

# Multiple Linear Regression II



Image source <https://commons.wikimedia.org/wiki/File:Autobahn&#x27;C7%AC;A4;80-A201.jpg>

## Lecture 8

Survey Research & Design in Psychology  
James Neill, 2016  
[Creative Commons Attribution 4.0](#)

---

---

---

---

---

---

---

---

### Overview



1. Summary of MLR I
2. Semi-partial correlations
3. Residual analysis
4. Interactions
5. Analysis of change
6. Writing up an MLR
7. Summary of MLR II
8. MLR II quiz – Practice questions

2

---

---

---

---

---

---

---

---

### Readings

1. As per previous lecture, plus
2. **Howitt & Cramer (2011/2014)**
  - Ch 38/39: Moderator variables on relationships between two variables

3

---

---

---

---

---

---

---

---

## Summary of MLR I

### Purpose of MLR:

To examine linear relations between:

- two or more predictors (IVs; X) and
- a single outcome variable (DV; Y)

### Model:

Establish a model:

- Path diagram and/or Venn diagram
- Express as one hypothesis per IV

4

---

---

---

---

---

---

---

---

## Summary of MLR I

### Check assumptions:

- Level of measurement
- Sample size
- Normality
- Linearity
- Homoscedasticity
- Collinearity
- Multivariate outliers
- Residuals should be normally distributed

5

---

---

---

---

---

---

---

---

## Summary of MLR I

### Choose type:

- Standard
- Hierarchical
- Stepwise, Forward, Backward

6

---

---

---

---

---

---

---

---

## Summary of MLR I

### Interpret output:

- **Overall:**
  - $R$ ,  $R^2$ , Adjusted  $R^2$
  - Changes in  $R^2$  (if hierarchical)
  - Significance ( $F$ ,  $p$ )
- **For each IV:**
  - Standardised coefficient
    - size, direction and significance
  - Unstandardised coefficient
    - report equation (if useful)
  - Semi-partial correlations ( $sr^2$ )

7

---

---

---

---

---

---

---

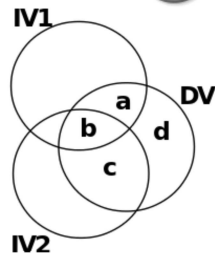
---

## Semi-partial correlation



Venn Diagrams depict variance and shared variance:

- $a + b + c + d =$  variance in DV
- $a + b + c = R$  (variance in the DV explained by IV1 and IV2)
- $a + c =$  uniquely explained variance
- $b =$  non-uniquely explained variance



$a$  and  $c$  are **semi-partial correlations ( $sr$ )**:

- $a = sr$  between IV1 and DV after controlling for (partialling out) the influence of IV2
- $c = sr$  between IV2 and DV after controlling for (partialling out) the influence of IV1

8

---

---

---

---

---

---

---

---

## Semi-partial correlations in MLR

When interpreting MLR coefficients:

- Draw a Venn diagram
- Compare 0-order ( $r$ ) and semi-partial correlations ( $sr$ ) for each IV to help understand relations amongst the IVs and the DV:
  - A semi-partial correlation ( $sr$ ) will be less than or equal to the correlation ( $r$ )
  - If a  $sr$  equals a  $r$ , then the IV independently predicts the DV
  - To the extent that a  $sr$  is less than the  $r$ , the IV's explanation of the DV is shared with other IVs
  - An IV may have a significant  $r$  with the DV, but a non-significant  $sr$ . This indicates that the unique variance explained by the IV may be 0.
- Compare the relative importance of the predictors using betas and/or  $srs$  (latter preferred)

9

---

---

---

---

---

---

---

---

## Semi-partial correlations ( $s_r$ ) in MLR

- SPSS provides semi-partial correlations (labelled “part”)
- Square these to get  $s_r^2$ .
- The  $s_r^2$  indicates the % of variance in the DV which is uniquely explained by an IV.

