Formelsammlung Statistik

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Inhaltsverzeichnis

1 Grundlagen

1.1 Mengenlehre und DE MORGANsche Regeln

$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$

und

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$$

1.2 Kombinatorik

1.2.1 Fakultät

$n! = 1 \cdot 2 \cdot \ldots \cdot (n-1) \cdot n$

$0! = 1$

1.2.2 Binomialkoeffizient

$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \quad k, n \in N, \quad k, n \geq 0.$

$$\binom{n}{0} = 1.$$

1.2.3 Zufallsstichproben

Anzahl der möglichen Stichproben vom Umfang n aus einer Grundgesamtheit vom Umfang N:

	Ohne Zurücklegen	Mit Zurücklegen
Mit	$\frac{N!}{(N-n)!}$	N^n
Ohne Berücksichtigung der Reihenfolge	$\binom{N}{n}$	$\binom{N+n-1}{n}$

1.3 Definition der Wahrscheinlichkeit

(Symmetrieprinzip oder **Prinzip nach LAPLACE**) Jedes Ergebnis A aus der Ergebnismenge Ω sei gleich häufig. |A| ist die Zahl der Ergebnisse,

die durch A belegt werden (Anzahl der günstigen Ergebnisse), $|\Omega|$ ist die Zahl aller möglichen Ergebnisse. Es ist

$P(A) = \frac{|A|}{|\Omega|}$.

Axiome der Wahrscheinlichkeiten (Kolmogoroff): Gegeben sind zwei Ereignisse A,B $\subset \Omega$.

1. $P(A) \geq 0$. **Nichtnegativität**
2. $P(\Omega) = 1$. **Normiertheit**
3. $P(A \cup B) = P(A) + P(B)$, falls A und B disjunkt sind.

1.4 Additionssatz

Für zwei Ereignisse A, B aus Ω gilt :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Für drei Ereignisse A, B, C aus Ω gilt analog :

$$\begin{aligned} P(A \cup B \cup C) = \quad & P(A) + P(B) + P(C) - P(A \cap B) \\ & - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C). \end{aligned}$$

Falls die Ereignisse **disjunkt** sind, gilt

$$P(A \cup B) = P(A) + P(B).$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C).$$

1.4.1 Bedingte Wahrscheinlichkeit

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

1.4.2 Unabhängigkeit von Ereignissen

Ein Ereignis A ist unabhängig von B, wenn

$$P(A|B) = P(A|\overline{B}) = P(A)$$

1.4.3 Totale Wahrscheinlichkeit

Sei $A_1 \ldots A_k$ eine disjunkte Zerlegung von Ω. Dann gilt für B ⊂ Ω: $P(B) = \sum_{i=1}^{k} P(B|A_i) \cdot P(A_i)$.

1.5 BAYES Theorem

Für zwei Ereignisse A und B mit $P(B) > 0$ lässt sich die Wahrscheinlichkeit von A unter der Bedingung, dass B eingetreten ist, angeben durch die Wahrscheinlichkeit von B unter der Bedingung, dass A eingetreten ist:

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

Hierbei ist

$P(A \mid B)$ die (bedingte) Wahrscheinlichkeit des Ereignisses A unter der Bedingung, dass B eingetreten ist,

$P(B \mid A)$ die (bedingte) Wahrscheinlichkeit des Ereignisses B unter der Bedingung, dass A eingetreten ist,

$P(A)$ die A-priori-Wahrscheinlichkeit des Ereignisses A und

$P(B)$ die A-priori-Wahrscheinlichkeit des Ereignisses B.

Endlich viele Ereignisse:

Wenn $A_i, i = 1, \ldots, N$ eine Zerlegung der Ergebnismenge in disjunkte Ereignisse ist, gilt für die A-posteriori-Wahrscheinlichkeit $P(A_i \mid B)$

$$P(A_i \mid B) = \frac{P(B \mid A_i) \cdot P(A_i)}{P(B)} = \frac{P(B \mid A_i) \cdot P(A_i)}{\sum\limits_{j=1}^{N} P(B \mid A_j) \cdot P(A_j)}$$

Den letzten Umformungsschritt bezeichnet man auch als Marginalisierung.

Da ein Ereignis A und sein Komplement $\overline{A}$ stets eine Zerlegung der Ergebnismenge darstellen, gilt insbesondere

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B \mid A) \cdot P(A) + P(B \mid \overline{A}) \cdot P(\overline{A})}$$

2 Zufallsvariablen und Verteilungsmodelle

2.1 diskrete Zufallsvariablen

Ein Merkmal X, das aufgrund zufälliger Ereignisse eine (endliche) Menge von Ausprägungen x_1, x_2 ... annehmen kann, nennt man diskrete Zufallsvariable X.

2.1.1 Eindimensionale Zufallsvariablen

Wahrscheinlichkeitsfunktion:

$$f(x) = \begin{cases} P(X = x_i) = p_i, & x = x_i \in \{x_1, x_2, ..., x_k..\} \\ 0 & sonst \end{cases}$$

Verteilungsfunktion:

$$F(x) = P(X \leq x) = \sum_{i:x_i \leq x} f(x_i).$$

Normiertheit:

$$\sum_{i=1}^{k} p_i = 1.$$

Erwartungswert

$$E(X) = \mu = \sum_{i=1}^{k} x_i \cdot p_i = \sum_{i=1}^{k} x_i \cdot f(x_i),$$

Varianz

$$Var(X) = \sum_{i=1}^{k} (x_i - E(X))^2 \cdot f(x_i).$$

bzw. mit dem Verschiebungssatz

$$Var(X) = \left(\sum_{i=1}^{k} x_i^2 \cdot f(x_i)\right) - E(X^2) =$$

Standardabweichung

$$\sigma = +\sqrt{Var(X)}.$$

Varianz der Summe unabhängiger Zufallsvariablen

$$Var(X+Y) = Var(X) + Var(Y).$$

2.1.2 Mehrdimensionale Zufallsvariablen

Einzelwahrscheinlichkeit

$$P(X = x_1) = f_X(x_1) = \sum_{j=1}^{m} f_{X,Y}(x_1; y_j)$$

Kovarianz

$$covXY = \sum_{i=1}^{n}\sum_{j=1}^{m}(x_i - E(X))(y_j - E(Y)) f_{X,Y}(x_i; y_j)$$

bzw. mit dem Verschiebungssatz

$$covXY = \sum_{i=1}^{n}\sum_{j=1}^{m} x_i \cdot y_j \cdot f_{X,Y}(x_i; y_j) - E(X) \cdot E(Y)$$

Korrelationskoeffizient $\mathbf{r_{xy}}$ nach Bravais-Pearson

für metrisch skalierte Merkmale zweier statistischer Variablen x und y

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2} \cdot \sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}},$$

mit $\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$ als dem arithmetischen Mittel des Merkmals x. Mit Hilfe des Verschiebungssatzes:

$$r = \frac{\sum_{i=1}^{n} x_i \cdot y_i - n \cdot \bar{x} \cdot \bar{y}}{\sqrt{(\sum_{i=1}^{n} x_i^2 - n \cdot (\bar{x})^2) \cdot (\sum_{i=1}^{n} y_i^2 - n \cdot (\bar{y})^2)}}$$

Rangkorrelationskoeffizient nach Spearman

- für Variablen, die stark von der Normalverteilung abweichen
- sowie ordinalskalierte Variablen

Nach Ordnung der einzelnen Beobachtungen von x bzw. y der Größe nach wird

jedem Wert wird seine Rangzahl $rg(x_i)$ und $rg(y_i)$ zugewiesen. Damit:

$$r_{SP} = \frac{\sum_i (rg(x_i) - \overline{rg(x)})(rg(y_i) - \overline{rg(y)})}{\sqrt{\sum_i (rg(x_i) - \overline{rg(x)})^2} \sqrt{\sum_i (rg(y_i) - \overline{rg(y)})^2}}$$

2.2 diskrete Verteilungsmodelle

2.2.1 Binomialverteiung

Für eine binomialverteilte Zufallsvariable X mit den Parametern n und θ ($0 \leq \theta \leq 1$) lautet die Wahrscheinlichkeitsfunktion

$$P(X = x) = b(x|n;\theta) = \begin{cases} \binom{n}{x} \theta^x (1-\theta)^{n-x} & \text{falls } x = 0, 1, \ldots, n \\ 0 & \text{sonst.} \end{cases}$$

Erwartungswert

$$E(X) = n \cdot \theta$$

Varianz

$$Var(X) = n \cdot \theta \cdot (1 - \theta)$$

2.2.2 Hypergeometrische Verteilung

Eine Zufallsvariable X ist **hypergeometrisch** verteilt mit den Parametern

N (Grundgesamtheit), M ("Kugeln der ersten Sorte") und n (Stichprobenumfang),

wenn ihre Wahrscheinlichkeitsfunktion lautet

$$P(X = x) = h(x|N;M;n) = \begin{cases} \frac{\binom{M}{x} \cdot \binom{N-M}{n-x}}{\binom{N}{n}} & \text{für x = 0, 1, ... , n} \\ 0 & \text{sonst} \end{cases}$$

Erwartungswert

$$E(X) = n \cdot \frac{M}{N} = n \cdot \Theta$$

Varianz

$Var(X) = n \cdot \frac{M}{N} \cdot \left(1 - \frac{M}{N}\right) \frac{N-n}{N-1}$.

Der Bruch $\frac{N-n}{N-1}$ wird *Korrekturfaktor* genannt.

2.2.3 Poissonverteilung

Wahrscheinlichkeitsfunktion ($\lambda > 0$)

$$P(X = x) = p(x|\lambda) = \begin{cases} \frac{e^{-\lambda} \cdot \lambda^x}{x!} & \text{für x = 0, 1, ...} \\ 0 & \text{sonst} \end{cases}$$

Erwartungswert und **Varianz**

$E(X) = Var(X) = \lambda$

2.3 stetige Zufallsvariablen

Eine stetige Zufallsvariable kann in jedem beschränkten Intervall unendlich viele Ausprägungen annehmen.

Ihre Verteilung lässt sich durch eine Dichtefunktion f(x) beschreiben.

(f(x) ist hier keine Wahrscheinlichkeit, sondern eine Dichte !)

Verteilungsfunktion

$$P(X \leq a) = F(a) = \int_{-\infty}^{a} f(x)dx$$

- Es gilt: P(X = a) = 0.
- Wegen P(X = a) = 0 ist P(X ≤ a) = P(X < a) und P(X > a) = P(X ≥ a)

Die **Dichtefunktion f(x)** ist die erste Ableitung der Verteilungsfunktion, falls diese an der Stelle x differenzierbar ist.

- Die Dichtefunktion f(a) kann auch größer als 1 werden.
- Ausgehend von $P(X \leq x) = p$ ist das p-Quantil x(p) der Wert x, der zu einer gegebenen Wahrscheinlichkeit p gehört. Speziell x(0,5) ist der Median.

Erwartungswert

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx,$$

falls E(X) existiert, d.h. nicht unendlich wird.

