NPS ARCHIVE
1968
ALLEN, T.

AIRCRAFT SURVIVABILITY INDEX FOR LOW ALTITUDE PENETRATION OF AN AIR DEFENSE COMPLEX

by

Thomas Harrison Allen, Jr.

UNITED STATES
NAVAL POSTGRADUATE SCHOOL

THESIS

AIRCRAFT SURVIVABILITY INDEX FOR LOW ALTITUDE
PENETRATION OF AN AIR DEFENSE COMPLEX

by

Thomas Harrison Allen, Jr.

December 1968

This document has been approved for public release and sale; its distribution is unlimited.

Library
U.S. Naval Postgraduate School
Monterey, California 93940

AIRCRAFT SURVIVABILITY INDEX FOR LOW ALTITUDE PENETRATION OF AN AIR DEFENSE COMPLEX

by

Thomas Harrison Allen, Jr.
Lieutenant Colonel, United States Marine Corps
B.S., Naval Academy, 1953

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL

December 1968

NPS ARCHIVE
1968
ALLEN, T.

ABSTRACT

This paper develops a model for computing the probability of kill for an air defense complex composed of antiaircraft automatic weapons, radar controlled guns, and missile batteries. Two dimensional terrain was used to evaluate the model. The probabilities were determined at major terrain points along the route of approach to the vital area for altitudes of up to 3000 feet above terrain. The curves of probability of kill versus altitude were found to be dependent on terrain, air defense tactics, and weapon system parameters. A survivability index is calculated by combining the probabilities of kill with a pilot visual navigational probability. The resulting curves of survivability index versus altitude were found to be nonlinear requiring a nonlinear programing technique to solve for the altitude of optimal survivability index within aircraft flight path constraints. The nonlinear solution was not included in this work.

Library
U.S. Naval Postgraduate School
Monterey, California 93940

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

ACKNOWLEDGEMENTS

I wish to express my appreciation to Lieutenant Colonel W. S. Miller, Jr., USMC and Major E. E. Brown, USMC for their assistance in the early development of this work. Their efforts in pioneering the way on this problem made possible my contribution.

I also wish to thank Professor W. Peyton Cunningham for his guidance and council during the preparation of this paper.

CHAPTER I

During the first year of the United States' involvement with the war in South East Asia, air power was employed extensively against many targets in North Vietnam. The commanders of attack aviation units were required to plan and execute strikes against the same targets over and over again. Initially air defense of these targets was sparse to nonexistent. As time passed, air defense weapons were supplied from communist bloc countries until the problem presented to the attack force commander was what to do about higher and higher attrition rates per raid. Specifically, strike planners were seeking a solution to the problem of finding the route of approach and altitude to fly into the target which would minimize aircraft losses from air defense means. One method of finding an answer to the problem is to analyze the losses from previous raids. This method is costly in lives and equipment and may not have produced data describing results from many altitudes and routes into the target. Using previous experience alone, one may never discover the optimal route. A major factor in the effectiveness of land air defenses is the terrain. Very often terrain provides a natural route of approach which in itself minimizes the capability of the air defense efforts. Sometimes this fact is obvious and in other situations it is not. It would be desirable in such a situation to provide the attack air force commanders with a method that determines the optimal approach altitude and route that minimizes the air defense effort. The method would generate an actual flight profile for the pilot. As new data on the enemy's air defenses are ascertained, updated flight profiles would be provided in the time period between strikes. The method referred to here is a digital computer model which evaluates all possible routes of approach into a target. For each route, the model evaluates the terrain, the air defenses situated therein, and the pilot navigational problem for all feasible altitudes up to a maximum value, and then determines the altitude along the route which maximizes aircraft survivability. With a functioning computer program which computes such a flight profile, new information could be inserted into the program and new results obtained in a matter of hours. Such a tool would be of tremendous value to attack air force commanders.

The development of a mathematical model and computer solution for generating a flight profile which maximizes strike aircraft survivability was accomplished in a thesis by Lieutenant Colonel W. S. Miller, Jr., USMC, and Major E. E. Brown, USMC, reference 1. In this work a model for calculating probabilities of detection of strike aircraft over two dimensional terrain was combined with calculated probabilities of detection of ground navigational targets to generate a functional relationship of cost versus altitude of the flight path above terrain. The term cost is defined to be a probability index which is a linear combination of probability of radar detection and the probability that the ground navigational target was not detected at each altitude above terrain from 100 feet to 3,000 feet. Using this function as an objective function and calculating aircraft flight path constraints for each terrain point, the problem of determining that altitude which minimized the cost was solved as a linear program. The probability of detection mentioned above was calculated by computing at each terrain point and altitude the ratio of terrain visable to the total terrain within the coordinate system. As altitude increased, less terrain was masked from the point in question. Therefore, the probability of detection was found to increase in approximately a linear fashion. This model assumed that a radar within the air defense system could be placed at any and all points along the terrain under the aircraft flight path. In other words, radar locations were considered to be uniformly distributed over the entire route of approach. To be sure, this method of calculating the probability of detection produces an indication of the effect of altitude on terrain masking in evaluating air defense capability.

This paper extends the work of Miller and Brown. Since the uniform distribution of radar sites is not realistic and the method of calculating probability of radar detection is at best only an indication of air defense capability, a mathematical model and computer program will be developed to calculate the probability of kill for three types of air defense weapons sited in the terrain. Specifically, the problem to be solved will be to emplace an air defense complex in a section of terrain in order to calculate at each terrain point and altitude above terrain from 100 feet to 3,000 feet along the aircraft flight path the cumulative probability of kill for an aircraft flying straight and level up to each terrain point. The terrain points considered are those within the maximum effective range of the longest ranged air defense unit to the defended area. The problem reduces to finding the length of course line exposed to radar detection and within the effective range of the weapon systems in order to determine the number of rounds that may be fired from which the probability of kill is determined. The model evaluates the terrain masking on detection range and firing time. The problem is developed for three dimensional terrain. The computer model was programmed for three dimensional terrain. No digitalized three dimensional terrain was found for use in this model. The computer program was accordingly verified using only two dimensional terrain which was available. That portion of the program which evaluates the three dimensional terrain was not verified.

CHAPTER II

THE MATHEMATICAL MODEL

A mathematical model will be designed to compute the probability of kill for an approaching aircraft penetrating an air defense complex composed of heavy antiaircraft gun batteries, light antiaircraft automatic weapon batteries, and surface to air missile batteries. The purpose of this model is to determine a relationship of altitude versus probability at each major terrain point through the air defense complex. The model is designed to be general. No specific weapon system parameters in existence are used in the calculations. Values for the parameters are of the order of those found in actual systems. The general nature of the model was selected in order to permit its use with any system and to prevent the necessity of classification under security regulations.

PROBLEM FORMULATION

Since it is necessary to compute the probability of kill at each terrain point and at altitudes of every 100 feet above terrain to 3,000 feet, a concept for what is meant by a kill probability at a large number of specific points in space along a route of approach must be defined. The probability of kill by a weapon system is not defined at a point. Probability is calculated from the number of rounds the weapon system is capable of firing while the target is within range. The kill probability at any particular terrain point and altitude, P_{Kki}, is defined to be the cumulative probability of kill resulting from the fire of all batteries of each type that bear on the target from each battery's maximum range to the terrain point in question. The amount of fire that each battery may produce is further constrained by terrain masking at the lower altitudes as it may occur. For example, at some terrain point k and altitude i, a line is drawn back along the route of approach to the beginning of the coordinate system or to the point of contact with a terrain formation, whichever is the shorter. Figure 1 is a diagram illustrating a two dimensional view of the terrain showing terrain points with altitude points above the terrain and the course lines associated with the (k,i)th point, representing the kth terrain point and ith altitude. In the figure, point F represents the 4th terrain point and the 1st altitude. Drawn from this point is a line back along the course line of aircraft flight as previously described. In this case, the line intersects a terrain formation at point G. The model assumes that when terrain interrupts a continuous flight, the aircraft will fly over the obstruction and continue on at the prescribed altitude. The line from G to F will represent a portion of the aircraft flight path. If a battery were located at point L, a probability of kill will be calculated from the number of rounds fired if sufficient time permits the firing of any rounds from any battery that bears on a target flying

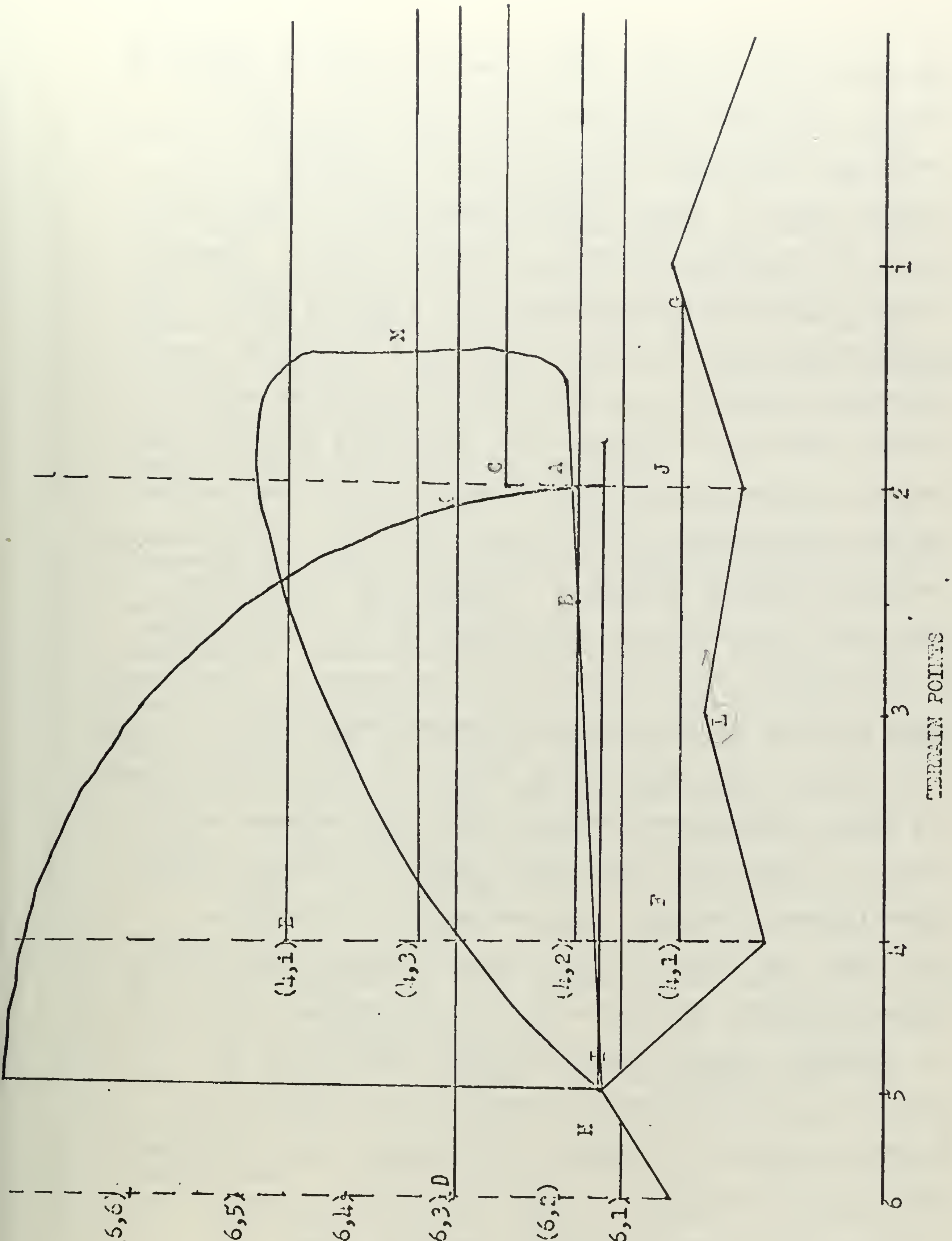


FIGURE 1

the course from G to F. Such a battery would be located at point L. The value of probability calculated here will be a number greater than or equal to zero for altitude 1 at terrain point 4. These numbers become part of some functional relationship existing at terrain point 4. Another example of discontinuous flight is at points H and I. The uninterrupted case is seen at point A, C, D, and E. Here no terrain obstruction enters into the situation and the factors effecting the probability calculated depend on the length of course lying in the detection radar beam pattern and the weapon system effectiveness envelope. As an example, consider a battery located at terrain point 5. If the model were evaluating the probability of kill contributed by this battery up to point (4,2), the number of rounds that could be fired as the aircraft traveled from point B to point (4,2) would be the determining factor. B is the point where the flight path passes into the radar beam pattern while point A is where the flight path enters the weapon system effectiveness envelope. The portion of the flight in both radar and battery effectiveness envelope only, enables firing to occur. Since the altitude of approach is so low, the aircraft cannot first be detected until point B. At point (4, 3) the battery at terrain point 5 detects the targets at point M in advance of its entry into the battery effectiveness envelope. A normal fire mission can be conducted in this case with adequate warning time.

Coordinate System

In order for calculations to be carried out by the computer a coordinate system must be placed on the terrain section of interest. No existing map coordinate system will suffice. The coordinate square must be small enough to provide an acceptable approximation of the terrain contours. Figure 2 is a diagram from a top view of the terrain and battery situation. This view shows two dimensions, distance along the course line, and offset distance from the course line for terrain and battery positions.

The grid system shown in Figure 2 shall be the size necessary to cover the section of terrain holding the air defense complex of interest. A route of approach to be analyzed is selected and the grid system is placed over this route on an appropriate map with the X axis parallel to the course direction and centered over the line. At the intersection of each grid line, the altitude taken from the terrain contour lines is recorded along with the X and Y coordinates. The altitude above sea level becomes the H coordinate. All distances in this model are in feet. The X, Y, and H values are then punched on IBM cards for placing on computer tape as data input. The job of preparing map data for transfer to computer tape is tedious and time consuming. For an individual to attempt the task would be impractical. A team of trained operations plotters equiped with the proper equipment would be capable of accomplishing the task. Since no three dimensional data was available,

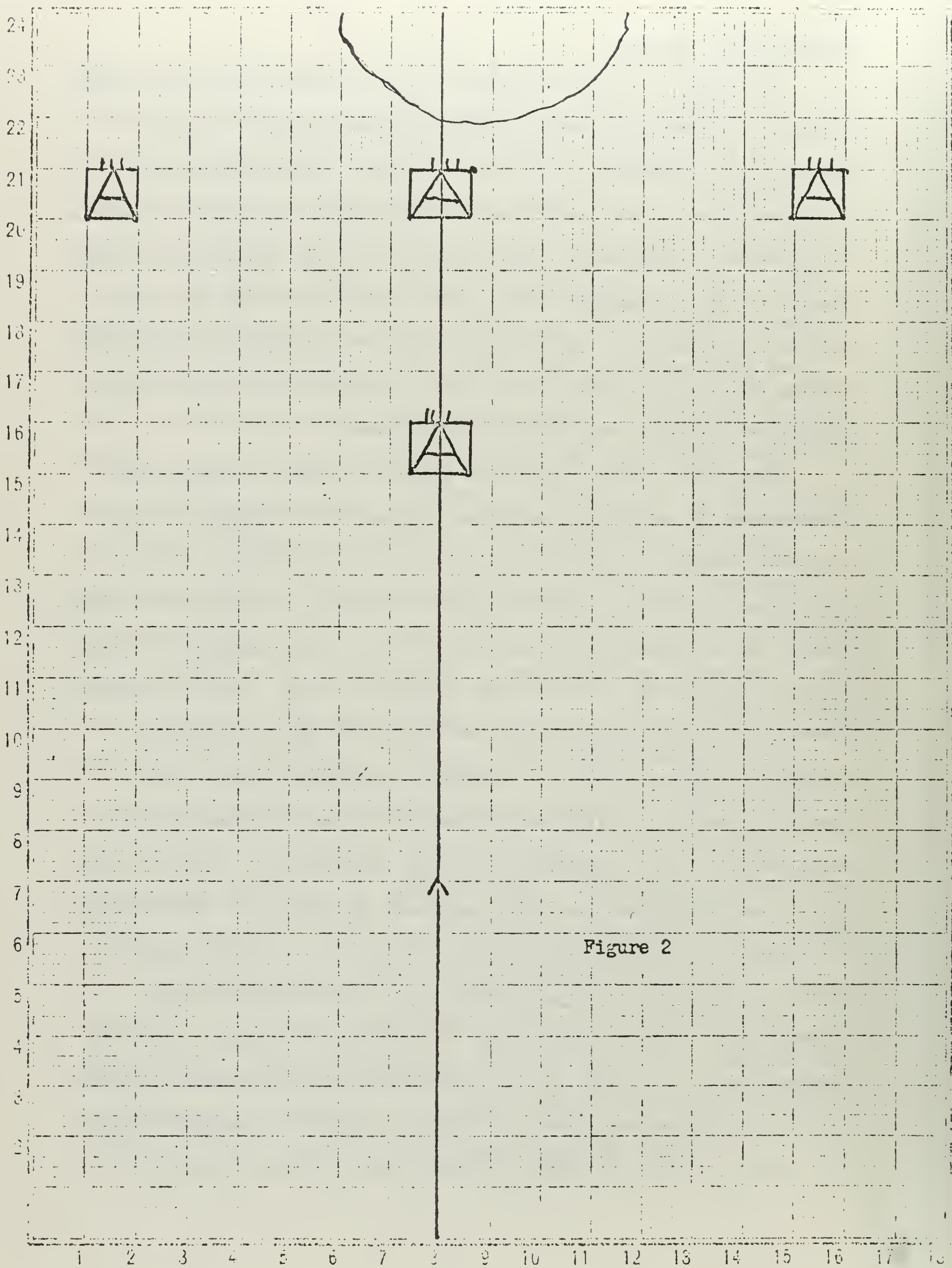


Figure 2

the computer model was solved using two dimensional data. The model was verified with this data except for the line of sight calculations for batteries not on the target flight line. The two dimensional terrain was taken from reference 1. Each major terrain point was selected as either a peak or a valley. A sample of the terrain is shown in Figure 3 and 4. Figure 3 is the detailed profile of the terrain. Figure 4 is the simplified version obtained by drawing straight lines between major terrain points. For a sample of the terrain actually used in the model see Figure 5 through 7. The coordinates of the major peaks and valleys in the X direction and altitude in the H direction are recorded in Appendix 2. The offset distance Y is zero for all computations but is carried along in the model for possible future use. Coordinates of the battery positions used are found in Appendix 2. Referring again to Figure 1, it should be noted that the X coordinate has its lowest value at the entrance of the air defense complex and increases to the maximum X coordinate value at the defended area.

Air Defense Complex

The three general types of weapons systems are the antiaircraft automatic weapon battery, the medium to heavy antiaircraft artillery battery, and the surface to air missile battery. These three are used since they cover the spectrum of ground to air weapon systems to be encountered in the reasonable future.

DETAILED TERRAIN PROFILE

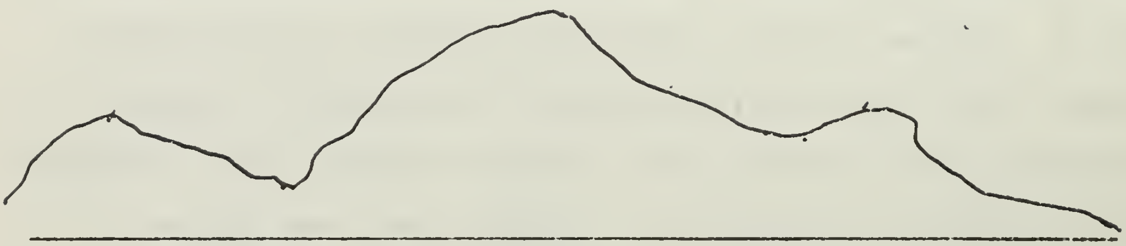


FIGURE 3

SIMPLIFIED TERRAIN PROFILE

FIGURE 4

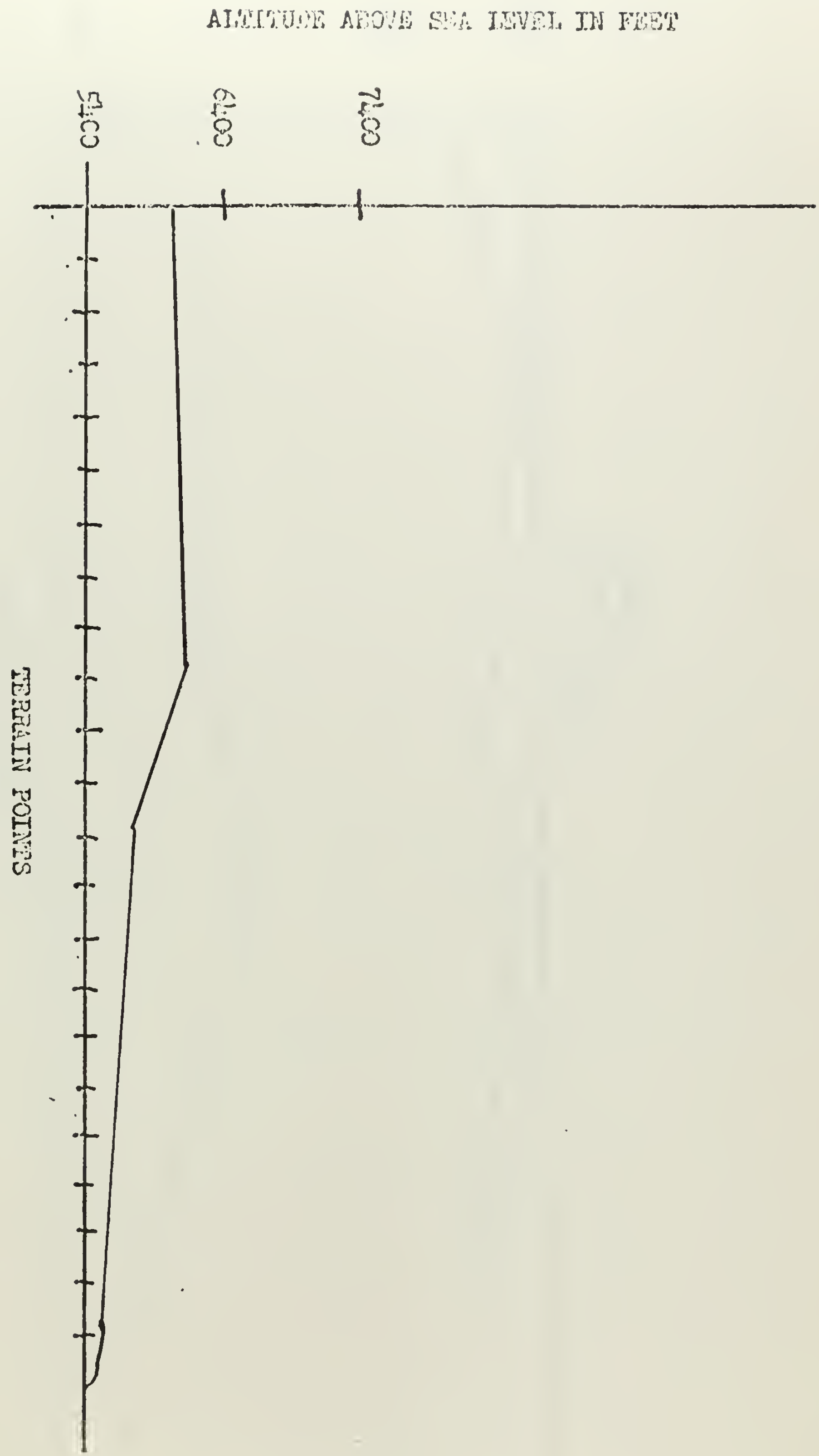
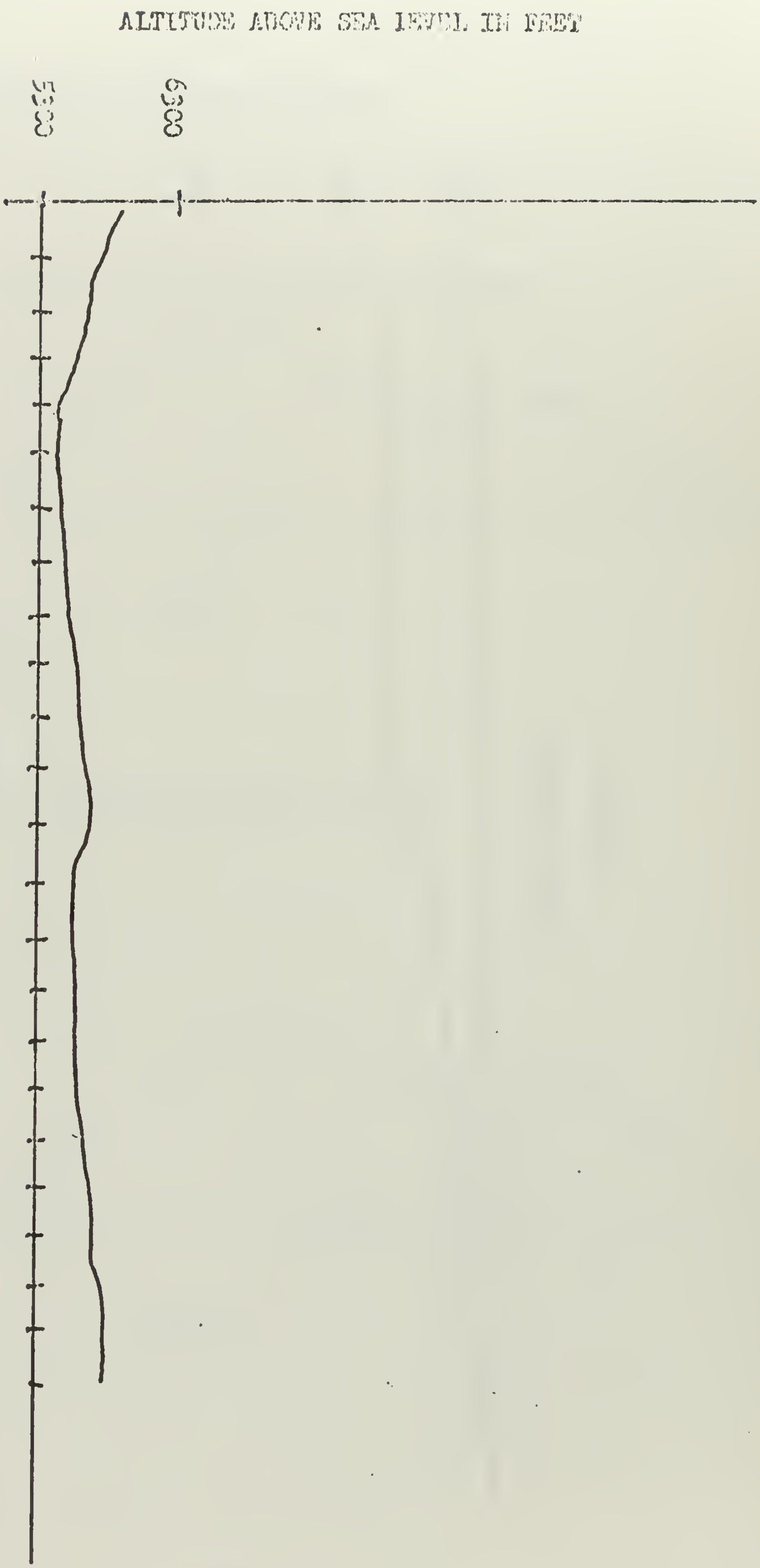

ALTITUDE ABOVE SEA LEVEL IN FEET
5400
6400
7400
TERRAIN POINTS

FIGURE 5



TERRAIN POINTS

FIGURE 6

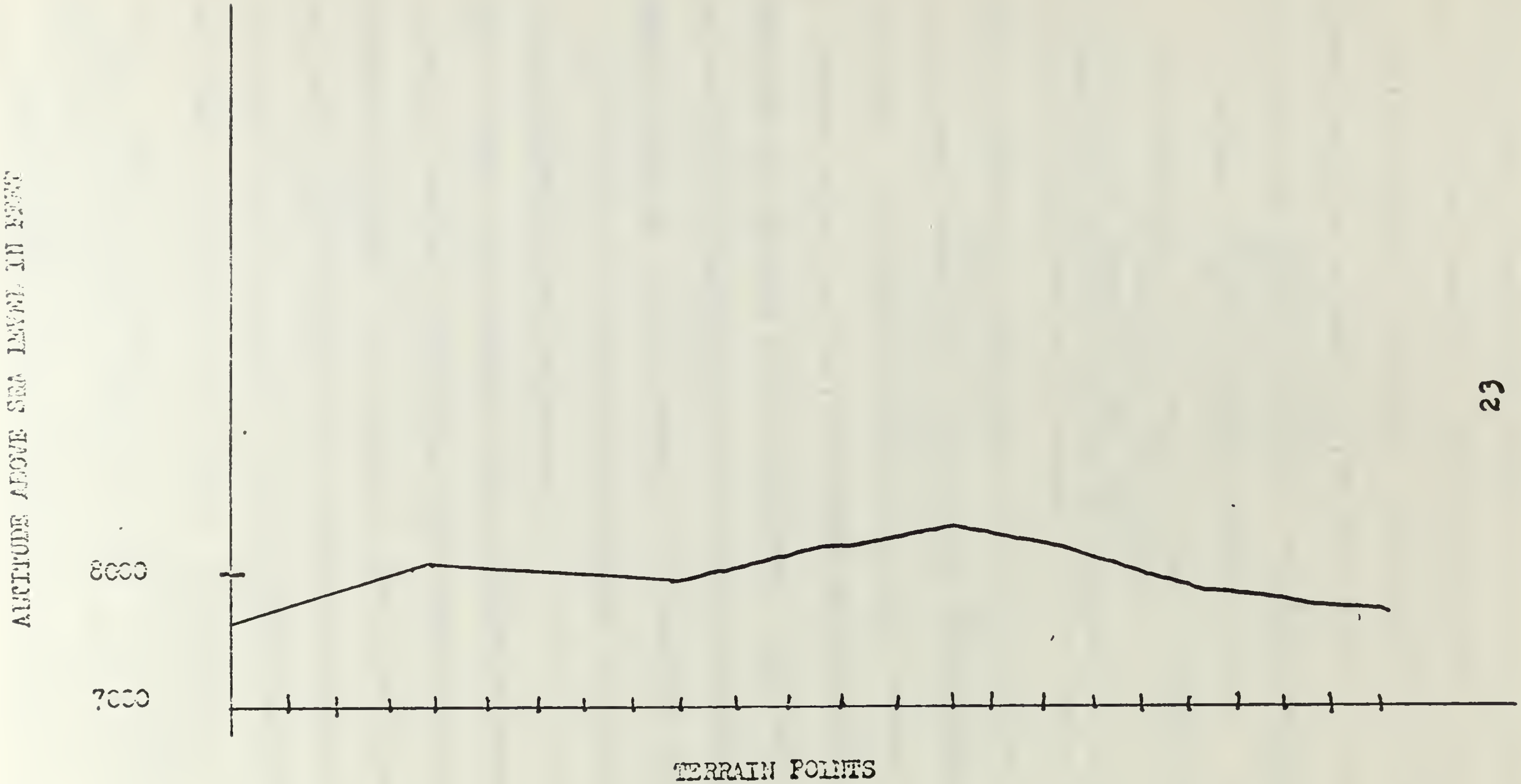


FIGURE 7

The automatic weapons battery is defined to be a 10 gun unit of rapid firing weapons, five mounts, two guns to a mount. The fire control is by optics with target course and speed estimated by the crew. A computer is used to generate predicted azimuth and elevation for positioning the gun. Early warning is by radar with target acquisition by optical means. The medium to heavy antiaircraft artillery battery is composed of 4 guns with early warning radar, gun laying radar, and computer generated gun positioning signals via appropriate servo systems.