10

---

---

---

---

---

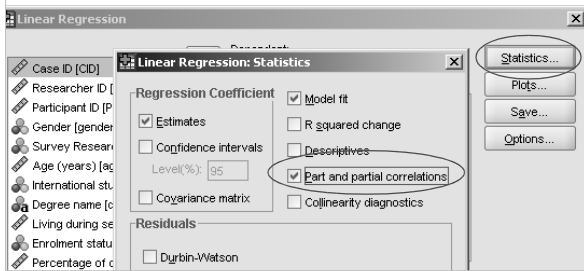
---

---

---

## Semi-partial correlations for MLR in SPSS

In Linear Regression - Statistics dialog box, check “Part and partial correlations”




---

---

---

---

---

---

---

---

## Semi-partial correlations for MLR in SPSS - Example

Coefficients<sup>a</sup>

Model		Correlations		
		Zero-order	Partial	Part
1	Zscore: GA: General Ability Total	.285	.107	.090
	Zscore: PSA: Perceptual Speed Ability Total	.419	.319	.280
	Zscore: PMA: Psychomotor Ability Total	.275	.034	.028
	QBK: Total Knowledge of Text Editing Keys (% correct)	.386	.368	.329

a. Dependent Variable: Typing Test: Speed (Words Per Minute)

Semi-partial correlations ( $s_r$ ) indicate the relative importance of each of the predictors;  $s_r^2$  indicate the % of variance uniquely explained by each predictor.

---

---

---

---

---

---

---

---

## Semi-partial correlations (*sr*) in MLR: Write-up example

- $R^2 = .32$
- 16% is uniquely explained by the IVs
- 16% is explained by the combination of the IVs

Independent variables	$sr^2$
Not Coping (NC)	.07
Somatic Coping (SC)	.06
Wishful Thinking (WT)	.02
Worry (W)	.01

13

---

---

---

---

---

---

---

---

---

---

## Summary:

### Semi-partial correlation (*sr*)

1. In MLR, *sr* is labelled "part" in the regression coefficients table SPSS output
2. Square these values to obtain  $sr^2$ , the unique % of DV variance explained by each IV
3. Discuss the extent to which the explained variance in the DV is due to unique or shared contributions of the IVs

14

---

---

---

---

---

---

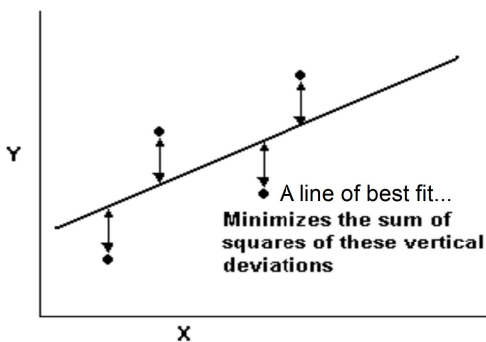
---

---

---

---

## Residual analysis




---

---

---

---

---

---

---

---

---

---

## Residual analysis

Assumptions about residuals:

- Random noise (error)
- Sometimes positive, sometimes negative but, on average, 0
- Normally distributed about 0

16

---

---

---

---

---

---

---

---

## Residual analysis

Linear Regression: Plots

DEPENDENT  
\*ZPRED  
\*ZRESID  
\*DRESID  
\*ADJPRED  
\*SRESID  
\*SDRESID

Scatter 1 of 1  
Previous Next

Y: \*ZRESID  
X: \*ZPRED

Standardized Residual Plots  
 Histogram  
 Normal probability plot  
 Produce all partial plots

