EARLY GEOPHYSICAL PAPERS

Ex J B Hersey

E. T. Bruce

EARLY GEOPHYSICAL PAPERS
of the
SOCIETY OF EXPLORATION GEOPHYSICISTS

Compiled and Printed
1947

Second Printing
1956

Papers presented at meetings of the Society of Petroleum Geophysicists 1929 through 1935, reprinted from PHYSICS and BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS, and JOURNAL OF THE SOCIETY OF PETROLEUM GEOPHYSICISTS.

SOCIETY OF EXPLORATION GEOPHYSICISTS
P. O. Box 1614
Tulsa, Oklahoma

EARLY GEOPHYSICAL PAPERS

of the

SOCIETY OF EXPLORATION GEOPHYSICISTS

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FOREWORD

The Society of Exploration Geophysicists was organized early in 1930 but Vol. I, No. 1, of its present journal, GEOPHYSICS, is dated January, 1936. In the period between these dates, many geophysical papers were presented at the regular annual meetings held jointly with the meetings of the American Association of Petroleum Geologists and at occasional other meetings. These papers were published in various journals, including special geophysical numbers of the *Bulletin of the American Association of Petroleum Geologists, Physics,* as "Special Papers" published in mimeograph form by the Society and in an earlier publication of the Society (before a change to its present name) which was called *Journal of the Society of Petroleum Geophysicists.*

Since the appearance of the first number of GEOPHYSICS, the membership of the Society has increased approximately five-fold, which means that these earlier scattered publications have not been readily available to a large proportion of the present membership. These papers contain much of the early history and technical fundamentals of the present geophysical exploration industry. Therefore, the Executive Committee of the Society, at its last regular meeting, authorized the republication of these old papers into a single volume. It is hoped that they will be found of interest and value to the present members of the Society and other geophysicists throughout the world, and particularly to the younger members of the profession to whom this early work is largely unknown.

Acknowledgment is made of permission granted by the American Association of Petroleum Geologists and by the American Institute of Physics to reprint papers originally published in their journals.

L. L. NETTLETON, *Editor*
Geophysics (1945-1947)

A UNIVERSAL DIP CHART

FOR USE WITH SEISMIC METHOD OF GEOPHYSICAL PROSPECTING (REFRACTIONS)

O. C. LESTER, JR.* and S. W. WILCOX†

The purpose of this paper is to present and discuss a dip chart or family of curves, by means of which the dip of subsurface beds may be determined by inspections of the curves, after the apparent velocity has been determined from a refraction profile. The theory is based on the assumption of rectilinear propagation of sound waves, as discussed by D. C. Barton, C. A. Heiland and others in "Geophysical Prospecting—1929."‡

Assuming the dip equations as derived by Schweydar# as a starting point, we have:

Schweydar Equations 15 and 17. "Geophysical Prospecting—1929", C. A. Heiland.

$$\frac{1}{V_{2+}} = \frac{1}{V_2} \left(\cos \alpha - \frac{1}{q} \sqrt{1 - q^2 \sin \alpha}\right) \qquad \text{\#\# 15}$$

$$\frac{1}{V_{2-}} = \frac{1}{V_2} \left(\cos \alpha + \frac{1}{q} \sqrt{1 - q^2 \sin \alpha}\right) \qquad \text{\#\#\# 17}$$

Note: See Article on Seismic Prospecting, by C. A. Heiland, "Geophysical Prospecting—1929." Page 639. Equations 15 and 17.

Note: This equation (15) as given in the book is in error due to misprint. The term before the parenthesis should be

$\frac{1}{V_2}$ rather than $\frac{1}{V_{2+}}$ as given.

Note: Notation in book apparently reversed in equations 15 and 17 as to direction of dip.

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*Geophysical Research Corporation, Los Angeles, Calif.

†Seismograph Service Corporation, Tulsa, Okla.; formerly with Geophysical Research Corporation.

‡American Institute of Mining and Metallurgical Engineers.

where according to notation used:

V_1 = true velocity of low speed medium
V_2 = true velocity of high speed medium
V_{2+} = apparent velocity of high speed medium (down dip)
V_{2-} = apparent velocity of high speed medium (up dip)
$\propto$ = angle between high speed bed and horizontal (dip of bed)
$q = \cos i = \sin \Theta$
where Θ = critical angle between low and high speed beds. ($\sin \Theta = \frac{V_1}{V_2}$)

Equation 15 may be written:

$$\frac{1}{V_{2+}} = \frac{1}{V_2} \left(\text{Cos} \propto - \frac{1}{\sin \Theta} \sqrt{1 - \sin^2 \Theta \sin \propto}\right)$$

Hence
$$\frac{1}{V_{2+}} = \frac{1}{V_2} \left(\cos \propto - \frac{\sqrt{V_2^2 - V_1^2 \sin \propto}}{V_1}\right)$$

and
$$V_{2+} = \frac{V_2}{\cos \propto - B \sin \propto}$$
where $B = \cot \Theta$, which is constant for two given beds regardless of dip.

Equation 17 may be written similarly, as:

$$V_{2-} = \frac{V_2}{\cos \propto + B \sin \propto}$$

For general cases, let V' represent the apparent velocity (either up or down dip)

Then
$$V' = \frac{V_2}{\cos \propto \pm B \sin \propto} \qquad \text{Equation A.}$$

The negative sign should be used for apparent velocities shooting up dip, and the positive sign for down dip.

Investigation of this equation will show that for $\propto = 0$, $V' = V_2$; which is, of course, true for a horizontal marker.

Also, for $\propto = \Theta$, $V' = \propto$

In equation A, if the apparent velocity V' be plotted against the tangent of the angle of dip, the resulting curve will be of the form Fig. 1, and the angle of dip can be read directly for a given apparent velocity.

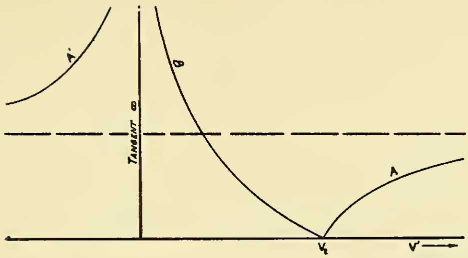


Fig. 1

As can be seen from the above curve (B) in Figure 1, the apparent velocity (shooting down dip) follows a smooth curve for increasing values of α.

The curve (A), however, indicates a rapidly increasing velocity (up dip) for increasing values of α. This velocity approaches infinity for the value of $\alpha = \Theta$, becomes discontinuous and then negative. A negative velocity would be represented on the time-distance curve by a velocity line sloping down toward the abscissa away from the origin, e.g.:

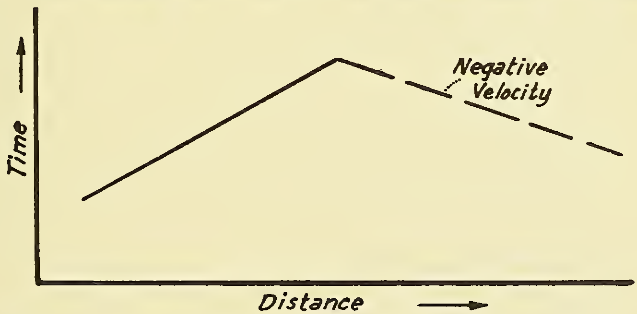
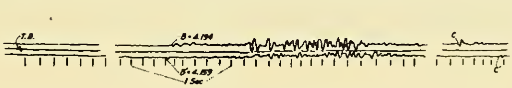


Fig. 2

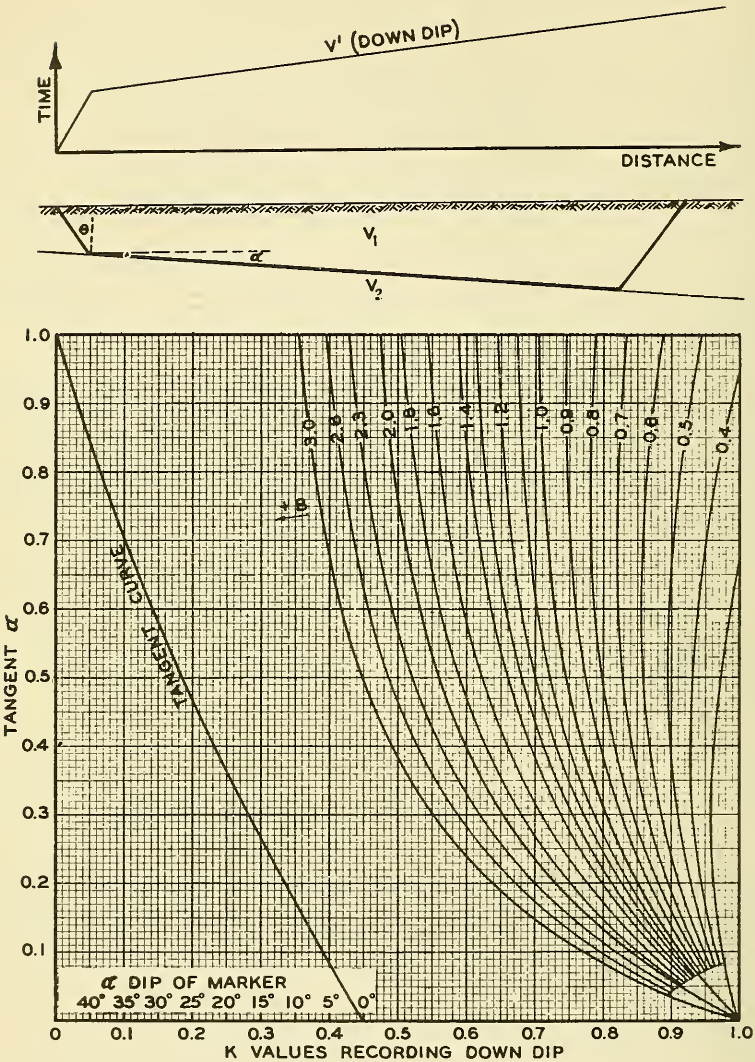
Physically, a negative velocity would result from shooting up a dip of such magnitude that the vertical speeding up would be sufficient to cause a shorter total travel time for a longer horizontal distance. Such a condition has been met in several instances in shooting a radial profile in toward a salt dome. An indisputable example is shown here:



Timing units on original record 0.02 sec.

Fig. 3

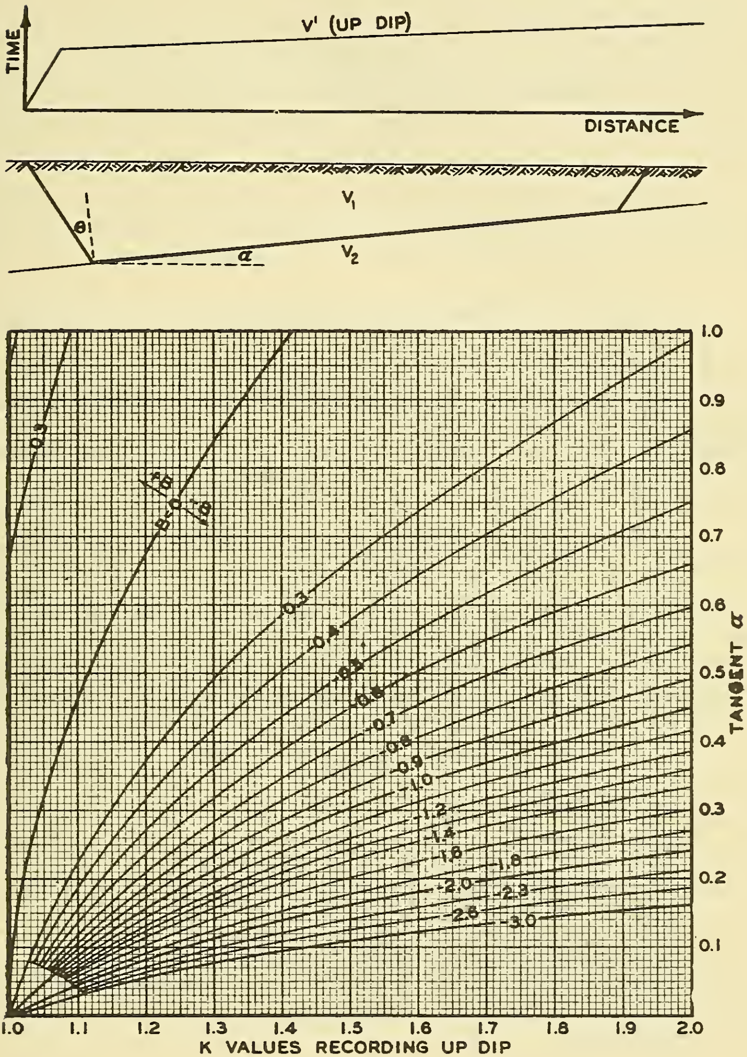
Actual Record (reduced with pantograph)



DOWN DIP CURVES

For V^1 values less than V_2

$$K = \frac{V^1}{V_2} = \frac{1}{\cos \alpha \pm B \sin \alpha}$$



UP DIP CURVES — Fig. 4

For V^1 values greater than V_2

V_1 = Velocity of upper layer (low speed medium)
V_2 = Velocity of lower layer (high speed medium)
V^1 = Apparent velocity of high speed medium
B = Cot Θ the parameter   Θ = Critical angle

The two traces of this record, representing two seismograph positions on a refraction profile with a chained difference in distance of 500′, from the shot point, were recorded simultaneously with the same oscillograph. The longer trace, which can be identified by the air waves (C, C′), definitely shows a shorter arrival time of the intial energy (B, B′).

This so-called negative velocity, while definitely shown to exist, is considered to be of academic, rather than practical interest, and that portion of the curves will not be discussed in the following:

A UNIVERSAL DIP CHART

The curve (Fig. 1) obtained from equation A is, of course, applicable to only one pair of beds of velosity V_1 and V_2. This necessitates the construction of a new curve for each pair of velocities under consideration. If, however, the equation A be writtes as:

$$K = \frac{V^1}{V_2} = \frac{1}{\cos \alpha \pm B \sin \alpha}$$

A family of curves may be drawn, using B as a parametér, which will be applicable to all velocity ratios. Such a family of curves is shown in Fig. 4 (Minus the negative velocity portion). A tangent curve is given (at left) for convenience in converting tangents to actual dip.

These curves may be used for the following purposes:

(See Figure 4)

1. Given apparent velocity up dip—to find the angle of dip α. V_1 and V_2, known.

2. Given apparent velocity down dip—to find the angle of dip α. V_1 and V_2, known.

3. Given apparent velocities both up and down dip (from reversed profiles)—to determine true velocity V_2, and angle α.

For case numbers:

1. a) Determine $B = \cot \sin^{-1} \frac{V_1}{V_2}$

   b) Determine $K = \frac{V^1}{V_2}$

   c) Select up dip curve (parameter) corresponding to above value of B. (Interpolating between curves if necessary). Project above value of K, parallel to the ordinate, to this curve. Project this point of intersection parallel to the abscissa to tangent table. The tangent curve is given on left page for convenience in converting this tangent to degrees dip.

2. Same procedure as (1), using down dip group of curves.

3. a) Assume an approximate value of V_2 as the arithmetical average of velocities up and down dip.

   b) Determine tentative values of K (up dip and down dip) from the relation $K = \frac{V^1}{V_2}$.

   c) Determine tentative value of B from $B = \cot \sin^{-1} \frac{V_1}{V_2}$

From tentative values of B and K above, read from curves the values of tan $\propto$ for both up dip and down dip cases. Assume average value of these tangents and read curves back to new values of K for average of tan $\propto$. Two values of V_2 can now be determined from the values of K $(K = \frac{V^1}{V_2})$ and the apparent velocities. These two values of V_2 may be averaged, a new value of B determined, and the process repeated until the values of V_2 agree. In most cases, the second trial will give values closely identical.

An explicit solution of V_2 in terms of V_{2+}, V_{2-}, and $\propto$ will be found in equation 19, Page 639, "Geophysical Prospecting—1929."

An explicit solution of V_2 in terms of V_1, V_{2+}, and V_{2-} as reduced from equation A, is given here:

$$V_2^2 = \left[\frac{V_{2-} - \sqrt{V_{2+}^2 - V_1^2} + V_{2+} \sqrt{V_{2-}^2 \; V_1^2}}{V_{2+} + V_{2-}}\right]^2 + V_1^2$$

A SUGGESTED METHOD OF APPROACH FOR THE DETERMINATION OF SALT DOME OVERHANG

By O. C. LESTER JR., Houston, Texas

The purpose of this paper is to present for consideration and discussion a method of seismic attack on the problem of salt dome, or associated caprock, overhang such as has been indicated by recent developments at Barber's Hill and Allen domes.

The original Gulf Coast geophysical problem was one of reconnaissance—the search for a salt plug of relatively shallow depth which **might** lie anywhere within a few hundred thousand acres. Next came the slightly more detailed problem of searching for deeper and deeper domes within the same large areas in which the shallow domes had been found.

As the location of more and more of these domes became an established fact, further and far more intricate problems were presented in the detailing of them, with primary attention paid to contouring the top (either caprock or salt), and outlining the boundary of the steep plunge from top to flanks. The varying degrees of success with which each of these problems has been met by geophysics, in numerous instances, is well known to all connected with the oil business. However, the significant point is, that each successive problem has been met, and a reasonable solution offered.

The problem now presented is one of salt dome detail under the surface of the dome; and the attack must be made **through** the top of the dome, or by approach on its flank, under the top.

A method of approach is suggested here, which involves the reflection type of seismic shooting, and will be submitted in a general way for your consideration.

In attempting to obtain detailed subsurface information by geophysical means we are confined to physical operations at the surface of the ground. This fact, coupled with the necessity of obtaining information at depths of several thousand feet requires, in refraction work, that shot and recording points be placed at distances several miles apart, so that energy (or seismic waves) may attain the depth desired. For investigating in detail the vertical changes in a roughly horizontal bed, the reflection method, which requires the seismic waves to contact the bed in question only at the point where information is desired, obviously possesses an inherent advantage over a method which requires a great horizontal travel through the bed itself. (See Fig. 1 and 2.).

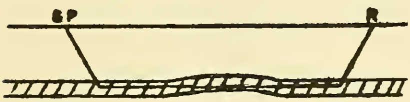


Fig. 1

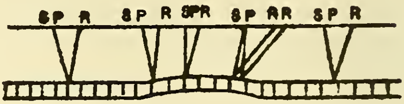


Fig. 2

In areas where correlation of reflecting horizons is impracticable over any great distance, this difficulty may be overcome by use of the "dip" method, or more detail shooting. The so-called dip method is essentially a short distance correlation between successive recording points of one shot point (setup indicated by two recording positions in Fig. 2). Correlation of reflections can easily be established between records obtained from positions thus closely spaced, and individual depth calculations may be used to determine the slope of the reflecting bed. This slope in turn establishes the position of the bed with respect to shot and recording points.

Assuming that depths, and hence slopes, by short distance correlation, can be determined by the reflection method, the new problem by its very nature should lend itself readily to solution. This statement is made because of the sloping nature of the sediments close to the flank of a dome. Obviously the point of reflection from a horizontal bed will lie vertically under a point midway between shot and recorder. If, however, this bed be sloping, the reflection point will be moved out from the vertical and will lie at that point where lines drawn from shot and recorder will make equal angles with the normal to the bed at the point of reflection. (See Figs. 3 and 4)

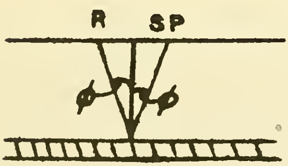


Fig. 3

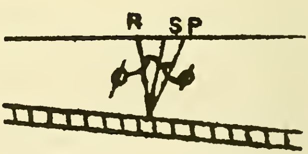


Fig. 4

With this in mind, and referring to the theoretical diagram, Plate 1, the method of attack is to shoot a line of reflection profiles radially into the dome, following the deeper sediments up slope as far as possible. As the angle of slope of the sediments increases, the reflection point moves further in toward the dome from the vertical, allowing the reflected energy to pass under the overhanging upper portion of the salt or caprock, thus securing data on the position of sediments actually underlying thick sections of salt.

For cases of relatively thin overhangs or fingers of caprock or salt, extending some distance out from the main salt body, there exists also the possibility of working through them as though they were ordinary beds, of varying thickness.

It may be well to point out in passing, that the entire success or failure of such a determination need not rest on an accurate quantitative analysis. For example: An edge well is drilled into caprock or salt. By means of such a procedure as outlined above, the presence of sediments is shown beneath the depth of the well merely by the existence of reflections at greater depth, on the record. (Obviously no reflections would be expected within the salt core itself.) While quantitative data as to the exact slope and depth of these sediments is, of course, greatly to be desired, the proving of the mere existence of sediments beneath such a well is half the story, and certainly valuable information.

Reference is given here to an actual case where reflection work has been done on the edge of a dome. At the time of this work, the question of overhang was not under consideration. A subsequent well, however, as seen on the accompanying section (See Plate 2), was drilled into caprock within the area below which sediments are definitely shown to exist. This would strongly indicate the existence of overhang and would suggest the value of further study and analysis of the data with this new idea in mind. This further study has been started, and while not yet complete, there is every indication that further analysis will bear out the conclusions drawn and present a very interesting picture.

As mentioned previously refraction work, by its inherent nature, can hardly be expected to contribute any detailed information to such a problem as this. It seems quite probable, however, that interesting and worth while general conclusions may be drawn in some instances from a closer study of refraction work, especially where any considerable amount of it has been done on a single dome. Especial attention should be paid in such study to relative "leads" on long and short lines, where shot through equivalent sections of salt. Obviously, for the commonly assumed salt dome shape where the salt section broadens continuously toward the base, a longer travel path with its greater penetration should receive a greater relative speeding up or lead than a shorter, shallower path. Where refraction lines of varying lengths are available on a dome previously shot, a study of this sort may serve to point out some very interesting general conclusions as to the change in horizontal salt section with depths.

In several cases already investigated from previous refraction data, a considerable variance has been found to exist between relative leads on long and short lines. At least one case has been noted where salt leads of considerable magnitude were obtained on a short line refraction fan, while a long line fan across the same dome showed so little lead as to question the probability that the dome would have been picked up in this manner. Assuming the data itself to be satisfactory, the logical analysis is that in this particular case the salt section likely decreases with depth, at least within the limits of penetration of the lines used. Though no detailed conclusions may be drawn from such data as to the particular depth at which the change takes place, nor accurate estimate be made as to the relative horizontal sections of salt, at least a more favorable prospect can be selected for further work of detailed nature. Conversely, and as an indication that all domes should not be considered as necessarily overhanging, are cases of domes which show relatively as great or greater leads when shot with long lines as with short.

A further general analysis may be made by a comparison of the "shadow" or edge leads or "sideswipe" as shown by different salt domes. More specifically, if a line shot tangent to the surface projection of a dome shows more or less relative lead than a similar line on another dome, the line of less lead may reasonably be supposed to have had less section to "sideswipe" than the lines of equal penetration, or approximately the same length.

It may be of interest to note that the refraction work on the dome discussed above showed an extremely sharp cut-off of lead on adjacent lines of a fan spreading over the edge (See Fig. 5),

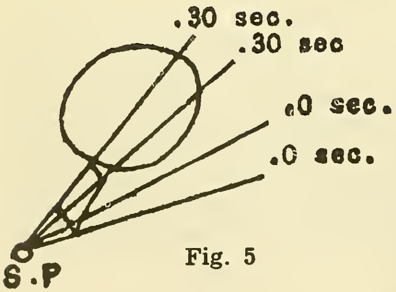


Fig. 5

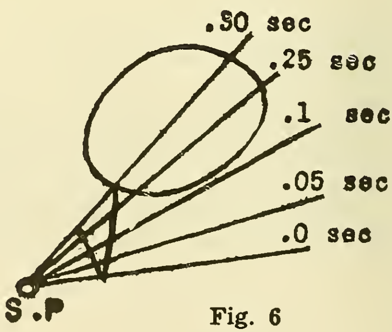


Fig. 6

rather than the gradual diminution of lead often obtained in this type shooting. (See Fig. 6).

Such cases as this are, I believe, very worthy of general discussion, and such discussion should be in no way detrimental to the interests of the companies concerned. It may well be that some contribution may be made toward the knowledge of origin, growth and shape of salt domes by a study and correlation of such technical data, if it is made available. Needless to say, this would not require any information with regard to location, nor specific mention as to the particular dome or prospect.

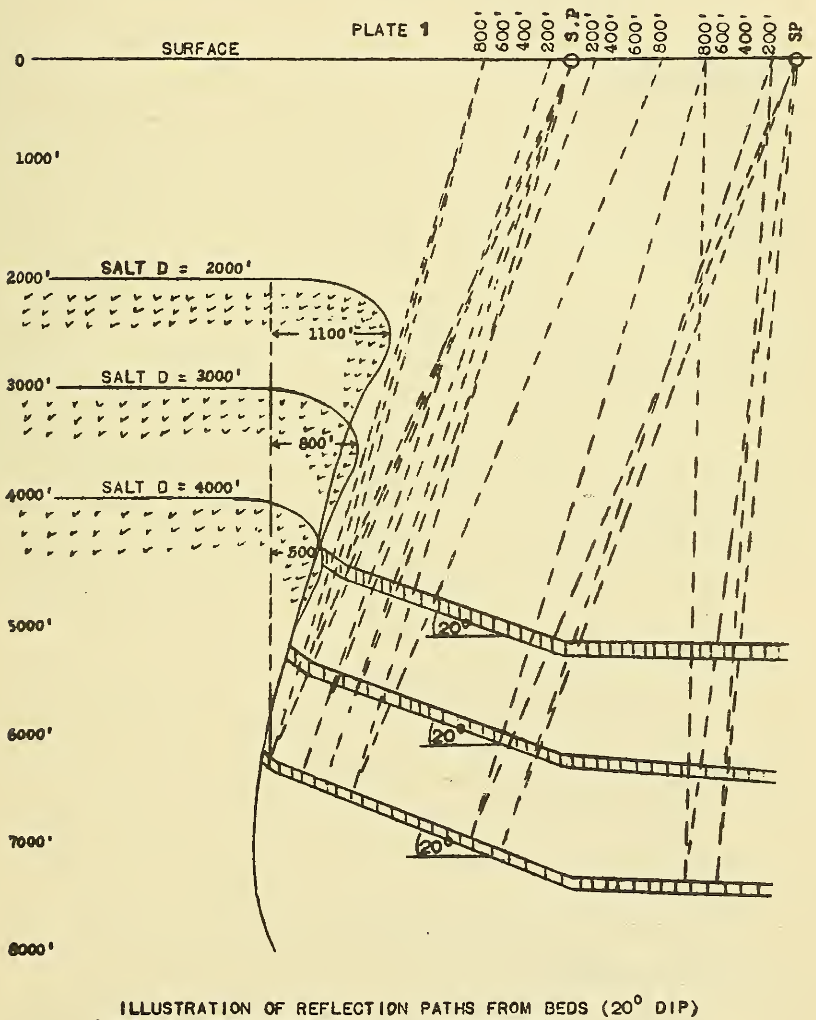


ILLUSTRATION OF REFLECTION PATHS FROM BEDS (20° DIP) PASSING UNDER OVERHANGING SALT OR CAPROCK.
DIAGRAM TO SCALE INDICATES APPROXIMATE MAGNITUDE OF OVERHANG WHICH CAN BE PASSED UNDER, FOR THIS CONDITION.

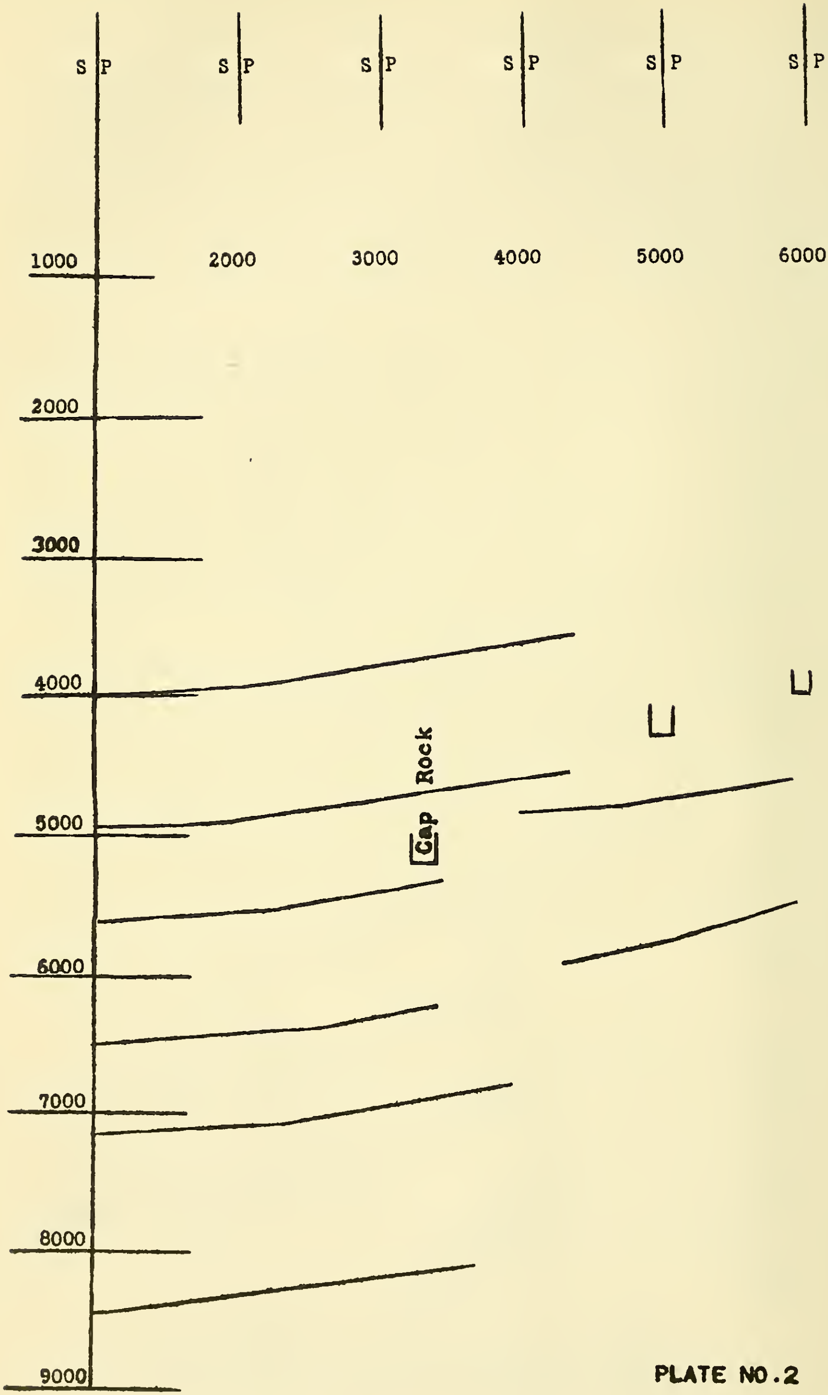


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PLATE NO. 2

A Semiportable Alternating-Current Susceptibility Meter*

By WILLIAM M. BARRET**
Shreveport, Louisiana

(Received May 11, 1932)

This paper describes a semiportable alternating-current instrument that has been developed recently for measuring the magnetic susceptibility of geological samples. Its construction, basic theory, and operation are outlined briefly.

INTRODUCTION

EVERY recognized geophysical method depends for its operation on the nonhomogeneous character of the lithosphere. The various rocks that compose this outer shell of the earth exhibit differences in elasticity, density, electrical conductivity, and magnetic susceptibility. These variations give rise to the seismic, gravimetric, electric, and magnetic methods, respectively.

An intelligent interpretation of the anomalies related to these various methods must therefore be predicated on an intimate knowledge of the requisite physical constants of the involved media.

Numerous laboratory methods have been suggested for determining the magnetic susceptibility[1] of geological samples, though unfortunately, these methods have usually proved ill-suited to the requirements of the practicing geomagnetician. During the past year there has been developed in the laboratory of William M. Barret, Inc., an instrument that offers marked advantages from the standpoint of portability, rapidity of operation, ease of manipulation, and increased sentitivity and precision. This instrument, which has been designated an alternating-current *Susceptimeter*, is self-contained and portable, except for the fact that external power sources are required for its operation. It is the purpose of this paper to present briefly the essential characteristics of this instrument.

DESCRIPTION

The Susceptimeter, Fig. 1, consists primarily of a modified form of inductance bridge,[2] and an alternating-current galvanometer to indicate the condition of balance. The bridge is energized with 60-cycle, 110-volt alternating current, and in order to have no current flowing through the galvanometer the four arms of the bridge must be balanced for capacitance, inductance, and resistance. When these three elements are correct, the bridge is balanced and the galvanometer is at rest. Now, if in one arm of the bridge we insert a sample, whose susceptibility differs from that of air, into an appropriate test

* Read before the American Association of Petroleum Geologists at the Oklahoma City meeting, March 25, 1932, and printed in *Physics* by permission of the A. A. P. G.

** Geophysicist, William M. Barret, Inc., Giddens-Lane Building.

[1] Throughout this paper the term "susceptibility" will refer to volume-susceptibility.

[2] For a detailed discussion of a.c. bridge methods the reader is referred to B. Hague, *Alternating-Current Bridge Methods* (Sir Isaac Pitman & Sons, Ltd., New York).

coil, the balance is disturbed because the inductance of the test coil has changed. The balance is restored by altering the inductance of a variable inductor placed in the arm with the test coil. When this operation is completed, the reading of the inductor dial indicates, by means of a calibration curve, the susceptibility of the sample. These variations in inductance are frequently so very slight that the effect must be amplified by vacuum-tubes, and even then only the most sensitive galvanometers will prove suitable.

Electric Circuits

The electric circuits of the Susceptimeter are shown in Fig. 2. The ratio arms, R_1 and R_2, consist of equal pure resistances, while one inductive arm includes the test coil L_1 and variable inductor L_2, and the other contains the

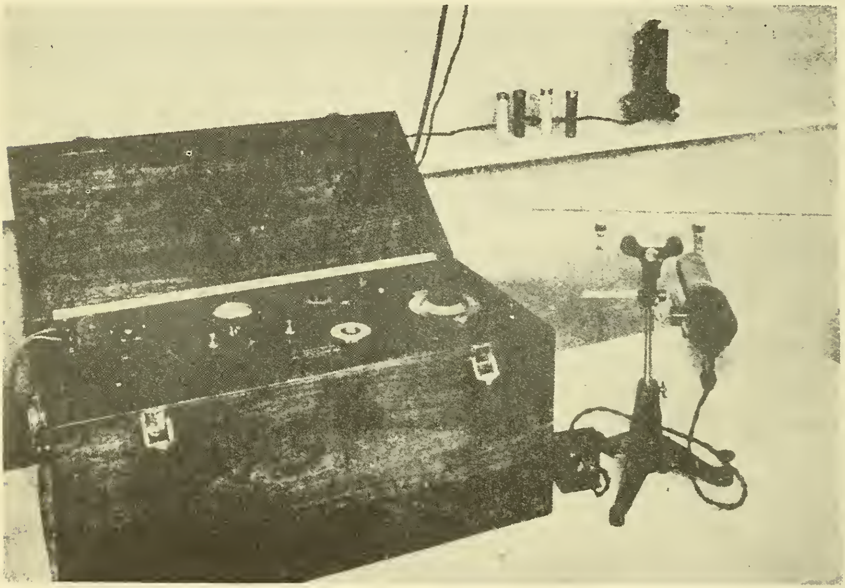

Fig. 1. Susceptimeter arranged for use with external vibration galvanometer. Several sample containers, filled with pulverized materials, may be seen near the galvanometer. These glass containers are provided with aluminum friction caps, and have outside dimensions of 2.7 cm dia. by 10 cm long.

tapped inductance L_3, and vernier variable inductor L_4. The resistance balance of the bridge is accomplished by means of r_5, and this balance is maintained for different positions of the inductance switch, S_1, by the loading resistors r_1 and r_2.

Low potential alternating current is supplied the bridge through the transformer T. The selector switch, S_4, and variable resistor, r_4, provide means for adjusting the bridge current, while its value may be determined with the ammeter A.

The indicating device consists of a pointer type, alternating-current galvanometer, energized by a three-stage, resistance-capacitance-coupled, vacuum-tube amplifier, which is provided with an appropriate output circuit that is designed to prevent the flow of direct current through the galvanometer circuit. The shunt r_3 permits critical damping of the galvanometer, and also furnishes a convenient means for adjusting its sensitivity. An external storage battery, B_1, supplies filament power for the amplifier, while the grid-bias battery, B_2, and plate battery, B_3, are included within the instrument case.

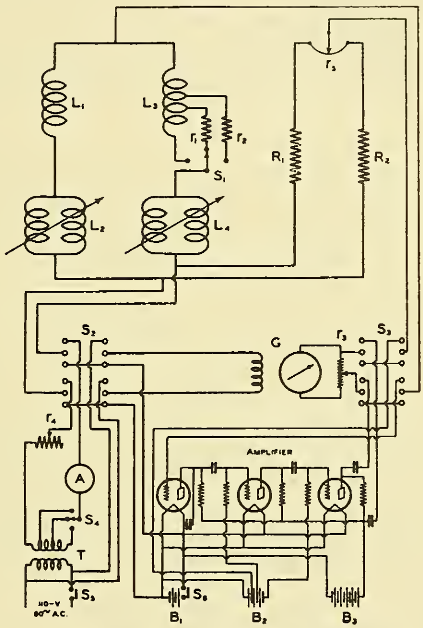

Fig. 2. Electric circuits of the Susceptimeter.

The switch S_2 makes it possible to energize simultaneously the bridge and galvanometer field winding with 60-cycle alternating current, or 6-volt direct current, while S_3 is for the purpose of connecting the galvanometer directly to the output junctions of the bridge, or including the amplifier in the indicator circuit. The switches S_5 and S_6 control the external alternating-current and direct-current power sources, respectively.

Theory

Let R_1 and R_2 be the resistances of the ratio arms; R_3 the resistance of the arm containing the inductances L_1 and L_2; R_4 the resistance of the arm containing the inductances L_3 and L_4; then, the condition imposed for a balance with direct current is $R_1R_3 = R_2R_4$ and by varying r_5 a balance can be obtained for steady currents.

When excited with sinusoidal currents, the general equation for balance with the inductance bridge is $LR_2 = L'R_1$, where L is the effective inductance of the arm containing L_1 and L_2, and L' is the effective inductance of the arm containing L_3 and L_4. The inductive balance is obtained by a proper adjustment of the vernier inductor L_4.

To avoid uncertainties occasioned by possible changes in the value of R_1 or R_2, the Susceptimeter circuit is based on the substitution method. When a sample, having a susceptibility differing from that of air, is placed in the test coil L_1, the inductance of this coil is changed, and to rebalance the bridge it is only necessary to alter the inductance of L_2, which is included in the arm with the test coil, by a like amount.

It is now required to express the relation between the susceptibility of the sample and the change in inductance of L_1 caused by its insertion. With no sample

$$N\phi = L_1 I \qquad (1)$$

where N is the number of effective turns on L_1; ϕ is the total flux threading N effective turns; L_1 is the inductance of the test coil in c.g.s. units; I is the current through the test coil in c.g.s. units. Then, with sample inserted,

$$N(\phi + \Delta\phi) = I(L_1 + \Delta L_1) \qquad (2)$$

and dividing (2) by (1)

$$\Delta\phi/\phi = \Delta L_1/L_1. \qquad (3)$$

Also, with no sample present

$$\phi = A_s H_s + A_a H_a \qquad (4)$$

where A_s is the cross-sectional area in cm^2 of the sample; A_a is the cross-sectional area in cm^2 of the annular space between the sample area and the mean area of the test coil; H_s is the effective magnetizing force in gilberts per cm in the sample; H_a is the effective magnetizing force in gilberts per cm in the annular space. When a sample of permeability μ_s is introduced into the test coil

$$\phi + \Delta\phi = \mu_s A_s H_s + A_a H_a \qquad (5)$$

and subtracting (4) from (5) $\Delta\phi = A_s H_s(\mu_s - 1)$.

Expressing μ_s in terms of the susceptibility, κ_s, we may write $\Delta\phi = A_s H_s \cdot 4\pi\kappa_s$. Substituting this value of $\Delta\phi$ in Eq. (3) and solving for κ_s, we have

$$\kappa_s = \Delta L_1 \phi/(A_s H_s \cdot 4\pi L_1) = C \cdot \Delta L_1 \qquad (6)$$

where C is a constant for the fixed frequency, and for the current range available.

In Eq. (6), ΔL_1 is determined by means of the variable inductor L_2; the product $A_s H_s$ is measured with a search coil and ballistic galvanometer; L_1 is measured in place with an external inductance bridge, and ϕ is calculated from flux measurements made through various sections of the test coil.

Operation

In operation, the Susceptimeter circuits are first energized with direct current, and a resistance balance obtained by varying r_5. Following this, the bridge is excited with alternating current, and the galvanometer and amplifier connected in the indicator circuit. With no sample in the test coil L_1, and with the variable inductor L_2 adjusted for maximum inductance (dial reading 0), the vernier inductor L_4 is used to bring the galvanometer pointer to mid-scale position.

If a specimen, having a susceptibility greater than air, be now introduced into the test coil, the galvanometer pointer will move toward the positive side of its scale, and to rebalance the bridge it will be necessary to reduce the inductance of L_2. This is accomplished through the aid of a micrometer drive mechanism, provided with a vernier dial, which makes it possible to adjust and read the position of the variable inductor to within 1/100 scale division. The proper decrease in L_2 is obtained when the galvanometer pointer again

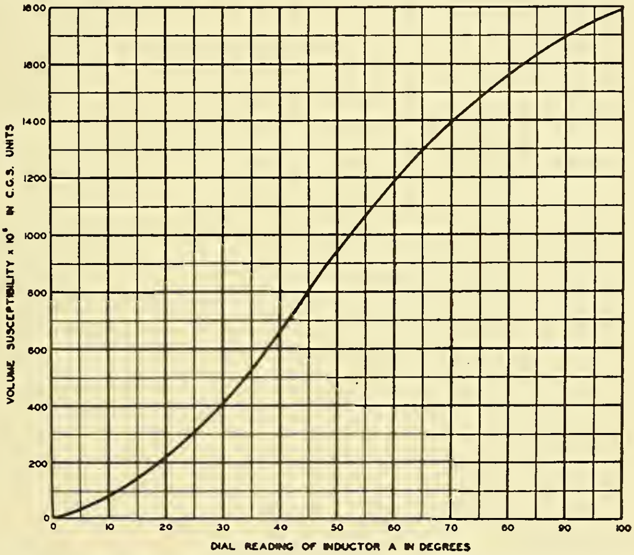

Fig. 3. Normal calibration curve for first selector switch position. For second or third position, a constant additive correction is applied, its value depending on switch position used.

assumes its equilibrium position. The dial reading of the variable inductor is then a measure of the susceptibility of the unknown sample, and by employing a calibration curve similar to Fig. 3, the dial readings may be conveniently converted to susceptibility values. To extend the range of the instrument, the tapped inductance L_3, and selector switch S_1 are used. When the second or third selector switch position is required, a constant additive correction is applied to the normal calibration curve, the value of the correction depending on the switch position used.

Should a diamagnetic specimen be inserted in the test coil, the galvanometer pointer moves in the direction of the negative part of its scale, and for this condition, the initial balance is obtained with the inductor L_2 set at some arbitrary scale position, and then its inductance increased until the galvanometer again indicates a balance. The same calibration curve employed for paramagnetic materials will serve for diamagnetic substances, simply by noting the initial and final scale positions of L_2 and reading the respective difference, in terms of κ, from the curve.

General

For convenience, the samples are used in the pulverized state, and when carefully prepared their density seldom differs markedly from that of the original material. Belz[3] and Wilson[4] have found that equal masses of several paramagnetic substances have the same magnetic susceptibility in the solid as in the powdered form.

The susceptibility is a function of the effective magnetizing force[5] for many materials, and for this reason it is well to use a fixed value of field strength for comparative studies. Within narrow limits, the magnetizing force applied to the sample is directly proportional to the bridge current.

As most electrical methods for measuring the susceptibilities of rocks have employed direct current, the question naturally arises as to the authenticity of determinations conducted with alternating current. Two possible sources of error are at once apparent, i.e., those caused by virtue of the specific inductive capacity of the sample, and its electrical conductivity. In order to determine the influence of the dielectric constant, a specimen of distilled water was examined. While this specimen exhibited a high dielectric constant and low conductivity, no observable effect was noticed. Also, it was found that a saturated solution of sodium chloride, having a high conductivity, failed to produce a sensible deflection of the indicator, which is in agreement with the similar investigations of Rücker.[6] As the values of dielectric constant and conductivity of these samples are much higher than those customarily encountered in practice, it is felt that these factors may be safely ignored.

The susceptibilities of ferromagnetic substances are extremely high for very low frequencies, and unity for all materials in the magnetic fields of light and infrared waves.[7] However, Brown[8] has shown that the susceptibility of iron in oscillating magnetic fields of less than 300,000 cycles per second is approximately constant and similar to that in stationary fields.

While variations in κ occur over tremendous changes in frequency, nevertheless, for routine measurements with geological specimens, where the numerical values of susceptibility, dielectric constant and conductivity are usually low, the discrepancy introduced by using 60-cycle current is negligible. In fact two important technical advantages result from the application of alternating current, (1) freedom from extraneous magnetic disturbances that react unfavorably on many direct-current and magnetometric methods, and (2) the susceptibility measurements are independent of the previous magnetic history of the sample.

[3] M. Belz, Phil. Mag. [6] **44**, 479–501 (1922).

[4] E. Wilson, Proc. Roy. Soc. **A96**, 429 (1919).

[5] P. Weiss, *Recherches sur l'aimantation de la magnétite cristallisée.* L'Eclairage Electrique 7, 487 (1896).

[6] A. W. Rücker, Proc. Roy. Soc. London **48**, 515 (1891).

[7] W. Arkadiew, Phil. Mag. [6] **50**, 157–163 (1925).

[8] R. Brown, Jour. Frank. Inst. **183**, 41 (1917).

BULLETIN

of the

AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS

Volume 15 | NOVEMBER 1931 | Number 11

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Symposium on Geophysics

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NOVEMBER 1931

Symposium on Geophysics

EDITOR'S FOREWORD

In 1929, five major articles on geophysics were published in the *Bulletin.* In 1930, fifteen geophysical papers were printed, including two on geothermal gradients. Eight of these fifteen papers appeared in the September number, but the other seven were distributed through five other monthly issues. This year we have made an effort to concentrate all the strictly geophysical contributions in the November and December numbers.

These papers, which follow, were presented in March, 1931, before the Society of Petroleum Geophysicists at the annual convention of The American Association of Petroleum Geologists at San Antonio. Together with a few others, not now available, they constituted the geophysical group of the Association's technical program. They were solicited largely through the efforts of the program subcommittee, consisting of H. A. Aurand, O. C. Lester, D. M. Collingwood, George M. Bevier, Paul B. Whitney, and G. H. Westby, chairman, all members of both the Society and the Association. To these gentlemen, and especially to Mr. Westby, we wish to express our thanks for their coöperation in making the program a success.

Geophysics is an important tool for helping to decipher the intricate and hidden conditions of the earth's interior. It is an important branch of geological investigation. The real significance of its data can not be satisfactorily interpreted except in the light of geology. We therefore concur with Mr. Westby in his invitation, included in his introduction, for coöperation between geophysicists and geologists; and we wish to add encouragement to the geophysicists to contribute more papers on this important subject.

F. H. LAHEE

INTRODUCTION

During the past year petroleum geophysics in the United States has entered a new, highly important phase of its gradual development and increasingly satisfactory application to the mapping of subsurface geological structure. With the advent of seismic reflection shooting, the locale of most intense geophysical prospecting has shifted from the Texas and Louisiana salt-dome areas to the Mid-Continent states of Oklahoma and Kansas. With this change has come a wider appreciation by geologists and executives of the utility of geophysics. In addition, more geologists now realize that geophysics is not a separate field, but a means of amplifying their ability to determine geological conditions. In short, we have passed from the more or less mysterious isogam and gamma stage to the foot stage in geophysical interpretation.

To date it has been difficult to induce most geophysicists to participate in the presentation or discussion of papers. This attitude is a result of the youthfulness of the science. A change may be expected with age. The present period in the science of petroleum geophysics is a recapitulation of the history of geological science in the oil industry prior to the formation of The American Association of Petroleum Geologists. It is to be hoped that the geologists, with their experience in the value of coöperation, will imbue geophysicists with their ideas.

To forward this spirit of coöperation and to acquaint more geologists with geophysical methods, The American Association of Petroleum Geologists, through the Society of Petroleum Geophysicists, solicited geophysical papers to be presented at the San Antonio meeting and later to be published in the *Bulletin*. The following papers, which were presented at San Antonio, represent a fair cross section of the present development of petroleum geophysics. Some of them present new, unorthodox ideas and, though subject to discussion, should stimulate new thought. Such discussion is very necessary, is cordially invited, and if given will accomplish one of the objects in the presentation of these papers.

To the various authors and their respective companies, we are greatly indebted for the release of material and the preparation of the papers.

G. H. WESTBY, *chairman*
San Antonio Program Sub-Committee on Geophysics

APPLICATION OF SEISMOGRAPHY TO GEOLOGICAL PROBLEMS[1]

EUGENE McDERMOTT[2]
Dallas, Texas

ABSTRACT

This paper contains an outline of the principles and methods of applied seismography. The science of seismography is predicated on the elastic properties of earth materials. The refraction and reflection of elastic waves and the conditions governing these are considered. These two phenomena are closely related. The application of the seismic method to salt-dome exploration on the Gulf Coast and the development of the refraction method for general structure determination is followed by an explanation of the reflection method. Several actual reflection records indicate the method of identifying reflections.

INTRODUCTION

The purpose of the writer is to present to the geologist rather than the practicing geophysicist the theory and method of applied seismography. The science of seismography derives its value from the fact that earth materials are elastic and this elasticity has extensive variations. The term seismography as used herein applies to the science which utilizes elastic waves that are generated artificially, as by an explosive charge.

GENERAL THEORY

In general, all materials are elastic, but in varying degrees. By elasticity is meant the resistance a material offers to changing its volume or form when subjected to stress. When stressed a material yields. This yield or change in volume or form is known as a strain. Elasticity may now be defined more accurately as the ratio of this stress to the resulting strain. This strain is propagated through the material with a definite velocity which is a function only of the elasticity and density of the material. This wave of strain is propagated in all directions in a straight line if the material is homogeneous or, more accurately stated, the wave front is spherical. If the material is not homogeneous the path is curved, that is, the wave is gently refracted, and the wave front is no longer

[1]Read before the Association at the San Antonio meeting, March 20, 1931. Manuscript received, June 4, 1931.

[2]Geophysical Service, Inc., 1311 Republic Bank Building.

spherical. At the contact plane between materials of definitely different physical constants, both refractions and reflections occur simultaneously. Part of the energy in the wave is transmitted through the second material in a direction different from its course in the first material, at the same time that some of the energy in the wave is thrown back as a reflection.

A very instructive illustration of these phenomena is given in Figure 1. In the upper part of the figure a rope fastened to a wall at *B* is held at the other end in the hand. A quick movement of the hand at *A* starts a wave down the rope. This wave travels at a definite velocity for a

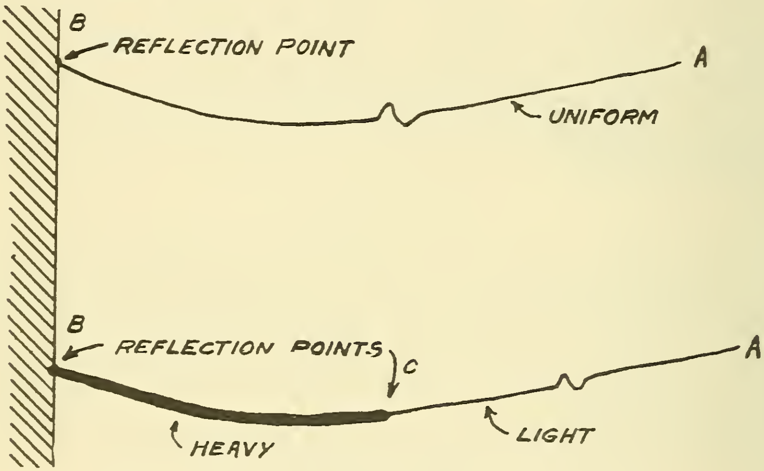

FIG. 1.—Refraction and reflection of wave in elastic rope.

specified weight of rope. On reaching *B*, most of the energy in the wave is reflected back toward *A*. A small part of the energy is refracted and actually causes the wall to move, though very slightly. This illustrates a very efficient reflecting contact between two media.

If, as in the lower part of Figure 1, two ropes of different weights are fastened together at *C*, a somewhat more complicated example of the same phenomena presents itself. In view of this difference in weight, the velocity of a wave is different in the two ropes. A wave travelling down the light rope from its starting point at *A* is partly refracted and partly reflected at the junction of the two ropes, *C*, as well as at *B*. On reaching *C* a reflected wave starts back toward *A* and a refracted wave starts travelling toward *B* in the heavy rope at a slower speed than the original wave travelled in the light rope. On reaching *B* this wave is mostly reflected as in the previous illustration. B is, of course, a much better reflecting point than C, because the wall is much more rigid than the rope. It should be borne in mind that at a junction point where refractions and reflections occur, the sum total of the energy in the refracted and reflected waves equals the energy in the original wave. A good reflector point, therefore, means one at which a large part of the arriving energy is reflected back while a small part travels on as a refracted wave.

In the previous illustration we dealt with points of contact between two media. Practically, we are concerned with planes of contact between solid earth materials of different elastic properties. Though this case is obviously more complicated, the same fundamental theory applies. The device of the ropes admirably illustrates the occurrence of reflections, but does not demonstrate the special case of refractions, of greatest value in seismography.

Figure 2 illustrates a simple condition of a soft low-speed and homogeneous earth material overlying a hard high-speed rock which is also homogeneous. The detonation of a charge of explosive at the surface of the ground generates an elastic wave of spherical wave front. What happens to the energy in various parts of this wave front is to be considered. Instead of considering the wave front, it is simpler to use rays which are normal to the wave front. There are, of course, an infinite number of these rays radiating into the earth from the shot point A.

As stated previously, the velocity of a wave through any medium depends only on the physical constants of the medium, specifically the elasticity and the density. It is true that the velocity does depend to a certain extent on the coefficient of absorption of the medium, but this is in general of second order in importance and is here neglected. The coefficient of absorption determines the loss of energy in a wave caused by molecular friction.

The velocity of a disturbance may be expressed simply:

$$V = \sqrt{\frac{E}{D}}$$

where V is the velocity, E is the appropriate coefficient of elasticity, and D is the density. In non-rigid media, such as fluids, E is simply the incompressibility, whereas in rigid media E is a function of both the incompressibility and the rigidity coefficient and may be expressed thus:

$$V = \sqrt{\frac{I + {}^4/_3 K}{D}}$$

where I is the coefficient of incompressibility, K is the rigidity coefficient, and D has the same meaning as in the first equation.

The foregoing expressions are for longitudinal waves. In longitudinal waves the vibration of the particles is in the direction of wave propagation. Transverse waves also are generated in which the particles of the medium vibrate at right angles to the direction of travel of the disturbance. In all that follows, longitudinal waves only are considered. The velocity of the transverse wave, however, is determined from the following relation:

$$V = \sqrt{\frac{K}{D}}$$

where K is the coefficient of rigidity.

As the speed of propagation depends only on these physical constants of a medium, the longitudinal wave travels in all directions at the same speed in a homogeneous medium. It is obvious that a homogeneous medium must be one which has the same physical constants at all points and in all directions in the medium. By a non-homogeneous medium is meant one in which the physical properties and the velocity of a disturbance vary in different parts of the medium. The earth is such a medium. It is the non-homogeneity of the earth which gives value to the science of seismography. Variations in density are slight compared with variations in elasticity. Hard limestones occur overlain by relatively soft shales. The difference in elasticity of these two materials makes possible definite refractions and reflections. Hard limestones exist in which the velocity is ten times the velocity at the surface. This means that the elasticity of such a limestone must be one hundred times the elasticity of the surface materials, as there is very little difference in density. Even in undifferentiated alluvial sediments the velocity increases with depth as a result of overburdening. Such a gradual change of velocity has no practical value. It is the abrupt changes that are valuable.

Referring again to Figure 2, the simplest example is that of the ray directed vertically downward, AB. On reaching B, some of the energy in the wave is reflected vertically upward while the remaining part generates a wave in the lower medium which travels vertically downward, BC, and, of course, at a greater speed. This is the only example in which the direction of the refracted wave is the same as that of the incident wave.

Now consider a ray travelling obliquely downward, as AF. The refracted ray is bent as shown at FG so that the ratio of sine a to sine c

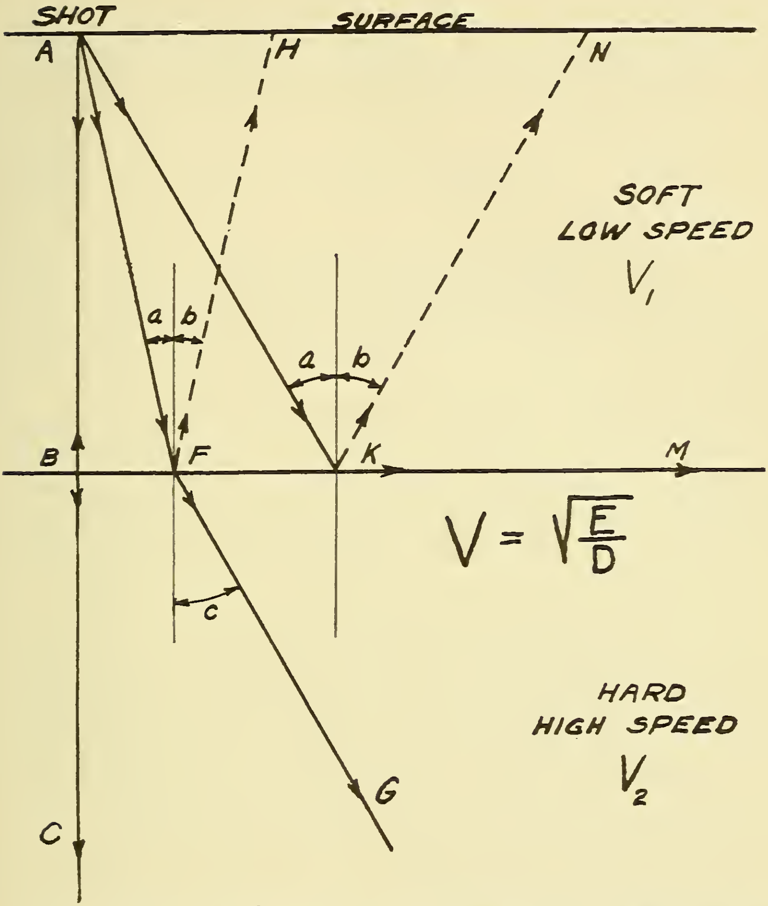

FIG. 2.—Refraction and reflection of waves in passing from low-speed to high-speed medium.

equals the ratio of velocity in the upper medium to velocity in the lower medium. The direction of the reflected ray is such that the angles a and b are equal. Refracted and reflected rays must always obey these two simple fundamental laws, which may be summarized:

$$\frac{\text{Sine } a}{\text{Sine } c} = \frac{V_1}{V_2}$$

$$a = b$$

where V_1 is the velocity in the upper medium and V_2 in the lower. Angle a is known as the angle of incidence, angle b the angle of reflection, and angle c the angle of refraction. Increasing the angle of incidence finally leads to the limiting case beyond which all the energy is reflected and none is refracted. Such an angle of incidence is known as the critical angle. In this case:

$$\text{Sine } a = \frac{V_1}{V_2}$$

This limiting case is illustrated by the ray AK. The refracted ray KM travels along the contact plane between the two media. This particular refracted ray is the one so much used when using the refraction method for determining structure and is referred to later where that method is discussed.

In actual practice homogeneous strata depicted in Figure 2 are not encountered. There is in all cases an increase of velocity with depth, as a result of which the rays are slightly curved due, of course, to a gradual refraction. In the diagrams, with one exception, this curvature is neglected for the sake of simplicity. No serious inaccuracy can result from this procedure.

REFRACTION METHOD; SALT-DOME EXPLORATION

Seismography found its first practical application in the United States in the field of salt-dome exploration on the Gulf Coast of Texas in 1924. It was a logical and fortunate introduction of a method that was later to extend its usefulness to exploring other types of geologic structure. The first domes sought were very shallow, as deep domes, later discovered, were not at the time known to exist. Naturally those known at the time were only the shallow domes which had given evidence of their presence by surface indications.

The problem was ideal for the seismograph. The relatively large masses of high-speed salt were intruded in uniform and low-speed sedimentary rocks. The velocity in salt may be almost as much as three times the velocity in the rocks which it has displaced. The velocity in salt is 16,000 feet per second, whereas the normal velocity in the surface sediments may be as low as 5,500 feet per second. Time anomalies of ½ second were not exceptional, because of the presence of the salt plug.

In the upper part of Figure 3 the path of the initial impulse arriving at a recording instrument from the shot point is indicated. The curvature of the path is caused, as before mentioned, by the fact that the velocity in the sediments increases gradually with depth. If there is a buried salt dome between the shot and recording points, the first disturbance to reach the recorder is one which has travelled in part through

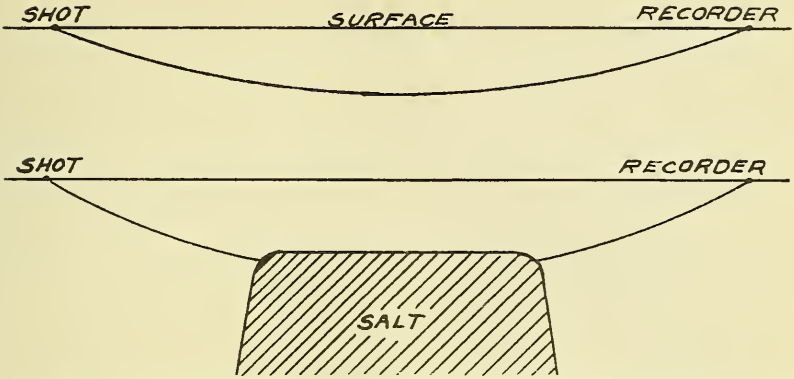

FIG. 3.—Shortest time paths (upper) in sediments and (lower) with salt dome present.

the high-speed salt. For a dome a mile in diameter, which is a fair average, the saving in time is approximately 0.6 second; obviously, the presence of the dome is easily detected.

As the method was applied to progressively deeper domes, the time anomalies to be expected became correspondingly less. It became necessary to increase the accuracy of timing to at least 0.01 second. Whereas the distance between shot and recorder points was previously determined by timing the air wave travelling from shot to recorder, it now became necessary carefully to survey this distance, especially as, for the deeper domes, it was necessary to place the recorder farther away from the shot point. Though a mile or two had been sufficient for the very shallow domes, it was found necessary to use 6-8 miles for the deep domes. This, of course, meant larger charges of dynamite, charges as large as 1,000 pounds being used for a single shot. The large charge required and the increased accuracy necessary in determining the distance between shot and recorder made the work increasingly expensive; also, the smaller time anomalies exhibited by the deeper domes made the results less reliable. However, in a period of 7 years many domes have been discovered by the seismograph.

REFRACTION METHOD; STRUCTURE DETERMINATION

In the study of the comparatively simple problem of searching for salt domes, the attempt was cautiously made to extend the range of usefulness of the seismograph method. Having found a dome, it was desirable to define it in as much detail as possible. It was possible to contour the top of the dome and determine its top edge.

The essentials of this method are illustrated in Figure 4. The refracting layer indicated may be a part of the top of the salt dome. It was mentioned previously that beyond a certain angle of incidence all the energy in an arriving disturbance is thrown back as reflected energy and none is refracted into the second medium. This angle of incidence is known as the critical angle. The ray shown in Figure 4 is assumed to be arriving at the contact of the salt and sediments at this angle. A part of the energy is consequently refracted along the contact surface. Some of this energy returns to the surface and registers on the recorder as shown. If the distance between the shot and recorder points is known and the velocity in the refracting layer is also known, by measuring the time necessary to travel over the path indicated it becomes a very simple problem in geometry to determine the depth to the refracting salt layer. By placing several recorders in a line at known distances from the shot point, called profiling, it is possible to determine the velocity of the refracting stratum. If the layer is not horizontal, it is still possible, by reversing the profile direction, to determine its velocity and at the same time determine the slope. By this procedure the configuration of the top of the salt may be accurately determined. Obviously, the method may be extended to any high-speed stratum other than salt domes. The thicker and more uniform the stratum, the greater the ease with which it may be worked. The method has in fact been extended to the determination of the configuration of several successive strata. The method is particularly adaptable to such regions as West Texas, though less applicable to a region such as Oklahoma.

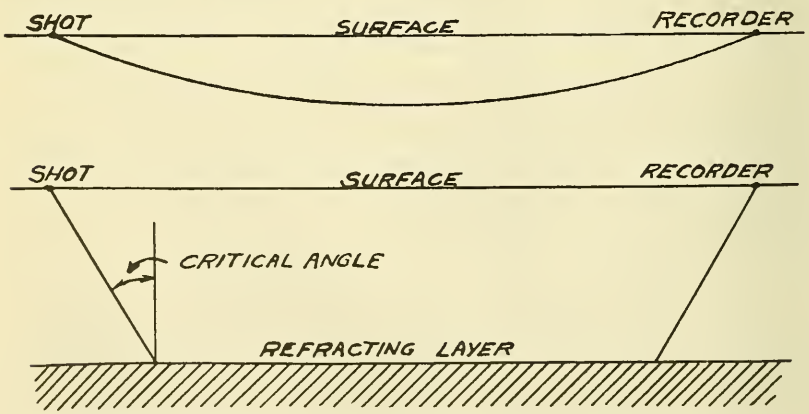

Fig. 4.—Path of wave at critical angle refraction as used to determine structure by refraction method.

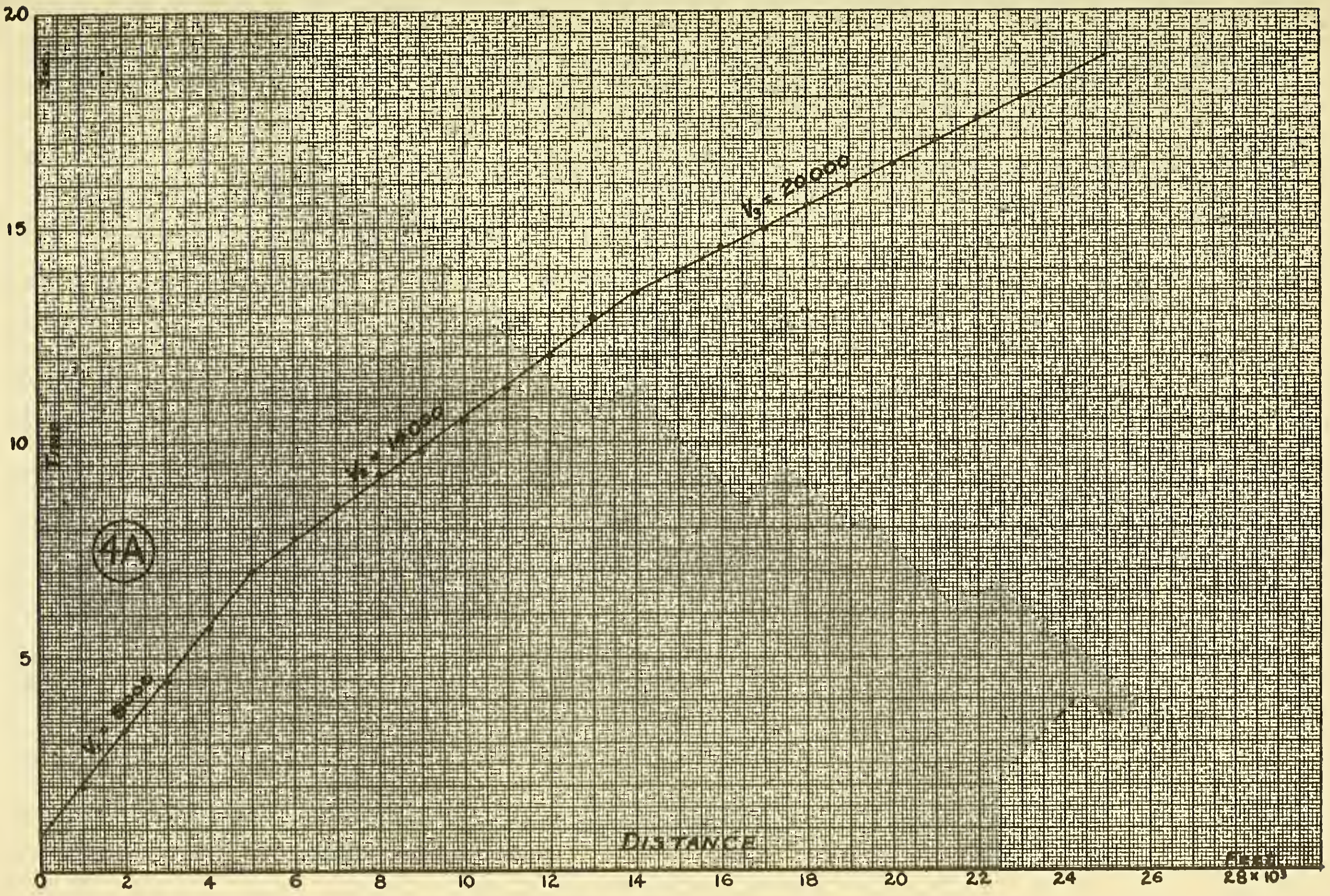

Fig. 4A.—Time-distance curve for movable recorder in Figure 4.

If, in the method of profiling mentioned, the distances from the shot to the recorder points are plotted against the times of arrival for these distances, the graph shown in Figure 4A results. Three lines are shown on this graph. The points falling on the first line are the directly transmitted disturbances, whereas the second and third lines are the result of refractions from each of two high-speed strata. The slopes of these lines represent the velocities in each of the three media.

There is at the surface a thin layer of weathered sediments the velocity of which is very low. It is necessary to determine the thickness of this and the elevations of the shot and recorder points in order to secure the greatest accuracy.

REFLECTIONS

As mentioned previously, refractions and reflections are closely related. One phenomenon rarely occurs without the other. Therefore, it is merely necessary to use the proper instruments and technique in order to separate and make usable the reflected energy that manifestly must be present whenever a disturbance is initiated. Reflections have been identified and have proved more valuable and certainly have permitted greater accuracy than refractions. In the refraction method, only the initial arrival of energy at the recorder was used, because other than initial arrival of energy is more difficult to interpret. The use of reflections, therefore, obviously means a refinement of instruments and method.

In the refraction method the distance between shot and recorder points is several times the depth of the refracting horizon, but in the reflection method this distance is only a fraction of the depth to the reflecting stratum, as shown in Figure 5. Here is shown the path of a reflected disturbance. It should be noticed that the projection of the point of reflection on the surface is midway between the shot and recorder points when the reflecting layer is horizontal. This satisfies the law of reflections that the angle of incidence *a* must equal the angle of reflection *b*. By measuring the time between the explosion of the charge and the arrival of the reflected disturbance at the recorder and knowing the average velocity through the medium, the length of the travelled path may be determined. By measuring the distance between the shot and recorder points, the depth may easily be computed as follows:

$$Z = \frac{1}{2} \sqrt{V^2 T^2 - X^2}$$

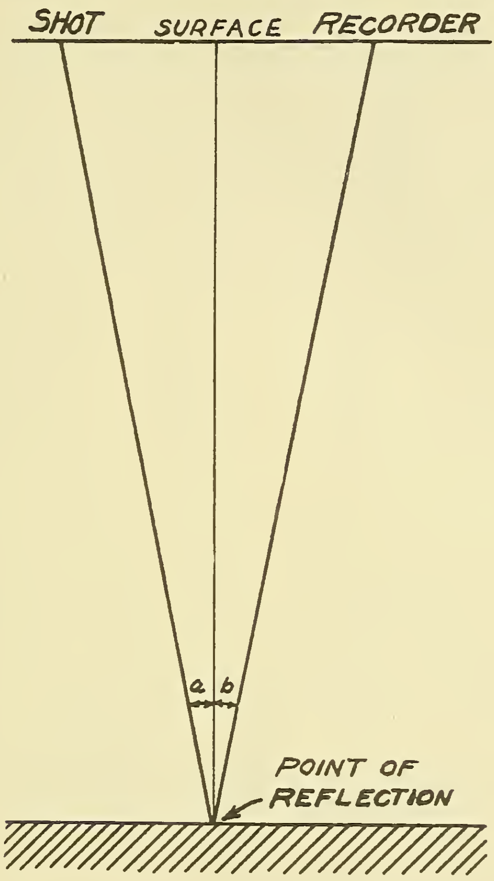

Fig. 5.—Path of reflected wave.

where Z is the depth, V is the average velocity from the surface to the reflecting stratum, T is the time of travel mentioned, and X is the distance between shot and recorder points.

The average velocity through the medium above the reflecting layer is a function of the depth of this layer. As a result, the path of the disturbance is not a straight line as shown in Figure 5, but is curved as shown by the full line in Figure 6. For simplicity the straight path is used in all of the diagrams. The average velocity may be accurately determined by lowering a recording instrument into a well and shooting charges on the surface. A velocity so determined may be used in a large area. As a

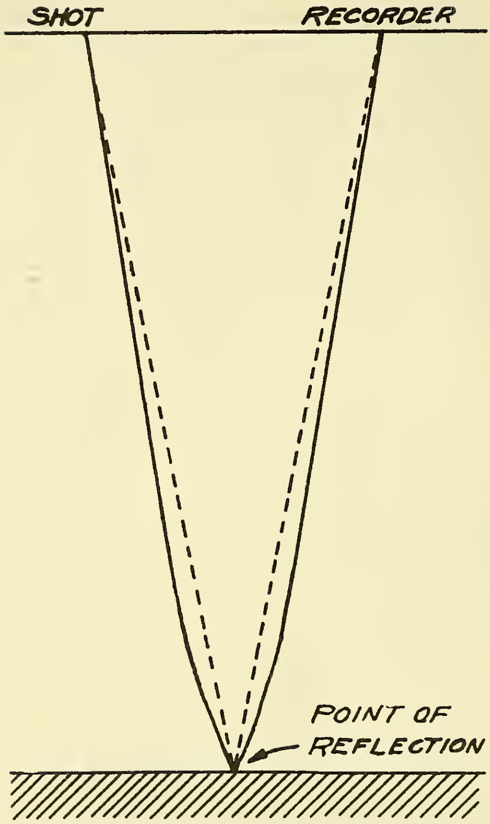

FIG. 6.—Curvature of path due to increasing velocity with depth.

matter of fact, as the accurate determination of relative depths is of greatest importance, it is generally possible to assume an approximate velocity which meets the general requirements of accuracy for absolute depths. Where a well is available, it is possible to check this velocity without lowering a recording instrument into it provided the reflecting strata are not too close together, and the reflections may therefore be definitely identified. In general, in the course of a shooting program, wells are thus checked as they are encountered. An approximate velocity may be determined by arranging several instruments in profile. The foregoing equation of the path of the disturbance is obviously a linear equation between X^2 and T^2. Thus, if X^2 is plotted against T^2, the straight line obtained has a slope equal to V^2. The velocity may thus be deter-

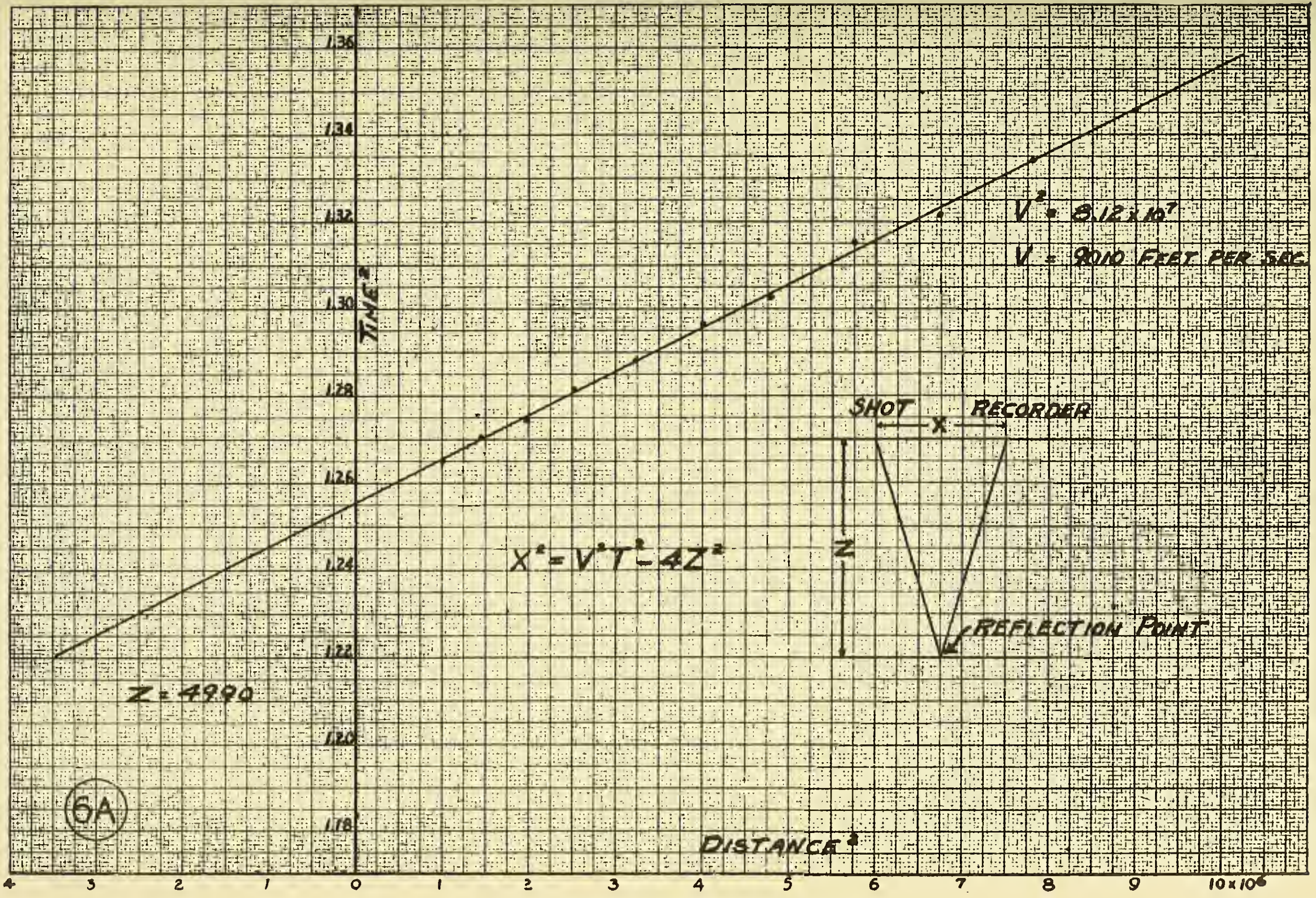

FIG. 6A.—Velocity determination from profile mined as shown in Figure 6A. The method is not very accurate, as second order quantities in time are being used. All of these methods, however, combined with experience, permit a fair estimate of the velocity.

The accuracy of the method requires that allowance be made for the thin weathered layer at the surface, as shown exaggerated in Figure 7. A representative thickness for this weathered layer is approximately 30 feet. As the velocity in it is very low—2,000 feet per second as compared with 8,000 in the unweathered layer below it—it can not be averaged with this layer but must be separated, as it varies considerably in thickness. This is accomplished by shooting a small charge at the shot and recorder points. The thickness of this layer and the time consumed in it are thus determined by the refraction obtained from the un-

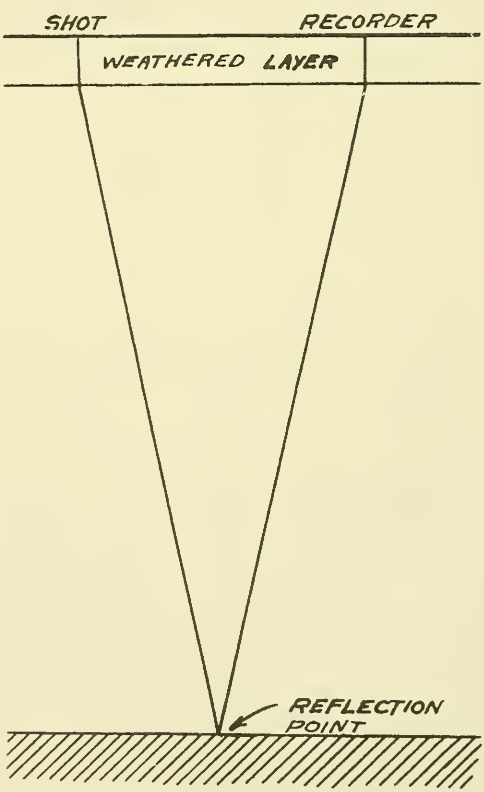

FIG. 7.—Weathered layer (exaggerated).

weathered layer. This is the refraction method applied on a miniature scale. If not accounted for separately in some such manner as this, it may easily introduce an error in depth determination of 100 feet or more. By taking this layer into account and also by determining elevations of shot and recorder points, the general, relative accuracy between two datums obtained by this method is found to be approximately 0.5 per cent. In order to accomplish this, all time measurements must be accurate to 0.001 second. The determination of distance between shot and recorder points need not be very accurate.

In most places several reflecting strata are present in the subsurface, as shown in Figure 8. The recorder responds to each of these reflections as it arrives.

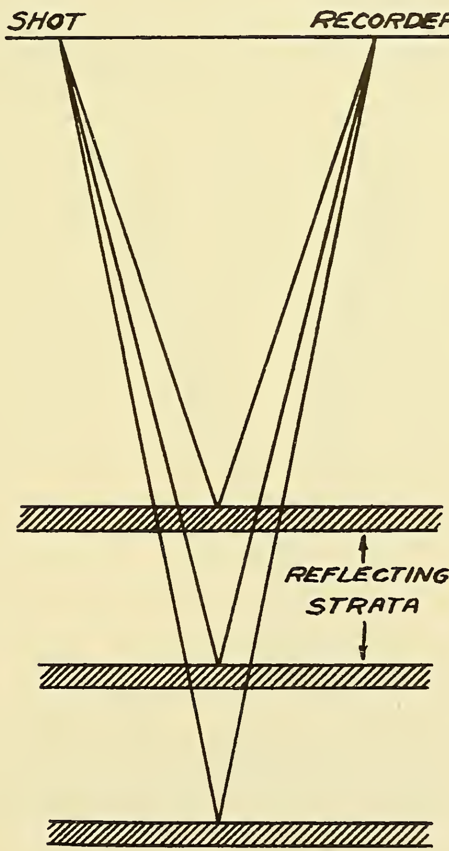

Fig. 8.—Multiple reflections.

The question of identifying reflections naturally arises. The most valuable aid in this identification is the use of several recording instruments, the movements of which are photographed simultaneously on the same strip of paper. These recorders are arranged as shown in Figure 9. The output of each of these recorders is amplified electrically by the desired amount, thus being made to actuate a galvanometer. The motions of the several galvanometers are photographed on the same strip of paper. This strip also has timing lines photographed on it so that the time of arrival of each reflection may be determined. Obviously, in Figure 9 the lengths of the four reflection paths from the shot point to the four recorders are not very different. If the recorders are fairly close to the shot point, the time differences between these paths may not be more than a few thousandths of a second and in general are not more than

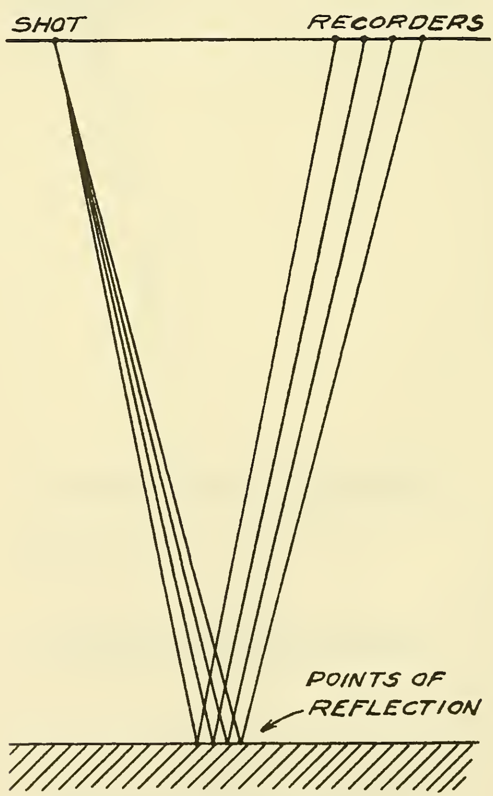

Fig. 9.—Multiple recorders.

0.004 or 0.005. This relation of time of arrival is the principal criterion for the identification of reflections. The differences in arrival time of any other type of wave are considerably greater. For example, the increment of time for the directly transmitted wave is approximately 0.02 second. For refractions from shallow beds to be confused with the reflections would require exorbitantly high velocities in these shallow beds. Beds showing such velocities do not exist.

As a matter of fact there is no great difficulty in identifying reflections on the record. Any difficulties that may present themselves are ordinarily those caused by the presence of too many reflecting strata too close together and the consequent appearance on the record of too many reflections. This, of course, can not be altered without altering the sub-

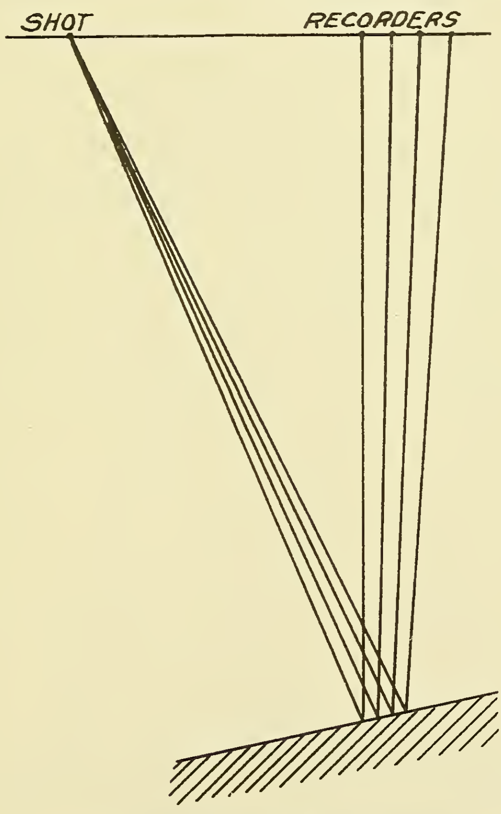

FIG. 10.—Reflections from sloping surface.

surface itself. In general, it is possible to predict the ease or difficulty with which the reflection method may be applied by a study of the geologic column. Well defined hard limestones with shale and sand, especially shale, above them, may be depended on to give good reflections, whereas poorly defined, broken, and siliceous limestones without definite shale breaks may offer difficulties.

Where the reflecting stratum is sloping, as in Figure 10, if the slope is sufficiently great, the reflection may arrive at the recorder farthest from the shot point sooner than at the nearest recorder. Where slopes are sufficiently great, this method may be used to determine slope and, where correlation is difficult, it may be used as an aid to correlation.

Where the presence of a fault is suspected it may be possible to arrange the recorders and shot point so that the fault bears the relation

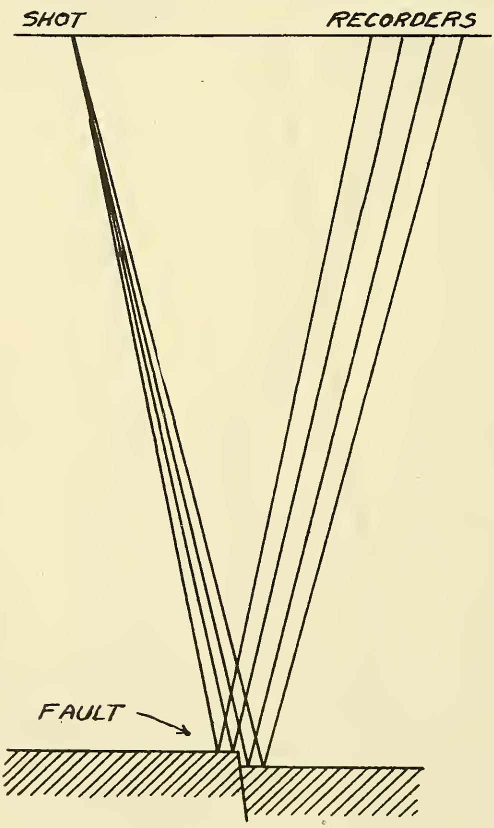

FIG. 11.—Reflections across fault.

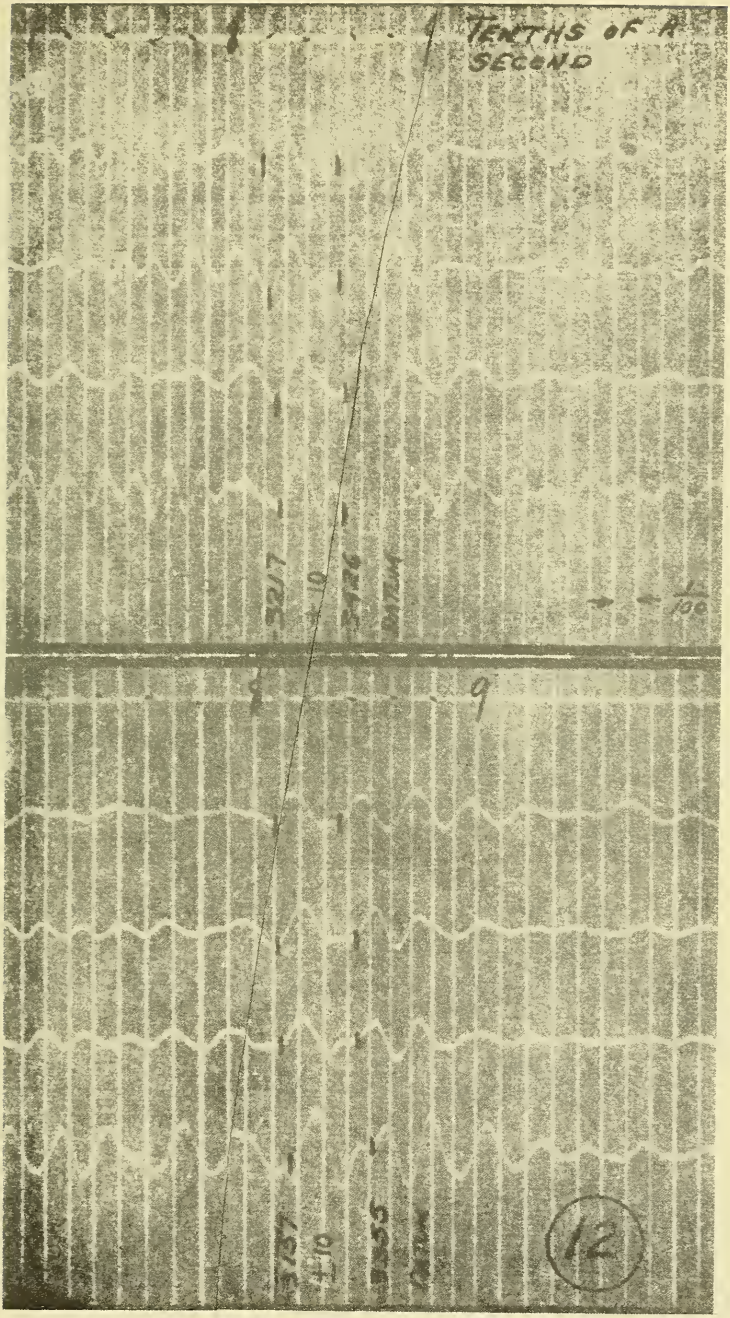

Fig. 12.—Correlation of reflections.

shown in Figure 11. The profile may then be reversed, further to check the fault. As a result of the abrupt change in length of the reflection paths due to the fault, the reflections on the record show a similar abrupt shift.

The average amount of dynamite used to obtain reflections is approximately 3 pounds per shot. In some places the cost of electric blasting caps may actually equal the cost of dynamite, but generally the cost is 50 per cent of the dynamite cost. It may safely be predicted that in the near future the average charge of dynamite for depths of 6,000 feet will not be more than one pound. Even at present the cost of dynamite is only 3 per cent of the total operating costs. As a result of the small charges necessary, damages resulting from the explosion are negligible.

As previously mentioned, the essential problem is that of identifying the reflections from shot point to shot point. In those places where there is but one outstanding reflection, this is a simple matter. Generally, where there is more than one reflecting stratum, the group of reflections has a characteristic appearance, as shown by the group of two reflections obtained from each of two shot points which were located a mile apart (Fig. 12). The vertical lines on these records mark off hundredths of a second and, by interpolating, the arrival of the reflection pulses may be determined to 0.001 second. The four traces are actuated by the four recording instruments already described. Although the frequency indicated in Figure 12 is 50 cycles, this may vary between 15 and 100, depending on the nature of the surface and subsurface rocks and the depth of the reflector.

DISCUSSION

B. B. Weatherby, Tulsa, Oklahoma (written discussion received, June 15, 1931): This paper should be of considerable interest to all geologists, but particularly to those of the Mid-Continent area. As a group, Gulf Coast geologists are better acquainted with the method, mainly because it was forced upon them some years ago after most of the surface had been worked for dome indications.

Very little has been published previously concerning reflection work, which is now becoming better known in the Mid-Continent area. As a matter of fact, until recently, many geophysicists denied the existence of reflections. It was natural that after several notable successes the method should come to the front. Inasmuch as the seismograph is a tool by which the geologist can greatly increase his success, particularly when his usual methods are becoming more and more difficult, it behooves him to learn all he can about the comparatively new reflection method.

This article is a good beginning in this direction. As the energy paths in seismology are very similar to those in optics, it is advisable to read also the chapters on the refraction and reflection of light in any standard physics text book.

Where it can be used, the reflection method has one great advantage over the fan shooting as it was conducted in the Gulf Coast. The older type of coast shooting denoted only the presence or absence of salt domes. Consequently, most of the data obtained were valueless to the geologist. The reflection method, however, gives datum figures on specific beds, and, in conjunction with existing subsurface control, permits the geologist to determine the geology of large areas where, at present, the best that can be done is to interpolate between wells. Consequently, it not only is a great asset in the discovery of petroliferous structures, but it is also increasing geological knowledge to a marked extent.

G. H. WESTBY, Bartlesville, Oklahoma (written discussion received August 20, 1931): During the past year many Mid-Continent geologists were called on to re-examine their determinations of formation datums in wells to permit a closer correlation with seismic reflection data. Errors were found in some determinations or crooked holes were suspected. Most of the seismic-geologic correlations could not be improved. It is probable that some of these men who feel certain of most of their well datums would not agree with Mr. McDermott's statement that generally the relative accuracy between two datums obtained by the seismic method is approximately 0.5 per cent. Despite this disagreement, and fully realizing the possibilities of error in seismic work, most men who have worked with the reflection method feel that it is second only to the use of well samples for the determination of subsurface structure. With this in mind, it seems pertinent to append to Mr. McDermott's excellent paper a short discussion of the present possibilities of error in the reflection method.

Most of the relative errors in the determination of formation datum points by the reflection method are results of the following causes.

I. Use of erroneous average velocity in calculations

II. Errors in the determination of the travel time of reflected or direct impulses

III. Insufficient data accurately to correlate reflections from point to point

I. The use of erroneous average velocity in calculation may be caused by several conditions.

1. Inability to detect and allow for rapid variation in average velocity.

The average velocity to a certain stratum is a function of the stratigraphic section above that stratum. Abrupt lateral variation in character of beds or variation in thickness of limestones affects the velocity. Though variations in velocity from this cause are less than might be expected, it remains as a source of appreciable error.

Seismic Viola datums are in many places affected by changes in the thickness of the Hunton or Mississippian limestone. For example, a change in the thickness of the Hunton limestone from 100 to 300 feet between two seismic stations introduces a relative error of approximately 60 feet in the Viola datums of these points. If the top of the Mississippian or Hunton affords a good reflection, the influence of the thickness of these limestones on the velocity to the Viola can be appreciated and partly eliminated. Where the Mississippian limestone thins over structure, the ordinary effect is to indicate a relief on the

Viola which is less than actual relief. If the changes occur in limestones higher in the section, a more serious problem is ordinarily presented and error occurs which may escape detection.

2. Inability to make corrections locally for gradual changes in average velocity.

From the Seminole area to a point nearly 80 miles north, the average velocity to a depth of 3,500 feet increases approximately 1,000 feet per second. This change is a function of sedimentation and is probably the result of increase in limestone content of the section toward the north. On the assumption that this change is uniform, the use of one velocity chart for calculation of depth points in an area four townships long north and south results in contouring on an inclined plane and the relative error between points on the north and south sides of the area is approximately 150 feet.

3. Inability to construct a velocity depth curve which will hold for varying depths to a certain formation on anticlines and in synclines.

This inadequate knowledge of the average velocity gradient presents only a small error unless the relief is great.

4. The use of a varying shot point to detector distance.

The average velocity to a certain depth is a function also of the distance between shot point and detector. This is a result of change of path by refraction. It can be eliminated by using a constant or nearly constant shooting distance.

5. The use of a constant average velocity.

Where the actual structural relief is great, considerable error may be introduced. If a velocity is used which is less than actual, a diminishing in structural relief will result. However, under some conditions, a constant average velocity affords better results than the use of an increase of average velocity with depth.

6. The assumption of a 2,000-foot velocity for the weathered zone and an 8,000-foot velocity for the unweathered.

These are approximations which may be locally in error.

To evaluate properly the foregoing possibilities of error, it must be noticed that only possibilities 1 and 6 might cause a false determination of the high point of structure. The other possibilities would distort the shape of a structure and give a false impression of the magnitude of the regional structure.

II. Error in the determination of the travel time of reflected or direct impulses may be caused by the following.

	Seconds
1. Inability to read records more closely than—	
a. Error in reading weathering record time break	±0.001
b. Error in reading weathering shot arrival time	±0.002
c. Error in reading time break of reflection record	±0.001
d. Error in selecting and reading time of reflected impulse	±0.001
Total	±0.005

To this figure may be added ±0.003 which from experience is the probable error in the weathering correction method. However, as the probable error of the result is equal to the square root of the sum of the squares of the individual errors, the probable error from these causes is ±.004 sec. or ± 25′ at 5,000 feet.

2. Inability to determine the same point on reflected impulse from record to record.

It is to be noticed that the selection of arrival time of the reflections as shown in figure 12 is not the moment of first arrival of the reflected impulse, but a point somewhat later. It is difficult to select this same point on the reflected impulse in every record. An error of a full period, or about 120 feet in depth, is most frequently made, but ordinarily can be detected in contouring. An error of a half period, or about 60 feet, may pass unnoticed.

3. Variation from true time of point selected for arrival of a reflected impulse when interference with the preceding impulse occurs.

In the lower illustration of Figure 12, the second impulse on each trace interferes with the preceding impulse and the reflection is selected at the interference pattern. The position of this point may vary depending on the ratio of the amplitude of the preceding impulse to the incoming impulse. Theoretically this point may vary 0.005 second from the true time. This is partly an instrumental characteristic and may change with different seismic equipment.

4. Discrepancy between time signal of explosion and actual explosion of dynamite.

This may be the result of defective blasting caps or blasting equipment and can be easily eliminated. However, such errors may exist for some time before being detected.

5. Errors in time-measuring device of seismic equipment.

Because of the present excellence of most of the timing instruments this error is now negligible. Formerly it was a serious and abstruse error.

In summary, each of the foregoing possibilities of error might give a false position for the crest of a structure.

III. Insufficient data accurately to correlate reflections from point to point.

A reflected wave shows on the seismic record as an impulse. In appearance on the record there is nothing to distinguish an impulse received from the Checkerboard limestone from an impulse received from the Dewey limestone. If two reflecting horizons occur close together, as shown in Figure 12, this "character" of impulse is retained as long as the upper limestone and the intervening shale remain constant in thickness. If either of these changes appreciably the "character" changes. "Character" correlation by itself, therefore, may be misleading.

Reflections are ordinarily obtained from limestones in excess of 30 feet which are overlain by shale. It is customary to obtain several reflections on each record and to correlate these groups of reflections from one position to another by means of the intervals. It happens, for no reason known at present, that a reflection which is strong on one record may fail to appear on the next. The correlation of these seismic logs is similar to the correlation of well logs on which some lime tops only were indicated with driller's log accuracy. It is a difficult task where intervals vary within short distances. Where several reflections occur close together, errors in correlations may be made if all reflections do not occur on every record. It is impossible to indicate what error might be possible from mistakes in correlation, but it is necessary in evaluating a seismic structure to realize such possibilities.

The percentage of relative error is here understood as the ratio of the absolute error in determining the difference in datum between two points to the mean depth of the two points. This is obviously a function also of the distance between the two points, since lateral velocity variation is a function of distance. With this qualification, it is true that, in a small area with many wells where seismic depth points are close together, the relative error may be only 0.5 per cent. However, with the usual mile control in an area with few wells, the relative error is probably of the order of 1 to 1.5 per cent for the better points. It is of indeterminate amount where reflections are poor and correlations questionable.

Despite the foregoing possibilities of error, it must be reiterated that most men who have followed the results of seismic work during the past year are convinced of its great value. There is reason to expect improvements in equipment and technique which will increase the present efficiency.

In order to obtain knowledge of the velocity variations throughout the Mid-Continent region so that correct geologic seismic correlation can be made, it is suggested that seismic data at drilled wells be exchanged by companies engaged in reflection shooting. This type of coöperation would be similar to an exchange of well samples.

PAUL WEAVER, Houston, Texas (written discussion received, August 21, 1931): Mr. McDermott calls attention to the importance of determination of the thickness of the "weathered layer" in accurate reflection work. In many places the thickness of this layer is greater than that which the geologist calls the "zone of weathering," and pending a more thorough analysis of the problem of the low velocity of seismic waves near the surface, it seems advisable to use the expression suggested by McCollum, "surface correction zone," because for it a "surface correction" is applied to the travel time. This zone has a low velocity, even in areas where diluvium, alluvium, et cetera, are thin or absent, such velocity extending much deeper than the deepest level of the ground water. The velocity is much lower than that of the beds immediately below, even when the geologist fails to detect a change.

It is suggested that, as overlying beds are removed by erosion, the lower beds thus released from pressure may manifest an "elastic rebound," and cause a change in density, which manifests itself in a greater porosity near the surface than in the same lower beds where under greater cover.

I hope that seismologists will measure the thickness of this "surface correction zone," and compare the number of feet thus obtained with the number determined for the depth to lowest ground water, that is, the zone of weathering which the geologists consider.

BELLE ISLE TORSION-BALANCE SURVEY, ST. MARY PARISH, LOUISIANA[1]

DONALD C. BARTON[2]
Houston, Texas

ABSTRACT

The geological prediction was made that the Belle Isle salt dome might be much larger than supposed. The torsion balance was used to corroborate that prediction. From the data of the torsion-balance survey, calculations were made of the probable depth, thickness, and edge of the cap rock. Seven sulphur tests were drilled on locations made on the basis of the calculated conformation of the cap. The predictions in regard to depth were shown to be 73 per cent correct, and in regard to the thickness of the cap, 58 per cent correct. The causes of the errors were probably the unfavorable marshy terrane, the assumption of a slightly too small relative density for the cap rock, and the irregularity of the conformation of the dome.

INTRODUCTION

This paper records an actual ordinary torsion-balance survey and the degree of success which was obtained in an attempt to base quantitative predictions on the torsion-balance data. The choice of this survey for publication was not made on the basis of exact verification by subsequent drilling. The accuracy of the predictions may seem somewhat disappointingly poor, if the actual cap rock which was encountered in the subsequent test well is compared with the predicted cap rock (Fig. 3). The survey, however, accomplished successfully the qualitative and semiquantitative tasks assigned to it. This survey is more or less representative of the degree of success attained by the average successful geophysical surveys, although it was made under adverse terrane conditions; most of the stations were on unstable marsh, and a lake precluded stations in a wide central area.

The cap rock as a whole is thinner, deeper, and steeper than was predicted and the inner part of the cap was very much thinner than was shown in the calculated west-east and southwest-northeast sections. These errors are in part caused by two factors: the absence of stations in

[1]Read before the Association at the San Antonio meeting, March 21, 1931. Manuscript received, March 20, 1931. Data released for publication through the courtesy of the Freeport Sulphur Company and Belle Oil Corporation.

[2]Consulting geologist and geophysicist.

South Pond, and the fact that, mathematically, a limited series of bodies may produce the same gradient profile, particularly if the probable error of observation at each station is moderately large and if, as at Belle Isle, the available gradient profile does not sufficiently cover the anomaly.

Location.—Belle Isle salt dome is in T. 17-18 S., R. 10 E., in St. Mary Parish, Louisiana, 15 miles south-southwest of Morgan City. Belle Isle properly is a triangular island of dry land which rises out of the flat low marshes to a maximum elevation of 80 feet. It lies in the sea marsh at the shore of Atchafalaya Bay. South Pond (Belle Isle Lake) marks the center of the dome.

History.—The exploration of Belle Isle began in 1896 when Captain Lucas drilled his well on the west edge of the island. The presence of the salt was disclosed by Lucas' second well, which was drilled in 1897 in the northern part of the island. After a series of wells were drilled to outline the salt, two unsuccessful attempts were made to sink shafts into the salt and to mine the salt.

Several tests were drilled for oil in the period 1906-1916.

Six sulphur tests were drilled during 1916-17 under the direction of Captain Lucas. In 1921, the Union Sulphur Company drilled more sulphur tests on the dome and oil tests on the north and east flank.

The top of the cap-salt core of a salt dome had been shown by the drilling prior to 1929 to be coincident with the island of Belle Isle (Fig. 1). That drilling had been confined to the area of the island and to the edge of the marsh on the east side and north point of the island. The edge of the dome had been delimited by deep flank wells only on the east and north. The physiography indicated that the northwest edge of the island probably marked the northwest edge of the dome. The position of the south edge of the dome was indefinite. There is a south dip on the top of the salt from the center of the island to Syndicate well No. 5 at the head of South Pond and the assumption was made by many geologists that the south edge of the dome lay in the pond, 1,000-1,500 feet south of that well. The belief was held by some geologists that South Pond reflected the presence of the dome and the south edge of the dome probably lay south of the pond. The salt domes of the Five Islands have a common individuality that is not shared with the other domes of the Gulf Coast. One of the features of that common individuality is a diameter of approximately 2 miles. South Pond is an exceptional lake in the marshes of St. Mary Parish and its position on the indefinitely delimited side of the Belle Isle dome seemed definitely suggestive.

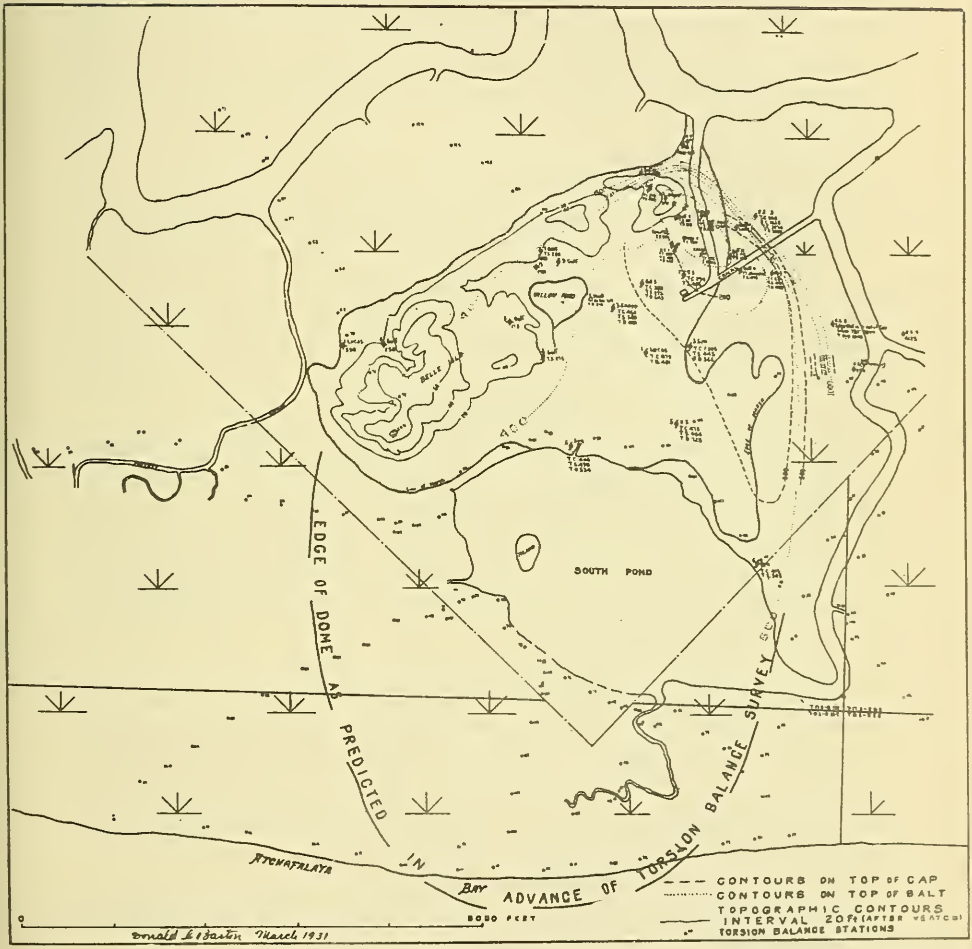

FIG. 1.—Known and predicted conformation of Belle Isle salt dome in advance of torsion-balance survey.

In a report made to the Freeport Sulphur Company in 1928, the writer stated that the dome probably extended under the pond, as far as the shore of Atchafalaya Bay and possibly slightly out under the bay, and that more than half of the probable cap-rock area had not been explored. The company made a torsion-balance survey of the dome and in 1929-30 drilled seven cap-rock tests in the southwest quadrant of the dome. The locations for the tests were made on the basis of the torsion-balance survey.

The results of the Freeport Sulphur Company's tests, as well as of the Union Sulphur Company's tests and of earlier cap-rock tests, condemn the dome as a favorable sulphur prospect, although there probably remains a narrow band of untested cap-rock on the northwest flank of the dome. The top of the dome has been condemned for cap and supercap oil production. The flank sands are practically untested. Good showings of 37° Bé. gravity oil have been found in the salt—in Knapp No. 1 at 125 feet and from 1,500 to 3,171 feet—and suggest that oil sands may be present in considerable depth.

Geologic situation.—The Belle Isle salt dome rises through an enormously thick stratigraphic section of late Tertiary sediments. Paleontological data are not available for the few moderately deep wells at Belle Isle, but from more recent deep drilling in this general area, it seems probable that the Pleistocene and Pliocene beds together have a thickness of more than 6,000 feet at Belle Isle. The thickness of the Miocene and older formations is not known.

TORSION-BALANCE SURVEY

Tasks.—The tasks which were attempted by use of the torsion balance in the survey of the dome were: (1) to verify or disprove the predictions of the extension of the dome to the shore of Atchafalaya Bay; (2) to determine whether a sufficiently thick cap rock was present at a sufficiently shallow depth and on a sufficiently large area to make the unexplored area a favorable sulphur prospect, and (3) to determine the edge of the cap so that locations could be made for wells for the purpose of exploring the cap near its edge.

Survey.—The field survey was made under the direction of the writer by C. I. McGlothlin, torsion-balance observer of the Freeport Sulphur Company, during the winter of 1928-29. Two large Süss (visual) torsion balances were used. Observations in general were made only during the day time; the observation at each station was continued until five successive good readings had been taken.

Most of the terrane was poor. Except for a few stations on the dry land of the island, the stations were on the sea marsh, which in general is covered with coarse, hummocky grass. The marsh is so soft that an 11-foot length of 2-inch pipe can be pushed into the marsh, level with the surface, by two men and pulled up by three men. Each foot of the instrument tripod was placed on the top of such an 11-foot length pushed down flush with the surface of the marsh. The instrument house was placed directly on the marsh. At times the foot of the instrument was under water. It was impracticable to clear off and level the station site; and on account of the grass hummocks and the soft character of the muddy ground between hummocks, the conclusion was reached that less error would be introduced by neglecting the terrane correction of the marsh stations than by taking levels and calculating the correction.

The instruments and shelter houses were carried by hand. On account of the difficulty of movement across the marsh and the many readings which had to be taken at each station, only one station per day per instrument could be obtained in the marsh.

A long east-west line of stations on the beach, and eight radial lines of stations were occupied. An average station interval of 400 feet was used. Although stations in South Pond would have been desirable, no attempt was made to occupy them because of the difficulties in occupying stations in the water.

Results of survey.—The results of the field survey are shown in Figure 2. The results of the preliminary line of stations which were occupied along the beach showed that the dome extends south of South Pond but does not reach the beach, that probably considerable cap rock might be expected on the south end of the dome, and that it would be advantageous for the Freeport Sulphur Company to make a detailed torsion-balance survey of the dome.

The results of the west, southwest, south, and southeast radial lines of stations showed that the Belle Isle salt dome is elliptical in plan, that its major axis has a north-northeast strike and a length of approximately 2 miles, and that much cap rock might be expected in a broad area south, southwest, and west of South Pond, probably at moderate depth. The approximate position of the edge of the dome is delineated in the gradient arrow map (Fig. 2) by the zone of maximum gradient which from tangency to the east side of the island swings clockwise across the outlet to South Pond, then southwest and west of South Pond to tangency to the northwest edge of the island.

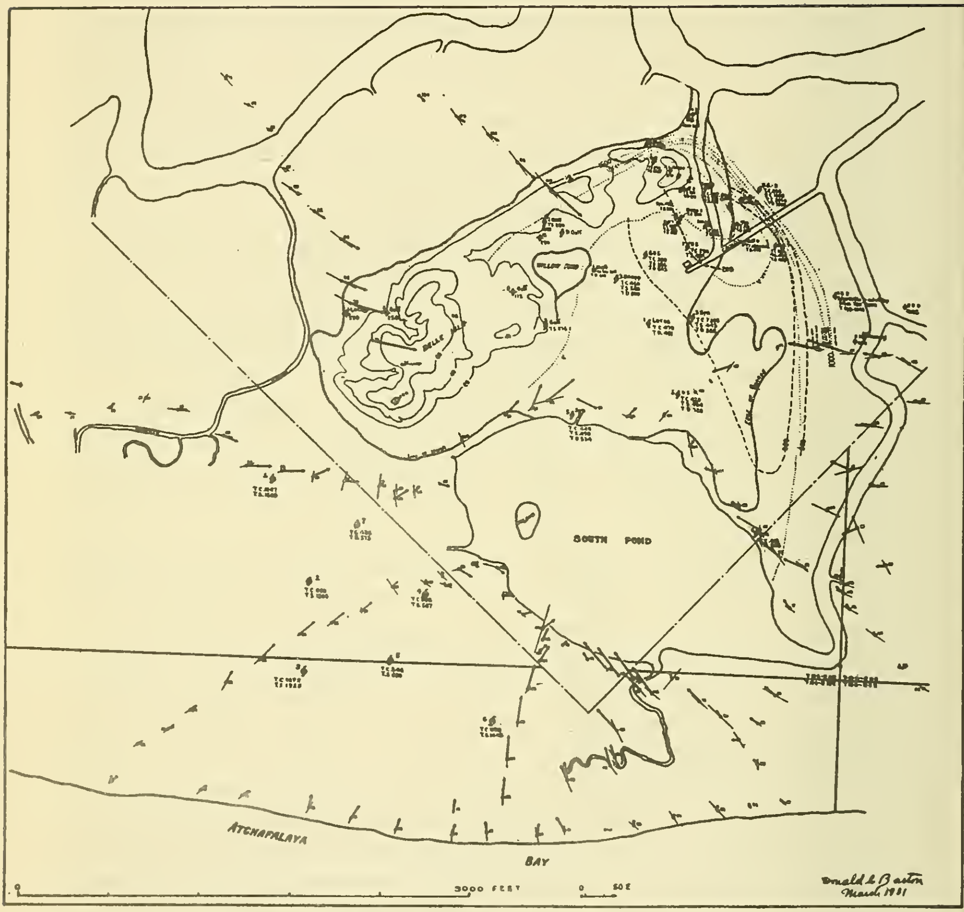

FIG. 2.—Gradient arrow map of Belle Isle salt dome.

The results of the field survey with the torsion balance, therefore, qualitatively verified the prediction of a large unexplored area of cap rock. The next task was to make a quantitative calculation of the amount and depth of the cap rock and the position of its edge.

Cap-rock calculations.—The calculation of the amount, depth, and edge of the cap rock was made by a series of trial and error calculations, profile by profile. A trial cross section of the dome along the selected profile was sketched; its gradient profile was calculated and then compared with the observed gradient profile; the trial cross section was remolded or replaced by a new sketch cross section; the gradient profile was calculated and compared with the observed gradient profile; the cycle was continued until a cross section was found whose calculated gradient profile fitted the observed gradient profile more closely than did the gradient profile of any of the other trial cross sections.

The calculations in connection with the torsion-balance survey at Belle Isle were made graphically by means of one of the writer's graphic charts (8).[1] The principle of these charts is that space is divided into rectangular prisms at right angles to the vertical plane of the section; the length of the prisms is calculated so as to vary by some simple law to approximate the dimensions of the structure at right angles to the plane of the section; the cross section of each prism is so calculated that each prism produces a gradient of 1E at the origin; if the cross section of a structure is sketched on transparent paper and superimposed on the chart, it is necessary only to count squares and multiply by the specific gravity in order to obtain the gradient at the origin. The actual calculations in connection with the Belle Isle survey were made by the writer's assistant, Mrs. Elizabeth B. Summers.

The specific gravity relations which were used were those which seemed geologically the most probable on the basis of the laboratory determinations of the specific gravity of cores from the cap rock at Bryan Heights, Hoskins Mound, and other domes and from cores of sediments from depths less than 3,500 feet and on the basis of experience in previous calculations in connection with other domes. Those assumed density relations are given in Table I.

For the purposes of checking, several other sets of assumptions were tried. Two of the more important sets of those assumptions are given in Table II.

[1]Numbers in parenthesis refer to list of references at end of article.

TABLE I

GEOLOGICALLY MOST PROBABLE SPECIFIC GRAVITY RELATIONS AT BELLE ISLE

Depth in Feet	*Specific Gravity*			*Relative Density*	
	Sediments	*Cap*	*Salt*	*Cap*	*Salt*
0- 300	1.90		2.20		+0.30
300- 700	2.00	2.60	2.20	+0.6	+0.20
700-1,200	2.10	2.70	2.20	+0.6	+0.10
1,200-1,600	2.15	2.75	2.20	+0.6	+0.05
1,600-4,400	2.20	2.80	2.20	+0.6	+0.00
4,400-8,000	2.30		2.20		−0.10

TABLE II

ALTERNATIVE ASSUMPTIONS IN REGARD TO THE SPECIFIC GRAVITY RELATIONS AT BELLE ISLE

Depth in Feet	*Specific Gravity*						*Relative Density*					
	Sediments		*Cap*		*Salt*		*Cap*			*Salt*		
	A	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*Geol. most probable*	*A*	*B*	*Geol. most probable*
300- 600	2.00	1.95	2.60	2.55	2.30	2.25	+0.6	+0.6	+0.6	+0.30	+0.30	+0.20
600-1,000	2.10	2.05	2.70	2.65	2.33	2.30	+0.6	+0.6	+0.6	+0.23	+0.25	+0.10
1,000-1,600	2.15	2.10	2.75	2.70	2.35	2.30	+0.6	+0.6		+0.20	+0.20	+0.06
1,600-3,000	2.20	2.15	2.80	2.75	2.30	2.25	+0.6	+0.6		+0.10	+0.10	0.00
3,000	2.25	2.20			2.30	2.25				+0.05	+0.05	−0.03

The assumptions (*A* and *B*) of Table II gave a very much better agreement of the calculated with the observed gradient profiles than did the geologically more probable assumptions of Table I. The assumptions of Table II evolved more or less mathematically in the process of the calculations. Geological analyses of these assumptions showed that geologically they were possible but somewhat improbable. Assumption *A* makes the specific gravity of the sediments slightly lower than seems probable and necessitates a content of approximately 15 per cent of anhydrite in the salt. Assumption *B* uses the geologically most probable densities for the sediments but uses a density for the salt which connotes an average anhydrite content of approximately 24 per cent. But the average anhydrite content of the salt in the near-by Weeks and Avery Island salt mines is approximately 1 per cent, and in the two analyses of Belle Isle salt which are available, the percentage of *NaCl* in the one analysis is 92.8 and in the other, 96.4. The density used for the salt in Table I connotes an anhydrite content of 7 per cent. An average anhydrite content of considerably more than 7 per cent in the salt theoretically is not impossible, but has not been encountered in the shaft or the wells into the shale. The assumptions of Table II, therefore, were discarded on account of their geologic improbability and the geologically most probable assumptions of Table I were used, although mathematically they did not give as good results as did those discarded assumptions.

The known data in regard to the depth of the top of the cap and of the salt were utilized and before the calculations were made for profiles in the unexplored areas, sections were calculated through the areas in which the conformation of the salt and cap were best known.

The calculated structural sections for the profiles in the southwest quadrant are shown in Figure 3. Each structural section is the final choice out of a long series of sections calculated for that particular profile. If the structural sections which were calculated with the discarded density assumptions of Table II had been retained, the predictions regarding the position of the edge of the cap and depth to the top of the cap would have been the same, but the thickness of the cap would have been predicted as approximately 100 feet.

The calculated structure contours on the top of the cap, or, in the absence of the cap, on the salt, are shown in Figure 4. The structure contours were made to conform to the known data from the drilling in the northeast quadrant.

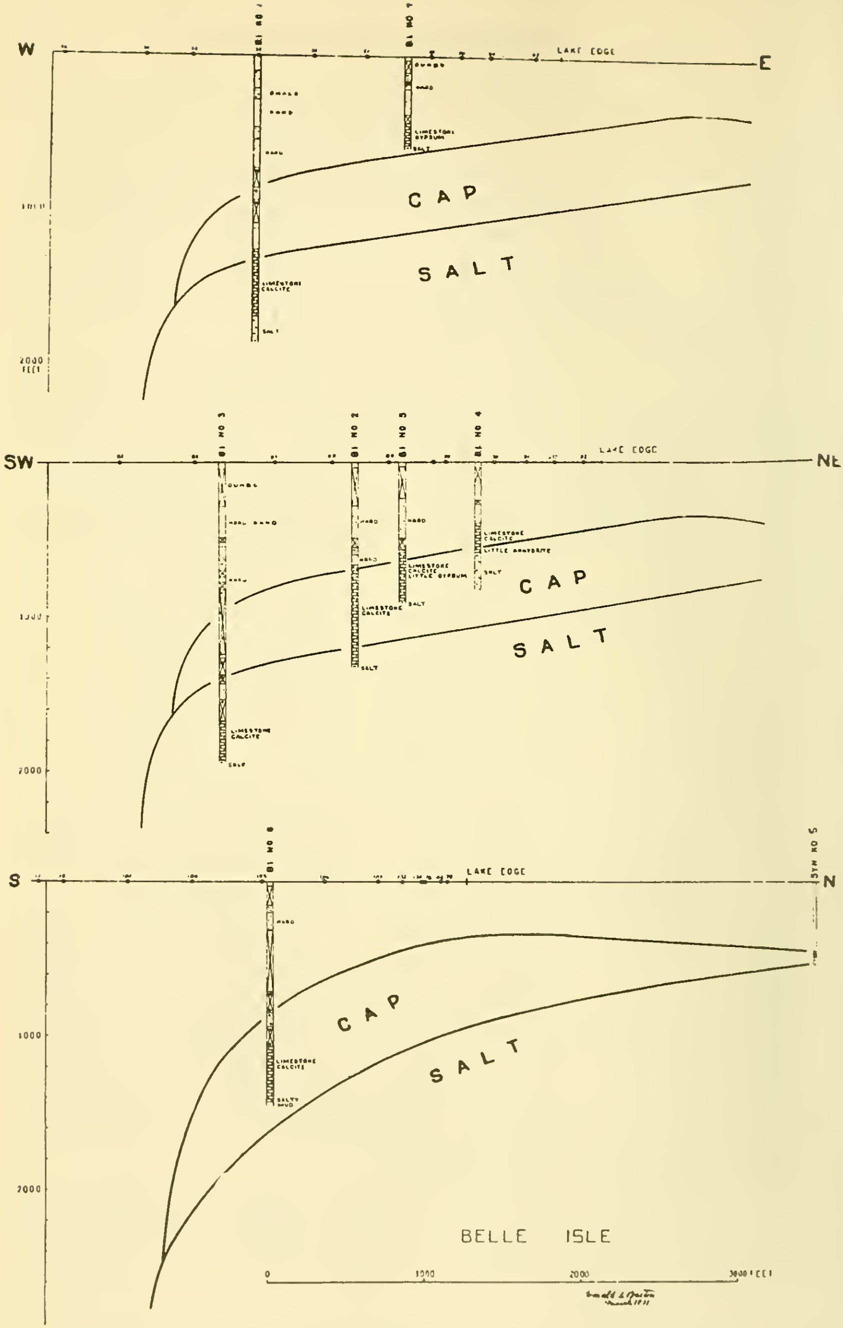

Fig. 3.—Calculated cross sections of southwest quadrant of Belle Isle salt dome and graphic logs of subsequent sulphur tests.

FIG. 4.—Structure contours on top of cap as predicted from calculations versus structure contours determined by results of drilling.

The following additional verbal predictions were made.

Over South Pond the top of the cap seems to lie at a level of about 400 feet below the surface. There seems to be a moderate amount of cap present, of the order of 100 to 300 feet in thickness, but on account of the absence of stations in the lake, our estimates of the thickness are only very crude.

In the southwest quadrant, west and southwest of South Pond, a moderately thick cap rock seems to slope gently west and southwestward. The thickness of the cap, according to our estimates, is of the order of 500 feet. The actual thickness may be less or greater. There are slightly greater possibilities for the actual thickness to be somewhat less than for it to be greater.....

The odds that the cap is appreciably thinner than on the sections but still of very considerable thickness are 35 out of 100.

The accuracy of the observations on which we had to base our calculations is fair, but is not as good as it was at Hoskins Mound or Bryan Heights. Practically all the stations were taken on the marsh where it is impossible to get good station sites or to make first-class observations. The terrane is so soft and unstable that it is practically impossible to get readings which will check as well as we would like..... Slight indefiniteness in our knowledge of the actual gradient at each station throws a corresponding indefiniteness into the results of our calculations.

The accuracy of the calculations is less for the relations of the salt than for the cap and the accuracy for either decreases rapidly with increase of depth below 1,500 feet.

VERIFICATION

Seven wells were drilled by the Freeport Sulphur Company in the previously wholly unexplored area of the southwest quadrant. The location of each test was based on the calculated structure contours of Figure 4 and the calculated cross sections of Figure 3.

The verification of the calculated predictions in regard to the cap rock qualitatively was good, although quantitatively there was an average error of 27 per cent in the predicted depth and of 42 per cent in the predicted thickness of the cap (Table III). The partial graphic well logs plotted in the profiles of Figure 3 and the dashed structure contours in the southwest quadrant of the dome in Figure 4 show graphically the degree of verification.

The errors, in the main, are two: that the actual thickness of the cap is only two-thirds the predicted thickness and that the actual depths to the top of the cap were greater than the predicted depths on two profiles and less on the intervening profile.

The calculated predictions, in spite of their error of 27-40 per cent, accomplished the purposes for which they were made. The cap was present in substantially the amount and position which were predicted and the predictions were sufficiently accurate to permit tests to be located in advance of exploration.

TABLE III

VERIFICATION OF PREDICTED DEPTH AND THICKNESS OF THE CAP ROCK
(Wells of Freeport Sulphur Company)

Well Number	*Depth to Top of Cap in Feet*			*Thickness of Cap in Feet*			*Total Depth*	*Character of Cap Rock*
	Predicted	*Actual*	*Error*	*Predicted*	*Actual*	*Error*		
1	850	1,247	−397	470	442	+ 28	1,853	Hard lime rock with calcite veins, mostly impervious
2	690	660	+ 30	510	644	−134	1,321	Same as foregoing
3	1,180	1,672	−492	410	253	+157	1,940	Same as foregoing
4	550	393	+157	470	174	+296	828	Limestone and some gypsum and anhydrite
5	650	546	+104	500	327	+173	880	Same as foregoing
6	860	1,058	−198	740	387	+353	1,445	Broken lime rock and calcite; voids filled with sandy shale
7	620	426	+194	730	149	+581	580	Broken lime rock, calcite and gypsum; voids filled with sandy shale

Percentage errors in predicted depth to the top of the cap:
46, 4, 41, 28, 16, 23, 31 — Numerical mean, 27 per cent

Percentage error in predicted thickness of the cap:
6, 26, 38, 63, 34, 47, 79 — Numerical mean, 42 per cent

CAUSES OF ERRORS

The errors in the calculated predictions probably arise from a series of causes.

The unfavorable conditions under which the observations were made are sufficient to account for considerable error in the magnitude of the observed gradient values. Because of the marsh terrane, it was impracticable to maintain our normally rigorous limits of error in the readings. The impracticability of taking the terrane levels and of making the terrane corrections probably led to the introduction of some error into the observations. An error of even 1 E with the same sign at three consecutive stations critically placed at the edge of the dome easily might cause considerable errors in the calculated position of the cap and in the calculated depth to the top of the cap on the flank. The error shown by well No. 1 on the west radius could be caused by an error of +2 E in the observed values of the gradient in the two stations above the predicted edge of the cap on that west radius. Errors in the observations at those stations are especially probable as the stations are near a bayou.

The error in the predicted thickness of the cap probably indicates that the assumed relative density is too small. The relation holds very roughly that if the relative density times the reciprocal of the area of cross section of a very long horizontal body is a constant, the gradient is constant. If, in the present calculations of the Belle Isle survey, too small a relative density were used for the cap rock, the predicted thickness of the cap would be too great. The cap proved on the whole to be tight and to have little of the porosity commonly present in the lime rock of the cap. The assumed specific gravity of the sediments also may have been slightly too high and actual specific gravity of the sediments may be 1.9 to a depth of 700 feet and 2.0 from 700 to 1,200 feet instead of 1.9 from the surface to 300 feet, 2.00 from 300 to 700 feet, and 2.1 from 700 to 1,200 feet, and the relative density might be 0.8 instead of 0.6. A difference between an actual relative density of 0.8 and the assumed relative density of 0.6 would be sufficient, substantially, to have caused the error in the prediction of the thickness of the cap.

Irregularities of mass at right angles to the plane of the section can not be detected by this type of calculation. The gradient profile is essentially the same, whether the mass is concentrated in the vertical plane of the section, or a slightly greater mass is uniformly and symmetrically disposed at right angles, or a certain mass is symmetrically concentrated at two points on opposite sides of the plane of the section. The upper part of the Belle Isle dome does not have a simple geometric form; several centers of uplift seem to be present; two are reflected in the topography by the north hill and by the west hill. The drilling reveals another on the southwest radius, and the torsion-balance data suggest another on the east-southeast radius. The flank of the cap is scalloped correspondingly, convex around the centers of uplift and concave between. The scalloping is reflected in the topography of the northwest flank of the island. The west radius lies in the concavity between the respective areas of the northwest and southwest centers of uplift. In the effect on the gradient, the excess of mass in the convexity of the area of these centers of uplift partly compensates for the deficiency of mass in the intervening concavity. The calculations, therefore, predict too much cap rock in the area of the concavity. Conversely, the calculations predict too little cap rock for a line of profile down a convex zone.

CONCLUSION

The Belle Isle torsion-balance survey affords an example of an average successful application of the torsion-balance method to the quantitative problem of the determination of the dimensions of the upper part of a shallow salt dome. The quantitative predictions were in error by 27-40 per cent, but, qualitatively, the work undertaken was accomplished completely by the balance. In other surveys with the torsion balance, equally good and better results have been obtained. In a survey to determine whether or not at Bryan Heights cap rock might be present on the flanks, the predicted depths proved to be in error by +200-300 feet at depths of 1,500 feet; however, in our report, we had estimated the probable accuracy of our predictions at approximately 40 per cent, but, rather to our surprise, the subsequent drilling proved the accuracy of the predictions to be nearly 60 per cent and cap-rock masses which we somewhat hesitatingly predicted proved actually to be present. In the Hoskins Mound survey, the errors in the predicted depth at four points were proved to be respectively 7, 5, 2, and 0.7 per cent (9).

The accuracy of quantitative calculations of this type decreases with the depth to the top of the anomalies, provided that the density situation remains constant. Geologically, the density may change with depth, and through the complication of the density situation, the accuracy of quantitative calculations through a certain vertical zone may increase with depth. The cap rock at all depths in the Gulf Coast is heavier than the surrounding sediments, but, compared with the salt, the sediments are slightly lighter or of almost the same density at the surface and become progressively denser with depth and at great depth are heavier than the salt. At certain depths, therefore, there is a neutral zone in which the salt can not be detected by gravitational measurements. Below those depths the presence of the salt can be detected by gravitational measurements and the accuracy of quantitative calculations in regard to dimensions of the salt through a certain vertical zone increases with depth, although on account of the law of decrease of accuracy with depth, the absolute accuracy may be very low. Practically, the dimensions of the cap rock can be calculated with fair accuracy for depths less than 3,000 feet and for any domes in the Gulf Coast area; but, on account of the enormously greater thickness of light sediments near the coast and their low specific gravity, predictions in regard to the dimensions of the salt of the domes near the coast are nearly valueless. Surveys across the inland domes of the coastal group suggest that calculated predictions in regard to the salt may be of some use.

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SOME RESULTS OF MAGNETOMETER SURVEYS IN CALIFORNIA[1]

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ABSTRACT

The writer describes magnetic conditions associated with some typical structural features in California,—an anticline, a syncline, an outstanding magnetic feature in the San Joaquin Valley, a buried fault near Mount Diablo, a part of Ventura County, and Kettleman Hills.

INTRODUCTION

Magnetometer work was begun in California by the Standard Oil Company of California on January 1, 1927. Acknowledgment is here made to officials of the company for permission to publish, to H. N. Herrick of the Research and Development Department of the company for his assistance and criticism in the preparation of this paper, and to all members of the geophysical field parties in California.

General conditions for use of magnetic geophysical methods in California are good, as there is marked variation in the magnetic susceptibility of the sedimentary rocks of economic interest. In the Tertiary rocks, the magnetic susceptibility varies from 14×10^{-6} in the Saugus of the Upper Pliocene to $4{,}120 \times 10^{-6}$ in the vivianitic sandstone of the McKittrick group of the Pliocene. This variation is sufficient to give a definite magnetic contrast at several horizons. "Magnetic marker beds," such as this vivianitic sandstone, beds of volcanic tuff and interbedded basaltic flows, extending thoughout considerable areas, have been found, which are sufficiently thick and magnetic to cause anomalies of several hundred gammas at surface exposures and recognizable indications under deep cover.

The Cretaceous rocks, especially the Knoxville, on the average, are somewhat more uniformly and strongly magnetic than the Tertiary. The Jurassic (Franciscan) is much more magnetic than either the Ter-

[1]Read before the Association at the San Antonio meeting, March 21, 1931. Manuscript received, February 19, 1931. Published by permission of the Standard Oil Company of California.

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tiary or Cretaceous and contains irregular masses of basic volcanic rocks, in many places altered to serpentine, which have a magnetic susceptibility as great as $7,000 \times 10^{-6}$, so large as to obscure results on younger beds in the vicinity.

Of the pre-Jurassic rocks in California, the granites, both of the Sierra Nevada and the Coast Ranges, generally have a susceptibility less than 200×10^{-6}, but in many places they contain large bodies of darker rock of gabbroid appearance that are intensely magnetic—sufficiently so that a small hand specimen deflects a compass needle. Many of the metamorphic rocks of the Sierra Nevadas also are extremely and irregularly magnetic. This must be kept constantly in mind in attempting magnetometer work in the San Joaquin Valley, particularly.

The chief practical application of magnetic methods in California has been in tracing known structural features, such as faults and "magnetic marker beds" which present eroded edges on the flanks of folds, from places where they can be seen into regions covered by alluvium. It is not safe in the present stage of the art to attempt to interpret magnetic data without some support either from surface geology or from well logs. It is necessary always to keep in mind that the observed results are affected by all the material beneath, so that many anomalies caused by the formations of principal interest are obscured by unrelated conditions below them in rocks of greater magnetic permeability. For example, it would be very poor engineering to attempt to interpret in terms of structure either a magnetic or a gravity survey of an area known to be underlain by Franciscan rocks at a depth of less than 5,000 feet, because very magnetic and dense bodies of basic igneous rock are probably distributed irregularly within.

Most work consists of measurements and mapping of the vertical component of the earth's magnetic field. For recognizing faults not known by surface evidence, and for other special problems, measurements of horizontal intensity also are required, but are made only where the vertical data indicate that they are necessary for use in interpretation. Ordinary common sense should be used in laying out magnetic surveys, so that time will not be wasted in attempting to solve problems to which the method is unsuited. For most useful results, the following conditions must exist.

1. The rocks of economic interest in the problem must have magnetic contrast, that is, must contain some beds known to be markedly more or less magnetic than the average.

2. The feature sought must have broken or deformed the strata so as to have caused a considerable difference in elevation of some bed of marked magnetic characteristics within the limits of the survey. As an illustration, it is a waste of time to try to trace magnetically a fault of small vertical displacement in the Kern River formation, which has little magnetic contrast.

Field parties.—Our practice in California is to have a party consisting of two geologists, one car, and two vertical magnetometers. This is found more economical because two readings with different instruments can be taken at the same time within a short distance of each other and their values averaged if within a few gammas. This method establishes the value of the point without further checking, and informs the operator immediately if either balance is out of adjustment.

INTERPRETATION

Presentation of results.—In order to interpret the results of a magnetic survey, it is necessary to show the values determined by the work in their true relation to topographic and known geological features on a map of the area surveyed. We have used the following methods.

1. Peg model of vertical intensities. If enough points have been occupied, the tops of the pegs show a magnetic surface from which much can be inferred about the course of faults, magnetic marker beds, and other structural features (Fig. 1). A model of this type is inconvenient to make, transport, or file, and can be shown in the report only by photographing it.

2. Celluloid profile model of vertical intensity. This is made by placing a large sheet of celluloid over a map and cementing to the celluloid cover profiles cut from the same material representing the vertical intensities to scale above each station.

This type of model is more useful than the foregoing because the celluloid is transparent and does not obstruct a view of the map as the pegs do. It is serviceable as a guide in preparing magnetic contour maps because the magnetic surface can be seen without use of the imagination. It has the disadvantages that it can not be easily transported and can be included in reports only by photographing it (Fig. 4).

3. Magnetic contour maps. In this method of presentation, contour lines are drawn connecting points of equal vertical intensity exactly as contour maps of topography are made. Contour maps are easily made and do not greatly detract from the original map on which the contours are drawn. The contours show all the major features and a great many of the smaller features.

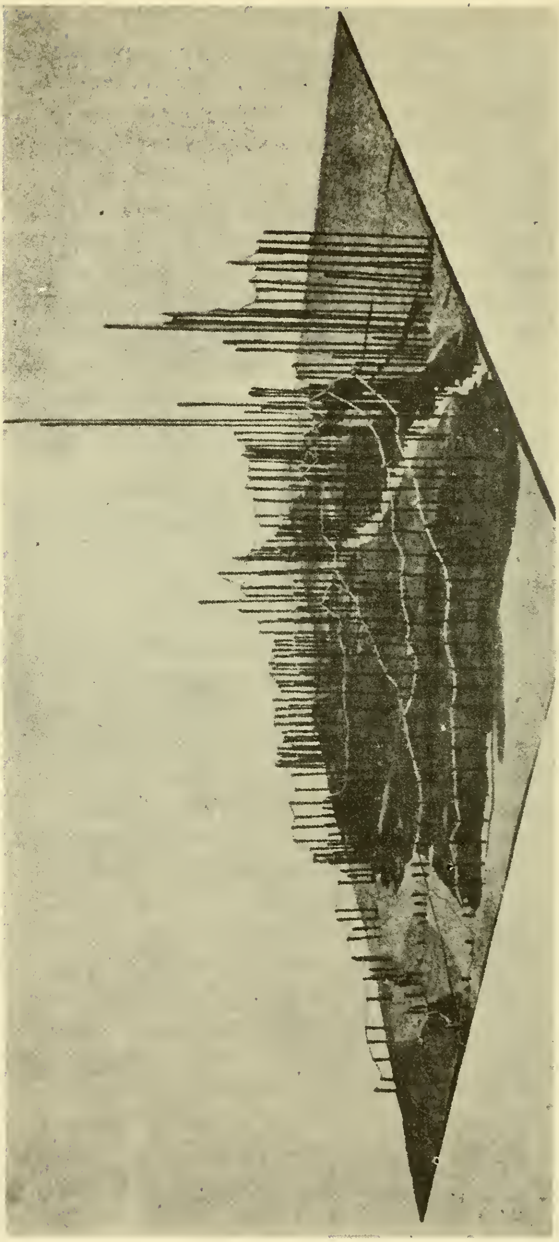


FIG. 1.—Peg model of vertical magnetic intensities from Salinas River to Peachtree Valley across San Andreas fault, southeast of King City, Monterey County, California.

This is probably the best method of showing actual results, and is undoubtedly the best when it is desirable to show the regional trend of folding. During the last few years all the results of our work have been reported in the form of contour maps, as they can be readily extended from area to area throughout large expanses of country and placed on a regional map with some very interesting results.

RESULTS OF MAGNETOMETER SURVEYS IN CALIFORNIA

It would be impossible to publish the results of all the magnetometer surveys we have made in California during the last four years; therefore, only a few of them are offered here. They are chosen because the results are type pictures of known geological conditions, such as the Raven Pass anticline and the White Creek syncline, and the others submitted show the interpretation of magnetometer surveys as carried from known geological features to the unknown.

The following work on known structures was done to test the vertical and horizontal magnetometers so as to use the resulting data in interpreting future results in areas covered by alluvial deposits.

RAVEN PASS ANTICLINE

This anticline trends almost N. 45° W. for a considerable distance, but the location chosen for the magnetometer survey was at the junction of Sections 29, 30, and 31, T. 26 S., R. 18 E. (Fig. 2).

The oldest beds exposed along the axis are the Cretaceous, with the younger Vaqueros and Monterey shale resting unconformably on both flanks. The anticline is apparently symmetrical, the dips ranging from 40° to 50° on both flanks. From surface observations there does not seem to be any faulting in the Cretaceous. As previously mentioned, the Cretaceous is strongly magnetic and the Miocene shales weakly magnetic. Therefore, an anticline in these contrasting formations should give a type picture of the behavior of the vertical intensities and the vectors of disturbance.

Results.—Both the vertical intensities and the magnetic vectors of disturbance were plotted. The vertical intensity curve rose from an intensity of 15 gammas over the Monterey shale on the extreme northeast end of the line to 110 gammas at the axis. From the axis southwestward, the intensity curve fell to 50 gammas over the Monterey shale. A short distance southwest of the axis, the vertical intensity curve gave a sharp "kick" to a maximum of 125 gammas, which strongly suggested a small fault. As stated before, no fault could be traced on

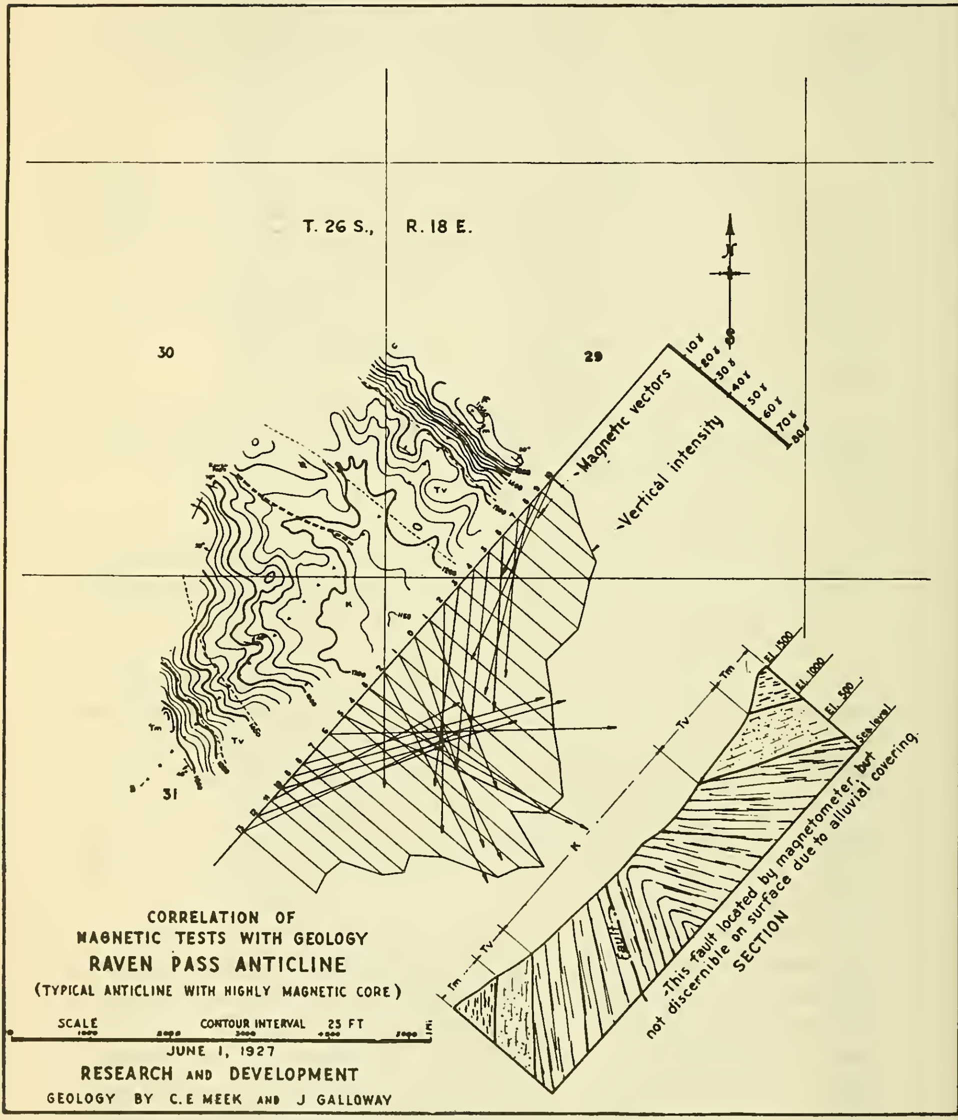

T. 26 S., R. 18 E.
30
29
31
Magnetic vectors
Vertical intensity
Tv
Tm
K
El. 1500
El. 1000
El. 500
Sea level
Fault
This fault located by magnetometer, but not discernible on surface due to alluvial covering.
SECTION
CORRELATION OF
MAGNETIC TESTS WITH GEOLOGY
RAVEN PASS ANTICLINE
(TYPICAL ANTICLINE WITH HIGHLY MAGNETIC CORE)
SCALE
CONTOUR INTERVAL 25 FT
JUNE 1, 1927
RESEARCH AND DEVELOPMENT
GEOLOGY BY C. E MEEK AND J GALLOWAY

Fig. 2

the surface because of the loose soil covering the surface. An erratic dip of 40° N. in the south flank lends support to this small fault.

All the vectors of disturbance, when plotted at each station, point toward the axis of the fold, the angle of each vector with the horizontal becoming steeper as the axis is reached until, at the axis, the vectors are almost perpendicular.

WHITE CREEK SYNCLINE

This syncline, 20 miles northwest of Coalinga, trends N. 60° W. for a considerable distance, but the locality chosen for the magnetometer line was in the SE. ¼, Sec. 23, T. 19 S., R. 13 E. (Fig. 3).

The Cretaceous beds are folded into an asymmetrical syncline with the north flank dipping 65° and the south 45°. Folded in the trough of these older beds and overlapping them is the Etchegoin. The axis of the syncline in the Etchegoin can be determined on the ground because of the outcropping on both sides of the axis of the characteristic *Glycimeris* bed. The Etchegoin is moderately magnetic and the Cretaceous is strongly magnetic.

Results.—The plotting of the vertical intensities and the magnetic vectors of disturbance showed a condition the reverse of those plotted from the Raven Pass anticline. The vertical intensities, plotted in gammas, showed a curve with its lowest point at the axis of the syncline where the Etchegoin would be at its thickest over the magnetic Cretaceous. The highest points in the curve naturally occurred at the end of the line directly above the Cretaceous.

The vectors of disturbance point vertically down at the axis; and outward from this point toward the more magnetic Cretaceous beds.

WALNUT CREEK FAULT (FIG. 4)

The Walnut Creek area comprises an alluvial valley bounded on the west by Martinez Ridge, and on the south by Shell Ridge, both of which expose Tertiary rocks. The structural features consist of two major faults traversing the valley, approximately at right angles to each other, and several minor faults. On the northeast side Cretaceous rocks are exposed and the valley is surrounded on the other two sides by the Eocene rocks. This is an ideal condition, as the Cretaceous in Walnut Creek is very magnetic, the Eocene is weakly magnetic, and these formations are cut by several faults.

The work was begun by running lines with the magnetometer across the supposed location of a major fault, marked on the map as the Martinez-Shell Ridge fault. The position on this is established by surface

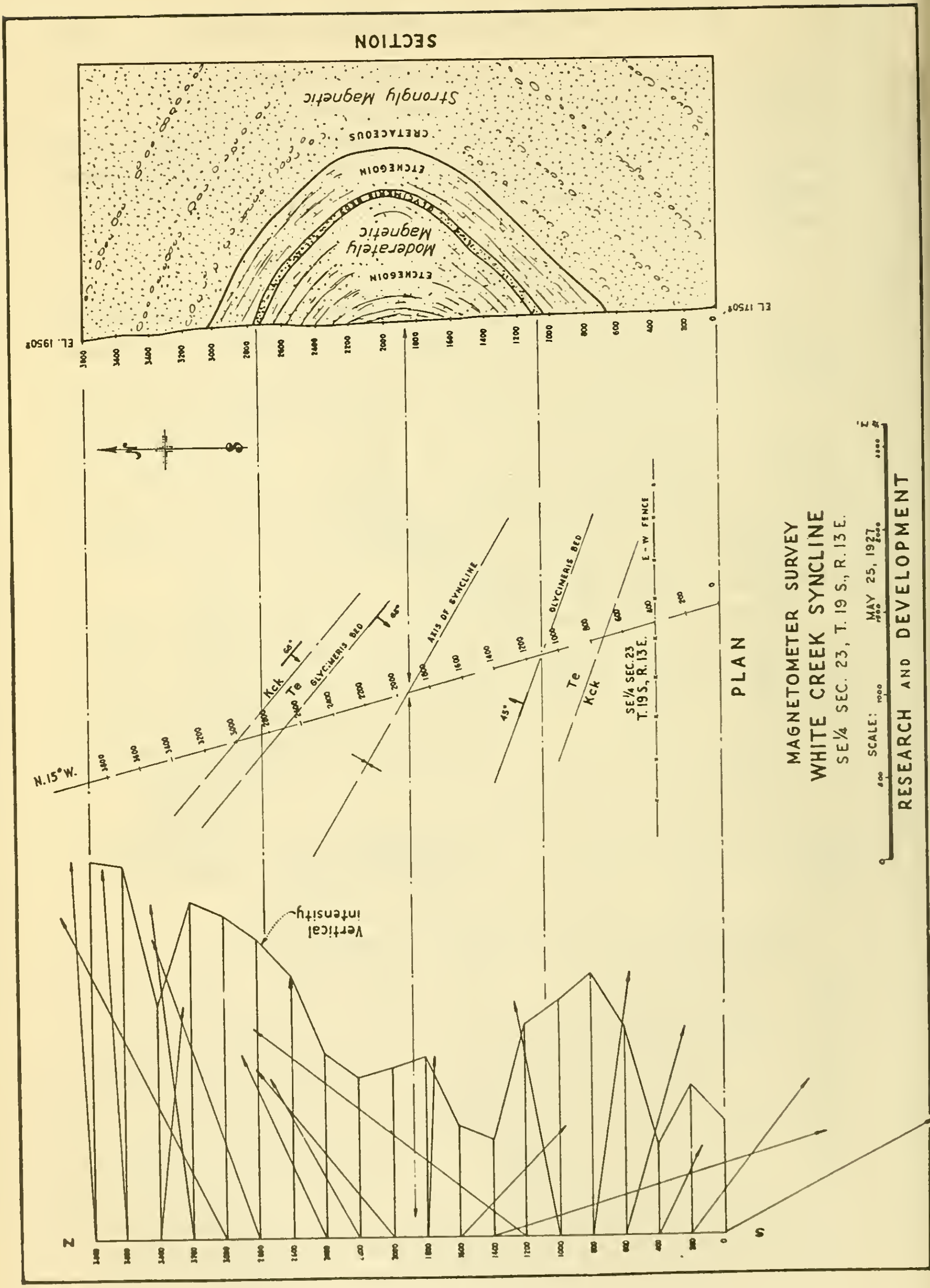
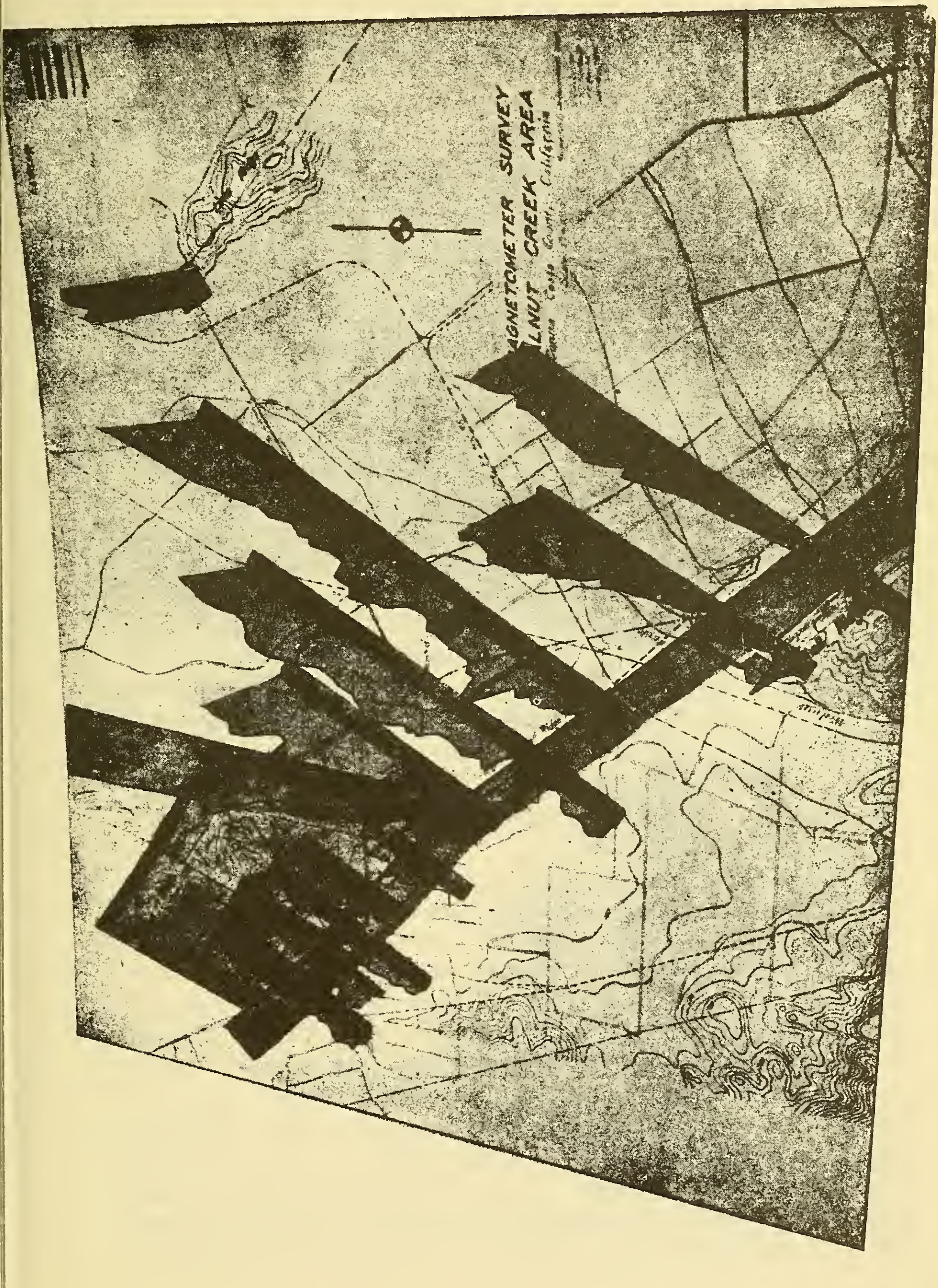

SECTION
Strongly Magnetic
CRETACEOUS
ETCHEGOIN
Moderately Magnetic
ETCHEGOIN
EL. 1950'
EL. 1750'
PLAN
N.15°W.
AXIS OF SYNCLINE
GLYCIMERIS BED
GLYCIMERIS BED
E-W FENCE
Kck
Te
Te
Kck
45°
SE¼ SEC. 23
T. 19 S., R. 13 E.
Vertical intensity
N
S
MAGNETOMETER SURVEY
WHITE CREEK SYNCLINE
SE¼ SEC. 23, T. 19 S., R. 13 E.
MAY 25, 1927
SCALE:
RESEARCH AND DEVELOPMENT

Fig. 3


AGNETOMETER SURVEY
LNUT CREEK AREA

evidence on both sides of the valley, but it is concealed by a deep cover of alluvium for several miles in the location chosen for an accurate determination of the trace of this fault. The procedure was to run lines at intervals of 1/4 mile across the approximate location of the fault. The average length of the lines is approximately 1 mile. Magnetometer readings were recorded at each 200 feet. The course of the lines was N. 27° E., approximately at right angles to the strike of the fault.

As the work continued, it became increasingly evident that the Cretaceous area northeast of the fault showed much stronger vertical intensity than the Eocene on the southwest. Celluloid profiles showing vertical intensity plotted to scale were affixed to the map along the lines of survey, and the surface, outlined by the edges of these profiles, indicated a sort of magnetic plateau over the supposed Cretaceous area. As the valley floor is concealed beneath alluvium, a short line was run across the point of Lime Ridge where there are outcrops of rocks known to be Cretaceous, and high vertical magnetic intensities were found corresponding with those in the doubtful region. A sharp depression in the magnetic profiles near the assumed position of the fault was followed to the hills and proved to occur at the known location of the fault. This sharp depression in the magnetic profile has therefore been assumed to indicate the trace of the fault throughout the part covered by alluvium. The shape of the curve at this point is characteristically the same on all the profiles. On the basis of this indication the writer has drawn in the fault line, which in general lies close to that originally assumed, but has two offsets on the north which may have resulted from cross faults supposed to exist at this location, provided the assumptions here followed are correct.

Several of the survey lines have crossed small faults, with much smaller indications. Seemingly there is no property of a fault as such that affects the magnetic instruments, and a fault will probably be indicated by them only where rocks of different magnetic properties have been brought opposite each other, or where a magnetic bed has been offset horizontally in crossing the fault.

Magnetic marker beds.—An inspection of the magnetic profiles shows high points at intervals, which are found to be in definite lines. It is natural to assume that these lie over buried outcrops of beds of stronger magnetic properties than those of adjacent rocks. In order to verify this conclusion some lines were run with observations at short intervals across exposed beds in line with these series of high points on the profiles. It was found that some beds show very high values, which distinguish them strikingly from the rest of the formation. Two such beds were observed in the Cretaceous, of which the course is indicated by lines connecting high points in the profiles (Fig. 4), and one such bed was found in the Briones. As some of the magnetic anomalies by which these beds differ from the associated rocks amount to as much as 300 gammas, nearly one hundred times the least reading possible with the instruments, they are very conspicuous markers, easily recognizable with the magnetometers. The effect of these magnetic beds is very large and sharply marked where they crop out, and becomes progressively less intense and more widely distributed as the depth of cover increases. Their effect is clearly evident under the deepest cover existing in the Walnut Creek Valley. This depth is not known, but, as indicated by water-well evidence, is probably more than 150 feet.

POSO CREEK, SAN JOAQUIN VALLEY, CALIFORNIA (FIG. 5)

In the vicinity of Poso Creek, the magnetometer outlined an outstanding magnetic "high" indicating that a strongly magnetic mass, such as gabbro, has been uplifted close to the surface. Confirmation of this fact is supported by the drilling of a well on the flank of this magnetic "high" which entered plutonic rocks at 2,700 feet. The uplifting of this gabbroid mass caused a fault on its north side. Figure 5 shows that in working with such large bodies of magnetic material the effect of the deeper rock can be corrected out. This picture presents the small anomaly, caused by a fault, separated from a large regional variation, caused by a very magnetic large mass in the underlying basement complex.

VENTURA COUNTY

The survey shown on Figure 6 comprises an area of about 35 square miles south and east of Oxnard, California. It is a relatively sandy flat basin flanked on the northeast by the Camarillo Hills and by Round Mountain, an isolated igneous hill in the southeastern part.

There is no doubt that the volcanic rocks are the chief causes of magnetic disturbances in this area. The effect of concentrations of magnetic minerals in terrace material is small and local—in few places more than 20 gammas—and does not mask the major disturbances.

Results of the survey are shown on Figure 6, which gives contours of vertical intensity at 10-gamma intervals. As volcanic rocks are exposed on the southeast rim of the basin, and not at the north, the large disturbances in this part of the Ventura basin are probably the result of underlying bodies of volcanic rock. This interpretation was later sup-

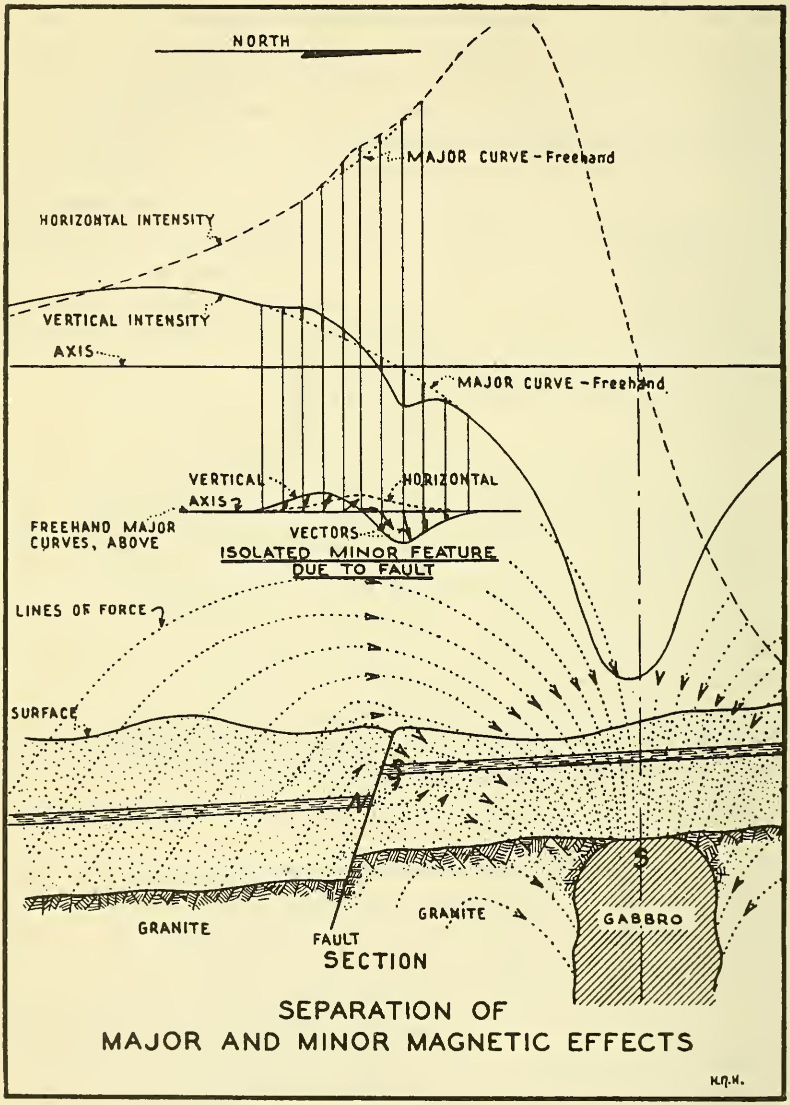


Fig. 5

ported by the results of the drilling of The Texas Company's Eastwood No. 1, 3 miles east of the town of Hueneme, which entered basalt at 1,915 feet. As this assumption was proved correct, it follows that the

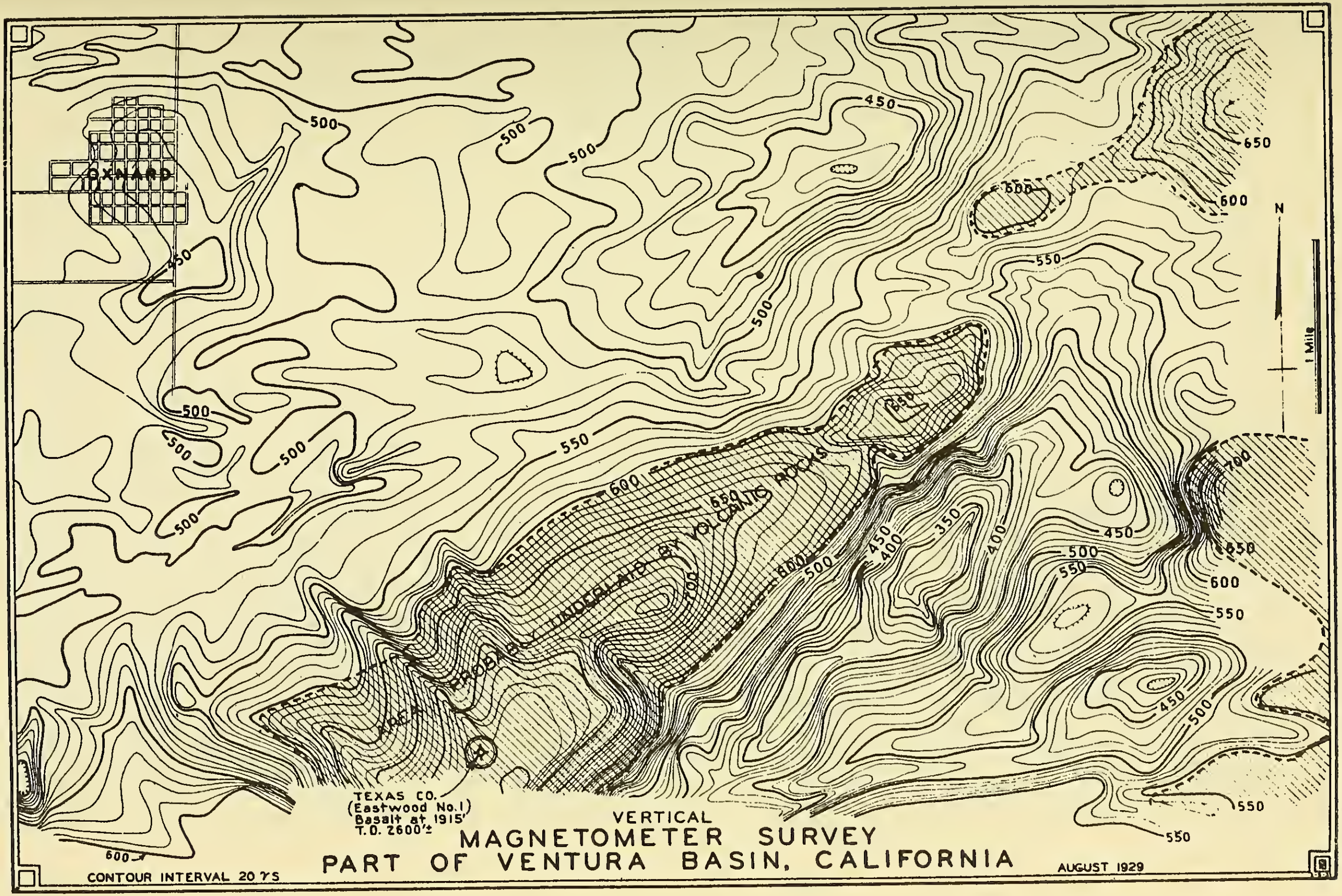

OXNARD
N
1 Mile
PROBABLY UNDERLAID BY VOLCANIC ROCKS
TEXAS CO.
(Eastwood No. 1)
Basalt at 1915'
T.D. 2600'±
VERTICAL
MAGNETOMETER SURVEY
PART OF VENTURA BASIN, CALIFORNIA
CONTOUR INTERVAL 20 γS
AUGUST 1929

FIG. 6

limits of the volcanic rocks are indicated by the edge of the area of great magnetic intensity as outlined on the map. The magnetic susceptibility of the basalt from the well was determined to be as great as 677×10^{-6}.

KETTLEMAN HILLS, CALIFORNIA

The Kettleman Hills structure is divided into three separate folds, known as the North, Middle, and South domes. The Lost Hills field is a continuation of the South dome structure and is intimately related to the Kettleman Hills structure. Magnetometer surveys have been made on all the structures. The North and Middle dome surveys are treated separately from the South dome-Lost Hills structure.

NORTH AND MIDDLE DOME

On the North dome the average dip is 35°-40° on the southwest flank and 25°-30° on the northeast. Thus the axial plane probably dips toward the northeast slightly. The beds of the Middle dome dip approximately 30° on both flanks. The trend of the two domes is about N. 40° W. and they lie *en échelon.*

There is a very intricate system of faulting, both lateral and cross, along the crest of the structures. Displacements range from a few feet to more than 100 feet in some places. This has resulted in raising or lowering blocks out of their original position. This minor faulting with its consequent disturbance of the magnetic vivianitic sandstone bed, together with its erosion, is largely responsible for some confusion in the magnetic picture.

The Paso Robles formation crops out as an almost complete belt around these domes with the Etchegoin formation exposed beneath. The Etchegoin is exposed along the crest for 1½ miles on both sides of the axis. The Paso Robles is composed of fresh-water beds of ill sorted and little consolidated sands and gravels. The Etchegoin underlying the Paso Robles and occupying the crest of the anticline consists of clays and sands with a coarse blue vivianitic sandstone as the most important member for the magnetic survey.