Varianz

$$Var(X) = \int_{-\infty}^{\infty} (x - E(X))^2 \cdot f(x)dx$$

wobei auch hier der Verschiebungssatz angewendet werden kann:

$$Var(X) = \left(\int_{-\infty}^{\infty} x^2 f(x)dx \right) - (E(X))^2$$

2.4 stetige Verteilungsmodelle

2.4.1 Stetige Gleichverteilung (Rechteckverteilung)

Dichtefunktion der Gleichverteilung im Intervall [a,b]

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{für } a \leq x \leq b \\ 0 & \text{sonst.} \end{cases}$$

Erwartungswert

$$E(X) = \frac{a+b}{2}$$

Varianz

$$Var(X) = \lambda \int_{a}^{b} x^2 \cdot \frac{1}{b-a} dx = \frac{(b-a)^2}{12}$$

2.4.2 Exponentialverteilung

Dichtefunktion der Exponentialverteilung

$$f(x) = \begin{cases} \lambda \cdot e^{-\lambda x} & \text{für } x \geq 0 \\ 0 & \text{für } x < 0 \end{cases}$$

Erwartungswert

$$E(X) = \lambda \int\limits_0^\infty x \cdot e^{-\lambda x} dx = \frac{1}{\lambda}$$

Varianz

$$Var(X) = \frac{1}{\lambda^2}$$

2.4.3 Normalverteilung

Für eine Zufallsvariable $X \propto N(\mu, \sigma^2)$ lautet die **Dichtefunktion der NV**

$f(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ für $x \in \mathbb{R}$

Normierung mit $z = \frac{x-\mu}{\sigma}$ ergibt die **Standardnormalverteilung** mit der Dichtefunktion $\phi_x(z) \propto N(0,1)$:

$$\phi_x(z) = \frac{1}{\sqrt{2 \cdot \pi}} \cdot e^{-\frac{1}{2}z^2}$$

Anm.:Es wird auch die Schreibweise $\phi_x(z|\mu, \sigma^2)$ anstelle $N(\mu, \sigma^2)$ verwendet

Erwartungswert

$$E(X) = \mu$$

Varianz

$$Var(X) = \sigma^2$$

p-Quantil

Der zu einer gegebenen Wahrscheinlichkeit p zugehörige z-Wert z(p)

$$P(Z \leq z(p)) = p$$

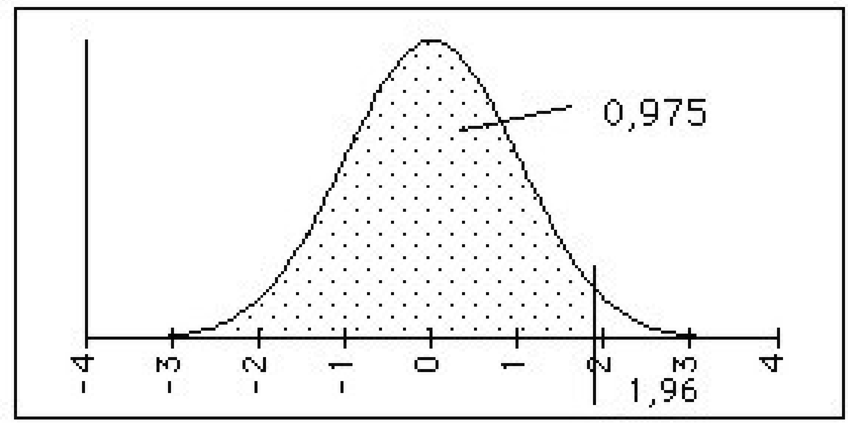

Abb. 1 97,5%-Quantil der Standardnormalverteilung

Beispielsweise ist z(0,975) = 1,96.

Linearkombinationen normalverteilter Zufallsvariablen

Für n normalverteilte Zufallsvariablen $X_i (i = 1,...,n)$, mit $X_i \propto N(\mu_i; \sigma_i^2)$ ist die Linearkombination

$$Y = a_0 + a_1 X_1 + a_2 X_2 + ... + a_n X_n = a_0 + \sum_{i=1}^{n} a_i X_i$$

ebenfalls normalverteilt mit dem Erwartungswert

$$E(Y) = a_0 + \sum_{i=1}^{n} a_i E(X_i) = a_0 + \sum_{i=1}^{n} a_i \mu_i$$

Falls die X_i $(i = 1,...,n)$ stochastisch unabhängig sind, gilt für die Varianz

$$Var(Y) = \sum_{i=1}^{n} a_i^2 \cdot (X_i) = \sum_{i=1}^{n} a_i^2 \sigma_i^2$$

Die Varianz muss größer Null sein, deshalb muss zudem $a_j \neq 0$ für mindestens ein $j \in \{1, \ldots, n\}$ gelten.

Verteilung des Stichprobendurchschnitts

Sind die n Zufallsvariablen X_i (i = 1, ... , n) sämtlich normalverteilt mit gleichem μ und gleichem σ^2, ist die Linearkombination

X1pt‾ mit $a_0 = 0$, $a_1 = a_2 = ... = a_n = 1/n$, also :$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$

normalverteilt dem **Erwartungswert**

$$E(\bar{X}) = \frac{1}{n}\sum_{i=1}^{n}\mu = \mu$$

und, falls die Xi (i = 1, ... , n) stochastisch unabhängig sind, mit der **Varianz**

$$Var(\bar{X}) = \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2 = \frac{\sigma^2}{n}$$

2.4.4 CHI-Quadrat-verteilung

Die $X_1, X_2, ...X_n$ seien unabhängige standardnormalverteilte Zufallsvariablen.

Dann ist die Verteilung der Zufalllsvariablen $Z = X_1^2 + X_2^2 + ... + X_n^2$

chi-quadrat verteilt mit n Freiheitsgraden $Z \propto \chi^2(n)$

Erwartungswert:

$$E(Z) = n$$

Varianz

$$Var(Z) = 2n$$

Anm.: Die Gruppe der Hypothesentests mit χ^2-Verteilung bezeichnet man als χ^2**-Test**.

Hierunter sind mehrere Tests zu verstehen:

Verteilungstest oder Anpassungstest: Hier wird geprüft, ob vorliegende Daten auf eine bestimmte Weise verteilt sind.

Unabhängigkeitstest: Hier wird geprüft, ob zwei Merkmale stochastisch unabhängig sind.

Homogenitätstest: Hier wird geprüft, ob zwei oder mehr Stichproben derselben Verteilung bzw. einer homogenen Grundgesamtheit entstammen.

2.4.5 t- (Student-) Verteilung

Für die unabhängigen Variablen X (standardnormalverteilt) und $Z(Z \propto \chi^2(n))$ ist die Variable

$$T = \frac{X}{\sqrt{Z/n}}$$

t-verteilt ($T \propto t(n)$) mit n Freiheitsgraden.

Erwartungswert

$E(T) = 0$ für $(m \geq 2)$

Varianz

$Var(T) = \frac{n}{n-2}$ für $(n \geq 3)$

2.4.6 Fisher- Verteilung

Für die unabhängigen Variablen $X \propto \chi^2(m)$ und $Y \propto \chi^2(n)$ ist die Verteilung der Variablen

$$Z = \frac{X/m}{Y/n}$$

Fisher- oder F-verteilt ($Z \propto F(m,n)$) mit den Freiheitsgraden m und n.

Erwartungswert

$E(T) = \frac{n}{n-2}$ für $(n \geq 3)$

Varianz

$Var(Z) = \frac{2n^2(n+m-2)}{m(n-4)(n-2)^2}$ für $(n \geq 3)$

2.5 Approximation von Verteilungen

Gesuchte Verteilung	**Approximation durch**		
$P(X \leq x)$	Binomial	Poisson	Normal
Binomial $B(x\|n\theta) \approx$	---	$P(x\|n\theta)$ falls $n \geq 50$ und $\theta \leq 0,05$	$\Phi(x+0,5\|n \cdot \theta; n \cdot \theta \cdot (1-\theta))$ falls $n > \frac{9}{\theta(1-\theta)}$
Hypergeometrische $H(x\|N;M;n) \approx$	$B(x\|n\frac{M}{N})$ falls $\frac{n}{N} < 0,05$	über Binomialverteilung	$\Phi(x+0,5\|n \cdot \frac{M}{N}; n \cdot \frac{M}{N} \cdot (1-\frac{M}{N}) \cdot \frac{N-n}{N-1}$ falls $n > \frac{9}{\frac{M}{N} \cdot (1-\frac{M}{N})}$ und $\frac{n}{N} < 0,05$
Poisson $P(x\|\lambda) \approx$	---	---	$\Phi(x+0,5\|\lambda;\lambda)$ falls $\lambda > 9$
χ^2-Verteilung $\chi^2(x\|n) \rightarrow P(\sqrt{2X} \leq \sqrt{2x}) \approx$	---	---	$\Phi(\sqrt{2x}\|\sqrt{2n-1};1)$ falls $n > 30$
t-Verteilung $t(x\|n) \approx$	---	---	$\Phi(x\|0;1)$ falls $n > 30$
F-Verteilung $F(x\|m;n) \approx$	---	---	$\Phi(x\|0;1)$ falls $m > 30$ und $n > 30$

2.6 Grenzwertsatz

2.6.1 Gesetz der großen Zahlen

Für ein beliebig kleines c > 0 gilt

$P(|\bar{X}_n - \mu| \leq c) \rightarrow 1$ für $(n \rightarrow \infty)$

2.6.2 Theorem von Bernoulli

Die relative Häufigkeit, mit der ein Ereignis A bei n unabhängigen Wiederholungen

eines Zufallsereignisses eintritt, konvergiert nach Wahrscheinlichkeit gegen P(A)

2.6.3 Hauptsatz der Statistik

Für eine Zufallsvariable X mit der Verteilungsfunktion F(x) gilt für die Verteilungsfunktion $F_n(x)$

für die unabhängigen wie identisch wie X verteilten $X_1 \ldots X_n$ (x∈ R)

$$P(sup|F_n(x) - F(x)| \leq c) \rightarrow 1 \text{ für } (n \rightarrow \infty)$$

(sup: Maximale Abweichung zwischen $\hat{F}_n(x)$ und $\hat{F}(x)$).

2.6.4 Zentraler Grenzwertsatz

Für unabhängig identisch verteilte Zufallsvariablen $X_1 \ldots X_n$ mit $E(X_i) = \mu$

und $Var(X_i) = \sigma^2 > 0$ konvergiert die Verteilungsfunktion $F_n(z) = P(Z_n \leq z)$

der standardisierten Summe

$Z_n = \frac{X_1 + .. + X_n - n \cdot \mu}{\sqrt{n}\sigma} = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{X_i - \mu}{\sigma}$

für n $\rightarrow \infty$ an jeder Stelle $z \in \mathbb{R}$ gegen die Verteilungsfunktion $\phi_x(z)$ der Standardnormalverteilung

$F_n(z) \Rightarrow \phi_x(z)$

2.6.5 Grenzwertsatz von De Moivre

Die Verteilung der standardisierten absoluten Häufigkeit $\frac{H_n - n \cdot \pi}{\sqrt{n\pi(1-\pi)}}$ der Standardnormalverteilung

konvergiert für n $\rightarrow \infty$ gegen eine Standardnormalverteilung.

3 Deskriptive Statistik

3.1 Skalenniveaus

3.1.1 Nominalskala

Die Ausprägungen des nominalskalierten Merkmals können nicht **geordnet** werden,

man kann sie nur **vergleichen** und **abzählen**.

Es handelt sich um **qualitative** Merkmale. Erhalten die Ausprägungen Ziffern zugeordnet,

handelt es sich nur um eine **Verschlüsselung** (Codierung): 1 = männlich, 2 = weiblich.

3.1.2 Ordinalskala

Zwischen den Ausprägungen des ordinalskalierten (rangskalierten) Merkmals existiert eine Beziehung

der Form mehr oder weniger, $<$ oder $>$, besser oder schlechter.

Eine Quotientenbildung macht wenig Sinn (Beispiel Noten: 1, 2, 3, 4, 5).

3.1.3 Intervallskala

Die Abstände zwischen den Ausprägungen des (quantitativen) Merkmals der Intervallskala

können gemessen werden. Es handelt sich bei den Ausprägungen um (reelle) Zahlen.

Beispiel: Kinderzahl, Temperatur.

3.1.4 Verhältnissskala

Sowohl die Abstände als auch Verhältnisse zwischen den Ausprägungen des (quantitativen) Merkmals

können gemessen werden. Es handelt sich bei den Ausprägungen um (reelle) Zahlen. Beispiel: Einkommen.

3.1.5 Zweig-Blätter-(stem-leaf-) Diagramm

Die linke Spalte enthält als „Stämme“ die Äquivalenzklassen, in die die auf der rechten Seite als „Blätter“

dargestellten Merkmale eingeteilt werden. Beispiel: Gegeben sind die Werte 0,3 0,4 2,5 2,5 2,6 2,7 2,8 3,5 3,7.

Wählt man die natürlichen Zahlen als Klasseneinteilung, ergibt sich folgendes Stamm-Blatt-Diagramm:

3	5	7			
2	5	5	6	7	8
1					
0	3	4			

3.2 Lageparameter

3.2.1 Arithmetisches Mittel

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

3.2.2 Median (Zentralwert)

Sind die Beobachtungswerte der Größe nach geordnet, ist der Median z die Stelle, die die Teilgesamtheit in zwei gleiche Hälften teilt.

$$z = \begin{cases} x_{[\frac{n+1}{2}]} & \text{n ungerade} \\ \frac{1}{2}(x_{[\frac{n}{2}]} + x_{[\frac{n}{2}+1]}) & \text{n gerade} \end{cases}$$

3.2.3 geometrisches Mittel

$$\bar{X}_{geom} = \sqrt[n]{\prod_{i=1}^{n} x_i}$$

3.2.4 harmonisches Mittel

$$\bar{X}_{harm} = \frac{n}{\sum\limits_{i=1}^{n} \frac{1}{x_i}}$$

3.2.5 Modalwert

Der am häufigsten auftretende Wert

3.2.6 Varianz

Grundgesamtheit:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

Stichprobe:

$$\bar{s}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{X})^2$$

3.2.7 Verschiebungssatz

Für jedes $c \in \mathbb{R}$ gilt

$$\sum_{i=1}^{n}(x_i - c)^2 = \sum_{i=1}^{n}(x_i - \bar{X})^2 + n(\bar{X} - c)^2$$

Damit erhält man als Varianz

$$s^2 = \frac{1}{n}(\sum_{i=1}^{n} x_i^2 - n \cdot \bar{X}^2)$$

3.2.8 Variationskoeffizient

$$v = \frac{\bar{s}}{\bar{X}}, \bar{X} > 0$$

3.2.9 Konzentrationsmasse

Konzentrationsrate

Die Konzentrationsrate[1] CRn ist die Summe der Marktanteile der n größten Unternehmen eines relevanten Marktes. Im GWB[2] werden die Raten CR_1, CR_3 und CR_5 herangezogen.

