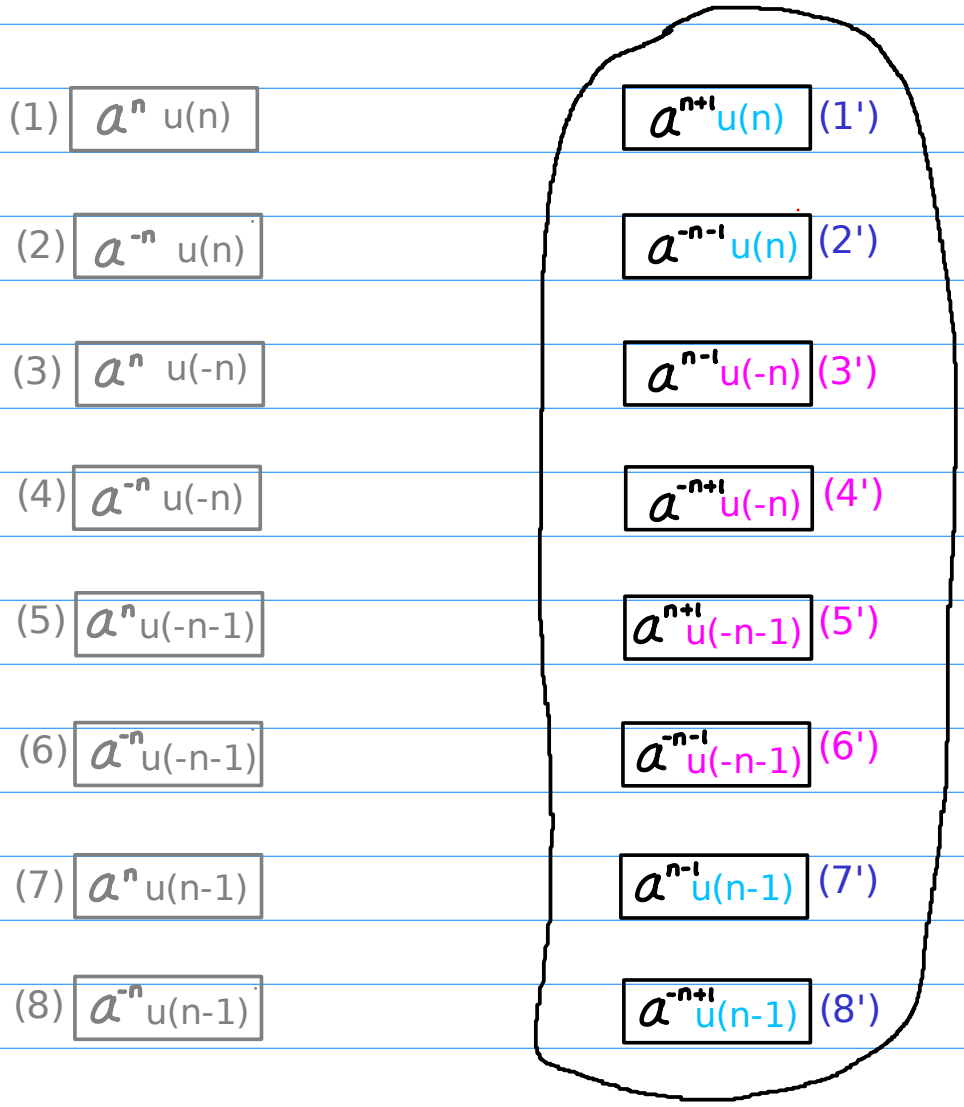
×

u(n)	u(-n-1)
u(n-1)	u(-n)

(1)	a^n u(n)	a^{-n} u(n)		(1)
(7)	a^n u(n-1)	a^{-n} u(n-1)		(7)
(5)	a^n u(-n-1)	a^{-n} u(-n-1)		(5)
(3)	a^n u(-n)	a^{-n} u(-n)		(3)

(1)	a^n u(n)	a^{-n} u(n)		(2)
(3)	a^n u(-n)	a^{-n} u(-n)		(4)
(5)	a^n u(-n-1)	a^{-n} u(-n-1)		(6)
(7)	a^n u(n-1)	a^{-n} u(n-1)		(8)

Intra-Permuations over (1') ~ (8')



D. Flipping2

Base Inverting

Shifted Range Flipping = Exponent Shifting2 + Range Flipppi

E. Shifting2 = Exponent Shifting2 + Range Shifting

Shifted Range Flipping = Exponent Shifting2 + Range Flipping

Range Complementing

F. Complementary Inverting

Base Inverting

Range Complementing

a^n × $R(n)$

a^{n+1}	a^{-n-1}
a^{n-1}	a^{-n+1}

×

u(n)	u(-n-1)
u(n-1)	u(-n)

(1') a^{n+1} u(n) a^{-n-1} u(n)

(7') a^{n-1} u(n-1) a^{-n+1} u(n-1)

(5') a^{n+1} u(-n-1) a^{-n-1} u(-n-1)

(3') a^{n-1} u(-n) a^{-n+1} u(-n)

~~(2') a^{n-1} u(n) a^{-n+1} u(n)~~

~~(8') a^{n+1} u(n-1) a^{-n-1} u(n-1)~~

~~(6') a^{n-1} u(-n-1) a^{-n+1} u(-n-1)~~

~~(4') a^{n+1} u(-n) a^{-n-1} u(-n)~~

(1') a^{n+1} u(n) a^{-n-1} u(n)

(3') a^{n-1} u(-n) a^{-n+1} u(-n)

(5') a^{n+1} u(-n-1) a^{-n-1} u(-n-1)

(7') a^{n-1} u(n-1) a^{-n+1} u(n-1)

~~(2') a^{n-1} u(n) a^{-n+1} u(n)~~

~~(4') a^{n+1} u(-n) a^{-n-1} u(-n)~~

~~(6') a^{n-1} u(-n-1) a^{-n+1} u(-n-1)~~

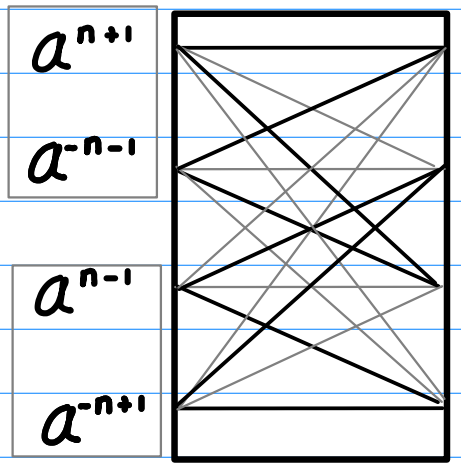
~~(8') a^{n+1} u(n-1) a^{-n-1} u(n-1)~~

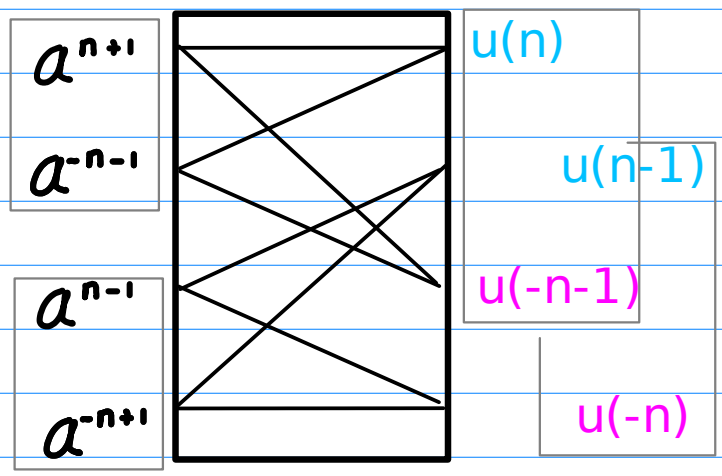
a^n × $R(n)$

a^{n+1}	a^{-n-1}
a^{n-1}	a^{-n+1}

×

u(n)	u(-n-1)
u(n-1)	u(-n)





n ← n+2 or
n ← n - 2

(1') (3') (5') (7')

a^{n+1}	u(n)	a^{-n-1}	u(n)
a^{n-1}	u(-n)	a^{-n+1}	u(-n)
a^{n+1}	u(-n-1)	a^{-n-1}	u(-n-1)
a^{n-1}	u(n-1)	a^{-n+1}	u(n-1)

(2') (4') (6') (8')

a^{n-1}	u(n)	a^{-n+1}	u(n)
a^{n+1}	u(-n)	a^{-n-1}	u(-n)
a^{n-1}	u(-n-1)	a^{-n+1}	u(-n-1)
a^{n+1}	u(n-1)	a^{-n-1}	u(n-1)

permutation over (1) ~ (8)

A. Flipping
    Base Inverting
    Range Flipping

B. Range Shifting = Range Flipping + Range Complementing
    Range Flipping
    Range Complementing

C. Complementary Inverting
    Base Inverting
    Range Complementing

permutation over (1') ~ (8')

D. Flipping2
    Base Inverting
    Shifted Range Flipping = Exponent Shifting2 + Range Flippping

E. Shifting2 = Exponent Shifting2 + Range Shifting
    Shifted Range Flipping = Exponent Shifting2 + Range Flipping
    Range Complementing

F. Complementary Inverting
    Base Inverting
    Range Complementing

Range Shifting = Range Flipping + Range Complementing
Shifted Range Flipping = Exponent Shifting2 + Range Flippping

Shifting2 = Shifted Range Flipping + Range Complementing
    = Exponent Shifting2 + Range (Flippping+Complementing)
    = Exponent Shfiting2 + Range Shifting

permutation over (1) ~ (8)

A. Flipping
- **Base Inverting**
- **Range Flipping**

(1) - (4) (5) - (8)
(2) - (3) (6) - (7)
(3) - (2) (7) - (6)
(4) - (1) (8) - (5)

B. Range Shifting
- **Range Flipping**
- **Range Complementing**

(1) - (7) (5) - (3)
(2) - (8) (6) - (4)
(3) - (5) (7) - (1)
(4) - (6) (8) - (2)

C. Complementary Inverting
- **Base Inverting**
- **Range Complementing**

(1) - (6) (5) - (2)
(6) - (1) (6) - (1)
(2) - (5) (7) - (4)
(5) - (2) (8) - (3)

permutation over (1') ~ (8')

D. Flipping2
- **Base Inverting**
- **Shifted Range Flipping**

(1') - (4') (5') - (8')
(2') - (3') (6') - (7')
(3') - (2') (7') - (6')
(4') - (1') (8') - (5')

E. Shifting2
- **Shifted Range Flipping**
- **Range Complementing**

(1') - (7') (5') - (3')
(2') - (8') (6') - (4')
(3') - (5') (7') - (1')
(4') - (6') (8') - (2')

F. Complementary Inverting
- **Base Inverting**
- **Range Complementing**

(1') - (6') (5') - (2')
(6') - (1') (6') - (1')
(2') - (5') (7') - (4')
(5') - (2') (8') - (3')

permutation over (1) ~ (8)

	A	B	C
Base Inverting	x		x
Range Flipping	x	x	
Range Complementing		x	x

permutation over (1') ~ (8')

	D	E	F
Base Inverting	x		x
Shifted Range Flipping	x	x	
Range Complementing		x	x

Range Shifting = Range Flipping + Range Complementing
Shifted Range Flipping = Exponent Shifting2 + Range Flippping

Shifting2 = Shifted Range Flipping + Range Complementing
= Exponent Shifting2 + Range (Flippping+Complementing)
= Exponent Shfiting2 + Range Shifting

Over (1) ~ (8)

Base Inverting $a^n \Longleftrightarrow a^{-n}$

Range Flipping $R(n) \Longleftrightarrow R(-n)$

Range Complementing $R(n) \Longleftrightarrow \overline{R(n)}$

Over (1') ~ (8')

Base Inverting $a^n \Longleftrightarrow a^{-n}$

Shifted Range Flipping $a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$

Range Complementing $R(n) \Longleftrightarrow \overline{R(n)}$

A.I Flipping
Base Inverting
Range Flipping

$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

B.I Range Shifting
Range Flipping
Range Complementing

$$R(n) \Longleftrightarrow R(-n)$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

C.I Complementary Inverting
Base Inverting
Range Complementing

$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n} \overline{R(n)}$$

D.I Flipping2
Base Inverting
Shifted Range Flipping

$$a^n \Longleftrightarrow a^{-n}$$
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

E.I Shifting2
Shifted Range Flipping
Range Complementing

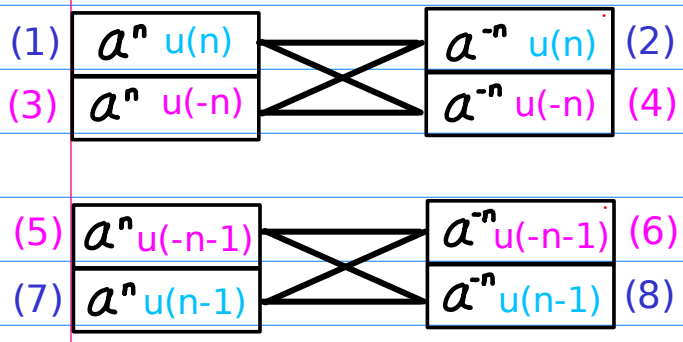
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} R(-n)$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

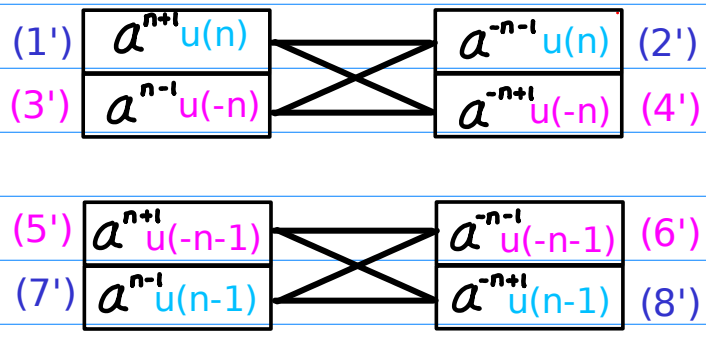
F.I Complementary Inverting
Base Inverting
Range Complementing

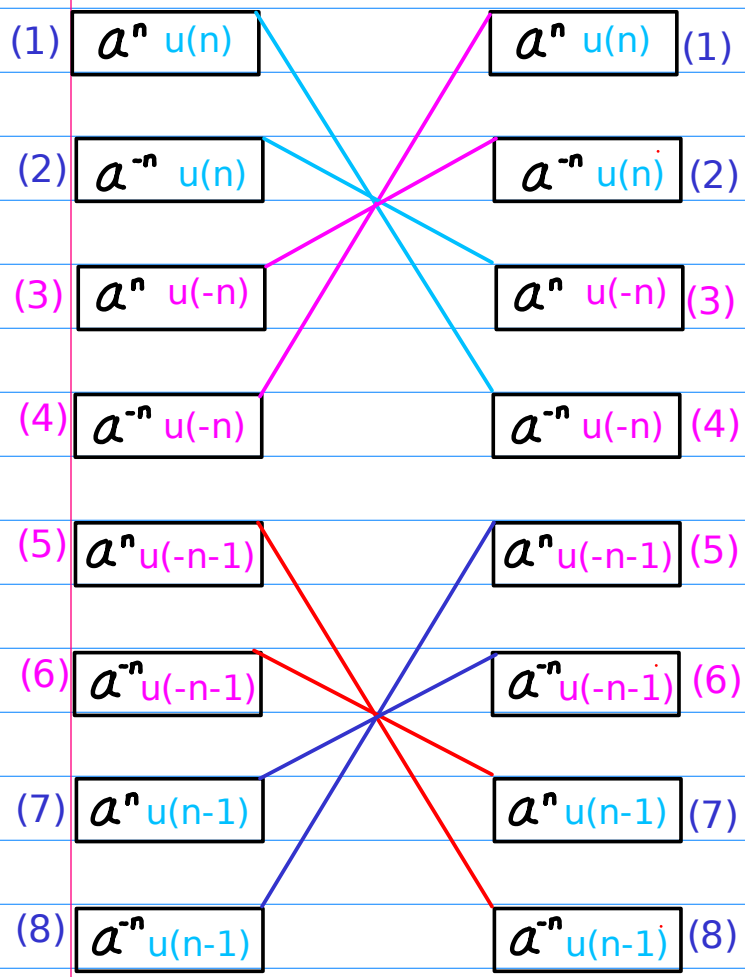
$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n} \overline{R(n)}$$

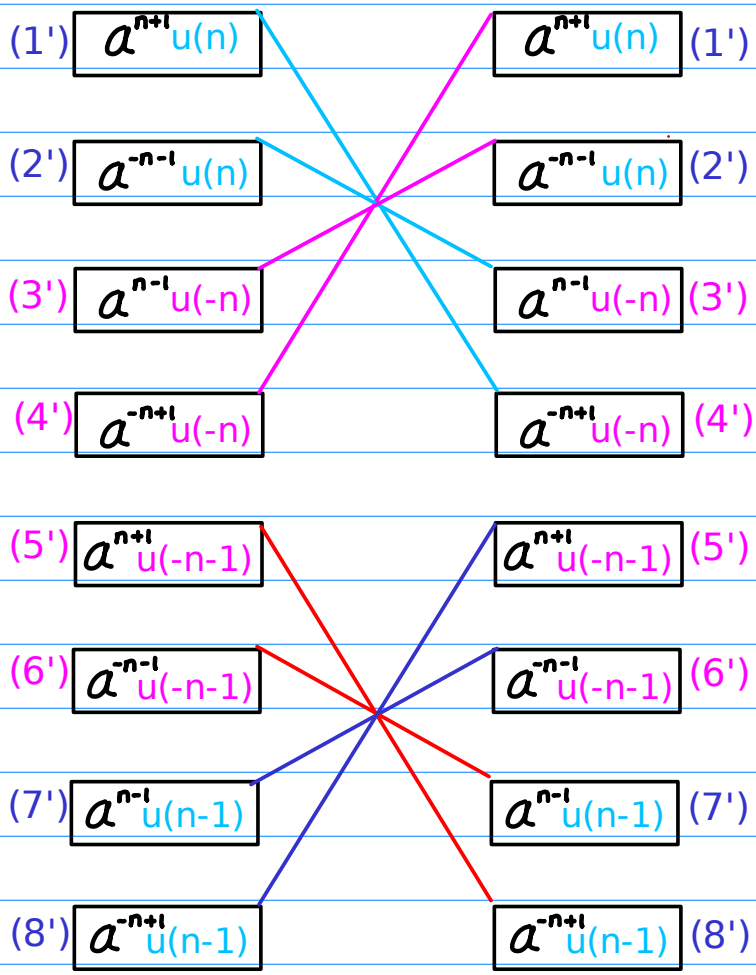
A.I Flipping
Base Inverting
Range Flipping

D.I Flipping2
Base Inverting
Shifted Range Flipping









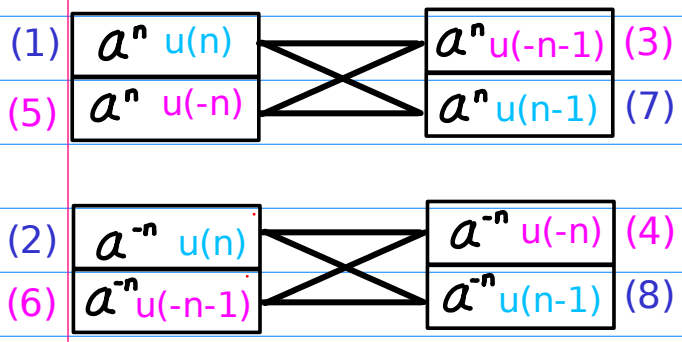
(1) - (4)
(2) - (3)
(3) - (2)
(4) - (1)

(5) - (8)
(6) - (7)
(7) - (6)
(8) - (5)

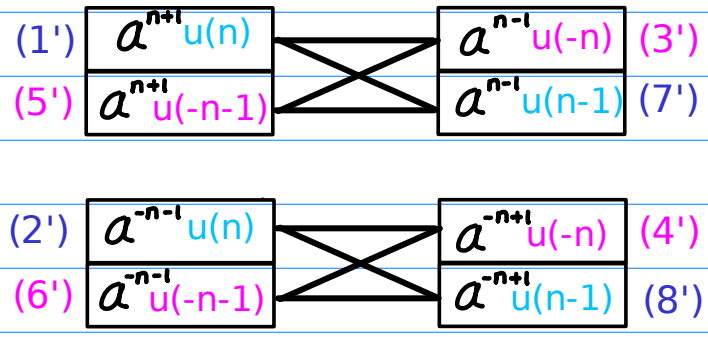
(1') - (4')
(2') - (3')
(3') - (2')
(4') - (1')

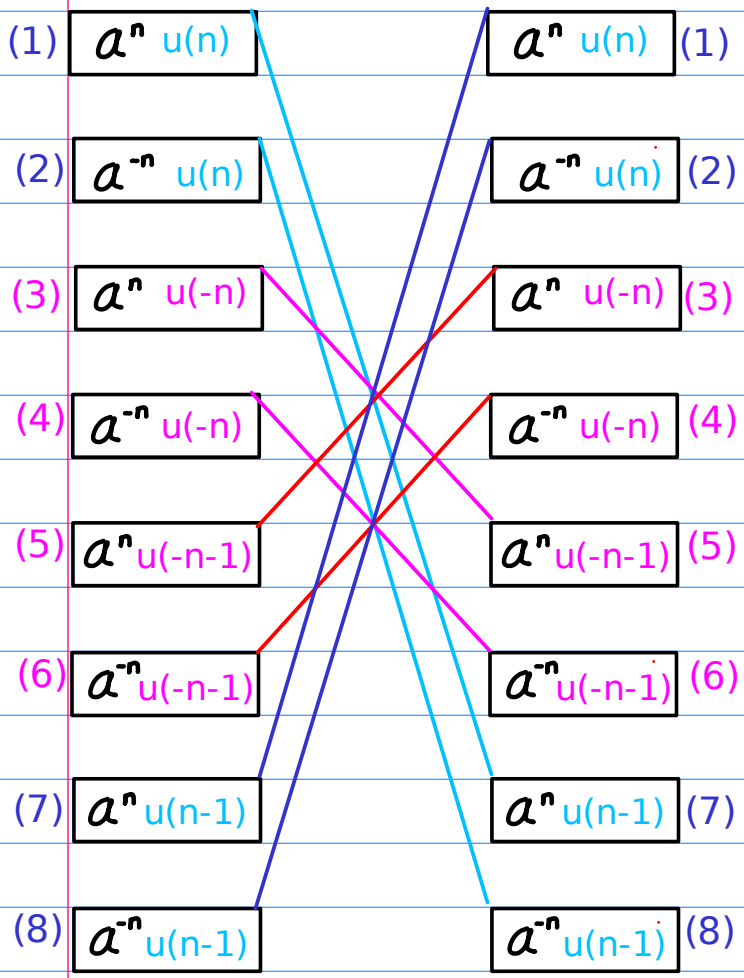
(5') - (8')
(6') - (7')
(7') - (6')
(8') - (5')