The surface to air missile system is a low to medium altitude system. It is capable of detecting targets within its early radar beam pattern which is approximately from 1° elevation to 50° for volume coverage. The radar is assumed to possess effective moving target indication at the low altitudes. The tracking radars are also assumed to have low altitude capability. Since only one aircraft is attacking the complex in this model it is not necessary to specify the type of guidance with respect to the number of targets the system may attack at one time.

The batteries are placed in the 2 dimensional terrain on the course line at various terrain points. The effect of terrain and increasing altitude, and the cumulative effect of more than one battery firing on a target is obtained from the various battery locations. No battery is permitted to fire at outbound targets. Each battery may fire from its maximum range or detection range up to its minimum range or to its maximum turning rate in azimuth to the terrain point in question, whichever occurs first. No battery is permitted to fire on targets at a lower altitude than its own since this is not feasibly done.

Weapon Target Kinematics

In order to determine the length of target course line exposed to the fire of any battery, it is necessary to define the parameters of weapon target dynamics. Figure 8 is a three dimensional diagram of the problem to be solved. The line C D is the path the aircraft flies as it passes into the range of battery J as evaluated at the (k, i)th point. Line A B is the projection of target course on a horizontal plane at the same altitude as the jth battery. The ΔH, the difference in altitude between the jth battery and the ith altitude of approach, is given by

$$\Delta H = |A_{Hk,i} - B_{Hj}| \qquad \begin{array}{l} K = 1, \ldots, N \\ i = 1, \ldots, 30 \\ j = 1, \ldots, N_1 \end{array} \quad . \tag{1}$$

The difference in position of the kth terrain point and the jth battery along the course line is given by

$$\Delta X = |X_{k,i} - B_{xj}| \qquad \begin{array}{l} K = 1, \ldots, N \\ i = 1, \ldots, 30 \\ j = 1, \ldots, N_1 \end{array} \quad . \tag{2}$$

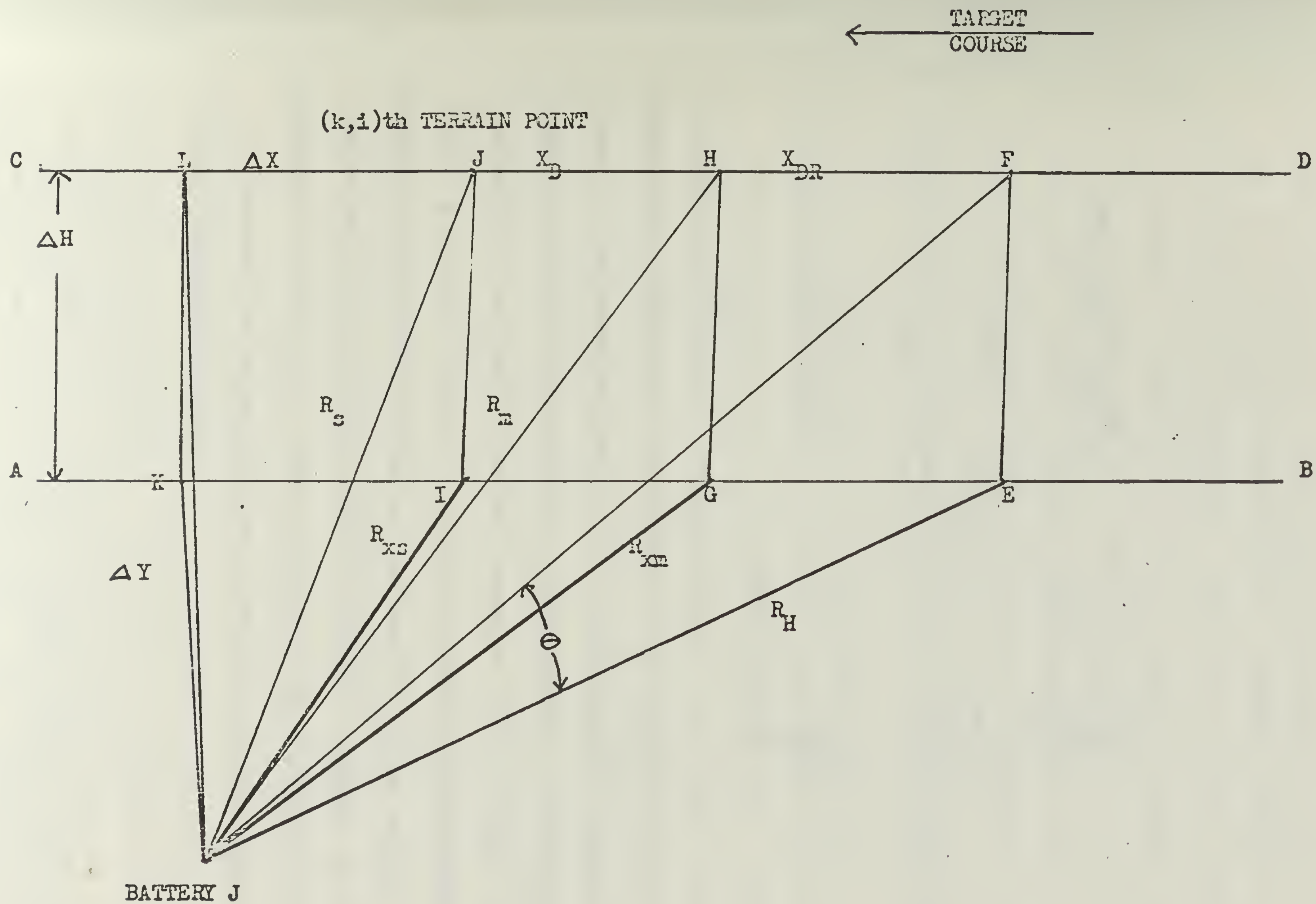

TARGET COURSE
(k,i)th TERRAIN POINT
C
L
ΔX
J
X_D
H
X_{DR}
F
D
ΔH
R_s
R_m
A
K
I
G
E
B
R_{xs}
ΔY
R_{xm}
θ
R_H
BATTERY J

FIGURE 8

Likewise for the offset distance of the jth battery

$$\Delta Y = |Y_{k,j} - B_{Y,j}| \qquad \begin{array}{l} k = 1, \ldots, N \\ i = 1, \ldots, 30 \\ j = 1, \ldots, N_1 \end{array} , \tag{3}$$

where

$$AH_{k,i} = H_k + 100\,i \qquad \begin{array}{l} i = 1, \ldots, 30 \\ k = 1, \ldots, N \end{array} \tag{4}$$

N = number of terrain points where a probability versus altitude function will be calculated

30 = number of altitude points above terrain for which probability calculations will be accomplished

N_1 = number of weapon batteries employed in the terrain.

The other parameters are defined as follows:

R_s = slant range from the battery to the (k,i)th point

R_{xs} = horizontal component of R_s

R_m = maximum effective range of the weapon system

R_{xm} = horizontal component of R_m

R_h = horizontal component of the range at which the lower portion of the radar beam, at the estimated half power point of the radar beam pattern, intersects the target course line

X_D = the distance J L along the course line from the maximum open fire range of the system to the (k, i)th point being evaluated. This parameter is the key distance which will determine the probability to be developed later.

X_{DR} = the warning distance, H J from maximum detection range to maximum open fire range

Θ = the estimated elevation of the half power point of the lower edge of the radar beam pattern.

Calculating Equations of the Model

In this section all calculating equations will be explained or derived as appropriate.

Prevent Outbound Firing. In a high threat operating condition when enemy attack is probable or imminent, air defense commanders are more concerned with the inbound raid than outbound raids. If a target passes through the fires of a particular battery, the probability of an outbound kill is less in most cases than an inbound raid. The danger of firing at outbound raids results from not being ready to engage the next inbound raid. Therefore the policy of not firing past the position of any battery is designed into the model. To insure no outbound firing ΔX is calculated in equation (2) without the absolute value notation.

$$\Delta X = \begin{cases} X_{k,i} - B_{xj} & ; \ \Delta X < 0 \\ 0 & ; \ \Delta X > 0 \end{cases} \qquad (5)$$

Since any terrain point with a greater X coordinate value lies in an outbound direction from the jth battery, $\Delta X > 0$ in this case. Firing is permitted up to the minimum range of the Jth battery, therefore setting $\Delta X = 0$ permits later calculations for X_D up to B_{xj} rather than to $X_{k,i}$. Any terrain whose X coordinate is less than B_{xj} will produce a $\Delta X > 0$ and ΔX will be set equal to $/X_{ki} - B_{xj}/$.

Determine Slant Range. The slant range to the (k, i)th point is given by

$$R_s = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta H^2} \tag{6}$$

In Figure 8, it can be seen that R_s is the vector sum of $\Delta X, \Delta Y$, and ΔH. Therefore equation (6) will hold. For any battery j whose maximum range $R_m < R_s$ will not contribute any probability of kill. Therefore no calculations for the jth battery will be carried out in this case.

Determine if Target Elevation is Negative. Since firing at elevations below 0° is not feasible for many reasons, this model excludes such cases. After solving for ΔH in (1), if $\Delta H < 0$, excludes the jth battery from a probability calculation. Figure 9 illustrates this case.

Minimum Firing Range. In certain cases which arise in this model the (k, i)th point lies over the jth battery or is inside the dead zone of battery effectiveness. The model must determine when this situation occurs and permit firing up to the minimum firing range for the ith altitude of approach. Figure 10 is a diagram of the minimum range

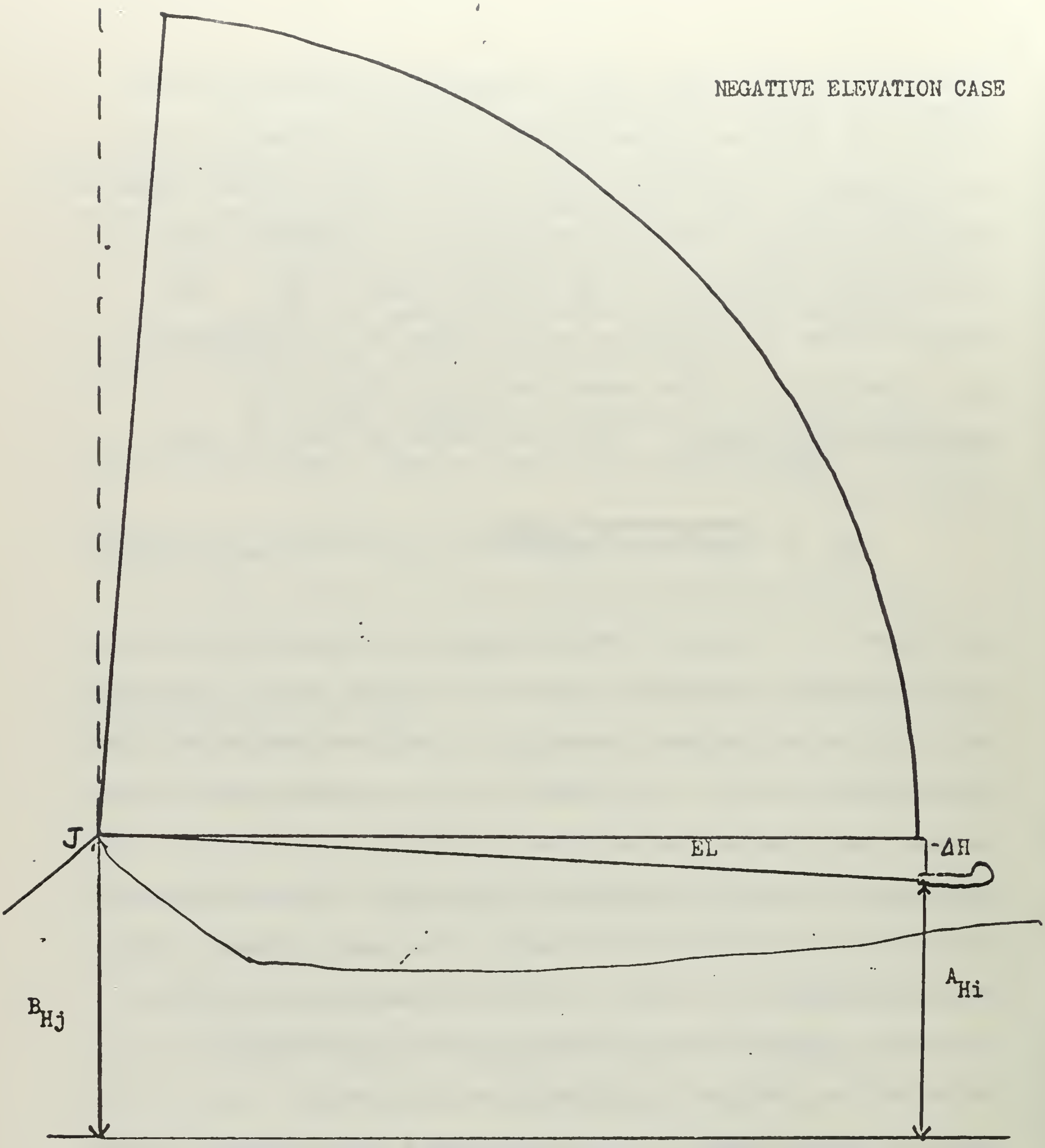


Figure 9

situation. R_F is the minimum firing range for the system. Angle B is the maximum elevation angle which is the minimum of the maximum radar antenna elevation angle and maximum elevation angle of the gun or launcher.

$$R_{XF} = R_F \cos B \tag{7}$$

R_{XS} as previously defined is given by

$$R_{XS} = \sqrt{R_S^{2} - \Delta H^{2}} \tag{8}$$

from Figure 8. If $R_{XS} < R_{XF}$, then firing must terminate at a horizontal range of R_{XF} rather than R_{XS}. Therefore equation (6) becomes

$$R_S = \begin{cases} \sqrt{R_{XS}^{2} + \Delta H^{2}} & ; \quad R_{XS} > R_{XF} \\ \sqrt{R_{XF}^{2} + \Delta H^{2}} & ; \quad R_{XS} < R_{XF} \end{cases} \tag{9}$$

<u>Maximum Azimuth Tracking Rates.</u> When the jth battery is at some distance ΔY from the course line, it is possible for the maximum tracking rate of the system to be exceeded before the (K, i)th point is reached. If this should occur the distance X_D would be decreased. The amount of this decrease is calculated by finding that point on the course line where maximum tracking occurs and determining if it lies between (k, i)th point and the crossover point which is where $\Delta X = 0$ or before the (k, i)th point is reached. If the point of maximum tracking rate occurs before (k, i)th point on the course line then X_D is reduced by the distance between the

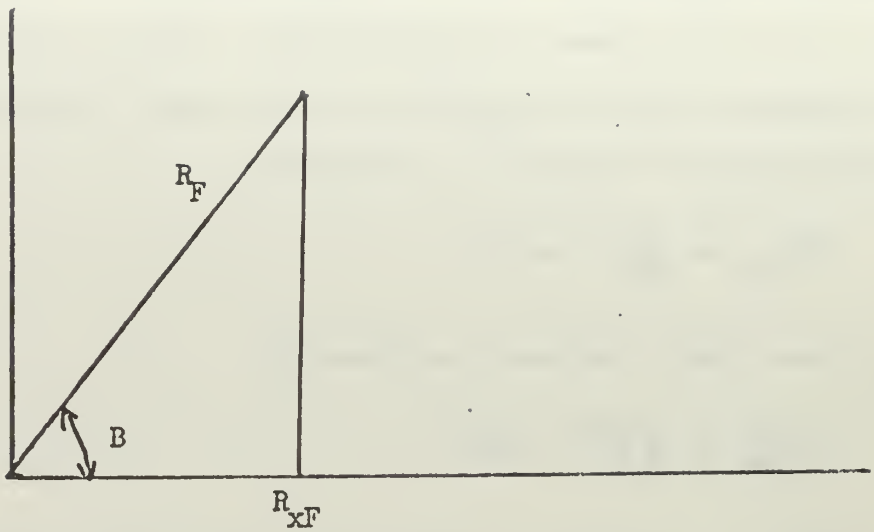


FIGURE 10

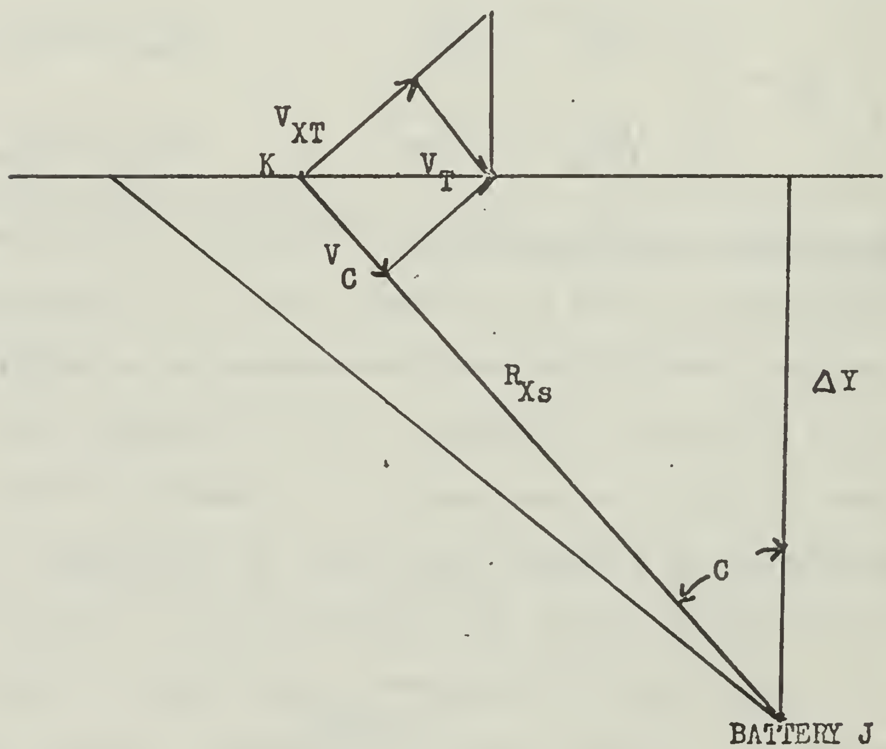


FIGURE 11

point of maximum tracking rate and (k, i)th point. Figure 11 is a diagram of the azimuth tracking rate problem on the horizontal plane. Point K is the point where firing must terminate projected on the horizontal plane at the same altitude as battery J. The tracking rate must first be determined at this point. Let $\dot{A}Z$ be the azimuth tracking rate. Then

$$\dot{A}Z = \frac{V_{XT}}{R_{XS}} ,$$

where V_{XT} = tangential velocity of the aircraft at point K and R_{XS} = horizontal component of the range to point K and was calculated in equation (8). From Figure 11 it is clear that V_{XT} is given by

$$V_{XT} = \frac{V_T \Delta Y}{R_{XS}} .$$

Substituting this result into the equation above yields

$$\dot{A}Z = \frac{V_T \Delta Y}{R_{XS}^2} . \tag{10}$$

Now substituting equation (8) into (10) results in

$$\dot{A}Z = \frac{V_T \Delta Y}{R_S^2 - \Delta H^2} \tag{11}$$

If $\dot{A}Z_M$ = maximum tracking rate for the system engaged, and $\dot{A}Z < \dot{A}Z_M$, no constraint due to tracking rate is active. If however $\dot{A}Z > \dot{A}Z_M$, then the constraint is active and firing must terminate at the point where the maximum rate occurred. To find this point solve equation (10) for R_{XS} when $\dot{A}Z$ is replaced by $\dot{A}Z_M$ resulting in

$$R_{XS} = \sqrt{\frac{V_T \Delta Y}{\dot{A}Z_m}} \quad . \tag{12}$$

Using the value of R_{XS} determined in (12) a new value for R_S is found by solving equation (8) for R_S as follows:

$$R_S = \sqrt{R_{XS}^2 + \Delta H^2} \tag{13}$$

Equation (13) has changed the (k, i)th point to reflect the constraint due to the maximum azimuth tracking rate.

Maximum Elevation Tracking Rate. The same problem occurs in elevation as in azimuth for the possibility of exceeding tracking rates at low altitudes. In some cases the elevation tracking rate is exceeded at a range further from the (K, i)th point than minimum range. It is necessary to compute the elevation tracking rate and determine if it is an active constraint. Figure 12 is a three dimensional diagram illustrating the vector component of target velocity tangential to the battery target line of sight. The tangential velocity in elevation is given by

$$V_{XTE} = \frac{V_T \sqrt{\Delta H^2 + \Delta Y^2}}{R_S} \tag{14}$$

where the

$$\sin A = \frac{\sqrt{\Delta H^2 + \Delta Y^2}}{R_S} \quad .$$

Since the velocity V_{XTE} is also opposite the angle A in the smaller triangle

$$V_{XTE} = V_T \sin A$$

Using the relationship as before that $\dot{E}L = \frac{V_{XTC}}{RS}$, results in

$$\dot{E}L = \frac{V_T \sqrt{\Delta H^2 + \Delta Y^2}}{R_S^2} \tag{15}$$

If $\dot{E}L > \dot{E}L_M$, where $\dot{E}L_M$ is the maximum tracking rate in elevation of the particular system, then it is necessary to find the range at which the maximum rate occurs. Solving (16) for R_{SE} gives

$$R_{SE} = \sqrt{\frac{V_T \sqrt{\Delta H^2 + \Delta Y^2}}{\dot{E}L_m}} \tag{16}$$

The new value of R_S is determined from

$$R_S = MAX\ (R_{SA},\ R_{SE})$$

and the new ΔX is determined by

$$\Delta X = \sqrt{R_S^2 - \Delta H^2 - \Delta Y^2} \tag{17}$$

At this point the conditions for terminating the firing at or before the (j, i)th point have been found. It is now necessary to find the point of open firing and then evaluate the effect of terrain on the length of course line between the open fire point, cease fire point, and the battery location.

Open Fire Point. Referring to Figure 8, it is seen that the open fire point is at H. The distance X_D is from the cease fire point which may or may not be the (k, i)th point to H. Depending upon the value of H, the distance X_{DR} may have positive, negative, or zero values. Point F must be found as it is the point where the detection radar first intersects the target course line. Figure 13 diagrams the situation. Lines 1 and 2 represent different altitudes of approach and the different points of detection in range. The horizontal range to the point of detection is found as follows:

$$R_h = \frac{\Delta H}{\tan \theta} \quad . \tag{18}$$

This relationship will be a good approximation since the ranges where the beam pattern begins to curve are far beyond the maximum effective range of any weapon system. The horizontal component of the maximum range of the system is determined to be

$$R_{xm} = \sqrt{R_m^2 - \Delta H^2 - \Delta Y^2} \quad . \tag{19}$$

The distance $X_D \neq X_{DR}$ on the target course line must be above all terrain points and must be visible over its entire length from the battery position in question. The effect of terrain on low altitude approaches on the distance $X_D \neq X_{DR}$ may be such that its value is decreased to less than zero when no firing is possible. A method of evaluating the terrain's effect on the problem will be developed.

