

Arrays (1A)

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-
- **Array declaration**
 - **Accessing array elements**

Computing the mean of N numbers

The mean of N numbers

$$m = \frac{1}{N} \sum_{i=0}^{N-1} x_i$$

$$m = \frac{1}{5} \sum_{i=0}^4 x_i = \frac{1}{5} (x_0 + x_1 + x_2 + x_3 + x_4)$$

5 integer variables

x[0]

x[1]

x[2]

x[3]

x[4]

5 indices

0

1

2

3

4

Definition of an Array

```
int x[5];
```

Array Type

int [5]

Array Name

x

5 integer variables

x[0]

x[1]

x[2]

x[3]

x[4]

5 indices

0

1

2

3

4

Element Type

```
int x[5];
```

Array Type

`int [5]`

Array Variable (constant)

`x`

Value: the starting address of 5 consecutive int variables

```
int x[5];
```

Element Type

`int`

Element Variable

`x[i]`

Index Variable

`i`

Using an Array

```
int x[5];
```

Array Type
Array Variable
(constant)

int [5]
x

```
int x[5];
```

[5] is declared and
[0], [1], [2], [3], [4] are used

Integer
Variables

```
x[i]
```

Element Type
Element Variable
Index Variable

int
x[i]
i

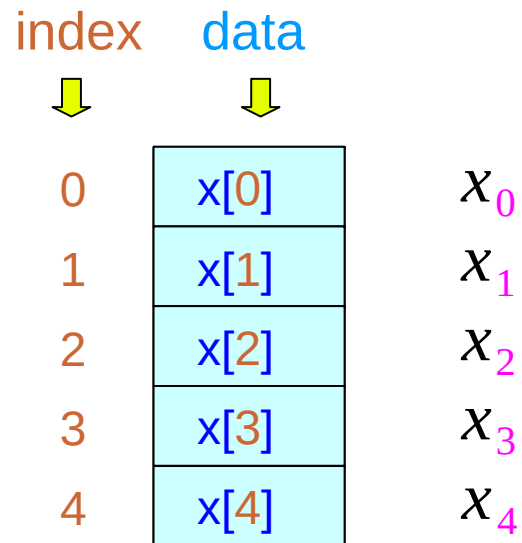
meaningful only for
 $i = 0, \dots, 4$

Accessing array elements – using an index

```
int    x[5];
```

x is an array of 5 integer elements

5 **int** variables



Computing the sum of n numbers (1)

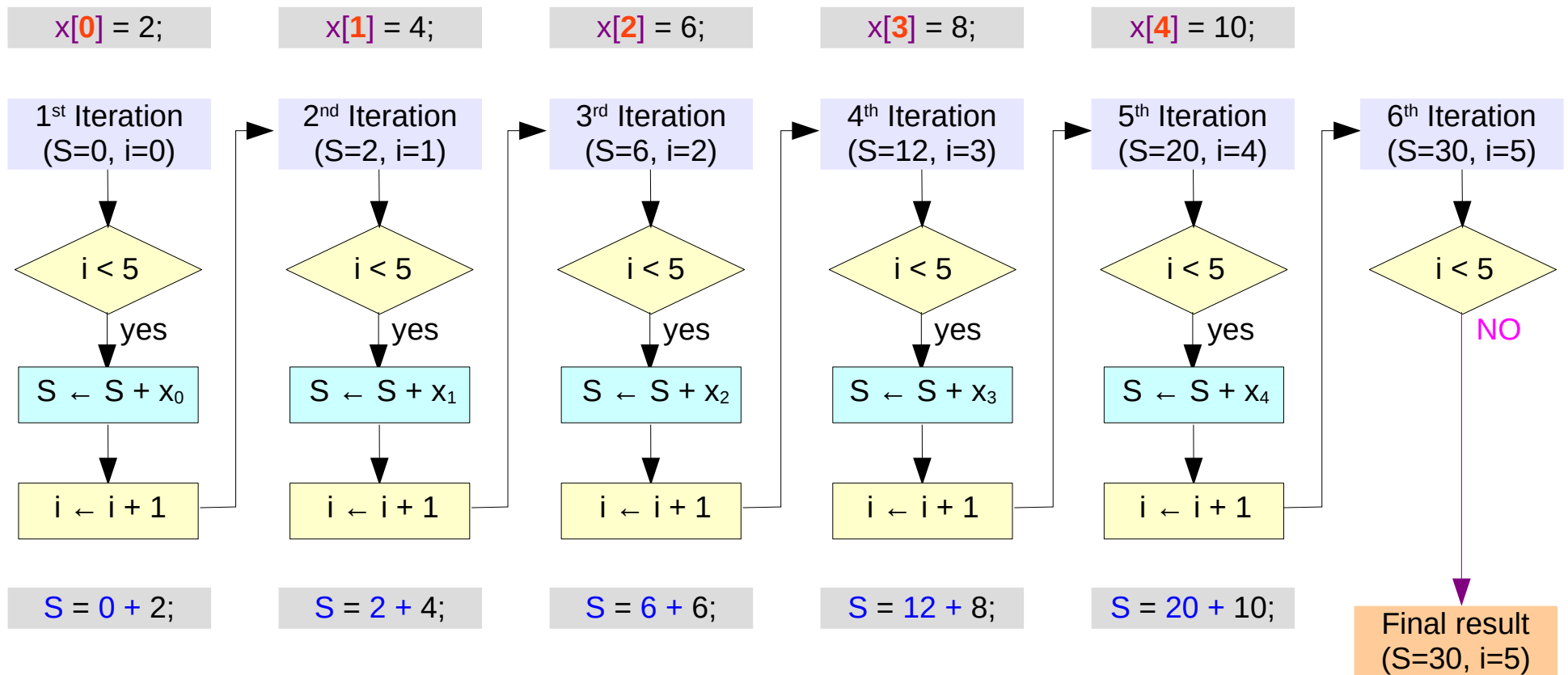
```
sum = 0;  
sum = sum + x[0];  
sum = sum + x[1];  
sum = sum + x[2];  
sum = sum + x[3];  
sum = sum + x[4];
```

treated as an **int** variable

```
sum : 0;  
sum : x0  
sum : x0 + x1  
sum : x0 + x1 + x2  
sum : x0 + x1 + x2 + x3  
sum : x0 + x1 + x2 + x3 + x4
```

```
sum = 0;  
for (i=0; i<5; ++i)  
    sum = sum + x[i];
```

Computing the sum of n numbers (2)



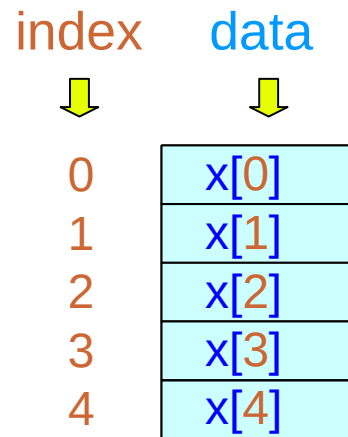
```
S = 0;  
for (i=0; i<5; ++i)  
    S = S + x[i];
```

Accessing array elements – using an address

```
int      x[5];
```

x holds the starting address
of 5 consecutive **int** variables

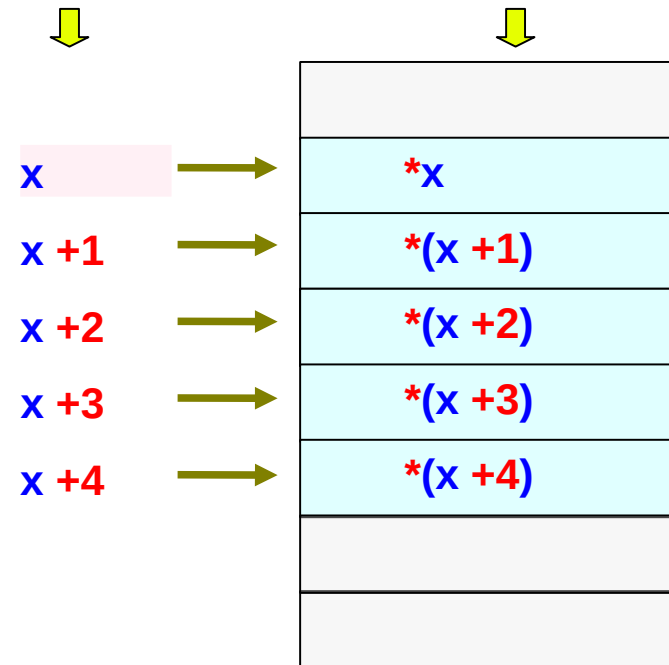
5 int variables



cannot change
address **x**
(constant)

address

data



Index and address notations

int x[5];

x holds the starting address
of 5 consecutive int variables

x[i] or *(x+i)

cannot change address x (constant)
assigned by the gcc compiler

i : an index variable [0 .. 4]
x[i] : the (i+1)th element variable

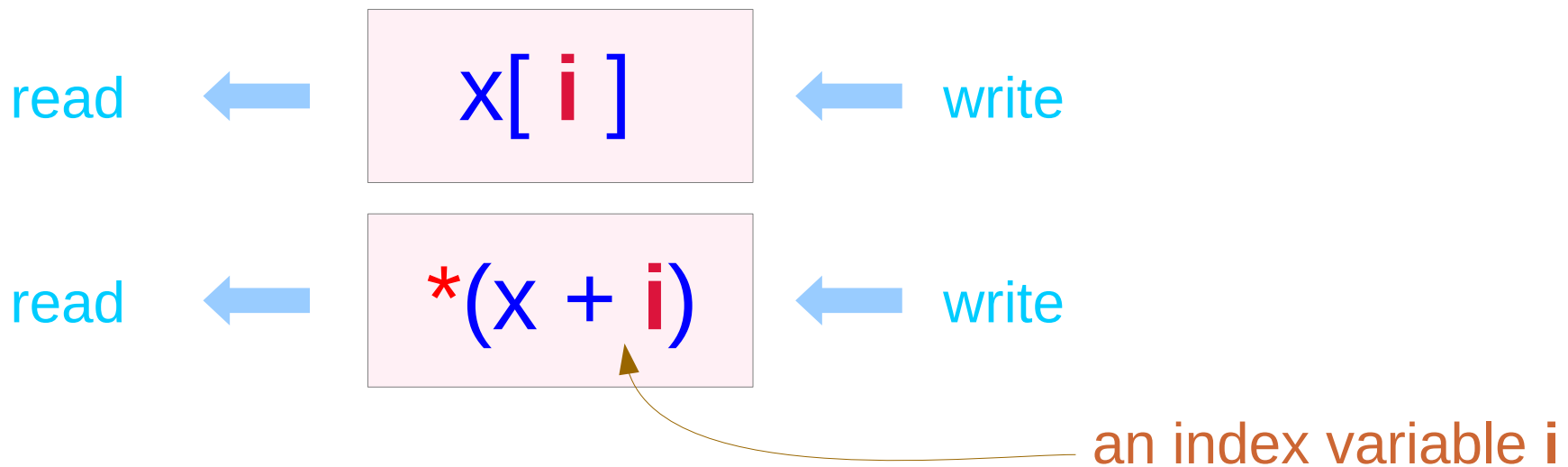
x : the starting address
x+i : the address of the (i+1)th element
*(x+i) : the (i+1)th element variable

A variable expressed by another variable

```
int    x[5];
```

x holds the starting address
of 5 consecutive **int** variables

treated as an **int** variable



-
- **Two aspects of a 1-d array variable**

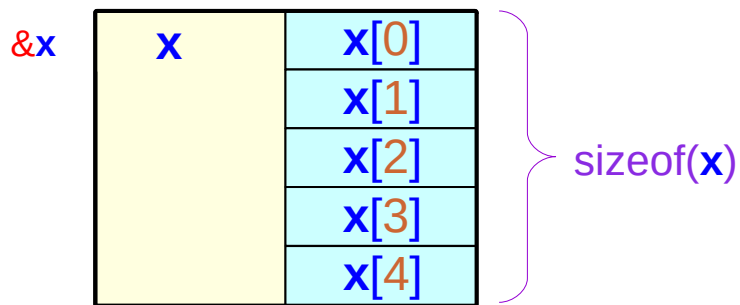
Value and size of an array variable

```
int x [5] ;
```

x : an array variable name (constant)

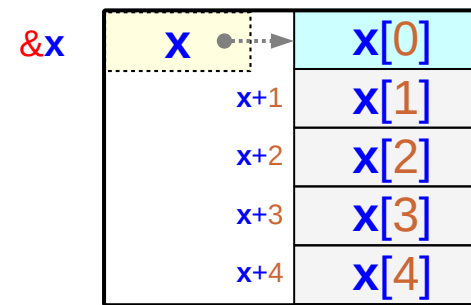
`sizeof(x)` : the *total size* of
5 consecutive **int** variables

`value(x)` : the *starting address* of
5 consecutive **int** variables



$\text{sizeof}(x) = 5 * \text{sizeof}(\text{int})$

subarray partitioning



$\text{value}(\&x) = \text{value}(x)$

address replication

An array variable as a virtual pointer

```
int x [5] ;
```

equivalence relations

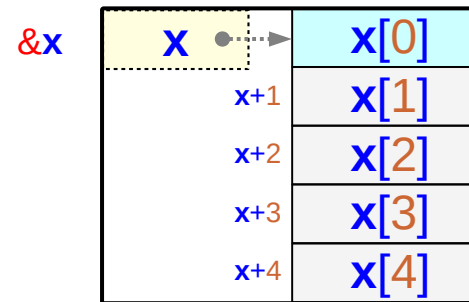
$$*(x+0) \equiv x[0]$$
$$(x+0) \equiv \&x[0]$$


$value(x) = value(\&x[0])$

x can be viewed as a pointer because x holds the address of the 1st array element $x[0]$

x : an array variable name (constant)

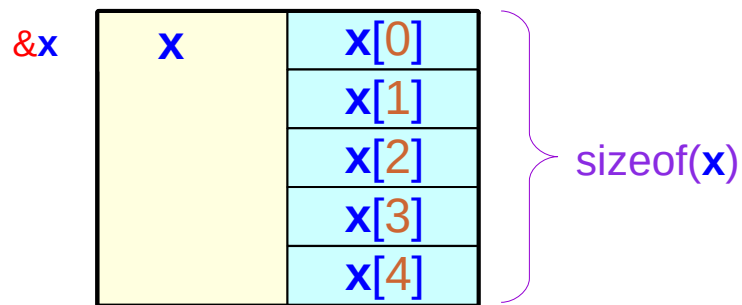
$value(x)$: the *starting address* of 5 consecutive **int** variables



The address of an array variable

```
int x [5] ;
```

`sizeof(x)` : the *total size* of
5 consecutive **int** variables



`x` : an array variable name (constant)

when an array `x` is referenced,
the address `&x` of the array `x`
is the same as the address
of the 1st element `&x[0] = x`
: **address replication**

`value(&x) = value(x)`

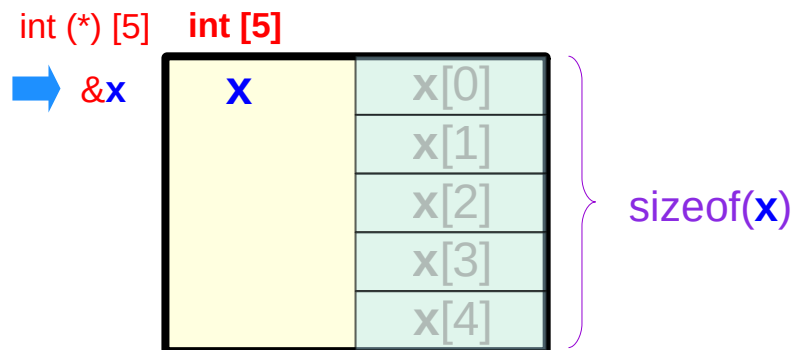
Outside and inside array types

```
int x [5] ;
```

outside of an array x

when an array is referenced

x has an array type `int [5]`



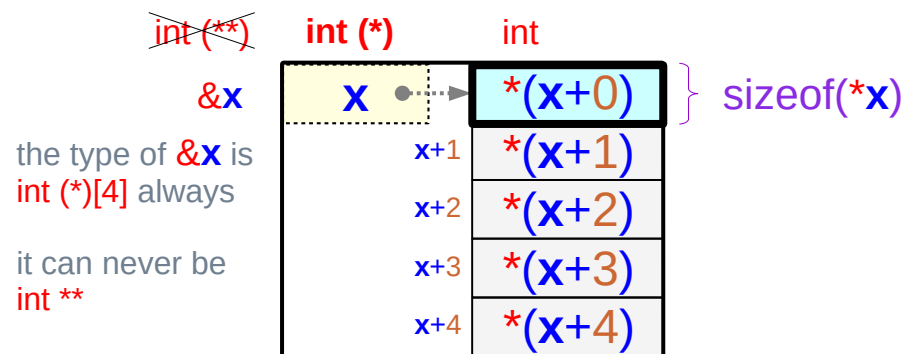
Abstract Data x
an array

x : an array variable name (constant)

inside of an array x

when an element of an array is referenced

x can be viewed as a pointer type `int (*)`



Primitive Data *x = x[0]
an array element

Abstract data x and virtual pointer x

```
int x [5] ;
```

x : an array variable name (constant)

outside of an array x

when an array is referenced

Abstract Data x – an array

Address $value(\&x) = value(x)$
 Size $sizeof(x) = 5 * sizeof(int)$
 Type $int [5]$

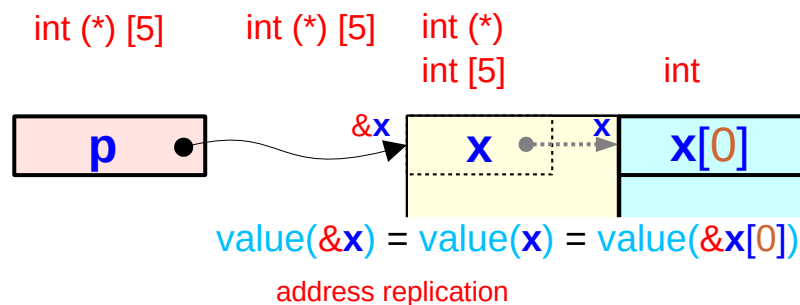
inside of an array x

when an element of an array is referenced

Virtual Pointer x – pointer to the 1st element

Address $value(x) = value(\&x[0])$
 Size $sizeof(x) = 5 * sizeof(int)$
 Type $int (*)$