1 `http://de.wikipedia.org/wiki/Konzentrationsrate`
2 `http://de.wikipedia.org/wiki/Gesetz_gegen_Wettbewerbsbeschr%25C3%25A4nkungen`

Lorenzkurve

Für eine geordnete Urliste $x_1 \leq x_2 \ldots \leq x_n$ trägt man die kumulierte relative Merkmalssumme

$$q_i = \frac{\sum_{k=1}^{j} x_i}{\sum_{k=1}^{n} x_i}$$

über den Anteil der Merkmalsträger $p_j = \frac{j}{n}$ auf.

Liegen die Merkmale in gruppierter Form vor, trägt man die kumulierte relative Merkmalssumme

$$q_i = \frac{\sum_{k=1}^{j} x_i}{\sum_{k=1}^{n} x_i}$$

über der Häufigkeit $p_i = \frac{1}{n} \cdot \sum_{j=1}^{i} h_j$ auf.

Zwischen (0;0) und (1;1) wird die Winkelhalbierende des Koordinatensystems eingetragen.

Gini-Koeffizient

Als Ginikoeffizient G bezeichnet man das Verhältnis der Fläche zwischen Winkelhalbierender und der Lorenzkurve

zur Gesamtfläche unter der Winkelhalbierenden (= 1/2).

Die Fläche unterhalb der Lorenzkurve kann man einfach aus Teil-Trapezflächen zusammensetzen:

$$G = \left(\frac{1}{2} - \frac{1}{2} \sum_{i=1}^{n} \Delta p_i \cdot (q_{i-1} + q_i) \right) : \frac{1}{2} = 1 - \sum_{i=1}^{n} (p_i - p_{i-1}) \cdot (q_{i-1} + q_i)$$

($p_0 = 0$; $q_0 = 0$):

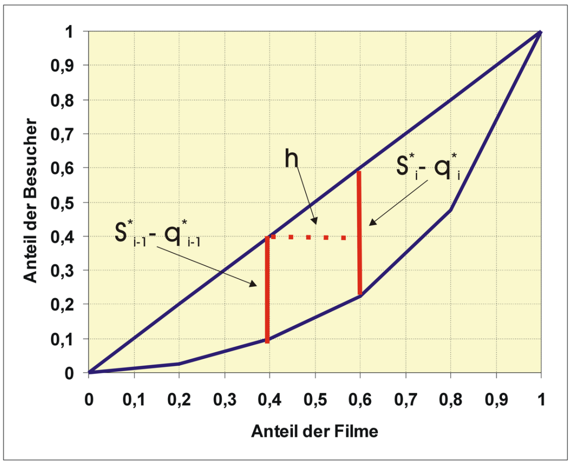

Abb. 2 Ginikoeffizient: Ermittlung einer Trapezfläche für i=3

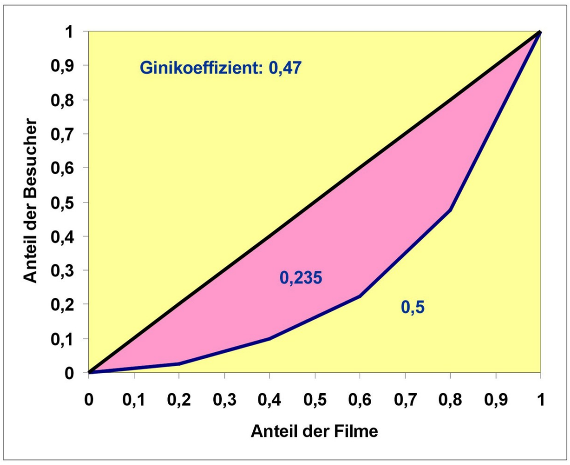

Abb. 3 Ginikoeffizient

Herfindahl-Index

$$H = \sum_{i=1}^{n} p_i^2, \text{wobei } p_i = \frac{x_i}{\sum\limits_{j=1}^{n} x_j}$$

4 Parameterschätzung

4.1 Konfidenzintervall für den Erwartungswert μ

4.1.1 Normalverteiltes Merkmal mit bekannter Varianz

Das Zufallsintervall enthält mit einer Wahrscheinlichkeit 1-α den Parameter:

$$P\left(\bar{x} - z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \leq -\mu \leq \bar{x} + z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$

Konfidenzintervall

$$\left[\bar{x} - z(1 - \tfrac{\alpha}{2})\frac{\sigma}{\sqrt{n}}; \bar{x} + z(1 - \tfrac{\alpha}{2})\frac{\sigma}{\sqrt{n}}\right].$$

(Quantil z aus Normalverteilungstabelle)

4.1.2 Normalverteiltes Merkmal mit unbekannter Varianz

Für normalverteilte Merkmale und unbekannter Varianz muss die Varianz durch s^2 geschätzt werden.

$$P\left(\bar{x} - t(1 - \tfrac{\alpha}{2}; n-1)\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t(1 - \tfrac{\alpha}{2}; n-1)\frac{s}{\sqrt{n}}\right) = 1 - \alpha.$$

Konfidenzintervall

$$\left[\bar{x} - t(1 - \tfrac{\alpha}{2}; n-1)\frac{s}{\sqrt{n}} \ ; \ \bar{x} + t(1 - \tfrac{\alpha}{2}; n-1)\frac{s}{\sqrt{n}}\right].$$

(Quantil $t(1 - \frac{\alpha}{2}; n-1)$ aus der t-Verteilungstabelle bei Freiheitsgrad n-1).

4.1.3 Merkmal mit unbekannter Verteilung und bekannter Varianz

Konfidenzintervall

$\left[\bar{x} - z(1 - \frac{\alpha}{2})\frac{\sigma}{\sqrt{n}}; \bar{x} + z(1 - \frac{\alpha}{2})\frac{\sigma}{\sqrt{n}}\right]$. für n > 30.

4.1.4 Merkmal mit unbekannter Verteilung und unbekannter Varianz

Konfidenzintervall

$\left[\bar{x} - z(1-\frac{\alpha}{2})\frac{s}{\sqrt{n}}\ ;\ \bar{x} + z(1-\frac{\alpha}{2})\frac{s}{\sqrt{n}}\right]$. für n > 50

4.2 Konfidenzintervalle für den Anteilswert einer dichotomen Grundgesamtheit

4.2.1 Modell mit Zurücklegen

Beschreibung durch den geschätztem Anteilswert $\hat{p} = \frac{x}{n}$. Für n > 100 und $n\hat{p}(1-\hat{p}) \geq 9)$

erhält man das 1-α-Konfidenzintervall für p durch eine Approximation der Binomialverteilung mit Hilfe der Normalverteilung:

$$\left[\hat{p} - z(1-\frac{\alpha}{2})\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\ ;\ \hat{p} + z(1-\frac{\alpha}{2})\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right].$$

4.2.2 Modell ohne Zurücklegen

Für $n > \frac{9}{p(1-p)}, n > 100 n/N \leq 0,05$ kann die hypergeometrische Verteilung durch die Normalverteilung approximiert werden:

$(1-\alpha)$-Konfidenzintervall für θ:

$$\left[p - z\left(1-\frac{\alpha}{2}\right)\sqrt{\frac{p(1-p)}{n}}\sqrt{\frac{N-n}{N-1}}\ ;\ p + z\left(1-\frac{\alpha}{2}\right)\sqrt{\frac{p(1-p)}{n}}\sqrt{\frac{N-n}{N-1}}\right].$$

5 Hypothesentests

5.1 Vorgehen beim Hypothesentest

I. Feststellung der Verteilung des Merkmals in der Grundgesamtheit

II. Aufstellen der Nullhypothese

III. Festlegen der Testfunktion T

IV. Festlegen des Annahmebereichs ("Nichtablehnungsbereichs") (für ein zu bestimmendes Signifikanzniveau)

Fällt die Prüfgröße $\overline{x}$ in den Bereich $[\overline{x}_{\text{u}}; \overline{x}_{\text{o}}]$,

wird H_0 nicht abgelehnt. Es soll sein

$$P(\bar{x}_u \leq \bar{X} \leq \bar{x}_o) = 1 - \alpha$$

(beachte: ein- oder zweiseitig)

α : Signifikanzniveau oder α-Fehler

V. Stichprobe erheben

VI. Entscheidung treffen

	$\mathbf{H_0}$ **ist wirklich wahr**	$\mathbf{H_1}$ **ist wirklich wahr**
H_0 wird beibehalten	richtige Entscheidung (1-α)	Fehler 2. Art (β-Fehler)
H_1 wird angenommen	Fehler 1. Art (α-Fehler)	richtige Entscheidung (1-β)

5.2 Tests auf Lageparameter (Erwartungswert, Median, Anteilswert)

5.2.1 Test auf Erwartungswert

Test	H_0	H_1
zweiseitig	$\mu = \mu_0$	$\mu \neq \mu_0$
rechtsseitig	$\mu \leq \mu_0$	$\mu > \mu_0$
linksseitig	$\mu \geq \mu_0$	$\mu < \mu_0$

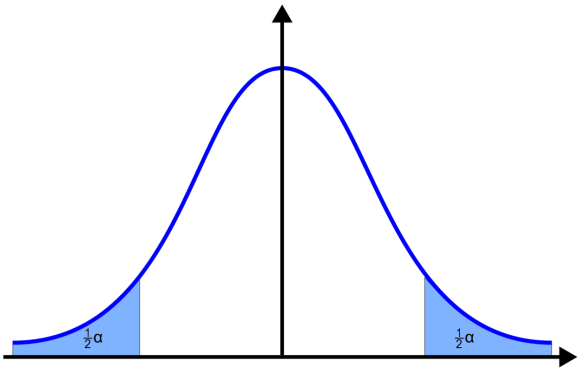

Abb. 4 Zweiseitiger Test für $\overline{x}$

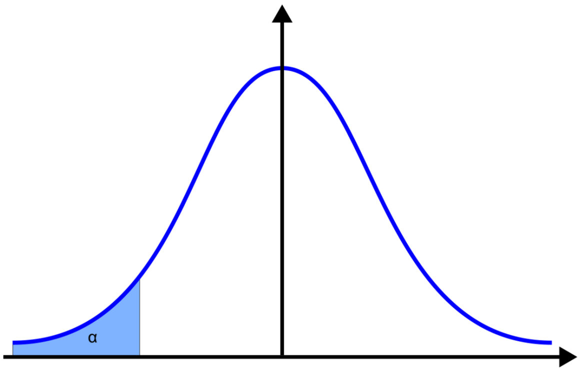

Abb. 5 linksseitiger Test für $\overline{x}$

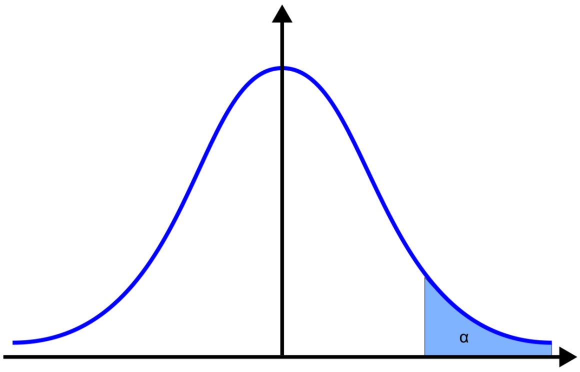

Abb. 6 Rechtsseitiger Test für $\overline{x}$

1. X ist normalverteilt, σ ist bekannt bei beliebigem n bzw. näherungsweise normalverteilt bei n > 30

Testfunktion

$T = \frac{\bar{X}_n - \mu_0}{\sigma} \cdot \sqrt{n} \sim N(0;1)$ (Gauß-Test):

	Ablehnungsbereich
zweiseitig	$\lvert T \rvert > z_{1-\alpha/2}$
rechtsseitig	$\lvert T \rvert > z_{1-\alpha}$
linksseitig	$\lvert T \rvert < -z_{1-\alpha}$

2. X ist normalverteilt, σ ist unbekannt bei beliebigem n

Testfunktion

$T = \frac{\bar{X_n} - \mu_0}{S} \cdot \sqrt{n} \sim t(n-1)$ (t-Test).