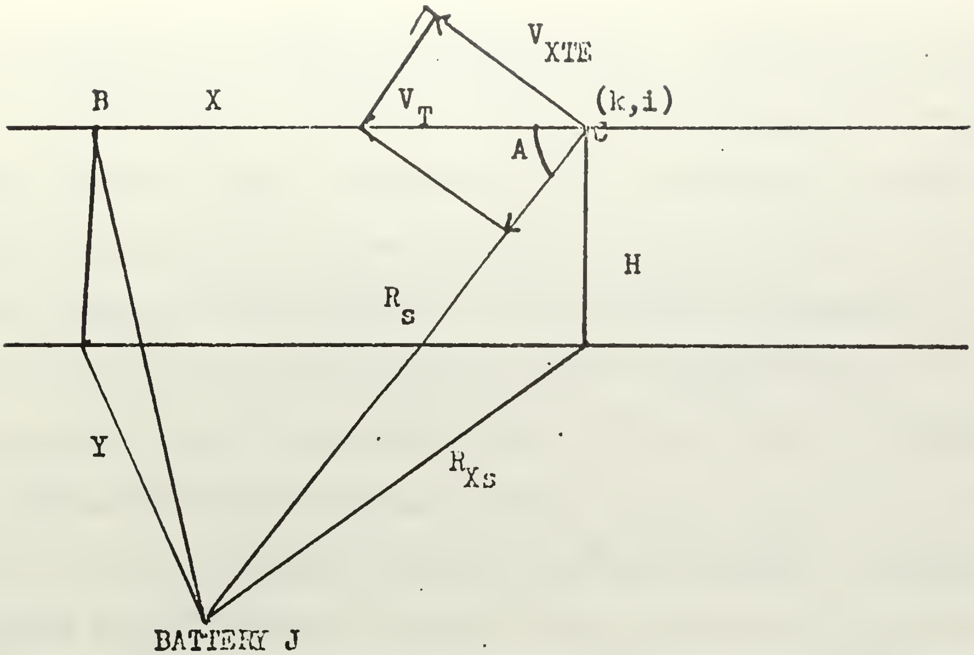


FIGURE 12

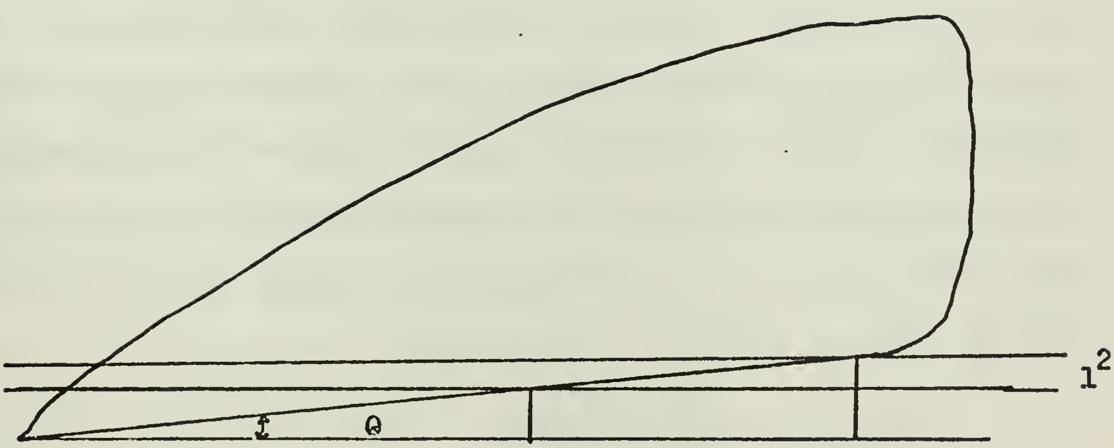


FIGURE 13

Terrain Evaluation. The method to be used for determining if line of sight exists from the point of radar detection to the battery position is to compute the slope of the line from this point to the battery as well as the slope from each intervening terrain point to the point of radar detection. If the slopes of the terrain points are greater than the slope to the point of radar detection then line of sight exists. If one or more terrain points extend above the line joining the battery and the point of radar detection, the line of sight does not exist for the length of the course line beyond the masking terrain points. A new point of radar detection is then found. A new slope line is computed from the masking terrain point at the altitude of approach and the battery. As before, all remaining intervening terrain point slopes are examined to determine if any more are masking the target course line. If another point has a slope less than the battery radar detection point line, then this higher terrain point becomes the new radar detection point. Depending on the terrain, the distance $X_D \neq X_{DR}$ may be uneffected or reduced to zero. If X_{NK} is the coordinate of the radar detection point then the slope to the battery position is given by

$$S_{LPB} = \frac{A_{Hi} - B_{Hj}}{B'_{xj} - X_{NK}} \tag{20}$$

The slope of each terrain point to radar detection point is

$$S_{LPm} = \frac{A_{H,i} - H_{m,l}}{X_{m,l} - X_{NK}}$$ (21)

where

$A_{H,i}$ = altitude of (k, i)th terrain point

$H_{m,l}$ = altitude of the (m,l)th terrain point

m = mth terrain point between the battery and the radar detection point

l = coordinate index of the offset distance from the course line

$X_{m,l}$ = X coordinate of the intervening terrain points.

As long as the engaged battery is located under the target course line, the determination of major terrain points that may mask the line of sight is a relatively easy matter and the slope calculations are simple. Figure 14 is a diagram of the calculation of slope. Points 1, 2, 3, and 4 represent terrain points which may be typically found between battery j and the radar detection point L. As shown in the diagram, terrain 3 and 4 have greater slopes than the line B_{Xj},L. They do not mask the line of sight. Terrain point 2 has a smaller slope than B_{Xj},L and does mask the line of sight. Therefore, point Q is the new radar detection point at which a new comparison slope must be calculated. Let Z be the distance from Q to B_{Xj} on the horizontal. Then

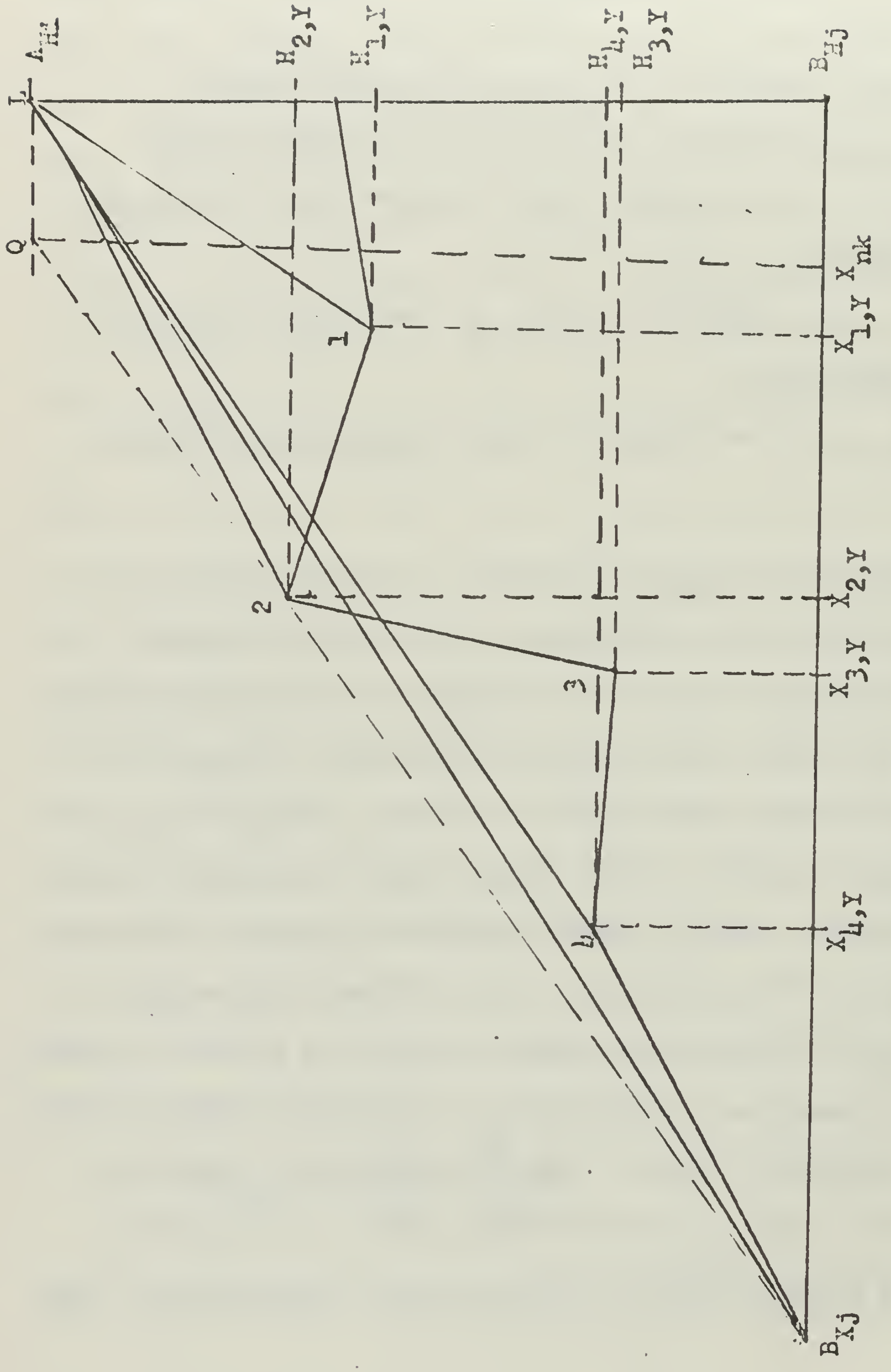


FIGURE 14

$$Z = \frac{(A_{Hi} - B_{Hj})(B_{xj} - X_{2,1})}{H_{2,1} - B_{Hj}} \quad . \tag{22}$$

This equation is derived from similar triangles in Figure 14. The value of the X coordinate of point Q is given by

$$X_{NK} = B_{xj} - Z \quad . \tag{23}$$

When the engaged battery is located at some offset distance, Y from the target course line, a procedure must be found to determine if line of sight exists between the battery j and all the terrain that lies between the lines joining the cease fire point, and radar detection point on the target course line. Figure 15 illustrates this problem. Notice the coordinate grid. At each intersection of a grid coordinate, an IBM card recorded the X, Y grid index along with the altitude at the intersection. Notice also that the X coordinate is parallel to the target course line. As previously indicated, the X coordinate increases in the direction of target flight. Point J is the battery position. In order to determine if line of sight exists from J to X_{NK} the slope of all terrain located on the line J, X_{NK} will be computed and compared to the slope of the line J, X_{NK}. In order to find the vertical and horizontal differential for all points on line J, X_{NK}, a series of triangles will be solved for the hypotenuse such as a, b, c in Figure 15. The distance ac is the horizontal distance for the slope

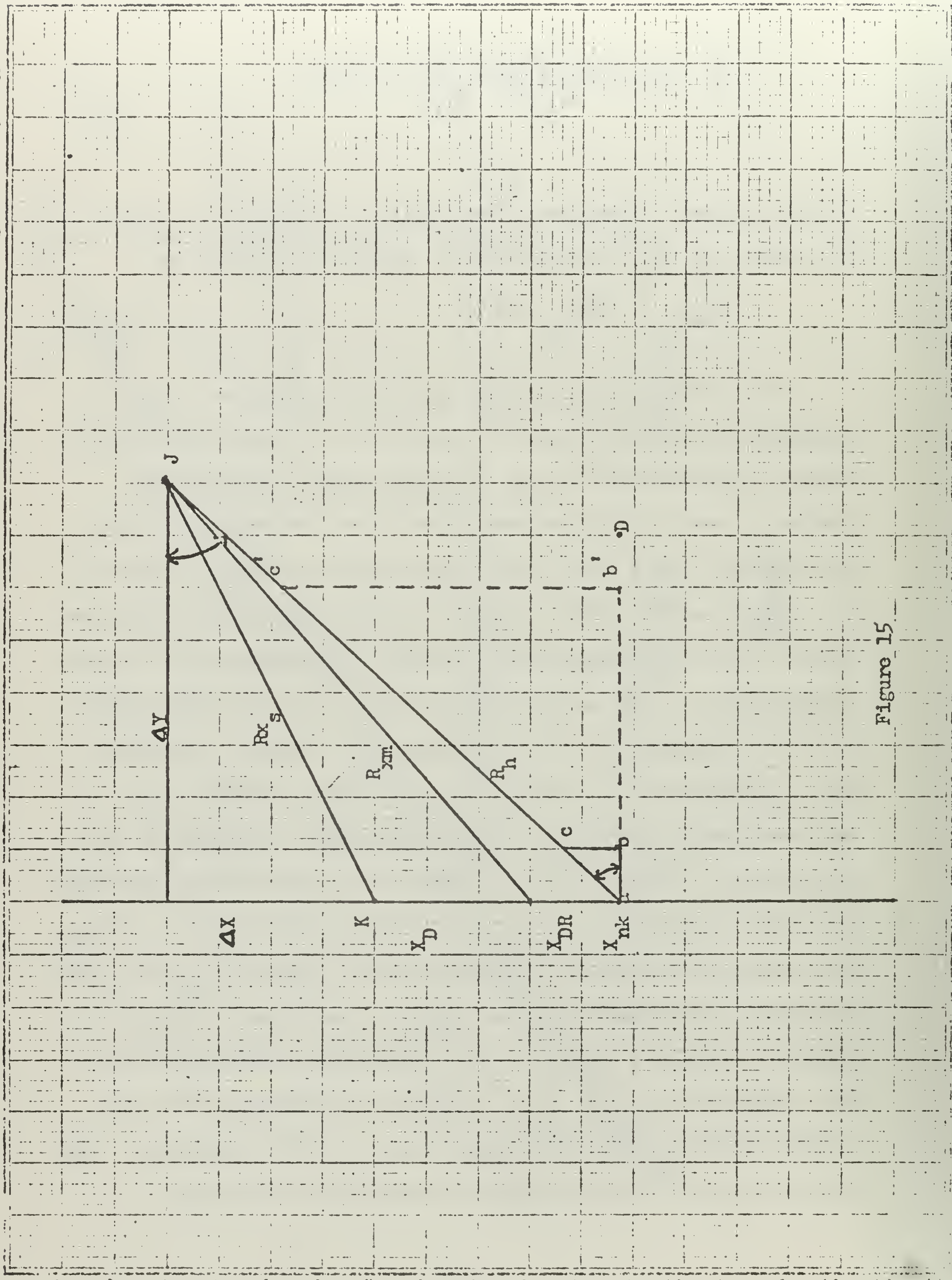


Figure 15

calculation. The distance bc is, if added to X_{NK}, the X coordinate of the closest point to P. Using the altitude of point P which has been recorded as previously described in digitalizing the terrain, the slope of the line X_{NK},C is calculated. The distance bc is given by

$$bc = ab \tan a \tag{24}$$

where

ab = distance between Y grid lines

$$\tan a = \frac{B_{xj} - X_{NK}}{Y}$$

so that

$$bc = ab \left(\frac{B_{xj} - X_{NK}}{\Delta Y} \right) \tag{25}$$

and

$$ac = \sqrt{(ab)^2 + (bc)^2} \tag{26}$$

For each successive triangle, ab is replaced by Nab where

$$N = (B_{yj} - Y_{k,1}) \,/\, Yk - 1$$

and

B_{Yj} = Y coordinate of battery j

$Y_{k,1}$ = Y coordinate of the target course line.

Substituting as appropriate,

$$ac = \sqrt{\left[\frac{(B_{xj} - Y_{K,1})}{Y_K} - 1\right]^2 (ab)^2 \left[1 + \left(\frac{B'_{xj} - X_{NK}}{\Delta Y}\right)^2\right]} \tag{27}$$

$$bc = \frac{ab}{\Delta Y_{YK}}\left(B'_{yj} - Y_{K,1}\right)\left(B'_{xj} - X_{NK}\right) \; . \tag{28}$$

Solving (27) and (28) for each value of N, enables the slope to be found at each Y_k interval and establishes line of sight along the particular line being examined. If any terrain slope is less than the comparison slope, then line of sight does not exist on that particular line from J to X_{NK}. In this case X_{NK} is increased by one X coordinate grid distance, X_k and the slope calculation process is again repeated. The value of X_{NK} which remains after each slope line is compared with terrain determines the actual radar detection point. In the three dimensional case it is possible to track a target up to a terrain formation masking a portion of the target course line between the cease fire and radar detection point, and either continue tracking through the mask in memory track, enabling relock, or to lose lock and require reacquisition on the other side of the mask. In the relock situation, the previous radar detection point X_{NK} stands. If reacquisition is necessary, then a new value of X_{NK} is computed from the point where the terrain ceased to mask the course line. The value that X_{NK} takes on is determined by

$$T = \frac{X_p}{V_T} \tag{29}$$

where

T_x = time for aircraft to pass terrain mask

X_p = distance on course line masking line of sight to the battery

V_T = aircraft speed

If $T_x \leq T_m$, where T_m is the memory tracking period, then X_{NK} remains unchanged. If $T_X > T_m$, X_{nk} receives a new value as shown below,

$$X_{NK} = X_K N_2 + X_{NK} \tag{30}$$

where

X_k = distance between X grid lines

and

N_2 = number of grid lines spanned by the terrain mask

Determination of Length of Course Line for Firing. With the computation of X_{nk} and the cease fire point after considering maximum effective range of the system, maximum radar detection range, and effects of possible terrain masking, the distance $B_{Xj} - X_{nk}$ is available for firing after the system delays have been accounted for. The system delays assumed in this model are as follows:

(1) Reaction time, t_r, which is the average of time for the radar operator to recognize target pips on the plan position indicator of the early warning radar set in the battery operations center, identification of the target and decision to engage the target.

(2) Transfer action time, t_a, which is the time for the fire control operators to receive an assignment, lock the tracking radar on the target account for computer settling time, and gun crew reaction time to fire the first round.

(3) Time of flight of the first round of the system to reach the target.

Average values for (1) and (2) are assumed but time of flight is calculated from system parameters. An equation for solution of time of flight for gun systems is given by

$$t_f = \frac{R_{OF}}{V_o - K_p R_{OF}} \tag{31}$$

where

R_{of} = range to open fire

V_o = projectile muzzle velocity

k_p = projectile drag

If the k_p is known, (31) is a useful formula. If a system graph of time of flight versus slant range is available, a regression equation is also useful. This model uses a regression equation approximateing a typical system curve of slant range versus time of flight. The concept is to place the first round on the target at the maximum effective range of the system. If sufficient time exists for the target to travel from the point of detection to open fire range, then the system is assumed to expend the time

$$t_T = t_r + t_a + t_f$$

prior to the targets arrival at the open fire range. If the detection range is less than maximum effective range, then the time t_T will reduce the time for firing. If on the other hand only a part of the time t_T is expended prior to open fire range, then some proportion of the t_T will reduce the firing time.

$$D_t = t_T V_T \qquad (32)$$

where

D_t = the distance the target travels during the system delay time

As a result of terrain evaluation, X_{nk}, as was seen, may have a new value. Therefore, the range to the maximum radar detection point is given by

$$R_h = \sqrt{(B_{xj} - X_{NK})^2 + \Delta Y^2} \qquad (33)$$

This relationship is illustrated in Figure 15. Since detection range may have decreased to be less than R_{Xm}, the open fire range, as a result of terrain restrictions, R_{Xm} is now given by

$$R_{xm} = \mathrm{MIN}(R_h, R_{xm}) \qquad (34)$$

Actually, R_{Xm} is the horizontal range component. The slant range to open fire becomes

$$R_{OF} = \sqrt{R_{xm}^2 + \Delta X^2 + \Delta Y^2} \quad . \tag{35}$$

The distance along the course line from the radar detection point to open fire is given by

$$X_{DR} = \begin{cases} \sqrt{R_h^2 - \Delta Y^2} - \sqrt{R_{xm}^2 - \Delta X^2} ; & R_h > R_{xm} \\ 0 ; & R_h \leq R_{xm} \end{cases} \tag{36}$$

Now the distance on the course line involved in reaction time t_T may be all within X_{DR} if $X_{DR} > t_T$, or part of t_T may be in X_{DR} if $X_{DR} < t_T$. If $X_{DR} = 0$ then t_T will reduce X_D by the amount $t_T V_T$. X_D will also be reduced by the time of flight for the last round fired since any round fired after the point (k, i) is reached does not contribute to the probability of kill up to point (k,i). Therefore, X_D is given by

$$X_D \begin{cases} \sqrt{R_{xm}^2 - \Delta Y^2} - \Delta X - X_T - t_f V_T ; & X_{DR} \leq 0 \\ \sqrt{R_{xm}^2 - \Delta Y^2} - \Delta X - X_T - t_f V_T ; & 0 < X_{DR} < X_T \\ \sqrt{R_{xm}^2 - \Delta Y^2} - \Delta X - t_f V_T ; & X_{DR} \geq X_T \end{cases} \tag{37}$$

where

$$X_{T1} = X_T - X_{DR}$$

$$X_T = (t_r + t_a)\, V_T$$

Having derived the equation for X_D, the length of course line available for firing, it only remains to determine the number of rounds that will be fired in the time the target passes over X_D. The time for target passage is

$$T = \frac{X_D}{V_T} \tag{38}$$

Then the number of rounds fired is given by

$$N_r = \frac{T}{T_{1/r}} \tag{39}$$

where $t_{1/r}$ = time to fire one round and the rate of fire is $\frac{1}{t_{1/r}}$.

If $X_{DR} < X_T$, then the entire delay time including the time of flight of the first round occurs during the early warning time period and (39) becomes

$$N_r \begin{cases} \frac{T}{t_{1/r}} + 1 & ; X_{DR} \geq X_T \\ \frac{T}{t_{1/r}} & ; X_{DR} < X_T \end{cases} \tag{40}$$

<u>Probability Model</u>. In Chapter I, it was stated that the model was being developed for three different air defense systems. In the development of the model, the general scheme was to permit all calculations to apply to each type of weapon system except for the parameters used in the computer model. Except for the regression equations used for determining time of flight, this generality has been successful. However, the models must be separated for probability calculations into a separate one applying to each weapon

system. In the case of the two types of gun systems, a common model is still possible with one or two exceptions. The radar controlled antiaircraft gun system will be delt with first. The range at which each round is fired is computed as follows:

$$R_{fi} = \sqrt{(X_0 + \Delta X - V_T t_{1/r}\, i)^2 + \Delta H^2 + \Delta Y^2} \qquad i = 1, 2, \ldots, N_r \tag{41}$$

This model assumes that the area a gun fires into is defined by a circular normal probability curve. The square of the radius of the circle described thusly has been determined empirically in references 2, 3, and 4 is given by

$$\sigma_i^2 = (.05 V_T)^2 + R_{fi}^2 \; 10^{-6} \left(\frac{\sigma_E + V_T}{1000}\right)^2 + (10 t_f)^2 \tag{42}$$

where

σ_E = angular error of the gun in milliradians.

σ_i is the result of all the errors introduced by the fire control, data transmission, orientation, and alignment of the battery. Equation (42) is based upon the following theoretical equation:

$$\sigma^2 = S_r^2 + S_T^2 + S_{FR}^2$$

where

S_r = quasi static errors consisting of

(1) Boresighting and alignment

(2) Range errors (radar developed), radar timing errors due to mechanical alignment

(3) Servo lag errors

(4) Computer error

(a) static errors
(b) ballistic match errors
(c) prediction errors

(5) Battery emplacement errors

S_t = Tracking perturbation

S_{fr} = Flight roughness

The constants and results of using equation (42) have been verified in actual test firings according to reference 2.

Target Area. The average cross sectional area of the target's vulnerable sections is denoted by A in square feet. The vulnerable cross sectional area changes with different positional attitudes. An average value is assumed in the following calculations. In addition a method for calculating the area as a function of target attitude was also implemented. This area is computed from

$$A_i = \frac{X_i}{R_i} A_X + \frac{Y}{R_i} A_Y + \frac{H}{R_i} A_H \qquad (43)$$

where

A_i = percentage of area presented for the ith round fired

R_i = the range of the ith round

X_i = the X component of the range to the target at the ith round fired

A_X = cross sectional area of the standard target defined to be 400 ft^2 in the X coordinate.

A_Y = same as above for the Y coordinate set at 800 ft^2

A_H = same as above for the H coordinate set at 150 ft^2

Y_i = the Y component of the range to the target at the ith round fired

H_i = the H coordinate of the range to the target at the ith round fired

The standard target may then be compared with any other target for converting probabilities.

The probability that one round from the gun hits the target is the ratio of the vulnerable cross sectional area of the aircraft to the area the weapon fires into as follows:

$$P_i = \frac{A}{2\pi\sigma_i^2}, \quad i = 1, \ldots, NR \tag{44}$$

The probability required is the probability that one round kills the target. If the probability of kill given a hit is P[K/H] then the probability of kill, assuming independence, is P[K/H] P[H]. Using this fact equation (44) becomes

$$P_i = \frac{A}{2\pi\sigma_i^2} \quad P[K|H] \tag{45}$$

If N_r is the number of rounds fired, assuming independence of rounds fired from the gun, the probability of kill is given by

$$P_K = 1 - \prod_{i=1}^{N_r} (1 - P_i), \tag{46}$$

where

N_g = number of guns in the battery. With all guns firing the probability that the aircraft is killed, assuming independence of guns in the battery, is given by

$$P_{Ki} = 1 - (1 - P_K)^{N_G} \tag{47}$$

The second type of weapon system is the automatic weapon battery. This battery does not possess gun laying radar. It depends on the optical means for acquiring the target. Early warning radar is assumed available. All equations in the previous model are applicable except equation (42). The total error for a gun with optical sights is assumed circular normal with the square of the radius of the circle of error given by

$$\sigma_i^2 = (.1V_T + f)^2 \tag{48}$$

This equation like (42) has been empirically determined and verified according to reference 2.

In the surface to air missile battery, the third type of weapon system, a simplified model is assumed. Specific models are not feasible in order for the computer model to be applicable to any type of missile system. Therefore, the single shot kill probability is p. Then the battery kill probability is given by

$$P_{kj} = 1 - (1 - p)^{N_r} \quad (49)$$

N_r was calculated previously. However, in the missile battery the number of rounds fired at any target is constrained by the basic load of missiles. Further the high single shot kill probability of a missile round precludes the firing of more than 2 or 3 missiles per target in most cases. This model insures that the value of N_r permitted is as follows:

$$N_r = \begin{cases} 2 & N_r > 2 \\ 1 & N_r = 1 \\ 0 & N_r < 1 \end{cases} \quad (50)$$

Any particular battery firing policy may change the value of N_r as the tactical situation may dictate.

For each terrain point k and altitude i, P_{kj} is calculated for all batteries. Then the overall kill probability for all batteries is given by

$$P_{TKj} = 1 - \prod_{j=1}^{N_1} (1 - P_{Kj}) \tag{51}$$

$$K = 1, 2, \ldots, N$$
$$i = 1, 2, \ldots, 30$$
$$j = 1, 2, \ldots, N_1$$

The assumption of independence is quite valid for equation (51) since all batteries act independently in actual firing. Equation (46) also requires the assumption of independence between each round fired. If the weapon is a radar controlled gun using equation (42) to model the area within which all rounds are fired, the probability of a hit for each round P_i, must be independent for each value of i for the assumption to hold in (46). When examining the factors which determined (46), it is seen that certain of the errors are the same from round to round or are corrolated errors. These errors tend to break down the assumption of independence. Most of the errors are random in nature from round to round and are the factors supporting the assumption of independence. It is not known how serious the correlated errors effect the results of equation (46). If independence were not assumed, (46) would be modified as follows:

$$P_{Kij} = 1 - \left[(1-P_1)(1-P_2 | \bar{P}_1)(1-P_3 | \bar{P}_1 \bar{P}_2) \ldots (1-P_N | \bar{P}_1 \ldots \bar{P}_{N-1})\right] \tag{52}$$

The difference between the probabilities in (46) and the conditional probabilities in (52) constitutes the error if if is significant. All models found in the literature use the independence assumption. Investigation of the probabilities in (52) to determine how bad the assumption of independence really is may prove interesting.

CHAPTER III

THE COMPUTER MODEL

In order to perform the calculations necessary to produce a relationship of altitude versus probability at each terrain point within the range of any battery in the defensive complex, a computer program capable of performing the calculations is essential. If the resulting computer program is to have any lasting value, the program itself must be documented in order to make clear how the program was designed and how the problem was solved by the computer. An explanation of the Fortran variables is found in Appendix III with the Fortran program located in Appendix IV.

The program was originally designed to produce as an output an array of probabilities and corresponding altitudes for each terrain point examined. It was expected that the results would be punched on an output deck for use as an input to the computer program in reference 1. The output deck from this program would then replace the probability calculations in the other program. This replacement has not been possible for reasons to be explained in a later chapter.

Computer Inputs

The inputs to the computer are of two types. The terrain digitalization and the weapon system parameters. The three arrays $X_{k,i}$, $Y_{k,i}$, and $H_{k,i}$ are the coordinates of the major terrain points being examined. In the two dimensional problem this is all the data needed. If the three dimensional data is available, three more arrays would be needed to record the terrain contours through out the area of interest. The arrays $HH_{I,K}$ for altitude, $XX_{I,K}$, X coordinate, and $YY_{I,K}$ the Y coordinate of each recorded terrain point would also have to be inputs to the program.

As previously explained, the model was developed for any weapon system. If the model is to represent any specific weapon system the input parameters necessary to define the system must be clearly understood. An explanation of the input parameters is contained later in this chapter. The parameters are defined in the Fortran code notation to facilitate their incorporation into the computer program.

<u>Description of Computer Solution</u>.

Figures 16, 16a, and 17 are a block diagram of the computer program in general terms. The following description will relate to the block diagram.

<u>Block #1</u>. All data is read into the computer with units in feet and seconds,,angles are in degrees.

<u>Block #2</u>. Most parameters of angular measure are given in degrees. The program converts all degrees to radiams.

<u>Block #3</u>. The program is solved once for each N_1 batteries at each of 30 altitudes and for each of N terrain points. This constitutes 30 N_1N iterations which in the sample used in the actual program is 30 x 11 x 53 = 18,490 iterations.