← --- the same --- →
 ← --- the same --- →
 ← --- different --- →



$*(x+0) \equiv x[0]$
 $(x+0) \equiv \&x[0]$

equivalence relations

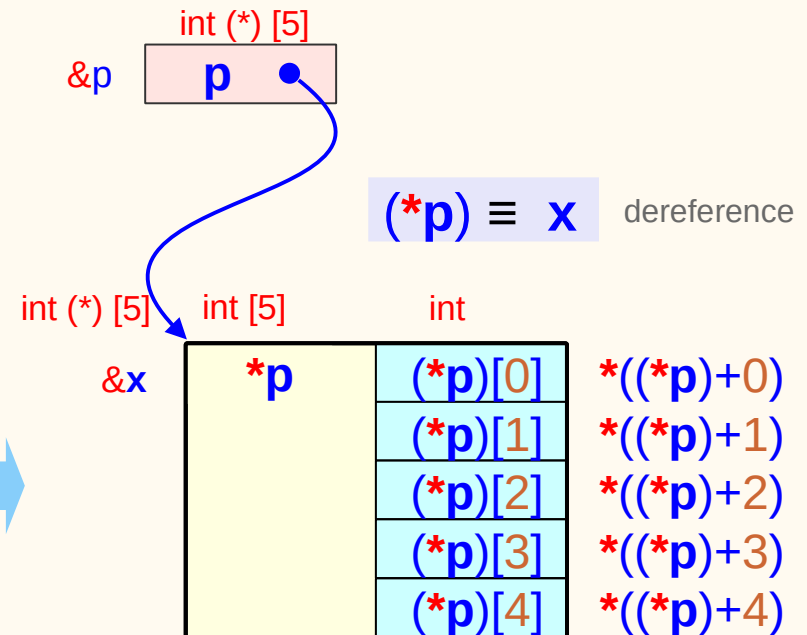
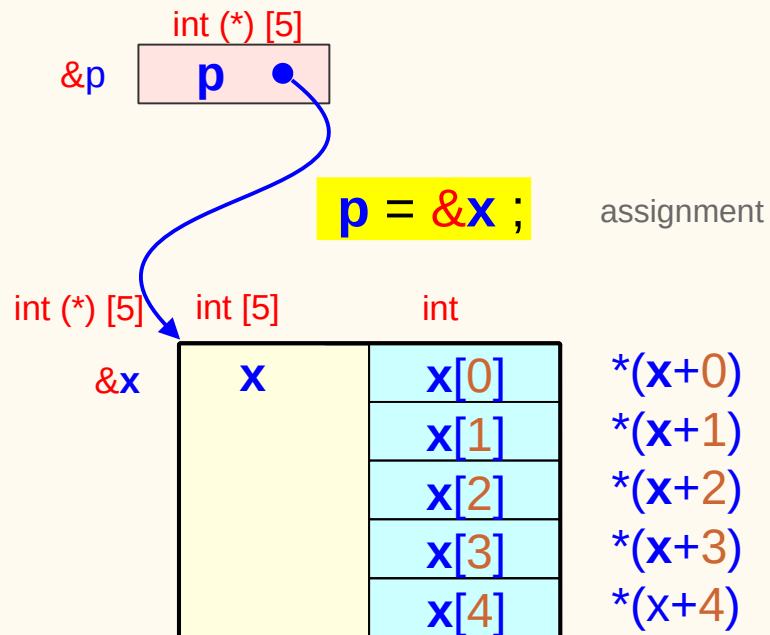
Using a 1-d array pointer **p**

```
int x [5] ;
```

```
int (*p) [5] = &x ;
```

referencing the outside of an array **x**

when an array is referenced



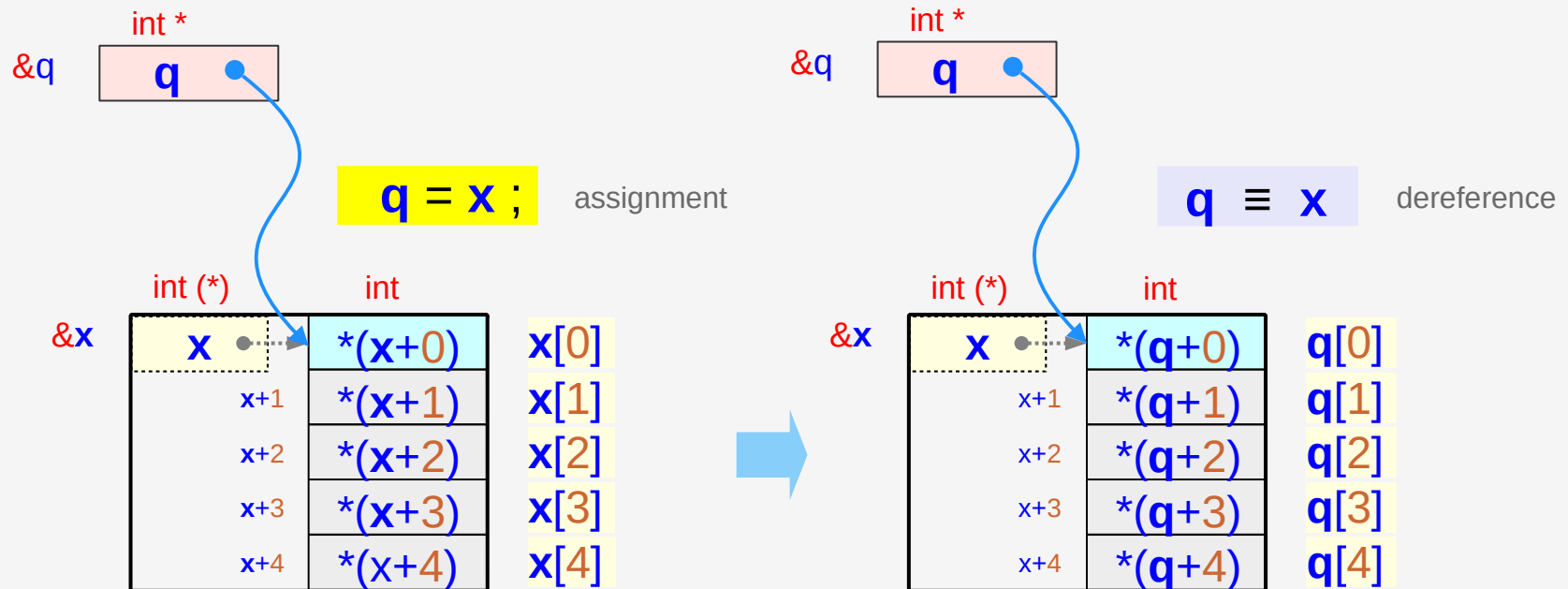
Using a 0-d array pointer q

```
int x [5] ;
```

```
int (* q) = x ;
```

referencing the inside of an array x

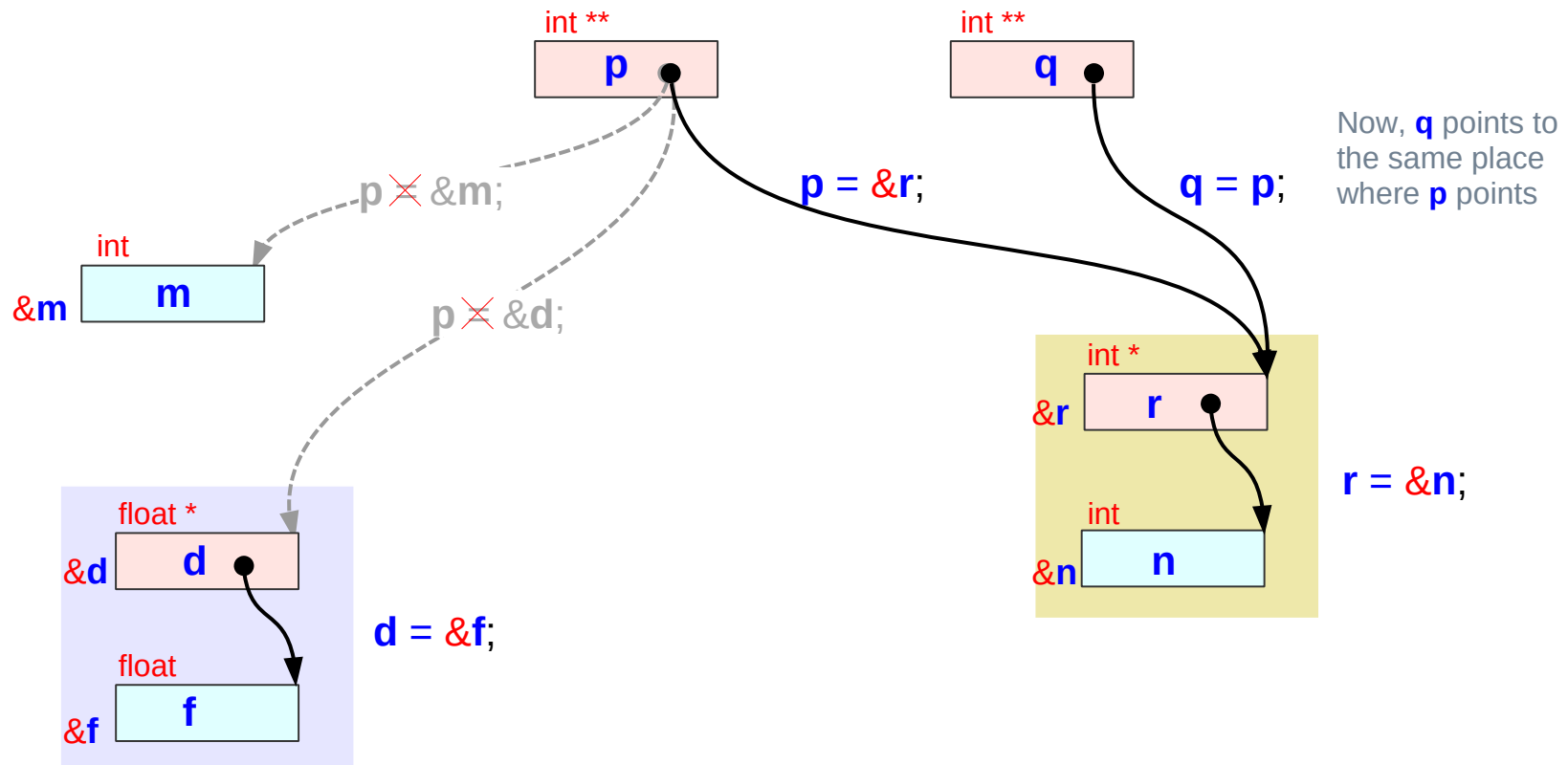
when an element of an array is referenced



Double pointer variable assignments

```
int **p, **q, *r, m, n ;
```

```
float *d, f ;
```

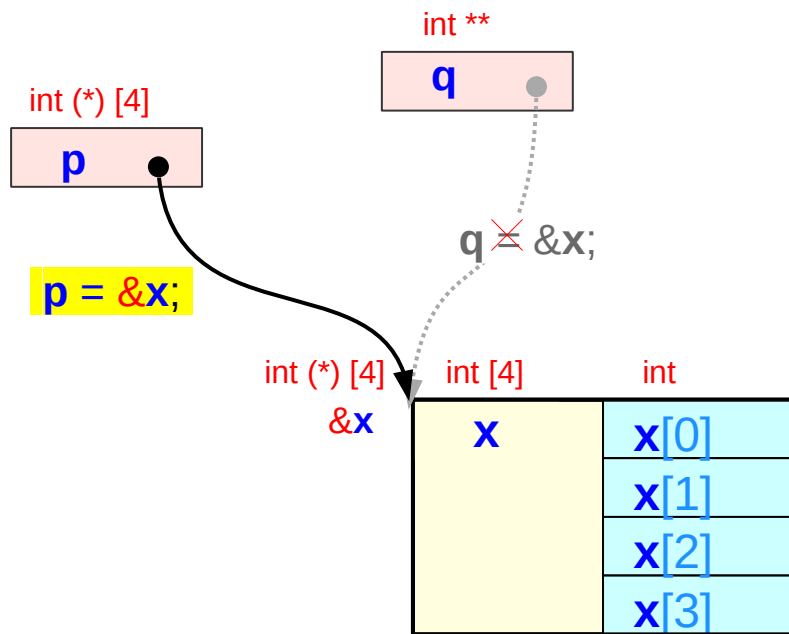


Double pointer variable assignments

```
int (*p) [4];
```

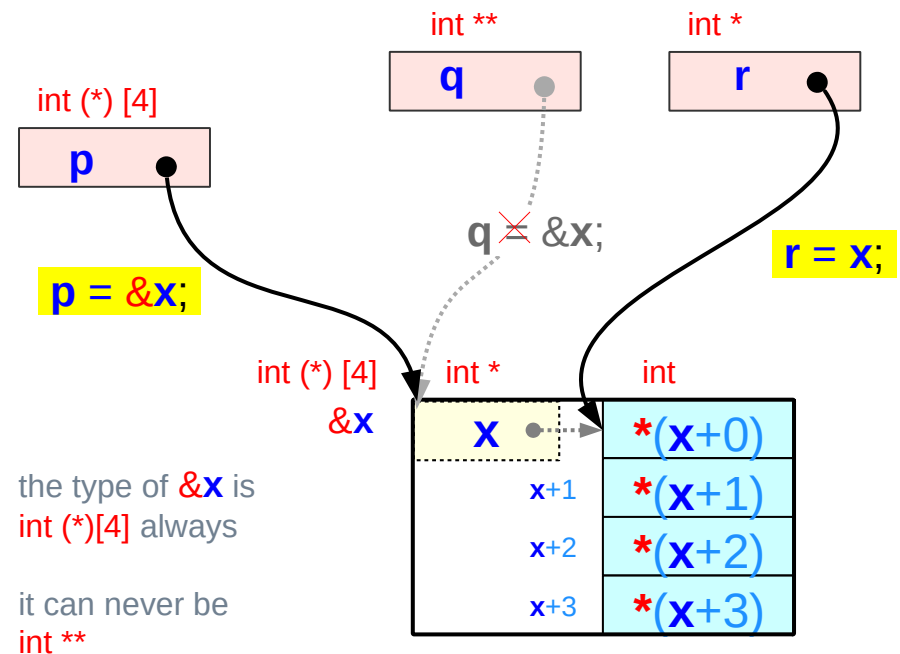
```
int ** q;
```

```
int *r;
```



when an array `x` is referenced, `x` has **outside array type** `int [4]`

thus, a pointer to `x` has `int (*) [4]` type



when an element of an array `x` is referenced, `x` has **inside array type** `int (*)`

within an array `x`, the array type `int [4]` can be relaxed to `int (*)`

-
- **Initializing an array**
 - **Copying and comparing arrays**

Using arrays

initialization

```
int a [3] = { 1, 2, 3 };
```

≡

```
int a [] = { 1, 2, 3 };
```

accessing elements

```
a [0] = 100;
```

```
a [1] = 200;
```

```
a [2] = 300;
```

```
a [m] = 100 * m;
```

$m = 0, 1, 2$

a function argument

```
func( a );
```

```
func( int x [ ] ) { ... }
```



Array initialization (1)

```
int a [5] ;
```

uninitialized values (garbage)

$a[0] = ?$, $a[1] = ?$, $a[2] = ?$, $a[3] = ?$, $a[4] = ?$;

```
int a [5] = { 1, 2, 3 } ;
```

= { 1, 2, 3, 0, 0 }

$a[0] = 1$, $a[1] = 2$, $a[2] = 3$, $a[3] = 0$, $a[4] = 0$;

```
int a [5] = { 0 } ;
```

= { 0, 0, 0, 0, 0 }

$a[0] = 0$, $a[1] = 0$, $a[2] = 0$, $a[3] = 0$, $a[4] = 0$;

all elements with zero

Array initialization (2)

```
int a [5] = { 1, 2, 3, 4, 5 } ;
```

sizeof(a) = 5*4 = 20 bytes

```
int b [ ] = { 1, 2, 3, 4 } ;
```

sizeof(b) = 5*4 = 20 bytes



4

```
int b [ ] ;
```

must have initialization data

Array initialization (3)

```
int c [3][4] = { { 1, 2, 3, 4},  
                { 5, 6, 7, 8},  
                {9,10,11,12} };
```

sizeof(c) = 3*4*4 = 48 bytes

```
c[0][0] = 1, c[0][1] = 2, c[0][2] = 3, c[0][3] = 4,  
c[1][0] = 5, c[1][1] = 6, c[1][2] = 7, c[1][3] = 8,  
c[2][0] = 9, c[2][1] = 10, c[2][2] = 11, c[2][3] = 12;
```

```
int d [ ][4] = { { 1, 2, 3, 4},  
                { 5, 6, 7, 8},  
                {9,10,11,12} };
```

sizeof(c) = 3*4*4 = 48 bytes

```
d[0][0] = 1, d[0][1] = 2, d[0][2] = 3, d[0][3] = 4,  
d[1][0] = 5, d[1][1] = 6, d[1][2] = 7, d[1][3] = 8,  
d[2][0] = 9, d[2][1] = 10, d[2][2] = 11, d[2][3] = 12;
```

```
int d [ ][ ] = { { 1, 2, 3, 4},  
                { 5, 6, 7, 8},  
                {9,10,11,12} };
```

Only the first dimension
can be unsized

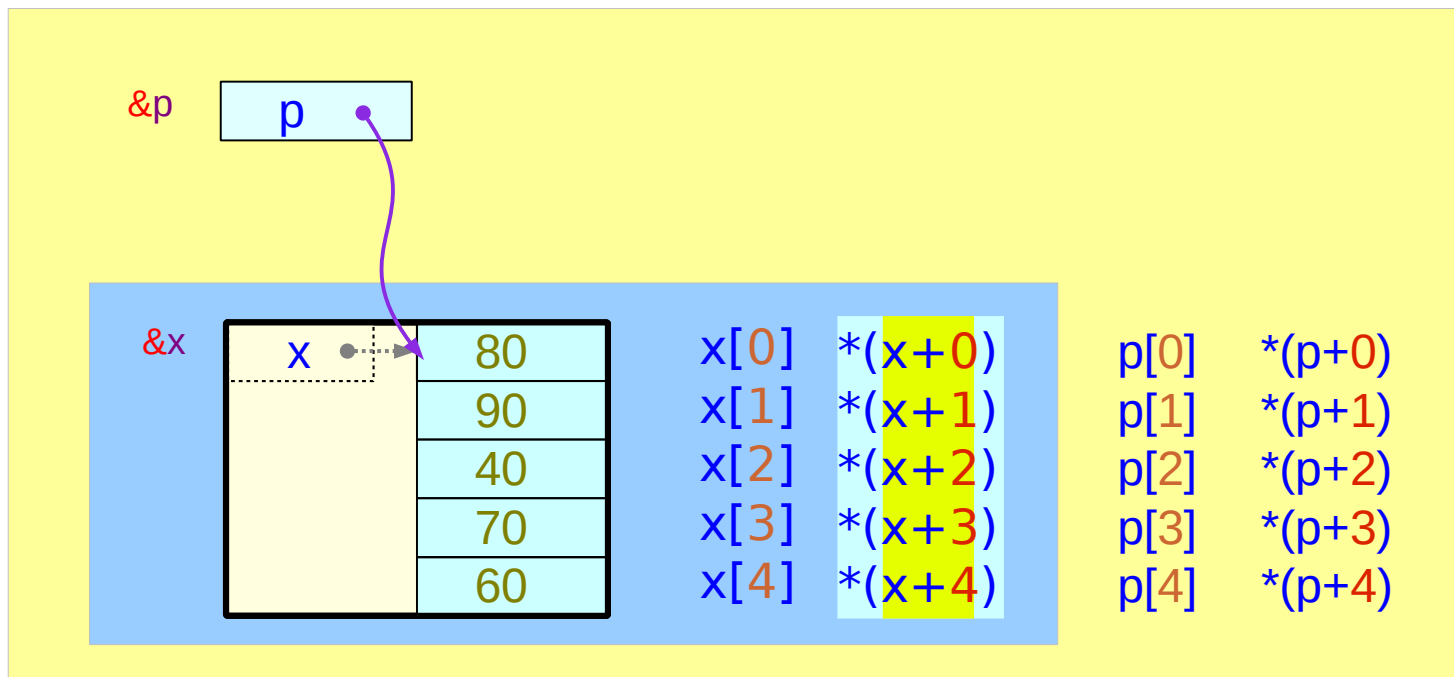
Accessing an array with a pointer variable

```
int x [5] = { 80, 90, 40, 70, 60 } ;
```

```
int *p = x;
```

x is a constant variable and cannot be changed

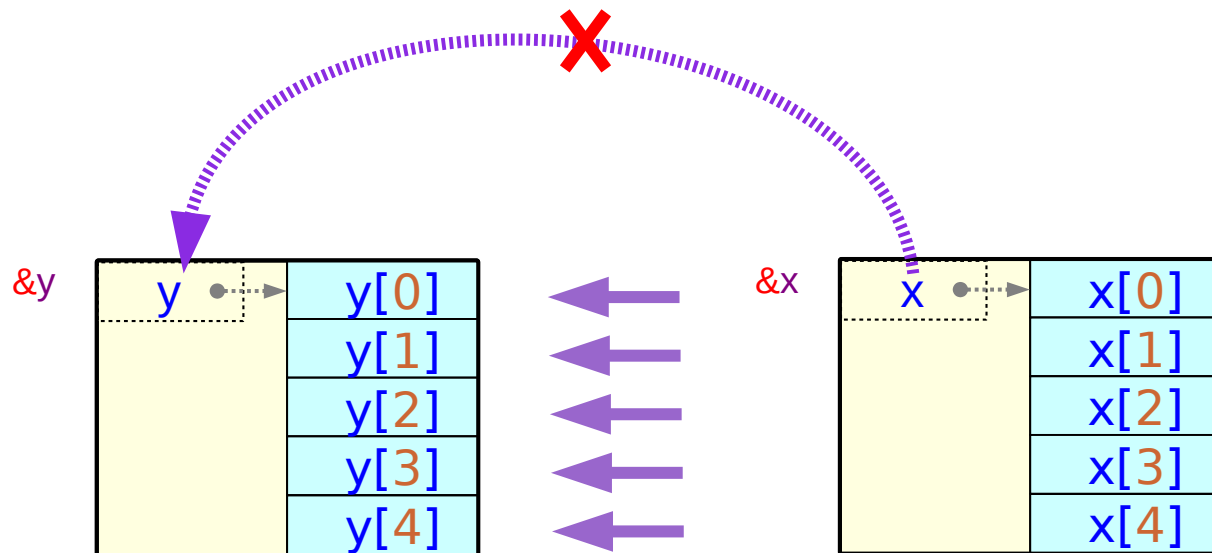
p is a variable can point to other addresses



Copying an array to another array

```
int x [5] = { 1, 2, 3, 4, 5 };  
int y [5] ;  
y = x;
```

y is a constant variable and cannot be assigned (changed)



```
for (i=0; i<5; ++i)  
    y[i] = x[i];
```