B.I Range Shifting
Range Flipping
Range Complementing



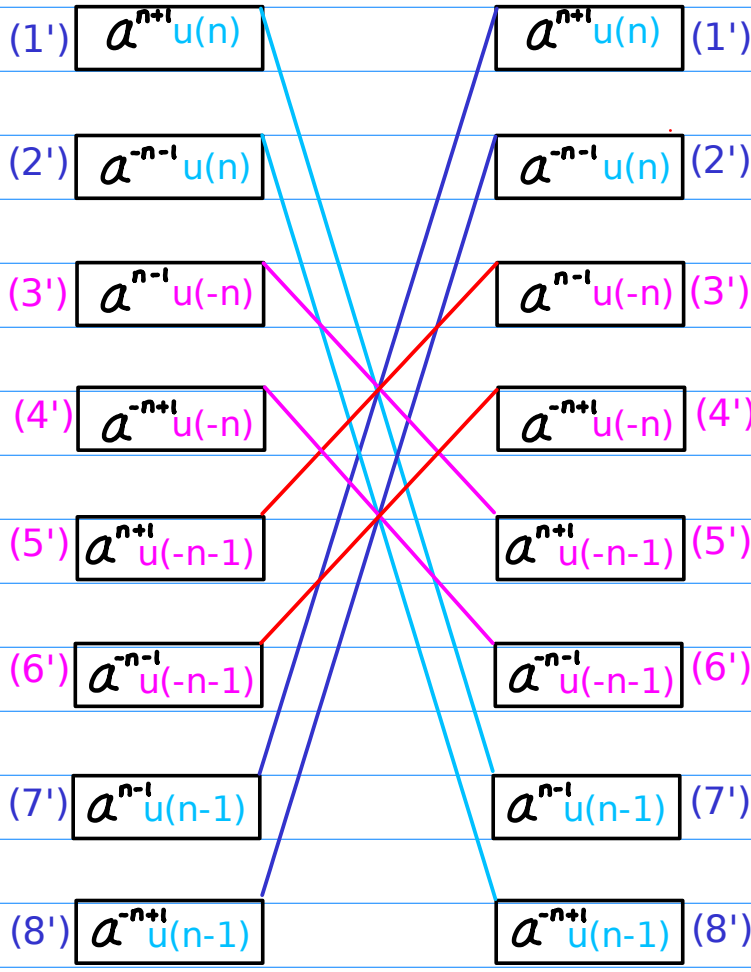
E.I Shifting2
Shifted Range Flipping
Range Complementing





(1) - (7)
(2) - (8)
(3) - (5)
(4) - (6)

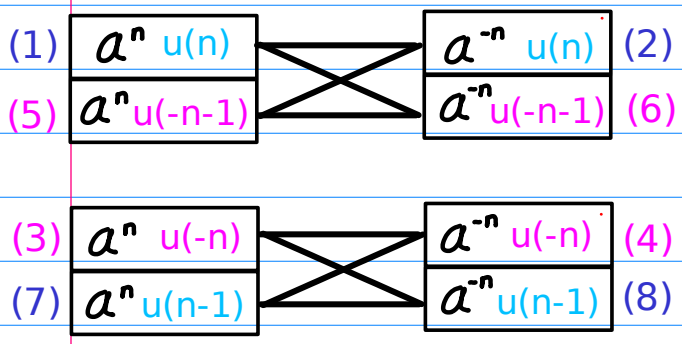
(5) - (3)
(6) - (4)
(7) - (1)
(8) - (2)



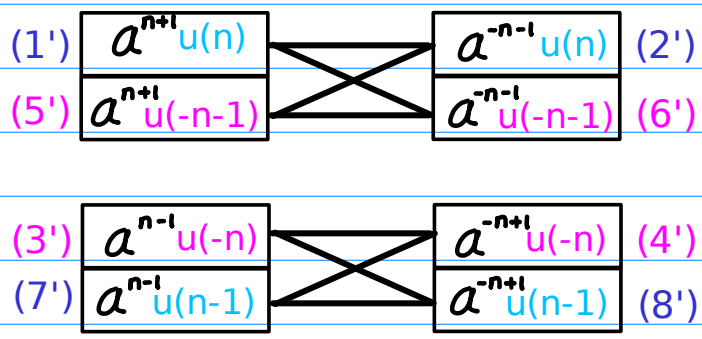
(1') - (7')
(2') - (8')
(3') - (5')
(4') - (6')

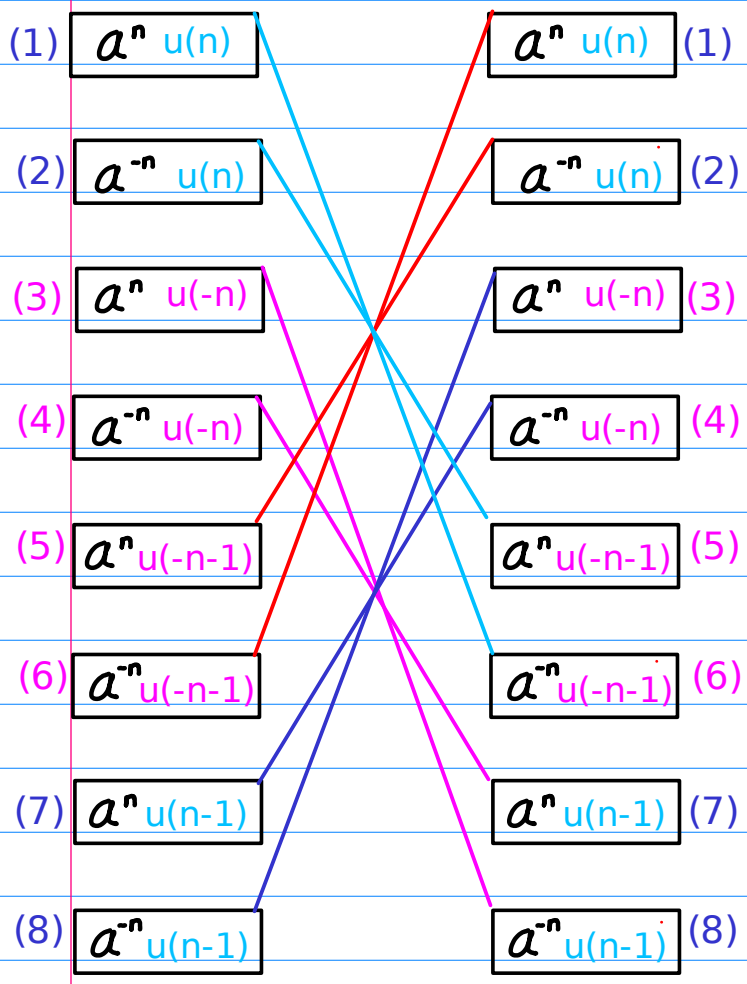
(5') - (3')
(6') - (4')
(7') - (1')
(8') - (2')

C.I Complementary Inverting
Base Inverting
Range Complementing



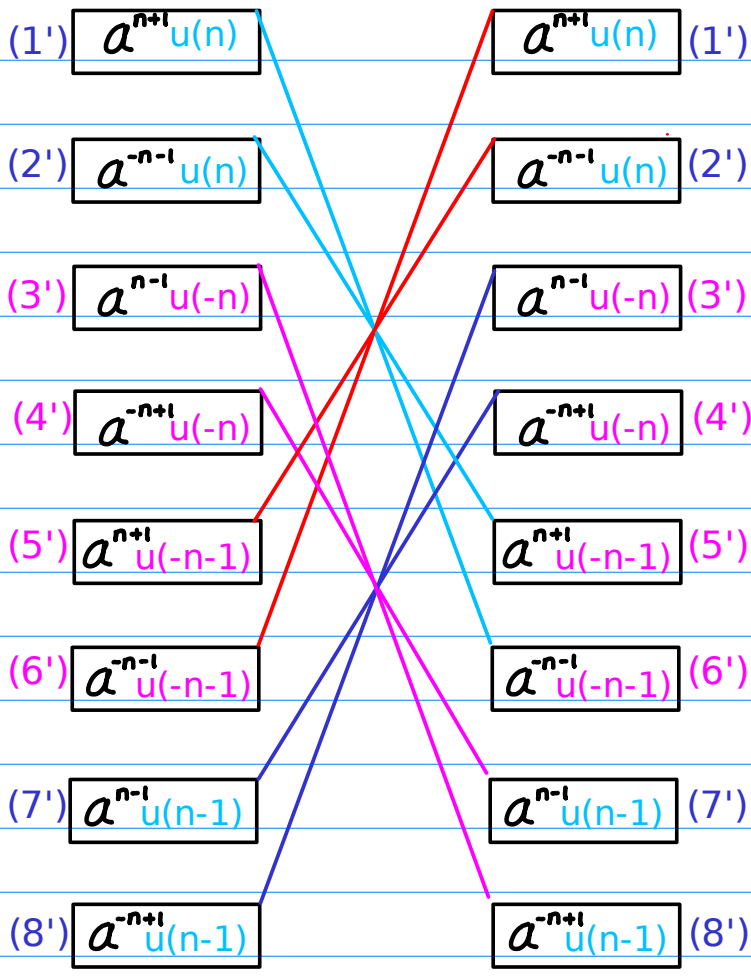
F.I Complementary Inverting
Base Inverting
Range Complementing





(1) - (6)
(6) - (1)
(2) - (5)
(5) - (2)

(5) - (2)
(6) - (1)
(7) - (4)
(8) - (3)

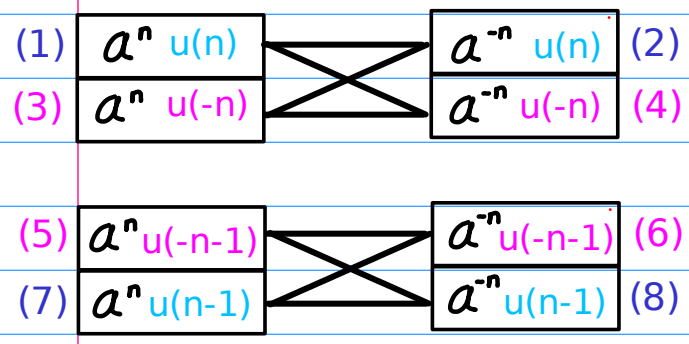


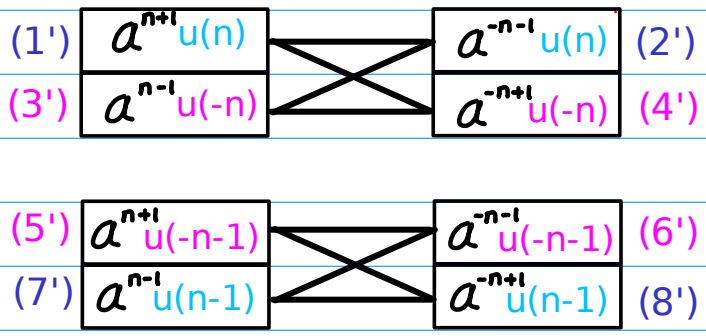
(1') - (6')
(6') - (1')
(2') - (5')
(5') - (2')

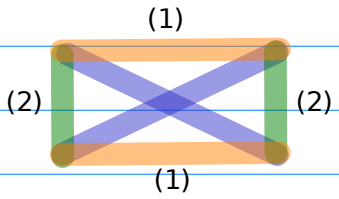
(5') - (2')
(6') - (1')
(7') - (4')
(8') - (3')

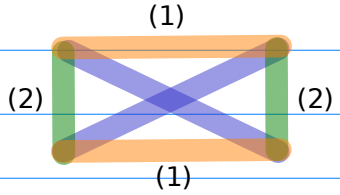
A.I Flipping
(1) Base Inverting
(2) Range Flipping

D.I Flipping2
(1) Base Inverting
(2) Shifted Range Flipping









$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$a^n R(n) \overset{(2)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

b^n	b^{-n}
a^n	a^{-n}
a^{-n}	a^n

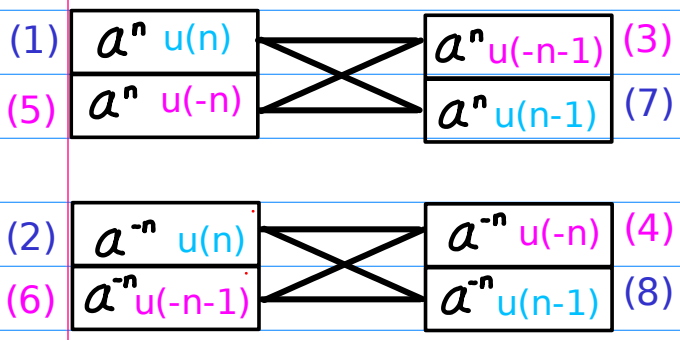
b^n	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

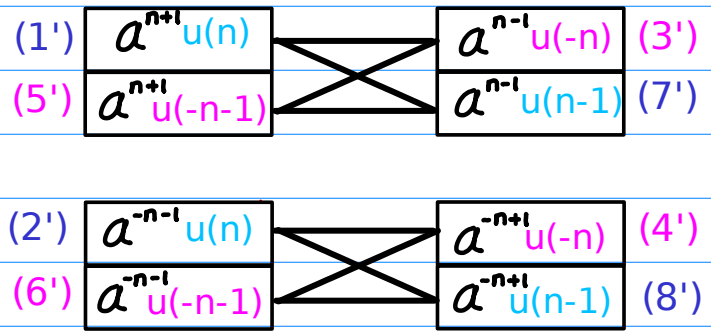
$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

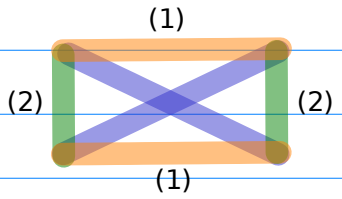
$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

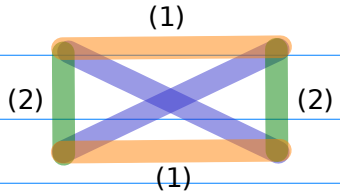
B.I Range Shifting
(1) Range Complementing
(2) Range Flipping

E.I Shifting2
(1) Shifted Range Flippin
(2) Range Complementing









$$R(n) \overset{(1)}{\Longleftrightarrow} R(-n)$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

$$a^n R(n) \overset{(1)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

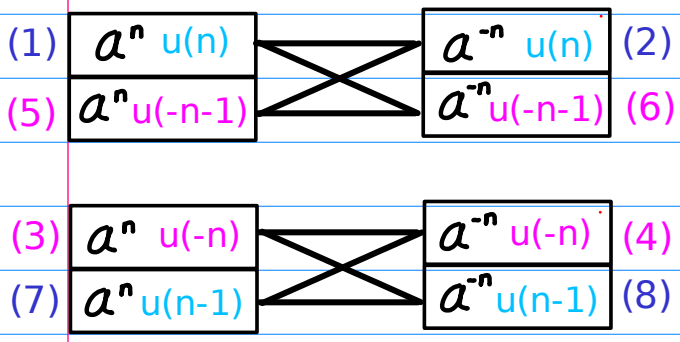
b^n	$b^{sh2(n)}$
$a^{(n+1)}$	$a^{(n-1)}$
$a^{-(n+1)}$	$a^{-(n-1)}$
$a^{(n-1)}$	$a^{(n+1)}$
$a^{-(n-1)}$	$a^{-(n+1)}$

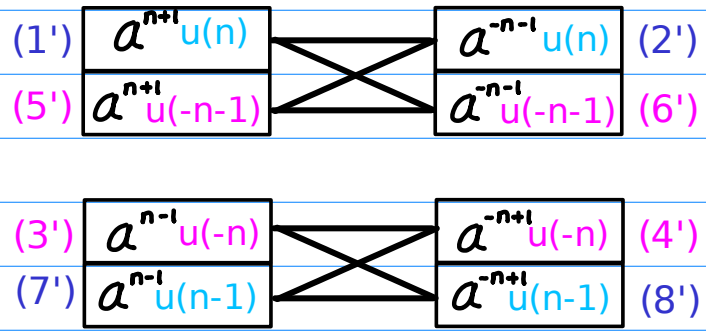
$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

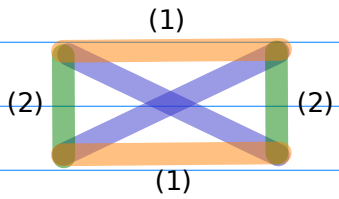
$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

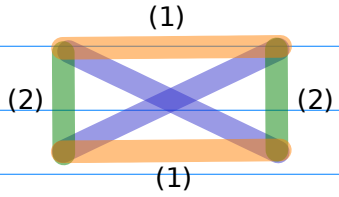
C.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing

F.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing









$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

b^n	b^{-n}
a^n	a^{-n}
a^{-n}	a^n

b^n	b^{-n}
$a^{(n+1)}$	$a^{-(n+1)}$
$a^{-(n+1)}$	$a^{(n+1)}$
$a^{(n-1)}$	$a^{-(n-1)}$
$a^{-(n-1)}$	$a^{(n-1)}$

$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

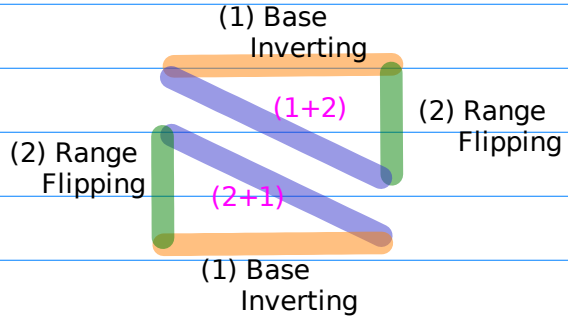
$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

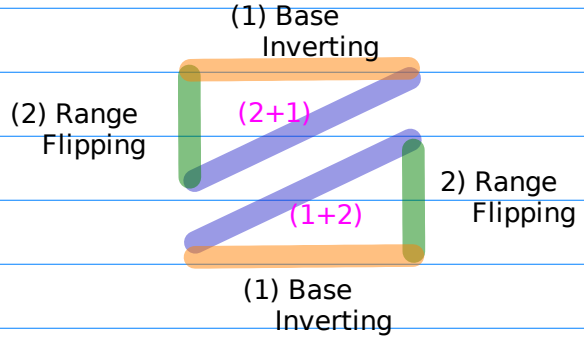
A.I Flipping
(1) Base Inverting
(2) Range Flipping

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$
$$\xrightarrow{(2)} a^{-n} R(-n)$$

$$a^n R(n) \xrightarrow{(2)} a^n R(-n)$$
$$\xrightarrow{(1)} a^{-n} R(-n)$$



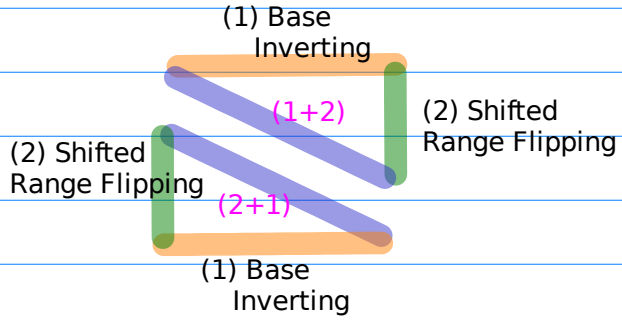


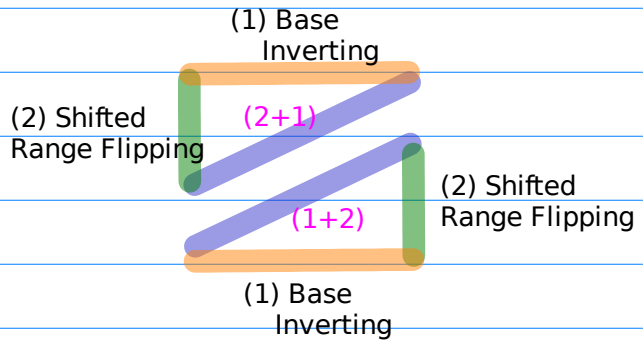
D.I Flipping2
(1) Base Inverting
(2) Shifted Range Flippin

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$a^n R(n) \overset{(2)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$
$$\xrightarrow{(2)} a^{-sh2(n)} R(-n)$$

$$a^n R(n) \xrightarrow{(2)} a^{sh2(n)} R(-n)$$
$$\xrightarrow{(1)} a^{-sh2(n)} R(-n)$$





B.I Range Shifting
(1) Range Complementing
(2) Range Flipping

$$R(n) \overset{(1)}{\Longleftrightarrow} R(-n)$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

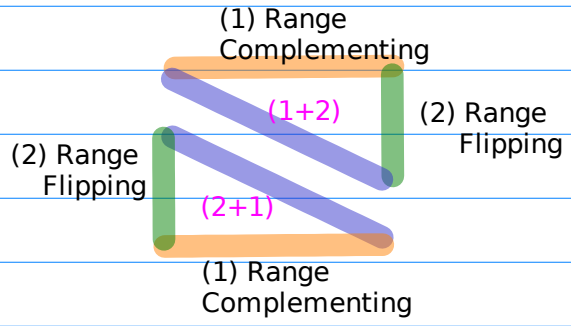
$$a^n R(n) \overset{(1)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