Block Diagram of Computer Program

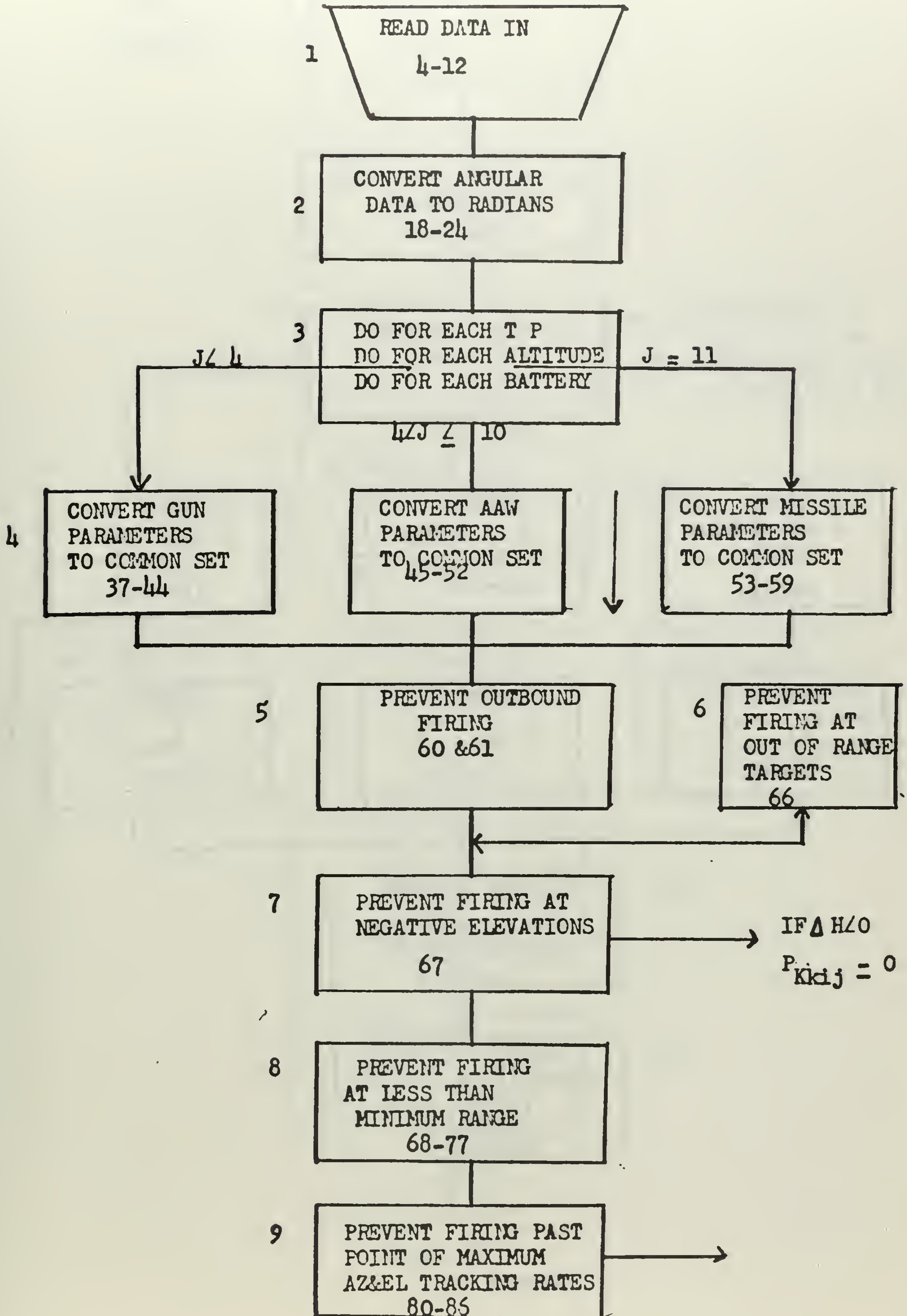
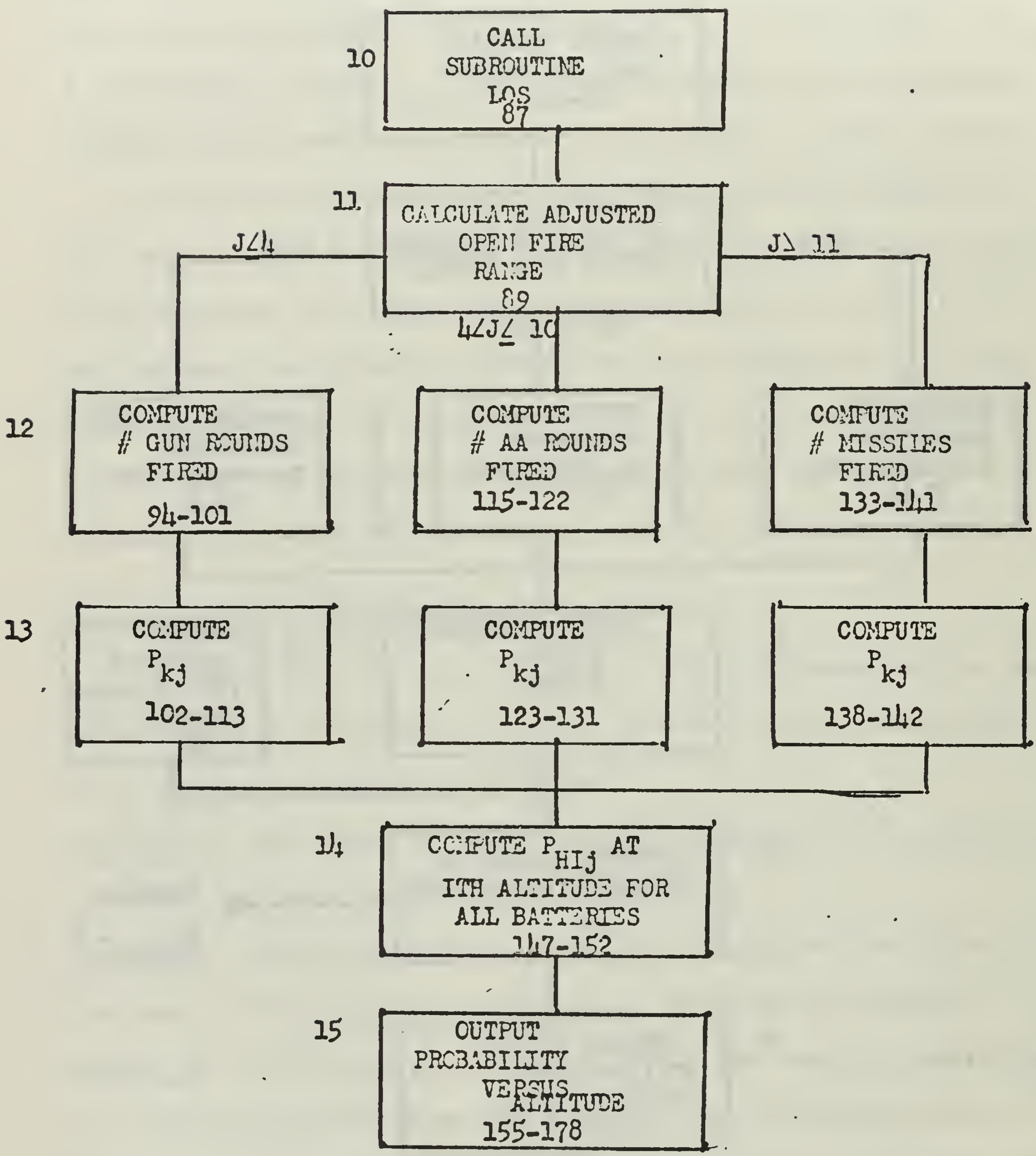


Figure 16



10
CALL SUBROUTINE LOS 87
11
CALCULATE ADJUSTED OPEN FIRE RANGE 89
J<4
J>11
4<J<10
12
COMPUTE # GUN ROUNDS FIRED 94-101
COMPUTE # AA ROUNDS FIRED 115-122
COMPUTE # MISSILES FIRED 133-141
13
COMPUTE P kj 102-113
COMPUTE P kj 123-131
COMPUTE P kj 138-142
14
COMPUTE P HIj AT ITH ALTITUDE FOR ALL BATTERIES 147-152
15
OUTPUT PROBABILITY VERSUS ALTITUDE 155-178

Figure 16a

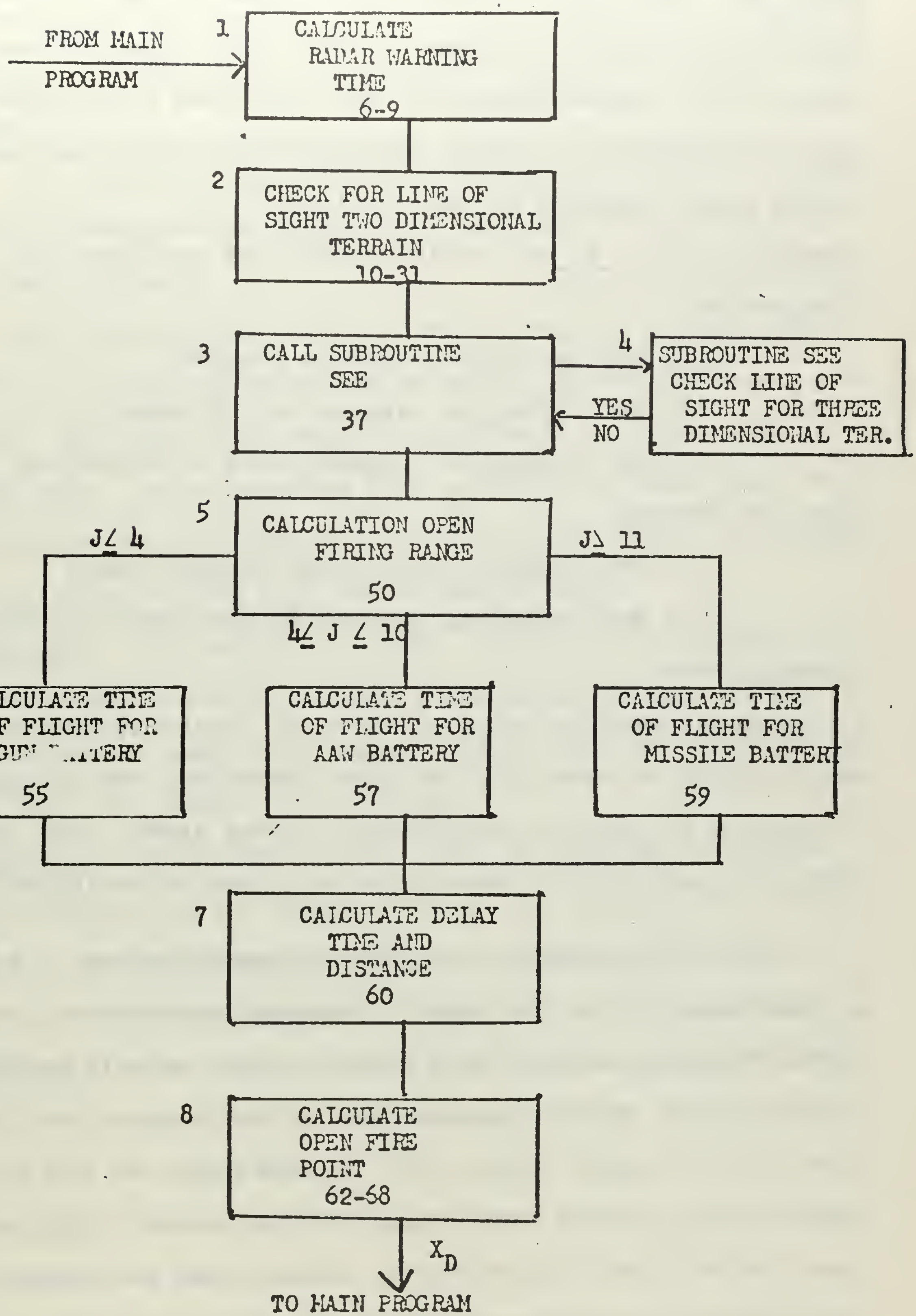

FROM MAIN PROGRAM
1
CALCULATE RADAR WARNING TIME
6-9
2
CHECK FOR LINE OF SIGHT TWO DIMENSIONAL TERRAIN
10-31
3
CALL SUBROUTINE SEE
37
4
SUBROUTINE SEE CHECK LINE OF SIGHT FOR THREE DIMENSIONAL TER.
YES
NO
5
CALCULATION OPEN FIRING RANGE
50
J ∠ 4
4 ∠ J ∠ 10
J > 11
6
CALCULATE TIME OF FLIGHT FOR GUN BATTERY
55
CALCULATE TIME OF FLIGHT FOR AAW BATTERY
57
CALCULATE TIME OF FLIGHT FOR MISSILE BATTERY
59
7
CALCULATE DELAY TIME AND DISTANCE
60
8
CALCULATE OPEN FIRE POINT
62-68
X_D
TO MAIN PROGRAM

Figure 17

Block #4. In order to permit the program to use the same code for solving the problem for all three types of weapon systems, the system parameters are converted to a common set which represents the values corresponding to the particular system being examined at any one time.

Blocks #5, 6, 7, 8, 9. Antiaircraft fire has been constrained by

(1) Firing only at inbound targets

(2) Not firing at targets out of range

(3) Not firing at targets lower in elevation than the battery

(4) Not firing at less than minimun range

(5) Not exceeding maximum azimuth and elevation tracking rates.

A section of computer code as shown is Figure 16 provides the calculations to determine, at each iteration, the occurance of any one of the five constraints listed above. The overall effect of each of the constraints will now be explained.

(1) The program is designed to permit firing up to minimum range or to the point of maximum azimuth and elevation tracking rates. If a terrain point is well behind the battery or in the opposite end of the course from the direction of target attack, the minimum point is thusly established. On the other hand, if the terrain point in question is in the direction of attack, then the terrain point itself is the cease fire point unless the above constraints act to increase the cease fire range.

(2) Firing at targets that will always be out of range is prevented by assigning zero to the probability of kill if the range to the terrain point and altitude is beyond system range.

(3) If a target is lower in elevation than the battery, zero is assigned the probability of kill for that battery at that altitude.

(4) When either minimum range, maximum azimuth and elevation tracking rates are exceeded, whichever occurs first, the cease fire range is increased to that point.

Block #10. The subroutine LOS is called at this point which calculates the distance of course line available for firing. The function of LOS will be explained later.

Block #11. The length of target course line available for firing resulting from terrain considerations may produce a new value for open fire range. Based upon the value of X_D, the open fire range is calculated.

Block #12. Depending on what type of battery is engaged, the program will go through one of the three paths in this block. The probability of kill depending on the number of rounds fired is calculated for one of the three systems.

Block #14. The overall kill probability is computed here for one iteration which includes all weapons at one terrain point and one altitude.

Block #15. After all iterations are completed the program is capable of plotting the curves of altitude versus probability. Any other output is available when desired.

Subroutine LOS

Block #1. Receiving all necessary data from the main program, all calculations planned are possible. Radar warning time is determined prior to terrain evaluation. The value for the point of first detection is the result of this block.

Block #2. Here the program determines which terrain points lie between the battery and the point of first detection. If any terrain interrupts the line of sight, a new value for radar detection point is calculated. The computations are for two dimensional terrain which mean terrain for batteries on the target line of sight.

Block #3, 4. If three dimensional terrain is used, subroutine SEE is called where the point of radar detection, as a result of terrain, is found.

Block #5. Open fire range without regard to delay is calculated for use in determining time of flight.

Block #6. Again the program branches into three paths, one for each weapon system. This is necessary since each system has a different function of slant range versus time of flight.

Block #7. The delay time which is the sum of crew reaction time, system delay time, and time of flight of the first round fired is calculated in this section. This value is not used in the case where early warning time is less than total delay time.

Block #8. After all the above calculations are completed, it is possible to find the actual point of open fire. This point is the range at which the first round is placed during delay time to maximize the number of rounds fired. Three possible situations arise in calculations for the open fire point as shown in Chapter II, equation (35). These are

(1) Adequate warning time exists for all delay time to occur during the early warning period.

(2) Only part of the delay time occurs during the early warning period and the open fire point is reduced.

(3) All of the delay time must fall during possible firing time due to the radar detection point being less than maximum effective range. The subroutine sends X_D back to the main program.

A Description of Input Parameters

At some later time, it may be desirable to compute the results from the model for an actual air defense system. In order to do this, an explanation of the input parameters is essential. A description of how to insert new inputs is provided below. Statement number 4 in Appendix III is the data statement. Contained in this statement are all the system parameters. The program is convertable to any specific weapon systems by changing the values in the program data statement to conform to those of the new systems. Included with the description of each parameter is the value assumed in the current form of the program.

Parameter	Description
NAP	Number of altitude points above terrain examined. NAP = 30
NTT	Number of terrain points examined. NAP = 84
NFU	Number of batteries placed in the terrain. NFU = 11
ELRMG	Maximum elevation tracking rate for the radar controlled gun system. ELRMG = 10°/sec
ELRMA	Maximum elevation tracking rate for the automatic weapon battery. ELRMA = 20°/sec
ELRMM	Maximum elevation tracking rate for the missile battery. ELRMM = 15°/sec
AZRMG	Maximum Azimuth tracking rate for radar controlled gun. AZRMG = 12°/sec
AZRMA	Maximum azimuth tracking rate for automatic weapon battery. AZRMA = 20°/sec
AZRMM	Maximum azimuth tracking rate for the missile system. AZRMM = 15°/sec
RMG	Maximum effective range for radar controlled gun battery. RMG = 45,000 feet
RMA	Maximum effective range AAW battery. RMA = 6,000 feet
RMM	Maximum effective range missile system. RMM = 105,600 feet
THETI	Elevation angle for lower portion of detection radar beam pattern at half power point. THETI = 1.1°2

Parameter	Description
THET 2	Elevation angle for detection radar, AAW battery. THET 2 = .5
THET 3	Elevation angle for detection radar, missile battery. THET 3 = 1.°0
API	The constant term in the linear equation for slant range versus time of slight, gun battery. API = 0
CONT	The constant in the exponential equation for time of flight for AAW Battery. CONT = .0002
YK	Distance between Y grid lines, three dimensional terrain. YK = 500 feet.
VT	Target speed. VT = 350 ft/sec
TRG TRA TRM	Crew reaction time. This time is measured from time of first decision of detection of target to track radar lock on. This is an average figure. TRG = gun system, TRA = AAW system, and TRM = missile system. TRG = 30 sec. TRA = 10 sec. TRM = 30 sec.
ATG ATA ATM	System delay time measured from radar lock on to first round fired for gun battery, AAW battery, and missile battery. ATG = 20 sec. ATA = 5 sec. ATM = 10 sec.
LT 1	Total number of gun batteries. LT 1 = 4
LT 2	Total number of AAW batteries plus gun batteries. LT 2 = 10
LT 3	Total number of gun batteries plus AAW batteries plus missile batteries. LT 3 = 11

A	Cross sectional area of target. The average value as target aspect changes. A = 350 feet
SIG	Angular error of radar controlled AA gun when firing at a fixed point. SIG = 1.5 mille radians
GUNS	The number of guns per gun battery. GUNS = 4
TCRA	Cyclic rate of fire, AAW battery. TCRA = 6 rounds/sec
PKSM	Single shot kill probability for missile system. PKSM = .48
NR	The firing policy of the missile battery, NR rounds per target. NR = 2
TIM	Time for one round fired from the gun battery. TIM = 20 sec
AGUN	Number of guns per AAW battery. AGUN = 4
TX	Memory track period for all systems. TX = 15 sec
LY	Index number for the Y coordinate of the target course line terrain points. LY = 1
RMFG	Minimum firing range, gun battery. RMFG = 500 feet
RMFA	Minimum firing range for AAW battery. RMFA = 500 feet
RMFM	Minimum firing range for missile system. RMFM = 9500 feet
FE	Maximum elevation angle for all track radars. FE = 87°
YM	Slope of the linear regression equation for time of flight of the gun battery. YM = .0007

ZM	Constant in linear regression equation for time of flight of missile system. AM = 2,000
SM	Slope of linear regression equation. SM = 1200
PG	Probability of a kill given a hit for the gun battery. This is the lithality factor. PG = .8
PA	Probability of kill given a hit for the AAW battery. PA = .3

Certain other parameters may be found in the data list. If they are not included in the list above, they are not in use in the final form of the program and should be disregarded.

Terrain Data Input. Statement number 7 in the main program is the read statement which picks up the data punched on 84 data cards. Here the variable NTT is set equal to 84. If in a subsequent use of the program more terrain points were required, changing the value of NTT and providing the same number of data cards provides the change. Statement number 8 reads in the coordinates of the Batteries. It is essential that the first system type be radar controlled gun batteries up to LT 1. Then the next type must be AAW batteries up to LT 2 which was defined to be LT 1 ≠ the number of AAW batteries. The last system in order is the missile battery. Generally only one missile battery will be in range of a single azimuth of approach unless the azimuth is in the overlapping zones of fire of two adjacent batteries. Setting the values of LT 1, Lt 2, and LT 3 along with the proper ordering of the data cards of each type of weapon system constitutes the conversion to a new defensive complex.

Three Dimensional Data Input. If three dimensional data is to be used, three new arrays must be introduced for the X, Y, and H coordinates of each recorded terrain point. When this is done, statement 6 in subroutine SEE must be replaced by a statement using the new altitude array for H(J,ly) presently in the program.

CHAPTER IV

PROGRAM VERIFICATION, RESULTS, AND ANALYSIS

The underlying factor in the model design from conception has been that the air defense capability is maximized. This means that the attacking aircraft situation is the worst possible condition for survival. This fact is apparent when considering the general model structure presented in Chapter II. It should be recalled that the aircraft was required to penetrate the defense from out of range to the defended area flying a straight and level course, with a raid size of one aircraft, and no electronic counter measures. In addition, the defense was assumed to be alerted, manned, and concentrating on the single intruder. The aircraft survivability index calculated in the model when submitted to a programing algorithm for minimization within the aircraft flight constraints finds the flight path which minimized the probability of kill under the worst possible attacking conditions. In actual application, anything the aircraft may do relieving the sever restrictions, wholly or in part, increases the survivability index. The act of following the programed output recommended flight path, penetrating the defense with more than one aircraft in the raid, and using electronic counter measures will increase survivability. These facts must be considered when evaluating the model's results.

With the development of the model and the program for the computer, a verification of its main computing aspects is in order. Following this, the computer results will be presented and analyzed.

Verification of Computer Results

The two main problems solved by the computer within the mathmetical model are the calculations for determining line of sight, in two dimensional terrain only, and the calculations for the probability of kill given a positive course line. All other computations within the program support these main requirements.

Terrain Evaluation. In order to verify the computer solution of the line of sight, a sample of computer print out contained in Figures 18 and 19 is presented. Figure 18 is a case where the coordinates of the point of maximum radar detection, X_{nk} = 26140. The range of first detection is 163,060 feet since the coordinates of Battery 6 are given as B_{x6} = 189,200 on the X axis and 6700 feet altitude. The comparison slope S_{LPB} = .00873, kk = 46, and II = 5. II is the terrain point number just greater than X_{nk}. KK is the terrain point number just less than B_{x6}. Thus, the program has determined the number of intervening terrain points to examine for slope comparison. As explained in Chapter II, if any slope S_{LP} is less than S_{LPB} then line of sight as corrected by the distance Z exists. Observing Figure 18 again, it is seen that $S_{LP}(5)$ through $S_{LP}(29)$ are greater

```
XNK=
XNK=       26140.     BX(J)=      189200. RXM=          5829.
SLPB=       0.00873  KK=  46II=   5
SLP(   5)=0.74229542  X(M,LY)=       30000.
SLP(   6)=0.12384624  X(M,LY)=       48854.
SLP(   7)=0.09659945  X(M,LY)=       52714.
SLP(   8)=0.07077888  X(M,LY)=       56446.
SLP(   9)=0.06196354  X(M,LY)=       58837.
SLP(  10)=0.05894263  X(M,LY)=       60886.
SLP(  11)=0.06192517  X(M,LY)=       62200.
SLP(  12)=0.05306274  X(M,LY)=       65245.
SLP(  13)=0.05455366  X(M,LY)=       68374.
SLP(  14)=0.03786973  X(M,LY)=       72325.
SLP(  15)=0.05154296  X(M,LY)=       78000.
SLP(  16)=0.04306990  X(M,LY)=       84023.
SLP(  17)=0.04392700  X(M,LY)=       85762.
SLP(  18)=0.03496312  X(M,LY)=       92782.
SLP(  19)=0.02833275  X(M,LY)=       97683.
SLP(  20)=0.02452741  X(M,LY)=       98753.
SLP(  21)=0.02195990  X(M,LY)=      100321.
SLP(  22)=0.02296912  X(M,LY)=      101633.
SLP(  23)=0.01907787  X(M,LY)=      105080.
SLP(  24)=0.02104160  X(M,LY)=      106600.
SLP(  25)=0.02184661  X(M,LY)=      115811.
SLP(  26)=0.01806961  X(M,LY)=      118118.
SLP(  27)=0.01997686  X(M,LY)=      120900.
SLP(  28)=0.01745568  X(M,LY)=      127540.
SLP(  29)=0.01071613  X(M,LY)=      132802.
SLP(  30)=0.00292461  X(M,LY)=      134531.
XNK=     118862.BX(J)=     189200.M= 30        Z=   70338.14
SLP(  31)=0.06842137  X(M,LY)=      137891.
SLP(  32)=0.06320457  X(M,LY)=      142800.
SLP(  33)=0.05109691  X(M,LY)=      147709.
SLP(  34)=0.04103781  X(M,LY)=      149151.
SLP(  35)=0.04532397  X(M,LY)=      150589.
SLP(  36)=0.02614986  X(M,LY)=      158059.
SLP(  37)=0.00723276  X(M,LY)=      159925.
XNK=     152203.BX(J)=     189200.M= 37        Z=   36996.74
SLP(  38)=0.08452289  X(M,LY)=      161609.
SLP(  39)=0.04290301  X(M,LY)=      163974.
SLP(  40)=0.05649878  X(M,LY)=      165000.
SLP(  41)=0.03155532  X(M,LY)=      167890.
XNK=     156523.BX(J)=     189200.M= 41        Z=   32676.86
SLP(  42)=0.05056256  X(M,LY)=      172800.
SLP(  43)=0.06191137  X(M,LY)=      178700.
SLP(  44)=0.02729561  X(M,LY)=      182901.
XNK=     176450.BX(J)=     189200.M= 44        Z=   12750.32
SLP(  45)=0.12489532  X(M,LY)=      184000.
SLP(  46)=0.07592067  X(M,LY)=      186737.
XNK=     183740.BX(J)=     189200.M= 46        Z=    5459.86
XC=                   -1953.7 BIRY=  6TERRAIN POINT  62 ALTITUDE    7
```

Figure 18

Computer Line of Sight Computations

```
XNK=      145C92.       BX(J)=      196703. KXM=        44989.
SLPB=       C.C1955 KK= 47II= 33
SLP( 33)=C.56328574 X(M,LY)=     147709.
SLP( 34)=0.30624897 X(M,LY)=     149151.
SLP( 35)=C.2616C727 X(M,LY)=     150589.
SLP( 36)=0.C79C4810 X(M,LY)=     158059.
SLP( 37)=C.02CC2321 X(M,LY)=     159925.
SLP( 38)=C.C4813284 X(M,LY)=     161609.
SLP( 39)=C.02674535 X(M,LY)=     163974.
SLP( 40)=C.C3631744 X(M,LY)=     165000.
SLP( 41)=C.C2171263 X(M,LY)=     167890.
SLP( 42)=C.C297C284 X(M,LY)=     17280C.
SLP( 43)=C.C4C85362 X(M,LY)=     17870C.
SLP( 44)=C.C19C4319 X(M,LY)=     182901.
XNK=     148515.BX(J)=     196703.M= 44       Z=   48187.61
SLP( 45)=C.C265749C X(M,LY)=     184000.
SLP( 46)=C.C2C43361 X(M,LY)=     186737.
XNK=     152598.BX(J)=     196703.M= 46       Z=   44105.12
SLP( 47)=C.C3887753 X(M,LY)=     18920C.
XC=                     483C.9 BIRY=  1TERRAIN POINT  62 ALTITUDE    7
                        5C89.                                    2461.59
                        3C59.                                    1085.10
                        128C.                                     442.65
                        (1-P1)(1-P2)...(1-PN)=  0.8468

                        PK( 1)= .485858563
NUMBER OF RCUNDS FIRED=        2.0000      T=      14.            TF=       3(
```

Figure 19

than S_{LPB}. When $S_{LP}(30)$ is compared to S_{LPB}, it is found to be less and thus masking line of sight. $Z = 70338.14$ was calculated from equation (22) Chapter II. A new value for X_{nk} is given as $X_{nk} = 118862$ from equation (23). Using the assumption that targets fly over the terrain when the flight path passes through terrain, a new comparison slope is calculated and X_{nk} is changed from 26140 to 118862. Three more terrain obstructions exist such that the final value of $X_{nk} = 183740$. From equation (37), $X_D = -1953.7$ which indicates that no time is available to fire any rounds and battery 6 contributes no probability of kill at terrain point 62, and altitude 7. Figure 19 illustrates a case where terrain masking occurs once but a distance of 4830.9 feet for X_D remains for firing remembering that the time for firing the first and last round have already been removed from X_D. Therefore three rounds are fired, one more than shown on Figure 19 which is the last round fired. The slope comparison calculation is performed 18,490 times less those cases where negative elevation angles occur and the terrain point is out of range of the battery being evaluated.