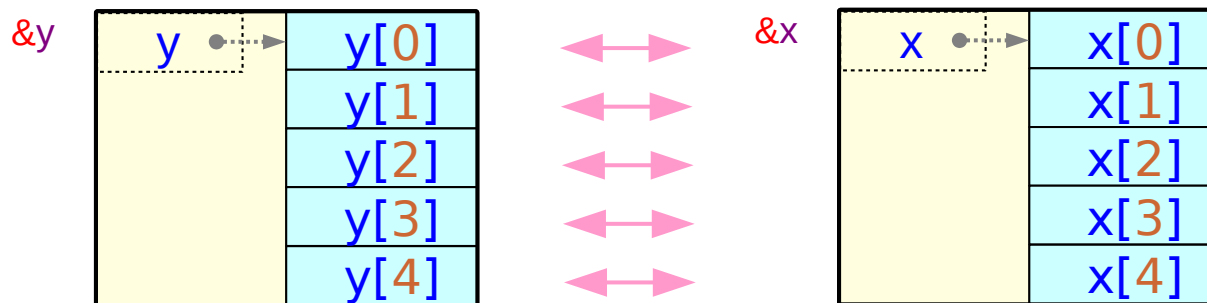
must copy each element

Comparing an Array with another Array

```
int x [5] = { 1, 2, 3, 4, 5 };  
int y [5] = { 1, 2, 3, 4, 5 };  
x == y
```

```
EQ &= (y[i] == x[i]);
```

```
EQ = EQ & (y[i] == x[i]);
```



```
EQ=1;
```

```
for (i=0; i<5; ++i)
```

```
    EQ &= (y[i] == x[i]);
```

must compare each element

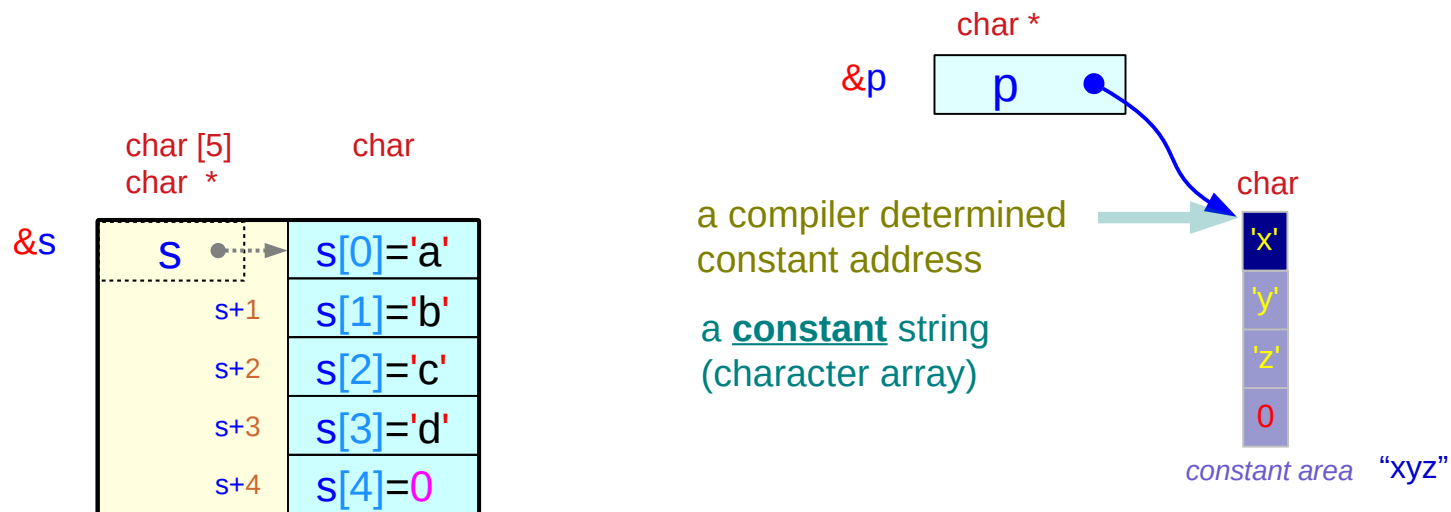
-
- **A string and a character array**

Initialized character arrays and pointers (1)

```
char s [5] = { 'a', 'b', 'c', 'd', 0 } ;
```

```
char s [5] = "abcd" ;
```

```
char *p = "xyz" ;
```



can change the value of any element

```
*s = 'm' ;  
s[0] = 'm' ;
```

cannot change the value of any element of a **constant** array

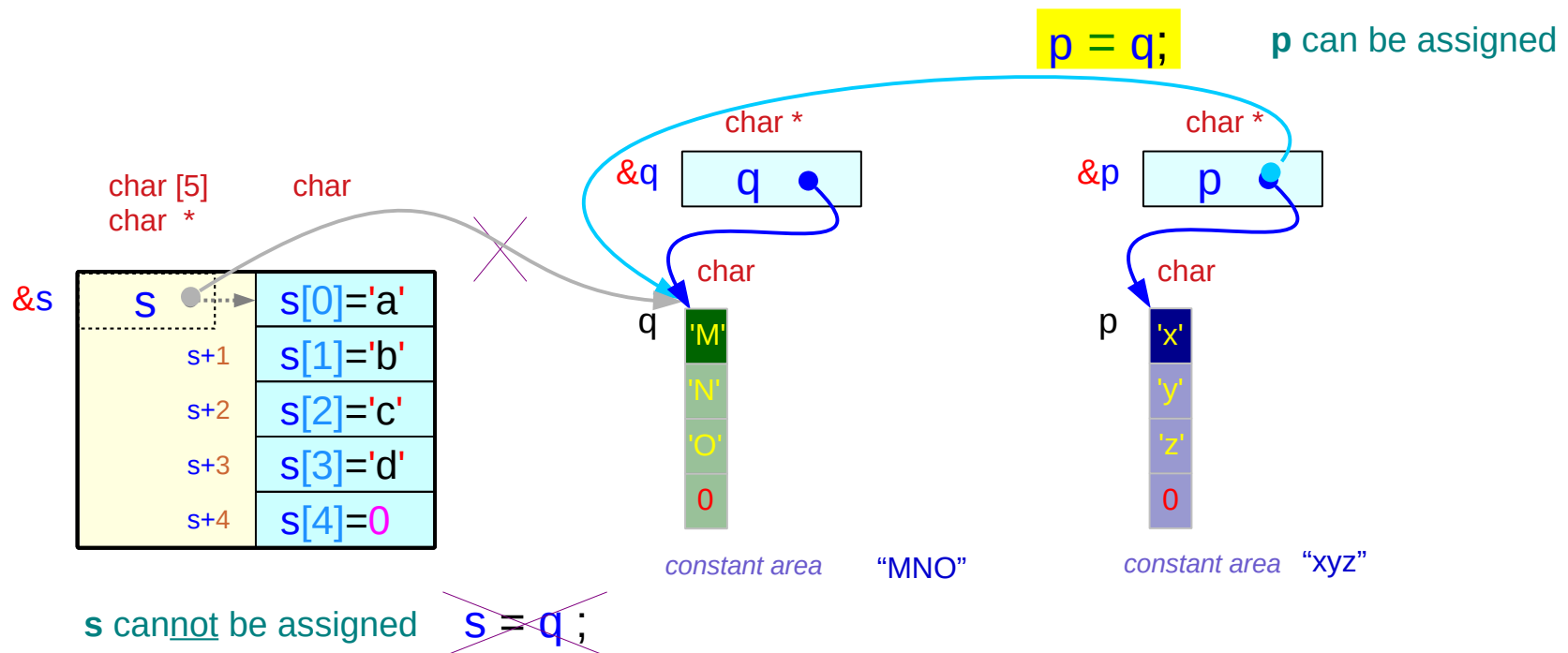
```
*p = 'm' ;  
p[0] = 'm' ;
```

Initialized character arrays and pointers (2)

```
char s [5] = { 'a', 'b', 'c', 'd', 0 } ;
```

```
char s [5] = "abcd" ;
```

```
char *p = "xyz", *q = "MNO" ;
```

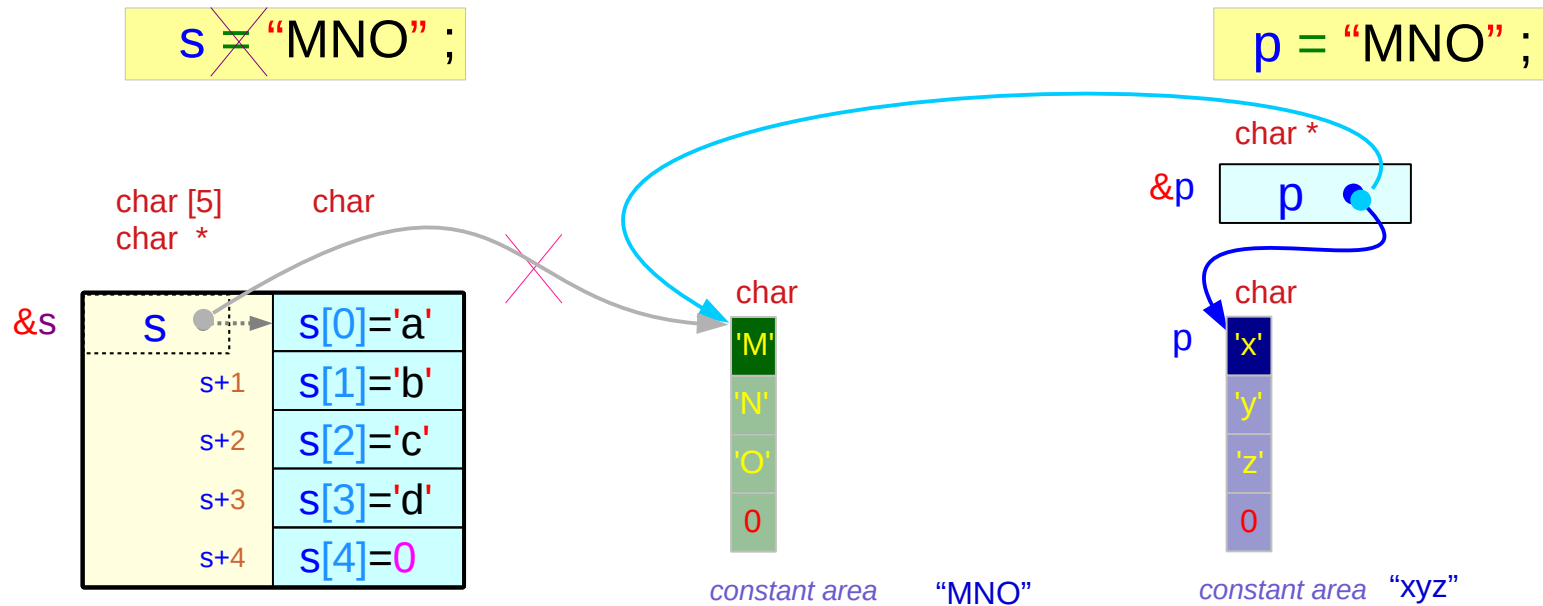


Assigning a constant character string

```
char s [5] = { 'a', 'b', 'c', 'd', 0 } ;
```

```
char s [5] = "abcd" ;
```

```
char *p = "xyz" ;
```



Copying a string

```
char s [5] = { 'a', 'b', 'c', 'd', 0 } ;
```

```
char s [5] = "abcd" ;
```



```
char *p = "xyz" ;
```

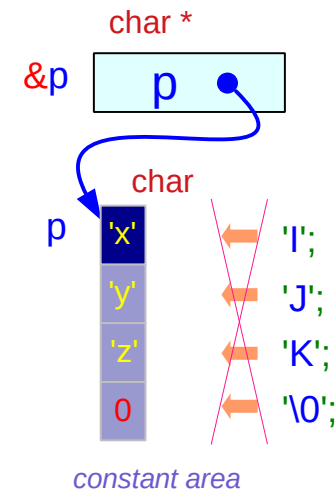
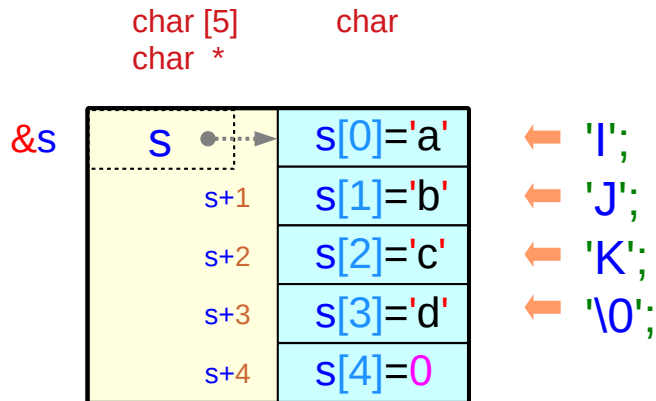
```
strcpy (s, "IJK") ;
```

s points to non-constant string

```
strcpy (p, "IJK") ;
```



p: points to a constant string



Character arrays and a string pointer

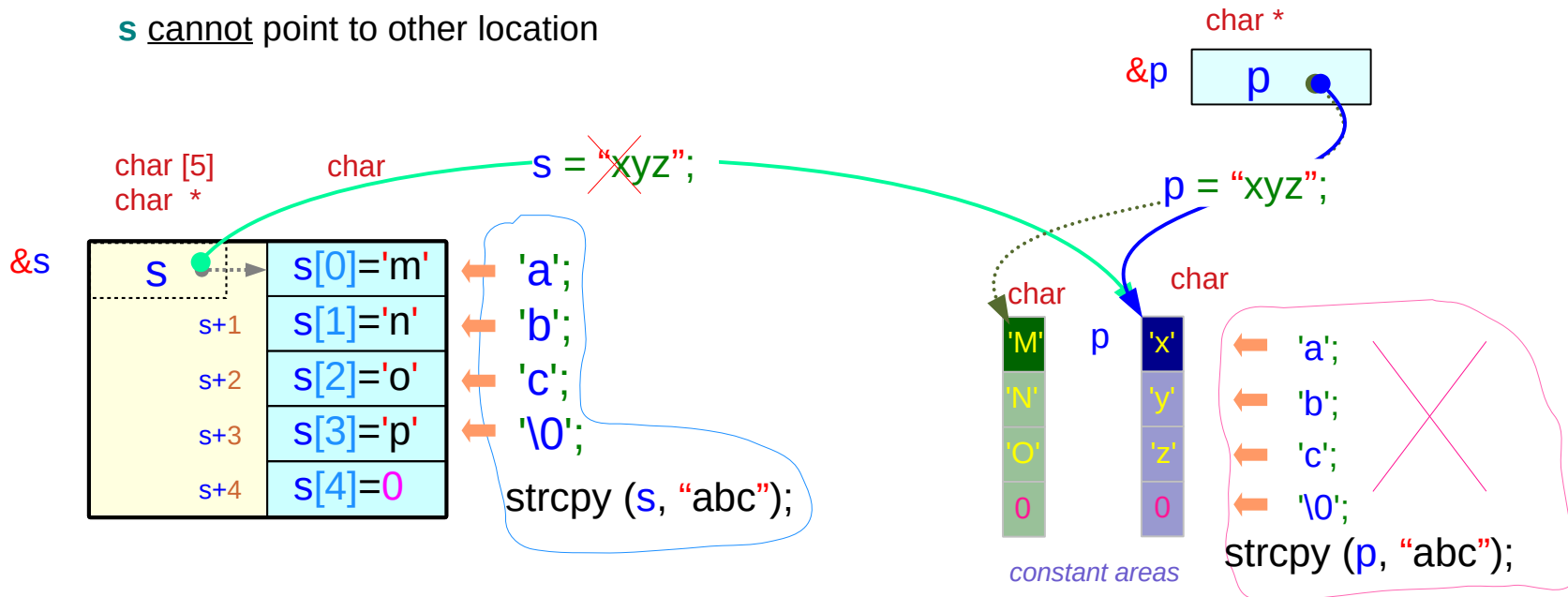
```
char s [5] = "mnop" ;  
const char *p = "MNO" ;
```

```
s = "xyz" ;  
p = "xyz" ;
```

```
strcpy (s, "abc") ;  
strcpy (p, "abc") ;
```

`char * const s`

`s` cannot point to other location

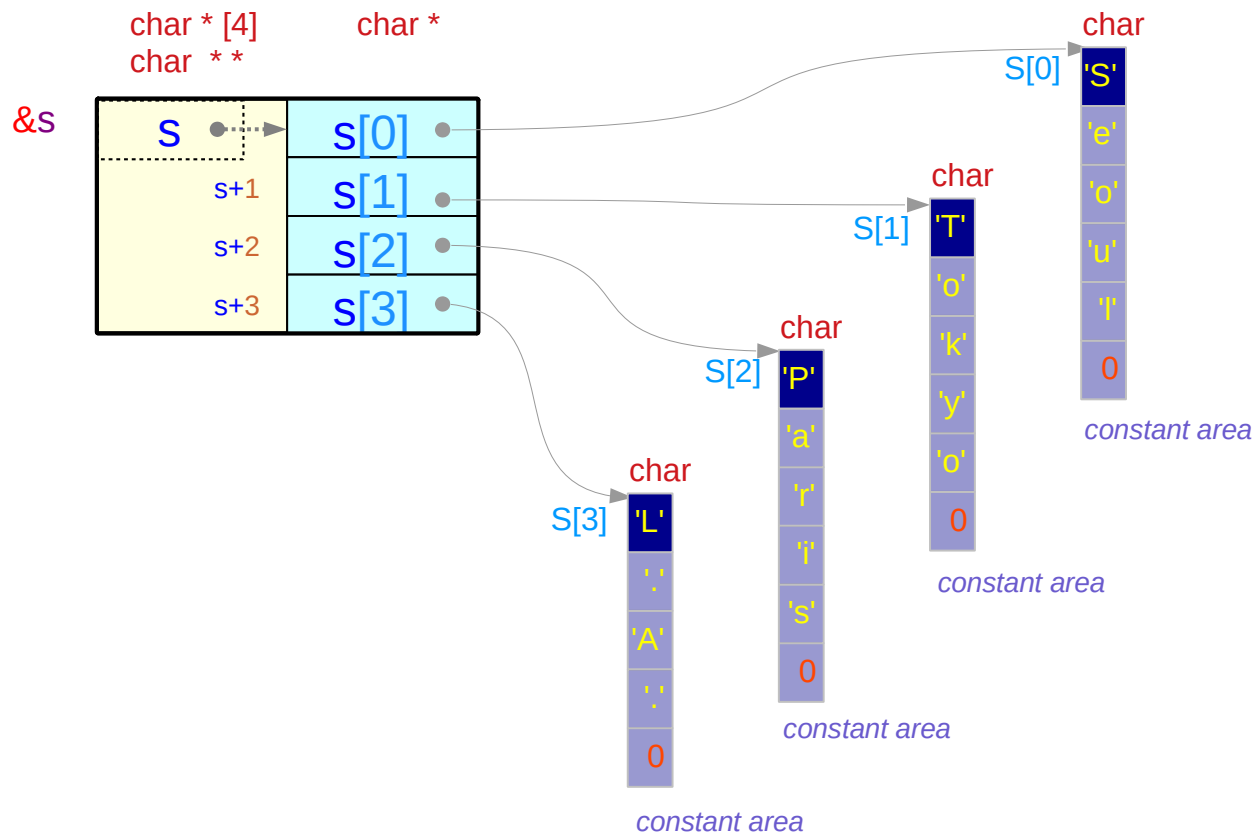


`const char * p`

`p` points to a string constant
which cannot be changed

Arrays of Pointers

```
char * S [4] = { "Seoul", "Tokyo", "Paris", "LA" } ;
```



A possible memory layout
(little endian system)

MSB			LSB
'u'	'o'	'e'	'S'
'o'	'T'	0	'l'
0	'o'	'y'	'k'
'i'	'r'	'a'	'P'
'l'	'L'	0	's'
	0	'A'	

-
- **Unsize array notations for 1-d arrays**

Unsize array notation `x[]`

1. An array definition with **initializers**

```
int x [ ] = { 1, 2, 3 } ;
```



```
int x [3] ;
```

```
x[0] = 1, x[0] = 2, x[0] = 3 ;
```

2. A formal **parameter** definition in a function

```
func( int x [ ] ) { ... }
```



```
int (*x)
```

compatible



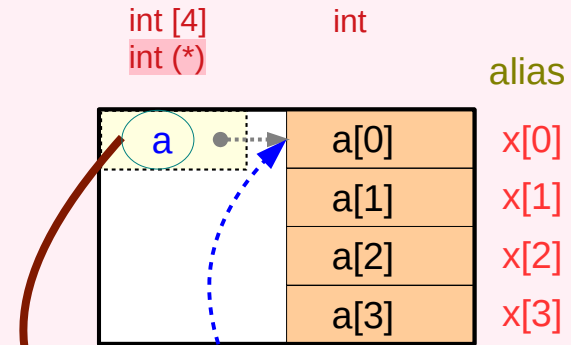
```
int * x
```


Passing 1-d Arrays – using 0-d array pointer

```
int a[4] = { 1, 2, 3, 4 };
```

```
func( a );
```

```
func( int x [ ] ) {  
    ...  
}  
or  
func( int (*x) ) {  
    ...  
}
```



int *
&x x = a

can change the original array **a** elements by the alias **x**

Passing 1-d Arrays – using 1-d array pointer

```
int a[4] = { 1, 2, 3, 4 };
```

```
func( &a );
```

```
func( int x [ ][4] ) {
```

```
    ...
```