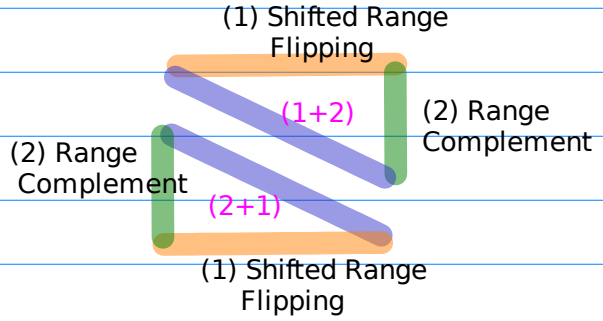
$$a^n R(n) \overset{(1)}{\rightarrow} a^{-n} R(n)$$
$$\overset{(2)}{\rightarrow} a^{-n} \overline{R(n)}$$

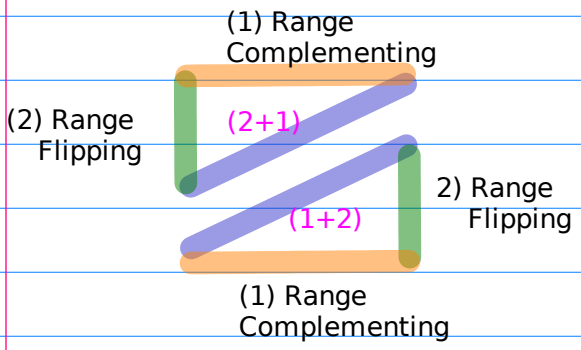
$$a^n R(n) \overset{(1)}{\rightarrow} a^{-n} R(n)$$
$$\overset{(2)}{\rightarrow} a^{-sh2(n)} R(-n)$$

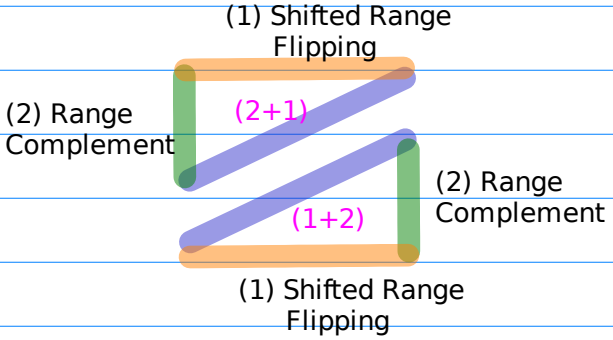
$$a^n rng(n) \overset{(2)}{\rightarrow} a^n \overline{R(n)}$$
$$\overset{(1)}{\rightarrow} a^{-n} \overline{R(n)}$$

$$a^n R(n) \overset{(2)}{\rightarrow} a^{sh2(n)} R(-n)$$
$$\overset{(1)}{\rightarrow} a^{-sh2(n)} R(-n)$$









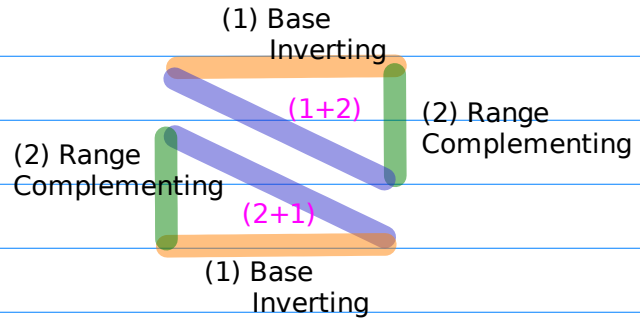
E.I Shifting2
(1) Shifted Range Flippin
(2) Range Complementing

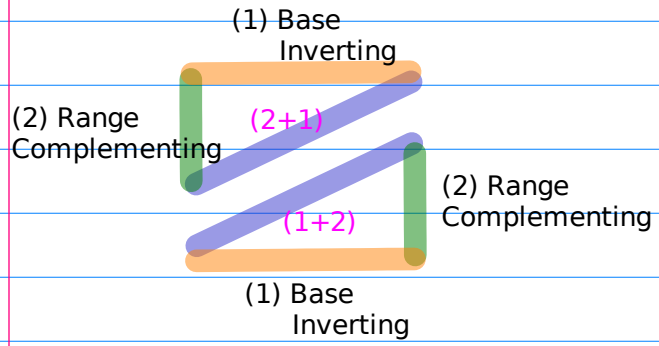
C.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^n R(n) \overset{(1)}{\Longrightarrow} a^{-n} R(n)$$
$$\overset{(2)}{\Longrightarrow} a^{-n}\overline{R(n)}$$

$$a^n R(n) \overset{(2)}{\Longrightarrow} a^n \overline{R(n)}$$
$$\overset{(1)}{\Longrightarrow} a^{-n}\overline{R(n)}$$



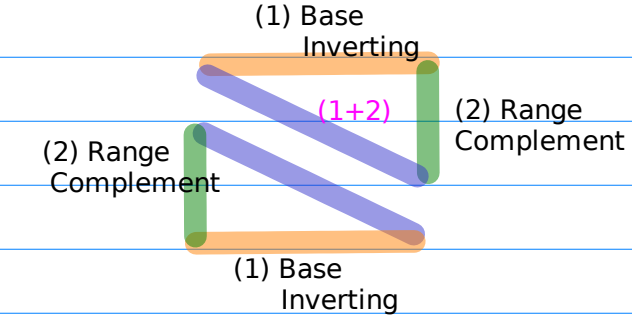


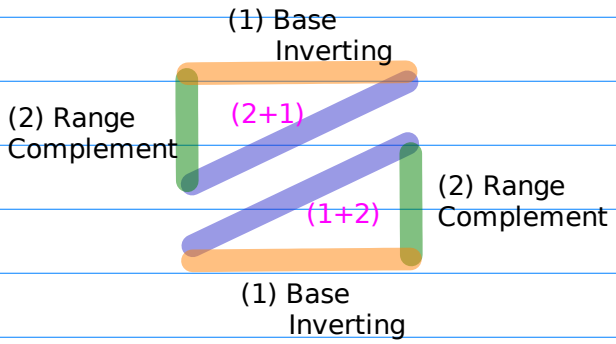
F.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

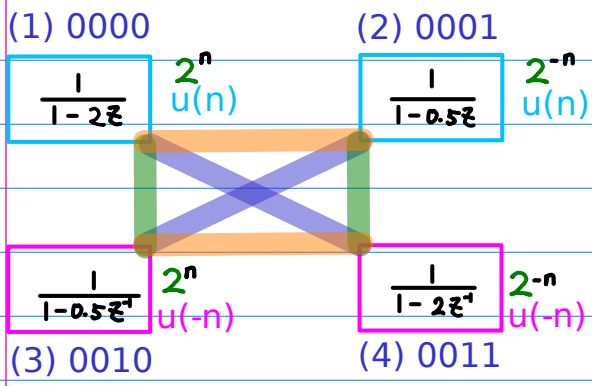
$$a^n R(n) \overset{(1)}{\Longrightarrow} a^{-n} R(n)$$
$$\overset{(2)}{\Longrightarrow} a^{-n}\overline{R(n)}$$

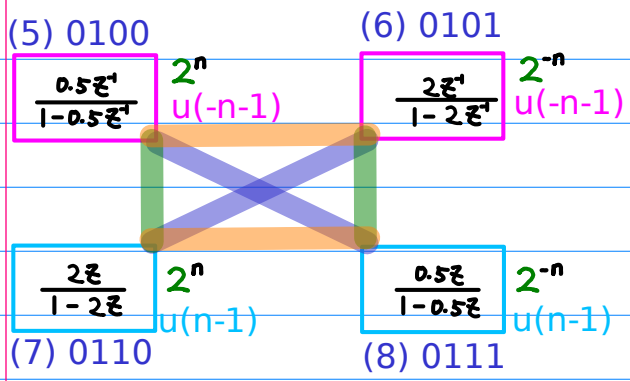
$$a^n R(n) \overset{(2)}{\Longrightarrow} a^n \overline{R(n)}$$
$$\overset{(1)}{\Longrightarrow} a^{-n}\overline{R(n)}$$



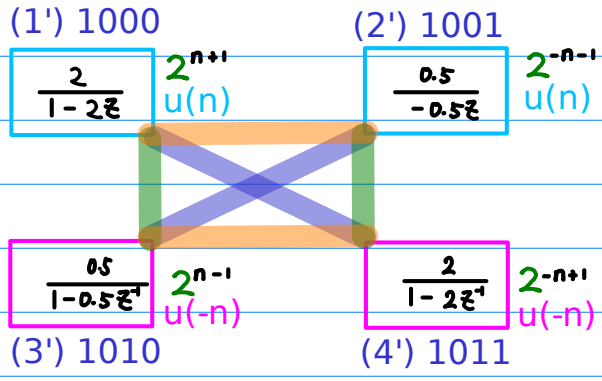


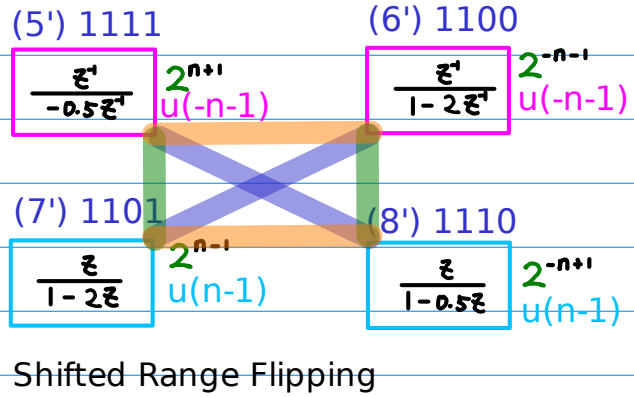
A.I Flipping
Base Inverting
Range Flipping





D.I Flipping2
Base Inverting
Shifted Range Flipping



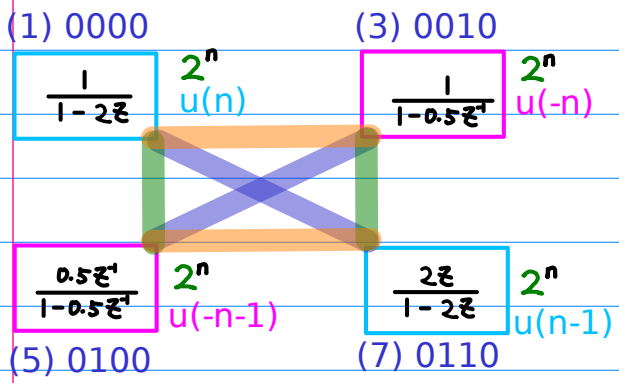


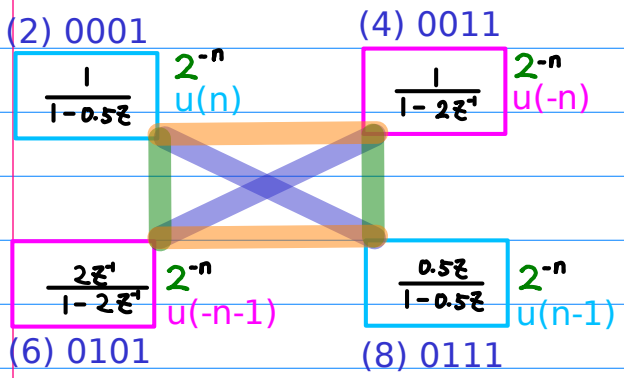
Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

$$a^n \quad \Longleftrightarrow \quad a^{-n}$$
$$R(n) \quad \Longleftrightarrow \quad R(-n)$$
$$a^n R(n) \quad \Longleftrightarrow \quad a^{-n} R(-n)$$

$$a^n \quad \Longleftrightarrow \quad a^{-n}$$
$$a^n R(n) \quad \Longleftrightarrow \quad a^{sh2(n)} R(-n)$$
$$a^n R(n) \quad \Longleftrightarrow \quad a^{-sh2(n)} R(-n)$$

B.I Range Shifting
Range Flipping
Range Complementing

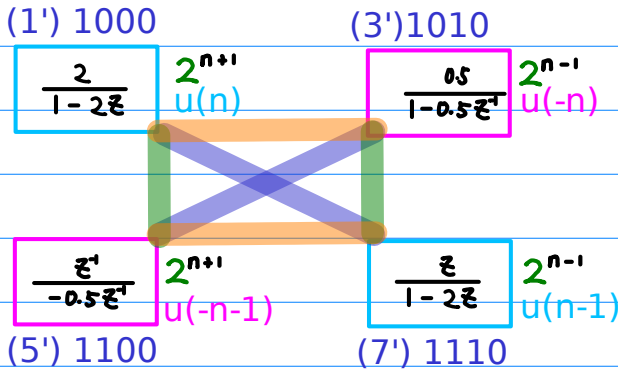


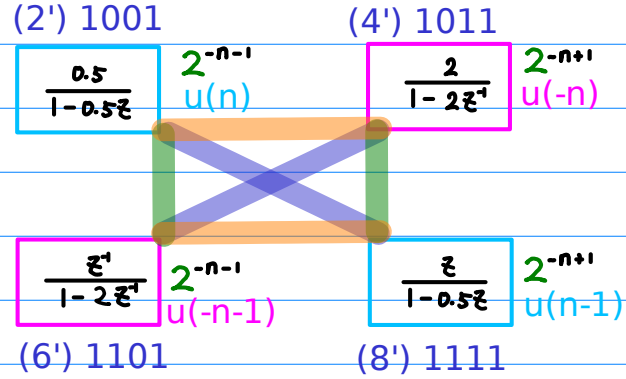


Range Shifting
= Range Flipping
+ Range Complementing

$$R(n) \Longleftrightarrow R(-n)$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

E.I Shifting2
Shifted Range Flipping
Range Complementing





Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

$$a^n R(n) \Longleftrightarrow a^{sh2(n)} R(-n)$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

C.I Complementary Inverting
Base Inverting
Range Complementing

(1) 0000 $\frac{1}{1-2z}$ 2^{n} u(n)

(2) 0001 $\frac{1}{1-0.5z}$ 2^{-n} u(n)

(5) 0100 $\frac{0.5z^{-1}}{1-0.5z^{-1}}$ 2^{n} u(-n-1)

(6) 0101 $\frac{2z^{-1}}{1-2z^{-1}}$ 2^{-n} u(-n-1)

(3) 0010 $\frac{1}{1-0.5z^{-1}}$ 2^{n} u(-n)

(4) 0011 $\frac{1}{1-2z^{-1}}$ 2^{-n} u(-n)

(7) 0110 $\frac{2z}{1-2z}$ 2^{n} u(n-1)

(8) 0111 $\frac{0.5z}{1-0.5z}$ 2^{-n} u(n-1)

$$a^{n} \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^{n}R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

F.I Complementary Inverting
Base Inverting
Range Complementing

(1') 1000 $\frac{2}{1-2z}$ 2^{n+1} u(n)

(2') 1001 $\frac{0.5}{1-0.5z}$ 2^{-n-1} u(n)

(5') 1100 $\frac{z^{-1}}{1-0.5z^{-1}}$ 2^{n+1} u(-n-1)

(6') 1101 $\frac{z^{-1}}{1-2z^{-1}}$ 2^{-n-1} u(-n-1)

(3') 1010 $\frac{0.5}{1-0.5z^{-1}}$ 2^{n-1} u(-n)

(4') 1011 $\frac{2}{1-2z^{-1}}$ 2^{-n+1} u(-n)

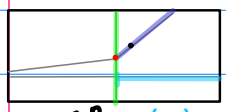
(7') 1110 $\frac{z}{1-2z}$ 2^{n-1} u(n-1)

(8') 1111 $\frac{z}{1-0.5z}$ 2^{-n+1} u(n-1)

$$a^{n} \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^{n}R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

A.II Flipping
Base Inverting
Range Flipping

(1) 0000



a^n u(n)

(4) 0011

a^{-n} u(-n)

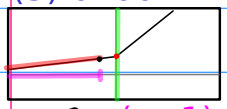
(3) 0010

a^n u(-n)

(2) 0001

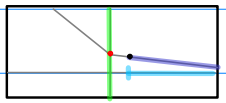
a^{-n} u(n)

(5) 0100



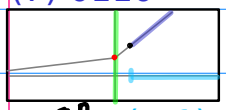
a^n u(-n-1)

(8) 0111



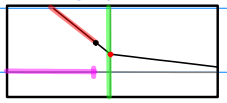
a^{-n} u(n-1)

(7) 0110



a^n u(n-1)

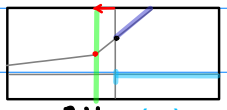
(6) 0101



a^{-n} u(-n-1)

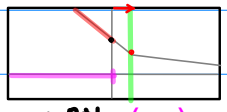
D.II Flipping2
Base Inverting
Shifted Range Flipping

(1') 1000



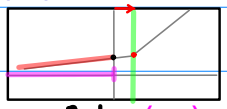
a^{n+1} u(n)

(4') 1011



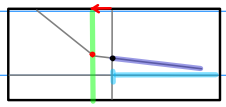
a^{-n+1} u(-n)

(3') 1010



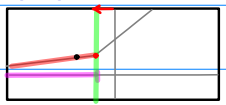
a^{n-1} u(-n)

(2') 1001



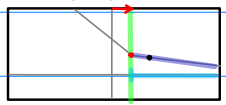
a^{-n-1} u(n)

(5') 1100



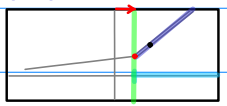
a^{n+1} u(-n-1)

(8') 1111



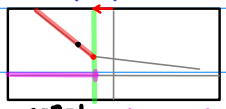
a^{-n+1} u(n-1)

(7') 1110



a^{n-1} u(n-1)

(6') 1101



a^{-n-1} u(-n-1)

Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

$$a^n \Longleftrightarrow a^{-n}$$

$$R(n) \Longleftrightarrow R(-n)$$

$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

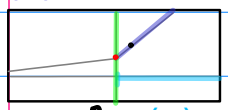
$$a^n \Longleftrightarrow a^{-n}$$

$$a^n R(n) \Longleftrightarrow a^{sh2(n)} R(-n)$$

B.II Range Shifting
Range Flipping
Range Complementing

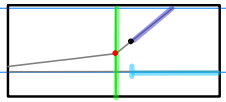
E.II Shifting2
Shifted Range Flipping
Range Complementing

(1) 0000



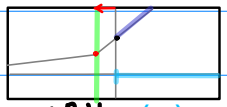
a^n u(n)

(7) 0110



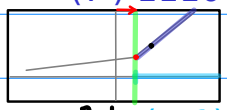
a^n u(n-1)

(1') 1000



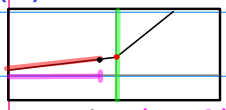
a^{n+1} u(n)

(7') 1110



a^{n-1} u(n-1)

(5) 0100

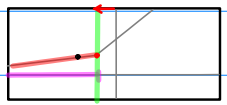


a^n u(-n-1)

(3) 0010

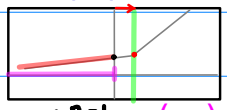
a^n u(-n)

(5') 1100



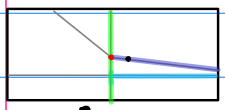
a^{n+1} u(-n-1)

(3') 1010



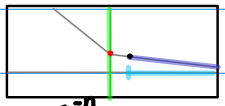
a^{n-1} u(-n)

(2) 0001



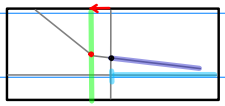
a^{-n} u(n)

(8) 0111



a^{-n} u(n-1)

(2') 1001

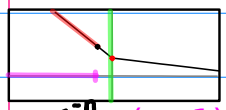


a^{-n-1} u(n)

(8') 1111

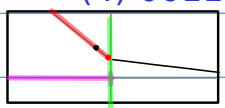
a^{-n+1} u(n-1)

(6) 0101



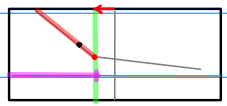
a^{-n} u(-n-1)

(4) 0011



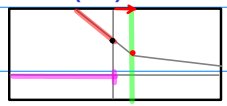
a^{-n} u(-n)

(6') 1101



a^{-n-1} u(-n-1)

(4') 1011



a^{-n+1} u(-n)

Range Shifting
= Range Flipping
+ Range Complementing

$$R(n) \Longleftrightarrow \overline{R(-n)}$$

$$R(n) \Longleftrightarrow R(-n)$$

$$R(n) \Longleftrightarrow \overline{R(n)}$$

Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

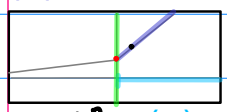
$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} \overline{R(-n)}$$

$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$$

$$R(n) \Longleftrightarrow \overline{R(n)}$$

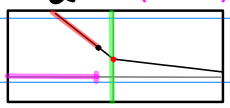
C.II Complementary Inverting
Base Inverting
Range Complementing

(1) 0000



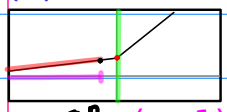
a^{n} u(n)

a^{-n} u(-n-1)



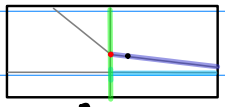
(6) 0101

(5) 0100



a^{n} u(-n-1)

(2) 0001

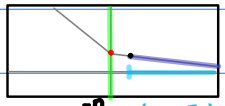


a^{-n} u(n)

(3) 0010

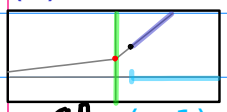
a^{n} u(-n)

(8) 0111



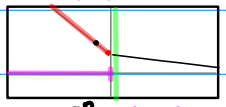
a^{-n} u(n-1)

(7) 0110



a^{n} u(n-1)