Probability Calculations. A sample of computer output of probability calculations is found in Figures 19, 20, and 21. Figure 19 is a sample of a radar controlled gun battery. As was seen, three rounds were fired with a probability of kill of $P_k(1) = .46$. This is the battery kill probability resulting from 12 rounds fired, 3 per gun at a range of

Computer Probability Calculations
Automatic Weapon Battery

0.00001618	1032951.29	4840.
0.00001641	1018093.25	4805.
0.00001666	1003343.25	4770.
0.00001690	988701.29	4735.
0.00001715	974167.38	4700.
0.00001741	959741.52	4665.
0.00001768	945423.70	4630.
0.00001795	931213.92	4595.
0.00001822	917112.19	4560.
0.00001850	903118.51	4525.
0.00001879	889232.87	4490.
0.00001909	875455.27	4456.
0.00001939	861785.72	4421.
0.00001970	848224.22	4386.
0.00002002	834770.76	4351.
0.00002034	821425.34	4316.
0.00002068	808187.97	4281.
0.00002102	795058.64	4246.
0.00002137	782037.36	4211.
0.00002173	769124.13	4176.

(1-P1)(1-P2)...(1-PN)= 0.9987

PK(5)= .006444904 0.0064449

Figure 20

Computer Probability Calculations
Gun Battery

```
D=                    33762.9 BTRY=  4TERRAIN POINT  62 ALTITUDE  23
                      33993.                         96474.10
                      31898.                         84984.61             0.00046192  4
                      29803.                         74229.16             0.00052437  4
                      27709.                         64207.75             0.00062235  4
                      25617.                         54920.39             0.00069465  4
                      23526.                         46367.07             0.00081142  4
                      21436.                         38547.30             0.00096211  4
                      19348.                         31462.57             0.00115606  4
                      17264.                         25111.39             0.00141639  4
                      15184.                         19494.25             0.00177463  4
                      13110.                         14611.16             0.00228598  4
                      11047.                         10462.11             0.00304906  4
                       9000.                          7047.10             0.00425957  4
                       6984.                          4366.14             0.00632365  4
                       5039.                          2419.22             0.01021659  4
                       3289.                          1206.35             0.01842054  4
                       2290.                           727.52             0.03694769  4
                      (1-P1)(1-P2)...(1-PN)=  0.8572                      0.06125369  4

                      PK( 4)= .46033751
UMBER OF ROUNDS FIRED=    17.0000      T=      96.           TF=      31.5000
```

Figure 21

5089, 3059, and 1280 feet. Due to the short range, the P_i are .04, .08, and .1. Looking at Figure 21, a run of 17 rounds was fired. Here the altitude eliminates terrain masking with a course length of X_D = 33762.9 feet. However, the range has increased to 23,526 feet so that the P_i vary from .00046 to .061. The battery kill probability, $P_k(4)$ = .46. This probability is the same as for battery 1 in the earlier example. The short range in the other example accounts for the high probability for only 12 rounds fired. Figure 20 is the calculations for an automatic weapon battery. Fifty rounds were fired in this case. Actually more rounds could have been fired but the constraint on this weapon system limits firing to 50 rounds. The range is about 4500 feet on the average with P_i varying from .000016 to .0000217. The battery kill probability is $P_k(5)$ = .0064.

Program Results. Appendix V is a set of sample curves which plot altitude versus cumulative probability of kill for specific terrain points. The program prints out the curve for each terrain point examined. Included in the set are illustrative examples of program results.

Appendix VI is a set of curves plotting probability cost index versus altitude above terrain. This curve is obtained by the following relationship:

$$P_1 = P_{HI,K,i} + (1 - P_{D1}) \qquad (53)$$

$$k = 1, \ldots, N$$
$$i = 1, \ldots, 30$$

where

P_{Di} = the probability that the aircraft pilot detects the navigational target.

The values for P_{Di} were obtained from the computer program VISTRAC in reference 1.

Analysis of Results

The purpose of the mathmetical and computer models of the air defense complex is to compute and plot a curve of probability of kill versus altitude at each terrain point. Having obtained these curves, it is necessary to analyze them in order to determine the reasons for their shape. With this information, the application for the model's output can then be considered.

Terrain Effects. One of the most significant contributors to the shape of these curves is the terrain within which the air defense is situated. As was seen in Chapter II, Figures 5 through 7, the terrain has different formations producing different curves. The curves in Appendix V illustrate this fact.

Due to inaccessibility, many prominent terrain points are not occupied by firing batteries and only cause serious masking to batteries located on accessible terrain with less altitude than some others. The result is that at low altitudes of approach, some batteries may be masked and contribute no probability of kill to the target. Whereas at higher altitudes the target is unmasked soon enough to be fired upon by the battery. Figure 22 illustrates this concept. The line of sight to altitude 1 intersects the course line at A and X_{D1} = BE. From this result it is clear that the probability of kill at altitude 2, $P_{Kk2} > P_{Kk1}$ since $X_{D2} > X_{D1}$ due to increasing the altitude of approach. With the effect of terrain masking above, more rounds will be fired as altitude is increased.

It can be observed in Appendix V that some curves appear to have discrete jumps or regions of discontinuity. This is caused by terrain effects as well. These jumps occur, when at the lower altitudes, a particular battery with high single shot kill probability is masked. At the mext altitude of approach considered, the battery is suddenly unmasked and a large jump in probability is the result.

Effect of System Parameters. The battery effectiveness envelope is another influencing factor to curve shaping. This envelope was shown in Figure 1, Chapter II. In Figure 1, notice the curve of the envelope as altitude increases. This curve is an assumption which approximates the effectiveness of the average type system in use. If this model is used to simulate a weapon system with a radically different envelope, then a modification may be in order. The effect of the assumed envelope is also illustrated in Figure 22. At altitude 3, the line of sight intersects the target course line at G, but is beyond the range of the gun system. Therefore, X_{D3} = CF since C is the point where the target course line intersects the weapon system envelope.

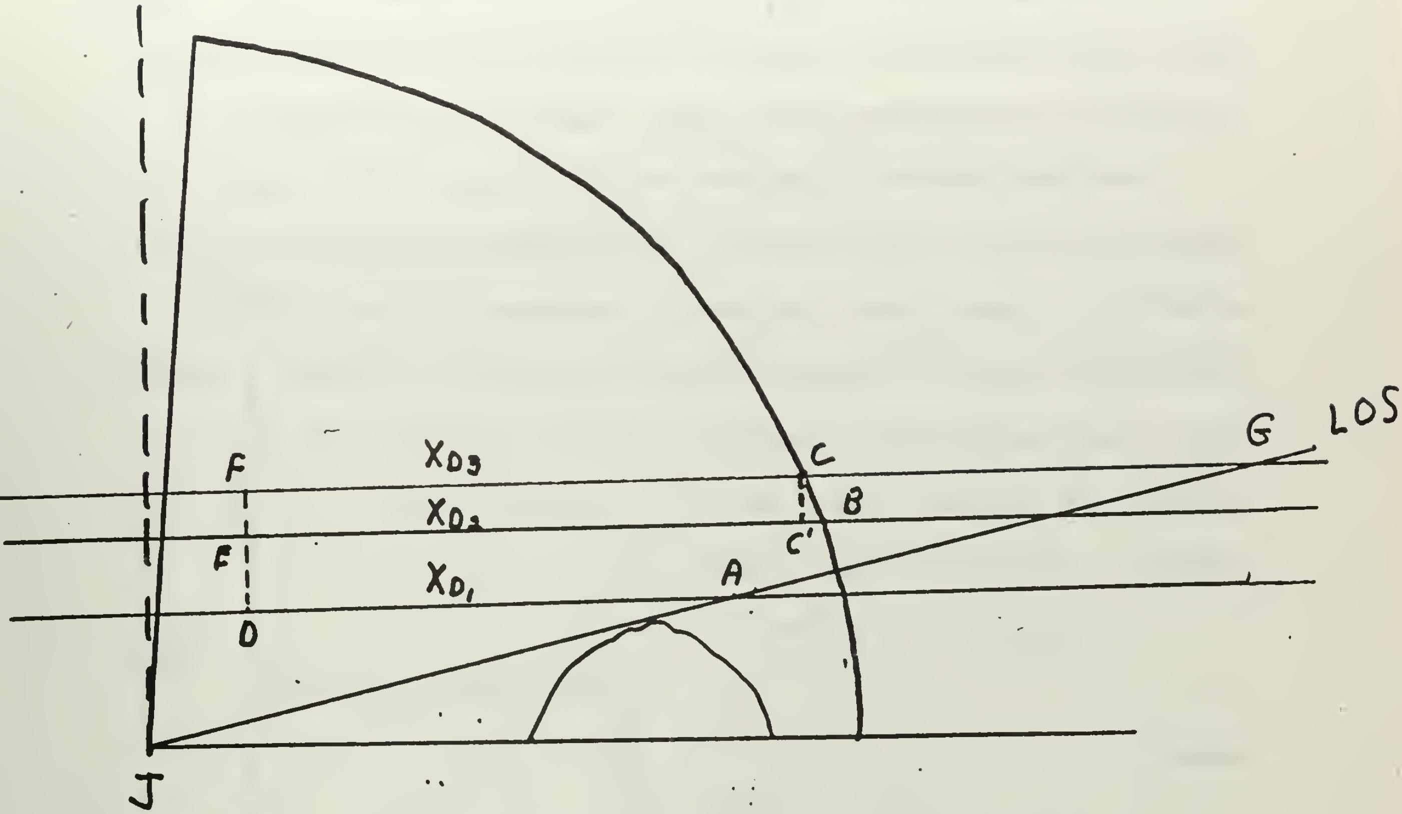


Figure 22

It is obvious that CF < BF by the amount of BC' from which it is determined that $X_{D3} < X_{D2}$ and accordingly $P_{Kk3} < P_{Kk2}$. This then illustrates why the curves tend to slope down as altitude is increased over some regions of altitude.

Another system parameter which effects the output curves is radar performance. In Chapter II, the half power point of a radar beam pattern is assumed to be the conservative limit of usable volume coverage provided. Therefore, any target in this model falling within the beam pattern is assumed detected with probability 1. This assumes a lateral range curve to be

$$P(X) = 1 - e^{-F(X)} \tag{54}$$

where

$$F(X) = \begin{cases} \infty \; ; & X \leq R \\ 0 \; ; & X > R \end{cases} \tag{55}$$

Substituting (55) into (54) results in

$$P(X) = \begin{cases} 1 \; ; & X \leq R \\ 0 \; ; & X > R \end{cases} \quad . \tag{56}$$

These equations are found in reference 11. Figure 23 is a plot of such a lateral range curve. The model assumed no maximum value for R as a parameter input. The maximum value R can take on is limited by the length of the terrain coordinate system. Maximum R used for the computer program is 47 miles. A nominal value for battery acquisition radars is 65 miles. If a coordinate system were longer than the

Lateral Range Curve

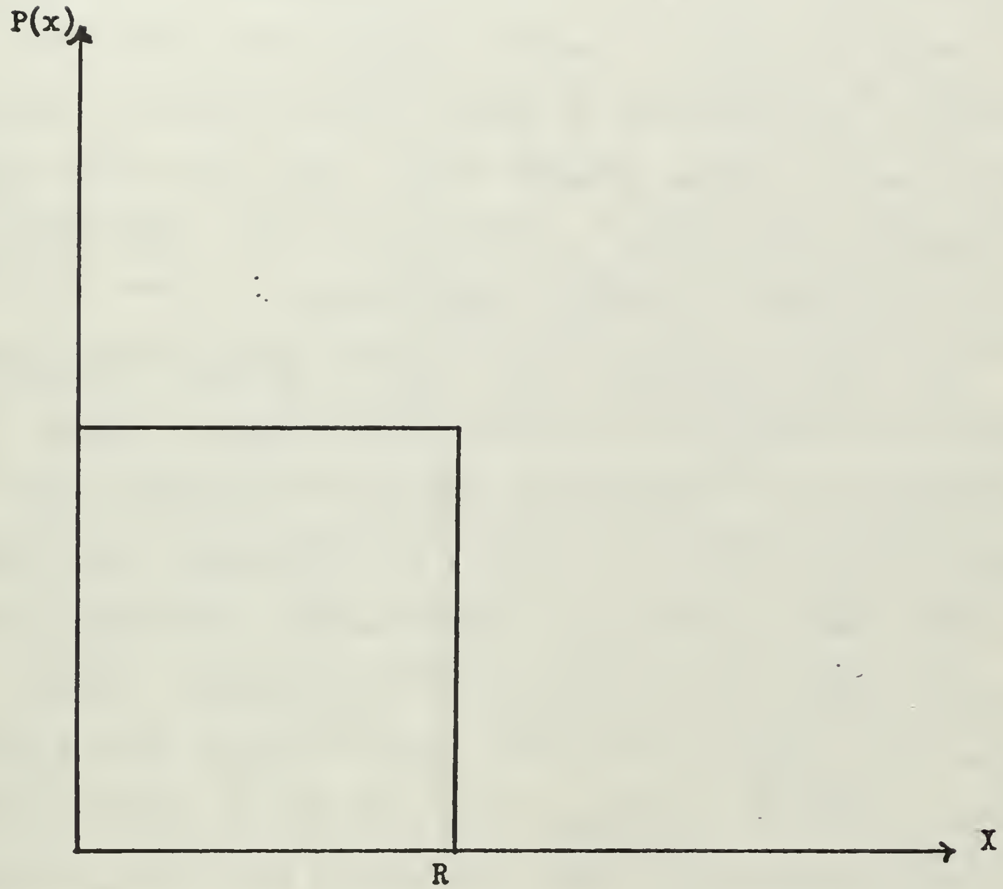


Figure 23

maximum rated range of a particular system, then a value of R for radar detection would have to be assigned. Further, if the radar detection range normally used in the program were at the maximum rated range, the assumption of equation (56) may introduce significant error and a different radar detection rate would have to be incorporated into the model. The reason this factor was not developed was due to the extremely low altitudes of target approach where a detection range does not become greater than a value much less than the maximum rated range of the radar. Such a development would constitute a needless complication.

Two other parameters were used to perform a sensitivity analysis to determine the effect of a change in these parameters on the probability curves. The minimum and maximum range of the AAW battery was originally 500 feet and 6000 feet respectively. These values were changed to 1500 and 9000 feet for one run of the computer. Curve number 8 in Appendix V is the result of using both parameters plotted on one graph. Curve I is the result of the increased parameters. These two curves are not radically different, but the increased range did cause a general increase in probability for most values of altitude. The rate of fire and total number of rounds fired per run per gun is a more powerful factor in the model. Curve 5 in Appendix V is a curve for AAW batteries firing up to 200 rounds per gun per run at 1000 rounds per minute with 10 guns firing independently per battery. The result of this situation is a very high probability curve which is obvious since if sufficient rounds are fired high probabilities will result. Practical constraints will not permit this type of performance in the usual case.

Battery Emplacement Effects. The probability curves generated by the terrain in the computer model are found to be quite different from terrain point to terrain point. A few curves from adjacent terrain points are similar in the case where terrain does not change rapidly. Except for this case, the curves have different shapes. A different curve will obviously result if a different battery emplacement scheme is implemented since new battery to terrain masking situations occur. As the program calculates probabilities for the different terrain points, new situations occur such as a battery on a high peak is in range or a battery behind a peak is unmasked. This variation in terrain conditions is why the curves are not exactly related even though the probabilities calculated are cumulative. The curves for terrain points 47 and 69, curves 3 and 4, Appendix V, when compared, are found to be quite different. An additional factor to be kept in mind is that the altitudes above terrain are not at the same absolute altitude. Altitude 600 at terrain point 47 is not necessarily the same altitude at terrain point 69. There is no intention that there be any relationship between the different terrain points. The probability curve for each terrain point is separate and distinct from any other unless, of course, ther is little terrain change between them except the X coordinate distance.

Model Application

In Chapter I, it was pointed out that the work done in reference 1 found the relationship between probability of radar detection and altitude above terrain to be approximately linear. As a result, the computer program incorporated a linear fit to the data. Following this the probability of pilot navigation was added to the linear function. The functional relationship was necessary for use in the simplex algorithm for linear program solution for the optimal altitude of approach. At the beginning of this work, the concept was to develop the air defense model as an input to the program in reference 1. After examining the output curves of Appendix V a linear fit did not appear feasible for the data. In order to determine how bad a linearity assumption would be, a regression equation was calculated from the data of terrain point 62 in Appendix V. Curve number 4 is a plot of the equation. The function derived is given by

$$Y = .128 + .000369\ X \tag{57}$$

To submit the output data to a linear fit would introduce serious misrepresentation of the model capability. This fact is obvious from the results of the linear fit to the data of terrain point 62.

Probability Cost Index. The curves in Appendix VI are a probability cost index. The values are not true probabilities since they are the sum of two probabilities as shown in equation (53). The term cost is defined to be a measure of the material effort necessary to penetrate the defense, reach the target, and deliver the ordnance. Since either one of the events constitutes failure in the mission, the higher the value of the cost index the greater the chance that the mission will be a failure. Consequently, the survivability of the aircraft in the penetration effort is maximized when the cost index is minimized.

When examining the curves in Appendix VI, it is observed that each curve contains a global minimum. The curves then possess a point which minimizes the cost index and provides the optimal survivability index number at the associated altitude.

CHAPTER V

CONCLUSIONS AND EXTENSIONS

The development of the air defense model within the assumptions and limitations is complete. As a result of the output from the computer model it is concluded:

(1) That the probability calculations developed a result that varies with terrain, air defense systems, and employment tactics. Consequently, no assumption of linearity between probability of kill and altitude above terrain is justified. In fact, an investigation of the degree of polynomial that a least square fit would require to model the curves in Appendix VI is necessary. Such an investigation would be required to discover how the degree would vary with terrain points using the same system parameters. It would also have to determine how the degree would vary with different air defense systems and terrain. Before an algorithm can be found to solve for the minimum point on the cost curve, the curve itself must be functionally modeled.

(2) That no practical results would be obtained from inserting the output from the air defense model into the program in reference 1. If such an attempt were made, the simplex algorithm used in reference 1 would obtain a solution, but it would not be known if the solution were the

global minimum. Further, the linear regression curve would grossly missrepresent the probabilities calculated by the air defense model.

(3) That further investigation is necessary to discover the feasibility of fitting the curves in Appendix VI to some degree polynomial such that the degree chosen is acceptable for the curves of all the terrain points. If a single degree polynomial is not feasible, then a method of finding the best fit for each curve over a specified range of altitudes must be adopted.

(4) That having determined the functional relationship for the probability cost curve, a non linear programing algorithm must be found that solves for the global minimum of each curve within the aircraft flight path constraints used in reference 1.

Air Defense Model Extensions

Two major assumptions in the air defense model should be investigated further. The assumption of independence between rounds fired successively from the same gun may introduce serious error. Research into the determination of the relative weights given to correlated and uncorrelated error should give an indication of how unrealistic the assumption of independence really is. If independence is unacceptable then means for finding the conditional probabilities associated with dependently fired rounds should be studied.

An additional problem for further study is the assumption that the gun fire shot pattern is circular normal. Much evidence points to the fact that the shot pattern is actually elliptical normal. Empirical equations for the elliptical normal error pattern such as equation (42) and (48), Chapter II in the circular normal case should be used if such equations exist.

The final extension is in the area of three dimensional terrain. As was previously pointed out, the Fortran code written for evaluation of line of sight in three dimensional terrain was not verified. With the availability of digitalized terrain this portion of the program should be verified. However, one serious disadvantage in the concept used in this model is the requirement that the target course line be parallel to the X axis of the coordinate system. The problem lies in the necessity of assigning new altitude values for each different route of approach into the target. Another method for determining line of sight between 2 points in 3 dimensional terrain is found in reference 6. This method requires a least square fit to the terrain points lying on the line between the two points for which line of sight is being determined. Then the method determines if the curve extends above the line of sight.

APPENDIX I

BIBLIOGRAPHY

m 5927

1. Miller, W. S., Jr. and Brown, E. E, Aircraft Survivability Index for Low Altitude Penetration, Thesis, U. S. Naval Postgraduate School, Monterey, California, June, 1968.

2. U. S. Army Material Command, Conventional AA Weapons (Effectiveness) Free World, U. S. A. Foreign Science and Technology Center, December 1965.

3. Shank, E. B. and Henderson, A. S., A Comparative Evaluation of Several Forward Area Air Defense Systems, Weapon Systems Laboratory Aberdeen Proving Ground, Maryland, Memorandum no. 1930, of January 1964.

4. Ordnance Corps Development and Proof Services, A Study of Errors Associated with the M33 Fire Control System, Aberdeen Proving Grounds, Maryland, July, 1957.

5. Vukmir, V., Sudeimer, R. H., Lum, G. H., Mostler, L. P., Ecker, H. A., Bell, R. J., and Libby, T. L., Analytical Methods and Specific Simulation Models for Determining Penetration Effectiveness of Manned Aircraft, Air Force Systems Command, Wright Patterson Air Force Base, Ohio, April, 1964.

6. Doughterty, C. B. and Tuck, R. E., Effectiveness of Guns, Rockets, and Unorthodox Weapons for Low Altitude Air Defense in the ZI, Rand Research Memorandum, Rm-1172, of 15 January, 1954.

7. Harrison, J. L., A Computer Simulation of an Aircraft Penetration Over Hostile Terrain, Thesis, U. S. Naval Postgraduate School, Monterey, California, 1965.

8. Green, J. R., and Martin, R. H., An Operations Model for Estimating the Effectiveness and Attrition of Tactical Aircraft When Penetrating Defenses at Low Altitude, Rand Memorandum Rm-3295-PR, December 1962.

9. Sodergren, P. R., Naval AA Gunfire Dispersion for Point Defense, The Johns Hopkins University, Applied Physics Laboratory, 1964.

10. Sesse, Mathematical Model Reconnaissance and Penetration Study, Volume IIB- PENSAM Specifications, U. S. Air Force Systems Command, Wright Patterson Air Force Base, Ohio.

11. Koopman, B. O., Search and Screening, OEG Report No. 56, Washington, D. C., 1946.

APPENDIX II

TERRAIN AND BATTERY COORDINATES

COORDINATES OF MAJOR TERRAIN POINTS

X	H	Y
0.0	6066.	0.0
6706.	6123.	0.0
9078.	5757.	0.0
19087.	5426.	0.0
30000.	5258.	0.0
48854.	5310.	0.0
52714.	5556.	0.0
56446.	5978.	0.0
58837.	6097.	0.0
60886.	6075.	0.0
62200.	5990.	0.0
65245.	6048.	0.0
68374.	5819.	0.0
72325.	6374.	0.0
78000.	5450.	0.0
84023.	5630.	0.0
85762.	5504.	0.0
92782.	5793.	0.0
97683.	6096.	0.0
98753.	6342.	0.0
100321.	6494.	0.0
101633.	6389.	0.0
105080.	6617.	0.0
106600.	6430.	0.0
115811.	6164.	0.0
118118.	6461.	0.0
120900.	6230.	0.0
127540.	6353.	0.0
132802.	6980.	0.0
134531.	7806.	0.0
137891.	6821.	0.0
142800.	6610.	0.0
147709.	6649.	0.0
149151.	6880.	0.0
150589.	6685.	0.0
158059.	7098.	0.0
159925.	7826.	0.0
161609.	7328.	0.0
163974.	7618.	0.0
165000.	7400.	0.0
167890.	7628.	0.0
172800.	7300.	0.0
178700.	6750.	0.0
182901.	7403.	0.0
184000.	7180.	0.0
186737.	7342.	0.0
189200.	6700.	0.0
196703.	7114.	0.0
200620.	7202.	0.0
202852.	7150.	0.0
205523.	7539.	0.0
206652.	7309.	0.0
210233.	7883.	0.0
212183.	7950.	0.0
214161.	7324.	0.0
219152.	8045.	0.0
222800.	7940.	0.0
224400.	8160.	0.0
226653.	8321.	0.0
230139.	7868.	0.0
242244.	7244.	0.0
244497.	7423.	0.0
246100.	7130.	0.0
251296.	7068.	0.0
257179.	6792.	0.0
259793.	6919.	0.0
262358.	6750.	0.0
264328.	7021.	0.0
265230.	7460.	0.0

COORDINATES OF MAJOR TERRAIN POINTS CONTINUED

265927.	7496.	0.0
268257.	7028.	0.0
275105.	6983.	0.0
279714.	7203.	0.0
281700.	7300.	0.0
285209.	8045.	0.0
286963.	8012.	0.0
288463.	7770.	0.0
290001.	7265.	0.0
291700.	7162.	0.0
294290.	7363.	0.0
295721.	7773.	0.0
297609.	7534.	0.0
300001.	7600.	0.0
302107.	7627.	0.0

COORDINATES OF BATTERY POSITIONS

X	Y	H
196703.	0.0	7114.
212183.	0.0	7950.
224400.	0.0	8160.
265927.	0.0	7496.
159925.	0.0	7826.
189200.	0.0	6700.
226653.	0.0	8312.
259793.	0.0	6919.
286963.	0.0	8012.
290001.	0.0	7265.
285209.	0.0	8045.

APPENDIX III

GLOSSARY OF FORTRAN VARIABLE NAMES

The following is a list of Fortran variable names in the order of appearance. This list excludes the system parameters which were presented in Chapter III.