The "magnetic marker bed" or series of beds of the vivianitic sandstone consists of a blue sand ranging from fine to coarse. It is found so poorly cemented that it is possible to rub it apart with the fingers, though in other places it is hard and resistant. The blue color seems to be caused by a thin coating of hydrous-ferrous phosphate on the individual grains. It is very magnetic and tests made in the field prove that it is polarized. Tests around an outcrop of this sandstone showed readings with a range of 300 gammas in a distance of less than 200 feet.

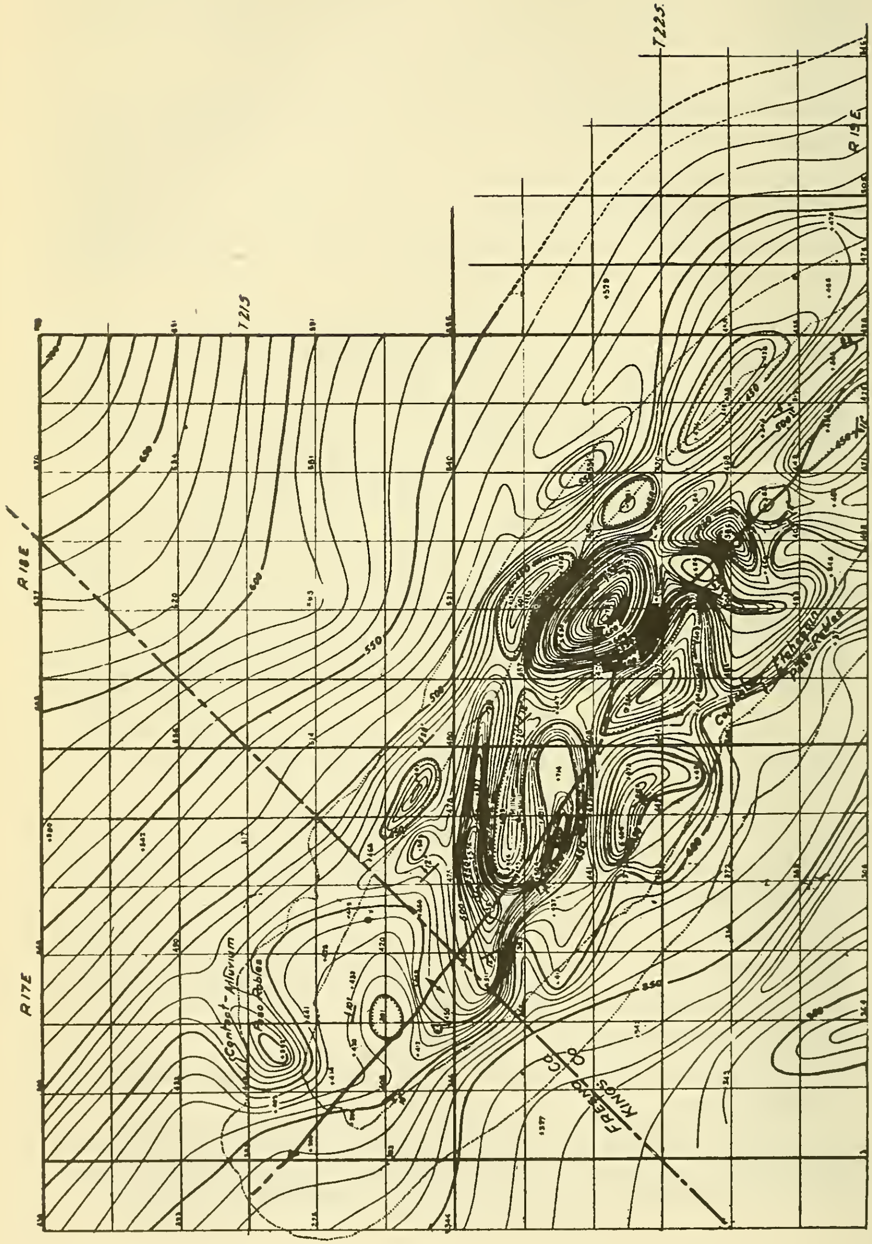
Results of magnetic work.—The magnetic picture (Fig. 7) presented of the North and Middle domes of Kettleman Hills is an irregular one and only general relations may be interpreted from it. We have definitely established the fact that the vivianitic beds cropping out at the surface are definitely polarized and that the presence or absence of these beds controls the magnetic results obtained. Unfortunately, the vivianitic horizon does not occur along definite lines of stratification, but is distributed in lenses and patches throughout the whole series comprising the top of the Etchegoin formation. It is this irregularity of the vivianite which gives such surprising and irregular results. Magnetic anomalies are very large and many abrupt changes of more than 300 gammas were obtained within a radius of a few hundred feet. Differences of readings of more than 100 gammas were obtained on the same outcrop, seeming to indicate that the concentration of magnetic material is decidedly lacking in uniformity and that mass also has some effect. Erosion and some minor faulting cause the operator in some places to be above and in some places to be below the beds furnishing the "kick."

In general, the magnetic contours trend northwest and follow structural contour lines. On the North dome is a series of "highs" along the upturned edges of the blue sandstone with alternate "highs" and "lows" along the axis, depending on whether patches of vivianite are present or not. The beds on the northeast flank of the structure show a well defined series of "highs," but the relationship is not so clear on the southwest flank, nor does the closure north of the Millham well conform to the closure as determined by the geology. There is a well defined series of "lows" along the northeast edge of the structure which may be indicative of fault conditions.

The Middle dome presents a more regular picture with a series of "highs" along the axis. These "highs" even follow the offset or warp of the axis between the two domes. The greater regularity, as shown on the Middle dome, is caused, probably, by the more moderate relief in the topography and greater protective covering over the magnetic beds. It is also possible that the magnetic content of these beds is more uniform than that of those farther north. The results suggest that the magnetic beds are continuous across the axis here.

From the foregoing described results the following conclusions can be drawn.

1. The North and South domes of the Kettleman Hills structure are marked by a zone of intense magnetic disturbance, which conspicuously distinguishes them from relatively featureless surroundings.


T22S
R19E
T21S
R18E
R17E
550
Contact-Alluvium
Paso Robles
FRESNO CO.
KINGS CO.

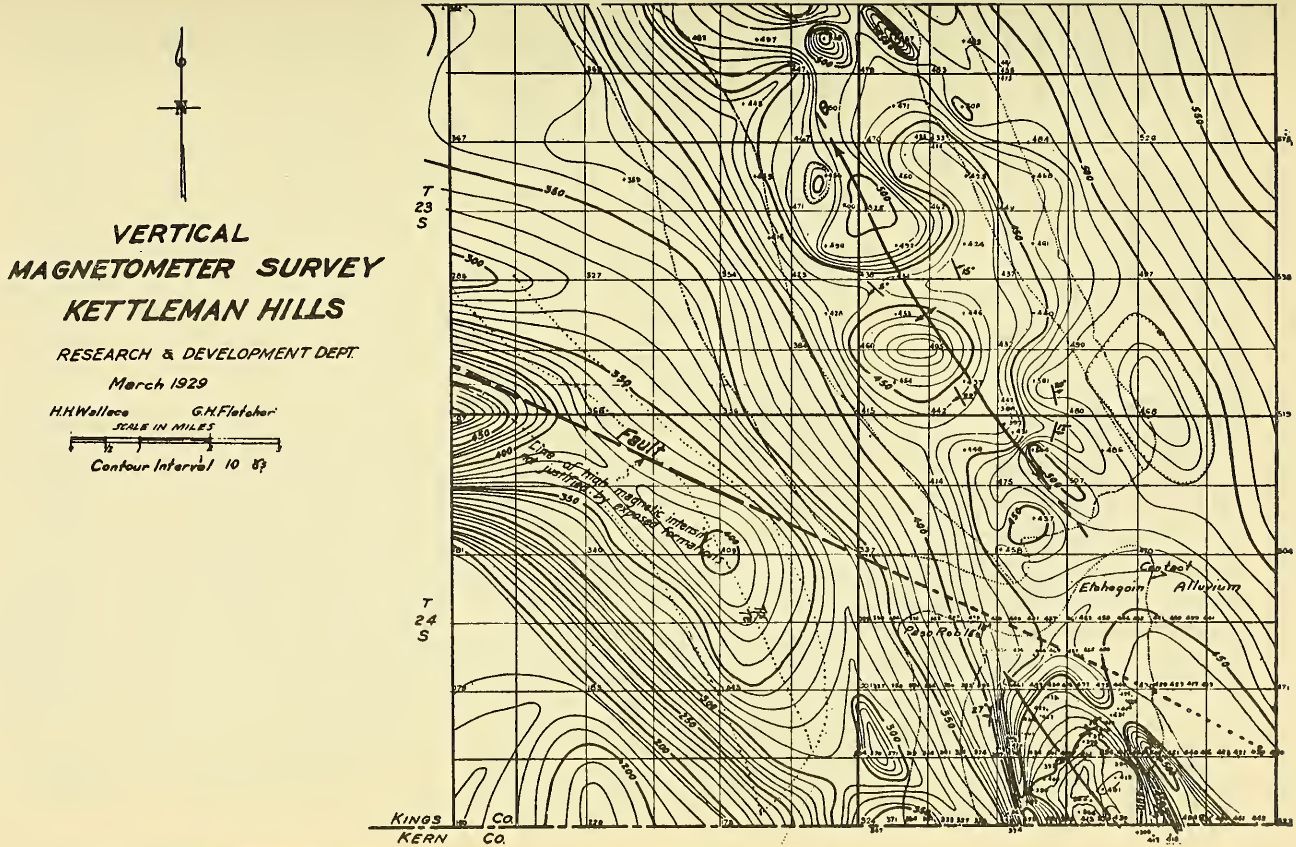


FIG. 7.—Vertical magnetometer survey, Kettleman Hills, California. Upper part shows North dome; lower part Middle dome, and north end of South dome.

2. The North dome shows low intensity along its axis surrounded by high intensities on the flanks over eroded edges of magnetic beds.

3. The Middle dome shows high intensity at the axis, as would be expected from a magnetic bed arched continuously across the structure and not eroded. The obvious inference is that the Middle dome is structurally lower than the North dome.

4. The magnetically active horizon has been identified as the vivianitic sandstone of the upper Etchegoin (McKittrick group) of Pliocene age.

5. It has been discovered that this sandstone is also strongly polarized along definite lines.

SOUTH DOME-LOST HILLS STRUCTURE

The area surveyed is a strip extending northwest-southeast from the northern end of the Lost Hills oil field, Kern County, to and including South dome of Kettleman Hills, Kings County, on the western side of the San Joaquin Valley, California. It ranges from 2 to 4 miles in width and has a length of 14 miles.

The country covered by the survey is one of relatively small relief. The central part is a gently rolling, alluvium-covered plain, slightly sloping downward toward the northeast. At either end of the area are low, rolling hills rising from 50 to 200 feet above the general level of the country. The Lost Hills, from the west side, appear as the crest of a gradually rising plain, but on their eastern side they rise more abruptly and are more prominent features of the landscape.

The only formations exposed are Etchegoin (Pliocene), San Joaquin clays and Tulare (Upper Pliocene), and alluvium.

The folding in the Lost Hills-South dome district is a segment of the long, anticlinal fold, extending northwest-southeast for 60 miles. Folding along this line has also caused the North and Middle domes of Kettleman Hills and the Coalinga anticline. Recurrent movements along the line have occurred since its probable beginning, at the end of Cretaceous time. Gentle folding was repeated at the end of the Eocene. More intense folding recurred at the close of Miocene and Pliocene time.

Surface evidence of the structure is incomplete, because of the mantle of alluvium in the central part of the area. The topographic "high" and surface exposures show the anticlinal origin of the Lost Hills. The trend of the *Mulinea* bed, at the top of the Etchegoin, and the dips obtainable at the South dome, disclose the northwestward plunge at that point.

Results of magnetometer survey.—The survey shows a magnetic "low" extending from the Lost Hills to the South dome. A very marked "high" is east of the "low" and extends parallel with it. On the western side of the area, the magnetic "high" is not so great in the south, but northwestward the intensity increases and the changes from "highs" to "lows" become much sharper and more pronounced. This is because the sandstone beds in the Upper Etchegoin cause the magnetic "highs," and in the south the mantle of alluvium is probably thicker, causing a greater masking effect on the magnetic beds. At the South dome the parallel "highs," on either side of the area, swing around and close, conforming to the trend of the *Mulinea* bed.

At the South dome the magnetometer results are substantially the same as those shown by surface and subsurface geology. The trend of the magnetic anomalies conforms with the strike of the beds, and the manner in which the "high" closes on the north is especially convincing. The alternating long, narrow "highs" and "lows," in the northwest, are undoubtedly caused by interbedded sandstones and sandy shales and clays. The magnetic "highs" are on sandstone beds.

The structural axis at the South dome follows a line of magnetic "lows." The "lows" fall midway between magnetic "highs" caused by magnetic sandstone beds near the Etchegoin-San Joaquin clays contact. Erosion has removed these magnetic beds from along the axis of the anticline, leaving their truncated edges on the flanks of the structure, with less magnetic beds in the center. This explains the relationship of the anomalies to the structure in this area.

By analogy with conditions existing at the South dome, the axis of the structure can be located southward, connecting with the Lost Hills field. Immediately north of the Lost Hills oil field, the magnetic "low" swings sharply westward and turns northwest to the South dome. As the magnetic anomalies are similar to those at the South dome, the axis of the anticline probably follows the magnetic "low."

From the foregoing evidence the following conclusions can be drawn.

1. It is possible to show the location of the axis of the Lost Hills-South dome anticline.

2. The structure is continuous from the Lost Hills to the South dome.

3. The position of the axis, according to the magnetometer, corresponds fairly well with the geologic axis, excepting north of the Lost Hills, where it deviates and causes an offset in the trend of the axis.

4. A pronounced feature of the survey is that the trend of the magnetic anomalies conforms with the strike of the beds in the South dome, and the manner in which the "high" closes on the north is most convincing.

5. The buried outcrop of an eroded magnetic bed can be traced completely around the South dome, including the Lost Hills field.

6. The survey shows a magnetic "low" extending from the Lost Hills to the South dome, with a marked "high" east of the "low" and paralleling it.

7. The magnetic anomalies at the South dome have a higher intensity than the rest of the area, because the sandstone beds in the Upper Etchegoin cause the magnetic "highs," and at the south the mantle of alluvium is thicker and the Etchegoin thins out, thus decreasing the intensity.

MAGNETIC DISTURBANCE CAUSED BY BURIED CASING[1]

WILLIAM M. BARRET[2]
Shreveport, Louisiana

ABSTRACT

A vertical string of casing becomes magnetized by induction under the influence of the terrestrial magnetic field. For theoretical investigation we may treat such a casing as a bar magnet having an exaggerated ratio of length to diameter. Formulae are derived, based on the fundamental law of Coulomb, to express the anomalous components of the magnetic elements in different horizontal planes and at different radial distances from the casing head. Curves representing these equations are shown. Graphical illustrations of the composition of these anomalous components and the normal components of the earth's magnetizing field are included, together with typical experimental data obtained in the field.

INTRODUCTION

In the investigation of magnetic phenomena directly associated with oil- and gas-producing fields, as well as in attempting to map geologic structure along the boundaries of such areas, the magnetic distortion traceable to buried casing becomes of vital importance to the geophysicist.

As many of the empirical data that furnish valuable criteria for interpretative analysis must necessarily be obtained in regions where subsurface features have been disclosed by existing wells, it seems that a thorough understanding of the disturbing effect of this metal is essential to a proper appreciation of the experimental results.

In this paper it is intended to present briefly the theoretical aspects of the problem and to include such field observations as are considered representative of the cases thus far investigated.

The writer wishes to acknowledge his indebtedness to Randolph H. Mayer for his valuable assistance during the preparation of this paper, and to express his appreciation to John S. Ivy for the data furnished regarding the location and completion dates of wells in the Sligo field.

[1]Read before the Association at the San Antonio meeting, March 21, 1931. Manuscript received, February 1, 1931.

[2]Geophysicist, William M. Barret, Inc., Giddens-Lane Building. Introduced by D. M. Collingwood.

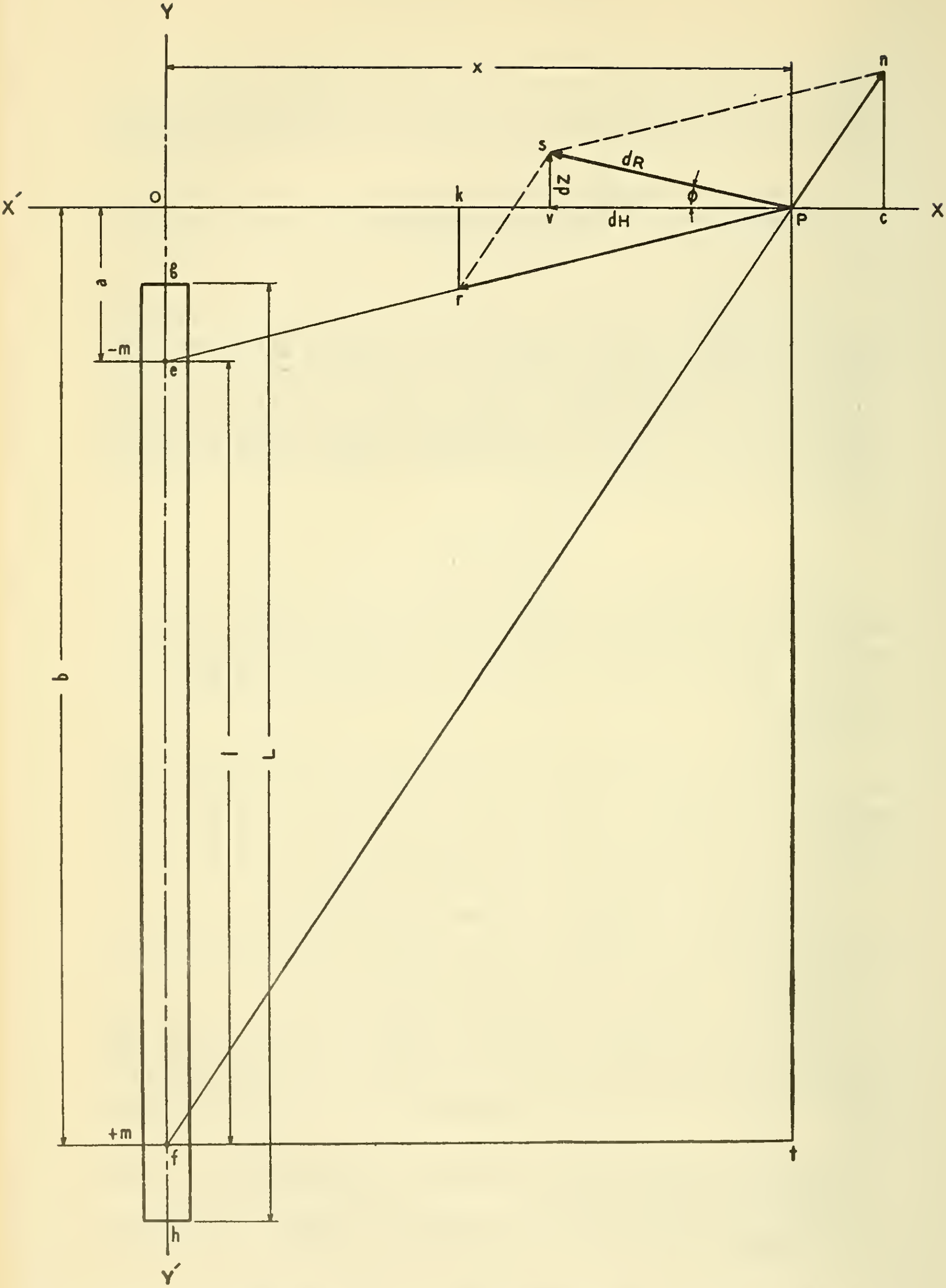


FIG 1.—Vector representation of the anomalous magnetic field at point P, caused by vertical string of magnetized casing *gh*.

THEORETICAL CONSIDERATIONS

The magnitude and direction of the resultant magnetic field near the casing head of a vertical string of pipe depends on the physical composition and proportions of the metal, the mechanical and thermal stresses involved, the perpendicularity of its vertical axis, the intensity and direction of the earth's magnetizing field, and the physical and magnetic characteristics of the subsurface media.

Referring to Figure 1, let us consider the vertical string of casing *gh*, having its major axis lying in YY'. It is evident that by induction the terrestrial magnetic field will establish a negative pole near the upper end of the casing and a positive pole near its lower end. Assuming the surrounding media to have unit permeability, and denoting the positive and negative pole strengths by $+m$ and $-m$ c. g. s. units respectively, we have

$$+m = -m = \frac{\pi}{4}(d_2^2 - d_1^2)\, kZ$$

$$= AkZ \qquad (1)$$

where d_2 is the external diameter of the casing in centimeters;
d_1 is the internal diameter of the casing in centimeters;
A is the cross-sectional area of the casing in square centimeters;
k is the susceptibility of the casing material;
Z is the vertical component of the earth's magnetic field in gauss.

For our purposes we may regard the casing as a bar magnet having the length L centimeters and distance between poles l centimeters. The magnetic moment in c. g. s. units will be given by

$$M = ml \qquad (2)$$

Let us consider the strength and direction of the anomalous field at the point P, lying in OX, caused by the vertical string of magnetized casing *gh*. The resultant force Ps at the point P will be determined in magnitude and direction by the vectors Pr and Pn, the former representing the attractive reaction on a unit positive pole placed at P, caused by $-m$, and the latter representing the repulsive reaction of the pole, $+m$. The solution of the problem is simplified by solving for the vertical and horizontal components dZ and dH, after which the resultant force dR and the angle of inclination $d\phi$ may be readily determined.

Denoting the permeability of the media surrounding the casing by μ, we have by Coulomb's law

$$Pn = \frac{m}{\mu \overline{fP}^2} = \frac{m}{\mu (b^2 + x^2)}$$

$$Pr = -\frac{m}{\mu \overline{eP}^2} = -\frac{m}{\mu (a^2 + x^2)}$$

As the triangles Pcn and fOP are similar, we may write

$$\frac{Pn}{cn} = \frac{fP}{Of} = \frac{(b^2 + x^2)^{\frac{1}{2}}}{b}$$

or $$cn = \frac{Pn \cdot b}{(b^2 + x^2)^{\frac{1}{2}}}$$

Substituting the value of Pn and reducing, we have

$$cn = \frac{bm}{\mu (b^2 + x^2)^{\frac{3}{2}}} \tag{3}$$

Similarly, it is seen that

$$\frac{Pr}{kr} = \frac{eP}{Oe} = \frac{(a^2 + x^2)^{\frac{1}{2}}}{a}$$

and $$kr = \frac{Pr \cdot a}{(a^2 + x^2)^{\frac{1}{2}}}$$

$$= -\frac{am}{\mu (a^2 + x^2)^{\frac{3}{2}}} \tag{4}$$

Combining expressions (3) and (4) we find the vertical component of the anomalous field at the point P to be

$$dZ = vs = \frac{bm}{\mu (b^2 + x^2)^{\frac{3}{2}}} - \frac{am}{\mu (a^2 + x^2)^{\frac{3}{2}}} \tag{5}$$

In like manner it may be shown that the horizontal component of the anomalous field at the point P is

$$dH = Pv = \frac{xm}{\mu (b^2 + x^2)^{\frac{3}{2}}} - \frac{xm}{\mu (a^2 + x^2)^{\frac{3}{2}}} \tag{6}$$

Also, the resultant may be found from (5) and (6), thus

$$dR = Ps = \left[(dZ)^2 + (dH)^2 \right]^{\frac{1}{2}}$$

$$= \frac{m}{\mu\,(a^2 + x^2)\,(b^2 + x^2)} \Big[(a^2 + x^2)^2 + (b^2 + x^2)^2 -$$

$$2\,(ab + x^2)\,(a^2 + x^2)^{\frac{1}{2}}\,(b^2 + x^2)^{\frac{1}{2}} \Big]^{\frac{1}{2}} \qquad (7)$$

The foregoing equations give values for the vertical, horizontal, and resultant forces of the anomalous field in gauss when the distances are expressed in centimeters and the pole strengths in c. g. s. units.

The angle between the resultant dR and the $OX-$ axis is termed the inclination, or dip, and may be found as follows:

$$\tan d\phi = \frac{dZ}{dH} = \frac{b\,(a^2 + x^2)^{\frac{3}{2}} - a\,(b^2 + x^2)^{\frac{3}{2}}}{x\left[(a^2 + x^2)^{\frac{3}{2}} - (b^2 + x^2)^{\frac{3}{2}}\right]} \qquad (8)$$

In Figure 2 are shown curves representing equations (5) to (8) inclusive. These data were prepared from an investigation of a 206-foot vertical string of 10-inch casing, having its upper end level with the earth surface. The pole strength of the casing was determined experimentally by measuring the vertical intensity anomaly directly above the pipe and solving for m in equation (5), it being assumed that $l = 0.833\ L$, and that the media surrounding the casing were of unit permeability. At a point 102.4 centimeters above the upper end of the pipe the anomaly was found to be 32,402 gammas.[1]

An examination of these curves reveals certain interesting characteristics. It is seen that the dZ and dR curves reach their maximum positive values directly over the casing, at which point they are numerically equal, because for this position the dH component is zero. At a radial distance of approximately 100 feet from the axis of the casing, dZ becomes zero and at 157 feet attains its maximum negative value of -102 gammas; at 260 feet the effect is reduced to -78 gammas. At a distance of 17 feet from the casing the dH component reaches its maximum amplitude of 12,500 gammas and at 75 feet appears equal to dR, because of the scale chosen. When the distance is increased to 260 feet the dH and dR values are 75 gammas and 108 gammas, respectively.

[1] 1 gamma = 10^{-5} gauss.

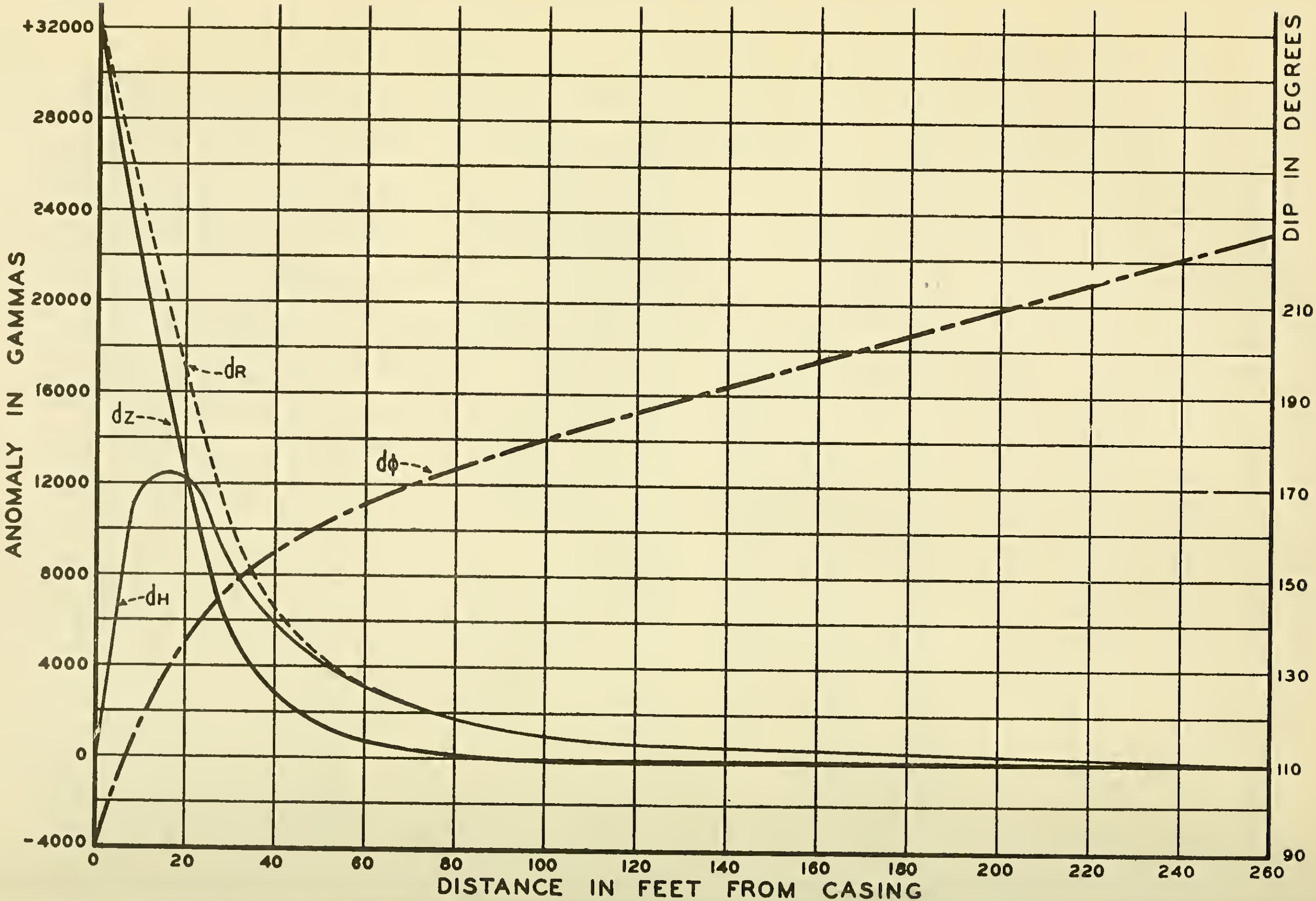


FIG. 2.—Theoretical curves representing equations (5) to (8) inclusive, and showing surficial variation of anomalous magnetic field caused by a 206-foot vertical string of 10-inch casing.

Directly over the casing the dip is 90°, indicating that the resultant vector is vertically downward and as the distance increases the dR vector rotates in a clockwise direction, becomes horizontal when $d\phi$ is 180°, and has moved 46° into the second quadrant of the rectangular coördinate system at a radial distance of 260 feet. The magnetic declination, which may be defined as the angle between the astromonic meridian and the magnetic meridian, will clearly depend on the azimuth selected for the profile, although the shape of the other curves will be the same regardless of the azimuth chosen.

The composition of these anomalous components with the normal terrestrial components determines the absolute values of the magnetic elements in the region surrounding the casing head. Let us next consider these resultant effects, which are graphically illustrated for a magnetic north-south traverse in Figure 3. The Z component at any point, as would be anticipated, represents the algebraic sum of the anomalous vertical intensity and the normal vertical intensity which would exist but for the presence of the magnetized casing. The normal vertical field strength, determined before the casing was set, was found to be 49,552 gammas and the maximum departure from this normal value occurs directly over the casing, where it becomes 81,954 gammas. The normal value of the resultant R was 55,489 gammas and its peak intensity of 87,891 gammas is coincident with the Z curve. The north and south parts of the R, H, and ϕ profiles are not symmetrical with respect to the casing axis, because these curves are determined by the vector composition of the anomalous and normal components. The maximum departures of the horizontal element, from the normal value of 24,974 gammas, occur in the immediate vicinity of the casing head, where the H curve indicates a variation of 25,000 gammas from a point 13 feet south to a position 15 feet north of the pipe. As for each azimuth the dH component is directed toward the casing, it is understood that the numerically lower values of H, for the north part of this profile, represent a subtractive effect, but south, the components are additive. Points of maximum and minimum for the angle of dip, which has a normal value of 63° 15′, occur adjacent to the casing, the relatively larger angles of inclination for the north part of the ϕ profile being traceable to the decrease in horizontal intensity. The different curves indicate that the magnetic elements have returned to substantially normal values at 260 feet in either direction from the casing. The declination profile is not shown because this element is subject to no variation, other than the

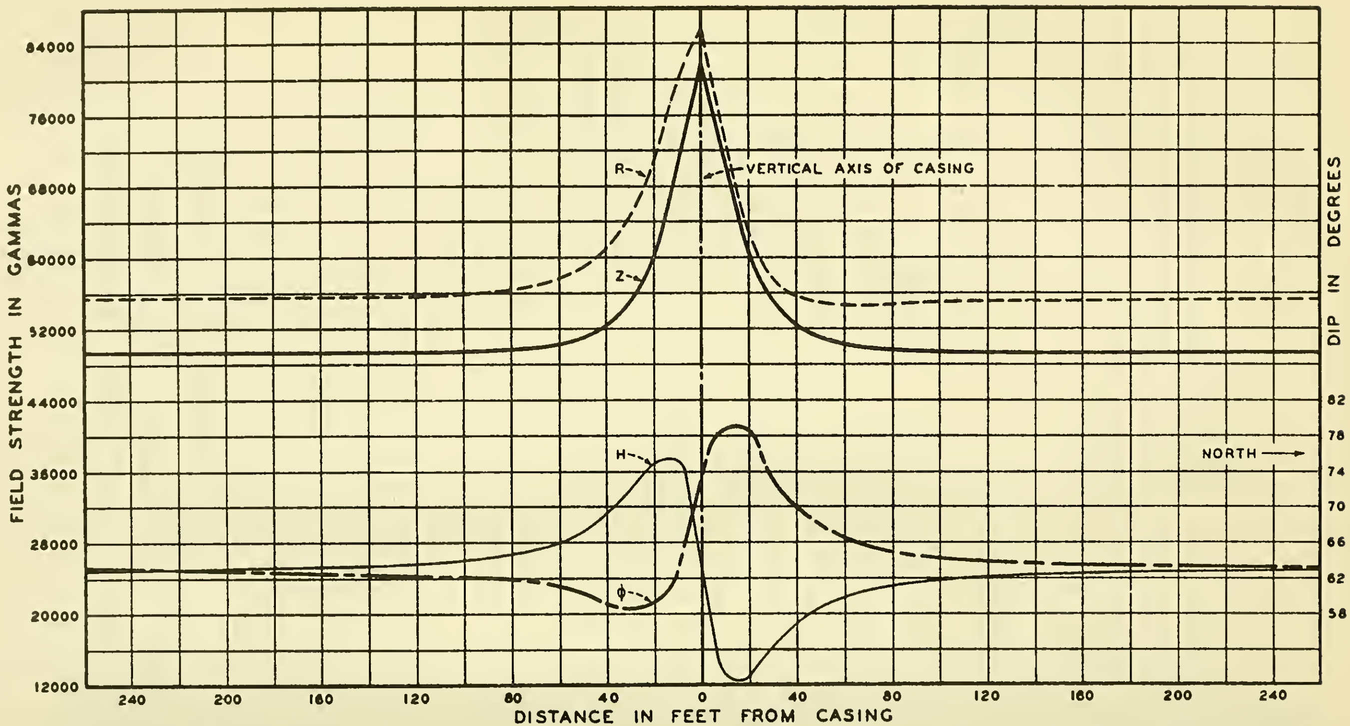


FIG. 3.—Composition of anomalous and normal values for a magnetic north-south traverse across a 200-foot vertical string of 10-inch casing.

constant areal value of 7° 50′ east, along the magnetic north-south azimuth.

Figure 4 shows the composition of the anomalous and normal components for a magnetic east-west traverse. The symmetry of the Z, R, H, and ϕ curves, as well as the relatively minor disturbance of the horizontal component and the dip, are apparent. The declination curve indicates a variation of 52° 30′ from a point 15 feet west of the casing to a corresponding position east.

EXPERIMENTAL RESULTS

To check the validity of the derived equations it was decided to take a series of actual observations along a magnetic north-south profile across the 206-foot string of casing represented by the foregoing theoretical curves. Vertical intensity measurements were chosen for the field investigations for the reason that, in the light of applied geomagnetics, this component assumes major importance. Further, it was felt that a comprehensive study of this single element would serve to test the authenticity of the various formulae in view of the mathematical relation between the several components. We therefore confine our comparison of the theoretical and experimental data to the vertical element.

For these measurements a Schmidt vertical field balance was used, the magnetic system being maintained at a constant elevation of 102.4 centimeters above a horizontal plane, through the upper end of the casing, to conform with the distance Og of Figure 1. The results for the north and south parts of the traverse agreed within the instrumental order of accuracy, mean values for the various positions being shown in Table I.

TABLE I

Distance in Feet	*DZ in Gammas*
0	+32,402
3.2	+19,470
6.6	+ 6,731
12.0	+ 1,988
22.0	+ 434
32.1	+ 158
46.0	+ 37
56.0	− 6
81.0	− 39
118.5	− 29
147.0	− 14
260.0	0

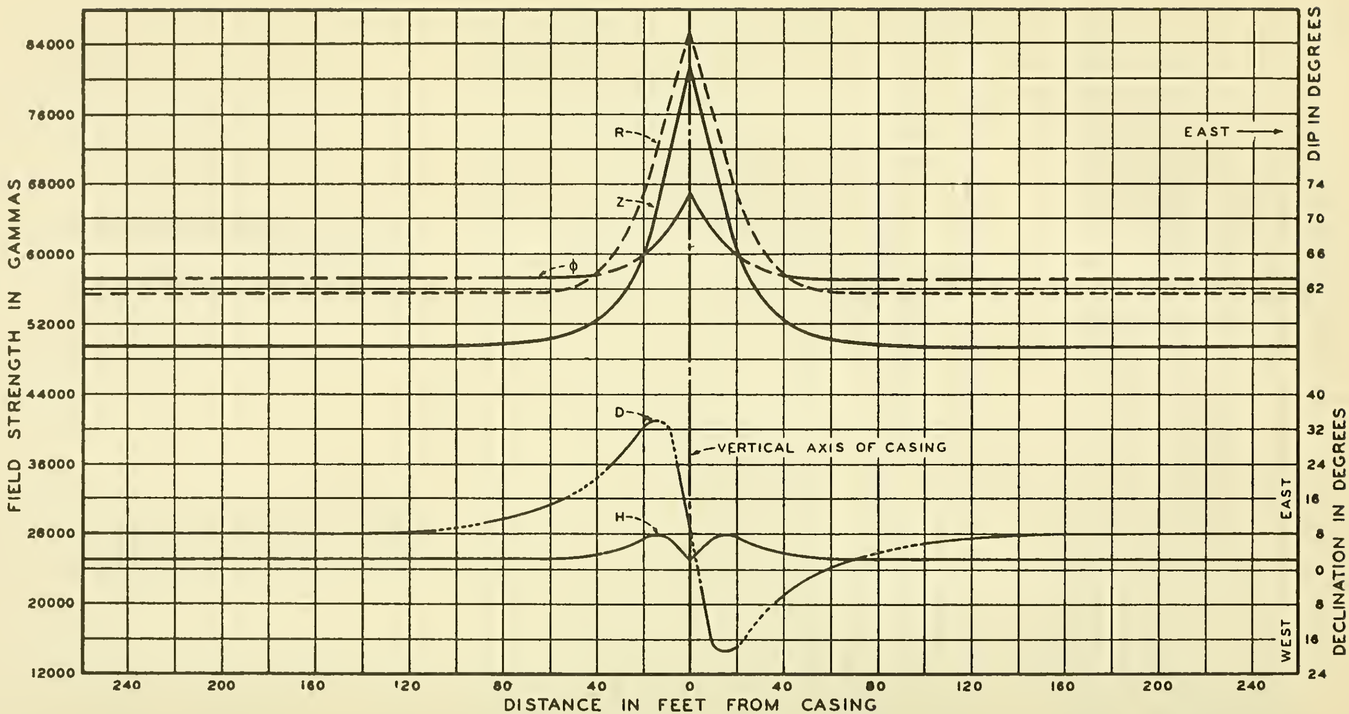


FIG. 4.—Composition of anomalous and normal values for a magnetic east-west traverse across a 206-foot vertical string of 10-inch casing.

A graphical representation of these data is illustrated by the heavy curves of Figure 5, an enlarged scale for the lower values of *DZ* being included for the purpose of disclosing the negative part of the curve. For these profiles the line of zero anomaly indicates the normal vertical intensity for the area, and it is seen that the *DZ* curve intersects this reference line at 54 feet, returning to normal value at approximately 200 feet. It is evident, from an inspection of equation (5), that a change in the elevation of the magnetic system of the variometer, with respect to the casing, would result in a variation of the vertical intensity anomaly. This was verified for the position 32.1 feet north of the casing head, where it was found that a change in elevation of 44 centimeters caused *DZ* to increase 34 gammas, the higher intensity corresponding with the lower elevation.

Let us now proceed to examine the disturbance of the vertical field produced by a 4,609-foot string of casing, consisting of 600 feet of 10-inch, 3,206 feet of 6⅝-inch, and 1,609 feet of 4¾-inch, the 6⅝-inch extending from the upper end of the surface casing and overlapping the 4¾-inch by 206 feet. The upper end of the casing projected 4.5 feet above the ground and was provided with a cap. As before, a magnetic north-south traverse was chosen and measurements of the vertical anomaly recorded at selected positions, the magnetic system of the balance being kept at an elevation of approximately 57 centimeters below a plane through the upper end of the pipe. These observations were made in a region where the normal vertical field strength was 50,065 gammas, the mean departures from this normal value, because of the presence of the casing, being indicated in Table II.

TABLE II

Distance in Feet	*DZ in Gammas*
0	
5	+35,242
10	+10,674
15	+ 5,512
20	+ 2,990
30	+ 1,219
50	+ 300
100	+ 53
150	+ 11
200	0
250	0
1,000	0

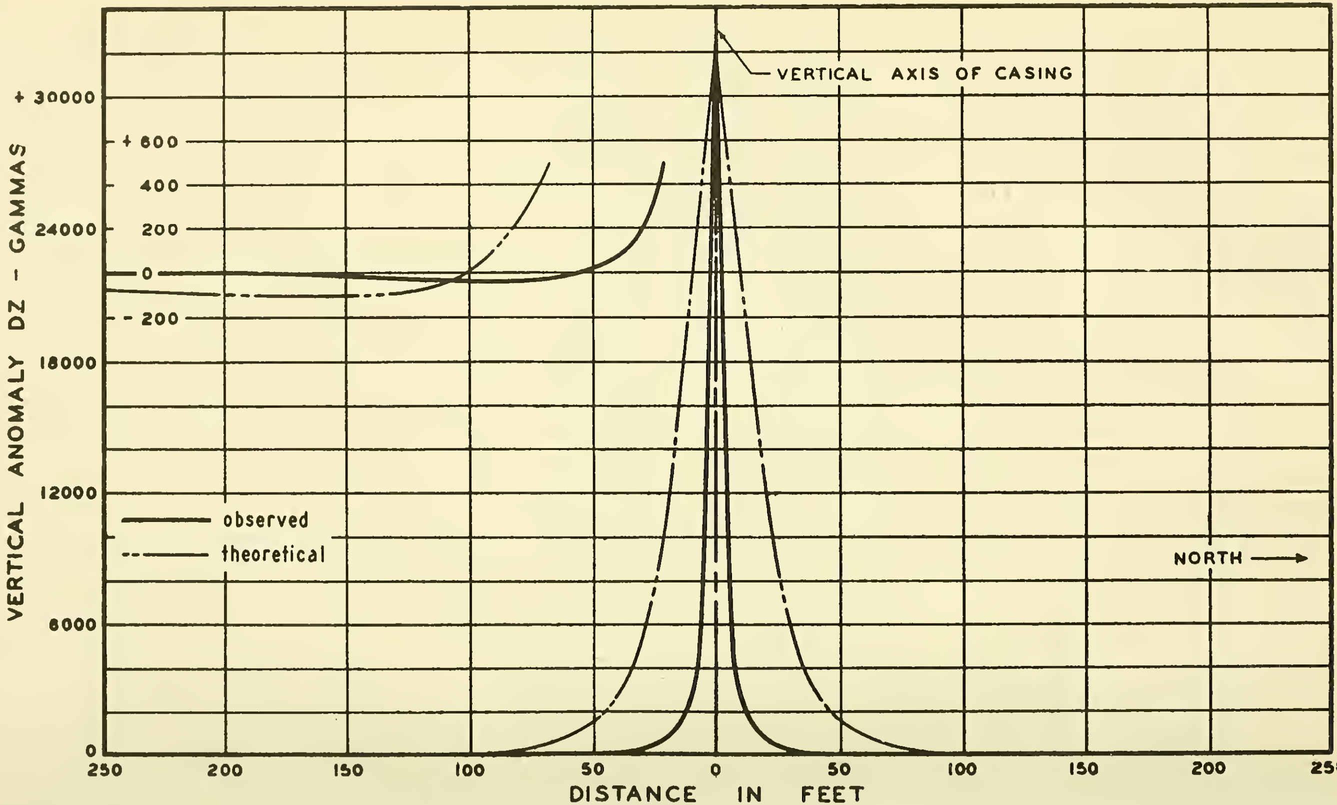


FIG. 5.—Comparison of observed and theoretical values for a magnetic north-south traverse across a 206-foot vertical string of 10-inch casing. Low values of DZ are indicated by supplementary graphs for the purpose of disclosing the negative parts of the curves.

It will be observed that no reading was obtained for the position directly above the casing. This was because of the fact that instrumental equipment was not available for measuring anomalies of this order. These data, with the exception of the last reading, are represented by the heavy curve of Figure 6, which illustrates the rapid rise of the magnetic gradient in the immediate vicinity of the casing and the disappearance of the disturbance at 200 feet. Two significant facts bear particular emphasis at this point: (1) the magnetic distortion caused by this string of casing, which is more than 22 times the length of the previously mentioned short pipe, disappears at very nearly the same radial distance; (2) there is no indication of a zone of negative anomalies as observed in the former case.

COMPARISON OF THEORETICAL AND OBSERVED DATA

For our theoretical treatment of the disturbing effect of magnetized casing we have based our discussion upon these fundamental conceptions: (1) the vertical string of casing may be regarded as a bar magnet having two *definite* magnetic poles; (2) the symmetry of the normal earth's field suffers no distortion by virtue of the presence of the magnetized pipe; (3) the direction and magnitude of the composite magnetic field near the casing head is determined by the vector composition of the secondary induced field and the normal terrestrial field. Though the validity of the last condition may be readily demonstrated, the two preceding assumptions justify further elaboration.

According to Jeans,[1] "The two ends of a magnet—or, more strictly, the two regions in which the magnetic properties are concentrated—are spoken of as the poles of the magnet." Many investigators have devoted an imposing amount of theoretical discussion and research to the study of the distribution of magnetism in bar magnets of different dimensions, and it has been determined that under some conditions a regularly magnetized bar may be treated as having two definite points, one near each end, at which the positive and negative effects may be considered concentrated. However, in any case involving the assumption of magnetic distribution for the purpose of accurately interpreting the action of a magnet at an external point, it is required, as pointed out by Gray,[2] that the point be removed a distance which is great in comparison with any dimension of the magnet.

[1] J. H. Jeans, *The Mathematical Theory of Electricity and Magnetism* (Macmillan, New York), p. 364.

[2] A. Gray, *Absolute Measurements in Electricity and Magnetism* (Macmillan, New York), p. 86.

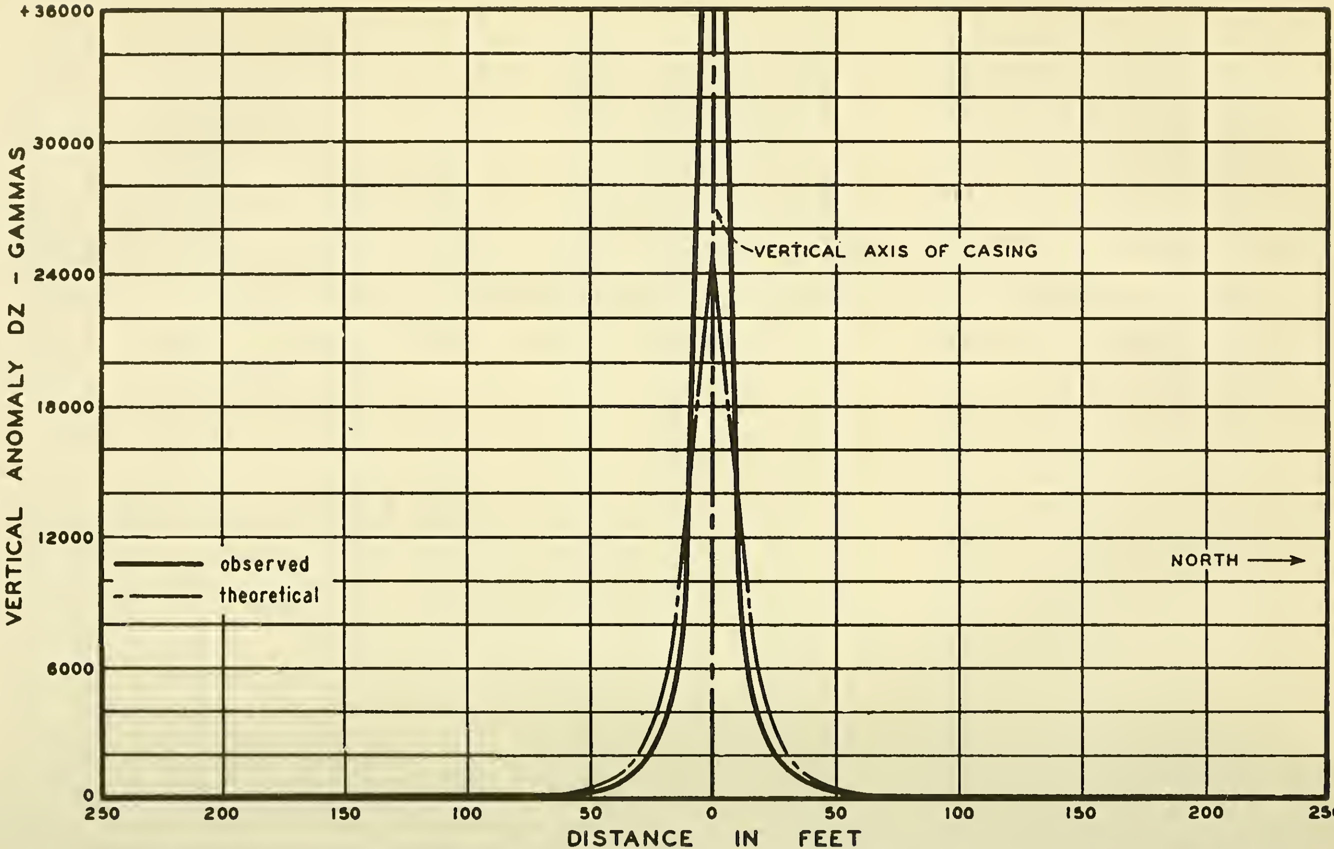


FIG. 6.—Comparison of observed and theoretical values for a magnetic north-south traverse across a 4,609-foot vertical string of casing.

As a first approximation, for the mathematical solution of the magnetic phenomena associated with the short string of casing, it was decided to regard the positive and negative effects as concentrated at the points f, e (Fig. 1), and to assume the ratio[1] of l/L to be 0.833. By reference to Figure 5, it is seen that the essential characteristics of the calculated and experimental curves are in fair conformity, equal peak values being chosen for the purpose of computing the pole strength. The divergence toward the base of the curves may be largely ascribed to our assumptions regarding the distribution of magnetism within the casing, subsequent investigation leading to the opinion that more consistent results could be obtained if the poles were considered located at the extremities of the pipe.

For the purpose of demonstrating the agreement between the theoretical and observed values when $l = L$, we place $dZ = o$ in equation (5) and find that the calculated curve crosses the zero reference line when

$$x = \pm \sqrt{\frac{a^2 b^{\frac{2}{3}} - a^{\frac{2}{3}} b^2}{a^{\frac{2}{3}} - b^{\frac{2}{3}}}} \qquad (9)$$

Also, by differentiating (5) we find dZ has a maximum value at $x = o$, and a minimum when

$$x = \pm \sqrt{\frac{a^2 b^{\frac{2}{5}} - a^{\frac{2}{5}} b^2}{a^{\frac{2}{5}} - b^{\frac{2}{5}}}} \qquad (10)$$

Substituting numerical quantities we find that dZ has a maximum value of 32,402 gammas directly over the casing, and becomes zero when $x = 54$ feet. The minimum value of -5 gammas occurs at a distance of 101 feet from the pipe and at 200 feet the effect is -3 gammas. These data are in excellent conformity with the observed quantities, with the exception of the numerical value of the negative inflection point.

So far, our comparison has been restricted to a short length of pipe in order to eliminate, as far as possible, the many uncertain factors associated with a relatively long string of casing. For this latter case, the relation between the calculated and the observed curves is illustrated in Figure 6, the required constants for the theoretical profile being found by solving simultaneously equation (5) for mean values of m and l, the corresponding anomalies and distances being obtained from the field data. The assumption was made that we may ignore the effect of the

[1]This ratio is customarily supposed to lie between 0.80 and 0.875 for the ordinary type of bar magnet.

FIG. 7.—Showing locations of wells and magnetic stations in Sligo area, Bossier Parish, Louisiana.

positive magnetic pole because of its remote depth. It is seen that the lower parts of the curves are in good agreement, negative anomalies being absent in each case, and that the calculated value is decidedly too small for the position directly above the casing. The failure to observe negative values in the field supports the supposition that the effect of the lower pole may be disregarded, though it is well to mention that this same effect would be noticed had the casing extended a sufficient distance above the ground to establish the upper pole at a position above the magnetic system of the variometer. For points near the casing head the discrepancy between the curves is exaggerated because of the tremendous ratio of L to x.

It has been repeatedly argued that the pronounced magnetic "highs" associated with certain producing fields may be largely attributed to magnetized casing. As we have seen that theory and experiment are in complete accord regarding the *highly localized* nature of the disturbance traceable to this cause, we are naturally led to a consideration of the possible *areal* distortion of the magnetic elements in conjunction with producing fields. This, in fact, brings to the attention the third assumption upon which the theoretical treatment has been based, that is, the symmetry of the normal earth's field suffers no distortion by virtue of the presence of the magnetized pipe. While this interpretation is supported by its general acceptance for mathematical analysis,[1] the most conclusive evidence is that results derived in this manner may be verified by precise experiments. Only observational data regarding the possible areal effect will be considered here.

In December, 1928, at the writer's direction, a vertical intensity survey was made of the Sligo field, located in T. 17 N., R. 12 W., Bossier Parish, Louisiana. At that time there were 17 producing wells in the field, this number being increased to 45 by January, 1931, when a second series of measurements were made at the same group of stations. It is felt that this field offered exceptional opportunities for determining the possibility of an areal disturbance for the reason that only minor magnetic reflections of the subsurface relief were obtained; hence any extraneous influence would be easily recognized. The locations of the wells, together with a selected series of magnetic stations, are shown in Figure 7, and for each survey the anomalies, with respect to station No. 1, are

[1]A. Gray, *op. cit.*, p. 50.
W. S. Franklin and B. MacNutt, *Advanced Theory of Electricity and Magnetism* (Macmillan, New York), p. 85.

indicated in Table III, the respective differences between the observations being included to facilitate comparison.

TABLE III

Station Number	*DZ in Gammas* *1928*	*DZ' in Gammas* *1931*	*DZ-DZ'*
1	0	0	0
2	− 4	−10	− 6
3	− 5	−12	− 7
4	−10	− 6	+ 4
5	+ 9	+ 9	0
6	+36	+16	−20
7	+32	+41	+ 9

It is seen that, with the exception of station No. 6, the variation between the surveys does not exceed the probable instrumental error, the difference at 6 being of little consequence, as this position is well removed from the area of potential disturbance. It is interesting to note, in connection with these data, that a circle having a diameter of 3 miles may be circumscribed about the field so as to enclose every well, and that the mean variation between the two series of observations, for the three stations within the circle, is only 1 gamma. Certainly, these results do not indicate that the original pattern of the vertical field has been altered by the presence of the additional wells.

CONCLUSION

We have seen that it is possible, with our derived formulae, to delineate with fair approximation the distribution of the magnetic elements within the surficial zone that surrounds an element of magnetized casing. These equations were not developed for the purpose of determining the appropriate correction belonging to an effected observation, but rather to explain and amplify our experimental evidence relating to the phenomena. By a more elaborate process we might define the observed effects with precise fidelity, although it is well to remember that the geophysicist is primarily interested in the attenuation of the disturbance and for this purpose the present derivations are entirely adequate.

In concluding, the writer wishes to summarize the following pertinent deductions.

1. The magnetic disturbance caused by an isolated string of magmetized casing is of a highly localized character.

2. The maximum effect, which occurs in the immediate proximity of the casing head, varies as the physical proportions of the pipe, increasing very nearly as the cross-sectional area of metal at the casing head.

3. From the present investigation it seems that the anomalous vertical effect disappears at a radial distance of approximately 200 feet from the casing head, and it is the writer's opinion that, for any commercial-size casing, this disturbance may be ignored at a distance of 500 feet.

4. In producing fields there appears to be no areal deformation of the vertical element, aside from the influence within the concentric zones surrounding the casing heads.

BRUNTON COMPASS ATTACHMENT FOR MEASUREMENT OF HORIZONTAL MAGNETIC INTENSITY[1]

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ABSTRACT

Utilizing the Brunton compass in common use by geologists, an attachment has been devised making it possible to measure the horizontal intensity of the earth's field. The operation of the instrument, the theory and derivation of equations, and the results of several surveys with the instrument are presented.

INTRODUCTION

As the Brunton compass is in almost universal use by geologists and engineers, an attachment which enlarges the usefulness of the compass is important. The attachment described herein is designed to measure the absolute horizontal intensity of the earth's magnetic field with sufficient accuracy to be of aid in some classes of geophysical work. The instrument is not intended for close or accurate magnetic investigations, but should be used where the anomalies exceed 250 gammas. The accuracy of the instrument in magnetic investigations is comparable with the accuracy obtained by the Brunton in traverse work.

CONSTRUCTION OF INSTRUMENT

The attachment consists of the compass holder and an auxiliary magnet arm.

The compass holder is constructed of non-magnetic metal, with slots which fit corresponding projections on the bottom of the Brunton compass. Two clamping screws prevent the compass from moving or falling off the compass holder when the attachment is in use.

The auxiliary and detachable magnet arm is graduated in millimeters and carries a magnet holder with a vernier reading device. The magnet holder is moved along the auxiliary arm by means of an adjustable rack and pinion. The auxiliary magnet arm makes an angle of 120° (measured in a clockwise direction) from the north index of the compass box. In

[1]Read before the Association at the San Antonio meeting, March 21, 1931. Manuscript received, March 14, 1931.

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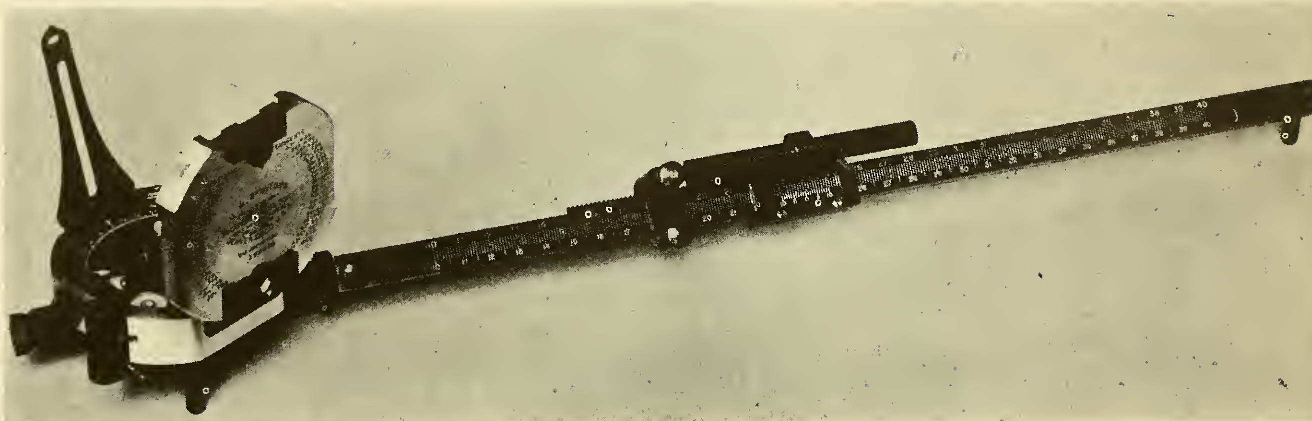

Fig. 1.—Brunton compass attachment for measurement of horizontal magnetic intensity.

using the instrument as a magnetometer, zero of the graduated circle is set exactly beneath the north index of the compass box.

OPERATION OF INSTRUMENT

The instrument may be used on a flat board or table or on an 18 × 24-inch planetable, or it may be mounted on a special tripod. The writer has found the use of the 18 × 24-inch planetable the most satisfactory. As the planetable is usually part of the geologist's equipment and if properly constructed is made of non-magnetic material, its use is recommended.

The steps of setting up the instrument and determination of horizontal intensity are as follows.

1. Place the ring on the base of the Brunton compass in the circle on the compass holder so that the projecting screw on the base in the northeast quadrant fits into the locating hole. Then clamp the compass firmly in the holder by means of the clamps and clamp screws engaging the slots on either side of the compass box.

2. Attach the auxiliary arm to the compass holder and clamp it by means of the clamping screw, move the magnet carrier with the rack and pinion into the approximate working position, and clamp the rack to the arm with the lower screw.

3. Level and approximately orient the instrument on the planetable or tripod with the aid of the level bubbles in the compass.

4. With the zero line of the graduated circle opposite the index, turn the whole instrument in a horizontal plane until the north end of the needle points exactly to the north point on the graduated circle. A small reading glass (containing no magnetic material) is of assistance. The auxiliary magnet should be at a distance of about 20 feet so that it can have no possible effect on the needle.

5. After orienting as in step 4 with the needle at rest, place the auxiliary magnet in the magnet holder with the south-seeking end of the magnet toward the compass. This should be done carefully so as not to disturb the level or orientation of the instrument.

6. Move the magnet carrier (with magnet in place) by means of the rack-and-pinion device until the north end of the needle is exactly at the 30° mark. In this position the needle is perpendicular to the axis of the auxiliary magnet and the magnet is in position known as the first position of gauss. It is suggested that the operator hold one hand on the auxiliary arm between the compass and the magnet while moving the magnet holder so that the orientation shall not be disturbed.

7. Read the distance, r, on the auxiliary magnet arm by means of the vernier to the nearest one hundredth of a centimeter.

At the conclusion of an observation, the compass box may be closed and the instrument may be carried already set up to the next station.

THEORY

The general theory of such a set-up can be obtained from any good book of physics.[1]

The equation which applies to the instrument when used as above is as follows:

$$H = \frac{4M}{r^3}\left(\frac{1}{1 - \frac{L^2}{2r^2} + \frac{L^4}{16r^4}}\right) \qquad (1)$$

where H is the horizontal intensity at the desired point; M is the magnetic moment of the auxiliary magnet; r is the distance between the center of the needle and the center of the magnet; and L is the length of the auxiliary magnet.

If the values of r are large in comparison with L, equation (1) reduces to

$$H = \frac{4M}{r^3} \qquad (2)$$

For anomalies of fair magnitude, equation (2) is sufficiently accurate.

As in any survey L and M are constant, a table or graph of the value of H for any value of r can be computed and used where absolute values are desired. The magnetic unit must, of course, first be determined by any of the customary methods.

When effects, such as temperature and diurnal variation, are less than the observational errors of the instrument, they may be neglected.

PROBLEMS TO WHICH APPLICABLE

With the vernier arrangement it is possible to read r to the tenth of a millimeter. With the strength of magnet commonly used it would be impossible to obtain the intensity closer than 25 gammas. Repeated set-ups by a competent observer at the same station checked within 0.025 centimeters of the mean value, which is equivalent to a departure

[1] William Watson, *Text Book of Physics* (Longmans, Green and Company, 1919).

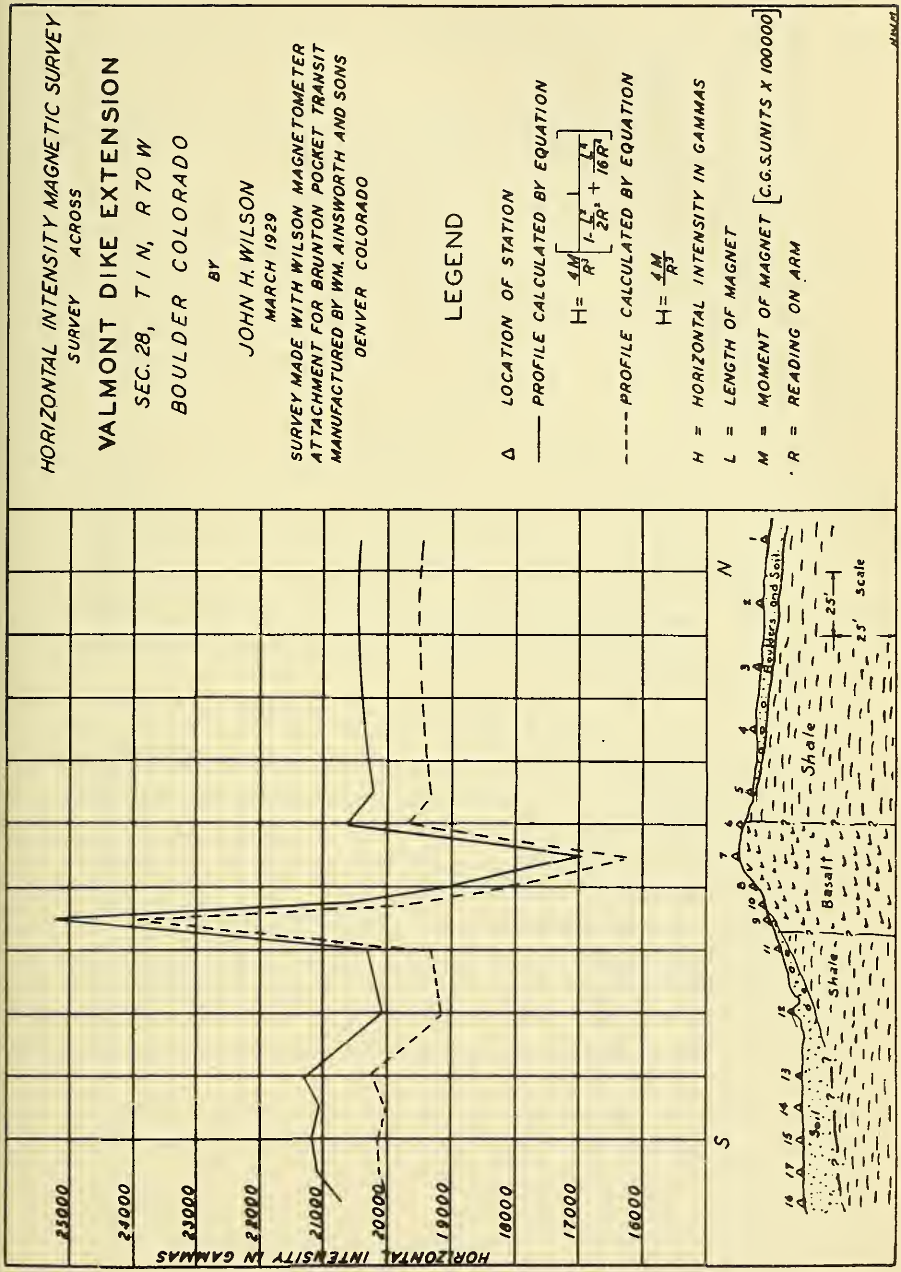

FIG. 2.—Results of survey with instrument near Boulder, Colorado.

of about 70 gammas from the mean. In practice it has been found that the instrument does not give satisfactory results on anomalies of less than 250 or 300 gammas.

The instrument is, therefore, applicable to investigation of larger anomalies. It gives fair results over the larger anomalies found in oil work, over basic dikes, and over magnetic ore bodies.

The instrument is so designed as to work in a range from 5,000 to 100,000 gammas absolute horizontal intensity. Without use of magnets of different strength, the instrument could not be used in regions where the intensity fell below 5,000 gammas or rose above 100,000 gammas.

It is not recommended that petroleum geologists use the instrument for regular reconnaissance work. It is best adapted to horizontal intensity work on large vertical anomalies which have already been located and to the investigation of igneous intrusions.

Mining geologists should find the instrument of aid in the tracing of some types of dikes, the limits of intrusive bodies, and the investigation of iron ore deposits. However, the instrument can not be expected to work on sulphide ore bodies not associated with magnetic minerals.

The instrument is well adapted to illustrate methods of measuring the intensity of magnetic fields, the moments of magnets, magnetic couples, and other magnetic phenomena. It would be of considerable use to a teaching physicist.

SOME RESULTS OBTAINED

In Figure 2 are shown the results of a survey across a small dike of basaltic rock near Boulder, Colorado. The dike could readily be traced with this instrument. Calculation of results by equations (1) and (2) are shown. It may be observed that for location work the simplified equation could be used satisfactorily.

In Figure 3 are shown the results of a horizontal intensity survey across the Mankato "high" in Jewell County, Kansas, which had a vertical anomaly of approximately 900 gammas. Below the profiles are shown the vectors calculated from the horizontal and vertical anomalies. It should be noticed that the vectors are calculated in the magnetic meridian, whereas the section was surveyed in true north and south directions. The results indicate that survey with this instrument is satisfactory on anomalies of this size and that they may give valuable information as to whether polarization is present, and if so, whether the magnetic axis is vertical or inclined, and whether the magnetic axis of the disturbing body should be regarded as of finite or infinite length.

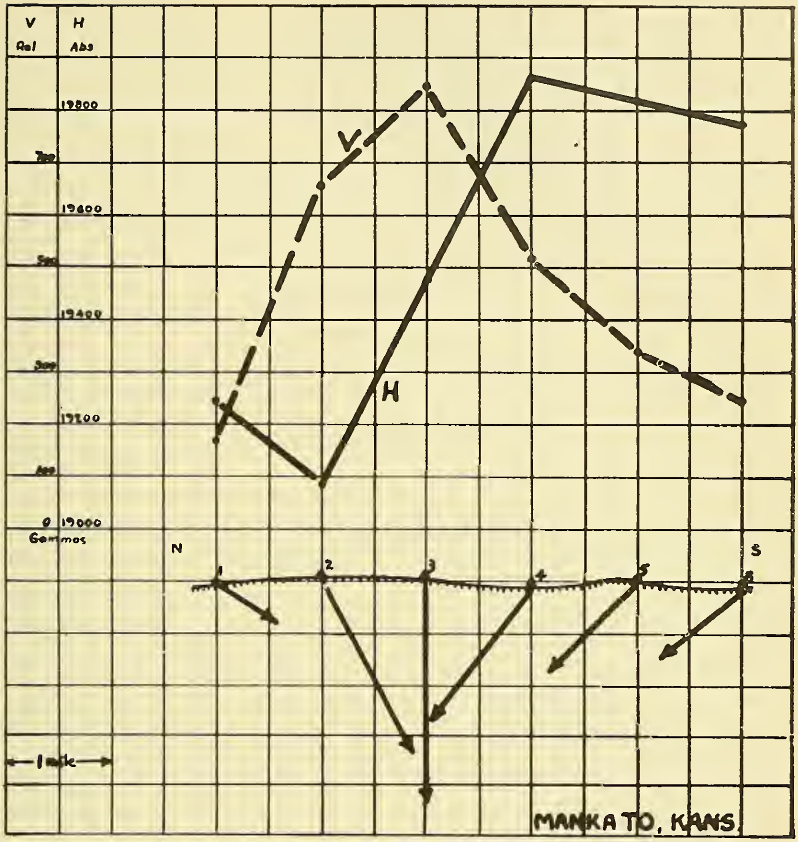

FIG. 3.—Profiles across "high" near Mankato, Kansas. Station 1 is at NE. cor. Sec. 1, T. 3 S., R. 9 W. Vertical intensity obtained with Schmidt magnetometer; horizontal intensity, with Brunton compass attachment.

By triangulation procedure, depth to the disturbing mass could be calculated under some conditions.

BULLETIN

of the

AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS

Volume 15 DECEMBER 1931 Number 12

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Volume 15 Number 12

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AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS

DECEMBER 1931

UTILIZATION OF EXISTING WELLS IN SEISMOGRAPH WORK[1]

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ABSTRACT

The writers describe a method of utilizing a well in a seismic exploration of the surrounding area for deep domes, with particular reference to a supposed dome near the well which, for any reason, may have missed its objective. Most of the paper is devoted to methods of utilizing a well near the flank of a salt dome for the purpose of making a seismic profile deep down the flanks. These methods are especially adapted to determining the extent of "mushrooming" such as that existing at Barbers Hill and the Allen dome.

The present economic status of the oil industry, regardless of its cause, is having the effect of directing considerable attention to improved efficiency in prospecting and production methods. In this connection, geophysicists have devoted much thought and effort to developing more accurate means for determining well locations. Some of the more recent seismic methods which have been proved successful are explained here in a qualitative manner with the intention of avoiding burdensome mathematics.

EXPLORING FOR VERY DEEP DOMES

During the past year or more, use has been made of existent wildcat wells for the purpose of making a seismic exploration of the surrounding

[1]Read before the Association at the San Antonio meeting, March 21, 1931. Manuscript received, May 28, 1931. Published by permission of McCollum Exploration Company.

[2]President and technical director, McCollum Exploration Company, Esperson Building. Introduced by George M. Bevier.

[3]Geophysicist, McCollum Exploration Company.

territory for very deep domes. The principle of the method used is shown in Figure 1. Here, a detector of seismic waves is lowered to the bottom

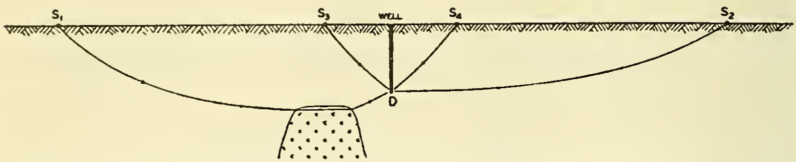

FIG. 1.—Diagram illustrating use of well for exploring surrounding region for deep domes. Patent applied for.

of a wildcat well which may have missed the dome. With the detector in this position, a series of shots is fired on the surface, on a circle having the center approximately at the well and the circumference at a distance ranging from 5 to 7 miles from the well. It is obvious that if a dome exists anywhere within a radius of several miles of the well, as shown in Figure 1, the shots from one direction, as S_1, will show greater velocity than from S_2 and other directions, thereby indicating the presence and position of the dome. If the top of the dome is no more than 2,000 feet below the bottom of the well, actual salt occurrences will be recorded on the detector, thereby giving a very definite lead in the direction of the dome. If the dome is much deeper, however, it can be detected because of the increased velocity in the shale overlying the dome, as this shale is more consolidated and is under abnormally great pressure. In the latter condition, the increase in velocity, especially if the dome is extremely deep, may be small enough so that, in order to make certain of exact shooting distances, it may be necessary to determine whether the bottom of the hole is directly under its surface location. This can be accomplished by the seismograph by shooting four additional shots in four different directions approximately a mile from the well, as indicated by S_3 and S_4 in the section shown in Figure 1. If the bottom of the hole deviates in any direction from vertical, this is revealed by the difference in time of arrival of the sound wave from the four different directions. By this method the direction and amount of drift of the hole can be determined.

It is obvious that by this method the intervening region between the well and the distant shots, particularly that part within 2 or 3 miles of the well, can be explored for deep domes to a depth considerably in excess of the depth of the well used for the detector.

Practical experience with this method has demonstrated its usefulness and there is no doubt that the method can be made very effective in exploring the Gulf Coast for domes to a substantially greater depth than has heretofore been possible by the usual surface methods of operation. This method has been used only in connection with wildcat wells. It is now proposed greatly to extend its usefulness by utilizing existing wells that have been drilled off the flanks of known domes. Literally hundreds of such wells are available at the present time for this purpose, because any well open to a depth ranging from 4,000 to 6,000 feet, or deeper, can be used. It does not matter whether the well is pumping or whether it is a dry hole. If a flowing well is used, it may be necessary to restrict its flow during the period it is being used for the test. The proposal here is to select any suitable well off the flank of the dome, as at *A* in Figure 2, and, after placing the detector down near the bottom of the

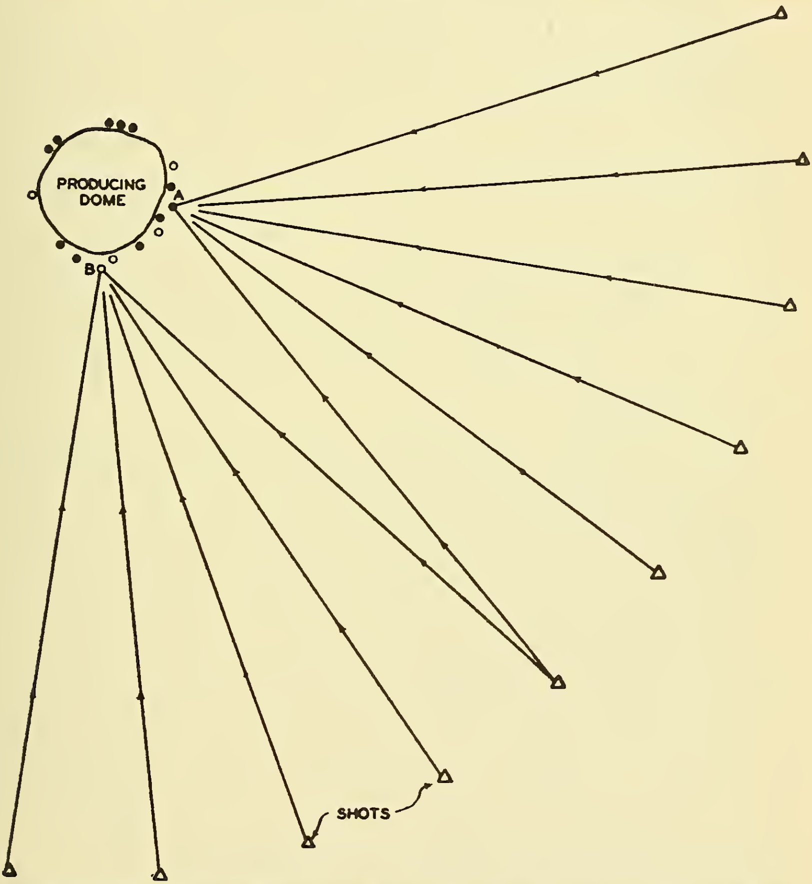

Fig. 2.—Plan diagram illustrating method for deep exploration of region surrounding known domes.

well, shoot a fan of perhaps a 60°-90° angle off the same side of the dome as the well. Then another well, as at *B*, would be used, as shown. On this basis, in order to explore the territory entirely surrounding the dome, it would be necessary to have five or six wells located at suitable places around the dome. It is estimated that by this method a thorough exploration of the entire territory within 3 or 3½ miles from the flank of the known dome can be made down to a depth ranging from 1,000 to 2,500 feet deeper than the depth of the well used in making the test.

Such a deep exploration of the area surrounding known domes might reveal several deep twin domes.

DEEP PROFILING OF SALT DOMES

The problem of profiling deep down the flanks of salt domes has heretofore presented very serious difficulties, and because of these difficulties few attempts have been made to profile more than the top of the dome and that part extending a very short distance over the shoulder. The recent successful use of the described method of placing detectors in wells and exploring the surrounding country for deep domes has opened the way for the development of an accurate method of profiling down the sides of salt domes to depths ranging from 5,000 to 6,000 feet. A method for accomplishing this purpose has always been considered extremely desirable, but it has recently become of vastly greater importance with the discovery of the mushrooming tendency on some salt domes in the Gulf Coast region.

The general method of procedure is simple, as shown by an inspection of Figure 3. Here a shot is placed as shown at some convenient point on the opposite side of the dome from the well which is being used in the exploration; a detector is let down in the well to any suitable position such as D_1, and a shot made. The detector is then lowered in the well to the point D_2, and so on, shots being made with the detector spaced at suitable intervals down to the bottom of the well. It is readily understood that the total time of travel of the shot to different detector positions is a function of the spacing between the well and the salt mass. With a knowledge of the velocity of sound in rock salt and its velocity in the deep shales, together with the geometrical relationships involved, it is possible to calculate the form of the profile of the dome opposite the well with considerable accuracy. In this way it can be determined with certainty whether or not the dome slopes continually outward or whether it recedes with increasing depth, as shown in Figure 3.

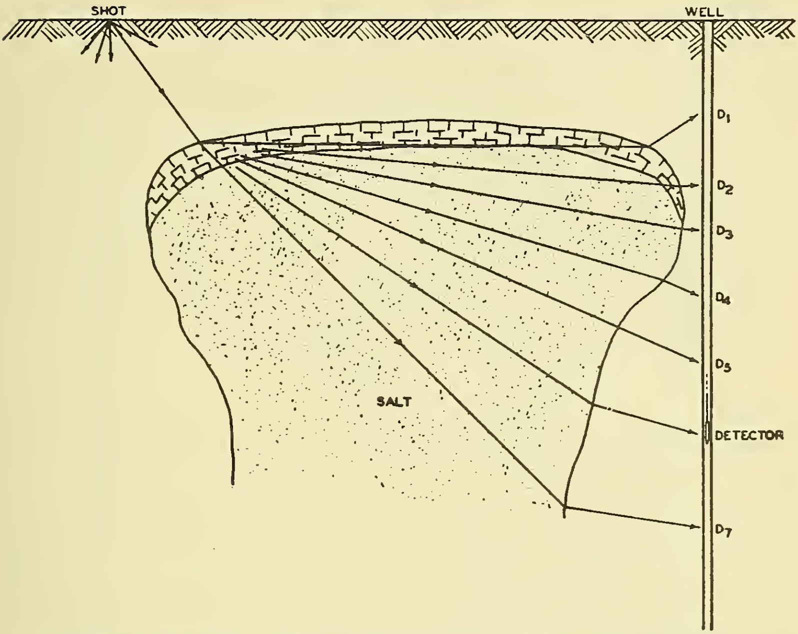

FIG. 3.—Sectional diagram showing use of well for profiling mushroomed flank of salt dome. Patent applied for.

If mushrooming is found to exist, the position of the flank of the dome, relative to the well, can be determined with considerable accuracy to a depth almost as great as that of the well.

Because of the very many deep or moderately deep wells around salt domes in the Gulf Coast region, many of which are, or could be made, available for work of this kind, this method may have extensive application. Any well open to the desired depth could be used and no injury to producing wells would result.

A further application of this method consists in the determination of the position of a fault plane at different depths, as shown in Figure 4. Here, for example, a well which was drilled too far back from the fault plane to obtain oil is used in determining the position of the fault plane for the purpose of making a second location.

GENERAL PROBLEM OF DETECTION OF MUSHROOMING

Use of the method discussed for profiling down the sides of salt domes determines not only the existence of mushrooming, should this exist, but

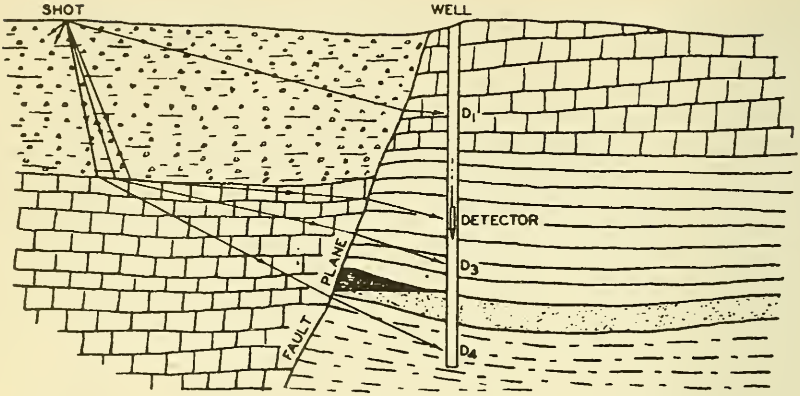

FIG. 4.—Diagram showing use of well for determining position of fault plane at different depths. Patent applied for.

also determines, with sufficient accuracy for all practical purposes, the extent of the mushroom condition. This can be done by the method previously outlined only in a zone immediately adjoining a well being used for the test. It is possible, however, by an extension of this method, to determine definitely the existence of mushrooming on a wide zone on the opposite side of the dome from that on which the well is located. The procedure is illustrated in Figure 5. Here the detector is placed at any suitable depth in the well and a shot is fired 4 or 5 miles away from the dome on the same side of the dome as the well. This shot point is designated as Shot Point 1 in Figure 5, its direction being indicated by the arrow. When the shot is several miles away from the dome, experience has shown that under these conditions the wave front, instead of being spherical, becomes somewhat ellipsoidal, the deeper parts of the wave front being far in advance of the shallower parts, as shown by the curved line representing such a wave front in Figure 5. Because of this bulging of the wave front with increasing depth, that part of the wave front which first reaches the detector at D_1 is far in advance of the position of the same wave front which reaches a series of detectors on the surface at D_2, D_3, D_4. If the detectors at all four of these positions be connected with a recorder, the amount of advance of the deeper parts of the wave front in excess of the surface part can be recorded accurately on a film.

Another shot is next placed at Shot Point 2 on the opposite side of the dome, and a detector, D_5, also is placed on the surface at a suitable point approximately symmetrical with respect to D_3 in relation to the

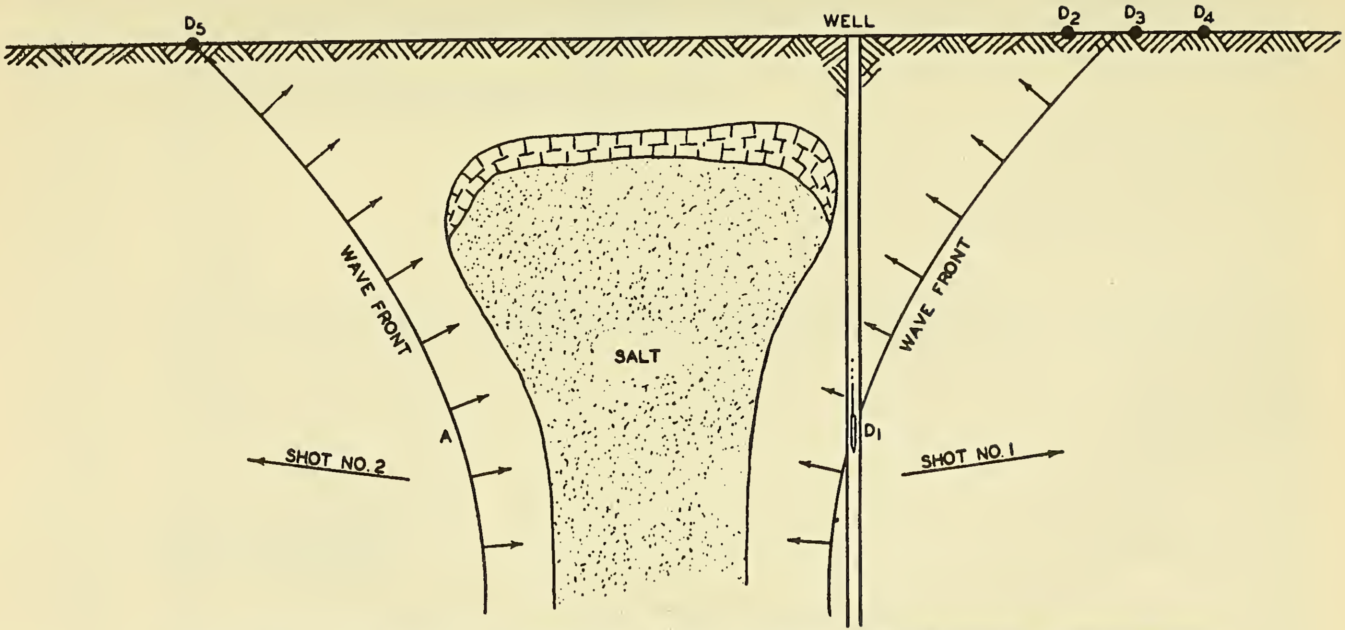

Fig. 5.—Diagram showing method of exploring side of dome opposite well for mushrooming. Patent applied for.

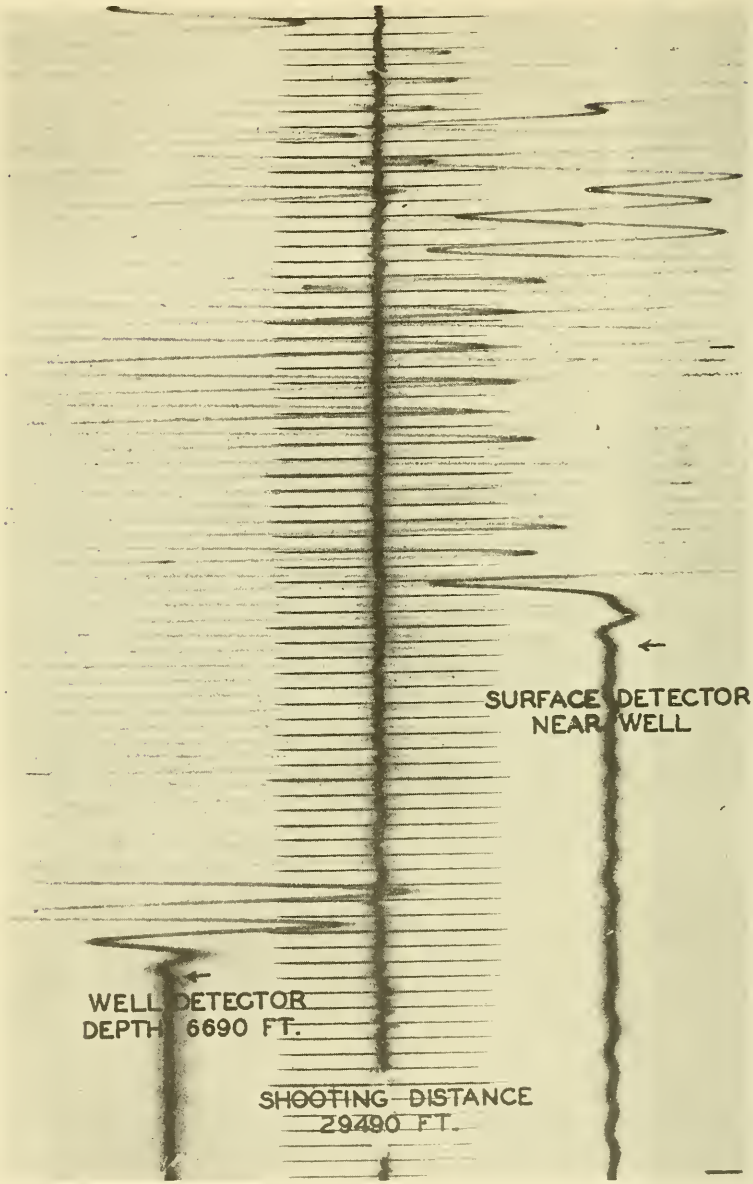

FIG. 6.—Typical record made with special seismic wave detector in well at depth of 6,690 feet (left trace). Notice greater velocity as compared with detector on surface near well (right trace). Shooting distance was 29,490 feet. Middle trace was not used.

dome. This shot is recorded on D_5 at the surface, also at D_1 in the well on the opposite side of the dome. By assuming the wave front to be the same from Shots 1 and 2, it is possible, after determining the time of arrival of the wave at D_5, to calculate the position of the deeper parts of the wave front; therefore, to determine definitely the position of a part, such as A, at a definite known time. From this, and the recorded time of the arrival of the same wave at D_1, the interval time between the positions A and D_1 is obtained. With the knowledge of this interval time and the velocity of sound in the shales and salt, and with the profile previously made of that side of the dome next to the well by the method shown in Figure 3, calculation can be made of the position of a point at a definite depth on the flank of the dome opposite the well.

Not only can this operation be completed on a line directly opposite the well, but similar lines can be shot, making an angle ranging from 30° to 45° on either side of the diametral line. Therefore, by the use of a single well, not only can the flank of the dome next to the well be accurately profiled, but it is possible to determine definitely the existence or non-existence of mushrooming anywhere within a 90° angle on the opposite side of the dome.

It is obvious, therefore, that with two, or at the most, three wells, properly selected around any dome, the entire periphery of the dome can be explored thoroughly for mushroom conditions.

ANALYSIS OF SOME TORSION-BALANCE RESULTS IN CALIFORNIA[1]

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Beverly Hills, California

ABSTRACT

The writer discusses torsion-balance results obtained in the Los Angeles basin. It is shown that here the distribution of gravity which is observed in the vicinity of an oil field is in no way connected with the inherent density of the beds. One phase of the mathematical theory of geophysics dealing with the gravity distribution set up by the folding of an otherwise uniformly dense bed is discussed, and it is shown that, in the region in question, the distribution of gravity is entirely the result of this folding. An approximate method of interpreting gravitational distribution in terms of subsurface structure is described.

Though the practical value of geophysics as a means of direct assistance in the solution of geological problems which confront many branches of oil exploration is far from being realized by many operators in California, this branch of geophysics has made great strides within the last few years, and is now equipped to give remarkably satisfactory subsurface information. The geological conditions in California are admittedly more complex than those of the coastal region of Texas and Louisiana, where geophysics was first used in the oil industry in the United States, but this complexity does not preclude the possibility of good results. After the difficulties have been overcome, more information may be deduced from geophysical studies in this region than in regions of simpler geology. The complexity of conditions in California, however, makes it necessary to leave the interpretation of the results to experts, in the same manner that micropaleontology is now left in the hands of a few men who really understand the work. When this is done the old record of geophysics will be erased and a new record established, which will encourage companies to spend their money on geophysical programs and subsequent leasing and drilling. There continue to exist erroneous ideas as to the nature of the properties of materials which are used in geophysical determinations, so that when the fundamental theory

[1]Presented before the Association at the San Antonio meeting, March 21, 1931. Manuscript received, February 12, 1931.

[2]Geologist and engineer, 438 South Maple Drive. Introduced by Paul B. Whitney.

of geophysics is examined by men who hold such erroneous ideas the result is confusion, and geophysical methods are hindered in getting the endorsement they merit.

It has generally been assumed that the distribution of gravity measured by the torsion balance depends on the configuration of denser or lighter rocks below the surface of the ground; in other words, it depends on the distribution of subterranean mass caused by the inherent density of the contorted beds, and as no inherent density differences are detectable within moderate depths in many places in California, this has been taken as *prima facie* evidence that the torsion balance was not adaptable to California conditions. When it was shown that all California oil fields do affect the distribution of gravity in their vicinity, the effect was attributed to slight inherent density changes, and the method of determining densities from core samples was questioned. Simple reasoning will show that the distribution of gravity in many oil fields in California has little if anything to do with the inherent density of the beds below the surface.

Figure 1 represents a cross section of an oil field in the Los Angeles basin. AA' is the observed gradient profile curve. The maximum ordinates, b and b', are 1,600 feet apart. BB' is the calculated gradient profile curve where a heavier bed of the same cross section just reaches the level surface of the ground at the crest of the fold, the ordinates of the profile curve being proportional to the difference in density of the heavier bed and the surface fill. This profile curve does not reach its maxima within the limits of the sketch. If this heavier bed had occupied a lower position in the series, or a higher position, and had been subsequently eroded to a level surface, the maxima on the profile curve would be still farther apart. Therefore, as the first case considered does not produce a distribution of gravity comparable with that observed in the field, no heavier bed in any position, nor any combination of lighter or heavier beds, nor increasingly lighter or heavier beds, can produce results in any way comparable with those observed. Hence it is proved that the observed distribution of density represented by the profile curve AA' does not depend on the inherent density change of the subsurface formations. Also, as all the oil fields in the Los Angeles basin cause a similar distribution of gravity, the proof may be extended to include them.

It seems probable that the causes which underlie the distribution of gravity over many structures in California must generally be sought in the compaction and rarefaction of the beds caused by folding. The forces of earth movement which give rise to structures generally remain

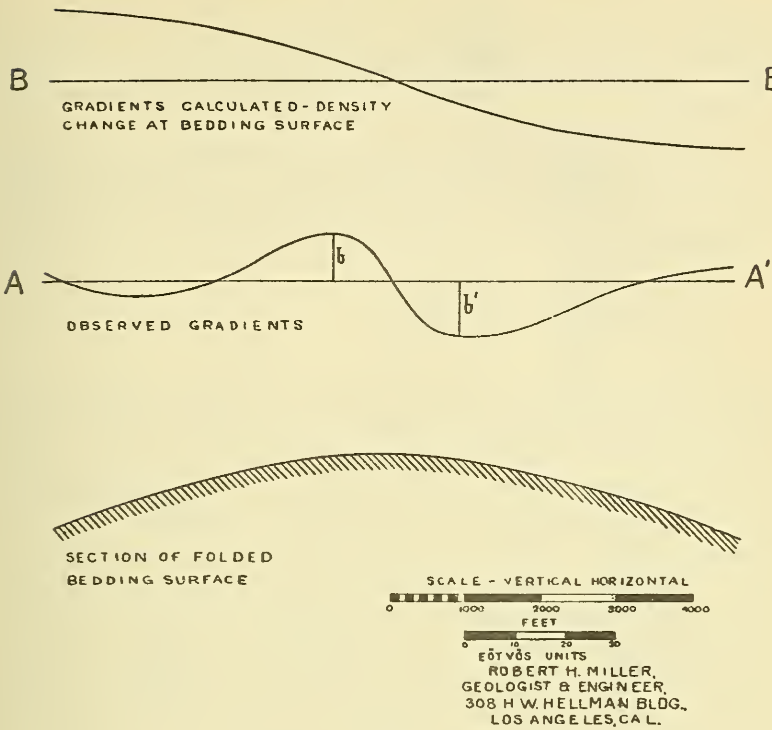

FIG. 1

active through a long period of time, and deposition and erosion occur contemporaneously in many places so that the formations at depth may be more folded than those at the surface. The effects of distance, however, more than offset the greater degree of folding that may occur at depth, so that only the folding of the surface formations has an appreciable effect on the distribution of gravity at the surface. When a whole series of formations is folded, the series may be divided into members that react to folding in the manner of rigid bodies and those that flow plastically or slip on cleavage planes; otherwise the strain in the materials would exceed all possible limits. It is not necessary to assume that sand beds exhibit the properties of rigid bodies and that under linear compression shales flow plastically, but it is a fact that surface sand formations, within 1,000 or 2,000 feet of the surface, when folded, assume a form that only rigid bodies would acquire under the circumstances. If allowances are made for the effects of friction, this form may be expressed as a sine function. It will be shown that the distribution of gravity observed throughout many California oil fields is mainly the result of the folding of the surface formations which behave like a rigid surface crust.

AB (Fig. 2) represents a vertical section of a sand formation through one of the Los Angeles basin oil fields drawn to scale. Below this sand formation are blue shales. The sine function assumed by this bed may be expressed by

$$y = 1{,}000 \left[1 - \cos \frac{x\pi}{9{,}000} \right]$$

where x and y represent, respectively, distance and height in feet from the origin O, of points on the lower surface of the bed. The points a and b are points of inflection at which there is no bending. Between a and b the arch is convex upward so that the part near the upper surface is extended, and the part near the lower surface is compressed. Between A and a, B and b, the arch is convex downward and the reverse is true. There is, therefore, a part of the bed between the upper and the lower surface which is neither extended nor compressed. This part of the bed is named the neutral surface, and its vertical section is the neutral axis. One of the consequences of bending is that at any point in a normal section the strain is proportional to the distance of that point from the neutral surface. As the change of density is equal to the strain, the terms may be used synonymously. If compression is considered as a positive density change and rarefaction as a negative density change, and if points above the neutral surface are considered positive and points below negative, the distance of a point from the neutral surface in a normal section, where the density change is δ, is given by a, where

$$a = \frac{\rho\delta}{1 + \delta}$$

ρ being the radius of curvature of the neutral axis at that section, being positive where the axis is concave upward and negative where the axis is concave downward. As the radius of curvature of a sine curve passes from a minimum at the points A, B, and C, to infinity at the points of inflection a and b, there is a deficiency of mass concentrated above the neutral surface at C, and below the neutral surface at A and B, and an excess of mass below the neutral surface at C and above the neutral

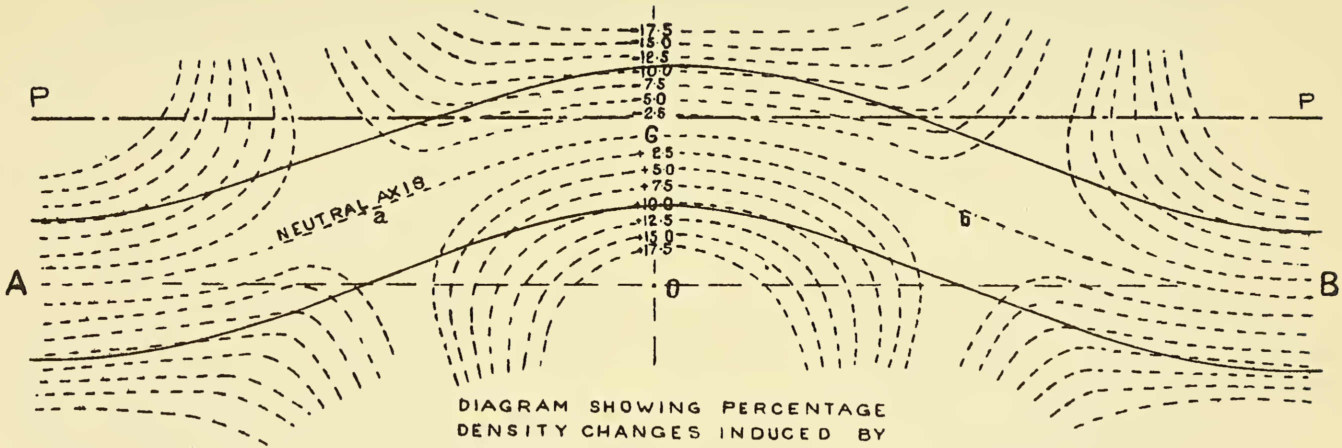

DIAGRAM SHOWING PERCENTAGE DENSITY CHANGES INDUCED BY FOLDING.

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FIG. 2

surface at A and B. The dashed lines represent surfaces of equal density, the figures referring to the percentage changes. It is thus evident that the mass within the bed is redistributed into six zones, occurring in pairs. The zones of each pair are of opposite nature and in the same vertical line. It is evident that the distribution of gravity at the surface caused by these zones depends on their magnitude and position relative to one another, which is a direct function of the fold, and their position as a group relative to the surface of the ground, which is a function of subsequent erosion, deposition, or earth movement. On the assumption that the normal density of the sand bed in Figure 2 is 1.8, and that subsequent to folding it has been buried to its crest under material of the same density, it is seen that the distribution of gravity would be represented by the calculated gradient profile curve BB' (Fig. 3). If erosion then took place to the plane PP (Fig. 2), the position of the base of the sand formation now coinciding with the blue shale contact of this particular field, the calculated distribution of gravity would be represented by the gradient profile curve CC (Fig. 3). The close similarity of this curve and the observed gradient profile curve in this field (curve AA, Fig. 3) is sufficient proof that the compaction and rarefaction of the surface formations resulting from folding is the direct cause of the observed distribution of gravity.

The practical value of this theory as a means of interpreting torsion-balance results in terms of subsurface structure can be illustrated best by considering an actual example. Curve HK (Fig. 4) is the actual gradient profile bed across an area in the Los Angeles basin drawn to the scales shown. This curve is complex and is obviously the composite of several simpler curves. On analysis the curve is found to be composed of three simpler curves, LM, NQ, and SV; that is to say, the ordinate of any point on curve HK is the algebraic sum of the ordinates of the corresponding point on curves LM, NQ, and SV. It is evident from what has been said before that curve LM represents the distribution of gravity set up by the zone of compression below the crest of an anticline, and curve NQ represents the distribution set up by the remnant of a zone of extension above the crest of the anticline. Curve SV represents the distribution of gravity set up by the zones on the flank of the anticline, the upper zone of compression masking the lower zone of extension. As there is no disturbance on the other side of the anticline corresponding with SV, there is clearly a fault on that side against which the beds were folded. Referring to Figure 2, it is seen that, in the zone of compression at the axis of the anticline, the surfaces

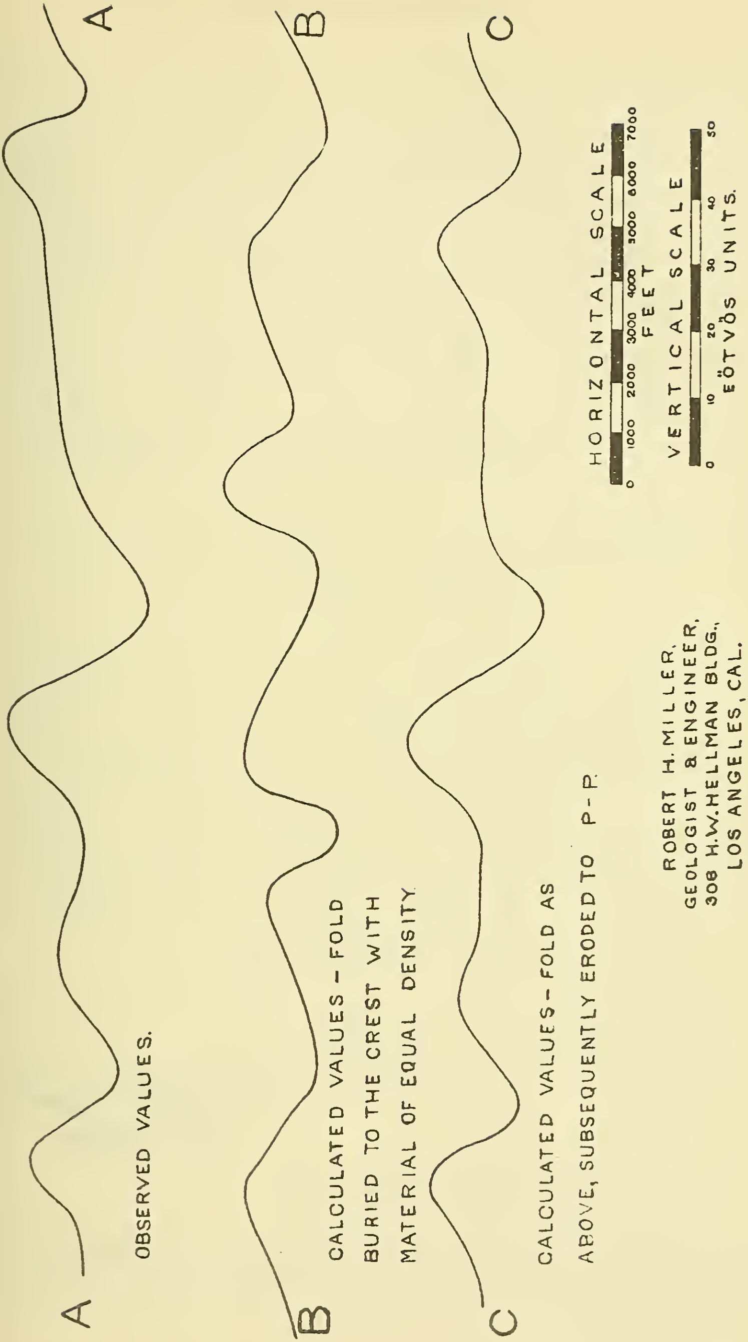
A
A
OBSERVED VALUES.
B
B
CALCULATED VALUES – FOLD
BURIED TO THE CREST WITH
MATERIAL OF EQUAL DENSITY
C
C
CALCULATED VALUES – FOLD AS
ABOVE, SUBSEQUENTLY ERODED TO P-P.
HORIZONTAL SCALE
0 1000 2000 3000 4000 5000 6000 7000
FEET
VERTICAL SCALE
0 10 20 30 40 50
EÖTVÖS UNITS.
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FIG. 3

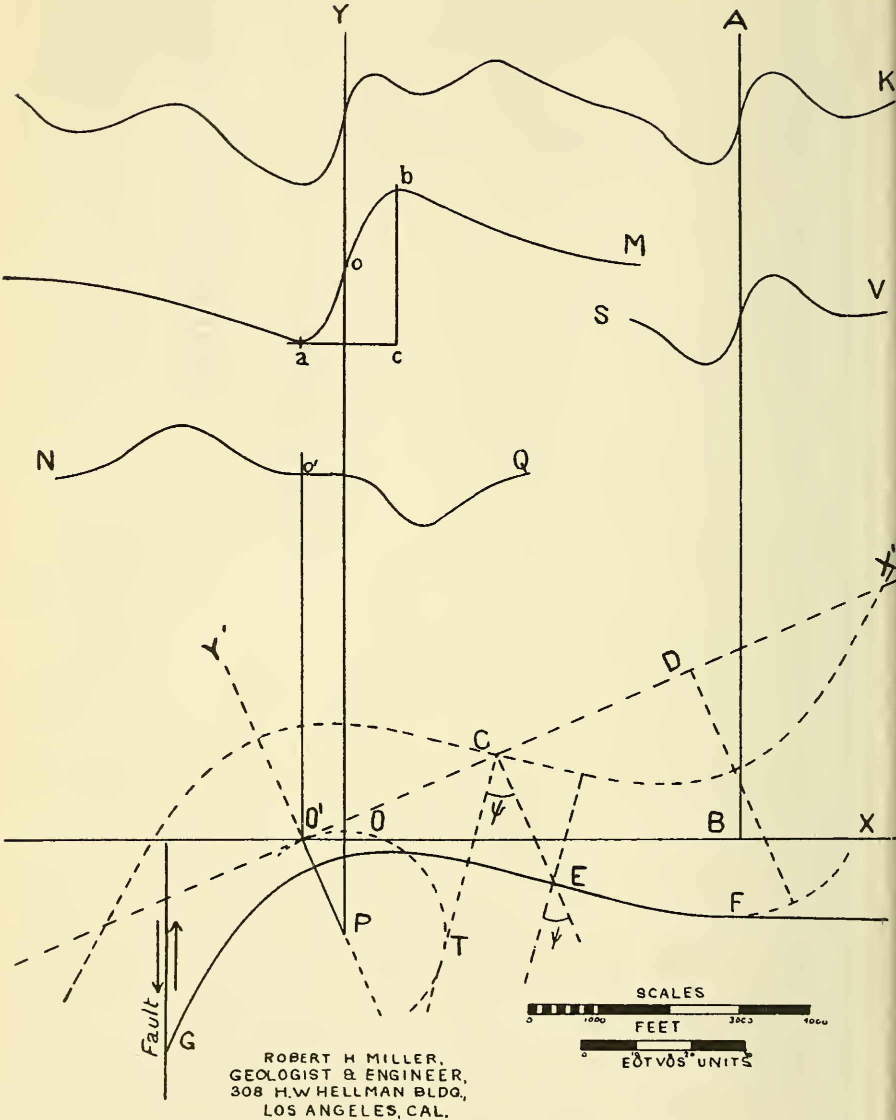

FIG. 4.—Graphical construction for the interpretation of results. For explanation see text.

of equal density present approximately concentric cylindrical surfaces in the immediate vicinity of the axis and that the center of curvature of these parts of these surfaces is approximately equidistant from the horizontal line tangent to the neutral axis and to the normals through the points of inflection a and b. Therefore, close to the axis, the distribution of gravity caused by the disturbing mass in this zone is approximately the same as if the mass were concentrated at this point, namely, the center of the circle touching the normals through the points of inflection and the ground surface, to approximate still further. Thus, in curve LM (Fig. 4) the two maxima are the distance ac apart, so that if OX represents the surface of the ground, the point would be located at P, a distance ac below the surface. The point of symmetry of curve NQ, representing the distribution of gravity caused by the remnant of a zone of extension at the axis of the anticline, is offset toward the left from that of curve LM, indicating that the beds have been dragged down by the fault at the time the anticline was formed, and that the sine curve which is the figure of the anticline must be referred to the new axes $X'O'Y'$ instead of the horizontal and vertical axes XOY. The construction is evident from the figure. On account of the rotation of the axes, the period of the sine curve representing the anticline is slightly greater than OB, because at the point F, vertically below the center of the zones represented by the profile curve SV, the beds are horizontal, and not parallel with the new axis $O'X'$. The period of the sine curve is estimated to be $O'D$, and C is the point of inflection of the sine curve referred to the axes $X'O'Y'$ (shown by the dashed line). The tangent CT to the circle $O'T$, with P as center, gives the inclination ψ of the normal through the point of inflection. Now the equation of this sine curve is

$$y' = a\left[1 - \cos\frac{x'\ \pi}{O'\ D}\right]$$

where a is a constant representing the half-amplitude of the sine curve and $O'D$ is the half-period, equal to 6,000 feet, and

$$\tan\psi = \frac{d\,y'}{d\,x'} = \frac{a\pi}{6{,}000} \qquad (1)$$

By measurement the angle TCE (ψ) is 39.5° and the tangent is 0.83, and x equals $O'C$ and is 3,000 feet. Substituting these values in equation (1), a is found to be 1,600 feet, approximately. The section of the anticline, therefore, represented by the profile curve *HK*, is a sine curve having a period equal to *O'D* and a half-amplitude of 1,600 feet. This curve is represented by curve *GEF* in the figure. As the point *G* has been dragged below its original level *F*, it is evident that the fault is down-thrown on the left. As the earth movements which caused the anticline evidently caused motion of the beds from left to right, friction and inertia must have caused the horizontal dimensions to be foreshortened to an extent varying inversely with the distance from the fault, so that the fault is nearer the axis of the anticline than is indicated by this theoretical analysis, and the left limb of the fold is somewhat steeper.

Though this discussion has been devoted solely to the distribution of gravity in the Los Angeles basin, where the density conditions are exceptional, an accurate interpretation can be made of the data acquired from measuring any of the other properties of materials which are utilized in geophysics in any region, provided that the data which have been obtained are complete.

DISCUSSION

Frank A. Moss, Los Angeles, California (written discussion received, March 12, 1931): The author of the preceding paper offers an interesting explanation for gradient profiles in which the points of maximum gradient are too close together to be caused by density differences in beds extending across a fold. In a loosely consolidated bed of sand, folding reasonably could be expected to cause an increase of density as postulated, but it is questionable whether there would be a decrease in density due to tension. However, there still would be gravity "highs" over the crest of the fold and over the flanking synclines.

Other possibilities to be considered in the example given are (1) that the gravity "highs" on either side of the fold could be caused by heavy beds truncated and covered by alluvium and (2) that a combination of lenses of heavy and light materials might distort the effect of inherent changes in density sufficiently to produce the results observed.

These theoretical considerations do not apply to folds in the western side of the San Joaquin Valley, where the beds involved in the folding are fairly well indurated and would be sheared rather than appreciably compressed. Observed variations in gravity can be related satisfactorily to measurable changes in density of the underlying materials.

Donald C. Barton, Houston, Texas (written discussion): Geologically, it seems to me that we must examine more closely the reasoning leading to the assumptions that a soft, poorly consolidated California sediment will act as if it were a rigid body. Without more definite corroboratory data, I would be unwilling to postulate the compression-rarefaction assumed by Mr. Miller.

Geophysically, I can not reach Mr. Miller's conclusions. Mathematically, it commonly is possible to obtain a considerable series of tentative structural profiles all of which produce gradient profiles which check given observed gradient profiles within the limits of the probable error. A close fit of a calculated to an observed gradient profile is not necessarily evidence that the trial structure section is correct. In the Belle Isle survey, I found that geologically improbable assumptions and a trial structure section now proved to be incorrect gave a very much better fit of calculated to observed gradient profiles than assumptions and structure sections 75 per cent of which have been confirmed by subsequent drilling.

A year ago I had the pleasure of studying a rather extensive torsion-balance survey which disclosed the structure of Figure 3 with fair detail and which also gave a fair picture of the gravitational situation of the surrounding area. This study of Mr. Miller's in a preliminary form also was seen by me. My study of that torsion-balance survey seemed to show that the gravitational picture over the particular structure could not be interpreted intelligently if it and the structure were studied separately from the anomaly of the major structural features of that part of the basin. Some of the smaller irregularities in that profile furthermore seemed to me referable to surficial irregularities in density.

Although I can not accept the conclusions expressed in this paper, studies of this type are necessary if we are to improve our technique of interpretation of torsion-balance surveys, and one of the problems which must be investigated is the effect of structural compaction.

THE AMERICAN PHYSICAL SOCIETY

GEOPHYSICS

1931

PAPERS PRESENTED BEFORE THE SOCIETY OF PETROLEUM GEOPHYSICISTS AT THE JOINT SESSION WITH THE AMERICAN PHYSICAL SOCIETY AT NEW ORLEANS, DECEMBER 29 AND 30, 1931

TRANSACTIONS
SOCIETY OF PETROLEUM GEOPHYSICISTS
VOL. II

Reprinted from PHYSICS
MARCH 1932

Published, March, 1932

Composed, Printed and Bound by
The Collegiate Press
George Banta Publishing Company
Menasha, Wisconsin

CONTENTS

SOME ASPECTS OF ELECTRICAL PROSPECTING APPLIED IN LOCATING OIL STRUCTURES

By Leo J. Peters and John Bardeen
The Gulf Companies, Research Department, Pittsburgh, Pa.

(Received February 2, 1932)

Abstract

Electrical prospecting is defined as the science and the art of determining the variations of the electrical constants (resistivity, magnetic permeability and the dielectric constant) of the earth's crust and of interpreting these variations in terms of geological structure. The most successful systems of prospecting are based on the study of resistivity variations. The basic assumption made is that in general changes of resistivity follow the bedding planes. Electrical methods of exploration may be divided into two classes, direct current methods and alternating current methods. In part II the fundamental theory of direct current method is discussed and a typical survey is described. Part III deals with the theory of alternating current methods, with particular reference to the optimum frequency to be used. It is shown that in general very low frequencies are desirable. Two alternating current surveys made by the Swedish American Prospecting Corporation are briefly described. In the conclusion (part IV) some of the difficulties of electrical prospecting are discussed. The depth to which investigations may be carried is limited. In the present stage of the art, it would take exceptionally favorable conditions to obtain reliable information much in excess of 2,000 feet. However, it is stated that improvements in methods of interpretation and in field technique should give electrical methods a definite field of usefulness in prospecting for oil.

I. Introduction

ELECTRICAL prospecting may be defined as the science and the art of determining the variations of the electrical constants of the earth's crust and of interpreting these variations in terms of geological structure or in terms of mineralization of a region. The three electrical constants whose variation may be determined are the resistivity, the magnetic permeability and the dielectric constant.

The resistivities of earth materials vary from a fraction of an ohm·cm to 10^{17} ohms·cm. The materials having values below 100 ohms·cm are confined largely to mineral ores such as marcasite, galena, molybdenite, chalcopyrite and iron pyrite. In the study of oil structures the resistivities encountered vary from 100 to 10^{17} ohms·cm. In the Texas Gulf Coast region the common variation is from 200 to 20,000 ohms·cm with the exception of rock salt of which the resistivity has a value of the order of 10^{17}.

The dielectric constant of earth materials has a value which ranges from 1 to 10. The variation in magnetic permeability, except in rare cases, is negligible.

Systems of electrical prospecting are, therefore, confined to studying the manner in which the resistivity or the dielectric constant varies in the earth's crust. The relative effect which these two constants have on the measurable electromagnetic quantities at the surface of the earth will be discussed later. However, it can be stated that up to the present time, the most successful systems of prospecting have been based upon a study of the resistivity variations in the crust of the earth.

The problems of any scheme of geophysical prospecting may be stated as follows:

First, from mathematical and physical considerations it is decided what data must be taken at the surface of the earth in order to predict the distributions of a chosen physical constant or constants in the earth's crust.

Second, instruments and field technique are developed to obtain these data.

Third, mathematical tools are developed which translate the quantities measured at the surface of the earth into the distribution in the earth's crust of the physical constant or constants under consideration. When these physical and mathematical problems are solved there remains the problem of finding out what relation (if any) the distributions of the physical constants which are determined bear to geology and of interpreting the measurements in terms of geology. Furthermore, to be economically justifiable the overall cost from field measurements to translation into geology must be commensurate with the value of the information obtained.

An ideal system of prospecting for oil would be one which directly indicated the presence of oil. Many attempts have been made to devise an electrical system which would meet this ideal requirement. However, these attempts have met with little or no success. In the present state of the art, the practical systems of prospecting for petroleum are those which are designed to give information concerning subsurface geology.

The basic assumption in mapping geological structures by electrical methods is that in general changes of resistivity follow the bedding planes. Thus a clay bed or shale bed may have a lower resistivity than surrounding limestones or sandstones. These beds usually are parallel with or are only slightly inclined to the surface of the earth. In general, the lateral changes in resistivity are neglected, and the changes in resistivity with depth are determined. If many such determinations are made over an area, the resistivity depth curves may be correlated and the subsurface topography may be obtained.

Generally one or more beds which have a large resistivity contrast with the beds above or below them are used as marker beds in mapping geological structures. It is obvious that the bed which is being used as a marker must conform to the bed carrying the oil in order to be of any practical use as a marker bed in exploring for petroleum. In many localities the relatively shallow beds are more or less parallel to the deeper ones. However, in many other places major unconformities occur and the shallow beds are no longer substantially parallel to the deeper ones. In such cases it is necessary to follow a marker bed which is below the unconformity and is conformable to the oil bearing horizon. The depth to which it is possible to work with any geophysical scheme is, therefore, an important consideration.

The depth which can be obtained reliably in electrical prospecting is a much debated question. Up to the present time the depth at which structures have been reliably mapped by electrical surveys has not exceeded 1500 feet. Electrical surveying has gone but a little deeper than the core drill.

While electrical prospecting is thus a competitor of the core drill, it may be used as an aid in making correlations between core drill holes and also between wells in cases where geological correlation is difficult. This is accomplished by making resistivity measurements in the holes at a series of depths and correlating on a resistivity basis.

Electrical methods of geophysical exploration may be divided into two classes, direct current methods and alternating current methods. The first of these also includes the use of very low frequency alternating currents which are commonly used to eliminate the effects of polarization. The frequency, however, is low enough so that the effect of currents induced by the varying magnetic field may be neglected. Induced currents play the major role in the second, or alternating current method of prospecting.

II. Direct Current Methods of Prospecting

There are a great many direct current methods available for the determination of the changes in resistivity with depth, but the same fundamental

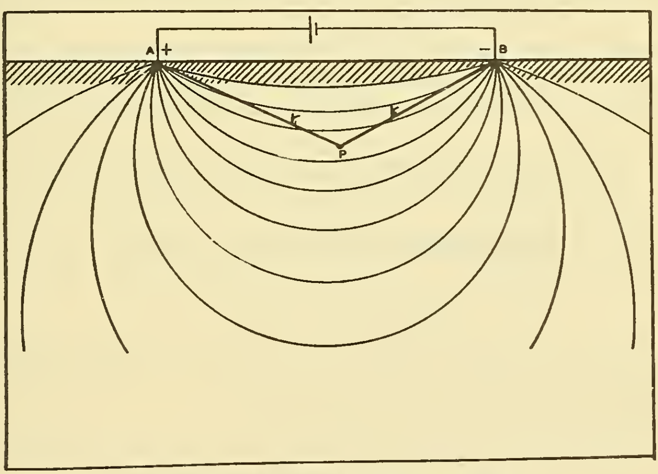

Fig. 1. Lines of flow in a homogeneous earth.

principles apply to all of them. A common practice is to pass the current into the earth by means of two electrodes on the surface, as shown in Fig. 1. If the earth is homogeneous, the lines of flow are arcs of circles connecting the electrodes. The potential on the surface of the earth is then called the normal potential. Any changes of resistivity in the subsurface beds alter this normal potential. Thus measurements of potential on the surface of the earth give an indication of these changes of resistivity. Most prospecting methods differ only in the spacing of the electrodes and in the manner in which the potential is measured. The same mathematical theory thus applies to all of them.

The theory is simplified by the fact that problems of steady flow in continuous media are the same mathematically as similar electrostatic problems. A current I entering an earth of conductivity σ at an electrode A may be replaced by a charge of $q = I/2\pi\sigma$ at A. The potential at any point in the earth, due to this electrode is $V = I/2\pi\sigma r$, where r is the distance from the electrode.

If several electrodes A_n are on the surface of the earth with currents I_n entering the earth, the potential at any point P in the earth is

$$V = \frac{1}{2\pi\sigma} \sum \frac{I_n}{r_n} \tag{1}$$

where r_n is the distance from P to A_n. It may be noted that algebraic values of the current must be used in this sum. Thus the potential at P in Fig. 1 for the current flow between two electrodes is

$$V = \frac{I}{2\pi\sigma}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{2}$$

This additive property of the potentials is general and is not confined to a homogeneous earth. Thus we may simplify the theoretical discussion by considering the potentials due to a single electrode.

In order to illustrate the type of departure from normal potential that may be expected, a simple problem will be solved. The earth is of conductivity σ_1, to a depth a and is of conductivity σ_2, below this depth. A current I flows

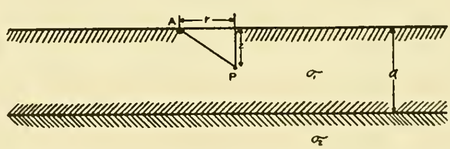

Fig. 2. Coordinate system for two-layer problem.

into the earth at the origin of cylindrical coordinates r, z, ϕ. (See Fig. 2.) Let the potential in the surface layer be V_1 and the potential for $z > a$ be V_2. The conditions to be satisfied by the potential are:

(1) The differential equations expressing the continuity of the currents

$$\nabla^2 V_1 = \frac{\partial^2 V_1}{\partial r^2} + \frac{1}{r}\frac{\partial V_1}{\partial r} + \frac{\partial^2 V_1}{\partial z^2} = 0 \tag{3.1}$$

$$\nabla^2 V_2 = 0. \tag{3.2}$$

(2) The continuity of current and potential at the boundary

$$V_1 = V_2 \text{ at } z = a \tag{3.3}$$

$$\sigma_1 \partial V_1/\partial z = \sigma_2 \partial V_2/\partial z. \tag{3.4}$$

(3) No current flow out of the surface of the earth

$$\partial V_1/\partial z = 0 \text{ at } z = 0. \tag{3.5}$$

(4) The conditions near the electrode

$$V_1 \to \frac{I}{2\pi\sigma_1(r^2 + z^2)^{1/2}} \text{ as } r_1 z \to 0. \tag{3.6}$$

(5) The conditions at ∞

$$\begin{aligned} V_1 &\to 0 \text{ as } \quad r \to \infty \\ V_2 &\to 0 \text{ as } r_1 z \to \infty. \end{aligned} \tag{3.7}$$

This problem has usually been solved by the method of images, which results in an infinite series. An integral solution which is often more useful[1] may be obtained from the following solution of $\nabla^2 V = 0$

$$V = \int_0^\infty [f_1(\lambda)e^{\lambda z} + f_2(\lambda)e^{-\lambda z}]J_0(\lambda r)d\lambda \tag{4}$$

where f_1 and f_2 may be any functions of λ subject to the condition that the integral converges. Thus if

$$V_1 = \frac{I}{2\pi\sigma_1}\int_0^\infty [(A(\lambda) - 1)e^{\lambda z} + A(\lambda)e^{-\lambda z}]J_0(\lambda r)d\lambda \tag{5}$$

and

$$V_2 = \int_0^\infty B(\lambda)e^{-\lambda z}J_0(\lambda r)d\lambda \tag{6}$$

where $A(\lambda)$ and $B(\lambda)$ are arbitrary functions of λ all the conditions (3) will be satisfied except the conditions at the boundary $z = a$. Of course, $A(\lambda)$ and $B(\lambda)$ must be chosen in such a manner that the integrals converge. These functions are determined by substituting V_1 and V_2 given by Eqs. (5) and (6) in the boundary conditions (3.3) and (3.4). The solution for $A(\lambda)$ is

$$A(\lambda) = \frac{1}{1 + ke^{-2\lambda a}} \tag{7}$$

where

$$k = \frac{\sigma_2 - \sigma_1}{\sigma_2 + \sigma_1}$$

On the surface of the earth, $z = 0$, the potential is

$$V_1 = \frac{I}{2\pi\sigma_1}\int_0^\infty \left[\frac{1 - ke^{-2\lambda a}}{1 + ke^{-2\lambda a}}\right]J_0(\lambda r)d\lambda \tag{8}$$

[1] This method is especially suitable when a number of horizontal layers are present.

$$= \frac{I}{2\pi\sigma_1 r} \int_0^\infty \left[\frac{1 - ke^{-2\beta a/r}}{1 + ke^{-2\beta a/r}}\right] J_0(\beta) d\beta \tag{9}$$

where

$$\beta = \lambda r.$$

When a/r is large, the integral becomes

$$V_1 = \frac{I}{2\pi\sigma_1 r} \int_0^\infty J_0(\beta) d\beta = \frac{I}{2\pi\sigma_1 r} \quad r \ll a \tag{10}$$

which is the normal potential for a homogeneous medium of conductivity σ_1. When a/r is small, the exponentials are approximately equal to unity, and the potential becomes

$$V_1 = \frac{I}{2\pi\sigma_1 r} \frac{1 - k}{1 + k} = \frac{I}{2\pi\sigma_2 r} \quad r \gg a. \tag{11}$$

This is the normal potential for a homogeneous medium of conductivity σ_2.

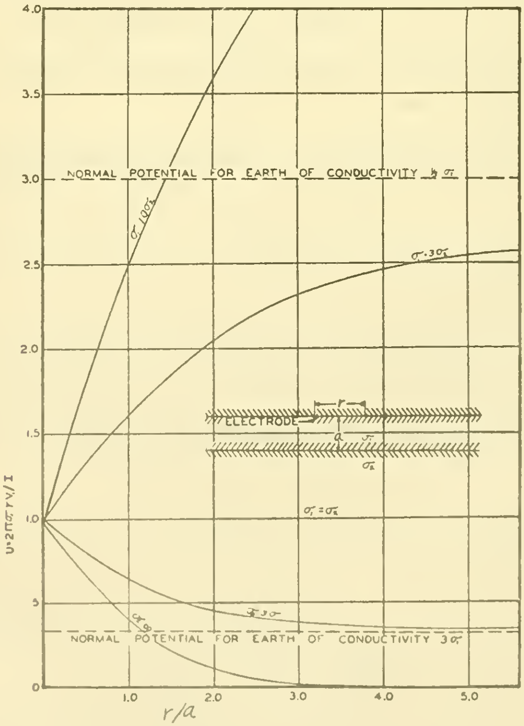

Fig. 3. Theoretical potential curves for two conducting layers.

This example illustrates a fundamental property of this type of prospecting. The potentials at points further and further from the electrode are influenced more and more by the deeper layers. The surface layers become less important. In order to illustrate this effect values of $u = 2\pi\sigma_1 r V_1/I$ were plotted against values of r/a for various ratios of σ_2/σ_1 (Fig. 3). The potentials are multiplied by r in order to eliminate the normal drop of potential with distance. If the ground is homogeneous the potential curve is a horizontal line, the ordinate of which depends on the conductivity. Thus for an earth of conductivity σ_1, $u = 1$ for all values of r. For any other conductivity σ_2, $u = \sigma_1/\sigma_2$ for all r. In Fig. 3 the earth is of conductivity σ_1 to a depth a, below which the conductivity is σ_2. The curves for u start at $u = 1$ for $r = 0$ then approach the lines $u = \sigma_1/\sigma_2$ asymptotically. These curves illustrate the gradually increasing effect of the lower medium as r/a increases.

Fig. 4 shows similar curves for a slightly more complicated case. There is a surface layer of conductivity σ_1, then a second layer of conductivity σ_2, and finally a medium whose conductivity is the same as that of the surface layer,

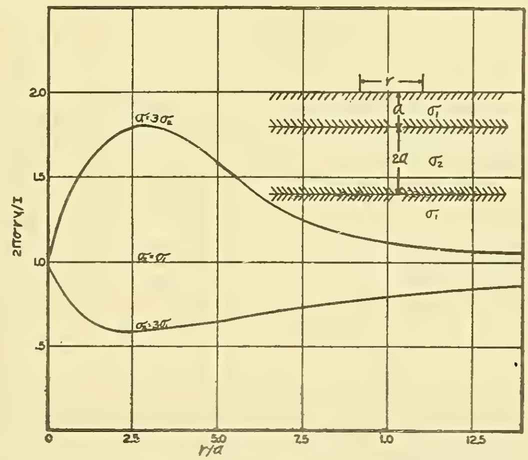

Fig. 4. Theoretical potential curves for three conducting layers.

σ_1. The thickness of the surface layer is a and that of the second layer $2a$. Two curves are shown in Fig. 4: (a) for $\sigma_2 = 3\sigma_1$, and (b) for $\sigma_1 = 3\sigma_2$. In curve (a) $u = 1$ at $r = 0$, and then u decreases for increasing r reaching a minimum at about $r = 2.5a$ where the second layer has most effect and finally approaching the line $u = 1$ asymptotically. Curve (b) rises from $u = 1$ when $r = 0$ to a maximum and then decreases rapidly as it approaches its asymptote $u = 1$. Thus the effect of the second layer on the potential is small near the electrode. It is a maximum at a distance equal to roughly the depth of the layer, and the effect diminishes as r increased beyond this point. These simple cases are sufficient to show how the potential due to a single electrode is affected by vertical changes in conductivity.

In practice it has usually been found more convenient to measure potential differences rather than absolute potential. One method is to put the current electrodes a long distance apart and measure potential differences in the neighborhood of one electrode so that the potentials due to the distant electrode may be neglected. The potential difference is measured between two search electrodes at distances r and $2r$ from the current electrode. If the earth is homogeneous and of conductivity σ, the potential difference between the search electrodes is:

$$V_r = V(r) - V(2r) = \frac{I}{2\pi\sigma}\left[\frac{1}{r} - \frac{1}{2r}\right] = \frac{I}{4\pi\sigma r} \cdot \tag{12}$$

The resistivity ρ may, therefore, be determined as

$$\rho = 1/\sigma = 4\pi r V_r/I. \tag{13}$$

If the earth is not homogeneous, Eq. (13) may be used to define the apparent resistivity. Let $V_1(r)$ be the potential of the current electrode. Then the apparent resistivity is

$$\rho_a = \frac{4\pi r}{I}\left[V_1(r) - V_1(2r)\right]. \tag{14}$$

This function is not independent of r except in the special case of a homogeneous earth. The variation of the apparent resistivity with r is used to determine the changes of resistivity with depth. As may be expected from the potential variation, the apparent resistance is equal to the resistance of the upper layer when r is small. As r increases the apparent resistance approaches the resistivity of the second layer. Long before it reaches this value, a third layer may affect it, and then the apparent resistivity will have a tendency to approach the resistivity of this layer. When a great many layers are present the resultant curve may be rather complicated, and the interpretation difficult.

A more common method of prospecting which leads to the same value for the apparent resistivity was developed by Wenner and Gish and Rooney. The two current electrodes and the two search electrodes are placed in a line equidistant from one another.

$$\underset{A}{\cdot}\!\leftarrow r \rightarrow\!\underset{C}{\cdot}\!\leftarrow r \rightarrow\!\underset{D}{\cdot}\!\leftarrow r \rightarrow\!\underset{B}{\cdot}$$

The current enters the earth at A and leaves at B. The potential difference is measured between C and D. The common spacing is r. The apparent resistivity is again given by Eq. (14), where V_1 again represents the potential due to a single electrode. The advantages of the method are that the electrode spacing need not be excessively great, and that possible errors due to neglecting part of the field do not enter. However, to obtain values of the apparent resistivity for different values of r, all four electrodes must be moved instead of only two as in the previous method. This method is often called the Gish-Rooney method of electrical prospecting.

In order to illustrate the results that may be expected under favorable conditions, a typical Gish-Rooney survey which was taken along a line of core drill holes is shown in Figs. 5 and 6. Fig. 5 shows the curves of apparent resistivity at the six stations shown on the profile of Fig. 6. These curves show a layer of very high resistivity which is at the surface at stations *A* and *B* on the west, and then dips to the east. A layer of low resistivity is on the surface at stations *C*, *D*, *E* and *F*, below which there is the high resistivity layer. There is apparently a very thick layer of low resistivity below this high resistivity layer. A rough qualitative interpretation of these curves is shown

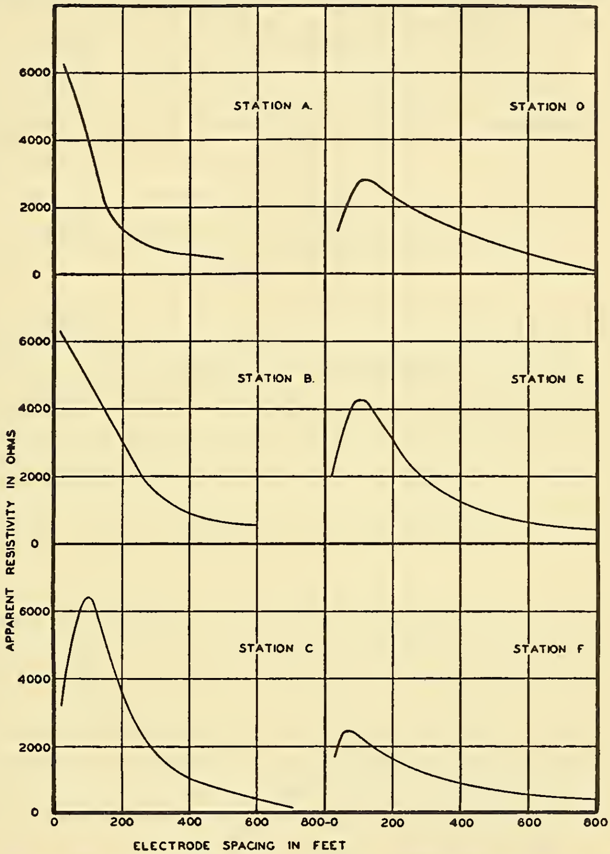

Fig. 5. Apparent resistivity curves obtained by field measurements along profile of Fig. 6.

in Fig. 6. It may be seen that it checks with the core drill holes which show a monocline dipping to the east.

Another method of utilizing the apparent resistivity, the ground resistivity map, has been developed by Schlumberger. An electrode spacing is chosen which is approximately equal to the desired depth of investigation. The apparent resistivities with this electrode spacing are measured at various points on the surface of the earth. A contour map is then made of these values. This method is particularly suitable in a country with a shallow overburden which covers a thick sediment with a large change in resistivity. Then the contours will roughly approximate the topographical contours on the sediment. Suppose that the sediment has a low resistivity. Then when it is near the surface, the apparent resistivity will be low. When the sediment is deeper, the apparent resistivity will be higher.

The apparent resistivity map, of course, gives only a qualitative interpretation of the electrical data. The methods at present in use for interpreting Gish-Rooney surveys give only approximate depths to the points at which sharp changes occur in the resistivity of the earth. These approxima-

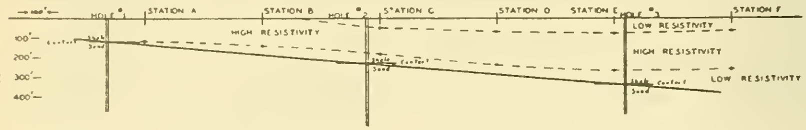

Fig. 6. Qualitative interpretation of Gish-Rooney survey shown in Fig. 5, giving correlation with core drill.

tions become worse and worse as the number of resistivity contrasts become greater. A rigorous solution of the problem of finding the variation of resistivity with depth from surface potential measurements under the assumption that the resistivity is substantially a function of the depth only could be based upon solutions of the potential problem given by the theory of images. However, these solutions are cumbersome to handle when the number of layers which must be dealt with exceeds two. The integral solution given in this paper is much easier to handle in the case of multilayer problems and should aid in placing the interpretation of direct current surveys on a more exact basis.

III. Electromagnetic Methods

Electromagnetic methods depend primarily on the induction of currents in the earth by means of a primary current on the surface. Inhomogeneities in the earth will affect the resultant field on the surface. In oil prospecting these inhomogeneities take the form of practically horizontal beds with different conductivities. Many different schemes have been used with varying success. The factors which should be taken into consideration are:

(1) The primary field should be as simple as possible so that the associated theoretical problems may be solved.

(2) The frequency should be chosen so that the inhomogeneities which are being followed will have a maximum measurable effect on the field.

(3) The field components should be measured at points where the beds produce the greatest measurable anomaly in the field components. Thus both the magnitude of the change and the percentage change in the field should be considered.

The quantities which may be measured are the magnitude and time phase of the components of the electric and magnetic vectors on the surface of the earth.

There are two simple primary fields for which the theoretical problems have been solved; the field of a circular coil, which may be considered to be an oscillating magnetic dipole with its axis perpendicular to the plane of the coil and the field of a long horizontal wire. Most electromagnetic methods have made use of one of these.

The basic equations that govern alternating currents are Maxwell's equations. The field components must satisfy the wave equation:[2]

$$\nabla^2 A_1 - \mu\sigma \frac{\partial A_1}{\partial t} - \mu p \frac{\partial^2 A_1}{\partial t^2} = 0. \tag{15}$$

If the impressed forces are simple harmonic functions of time, A_1 can be represented as

$$A_1 = \text{real part of } Ae^{-i\omega t}$$

where A is in general complex. Then Eq. (15) becomes

$$\nabla^2 A + i\omega\mu A(\sigma - i\omega p) = 0. \tag{16}$$

This equation shows the relative importance of the conductivity and the relative permittivity or dielectric constant in determining the current flow in the earth. If $\sigma \gg \omega p$ the conductivity will be the determining factor. If $\omega p \gg \sigma$ the dielectric constant will be the determining factor. If the conductivity is 10^{-4}ohms·cm and the dielectric constant is 10, then the two are of equal importance when the frequency is

$$f = \frac{10^{-4}}{2\pi \times 10 \times 8.85 \times 10^{-14}} = 1.8 \times 10^7.$$

It will be shown later, however, that the high frequency waves are very highly damped by the surface layers, and are thus not important in oil prospecting. At the lower frequencies, the conductivity governs the distribution of current in the earth. Methods of locating oil directly by means of its dielectric constant are thus not very hopeful.

In order to show the importance of using a low frequency, a particularly simple situation has been assumed. The primary field is due to a vertical oscillating electric dipole on the surface of the earth.[3] The earth is homogeneous and of conductivity σ to a depth h, below which there is a layer of very high conductivity which we shall assume to be infinite. The problem to be solved is—at what frequency will the highly conducting layer have the biggest influence on the field at the surface of the earth?

The field may be conveniently represented by the Hertzian function Π, which satisfies the wave equation

[2] The rationalized practical system of units is used. Magnetic intensity H is expressed in ampere turns per cm. Magnetic flux density B is expressed in webers per $\text{cm}^2(=10^8$ Maxwell's per $\text{cm}^2)$. Permeability $\mu = B/H = 4\pi \times 10^{-9}$ for free space. Electric intensity E is expressed in volts per cm. Displacement D is expressed in coulombs per cm^2. Permittivity $p = D/E = 8.85 \cdot 10^{-14}$ for free space. Conductivity σ is expressed in mhos per cm.

[3] Riemann-Webers, Diff. Gleich. der Phys. (Seventh ed. 1925) p. 542.

$$\nabla^2\Pi + k_1^2\Pi = 0 \begin{cases} k_1^2 = k_0^2 = \mu p_0\omega^2 = \omega^2/c^2 \text{ in the air} \\ k_1^2 = k^2 = \mu p\omega^2 + i\mu\sigma\omega \text{ in the earth} \end{cases} \tag{17}$$

where c is the velocity of light.

Cylindrical coordinates (r, z, ϕ) with the origin at the dipole will be used. The electric intensity E and the magnetic intensity H are:

$$\begin{aligned} E_\phi &= H_z = H_r = 0 \\ E_r &= \partial^2\Pi/\partial r\partial z \\ E_z &= -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Pi}{\partial r}\right) \\ H_\phi &= \frac{ik_1^2}{\mu\omega}\frac{\partial\Pi}{\partial r}. \end{aligned} \tag{18}$$

We shall take the Hertzian function for the air to be Π_0, and the Hertzian function for the earth to be Π. The boundary conditions then are

$$\left.\begin{aligned} k_0^2\Pi_0 &= k^2\pi \\ \partial\Pi_0/\partial z &= \partial\pi/\partial z \end{aligned}\right\} \text{ at } z = 0 \tag{19}$$

and

$$\frac{\partial\Pi}{\partial z} = 0 \text{ at } z = -h. \tag{20}$$

The Hertzian function for a dipole in free space is

$$\Pi_d = \int_0^\infty \frac{\lambda}{(\lambda^2 - k_0^2)^{1/2}} e^{-z(\lambda^2-k_0^2)^{1/2}} J_0(\lambda r)d\lambda. \tag{21}$$

We, therefore, take for Π_0

$$\Pi_0 = \int_0^\infty \left[\frac{\lambda}{(\lambda^2 - k_0^2)^{1/2}} - f(\lambda)\right] e^{-z(\lambda^2-k_0^2)^{1/2}} J_0(\lambda r)d\lambda \quad z > 0 \tag{22}$$

and for Π

$$\Pi = \int_0^\infty [g_1(\lambda)e^{z(\lambda^2-k^2)^{1/2}} + g_2(\lambda)e^{-z(\lambda^2-k^2)^{1/2}}]J_0(\lambda r)d\lambda. \tag{23}$$

The functions f, g_1, and g_2 are chosen so as to satisfy the boundary conditions Eqs. (19) and (20). The solution for Π_0 is

$$\Pi_0 = \int_0^\infty \frac{2k^2\lambda e^{-z(\lambda^2-k_0^2)^{1/2}} J_0(\lambda r)d\lambda}{k^2(\lambda^2 - k_0^2)^{1/2} + k_0^2(\lambda^2 - k^2)^{1/2}\tanh h(\lambda^2 - k^2)^{1/2}}. \tag{24}$$

The normal field for a homogeneous earth of conductivity σ is

$$\Pi_n = \int_0^\infty \frac{2k^2\lambda e^{-z(\lambda^2-k_0^2)^{1/2}} J_0(\lambda r)d\lambda}{k^2(\lambda^2 - k_0^2)^{1/2} + k_0^2(\lambda^2 - k^2)^{1/2}}. \tag{25}$$

The change in the field due to the layer of high conductivity is:

$$\Pi_0 - \Pi_n = \int_0^\infty \phi(\lambda) e^{-z(\lambda^2 - k_0^2)^{1/2}} \lambda J_0(\lambda r) d\lambda \tag{26}$$

where

$$\phi(\lambda) = \frac{2k^2 k_0^2 (\lambda^2 - k^2)(1 - \tanh h(\lambda^2 - k^2)^{1/2})}{[k^2(\lambda^2 - k_0^2)^{1/2} + k_0^2(\lambda^2 - k^2)^{1/2}][k^2(\lambda^2 - k_0^2)^{1/2} + k_0^2(\lambda^2 - k^2)^{1/2} \tanh h(\lambda^2 - k^2)^{1/2}]} . \tag{27}$$

In order to calculate the effect of the layer on the normal field, one could calculate the components of this change in field and compare them with the normal field components. A simpler way to compare them is to compare the power flow associated with the change in field to the normal power flow. The changes in the field components are:

$$\begin{aligned} (E_r)_0 - (E_r)_n &= \int_0^\infty \phi(\lambda)(\lambda^2 - k_0^2)^{1/2} e^{-z(\lambda^2 - k_0^2)^{1/2}} \lambda^2 J_1(\lambda r) d\lambda \\ (H_\phi)_0 - (H_\phi)_n &= \frac{ik_0^2}{\mu\omega} \int_0^\infty \phi(\lambda) e^{-z(\lambda^2 - k_0^2)^{1/2}} \lambda^2 J_1(\lambda r) d\lambda . \end{aligned} \tag{28}$$

The power flow of this field through a horizontal plane in the air is:

$$\begin{aligned} P &= \pi \int_0^\infty [(E_r)_0 - (E_r)_n][(H_\phi)_0 - (H_\phi)_n]^* r dr \\ &= \frac{\pi k_0^2}{\mu\omega} \int_0^{k_0} (k_0^2 - \lambda^2)^{1/2} | \phi(\lambda) |^2 \lambda^3 d\lambda . \end{aligned} \tag{30}$$

The asterisk indicates that the complex conjugate of $(H_\phi)_0 - (H_\phi)_n$ is to be taken.

For all but very high frequencies, k^2 is very much greater than k_0^2. Under this condition, the power flow is approximately

$$P = \frac{4\pi k_0^6 [(1 - R)^2 + I^2]}{(2)^{1/2} \mu\omega | k | (1 - R + I)} \log \frac{1}{R - I} \tag{31}$$

where R and I are real, and

$$\tanh h | k | \frac{1 + i}{2^{1/2}} = R + iI . \tag{32}$$

The normal power flow from a homogeneous earth of conductivity σ is

$$P_n = \frac{4\pi k_0^2 | k |^4}{\mu\omega} \int_0^{k_0} \frac{(k_0^2 - \lambda^2)^{1/2} \lambda^3 d\lambda}{| k^2(\lambda^2 - k_0^2)^{1/2} + k_0^2(\lambda^2 - k^2)^{1/2} |^2} \tag{33}$$

which becomes for $k^2 \gg k_0^2$

$$P_n' = 8\pi k_0^5 / 3\mu\omega . \tag{34}$$

The ratio of the power flow of the change in field to the normal power flow is:

$$P/P_n = \frac{3}{2}\,\frac{k_0}{2^{1/2}|\,k\,|\,u}\,g(u)\,\frac{2.81 \times 10^{-3}}{h\sigma}\,g(u) \tag{35}$$

where

$$g(u) = u\,\frac{(1-R)^2+I^2}{1-R+I}\,\log\frac{1}{R-I} \tag{36}$$

$$u = h(\mu\sigma\omega)^{1/2} \tag{37}$$

$$\tanh u\,\frac{1+i}{2^{1/2}} = R + iI. \tag{38}$$

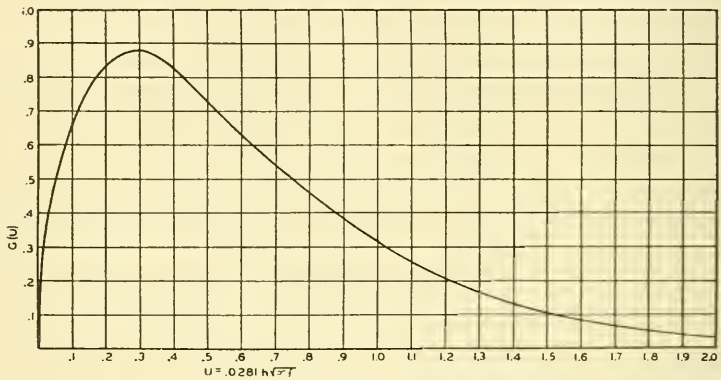

Fig. 7. Power ratio curve for vertical electric dipole.

Fig. 7 gives a graph of $g(u)$ which shows a maximum at $u=0.29$. This gives the optimum frequency in terms of the depth of investigation and the conductivity of the upper beds. This frequency is

$$f = \frac{10^9\,u^2}{8\pi^2 h^2\sigma} = \frac{(0.29)^2\cdot 10^9}{8\pi^2 h^2\sigma} = \frac{10^7}{h_m{}^2\sigma} \tag{39}$$

if h is measured in meters. If $\sigma = 10^{-4}$ and $h=100$ meters, the best frequency to use is about 100 cycles. If $h=300$ meters the optimum is about 10 cycles. At higher frequencies the effect of the conducting layer on the field becomes smaller. Thus for $\sigma=10^{-4}$ and $h=100$ meters, the effect of the layer with a frequency of 2500 cycles is only about one tenth of the effect at 100 cycles. At very low frequencies only a small percentage of the power input enters the ground. At high frequencies, most of the power that enters the ground is absorbed by the surface layers.

A similar investigation has been carried out to determine the effect of a highly conducting layer on the field of a long horizontal wire carrying an alternating current. As in the previous case, the earth is assumed homogeneous with conductivity σ to a depth h, below which the conductivity is infinite.

The wire is on the z-axis of rectangular coordinates, the y-axis is vertical and the x-axis is horizontal and perpendicular to the wire. Maxwell's equations for the conducting medium reduce to:

$$\begin{aligned} E_x &= E_y = H_z = 0 \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= (\sigma - i\omega p)E \\ \frac{\partial E_z}{\partial y} &= i\omega\mu H_x \\ \frac{\partial E_z}{\partial x} &= - i\omega\mu H_y \end{aligned} \tag{40}$$

or

$$\nabla^2 E_z + k^2 E_z = 0.$$

The equations for the air may be obtained by setting $\sigma = 0$ and $p = p_0$. Let us take

$$\begin{aligned} E_z &= E_0 \text{ in the air} \\ E_z &= E \text{ in the earth.} \end{aligned} \tag{41}$$

The boundary conditions then are

$$\left.\begin{aligned} E_0 &= E \\ \partial E_0/\partial y &= \partial E/\partial y \end{aligned}\right\} \text{ at } y = 0 \tag{42}$$

$$E = 0 \text{ at } y = -h \tag{43}$$

and

$$E_0 \to k_0 \log (x^2 + y^2)^{1/2} \text{ as } x, y \to 0. \tag{44}$$

The solution of these equations for E_0 is

$$E_0 = k_0 \int_0^\infty \frac{2e^{-y(\lambda^2 - k_0^2)^{1/2}} \cos \lambda x d\lambda}{(\lambda^2 - k_0^2)^{1/2} + (\lambda^2 - k^2)^{1/2} \coth h(\lambda^2 - k^2)^{1/2}}. \tag{45}$$

The normal field for a homogeneous earth is

$$E_n = k_0 \int_0^\infty \frac{2e^{-y(\lambda^2 - k^2)^{1/2}} \cos \lambda x d\lambda}{(\lambda^2 - k_0^2)^{1/2} + (\lambda^2 - k^2)^{1/2}} \tag{46}$$

Thus the change in the field due to the conducting layer is

$$E_0 - E_n = k_0 \int_0^\infty \phi(\lambda) e^{-y(\lambda^2 - k_0^2)^{1/2}} \cos \lambda x d\lambda \tag{47}$$

where

$$\phi(\lambda) = \frac{2(\lambda^2 - k^2)^{1/2}(1 - \coth h(\lambda^2 - k^2)^{1/2})}{[(\lambda^2 - k_0^2)^{1/2} + (\lambda^2 - k^2)^{1/2}][(\lambda^2 - k_0^2)^{1/2} + (\lambda^2 - k^2)^{1/2}\coth h(\lambda^2 - k^2)^{1/2}]} \quad (48)$$

and the change in magnetic intensity is:

$$(H_x)_0 - (H_x)_n = \frac{ik_0}{\mu\omega}\int_0^\infty (\lambda^2 - k_0)^2\phi(\lambda)e^{-y(\lambda^2 - k_0^2)^{1/2}} \cos \lambda x d\lambda. \quad (49)$$

The power flow through a horizontal plane in the air per centimeter of wire length is

$$P = \tfrac{1}{2}\int_\infty^\infty [E_0 - E_n][(H_x)_0 - (H_x)_n]^* dx = \frac{k_0^2\pi}{4\mu\omega}\int_0^{k_0} |\phi(\lambda)|^2(k_0^2 - \lambda^2)^{1/2}d\lambda. \quad (50)$$

When $k^2 \gg k_0^2$ this integral reduces to

$$P = \frac{\pi^2 k_0^4}{4\mu\omega |k|^2}[(1 - R)^2 + I^2] \quad (51)$$

where R and I are real and

$$R + iI = \tanh h|k|\left(\frac{1 + i}{2^{1/2}}\right). \quad (52)$$

The normal power flow through the plane for a homogeneous earth is:

$$P = \frac{\pi^2 k_0^4}{4\mu\omega |k^2|}\cdot \quad (53)$$

As the frequency approaches zero, the ratio $k_0^4/\omega|k|^2$ approaches zero, so that both the normal power flow and the power flow of the change in field due to the layer approach zero. The ratio, however, is

$$P/P_n = (1 - R)^2 + I^2 \quad (54)$$

where

$$R + iI = \tanh u \frac{(1 + i)}{(2)^{1/2}}$$

$$u = h|k| = 0.0281\, h_{\text{meter}}(\sigma f)^{1/2}.$$

The ratio p/p_n is shown graphically in Fig. 8. It is a maximum at zero frequency, and then drops off as the frequency increases, approaching zero at very high frequencies. In this case the optimum frequency depends primarily on the measuring apparatus. The frequency should be chosen as low as the apparatus permits the field quantities to be measured accurately.

The curve in Fig. (8) shows the absorption of the surface layer. If the conductivity of the surface layer is 10^{-4} ohms and the depth to the conducting layer is 100 meters, the effect of the layer at a frequency of 3000 cycles is only about one twentieth of the effect at 100 cycles. This curve illustrates the importance of using a low frequency alternating current.

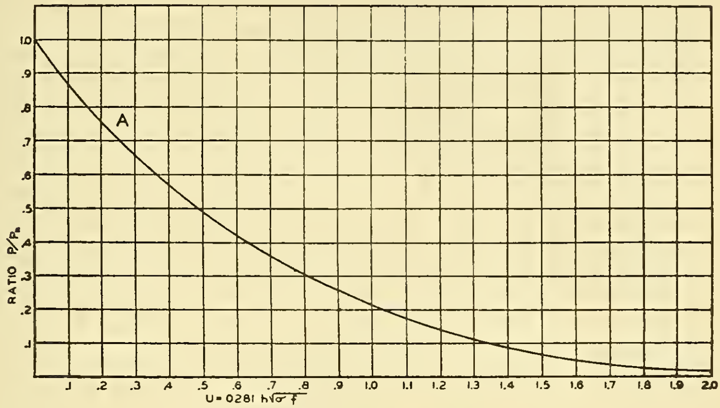

Fig. 8. Power ratio curves for long line.

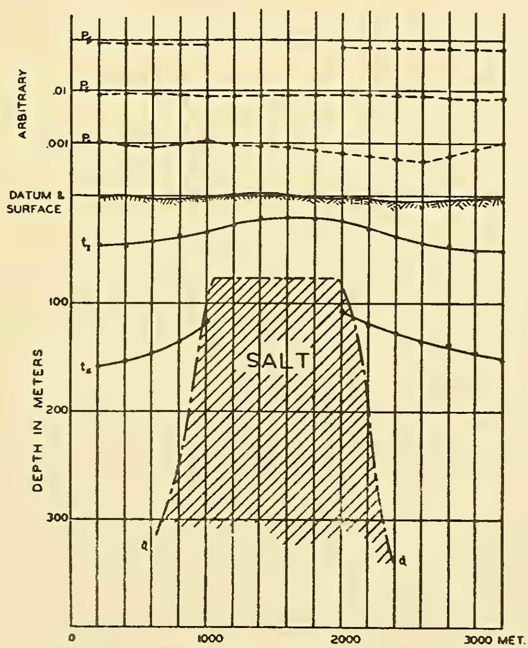

Fig. 9. Electromagnetic survey across a salt dome.

In order to illustrate the results that may be obtained in the field by a good electrical survey under favorable conditions, two surveys made by the Swedish American Prospecting Corporation for the Gulf Oil Corporation will be described. The method of survey used was the Sundberg method,[4] which will be briefly described.

The primary field is the field of a long horizontal wire carrying an alternating current with a frequency of 100 to 500 cycles. Measurements of the direction and time phase of the magnetic intensity vector are made at a series of points along transverse lines. These measurements are made by means of a search coil connected to a compensating arrangement.

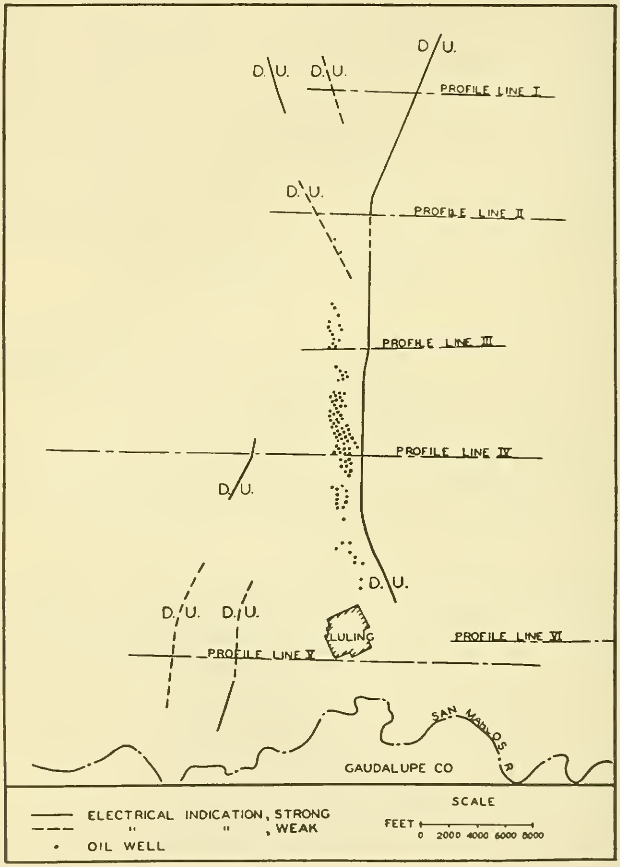

Fig. 10. Fault traced by electrical methods where only one well was drilled in the field.

The induced currents are assumed to be confined to a series of beds which have a much higher conductivity than the surrounding beds. Each of these beds is characterized by a certain induction factor p which depends on its conductivity and thickness, and also on the frequency of the exciting current. From the measurements made at the surface, the induction factors and the

[4] T. Zuschlag, Tech. Pub. No. 313 Inst. of Min. and Met. Eng.

depth of the beds are determined. The conducting beds which are followed are called marker beds.

Fig. 9[5] shows the results of a survey across a shallow salt dome. Three conducting beds were assumed, one at the surface, one at an intermediate depth, and the last at about 400 feet. The intermediate bed is arched over the dome. The deeper conductor is missing where it is pinched out by the salt, and locates the flanks of the dome.

Fig. 10 gives the electrical indication of the Bruner fault zone, Texas. A clay bed at about 500 feet was used as a marker. When the survey was started there was only one producing well in the area. Later production has followed parallel to the electrically indicated fault. The electrical picture was not only qualitively right, but later drilling showed a constant interval between the clay bed and the Austin chalk, which is the producing horizon. Thus, a very good shallow picture of this fault was obtained electrically.

IV. Conclusion

Up to the present point we have discussed in a general way the problems encountered in electrical prospecting under very simple geological conditions. Experience in the field has shown that in general good marker beds can be found within the depth to which investigation can be carried electrically. However, in countries where the shallow sediments have been laid down in the form of lenses, these beds may not be continuous enough to give a structure picture.

On monoclinal slopes where successive beds are outcropping so that the surface material is changing in character, the lateral variation in conductivity of the surface material is large. This resistance variation of the near surface material makes the problem of interpreting electrical measurements somewhat difficult. A condition of this kind was encountered in certain parts of the Balcones fault zone of Texas. The variation in surface resistivity resulted in many apparent fault indications and it was difficult to distinguish these apparent indications from the true ones.

The depth to which geophysical investigations can be carried is of considerable importance in regions where the lower beds which contain the oil are not laid down conformably with the shallow beds. If the beds in which oil must be sought lie below an uncomformity which occurs at a depth of 5000 feet, then it is obvious that a prospecting scheme cannot be used which does not give information below 5000 feet.

Many extravagant claims have been made for the possible depths to which it is possible to work by electrical methods. In direct current cases, if the earth were homogeneous except for two or three sharp breaks in conductivity, it would be possible to map to a large depth. However, even in this ideal case as the depth of investigation is increased, there would be a gradual loss in detail. Actually conditions are much more complicated. As the depth of investigation is increased, more and more discontinuities enter both later-

[5] Surveys shown by Figs. 9 and 10 have previously been published by Zuschlag in A.I.M.M.E. Technical Publication No. 313.

ally and vertically. Consequently, the difficulties of interpreting the measurements become more and more difficult. Up to the present time information of immediate value in oil geology has not been obtained below a depth of about 2000 feet.

In the alternating current case there is a definite depth of current penetration for a given frequency and a given earth resistivity. In attempting to work at deeper and deeper depths, the operating frequency must be lowered continually. In addition to the difficulties mentioned in the direct current case, one is confronted with the problem of measuring very weak electric and magnetic fields of low frequency. In some cases where the resistivity of the surface materials is high the alternating current field is less influenced by lateral irregularities in the surface layer than the direct current field. In the present state of the art, it would take exceptionally favorable conditions to obtain reliable information at a depth much in excess of 1500 feet.

At the present time electrical methods of prospecting for oil seem to be in disrepute. This is partly due to cost of electrical surveys as compared with other geophysical methods and partly due to the failure of the extravagant claims made for the process to materialize. However, the electrical method of prospecting for oil cannot be forgotten because it is one of two prominent geophysical methods in which it is possible to control the field being employed. Improvements in methods of interpretation and in field technique should give electrical methods a definite field of usefulness in prospecting for oil.

A NEW INSTRUMENT FOR MEASURING VERY SMALL DIFFERENCES IN GRAVITY

By Kenneth Hartley

Hartley Gravity Balance Corporation, Houston, Texas

(Received January 2, 1932)

Abstract

Description of a new portable instrument for measuring relative values of gravity to within two or three parts in ten million, specially designed for geophysical exploration. The principles of the design are analysed and the methods for eliminating effects of elastic hysteresis, temperature changes, variations in barometric pressure, etc., are discussed. Also effects of initial stresses in materials, defects in alignment of locking mechanism, inaccurate leveling, etc. These difficulties are serious but seem to have been overcome. Results of preliminary field measurements near Houston, Texas, are presented and some comparison between the type of information given by this instrument and by the torsion balance.

UNTIL now the only successful method of measuring gravity has been with some form of pendulum, in other words determining the acceleration produced, or, we may say, obtaining the value of gravity in terms of inertia. There is only one other possible method and that is to measure it in terms of the elasticity of a spring which can be calibrated in some way. This has the advantage of eliminating time measurements but introduces some other difficulties that are avoided by the pendulum.

In all previous attempts that I know of to use a spring the effort has been to obtain extreme sensitivity by some arrangement approaching unstable equilibrium, and one reason for their failure is that this unstable condition also exaggerates all the causes of error and uncertainty in the indications of the apparatus. In the new instrument the effort has been in the other direction, that is to obtain the maximum stability, the small displacements are then amplified by optical means with less uncertainty.

The fundamental idea of the design is that if we can apply to a suspended mass an upward force almost, but not quite, equal to the pull of gravity and maintain this force *constant* then it will be easy to measure changes in the small additional force required to hold the system in equilibrium. The first problem then is to apply a force that can be maintained constant to the required degree of accuracy. The value of such an instrument for geophysical prospecting depends on its ability to measure very minute differences in the gravitational force, so the instrument has been designed to have the greatest sensitivity that can be of any practical use. Since the change in the gravitational attraction due to a difference of one foot in elevation is almost exactly one part in ten million this was taken as the maximum sensitivity that could possibly be useful; for most purposes three or four parts in ten million will be all that is desired. This is about ten times the accuracy of the best work done with the pendulum apparatus of the United States Geodetic Survey.

Figure 1 is a simplified diagram of the essential parts of the apparatus. The main spring, which carries about 99.9 percent of the load, is formed from heavy wire and not stressed to more than 20 percent of its apparent elastic limit, so as to give the greatest possible dependability. A very light beam, hinged at the left side, is flexibly attached between the spring and the weight

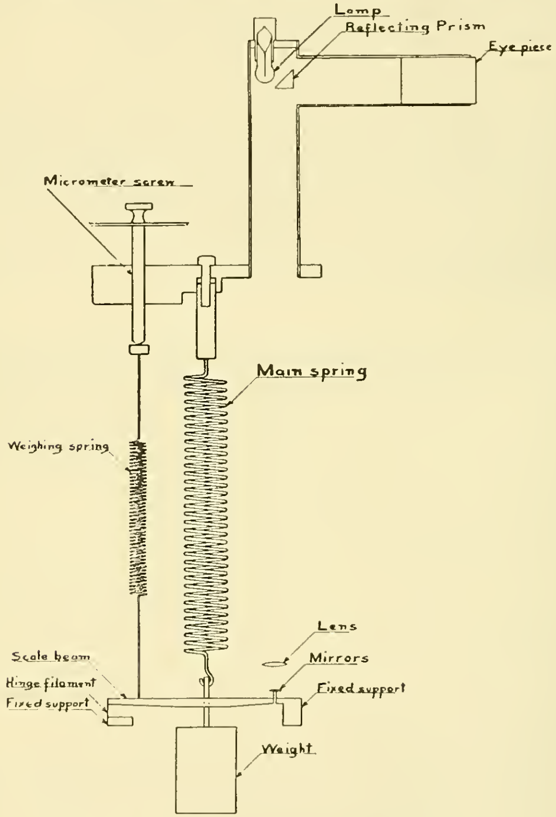

Fig. 1. Schematic diagram of gravimeter.

and carries two small mirrors at the free end. These mirrors act on the principle of the optical lever and are so mounted that a vertical displacement of the beam causes them to turn in opposite directions. This doubles the amplification but a much more important fact is that it avoids the necessity for maintaining a fixed point of reference in the telescope; when the two mirrors are in the same plane the two images of the lamp filament as seen in the eye-piece will be in one straight line no matter how much the eye-piece and other optical parts may be displaced. A very light weighing spring is attached to the beam between the center and the hinged end and the tension in this spring is controlled by a micrometer screw above.

If the instrument is adjusted at a certain location so that the two images of the lamp filament are in line and it is then taken to a different location, the difference in gravity will cause a slight displacement of the images which are then brought back to the zero position by altering the tension in the weighing spring; the difference between the two readings of the micrometer dial is then a measure of the difference in the force of gravity at the two places.

The difficulties involved in carrying out this idea to the necessary degree of precision have been greatly increased by the requirement that the instrument be rugged enough to stand transportation over rough roads and small and light enough to be carried by two men where there are no roads.

Suppose a mass of 100 grams or more suspended by a spring of such stiffness that the total elongation, from no load to full load, is 100 millimeters; then, to maintain the tension constant to one part in ten million would require; first, that the length be controlled to within one one-hundred-thousandth of a millimeter; second, that the temperature be maintained constant for the whole period required for a series of observations so that the elongation of the spring due to temperature effect minus the linear expansion of the supporting frame will be within the same limit; third, that effects of elastic hysteresis be eliminated not only from the main spring but from the flexible supports of the beam and mirrors also.

In the present instrument the distance between the thin ribbons supporting the mirrors is one millimeter and the distance from the mirrors to the eye-piece is 600 millimeters, so that the amplification for each mirror is 1200 times, but as the two mirrors move in opposite directions the relative displacement of the images is amplified 2400 times. As the beam ratio is 1.8 the total amplification is 4320, but as this is observed through an eye-piece with a magnification of 14 the apparent amplification is over 60,000, so that a movement of the suspended mass of one one-hundred-thousandth of a millimeter will look like six-tenths of a millimeter and be easily observed. If the force of gravity is increased by one part in ten million, part of the increase is used in stretching the main spring and part in bending the small filaments that support the beam and mirrors, so the sensitivity in terms of force is not so great as this, but the first requirement is accomplished—the maintenance of the length of the main spring to within one part in ten million.