Continue  
Cancel  
Help

---

---

---

---

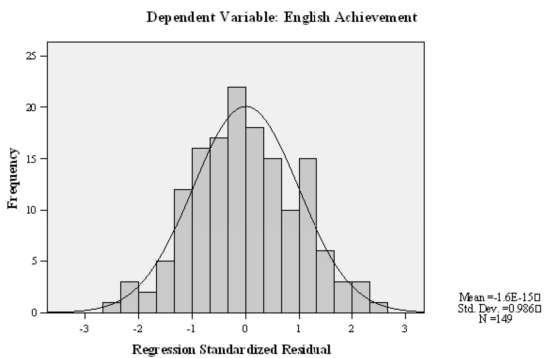
---

---

---

---

## Residual analysis: Histogram



---

---

---

---

---

---

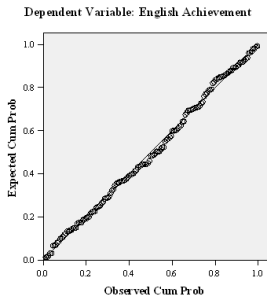
---

---

“The **Normal P-P (Probability) Plot of Regression Standardized Residuals** can be used to assess the assumption of normally distributed residuals. **If the points cluster reasonably tightly along the diagonal line (as they do here), the residuals are normally distributed.** Substantial deviations from the diagonal may be cause for concern.”  
 Allen & Bennett, 2008, p. 183

## Residual analysis: NPP

Normal P-P Plot of Regression Standardized Residual




---

---

---

---

---

---

---

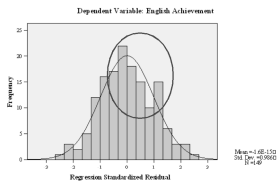
---

---

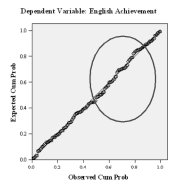
---

## Residual analysis: Histogram and NPP

Histogram



Normal P-P Plot of Regression Standardized Residual




---

---

---

---

---

---

---

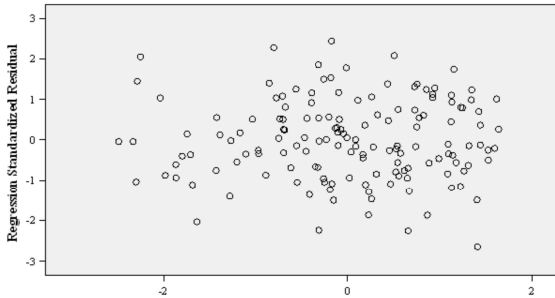
---

---

---

## Residual analysis: Scatterplot of predicted vs. residual values

The absence of any clear patterns in the spread of points indicates that the MLR assumptions of normality, linearity and homoscedasticity are met. (Allen & Bennett, 2008, p. 183)




---

---

---

---

---

---

---

---

---

---

### Why the big fuss about residuals?

↑ assumption violation

→

↑ Type I error rate  
(i.e., more false positives)

22

---

---

---

---

---

---

---

---

### Why the big fuss about residuals?

- Standard error formulae (which are used for confidence intervals and sig. tests) work when residuals are well-behaved.
- If the residuals don't meet assumptions these formulae tend to underestimate coefficient standard errors giving lower  $p$ -values and more Type I errors.

23

---

---

---

---

---

---

---

---

### Summary: Residual analysis

1. Residuals are the difference between predicted and observed Y values
2. MLR assumption is that residuals are normally distributed.
3. Examining residuals also helps to assess:
  1. Linearity
  2. Homoscedasticity

24

---

---

---

---

---

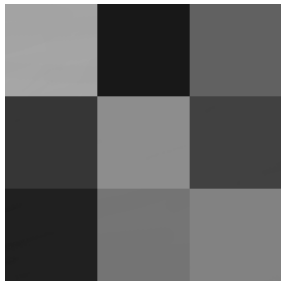
---

---

---



## Interactions



---

---

---

---

---

---

---

---

## Interactions

- Additivity - when IVs act independently on a DV they do not interact.
- Alternatively, there may be interaction effects - i.e. the magnitude of the effect of one IV on a DV varies as a function of a second IV.
- Also known as a moderation effect.