K(K,LY)	THE X COORDINATE OF THE KTH TERRAIN POINT WITH THE LY TH Y COORDINATE INDEX
H(K,LY)	THE H COORDINATE OR ALTITUDE OF THE KTH TERRAIN POINT WITH LY TH Y COORDINATE INDEX
Y(K,LY)	THE Y COORDINATE OF THE KTH TERRAIN POINT WITH LY TH Y COORDINATE INDEX, SET EQUAL TO 0 FOR TWO DIMENSIONAL TERRAIN
BX(J)	X COORDINATE OF THE JTH BATTERY
BY(J)	Y COORDINATE OF THE JTH BATTERY SET EQUAL TO 0 FOR 2 DIMENSIONAL TERRAIN
BH(J)	H COORDINATE OF THE JTH BATTERY
RANGE(I)	AN ARRAY USED WITH SUBROUTINE UTPLOT
AH(I)	ALTITUDE ABOVE TERRAIN FOR THE ITH POINT
DELTX	THE DIFFERENCE IN X COORDINATES OF THE KTH TERRAIN POINT AND THE JTH BATTERY
DELTY	THE DIFFERENCE IN Y COORDINATES OF THE KTH TERRAIN POINT AND JTH BATTERY
DELTH	THE DIFFERENCE IN THE H COORDINATES OF THE KTH TERRAIN POINT AND THE JTH BATTERY

ELRX	MAXIMUM ELEVATION TRACKING RATE FOR THE SPECIFIC BATTERY BEING EXAMINED
AZRX	MAXIMUM AZIMUTH TRACKING RATE OF THE SPECIFIC BATTERY BEING EXAMINED
RMF	MINIMUM RANGE OF THE SPECIFIC BATTERY BEING EXAMINED
RM	MAXIMUM RANGE OF THE BATTERY BEING EXAMINED
THETA	DETECTION RADAR LOWER EDGE OF BEAM PATTERN ELEVATION ANGLE IN DEGREES OF THE BATTERY BEING EXAMINED
AT	SYSTEM DELAY TIME FOR THE BATTERY BEING EXAMINED
TR	CREW REACTION TIME FOR THE BATTERY BEING EXAMINED
RS	SLANT RANGE TO THE (K,I)TH POINT
HF	HORIZONTAL COMPONENT OF MINIMUM RANGE
RXS	HORIZONTAL COMPONENT OF RS
HFT	HORIZONTAL COMPONENT OF THE RANGE OF THE POINT WHERE THE LINE DRAWN FROM THE MAXIMUM ELEVATION ANGLE INTERSECTS THE TARGET COURSE LINE
RSA	THE SLANT RANGE WHERE MAXIMUM AZIMUTH RATE OCCURS
RSE	THE SLANT RANGE WHERE MAXIMUM ELEVATION RATE OCCURS

XD	THE DISTANCE ON THE TARGET COURSE LINE AVAILABLE FOR FIRING ALL BUT FIRST AND LAST ROUNDS
ROF	OPEN FORE SLANT RANGE
T	TIME AVAILABLE FOR FIRING EXCLUDING FIRST AND LAST ROUNDS
NS,XNR	NUMBER OF ROUNDS FIRED PER RUN
RH	HORIZONTAL RANGE TO MAXIMUM RADAR DETECTION POINT ON COURSE LINE
TF	TIME OF FLIGHT
GUNS, NGUNS	NUMBER OF GUNS PER BATTERY, RADAR CONTROLLED GUN BATTERY
R(II)	RANGE OF II TH ROUND FIRED
SIG(II)	THE FIRING ERROR RADIUS OF THE II TH ROUND FIRED
SIGM	FIRING ERROR OF THE RADAR CONTROLLED GUN
EP(II)	THE PROBABILITY OF KILL OF THE II TH ROUND FIRED
TW	$\prod_{L=I}^{NS}$ (1-EP(II)) FOR THE RADAR CONTROLLED BATTERY
PD(J)	PROBABILITY OF KILL OF THE JTH BATTERY
TY	$\prod_{L=I}^{NS}$ (1-EP(II)) FOR AAW BATTERY
NT, XNR	NUMBER OF MISSILES FIRED
TP	$\prod_{L=I}^{NFU}$ (1-PK(J)) THE PROBABILITY THAT NO ROUND KILLS THE TARGET

PHI(K,I)	PROBABILITY THE AIRCRAFT IS KILLED AT THE (K,I)TH POINT IN SPACE

SUBROUTINE LOS

RXM	HORIZONTAL COMPONENT OF MAXIMUM FIRING RANGE
XNK	X COORDINATE ON TARGET COURSE LINE OF FIRST RADAR DETECTION POINT
KK	X COORDINATE INDEX OF LAST TERRAIN POINT BETWEEN POINT IN SPACE FOR CALCULATION AND THE BATTERY
II	X COORDINATE INDEX OF FIRST TERRAIN POINT BETWEEN POINT IN SPACE FOR CALCULATION AND THE BATTERY
SLPD	THE SLOPE OF THE LINE BETWEEN THE POINT OF FIRST RADAR DETECTION AND THE BATTERY
SLP(M)	THE SLOPE OF EACH OF THE M TERRAIN POINTS THAT MAY CAUSE TERRAIN MASKING ON EACH FIRING RUN
Z	DISTANCE ON TARGET COURSE LINE FROM POINT OF TERRAIN UNMASKING TO THE BATTERY
XDR	THE DISTANCE BETWEEN POINT OF FIRST RADAR DETECTION AND OPEN FIRE POINT, ALSO CALLED WARNING TIME DISTANCE
TT	TOTAL DELAY DISTANCE ON TARGET COURSE LINE

SUBROUTINE SEE

EYE INDICATION OF WHETHER LINE OF SIGHT EXISTS FOR EACH CALLING OF THE SUBROUTINE SEE

EYE = 0 INDICATES LINE OF SIGHT EXISTS

EYE = 1.0 INDICATES LINE OF SIGHT DOES NOT EXIST

APPENDIX IV

FORTRAN PROGRAM

```
      IMPLICIT REAL*8(A-H,O-Z),INTEGER*4(I-N)
      REAL*4AX1(30),PH1(30),RANGE(4)
      COMMON    AH(30),H(120,1),X(120,1),Y(120,1),BX(20),BY(20),BH(20),P
     1K(30),PHI(120,30),AXH(120,30),SLP(90),P(200),SIG(200),FP(200),PD(4
     13)
      DATA NAP/30/,NTT/84/,        ELRMG/10.0/,ELRMA/20.0/,ELRMM/15.0/,AZ
     1RMG/12.0/,AZRMA/20.0/,AZRMM/15.0/,RMG/45000.0/,RMA/ 9000.0/,RMM/10
     15600.0/,THET1/1.12/,THET2/.50/,THET3/1.0/,API/0.0/,GR/25./,CONT/.0
     1002/,NY/1/,YK/500.0/,VT/350.0/,TRG/30.0/,TRA/10.0/,TRM/30.0/,ATG/2
     10.0/,ATA/5.0/,ATM/10.0/,LT1/4/,LT2/10/,        TIR/6.0/,A/350.0/,SIG
     1M/ 1.5/,GUNS/4.0/,TCRA/10.0 /,PKSM/0.48/,NR/2/,T1M/20.0/,AGUN/5.0
     1/,TX/15.0/,LY/1/,RMEG/3000./,RMEA/1500./,RMEM/9500./,FE/87 /,YM/.0
     1007/,ZM/2000./,SM/1200./,RG/0.8/,NT1/36/,NFU/11/,LT3/11/,ZK/.2/,AX
     1/400./,AY/150./,AZ/800./,PG/ 8/,PA/.3/
      CALL FP
      PRINT 24
   24 FORMAT(1H1)
      READ 5,(((X(I,J),H(I,J),Y(I,J)),I=1,NTT), J=1,1)
    5 FORMAT(3F10.0)
      READ 5,((BX(L),BY(L),BH(L)),L=1,NFU)
      READ 3,(PD(I),I=1,NAP)
    3 FORMAT(F10.4)
C     COMPUTE PK FOR EACH TERRAIN POINT
      THET1=THET1*3.14159265/180.
      THET2=THET2*3.14159265/180.
      THET3=THET3*3.14159265/180.
      AZRMG=AZRMG*3.14159265/180
      AZRMA=AZRMA*3.14159265/180.
      AZRMM=AZRMM*3.14159265/180
      FE=FE*3.14159265/180.
      RANGE(1)=3000.
      RANGE(2)=0.
      RANGE(3)=1 0
       RANGE(4)=0.
      DO 20 K=NT1,NTT
C     COMPUTE PK FOR EACH ALTITUDE TO 3000 FEET
      DO 20 I=1,NAP
      BB=I
      AH(I)=H(K,LY)+BB*100.
C     COMPUTE PK FOR EACH FIRE UNIT
      DO 30 J=1,NFU
      IF(J.LE.LT1) GO TO 140
      IF(J.LE.LT2) GO TO 150
      IF(J.LE.LT3) GO TO 160
  140 ELRX=ELRMG
      AZRX=AZRMG
      RME=RMEG
      RM=RMG
      THETA=THET1
      AT=ATG
      TR=TRG
      GO TO 105
  150 ELRX=ELRMA
      RME=RMEA
      RM=RMA
      AZRX=AZRMA
      THETA=THET2
      AT=ATA
      TR=TRA
      GO TO 105
  160 ELRX=ELRMM
      RME=RMEM
      RM=RMM
      AZRX=AZRMM
      THETA=THET3
      AT=ATM
      TR=TRM
C     PREVENT OUTBOUND FIRING
  105 DELTX=(X(K,LY)-BX(J))
      DELTX=DABS(X(K,LY)-BX(J))
      IF(DELTX.GT.0.) DELTX=0
      DELTH=DABS(AH(I)-BH(J))
       DELTY=DABS(Y(K,LY)-BY(J))
      RS=DSQRT(DELTX*DELTX+DELTY*DELTY+DELTH*DELTH)
      IF(RS.GT.RM) GO TO 100
C     DETERMINE IF ELEVATION IS NEGATIVE
      IF(AH(I)   -BH(J)) 100,15,15
C     CHECK FOR MINIMUM FIRING RANGE
   15 HE=RME*DCOS(FE)
      RXS=DSQRT(DELTX*DELTX+DELTY*DELTY)
      HET=DELTH/DTAN(FE)
      IF(RXS.GT.HET) GO TO 50
      IF(HET.GT.HE) GO TO 49
      RXS=HE
      RS=DSQRT(RXS*RXS+DELTH*DELTH)
      GO TO 48
   49 RXS=HET
   48 DELTX=DSQRT(RXS*RXS-DELTY*DELTY)
C  50 PRINT 162,DELTX ,HE,HET,RXS
C 162 FORMAT(10X,'DELTX=',1X,F10.4,1X,3G10.2 )
   50 CONTINUE
      IF(DELTY.EQ.0.) GO TO 166
C     CHECK FOR MAXIMUM AZIMUTH TRACKING RATES
      AZR=VT*DELTY/(RS*RS-DELTH*DELTH)
      RSA=DSQRT(VT*DELTY/AZRX+DELTH*DELTH)
      IF(AZR.GT.AZRX) RS=RSA
C     CHECK FOR MAXIMUM ELEVATION RATE
  166 ELR=(VT*DSQRT(DELTH*DELTH+DELTY*DELTY))/(RS*RS)
      RSE=DSQRT((VT*DSQRT(DELTH*DELTH+DELTY*DELTY))/ELRX)
      IF(ELR.GT.ELRX) RS=DMAX1(RS,RSE)
      DELTX=DSQRT(RS*RS-DELTH*DELTH-DELTY*DELTY)
```

```
      CALL LOS(RM,K,I,J,LY,DELTX,DELTY,DELTH,APC,YM,THETA,TR,AT,VT,YK,XD
     1,TF,TX,NTT,LT1,LT2,LT3,CONT,ZM,SM,RH,RXM,RSF)
C     PRINT 522,XD,J,K,I
C 522 FORMAT(10X,'XD=',1X,F20.1,1X,'BTRY=',I3,'TERRAIN POINT',1X,I3,
C    11X,'ALTITUDE',1X,I3)
      IF(XD) 100,100,521
  521 RSF=DSQRT((DELTX+XD)**2+DELTY*DELTY+DELTH*DELTH)
      RDF=RSF
      IF(J.LE.LT1) GO TO 500
      IF(J.LE.LT2) GO TO 600
      IF(J.LE.LT3) GO TO 700
  500 T=(XD/VT )
      NS=T/TIR
      IF(RH.GT.RXM) NS=NS+1
      XNR=NS
      IF(NS.LE.0) GO TO 100
      NGUN=GUNS
      TW=1.0
      NS=NS+1
      DO 135 II=1,NS
      WI=II-1
      IF((XD+DELTX).LT.(WI*VT*TIR)) GO TO 135
      R(II)=DSQRT((XD+DELTX-WI*VT*TIR)**2+DELTH*DELTH+DELTY*DELTY)
      SIG(II)=(.05*VT)**2+(R(II)**2*10.0E-6)*(SIGM+VT/1000.0)**2+(10.0*(
     1API+YM*R(II)))**2
      FP(II)=(A/(2.0*3.14159265*SIG(II)))*PG
      TW=TW*(1.0-FP(II))
      RR=R(II)
      SIG1=SIG(II)
      FP1=FP(II)
  135 CONTINUE
      PK(J)=(1.0-(TW)**NGUN)
C     PRINT 411,TW,J,PK(J)
C 411 FORMAT(30X,'(1-P1)(1-P2)...(1-PN)=',G12.4,//,
C    130X,'PK(',I2,')=',1X,F10.9)
C     PRINT 501,XNR,T,TF
C 501 FORMAT(10X,'NUMBER OF ROUNDS FIRED=',1X,F10.4,5X,'T=',1X,G10.2,5X,
C    1'TF=',1X,F10.4)
      GO TO 30
C     AUTOMATIC WEAPON BATTERY
  600 T=(XD/VT)
      NS=T*TCRA
      IF(RH.GT.RXM) NS=NS+1
      IF(NS.GT.50 ) NS=50
      XNR=NS
      TY=1.0
      NAGUN=AGUN
      NS=NS+1
      DO 144 II=1,NS
      WI=II-1
      IF((XD+DELTX).LT.(WI*VT*1.0/TCRA)) GO TO 144
      R(II)=DSQRT((XD+DELTX-WI*VT*1.0/TCRA)**2+DELTH*DELTH+DELTY*DELTY)
      SIG(II)=(.36*VT*R(II)*10.0E-3)**2
      FP(II)=(A/(2.0*3.14159265*SIG(II)))*PA
      TY=TY*(1.0-FP(II))
  144 CONTINUE
      PK(J)=(1.0-(TY)**NAGUN)
C     PRINT 410,TY,J,PK(J)
C 410 FORMAT(30X,'(1-P1)(1-P2)...(1-PN)=',G12.4,//,
C    130X,'PK(',I2,')=',1X,F10.9)
C     PRINT 640,XNR,PK(J)
C 640 FORMAT(10X,'XNR=',F20.0,1X,'PK(J)=',1X,F20.7,10X,'AA BTRY')
      GO TO 30
C     MISSILE MODEL
  700 T=(XD/VT)
      WR=NR
      NT=NR
      IF(T.GT.(WR*TIM)) GO TO 710
      NS=T/TIM
      XNR=NS
      TEMP=1.0
      XNR=DMAX1(TEMP,XNR)
      NT=XNR
  710 PK(J)=1.0-(1.0-PKSM)**NT
C     PRINT 605,T,TF,FFP
C 605 FORMAT(10X,'T=',F10.1,'TF=',F10.1,'FFP=',G10.2)
C     PRINT 705, NR,T,PK(J)
C 705 FORMAT(10X,' NR=',1X,I3    ,1X,'T=',1X,G10.2,5X,'MISSILE',G10.4)
      GO TO 30
  100 PK(J)=0.
   30 CONTINUE
      TP=1.0-PK(1)
      NL=NEU+1
      DO 190 JI=2,NL
      TP=TP*(1.0-PK(JI))
  190 CONTINUE
      PHT(K,I)=1.0-TP
      AXH(K,I)=AH(I)
   20 CONTINUE
   10 CONTINUE
      DO 750 I=NT1,NTT
      DO 751 J=1,30
      AA=J
      ALT=AA*100.
      NLT=ALT
      PHI(J)=(PHI(I,J)+(1.0-PD(J)))
  751 CONTINUE
      PRINT 752
  752 FORMAT(1H1)
```

```
  750 CONTINUE
      DO 176 K=NT1,NTT
      DO 175 I=1,30
      AA=I
      AX1(I)=AA*100.
      PH1(I)=PHI(K,I)
  175 CONTINUE
      PRINT 177,K
  177 FORMAT(1H1,52X,'ALTUTIDE VS PROB',//,54X,'TERRAIN PT',2X,I3,//)
      CALL UTPLOT(AX1,PH1,30,RANGE,1,0)
  176 CONTINUE
  181 CONTINUE
      END
```

FORTRAN IV G LEVEL 1, MOD 2 LOS DATE = 68328 17/51/55

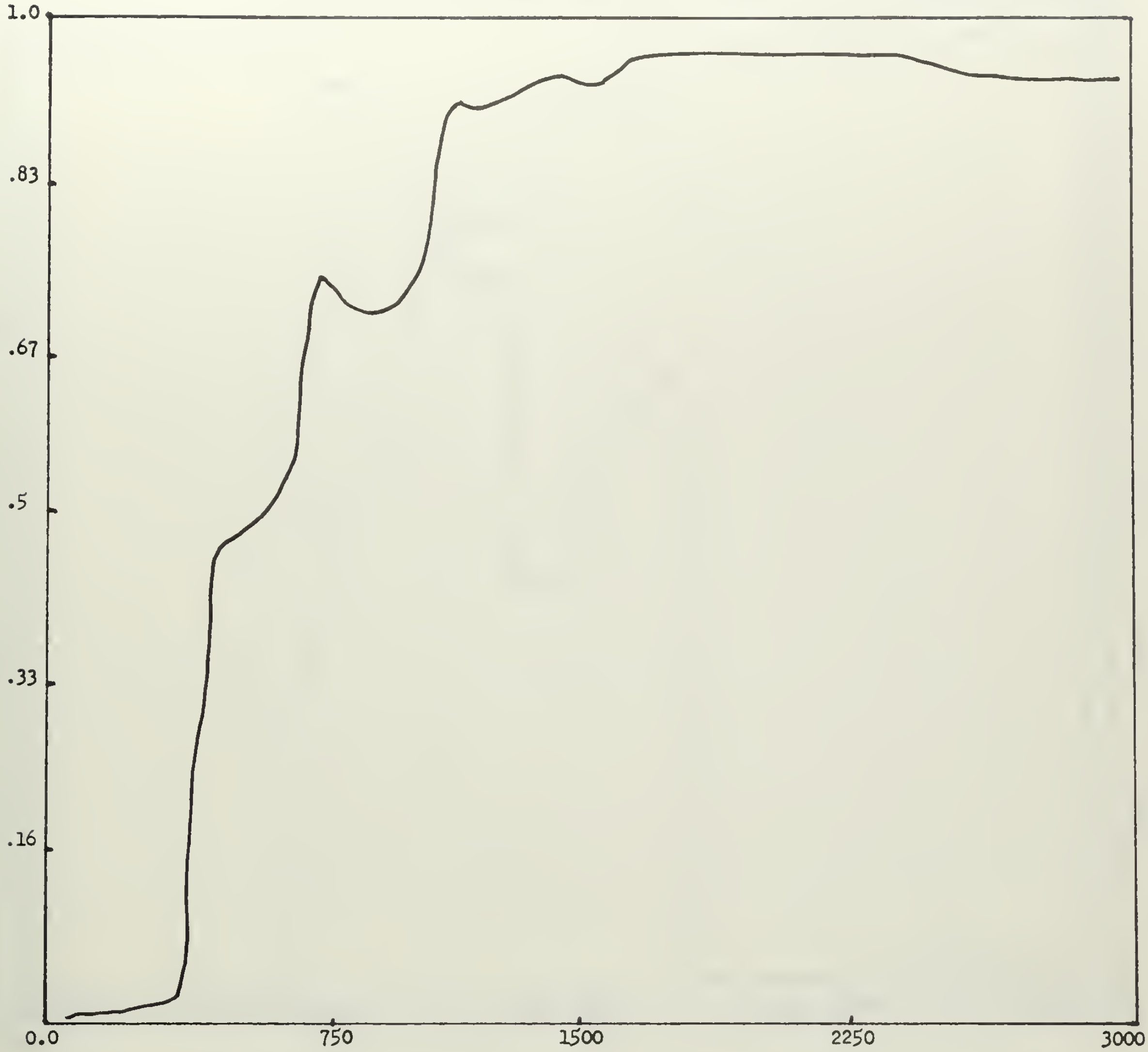
```
      SUBROUTINE LOS(RM,K,I,J,LY,DELTX,DELTY,DELTH,API,YM,THETA,TR,AT,VT
     1,YK,XD,TE,TX,NTT,LT1,LT2,LT3,CONT,ZM,SM,RH,RXM,RSE)
      IMPLICIT REAL*8(A-H,O-Z),INTEGER*4(I-N)
      COMMON     AH(30),H(120,1),X(120,1),Y(120,1),BX(20),BY(20),BH(20),P
     1K(30),PHI(120,30),AXH(120,30),SLP(90),R(200),SIG(200),EP(200)
C     CALCULATE DISTANCE ON COURSE LINE FOR FIRING
C     CALCULATE RADAR WARNING TIME
      BK=.21'
      ZK=.24
      RXM=DSQRT(RM*RM-DELTH*DELTH-DELTY*DELTY)
      RH=DSQRT((DELTH/DTAN(THETA))**2-DELTY**2)
      XNK=BX(J)-DSQRT(RH*RH-DELTY*DELTY)
      IF(XNK.LT.0.) XNK=0.
C     PRINT 6,K,I,J
C   6 FORMAT(10X,'TP=',1X,I3,1X,'ALT=',2X,I3,1X,'BTRY=',1X,I3)
C     PRINT 153,XNK,BX(J),RXM
C 153 FORMAT(10X,'XNK=',1X,F10.0,5X,'BX(J)=',1X,F10.0,1X,'RXM=',1X,F10.0
C    1)
      IF(XNK.EQ.BX(J)) GO TO 23
      NXK=XNK
      IF(BY(J).NE.0.) GO TO 155
      II=0
      DO 31 M=1,NTT
      IF(II.NE.0) GO TO 32
      IF(XNK.LT.X(M,LY))II=M
   32 IF(BX(J).GE.X(M,LY)) GO TO 31
      KK=M-2
      GO TO 36
   31 CONTINUE
   36 SLPB=(AH(I)-BH(J))/(BX(J)-XNK)
C     PRINT 7,SLPB,KK,II
C   7 FORMAT(10X,'SLPB=',1X,F10.5,1X,'KK=',I3,'II=',I3)
      DO 300 M=II,KK
      IF(KK-II) 23,10,10
   10  SLP(M)=(AH(I)-H(M,LY))/(X(M,LY)-XNK)
C     PRINT 20,M,SLP(M),X(M,LY)
C  20 FORMAT(10X,'SLP(',I3,')=',F10.8,1X,'X(M,LY)=',F10.0)
      IF(SLPB.GT.SLP(M)) GO TO 310
      GO TO 300
  310 Z=(AH(I)-BH(J))*(BX(J)-X(M,LY))/(H(M,LY)-BH(J))
      XNK=BX(J)-Z
      SLPB=(AH(I)-BH(J))/(BX(J)-XNK)
C     PRINT 21,XNK,BX(J),M
C  21 FORMAT(10X,'XNK=',F10.0,'BX(J)=',F10.0,'M=',I3)
C     PRINT 8,SLPB,AH(I)
C   8 FORMAT(10X,'SLPB=',F10.8,'AH(I)=',F10.0)
  300 CONTINUE
C     PRINT 143,XNK
C 143 FORMAT(10X,'XNK=',1X,G10.4)
      GO TO 320
  155 KK=(X(K,LY)-XNK)/YK+1.0
      XMK=0.
      MN=0.
      Z1=0.
      DO 320 M=2 ,KK
      CALL SEE(NXK,K,I,J,EYE,LY,YK)
      IF(EYE) 322,322,340
  340 IF(MN.EQ.(M-1))GO TO 350
      GO TO 355
  350 TS=Z1/VT
      IF(TS.GT.TX) XNK=XNK+YK*XMK
  355 Z1=Z1+YK
      MN=M
      NXK=XNK
  322 XMK=XMK+1.0
  320 CONTINUE
      RH=DSQRT((BX(J)-XNK)**2-DELTY*DELTY)
      RXM=DMIN1(RXM,RH)
      ROE=DSQRT(RXM*RXM+DELTH*DELTH+DELTY*DELTY)
      R(1)=ROE
      IF(J.LE.LT1) GO TO 500
      IF(J.LE.LT2) GO TO 600
      IF(J.LE.LT3) GO TO 700
  500 TE=API+YM*ROE
      GO TO 30
  600 TE=DEXP(CONT*ROE)
      GO TO 30
  700 TE=(ROE-ZM)/SM
   30 TT=(TR+AT+TE)*VT
      XDR=RH-RXM
      IF(XDR.EQ.0.) GO TO 235
      IF(TT.LT.XDR) GO TO 55
      TT=TT-XDR
      GO TO 235
   55 XD=RXM-DELTX-TE*VT
      GO TO 233
  235 XD=RXM-DELTX-TT-TE*VT
  233 CONTINUE
C     PRINT 20,   RH,RXM,XD,XDR,TT
C  20 FORMAT(10X,                        5G10.4)
      GO TO 323
   23 XD=0.
  323 RETURN
      END
```

```
      SUBROUTINE SEE(NXK,K,I,J,EYE,LY,YK)
      IMPLICIT REAL*8(A-H,O-Z),INTEGER*4(I-N)
      COMMON    AH(30),H(120,1),X(120,1),Y(120,1),BX(20),BY(20),BH(20),P
     1K(30),PHI(120,30),AXH(120,30),SLP(90),P(200),SIG(200),EP(200)
      XNK=NXK
      RMRX=DSQRT((BY(J)-Y(K,LY))**2+(BX(J)-XNK)**2)
      SLPB=(H(J,LY)-BH(J))/RMRX
      N=((BY(J)-Y(K,LY))/YK)-.9999
      NYK=YK
      LYK=LY
      SS=(BY(J)-LYK)/RMRX
      PRINT 40,J,SS
   40 FORMAT(10X,'BTRYIS',I3,10X,F10.4)
      DO 10 II=1,N
      XK=YK*DTAN(DARCOS((BY(J)-LYK)/RMRX))
      LXK=(XK+XNK)/YK
      XXK=LXK
      DX=XK+XNK-XXK
      IF(DX.LT..5) GO TO 50
      LXK=LXK+1
   50 LYK=LYK+YK
      XI=I+1
      SLPJ=(H(K,LY)-H(LXK,LYK))/DSQRT(YK*YK+XK*XK)
      IF(SLPB.GT.SLPJ) GO TO 100
      YK=YK*XI
   10 CONTINUE
      EYE=0.
      GO TO 30
  100 EYE=1.0
   30 CONTINUE
      RETURN
      END
```