(4) 0011



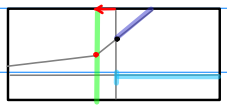
a^{-n} u(-n)

$$a^{n}R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^{n} \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$

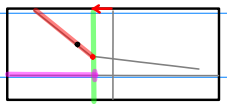
F.II Complementary Inverting
Base Inverting
Range Complementing

(1') 1000



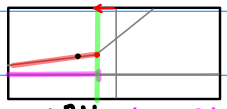
a^{n+1} u(n)

a^{-n-1} u(-n-1)



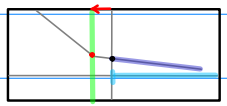
(6') 1101

(5') 1100



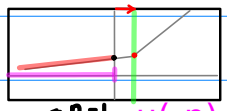
a^{n+1} u(-n-1)

(2') 1001



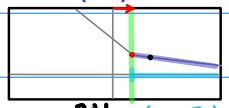
a^{-n-1} u(n)

(3') 1010



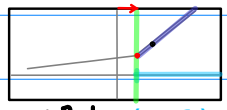
a^{n-1} u(-n)

(8') 1111



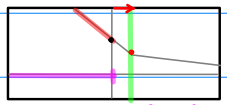
a^{-n+1} u(n-1)

(7') 1110



a^{n-1} u(n-1)

(4') 1011



a^{-n+1} u(-n)

$$a^{n}R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^{n} \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$

A. Flipping

- Base Inverting
- Range Flipping

B. Range Shifting = Range Flipping + Range Complementing

- Range Flipping
- Range Complementing

C. Complementary Inverting

- Base Inverting
- Range Complementing

D. Flipping2

- Base Inverting
- Shifted Range Flipping = Exponent Shifting2 + Range Flipppin

E. Shifting2 = Exponent Shifting2 + Range Shifting

- Shifted Range Flipping = Exponent Shifting2 + Range Flipping
- Range Complementing

F. Complementary Inverting

- Base Inverting
- Range Complementing

Base Inverting

$2^{n+1} \Longleftrightarrow 2^{-(n+1)}$

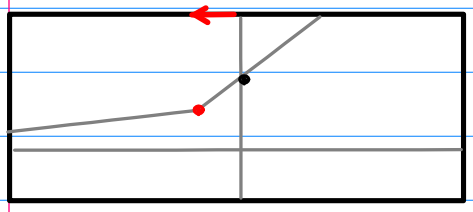
$2^{n-1} \Longleftrightarrow 2^{-(n-1)}$

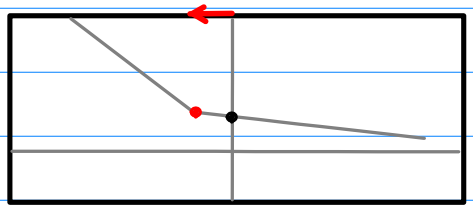
$2^{n} \cdot 2^{+1} \Longleftrightarrow 2^{-n} \cdot 2^{-1}$

$2^{n} \cdot 2^{-1} \Longleftrightarrow 2^{-n} \cdot 2^{+1}$

as a scaling

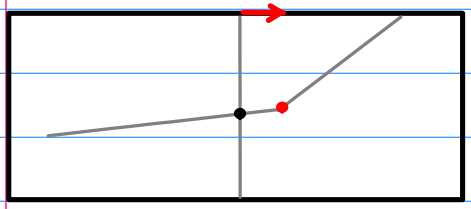
2^{n+1}

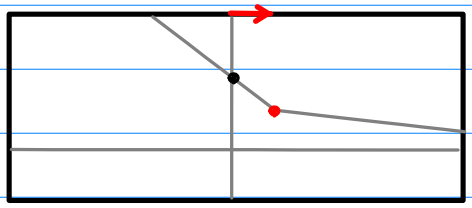




$2^{-(n+1)}$

2^{n-1}





$2^{-(n-1)}$

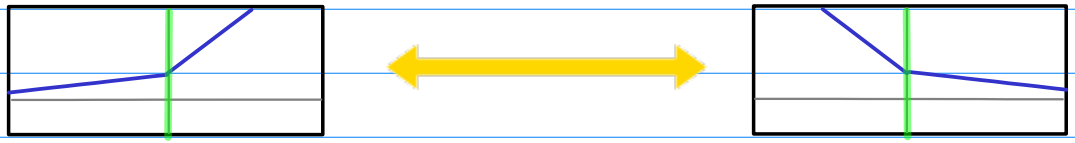
b^{n}	b^{-n}
a^{n}	a^{-n}
a^{-n}	a^{n}

b^{n}	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

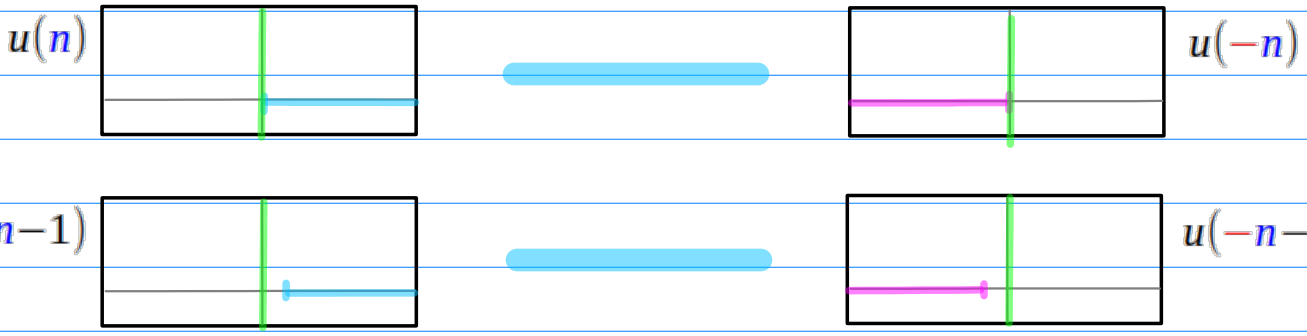
A.2a Flipping
. Base Inverting
Range Flipping

$a^n R(n) \quad \Leftrightarrow \quad a^{-n} R(-n)$

1) Base Inverting $a^n \Leftrightarrow a^{-n}$



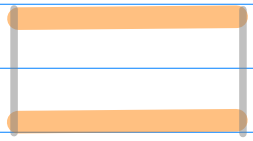
2) Range Flipping $R(n) \Leftrightarrow R(-n)$



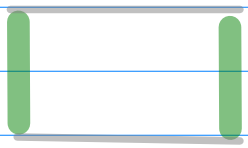
3) Flipping $a^n R(n) \Leftrightarrow a^{-n} R(-n)$

1) Base Inverting

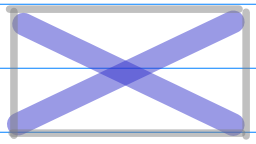
a^n a^{-n}



2) Range Flipping



3) Flipping

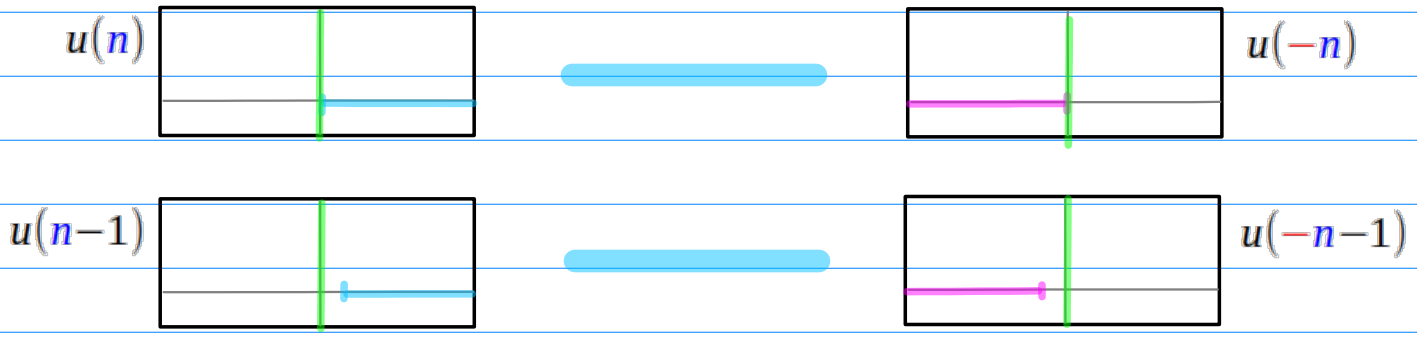


Range Shifting over (1) ~ (8)

$$R(n) \Leftrightarrow \overline{R(-n)}$$

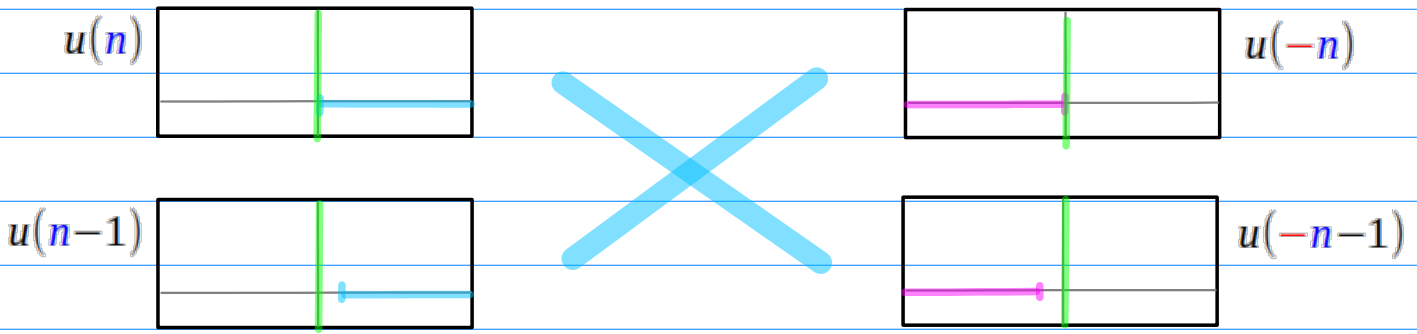
(1) Range Flipping

$$R(n) \Leftrightarrow R(-n)$$



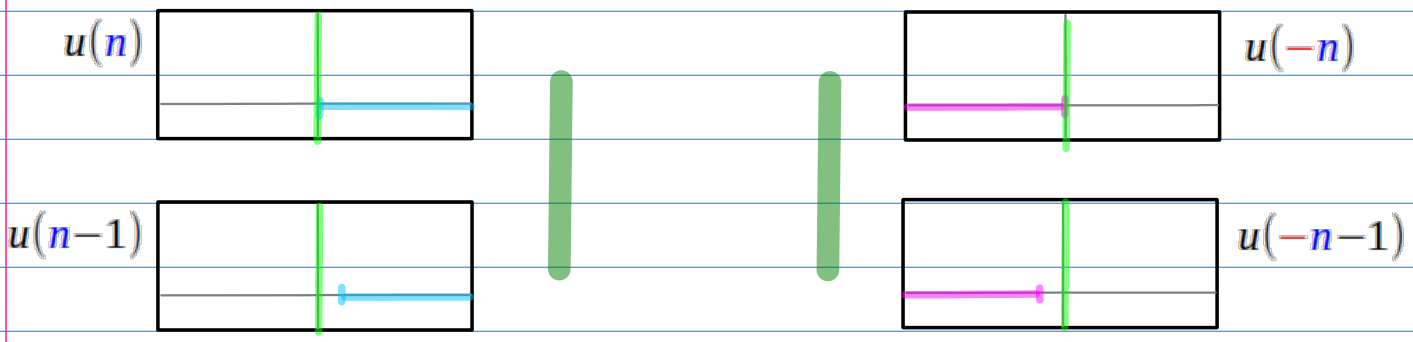
(2) Range Complementing

$$R(n) \Leftrightarrow \overline{R(n)}$$

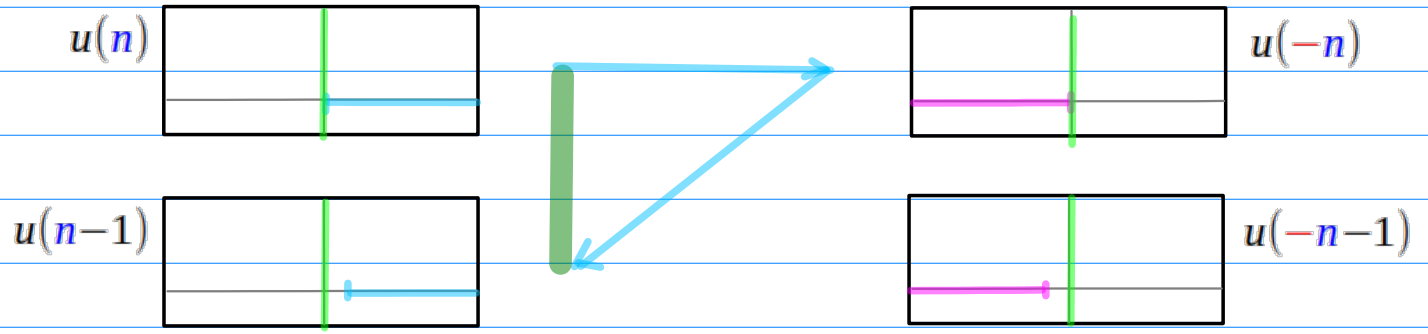


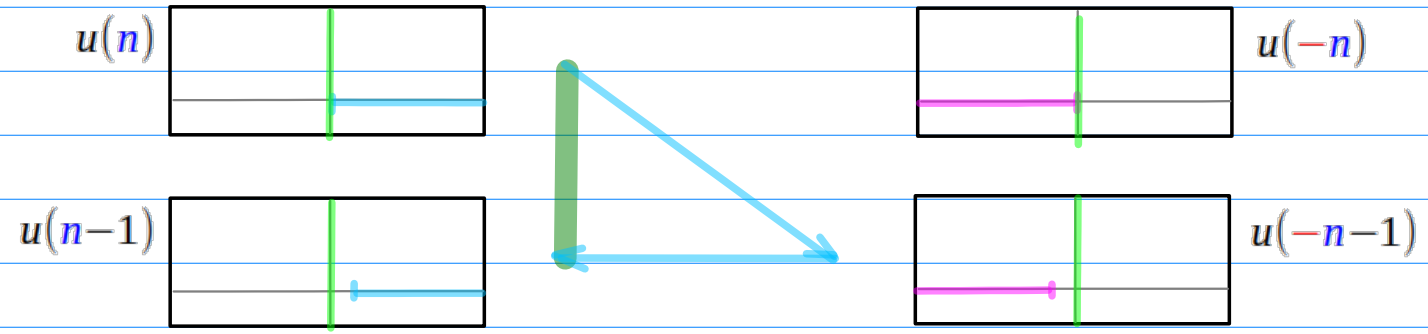
(1+2) Range Shifting

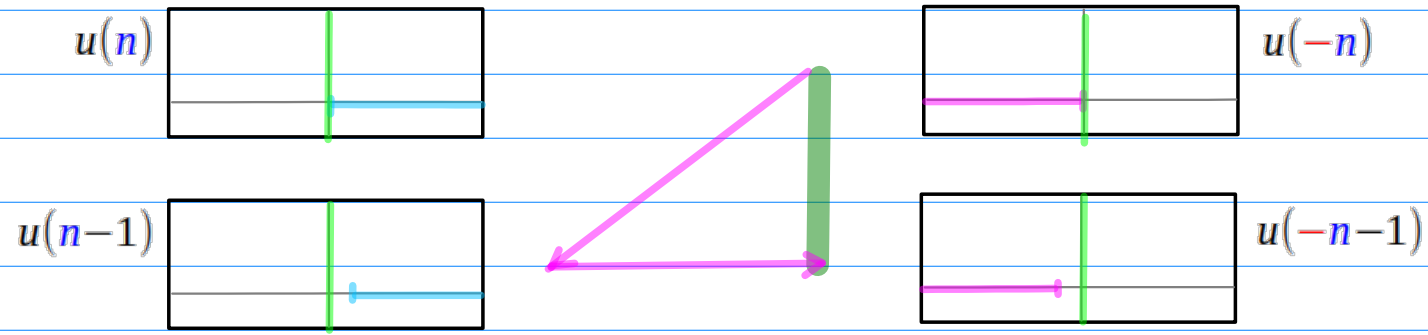
$$R(n) \Leftrightarrow \overline{R(-n)}$$

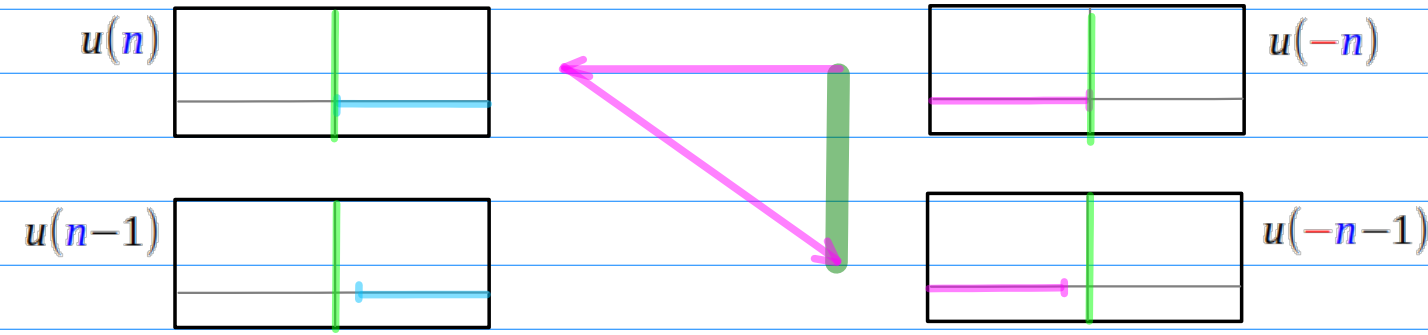


Range Shifting = Range Flipping + Range Complementing

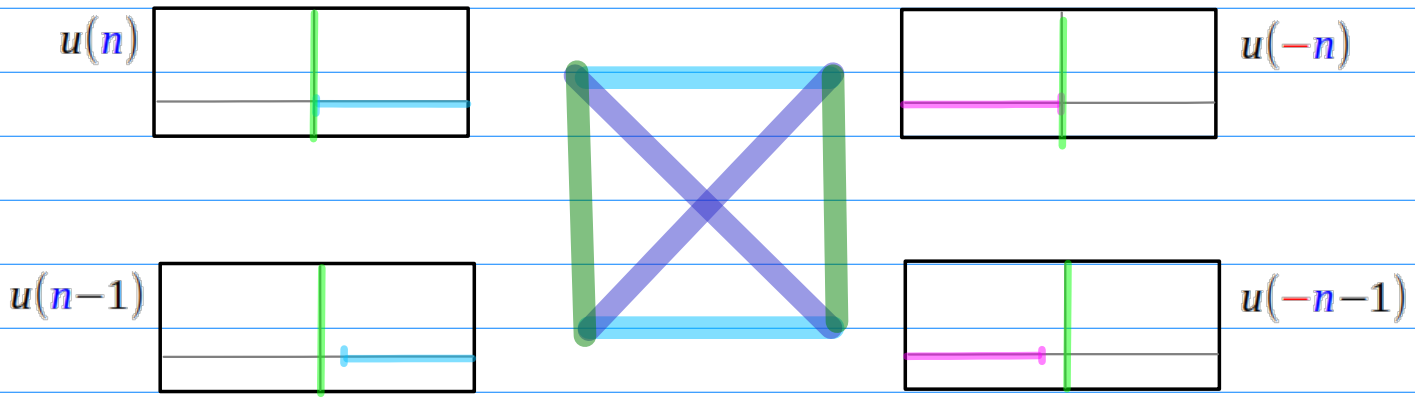








Range Shifting, Range Flipping, Range Complementing



- **1) Range Flipping** $R(n) \Longleftrightarrow R(-n)$
- **2) Range Complementing** $R(n) \Longleftrightarrow \overline{R(n)}$
- **3) Range Shifting** $R(n) \Longleftrightarrow \overline{R(-n)}$

$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

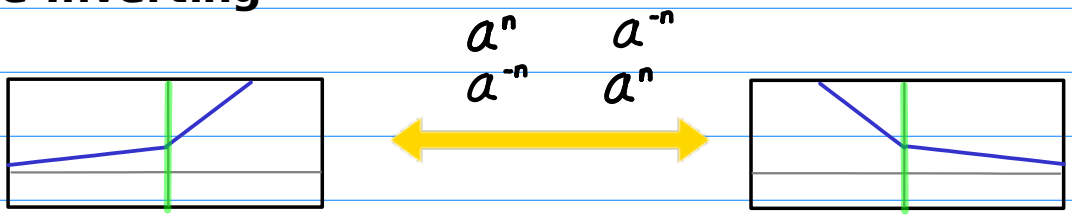
$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

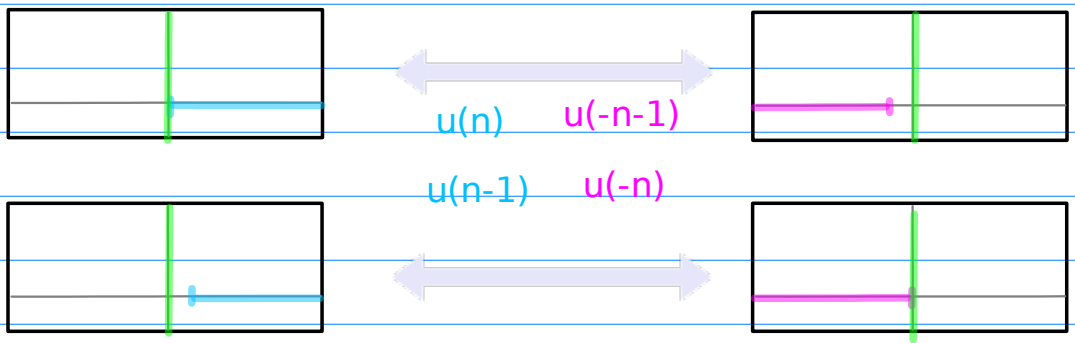
Complementary Inverting over (1) ~ (8)