I expected to be able to make the beam and mirror system so flexible that the resistance to movement would not be any greater than that of the main spring, but I have not succeeded in doing so. Although the filaments that carry the mirrors are only 0.01 mm thick, it is the stiffness of these filaments that limits the sensitivity of the instrument. The present instrument is satisfactory for readings of one part per million but is difficult to read any closer than that so a new beam system is being made which is expected to be at least four times as sensitive as the present one.

The temperature effect on the spring is very large and unless accurately controlled will completely mask the small variations in force that are to be measured. The ideal material for the spring would be a nonmagnetic metal with a high elastic limit but a low value for Young's modulus and a low temperature coefficient. This combination might be found in one of the less common metals or alloys but information on the elastic properties of these metals, particularly temperature effects, was not to be had. Assuming that the temperature coefficient of elasticity would probably be smaller in a metal with very high melting point led to the consideration of tantalum, molybdenum, etc., and several springs were made of approximately the dimensions that would be required and tested in that form. The final choice was an alloy of tantalum and tungsten which seems to be fairly satisfactory. The spring is wound with a high initial tension so that when carrying its full load the coils are separated only about half a millimeter, making the total length of the stretched spring less than ten centimeters. The frame of the instrument is made of an aluminum alloy, which has the highest coefficient of expansion of any suitable metal, and is constructed on the principle of the old grid-iron compensated pendulum so that the total length of the high expansion members is about 80 centimeters, the low expansion rods are invar steel. This compensation is mainly for the purpose of avoiding too great a change in the relative position of the parts when the working temperature is altered. In order that the whole instrument be in a condition of thermal equilibrium and free from temperature stresses it must be kept at a constant temperature day and night during a whole series of observations; it is therefore enclosed in a new type of portable thermostat, designed on the principle worked out at the Bureau of Standards two or three years ago, which controls the temperature to within one ten-thousandth of a degree. The heating is electrical and is designed to work with a six volt storage battery.

A study of the action of this instrument while the temperature is either rising or falling will quickly convince any one that any plan for making a temperature correction, instead of maintaining a constant temperature, is wholly out of the question. In the first place, even with the best possible compensation the temperature would have to be measured to within one hundredth of a degree, and in the second place, it is impossible to have the entire apparatus at the same temperature unless it is kept constant for a considerable time. In the present instrument if the temperature is altered by two or three degrees it requires at least five hours to reach a condition of equilibrium and if the change is much greater it may take all day, but with thermostat control the temperature effects are also brought within one part in ten million.

The change in density of the air, due to varying barometric pressure, may alter the buoyant force on the suspended mass by several times the quantity that we are attempting to measure, so the case is closed air-tight and the two operating rods are surrounded by a new type of mercury seal which prevents the passage of any air in or out even with a difference of pressure of more than two inches of mercury.

The effects of elastic hysteresis are eliminated by locking the weight and spring in the zero position between observations, so that the tension on the spring is never varied by more than the difference between the force of gravity at two successive stations. Any hysteresis effect is then only a small fraction of this amount. This sounds simple but it is difficult to carry out for it requires that the suspended mass be locked to within less than one thousandth of a millimeter of the true position. To meet this requirement and to avoid any strain on any part of the spring from jolts in transportation has necessitated a very elaborated locking mechanism.

In connection with this is another matter that no one seems to think of unless he has been working with a seismograph; that is vibration. The ground is constantly vibrating, and a weight hung on a spring is an excellent seismograph if its motions can be closely observed, so some way must be found to reduce the sensitivity to vibrations while maintaining the sensitivity to changes in the static force. In the present instrument this is accomplished to a certain extent by excessive air damping but this is only satisfactory under very favorable conditions so some modifications are being planned for the beam system which will give a more favorable ratio between these two effects. In the city traffic vibration is the chief difficulty and in the country it is the wind in the trees, so it is impossible to use the instrument in very windy weather but with a light wind readings can be made by watching for a favorable moment, the mirrors may be almost stationary for several seconds at a time and an experienced operator can utilize these moments to secure readings to about one part in a million.

One more difficulty encountered in the construction of the instrument was due to initial strains in the frame. Any change in temperature would alter the elastic forces present and cause the frame to twist enough to throw the beam system badly out of adjustment. To avoid this as far as possible the beam and mirror system is carried by a separate frame entirely independent of that which carries the rest of the mechanism. This frame is made of annealed aluminum tubes very accurately fitted into sockets in the top and bottom plates, and clamped in position by a method designed to avoid putting any strain in the tubes when they are tightened up.

It was found, in the first experimental instrument, that the temperature stresses together with stresses due to imperfect construction of the locking mechanism would pull the beam and mirrors slightly out of alignment when it was locked. This caused distortion of the filament supports of the mirrors and of a light guide spring that was used in that instrument and when released the filaments would not immediately return to their relaxed condition but there would be a slow drift of the readings extending over a period of from half an hour up to several hours before the correct reading was obtained. In the latest instrument this effect is almost eliminated but it is sometimes necessary to wait about ten minutes.

These effects due to elastic hysteresis have been the most serious difficulties encountered, and there is still room for improvement in this direction, but measurements are now being made than can be checked, day after day, to $\pm 1 \cdot 10^{-6}$ and, as soon as a small alteration in the mirror system can be completed, which will give a better ratio between the effect of gravity and the effect of vibrations, I expect to get an accuracy four or five times as great.

The instrument must be very accurately leveled, that is, the axis of the instrument must be in the direction of the gravitational force. An angle of ninetv seconds between the axis of the instrument and the true vertical would

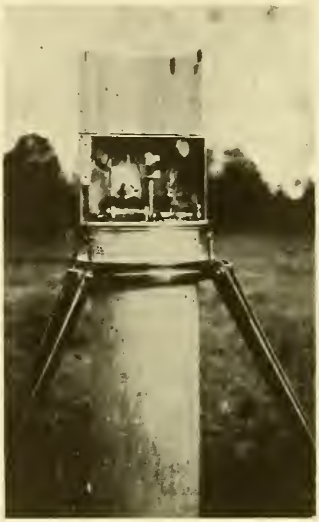

Fig. 2. The gravimeter controls and adjustments.

theoretically cause an error of one part in ten million, but it would cause a much greater difference in the actual measurement because of lateral strains in the filaments carrying the beam and mirrors. The present instrument is

Fig. 3. The gravimeter in the field.

easily adjusted to thirty seconds and that seems to be sufficiently close for the purpose.

The dial readings give a direct measure of the differences in gravity and do not require any correction except to multiply by the calibration factor for the instrument and subtract the effect of elevation, which is one part in ten million for every foot. These differences in elevation can be determined to within three or four feet with the Paulin Altimeter, provided that it is used according to a particular system that has been worked out for this purpose, but if greater accuracy is desired the land must be surveyed. These facts make the use of the gravity balance much simpler than any other geophysical instrument except the magnetometer.

Recent field work has shown that in reasonably good weather it is easy to make three stations per hour. The torsion balance makes three stations per day and the Government pendulum apparatus three or four stations per month. This measurement which takes only a few minutes appears to have four or five times the accuracy of a single run of the pendulum which takes twelve hours. The information obtained is exactly the same as with the pendulum, the total intensity of the gravitational field. For this reason the presence of a heavy mass near by on the surface or at small depth has no effect on the reading; the presence of such a mass merely alters the direction of the

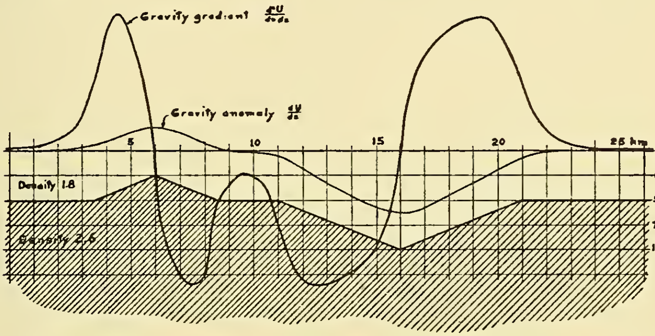

Fig. 4. Gradient and gravity curves for assumed structure.

resultant force and as the instrument is set exactly in line with this direction the measurement is the same. These surface masses make so much trouble for the torsion balance operator that it seems to be hard for him to believe that they do not disturb this instrument.

The relation between the gravity anomaly, which is measured directly by the gravity balance, and the gravity gradient, which is measured by the torsion balance, is very clearly shown in a diagram in Ambronn's Elements of Geophysics, which he has taken from a paper by von Eötvos. Figure 4 is derived from this diagram; the heavy curve representing the values of the gravity gradient and the light curve the gravity anomaly caused by a mass of heavy rock at varying depths below the surface. It will be noted that the curve representing the magnitude of the anomaly follows very closely the outline of the rock mass below while the curve representing the magnitude of the gradient bears no resemblance whatever to this outline and requires some study to interpret its meaning. In actual field work these curves are not obtained but only a rather small number of points on them. A comparatively small number of gravity measurements would give the anomaly curve with a fair degree of accuracy but the same number of ordinates to the gradient curve would leave it still extremely uncertain and the results consequently very difficult to interpret and the interpretation of very doubtful validity.

While the new instrument is still in the experimental stage and there will undoubtedly be improvements in details of construction, it is not probable that it can ever be used effectively by an unskilled operator. The method of operation is extremely simple but it seems to be necessary for the operator to have had some kind of experience with precision measurement in order to guard against deceptive appearances and be sure that the instrument is really in equilibrium when the reading is made.

CHARTS FOR TORSION BALANCE READINGS*

By M. M. Slotnick

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(Received February 1, 1932)

THE calculations necessary to determine the values of the gradient and curvature quantities from the readings taken at a torsion balance station involve either slide-rule or logarithmic manipulations which are somewhat

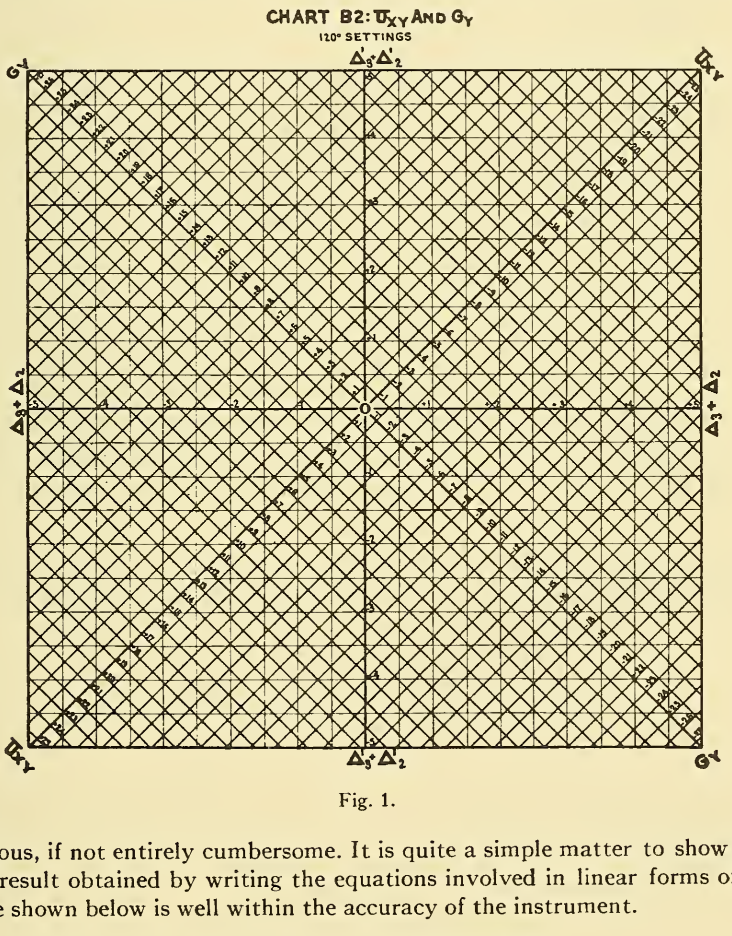

Fig. 1.

tedious, if not entirely cumbersome. It is quite a simple matter to show that the result obtained by writing the equations involved in linear forms of the type shown below is well within the accuracy of the instrument.

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A pair of charts can readily be made for each instrument when used in 72° azimuth settings and another pair for 120° azimuth settings. The charts appended were made for field use for a Bamberg instrument of the

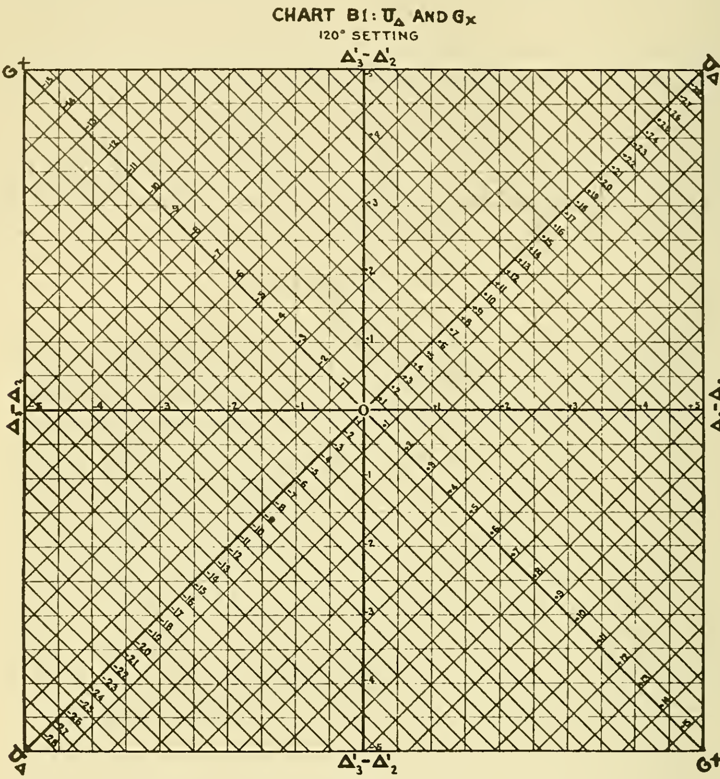

Fig. 2.

Humble Company equipped with Blau wires and are for the latter type of settings. For this particular instrument the equations in this case are:

$$G_x = 1.59[(\Delta_3' - \Delta_2') - (\Delta_3 - \Delta_2)],$$
$$U_\Delta = 2.93[(\Delta_3' - \Delta_2') + (\Delta_3 - \Delta_2)],$$
$$G_y = -2.75[(\Delta_3' + \Delta_2') - (\Delta_3 + \Delta_2)],$$
$$U_{xy} = -2.56[(\Delta_3' + \Delta_2') + (\Delta_3 + \Delta_2)].$$

The first pair of equations is charted on B1 and the second on B2. An examination of the charts will show at once the ease and accuracy with which they are read.

In accordance with the usual notation we have

$$\Delta_k = n_k - n_0, \quad k = 1, 2, 3,$$

where n_k is the reading of the instrument in its kth position, and

$$n_0 = \frac{1}{3}(n_1 + n_2 + n_3).$$

The unprimed letters refer to the quantities bearing on the one beam, and the primed to those on the other beam.

The originals of the charts are 20″×20″ with the horizontal and vertical scales running from −10 to +10 units at intervals of 0.1, a range well within that usually necessary.

It has been found that nomographs are not quite as convenient as these charts for field use.

THE EFFECTS OF HEAT TREATMENT ON FINE METALLIC SUSPENSIONS

By N. N. Zirbel

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(Received February 15, 1932)

Abstract

When a suspended system is supported by a fine wire the equilibrium position usually changes slowly for a long time after the load is applied. The equilibrium position also changes with temperature. It is found that both of these disturbing factors can be eliminated by a suitable heat treatment of the wire. Observations have been made on tungsten and platinum-iridium wires of sizes suitable for use in the Eötvos torsion balance.

Introduction

MANY types of physical apparatus involve the use of a rotating system suspended by a thin filament of metal or other material. Numerous irregularities in the behavior of an arrangement of this sort are often observed, such as changes in the equilibrium position with time and temperature, changes in the torsion constant of the suspending filament and failure of the suspended system to return to its initial position after a displacement. These irregularities are especially serious in the Eötvos torsion balance, in which the suspension is very thin and is heavily loaded. Investigations of various sorts on the behavior of suspension wires have been published by Shaw and Lancaster-Jones,[1] Boys,[2] Threlfall,[3] Königsberger[4] and others. The present work is devoted to an attempt to remove by suitable heat treatment the changes in equilibrium position with time and temperature. On this particular phase of the subject very little has been published.

Apparatus and Procedure

Observations have been made on several tungsten and platinum-iridium wires of diameters and lengths suitable for torsion balance use. The furnace used to anneal the wires and for the time drift and temperature coefficient determinations was a brass tube wound with an electrical heating coil and well insulated with asbestos. A glass window inserted in the side allowed the angular position of the suspended system to be observed with a lamp and scale, and a thermometer inserted at the top allowed the temperature to be read. In all the work the scale distance was 120 cm and the scale divisions millimeters.

[1] H. Shaw and E. Lancaster-Jones, Proc. Phys. Soc. London 35, 151 (1923).

[2] V. C. Boys, Phil. Mag. 23, 489 (1887).

[3] Richard Threlfall, Phil. Mag. 30, 99 (1880).

[4] J. Königsberger, Zeits. f. Physik 40, 729 (1927).

The suspended system used was a cylindrical brass weight with a mirror attached since this design would minimize the effects of convection currents. The weight of the suspended system was about half that required to break the wire. The upper end of the suspension was fastened to a brass rod which extended through the top of the furnace and was clamped outside so that a twisting or slipping of the furnace would not affect the position of the suspended system.

In the case of tungsten wires carbon dioxide gas was kept flowing through the furnace during all annealings at temperatures higher than 200°C. In order to anneal at temperatures higher than 500°C the wire was suspended in a glass tube with a weight of 6 to 10 grams at the lower end dipping in a copper sulfate solution so that a current could be passed through the wire.

The temperature coefficient, which is defined as the change in the equilibrium position per unit temperature change, was measured by heating the furnace about 20°C above room temperature and allowing it to cool slowly; the scale reading being noted at the beginning and at various temperatures during the cooling. Then a graph was plotted showing the scale reading as a function of the temperature. Since the relation between the scale reading and the temperature is linear, the temperature coefficient can be easily obtained from the graph. It was not possible to make readings during the heating because convection currents disturbed the suspended system in an irregular way and the alternating magnetic field of the heating coil exerted a torque on it. The possibility of convection currents affecting the position of the suspended system even during the slow cooling was investigated by making two sets of readings on the same wire, one set using the furnace in the usual manner, the other set by changing the temperature of the whole room very slowly. The corresponding readings were the same, within experimental error, showing that the effect of convection currents was not appreciable. For a typical temperature coefficient determination see Fig. 1. The coefficient is reckoned positive if an increasing temperature causes a clockwise rotation of the suspended system, as viewed from above, and negative if the rotation is counter-clockwise.

Experimental Results

The first observations were made at room temperature on changes in the equilibrium position with time. In one typical case the rate of drift immediately after loading the wire was 4.4 mm per hour. After twenty-four hours the motion was still easily observable but much slower, about 0.7 mm per hour. In some cases a change in the direction of the drift would occur. It seems that this time drift is caused by a slow release of strains of some sort in the wire and that in a sufficiently long time a stable condition would be reached. Just how long a time would be required is not known but Shaw and Lancaster-Jones[1] have given a case in which the drift was still going on after 50 days.

The next observations made at about 200°C showed that at this temperature the untwisting process was greatly accelerated so that the stable position was reached in about 7 hours instead of many days. At higher temperatures still less time was required. During this baking process the total rotation was often 180° or more. It was also found that if a wire from which the time drift had been removed by baking were strained beyond the elastic limit by a longitudinal stress, then the time drift again appeared and could

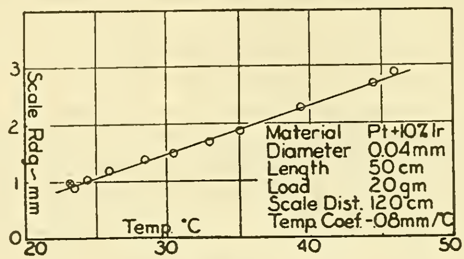


Fig. 1.

be removed only by further baking. The same result followed when the brass cylinder attached to the lower end of the wire was twisted through about half a revolution and held in this twisted position for several hours.

The time drift is thus quite easily removed. There remains, however, the change in equilibrium position with the temperature of the wire, as shown in Fig. 1.

The next problem is then to study the effect of heat treatment on this temperature coefficient, which in Fig. 1 is -0.08 mm per °C. Fig. 2 shows

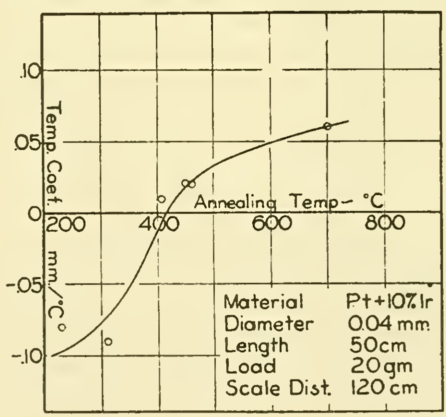


Fig. 2.

the results of such a study. The complete procedure in obtaining these results is as follows. The wire is hung up with a load of 20 grams and baked at 230°C for 6 hours. This baking removes the time drift. The temperature coefficient is then obtained by taking the readings shown in Fig. 1 and is found to be -0.08 mm per °C. This value gives the first point on Fig. 2. The wire is then baked at 310°C for 2.5 hours. The temperature coefficient is then again measured between 20°C and 50°C and found to be −0.09 mm per °C. The process is then continued with successively higher baking temperatures. Fig. 2 shows that the temperature coefficient between 20° and 50°C becomes

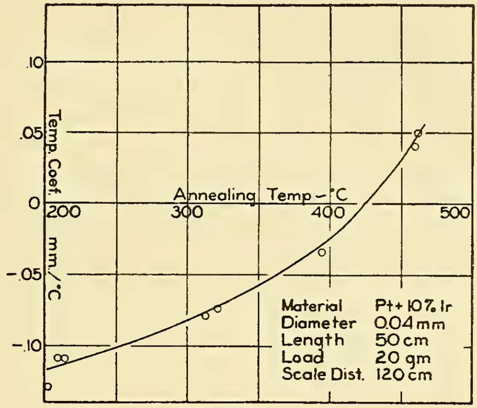


Fig 3.

zero after baking at a certain critical temperature of about 425°C and takes on increasing positive values after baking at higher temperatures. Fig. 3 shows similar and somewhat more consistent results obtained with another length of the same wire. The critical temperature is about the same in the

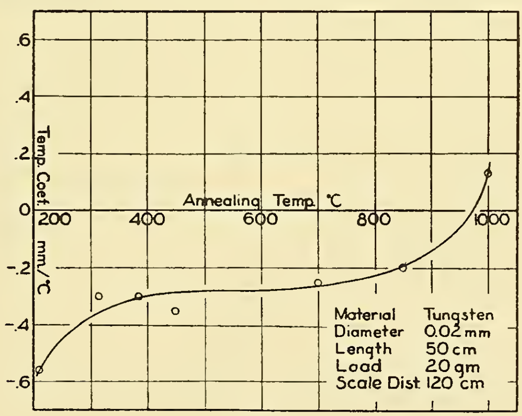


Fig. 4.

two cases. Fig. 4 shows that for tungsten the critical temperature is considerably higher. These latter results are subject to considerable error because temperatures higher than 500°C were estimated roughly by the color of the hot wire.

It has been found that the duration of time during which the wire is baked at a certain temperature is of little importance so far as the effect on the temperature coefficient between 20° and 50°C is concerned. In the results shown, this time varied from 3 minutes to 7 hours. It has also been found that after a wire has once been heated to a certain temperature, subsequent bakings at lower temperatures have no effect. That is, the curves shown can be traversed in one direction only, and after a wire has once been carried above the critical temperature no later heat treatment can diminish its temperature coefficient. Finally, measurements made on the two halves of the same wire show that the temperature coefficient is uniformly distributed along the wire and is not due to any local irregularity.

We have no explanation for the peculiar behavior shown in the last three figures. However, incomplete results on a fourth sample from an entirely different source show the same sort of behavior, and we believe that the effect, whatever its cause, is genuine. Suitable precautions have been taken to avoid the effect of convection currents and other disturbing factors. The fact that in all cases so far observed, involving wires from three sources, the temperature coefficient is initially in the same direction is also difficult to explain and may be merely a coincidence.

In conclusion, the writer wishes to express his thanks to Professor H. A. Wilson, under whose direction this work was done.

ON THE CORRELATION OF ISOGEOTHERMAL SURFACES WITH THE ROCK STRATA

By C. E. Van Orstrand[1]

United States Department of the Interior

(Received February 1, 1932)

Abstract

An instance of regional variation in Oklahoma and two cases of local variation, one at Long Beach, California, and the other at Salt Creek, Wyoming, have been selected for consideration from a large number of geothermal surveys conducted during the past few years by the U. S. Geological Survey and the American Petroleum Institute. The causes of local and regional variations are unknown. Possible explanations such as radioactivity, proximity to crystalline rocks, and transfer of heat along the strata are given careful consideration in attempting to explain the observed relations between the strata and the isogeothermal surfaces.

Regardless of the source of the earth's internal heat, the mass of the earth is so large, and the time during which it has been losing heat is so long, that whatever the law of increase of temperature with depth may be, the theoretical temperature distribution to depths of one or two miles should be, to a very high degree of precision, a straight line; but, in a recent summary of the data from 400 deep wells, distributed in 18 states, only 5 percent of the depth-temperature curves could be classified as linear; 36 percent were concave to the depth axis; and 59 percent were convex to the depth axis. A curve of the latter type is shown in Fig. 1. The causes of these anomalies are not definitely known. Diminution of thermal conductivity with depth resulting from a corresponding diminution in the moisture content of the rocks may be a partial explanation of the observed convexity of the curves; and the increased conductivity of the more dense crystalline or basaltic rocks beneath thin sediments is a possible explanation of the concave curves. The curves tend to be more or less of the same type in each individual field. Notwithstanding these anomalies, however, the curves obtained under ideal conditions are exceptionally smooth and uniform, and the resulting isothermal surfaces determined from them are likewise smooth and uniform.

Regional Variation of Isogeothermal Surfaces

The general trend of the 100°F isogeothermal surface[2] extending over a distance of 100 miles from Okemah to Oklahoma City, Oklahoma, is shown in Fig. 2. That these surfaces are closely related to the strata is evidenced by

[1] Published by permission of the Director of the U. S. Geological Survey.

[2] John A. McCutchin, Determination of geothermal gradients in Oklahoma. Bull. Am. Assoc. Petroleum Geologists 14, 535–555 (1930); Determination of geothermal gradients in oil fields located on anticlinal structures in Oklahoma. Am. Petroleum Institute, Production Bull. No. 205, 19–29 (1930).

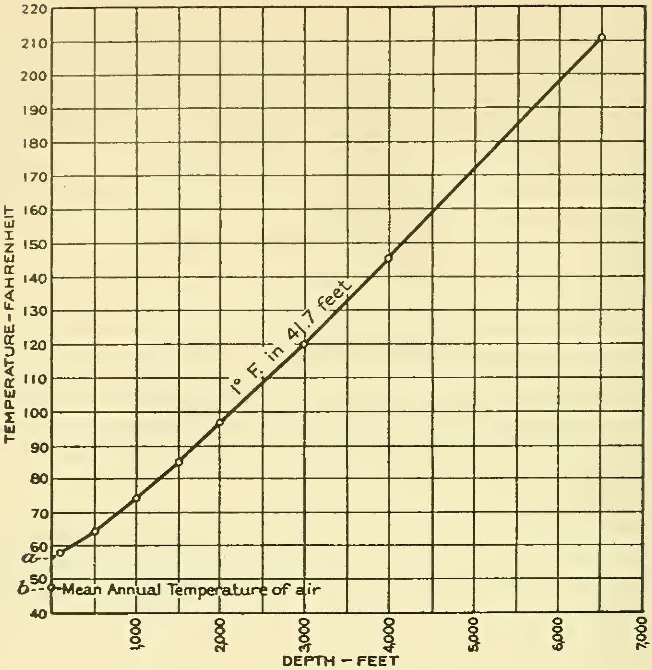


Fig. 1. Depth-temperature curve of deep well in S. W. 1/4, Sec. 34, T. 2 N., R. 69 W., Longmont, Boulder County, Colorado.

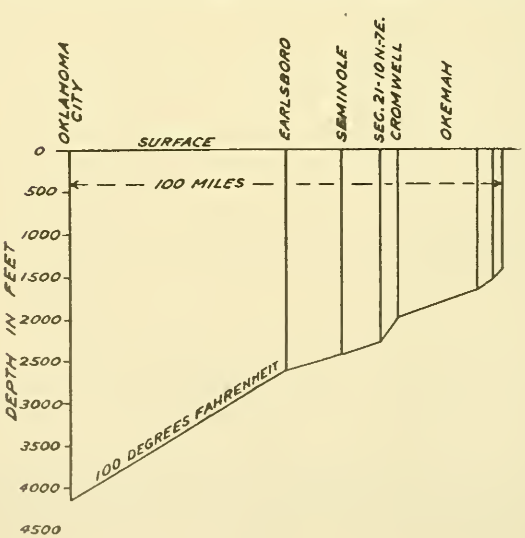


Fig. 2. Depths at which temperature of 100° Fahrenheit is reached in Central Oklahoma.

the fact that the crystalline rocks are found at a depth of about 2,500 feet at Okemah and at about 10,000 feet at Oklahoma City. The observations show that the isogeothermal surfaces tend to parallel the granitic bed at the base of the sedimentary strata. The irregularities which appear at several points along the general trend are apparently due to local uplifts on some of which oil fields are located.

Local Variations of Isogeothermal Surfaces

The Salt Creek and Long Beach oil domes, Figs. 3 and 4, are excellent examples of local uplift in which the isogeothermal surfaces stand in a definite

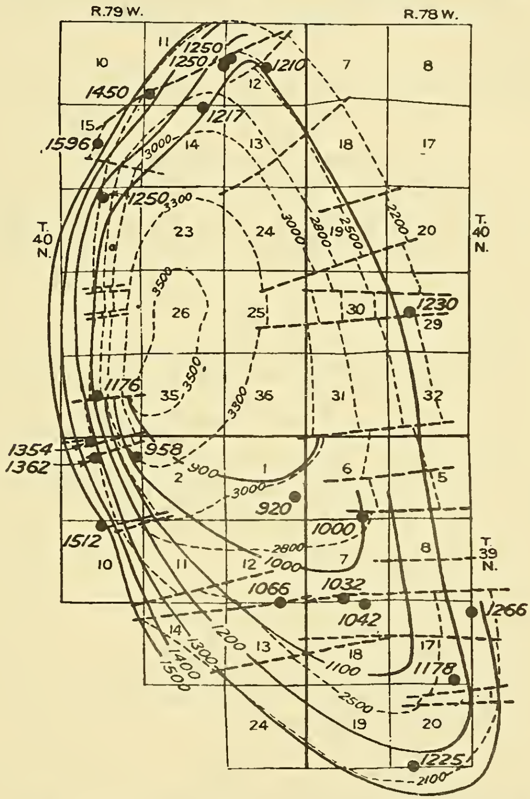


Fig. 3. Salt Creek Dome, Natrona County, Wyoming.

relation to the strata. In Fig. 3, the closed broken lines represent elevations on the "Second Wall Creek oil sand". Faults are represented by broken lines intersecting the contours. The small dots or disks represent well locations and the accompanying numbers represent the depths at which a temperature of 80°F is reached. The heavy continuous lines represent contours on the 80°F isogeothermal surface. Fig. 3 shows clearly that the highest point on the isothermal surface corresponds very closely to the highest point on the "Second Wall Creek sand". Extending laterally from this point, the 100 and 120°F isothermal surfaces tend to parallel the surface of the sand as shown in

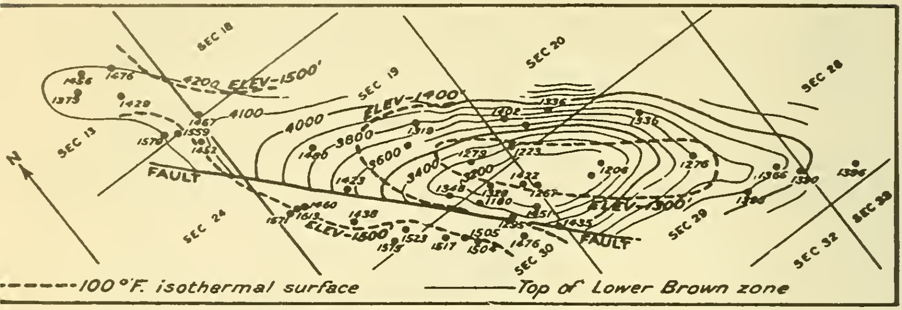


Fig. 4. Long Beach Dome, Los Angeles County, California.

Fig. 5.[3] In general, the isothermal surfaces are flatter than the surfaces of the strata and the flatness increases with diminishing depth until a point is reached near the surface of the ground at which the isothermal surfaces tend more or less to conform to surface topography. As the effects of surface

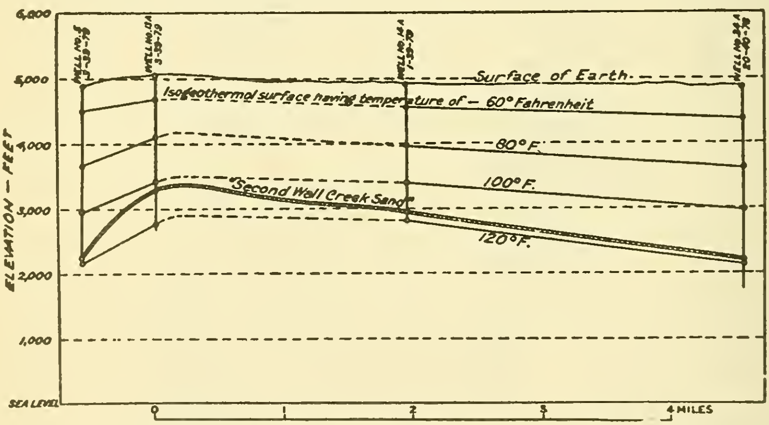


Fig. 5. Cross-section of Salt Creek Dome, Wyoming.

topography, however, are practically negligible in this field, we must look to other causes for an explanation of the marked rise of the thermal surfaces as they pass over the dome.

The Long Beach dome[4] presents a problem in which surface topography is

[3] W. T. Thom, Jr., Relation of earth temperatures to burried hills and anticlinal folds. Economic Geology 20, 524–530 (1925).

[4] A. J. Carlson, Geothermal variations in oil fields of Los Angeles Basin, California. Bull. Am. Assoc. Petroleum Geologists 14, 997–1011 (1930); Geothermal conditions in oil producing areas of California. Am. Petroleum Institute, Production Bull. No. 205, 109–139 (1930).

an important factor. As shown in Fig. 4, the isothermal surfaces are exceptionally smooth and uniform and follow the top of the upper oil producing

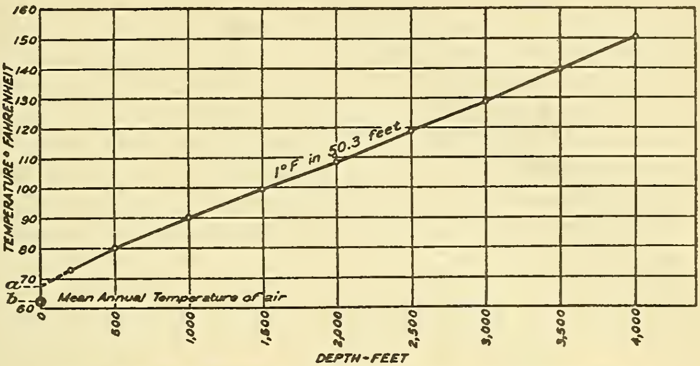


Fig. 6. Depth-temperature curve, deep well No. 2, Tucker and Johnson lease, Long Beach, California.

strata, the Lower Brown Zone, with marvelous precision. That uniformity in these surfaces was to be expected is suggested by the depth-temperature

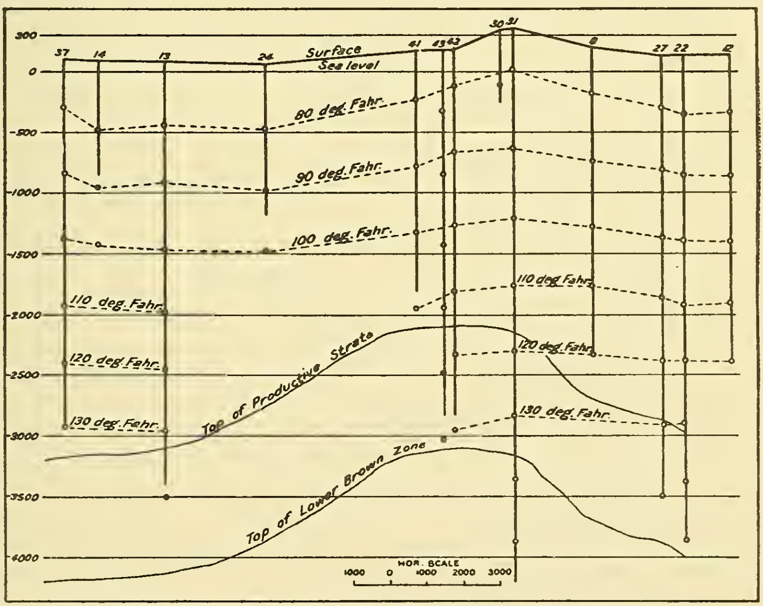


Fig. 7. Longitudinal section of Long Beach oil field. (See 4, first reference.)

curve of deep well No. 2, Fig. 6. This curve like many others in the Long Beach field, shows a slight rise in temperature at a depth of about 1000 feet, beyond which the curve is slightly convex towards the depth axis. Another point of interest is the fact that the excess of annual mean soil temperature just beneath the surface of the ground over annual mean air temperature just above the surface of the ground (*a–b*, Fig. 6), amounts to more than 7°F. This large discontinuity in the depth temperature curves at the surface of the ground is typical of oil field areas in Southern California.

Correction for Surface Topography

The longitudinal section of the Long Beach field, Fig. 7, shows at once that the total rise in the isothermal surfaces as they pass over the dome and beneath the hill can not be explained on the basis of normal cooling; for, the

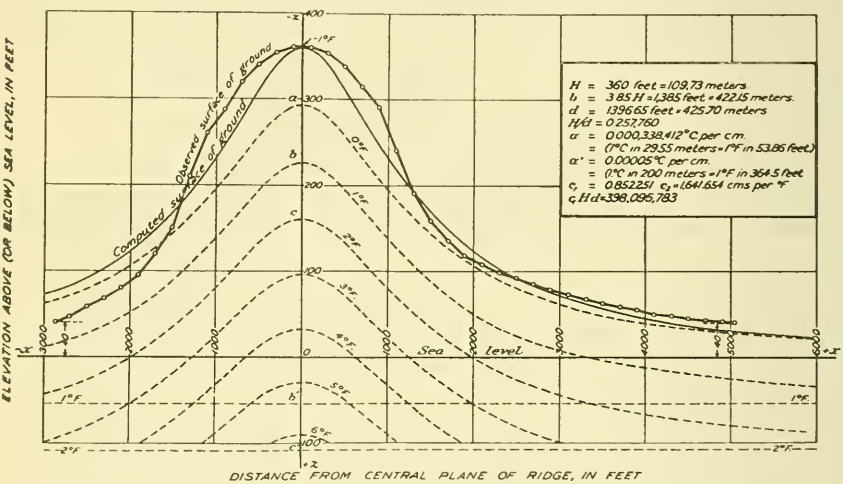


Fig. 8. Transverse section of Long Beach Dome showing computed isotherms.

fact that the thermal surfaces are steeper and show a greater rise from the lowest to the highest points than the topographic surface above them proves conclusively that we are not dealing with a normal flow of heat towards the surface of the earth. In order to form an estimate of the discrepancy resulting from this abnormality, use will be made of the equations given by Lees.[5]

Referring to Fig. 8, let the axis of x be taken at sea level, with the origin *o* directly beneath the apex of the symmetrical hill. Elevations above sea level are negative; below, positive. As the hill or mountain is assumed to be a ridge which is very long in comparison with its cross-section, the problem is reduced at once to the flow of heat in two dimensions. The fundamental equation to be solved is

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} + \frac{w}{k} = 0 \tag{1}$$

[5] Charles H. Lees, On the shapes of the isogeotherms under mountain ranges in radioactive districts. Proc. Roy. Soc. London **A83**, 339–346 (1910).

subject to the condition

$$k\frac{\partial v}{\partial n} = h(v - v_a) \tag{2}$$

which represents the loss of heat from the surface of the earth at any point on the plain or mountain slope.

In these equations, v = temperature at point x, z; v_a = temperature of air in contact with the surface of the ground at the given point; w = number of calories of heat generated per second per cubic centimeter by radioactive processes; k = coefficient of thermal conductivity of the rocks; h = coefficient of emissivity; n = numerical magnitude measured inward along the normal to the surface of the ground.

Now let us put, also, v_0 = annual mean temperature of soil just beneath the plain surface of the ground; α = temperature gradient beneath the plain; α' = temperature gradient in the air along the mountain slope.

$$\begin{aligned} c_1 &= (\alpha - \alpha')/\alpha \\ c_2 &= (v - v_0)/\alpha \end{aligned} \tag{3}$$

H = elevation of top of hill referred to level of plain; b = half-breadth of hill at elevation h nearly one-half the height of the hill; d = distance from apex of hill to point on contour at elevation h above the plain. (See Fig. 9).

$$\begin{aligned} \beta &= w/2k(\alpha - \alpha') \\ \frac{h}{H}\,\frac{(1+\beta h)}{(1+\beta H)} &= \tfrac{1}{2}. \end{aligned} \tag{4}$$

The quantity β is dependent on radioactivity. Eq. (4) is an arbitrary assumption introduced for convenience in handling the equations.

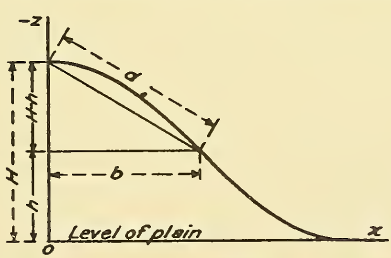


Fig. 9. Transverse section of imaginary hill or mountain.

Without going through the necessary steps of integration of Eqs. (1) and (2) and the subsequent transformation of the integrals in order to take into account the cross-section of the mountain, we write at once the equation to the contour of the transverse section,

$$z(1 - \beta z) + Hd(1 + \beta H)\frac{(z + H + d)}{x^2 + (z + H + d)^2} = 0 \tag{5}$$

and the corresponding equation to the isogeotherms,

$$v - v_0 = \alpha z - \frac{wz^2}{2k} + (\alpha - \alpha')\frac{Hd(1 + \beta H)(z + H + d)}{x^2 + (z + H + d)^2}\cdot \tag{6}$$

As Eqs. (5) and (6) are cubics in z, it will be more convenient for purposes of calculation to put them in the respective forms:

$$x^2 = -\left[z + H + d\right]\left[(z + H + d) + \frac{Hd}{z}\right] \tag{7}$$

$$x^2 = -\left[z + H + d\right]\left[(z + H + d) + \frac{c_1 Hd}{z - c_2}\right] \tag{8}$$

in which the terms dependent on radioactivity have been neglected.

Putting $x=0$ in (8),

$$z^2 + (H + d - c_2)z - (H + d)c_2 + c_1 Hd = 0 \tag{9}$$

and assigning particular values to c_2, which according to Eq. (3) represents the depth beneath the plain at which a particular temperature v is reached, Eq.

TABLE I. *Depth of isogeotherms beneath level of plain and rise of isogeotherms beneath apex of hill.*

Isogeotherms		Depth of isogeotherms beneath level of plain c_2		Rise of isogeotherms beneath apex of hill		Isogeotherms Degrees		Depth of isogeotherms beneath level of plain c_2		Rise of isogeotherms beneath apex of hill	
°C	°F	Meters	Feet	Meters	Feet	Cent.	Fahr.	Meters	Feet	Meters	Feet
0.0	0	0.0	0	89.2	293	38.9	70	1149.2	3770	24.0	79
1.1	2	32.8	108	81.8	269	41.7	75	1231.2	4039	22.8	75
2.2	4	65.7	215	75.8	249	44.4	80	1313.3	4309	21.8	71
3.3	6	98.5	323	70.7	232	47.2	85	1395.4	4578	20.9	68
4.4	8	131.3	431	66.3	218	50.0	90	1477.5	4847	20.0	66
5.6	10	164.2	539	62.5	205	55.6	100	1641.6	5386	18.5	61
6.7	12	197.0	646	59.1	194	61.1	110	1805.8	5925	17.1	56
7.8	14	229.8	754	56.1	184	66.7	120	1970.0	6463	16.0	52
8.9	16	262.7	862	53.5	175	72.2	130	2134.2	7002	15.0	49
10.0	18	295.5	969	51.1	167	77.8	140	2298.3	7540	14.1	46
11.1	20	328.3	1077	48.9	160	83.3	150	2462.5	8079	13.3	44
13.9	25	410.4	1346	44.1	145	88.9	160	2626.6	8618	12.6	41
16.7	30	492.5	1616	40.3	132	94.4	170	2790.8	9156	12.0	39
19.4	35	574.6	1885	37.1	122	100.0	180	2955.0	9695	11.4	38
22.2	40	656.7	2154	34.4	113	105.6	190	3119.1	10233	10.9	36
25.0	45	738.7	2424	32.1	105	111.1	200	3283.3	10772	10.5	34
27.8	50	820.8	2693	30.0	98	116.7	210	3447.5	11311	10.0	33
30.6	55	902.9	2962	28.2	93	122.2	220	3611.6	11849	9.6	32
33.3	60	985.0	3232	26.7	87	127.8	230	3775.8	12388	9.3	30
36.1	65	1067.1	3501	25.2	83	133.3	240	3940.0	12926	8.9	29

$\alpha = 0.000\,338\,412$°C per cm (=1°C in 29.55 meters = 1°F in 53.86 feet).
$\alpha' = 0.000\,05$°C per cm (=1°C in 200 meters = 1°F in 364.5 feet).[6]
H = 360 feet = 109.73 meters.
$b = 3.85H = 1385$ feet = 422.15 meters.
$d = 1396.65$ feet = 425.70 meters. $H/d = 0.257\,760$.
$c_1 = 0.852\,251$ $c_2 = 1641.654$ cm per degree Fahrenheit.

[6] W. J. Humphreys, Physics of the Air. Sec. ed., p. 40 (1929).

(9) gives the elevation beneath the apex of the hill of that particular isotherm which is asymptotic to c_2. The difference between z and c_2 is equal to the rise of the isothermal surface above its asymptotic plane. Thus in Fig. 8, oa represents the rise of that isothermal surface which is just beneath the surface of the plain, ($c_2=0$), and which has ox extended for an asymptote; bb' represents the rise of that surface whose asymptote is perpendicular to the axis of the hill at b'; and so on. These values are tabulated in Table I. The table applies to any other hill of the same cross-section and the same geothermal gradients (α and α') beneath the plain and along the mountain slope. To determine the temperature represented by any isotherm, it is merely necessary to add the tabular value in the table, or Fig. 8, to the annual mean temperature of the ground just beneath the surface of the plain.

In order to compare the theoretical rise of the isogeothermal surfaces with the observed, the gradient beneath the plain was assumed to be, $\alpha = 0.000{,}338{,}412$, corresponding to a rate of temperature increase of 1°C in 29.55 meters, or 1°F in 53.86 feet. This value is an approximate average of the gradients for 42 wells in the Long Beach field. From the same wells, the average annual mean temperature just beneath the surface of the ground was found to be, $v_0 = 71.6 \pm 0.2$°F. Hence, the 80°F isotherm corresponds to a temperature difference in Table I of 8.4°F. Interpolating, we find the rise of the isotherm to be 215 feet. The last two columns of Table II were obtained by a repetition of this process, using for the last column a table similar to Table I. In column 4, the level of the plain was assumed to coincide with sea level; in the last column of the table, it is taken to be 40 feet above sea level. The data for the transverse section of the hill, Fig. 8, were kindly forwarded to me by Dr. A. J. Carlson. The observed differences in elevation of the surface of the ground and the isotherms, column 3, Table II, are taken from his paper on "Geothermal variation in the oil fields of Los Angeles Basin, California."[4]

Comparison of column 3 of Table II with the last two columns shows that the observed heights are somewhat more than twice the computed

TABLE II. *Rise of isogeotherms.*

Isogeotherms °F	Depth beneath plain feet	Differences in elevation		
			Assumed	Assumed
		Observed feet 296	Computed feet 360	Computed feet 320
80	453	492	215	189
90	991	348	166	144
100	1530	274	136	118
110	2068	223	116	100
120	2607	182	100	86
130	3146	200	89	76
140	3684	141	80	68
150	4223	—	72	62
Lower Brown Zone	—	1100	—	—

heights. In making this comparison, I have attempted not to underestimate the computed heights. For example, the assumed elevations of the top of the hill above the level of the plain, namely 360 and 320 feet, instead of 296 feet given by Dr. Carlson, gives a rise of isothermal surface which is too high. Likewise, the substitution of a long ridge for a hill tends to make the computed values too high. An important possible exception, however, should be noted. Let us assume that the gradient beneath the plain is the steepest in the field corresponding to a rate of 1°F in 48.5 feet instead 1°F in 53.86 feet as used in constructing Table I. Hence we have

$$\alpha = 0.000\ 375\ 811 \quad c_1 = 0.866\ 954$$

and substituting in Eq. (9), putting $c_2=0$ in order to represent the 0°F isotherm, we find 299 feet instead of 293 feet as recorded in Table I. This difference of 6 feet is rather insignificant in comparison with the large difference of more than 100 feet for which we require an explanation. Using the value $\alpha'=0.00006$ instead of $\alpha'=0.00005$, the corresponding rise is 288 feet instead of 299 feet. This shows that the value of α' is of importance in determining the computed heights. Taken as a whole, however, the evidence substantiates the conclusion that the differences in elevation between the highest and lowest points on the observed isotherms are abnormally large.

Comments on the Data of Observation

The tests in the Salt Creek field were made under conditions such that a close approximation to true rock temperatures is to be expected. Values of a in the equation

$$y = a + bx$$

in which y = temperature at depth x; a = annual mean temperature just beneath the surface of the ground; b = gradient, vary from 46.9 to 52.3°F. The mean from 19 wells is 49.8 ± 0.2°F. The probable error r of an observation of weight unity is 1.0°F. Values of the excess of soil temperature over air temperature ($a-b$, Figs. 1 and 6) range from 0.2 to 5.6°F. The mean is 3.10 ± 0.2°F. $r = \pm 1.0$°F.

At Long Beach, the temperatures just beneath the surface of the ground from 42 wells fall between the limits 68.4 and 77.2 with a mean value of 71.6 ± 0.2°F. $r = \pm 1.4$°F. The maximum value of the excess of soil temperature over air temperature is 14.6, the minimum, 5.8°F. The mean is 9.0 ± 0.2°F. $r = \pm 1.4$°F.

The large range in the preceding values suggests the possibility that the temperatures in some of the wells that had been pumped are too high. The resulting error is a maximum at or near the surface of the ground and gradually diminishes with more or less regularity to zero at the source of the rising column of oil or water. An obvious effect of this abnormality is to elevate the isotherms in amount roughly proportional to their elevation above the fluid bearing sand, and diminish the gradients or increase the reciprocal gradients in the same relative proportion. To ascertain the possible magnitude of this

TABLE III. *Mean values of a, b, and 1/b.*

100–1000 feet								100–2000 feet				
No. of Obs.	a	r	r_0	b	$r \times 10^5$	$r_0 \times 10^5$	$1/b$	No. of Obs.	b	$r \times 10^5$	$r_0 \times 10^5$	$1/b$
5	72.8	1.2	0.5	0.01781	58	26	56.1	5	0.01776	44	20	56.3
35	71.3	1.4	0.2	.01885	9	2	53.0	35	.01878	59	10	53.2
10	73.4	1.6	0.5	0.01804	87	28	55.4	10	0.01804	63	20	55.4
30	70.9	1.0	0.2	.01895	88	16	52.8	30	.01885	55	10	53.0
10	73.4	1.6	0.5	0.01804	87	28	55.4	10	0.01804	63	20	55.4
10	70.9	1.0	0.3	.01919	81	26	52.1	9	.01923	34	11	52.0
10	71.2	1.1	0.3	.01855	108	34	53.9	10	.01849	69	22	54.1
10	70.4	1.1	0.3	.01912	74	23	52.3	9	.01882	38	13	53.1

source of error, Table III has been computed. The table is based on the data from 40 wells, two of the poorest values being omitted. In the first two lines, the mean values for 5 wells highest on the structure are compared with the means from the remaining 35 wells. Similarly, in the third and fourth lines, the means for 10 wells highest on the structure are compared with the means from the remaining 30 wells, while in the final tabulation the wells have been grouped in four successive groups as regards elevation on structure, the first

TABLE IV. *Gradients and reciprocal gradients* ($H = 360$ *feet; and* $H = 320$ *feet*).

In air on slope of hill			Beneath adjacent plain			Beneath summit of hill		
Gradients (α')	Reciprocal gradients		Gradients (α)	Reciprocal gradients		Gradients (α_s)	Reciprocal gradients	
°C per cm	Feet per °F	Meters per °C	°C per cm	Feet per °F	Meters per °C	°C per cm	Feet per °F	Meters per °C
0.00006	303.8	166.7	0.00036	50.6	27.8	0.00028 27	64.5	35.4
0.00006	303.8	166.7	0.00030	60.8	33.3	0.00023 81	76.5	42.0
0.00005	364.5	200.0	0.00036	50.6	27.8	0.00028 01	65.1	35.7
0.00005	364.5	200.0	0.00030	60.8	33.3	0.00023 56	77.4	42.5
0.00006	303.8	166.7	0.00036	50.6	27.8	0.00028 64	63.6	34.9
0.00006	303.8	166.7	0.00030	60.8	33.3	0.00024 11	75.6	41.5
0.00005	364.5	200.0	0.00036	50.6	27.8	0.00028 40	64.2	35.2
0.00005	364.5	200.0	0.00030	60.8	33.3	0.00023 87	76.4	41.9

group as heretofore being the highest. The table shows clearly that the values of a and $1/b$ in the first group of each set are always the highest. However, minor irregularities of this kind can not always be attributed to pumping operations. In general, the depth-temperature curves become more irregular as the surface of the ground is approached. It is for this reason that a map showing the isothermal surfaces is much more reliable than a map showing the distribution of reciprocal gradients. In addition to the preceding irregularities, the variation of the annual mean temperature of the air over the surface of the hill must be taken into account. Thus, the value of a for six wells on the south side of the hill is 73.0 ± 0.8; the same for 15 wells on the north side is 71.4 ± 0.3, making a possible temperature difference on the two slopes of 1.6 ± 0.9°F. Other observers have likewise found maximum temperatures on southern slopes.[7]

As a further explanation of the slight increment in the observed values of $(1/b)$ as the crest of the hill is approached, a comparison must be made between the assumed gradient (α) beneath the plain and the theoretical gradient (α_s) beneath the summit of the hill. Using the value of H/d given in Table I and Fig. 8, and making the appropriate substitution in the equation

$$\alpha_s = \alpha - H(\alpha - \alpha')/d$$

we obtain the values of α_s tabulated in the first part of Table IV. In the second part of the table, (α_s) is given for $H = 320$ feet, $H/d = 0.245\ 240$.

Interpolating for our assumed reciprocal gradient beneath the plain, 1°F in 53.86 feet, we find the value 1°F in 68.3 feet at the summit of the hill. That is, the computed variation from the plain to the summit of the hill is four or five times the differences, 3.1 and 2.3, shown between the highest and lowest groups on the structure in the last set of values in Table III. In one way only does it seem possible to bring theory and observation into agreement. Substitution in the equation,

$$\frac{\partial v}{\partial z} = \alpha - (\alpha - \alpha')\frac{Hd}{(z + H + d)^2} \tag{10}$$

which represents the gradient at any elevation (z) beneath the apex of the hill, we find for

$$\alpha = 0.00033\ 8412 (1° \text{ F in } 53.86 \text{ feet}),\quad \alpha' = 0.00005,\ H = 360 \text{ feet},$$

the values,

$1/b = 1$°F in	62.5 feet	$z = 0$ meters
= "	55.9 "	$z = 500$ "
= "	54.8 "	$z = 1000$ ".

At $z = o$, 360 feet beneath the top of the hill, the theoretical difference of 8.6 feet between the assumed and computed values of $(1/b)$ greatly exceeds the observed differences, 3.1 and 2.3 feet, Table III; but at $z = 500$ meters, corresponding to a depth of 2000 feet beneath the top of the hill, the observed differences slightly exceed the computed value of 2.0 feet. At greater depths, Eq. (10) shows that the gradients beneath the plain and the hill approach coincidence.

To sum up the evidence at Long Beach, the elevation of the isotherms with reference to the topographic surface, and the small variation of gradient over the structure in comparison with the theoretical variation proves con-

[7] Edith M. Fitton, Soil temperatures in the United States. Monthly Weather Review 59. 9 (1931).

clusively that the temperature distribution in this field is highly abnormal. The only exception appears to be the possible agreement of the observed and computed gradients at depths of about 2000 feet.

At Salt Creek, the evidence of variation of temperature with structure is conclusive. At Long Beach, the evidence is confused on account of the fact that in some of the wells the temperatures at the higher levels were increased by pumping operations while in a few of the remaining wells, the temperatures may have been lowered as a result of the wells being on a vacuum. However, proceeding on the assumption that we have a normal distribution of temperatures, and taking into account the fact that the abnormal temperatures in some of the wells have tended to increase the values of $(1/b)$, it follows that the observed variation in the values of $(1/b)$ as we pass from the plain to the summit of the hill, should have exceeded slightly the theoretical variation; but, just the converse is true—the observed variation is only a small fraction of the theoretical variation. Hence we are driven to the conclusion that abnormal temperature conditions in this field have tended to neutralize the effects of surface topography.

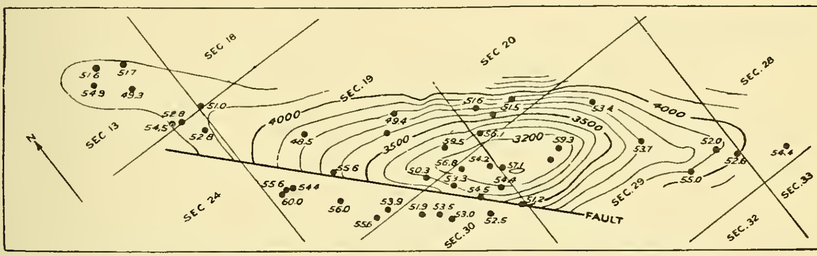


Fig. 10. Long Beach Dome. The numbers opposite well locations represent feet per degree Fahrenheit, 100–2000 feet.

Causes of Temperature Variations

Depth to the crystalline rocks and flow of heat along the strata appear to be possible explanations of the close relationship that apparently exists between the isotherms and the general trend of the strata in central Oklahoma (See Fig. 2).

A similar explanation may apply at Salt Creek, which is an exceptionally well defined local uplift. Steep dips occur on the western side of the field (See Figs. 3 and 5) and very hard granite was found by The Midwest Refining Company at a depth of 5420 feet near the top of the dome in the S.W. 1/4 N.W. 1/4, sec. 35, T. 40 N., R. 79 W. This evidence suggests the possibility that the depth to the granite is least on top of the dome, and that the direct contact of the granite with the hot crystalline rocks at great depths serves to maintain lines of flow from an almost inexhaustible heat source to the top of the dome. Additional evidence in substantiation of this conclusion is to be found in the fact that the values of $1/b$ vary from 1°F in about 29 feet on top of the dome to 1°F in about 56 feet in Sec. 15, T. 40 N., R. 79

W. on the outer edge of the dome. Not only is the great range in values of $1/b$ in agreement with our hypothesis, but the further fact that abnormally high temperatures are found immediately above the granitic mass leaves little to be desired in explanation of the observed distribution of temperatures.

On account of the great depth to the crystalline rocks at Long Beach, possibly 20,000 feet, it seems unlikely that depth to the crystalline rocks, which appears to be an important factor at Salt Creek and in Central Oklahoma, will provide an explanation of the temperature variations found in this field. Steeply dipping strata are present, but about the only other possibilities to which we can appeal are recent erosion, abnormal thermal conductivities and generation of heat in the strata. I shall not here attempt to discuss the merits of these possibilities.

To estimate the effects of radioactivity on the isogeotherms at Long Beach, let us expand Eq. (4) into the form

$$h = \frac{H}{2}\left[1 + \frac{\beta}{2}H + \cdots\right]$$

and then putting

$$H = 360 \text{ feet} = 10973 \text{ cms} \quad w = 1.5 \times 10^{-13}$$
$$\alpha = 0.00036 \quad \alpha' = 0.00006 \quad k = 0.004$$

we have

$$h = \frac{H}{2} = 5486 \text{ cms} \quad \beta = \frac{w}{2k(\alpha - \alpha')} = 6.25 \times 10^{-8}$$

and the new value of h is found to be 5488 cms an increase of only 2 cms. Using Joly's[8] recent value, $w = 42.1 \times 10^{-14}$, we have

$$\beta = 1.75 \times 10^{-7} \quad h = 5491 \text{ cms}$$

an increase of 5 cms. The resulting change in d (Fig. 9) is obviously negligible, consequently Eq. (6), which contains (d), remains unchanged.

Another method of procedure consists in computing the rise in the 0°F isotherm beneath the apex of the hill. Putting $x = o$ and $z = o$ in (6) and using the same constants as before, including Joly's value of w, we find

$$v + \Delta v = (\alpha - \alpha')(1 + \beta H)\frac{H}{1 + \frac{H}{d}}$$

$$= 2.617 + 0.005 = 2.622° \text{ C}$$

from which it again appears that the term due to radioactivity is negligible in comparison with the observed variations in temperature across the structure.

[8] John Joly. The surface-history of the earth. Second edition, p. 60 (1930). The Clarendon Press, Oxford.

Some General Suggestions

Our observations are not yet sufficiently accurate nor sufficient in number to permit making definite generalizations. The chief source of inaccuracy is unstable temperature equilibrium in wells. Notwithstanding these irregularities, however, the observations show conclusively that the isogeothermal surfaces rise in passing over a large number of salt domes and anticlinal structures. There is no well established evidence of depressed isogeotherms in any oil field, but in a considerable number of fields, the rise in temperature is not very marked. In general, the observations suggest the conclusion that a relation exists between the high temperatures and positive gravity anomalies that prevail over the crests of many of the oil field anticlines.

The causes of temperature variations given in the preceding paragraphs are to be regarded merely as preliminary suggestions. The complete explanation will undoubtedly involve isostasy, geological history, and the sources of the earth's internal heat.

GEOTHERMAL GRADIENT DETERMINATIONS IN THE LAKE SUPERIOR COPPER MINES

By L. R. Ingersoll
University of Wisconsin

(Received February 10, 1932)

Abstract

The Michigan College of Mining and Technology, in cooperation with the Calumet and Hecla Copper Company and the author, is carrying out a program of temperature measurements in the deep copper mines of Northern Michigan, extending the previous work of Agassiz and others. Temperatures are measured with mercury thermometers mounted in bakelite tubes, placed in drill holes in mine workings where the rock has been freshly exposed, special attention being given the effects of drilling, blasting, and other heat conduction considerations. Present results give as the average gradient from the surface to 5679 feet below (temp. 95.3°F), 1°F in 108.5 feet (0.0168° C/meter). The gradient is more nearly uniform than has sometimes been supposed. A preliminary attempt has been made at calculating the previous "thermal history" of this region. Diffusivity of specimens of the rock measures 0.0075 c.g.s., and on this basis calculations of theoretical temperature-depth curves have been made for 25 different assumptions of previous temperature conditions, and compared with the actual curve. Results as yet are inconclusive but indicate that at least 30,000 years have elapsed since the last glacial epoch, a longer period than usually assumed.

Ever since the days of Kelvin's theoretical discussion of the age of the earth there has been interest in measurements of the earth's temperature gradient, and while radioactive discoveries have considerably diminished the importance of calculations on this basis there are nevertheless many other interests—some of them new—attaching to geothermal measurements. As a result a mass of thermal data has been and is being collected from deep mine, well and drill hole measurements in all parts of the world. The copper mines of the Keweenaw peninsula in northern Michigan offer a particularly inviting opportunity for such study because of their great depth and freedom from local heat sources. The pioneer measurements of Agassiz,[1] which resulted in an apparently unusually low gradient, are too well known to require comment, and Lane,[2] Van Orstrand[3] and others have also made valuable contributions along this line.

Certain favorable opportunities, depending partly on the increasing depth of the mines, have for some time called for a reopening of this study and accordingly Professor James Fisher of the Michigan College of Mining and Technology and Mr. Harry Vivian, Chief Engineer of the Calumet and Hecla

[1] Alexander Agassiz, Am. Jour. Sci. (3) 50, 503 (1895): Brit. Assoc. Adv. Sci. Report of 71st meeting, p. 65 (1901).

[2] A. C. Lane, Bull. Geol. Soc. of Am. 34, 703 (1923).

[3] C. E. Van Orstrand, Am. Jour. Sci. 15, 495 (1928). See also N. H. Darton, U. S. Geol Survey, Bulletin 701, p. 50.

Consolidated Copper Company, are cooperating with the writer in a program of work which may extend over a period of years. The Michigan College is generously supplying the financial aid and the Calumet and Hecla Company the skilled technical assistance necessary to the success of the work. The measurements in all cases are to be guided by heat conduction considerations to the end that they may represent as closely as possible the actual virgin temperature at the spot, unaffected by mining operations. The work must accordingly proceed slowly, waiting frequently for favorable opportunities for temperature measurement; but while it has been going only about a year some preliminary measurements of value have been obtained as well as other results which may be of interest to the physicist.

Method of Temperature Measurement

General procedure

The measurements so far have been confined to the Calumet and Hecla mine in which most of the Agassiz measurements were also made. A somewhat different procedure has been adopted, however. In the Agassiz work thermometers were sealed for weeks at the bottom of drill holes 10 feet deep, in shafts or passages which had, themselves, been exposed to ventilation in some cases for many months. A little calculation on the basis of heat conduction theory will serve to show that the results under such conditions are very likely to be influenced by the ventilation, and only in the most favorable cases can be depended on to give virgin rock temperatures.

In the present work the measurements have all been carried out in new workings, i.e., drifts or other cuttings which have been advancing steadily a number of feet a week and which are well removed from other parts of the mine. At such a "temperature station" a hole is drilled a few feet back from the breast and in rock whose face has been exposed only a few days. The hole is usually 7 feet deep, but in some cases special 14 foot holes are run. Temperatures are taken with two or more thermometers located at the bottom of the hole. It may be remarked that, save for the water used in drilling, which quickly drains out, the hole has been found practically dry in all cases.

Thermometers

After careful consideration of the advantages and disadvantages of electrical temperature measuring instruments, mercury thermometers have been finally chosen for this work, at least for the type of measurements being made at present. They have been specially made by Henry J. Green, reading 0−40°C in 1/10°, and frequent zero tests and comparisons with one of the number calibrated by the Bureau of Standards give maximum errors of the order of 0.04°C. They are mounted in bakelite tubes 1 inch in diameter and 14 inches long with the thermometer bulb specially insulated thermally. This is a vital matter to which a great deal of attention has been given. It is necessary that after pulling from a drill hole the mercury thread show no change for something like a minute, so as to give the operator plenty of time to make his readings. At the same time there must be combined with this lag or "delay action" a reasonably rapid action of the thermometer, once the bulb starts to change in temperature, as otherwise it would occupy too much time in coming to the temperature of the hole.

After some calculation and much experimentation a very satisfactory solution of the problem has been reached, involving insulating the bulb in vulcanite, wax and dynamo paper with waterproof lacquer as shown in Fig. 1. If such a thermometer is subjected to even such a radical temperature change as is involved in plunging into ice water, it is a full minute before the mercury thread shows a change of 1/10°. Having started to move, however, it falls several degrees a minute and comes down (asymptotically) to the

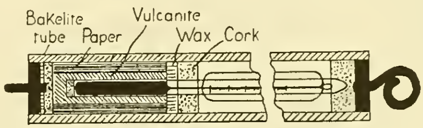


Fig. 1. Section of mounted thermometer (Overall length, 35 cm).

temperature of the bath—within 0.02°, say—in about half an hour. The final design of these thermometers has proved very satisfactory indeed for the work in hand. Difficulties experienced with the early ones over the separation of the mercury thread have been entirely overcome in the later ones by having them made with about one atmosphere of inert gas pressure over the mercury. This has the further advantage of relieving the bulb of practically all pressure, inside or outside, and doubtless tends to render the zero still more stable.

Temperature readings

As the holes are drilled in the usual manner with air-operated machines, with a flow of water through the drill steel, there is almost no heating effect due to drilling. Temperature readings taken immediately after the hole is completed are sometimes a degree high, but after a few hours all evidence of drilling heat has usually disappeared. To be on the safe side, however, little reliance is placed on readings made less than 24 hours after drilling, and the whole question of possible small errors due to heat of drilling is one on which considerably more study is to be expended.

Thermometers are used in tandem groups of two or three, run into the hole with a stout wire, with the last one carrying a rubber gasket. The mouth of the hole is stopped with a wooden plug. While the thermometers reach equilibrium in half an hour they are left for at least an hour and then quickly withdrawn and read. They are then replaced in the hole, to be read again about every two hours until several sets have been taken. This procedure is repeated a couple of days later, with occasional readings continuing for several weeks or months. In addition, air, rock face and psychrometer readings are occasionally taken in the drift to furnish general information as to the thermal conditions at the station.

Plots of these temperatures with time usually give an approximately straight line, sloping slowly downwards, showing the gradual cooling—of the order of a degree or two a month—which takes place as heat is conducted from the rock into the slightly cooler drift. A slight extrapolation of the curve gives the initial (virgin) rock temperature with an accuracy believed to be of the order of 1/10°C. To make sure that drilling and blasting effects are really negligible, as any calculations which can be carried out would indicate, an occasional 14 foot hole has been run in at the bottom of a short cross-cut such as occasionally occurs in drifting. In such a hole the tests made so far indicate a steady temperature for weeks and months and go far towards removing all doubt in one's mind as to any transient effects. The agreement which has been found so far between measurements in such deep holes and the standard 7 foot ones is good, but much more work remains to be done along this line before complete confidence can be placed in the results, however satisfactory they appear to be as a whole.

Results

The results to date are shown in Fig. 2. The points determined in the present work are seen to lie very well on a smooth curve which is almost a straight line. When this is continued to the surface level, making such use as seems justifiable of the Agassiz measurements, and especially of the point[4] indicating average surface temperature, we have the geothermal curve for this particular spot—insofar as it can be considered to be established by these measurements; it is unfortunate that there have been no recent opportunities for reliable measurements in the upper mine levels. The absence of local heat sources in this region, as well as general geological considerations, would lead one to believe that this curve will probably be found to be typical of this region, but measurements in other mines will be made as opportunities offer. A comparison with all the Van Orstrand drill-hole data for this locality must also be made. It may be noted in passing that the average gradient found here of 1°C in 59.5 meters is less than half as steep as that taken by Kelvin as the average for the whole earth, viz., 1°C in 27.76 meters.

Theoretical interpretation of results

A preliminary attempt has been made at the interpretation of this curve in terms of the previous thermal history of this region, involving particularly the time since the last ice age. In order to do this it is first necessary to determine the thermal diffusivity (thermal conductivity divided by the product of specific heat and density) of the rock material. B. O. Peirce[5] made a number of careful tests of the conductivity of specimens of rock from the Calumet and Hecla mine many years ago, but unfortunately several assumptions are involved when it comes to getting diffusivities from these results. Accordingly, a few tests on specimens brought from two regions in the mine

[4] This is the point taken by Lane (reference 2, p. 703–705) as the average surface temperature at Calumet. It was arrived at in several ways, all of which give nearly the same result and is believed to be a very good average.

[5] B. O. Peirce, Proc. Am. Acad. of Arts and Sci. 38, 652 (1903).

have been made by a method developed by the writer[6] some years ago. The specimen of rock, which must be in the form of a slab say 6 cm thick and 25 or more cm square, is mounted in a paraffined wood frame which thermally insulates the edges, leaving only the faces exposed. A fine thermocouple is located in a small hole drilled into the center of the slab, everything being

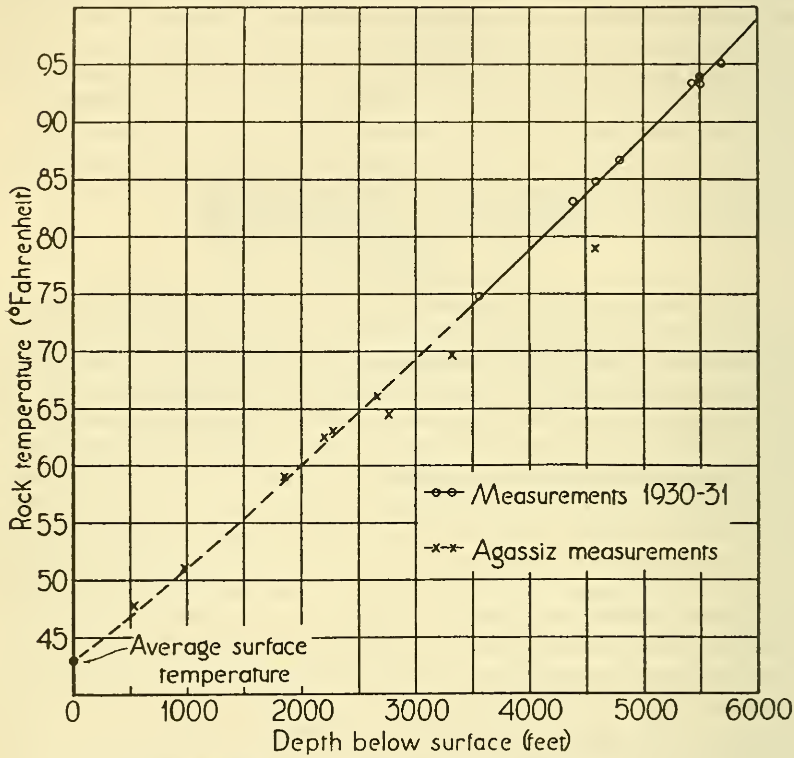


Fig. 2. Geothermal measurements in the Calumet and Hecla mines. Average temperature gradient, 0–5500 ft. depth, 1°F in 108.5 ft. or 0.00922°F/ft., or 1°C in 59.5 meters, or 0.0164° C/m. Gradient, 3500–5500 ft. depth, 1°F in 103.1 ft. or 0.00970°F/ft.

carefully waterproofed. When such a slab has been left in a constant temperature region for 24 hours and is then suddenly plunged into stirred ice water so that the faces are quickly lowered to and kept at zero, the diffusivity can be readily calculated from the time it takes for the center temperature to fall half way from its original value to the zero point. Two specimens of trap rock tested in this way gave values for the diffusivity of 0.0077 and 0.0074 respectively. Since the former specimen was from the vein and the latter

[6] For a brief account of this method see L. R. Ingersoll and O. A. Koepp, Phys. Rev. 24, 92 (1924). For the underlying theory see Ingersoll and Zobel, "Mathematical Theory of Heat Conduction" (Ginn) p. 105 ff.

from the surrounding trap rock, which composes the great bulk of the material, the weighted average is taken as 0.0075 c.g.s. units.

With the above value, calculations[7] have been made of theoretical temperature-depth curves for some 24 different assumed "thermal histories" of this region. The simplest assumption is that the surface temperature has been about as at present for 10,000 years and that previous to this the surface was covered with ice. In other assumptions the time has been changed and in still others several glacial overflows are taken account of. The periods of time have been varied between wide limits. The equation for calculating the points of these curves is

$$\theta = \frac{x}{2h(\pi)^{1/2}} \int_0^t F(\lambda) e^{-x\ /4h^2(t-\lambda)} \cdot (t-\lambda)^{-3/2} d\lambda$$

where x is the distance below the surface, at which the temperature θ is to be calculated, h^2 the diffusivity, and $F(\lambda)$ the surface temperature function.

These curves having been calculated and corrected for the general average gradient slope, a careful comparison has been made between them and the actual curve as given in Fig. 2. The results are a little disappointing at first as the curve is so nearly a straight line anyway and so much depends on the justification for certain assumptions which cannot as yet be proved satisfactorily, e.g., that the diffusivity of the rock is unchanged for the first mile or more in depth. However, certain assumptions fit much better than others and point on the whole to a somewhat longer period—say 30,000 years—since the last glacial overflow for this region, than is usually assumed.

In addition to the acknowledgements at the beginning of this article, special thanks are due to Mr. James McNaughton, President of the Calumet and Hecla Company, for placing at the service of this investigation the facilities of the mines, and particularly to Messrs. H. E. Jefferson, H. S. Donald and R. F. Wilson for their care in making the measurements, as well as for many valuable suggestions.

[7] I am glad to acknowledge the assistance of the following of my students in making these long and arduous calculations: Miss M. C. Wolf and Messrs. R. O. Anderson, R. E. Erickson, R. G. Herb, W. L. Hole, R. W. Prucha and M. T. Rodine.

VELOCITY OF ELASTIC WAVES IN GRANITE

By L. Don Leet and W. Maurice Ewing
Harvard Seismograph Station and Lehigh University

(Received January 14, 1932)

Abstract

The velocity of elastic waves in granite was determined at Quincy and Rockport, Massachusetts, and Westerly, Rhode Island. The waves measured were generated by dynamite explosions. They were recorded by portable seismographs at distances ranging from fifty feet to four thousand six hundred feet. The observed velocities for longitudinal waves were:

Quincy......16,260 ± 70 ft./sec. or 4.96 ± 0.02 km/sec.*
Westerly....16,400 ± 120 ft./sec. or 5.00 ± 0.04 km/sec.
Rockport....16,670 ± 40 ft./sec. or 5.08 ± 0.01 km/sec.
Average.....16,530 ± 90 ft./sec. or 5.04 ± 0.03 km/sec.

A three-component seismograph, used only at Quincy, recorded transverse waves, the velocity of which was 8150 ± 90 ft/sec., or 2.48 ± 0.03 km/sec. From the two velocities determined at Quincy and the density of specimens taken from the shooting location, 2.65 grams/cm³, values for the bulk modulus, k, compressibility, β, rigidity, μ, Poisson's Ratio, σ and Young's Modulus E, were obtained as follows: $k = 44 \pm 1 \times 10^{10}$ dynes/cm²; $\beta = 2.28 \pm 0.05 \times 10^{-12}$ cm²/dynes; $\mu = 16.3 \pm 0.4 \times 10^{10}$ dynes/cm²; $\sigma = 0.333 \pm 0.005$; $E = 43 \pm 1 \times 10^{10}$ dynes/cm². The form of the time-distance curves, straight lines through the origin, indicated that the waves did not penetrate deeply. Accordingly, the values obtained are for pressures of only a few atmospheres. The bearings of these results upon earlier investigations of the elastic constants of granite are discussed. Although direct comparisons between laboratory and field results are not conclusive, they indicate that the Adams and Williamson curve is incorrect for pressures below 2,000 megabars, and that there is no marked difference between dynamically and statically determined compressibilities of granite.

I. Location

DETERMINATIONS of the velocity of elastic waves in granite were made in three well-known granite quarrying areas, Quincy and Rockport, Massachusetts, and Westerly, Rhode Island. Within each area a zone of practically no topographic irregularity was selected. The continuity of the granite in each zone was proven by numerous quarries and outcrops. Maps of these zones appear in Fig. 1. The observing stations and shot locations shown there were mapped by plane table on a scale of 300 feet to the inch.

II. Method

The elastic waves were produced by the explosion of charges of dynamite which were always placed in contact with the granite. Sixty percent quarry gelatine was used. Their arrival at a point on the granite at any desired distance from the explosion was recorded by a seismograph placed at that point.

* The ± values given in this paper are probable errors.

The distance travelled by the waves and the time required constituted the observed data. Distances were scaled from the maps reproduced in Fig. 1.

The time plotted against the distance yielded a time-distance graph which was a straight line through the origin. The velocity, the reciprocal slope of this line, was computed by a least square solution.

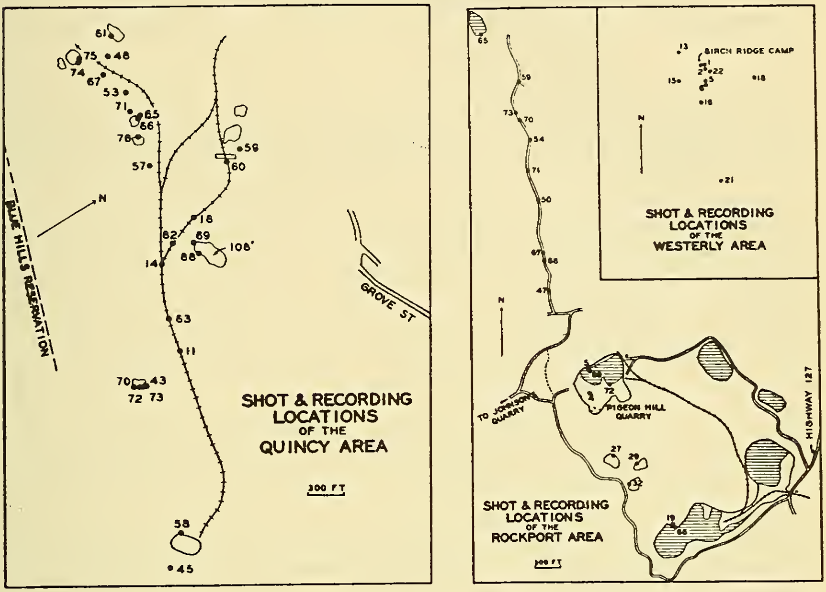


Fig. 1.

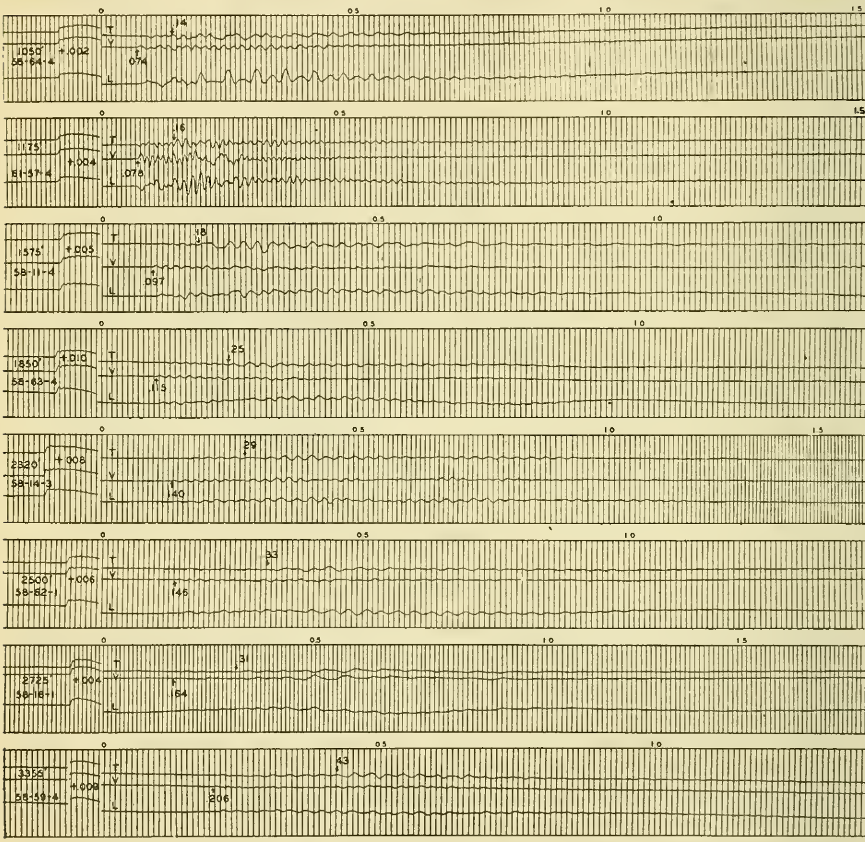
III. Instruments

The seismograph used at Quincy consisted of three components, two horizontal and one vertical, mounted on a single base. The instrument was always oriented so that one of the horizontal components recorded vibrations on a line from shot to recorder, while the other, at right angles to this, recorded vibrations transverse to that line. These are designated respectively as the longitudinal and transverse components. The longitudinal component had a free period of 0.77 sec., the transverse, of 0.62 sec., and the vertical, of 0.55 sec. Each was critically damped by a magnetic device of conventional design. The static magnification, effected by mechanical and optical means, was about one thousand.

At Westerly and Rockport, a vertical component seismograph was used. It had a free period of approximately two tenths of a second and was critically damped by a piston working in a cup of oil. The static magnification, also mechanical and optical, was about twelve thousand.

Both of the instruments recorded photographically on a strip of bromide paper one inch wide, moving at a speed of about twenty centimeters per second. A vacuum-tube driven tuning fork controlled the interruption of a beam of light, thereby producing timing lines, which extended the full width of the record, at intervals of 0.01 sec.

The electrical circuit which was used to detonate the dynamite from the recording location also served for the registration on the record of the instant of the explosion. A modified polarized relay in this circuit was actuated both when the firing circuit was closed and when it was broken by the explosion. A mirror attached to the moving vane of the relay and in the path



of the beam from light-source to seismograph effected the actual registration. In Fig. 2, the upward jog in all seismograph lines marks the deflection caused by closing the firing circuit, and the downward jog a few hundredths of a second later, the instant of the explosion.

IV. Firing Positions

At Quincy and Rockport, several abandoned water-filled quarries were used as firing locations. The charges were placed against the quarry walls and in some cases as much as thirty feet beneath the surface of the water, and one hundred feet beneath the surface of the ground. This arrangement had the important advantages that the explosions were confined by the water and that the energy entered sound, unweathered granite. The efficient confinement of the explosions by the water made it possible to use relatively small

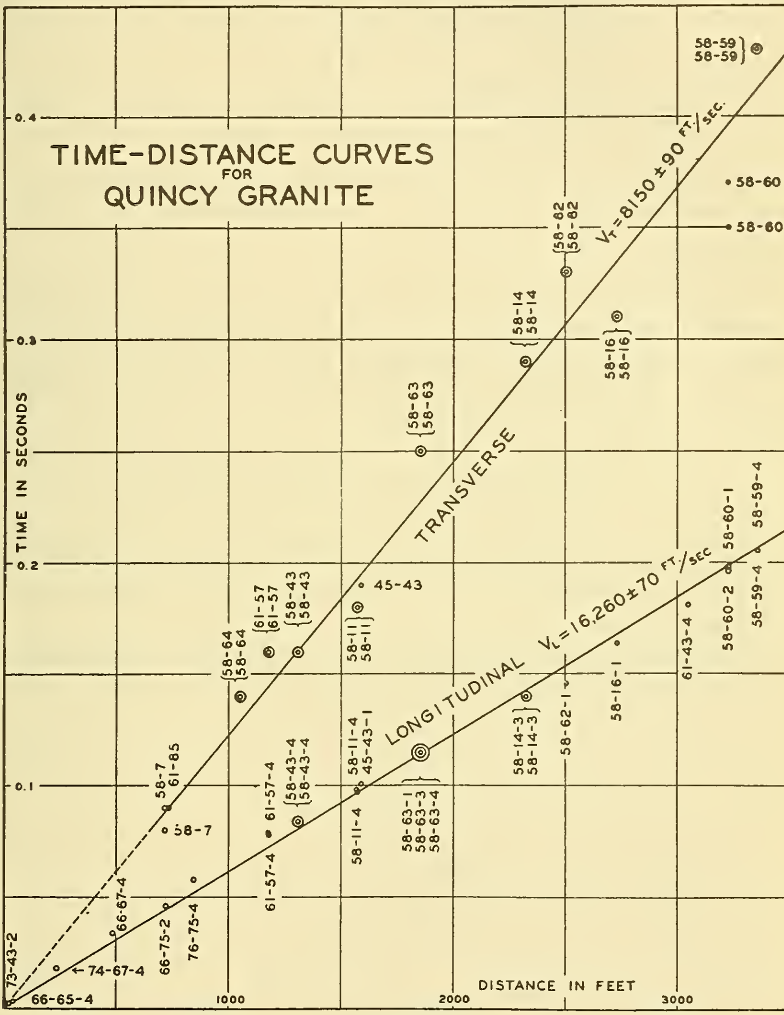


Fig. 3.

charges. About five hundred pounds of dynamite sufficed for the entire investigation.

At Westerly, shooting positions were chosen where the granite was covered with several feet of glacial drift. There the charges were placed in holes dug down to the granite.

V. Quincy Results

Specimen records from Quincy are reproduced in Fig. 2. The traces marked "T", "V", and "L" are the transverse, vertical, and longitudinal components respectively. Each record is identified by figures indicating the distance, shot position, recording position, and weight assigned it in the least square solution. For example, the first record in Fig. 2 is for a distance of 1050 feet. The figures 58–64–4 indicate that the shot was fired at position 58 (see map, Fig. 1), recorded at position 64, and accorded a weight of 4 in the least square solution. Weights were assigned on the basis of amplitude of trace and general photographic quality, which determined the accuracy with which the time could be read.

The interval between the instant of the explosion and the "0" timing line, the travel-time of the longitudinal wave on the vertical component, and the travel-time of the transverse wave on the transverse component are shown on each record.

The time-distance graph for Quincy is shown in Fig. 3. The fact that it is a straight line through the origin indicates that the waves did not penetrate deeply. It is probable that the penetration was not more than twenty meters, and certain that it did not exceed 200 meters.* The lines drawn through the points were determined by least square solutions. Their reciprocal slopes give:

$$V_L = 16{,}260 \pm 70 \text{ ft./sec. or } 4.96 \pm 0.02 \text{ km/sec.}$$
$$V_T = 8{,}150 \pm 90 \text{ ft./sec. or } 2.48 \pm 0.03 \text{ km/sec.}$$

where V_L and V_T are the velocities of longitudinal and transverse waves respectively.

The density of specimens taken from the shooting location was 2.65 ± 0.02 grams/cm.[3]

The elastic constants of the granite can be obtained by substituting these values in the following equations.[8]

$$V_L = \left(\frac{\lambda + 2\mu}{\rho}\right)^{1/2} \qquad (1)$$

$$V_T = (\mu/\rho)^{1/2} \qquad (2)$$

$$k = \lambda + 2/3\mu \qquad (3)$$

$$\beta = 1/k \qquad (4)$$

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu} \qquad (5)$$

$$\sigma = \frac{\lambda}{2(\lambda + \mu)} \qquad (6)$$

where k is the bulk modulus, β the cubical compressibility, μ the rigidity, E Young's modulus, σ Poisson's ratio, and ρ the density.

* This point is being investigated further.

The values obtained for these constants are:

$$\mu = 16.3 \pm 0.4 \times 10^{10} \text{ dynes/cm}^2$$
$$\lambda = 33 \pm 1 \times 10^{10} \text{ dynes/cm}^2$$
$$k = 44 \pm 1 \times 10^{10} \text{ dynes/cm}^2$$
$$\beta = 2.28 \pm 0.05 \times 10^{-12} \text{ cm}^2\text{/dynes}$$
$$\sigma = 0.333 \pm 0.005$$
$$E = 43 \pm 1 \times 10^{10} \text{ dynes/cm}^2.$$

VI. Rockport Results

Specimen records from Rockport are reproduced in Fig. 3 and the time-distance graph is shown in Fig. 5.

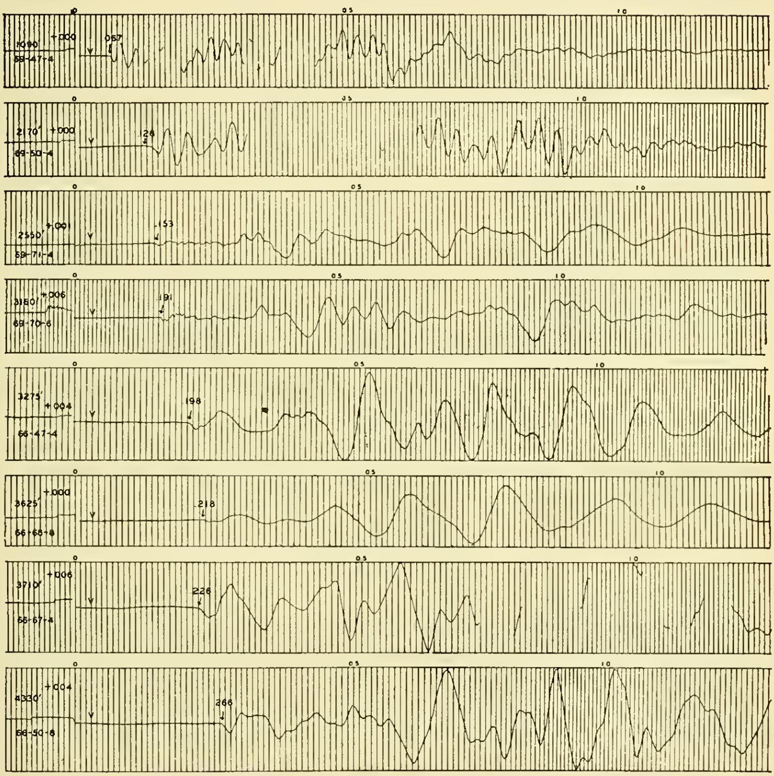


Fig. 4. Specimen Rockport records, vertical component.

The velocity for longitudinal waves obtained from these data was 16,670 ±40 ft./sec. or 5.08 ± 0.01 km/sec.

VII. Westerly Results

The time-distance graph for Westerly is shown in Fig. 5. The velocity for longitudinal waves obtained from these data was 16,400 ± 120 ft./sec. or 5.00 ± 0.04 km/sec.

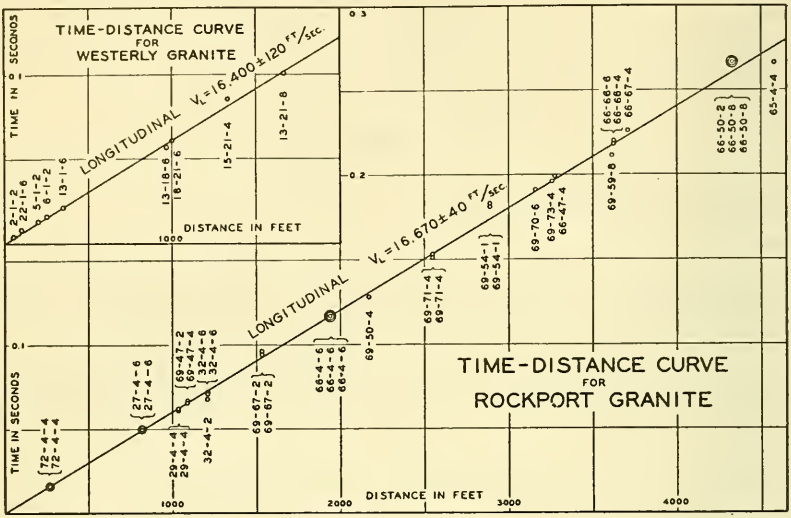


Fig. 5.

VIII. Average Longitudinal Velocity

The weighted average of these three longitudinal velocities is 16,530 ± 90 ft./sec. or 5.04 ± 0.03 km/sec.

The close agreement in velocity found here for three well-separated and different types of granite indicates that minor fluctuations in composition do not seriously affect the velocity.

IX. Analyses

Chemical

The following analyses of specimens from the regions investigated were made by Mr. Forest A. Gonyer, chemist of the Harvard Department of Mineralogy.

Mineralogical

The following mineralogical analyses were made by Mr. Chalmer J. Roy, Assistant in the Department of Geology, who was a member of the party throughout the investigation.

The Westerly analyses were made from thin sections. The Rockport and Quincy determinations were made by gravity separation methods from initial samples of 1000 grams.

TABLE I.

	Westerly Fine grain	Westerly Coarse grain	Quincy	Rockport
SiO_2	74.00	70.30	72.86	74.37
TiO_2	0.10	0.31	0.18	0.18
Al_2O_3	13.86	15.10	11.74	12.88
Fe_2O_3	1.18	1.58	2.99	0 94
FeO	0.71	0.99	1.56	1.56
MnO	none	0.04	0.03	0.03
MgO	0.11	0.50	0.19	0.23
CaO	0.75	1.25	0.14	0.52
Na_2O	2.88	3.69	4.36	4.11
K_2O	6.20	5.58	5.28	4.92
H_2O-	0.09	none	none	none
H_2O+	0.31	0.40	0.41	0.31
CO_2	none	none	none	none
P_2O_5	none	none	none	none
SO_3	none	none	none	none
	100.19	99.74	99.74	100.05

TABLE II.

	Westerly Fine grain	Westerly Coarse grain	Quincy	Rockport
Feldspar	45% (orthoclase)	50% (orthoclase)	60.10% (mostly Perthite)	59.48% (mostly Perthite)
Quartz	35%	35%	28.88%	35.14%
Others	20% (muscovite 15 biotite 2 magnetite 3)	15% (muscovite 3 biotite 7 magnetite 3 zircon and apatite 2)	11.02% (riebeckite aegerite fluorite titanite)	5.58% (hornblende)
	quartz and feldspar crystals range from 1/60 mm to 2 mm in diameter, average ½ mm *ca.*	quartz and feldspar crystals range from ⅓ mm to 4 mm in diameter, average 1½ mm *ca.*		

X. DISCUSSION

La Courtine

At La Courtine, Maurain, Eblé, and Labrouste[9] registered seismic waves generated by explosions and transmitted through a granitic terrane.

They recorded longitudinal waves at distances of 5.6 km, 7.7 km, and 13.9 km. When distances of this magnitude are used, a good velocity determination can be made with less accurate timing than is necessary with shorter distances such as were used in the present work. This advantage, however, is offset by the introduction of uncertainty as to the uniformity of terrane throughout the range. At La Courtine the presence of both gneiss and granite in the path of the waves lessened the certainty with which the velocities obtained may be assigned to granite.

The method of timing the blasts introduced further uncertainty. The instant of the blast, because of the failure of an electric circuit designed to register it on the records, was determined visually by an observer equipped with a chronometer. *An allowance of 0.2 sec. was made for his time of reaction.*

The mean longitudinal velocity obtained from five records in the La Courtine experiments was 5.524 km/sec. The five values, however, varied so much among themselves that they were segregated into two groups, giving averages of 5.905 km/sec. and 5.270 km/sec. respectively. The former was assigned tentatively to granite, and the latter to gneiss. An examination of their geologic map shows that this assignment is not valid because both granite and gneiss were traversed in every case.

In spite of the use of "un amortissement trés faible", the La Courtine investigators report, in addition to longitudinal waves, "autres, environ deux fois moins rapide, d'amplitude plus grande, se manifestent sur les trois composantes; nous les désignerons par le symbole '*L*' appliqué de maniére générale en sismologie aux ondes de plus grande amplitude (longues ondes). De plus, a La Courtine, station la plus rapprochée des explosions, ont été enregistrés deux fois trés faiblement sur la composante horizontale transversale, et une fois en méme temps sur la composante verticale, des mouvements trés faibles oui paraissent ainsi correspondre a des ondes a vibrations transversale de vitesse intermediaire entre celles dex deux categories précedentes; nous les désignerons par '*S*' (secondes ondes)." The use of very feeble damping must, of course, have made the identification of these later phases somewhat uncertain. In the light of experience with transverse waves from explosions, which has been accumulated since the completion of this pioneering experiment, it seems clear that the waves designated as "*L*" were actually transverse. (See, for example, refs. 3, 7). The velocity of these waves, as determined from the mean of eight observations, was 2800 m/sec., making the ratio of the velocities of longitudinal and transverse waves 1.97. This is practically the same as the value of 2.00 obtained in the present investigation.

Thus, the velocities obtained by Maurain, Eblé, and Labrouste for both transverse and longitudinal waves, 2.800 km/sec. and 5.524 km/sec., respectively, are about 12 percent higher than the values for Quincy granite. Since both the method of timing the blasts and the presence of large areas of gneiss in the La Courtine region could introduce systematic errors, this divergence is not surprising.

Comparison of laboratory and field results

There is a fundamental problem involved in a consideration of the utility of comparing small-scale laboratory determinations of rock constants with field results. For many years it has been customary to use constants determined in the laboratory for computing elastic-wave velocities characteristic of certain rocks. On the basis of such comparisons deductions have been made concerning rock types represented by velocities observed in studies of near earthquakes.[6 et al] When the magnitude of the laboratory specimens in past experiments is compared with the volume and field relationships of materials of which they are called representatives, however, this appears to be a very curious procedure indeed.

A case in point is the investigation of Adams and Coker,[1] with particular reference to the granites which they measured, since those results bear directly on the present work. Their measurements were made on granites from six widely separated localities. With the exception of Quincy, from which there were two, a single rock sample was used from each place, and from one to four test specimens were cut from each sample. The test pieces were about one inch in diameter and 3 inches long. The report states that "the rocks in all cases were air dry, having been allowed to remain in the laboratory for several weeks after they had been cut, before measurements were made". Examination of individual measurements reported shows that constants vary from specimen to specimen, and in different directions in given specimens, up to as much as 50 percent. The lateral extension curve for one Quincy specimen has a definite kink, and Poisson's Ratio at 2000 pounds of pressure is nearly 40 percent less than the average given for the specimen under pressures up to 9000 pounds. All of this raises a serious question as to the significance of the average results as representative of the Quincy granite in place.

Laboratory determinations of the constants of a sufficiently large number of samples from a given region would, of course, give a much more representative figure. It is difficult to see, however, how past determinations reported in the literature can be said to represent general averages in any sense, entirely aside from any question as to methods and accuracy of measurements. Their acceptance as a basis for deductions from studies of near earthquakes seems to imply an illusion that granite bodies, whether shallow or deep, are homogeneous masses of uniformly solid, flawless rock. Since they are anything but that, it should not seem strange if actual velocities of waves propagated through several thousand feet of granite in the field differed from those computed from the constants of isolated 1 in.$\times$3 in. specimens of the same granite.

Realizing this, we have nevertheless included the following comparisons on the possibility that they may record relationships between laboratory and field results which are interesting even if not particularly significant.

Relation between compressibilty of granite and pressure

Adams and Williamson[2] have determined the compressibility of several rocks and minerals by a static method at pressures of 2,000 and 10,000 megabars. In Fig. 6 a graph showing the variation of compressibility with pressure is reproduced from their report. The dotted line and marked point have been added. It can be seen that according to this graph the statically determined compressibility of granite changes very rapidly at pressures under 2000 megabars. Independent measurements of the compressibility of quartz and the feldspars which are the chief constituents of granite fail to show any such anomalous increase at pressures less than 2000 megabars, so the reported behavior of granite stands out as an exception to the law demonstrated by Adams and Williamson "that at moderately high pressures the compressibility of a fresh holocrystalline rock is an additive function of the compressibility of its minerals", and resulted in their modifying it to apply only "provided the pressure is not too low". The following quotations summarize the evidence which Adams and Williamson had for this change and show the difficulty which they encountered in explaining it.

In the discussion on page 523 of their report, accompanying the graph shown in Fig. 6 here, is the following statement: "The results of Adams and Coker for six different granites provide the basis for the estimate of the range

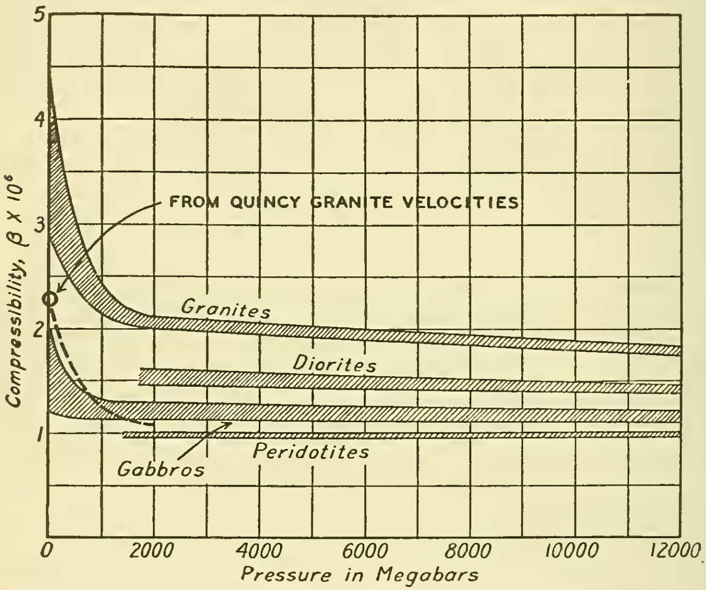


Fig. 6. Compressibility as a function of pressure, according to Adams and Williamson. The circle and dotted line have been added to their graph.

of variation for granites, at low pressures. The corresponding variation for the gabbros at low pressures is admittedly a very rough estimate representing mainly the opinion of the authors."

Thus, the only evidence for the anomalous behavior of the compressibility of rocks at low pressures seems to be the results of Adams and Coker on six granites. Even a futile attempt to find a consistent explanation for the anomaly apparently did not lead Adams and Williamson to doubt the validity of the earlier results as average figures for low pressures. Porosity was looked upon as a possible cause, but after considering it at some length, they concluded: "None of the rocks has enough porosity to explain, on the basis of detached voids or channels, more than a few percent of increase of compressibility. . . . The granites, moreover, although containing less than one percent of pore space, show an abnormally high compressibility at very low pressures; the value at a pressure of only a few hundred megabars being nearly double that at 1000 megabars."

After suggesting two more possible explanations and citing examples both contradictory and confirmatory, they proceed to a fourth: "Very often the abnormally high compressibility at low pressures is associated with coarseness of grain—for example, the granites and marbles, which are comparatively coarse-grained, show a high initial compressibility. The Sudbury diabase, however, is moderately coarse-grained and does not show much of this effect, while the serpentine, which is very fine-grained, shows a considerable decrease in compressibility and this tendency to decrease persists even at high pressures".

In this same connection, it is interesting to note that Adams and Coker[1] had investigated the effects of grain size, including a granite in the trial runs, and concluded: "It will thus be seen that there is no correspondence between the coarseness of grain and the magnitude of the variations in the readings obtained."

This situation, which clearly proved rather difficult, and hinges essentially on the high values for the compressibility of granite obtained by Adams and Coker, is left by Adams and Williamson as follows: "About all that can be said concerning the way in which the compressibility of rocks changes with pressure is: For pressures above 2000 megabars the compressibility does not change very much; at pressures of only a few hundred megabars the compressibility is likely to be notably higher; and that the change of compressibility at low pressures, while connected with the admixture of minerals of different compressibility and with a looseness of structure existing in some coarse-grained rocks, cannot with certainty be predicted in advance."

The compressibility of Quincy granite, 2.29×10^{-12} cm^2/dynes, reported earlier in this paper, has an important bearing on the question under discussion. The issue is somewhat confused by the fact that some writers contend that high-stress, statically determined compressibilities may be considerably different from those determined dynamically, which involve low stresses. If, for the moment, we assume the equivalence of statically and dynamically determined values, it is at once apparent that the compressibility of Quincy granite as plotted in Fig. 6 agrees with the high-pressure values of Adams and Williamson and eliminates the anomalous change at low pressures advocated by them. If, on the other hand, we deny this equivalance, it seems that it is almost inescapable to assume a variation in dynamically determined compressibility similar to that shown by Adams and Williamson. The dotted curve in Fig. 6 represents this assumed variation. According to this curve, the seismically effective compressibility at 2000 megabars would be roughly one half that observed at Quincy for pressures of a few megabars. Barring compensating changes in density and rigidity, such a decrease in compressibility would give a velocity of approximately 7.5 km/sec. for longitudinal waves in granite at 2000 megabars pressure. This is far in excess of the figure 5.6 km/sec. observed by Jeffreys[6] and others as a maximum velocity in the "granitic layer" to depths where the pressure is greater than 2000 megabars. We are accordingly forced to conclude that either the "granitic layer" is not granitic, or granite does not exhibit such rapid changes in compressibility at pressures below 2000 megabars. It appears that, whether or not we consider statically and dynamically determined compressibilities equivalent, the observed compressibility of Quincy granite is not comparable with the low-pressure part of the Adams and Williamson curve.

The good agreement between the compressibility measured at Quincy by a dynamic method and that measured statically by Adams and Williamson suggests that the marked discrepancy between static and dynamic values which has been reported[4,5] does not exist for granite. It should be noted that though the only quantities which they measured were the compressibility and density, on page 520 of their paper they have computed the velocities of seismic waves in the rocks they studied. The values which they show for the velocities depend upon the value 0.27 which was assumed for Poisson's ratio. If the value 0.333, which was determined at Quincy, were substituted in their computations, the resulting velocities would be in agreement with those determined in the present investigation.

XI. Acknowledgments

The investigation was conducted by the Harvard Seismograph Station during the summer of 1931. The instruments used were built and patented by H. Gordon Taylor in the name of the Geophysical Exploration Company. They were purchased through generous gifts from Mr. Edward Mallinckrodt, Jr., chairman of the Committee to Visit the Division of Geology at Harvard, and Professor R. A. Daly. The cost of the balance of the field equipment and all operating expenses were met by a grant from the Milton Fund of Harvard University.

We are indebted to the Atlas Powder Company for permission to shoot on their property at Quincy, to the Rockport Granite Company for permission to use their property at Rockport, and to Mr. John H. Austin for permission to run our Westerly profile on his property. Their ready cooperation was an important factor in making possible the rapid and successful completion of the program.

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ASYMMETRY OF SOUND VELOCITY IN STRATIFIED FORMATIONS

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(Received February 23, 1932)

Abstract

In the course of explorations of subsurface geology by the seismograph the authors have frequently observed the pronounced effect of stratification on the velocity of seismic waves in shales, and this effect has often been utilized in practical seismography. Recently an opportunity was afforded for securing additional quantitative data on the velocity normal to and parallel to the bedding planes. The paper points out that the velocity parallel to the planes of stratification is, in some instances, as much as fifty percent higher than the velocity in a direction normal to the bedding planes. It is shown also that inclined stratified beds exhibit a higher apparent point-to-point velocity when sound travels in an up-dip direction than when traveling down-dip. The paper describes a procedure whereby this effect may be utilized for determining the direction and approximate magnitude of the dip in such stratified deposits. The method has proved to be of considerable practical importance where the stratified formations are obscured by overlying deposits.

General Outline

ONE of the earliest applications of the seismograph was its use in profiling salt domes, limestone horizons, or other high velocity rock formations buried beneath sands, clays, or shales of lower velocity. In this method of profiling, the portion of the sound wave utilized travels substantially along the path shown diagrammatically in Fig. 1. As is well understood, the time interval between the arrival of the wave at two stations some distance apart

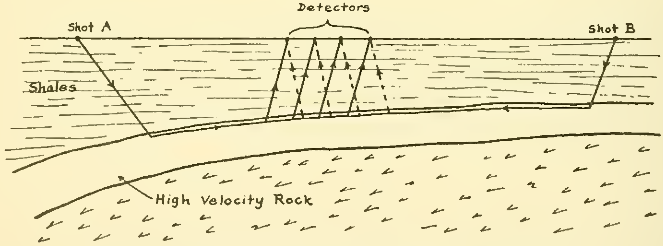

Fig. 1. Characteristic geology for diffraction profiling.

along the line of propagation of the wave will be dependent on whether the wave is being propagated upslope or downslope. By measuring this velocity asymmetry the direction and magnitude of the slope of the high velocity bed can be determined.

Commonly spoken of as diffraction or refraction shooting, this method of profiling found wide application in many parts of the world, and still possesses much merit as a means of subsurface investigation in many localities. Its technique is now the most elementary part of the stock-in-trade of the oil seismologist. The procedure for obtaining a complete cross-sectional profile usually consists in shooting a line of detector stations from both directions. Generally the overall time of travel of the sound wave is recorded from each shot to its appropriate stations, and complementary time-distance curves are plotted for the series of shots from each direction.

Whatever may be the subsequent steps in making use of the primary time-distance data, all the calculations must necessarily be based on the deviation,

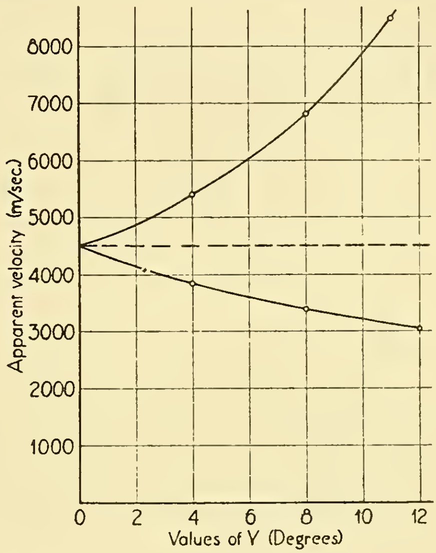

Fig. 2. Chart for showing relationships between apparent velocities when shooting upslope and downslope. Y**=angle of slope;** V_1 **taken as 2500 m/sec.;** V_2 **taken as 4500 m/sec.**

or asymmetry, of the apparent velocity relative to the true velocity of the rock which is being profiled. This asymmetry of the upslope and downslope values is best shown by comparing the station-to-station velocities derived by dividing the horizontal distance from one station to the next by the interval time, Δt. Fig. 2 shows the striking asymmetry which results from the slope effect in an ordinary case, while in salt dome work it is well-known that infinite apparent velocities are not uncommon.

The above described method, as is well understood, gives the slope of the *contact surface* between the high velocity bed and the overlying low velocity deposits. In many cases this contact surface, where it has not been subject to irregular erosion, will conform to the structural outlines of the subsurface geology. In such cases the simple method above outlined is adequate to give the desired information on the subsurface geology. There are many cases, however, where this situation does not exist, and in such cases the method fails to reveal the subsurface picture. Fig. 3 shows a typical case of this geology. Here the shale, 1, in which there exist more or less definite bedding planes, has been folded into a definite structural relief. Subsequent to this folding, extensive erosion took place leaving a surface shown by the line 2–3, and on this surface later deposits, 4, were laid down. In general, under these conditions, the shale, 1, practically always has a higher velocity than the recent deposits, 4. However, if we attempt to apply the simple method above described, we shall derive not the structural picture but merely a poor approximation to the erosional surface between the shale and the recent deposits. To further increase the difficulty there is often no definite reflecting

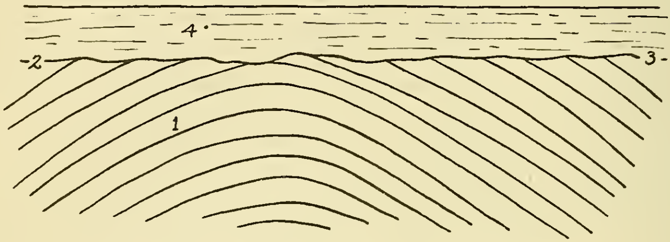

Fig. 3. Anticline hidden under unconformable beds.

surface at greater depth which can be used to define the structure by reflection shooting. Under such conditions the true structural picture cannot be revealed by any method of subsurface exploration heretofore published. These conditions are very frequently encountered and the purpose of this paper is to describe a method of exploration that is often capable of giving a correct structural picture where this situation exists. This method has been called "dip shooting" to distinguish it from the slope shooting heretofore used by geophysicists.

Origin of Dip Shooting

The method of profiling stratified formations which this paper outlines does not supercede any other method, but is simply a special development which is useful and applicable in certain cases which cannot be handled in other ways. The method is based upon an asymmetrical velocity effect which is analogous to the slope asymmetry which forms the foundation of refraction profiling. This "dip effect", as it may be called, has been observed and utilized in places as widely separated, geographically and geologically, as Canada and Venezuela. So far as our observations show, the phenomenon is common to most stratified rocks and even to some which are not ordinarily classed as stratified.

Although the dip effect is analogous to the asymmetry produced by slope, it may arise from a different cause. The velocity asymmetry associated with stratification is often due to the effect of the bedding planes, dipping with respect to the surface, and not to a sloping subsurface.

On this fact depend two important corollaries: First, the method necessarily gives a structural rather than a topographic profile; for whereas the refraction method yields a structural profile under certain conditions, it is obvious that in the case of an eroded and buried land surface, for example, it merely records the old topography. Second, the method does not presuppose the existence of any cover over the formation being investigated. In this case investigation with the seismograph sounds rather paradoxical, but the condition, nevertheless, is a real one and is frequently met with in tropical work. For example, shale areas which have been folded and subsequently leveled may be covered with just sufficient surface soil to make pitting difficult and expensive while heavy vegetation obscures all the outcrops. Here the seismograph, using the dip method, can profile the structure or work out the contours with almost as much facility as surface geology displays in exposed regions. On the other hand, the presence of considerable recent overburden does not interfere with the use of the method.

A third point which may be noted here as bearing on the theory of interpretation of the records, which we shall refer to later, is that the *refraction* system of profiling does not concern itself with the sound wave as it travels through the rock being profiled, but only with its emergence into the overlying formation. On the other hand, the dip method must take into account the phenomena attending the travel of the wave at a considerable depth within the body of the stratified rock itself.

Evidence of Velocity Asymmetry

The idea that stratification might influence velocity in a way to be of practical value was first suggested to the authors by J. E. Brantly. It required considerable field work and the accumulation of extensive data, however, before it was possible to sift the velocity variations due to stratification from the variations due to several other causes. Some of these other causes will at once occur to anyone who has used the seismograph. Among them we might list such factors as varying velocity in the overburden, dip in the overburden compounding with the dip effect from the subsurface, differences of surface elevation, slope of the subsurface as distinguished from its dip in cases where the subsurface was eroded before being buried, etc.

After making due allowance for such contributory causes as the foregoing, however, it was found that the sound wave travels faster parallel to the bedding planes than it does perpendicular to them.

There are two lines of evidence to show that this is the case. The first is direct velocity measurement both parallel and normal to the bedding. Measurements of this kind were carried out recently on some exposures of the Lorraine Shales in the Province of Quebec, Canada. These shales have been highly folded along the Lake Champlain-Quebec Fault, and in a number of places are to be found standing on edge. This afforded an opportunity to measure the velocity perpendicular to the laminations of the shale and compare it with the velocity of the same formation in a horizontal position nearby. The following are some of the velocity readings.

TABLE I. *Showing velocity of sound parallel and normal to bedding.*

Velocity parallel to bedding planes. (Meters per sec.)	Velocity normal to bedding planes. (Meters per sec.)	Ratio
4450	3205	1.39
4655	3310	1.41

From the above figures it will be seen that the velocity parallel to the bedding is in the neighborhood of 40 percent higher than it is in a direction at right angles.

The second type of evidence which shows the existence of this asymmetrical effect is the difference in apparent interval velocity found in shooting up and down the dip of stratified rocks which have been tilted out of the horizontal position. Before presenting the field observations which illustrate this aspect of the problem, let us consider what theoretical explanation can be advanced to account for velocity variations as produced by dip.

It is admittedly very difficult to make an exact analysis of wave paths in a non-homogeneous medium like shale, and we do not attempt to offer any comprehensive solution here. Even a preliminary study of the problem, however, suggests that in addition to the refraction produced by more or less alternating layers of different velocity, there is also refraction due to increase of velocity with depth in most cases. Furthermore, it seems logical to suppose that considerable absorption of the wave energy would occur when the wave path cuts across the bedding, and that this absorption would tend to disappear when the path parallels the stratification.

Consideration of these and similar factors affecting the travel of the sound wave leads to the conclusion that we can make a useful approach to the problem by studying some of the simpler typical subsurface conditions such as are often met with in the field. From these simple cases it should be possible to extend the analysis to more complicated conditions with the aid of further experimental work. It is to be hoped, therefore, that additional data bearing on the subject will be forthcoming from localities where the dips are more accurately known than in many areas which the authors have investigated.

To return, however, to those instances where it can be readily seen that dipping strata would give rise to asymmetrical velocities, the three clearest cases, perhaps, are the following:

CASE 1

The first example in which dip will obviously give rise to velocity asymmetry is illustrated diagrammatically in Fig. 4. Almost any stratified formation has occasional hard layers which give a somewhat higher velocity than the average. These layers may be of the nature of "markers" which are readily distinguished by some peculiarity, or they may be almost unrecognizable to the eye while still possessing abnormal velocity characteristics. Such layers act as conductors for the sound wave because they offer the path of least time.

In the diagram, Fig. 4, layers of the above-the-average velocity are represented at *a-a*. In shooting a line of stations in an up-dip direction, the time intervals between stations 1-2 and 2-3 will be short, while between stations 3-4 and 4-5 the intervals will be longer, because these stations are beyond the zone of influence of the fast layer "*a*". However, on advancing farther down the line, other laminations of higher-than-average velocity will be encountered and the intervals will again be shortened. Conversely, when shooting down-dip, many of the intervals will be lengthened by these same layers.

Now it is expectable that the flatter the dip (until it becomes nearly horizontal), the more continuous will be the effect on the time intervals. Therefore, moderate dips, such as are usually found in practice, will show this type of velocity asymmetry to the best advantage, and those formations which exhibit the least uniform stratification will generally be the ones whose dip it will be easiest to determine.

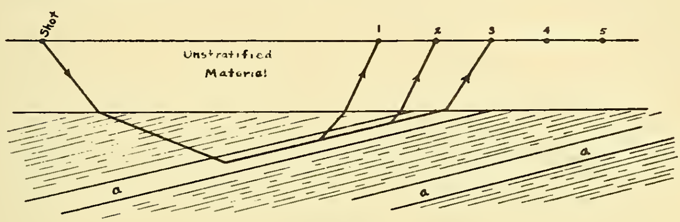

Fig. 4.

It may be objected that the foregoing analysis is based on the slope-effect and not on dip. To a certain extent this is true. In extreme cases, like that shown in Fig. 4 where the high velocity layers are unusually thick and clearly defined, the interpretation can be based purely on the theory of refraction shooting. But in a majority of cases the layers of higher velocity are relatively thin, rather closely spaced, and seldom recognizable in the outcrops. Thus it seems to be the aggregate effect of many sloping layers which generally produces asymmetrical velocities such as those tabulated in the latter part of this paper. Consequently, there is no choice but to make the interpretation in terms of dip rather than in terms of slope.

Case 2

In Fig. 5a the wave is shown traveling in an up-dip direction. Exaggerating conditions somewhat for the sake of clearness, we may assume that the wave, taking the path of least time, descends fairly deeply into the shale and is steadily refracted on a curved path until the critical angle is reached, when it follows the bedding and finally emerges at the surface giving the time interval Δt_1. In shooting down dip, the principle of reversibility of wave paths tells us that, between identical surface points, the path will be the same from either direction and there can be no difference in the recorded times. Similarly, if the subsurface dip is everywhere uniform, equal shooting distances will give identical paths even though the surface points are not the same.

From this it follows that a perfectly uniform dip, in this sense, will not result in any velocity asymmetry.

On the other hand, if the dip is non-uniform as illustrated in Fig. 5 and as is most commonly the case under actual conditions, especially close to an anticline, the wave paths will not be the same provided the surface points are shifted. Thus, in Fig. 5b, the shooting distance, L, is the same as in 5a

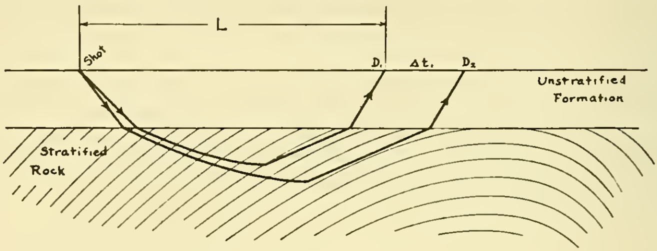

Fig. 5a.

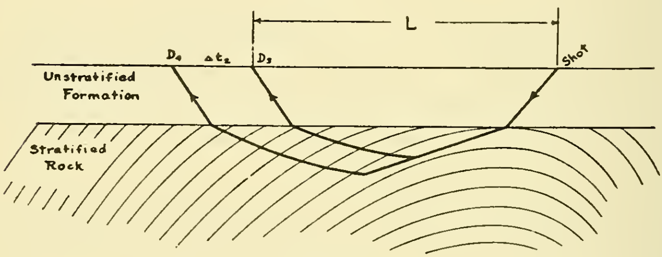

Fig. 5b.

but the setup is shifted slightly so as to intercept different conditions of dip. Consequently, the wave paths are not the same for the two cases, and both the overall times and the station-to-station time intervals will be different. In other words, Δt_2 will not, in general, have the same value as at Δt_1, and the apparent velocities will show an asymmetrical effect.

Case 3

A third type of velocity asymmetry is often due to a relationship which frequently exists between the hardness and the age of rocks and shales. In shooting up-dip (except in the case of overturned folds), the wave is traveling from younger to older beds. Now in many cases it has been found that the hardness of rocks increases with their age and the velocity closely corresponds to the hardness. Consequently, in shooting from a given shot point to a detector setup located in an up-dip direction, the tendency is for the velocity to increase, and the converse is true in shooting down-dip. This tendency, while by no means invariable, must nevertheless be taken into consideration as a causative factor. Whenever this effect exists it gives a true indication of the direction of the dip of the bedding planes.

The reader's attention has already been called to the complex nature of the problem under discussion. It is well to emphasize again, however, that many factors affecting the velocity of sound in stratified rocks have not yet been mentioned. Indeed, many factors doubtless exist which have not yet been recognized. Therefore, there is still room for much experimental investigation along these lines. At the present time, lacking a theoretical explanation which covers all phases of the phenomenon, the existence of the dip-effect and the validity of its use as a working method are best evidenced by numerous field observations; a few of which follow.

Examples of the Dip Effect

Table II. *Material: Tertiary shale in Venezuela. Shooting distance: 1500 meters. Station spacing: 250 meters.*

NW Shots		SE Shots	
Stations	Velocities	Stations	Velocities
112–111	2450	106–107	2660
111–110	2360	107–108	2720
110–109	1980	108–109	3120
109–108	2080	109–110	2600
108–107	2320	110–111	2500
107–106	2400	111–112	2720
106–105	2400	112–113	2200
105–104	2600	113–114	2180
Average	2324		2588

The velocity values shown in Table II were recorded in the course of surveying a line which was laid out approximately at right angles to the general strike of the area. The total distance covered by the stations is two kilometers, and the stations have been so chosen as to represent the same portion of the subsurface in the northwest and southeast shot groups, after making allowance for the probable amount of throwback of the reference point. The shale, in this case, was covered by clay, but the evidence of various nearby test wells pointed to the cover being fairly uniform in thickness. Consequently, the difference in the velocity averages indicates a regional dip to the southeast. The variations of the individual velocity values show the effect of local folding.

We have already mentioned the fact that some relatively unstratified formations show asymmetry due to dip. The following data were gathered in areas of indistinctly bedded sands and clays in Western Venezuela.

The values in Table III cover a distance of ten kilometers with an allowance for throwback as in the case of Table II. The difference between the average velocities is small, but in view of the circumstances a larger difference would hardly be expected. These shots were taken at a distance of about twenty kilometers out from the foothills of the Perija mountains. The dip represented by the averages is simply the normal depositional dip of water borne sediments laid down along the mountain front. The dip is toward the

TABLE III. *Material: Tertiary sandstones, sands and clays. Shooting distance: 6000 meters. Station spacing: 500 meters.*

E Shots Stations	Velocities	W Shots Stations	Velocities
165–163	3290	127–129	3420
163–161	2980	129–131	2980
161–159	3120	131–133	3090
159–157	3120	133–135	3050
157–155	3050	135–137	2970
155–153	3420	137–139	2990
153–151	3380	139–141	2940
151–149	3160	141–143	2910
149–147	3050	143–145	3010
147–145	3900	145–147	2600
145–143	2940	147–149	3730
143–141	3210	149–151	2870
141–139	3290	151–153	3470
139–137	3180	153–155	2910
137–135	3080	155–157	3120
135–133	3090	157–159	2980
133–131	2930	159–161	3290
131–129	3080	161–163	3160
129–127	3080	163–165	3200
127–125	3290	165–167	3330
Average	3182		3101

east, that is, away from the mountains, and would doubtless be steeper if investigated nearer the foothills. To illustrate this latter point we may average the western half of the group of velocities, Stations 125 to 147, with these results: East shot average, 3117 meters per second; West shot average, 2996 meters per second; showing a greater difference than for the whole group.

TABLE IV. *Material: "Mottled clay", Venezuela. Shooting distance: 500 meters. Station spacing: 500 meters.*

	E Shots Apparent velocities	W Shots Apparent velocities
	2840	2480
	2870	2480
	2340	2500
	2530	2250
	2810	2290
	2400	2420
	2780	2170
	2600	2140
Average	2646	2341

Table IV shows the results of some scattered shots on two lines at right angles to the Perija Mountain front. The material is what is known as the "Mottled clay" of the Lake Maracaibo region. It is a compact clay with very little admixture of sand and has no recognizable bedding. In the locality where the above data originated, however, there were good reasons to suppose that the regional dip was eastward, in a direction away from the hills. Drilling subsequently indicated that such was the case for the underlying formations, but the seismograph results appeared to be the only definite evidence that the same dip existed in the mottled clay cover. Shots at 1000 meters distance confirmed the figures obtained from the 500 meter shots, as the following tabulation will show.

TABLE V. *Material: "Mottled clay", Venezuela. Shooting distance: 1000 meters. Station spacing: 500 meters.*

	E Shots Apparent velocities	W Shots Apparent velocities
	2320	2360
	2660	2550
	2780	2550
	2690	2580
	2940	2580
	2800	2580
		2230
		2170
		2320
Average	2698	2436

Tests for Known Structures

Perhaps the best confirmation of any geophysical theory is its test on known structures. We have carried out several tests of this sort with very gratifying results. One of the earliest, and at the same time, one of the most comprehensive trials of the dip method of shooting was made on a group of

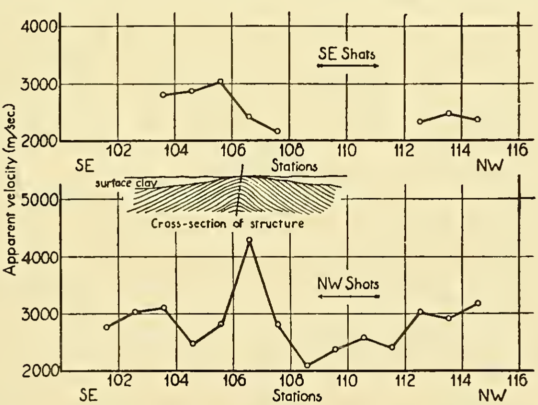

Fig. 6. Dip-velocity curves across structure miocene shale—Venezuela.

shale structures, arranged in echelon, near Maracaibo, Venezuela. These folds, outcropping through the mottled clay cover, had been worked out by surface geology, and their axes fairly well located. The seismograph was employed to check the location of the recognized folds and to search for others which were not exposed on the surface.

The results tallied with the geological picture in a manner which could hardly be questioned. It is not possible, here, to present all the data which were gathered, but one example may not be out of place to illustrate this special application of the method to exposed or very thinly covered folds which have been planed by erosion before being buried. Fig. 6 shows the apparent velocities given by shots from opposite directions plotted in the form of velocity curves. The line of survey crossed one of the known shale structures already mentioned. The cross-section of the fold is shown diagrammatically at its proper location near Station 106. The northwest flank of the anticline is considerably steeper than the southeast flank. In shooting from the NW, note how the velocity rises to an abnormally high value due to the steep dip of that flank. A corresponding minimum value is given by the southeast shots, and the reverse occurs in both cases on the other side of the fold after crossing the crest. Doubtless a part of the velocity asymmetry is due to slope effect on the surface of the shale under the clay cover, but repeated observations seem to demonstrate that this will not account for all of the velocity variation which is found in such cases.

Application of Dip Shooting

The main essential in utilizing the dip effect is to shoot all stations from opposite directions. Generally the stations are set along a straight line which is laid out, as nearly as may be, at right angles to the strike of the area in order to get the maximum effect from the dip. Shooting distance, station spacing, etc., must be determined according to what is known of the surface and subsurface conditions, as with any other method. Often special preliminary shooting is necessary to reveal this information.

It is possible that satisfactory results might be obtained from the use of overall velocities, derived from the total time of travel of the wave from shot to detector, but the authors have always preferred to employ station-to-station velocities because they not only localize the dips better, but also bring out the velocity asymmetry more sharply.

In accordance with this practice, then, the interval velocities given by the shots from both directions may be tabulated and the complementary values compared. It is necessary to make allowance for the throw-back of the reference point as is customarily done in all profile work, although here the exact amount to allow for is indeterminate on account of the variable form of the wave path. The fact that the wave may penetrate to an appreciable depth in the shale, and that all parts of the path are generally curved, renders it impossible to do more than make an approximate calculation of the throw-back, but this will usually suffice for practical purposes.

The velocity asymmetry observed on any setup may, of course, be due to two causes. First, the slope of the contact between the shales and the overlying formations, and second, the velocity asymmetry in the shales themselves. In order to determine and eliminate the former, it is usually best to shoot a correction shot on each setup from a point symmetrically placed with respect to two detector stations, as, for example, at a point midway between them. If the spacing of the stations has been properly chosen with respect to the local geological conditions, which can always be accomplished, the required correction can easily be made.

The most difficult phase of the work is to translate the velocity readings into terms of degrees of dip. There seems to be no theoretical basis for a rigorous mathematical treatment which would accomplish this. Consequently, the best solution is probably an empirical calibration of the velocity values on some structure where the dips are accurately known. In most cases, however, it is not of great importance to know the absolute dip. For general profiling and reconnaissance the direction and relative steepness of the dips is all that is required. The use of relative values permits comparison of the size of different structures, shows which side of the fold is steeper, and, of course, locates the crest of the fold as accurately as if precise quantitative dips were obtained.

Summary

A large amount of accumulated data similar to that herein presented demonstrates that the method described above provides a satisfactory means of outlining structure in stratified and other asymmetrical deposits where conditions are such that the usual methods of refraction and reflection cannot be applied.

THE CALCULATION OF THE MOTION OF THE GROUND FROM SEISMOGRAMS

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(Received January 30, 1932)

Abstract

The equation of motion of a mechanical seismograph is $-\ddot{x}=\ddot{y}+2k\dot{y}+p^2y$ where x is the ground displacement and y the seismograph deflection. This equation may be solved for y when x is supposed known or for x when y has been observed as a function of the time. In this paper both of these ways of solving the equation are considered. The motion of the seismograph due to a train of waves starting at $t=0$ is considered and also the motion due to the arrival of a single wave. In each case seismographs with several periodic times and either undamped or critically damped are considered. Curves are given showing the motion of the ground and the calculated motion of the seismograph. The motion of the ground corresponding to several simple assumed seismograms is also worked out and shown by means of curves. The motion corresponding to a given seismogram depends greatly on the periodic time and damping of the seismograph. Finally the ground motion is deduced from two actual seismograms due to dynamite explosions. An integraph is described which enables the calculations to be done more quickly.

IN THE case of seismograms obtained with mechanical seismographs having a definite periodic time and damping coefficient the motion of the ground can be deduced from the seismograms without serious difficulty. This of course is well-known, but so far as the writer is aware few if any results of such calculations have been published, at any rate for seismograms obtained in geophysical prospecting.[1]

The seismograph consists essentially of a heavy mass supported on springs, or in some other way, so that it is free to oscillate. The motion of the ground relative to this mass is magnified and recorded.

Consider a mass m supported by a spiral spring from a stand resting on the ground. Let y denote the upward displacement of the mass m, relative to the stand, from its equilibrium position, so that y is equal to the decrease in the length of the spring due to the upward displacement of the mass relative to the stand. When y is zero the force exerted by the spring is equal and opposite to the weight of the mass.

Let x denote the upward displacement of the ground and stand due to a wave in the ground relative to axes supposed fixed. The equation of motion of the mass m is then

$$m(\ddot{x}+\ddot{y}) = -mp^2y - 2mk\dot{y}.$$

[1] H. Arnold, Zeits. f. physik. Erdkunde 10, 269–317, gives calculations of the ground motion from several seismograms.

Here p^2 is the restoring force per unit mass exerted by the spring when its length is increased by unity and $2k$ is the damping force or viscous resistance, per unit mass, to the motion of the mass m when moving relative to the stand with unit velocity. Hence we have

$$\ddot{y} + 2k\dot{y} + p^2 y = -\ddot{x}.$$

We see from this equation that if initially $x=0$, $\dot{x}=0$ and the mass is at rest in its equilibrium position, then when the ground begins to move we shall have $y=-x$ because at the start y and $\dot{y}$ are zero so that $\ddot{y}=-\ddot{x}$. This merely means that at the start the mass m remains at rest relative to the fixed axes. But y will only be equal to $-x$ during an interval small compared with the natural period of vibration of the mass m.

The above equation may be regarded as giving y when x is known or as giving x when y is known. In previous discussions of the theory of seismographs, which the writer has seen, the motion of the ground or x has been assumed known and the resulting values of y have been calculated.

The solution of the equation $\ddot{y}+2k\dot{y}+p^2y=-\ddot{x}=F(t)$ where $F(t)$ denotes a function of the time t and when y and $\dot{y}$ are zero at $t=0$ is

$$y = \frac{1}{\mu}\int_0^t e^{-k(t-\eta)} \sin \mu(t-\eta)F(\eta)d\eta$$

where

$$\mu = (p^2 - k^2)^{1/2}.$$

By means of this equation it is easy to calculate the seismograph deflections y as a function of t for any simple values of $F(t)$.

In homogeneous ground an explosion of dynamite would be expected to produce single waves, or short trains of waves, so that it is important to consider the motion of the seismograph due to single waves. To represent a single wave we may suppose that $x=A(1-\cos\omega t)$ from $t=0$ to $t=2\pi/\omega$ and that $x=0$ when $t<0$ and when $t>2\pi/\omega$. This gives the wave shown in Fig. 1.

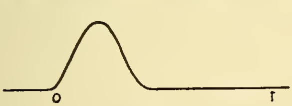

Fig. 1

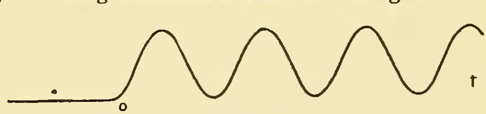

Fig. 2.

Before considering this case it will be convenient to consider the case when $x=A(1-\cos\omega t)$ when $t>0$ and $x=0$ when $t<0$ which gives the series of waves shown in Fig. 2.

The equation $x=A(1-\cos\omega t)$ gives $\ddot{x}=A\omega^2 \cos \omega t$ so that the expression for y becomes

$$y = -\frac{A\omega^2}{\mu}\int_0^t e^{-k(t-\eta)} \sin \mu(t-\eta) \cos \omega\eta d\eta.$$

We shall consider two cases. One when the seismograph is undamped so that $k=0$ and one when the seismograph is critically damped so that $\mu=0$.

In the first case with $k=0$ we get

$$y = \frac{A\omega^2}{\omega^2 - \mu^2}(\cos \omega t - \cos \mu t)$$

and in the second case with $\mu=0$ we get

$$y = \frac{A\omega^2}{\omega^2 + k^2} \cos (\omega t + \alpha) + \frac{A\omega^2 e^{-kt}}{(\omega^2 + k^2)^2}\left\{ k^2(1 + kt) - \omega^2(1 - kt)\right\}$$

where

$$\tan \alpha = \frac{2\omega k}{\omega^2 - k^2}\cdot$$

We shall now consider three cases with the undamped seismograph (a) when the period of the seismograph is greater than that of the waves in the ground (b) when the period of the seismograph is equal to that of the waves and (c) when the period of the seismograph is less than that of the waves.

In case (a) let $\omega=2\pi/12$, $\mu=2\pi/36$ and $A=1$ so that

$$y = -(8/9)(\cos (30t)^\circ - \cos (10t)^\circ)$$
$$x = 1 - \cos (30t)^\circ.$$

Fig. 3 shows the curves given by these equations. We see that the movement of the ground sets up an oscillation of the seismograph so that the deflections

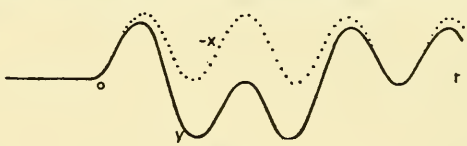

Fig. 3.

obtained are the resultants of the oscillations of the ground and of the seismograph.

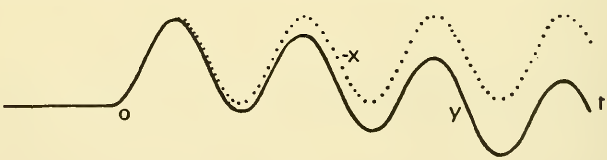

Fig. 4.

Fig. 4 shows the curves obtained with $\omega=2\pi/12$ and $\mu=2\pi/120$ so that $y=-\cos(3t)^\circ+\cos(30t)^\circ$ $x=1-\cos(30t)^\circ$. When, as in this case, the period of the seismograph is ten times that of the waves, the undamped seismograph records the first wave fairly accurately.

Now consider case (b) when the period of the seismograph is equal to that of the waves. If we put $\omega=\mu+\epsilon$ in the equation

$$y = \frac{A\omega^2}{\omega^2 - \mu^2}(\cos \omega t - \cos \mu t)$$

we get when ϵ is very small $y = -A\mu t/2 \sin \mu t$. If $A = 1$ and $\omega = \mu = 2\pi/12$ this gives

$$-y = \frac{\pi}{12} t \sin (30t)^\circ$$

and

$$x = 1 - \cos (30t)^\circ.$$

Fig. 5 shows the curves for y in this case. The amplitude of the first oscillation is only about one half that of the ground but the amplitude rapidly increases as the successive waves arrive.

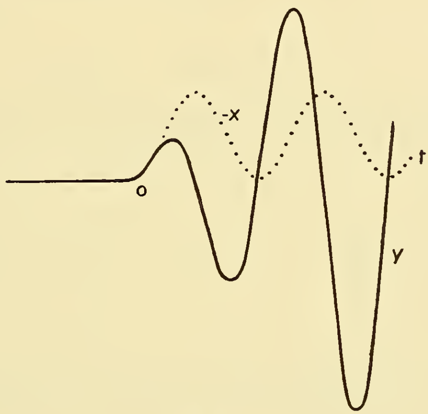

Fig. 5.

We see from this that for the purpose of detecting the arrival of the beginning of a train of waves, it is not a good plan to use a seismograph having a free period equal to that of the waves. A seismograph with a longer period gives twice as great a deflection due to the first wave as one with a period equal to that of the waves. Of course at the start $-y = x$ in all cases so that the initial sensibility is independent of the periods but it is advantageous to get a larger deflection due to the first wave since a large deflection is less likely to be overlooked or obliterated by the small oscillations due to local disturbances.

Now consider the third case when the period of the seismograph is smaller than that of the waves. The equation

$$y = \frac{A\omega^2}{\mu^2 - \omega^2}(\cos \mu t - \cos \omega t)$$

shows that making μ/ω large will diminish the maximum values of y very greatly. For example if $\mu/\omega = 10$ we get

$$y = (A/99)(\cos 10\omega t - \cos \omega t)$$

so that the maximum amplitude is only about one percent of that of the waves in the ground. In this case we get short oscillations due to the term $\cos 10\omega t$ superposed on the curve given by $-(A/99) \cos \omega t$. Hence a smooth curve drawn across the small oscillations would give $-(A/99) \cos \omega t$ which resembles the curve given by $x = A(1 - \cos\omega t)$ although displaced downwards and with only one percent of the amplitude. This suggests that the results obtained with high frequency undamped seismographs can be improved by drawing a smooth curve through the small oscillations of frequency equal to that of the seismograph.

Fig. 6 shows the curves given by

$$x = 1 - \cos (30t)^\circ$$

and

$$-y = (\tfrac{1}{8})(\cos (90t)^\circ - \cos (30t)^\circ)$$

which are those for the case $A = 1$, $\omega = 2\pi/12$ and $\mu = 2\pi/4$ in which the period of the seismograph is one third that of the waves.

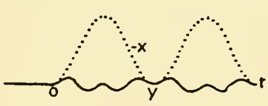

Fig. 6.

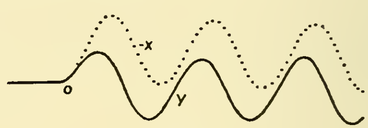

Fig. 7.

Now consider the case of a critically damped seismograph. In this case if $\omega = 2\pi/12$ so that $x = 1 - \cos(30t)^\circ$ and $k = p = 2\pi/36$ so that the seismograph period is three times that of the waves then the equation for y gives

$$y = \frac{9}{10} \cos (30t + 36{\cdot}87)^\circ + \frac{5\pi t - 72}{100} e^{-\pi t/18}.$$

Fig. 7 shows the curves given by these equations. Comparing Fig. 7 with Fig. 3 we see that the critical damping cuts out the oscillations of the seismograph, set up at the start, so that we do not get interference between two

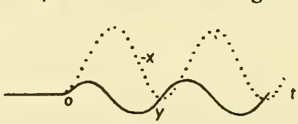

Fig. 8.

vibrations of different frequencies. The damping however diminishes the interval at the start in which x and $-y$ agree. Thus without damping in Fig. 3 the two curves agree from $t = 0$ to $t = 4$ but with critical damping in Fig.

7 they only agree from $t=0$ to $t=1$. We see that to obtain the actual motion of the ground it is necessary to use a much longer period with a critically damped seismograph than with an undamped one.

Fig. 8 shows the curves for x and $-y$ when $\omega=p=k$ so that the periods of the waves and of the seismograph are equal. The equations for x and y in this case are

$$x = 1 - \cos (30t)^\circ$$
$$y = -\tfrac{1}{2}\sin (30t)^\circ + (\pi t/12)e^{-\pi t/6}.$$

The seismograph is less sensitive and x and $-y$ do not agree as well as when the period of the seismograph is greater than that of the waves.

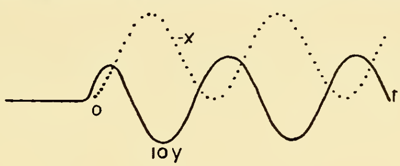

Fig. 9.

If the period of the seismograph is one third that of the waves and the damping critical then we may take

$$x = 1 - \cos (30t)^\circ$$

so that $\omega=\pi/6$ and $k=\pi/2$. The equation for y is then

$$y = -\frac{1}{10}\cos (30t - 36\cdot 87)^\circ + \frac{5\pi t + 8}{100}\, e^{-\pi t/2}.$$

Fig. 9 shows the curves for x and $-10y$ in this case. $-10y$ is plotted instead of $-y$ because the maximum value of y is only one twentieth that of the maximum value of x.

Comparing Fig. 9 with Fig. 7 we see that the deflections of a critically damped seismograph with period three times that of the waves are somewhat similar in form but about ten times larger than those of a critically damped seismograph with period one third that of the waves.

It appears that, while in all cases the initial deflections are equal and opposite to the motion of the ground, the only case in which this remains true approximately for more than a fraction of a wave is that in which the period of the seismograph is large compared with that of the waves and the seismograph is undamped.

The ratio of the maximum value of y to the maximum value of x is about $\omega^2/(\omega^2-\mu^2)$ or if T is the period of the waves and T' that of the seismograph it is $T'^2/(T'^2-T^2)$. This is equal very nearly to unity when T'/T is large and is nearly equal to $(T'/T)^2$ when T'/T is small. When T' and T are nearly equal it becomes large but not for the first wave.

Very short period seismographs are difficult to make sufficiently sensitive because of the small value of the ratio of the maximum value of y to that of x. A critically damped seismograph with a period very small compared with that of the waves in the ground gives a deflection very nearly proportional to the acceleration of the ground. The equation

$$y = \frac{1}{\mu}\int_0^t e^{-k(t-\eta)}\sin \mu(t - \eta)F(\eta)d\eta$$

when $\mu = 0$ for critical damping becomes

$$y = \int_0^t e^{-k(t-\eta)}(t - \eta)F(\eta)d\eta.$$

If the period of the seismograph is very small so that k is large then the factor $e^{-k(t-\eta)}$ is negligible except when η is nearly equal to t. If we suppose that $F(\eta)$ varies slowly and so can be regarded as constant during the short interval when η is nearly equal to t then

$$y = F(t)\int_0^t e^{-k(t-\eta)}(t - \eta)d\eta$$

or

$$y = \frac{F(t)}{k^2}(1 - e^{-kt}(1 + kt))$$

or when t is not very small $y = F(t)/k^2$. Thus in this case since $F(t) = -\ddot{x}$ we have $y = -\ddot{x}/k^2$.

If a very short period critically damped seismograph can be made sufficiently sensitive, it will indicate the acceleration of the ground with considerable accuracy. Whenever a sudden change in the acceleration of the ground occurs, the change in the seismograph deflection will be proportional to the change in the acceleration multiplied by the factor $1 - e^{-kt}(1 + kt)$ where t is the time since the sudden change took place, which very soon becomes equal to unity when k is large.

Now consider the deflection of the seismograph due to a single wave. Let $x = A(1 - \cos \omega t)$ from $t = 0$ to $t = 2\pi/\omega$ and $x = 0$ when $t < 0$ and when $t > 2\pi/\omega$. In this case y is given by

$$y = -\frac{A\omega^2}{\mu}\int_0^t e^{-k(t-\eta)}\sin \mu(t - \eta)\cos \omega\eta d\eta$$

from $t = 0$ to $t = 2\pi/\omega$ and by

$$y = -\frac{A\omega^2}{\mu}\int_0^{2\pi/\omega} e^{-k(t-\eta)}\sin \mu(t - \eta)\cos \omega\eta d\eta$$

when t is greater than $2\pi/\omega$ because the acceleration is zero when $t > 2\pi/\omega$.

First suppose that the seismograph is undamped so that $k = 0$ and then when $t > 2\pi/\omega$ we have

$$y = -\frac{A\omega^2}{\mu}\int_0^{2\pi/\omega} \sin \mu(t - \eta)\cos \omega\eta d\eta$$

which gives

$$y = \frac{2A\omega^2}{\omega^2 - \mu^2} \sin \frac{\mu\pi}{\omega} \sin \mu(t - \pi/\omega).$$

From $t=0$ to $t=2\pi/\omega$ the equation for y is

$$y = \frac{A\omega^2}{\omega^2 - \mu^2}(\cos \omega t - \cos \mu t)$$

as for $x = A(1 - \cos \omega t)$ when $t > 0$.

The curves for y and x in the three cases

$$A = 1, \quad \omega = \pi/6, \quad \mu = \pi/18$$
$$A = 1, \quad \omega = \pi/6, \quad \mu = \pi/6$$
$$A = 1, \quad \omega = \pi/6, \quad \mu = \pi/2$$

are shown in Fig. 10, Fig. 11, and Fig. 12.

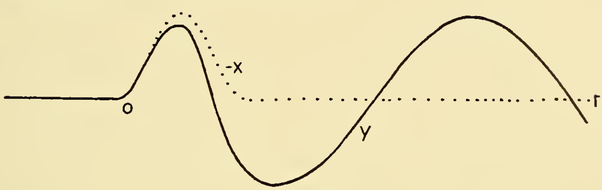

Fig. 10.

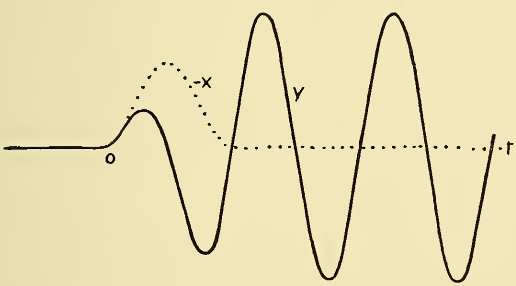

Fig. 11.

Fig. 12.

In Fig. 10 the period of the seismograph is three times that of the wave and a large oscillation is started in the seismograph. In Fig. 11 the two periods are equal. In Fig. 12 the period of the seismograph is one third that of the wave and the wave ends with the seismograph in equilibrium so that $y=0$ after the wave has passed.

Now suppose that the seismograph is critically damped so that $\mu=0$ and $k=p$. In this case the equation for y when $t>2\pi/\omega$ is of the form $y=a+b(t-2\pi/\omega)e^{-k(t-2\pi/\omega)}$ and the curves from $t=0$ to $t=2\pi/\omega$, of course, are the same as with $x=A(1-\cos \omega t)$ when $t>0$. Figs. 13, 14 and 15 show the curves for $-y$ and x in the three cases

$$A = 1, \quad \omega = \pi/6, \quad \mu = \pi/18$$
$$A = 1, \quad \omega = \pi/6, \quad \mu = \pi/6$$
$$A = 1, \quad \omega = \pi/6, \quad \mu = \pi/2.$$

We see that with critically damped seismographs the shape of the curves obtained depends less on the period of the seismograph than with undamped seismographs. When the period of the seismograph is much smaller than that of the wave the deflections are very small.

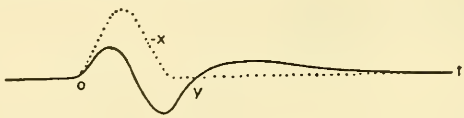

Fig. 13.

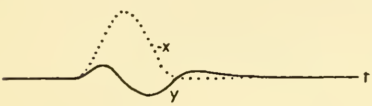

Fig. 14.

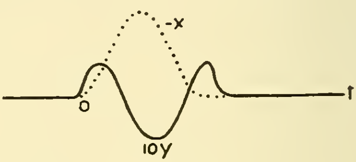

Fig. 15.

It appears that the only case in which the deflection of the seismograph is approximately equal and opposite to that of the ground during the passage of one or more waves is that of an undamped seismograph with period much larger than that of the waves. In this case, however, a single wave sets up an oscillation which continues indefinitely.

The principal conclusion which can be drawn from the above results is that seismographs do not register the actual motion of the ground so that arguments based on wave forms or the apparent arrival of waves later than the first wave must be used with great caution.

Critically damped seismographs are obviously better when it is desired to observe the arrival of several waves separated by appreciable intervals.

The inverse problem of calculating the motion of the ground from the recorded deflections of the seismograph will now be considered. The equation $-\ddot{x}=\ddot{y}+2k\dot{y}+p^2y$ on integrating gives $-\dot{x}=\dot{y}+2ky+p^2\int_0^t ydt+C$. We may suppose that $y=0$ at $t=0$ so that $-\dot{x}_0=\dot{y}_0+C$. But at $t=0$ $-\dot{x}_0=\dot{y}_0$ so that $C=0$. Also when t is very large we may suppose $\dot{x}=\dot{y}=y=0$ so that $\int_0^\infty ydt=0$. Integrating again we get

$$-x = y + 2k\int_0^t ydt + p^2\int_0^t\left\{\int_0^t ydt\right\}dt + D.$$

If $x=y=0$ at $t=0$ then $D=0$. Let $\int_0^t ydt=a_1$, and $\int_0^t\{\int_0^t ydt\}dt=a_2$ so that $-x=y+2ka_1+p^2a_2$. When t is large we may suppose that $x=y=0$ so that we must have $2ka_1+p^2a_2=0$. But $a_1=0$ when $t=\infty$ so that $a_2=0$ when $t=\infty$.

In the equation $-x=y+2ka_1+p^2a_2$ since $p^2=4\pi^2/T^2$ where T is the natural period of the seismograph, when undamped, we see that the correction p^2a_2 which must be added to y to get the true motion of the ground is inversely as T^2 for all values of the time t. If T is very large this correction may be negligible. In the same way the damping correction $2ka_1$ is proportional to the damping coefficient. Thus for an undamped seismograph of very long period $-x=y$ so that the seismograph gives the ground motion exactly. Any increase of the damping and any reduction of the period T increases the corrections and so increases the difference between the seismograph deflections and the ground motion. If $k=p$ the seismograph is critically damped.

To find a_1 it is convenient to make a tracing of the seismogram on graph paper and draw a curve for the area by counting the squares between the y curve and the zero line $y=0$. The zero line is usually not marked on the seismograms so that it is necessary to draw it. A small error in the position of the zero line makes a large error in a_1. Thus if it is drawn at $y=-\frac{1}{2}$ instead of at $y=0$ we get a_1 too large by $\frac{1}{2}t$. Such errors usually occur but are easily allowed for when it is remembered that $a_1=0$ for large values of t. If a_1 does not finally approach zero as t increases then by slightly shifting the zero line it is made to do so. Very often it is found that the area curve fluctuates about a straight line through the origin finally approaching this line. In such cases it is clear that the zero line is too high or too low. The line through the origin may then be taken to be the zero line for the area. The integral a_2 is then obtained from the curve giving the area in the same way. If it does not finally approach zero it must be made to do so by a suitable shift of its zero line. If an error Δa is made in the area a_1 at time t_1 then this produces an error $\Delta a(t-t_1)$ in the value of a_2 at a time t later than t_1. Such errors are usually appreciable and cause the curve obtained for this integral to drift up and down. It is therefore usually necessary to draw a new zero line for this curve. This zero line must be a smooth curve such that the value of the integral oscillates more or less equally above and below it.

Instead of endeavouring to correct such errors by shifting the zero lines, a process which depends a good deal on arbitrary judgment, one may carry out the calculation of x using the uncorrected curves for the area and the integral. When this is done the curve obtained for the motion of the ground usually appears to oscillate about a smooth curve which may drift very far from the zero line. This smooth curve may be drawn and taken to be the true zero line for the ground motion. These methods of correcting the calculated ground motion are necessary unless the areas are very accurately determined. They obviously result in the removal from the ground motion of any long period oscillations having periods long compared with the period of the seismograph. The constant p is nearly equal to $2\pi/T$ unless the damping is large, T being the periodic time of the seismograph. In getting the areas of the curves it is convenient to use a time scale such that $p=1$.

Before considering results obtained with seismograms some simple theoretical cases will be considered. Suppose $y=0$ when $t<0$ and $y=a \sin pt$ when $t>0$ and that $k=0$. In this case

$$-x = a \sin pt + ap^2 \int_0^t \left\{ \int_0^t \sin pt\, dt \right\} dt$$

which gives

$$-x = a \sin pt + ap \int_0^t (1 - \cos pt) dt$$

so that

$$-x = a \sin pt + pat - a \sin pt = pat.$$

Thus in this case the ground moves with uniform velocity ap. The sudden starting of this motion at $t=0$ starts the seismograph oscillating freely with amplitude a but has no further effect on the motion. If a ground wave ar-

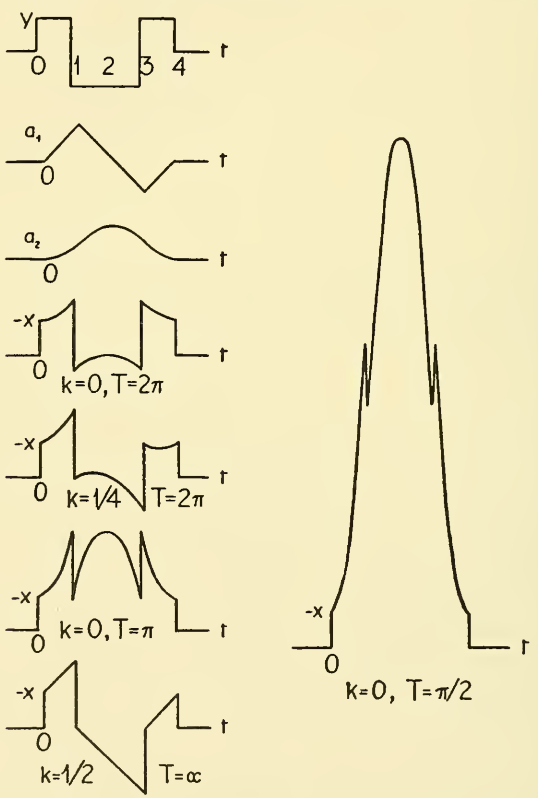

Fig. 16.

rives the period of which is long compared with that of the seismograph it will produce a nearly uniform motion of the ground for a short time which will start the seismograph oscillating with its natural period.

Suppose that $y=0$ when $t<0$ and $y=a\cos pt$ when $t>0$ and $k=0$. In this case $-x=a\cos pt+ap^2\int_0^t\{\int_0^t\cos pt\,dt\}dt$ which gives $-x=a$. Thus in this case the ground suddenly moves a distance a at $t=0$ and then remains at rest.

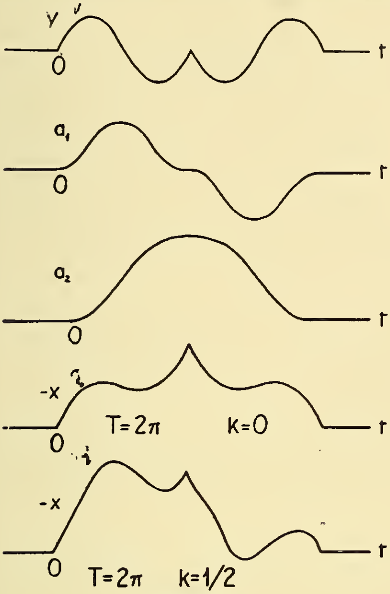

Fig. 17.

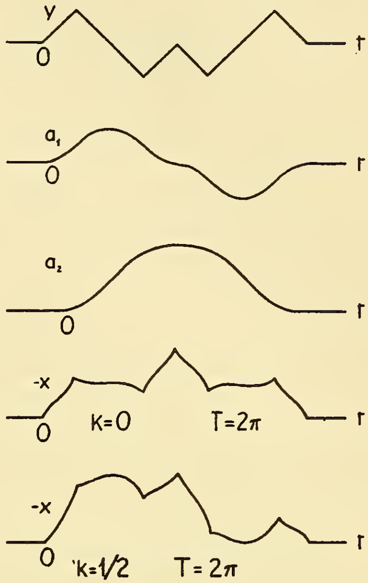

Fig. 18.

Figs. 16, 17, 18, show the curves for y, a_1, a_2, and $-x$ in some simple theoretical cases which need no explanation. The seismograph of course always records sudden movements correctly. Fig. 19 shows a seismogram of the waves due to an explosion of 1000 pounds of dynamite at a distance of 9870 meters. The seismograph was of the vertical mechanical type. The calculated

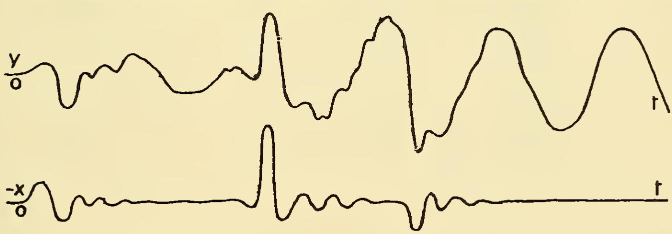

Fig. 19.

curve for the ground motion (x) is also shown. It appears that the explosion waves consist of a rapidly damped oscillation of period about 0.1 sec. Several such waves come in at different times and all have about the same shape and period. In this case there were no geological anomalies in the neighborhood, but the wave velocity increased more or less uniformily with the depth. The waves therefore travelled along nearly circular paths and may have been reflected from the ground surface as shown in Fig. 20. In this way the arrival of several waves at different times may be explained. Fig. 21 shows a seismogram of the waves from 1000 pounds of dynamite at 9590 meters. In this case

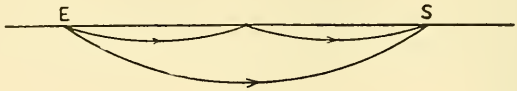

Fig. 20.

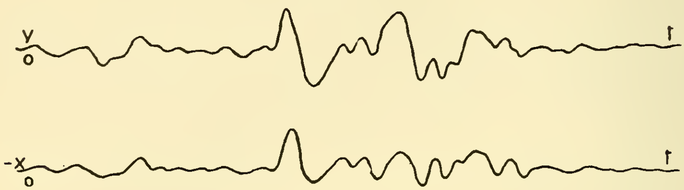

Fig. 21.

there was a salt dome in the path of the waves. The curve for x the ground motion is also shown. The ground motion can be regarded as mainly due to several rapidly damped oscillations which, however, are not as well separated as in the previous case.

An integraph[2] has been constructed which enables the curves for a_1 and a_2 to be drawn quickly. Fig. 22 is a diagram of this integraph. FF' is a brass block which rolls on two wheels WW' and carries a pencil at its middle point x. The points FF' on this block are connected to two parallel bars AD and $A'D'$ by means of two Peaucellier linkages which allow F and F' to move along straight lines perpendicular to AD and $A'D'$ respectively. The bars

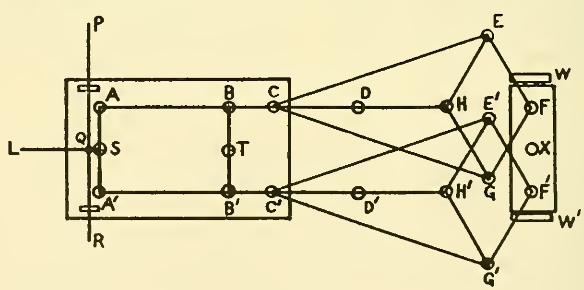

Fig. 22.

AD and $A'D'$ are connected by two links AA' and BB' which are supported on pivots at S and T fixed to a base. The link AA' can be turned about S by means of a liver SL. This lever also moves a bar RQP which can slide on the

[2] Intergraphs based on the same general principle are described in Les Intergraphs. Abdank-Abakauowiz. Paris, (1886).

base parallel to itself. When AA' is turned the block FF' also turns so that it is always parallel to AA'. When the lever SL is turned through an angle θ the end P of the bar PR moves a distance $l \tan \theta$ where l is the distance from S to PR.

The integraph is moved along parallel to the zero line of the curve of which the area is desired and the point P is made to follow the curve by moving the lever SL. When the base is moved a distance dx the pencil moves along a distance ds such that $ds \cos \theta = dx$. Let $ds \sin \theta = dz$ so that $dz/dx = \tan \theta$. If the first curve is given by $y = f(x)$ then $y = l \tan \theta$. Hence $y = ldz/dx$ or $ydx = ldz$ so that $lz = \int ydx \cdot z$ is the distance the pencil moves side ways so that the pencil draws a curve giving the area of the first curve.

In the instrument which was made the lengths AA', BB', FF' were each two inches and l was made one cm. The links were made of stainless steel 3/8 by 1/16 inch. The wheels were 7/8 inch ball bearings. The block FF' was $3 \times 1 \times 1$ inches.

This instrument enables the curves for a_1 and a_2 to be drawn quickly and with sufficient accuracy for most purposes. In using it one side of the base is kept against a straight edge fastened to the drawing board.

J B Hersey

THE JOURNAL

of the

Society of Petroleum Geophysicsts

(Division of Geophysics, American Association of Petroleum Geologists)

JULY - 1935

Vol. VI No. 1

Volume VI JULY, 1935 Number 1

THE JOURNAL

of the

SOCIETY OF PETROLEUM GEOPHYSICISTS

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THE JOURNAL
of the
Society of
Petroleum Geophysicists
(*Division of Geophysics, American Association of Petroleum Geologists*)

VOLUME VI JULY, 1935 NUMBER 1

NOTES ON THE EARLY HISTORY OF APPLIED GEOPHYSICS IN THE PETROLEUM INDUSTRY

E. DeGOLYER

The writer first became interested in the possibility of using applied geophysics as an aid to prospecting for petroleum deposits during the summer of 1914 as the result of a conversation in London with the late P. C. A. Stewart, regarding the difficulties of prospecting for new salt domes in the flat Gulf Coastal plain of Texas and Louisiana and in the Isthmus of Tehuantepec, Mexico. Stewart vaguely remembered that some success was said to have been had in determining underground geology by gravity surveys in the great Hungarian Plain. Inquiry was made and we were told that such was indeed the case, that the surveys had been made with a new instrument, the torsion balance, invented by Baron Roland von Eötvös and that the only existing balances were within the territory of the Central Powers. The Great War had just started.

The torsion balance is essentially a development into a comparatively robust field instrument of the more delicate and older Coulomb torsion balance which has been used in physical laboratories since the eighteenth century for investigating and demonstrating the laws of gravitational attraction. In 1888, the eminent Hungarian physicist, Baron Roland von Eötvös, Professor of Physics in the University of Budapest, demonstrated how this balance, which has since been known universally as the Eötvös torsion balance, could be used for more extensive studies of gravity variation than had hitherto been possible by the use of pendulum stations. The first primitive field instrument was completed by 1890. Various determinations were made in the laboratory and in the field, and the instrument was modified and refined both as to mechanical system, insulating housing, and recording until, in 1902, Eötvös introduced the double-beam instrument which is essentially the instrument of today.

The first exhaustive field investigation by Eötvös was carried out in 1901 on the ice of Lake Balaton, in order to avoid the computation of terrain effects, but surveys were soon extended into the Great Hungarian Plain where such corrections were necessary.

Eötvös designed and built his instrument for geodetic research; geodesy and physics being his main interest in the instrument, the methods, and the field surveys throughout his entire life, except that during the first decade of the present century, he demonstrated the possibility of interpreting buried regional geological structure from a consideration of gravity anomalies. He apparently believed that the precision of his instrument was great enough and the gravity expression of smaller structures such as anticlines and domes was sufficiently definite for them to be mapped by torsion balance surveys.

The more definite understanding of the possibilities of the instrument as a geological tool was mainly the realization of the geologist Hugo V. Boeckh who, in 1917, first called attention to the fact that anticlines and domes with light or heavy cores could be located by means of the torsion balance, citing surveys and the existence of such anticlines at Gbely (Egbell). The American geologist, Eugene Wesley Shaw, suggested the possibility of using gravity anomalies in a search for salt domes in the same year. In 1918, Schweydar, with the advice of Boeckh, employed the method to delineate the boundary of an explored German salt deposit.

The possible use of the balance was again discussed in early 1919 with the late Dr. Th. Erb, when in charge of exploration throughout the world for the Dutch-Shell group. Dr. Erb said that his group had investigated the method and that it appeared to be theoretically sound but that they had not yet made a practical test.