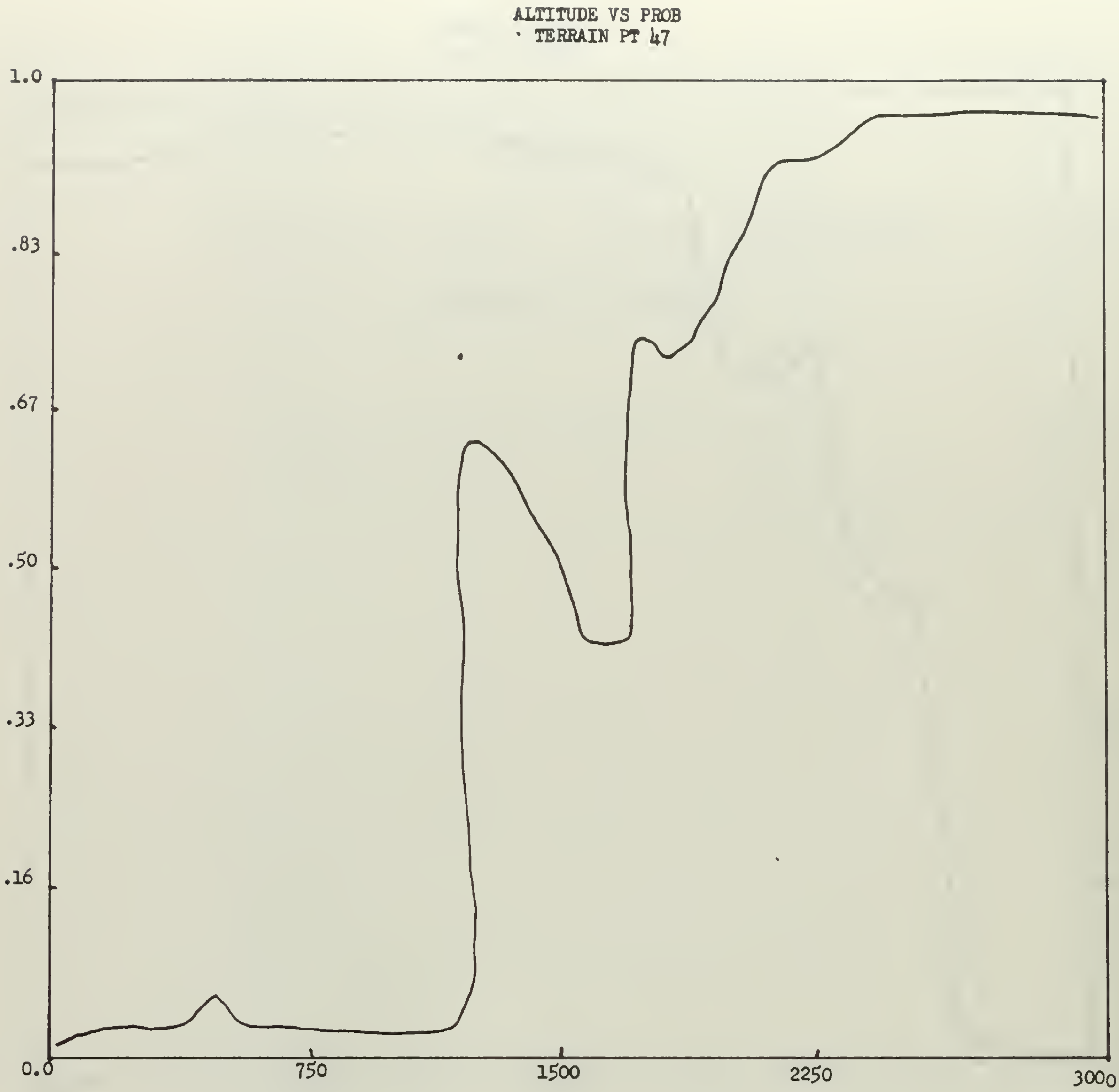
APPENDIX V

PROBABILITY VERSUS ALTITUDE FOR SELECTED TERRAIN POINTS

ALTITUDE VS PROB
TERRAIN PT 41

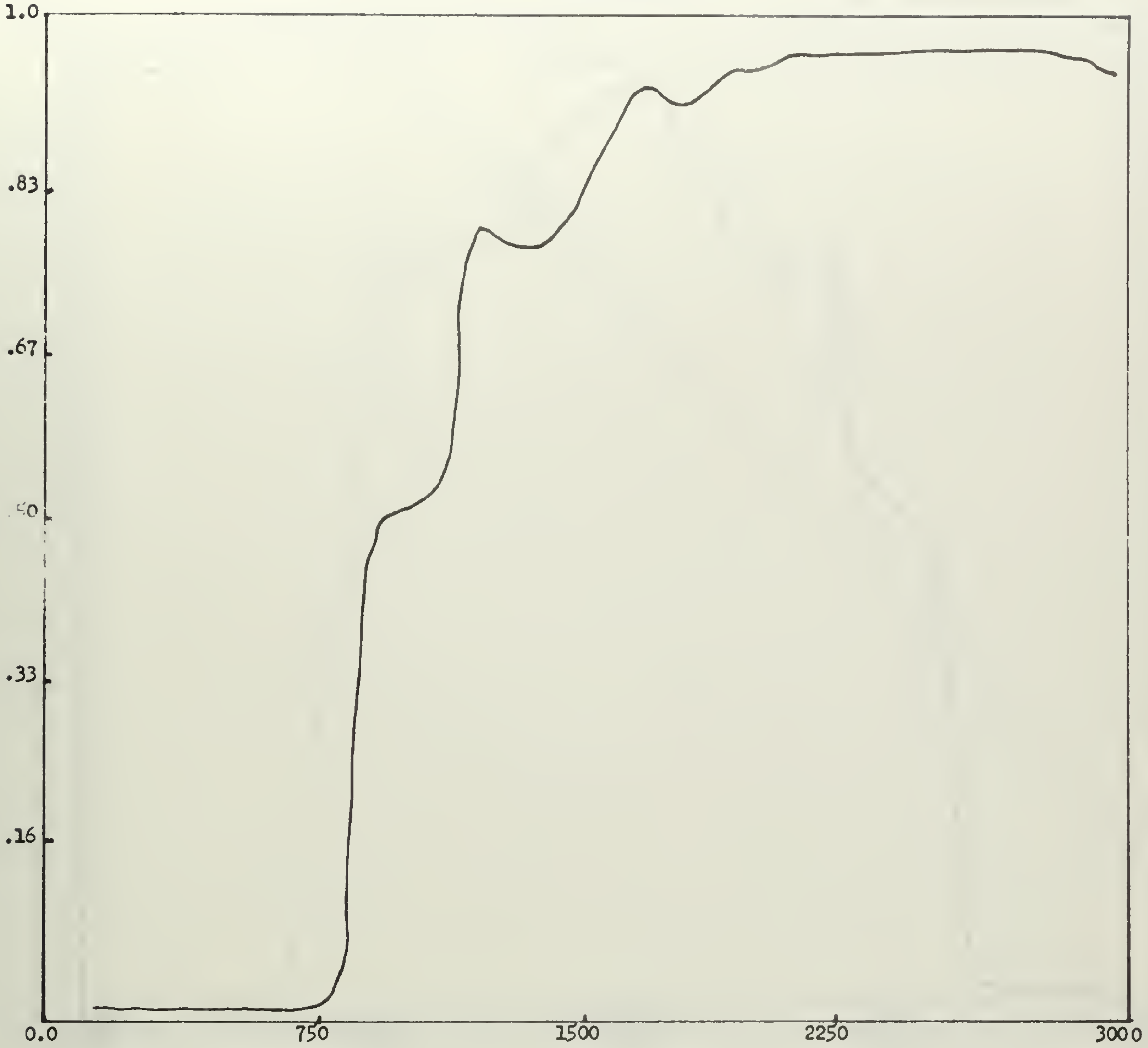


CURVE 1

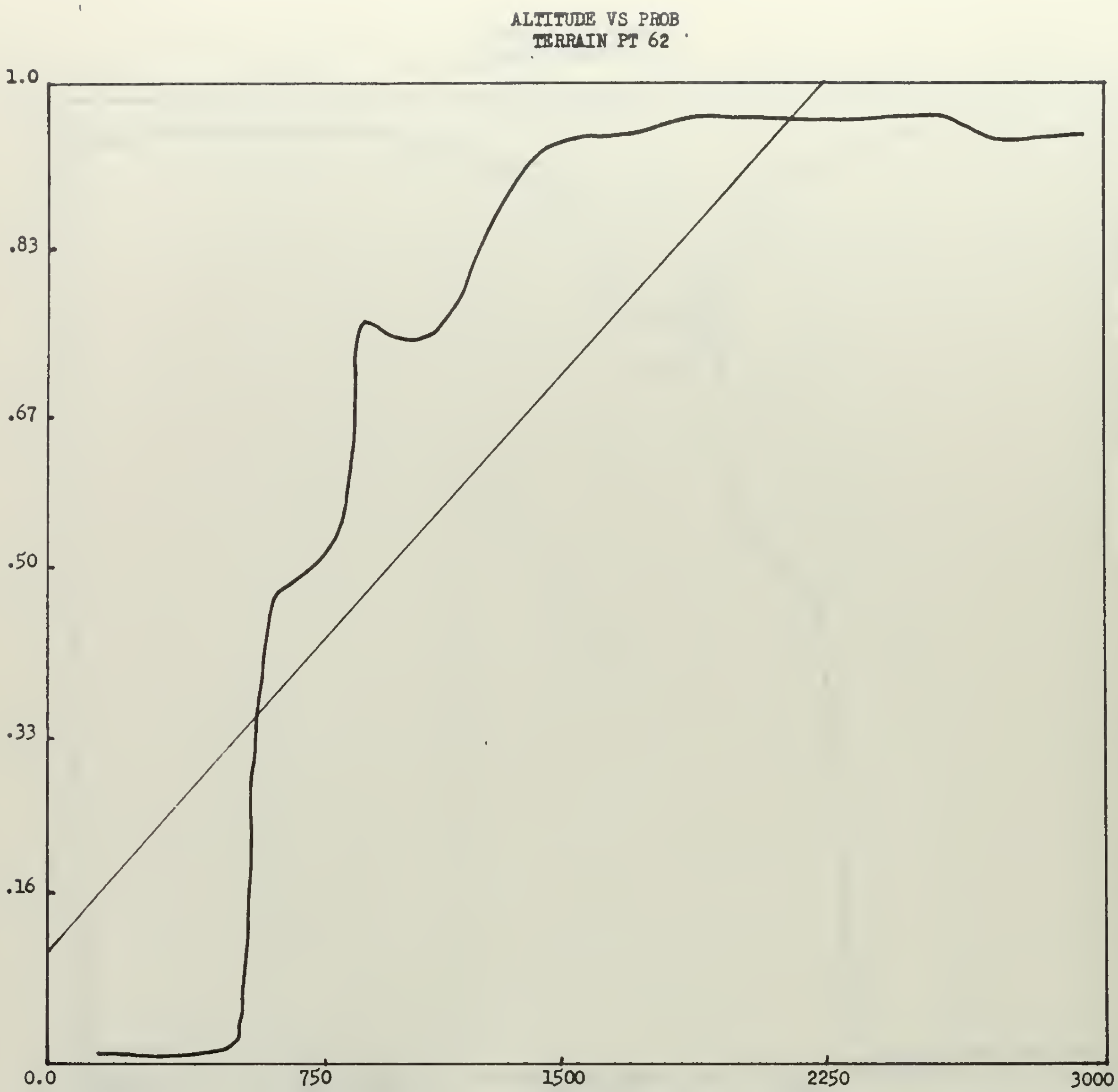

ALTITUDE VS PROB
TERRAIN PT 47
1.0
.83
.67
.50
.33
.16
0.0
750
1500
2250
3000

CURVE 2

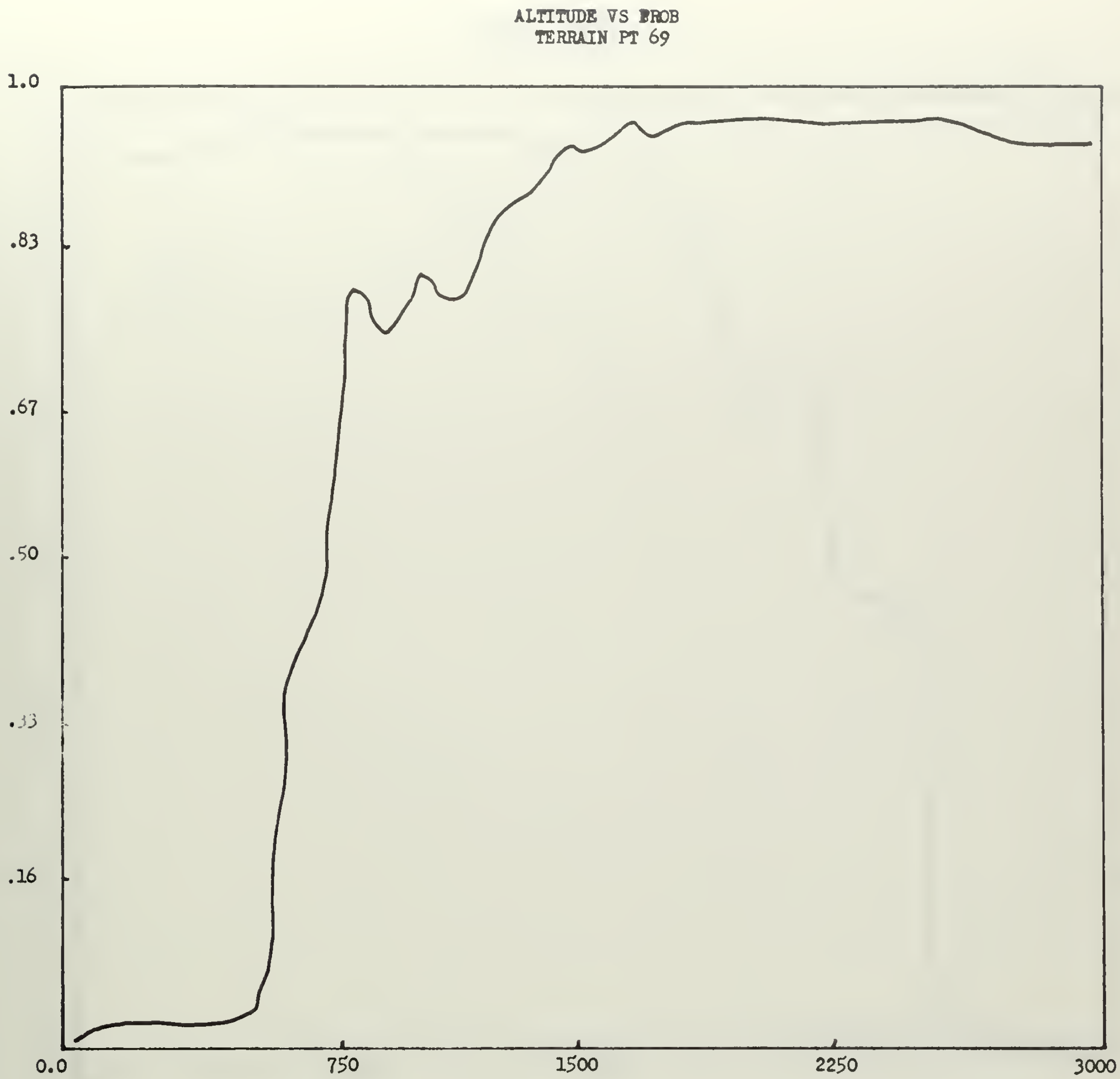
ALTITUDE VS PROB
TERRAIN PT 48



CURVE 3

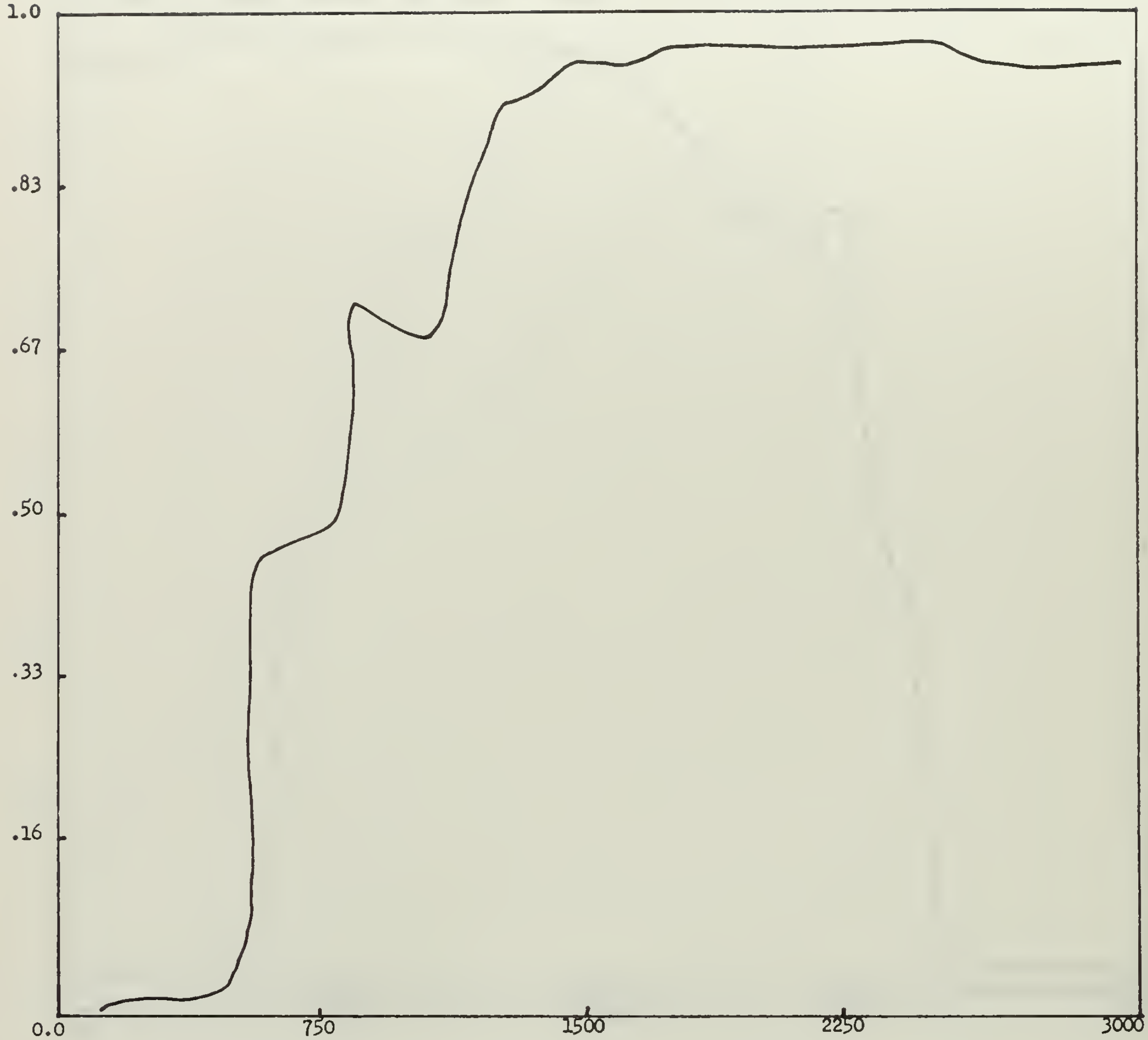

ALTITUDE VS PROB
TERRAIN PT 62
1.0
.83
.67
.50
.33
.16
0.0
750
1500
2250
3000

CURVE 4

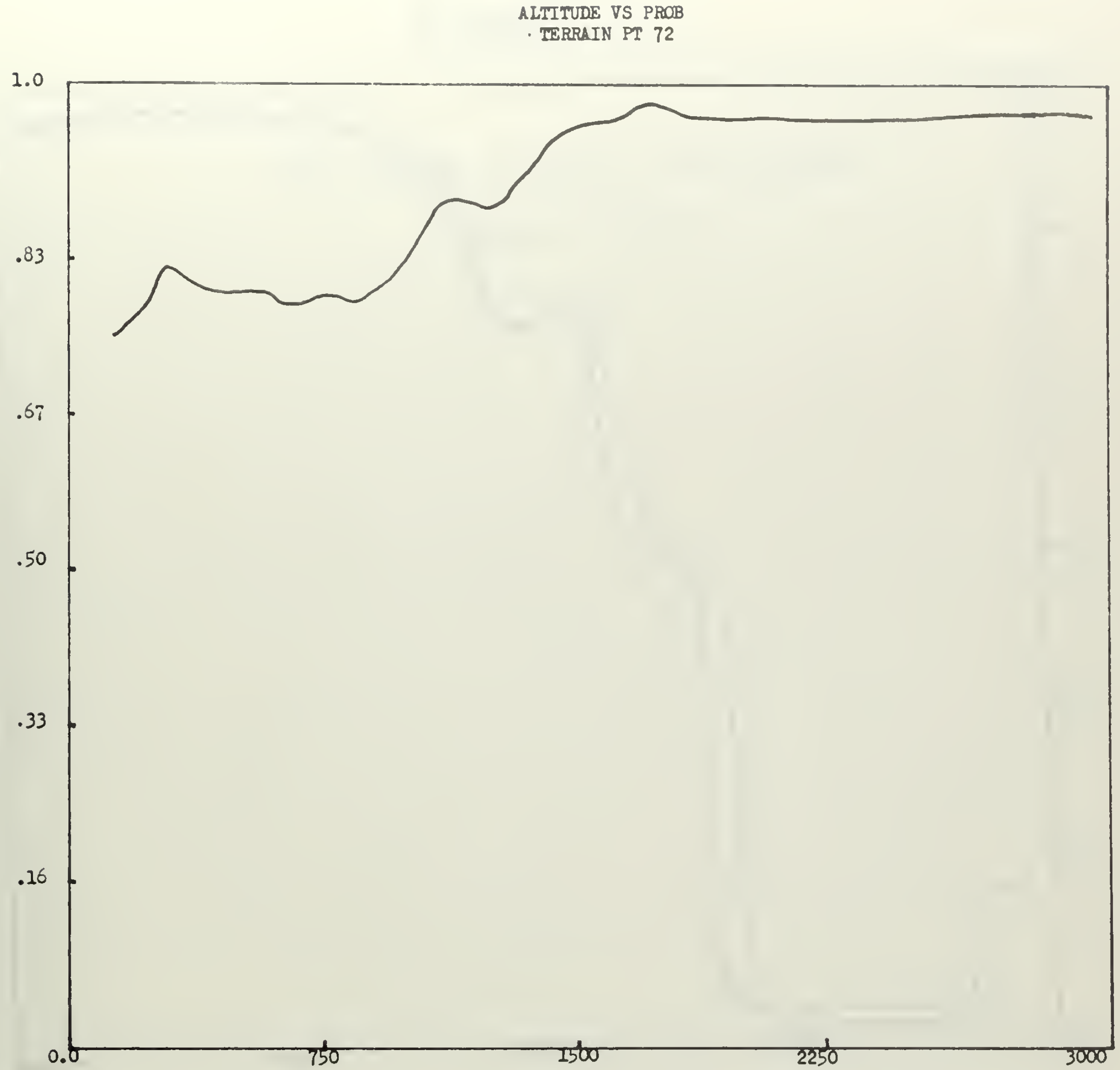

ALTITUDE VS PROB
TERRAIN PT 69
1.0
.83
.67
.50
.33
.16
0.0
750
1500
2250
3000

CURVE 5

ALTITUDE VS PROB
TERRAIN PT 82

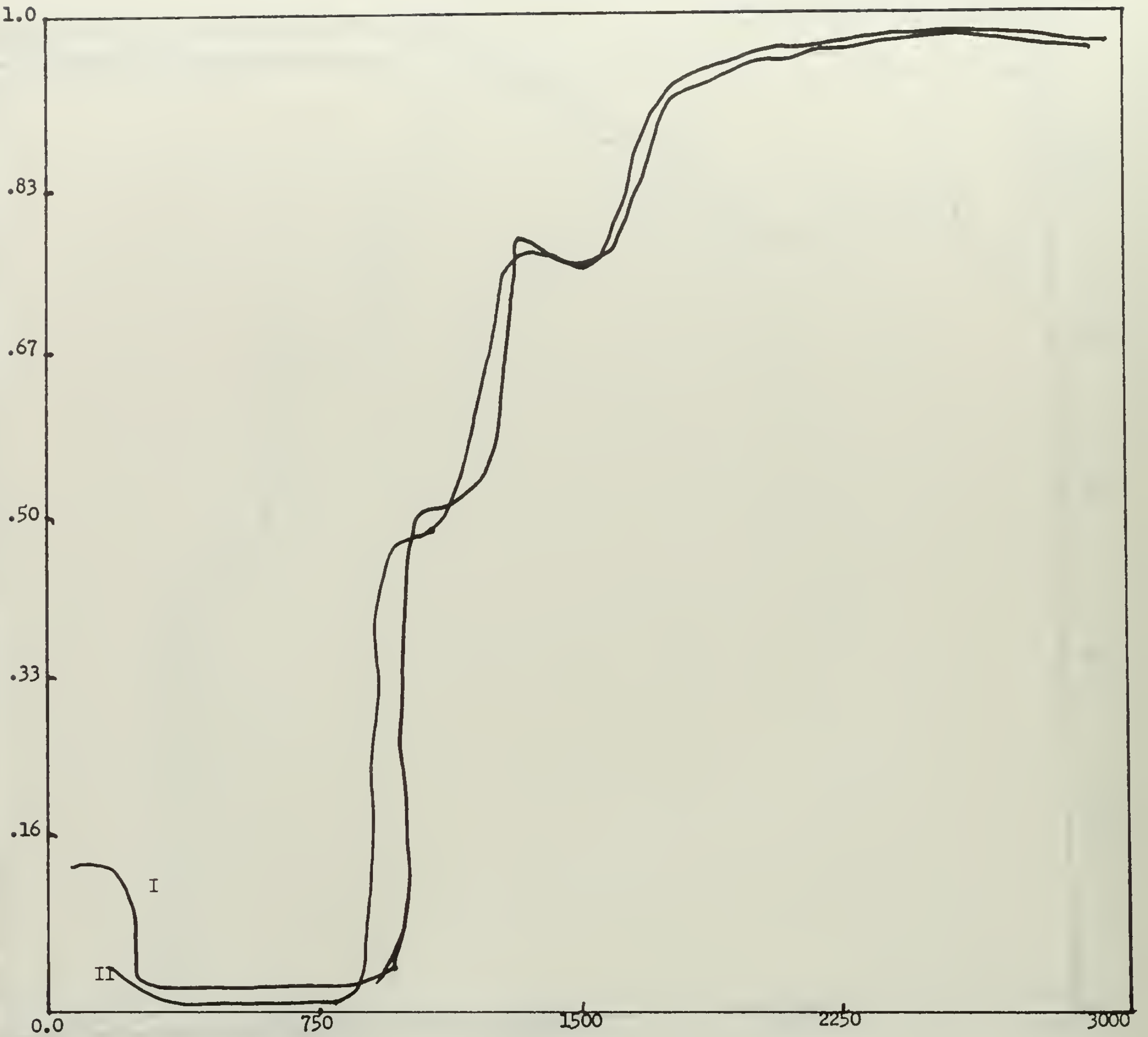


CURVE 6


ALTITUDE VS PROB
TERRAIN PT 72
1.0
.83
.67
.50
.33
.16
0.0
750
1500
2250
3000

CURVE 7

ALTITUDE VS PROB
TERRAIN PT 68

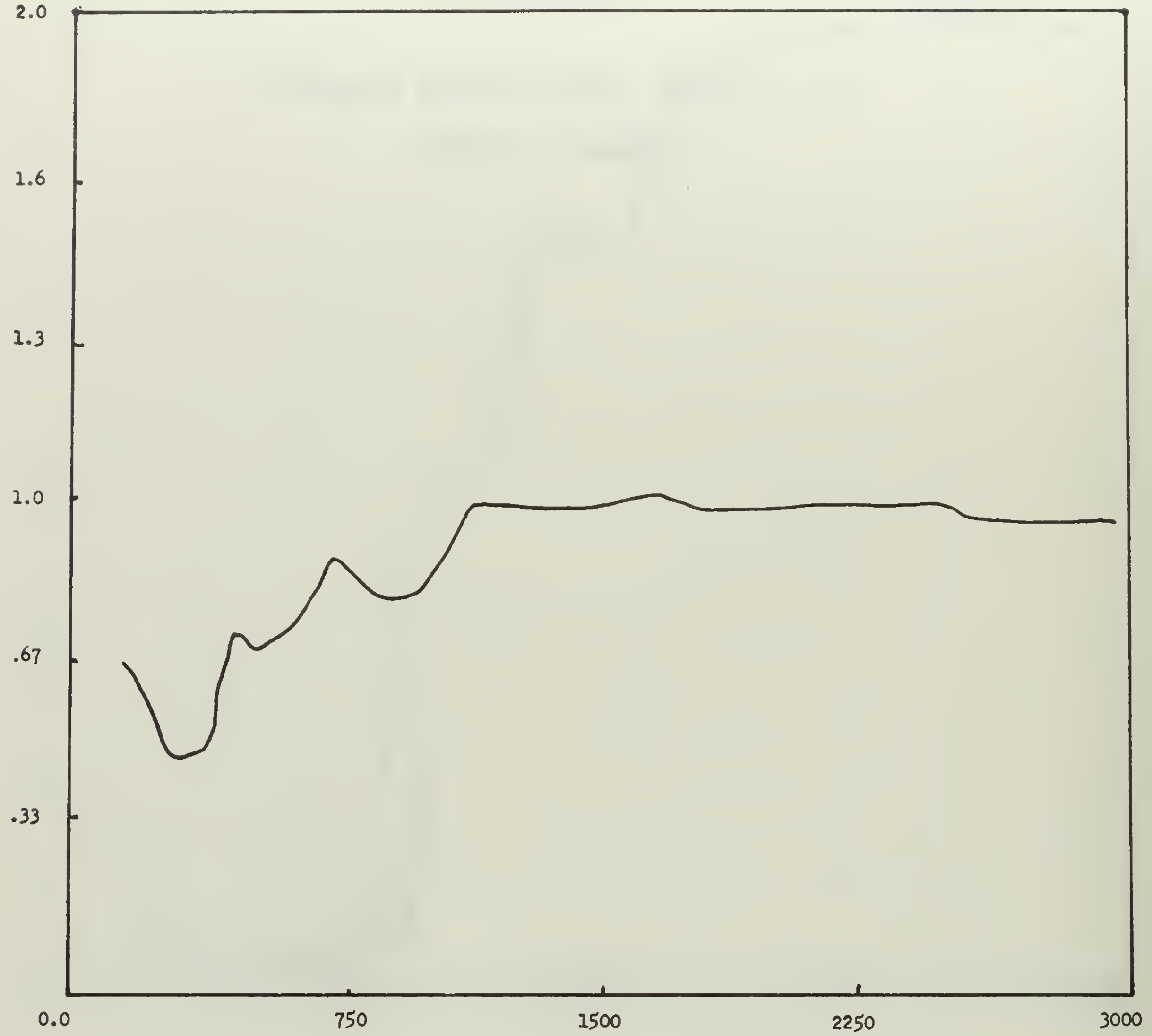


CURVE 8

APPENDIX VI

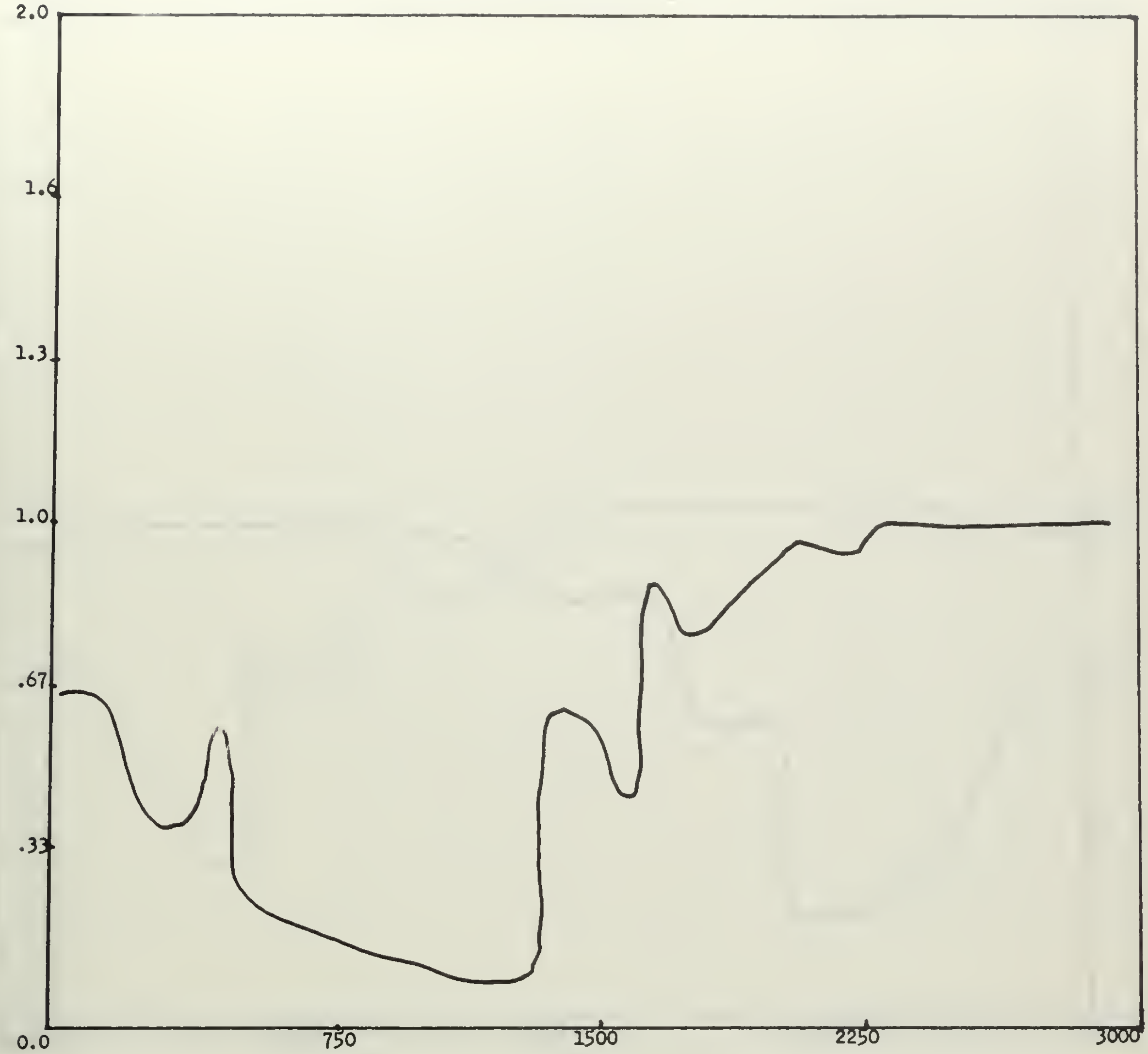
COST VERSUS ALTITUDE FOR SELECTED TERRAIN POINTS