$a^{n}R(n) \Leftrightarrow a^{-n}\overline{R(n)}$

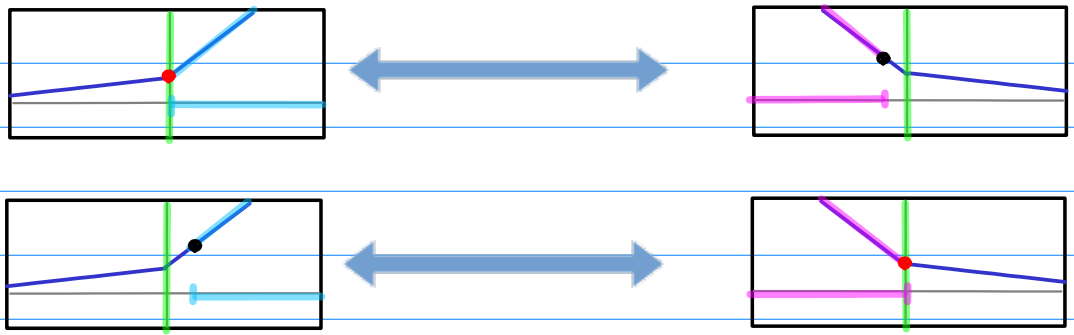
(1) Base Inverting



(2) Range Complementing



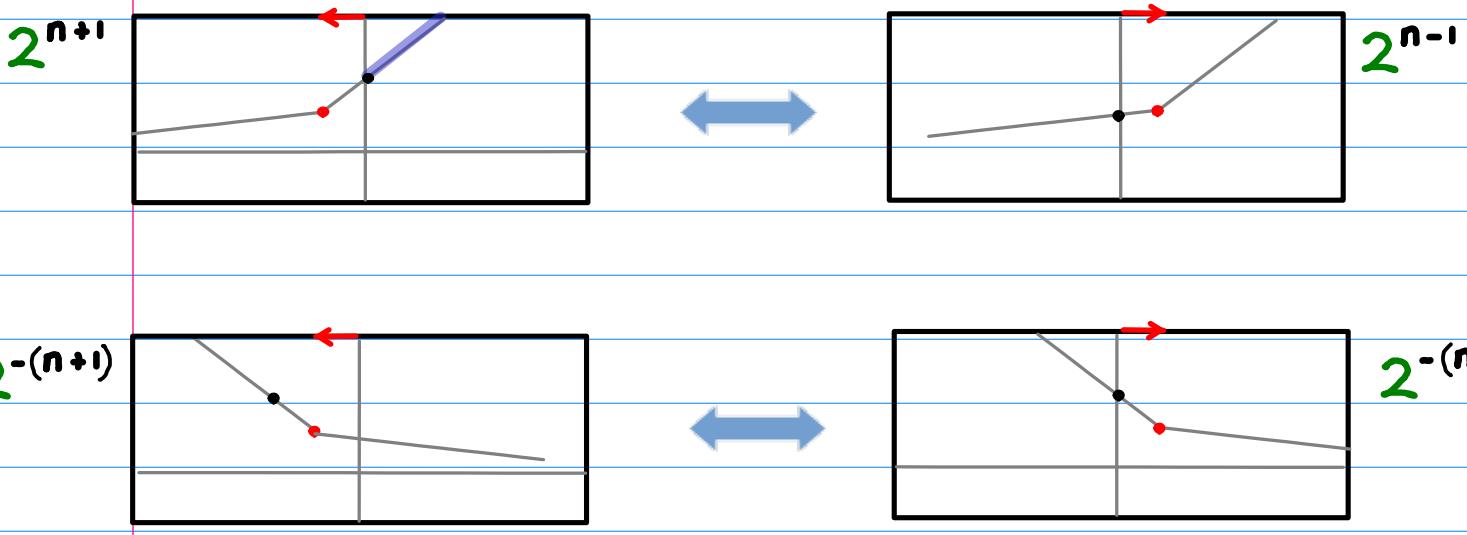
(1+2) Complementary Inverting



Exponent Shifting2 sh2(n)

$2^{n+1} \Longleftrightarrow 2^{n-1}$

$2^{-(n+1)} \Longleftrightarrow 2^{-(n-1)}$



Base Inverting

$$a^{n} \Longleftrightarrow a^{-n}$$

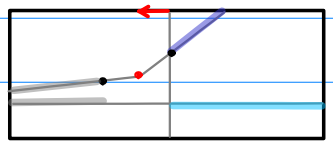
Shifted Range Flipping

$$a^{n}R(n) \Longleftrightarrow a^{sh2(n)}R(-n)$$

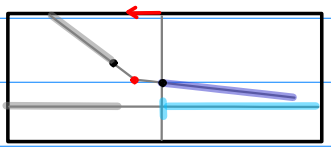
Exponent Shifting2 **Range Flipping**

Exponent Shifting2	
b^{n}	$b^{sh2(n)}$
$a^{(n+1)}$	$a^{(n-1)}$
$a^{-(n+1)}$	$a^{-(n-1)}$
$a^{(n-1)}$	$a^{(n+1)}$
$a^{-(n-1)}$	$a^{-(n+1)}$

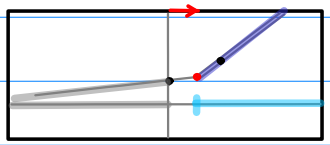
$$a^{n}R(n) \Longleftrightarrow a^{-sh2(n)}R(-n)$$



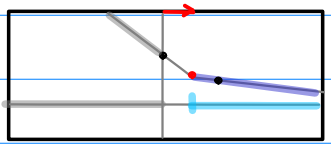
a^{n+1}



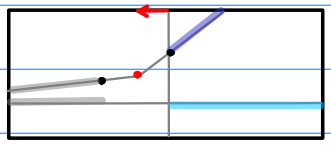
a^{-n-1}



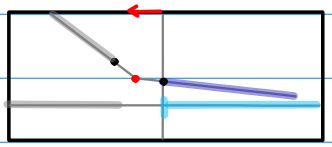
a^{n-1}



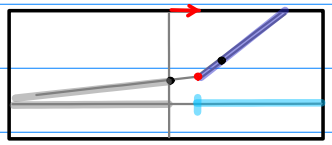
a^{-n+1}



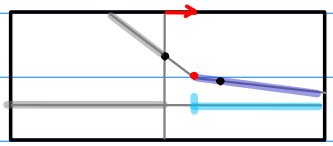
a^{n+1} u(n)



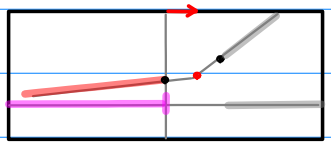
a^{-n-1} u(n)



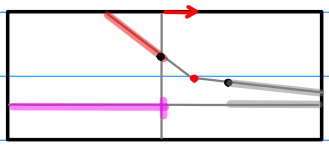
a^{n-1} u(n-1)



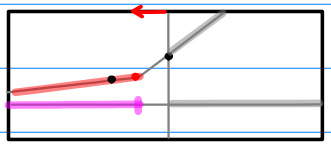
a^{-n+1} u(n-1)



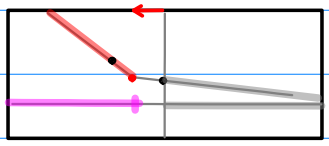
a^{n-1} u(-n)



a^{-n+1} u(-n)



a^{n+1} u(-n-1)



a^{-n-1} u(-n-1)

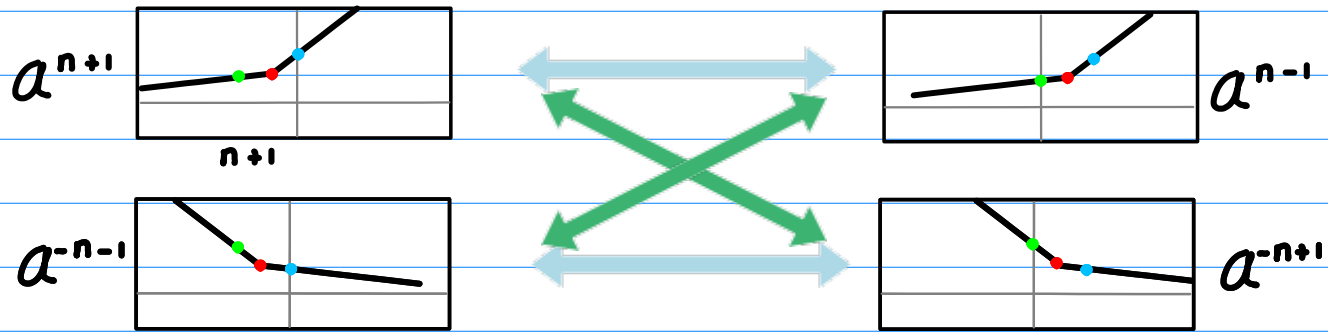
Shifted Range Flipping

$$a^{n}R(n) \quad \Longleftrightarrow \quad a^{sh2(n)}R(-n)$$

Exponent Shifting2 + Base Inverting

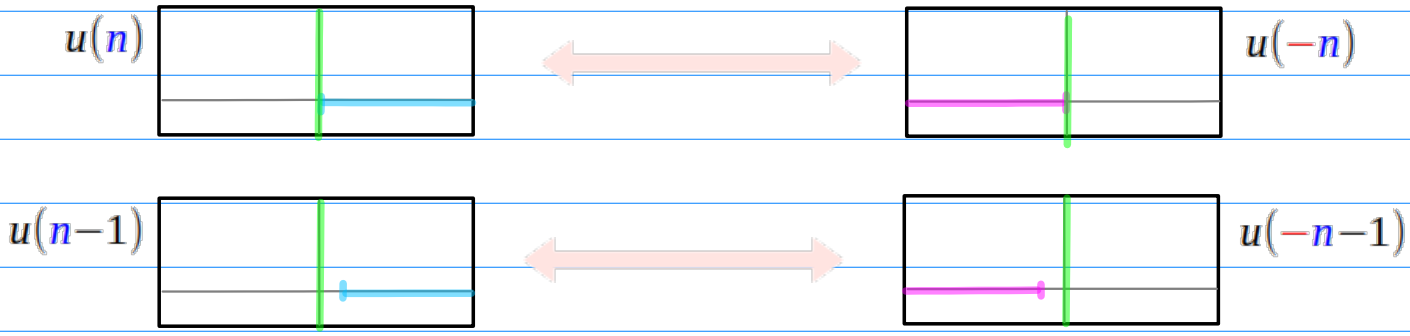
$$a^{n} \quad \Longleftrightarrow \quad a^{sh2(n)}$$

$$a^{n} \quad \Longleftrightarrow \quad a^{-sh2(n)}$$



Range Flipping

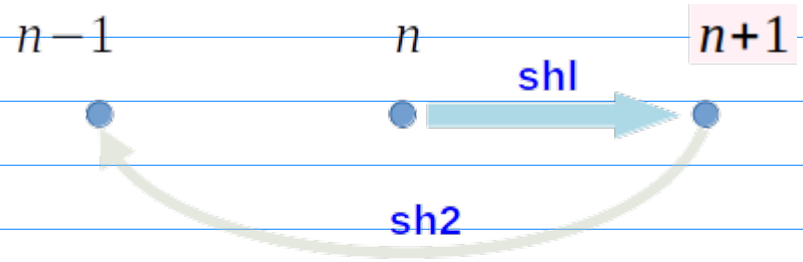
$$R(n) \quad \Longleftrightarrow \quad R(-n)$$

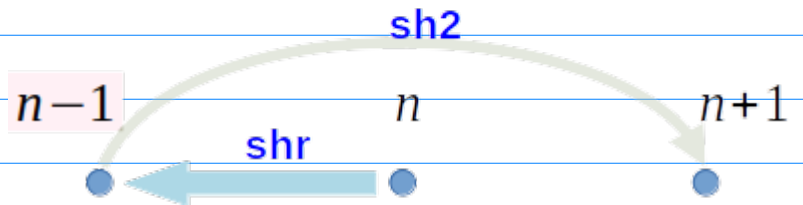


b^{n}	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

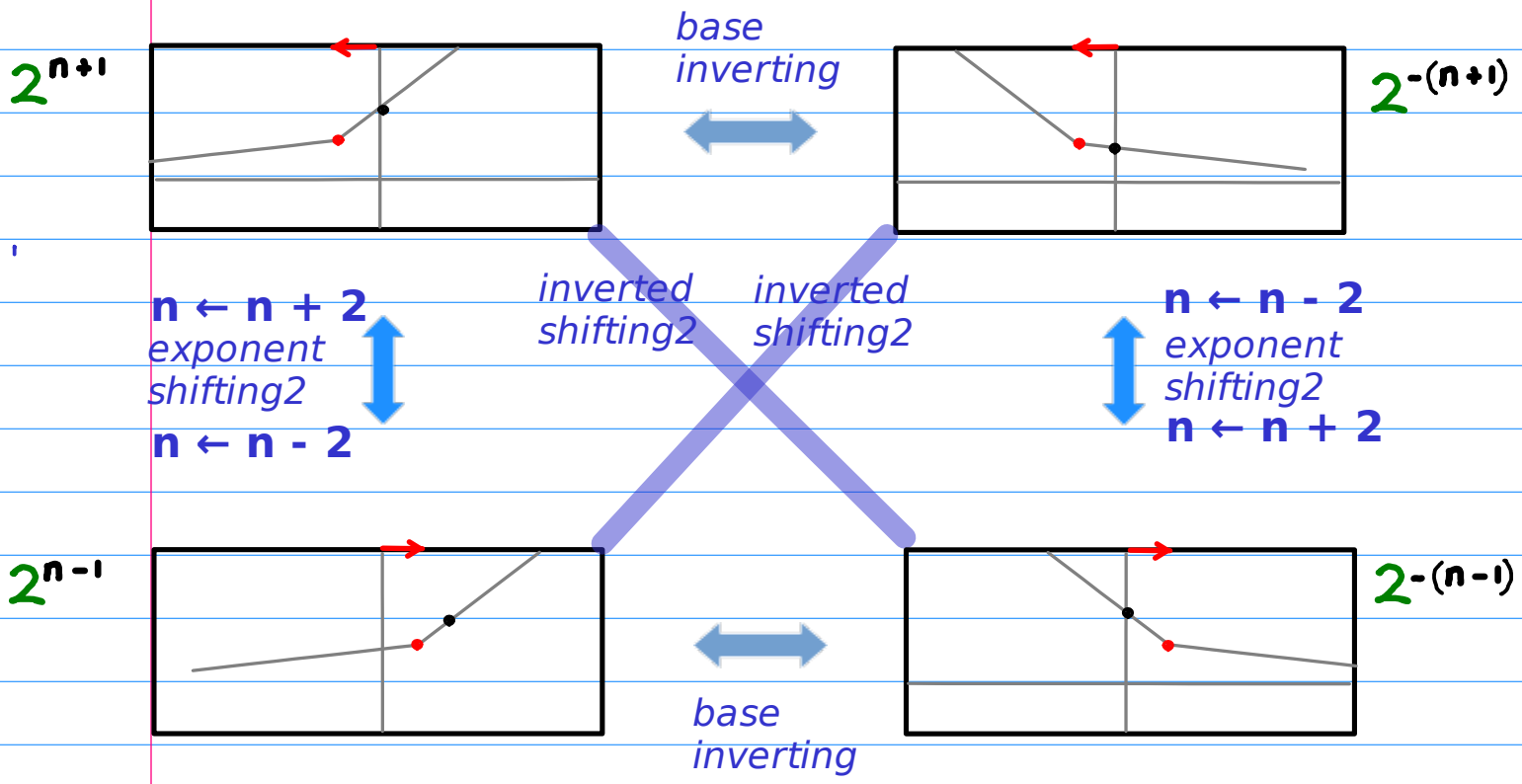
Exponent Shifting2 sh2(n)





n		$sh2(n)$
$n+1$	$n \leftarrow n-2$	$n-1$
$n-1$	$n \leftarrow n+2$	$n+1$
$-(n+1)$	$n \leftarrow n-2$	$-(n-1)$
$-(n-1)$	$n \leftarrow n+2$	$-(n+1)$

Inverted Shifting2 -sh2(n)



Inverted Shifting2

b^n	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

$$a^n \Leftrightarrow a^{-n}$$

$$a^n R(n) \Leftrightarrow a^{sh2(n)} R(-n)$$

Exponent Shifting2 **Range Flipping**

$$a^n R(n) \Leftrightarrow a^{-sh2(n)} R(-n)$$

Inverted Shifting2

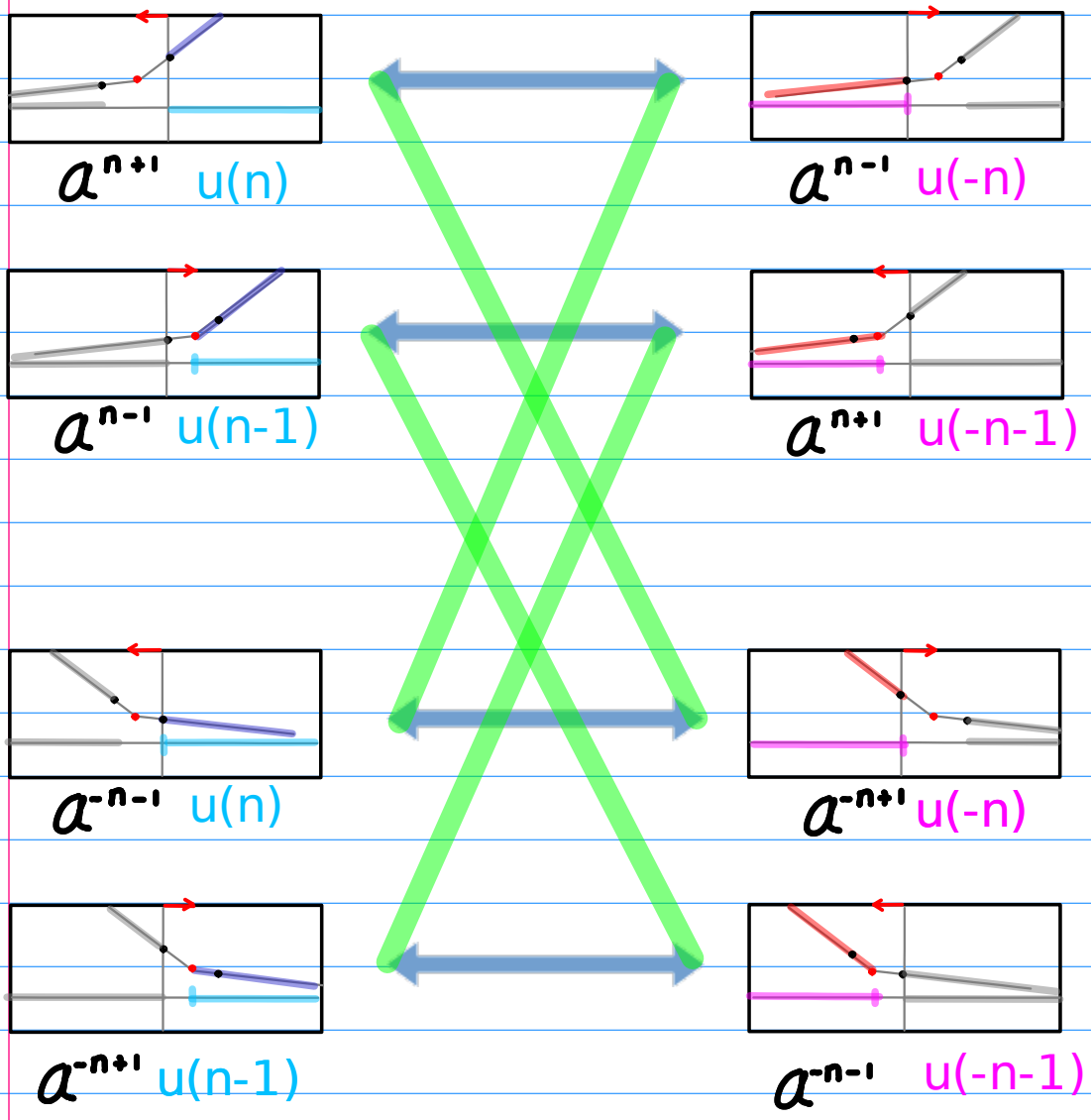
Shifted Range Flipping

$$a^{n} R(n) \Longleftrightarrow a^{sh2(n)} R(-n)$$

Flipping2

$$a^{n} R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

Flipping2 = Base Inverting + Shifted Range Flipping
= Base Inverting + Exponent Shifting2 + Range Flipping