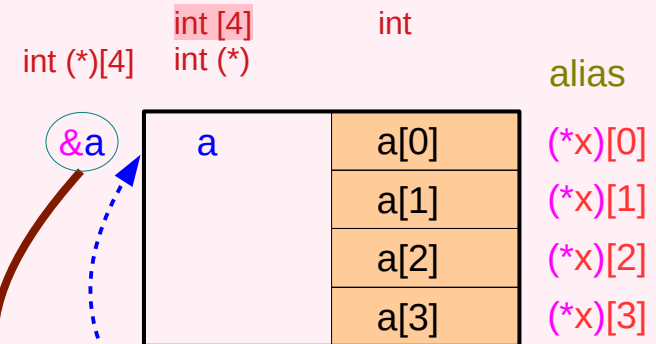
```
}
```

or

```
func( int (*x) [4] ) {
```

```
    ...
```

```
}
```




```
&x x = &a
```

can change the original array `a` elements by using the alias `(*x)`

Passing an individual element by value

```
int a[4] = { 1, 2, 3, 4 };  
  
func(a[3]);
```




```
func(int x) {  
    ...  
}
```

int
&x x = a[3]

cannot change the original array element a[3]

```
int a[4] = { 1, 2, 3, 4 };  
  
func(&a[3]);
```



```
func(int *x) {  
    ...  
}
```

int *
&x x = &a[3]

can change the original array element a[3] by the alias *x

Passing an individual element by value

```
int a[4];
```

```
func( a );
```

```
func( int x [4] ) { ... }
```

```
func( int x [ ] ) { ... }
```

```
func( int (*x) ) { ... }
```

```
int c[3][4];
```

```
func( c );
```

```
func( int c [3][4] ) { ... }
```

```
func( int x [ ][4] ) { ... }
```

```
func( int (*x)[4] ) { ... }
```

Passing an individual element by value

```
int a[4];
```

```
func( &a );
```

```
func( int (*x) [4] ) { ... }
```

```
func( int x[ ] [4] ) { ... }
```

```
int c[3][4];
```

```
func( &c );
```


```
func( int (*x) [3][4] ) { ... }
```

```
func( int x[ ] [3][4] ) { ... }
```



-
- **Unsize array notations for 2-d arrays**

Unsize array notation `x[][N]`

1. An array definition with **initializers**

```
int x[ ][3] = { {1, 2, 3}, {4, 5, 6} };  int x[2][3] ;  
x[0][0] = 1, x[0][1] = 2, x[0][2] = 3,  
x[1][0] = 4, x[1][1] = 5, x[1][2] = 6 ;
```

2. A formal **parameter** definition in a function

```
func( int x[ ][3] ) { ... }  int (*x)[3]  
  
not compatible  int ** p
```

Passing 2-d Arrays – using 1-d array pointer

```
int b[2][3] = { {1, 2, 3}, {4, 5, 6} };
```

```
func( b );
```

```
func( int y [ ][3] ) {
```

```
    ...
```

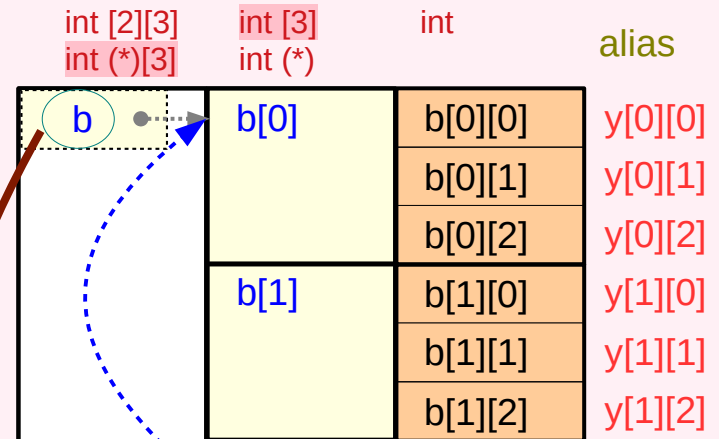
```
}
```

or

```
func( int (*y) [3] ) {
```

```
    ...
```

```
}
```



&y y = b

can change the original array **b** elements by using the alias **y**

Passing 2-d Arrays – using 2-d array pointer

```
int b[2][3] = { {1, 2, 3}, {4, 5, 6} };
```

```
func( &b );
```

```
func( int y [ ][2][3] ) {
```

...

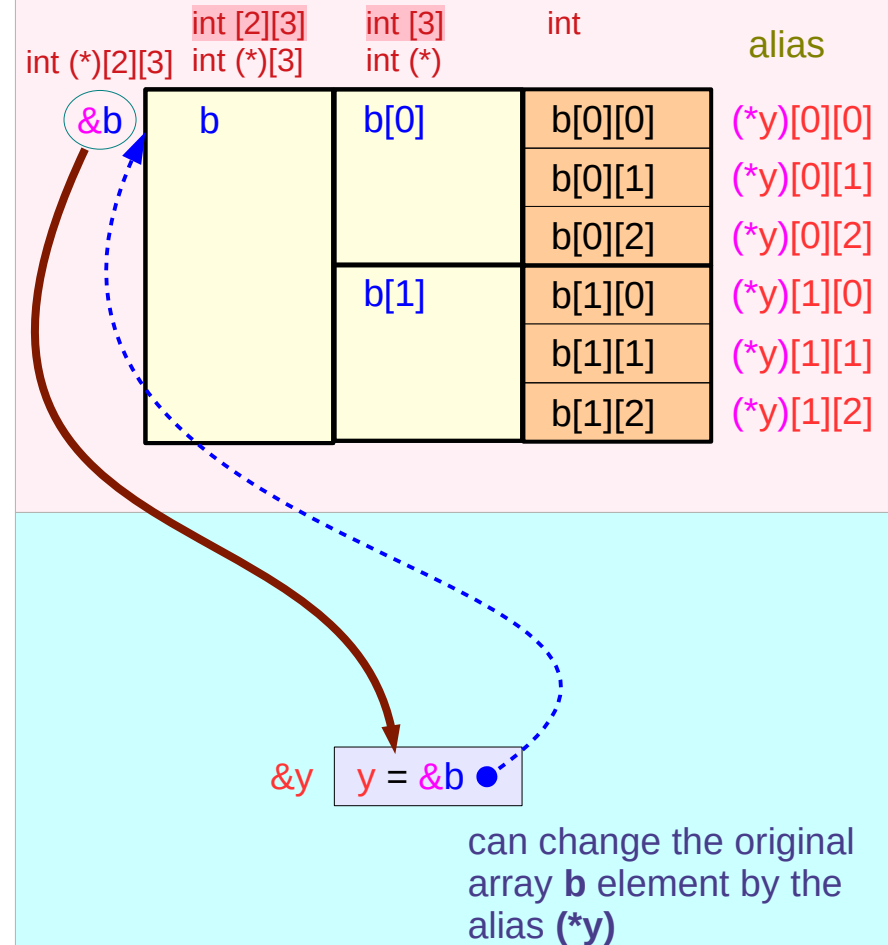
```
}
```

or

```
func( int (*y) [2][3] ) {
```

...

```
}
```



Passing an individual element by reference

```
int b[2][3] = { {1, 2, 3}, {4, 5, 6} };  
  
func(b[0][1]);  
  
func(int y) {  
    ...  
}
```

`&y` `y = b[0][1]`

cannot change the original array element `b[0][1]`

```
int b[2][3] = { {1, 2, 3}, {4, 5, 6} };  
  
func(&b[0][1]);  
  
func(int *y) {  
    ...  
}
```

`&y` `y = &b[0][1]`

can change the original array element `b[0][1]` the alias `*y`

-
- **Type definitions and 2-d arrays**

Type definitions

```
typedef int int_type ;
```

type alias **int_type** variable **i**

```
int_type i;
```



```
int i ;
```

```
typedef int * iptr_type ;
```

type alias **iptr_type** variable **ip**

```
iptr_type ip;
```



```
int * ip ;
```

```
typedef int arr_type [4] ;
```

type alias **arr_type** variable **a**

```
arr_type a;
```



```
int a [4] ;
```

Array Type Definition

```
typedef int arr_type [4];
```

```
arr_type a;
```

≡

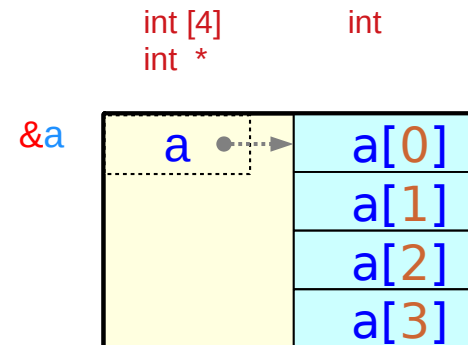
```
int a [4];
```

```
a [0] = 100;
```

```
a [1] = 200;
```

```
a [2] = 300;
```

```
a [3] = 400;
```



Pointer to Array Type Definition

```
typedef int arr_type [4];  
arr_type a, *q;
```

≡

```
int a [4], int (*q) [4];
```

```
typedef int (*arr_ptr) [4];  
arr_ptr p;
```

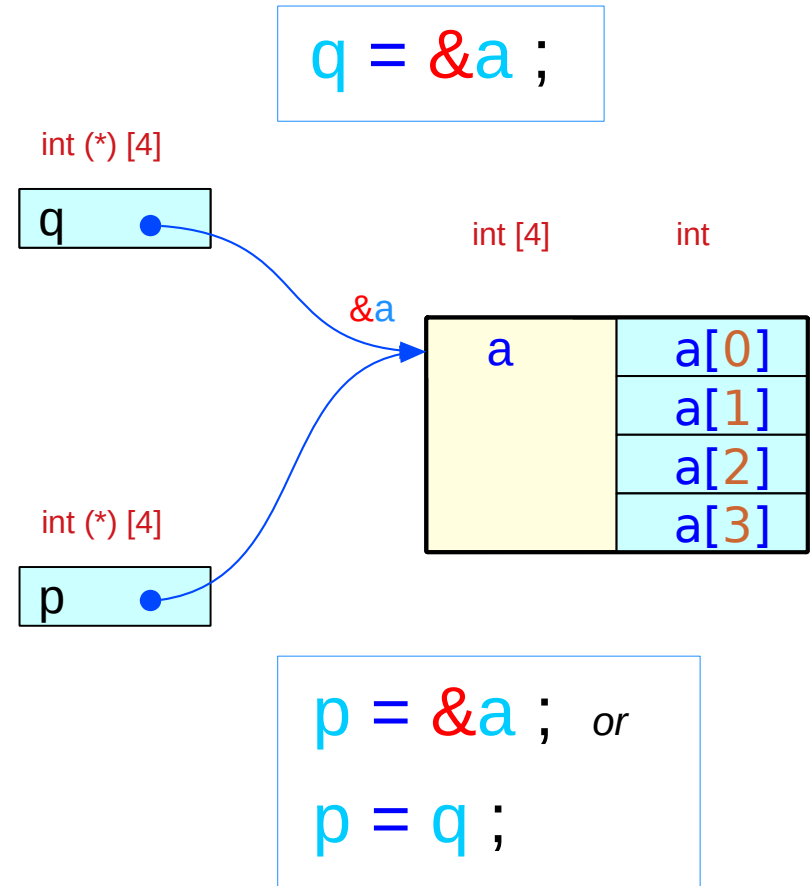
≡

```
int (*p) [4];
```

Pointer to Array Type Assignment

```
typedef int arr_type [4];  
arr_type a, *q;
```

```
typedef int (*arr_ptr) [4];  
arr_ptr p;
```

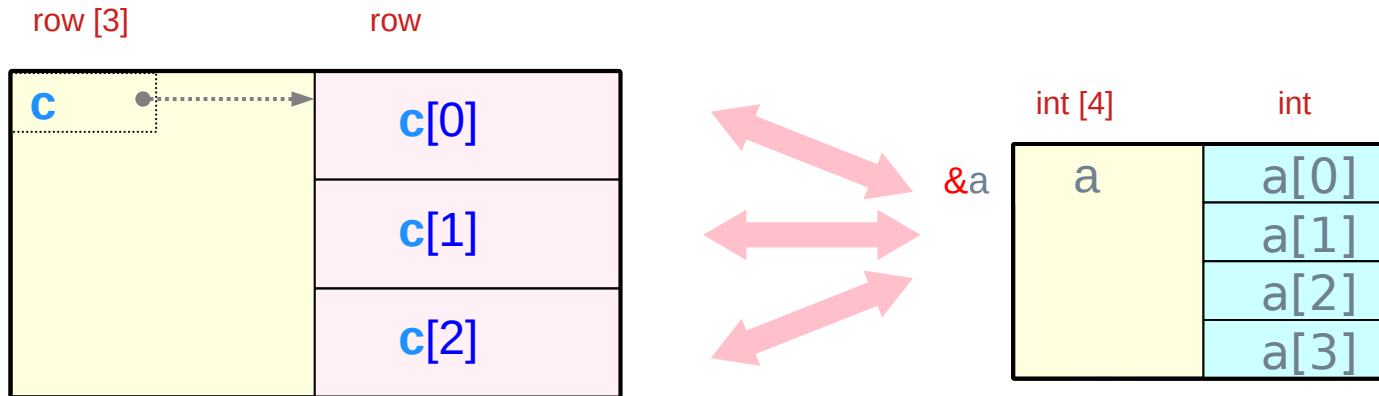


Nested array declared explicitly

```
typedef int row [4] ;  
row c [3] ;
```

≡

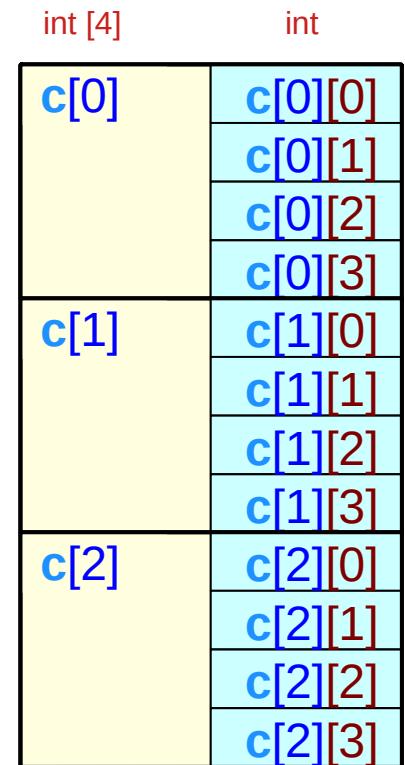
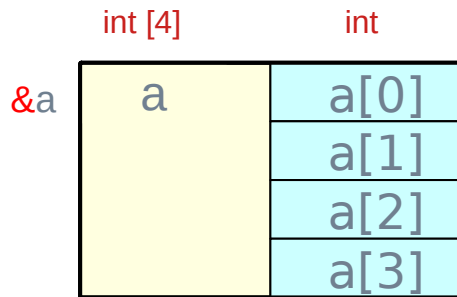
```
int c [3] [4] ;
```



each element **c[i]** has the type of **row (int [4])**

Nested array declared explicitly

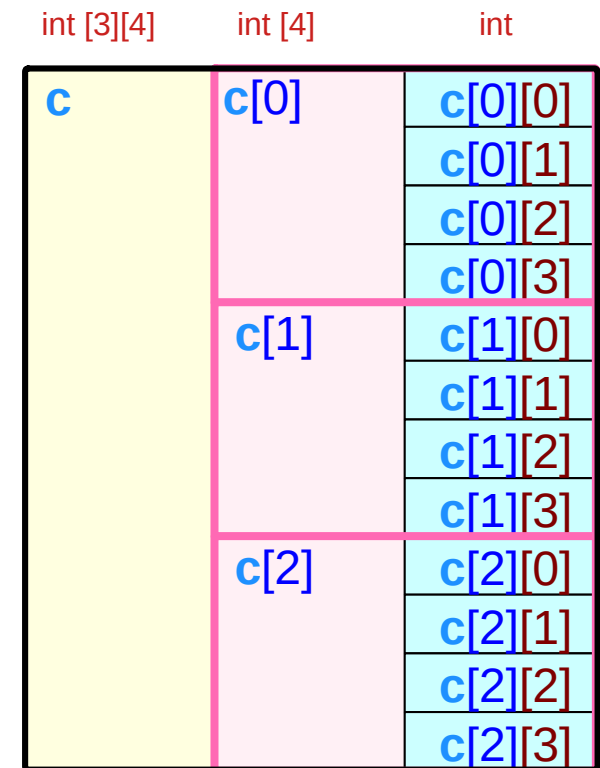
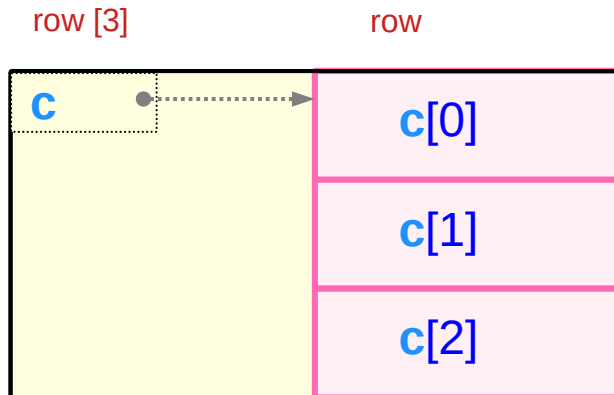
```
typedef int row [4] ;  
row c [3] ;
```



each element $c[i]$ has the type of row (**int [4]**)

Nested array declared explicitly

```
typedef int row [4] ;  
row c [3] ;
```



-
- **2-d arrays and element addresses**

2-d array definition

```
int c [3][4];
```

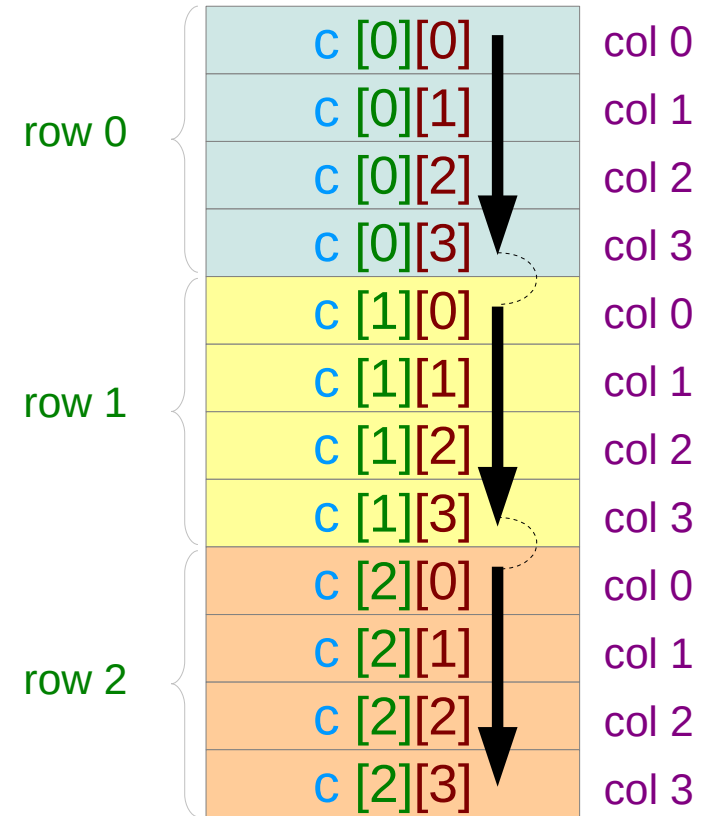
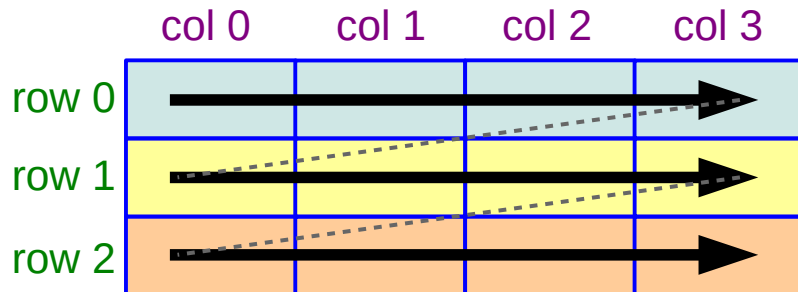
A matrix view

	col 0	col 1	col 2	col 3
row 0	c [0][0]	c [0][1]	c [0][2]	c [0][3]
row 1	c [1][0]	c [1][1]	c [1][2]	c [1][3]
row 2	c [2][0]	c [2][1]	c [2][2]	c [2][3]

