## DFT Frequency (4A)

- Angular Frequency
- Negative Frequency
- Fundamental Frequency
- Harmonic Frequency
- Sampling Frequency
- Normalized Frequency

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## Angular Frequency

$$
\text { Frequency } \quad f=\frac{1}{T} \quad(H z: \text { cycles per second })
$$

$1 \mathrm{~Hz} \rightarrow$ event repeats once per second

$$
\begin{aligned}
& \text { Angular } \\
& \text { Frequency }
\end{aligned} \quad \omega=\frac{2 \pi}{T} \quad \text { (radians per second) }
$$

One revolution $=2 \pi$ radian $\quad$ speed $=\frac{\text { displacement }}{\text { time }}$

$$
\begin{aligned}
\omega & =2 \pi f=2 \pi \frac{1}{T} \quad \\
\theta=\omega t & =2 \pi f t
\end{aligned} \quad \square \text { Angular Speed }
$$

## Negative Frequency

An angle can be measured either by a positive angle or a negative angle.
c.c.w $\theta_{1}$

$$
\begin{aligned}
& e^{+j \omega_{1} t}=\cos \left(+\omega_{1} t\right)+j \sin \left(+\omega_{1} t\right) \\
& e^{-j \omega_{2} t}=\cos \left(-\omega_{2} t\right)+j \sin \left(-\omega_{2} t\right)
\end{aligned}
$$

c.w $\quad \theta_{2}$

|  | c.c.w $(+)$ | Positive Angle | $\theta_{1}=\omega_{1} t$ | $\left(\theta_{1}>0\right)$ |
| :--- | :--- | :--- | :--- | :--- |
| Angle | c.w $(-)$ | Negative Angle | $\theta_{2}=\omega_{2} t$ | $\left(\theta_{2}<0\right)$ |
| Angular <br> Speed | c.c.w $(+)$ | Positive Frequency | $\omega_{1}=+\frac{2 \pi}{T}$ | $\left(\omega_{1}>0\right)$ |
|  | c.w $(-)$ | Negative Frequency | $\omega_{2}=-\frac{2 \pi}{T}$ | $\left(\omega_{2}<0\right)$ |

## DFT Matrix

$W_{8}^{0} \quad W_{8}^{0} \quad W_{8}^{0} \quad W_{8}^{0} \quad W_{8}^{0} \quad W_{8}^{0} \quad W_{8}^{0} \quad W_{8}^{0}$ $W_{8}^{0} \quad W_{8}^{1} \quad W_{8}^{2} \quad W_{8}^{3} \quad W_{8}^{4} \quad W_{8}^{5} \quad W_{8}^{6} \quad W_{8}^{7}$ $W_{8}^{0} \quad W_{8}^{2} \quad W_{8}^{4} \quad W_{8}^{6} \quad W_{8}^{0} \quad W_{8}^{2} \quad W_{8}^{4} \quad W_{8}^{6}$ $W_{8}^{0} \quad W_{8}^{3} \quad W_{8}^{6} \quad W_{8}^{1} \quad W_{8}^{4} \quad W_{8}^{7} \quad W_{8}^{2} \quad W_{8}^{5}$ $W_{8}^{0} \quad W_{8}^{4} \quad W_{8}^{0} \quad W_{8}^{4} \quad W_{8}^{0} \quad W_{8}^{4} \quad W_{8}^{0} \quad W_{8}^{4}$ $W_{8}^{0} \quad W_{8}^{-3} \quad W_{8}^{-6} \quad W_{8}^{-1} \quad W_{8}^{-4} \quad W_{8}^{-7} \quad W_{8}^{-2} W_{8}^{-5}$ $W_{8}^{0} \quad W_{8}^{-2} \quad W_{8}^{-4} \quad W_{8}^{-6} \quad W_{8}^{0} \quad W_{8}^{-2} W_{8}^{-4} W_{8}^{-6}$
$W_{8}^{0} \quad W_{8}^{-1} W_{8}^{-2} W_{8}^{-3} W_{8}^{-4} W_{8}^{-5} W_{8}^{-6} W_{8}^{-7}$

$$
W_{N}^{n k \pm N}=W_{N}^{n k}
$$

$$
W_{8}^{n k}=e^{-j\left(\frac{2 \pi}{8}\right) n k}
$$

* still symmetric matrix
c. w (-)
c.c. ${ }^{(+)}$

Exponents and Strides
$n=0 \quad n=1 \quad n=2 \quad n=3 \quad n=4 \quad n=5 \quad n=6 \quad n=7$