	Ablehnungsbereich
zweiseitig	$\lvert T\rvert > t_{1-n,1-\alpha/2}$
rechtsseitig	$\lvert T\rvert > t_{n-1,1-\alpha}$
linksseitig	$\lvert T\rvert < -t_{n-1,1-\alpha}$

3. X ist näherungsweise normalverteilt, σ ist unbekannt bei n > 30

Testfunktion

$T = \frac{\bar{X_n} - \mu_0}{S} \cdot \sqrt{n} \approx N(0;1)$ (Gauß-Test) .

	Ablehnungsbereich
zweiseitig	$\lvert T\rvert > t_{1-n,1-\alpha/2}$
rechtsseitig	$\lvert T\rvert > t_{n-1,1-\alpha}$
linksseitig	$\lvert T\rvert < -t_{n-1,1-\alpha}$

5.2.2 Vorzeichentest

Einstichprobenproblem

	Einseitig		**Zweiseitig**
H_0	$P(X \geq \theta_0) \geq 1/2$	$P(X \geq \theta_0) \leq 1/2$	$P(X \geq \theta_0) = 1/2$
H_1	$P(X \geq \theta_0) < 1/2$	$P(X \geq \theta_0) > 1/2$	$P(X \geq \theta_0) \neq 1/2$
H_0	$\theta \geq \theta_0$	$\theta \leq \theta_0$	$\theta = \theta_0$
H_1	$\theta < \theta_0$	$\theta > \theta_0$	$\theta \neq \theta_0$

Die Stichprobenwerte, die größer als der hypothetische Median θ_0 sind, bekommen ein "+" zugeordnet;

Werte, die kleiner sind, ein "-". Die Anzahl der positiven Vorzeichen wird gezählt und dient als Teststatistik.

Zweistichprobenproblem

Die n Beobachtungspaare dürfen nicht voneinander abhängen, d.h. das Wertepaar (x_{1i}, x_{2i}) muss unabhängig

vom Wertepaar $(x_{1j}, x_{2j}), \forall i \neq j$ sein.

Besitzen beide Grundgesamtheiten den gleichen Median, gilt $P(X_{11} > X_{12}) = P(X_{11} < X_{12})$.

Folgende Hypothesen können mit dem Vorzeichentest geprüft werden:

	Einseitig		**Zweiseitig**
H_0	$P(X_1 \geq X_2) \geq 1/2$	$P(X_1 \geq X_2) \leq 1/2$	$P(X_1 \geq X_2) = 1/2$
$H_1:$	$P(X_1 \geq X_2) < 1/2$	$P(X_1 \geq X_2) > 1/2$	$P(X_1 \geq X_2) \neq 1/2$

Die Wertepaare der Stichproben, bei denen $x_{i1} > x_{i2}$ gilt, bekommen ein "+" zugeordnet; Wertepaare, für die $x_{i1} < x_{i2}$ gilt, ein "-". Die Anzahl der positiven Vorzeichen wird gezählt und dient als Teststatistik. Die Teststatistik entspricht der Anzahl der positiven Vergleiche (Differenzen der Werte bzw. Ränge):

$$V = \sum_{i=1}^{n'} \mathrm{I}(x_{i1} > x_{i2}) \sim B(\pi = 0{,}5, n')$$

mit

$$\mathrm{I}(x_{i1} > x_{i2}) = \begin{cases} 1, & \text{wenn} x_{i1} > x_{i2} \\ 0, & \text{sonst} \end{cases}$$

Für das Einstichprobenproblem sind die Werte der zweiten Stichprobe durch den hypothetischen Median zu ersetzen.

Bei Gültigkeit der Nullhypothese H_0 ist die Summe der positiven Differenzen binomialverteilt mit $\pi = 0{,}5$,

da der Median dem 50 %-Quantil entspricht. n' bezeichnet den nach Behandlung von Ties (Nulldifferenzen, Rangbindungen, s.u.)

verbleibenden Stichprobenumfang. Bei Gültigkeit der Nullyhypothese ist die Verteilung der Prüfgröße symmetrisch.

Approximation durch die Normalverteilung

Mit $n \to \infty$ nähert sich die Binomialverteilung einer Normalverteilung mit $N(np, np(1-p))$,

als Faustregel $np(1-p) \geq 9$ $(H_0 : p = 1/2)$.

Mit $\frac{1}{4}n \geq 9$ bzw. $n \geq 36$ ist die z-standardisierte Größe

$z_V = \frac{\sum_{i=1}^{n'} - \frac{1}{2} \cdot n'}{\frac{1}{2}\sqrt{n'}} \approx N(0,1)$

näherungsweise standardnormalverteilt.

Bindungen (Nulldifferenzen) Sind im Zweistichprobenproblem die Werte von Beobachtungen von der ersten zur zweiten Stichprobe unverändert

oder im Einstichprobenproblem einige Werte gleich dem Median, ergeben sich Nulldifferenzen bzw. Bindungen (Ties),

die man so behandeln kann:

- Beobachtungen mit Rangbindungen werden eliminiert, d.h. der Stichprobenumfang wird reduziert.
- Die Beobachtungen werden zu gleichen Teilen den Gruppen zugeordnet. Bei ungerader Anzahl von Bindungen wird ein Beobachtungspaar eliminiert.
- Die Beobachtungen werden jeweils mit einer Wahrscheinlichkeit von 0,5 einer der beiden Gruppen (+ oder -) zugeordnet.

5.2.3 Test auf Anteilswert (Binomialtest)

Der Anteilswert θ wird geschätzt durch

$$\hat{\theta} = p = \frac{x}{n}$$

Mit dem Binomialtest können folgende Hypothesenpaare für θ getestet werden:

Test	H_0	H_1
zweiseitig	$\theta = \theta_0$	$\theta \neq \theta_0$
rechtsseitig	$\theta \leq \theta_0$	$\theta > \theta_0$
linksseitig	$\theta \geq \theta_0$	$\theta < \theta_0$

für n $>$ 30 , nθ_0 $\geq$ 10 n(1-θ_0) $\geq$ 10

kann man durch die Gauß-Verteilung approximieren:

Testfunktion

$T = \frac{\theta - \theta_0}{\sqrt{\theta_0(1-\theta_0)}} \cdot \sqrt{n} \approx N(0;1)$ (Gauß-Test) .

	Ablehnungsbereich
zweiseitig	$\|T\| > z_{1-\alpha/2}$
rechtsseitig	$\|T\| > z - 1 - \alpha$
linksseitig	$\|T\| < -z - 1 - \alpha$

für n $<$ 30 oder nθ_0 $<$ 10 oder n(1-θ_0) $<$ 10

ist der exakte Binomialtest anzuwenden:

Testfunktion

Die Teststatistik X gibt an, wie oft das Merkmal in einer zufälligen Stichprobe vom Umfang n aufgetreten ist.

Unter der Nullhypothese $H_0 : \theta = \theta_0$ ist die Teststatistik $B(\theta_0, n)$-verteilt, das heißt

$$P(X = i) = B(i|\theta_0, n) = \binom{n}{i} \theta_0^i (1 - \theta_0)^{n-i}$$

Ablehnungsbereich

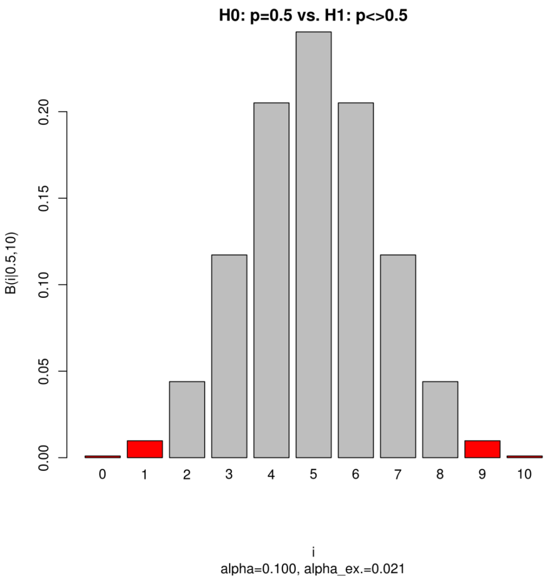

Abb. 7 Teststatistik für den Binomialtest, die roten Balken gehören zum kritischen Bereich.

Da die Teststatistik diskret verteilt ist, kann das vorgegebene Signifikanzniveau α in der Regel nicht eingehalten werden.

Daher wird gefordert, die kritischen Werte so zu wählen, dass für ein möglichst großes *exaktes* Signifikanzniveau α_{ex} gilt $\alpha_{\text{ex}} \leq \alpha$.

Für den zweiseitigen Test werden daher als kritische Werte das größte c_1 und das kleinste c_2 bestimmt, für die gilt

- $\sum_{i=0}^{c_1} B(i|\theta_0, n) \leq \alpha/2$ und
- $\sum_{i=c_2}^{n} B(i|\theta_0, n) \leq \alpha/2$.

Das exakte Signifikanzniveau ergibt sich als

$\alpha_{\text{ex}} = \sum_{i=0}^{c_1} B(i|\theta_0, n) + \sum_{i=c_2}^{n} B(i|\theta_0, n)$.

Für die beiden einseitigen Tests wird analog verfahren.

Test	**Kritische Werte**	**Kritischer Bereich**	**Grenze(n)**
zweiseitig	c_1+1 und c_2-1	$\{0,\dots,c_1\} \cup \{c_2,\dots,n\}$	
rechtsseitig	$c-1$	$\{c,\dots,n\}$	$c =$ kleinster Wert, für den $\sum_{i=c}^{n} B(i|\theta_0, n) = \alpha_{\text{ex}} \leq \alpha$

Test	Kritische Werte	Kritischer Bereich	Grenze(n)
linksseitig	$c+1$	$\{0,\ldots,c\}$	$c =$ größter Wert, für den $\sum_{i=0}^{c} B(i\|\theta_0,n) = \alpha_{\text{ex}} \leq \alpha$

5.3 Tests auf Streuung

5.3.1 Test auf Varianz

Test	H_0	H_1
zweiseitig	$\sigma^2 = \sigma_0^2$	$\sigma^2 \neq \sigma_0^2$
rechtsseitig	$\sigma^2 \leq \sigma_0^2$	$\sigma^2 > \sigma_0^2$
linksseitig	$\sigma^2 \geq \sigma_0^2$	$\sigma^2 < \sigma_0^2$

1. X ist normalverteilt, μ ist unbekannt, n beliebig

Testfunktion

$$T = \frac{(n-1)S^2}{\sigma_0^2} = \frac{1}{\sigma_0^2}\sum_{i=1}^{n}(X_i - \bar{X}^2)^2 \sim \chi^2(n-1)$$

	Ablehnungsbereich
zweiseitig	$T < \chi^2_{n-1,\alpha/2}$ oder $T > \chi^2_{n-1,1-\alpha/2}$
rechtsseitig	$T > \chi^2_{n-1,1-\alpha}$
linksseitig	$T < \chi^2_{n-1,\alpha}$

2. X ist normalverteilt, μ ist bekannt, n beliebig

Testfunktion

$$T = \frac{(n-1)\tilde{S}^2}{\sigma_0^2} = \frac{1}{\sigma_0^2}\sum_{i=1}^{n}(X_i - \mu)^2 \sim \chi^2(n)$$

	Ablehnungsbereich
zweiseitig	$T < \chi^2_{n,\alpha/2}$ oder $T > \chi^2_{n,1-\alpha/2}$
rechtsseitig	$T > \chi^2_{n,1-\alpha}$
linksseitig	$T < \chi^2_{n,\alpha}$

5.4 Tests auf Zusammenhangs- und Assoziationsparameter

5.4.1 Chi-Quadrat-Unabhängigkeitstest

Nullhypothese

H_0: Die Merkmale X und Y sind stochastisch unabhängig.

Die Beobachtungen der Merkmale X und Y liegen paarweise in m bzw. r Klassen vor.