2-d array stored as a linear array

```
int c [3][4];
```

row major order



Element address $c[i] + j$

$$X[i] \equiv *(X+i)$$

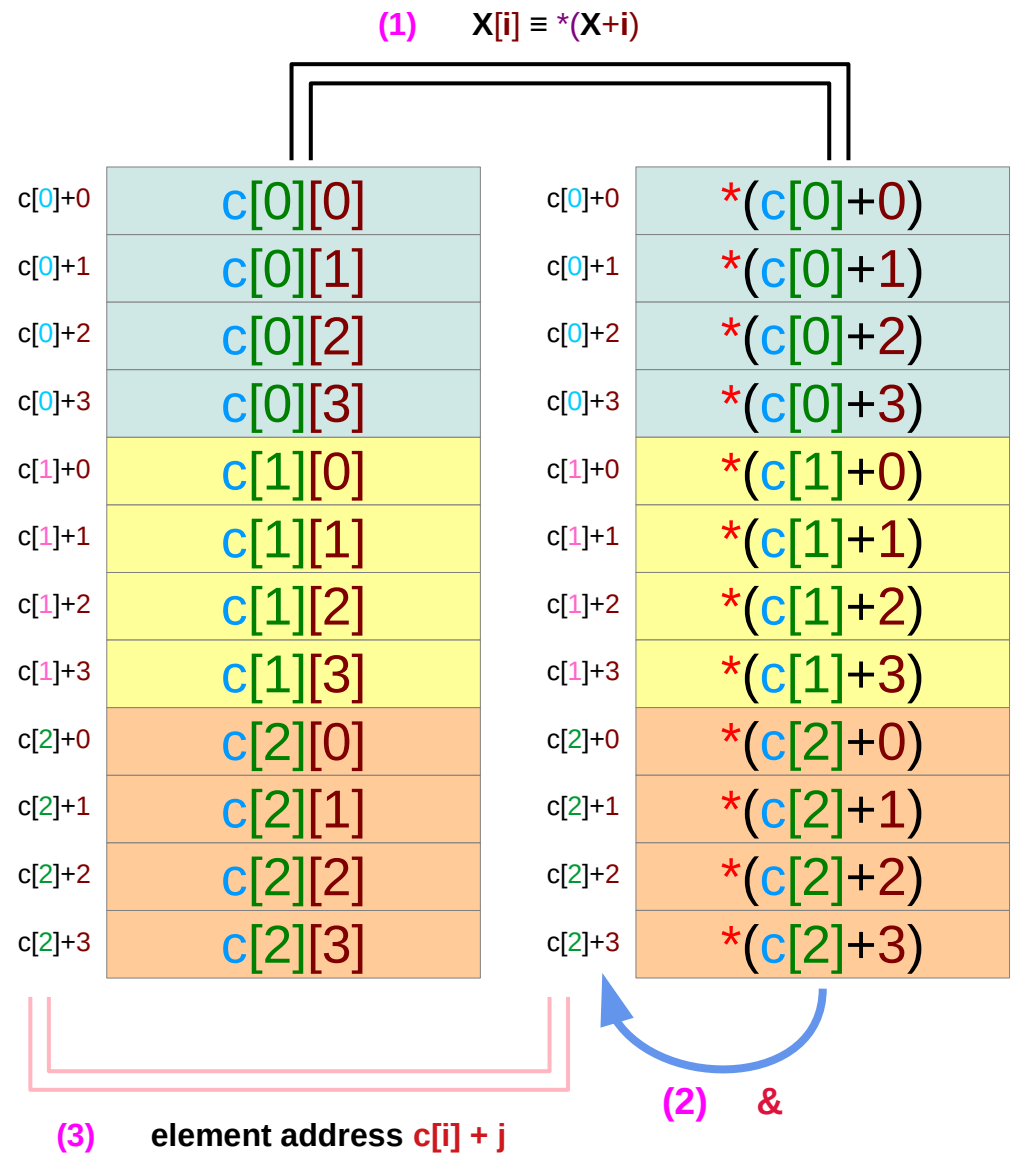
$$X[j] \equiv *(X+j)$$

$$\text{let } X \equiv c[i] \quad (1)$$

$$c[i][j] \equiv *(c[i]+j)$$

$$\&c[i][j] \equiv c[i]+j \quad (3)$$

the address of $c[i][j]$ is $c[i]+j$



Row address $c[i]$

$$c[i][j] \equiv *(c[i]+j)$$

$$\&c[i][j] \equiv c[i]+j$$

the address of $c[i][j]$ is $c[i]+j$

↓ let $j = 0$

$$c[i][0] \equiv *c[i]$$

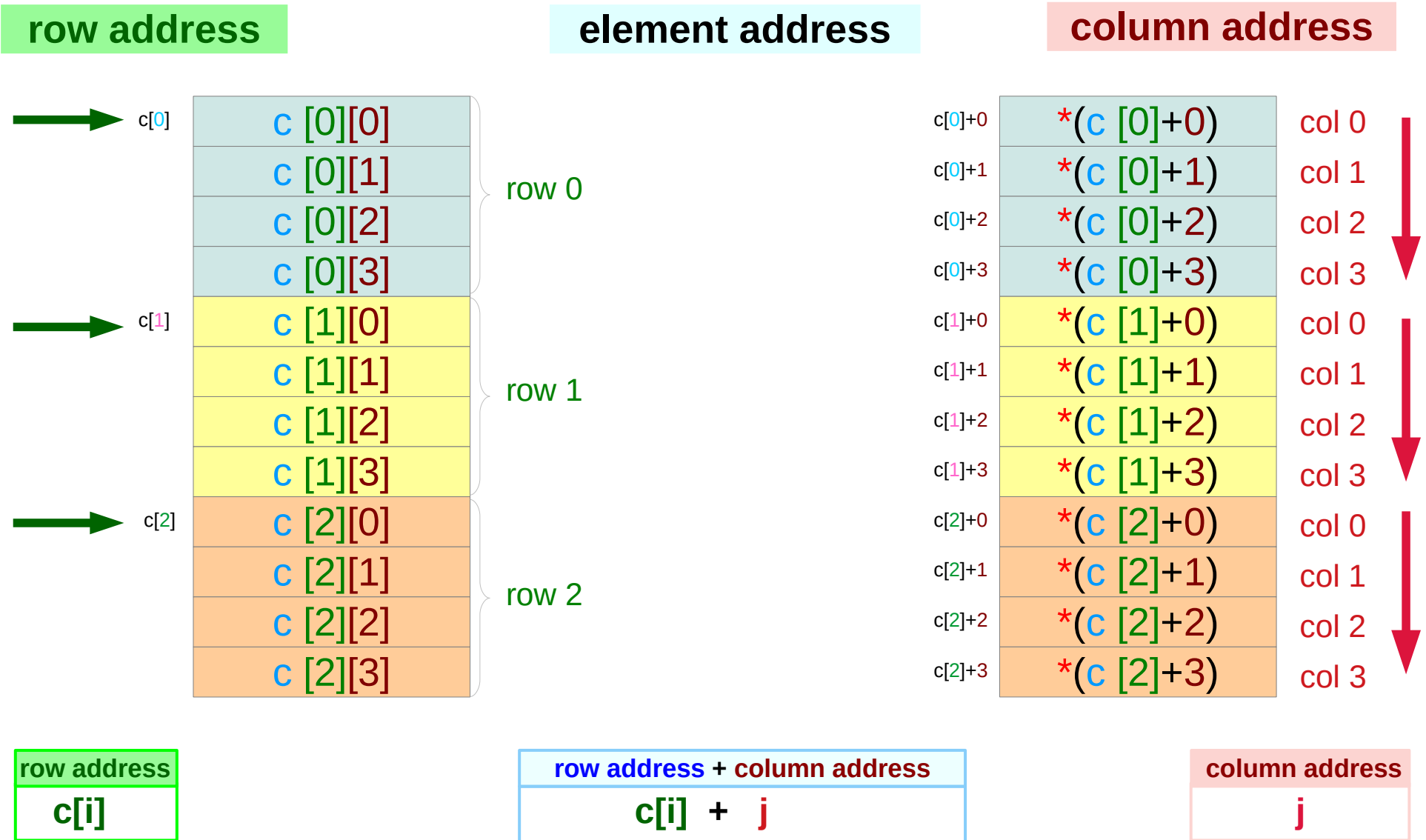
$$\&c[i][0] \equiv c[i]$$

row address : the address of the 1st element of each row

$c[0]$	$c[0][0]$
	$c[0][1]$
	$c[0][2]$
	$c[0][3]$
$c[1]$	$c[1][0]$
	$c[1][1]$
	$c[1][2]$
	$c[1][3]$
$c[2]$	$c[2][0]$
	$c[2][1]$
	$c[2][2]$
	$c[2][3]$

$c[0]$	$*(c[0]+0)$
	$*(c[0]+1)$
	$*(c[0]+2)$
	$*(c[0]+3)$
$c[1]$	$*(c[1]+0)$
	$*(c[1]+1)$
	$*(c[1]+2)$
	$*(c[1]+3)$
$c[2]$	$*(c[2]+0)$
	$*(c[2]+1)$
	$*(c[2]+2)$
	$*(c[2]+3)$

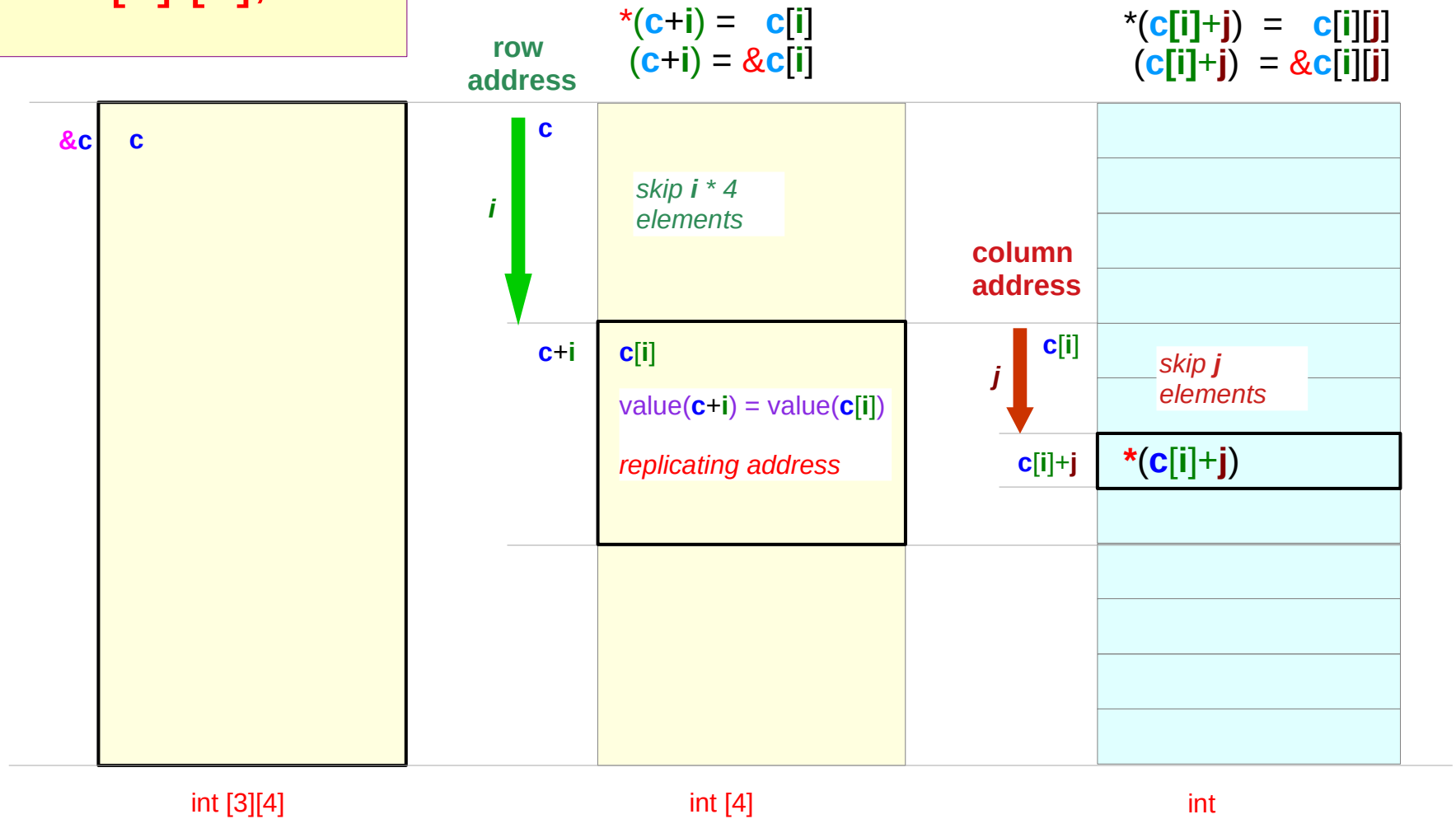
Row Address $c[i]$ and Column Address j



Element address $c[i] + j$

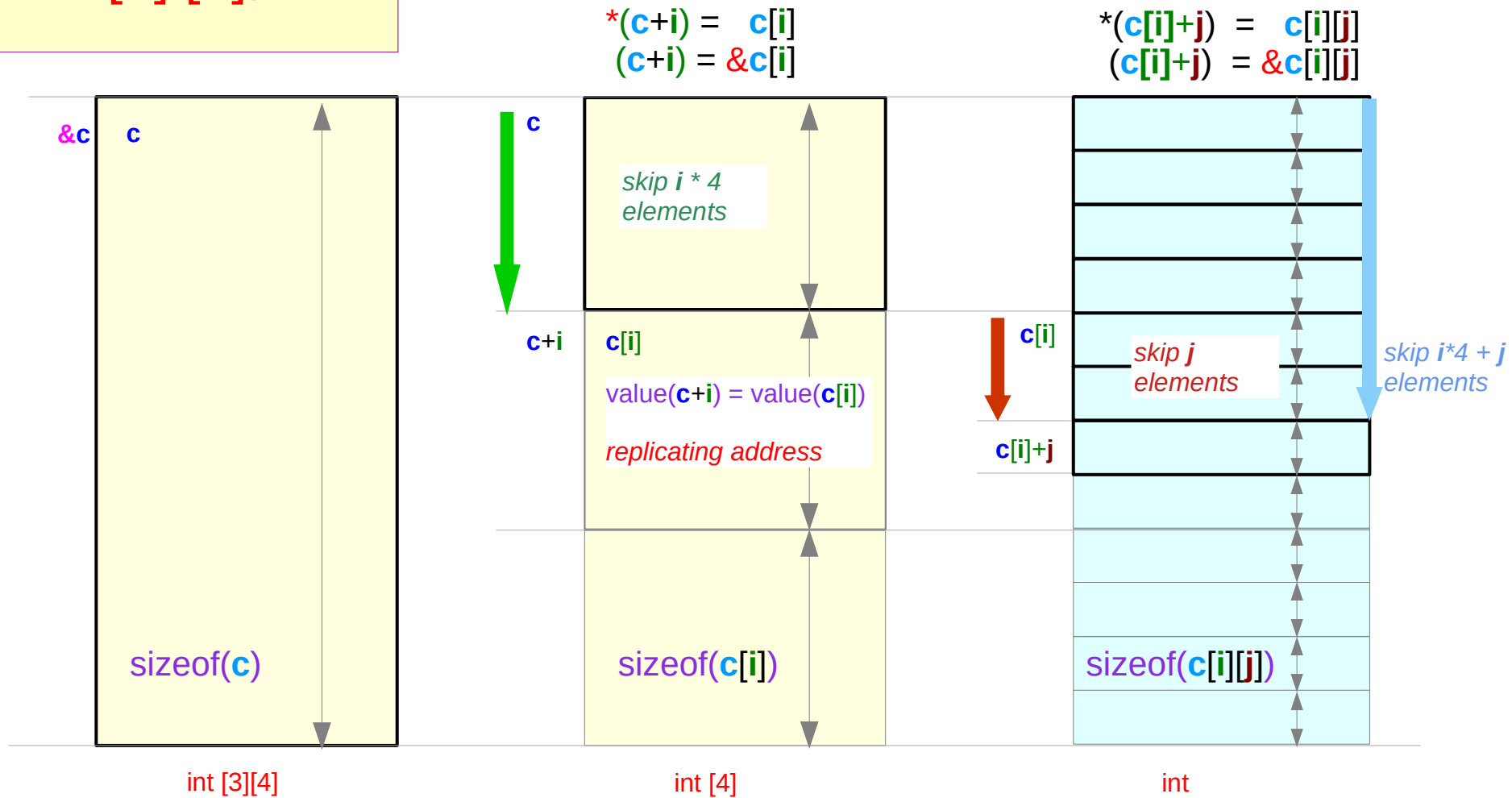
```
int c [3] [4];
```

row address + column address



Sizes of `c`, `c[i]`, `c[i][j]`

```
int c [3] [4];
```



Address of $c[i][j]$

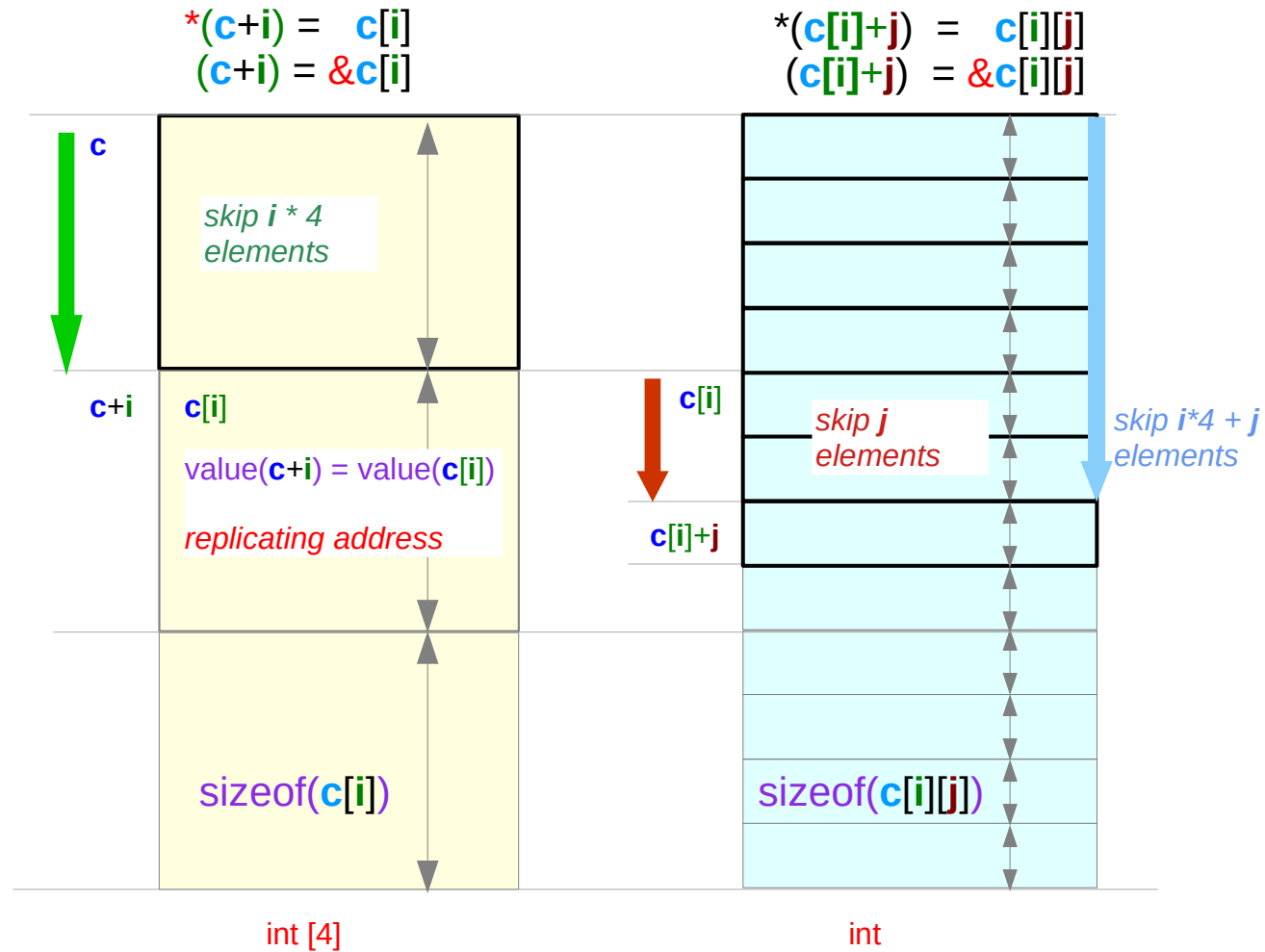
```
int c [3] [4];
```

value($c + i$) =
value(c) + $i * \text{sizeof}(*c)$

value($c[i] + j$) =
value($c[i]$) + $j * \text{sizeof}(*c[i])$

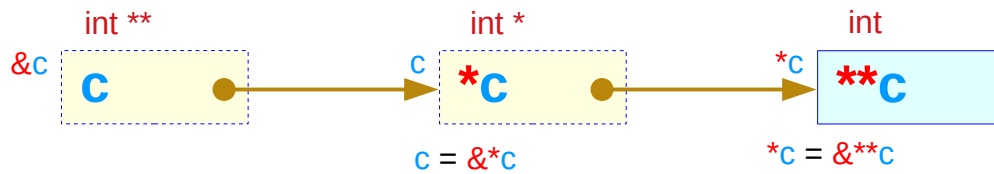
value($c + i$) = value($c[i]$)
address replication

$\&c[i][j] = \text{value}(c[i] + j)$
 $= \text{value}(c[i]) + j * \text{sizeof}(*c[i])$
 $= \text{value}(c + i) + j * \text{sizeof}(*c[i])$
 $= \text{value}(c) + i * \text{sizeof}(*c)$
 $\quad + j * \text{sizeof}(*c[i])$
 $= \text{value}(c) + i * 4 * 4 + j * 4$