26

---

---

---

---

---

---

---

---

## Interactions: Example 1

Some drugs interact with each other to reduce or enhance other's effects e.g.,

Pseudoephedrine → ↑ Arousal

Caffeine → ↑ Arousal

Pseudoeph. X Caffeine → ↑↑↑ Arousal



X



27

---

---

---

---

---

---

---

---

### Interactions: Example 2

Physical exercise (IV1)  
and  
Natural environments (IV2)  
may provide  
multiplicative benefits  
in reducing stress e.g.,

Natural environment  $\rightarrow \downarrow$  Stress

Physical exercise  $\rightarrow \downarrow$  Stress

Natural env. x Phys. ex.  $\rightarrow \downarrow\downarrow\downarrow$  Stress

28

---

---

---

---

---

---

---

---

### Interactions: Example 3

University student satisfaction (IV1)  
and  
Level of coping (IV2)  
may predict level of stress e.g.,

Satisfaction  $\rightarrow \downarrow$  Stress

Coping  $\rightarrow \downarrow$  Stress

Satisfaction x Coping  $\rightarrow \downarrow\downarrow\downarrow$  Stress

(Dissatisfaction x Not coping  $\rightarrow \uparrow\uparrow\uparrow$  Stress)

29

---

---

---

---

---

---

---

---

### Interactions

Test interactions in MLR by computing  
a cross-product term e.g.,:

- Pseudoephedrine (IV1)
- Caffeine (IV2)
- Pseudoephedrine x Caffeine (IV3)

30

---

---

---

---

---

---

---

---

## Interactions

$$Y = b_1x_1 + b_2x_2 + b_{12}x_{12} + a + e$$

- $b_{12}$  is the product of the first two slopes ( $b_1 \times b_2$ )
- $b_{12}$  can be interpreted as the amount of change in the slope of the regression of Y on  $b_1$  when  $b_2$  changes by one unit.

31

---

---

---

---

---

---

---

---

## Interactions

- Conduct Hierarchical MLR
  - Step 1:
    - Pseudoephedrine
    - Caffeine
  - Step 2:
    - Pseudo x Caffeine (cross-product)
- Examine  $\Delta R^2$ , to see whether the interaction term explains additional variance above and beyond the additive effects of Pseudo and Caffeine.

32

---

---

---

---

---

---

---

---

## Interactions

Possible effects of Pseudo and Caffeine on Arousal:

- None
- Pseudo only ( $\uparrow/\downarrow$ )
- Caffeine only ( $\uparrow/\downarrow$ )
- Pseudo + Caffeine ( $\uparrow\uparrow/\downarrow\downarrow$ )
- Pseudo x Caffeine ( $\uparrow\uparrow\uparrow$  (synergistic))
- Pseudo x Caffeine ( $\downarrow\downarrow\downarrow$  (antagonistic))

33

---

---

---

---

---

---

---

---

## Interactions – SPSS example

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	R Square Change
	1	.543 <sup>a</sup>	.295	.288	17.34399
2	.547 <sup>b</sup>	.299	.289	17.33241	.004

a. Predictors: (Constant), Caffeine, Pseudoephedrine

b. Predictors: (Constant), Caffeine, Pseudoephedrine, PseudoCaffeine interaction

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	57.705	5.109		11.296	.000
	Pseudoephedrine	11.511	1.510	.464	7.625	.000
	Caffeine	4.735	1.780	.162	2.660	.008
2	(Constant)	42.730	14.137		3.023	.003
	Pseudoephedrine	18.137	6.025	.732	3.010	.003
	Caffeine	10.126	5.069	.346	1.998	.047
	PseudoCaffeine interaction	-2.313	2.036	-.380	-1.136	.257

---

---

---

---

---

---

---

---

---

---

## Interactions

- Cross-product interaction terms may be highly correlated (collinear) with the corresponding simple IVs, creating problems with assessing the relative importance of main effects and interaction effects.
- An alternative approach is to conduct separate regressions for each level of one of the IVs.