ALTITUDE VS COST
TERRAIN PT 41

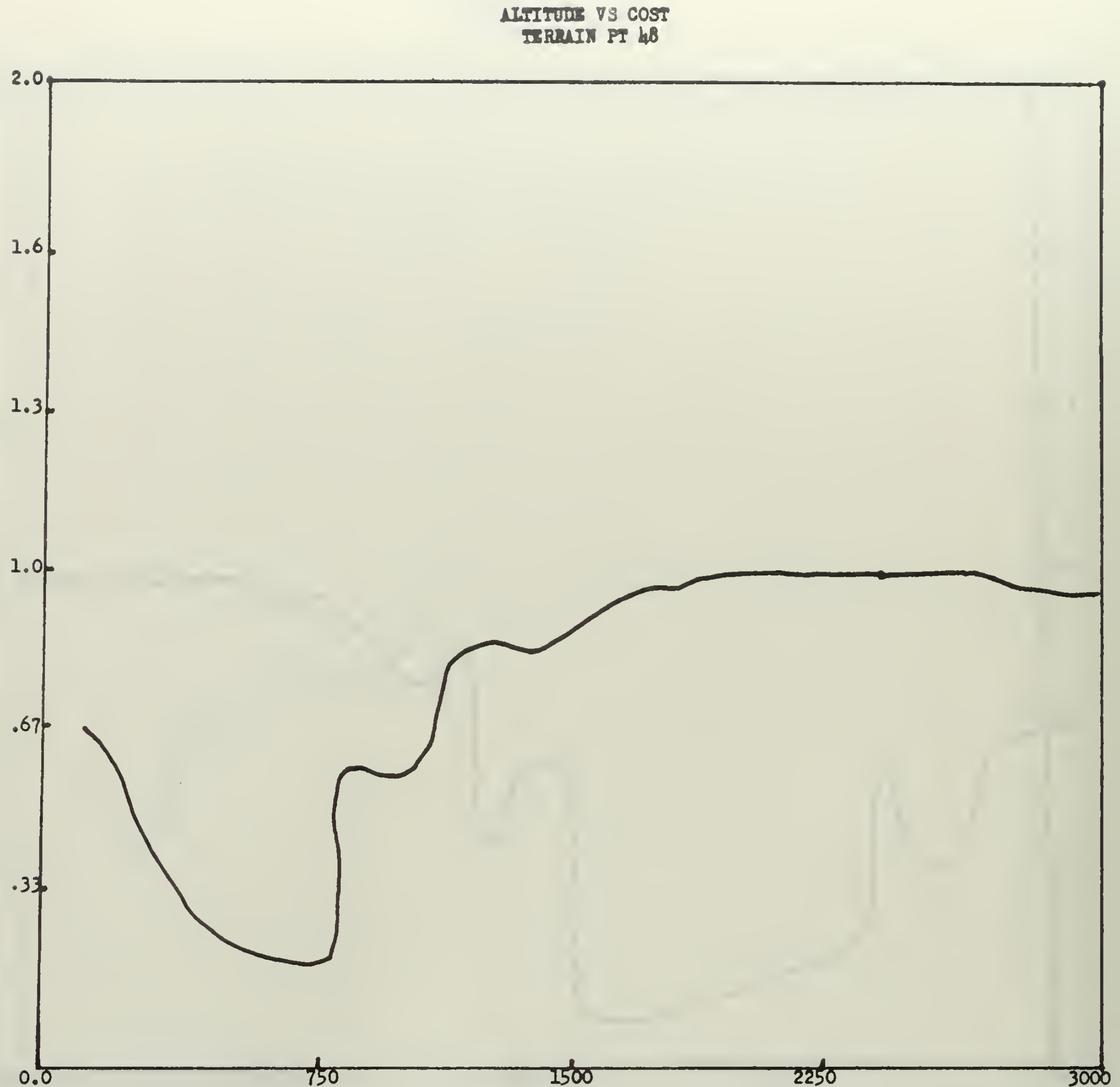


CURVE 1

ALTITUDE VS COST
TERRAIN PT 47

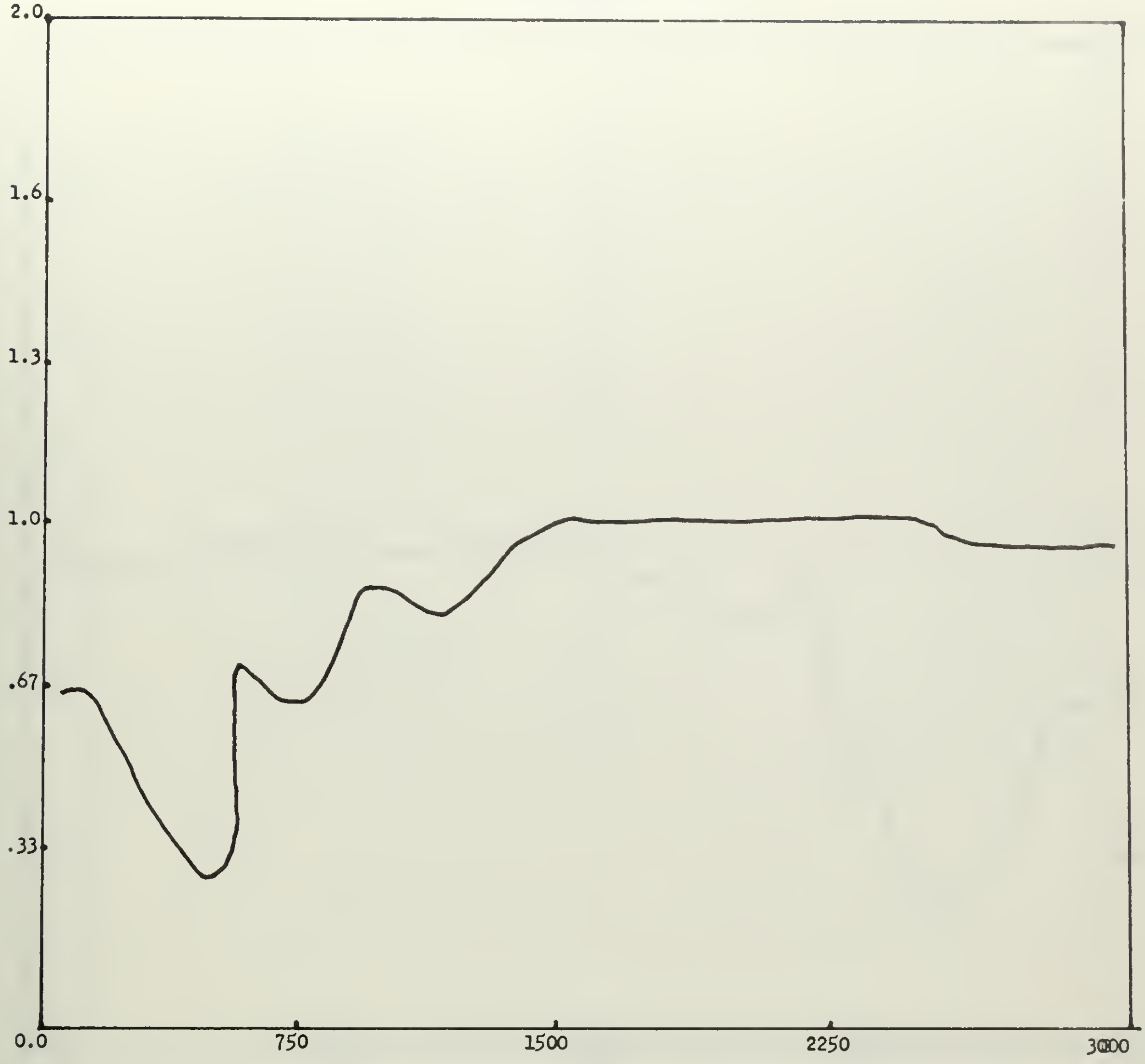


CURVE 2


ALTITUDE VS COST
TERRAIN PT 48
2.0
1.6
1.3
1.0
.67
.33
0.0
750
1500
2250
3000

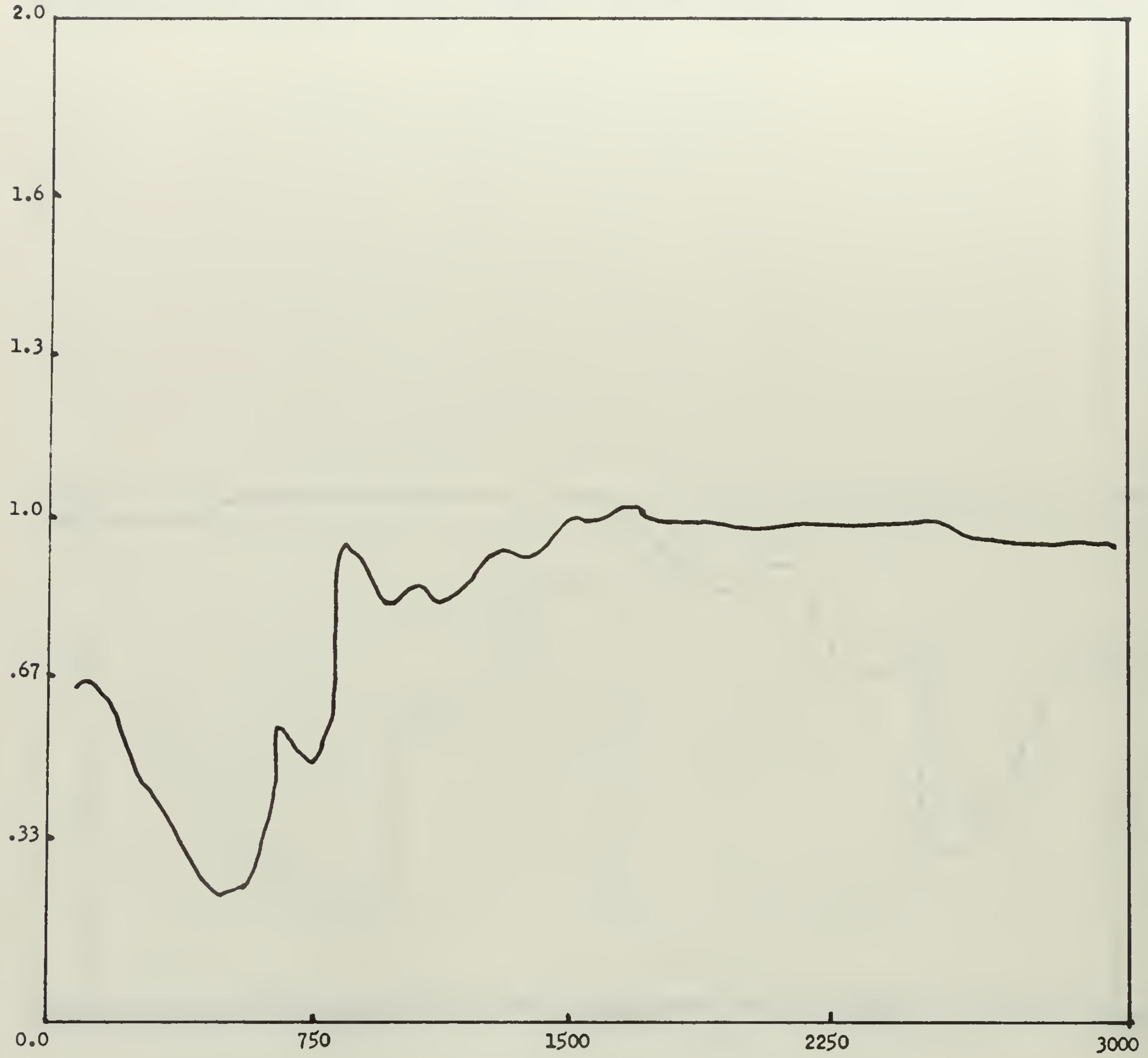
CURVE 3

ALTITUDE VS COST
TERRAIN PT 62



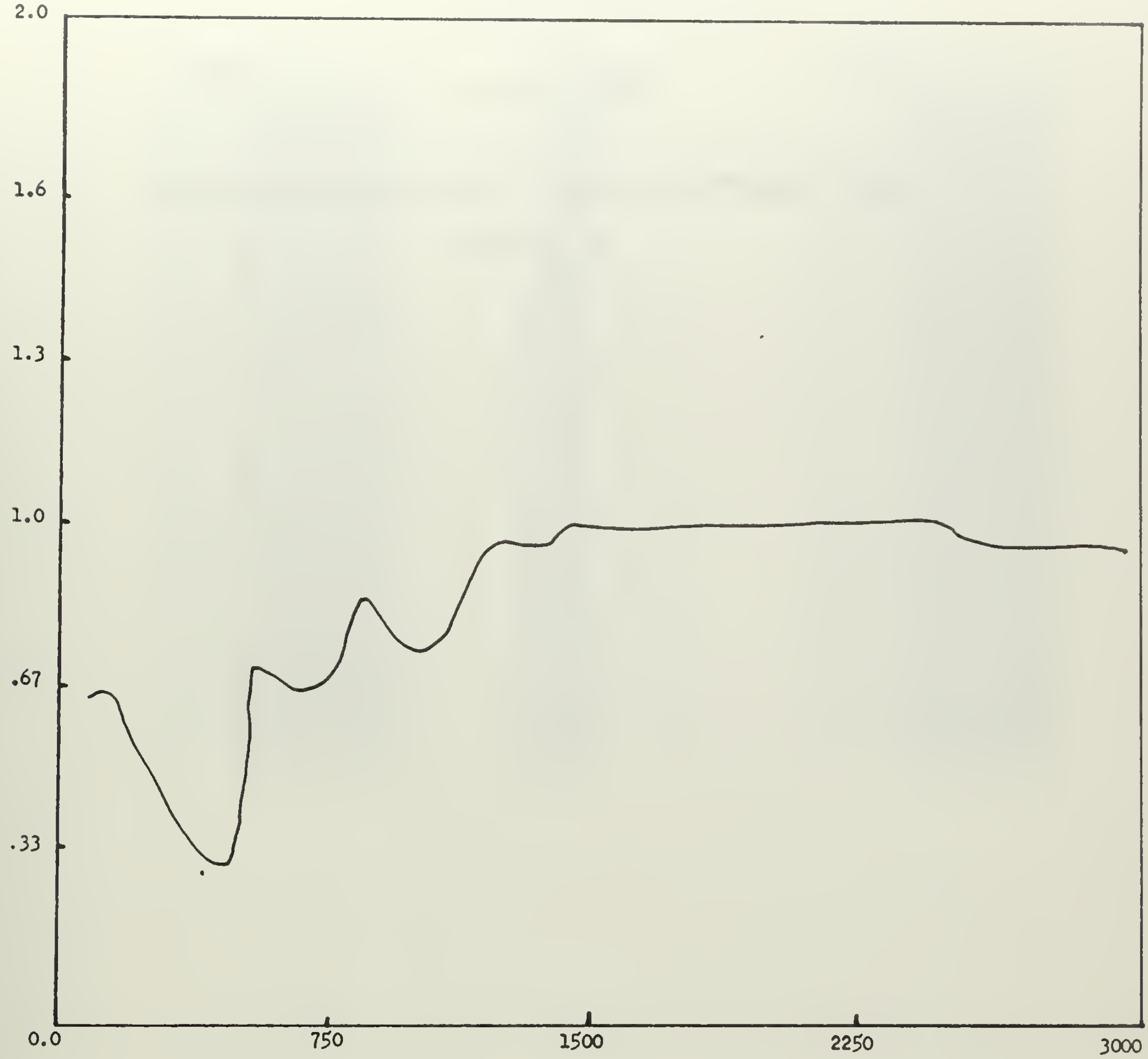
CURVE 4

ALTITUDE VS COST
TERRAIN PT 69



CURVE 5

ALTITUDE VS COST
TERRAIN PT 82



CURVE 6

APPENDIX VII

DATA POINTS FOR THE CURVES IN APPENDICES

V AND VI

TERRAIN POINT 41

PROB	ALTITUDE	COST
0.0027158	100	0.6375157
0.0027110	200	0.4982110
0.0954828	300	0.4660828
0.4619637	400	0.7422637
0.4746011	500	0.7152010
0.5874727	600	0.7920726
0.7439264	700	0.9031264
0.7306477	800	0.8490477
0.7106845	900	0.8013845
0.8248070	1000	0.8931070
0.9328496	1100	0.9937496
0.9329319	1200	0.9874319
0.9559594	1300	1.0060587
0.9597607	1400	1.0072603
0.9563827	1500	1.0017824
0.9864236	1600	1.0305233
0.9782999	1700	1.0090990
0.9858453	1800	1.0098448
0.9815221	1900	1.0030212
0.9825184	2000	1.0035181
0.9809653	2100	0.9981653
0.9793320	2200	0.9940320
0.9776287	2300	0.9920287
0.9758614	2400	0.9901614
0.9740362	2500	0.9847361
0.9721588	2600	0.9792588
0.9702350	2700	0.9763350
0.9683002	2800	0.9736002
0.9663471	2900	0.9704471
0.9643678	3000	0.9676678

TERRAIN POINT 47

PROB	ALTITUDE	COST
0.0	100	0.6348000
0.0	200	0.4955000
0.0	300	0.3706000
0.2796825	400	0.5599825
0.0042940	500	0.2448940
0.0027300	600	0.2073299
0.0055344	700	0.1647344
0.0027246	800	0.1211246
0.0027211	900	0.0934210
0.0027170	1000	0.0710170
0.0027124	1100	0.0636123
0.0027072	1200	0.0572072
0.0054274	1300	0.0555274
0.5450826	1400	0.5925826
0.5536741	1500	0.5990741
0.3935170	1600	0.4376170
0.8372307	1700	0.8680307
0.7275365	1800	0.7515365
0.8045451	1900	0.8260451
0.8778201	2000	0.8988200
0.9350527	2100	0.9522527
0.9264631	2200	0.9411631
0.9693650	2300	0.9837649
0.9780607	2400	0.9923607
0.9817001	2500	0.9924001
0.9815891	2600	0.9886891
0.9841095	2700	0.9902095
0.9792450	2800	0.9845450
0.9829284	2900	0.9870284
0.9814092	3000	0.9847092

TERRAIN POINT 48

PROB	ALTITUDE	COST
0.0027316	100	0.6375316
0.0027297	200	0.4982296
0.0045465	300	0.3751464
0.0027241	400	0.2830241
0.0027205	500	0.2433205
0.0027164	600	0.2073163
0.0027117	700	0.1619117
0.4124928	800	0.5308927
0.4975135	900	0.5882134
0.5125919	1000	0.5808919
0.6919966	1100	0.7528966
0.7918245	1200	0.8463245
0.8012550	1300	0.8513550
0.7841540	1400	0.8316540
0.8661683	1500	0.9115682
0.9242823	1600	0.9683823
0.9468088	1700	0.9776088
0.9326292	1800	0.9566292
0.9726202	1900	0.9941202
0.9689952	2000	0.9899952
0.9749367	2100	0.9921367
0.9843206	2200	0.9990206
0.9811772	2300	0.9955771
0.9842251	2400	0.9985251
0.9827261	2500	0.9934261
0.9811880	2600	0.9882880
0.9795651	2700	0.9856650
0.9778711	2800	0.9831711
0.9761124	2900	0.9802124
0.9742950	3000	0.9775949

TERRAIN POINT 62

PROB	ALTITUDE	COST
0.0027238	100	0.6375238
0.0027202	200	0.4982201
0.0027160	300	0.3733160
0.0027113	400	0.2830112
0.4737283	500	0.7143282
0.4753606	600	0.6799605
0.4886442	700	0.6478442
0.6693962	800	0.7877962
0.7629707	900	0.8536707
0.7530464	1000	0.8213464
0.7393761	1100	0.8002760
0.8401864	1200	0.8946863
0.9071808	1300	0.9572808
0.9384188	1400	0.9859188
0.9611555	1500	1.0065546
0.9649823	1600	1.0090818
0.9611157	1700	0.9919156
0.9878440	1800	1.0118437
0.9806270	1900	1.0021267
0.9793023	2000	1.0003023
0.9825637	2100	0.9997637
0.9825942	2200	0.9972941
0.9810450	2300	0.9954450
0.9794154	2400	0.9937154
0.9777154	2500	0.9884154
0.9759512	2600	0.9830511
0.9741287	2700	0.9802287
0.9722538	2800	0.9775538
0.9703322	2900	0.9744322
0.9683934	3000	0.9716933

TERRAIN POINT 68

PROB	ALTITUDE	COST
0.1368645	100	0.7716645
0.0027315	200	0.4982314
0.0027295	300	0.3733295
0.0041496	400	0.2844496
0.0027239	500	0.2433239
0.0027203	600	0.2073202
0.0027161	700	0.1619161
0.0027114	800	0.1211113
0.4335030	900	0.5242029
0.4805196	1000	0.5488196
0.4941302	1100	0.5550302
0.6767096	1200	0.7312096
0.7700044	1300	0.8201044
0.7620402	1400	0.8095402
0.7501766	1500	0.7955766
0.8461959	1600	0.8902959
0.9112051	1700	0.9420050
0.9404540	1800	0.9644540
0.9631043	1900	0.9846043
0.9668788	2000	0.9878788
0.9629577	2100	0.9801577
0.9883308	2200	1.0030308
0.9815130	2300	0.9959130
0.9797094	2400	0.9940094
0.9829618	2500	0.9936618
0.9826244	2600	0.9897244
0.9810769	2700	0.9871768
0.9794487	2800	0.9847487
0.9777501	2900	0.9818500
0.9759870	3000	0.9792870

TERRAIN POINT 69

PROB	ALTITUDE	COST
0.0027225	100	0.6375225
0.0027187	200	0.4982187
0.0027143	300	0.3733143
0.0027094	400	0.2830094
0.0066825	500	0.2472824
0.3684522	600	0.5730522
0.3379564	700	0.4971564
0.8041257	800	0.9225257
0.7355692	900	0.8262691
0.8131848	1000	0.8814847
0.7832339	1100	0.8441339
0.8648104	1200	0.9193104
0.8990118	1300	0.9491118
0.9483667	1400	0.9958667
0.9563300	1500	1.0017300
0.9776988	1600	1.0217981
0.9644131	1700	0.9952130
0.9816354	1800	1.0056353
0.9819365	1900	1.0034361
0.9795864	2000	1.0005856
0.9827996	2100	0.9999996
0.9820328	2200	0.9967327
0.9804507	2300	0.9948506
0.9787943	2400	0.9930943
0.9770698	2500	0.9877698
0.9752833	2600	0.9823833
0.9734408	2700	0.9795408
0.9715480	2800	0.9768479
0.9696105	2900	0.9737105
0.9676816	3000	0.9709816

TERRAIN POINT 82

PROB	ALTITUDE	COST
0.0027197	100	0.6375197
0.0027155	200	0.4982154
0.0027107	300	0.3733107
0.0316351	400	0.3119351
0.4451180	500	0.6857180
0.4572794	600	0.6618794
0.5182828	700	0.6774828
0.7186771	800	0.8370770
0.7030515	900	0.7937515
0.6739765	1000	0.7422764
0.8060656	1100	0.8669655
0.9227645	1200	0.9772644
0.9237672	1300	0.9738672
0.9685816	1400	1.0160809
0.9673398	1500	1.0127392
0.9683690	1600	1.0124683
0.9843186	1700	1.0151186
0.9753590	1800	0.9993590
0.9849095	1900	1.0064087
0.9802112	2000	1.0012112
0.9824312	2100	0.9996311
0.9808694	2200	0.9955694
0.9792317	2300	0.9936317
0.9775244	2400	0.9918243
0.9757534	2500	0.9864534
0.9739249	2600	0.9810249
0.9720447	2700	0.9781446
0.9701182	2800	0.9754182
0.9681811	2900	0.9722811
0.9662319	3000	0.9695319

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2.	Library Naval Postgraduate School Monterey, California 93940	2
3.	Director of Systems Analysis Division (OP 96) Office of Chief of Naval Operations Washington, D. C. 20350	1
4.	Commandant of the Marine Corps (Code A03C) Headquarters, U. S. Marine Corps Washington, D. C. 22214	1
5.	Professor W. Peyton Cunningham Department of Physics Naval Postgraduate School Monterey, California 93940	2
6.	James Carson Breckinridge Library Marine Corps Development and Education Command Quantico, Virginia 22134	1
7.	Lt. Col. T. H. Allen, Jr., USMC Naval Postgraduate School Student Mail Center Number 1773 Monterey, California 93940	1
8.	Lt. Col. W. S. Miller, Jr., USMC 4120 Darbydale Avenue Woodbridge, Virginia 22191	1
9.	Major E. E. Brown, USMC 427 Blair Road Vienna, Virginia 22180	1
10.	Major Paul C. Palmer, USA Naval Postgraduate School Student Mail Center Number 1295 Monterey, California 93940	1

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

AIRCRAFT SURVIVABILITY INDEX FOR LOW ALTITUDE PENETRATION OF AN AIR DEFENSE COMPLEX

4. DESCRIPTIVE NOTES *(Type of report and, inclusive dates)*
Thesis

5. AUTHOR(S) *(First name, middle initial, last name)*

Thomas Harrison Allen, Jr., Lieutenant Colonel, USMC

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1968	130	11

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

10. DISTRIBUTION STATEMENT

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

This paper develops a model for computing the probability of kill for an air defense complex composed of antiaircraft automatic weapons, radar controlled guns, and missile batteries. Two dimensional terrain was used to evaluate the model. The probabilities were determined at major terrain points along the route of approach to the vital area for altitudes of up to 3000 feet above terrain. The curves of probability of kill versus altitude were found to be dependent on terrain, air defense tactics, and weapon system parameters. A survivability index is calculated by combining the probabilities of kill with a pilot visual navigational probability. The resulting curves of survivability index versus altitude were found to be nonlinear requiring a nonlinear programing technique to solve for the altitude of optimal survivability index within aircraft flight path constraints. The nonlinear solution was not included in this work.

DD FORM 1 NOV 65 1473 (PAGE 1)
S/N 0101-807-6811

UNCLASSIFIED
Security Classification

A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
TERRAIN DIGITALIZATION						
VISUAL TARGET RECONNAISSANCE AND ACQUISITION						
GUNFIRE PROBABILITY CALCULATIONS						

DD FORM 1 NOV 65 1473 (BACK)
S/N 0101-807-6821

thesA3799
Aircraft survivability index for low alt
3 2768 000 98969 3
DUDLEY KNOX LIBRARY