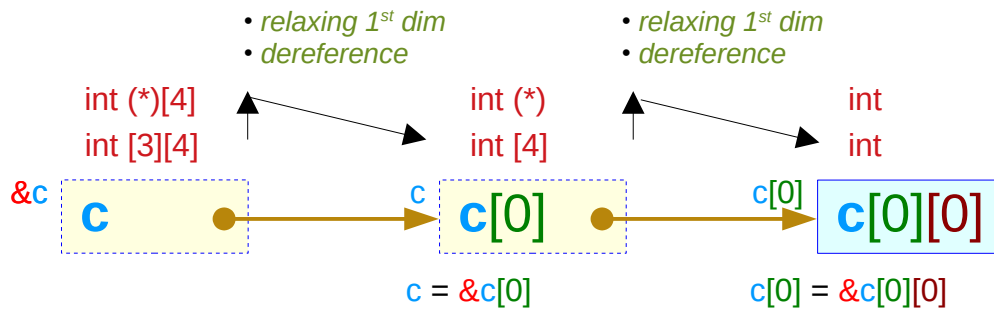


-
- **Two types of a 2-d array c**
 - **Two types of 1-d sub-arrays $c[i]$**

Chains of dereferences

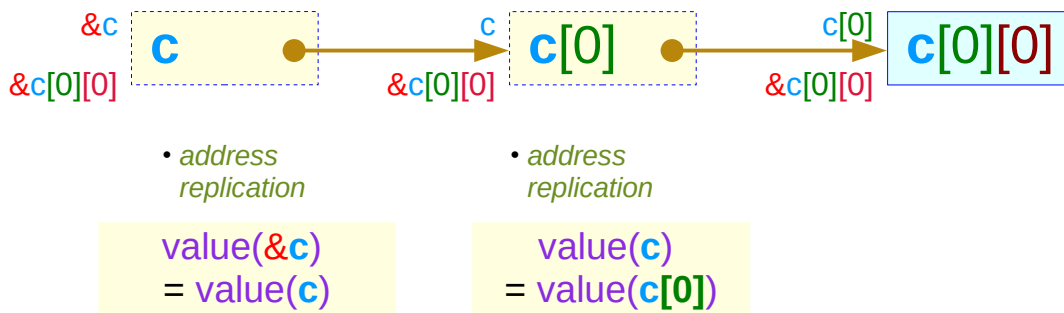


a chain of dereferences using *



← virtual array pointer type
 ← abstract data (array) type

a chain of dereferences using []

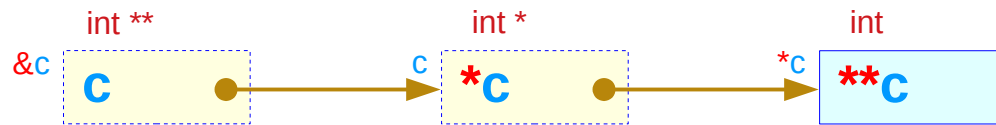


replicating a physical address

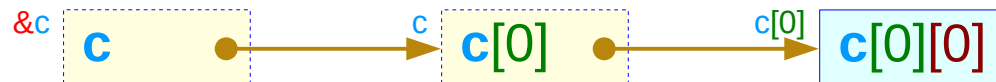
conditions for **c** and **c[0]**
 to start at the same address $\&c[0][0]$

address(pointer) = value(pointer)

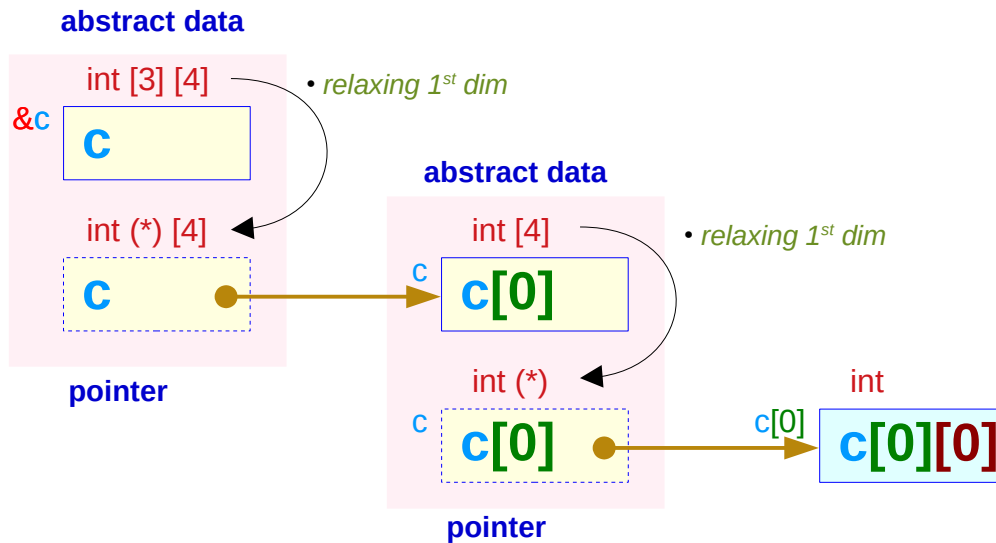
Chains of dereferences with correct types



a chain of dereferences using *



a chain of dereferences using []



with correct types of referencing

value(&c)
= value(c)
• address replication

value(c)
= value(c[0])
• address replication

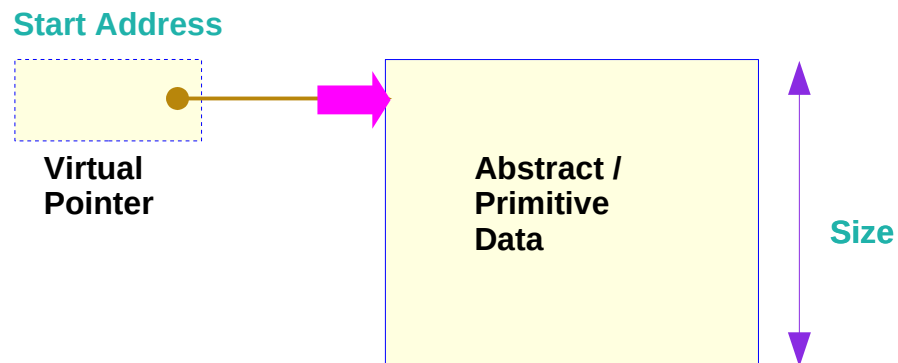
Two types of an array – **c**, **c[i]**, **c[i][j]**

```
int c[3] [4] ;
```

		c	c[i]	c[i][j]
abstract / primitive Data	Size	2-d array int [3][4]	1-d array int [4]	integer int
virtual pointer	Start Address	1-d array pointer int (*)[4]	0-d array pointer int (*)	

Two types of an array – pointer to abstract data

```
int c[3][4];
```

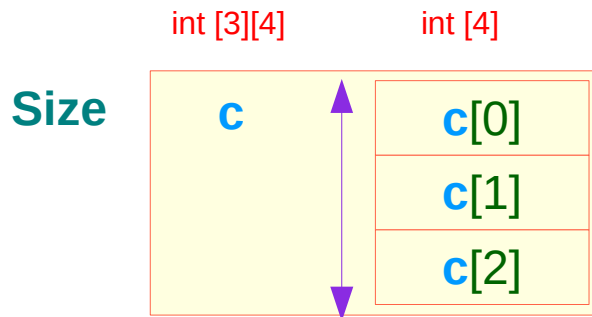


Virtual Pointer		Abstract / Primitive Data
<code>int (*)[4]</code>		<code>int [4]</code>
<code>c</code>	● →	<code>c[0]</code>
<code>int (*)</code>		<code>int</code>
<code>c[0]</code>	● →	<code>c[0][0]</code>
<code>c[1]</code>	● →	<code>c[1][0]</code>
<code>c[2]</code>	● →	<code>c[2][0]</code>
Start Address		Size

Two types in a 2-d array `c`

`int c[3][4];`

Abstract data `c`



`int (c[3]) [4] ;`

3 element array `c`

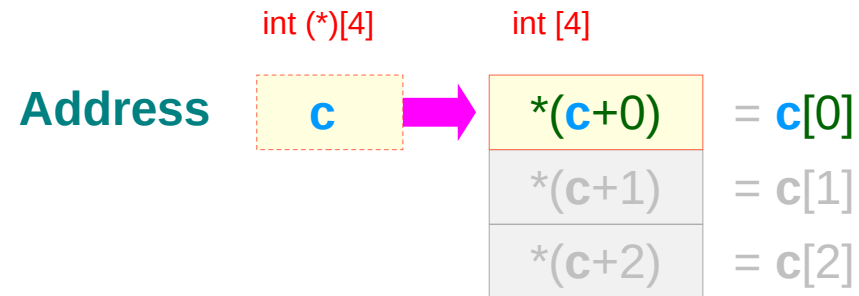
type `c[3]` ;

`c` 2-d array

type : `int [3][4]`

size : `3 * 4 * 4`

Pointer `c`



`int (c[3]) [4] ;`

each element `c[i]` has
the array type `int [4]`

type `c[3]` ;

`c` 1-d array pointer

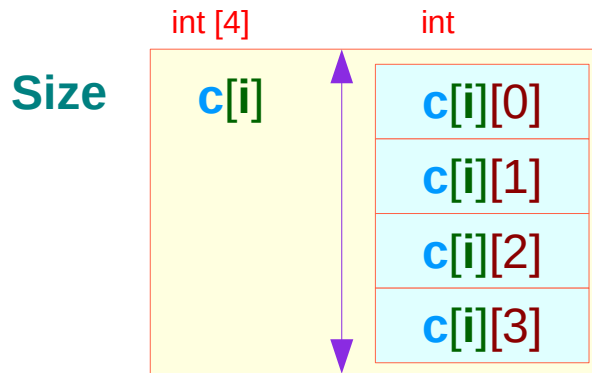
type : `int (*)[4]`

value : `&c[0][0]`

Two types in a 1-d array $c[i]$

`int c[3][4];`

Abstract data $c[i]$



`int (c[3]) [4] ;`

4 element array $c[i]$

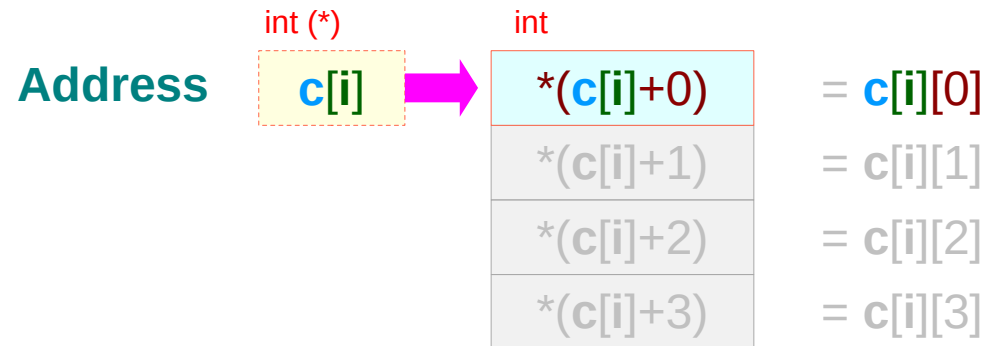
`int c[i] [4] ;`

$c[i]$ 1-d array

type : `int [4]`

size : `4 * 4`

Pointer $c[i]$



`int (c[3]) [4] ;`

each element $c[i][j]$ has
the array type `int`

`int c[i] [4] ;`

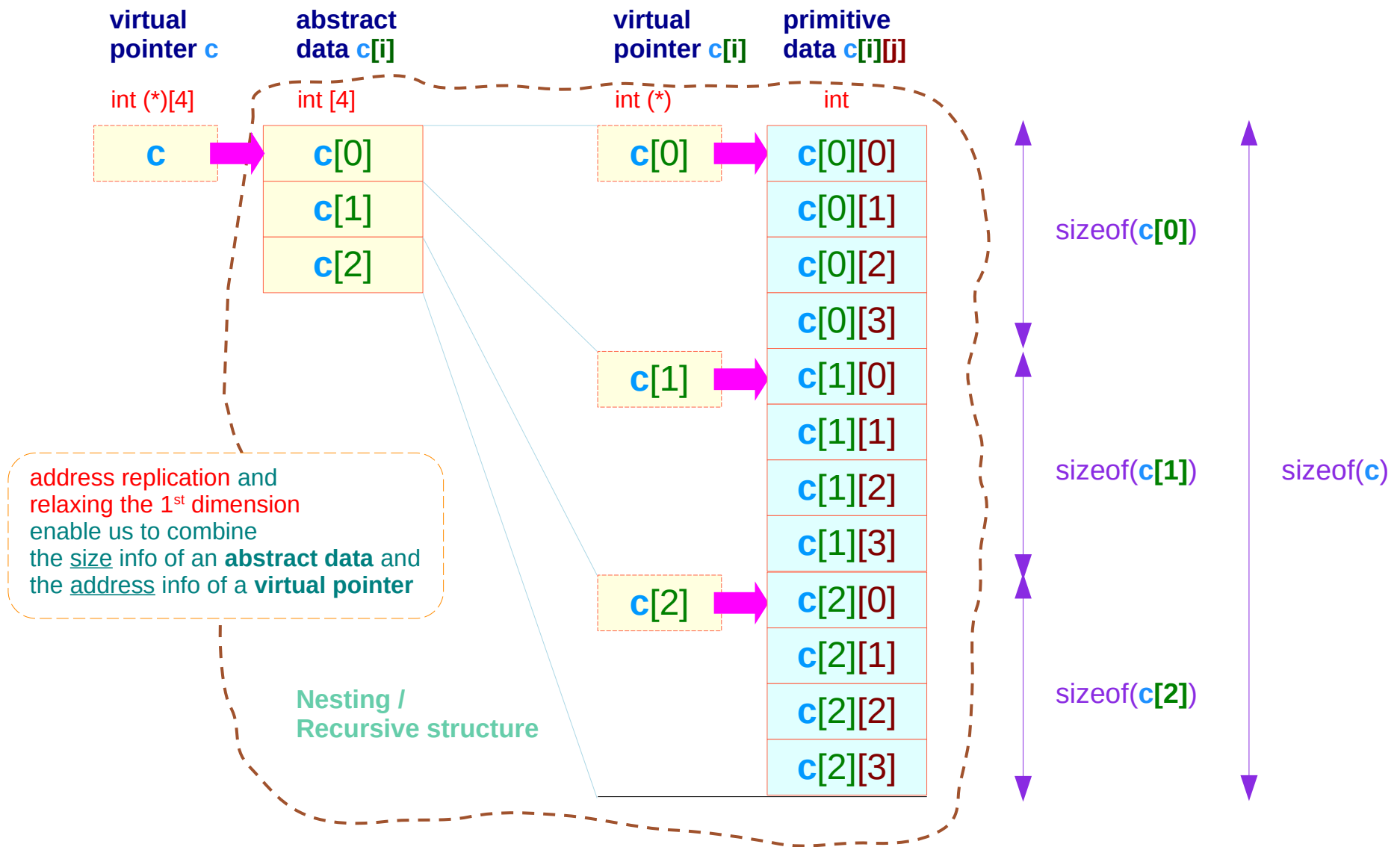
$c[i]$ 0-d array pointer

type : `int (*)`

value : `&c[i][0]`

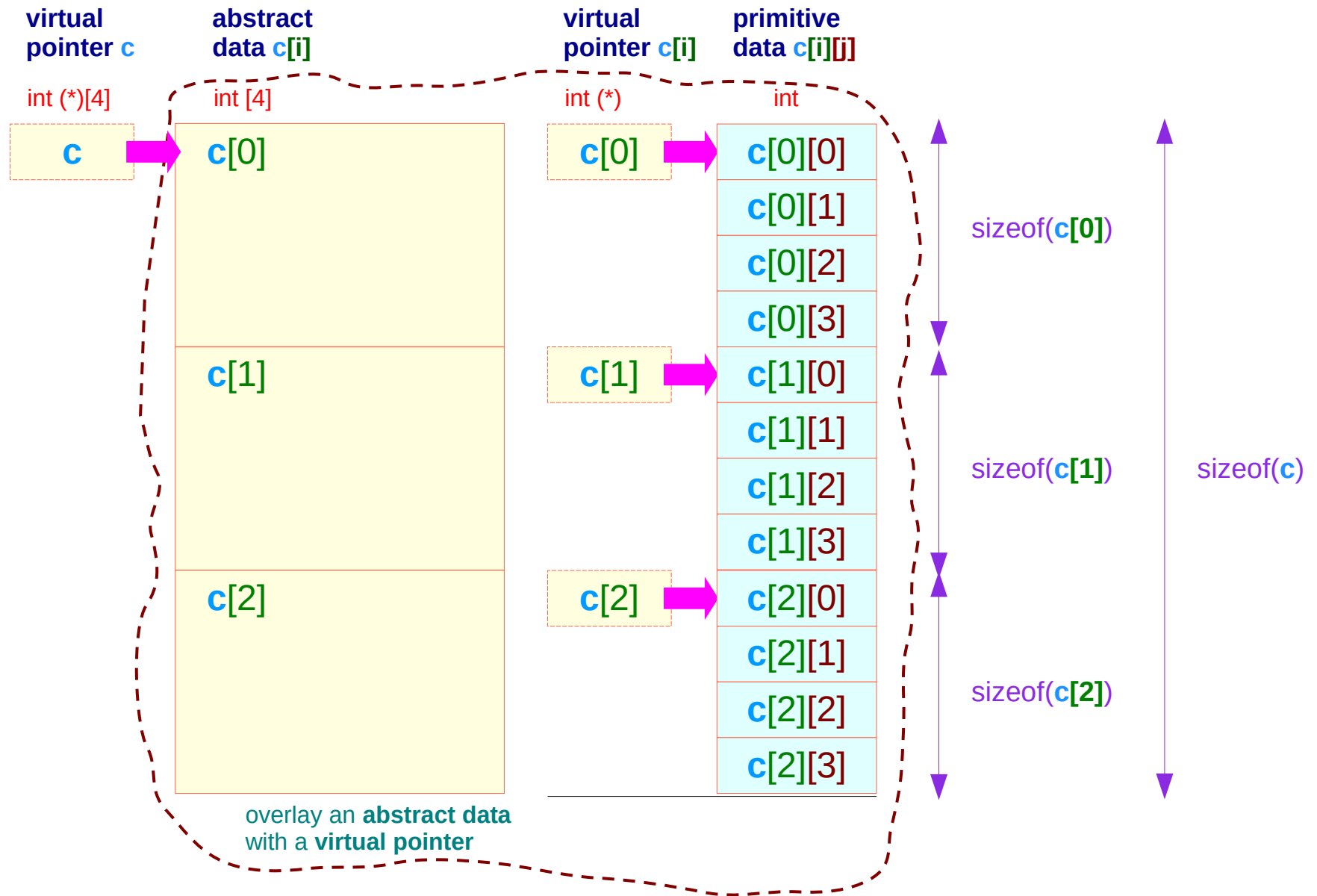
Combining size and address information (1)

`int c[3][4];`



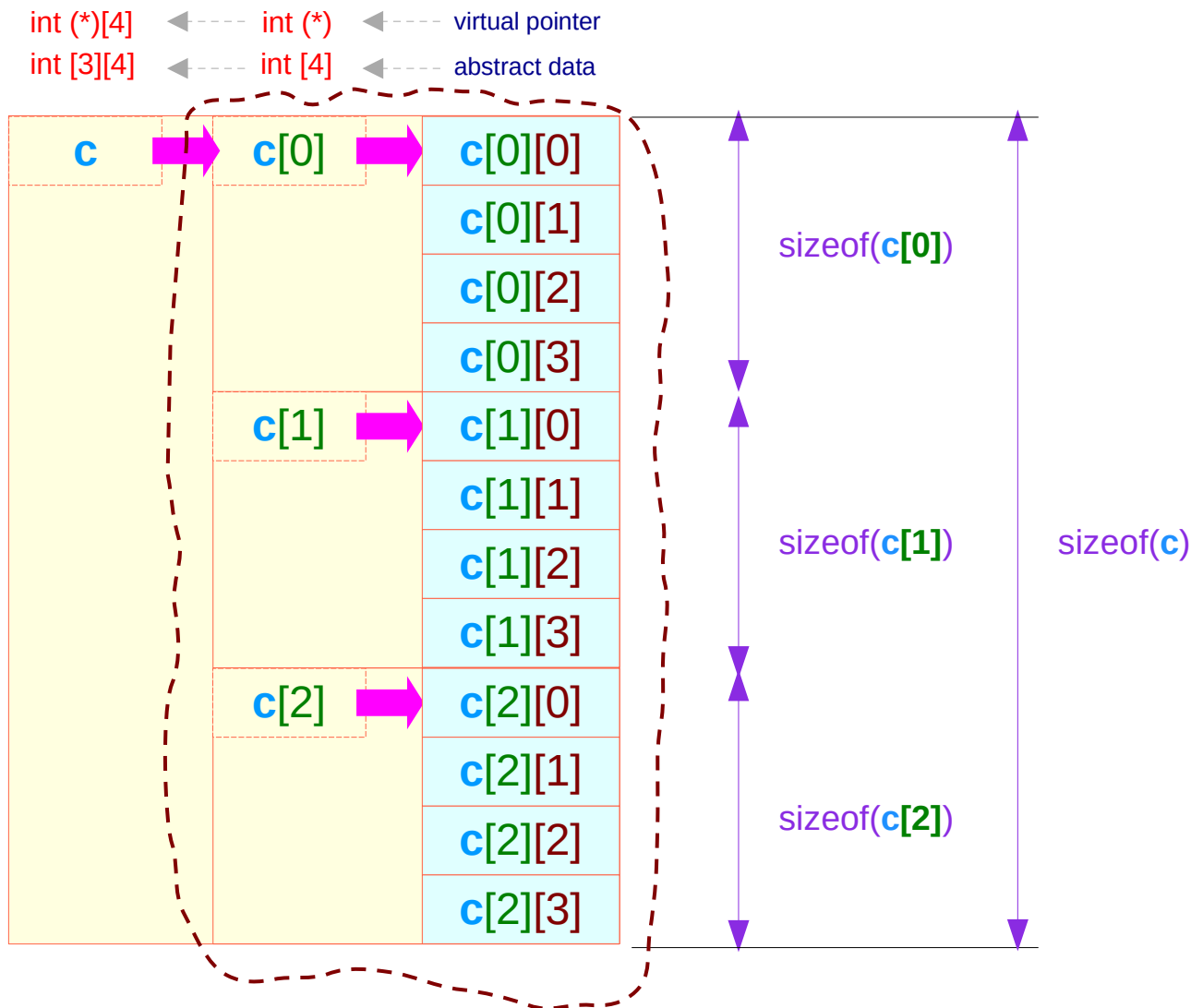
Combining size and address information (2)

`int c[3][4];`



Combining size and address information (3)

`int c[3][4];`

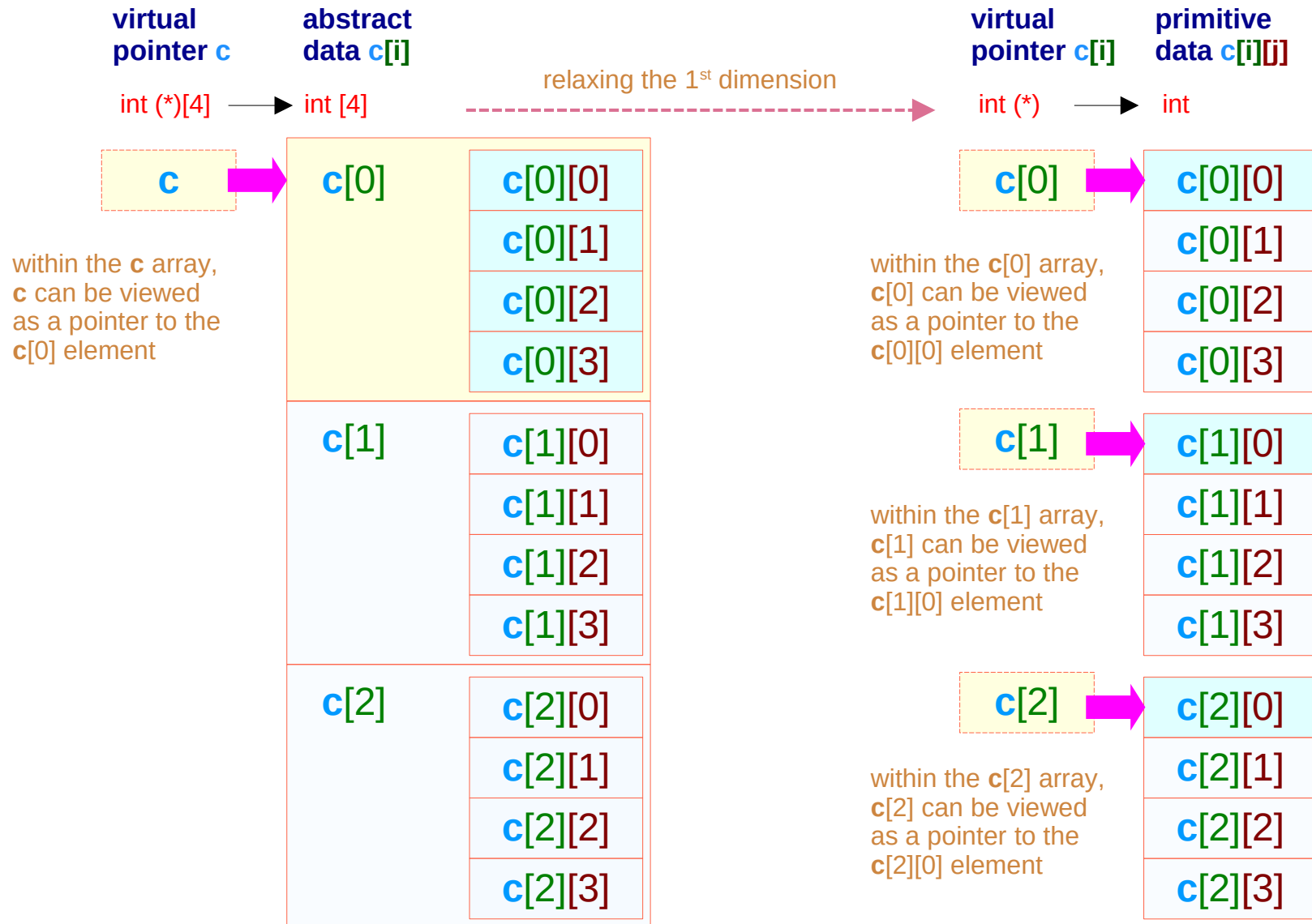


When `c[i]` is referenced outside, `c[i]` has the array type `int[4]`

When `c[i]` is referencing the 1st element inside, `c[i]` has the pointer type `int (*)`

Recursive pointer-to-data views

`int c[3][4];`



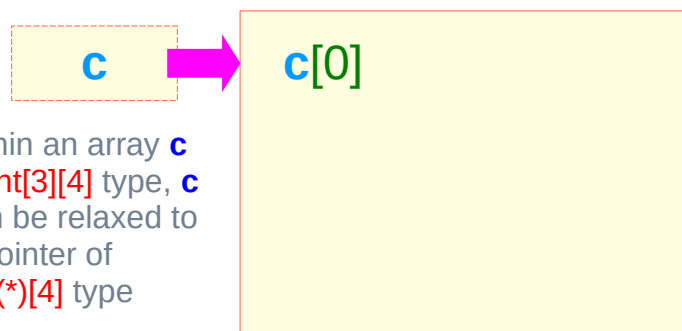
Type, address, and value of **c** and **c[i]**

```
int c [3] [4];
```

virtual pointer **c** abstract data **c[i]**

int (*)[4]

int [4]



within an array **c** of **int[3][4]** type, **c** can be relaxed to a pointer of **int (*)[4]** type

$$c[i] = *(c + i)_{4.4}$$

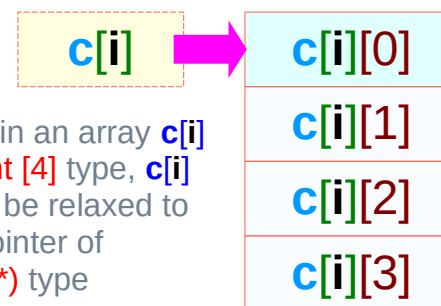
Math Expression

$$\begin{aligned} \text{value}(c[i]) &= \text{value}((c + i)_{4.4}) \quad \text{address replication} \\ &= \text{value}(c) + i * 4 * 4 \\ &= \text{value}(c) + i * \text{sizeof}(*c) \end{aligned}$$

virtual pointer **c[i]** primitive data **c[i][j]**

int (*)

int



within an array **c[i]** of **int [4]** type, **c[i]** can be relaxed to a pointer of **int (*)** type

$$c[i][j] = *(c[i] + j)_{1.4}$$

Math Expression

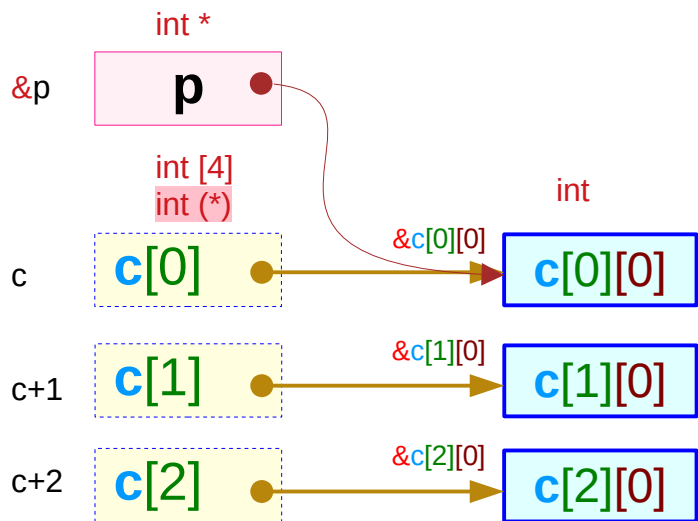
$$\begin{aligned} \text{value}(c[i][j]) &= \text{value}((c[i] + j)_{1.4}) \quad \text{address replication} \\ &= \text{value}(c[i]) + j * 1 * 4 \\ &= \text{value}(c[i]) + j * \text{sizeof}(*c[i]) \end{aligned}$$

Type, address, and value of $c[i]$

`int c[3][4];`

`int (*p) = c[0];`

real pointer
real memory location



non-real pointer
no memory locations

primitive data

row addresses

$\text{type}(c[i]) = \text{int}[4]$ abstract data type
 $\text{int} (*)$ virtual pointer type

address	variable	value
$\text{value}(c)$	$= \text{value}(c[0])$	$= \text{value}(\&c[0][0])$
$\text{value}(c+1)$	$= \text{value}(c[1])$	$= \text{value}(\&c[1][0])$
$\text{value}(c+2)$	$= \text{value}(c[2])$	$= \text{value}(\&c[2][0])$

address replications

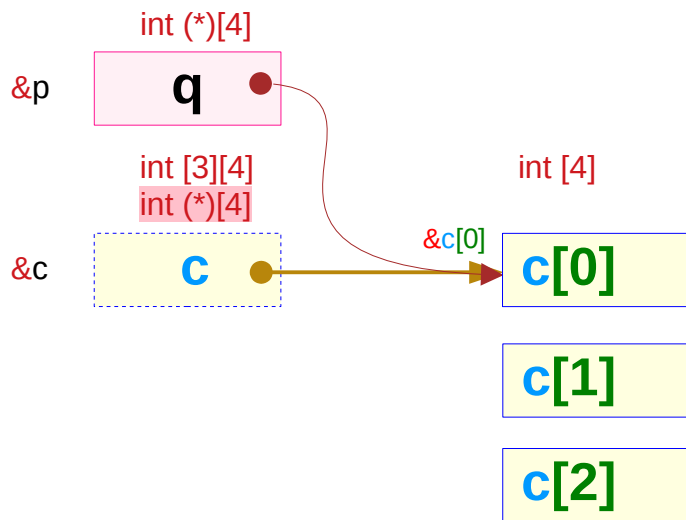
$\text{value}(\&p) \neq \text{value}(p) = \text{value}(\&c[0][0])$

Type, address, and value of **c**

`int c [3] [4];`

`int (*q) [4] = c ;`

real pointer
real memory location



non-real pointer
no memory locations

row addresses

$\text{type}(\mathbf{c}) = \begin{matrix} \text{int [3][4]} & \text{abstract data type} \\ \text{int (*)[4]} & \text{virtual pointer type} \end{matrix}$

address	variable	value
$\text{value}(\&\mathbf{c})$	$= \text{value}(\mathbf{c})$	$= \text{value}(\&\mathbf{c}[0][0])$

address replications

$\text{value}(\&\mathbf{q})$	$\neq \text{value}(\mathbf{q})$	$= \text{value}(\&\mathbf{c}[0][0])$
------------------------------	---------------------------------	--------------------------------------

Size view of abstract data c , $c[i]$

size of abstract data c

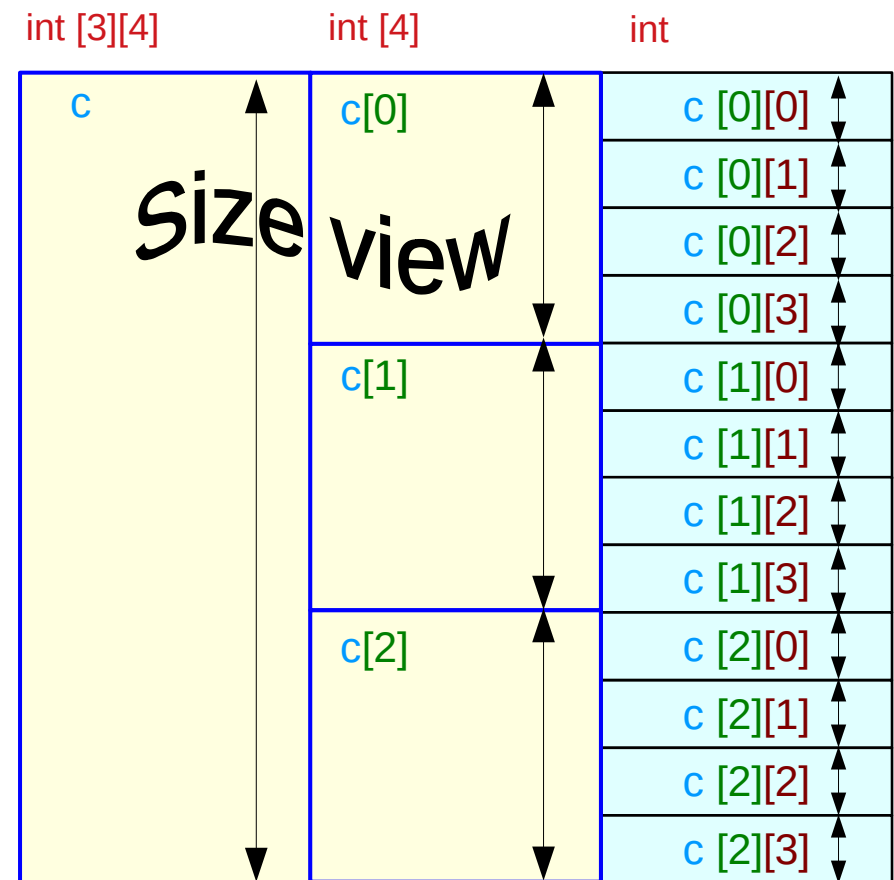
$$\begin{aligned}\text{sizeof}(c) &= \text{sizeof}(*c) * 3 \\ &= \text{sizeof}(c[i]) * 3\end{aligned}$$

size of 4 elements
 $c[0]$, $c[1]$, $c[2]$

size of abstract data $c[i]$

$$\begin{aligned}\text{sizeof}(c[i]) &= \text{sizeof}(*c[i]) * 4 \\ &= \text{sizeof}(c[i][j]) * 4\end{aligned}$$

size of 4 elements
 $c[i][0]$, $c[i][1]$, $c[i][2]$, $c[i][3]$



Address views of virtual pointer \mathbf{c} , $\mathbf{c}[i]$

$\text{value}(\mathbf{c}) = \text{value}(\mathbf{c}[0]) = \text{value}(\&\mathbf{c}[0][0])$
 $\text{value}(\mathbf{c}[1]) = \text{value}(\&\mathbf{c}[1][0])$
 $\text{value}(\mathbf{c}[2]) = \text{value}(\&\mathbf{c}[2][0])$



address replications

$\text{value}(\&\mathbf{c}) = \text{value}(\mathbf{c})$

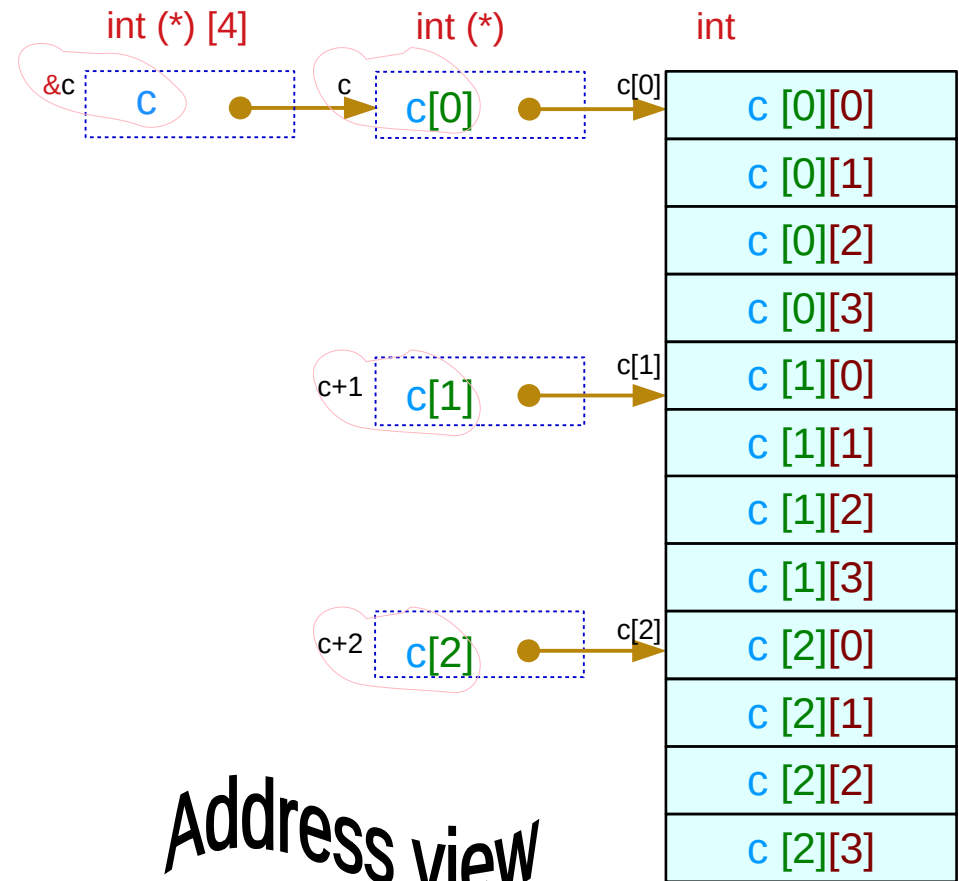
$\text{value}(\&\mathbf{c}[0]) = \text{value}(\mathbf{c}[0])$

$\text{value}(\&\mathbf{c}[1]) = \text{value}(\mathbf{c}[1])$

$\text{value}(\&\mathbf{c}[2]) = \text{value}(\mathbf{c}[2])$

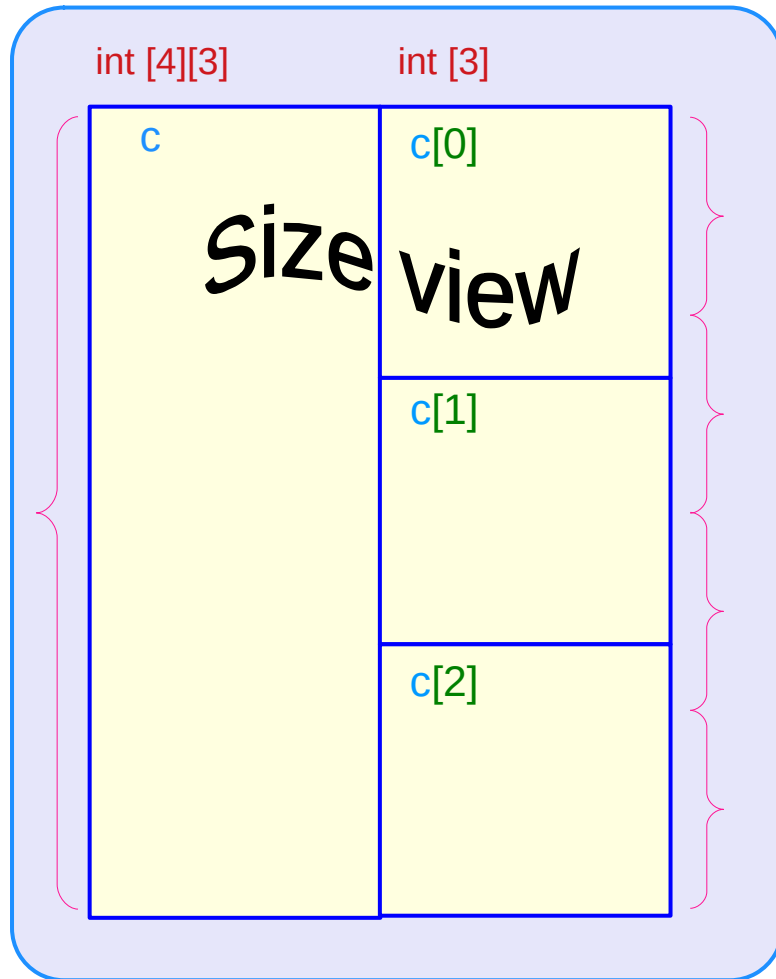
no real pointer can satisfy
these conditions

no physical memory location

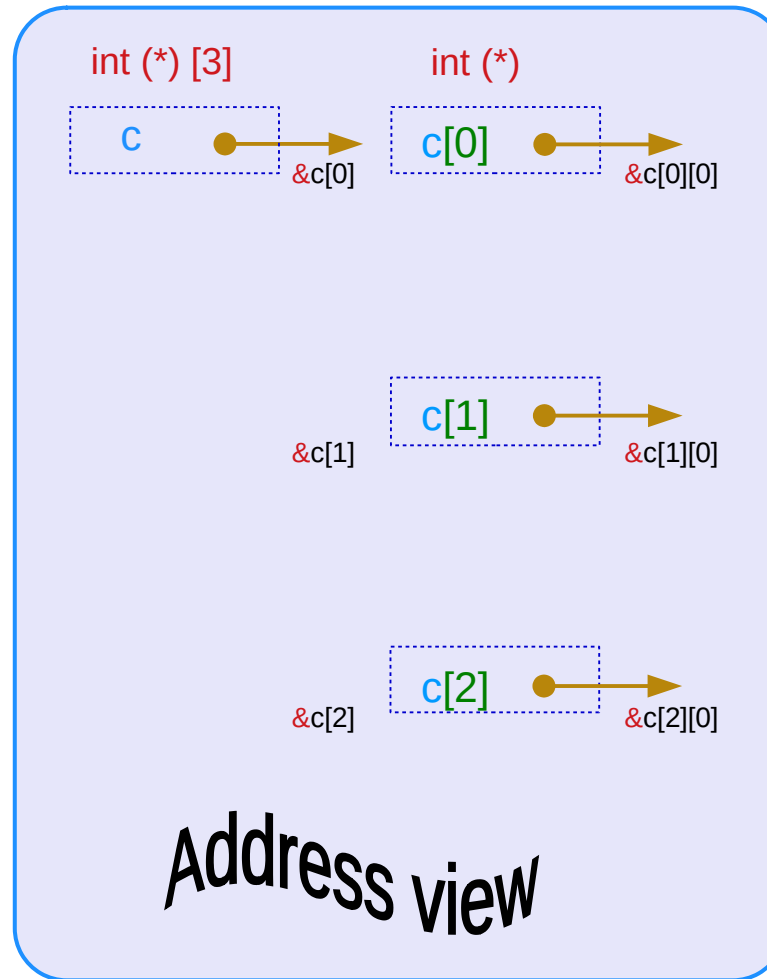


Address view

Size and address views of `c`, `c[i]`



$$\text{sizeof}(c) = \text{sizeof}(c[i]) * 3$$



$$\begin{aligned} \text{value}(c) &= \text{value}(c[0]) = \text{value}(\&c[0][0]) \\ \text{value}(c[1]) &= \text{value}(\&c[1][0]) \\ \text{value}(c[2]) &= \text{value}(\&c[2][0]) \end{aligned}$$

int
<code>c [0][0]</code>
<code>c [0][1]</code>
<code>c [0][2]</code>
<code>c [0][3]</code>
<code>c [1][0]</code>
<code>c [1][1]</code>
<code>c [1][2]</code>
<code>c [1][3]</code>
<code>c [2][0]</code>
<code>c [2][1]</code>
<code>c [2][2]</code>
<code>c [2][3]</code>