 $\mathbf{k}=3 \quad 0 \quad-3^{-3}-3^{-6}-3^{-1}-3^{-4}-3^{-7}-3^{-2}-3^{-5}$ $\mathbf{k}=4 \quad 0 \quad-4{ }^{-4}-4{ }^{0}$ $\mathbf{k}=5 \quad 0 \quad+3^{+3}+3{ }^{+6}+3{ }^{+1}+3{ }^{+4}+3+7+3+2+3^{+5}$

$$
\mathbf{k}=6 \quad 0 \quad+2+2+2{ }^{+4}+2{ }^{+6}+20^{0}+2+24^{+4}+2+6
$$

$$
\mathbf{k}=\mathbf{0} \quad 0_{+1}^{+1}+1{ }^{+2}+1{ }^{+3}+{ }^{+4}+1{ }^{+5}+1+6+1
$$

## Fundamental and Harmonic Frequencies

Exponents and Strides of DFT Matrix

|  |  |
| :---: | :---: |
| 0 |  |
| 1 |  |
| $=2$ | $02^{-2}-2{ }^{-4}-22^{-6}-2{ }^{0}-2{ }^{-2}-2{ }^{-4}-2{ }^{-6}$ |
| k=3 | $03^{-3}-3{ }^{-6}-38^{-1}-34^{-4}-37^{-7}-3{ }^{-2}-3{ }^{-5}$ |
| $\mathrm{k}=4$ |  |
| =5 | $0{ }^{0} 3^{+3}+3{ }^{+6}+3{ }^{+1}+3{ }^{+4}+3 \stackrel{+7}{+3}+{ }^{+2}+3{ }^{+5}$ |
| =6 | $\left.0{ }^{0} 2^{+2}+2^{+4}+2^{+6}+2\right)^{0}+2+2{ }^{+2}+4+2{ }^{+6}$ |
| k=7 | ${ }_{+1}{ }^{+1}+1{ }^{+2}+1{ }^{+3}+1{ }^{+4}+1 \stackrel{+5}{+1}{ }^{+6}+1+7$ |


| stride | angular speed |  |  | Measuring Frequency | Harmonic <br> Frequency |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | cw | 0 | - | 0 |  |
| -1 | cw | $-1 \omega$ | $\square$ | +1f | Fundamental $\rightarrow$ $1^{\text {st }}$ harmonic |
| -2 | cw | $-2 \omega$ |  | +2f | $2^{\text {nd }}$ harmonic |
| -3 | cw | -3 $\omega$ |  | +3f | $3{ }^{\text {rd }}$ harmonic |
| -4 | CW | $-4 \omega$ | $\square$ | +4f | $4^{\text {th }}$ harmonic |
| +3 | CCW | +3 $\omega$ | $\square$ | -3f | $5^{\text {th }}$ harmonic |
| +2 | ccw | $+2 \omega$ | - | -2f | $6{ }^{\text {th }}$ harmonic |
| +1 | ccw | +1 $\omega$ | - | -1f | $7^{\text {th }}$ harmonic |



$$
T=N \top
$$

Fundamental
Frequency

$$
f_{0}=\frac{1}{T}
$$

Harmonic
Frequency

$$
f_{k}=k \cdot f_{0} \quad(k=1,2, \cdots)
$$

## Sampling Frequency



$$
\begin{array}{ll}
\text { Sampling Time } & \tau \\
\text { Sampling Frequency } & f_{s}=\frac{1}{\tau} \quad \text { (samples per second) } \\
\text { Period } & T=N \top \\
\text { Fundamental Freq } & f=\frac{1}{T} \quad \text { (cycles per second) } \\
& f=\frac{f_{s}}{N} \quad\left(=\frac{1}{N_{\mathrm{T}}}\right)
\end{array}
$$

Sinusoid with Fundamental Frequency


## Normalized Frequency

Sampling Time $\tau$
$\longrightarrow$ T


Sampling Frequency

$$
f_{s}=\frac{1}{\tau}
$$

(samples per second)
$\nabla T=N_{\top}$

Normalized Frequency

$$
\frac{f_{n}}{f_{s}}=\frac{n}{N} \quad \text { (cycles per sample) }
$$

Fundamental
Frequency

$$
f_{0}=\frac{1}{T}=\frac{1}{N \tau}
$$

$$
f_{0}=\frac{f_{s}}{N}
$$

$$
\frac{1}{N}
$$

Harmonic
Frequencies

$$
\begin{aligned}
f_{1} & =1 \cdot f_{0}=\frac{1}{N} f_{s} \\
f_{2} & =2 \cdot f_{0}=\frac{2}{N} f_{s} \\
f_{3} & =3 \cdot f_{0}=\frac{3}{N} f_{s} \\
& \cdots \\
f_{N-1} & =(N-1) \cdot f_{0}=\frac{(N-1)}{N} f_{s}
\end{aligned}
$$

| $\frac{1}{N}$ |
| :---: |
| $\frac{2}{N}$ |
| $\frac{3}{N}$ |
| $\cdots$ |
| $\frac{(N-1)}{N}$ |

$$
\frac{2}{N}
$$

$$
\frac{3}{N}
$$

Normalized
Frequencies

## References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann