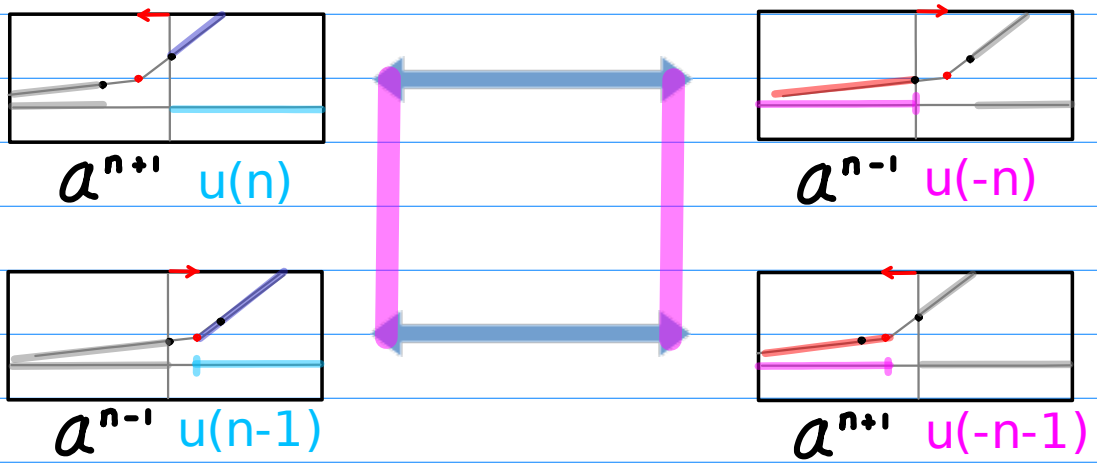
Shifted Range Flipping

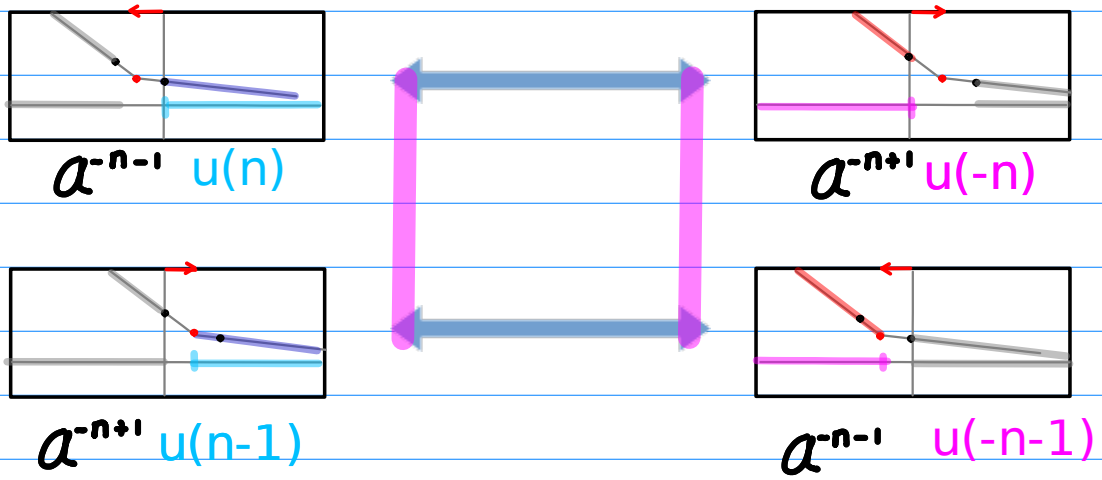
$$a^{n}R(n) \Leftrightarrow a^{sh\,2(n)}R(-n)$$

$$a^{n}R(n) \Leftrightarrow a^{sh\,2(n)}\overline{R(-n)}$$

Shifting2

shifting2 = Shifted Range Flipping + Range Complementing
= Exponent Shifting2 + Range (Flipping + Complementing)
= Exponent Shifting2 + Range Shifting



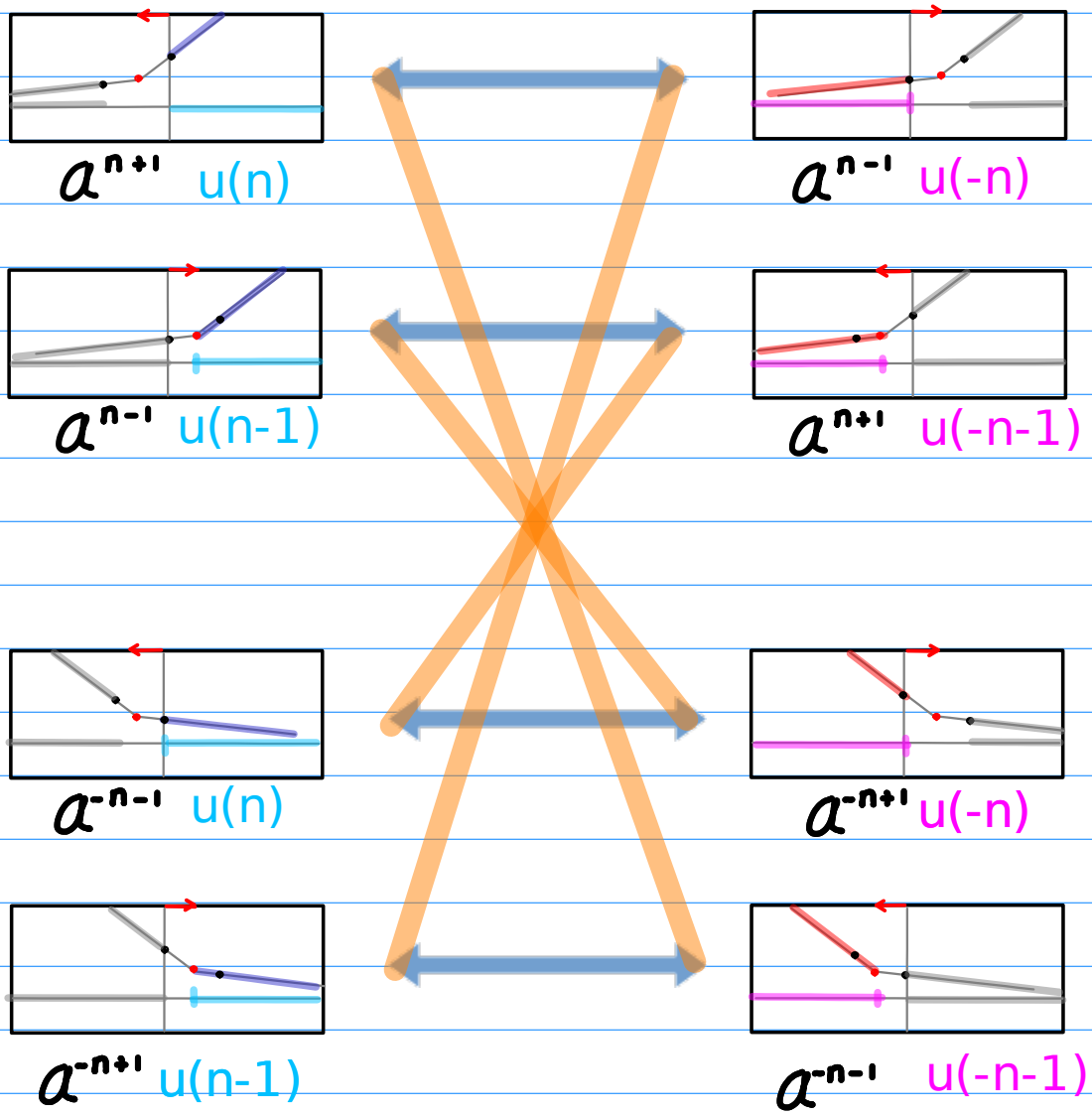


Shifted Range Flipping

Complementary Inverting

$$a^{n}R(n) \Longleftrightarrow a^{sh\,2(n)}R(-n)$$

$$a^{n}R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$



D. Flipping2

$$a^{n}R(n) \Longleftrightarrow a^{-sh2(n)}R(-n)$$

$$a^{n} \Longleftrightarrow a^{-n}$$

$$a^{n}R(n) \Longleftrightarrow a^{sh2(n)}R(-n)$$

n		$sh2(n)$
$n+1$	$n \leftarrow n-2$	$n-1$
$n-1$	$n \leftarrow n+1$	$n+1$
$-(n+1)$	$n \leftarrow n+1$	$-(n-1)$
$-(n-1)$	$n \leftarrow n-1$	$-(n+1)$

$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

E. Shifting2

$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)}\,\overline{R(-n)}$$

$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$$

$$R(n) \Longleftrightarrow \overline{R(n)}$$

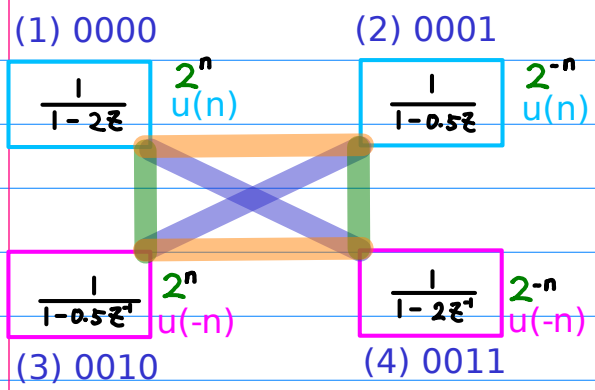
F. Complementary Inverting

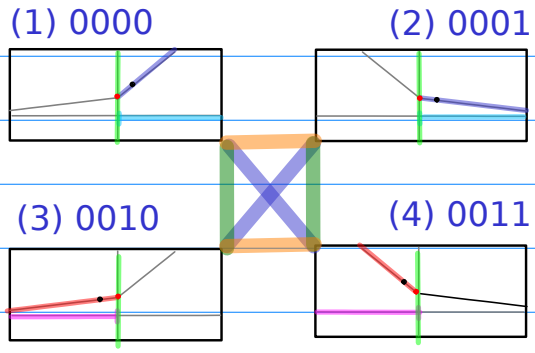
$$a^n R(n) \quad \Longleftrightarrow \quad a^{-n}\overline{R(n)}$$

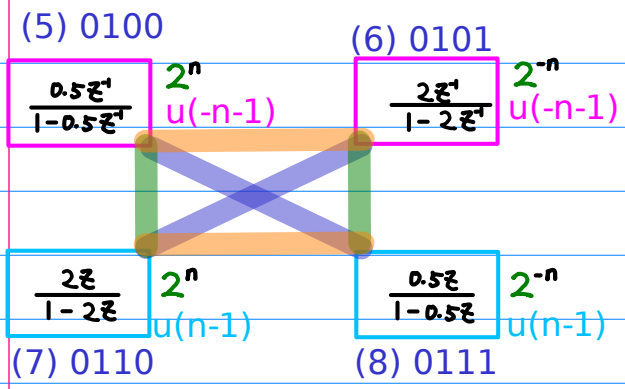
$$\begin{aligned} a^n &\Longleftrightarrow a^{-n} \\ R(n) &\Longleftrightarrow \overline{R(n)} \end{aligned}$$

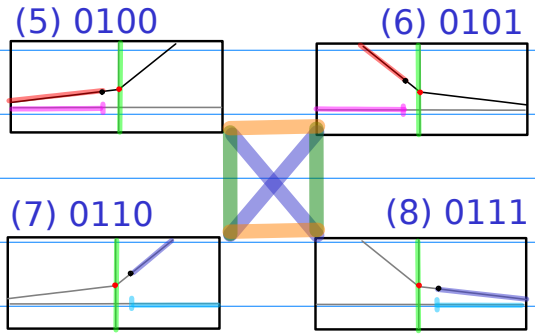
$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

A.1 Flipping
Base Inverting
Range Flipping









Base Inverting

$0\ b_2\ b_1\ b_0 \rightarrow 0\ b_2\ b_1\ \overline{b_0}$

(1) 0 0 **0** 0 — 0 0 **0** 1 (2)

Range Flipping

$0\ b_2\ b_1\ b_0 \rightarrow 0\ b_2\ \overline{b_1}\ b_0$

(3) 0 0 **1** 0 — 0 0 **1** 1 (4)

(5) 0 1 **0** 0 — 0 1 **0** 1 (6)

Flipping

$0\ b_2\ b_1\ b_0 \rightarrow 0\ b_2\ \overline{b_1}\ \overline{b_0}$

(7) 0 1 **1** 0 — 0 1 **1** 1 (8)

$$a^n \Longleftrightarrow a^{-n}$$

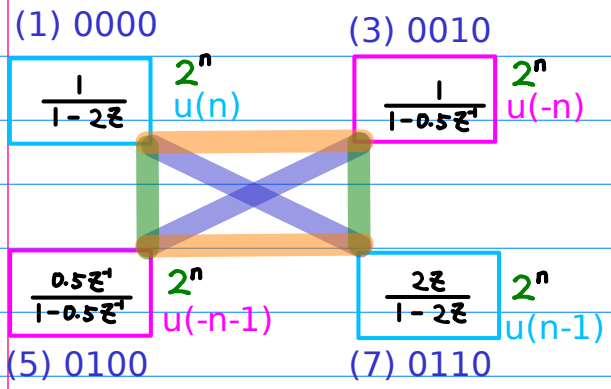
$$R(n) \Longleftrightarrow R(-n)$$

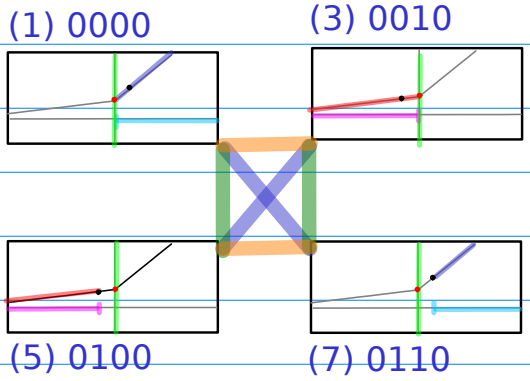
$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

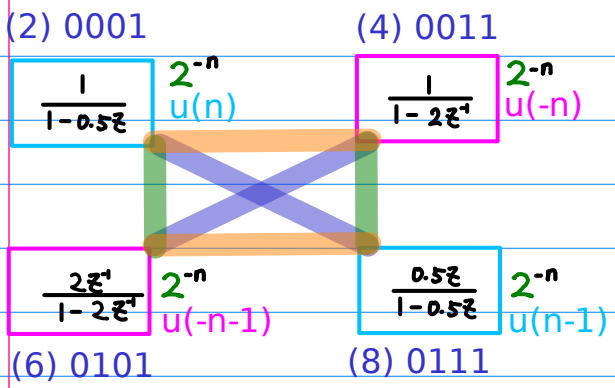
B.1 Range Shifting = Range Flipping + Range Complementing

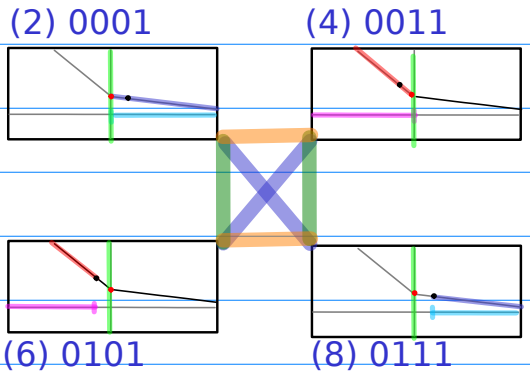
Range Flipping

Range Complementing









Range Flipping

$$0\ b_2\ b_1\ b_0 \longrightarrow 0\ b_2\ \overline{b_1}\ b_0$$

(1) 0 0 0 0 — 0 0 1 0 (3)

Range Complementing

$$0\ b_2\ b_1\ b_0 \longrightarrow 0\ \overline{b_2}\ b_1\ b_0$$

(5) 0 1 0 0 — 0 1 1 0 (7)

(2) 0 0 0 1 — 0 0 1 1 (4)

Range Shifting

$$0\ b_2\ b_1\ b_0 \longrightarrow 0\ \overline{b_2}\ \overline{b_1}\ b_0$$

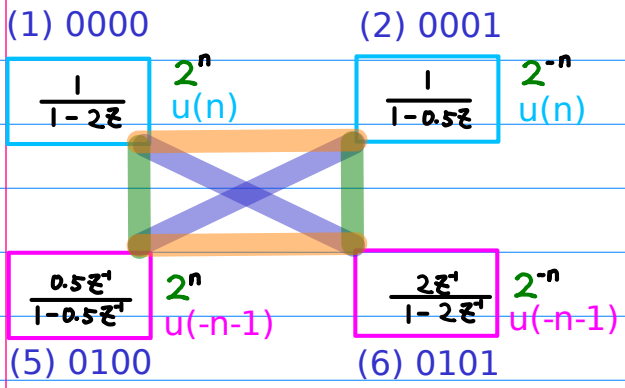
(6) 0 1 0 1 — 0 1 1 1 (8)

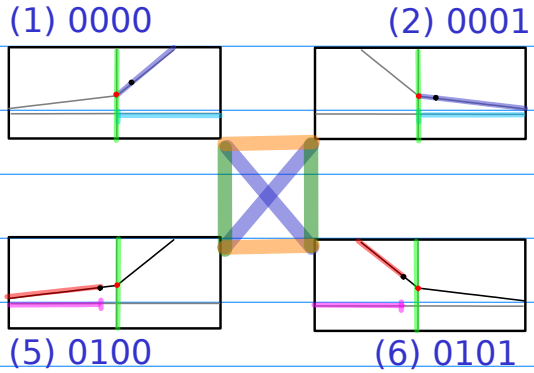
$$R(n) \Longleftrightarrow R(-n)$$

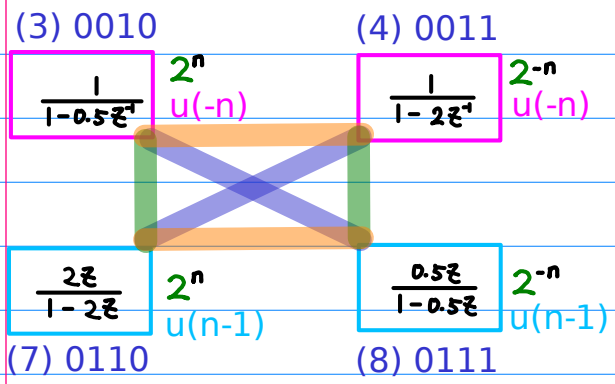
$$R(n) \Longleftrightarrow \overline{R(n)}$$

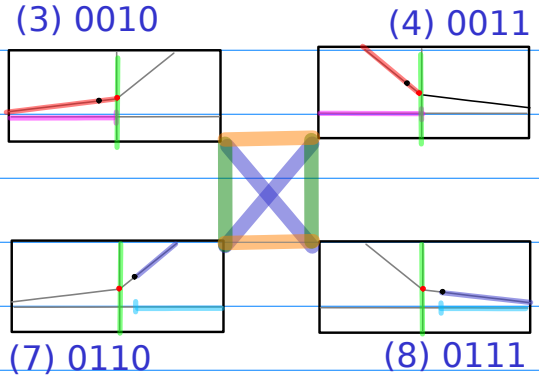
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

C.1 Complementary Inverting
Base Inverting
Range Complementing









Base Inverting

(1) 0 0 0 0 — 0 0 0 1 (2)

Range Complementing

(5) 0 1 0 0 — 0 1 0 1 (6)

(3) 0 0 1 0 — 0 0 1 1 (4)

Complementary Flipping

(7) 0 1 1 0 — 0 1 1 1 (8)

$$a^n \Longleftrightarrow a^{-n}$$

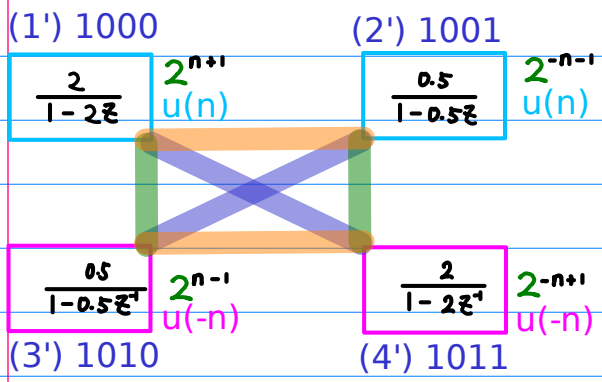
$$R(n) \Longleftrightarrow \overline{R(n)}$$

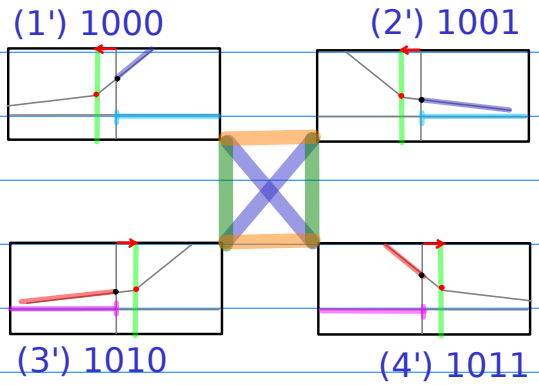
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

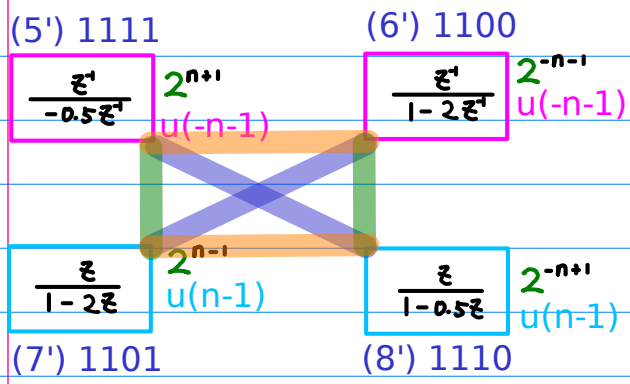
D.1 Flipping2

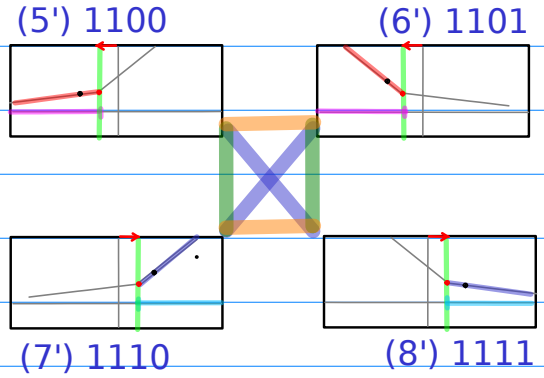
Base Inverting

Shifted Range Flipping = Exponent Shifting2 + Range Flipping









Base Inverting

Shifted Range Flipping

Flipping2

(1) 1 0 0 0 1 0 0 1 (2)

(3) 1 0 1 0 1 0 1 1 (4)

(5) 1 1 0 0 1 1 0 1 (6)