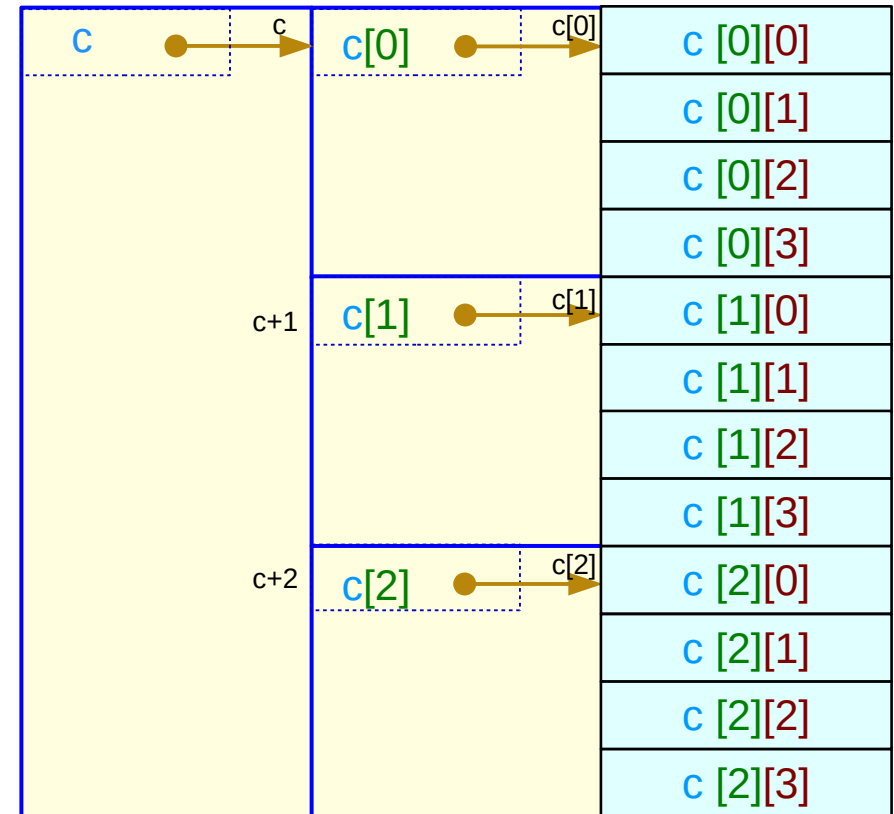
Combining size view and address view

Size view + Address view

$\text{sizeof}(c) = \text{sizeof}(c[i]) * 3$
 $\text{sizeof}(c[i]) = \text{sizeof}(c[0][0]) * 4$

$\text{value}(c) = \text{value}(c[0]) = \text{value}(\&c[0][0])$
 $\text{value}(c[1]) = \text{value}(\&c[1][0])$
 $\text{value}(c[2]) = \text{value}(\&c[2][0])$

Abstract data \longrightarrow $\text{int } [4][3]$ \longrightarrow $\text{int } [3]$ \longrightarrow int
Pointer \longrightarrow $\text{int } (*) [3]$ \longrightarrow $\text{int } (*)$



Determining types of sub-arrays

from the declaration of an array

Types of array names

```
int a[4];
```

a is the name of the **1-d** array

int [4]

`sizeof(a)` = 4 * 4

[3] is declared;
[0], [1], [2] are used

```
int c[3][4];
```

c[i] is the name of the **1-d** subarray

int [4]

`sizeof(c[i])` = 4 * 4

```
int c[3][4];
```

c is the name of the **2-d** array

int [3][4]

`sizeof(c)` = 3 * 4 * 4

Values of array names



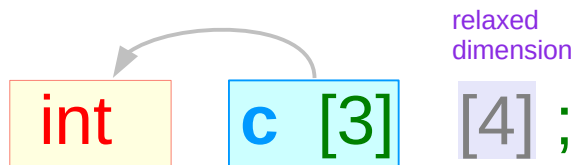
the value of **a** is the starting address of an array with 4 elements of **int** type

int (*)

a: pointer to the first element

a = &a[0]

[3] is declared;
[0], [1], [2] are used

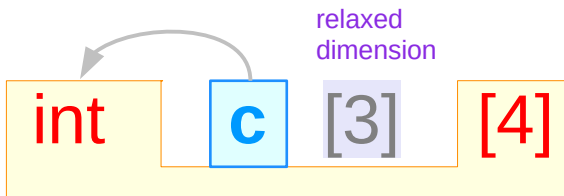


each value of **c[i]** is the starting address of an array with 4 elements of **int** type

int (*)

c[i]: pointer to the first element

c[i] = &c[i][0]



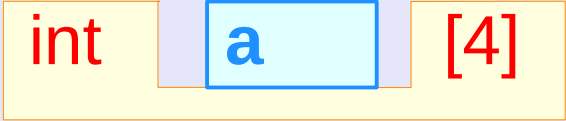
the value of **c** is the starting address of an array with 3 elements of **int [4]** type

int (*) [4]

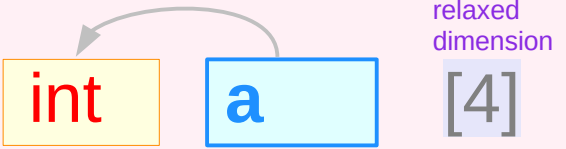
c: pointer to the first element

c = &c[0]

Array and pointer types in a 1-d array

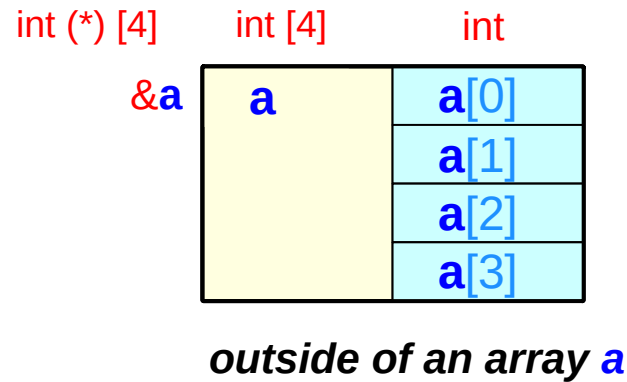


a 1-d array
type : `int [4]`
size : `4 * 4`



a 0-d array pointer
type : `int (*)`
value : `&a[0]`

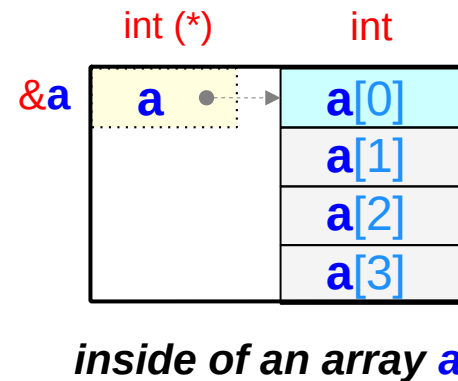
a points to the 1st `int` element
there are 4 `int` elements



`int (*) [4]` `int [4]` `int`

`&a` **a** `a[0]`
`a[1]`
`a[2]`
`a[3]`

outside of an array a

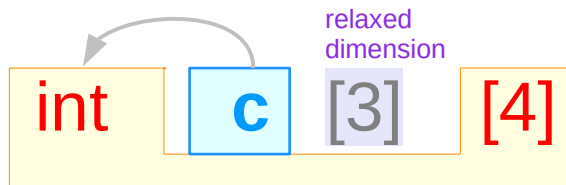
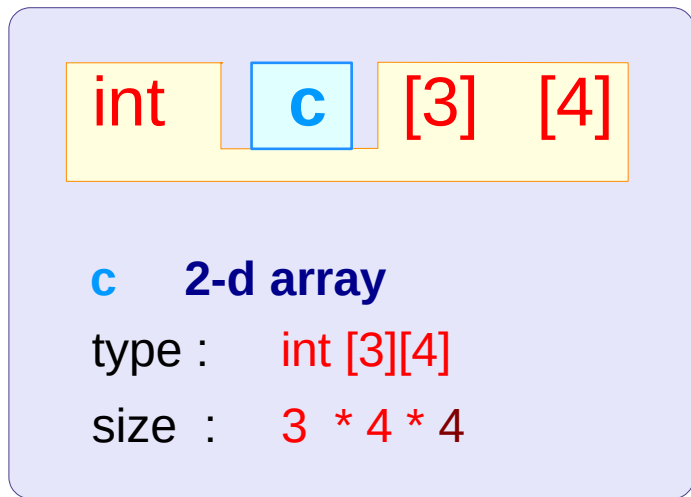


`int (*)` `int`

`&a` **a** `a[0]`
`a[1]`
`a[2]`
`a[3]`

inside of an array a

2-d array type



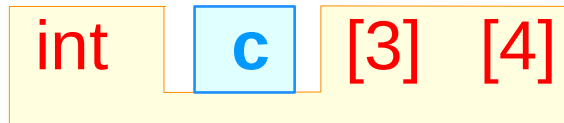
c 1-d array pointer
 type : `int (*) [4]`
 value : `c = &c[0][0]`

`int (*) [3][4]`
 &c

<code>int [3][4]</code>	<code>int [4]</code>	<code>int</code>
c	c[0]	<code>c[0][0]</code>
		<code>c[0][1]</code>
		<code>c[0][2]</code>
		<code>c[0][3]</code>
	c[1]	<code>c[1][0]</code>
		<code>c[1][1]</code>
		<code>c[1][2]</code>
		<code>c[1][3]</code>
	c[2]	<code>c[2][0]</code>
		<code>c[2][1]</code>
		<code>c[2][2]</code>
		<code>c[2][3]</code>

outside of an array **c**
 (c as an abstract data)

1-d array pointer type



c 2-d array

type : `int [3][4]`

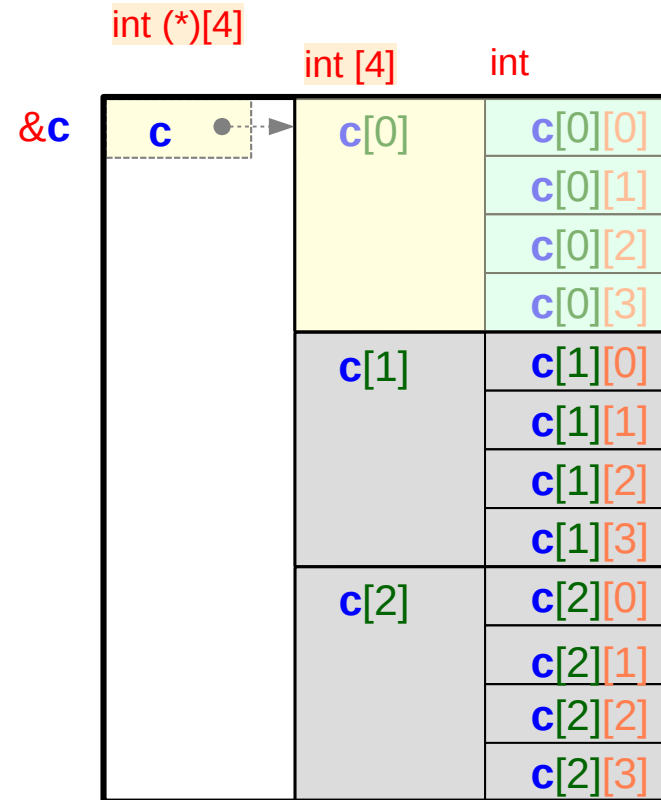
size : `3 * 4 * 4`

c 1-d array pointer

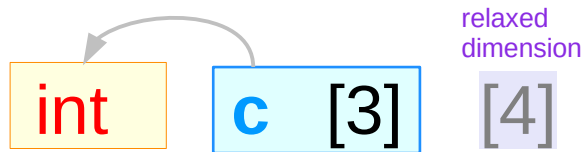
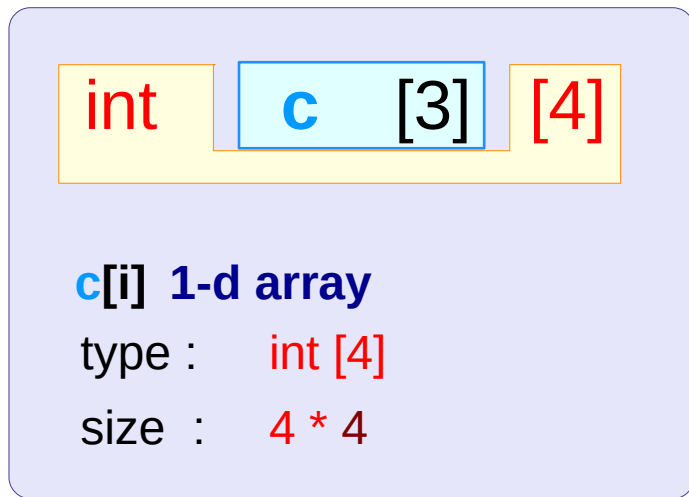
type : `int (*) [4]`

value : `c = &c[0][0]`

c points to the 1st `int [4]` element
There are 3 `int [4]` elements



1-d array type

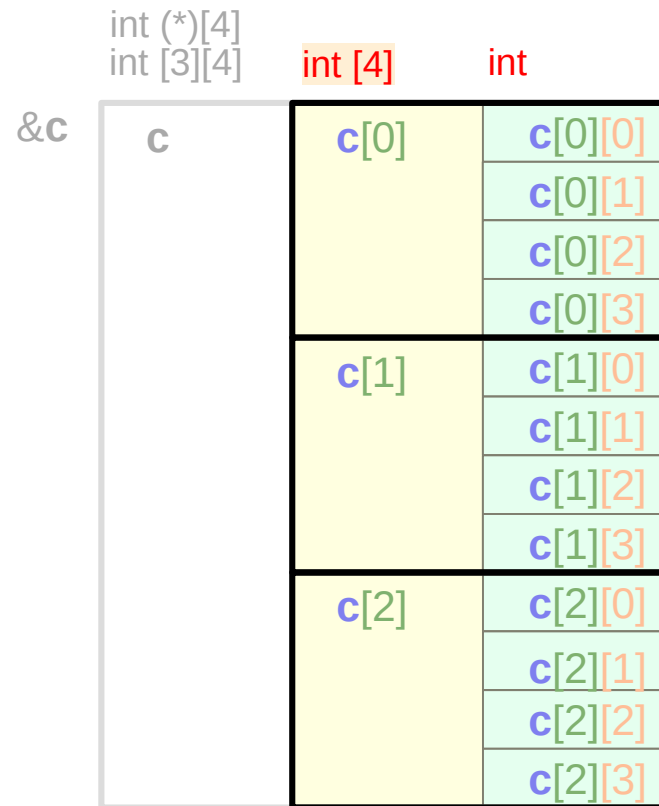


c[i] 0-d array pointer

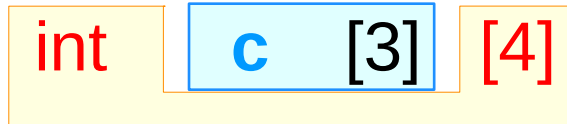
type : `int (*)`

value : `c[i] = &c[i][0]`

`c[i]` points to the 1st `int` element
There are 4 `int` elements



0-d array pointer type



c[i] 1-d array

type : int [4]

size : 4 * 4

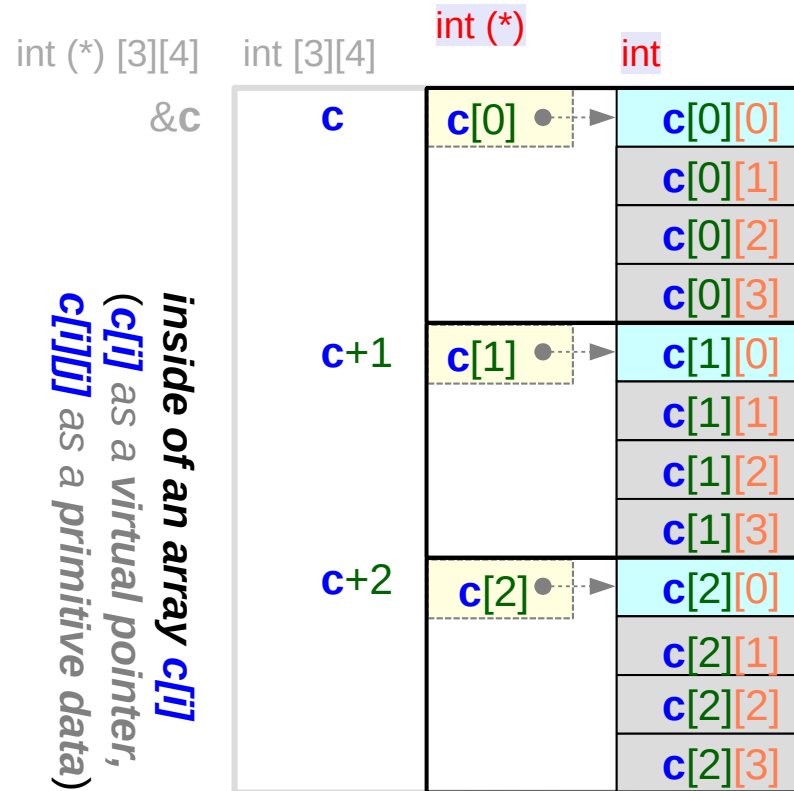
relaxed dimension

c[i] 0-d array pointer

type : int (*)

value : $c[i] = \&c[i][0]$

c[i] points to the 1st int element
There are 4 int elements



Types in a 2-d array

int c [3] [4]

c 2-d array

type : int [3][4]

size : 3 * 4 * 4

relaxing the 1st dimension

int c [3] [4]

c 1-d array pointer (virtual)

type : int (*) [4]

value : &c[0][0]

int c [3] [4]

c[i] 1-d array

type : int [4]

size : 4 * 4

relaxing the 1st dimension

int c [3] [4]

c[i] 0-d array pointer (virtual)

type : int (*)

value : &c[i][0]

Limitations

No index Range Checking

Array Size must be a constant expression

Variable Array Size

Arrays cannot be Copied or Compared

Aggregate Initialization and Global Arrays

Precedence Rule

Index Type Must be Integral

References

- [1] Essential C, Nick Parlante
- [2] Efficient C Programming, Mark A. Weiss
- [3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
- [4] C Language Express, I. K. Chun
- [5] <https://pdos.csail.mit.edu/6.828/2008/readings/pointers.pdf>