Es gibt insgesamt n paarweise Beobachtungen von X und Y, die sich auf $m \cdot r$ Kategorien verteilen. Aufstellung z. B. in einer Häufigkeitstabelle:

	Merkmal Y						**Summe** Σ
Merkmal X	1	2	...	k	...	r	$n_{j.}$
1	n_{11}	n_{12}	...	n_{1k}	...	n_{1r}	$n_{1.}$
2	n_{21}	n_{22}	...	n_{2k}	...	n_{2r}	$n_{2.}$
...	...	...	...	...	...	...	...
j	...	...	...	n_{jk}	...	...	$n_{j.}$
...	...	...	...	...	...	...	...
m	n_{m1}	n_{m2}	...	n_{mk}	...	n_{mr}	$n_{m.}$
Summe Σ	$n_{.1}$	$n_{.2}$	...	$n_{.k}$	...	$n_{.r}$	n

Absolute Randhäufigkeiten $n_{j\cdot}$ bzw. $n_{\cdot k}$

$n_{j\cdot} = \sum_{k=1}^{r} n_{jk}$ und $n_{\cdot k} = \sum_{j=1}^{m} n_{jk}$

Prüfgröße für den Unabhängigkeitstest:

$$\chi^2 = \sum_{j=1}^{m}\sum_{k=1}^{r} \frac{(n_{jk} - n_{jk}^*)^2}{n_{jk}^*}.$$

Mit :$n_{jk}^* = \frac{n_{j\cdot} \cdot n_{\cdot k}}{n}$,

H_0 wird abgelehnt, wenn $\chi^2 > \chi^2(1-\alpha;(m-1)(r-1))$ ist.

5.5 Anpassungs- oder Verteilungstests

5.5.1 Chi-Quadrat-Anpassungs- oder Verteilungstest

Die Wahrscheinlichkeiten eines Merkmals X seien in der Grundgesamtheit unbekannt.

Nullhypothese: H_0: Das Merkmal X besitzt die Wahrscheinlichkeitsverteilung $F_0(x)$

Für n unabhängige Beobachtungen $x_1, \ldots, x_n$ des Merkmals X wird die Zahl

der Beobachtungen in der j-ten Klasse ist die beobachtete Häufigkeit N_j.

Im Vergleich dazu wird die hypothetische Verteilung bestimmt aufgrund der Wahrscheinlichkeit p_{0j},

dass eine Ausprägung von X in die Kategorie j fällt. Die unter H_0 zu erwartende Häufigkeit ist:

$$n_{0j} = p_{0j} \cdot n$$

Die Prüfgröße (Größe der Abweichung)

$$\chi^2 = \sum_{j=1}^{m} \frac{(N_j - n_{0j})^2}{n_{0j}}$$

ist bei ausreichend großen N_j annähernd chi-Quadrat-verteilt mit $m-1$ Freiheitsgraden.

H_0 wird abgelehnt, wenn $\chi^2 > \chi^2_{(1-\alpha;m-1)}$ gilt.

5.5.2 Kolmogorow-Smirnow-Anpassungstest

Test auf Übereinstimmung zweier Wahrscheinlichkeitsverteilungen.

Man betrachtet ein statistisches Merkmal X, dessen Verteilung in der Grundgesamtheit unbekannt ist.

$H_0 : F_X(x) = F_0(x)$ (Die Zufallsvariable X besitzt die Wahrscheinlichkeitsverteilung F_0.)

$H_1 : F_X(x) \neq F_0(x)$ (Die Zufallsvariable X besitzt eine andere Wahrscheinlichkeitsverteilung als F_0.)

Der Kolmogorow-Smirnow-Test vergleicht die empirische Verteilungsfunktion F_n mit F_0 mittels der Teststatistik

$d_n = \|F_n - F_0\| = \sup_x |F_n(x) - F_0(x)|$, (sup: Supremum)

Die Teststatistik ist unabhängig von der hypothetischen Verteilung F_0.

Ist der Wert der Teststatistik größer als der entsprechende tabellierte kritische Wert, so wird die Nullhypothese verworfen.

Einstichprobenproblem

Von einer reellen Zufallsvariablen X liegen n aufsteigend sortierte Beobachtungswerte x_i $(i = 1, \ldots, n)$ vor.

Von diesen Beobachtungen wird die relative Summenhäufigkeit $S(x_i)$ mit der entsprechenden hypothetischen

Verteilung der Grundgesamtheit $F_0(x_i)$ verglichen. Voraussetzung: F_0 ist stetig.

Für jedes $i = 1, \ldots, n$ werden die absoluten Differenzen

$d_{oi} = |S(x_i) - F_0(x_i)|$ und :$d_{ui} = |S(x_{i-1}) - F_0(x_i)|$

berechnet, wobei $S(x_0) := 0$ gesetzt wird. Wenn die größte Differenz $d_{\max}$ aus allen Differenzen d_{oi}, d_{ui}

einen kritischen Wert d_α übersteigt, wird die Hypothese abgelehnt.

Bis n=40 greift man auf Tabellen zurück (s. Anhang). Für größere n werden sie über $d_\alpha = \frac{\sqrt{\ln\left(\frac{2}{\alpha}\right)}}{\sqrt{2n}}$ angenähert.

Zweistichprobenproblem

Liegt nun zusätzlich zur Zufallsvariablen X eine entsprechende Zufallsvariable Y vor (mit m geordneten Werten y_i),

so kann durch den Zweistichprobentest überprüft werden, ob X und Y derselben Verteilungsfunktion folgen.

Von beiden Beobachtungen werden die die Differenzen der relativen Summenfunktionen $S_X(x_i)$ bzw. $S_Y(y_i)$ ermittelt:

$d(z) = |S_X(z) - S_Y(z)|$ und :$d_{max} = \sup_z d(z)$.

Die Nullhypothese wird abgelehnt, falls d_{max} den kritischen Wert $d_{krit}(\alpha, n, m)$ überschreitet.

Für kleine Werte von n und m greift man auf Tabellen zurück.

Für große Werte von n und m wird die Nullhypothese abgelehnt, falls

$$\sqrt{\frac{nm}{n+m}} d_{max} > K_\alpha$$

wobei K_α für große n und m näherungsweise als $K_\alpha = \sqrt{\frac{\ln\left(\frac{2}{\alpha}\right)}{2}}$ berechnet werden kann.

6 Varianzanalyse

univariate Varianzanalyse (ANOVA)

Man untersucht man den Einfluss einer unabhängigen Variable (Faktor) mit k verschiedenen Stufen (Gruppen) auf die Ausprägungen einer Zufallsvariablen.

Dazu werden die k Mittelwerte der Ausprägungen für die Gruppen miteinander verglichen, und zwar vergleicht man die Varianz zwischen den Gruppen mit der Varianz innerhalb der Gruppen.

Weil sich die totale Varianz aus den zwei genannten Komponenten zusammensetzt, spricht man von Varianzanalyse.

Die einfaktorielle ANOVA ist die Verallgemeinerung des t-Tests bei mehr als zwei Gruppen. Für k=2 ist sie äquivalent mit dem t-Test.

Es sei μ_i der Erwartungswert der abhängigen Variable in der i. Gruppe.

$H_0 : \mu_1 = \mu_2 = ... = \mu_k$ (Es besteht kein Unterschied zwischen den Erwartungswerten der Gruppen.)

$H_1 : \exists i, j : \ \mu_i \neq \mu_j$ (Es besteht zwischen *mindestens* zwei Erwartungswerten ein Unterschied.)

$\rightarrow$ Wir wissen dann nur mit einer bestimmten Wahrscheinlichkeit, dass mindestens zwei Ausprägungen einen bedeutsamen Unterschied aufweisen.

Effektdarstellung :

$$X_{ij} = \mu + \alpha_i + \varepsilon_{ij}, \quad i = 1, \ldots, k, \ j = 1, \ldots, n_i.$$

Darin sind:

X_{ij}: Zielvariable; annahmegemäß in den Gruppen normalverteilt

k: Anzahl der Faktorstufen des betrachteten Faktors

n_i: Stichprobenumfänge für die einzelnen Faktorstufen

$\boldsymbol{\mu}$: arithmetisches Mittel der Erwartungswerte in den Gruppen

$\boldsymbol{\alpha}_i$: Effekt der i-ten Faktorstufe

$\boldsymbol{\varepsilon}_{ij}$: Störvariablen, unabhängig und normalverteilt mit Erwartungswert 0 und gleicher (unbekannter) Varianz $\boldsymbol{\sigma}^2$.

Erwartungswert in der *i*. Gruppe: $\mu_i = \mu + \alpha_i$

$$\sum_{i=1}^{k} n_i \alpha_i = 0.$$

Betrachtung der **Quadratsummen** (Variabiliät)

Die gesamte Variabilität QST (gesamte quadratische Abweichung vom Mittelwert) lässt sich in zwei Teile zerlegen:

$$QST = \sum (X_{ij} - \overline{\overline{X}})^2 = QSA + QSE$$

Der erste Teil QSA (Gruppenzugehörigkeit) entspricht der ('Inter-')Variabilität zwischen den Gruppen.

$$QSA = \sum_i n_i (\overline{X}_i - \overline{\overline{X}})^2,$$

Der Rest QSE entspricht der Variabilität innerhalb der Gruppen (gesamte 'Intra'-Abweichung von den Mittelwerten in den Gruppen, der 'Zufall'):

$$QSE = \sum_{i,j} (X_{ij} - \overline{X_i})^2.$$

Die zwei Quadratsummen QSA und QSE sind stochastisch unabhängig.

Im Fall von *k* Gruppen mit gleichem Umfang *b=n/k* gilt unter der Nullhypothese außerdem:

QSA/σ^2 folgt einer Chi-Quadrat-Verteilung mit *k*-1 Freiheitsgraden,

und

QSE/σ^2 folgt einer Chi-Quadrat-Verteilung mit *n*-*k* Freiheitsgraden.

mittlere Quadratsummen:

$MQSA = \frac{1}{k-1} QSA$, und :$MQSE = \frac{1}{n-k} QSE$.

Prüfgröße:

$$F = \frac{MQSA}{MQSE}.$$

Gruppen gleicher Größe

Im Falle Gruppen gleicher Größe ist F unter der Nullhypothese F-verteilt

mit $k-1$ Freiheitsgraden im Zähler und $k \cdot (b-1)$ Freiheitsgraden im Nenner.

Wenn die Prüfgröße

$$F = \frac{MQSA}{MQSE} = \frac{\frac{1}{k-1} \cdot b \cdot \sum_i (\overline{X}_i - \overline{\overline{X}})^2}{\frac{1}{k \cdot (b-1)} \cdot \sum_{i,j} (X_{ij} - \overline{X_i})^2}.$$

signifikant (d.h. $F > F_{krit}(\alpha, k-1, k \cdot (b-1))$ wird, unterscheiden sich mindestens zwei Faktoren ('Gruppen') voneinander.

In Post-Hoc-Tests kann dann berechnet werden, zwischen welchen einzelnen Gruppen der Unterschied liegt.

7 Regressionsrechnung

7.0.3 Lineare Regression

Methode der kleinsten Quadrate

$$RSS = \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} (y_i - (a + bx_i))^2 \rightarrow min!$$

bezüglich a und b.

Nach Ausmultiplikation, Ableiten und Nullsetzen

$$\frac{\partial S}{\partial a} = -2\sum_{i=1}^{n} y_i + 2na + 2b\sum_{i=1}^{n} x_i = 0,$$

$$\frac{\partial S}{\partial b} = -2\sum_{i=1}^{n} x_i y_i + 2a\sum_{i=1}^{n} x_i + 2b\sum_{i=1}^{n} x_i^2 = 0,$$

erhält man die gesuchten Regressionskoeffizienten als die Lösungen

$$b = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}$$

und

$a = \bar{y} - b\bar{x}$,, wobei $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$.