In 1920, I learned that one could then purchase or contract for the construction of torsion balances. After lengthy consideration, a joint field research was arranged between the Amerada Petroleum Corporation and the Mexican Eagle Oil Company. Two instruments were contracted with Ferdinand Süss, Budapest, and construction was commenced in August, 1921. Donald C. Barton was sent to Budapest in May, 1922, to receive the finished instruments, which had been standardized by Dr. Pekar of the Eötvös Institute and to be instructed in their operation. The instruments were received in New York on September 5, 1922, and were in Houston in early November. The first survey, one of the Spindletop salt dome, was made in early December, 1922. This was the first or one of the first surveys of an oil pool made by geophysical methods in the United States and appeared to be a brilliant success though it now seems, in the light of our more extensive knowledge of gravity variations, to have been in the nature of a lucky accident, since it was a very definite gravity maximum, one of the very few in the entire coastal regions.

The method was apparently taken up by the petroleum companies in the early 1920s, first by the Anglo-Persian and the Dutch-Shell groups. The *Mexican Financier* for January 1, 1922, contained an advertisement of the Eötvös balance and within the early months of the year the Dutch-Shell group was making a survey of the Hurgada field in Egypt, using Bamberg instruments. By the latter part of the year they were possibly being experimented with in California and a Dutch-Shell instrument was sent to the Gulf Coast about the time that the Amerada-Mexican Eagle instruments arrived and to Mexico within a very short time thereafter.

With the apparently brilliant success of the initial survey in indicating Spindletop as a definite gravity maximum, surveys were extended to other known domes, for the most part with vague and indifferent results. These experimental surveys generally covered only small areas and were entirely inadequate to test the value of the instrument or of the method. Various prospects were surveyed with results none too definite and several of them were drilled and failed.

Just as we were about to abandon the instruments and method, a survey of the Nash area in southern Fort Bend County, Texas, gave a gravity maximum as brilliant and definite as that for Spindletop. A well was drilled in November, 1924, and encountered cap rock at 648 feet and salt at 943 feet. This was the first successful geophysical prospect to be proved in the United States. Oil was discovered on the flank of this dome January 3, 1926, and this was probably the first oil pool to be discovered by geophysical methods in the entire world.

During 1922, as consulting geologist to the Mexican Eagle Oil Company, I had recommended that the company make an attempt to locate the extensions of the buried Tomasopo ridge by the use of refraction seismic surveys. A German crew was engaged from Dr. L. Mintrop of the Seismos Company and it worked without very satisfactory results, attempting to map the extension of the Tomasopo ridge in late 1923.

About the same time, a Mintrop crew was engaged by the Marland Company, as a result of Dr. Van der Gracht's recommendations. Various tries in the Mid Continent from northern Oklahoma to the possible extension of the fault line play north of Mexia gave results of interest but of doubtful value. A discussion in late October, 1923, with F. Park Geyer, then chief geologist of the Marland Company, indicated that the method might be developed into one of usefulness but had not yet been of practical value.

In March, 1924, John F. Weinzierl appeared on the Gulf Coast with the first seismic crew, a Mintrop crew working under Weinzierl and under the general direction of Alexander Deussen for the Marland Company.

A Mintrop crew was also engaged by the Gulf Production Company in 1924 and before the end of the year, the Orchard dome in Fort Bend County, Texas, had been found; the first seismic discovery on the Coast and possibly the first in the world.

In view of my own unconvincing experience with the seismograph in Mexico, and that reported by Geyer, for the Marland Company, in Oklahoma and north Texas, and in view of the recent success of the torsion balance at Nash, I was inclined to be skeptical with regard to the possible value of the seismic method. Repeated successes of the Seismos crews for the Gulf, however, soon convinced me that the method was one to be reckoned with. Its results were positive, the solution obvious, and the speed of coverage very great, in all of which respects it was superior to the torsion balance in a highly competitive search for shallow salt domes.

In searching for some one competent to develop seismic methods, I became acquainted with Dr. J. C. Karcher who, as early as 1919, while a fellow in the Graduate School of the University of Pennsylvania, had applied for patents on the reflection method and who, with the late W. P. Haseman, had been conducting field work in Oklahoma as early as 1921. Karcher was engaged to head the work for a new company, the Geophysical Research Corporation, organized in May, 1925. He designed and built instruments and was in the field with a crew for the Gulf early in 1926. It seems probable that Udden's famous "Suggestions of a New Method for Making Underground Observations," as published in early 1920, Vol. 4, *Bulletin of the American Association of Petroleum Geologists*, was inspired by Karcher's early ideas on the subject since one of Udden's sons was a fellow in physics at the University of Pennsylvania, and a colleague of Karcher, who had communicated ideas on the subject to him.

During 1925, practically all of the commercial work was being done by Seismos. In November of that year three crews were operating for the Gulf and one crew for Marland. The Humble was developing its own instruments and McCollum was in the field during the latter part of the year. The Seismos crews were using mechanical seismographs, surveying positions, and estimating time of shot explosions by sound through the air. Karcher introduced the electrical seismograph, determined the time of shot explosion by radio and used the air sound waves for surveying position, innovations which became standard practice within the year.

The real success of the refraction type of seismic surveys, and one of the most brilliant successes for geophysical prospecting yet scored by any method was the intensive campaign of searching for shallow salt domes which swept the Gulf Coast of Texas and Louisiana from 1924 to 1930. This campaign was initiated with the appearance in the Gulf Coast, in March, 1924, of the Mintrop crew working for the Marland Oil Company. This crew was soon followed by another Mintrop crew for the Gulf Production Corporation under the general direction of L. P. Garret.

Mr. Garret had conceived the idea of using seismic refraction surveys as a tool to prospect for salt domes as early as 1905–06 and together with Mr. Robert Welch of Houston, had made inquiries as to instruments and methods. Instruments were expensive and not really adapted to work on the scale required and so the matter was dropped. When the Mintrop crews appeared in the United States, however, they found in Mr. Garret one who was already quite convinced as to the possible value of the method, and one who was in position to use them broadly and earnestly. The Gulf Coast with its types of rocks-clays, shales, and sands, having a linear velocity of sound transmission of around 6,000 feet per second and rock salt with a corresponding velocity of 15,000–16,000 feet per second, was an ideal region for the method. The outstanding success of the method and indirectly of the introduction of geophysical methods into the oil business, owes much to the peculiar suitability of the area where it was tried and to the vigorous support given by Mr. Garret.

The seismic or sonic method was by no means new. A little less than a century ago, Robert Mallet, one of the fathers, if not *the* father of the modern science of seismology, in a paper delivered before the Royal Irish Academy—*On the Dynamics of Earthquakes: Being an Attempt to Reduce Their Observed Phenomena to the Known Laws of Wave Motion in Solids and Fluids*,[1] said:

However well modern geologists have surveyed and mapped the formations constituting the land which we can see and handle, of the nature of the bottom of the great ocean we know nothing; no human eye has or ever can behold it; we cannot even reach its deep abysses with the sounding line; yet the ocean covers nearly three-fourths of our entire globe, and of this vast area the geology is an utter blank. If, however, we are enabled hereafter to determine accurately the time of earthquake shocks, in their passage from land to land, under the ocean bed, we shall be enabled almost with certainty to know the sort of rock formation through which they have passed, and hence to trace out at least approximate geological maps of the floor of the ocean. For, knowing the time of transit of the wave, we can find the modulus of elasticity which corresponds to it, and finding this, discover the particular species of rock formation to which this specific elasticity belongs.

After further emphasis on the desirability of acquiring such information, he continued:

While the facility with which one class of our data may be ascertained will be disputed by none, it may, perhaps, occur to some that, as earthquakes are happily rare, and give no notice of their advent, and moreover, are times of such consternation, so but little accuracy is to be hoped for in observations, as to the speed or circumstances of the shock, made during such visitations. This might be partly true, were we dependent upon the nerve or watchfulness of individual observers; but already attention has been given to the contrivance of self-acting instruments (and instruments, though by no means well devised nor self-registering, have been already in use in Scotland, and perhaps elsewhere) for the registration of earthquake shocks; and there can be no doubt that, by *earthquake observatories* established, with suitable instruments, at distant localities, in South America or Central Asia for instance, where earthquakes, greater or less, are of almost daily occurrence, a very complete knowledge of the time of wave transit, and of the amplitude and altitude of the earth waves for given districts, would be soon obtained. No instruments for ascertaining the latter have been yet proposed, but they do not seem by any means difficult to devise.

He minimized the apparent delays in waiting for natural earthquakes, and suggested a workable substitute as follows:

But another, and much more rapid, and perhaps even certain,

[1] *Irish Acad. Trans.*, XXI, pp. 50–106, Dublin, 1848.

method, remains to be noticed, for obtaining part of our data as to the *specific period of wave transit*, viz., *by direct experiment*, which in all matters of inductive science may be pronounced, whenever it is possible, better than mere observation.

I have already stated that it is quite immaterial to the truth of my theory of earthquake motion what view be adopted, or what mechanism be assigned to account for the original impulse; so, in the determination of the time of transit of the elastic wave through the earth's crust, if we can only produce a wave, it is wholly immaterial in what way, or by what method, the original impulse be given.

Now the recent improvements in the art of exploding, at a given instant, large masses of gunpowder, at great depths under water, give us the power of producing in fact, an artificial earthquake at pleasure; we can command with facility a sufficient impulse to set in motion an earth wave that shall be rendered evident by suitable instruments at the distance, probably, of many miles, and there is no difficulty in arranging such experiments, so that the explosion shall be produced by the observer of the time of transit himself, though at the distance of twenty or thirty miles, or that the moment of explosion shall be fixed, and the wave period registered by chronometers, at *both* extremities of the line of transit.

In presenting this paper, Mallet stressed the necessity for experiments to determine the time of wave transit through rocks of different type, and three years later we find him giving notice of experiments in progress for the direct measurement of the velocity of transit of earthquake waves. He describes experiments to be carried out over a measured mile with a charge of "a few pounds of gunpowder buried four or five feet in the ground" connected by a wire circuit with a battery so that it could be fired by the experimenter at the other end of the line with his crude seismometer for observing the arrival of the earth wave. A chronograph was to be attached to the battery for determining the time of transit of the wave.

The author stated that if the elasticity of the earth's crust were known, it would be possible to determine the point from which an earthquake shock originated, and also to form an estimate of the nature of the intervening formations— whether solid or plastic— through depths of perhaps hundreds of miles, and even under the ocean.[2]

In 1849, Mallet actually carried out experiments in the field at Killiney Bay on the coast of County Dublin, Ireland, and at the nearby Dalkey Island. At Killiney Bay he had a widely extended

[2] *The Athenium*, Vol. 22, Num. 1144, p. 991, London, Sept. 29, 1849.

beach and attempted to determine the velocity of the transit of elastic waves through sand; at Dalkey Island, velocity in granite. He used a battery circuit for firing black powder, buried half a mile distant from the observers station, and recorded with chronographs at each end of the line. The arrivals of waves through the earth were detected by using a crude seismoscope or seismograph consisting essentially of a tray of mercury from the surface of which was reflected a spot image kept under observation through a small telescope by the observer. Disturbance of the mercury by earth movement destroyed the reflected image.[3]

In 1850, similar observations were made for granite on Dalkey Island over a distance of 1,166 feet.

The results of these experiments were to determine a velocity of transit through sand of 824 feet per second and through granite of 1,306 and 1,664 feet per second.

In 1856 Mallet carried on similar experiments in government quarries at Holyhead in North Wales, determining similar velocities for quartz and slate (metamorphics) of 1,088 feet per second.[4]

The rock velocities observed by Mallet are all extremely low and it is practically certain that he was observing the surface or Rayleigh wave. It is remarkable, however, that in his experiments he was doing essentially what the geophysicist engaged in refraction work does today i.e., discharge an explosive and attempt to determine the time of transit of waves generated by the explosion to points a measured distance away. It is a tribute to his genius that he had so clear a conception of the value of the method as a geologic tool.

On April 2, 1914, Reginald Fessenden filed the original Sonic Sounder application for determining ocean depths, the first and a clear use of the reflection method. The U. S. Hydrographic Bulletin for May 13, 1914, gives an account of the location of an iceberg off the Newfoundland Banks, at a distance of two and one-half miles, by acoustic methods and on January 15, 1917, Fessenden filed his patent application on Methods and Apparatus for Locating Ore

[3] An exceedingly detailed account of these observations is given in Second Report on the Facts of Earthquake Phenomena, Brit. Assoc. for the Advancement of Science, Report of Meetings, 1851, Vol. 21, pp. 272–320.

[4] Account of Experiments made at Holyhead (North Wales) to Ascertain the Transit-Velocity of Waves, Analogous to Earthquake Waves Through the Local Rock Formations. *Phil. Trans. Royal Soc.* of London, Vol. 151, Part 3, pp. 655–679, London, 1861.

Bodies, a division of his original Sonic Sounder application. The claims of this patent cover clearly both refraction and reflection as used to locate geologic formations.

Dr. L. Mintrop in 1919 applied for patents on the refraction seismic method in Germany.

On July 9, 1920, Prof. John William Evans, F.R.S., and Willis Bevan Whitney, applied for British patents on the seismic method.

On August 14, 1922, Burton McCollum applied for the first of a series of patents on the seismic method.

These, in brief, are the outstanding earlier patent applications. Mallet clearly was the discoverer of the refraction method; Fessenden of the reflection method. Mintrop worked with some success on a group of extremely high grade prospects selected by Garret in an area ideally suited to the refraction method, while Karcher improved considerably the technique of the refraction method, and later reduced the reflection method to practice. It should be emphasized that the refraction and reflection methods are quite different. The instrumental technique is different. The field set-up is different. The problem is different. The solution is different and the results secured are different. Mintrop's brief "On the History of the Seismic Method for the Investigation of Underground Formations and Mineral Deposits"[5] avoids this point. He barely mentions reflections in his patent and apparently did not develop nor use the reflection method.

After several years of preliminary experimentation the Geophysical Research Corporation mapped the Maud pool in T. 8 N., R. 5 E., Oklahoma, in 1927, upon which the discovery well was subsequently drilled by the Gypsy Oil Company. In 1928, a supposed structure was drilled in T. 7 N., R. 3 E., Oklahoma, resulting in a small well. In 1930, three geophysical highs, found by the reflection method in the Seminole plateau, Oklahoma, were tested with excellent results. The Edwards area in Sec. 22 and 23, T. 9 N., R. 5 E.; a northwest extension of the Carr City pool, Sec. 3, T. 8 N., R. 5 E.; and the Chase area in Sec. 29, T. 9 N., R. 6 E., were all successfully explored, technically and practically, and established the value of the reflection method as an exploratory tool beyond any doubt.

Late in 1929, the dip method, a gradient method of reflection shooting differing somewhat from the correlation method, was developed by the Geophysical Research Corporation. The Darrow dome, Louisiana, which had previously been discovered by other methods

[5] Publications of the Seismos Company, II., Hannover, Germany, 1930.

was the first to be detailed by this method. The dip method has been used chiefly for detailing known salt domes in the Gulf Coast and is today almost the only method of seismic exploration being used in that area. It is used as an exploratory metho l in a search for the so-called deep domes. The Valentine or Harang dome, Louisiana, is the only dome actually discovered by the method which has been proved by the drill to be an actual salt dome, though many structures, of the so-called "deep-seated" type have been found by it and, with deeper drilling, the salt cores will probably be found.

Such in brief are the outstanding events in the early history of applied geophysics in the oil industry as they have come to the knowledge of the writer. The technique has developed rapidly and is still being improved substantially. Much remains to be done. It is a notable fact that, since the discovery of the great East Texas field some five years ago, notwithstanding that exploratory drilling of geophysical structures has been at a maximum, actual discovery of oil in the United States has not been able to keep pace with consumption. This condition suggests that, without the aid of geophysics, the industry would have fallen even further in arrears. The aid of the geophysicist is and must be the hope of the future in further search for oil.

ON THE STRATEGY AND TACTICS OF EXPLORATION FOR PETROLEUM[1]

E. E. ROSAIRE[2]

The marked increase in applications of geophysical methods of exploration since 1930 has resulted in the recent grouping of many men relatively unfamiliar with the history of geophysics. Since these men, generally, are specialists in only one exploration method, they are prone to considerable discussion as to the relative merits of various geophysical methods, and too frequently deprecate the value of methods other than their own individual specialty. Pride in, and prejudice for a chosen arm are to be expected of second lieutenants, but are quite unseemly in a chief of staff.

One purpose of this paper is to review the history of the use and development of geophysical methods in the exploration for petroleum as exemplified in one particular, successful strategy. A second purpose is to examine the choice and sequence of methods to be used in an exploration campaign. A third purpose is to determine the relative importance of problems of tactics, such as research in methods of exploration, and problems of strategy, such as the more extended use of the methods already developed. The justification for this paper to the writer at least, has been his own past confusion as to the relative merits of various exploration methods, and his observation that their proper use now is the exception rather than the rule.

The policy of an oil company, at various times, may vary considerably. Shortly after the discovery of the East Texas field, the general policy was to decrease exploration for new fields. Recently, however, the general policy has been that of searching intensively for new fields. Policy, then, may be defined as the recognition and statement of a broad problem intimately associated with the future of the organization.

With the problem recognized and stated, the next question is that of the choice of approach to the solution. Thus, with an oil company committed to a policy of increasing reserves, strategy calls for a choice between the tactics of purchase or discovery, or the use of

[1] Address of Retiring President, presented at the Annual Meeting, Society of Petroleum Geophysicists, March 22, 1935, at Wichita, Kansas.

[2] Independent Exploration Company, Houston, Texas.

both. Purchase may be on a minor scale, such as the acquisition of proved or semi-proved acreage; or on a larger scale, such as the Tide Water Oil Company's recent purchase of the producing properties owned by the Simms Oil Company. Similarly, discovery may be on a minor scale, in the nature of the development of new producing horizons in a field already in production, such as the recent discovery of deeper production (Cockfield) at Raccoon Bend, Austin County, Texas, by the Humble Oil and Refining Company. Discovery may also take the form of improvements in recovery methods, as in the development of the water drive at the Bradford Oil Field in Pennsylvania, and in the use of acid treatment in the producing lime horizons of Michigan. The development of cracking methods in the refining of crude oil is an equivalent of discovery. Finally, discovery may take the form most familiar to the geophysicist, i.e., the search for and development of production on previously dry or undiscovered structures.

Strategy, then, is the approach to the solution of a problem engendered by policy, and tactics is the means of solution.

With a chosen policy of acquiring additional reserves, and a chosen strategy of searching for and developing production on untested structures, the problem of finding these structures is one of tactics.

The strategist, however, may be stalemated, as under conditions where there is no known answer to the problem, or where the only answer is prohibitively expensive. Two such stalemates come to mind: One, the trench deadlock in the World War in 1916; and, the second, the high cost of finding new production in 1914.

These two cases are interesting because of the parallelism in question and answer. In each case, the answer was theoretically attainable, but only at a cost obviously so great that the practical attainment of the solution might well be a Pyrrhic victory. Further, in each case, the tactical means so badly needed had already been conceived, and in one case proposed.

The trench deadlock was solved by the tank. Yet the idea of a mobile fortress was as old as the armored chariots of the Egyptians and the war elephants of the Carthaginians. In 1916, the mechanism required was already at hand in the shape of the caterpillar tractor in use in the grain fields of America. The major credit for the introduction of the tank is generally given to Lloyd George, who, however, did not invent the tank. His contribution was in the field of strategy, and consisted primarily of intangibles. For the successful introduction of the new tactics at hand, the fundamental need was for some strategist of prominence and position to realize that the existing stalemate could not be solved by the tactics then recognized, and who would further cast about for tactics which, even though new and unrecognized, had possibilities of breaking the stalemate. The inertia of accepted thought too frequently exists with discouraging ponderosity.

In the case of the petroleum industry, a somewhat similar situation existed in 1914. New exploration methods had appeared on the scene some time before, primarily those of surface mapping, core drilling and subsurface stratigraphical studies. However, after intensive application of these methods, only marginal prospects remained, for which the cost of development was obviously high in view of the hazard involved.

As a result of the indicated hazard, the strategy of exploration approached a stalemate. New tactics were needed. And yet, in 1896, von Eötvös had developed the torsion balance, and had used it in 1902 to indicate the subsurface extension of the Jura Mountains. Even prior to this development, as far back as 1846, Mallet had proposed the use of artificially excited seismic waves to explore the subsurface of the earth, and had gone so far as to secure field data. He anticipated and used the fundamentals of applied seismology, for he transmitted electrically an instant of explosion, observed the arrival of the resulting seismic wave with an inertia type seismometer, and used a chronometer to indicate the interval of time between. Further than these, little more, fundamentally, is done today.

In neither of these two examples was the development of new tactics required. The fundamental need was that strategy recognize the fact that the law of diminishing returns had run its course in the case of the accepted tactics of the day. Both stalemates were strategical rather than tactical, and the rates with which the newer tactics were applied in both cases are probably good measures of the relative inefficacy of the older competing tactics. Further, the spectacular results which followed strongly suggest the almost complete absence of really long range strategy in the years prior to the development of these stalemates.

Case treatments of all the strategies that blossomed with the advent of geophysical prospecting would be interesting and instructive. The time and detailed information required for such a complete study has not been available. However, the writer's prior and early service with the Geophysical Research Corporation has presented some personal opportunity for observing, in the history, aspirations, and successes of the Geophysical Research Corporation, one particular strategical approach, that of E. L. DeGolyer. His has been chosen, not only because it is more familiar and so more personally interesting to the writer, but also because therein are found excellent examples of effective short and long range strategies.

The chronology that follows is certainly open to the charge of being incomplete, but is not intentionally biased. With apologies for the gaps, and with a premium upon brevity, DeGolyer's strategy appears as follows.

Some time before 1914, study had convinced him that the high cost of crude was fundamentally due to the increasing cost of successful exploration. In that year, in London, he heard of a device which had possibilities of mapping subsurface structure while operating at the surface of the earth. The trail led to two torsion balances, one in Germany, the other in Austria. By that time, the war had intervened, and nothing further could be done. After the cessation of hostilities, arrangements were made with the firm of Ferdinand Süss, at Budapest, for the delivery of two such devices, and for the training of an observer in their manipulation. By arrangement, one was to be used in Mexico by the Mexican Eagle, and the other in the Gulf Coast by the Amerada.

Dr. Donald C. Barton was chosen to receive this training, and on his arrival in Europe, found the delivery dates postponed. This additional time was spent, at DeGolyer's suggestion, in checking into the state of applied geophysics on the Continent. Barton was able to secure some information on the possible use of the refraction seismograph as an exploration instrument. Meanwhile, DeGolyer, through his association with the Mexican Eagle, had witnessed an unsatisfactory demonstration of the seismograph in Mexico.

On his return to the United States, Barton made the first torsion balance survey at Spindletop, in 1922. There he was able to observe a marked anomaly with the torsion balance, but other known domes did not consistently show the same pronounced effects. However, a success was scored with the confirmation by the drill of the gravity anomaly at the Nash salt dome in Fort Bend County, Texas, although at least the next two tries were failures.

About this time (1924), Mintrop appeared on the scene, and under the patronage of the Marland Oil Company and the personal supervision of Alexander Deussen, introduced the refraction seismograph into the Gulf Coast. In the exploration of several prospects recommended by Deussen, the device consistently failed to indicate any marked anomalies. One of the prospects was Sheppards Mott, in Matagorda County, Texas, and another later proved to be the Rabbs Ridge, or Thompson oil field in Fort Bend County. The failures of the device to register then at those two prospects is understandable now that they are recognized as very deep-seated salt domes, but Deussen could not reconcile himself to the possibility that all of his carefully studied and chosen prospects were normal areas. Finally, in desperation, he tried the device out in another area where, unknown to the operators, the shallow salt dome near New Iberia, Louisiana, had recently been discovered as a result of drilling a surface prospect. We can probably appreciate Deussen's sigh of relief when the all too familiar report "No salt dome present" was made.

About this time, a second Mintrop seismograph crew appeared in the Gulf Coast, operating under the auspices of the Gulf Production Company. Possibly, as H. C. Cortes suggests, the experience gained at the cost of the Marland failures had borne fruit by this time, for the method showed positive evidence of shallow salt domes in quick succession on the Gulf's chosen surface prospects at Orchard, Fannett, Hawkinsville, and Starks.

This spectacular performance, together with the quicker and greater operating coverage of the seismograph (as compared with that characteristic of the torsion balance), led DeGolyer to reconsider his former unfavorable opinion of the device and to change his strategy. Investigation showed that Dr. J. C. Karcher alone remained available of four American physicists who had participated in the first seismograph exploration in the United States in 1919. With him as a nucleus, the Geophysical Research Corporation was organized and staffed with men trained in operating electrically rather than mechanically. This organization invaded a field dominated by a successful rival, and quickly set the pace. In less than a year, positive results were registered in the discovery of the Moss Bluff salt dome, Liberty County, Texas, and the Port Barre salt dome, St. Landry Parish, Louisiana, in May and June, 1926.

As a strategist, DeGolyer properly allowed his tacticians a free hand. Rather than a taskmaster, he was a taskmaker. For instance, before his organization had thoroughly mastered the refraction seismograph, he set them the problem of quantitatively anticipating the drill. In little more than two years after the formation of the Geophysical Research Corporation, the answer was provided by the introduction of the reflection method on a commercial basis on the Seminole Plateau, Oklahoma. The solution to this problem, however, was much harder than DeGolyer anticipated, for in 1927 the prevalence and importance of crooked drill holes had not yet been appreciated. The fact that the sub-sea datums available as yardsticks were not infrequently in error by as much as one to three hundred feet was a temporary but serious handicap in the task of establishing this new method as a valuable exploration tool.

DeGolyer's strategy was successful because, first, after realizing and evaluating his existing tactical limitations, and finding one answer (the torsion balance), he arranged for and secured tactics which furnished a better answer to the immediate problem by shifting to the refraction seismograph. Second, not content with tactical developments which were highly successful in answering the immediate problem, he went further and instigated the development of the reflection seismograph, a method which closely approximated his ideal of the basic exploration method, and remains today as the exploration method with a resolving power bettered only by the drill.

This latter point is an example of really long range strategy and undoubtedly eliminated another strategical stalemate. The justification for developing new tactics, rather than switching back to the torsion balance, was that an exploration method with a resolving power uncontrollably greater than the range of exploitation can be economically unsound. The torsion balance lost prestige when, for appreciable periods of time, the gravity predictions at Lost Lake, Roanoke, and Eureka remained unconfirmed and apparently disproved because of the existing limitations on drilling depths. Therefore, DeGolyer was justified in choosing to seek an exploration method with a resolving power which could be set by the operator, and so could keep proper pace with the exploitation methods.

THE CHOICE AND SEQUENCE OF TACTICS

Since 1912, when only surface mapping, core drilling and subsurface studies were available, five new exploration tactics have been developed, i.e., airplane photography, and the recognized methods of exploring by seismic, magnetic, gravimetric and electric methods. Any one of these methods can be used to demonstrate, quite convincingly, the existence of an anomaly at one or more producing fields. In planning a campaign, then, what should be the initial choice and the sequence of tactics?

This is not a simple problem. Thus, in 1923, an examination of the history of the discovery of salt domes in the Gulf Coast would have shown that the exploration method with the greatest record of accomplishment was that attained by drilling prospects resulting from the search for gas seepages, topographic anomalies and paraffin dirt beds. Yet this method had stagnated,[3] and two years later the introduction of geophysical tactics revolutionized the strategy of exploration. In 1929, the refraction seismograph had attained a remarkable record of discoveries in the Salt Dome Provinces of the Gulf Coast Embayment, and yet a year later the method had become practically obsolete. In considering a choice of tactics, the important question is not only whether the method under consideration will reasonably define an anomaly at some known producing field, but also to what extent the law of diminishing returns has run for that particular method in the province under consideration. No matter how strikingly an airplane photograph may show the areal geology, the method will contribute little if the surface geology was carefully, even though laboriously, mapped at an earlier date.

Assuming an ultimate recovery of twenty thousand barrels to the acre, fifty thousand acres, or about two and one-fourth townships would produce one billion barrels of oil. Ten billion barrels of oil, under such conditions, would be recovered from about twenty-five townships, or a square about thirty miles on a side. The area of such a square is negligible in comparison with the thousands of square miles of potential oil-producing territory in the continental United States of America.

Although from an individual operator's standpoint, the problem is that of finding a structure, from the standpoint of an exploration campaign, one major problem is that of eliminating unfavorable areas, or rather, selecting areas where the great odds against random drilling are minimized. Bearing in mind that good strategy requires the use of exploration methods with greater resolving power than those previously used, obviously the preliminary phase of the exploration should be the critical examination of existing sub-marginal prospects.

The resolving power of a given exploration method might be likened to the cone of error for a rifle. As the range increases, the base of the cone of error increases. As long as the target is greater than the base

[3] *Geology of Salt Dome Oil Fields*, DeGolyer *et al.*, p. 776, *Occurrence of Sulphur Waters in the Gulf Coast of Texas and Louisiana, and their Significance in Locating New Domes*, by W. F. Henninger.

of the cone of error, a satisfactory percentage of hits will be registered. However, when the base of the cone of error becomes greater than the target, the percentage of hits can be expected to fall off quite rapidly. At such an extended range the percentage of hits can best be raised by the substitution of another rifle with a smaller cone of error at the range now in question, rather than by trusting wholly to chance and continuing the use of the original rifle.

The primary function of geophysics was the evaluation of existing sub-marginal prospects. The discovery of prospects by these newer tactics was a later function. In the usual course of events, discovery at a premium, i.e., at a minimum cost, is naturally expected to follow from the use of geophysical methods to evaluate existing sub-marginal prospects, rather than in their use to discover as well as to evaluate prospects.

Such was the case for each exploration campaign, except one,[4] in which geophysical methods made spectacular records. The first discovery for the torsion balance was at the Nash salt dome in Fort Bend County, Texas, a sub-marginal surface prospect. The first discoveries of shallow salt domes in the Gulf Coast Province for the refraction seismograph followed in rapid succession as the sub-marginal surface prospects at Orchard, Fannett, Hawkinsville and Starks were examined. In East Texas, most of the refraction discoveries followed the examination of surface prospects, while the early successes of the reflection method on the Seminole Plateau were almost exclusively due to the examination of sub-marginal prospects. And in every case to date, with one exception,[5] the discoveries following the use of the reflection method in the Gulf Coast have been due to the examination of sub-marginal prospects indicated by one or more of the exploration methods in previous use, i.e., surface indications, the refraction seismograph and the torsion balance. Finally, the

[4] This one geophysical exploration campaign attended by discovery at a premium, and unassisted by previous methods, was the refraction seismograph exploration made by the Geophysical Research Corporation for the account of the Louisiana Land and Exploration Company in the lakes and bays of Southern Louisiana. This spectacular success followed the extensive initial use of a method with high resolving power in virgin territory, wherein the number of prospects existing was undoubtedly abnormally high. However, after the discovery of the Lake Barre salt dome, in this same campaign, gas seeps were found to have been known there for many years before.

[5] And in the case of this one (unassisted) discovery by the reflection seismograph, at the Valentine salt dome in LaFourche Parish, Louisiana, favorable surface indications were later found to have been known for several years over the dome itself.

first oil field credited to geophysics in the Rocky Mountain Province resulted from the use of the reflection seismograph at the sub-marginal surface prospect at the Quealy dome, Albany County, Wyoming.

Actual experience shows, then, that the geophysical discoveries at a premium have, in general, been due to the use of these tactics (characterized by a higher resolving power) in the re-examination of sub-marginal prospects carried over from the earlier campaigns with tactics of a lower resolving power. However, this increase in resolving power was attained only at a marked increase in operating cost. Thus, if the recognized methods of exploration are listed (approximately) in order of increasing cost, they are also listed (approximately) in order of resolving power, as follows:

Library methods[6]
Surface geological methods
Airplane photographs
Core drilling (cost listing probably markedly incorrect)
Magnetometer
Gravity meter
Torsion balance
Refraction seismograph
Reflection seismograph
Drill (including subsurface paleontology, subsurface stratigraphy and electrical logging).

From past experience, then, and for an exploration campaign, rather than a prospecting sortie, empirical rules can be drawn, that for the general case of discovery at a minimum cost:

1. Prospects should be first located by the appropriate method of lowest operating cost. These prospects should be evaluated and culled by the successive use of the appropriate methods of higher cost (successive high-grading).
2. Prospect development in a particular area should not be initiated by any given method until the area has been previously explored by the appropriate methods of lower cost.

Thus, if in a given area there are "N" appropriate methods, A, B, C, D, etc., so arranged in order of increasing cost and resolving power, the sequence is:

[6] Including all the ethical and unethical methods of securing and making use of "other peoples' data."

Prospect Development	*Prospect Evaluation and Culling*	*Final Evaluation by Drill Prior to Exploitation*
A	B C D E F · · ·	N
B	C D E F · · ·	N
C	D E F · · ·	N
D	E F · · ·	N
E	F · · ·	N
F	N	
N (Active wildcat drilling)		

The justification for successive prospect evaluation is that the experience gained thereby will be valuable in the later phase of prospect discovery by the same method. Thus, if, in a given area, the appropriate exploration methods are:

Library methods
Surface methods
Torsion balance
Reflection seismograph
Drill

there may be a strong temptation to skip either or both the surface methods and the torsion balance in prospect evaluation, particularly if experience shows that not all prospects shown by the reflection seismograph are indicated by one or both of these methods. However, even if there existed only incomplete correspondence between these methods (i.e., that the torsion balance, say, did not indicate anomalies at all reflection seismograph structures), the method of lower resolving power should be used in sequence for prospect evaluation, not only because of its lower operating cost, but also because the experience and data secured in the course of prospect evaluation by this method of lower resolving power would be useful in the later stage of prospect development by that same method.

These conclusions suggest that an exploration department should be completely integrated, consisting of the use of geological and geophysical methods under the supervision of one head. Also, proficiency in the effective use of one method of exploration should not be considered a valid reason for neglecting the use of other methods, for the result may be either an inherently greater hazard in exploitation, or an inherently greater cost in discovery. Thus, if an exploration department in the Gulf Coast relied wholly upon geological methods, or, perhaps, was satisfied with torsion balance data alone prior to drilling, then prospect exploitation would be burdened with an unduly great hazard. On the other hand, if these methods, of resolving power less than the reflection seismograph, were neglected in prospect discovery, the cost of prospect discovery would probably be unduly great.

EXPLORATION STATUS OF THE VARIOUS PETROLEUM PRODUCING PROVINCES

It may be of interest to examine several petroleum producing provinces from the standpoint of the appropriate methods of exploration and the present exploration practice.

GULF COAST

There has been intensive use of library methods, surface methods, airplane mapping, the refraction seismograph, torsion balance, and the reflection seismograph. At present, the most effective sequence of tactics seems to be, for a company which has recently initiated an exploration campaign,

1. Library methods
2. Torsion balance
3. Reflection seismograph
4. Drill

There is no doubt, however, that the law of diminishing returns is beginning to operate against the present general reliance upon library methods and the torsion balance for prospect discovery, and that the next and rather imminent phase will be prospect discovery and evaluation by the reflection seismograph. This phase, naturally, will be materially more expensive, and will be required when prospect discovery can no longer be made by any of the lower cost methods. There is no doubt but that the prospects now being discovered by the torsion balance are approaching the sub-marginal stage, and the time will soon come when many companies will find it cheaper to discover prospects by simply filling in the gaps between the areas already covered by the reflection seismograph in previous prospect evaluation. At present, the reflection seismograph is mapping at depths from 8,000 feet to 12,000 feet sub-sea, to keep pace with and to reasonably anticipate drilling practice. There is considerable doubt that gravity methods would be materially revived even if deeper objectives were found, for torsion balance data, once properly taken, hardly justifies repetition, and so belongs properly in the past and in the library.

EAST TEXAS

The area has been intensively explored by surface methods, refraction seismograph, the torsion balance, and the reflection seismograph.[7] At present, even this last method yields only marginal prospects. The discovery of a new objective, such as production in the Trinity Sand series, can result only in a revival of deeper exploration by the reflection seismograph, for the results to date of all methods used in that area should now be listed as future library methods.

CENTRAL OKLAHOMA

The appropriate methods in the past have been surface methods, subsurface methods, and the reflection seismograph. The stage of prospect discovery by the reflection seismograph has been current for several years, and is drawing to a close, with the prospects for discovering new objectives rather poor.

EASTERN OKLAHOMA
WEST VIRGINIA
OHIO-INDIANA
ILLINOIS

In all four of these provinces, the use of surface and subsurface methods is generally assumed to have practically exhausted the possibilities prior to the advent of geophysics. Although there is thought to be little hope of new objectives, still geophysical methods have not been intensively applied.

CALIFORNIA

Most of the existing known structures have, in general, been so highly folded that nearly all possibilities appear to have been exhausted prior to the introduction of geophysics. There exists, at present, an almost hysterical exploration activity, based on the use of the reflection seismograph to discover as well as to evaluate deep-seated prospects, with no great encouragement shown to date. Deeper objectives are being discovered with almost monotonous regularity in the known fields, but these newly recognized horizons may be too deeply buried if future discoveries are not as highly folded as the known fields. Perhaps the discovery of the Bosco oil field in South Louisiana will prove to be the result of good strategy on the part of the Superior Oil Company of California.

[7] At least to the Georgetown Limestone.

PERMIAN BASIN

Surface and subsurface methods, the magnetometer, the torsion balance and the refraction seismograph have been used successfully for prospect discovery in this province. The present stage seems to be the early phase of the discovery as well as the evaluation of prospects by the reflection seismograph. An interesting situation prevails in that seismic reflections are not as readily returned from the Big Lime (the present production objective) as they are from a deeper horizon, probably Pennsylvanian in age.

EDWARDS PLATEAU

This province is one of the most intriguing and yet most baffling existing. Discoveries to date have been due to random wildcatting and the use of surface geology. All other recognized exploration methods have been tried and found wanting. Development in the area is attended by an unusually great hazard because of the absence of any method with satisfactory resolving power which is appropriate for use ahead of the drill.

ROCKY MOUNTAIN PROVINCE

The possibilities of surface methods in the area were practically exhausted several years ago. At present the area is yet in the stage of sub-marginal surface prospect evaluation by the reflection seismograph. There have been trials of geophysical methods with resolving powers lower than that of the reflection seismograph, but with little success except for the torsion balance, which has shown possibilities of locating structure, but little possibility of detailing structure. If the present interest in the producing possibilities of the Sundance horizon persists, the reflection seismograph may yet be used intensively to discover as well as to evaluate prospects.

SOUTH TEXAS

Although surface prospects are still being found, the most promising tactics now seem to be prospect discovery by gravity methods, followed by evaluation with the reflection seismograph. Although there is some use of the reflection seismograph to discover prospects, the attempt is probably premature, and so may prove to be unduly expensive.

SOUTHERN OKLAHOMA

Prospect discovery by library methods, surface methods, and perhaps airplane mapping would seem to be the most favorable tactics, for, except in local areas such as near the Fitts Pool, prospect discovery by the reflection seismograph seems to be a little premature.

MICHIGAN

The only appropriate tactics recognized at present seem to be surface and subsurface methods. Due to the thick cover of glacial drift, the resolving powers of those methods are none too high, so that the hazard of exploitation is very real and very great. There is apparently a great need here for some method with a resolving power greater than the geological methods available. Although to date geophysical methods have not proved very successful, this conclusion seems to be based upon rather half-hearted trials, since recent experiments with the reflection seismograph in the area have shown favorable results.

EASTERN COLORADO

The exploration of the Los Animas Arch is an interesting demonstration of the use of methods of varied cost and resolving power. At the southern part of the area, surface geology is the appropriate method of lowest operating cost. To the north, as the Arch plunges, surface geology does not have sufficient resolving power, and core drilling succeeds it as the appropriate method of lowest operating cost. Still farther to the north, however, neither surface geology nor core drilling has sufficient resolving power, and recourse is had to the reflection seismograph as the appropriate method of lowest operating cost. The magnetometer anomalies present seem to result from compositional rather than structural irregularities in the basement.

NEW YORK–PENNSYLVANIA

The spasmodic exploration being carried out seems to consist primarily of prospect discovery by the reflection seismograph and by the drill.

THE GENERAL SITUATION, PRESENT AND FUTURE

In general, then, the petroleum producing areas so far generally considered to have possibilities for discovery, are in, or are being reduced to, the stage of prospect discovery either by the reflection seismograph or by the drill.

However, there are rules for playing football, but no rules for winning. The rules proposed for the choice and sequence of tactics are obviously subject to modification. As principles, however, they appear to the writer to be self-evident. It is somewhat surprising, therefore, to find them more honored in the breach than in the observance.

The usual sequence for initiating exploration in a given province is as follows. A wildcatter drills a well in a geological area not looked on with general favor. His reasons are usually best known to himself, but are frequently based on a minimum expenditure for leases. When the occasional strike is made (and in the case of prospect discovery by the drill, the odds, small as they may be, still favor the discovery of large rather than small features, as in the case of East Texas and Conroe), the oil fraternity moves in, and the exodus is usually proportional to the size of the strike and to the previously existing unfavorable opinion of the area. The exploration chief of staff is swamped (for the time being, at least) with demands for prospects to lease, explore and drill, and since too frequently the larger strikes are made in areas not in favorable regard, he is then in no position to make immediate and well-considered recommendations. In the absence of good strategy, the chiefs of line take over, and the results are frequently expensive and even disastrous. Very often a blanket exploration campaign is prematurely initiated with the method of highest available resolving power, with the average success and cost of a frontal attack in force upon a strong point. Tactics, at best, can hardly substitute effectively for strategy.

And so what of today? In 1912, the problem was one of discovering reserves of potentially high priced crude. In 1935, with East Texas still a market-factor field, the problem is one of locating reserves of potentially low priced crude, with tactics of much higher operating cost. More than ever, the problem is one of strategy. By virtue of past experience, even if not because of continued success, the position of chief of staff or exploration frequently has been assigned to a tactician, whose early experience was generally gained primarily in walking outcrops or piroguing to gas seeps, and whose interest in these newer tactics is academic, or even hostile. Proficiency in tactics is a necessary but not sufficient prerequisite for proficiency in strategy. Now, the problem is one of discovering and evaluating structures at depths of one to three miles. Surface geology and airplane mapping may occasionally be more than good honest fun, but in these days they are sporting and so hardly war. The higher cost of these new tactics (together with the expected low price of the oil that may be produced as a result) puts a premium upon good strategy, and good strategy requires an excellent working knowledge of the newer tactics. Geology remains one of the ends, but no longer the primary means of discovery. In that recent "Stone Age" of optical exploration, strategy and tactics were practically one and the same. The giants of those days were not called on to ride airplanes, have diurnal worries, eliminate temperature coefficients, and see that dynamite was kept harmless. Although the tacticians of today are shouldering these necessary evils, the successful strategist of today must take them in his stride.

And what of tactics in the future? Well, some modern strategists are bound to break with tradition and, disregarding the areal condemnations of Stone Age, optically-minded strategists, subject some of the featureless Tertiary covered areas, now in disfavor, to examination by modern tactics, and thereby enjoy the thrill of organizing a campaign from the library stage on.

In those areas which are as yet unexplored by any geophysical method, and which so have possibilities of shallow production, perhaps the electrical method may some day actually locate petroleum in situ. As long as marginal prospects can still be discovered by the torsion balance, the gravity meter, the magnetometer and the refraction seismograph, the improvement of these methods is justified. Further, as long as prospects remain to be evaluated or discovered by the reflection seismograph, its improvement is justified. Finally, as long as areas remain unexplored because of problems which cannot be solved even by the newer tactics, careful, cautious research on still newer methods is justified.

And beyond that? Well, with hydrogenation of coal in the pilot plant stage, with the development of oil shale refining in its infancy, and with the decrease in the cost of manufacturing alcohol to be expected from probable advances in chemistry, the pessimistic strategist in exploration may be like some other strategist of the not far distant past, who, envisioning faintly the United States of today, but without anticipating the automobile, wondered and worried about the future distribution of corn and wheat between man, mule, and moonshine.

EXPLOSIVES AND ELECTRIC BLASTING CAPS FOR GEOPHYSICAL PROSPECTING

G. H. LOVING and G. H. SMITH[1]

INTRODUCTORY REMARKS

In the early days of seismograph explorations, the explosives and electric blasting caps were of the same design as those supplied for ordinary types of blasting. During the succeeding years, the instruments and shooting practices employed in geophysical prospecting were constantly improved with the result that the usual products of the explosives manufacturers failed to satisfy the highly specific requirements of this kind of work. It is the purpose of this paper to present the pertinent characteristics of the special types designed for the seismograph trade and to enumerate the more important steps taken during the development program. For the sake of clarity, the data on explosives and electric blasting caps will be given separately.

EXPLOSIVES FOR GEOPHYSICAL PROSPECTING

Before mentioning the properties and physical characteristics of the explosives now used in seismograph work, the factors, influencing the development of the special types brought out for the geophysical trade within the last few years, will be reviewed. It will be understood, of course, that the discussion applies only to exploration work as carried on in the United States.

Several years ago, the refraction method of prospecting was the accepted type. With this procedure, the explosives employed to initiate the seismic wave were stacked on the surface of the ground, sometimes as much as several thousand pounds of powder being used per shot. The geophones for recording the earth vibrations were placed on the arc of a circle several miles distant from the shot point. Since the dynamite was not loaded in holes or under water and, in many instances, not even removed from the boxes, the only prerequisite was that it detonate with the desired velocity. Later modifications included the use of shallow holes, dug by hand and loaded with large diameter cartridges, using water tamping, or well-drilled, water-filled holes into which the explosive was loaded by means of tin tor-

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pedoes. Less powder was consumed in these methods of loading and shooting, but no additional explosive requirements were introduced, except that of suitable water resistance. Subsequently, the reflection method of prospecting, employing deep holes and relatively small charges, has become very popular and, at present, is in almost universal use.

In the reflection system of seismic exploration, the shot holes are generally 3 to 5 inches in diameter and from 25 to 500 feet deep, the usual depth being approximately 75 feet. In following the normal shooting practice, the explosive charge may vary from an electric blasting cap alone to 100 pounds of dynamite, but generally ranges from 1 to 10 pounds. Several charges may be fired in the same hole to obtain check records or to secure data on the strata in different directions from the shot point. The holes are not cleaned out between shots and considerable difficulty is occasionally experienced in loading successive charges, due to sand, muck, or stones obstructing the hole. Water is used as stemming to confine the shots, the depth of water being as much as several hundred feet in very deep holes. Most charges are detonated almost immediately after loading, but occasionally the explosive may be left in the hole for several days before shooting. From this brief description of conditions, it can be seen that this type of shooting requires an altogether different explosive, at least as regards physical characteristics, than does the refraction method.

As stated before, refraction shooting demands only that the explosive detonate at the desired speed and, in certain instances, be of good water resistance. With the reflection method, these two qualities are essential and, in addition, the dynamite must be capable of shooting under fairly high water pressures, and be of such consistency or stiffness that the cartridge will not bulge or buckle when forced down a hole which has become obstructed from a previous shot.

The most popular explosive for this work has been 60 per cent ammonia gelatin. It detonates at the rate of about 4,700 meters or 15,400 feet per second; it is very water resistant; and it is entirely satisfactory under wet conditions which are not too severe. Until quite recently, the 60 per cent ammonia gelatin for the seismograph trade was identical with that furnished the mining, contracting, and other explosives consumers. In other words, it was the standard 60 per cent ammonia gelatin packed by machine into light weight, paper shells. For machine packing, a gelatin dynamite must be of the proper consistency to allow extrusion through a nipple into a shell in much the same manner as sausages are filled. This method of packing has its limitations so far as producing a firm cartridge is concerned, in that, for efficient operation, the gelatin must be relatively soft and plastic. While the standard explosive proved well adapted to the old refraction method, it was not satisfactory for reflection shooting because the cartridges lacked the necessary stiffness and rigidity to withstand the rough treatment used in forcing them down ragged or obstructed holes. To overcome this deficiency, the gelatin was manufactured on the stiffest formula compatible with the extrusion process and was packed into specially designed heavy paper shells which offered more resistance against bulging or buckling under pressure. Even with this special packed explosive, some trouble was experienced, particularly during the hot summer months, due to the inherent softness of a gelatin dynamite. Trouble in loading difficult holes was minimized to some extent by placing the cartridges in tin torpedoes before lowering the charge into the shot holes. While this method was of some benefit, it did not prove to be the complete answer to the loading problem, since, occasionally, the pressure exerted on the tamping pole was sufficient to expand the explosive cartridge, causing the tin torpedo to burst and become lodged at some point midway down the hole. In addition, the cost of these torpedoes increased the explosive expense of seismograph crews.

More recently, another specially designed 60 per cent explosive has been developed for that seismograph trade desiring a powder even harder and firmer than the stiff, heavily wrapped 60 per cent ammonia gelatin referred to before. This explosive differs from the preceding type in that it is hand packed. As a general rule, this type has proved very satisfactory under severe loading and hot weather conditions.

One other type of gelatin has been developed recently for certain shooting conditions encountered in seismograph prospecting. In the Kansas area, it has been found that very small charges, ranging from an electric blasting cap alone to one-half pound of powder, will set up seismic vibrations of the required magnitude for efficient recording. The method of loading usually employed in that district is to mold a small piece, say one-sixteenth of one pound, of gelatin around the detonator and wrap the combination in a piece of paper, which in turn is held in place by means of tape or a rubber band. Obviously, neither of the types of gelatin previously discussed would prove altogether satisfactory for molding purposes. For this reason, another 60 per cent ammonia gelatin characterized by its sticky, rubbery, plastic nature has been developed for this particular type of loading.

The several gelatins just discussed fulfill the qualifications normally required by the reflection method of geophysical prospecting. Occasionally, however, shots are made in very deep water-filled holes, say 200 feet or more in depth. Under such high water pressures, the powders under discussion may not always detonate completely, the visual evidence of this condition being the evolution of brown vapors from the mouth of the hole. In this connection, probably the most satisfactory explosive for shooting under unusual depths of water is 60 per cent Hi-Velocity gelatin, a patented product not made by all manufacturers. This explosive was developed several years ago for submarine blasting and has rapidly become popular for especially severe shooting conditions.

Needless to say, the four types of gelatin which have been described possess almost identical explosive properties in speed and strength. These explosives are packed in any size ranging from $1\frac{1}{4}$ inches$\times$8 inches or one-half pound, to 3 inches$\times$12 inches or 5 pound cartridges, the most popular being the 1, $2\frac{1}{2}$, and 5 pounds cartridges, which are 2 inches$\times$6 inches, 2 inches$\times$16 inches, and 3 inches$\times$12 inches, respectively, in size. Cartridging to a known weight at the explosives plant relieves shooting crews of this responsibility.

ELECTRIC BLASTING CAPS FOR GEOPHYSICAL PROSPECTING

In the usual types of work requiring electric blasting caps, the detonator serves only one purpose, that of initiating the explosive charge in which it is embedded. On this basis, the cap is not required to satisfy any precise limitations as to speed or uniformity of detonation. In seismograph explorations, however, these points are of prime importance and have prompted the development of an electric blasting cap of special design. Before discussing this work, it seems well to give a brief description of the construction of an electric blasting cap and its function in geophysical prospecting.

The component parts of an electric blasting cap are; (1) a metal container or shell, (2) the pressed explosive charge or charges, (3) a firing circuit, surrounded by a material which ignites when current is applied to the cap and thus accomplishes initiation of the pressed charge, and (4) the waterproofing column and sulfur seal. The shell is of such simple construction that no further mention seems necessary. The character of the pressed explosive material may vary widely, however, depending upon the design of the electric blasting cap. In some types, only a single charge is employed, while in others two loads are used, first, a base charge of a high speed, high strength compound, and, second, a superimposed primer whose function is to initiate detonation of the high velocity load. The firing circuit consists of two leg wires, connected within the cap by a very fine wire known as the bridge wire. When the leg wires are connected to a source of electrical current, the bridge wire fuses and causes ignition of the surrounding material. This ignition composition may be either a loose charge, introduced into the shell in the same manner as the pressed leads, or a solid pellet formed around the bridge wire. The waterproofing column is located above the electrical circuit and prevents water from entering the interior of the cap. The sulfur seal fastens the waterproofing compound and leg wires in place.

In establishing the time vs. distance values in seismograph exploration, it is essential that a record be made instantly of the dynamite explosion which initiates the earth vibrations picked up by the geophones. To obtain this record, the seismograph set-up includes a time circuit and suitable means for breaking this circuit simultaneously with the dynamite explosion.

Several methods are employed for producing this time break. In one of them a wire is wrapped around the dynamite charge and is broken by the explosion. This system has one very desirable feature in that the time and electric blasting cap circuits are entirely separate; therefore, the speed of the cap is not a factor in the time values. On the other hand, such a procedure necessitates two sets of wires leading to the dynamite charge. This is disadvantageous to a certain extent, since, with the reflection type of seismograph work now almost universally employed, the relatively deep shot holes are at best none too easy to load and the use of two circuits increases the possibility of broken wires and poor connections.

The second time break method utilizes two electric blasting caps. One detonator is lowered into the shot hole with the dynamite charge and serves only to initiate its detonation. The second electric blasting cap, which is hooked in series with the first cap and is placed on the ground adjacent to the hole, is brought in firm contact with a wire connected in the time circuit, this wire being broken when the shot is fired. This procedure, which has the disadvantage that two caps are used for each shot, requires, for proper performance, that there be little or no difference between the detonation rates of the two electric blasting caps involved.

The third time break system is of fairly recent development and has proved so efficient that it has replaced, to a great extent, the two methods previously mentioned. With this type, only one cap is used, this detonator having two functions, first, the detonation of the dynamite charge and, second, the production of the desired time break. The latter step is accomplished by so arranging the time circuit that the leg and bridge wires of the electric blasting cap form a part of this hook-up. On this basis, the time signal is given when the bridge wire fuses or breaks.

With the single cap set-up just described, the elapsed period, or lag, between the breaking of the bridge wire and the detonation of the main charge of the electric blasting cap, is the important time value. Tests by the companies engaged in geophysical prospecting demonstrated that the lag with standard electric blasting caps was of such magnitude as to render the caps unsatisfactory. Laboratory and field tests were conducted, therefore, on the development of a detonator with the necessary qualifications.

In the laboratory investigation, a cathode-ray oscillograph was employed, this instrument being exceptionally well adapted to the measurement of small intervals of time. The experiments covered numerous designs of electric blasting caps and involved evaluation of the characteristics of different types of bridge wire and explosive compounds. The effect of firing amperage was also determined. This latter phase of the program developed some very interesting information on the relationship between amperage and time lag. For example, it was found that, if a low current, say 0.75 amperes, were used in firing any of the types of electric blasting caps now being manufactured, the severance of the bridge wire and detonation of the main cap charge took place simultaneously. As the amperage was increased, a measurable lag occurred.

It is to be understood, of course, that the above discussion refers to the time lag between bridge wire severance and cap detonation. If the lag is taken as applying to the total firing time of the cap, the picture is changed entirely. On the latter basis the detonation rate of the electric blasting cap was found to become greater as the amperage was increased.

During the oscillograph tests, there was developed a combination of bridge wire and cap charge which gave promise of having desirable characteristics of speed and uniformity of detonation. A slight lag was noted at the higher firing amperages, but this value was so small and so uniform as to be considered negligible in its effect on the accuracy of seismograph work. Detonators of this type were then submitted to field trials under actual conditions of geophysical prospecting. The results obtained, which were in direct confirmation of the laboratory data, demonstrated that the special electric blasting cap would meet the requirements exacted by exploration use.

In closing, it seems well to point out that an electric blasting cap must possess, in addition to proper uniformity of detonation, other essential qualities, namely, excellent water resistance and detonating power. These two features are of primary importance, in view of the deep, water-filled holes employed in reflecting shooting and the previously described special seismograph gelatins which are, by virtue of their necessarily stiff character, less sensitive than normal powders. In regard to the latter point, it is considered that a composition cap, containing a high velocity, high strength base charge, is particularly adapted to geophysical prospecting.

NOTE ON THE THEORY OF SEISMIC PROSPECTING*

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INTRODUCTION

As is well known, the field procedure in refraction prospecting consists of the establishment of shot points, i.e., sources of elastic disturbances, and seismic detector locations. The impulse is started at a shot point by setting off an explosive. The impulse energy spreads in all directions. These impulses travel along various paths to the detector locations. What is recorded is the time required for the impulse to travel from the shot point to each detector location.

The only character of wave transmission that is used is the transmission velocity. Fortunately for prospectors, the velocities usually increase with depth. Again fortunately the velocity of propagation in any given stratum usually does not vary very much as one moves over an area being surveyed.

Although we do not consider anything below the bottom of the first stratum in this paper, still the general method adopted here can be profitably applied to deeper layers.

The theory of refraction prospecting is well worked out for the case where the detectors are on lines through the shot point. According to this theory if one graphs the travel time as ordinate and the distance as abscissa for such a set of detector locations the graph is a polygonal line through the origin. Starting from the origin the graph is straight until at a certain critical distance we meet the first corner. Then the graph proceeds again until at another critical distance we meet another corner. At each corner the slope is diminished in numerical value.

Imagine a single shot point S and, instead of locating detectors on just one line through S, locate detectors on all lines through S. For each direction through S there is a time-distance graph. Select any value of the travel-time, say T_0. Then for each direction through S there is a distant point, S_0, such that the travel-time from S to S_0 is T_0. If we hold S and T_0 constant and vary the direction of the line through S, S_0 will trace a path on the surface of the earth which we

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call an *iso-time* curve because the travel-time from S to each point to this curve is the fixed value, T_0. If one takes several values of the time one gets several *iso-time* curves.

Another type of curve that we study is the curve of critical points. The critical point S_0' is the point at such a distance from S that the first corner appears on the time-distance graph. As the direction through S varies, S_0 traces out a closed curve surrounding S which we call the *critical point curve.*

Our treatment is limited to the case of two strata. One stratum is bounded by two planes the upper of which is the earth's surface. The lower stratum is bounded only by the lower boundary of the upper stratum and extends to great depths below.

The work that follows can be put into analytical form easily but the geometrical constructions bring out the descriptive features more clearly. Besides, the geometrical constructions all correspond directly to a physical picture, whereas the details of analysis do not have counterparts that are helpful in comprehending the situation.

We also show some properties of the critical distance from the shot point where the direct wave and the refracted wave hit the detector at the same time, i.e., where the first corner appears in the refraction profile.

ISO-TIME CURVES

In Fig. 1 S is the shot point on the plane surface of the earth II'. R_1 and R_2 are critical points, in the cross-section shown, where critical refractions begin in the plane interface RR'. Let V_1 and V_2 be the velocities in the two sections. Then as is well known V_1/V_2 is the sine of the angle between R_1S and the normal to RR'. Evidently SR_1R_2 is the section of a cone with a vertex at S and axis SS', where S' is the image of S in RR', and with a circular base in the refracting plane. The impulse travels, according to our present interpretation, from S down to R_1 and then along the interface and up from each point of the refracting interface to the right of R_1. The refracted rays travelling up are parallel in our cross-section. These rays and their normals are indicated as dotted lines. The normals to the rays are the wave fronts of the disturbance.

A wave front is an iso-time surface, i.e., two detectors at any two points on the same wave front would detect the disturbance at the same time. The refracted returning wave front is a part of the surface of a cone. The intersection of this cone surface with the plane surface of the earth forms an iso-time curve which is a conic section, usually an ellipse. The wave front surface is not all conical but is partly a sector of a sphere. The spherical part corresponds to the wave front of the returning reflected wave incident on the interface at a smaller angle than α. The spherical part of the surface fits on to the conical part the way a sphere would rest in a cone with vertex down. Evidently this surface, partly spherical and partly conical may intersect the surface of the earth in a curve that is partly an

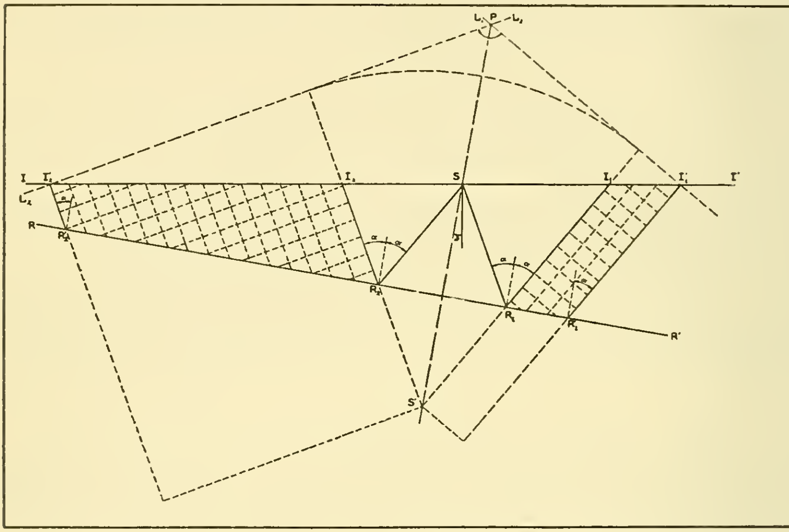


FIG. 1

ellipse and partly a circle. In fact there is a curve, usually an ellipse, which is the curve where the cone whose vertex is S', whose axis is $S'S$, and whose vertex angle is 2α intersects the surface of the earth. Inside of this curve the iso-time lines are circles and outside the iso-time lines are conic sections.

Let β be the vertex angle of the cone of wave fronts. Then $\beta/2 = 90-\alpha$. Hence $\cos \beta/2 = \sin \alpha = V_1/V_2$. Evidently then the shape of the cone, i.e., β, depends only on V_1/V_2 and not on the dip angle ϑ.

Observe what happens when the dip angle ϑ increases. First when $\vartheta = 0$ the axis of the cone is vertical and the iso-time curves are circles. As ϑ increases from 0 to α the conic sections which are ellipses have greater and greater eccentricities. When $\vartheta = \alpha$ the iso-time curves

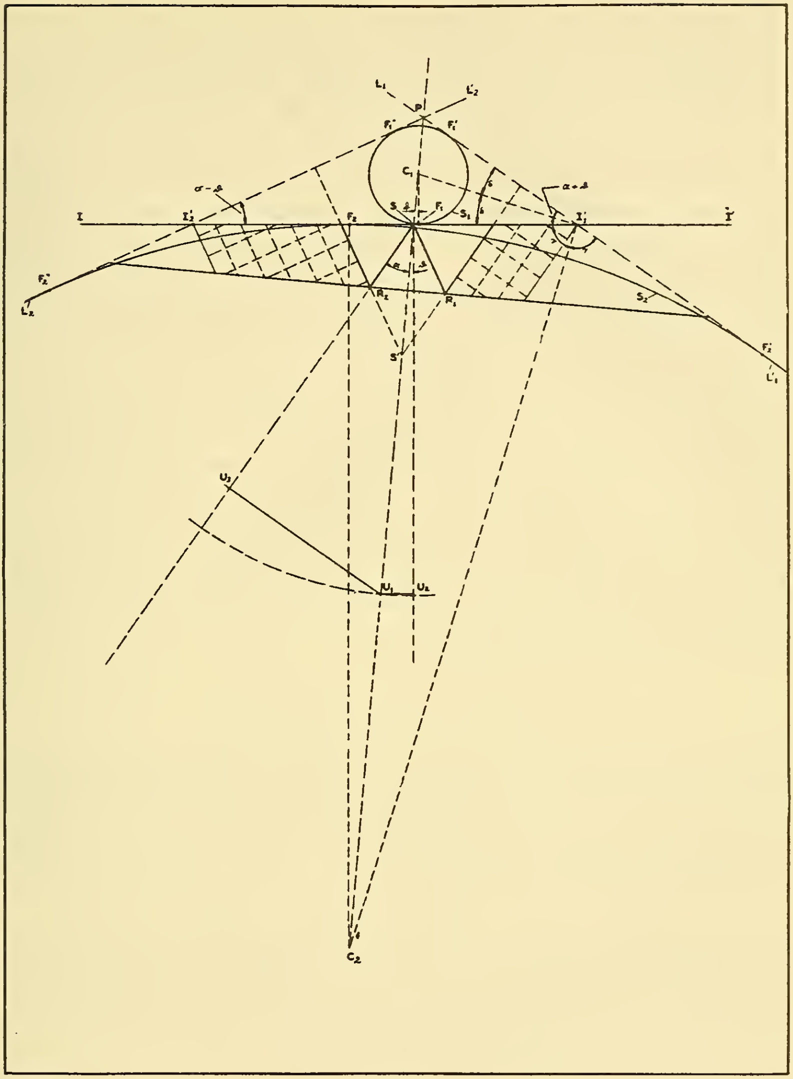


FIG. 2

are parabolas. When ϑ is greater than α the iso-time curves have two branches and are hyperbolas.

As is well known (see Osgood and Graustein's' *Plane and Solid Analytical Geometry*, 1927, p. 144) the foci can be constructed by constructing the two greatest spheres tangent to the cone and to the plane (surface of the earth) on both sides of this plane. The two points of contact of the spheres with the plane are the foci. The case where ϑ is greater than or equal to α need not be considered although the treatment is much the same as for the case $\vartheta < \alpha$.

In Fig. 2 we show the construction of the foci. The cross-section there shown is perpendicular to the strike of the refracting interface. S_1 and S_2 are the two spheres. Evidently their centers are on the axis of the cone. The center C_1 of S_1 is also on the line bisecting the angle $SI_1'P$. The center C_2 of S_2 is on the line bisecting the angle $L_1'I_1'S$. Hence F_1 is the foot of the perpendicular dropped from C_1 to II' and F_2 is the foot of the perpendicular dropped from C_2 to II' As the conical wave front proceeds (with the normal velocity V_1) these spheres expand and the foci move outwards but the ellipses all have the same eccentricity. The semi-minor axis of the ellipse is b. Then $b=\sqrt{(\overline{I_1'I_2'}/2)^2-(\overline{F_1F_2}/2)^2}$. The eccentricity is $e=\overline{F_1F_2}/\overline{I_1'I_2'}$. The points I_1' and I_2' move outwards with the velocities $V_1/\sin(\alpha+\vartheta)$ and $V_1/\sin(\alpha-\vartheta)$, respectively. The distance from S to the center of the ellipse is $\overline{I_2'S}-\overline{I_1'I_2'}/2$ which is $\overline{I_2'S}-\overline{I_1'S}/2-\overline{I_2'S}/2=(\overline{I_2'S}-\overline{I_1'S})/2$ So the velocity of the center is

$$v=\frac{1}{2}\left(\frac{v_1}{\sin(\alpha-\vartheta)}-\frac{v_1}{\sin(\alpha+\vartheta)}\right)$$

which is a constant. The foci also move with constant velocities. The distances of the foci, the center, and the vertices I_1' and I_2' from S can readily be computed using the constancy of the velocities. In fact the distance of the center from S is

$$D_c(T)=v(T-k)$$

where k is such that when $T=k$, $D_c(T)=0$. k can readily be computed. When $D_c(T)=0$ the vertex of the cone coincides with S. Fig. 3 shows the diagram with which k is computed. One can see directly that

$$\begin{aligned} k&=\frac{H}{V_1\cos\alpha}(1+\cos 2\alpha)\\ &=\frac{H}{V_1\cos\alpha}(1+\cos^2\alpha-\sin^2\alpha)\\ &=\frac{2H}{V_1}\cos\alpha. \end{aligned}$$

Without going through all the steps in the simple deductions we may write down the following formulae for the positions of the center, foci, and vertices of the iso-time ellipses. S is the origin. Distances to the right of S are positive and to the left are negative.

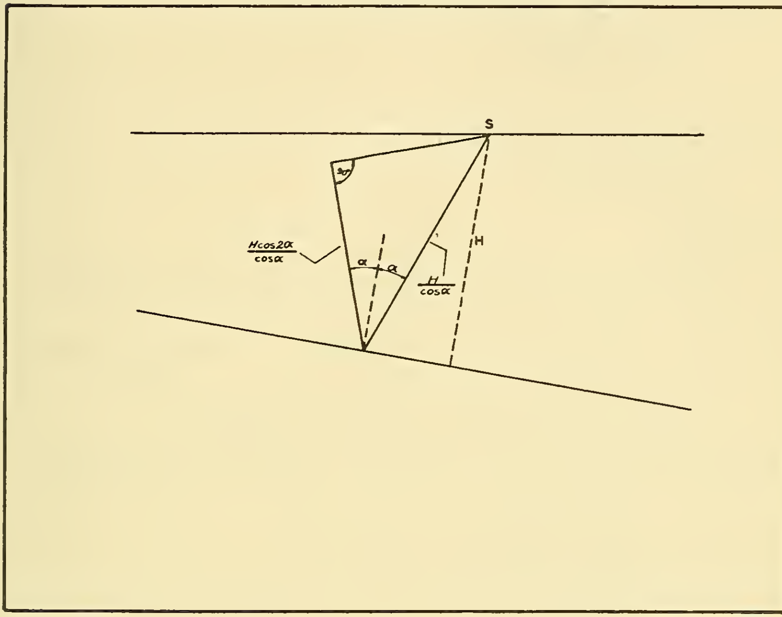


FIG. 3

Distance from S to center

$$= D_c(T) = \frac{-V_1}{2}\left(\frac{1}{\sin(\alpha-\vartheta)} - \frac{1}{\sin(\alpha+\vartheta)}\right)\left(T - \frac{2H}{V_1}\cos\alpha\right)$$

$$= \frac{-V_1 T \cos\alpha \sin\vartheta}{\sin^2\alpha - \sin^2\vartheta} + \frac{H \cos^2\alpha \sin\vartheta}{\sin^2\alpha - \sin^2\vartheta}$$

$$= \frac{-\cos\alpha \sin\vartheta}{\sin^2\alpha - \sin^2\vartheta}\,(V_1 T - H\cos\alpha).$$

Distance from S to right vertex

$$= D_1(T) = \frac{V_1}{\sin(\alpha+\vartheta)}\left(T - \frac{2H\cos\alpha}{V_1}\right).$$

Distance from S to left vertex

$$=D_2(T)=-\frac{V_1}{\sin(\alpha-\vartheta)}\left(T-\frac{2H\cos\alpha}{V_1}\right).$$

Distance between center and vertex

$$=\frac{D_1(T)-D_2(T)}{2}=A(T).$$

Distance from center to focus

$$=eA(T)=C(T).$$

In Figure 2,

$$\overline{F_1'I_1'}=\overline{F_1I_1'} \text{ and } \overline{F_2'I_1'}=\overline{F_2I_1'}. \text{ But } \overline{F_2I_1'}-\overline{F_1I_1'}=2C(T).$$

So $(\overline{F_2'I_1'}-\overline{F_1'I_1'})/2=C(T)$. Also $(\overline{F_2'I_1'}+\overline{F_1'I_1'})/2=A(T)$. Hence

$$e=(\overline{F_2'I_1'}-\overline{F_1'I_1'})/(\overline{F_2'I_1'}+\overline{F_1'I_1'})=\sin\vartheta/\sin\alpha.$$

We may thus write down the equation of the ellipses in the form

$$\frac{\left[x(T)+\dfrac{\cos\alpha\sin\vartheta}{\sin^2\alpha-\sin^2\vartheta}(V_1T-H\cos\alpha)\right]^2}{(V_1T-2H\cos\alpha)^2\left(\dfrac{1}{\sin(\alpha+\vartheta)}+\dfrac{1}{\sin(\alpha-\vartheta)}\right)^2}$$
$$+\frac{[Y(T)]^2}{(V_1T-2H\cos\alpha)^2\left(\dfrac{1}{\sin(\alpha+\vartheta)}+\dfrac{1}{\sin(\alpha-\vartheta)}\right)^2\left(1-\dfrac{\sin^2\vartheta}{\sin^2\alpha}\right)}=1$$

which reduces to

$$\frac{((\sin^2\alpha-\sin^2\vartheta)X(T)+\cos\alpha\sin\vartheta(V_1T-H\cos\alpha))^2}{\sin^2\alpha}$$
$$+(\sin^2\alpha-\sin^2\vartheta)(Y(T))^2=4\cos^2\vartheta(V_1T-2H\cos\alpha)^2.$$

Perhaps the best way to picture what is happening is to imagine the cone with a fixed axis moving up with a constant velocity so that the velocity of the vertex moving along this axis is $v_1/\cos\alpha$. Then the sections of this cone with the plane surface of the earth expand with increasing time in a uniform manner. This picture is a good one to keep in mind as it involves merely the propagation of the wave front.

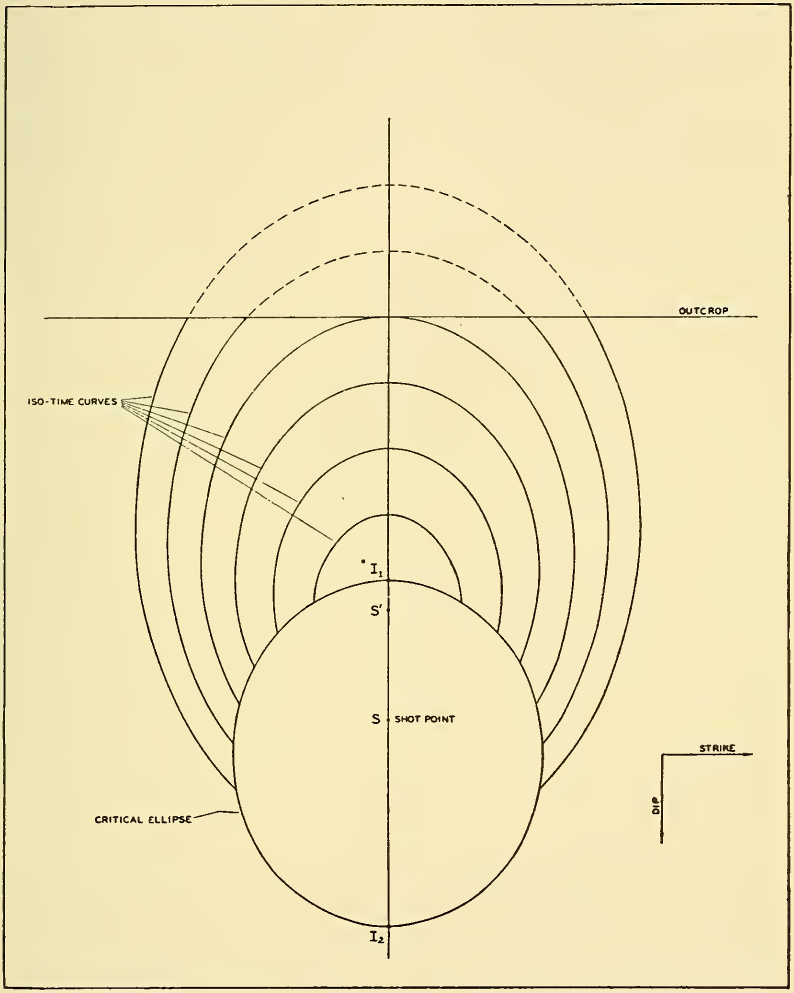


FIG. 4

In Fig. 4 we have shown the conic section K inside of which the iso-time curves are circles and outside of which the iso-time curves are conic sections for the case $\alpha = 30°$, $\vartheta = 20°$, and $H = 1000$ feet.

Critical Point Curves: As is well known, for points very near to the shot point the impulses that come over direct paths come sooner than the refracted impulses. As we go out farther and farther from the shot point the travel times for the direct impulse and the refracted impulse come closer together until at some point, Q, they coincide, i.e., Q is a point such that the direct impulse and the refracted impulse from S arrive simultaneously. As we vary the direction out from the shot point, Q traces out the locus that we are concerned with here.

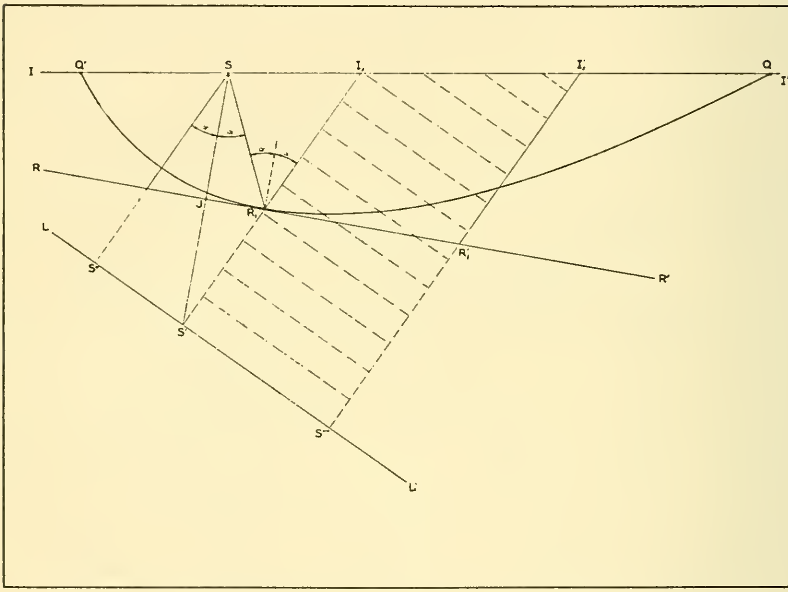


FIG. 5

As in the preceding section our construction is here geometrical. Fig. 5 shows a section of the construction. In Fig. 5 the line LL' is parallel to the refracted wave fronts. Each refracted ray reaches the surface II' at the time it would have reached that same point I_1' had it started from the foot of the perpendicular to LL' drawn from this point I_1', i.e., $\overline{SR_1}/v_1+\overline{R_1R_1'}/v_2+\overline{R_1'I_1'}/v_1=\overline{S'''I_1'}/v_1$. So we have reduced the problem to the problem of finding the point Q on II' such that the distance $\overline{SQ}$ is equal to the distance from Q to LL'. In the plan of the figure (Fig. 5) we have drawn the parabola QR_1Q' with focus at S and directrix LL'.

For other directions than the one considered above, the directrices all pass through the point S_1 and they all make an angle $90+\alpha$ with $S'S$. Hence the distance SS'' remains fixed. Hence the parabolas are all of the same size. Furthermore, all of the parabolas pass through points of critical reflection like R_1. In fact the surface of revolution, formed by rotating the parabola, not about its own axis (unless $\vartheta=0$), but about SS', is a surface of critical points. Where this surface intersects the surface of the earth is the curve of critical points that we wished to construct.

It might be observed that this surface of revolution has an inner part (corresponding to Q') and an outer part (corresponding to Q) and that for us the only significant part is the outer part.

THE DISCOVERY BY REFLECTION SEISMOGRAPH OF A SMALL PRODUCING STRUCTURE IN OKMULGEE COUNTY, OKLAHOMA

G. H. WESTBY[1]

INTRODUCTION

Most of the reflection seismic work in Oklahoma which has resulted in the discovery of oil fields has been done in areas of structural relief of more than fifty feet of closure. It is well known that in older producing areas of the state, such as Okmulgee County, many prolific small oil pools occur which have structural relief of less than this. To locate such pools satisfactorily necessitates a refinement of method and technique which will reduce the normal seismic error to its minimum. In addition, extreme detail is required to delineate these small sharply dipping structures. It is the purpose of this paper to present from inception to completion a program of subsurface and reflection seismic study which has resulted in the discovery of a typical small oil field of this district.

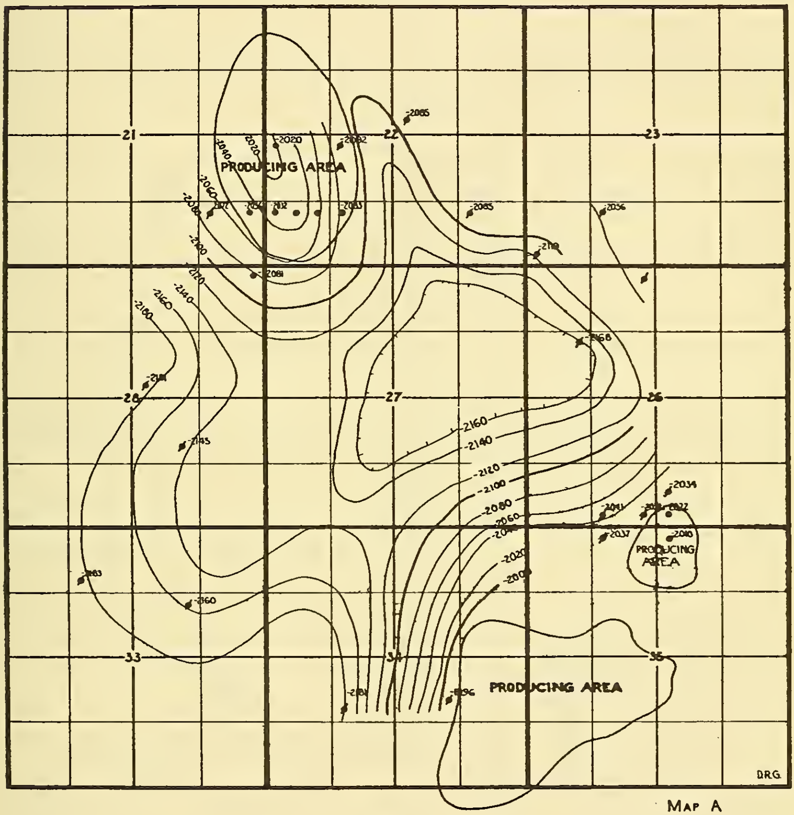
HISTORY OF DEVELOPMENT AND SUMMARY

J. L. Gartner of Tulsa, from a subsurface study of T. 15 N., R. 11 E. concluded that a structure must exist near the common boundary of Secs. 27 and 28. A reproduction of his subsurface map on the Viola limestone which occurs immediately above the Wilcox sand is shown as Map A in Fig. 1. Gartner and Max B. Andreae acquired leases in this area and approached the Seismograph Service Corporation early in 1933 to shoot this area and accept a part interest in the leases as part payment for the seismograph work. Seismic work was undertaken on this basis. This work was not definite but did tend to show that a point one-fourth mile north of the southeast corner of Sec. 28 was considerably higher than the well in the SE. NW. SE. of Sec. 28. In August, 1934, this work was repeated under the direction of the writer with the result shown by the reflection seismic map B in Fig. 2. This work appeared entirely satisfactory and confirmed and clarified the earlier work. The map indicated a structure of the type common in this area with suitable closure to produce. Since Seismograph Service Corporation had interest in these leases, Gartner and

[1] Seismograph Service Corporation, Tulsa, Oklahoma.

Andreae requested them to take over the drilling of this well. The Lease Investment Company was formed to acquire this interest and drill a well. The well was located in the center of the NW. SW. SW. of Sec. 27, T. 15 N., R. 11 E. It found the deeper Pennsylvanian

SUBSURFACE MAP ON VIOLA DATUM
T15N R11E
OKMULGEE CO. OKLA.



MAP A

FIG. 1

formations high and was completed as a commercial oil and gas well in the Taneha sand of Pennsylvanian age at a depth of 2,055 feet. A second well was located in the SW. SW. SW. Sec. 27 to go to the Wilcox sand. This well was completed in the Wilcox sand for forty barrels per day but the sand was considerably lower than was antici-

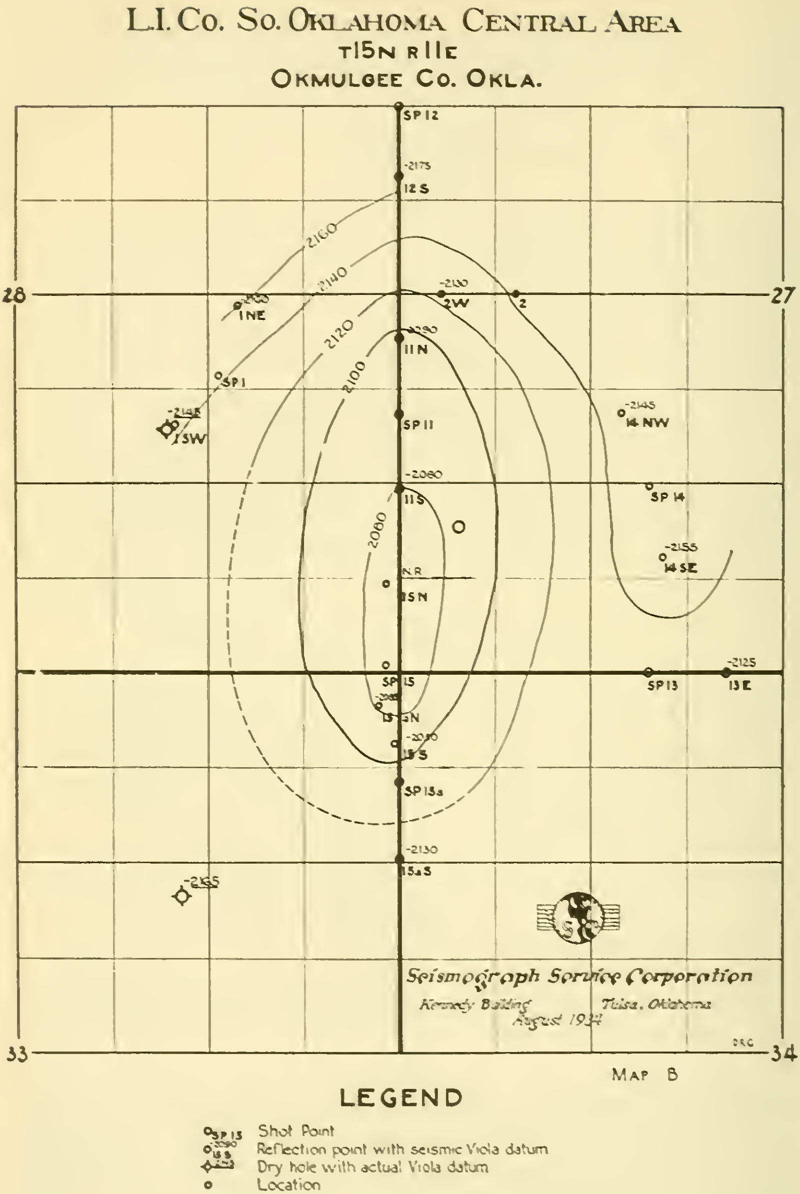

L.I. Co. So. Oklahoma Central Area
T15N R11E
Okmulgee Co. Okla.
SP 12
-2175
12 S
2160
2140
2120
2100
2080
28
27
-2100
2W
2
1NE
-2090
11 N
SP 1
-2145
14 NW
-2145
15W
SP 11
-2060
11 S
SP 14
-2155
14 SE
N.R.
15 N
SP 15
SP 13
-2125
13 E
-2090
15 S
SP 15a
-2130
15a S
-2165
Seismograph Service Corporation
Kennedy Building Tulsa, Oklahoma
August 1934
33
34
Map B
LEGEND
Shot Point
Reflection point with seismic Viola datum
Dry hole with actual Viola datum
Location

Fig. 2

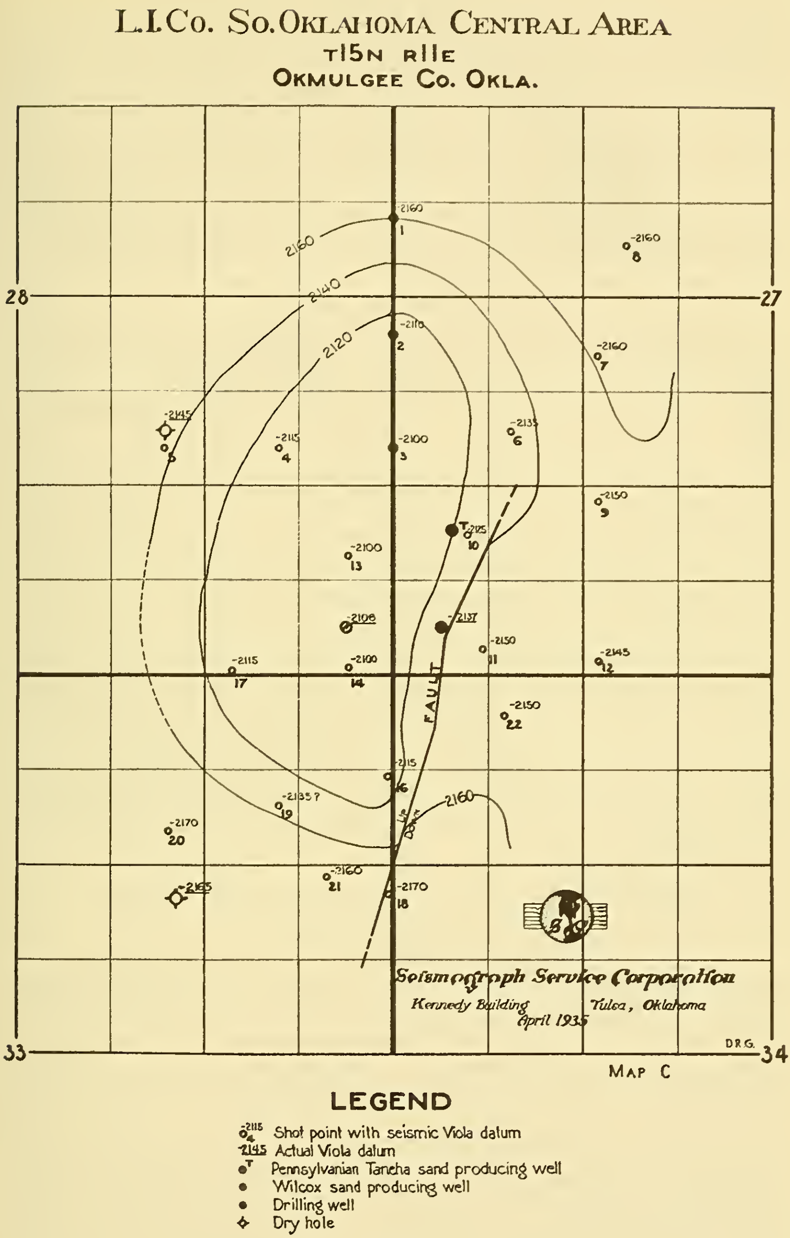

L.I.Co. So.Oklahoma Central Area
T15N R11E
Okmulgee Co. Okla.
28
27
33
34
2160
2140
2120
FAULT
Seismograph Service Corporation
Kennedy Building
Tulsa, Oklahoma
April 1935
DRG.
Map C
LEGEND
Shot point with seismic Viola datum
Actual Viola datum
Pennsylvanian Taneha sand producing well
Wilcox sand producing well
Drilling well
Dry hole

FIG. 3

pated from the seismograph work. The reflection results were rechecked and found to be satisfactory. It was concluded that the geological assumption of uniform dip between seismic datum points was in error. Further seismic work confirmed this belief and resulted in the map C shown in Fig. 3. A well was located in the SE. SE. SE. Sec. 28 and was drilled to the Wilcox. The seismic work was checked perfectly and as this is being written the well has just commenced to make a considerable amount of gas and oil from the Wilcox sand.

TOPOGRAPHY AND GEOLOGY

The topographic relief in the area shot is about 50 feet or about equal to the structural closure. The creek shown on map D in Fig. 4 has a flood plain averaging one-third mile in width over the prospect.

The Checkerboard limestone outcrops to the northeast and west of the prospect. It was found also in a few shot holes. Its position in these shallow holes indicates a nosing over the north end of the prospect. Out of the alluvium filled flood plain Pennsylvanian shales are found beneath a thin mantle of soil.

The Glenn sand, here water bearing, lies at about 1,800 feet. The Taneha sand containing oil and gas occurs at a depth of 2,050 feet. The Wilcox sand is the chief source of production in this area. Its average yield is about 5,000 barrels to the acre but on good structures this average has exceeded 30,000 barrels per acre. A partial section of the lower formations is shown in Fig. 5. Over the top of the structure thinning in the Mississippi lime permits accentuation of structure in the Wilcox sand.

EQUIPMENT AND METHODS USED IN SEISMIC WORK SHOWN ON MAP B IN FIG. 2

A standard Seismograph Service Corporation crew was employed. Six electro-magnetic pickups were used, each connected through a three stage amplifier. Recording was accomplished by a moving coil oscillograph. Timing lines of one one-hundredth second were placed on the record by means of a tuned light shutter controlled by a tuning fork. Telephone communication by means of a line from shot point to recorder transmitted the shot moment and instructions from the observer to the shooter. An example of records taken with this equipment is shown in Fig. 6.

Shot holes were dug to the base of the alluvium or to a similar

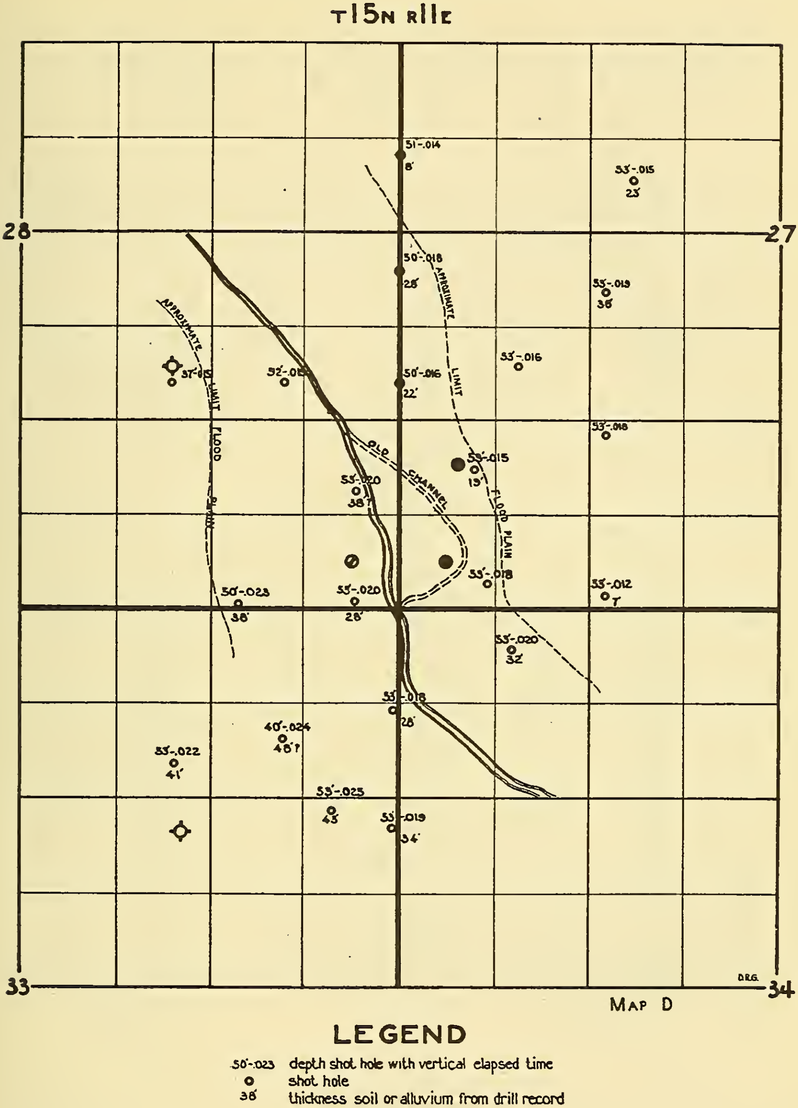


Fig. 4

depth in the bed rock and terminated in shale. The six pickups were placed in a line away from the shot point at a distance of from 800 to 1,300 feet. The pickups in this case were buried to a depth of 2 feet.

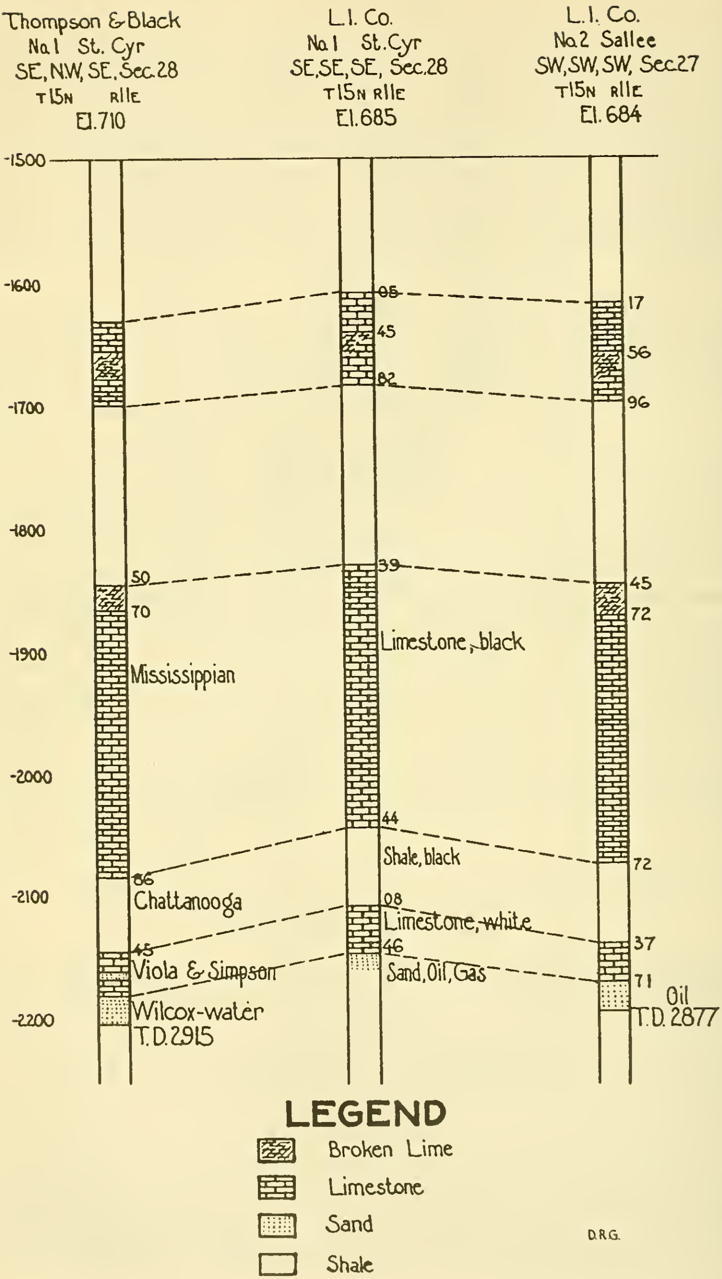

Thompson & Black
No. 1 St. Cyr
SE, NW, SE, Sec. 28
T15N R11E
El. 710
L. I. Co.
No. 1 St. Cyr
SE, SE, SE, Sec. 28
T15N R11E
El. 685
L. I. Co.
No. 2 Sallee
SW, SW, SW, Sec. 27
T15N R11E
El. 684
-1500
-1600
-1700
-1800
-1900
-2000
-2100
-2200
05
45
82
17
56
96
50
70
39
45
72
Limestone, black
Mississippian
44
Shale, black
72
86
Chattanooga
08
Limestone, white
37
45
46
Viola & Simpson
Sand, Oil, Gas
71
Wilcox-water
T.D. 2915
Oil
T.D. 2877
LEGEND
Broken Lime
Limestone
Sand
Shale
D.R.G.

Fig. 5

Where possible each shot hole was utilized for shooting in two directions. Shots for the determination of the surface correction zone were placed at a distance of one hundred feet on either side of the geophone spread. Simplified refraction formulae were used in calculation of the

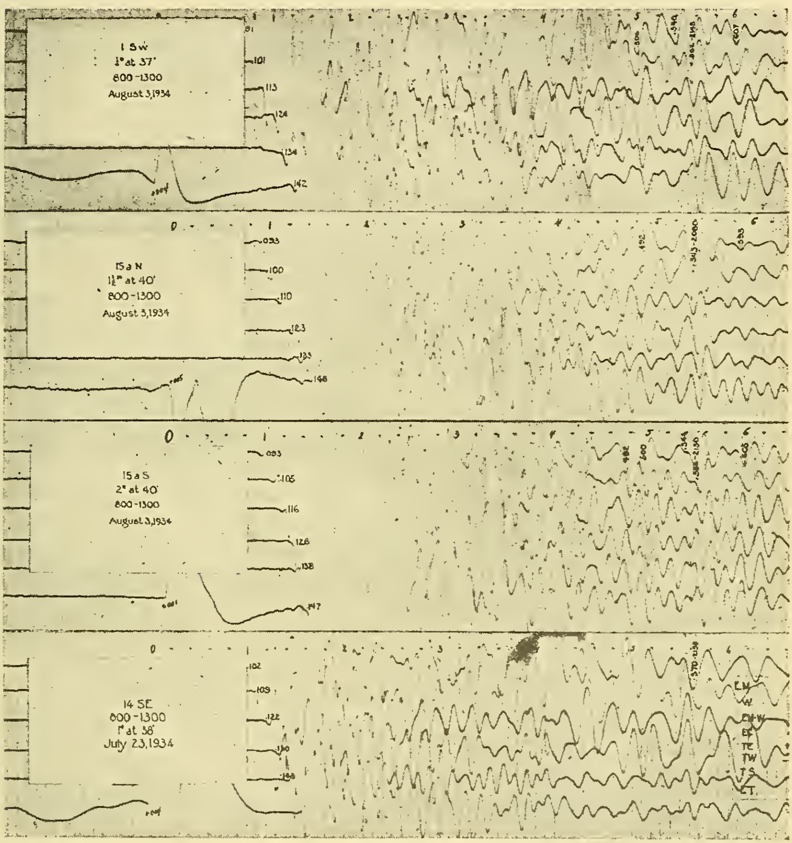


FIG. 6

thickness and time occupied in this zone. These correction shots were buried at a depth of about 10 feet in the alluvium and 5 feet in the Pennsylvanian shale. This procedure as outlined is more or less standard correlation shooting technique as was practiced in Oklahoma during 1933 and 1934. All calculations and maps were made in time originally but were translated into feet for this paper.

The correlation of reflections with known formations and calibration of the velocity was made by shooting across the well in the SE. NW. SE. of Sec. 28. A partial graphic log of this well is shown in Fig. 5. The four most important records obtained in this original shooting are shown in Fig. 6. No. 1–SW. was shot across the described well. After study of the possible velocities in the area, it was determined that the reflection whose trough is at .506 was from a member of the Dutcher limestone, and that the reflection whose trough is at .562 was from either the base of the Mississippi lime or the Viola limestone. In either case, structure would be satisfactorily portrayed, since the shortening over structure is usually at the top of the Mississippi lime and there is little change in the thickness of the Chattanooga shale. The reflection whose trough is at .607 was assumed to be from the Arbuckle limestone.

The record 15–A–N, and its correlation with 1–SW. which appears quite clear, indicated with considerable definiteness the presence of the structure. The steep dip off to the south, as evidenced by correlation with record 15–A–S., which shows a drop of 50 feet in about 1,100 feet, and the dip to the east shown by correlation with record 14–SE. are typical of these small structures. Even without corrections, the arrival times of the reflections show the presence of structure.

Since well control was available, both to the southeast and the southwest, indicating low areas (see Fig. 1), it was thought unnecessary to do more work in these directions. From the control obtained one must contour this small structure with the north-south section line as an axis of symmetry.

Since this structure appeared more or less typical and seemed to fit in very well with the subsurface data drawn by Gartner, it was thought to be adequate evidence on which to drill a well. Although there are twelve reflection points determined in about 320 acres, this control is now recognized to be what might be called a detailed reconnaissance for this area.

It must be remembered also that this type of shooting is subject to all the errors incurred in reflection seismic work, which have been discussed by the writer,[1] and is much inferior to the more recent method of shooting this type of structure which has been developed. Nevertheless, the accuracy was deemed entirely satisfactory for this particular area. Accordingly, a test well, intended to go to the Wilcox

[1] E. McDermott, Application of Seismography to Geological Problems, *A.A.P.G Bull.*, 15 (1931) pp. 1311–1334.

sand, was located in the NW. SW. SW. of Sec. 27. This location was chosen because the structure was symmetrical about the road as an axis and it was thought wise to locate on the far side of the axis from the dry hole in Sec. 28. The Lease Investment Co. well found the Taneha sand considerably elevated and with commercial amounts of gas and oil, and was completed in this sand. The structural attitude of the Pennsylvanian beds and their commercial oil and gas content seemed to verify the seismic work, though no reflections as shallow as the Taneha were worked, and a south offset location was made in the SW. SW. SW. of Sec. 27. This well found the Pennsylvanian beds 10 feet lower than in the first well, and with some oil, but not enough for a commercial well. It was continued on to the Wilcox and was completed in this sand for a 40 barrel pumping well. Unfortunately, however, the Viola datum on this well was −2137, which was considerably lower than had been anticipated. The seismograph data were rechecked and found to be entirely satisfactory and convincing, as may be seen from the records shown in Fig. 6.

Since an oil well in the Wilcox in this area is usually indicative of a suitable structure with closure, there was good evidence that the geological assumptions from seismic work were in error. In order to determine the position of the structure and the reason for the apparent inaccuracy of the first map, it was decided to carry out a detailed seismic survey of this limited area, employing every means for increasing our accuracy of depth determination.

METHOD ADOPTED FOR DETAILED SEISMIC SURVEY

In the second approach to the problem, two factors to increase accuracy were stressed. First, it was necessary to eliminate all possible small errors in determination of time, such as time break, cap lag or surface correction zone determination; and second to survey continuously so that small faults or areas of sharp dip were not missed. The disadvantage of the former method in determining a depth point lay in the necessity of determining such a depth point with only one set of conditions in the surface correction zone. To increase the accuracy of each depth point, it was deemed advisable to secure several measurements at or near one point, each determined with different surface correction zone conditions. The method adopted was a refinement of a continuous profile scheme used in other detailed field problems. Shot points were located 800 feet apart. These holes were drilled into the Pennsylvanian shale to a depth of about 50 feet. In the alluvium filled valley the alluvium was penetrated and the shale entered for a short distance. Seven pickups were used, as shown in Fig. 7, and this arrangement was left in place while shots were made from both shot points. Shooters took special care to get the shots exactly on the bottom of the hole. Vertical time was measured at the shot hole but was not used in the calculation. This method carries the correlation mechanically and was used here to ascertain the presence of small faults or steep dips which might exist over very short distances. It was entirely successful in both respects.

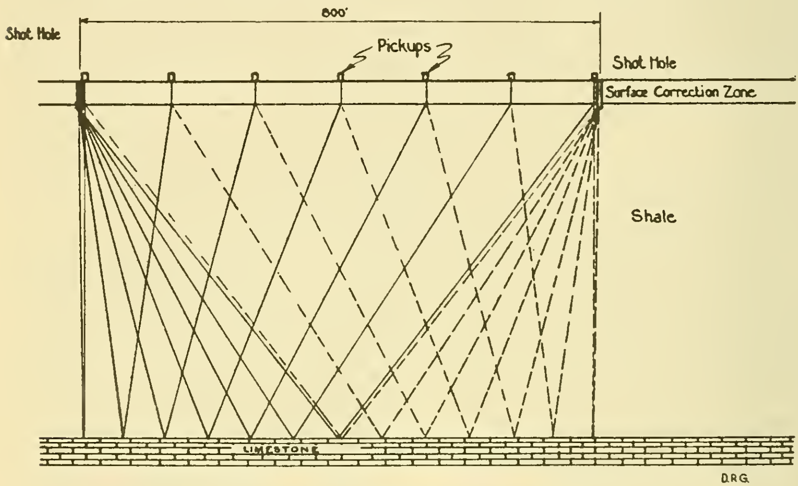


FIG. 7

Data for the surface correction zone were determined from the arrival time of the initial energy on the reflection record. At each shot point the first two traces on the record were used in each direction to obtain an average determination for the depth below this shot point. This scheme gave depth points which were all within a circle of 133 feet in radius about the shot point. These were averaged for the depth calculation beneath the shot point. In shot point No. 3, shown on map C in Fig. 3, shooting was completed in five different directions interlocking with records shot from shot points 2, 4, 6, 10, and 13. Not less than three records were shot in each of these five directions from shot point 3, so that the resultant average from shot point 3 was determined from more than fifteen records, on each of which two traces were used. At this shot point the maximum variation between determinations of the depth in the five different directions was about 22 feet or .004 second, which represents a variation of .002 second or about 10 feet from the average.

SURFACE CORRECTION ZONE

In this area the time in the surface correction zone of low velocity varies from a minimum of .010 second on the north and east to a maximum of .025 second on the southwest. This variation is illustrated by the vertical elapsed time at the shot hole, as shown on map D in Fig. 4, which shows a vertical time of only .012 second for a hole depth of 53 feet at shot point 12 to a vertical time of .025 second for the same hole depth at shot point 21. It is noteworthy that the variation in surface correction zone time is more than the corrected time relief of the structure.

On the map showing the vertical time in the shot holes, a figure has been added showing from the driller's logs the thickness of the alluvium or soil cover, or the depth to bed rock. Except in one or two instances, a close correlation may be seen between the increase in time and the increase in thickness of the alluvium. The low velocity shown in the shot holes 19, 20, and 21 is entirely due to a thickening of the alluvium. It is worthy of note that shot point 11 shot east has a surface correction zone time of .010 second for the average of the first two traces, while shot west, it has an average of .019 second. With the corrections applied, the determinations for the depth of the Viola limestone at shot point 11 differ only .002 second from the west side to the east side. This indicates quite clearly that an adequate method has been used to correct for these abrupt variations in the alluvium thickness. The flood plain of the small creek has its boundary about 100 feet east of shot point 11.

DISCOVERY OF STEEP DIP AND FAULT

A small fault was discovered between shot points 10 and 11, and was again crossed between 11 and 14, 16 and 22, and 16 and 18. Fig. 8 shows the record obtained showing a possible fault with small throw lying between shot points 10 and 11. While the amount of displacement on the fault is so small that it might have been questioned as due to effect of variation in the surface correction zone, it was picked up so definitely on the mentioned records that it is believed present.

The close approach of the syncline on the east to the high point at the center of the structure indicates also that an anomalous condition exists which is the result of this fault. It was impossible, due to

FIG. 8

bad surface conditions and the presence of a creek branch near the well, to shoot exactly at this point below the well. For this reason, we are not entirely certain whether the well is on the up throw or the down throw side of this small fault, but suspect from a calculation of each trace of the records shot between 11 and 14 which shows a steep east dip east of 14, that the well is on the up throw side.

COMPARISON OF ORIGINAL WORK WITH DETAILED WORK

An examination of the two maps reveals that in most places where seismograph points have been duplicated, the two methods have yielded almost identical results. This is not true, however, on the road near the axis of the structure where the original work shows points about 20 feet higher or slightly more than .003 second difference in corrected time. Since we are now comparing absolute depths, we are depending upon a good calibration shot across the dry hole as well as satisfactory work along the axis of the structure. With the correlation type shooting, only one determination under one single weathering condition is obtainable at every reflection point. With the detailed method, several reflection points are averaged to determine the datum at the shot point, as described above. In short, a comparison of the maps indicates that the original work was entirely satisfactory work of its type, but was not spaced closely enough to secure adequate data on this area of small structure and steep dips.

IDENTIFICATION OF THE REFLECTION

When the well, St. Cyr No. 1, reached the Viola formation, velocity tests were made by lowering the detector into the well and shooting on the surface. The total depth of the hole at this time was 2,801 feet, or a distance of 8 feet in the Viola limestone. Velocity tests with the shot point at a distance of 250 feet from the well were entirely unsatisfactory, due to energy traveling directly to the casing and creating a disturbance on the geophone which obviated any possibility of getting satisfactory velocity information. Accordingly, two new shot points were chosen, one 500 feet southwest of the well and one 500 feet northeast. Unfortunately, these were drilled only to a depth of 30 feet, but good velocity records were obtained nevertheless (see Fig. 9). This velocity information shows the average overall velocity to the Viola limestone to be approximately 10,600 feet per second. The reflection on the record 1–SW. commencing at .548 second, must therefore be identified as being from the base of the Mississippi limestone or its contact with the Chattanooga shale. A velocity survey made six miles to the east of this well reveals similar data. Since our velocity information is never accurate for the identification of a reflection closer than ±20 feet, it would be entirely possible for this reflection to come from a harder bed of limestone in the base of the Mississippi lime. However, the writer was present at the drilling of the Mississippi lime in both wells in this vicinity and found no evidence either in the samples or in the drilling to warrant the belief that

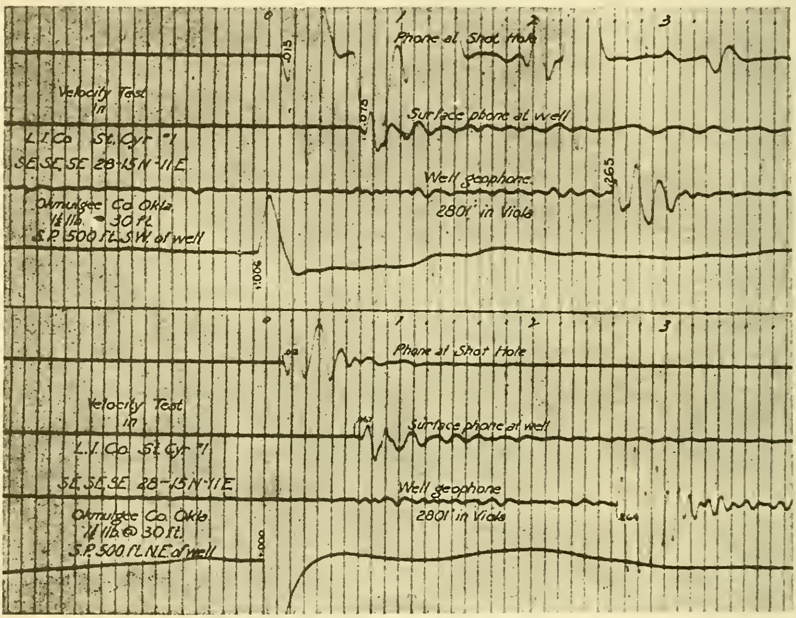


Fig. 9

a reflection could be coming from a harder limestone stratum in the base of the Mississippi lime section. It is believed probable, therefore, that this reflection is coming from the contact of the Mississippi lime and the Chattanooga shale which theoretically should afford an especially good reflecting interface. The Viola limestone reflection therefore arrives in phase with this reflection and can not in this area be readily separated from it.

RESULTS OF ST. CYR NO. 1 WELL

The St. Cyr No. 1 well encountered the Viola limestone at a depth of 2,793 feet, or a sub sea datum of −2,108. This is within 8 feet of the predicted datum. The uniformity of the small structure, as portrayed by the detailed seismic work, suggested satisfactory accuracy previous to the drilling. At the present time St. Cyr No. 1 well is in the process of being completed as a flowing oil and gas well from the Wilcox sand.

CONCLUSIONS

We may conclude from the foregoing that the seismic reflection method, if correctly applied, is entirely suitable for the discovery and delineation of the small Wilcox structures as found in Okmulgee County. The great necessity for detailed work on a problem of this type must be again emphasized. Our seismic work may be entirely satisfactory as to quality but deficient in quantity and correct geologic interpretation. Small subsurface faults may be easily located by a method of continuous profiling if the fault is a clean break with little or no deformation of the formations contiguous to it. All small sources of error must be carefully examined, cap lag, or time break trouble entirely eliminated, and definite determinative initial impeti secured to insure satisfactory surface correction zone data. Absolute determination of average overall velocity to the formation is entirely unnecessary so long as the proper correlation of reflections is made. From the method used in this area and the care which has been taken in all corrections it is believed each point shown on the detailed map represents an accurate depth to the Viola limestone within 10 feet. It is intended to use this area as a field laboratory and to carry out further research in regard to variation of velocity on structure and variation of surface correction zone. The writer invites personal communications suggesting problems which may be attacked with this particular set of conditions.

PORTABLE DYNAMITE STORAGE MAGAZINES

J. W. FLUDE[1]

The proper storage of a seismograph crew's supply of dynamite is a problem that has confronted many a shooter but, I am afraid, one which, too often, has been of small concern to his superiors. Here are the potential makings of a major accident, and yet in the past, rather than solve the problem, the easiest way was chosen. Destruction of property, accidents to the public, accidents to employees were possibilities, yet for years no one to my knowledge would spend the small amount of money necessary for protection.

There are several possible sources of danger from poor storage. Fire is one of these. If stacked in the open fire might only destroy the dynamite. On the other hand, dynamite in cases might explode if ignited, and if the quantity were large, bills would be promptly presented for all the windows in the county plus the value of all eggs which failed to hatch. There would be doctor bills for prospective mothers and others, and then, to help swell the total, there would be assorted bills for house foundations, roofs, broken dishes, and so forth.

The danger from theft is two-fold. We know of accidents to children who have found blasting caps and thrown them in bonfires or hit them with hammers, and similar accidents to the public could easily result. Last year a quantity of dynamite was found under a wharf at the port of Houston. The local papers claimed that this dynamite had been stolen from a seismograph crew at Victoria, Texas. According to the Atlas Powder Company, the dynamite, as shown by the size and strength did not come from a seismograph crew, although it *had* been stolen. The quantity of explosives a crew usually has in storage could be the cause of a calamity if it fell into the hands of an insane person or a fanatic of some sort. This is a responsibility we should not assume too lightly.

Lightning could easily detonate poorly stored explosives with the effects noted above.

Unless kept under a roof, rain also furnishes a hazard since water causes the explosive particles to separate from the filler and collect in larger grains, and then the powder becomes nearly as dangerous to handle as nitroglycerin.

[1] Independent Exploration Company, Houston, Texas.

Stray shots from firearms are another source of danger to poorly stored explosives.

Aside from the dangers, considerable time is often spent by the shooter in finding someone willing to allow him to use some old barn or house as a magazine. Sometimes he is forced to hide it in the woods or leave it in an open field.

Let's pause to consider how we stored our dynamite in the past, or for that matter, how a large quantity of it is stored today.

My first assignment in geophysics placed me on a party shooting on water. We lived on a houseboat. Tied to the houseboat at all times was an ordinary uncovered barge. On this barge was carried up to five tons of dynamite together with about ten drums of gasoline, a drum of cylinder oil, coal for a cook stove, batteries, and a lot of unassorted junk. The caps were kept inside the houseboat.

This was during the winter. One day after the first thunderstorm, someone in authority had the happy thought to have the barge moved about a quarter of a mile away.

Some water crews left their powder in the marsh. There, of course, it was exposed to all these hazards, but the consequences of an explosion would not have been so serious.

Nowadays the crew's explosives are frequently stored in an old house, cabin, or barn, and very often quite close to one or more homes. Usually the caps are on one side of the room and the dynamite on the other, but of course some shooters are not that particular. Now and then the door is locked, usually with a cheap padlock.

I have seen explosives left in peculiar places, even under a railway bridge; but if you think that odd I should like to ask any man in charge of a seismograph party if he knows exactly where his powder is stored. Let us hope it is in a specially prepared magazine.

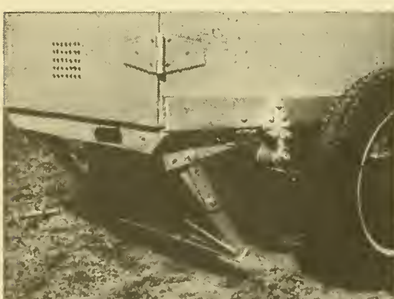
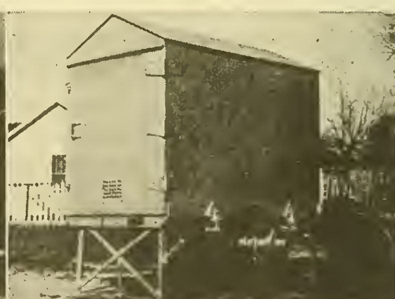
While the idea of portable dynamite magazines is not at all new, all credit for being the first to adopt them as standard equipment should go to Mr. Stewart Sherar and the Humble Company. This company now has one with every crew, as has the Independent Exploration Company. The Western Geophysical Corporation has at least one and is reported to be building others. The Texas Body and Trailer Company of Houston has built twenty-six magazines to date and others have been or are being built from photographs of the first ones.

The portable magazine successfully solves most problems of storage. Permission is easily obtained to leave it in an open field.

Favorable locations can be found with little trouble. About the only conditions are that the chosen spot should not have high grass and, according to the Standard Table of Distances, it should be not less than 1,300 feet from habitations, 780 feet from any railway, and 390 feet from any public highway.


A ground connection makes it a veritable lightning rod and eliminates this hazard. In addition, it can be securely locked to prevent theft, can be fairly well ventilated, is weatherproof, and, finally, is sturdy enough to stop ordinary lead bullets.

At this time there are two types of portable magazines in use. One is mounted on a two- and the other on a four-wheel chassis. The

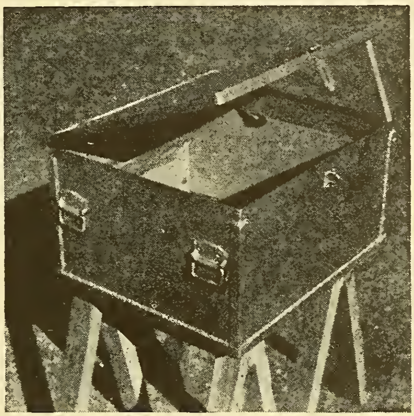



dimensions of these magazines differ slightly, and a good size would be 41 inches wide, 80 inches long and 40 inches high to the eaves. It should have a gable roof with about a 4 inch rise.

All surfaces are of 10-gauge hot rolled steel, blue annealed. The floor frames are $2 \times 2 \times \frac{1}{4}$ inch thick angle iron. The posts and top framing are $1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{4}$ inch thick angle iron.

The body is electric welded throughout. There are ventilating holes at the bottom at one side and at the top on the opposite side and these are backed by baffle plates inside to stop stray bullets. The floor is covered with wood or with pressed wood. The door is fitted with two hasps and padlocks. It is painted with aluminum paint.

The two-wheel type chassis is built from heavy standard trailer parts. Folding legs are added to hold the trailer upright. The four wheel type uses a standard Model A Ford Chassis with extra spring leaves added. The capacity of this magazine is about one ton and



its weight is about nineteen hundred pounds. The two-wheel trailer costs $340 F.O.B., Houston, while the four-wheel trailer costs $210 F.O.B., Houston.

A few precautions might well be mentioned. A number of tires and wheels have been stolen. It is wise to provide heavy wooden horses for the four-wheel magazine and put the wheels inside while on location. The two-wheel magazine stands on its own legs.

It is against the law to carry explosives in a trailer or in a vehicle which is towing a trailer. Dynamite can be carried in one vehicle, the magazine towed with another and used en route to carry office equipment or personal baggage. It is wise to make sure that your automobile insurance permits you to tow a trailer.

Another valuable piece of safety equipment is a steel, weatherproof cap magazine. This box measures $21\frac{1}{2} \times 26 \times 14$ inches high inside steel dimensions, and is lined with $\frac{3}{4}$ inch oak. It also is made of 10-gauge steel and is electrically welded. It is fitted with handles and has a hinged lid which locks with a padlock.

It holds one full case of 30 foot caps with enough room to take care of a few extra cartons. It weighs about 160 pounds and costs less than $25.

In use it is left near the dynamite magazine, the Standard Table of Distances, requiring that it be not less than 100 feet away. It can be chained to a tree or any other available anchor.

Public liability insurance rates are comparable with those for workmen's compensation. While I am quite sure the statistics on accidents fail to reveal the reason for this, the insurance companies can hardly be blamed unless the sources of possible enormous losses are eliminated. From an economic and from a humane standpoint I urge the use of portable magazines.

MEMBERSHIP APPLICATIONS

APPROVED FOR PUBLICATION

The Executive Committee has approved for publication the names of the following candidates for membership in the Society. This publication does not constitute an election, but places the name before the membership at large. If any member has information bearing on the qualifications of these nominees, he should send it promptly to the Secretary. (Names of sponsors are placed beneath the name of each nominee.)

FOR ACTIVE MEMBERSHIP

Malcolm C. Baker, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
James Andrew Brooks, Jr., Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Olin Derryl Brooks, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Charles Hewitt Dix, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Robert Summers Duty, Jr., Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Haakon Muus Evjen, Houston, Texas
F. Goldstone, L. W. Blau, Paul Weaver
Jerome Franklin Freel, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
George Alvin Garrett, Houston, Texas
E. E. Rosaire, J. W. Flude, F. M. Kannenstine
Francis Alex Gibson, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Henry Volandis Goss, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Gerald Lawrence Jackson, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Daniel Wilson Johnson, Jr., Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
William Stephen Levings, Washington, D.C.
B. B. Weatherby, L. Y. Faust, G. H. Westby
Paul Shean Lewis, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Wm. Bradley Lewis, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
John Mueller Lohse, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Thomas Albert Manhart, Tulsa, Oklahoma
J. L. Greenfield, G. H. Westby, Bela Hubbard
George W. Martin, Jr., Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell

Aubret Britton McCollum, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
John Helmer Neidert, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Cedric James Newby, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Thomas Alonzo Ogg, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Decatur O'Brien, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Homer Glover Patrick, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
William Monroe Rust, Jr., Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
William E. Steele, Jr., Houston, Texas
E. E. Rosaire, J. W. Flude, H. E. Banta
Hubert Robert Thornburgh, Houston, Texas
F. Goldstone, A. E. Smith, G. H. Westby
Earl Milton Wolters, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
John Price Woods, Houston, Texas
L. W. Blau, Paul Weaver, F. Goldstone

FOR ASSOCIATE MEMBERSHIP

Carl Fred Barnhart, Tulsa, Oklahoma
Andrew Gilmour, O. C. Lester, Jr., C. M. Ross
James Dungan Berwick, Tulsa, Oklahoma
Geo. E. Wagoner, August F. Beck, Stuart Sherar
Robert Elkins Carson, Houston, Texas
E. E. Rosaire, J. W. Flude, T. I. Harkins
George William Curtzinger, Dallas, Texas
Eugene McDermott, J. C. Karchen, H. B. Peacock
Roswell Stewart Epperson, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
William Sidney Ferguson, Houston, Texas
E. E. Rosaire, J. W. Flude, T. I. Harkins
Richard Silvio Gozzaldi, Houston, Texas
E. E. Rosaire, J. W. Flude, T. I. Harkins
James David Grant, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
James Howard Dunaway, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Jerome Glen Jerome, Houston, Texas
E. E. Rosaire, J. W. Flude, T. I. Harkins
Stanley C. Peters, Houston, Texas
Dave P. Carlton, J. E. La Rue, Lynn G. Howell
Frederick Arthur Tompkins, Rochester, New York
W. T. Born, B. Hubbard, G. H. Westby

CHANGE IN STATUS

FROM ASSOCIATE TO ACTIVE

As recommended by the Executive Committee under Article III, Section B-6 of the Constitution.

J. E. Duncan, Manitoba, Canada
E. G. Brydon, Houston, Texas
J. O. Hoard, Houston, Texas
B. P. Howerton, Houston, Texas
R. Saibara, Houston, Texas

COMPLETE LIST OF MEMBERS, HONORARY MEMBERS, AND ASSOCIATE MEMBERS

IN GOOD STANDING ON

MARCH 21ST, 1935

HONORARY MEMBERS

E. L. DeGolyer, Esperson Bldg., Houston, Texas
L. Mintrop, University of Breslau, Breslau, Germany

ACTIVE MEMBERS

Allen, Thomas L., Drawer 1477, San Antonio, Texas
Adler, Joseph L., Dr., 2011 Esperson Bldg., Houston, Texas

Barnette, L. Atmar M., 1331 Tulane St., Houston, Texas
Barret, Wm. M., Giddens-Lane Bldg., Shreveport, Louisiana
Barton, Donald C., Dr., Humble O. & R. Co., Box 2180, Houston, Texas
Barthelmes, Albert J., Dr., Shell Petroleum Corp., Box 2099, Houston, Texas
Banta, H. E., Dr., 2011 Esperson Bldg., Houston, Texas
Baker, Arnold G., Box 801, Tulsa, Oklahoma
Bennett, Roy F., 2011 Esperson Bldg., Houston, Texas
Beck, August, Humble O. & R. Co., (Box 801, Tulsa) Houston, Texas
Bevier, Geo. M., 1517 Shell Bldg., Houston, Texas
Becker, H. C., Box 2040, Tulsa, Oklahoma
Blau, L. W., Dr., Box 2180, Houston, Texas
Blondeau, E. E., Box 2040, Tulsa, Oklahoma
Bowman, Wayne F., Tide Water Oil Co., Esperson Bldg., Houston, Texas
Boucher, F. G., Box 2180, Houston, Texas
Borman, Frank W., Box 2040, Tulsa, Oklahoma
Brown, Hart, 1405 Esperson Bldg., Houston, Texas
Boos, C. Maynard, 2011 Esperson Bldg., Houston, Texas
Born, Wm. T., Box 2040, Tulsa, Oklahoma
Born, Milton C., Box 2040, Tulsa, Oklahoma
Bryan, A. B., Dr., Box 801, Tulsa, Oklahoma

Carlton, Dave P., Box 2180, Houston, Texas
Caster, E. L., Arkansas Fuel Oil Co., Shreveport, Louisiana
Collingwood, D. M., Sun Oil Co., Box 2880, Dallas, Texas
Cortes, Henry C., Magnolia Petroleum Co., Box 111, Houston, Texas
Copeland, Jefferson L., American Seismograph Co., 1707 Ramsey Tower, Oklahoma City, Oklahoma
Cram, Ira H., Pure Oil Company, Tulsa, Oklahoma
Crawford, Paul D., 1430 Milam Bldg., San Antonio, Texas
Crowell, J. H., 2011 Esperson Bldg., Houston, Texas

Davis, Donald M., Pure Oil Co., Houston, Texas
Dahlberg, R. S., Jr., Box 801, Tulsa, Oklahoma

Deussen, Alexander, 1606 Shell Bldg., Houston, Texas
Donnally, Chester J., 950 S. Flower St., Los Angeles, California

Eby, Albert N., 2011 Esperson Bldg., Houston, Texas
Eby, J. Brian, Dr., 1615 Sterling Bldg., Houston, Texas
Eckhardt, E. A., Dr., Gulf Research & Development Corp. P. O. Drawer 2038, Pittsburgh, Pennsylvania
English, Walter A., Superior Oil Co., Gulf Bldg., Houston, Texas
Ewing, Maurice, Dr., Lehigh University, Bethlehem, Pennsylvania

Falkenhagen, H. M., 2011 Esperson Bldg., Houston, Texas
Faust, Lawrence Y., Box 2040, Tulsa, Oklahoma
Fecht, Arthur J., McCollum Exploration Co., 611 Esperson Bldg., Houston, Texas
Ferguson, John L., Amerada Petroleum Co., Tulsa, Oklahoma
Flude, John Wm., 2011 Esperson Bldg., Houston, Texas
Freeman, Lawrence, Phillips Petroleum Company, Bartlesville, Oklahoma
Frosch, Alex, Box 2180, Houston, Texas

Gardner, Derry H., Box 2180, Houston, Texas
Gella, Norbert, Casilla 3095, Santiago de Chile, S.A.
Gemmer, R. W., Box 801, Tulsa, Oklahoma
Gibbon, H. A., Box 2180, Houston, Texas
Gilmore, Homer, 724 West Symmes, Norman, Oklahoma
Gilmour, Andrew, Box 2040, Tulsa, Oklahoma
Goldstone, Frank, Shell Petroleum Corp., Houston, Texas
Golden, John M., 2011 Esperson Bldg, Houston, Texas
Green, W. G., 709 Kennedy Bldg., Tulsa, Oklahoma
Greenfield, John L., 1612 S. College, Tulsa, Oklahoma

Harkins, T. I., 2011 Esperson Bldg., Houston, Texas
Harrington, George, Barnsdall Oil Co., Tulsa, Oklahoma
Harris, Sidon, Western Geophysical Co., Los Angeles, California
Heidecke, Otto, Bakenstrasse 2, Halberstadt, Germany
Heiland, C. A., Prof., Colorado School of Mines, Golden, Colorado
Henderson, Homer I., McCollum Exploration Co., Houston, Texas
Harding, Robert L., Box 2040, Tulsa, Oklahoma
Hibbler, Alfred J., The Texas Co., Houston, Texas
Hlauschek, Hans, Prague Smichov Nabr. Legii 10, Praha, Czechoslovakia
Horn, John W., Arkansas Natural Gas Co., Shreveport, Louisiana
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Howell, Lynn G., Box 2180, Houston, Texas
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Hubbard, Bela, c/o Standard Oil Co., of New Jersey, 30 Rockefeller Plaza, New York City
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Hall, Dollie Radler, Box 2040, Tulsa, Oklahoma

Innes, Arland I., Geophysical Research Corp., Box 2040, Tulsa, Oklahoma
Ivy, John S., 921 Rusk Bldg., Houston, Texas

Jackson, Ralph S., Independent Exploration Co., Houston, Texas
Judson, Sidney, Texas Gulf Producing Co., Esperson Bldg., Houston, Texas

Kannenstine, Fabian M., Dr., 2011 Esperson Bldg., Houston, Texas
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Kauenhowen, Walter, Deutsche Vakuum Oel A. G., Semperhaus B-111, Hamburg, Germany
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Kerns, A. D., Box 2040, Tulsa, Oklahoma
Kerns, George P., Box 2040, Tulsa, Oklahoma
Kendall, G. D., Box 2040, Tulsa, Oklahoma
Kidd, Robt. L., Empire Oil & Refining Co., Bartlesville, Oklahoma
Klipsch, Paul W., 2011 Esperson Bldg., Houston, Texas

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Le May, R. A., 2011 Esperson Bldg., Houston, Texas
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McCollum, Burton, 611 Esperson Bldg., Houston, Texas
McDermott, Eugene, 1311 Republic Bank Bldg., Dallas, Texas
Mounce, W. D., Box 2180, Houston, Texas
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Mott-Smith, Morton, Dr., 2011 Esperson Bldg., Houston, Texas
Muzzy, David S., Jr., Shell Petroleum Corp., Houston, Texas

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Petty, Dabney E., P. O. Drawer 1477, San Antonio, Texas
Petty, O. S., P. O. Drawer 1477, San Antonio, Texas
Pollard, J. C., Magnolia Petroleum Co., Houston, Texas

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Schumacher, J. P., 1406 Shell Bldg., Houston, Texas
Schwennesen, A. T., Petroleum Exploration Co. of Texas, Houston, Texas
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Smith, Aylwin L., 2011 Esperson Bldg., Houston, Texas
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Stanton, Austin N., 3012 West Cantey St., Ft. Worth, Texas
Statham, Louis, 1706 Waugh Drive, Houston, Texas
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Stiles, Elizabeth (Miss), 2011 Esperson Bldg., Houston, Texas
Storm, Alfred E., 1311 Republic Bank Bldg., Dallas, Texas
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Sweet, Elliott, American Seismograph Co., 1707 Ramsey Tower, Oklahoma City, Oklahoma

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Taylor, William H., Petty Geophysical Corp., San Antonio, Texas
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Wagoner, Geo. E., Box 801, Tulsa, Oklahoma
Wantland, Dart, Colorado School of Mines, Golden, Colorado
Warrick, Thomas R., c/o Standard Oil of Venezuela, Caripito, Venezuela, S.A.
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Watt, J. S., Box 2180, Houston, Texas
Walling, Orville D., Western Geophysical Co., 950 S. Flower St., Los Angeles, California
Weatherby, B. B., Dr., Box 2040, Tulsa, Oklahoma

Weaver, Paul, Drawer 2100, Houston, Texas
Weinzierl, Jno. F., 607 Petroleum Bldg., Houston, Texas
Westby, G. H., 709 Kennedy Bldg., Tulsa, Oklahoma
Wilhelm, A. K., Empire Oil & Refining Co., Bartlesville, Oklahoma
Williston, Samuel H., Roosevelt Apts., Aberdeen, Washington
Wilson, John H., Colorado Geophysical Corp., Midland Savings Bank Bldg., Denver, Colorado
Wolf, Alexander, The Texas Co., Houston, Texas

Young, Karl E., 713 Esperson Bldg., Houston, Texas

Zimmerman, C. C., The Texas Co., Houston, Texas
Zimerman, Sam, Box 158, Alma, Michigan

ASSOCIATE MEMBERS

Black, J. P., Republic Prod. Co., Petroleum Bldg., Houston, Texas
Blessing, Wendell, 1207 Sterling Bldg., Houston, Texas
Brydon, Earl G., 2011 Esperson Bldg., Houston, Texas
Broussard, D. F., 2011 Esperson Bldg., Houston, Texas
Burton, Raiford H., Box 2040, Tulsa, Oklahoma

Duncan, J. E., 970 McMillan Ave., Winnipeg, Manitoba, Canada

Hoard, J. O., 320 W. Polk Avenue, Houston, Texas
Howerton, Bert P., 2011 Esperson Bldg., Houston, Texas
Hamilton, Wm. B., 2011 Esperson Bldg., Houston, Texas

Jarnagin, R. V., Jr., Independent Exploration Co., Houston, Texas

Manning, Earl L., 2011 Esperson Bldg., Houston, Texas
Manning, W. G., Independent Exploration Co., Houston, Texas
Morris, Euclid, 2011 Esperson Bldg., Houston, Texas

Parker, Lewis A., 2011 Esperson Bldg., Houston, Texas

Rosaire, Forrest C., 2011 Esperson Bldg., Houston, Texas

Saibara, Robert, 2011 Esperson Bldg., Houston, Texas
Stubbe, Gerhard, American Askania Corp., Houston, Texas

Williams, E. Darrell, Box 2040, Tulsa, Oklahoma
Wall, Thomas E., American Seismograph Co., 1707 Ramsey Tower, Oklahoma City, Oklahoma.

THE SOCIETY OF PETROLEUM GEOPHYSICISTS

CONSTITUTION AND BY-LAWS

(As amended 1935)

I NAME

This Association shall be called the "Society of Petroleum Geophysicists."

II OBJECT

The object of this Association is to promote the science of geophysics especially as it relates tc petroleum geology and to the discovery and production of oil and natural gas and associated minerals.

III MEMBERSHIP

A. Members

1. Any geophysicist of recognized standing shall be eligible to membership, whether or not he is engaged in petroleum geophysics or in geophysical prospecting.
2. Any physicist, mathematician, geologist, or engineer of recognized standing who is investigating a geophysical problem or problems shall be eligible to membership.
3. Any geologist of recognized standing who is not a geophysicist of recognized standing, but who is acting as chief, or division chief in charge of geophysical surveying, shall be eligible to membership.

B. Associates

1. Any geophysicist, geologist, engineer, mathematician, or physicist who is a graduate of an institution of recognized scientific standing who is engaged in geophysics and who is not eligible to membership shall be eligible to associate membership.
2. Any person who is not a graduate of an institution of recognized scientific standing and who is not eligible to membership and who has shown ability in a routine phase of geophysical work and who is in a position of responsibility in geophysical surveying shall be eligible to associate membership.
3. Any representative or employee of a firm or corporation engaged in supplying equipment or supplies to the geophysical prospecting industry, provided such individual is the graduate of an institution of recognized scientific standing, or provided otherwise that such individual is qualified by adequate technical experience shall be eligible to associate membership.
4. Associate members shall be known as associates.
5. Associates shall enjoy all the privileges of membership in the Society, save that they shall not hold office, sign applications for membership, or vote; neither shall they have the privilege of advertising their affiliation with the Society in professional cards or professional reports or otherwise.
6. The executive committee may advance to active membership, without the formality of application for such change, those associates who have, subsequent to election fulfilled the requirements for active membership.

C. Election to Membership

1. Every candidate for admission as a member or associate shall submit a formal application on an application form authorized by the executive committee signed by him, and endorsed by not less than three members who are in good standing, stating his training and experience and such other facts as the executive committee shall from time to time prescribe. Provided the executive committee, after due consideration, shall judge that the applicant's qualifications meet the requirements of the constitution, they shall cause to be published in an appropriate publication the applicant's name and the names of his sponsors. If, after at least thirty days have elapsed since such publication, no reason is presented why the application should not be admitted, he shall be deemed eligible to membership or to associate membership, as the case may be, and shall be notified of his election.
2. An applicant for membership, on being notified of his election in writing, shall pay full membership dues for the current year and on making such payment shall be entitled to receive the regular Society publications for that year. Unless payment of dues is made within thirty (30) days by those living within the continental United States and within ninety (90) days by those living elsewhere, after notice has been mailed of his election, the executive committee may rescind the election of the applicant. Upon payment of dues, each applicant for membership shall be furnished with a membership card for the current year, and until such written notice and card are received, he shall in no way be considered a member of the Society.

D. Honorary Members

1. The executive committee may from time to time and by unanimous action elect as honorary members persons who have contributed distinguished service to the cause of geophysics. Honorary members shall not be required to pay dues.

E.

1. Membership of any class shall be contingent upon conformance with the code of ethics of the American Association of Petroleum Geologists and with the established principles of professional ethics.

IV

A. Resignation—Suspension—Expulsion

1. Any member or associate may resign from the Society at any time. Such resignation shall be in writing and shall be accepted by the executive committee, subject to the payment of all outstanding dues and obligations of the resigning member or associate.
2. Any member or associate who is more than one year delinquent (in arrears) in payment of dues shall be suspended from the Society. Any delinquent or suspended member or associate, at his own option, may request in writing that he be dropped from the Society and such request shall be granted by the executive committee. Any member or associate more than two years in arrears shall be dropped from the Society. The time of payment of delinquent dues for either one year or two years may be extended by a unanimous vote of the executive committee.

3. Any member or associate who resigns or is dropped under the provisions of Sections 1 and 2 of this article ceases to have any rights in the Society and ceases to incur further indebtedness to the Society.
4. Any person who has ceased to be a member or associate under Section 1 or Section 2 of this article may be re-instated by a unanimous vote of the executive committee subject to the payment of any outstanding dues and obligations which were incurred prior to the date when he ceased to be a member or associate of the Society.
5. Any member or associate who, after being granted a hearing by the executive committee, shall be found guilty of a violation of the code of ethics of this Society or shall be found guilty of a violation of the established principles of professional ethics, or shall be found guilty of having made a false or misleading statement in his application for membership in the Society, shall be asked to resign from the Society by a unanimous vote of the executive committee. The decision of the executive committee in all matters pertaining to the interpretation and execution of the provisions of this section shall be final.

V OFFICERS AND THEIR DUTIES

A. Officers

1. The officers of the Society shall be a president, a vice-president, a secretary-treasurer, and an editor. These, together with the past president, shall constitute the executive committee and managers of the Society.
2. A ticket of nomination for officers and representatives shall be prepared by a Committee on Nominations, which shall consist of the president, and the two qualified past presidents in order to precedence. Favorable action of this committee shall be two out of three votes. The ticket decided on by this committee, as well as any ticket presented in writing and signed by twenty other members in good standing, shall be submitted to the Secretary-Treasurer by December 31st. At least three candidates for president shall be named on the ticket. This shall be placed on a ballot and one mailed to each member in good standing on December 31st by the Secretary-Treasurer, not later than January 10th. A blank space shall be left on this ballot for writing in nominations for each place. Before ballots are mailed the Secretary-Treasurer shall secure written acceptance of nomination from each candidate.

   The nominations for president shall be alphabetically arranged, except the retiring vice-president, who shall automatically occupy first place on the ticket. Each member shall express a first choice for president, and a second choice for another candidate for president. First choice votes shall count double and second choice shall count single. The candidate receiving the highest number of votes so cast and counted shall be elected president, and the candidate receiving the second highest number of votes so cast and so counted shall be elected vice-president, except where the choice conflicts with Section Three of Article V, in which case the next highest candidate shall be elected vice-president.

   Such ballots, to be counted as valid, must be received by the Secretary-Treasurer at his official recognized address not less than ten days prior to the Annual Meeting. The ballots shall be mailed to the Secretary-Treasurer in envelopes which shall carry the written signature and the typewritten name of the member. These ballots will be opened and counted by the nominating committee immediately before the Annual Meeting. The secretary-treasurer, before the ballots are opened, shall approve by initialing only those ballots received from members in good standing at the time the ballots were received. The announcement of the election will be made as the first of the new business considered at the Annual Meeting. A majority vote of said ballots received in accordance with this action, except where otherwise designated, shall be considered as the will of the Society. In the event of a tie, the favoring secret vote of the nominating committee shall decide.

3. No one shall hold the office of president for two consecutive years and no one shall hold any other office for more than two consecutive years except the editor, who shall not hold office for more than six consecutive years.

B. Duties of Officers

1. The president shall be the presiding officer at all the meetings of the Society, shall take cognizance of the acts of the Society and of its officers, shall appoint such committees as are required for the purposes of the Society, and shall delegate members to represent the Society. He may, at his option, serve on, and may be chairman of, any committee.
2. The vice-president shall assume the office of president in case of a vacancy from any cause in that office and shall assume the duties of president in case of the absence or disability of the latter. He shall also act as chairman of the Program and Arrangements Committee of the Society, and as the junior delegate of the Society on the Business Committee of the American Association of Petroleum Geologists.
3. The secretary-treasurer shall assume the duties of the president in case of the absence of both the president and vice-president. He shall have charge of the financial affairs of the Society and shall annually submit reports as secretary-treasurer covering the fiscal year, which he shall arrange to have published in the next regular issue of the recognized organ of the society. He shall receive all funds of the Society, and, under the direction of the executive committee, shall disburse all funds of the Society. He shall cause an audit to be prepared annually by a public accountant at the expense of the Society. He shall give a bond, and shall cause to be bonded all employees to whom authority may be delegated to handle Society funds. The amount of such bonds shall be set by the executive committee and the expense shall be borne by the Society. The funds of the Society shall be disbursed by check as authorized by the executive committee.
4. The editor shall be in charge of the editorial business, shall submit an annual report of such business, shall have authority to solicit papers and material for the regular Society publication and for special publications, and may accept or reject material offered for publication. He may appoint associate, regional, and special editors. He shall serve as senior delegate to the Business Committee of the American Association of Petroleum Geologists.
5. In the event of absence, disability or resignation of any member of the executive committee, the vacant place in the committee shall be automatically filled by the next preceding qualified past-president who shall be known as Prior Past-President.
6. The officers shall assume the duties of their respective offices immediately after the annual meeting following their election.

VI EXECUTIVE COMMITTEE—MEETINGS AND DUTIES

A. Executive Committee

1. The executive committee shall consist of the president, past-president, vice-president, secretary-treasurer, and editor.

B. Meetings and Duties

2. The executive committee shall meet preceding the annual meeting and at the call of the president may hold meetings when and where thought advisable, to conduct the affairs of the Society. A joint meeting of the outgoing and incoming executive committees shall be held immediately after the close of the annual Society business meeting. Members of the Executive committee may vote by proxy, mail or person on all matters which call for favorable or unanimous action.
3. The executive committee shall consider all nominations for membership and pass on the qualifications of the applicants; shall have control and management of the affairs and funds of the Society; shall determine the manner of publication; and shall designate the place of the annual meeting. They are empowered to establish a business headquarters for the Society, and to employ such persons as are needed to conduct the business of the Society. They are empowered to make investments of both general and special funds of the Society. Trust funds may be created giving to the trustees appointed for such purpose such discretion as to investments as seems desirable to the executive committee to accomplish any of its objects and purposes, but no such trust funds shall be created unless they are revocable upon ninety (90) days' notice. Favorable action shall consist of a favoring three out of five votes. Committee members may vote by person, proxy or in writing.

VII

A. Meetings

1. The Society shall hold at least one stated meeting each year, which shall be the annual meeting. This meeting shall be held at a time and place designated by the executive committee. At this meeting the proceedings of the preceding meeting shall be read, Society business shall be transacted, scientific papers shall be read and discussed, and results announced of the mail ballot for officers for the ensuing year.

VIII

A. Amendments

1. Amendments to this constitution may be proposed by a resolution of the executive committee, by a constitutional committee appointed by the president, or in writing by any ten members of the Society. All such resolutions or proposals must be submitted at the annual meeting of the business committee of the Society as provided in the by-laws, and only the business committee shall make recommendations concerning proposed constitutional changes at the annual Society business meeting. If such recommendations by the business committee shall be favorably acted on at the annual Society business meeting, the secretary-treasurer shall arrange for a ballot of the membership by mail within thirty (30) days after said annual Society business meeting, and a majority vote of the

ballots received within ninety (90) days of their mailing shall be sufficient to amend. The legality of all amendments must be determined by the executive committee prior to balloting.

B. Parliamentary Regulations

1. The rules contained in "Roberts Rules of Order, Revised" shall govern the Society in all cases to which they are applicable, and in which they are not inconsistent with the by-laws or the special rules of order of this Society.

BY-LAWS

I

A. Dues

1. The fiscal year of the Society shall correspond with the calendar year.
2. The annual dues of members of the Society shall be five dollars ($5.00). The annual dues of associates for not to exceed six years after election shall be four dollars ($4.00); thereafter, the annual dues of such associates shall be five dollars ($5.00). The annual dues are payable in advance on the first day of each calendar year. A bill shall be mailed to each member and associate before January first of each year, stating the amount of the annual dues and the penalty and conditions for default in payment. Members or associates who shall fail to pay their annual dues by March fifteenth shall not receive further copies of the regular Society publication, nor shall they be privileged to buy Society special publications at prices made to the membership, until such arrears are met.

II

A. Publications

1. The proceedings of the annual meeting, the constitution and by-laws, and the papers presented at such meeting shall be published at the discretion of the executive committee in the regular publication or in such other form as the executive committee may decide best meets the needs of the membership of the Society.
2. The payment of annual dues for any fiscal year entitles the member or associate to receive without further charge a copy of the regular publication of the Society for that year.
3. The executive committee may authorize the printing of special publications to be financed by the Society from its general, publication or special funds and offered for sale to members and associates in good standing at not less than the cost of publication and distribution.

III

A. Regional Sections, Technical Divisions, and Affiliated Societies

1. Regional sections of the Society may be established provided the members of such sections are members of the Society and shall perfect an organization and make application to the executive committee. The executive committee shall submit the application for action first by the Business Committee and second to the Society for a letter ballot as for a constitutional amendment, and provided that the Society may revoke the charter of any regional section by similar action.

2. Subject to the same requirements as for a Constitutional amendment, and with legal advice, the executive committee may arrange for the affiliation with the Society of duly organized groups or societies, which by object, aims, constitution, by-laws, or practice are developing the study of geophysics or petroleum technology. In like manner and with like advice, the executive committee may arrange conditions for dissolution of such affiliations. Affiliation with the Society need not prevent affiliation with other scientific societies. Members of affiliated societies who are not members of the Society, shall not have the privilege of advertising their affiliation with the Society on professional cards or otherwise.

IV

A. District Representatives

1. The executive committee may cause to be elected district representatives from districts which it shall define by a local geographic grouping of the membership. Such districts shall be redesignated and redefined by the executive committee as often as seems advisable. Each district shall be entitled to one representative for each seventy-five members, but this shall not deprive any designated district of at least one representative. The representatives so apportioned shall be chosen from the membership of the district by a written ballot arranged by the executive committee. They shall hold office for two years, their term of office expiring at the close of the annual meeting. In so far as possible, these two-year terms shall be overlapping.

V

A. Business Committee

1. The Executive committee may institute a business committee to act as a council and advisory board to the executive committee and the Society. This committee shall consist of the executive committee, the prior past president, and the district representatives. Favorable action by the business committee shall consist of a favorable two-thirds vote. The secretary-treasurer shall act as secretary of the business meeting. Members of this committee may vote in person, by proxy, or in writing. The business committee shall meet the day before the annual meeting at which time all proposed changes in the constitution or by-laws shall be considered, all old and new business shall be discussed, and recommendations shall be voted for presentation at the annual meeting.

VI

A. Amendments

1. These by-laws may be amended by favorable vote of the business committee, providing that such changes have been recommended and legality approved by the executive committee. Such amendments shall be published in the next regular number of the recognized organ of the Society.

ANNUAL MEETING, 1935
SOCIETY OF PETROLEUM GEOPHYSICISTS

WICHITA, KANSAS

THURSDAY, MARCH 21, AVIATION ROOM

(THURS., 1:45–4:45 P.M., Society of Petroleum Geophysicists, E. E. Rosaire, presiding.)

1. John W. Flude, Portable Storage Magazines for Dynamite and Caps.
2. Roy Hightower, Premature Explosions in Seismic Explorations.
3. G. H. Loving and G. H. Smith, Explosives and Electric Blasting Caps for Geophysical Prospecting.
4. P.W. Klipsch, Performance of Oscillograph Amplifiers to Transient Shock Excitation.

A qualitative answer is offered to the question. "What is desirable transient response?" The correlation between transient and steady state performance is discussed with the purpose of gaining a knowledge of the former from a measurement of the latter. By separating the elements of an amplifier system and measuring steady state performance of each component, any corrections necessary may be determined and applied to the proper element. The basis for design and adjustment is improved and testing is facilitated by the methods outlined.

5. E. E. Rosaire, Strategy and Tactics in Geophysical Exploration for Petroleum.

A review of geophysical prospecting for petroleum with emphasis upon limitations of the tactics developed. An attempt is made to examine the strategical requirements of today with regard to the need for developing new exploration methods.

FRIDAY, MARCH 22

(FRI., 9:00–12:30 P.M., Society of Petroleum Geophysicists, F. Goldstone, presiding.)

1. Donald C. Barton, Delimitation of Cap by Torsion Balance, Hoskins Mound, Brazoria County, Texas.

The survey and the quantitative calculations were made for the Freeport Sulphur Company to determine the limits and depth of the cap in the unexplored parts of the Hoskins Mound dome. The crest and the southwest quadrant of the dome were known from exploratory wells and from producing wells. No wells had been drilled below the 1,000-foot contour from the northwest radius clockwise to the southeast radius. Thirteen radial torsion balance profiles were run. Office trial-and-error calculations were made by the ordinary formula for the gradient effect of a finite horizontal prism to determine the most probable form of the cap which would produce the observed gradient anomaly. The trial forms of the cap were tied to part of the known drilling data. A net of subsequent wells has shown the degree of accuracy of those predictions.

2. Paul Weaver, Irregularities in Torsion-Balance Surveys from Near-Surface Effects.

Examples are given showing that even in flat country irregular torsion balance readings have been obtained and suggestions are made for minimizing these effects and possible contributing causes are discussed.

3. E. W. K. Andrau, Schlumberger Correlations and Tectonic Problems on Gulf Coast Salt Domes.

Schlumberger electrical well surveying enables the petroleum engineer to improve his degree of accuracy when correlating wells. This is illustrated by cross sections, which also show the accuracy with which faulting can be determined and predicted in future wells. The economic importance of faults and their influence on oil and gas accumulations are being stressed.

The type of faulting on Gulf Coast salt domes is discussed and in addition some ideas are given in regard to salt-dome tectonics in general.

It is pointed out that stratigraphic irregularities, hitherto explained by lensing, should be reconsidered from a tectonic point of view. Tectonics in the evolution of our geologic knowledge of Gulf Coast salt domes has not received its share of attention.

4. M. Ewing, A. P. Crary, and J. W. Peoples, Seismic Tests on Anthracite.(By Title)

Refraction profiles 400 feet long were run over anthracite outcrops and over shales between veins. A profile directly on the anthracite showed a velocity of 5,000 feet per second. The velocities so obtained were too nearly equal for prospecting.

5. C. A. Heiland, Confirmation of Reflection Work in Canada by Drilling.

The paper brings out the fact that although the reflection geophysicist may be inclined to consider his work a matter of routine physical measurements, he must be able to recognize geologic situations that are apt to interfere with correct findings, and to adapt his field and interpretative technique to same.

The area covered by the survey is situated near Duvernay, on the Saskatchewan River, in Alberta. A number of shallow and two deep wells had been drilled there, and a magnetic survey been made. The stratigraphic and structural conditions deduced from the preliminary work are discussed in detail.

The problems confronting the reflection survey arose from the variations in dimensions and petrographic character of the strata composing the glacial drift. They affected, (a) the near surface time delay, (b) the accuracy of average velocity determinations, and (c) the character of seismic impulses. How these difficulties were analyzed and partially overcome is discussed in the sections on field procedure, interpretation, and results.

A graphical method of logging and correlating impulses of adjacent stations was employed. Traverses and contour maps thus obtained indicated that the wells referred to had been drilled off structure.

6. G. H. Westby, Effect of Surface and Near-Surface Beds in Reflection Seismic Results.

7. J. Brian Eby and Robert P. Clark, Relation of Geophysics to Salt-Dome Structures.

This paper presents eighteen geophysical maps covering nine salt domes and deep-seated structures of the Texas and Louisiana Gulf Coast. Two shallow domes, namely, Moss Bluff and Fannett, are given. Fannett is a gravity maximum, and Moss Bluff is a combination of a gravity minimum and maximum. The Sugarland dome is given as an example of a medium-depth structure, which is indicated by the torsion balance as a minimum. Six deep-seated structures are shown, including Sheppards Mott, Pledger Mykawa, Tomball, Livingston, and English Bayou-Gillis. The deeper seated structures invariably show a gravity minimum influence.

A refraction seismograph map of Moss Bluff is given, and reflection seismograph maps are given for Sheppards Mott, Tomball, and English Bayou-Gillis. A magnetic map of the Fannett field is likewise presented.

8. Symposium of Geophysical Activity Throughout the United States and Som Foreign Countries.

G. H. Westby, Henry Salvatori, Joseph L. Adler, H. B. Peacock, John H Wilson, E. G. Leonardon, O. C. Lester, Jr.

THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS

GEOPHYSICS

1932

INCLUDING PAPERS PRESENTED BEFORE THE SOCIETY OF PETROLEUM GEOPHYSICISTS AT THE ANNUAL CONVENTION OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS AT OKLAHOMA CITY, MARCH 25, 1932; REPRINTED FROM THE BULLETIN OF THE ASSOCIATION, VOL. 16, NUMBER 12, DECEMBER, 1932

TRANSACTIONS
SOCIETY OF PETROLEUM GEOPHYSICISTS
VOL. III

PUBLISHED BY
THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
TULSA, OKLAHOMA, U. S. A.

CONTENTS

MAGNETIC RESURVEY OF OKLAHOMA CITY FIELD[1]

O. C. CLIFFORD, JR.[2]
Enid, Oklahoma

ABSTRACT

Comparative results of two magnetometer surveys, one in December, 1927, before the discovery of oil, and one in January, 1932, after production had reached maturity, show that the vertical component of the earth's magnetic field has been changed in detail by the presence of production equipment. The generalized magnetic pattern remains similar, however, for both surveys.

The first problem outlined at the initiation of an exploratory program of earth magnetics 5 or 6 years ago was, almost without exception, the definition of the vertical component pattern across well known and highly developed producing structures. The results of such surveys did not, in general, fit into the preconceived pattern as anticipated by the geologist-geophysicist. There were, however, such noteworthy exceptions as the buried extension of the Wichita Mountains in the Texas Panhandle and the southern portion of the Granite ridge in the vicinity of Eldorado, Kansas. With the knowledge that the magnetic pattern over these types of structure was so readily reconciled with the configuration of the granite, attempts were made at reconciling the known geological features in other areas with the measured magnetic anomaly, but these attempts proved, too frequently, unsuccessful. The geologist-geophysicist faced with the necessity for an explanation of phenomena with which he was little acquainted, but concerning which he had definite convictions, expounded such hypotheses as the hysteresis induced by buried casing, the possibility of polar reversibility under the impact of cable tools, *et cetera ad infinitum*, being limited only by an inadequate vocabulary.

[1] Read before the Association at the Oklahoma City meeting, March 25, 1932.

[2] Geologist-geophysicist.

During the past 6 years the study of micro-geomagnetics has progressed beyond the limits of early credulity and, with the publications of Heiland, Wilson, Barret, Lynton, Spraragen and others, many of these problems are nearing solution. Though other geo-

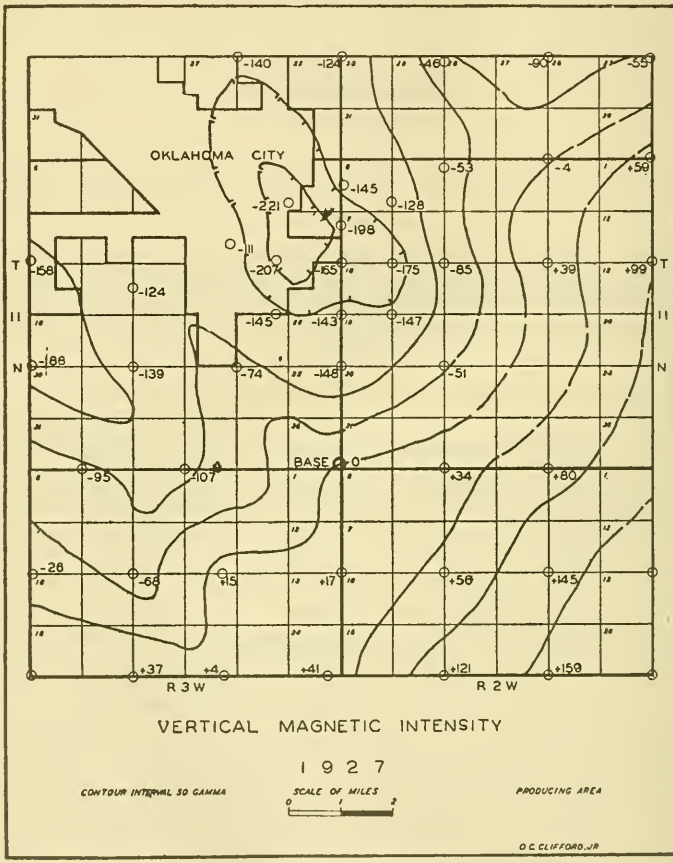


FIG. 1.—Map showing vertical magnetic intensity, 1927.

physical methods have rightfully supplanted magnetic methods throughout most of the Mid-Continent and Gulf Coast areas, magnetometers must still be considered of exploratory value for specialized structural types and for rapid, inexpensive surveys of regional character.

There has been no material published, with one exception, on the measured change in the vertical component of the magnetic field in an area surveyed both before the discovery of production and after development had reached maturity. Barret[1] has published an interesting summary of such work in the Sligo field, Bossier Parish, Louisiana, which must be considered inconclusive because of the small number of control points. With the permission of the management of the Indian Territory Illuminating Oil Company, the results of two magnetic surveys in the vicinity of Oklahoma City are presented.

In December, 1927, a magnetic survey of the vertical intensity anomaly was completed in the vicinity of Oklahoma City, with the use of a Schmidt vertical intensity variometer manufactured by the American Askania Corporation. The results of this survey contoured on a 50-gamma interval are shown in Figure 1. The dominant feature is an area of negative relief centered around the west half of Sec. 12, T. 11 N., R. 3 W. Here a 110-gamma closure is the maximum observed. No attempt is made to interpret this feature with respect to the existence of Permian, Pennsylvanian, Ordovician, or pre-Cambrian structure. The structure was undoubtedly there and the magnetic picture appeared as shown.

The discovery well of the present Oklahoma City field was spudded in near the center of the SE. ¼ of the SE. ¼ of Sec. 24, T. 11 N., R. 3 W., June 12, 1928, and was completed December 4, 1928. An intensive drilling campaign followed and by January 1, 1932, a total of 867 wells had been completed, of which 5 had been abandoned. This represents, according to Charles,[2] a development of 80 per cent of all acreage which is thought to have possibilities of becoming productive. The structural conditions of Oklahoma City have been described by Charles[3] and others.

In mid-January, 1932, a resurvey of the vertical component of the earth's magnetic field was made. The results are shown in Figure 2, also contoured on a 50-gamma interval.

There appears, at first glance, to be a marked discrepancy between the work of 1927 and that of 1932. This becomes less noticeable

[1] William M. Barret, "Magnetic Disturbance Caused by Buried Casing," *Bull. Amer. Assoc. Petrol. Geol.*, Vol. 15, No. 11 (November, 1931), pp. 1387–88.

[2] H. H. Charles, verbal communication.

[3] H. H. Charles, "Oklahoma City Oil Field," *Bull. Amer. Assoc. Petrol. Geol.*, Vol. 14, No. 12 (December, 1930), pp. 1515–33.

on closer scrutiny. The dominant feature of the later survey, also, is an area of negative relief, but this has been shifted from the west half of Sec. 12, T. 11 N., R. 3 W., to the SE. ¼ of Sec. 10, T. 11 N., R. 3 W. The maximum closure appears to be approximately the same,

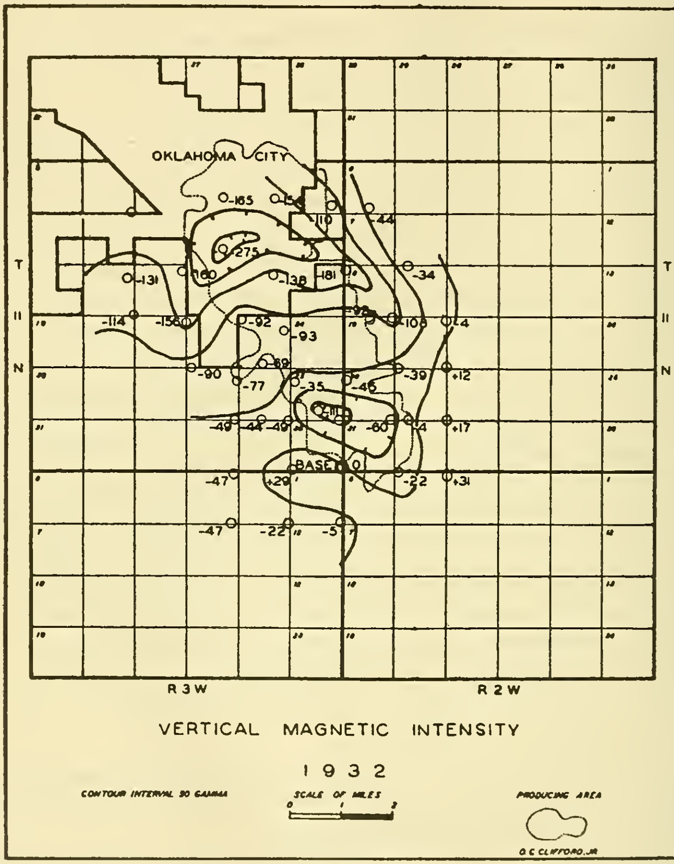


Fig. 2.—Map showing vertical magnetic intensity, 1932.

though it may be greater in the later work. In the recent survey, the contours lie closer together and the general pattern practically duplicates that of 1927, but the plane of the feature appears to have been tilted toward the west.

The base point, SE. corner, Sec. 36, T. 11 N., R. 3 W., is identical for both surveys and is thought to be in an undisturbed state, as no well is closer to it than 3/8 mile. Between the base and a point two miles farther east, also in an undisturbed condition, there is a marked accordance of anomaly, a difference of plus 34 gammas being measured in the first survey and a difference of plus 31 gammas in the second survey. The stations south and west of the base point, though they do not coincide, either in position or value, do roughly correspond. It appears, then, that in the undisturbed area on the south there is a pronounced accordance of anomaly value which gives a reasonable check on the comparative accuracy of the two surveys.

The most pronounced discordance between the two surveys appears one mile north of the base station. In the original survey this point falls on a gradual northward decline of anomaly which continues uninterrupted for 5 miles. In the more recent work, there is a sharp decline of intensity in the first mile, followed by an almost equally sharp rise in the succeeding mile. Though every effort was made to keep pronounced effects of extraneous material from the survey, it is thought that this discrepancy represents such effects. When the instrument gave readings off the field of view, it was moved until large effects appeared vitiated.

The changed aspect of the vertical component map may be attributed to one or all of several factors. No attempt is made to evaluate the relative importance of these. They are: (1) the recent survey was more compactly observed and for that reason the intensity changes may appear more accentuated than in the original survey; (2) the physical development produces major changes in the magnetic field; (3) the actual extension of Oklahoma City proper, both laterally and vertically, has produced immeasurable changes in the magnetic field; and (4) secular variation causes slight differences.

The results from one resurvey of this type can not be considered conclusive. It is reasonable to expect, however, that similar attempts in other areas will produce similar results. The results of the present survey tend to show that, with care in the selection of observation points, the general picture of an area will show little difference before and after the development of production; that, if the original magnetic structure had any considerable relief, it will retain not only the same approximate closure, but also closure in the same direction. The early magnetic work in existing fields can thus be expected to have shown a rough approximation to the pre-development state.

It is hoped that similar surveys will be undertaken in areas which were magnetically defined prior to the discovery and development of production. A resurvey of Hobbs field, New Mexico, and various of the Permian basin fields of Texas should show interesting relationships.

MAGNETIC VECTOR STUDY OF REGIONAL AND LOCAL GEOLOGIC STRUCTURE IN PRINCIPAL OIL STATES[1]

W. P. JENNY[2]
Dallas, Texas

ABSTRACT

The local magnetic vectors at the United States Coast and Geodetic Survey stations in Louisiana, Texas, Arkansas, Oklahoma, Kansas, Mississippi, Alabama, and California have been computed by deducting the normal values of the earth's magnetic field from the absolute measurements, at each station, of the declination and of the vertical and horizontal magnetic intensities.

The local magnetic vectors have been plotted as vector triangles at the respective stations on the vector maps of the different states.

These vectors indicate the intensity and the direction in space of the magnetic lines of force, as due to local magnetic anomalies mainly within the first 15,000 feet of subsurface.

As local magnetic anomalies are, with negligible exceptions, the result of geologic features, either of structural or petrographic character, a large amount of regional and local geologic information is obtained by a study of these maps.

The main magnetic anomalies of the different states have been interpreted in terms of geology, though no attempt has been made to exhaustively interpret all vectors.

The vector maps show at a glance which areas are of interest for magnetometer surveys, what size anomalies may be expected, how far a magnetometer survey has to be extended, or where it is best commenced to cover a certain area.

As the vectors are of sufficient accuracy, they allow a checking and tying-in of scattered magnetometer surveys.

INTRODUCTION

Extensive studies of the anomalies of the magnetic vertical intensity and their correlation with geology were made some years ago in Europe,[3] in the Mid-Continent and Gulf Coast areas[4] and in California.[5]

[1] Read in part before the Association at the Oklahoma City meeting, March 25, 1932. Manuscript received, May 18, 1932.

[2] Geologist and geophysicist, 6241 Richmond Avenue.

[3] A. Nippoldt, "Karten der Verteilung des Erdmagnetismus und seiner oertlichen vertikalen Stoerungen in Europa," *Archiv des Erdmagnetismus*, Heft 6 (Julius Springer, Berlin, 1927).

[4] Magnetometer survey of seven states in the Mid-Continent and Gulf Coast regions, by L. Spraragen, series of articles published in the *Oil and Gas Journal* (1928–29).

[5] G. B. Somers, "Anomalies of Vertical Intensity Compared with Regional Geology for the State of California," *Colorado School of Mines Magazine* (September, 1929).

The investigations were based on the measurements made by the surveys of the respective European states, and by the United States Coast and Geodetic Survey.[1]

Though by these studies a definite connection between broad structural features—such as the West Texas Permian basins, the Central Mineral region, the main geological trends in California and Europe—and magnetic anomalies of the vertical intensity could be definitely established, such studies could only incidentally yield any information about the type of structure which is of special interest

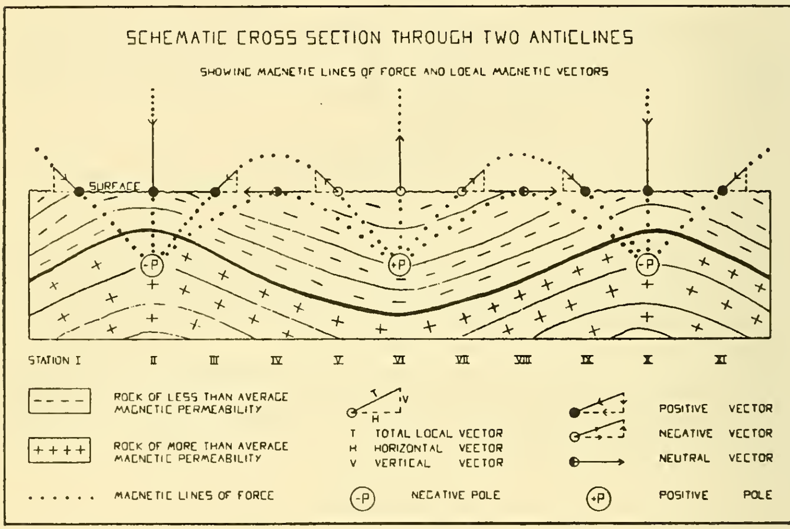


Fig. 1

to the commercial geologist, because the distances between the magnetic stations, ranging from 20 to 50 miles, were far too large.

It is quite evident that two, three, and more magnetic "highs" and "lows" may occur between stations placed at such distances and the lines of equal magnetic intensity, drawn on the base of such stations, as a rule veil, rather than set forth, the local magnetic anomalies which are due to small "structures."

Study of Figure 1 will make the previous statement clear. This cross section through two anticlines separated by a syncline demonstrates schematically the paths of the magnetic lines of force and the

[1] "United States Magnetic Tables and Magnetic Charts for 1925 by U. S. Department of Commerce," *U. S. Coast and Geodetic Survey Serial 453.*

local magnetic vectors as due to the local "structures" only, after the normal magnetic field of the earth has been eliminated. The total local vector is shown to be resolvable into two components—the local vertical and the local horizontal magnetic vector.

We have assumed that the material of the anticlines is of more than average magnetic permeability and the material of the syncline of less. Thus the magnetic "high" corresponds with a structural "high," and the magnetic "low" with a structural "low." This is the ordinary condition, for example, if two granite ridges are embedded in shales and limestones. Should the material of the anticlines be of less than average magnetic permeability and the material of the syncline of more, then the shape of the magnetic lines of force remains the same, but their direction is opposite to the direction shown in the picture. Thus a magnetic "low" would correspond with a structural "high" and vice versa. This condition exists in many places, for example in Kansas, where granite plugs or ridges pierce through the overlying highly magnetic schists.

For a clearer understanding, let it be assumed that the magnetic effects of the two anticlines and the syncline are equivalent to the effects of two negative (−P) poles, and one positive (+P) pole, as shown in Figure 1.

Above the negative poles the magnetic intensity is directed perpendicularly downward toward the poles (positive vector). Above the positive pole the magnetic intensity is directed perpendicularly upward, away from the pole (negative vector). Along the surface, half way between the positive and negative poles, the two vertical tendencies compensate each other and only a horizontal component remains, directed from the positive toward the negative pole (neutral vector). Between the vertical and horizontal directions of the intensity there occurs a gradual change, as shown in the picture. As long as the vector is still directed downward, it may be called a positive vector; if the direction is upward, a negative vector.

It might happen now that stations I and III had been occupied by the Coast and Geodetic Survey. As both stations have the same magnetic vertical intensity they lie on the same isogam, which would pass over the entire structure without taking the slightest notice of it. The same would happen, if also stations III and IX had been surveyed. Any other combination of the vertical intensity of two or three stations give widely varying indications of some anomaly. We should consider, however, that most of the "structures," which create the local anomalies, lie at depths ranging from 2,000 to 15,000 feet and that the magnetic effect is felt within a horizontal distance from the edge of a "structure" of only about twice its depth. Therefore it is reasonable to assume that, within a distance of 20–50 miles, only one station is occupied, as a rule, on a particular "structure" and there is little use in combining the measurements of the different stations, unless broad structural features are suspected.

Contrary to the opinion of some magneticians,[1,2] the writer holds that at such large distances between stations, much more detailed information may be obtained by studying the horizontal intensity than by studying the vertical intensity, and thinks that this results clearly from a study of the horizontal intensities in Figure 1, especially if it be realized that the anomalies are 3-dimensional.

The best information is of course obtained by a combination of the horizontal and the vertical intensities of a magnetic vector in space.

The Coast and Geodetic Survey has measured the declination and the horizontal and vertical intensities at the occupied stations. If the normal declination and the horizontal and vertical intensity be figured for each station and these normal values are deducted[3,4] from the actually measured values, the local deviation of the declination and the local vertical and south-north horizontal component of the total local magnetic vector are obtained as the difference. From the deviation of the declination may be easily figured also the local west-east component of the total local magnetic vector.

The average absolute accuracy of the components of these local magnetic vectors empirically lies at ±40 gammas, the average accuracy relative to neighboring stations at ±20 gammas.

The local magnetic vectors have been plotted at their respective stations on the maps of the principal oil-producing states, in the shape of vector triangles. The two dashed lines represent the horizontal and the vertical vectors; the hypotenuse is the total vector. If the vector triangle be turned through 90° around the horizontal vector, which is of course the dashed line beginning at the station point, the direction in space of the total vector may easily be visualized and the direction marked by a pin, if the map is mounted on cardboard.

[1] A. Nippoldt, *op. cit.*

[2] G. B. Somers, *op. cit.*

[3] A. Nippoldt, *op. cit.*

[4] L. A. Bauer, "Chief Results of a Preliminary Analysis of the Earth's Magnetic Field for 1922," *Terr. Mag.*, Vol. 28 (1923), pp. 1–28.

In studying these local vector maps, it should be considered that, as shown in Figure 1, there exists somewhere along the prolongation of the horizontal vector either a magnetic "high" or "low," which corresponds with some kind of geologic feature, either of structural or petrographic character.

The great and irregular variations of the total intensity and of the directions of most of the horizontal vectors should make it clear that most of these features necessarily must be localized.

This is the reason why no attempt has been made to connect the stations by certain curves, indicating, for example, equal total intensities, equal angles of inclination or equal vertical and horizontal intensities, though by help of the vectors the last two sets of curves could, of course, be much more intelligently computed than from the vertical or horizontal components alone.

A few of the regional geologic "structures" can be quantitatively interpreted by the help of these vectors.

Due to the long distances between stations, it has been necessary to use the most reasonable geological interpretation of the vector maps. Though two, three, or more local "highs" and "lows" are possible between two neighboring stations, it is probable that regional geologic "structures" extend at right angles to the horizontal magnetic components, where the majority of the latter are nearly parallel with one another over a large area, that is, a common geologic feature is probably responsible for a series of vectors, if they are all directed toward, or away from, the same point.

Due to the long distances between stations, the interpretation was mainly restricted to regional magnetic anomalies, though it is expected it will be found that the main value of the vector maps lies just in these areas, where the magnetic anomalies are so localized, or so shallow that no relation between the individual vectors can be established.

For all those areas where the vectors indicate regional magnetic tendencies, we may reasonably assume that the vectors are mainly due to deeply buried regional structure. But if the main magnetic influence in these areas originates at great depths and if the total intensity of the vectors amounts to only a few hundred gammas, we must conclude that the average magnetometer surveys can not possibly yield much information with regard to commercial structure, unless the deeper structure is truly reflected within the shallower beds. The results of the average magnetometer surveys are comparable with the results which we would obtain with torsion balances, if their accuracy were reduced to ± 10 Eötvös. If, however, the accuracy of the magnetometer surveys is greatly increased, these surveys may yield important information about structure at commercial depths also in these areas where the magnetic anomalies as located by the average magnetometer survey are unquestionably due to deeply buried regional structure.[1]

As the vectors are of sufficient accuracy, they allow a checking and tying-in of scattered magnetometer surveys. These stations should prove of the same value to 3-dimensional magnetometer surveys as pendulum stations to gravimetric surveys.

The vector maps show at a glance which areas are of interest for magnetometer surveys, what size anomalies may be expected, how far a magnetometer survey has to be extended or where it is best commenced to cover a certain anomaly.

In the following study of the vector maps no attempt has been made for an exhaustive interpretation of all vectors. The reader may easily perceive an abundance of further detailed information, which may interest him, especially in connection with scattered magnetometer surveys and detailed geologic information.

LOUISIANA[2]

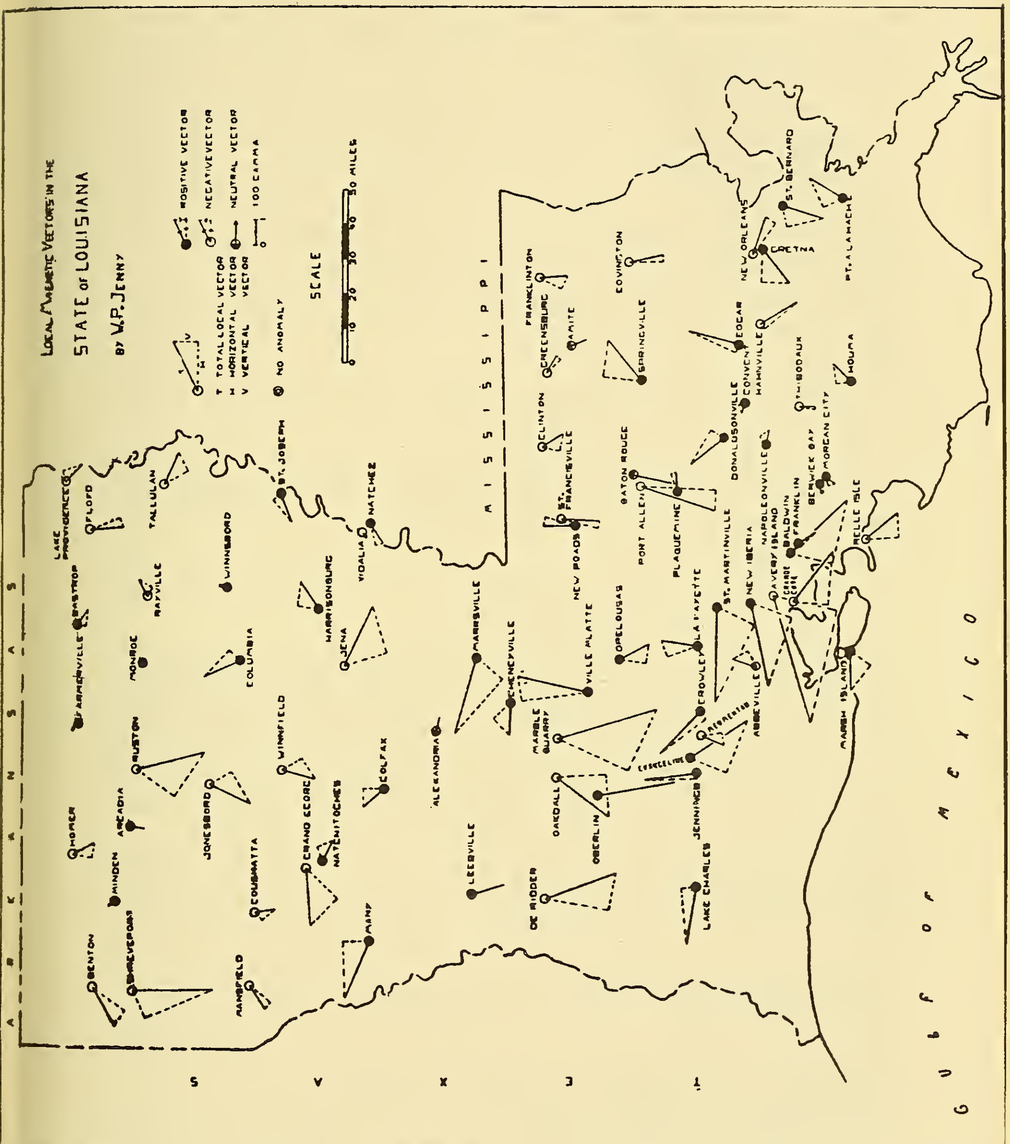
The vector map of this state (Fig. 2) gives a true picture of the known main structural trends and further indicates many distinct "structures."

Sabine uplift.—The horizontal components of the two negative vectors at Benton and Shreveport concur at a point about 10 miles southwest of Shreveport, at which point therefore a magnetic "high" is suggested. A line connecting this "high" with the "high" indicated a few miles south of Many coincides well with the axis of the Sabine uplift.

North Louisiana geosyncline.—The north Louisiana geosyncline is expressed by the negative vectors at Ruston, Jonesboro, and Winnfield.

[1] D. M. Collingwood, "Magnetics and Geology of Yoast Field, Bastrop County, Texas," *Bull. Amer. Assoc. Petrol. Geol.*, Vol. 14, No. 9 (September, 1930), pp. 1191-97.
E. D. Lynton, "Some Results of Magnetometer Surveys in California," *Bull. Amer. Assoc. Petrol. Geol.*, Vol. 15, No. 11 (November, 1931), pp. 1351–70.
A. Nippoldt, *Verwertung magnetischer Messungen zur Mutung* (Berlin, 1930), p. 58.

[2] The local magnetic vectors are based on the information given in "United States Magnetic Tables and Magnetic Charts for 1925," *op. cit.*


LOCAL MAGNETIC VECTORS IN THE
STATE OF LOUISIANA
BY V.P. JENNY
POSITIVE VECTOR
NEGATIVE VECTOR
NEUTRAL VECTOR
100 GAMMA
T TOTAL LOCAL VECTOR
H HORIZONTAL VECTOR
V VERTICAL VECTOR
NO ANOMALY
SCALE
0 10 20 30 40 50 MILES
A R K A N S A S
M I S S I S S I P P I
T E X A S
G U L F O F M E X I C O
SHREVEPORT
BENTON
MANSFIELD
MINDEN
HOMER
ARCADIA
RUSTON
MONROE
FARMERVILLE
BASTROP
RAYVILLE
FLOYD
LAKE PROVIDENCE
TALLULAH
WINNSBORO
ST. JOSEPH
COLUMBIA
JONESBORO
WINNFIELD
COUSHATTA
GRAND ECORE
NATCHITOCHES
MANY
COLFAX
JENA
HARRISONBURG
VIDALIA
NATCHEZ
ALEXANDRIA
MARKSVILLE
CHENEYVILLE
LEESVILLE
DE RIDDER
OAKDALE
OBERLIN
MARBLE QUARRY
VILLE PLATTE
OPELOUSAS
NEW ROADS
ST. FRANCISVILLE
CLINTON
GREENSBURG
AMITE
FRANKLINTON
COVINGTON
SPRINGVILLE
BATON ROUGE
PORT ALLEN
PLAQUEMINE
DONALDSONVILLE
CONVENT
EDGAR
HAHNVILLE
NEW ORLEANS
GRETNA
ST. BERNARD
PT. A LA HACHE
HOUMA
THIBODAUX
MORGAN CITY
BERWICK BAY
FRANKLIN
BALDWIN
NAPOLEONVILLE
AVERY ISLAND
NEW IBERIA
ST. MARTINVILLE
LA FAYETTE
CROWLEY
ABBEVILLE
JENNINGS
LAKE CHARLES
MARSH ISLAND
BELLE ISLE

Area east of geosyncline.—East of the geosyncline are found two trends of magnetic "highs," which parallel the axis of the Sabine uplift: (1) the Farmerville-Columbia-Harrisonburg trend and (2) the Bastrop-Winnsboro trend.

On the second trend lie the gas fields of Monroe and Richland. The magnetic "high" indicated slightly north of Bastrop is interpreted as an uplift in the deep basement rocks, which uplift accounts for the Monroe gas field. From magnetometer work we know that half way between Rayville and Winnsboro a similar magnetic "high" and corresponding uplift accounts for the Richland gas field.

The magnetic "high" as indicated south of Winnsboro therefore suggests other possibilities of uplifts along the known trend toward the south.

Central Louisiana.—A magnetic "high" south of Colfax is indicated.

The concurrence of the horizontal vectors of Jena, Alexandria, and Marksville at a point about 10 miles northeast of Marksville suggests a magnetic and possibly structural "high" of large dimensions at that point.

Other magnetic "highs" are suggested between Leesville and De Ridder and southeast of Cheneyville.

At St. Francisville and New Roads there is a striking example of two stations which may reasonably be assumed to lie on the same "structure." The magnetic lines of force come out of the ground at a low angle at St. Francisville, assume a horizontal direction between the two stations, and penetrate into the ground at a low angle at New Roads, directed toward a magnetically positive "structure" on the south.

The distances are too great to say that this "structure" is the eastern extension of a west-east ridge or anticline as indicated between Ville Platte and Opelousas, but this possibility is strongly suggested.

Southern Louisiana.—There is a northwesterly trend of magnetic "highs" from Morgan City to St. Martinsville; this trend parallels a line through the Five Island domes, about 20 miles west, and may correspond with the so-called Iberian structural axis.

No continuation of this trend beyond St. Martinsville can be preceived from the vectors. An east-west trend of magnetic "highs" from St. Martinsville to Lake Charles, however, is clearly noticeable. This trend passes south of Lafayette, northeast of Crowley, north of Evangeline, south of Lake Charles, and corresponds with the well known structural axis in southern Louisiana along which, besides several unproved prospects, lie the domes and oil fields of Jennings, Roanoke, Welsh, and Iowa. The indicated "high" south of Lake Charles suggests the conclusion which has been drawn from recent seismograph work in this area, that the structural axis turns southward past the Iowa dome and continues south of Lake Charles toward the Lockport dome.

The large negative vectors at DeRidder, Oakdale, and Marble Quarry and the horizontal vector at Oberlin suggest a west-east magnetic "high" trend south of Oberlin. Due to the large intensity of these vectors, it does not seem justifiable to connect them with the magnetic "high" trend of Lafayette-Lake Charles but it may be concluded that the magnetic "high" or anticlinal structure indicated between Ville Platte and Opelousas extends westerly past the Texas border. A corresponding gravimetric high trend is indicated by torsion-balance work and is usually interpreted as due to a deep petrographic or structural regional feature.

TEXAS[1]

REGIONAL STRUCTURE

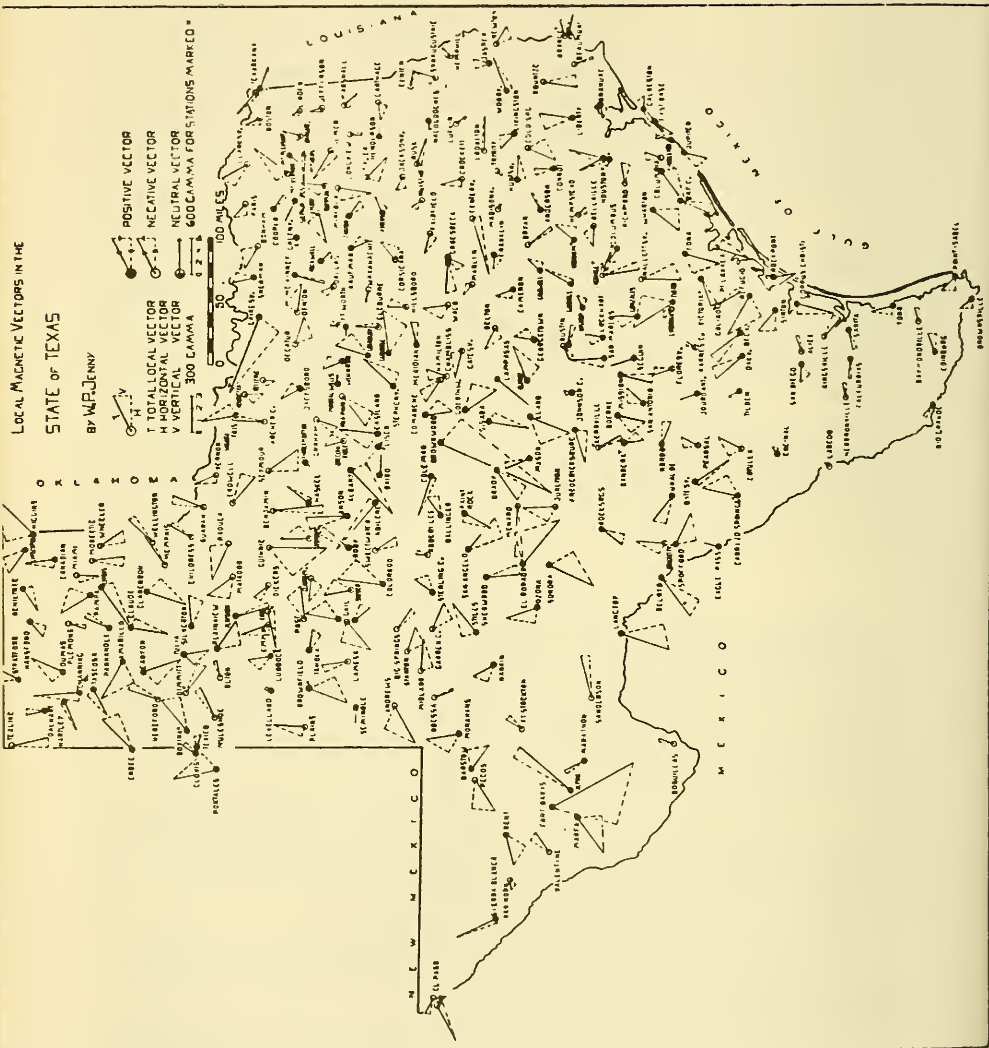
Delaware basin.—The magnetic center of the Delaware basin seems to lie between Pecos and Barstow. The negative vector at Pecos is directed toward the west, and at Barstow, toward the east or toward the Central Basin platform, which is indicated by the positive vector at Monahans.

Main Permian basin.—In the southwest part of the main Permian basin the negative vectors from Odessa to Big Springs are directed toward the southeast, and the negative vectors at Andrews and Lamesa toward the northwest, which means that the magnetic axis of a basin lies between these two sets of stations.

The Bend arch.—The Bend arch is indicated by the positive vectors at Brownwood and neighboring stations, with a north extension to Jacksboro.

Central Mineral region.—The horizontal components of the positive vectors at Llano and Burnet and the neutral vector at Lampasas all concur at a point half-way between these three stations. This

[1] The local magnetic vectors are based on the information given in "United States Magnetic Tables and Magnetic Charts for 1925," *op. cit.*


LOCAL MAGNETIC VECTORS IN THE
STATE OF TEXAS
BY W.P. JENNY
T TOTAL LOCAL VECTOR
H HORIZONTAL VECTOR
V VERTICAL VECTOR
300 GAMMA
POSITIVE VECTOR
NEGATIVE VECTOR
NEUTRAL VECTOR
100 MILES
LOUISIANA
OKLAHOMA
NEW MEXICO
MEXICO
GULF OF MEXICO

point should correspond with the center of a large magnetic "high," due to the Llano-Burnet uplift.

East Texas syncline.—The East Texas syncline is indicated by many negative vectors in the northeastern part of the state.

Northwest Texas.—A basin of considerable extent is indicated in northwest Texas, with its magnetic center half-way between Hereford, Dimmitt, and Bovina. The rim of this basin probably extends from Bovina northward toward Endee, eastward and southward toward Canyon, Tulia, and Plainview, and westward north of Olton and Muleshoe back to Bovina. This basin may hold oil possibilities similar to the Delaware or Black Shale basins.

Gulf Coast.—Many of the horizontal magnetic components along the Gulf Coast are nearly perpendicular to the coast; we may therefore expect regional "structures" parallel with the coast.

Especially in southwest Texas such "structures" seem to be clearly indicated. We may further follow up the regional magnetic high trend between Ville Platte and Opelousas, as discussed in the last paragraph on "Louisiana," south of Newton, Woodville, Cold Spring, Conroe, Hempstead, Bellville, Columbus. This trend may prove of interest in the light of recent developments south of Conroe.

LOCAL STRUCTURE

Anticlinal structure at Big Lake is indicated by a positive vector at Rankin and at Stiles.

A southwest-northeast ridge is indicated by the positive vectors at Sherwood and San Angelo.

A magnetic "high" is indicated between Ozona, Eldorado, and Sonora. With this "high" may correspond one "structure" or more, which, in the light of recent development west of Ozona, may prove of great interest.

The great variety in the magnetic field along the coast should prove that magnetometer surveys there may be not only of a regional, but also of a local, structural interest in many places.

The Refugio anticline and the Pierce-Junction and the Liberty domes are indicated by positive vectors.

It is possible that these magnetic "highs" are the result of the uplifted strata at relatively shallow depths, though the possibility also exists that they are due to uplifts in the basement. In any case, more detailed magnetic investigations may prove of great value, especially in southwest Texas.

ARKANSAS[1]

Arkansas has been divided into four main geological sections: (1) the Gulf Coastal Plain, south and east of a line passing through DeQueen, Arkadelphia, Little Rock, and Pocahontas; (2) the Arkansas Valley, between a line from Van Buren to Heber Springs on the north and a line from Waldron to Little Rock on the south; (3) the Ouachita Mountain region, between the Arkansas Valley and the Gulf Coastal Plain; (4) and the Ozark region, north of the Arkansas Valley.

Gulf Coastal Plain.—The southern part of the Gulf Coastal Plain from DeQueen to Lake Village is indicated as a large negative anomaly, interrupted by a north-south trend of magnetic "highs" from Hamburg to Star City, which seems to be the northward continuation of the magnetic high trend from Winnsboro to Bastrop in Louisiana.

The well known local anomaly at Rison is due to peridotite intrusions with as much as 20 per cent magnetite content.

The positive vectors from Stuttgart to Forest City seem to belong to a large northwest-southeast magnetic high trend, toward which are also directed the negative vectors at Wynne and Helena. The large irregular positive and negative vectors farther north suggest very localized and relatively shallow sources of magnetism.

Ouachita Mountain region.—In the Ouachita Mountain region it is interesting to note that the two positive vectors at Mena and Mount Ida are almost horizontal and directed toward the Arkansas Valley, instead of south toward the axis of the Ouachita Mountains, as would be expected if the Ouachita Mountains were the result of uplift in the Basement complex.

If it be assumed that the Basement complex is the cause of the magnetic anomalies in this region, the magnetic conditions suggest that this complex lies deeper below the Ouachita Mountains than below the Arkansas Valley, because the magnetic lines of force seem to come out of the ground at DeQueen and Murfreesboro, to assume a horizontal direction over the mountain range, and to penetrate into the ground in the Arkansas Valley.

Thus the magnetic conditions seem to be in favor of the theory that the Ouachita Mountains are the result of a huge overthrust, as explained by W. A. J. M. van Watershoot van der Gracht.[2] Similar

[1] The local magnetic vectors are based on the information given in "United States Magnetic Tables and Magnetic Charts for 1925," *op. cit.*

[2] W. A. J. M. van Watershoot van der Gracht, "Permo-Carboniferous Orogeny in South-Central United States," *Bull. Amer. Assoc. Petrol. Geol.*, Vol. 15, No. 9 (September, 1931), pp. 991–1057.

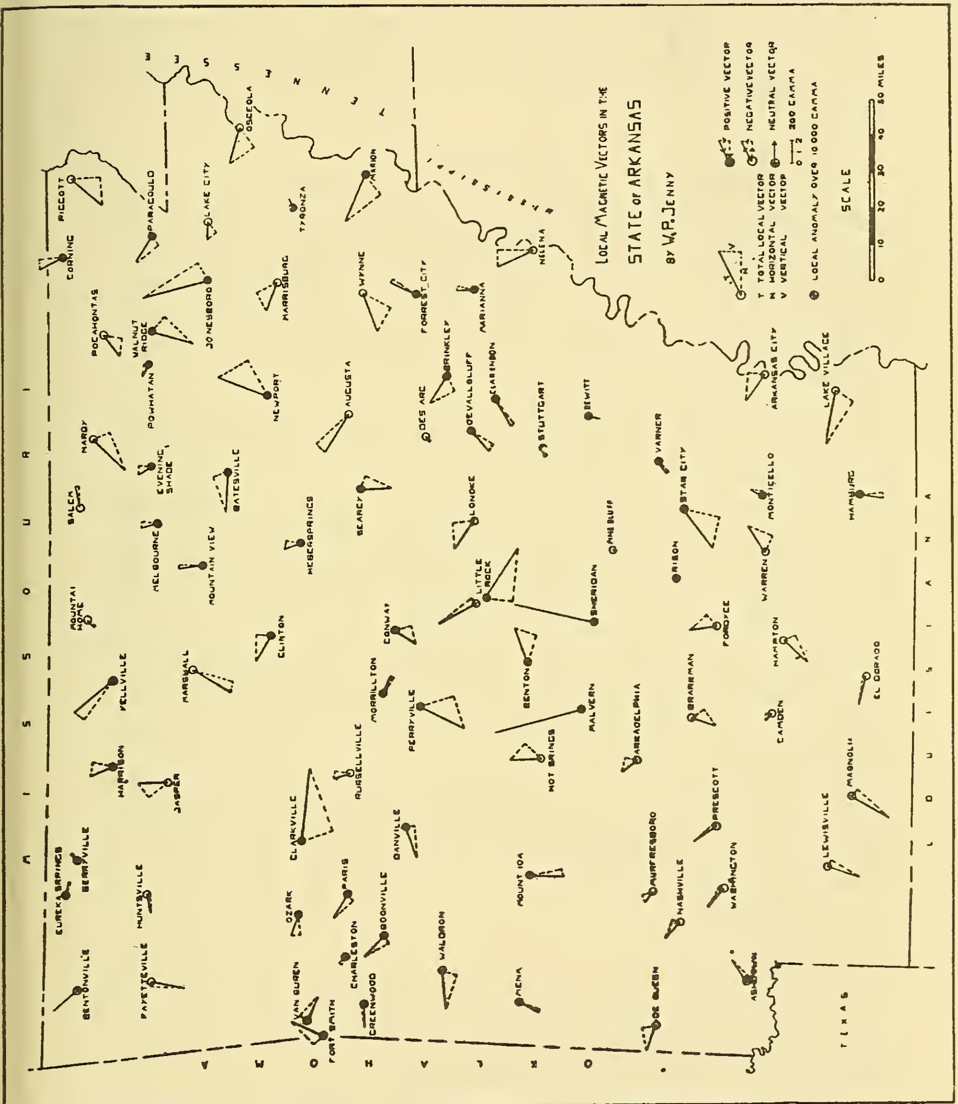

LOCAL MAGNETIC VECTORS IN THE
STATE OF ARKANSAS
BY W. P. JENNY
POSITIVE VECTOR
NEGATIVE VECTOR
NEUTRAL VECTOR
200 GAMMA
T TOTAL LOCAL VECTOR
H HORIZONTAL VECTOR
V VERTICAL VECTOR
LOCAL ANOMALY OVER 10 000 GAMMA
SCALE
0 10 20 30 40 50 MILES
MISSOURI
TENNESSEE
MISSISSIPPI
LOUISIANA
TEXAS
OKLAHOMA
LITTLE ROCK

FIG. 4

magnetic conditions exist in the region of the nappes of the European Alps.

Arkansas Valley and Ozark Mountain regions.—The magnetically positive zones of the Arkansas Valley and Ozark Mountain regions are separated by a negative zone extending from Fayetteville tc Marshall.

OKLAHOMA[1]

Eastern Oklahoma.—In eastern Oklahoma is to be observed the same arrangement of the magnetic anomalies as in western Arkansas; that is, the positive vector at Miami is indicative of the Ozark Mountains, the negative vectors near Westville are indicative of the magnetic "low" between the Ozark uplift and the Arkansas Valley, which latter "structure" is represented by the positive vectors at Sallisaw, Muskogee, Stigler, and Poteau.

The few stations in the general area of the Ouachita Mountains seem to confirm the interpretation given for the magnetic anomaly in the Ouachita region of Arkansas.

On the basis of the west-east direction of the horizontal magnetic components, it seems possible to follow a positive magnetic trend from Nowata to Pryor and a second trend from Bartlesville through Tulsa, Sapulpa, and Okemah to Holdenville, along which trends lie the main oil "structures" of northern Oklahoma.

Arbuckle Mountains.—The Arbuckle Mountains are indicated by a strong positive vector at Tishomingo. The positive vector at Durant might indicate the axis of the Arbuckle Mountains as extending southeast and northwest.

Wichita Mountains.—The Wichita Mountains lie along a magnetic high trend beginning at Marietta and passing north of Waurika, Walters, and Hobart to Sayre. The oil fields of southern Oklahoma lie along this line.

A magnetic high trend is indicated between Sulphur and Oklahoma City.

Another high trend lies east of Rush Springs, Chickasha, and Minco.

Another possible high trend lies between Alva and Carmen, between Trail and Taloga, and east of Hammon.

The southern extension of the Nemaha mountains might be indicated by the positive vectors at Medford, Pond Creek, and Enid.

[1] The local magnetic vectors are based on the information given in "United States Magnetic Tables and Magnetic Charts for 1925," *op. cit.*

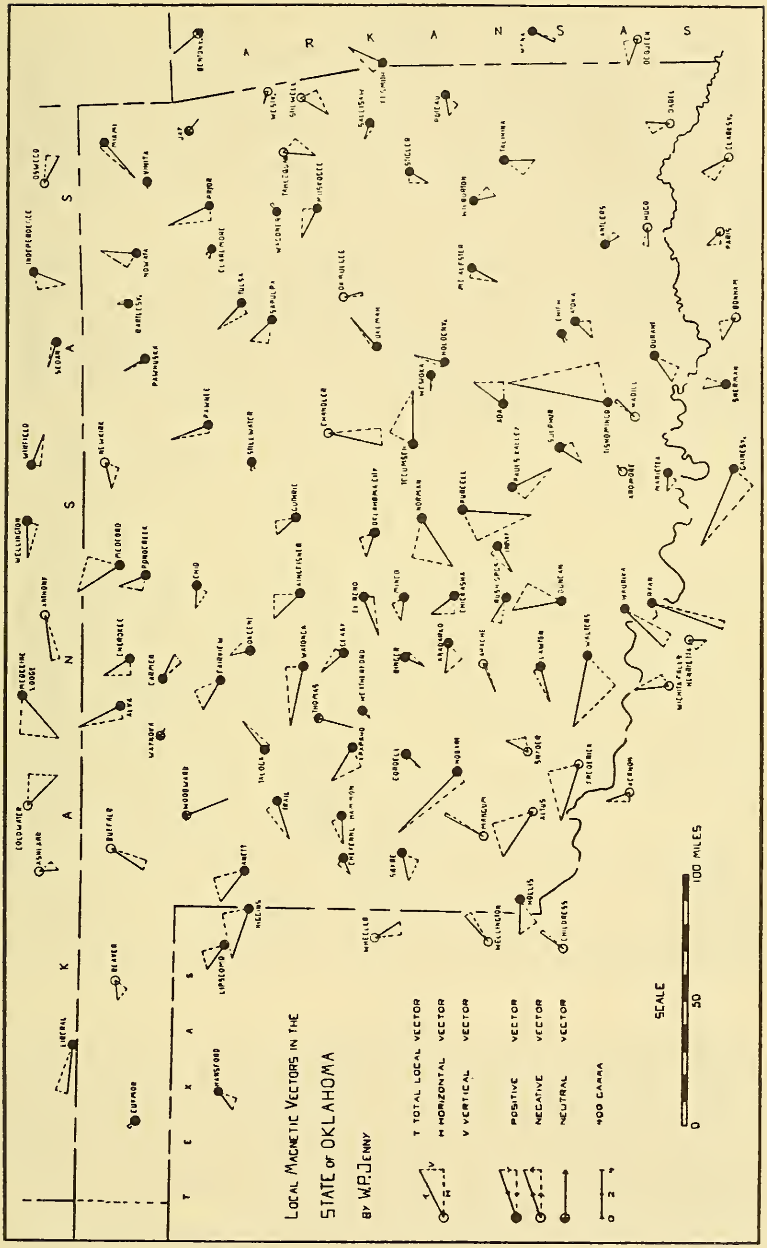

LOCAL MAGNETIC VECTORS IN THE
STATE OF OKLAHOMA
BY W.P. JENNY
T TOTAL LOCAL VECTOR
H HORIZONTAL VECTOR
V VERTICAL VECTOR
POSITIVE VECTOR
NEGATIVE VECTOR
NEUTRAL VECTOR
400 GAMMA
SCALE
0
50
100 MILES
K A N S A S
A R K A N S A S
T E X A S

FIG. 5

KANSAS[1]

Nemaha Granite ridge.—The outstanding feature of eastern Kansas is the Nemaha Granite ridge, which extends from Seneca in a southwesterly direction toward Winfield. The top of the granite lies at +560 feet near Seneca and at −2,200 feet near Winfield, which indicates a southward dip of the ridge. From the *Kansas State Geological Survey Bulletin 13* we quote:

> The earth movement, which elevated the granite appears to have taken place in the post-Mississippian and pre-Pennsylvanian time, for the upper part of the Ordovician and all of the Mississippian has been stripped away in the region of Eldorado and Augusta. North of Butler County, along the axis of the uplift, all of the sedimentary rocks older than the Pennsylvanian were removed so that the Pennsylvanian rests directly on the granite ridge.

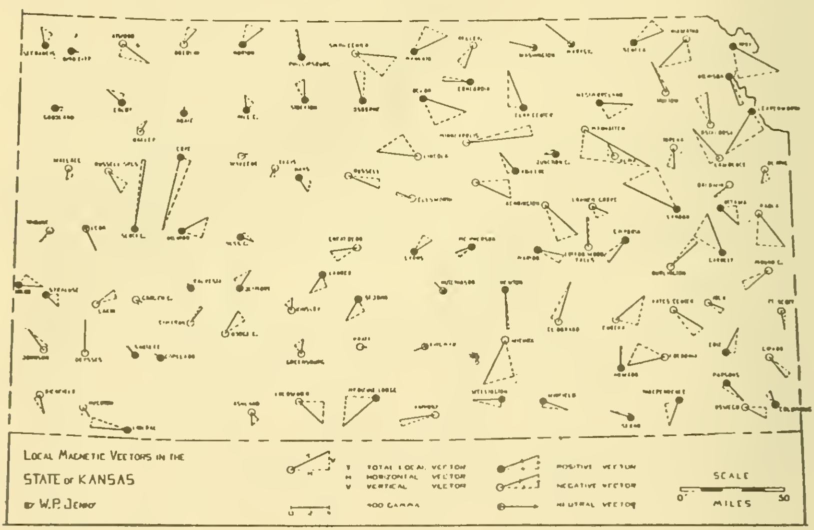


FIG. 6

If, therefore, the granite is embedded in lower Ordovician in the southern part of the ridge and in the Pennsylvanian in the northern part, we may expect this ridge to be indicated magnetically as a "low" on the south and as a "high" on the north if the Ordovician is of a higher and the Pennsylvanian of a lower magnetic permeability than the granite.

[1] The local magnetic vectors are based on the information given in "United States Magnetic Tables and Magnetic Charts for 1925," *op. cit.*

But, though in central and western Kansas magnetic "lows" seem actually to correspond in some places with structural "highs," the vector map seems to be in favor of the assumption that in the eastern part of the state magnetic "lows" correspond with structural "lows," and magnetic "highs" with structural "highs."

The axis of the Nemaha ridge lies slightly east of Seneca, 5 miles west of Alma and Council Grove, slightly west of Cottonwood Falls and Eldorado, and between Winfield and Wellington. The ridge has a very steep eastern flank but slopes much more gently toward the west. Thus, the center of mass of the ridge lies considerably west of the structural axis, which would explain a corresponding shift of the magnetic axis.

We think that the axis of the ridge is indicated by the positive vectors at Seneca, Council Grove, and Winfield-Wellington, and by the negative vectors at Alma and Eldorado, both of which point toward the west, so that the magnetic lines of force, emerging at Alma and Eldorado, may enter the positive ridge a few miles west of the stations.

Drilling has revealed a structural low trend along the east flank of the ridge and another extended "low," with a west-east axis, between Eureka-Emporia and Yates Center-Burlington. The first trend is indicated magnetically by the negative vectors at Hiawatha, Holton, Alma, and Eldorado. In the second trend, it is clear that the negative vectors at Eureka, Yates Center, Iola, Burlington, Cottonwood Falls, and the positive vector at Emporia may all be assumed to emerge from the same magnetic "low."

We have pointed out already that in central and western Kansas both magnetic "lows" and "highs" may correspond with structural "highs."

The large and irregular vectors should yield considerable information, if the vector map is combined with magnetometer surveys and detail geologic maps.

MISSISSIPPI[1]

Many of the magnetic horizontal components of the vectors in the northern half of Mississippi have a northeast or a southwest direction, which may indicate a general northwest-southeasterly trend of the deeper "structure." We might suspect two magnetic high trends: (1) Butler-Philadelphia-Kosciusko-Greenwood-Rosedale on the south and

[1] The local magnetic vectors are based on the information given in "United States Magnetic Tables and Magnetic Charts for 1925," *op. cit.*

(2) Carrollton-Columbus-Pittsboro-Water Valley, between Helena and Tunica on the north, and interpret them as indicative of the generally assumed connection of the Appalachian with the Ouachita Moun-

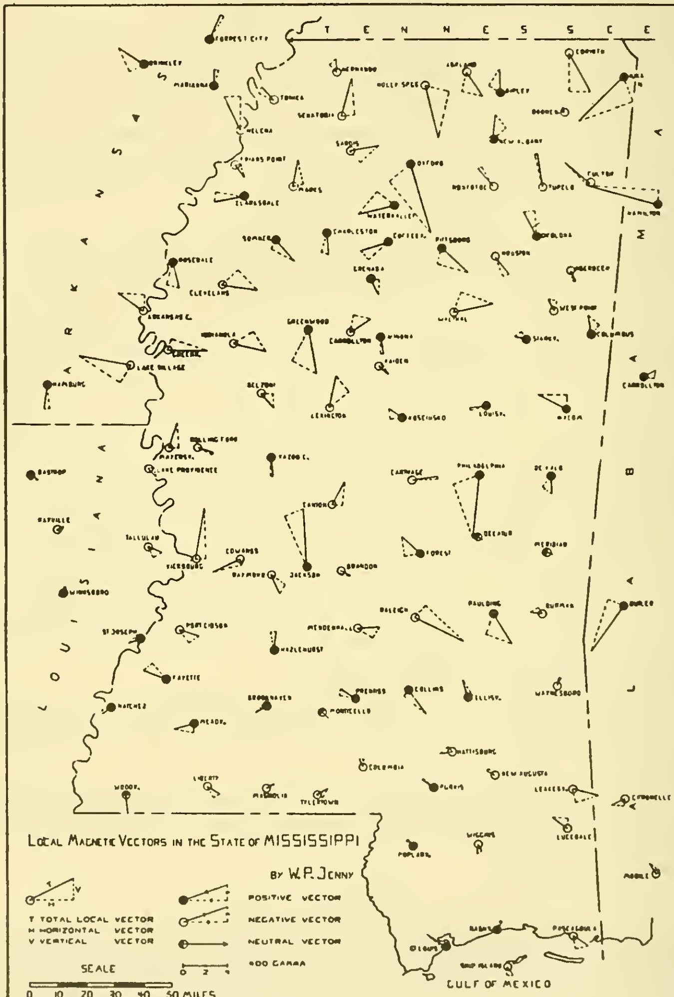


Fig. 7

tains. The distances between the vectors and their irregularities are, however, too great to allow any definite statement, without additional magnetic and geologic data.

The positive vector at Jackson indicates a magnetic "high" slightly southeast of the town. This "high" is due to an igneous plug, supposedly nepheline-syenite, and is in agreement with magnetometer surveys of the well known gas field. An interesting west-east trend of magnetic "highs" extends from Fayette to Ellisville.

In general, there are hardly more than two vectors in Mississippi, which can be definitely attributed to the same structure. This circumstance may be interpreted as due to local and relatively shallow sources of magnetism, which should be favorable for detailed investigations by the magnetometer.

ALABAMA[1]

From the northeastern corner of Alabama, a magnetic high trend, extending southwest to Greensboro, indicates the southern extension of the Appalachian Mountains.

The vectors of the stations surrounding Greensboro are all positive and indicate by their direction that a large continuous positive anomaly exists, covering that part of the state, which is explained as the "southernmost tip of the folded Appalachian Mountains" by D. R. Semmes.[2]

This "high" is an example of a large magnetic anomaly, corresponding with an extended structural feature, which could be correctly interpreted by the study of the vertical intensity alone.[3]

Another outstanding feature is the "low" at Wetumpka. This "low" may lie along a low trend Center-Anniston-Talladega-Rockford-Wetumpka, with the high trend Guntersville-Greensboro on the west and another high trend on the east, which would pass near Ashland, Dadeville, and between Tuskegee and Wetumpka.

In the southern part of the state a low trend is indicated between Georgiana and Greenville, extending northwestward toward Camden and southeastward toward Elba. Another possible low trend lies east of Tuskegee, Union Springs, and Troy.

The positive vectors at Opp, Geneva, and possibly also at Ozark and Abbeville appear to be due to localized "structures," which might be of interest for oil and gas possibilities, because they lie in the Gulf Coastal belt.

[1] The local magnetic vectors are based on the information given in "United States Magnetic Tables and Magnetic Charts for 1925," *op. cit.*

[2] D. R. Semmes, *Geol. Survey of Alabama Bull. 22.*

[3] L. Spraragen, *op. cit.*

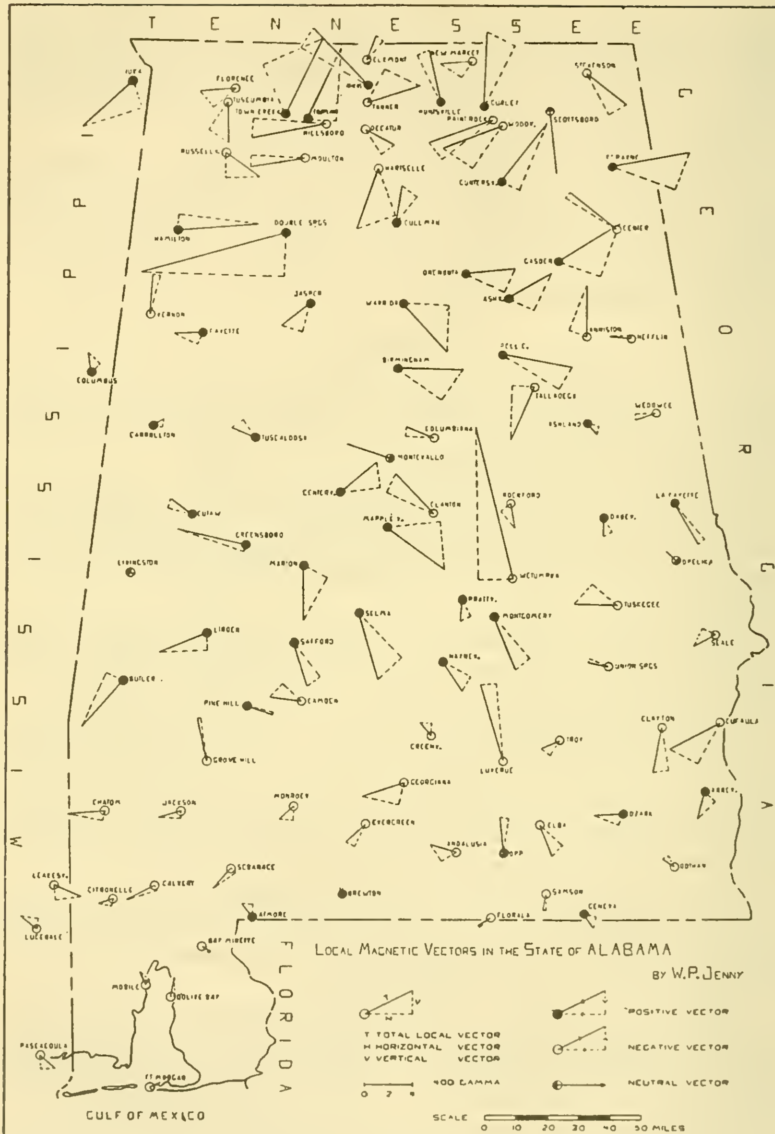

TENNESSEE
MISSISSIPPI
GEORGIA
FLORIDA
GULF OF MEXICO
LOCAL MAGNETIC VECTORS IN THE STATE OF ALABAMA
BY W.P. JENNY
T TOTAL LOCAL VECTOR
H HORIZONTAL VECTOR
V VERTICAL VECTOR
POSITIVE VECTOR
NEGATIVE VECTOR
NEUTRAL VECTOR
400 GAMMA
SCALE
0 10 20 30 40 50 MILES

FIG. 8

THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS

GEOPHYSICS

1932

INCLUDING PAPERS PRESENTED BEFORE THE SOCIETY OF PETROLEUM GEOPHYSICISTS AT THE ANNUAL CONVENTION OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS AT OKLAHOMA CITY, MARCH 25, 1932; REPRINTED FROM THE BULLETIN OF THE ASSOCIATION, VOL. 16, NUMBER 12, DECEMBER, 1932

TRANSACTIONS
SOCIETY OF PETROLEUM GEOPHYSICISTS
VOL. III

PUBLISHED BY
THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
TULSA, OKLAHOMA, U. S. A.

Published, December, 1932

Composed aud Printed by
George Banta Publishing Company
Menasha, Wisconsin, U. S. A.

An interesting feature is the indicated high trend between D'Olive Bay-Mobile-Lucedale on the south and Bay Minette-Citronelle-Leakesville on the north. This trend is parallel with the Hatchetigbee anticline, about 50 miles farther north.

The Hatchetigbee anticline is not indicated by the vectors, because the stations are not favorably located. It is parallel with another magnetic high trend about 30 miles farther north, extending from Butler toward midway between Grove Hill and Pine Hill. This last trend is known from magnetometer work in combination with the vector map.

It seems, therefore, that there are three parallel pronounced anticlinal trends, south of, and practically at right angles to, the Appalachian trend from Guntersville to Greensboro.

In the writer's opinion, the three anticlinal trends in southwestern Alabama represent a mountain system distinctly different from the Appalachians. If the Appalachians extend farther than Greensboro and Linden, they are expected to disappear below the anticlinal trend at Butler. It is further suggested that the three anticlinal trends in southwestern Alabama possibly represent an eastern extension of the Ouachita Mountain system, inasmuch as the high trend at Butler might be followed through Philadelphia-Koskiusko-Greenwood-Rosedale into the Ouachita Mountains, and the high trend at Mobile through Ellisville-Fayette-Winnsboro-Bastrop-Hamburg-Star City.

In the northern part of Alabama we observe a pronounced magnetic high trend Hamilton-Double Springs-Cullman. This trend seems to join the high trend Guntersville-Greenboro, and to extend eastward between Stevenson and Fort Payne. In the writer's opinion, the trend north of Fort Payne to Hamilton is representative of the main Appalachian trend, whereas the trend Guntersville-Greensboro is a southwestward extension, branching off the main trend at Guntersville.

A possible westward continuation of the main trend of the Appalachians seems obscured by the magnetic influence of the Cincinnati uplift, which is indicated in the northeastern corner of Mississippi by the "high" at Iuka. On the basis of magnetics only, we might suspect a high trend from Oxford toward a location midway between Helena and Tunica, with a westward continuation toward Brinkley and the Arkansas Valley. A continuation of the Arkansas Valley magnetic high trend into Oklahoma might be suspected south of Muskogee-Guthrie-Kingfisher-Watonga.

Additional magnetic and geologic information is needed, however, to clear up the possible existence and geologic meaning of this suspected trend.

CALIFORNIA[1]

It seems probable that the main magnetic effects in California are due to deep regional structure with a northwest-southeasterly trend. This interpretation explains the southwest and northeast directions of the most of the horizontal components, and is well established by the known regional geology.

A magnetic high trend extends all along the California Valley from Red Bluff in the north through Stockton to Bakersfield in the south.

A second parallel high trend may be noticed east of Bradley, San Lucas, Soledad, and Salinas.

Farther south it seems possible to construct a high trend from Oceanside to Santa Monica and another parallel trend from San Clemente to San Nicholas.

The magnetic stations are considerably denser in the valleys than in the mountainous regions of the Sierra Nevada and Coast Ranges or in the Mojave Desert. Inasmuch as the valleys represent sedimentary basins, whereas the mountainous regions are mainly composed of volcanic rocks, schists, gneisses, and highly metamorphosed sediments, it is surprising to note that the average vector in the valleys is of about the same magnitude as the average vector in the mountainous regions. This circumstance may no doubt be explained in part by the geographic location and distribution of the stations, but it may possibly also have a deeper meaning, which the writer will try to explain.

If the magnetic field is due to regional "structures" and if the cross sections in Figure 2 of "Decline of Great Basin"[2] are true, we should logically expect a magnetic high trend about 20 or 30 miles west of the San Andreas fault, over the Santa Lucia "blocks," another high trend over the Sierra Nevada "block," and a low trend in the California Valley, or sedimentary basin, between these two "blocks."

The cross section mentioned passes approximately through Soledad and slightly north of Madera, on our vector map. We notice a considerable difference between the actual magnetic data and those that would be assumed if based on the cross section.

[1] The local magnetic vectors are based on the information given in "United States Magnetic Tables and Magnetic Charts for 1925," *op. cit.*

[2] J. Edmund Eaton, "Decline of Great Basin, Southwestern United States," *Bull. Amer. Assoc. Petrol. Geol.*, Vol. 16, No. 1 (January, 1932), p. 5.

The magnetic high trend indicated between Salinas and Hollister lies approximately along the San Andreas fault, that is, along the east flank of the Santa Lucia "blocks," and the high trend in the California

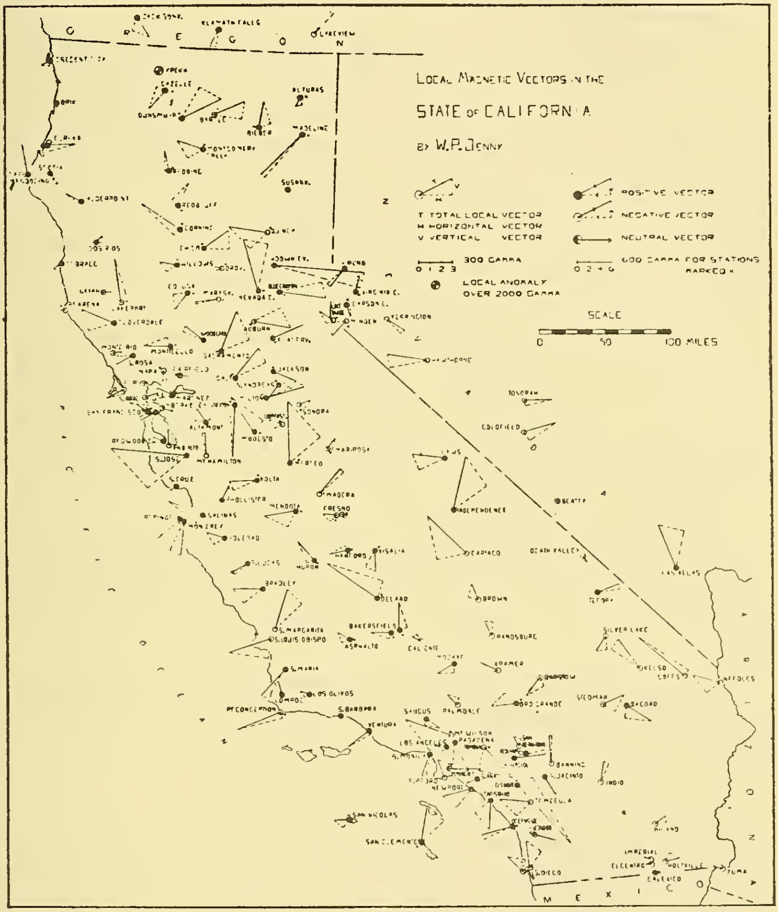


FIG. 9

Valley, a few miles northeast of Mendota, practically lies above the western edge of the Sierra Nevada "block," as shown in the figure mentioned.

There are not enough stations on the Sierra Nevada or Santa Lucia "blocks" to make possible any definite statement regarding their magnetic behavior. From the magnetic vectors at Bradley, San Lucas, Soledad, and Salinas it seems, however, permissible to conclude that the Santa Lucia "blocks" would act somewhat neutral magnetically. If this should be the fact, then these "blocks" could not possibly reach great depths.

It is interesting to note that the two pronounced magnetic high trends occur where they are least expected, that is, along the east flank of the Santa Lucia "blocks" and along the west flank of the Sierra Nevada "block." If we interpret these high trends in the usual way, we should expect an anticlinal trend in the Basement complex below the California Valley, or along the west edge of the Sierra Nevada "block," and another anticlinal trend in the Basement complex approximately below the San Andreas fault, or along the east edge of the Santa Lucia "blocks."

To interpret the magnetic data properly, we have to assume that the Santa Lucia "blocks" and the Sierra Nevada "block" are floating on a Basement complex, which is greatly depressed below the "blocks" and elevated along the edges of the "blocks." This might be explained either by assuming that these two "blocks" are laccoliths, or by the assumption of huge overthrusts.

The mechanics of the "progressive breaking down of the crystalline Pacific borderland," as explained by the arrows in Figure 2 and in the text of "Decline of Great Basin,"[1] are not sufficiently free from possible objections, to speak plainly against the foregoing interpretation.[2]

In connection with a detail geologic map and scattered magnetometer surveys, the vector map of California may prove of value also for the study of local anomalies.

SUMMARY OF REGIONAL ANOMALIES

Figure 10 summarizes the regional magnetic trends and areas of magnetic anomalies, which have been mentioned before in the discussion of the respective states. We have discriminated between probable and possible anomalies. The probable anomalies are fairly well established by the magnetic vectors; the possible anomalies, however, may upon more detailed investigations prove to be more localized and not as continuous as shown on the map, though the vectors

[1] J. Edmund Eaton, *op. cit.*

[2] Friedrich Noelke, "Geotektonische Hypothesen," *Sammlung geophysikalischer Schriften*, No. 2 (Berlin, 1924), p. 70, b.

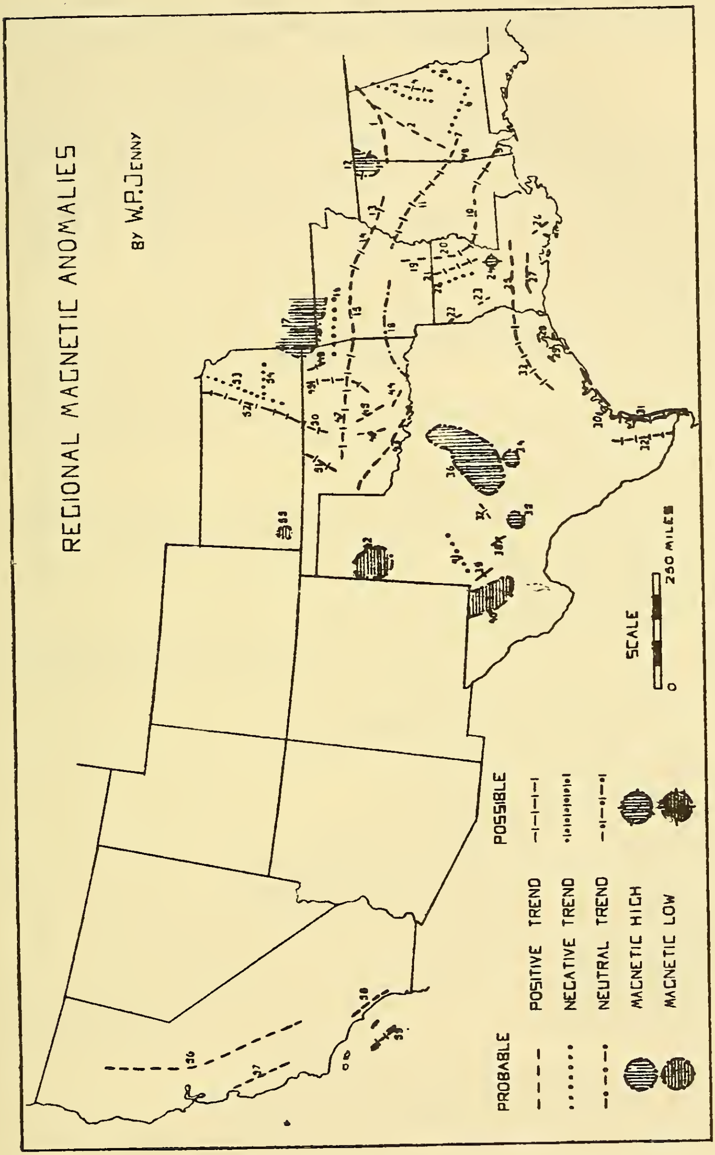

REGIONAL MAGNETIC ANOMALIES
BY W.P.JENNY
PROBABLE
POSSIBLE
POSITIVE TREND
NEGATIVE TREND
NEUTRAL TREND
MAGNETIC HIGH
MAGNETIC LOW
SCALE
0
250 MILES

FIG. 10

so far computed seem to be in favor of continuous trends, or at least do not disprove the trends based on regional magnetic or geologic evidence.

LIST OF REGIONAL ANOMALIES (FIG. 10)

1. High trend beginning between Stevenson and Ft. Payne and extending to Hamilton
2. High trend Guntersville-Greensboro
3. Low trend Center-Wetumpka
4. High trend Ashland to a location between Tuskegee and Wetumpka
5. Low trend Tuskegee-Troy
6. Low trend Camden-Elba
7. High trend Butler to a location between Grove Hill and Pine Hill
8. Hatchetigbee anticline
9. High trend north of Mobile
10. Trend of "highs" from Ellisville to Fayette
11. Possible high trend Butler-Philadelphia-Greenwood-north of Rosedale-between De Witt and Varner
12. High area near Iuka
13. Possible high trend Oxford, Water Valley to a location between Helena and Tunica
14. High trend Marianna-Brinkley
15. High trend of Arkansas Valley
16. Low trend Marshall-Fayetteville-Tahlequah
17. "High" at Miami and Ozark region
18. Neutral trend parallel with axis of Ouachita Mountains from area between Hugo and Antlers toward area south of Mena and Mount Ida to Sheridan
19. High trend Hamburg-Star City
20. High trend Winnsboro-Bastrop
21. Possible high trend Farmerville-Harrisonburg
22. Low trend Ruston-Winfield
23. "Highs" south of Shreveport and Many, with connecting trends parallel with axis of Sabine uplift
24. Possible "high" northeast of Marksville
25. High trend between Ville Platte and Opelousas
26. High trend between Morgan City and Napoleonville
27. High trend Lafayette-Lake Charles
28. "High" south of Liberty, regional trend
29. "High" south of Houston, regional trend
30. "High" at Refugio, regional trend
31. Possible high trend Point Isabel-Corpus Christi
32. Possible high trend east of Edinburgh-Falfurias-San Diego
33. Possible high trend south of Newton-Conroe-Columbus
34. "High" at Lampasas, Llano, Burnet, corresponding with the Llano-Burnet uplift
35. "High" at Sonora, Ozona, Eldorado
36. "High" from Brady to Jacksboro, corresponding with the Bend arch
37. Possible high trend north of San Angelo
38. "High" indicated by the vectors at Rankin and Stiles, probably corresponding with the Big Lake anticline
39. "High" south of Monahans, with trend corresponding with Central Basin platform
40. "Low" near Pecos, corresponding with Delaware basin
41. Low trend, corresponding with the axis of main Permian basin
42. "Low" between Bovina, Tulia, Canyon, interpreted as basin
43. High trend Marietta-Lawton-Hobart-Sayre, corresponding with Wichita Mountains
44. High trend Tishomingo-Durant, corresponding with Arbuckle Mountains
45. High trend Sulphur-Oklahoma City
46. High trend Rush Springs-Minco
47. Possible high trend Fort Smith-Muskogee-Watonga

48. Possible high trend Nowata-Pryor
49. Possible high trend Bartlesville-Tulsa-Holdenville
50. Possible high trend east of Medford-Enid, corresponding with possible south extension of Nemaha Mountains
51. Possible high trend east of Alva-Trail
52. Possible high trend corsponding with Nemaha Mountains
53. Low trend east of Nemaha Mountains
54. Low trend between Yates center and Burlington
55. "Low" at Ulysses
56. High trend along California Valley
57. High trend along Salinas Valley
58. High trend from Oceanside to Santa Monica
59. Possible high trend San Clemente-San Nicholas

APPLICATION OF REFLECTION SEISMOGRAPH[1]

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ABSTRACT

This paper describes in detail a reflection seismograph survey of western Henderson and eastern Navarro counties, and covers the area just north of the Mexia-Powell oil field. A number of faults are located and closure is indicated on two. A cross section of the area shows the relation of the reflection records to the subsurface structure. Reflections were obtained throughout the area on the Pecan Gap and a basal member of the Austin chalk. Contour maps on these two reflection horizons are included.

INTRODUCTION

The area surveyed in this application of the reflection seismograph lies just north of the Mexia-Powell fault-line fields in eastern Navarro and western Henderson counties, Texas, and has long been considered a favorable area in which to explore for a possible extension of this line of fields. At least twenty dry holes have been drilled in this area. A seismograph survey such as the one under consideration would have made possible the selection of the best location, structurally, so that one, or at most two, wells would have thoroughly tested the area for the presence of oil.

A total of approximately seventy-five depth determinations were made in a period of about two weeks, resulting in a cost per acre of somewhat less than 10 cents.

The author has attempted to present the data in the form of two contour maps, one of the top of the Pecan Gap chalk and the other of the basal member of the Austin chalk, a cross section with corresponding reflection records from which the depths to these two reflecting horizons were made, an enlarged view of a typical record showing the relation between the reflection events and the section, and has endeavored to make these as nearly self-explanatory as possible.

[1] Presented before the Association at the Oklahoma City meeting, March 25, 1932. Manuscript received, June 4, 1932.

[2] Geophysical Service, Inc., 1311 Republic Bank Building.

PROCEDURE

As the trend of any faults that might exist was assumed to be the same as that of the field on the south, east-to-west lines of depth determinations were laid out, taking advantage of roads available. When the data indicated the presence of a fault, the layout was changed so as to increase the density of shot locations in the neighborhood of the fault and follow its trend. The program was essentially of a reconnaissance nature.

CORRELATION OF REFLECTIONS

Two dominant reflections appear on the records. The interval between them corresponds to that between the top of the Pecan Gap chalk and the basal Austin. A standard Cretaceous velocity chart was used in the computation of the depth determinations. Although this could not be expected to give correct absolute depths beyond approximately 100 feet, as it was constructed from well determinations in another part of East Texas, interval and relative-depth determinations made from it could be relied upon to be correct to 0.5 per cent. The two contour maps show the interval between the Pecan Gap and the Austin to be approximately constant but indicate some regional variation.

A typical reflection record showing the relation of the reflection events to the geologic section is shown in Figure 1. The fact that a dominant reflection is obtained from near the base of the Austin chalk would indicate that the basal member of this body of chalk is considerably harder than the rest of the chalk section. Occasionally a reflection is obtained from the upper part of the chalk section, but it is never as pronounced as the basal reflection nor does it appear on the records with any regularity. In view of the dominant character of the two reflections and the regularity of interval between them, the possibility of miscorrelation is extremely small.

The two contour maps, Figures 2 and 3, were constructed from reflection depth determinations entirely. No well data were used. Although the contour maps may not check similar maps made from well data in absolute value, they should check as regards relative data. Most of the wells drilled in this area are shown on the contour maps.

A cross section showing two faults is presented in Figure 4. This cross section is along line *AA* indicated on the contour maps. The reflection records from which the depth determinations were made are shown opposite the cross sections. Each record corresponds with the depth determinations opposite it on the cross section. The two reflections, Pecan Gap and basal Austin member, are marked on the records and the datum values shown. These datum values are shown to scale on the cross section.

Due to the greater depth of the Austin, the reflection from it should be smaller in amplitude than that from the Pecan Gap. For a reasonably large Pecan Gap reflection the Austin reflection amplitude

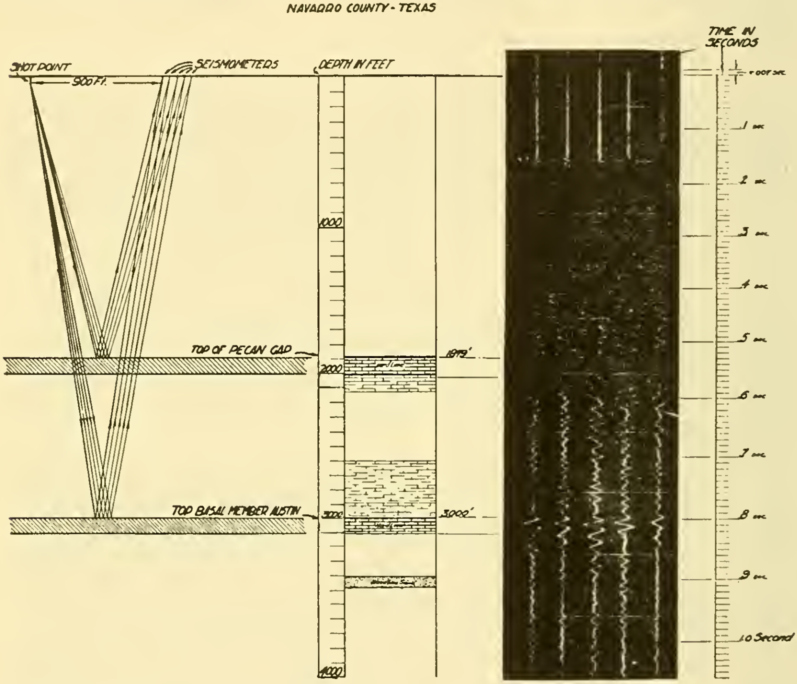


FIG. 1.—Typical reflection record showing relation of reflections to geologic section.

is almost too small to use. It is, therefore, general practice to take several records with different dynamite charges. In this way both large- and small-amplitude records are obtained for the two reflections. The small-amplitude Austin reflections shown in Figure 4 have been further substantiated by larger amplitude records.

Attention is called to the absence of the Austin reflection on record F.P.210. It so happens that the normal reflection point on the Austin falls in the Austin fault plane. In view of the extraneous disturbance in this record at the time of normal occurrence of the Austin reflec-

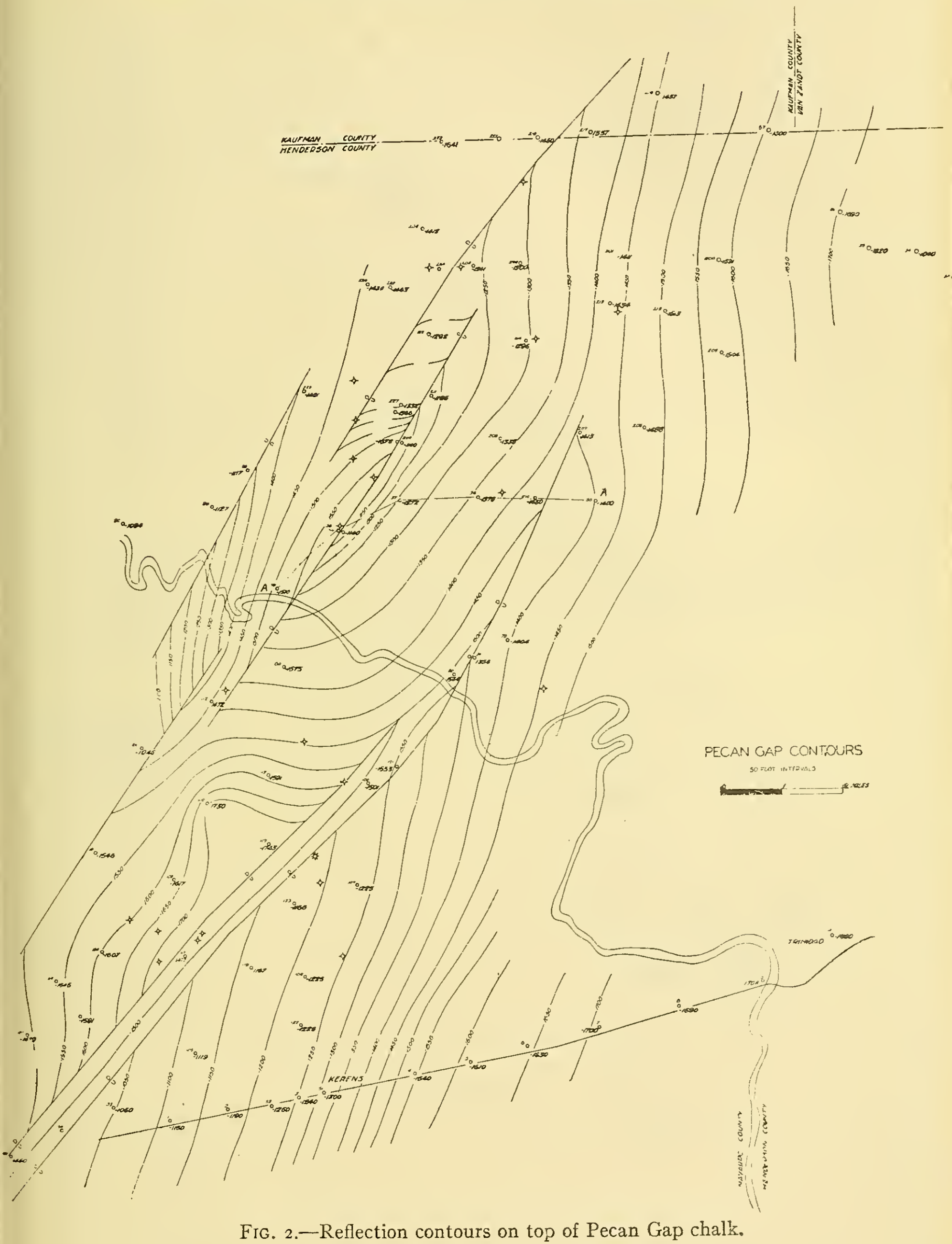


FIG. 2.—Reflection contours on top of Pecan Gap chalk.

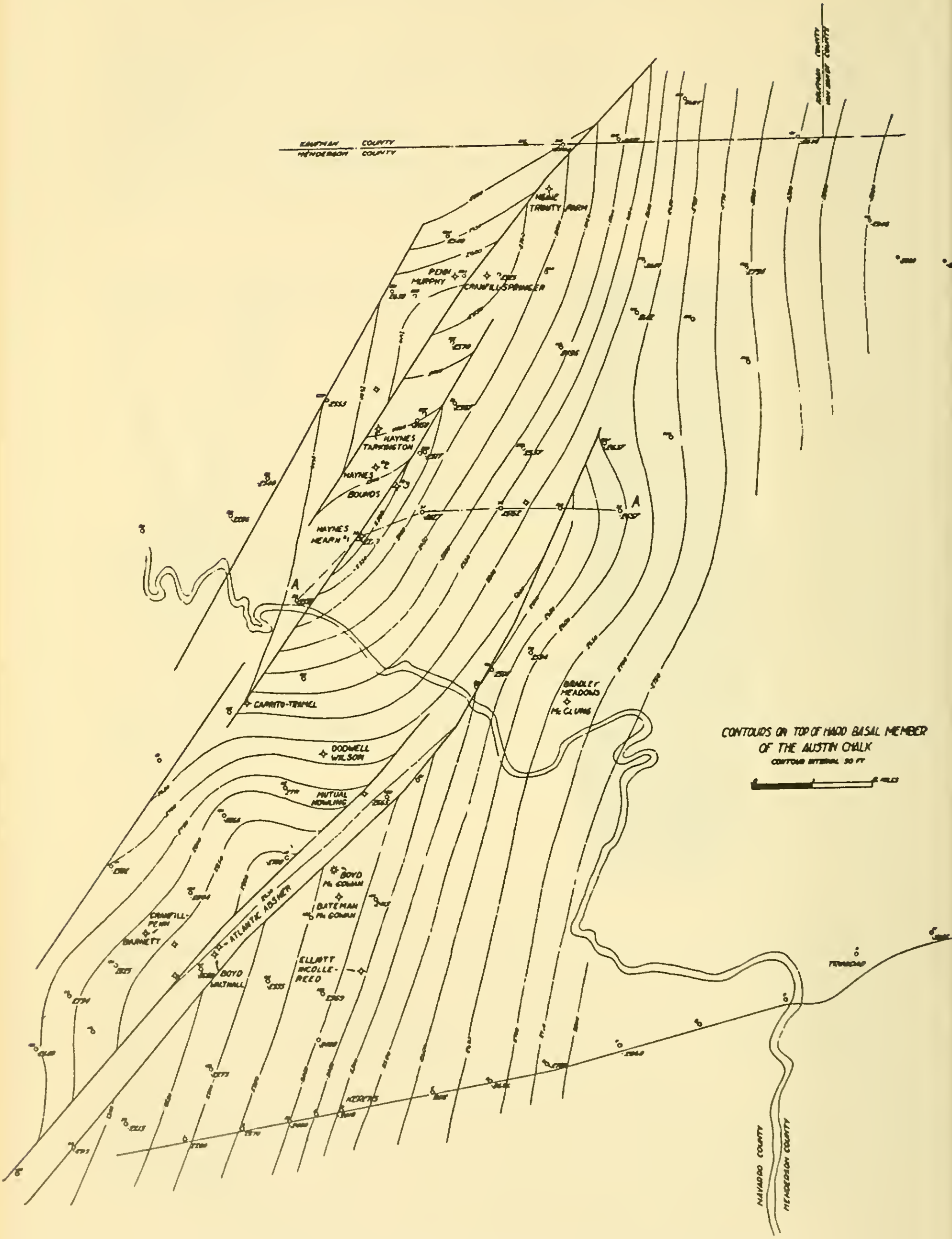


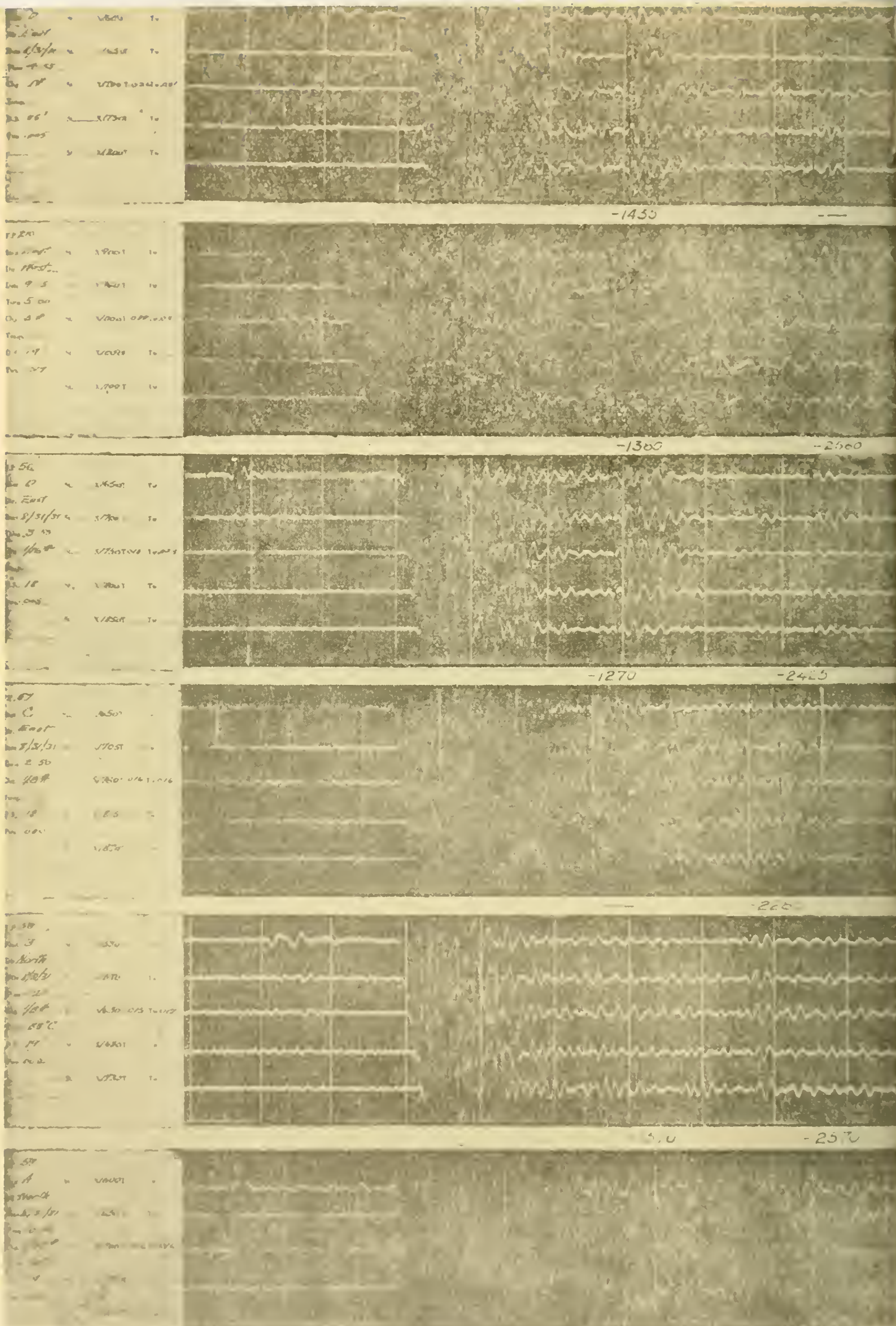
FIG. 3.—Reflection contours on top of basal member of Austin chalk.

tion, the absence of the reflection in this instance can hardly be called conclusive evidence of the presence of the fault.

Due to the absence of the Pecan Gap reflection, record F.P.58 is of special interest. As the amplitude of the Austin reflection on this record is larger than on any of the other records, the Pecan Gap reflection under normal conditions should be very large indeed. There can be little doubt that this must be the result of very abnormal subsurface conditions. It is seen from the cross section and contour maps that the normal reflection point for the Pecan Gap lies in the fault plane of the Pecan Gap. This fault, it should be noted, has been located by several other determinations. In many cases it is impossible to get any reflections very close to a fault, due probably to minor fracturing in the proximity of the fault plane. The presence of the Austin reflection in this case would indicate that the fault must be a fairly clean break without much minor fracturing.

CONCLUSION

It is thought that the reflection seismograph may economically justify its use in many cases by permitting the location of a test well in the most advantageous position structurally and thus save the cost of many unnecessary wells.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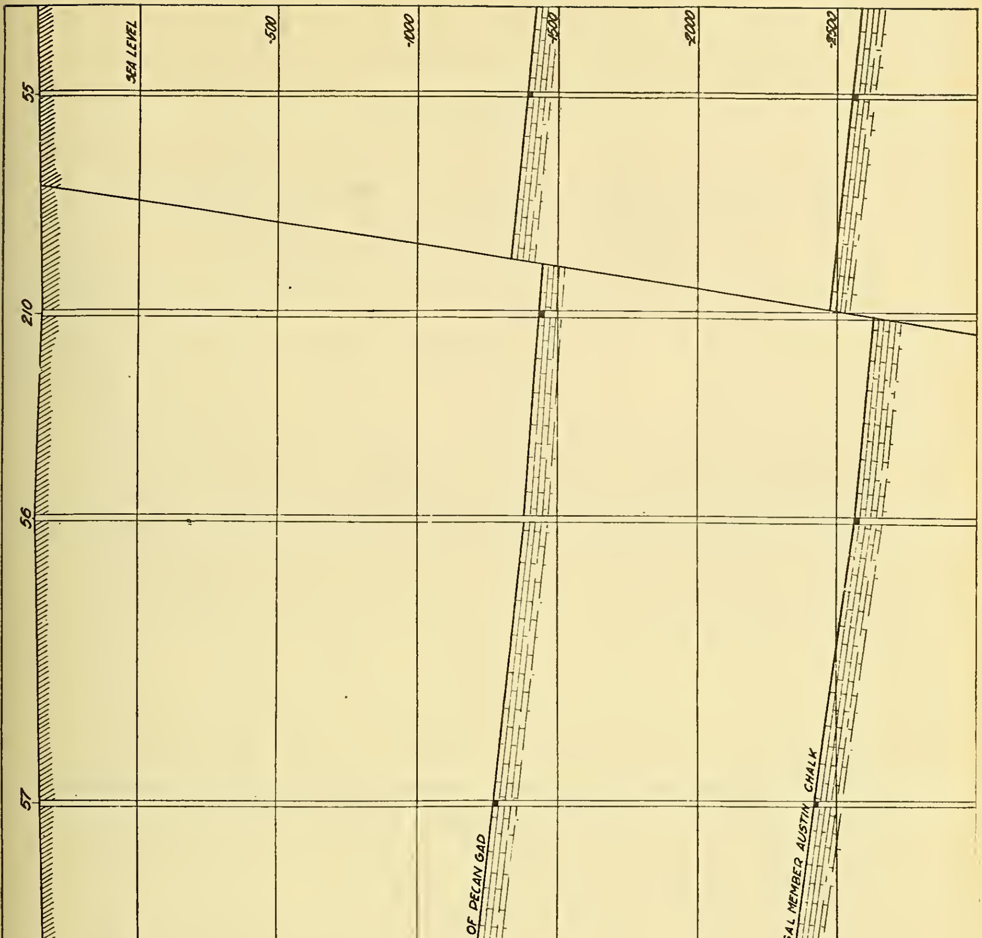

FIG. 4—Cross section *AA* and corresponding reflection records. Horizontal length of section, 39,500 feet.

USE OF RECORD CHARACTER IN INTERPRETING RESULTS AND ITS EFFECT ON DEPTH CALCULATION IN REFRACTION WORK[1]

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ABSTRACT

In seismograph exploration, the author has encountered acoustic horizons which do not transmit seismic energy readily. The first impulse on the seismogram has a short period with small amplitude, and requires considerable energy to make it apparent at long distances. This horizon is underlain by one of slightly higher velocity which transmits the energy readily and has a distinctly different wave form. On account of the difficulty of energy transmission, the horizon which has the small "forerunner" wave might be interpreted as a lens and not a continuous horizon. By making use of the character of the wave form on the seismograms, the two horizons may be definitely identified, and the one which is a poor conductor of energy definitely proved to be a continuous acoustic horizon.

INTRODUCTION

This paper covers a short discussion of a problem encountered in refraction work done in Venezuela during 1931. The work was done by the International Petroleum Engineering Corporation of New York, using electrical seismographs and radio equipment built by the Petty Geophysical Engineering Company of San Antonio, Texas. The writer was in charge of the party in the field.

The problem to be considered involved an interpretation of results in an area where an acoustic horizon transmitted the seismic energy poorly. Referring to profile *A*, Plate 1, it is evident that there are four acoustic horizons involved, with velocity values of approximately 2,000, 2,160, 2,580 and 6,510 meters/second, the first three being sedimentary, while the fast one represents the igneous basement. There were several other profiles shot in the same general area as profile *A* and all of them had the same general velocity values. Profiles *B*, *C*, and *D* were approximately 50 kilometers from profile *A*, and the sedimentary horizons had become thinner in this distance. Several profiles were shot in the distance between the two areas, and

[1] Published by permission of International Petroleum Engineering Corporation, 80 Maiden Lane, New York, N. Y. Read before the Association at the Oklahoma City meeting, March 24, 1932.

[2] Field manager, Petty Geophysical Engineering Company. Introduced by L. W. Blau.

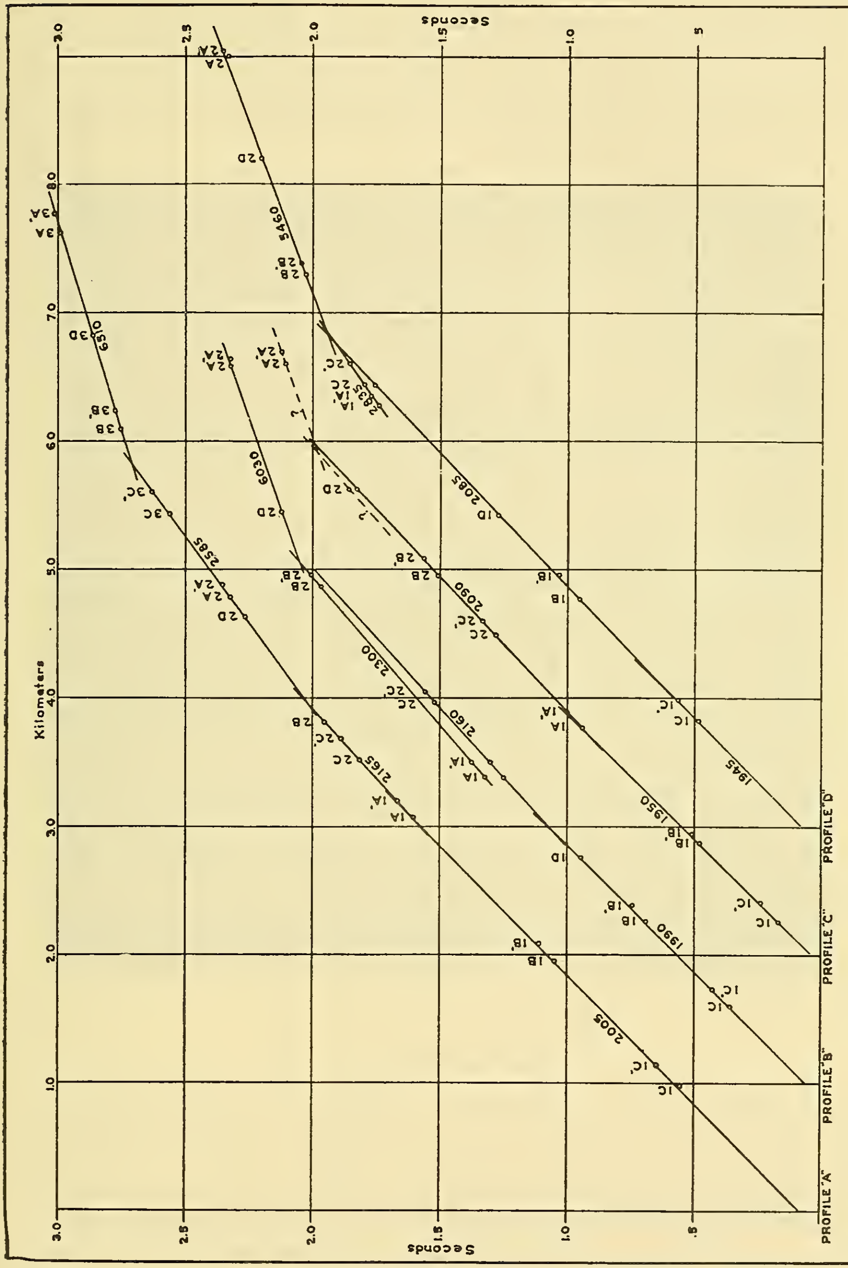

PLATE I - TIME-DISTANCE CURVES

PLATE I

their results showed a gradual thinning of the sedimentary beds and a gradual decrease in the velocity values. The seismograms shown in Plate 2 are the ones from which profile *B* is plotted, Plate 3 shows the seismograms from which profile *C* is plotted, and Plate 4 shows those from which profile *D* is plotted.

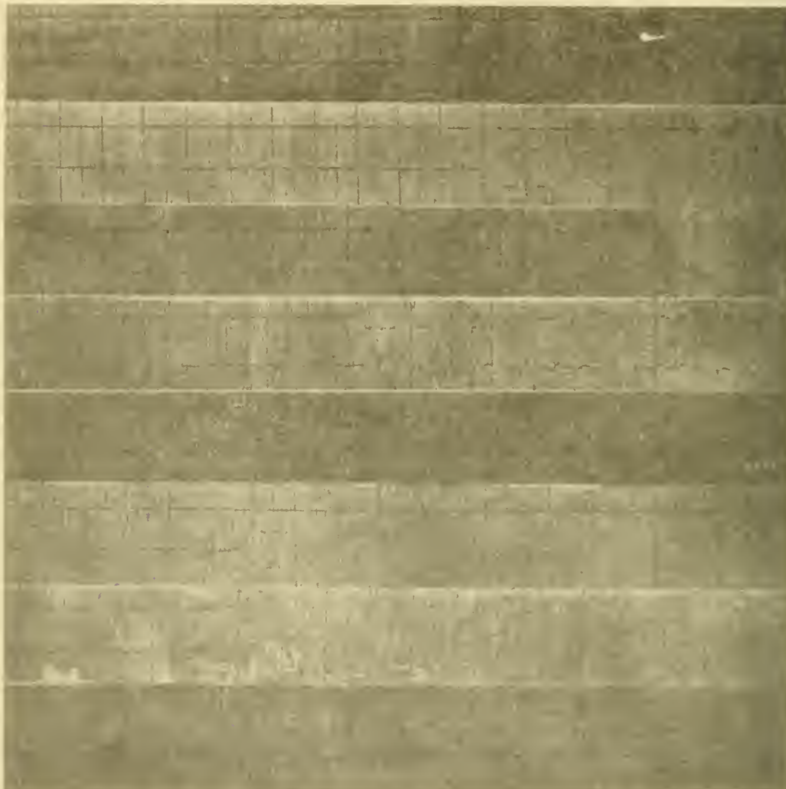

PLATE 2.—Seismograms for profile *B*.

STUDY OF RECORDS

A study of the three sets of seismograms reveals four distinct record characteristics. The first five seismograms of profile *B*, Plate 2, break sharply and plot on the surface velocity curve; the next four seismograms do not break sharply but have a small wave of short period for the first impulse followed by a wave with a sharp break and large amplitude. The small impulse and the sharp break were both considered and are plotted on the profile. The sharp break with large amplitude is the first impulse on records $2B$ and $2B'$, while on the longest three records the sharp break is preceded by an absorbed impulse which is characteristic of the igneous basement. Several other profiles in the same general area as profile B all had a small wave of short period on the second horizon which required large dynamite charges to bring out. A curve drawn through the points represented by the first small impulse or "forerunner" does not intersect the curve drawn through the points representing the sharp breaks of the third horizon until after the basement velocity curve intersects both of them. Should the small wave or "forerunner" be interpreted as a thin lens-like formation overlying the horizon which gives the sharp break, or is it a separate acoustic horizon? If the former, it might be disregarded as far as depth calculation is concerned, but if it is the latter its thickness must be considered.

THEORETICAL CONSIDERATION

If the conditions shown in Figure 1 be assumed, it is possible to derive equations for the time-distance curves which these velocities

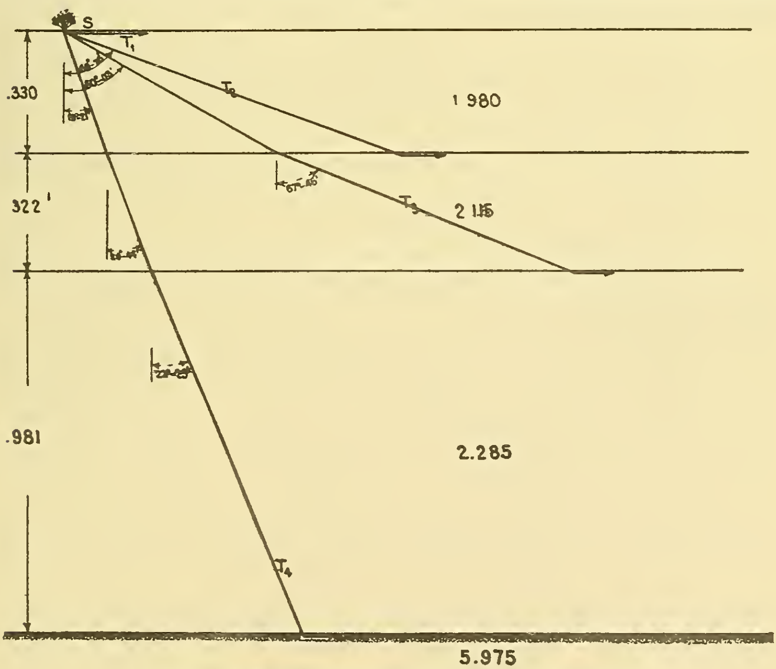

FIG. 1.—Assumed conditions of depth and velocity.

and thicknesses will give. The distance of the receiving station from the shot point will be designated as x, and the equations for the time through each path derived. For the surface layer, the general equation for the time is,

$$T_1 = \frac{x}{1.980}$$

The general equation for the time through the second horizon is,

$$T_2 = \frac{2 \times .330}{\cos 69°25' \times 1.980} + \frac{x - 2 \times .330 \times \tan 69°25'}{2.115}$$

For the third horizon, it is,

$$T_3 = \frac{2 \times .330}{\cos 60°03' \times 1.980} + \frac{2 \times .322}{\cos 67°46' \times 2.115}$$
$$+ \frac{x - 2 \times .330 \times \tan 60°03' - 2 \times .322 \times \tan 67°46'}{2.285}$$

and for the fourth horizon, it is,

$$T_4 = \frac{2 \times .330}{\cos 19°21' \times 1.980} + \frac{2 \times .322}{\cos 20°44' \times 2.115} + \frac{2 \times .981}{\cos 22°29' \times 2.285}$$
$$+ \frac{1}{5.975}[x - 2 \times .330 \times \tan 19°21' - 2 \times .322 \times \tan 20°44'$$
$$- 2 \times .981 \times \tan 22°29'].$$

These are all straight line equations which may be reduced to the simple forms which follow:

$$T_1 = \frac{x}{1.980}$$

$$T_2 = .948 + \frac{x - 1.757}{2.115}$$

$$T_3 = 1.473 + \frac{x - 2.720}{2.285}$$

$$T_4 = 1.608 + \frac{x - 1.288}{5.975}$$

The curves represented by these equations are shown in Figure 2. It is apparent that the wave from the third horizon would never be the first impulse, since the intersection of the T_2 and T_3 curves is past the point where the T_4 wave arrives first. The only manner by which the third horizon velocity may be determined is to select the secondary waves on records shorter than 4.174 kilometers, which is the value of x where the T_2 and T_4 curves intersect in this particular case.

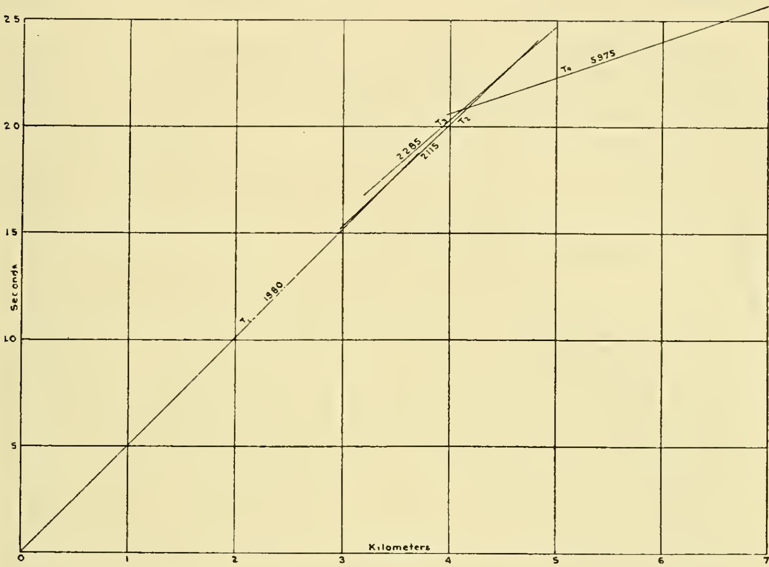

FIG. 2.—Time-distance curve for assumed conditions.

This theoretical consideration seems to indicate that the small wave or "forerunner" represents a distinct acoustic horizon and not a lens-like formation. If this be true, it should always give the first seismic impulse on the seismogram, provided there is enough energy available to bring it out. In Plate 3, there was more energy available on account of the shorter distances and the amplitude of the first impulse is quite large on all the seismograms except *2D*. The period of the wave is quite short, however, indicating that it is the impulse characteristic of the second horizon. On Plate 4, records *1A* and *1A′* have a sharp break as the first apparent impulse, but a study of these seismograms under a glass shows a slight movement ahead of the apparent initial impulse. Record *2C* was only about 100 meters longer than $1A'$ but had four times as much dynamite, and the small forerunner wave is very evident on this record. This proves conclusively that the formation which transmits the small forerunner type of wave is a continuous acoustic horizon which does not transmit the seismic energy readily.

CONSIDERATION OF DIFFERENT PHASES OF THE PROBLEM

The problem presented here might be thought of from two different angles. If all horizons had been of equal energy-carrying ability,

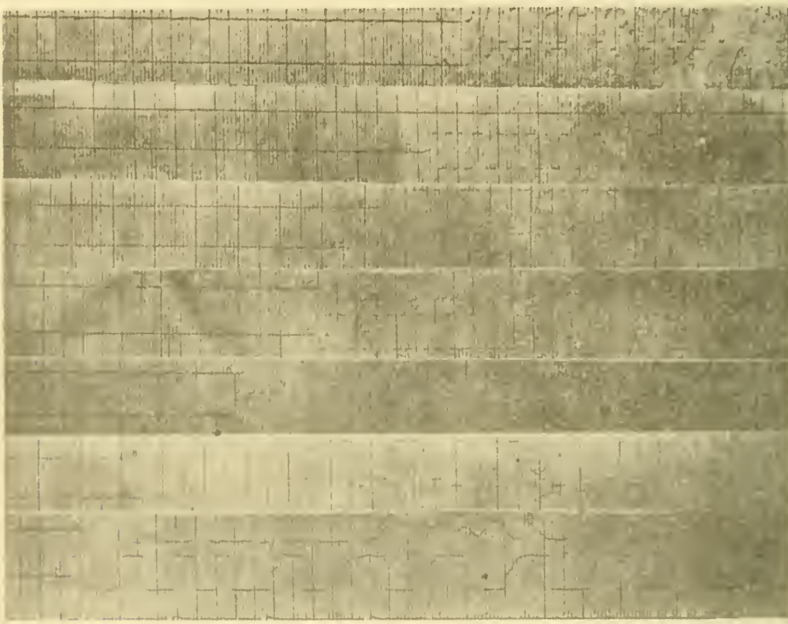

PLATE 3.—Seismograms for profile *C*.

the third one would have been difficult to detect, due to the fact that the particular thicknesses and velocities are such that the third horizon impulse would never have been first. On the other hand, it would be easy to neglect the beds which transmitted energy poorly unless sufficient dynamite was used to make the first impulse large enough to be seen. In the area where this work was done, the dynamite charge was chosen that would give just enough amplitude to the second horizon wave to make it apparent, yet not so large that it obscured the large secondary impulse from the third horizon. In this manner, it was possible to choose a primary wave and designate it as the second horizon impulse and also choose a secondary wave and consider it as a primary wave from the third horizon. If the second hori-

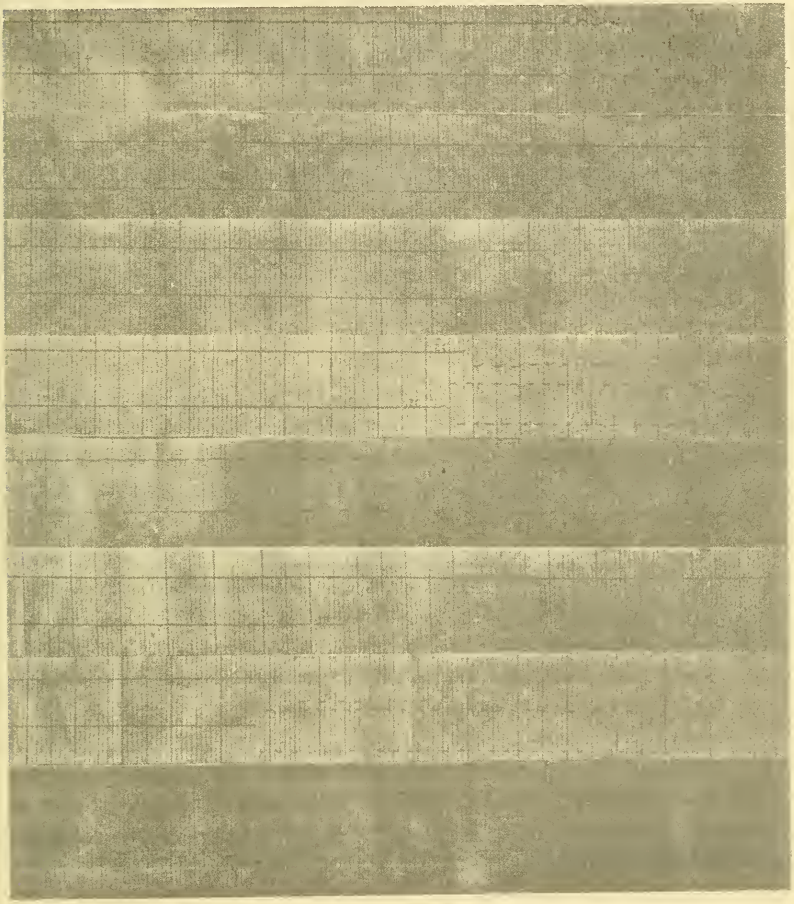

PLATE 4.—Seismograms for profile *D*.

zon had been omitted from the calculations, the total depth to the basement would have been too small and the surface horizon would appear much thicker. Omission of the third horizon from the calculation would also have made the total depth to the basement too small but would not have changed the thickness of the surface horizon.

REASON FOR POOR ENERGY-CARRYING CHARACTER

The reason for poor energy transmission is probably due to alternating beds of hard and soft material, perhaps sandstones and clays. There is an area in the Gulf Coast where a poor energy-carrying horizon is found, and it is especially noticeable in Harris County, Texas. The movement on the seismogram is a small wave whose beginning is scarcely perceptible and is sometimes mistaken for a salt dome "forerunner." Recent work by the reflection method in the Gulf Coast region shows several reflections from the horizon which transmits the refracted wave poorly, indicating a change in density which would logically be explained by alternating beds. If the alternating beds which compose the acoustic horizon had slightly different velocity values, the refracted wave would gain a slight amount in the faster bed, and be gradually damped out.

CONCLUSION

The seismic wave accurately shows the subsurface conditions. The character of the wave is affected by the various strata through which it passes, and a study of the wave character is important in the correct interpretation of seismic results. The possibilities of the seismograph as an accurate exploratory instrument have not been fully realized, and this paper is presented with the hope that further study along similar lines will broaden the scope of usefulness for this in strument.

SEISMOLOGICAL DISCOVERY AND PARTIAL DETAIL OF VERMILION BAY SALT DOME, LOUISIANA[1]

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Houston, Texas

ABSTRACT

The Vermilion Bay salt dome in Iberia Parish, Louisiana, was one of the first discovered by a geophysical survey of water-covered areas. The field organization and equipment of such a crew is described. The discovery and detail data are presented, along with the original interpretation of these data. The results of drilling are shown, and the agreement and disagreement of these tests with the original interpretations discussed to some extent.

LOCATION

Vermilion Bay is one of the major features of the Louisiana coast line. The Vermilion Bay salt dome is located in the southeastern corner of the bay, but in the extreme southern portion of Iberia Parish, Louisiana. It is best reached by boat along the Boston Canal from Abbeville, or by car to Cypremort Point, and thence by boat to the dome.

DISCOVERY

This salt dome was discovered on September 25, 1927, in a reconnaissance refraction exploration conducted for the Louisiana Land and Exploration Company by a Geophysical Research Corporation party[4] under the direction of the junior writer.

The discovery of this dome was of importance beyond the usual, for several reasons. Among these may be listed the following.

1. This salt dome was discovered as a result of the use of the seismograph in exploring large bodies of water. The Louisiana Land and Exploration Company was the first to launch a major exploration campaign with this definitely in view, using for this purpose two Geophysical Research Corporation parties,[5] equipped for water surveys.

[1] Read before the Association at the Oklahoma City meeting, March 25, 1932. Published by permission of the Louisiana Land and Exploration Company, and the Geophysical Research Corporation.

[2] Consulting geophysicist, 2210 Esperson Building.

[3] Geophysical Research Corporation.

[4] Within a month, this same crew had also discovered the Four Isle and Dog Lake salt domes, in southwest Terrebonne Parish.

[5] The other crew, under the direction of C. G. Rosaire, was the first to discover a salt dome during a water survey, since his discovery of the Calcasieu Lake salt dome antedated O. C. Lester's discovery of the Vermilion Bay dome by several days.

By virtue of their being first to embark upon this virgin field, these two refraction crews staged what is probably the most spectacular campaign in the history of geophysical exploration, discovering no less than eleven salt domes in 9 months. These successes, starting similar campaigns by other companies, caused the intensive exploration (by seismograph crews) of southeastern Louisiana, and further resulted in the discovery of six additional salt domes in this same area, all within a year after the discoveries of the Calcasieu Lake and Vermilion Bay salt domes.

2. This successful application of the seismograph to water explorations undoubtedly brings part, at least, of the Gulf of Mexico within the available petroleum reserves of this geological province, if or when the value of crude oil justifies the solution of such practical and legal complications as are apparent.

OPERATIONS

Vermilion Bay varies in mean depth from 10 to 13 feet in an area of approximately 125,000 acres (Fig. 1). The operations in this instance, as in other water explorations of similar magnitude, were conducted entirely from boats. The crew was quartered in a houseboat with auxiliary barges for explosives and supplies, while the recording apparatus was mounted in fishing luggers 40 to 75 feet in length. A similar lugger was used in locating, planting, and firing the charges of explosives.

The explorations were conducted by the usual method of refraction fans. Communications and transmissions of time signals were effected by two-way radio operations on 180 meters, using 5-watt I C W transmitters. Distances from shot to recorders were determined from the air travel times of the explosive sound waves.

Though the analysis of such exploration data is comparatively simple, its acquisition involves the solution of many practical difficulties. Adverse winds may carry the sound wave above the recorders, or may lower the water level to such an extent that operations of any kind are impossible over the subsequent mud flats; and may even maroon the recording boats for as much as a week. In the latter case the resulting diet of canned tomatoes and oysters becomes rather monotonous after a few days. On the other hand, moderately rough water is no great handicap, since seasick observers have taken excellent records with the seismographs planted in water and mud at depths of 15–20 feet. Also, as this coast is not infrequently visited by tropical hurricanes, the field crews watch the Coast Guard Weather Reports (received by radio) and occasionally have to run for land, returning later to salvage gasoline drums and boxes of dynamite scattered over the shore by sudden and severe squalls.

Even under such conditions, the explorations averaged 200,000 acres per month; and as much as 15,000 or more acres have been surveyed in a single operating day.

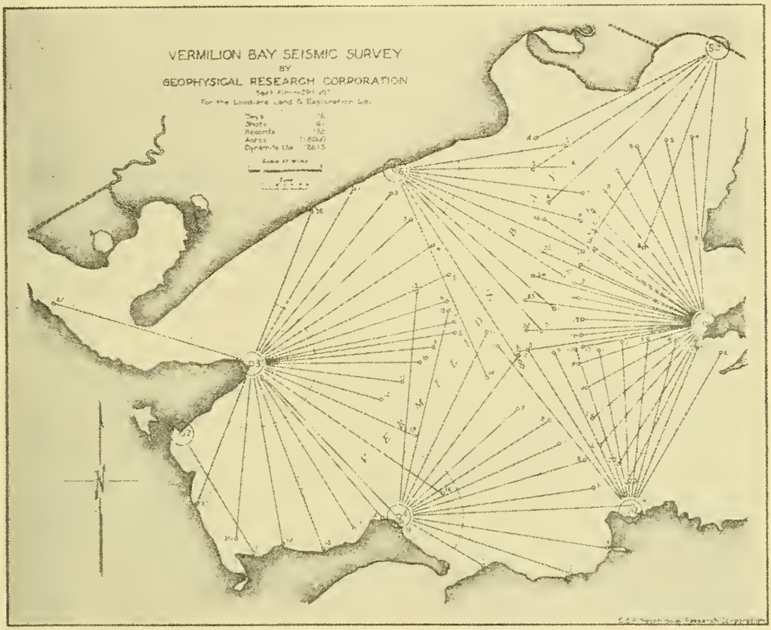

FIG. 1.—Refraction fan layout for exploration of Vermilion Bay.

In the survey of Vermilion Bay, three recording boats were used. Each was equipped with a radio transmitter and receiver, an oscillograph, and a velocity meter (geophone) recording through a vacuum tube amplifier. The timing device was an electrically operated 50-cycle tuning fork. The air wave was picked up by a carbon-button unit. The resulting records were developed immediately, thus guiding the observers in the size of the next shot. The instant of explosion was determined by the breaking of a wire wrapped around part of the charge of explosive, thus terminating the operation of the shot transmitter. In general, the charge of explosive was simply lowered to the bottom of the water, or buried in a hole on shore. A smaller simultaneously detonated surface charge was used for the air wave distance determination. Travel times were read to hundredths of a second.

Distance determinations were made from the travel times for the air waves, corrections being used for wind velocities and directions, determined by compass and hand anemometers at the time of each shot. For short-shot lengths, up to about 5 miles, these were relatively accurate to about 200 feet. (At greater distances over land, large errors

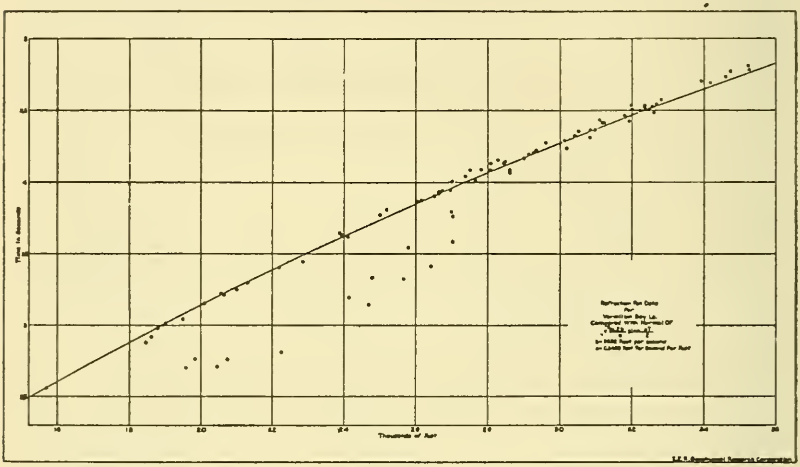

FIG. 2.—Travel time data for Vermilion Bay.

appeared in this method, as for instance, when an error of 2,000 feet appeared in a shot line 40,000 feet long.)

Distance determinations and final travel-time counts were made at headquarters in the houseboat. The travel-time data (Fig. 2) were plotted, and the normal relationship indicated by drawing the obvious "normal" travel-time curve. Travel times appreciably shorter than normal were then considered as suggesting the existence of a salt dome, and bona fide time differences checked by a cross fan.

INTERPRETATIVE PROCEDURE

Long distance refractions.—The data collected in Vermilion Bay may be taken as typical of a shallow Gulf Coast salt dome. For convenience of study, the time intervals are plotted, on each line, from an arc indicating normal arrival times. If minus time differences are plotted out from the shot point, an approximation is made to the outline of the wave front at the recording points. In the case of the Vermilion Bay data, the resulting pattern of time differences unmistakably indicated the presence of a previously unknown salt dome.

The smooth curve used as the "normal" in Figure 1 is calculated from the travel-time relationship

$$X = \frac{2b}{a} \sin h \frac{aT}{2}$$

for a linear increase of velocity with depth. The constants b (surface velocity) and a (increase in velocity per foot of depth) were determined by trial, and found to be

$$b = 5{,}633 \text{ feet per second}$$

$$a = 0.5483 \text{ foot per second.}$$

The penetration, Z, for such a curve is

$$Z = \sqrt{\frac{x^2}{4} + \frac{b^2}{a^2}} - \frac{b}{a}.$$

Short refraction data.—A short negative profile was shot north from *SP* 1, to serve as a "normal" in mapping the top of the dome. This is shown in Figure 3.

An interesting feature of this profile is that the extrapolated travel-time curve indicates a negative time for zero distances. This is to be expected for explosion-generated data, and indicates the abnormally high velocity of sound in the immediate neighborhood of the explosion. However, in comparison with the greater number of such short refraction profiles, these data are exceptional, since the usual data indicate a plus time at zero distance, due to the general existence of a low-speed layer at the surface of the ground (weathered or aerated zone). In the case of the Vermilion Bay data, there is apparently no aerated zone,[1] so that there is opportunity to observe the effect of the abnormally high sound velocities in the immediate vicinity of the explosion.[2]

[1] This is not necessarily an obvious conclusion, since in some cases of water-covered marsh, definite geophysical evidence of an aerated zone has been found on the shorter travel-time curves. As Paul Weaver has pointed out in "Geophysics of the Soil," this is to be expected in the case of thick layers of soil being formed from rotting vegetation.

[2] A. B. Wood, *Textbook on Sound*, p. 266.

This short profile can also be fitted by a travel-time curve such as that approximated by the longer refraction data. For the shorter data, the constants are

$$b = 4{,}990 \text{ feet per second}$$
$$a = 1.25 \text{ feet per second per foot.}$$

The difference in the values of these "constants" for the two sets of data is significant, since it is an indication of the extent to which

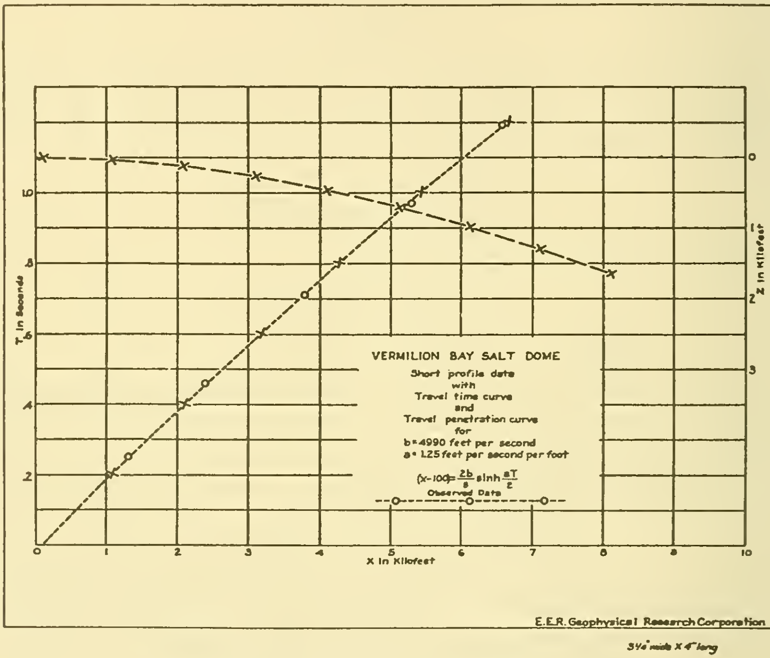

FIG. 3.—Short refraction data for Vermilion Bay.

these data can be approximated by a linear increase of velocity with depth.

Partial refraction detail.—The location of the dome by refraction fans was recognized as too qualitative for drilling. Therefore, with the thought of securing more definite information as to the depth and location of the top and flanks, the following detail survey was carried out.

Figure 4 shows the location of these refraction profiles, together with the resulting interpretation of the outline. The profile data are shown in Figure 5 and Figure 6. The solid part of the travel-time curves are taken from the negative profile data described in Figure 3.

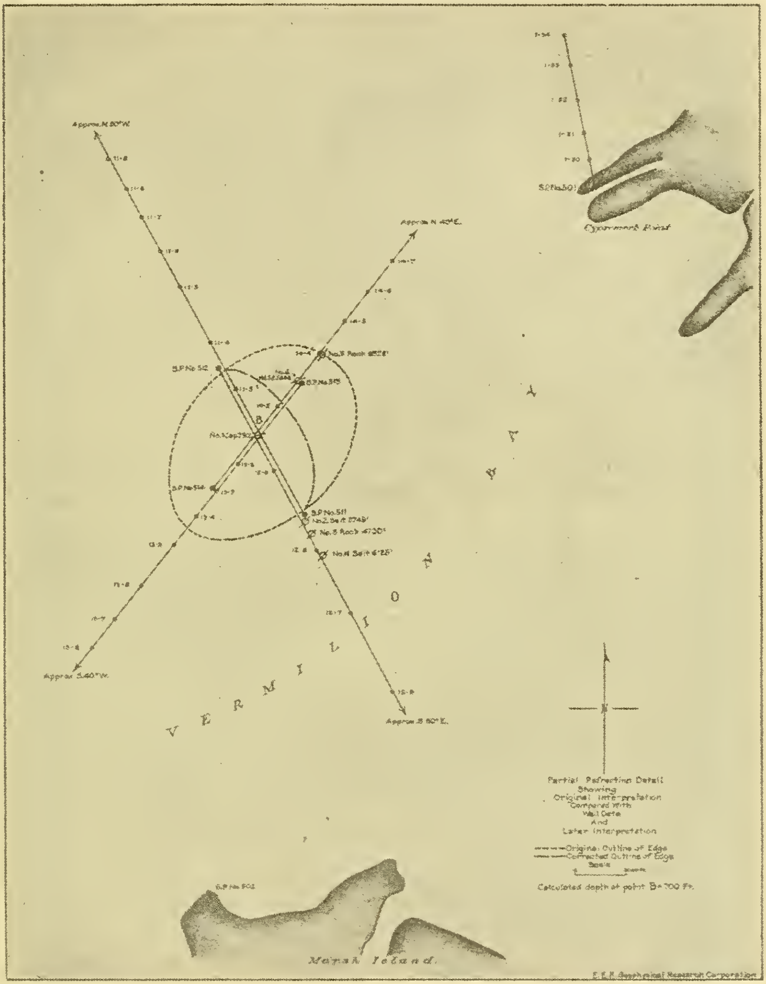

FIG. 4.—Refraction profile layout for partial detail of Vermilion Bay salt dome.

All four of these profiles show the usual features of a refraction profile characteristic of a shallow salt dome. The most essential feature is the more or less sudden decrease in apparent velocity shortly after the recording points pass well beyond the salt flank. The junior writer, at the time of this detail, made a depth calculation of 800 feet to cap or salt, at the intersection of the profiles. The agreement with the results of the well later drilled at this point was very satisfactory.

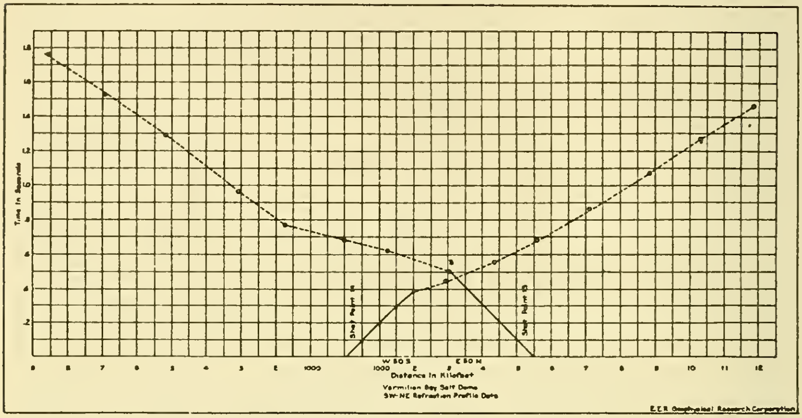

FIG. 5.—Southwest-northeast refraction profile data, Vermilion Bay salt dome.

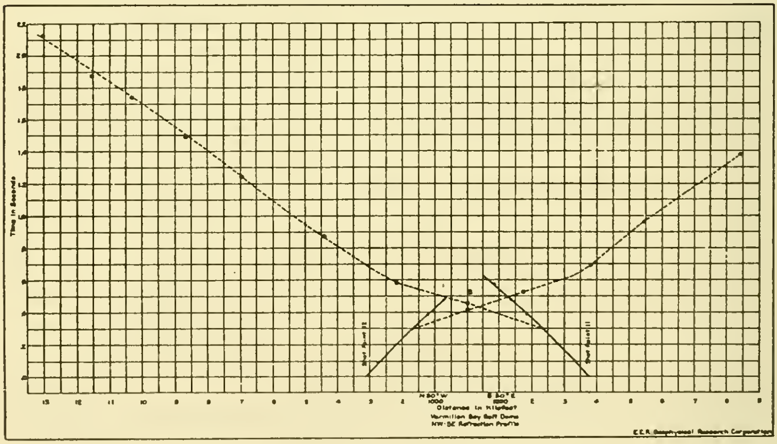

FIG. 6.—Northwest-southeast profile data, Vermilion Bay salt dome.

The original interpretation of the outline of the dome (dotted) was made in the New York office, not in the field. The southeast flank location was good, as shown by the findings in the wells drilled on the line. The only other check of this outline was made on the northeast flank, with disappointing results. Well No. 5, supposedly a location similar to Well No. 2, was abandoned in rock at 6,500 feet. Well No. 6, well inside this estimated edge, was abandoned at 5,824 feet without finding dome materials, when cap or salt was expected not deeper than 900 feet. A recent revision of the interpretation on this flank was made by the junior writer, and led to the location of this edge as shown by the dashed line. As no further wells have been drilled since No. 6, the original interpretation of the northwest and southwest flanks, and the re-interpretation of the northeast flank remain unchecked to date.

With the experience accumulated, since this detail, on several other domes, the writers are in a position to comment on the job. It is obviously incomplete, since four points, no matter how well determined, can hardly furnish satisfactory control on the periphery of a salt dome like that in Vermilion Bay. Later practice was to locate eight points, reversing all profiles. These additional points, determined on stronger evidence, resulted, in the case of the Lake Barre salt dome, in commercial production by the first flank test, followed by a string of nine successive flank producers around the periphery of the dome.

In addition, the profile data on the northeast flank somewhat definitely indicate that this flank slopes less steeply than any of the other three. Failure to realize the amount of this slope (as shown by the apparent velocities) undoubtedly played a major part in the erroneous location of this flank.

Finally, these conclusions were reached on profile data with too low a survey density. Recorder intervals of 1,500 feet can hardly be expected to locate salt dome flanks to plus or minus 250 feet. In the later details, these recorder intervals were cut down to as little as 250 feet, along the profile.

In presenting the data for publication, attention is called to the fact that curved lines have been used to approximate the travel-time curves. At the time of this survey, field practice was to use successive, intersecting straight lines, but for the past 3 years, curved path interpretations have entirely replaced the older, empirical, strong-arm methods of attack in almost all phases of refraction surveys. In view of this, the writers feel justified in restricting their case treatment to the earlier predictions, and so indicating, to one experienced in the art, the contrast between the old and the new interpretation methods.

SEISMIC WEATHERED OR AERATED SURFACE LAYER[1]

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ABSTRACT

The existence of a comparatively thin surface layer with a low velocity characteristic has been recognized in seismic work for several years, and has been generally referred to as the "weathered layer." This low velocity surface characteristic has been found to be almost universally present regardless of the nature of the surface deposits, and does not conform to the geologic weathering of the area.

The purpose of the writer is to offer as an explanation of this phenomenon, the mixture of air in a free state with surface materials. Theoretical calculations, if the earth is assumed to be a fluid, indicate that velocities less than that of sound in air should be obtained from such mixtures. This is borne out by experimental data, as far as available, indicating that this so-called weathered layer might properly be termed an "aerated" layer.

It must be said at the outset that the conclusions of this paper are based on meager field data. The primary argument arises from theoretical study and postulates certain effects. These effects, however, have been confirmed by experiment in so far as such experimental data are available. The purpose of the discussion is, first, to present a probable explanation of a phenomenon which has been recognized and dealt with—though unexplained; and second, as a result of such explanation, to suggest a more descriptive and less confusing name than that of "weathered."

The existence of a comparatively thin surface layer with a low velocity characteristic has been recognized in seismic work for several years, and it has been generally referred to as the "weathered layer." This condition has been found to be almost universally present, regardless of the nature of the surface deposits, and has been recognized from its low velocity characteristic rather than by any physical or geological difference between it and the materials immediately below. The drilling of deeper shot holes, below the depth of this low velocity horizon, has failed to indicate any visible differentiation between it and adjacent materials.

[1] Read before the Association at the Oklahoma City meeting, March 25, 1932.

[2] Geophysical Research Corporation, Box. 2040.

Since the recognition of this stratum, various methods have been applied to correct for the abnormally slow travel time through it (particularly in reflection work), but little or no attempt has been made to explain its nature, or to determine the cause of its important characteristic.

Data taken in various places, over a wide area, from Kansas and Oklahoma to the gulf coasts of Mississippi, Louisiana, and Texas, have indicated that this layer *may* vary in thickness from practically zero to more than 150 feet, though the general average is approximately 50 feet. The velocity of sound in this "weathered" material, though variable, is always comparatively low and a general average might be placed between 2,000 and 2,500 feet per second; while the average velocity of sound in the same materials immediately below is found to be generally about 5,600 feet per second.

As this top layer is at the surface of the ground and exposed, as its low velocity characteristic exists almost universally regardless of composition, and as this low velocity characteristic could result from a broken, disintegrated or loosely bedded condition, the term "weathered layer" is perhaps natural. This term has been in use for some time for the sake of convenience and for lack of a better one, though it has been granted that this layer does not, in many cases, conform to the geologic term "weathered." In the Gulf Coast region, for example, the geophysical weathered layer is exceptionally uniform and thin, though the geologic term "weathered" could be applied to the sediments at considerable depths.

The suggestion that the "weathered layer" might be associated with the regional ground-water table has often been made, but though in some areas the thickness of this material seems to be determined by the depth of the water table, this has not been found to be true in a sufficient number of cases to permit a general conclusion.

The extremely low velocities (approaching, or even less than, the velocity of sound in air) that have been obtained in some areas, led to the idea of air inclusion in the surface materials as the cause.[1]

With this idea in mind it was predicted that short (shallow) refraction profiles would show lower and lower velocities with shallower depth penetrations, approaching if not actually equaling the velocity

[1] The discussion here refers to air mixture in a free state—not in solution. As $V=\sqrt{\frac{E}{P}}$ air or gases may be considered together, as opposed to fluids or solids, because of the order of magnitude of difference in E and P for these two classes of materials. E = elasticity: P = density.

of sound in air, as the air became present in larger and larger proportions near the surface. The few profiles in which velocities lower than that of sound in air had been indicated, were considered interesting but somewhat anomalous, and were strongly suspected of containing some unseen error. A discussion of sound velocities in air-water mixtures,[1] which was pointed out by E. E. Rosaire, was the direct cause of the experiment here discussed. This experiment was carried out in the Gulf Coast region near Houston, Texas.

On the assumption that Gulf Coast gumbos may be treated as fluids, within limits, calculation of theoretical velocities to be obtained from earth-air mixtures in varying proportions were carried out from the equations given in the article mentioned,[2] namely:

$$V = \sqrt{\frac{E}{P}} = \sqrt{\frac{E_1E_2}{\{XE_2 + (1 - X)E_1\}\{XP_1 + (1 - X)P_2\}}} .$$

Where:

P_1 = density of air = .0012
E_1 = elasticity of air = 1.2×10^6
P_2 = density of earth, assumed as 1.9
E_2 = elasticity of earth, computed as 5.58×10^{10}
X = proportion air to total by volume
$1 - X$ = proportion earth to total by volume
V = velocity of sound in mixture

If the values of the composite velocities be plotted against proportions of air to earth, a curve of the form in Figure 1 is obtained.

Though this curve should perhaps be considered only qualitatively because of the assumption of earth as a fluid, it indicates the probability of obtaining velocities from earth-air mixtures, whose magnitudes may be actually less than that of sound in air alone. This prediction has been verified experimentally by shooting a short detailed refraction profile, extreme precautions being taken to muffle the shot point and hence eliminate any effect of the air wave. The recording was begun at a distance of 5 feet from the shot point and carried out on the surface to a distance which definitely indicated the "unweathered" velocity on the time-distance curve. This curve is shown as curve (A) in Figure 2.

[1] A. B. Wood, *A Textbook on Sound,* pp. 327–28.

[2] A. B. Wood, *op. cit.*

As can be seen from the curve, a velocity within the weathered layer is indicated (slope of curve near origin) as low as 550 feet per second, while the velocity of sound in air alone is approximately 1,100 feet per second. Assuming the low velocity to be due to the inclusion of air as postulated, we would naturally expect the velocity to be

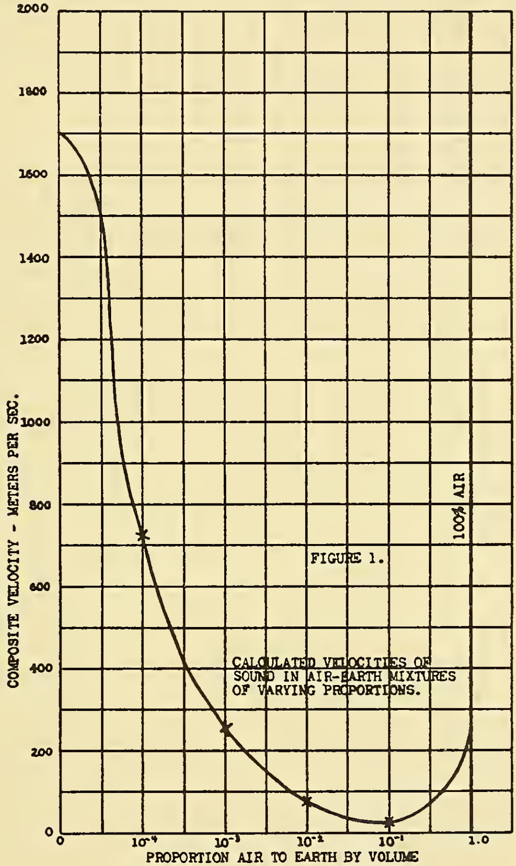

FIG. 1

lowest nearest the surface, where the proportion of air should be greatest. Referring this velocity (550 feet per second) to the curve (Fig. 1), the corresponding air concentration is 0.001 part by volume, which seems probable in the near-surface materials. A computation of the depth corresponding with the "break-point" of the time-distance curve indicates 7½ feet as the thickness of the "aerated layer."

After completion of this profile, a hole was dug at the shot point and the top of the water table was determined to be at a depth of slightly less than 8 feet. On the assumption that below the water table all air in a free state is displaced by water, whose density and elasticity are of the same order of magnitude as the earth under consideration, and hence has no material effect on the velocity of transmission of sound through such substance, both the shot point and recorders were lowered to the top of the water table. At this depth the shorter or "weathered" portion of the profile was re-shot with the result shown in curve (*B*), Figure 2.

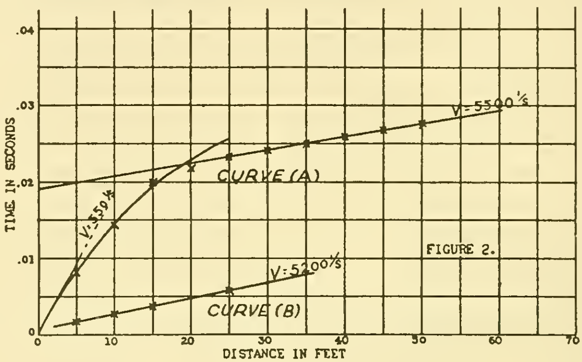

FIG. 2

On examination, this curve indicates an "unweathered" velocity (5,200 feet per second), which passes through the origin with only that curvature which is to be expected with a slight increase of velocity with depth. At the same recording distances from the shot point, the "weathered" portion of the time-distance curve has been eliminated with the elimination of the "aerated layer." The obvious conclusion is that where the water table is sufficiently shallow the thickness of the aerated layer is determined by its position. However, if the water table be at considerable depth, the "weathered" thickness is determined by the depth to which any appreciable amount of air penetrates.

This explains, in a measure, the sometimes extreme variation of "weathered" thickness with different surface materials. It would also predict a change of weathered thickness with wet and dry seasons.

Though these experiments have not been carried out to such an extent as to make the conclusions indisputable, it may be said that all the experimental data obtained favor the theoretical predictions resulting from the assumption of an "aerated" layer.

ACCURACY OF DETERMINATION OF RELATIVE GRAVITY BY TORSION BALANCE[1]

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ABSTRACT

Determination of relative gravity by the Eötvös torsion balance has been shown by Oltay and others to be as accurate as the determination by the invariable pendulum. In the present paper, the probable error of the torsion balance determination of relative gravity is calculated from the error of closure of 45 traverses comprising 2,800 stations, most of them in the Gulf Coast region. The probable error of each individual observation in those surveys is calculated to have been ±2.2 Eötvös units for the traverses taken together, or from ±1.9 *E* for the traverses with the larger station interval to ±5 *E* for the traverses for the small station interval. But the greater error of the individual observations in the traverses which have the shorter station interval has been compensated, intentionally, by that shorter interval, and the probable error of the determination of relative gravity by those 45 traverses is approximately 0.4 milledyne per 10 kilometers of traverse without regard to the magnitude of the probable error of the individual observations or of the station interval. The probable error of the determination of relative gravity between key points in good torsion balance surveys presumably is approximately ±2.5 to ±3.5 *E* per 10 kilometers airline distance between the two places. If pendulum determinations of relative gravity are used to supplement and to increase the accuracy of torsion balance surveys, there is a minimum interval at which the pendulum stations should be placed, for at lesser intervals the determination of relative gravity by the torsion balance is more accurate than that by the pendulum. That minimum interval ranges from 100 to 200 kilometers for the pendulum observations of the first quarter of the century to 8 to 50 kilometers for first class modern pendulum observations.

The relative accuracy of the determination of relative gravity by good torsion balance surveys in favorable terrane has been shown by Oltay, Jung, Numerov, and others to be comparable with the accuracy of the determination of relative gravity by the invariable pendulum during the past quarter century. The differences between the relative gravity by the torsion balance and pendulum on six traverses in the Arad district of the Hungarian plain were respectively 0.001, 0.001, 0.004, 0.000, 0.005, 0.000 cm. sec.$^{-2}$ The respective lengths of the traverses were 12, 9, 13, 20, 42, and 50 kilometers.[3] The probable error of the pendulum observations was calculated to

[1] Read before the Association at the Oklahoma City meeting, March 25, 1932.

[2] Consulting geologist and geophysicist, Petroleum Building.

[3] Karl Oltay, "Die Genauigkeit der mit der Eötvösschen Drehwage Durchgefuhrten Relativen Schwerkrafts-messungen," *Geodätische Arbeiten d. Baron R. v. Eötvösschen Geophysischen Forschungen*, No. III (Budapest, 1928).

be ±0.001 cm. sec.$^{-2}$ In the surveys in the Ries area of Bavaria, the differences between the determination of relative gravity by the torsion balance and the pendulum for six different lines were 0.003, 0.000, 0.000, 0.002, 0.003, 0.001 cm. sec.$^{-2}$ The lines ranged from 4 to 6 kilometers in length.[1] In the Ural Emba district of Russia, the differences were 0.001, 0.002, 0.007, 0.004, 0.003, cm. sec.$^{-2}$ The length of the traverses ranged from 9.5 to 15 kilometers.[2] In the Grosny district of Russia, the differences were 0.001, 0.002, cm. sec.$^{-2}$ and the lengths of the traverses ranged from 9.5 to 19 kilometers. As the probable error of the pendulum observations was ±0.001 to ±0.002, and as errors of three times the probable error should be expected to be fairly common, the torsion balance surveys gave as accurate a determination of relative gravity as did the pendulum observations. But from the data, it is impossible to tell whether the determinations of relative gravity by those torsion balance surveys were more accurate than those by the pendulum.

The accuracy of the torsion balance determinations of the gradient has been checked by the writer by calculation of the normal northward (planetary) gradient from torsion balance observations and comparison of that value with the theoretical value. The theoretical value was 7.14 E north in comparison with the value of 7.4 E., N. ¾°E., which was calculated from the torsion balance observations.[3]

The present paper reports the determination of the effective accuracy of the (horizontal) gradient which is observed with the Eotvös torsion balance and of the accuracy of relative gravity which is calculated from the gradient.

The data on which the study is based consist of the error of closure in relative gravity in forty-five closed traverses from commercial torsion balance surveys. The length of the traverses ranged from 3 to 144 kilometers and in twenty-eight of the traverses, the lengths were evenly distributed over the range of 10 to 40 kilometers. The interval between stations on thirty-two of the traverses was 300–500 meters, on five traverses, 150± meters, and on eight traverses, 600–800 meters. The instruments which were used on the surveys mainly

[1] Karl Jung, "Drehwagemessungen im Ries bei Nordlingen," *Zeitschrift für Geophysik*, Vol. 7 (1931), pp. 18–19.

[2] B. Numerov, "Results of the General Gravity Survey in the Emba District," *Zeitschrift für Geophysik*, Vol. 5 (1929), pp. 268–70; and "Results of Gravitational Observations in the Region of Grosny in 1928," *ibid.*, Vol. 5 (1929), pp. 271–75.

[3] Donald C. Barton, "Gravity Measurements with the Eötvös Torsion Balance," in "Physics of the Earth," Vol. II, "The Figure of the Earth," *Bull. National Research Council*, No. 78 (February, 1931), pp. 186–87.

were the small type of the Süss "Original Eötvös" torsion balances and, on a few traverses, large Askania (automatic) instruments. The calculation of relative gravity around each traverse was made by a method in which superposition of a transparent graph over a gradient arrow gives the spacing of the isogams at that station; the isogams are sketched forward toward the next station and are bent as the experience of the geophysicist and the exigencies of the adjacent gradient arrows seem to demand. Single abnormal gradient arrows were neglected or given reduced weight according to subjective estimation of their value. The location of most of the surveys was in the Texas-Louisiana Gulf Coast within 150 miles of the coast, but a few of the surveys were farther inland and some of the surveys were located outside of Texas and Louisiana. Most of the terrane was good, but a few of the surveys were made in areas in which caliche was present in the subsoil and other surveys were made in river bottoms. The magnitude of the gradient most commonly was less than 20 *E*. The station sites on a few of the surveys were prepared with the meticulous care of the European custom, but on most of the surveys, slightly less care was used. Probably two-thirds of the observations were made during the day-time and one-third at night. The general rate of occupation of stations was two stations per day per instrument and two stations per night per instrument if observations were taken at night, although in about a quarter of the surveys, the rate of observation was one station during the day and two at night per instrument. These surveys were made in commercial oil prospecting in which, generally, speed and moderate accuracy are regarded as more important than high scientific accuracy, and the purpose of the surveys was qualitative or only roughly quantitative rather than accurately quantitative. The surveys were made by many different torsion balance parties. Most of the observers were not highly trained scientists.

The precision of scientific measurements of such quantities as gravity in America is stated in terms of what is called the "probable error"; in Continental Europe, the "mean square error" is used; the "probable error" is 0.6745 times the "mean square error." The term "probable error" is a technical term and is defined as the value such that half of the errors which are likely to occur in a large number of measurements of the quantity will be larger and half smaller; that is, if the determination of gravity at a station is stated in an American publication to be 978.569 ± 0.001 cm. sec.$^{-2}$, it is understood that there are 50 chances out of 100 that algebraically, the actual error is less than −0.001 or greater than +0.001 and that there are 50 chances out of 100 that algebraically, the actual error will be between −0.001 and +0.001. According to the theory of the "probable error," the actual error will be five times the "probable error" 1 time in 1,000, four times the "probable error" 7 times in 1,000, three times the "probable error" 43 times in 1,000, twice the "probable error" 177 times in 1,000. The "probable error," therefore, affords a method of comparing the precision of different measurements and of determining the probable range within which the actual error of a measurement of quantity may lie.

The "probable error"[1] of a series of measurements depends upon the "probable error" of each measurement, the number of measurements, and the character of the series of measurements. If PE is the "probable error" of the series, if $pe-1$, $pe-2 \cdots pe-n$ are the respective "probable errors" of n measurements numbered, 1, 2, · · · to n and if the series consists of repeated measurements of a single quantity, the "probable error" of the arithmetic mean of the measurements is:

$$PE = \frac{pe}{\sqrt{m}} \tag{1}$$

if the pe's are all the same. But if the series consists of a series of consecutive applications of the measuring device, as for example, the consecutive use of a surveyor's tape, in measuring the length of a line which is longer than the tape:

$$PE = \sqrt{(pe_1)^2 + (pe_2)^2 + \cdots (pe_n)^2} \tag{2}$$

or if the pe's are all the same:

$$PE = \sqrt{n}\, pe \tag{3}$$

The "probable error" (pe) of a mean in terms of the actual error (ae) of each measurement is given by the formula:

$$PE = 0.6745\sqrt{\frac{(ae_1)^2 + (ae_2)^2 + \cdots + (ae_n)^2}{n}} \tag{4}$$

The accuracy of torsion balance surveys can be expressed by a probable error which can be calculated by the formula:

[1] Mansfield Merriman, *A Textbook on the Method of Least Squares*, 8th ed. (1911), pp. 66–79.

$$(5)\ PE_{U_{sz}} = 0.6745\sqrt{\frac{\left(\frac{EC_1}{L_1}\right)^2 N_1 + \left(\frac{EC_2}{L_2}\right)^2 N_2 + \cdots \left(\frac{EC_n}{L_n}\right)^2 N_n}{n}}$$

where:

EC_1, EC_2, $\cdots$ EC_n are the respective errors of closure in Δg of n closed torsion balance traverses numbered from 1, 2, $\cdots$ to n;

N_1, N_2, $\cdots$ N_n are the respective number of stations in those traverses;

L_1, L_2, $\cdots$ L_n are the respective lengths of those traverses in centimeters;

n is the number of traverses; and

$PE_{U_{sz}}$ is the probable error in the dimensions of the gradient.

The formula is adapted from Merriman's formula:[1]

$$(6)\qquad PE = 0.6745\sqrt{\frac{1}{n}\left(\frac{d_1^2}{L_1} + \frac{d_2^2}{L_2} + \cdots + \frac{d_n^2}{L_n}\right)}$$

where PE is the probable error of each measurement with a surveyor's chain. It is assumed that the length of (n) lines (1, 2, $\cdots$ n) has been measured with the chain and is expressed in chain lengths; and that two measurements of each line were made. The respective differences between those duplicate measurements of each line are represented by d_1, d_2, $\cdots$ d_n and the length of the respective lines in chain lengths by L_1, L_2, $\cdots$ L_n. PE, then, is the probable error of any one of the many single measurements of the chain. The probable error of the measurement of each line, then, can be calculated by formula (3) from the PE which has been obtained by formula (6), and from the number of individual consecutive measurements with the chain which were necessary to measure the length of the line.

The measurement of Δg by the torsion balance around a series of closed traverses is comparable to that duplicate measurement of each of a series of lines with a surveyor's chain. Each duplicate measurement of the length of the line may be regarded as a closed traverse which will consist of as many single measurements with the chain as the length of the line in chain lengths. A torsion balance traverse which is composed of N successive stations may be regarded as N consecutive measurements of a line comparable with those consecutive applications of the surveyor's chain to obtain the length of those lines. The error of closure of Δg, (EC), of a closed torsion balance

[1] *Op. cit.*, p. 103.

traverse is comparable with the difference, (*d*), between the duplicate measurements of the length of a line with the surveyor's chain; and the number of torsion balance stations, *N*, is comparable with the number of applications of the surveyor's chain.

The formula (6), therefore, may be re-written in the notation which we have assumed for the torsion balance surveys, to read:

$$(7) \qquad PE = 0.6745\sqrt{\frac{1}{n}\left(\frac{\overline{EC}_1^2}{N_1} + \frac{\overline{EC}_2^2}{N_2} + \cdots \frac{\overline{EC}_n^2}{N_n}\right)}$$

The error of closure, *EC*, is in terms of Δg (that is, cm. sec.$^{-2}$) and the probable error (*PE*), therefore, would be given in terms of Δg per unit of measurement which according to our preceding assumptions is the station interval. The latter rather commonly is fairly constant within a traverse, but is variable from traverse to traverse. The calculation of Δg over a station interval has the mathematical form of a gradient times distance, that is, times the length of the station interval; and, therefore, any error in Δg over a station interval will tend to be lineally proportional to the length of the station interval. The calculated "probable error" in terms of Δg and of the inconstant station interval would be useless. The "probable error," however, may be reduced to the dimensions of a gradient and then will be independent of the length of the station interval. All error in the calculation of Δg over a station interval may be assumed to be in the gradient, if, as we usually assume, the stations are close enough together so that the variation of the gradient is linear from station to station. The probable error in Δg over a station interval, then, will vary linearly with the length of a station interval and may be expressed as the product of the length of the station interval times that probable error in the gradient. Conversely, the probable error of the gradient may be expressed as the quotient of the probable error in terms of Δg over the station interval divided by the length of the station interval. If the terms

$$\frac{\overline{EC}^2}{N}$$

of formula (7) are weighted in terms of the square of the reciprocal of the length of the station interval, *SI*, to read

$$\frac{\overline{EC}_1^2}{N_1\overline{SI}_1^2},\ \frac{\overline{EC}_2^2}{N_2\overline{SI}_2^2},\ \frac{\overline{EC}_n^2}{N_n\overline{SI}_n^2}$$

the errors of the torsion balance surveys, then, are expressed in terms of gradient and not of Δg and the probable error of the torsion balance measurements will be in terms of gradient. But

$SI = \frac{L}{N}$ and the other terms $\frac{\overline{EC^2}}{N \cdot (SI)^2}$ can be written:

$$\frac{\overline{EC^2}}{N\frac{L^2}{N^2}} \text{ or } \frac{N \cdot \overline{EC^2}}{L^2}.$$

If the terms $\frac{\overline{EC^2}}{N}$ of formula (7) are replaced by $\frac{N \cdot \overline{EC^2}}{L^2}$ formula (5) is obtained, which gives the probable error in the torsion balance observations in terms of gradient; that is, in Eötvös units, although the observed errors are in terms of Δg; that is, of cm. sec.$^{-2}$

Formula (5) involves the two assumptions, first, that the error in calculation of Δg over a station interval is a linear function of the length of the station interval and of the error in the gradient and, second, that the error is independent of the magnitude of the gradient. Neither assumption is exactly true but in the present study no attempt has been made to analyze that more complicated variation of the error of torsion balance measurements.

The original data and the calculated values of

$$\frac{EC^2 \cdot N}{L^2}$$

are given in Table I.

The "probable error" of the determination of the gradient at those twenty-eight hundred odd stations, therefore, is of the general magnitude of ± 2.2 E, but the "probable error" of the determination of the gradient may be different in different areas. If the surveys of Table I are grouped by the length of the interval between stations, the "probable error" of the gradient varies considerably between groups. The various "probable errors" are given in Table II. They range from ± 1.5 E to ± 5.0 E. The corresponding "probable error" in the individual traverses must have a yet higher range. The figure, ± 2.2 E, therefore, itself has a large "probable error" and only its general magnitude is of significance. Intuitively, the writer for years has estimated the common error of torsion balance observations of the gradient in the Gulf Coast region as of the magnitude of 1.5 to 2.5 E.

TABLE I

ORIGINAL DATA AND CALCULATED VALUES OF $\frac{EC^2N}{L^2}$

Error of Closure EC in 10^{-4} Cm. Sec.$^{-2}$	*Number of Stations N*	*Length of Traverse L in Km.*	$\frac{EC^2N}{L^2}$	*Error of Closure EC in 10^{-4} Cm. Sec.$^{-2}$*	*Number of Stations N*	*Length of Traverse L in Km.*	$\frac{EC^2N}{L^2}$
7.6	71	38.4	2.8	2.0	30	11.6	0.9
3.8	72	45.1	0.5	2.9	90	32.0	0.7
11.4	37	19.5	12.6	7.6	65	25.9	5.8
7.6	44	30.5	2.7	11.4	178	28.3	29.0
11.4	45	36.0	5.2	2.4	201	105.2	1.0
11.4	51	29.6	7.6	8.0	75	41.1	2.9
7.6	56	28.7	4.0	3.0	102	23.1	1.7
7.6	64	36.6	2.9	5.0	97	21.3	5.4
0	123	37.2	0	8.0	64	14.0	21.0
3.8	65	19.5	0.2	6.0	80	19.5	0.7
7.6	82	23.2	8.9	5.0	81	24.4	3.4
7.6	73	22.6	8.3	7.6	61	35.6	2.8
13.3	201	69.5	7.4	15.2	79	43.6	9.6
36.1	164	65.5	50.5	11.4	56	28.0	9.3
36.1	171	53.9	76.5	3.8	24	11.3	2.2
5.7	64	27.4	2.8	5.7	36	17.0	4.1
4.8	50	13.4	6.4	1.9	39	17.7	0.4
8.0	79	58.5	1.5	5.7	32	15.5	4.4
4.8	26	3.0	66.5	13.3	130	72.2	4.4
1.6	35	4.9	3.8	7.6	113	51.8	2.4
1.6	38	4.6	4.7	17.1	100	48.9	12.3
4.8	71	9.4	18.5	0	62	37.6	0
$n=45$ $PE_{U_m}=0.6745\sqrt{418.7.45}=2.2E$				0	349	143.8	0
							418.7

TABLE II

PROBABLE ERROR OF THE DETERMINATION OF THE GRADIENT IN SURVEYS WITH DIFFERENT STATION INTERVALS

Station interval in meters....	150	300	400	500	600	700	800
Number of traverses........	5	12	8	12	6	2	1
Probable error.............	±5.0	±3.4	±2.8	±1.9	±1.9	±1.5	±2.3

TABLE III

PROBABLE ERROR OF DETERMINATION OF Δg BETWEEN PLACES BY GOOD TORSION BALANCE SURVEYS IN 10^{-4} DYNES (0.1 MILLEDYNE)

Distance between places in kilometers.	1	5	10	20	40	80	100
Probable error of: single direct traverse; or single traverse distance measured along traverse; or two more or less indirect traverses.....	±12	±3	±4	±6	±8	±11	±13
Several traverses..................	±0.8	±2	±3	±4	±6	±8	±9

That "probable error," $\pm 2.2\ E$, is the composite error of a considerable number of factors. It arises: (a) from all instrumental errors in the determination of the crude gradient at the station, (b) from all errors in applying corrections to the crude gradient, (c) from errors in plotting gradient arrows on the map and from the crudeness of gradient arrows on a scale of 1 mm. $= E$ as the starting point for the calculation of relative gravity, (d) from inaccuracy in the use of the writer's method of calculating relative gravity, and (e) from error arising from the fact that the interval between stations is not sufficiently close to give a true picture of the variation of the gradient. It probably also varies with the magnitude of the gradient; the data are not sufficient to study that variation. An actual error five times the "probable error" should occur once in one thousand times, according to the theory of the "probable error." Divergent gradients which depart vectorially much more than $5 \times \pm 2.2\ E$ are moderately common. Such plainly aberrant gradient arrows were disregarded by the geophysicist in the calculation of relative gravity in the traverses from which the data of Table I were taken, and their weight, therefore, was not felt in the calculation of that "probable error."

The "probable error" of the determination of relative gravity by good torsion balance surveys can be stated in a simpler and more practical form. The "probable error" of the determination of relative gravity by the torsion balance can be controlled within certain limits by variation of the station interval. Formula (3) can be rewritten:

$$\text{(8)} \qquad \begin{aligned} &\text{from:}\ PE = \sqrt{n} \cdot pe \\ &\text{to:}\ PE_{\Delta g} = \sqrt{n} \cdot SI \cdot pe_{U_{xz}} = \frac{L}{\sqrt{n}} pe_{U_{xz}} \end{aligned}$$

for the special case of the torsion balance. If the number of stations in a traverse of constant length is varied as the square of the variation of the "probable error" of the individual observations, the "probable error" of the determination of Δg for the traverse remains constant. In practice in good torsion balance surveys, the station interval between stations is decreased in areas of irregular and erratic gradient arrows and is increased in areas of consistent gradient arrows. The "probable error" of good ordinary torsion balance surveys in practice, therefore, should be independent of the station interval and should vary as the square root of the length of the traverse. If $(SI \cdot pe_{U_{xz}})$ in formula (8) is maintained constant, then (n) will vary directly with the length of the traverse, and $PE_{\Delta g}$ will vary as the square root of the length of the traverse. As $SI \cdot pe_{v_{st}}$ somewhat commonly in practice is held approximately constant, the "probable error" of the determination of Δg by ordinary good torsion balance surveys can be expressed in terms of Δg per some standard length of traverse.

That this is so, can be seen by analysis of the error of closure of those 45 traverses. The error of closure of each closure of Δg in each of those traverses can be scaled up or down by the formula

$$EC_{10} = \sqrt{\frac{10}{L}} \cdot EC \qquad (9)$$

to the error of closure (EC_{10}), which the traverse would have had, if its length had been 10 kilometers instead of (L) kilometers, and if there had been no change in the station interval. The resulting errors of closure per 10-kilometer traverses have been sorted by length of the station interval and are given in Table IV. The respective errors of closure and the respective "probable errors" of Δg for the traverses of 100, 200, 300, 400, 500, 600 meters station interval, can be seen to

TABLE IV

ERRORS OF CLOSURE SCALED TO A 10 KM. LENGTH OF THE TRAVERSE

Station interval in meters	100	200	300	400	500	600	700	800
Error of closure of Δg in 10^{-4} dynes	8.7	6.8	15.6	14.1	8.2	7.3	4.3	6.0
	4.9	6.8	5.0	5.0	7.7	6.6	3.3	
	3.4	4.3	5.0	4.7	6.8	4.9		
	2.3	3.4	4.1	3.4	4.5	4.0		
		2.0	3.2	1.9	4.4	4.0		
			2.7	1.6	3.9	1.8		
			0.0		3.9	0.0		
					3.6	0.0		
					3.3			
					1.6			
					1.4			
"Probable error" of traverse..........	±3.6	±3.4	±4.6	±4.4	±3.4	±3.2		

"Probable error" for all 45 traverses: ±3.9

be independent of the length of the station interval. That "probable error" ranges only from 0.32 milledyne for the traverses with a station interval of approximately 600 meters to 0.46 milledyne for the traverses of approximately 300 meters; and for the traverses of approximately 100, 200, 500, and 600 meters length, that "probable error" ranges only from 0.32 to 0.36 milledyne. The "probable error" of the individual observations, however, was considerably larger in the traverses with the shorter station interval (Table II). That larger "probable error" of the individual observations in the traverses with the shorter station interval has been compensated by the use of that shorter station interval; and the "probable error" of the traverse as a whole is no greater for those traverses with the shorter station interval than for those with the longer station interval. The "probable error" of determination of relative gravity by those 45 traverses is 0.4 milledyne per 10 kilometers of traverse without regard to the station interval; and the actual error of closure, with two exceptions, is less than 0.9 milledyne and with only five exceptions, is less than 0.75 milledyne per 10 kilometers of traverse without regard to the station interval.

The "probable error" of the determination of relative gravity between two places by torsion balance surveys depends in part on the number of torsion balance traverses which connect the two places, decreasing inversely as the square of the number of equally good traverses. Key places in torsion balance surveys in general are connected by at least two traverses, and in many surveys are connected by a net of traverses. Different weights in general should be given the different traverses in the calculation of the relative gravity between the two places and in the calculation of the "probable error" of that determination of the relative gravity. But that "probable error" in practice probably will be $1/\sqrt{2}$ to $1/\sqrt{3}$ times the "probable error" of the determination of the relative gravity by a single traverse. The "probable error" of the determination of relative gravity between key places within the surveys of those 45 traverses, therefore, should be $(1/\sqrt{2}$ to $1/\sqrt{3})$ times 0.4 milledyne or 0.30 to 0.25 milledyne for places 10 kilometers apart along the route of the torsion balance traverses. For an airline distance of 10 kilometers between the two places, the corresponding "probable error" in practice should be 0.7 to 1.0 times the "probable error" of the determination of Δg by an airline torsion balance traverse between the two places and, therefore, should be 0.3 to 0.4 milledyne.

Shrewd adjustment of errors in those 45 traverses presumably has reduced the "probable error" of those determinations of relative gravity below the figures of the preceding paragraph. The error of closure in Δg in torsion balance traverses commonly is caused not only by the cumulative effects of small, or moderately small, errors at each set-up but also by a few large errors. A skilled interpreter commonly can recognize the weak spots in which those errors are mostly likely to have come in; by disregarding or adversely weighting the effect of single seriously abnormal gradient arrows, or less commonly, pairs of gradient arrows, he attempts to eliminate such errors in the first calculation of relative gravity; in the subsequent adjustment of the traverse or traverses, somewhat commonly much of the error of closure may be eliminated by a slight change in his weighting of those few irregular gradient arrows. The variation of the gradient from station to station in many places is not linear and the geophysicist has a certain latitude in his subjective estimation of the probable variation of gravity between the two stations. The error of closure in many traverses can be almost eliminated by taking those slightly different alternative choices in the spacing and the running of the isogams. The subjective weighting of observations, of course, is open to danger of error and in the hands of an inexpert geophysicist, leads to much more erroneous results than the automatic mathematical distribution and adjustment of the error. But the writer believes that the expert interpreter can obtain better accuracy by exercising subjective judgment in throwing as much of the errors of closure as possible into the weak places in lines rather than by automatic mathematical distribution of the errors. The "probable error" of the determination of relative gravity in those 45 traverses presumably, therefore, is slightly less than ± 0.25 milledyne per 10 kilometers of double traverse and less than ± 0.35 milledyne between places 10 kilometers apart, airline distance.

That "probable error," 0.25 milledyne per 10 kilometers of double traverse or 0.35 milledyne per 10 kilometers of airline distance, presumably is characteristic of most good torsion balance surveys in what a torsion balance operator would call good, fair, and slightly poor torsion balance terrane. Those 45 traverses lay in all types of terrane in the Gulf Coastal Plain of Texas and Louisiana and a few of them were on the llanos of Venezuela and elsewhere than in the Gulf Coastal Plain. Less than one-fourth of the traverses were in the Coastal Prairies of Texas and Louisiana, which are especially favorable for a low "probable error" of observation. The 45 traverses, therefore, are fairly characteristic of torsion balance work in areas which are commonly regarded as good, fair, or slightly poor for torsion balance observations. Mathematically the same grade of accuracy could be maintained by use of a sufficiently close net of stations to compensate the "probable error" in the individual observations, but the number of stations necessary increases as the square of the increase in the "probable error" of the individual observations, and, therefore, the cost and tediousness of too close a net of stations puts a practical limit to the maintenance of that accuracy of ±0.35 milledyne between places 10 kilometers apart, airline distance, or for the maintenance of any particular grade of accuracy. In reconnaissance in oil work, a station interval less than 250 meters commonly is impracticable, and if the "probable error" of the individual gradient observations rises above ±2 *E*, it becomes impracticable to maintain an accuracy of ±0.35 milledyne in the determination of relative gravity between places approximately 10 kilometers apart, airline distance. In some areas in South Texas, in which the caliche lies close to the surface, the "probable error" of individual observations probably is greater than ±7 *E;* and in certain areas in East Texas, the "probable error" of individual observations probably is greater than ±5 *E*. The "probable error" of the determination of gravity between two places approximately 10 kilometers apart in such areas will be very much greater than ±0.35 milledyne.

Knowledge of the "probable error" of the determination of Δg by torsion balance surveys is important in certain types of combinations of torsion balance and pendulum surveys. According to a rather common suggestion from "pure" geophysicists, pendulum observations of relative gravity at a net of key stations should be used to provide a framework of accurate gravity benchmarks to which the supposedly less accurate torsion balance surveys would be tied and adjusted. But the accuracy of the torsion balance surveys will be increased thereby only if the "probable error" of the determination of relative gravity by the pendulum between two stations is less than that by the torsion balance surveys. The determination of relative gravity by the relative pendulum is independent of the distance between stations; the value of Δg between two stations is simply the difference between the observed values of relative gravity at the two stations, and the "probable error" of Δg between the two stations is the square root of the sum of the squares of the respective "probable errors" of the individual observations. The "probable error" of the determination of Δg between two places by good torsion balance surveys varies, in practice, with the square root of the distance between the two places. The torsion balance surveys will measure Δg more accurately than the pendulum if the two stations can be connected by very short torsion balance traverses, and the pendulum will measure Δg more accurately than the torsion balance if the two stations can be connected only by very long torsion balance traverses.

If the pendulum stations are placed closer together than a certain minimum distance, the "probable error" or the pendulum determination of Δg between adjacent stations will be greater than the "probable error" of the torsion balance determination of that Δg; and adjustment of the torsion balance survey to the pendulum values of Δg will decrease the accuracy of the torsion balance survey. If the distance between the adjacent pendulum stations is greater than that minimum distance, the "probable error" of the pendulum determination of Δg between adjacent stations will be less than that of the torsion balance determination; and the accuracy of the torsion balance survey will be improved by adjusting its results to the pendulum results.

The accuracy of the determination of relative gravity by the relative pendulum varies. The "probable error" of the pendulum observations of the latter part of the past century presumably was, in general, many milledynes. The probable error of the observations of the recent past in general has been ± 1 milledyne, although a few poorer but acceptable stations have had a "probable error" of ± 2 milledynes. The better modern observations are reported to have a "probable error" of ± 0.3 to ± 0.5 milledyne. The geophysical department of one oil company asserts that its pendulums are reoccupying stations with a "probable error" of observation less than ± 0.1 milledyne. The "probable error" of the determination of the elevation or relative elevation of the pendulum station is relatively unimportant in connection with the pendulum observations whose "probable error" is ± 1 milledyne or more, but is, of course, of very considerable importance in connection with observations whose "probable error" is less than ± 0.3 milledyne, for the variation of (g) with elevation is ± 0.308 milledyne per meter. The "probable error" in feet of first class (but not precise) levelling is approximately ± 0.05 (number of miles), but only 1.5 miles of closed traverse, or 3 miles of unrepeated traverse can be run per day in levelling of that accuracy and, therefore, most commercial levelling in connection with commercial geophysical work probably will be speedier and less accurate and its "probable error" may have to be taken seriously into account. The "probable error" of the station determination of gravity will be the square root of the sum of the square of the "probable error" of the pendulum determination of gravity, plus the square of the "probable error" of the determination of the elevation. The "probable error" of the determination of Δg between two places will be the sum of the squares of the respective "probable errors" of the two pendulum observations and of the two determinations of the elevation. The "probable error" of the pendulum determination of gravity at each key station can be decreased by reoccupation of that station or by the occupation of additional closely adjacent stations which are tied together by torsion balance determinations of the value of Δg between the key station and each of the subsidiary stations. The decrease in the "probable error" will vary as the reciprocal of the square root of the number of stations.

The minimum interval between pendulum observations which are to supplement ordinary, good torsion balance surveys in good, fair, or even slightly poor torsion balance terrane is given in Table V.

TABLE V

MINIMUM INTERVAL AT WHICH PENDULUM OBSERVATIONS SHOULD BE USED TO TORSION BALANCE SURVEYS

	"Probable Error" of Torsion Balance Determination of Δg in 10^{-4} Dynes for Places 10 Km. Apart Airline Distance		*"Probable Error" of Pendulum Determination of Δg in 1×10^{-4} Dynes*							
			±10		±5		±2½		±1	
			Interval in Kilometers							
	*A**	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*
Poorer "Good" surveys......	7	5	40	75	10	20	2½	5	0.3	0.5
Average "Good" surveys......	4	3	120	220	30	55	8	14	1.0	1.5
Better "Good" surveys......	2	1½	500	650	120	210	30	55	4.5	6

* *A*. The two places are connected by a single direct or two indirect torsion balance traverses.
B. The two places are connected by several traverses.

In surveys primarily by the pendulum, or in pendulum surveys which are supplemented by sketchy torsion-balance surveys, the intervals between the pendulum stations very much less than the minimum intervals of Table IV may be used in order to increase the accuracy of the survey.

CURVATURE OF EQUIPOTENTIAL SURFACES[1]

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ABSTRACT

If one is to have a complete and satisfying understanding of the theory of the torsion balance, it is essential to have clearly in mind a physical picture of the quantities involved. Most authors assume either that their readers are well versed in the differential geometry of surfaces in a Euclidean three-dimensional space or that a discussion of surface curvature is far beyond the reader's ability to grasp. Too often is neither of these true. The writer attempts to present a mathematical discussion of this matter in which only the bare fundamental concepts of the differential calculus are needed.

INTRODUCTION

In this paper, the writer attempts to develop the mathematical theory of the curvature of the equipotential surfaces due to the gravitational field, with the purpose of producing a clear "physical" picture of this quantity. It is the writer's impression that such a development, although fundamental to the mathematician, is lacking in the usual torsion balance literature available to the geophysicist. Our goal is to find the curvature relations used in torsion balance work. The mathematical needs for this subject are not too great—and for further simplification, the demands of mathematical rigor are, at times, sacrificed in this paper

DEFINITIONS

Consider an arbitrary point P on a surface S, which is "smooth" in the neighborhood of P in the sense that the equation of S may be expressed as a series when the surrounding space is referred to a convenient cartesian coördinate system. The *tangent plane* to S at P is that plane in which all the lines tangent to S at P lie. The straight line perpendicular to this plane at P is the *normal* to the surface S at that point.

COÖRDINATE SYSTEM

We now proceed to choose, as is our privilege, a coördinate system in such a manner that the mathematical work involved is materially

[1] Read before the Association at the Oklahoma City meeting, March 25, 1932.

[2] Geophysics department, Humble Oil and Refining Company. Introduced by L. W. Blau.

simplified. Let the normal to S at P, oriented in either of its directions, be the z axis. In the tangent plane to S at P let any two mutually perpendicular lines be chosen as the x and y axes. Both of these axes are obviously perpendicular to the z axis. Let us also point out

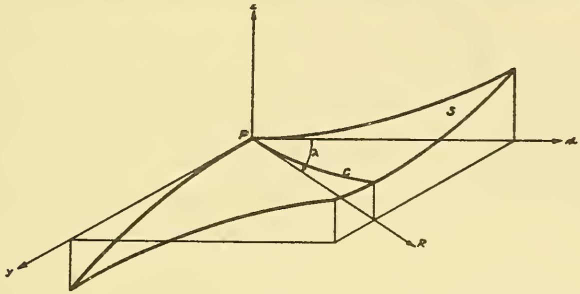


FIG. 1

that the tangent line at P to any curve on S passing through P lies in the (x, y) plane. The situation thus far described is schematically shown in Figure 1, where, for convenience, only part of the surface S in the neighborhood of P is indicated.

EQUATION OF S

Suppose that the equation of S, referred to the chosen coördinate system, is

(1) $$z = z(x, y),$$

and that this expression is expanded in series about P; that is,

(2) $$z = a + (px + qy) + \tfrac{1}{2}(rx^2 + 2sxy + ty^2) + \cdots,$$

where

$$p = \frac{\partial z}{\partial x}, \qquad q = \frac{\partial z}{\partial y},$$

$$r = \frac{\partial^2 z}{\partial x^2}, \qquad s = \frac{\partial^2 z}{\partial x \partial y}, \qquad t = \frac{\partial^2 z}{\partial y^2},$$

the values of these derivatives being taken at the point P, whose coördinates are, of course, (0, 0, 0). The series (2) has been written only to the second order terms, which are all that are required.