---

---

---

---

---

---

---

---

---

---

## Summary: Interactions

1. In MLR, IVs may interact to:
  1. Increase the IVs' effect on DV
  2. Decrease the IVs' effect on DV
2. Model interactions with hierarchical MLR:
  1. Step 1: Enter IVs
  2. Step 2: Enter cross-product of IVs
  3. Examine change in  $R^2$

---

---

---

---

---

---

---

---

---

---

## Analysis of Change



Image source: <http://commons.wikimedia.org/wiki/File:PMinspiret.jpg>, CC-by-SA 3.0

---

---

---

---

---

---

---

---

## Analysis of change

- MLR can be used to analyse the variance of changes in outcome measures over time.
- There are two ways this can be done:
  1. Compute difference (or change) scores in the DV (Change in DV = Post-test score minus Pre-test score) and use this as the DV in a standard MLR.
  2. Hierarchical MLR:
    1. "Partial out" baseline measures from post-scores: Enter Post-test score as the DV and the Pre-test score as the IV in Step 1.
    2. Seek to explain remaining variance in the Post-test DV by entering the IVs of interest in Step 2 and examining the change in  $R^2$ .

38

---

---

---

---

---

---

---

---

## Analysis of change

Example research question:

Does the quality of social support (IV2) explain changes in mental health (MH) between the beginning (IV1) and end (DV) of an intervention ?

39

---

---

---

---

---

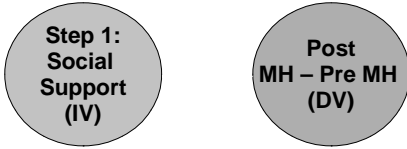
---

---

---

## Analysis of change: Option 1

Calculate the difference scores which indicate the change in mental health.



Use regression to determine the extent to which the IV can explain changes in the DV.

40

---

---

---

---

---

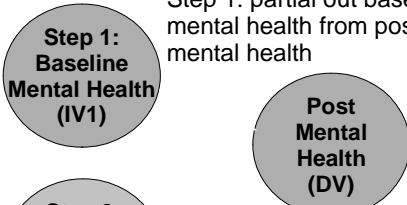
---

---

---

## Analysis of change: Option 2

Step 1: partial out baseline mental health from post mental health



Step 2: analyse the effect of Social Support on the change (remaining variance) in Mental Health

41

---

---

---

---

---

---

---

---

## Analysis of change: Option 2

### Hierarchical MLR

- DV = Mental health after the intervention
- Step 1
  - IV1 = Mental health before the intervention
- Step 2
  - IV2 = Support from group members

42

---

---

---

---

---

---

---

---

## Analysis of change: Option 2

### Results of interest

- Change in  $R^2$  – amount of additional variance in the DV (MH2) explained by the predictor (IV2) added in Step 2 (after having baseline IV1 variance has been accounted for in Step 1)
- Regression coefficients for the predictor (IV2) in Step 2 – indicates variance explained in the DV (MH2) by the IV (SS) after controlling for baseline MH (MH1) in Step 1

43

---

---

---

---

---

---

---

---

## Summary: Analysis of change

1. Analysis of changes over time can be assessed by
  1. Standard regression
    1. Calculate difference scores (Post-score minus Pre-score) and use as a DV
  2. Hierarchical MLR
    1. Step 1: "Partial out" baseline scores
    2. Step 2: Enter other IVs to help predict variance in changes over time.

44

---

---

---

---

---

---

---

---

## Writing up an MLR



Image source: <http://commons.wikimedia.org/wiki/File:Writing.svg>, Public domain

---

---

---

---

---

---

---

---

### Writing up an MLR (Intro)

- Explain the constructs of interest and why they are important
- Explain how / why the IVs may be related to the DV drawing on theory and research
- State hypotheses (one per IV)

46

---

---

---

---

---

---

---

---

### Writing up an MLR (Results)

- Establish the purpose of the analysis
- Provide univariate descriptive statistics (*M*, *SD*, Skewness, Kurtosis) for the variables of interest
- Describe type of MLR and testing of assumptions
- Table of correlations
- Overall model:  $R^2$ , Adjusted  $R^2$ ,  $F$ ,  $p$
- Table of regression coefficients (unstandardised and standardised),  $t$ ,  $p$ , and  $sr^2$  values

47

---

---

---

---

---

---

---

---

### Writing up an MLR (Discussion)

- Summarise the results
- Consider what the results indicate about the overall research question and each of the hypotheses, paying attention to:
  - Direction
  - Size
  - Statistical significance
- Interpret the results in the context of relevant psychological theory and research
- What are the implications/recommendations?