Mit dem Verschiebungssatz:

$$b = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Schätzungen ŷ

$$\hat{y}_i = a + bx_i$$

Residuen r_i :

$$\begin{array}{rcccl} y_i & = & a + bx_i + d_i & = & \hat{y}_i + d_i \\ \Rightarrow d_i & = & y_i - \hat{y}_i & & \end{array}$$

Stichprobenvarianz der Residuen:

$$s^2 = \frac{1}{n-2}\sum_i d_i^2$$

Bestimmtheitsmaß

$$r^2 = \frac{\frac{1}{n}\sum_{i=1}^n(\hat{y}_i-\bar{y})^2}{\frac{1}{n}\sum_{i=1}^n(y_i-\bar{y})^2} = \frac{(\sum_{i=1}^n(x_i-\bar{x})(y_i-\bar{y}))^2}{\sum_{i=1}^n(x_i-\bar{x})^2\sum_{i=1}^n(y_i-\bar{y})^2},$$

mit dem Verschiebungssatz :

$$r^2 = \frac{(\sum_{i=1}^n x_i y_i - n\cdot\bar{x}\cdot\bar{y})^2}{(\sum_{i=1}^n x_i^2 - n\cdot\bar{x}^2)(\sum_{i=1}^n y_i^2 - n\cdot\bar{y}^2)}.$$

$$0 \le r^2 \le 1$$

Varianz der Residuen

$$s^2 = \frac{1}{n-2}(1-r^2)\cdot\sum_{i=1}^n(y_i-\bar{y})^2$$

7.0.4 Variablentransformation

Funktion	**u**	**v**
$y = a + b\cdot x^n$	x^n	$v = a + b\cdot u$
$y = \frac{a}{b+x}$	x	$v = \frac{1}{y} = \frac{b}{a} + \frac{1}{b}\cdot u$
$y = a\cdot x^b$	$ln(x)$	$v = ln(y) = ln(a) + b\cdot ln(x)$
$y = a\cdot b^x$	x	$v = ln(y) = ln(a) + x\cdot ln(b)$
$y = a\cdot e^{b\cdot x}$	x	$v = ln(y) = ln(a) + b\cdot x$

Die Methode kann auf weitere Parameter erweitert werden.

8 Zeitreihenanalyse

8.1 Komponentenunterteilung bei Zeitreihen

Mögliche Aufteilung einer Zeitreihe in Komponenten:

- Trend Q
- Konjunkturelle Schwankung K
- Saisonale Schwankung S
- Restschwankung r

Bei Unabhängigkeit dieser Komponenten kann man ein additives Modell annehmen:

$$y = Q + K + S + r$$

Nehmen beispielsweise zyklische Schwankungen mit steigendem Trend zu, könnte ein multiplikatives Modell

$$y_t = Q_t \cdot K_t \cdot r_t$$

angebracht sein. Variablentransformation durch Logarithmieren

$$\log y_t = \log Q_t + \log S_t + \log r_t$$

8.1.1 Schätzung des Trends durch Regression

"'Regressionsmodell"'

$\hat{y}_t = a + bt$ bzw. $y_t = a + bt + d_t$ $(t = 1, 2, \ldots, T; y_t = y_1, y_2, \ldots, y_T)$

mit den Lösungen

$$b = \frac{\sum_{t=1}^{T}(t-\bar{t})(y_t-\bar{y})}{\sum_{t=1}^{T}(t-\bar{t})^2}$$

$$= \frac{\sum_{t=1}^{T} t \cdot y_t - T \cdot \bar{t} \cdot \bar{y}}{\sum_{t=1}^{T} t^2 - T \cdot \bar{t}^2}$$

$$= \frac{\sum_{t=1}^{T} t y_t - \frac{T(T+1)}{2}\bar{y}}{\frac{1}{12}(T^3 - T)}$$

und

$$a = \overline{y} - b \cdot \bar{t}$$

$$= \overline{y} - b \cdot \frac{T+1}{2}$$

Die Trendwerte Q_t sind dann

$$Q_t = \hat{y}_t = a + bt$$

Nichtlinearer Trendverlauf: Lösung über Variablentransformation oder Anwendung eines nichtlinearen Regressionsansatzes

8.1.2 Schätzung der Saisonkomponente

Additives Modell

$$y_t = Q_t + S_t + r_t$$

Nach Schätzung der Trendkomponente Q_t bleibt noch die Abweichung

$$d_t = y_t - Q_t$$

und

$$d_t = S_t + r_t$$

d_t: trendbereinigter Zeitreihenwert

Bestimmung der saisonalen Komponente S_t über Fourieranalyse oder (einfacher)

Bildung des arithmetischen Durchschnitts aller Werte d_t, die die gleiche Saison betreffen,

als Schätzung für die saisonale Komponente. Dann bleibt die nichterklärte Restschwankung

$$r_t = y_t - Q_t - S_t$$

Prognose für den Zeitpunkt T+k (mit S_t als Wert in der Saison T+k)

$$\hat{y}_{T+k} = Q_{T+k} + S_{T+k},$$

8.1.3 Schätzung der glatten Komponente mit gleitenden Mittelwerten

Lässt sich die Trendkomponente des Zeitreihenmodells offensichtlich durch keine funktionale lineare oder

nichtlineare Beziehung darstellen, kann man eine **glatte Komponente** mit Hilfe gleitender Mittelwerte bestimmen.

einfacher gleitender Mittelwert

Beispiel: Mittelwert dritter Ordnung:

$$Y_k = \frac{1}{3} \cdot (Y_{k-1} + Y_k + Y_{k+1}) \frac{1}{3} \sum_{i=k-1}^{k+1} Y_i$$

Die Ordnung des Mittelwerts sollte so gewählt werden, daß möglichst genau eine Periode umfasst wird.

Zur Prognose über den Beobachtungszeitraum hinaus sind gleitende Mittelwerte bedingt geeignet,

da die Randwerte der Zeitreihe nicht geschätzt werden.

gewichteter gleitender Mittelwert

Beispiel: Mittelwert dritter Ordnung mit z.B. $w_1 = \frac{1}{4}, w_2 = \frac{1}{2}, w_1 = \frac{1}{2}, \sum_i w_i = 1$

$$Y_k = w_1 \cdot Y_{k-1} + w_2 \cdot Y_k + w_3 \cdot Y_{k+1}$$

8.1.4 Exponentielle Glättung

Gewichtung durch den Glättungsfaktor α mit $0 \leq \alpha \leq 1$:

Geglätteter Schätzwert $y*_t$ als gewichteter Durchschnitt aus dem aktuellen Zeitreihenwert y_t

und dem Schätzwert der Vorperiode $y*_{t-1}$ ($y*_0$ geeignet wählen):

$$y_t^* = \alpha \cdot y_t + (1 - \alpha) \cdot y_{t-1}^*.$$

Auflösung der Rekursivität:

$$y_t^* = \alpha y_t + \alpha(1-\alpha) y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + ... + \alpha(1-\alpha)^{t-1} y_1 + (1-\alpha)^t y_0.$$

Für die Wahl des Glättungsfaktors wird häufig 0,2 bis 0,3 empfohlen. Man kann aber auch mit Hilfe der Regressionsanalyse den Glättungsfaktor schätzen.

Exponentielle Glättung bei trendbehafteten Werten

Bei Trend werden die Zeitreihenwerte systematisch unter- bzw. überschätzt. Abhilfe bieten ggf. gleitende Durchschnitte zweiter Ordung.

Die bereits einmal geglätteten Werte erneut einer Glättung unterzogen. Man erhält den Schätzwert y^{**}, der sich analog zu oben berechnet aus

$$y_t^{**} = \alpha \cdot y_t^* + (1-\alpha) \cdot y_{t-1}^{**}$$

Für einen brauchbaren Prognosewert für Periode t+1 muss man dann bestimmen

$$\widehat{y}_{t+1} = 2 \cdot y_t^* - y_{t-1}^{**}$$

9 Symbolverzeichnis

9.1 allgemein

Symbol	Verwendung
A,B	Ereignisse
$\Omega = \{A,B,C...\}$	Ereignisraum
$\|A\|$	Anzahl der Ereignisse A
P(A)	Wahrscheinlichkeit für das Eintreten von A
$\cup \cap$	Und-Verknüpfung (Konjunktion) / Oder-Verknüpfung (Disjunktion)
$P(A \mid B)$	Bedingte Wahrscheinlichkeit (A wenn B)
E(X)	Erwartungswert von X
f(X)	Wahrscheinlichkeitsfunktion
F(X)	Verteilungsfunktion
N	Grundgesamtheit
n	Stichprobe
X	Zufallsvariable
$Var(X)$	Varianz von X
Θ	Anteilswert einer Grundgesamtheit

9.2 Verteilungsmodelle

Symbol	Verwendung
$b(x\|n;\theta)$	Binomialverteilung
F(m,n)	Fisher-Verteilung
$h(x\|N;M;n)$	Hypergeometrische Verteilung
$p(x\|\lambda)$	Poissonverteilung
$N(\mu,\sigma^2)$	Normalverteilung
t(n)	t- (Student-) Verteilung
$\phi_x(z)$	Standardnormalverteilung

10 Tabellen

Binomialverteilung (Wahrscheinlichkeitsfunktion)								
		p=	...					
n	x	0,01	0,05	0,1	0,2	0,25	**0,3**	0,5
2	0	0,9801	0,9025	0,81	0,64	0,5625	0,49	0,25
	1	0,0198	0,095	0,18	0,32	0,375	0,42	0,5
	2	0,0001	0,0025	0,01	0,04	0,0625	0,09	0,25
3	0	0,9703	0,8574	0,729	0,512	0,4219	0,343	0,125
	1	0,0294	0,1354	0,243	0,384	0,4219	0,441	0,375
	2	0,0003	0,0071	0,027	0,0960	0,1406	0,189	0,375
	3	0	0,0001	0,001	0,008	0,0156	0,027	0,125
4	0	0,9606	0,8145	0,6561	0,4096	0,3164	0,2401	0,0625
	1	0,0388	0,1715	0,2916	0,4096	0,4219	0,4116	0,25
	2	0,0006	0,0135	0,0486	0,1536	0,2109	0,2646	0,375
	3	0	0,0005	0,0036	0,0256	0,0469	0,0756	0,25
	4	0	0	0,0001	0,0016	0,0039	0,0081	0,0625
5	0	0,951	0,7738	0,5905	0,3277	0,2373	0,1681	0,0313
	1	0,048	0,2036	0,3281	0,4096	0,3955	0,3602	0,1563
	2	0,001	0,0214	0,0729	0,2048	0,2637	0,3087	0,3125
	3	0	0,0011	0,0081	0,0512	0,0879	0,1323	0,3125
	4	0	0	0,0005	0,0064	0,0146	0,0284	0,1563
	5	0	0	0	0,0003	0,001	0,0024	0,0313
6	0	0,9415	0,7351	0,5314	0,2621	0,178	0,1176	0,0156
	1	0,0571	0,2321	0,3543	0,3932	0,356	0,3025	0,0938
	2	0,0014	0,0305	0,0984	0,2458	0,2966	0,3241	0,2344
	3	0	0,0021	0,0146	0,0819	0,1318	0,1852	0,3125
	4	0	0,0001	0,0012	0,0154	0,033	0,0595	0,2344
	5	0	0	0,0001	0,0015	0,0044	0,0102	0,0938
	6	0	0	0	0,0001	0,0002	0,0007	0,0156
7	0	0,9321	0,6983	0,4783	0,2097	0,1335	0,0824	0,0078
	1	0,0659	0,2573	0,372	0,367	0,3115	0,2471	0,0547
	2	0,002	0,0406	0,124	0,2753	0,3115	0,3177	0,1641
	3	0	0,0036	0,023	0,1147	0,173	0,2269	0,2734
	4	0	0,0002	0,0026	0,0287	0,0577	0,0972	0,2734
	5	0	0	0,0002	0,0043	0,0115	0,025	0,1641
	6	0	0	0	0,0004	0,0013	0,0036	0,0547
	7	0	0	0	0	0,0001	0,0002	0,0078