Since the surface S passes through P:(0, 0, 0), equation (2) must be satisfied by its coördinates. Accordingly, $a = 0$. Again, the slope of the curve on S at P, which is the section of the surface made by the (x, z) plane, is the value of p there, and consequently this is zero. Similarly, $q=0$. Thus, by virtue of our choice of axes, the series (2) reduces to

$$z = \tfrac{1}{2}(rx^2 + 2sxy + ty^2) + \cdots . \tag{3}$$

CURVATURE OF A PLANE SECTION OF S AT P

Let C be the curve of intersection of any plane zPR through the z axis with the surface S, and let the angle between this plane and the (x, z) plane be λ. Obviously, then, the curve C is a plane curve and PR, the tangent line to it at P, lies in the (x, y) plane, with the angle xPR equal to λ. The lines Pz and PR are at right angles to each other, and we choose these lines as cöordinate axes in their plane zPR. Since

$$x = R\cos\lambda,\ \ y = R\sin\lambda, \tag{4}$$

the equation of C in terms of the coördinates (R, z) is, by virtue of (3):

$$z = \tfrac{1}{2}R^2(r\cos^2\lambda + 2s\sin\lambda\cos\lambda + t\sin^2\lambda) + \cdots . \tag{5}$$

We recall at this stage the theorem of the elementary differential calculus that the curvature of the curve whose equation is $y=f(x)$ at a point is the value of

$$k = \frac{\dfrac{d^2y}{dx^2}}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}} \tag{6}$$

at that point. In equation (5), z is expressed as a function of R, and, at P, $z=R=0$. The quantities r, s, t are constants, inasmuch as they represent the values of the second derivatives of z with respect to x and y at P; and λ is a constant for the particular section C chosen. Consequently, the curvature of C at P is the value of

$$k = \frac{\dfrac{d^2z}{dR^2}}{\left[1+\left(\dfrac{dz}{dR}\right)^2\right]^{3/2}}, \tag{7}$$

when $z=R=0$.

Differentiating (5), we obtain

$$\frac{dz}{dR} = R(r\cos^2\lambda + 2s\sin\lambda\cos\lambda + t\sin^2\lambda) + \cdots , \tag{8}$$

and

(9) $$\frac{d^2z}{dR^2} = (r \cos^2 \lambda + 2s \sin \lambda \cos \lambda + t \sin^2 \lambda) + \cdots .$$

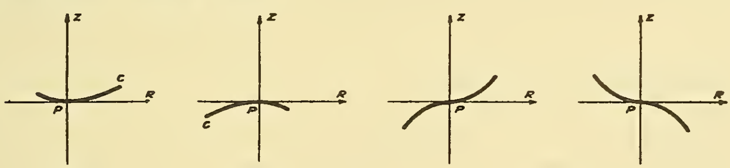
The terms of each of these series beyond those written involve R to at least the first power. At P, then, where $z = R = 0$,

(10) $$\frac{dz}{dR} = 0, \quad \frac{d^2z}{dR^2} = r \cos^2 \lambda + 2s \sin \lambda \cos \lambda + t \sin^2 \lambda.$$

It is these values, as we have seen, that we must use in (7) to obtain the curvature of C at P, and substituting these, we have

(11) $$k = r \cos^2 \lambda + 2s \sin \lambda \cos \lambda + t \sin^2 \lambda.$$

In ordinary work in the elementary differential calculus, the value of the curvature of a curve at a point is taken as the absolute value of (6) and no attention is paid to the fact that the numerator, $\frac{d^2y}{dx^2}$, may be negative as well as positive. However, in surface theory, the algebraic value of the curvature is important, and our definition of the curvature of the plane section C of the surface S at the point P, as embodied in (7), places due regard on the *algebraic* value of $\frac{d^2z}{dR^2}$. We append a schematic series of figures here to show, somewhat more clearly, what is meant.



2a 2b 2c

FIG. 2

Figure 2*a* indicates the form of the curve C in the neighborhood of P, if, for the value of λ defining C, $\frac{d^2z}{dR^2} > 0$. Figure 2*b* indicates the same thing if $\frac{d^2z}{dR^2} < 0$. Finally, if for that value of λ defining C, $\frac{d^2z}{dR^2} = 0$ and $\frac{d^3z}{dR^3} \neq 0$ at P, the form of C is as shown in one or the other of the figures 2*c*.

VARIATION OF CURVATURE WITH λ

Equation (11) expresses the algebraic value of the curvature at the point P of that plane normal section C of the surface S, made by the plane through the z axis whose angle with the (x, z) plane is λ. Varying the value of λ from 0° to 180° continuously corresponds to twisting the normal plane continuously around the z axis from the (x, z) plane, $(\lambda=0^\circ)$, through its position of coincidence with the (y, z) plane, $(\lambda=90^\circ)$, and finally back to the (x, z) plane for which $\lambda=0^\circ$ or 180° (from 180° to 360°, the sections obtained as λ varies from 0° to 180° are repeated).

The first fact to notice is that if for all values of λ from 0° to 180°, the curvature (11) is always positive, or always negative, the surface in the neighborhood of P lies on one side of its tangent plane. If, however, as λ varies in this range, the curvature changes sign, part of the surface lies on one side of the tangent plane to S at P and part on the other. The latter situation is the one shown schematically in Figure 1.

Suppose that we fix our attention to the curvature k_1 at P of the section for which $\lambda=\lambda_1$; that is,

$$k_1 = r\cos^2\lambda_1 + 2s\sin\lambda_1\cos\lambda_1 + t\sin^2\lambda_1. \tag{12}$$

Consider the curvature k_2 at the same point, of the section whose plane is at right angles to that of the first section; that is, that for which $\lambda=\lambda_2=\lambda_1\pm 90^\circ$. Substituting this value for λ in (11) yields:

$$k_2 = r\sin^2\lambda - 2s\sin\lambda\cos\lambda + t\cos^2\lambda. \tag{13}$$

The sum of (12) and (13) is:

$$k_1 + k_2 = r + t, \tag{14}$$

a constant. To put this interesting result in words: *The algebraic sum of the curvatures at a non-singular point of an analytic surface of any two normal plane sections at right angles to each other is constant.* One-half of this constant is called the *mean curvature* of the surface at the point.

The next question to be raised in regard to (11) is whether, as λ varies from 0° to 180°, the curvature k reaches extreme (maximum and minimum) values; and, if so, for what values of λ? A maximum or minimum value of k is obtained whenever $\frac{dk}{d\lambda}=0$ and $\frac{d^2k}{d\lambda^2}\neq 0$.

Differentiating (11), we obtain:

(15) $$\frac{dk}{d\lambda} = (t - r) \sin 2\lambda + 2s \cos 2\lambda.$$

If this derivative is set equal to zero, and solved for λ, we find:

(16) $$\tan 2\lambda = \frac{2s}{r - t} .^{*}$$

There are two values of λ lying between 0° and 180° which satisfy equation (16), and their difference is 90°.[1] That is to say, if λ_1 is one solution of (16) lying in value between 0° and 180°, then $\lambda_1 \pm 90°$ is

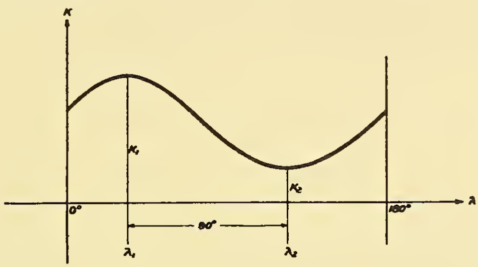


FIG. 3

the other, that one being chosen which lies in value in the same range. This result may be stated in the form of a theorem: *At every non-singular point (not umbilical) of an analytic surface there are two normal sections of the surface, at right angles to each other, for which the curvatures attain extreme values.*

Combining this theorem with the preceding one, in which we have seen that the sum of the curvatures for two normal sections at right angles to each other is a constant (*cf.* (14)), we must conclude that at one of these sections the curvature is a maximum and at the other it is a minimum.

* To show that $d^2k/d\lambda^2 \neq 0$ when $dk/d\lambda = 0$, we note that, since

$$\frac{d^2k}{d\lambda^2} = 2 \cos 2\lambda [(t - r) - 2s \tan 2\lambda],$$

its value when $dk/d\lambda = 0$ is obtained by substituting the value of λ of equation (16) in this last equation. After simplification, this reduces to

$$\pm 2\sqrt{(r - t)^2 + 4s^2},$$

which can vanish when and only when $r = t$ and $s = 0$. A point at which this occurs is an *umbilical* point; and, referring back to equation (11), we see that the curvatures of the normal sections to a surface at a point of this type are all equal. Such points will be excluded from this discussion.

[1] If $r = t$ and $s \neq 0$, the values of λ are 45° and 135°.

If the curvatures k of the plane normal sections be plotted against λ for $0° \leqq \lambda < 180°$, the curve would have the characteristic form shown in Figure 3. If the curvature changes sign, as λ varies, the corresponding curve crosses the λ-axis twice.

It has been shown that the difference in the values of λ for which k is a maximum and that for which it is a minimum is 90°. Moreover, the algebraic sum of the ordinates (k) for any two values of λ whose difference is 90° is a constant.

The directions on the surface at P in which these extreme values of the normal curvature occur are called the *principal directions*. The reciprocals, ρ_1 and ρ_2, of the extreme values of the curvatures, k_1 and k_2, are called the *radii of principal curvatures:*

$$k_1 = \frac{1}{\rho_1}, \quad k_2 = \frac{1}{\rho_2} \cdot$$

The *total* or *Gaussian curvature* of the surface at the point is defined as the product k_1k_2; and one-half the sum: $\frac{1}{2}(k_1+k_2)$ is the *mean curvature* of the surface at the point.

Many further interesting properties of the curvature of a surface can readily be deduced from the analytical method here developed. However, it is not necessary for our purpose to delve into these. It is well to point out before leaving the matter, that, though, initially, we chose the x and y axes as any two mutually perpendicular lines in the tangent plane to S at P, we may now, in the light of our results, choose them in the two principal directions. Analytically, this means that the solutions of (16) are $\lambda = 0°$ and $\lambda = 90°$; that is, $s = 0$ in the series expansion (3).

The equation (16) leads to the results:

$$(17) \qquad \sin 2\lambda = \frac{\pm 2s}{w}, \quad \cos 2\lambda = \frac{\pm (r-t)}{w},$$

where

$$(18) \qquad w = \sqrt{(r-t)^2 + 4s^2}.$$

The positive signs in (17) yield the value of λ for which the curvature attains one extreme value, and the negative signs yield the other value of λ, differing from the first by 90°, for which the curvature attains the other extreme value. If we set either one of these values of λ for λ_1 in (12) and (13), the k_1 and k_2 so obtained will be the values of these extreme curvatures.

Let us note that the difference of equations (12) and (13) may be written

(19) $$k_1 - k_2 = (r - t)\cos 2\lambda + 2s\sin 2\lambda,$$

and, using the results of (17), we find that the extreme curvatures satisfy the relation

(20) $$k_1 - k_2 = \pm\left[\frac{(r-t)^2 + 4s^2}{w}\right] = \pm\, w.^*$$

APPLICATION TO EQUIPOTENTIAL SURFACES

The torsion balance deals with the equipotential surfaces, which are the surfaces perpendicular to the lines of force of the earth's gravitational field. Let the equation of the equipotential surface through the center of gravity of the suspended system of a torsion balance be

(21) $$U(x, y, z) = \text{constant},$$

with respect to the usual coördinate system used; namely, the z axis is the direction of the force of gravity at the center of gravity of the suspended system (taken as the origin) directed positively downward, the x and y axes are the lines through the origin, tangent to the equipotential surface, bearing north and east respectively. The value of gravity at the point is, then,

(22) $$g = \frac{\partial U}{\partial z}\,\cdot$$

The torsion balance observes the values of

(23) $$\frac{\partial^2 U}{\partial x \partial z} = \frac{\partial g}{\partial x}, \quad \frac{\partial^2 U}{\partial y \partial z} = \frac{\partial g}{\partial y},$$

and

(24) $$U_\Delta = \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}, \quad U_{xy} = \frac{\partial^2 U}{\partial x \partial y},$$

* It will be noticed that the mathematical analysis does not differentiate between k_1 and k_2, as to which is the maximum and which the minimum, and cannot do so. This becomes obvious when we consider that if, initially, the z axis had been oriented in the opposite direction, as it well might have, the maximum curvature would become the minimum and vice versa. This corresponds with a reflection of the curve of Figure 3 in the λ-axis; for what was originally z is now $-z$ and the equation (10) changes sign. In short, equation (20) states that the numerical value of the difference between the principal curvature is w. In the numerical work used in torsion balance calculations, however, the curvatures assume, tacitly, numerical algebraic values and the corresponding λ to each of these can be assigned. This will be indicated briefly in the last paragraph of the paper.

at the point in question. The first pair of equations deals with the gradient of gravity and those of the latter pair are referred to as the curvature quantities.

Our problem is to find the values of p, q, r, s, and t for our surface defined in the form (21). This equation defines z implicitly as a function of the two independent variables x and y. Recalling that the center of gravity of the suspended system bears the same relation to the equipotential system through it as does the point P to the surface S previously described, we conclude that

$$\frac{\partial z}{\partial x} = p = 0, \text{ and } \frac{\partial z}{\partial y} = q = 0.$$

Since, however,

$$\frac{\partial U}{\partial x} + \frac{\partial U}{\partial z}\frac{\partial z}{\partial x} = 0, \quad \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}\frac{\partial z}{\partial y} = 0,^* \tag{25}$$

it follows that at the origin, $\dfrac{\partial U}{\partial x} = \dfrac{\partial U}{\partial y} = 0$.

Similarly, since at the origin, $p = q = 0$, we find that

(26)

a. $$\frac{\partial^2 U}{\partial x^2} + \frac{\partial U}{\partial z}\frac{\partial^2 z}{\partial x^2} = 0,$$

b. $$\frac{\partial^2 U}{\partial y^2} + \frac{\partial U}{\partial z}\frac{\partial^2 z}{\partial y^2} = 0,$$

c. $$\frac{\partial^2 U}{\partial x \partial y} + \frac{\partial U}{\partial z}\frac{\partial^2 z}{\partial x \partial y} = 0.$$

By definition

$$r = \frac{\partial^2 z}{\partial x^2}, \quad s = \frac{\partial^2 z}{\partial x \partial y}, \quad t = \frac{\partial^2 z}{\partial y^2}; \text{ and } \frac{\partial U}{\partial z} = g.$$

Hence

$$r - t = \frac{U_\Delta}{g} \tag{27}$$

and

* The method of obtaining these derivatives of z with respect to x and y when z is defined implicitly, as in (21), is fully discussed in Goursat-Hedrick, ***Mathematical Analysis***, Vol. 1 (Ginn & Co., 1904), p. 42.

(28) $$s = \frac{-U_{xy}}{g},$$

where we have used (22), (23), and (24). The first of these is the difference of (26a) and (26b) and the second is equivalent to (26c).

We must express all our results concerning curvature in terms of U_Δ and U_{xy}, which are the quantities observed by the torsion balance. Equation (18) becomes

(29) $$w = \frac{1}{g}\sqrt{U_\Delta^2 + 4U_{xy}^2}$$

and equations (17) and (20);

(30) $$\sin 2\lambda = \frac{\mp 2U_{xy}}{\sqrt{U_\Delta^2 + 4U_{xy}^2}}, \quad \cos 2\lambda = \frac{\pm U_\Delta}{\sqrt{U_\Delta^2 + 4U_{xy}^2}},$$

and

(31) $$(k_1 - k_2) = \pm \frac{1}{g}\sqrt{U_\Delta^2 + 4U_{xy}^2},$$

respectively.

If, in (31), we use the positive sign, as is customary, then k_2 becomes the curvature which is algebraically less in value than k_1, and the value of λ corresponding to this least curvature is defined without ambiguity in sign:

(32) $$\sin 2\lambda = \frac{2U_{xy}}{\sqrt{U_\Delta^2 + 4U_{xy}^2}}$$

and

(33) $$\cos 2\lambda = \frac{-U_\Delta}{\sqrt{U_\Delta^2 + 4U_{xy}^2}}$$

This is the value of λ usually used in torsion balance work. Also the value of

(34) $$R = |\, g(k_1 - k_2) \,| = \sqrt{U_\Delta^2 + 4U_{xy}^2}$$

leads to

(35) $$R = \frac{-U_\Delta}{\cos 2\lambda}.$$

ADVANCES IN TECHNIQUE AND APPLICATION OF RESISTIVITY AND POTENTIAL-DROP-RATIO METHODS IN OIL PROSPECTING[1]

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ABSTRACT

The successes which have been obtained with electrical methods of prospecting in detecting metallic ore bodies have led to repeated attempts to use them also in oil prospecting, both in structural work and in locating the oil itself. These attempts have not met with as much success as the application of electrical methods to the location of ore bodies; however, the interest in the possibilities of electrical prospecting as applied in oil work has been aroused again of late, due to the perfection of the resistivity and potential-drop-ratio method in regard to the determination of the depth to geologic bodies.

The object of the writer is to give a summary of the whole field of the resistivity methods, with particular reference to the recent developments. The factors affecting the resistivity of rocks, and methods for the determination of resistivities of rocks and formations are described. Then follows a description of the various surface-potential methods. First, the resistivity methods proper are discussed, with reference to electrode arrangement, apparatus, and methods of interpretation used. Second, the potential-drop-ratio methods are treated, also with regard to the electrode arrangements, apparatus, and methods of interpretation. The opinions of various authors are discussed who have expressed their views about the possibilities of locating oil directly by electrical methods, and examples are presented of results which have been obtained thus far with resistivity methods, both in structural prospecting and in attempting to locate the oil directly.

A. Principles and History of Resistivity Prospecting

In prospecting for oil structure, the four major geophysical methods—gravitational, magnetic, seismic, and electrical—have been widely used. Both the seismic and electrical methods have a distinct advantage over the gravitational and magnetic methods: the possibility of controlling the depth of penetration. This is an important factor in the interpretation of the results, as not only the physical characteristics but also the depths of geologic formations are obtained.

Thus, the seismic and electrical methods as applied in oil prospecting have a number of features in common; on the other hand, they

[1] Read before the Association of the Oklahoma City meeting, March 25, 1932. (Series of publications No. 43, Dept. of Geophysics, Colorado School of Mines.)

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also differ in a number of respects. The seismic method is essentially a dynamic method (velocity measurements) while the electrical method is essentially a static method (determination of potentials and electromagnetic fields), disregarding for the moment that the element of time enters into the latter through frequency and phase. The seismic and electrical methods each have a dual advantage; namely, that two types of the energy may be observed; in seismic methods, the refracted and reflected energy; in electrical methods, the potential distribution and the electromagnetic field of the ground currents. As far as seismic methods are concerned, the variety of application is very nearly exhausted with the foregoing. In electrical methods, added advantage is obtained by varying the type of application of primary energy: first, in regard to frequency; second, by employing either galvanic or inductive coupling.

Notwithstanding such advantages, electrical methods, as a whole, have not found such wide application in oil work as seismic prospecting. One reason for this is their more complicated field technique when the horizontal radius of operation is increased. Another reason is the limit in depth penetration when the frequency is increased. A third reason is the following: in most geophysical methods, the sought formations must have a thickness comparable with their depth. This holds for the seismic refraction method and the electrical-potential-methods. Noted exceptions are the seismic reflection-method, in which even a comparatively thin formation may still be noticed, provided it has good reflecting characteristics, and the inductive electrical method, in which a thin layer may also be determined, provided it excels in conductivity. However, there again, the electrical methods are at a disadvantage, as the depth penetration, according to practical experience, of the inductive method is not nearly as great as that of the seismic reflection method. A fourth reason, finally, for the more limited application of electrical methods is purely geological; namely, that the factors which can modify the electrical characteristics of formations in both horizontal and vertical direction (such as pore volume and electrolytic conductivity of moisture) are more numerous than similar factors affecting their elastic characteristics. Theoretically, all these obstacles could be amply compensated if it should be possible to perfect electrical methods to such a degree as to enable the direct location of petroleum. However, the reasons that are a great handicap to such effort are again purely geological in nature; namely, the variable factors influencing the electrical characteristics of formations as already stated are those which in most cases mask the effect of the oil.

On the other hand, it must be admitted that notable advances have been made in approaching the solution of the problem, at least for favorable conditions. Furthermore, remarkable progress has been made in reducing the difficulties which have been outlined in the preceding paragraph; that is, attempts have been made to increase the depth penetration, and to perfect the technique in such a way as to obtain a more interpretable type of indication.

It is quite logical that the resistivity and surface-potential methods offered the greatest possibilities in this respect; for the first necessity in attempting an increase in depth penetration is a decrease in working frequency. For inductive methods, this would materially reduce the current strength induced in subsurface conductors; for electromagnetic methods, it would embody a complication in methods of observation (replacement of the telephone by the galvanometric-rectifier method, or, if the telephone is to be retained, the use of a frequency converter).

From what has gone before it is obvious that an increase in depth penetration should be concomitant, if possible, with a greater precision in determining effects from such depths. Therefore, efforts have been made to find such electrode arrangements which would give the most distinct indication of formation boundaries, as the original Wenner 4-terminal, and similar electrode assemblies do not always give readily interpretable curves. Probably the most successful accomplishment along this line has been the perfection of the potential-drop-ratio method which required an appreciable modification in the technique of the resistivity measurement which had been employed up to that time. The potential-drop-ratio method permits the determination of small differences in apparent resistivity with a great deal of precision, requires no connection of the ratiometer to the power electrodes, and gives well interpretable curves.

There is a great deal of literature on the resistivity method (see reference list at the end of this article), chiefly referring to the application of the method in mining and civil engineering. In the geophysical laboratory of the Colorado School of Mines, extensive studies have been made on the application of resistivity methods, not only to the problems already mentioned, but also to structural oil prospecting. The Gish-Rooney equipment was perfected, model experiments were made, a Racom outfit was acquired through the courtesy of the

Swedish American Prospecting Corporation, and a number of encouraging results were obtained on geologic structure.

The object of the writer is to give a preliminary account of some of the results obtained, and to present a review of the entire field, covering both technique and results obtained by other authors.

As our activities are not limited to research in electrical prospecting alone, the work has been carried through a comparatively great length of time, and various associates of the writer have been engaged in the work. Grateful acknowledgment is made for the contributions of data to T. A. Manhart, C. D. Keen, J. A. Malkovsky, D. H. Griswold, to the Swedish American Prospecting Corporation for part of the equipment, and to Harry Aurand and R. Clare Coffin, of the Midwest Refining Company, for permission to use some of the data and experience which this company has obtained in resistivity work.

As the name indicates, resistivity methods are electrical methods in which the variation of resistivity is measured on the surface. Numerous methods are available for measuring resistivities, either in the well known resistivity bridges for D. C. or A. C., or by the use of ohmmeters, or finally, by separate measurements of voltage drop and current. As for known or constant current, the resistivity follows immediately from the potential distribution, all methods for the determination of potential differences or potential ratios on the surface, the so-called surface-potential methods, may also be included under the heading of resistivity methods.

It is not surprising that attempts were made rather early to use the simple idea of resistivity measurements in prospecting. As early as 1900, Brown and McClatchey applied for a patent on resistivity measurement in this country, and at the same time Daft and Williams, in their English and American patents, suggested the use of potential-difference observations for resistivity work. W. Petersson (ref. list No. III_1) describes a qualitative resistivity method for the location of ore, consisting of a buzzer and telephone, the intensity of the sound received being an indication of ground conductivity. The method was used successfully in Sweden in 1906.

The early attempts at the resistivity method were, however, only of a qualitative nature, as neither potential differences for fixed electrode spacings, nor even equipotential lines were observed.

The credit for making the first systematic studies of ground resistivities goes to C. Schlumberger, who began work on the method in about 1913. The first year in which he applied the method of resistivity mapping was 1920, and numerous structural studies for commercial purposes were carried out by this company in subsequent years.

Schlumberger obtained a French patent on his resistivity method on September 15, 1925.

Considerable interest was attracted at the same time to an identical method, described by Gish and Rooney, December, 1925 (ref. list No. III_2), which was used in determining earth resistivities for the study of earth-current and related magnetic phenomena. Their method was based on Wenner's method of measuring earth resistivity, published in 1915 (ref. list No. II_1).

The Gish-Rooney method was taken up with considerable interest by a number of mining companies; primarily, the Michigan School of Mines has been largely responsible for the advances of this method in mining. The papers written by W. O. Hotchkiss, J. Fisher, and W. J. Rooney (ref. list No. III_8) in 1929, report on the results obtained in the Michigan copper and iron country.

At about the same time, the U. S. Bureau of Mines became interested in resistivity work, and coöperated with A. S. Eve and D. A. Keys in the experimentation with electrical methods. Their studies have continued up to the present time, and the results are recorded in numerous papers, written by F. W. Lee, A. S. Eve, D. A. Keys, and J. H. Swartz (ref. list Nos. II_3, I_6, $III_{3,4,9,19,29,30,33,37}$).

A number of fundamental papers also appeared on the theoretical foundation of the method, written by J. N. Hummel, W. Weaver, D. O. Ehrenburg, Lancaster-Jones, Tagg, R. J. Watson, T. Roman, L. J. Peters, J. Bardeen, S. Stefanesco, and C. and M. Schlumberger (ref. list, section II).

Meanwhile, in 1928, an important application of the resistivity method had been found, the determination of depth to bedrock in dam sites. Papers by I. B. Crosby and E. G. Leonardon (ref. list No. $III_{6,12,18,20,26,32,35}$) deal with this application.

As far as the use of resistivity methods in oil work is concerned, the Schlumberger Company had proved the practicability of the method in structural studies since 1921. Work was then begun in the Pechelbronn oil region and continued until 1926. In Roumania, commercial work was carried out, among others, for the Steaua Romana in 1923–1926. Salt domes were located in the Alsace region in 1926–1927.

Resistivity prospecting for oil structures was begun in the United

States by 1925, when the Schlumberger Company was engaged to work for the Roxana Petroleum Corporation and the Shell Company of California. The work for the latter continued to about 1929.

A number of oil companies have carried on extensive studies with their own electrical resistivity and potential equipment, such as the Sun Oil Company, the Pure Oil Company, and the Midwest Refining Company.

In addition to the Schlumberger Company, several other geophysical prospecting companies have since taken up studies, such as the McCollum Exploration Company, the Swedish-American Prospecting Corporation, the Radiore Company, the Elbof Company, the Geophysical Service Inc., and the International Geophysics, Inc. Very little has been published on the successes of structural resistivity prospecting; practically everything is by members of the Schlumberger Company (ref. list No. $III_{7,10,12,16,18,20}$).

One of the outstanding developments in resistivity-prospecting methods in late years was the perfection of the potential-drop-ratio method. This was suggested by three authors at almost the same time. Although a 3-contact ratio arm bridge was devised by A. B. Edge as early as 1925 and was used in northern Rhodesia during that year, and although an A. C. potential ratiometer was used, up to the end of 1929, by the I.G.E.S. in Australia, it was not until January, 1931, that the method was first described by Edge (ref. list No. IV_4). Already before this publication appeared, Koenigsberger (ref. list No. IV_1) had suggested a potential-ratio method in 1930. Two months after Edge's article had been published, H. Lundberg and Th. Zuschlag published their description of the Racom (ref. list No. IV_5).

The potential-drop-ratio method, on account of its sensitivity and accuracy, gives very good results when suitable instruments are used, and it is believed that it is only in its initial stage of development.

There follows now a discussion of (1) the resistivity of formations and methods for its determination; (2) a description of the technique of resistivity and potential-drop-ratio methods; and (3) a discussion of the results obtained in structural work.

B. Resistivity of Formations and Rocks

With the exception of the few rocks which contain some sort of a metallic mineralization (mostly sulphidic), and the conductivity of which is, therefore, of a metallic nature, the conductivity of all the other rocks and formations is of a purely electrolytic character. Therefore, all these rocks are highly resistant when dry; or, in other words, the conductivity of a rock is proportional to the amount of water which it contains, and to the quantities of ionized salts dissolved in the water.

I. FACTORS AFFECTING ROCK RESISTIVITY

From what has been previously stated, it is obvious that the resistivity of the formations which are of importance in electrical prospecting is affected primarily by the following factors: (1) the resistivity of the mineral constituents; (2) the resistivity of the water filling the pores; (3) the pore volume; (4) the shape of the pores; (5) the temperature; and (6) the pressure at the particular depth of the formation under investigation.

As far as the resistivity of the mineral constituents is concerned, which make up most of the rocks encountered in oil resistivity prospecting, this factor is of comparatively small influence. As has been stated before, only rocks with a metallic mineralization have a comparatively low resistivity, while the resistivity of the mineral matter in most barren sedimentary and igneous rocks is so high (of the order of 10^6–10^{14} ohms cm.$^{-3}$), that they may be considered as insulators for the purposes of geophysical prospecting; that is to say, in most sedimentary rocks, the influence of the pore volume and of the conductivity of the waters filling the pores is so much more predominant a factor in determining the conductivity of the rock, that the influence of the resistivity of the mineral matter may be disregarded in comparison.

The resistivity of the water filling the pores in such rocks as are of importance in resistivity oil prospecting is of paramount importance.

Following Sundberg (ref. list No. $I_{9,10}$), the impregnating waters may be divided into the following groups.

A. *Surface waters* (waters from the surface of the ground down to and including the ground water).

(a) Fresh water, such as rain, snow, lake, river, and ground water, and the normal moisture in the upper soil. The percentage of dissolved substances ranges from 0.01 to 0.1 at an average. These dissolved substances are ordinarily carbonates and silicates and small amounts of chlorides and nitrates of calcium, magnesium, sodium, and potassium. The resistivity of such surface waters varies at an average between 30,000 and several hundred thousand ohms cm^{-3}.

(b) Mineral waters with varying concentrations. Such waters are ordinarily locally limited, and are not commonly encountered on the surface of oil-bearing structures; they are not of as great importance for our problem as those previously mentioned, and the deep waters to be discussed now.

B. Deep waters.

(a) Connate waters.
These waters are ordinarily encountered in oil fields. As the name indicates, their origin dates back to the time when the formations were laid down in sedimentary basins. They contain chlorides of sodium, potassium, magnesium, and calcium. They differ from sea water, however, by the abundance of calcium chloride and by the absence of sulphates. They are distinguished by a very wide range of concentrations, from dilute to concentrated solutions. Consequently, their resistance also varies within wide limits, to as low as a few ohms cm^{-3}.

(b) Mine waters.
These waters are ordinarily solutions of metal sulphates, but they also contain carbonates of sodium, calcium, and magnesium. They vary widely in resistivity which may be as low as 30 ohms cm^{-3}.

The most important chemical constituent of waters of all kinds which determines their conductivity is the chlorine content. Figure 1 *b* illustrates the relation between resistivity and chlorine content for three ranges of concentration.

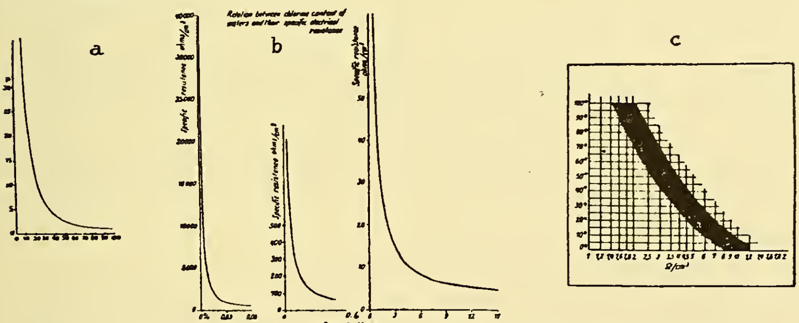

FIG. 1.—Factors controlling resistivity of rock waters (after Sundberg).
a: Relation between resistance factor $P=\rho_x/\rho$ and percentual volume v of water in rocks. (ρ_x = resistivity considering grain arrangement; ρ = resistivity not considering grain arrangement.)
b: Relation between chlorine content of waters and their specific electric resistance.
c: Relation between temperature and specific electric resistance for electrolytes.

The next outstanding factor determining the resistivity of rocks and formations is the pore volume. The pore volume depends on the size and shape of the grains, and their mutual arrangement. The porosities of such formations as are ordinarily encountered in oil prospecting cover a wide range, approximately between 10 and 45 per cent.

The pore volume, of course, is equal to the maximum amount of water which is possible in a rock. This condition is generally assumed to exist below the ground-water level, except where the water has been replaced by gas or oil. Above the ground-water level, there are considerable variations, which account for the varying values of resistivities that are often encountered near the surface in resistivity work. The moisture content of the soil is generally very low near the surface on account of evaporation, then increases rapidly with depth, then decreases and reaches a maximum in the ground-water level.[1] At the immediate surface, the moisture, and therefore the resistivity, varies considerably with the meterological factors. During rainfalls, the resistivity will usually be very great on account of the purity of waters; during droughts, highly conductive waters migrate upward, resulting in a decrease of the resistivity of the formations concerned.

Due to the fact that the conductive medium, the water, in a rock is not everywhere of the same section, but varies from a very thin section where the grains are touching each other to a greater section where grains are not in contact, the relation between pore volume and rock resistivity is not altogether simple; it may, however, be worked out for certain simplified assumptions; for instance, for the case that the grains are spheres of equal size or for the case that the pores have the shape of three mutually perpendicular and parallel systems of tubes. Figure 1 *a* shows the relation between a resistance factor *P*, which is the ratio of the actual water resistivity depending on porosity to the water resistivity, without regard to porosity (Chapter B, II, d), and the water volume of a rock in percentage.

Finally, the temperature and the pressure influence the electrolytic resistivity of rocks. Figure 1 *c* shows the influence of temperature; an increase in temperature decreases the resistivity considerably; the relation shown in the figure holds for any kind of an electrolyte, and indicates that a rise in temperature by only 25° C. from zero is sufficient to increase the conductivity of a rock approximately 100 per cent.

The effect of pressure is much smaller and is illustrated in Figure 2. The effect is different on concentrated solutions from what it is on solutions of smaller concentrations. At any rate, with the maximum

[1] For details, see ref. list III, 38.

effect on a concentrated solution, a pressure corresponding with a depth of burial of nearly 10 kilometers would bring about a change in resistivity of only 10 per cent.

More details on the effect of water on rock resistivities may be obtained from K. Sundberg's papers (ref. list $I_{9,10}$).

II. DETERMINATION OF RESISTIVITIES

The experimental determination of the resistivity of rocks or formations is by no means a very easy matter. This is chiefly due to the

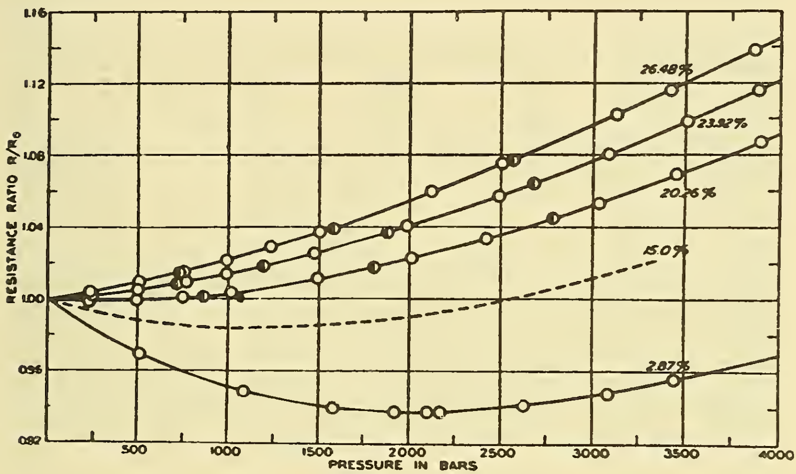

FIG. 2.—Relation between pressure and resistivity of electrolytes (after L. H. Adams and R. E. Hall).

fact that a sample can not well be kept in the same condition after being removed from its location as it has been in before it was disturbed, chiefly in reference to its mechanical constitution and its moisture content. Therefore, experiments to test the resistivity of rocks are usually made, whenever possible, on outcrops, or by means of special equipment, in wells (Schlumberger method). If there is no possibility of avoiding the testing of samples, the tests should be made in a portable apparatus immediately after the sample has been taken, or else, the resistivity of a formation may finally be computed from the pore volume of the constituent rock and the conductivity of the impregnating water (Sundberg method).

a. DETERMINATION OF RESISTIVITIES ON SAMPLES

A simple arrangement for the determination of resistivities of solid rock specimens is shown on the right side of the panel illustrated in Figure 3.

The arrangement consists of a frame supported on springs; one round electrode is connected to the bottom of this frame, the other is attached to the bottom of a spindle. The sample to be tested is placed between the electrode on the spindle and that on the frame,

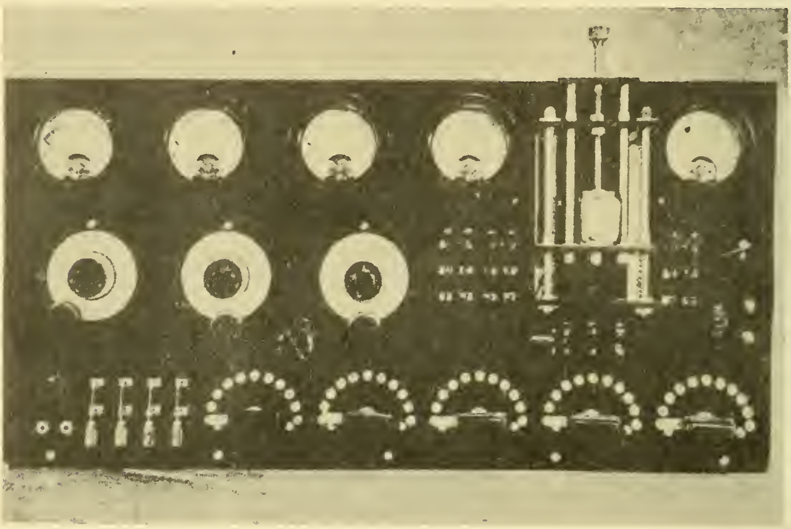

FIG. 3.—A.C. and D.C. resistivity bridge (C S M).

with some tin foil between sample and electrodes. Mercury electrodes may be used on drill core samples instead of the solid metal plates. This arrangement may be placed in a Wheatstone bridge.

The samples should, preferably, have a simple geometric shape. If l is the length, s the section of the sample, R the observed resistance and ρ the resistivity to be determined, $\rho = \frac{R \cdot S}{l}$.

The resistivity is usually expressed in ohm·cm (see ref. list I, 12). In some publications ohm·meters are used; resistivities expressed in the latter units are one hundredth as large as resistivities expressed in the former units.

The bridge shown in Figure 3 has on the left side the controls for high frequency tests, and on the right are the meters and switches for D.C. testing. The bridge is A.C. operated, and consists of an oscillator, the frequency of which may be varied within wide limits in the high frequency range. A pickup coil with variable coupling transfers the energy of the oscillator to the testing circuit, which is essentially an A.C. Wheatstone bridge. Non-inductive resistors are used; the switches for the 10, 100, 1,000, 10,000 and 100,000 ohm ranges are seen on the panel. The same resistors are used when the bridge is operated on D.C. The binding posts on the extreme right edge of the panel are for making connections to the testing battery or a low frequency oscillator or commutator.[1]

The bridge is only for small samples of regular shape, such as diamond drill cores and the like. For larger specimens of consolidated rocks, we use a different bridge which consists of a frame with two spindles to hold the specimen, and four arms spaced at equal intervals which carry contact points at their end. The Gish-Rooney (or Wenner) method is applied in testing the specimen, and the Gish-Rooney equipment as used in the field can be connected directly to this bridge.

M. W. Pullen (ref. list No. I_6) uses also direct and alternating current in his resistivity tester. The resistivities are measured by comparing the resistance of the specimen with a known resistance, that is, by switching the current from one circuit containing this specimen over to the other circuit containing the known resistance. The electrodes used were chiefly mercury pools, held in place upon the specimen by wax dams if the specimen had an irregular shape, or by a bakelite cup and a paper collar for drill cores. To eliminate surface leakage of current across the specimen, guard rings were applied. In using continuous current, the observed resistivity values were found to change with time on account of the variable polarization. Therefore, discontinuous current was found to be more satisfactory, which was obtained by shortening the regular 60-cycle A.C. from a light circuit through a potentiometer (for obtaining different test voltages) and using a copper oxide rectifier to make the use of a direct current galvanometer possible (Fig. 4 *a*).

For large specimens of irregular shape, not only the Gish-Rooney four-electrode method may be used, but also two like electrodes of regular geometric shape may be employed, provided their distance is small as compared with the dimensions of the sample. The theory

[1] This resistivity bridge was developed largely after the bridge used by the Radiore Company.

of such an arrangement (Sundberg, ref. list No. I_9) shows that for square electrodes with sides of 0.37 cm. the measured resistance is equal to the specific resistance of the specimen.

The bridges described before may only be used for solid specimens. They are not applicable for soils. For testing of soil samples in

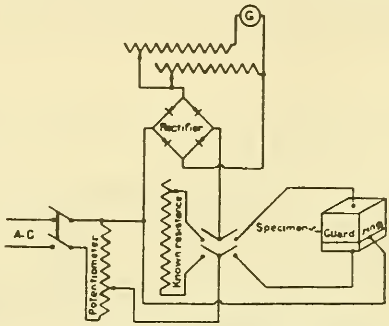

FIG. 4*a*.—Circuit of A.C. rock-resistivity tester, operated from light socket (after M. W. Pullen).

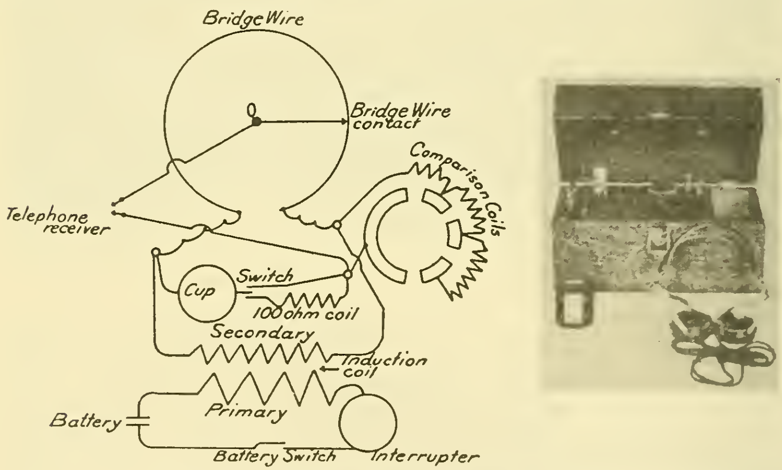

FIG. 4*b*.—Leeds and Northrup soil-resistivity tester (after R. O. E. Davis).

the field, the Leeds and Northrup soil bridge may be applied. The instrument (known as the Wheatstone-Kirchhoff bridge) as shown in Figure 4 *b*, is a modified form of a Wheatstone bridge. As a source of energy, a buzzer operated by a dry cell is used; consequently, a telephone may be employed to indicate the balance. The soil to be tested is placed in a cup; this represents the variable resistance in one arm of the bridge; the fixed comparison coils are in the other arm of the bridge; both connect to the variable slide wire and the secondary of the buzzer coil. The comparison coils are made up of three fixed resistances of 10, 100, and 1,000 ohms. An additional resistance of 100 ohms may be used in series with the cup when its resistance is low. The balance is adjusted on the slide wire, with proper setting of the fixed resistances, and the reading on the slide wire is multiplied by the resistance of the comparison coil used.

The use of the cup is very convenient as not only soils, but also electrolytic solutions may be tested in it. It may be standardized by using solutions of known concentrations. The results must be re-

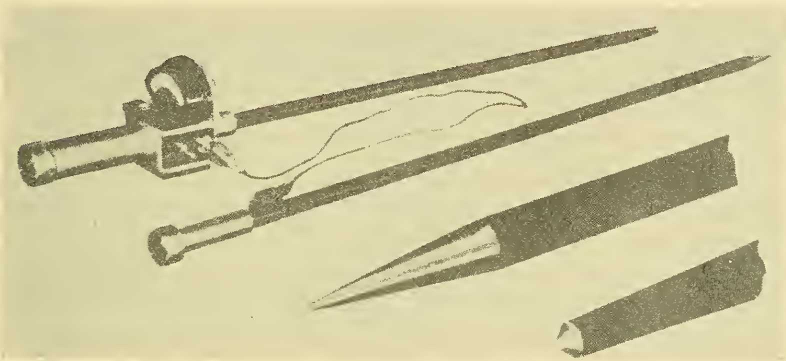

FIG. 5.—Shepard earth-resistivity meter.

ferred to a normal temperature. The publication by R. O. E. Davis (ref. list No. I_1) gives a number of tables on resistivities of soils and solutions which will be found valuable when using this bridge.

b. DETERMINATION OF RESISTIVITIES ON OUTCROPS

On account of the difficulties which are inherent in the accurate determination of the resistivity of samples and which have been discussed before, determinations of the resistivities on outcrops, or, generally speaking, on the ground surface are ordinarily much more satisfactory.

Quite a few methods may be used for this purpose. For approximate tests, the Shepard earth-resistivity meter may be employed (Fig. 5). It is an instrument working on D.C.; the ohmmeter and the battery are mounted on one probe; the probes are made of iron, and the cathode is larger than the anode, reducing polarization to a great extent. For this purpose, the electrodes have a large ratio of areas, as seen in the figure.[1]

For more accurate work on the surface, the Megger and McCollum's earth-resistivity testers, or the 4-terminal Gish-Rooney equipment is used, which will be described later.

J. Koenigsberger (ref. list No. I_5) uses two circular iron electrodes about 25 cm. in diameter, and about 5 mm. in thickness, provided with a contact substance which is either hematite powder or a clayey paste with an aqueous solution of $FeCl_3$, or $FeSO_4$, or of 10 per cent $NaCl$. Where the necessity arises of using larger electrodes, iron screens may be employed. As a source of energy, a buzzer is used at a frequency from 100–400 cycles; the electrodes are placed in a Wheatstone bridge with a telephone as indicator. Koenigsberger gives the formulae to be used with this arrangement of the electrodes.

c. DETERMINATION OF RESISTIVITY IN WELLS

This is the only method for obtaining accurate resistivity values for any formation below the surface. The equipment and technique has been perfected by the Schlumberger Company (ref. list No. $I_{8,11}$), and excellent results have been obtained with it. The arrangement is shown in Figure 6. It consists of four electrodes, two of which are supplied with current, while the potential difference between the two intermediate electrodes is observed; one current electrode is grounded at the surface. The arrangement of the three other electrodes always remains fixed at any depth. With the symbols used in the figure, the resistivity at any depth is given by the relation

$$\rho = \frac{4\pi\Delta V}{i} \cdot \frac{rr'}{r' - r}$$

if i is the current read on a milliammeter in the current circuit and ΔV is the potential difference read on a potentiometer between the electrodes 2 and 3. Excellent results have been obtained by this method, and have occasionally resulted in the detection of oil and coal horizons which had been overlooked in drilling. The results, together with some outstanding examples, will be discussed later.

[1] *See:* S. Ewing: "Soil Survey Methods," *Oil and Gas Journal*, April 21, 1932, p. 42. C. R. Weidner and L. E. Davis: "Relation of Pipe Line Currents and Soil Resistivity to Corrosion," *The Oil Weekly*, December 4, 1931, p. 26.

d. DETERMINATION OF RESISTIVITIES FROM WATER ANALYSES AND POROSITY DETERMINATIONS

If it is not possible to determine underground resistivities by the Schlumberger method, that is, if there is casing in the well at the depth to be investigated, or if the wells are dry, sufficiently accurate estimates of resistivities of formations may often be obtained by a method suggested by Lundberg (reference list No. $I_{9,10}$), provided water analyses are available for the formations under investigation, which is often the case. If the water analyses can, in addition, be supplemented by a determination of the pore volume of the forma-

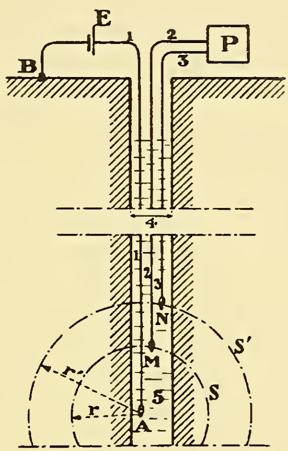

Fig. 6.—Schlumberger's method of measuring resistivities in wells (after C. and M. Schlumberger).

tion from an investigation of drill cores, for instance, a fairly accurate determination of the resistivity of the formation can be made.

If the porosity of a sample is known, the resistance factor *P* may be found from the diagram in Figure 1 *a*, if it is assumed that all the pores are filled with water; this is ordinarily true, as already stated, below the ground-water level. If the chemical composition of the waters at a certain depth is known, and if it is assumed, with sufficient accuracy, that only sodium chloride is in solution, the diagrams in Figure 1 *b* may be used to determine the resistivity of the water; to convert the results into actual resistivities at a certain depth, the temperature correction illustrated in Figure 1 *c* may be applied.

The resistivity factor *P* is of the order of 50 to 100 for dense limestones and sandstone, 20 to 40 for clays and sands, and 2 to 20 for porous sands, clays, and soils.

C. Resistivity Methods

In the two sections that follow, the principles, apparatus, and electrode arrangements used in the resistivity and potential-drop-ratio measurements are discussed first. After that, some outstanding results are described which have been obtained with both methods on geologic structure of various types, as well as in direct prospecting for oil.

Though the purposes for which the resistivity and potential-drop-ratio methods are used are usually identical, these two methods differ essentially in electrode arrangement and technique of measurement.

In the resistivity methods proper, current is usually supplied to two points, and the potential distribution between these two points is investigated by either one or two electrodes. The current is determined which flows between the two outside current terminals, and the potential difference is observed which exists either between the two potential electrodes, or between the one potential electrode and one neighboring current electrode. The ratio of voltage and current, multiplied by a constant depending on the mutual arrangement of the electrodes, gives the resistivity of the surface formation, if only one formation exists within the reach of the whole contacting arrangement. If more formations are present the "apparent" resistivity is measured as a function of the electrode spacing, and thus, as a function of the depth to the interfaces of the formations involved.

In the potential-drop-ratio methods, current is again supplied to two points in the area under investigation; but this time, the investigation of the potential distribution takes place outside of the pole doublet and is usually made in the vicinity of one electrode. The drop of the potential in the vicinity of this one electrode is intimately related to the existence of resistivity changes with depth. Experience has shown that the most interpretable results are obtained by not investigating the drop of the potential itself, but by determining the ratio of potential drops in two adjacent intervals. That is to say, the testing device in this method requires only 3 electrodes, and is independent of any connection to the current circuit or to the source of energy, which gives these methods a distinct advantage over the resistivity methods proper.

We shall first discuss briefly the electrode arrangements and apparatus used in the resistivity methods proper.

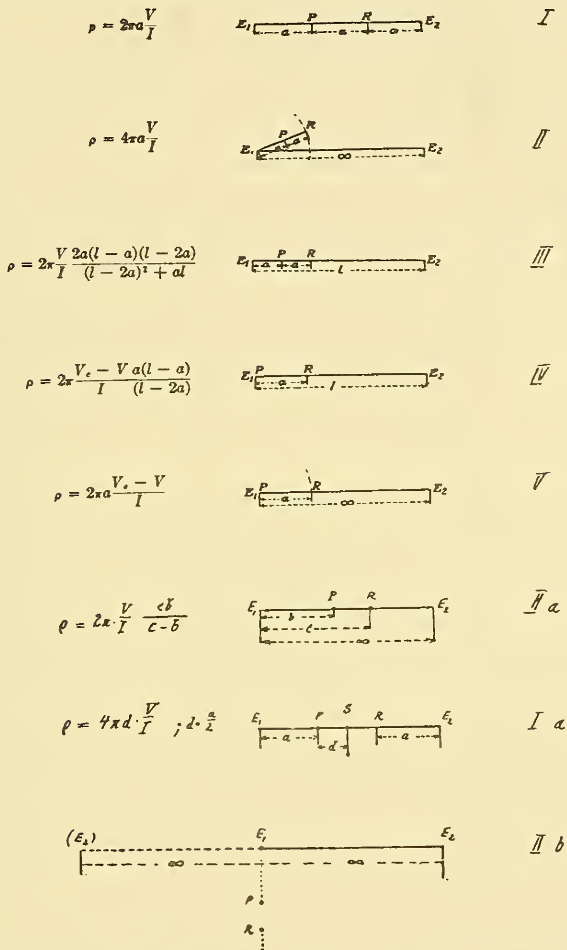

FIG. 7.—Electrode arrangements in resistivity prospecting.

I. ELECTRODE ARRANGEMENTS

a. POSSIBLE ARRANGEMENTS

Five possible electrode arrangements which can be used in resistivity work are described in detail, and formulae are given for each arrangement, in J. N. Hummel's publication (reference list No. II_{12}) and Figure 7. They are summarized briefly as follows.

(1) Wenner's arrangement, also called simply the 4-terminal method. (This is the electrode arrangement most widely used.) The potential electrodes are arranged in line with the current electrodes, and the spacing of all 4 electrodes is equal. That is, the distance between the outside terminals is $3a$ if a is the electrode interval.

(2) Single-current-probe method: One current electrode is far apart from the other, and the two potential electrodes are used in the vicinity of one electrode. If their interval is a, they are so used in the vicinity of the current electrode that the distance of the current electrode from the potential electrode next to it is also always kept equal to a. The potential electrodes need not be in a straight line with the current electrodes.

(3) This third possible electrode arrangement is practically identical with that described in No. 2, except that the second current electrode is no longer in infinity, and that the potential electrodes are now in a straight line with the current electrodes.

(4) The fourth method may be called the single-potential-probe method. Again two current electrodes are used, but only one potential electrode, the position of which is varied between the two current electrodes.

(5) The fifth method is identical with the fourth, but this time one current electrode is at infinity. This method may thus be called the single-current and the potential-electrode method.

Two more methods are possible and have been used, both being modifications of the second and the first method already described.

The first of these is the single-probe method by Eve and Keys and is similar to the second method (Fig. 7, *II a*), with the exception that the interval between the two potential and one current electrode is no longer equal; only the spacing between the potential electrodes remains constant.

The second of these additional methods is a modification of Wenner's and Gish-Rooney's 4-terminal method. An additional electrode is provided between the potential electrodes; measurements take place as before, that is on four electrodes. Two sets of results, however, are now obtained: one (at the right), using the electrodes *R* and *S;* the second (at the left), using the electrodes *P* and *S*. This is called Lee's method (see ref. list No. III, 37) of partitioning.[1]

[1] A third additional method may be mentioned here, which is the "Potential center displacement" method of S. H. Williston and C. R. Nichols, described by F. H. Lahee in the third (1931) edition of his *Field Geology* (p. 694). The electrode arrangement is that shown in IV in Figure 7. The object of the method is the measurement of the displacement of the potential center from its geographic center between the two primary electrodes which it would occupy in homogeneous ground. In surveying an area, the whole contacting arrangement is stepped forward as in resistivity mapping, the electrode base remaining constant. The depth of penetration is roughly equal to half the length of this base; an area may be covered with different exploration depths as in resistivity mapping. The potential center displacements are at a maximum over the lateral edges of resistant bodies; zero displacements between two opposite maxima would thus indicate the crest of an anticline, etc.

In Figure 7, these electrode arrangements are shown. Also the formulae are given for the computation of the resistivities of the ground if no unhomogeneity is within the reach of the measuring arrangement. Otherwise, the resistivity values obtained by the application of the formulae represent apparent resistivities only, from which the true resistivities, as well as the depth to the formation boundaries, may be obtained by applying methods to be described later.

b. CUSTOMARY ELECTRODE ARRANGEMENTS

Of the many electrode arrangements previously described, only a few are customarily applied and it all depends on the purpose for which a resistivity investigation is made, which method is used.

There are two distinct applications of the resistivity method. First is the method of resistivity mapping. The object of this method is to obtain a contour map of the area, showing lines of equal resistivity. These lines represent the ground resistivity only to a definite depth, and, to facilitate the interpretation of the results, two or more contour maps may be made for the same area, representing the resistivity down to two depths of investigation. This can be accomplished simply by working with two fixed electrode separations, as described hereafter. The second method is the so-called method of electrical drilling. The results are to give the vertical variation of the resistivity at one point only and the depth at which any changes occur. The only way to accomplish this is, of course, to survey along a line beginning at the point where the vertical resistivity variation is to be tested; and this line is the longer, the greater the desired depth penetration. It is thus seen that accurate results in this method depend altogether on the question whether or not the vertical differentiation of the resistivity carries unchanged in a horizontal direction below the last point of measurement.

b 1. RESISTIVITY MAPPING

In this method, usually the 4-terminal electrode arrangement is employed and the whole contacting outfit is carried over the area to be surveyed with a constant electrode separation.[1] If it is desired to obtain two equi-resistivity maps covering two depths of penetration, two electrode separations are used at each point of observation. The results of some equi-resistivity surveys are shown in Figures 26, 28, 29, 30.

[1] As a matter of interest it may be mentioned that the equiresistivity method has recently been used for oil prospecting in shallow water (ref. list No. III, 41).

b 2. VERTICAL ELECTRICAL DRILLING

For this purpose, several methods of those already enumerated may be used.

(1) The method that has been most commonly used is probably the 4-terminal Gish-Rooney-Wenner method (Fig. 7 *I*). The electrodes are arranged always symmetrically around the point to be drilled electrically; that is to say, this point is located where the electrode S in the Lee partitioning method would be placed. By applying this modification of the Gish-Rooney method, one readily obtains the distribution of resistivity at the north and at the south (or any other two opposite directions) from the midpoint; that is to say, it is possible to determine by this method if there is a dip of the formations, or if formations come in north of this point that may be absent in the south.

(2) The method that has been used perhaps as frequently as the 4-terminal method for electrical vertical drilling is the single-current-probe method, that is, the investigation of the potential distribution in the vicinity of one current electrode only. For this purpose, either the electrode arrangement shown in *II* in Figure 7 may be used (ER equal to $2a$) or the modification shown in *IIa*. As indicated in Figure 7 *II*, it is not necessary that the two potential electrodes be in line with the current electrodes; in fact, to eliminate completely the influence of the second current electrode, which is assumed to be at infinity, the line of measuring the potential should be located on a circle through E_1 with the distance E_1E_2 as radius around E_2; or, if that is not feasible, a close enough approximation may be obtained by running the potential measurements on a line at right angles to E_1E_2. This is the method employed by Ehrenburg and Watson (with $EP = a$, $ER = 2a$) (reference list No. II_{11}). Gilchrist has applied (reference list No. III_{34}) a modification of this last method by using two electrodes E_2 at either side of E_1, and passing currents of equal strength through both circuits (Fig. 7 *II b*). Instead of using only one current base E_2E_2 at right angles to the potential line, any number of current bases, arranged in the form of a symmetrical star, may be employed.

II. APPARATUS FOR RESISTIVITY WORK

We now come to a description of the equipment used in resistivity work. In order not to make this paper unduly long, rather extensive use of figures will be made instead of giving lengthy descriptions of the apparatus. Further details may be obtained from the literature given in the reference list.

a. GENERAL: POWER SOURCES

As a source of power, one Radio B battery, or two, may be sufficient, depending on the length of the current basis, of course. If very great depth penetration is required, a D.C. generator is used, driven by a gasoline engine. When using a Megger, a hand-driven D.C. generator is used, which is incorporated in the meter box. Due to the fact that polarization must be eliminated, if non-polarized electrodes are not used, a commutator is provided in the measuring circuit, which brings it about that in reality the current used is not strictly

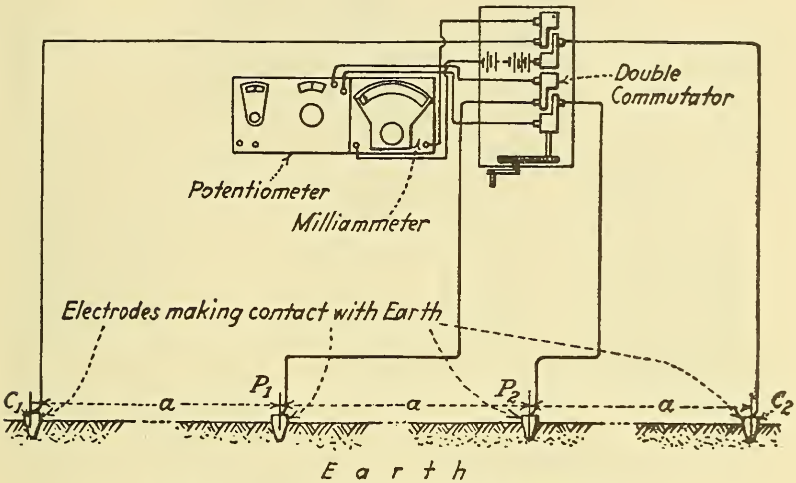

FIG. 8.—Schematic diagram of Gish-Rooney circuit.

D.C., but an alternating current of low frequency. Induction effects are, therefore, sometimes observed between the current and the potential leads; besides, not the ohmic resistance of the ground is obtained, but the impedance for the particular frequency employed, which for the commutators in Gish-Rooney outfits is approximately 16, and nearly 50 in Meggers.

The only way to avoid polarization and induction effects is by using non-polarized electrodes, which consist of cups filled with copper sulphate and provided with a permeable bottom, made either of porous clay or of wood.

The induction between leads is minimized when using the potential measuring line at right angles to the current basis, as already described.

There is no reason why alternating current could not be used in resistivity work, the same as it is done in the potential-drop-ratio method. For instance, the current base could be supplied with A.C. of moderate frequency by a buzzer, or vacuum-tube oscillator, or by

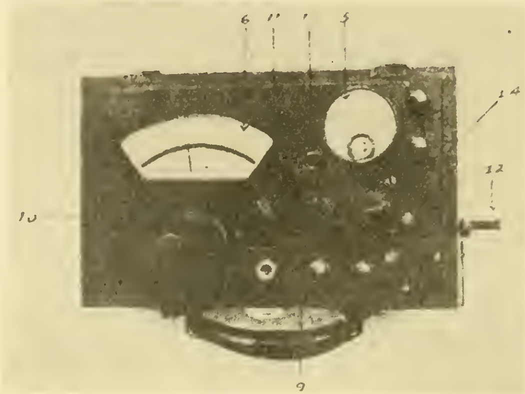

FIG. 9.—McCollum earth-current meter (after McCollum).

EARTH-CURRENT METER

5, milliammeter; 6, voltmeter; 9, commutator switch; 10, short-circuiting switch; 11, voltmeter switch; 12, commutator crank; 13, ammeter switch; 14, resistance switch.

EARTH-CURRENT METER CONTACTORS

1, trench contactor; 2, cantilever contactor; 3, single terminal electrode with extension rod; 4, single terminal electrode (disassembled).

taking, if available, current from the light circuit, and feeding it to the ground through a transformer. The only difficulty would be to obtain an accurate reading of the current; however, the absolute value of the current is not of great importance when using the methods *II*, *II a*, or *II b*. To measure the potential difference between the potential electrodes, it would be feasible to have a compensator, and a lead to a pickup coil coupled to the generator or the power leads; the object of this lead being to carry the reference e.m.f. and the generator phase to the point of observation.

b. ORIGINAL GISH-ROONEY APPARATUS

This apparatus has been described so often in the literature that it is hardly necessary to go into more details. It contains a potentiometer for measuring the potential difference between the two potential electrodes, a milliammeter for measuring the current, and a double

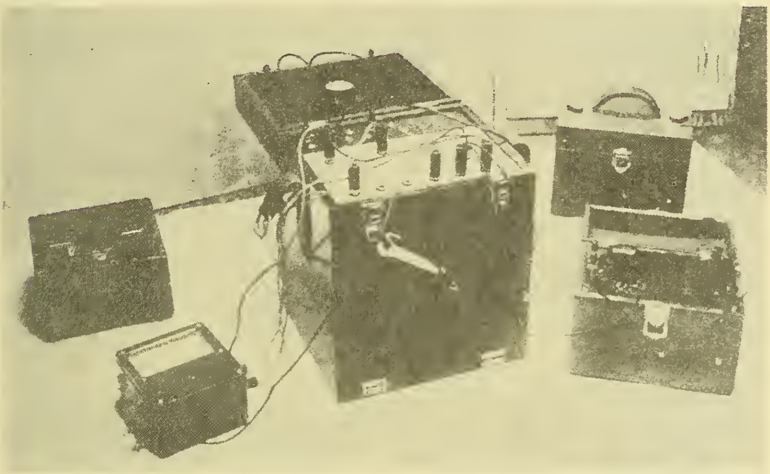

FIG. 10*a*.—First model of CSM Gish-Rooney outfit, built after MSM model. Hand-cranked commutator box which contains also batteries, shown in center; milliammeter at left and potentiometer at right.

commutator, the purpose of which is to pass current to the ground in alternating direction, yet pass the current and the potential to be measured always in the same direction through the meters. Figure 8 shows a sectional view of the apparatus.

c. MC COLLUM EARTH-CURRENT METER

Although this instrument was developed altogether independently from the Gish-Rooney apparatus and for a different purpose, it is, nevertheless, of practically the same construction as the Gish-Rooney apparatus. It is shown in Figure 9. It has a voltmeter and milliammeter like the Gish-Rooney apparatus, and also a commutator. It differs somewhat from the Gish-Rooney as a number of ranges are provided for both the milliammeter and the voltmeter, and as non-polarized electrodes are provided (reference list No. I_2).

d. MODIFIED GISH-ROONEY APPARATUS

A number of oil and mining companies have designed their own Gish-Rooney equipment, which differs slightly from the original in one way or another. The Michigan School of Mines has obtained much experience in the design of this equipment, and their apparatus

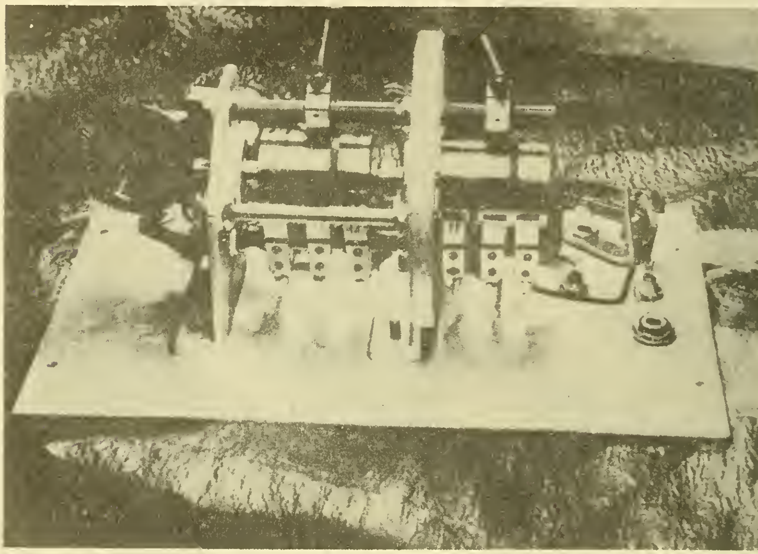

FIG. 10*b*.—Construction of commutator in instrument shown in Figure 10*a*.

was used as model for one of the instruments built in the instrument shop of the Colorado School of Mines (Fig. 10 *a* and Fig. 10 *b*).

Figure 11 shows the latest form of a Gish-Rooney equipment built at the Colorado School of Mines. On the left side of the figure, a box is shown containing the potentiometer, milliammeter, switches and jacks, and below the panel, the commutator, which is driven by a small D.C. motor, operated from a storage battery. The battery box is seen between the reels and the meter box.

e. MEGOHMERS

These instruments have been used for years for the testing of grounds and insulation in electrical power-plant-, railway-, and radio-engineering. They differ from the Gish-Rooney equipment chiefly in that the potentiometer and the milliammeter are combined in one

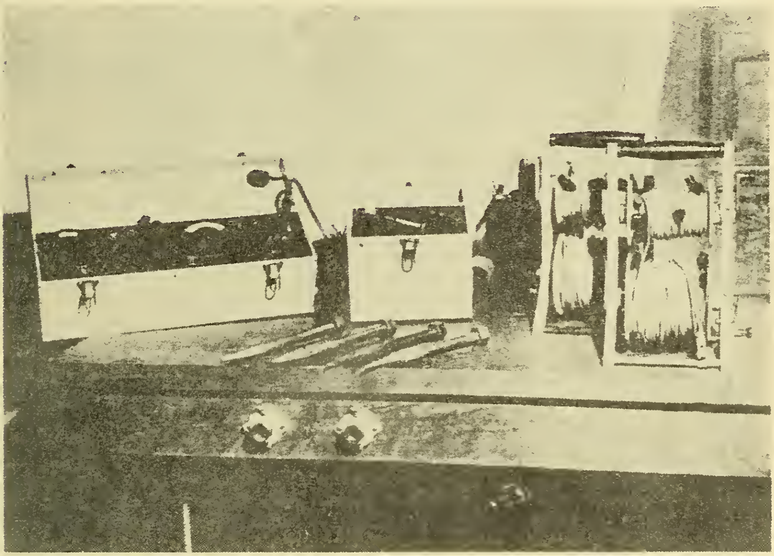

FIG. 11.—Latest CSM model of Gish-Rooney equipment with motor-driven commutator.

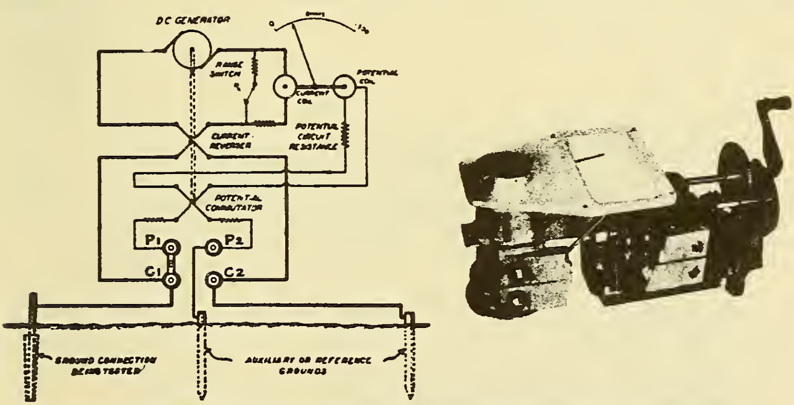

FIG. 12.—View and circuit diagram of Megger manufactured by Biddle.

instrument, the ohmmeter, the moving coil of which has two windings, the voltage and the current winding. They also differ from the Gish-Rooney equipment as the power is not taken from batteries, but from a hand-driven D.C. generator, on the shaft of which a double commutator, similar to the one used in the Gish-Rooney apparatus, is mounted.

The most important manufacturers of Megohmers or Meggers are: Evershed and Vignoles, Acton, England; James G. Biddle, Philadelphia; and Herman H. Sticht and Company, New York.

The accompanying figures show a number of Meggers, which are all very similar in principle. Details on the construction of these instruments may be obtained by writing to the manufacturers and asking for their catalogs. Figure 12 shows wiring diagram and sectional view of a Megger manufactured by Biddle, and Figures 13 *a* and 13 *b* show Meggers manufactured by the Sticht Company.

III. INTERPRETATION OF RESULTS

The fundamental principle underlying the interpretation of the resistivity results is comparatively simple.

The depth reached depends on the spacing of the potential electrodes (or their distance from a current electrode in methods where stationary current electrodes are used).

In other words, if the observed resistivity values are plotted as a function of the electrode separation (in the 4-terminal method), or as a function of the distance of the potential electrodes from the near current electrode in single-probe methods, then the *electrode separation* or electrode distance *at which* marked *changes in apparent resistivities occur* is *approximately equal to the depth* to the resistivity discontinuities underground.

This statement is made merely to illustrate the nature of the fundamental principle underlying resistivity measurements. In practice, however, there are more difficulties, as the presence of more than one discontinuity of resistivity is considerably complicating.

Actual experience shows that this simple depth rule works surprisingly well in mining when insulated bodies of good conductivity are encountered. In application to stratified ground, severe complications arise, chiefly with the Gish-Rooney method, and particularly, when there is more than one layer present.

In the practice of resistivity work, therefore, we use several lines of attack in the interpretation of results. As in other types of geophysical work, both qualitative and quantitative methods may be applied. The qualitative method usually precedes the quantitative method in application, and the quantitative method is applied if the

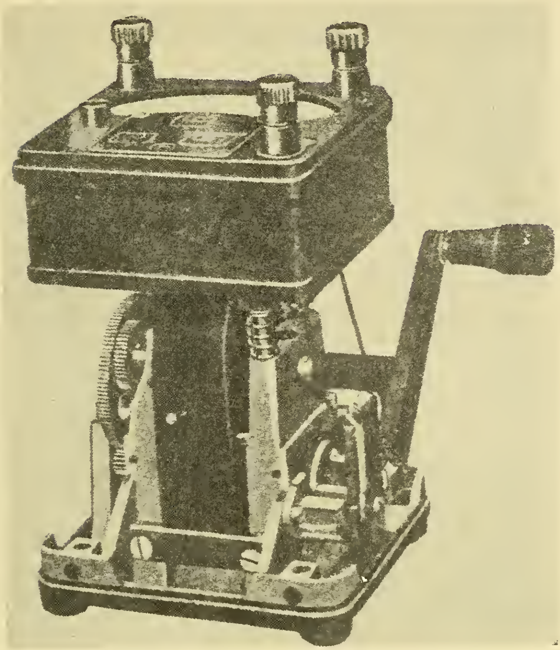

FIG. 13*a*.—Model D M Megohmer, manufactured by Sticht.

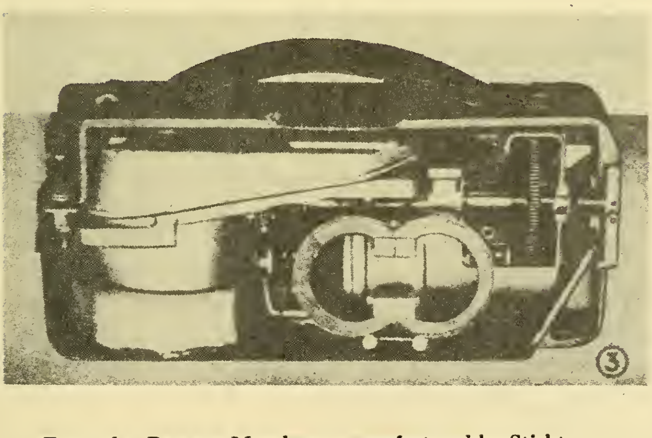

FIG. 13*b*.—Paragon Megohmer, manufactured by Sticht.

nature of the indications, derived from their qualitative analysis, warrants it.

Due to the nature of resistivity work, qualitative methods are usually applied in resistivity mapping. The area is covered with a 4-terminal contacting outfit, and resistivities are measured; the results are plotted and points of equal resistivity are connected by lines which resemble contour lines, and which are interpreted accordingly. The interpretation assumes a somewhat more quantitative nature if, at every point, instead of one electrode spacing, two electrode spacings are used, so that two contour maps may be drawn for any one area, covering two depths of penetration. Finally, considerations of a quantitative nature enter also when the contact plane of two formations of different resistivities is considered, and if one, two, or three of the contacts are in one, and the remainder in the second formation. The theoretical computations for this case have been carried out by Tagg (ref. list No. II_{10}) and Hedstrom (ref. list No. IV_7) and the results are shown in Figure 14. The curves illustrate how the apparent resistivity is influenced, for various values of resistivity ratios of the formations involved, by the distance of the center of the contacting arrangement from the boundary. The curves of Figure 14 are computed for four electrodes at right angles to the formation boundary, and show, therefore, four discontinuities. If the contacting arrangement is parallel with the fault plane, the apparent resistivity changes gradually as the boundary is approached. Tagg has also computed the apparent resistivity curves for this case (ref. list No. II_{10}).

Outside of the conditions occurring in the immediate vicinity of a formation boundary, the method of resistivity mapping involves essentially only qualitative methods of interpretation, which consist, as stated before, of an interpretation of the equi-resistivity lines (Figs. 28 and 29), or of a resistivity profile (Figs. 26 and 30). In interpreting a resistivity profile, it is important to take into consideration the influence of the electrode separation, and thus of the depth penetration, for which Figure 26 is an example. Finally, in the interpretation of results obtained by resistivity mapping, tank experiments may prove to be of great value, inasmuch as they are very easily performed. The geologic bodies are usually represented by metallic conductors, and these are arranged at various depths or with varying angles of dip and strike as the case may require. Then the 4-terminal contacting arrangement is carried across the body at the surface of the water or solution in the tank, and the apparent resistivities are measured

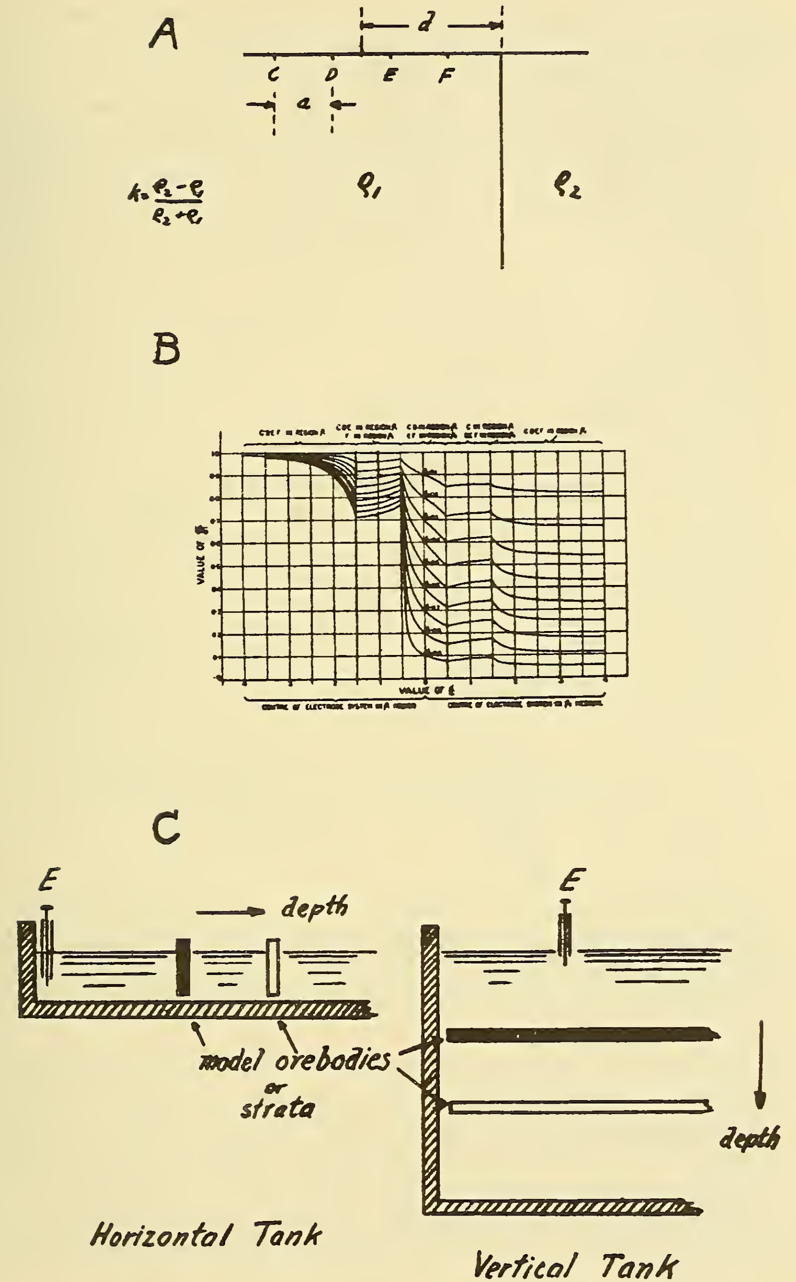

FIG. 14.—*A:* Arrangement of 4 electrodes crossing vertical boundary (after Tagg). *B:* Apparent resistivities for various electrode positions (after Tagg). *C:* Tanks used for model resistivity experiments.

and plotted. Figure 15 shows, as an example, the results of tank experiments made at the Colorado School of Mines with the objective to study the effect of dip on the equi-resistivity profiles.

The quantitative methods of interpretation are chiefly used in interpreting the results of electrical vertical drilling. The quantitative methods may be of a direct and of an indirect nature. In the direct methods, we determine the depth to formation boundaries directly

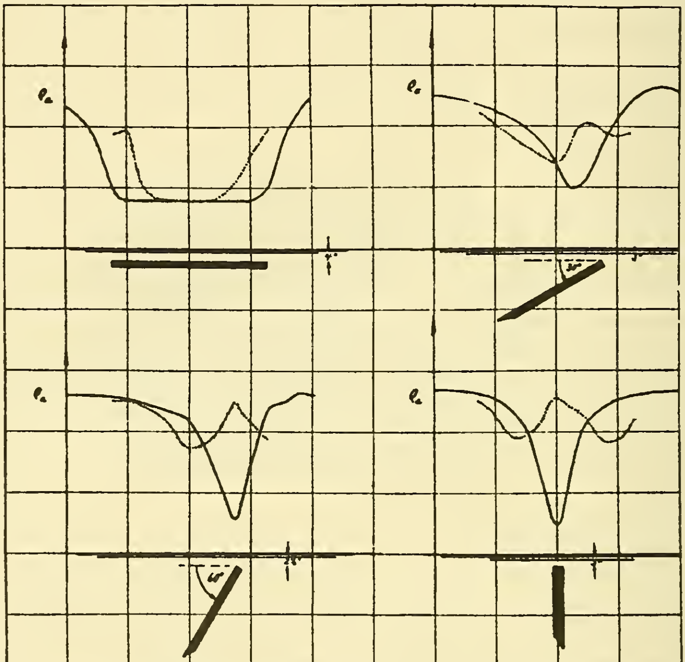

FIG. 15.—Studies of effect of dip of formation models in water tank, using 4-terminal contacting arrangement with constant electrode separation of 8 inches. *Solid lines:* Apparent resistivity when line of electrodes is parallel with strike of formation. *Dotted lines:* Apparent resistivity when electrode line is at right angles to strike.

from the curves. These direct methods are applicable in simple cases, that is, when only one formation boundary is present or possibly two. Otherwise, indirect methods are much safer; their principle is the assumption of certain formations in definite depths and a theoretical computation of the results to be expected. These results are then compared with the data obtained in the field, and the assumptions are modified until a satisfactory agreement between theoretical and field data is obtained. In doing so, all available geological data must, of course, be considered. It may then happen that the correct interpretation is the one which does not show as good agreement between the theoretical and the field data as another assumption which is less probable for geologic reasons. It should again be recalled, in this connection, that the possibility of quantitative interpretation in electrical vertical drilling depends altogether on whether the formations involved retain their resistivities in a horizontal direction. The determinations of theoretical effects, such as involved in the indirect methods of interpretation, may not only be made by means of computation, but also by means of model experiments. For this purpose tanks may be used, filled with water or weakly electrolytic solutions, and the geologic bodies may be represented by metal plates, et cetera (for ore bodies, water tables, and faults), or by sand or clay layers. When working with metal plates it is convenient to use a horizontal tank instead of a vertical tank, that is, a tank in which the model ore bodies are moved horizontally away from the electrodes to simulate changes in depth (Fig. 14 *C*). Instead of using tanks for the model experiments, pits may be dug into the ground and filled with such alternating layers of clays, sands, et cetera, as the case may require (reference list No. III_{33}).[1]

Summing up the methods of interpretation applied in resistivity work, we have:

- I. Qualitative interpretation: resistivity mapping
- II. Quantitative interpretation: electrical vertical drilling
  - a. Indirect methods
    - 1. Computations
    - 2. Model experiments
  - b. Direct methods

There exists a very extensive literature on the theory of quantitative interpretation in resistivity work. The references to this literature are given in section II of the bibliography attached to the end of this paper. It is believed to be fairly complete. A perusal of this literature will show that practically the only electrode arrangement for which a complete theoretical treatment is given is the 4-terminal Gish-Rooney method, and the papers which give this theory in probably the most complete manner are those by Hummel (ref. list No. II_{12}), Tagg (ref. list No. $II_{10,14,16}$), Roman (ref. list No. II_{15}), Peters and

[1] If sand and clay layers, et cetera, are used in a small laboratory tank, the latter has to be compensated to eliminate the effects of the walls (see T. A. Manhart, ref. list No. II_{17}).

Bardeen (ref. list No. II_5), and Ehrenburg and Watson (ref. list No. II_{11}).

Very little is available on the theory of interpretation of results obtained by the single-probe method. It seems, however, that the results obtained by this method are generally not nearly as difficult to interpret as those obtained by the 4-terminal Gish-Rooney method. For the single-probe method in its application, illustrated in Figure 7, *IIa*, the thumb rule that the depth of discontinuities of resistivity is approximately equal to the arithmetic means of the two potential probe distances at which a marked change in resistivity is observed generally works well. This holds in particular if the distance PR is much smaller than E_1P; otherwise, the rule that $h=\sqrt{2bc}$ should be applied. Thus, if E_1P is used equal to PR (method II), the depth of the geologic body becomes equal to the electrode separation at which its influence is noticed, or $h=a$.

It is beyond the scope of this paper to go deeply into the details of the theory of interpretation. However, several diagrams are given which represent the results of theoretical computations of the effect of vertical changes in resistivity on the 4-terminal method.

Figure 16 illustrates curves of apparent resistivity for the 2-layer case, and for various ratios of the resistivity of the lower layer divided by the resistivity of the upper layer. The corresponding values of k mean the ratio: $\frac{\rho'-\rho}{\rho'+\rho}$. The observed resistivities may be plotted as ordinates, the abscissas being electrode separations; for the purposes of interpretation, however, it is more instructive to make the depth axis the ordinate and plot the observed resistivity values as abscissas, in accordance with the statement previously made that, in a general way, marked changes in resistivities occur at electrode separations a which are equal to the depth h of the resistivity discontinuity underneath. (Or else, this depth h is by a constant ratio greater or smaller than a, which is taken into consideration in the diagrams, as not h, but $\frac{a}{h}$ is plotted.)

Thus, we see from Figure 16 that for all resistivity ratios the interface between surface layer and bottom layer coincides approximately with the inflection point in the curve. The abscissas, by the way, do not represent the observed apparent resistivities directly, but the ratio of the observed resistivities to the resistivity of the upper layer. For the reason just stated, the ordinates likewise do not give depth directly, but the electrode separation or depth of penetration divided by the thickness of the upper layer. As both the resistivity and the thickness of the upper layer are constants, this method of graphical representation does not change the shape of the curves.

It is seen from the figures that in this case the boundaries of the formations underneath are reflected as points of maximum curvature, or as third derivatives of the apparent resistivity with respect to the electrode separation.

The curve denoted by "*k* equal to one" is of very great importance in practical work; it is frequently encountered in applying resistivity

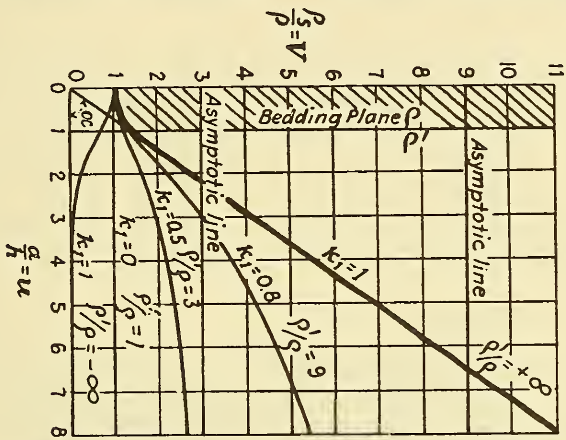

FIG. 16.—Curves of apparent resistivity over surface layer of thickness *h* for different ratios of resistivities of surface layer and infinite lower layer (after Hummel).

methods in civil engineering work and related geological problems where the thickness of the overburden above bedrock is to be determined. In such cases the resistivity of the bedrock is ordinarily so much greater than the resistivity of the overburden that the curves approach the theoretical case where the lower resistivity may be considered as infinite. For a comparison with the results obtained in practice, see the curves shown in Figure 27.

The diagrams shown in Figure 17 *a* and *b* represent the results of the theoretical analysis for the 3-layer case. In Figure 17 *a*, the second medium has the lowest resistivity; in Figure 17 *b*, the second medium has the highest resistivity. It is seen that in this case (3-layer problem) the peaks in the curves represent approximately the depth to

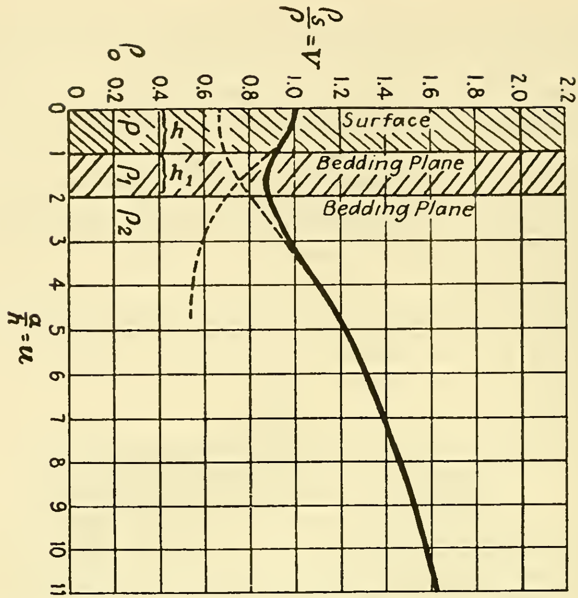

FIG. 17*a*.—Curve of apparent resistivity for case $h = h_1$ and ratio $\rho : \rho_1 : \rho_2 = 1 : 0.5 : 2$ (after Hummel).

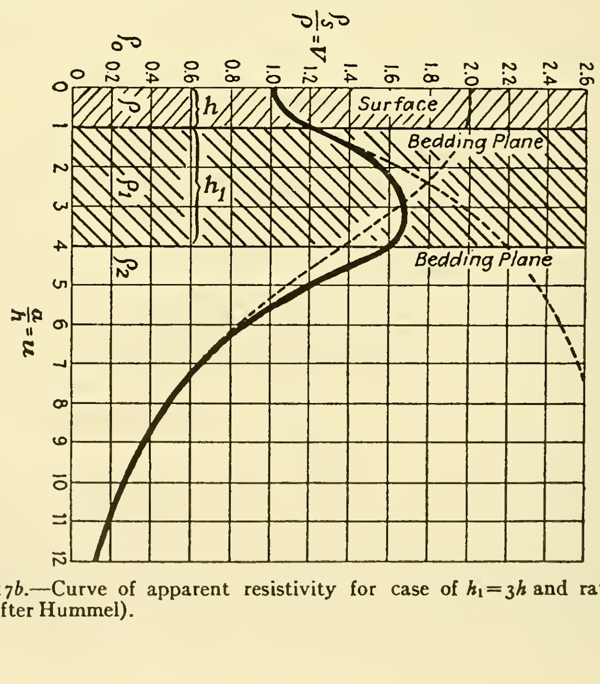

FIG. 17*b*.—Curve of apparent resistivity for case of $h_1 = 3h$ and ratio $\rho : \rho_1 : \rho_2 = 1 : 3 : 0$ (after Hummel).

the center of the second medium. It is of interest to compare the theoretical curve of Figure 17 *b* with the curve obtained in actual practice shown in Figure 34.

One can readily see that already in the 3-layer case the analysis of the results obtained by the 4-terminal method presents considerable difficulties, and it is not surprising that on that account the use of this method for electrical vertical drilling has been largely abandoned and has been replaced by the various types of the single-probe method and by the potential-drop-ratio method.

Only in the 2-layer case, a rigorous direct interpretation, and a determination of the depth to the interface are possible. How this can be done in favorable geological cases, has been shown by Tagg (ref. list No. $II_{10,14,16}$).

Tagg's method is based on the following consideration. If we measure the apparent resistivity as a function of horizontal distance, the following quantities are given: the apparent resistivities as a function of electrode separation, and the resistivity of the upper layer. The ratio of the upper to lower resistivity, and the depth of the boundary are unknown. Theoretically, therefore, the equations can be solved by determining the apparent resistivities at two electrode separations. Designating by k the resistivity ratio $\frac{\rho'-\rho}{\rho'+\rho}$ (Fig. 16), and by h the depth to the interface, it follows that from the surface resistivity, and the apparent resistivity for only one electrode separation, h as a function of k can be determined and graphically represented. By taking then the apparent resistivity for a *second* electrode separation, a second function of h of k becomes available and can be represented by a curve, and the intersection of these two curves gives the depth. In practice, the more electrode separations used, the greater are the number of curves of $h=f(k)$, and consequently the greater the accuracy in the determination of the point of intersection of the corresponding curves.

In order to show how the interpretation is made, a practical example is demonstrated in Figure 18. In the field, the curve shown in the upper part of Figure 18*D* was obtained, indicating immediately (by comparison with Figure 16) that a poor conductor is underlain by a good conductor and that k is negative. The resistivity of the top stratum is taken from the curve. If the data are not as good as they are in the illustrated case, several separate determinations may be made with small electrode separation. Then for definite electrode

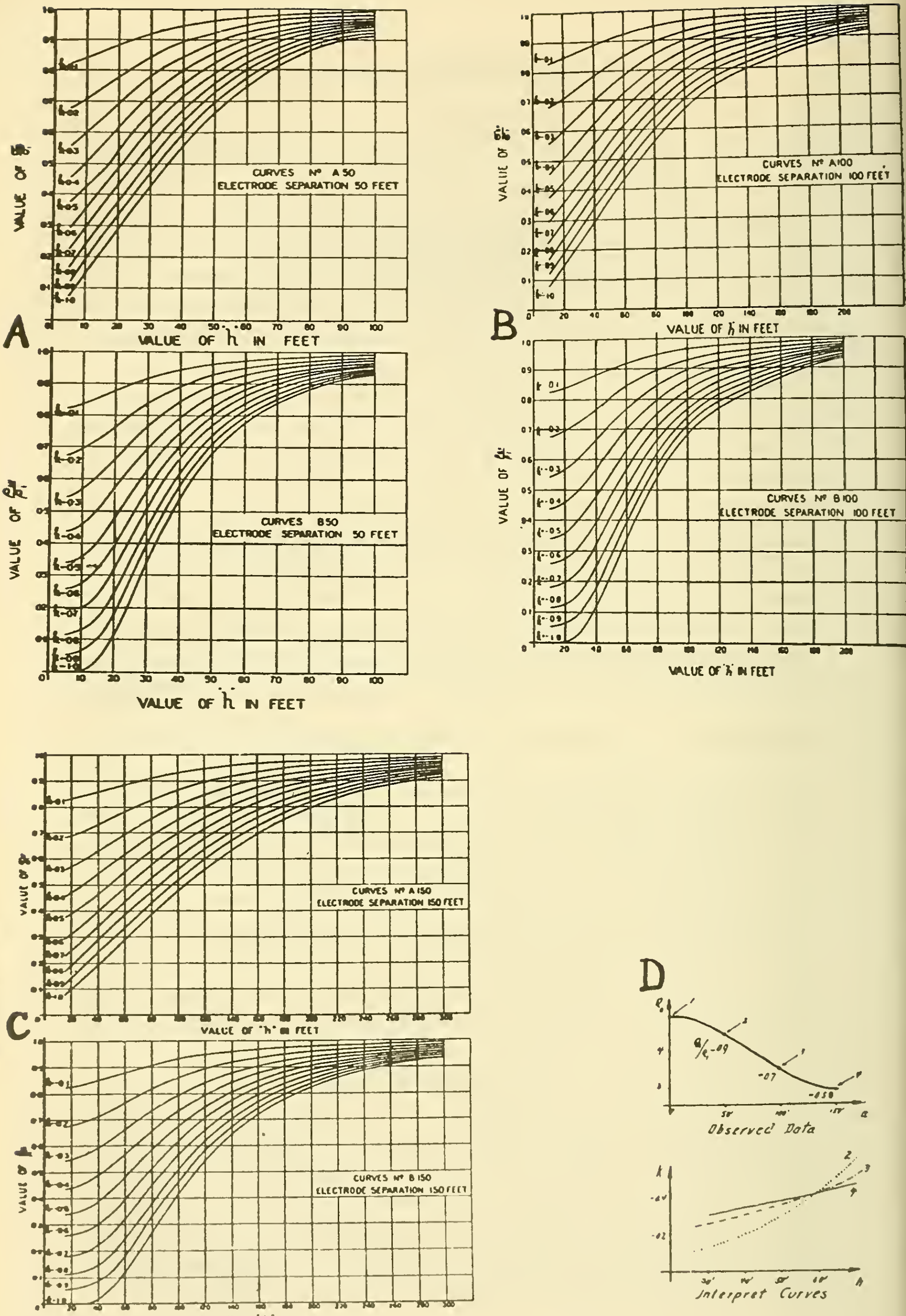

FIG. 18.—*A* to *C:* Apparent resistivities as function of depth to boundary and as function of *k*, for separations of 50,100, and 150 feet (after Tagg). *D:* Application of above diagrams in depth determinations.

separations, say for 50, 100, and 150 feet (at the points 2, 3, and 4) the ratio of the apparent resistivity to the resistivity of the upper medium is computed (Fig. 18*D*).

Diagrams have been prepared previously for this ratio $\frac{\rho_a}{\rho'}$ as a function of *h*, *a* being constant for each diagram (Fig. 18 *A–C*). (The appearance of the curves in these diagrams is much the same as those shown in the diagram of Figure 16, except that not *a/h*, but its reciprocal, *h/a* is used, and that *a* is a constant for each diagram.) In the example shown in Figure 18*D*, the ratio of apparent to surface resistivity is 0.9 for the separation 50 feet; hence, the lower diagram of Figure 18*A* is used, and the corresponding values of *h* and *k* which are intersected by the line 0.9 are noted and plotted (curve 2 in Figure 18*D*). Then the next point, $a = 100$, is taken, at which the ratio is 0.7; one proceeds as before, this time using the diagram in Figure 18*B*, and plotting *k* as a function of *h* for the ratio 0.7. This gives curve 2 in the interpretation diagram. Curve 3 is obtained in the same manner. All three curves intersect at a depth of nearly 58 feet, which is the desired depth of the formation boundary.

Under favorable conditions, this method works very well for the 2-layer case, and is often applicable in practice when the depth of overburden is to be determined. Its application, however, depends altogether on an accurate determination of the surface resistivity, which is not always easy to accomplish—or else, the resistivities determined at the surface do not represent at all the resistivity of the upper layer, the thickness of which is to be determined. In other words, the problem then is no longer a 2-layer problem.[1,2]

Schlumberger uses the following method of interpretation which deviates in a number of respects from that of Tagg's:

1) As in Tagg's method, the resistivity of the upper stratum is first determined from measurements with small electrode separations.

2) The resistivity of the lower layers is then obtained from exposures or measurements in wells (Schlumberger's "electrical coring"), or from assumptions based on past experience.

3) With these resistivities, a diagram is constructed showing the apparent resistivity as function of electrode separation in *several* curves for *a number* of values of thickness of the upper stratum.

[1] Compare also the discussion of Tagg's paper at the meeting of the American Institute of Mining and Metallurgical Engineers, February, 1932.

[2] The extension of Tagg's method to the three-layer case has recently been worked out by Manhart (ref. list No. II_{17}).

4) To find the thickness of the upper stratum, all that is necessary is to place the observed resistivity curve upon the one which has been computed theoretically in the manner described, and to select the

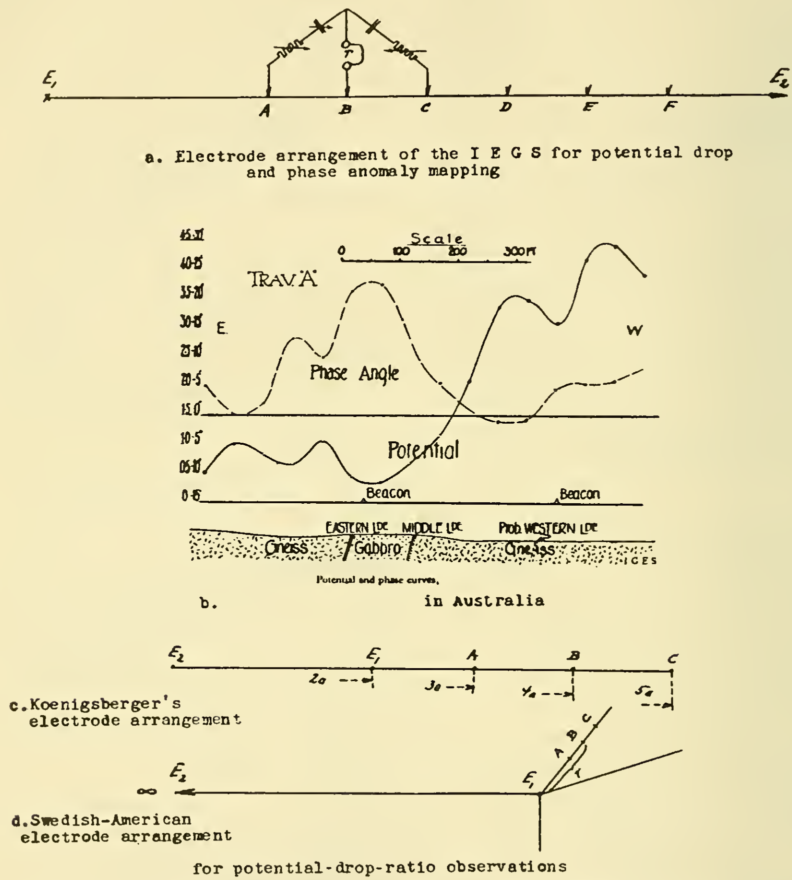

FIG. 19.—Electrode arrangements in potential-drop mapping and potential-drop-ratio work.

depth for which the observed and computed curves show the best agreement.

The same method is applicable to the three-layer case. Poldini (ref. list No. III_{39}) has recently demonstrated the application of this interpretation method in a number of cases.

D. Potential-Drop-Ratio Method

Although this method has been chiefly developed for the purpose of obtaining better interpretable curves in electrical vertical drilling, it may, like the resistivity method proper, be applied also in what may be termed potential-drop mapping, and phase-anomaly mapping.

As the name implies, the principle of measurement is the determination of the ratios of the potential difference between three points (Fig. 19, *a*). The instruments give the voltage difference $A-B$ divided by the difference $B-C$. It is therefore possible, by successive occupation of the points A, B, and C, and then of B, C, D et cetera, to determine the potential drop along a traverse line. This leads to the plotting of potential-drop curves and phase-anomaly curves (Fig. 19 *b*). It is seen that the potential decreases and the phase angle increases, above conductive bodies and vice versa.[1] However, this method of plotting potentials and phases does not furnish nearly as much quantitative data in respect to depth of formation boundaries as the equi-resistivity map method (for example, Fig. 26). The information obtained by a potential and phase profile is at the present stage of interpretative technique, comparable with the information conveyed by an equipotential line map. Electrical vertical drilling by means of the potential-drop-ratio method, however, furnishes a type of curve which is most readily interpreted with reference to the depth of formation boundaries (for example, Fig. 35). While the mapping of potential drops is chiefly applicable to the detection of abrupt changes in conductivity in a horizontal direction[1] (steeply dipping ore bodies, faults, vertical formation boundaries, and the like), the potential-drop-ratio method is to be given preference when the depth to horizontal formation boundaries is determined. Figure 20 shows this advantage of the potential-drop-ratio method by a comparison of the indications obtained by it with the type of curves furnished by: (*a*) equipotential line, (*b*) potential drop, and (*d*) apparent resistivity methods. It is seen that the interpretation of the potential-drop ratios becomes particularly easy if the curves shown under (*c*) are turned through 90°, that is, if the geologic section is plotted in the usual manner, with the exception that the vertical scale is stretched to 3/2 of the original scale, and the electrical curve is superimposed upon it. We shall return to this method again when dealing with the interpretation of the potential-drop-ratio indications.

[1] See also Hedstrom (ref. list No. IV_7).

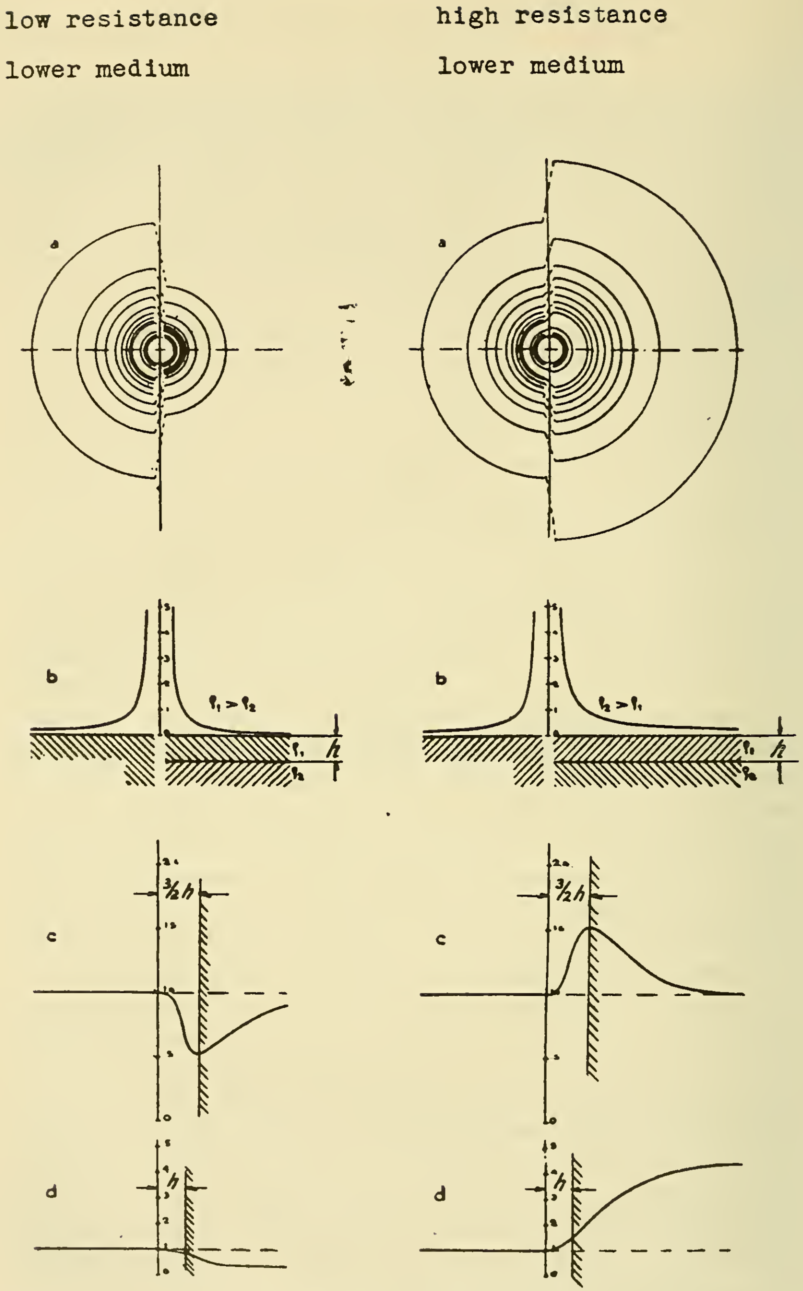

FIG. 20.—Comparison of potential indications:
a: Equipotential lines
b: Potential drop
c: Potential-drop ratio
d: Apparent resistivity
(after Lundberg & Zuschlag)

I. ELECTRODE ARRANGEMENTS

a. IN POTENTIAL-DROP AND PHASE-ANOMALY MAPPING

The arrangement customary in this work is shown in Figure 19*a*. Current from an A.C. source, that is, either from a generator or from a buzzer, is supplied to the ground at the points E_2 and E_1. The potential drops are measured between the power electrodes, at the points *A*, *B*, *C*, *D*, *E*, et cetera, and the bridge arrangement occupies first the points *A*, *B*, and *C*; then the points *B*, *C*, and *D*; then *C*, *D*, and *E*, et cetera. The procedure is accurate but laborious. For the reasons just stated, that is, that the potential-drop indications are more pronounced for vertical formation boundaries than for horizontal ones, this method of potential mapping has not found very much application in oil work. Only vertical electrical drilling by means of the potential-drop-ratio method has gained importance.

b. ELECTRODE ARRANGEMENTS FOR ELECTRICAL VERTICAL DRILLING

The two types which have been used to date are illustrated in Figure 19 *c* and *d*. The electrode set-up used by Koenigsberger differs from the arrangement employed by the Swedish American Prospecting Company in that the spacing of the current electrodes is changed, while in the Swedish American arrangement the current basis remains constant. Furthermore, in Koenigsberger's arrangement the current basis is short and only twice as long as the interval of the search electrodes *A*, *B*, and *C* while in the Swedish method the exploring electrodes are used in the vicinity of one current electrode and the other current electrode is practically at infinity. Finally, in the Swedish American arrangement the interval between the exploring electrodes remains fixed, while in Koenigsberger's method this interval increases with an increase in the length of the current basis. Hence, for purposes of interpretation, the depth reached in Koenigsberger's method is a function of the interval *a*, while in the Swedish method the depth is proportional to the distance of the center probe, *B*, from the power electrode *E*. In Koenigsberger's method the measurements are made in the extension of the current basis; in the Swedish American method, the traverses may be laid so as to radiate in all directions from the near power electrode; to eliminate the influence of the far current electrode completely, the most advantageous arrangement is at right angles to the current basis (strictly on the circumference of a circle with the radius E_1E_2 about the far current electrode).

II. APPARATUS

a. GENERAL: POWER SOURCES

Potential-drop-ratio measurements may be made with both D.C. and A.C. Koenigsberger has used both types of current, while the Swedish American employs only A.C. If D.C. is used, a number of B-batteries are sufficient, if the required depth penetration is not great. Otherwise, a gasoline engine D.C. generator is more advantageous for greater depth of exploration. The current should be interrupted by a commutator similar to the one employed in the Gish-Rooney apparatus, so that effects of polarization and stray currents are eliminated. As will be stated later, the Gish-Rooney equipment itself may be employed for potential-drop-ratio determinations (with slight modification) for simple problems, although the accuracy furnished by the potentiometer customarily employed in this apparatus is not sufficient for many purposes.

For the potential ratiometers as used by the Imperial Geophysical Survey and the Swedish American Prospecting Company, alternating current of about 500 cycles is required, which for small depths of penetration is furnished by a buzzer operated from a storage battery (Fig. 23) while for greater distances and unfavorable contact conditions, a 500 A.C. generator is preferable. It must be borne in mind, however, that the use of a stronger power source alone with average frequencies does not bring about a greater depth penetration, as the latter decreases with the frequency employed; hence, for great depth penetrations, the frequency has to be considerably lowered, which then also would require a change in the dimensions of resistances and inductances in the ratiometers, or else commutated D.C. should be used.

b. KOENIGSBERGER'S AND OTHER D.C. METHODS

Koenigsberger indicates in his publication (ref. list No. IV_2) in a general way a number of arrangements which he has used for the measurement of potential-drop ratios, but does not go into details with reference to apparatus employed. When using D.C., the potential differences $A-B$ and $B-C$ may, of course, be measured independently by a potentiometer and their ratio may be formed by computation. For this purpose, a regular Gish-Rooney equipment may be employed, with an electrode arrangement identical with that used in the single-probe method (Fig. 7, type *IIa*). The potential difference between the points B and C is measured in the usual way; then the electrode in *C* is left there and the electrode which was at *B* is moved to a next point, say *D*, which has the same distance from *C*, as *B* has from *C*. The curves shown in Figure 33 have been obtained in this manner. The method is not very accurate, as the surface and contact resistances enter considerably.

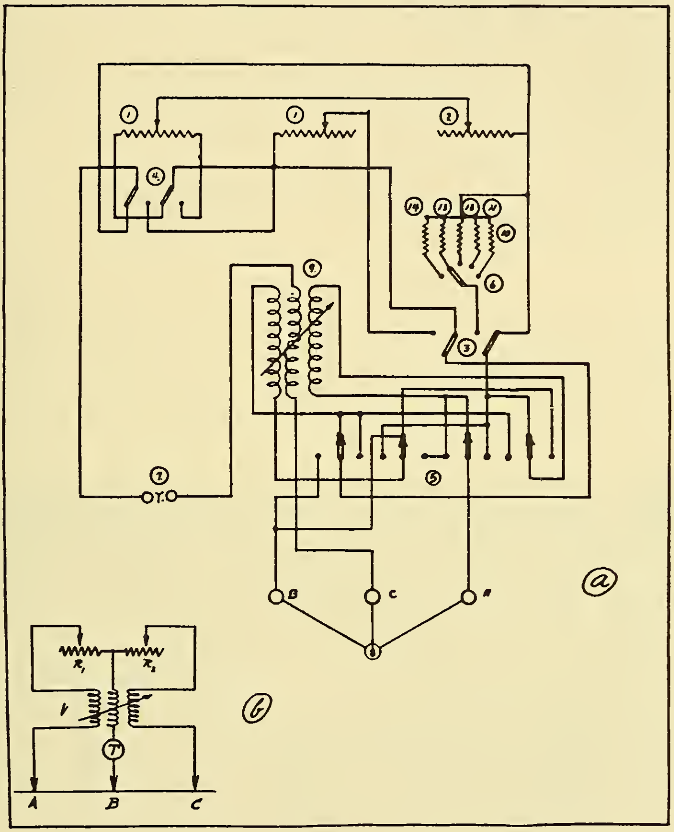

FIG. 21
a: Detailed circuit diagram of Racom compensator
b: Schematic circuit diagram of same

Koenigsberger uses also a modification of this method: The central electrode is left permanently connected to the potentiometer and the other electrodes, *B* and *C*, are connected alternately with it by means of a commutator. The ratio of the deflections gives the potential drop ratio; or else, it is possible to leave one electrode, say *C*, stationary, and change the position of the other until a balance is obtained.

A still better arrangement would be the bridge shown schematically in Figure 21*b*, when modified for direct current: the variometer *V* would be left out, and the telephone *T* would be replaced by a sensitive galvanometer.[1] To overcome the contact resistance at the probes, two settings of the resistances R_1 and R_2 would be required. If commutated D.C. is used, the commutator could be incorporated in the bridge. This would necessitate a lead from the bridge to the power source. In order to eliminate this lead, very low frequency A.C. could be supplied to the power stakes, and a rectifier could be used with the galvanometer.

In addition to D.C., Koenigsberger has also used A.C. with bridges similar to the arrangements now to be described.

b. SWEDISH AMERICAN RACOM

The name Racom is an abbreviation for ratio compensator. The principle of the instrument is a compensation of the difference of the potential P_{A-B} and P_{B-C} between the three points *A*, *B*, and *C*, by adjusting resistances in the two circuits *AB* and *BC* to balance on a telephone. Alternating current of about 500 cycles is used, and for small depths is furnished by a buzzer, operated from a storage battery, shown in Figure 23. A schematic circuit diagram is given in Figure 21, *b;* as seen from the figure, the current in the circuit *AB* is equal to $\frac{P_{A-B}}{R_A+R_1}$, and the current in the circuit *BC* is equal to $\frac{P_{B-C}}{R_C+R_2}$, where R_A and R_C are the contact resistances of the stakes *A* and *C*. As the resistances R_1 and R_2 are so adjusted that the telephone is silent, we obtain the equation for the potential-drop ratio $\frac{P_{A-B}}{P_{B-C}}=\frac{R_A+R_1}{R_C+R_2}$. The contact resistances may be eliminated by a second setting of R_1 and R_2. The phase differences are compensated by a suitable setting of the variometer, *V*. If the observed currents are too feeble to be heard on the telephone an amplifier is employed, as shown in Figure 22. A schematic diagram and a detailed circuit diagram are given in this figure. The amplifier is a 2-stage, resistance-

[1] A similar arrangement is used for pipe-line corrosion studies by Schlumberger (ref. list No. III_{40}).

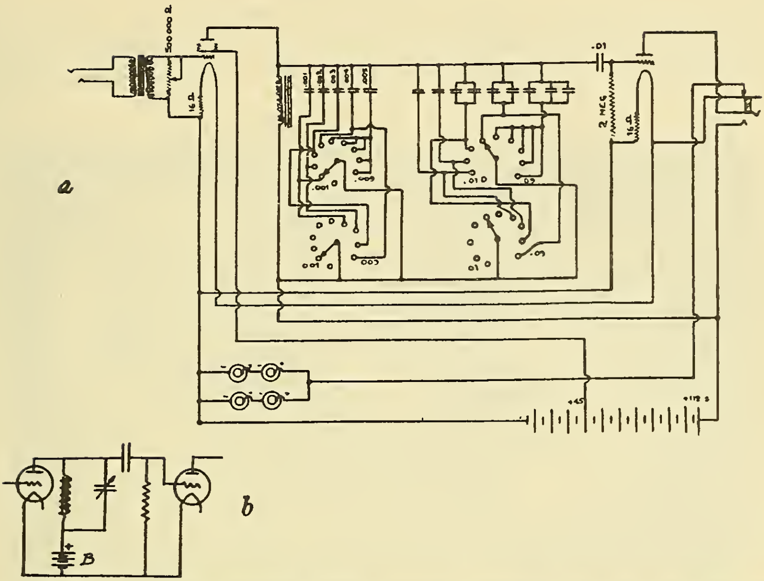

FIG. 22.—*a:* Wiring diagram, Racom amplifier. Courtesy Swedish American Prospecting Corporation. *b:* Schematic diagram.

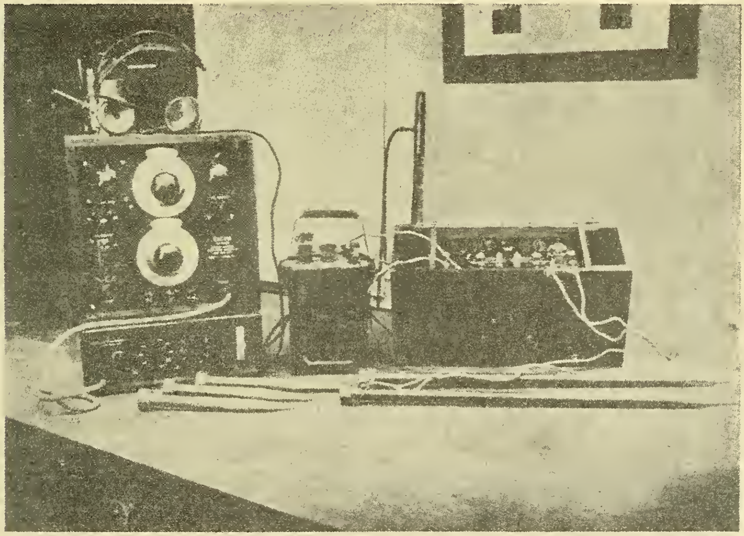

FIG. 23.—View of Racom with amplifier and buzzer.

impedance-coupled amplifier, incorporating a tuning of the choke for the elimination of harmonics so as to enable a better setting of the bridge.

Figure 23 shows a view of the Racom, with amplifier underneath the instrument box, and a buzzer on the side together with the power and the exploring electrodes.

For more details on the operation of the Racom and examples of a number of surveys, see Lundberg and Zuschlag's publication on the subject (ref. list No. IV_5).

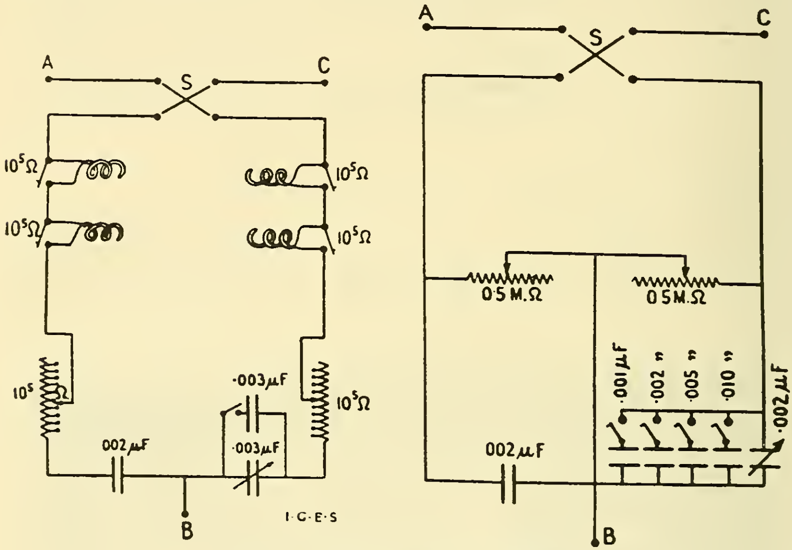

FIG. 24.—I.E.G.S. ratio arm bridges.

FIG. 24*a*.—Series ratio arm bridge. FIG. 24*b*.—Parallel ratio arm bridge.

d. I.G.E.S. RATIO COMPENSATORS

The potential ratiometers used by the Imperial Geophysical Experimental Survey in Australia are very similar in design to the Swedish American bridge just described. A schematized view of their bridge is given in Figure 19 *a*. It is seen that this bridge differs from the Racom type only by the fact that condensers are used instead of inductances for the determination of phase shifts. The wiring diagrams for a series and a parallel ratio arm bridge are shown in Figure 24. Figure 25 gives a view of the panel of one of these bridges. An amplifier is also used in conjunction with these bridges, and in the field is carried by means of straps on the operator's back. For details on the operation of these ratiometers and their theory, the publication by Edge and Laby (pp. 50–54, and 268–274) (ref. list No. IV_3) should be consulted.

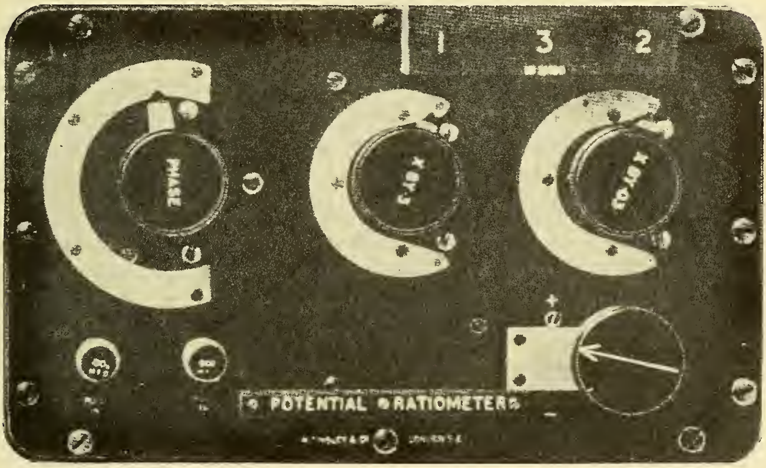

FIG. 25.—View of I.E.G.S. ratio compensator.

III. INTERPRETATION

Methods of interpretation of potential-drop-ratio results have been worked out in a general way for Koenigsberger's and for the Swedish American methods.

The results of the computations show that Koenigsberger's method is, in a number of ways, at a disadvantage as compared with the Swedish method, because his ratios are only opposite but not equal for good and poor conductors which have the same conductivity ratio. What is meant by this is the following: If we form, for a 2-layer problem, the ratio of the conductivity of the surface layer to the conductivity of the lower layer, then the potential-drop ratio in Koenigsberger's method is 3.57 if the conductivity ratio is one (homogeneous ground). However, if the conductivity ratio is 10:1, the potential ratio decreases only to 2.48, whereas, in the converse case (if the conductivity ratio is 1:10) the potential ratio is 8.70 (Fig. 40 *c*). Thus, it is seen that in his method the potential-drop-ratio responses are not symmetrical around the 3.57 position, and that the influence of good conductors below is much greater than the influence of poor conductors below. This would make this method a rather valuable accessory to the Racom, if it is desirable to emphasize in the results the effects of good conductors, such as ore bodies or formations which are filled with salt waters.

Figures which give the general values for the potential ratios in the Racom method have not been published, but it appears from the diagrams given by Lundberg (Fig. 20) that for the average conductivity ratios the potential-drop-ratio responses are approximately the same for a good conductor below as they are for a poor conductor below; for, as seen from Figure 20, for the assumed conductivity ratio, the potential ratio is 1.5 for a good conductor above and 0.5 for a good conductor below; or, the deflections in the Racom method are equal and opposite for equal and opposite conductivity ratios. Hence, when plotting in the Racom method the potential ratios on a vertical axis and the formation boundaries in a horizontal position, a poor conductor manifests itself by a deflection of the ratio curve toward the right, and a good conductor appears as a deflection of the ratio curve toward the left. For a poor conductor below, the ratio is greater than one, and for a good conductor below, the ratio is smaller than one.

The chief advantage of the potential-drop-ratio method lies in the ease with which the potential ratio indications, if plotted in a suitable manner, may be interpreted with reference to the depth of formation boundaries. Although the potential-drop ratios, and hence also the relations between depth of boundary and maximum of indication, depend on the conductivity ratios of the formations involved, yet several empirical rules have been found to hold very well for the average conductivity ratios encountered in the field.

In Koenigsberger's method, the depth corresponding with a ratio maximum is approximately $\frac{1}{2}$ *a* for conductivity ratios up to 1:10 or 10:1. In the Racom method, the depth to the formation boundary is, under similar conditions, equal to $\frac{2}{3}$ *r*. (The significance of the symbols is explained in Figure 19.) Hence, when we turn the ratio traverses through 90° and plot the curves vertically on a geologic section, we have to stretch the vertical scale of the section to 3/2 in the Racom, and double it in Koenigsberger's method. Or, if we want to leave the scale of the geologic section as it is, we have to reduce the distances in Koenigsberger's method to $\frac{1}{2}$ and the distances in the Racom method to $\frac{2}{3}$ of their original values. These relations are well explained in the practical examples illustrated in Figures 33, 35 and 40.

E. Opinions Regarding Direct Electrical Prospecting for Oil

In view of the recent developments of the technique of resistivity and potential-drop-ratio-measurements which permit (as has been shown and can be verified by an inspection, for example, of Figures 34 and 35) the accurate identification of certain geologic horizons under favorable circumstances, the question becomes, of course, acute again as to the extent to which oil deposits may be located directly by electrical measurement from the surface.

Very much has been written about this subject, probably altogether too much at a time when field technique and interpretation of electrical prospecting were not nearly far enough developed to enable anyone involved in the discussion to state definitely that certain things were altogether impossible. The writer believes that several authors who were involved in the animated discussion at that time wish now, in view of the recent developments, that they had not voiced their opinion in the matter.

Another factor which influenced the exchange of opinions in the problem of the possibility of locating oil directly was the fact that most of the involved parties were interested in competitive geophysical consulting companies. Hence, the writer doubts if very much could be gained by repeating in detail at this time all the arguments which have been advanced for and against the direct electrical location of oil. Only the main points will be discussed. At any rate, it is emphasized that it is not the object of this paper to take sides in an argument which is not as yet ripe for an altogether unbiased discussion. We shall not try to decide at this time if a direct location of oil by electrical prospecting is possible. We shall merely state, by discussing actual results obtained, what has been done and can probably be done with the present state of development of instrumental technique and interpretation. The whole discussion in this matter seems to have been started when the representatives of the Elbof electrical prospecting company claimed that they were in a position, by means of their method, to locate the oil directly. The chief factor which probably encouraged doubts and discussions of their statements was the fact that the type of indications which they obtained by means of their method, namely the deflection of current lines, did not lend itself very readily to such far-reaching interpretations. In fact, even under favorable geologic conditions, the deflections of the current lines were usually small and therefore the claimed effects could be readily disputed. In addition, this company made their claims on the basis of some work which was carried out at comparatively great depth and under complex geologic conditions, so that it was easy for their opponents to go so far as to state that there was no telling whether the observed deflections came from salt water or from oil.

The opponents of the Elbof theory, notably Ambronn, then went to the other extreme in trying to prove that it was theoretically impossible, under any circumstances, to detect oil directly with alternating current. Ambronn claimed that the conductivity ratio required for electrical prospecting by means of alternating current should be at least 1,000 for such a change to be detectable, a figure which has been proved to be much too high by actual experience obtained since with the potential and resistivity methods. In addition, Heine and Hunkel proved theoretically that the formulas used by Ambronn for his proofs were in error.

Lately, the Elbof company has also taken up, in addition to their electromagnetic method, the study of the potential-drop-ratio and resistivity methods; the results obtained in several areas have been published by Gella and Koenigsberger (ref. list III_{23}; IV_2). Unfortunately, in both publications, no curves are given and this is probably the reason why these more recent claims of the Elbof company and Koenigsberger, which are based on reliable and favorable technique, have passed almost unnoticed. The figures given for the results obtained with the potential drop ratio at the oil field of Oberg in Germany are given in such manner that the reader who is not familiar with the locality can not form an opinion as to the extent to which the claims advanced are justified. The writer has made an attempt to use as many of Koenigsberger's data as possible and to superimpose them upon the geologic section; the results are represented in Figure 40 and are discussed at the end of this paper.

While working with the potential-difference method, the Elbof company found that in certain areas where the oil-bearing strata are overlain by porous strata such as sands and sandstones, the gas had in many places migrated into them from the strata below, had displaced the salt water and had thus increased considerably the thickness of the strata acting as poor conductors. This may make it possible in some places to apply the electrical method to advantage, but it is, on the other hand, a condition which is by no means representative of all oil reservoirs.

A rather serious argument has been advanced lately by Hedstrom (ref. list No. III_{21}) against the direct electrical prospecting for oil. This author claims that from theory and experiment it follows that a formation which is practically an insulator does not show in the electrical results any differently from what it would if its conductivity were only 10 times smaller than that of the surrounding formation; hence, a poorly conductive sand or limestone with a conductivity only about 10 times less as compared with that of the surrounding strata, would not be distinguishable from the effect of an oil formation.

This argument, however, does not preclude the possibility of direct location of oil in an area where, by correlation with geologic data, a poor conductor is known to be oil bearing, and where it would thus be possible to trace this oil-bearing horizon in various depths of structure, and to determine where the filling of the pores with oil ceases and where the oil is replaced by salt water. If the argument just advanced is correct, it would seldom be possible to distinguish between a sand filled with gas or a dry sand, and a sand filled with oil.

One might finally say in commenting on Hedstrom's argument, that with a greater perfection of technique of electrical prospecting we may some day be able to distinguish the surface effects of poor conductors with a greater precision and obtain an electrical log in which the formation resistivities could be represented by their true and not their apparent resistivity, so that an oil sand would show somewhat like the sands illustrated in the electrical logs demonstrated in Figure 37 and Figure 38.

However, as already stated, it is beyond the scope of this paper to take sides in this argument, and we will merely confine ourselves to a discussion of the results thus far obtained in attempting to locate the oil directly at the end of the following section (F II).

F. Results Obtained With Resistivity and Potential-Drop-Ratio Methods

I. In Structural Work

Several outstanding examples are here described which illustrate both the possibilities and the limitations of the resistivity and potential-drop-ratio methods in determining geologic structure.

a. DETERMINATION OF THICKNESS AND CHARACTER OF SURFACE FORMATIONS

The chief potentialities of both the resistivity and the potential-drop-ratio methods lie in the determination of character and depth to formation boundaries of surface formations. In general, as in any other geophysical method, the thickness of geologic formations has to compare favorably with their depth, otherwise they may pass unnoticed in resistivity indications; on the other hand, conditions for recognizing formation boundaries are much more favorable in the potential-drop-ratio method than they are in the resistivity methods. In addition, a reason why these methods lend themselves so readily to the determinations of depths and thicknesses of surface formations is the fact that they often differ much more in their resistivity from the underlying series than these lower formations will differ between themselves.

A problem which often arises in practical geology is the determination of the thickness of glacial overburden. Both the method of resistivity mapping and the method of vertical electrical drilling can be applied to advantage, depending on the problem.

Thus far, the majority of examples for the determination of the thickness of surface formations have become known from applications in mining and civil engineering, but they would also be of importance in certain types of oil-structural work.

1. BY EQUI-RESISTIVITY MAPPING

A rather convenient way to map bedrock contours in a fairly large area, or to determine the depth to bedrock along a traverse line, is to carry a 4-terminal contacting arrangement with fixed electrode separation over the area and to observe and plot apparent resistivities in the manner shown in Figure 26. The results obtained with two electrode separations are illustrated. The solid contour represents the depth to bedrock, the dashed line is the indication obtained with the correct electrode separation, and the dotted lines show the apparent resistivities with too small electrode spacing. The geologic section was glacial drift above limestones and conglomerates. This figure shows that with an electrode separation larger than the greatest depth to bedrock, the bedrock contour may be mapped with a good deal of accuracy.

2. BY ELECTRICAL VERTICAL DRILLING

This method has been applied very frequently in late years in determining the depth to bedrock in tunnel and dam sites. Figure 27

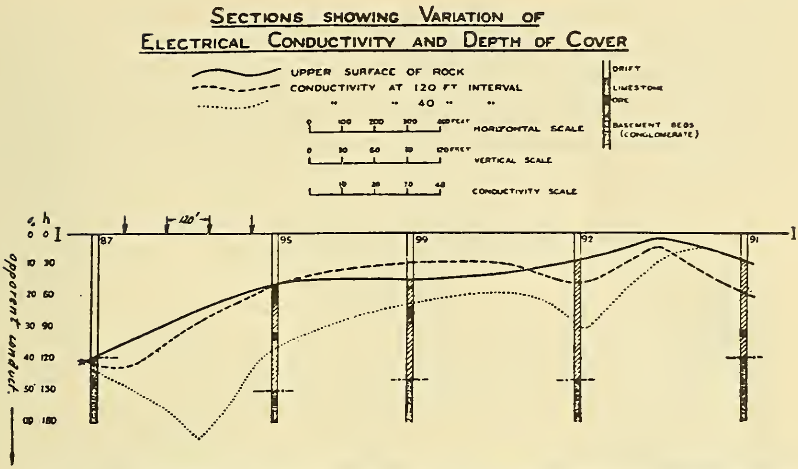

FIG. 26.—Resistivity mapping with two electrode separations to determine depth of cover (after Lancaster-Jones).

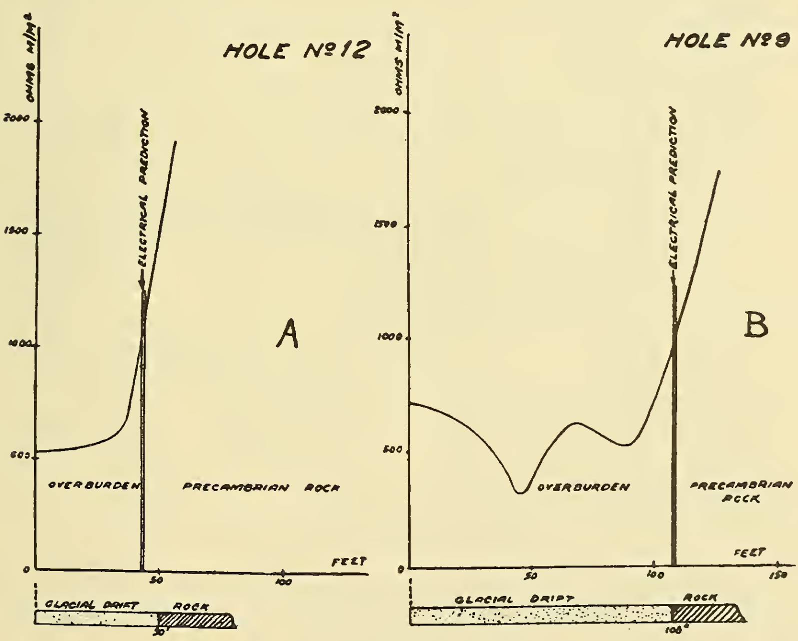

FIG. 27.—Results of electrical vertical drilling to determine thickness of cover (after Leonardon).

A : Diagram in case of fairly homogeneous overburden. Accuracy fairly satisfactory.

B : Form of results when overburden is composed of different layers.

shows two resistivity profiles with the corresponding drilling results. In interpreting the resistivity curves, reference should be made to the curves shown in Figure 16 for the case in which the resistivity of the lower layer is practically infinite. The two curves shown in Figure 27 also illustrate the effect of a homogeneous and an inhomogeneous overburden.

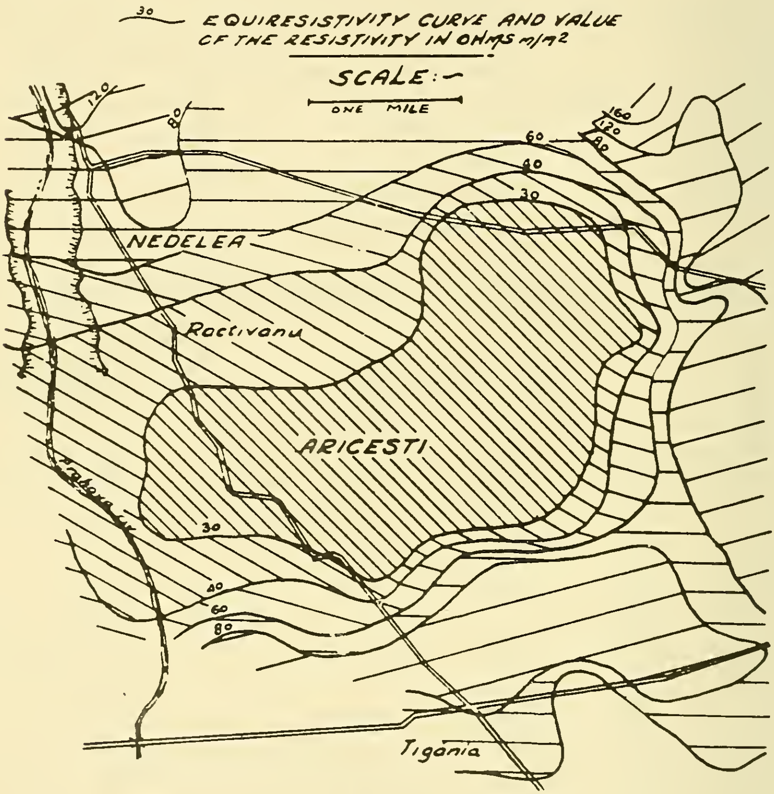

Fig. 28.—Resistivity map of the Aricesti anticline. Work carried out in 1923 (after Schlumberger).

3. BY POTENTIAL AND PHASE MAPPING

This method will probably find little application in oil structural work, although it can give valuable indications in determining the character of surface formations in suitable areas under a shallow cover. Figure 19 shows, as an example, the effect of a gabbro intrusion with two ore bodies on the contact zone upon the potential-drop and

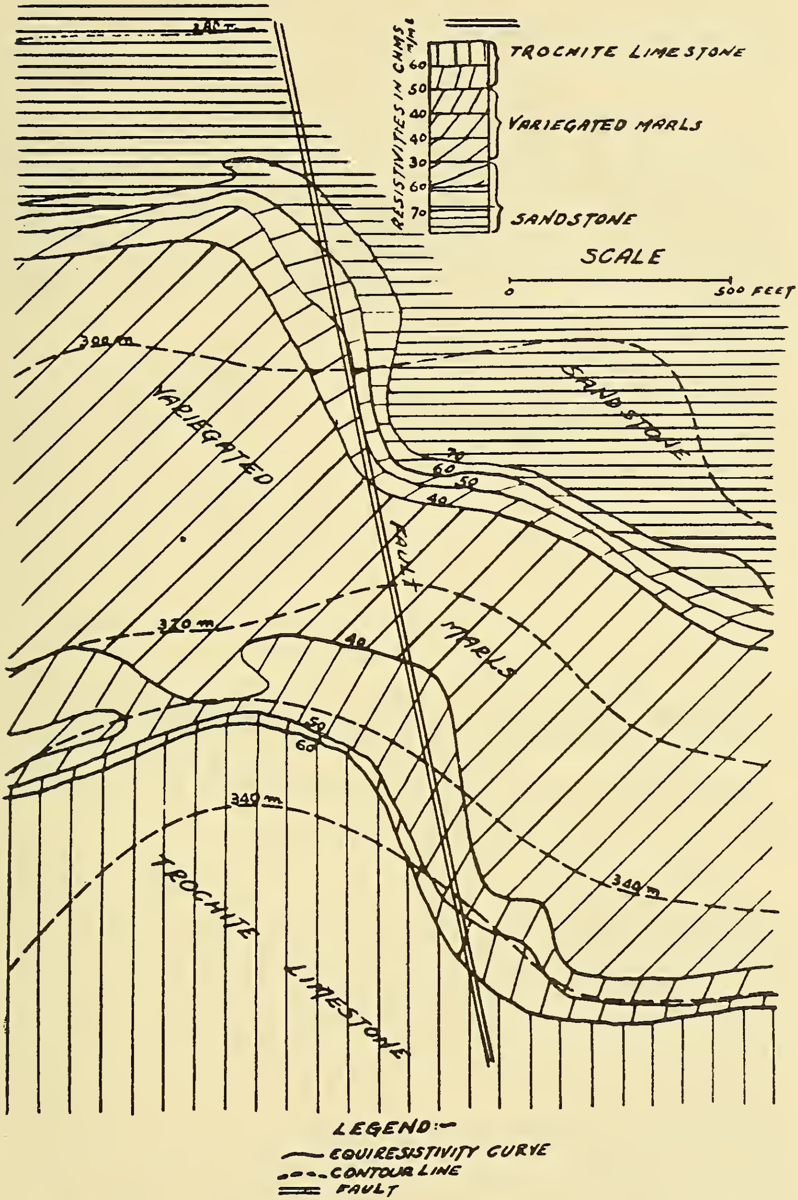

FIG. 29.—Resistivity map of fault in Triassic in Lorraine. Study made in 1928 (after Schlumberger).

phase curves. It will be noticed that the potential decreases above a good conductor, while the phase angle has a maximum, and vice versa.

b. LOCATION OF STRUCTURE

Both resistivity and potential-drop-ratio methods have found already several remarkable applications in locating geologic structure favorable for the accumulation of oil, such as anticlines, salt domes, and the like. They have also been applied very successfully in extending known geologic information into unknown territory, which is an application always recommended for any kind of geophysical method.

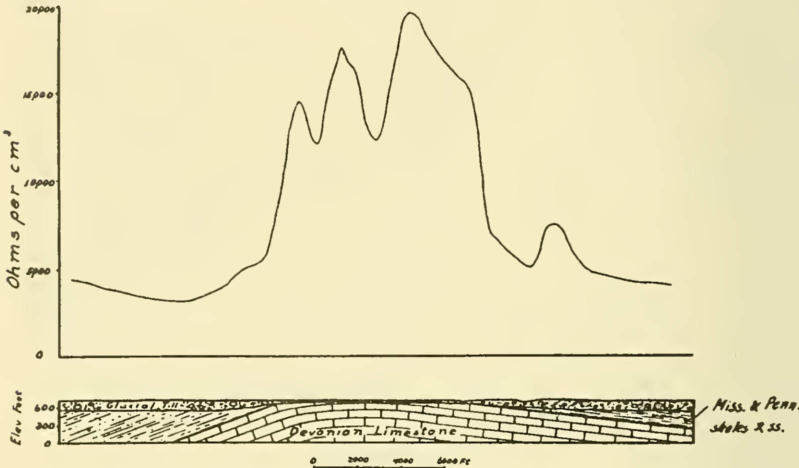

FIG. 30.—Resistivity profile across buried anticline, taken with electrode spacing *a* equal to 200 feet (after Hubbert).

1. BY RESISTIVITY MAPPING

The Schlumberger Company has located and mapped geologic structure of several types by the resistivity method. As examples, the anticline illustrated in Figure 28 and the fault illustrated in Figure 29 have been selected. The first figure shows an equi-resistivity map of the Aricesti anticline in Roumania which appears in the resistivity map as a body of lower resistivity. Very instructive is the location of a fault in Alsace-Lorraine by the method of resistivity mapping, as it illustrates how definite resistivity values may be identified with definite geologic horizons. The marls have the lowest resistivity, then follow the limestones, and then the sandstones in this area. The resistivity contours follow truly the trace of the fault which has displaced the formations under discussion in a horizontal direction.

Figure 30 shows another good example of structure located by means of a resistivity profile. The traverse was made with a 4-terminal contacting arrangement using an electrode interval of 200 feet. As this interval is greater than the greatest thickness of the glacial till in the area, the observed apparent resistivities represent chiefly the variations in resistivities below the glacial drift. Thus it was possible to get the effect of the anticline composed of Devonian limestone of high resistivity, flanked by Mississippian shales and sandstones of lesser resistivity. Hubbert states that the causes of the smaller variations in the resistivity peak are unknown; it is probable, however, that they

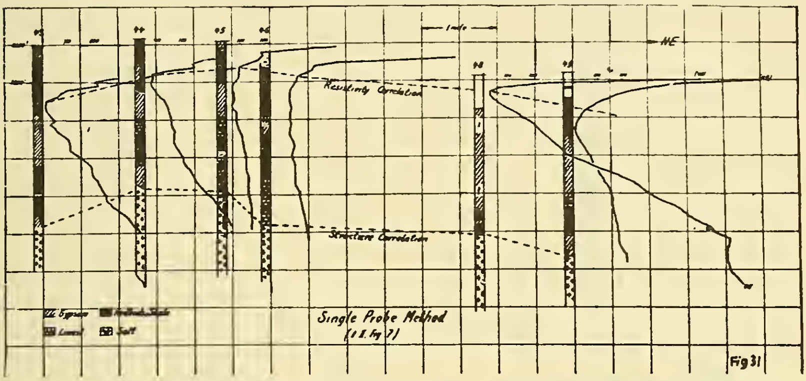

FIG. 31.—Results of electrical vertical drilling with single-probe method (II) and corresponding geologic section in an area in New Mexico. Courtesy of Harry Aurand, Midwest Refining Company.

are either due to variations in the composition of the glacial till, or due to the fact that this till occurs as filling in erosion channels on top of the structure.

2 BY ELECTRICAL VERTICAL DRILLING

There follows now a discussion of some examples of structural resistivity work, most of which were obtained recently and may not be found elsewhere in the geophysical literature. They illustrate well the great possibilities of the method of electrical drilling with the resistivity and potential-drop-ratio method under favorable conditions.

Figure 31 shows the results of several resistivity traverses which were very carefully measured every 10 feet by means of the single-probe method (No. II of Hummel in Figure 7 or the Ehrenburg-Watson method). The geologic section consists at the surface of alternating shale, gypsum, and limestone beds with an average thickness of 400 feet underlain by salt. The resistance of the gypsum is high, and that of the shales is low. Where the effect of the salt has been picked up, its resistance seems to be fairly low, such as shown in well No. 48, which is somewhat contrary to the experience obtained on salt domes. All resistivity traverses shown in Figure 31 have a similar appearance as far as the very high resistance surface layers are con-

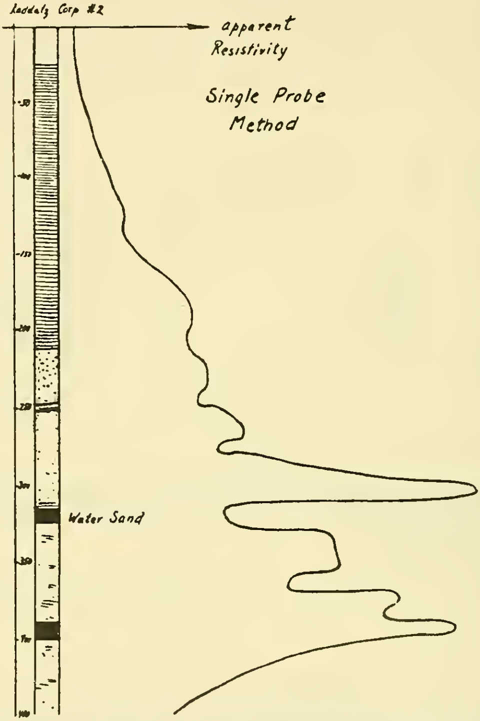

FIG. 32.—Results of electrical vertical drilling using single-probe method (IIa) and corresponding well log at Berthoud dome, Colorado. (Measurements made by Keen, Bishop, and Johnson.)

cerned. Then they all show a pronounced "low," corresponding with the influence of the red beds and shales close to the surface. After that, all curves go up again, depending on the amount of gypsum present in the section. Thus, while the curves of Nos. 45 and 46 are practically flat, they are steeper in 49, 44, 43, and steepest in 48. Just what takes place in No. 48 is not very well known on account of lack of knowledge of the well log; but it seems that the formations present at that locality, probably gypsum, are very poor conductors. A striking similarity prevails between the curves of 43, 44, 45, and 46. Particularly, 43 is very similar to 44, and 45 very similar to 46. Corresponding with the bottom of the first red-bed layer, all the curves mentioned have the same appearance, which becomes less pronounced toward the northeast, but may yet be recognized very faintly in curve 49. Therefore,

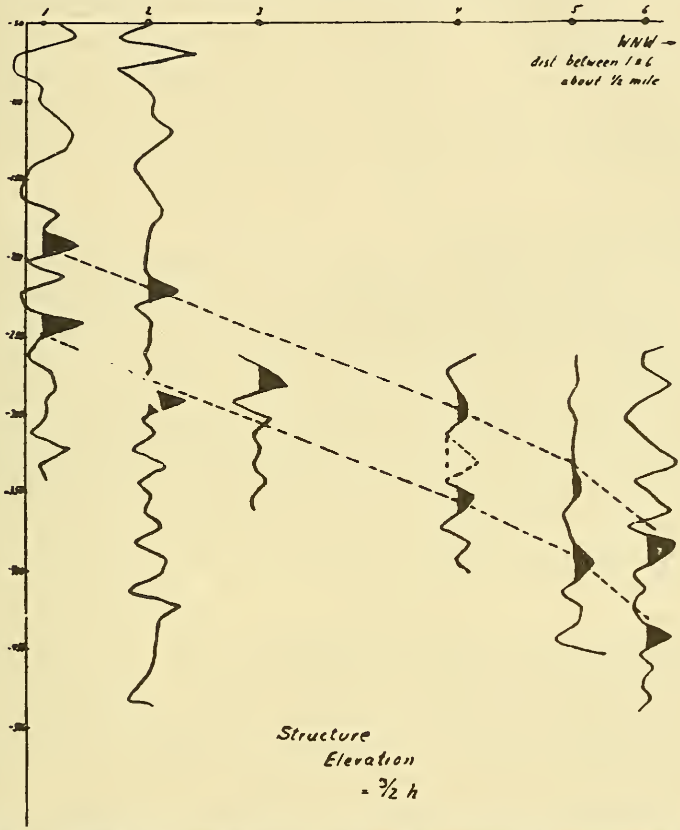

FIG. 33.—Results of potential-drop-ratio observations and corresponding structure at Berthoud dome, Colorado.

the writer has used that particular part of the curve as horizon marker for the resistivity correlation. It is seen that the resistivity contour thus obtained is not everywhere in accord with the logs, but that it checks very well with the structure correlation made on top of the salt beds. The next two figures, 32 and 33, show some results which were obtained with the single-probe and the potential-drop-ratio methods on the Berthoud dome, not far from Denver. Some oil and gas have been produced from this dome. The geologic section consists of Cretaceous shales and sands. The curve shown in Figure 32 was obtained by using the method No. *IIa* (Fig. 7), and is plotted together with the log of a well next to which the measurements were made. Pending further work in this area, the present interpretation is that the peaks in the curve are produced by the water sands, and that these sands contain water of artesian origin, which is encountered in geologic structure of many types in eastern Colorado near the Front Range of the Rocky Mountains. If nothing else, this curve shows that certain resistivity peaks appear at certain depths which can be correlated with definite geologic horizons, and which can be traced throughout the structure.

This makes it possible to determine structural contours in this area as shown in Figure 33. It should be understood that these results were obtained with an equipment which was not at all adequate for the purpose. Before the regular Racom equipment (already described) became available for field work, we made some preliminary field tests with the Gish-Rooney apparatus modified for the determination of potential-drop ratios. Hence, the results are masked to some extent by the variations in surface resistivities which can be eliminated to a much greater degree in the Racom. Nevertheless, the results are reliable enough to justify at least a preliminary interpretation. The potential-drop-ratio curves were turned 90° and plotted vertically. It is seen that certain peaks in the curves repeat themselves in every traverse, and these are indicated in solid black. Then the structural contour as known from the well logs was superimposed upon the potential-drop-ratio profiles, and was plotted on a scale 3/2 that of the potential profiles, in accordance with what has been previously stated about the interpretation of potential-drop-ratio measurements. It is seen that the known structural information and the peaks in the potential-drop-ratio profiles correspond fairly well. In this connection, it should be remembered that the indications obtained from a potential-drop-ratio log in electrical vertical drilling are altogether different from the indications obtained in a resistivity log. In the former, formation boundaries are marked by a peak in the drop-ratio curve (Fig. 20).

Although the results just described are not as good as they might have been if more adequate equipment had been used they proved definitely that the potential-drop-ratio method could be used to advantage in eastern Colorado, in close conjunction with geologic data, for the purpose of obtaining data about geologic structure.

Figure 34 shows again some resistivity curves, obtained on four traverses in the San Fernando Valley in California. The results of this work were published by J. J. Jakosky (ref. list III_{31}) in different form; it appears from the description of the work that some sort of a single-power-probe method was used. It may be possible that the 5-electrode system described by Jakosky[1] in a later paper was employed, on which no data are at present available, and which may be some sort of potential-drop-ratio method with an electrode arrangement similar to that shown in Figure 19 or may be the partition method (Fig. 7, *Ia*).

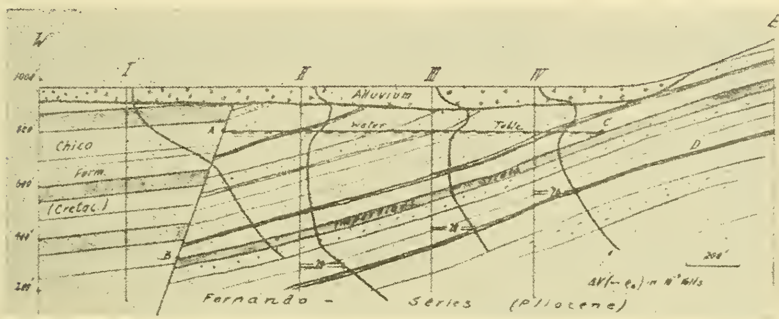

FIG. 34.—Results of four resistivity traverses (single-probe method) and corresponding geologic section in San Fernando Valley, California. (Plotted from data published by J. J. Jakosky.)

As stated by Lundberg and Zuschlag (ref. list No. IV_5) apparent resistivities may also be readily computed from the potential-drop ratios.

In Figure 34, the results obtained by Jakosky are shown, plotted in millivolt units which are directly proportional to the apparent resistivity. The geologic conditions are as follows. The larger part of the area is covered by a blanket of alluvial strata with a maximum thickness of nearly 80 feet. Below, in the eastern part of the area, are inclined strata of the Fernando group of the Pliocene. These formations are cut off by a fault in the western part of the area which has brought strata of the Cretaceous Chico formation in contact with the Pliocene in the east. These Cretaceous strata do not contain water, but the upper parts of the Pliocene are filled with highly conductive waters down to a depth where the impervious strata indicated as

[1] J. J. Jakosky, "Geophysical Examination of Meteor Crater, Arizona," *Geophysical Prospecting* (Amer. Inst. Min. Met. Eng., 1932), pp. 63–98.

such in the figure occur. As the Pliocene strata are cut off in the west by the fault, a conductive triangle, so to speak, occurs between the points *A*, *B*, and *C*. Below the line *BC*, the strata are less conductive, and the same is true of the strata above the water table. Due to the absence of water west of the fault in the Cretaceous section, these strata also are poor conductors.

The resistivity profiles *II*, *III* and *IV* are similar to each other, but differ greatly from the profile I. They indicate a high resistance peak, corresponding with the Quaternary top layer and the Fernando strata above the water table. Then the resistivities drop to very low value, corresponding with the depth of the conductive triangle which is involved in every case. Thus, the horizontal part of the resistivity curve indicating the part of this triangle traversed by each traverse, is very short in traverse *IV*, longer in *III*, and still longer in traverse *II*. As soon as the impervious strata are reached, the resistivity curves go up again, this rise occurring at the shallowest depth in traverse *IV*, being deeper at *III*, and the deepest in traverse *II*. In other words, the point where, on each curve, this rise begins, gives approximately the depth, and thus the dip of the impervious formations. The agreement with the geologic section goes still further. After the impervious layer has been traversed in each section, the recorded resistivities are in proportion with the resistivities of the section below this impervious series. Thus, if we take a certain point on each curve, for example, those corresponding with a deflection of 20 millivolts, and connect these points, the resulting line gives directly the dip of the strata below the impervious layers.

The resistivity curve obtained at the traverse *I* indicates formations of much higher resistivity than the curves obtained on the other traverses. The peak corresponding with the dry layer above the water table which was obtained on the other traverses is also missing. A distinct break occurs in this curve which is explained by the fact that the apparent resistivity drops when a depth is reached which is equal to the horizontal distance (at that depth) of a better conducting medium from the traverse line. We, therefore, interpret this break in the traverse *I* as the effect of the strata east of the fault plane.

In Figure 35, the results of two potential-drop-ratio traverses across an area in the Spring Coulee district in Alberta are shown. The geologic section is Cretaceous below glacial drift, the Cretaceous being alternating shales and sandstones of the St. Mary and the Fox Hills formations. The measurements are made with a Racom equipment by the Swedish American Prospecting Corporation. As seen from Figure 20, in electrical vertical drilling by the Racom method, formation boundaries are represented by peaks in the ratio curves. Plotting the data as done in Figure 35, that is, ratios higher than one, on the right, and ratios lower than one, on the left, a comparison with Figure 20 shows that a peak on the right means a poor conductor below that peak, and a peak on the left means a good conductor below the peak. Thus it is readily seen that the sandstones are poorer conductors than the shales, which is in accordance with an observation which was

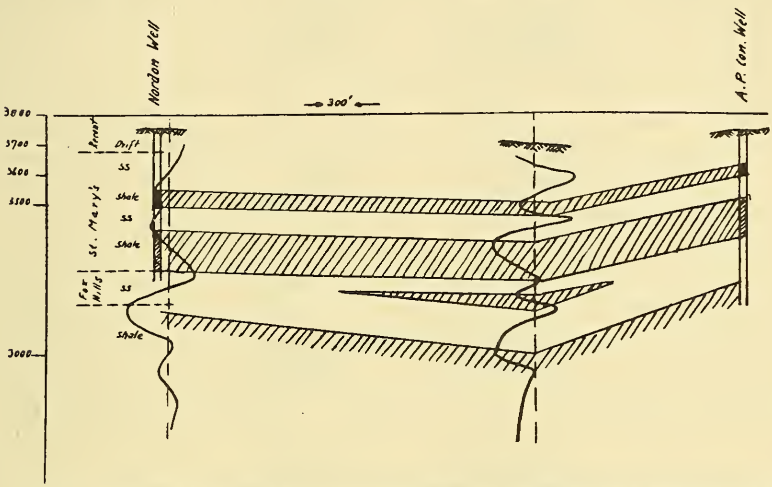

FIG. 35.—Results of electrical vertical drilling and corresponding geologic section at Spring Coulee, Alberta, Canada. Potential-drop-ratio method was used. (From pamphlet of Swedish American Prospecting Corporation.)

previously made when discussing the results represented in Figure 31. It is also seen from Figure 35 how well the electrical results check the well data and how the electrical vertical drilling by means of the potential-drop-ratio method may be used to advantage in carrying the structural data obtained from wells into unknown territory.

3. RESISTIVITY CORRELATION IN WELLS

In discussing the results which have been obtained on structure with the resistivity and potential-drop-ratio methods the experience obtained by measuring resistivities in wells should by no means be overlooked. In the first place, the results obtained by determining the resistivities *in situ* at the depths reached by the drill, give very important information for the interpretation of resistivity measurements made at the surface. Secondly, the systematic "electrical coring" makes it possible to correlate wells by their "resistivity log" in much the same manner as it is possible by the sample log, and gives

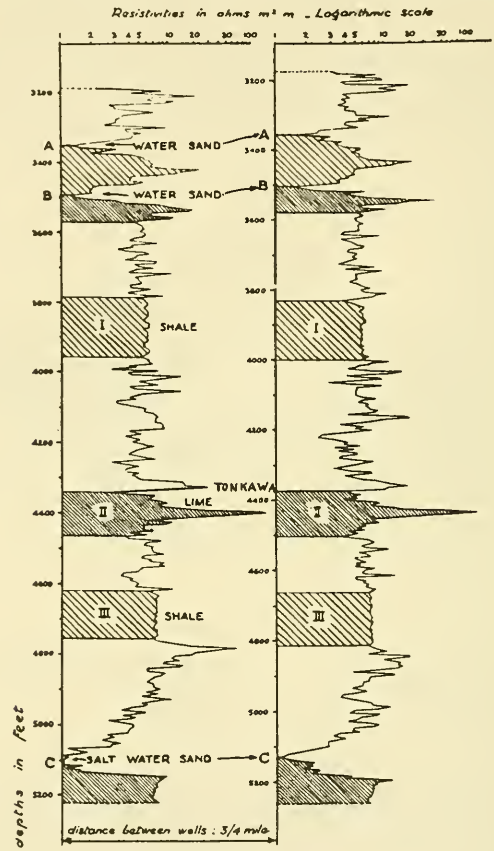

FIG. 36.—Electrical correlations between two wells, Oklahoma City, 1929. (Waters No. 3 and Mackey No. 1.)

not only a possibility for structural correlations, but at times enables one to recognize beds which may otherwise be overlooked.

Figure 36 represents two resistivity logs obtained with the Schlumberger method of electrical coring in two wells ¾ mile apart near Oklahoma City. In this figure, the resistivities are plotted in logarithmic scale. It is seen that the two curves are very similar in almost every minute detail. All other formations except the shales exhibit rather irregular characteristics which makes it possible to correlate definitely the shale horizons in the two wells. Water sands are characterized by very low resistivity; the resistivity of the water sand at the bottom of the holes is almost nil. The Tonkawa limestone shows by a pronounced and characteristic resistivity peak.

II. DIRECT LOCATION OF OIL

There are two possibilities of locating oil directly or of distinguishing oil-bearing formations from barren strata: first, by resistivity measurements in wells; and second, by resistivity measurement at the surface. The former method has proved its merit almost everywhere it has been applied, but the second possibility has not yet been established as a commercial method, and only with exceptionally favorable conditions have direct indications been obtained from oil-bearing formations where they were known to exist at shallow depths.

The possibility of obtaining a direct indication from an oil-bearing horizon in a well does not only mean the possibility of tracing this same oil formation in another well; but, as experience has shown, it is possible to determine where this oil formation becomes barren or changes into salt water in other wells (which is of practical importance in deciding the depth of water shut-off), as well as how thick and how prolific an oil sand is, the information on the productivity being a valuable guide which can not be obtained from the drill record.

1. RESISTIVITY MEASUREMENTS IN WELLS

Figure 37 illustrates without further comment how much more resistant oil-producing formations are than non-productive formations. This figure is an example of a resistivity log taken in the Seminole area, the two sands being saturated with heavy oil. It is seen that the resistivities measured are much higher for oil-bearing beds than for highly resistant, compact sedimentary formations. In case of doubt the ambiguity can be eliminated by simultaneous records of porosities, the technique of which is described in Schlumberger's paper on electrical coring (ref. list I_{11}).

Figure 38, representing several electrical logs taken in the Maracaibo field, not only illustrates very well the possibility of resistivity correlations of the oil beds directly in adjacent wells, but also shows how the resistivity of an oil bed changes if it passes from the productive to the non-productive stage. The upper tar sand and the first productive horizon both are seen to decline rapidly in resistivity toward the edge of the productive zone.

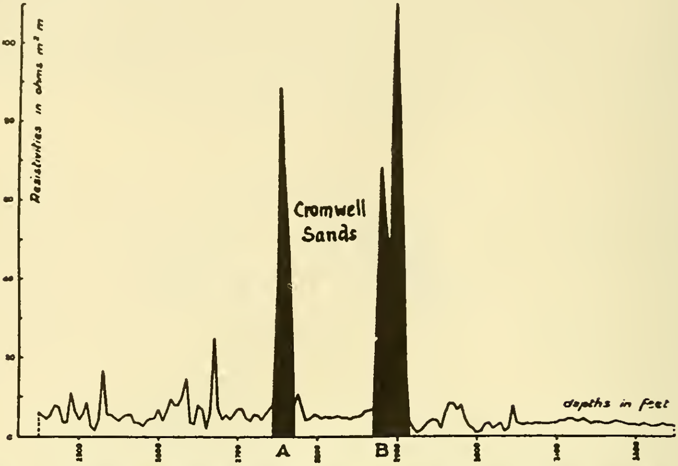

FIG. 37.—High resistivity of two producing sands, Wewoka field, Seminole area, Oklahoma, 1929. (Peney Reed No. 1.) (After Schlumberger.)

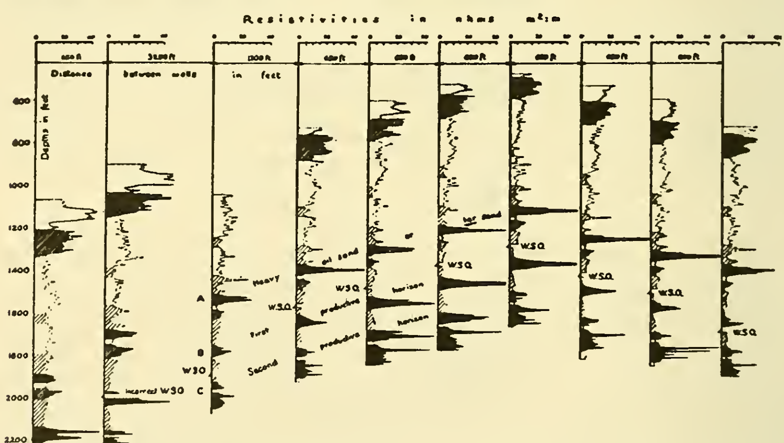

FIG. 38.—Electrical correlations between wells in the Maracaibo field, Venezuela. (After Schlumberger.)

2. ELECTRICAL VERTICAL DRILLING IN DIRECT PROSPECTING FOR OIL

If it is possible to recognize oil-bearing beds in wells in such indisputable manner, is it not possible to get the same or similar results by measurements from the surface? The results obtained thus far in approaching a solution of this problem will now be discussed.

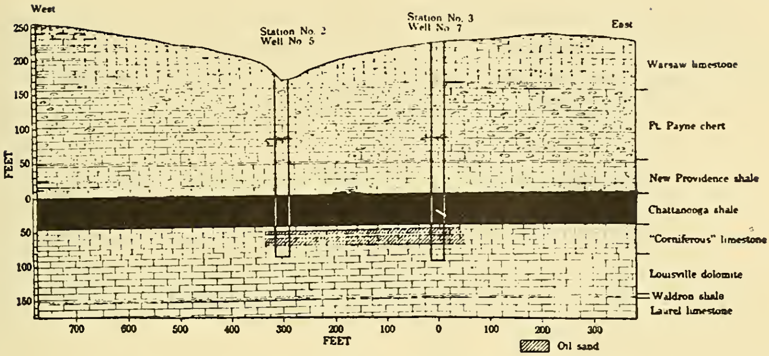

FIG. 39 *a*.—Geological section in Allen County, Kentucky.

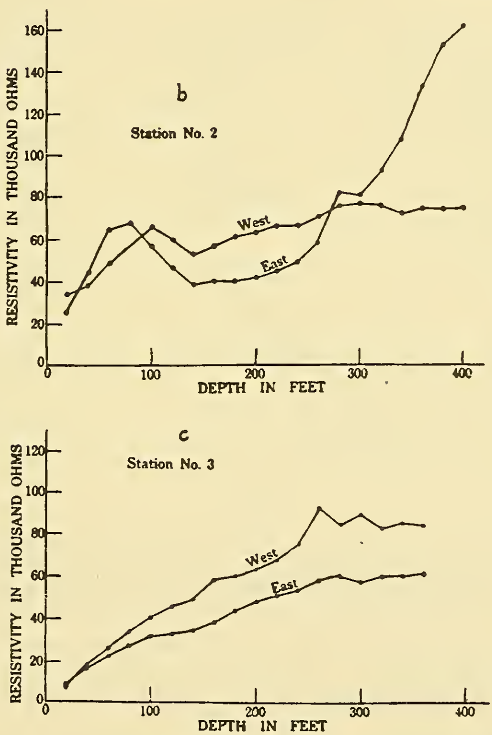

FIG. 39 *b* and *c*.—Resistivity traverses (partition method) at same locality (after Lee and Swartz), showing differences of resistivity between east and west at station 2 and station 3.

The writer knows of only two[1] instances where it is claimed that it was possible to recognize a direct effect of the oil in the resistivity data (in this paper we shall not deal with the claims which have been

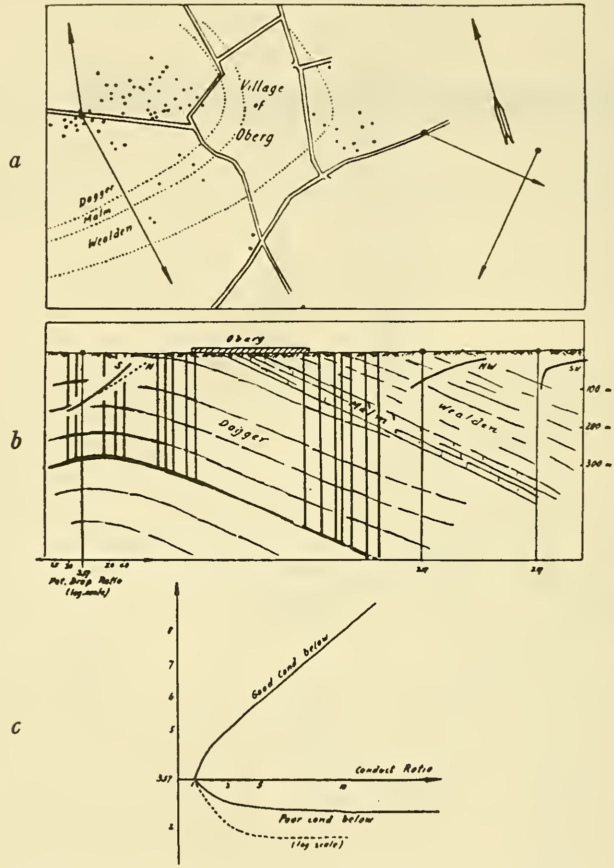

FIG. 40 *a* and *b*.—Results of some potential-drop-ratio measurements, and corresponding geologic section, in Oberg oil field, Germany. (Plotted after data published by J. Koenigsberger.) *c:* Potential-drop ratios as function of conductivity ratio.

made in that respect for the electromagnetic Elbof method). The first of the two instances in question has been published by Lee and Swartz, the second by Koenigsberger (ref. list Nos. III_{19}, III_{37}, and IV_2).