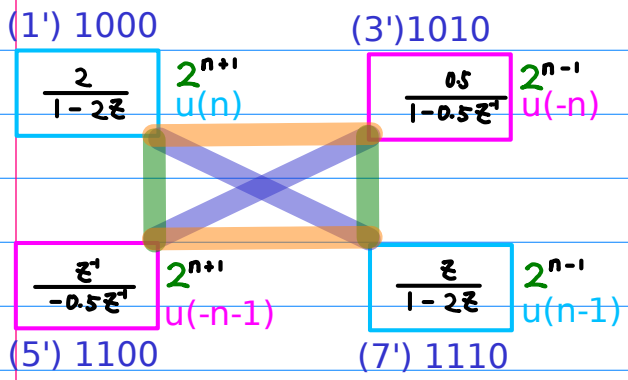
(7) 1 1 1 0 1 1 1 1 (8)

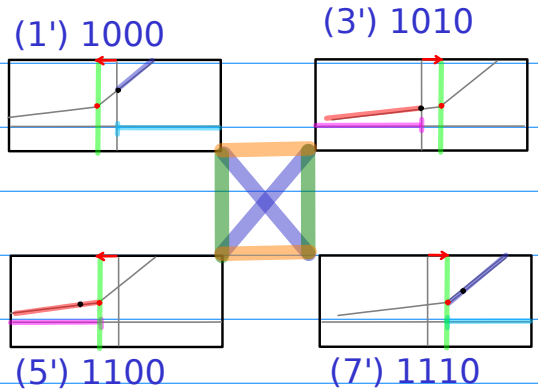
$$a^n \Longleftrightarrow a^{-n}$$

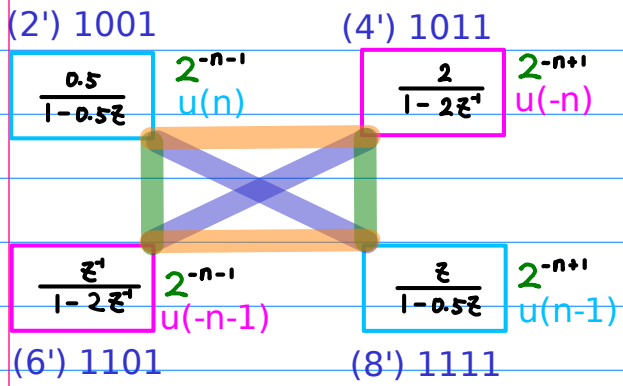
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} R(-n)$$

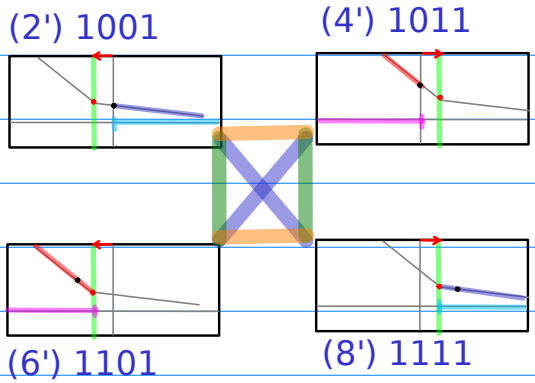
$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

E.1 Shifting2 = Exponent Shifting2 + Range Complementing
Shifted Range Flipping = Exponent Shifting2 + Range Flipping
Range Complementing









Shifted Range Flipping

Range Complementing

Shifting2

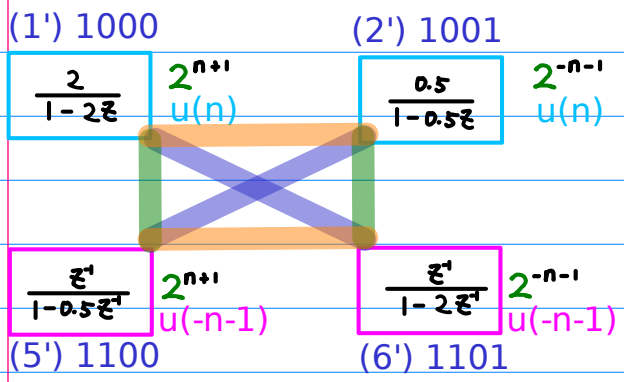
(1)	1	0	0	0		1	0	1	0 (3)
(5)	1	1	0	0		1	1	1	0 (7)
(2)	1	0	0	1		1	0	1	1 (4)
(6)	1	1	0	1		1	1	1	1 (8)

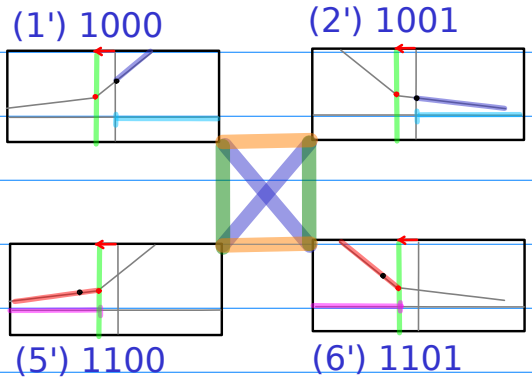
$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$$

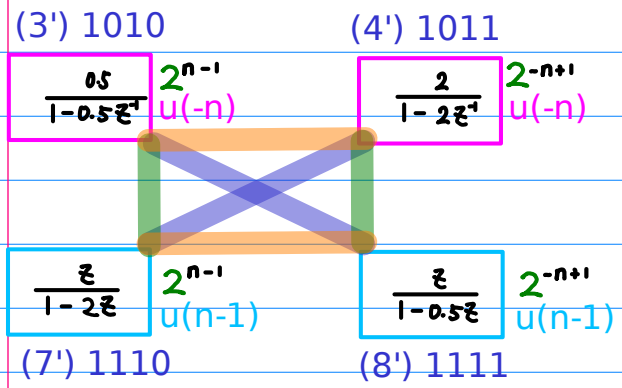
$$R(n) \Longleftrightarrow \overline{R(n)}$$

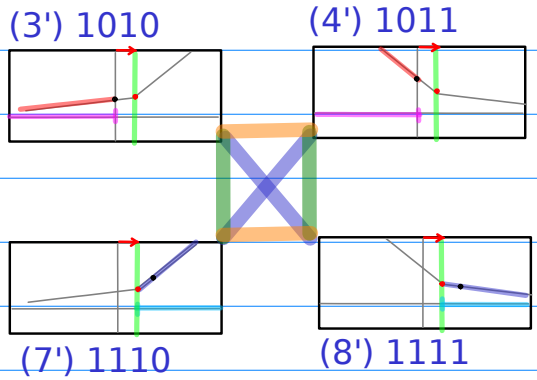
$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} \overline{R(-n)}$$

F.1 Complementary Inverting
Base Inverting
Range Complementing









Base Inverting

(1) 1 0 0 0 1 0 0 1 (2)

Range Complementing

(5) 1 1 0 0 1 1 0 1 (6)

(3) 1 0 1 0 1 0 1 1 (4)

Complementary Inverting

(7) 1 1 1 0 1 1 1 1 (8)

$$a^n \Longleftrightarrow a^{-n}$$

$$R(n) \Longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

A.2b Flipping
Base Inverting
Range Flipping

$$a^n R(n) \quad \Longleftrightarrow \quad a^{-n} R(-n)$$

1) Base Inverting

2) Range Flipping

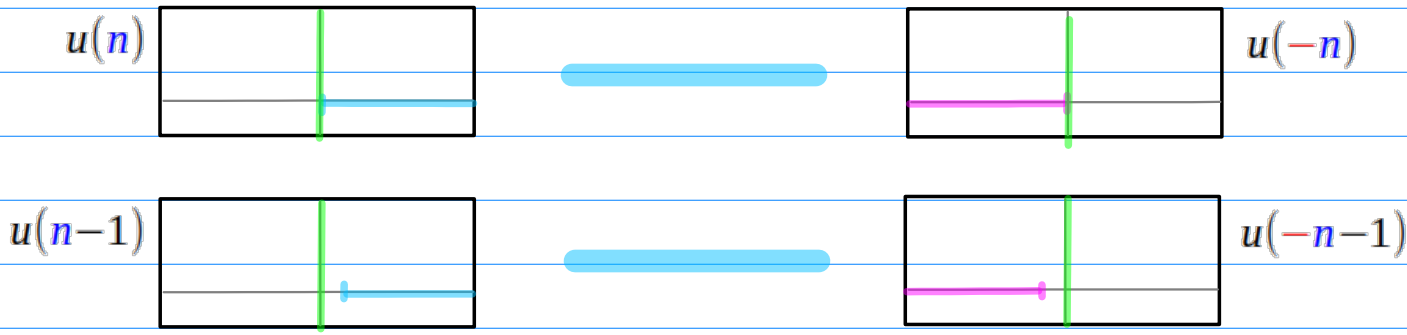
3) Flipping

B.2a Range Shifting
Range Flipping
Range Complementing

$$R(n) \Longleftrightarrow \overline{R(-n)}$$

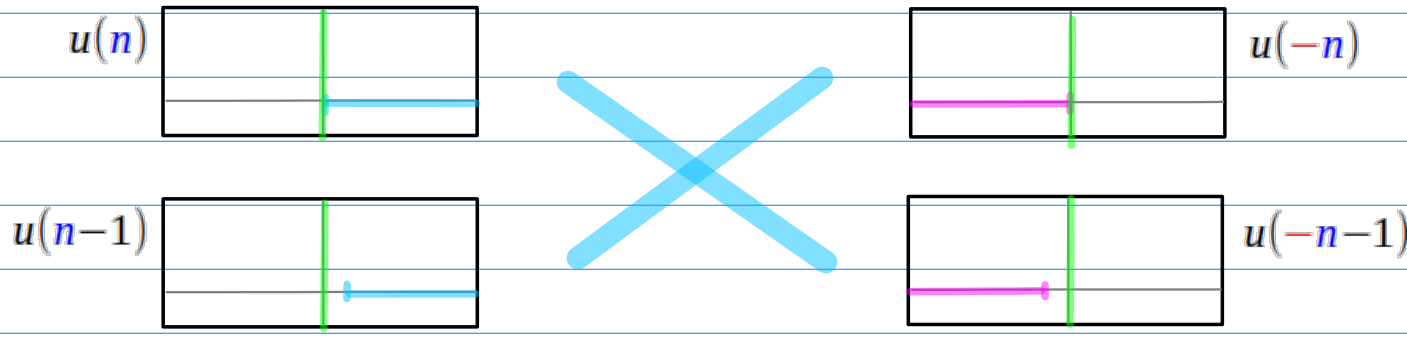
1) Range Flipping

$$R(n) \Longleftrightarrow R(-n)$$



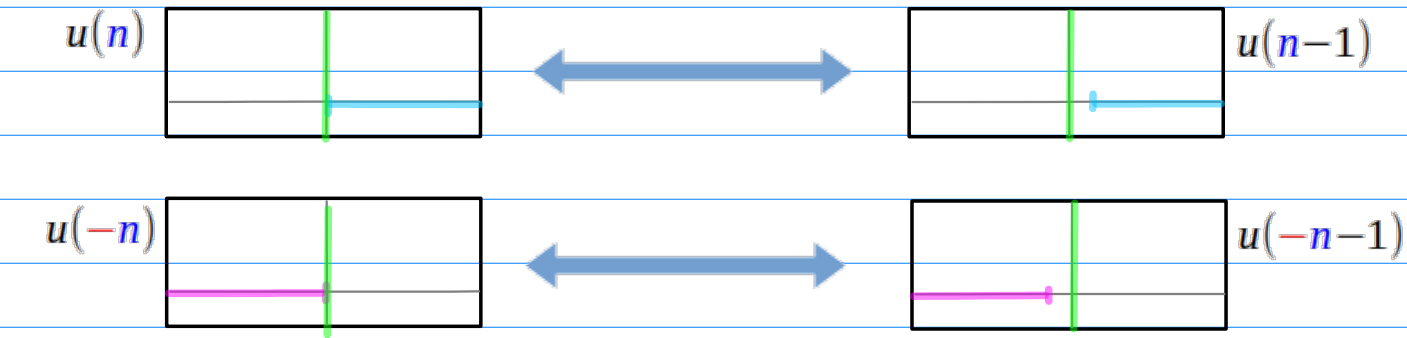
2) Range Complementing

$$R(n) \Longleftrightarrow \overline{R(n)}$$



3) Range Shifting

$$R(n) \Longleftrightarrow \overline{R(-n)}$$



B.2b Range Shifting
Range Flipping
Range Complementing

$$R(n) \Longleftrightarrow \overline{R(-n)}$$

1) Range Flipping

2) Range Complementing

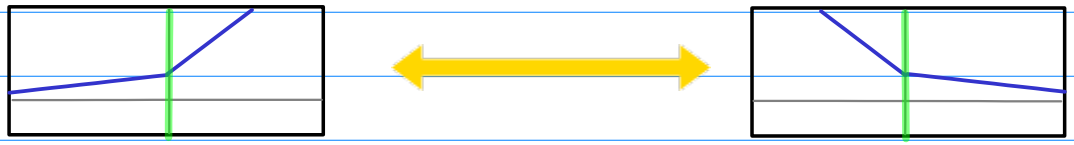
3) Range Shifting

C.2a Complementary Inverting
Base Inverting
Range Complementing

$$a^{n} R(n) \quad \Longleftrightarrow \quad a^{-n} \overline{R(n)}$$

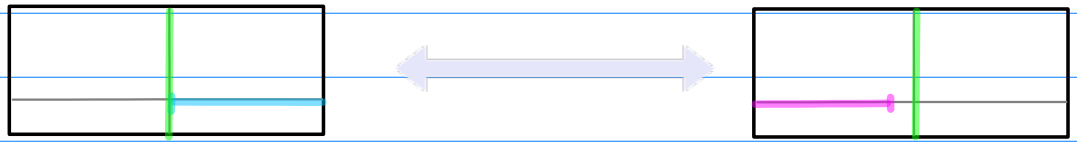
1) Base Inverting

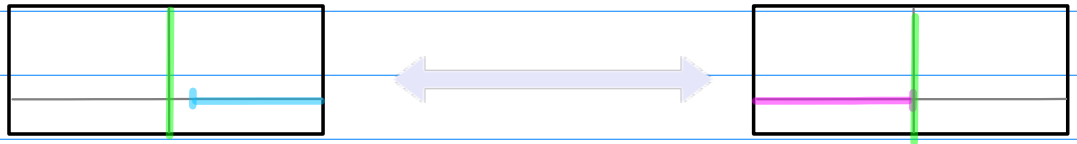
$$0 \; b_2 \; b_1 \; b_0 \rightarrow 0 \; b_2 \; b_1 \; \overline{b_0}$$



2) Range Complementing

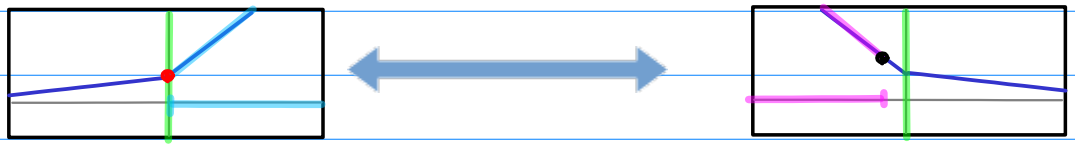
$$0 \; b_2 \; b_1 \; b_0 \rightarrow 0 \; \overline{b_2} \; b_1 \; b_0$$





3) Complementary Flippin

$$0 \; b_2 \; b_1 \; b_0 \rightarrow 0 \; \overline{b_2} \; b_1 \; \overline{b_0}$$



D.2a Flipping2

. **Base Inverting**

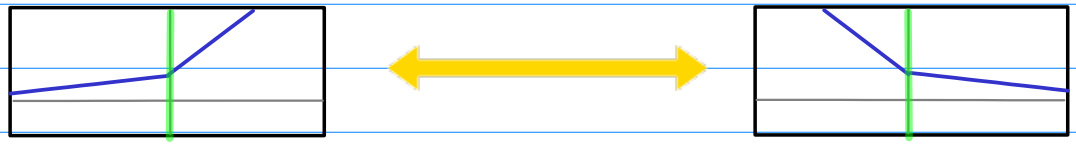
Shifted Range Flipping

$$a^n \quad \Longleftrightarrow \quad a^{-n}$$

$$a^n R(n) \quad \Longleftrightarrow \quad a^{sh(n)} R(-n)$$

1) Base Inverting

$$a^n \quad \Longleftrightarrow \quad a^{-n}$$

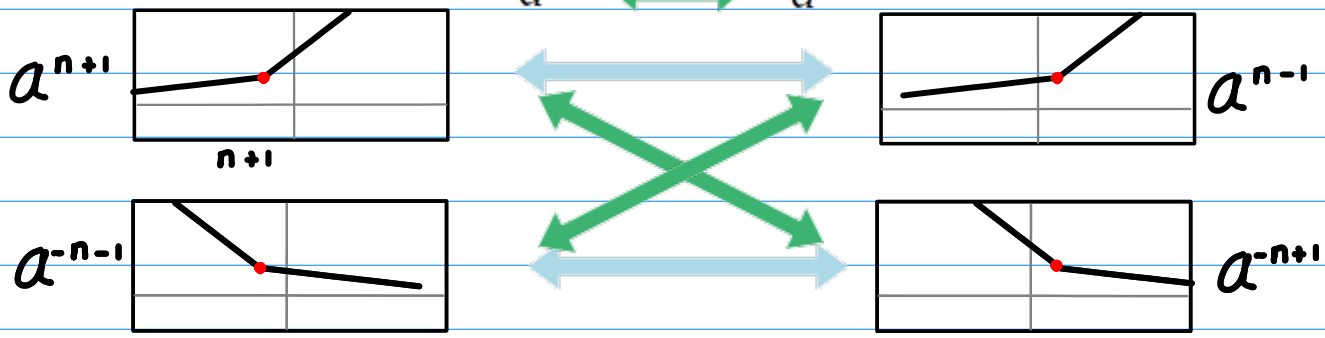


2) Shifted Range Flipping

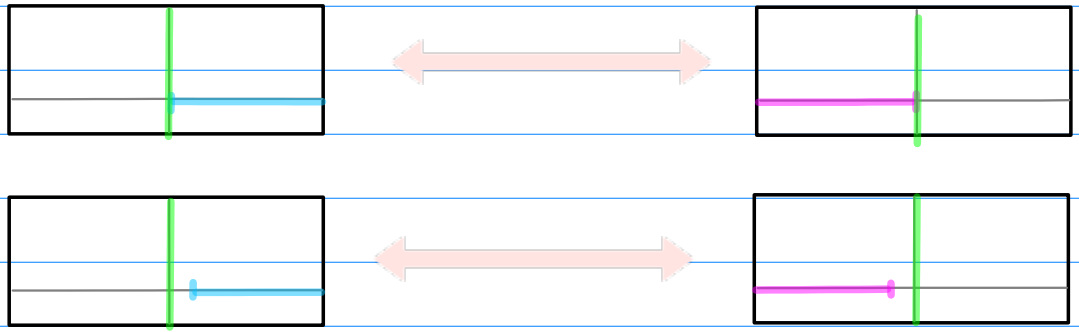
$$a^n R(n) \quad \Longleftrightarrow \quad a^{-sh2(n)} R(-n)$$

$$a^n \quad \Longleftrightarrow \quad a^{sh2(n)}$$

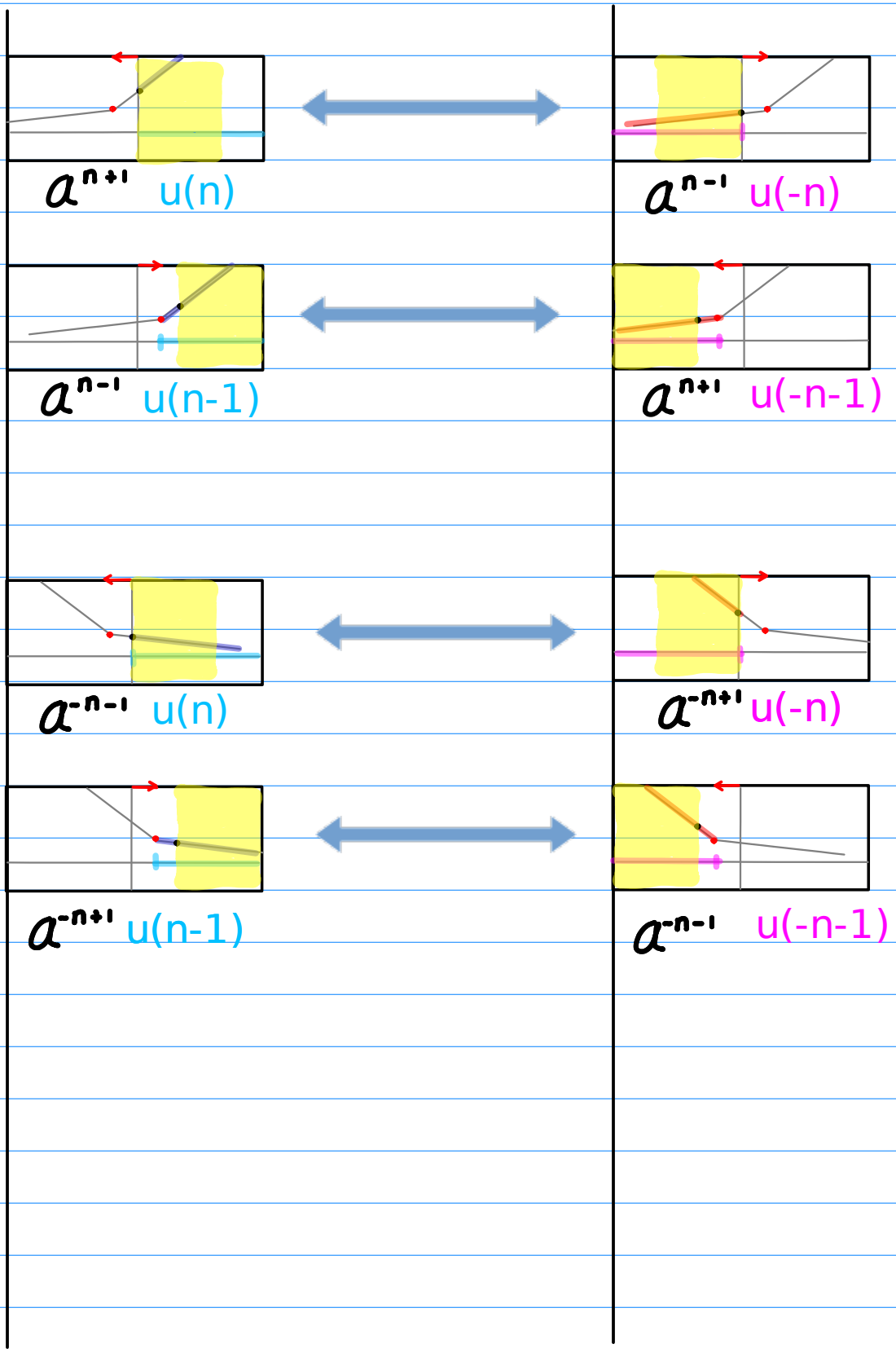
$$a^n \quad \Longleftrightarrow \quad a^{-sh2(n)}$$