Binomialverteilung (Wahrscheinlichkeitsfunktion)								
n	x	p= 0,01	... 0,05	0,1	0,2	0,25	**0,3**	0,5
8	0	0,9227	0,6634	0,4305	0,1678	0,1001	0,0576	0,0039
	1	0,0746	0,2793	0,3826	0,3355	0,267	0,1977	0,0313
	2	0,0026	0,0515	0,1488	0,2936	0,3115	0,2965	0,1094
	3	0,0001	0,0054	0,0331	0,1468	0,2076	0,2541	0,2188
	4	0	0,0004	0,0046	0,0459	0,0865	0,1361	0,2734
	5	0	0	0,0004	0,0092	0,0231	0,0467	0,2188
	6	0	0	0	0,0011	0,0038	0,01	0,1094
	7	0	0	0	0,0001	0,0004	0,0012	0,0313
	8	0	0	0	0	0	0,0001	0,0039
9	0	0,9135	0,6302	0,3874	0,1342	0,0751	0,0404	0,002
	1	0,083	0,2985	0,3874	0,302	0,2253	0,1556	0,0176
	2	0,0034	0,0629	0,1722	0,302	0,3003	0,2668	0,0703
	3	0,0001	0,0077	0,0446	0,1762	0,2336	0,2668	0,1641
	4	0	0,0006	0,0074	0,0661	0,1168	0,1715	0,2461
	5	0	0	0,0008	0,0165	0,0389	0,0735	0,2461
	6	0	0	0,0001	0,0028	0,0087	0,021	0,1641
	7	0	0	0	0,0003	0,0012	0,0039	0,0703
	8	0	0	0	0	0,0001	0,0004	0,0176
	9	0	0	0	0	0	0	0,002
10	0	0,9044	0,5987	0,3487	0,1074	0,0563	0,0282	0,001
	1	0,0914	0,3151	0,3874	0,2684	0,1877	0,1211	0,0098
	2	0,0042	0,0746	0,1937	0,302	0,2816	0,2335	0,0439
	3	0,0001	0,0105	0,0574	0,2013	0,2503	0,2668	0,1172
	4	0	0,001	0,0112	0,0881	0,146	0,2001	0,2051
	5	0	0,0001	0,0015	0,0264	0,0584	0,1029	0,2461
	6	0	0	0,0001	0,0055	0,0162	0,0368	0,2051
	7	0	0	0	0,0008	0,0031	0,009	0,1172
	8	0	0	0	0,0001	0,0004	0,0014	0,0439
	9	0	0	0	0	0	0,0001	0,0098
	10	0	0	0	0	0	0	0,001

Weitere Werte können mittels =BINOMVERT(x;n;p;0) bei Tabellenkalkulationsprogrammen

oder der R-Funktion[1] dbinom(x, n , p) bestimmt[2] werden

1 `http://de.wikibooks.org/wiki/GNU_R%20`
2 `http://netmath.vcrp.de/downloads/Systeme/R.html`

Standard-Normalverteilung (Verteilungsfunktion) $\phi_x(z)$										
z	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,5	0,504	0,508	0,512	0,516	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,591	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,648	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,67	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,695	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,719	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,758	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,791	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,834	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,877	0,879	0,881	0,883
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,898	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,937	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,975	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,983	0,9834	0,9838	0,9842	0,9846	0,985	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,989
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916
2,4	0,9918	0,992	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936
2,5	0,9938	0,994	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,996	0,9961	0,9962	0,9963	0,9964
2,7	0,9965	0,9966	0,9967	0,9968	0,9969	0,997	0,9971	0,9972	0,9973	0,9974
2,8	0,9974	0,9975	0,9976	0,9977	0,9977	0,9978	0,9979	0,9979	0,998	0,9981
2,9	0,9981	0,9982	0,9982	0,9983	0,9984	0,9984	0,9985	0,9985	0,9986	0,9986
3,0	0,9987	0,9987	0,9987	0,9988	0,9988	0,9989	0,9989	0,9989	0,999	0,999
3,1	0,999	0,9991	0,9991	0,9991	0,9992	0,9992	0,9992	0,9992	0,9993	0,9993
3,2	0,9993	0,9993	0,9994	0,9994	0,9994	0,9994	0,9994	0,9995	0,9995	0,9995
3,3	0,9995	0,9995	0,9995	0,9996	0,9996	0,9996	0,9996	0,9996	0,9996	0,9997
3,4	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9998
3,5	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998
3,6	0,9998	0,9998	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999

Zur Bildung von z ist der Wert von linker Spalte und oberer Zeile zu addieren.

Ablesebeispiel: $\phi_x(1,75) = 0,9599$

weitere Werte: =NORM.S.VERT(z;WAHR) bzw. (R-Aufruf[3]): pnorm(z)

3 `http://netmath.vcrp.de/downloads/Systeme/R.html`

χ^2-Verteilung (Quantile)											
n	α= 0,005	... 0,01	... 0,025	0,05	0,1	0,5	0,9	0,95	0,975	0,99	0,995
1	0,000	0,000	0,001	0,004	0,016	0,455	2,706	3,841	5,024	6,635	7,879
2	0,010	0,020	0,051	0,103	0,211	1,386	4,605	5,991	7,378	9,21	10,597
3	0,072	0,115	0,216	0,352	0,584	2,366	6,251	7,815	9,348	11,345	12,838
4	0,207	0,297	0,484	0,711	1,064	3,357	7,779	9,488	11,143	13,277	14,86
5	0,412	0,554	0,831	1,145	1,61	4,351	9,236	11,07	12,833	15,086	16,75
6	0,676	0,872	1,237	1,635	2,204	5,348	10,645	12,592	14,449	16,812	18,548
7	0,989	1,239	1,69	2,167	2,833	6,346	12,017	14,067	16,013	18,475	20,278
8	1,344	1,646	2,18	2,733	3,49	7,344	13,362	15,507	17,535	20,09	21,955
9	1,735	2,088	2,7	3,325	4,168	8,343	14,684	16,919	19,023	21,666	23,589
10	2,156	2,558	3,247	3,94	4,865	9,342	15,987	18,307	20,483	23,209	25,188
11	2,603	3,053	3,816	4,575	5,578	10,341	17,275	19,675	21,92	24,725	26,757
12	3,074	3,571	4,404	5,226	6,304	11,34	18,549	21,026	23,337	26,217	28,3
13	3,565	4,107	5,009	5,892	7,042	12,34	19,812	22,362	24,736	27,688	29,819
14	4,075	4,66	5,629	6,571	7,79	13,339	21,064	23,685	26,119	29,141	31,319
15	4,601	5,229	6,262	7,261	8,547	14,339	22,307	24,996	27,488	30,578	32,801
16	5,142	5,812	6,908	7,962	9,312	15,338	23,542	26,296	28,845	32	34,267
17	5,697	6,408	7,564	8,672	10,085	16,338	24,769	27,587	30,191	33,409	35,718
18	6,265	7,015	8,231	9,39	10,865	17,338	25,989	28,869	31,526	34,805	37,156
19	6,844	7,633	8,907	10,117	11,651	18,338	27,204	30,144	32,852	36,191	38,582
20	7,434	8,26	9,591	10,851	12,443	19,337	28,412	31,41	34,17	37,566	39,997
21	8,034	8,897	10,283	11,591	13,24	20,337	29,615	32,671	35,479	38,932	41,401
22	8,643	9,542	10,982	12,338	14,041	21,337	30,813	33,924	36,781	40,289	42,796
23	9,26	10,196	11,689	13,091	14,848	22,337	32,007	35,172	38,076	41,638	44,181
24	9,886	10,856	12,401	13,848	15,659	23,337	33,196	36,415	39,364	42,98	45,559
25	10,52	11,524	13,12	14,611	16,473	24,337	34,382	37,652	40,646	44,314	46,928
26	11,16	12,198	13,844	15,379	17,292	25,336	35,563	38,885	41,923	45,642	48,29
27	11,808	12,879	14,573	16,151	18,114	26,336	36,741	40,113	43,195	46,963	49,645
28	12,461	13,565	15,308	16,928	18,939	27,336	37,916	41,337	44,461	48,278	50,993
29	13,121	14,256	16,047	17,708	19,768	28,336	39,087	42,557	45,722	49,588	52,336
30	13,787	14,953	16,791	18,493	20,599	29,336	40,256	43,773	46,979	50,892	53,672
31	14,458	15,655	17,539	19,281	21,434	30,336	41,422	44,985	48,232	52,191	55,003
32	15,134	16,362	18,291	20,072	22,271	31,336	42,585	46,194	49,48	53,486	56,328
33	15,815	17,074	19,047	20,867	23,11	32,336	43,745	47,4	50,725	54,776	57,648
34	16,501	17,789	19,806	21,664	23,952	33,336	44,903	48,602	51,966	56,061	58,964
35	17,192	18,509	20,569	22,465	24,797	34,336	46,059	49,802	53,203	57,342	60,275
37	18,586	19,96	22,106	24,075	26,492	36,336	48,363	52,192	55,668	59,893	62,883
38	19,289	20,691	22,878	24,884	27,343	37,335	49,513	53,384	56,896	61,162	64,181
39	19,996	21,426	23,654	25,695	28,196	38,335	50,66	54,572	58,12	62,428	65,476
40	20,707	22,164	24,433	26,509	29,051	39,335	51,805	55,758	59,342	63,691	66,766

weitere Werte: =CHIQU.INV(n;p) bzw. (R-Aufruf[4]): qchisq(p,n)

4 `http://netmath.vcrp.de/downloads/Systeme/R.html`

Kritische Werte für den Kolmogorov-Smirnov- (KS-) Anpassungstest						
n	$D_{0,20}$	$D_{0,10}$	$D_{0,05}$	$D_{0,02}$	$D_{0,01}$	$D_{0,005}$
1	0,9	0,95	0,975	0,99	0,995	0,9975
2	0,68377	0,77638	0,84187	0,89998	0,92925	0,94995
3	0,56481	0,63604	0,70758	0,78452	0,82895	0,86419
4	0,49265	0,56521	0,62392	0,68884	0,73417	0,77628
5	0,44697	0,50944	0,56326	0,62715	0,66848	0,70533
6	0,41035	0,46799	0,51925	0,57738	0,61655	0,65277
7	0,38145	0,43606	0,48341	0,53841	0,57576	0,60966
8	0,35828	0,40962	0,45426	0,50652	0,54174	0,5742
9	0,33907	0,38746	0,43	0,47957	0,51327	0,54435
10	0,32257	0,36866	0,40924	0,4566	0,48889	0,51864
11	0,30825	0,35241	0,39121	0,43668	0,46766	0,49631
12	0,29573	0,33814	0,37542	0,41916	0,449	0,47664
13	0,28466	0,32548	0,36142	0,4036	0,43243	0,45914
14	0,27477	0,31416	0,34889	0,38968	0,41758	0,44345
15	0,26585	0,30397	0,33759	0,37711	0,40416	0,42927
16	0,25774	0,29471	0,32733	0,36569	0,39197	0,41637
17	0,25035	0,28626	0,31796	0,35526	0,38083	0,40458
18	0,24356	0,2785	0,30935	0,34568	0,37059	0,39374
19	0,23731	0,27135	0,30142	0,33684	0,36114	0,38373
20	0,23152	0,26473	0,29407	0,32864	0,35238	0,37445
21	0,22614	0,25857	0,28724	0,32103	0,34423	0,36582
22	0,22111	0,25283	0,28086	0,31392	0,33663	0,35776
23	0,21642	0,24746	0,2749	0,30727	0,32951	0,35021
24	0,21201	0,24241	0,2693	0,30102	0,32283	0,34313
25	0,20786	0,23767	0,26404	0,29515	0,31654	0,33646
26	0,20396	0,2332	0,25907	0,28961	0,3106	0,33016
27	0,20026	0,22897	0,25437	0,28437	0,30499	0,32421
28	0,19676	0,22497	0,24993	0,2794	0,29968	0,31857
29	0,19344	0,22117	0,24571	0,27469	0,29463	0,31322
30	0,19029	0,21756	0,2417	0,27021	0,28984	0,30813
31	0,18728	0,21412	0,23788	0,26595	0,28527	0,30328
32	0,18442	0,21084	0,23424	0,26188	0,28091	0,29865
33	0,18168	0,20771	0,23076	0,25799	0,27675	0,29423
34	0,17906	0,20471	0,22743	0,25428	0,27276	0,29
35	0,17655	0,20184	0,22424	0,25072	0,26895	0,28595

Für $n > 35$ kann näherungsweise die Formel $d_\alpha = \frac{\sqrt{\ln(\frac{2}{\alpha})}}{\sqrt{2n}}$ verwendet werden.