[1] As a possible third instance may be mentioned the measurements carried out by Swartz and others to determine the depth of rock asphalt in very shallow depth (ref. list No. III_{29}, p. 114). The observations were made in the same area in which indications from oil-bearing strata in 5–800 feet depth, to be described later, were obtained.

Koenigsberger has published results obtained along three potential-drop-ratio traverses in the Oberg field in Germany, of which only the ones measured on the extreme west and the ones measured on the extreme east side of the field lend themselves to graphical representation. The location of these traverses is shown in Figure 40 *a*. By designating with Koenigsberger the interval between the power electrodes by $2a$, results are available in this author's publications for intervals a equal to 100, 200, and 300 meters for the southern part, and for the intervals of 50, 100, and 200 meters for the northern part of the traverse on the west side of the field, while the distances used were 50, 100, and 200 meters in both traverses on the east side of the oil field. The productive zone is located on the west side of the area, and the oil is taken from a depth of approximately 300 meters from strata of the lower part of the middle Jurassic, while in the central part of this area the productive zone is nearly 500 meters in depth. At the locality at which the eastern traverses were made, the lower Dogger strata presumably contain salt water instead of oil.

From the foregoing figures for the distances used, it is seen that potential-drop ratios are given only for four (or three, respectively) depths in the western and for three depths in either traverse on the east. Koenigsberger states in his publication that the depth corresponding with the potential-drop-ratio indications is $h = \frac{1}{2} a$, and the observed potential-drop-ratio values have been plotted in the middle part of Figure 40 in accordance with this depth rule. As for normal ground the ratio is 3.57, values greater than this ratio have been plotted at the right, values smaller than this value were plotted at the left. As seen from Figure 40 *c*, the indications obtained from poor conductors are not nearly as pronounced as the indications from good conductors below; in order to equalize the indications somewhat, the logarithmic scale was employed to draw the curves shown in Figure 40 *b*.

Koenigsberger, in his article, now makes the statement that "the influence of insulating formations is quite evident from the fact that the ratio drops below 3.6 in the traverse on the west side."

The writer does not see how the observed values can possibly be correlated with the productive area, as the electrode spacing used was in all cases too small to penetrate down to the oil-bearing formations. On the other hand, it is true that some oil occurs in this field above the productive zone in the strata of the lower Dogger (that is, in depths less than 300 meters) and it is also possible that the diffusion of gas from the productive layers into the upper strata, which has been previously mentioned, has replaced all salt water and has made the strata poor conductors. At what depth this zone begins would be difficult to state as the curves are not complete enough to draw such conclusions. If this assumption is correct, it follows that the electrical characteristics of the Wealden can not be much different from those of the Dogger, as the curve observed in the northwest profile in the east is not much different from the curve observed in the west traverse. It actually appears from geologic reports that the Wealden is also oil bearing in this area and contains a heavy oil which is not exploited, and it is further seen from the results that, if such an oil horizon exists below the locality of the northwest part of the east profile, this horizon is at shallower depth at this point than it is in the west. The southwest part of the east traverse does not show an influence of poor conductors at the depths reached.

As a whole, the data published by Koenigsberger appear to be far too meager to permit very far-reaching conclusions. They do not seem to indicate that it would be possible with his method to differentiate a poorly conductive sedimentary stratum devoid of oil, from a stratum which is saturated with oil. It is possible that Koenigsberger has used greater electrode separations and thus greater depths of penetration in addition to those for which results have been published, and it is possible that the results obtained at greater depth substantiate his statements. However, the published data do not corroborate them, and the writer doubts very much that, if the illustrated results had been obtained in areas in which the presence of oil was not known, anybody would have seen anything unusual about them.

The second instance where a direct location of oil by a resistivity method has been described offers much more concrete information. The results in question were obtained on a site in Kentucky, the geology of which is shown in Figure 39 *a*. Lee's partition method was used, and the curves obtained when going to either west or east of both stations 2 and 3 are illustrated in Figure 39 *b*. The depth of the oil zone from the surface was in this place, however, much less than the depth of the oil zone on which Koenigsberger did his work; it was only 250–300 feet, while the depth of the productive oil formations in Koenigsberger's experiments was about three times as great. The oil zone occurs in the form of an oil sand between wells 2 and 3. Actually, east from station 2, the observed resistivity values were greater than those on the west, and the converse holds for station 3;

toward the east, the resistivities become smaller than the values obtained toward the west.

This is the only place at which, to the writer's knowledge, it has been definitely established that the oil can be located directly by surface resistivity measurements.

After this article was written, Swartz (ref. list No. III_{37}) published the results of further resistivity measurements in Kentucky. In the recent observations, the depth of the oil-bearing sands was much greater than in the determinations discussed above; it varied between 500 and 800 feet. It was found that generally the resistivity curves rose where oil and gas were present and fell where dry territory was encountered. One case is cited where predictions as to the occurrence of hydrocarbons made on the basis of resistivity determinations were checked later by drilling with striking accuracy. Furthermore, in the area of the Legrande pool, the boundary of the dry and producing area could be outlined from the resistivity data and checked the information obtained from wells. The striking fact about this boundary is that it is not marked by edge water, but by a decrease in porosity with corresponding decrease in the oil and gas content: another fact worth mentioning is that the best indications were obtained from a gas sand only 4 feet thick in a depth of about 640 feet; that is, the sand detected was only 0.6 per cent of the depth.

The author concludes that "for the partitioning method the shielding effect of such low resistivity beds as are encountered in oil and gas fields are negligible as far as the detectability of underlying gas and oil horizons is concerned," and continues as follows: "It is feasible to locate oil and gas zones, as well as salt water zones, *directly* by resistivity measurements on the surface of the ground. Whether such beds can be detected under all conditions and at all depths is, of course, not at present known. So far it is safe to say only that for depths up to 1,000 feet and under the physical and stratigraphic conditions here obtaining, such beds may be determined with excellent accuracy."

G. Summary and Conclusions

Resistivity and seismic refraction methods have several physical characteristics in common. They both involve the advantage of controlling the depth of penetration and thus a greater definiteness of interpretation than obtained in other geophysical methods. On the other hand, they are largely subject to the same limitations as other methods, namely, that the depth of geologic bodies must compare favorably with their size or thickness. The resistivity methods have been applied for some time in oil work, but interest in them has been aroused of late due to the perfection of technique which makes it possible to obtain a type of indication which may be interpreted more readily (the potential-drop-ratio method).

Although the determination of actual resistivities of samples is not a factor as important in resistivity work as the determination of related physical properties in other geophysical methods, remarkable progress has been made in the design of apparatus which permits the determination of resistivities on samples in the field and in the laboratory, and on outcrops of geologic formations.

The requirements on instruments and technique are not excessive in resistivity and potential-drop-ratio work; for moderate depth of penetration, moderate power sources are sufficient. The equipment for the determination of resistivities and potential-drop ratios is readily portable and easily operated. In both resistivity and potential-drop-ratio work, two distinct fields of application may be distinguished; first, the method of resistivity mapping, and second, the method of electrical vertical drilling. For the former, the 4-terminal Gish-Rooney equipment with one or two fixed electrode separations is usually employed, while for the electrical vertical drilling any one of the methods in resistivity work, and three potential electrodes near one power electrode in potential-drop-ratio work, is employed.

The results obtained thus far indicate that it is possible to locate readily geologic structure with the method of equi-resistivity mapping at not too great depth, and that detailed information may be obtained at any one point by the method of electrical vertical drilling with a fairly great degree of accuracy down to a depth of 1,500 feet with the present state of the technique, under favorable circumstances.

While remarkable progress has been made in identifying and correlating indications due to oil beds in wells, the location of oil directly by measurements from the surface is still in its very early stages of development, and there is only one case where the direct location of oil in very shallow depth from the surface by electrical measurement has been definitely established.

The resistivity methods depend probably more than any other geophysical method on a very good correlation of the geophysical data with known geological results, and their chief application is given where the problem is to extend the structural information in a known area into adjacent territory.

The chief obstacle which now faces the direct electrical location of oil is that, with our present technique, we can not detect a perfect insulator any better than a stratum, the conductivity of which is only approximately 10 times smaller than that of the adjacent beds. However, there is no reason why in the near future the methods can not be perfected to such an extent that, when a resistivity indication has been definitely established to be due to an oil bed in a certain depth, this indication can be traced into adjacent territory with the object of determining whether or not the oil is replaced by salt water. It is doubtful, however, whether or not the reverse will be accomplished so soon, as it is difficult, *at the surface*, to distinguish a dry resistant formation from an oil-bearing bed. In problems of the type just discussed, that is, in tracing a formation with the object of determining whether its contents change from oil to salt water, the potentialities of the resistivity and potential-drop-ratio methods will probably be utilized to the greatest advantage when the resistivity indications are combined with results obtained from other geophysical methods which would give the structural data alone, such as the seismic reflection method.

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USE OF GEOELECTRIC METHODS IN SEARCH FOR OIL[1]

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ABSTRACT

Factors which have adversely influenced opinions and general impressions regarding the value of geolectric methods as aids in the search for oil are thought by the author to be in considerable part of subjective rather than objective origin. However, geoelectric methods entered the field relatively late and with inadequate theoretical equipment for the best interpretation of results. Furthermore, sources of error which doubtless vitiated results have been in some cases overlooked because these did not come into serious account in the previous experience of exploration for ores. The principal sources of error for the resistivity method are pointed out. Although electromagnetic and resistivity methods have shown distinct promise in this field, it is, in the author's opinion, not possible with the data now available to make a reliable comparison, in terms of potential results per dollar, between these and gravimetric or seismic methods.

Published pronouncements on the success of geophysical methods as aids to the geologist in the search for oil have been numerous and various. It is not intended here to add to the number and it would be quite impossible to add to the variety. A careful quantitative appraisal of any one method would require the consideration of many factors and would accordingly call for a statistical analysis of a great mass of data. If all the data now resting in the files of various organizations were pooled, they would probably be adequate for such a study, at least in the case of those methods which have been used most extensively. Unless and until such a study can be made, a considerable diversity of opinion, as to the degree of success of a given method under the various conditions thus far encountered, may be expected to continue. Such studies of individual methods would constitute an important preparation for the much more difficult task of evaluating the relative merits and determining the particular province of each of the different methods. And then there would remain for consideration the advantage which accrues from the joint use of two or more methods.

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Statistical studies of these questions may seem so laborious as to be uneconomical. However, the variables which are not under man's control are here so numerous that the statistical along with all other means of attack must be used. It is feared that the importance of considering geophysical problems and data from a statistical viewpoint is not fully appreciated.

The physicist coming from his problems in the laboratory where accessory factors are largely under his control, on his early encounters with geophysical problems is likely to be either over-optimistic or pessimistic, according to temperament. Although his technique and thorough knowledge of the underlying principles of method and apparatus are needed, yet he is likely to be handicapped at the outset by a lack of that "statistical sense" which is so necessary in the successful geophysicist and which is generally acquired only with time and experience. Unfortunately, some persons, otherwise well qualified, seem unable to acquire this sense or judgment and as a result continue for years to harass editors and to clutter up the literature of geophysical science with pet theories and general conclusions based on inadequate data or even trivial manifestations.

One in whom this statistical sense is lacking is likely either to extol or to condemn a method after too brief a trial. In either case he may be unaware not only of the many phenomena of the earth which act as disturbing influences, but also of the fact that many suitably distributed data, or samples, are required under such circumstances before a reliable interpretation can be made or a decision either for or against a method is justifiable. Both experience and temperament go to determine the extent to which a judicious balance may be maintained between optimism and pessimism when evaluating methods or interpreting results.

The element of uncertainty in the final interpretation of a geophysical survey must be not only recognized but also admitted by the geologist and the geophysicist who share jointly the responsibility for the interpretation. The enthusiast can do real harm by leading the patron to expect too much of geophysical methods and thus add to the damage already wrought by self-styled "geophysicists" of either the unscrupulous or the incompetent variety. This probably applies with especial force in the case of geoelectric methods. The quack and the shyster seem to have a strong predilection for electrical vestments.

Another unfortunate circumstance is that electrical trappings are, in the minds of many laymen, endowed with mystical power. No doubt everyone who has worked with geoelectric methods has been amused by the curious spectator who loses all interest when he finds that the apparatus in use is not provided with a pointer and scale to indicate the number of barrels of oil which may be obtained from a well sunk at a particular spot. But it is not only the curious spectator who entertains this attitude. In recent years some hard-headed business men in an eastern oil field patronized a self-styled geophysicist who with a voltmeter and a few vacuum tubes connected in a meaningless circuit professed to perform this miraculous feat. The vacuum tube is the superb present-day garnishment for the doodle-bug and doubtless whets the appetite of those who have a taste for this morsel.

The bonafide geophysicist will, of course, not make such pretentious claims. He will recognize in his task a parallel to that of the weather-forecaster and will not shrink from using that qualifying word, "probable." He will speak only of indications, good, bad, or indifferent. These should be among the passwords, without which an entrance into the confidence of prospective patrons can not be won.

Although some geoelectric methods may under favorable conditions detect oil directly because of its high insulating property, geo-electric as well as other methods are efficacious in the location of oil only to the extent that they disclose subsurface structural features which in turn may indicate oil-bearing structure. That geoelectric methods are capable of disclosing hidden structure had been definitely shown before these methods were tried in the oil field, and since then a number of cases in which geoelectric surveys disclosed oil-bearing structure have been reported. Some of these are mere statements and have to be accepted on faith, but in several reports data are published which seem definitely to support the claims.

It is, of course, obvious that structure will be revealed by any geophysical method only when that property of the structure upon which the method depends presents a certain degree of contrast to the surroundings. Thus two different methods, as, for example, the gravimetric and the electric, may reveal entirely distinct structures, or one method may give indications of structure while the other does not. It must not be assumed, however, that the method which gives no indication is not of positive value, for if the information which it supplies is considered along with that obtained by the other method, then more definite conclusions as to the nature of the structure are likely to be justified.

The electrical conductivity, or the resistivity, of earth materials is the electrical property upon which those geoelectric methods which have shown some promise in the oil field depend. Methods employing radio waves would, of course, respond to differences in the dielectric constant and some entertain hopes for such methods. Perhaps, when the technique for sending directed beams of radio waves is sufficiently developed some success will be enjoyed by these methods.

The resistivity of earth materials varies between great extremes. Thus of the materials which come into account in the oil field, salt water, at one extreme, may have a resistivity as low as 10 ohm-centimeters, whereas the value for the igneous bed rocks, the other extreme, may run high in the millions of ohm-centimeters. Oil itself has very high resistivity, but to utilize this property for direct detection requires a relation between the extent of the pool and the depth from surface which probably occurs only rarely. Although some claims of direct detection have been made, the evidence presented is not conclusive.

The diversity in the resistivities of earth materials is far greater than that of any other property upon which geophysical methods depend. This is, however, not the advantage that it may at first thought seem. One reason for this is that the condition for detecting structure does not continue to improve indefinitely as the contrast in resistivity increases, but a limit is soon approached where "enough is as good as a feast." Most of the benefit is obtained when the resistivities differ by a factor of 100. Another reason is that this great diversity increases the chance that inhomogeneities within a structure and even small local differences in constitution may act as disturbing factors. Of course, if the data are obtained in such a way as to constitute an adequate sampling, then by suitable treatment in which these disturbing effects are regarded as accidental errors of observation they can be largely smoothed out. Such disturbing effects are doubtless met with to some extent in all geophysical measurements. Their magnitude is a determining factor both as regards the number and the distribution of observations required in a survey in order that reliable indications of the more general features may be obtained. This is also a factor which plays a part in determining the optimum sensitivity of the measuring instruments. Thus, if the accidental errors of observation are about one-tenth the magnitude of the disturbing effects, the measurements are doubtless of adequate precision. Greater precision may make the work unduly tedious and lead to confusion. This does not apply to systematic errors of measurement. Advantage may accrue if these are of an order of lower magnitude than that of the accidental errors.

The various electric methods which have been in use for one or another phase of geophysical exploration may be classed as follows: radio methods, electromagnetic methods, equipotential methods, resistivity methods, and miscellaneous electric methods. It appears that of these the electromagnetic and the resistivity methods have enjoyed a measure of success in the exploration for oil. When this is taken along with the fact that, in comparison with gravimetric and seismic methods, these electric methods are late comers in this field, their present prospects seem good.

The introduction of electrical methods into the more difficult phases of exploratory work has doubtless been hindered by the tardy development of a quantitative theoretical basis for the interpretation of the electrical survey data. In the case of seismic and gravimetric methods such a basis had been fairly well developed through their years of use in the basic studies of geophysics. The theoretical problems presented by the electric methods are more complicated, but these have been attacked in recent years by several investigators and the results obtained are of considerable practical value. This is especially true of those investigations which apply to resistivity methods. The detailed solutions here are restricted to the simpler ideal cases, but these provide norms with which survey results may be compared. It is to be expected that calculations will be extended to include other typical cases and that in the near future methods to facilitate such calculations will also be developed.

From the theoretical considerations it appears that geophysical measurements alone will admit of a unique quantitative interpretation only in special cases. Of the several interpretations which would be consistent with the measurements, some may be eliminated as being incompatible with facts known to the geologist. The number of possible interpretations is likely to be further reduced if surveys by more than one method are made. Thus, by the consideration of other information along with the survey data, it seems possible to decide upon one as the most probable interpretation. When there is added to this the data from a single test hole, the structure of the region covered by the survey should be determinable in many cases.

Interpretation of some aspects of geoelectric surveys has often been made with the aid of empirical rules, for which no satisfactory theoretical support has yet been found. A few remarks on one of these, for which the writer bears some responsibility, may be of interest. In that form of development of the resistivity method where four electrodes are set in line at equal intervals, it was at the outset tentatively considered that the distance between adjacent electrodes was a rough measure of the depth of earth involved in the measurement. This rule seemed to do better service than was expected. Such checks as it was possible to make strengthened confidence in it, and even to the present time some geophysicists with extensive experience in geoelectric work continue to use it even though they are aware that the theoretical considerations of recent years give no general support to such a rule. The observational data now available to most workers are probably inadequate to determine whether there is a conflict here between fact and theory. The theory seems sound, except for the assumption that earth materials are isotropic as regards the property of resistivity. There seems to be considerable evidence that this assumption is not justified, but that the resistance to electric flow in a horizontal direction is different from that in a vertical direction. A more general theory, embracing eolotropic materials, is, because of its difficulty, not likely to be developed unless overwhelming evidence is adduced to show that the simpler theory is inadequate. Hence at present it would seem best to use the theory now developed as à guide in the interpretation of resistivity surveys.

One who is confronted with the task of selecting an electric method for use in the search for oil will find his chief difficulty in making a choice between electromagnetic and resistivity methods. If he has had experience with one of these, he will probably do well to select that method, for in "the present state of the art" success depends to a considerable extent upon familiarity with the method and apparatus, and this is not gained in a day. The remarks which follow may help to elucidate this and, at the same time, call attention to some systematic errors which have not always been avoided in resistivity measurements.

In 1922 the writer gave consideration to various possibilities of obtaining a measure of the resistivity of large masses of undisturbed earth. Such data were desired in connection with investigations of the natural electric currents in the earth which were then being initiated in the Department of Terrestrial Magnetism of the Carnegie Institution of Washington. The method of measurement which was finally decided upon should be regarded as a direct-current method.[1] It was thought that with this method it should be easier to carry out surveys, especially when great depths of earth are to be included in the measurements, and that the interpretation should be simpler than for a number of other possibilities which were examined. This method is an improvement and extension of that developed by McCullom[2] and used by him in electrolysis surveys and which was in turn based on an alternating-current method suggested by Wenner.[3] The first preliminary survey, made in the early autumn of 1924 with the apparatus designed for large-scale measurements, showed that the method could be used for determining and locating structure under certain circumstances. Following the publication of a report on this work, the method was adopted in some form by several organizations which had previously made extensive use of equipotential methods. In response to requests in recent years, working drawings and other information required for the duplication of the apparatus have been supplied by the Department of Terrestrial Magnetism to a number of responsible individuals and research organizations. It appears from reports, published by some of these, that a few of the more essential features of the apparatus have at times been overlooked or misunderstood. The fact that this occurred, even where the work was in charge of capable physicists, indicates the need of the additional emphasis which a restatement of these features may give.

There are situations where the detection of the desired structure is so simple that relatively crude apparatus will suffice. However, when the exploration is to extend to greater depths, in the hundreds and into the thousands of feet, and under conditions which are made difficult by the relationship between the conducting and non-conducting strata, success is dependent upon careful attention to the following matters: (1) the polarization, contact potentials, and earth potentials of various origin of the pick-up electrodes must be largely eliminated; (2) the effects of induction between the two circuits must be made negligibly small; and (3) the insulation between the relatively high-potential energizing circuit and the low-potential pick-up

[1] *Terr. Mag.*, 30 (1925), pp. 161–88; *Bull. Nat. Res. Council*, 11, Pt. 2 (1926), pp. 86–91; *U. S. Patent No. 1, 813, 845* (1931).

[2] "Measurement of Earth Currents," *Elec. Ry. Jour.* (November 5, 1921).

[3] *Bull. Bur. Standards*, 12 (1916), pp. 469–78.

circuit must be maintained in a much better condition than is required in the more common electrical measurements or else some provision must be made in the design to obviate the effects of leakage currents.

In the apparatus developed at the Department of Terrestrial Magnetism by the writer, the contact potentials, etc., are eliminated by employing a double commutator which reverses the energizing current about twenty times per second and at the same time rectifies the pick-up current so that a direct-current potentiometer may be used in this part of the measurement. If this is to be satisfactorily accomplished, the commutator must be carefully made and properly adjusted. When the contact potentials, for example, are very large, 0.5 volt and upwards, the forced vibration of the pointer of the potentiometer-galvanometer becomes so great that settings are difficult. This difficulty is eliminated if a condenser of about 20 microfarads capacitance is thrown in series with one of the pick-up lines. The condenser should not be required when the contact potentials are much less than 0.5 volt unless the galvanometer has characteristics which make it unsuitable for this work or the adjustment or construction of the commutator is defective. If the latter be the cause, serious systematic errors may enter the measurements.

The effects of induction between the two circuits can be very serious, especially when measurements are being made with long lines. Some workers have attempted to correct for the effects by obtaining measurements at two different and known rates of commutation, but in the opinion of the writer this is not satisfactory either in theory or in practice. These induction effects may, however, be avoided if the potential between the pick-up points is measured only during that part of the cycle when the energizing current is constant. That this can be accomplished automatically if the commutator is designed and adjusted with this end in view has been demonstrated by many comparisons made by W. J. Rooney.[1] It is, of course, obvious that if the commutator in the present form should be used with lines many times the length of any employed heretofore in surveys, the time constant of the circuit may become so great that adjustment would no longer be possible. It should also be mentioned that an improperly adjusted commutator can falsify the potential measurements by effectively shunting the potentiometer during a part of the cycle.

Errors from defective insulation may at times be very great. During early experience in this work, errors of several hundred per cent

[1] *Terr. Mag.*, 35 (1930), pp. 61–72

were found on a few occasions. It is when these give rise to negative values in the calculated resistance that they are especially conspicuous. With some types of apparatus which have been used in resistivity surveys, such absurd results are not forced upon the attention of the observer, but the error is not less serious for his being unaware of it.

The errors which may arise from defective insulation on the field cables are nearly always additive. It is only for some unusual positions of the cables that they cause the measured values to be too small. However, in all our experience, errors from this source have been inappreciable except on a few occasions when old cables were used in rainy weather.

It is in the instruments that troublesome insulation defects may develop. The errors which result from these are in part dependent upon the relationships between the circuits in the instruments and upon the contact resistance of the electrodes. Since both these factors may be varied to some extent, this provides a means of detecting such errors. Four different circuit relationships are readily obtained by changing the connections of the field cables with the instruments. The values of resistivity obtained from measurements with these different combinations may all differ if leakage effects are present. Their mean is in general not free from leakage error as has been assumed by some. These errors are completely eliminated from this mean only when the contact resistances of the pick-up electrodes or those of the energizing electrodes are equal, and this holds only if the condition of the insulation remains constant during the series of measurements.

The uncertainty and inconvenience which attend attempts to eliminate these errors by such means can, however, be avoided by making suitable provisions in the apparatus. The device which has been adopted for this purpose is similar in conception to the "guard ring" which is often used, especially in instruments employed for measuring very small quantities of electricity. This consists essentially of an independent system of conductors so arranged as to intercept all possible paths by which current may leak from the energizing circuit to the pick-up circuit. If this guard system is connected to earth at a relatively neutral point, its potential is maintained at a value which differs so little from that of the pick-up circuit that this serves to reduce such errors to several orders' lower magnitude than those which would result if this device were not employed. In thousands of tests made by W. J. Rooney under a wide variety of conditions, no evidence of errors from this source has ever been found when the "guard" was in use.

The foregoing rather technical remarks apply when a commutator is used in the resistivity measurements. Resistivity surveys have, however, been carried out without employing a commutator but by using, instead, simple hand-operated switches. The errors which may enter into measurements made by such a method include those which have been discussed for the case where a commutator is used. Of these, that type which arises from the various extraneous potentials which act upon the pick-up electrodes is likely to be much more serious with this method than with the commutator method. This is partly because, even with the best technique, the time required for a single setting of the potentiometer and the interval between readings for reversed current are both so long in comparison with the corresponding cycle in the commutator method that the error in the result is likely to be considerably greater on this account alone. On the other hand, a single reading obtained by the use of a commutator is effectively a mean taken over a considerable number of cycles and on this account is of enhanced accuracy in cases where errors of a random nature may be appreciable. In the simple switching method, errors due to defective insulation may be controlled by the design and arrangement of the switches, whereas induction effects will depend upon the technique of measurement. As far as the last two sources are concerned, the results obtained with the simple method should be better than those obtained with an unsuitable commutator. The writer is, however, of the opinion that if the possibilities of the direct-current resistivity method are to be fully realized, suitable mechanical means of rapid commutation are required.

The search for oil often requires exploration to greater depths than was required in the earlier applications of geoelectric methods, namely, in the search for ores. Views as to the capability of geoelectric methods for such deep exploration vary, whether these be based upon theory or practice. Obviously the details of structure will become less distinct the greater the depth from the surface. Thus, a feature of the hidden structure can be detected at the surface only when it is of sufficient extent. The necessary relation between depth and extent is dependent upon the accuracy of the measurements and upon the magnitude of the uneliminated influences of the disturbing features of the earth. The accuracy of the measurements can be controlled by suitable design and operation of the measuring apparatus. The errors arising from disturbing features of the earth, such as topography and inhomogeneities within structure, are to a considerable extent dependent upon the number and the distribution of measurements and upon the method by which these data are treated. In order that corrections may be made for the more general features of topography, a further development of theory is required.

Although there is some evidence that features of structure located between 2,000 and 4,000 feet below the surface have been disclosed by resistivity surveys, this is by no means definitely established. General theory, as the writer understands it, does not exclude such a possibility. In fact, the likelihood of exploring to far greater depths seems so good to some of us that plans are being developed for a large-scale coöperative undertaking in which it is hoped to obtain by resistivity measurements information about the structure of the earth's crust down to a depth of about 50 miles. The results to be obtained from such a venture are, of course, not expected to be of practical value, except perhaps indirectly, but they should have important bearing on problems in several branches of the basic science.

The principal points which it has been attempted to bring out in this article may be briefly stated as follows. Although evidence is at hand showing that some geoelectric methods are aids in the search for oil, yet it can not be determined at this time whether the results per dollar from these methods compare favorably with those obtained from gravimetric or seismic methods.

An attitude of disfavor toward geoelectric methods has perhaps in considerable part sprung from subjective rather than objective sources. The widespread belief in the near-omnipotence of electricity has brought forth many impotent geoelectrical schemes and has provided easy prey for these as well as for the outright faker. Of course, an unfavorable reaction followed. Furthermore, a bonafide geoelectric method can not qualify as the idol of these "believers."

Geoelectric methods also entered this field with the handicap of inadequate theoretical equipment and with practical experience in a class of problems not adapted to bring out some details of apparatus and method important in the more difficult work to be encountered here. These conditions doubtless added considerably to the cost and lessened the value of the results obtained from some of the geoelectric surveys. And withal there is the seniority of gravimetric and seismic methods to be considered.

In principle it seems entirely feasible in many cases to determine from the data of resistivity surveys, taken along with other informa tion which can be supplied by the geologist and the petroleum engineer, the approximate depth and the features of the more general structure associated with petroleum resources.

Reports of actual tests seem to bear this out, perhaps rather too well in some cases. This applies especially to some recent tests which came to attention[1] as this conclusion was being written. These consisted of the correlation of hundreds of drill-hole records with results of resistivity surveys made by five different parties in as many different oil fields in Russia. According to the abstracts, one of the articles contains the conclusion that "more tests are required to draw definite conclusions on the degree of usefulness" of this electric method. This leaves one uninformed as to whether or not any indication that the method may prove useful was found. However, there is no ambiguity in the four other conclusions, which run as follows: "This method is of great importance in geological interpretation of the subsoil and should be used on a large scale"; "whether the mechanical coring can entirely be substituted by this method or not can not be definitely decided at present." One in which 180 drill-holes were investigated concludes that this "must be recognized as one of the most useful and necessary methods of determining the correlations in the stratigraphical conditions of oil deposits." In what appears to be the most extensive of the five investigations 220 drill holes were covered. In the report of this it is concluded that "the great advantage of electrical coring is established." The full reports may restrict some of these conclusions more than is indicated in the abstracts.

It is perhaps utopian to wish that these investigations had also included surveys by the other outstanding geophysical methods. An investigation of such scope should have considerably lessened the element of guesswork which is involved in making a choice from the several available methods. The writer is not impelled to venture such a guess here; he is, however, convinced that some geoelectric methods will in due time be accepted as useful aids in the search for oil.

[1] *Geophys. Abstr., No. 37* (1932), pp. 450–53.

CORRELATION BETWEEN RADON AND HEAVY MINERAL CONTENT OF SOILS[1]

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ABSTRACT

The presence of radon in the soil cannot be used as a criterion for locating oil fields, since the source of the radon lies in the radioactive minerals of the soil and not in the petroleum at greater depths. Formation contacts could be located by a radon survey even though obscured by top soil and vegetation, because there is quite likely to be a difference in heavy mineral character and content of two different formations. Likewise faults may be indicated if they bring two quite different formations into juxtaposition, but there is always danger of misinterpreting variations in the radon content of a soil which may be due simply to local variations in the amount of radioactive minerals in one and the same formation.

INTRODUCTION

In recent years radioactive methods of geophysical prospecting have attracted some attention. Most prominent among these methods has been that of measuring the radon content of soils[4] and attempting to deduce from these results the nature of the geological substrata. Radon is one of the constituents of the soil gas and is the immediate decay product of the metallic element radium. The radon itself is radioactive and its radioactivity per unit of weight is several times that of radium, thus enabling extremely small quantities of the gas to be detected. It is claimed that a fault will be evidenced by a higher radon concentration in the overlying soil,[5] and that the radon content of the soil above an oil bearing formation is higher than elsewhere.[6] Since there were no data available as to the success or failure of these radioactive methods in this country, an extensive study was made of the radon content of soils in regions whose geology and lithology were

[1] Manuscript received, June 23, 1932.

[2] Geologist, Gulf Companies.

[3] Physicist, Gulf Research Laboratory.

[4] "Radon Content of Soil Gas," H. G. Botset and Paul Weaver, *Physics*, Vol. 2 (1932), pp. 376–85.

[5] F. Müller, "Radioaktivitälsmessungen als geophysikalische Aufschlussmethode," *Zeits. für Geophysik*, Vol. 3, No. 7 (1927), pp. 330–36.

[6] L. N. Bogoyavlensky, "Radiometric Exploration of Oil Deposits," *Bull. Inst. Practical Geophysics* (Leningrad), No. 3 (1927), pp. 113–24.

quite well known. The authors acknowledge their indebtedness to A. E. Ruark, of the University of Pittsburgh, and Paul Weaver, Houston, Texas, for valuable suggestions in the conduct of this work.

APPARATUS AND FIELD METHODS

The measurements were made in soils which were developed over sedimentary formations consisting predominantly of sandy clays with

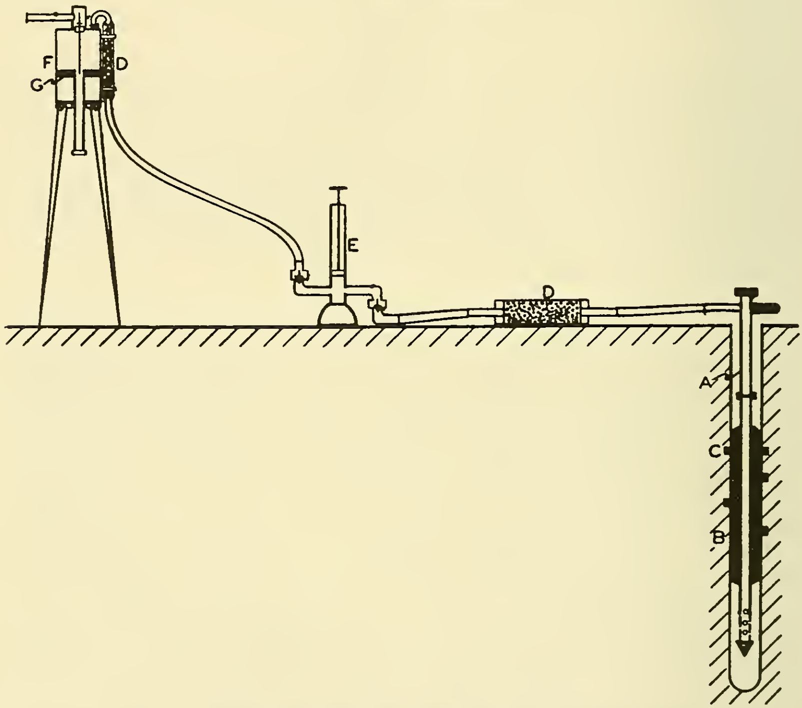

Fig. 1.—Diagrammatic section of radon measuring apparatus.

some slightly consolidated sands, all of Tertiary age. Most of the measurements were made in wooded areas, but some were in cultivated fields, and some in natural prairies.

The technique of making the measurements is a modification of the method developed by Ambronn.[1] The equipment used is shown diagrammatically in Figure 1. The hole into which the sampling tube is placed is made by first driving a steel rod into the ground about a foot and then continuing the hole with an earth auger. This is to

[1] R Ambronn, *Elements of Geophysics*, English translation by Cobb (1928), p. 119.

avoid compacting the soil around the hole and thus reducing its porosity, which would tend, in clayey soils, to make it more difficult to obtain a sample of soil gas. The sampling tube (*A*) consists of a 1¼-inch iron pipe, broken and coupled near the top to permit the insertion of nipples so that the tube can be extended when samples of soil gas from greater depths are desired. A wall packer (*B*) is welded on the outside of this tube about a foot from the bottom. The packer consists of a piece of tubing of the same diameter as the hole, around which an iron rod (*C*) of ¼-inch square cross section was wrapped spirally and welded in place. This rod cuts into the walls of the hole and forms a tight seal against leakage from the atmosphere. The gas sample is pumped through drying tubes (*D*) by the piston pump (*E*) into the ionization chamber (*F*) of the electroscope. The ionization chamber is a cylinder whose bottom is a movable piston (*G*). To empty the ionization chamber of gas the outlet cock on the top is opened and the piston pushed up. The outlet cock is then closed and the inlet opened. The gas sample being pumped in thus exerts an outward pressure on the system and any leakage occurring is outward. This prevents the dilution of the gas sample with atmospheric air. The minute traces of radon in the soil gas, by virtue of radioactive decay, produce an ionization of the gas in the ionization chamber and discharge the electrometer fiber which has been previously charged to a potential of about 200 volts. The rate of discharge is proportional to the amount of radon present in the gas. The sensitivity of the instrument is such that one part of radon can be detected in about 10^{12} parts of soil gas.

Several profiles were run across a known fault in the Balcones fault region. The measurements made in these profiles are shown in Figure 2. It is difficult to express the results in terms of quantity of radon per cubic centimeter of soil or per gram of soil mineral, because of the great variation in the porosity of the soil and because of another factor which may be even more variable, the so-called emanating power of the soil grains. Since the radon gas is the product of the decay of radium atoms, the radon must be formed on the surface of the soil grains or within the grains themselves. As the radon is a gas formed from a solid, only that part formed at the surface of the grains is able to get into the soil gas. The emanating power is the ratio of the amount of radon which gets into the soil gas to the total formed in the soil. This obviously depends, then, not only upon the size of the soil grains, but upon their shape (ratio of volume to surface) and upon

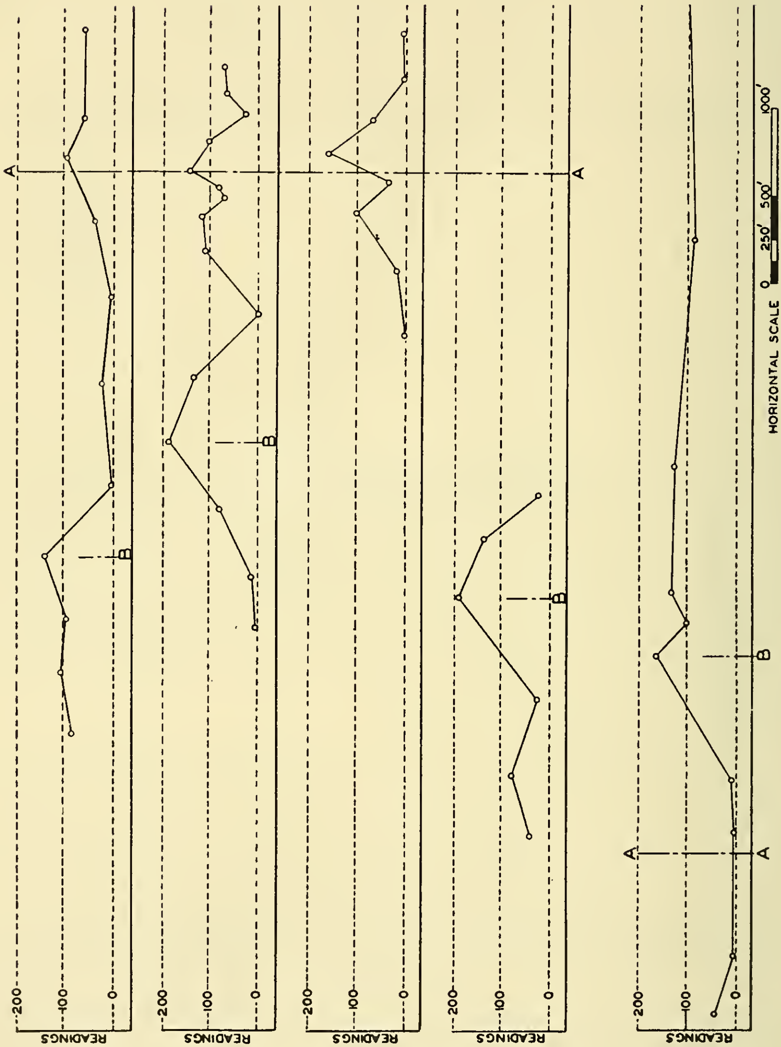

FIG. 2.—Radon profiles across a fault, *A*.

whether the parent radium has been deposited near the surface of the grain (secondary deposition), or whether it is distributed more or less uniformly throughout the grain. Naturally radium formed from uranium salts deposited chemically on the surface of the grains will have a large emanating power, while that formed in minerals in which the uranium was originally incorporated will have a lower emanating power. According to Satterly[1] the emanating power of a soil may vary from one-sixth to one-twentieth. Consequently, the ordinates of the curves presented here are expressed simply as electroscope readings, no account being taken of the variations in emanating power of different soils. These ordinates are divisions per second of movement of the electrometer fiber multiplied by 1,000. A calibration measurement on the electroscope made by using a known quantity of radon showed that an electroscope reading of 100, that is an actual rate of movement of the electrometer fiber of 0.1 division per second, was produced by $1{,}539 \times 10^{-12}$ curies of radon in the ionization chamber. The curie is the amount of radon in equilibrium with one gram of radium. If we assume a soil porosity of 33 per cent, $1{,}539 \times 10^{-12}$ curies in the electroscope would correspond to about 0.57×10^{-12} curies of radon per cubic centimeter of soil.

As will be seen from Figure 2, an attempt to locate the fault by radon measurements would be very misleading, since the radon values obtained at the points marked B, where there is no fault, are just as high as, or higher than, those obtained at A, where the fault is actually located.

In order further to study the radon content of soils, surveys were made over the South Liberty salt dome. This dome has at present very little elevation above the surrounding terrain. The center of the dome is crossed by a small river. The eastern edge of the dome is a producing oil field. It is claimed by Bogoyavlensky[2] that an oil field will be indicated by a change in the radioactivity of the soil. To test this statement a profile was run across the producing section of the dome, starting from the outside edge of production and extending toward the center of the dome some distance beyond the producing area. The result of this survey is shown in Figure 3. The holes from which samples were taken for these measurements were 5 feet deep. The higher radon concentration in the producing area is obvious. After these results were obtained the profile was extended eastward

[1] J. Satterly, *Proc. Cambridge Society*, Vol. 16 (1911–12), pp. 336, 356, 514.

[2] *Op. cit.*

off the dome for at least ½ mile, and the radon content was still practically as high as was found in the producing area. This seemed to indicate that the presence of oil had little to do with the radioactivity of the soil.

Knowing the association of radioactivity with heavy minerals,[1] it was decided to make another profile parallel to and near the first and to take samples of the soil from the bottom of each hole. These samples were taken to the laboratory for quantitative determination of their heavy mineral content.

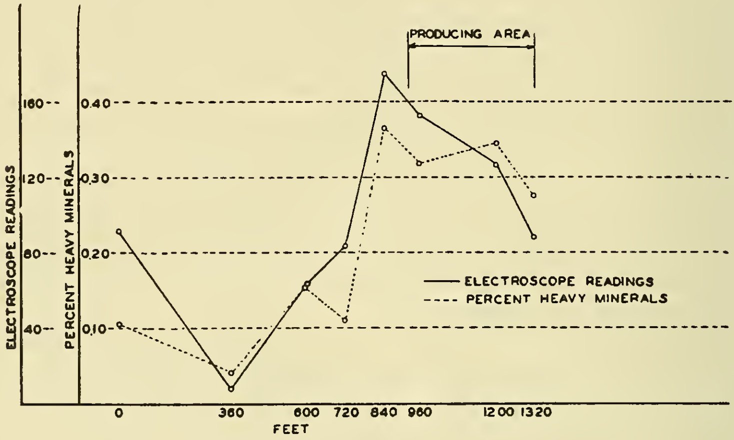

FIG. 3.—Radon profile across South Liberty salt dome.

HEAVY MINERAL SEPARATION

About 30 grams of each of these samples were ground to pass a 130-mesh sieve. The heavy minerals were first separated by using bromoform in a separatory funnel. This bromoform had a density, as determined by the pyknometer, of 2.5. It was found, however, that the heavy mineral fraction obtained by this separation contained quartz and calcite. Consequently, this heavy mineral fraction was again separated, Thoulet's solution (density 3.15) being used. This gave a very complete separation and was adopted as standard procedure.

[1] E. H. Büchner, *Jahrb. f. Radioakt.*, Vol. 10 (1913), p. 516.
W. Waters, *Phil. Mag.*, Vol. 19 (1910), p. 903.
R. H. Strutt, *Proc. Roy. Soc.*, Vol. 84 (1910), p. 377.
A. Gockel, *Die Radioaktivität von Boden und Quellen* (Braunschweig, 1914).
St. Meyer and Schweidler, *Radioaktivität* (Leipsic, 1927).

The following table taken from Gockel will indicate some of the heavy minerals associated with radioactivity.

TABLE I

Mineral	*Location*	*Radium per Gram of Mineral* $\times 10^{12}$ *Grams*
Zircon	Kimberley	19.1
Eudalite	Greenland	12.6
Orthite	Sweden	236.0
Gadolinite	Hitterö	156.0
Keilhanite	Alve, Norway	452.0
Niobite	Connecticut	97.0
Apatite	Canada	14.6
Cerite	Sweden	30.0

The average radium content of sedimentary rocks is about 1.4×10^{-12} grams[1] per gram of rock. Thus it is seen how considerably the radioactivity is concentrated in the heavy mineral constituents.

Petrographic examination of the heavy mineral residues from the South Liberty dome revealed the presence of zircon, rutile, brookite, tourmaline, magnetite, ilmenite, apatite, biotite, augite, and cyanite. Zircon, tourmaline, and magnetite are the most abundant in all the samples examined. If the radioactivity of the soil could be ascribed to the mineral composition of the soil itself rather than to any deep-seated source, zircon is the one mineral of all those observed that is the most likely to be radioactive. F. L. Hess[2] describes a considerable number of unusual rare-earth minerals occurring in some of the igneous rocks of the central mineral region of Texas. Since this area was probably the source of much of the material of the late Gulf Coast Tertiary, these minerals were undoubtedly present to some extent in the soils but not recognized in the heavy mineral residues examined. In these minerals are such rare-earth elements as thorium and uranium. Thorium is the parent of one series of radioactive substances and uranium is the parent of the radium series and probably also of the actinium series. Since these elements (uranium and thorium) are isomorphous with zirconium, it is only logical to look for a possible relationship between the radon content of the soil and the amount of zircon and other rare-earth minerals in the soil. Experimentally, only the total heavy mineral content of the soil was determined, since zircon was always present in large percentages in the residues and not all the possible sources of radioactivity could be differentiated and segregated.

[1] G. Kirsch, *Geologie und Radioaktivität* (Berlin, 1928), p. 46.

[2] "Minerals of the Rare-Earth Metals at Baringer Hill, Llano County, Texas." *U. S. Geol. Survey Bull. 340* (1907), pp. 286–94.

Figure 3 shows almost a parallelism between the radon content of the soil and its heavy mineral content. This is one of many profiles across the South Liberty dome, all of which are generally similar. Such an abundance of corroborative evidence leads one to conclude that the radon content of soils is directly related to and due to the presence of radioactive minerals in the soils.

CONCLUSIONS

The presence of radon in the soil cannot be used as a criterion for locating oil fields, since the source of the radon lies in the radioactive minerals of the soil and not in the petroleum at greater depths. Formation contacts could be located by a radon survey even though obscured by top soil and vegetation, because there is very likely to be a difference in heavy mineral character and content of two different formations. Likewise faults may be indicated if they bring two different formations into juxtaposition, but there is always danger of misinterpreting variations in the radon content of a soil which may be due simply to local variations in the amount of radio-active minerals in one and the same formation.

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THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS

GEOPHYSICS

1933

INCLUDING PAPERS PRESENTED BEFORE THE GEOPHYSICS DIVISION OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS IN ANNUAL CONVENTION AT HOUSTON, TEXAS, MARCH 24, 1933; REPRINTED FROM THE BULLETIN OF THE ASSOCIATION, VOL. 17, NUMBER 1 JANUARY, 1934

TRANSACTIONS

SOCIETY OF PETROLEUM GEOPHYSICISTS

VOL. IV

PUBLISHED BY

THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS

TULSA, OKLAHOMA, U. S. A.

Published, January, 1934

Composed and Printed by
George Banta Publishing Company
Menasha, Wisconsin, U. S. A.

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Symposium on Geophysics

FOREWORD

At the present time there are more than fifty reflection seismograph parties operating throughout the United States. The vast majority of this work is in Oklahoma and Texas. There is considerable gravimetric work being carried on, mainly in the Gulf Coast. While the magnetometer has lost ground compared with a few years ago, there are still a few parties in the field.

The total monthly outlay on reflection seismic operations must be more than a half million dollars, with every expectation that this expenditure will increase as the known reserves of oil decrease, and as the oil business becomes more profitable. The increased confidence in reflection seismic methods, both dip and correlation, will also contribute to this expanding program. In spite of several failures, some due to carelessness and others to lack of understanding of the limitations of the method, there are still some notable recent successes. The manner in which well spacing and townsite drilling was discussed at Tomball prior to the first well being spudded shows how great was the confidence of the oil fraternity in the work in this new area.

With the realization that this method was becoming successful and that the pioneer work was done, and with the desire to get their share of the juicy half million dollar plum, several consulting companies have been organized. Under competitive bidding, prices have been reduced to the point where there is barely an adequate return on the investment and certainly no money for research and development.

In a further effort to keep the program costs as low as possible, inadequate data have been frequently obtained, so that, even in the hands of a skilled seismologist, there has been insufficient material for a proper analysis. Small wonder then that he has made mistakes.

The foregoing statements are not so true of the magnetic and gravimetric work. Since the stations are in both cases appreciably cheaper than seismograph datums, it was easier to persuade the users of these methods to increase the density of observations.

If advances in the art are to be made by the consulting companies, they must have not only sufficient profits to invest in experimental work, but also capable personnel willing to carry on experimentation. Much work of the type reported on in this issue must be done so that in the future geophysics will be charged with fewer failures and more successes.

B. B. WEATHERBY

BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 18, NO. 1 (JANUARY, 1934), PP. 3-12

RELATIONS OF GEOPHYSICS TO GEOLOGY[1]

PAUL WEAVER[2]
Houston, Texas

ABSTRACT

The present aim of geophysical methods is to construct subsurface contour maps. Three principal methods, viz., measurement of gravity, of magnetic field, and of electric field, are forms of measurement of a potential function, and consequently have inherent limitations for construction of contour maps.

Another principal geophysical method is the seismic, which does not measure a potential function, and is inherently suitable for constructing contour maps.

Both types of methods have difficulties in constructing contours due to variations in physical character of rocks. Suggestions are made for coöperation of geology and geophysics to minimize these difficulties.

If we study current scientific literature, we observe that new phenomena are being discovered in all branches of physics by precise measurements aided by mathematical analyses, and that the tools and technique for applying these discoveries to commercial processes are being evolved at the greatest speed in history. Geophysics has kept pace in this general development of physics and attempts are being made to increase its applications to commercial processes in all directions. It is the purpose of this discussion to survey the commercial application of geophysics to the oil development problems and especially its relation to the similarly expanding efforts in geology.

In this rapid development of science, it is always necessary to examine critically each new process to be sure that it has a place in commercial work. Some of our advances are not yet necessary for the present state of commercial operation. We may illustrate in the following manner.

More than nine-tenths of the people in the world to-day probably think that the sun rises every morning in the east and sets every evening in the west. This interpretation of what they see is sufficient for their actions, and they need no other in their daily chores unless they be seafaring people who steer their own boat; in such a case they can determine their position at sea best by studying astronomy and learning that the earth moves, not the sun. Similarly, the average man in oil (including the geologist) will not learn the later developments in

[1] Read before the Association at the Houston meeting, March 24, 1933. Manuscript received, November 4, 1933. Presidential address, Society of Petroleum Geophysicists.

[2] Gulf Production Company.

physics, including geophysics, unless he needs them to solve a geological or a mechanical problem not readily understood by his former teaching and its interpretation of his former experience.

Geophysics, like geology and like chemistry, began with the finding of novelties. For example, the first chemists mixed together various combinations of substances and observed that, although usually the substances preserved their identity, and could subsequently be separated, in some cases, substances were changed by juxtaposition, and lost their identity, so that new names were necessary for the resultant new substances. Chemistry of twenty-five years ago was principally a memorizing of names of the substances which were formed by the juxtaposition, technically called a "reaction," of other substances. Now, however, chemistry not only foretells the products of a reaction, under different environments, but calculates numerically the rate at which the reaction is carried out and the heat evolved during the process.

In geology, both applied to mining and to oil, pioneers were looking for novel structures, such as anticlines and faults, which they had found to be present where known commercial deposits existed. Now, however, a change similar to that of chemistry has come in geology. In geology we now talk about the amount of closure on the surface beds of an anticline, the number of feet of throw and of hade of a fault.

Our latest development in geophysics has been an endeavor to give profiles and contour maps to the anomalies which we discover. Now this advance in geophysics requires very accurate measurements, which means a very careful calibration of the measuring device, especially in view of the fact that these measurements are carried on in the field and can not be exempt from the effects of variations in temperature and in other factors of operating environment.

Although further improvements are constantly being made, the instruments now available for geophysical measurements in most cases already have the precision necessary for the present state of the art.

It is also generally agreed that the technique of operation is advancing equally with the instrumental precision and is already being handled satisfactorily by the experienced operators.

The present high specialization in instruments and technique is gratifying, but does not insure that geophysics can successfully solve all the problems which it is now attempting. We must find out whether the problems are theoretically capable of solution, and I must assume in the discussion which follows, on the part of the geologists, a certain knowledge of mathematical theory, but I shall try to present analogies which will make the discussion nonmathematical as far as applications are concerned.

A geophysicist may be likened to a civil engineer who accompanies an explorer around the world. If the two are in a big city, such as New York, where there are many high buildings close together, the explorer may ask the engineer to calculate the height and capacity of a building on a crooked street where there is heavy traffic and where the building is sandwiched between others. Data for such calculations may comprise not only measurements along the street, but also measurements from some window of adjoining buildings, and the latter may be necessary for accurate results.

Now suppose the two pass across the country to the Colorado Rockies and the problem is to construct an aerial tramway from a mine. At the mine high on the mountain side, the engineer looks across a canyon to a sharp rock pinnacle which he can not occupy himself, and he wishes to determine the span from the mine to the pinnacle.

The explorer and the engineer continue westward in a car and cross the mountains to the edge of Death Valley. They have no map and the explorer wishes the engineer to determine the shortest distance across the Valley to a water hole.

These three problems are similar to problems which the geologists present to the geophysicists. The first problem is one where measurements can be taken a relatively short distance from the object to be measured, and where any point at which measurement is desired is accessible. There is some difficulty in determining vertical angles and in using the stadia on account of the crowded streets, but the problem can be worked out to any degree of accuracy.

The second problem is one where measurements are somewhat restricted, namely, to a certain position, but where clear atmosphere gives very great precision to such measurements as are made, and enables the engineer, with an accurate transit, to measure distance from a very short base line.

The third problem is the difficult one. As the engineer stands on the edge of the desert valley, instead of clear view to the opposite hills, he sees a mirage. If he is so fortunate as to obtain a vantage point from which part of the opposite range is clearly visible, he is too far to identify vegetation which might be characteristic of a water hole. In most cases, the best he can do is to indicate the nearest point of the mountains on the opposite side of the valley and can tell nothing as to whether they contain at that point the water which is necessary to the traveler. The desert interferes with accurate measurements of the distance across the valley, variations in temperatures and density of the air give rise to refraction, dust clouds decrease visibility, the party must continue because they have a limited time within which they must obtain water, and extended surveys are therefore not permitted. The engineer, however, must make some recommendation, as the party must go on, and this is the situation which is generally present in our geophysical problems of to-day.

The first successful geophysical application to commercial work was the determination of depth to the ocean bottom, and this problem is analogous to the measurement in the Colorado mountain because the water between the measuring device and the object whose position is sought by the measurements has only slight variation in physical properties, and a calibration chart for these variations can be readily made. There are some geophysical problems where these conditions also exist, but they are not the ones to which our greatest effort is being devoted at this time.

The principal geophysical methods which are being used are familiar to all of you. The measurements made by these methods are of the variation of gravity, of the variation of magnetic field strength, of the variation of electrical field strength, and of the variation in velocity of sound. They have been divided into methods which represent the measurement of a natural field and those which give the measurements of an artificial field, but in considering the relation of geophysics to geology, a different classification is necessary, and the geophysical methods may be divided into two classes, those which measure a potential function or some of its components, and those which determine a point.[3]

[3] The reader is referred for a complete discussion of a potential function to: W. D. MacMillan, *The Theory of the Potential* (McGraw-Hill Book Company, 1930). For the aid of those who are not at present interested further than in this present article, the following short summary may serve:

Bodies in space exert force upon other bodies at a distance. The amount of the force varies in some way with the distance, usually inversely as the square of the distance. Some kinds of force are attractive, some repellent. Now to define the potential, let us take one body, which we call A, at a fixed position. Let us move another body from a point *1* in space to a point *2*, and let this second body be so small that every part of it can be considered the same distance from A, so we will call it P, meaning a particle. It will require work to move it from *1* to *2* if they are at different distances from A. Then, the amount of this work is equal to the difference in the values of the potential function for *1* and for *2*.

The potential function is a number, since work is a number. It has no direction, but it changes in general from every point in space to the next one, and at different rates. We express this by saying that the potential function is a scalar function of position, and that its derivatives are the components of a vector.

Gravity, magnetic, and electric forces each vary inversely as the square of the distance, but differ in that the gravity force is always one of attraction, whereas the magnetic and electric may be either attractive or repellent. The potentials for these forces, therefore, "differ from one another only in the constant factor of proportionality

Gravity, magnetic, and electric geophysical methods are measurements of a potential function, but the seismic method is not. From the standpoint of geology and geophysics, measurements of the potential function and its components have the following implications.

1. Size (that is, mass) and distance cause most of the effect. The larger the mass, the greater the effect. The greater the distance, the less the effect.

2. All bodies have an effect, and this effect can become very small only if the body is very small, or very far away.

3. The shape of the body modifies the effect only slightly.

In reconnaissance geology, we wish to find the existence of large bodies, and so potential function methods are valuable in reconnaissance. In detail geology, we wish to construct a contour map of a comparatively small segment of a surface of a body, and therefore unless we can determine the shape of this segment very accurately, we can not construct accurate maps; thus, in this case the potential function methods are not easy to apply unless the part of the body which we wish to contour is very close to our observation point as compared with the rest of the body.

The seismic method is not a potential function method and it attempts to map contours on an underground bed, or beds, by determining surfaces which are the possible loci of the travel of a sound wave, the bed sought being the envelope of these loci. The seismic method is intended, therefore, to give a result which determines uniquely the depth to a bed, or beds, below the point of observation, and is probably the only method which gives theoretical possibilities of being the equivalent of core drilling; it has theoretical possibilities for constructing contour maps greater than those of core drilling, because the seismic method in order to determine the depth below a point of observation determines the amount and direction of slope at that point.

The statements made in the preceding paragraph are general, and, like all generalities, are wrong. In certain cases, measurements of the potential function can be used by the geologists for contouring because the number of possible distributions of matter which might give the measurements are limited by known facts regarding the geology. For example, a very strong gravitational minimum covering a limited area where sedimentary rocks are present to considerable depth must be caused by salt or by a large block of diatomaceous shale. There-

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which is associated with the potential; this factor of proportionality depending upon the forces under consideration and the system of units which is employed" (MacMillan, *op. cit.*, p. 283).

fore, the geologist can restrict the interpretation of such a sharp gravity minimum to a relatively simple picture of a salt dome, or an anticline, depending on the shape and sharpness of the minimum. Again, although the seismic method is supposed to give a unique solution, unknown variations in the formation traversed by the sound wave may give several alternative interpretations.

At the present time we are asking that the geophysical report for a salt dome shall include an estimate of the depth to the salt, the area of its cross section, and the existence of an overhang on some part of the periphery. When these additional questions are asked, the difficulty of obtaining the information by potential function methods becomes very great. We do not refer to the practical difficulty of making very accurate measurements, but to the theoretical difficulty, or what we shall call the resolving powers of the method. To refer to our desert analogy, as we look across the valley to the hills on the far side, we can appreciate that these hills have certain salients and alluvial fans, but we can not, even with a powerful telescope, see the minor topographic details because the air through which the light travels prevents obtaining a proper stereoscopic picture. We can see the same difficulty if we go up in an airplane on a day when there are no shadows. At slight elevations we can estimate visually the height of buildings, but at 10,000 feet, we can calculate the height of buildings only by photographs taken several hundred feet apart, and can do this accurately only when the photograph is very clear and there are no local hazy spots in the air between us and the ground.

The potential function methods, therefore, will always have a limit when we try to make profiles and contour maps of the top surface of buried structures because the resolving power of these methods is inadequate. Even where the structures which are being studied are shallow, this difficulty appears. For example, a single bed which is thin and a very good conductor can be mapped by electrical methods, but where there are three good conductors at fairly short intervals, even if all of them are shallow, the electrical problem becomes extremely difficult. Now the limit in resolving power of the potential function methods applies not only where there are a number of beds which we are trying to map, as in the electrical case just mentioned, but it applies also to the kind of surface which a single bed may have. For example, to distinguish between a fault and a short steep dip becomes an impossible problem in certain measurements.

There is a further difficulty in geophysical surveying by potential function methods. It is necessary to extend the surveys to a considerable distance because the rate of change of the values measured is important for the interpretation. In areas where there are a number of anomalies close together, it therefore results that the effect of each anomaly is obscured by effects from adjacent ones.

Potential function measurements can be made of numerical value where there is a large effect of a body of a known geometrically simple form near the points at which measurements are made. The successful contouring of the top of cap rock in some of the shallow salt domes by Barton is the most brilliant example of this method.

The geologist can aid very much in the interpretation of the potential function type of geophysical survey by obtaining as much information as possible regarding the type of structure to be expected in a particular area and by regional studies to indicate to the geophysicist the most probable locality for investigation for anomalies.

By the seismic method, it is possible theoretically to make accurate determinations from a few observations close together. This method, therefore, has an advantage over the potential function methods in requiring very much less areal extent to be investigated. However, in actual field surveys, the problem is not as simple as indicated by theory. Corresponding with the bending of light waves through the air over a desert, we have the bending of sound paths underground, due to irregular distribution of elasticity and density in sedimentary rocks. These variations are in part a result of depositional conditions and as such show a general stratification parallel to the bedding; and the geophysicist should call on the geologist for aid in studying these variations of sedimentation. But, in addition to such variations, there is some evidence gradually accumulating that uplift will effect changes in the elasticity and density sufficient to require consideration in the accurate determination of depths which are now being attempted with the reflection seismograph. This is one of the most important studies which geophysicists must make before contouring can be precise, and it is one where the aid of the geologist is especially important. At the present time we are assuming, for lack of complete data, that formations arched over a structure maintain the same physical constants as when they are in a normal position.

How can the geologists and the geophysicists set about finding the variations in physical character of the rock caused by structures of the type we are studying in the oil fields? In the first place, we can use wells which have already been drilled on other structures and study variations reported by the drillers. So-called concretionary zones in shales and gumbos, and indurated sand members, may be due to the uplift of beds containing solutions into an environment of less pressure and temperature, thereby causing precipitation. Folding and faulting may cause tension at some points, compression at others, and through long periods of time the physical constants of the rock change accordingly. We have practically no quantitative data on such changes as may occur in gentle folding and faulting although there is a considerable body of information regarding the effect of high pressures such as are found in structures in more intensely folded regions. A careful study of geophysical surveys, particularly reflection shooting and shooting in wells, from areas where the geology is known from a number of wells, and where samples are available from these wells for study of their physical characteristics, will contribute materially to the interpretation of results on new prospects. This study will also result in throwing much light on some of the tectonic questions which geologists have considered, such as the theory of elastic rebound. Correlation of geophysical results by reflection shooting in known oil fields with the geology should be the most obvious and simple test of geophysical results because this method is the only one generally used which is not a potential function method, and is therefore particularly easy to understand and to interpret from the theoretical side. It is, however, one which experience shows presents fully as many problems on the practical side as other methods, due to variations in the sedimentary rocks, but these variations are the ones where we find the geologists have already collected so much data that joint effort should give solutions to many of these practical problems.

In connection with the comparison of geophysical contour maps with geological contour maps based on a number of wells, there is a further caution necessary. Although reflection shooting is supposed to determine the slope and depth at a point, contours based on a number of such determinations are usually smoothed out and are so drawn as to give a most probable value to the contour. This is the proper procedure, if the determinations are supposed to be made on a warped surface, but if the horizon which is being contoured represents an unconformity, or a line of irregular cementation, the topography of the surface itself may be fine-textured. Unless the determinations with the seismograph on such a rough surface are made at exactly the same point at which the wells are drilled, there will be a chance for differences between the two sets of results at certain localities equal to the irregularity of the surface which is being mapped. These differences do not come about on account of the resolving power of the geophysical method, but are due to the fact that sufficient points have not been determined either from geophysical survey or from well data to give a picture of the real roughness of the surface being mapped.

For example, there are certain localities along the Dos Bocas-Alamo anticline in Mexico where the upper surface of the producing limestone shows deep, narrow cañons. These have been discovered because a very close spacing of wells happened to be made at their locality. If the wells had been spaced farther apart, the existence of such narrow cañons in the limestone would probably have been unsuspected. Similarly, the usual geophysical survey of this ridge would miss these cañons. It therefore is very important for the geologist to advise the geophysicist whether the history of an area gives any clues to the existence of very irregular formation boundaries, because such a condition may not be found by surveying unless very close spacing is used, which will not generally be the case in prospecting work.

In the foregoing discussion, an endeavor has been made to show the limitations in geophysical surveying which attempts to give contour maps of an area, and suggestions have been made for the coöperation of the geologist in reducing the amount of computation to be made by the geophysicist. It is not intended to suggest that the geologist should lead the geophysicist into a particular interpretation, but rather that he should suggest the most probable type of structure and variation in physical characteristics of the rocks on such structures so that the geophysicist can arrange his survey to give the greatest light with a minimum of expense and with the greatest probability.

In summary, with regard to geophysical methods which depend on the formulae of the potential function, profiles with numerical values or contour maps can only be made by geophysical surveying under certain favorable conditions. Although such numerical results are theoretically obtainable by the seismic method, which is not a potential function method, there are uncertainties in many localities regarding the regularity of the formations which give rise in turn to uncertainties in the contour maps made by the seismic method.

Geology and geophysics represent two aspects of scientific labor which have been well explained by J. B. Dumas:[4]

> The art of observation and that of experiment are very distinct. In the first case, the fact may either proceed from logical reasons or be mere good fortune; it is sufficient to have some penetration and the sense of truth in order to profit by it. But the art of experimentation leads from the first to the last link of the chain, without hesitation and without a blank, making successive use of reason, which suggests an alternative, and of experience, which decides on it, until, starting from a faint glimmer, the full blaze of light is reached.

[4] Letter to Louis Pasteur, quoted by R. Vallery-Radot, *The Life of Pasteur*, translation by Mrs. R. L. Devonshire (Garden City Publishing Company), p. 122.

If the difference in principle between geological and geophysical methods be that described by Dumas, we see that there follows an important difference in cost. Observation on natural phenomena is very much cheaper per acre than experiment, and the geologists' reports are therefore more readily available than the geophysicists'.

It seems that there would be greater joint progress if the distinction of Dumas could be eliminated, or at least shaded. If geology will add to its observation a portion of experimental effort, we shall probably obtain some such increase in specific discoveries as we have witnessed in the application of experimentation in geophysics. Similarly, the leisurely observation of a number of geophysical results from similar areas will increase the reasoning ability of our geophysicists. The more experienced of our geophysicists are probably more successful, now, as much because they have had opportunity for observation as because they have the greatest technical skill as experimenters.

At present, it seems we have a good chance to realize the ideal of Dumas; our hopes and the great possible rewards should reconcile us to the many tedious operations in the field and laboratory, which are yet before us in our efforts toward joint progress in geology and geophysics.

BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 18, NO. 1 (JANUARY, 1934), PP. 13-38, 14 FIGS.

SOME POSSIBLE APPLICATIONS OF GEOTHERMICS TO GEOLOGY[1]

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ABSTRACT

The generation and dissipation of heat are important factors in earth history. The present distribution of temperature down to the level of isostatic compensation can probably be determined with more accuracy than has heretofore been obtained by making use of the observations of temperatures in tunnels or across mountain ranges.

Recent geothermal surveys show that relatively high temperatures are generally associated with faults, salt domes, sand lenses, and anticlinal structures of both large and small closure.

Radioactivity and thermal conduction through oil-bearing strata are shown to be possible sources of temperature variations. Generation of heat by the oxidation of petroleum appears to be of minor importance as a heat source. The most potent source of heat is to be found in the hot rocks immediately beneath uplifts.

Ceothermal prospecting is a possibility, but as developed at present, it is much less efficient than other methods of geophysical prospecting.

COSMOGONY

Astrophysics (1)[3] teaches that the life history of the countless millions of stars distributed throughout celestial space is dependent largely on a single phenomenon—the generation and dissipation of heat. During the period of condensation of the nebula, heat is generated, until, in the interior of the resulting star, such as our sun, matter exists chiefly in the forms of electrons and protons, which may ultimately, as the temperature continues to rise, neutralize each other, leaving in their stead an electro-magnetic wave which carries off the enormous quantity of energy released by the merging together of the positive and negative charges of electricity. Such an hypothesis seems necessary to account for the long life of the stars. None of the known radioactive transformations liberates sufficient energy to maintain the sun's heat. As the loss of mass and generation of heat proceeds, a point is ultimately reached, depending on the size of the star, where its internal energy is radiated more rapidly than it is generated. Our sun has long ago passed the point of maximum temperature and is now

[1] Published with the permission of the director, United States Geological Survey. Read before the Association at the Houston meeting, March 24, 1933. Manuscript received, August 7, 1933.

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[3] This and the following references are in the Bibliography at the end of this article.

about half way down the scale of diminishing thermal energy. In the course of this process, which has required a few billions of years for its completion, the earth is supposed to have been separated from the sun and subsequently cooled to its present state.

In support of the preceding hypothesis is the fact that 61 of the 91 known chemical elements that enter into the constitution of the earth have been identified, with more or less certainty, in the sun (2). Of particular interest are the elements—helium, carbon, oxygen, nitrogen, and sulphur in the atomic state, and the compounds—the hydroxyl (OH); ammonia, or some other compound of nitrogen and hydrogen; a compound of carbon and nitrogen, probably cyanogen; and a compound of carbon and hydrogen (3). Further evidence in support of the hypothesis that the primitive earth was molten to the surface are the facts that it now acts as a magnet, and that the density increases, probably discontinuously, from about 2.7 at the surface to a mean value of about 5.5 for the entire earth.

According to Jeffreys (4), the loss of heat from the liquid surface of the earth must have been so rapid that radioactivity was a minor source of heat prior to the solidification of the outer shell. As the solidified state was approached, feeble convection currents carried off the radioactive heat and completed the transfer of the radioactive substances toward the surface of the earth. Evidence in substantiation of the hypothesis of an increasing radioactive content of the rocks as the surface of the earth is approached is contained in the following record (4) which gives the contents by weight of Finland granites of decreasing age.

	$Ra(\times 10^{12})$	$Th(\times 10^{-5})$	$K(\times 10^{-2})$
A	2.36	0.87	2.51
B and C	4.60	2.67	3.61
D	6.21	5.85	5.06

It has also been established that volcanic rocks contain more radium than plutonic rocks. Another reason for believing that radioactive substances are concentrated chiefly in the outer layers of the earth is the fact that if the deep layers were as radioactive as the surface layers, the rocky shell could never have solidified.

COOLING AFTER SOLIDIFICATION

By taking into account radioactivity, heat conduction, and an assumed initial temperature distribution based on the increase of the melting point per unit depth in the crust of the earth, Adams (6) and Jeffreys (7) have estimated the temperature at great depths in the crust of the earth (Table I). Their calculations show that cooling may

TABLE I

Temperatures in Interior of Earth

Depth, Kilometers	*Jeffreys °C.*	*Adams °C.*	*Depth, Kilometers*	*Jeffreys °C.*
0	0	0	300	2,090
25	600	560	350	2,290
50	800	860	400	2,490
75	940	1,090	450	2,670
100	1,080	1,290	500	2,840
125	1,220	1,470	550	3,010
150	1,360	1,620	600	3,170
175	1,490	1,760	650	3,330
200	1,620	1,900	700	3,490
225	1,750	2,030		
250	1,870	2,160		
275	1,980	2,290		
300	2,090	2,420		

have extended to somewhat more than 600 kilometers below the level of isostatic adjustment, which, according to Bowie (8), is now considered to be at a depth of about 96 kilometers, or 60 miles. Below the level of isostatic adjustment, the surfaces of constant pressure and constant temperature are perfect ellipsoids of revolution, while above this level these surfaces become more and more irregular as the surface of the earth is approached. One of the outstanding problems of the geologist is first to locate accurately the isogeothermal surfaces and second to relate them to heat sources past and present.

Neglecting the radioactivity of the sedimentary rocks, Jeffreys (9) concludes that nearly all of the heat that escapes from the surface of the earth is generated in a granitic layer which has a thickness of about 11 kilometers. Beneath this layer is an intermediate layer which is practically free from radium. It has a thickness of 22 kilometers. The discontinuities at 11 and 33 kilometers are supposed to be in agreement with seismological data. From 33 kilometers to 2,900 kilometers, another discontinuity, there appear to be no special changes in the properties of the rocks in so far as the transmission of earthquake waves are concerned. These results, unfortunately, are subject to correction on account of the difficulty of interpreting seismological data, but they are of interest to us in showing the possibility of correlating the temperatures and densities of the rocks in the outer shell of the earth. By taking into account a wide range of phenomena, including the evidence of seismology, chemistry, geothermics, cosmogony, geology, and so on, Daly (10) places the discontinuities at 30, 40, 60, 1,200, and 2,900 kilometers.

ISOSTASY

It has been found by geodetic observations that the crust of the earth extends to a depth of about 100 kilometers, or 60 miles, below sea-level. At this depth, the pressure is supposed to be constant, that is, the weight of each column of rock per unit area extending from the level of isostatic adjustment to the surface of the ground is constant, regardless of the height of the column. This condition implies that the density of a mountain mass is less than that of the rocks beneath the adjacent plains or beneath oceans. Two hypotheses have been proposed to explain the geodetic facts. According to Pratt, the crust of the earth is of uniform depth, the density varying inversely as the elevations of the earth's surface, while according to the Airy, or the "Roots of Mountains" theory (11), the crust of the earth is supposed to be of variable thickness, and the mountains are supposed to float in the deep-seated magma just as icebergs float in water. The magmatic

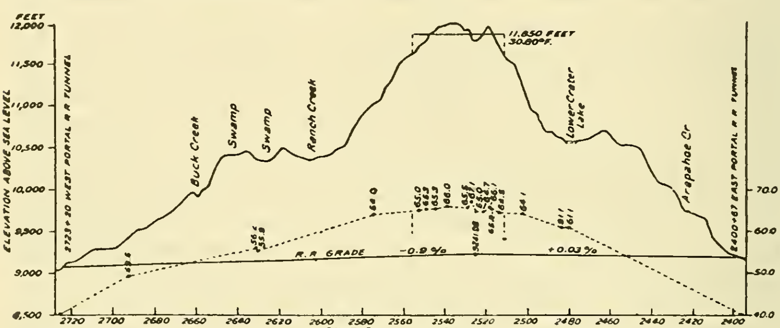

FIG. 1.—Observed temperatures in Moffat tunnel, Denver, Colorado.

rocks are supposed to be sufficiently plastic to yield to the pressure induced by the lighter strata above them and at the same time they are supposed to possess sufficient strength to carry the load imposed upon them.

Quite curiously, advocates of each hypothesis have appealed to geothermal data as a means of deciding the issue. Osmond Fisher (12) contended that the isogeotherms beneath a mountain may pass from the convex to the plane type and ultimately become concave upward. Lees (13) rejects this hypothesis. He does not believe that a mountain has solid roots extending downward into a plastic substratum and that the isogeotherms pass from the convex to the concave type.

In another publication, the writer will give a detailed application of Lees's equations to the temperature data obtained by the writer and Burgis G. Coy, resident engineer, in the Moffat tunnel. These observations are shown in Figure 1. The values, 40.0° F. at the entrances of the tunnel, and 30.80° F. at the apex of the mountain were obtained by adjusting a straight line to the observed annual mean temperatures of the air for 41 Weather Bureau stations surrounding the tunnel. From the data included between the dotted lines in Figure 1, a reciprocal gradient (1° F. in 75.8 feet) was computed for the apex of the mountain, and then having given the profile of the mountain and the diminution of air temperature along the mountain slopes, the reciprocal gradient (1° F. in 57.6 feet) beneath the adjacent plain was determined. The results of the calculations are shown in Figure 2. The computed surface of the mountain passes through the apex of the mountain and a point on the profile which is at an elevation above

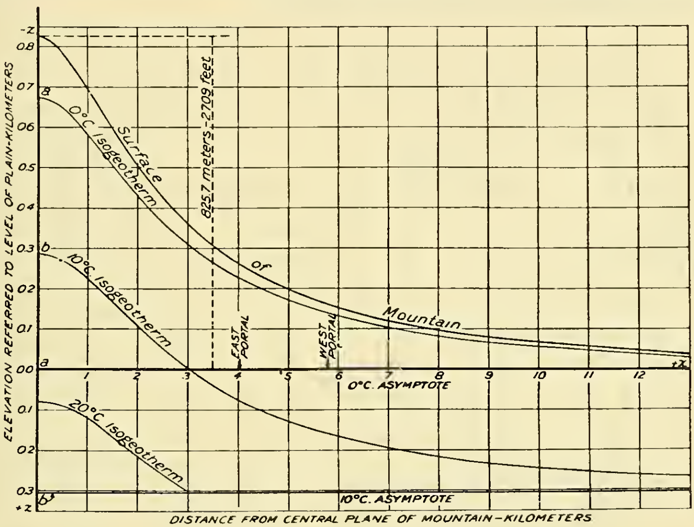

FIG. 2.—Computed profile of mountain and corresponding isogeotherms.

the plain equal to the half-height of the mountain. This approximation to the original profile of the mountain is sufficiently accurate for our purpose.

It is important to note that the profile of the mountain, the gradients at the apex of the mountain and beneath the adjacent plains, and the rate of diminution of air temperature along the mountain slope—four quantities in all—serve to fix the isogeotherms in space just as the two sides and the included angle serve to fix a plane triangle. The calculation is thus somewhat of the nature of a geothermal triangulation rather than an extrapolation to great depths. In Table II is given the rise of certain isogeotherms as they pass beneath the apex of the mountain. Thus, *a-a* in Figure 2 represents a rise of 2,207 feet. Similarly, the rise, *b-b*, is about 1,950 feet, and so on. The isotherms do not approximate to planes until a depth of 100 kilometers or more is reached. As this is the level of isostatic adjustment, it follows that the evidence supports the Pratt hypothesis. One defect has entered into the calculations. It has been assumed that the temperature gradient is constant. This gives a temperature at the depth of isostatic equilibrium which is about twice that obtained by Adams and Jeffreys as recorded in Table I. To obviate this difficulty, it can be assumed that the depth to the level of isostatic adjustment is known, and then from the equations can be determined the distribution of temperatures down to that depth which reduces the isotherms to planes at the isostatic level.

TABLE II

Rise of Isogeotherms Beneath Apex of Mountain

East Portal				*West Portal*			
Tangent Plane		*Rise of Isogeothermal Plane*		*Tangent Plane*		*Rise of Isogeothermal Surface*	
Depth, Kilometers	*Temp., °C.*	*Meters*	*Feet*	*Depth, Kilometers*	*Temp., °C.*	*Meters*	*Feet*
0	0	672.5	2,207	0	0	674.4	2,213
10	316	153.4	503	10	350	126.9	416
20	632	88.2	290	20	701	71.5	235
30	949	62.0	203	30	1,051	49.9	164
40	1,265	47.8	157	40	1,401	38.3	126
50	1,581	38.9	128	50	1,752	31.1	102
60	1,897	32.8	108	60	2,102	26.1	86
70	2,213	28.4	93	70	2,453	22.6	74
80	2,530	25.0	82	80	2,803	19.8	65
90	2,846	22.3	73	90	3,153	17.7	58
100	3,162	20.2	66	100	3,504	16.0	52

POROSITY AND TEMPERATURE

Laboratory tests prove that moisture greatly increases the conductivity of soils (14). The same condition apparently holds for solid rocks (15). A comparison of the depth-temperature curve (16) with the porosity curve (17) of a deep well, Ransom No. 1, Syracuse, Hamilton County, Kansas (Fig. 3), shows that whereas the porosity decreases with the depth, the temperature gradient increases with the depth. That the latter variation may be dependent on the moisture in the rocks is readily shown by considering that the product of the thermal gradient and the thermal conductivity represents the constant flow of heat through a vertical column of unit area. As the depth-temperature curve shows that the gradients increase with the depth, it follows from the relation of constancy that the conductivity, like the porosity, diminishes with the depth, that is, the conductivity diminishes with decreasing moisture content of the rock.

Another explanation of the curvature of the depth temperature curve shown in Figure 3 has been proposed by A. C. Lane (18). He emphasizes the fact that the upper portion of the depth-temperature curve must have risen in response to a gradual rise in the annual mean temperature of the air since the ice age. The fact, as is shown later, that the temperatures of the rocks near the surface of the ground are closely adjusted to the temperatures of the air above them, is substantial evidence in favor of Lane's hypothesis. Tests of the thermal

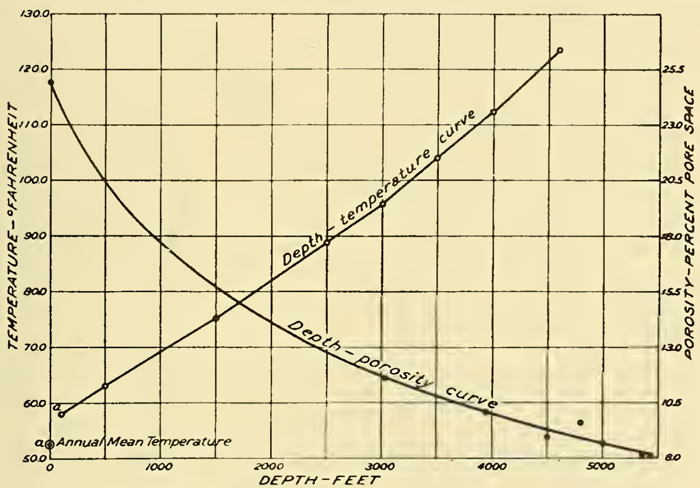

FIG. 3.—Depth-temperature curve and porosity curve.

conductivity of the rocks in their natural state at various depths in wells and mines would be of great value in the elucidation of this problem.

ERRORS IN GEOTHERMAL MEASUREMENTS

Accurate observations can be made in wells drilled with the standard rig after drilling has been discontinued for a day or two. The well must of course be free from flowing fluid at the time of the test. Wells that have been discharging fluid for a considerable time prior to the test are in unstable temperature equilibrium and the records obtained from them are not likely to be of much value.

The time required for a well drilled with a rotary outfit to reach thermal equilibrium has not been determined. It undoubtedly varies greatly with the rate of drilling, the hardness of the rocks, and many other factors. In Figure 4 are shown two curves which give some information in regard to the time required for a 2-inch diamond-drill hole to regain thermal equilibrium. The first curve represents the temperatures after drilling has been discontinued for about 30 days. The curves show clearly that the best records in a rotary hole are to be obtained a few hundred feet above the drill. This result is to be expected as the time during which the water has been in circulation is a minimum in this portion of the well.

Errors of an entirely different character appear in producing or depleted fields. Here the discharge of fluid from the producing sands

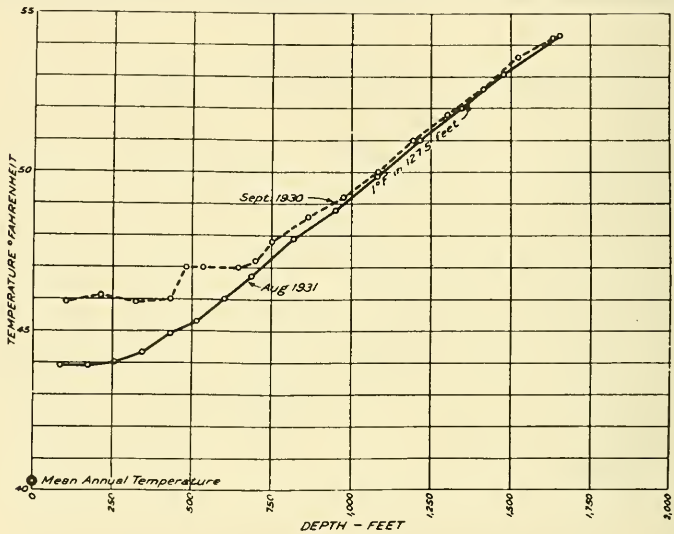

FIG. 4.—Depth-temperature curves. Diamond drill hole No. 79. Copper Range Consolidated Copper Company. Location, 1,740 feet south, 2,340 feet east, NW. corner, Sec. 1, T. 53 N., R. 35 W., Houghton County, Michigan.

tends to lower the temperatures immediately above and below the sand and to elevate those at the higher levels. Thus, wells that have been pumped rapidly for a long time, invariably show a marked rise of temperature near the surface of the ground. Furthermore, the replacement of gas and oil in the sands with water may so completely alter the thermal conductivity and heat-generating properties of the sands that the original temperatures are never restored.

RESULTS OF RECENT GEOTHERMAL SURVEYS

The American Petroleum Institute (19) has completed an extensive series of geothermal surveys extending over a period of about 5 years. More than 400 wells were tested in the states of Kansas, Oklahoma, Texas, New Mexico, and California. The writer has made tests in 250 or more wells, some of them in each of the preceding states and some in each of the other states listed in Table III, excepting as otherwise noted in the table.

TABLE III

Values of Excess of Soil Temperature over Air Temperature (e) and of Reciprocal Gradient ($1/b$)

Town or Field	*County*	*No. of Wells*	*e*	*r*	r_0	*1/b Ft.-°F.*	*Range of 1/b*	*Remarks*
Alabama								
Albany	Morgan	1	−0.5	—	—	386.2	—	12″ hole; no casing
Birmingham	Jefferson	7	+2.7	0.3	0.1	96.8	8.1	
Fayette	Fayette	1	+1.7	—	—	128.3	—	
Arkansas								
Eldorado	Union	7	+0.7	1.0	0.4	49.9	3.5	
California								
Bakersfield	Kern	5	−10.1	1.6	0.8	77.0	11.0	
Coalinga	Fresno	51	−8.2	1.5	0.2	54.4	9.0	
Fullerton	Orange	1	−9.4	—	—	49.8	—	
Grass Valley (19)*	Nevada	1	+0.2	—	—	168.6	—	
Huntington Beach	Orange	4	−8.0	1.3	0.7	42.4	3.1	
Kettleman Hills	Kings	5	−10.1	3.6	1.6	55.1	9.2	
Long Beach	Los Angeles	41	−10.0	1.4	0.2	53.6	5.5	
Santa Fe Springs	Los Angeles	31	−6.5	2.0	0.4	51.6	8.1	
Seal Beach	Los Angeles	1	−1.1	—	—	33.9	—	From 100 to 3,500 ft., 1/b =1° in 44.3 ft.
Whittier	Orange	4	−9.9	1.3	0.7	57.6	2.2	
Colorado								
Calhan	El Paso	1	−5.9	—	—	63.5	—	
Florence	Fremont	3	−3.8	0.8	0.4	52.5	13.8	
Fort Collins	Laramie	2	−5.7	0.5	0.3	56.9	5.5	
Longmont	Boulder	1	−8.4	—	—	56.4	—	
Illinois								
Bridgeport	Lawrence	4	−1.5	0.8	0.4	58.9	7.9	
Iowa								
Ames (20)	Story	1	−2.9	—	—	143.9	—	
Kansas								
Augusta	Butler	2	−1.8	0.4	0.3	50.3	2.0	
Eldorado	Butler	24	+0.1	0.7	0.1	49.3	3.7	
Florence	Marion	4	−1.7	0.7	0.4	63.7	8.7	
Haverbill	Butler	3	−1.7	0.5	0.3	54.7	5.1	
Syracuse	Hamilton	1	−4.3	—	—	82.0	—	
Kentucky								
Prater Creek	Floyd	1	−0.8	—	—	122.0	—	
Prestonburg (near)	Martin	1	+1.4	—	—	118.4	—	
Louisiana								
Blue Lake	Sabine	2	−3.3	0.2	0.1	55.8	0.5	
Caddo Lake	Caddo	1	+1.0	—	—	37.7	—	
Caddo Lake (near)	Caddo	3	+1.8	0.5	0.3	36.9	0.9	
Haynesville	Claiborne	1	+0.2	—	—	48.2	—	
Homer	Claiborne	8	+1.0	1.1	0.4	36.1	2.5	
Many	Sabine	6	+0.1	0.5	0.2	51.7	1.7	
Pine Island	Caddo	1	+0.4	—	—	35.1	—	
Sligo	Bossier	2	+0.9	0.2	0.1	38.3	0.7	
Zwolle	Sabine	3	−2.1	1.6	0.9	53.7	4.5	
Michigan								
Surrey Township	Clare	1	0.0	—	—	84.3	—	
Houghton	Houghton	2	−1.4	0.2	0.1	128.2	2.7	
Lake Linden	Houghton	1	−4.3	—	—	160.2	—	
Mississippi								
Jackson	Rankin	1	+2.1	—	—	34.4	—	

TABLE III (*Continued*)

Town or Field	*County*	*No. of Wells*	*e*	*r*	*r₀*	*1/b Ft.-°F.*	*Range of 1/b*	*Remarks*
Montana								
Anaconda (21)	Deerlodge	1	+2.1	—	—	38.0	—	
Conrad-Kevin-Sunburst	Teton and Toole	10	−4.0	0.8	0.3	84.0	25.8	
Nevada								
Virginia City (22)	Storey	1	+1.7	—	—	29.4	—	
New Jersey								
Franklin Furnace	Sussex	1	−0.4	—	—	276.1	—	
New Mexico								
Artesia	Eddy	4	−5.4	1.2	0.6	173.9	—	
Carlsbad	Eddy	1	−3.9	—	—	147.1	—	
Lovington	Lea	1	−1.7	—	—	79.9	—	Omitted on map, probably incorrect
Roswell	Chaves	2	−6.4	0.4	0.3	188.3		
North Dakota								
Lonetree	Ward	1	−6.1	—	—	63.4	—	
Oklahoma								
Ardmore	Carter	4	−0.3	0.7	0.3	135.0	18.9	
Ardmore-Healdton	Carter	9	0.0	0.3	0.1	117.4	41.7	
Ardmore-Hewett	Carter	8	+0.8	0.5	0.2	106.6	15.5	
Bessie	Washita	1	−2.7	—	—	141.5	—	
Billings	Noble	3	−2.4	0.8	0.6	54.8	4.1	
Blackwell	Kay	2	+2.1	0.8	0.5	47.6	2.7	
Braman	Kay	3	+0.7	2.5	1.5	48.7	12.3	
Burbank	Osage	3	−0.2	0.8	0.4	48.4	8.4	
Covington	Garfield	1	−2.0	—	—	51.7	—	
Cromwell	Seminole and Okfuskee	9	+1.3	0.4	0.1	51.4	1.7	
Davenport	Lincoln	1	+0.3	—	—	113.4	—	
Dilworth	Kay	11	−0.6	0.3	0.1	54.0	3.7	
Drumright	Creek	28	−0.8	0.8	0.2	78.8	18.9	
Glenn pool	Creek	19	+1.4	0.9	0.2	40.4	2.9	
Holdenville (23)	Hughes	1	−1.8	—	—	52.9	—	
Hubbard	Kay	1	−1.2	—	—	48.4	—	
Morris	Okmulgee	1	0.0	—	—	39.2	—	
Newkirk	Kay	1	−0.5	—	—	44.2	—	
Okemah	Okfuskee	4	−0.9	2.3	1.1	41.2	2.4	
Oklahoma City	Oklahoma	3	−3.1	1.7	1.0	137.6	16.5	
Papoose	Okfuskee	4	−1.0	1.5	0.8	49.1	3.8	
Perry	Noble	1	+1.2	—	—	55.0	—	
Sasakwa	Seminole	2	−0.2	0.8	0.5	67.3	14.5	
Seminole	Seminole	19	−0.7	0.9	0.2	82.6	15.9	
Shamrock	Creek	1	—	—	—	86.4	—	
Tonkawa	Kay	15	−3.1	1.4	0.4	55.7	3.9	
Walters	Cotton	10	−1.0	0.3	0.1	117.6	10.9	
Wewoka	Seminole	5	+0.6	0.3	0.1	48.3	3.6	
Yale	Payne	1	—	—	—	54.3	—	
Oregon								
Astoria	Clatsop	1	−0.9	—	—	66.5	—	
Burns	Harney	1	−7.9	—	—	17.1	—	100 to 300 ft.
Klamath Falls (near)	Klamath	3	−2.4	2.3	1.4	25.7	—	
Medford	Jackson	2	−4.3	0.9	0.6	67.5	14.5	
Merrill	Klamath	1	−2.8	—	—	40.5	—	
Vale	Malheur	1	−8.7	—	—	19.6	—	
Pennsylvania								
Gaines Junction	Tioga	1	−1.8	—	—	94.3	—	
Johnstown	Westmoreland	1	−0.7	—	—	108.8	—	
Long Bridge	Westmoreland	3	+2.1	0.3	0.2	71.0	3.5	
McDonald	Washington	1	−4.3	—	—	130.9	—	
McKeesport	Allegheny	1	+0.2	—	—	80.0	—	
Texas								
Bastrop	Bastrop	1	−7.0	—	—	48.2	—	
Big Lake	Reagan	19	−2.9	1.0	0.2	128.2	28.2	
Blue Ridge	Fort Bend	7	−1.7	0.7	0.3	34.8	13.7	

TABLE III (*Continued*)

Town or Field	*County*	*No. of Wells*	*e*	*r*	*r₀*	*1/b Ft.-°F.*	*Range of 1/b*	*Remarks*
Boggy Creek	Anderson	3	−2.1	0.5	0.3	37.7	4.7	
Bonham	Fannin	1	−2.0	—	—	43.9	—	
Callisburg	Cooke	1	−0.6	—	—	103.0	—	
Canadian	Hemphill	1	−2.3	—	—	125.9	—	
Dale	Caldwell	1	−1.1	—	—	46.6	—	
Damon	Brazoria	1	−4.8	—	—	69.7	—	
Del Rio	Val Verde	2	−9.1	4.5	3.2	71.8	13.3	
Smith Ranch	De Witt	1	−1.7	—	—	57.0	—	
Dickens	Dickens	1	−3.4	—	—	97.7	—	
Eastland	Eastland	1	−3.8	—	—	56.8	—	
Edwards	Val Verde	1	−1.9	—	—	79.9	—	
Electra	Wichita	1	−0.8	—	—	86.3	—	
Ennis	Ellis	1	−5.4	—	—	58.3	—	
Ferris	Ellis	1	−1.1	—	—	49.2	—	
Graford	Palo Pinto	1	−3.3	—	—	57.6	—	
Graham	Young	2	−4.6	2.4	1.7	60.5	10.9	
Grand Saline salt dome	Van Zandt	4	−2.7	1.3	0.6	42.3	—	
Greenville	Hunt	1	−1.7	—	—	51.2	—	
Guthrie	King	1	−3.1	—	—	93.6	—	
Humble oil field	Harris	30	+0.8	1.6	0.3	30.8	15.5	
Independent oil field	Upton	1	−5.0	—	—	107.9	—	
Leonard	Fannin	1	−2.4	—	—	55.6	—	
Long Point salt dome	Fort Bend	8	−1.2	0.3	0.1	33.2	9.4	
Luling	Caldwell	4	−4.0	2.5	1.3	48.8	14.1	
Lytton Springs	Caldwell	7	−1.2	0.4	0.2	39.4	1.3	
Mexia oil field	Limestone	6	−4.9	2.5	1.0	47.7	9.3	
Muenster oil field	Cooke	1	+3.2	—	—	73.5	—	
Odessa	Midland	1	−5.1	—	—	144.2	—	
Ozona	Crockett	1	−0.9	—	—	97.1	—	
Panhandle area	Gray-Roberts-Carson	6	−4.8	2.5	1.0	117.8	39.1	
Pierce Junction oil field	Harris	3	−6.3	3.8	2.2	59.7	0.7	
Powell oil field	Navarro	14	−15.0	6.0	1.6	52.1	15.5	
Ranger	Eastland	5	−2.8	0.6	0.3	47.1	4.0	
Rosewood	Upshur	1	−5.3	—	—	51.3	—	
Sherman	Grayson	2	1.1	0.0	0.0	65.9	5.0	
St. Jo	Montague	1	+1.8	—	—	86.2	—	
Thrifty	Brown	1	−4.1	—	—	61.0	—	
Waco	McLennan	2	−3.9	1.7	1.2	48.5	4.5	
Wolfe City	Hunt	2	−0.3	0.7	0.5	53.5	2.4	
Wortham	Freestone	2	−10.9	0.8	0.6	48.3	3.5	
Washington								
Ballard Station	King	1	+1.8	—	—	107.7	—	
Benton City	Benton	1	−5.5	—	—	44.1	—	
Moclips	Grays Harbor	1	+3.1	—	—	101.6	—	
West Virginia								
Bridgeport	Harrison	1	−2.3	—	—	85.7	—	
Fairmont	Marion	1	−1.5	—	—	80.4	—	
Grantsville	Calhoun	1	0.0	—	—	86.8	—	
Mannington	Marion	1	−0.3	—	—	72.9	—	
Spencer	Roane	2	−0.3	1.0	0.7	90.2	—	
Volcano	Wood	2	−0.3	0.4	0.3	114.8	2.7	
Wyatt	Harrison	1	−1.5	—	—	92.1	—	
Wyoming								
Big Muddy	Converse	4	−5.4	1.6	0.8	60.6	12.1	
Cody	Park	2	−7.9	0.5	0.3	56.5	10.3	
Grass Creek	Hot Springs	3	−2.0	0.3	0.2	50.7	1.1	
Lance Creek	Niobrara	3	−6.7	0.4	0.2	41.2	1.7	
Lost Soldier	Sweetwater	3	−3.3	0.7	0.4	22.3	8.7	
Pine Mountain	Natrona	1	+0.5	—	—	39.1	—	
Rawlins	Carbon	4	−2.3	0.3	0.1	58.8	22.9	
Rock River	Carbon	3	−3.0	1.2	0.7	66.6	16.5	
Salt Creek	Natrona	26	−3.4	1.3	0.2	40.1	14.5	
Teapot	Natrona	1	−2.8	—	—	44.8	—	
Thermopolis	Hot Springs	9	−5.8	0.6	0.2	22.3	1.1	
Thermopolis (near)	Hot Springs	1	−7.3	—	—	56.4	—	
Thermopolis (near)	Hot Springs	1	−4.8	—	—	68.6	—	

The quantity e in the table represents the excess of soil temperature over air temperature (a-a, in Figs. 3 and 13). A negative sign precedes this quantity when the rock temperatures are higher than the annual mean air temperatures. Conversely, a positive sign precedes the tabular value when the reverse condition exists. The probable error of an observation of weight unity is represented by r and the probable error of the mean is represented by r_0.

The tabular values of the reciprocal gradient $1/b$ were computed by reciprocating the mean values of the gradient b. To obtain the range of $1/b$, the value of $2r$ for b was added to and subtracted from b. The mean of the differences between $1/b$ and the reciprocal of the two preceding values was selected to represent the range of $1/b$. For example, the table shows that the values of $1/b$ for Birmingham, Alabama, fall between the limits 88.7 and 104.9. This method of procedure is a fairly satisfactory method of presenting the data in a very condensed form. For a small number of observations, the indicated range is frequently too large.

Each value of $1/b$ in the table is recorded on the map of the United States (Fig. 5). Depths are limited to 1,000 feet, or less.

The map shows clearly that the temperatures are uniformly low in the Appalachians. There are no areas of high temperatures. In the Rockies, the temperatures vary from the extremely high to the extremely low. Irregularities over large areas rather than uniformity is the rule. In southeastern New Mexico and western Texas, including the Panhandle, extraordinarily low temperatures prevail. In Wyoming, the temperatures vary from the extremely high to a moderate average. That is, Wyoming is a high-temperature area. Likewise, eastern Texas, Oklahoma, and Kansas, southern California and Arkansas, and northern Louisiana may be characterized as areas of high to moderately high temperatures. A more detailed study of the areal and local distribution of temperatures is given in *Problems of Petroleum Geology*.[4]

As a further means of summarizing and interpreting the data, let us consider the temperature distributions over regional and local areas.

Figure 6 is a reproduction of McCutchin's (25) map showing the general trend of temperatures over a distance of approximately 100 miles in central Oklahoma, extending in a southeasterly direction from Oklahoma City to Wewoka. Another map by McCutchin, not here represented, shows a similar trend of the isothermal surfaces extending from Oklahoma City in a northeasterly direction to Sapulpa.

[4] A sequel to *Structure of Typical American Oil Fields*. In manuscript.

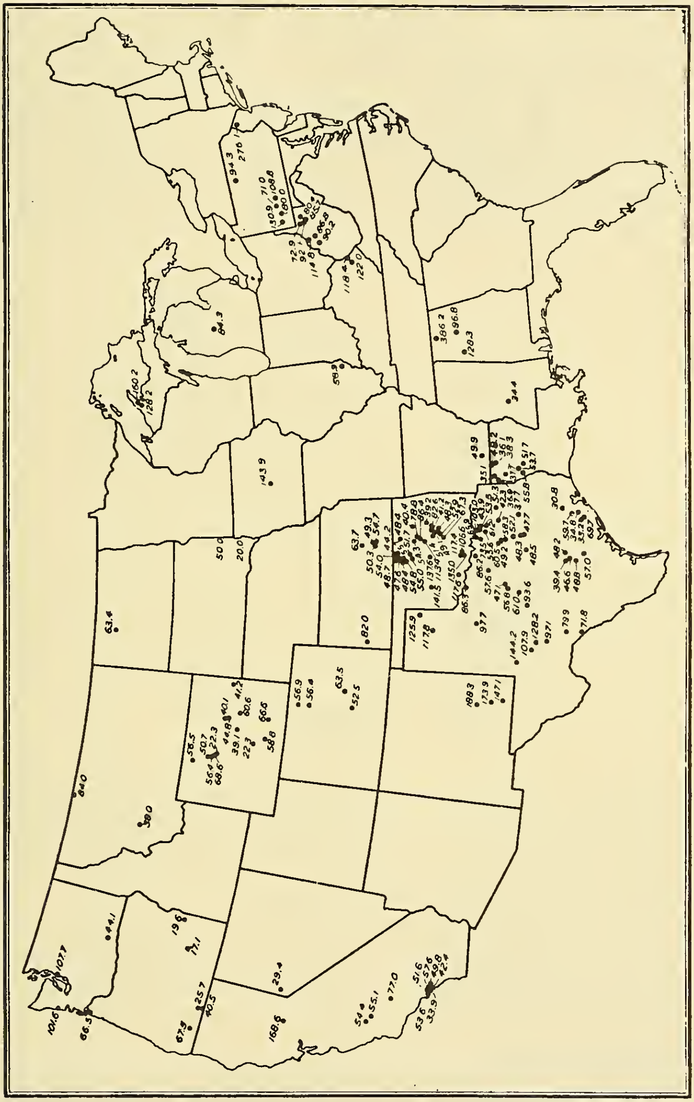

FIG. 5.—Values of $1/b$ expressed in feet per degree Fahrenheit.

Heald (19) has already emphasized the significance of these results in their relation to the general slope of the strata. He suggests that the isotherms may be a definite reflection of regional structure.

In order to summarize the evidence on the nature of the isotherms in local areas, five different types of structure have been selected as illustrations, namely, a fault, a salt plug, an unconformity of the granite ridge type, a sand lens, and an anticline with a large closure in which the circulation of water may be of importance.

Figure 7 shows a remarkable rise in the isotherms as the fault in the Powell oil field, Navarro County, Texas, is approached (26).

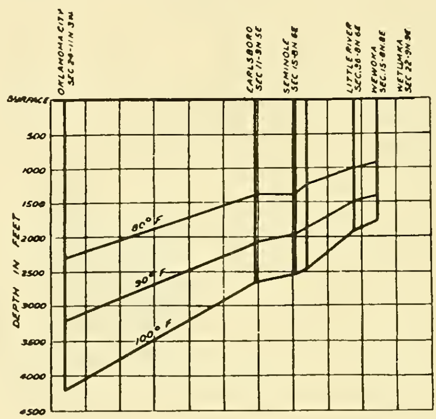

FIG. 6.—Regional variation in central Oklahoma.

As the wells in this field were in unstable temperature equilibrium, the rise is much less than is indicated on the map, but the fact that the 120° F. isotherm is within 400 or 500 feet of the producing sand justifies the belief that a real variation exists.

There appears to be no reason for questioning the large variation of temperature found by Hawtof (19) over the salt dome at Humble, Harris County, Texas (Fig. 8). According to Logan (27) this dome has a maximum diameter of 17,500 feet and a minimum diameter of 14,-000 feet at the 3,000-foot salt contour.

McCutchin's (25) map of the El Dorado, Kansas, field (Fig. 9) is of interest from two standpoints. In the first place, the isotherms dip a little more steeply than the strata, and in the second place, El Dorado is a good example of the granite-ridge type of structure. This field is also representative of a simple type of unconformity (28).

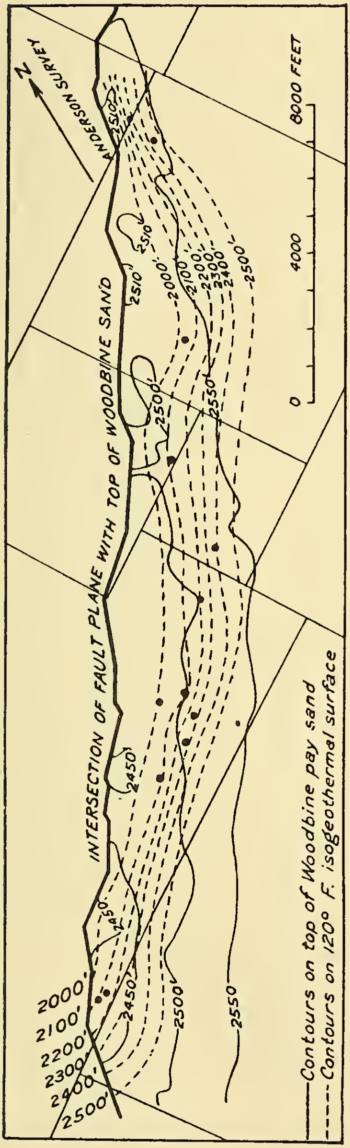

FIG. 7.—Local variation on fault, Powell oil field, Navarro County, Texas.

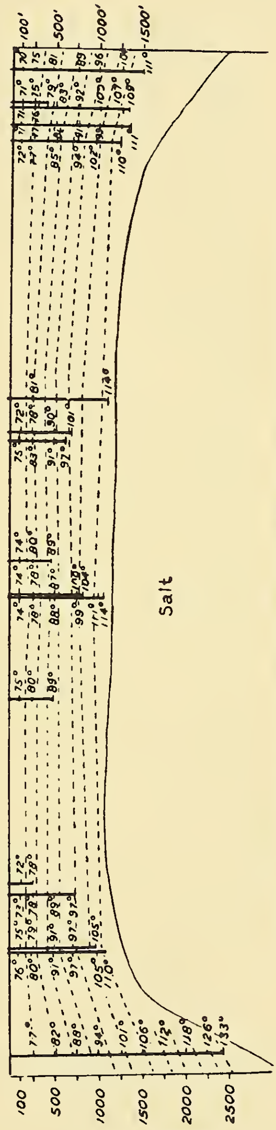

FIG. 8.—Local variation over salt dome, Humble oil field, Harris County, Texas.

Because of the difficulty experienced in obtaining wells located at a sufficient distance from edge water, the evidence on the variation of temperature over sand lenses is not very complete. However, Mc-

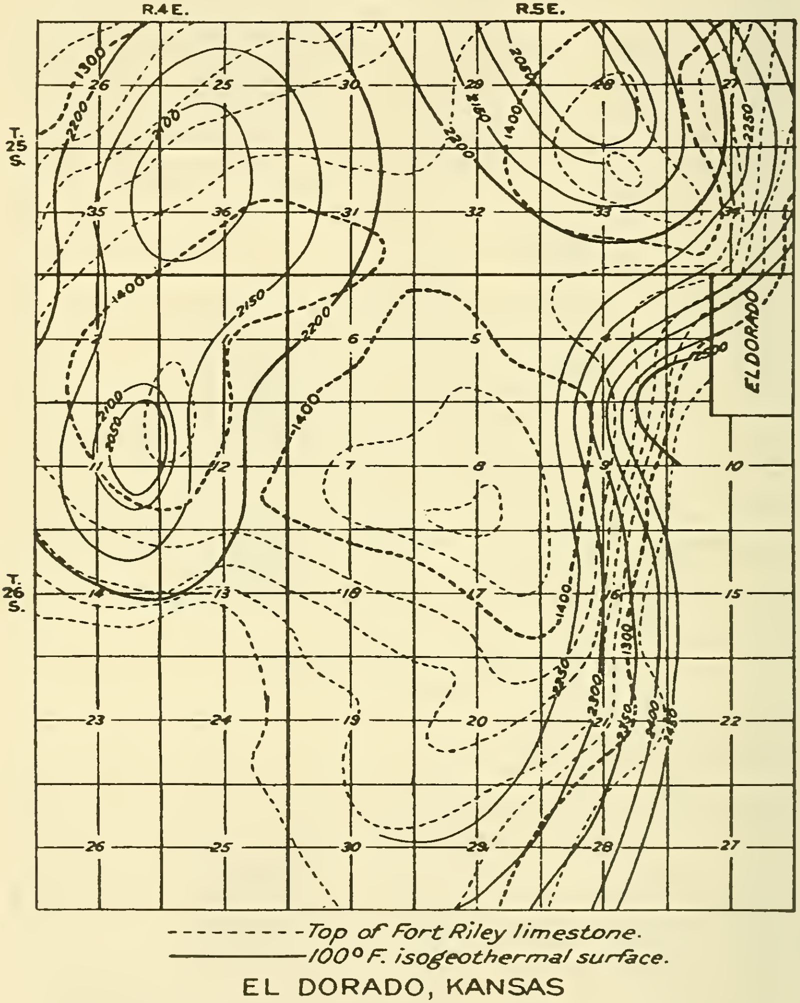

FIG. 9.—Local variation over granite ridge, El Dorado, Butler County, Kansas.

Cutchin's (29) observations in the Haverhill field, Butler County, Kansas (Fig. 10), located about 7 miles south of El Dorado, Kansas, show a well defined variation. The numbers on the right of the well locations (Fig. 10) represent the depths at which a temperature of

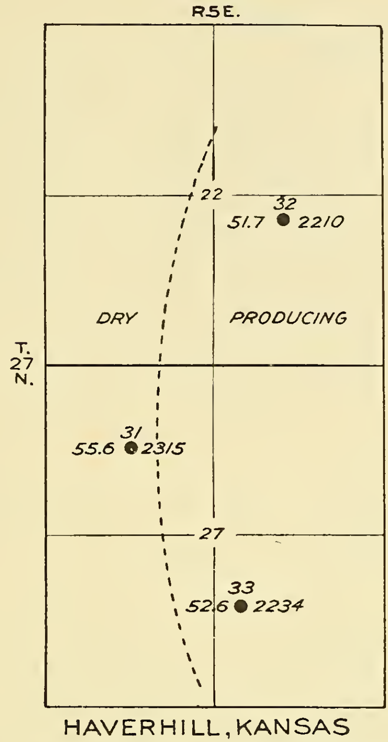

FIG. 10.—Local variation over sand lens, Haverhill, Butler County, Kansas.

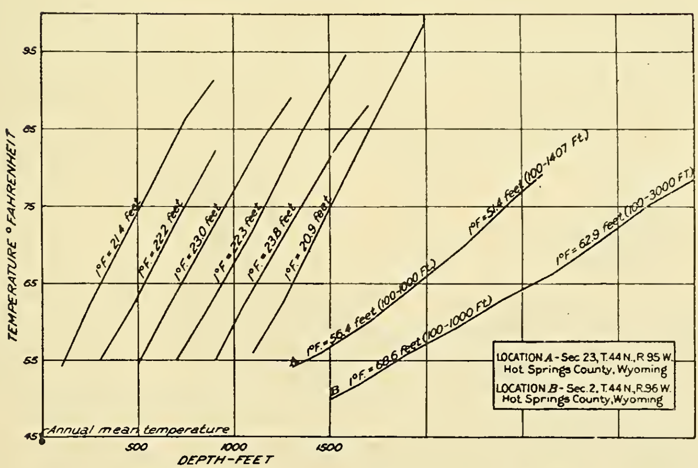

FIG. 11.—Local variation over structure in which movement of water may be of importance, Warm Springs domes, Hot Springs County, Wyoming.

100° F. is reached. The numbers on the left of the locations represent feet per degree Fahrenheit.

Very rapid rates of temperature increase have been found at Lost Soldier and Thermopolis, Wyoming. At Thermopolis, the writer made tests in 9 wells located on the tops of the two Warm Springs domes, Hot Springs County, Wyoming. The depth-temperature curves are practically the same in all of the wells. Six of the curves are shown in Figure 11. All of the curves in the figure have been drawn on the same scale, but the origin for each curve has been displaced to the right for the sake of clearness in representation. The two wells in which curves *A* and *B* were obtained are located 10 or 12 miles northwest of the Warm Springs domes. As the Big Horn hot springs, one of the largest hot springs in the world, is about 5 miles west of the oil fields, the movement of water over the domes may be of very great importance.

CAUSES OF TEMPERATURE VARIATIONS

Three distinct types of variation have been discovered, namely, variation in the vertical, and variations over both large and small areas.

The dependence of the distribution of temperatures in the vertical on the porosity of the rocks and on the rise of air temperatures since the ice age is discussed in previous paragraphs. The precision with which the rock temperatures are adjusted to the air temperatures above them is indicated by the fact that the mean of 514 values of *e* (Table III), distributed throughout the United States, exclusive of the Powell oil field, Texas, and the oil fields of California, is −1.54°F. The data from the Powell oil field are believed to be erroneous. The mean value of *e* for 144 wells in the California oil fields is −8.44° F. The cause of this large value is not known. By assuming it to be an error, and adding 1° F. to the observed annual mean temperature of the air, the following corrected reciprocal gradients are obtained.

Bakersfield	53.8	Long Beach	43.1
Coalinga	45.9	Santa Fe Springs	45.7
Fullerton	46.9	Seal Beach	42.6
Huntington Beach	34.6	Whittier	48.2
Kettleman Hills	42.6		

As these numbers are much less than the corresponding numbers in Table III, it follows that the evidence in support of the hypothesis of a variation of temperature over these structures is greatly strengthened if it is assumed that the large value of *e* is due to drilling and producing operations.

Another marked variation in the vertical is shown in Figure 4. Here the rise in the depth-temperature curve is almost imperceptible to a depth of about 300 feet. A similar condition exists at Birmingham, Alabama. Barring the possibility of abnormal conditions in the well, which seems rather improbable, the flattening of the curves may be due to ground water.

In summarizing the remaining evidence of the preceding sections, it can be said that it tends to show that regional variations exist; and that local variations have been found on five different types of structure on which oil fields are located. They are: faults, salt domes, granite ridges (or simple types of unconformity), sand lenses, and a structure with a large closure in which the temperature may be controlled in part by the movement of water. Elsewhere (31), the writer has discussed the evidence in regard to the variation of temperature over structures of both large and small closure in which the movement of water is probably of no importance.

With regard to regional variations, an hypothesis of vertical uplift proposed by Nevin and Sherrill (32) to explain the origin of certain local uplifts in north-central Oklahoma is of fundamental importance. According to their hypothesis, regional tilting of a rock mass by vertical forces has resulted in producing local uplifts over areas of weakness in the pre-Cambrian complex. By referring to Figure 6, it will be noted that the temperatures rise gradually as the depth to the granite diminishes, as we pass from Oklahoma City to Wewoka, and, furthermore, the highest temperatures are found over the oil fields, the areas in which, according to Nevin and Sherrill, the hot rocks have been pushed upward through the basement complex. The processes of uplift thus proposed by Nevin and Sherrill suffice to explain both local and regional variations of temperature in certain areas. Another important contribution to our hypothesis of regional variations was made a number of years ago by N. H. Darton (33). His map shows clearly that the temperature gradients in eastern South Dakota increase as the depth to the granite or quartzite diminishes. As shown on the map (Fig. 5), the reciprocal gradients vary from 1° F. in about 20 feet to 1° F. in more than 50 feet.

In order to reach a conclusion in regard to other causes of local variations, let us consider the possibilities of thermal conductivity, chemical reactions, and radioactivity.

First, let us consider the differences in the values of $1/b$ at Haverhill, Kansas (Fig. 10), from the standpoint of thermal conductivity and the generation of heat from chemical reactions.

The sand lens can be regarded as a buried disk that is a very poor conductor of heat, for, under the same conditions, water transmits 4 times and limestone 14 times as much heat as petroleum. The correct solution of the problem can be obtained by constructing the isogeotherms. By proceeding in this way, Strong (34) has shown that the general effect of a poorly conducting disk is to increase the gradients immediately above and below the disk.

Let us now attempt to estimate the rate at which heat must be developed by chemical reactions within the disk in order to maintain the differences in the values of the reciprocal gradients ($1/b$) which were found at Haverhill. The mean of the two reciprocal gradients on the producing area is 52.15 feet per °F, which compared with the one

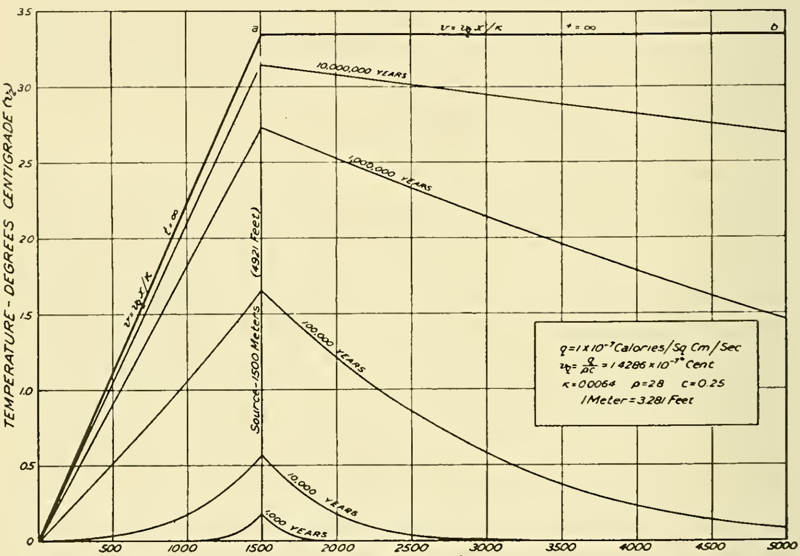

FIG. 12.—Rise in temperature due to heat source at 1,500 meters (4,921 feet).

obtained on the nonproducing area, 55.6 feet per °F., corresponds with a temperature difference of 2.97° F. (1.65° C.) at a depth of 762 meters, 2,500 feet. From the diagram (Fig. 12), (35) the maximum rise in temperature due to a heat source at a depth of 762 meters is about 1.7° C., and as is also shown in the diagram, this constant temperature difference can be maintained by the generation q of 1×10^{-7} calories per square centimeter per second, or, 3.16 calories per square centimeter per year (2,932 calories per square foot per year) in a thin stratum of rock. As the value, 1.7° C., is a sufficiently close approximation to the value 1.65° C., it may be assumed that the value of q has been determined with sufficient accuracy for our purpose. Let us now estimate for comparison with our calculated value of q, the quantity of heat that can be obtained by the oxidation of petroleum in its natural state. Let us assume that the heat-generating disk is 100 feet in thickness and that 1/3 of the entire volume is crude oil. Then a column of rock, 1 square centimeter in cross section and 100 feet in length, contains about 900 grams of oil, and as the heat of combustion of crude oil is about 11,094 calories per gram, it follows from our preceding value of q that all of the oil would be oxidized in $(11{,}094 \times 900)/3.16 = 3.2 \times 10^6$ years. As this result is not in agreement with geological facts, we conclude that the oxidation of oil may be a contributing factor, but it is not sufficient in itself to maintain the constant temperature differences which exist at Haverhill. Werner (36) regards petroleum deposits as special sources of heat. He believes that the

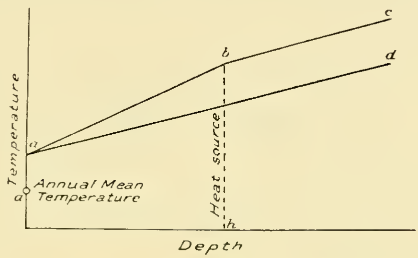

FIG. 13.—Sketch of depth-temperature curves. Heat source at *h*.

presence of carbonic acid in the gases is evidence of oxidation brought about probably by the presence of free oxygen in salt water.

In Figure 13, the line *ad* represents a depth-temperature curve on the basis of normal cooling. The excess of soil temperature over air temperature, *e*, is represented by *aa*. The line *abc* represents a depth-temperature curve when there is a source of heat at *b*. Curves of this type have been found at El Dorado, Kansas; Coalinga, California; Big Lake, Texas, and in some other fields. Ordinarily, however, the curvature is in the opposite direction.

It remains to consider the possibility that radium is a source of heat beneath anticlines. Three factors are involved, namely, transmission of heat along the strata, an excess of radium in granite as compared with the sediments, and the presence beneath the oil-bearing strata of a granitic mass that stands above the general level of the basement rocks. The problem is illustrated in Figure 14 in which the height of the granite ridge is represented by *ab*. Inspection of the figure shows that transmission of heat along the strata may result in a considerable excess of heat beneath the anticline. Two sources of heat are available. First, excess of radioactivity in the granite and, second, an increased flow of heat from rocks that have been displaced towards the surface of the earth.

Joly's (37) values for the heat developed by radioactivity are: 16.6×10^{-14} calories per second per gram for sediments; and 30×10^{-14} for granite. This leaves 13.4×10^{-14} calories per second per gram as a source of excess heat in the granite. This estimate is based on the activity of thorium and uranium. Carrying out the calculations, we find for the excess heat developed in the granite, the value, 0.348 calories per year per square centimeter per 1,000 feet of thickness (*ab*, Fig. 14) of granite standing above the general level of the basement floor. To make an accurate calculation, it is necessary to assume a great number of heat sources and draw the curves for each source as shown in Figure 12. The sum of all of the ordinates at any point represents the

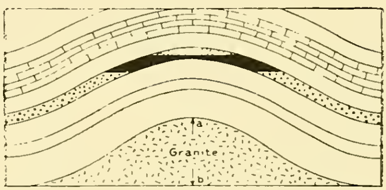

FIG. 14.—Sketch showing cross section of anticline.

temperature at that point. It will suffice for our purpose to assume that all of the heat is generated in one plane, or thin stratum; and since heat has been developed in granitic rocks since their solidification, it follows that the increment in temperature is represented by the straight line *oa* in Figure 12. The slope of the line *oa* is determined by the thermal constants of the rocks and the rate of generation of heat. Hence, it is independent of the depth to the heat source. This means that a granitic mass of height *ab* (Fig. 14), above the basement complex at El Dorado, Kansas, where the depth to the granite is only 3,000 feet, produces the same change in the gradient, other conditions being the same, as at Long Beach where the granitic mass may be buried to a depth of 20,000 feet. In other words, ridges in the basement floor are reflected by increased temperature gradients at the surface, and the change in the gradient is independent of the thickness of the sediments.

We can estimate the magnitude of the increment in the temperature or the gradient by comparing the value, 0.348 calories per year per square centimeter per 1,000 feet of thickness of granite with the value 3.156 calories per year per square centimeter for a thin stratum at a depth of 1,500 meters (Fig. 12). The comparison shows that a thickness of 9,000 feet of granite (*ab*, Fig. 14) produces the same increment in the gradient as the heat source represented in the figure. As the increments in temperature represented in the figure are of the same order of magnitude as those found on the crests of anticlines as compared with the temperatures at the same depth near edge water, it follows that some of the excess heat beneath anticlinal structures may be due to radioactivity.

SUMMARY AND SUGGESTIONS

The importance of geothermics in large-scale problems has been illustrated by attempting to estimate the depth to the level of isostatic adjustment from observations of temperature in Moffat tunnel. By assuming the depth to the level of isostatic compensation to be given, it may be possible to make a more accurate estimation of temperatures down to that depth than has heretofore been made.

The geothermal map (Fig. 5) probably represents high temperatures over small areas rather than average temperatures over large areas. For example, the areas surrounding salt domes are probably areas of average temperature, or at most, only moderately high temperatures.

Summarizing the results of recent geothermal surveys, the evidence shows that variations of temperature have been found to be associated with salt domes, sand lenses, faults, and structures with both large and small closure. The extent to which this generalization applies is not known.

Concerning causes of temperature variations, both local and regional variations are apparently dependent on the presence of granite. This result, as applied to regional variations, was anticipated to a certain extent by Lawson (38), who foresaw the possibility of correlating temperature gradients with the granitic layers in different parts of the world. As applied to local variations, the result is in agreement with an important suggestion by Clapp (39) that the importance of the shortening of the columnar sections over anticlines is just beginning to be appreciated. The result must not be interpreted as proving conclusively that radium is the source of all of the heat escaping from the surface of the earth. Normal cooling is still a possibility that the evidence has not yet excluded.

Chemical reactions in oil-bearing strata appear to be of minor importance as heat sources.

Further calculations and experiments are needed as a means of estimating the importance of oil-bearing strata in preventing the escape of heat from the earth, and incidentally causing changes in the thermal gradient.

Tests in one well or more drilled between Thermopolis and the Warm Springs dome would have given important evidence on the amount of heat transferred by water. With existing data (Fig. 11) and Darton's (40) description of the movement of the water, it may still be possible to reach conclusions of more or less importance.

Geothermal prospecting may succeed in certain areas. Under the most favorable circumstances it would be necessary to make tests in core drill holes extending through a period of several months in order to be sure that true rock temperatures are obtained. Ordinarily, the holes would have to be drilled to a depth in excess of 500 feet. A less depth would doubtless be required over shallow salt domes.

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BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 18, NO. 1 (JANUARY, 1934), PP. 39-62, 17 GRAPHS

EARTH RESISTIVITIES AT DEPTHS LESS THAN ONE HUNDRED FEET[1]

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ABSTRACT

Empirical data on the resistivities of different kinds of rocks to depths of 100 feet were obtained under known field conditions. The resistivity-depth relations are plotted in the conventional manner.

A series of graphs shows the resistivity of the rocks along three geologically known sections across the Cap-au-Gres fault in northeast Missouri, the adjacent rocks across the fault surface being different in each case.

INTRODUCTION

While prospecting for near-surface deposits of clay and hematite in central Missouri during 1930 and 1931, the writer's attention was turned to the use of the earth-resistivity method of geophysical prospecting. Being wholly unfamiliar with the technique, the writer found it necessary to read as much of the literature available on the subject at that time as was possible. It was disappointing, however, to find hardly a single systematic study of the elementary principles published which would be applicable to work at shallow depths.

Consequently, several weeks of field work was done simply to obtain, empirically, typical resistivity values over shale, sandstone, limestone, alluvium, a known thickness of one type of rock over another, and other relationships, at depths less than 100 feet. It was believed that a determination of these resistivity values in the field under known conditions would be of more practical value than observations on miniature "synthetic" laboratory structures. After obtaining data on known conditions, it was proposed to apply them to the unknown. Since similar data have not appeared in the literature to date, the writer offers his results in the hope that they may be of aid to others.

The work was carried on in 1930 and 1931, as previously stated, and was done wholly within Missouri. The specific location of each observation is indicated in its write-up.

Through the courtesy of the A. P. Green Fire Brick Company,

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Mexico, Missouri; Jones and Turner, mining contractors, Linn, Missouri; and the physics department, University of Missouri, the writer borrowed and assembled a set of equipment, and obtained data which would otherwise not be available. Professor W. A. Tarr, of the department of geology, University of Missouri, offered suggestions and criticisms.

APPARATUS AND METHOD

The method followed was the simple Wenner 4-electrode hook-up, described in some of the handbooks[3] on geophysical prospecting. In addition, a supplementary potential electrode, as proposed by Lee and Swartz,[4] was frequently used for tapping in the center of the circuit. As a ready-made resistivity measuring set was not available with the funds at hand for the work, a modified Gish-Rooney set was assembled by using a standard 0–75 milliammeter, student potentiometer, 90-volt radio B battery, and a locally constructed circuit reversing commutator driven by a 6-volt motor.

Ordinary surveyor's tapes were used to measure the electrode separation. They were laid off in feet, and as a convenience the computations of resistivity were reported, not in ohms per centimeter cube, the usual method, but in a field unit, ohms per foot cube. At each observation the electrode spacing in feet was selected, the current in milliamperes and the potential in millivolts read, and the field resistivity unit, r, computed from the equation:

$$r = a\frac{E}{I}$$

where a is the electrode separation in feet, and E and I, millivolts and milliamperes, respectively. It is obvious that the computation was made with the greatest ease in the field at once, and was available for use there on the ground. Since an intelligent selection of the location or procedure of the next set-up may be governed to a large extent by the results at the previous one, and since this advantage is lost if calculations are delayed until the office is reached, the advantage of a simple and easily calculated unit is apparent.

In accordance, all data embodied in this report are in terms of the aforementioned field resistivity unit.

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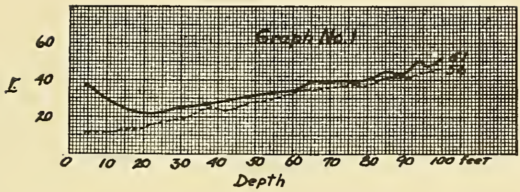
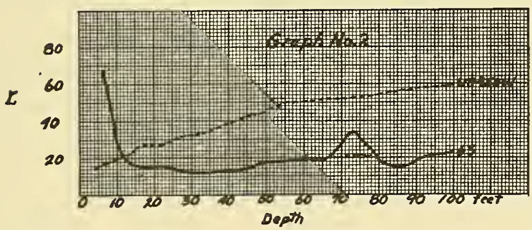
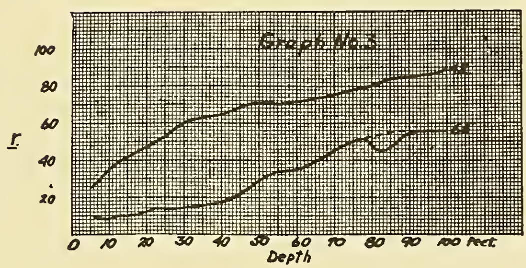
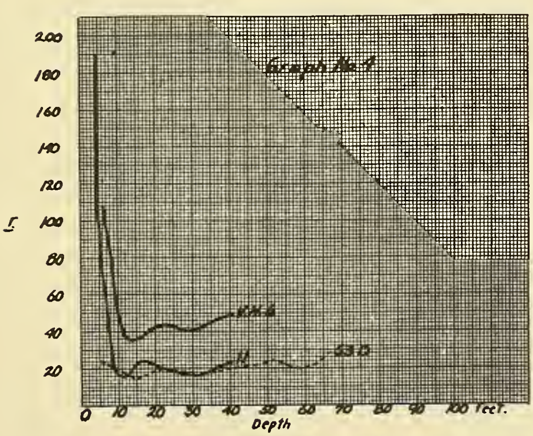

Graph No.1
60
40
20
r
0 10 20 30 40 50 60 70 80 90 100 feet
Depth


Graph No.2
80
60
40
20
r
0 10 20 30 40 50 60 70 80 90 100 feet
Depth


Graph No.3
100
80
60
40
20
r
0 10 20 30 40 50 60 70 80 90 100 feet
Depth


Graph No.4
200
180
160
140
120
100
80
60
40
20
r
0 10 20 30 40 50 60 70 80 90 100 feet.
Depth

TYPICAL RESISTIVITY CURVES

To obtain typical resistivities of the various kinds of rock at depths to 100 feet, a set-up was made over a particular kind of rock having a minimum thickness of at least 100 feet and readings were taken at increasing electrode spacings. The resistivity unit for each spacing (or depth) was computed and the results plotted graphically, as shown in the following curves. Depth in feet is shown on the abscissa and resistivity on the ordinate.

It may be pointed out that the resistivity indicated for the 50-foot depth, for example, is not for the rock at that depth alone, but includes all rock between, adjacent to the electrodes, and to a depth of 50 feet.

ALLUVIUM

Graph No. 1 shows the range of the resistivity of alluvium in the simple resistivity units up to 100 feet in depth. The stations were located on the Mississippi River flood plain near Foley, Lincoln County, Missouri. The alluvium of curve 51 is underlain by Jefferson City dolomite and that of 56 by St. Peter sandstone. The curves are essentially the same. The slight upturn of the upper one at less than 20-foot depths was due to dry soil on the edge of a corn field.

In all resistivity curves of fine alluvium obtained by the writer the range is low, usually under 75 units. The two curves given are typical.

In high or dry gravel banks the resistivity may be considerably higher.

The resistivity of sedimentary rocks seems normally to increase with depth, and the higher the resistivity, the faster the increase.

SHALE

Curve 63 on Graph No. 2 is typical of the resistivity of an ordinary shale, the Grassy Creek shale, located about 3.5 miles southwest of Foley, Missouri.

The resistivity of slightly sandy shale is shown on curve *VH* 450 *W*. It was taken in the Cherokee formation, 3 miles east of Linn, Osage County, Missouri. The curve is also typical and is similar to 63 and 65.

Curve 65, Graph No. 3, shows the resistivity of Maquoketa shale measured on a gentle slope above a creek bed a few hundred feet north of the location of curve 63. The higher trend of 65 over 63 is probably due to the fact that the Maquoketa is much more calcareous and sandy than the Grassy Creek.

Shale of still higher resistivity is shown by curve 42. However, it does not differ in composition from 65, but is the same Maquoketa measured on a high dry creek bank where little soil mantled the shale outcrop. The writer shows this as an excellent illustration of the effect of the position of a set-up with reference to the drainage and water table on the resistivity of the rocks. It seems unquestionably true that the higher resistivity of 42–65 is due to an unsaturated condition prevailing in the vertical high creek bank. As a precaution the measurement of 42 was taken sufficiently far back from the face of the bank to obviate any effects of a limited conductor, as would be the case if the set-up had been made on the brink. In that position half of the normal current conductor space would have been air instead of rock and a high resistivity would have resulted.

The measurement of shale under ordinary topographic conditions is represented by 65 and that curve is taken as normal. The other should be kept in mind because it may occur under some conditions of prospecting. A similar effect was also shown by limestone.

FIRE CLAY

Thirty feet of Pennsylvanian fire clay (Cherokee formation) lying below 15 feet of reworked glacial sand and clay is represented by curve 53 *D* on Graph No. 4. Its electrical properties are so similar to those of shale that the two rocks can not be differentiated by resistivity methods. Unfortunately also, from an economic standpoint, the overburden on this deposit had a resistivity similar to the clay, and the resistivity method was of no value in prospecting here.

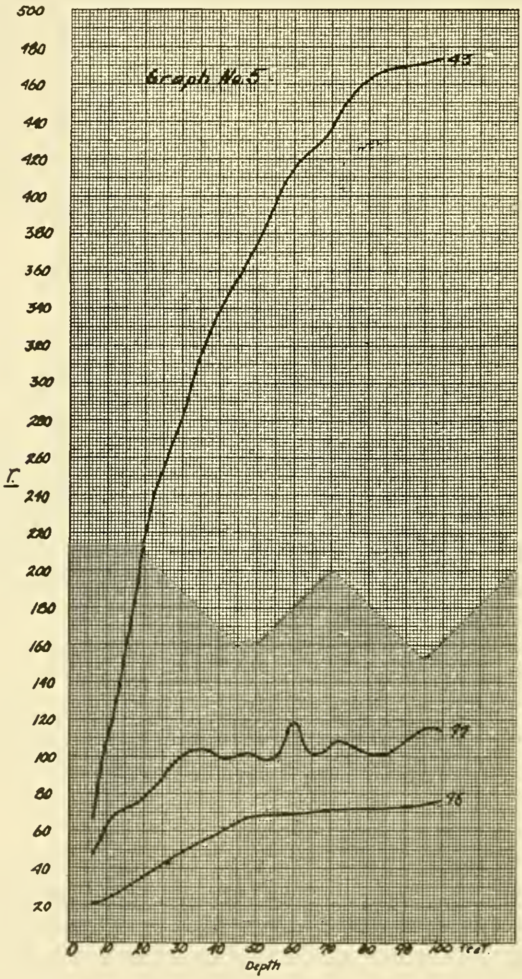
The resistivity of diaspore clay is shown in the flat part of curve *V. H.* 6. It is normally slightly higher than that for shale, but the difference is hardly great enough to enable one to differentiate between the two on this basis alone. The geology of the region and effects of the sandstone rim rock must be taken into consideration in the electrical prospecting for diaspore.

The steep parts of curves *V. H.* 6 and 11 represent resistivities to depths of about 12 to 13 feet, the transition from overburden to solid rock. Both locations, *V. H.* 6 for diaspore, and 11, a hematite prospect, were drilled and found to check electrical predictions exactly as far as overburden and solid rock relationship were concerned. However, the flat part of 11 went into only slightly ocherous, sandy shale and not into hematite which the lessees desired.

The high surface resistivities are due to the nature of the overburden, a cherty gravelly soil that drains readily, especially in the hills of Osage County, where these locations were run. Similarly, glacial gravels that are fairly clean from clay show high resistivities.

LIMESTONE

The resistivity of the Kimmswick limestone is shown on Graph No. 5 under three different conditions. First, curve 77, the limestone with no soil cover, was measured where it was cleanly exposed in the



bed of a small creek one mile north of Elsberry, Lincoln County. The electrodes were laid directly on the limestone in the shallow, almost quiet water of the stream.

The next set-up, 76, was made about 35 feet north of 77, where the same limestone was covered by about 3 feet of alluvium. The resistivity here is uniformly about 30 units less than that of 77. The writer sees no other explanation for the lowering of the resistivity than the effect of the low resistivity alluvium. The rock was the same, the stratigraphic horizon the same, the current lines parallel, and the separation of instrument locations less than the distance of maximum current path spread so that there must have been overlap of current paths of the two set-ups; the only difference between the two locations was the 3 feet of alluvium at 76. Other determinations, some of which will be shown later, substantiate this view. The writer believes that the effect due to the thickness and nature of the mantle rock is significant and important, and can not be ignored in careful work, especially at shallow depths.

In marked contrast to 76 is 43, taken over the same formation and at about the same stratigraphic horizon, but where the limestone is exposed in approximately a 75-foot vertical river bluff. The soil was thin and the hill well drained on all sides. The resistivity has reached a maximum of 475 field units.

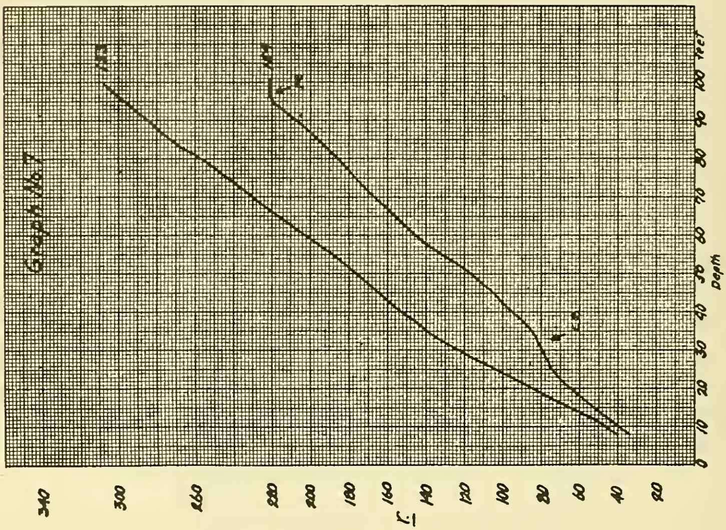
SANDSTONE

The St. Peter sandstone was studied under similar conditions to those of the Kimmswick limestone. Curve 47 of Graph No. 6 indicates the resistivity of the St. Peter in a creek bed where the electrodes were laid directly upon the sandstone in shallow water. Curve 46 indicates the resistivity of the same rock about 25 feet north of 47, where 4–5 feet of alluvium covers the sandstone. Electrodes were parallel in the two set-ups and conditions made as near identical as possible with the exception of the alluvium. The lowering of the resistivity in this case is similar to that with limestone excepting that it is more pronounced here. In fact, curve 46 could as well be taken for shale. In three other instances the writer found low resistivity values for sandstone, due once to clay impurities and for the other two presumably to a mantle of fine wet soil.

The same formation (St. Peter) at approximately the same stratigraphic horizon was run on a high bluff covered with a thin layer of sandy soil. The resistivity was high, as shown in curve 58. The writer again attributes the increase to the drainage of the sandstone, that is, lowering of the water table.

A 50-foot thickness of La Motte sandstone (Cambrian) resting on porphyry was run and its results shown in curve 152. It was located on the side of a moderately sloping stream valley mantled with about 4 feet of soil.

The location of 46 and 47 is on Big Sandy Creek, one mile west of Burr Oak, Lincoln County; 58 is half a mile west of Bethany Church, near Foley, Missouri; 152 is 5 miles west of Fredericktown, Missouri.


Graph No. 7
Depth
feet


Graph No. 6
Depth
feet

IGNEOUS ROCK

Curve 153 of Graph No. 7 shows the resistivity of a diabase dike 4 miles west of Fredericktown, Missouri. The instrument was then moved over to within 50 feet of the contact with the rhyolite porphyry and curve 154 run. The short flat parts occur in this curve where the current and potential electrodes moved over the contact, but the break is not sufficiently pronounced to be used to differentiate the rocks in an unknown area. The curves are given merely as examples of igneous rock resistivities.

Curve 161 of Graph No. 8 illustrates the data taken over bare rhyolite in Stouts Creek above Lake Killarney in Iron County. The resistivity is extremely high and variable.

Insufficient data were accumulated on igneous rock to justify drawing any definite conclusions, but the indication appears to be that the Gish-Rooney method of electrical prospecting can be used more satisfactorily in the stratified sediments than in the igneous.

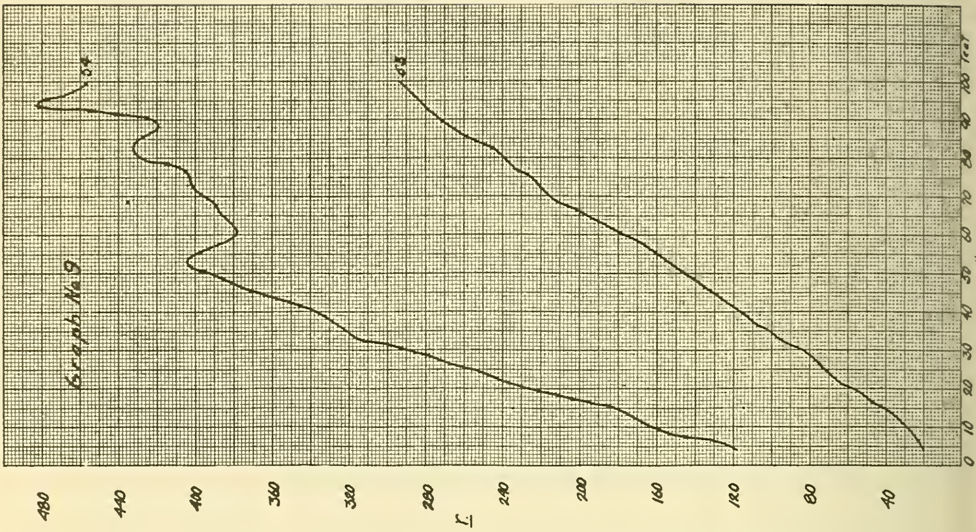
RESISTIVITY AT A QUARRY FACE

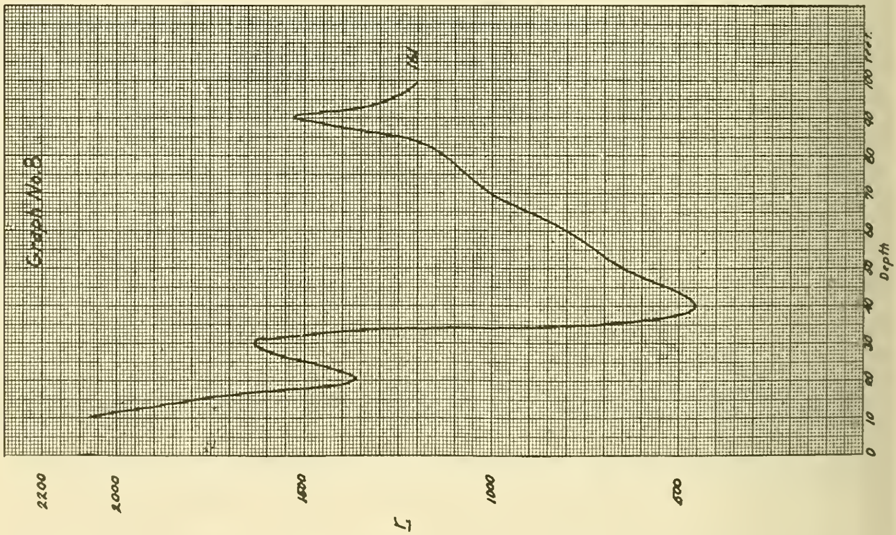
The occasion arises at times when one wishes to obtain, for the purpose of a standard or a known, the resistivity of a deposit which is exposed in a vertical face in a quarry or open mine. If a set-up is made at the edge of the face the figures obtained should theoretically not be the same as those from a set-up 150 or more feet back because half of the earth conductor has been removed at the quarry face. It would seem that the resistivity should be higher at the face because the cross sectional area of the conductor has been reduced. That this is actually true is shown by curves 53 and 54 on Graph No. 9. Curve 53 is the resistivity record of a deposit of Joachim dolomite-Plattin limestone 175 feet back of the face of a vertical bluff 100 feet high half a mile north of Foley, Missouri. Set-up 54 was next made within a few feet of the edge of the bluff and the resistivities found to be consistently 100–150 units higher than a normal set-up.

It is logical to assume that an ore deposit in this bluff would show a higher resistivity than the same ore would when wholly buried. A correction factor derived in the same manner as the foregoing should be obtained and applied wherever the same conditions are met in practice.

ADJOINING HIGH AND LOW RESISTIVITY FORMATIONS

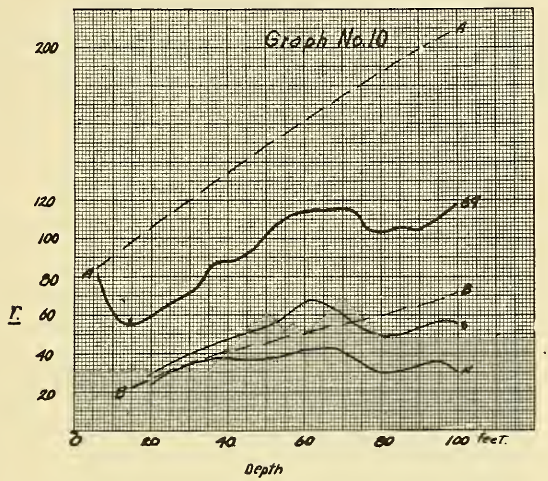
There may be a necessity to make a set-up in some regions on the edges of dipping beds that differ electrically, and if the line of electrodes is run across the strike it is possible that more than one forma-

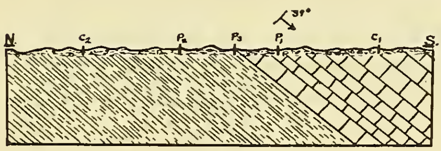

Graph No. 9
Depth
480
440
400
360
320
280
240
200
160
120
80
40


Graph No. 8
Depth
2200
2000
1500
1000
500

tion is included in the circuit. In other instances the width of outcrop of a formation may be so small that the current circuit and even the potential circuit extends beyond its limits and on other formations. One would not expect the measured resistivity to be that characteristic of one formation alone.

An actual test case of this type was run by making a depth determination from a set-up at the contact of two electrically different formations, the Grassy Creek shale and the Chouteau-Burlington





limestones, shown in section below Graph No. 10. The results of the measurement are shown in curve 69, which occupies a position essentially midway between what would be expected for limestone, line *AA*, and for shale, line *BB*. In other words, the resistivity at the contact of two formations should be an average of their resistivities in proportion to the volume of each functioning as a conductor.

It is interesting to note that the increase in resistivity is not in proportion to the increase in depth at this particular station, but that it flattens out. The explanation is that as the depth of current penetration increases, the volume of shale included in the expanding circuit increases relatively at the cost of volume of limestone because of the dip of the rocks, and the net result is the lowering of the rate of increase of total resistivity.

The circuit was also partitioned by using the auxiliary potential electrode, P_3, located at the formation contact. The data are represented by curves *N* and *S*, which show the values of the potentials on the north and south halves, respectively, of the potential circuit. They indicate nicely that the north half of the circuit is only about two-thirds as resistant as the south side, which is wholly in accord with the geology.

There is no indication of the prevailing structural condition in curve 69, and if the rocks were covered, this contact would be passed over unknowingly. A partitioned circuit, however, would give warning, at least, that the geology was changing, if not a fair indication of the nature of the components as a whole.

BEHAVIOR OF MULTIPLE ROCK SYSTEMS

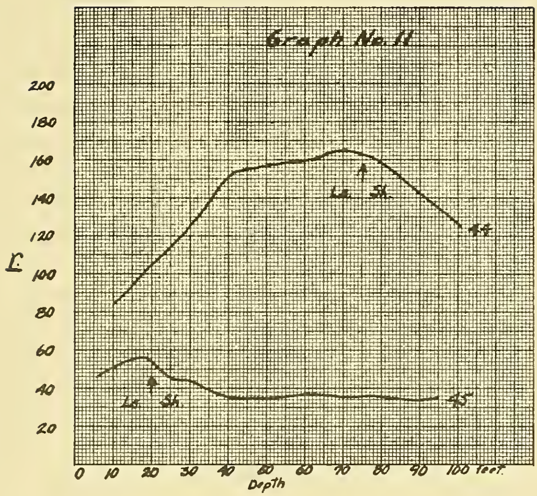
After a consideration of the resistivity of a single type rock, the next logical step is toward a system of two. Suppose that 25 feet of bed *A* underlie 75 feet of bed *B* and a depth determination is made to 100 feet. What will the resistivity figures for the 76- and 100-foot depths be? Will the 100-foot figure be the same as that for 100 feet of *B*, 100 feet of *A*, 75 feet of *B*, 25 feet of *A*, an average of 25 feet of *A* and 75 feet of *B*, or any other combination? The field answer is shown in the following paragraphs.

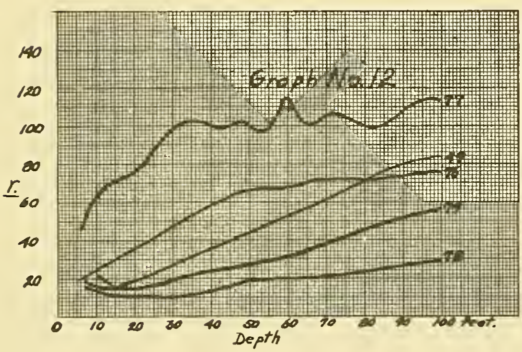
HIGH OVER LOW RESISTIVITY

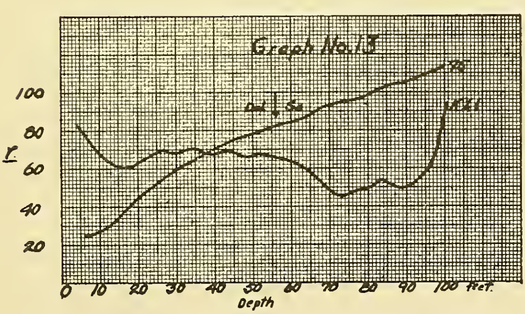
Graph No. 11 carries two resistivity curves of 20 feet and 75 feet of limestone over 80 feet and 25 feet of shale, respectively. In curve 45 the resistivity starts at 46 units and rises in a normal limestone slope to an 18-foot depth and then drops to a shale level. The electrical prediction of shale at 18 feet, that is, the break in the curve, with actual occurrence at 20 feet, is satisfactory.

In 44 the curve is typically that of limestone to a depth of 70 feet, where it falls consistently to that of shale. The electrical prediction of 70 feet is apparently 5 feet in error, but the field notes on this set-up state that the current electrodes were 3 feet below the instrument from the 100-foot mark out. Prediction is again within 2 feet of actuality.

The two curves are clearly illustrative of the fact that the break in the curve follows rather closely the change in the rocks. Instead of designating the maximum or minimum in the curve as the location of the geological change, it is often customary to split the slope on a curve


Graph No. 11
200
180
160
140
120
100
80
60
40
20
r
Ls. Sh.
Ls. Sh.
44
45
0 10 20 30 40 50 60 70 80 90 100 feet
Depth


Graph No. 12
140
120
100
80
60
40
20
r
77
0 10 20 30 40 50 60 70 80 90 100 feet
Depth


Graph No. 13
100
80
60
40
20
r
0 10 20 30 40 50 60 70 80 90 100 feet
Depth

if it is steep and mark the contact of the two formations at that point. Since the slope and the apparent magnitude of break in the curve will vary with the relative scale in plotting values on the axes, the depth predictions might vary slightly if either scale is exaggerated.

The writer finds that the highly conductive (low resistivity) beds apparently exert a greater effect on the current and show a thickness greater than is actually the case. Hence, it is usually more nearly correct to favor slightly the size of the beds of high resistivity at the expense of the low. An exception to this rule of thumb is in the case of overburden of high resistivity, in which case its thickness is usually slightly less than indicated electrically.

Generalizing, where bed *A* underlies bed *B* its resistivity is neither that of normal *A* or *B* at *A*'s depth below the surface but is roughly equal to the average resistivity of the two taken in proportion to the thickness of the two involved.

Stations 44 and 45 were located half a mile west of Dameron, Missouri.

LOW OVER HIGH RESISTIVITY

The curves on Graph 12 indicate the resistivities of increasing thicknesses of shale over limestone. The series was made by setting up near the base and at points consecutively higher on the side of a gently sloping, large hill of Maquoketa shale underlain with Kimmswick limestone. Electrode lines were parallel throughout, roughly following contours.

Curve 77, made with the electrodes on bare limestone in water, is repeated for reference. The resistivity rises with increasing depth of current to about 30 feet in depth, from which depth it remains fairly constant at 100–110 units. Curve 76, also repeated, is from measurements of the limestone under 3 feet of alluvium. Notice that it almost parallels the mean of 77 but is about 40 resistivity units lower, due, no doubt, to the alluvium.

Where 10 feet of shale covers the limestone its (the shale's) higher conductivity lowers the whole curve as shown in 49. The minimum point on the curve checks the 10-foot thickness nicely.

The curve of 20 feet of shale over limestone is shown by 79. Note that the higher conductivity of the shale has lowered the resistivity of the entire remaining thickness of limestone. The agreement between the actual thickness and that indicated electrically is excellent.

A 35-foot thickness of shale over 65 feet of limestone lowered the resistivity of the whole section to that of ordinary shale, as shown in curve 78. The curve indicates limestone at a depth of 30 feet, which is

5 feet in error. The curve as a whole is very poorly diagnostic and is not satisfactory. It is given to show what can and does occur in some determinations. The writer is inclined to believe that because of the fairly low resistivity of the limestone alone, and perhaps because of the nature of the contact with the shale, whatever it is, the total resistivity is low.

A second example of low resistivity rock over high is given in curve *KXI*, Graph No. 13. The log of the hole drilled by a clay auger beneath the place of set-up is:

	Feet
Gravelly soil and clay	0–10
Diaspore and burley clay	10–60
Plastic clay and rotten rock	60–95
"Cotton rock" (Jefferson City dolomite)	95—

The transition in this case from "plastic clay and rotten rock" to dolomite is sharp and clear. The electrical prediction of dolomite at 96 feet was very satisfactory.

Perhaps the upper contact of the dolomite was nonconductive or the resistivity of the dolomite high so that the influence of the clay on the total resistivity was not marked. The writer is not able to explain adequately the difference between the transitions on the two curves.

Station *KXI* was located over a diaspore pit 1.5 miles southeast of Swiss, Gasconade County, Missouri.

The stations shown on Graph No. 12 were located one mile northwest of Elsberry, Missouri.

EQUI-RESISTIVE FORMATIONS

A complete series of 2 formational resistivity curves will include those of 2 rocks of similar resistivity. Such is shown in curve 75 on Graph No. 13, at which location 56 feet of Joachim dolomite overlies the St. Peter sandstone. Since both rocks have high resistivity no marked break in the curve would be expected and there is none. A small irregularity occurs at 60 feet and may represent the disconformable contact, or, being so slight, may simply be caused by a variation in the rock. At any rate it is too small to be used as a mark separating formations in interpreting an unknown curve.

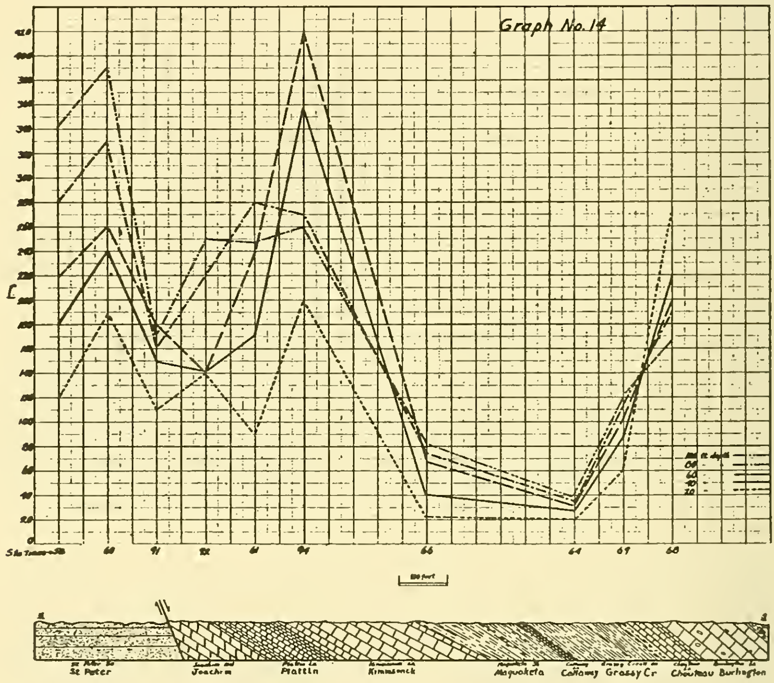
The location was half a mile west of Corinth Church, near Foley, Missouri.

RESISTIVITIES ACROSS FAULT

In the next study measurements were made across an upturned series of rocks and an included fault occurring in Lincoln County, Missouri. Two other applications of the earth resistivity method, traversing, and the location of a fault plane, were tested.

TRAVERSING

Stations were located along a straight line at known intervals and the resistivity of the rocks was measured at uniform depths. These resistivities were plotted against the station locations, giving a resistivity profile along the traverse. A change in the character of the



rocks was expected to show an irregularity in the resistivity profile. That such was the case is shown in the curves and cross section on Graph No. 14. Complete depth determinations were made at each station and the resistivities were plotted for 5 depths, 20, 40, 60, 80, and 100 feet.

LOCATION OF FAULT

By running traverses across a fault at three selected places a small variety of contacts were studied electrically. In one place the St. Peter sandstone was faulted against the Plattin limestone; in another, St. Peter sandstone was faulted against Maquoketa shale; and in the third, Jefferson City dolomite was brought against Kimmswick limestone.

The geologic section of the region and a brief description of the fault are given to aid in understanding the discussion which follows.

	Feet
Alluvium	
Pleistocene glacial debris	
Pennsylvanian	
Cherokee formation	0–50
Mississippian	
St. Louis limestone	75
Keokuk-Warsaw-Salem group	140
Burlington limestone	150
Chouteau limestone	55
Grassy Creek shale	140
Devonian	
Callaway limestone	12–15
Ordovician	
Maquoketa shale	150
Kimmswick limestone	140
Plattin limestone	125
Joachim dolomite	65
St. Peter sandstone	125
Jefferson City dolomite	125

The Cap-au-Gres fault, a normal fault having a stratigraphic throw of about 450 feet in the locality worked, cuts across Mississippi River at old Cap-au-Gres, Illinois, and extends into Illinois and Missouri, trending about N. 85° W. The rocks dip away very gently north of the fault plane but are dragged up in excess of 45° on the south, the downthrown side, flattening out within half a mile. This structure brings up the greater part of the geologic section within this small area.

SANDSTONE FAULTED AGAINST DOLOMITE

Graph No. 14 carries the resistivity profiles of the measurements at 5 different depths made at the set-ups along a line across the fault where the St. Peter sandstone was faulted against the Joachim dolomite. The traverse was also continued across the greater part of the upturned geologic section.

The stations at which the determinations were made were located at fairly regular intervals along a small valley in which the soil thickness was probably as uniform as could be found. The line of electrodes was always north-south, across the strike. Most of the stations were 100 feet apart, under which conditions the 100-foot maximum spacing of the potential electrodes formed a continuous length of potential measurement. The current circuits were overlapping, of course, in those cases. At any rate, the position of the stations is plotted to scale along the abscissa. Resistivity is shown on the ordinate. The geologic cross section along the traverse is drawn at the bottom of the graph in its proper position with respect to the ten stations.

For convenience in analyzing Graph No. 14 let us proceed along the traverse from north to south, left to right along the page.

The first stations, 58 and 60, are on the highly resistive St. Peter sandstone. There is a fairly uniform increase in resistivity with depth; the figures on the whole are high; the curves are roughly parallel; in other words, they are typical sandstone curves. The increase in resistivity of 60 over 58 is believed to be due to increased silicification near the fault plane, evidenced by the fact that the sandstone in its outcrops nearest the fault shows a little crushing and some cracks and veinlets filled with quartz and calcite.

A sharp drop in all profiles occurs at 91. The writer does not believe this to be due to low resistive Plattin limestone or Joachim dolomite, which would be taken in below at greater depth, but rather to possible clay gouge and similar material perhaps well saturated by movement of water along the plane.

Upon advancing to 92 the current is dominantly in Plattin limestone and the resistivity increases. It rises to a maximum over the center of the limestone series at 94.

The low profile over the Maquoketa and Grassy Creek shales needs no comment after one has seen their normal curves. Station 69 shows higher resistivity because the south current electrode has been extending over the Chouteau and Burlington limestone.

Further advance to 68 over limestone is indicated by a rise in the profile. This set-up is one of the most interesting made. Since the instrument was set up at approximately the center of the limestone formation, one would expect, without further analysis of the conditions, a normal increase in resistivity with depth. However, exactly the opposite is what happens. Each increasing depth shows a corresponding decrease in resistivity. But after consideration of the geology of the traverse the explanation is easily made. At a 20-foot depth the outer electrodes are separated 60 feet, still on limestone; but as the potential electrodes are separated wider the current electrode spacing is tripled, and they pass off the limestone at each side, going onto the Grassy Creek shale at the north and the intercalated shale and limestone of the Keokuk-Warsaw group at the south, whereby the resistivity is lowered. With increased spacing more shale is taken into the circuit, outweighing the limestone, and consequently the curve declined all of the way to the end.

The conclusion that a covered fault can be located by an earth resistivity method is self-evident. There is nothing original or new in the confirmation of that fact. The writer does, however, consider that the location of the plane between the sandstone and limestone because of the low resistivity of the fault zone is worth mentioning.

Another item of importance in practical prospecting, that having to do with the determination of the optimum spacing of electrodes for rapid traversing, can be derived, tentatively at least, from this graph. For instance, when a party is doing commercial traversing or reconnoitering, the element of time and expense enters in. It would usually be too costly to make detailed determinations at each station. Instead, only one depth reading or perhaps two which are most diagnostic would be made.

A glance at the profiles of the different depths, although of the same traverse, shows that they are not the same. Their variation, however, is not such that one contradicts the other, as might be argued, but instead, if the geology involved is studied, their interpretation is entirely logical.

Inspection of the graph shows that a 20-foot depth determination is hardly satisfactory because of the relatively great influence of local variations in the soil or mantle rock. At the other extreme, a 100-foot potential electrode spacing takes in 300 feet between current electrodes, which is so large that it usually includes more rocks in the circuit than is desirable. The 80-foot depth, as shown on the graph, is not much better than that of 100 feet. The 60- and 40-foot depths appear to be almost equal in their diagnostic values, perhaps the 60-foot one being a bit more pronounced. On the other hand, if one is prospecting for small ore deposits, the 40-foot spacing would be preferred because its current circuit takes in 60 feet less of rock that might alter the effect of the ore, than does that of the 60-foot spacing. The choice of the two should really be determined by the use to which the prospecting is to be put.

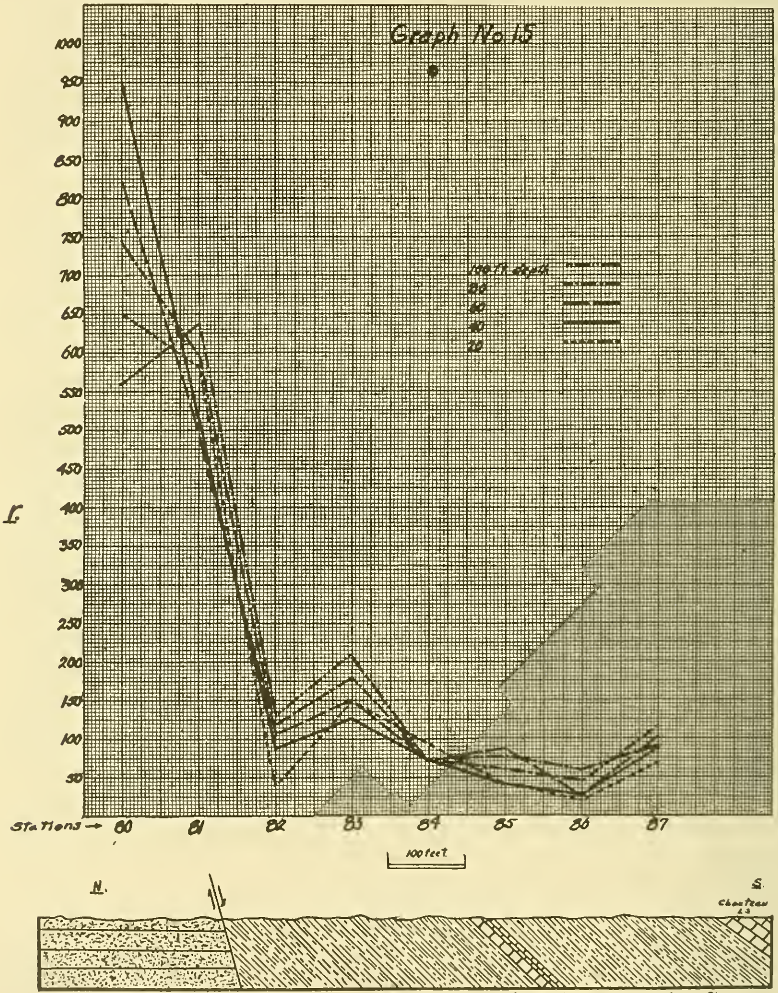
The section shown on Graph No. 14 is located on the C. T. Cannon farm, 3.5 miles southwest of Foley, Missouri.

SANDSTONE FAULTED AGAINST SHALE

A fault where sandstone is brought against shale is sharply indicated by earth-resistivity methods. The curve showing the extremes of high resistivity of sandstone and adjacent low resistivity of shale has an abrupt steep slope between the two that permits ready location of the contact of the two formations.

The resistivity curves on Graph No. 15 demonstrate nicely such a fault condition found near McClain's Creek Crossing, 5 miles west of Foley, Missouri. Determinations were made in the same manner as in Graph No. 14.

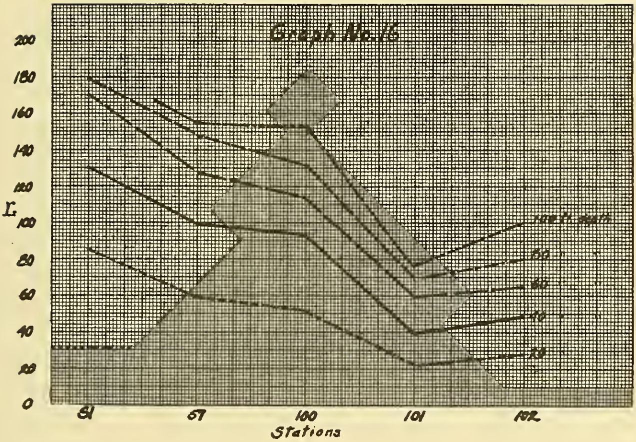
It is evident that a fault is easily detected where sandstone is faulted against shale.

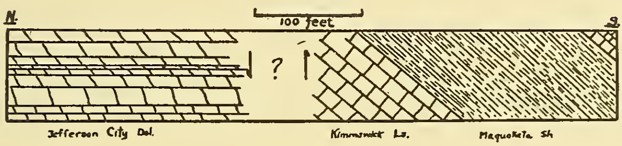


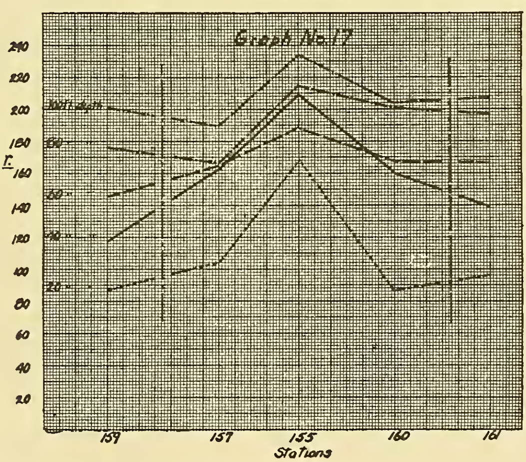
Again the 40-foot and 60-foot depth determinations appear to be most diagnostic for average changes in the rocks at a depth less than 100 feet.

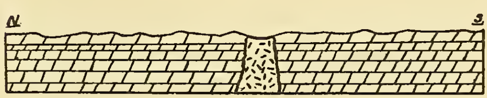
DOLOMITE FAULTED AGAINST LIMESTONE

Jefferson City dolomite is faulted against Kimmswick limestone at Lick Spring Farm, 3 miles south of Foley, Missouri. Graph No. 16


Graph No. 16
r.
200
180
160
140
120
100
80
60
40
20
0
100 ft. depth
80
60
40
20
81
67
100
101
102
Stations


N.
100 feet
S.
?
Jefferson City Dol.
Kimmswick Ls.
Maquoketa Sh


Graph No. 17
r.
240
220
200
180
160
140
120
100
80
60
40
20
100 ft. depth
80
60
40
20
159
157
155
160
161
Stations


N.
S.

shows the cross section and resistivity profile along a traverse crossing the fault there. The resistivities of the dolomite and the limestone are so nearly alike that no direct indication of the fault is given by this method. There was no evidence of clay gouge in the fault plane or variation in resistivity due to mineralization. The resistivity of the rocks is of little direct value in this case.