48

---

---

---

---

---

---

---

---



## Table of regression coefficients

Table 2

Standard Multiple Linear Regression of Non-productive Coping Strategies on Psychological Distress Reported by Adolescents During Outdoor Education Programs

Variables	PD <sup>a</sup> (DV)	NC	WT	W	SC	B	$\beta$	$sr^{2b}$
						(unique)		
Not Coping (NC)	.43					.41*	.29	.07
Wishful Thinking (WT)	.38	.36				.15*	.15	.02
Worry (W)	.36	.25	.40			.13*	.13	.01
Somatic Coping (SC)	.42	.22	.30	.38		.26*	.26	.06
Intercept						.60*		

Note. \* $p < .05$ ; <sup>a</sup>PD = Psychological Distress; <sup>b</sup> $sr^2$  = the squared semipartial correlations indicate the unique variance predicted by the independent variable.

---

---

---

---

---

---

---

---

---

---

## Summary: Writing up an MLR

### 1. Introduction:

1. Purpose
2. Describe model and hypotheses

### 2. Results:

1. Univariate descriptive statistics
2. Correlations
3. Type of MLR and assumptions
4. Regression coefficients
5. Equation (if appropriate)

50

---

---

---

---

---

---

---

---

---

---

## Questions



51

---

---

---

---

---

---

---

---

---

---

**MLR II Quiz –  
Practice question 1**

In an MLR, if the  $r$  between the two IVs is 1,  $R$  will equal the  $r$  between one of the IVs and the DV.

[Hint: Draw a Venn Diagram.]

- a) True
- b) False

52

---

---

---

---

---

---

---

---

**MLR II Quiz –  
Practice question 2**

In MLR, if two IVs are somewhat correlated with the DV and with one another, the  $srs$  between the IVs and the DV will be \_\_\_\_\_ in magnitude than the  $rs$ :

[Hint: Draw a Venn Diagram.]

- a) Equal
- b) Smaller
- c) Larger
- d) Impossible to tell

53

---

---

---

---

---

---

---

---

**MLR II Quiz –  
Practice question 3**

In MLR, the unique variance in the DV explained by a particular IV is estimated by its:

- a) Zero-order correlation square ( $r^2$ )
- b) Multiple correlation coefficient squared ( $R^2$ )
- c) Semi-partial correlation squared ( $sr^2$ )

54

---

---

---

---

---

---

---

---

**MLR II Quiz –  
Practice question 4**

Interaction effects can be tested in MLR by using IVs that represent:

- a) Cross-products between the IVs and DV in a hierarchical regression
- b) Cross-products of IVs
- c) Semi-partial correlations squared ( $sr^2$ )

55

---

---

---

---

---

---

---

---

**MLR II Quiz –  
Practice question 5**

To assess the extent to which social support from group members during an intervention program explain changes in mental health (MH) between the beginning and end of an intervention program, what MLR design could be used?

- a) Hierarchical with pre-MH in Step 1
- b) Hierarchical with cross-products of IVs in Step 2

56

---

---

---

---

---

---

---

---

**References**

- Allen, P. & Bennett, K. (2008). *SPSS for the health and behavioural sciences*. South Melbourne, Victoria, Australia: Thomson.
- Francis, G. (2007). *Introduction to SPSS for Windows: v. 15.0 and 14.0 with Notes for Studentware* (5th ed.). Sydney: Pearson Education.
- Howell, D. C. (2010). *Statistical methods for psychology* (7th ed.). Belmont, CA: Wadsworth.
- Howitt, D. & Cramer, D. (2011). *Introduction to statistics in psychology* (5th ed.). Harlow, UK: Pearson.

57

---

---

---

---

---

---

---

---

## Next lecture

- Statistical power
- Significance testing
- Inferential decision making
- Effect sizes
- Confidence intervals
- Publication bias
- Academic integrity

58

---

---

---

---

---

---

---

---

## Open Office Impress

- This presentation was made using Open Office Impress.
- Free and open source software.
- <http://www.openoffice.org/product/impress.html>



59

---

---

---

---

---

---

---

---