Werte berechnet nach George Marsaglia et al . Evaluating Kolmogorov's DistributionEvaluating Kolmogorov's Distribution[5]. *Journal of Statistical Software* , **8** (18):1-4Nov 2003

5 `www.jstatsoft.org/v08/i18/`

F-Verteilung (rechtsseitige Quantile)										
n_1	α	**n_2= 1**	**n_2= 2**	**... 3**	**4**	**5**	**6**	**7**	**8**	**9**
1	0,950	161,45	18,513	10,128	7,709	6,608	5,987	5,591	5,318	5,117
	0,975	647,79	38,506	17,443	12,218	10,007	8,813	8,073	7,571	7,209
	0,990	4052,2	98,503	34,116	21,198	16,258	13,745	12,246	11,259	10,561
	0,995	16211	198,5	55,552	31,333	22,785	18,635	16,236	14,688	13,614
2	0,950	199,5	19	9,552	6,944	5,786	5,143	4,737	4,459	4,256
	0,975	799,5	39	16,044	10,649	8,434	7,26	6,542	6,059	5,715
	0,990	4999,5	99	30,817	18	13,274	10,925	9,547	8,649	8,022
	0,995	20000	199	49,8	26,284	18,314	14,544	12,404	11,042	10,107
3	0,950	215,71	19,164	9,277	6,591	5,409	4,757	4,347	4,066	3,863
	0,975	864,16	39,165	15,439	9,979	7,764	6,599	5,89	5,416	5,078
	0,990	5403,4	99,166	29,457	16,694	12,06	9,78	8,451	7,591	6,992
	0,995	21615	199,17	47,467	24,259	16,53	12,917	10,882	9,596	8,717
4	0,950	224,58	19,296	8,941	6,094	4,818	4,099	3,637	3,313	3,073
	0,975	899,58	39,298	14,735	9,074	6,757	5,523	4,761	4,243	3,868
	0,990	5624,6	99,299	27,911	14,976	10,289	7,976	6,62	5,734	5,111
	0,995	22500	199,3	44,838	21,622	13,961	10,391	8,38	7,104	6,227
5	0,950	230,16	19,296	9,013	6,256	5,05	4,387	3,972	3,687	3,482
	0,975	921,85	39,298	14,885	9,364	7,146	5,988	5,285	4,817	4,484
	0,990	5763,6	99,299	28,237	15,522	10,967	8,746	7,46	6,632	6,057
	0,995	23056	199,3	45,392	22,456	14,94	11,464	9,522	8,302	7,471
6	0,950	233,99	19,33	8,941	6,163	4,95	4,284	3,866	3,581	3,374
	0,975	937,11	39,331	14,735	9,197	6,978	5,82	5,119	4,652	4,32
	0,990	5859	99,333	27,911	15,207	10,672	8,466	7,191	6,371	5,802
	0,995	23437	199,33	44,838	21,975	14,513	11,073	9,155	7,952	7,134
7	0,950	236,77	19,353	8,887	6,094	4,876	4,207	3,787	3,5	3,293
	0,975	948,22	39,355	14,624	9,074	6,853	5,695	4,995	4,529	4,197
	0,990	5928,4	99,356	27,672	14,976	10,456	8,26	6,993	6,178	5,613
	0,995	23715	199,36	44,434	21,622	14,2	10,786	8,885	7,694	6,885
8	0,950	238,88	19,371	8,845	6,041	4,818	4,147	3,726	3,438	3,23
	0,975	956,66	39,373	14,54	8,98	6,757	5,6	4,899	4,433	4,102
	0,990	5981,1	99,374	27,489	14,799	10,289	8,102	6,84	6,029	5,467
	0,995	23925	199,37	44,126	21,352	13,961	10,566	8,678	7,496	6,693
9	0,950	240,54	19,385	8,812	5,999	4,772	4,099	3,677	3,388	3,179
	0,975	963,28	39,387	14,473	8,905	6,681	5,523	4,823	4,357	4,026
	0,990	6022,5	99,388	27,345	14,659	10,158	7,976	6,719	5,911	5,351
	0,995	24091	199,39	43,882	21,139	13,772	10,391	8,514	7,339	6,541
10	0,950	241,88	19,396	8,786	5,964	4,735	4,06	3,637	3,347	3,137
	0,975	968,63	39,398	14,419	8,844	6,619	5,461	4,761	4,295	3,964
	0,990	6055,8	99,399	27,229	14,546	10,051	7,874	6,62	5,814	5,257
	0,995	24224	199,4	43,686	20,967	13,618	10,25	8,38	7,211	6,417
11	0,950	242,98	19,405	8,763	5,936	4,704	4,027	3,603	3,313	3,102
	0,975	973,03	39,407	14,374	8,794	6,568	5,41	4,709	4,243	3,912
	0,990	6083,3	99,408	27,133	14,452	9,963	7,79	6,538	5,734	5,178
	0,995	24334	199,41	43,524	20,824	13,491	10,133	8,27	7,104	6,314

F-Verteilung (rechtsseitige Quantile)										
n_1	α	**n_2= 1**	**n_2= 2**	**... 3**	**4**	**5**	**6**	**7**	**8**	**9**
12	0,950	243,91	19,413	8,745	5,912	4,678	4	3,575	3,284	3,073
	0,975	976,71	39,415	14,337	8,751	6,525	5,366	4,666	4,2	3,868
	0,990	6106,3	99,416	27,052	14,374	9,888	7,718	6,469	5,667	5,111
	0,995	24426	199,42	43,387	20,705	13,384	10,034	8,176	7,015	6,227
13	0,950	244,69	19,419	8,729	5,891	4,655	3,976	3,55	3,259	3,048
	0,975	979,84	39,421	14,304	8,715	6,488	5,329	4,628	4,162	3,831
	0,990	6125,9	99,422	26,983	14,307	9,825	7,657	6,41	5,609	5,055
	0,995	24505	199,42	43,271	20,603	13,293	9,95	8,097	6,938	6,153
14	0,950	245,36	19,424	8,715	5,873	4,636	3,956	3,529	3,237	3,025
	0,975	982,53	39,427	14,277	8,684	6,456	5,297	4,596	4,13	3,798
	0,990	6142,7	99,428	26,924	14,249	9,77	7,605	6,359	5,559	5,005
	0,995	24572	199,43	43,172	20,515	13,215	9,877	8,028	6,872	6,089
15	0,950	245,95	19,429	8,703	5,858	4,619	3,938	3,511	3,218	3,006
	0,975	984,87	39,431	14,253	8,657	6,428	5,269	4,568	4,101	3,769
	0,990	6157,3	99,433	26,872	14,198	9,722	7,559	6,314	5,515	4,962
	0,995	24630	199,43	43,085	20,438	13,146	9,814	7,968	6,814	6,032
20	0,950	248,01	19,446	8,66	5,803	4,558	3,874	3,445	3,15	2,936
	0,975	993,1	39,448	14,167	8,56	6,329	5,168	4,467	3,999	3,667
	0,990	6208,7	99,449	26,69	14,02	9,553	7,396	6,155	5,359	4,808
	0,995	24836	199,45	42,778	20,167	12,903	9,589	7,754	6,608	5,832
40	0,950	251,14	19,471	8,594	5,717	4,464	3,774	3,34	3,043	2,826
	0,975	1005,6	39,473	14,037	8,411	6,175	5,012	4,309	3,84	3,505
	0,990	6286,8	99,474	26,411	13,745	9,291	7,143	5,908	5,116	4,567
	0,995	25148	199,47	42,308	19,752	12,53	9,241	7,422	6,288	5,519
50	0,950	251,77	19,476	8,581	5,699	4,444	3,754	3,319	3,02	2,803
	0,975	1008,1	39,478	14,01	8,381	6,144	4,98	4,276	3,807	3,472
	0,990	6302,5	99,479	26,354	13,69	9,238	7,091	5,858	5,065	4,517
	0,995	25211	199,48	42,213	19,667	12,454	9,17	7,354	6,222	5,454
75	0,950	252,62	19,482	8,563	5,676	4,418	3,726	3,29	2,99	2,771
	0,975	1011,5	39,485	13,974	8,34	6,101	4,937	4,232	3,762	3,426
	0,990	6323,6	99,486	26,278	13,615	9,166	7,022	5,789	4,998	4,449
	0,995	25295	199,49	42,086	19,554	12,351	9,074	7,263	6,133	5,367
100	0,950	253,04	19,486	8,554	5,664	4,405	3,712	3,275	2,975	2,756
	0,975	1013,2	39,488	13,956	8,319	6,08	4,915	4,21	3,739	3,403
	0,990	6334,1	99,489	26,24	13,577	9,13	6,987	5,755	4,963	4,415
	0,995	25337	199,49	42,022	19,497	12,3	9,026	7,217	6,088	5,322
150	0,950	253,46	19,489	8,545	5,652	4,392	3,698	3,26	2,959	2,739
	0,975	1014,9	39,491	13,938	8,299	6,059	4,893	4,188	3,716	3,38
	0,990	6344,7	99,492	26,202	13,539	9,094	6,951	5,72	4,929	4,38
	0,995	25380	199,49	41,957	19,44	12,248	8,977	7,17	6,042	5,278

weitere Werte: =F.INV(α ;n1;n2) bzw. (R-Aufruf[6]): qf(α,n1,n2)

6 `http://netmath.vcrp.de/downloads/Systeme/R.html`

t-(Student-)Verteilung (Quantile)									
n	$\alpha =$ 0,75	... 0,90	0,95	0,975	0,99	**0,995**	0,999	0,9995	0,9999
1	1,00	3,078	6,314	12,706	31,821	63,657	318,31	636,62	3183,1
2	0,816	1,886	2,92	4,303	6,965	9,925	22,327	31,599	70,700
3	0,765	1,638	2,353	3,182	4,541	5,841	10,215	12,924	22,204
4	0,741	1,533	2,132	2,776	3,747	4,604	7,173	8,61	13,034
5	0,727	1,476	2,015	2,571	3,365	4,032	5,893	6,869	9,678
6	0,718	1,44	1,943	2,447	3,143	3,707	5,208	5,959	8,025
7	0,711	1,415	1,895	2,365	2,998	3,499	4,785	5,408	7,063
8	0,706	1,397	1,86	2,306	2,896	3,355	4,501	5,041	6,442
9	0,703	1,383	1,833	2,262	2,821	3,25	4,297	4,781	6,01
10	0,7	1,372	1,812	2,228	2,764	3,169	4,144	4,587	5,694
11	0,697	1,363	1,796	2,201	2,718	3,106	4,025	4,437	5,453
12	0,695	1,356	1,782	2,179	2,681	3,055	3,93	4,318	5,263
13	0,694	1,35	1,771	2,16	2,65	3,012	3,852	4,221	5,111
14	0,692	1,345	1,761	2,145	2,624	2,977	3,787	4,14	4,985
15	0,691	1,341	1,753	2,131	2,602	2,947	3,733	4,073	4,88
16	0,69	1,337	1,746	2,12	2,583	2,921	3,686	4,015	4,791
17	0,689	1,333	1,74	2,11	2,567	2,898	3,646	3,965	4,714
18	0,688	1,33	1,734	2,101	2,552	2,878	3,61	3,922	4,648
19	0,688	1,328	1,729	2,093	2,539	2,861	3,579	3,883	4,59
20	0,687	1,325	1,725	2,086	2,528	2,845	3,552	3,85	4,539
21	0,686	1,323	1,721	2,08	2,518	2,831	3,527	3,819	4,493
22	0,686	1,321	1,717	2,074	2,508	2,819	3,505	3,792	4,452
23	0,685	1,319	1,714	2,069	2,5	2,807	3,485	3,768	4,415
24	0,685	1,318	1,711	2,064	2,492	2,797	3,467	3,745	4,382
25	0,684	1,316	1,708	2,06	2,485	2,787	3,45	3,725	4,352
26	0,684	1,315	1,706	2,056	2,479	2,779	3,435	3,707	4,324
27	0,684	1,314	1,703	2,052	2,473	2,771	3,421	3,69	4,299
28	0,683	1,313	1,701	2,048	2,467	2,763	3,408	3,674	4,275
29	0,683	1,311	1,699	2,045	2,462	2,756	3,396	3,659	4,254
30	0,683	1,31	1,697	2,042	2,457	2,75	3,385	3,646	4,234
31	0,682	1,309	1,696	2,04	2,453	2,744	3,375	3,633	4,216
32	0,682	1,309	1,694	2,037	2,449	2,738	3,365	3,622	4,198
33	0,682	1,308	1,692	2,035	2,445	2,733	3,356	3,611	4,182
34	0,682	1,307	1,691	2,032	2,441	2,728	3,348	3,601	4,167
35	0,682	1,306	1,69	2,03	2,438	2,724	3,34	3,591	4,153
37	0,681	1,305	1,687	2,026	2,431	2,715	3,326	3,574	4,127
38	0,681	1,304	1,686	2,024	2,429	2,712	3,319	3,566	4,116
39	0,681	1,304	1,685	2,023	2,426	2,708	3,313	3,558	4,105
40	0,681	1,303	1,684	2,021	2,423	2,704	3,307	3,551	4,094

weitere Werte: =T.INV(α ;n) bzw. (R-Aufruf[7]): qt(α ,n)

7 `http://netmath.vcrp.de/downloads/Systeme/R.html`

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Version 3, 29 June 2007

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