$$R(n) \quad \Longleftrightarrow \quad R(-n)$$

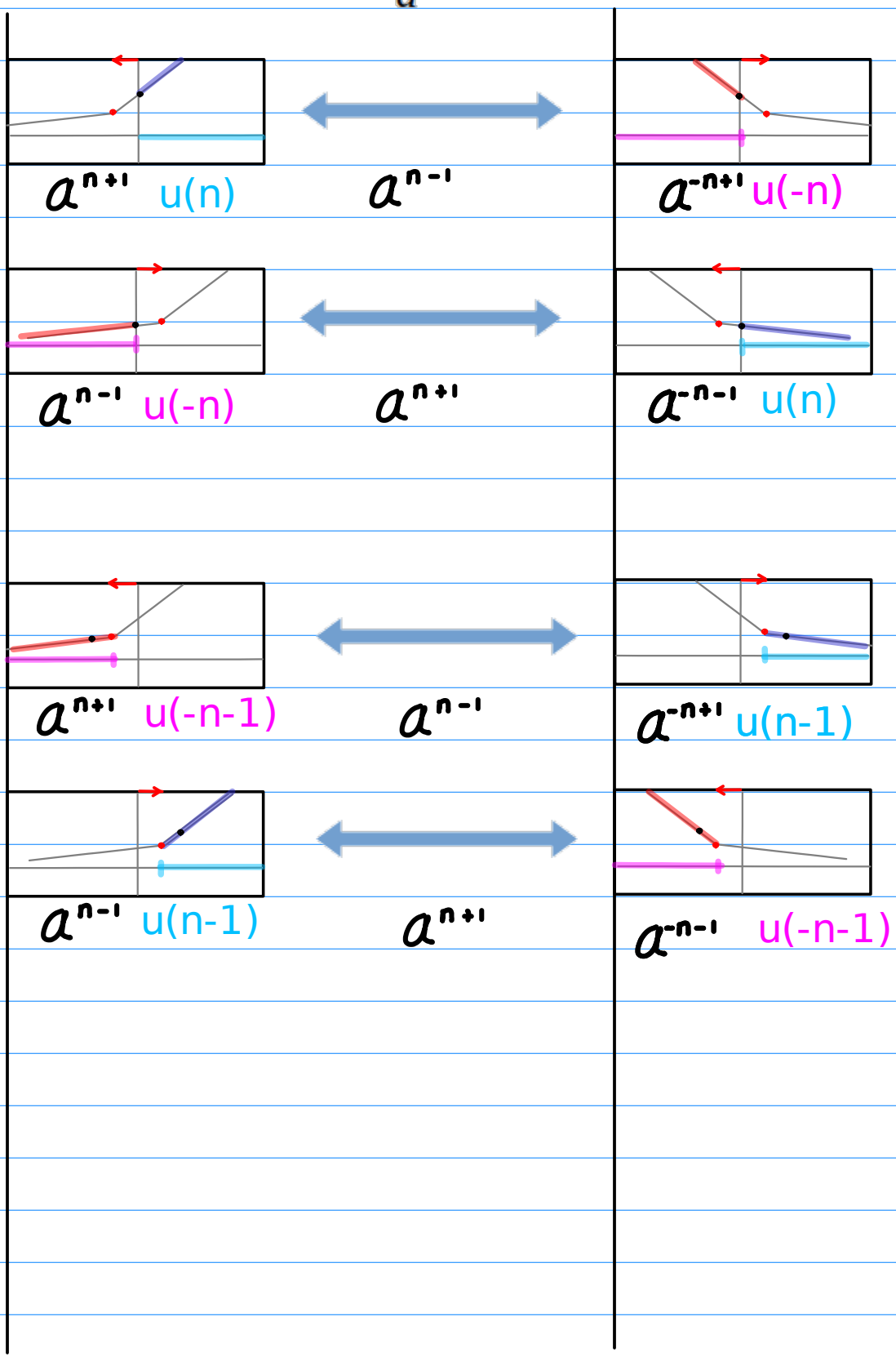


2) Shifted Range Flipping $a^{n}R(n) \Longleftrightarrow a^{sh2(n)}R(-n)$



3) Flipping2

$$a^{n}R(n) \quad \overset{a^{sh(n)}}{\Longleftrightarrow} \quad a^{-sh2(n)}R(-n)$$

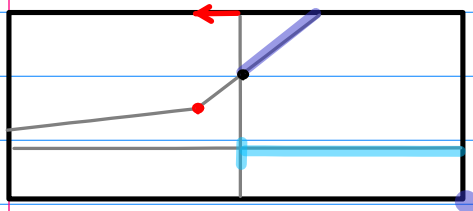


D. Flipping2

Base Invering

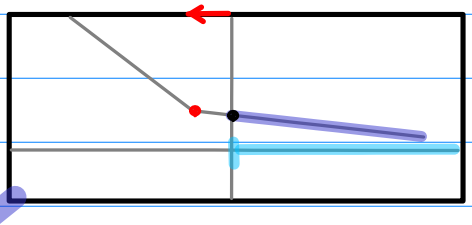
Shifted Range Flipping = Exponent Shifting2 + Range Flipping

(1') 1000



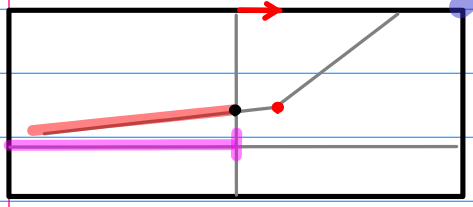
2^{n+1} u(n)

(2') 1001



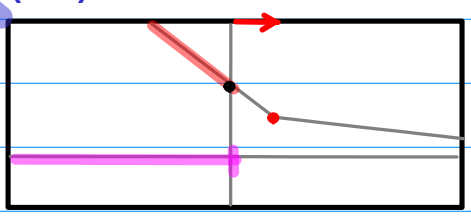
2^{-n-1} u(n)

(3') 1010



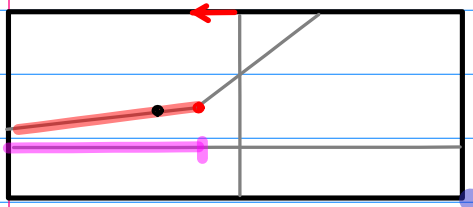
2^{n-1} u(-n)

(4') 1011



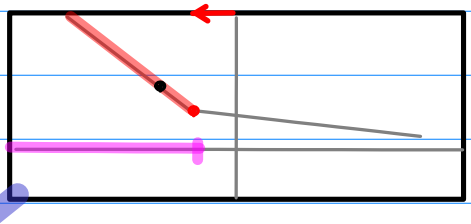
2^{-n+1} u(-n)

(5') 1111



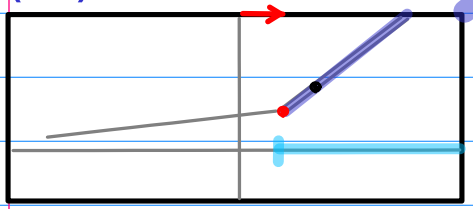
2^{n+1} u(-n-1)

(6') 1100



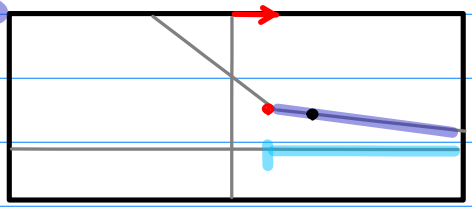
2^{-n-1} u(-n-1)

(7') 1101



2^{n-1} u(n-1)

(8') 1110



2^{-n+1} u(n-1)

D. Flipping2

Base Inverting

Shifted Range Flipping = Exponent Shifting2 + Range Flipping

1) Exponent Flipping = (Base Inverting, Exponent Shifting)

$2^{n+1} \Longleftrightarrow 2^{-(n-1)}$

$2^{n-1} \Longleftrightarrow 2^{-(n+1)}$

2) Range Flipping

$u(n) \Longleftrightarrow u(-n)$

$u(n-1) \Longleftrightarrow u(-n-1)$

3) Exponent Flipping + Range Flipping

$2^{n+1}\,u(n) \Longleftrightarrow 2^{-(n-1)}\,u(-n)$

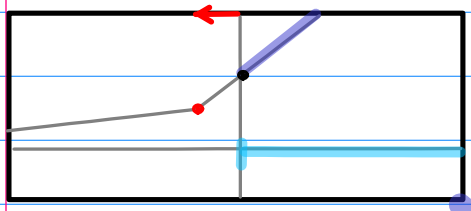
$2^{n+1}\,u(-n-1) \Longleftrightarrow 2^{-(n-1)}\,u(n-1)$

$2^{n-1}\,u(-n) \Longleftrightarrow 2^{-(n+1)}\,u(n)$

$2^{n-1}\,u(n-1) \Longleftrightarrow 2^{-(n+1)}\,u(-n-1)$

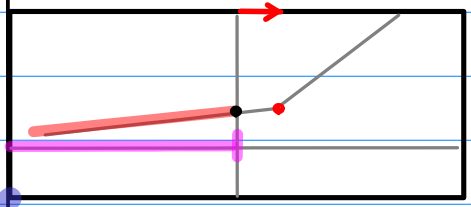
E. Shifting2 = Exponent Shfiting2 + Range Shifting
Shifted Range Flipping = Exponent Shifting2 + Range Flipping
Range Complementing

(1') 1000



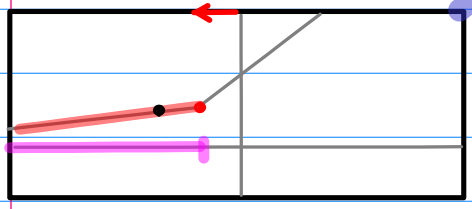
2^{n+1} u(n)

(3') 1010



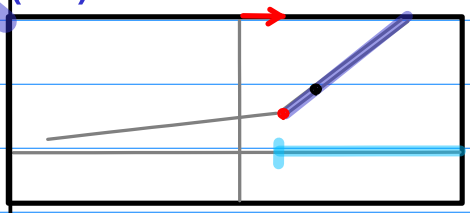
2^{n-1} u(-n)

(5') 1111



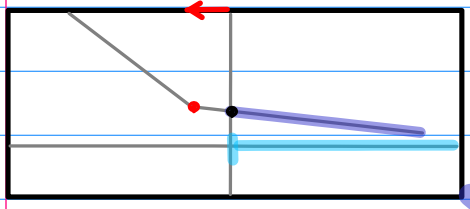
2^{n+1} u(-n-1)

(7') 1101



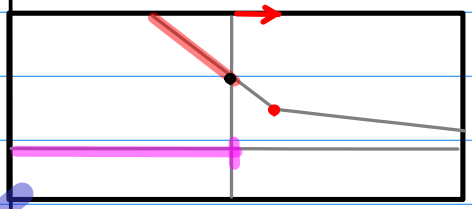
2^{n-1} u(n-1)

(2') 1001



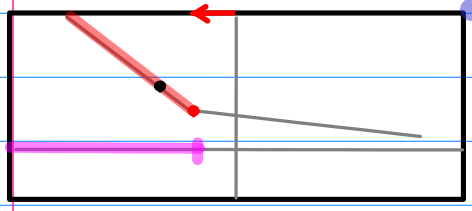
2^{-n-1} u(n)

(4') 1011



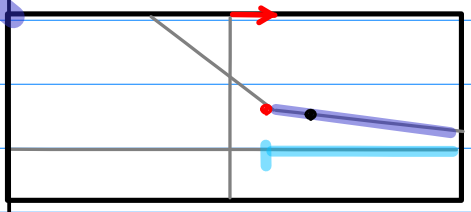
2^{-n+1} u(-n)

(6') 1100



2^{-n-1} u(-n-1)

(8') 1110



2^{-n+1} u(n-1)

E. Shifting2 = Exponent Shfiting2 + Range Shifting
Shifted Range Flipping = Exponent Shifting2 + Range Flipping
Range Complementing

1) Exponent Shifting = (Exponent Shifting, ID)

$2^{n+1} \Longleftrightarrow 2^{n-1}$

$2^{-(n+1)} \Longleftrightarrow 2^{-(n-1)}$

2) Range Shifiting

$u(n) \Longleftrightarrow u(n-1)$

$u(-n-1) \Longleftrightarrow u(-n)$

3) Shifting

$2^{n+1}\,u(n) \Longleftrightarrow 2^{n-1}\,u(n-1)$

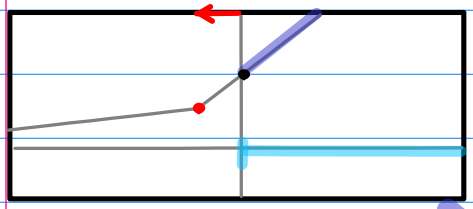
$2^{-(n+1)}\,u(n) \Longleftrightarrow 2^{-(n-1)}\,u(n-1)$

$2^{n-1}\,u(-n) \Longleftrightarrow 2^{n+1}\,u(-n-1)$

$2^{-(n-1)}\,u(-n) \Longleftrightarrow 2^{-(n+1)}\,u(-n-1)$

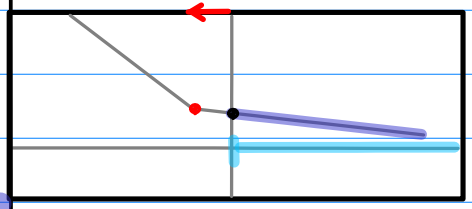
F. Complementary Inverting
Base Inverting
Range Complementing

(1') 1000



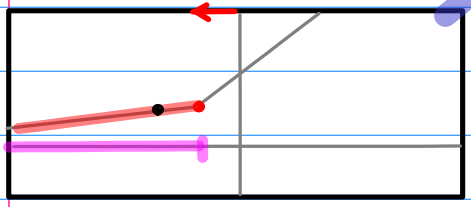
2^{n+1} u(n)

(2') 1001



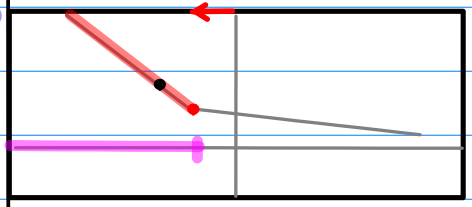
2^{-n-1} u(n)

(5') 1111



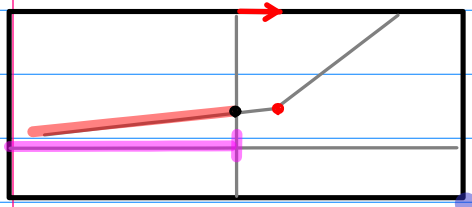
2^{n+1} u(-n-1)

(6') 1100



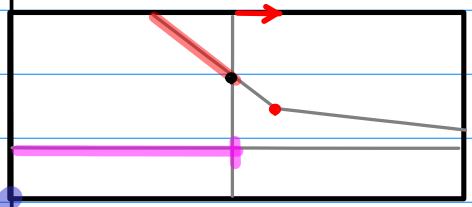
2^{-n-1} u(-n-1)

(3') 1010



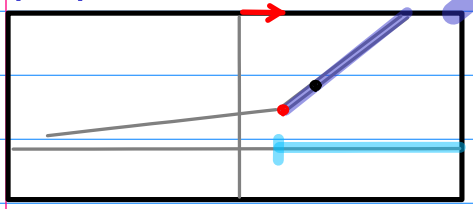
2^{n-1} u(-n)

(4') 1011



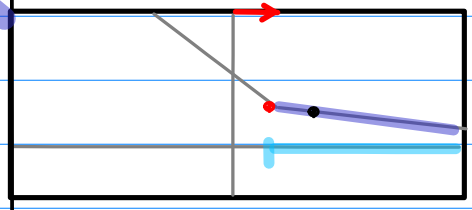
2^{-n+1} u(-n)

(7') 1101



2^{n-1} u(n-1)

(8') 1110



2^{-n+1} u(n-1)

F. Complementary Inverting
Base Inverting
Range Complementing

1) Base Invering = (Base Inverting, ID)

$2^{n+1} \Longleftrightarrow 2^{-(n+1)}$

$2^{n-1} \Longleftrightarrow 2^{-(n-1)}$

2) Range Complementing

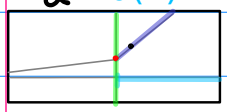
$u(n) \Longleftrightarrow u(-n-1)$

$u(n-1) \Longleftrightarrow u(-n)$

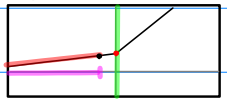
3) Complementary Inverting

2^{n+1} $u(n)$	$2^{-(n+1)}$ $u(-n-1)$	2^{n-1} $u(-n)$	$2^{-(n-1)}$ $u(n-1)$
$2^{-(n+1)}$ $u(n)$	2^{n+1} $u(-n-1)$	$2^{-(n-1)}$ $u(-n)$	2^{n-1} $u(n-1)$

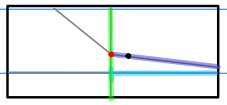
(1) a^n u(n)



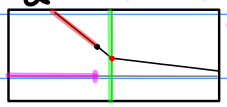
(5) a^n u(-n-1)



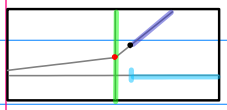
(2) a^{-n} u(n)



(6) a^{-n} u(-n-1)

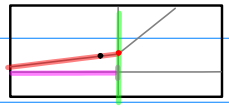


(7)



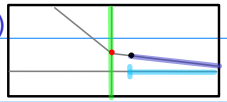
a^n u(n-1)

(3)



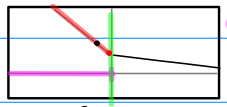
a^n u(-n)

(8)



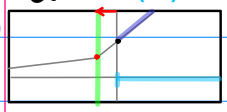
a^{-n} u(n-1)

(4)

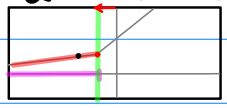


a^{-n} u(-n)

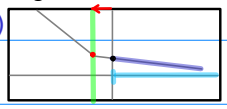
(1') a^{n+1} u(n)



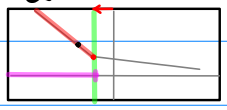
(5') a^{n+1} u(-n-1)



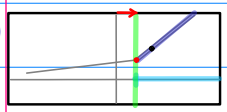
(2') a^{-n-1} u(n)



(6') a^{-n-1} u(-n-1)

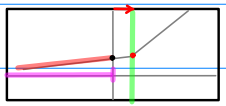


(7')



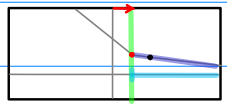
a^{n-1} u(n-1)

(3')



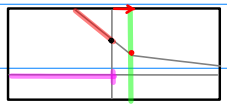
a^{n-1} u(-n)

(8')



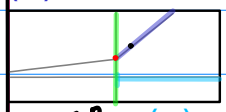
a^{-n+1} u(n-1)

(4')



a^{-n+1} u(-n)

(1) 0000

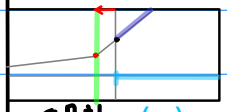


a^n u(n)

(3) 0010

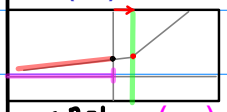
a^n u(-n)

(1') 1000



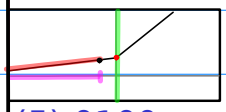
a^{n+1} u(n)

(3') 1010



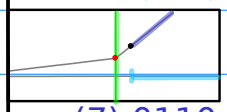
a^{n-1} u(-n)

a^n u(-n-1)



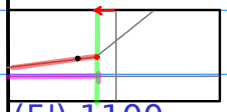
(5) 0100

a^n u(n-1)



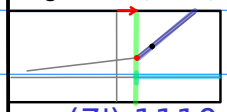
(7) 0110

a^{n+1}u(-n-1)



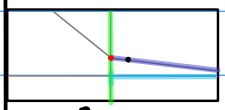
(5') 1100

a^{n-1} u(n-1)



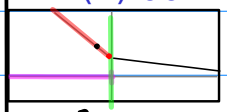
(7') 1110

(2) 0001



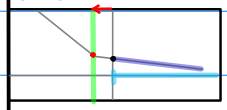
a^{-n} u(n)

(4) 0011



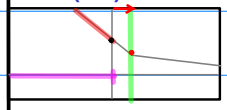
a^{-n} u(-n)

(2') 1001



a^{-n-1} u(n)

(4') 1011

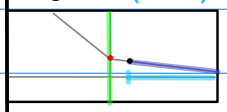


a^{-n+1} u(-n)

a^{-n} u(-n-1)

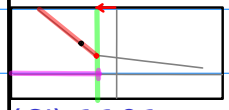
(6) 0101

a^{-n} u(n-1)



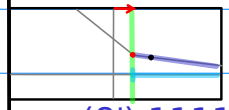
(8) 0111

a^{-n-1}u(-n-1)



(6') 1101

a^{-n+1}u(n-1)



(8') 1111