However, if the observer keeps his geologic section in mind, he may be able to approximate the position of the fault or at least be aided in his search by the location of the Maquoketa shale. He may be aided indirectly, although given no direct evidence.

This study serves to indicate that earth resistivity methods of prospecting are no panaceas for all geologic ills but simply additional tools useful in prying loose the sometimes tightly held secrets of Mother Earth, and that to gain most information from the electrical data of the rocks their geological relations must ever be kept in view.

A consideration of the three resistivity profiles across the fault plane leads to the following conclusions.

1. The mere presence of a fault plane does not necessarily cause a break in the resistivity profile. The resistivity at the plane is not necessarily a convenient and opportune variation from those of its flanks, regardless of what their values may be.

2. However, the location of the position of a fault plane is to be expected more usually as a result of fundamental differences in the resistivity of the two adjoining formations.

ALNOITE PIPE IN DOLOMITE

The writer's study of the resistivity of igneous rocks was too brief to be conclusive, as has been said before, but one set of curves, taken where an alnoite pipe is intruded into dolomite, is believed to be significant enough to be included because of the possible application to the location of intrusives into sediments. The pipe described by Singewald and Milton[5] is exposed for about 120 feet in length and 15 feet in width. It occurs in a small ravine eroded in a wide, gently sloping hillside which affords an excellent surface for earth resistivity tests.

The traverse was run from the dolomite on one side, across the pipe, and onto the dolomite on the other side. The line of electrodes was run in a north-south line which was fairly level and in a direction transverse to the long axis of the outcrop.

[5] Joseph T. Singewald, Jr., and Charles Milton, "An Alnoite Pipe, Its Contact Phenomena, and Ore Deposition Near Avon, Missouri," *Jour. Geol.*, Vol. 38 (1930), pp. 54–66.

Graph No. 17 carries the resistivity profiles along with the probable section sketched below it. Curve 159, which was taken on the north end of the traverse, 600 feet north of the pipe, is a normal dolomite curve. After advancing to 157 we find the resistivity to rise rapidly at lower levels and then flatten. Apparently a highly resistive body lies at the south.

Station 155 is directly over the pipe. Its entire resistivity determination, and particularly that for the 40-foot depth, is somewhat higher than that for the dolomite. Apparently alnoite is more resistive than dolomite.

Normal dolomite resistivities are resumed by moving over 160 and 161.

It is questionable whether the earth-resistivity method could be relied on to locate similar pipes or dikes in a similar dolomite. The increase in resistivity over this pipe is relatively small, and that amount of variation can be caused as well by such other factors as variations in soil and topography. In this case, however, there is hardly any doubt that the rise in resistivity is due to the igneous rock. If no other evidence of the existence of an intrusion had been at hand, the profile on Graph No. 17 would probably not have been interpreted as it has been.

However, the method could be used to supplement core drilling or to follow buried dikes from their exposures.

In view of the resistivity values obtained from these set-ups there is hardly any doubt that intrusions in shales could be detected readily.

CONCLUSIONS

The data obtained by the writer, using a modified Gish-Rooney earth-resistivity apparatus while working at depths less than 100 feet, lead to the following conclusions.

1. Damp alluvial and residual soil, shale, clay, and other argillaceous materials show a relatively low resistivity.

2. Gravelly deposits and soil, sandstone, limestone, and igneous rocks show a high resistivity.

3. The same formation shows a higher resistivity where located in a well drained place than where saturated with water.

4. The resistivity of a formation taken through an overlying one is roughly equal to an average of the resistivities of the two in the proportion to their thicknesses involved in the test.

5. If both current electrodes or potential electrodes are not set in the same formation, or if a foreign body lies between the electrodes,

the portions of the circuits on each side of the observer will be unsymmetrical or unequal electrically. The difference between the two is measured by connecting in a centrally located supplementary potential electrode placed at the instrument. This partitioning operation is important in prospecting where lateral changes in the rocks are significant or critical.

6. When rapid traversing is being done in search for deposits or lateral changes at depths less than 100 feet, an electrode spacing of 40 feet or 60 feet seems to be the optimum.

7. The location of a bed, deposit, structural change, or water table is due primarily to differences in inherent resistivity of the bodies rather than to a difference induced by the transition surface of the bodies or conditions. Hence, it is difficult to differentiate between two unlike adjacent bodies having a similar resistivity.

8. The routine of making measurements and obtaining resistivity data is the lesser part of the work of the resistivity prospector. The logical and proper interpretation of the resistivity "code" is by far the greater thought-provoking part of his task. For a most efficient and successful prosecution of the method, the observer should correlate the resistivities, so far as possible, as they are obtained in the field with the changes in topography, soil, water table, and probable changes in geology. For that reason, data which are obtained by an untrained observer in "cookbook" fashion can not be interpreted in the office to their fullest extent. It is likewise most essential that the observer have an understanding of the fundamentals of geology.

9. The earth-resistivity method can probably not be advantageously employed by one going into an area in which the geology is entirely unknown. It will probably find its greatest value in supplementing and filling gaps between core-drill prospects, locating the position of hidden structural changes thought or known to exist in the area, and rapidly, approximately, and cheaply determining depths and thicknesses of strata, particularly overburden over solid rock, and in use in some types of rapid reconnaissance work.

BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 18, NO. 1 (JANUARY, 1934), PP. 63-68, 3 FIGS.

RADIOACTIVITY OF SOIL GASES[1]

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ABSTRACT

The Alpha-ray activity of the radon in the gases drawn from the soil was measured by means of a Wulf-type electroscope having a 10-liter ionization chamber. The method of making the measurements is described. Indications that a fair sample of gases was obtained at a station were found by repeating measurements at the station on separate days. However, there are some conditions under which repeated measurements may not check. A typical line survey in the Gulf Coast region shows large variations in radioactivity of the gases in the soil. Similar variations are found in a faulted region where there seems to be little hope of correlating the positions of high radioactivity with the positions of the faults; this agrees with the work of Botset and Weaver.

INTRODUCTION

Radioactive minerals are found distributed throughout the upper crust of the earth and are thought to be concentrated for the most part in a limited layer (1)[3] of the order of tens of kilometers in thickness. Due to the presence of these minerals in the earth, a knowledge of the processes of radioactivity has been of great value in geophysics, especially in connection with the accounting for the generation of heat in the earth and with calculations of the ages of the various layers in the earth's crust.

In applied geophysics the properties of radioactive minerals have been of importance mainly in the location of the radioactive ores themselves. However, Ambronn (2), Link and Schober (3), and Mueller (4) have made surveys of the content of radon in the soil gases over subsurface features of the type represented by faults, in the close neighborhood of which an increase in radon content was found. Bogojavlensky (5) has made surveys of the intensity of penetrating rays coming from the radioactive minerals in the top layer of the earth and has reported finding radiations many times more penetrating than the known radiations from radioactive substances. These results can hardly be considered as well established. Bogojavlensky (6) has also claimed to have found an increase in intensity over pools of oil in

[1] Read before the Geophysics Division of the Association at the Houston meeting, March 24, 1933.

[2] Geophysics research department, Humble Oil and Refining Company. Introduced by L. W. Blau.

[3] Parenthetical numbers refer to Bibliography at end of this article.

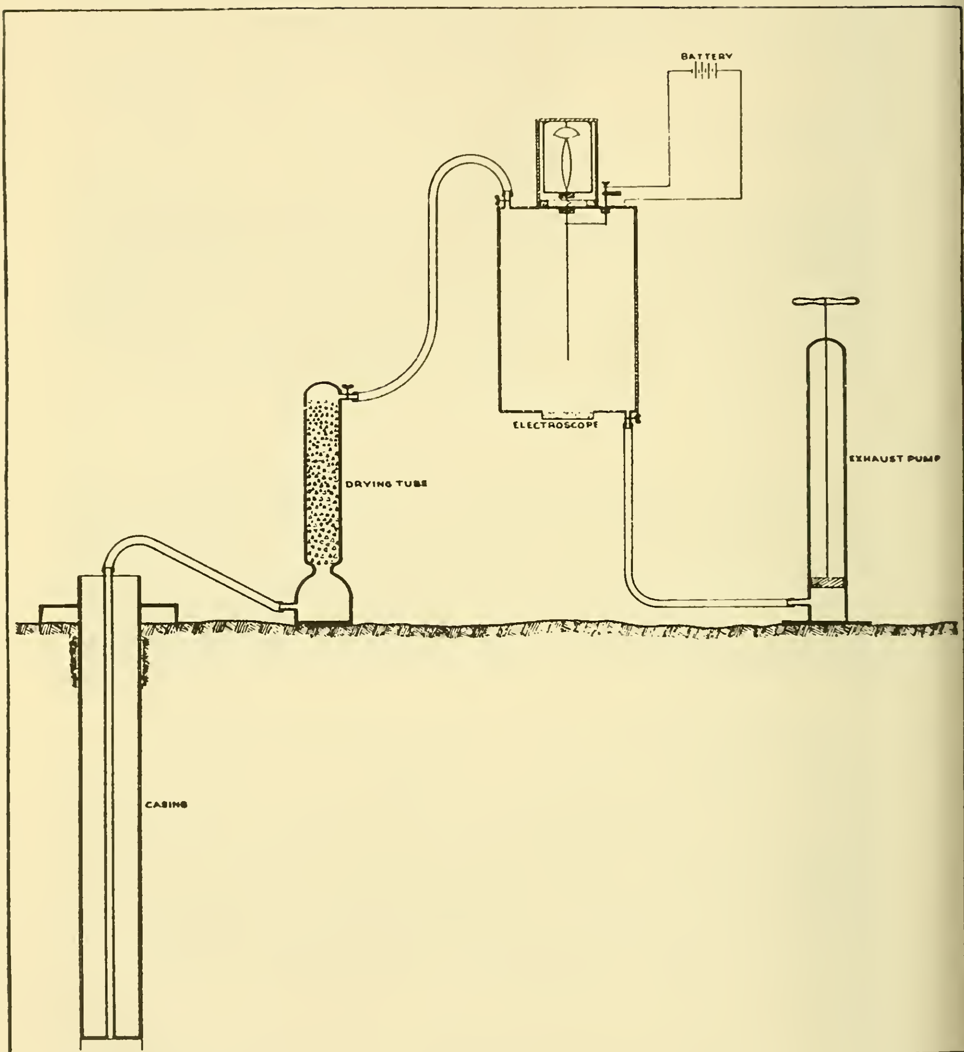

FIG. 1.—Apparatus for making relative measurements of radon content of soil gases.

one or two cases. This result can have little practical application in the direct location of oil. Practically all of the radiations from radioactive substances are absorbed by a few feet of soil and since oil usually occurs hundreds of feet below the surface of the earth, the radioactive minerals associated with oil are in practice much too deep to be detected.

EXPERIMENTAL

For the purpose of making relative measurements of the radon content of the soil gases, an apparatus resmbling that of Elster and Geitel (7) was constructed as is shown in Figure 1. The gases are drawn out from a hole in the soil about 5 feet deep through the inside tube of a 3.5-inch iron casing driven into the hole. The circular flange at the top of the casing driven into the ground helps to give a better seal at the surface of the ground. The gases are passed through calcium chloride drying tubes into the 10-liter ionization chamber of the electroscope which contains a phosphorus pentoxide drying dish. The gases are pulled through the system by a suction pump.

The electroscope for measuring the Alpha-ray activity of the gases is of the Wulf type. In the upper chamber, the platinized quartz fibers, whose deflection is read with a reading microscope, are suspended at the top from an insulating quartz fiber bow. The fiber system is insulated from the case at the lower end by means of an amber plug. The air in the upper chamber is dried with phosphorus pentoxide. The fiber system is connected to the rod electrode in the lower chamber, the rod being held in an insulating ambroid plug. The charging lever is shown at the top of the ionization chamber on the right. A bank of a dozen small 22.5-volt B batteries is used as a source of charging potential.

Although the Elster and Geitel method of filling the ionization chamber would not seem to be as satisfactory as the later method used by Ambronn (8), since there is a possibility that the original air in the chamber may not be completely pumped out and the quantity of this air remaining may vary with each filling of the chamber, still the consistency of the results obtained by repeating measurements with gases from different holes at a given point would seem to indicate that the method was reliable enough for this type of measurement, in which consistency of repeated readings rather than precision of any particular reading seems desirable.

The Wulf-type electroscope is quite suitable for field work in spite of the fact that it has a delicate suspension. This suspension will stand rather severe jars and the calibration remains essentially constant with time. The sensitivity is such that the readings can usually be taken over time intervals sufficiently short so that the temperature changes are too slight to change appreciably the deflections of the fibers by expansion or contraction of the frame holding the fibers.

In the field procedure, readings taken with a stop-watch, of the discharge of the fibers, usually over a range of four divisions on a 100-division scale in the eye-piece, are begun immediately after the soil gas is sucked into the chamber from a hole bored in the ground with an augur. The pumping is usually maintained uniform by making a given number of strokes of the pump. These readings are repeated until a fairly constant value is found. The thoron in the soil gases at some points causes a more rapid discharge in the beginning, but since it decays very rapidly with a half-life of 54.5 seconds, its effect is soon negligible. There is, of course, an increase in activity while the active deposit is coming into equilibrium with the radon, but this increase is fairly slow after a short time.

RESULTS

As an example of consistency which could be obtained under favorable conditions, the following data are presented which show the rates of discharge at one station on different days.

Date	*Rate of Discharge Divisions per Second*
Dec. 8	.22
Dec. 21	.12
Dec. 23	.13
Dec. 29	.11
Dec. 31	.11
....	.17

The first five readings were taken with gas taken from different holes spaced along a line with an interval between holes of the order of 4 feet. Even the discrepancy between the two extreme readings is not serious, since in a field survey, a "radioactive low" reading is of an entirely different order of magnitude from a "radioactive high" reading.

The following data were obtained in another area at several different stations, the interval between stations in general varying from about 200 to 1,500 feet. The readings at these stations were repeated after the capacity, and hence the sensitivity, of the electroscope had been changed. A new hole was drilled at each station within a few feet of the original hole for these check readings which were made several days after the first readings.

Station	First Reading (Time of Discharge of 4 Scale Divisions)	Second Reading After Change of Sensitivity (Time of Discharge of 4 Scale Divisions)
A	10 Sec.	24 Sec.
B	3.5	8.5
C	2	3.5
D	(Very wet hole)	>40
D	3.4	10
E	21	50
F	6	13
G	45	85

The check is very good (the ratio between readings being of the order of 2), especially since the error in measuring some of the shortest time intervals is appreciable.

However, consistent results are not always obtained. For example, in dry cracked ground the readings are increased after a rain has closed up the cracks. After a rain not only is there a possibility that a better seal can be obtained around the casing and that the gases in the ground can not diffuse into the atmosphere so freely, but also, as is well known, dry radioactive salts tend to retain the radon produced by their decay. The radon can be released by dissolving the radio-

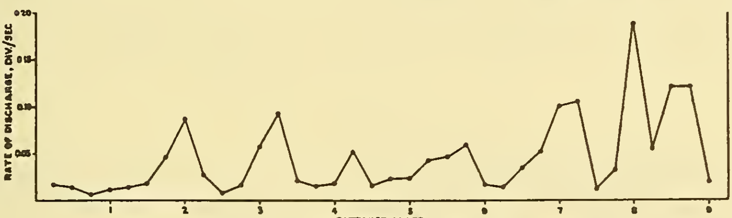

FIG. 2.—Typical line survey in Gulf Coast region.

active salts, and thus it may be that radon is set free in dry ground after a rain by this process of dissolving dry radioactive salts. However, if the ground becomes too wet, it becomes difficult to extract gas as the ground pores are filled with water for the most part. There are a number of factors which influence the radon content of the soil gases, and since these have been discussed at length by others, it seems hardly worth while to discuss them further.

A typical line survey of the Gulf Coast region is seen in Figure 2, in which the rate of discharge of the electroscope in divisions per second is plotted in the ordinate direction and the distance in miles is plotted in the abscissa direction. It will be seen that the radon content of the soil gases varies in a rather erratic fashion from point to point even though there is no apparent change in soil, except of a local nature. The curves in Figure 3 represent two line surveys in a faulted region. The positions of the supposed faults are shown beneath the curves. It is readily seen that any sort of correlation is difficult. The distribution seems erratic, as in the case of the previous curve for a supposedly non-faulted region. This lack of correlation between faults and radioactive "highs" is in agreement with the work of Botset and Weaver (9), who also worked in Texas. Along a line in one region, Clark and Botset (10) found a rather close correlation between the radon and the heavy mineral content of the soil.

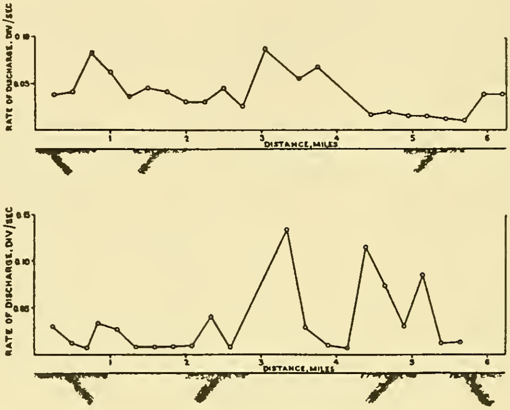

FIG. 3.—Line surveys in faulted area.

Although it is possible that the methods of radioactivity may be of some value in the study of local soil variations, at the present stage of our knowledge there seems to be little application for these methods in petroleum geophysics, at least in the Gulf Coast region.

In the work reported herewith the observations in the field were taken by Bruce Hill under the direction of L. W. Blau.

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BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 18, NO. 1 (JANUARY, 1934), PP. 69-96, 11 FIGS.

MAGNETIC AND TORSION-BALANCE SURVEY OF MUNICH TERTIARY BASIN, BAVARIA[1]

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ABSTRACT

The Munich Tertiary basin consists of a structural basin of Tertiary sediments. It is bounded on the east by the Bohemian crystalline massif; on the north by Mesozoic sediments which dip southward under it; and on the south, it has been overridden by the Alpine overthrusts. The whole Bavarian part of the basin has been covered by a close net of lines of observation of the vertical component of magnetism. Two torsion-balance lines were run north-south across the basin, and some additional cross lines were run. The geophysical survey indicates: (1) the surface of the crystalline basement dips southwestward across under the Tertiary basin; (2) the Bohemian crystalline massif seems to dip under the Austrian continuation of the Tertiary basin, to connect with the crystalline core of the Austrian Alps; (3) the Bohemian massif seems to have a sharp subsurface scarp on the southward prolongation of the Regensburg fault; (4) the basement under the Tertiary sediments is not the crystalline basement, but presumably the surface of a wedge of Mesozoic rocks; (5) the crystalline core of the Vendelician ridge must be south of the present Munich Tertiary basin; (6) a line of hitherto unsurmised structures with basaltic cores lies along the Danube from Vohburg to Ulm; (7) the cause of most of the residual magnetic anomalies is not clear, but the Passau axis of the maximum seems to reflect structure with the basement; (8) no folding is indicated parallel with the front of the Alps.

INTRODUCTION

This paper presents the more broadly interesting results of an extensive magnetic survey and a less extensive torsion-balance survey which were made of the Munich Tertiary basin by the Bayerische Mineral Industrie A. G. in the search for possibly petroliferous structure. The execution of the surveys was under the immediate supervision of E. Gundermann. The field torsion-balance observations of the reconnaissance traverses were made under contract by "Seismos" Gesell.G.m.b.H. of Hannover. The field magnetic observations were made by O. Keunecke, G. Meyer, and W Wolff, and G. Sander under the supervision of Gundermann. General direction from a distance and interpretation of the results of the geophysical surveys were by the writer. The whole program of the Bayerische Mineral Industrie was under the direct supervision of J. Elmer Thomas.

[1] Read before the Geophysics Division of the Association at the Houston meeting, March 24, 1933. Part of this paper was read before the XVI International Geological Congress at Washington, D. C., July, 1933. Published by permission of J. Elmer Thomas, Bayerische Mineral Industrie A. G. A more detailed description of the survey and more extensive discussion of the results is to be published by the *Geologische Abteilung*, Bergamt, Bayern.

[2] Consulting geologist and geophysicist, Petroleum Building.

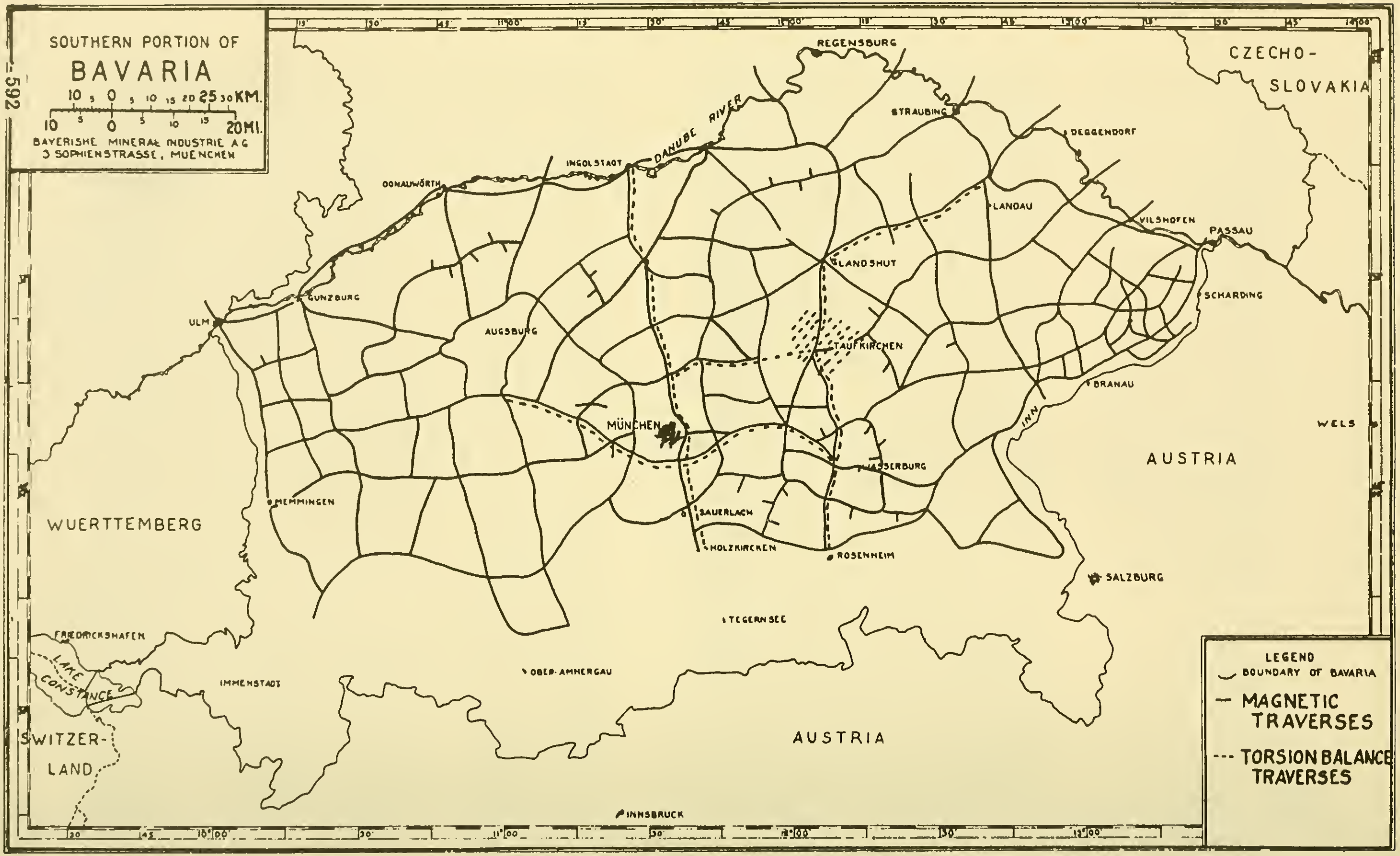


FIG. 1.—Map of B. M. I. magnetic and torsion-balance traverses.

The torsion-balance survey (Fig. 1) comprised 1,195 reconnaissance stations and 366 detail stations. The reconnaissance stations are disposed: (1) in two north-south lines, one from the Danube at Ingolstadt to the edge of the foothills at Holzkirchen, and the other from Landshut to the foothills at Rosenheim; and (2) in a composite east-west line through the central part of the basin. A station interval of 200 meters was used on the Ingolstadt-Holzkirchen traverse in order to detect structures with narrow anomalies as well as structures with large anomalies and to determine the degree of surface irregularity of density. The station interval subsequently was increased to 400 and 600 meters. A latitude correction of 8E was used in the routine calculation of the gradient. The gradient effect of the Alps was neglected; the gradient which is produced at Sauerlach by the topographic mass of the Alps was calculated to be $+1.4$ for U_{xz}; as that gradient varies slowly and smoothly and is small compared with the observed gradient, it need not be eliminated unless quantitative calculations of structural profiles are made or unless the position of structures near the foot of the mountains needs to be determined with high accuracy.

The magnetic survey (Fig. 1) comprised 14,743 stations with a Schmidt-Lloyd vertical variometer. Those stations are disposed in a criss-cross series of reconnaissance traverses which covered thoroughly the whole of the Munich Tertiary basin and which extended into the foothills of the Alps on the south, and across the Danube into the Frankish-Schwabian Jura on the north, and the area of the Bohemian massif on the northeast. A primary base line of magnetic benchmarks was run between Ingolstadt and Holzkirchen; the value of Δz was determined by rapid runs in an automobile between benchmarks until a consistent series of determinations was obtained; from that base line, a net of primary benchmarks was similarly established through the rest of the area. The traverse lines were adjusted to those primary benchmarks and to each other. A station interval of 200–300 meters was used on most of the traverses. Careful checking back and careful checking in on base stations and benchmarks was practiced. Corrections were applied for the fluctuations of the intensity of the earth's magnetic field as observed at Pottsdam. The following correction for the normal increase of intensity with magnetic latitude was applied in a routine manner in the station calculations: $+8\gamma$ per 1 minute of increase of (astronomic) latitude and -1.5γ per 1 minute of increase of longitude. The magnetic survey was run as an inexpensive and rapid method of possibly supplementing the slower and more costly torsion-balance survey. The expectations of it were not great.

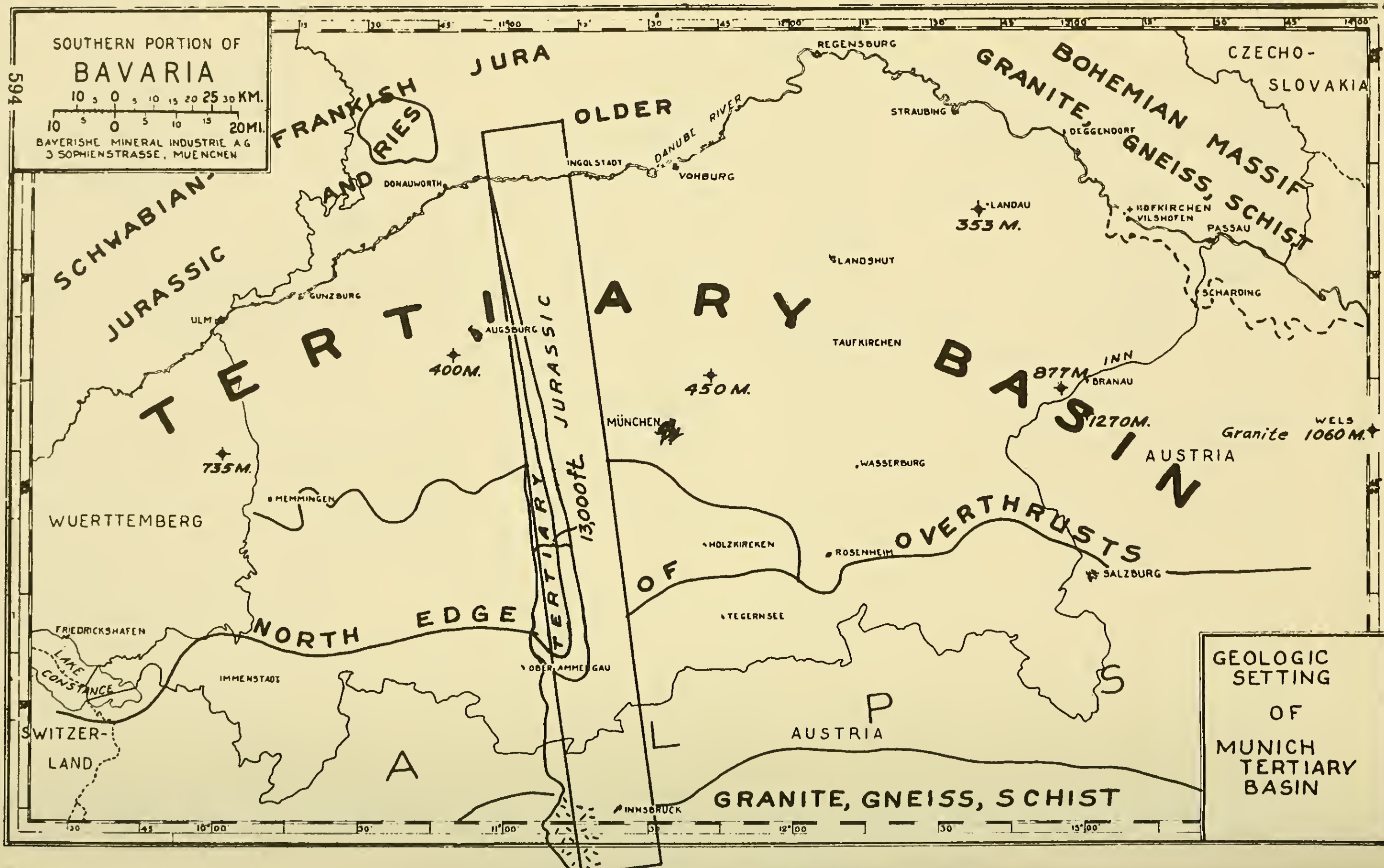


FIG. 2.—Sketch map of geologic setting of Munich basin.

GEOLOGICAL SETTING

The Munich Tertiary basin of southern Bavaria is a local widening of the belt of Tertiary formations which lies in front of the northern edge of the Alpine mountain system and which extends from western Switzerland through northern Switzerland, southern Wuerttemberg, southern Bavaria, into Austria, to connect with the Vienna Tertiary basin (Fig. 2).

North of the Munich Tertiary basin, there is the broad east-northeast-west-southwest zone of the Schwabian and Frankish Jura, an area of gently dipping Jurassic sedimentary beds. The dip in general is southerly, toward the basin, although at the eastern end of the zone, the Upper Jurassic beds dip gently eastward under a thin cover of Cretaceous, and into the syncline which extends northward from Regensburg. The Malm (Upper Jurassic) and under it, the Middle and Lower Jurassic and the Triassic formations, dip under the Tertiary beds of the Tertiary basin, but the southward extent of the several members of the Mesozoic under the Tertiary basin is unknown from geologic data. The Mesozoic beds of corresponding age which crop out in the Alps are of different aspect and were deposited in an ocean basin which had no connection with the basin in which the Mesozoic sediments of Germany were deposited; and a Mesozoic landmass must have separated the two ocean basins. That ancient Mesozoic landmass has been termed the Vindelician ridge. The Mesozoic formations which dip southward under the Tertiary beds, somewhere at the south must wedge out against the north flank of the ridge. The Tertiary beds in general transgress northward over the Mesozoic formations; in the east, probably over Cretaceous; but farther west over the Malm (Upper Jurassic). At the surface, from Vohburg west to Ulm, however, the Tertiary and Jurassic beds are presumed to be in fault contact.

The Jurassic and Triassic formations are composed of well consolidated, or fairly well consolidated beds presumably of high density. The Jurassic-Triassic stratigraphic section in order of increasing age and depth has the following character: Malm, massive limestone and dolomite; Dogger and Lias, shale, marl, limestone, and sandstone; Keuper, shale, marl, sandstone; Muschelkalk, massive limestone and anhydrite; Buntsandstein, massive sandstone.

The famous Ries area lies in the zone of the Frankish-Schwabian Jura. It is supposed to be the result of an enormous volcanic explosion; and in the general area around the Ries, the Jurassic beds have been cut by a considerable number of igneous intrusions.

On the east-northeast side of the basin, there is the Bohemian crystalline massif composed of granite, gneiss, and schist. From Regensburg to Hofkirchen the Tertiary beds are in fault contact with the crystalline rocks. But farther south, in the Passau district, the Tertiary beds are at least partly transgressive over the crystalline rocks and over a few small remnants of Jurassic; and there certainly is no fault contact between the Tertiary beds and the crystallines comparable with that between Regensburg and Hofkirchen. The Danube flows on the contact between the Tertiary beds and the crystallines from Regensburg to Hofkirchen; and across the crystallines from Hofkirchen to Passau.

On the south, the Tertiary basin is overthrust by the Alps, whose northward driven overthrusts have overridden the southern part of the basin; how far, is a question; and the Tertiary beds of its southern edge, and underlying Cretaceous beds, have been turned up on end.

Gumbel's Vindelician ridge, or divide of dry land, is supposed to have come into existence at the beginning of the Triassic, and, according to Beyschlag, to have continued in existence throughout the Mesozoic and into the beginning of the Tertiary. It is presumed to have extended from the Bohemian massif through the Swiss-Upper Bavarian upland area to the Plateau Centrale of France.

The thickness of the Tertiary sediments in the Munich basin is unknown from geologic data. The granite was reached at a depth of 1,060 meters in a test well at Wels in Austria. The deepest (Eisenhub) well, in Austria, south of Braunau and not far from the Bavarian border, went to a depth of 1,250 meters and stopped in the Oligocene. The Ochsenhausen well in Wuerttemberg, not far from the Bavarian border, went to a depth of 738 meters and stopped in the Upper Oligocene. The deepest wells in the Bavarian part of the basin are the Juhlbach wells west of Braunau, the deepest of which went to a depth of 877 meters. The next deepest wells (other than the other Juhlbach wells) had depths only of 430, 400, 353 meters.

MAGNETIC AND GRAVITY QUANTITIES REPORTED

The magnetic and gravity quantities which are reported in this paper are what may be called the "structural" variation of the vertical component *Z* of the terrestrial magnetic field and the "structural" variation of gravity.

The results of observations of such quantities as the intensity of *Z* or of gravity can be reported in various ways. In geodetic surveys the value of gravity commonly is given (1) as the observed value at the station, (2) corrected for topography and reduced to sea-level by one or more different formulae; (3) corrected also for isostatic compensation. In geodetic surveys, the absolute value of Z is reported for the date of the observation or it may be corrected to some standard date. The value of gravity varies with astronomic latitude and the value of Z with magnetic latitude. In geodetic work, commonly no correction is made for variation with latitude. A variation of gravity which can be calculated from the results of torsion-balance surveys is the variation of gravity at any equipotential surface, that is, level surface which lies approximately at the general elevation of the surface of the area, for (1) the horizontal gradient of gravity, which the torsion-balance measures, varies slowly with elevation and (2) torsion-balance surveys are made only in areas of slight relief. That variation of gravity would be comparable with the variation of geodetic observed values of gravity reduced by the "free air" reduction to a common elevation essentially that of the surface. Torsion-balance surveys such as those of the B.M.I. are made in search of structure; the effect of the variation of gravity with latitude ranges from 6 to 8 Eötvös units[3] in temperate latitudes and therefore seriously obscures the very many structural anomalies whose maximum gradient may range only from 7 to 15 Eötvös units. The effect of the latitudinal variation of gravity, therefore, is eliminated in the station calculations of the gradient. Relative gravity which is calculated from torsion-balance observations, therefore, is not directly comparable with any of the various types of values of gravity which are given in geodetic reports. The vertical component Z of the earth's magnetic field similarly varies with magnetic latitude; the latitudinal variation obscures the structural anomalies and therefore is eliminated in the routine of the station calculations in most magnetic surveys by oil companies. The observed variation of Z_s of such surveys, therefore, is not directly comparable with the variation of Z in geodetic magnetic surveys.

The observed variation of gravity (Δg_s) of this paper, therefore, is the horizontal variation of gravity at a common level, which is any level between 300 and 500 meters elevation above sea-level,—corrected for topography and for the variation with latitude. If the earth's crust and interior were perfectly homogeneous, no variation of gravity would be observed. Any inhomogeneity in the earth's interior could produce faint large-scale anomalies. The variation of density in the earth's crust and, mostly, in the upper kilometers of the earth's crust produces most of the observed anomalies of this paper and of surveys similar to the B.M.I. torsion-balance survey.

[3] 6 to 8×10^{-9} C.G.S. units per horizontal centimeter.

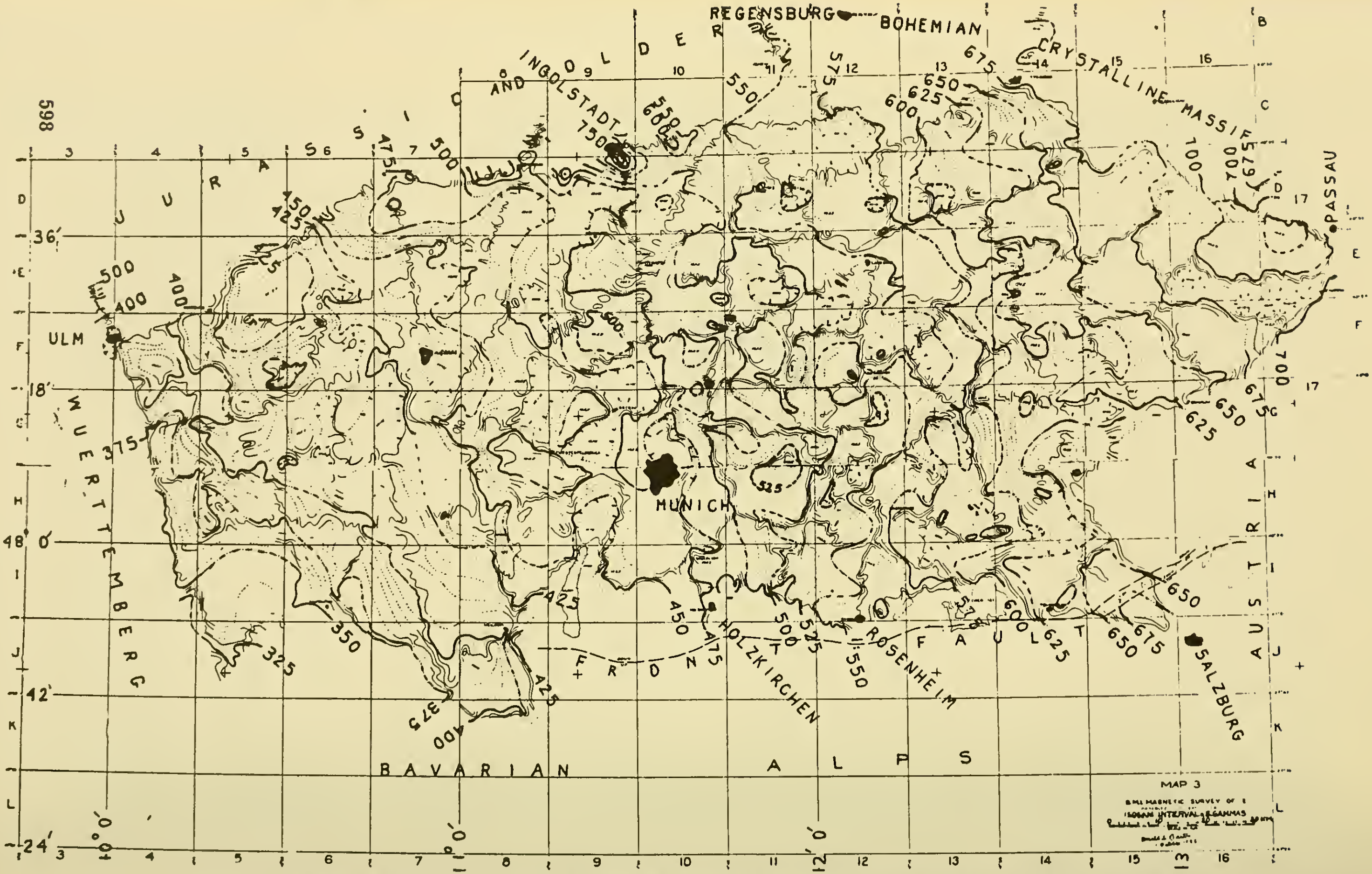


Fig. 3.—Observed variation of Δs_z, vertical component of terrestrial magnetic field. Isogam interval, 5 gammas.

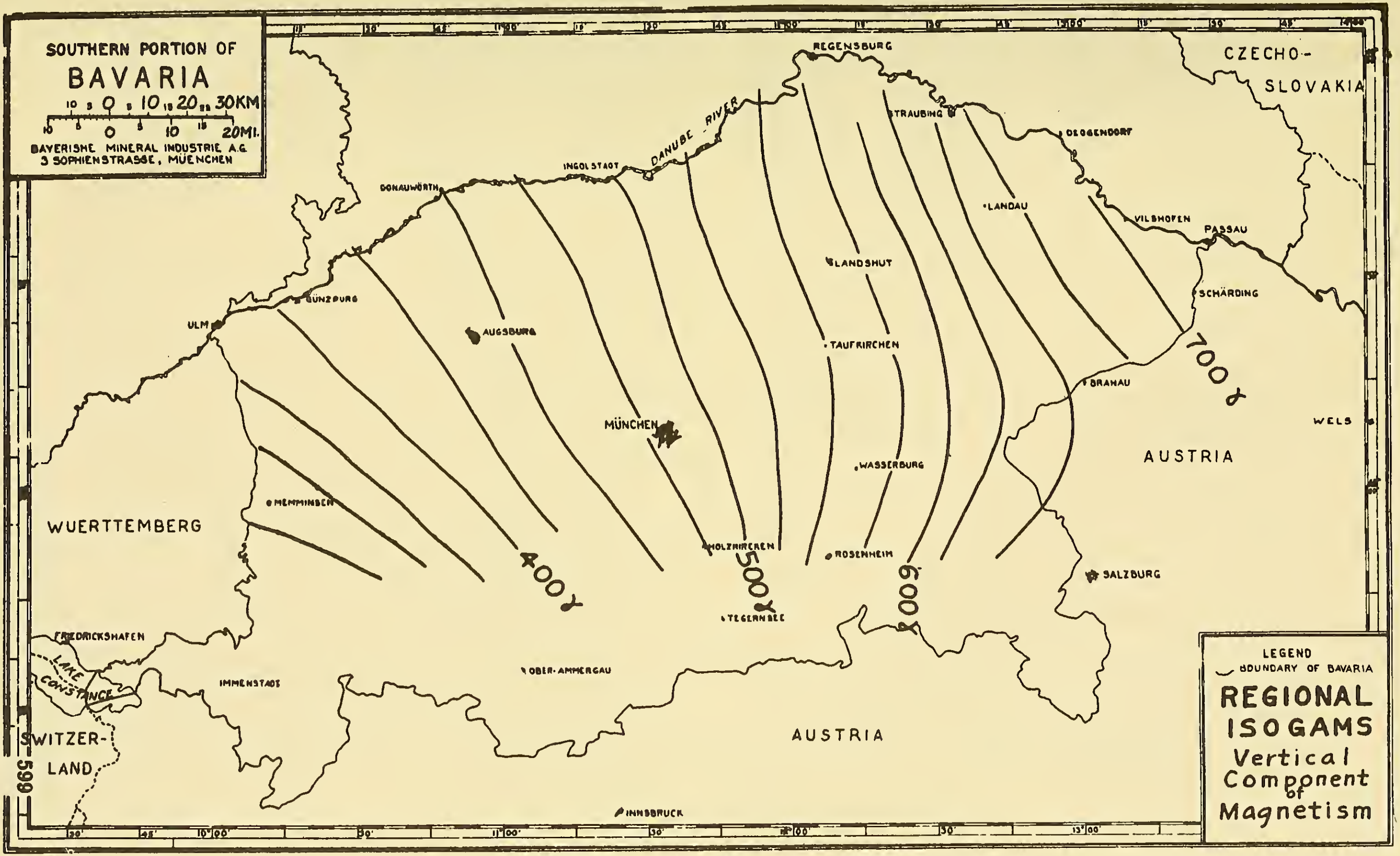


FIG. 4.—Regional variation of Δz_s.

The observed variation of Z_s, the vertical component of the earth's magnetic field, in this report gives the horizontal variation of Z corrected for latitudinal variation and as of the time when the first observation was made at the primary base station. The observed variation of Z_s is the effect of the variation of magnetic permeability in the earth's crust, and mostly of the upper 10 kilometers.

The numerical values of Δg and Δz of this paper are not absolute values, but are relative to some convenient arbitrarily chosen value at some base station.

The absolute value of gravity along the torsion-balance traverses can be calculated from the observed values of relative gravity of this paper (Fig. 5) by re-introducing the latitudinal effect and by tying those recorrected values to the known value of absolute gravity at Munich. To obtain the value of gravity at the surface, a correction for elevation would also have to be applied.

The absolute value of Z along the magnetic traverses can be calculated from the observed values of this paper (Fig. 3) by re-introducing the latitudinal effect and by tying those recorrected values to the known value of absolute Z, wherever known within the area.

REGIONAL VARIATION OF Z_s AND OF GRAVITY (g_s)

The regional variation of the vertical component of Z_s, the vertical component of the earth's magnetic field, is shown in Figure 4. The observed magnetic isogams are shown in Figure 3. The complex irregularity of their winding obscures their regional pattern. In Figure 4, the 25 isogams have been smoothed out without regard to local anomalies. The smoothing is not difficult over most of the area; but from Ingolstadt westward along the Danube, the large local anomalies obscure the regional picture; and for the area from Ingolstadt west, it is a question whether the northern ends of the regional isogams should not show a curvature westward.

The following characteristics of those isogams should be noticed: (1) their trend in general is north-northwest and south-southeast, approximately parallel with the southwest edge of the Bohemian massif; (2) they curve toward the southwest in the southeastern part of the area, in front of Salzburg; (3) the spacing between the 575, 600, and 625 isogams in the north half of their course is closer than normal; (4) the isogams in the southwest part of the area tend to ray into a west-northwest trend; (5) the failure of the northern ends of the 325–525 isogams to begin to curve westward is not necessarily significant, as the large local anomalies and the lack of data on the north obscure the pattern of the regional isogams.

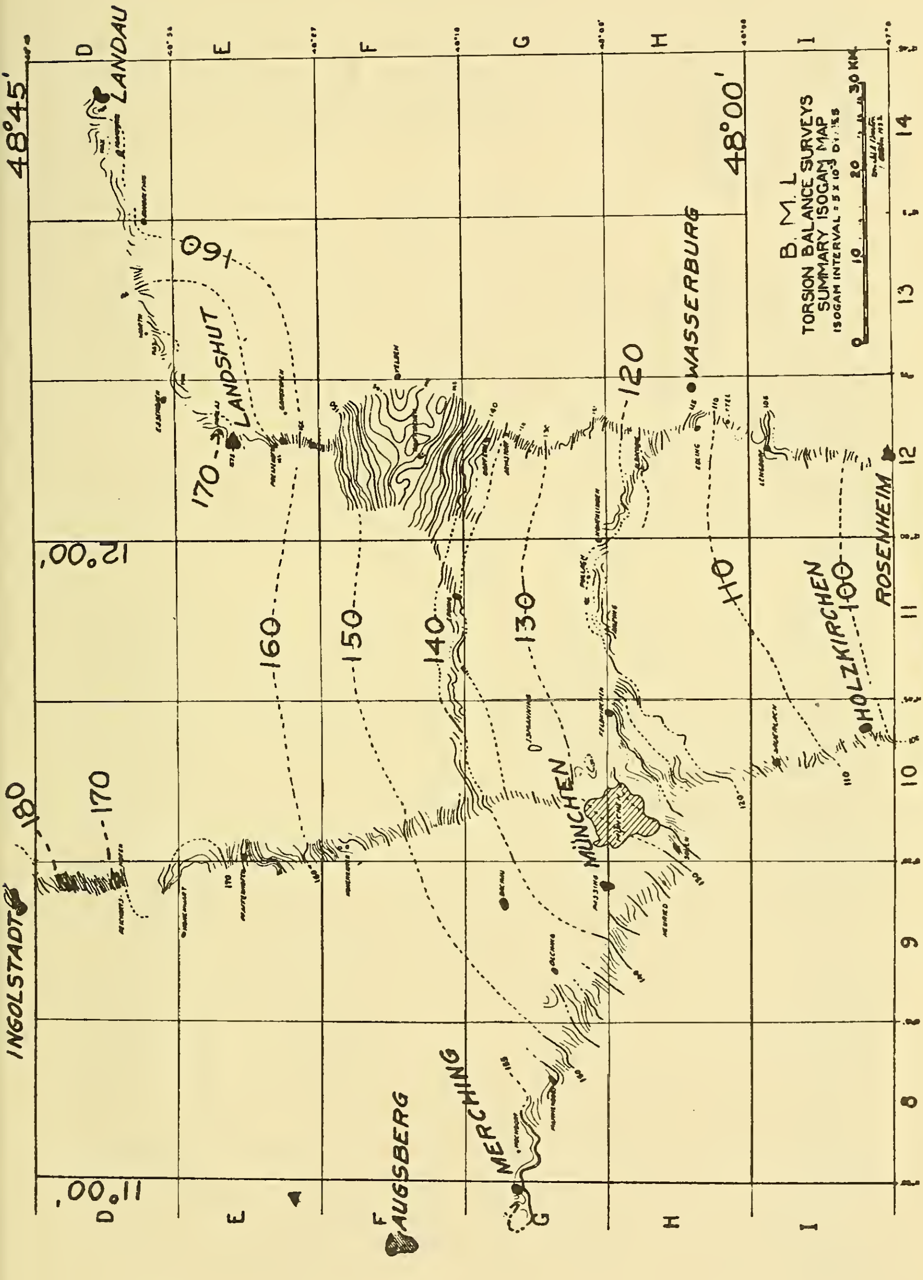


FIG. 5.—Observed variation of Δg.

The regional variation of gravity (Δg_s) is shown by the isogams of Figure 5. That regional variation shows the following significant characteristics: (1) a general, fairly uniform southward decrease in the intensity of gravity, (2) the suggestion of a broad, faint gravity nose through Landshut, and (3) slight curvature of the isogams as if around a basin, or else as if in conformity with another gravity nose through Augsberg.

INTERPRETATION

ISOSTATIC EFFECT

A southward decrease in Δg_s across the Munich basin would be produced by the root of the Alps if they are in isostatic equilibrium. According to the isostatic theory, the excess mass of the projection of the mountains above the general level must be compensated by an equivalent deficiency of mass in the root of the mountains. That subsurface deficiency of mass would produce a gravity minimum which would be concentric with the mountains and which will extend out from the edge of the mountains for a distance equivalent to two to three times the depth of the center of gravity of the deficiency of the root.

The isostatic effect of the Alps at a maximum could produce only a third of the observed variation of Δg_s; and no such isostatic effect need be postulated to explain the observed variation of Δg. The results of approximate calculations of the Δg_s which would be produced by an isostatically compensated Alpine mass are shown in Figure 6. The following assumptions were used: (1) The excess of mass of the Alps above the general level of the surrounding country is compensated by a corresponding deficiency in the mass of the root of the Alps; (2) the mean density of the Alps above ground is 2.5; (3) the Alps consist of a prism infinite in the east and west directions, 120 kilometers wide in north and south directions, rising 2.5 kilometers above the general level, and extending 25 or 50 kilometers below the general level; (4) the deficiency in mass is distributed uniformly through the vertical zone 0 to −25 kilometers; uniformly through the vertical zone 0 to −50 kilometers, uniformly through the vertical zone −25 to −50 kilometers, and uniformly through the vertical zone 0 to −100 kilometers. It is evident from Figure 6 that at a maximum, only a third of the observed variation of Δg_s north-south across the basin could be produced by an isostatic deficiency of mass in the root of the Alps. If any of the calculated Δg_s curves of the isostatic effect is subtracted from the curve of observed Δg_s, a resultant curve will be obtained which will indicate that at Rosenheim and Holzkirchen, the

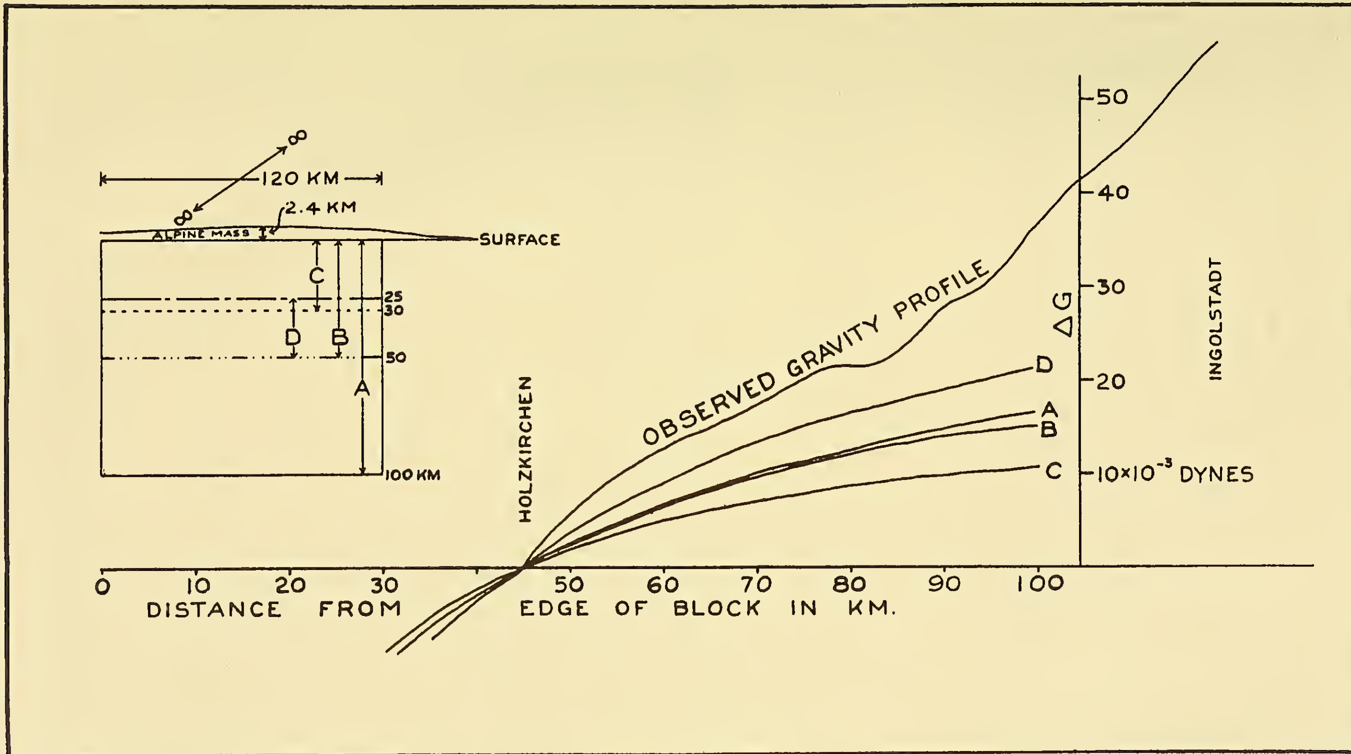


FIG. 6.—Observed variation of Δg_s *versus* calculated gravitational effect of isostatically compensated Alpine mass. Distribution of deficiency of density in root of Alpine mass is assumed to be: A = uniformly 0 to −100 kilometers; B = uniformly 0 to −50 kilometers; C = uniformly 0 to −30 kilometers; D = uniformly −25 to −50 kilometers (relative density 0 to −25 kilometers = *nil*).

basement must be rising rapidly southward. Rosenheim and Holzkirchen lie well north of the front overthrusts; the front of the Bavarian Alps has been formed by overthrusting from the south; experiments seem to indicate that the front of the extensive yielding in overthrusting lies along a 45° shear line; the weight of the overriding overthrust sheets should tend to depress the basement, even for a considerable distance out into the foreland; the great thickness of the upturned Tertiary beds in the edge of the Alps would indicate correspondingly great thickness of the undisturbed Tertiary beds at Holzkirchen and Rosenheim. A southward dip of the basement at those places, therefore, is geologically more probable than a southward rise. The observed southward decrease of Δg_s is satisfactorily explained by the dip of the basement under the Tertiary beds.

DIP OF BASEMENTS

The divergent regional variation of Δg_s and of Δz_s seems to indicate divergent slopes between the surface of the crystalline basement and some higher surface of discontinuity between generally heavier rocks below and generally lighter rocks above.

The generally southerly decrease of gravity presumably is produced by a southward dipping upper surface of a thick series of rocks heavier than the overlying rocks. Geologically, such a dip is probable. The Mesozoic beds immediately north of the Danube in general dip gently southward. The Tertiary section must thicken greatly southward to produce the very great thickness of Tertiary beds which are upturned at the front of the Alps. The basement on which the Tertiary beds rest must dip southward. The unconsolidated Tertiary beds are composed predominantly of sands, clays, and gravel and, therefore, are relatively light. The underlying, presumably mostly lower Mesozoic beds are much more consolidated, comprise much limestone and sandstone, and presumably must be relatively light. The surface on which the Tertiary beds rest, therefore, presumably is a surface of density discontinuity with heavier rocks below. Quantitatively, with reasonable density assumptions, the observed gradient profile across the basin is produced by a geologically reasonable slope of that basement to the geologically reasonable thickness of unconsolidated sediments at Rosenheim and Holzkirchen of 13,000 feet ($\pm$ 25 per cent).

The southerly decrease of Δg_s across the basin mathematically also could be produced in whole or in part by a progressive horizontal variation in the character of the beds from unconsolidated sand and clay at the south to massive limestone at the north in a vertical section 4,000 meters thick; geologically, such a gradation in the Munich basin is most improbable.

The west-southwesterly regional decrease of Z across the basin presumably must be produced by a west-southwesterly slope to the surface of the crystalline basement. Theoretically, that regional variation of Z_s could be produced also either by (1) a vertically uniform horizontal variation in magnetic permeability through a great vertical thickness, or (2) very special conformations of rock of high magnetic permeability, within the basement. Neither of those two possible causes of the observed southwesterly decrease in the intensity of ΔZ_s is probable geologically. Granitic-gneissic masses commonly have a higher magnetic permeability than most sediments. The surface on the crystalline rocks of the Bohemian massif dips under the Tertiary sediments. The most probable explanation of that observed regional variation of Z, therefore, is that the southwesterly slope of that surface presumably must continue across the basin under the Tertiary beds and under the Mesozoic beds.

The failure of the gravity picture to reflect the southwesterly dip of the surface of the crystalline basement across the basin and the failure of the magnetic picture to reflect the southerly dip of the contact between the Mesozoic beds and the Tertiary beds is the effect of the independence between the density and magnetic properties of rocks. The crystalline rocks of the basement must have a high density and a high magnetic permeability. The Mesozoic sediments, or at least the Jurassic-Triassic sediments, must have a relatively high density, probably about the same as that of the underlying crystalline rocks, and presumably a low magnetic permeability. The Tertiary sediments seem to have both a low density and a low permeability. The contact between the Tertiary and Mesozoic sediments therefore produces a gravity effect but no magnetic effect; and the contact between the sediments and the underlying crystalline basement produces a magnetic effect but no gravity effect.

The curvature of the magnetic isogams into a southwest trend in the southeastern part of the basin, in front of Salzburg, suggests the rise of the basement southeastward and the extension of the crystalline rocks of the Bohemian massif southwestward under the Austrian continuation of the basin, perhaps to connect with the crystalline core of the eastern Alps. The moderate depth of the granite at Wels in Austria shows that the crystallines extend out at least to the middle of the Austrian Tertiary belt.[4]

[4] That conclusion was reached before the writer came across Robert Schwinner's "Magnetismus in Bohmischer Masse und Ostalpen," *Gerlands Beitrage zur Geophysik*, Bd. 39, Heft 1 (1933), pp. 58–81.

The crystalline basement, somewhere on the western border of Bavaria or in Wuerttemberg, should begin to rise northwestward toward the granitic core of the Black Forest and Odenwald. The raying of the magnetic isogams in the southwestern part of the basin may indicate the beginning of the turn of the crystalline basement. Farther north, along the Danube, the corresponding bending of the isogams may have been lost in the large anomalies of the Alb and Donau magnetic axes.

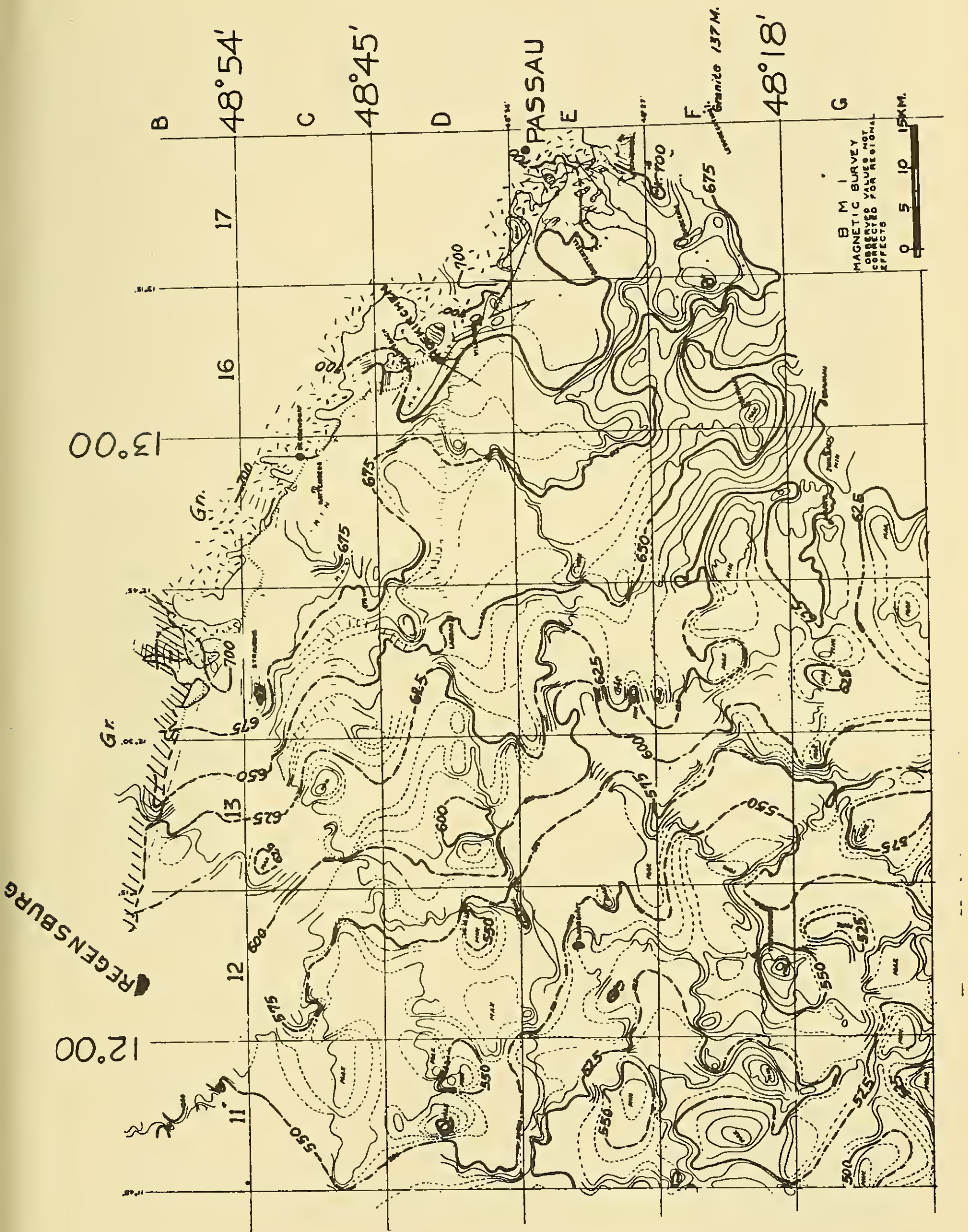
VINDELICIAN RIDGE

The crest of the much disputed Vindelician ridge would seem not to cross the Munich Tertiary basin. The crystalline basement very likely would have been arched up to form a crystalline ridge as the core to the Vindelician ridge. The results of the magnetic survey of the basin indicate the absence of any such subsurface crystalline basement ridge, and the presence of a generally plane west-southwesterly dip of the basement across the basins. The results of the torsion-balance survey indicate the absence of any large structural ridge east-west across the basin, and indicate the presence of a fairly uniformly southerly dip of the basement under the Tertiary beds. The presence of a Vindelician ridge across the basin presumably should be suggested at least faintly, either by one or the other or by both of the methods (torsion balance and magnetometric). The absence of the slightest suggestion of the presence of any such ridge across the basin indicates that presumably no Vindelician ridge crosses the Munich Tertiary basin.

The position of the crystalline core of the Vindelician ridge south of the present Munich Tertiary basin would be consistent with the results of the B. M. I. surveys and would be suggested by the presumable continuation of the Bohemian crystalline massif southwestward under the upper Austrian belt of Tertiary formations.

WEST FLANK OF BOHEMIAN MASSIF

One of the problems of the B. M. I. geophysical surveys was to determine the character and steepness of the dip of the surface of the crystallines from the Bohemian massif west-southwestward under the Tertiary beds. In the area north of Regensburg, a north-south fault of considerable throw marks the contact between crystalline rocks of the Bohemian massif on the east and the Mesozoic sediments on the west, and west of the fault there is a large north-south syncline in which the only Cretaceous beds north of the mountains outcrop, overlying the Jurassic beds. From the vicinity of Regensburg south-southeastward to Hofkirchen, a fault is postulated also for the south-


PASSAU
REGENSBURG
48°54'
48°45'
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0 5 10 15 KM.
Gn.
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west front of the Bohemian massif. Over that stretch, the Danube flows at the foot of the edge of the crystalline rocks of the Bohemian massif. But from Hofkirchen past Passau, the Danube flows across the crystalline rocks, and a cross (northeast-southwest) fault is shown by Schuster's map at Hofkirchen. The structural block southeast of Hofkirchen geologically seems to have been uplifted; or the structural block northwest of Hofkirchen and in front of the crystallines geologically would seem to have been dropped down.

The broad flat-topped magnetic maximum in the area south of Hofkirchen, and west and southwest of Passau, and the magnetic minimum which extends from the vicinity of Hofkirchen northwestward along the Danube are in agreement with the picture of the relatively uplifted block south and west of Passau and Hofkirchen and the relatively down-dropped block northeast of Hofkirchen (Fig. 7).

The throw of the Danubian fault from Regensburg to Hofkirchen, on the basis of the magnetic data would seem not to be large. That fault brings the sediments in fault contact with the crystalline rocks. If the throw of the fault were large, there would be a sharp jump in the magnetic intensity at the contact. A sharp jump was observed on a traverse across the contact a short distance northwest of Deggendorf; but on the other traverses across the contact, its presence was dubiously reflected in the observed magnetic picture. There are two contrasting *a priori* alternatives in regard to that Danubian fault: (1) the sediments may have transgressed up the southwesterly dipping surface of the crystalline rocks, and a fault of moderate throw (100–200 meters) may have dropped the basinward sediments down only that moderate distance; or (2) progressive movement along the fault may have carried the basement of the basinward block down concomitantly with the general subsidence of the basin and with the deposition of a thick prism of sediments in the basin. Under the second hypothesis, there would be great thickening of the Tertiary sediments only a short distance out from the fault. The slight magnetic anomaly at the contact indicates that the throw of the Regensburg-Hofkirchen fault is 100 or 200 meters rather than 1,000 or 2,000 meters.

Three westward facing structural scarps, or extra steep slopes, presumably fault scarps, on the subsurface flank of the Bohemian crystalline massif are suggested by closer spacing of the isogams along certain zones. Faint regional features are obscured by the numerous lesser features. But if the isogams are smoothed out as well as possible to show their normal or regional course without regard to the lesser, local irregularities, there seems to be less than normal spacing between the 575- and 625-gamma isogams, in the zone from Strauburg south-

southeastward along the 650-gamma isogam, and along a parallel slightly farther northeast and partly along the 675-gamma isogam and partly along the 700-gamma isogam. This latter zone of steeper than normal magnetic gradient suggests the presence of a structural (subsurface) fault scarp forming the west-southwestern edge of the uplifted basement block in front of Hofkirchen and Passau. The zone between the 575- and 625-gamma isogams is closely on the prolongation of the fault which extends northward from Regensburg. The closer than normal spacing of the isogams of that zone suggests that the Regensburg fault extends far southward under the cover of the Tertiary sediments; unfortunately the survey did not cover the area immediately south of Regensburg; and that zone of closer than normal spacing of the isogams cannot be followed in the data of the survey directly up to the south end of the surface exposure of the fault.

THICKNESS OF TERTIARY SEDIMENTS

The thickness of the prism of unconsolidated sediments from Ingolstadt southward across the basin to Holzkirchen can be calculated approximately from the results of the torsion-balance survey if the following approximately justifiable assumptions are made: (1) that increase in density between the unconsolidated and consolidated series is concentrated at the contact between the two series; (2) that the base of the unconsolidated series crops out at Ingolstadt; and (3) that the effect from isostatic compensation of the Alps is negligible. The calculation can be made graphically, or by ordinary formulae used in torsion-balance work. Our qualitatively quantitative calculations from the torsion-balance data indicate that the thickness of the unconsolidated sediments is as follows.

	Feet	*Per Cent*
At Munich	8,000	±25
Holzkirchen	13,000	±25
Landshut	1,500	±25
Taufkirchen		
Normally	6,000	±25
Top of prospective structure	5,200	±25
Rosenheim	13,000	±25

The errors probably are all of the same algebraic sign and approximately of the same relative magnitude.

Geologically, the unconsolidated sediments presumably are the Tertiary formations, and the Mesozoic are the consolidated sediments; but the consolidation, and consequently the density of the Cretaceous, may be closer to that of the Tertiary than to that of the Triassic and Jurassic; the Cretaceous should then be included with the unconsolidated sediments.

MAJOR STRUCTURES

HERCYNIAN RIDGES

A broad Hercynian (northwest-southeast) ridge through Landshut is suggested by the results of the torsion-balance survey, which partially corroborate the northwest-southeast positive anomaly on Schütte's map of the Bouguer anomalies from the pendulum determinations of gravity in southern Germany. The B. M. I. torsion-balance survey mapped the presence of a broad, somewhat complex northwest-southeast maximum through Landshut which would correspond with the Landshut part of that maximum on Schütte's map.[5] The B. M. I. torsion-balance and magnetic surveys indicate that the anomaly at Ingolstadt is produced by sharp local structure and suggest that the Ingolstadt and Landshut anomalies should not be connected, until further evidence indicates an actual connection. From the few data, the writer would infer that the Landshut ridge does extend northwestward but that it probably crosses the Danube somewhere east of Ingolstadt.

The results of the magnetic survey do not suggest the presence of the Landshut ridge.

The results of the two types of surveys seem to indicate: (1) that deformation of the crystalline basement is not sufficient to produce an appreciable effect in view of the great depth of the basement, and (2) that the Mesozoic formations and perhaps the surface of the Mesozoic basement under the Tertiary formations must be deformed sufficiently to affect the variation of gravity at the surface. The torsion-balance data of the single traverse are rather scanty for much interpretation of the ridge.

The results of the torsion-balance survey suggest that there may be a parallel and similar ridge through Augsburg.

REGENSBURG SYNCLINE

The southward prolongation of the Regensburg syncline under the Tertiary formations is suggested by the results of the single torsion-balance line eastward from Landshut to Landau. A trough of minimum is shown by Schütte's map of the Bouguer anomalies as lying in front of the Bohemian massif. North of Regensburg, the minimum tends to coincide approximately with the faulted Regensburg syncline. The results of the torsion-balance survey tend to corroborate the presence of that trough of minimum in the Landau area.

The results of the B. M. I. magnetic survey do not reflect the presence of the syncline.

[5] K. Schütte, *Karte der Schwereabweichungen Süddeutschland* (Munich, 1930).

North of Regensburg, the broad faulted syncline seems to have been formed by the down-warping of Mesozoic formations in reference to the area west and by down-faulting of those formations in reference to the Bohemian crystalline massif. The syncline, therefore, may not affect the surface of the crystalline basement to the same extent that it affects the Mesozoic formations or may not affect it at all. The results of the B. M. I. magnetic survey would seem to indicate that within the area of the Munich Tertiary basin, the Regensburg syncline does not appreciably affect the crystalline basement.

The question is open whether within the Munich Tertiary basin: (1) the Landshut ridge is the effect of positive arching along a Landshut structural axis; or (2) the effect of the negative down-warping of the Regensburg syncline. The isogams of the torsion-balance survey agree qualitatively with Schütte's isogams of the Bouguer anomalies. In the extreme eastern part of the basin, the isogams tend to bend into parallelism with the front of the Bohemian massif. In that area, the west-southwestward dip of the surface of the crystalline complex under the sedimentary formations may be producing an appreciable effect on the variation of gravity at the surface. It is possible also that the Mesozoic basement under the Tertiary formation may rise east-northeastward up toward the Bohemian massif. North of Regensburg, the syncline seems to have been produced by negative movement in front of the Regensburg fault. Similar structural movement south of Regensburg, within the Tertiary basin, would produce a Landshut ridge and a continuation of the Regensburg syncline, which in turn would produce the observed variation of gravity. The Landshut ridge, however, would not represent the effect of a positive axis of deformation.

BASALTIC STRUCTURES ALONG DANUBE

A line of previously unsurmised structures with basaltic cores was mapped along Danube River from Vohburg to Ulm. The Ingolstadt structure is one of the most pronounced of those structures and is the only one which is crossed by both the magnetic and the torsion-balance surveys. It has a sharp gravity maximum and a sharper magnetic maximum. The magnetic isogams are shown in Figure 8. The north-south profiles of relative gravity (Δg_s), of the horizontal gradient of gravity (U_{xz}), and of relative Z_s are shown in Figure 9. The amplitude of most of the residual anomalies within the area of the survey is ± 25 gammas from the regional value of Z_s of Figure 4, although a few residual anomalies have an amplitude of $+50$ or -50 gammas. The residual maximum of the Ingolstadt structure has an amplitude of 250 gammas. Part, approximately half, of the steep north gradient and steep southerly decrease of Δg is regional and has no relation to the Ingolstadt structure. The gravity maximum is not as sharp or as flat topped as the magnetic maximum. Their respective shapes indicate that the extra dense mass is not co-extensive with the magnetically extra-permeable mass, but is somewhat larger and ex-

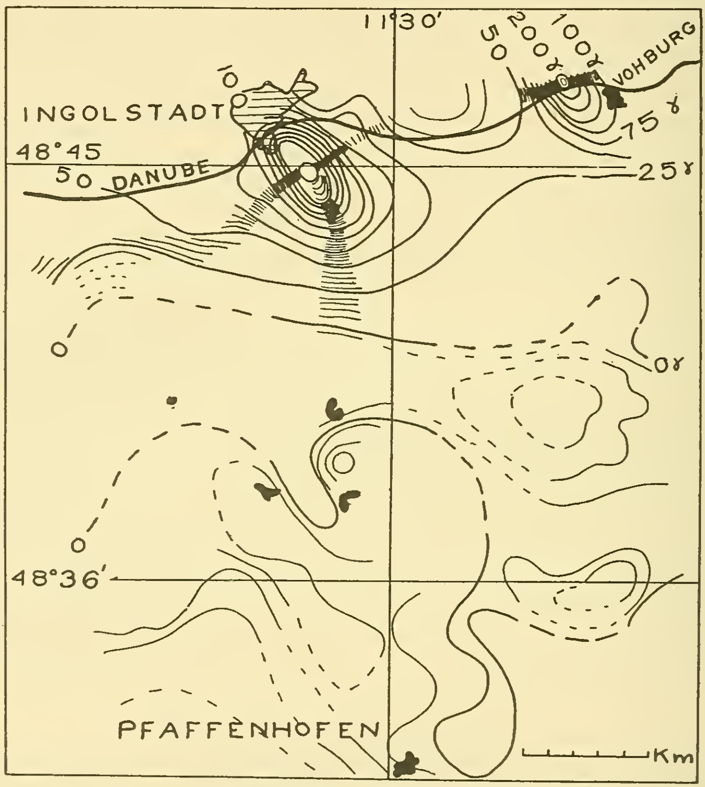

FIG. 8.—Observed variation of Δz, Ingolstadt area.

tends somewhat deeper. The form of the magnetic anomaly indicates that the extra-permeable mass has a crudely disc-like form. Crude calculations in which the magnetic profile was treated as if it were a gravity profile indicate a depth to its top of approximately 3,000 feet and a thickness of approximately 800 feet for the disc. The high magnetic permeability of the rock of the disc suggests a basaltic character.

The Vohburg (C-10, Fig. 8), Weichering (D-9), and Strass (D-8) magnetic maxima are similar anomalies and presumably likewise represent basaltic, probably laccolithic, intrusions (Figs. 3 and 10).

This Danube line of anomalies is continued toward the west by the Oberndorf (D-7), Bergau (E-F-5), and southeast of Ulm (F-4) maxima (Figs. 3 and 10). They are areally large and have respective amplitudes of +35, +35, and +20 gammas. They are no larger in area or amplitude than the larger maxima out in the Tertiary basin and may be produced by the same type of irregularity in magnetic permeability, whatever it may be. But on account of their association

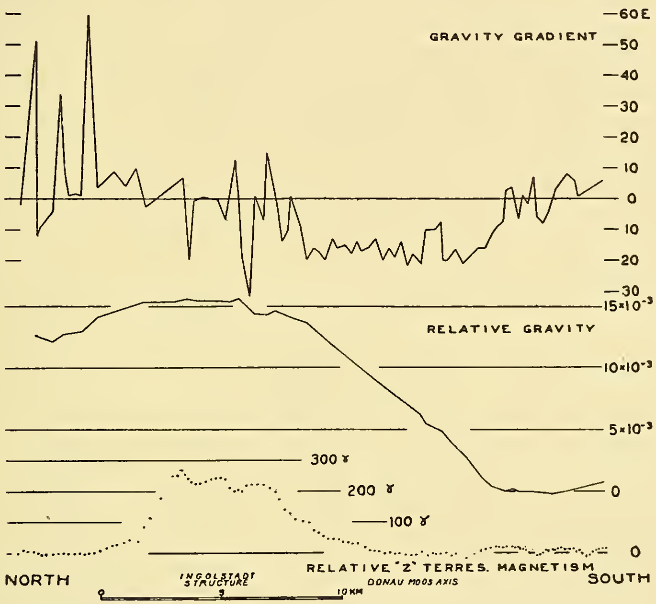

FIG. 9.—Profiles gravity gradient (U_{xz}), relative gravity (Ug_s), and relative magnetic vertical component (Δz_s), across Ingolstadt anomaly and Donau Moos axis.

with the Vohburg, Ingolstadt, Weichering, and Strass maxima, they may be the effect of deeper basaltic intrusions.

This Danube line of structures with basaltic cores lies along the postulated Danube fault, which seems to have guided the course of Danube River between Ulm and Vohburg and which separates the Tertiary from the Jurassic formations. The fault must be deep in origin and presumably must extend far down into the basement. Basalt or other igneous rock of high magnetic permeability seemingly must have come upward along the fault plane to form the laccolithic intrusions of Vohburg, Ingolstadt, Weichering, and Strass in the Jurassic or Triassic sediments.

FIG. 10.—Local anomalies of Δz, after elimination of regional anomaly of Figure 4. Residual isogams after elimination of southwest regional decrease. Isogam interval, 5 gammas. Dotted isogams based on Haussmann's data.

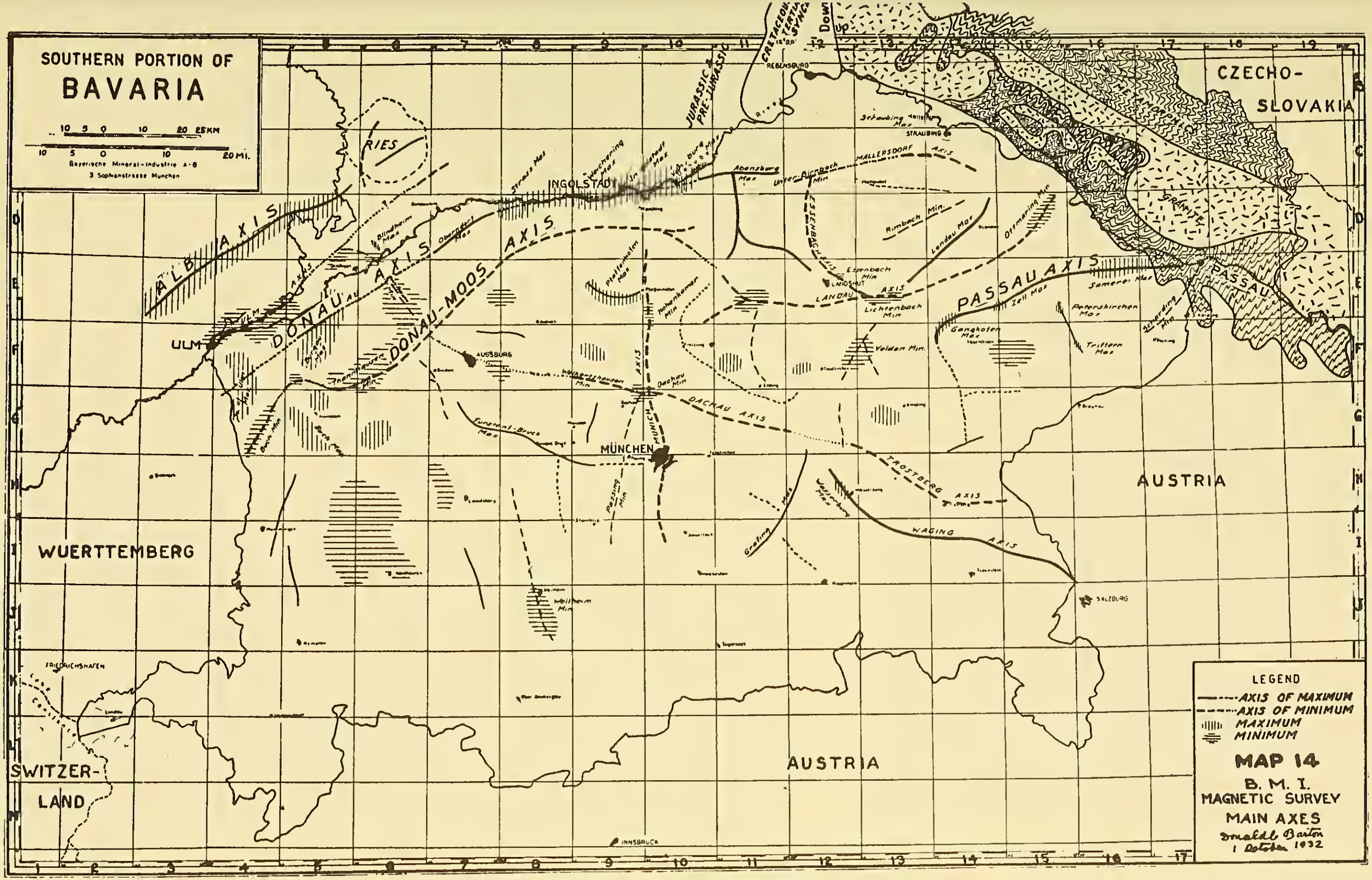

FIG. 11.—Axes indicated and suggested by B. M. I. magnetic survey.

ALB BASALTIC AXIS

The Alb magnetic maximum (Fig. 10) on the northwest edge of the area surveyed indicates the presence of an elongate intrusion or a line of intrusions parallel with the Danube line. The data mostly have been taken from Haussmann.[6] The results of the B.M.I. line from Ulm northwestward corroborate its presence although they place the southeast slope of the ridge of maximum north of Ulm rather than east of Ulm, as Haussmann delineates it. The writer agrees with Haussmann that the Alb axis of magnetic maximum must reflect the presence of a large basic igneous intrusion at depths of 3–8 kilometers. But Haussmann's magnetic stations are too widely scattered to show whether there is one large intrusion or whether there are a series of intrusions whose magnetic effects merge and give the effect of one large intrusion.

DONAU MOOS AXIS

The Donau Moos axis of magnetic minimum (Fig. 10) south of the Danube axis of maximum is not so sharp or definitely delineated. Yet it does seem to exist and to parallel that Danube axis. There is a suggestion that the line is made up of a series of local minima, which are paired with, and southeast of, the maxima of the Danube line. A volcanic plug which has a large vertical dimension may act as a bar magnet; if it is vertical or nearly vertical and has only induced magnetism, its upper pole will produce a maximum and its lower pole a much weaker minimum. If a volcanic plug dipped 75°–80° toward the southeast in Bavaria, and if it were polarized, the lower pole would tend to produce a weak minimum to the southeast of the maximum. This minimum does not indicate the presence of any structure separate from the volcanic plug and must not be confused with the ordinary minima which represent lows in the ordinary (positive) intensity. South of Ingolstadt, there is a gravity minimum which is fully as definite as the magnetic minimum. The two minima are offset from each other 5 or 6 kilometers, but they appear to belong together and are reflecting a common structural feature. But if the two are produced by a common structural feature, the minimum can not be produced by the lower pole of an Ingolstadt volcanic plug, but must be produced by some structural feature which is independent from, but perhaps associated with, the Ingolstadt structure. The gravity minimum, presumably, must indicate a syncline. It therefore would tend to indicate that the Donau Moos axis indicates a syncline (or graben).

[6] Karl Haussmann, "Magnetische Messungen um Ries und Dessen Umgeburg," *Abh. d. k. Preuss* (Akademie der Wissenschaff, 1904).

LESSER MAGNETIC ANOMALIES

The heterogeneous jumble of more or less irregular residual magnetic maxima and minima within the Tertiary basin (Fig. 8) at first sight gives little suggestion of any fundamental underlying structural plan in connection with the anomalies. It is probable that a considerable part of the anomalies are not produced by geologic structure.

Practically all detail magnetic surveys have many anomalies which seemingly are not produced by geologic structure. The data usually are not sufficient to determine what is the cause of these anomalies. Some of them presumably are apparent rather than real and are produced by observational errors, by accidental coincidence of several surficial anomalies of the same sign in consecutive stations, and by the incomplete covering of the area by the survey and the consequently inaccurate inference in regard to the course of isogams in the unmapped areas between surveys. Others of these anomalies presumably are produced by variations in the content of magnetite and ilmenite in the sediments. Many of the anomalies of Figure 1 presumably are of one of these types or a combination of these types.

A certain more or less definite pattern does stand out, however, after closer study of Figure 10 and suggests some fundamental underlying structural plan and cause for many of the anomalies. The magnetic axes are shown on Figure 11. Some of them, such as the Alb and Danube axes, plainly are actual axes and must reflect geologic structure in the subsurface. Others of the lines may be the result of subjective connection of accidentally aligned independent local anomalies.

The Passau axis, however, can be seen probably to be the effect of differences of magnetic permeability in the basement. The Passau axis of maximum is well delineated. Its amplitude is above the average of the anomalies other than those of the Alb and Donau axes. It has been crossed by sufficient traverses to indicate that it does exist. But unfortunately it does not cross or approach any of the torsion-balance work. The eastern end of the maximum, as the B. M. I. mapped it, extends into the area of the outcrop of the granite and the metamorphic rocks; and for a considerable distance in the area slightly farther west, the depth to the granite and the metamorphic rocks can not be great. A nose of metamorphic rocks projects from Passau eastward into the area of granite exactly on the prolongation of the Passau axis of maximum. From the B. M. I. magnetic data and the other data which are available, genetic connection between that nose of metamorphic rocks in the granite can not be proved. But it seems probable that the Passau axis of maximum is being produced by some such geologic feature within the crystalline basement. The center of gravity of the feature can be as deep as 5,000 meters, but can be shallower.

ABSENCE OF FOLDING PARALLEL WITH ALPINE FRONT

The determination of the presence or absence of folding parallel with, and in front of, the Alps was an important problem for these surveys. Northward thrust from the Alps against competent beds within the beds of the Tertiary basin should be expected to produce folds in front of, and parallel with, the front of the Alps. But thrust against a prism of wholly incompetent sediments should produce yielding along overthrusts but no folding ahead of the overthrusts.

The presence of folding in front of, and parallel with, the edge of the Alps is not indicated or suggested by the results of the torsion-balance and magnetic surveys, although there is suggestion of faulting. The failure of both the magnetic and torsion-balance surveys to indicate or suggest the presence of such folding is not conclusive proof of its absence. It is conceivable that there is no vertical variation of magnetic permeability. Such folding, therefore, would produce no magnetic anomalies. The absence of any vertical variation of density is possible, but increase of compaction, and therefore of density, with depth is probable. The increase of density with depth, however, might be so slight that the folding would produce very faint anomalies. The close spacing of the torsion-balance stations was used so that faint anomalies might possibly be detected. The writer was unable to recognize any anomalies which he would interpret as indicating such folding. The absence of indication or suggestion of the folding by the results of the geophysical surveys is evidence, therefore, although not finally conclusive evidence, that folds of moderate or large size and amplitude are not present in front of the Alpine overthrusts.

BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 18, NO. 1 (JANUARY, 1934), PP. 97-105, 4 FIGS.

MAGNETIC VECTOR STUDY OF KENTUCKY AND SOUTHERN MICHIGAN[1]

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Houston, Texas

ABSTRACT

The principle of the magnetic vector method is briefly explained.

The salient geological features of Kentucky are investigated as to their regional magnetic effects. It is found that the Western and Eastern geosynclines show up magnetically as negative anomalies, but that also the Cumberland River arch and the Lexington dome show up as negative anomalies. From this follows that some of the outcropping beds in Kentucky are of a higher magnetic permeability than the underlying beds and Basement complex.

The study of the magnetic vector map in Michigan reveals a number of magnetic high trends, which are shown to correspond to known structural trends, such as the Howell-Owosso anticline and the Muskegon anticline. The deepest area of the Michigan basin is indicated by a negative magnetic anomaly.

Practical value and application of the magnetic vector maps to field problems is explained by the example of the newly discovered Hart field. Apart from the valuable information obtained by magnetic investigations alone, such investigations are of great help in the interpretation of data gained by other geophysical methods.

INTRODUCTION

The principle of the magnetic-vector method, which has been fully explained elsewhere,[3] may be briefly summarized as follows.

In order to obtain the direction in space and the intensity of the magnetic lines of force as due to local structures, the "normal" values of the earth's magnetic field have been deducted from the absolute measurements at the United States Coast and Geodetic Survey's stations of the declination, and of the vertical and horizontal magnetic intensities.

The differences between the absolute and "normal" vertical and horizontal intensities have been combined into a vector triangle, indicating the direction and intensity of the local magnetic force. The vector triangles have been plotted on the maps at their respective stations, by turning the triangles through 90° around the horizontal component (which is the dashed line starting at the station point)

[1] Manuscript received, October 26, 1933.

[2] Geologist and geophysicist, 2102 Bissonnet Avenue.

[3] W. P. Jenny, "Magnetic Vector Study of Regional and Local Geologic Structure in Principal Oil States," *Bull. Amer. Assoc. Petrol. Geol.*, Vol., 16, No. 12 (December, 1932). "Structural Trends in Florida," *The Oil Weekly* (October 2, 1933).

into the plane of the map. A black station point means that the magnetic force is directed toward the station; a hollow station point means that the magnetic force is directed away from the station, as indicated by the arrows in the legend.

KENTUCKY

The salient geological features of Kentucky are the Cumberland River arch, the Western geosyncline, the Cincinnati arch, the Lexington dome, the Eastern geosyncline and the Pine Mountain thrust fault, as set forth on the inset of Figure 1. The most westerly corner of Kentucky is covered by Quaternary, Tertiary, and some Cretaceous. East of the Cumberland River arch only rocks of Pennsylvanian age and older are found, which are subdivided by the legend of the geologic map of Kentucky[4] as follows:

Pennsylvanian	Conemaugh Allegheny Pottsville
Mississippian	Chester Meramec Osage
Devonian	
Silurian	
Ordovician	Cincinnatian Champlainian

The Meramec crops out along a belt east and south of the Western geosyncline and is the outcropping formation along the axis of the Cumberland River arch west of the syncline.

The core of the syncline consists of Conemaugh.

The Lexington dome has its apex in the general neighborhood of Lexington, where the beds of the Champlainian crop out, surrounded by a vast area of Cincinnatian.

The whole Eastern geosyncline and surroundings are covered by the Pottsville to within a distance of about 30 miles east of the axis of the Cincinnati arch.

It is most characteristic that the Cumberland River arch and the Lexington dome show up as negative magnetic anomalies. From this we must conclude that the Chester or some beds of the Pennsylvanian are of a greater magnetic permeability than the Meramec and further that the Cincinnatian, Silurian, Devonian, and Osage are of a greater magnetic permeability than the Champlainian and lower formations.

The areas of the two geosynclines are characterized by magnetic negative anomalies.

W. R. Jillson, "Geologic Map of Kentucky," *Kentucky Geol. Survey* (1929).

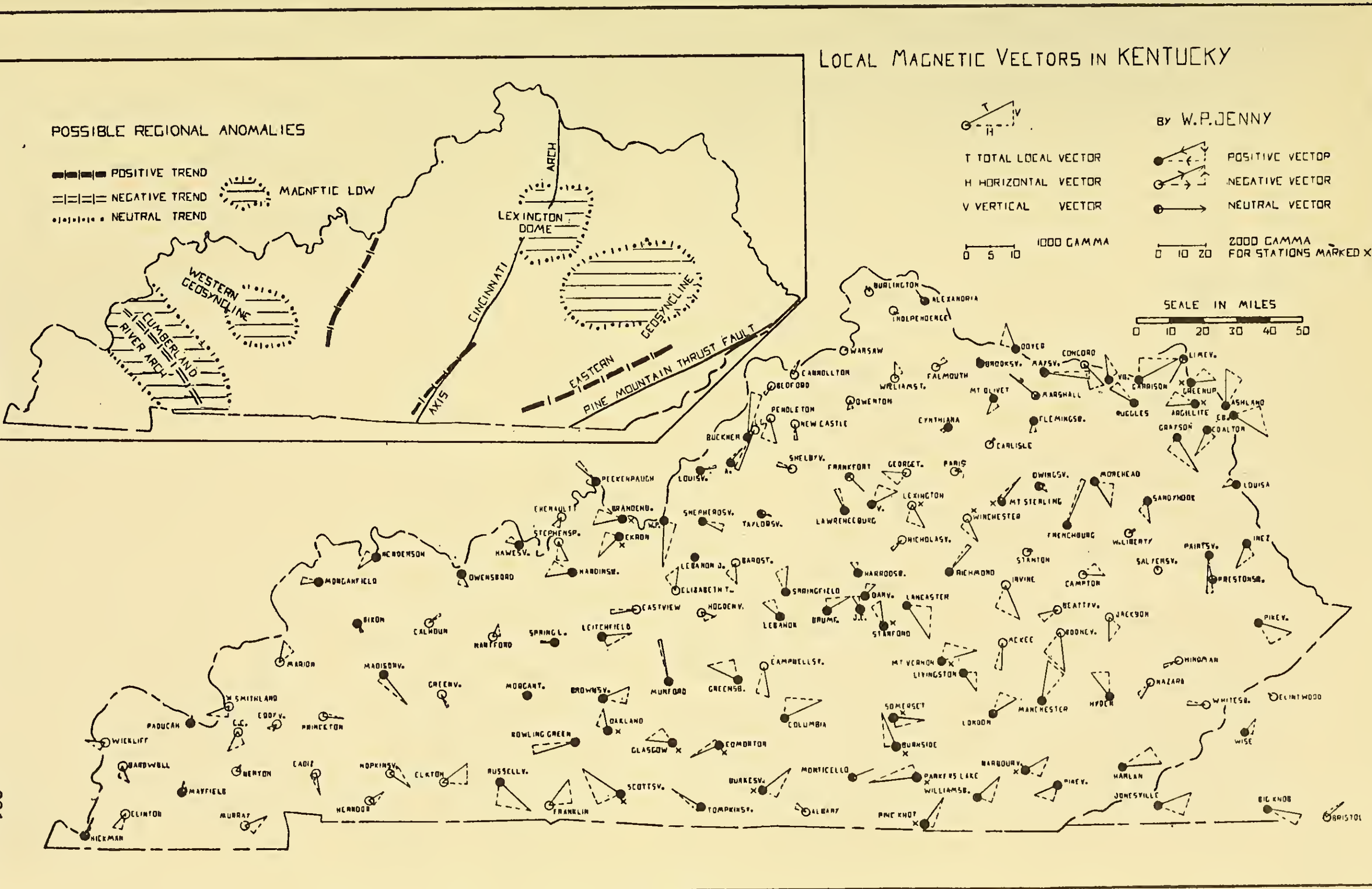

LOCAL MAGNETIC VECTORS IN KENTUCKY
BY W. P. JENNY
T TOTAL LOCAL VECTOR
H HORIZONTAL VECTOR
V VERTICAL VECTOR
POSITIVE VECTOR
NEGATIVE VECTOR
NEUTRAL VECTOR
0 5 10 1000 GAMMA
0 10 20 2000 GAMMA FOR STATIONS MARKED X
SCALE IN MILES
0 10 20 30 40 50
POSSIBLE REGIONAL ANOMALIES
POSITIVE TREND
NEGATIVE TREND
NEUTRAL TREND
MAGNETIC LOW
LEXINGTON DOME
ARCH
CINCINNATI AXIS
EASTERN GEOSYNCLINE
PINE MOUNTAIN THRUST FAULT
WESTERN GEOSYNCLINE
CUMBERLAND RIVER ARCH

FIG. 1

The southern part of the axis of the Cincinnati arch seems to be indicated by a positive magnetic trend.

Another regional positive trend about parallel with the Cincinnati arch may be perceived in the northern central part of Kentucky, starting between West Point and Brandenburg and extending southward to beween Morgantown and Brownsville.

A third regional positive trend, about parallel with the Pine Mountain thrust fault, possibly extends from Pine Knot to Hyden.

The interpretation of the magnetic vectors in Kentucky encounters increased difficulties, because the magnetic susceptibility of some of the outcropping beds appears to be so great that large magnetic "highs" and "lows" may be expected as a result of stratigraphic changes only, without any structural significance. Without adequate support from geologic data, it is therefore not possible to say whether regional trends are mainly due to structural or stratigraphic features and the same holds true for the large number of local anomalies indicated by the vectors.

The shallowness of the magnetically active horizons should, however, prove to be of advantage for the interpretation of local anomalies, if they are sufficiently detailed by magnetometer surveys.

SOUTHERN MICHIGAN

Southern Michigan is a large structural basin, the deepest area of which lies between Mount Pleasant, Saginaw, and Lansing. The beds dip from all sides at a rate of 25–50 feet per mile toward this central area.[5]

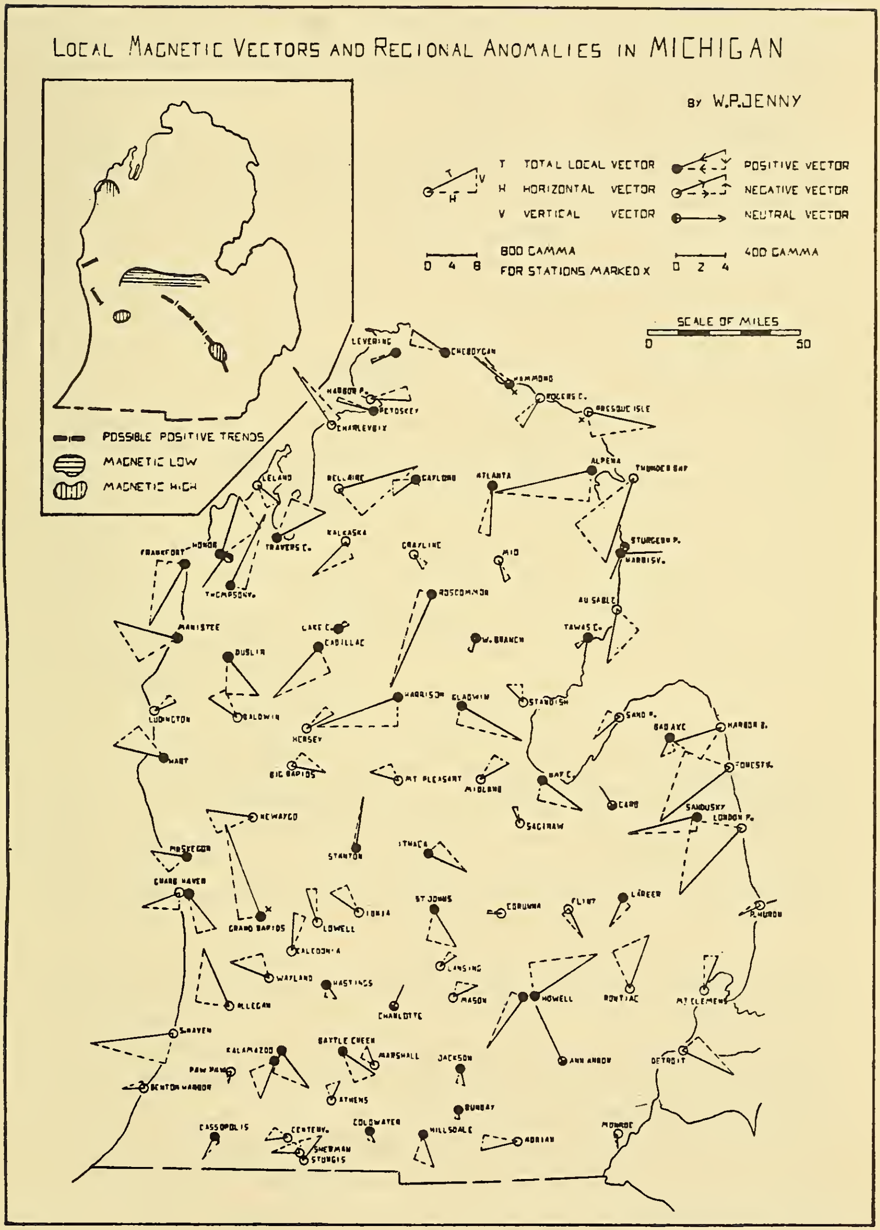
A sheet of unconsolidated glacial drift with an average thickness of 200 feet covers the whole of Michigan. Below this sheet the Pennsylvanian crops out in the central part of the basin and the Mississippian, Devonian, and Silurian surround this core in belts of varying widths.

Among the few minor structures known in Michigan[6] are: the Saginaw anticline east and north of Saginaw; the Howell-Owosso anticline, extending northwest from Ann Arbor toward Howell, Owosso, and St. Johns; the Muskegon anticline north and east of Muskegon; and the Mount Pleasant anticline east of Mount Pleasant.

Different theories have been advanced to explain the origin of these folds. They may be connected with the Cincinnati and LaSalle

[5] G. W. Pirtle, "Michigan Structural Basin and Its Relationship to Surrounding Areas," *Bull. Amer. Assoc. Petrol. Geol.*, Vol. 16, No. 2 (February, 1932), pp. 145–52.

[6] R. B. Newcombe, "Oil and Gas Development in Michigan," *Michigan Geol. Survey Pub. 37*, Pt. 3, Geol. Series 31 (1928). "Geology of Muskegon Oil Field, Muskegon, Michigan," *Bull. Amer. Assoc. Petrol. Geol.*, Vol. 16, No. 2 (February, 1932), pp. 153–68.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T TOTAL LOCAL VECTOR
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POSITIVE VECTOR
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NEUTRAL VECTOR
800 GAMMA
0 4 8
FOR STATIONS MARKED X
400 GAMMA
0 2 4
SCALE OF MILES
0
50
POSSIBLE POSITIVE TRENDS
MAGNETIC LOW
MAGNETIC HIGH
CHEBOYGAN
PRESQUE ISLE
CHARLEVOIX
PETOSKEY
ALPENA
THUNDER BAY
BELLAIRE
GAYLORD
ATLANTA
LELAND
TRAVERS C.
KALKASKA
GRAYLING
MIO
FRANKFORT
HONOR
THOMPSONV.
ROSCOMMON
AU SABLE
MANISTEE
LAKE C.
CADILLAC
W. BRANCH
TAWAS C.
DUBLIN
HARRISON
GLADWIN
STANDISH
LUDINGTON
BALDWIN
HERSEY
HART
BIG RAPIDS
MT PLEASANT
MIDLAND
BAY C.
CARO
SAGINAW
SANDUSKY
NEWAYGO
STANTON
ITHACA
MUSKEGON
GRAND HAVEN
GRAND RAPIDS
LOWELL
IONIA
ST JOHNS
CORUNNA
FLINT
LAPEER
P. HURON
CALEDONIA
LANSING
WAYLAND
HASTINGS
CHARLOTTE
MASON
HOWELL
PONTIAC
MT CLEMENS
ALLEGAN
S.HAVEN
KALAMAZOO
BATTLE CREEK
MARSHALL
JACKSON
ANN ARBOR
DETROIT
PAW PAW
BENTON HARBOR
ATHENS
CASSOPOLIS
COLDWATER
HILLSDALE
ADRIAN
MONROE
STURGIS

Fig. 2

anticlines, or they may be due to more local stresses and lateral pressures, as a consequence of the settling of the basin. Probably both factors should be considered and some unconformity between deeper and shallower structure may reasonably be expected, mainly as the result of salt flowage in the Salina.

Since the glacial drift and the shallow formations do not seem appreciably to influence the magnetic field, it appears that the magnetic anomalies may be mainly attributed either to the structures or to the

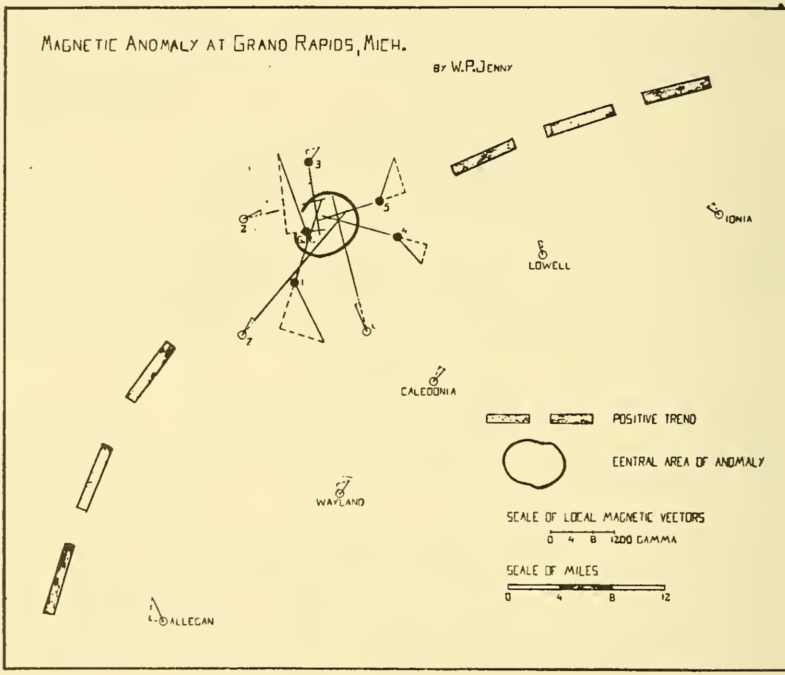

FIG. 3

mineral content of the pre-Cambrian. From a comparison of the vector map (Fig. 2) with known structures, we conclude that the majority of magnetic anomalies may safely be interpreted in terms of structures: positive vectors indicating anticlinal structures, negative vectors indicating synclinal structures.

The negative magnetic vectors at Midland, Mount Pleasant, Big Rapids, and Newaygo are indicative of the large depression in the heart of the basin.

The horizontal vector at Ann Arbor points toward a "high" near Howell as indicated by the two vectors. The negative vectors at Flint, Corunna, Lansing, and Mason all indicate a "high" trend extending from Howell toward St. Johns and possibly as far as Ithaca and Stanton. This magnetic trend should correspond with the known Howell-Owosso anticline and suggests a northwestward extension toward Stanton.

The positive vector east of Muskegon plainly suggests the known northwesterly anticlinal ridge, which at its intersection with a west-easterly ridge, formed the Muskegon anticline.

A large uplift is indicated by an anomaly of more than 1,500 gammas in the general neighborhood of Grand Rapids. The United States Coast and Geodetic Survey has made observations of the magnetic elements at 7 auxiliary stations near Grand Rapids. The local vectors at these auxiliary stations are represented in Figure 3 together with a few near-by vectors. The negative vectors at Allegan, Wayland, Caledonia, Lowell, and Ionia seem to indicate a regional magnetic "high" trend west, northwest, and north of these stations as indicated in the figure. Along this regional trend occurs the large positive anomaly of Grand Rapids. The vectors at Grand Rapids and at the 7 auxiliary stations all point toward a positive area with its center slightly east of Grand Rapids. It seems safe to interpret this anomaly as a large domal uplift along a regional anticlinal trend.

Another uplift is suggested by the vectors at Frankfort, Honor, and Thompsonville.

APPLICATION OF MAGNETIC VECTOR MAPS

In areas like Kentucky, where some of the outcropping beds appear to be of a higher magnetic permeability than the deeper strata, most of the vectors are necessarily connected with shallow and local anomalies. Some of these anomalies may correspond with actual structures, others with stratigraphic changes along the surface. Though only scant information as to regional and local structures may be gained by a study of this vector map, it will, however, be of great value for an intelligent planning of detailed investigations by the magnetometer.

For all these areas, like Michigan, where the deeper strata and the basement are of a higher magnetic permeability than the uppermost few thousand feet of sediments, the large majority of magnetic anomalies may be connected with structural features; some of the anomalies may, however, be caused by a variation of the magnetic permeability, brought about by changes in the mineral content of the magnetically active horizons. Due to the long distances between stations the magnetic vector maps at first sight mainly reveal only regional structural features. For those stations, which are sufficiently close to local structures, the magnetic information gained from the vector map may, however, be connected with the local structures. In this respect we have already mentioned the vectors at Refugio, Conroe, Livingston, Mineola (Texas), Bastrop, Evangeline (Louisiana), Jackson (Mississippi), Oklahoma City, Eldorado (Kansas), and others.[7]

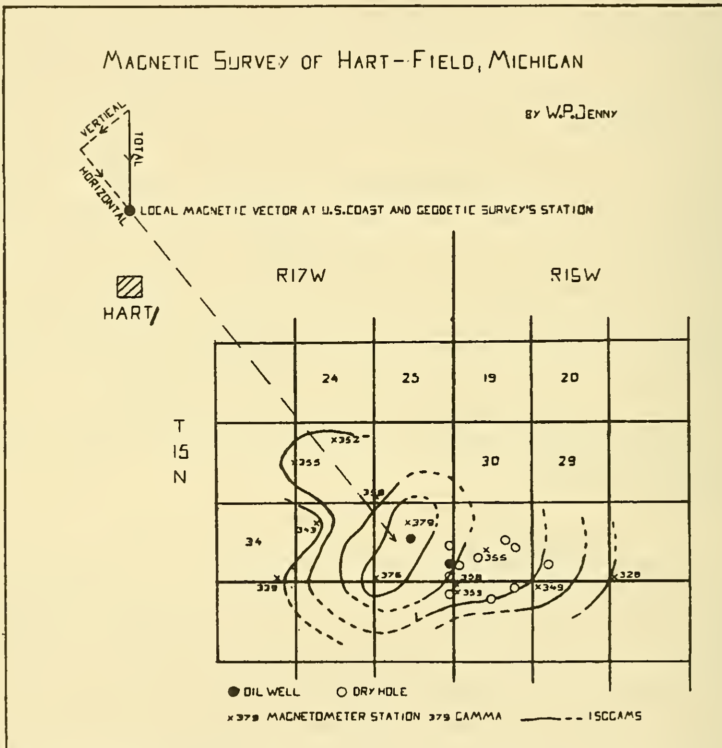

FIG. 4

The magnetic vector at Hart, Michigan, is another example where there exists a direct connection with a local commercial structure. The positive vector suggests a magnetic "high," southeast of Hart, which might correspond with an uplift with a possible northeast-southwesterly axis. Prompted by this suggestion, the writer has made a casual magnetic survey of the recently discovered Hart field in order to check the conclusions drawn from the vector map. Figure 4 shows

[7] W. P. Jenny, *op. cit.*

the locations of the first wells and the isogams based on 12 magnetometer stations. The isogams check fairly well with the contour lines drawn on the red beds of the Coldwater, but in contradiction to current geological conceptions, seem to be in favor of the local magnetic vector, suggesting a northeast-southwesterly trend of the structure. Though the production from the Traverse has been disappointing so far, there seems to be no doubt that the magnetic data indicate a considerable uplift.

It is, however, rather surprising that the magnetic "high" should so closely correspond with the structural "high" in the Traverse. The rule for Michigan will most probably be that the magnetic "high," which is caused by an anticlinal structure in the Basement complex, is considerably offset from the structural "high" in the Dundee or Traverse, mainly because of the several hundred feet of plastic salt and anhydrite beds, which lie between the upper and lower structure.

Since the gravimetric field may reasonably be assumed to show some influence of upper structures, it seems that a combination of gravimetric and magnetic data may lead to the discovery of commercial structures in such areas, which, like large parts of Canada, are covered by several hundred feet of unconsolidated glacial drift.

The application of three-dimensional magnetic prospecting is liable to improve a great deal the chance of locating commercial structures magnetically. By publishing these vector maps, it is not so much the writer's intention, however, to draw renewed attention to the magnetic method at the expense of other geophysical methods, but rather to encourage magnetic investigations as a great help for the interpretation of results gained especially by the gravimetric and seismic methods. Both these methods work, as a rule, on beds considerably above the magnetically active horizons, and in many cases, only the combination of their data with the data gained magnetically from deeper strata will lead to a proper geological interpretation of the gravimetric and seismic indications obtained in a certain area.

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VOL. 18, NO. 1 (JANUARY, 1934) PP. 106-118, 5 FIGS

GRANITE AND LIMESTONE VELOCITY DETERMINATIONS IN ARBUCKLE MOUNTAINS, OKLAHOMA[1]

B. B. WEATHERBY, W. T. BORN, and R. L. HARDING[2]
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ABSTRACT

Determinations of the velocity of elastic waves in the Tishomingo granite and Arbuckle limestone were made by seismic methods. The measurements on the granite were taken over both long and short spans in the same area, the former including distances of 500 to 2,500 feet, the latter, distances of 5 to 400 feet. Due to unusual precautions, the short-span data are accurate to 2 per cent. The long-span data give a straight line time-distance graph within observational errors and yield a value of 17,950 ft./sec. for the longitudinal wave velocity. The data taken over the shorter span show a definite curvature in the time-distance graphs, the longitudinal velocity varying from 14,880 to 17,150 feet per second, and the transverse velocity from 7,000 to 7,950 ft./sec. From these data the elastic constants of granite are calculated. Laboratory measurements were made on two small rods of granite from the same locality, by a method giving results accurate to less than 1 per cent. The velocities so obtained were somewhat higher. Velocity measurements were made along and across the bedding plane of Arbuckle limestone in a region where the dip of the bed approaches 90°. The velocity along the bedding plane was 17,400 feet per second and across the bedding plane 13,400 feet per second.

For some time the discrepancies between laboratory measurements of the elastic constants of rock samples and their measurements by seismic methods have been subject to discussion. At the present time, as far as the writers have been able to determine, there is no satisfactory explanation for this lack of agreement.

It was with the idea that an explanation might be found that the present work was undertaken. Two profiles were run on an extensive outcrop of the Tishomingo granite in the Arbuckle Mountains near Tishomingo, Oklahoma. Elastic waves, which were generated by explosions of dynamite, were recorded by an electrical seismograph, time and distance being the measured quantities. The first profile was started 500 feet from the shot point and was extended to 2,500 feet with the detectors placed at 200-foot intervals.

The time-distance graph for this profile is a straight line within the limit of error, which is considered to be 1 per cent. The velocity shown by this set of data for the longitudinal wave is 17,950 feet per second. Although impulses, which were probably due to transverse

[1] Read before the Geophysics Division of the Association at the Houston Meeting, March 24, 1933.

[2] Geophysical Research Corporation.

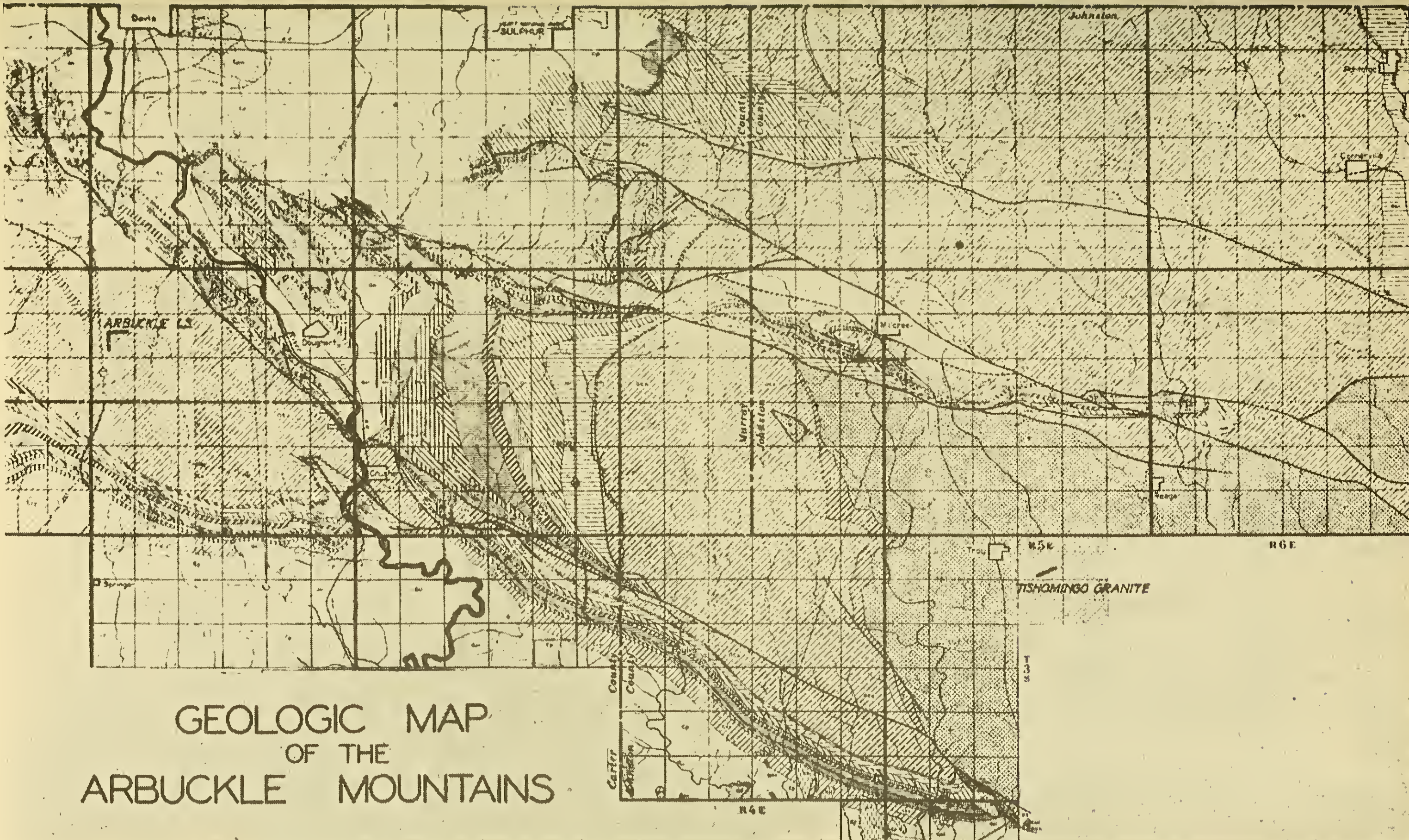


FIG. 1.—Geologic map of Arbuckle Mountains, showing recording locations. West-east distance, approximately 32 miles.

waves or Rayleigh waves, were observed on these records, they could not be interpreted with sufficient precision to draw any definite conclusions from them. Since it is necessary to obtain the transverse velocity in order to determine the elastic constants of the rocks by the seismic method, this profile shed no light on the problem.

The second profile, extending over a span of 400 feet, was laid out on a section of the granite which was free from fissures. The records taken over this profile showed both the longitudinal and transverse waves.

INSTRUMENTS

It was evident that it would be necessary to measure relatively small increments of time with a high degree of accuracy if a large number of points were to be obtained in this interval of 400 feet. If it be assumed, for the moment, that the velocity to be measured is of the order of 16,000 feet per second, and that the distance over which a velocity is to be determined be set at 100 feet, it would then be necessary to measure a time interval of the order of 0.0063 second. Since an accuracy of 2 per cent was desired, it was necessary to design recording and measuring instruments capable of detecting time differences as small as 0.00012 second.

The desired precision was attained by exercising extreme care in the selection of a perfectly matched set of detectors and recording channels.[3] The optical system was designed to give maximum sensitivity with a minimum width of the beam of light reflected from the galvanometer, thus producing a fine, clean-cut line on the record. The high frequency response of the recording systems was made as great as possible and a special fine-grain recording paper was used. The result was an extremely sharp, clean-cut, high amplitude break in the line corresponding with the time of arrival of the longitudinal energy. Time intervals of 0.01 second were placed on the record by means of an electrically driven tuning fork and a synchronized vibrating reed. This interval did not vary from an absolute value of 0.01 second by more than 0.04 of 1 per cent. Two complete sets of data were taken on this profile, one with a high speed oscillograph and the other using normal film speed. The purpose of the high film speed was primarily to obtain additional accuracy. The records were read with the aid of a micrometer microscope with which it was possible to read distances on the records corresponding to 0.00002 second.

[3] Theoretically this is not necessary where impulses corresponding with the first arrival of energy are observed. This added precaution was taken to insure uniformity in the later portions of the records.

METHODS

The section of granite selected for this work had a reasonably level and continuous surface for slightly more than 400 feet. No overburden of any sort was present.

A recording set-up comprising five detectors spaced 25 feet apart was placed at one end of this section of granite. It was found that it was quite essential to fasten the detectors rigidly to the granite since, when this was not done, relatively large errors were introduced by poor coupling. In order to accomplish this, plaster-of-Paris dams were built around the detectors and melted sulphur poured in. After the sulphur had solidified, the detectors were found to be cemented rigidly to the granite. Since it was necessary to use such extreme care in the mounting of the detectors, they were not moved during the course of the profile, but the shot point was moved instead. Shot points were placed every 100 feet, from 5 feet to 305 feet from the nearest detector. This gave data at 25-foot intervals, from 5 to 405 feet, with a check or overlap position on each record.

No indication of the instant of detonation was used in this work because of the possibility of small errors being introduced from this source. Times were measured from the instant of arrival of the wave to the nearest detector.

Two complete sets of data were taken on this profile. The first set was taken with a normal film speed in the oscillograph, that is with 0.01 second covering a distance on the record of about 0.75 centimeter. The second set was taken with a high film speed in the oscillograph, making the same time interval cover approximately 2.50 centimeters.

This procedure was carried out for both the longitudinal waves and the transverse waves. Separate sets of records were obtained for each type of wave. Specimens of the high and low speed longitudinal and transverse wave records are shown in Figure 2.

RESULTS

From an examination of Figure 3, it is seen that the time-distance graphs for both the longitudinal and transverse waves are slightly, but unmistakably, curved. If the observed points are plotted on a scale, of one half that shown in Figure 3, the graph then appears to be a straight line and it is only when an enlarged scale is used that the curvature becomes apparent.[4] From this it is evident that there is an

[4] Proof of the curvature of this line may be had by a least-square solution. Assuming a straight line, the residual is 0.00138 second while with the curve it becomes 0.00013 second.

increase in velocity with depth of penetration. As might be expected, the rate of change of velocity with depth is relatively small. From the curvature of the time-distance graph the penetration at 400 feet is estimated to be about 28 feet.

The longitudinal velocities calculated for four 100-foot sections of

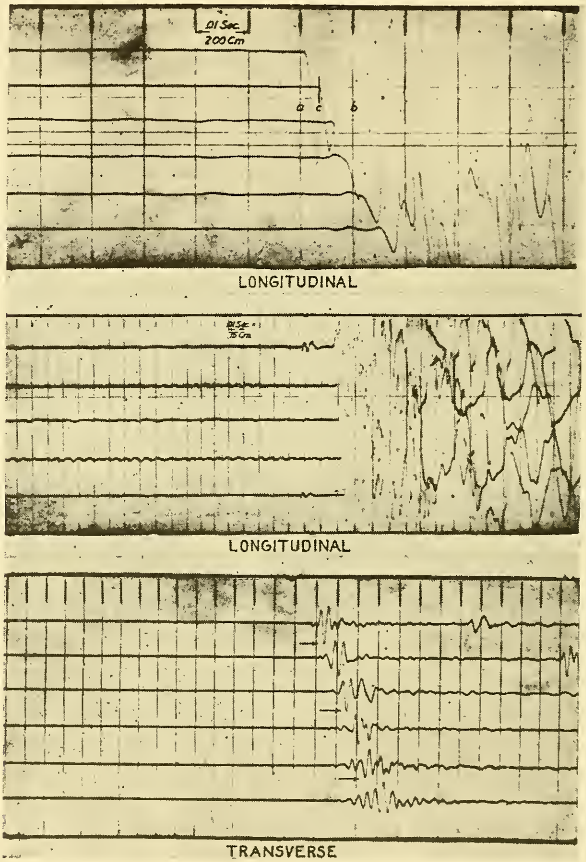


FIG. 2.—High and low speed longitudinal and transverse wave records.

this curve, range from 14,880 feet per second close to the shot point, to 17,150 feet per second at the far end of the curve. The corresponding transverse velocities are 7,000 feet per second and 7,950 feet per second. The longitudinal velocities are an average of 52 determinations; and the transverse, of 27 determinations.

The average density of the granite was obtained by determining the density of a large number of samples taken from the rock in the vicinity of the location used. The average value so obtained is 2.65

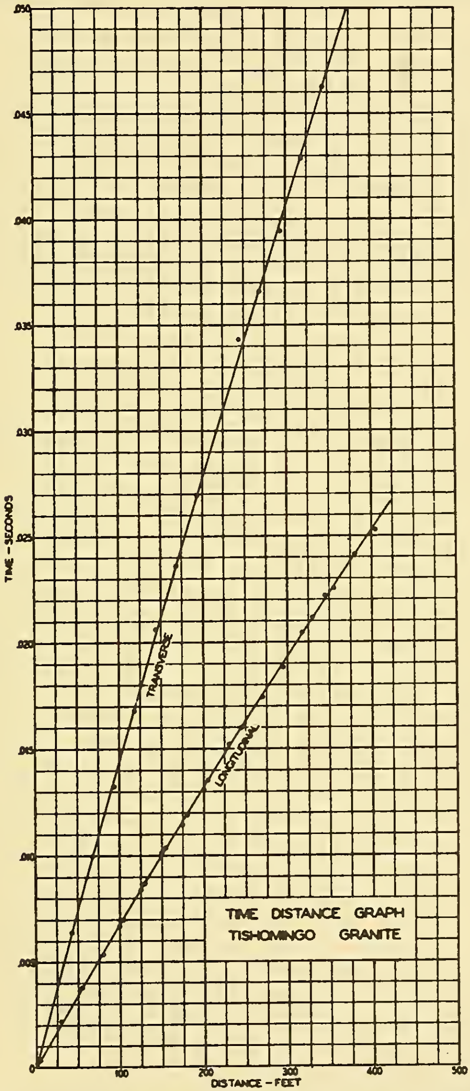


FIG. 3.—Time-distance graph, Tishomingo granite.

grams per cubic centimeter. Using this value, together with the observed velocities, the elastic constants may be calculated. The values thus obtained are shown in Table I.

TABLE I

		First Section 0–100 Feet	Last Section 300–400 Feet
Incompressibility	K	3.85×10^{11} dynes/cm².	5.17×10^{11} dynes/cm².
Rigidity	n	1.21×10^{11} dynes/cm².	1.56×10^{11} dynes/cm².
Compressibility	β	2.60×10^{-12} dynes/cm².	1.93×10^{-12} dynes/cm².
Poisson's ratio	σ	0.357	0.363
Young's modulus	E	3.29×10^{11} dynes/cm².	3.91×10^{11} dynes/cm².

Considering the fact that these values are representative of less penetration than the average values obtained by Leet and Ewing,[5] there is a remarkable agreement between their data and those presented here.

Although their average velocity values are somewhat higher, they are undoubtedly influenced by the high velocity portion of the time-distance curve since their profile extended to 4,000 feet.

SOURCES OF ERROR

In order to obtain an idea of the accuracy of the readings in this work, we may first consider the errors from a theoretical standpoint and then compare this with the observed variations.

Since the micrometer microscope makes possible the reading of distances on the records to 0.02 millimeter, and since the time interval measured (0.01 second) covered, on an average, 1 centimeter, a reading of a given point on the record could be made to 0.00002 second. In each determination of time, it was necessary to measure the two timing lines *a* and *b* and the instant of arrival of energy *c* (Fig. 2).

The maximum total error of reading would be obtained when the errors in all three readings were additive in the same direction and would therefore be .00006 second. However, it was not found possible to pick the instant of arrival *c* to this degree of precision. From an analysis of a large number of records it was concluded that the total error in picking the instant of arrival and in measuring the three necessary intervals on any one record did not exceed 0.00015 second. Since the total time corresponding with a distance of 100 feet is approximately 0.0063 second, the maximum error to be expected in reading any one trace would be approximately 2.5 per cent. In determining the average time for a given span a sufficiently large number of records were used so that the average value obtained is thought to be correct to within at least 2 per cent.

The distances along the profile used were laid out with a steel tape with a probable error negligible compared with the possible error made in measuring the time.

[5] L. Don Leet and W. Maurice Ewing, "Velocity of Elastic Waves in Granite," *Physics* (March, 1932), p. 160.

The frequency of the fork was maintained at 50 cycles per second within 0.03 per cent. The possible error in timing was therefore negligible.

Attention is directed to the fact that there are not as many supporting points on the time-distance graph of the transverse wave as there are on that of the longitudinal wave (Fig. 3). Due to the later arrival of the transverse energy in many cases there was interference between it and the longitudinal energy which made it difficult to pick corresponding points on successive traces. Consequently, only those traces were used where there appeared to be similar character. This is shown by the arrows on the transverse record of Figure 2.

LABORATORY MEASUREMENTS

Two samples of the granite were taken from this location for the purpose of making a laboratory determination of the velocity.

These samples were cut into two rods with a square cross section of 1.125 inches and lengths of 11.3 inches and 9.0 inches. A thin piece of steel was cemented to one end of the rod, which was supported by a clamp at its center. A permanent magnet around which a coil was wound was placed in line with the bar and near the steel on the end of the rod. The rod was then struck lightly on the opposite end, producing longitudinal vibrations which generated an E. M. F. of the same frequency in the coil. This E. M. F. and an E. M. F. from a calibrated vacuum tube oscillator were then impressed on the input of an amplifier, the output of which was rectified and the resulting beat frequency recorded. Then, this beat frequency and the frequency of the oscillator being known, the resonant frequency of the granite rod was determined. Substituting this value in the following equation gave the longitudinal rod velocity of the granite.

$$V=2fl$$

Where V is the longitudinal rod velocity
f is the natural frequency
and l is the length of the bar.

Figure 4 shows specimens of records obtained by this method.

The density of these granite samples was determined by weighing them in air and in water. Knowing the longitudinal wave velocity in the rod and the density of the material, the value of Young's modulus may be found from the relationship

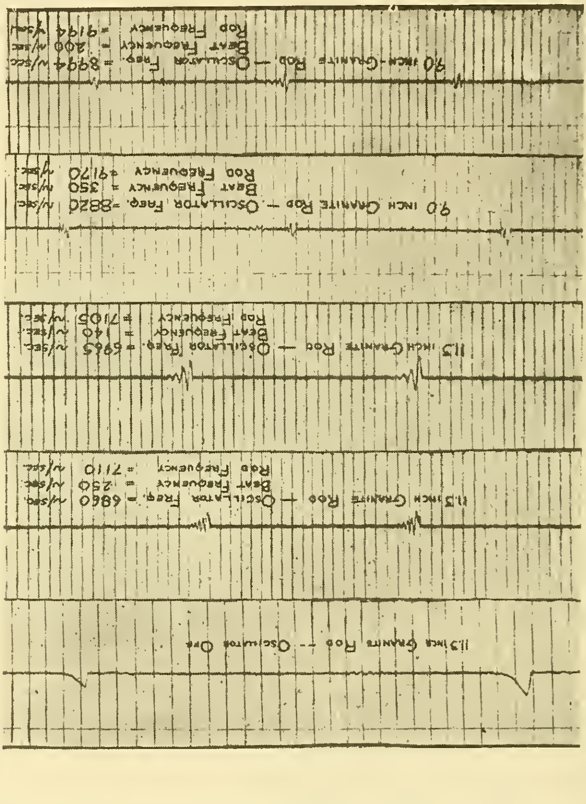
$$E=\rho V^2$$

Where E is Young's modulus
ρ is the density
and V is the longitudinal rod velocity.

Table II gives the values found for the two samples.

TABLE II

	Sample 1	*Sample 2*	*Average*
Dimensions	1.13 in. × 1.13 in. × 11.31 in.	1.13 in. × 1.13 in. × 9.0 in.	
Natural frequency	7,108 cycles/sec. (av.)	9,182 cycles/sec. (av.)	
Rod velocity	13,399 50 ft./sec.	13,773 100 ft./sec.	13,586 ft./sec.
Density	2.62 grams per cc.	2.68 grams per cc.	2.65 grams per cc.
Young's modulus	4.37×10^{11} dynes/cm.^{2}	4.72×10^{11} dynes/cm.^{2}	4.55×10^{11} dynes/c



Young's modulus is the only elastic constant which may be directly obtained by this method. The longitudinal rod velocity observed is characteristic of the material in the form of a rod only, and is therefore not directly comparable with the bulk velocity determined in the field measurements. If, however, the value of Poisson's ratio for the material is known, the corresponding bulk velocity may be calculated by means of the relationship

$$V_{LB} = V_{LR} \sqrt{\frac{1-\sigma}{1-\sigma-2\sigma^2}}$$

Where V_{LB} is the longitudinal bulk velocity
V_{LR} is the longitudinal rod velocity
σ is Poisson's ratio.

By using the value of Poisson's ratio obtained in the field measurements, namely, 0.36, the bulk velocities for the two samples were calculated to be 17,380 feet per second and 17,900 feet per second, respectively, the average being 17,640 feet per second.

SOURCES OF ERROR

The most important error in determining the longitudinal velocity in the rods used was in the determination of the recorded beat frequency. The oscillator calibration was known to one part in 5,000 so that calibration errors were negligible. The lengths of the rod samples were easily measured with sufficient accuracy so that any error thus introduced could also be neglected.

The equation relating the natural frequency of the rod to its length and the velocity of sound in the material is strictly accurate only if the length of the sample be very large compared with its lateral dimensions. In the case of relatively short rods, a correction must be applied to the experimentally determined value of natural frequency. The expression for this correction has been derived by Lord Rayleigh.[6] This correction was calculated for the particular samples used and was found to be considerably less than the experimental errors and so was neglected altogether.

The total error in the determination of the rod velocity of any particular sample was less than 1 per cent.

The temperature coefficient of frequency of the rock samples was not determined. The measurements were made at a temperature of 23° C.

COMPARISON OF FIELD AND LABORATORY DATA

The average bulk velocity for the longitudinal wave calculated from the laboratory data, namely, 17,640 feet per second, corresponds with the value of 17,950 feet per second observed over the long profile used in the field work and corresponds reasonably well with the value of 17,150 feet per second obtained for the 300 to 400-foot interval of the shot profile. To this extent, at least, the laboratory and field determinations may be said to agree. The values of Young's modulus obtained by the two methods do not, however, show any such agreement. Furthermore, the velocity of 14,880 feet per second characteristic of the 100-to-200-foot interval of the short field profile is de-

[6] Lord Rayleigh, *Theory of Sound*, Vol. 1, p. 251.

cidedly at variance with the velocity determined by the laboratory methods. It might reasonably be expected that the velocity in a small sample which is under no external pressure should check the velocity found for granite near the surface rather than that of the granite at some depth. It must be concluded, therefore, that there is a serious discrepancy between the two sets of data, which can not be accounted for by the experimental errors inherent in either method. The question, therefore, arises whether there is some factor influencing the results obtained by the laboratory method which does not affect the results obtained in field measurements. This point has been rather carefully investigated, but will be discussed only briefly.

One decided difference between the velocity measurements made in the field and those made in the laboratory is in the frequency used in making the measurements. The frequencies recorded in the field are approximately 100 cycles per second, while the natural frequencies of the rock samples tested in the laboratory are about 7,000 cycles per second. The question arises whether there is sufficient variation of velocity with frequency to account for a difference in the results obtained in the two cases. One possible effect which might cause a change of velocity with frequency is due to the fact that the attenuation constant of an elastic solid is a function of the frequency, and that the velocity at any frequency is dependent, to some extent, on the value of the attenuation constant.

The effect of attenuation upon the velocity may be disposed of very briefly. It may be shown that, if the attenuation in the rock sample under test is sufficiently low to permit the sample to oscillate vigorously enough to make the laboratory method practicable, then the effect of the attenuation upon the velocity will be negligible.

Another factor which might affect the results is the different amplitudes used in the two methods. Little information is available concerning the propagation of disturbances of finite amplitude in solids. It is believed, however, that the amplitudes used in both field and laboratory measurements are in general small enough so that the velocity is independent of the amplitude. The amplitudes in both cases are estimated to be of the order 10^{-6} cm. although, in the case of field measurements, the amplitude may be considerably larger at points near the shot point. A qualitative idea of the magnitude of the effect for this order of amplitude can be obtained by making use of the formula applicable to the transmission of plane waves of finite amplitude through gases. These calculations indicate that for the case assumed, the effect is entirely negligible. Calculation further shows that the amplitude would need to be of the order of 10^{-3} cm. to produce a variation in velocity of even 0.10 per cent. This effect may, therefore, also be neglected.

If, therefore, the samples used in making the laboratory determinations are truly representative of the bulk of the material in which the velocity is determined in the field, the results obtained in each case should be directly comparable, and the lack of agreement must, therefore, be interpreted as being due to an actual difference in the elastic constants of the samples used and the average value of those of the rock *in situ*. In the present investigation only two samples of the granite were used in making the laboratory determinations so that the "sampling error" may have been large. There is a further possibility that the elastic constants of the small samples may differ from those characteristic of the material before being cut out.

Another probable cause of the observed difference is a possible variation in the moisture content of the granite *in situ* and that of the samples used in the laboratory work. Recent work of the authors has shown that the presence of a small amount of moisture may seriously affect the elastic constants of rocks. This effect has also recently been reported by W. A. Zisman.[7]

Preliminary work by the writers indicates that the presence of moisture, in general, lowers the value of Young's modulus. The lower value of Young's modulus observed in the field determinations may, therefore, be due, in part at least, to larger percentage of moisture in the surface granite in the field as compared with that in the laboratory samples. Although no effort was made to dry out the samples used in the laboratory work, they were actually in an air-dry condition.

It is interesting to note that this explanation serves also as a possible explanation for the unexpectedly large curvature of the time-distance graph obtained from the field data. The increase of velocity with depth may prove to be partially due to a decrease in the moisture content of the rock below the surface.

Work is now in progress to test the validity of the explanations here advanced and it is hoped that they may be reported later.

ARBUCKLE LIMESTONE

Two profiles were laid out in the Arbuckle limestone with distances of 100–2,500 feet from the shot point. Detectors were placed every 200 feet along these spreads. A section of the limestone outcrop where the angle of dip of the bedding planes was approximately 90° was selected for the work. The lines of the two profiles were at right

[7] "Elastic Constants of Rocks and Their Relation to Seismic Wave Speeds," *Physical Review*, Vol. 43, p. 501.

angles to each other so that one set of data was taken along the bedding plane and the other across the bedding plane.

The time-distance graphs (Fig. 5) of these two profiles were drawn as straight lines since the accuracy of the data was not great enough to determine penetration.

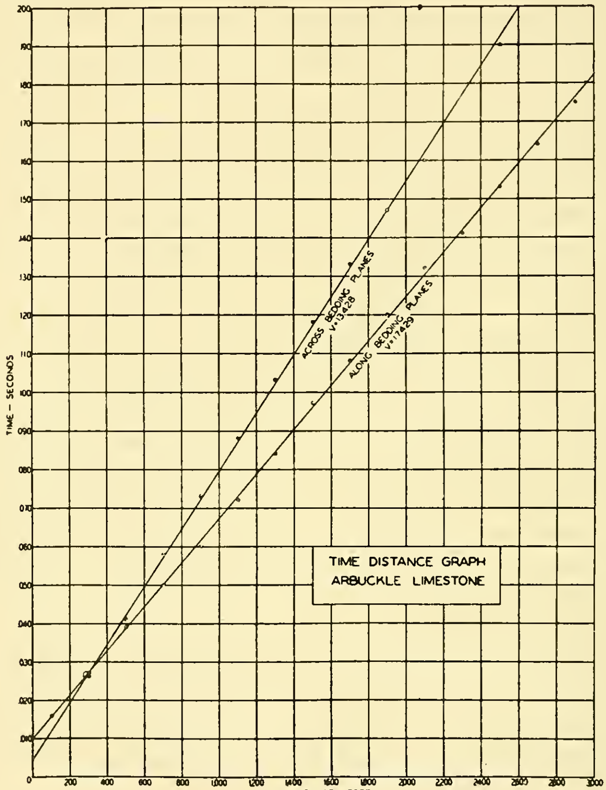


FIG. 5.—Time-distance graph, Arbuckle limestone.

The average velocity across the bedding planes of the limestone was found to be 13,430 feet per second and the velocity along the bedding plane was found to be 17,430 feet per second.

These results are in agreement with the data presented by McCollum and Snell.[8]

In conclusion, the writers wish to express their thanks to E. E. Blondeau and L. Y. Faust for their assistance in calculating and checking the data.

[8] Burton McCollum and F. A. Snell, "Asymmetry of Sound Velocity in Stratified Formations," *Physics* (March, 1932), p. 174.

BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 18, NO. 1 (JANUARY, 1934), PP. 119-132), 5 FIGS

APPLICATIONS AND LIMITATIONS OF DIP SHOOTING[1]

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Houston, Texas, and Houghton, Michigan

ABSTRACT

The history and methods of dip-shooting are outlined and the degree of accuracy obtainable by the method is investigated. Plotting of the misclosures on 151 dip-shooting traverses in the Gulf Coast to yield a frequency curve shows a probable error of about 87 feet, in traverses averaging 27,500 feet in length. This error is apparently independent of the length of the traverse, and within the range of spreads commonly used, that is, from 1,400 to 2,400 feet, independent of the spacing of the dip determinations. The smoothness of the frequency curve is broken by an excessive number of misclosures greater than 250 feet. These may be due to unusual structural conditions on the traverses concerned, such as terracing between dip points or faulting, or to gross personal errors. The principal common errors appear to lie in the drafting of the profiles, the determination of the time step-out, and the weathering correction. These errors on individual dips may considerably exceed the probable error, but on traverses of average length will tend to compensate one another. Other sources of error are discussed, but are believed to be of minor importance.

The purpose of this paper is to present an outline of the methods used in the determination of the dip of sound-reflecting strata in the Coastal Plain series of Louisiana and Texas by the reflection seismograph, and an analysis of the errors involved. Many data accumulated by Rosaire and Kannenstine and the Independent Exploration Company, of Houston, were available for this purpose. Especial acknowledgement must be made to J. D. Marr and C. B. Bazzoni for suggestions bearing on the research.

CONDITIONS GOVERNING CHOICE OF METHOD

The fundamental limitation on the use of reflections is that in themselves they tell little or nothing with regard to the nature of the reflector. Even in the "hard rock" areas, where definite lithological breaks occur, reflections must be identified by some auxiliary method, such as checking against the log of a well already drilled, or preferably, determining arrival half time by recording or shooting down a well.

In the upper portion of the Coastal Plain series no laterally persistent lithologic horizons have yet been determined, and so, *a priori*,

[1] Read before the Geophysics Division of the Association at the Houston meeting March 24, 1933. Manuscript received, September 30, 1933.

[2] Rosaire and Kannenstine, consulting geophysicists, Houston, Texas.

[3] Assistant professor of geology, Michigan College of Mining and Technology, Houghton, Michigan.

correlation of reflection depths holds forth little promise as a method of procedure in areas where oil production is sought from these strata. As a general rule, records of the reflections in these beds from two near-by shot points show little or no correspondence of reflection events. Records from adjacent shot points are no more correlatable than average well logs in those areas. In areas where the lower Tertiary is within reach of the drill, however, as on the so-called Conroe trend, it is sometimes possible to match reflections, and to contour depths on some persistent reflecting horizon with a considerable degree of confidence.

On the other hand, the salt-dome province of the Gulf Coast is in general characterized by localized structures of great relief. As a rule, there is very little surface topography, and the surface aerated layer is generally thin and uniform. Finally, the average velocity down through the geological column is a minimum, as compared with that of the Mid-Continent. All of these factors favor the accurate determination and use of the dip of the reflecting beds in mapping structure, a method which at present seems to be the one which holds the greatest promise for detailing sedimentary structure above the more deeply buried salt domes.

CORRELATION METHODS IN REFLECTION SURVEYS

Several investigators prior to 1926 had experimented in the use of reflections, but the general use of the method at present is undoubtedly closely associated with the success of the methods developed by the Geophysical Research Corporation.

This work was initiated in 1926 by J. E. Duncan, on the Nash salt dome in Fort Bend County under the supervision of J. C. Karcher. He continued his efforts there and in other areas in the Gulf Coast and in the Balcones fault zone, although his success was only promising, to say the least. However, due both to accumulated experience and perhaps to the chance to work on better defined problems, his results improved as he moved back into the hard-rock country, until he was getting fair reflections from the Viola limestone at Seminole. During the following two years, the Geophysical Research Corporation, in conjunction with the Amerada Petroleum Corporation, steadily worked on the problem, with regular improvements in technique. To-day's reflection technique is essentially that developed by the Geophysical Research Corporation during these two years, and on the whole, the use of dip shooting is really the only new departure from the interpretation methods established in Oklahoma from 1927 to 1929.

In justice to the investigators and their sponsors, the writers feel that some attention must be called to the fact that many scientists holding responsible positions in companies sponsoring major geophysical programs, refused flatly to admit that the method was based on fact. As one who personally tried to introduce the method into general consulting practice, the senior writer can definitely recall many times when reflections were not even considered on a par with the divining rod, for at least that device had a background of tradition.

DIP METHODS

In 1929, with the end of refraction explorations in sight, Eugene McDermott, then of the Geophysical Research Corporation, secured the approval of L. P. Garrett of the Gulf Production Company, to try reflections in the Gulf Coast. It must be admitted that no interpretation other than that developed in Oklahoma was contemplated, that is, the correlation of reflections from supposedly consistent beds. Some success was secured on two producing domes, Hankamer and Port Barre, but the method failed at the Clodine Prospect, and one dry hole was credited there to attempted correlations.

The next trial was at Darrow, then considered a very favorable refraction discovery which had been confirmed by the torsion balance. However, the area had already been investigated by two wells, both dry. So where even geologists walked softly, a reflection survey was started.

T. I. Harkins, in charge of the work, noticed that abnormal step-outs in the reflection time were rather characteristic of the area, and that these abnormal step-outs reversed. He correctly attributed this phenomenon to dipping beds, and pointed out that when recording up-dip the reflection arrival times for the farthest geophone should be abnormally short, and when recording down-dip they should be abnormally long. The method was then placed on a more quantitative basis, and is now the recognized method for mapping the younger formations of the Gulf Coast.

Since that time, the method has had some striking successes, and also some disappointments. However, these disappointments were largely the result of condemning reflection structures with single drill tests, which, to say the least, is somewhat rigorous. However, the background of success is rather satisfactory, since in several instances the reflection detail resulted in the location of discovery wells even after as many as three dry holes had supposedly condemned a prospect. On the other hand, at least three prospects have been drilled in spite of a condemnation by reflections, with resulting dry holes.

COMPUTATION OF SUBSURFACE DIP FROM REFLECTIONS

Figure 1 illustrates how dips are measured by reflections. By plotting the image of the shot point for the recording geophones, the problem is resolved into one of triangulation upon this image point by means of sound-wave paths. The distance between the recording geophones serves as a base line, and the other two sides of the triangle are the path lengths of the recorded reflections. Since only the travel times for these reflections are determined directly, the triangle is completed by multiplying these travel times by the appropriate velocity with which the sound waves have traveled. This computation is

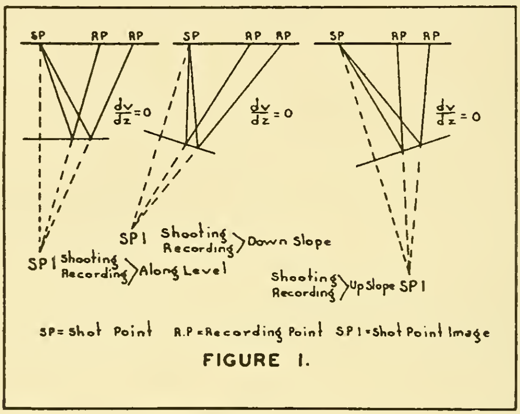

FIG. 1.—Geometrical solution of dip determination.

modified by certain corrections necessitated by a thin zone of low-velocity material almost invariably present at the earth's surface. The methods of correcting for this zone, known as the "aerated layer," are familiar to seismologists, but are too involved to describe here.

The velocity of the sound waves below the aerated layer can be measured in any of several ways. One method is to lower a recorder down a well, and record the travel time from an explosion at the surface. A second method is the analysis of refraction data from a profile of appropriate length and recorder density. Thirdly, the appropriate velocity may also be determined empirically by an analysis of the reflection times themselves, and due to the general lack of available refraction profiles or well recordings, this is the most widely used method in the Gulf Coast.

ACCURACY AND CONTROL

Dip reflection surveys for the purpose of local preliminary reconnaissance consist essentially of unclosed traverses, in general made up of straight lines intersecting approximately at the center of interest. This was the first type of survey originated, and is still used by many operators.

Shot points and recording positions alternate along some convenient straight line, so that records are taken on each geophone position from adjacent shot points. Using an appropriate velocity, the individual reflections are plotted on a one-one scale. After beginning at some convenient depth the profile of a "phantom" horizon is projected by use of the dips from near-by reflections. The elevations of this horizon are carried from one profile to another at the points where the traverses intersect, and the even hundred-foot elevations of the horizon are spotted on a map. By use of a contour interval of approximately 100 feet, the structure is then contoured on the resulting phantom horizon.

Such a survey is simple, and rarely leads to complications in interpretation, since with no closing traverses any error simply results in freak contouring. A simple survey of this type is quite satisfactory for symmetrical, unfaulted structure.

While, as pointed out later, errors inherent in the dip method tend to cancel each other over large areas, for accurate detail small closed traverses must be used, and the error in each must be distributed around it. We have therefore borrowed the torsion-balance technique, such as described by Roman.[4] This distribution of the apparent errors must precede any attempt at contouring. It can generally be made by inspection, or the whole traverse least squared by any one of a number of methods developed for use in torsion-balance surveys.

STATISTICAL ANALYSIS OF ERRORS

One hundred fifty-one closed traverses shot by the dip-reflection method, having a total length of 786 miles and involving 2,458 dip determinations, were used for the investigation. On these traverses the difference in elevation of an ephemeral horizon had, in large part, already been computed from the dips. The differences in elevation were summed up in a clockwise direction. An increase of elevation in that direction was given a positive sign, a decrease a negative sign. The residual found on closing the traverse has been termed the "misclosure." It is designated in feet and carries the sign resulting from an algebraic addition of the differences in elevation around the traverse.

The misclosure of each of the traverses used has been plotted with-

[4] Irwin Roman, "Least Squares in Practical Geophysics," *Geophysical Prospecting* (1932), p. 491.

out regard to sign to obtain the misclosure frequency curves reproduced in Figure 2. The number of occurrences of each misclosure has been laid off vertically. The amount of the misclosure to the nearest 10 feet is indicated along the horizontal axis.

A radical break in the original irregular curve will be noted at about 250 feet. This break may indicate that the errors in excess of 250 feet, which are obviously more frequent than errors near 250, are due to some special conditions or to gross personal errors occurring on a few traverses. The nature of such special conditions is discussed later, but it may be said here that they are probably geological.

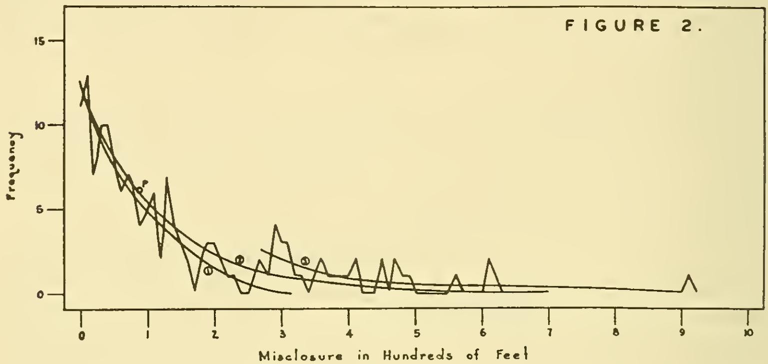

FIG. 2.—Frequency curve showing misclosure on 151 traverses. Horizontal co-ordinate of point *P* equals probable error.

To bring out this break, separate curves, numbered 1 and 3, have been fitted to the two portions of the original curve. Curve 1 may indicate the frequency of the usual errors encountered in dip shooting, while curve 3 may indicate in part the frequency of large errors due to special conditions. If that is the case, it would be advisable for the geophysicist to investigate any misclosure in excess of 250 feet which he may encounter in the course of his work, to discover if some peculiar geological condition such as faulting or terracing is not present. It may be, however, that the break noted in the curve is simply an accident due to insufficiency of data.

Curve 2 is an exponential curve representing the mean of curves 1 and 3. It approximates very closely the curve best fitting the original data. It is, moreover, capable of formulation as follows.

Equation (1) $x = 3.1 - 1.23 \log y$ where x equals the misclosure in hundreds of feet and y equals the frequency of occurrence.

From this equation the probable error in the traverses analyzed may be computed. The probable error has been defined as "that error which in a large number of cases is as often exceeded as not." Since the area under curve 2 is directly proportional to the number of cases considered, the value of the horizontal co-ordinate (x) corresponding with half the total area will be the value of the probable error. This value was found to be about 87 feet. The mathematical solution of the problem follows.

Differentiating equation (1), we obtain

Equation (2) $dx = -1.23\ (d \log y) = -1.23\ dy/y$

The area under the curve is the integral of ydx. As, by equation (2), dx equals $-1.23\ dy/y$, we have for the area A

Equation (3) $A = -1.23 \int_0^\infty ydy/y = -1.23\ (y_\infty - y_0)$

By equation (1) when x is equal to zero

Equation (4) $0 = 3.1 - 1.23 \log y$

(5) $\log y = 3.1/1.23$

(6) $y = e^{2.64} = 12.3$

Again by equation (1), when x is equal to ,

Equation (7) $\infty = 3.1 - 1.23 \log y$

(8) $\log y = \dfrac{3.1 - \infty}{1.23} = -\infty$

(9) $y = e^{-\infty} = 0$

Then by equations (3), (6), and (9), we have

Equation (10) $A = -1.23\ (y_\infty - y_0) = -1.23\ (0 - 12.3) = 15.1$

But the probable error is that value of x corresponding with half of the area, A. Let us designate this value of x as p. Then, substituting 7.55 for $1/2\ A$, y_p for y_0, and zero for y_∞ in equation (3) we derive

Equation (11) $7.55 = -1.23\ (0 - y_p) = 1.23\ y_p$

(12) $y_p = 6.14$

Substituting p for x and 6.14 for y_p in equation (1), we find,

(12) $p = 3.1 - 1.23 \log 6.14 = 0.87$

As this is the probable error sought, expressed in hundreds of feet, the probable misclosure deduced from the available data is about 87 feet.

The probable error or misclosure here computed will serve as a measure of the accuracy to be expected from the dip-reflection method in the Coastal Plain series on the Gulf Coast. Some of the sources of misclosure, together with the conditions affecting the accuracy of the dip method will be considered next.

CRITICAL ANALYSIS OF METHODIC ERRORS

Probably the greatest source of error is in the drawing of profiles. The average length of the traverses in the group considered was 27,500 feet. Dividing the probable misclosure by the average length of traverse we obtain the tangent of the probable angular misclosure, the net error being distributed over the entire length of the traverse.

$$87/27{,}500 = 0.00316 = \tan 0°.11'$$

This angular error is astoundingly small. Clearly in making up a profile a computer can not plot the dips to so great a degree of accuracy. Indeed, the difference between the dips of the several reflections obtained at different horizons from the same shot is usually several times this amount. In drawing the phantom horizon, however, the geophysicist, realizing which dips conform best to the structural picture he is making, will naturally select those yielding the least misclosure. This personal element doubtless accounts to some extent for the surprisingly small value of the probable error; but it must also be remembered that the error here determined takes into account the compensatory effect of numerous observations.

Another considerable error may enter into the computation of the dips through incorrect time determinations. Such errors may be due either to phase lag in the recording instrument, or to the personal element in picking the exact time on the record. Errors of more than 0.002 second due to these two causes are probably not unusual,[5] and might produce errors in the time difference of at least 0.0015 second. While such an error in determining the time step-out would produce an error in dip of more than 1°, the compensatory effect of the numerous observations made in any traverse will reduce the net error to a fraction of this amount.[6]

Other sources of error dependent upon inaccuracies of measurement lie in the determination of the thickness of the low velocity zone under the first and last geophones, computations based on an incorrect velocity for that zone, and use of an incorrect velocity in the underlying high-velocity zone. Of these three sources of error, the first is the most serious. Where great variability in the thickness of the low velocity zone exists, its relative thickness under each of the end seismometers must be determined as accurately as possible before any validity can be attached to the dips computed. A difference of 3.5 feet in the thickness of the low-velocity zone between the ends of the geophone spread may easily produce an error of about 1° in the computed dip if not properly corrected.

Errors in the velocity itself are less important. While the use of an erroneous velocity may produce an error in dip of several degrees, since shot points are located at equal distances on opposite sides of the seismometer position, the errors will be in opposite directions in the pair of dips so obtained from each reflecting horizon. They thus compensate each other almost exactly. If the error is great this alterna-

[5] C. B. Bazzoni, personal communication.

[6] Paul Weaver, paper read before the Houston Geological Society, September 21, 1933.

tion of direction becomes obvious in the profile and the velocity may be corrected accordingly. The phenomenon of alternate opposite errors in dip is commonly known as "wash-boarding."

A more serious velocity error enters where there is a rapid but uniform change of velocity horizontally. This results in a displacement of the image point in the direction of increasing velocity. In consequence, the computed dip of the sediments will be rotated downward away from the direction of increasing velocity, as compared with the true dip. Therefore, if the velocity in the sediments over a deeply buried salt dome increases over the uplift, the uncorrected dip picture will tend to exaggerate the structure. Refraction surveys have demonstrated that many of the more deeply buried domes produce such velocity increases in the overlying sediments, as at Iowa.

This type of velocity change will produce "wash-boarding" in those portions of the area where the velocities are notably different from the standard velocities used for various depths. But the error here referred to will not be compensated by the "wash-boarding," as it always rotates the dip downward away from the direction of increasing velocity, regardless of the direction of the shot.

In dip shooting it is assumed that the plane of the incident and reflected wave-path is vertical. Obviously, this is true only when the shot point and seismometers are lined up directly along the direction of dip. If they are lined up in any other direction the plane of the wave path lies not vertical but at right angles to the reflecting bed. The dip measured is then the component of the dip in that plane rather than in a vertical plane. The error introduced by this inclination of the reflection plane, generally known as "side-swipe," is negligible in the case of low dips, but where the dip is steep and the strike variable, it might produce a perceptible misclosure. This would be especially likely if the form of the closed traverse departed far from a parallelogram. In that case the errors made by shooting diagonally across the dip downward would not be compensated in shooting diagonally across the dip upward on the opposite side of the traverse, or *vice versa.*

A factor which might be expected to control the amount of misclosure is the spacing of the dip determinations. In drawing an ephemeral horizon the dip between the points where it has actually been determined must be interpolated. This interpolation is, of course, based on the assumption that no radical variations in dip occur between the actual determinations. This assumption is obviously a possible source of misclosure, and accordingly, closer spacing of dips should tend to reduce misclosure.

The writers attempted to ascertain whether the spacing factor exercised any control over the amount of misclosure in the traverses analyzed. The average spacing of dips on these traverses varied from 1,400 to 2,400 feet, with a very few lying outside of this range. For 107 of the traverses, selected at random, the amount of misclosure was plotted along the horizontal axis of a graph, while the average spacing of dip determinations in the same traverse was laid off vertically. The resulting diagram is shown in Figure 3.

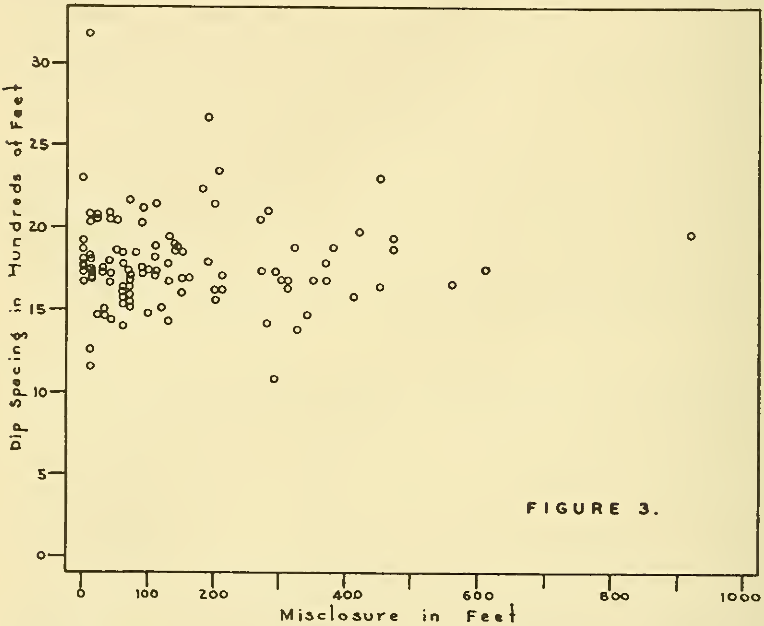

FIG. 3.—Comparison of average spacing of dips with misclosure of traverse. Chart shows no apparent relationship.

An inspection of the figure reveals a scattering of points through which no mean curve could justifiably be drawn. This negative result is probably explainable by the lack of variety in the several traverses considered. It serves to demonstrate, however, that for the dip spacing used in the work analyzed, the spacing exercised little if any effect on the amount of misclosure. Considering also the surprisingly small size of the probable angular misclosure obtained, this result suggests that no closer spacing is necessary, and indeed, that an even wider spread than the average one used might be practicable. The average spread on the traverses studied from the shot point to the middle geophone was 1,700 feet.

It has already been pointed out that due to the custom of transmitting the seismic waves in alternate directions around the traverse most of the errors discussed tend to cancel on alternate shots. A careful tabulation of the direction of shooting on the traverses inspected showed an almost equal distribution of the alternate directions around each traverse. Only the effect of "side-swipe" on asymmetrical traverses and rapid horizontal increase of velocity over deep salt domes seem likely to produce cumulative errors.

Cumulative errors should on the average produce maximum mis-

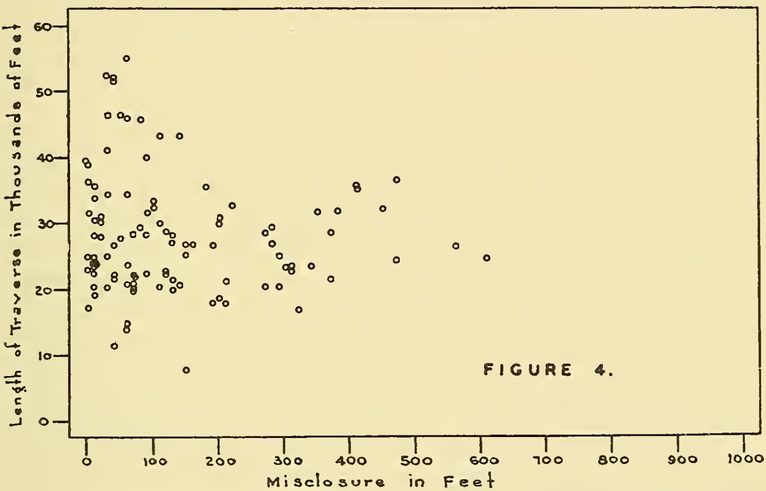

FIG. 4.—Comparison of length of traverse with amount of misclosure. Chart shows no apparent relationship.

closure on the longest traverses, while non-cumulative errors should tend to balance out more perfectly on the longer traverses. The same 107 traverses shown in Figure 3 were plotted to yield Figure 4. In the latter figure the amount of misclosure of each traverse has been laid off along the horizontal axis while the length of the traverse has been laid off vertically. An inspection of the figure shows a scattering of points indicating no readily perceptible relation between the length of the traverse and the amount of misclosure. It is therefore concluded that cumulative errors enter but slightly into the dip-reflection method, and that the length of the traverse has little to do with the amount of misclosure, except for the cases where the misclosure should be explained as follows.

STRUCTURAL SIGNIFICANCE OF LARGE MISCLOSURES

The most significant source of misclosure will, of course, be sought in structural conditions. We have seen, however, that only in cases of extreme misclosure, namely misclosures in excess of 250 feet, is it necessary to seek such a source. One structural condition which would readily give misclosure is a short sharp divergence from the normal dip, producing in effect a small structural terrace. Such a dip might yield a reflection on one side of a closed traverse and, due to the accident of dip spacing, fail to be detected where crossed by the other side of the traverse. Or the terrace might die out laterally and therefore intersect the traverse at one place only. In the latter case a large misclosure would result if no reflection were observed upon the terrace.

Another structural source of large misclosure may be the intersection of the traverse by a fault at one point only, the fault terminating somewhere within the traverse. Likewise a fault that continues across both sides of a traverse will produce misclosure if its throw differs markedly at the two points of intersection. In either case the fault must necessarily be of the hinge type. A fault having approximately equal displacement at its two points of intersection with the traverse cannot produce a misclosure large enough to be safely attributed to structural conditions.

In postulating faulting as the explanation of a large misclosure, due consideration must be given to the limitations set by common geological knowledge of the behavior of hinge faults. While there is, of course, some change in the rate of decrease of displacement in passing from the point of maximum displacement toward either end of the fault, this change in rate is very gradual. Consequently, large misclosures caused by hinge faulting should be observed on several adjoining traverse blocks, should differ more or less uniformly in value, and except where passing over the point of maximum displacement, should be of like arithmetical magnitude. In passing over the point of maximum displacement, however, the sign of the misclosure will change.

If the observer places himself on the upthrown side, faces the fault, sums up the changes in elevation around the traverses in a clockwise direction, giving a positive sign to increases in elevation and a negative sign to decreases, the following rule will apply: positive misclosures will lie on the right of the point of maximum displacement; negative misclosures will lie on the left. By means of this rule the upthrown and downthrown sides of the fault may be distinguished.

This rule may be used only where the point of maximum displacement lies within the area under examination. If the fault dies out within the area prospected the upthrown side may be distinguished by the following rule. If the observer places himself on the downthrown side of the fault, facing the fault, positive misclosures in the successive traverses which the fault cuts will increase toward the right, while negative misclosures will increase toward the left. If neither the end nor the point of maximum displacement of the fault lies within the traversed areas, the direction of throw can not be determined, as the points of maximum positive and negative misclosure lie approximately

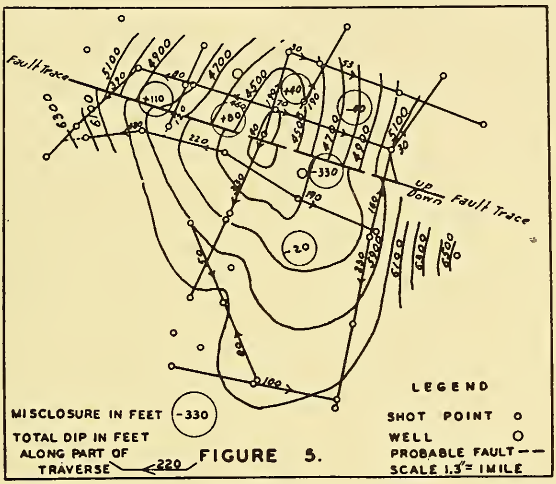

FIG. 5.—Structure contour map of faulted salt dome showing effect of differential displacement on closure of traverses. Contour elevations below sea-level.

midway between the point of maximum displacement and the ends of the fault. Therefore the misclosures will all be of like sign in the case postulated, and the amount of misclosure will decrease either toward the end of the fault, or toward the point of maximum displacement, or both, the two directions being indistinguishable under the circumstances.

In all of the work examined only two cases of misclosure which the writers considered justifiably attributable to faulting were found. One of these cases is used here as an illustration of the principles already outlined. A portion of the structure-contour map built up on dip-reflection data is shown in Figure 5. The significant closed traverses are also given, and the amount of misclosure is written in the center of each. The fault was first suspected because of the progressive values of the misclosure in the three traverses through which it is believed to pass. These are from west to east, plus 110 feet, plus 80 feet, minus 330 feet, suggesting a fault having its maximum displacement near the common side of the eastern and middle traverses and its upthrown side on the north.

Paleontological data and lithological logs from two wells, one on each side of the supposed fault, appear to indicate a maximum throw of about 1,200 feet. As the paleontological markers used have not been thoroughly tested in the region, however, this amount of throw may be erroneous. The horizon used was a calcareous bed characterized by the simultaneous first appearance of *Textularia mayori* and *Nonion scapha* in one well, and in the other well the simultaneous first appearance of the same two species in shale a little more than 30 feet below it. No species whatever of *Textularia* or *Nonion* were found above this horizon in either well. Correlated on the basis of the calcareous bed referred to, a good correspondence between the principal calcareous and sandy horizons in the lithologic logs of the two wells was found.

The two faults noted by the writers in the course of the research were rendered conspicuous by the large misclosures they produced on some of the traverses they crossed. Faults with less differential of displacement would easily pass unnoticed because of the small misclosure they produce. In conclusion, therefore, it may be said that faults having small differential of displacement will go undetected in dip-reflection work and may constitute one of its greatest sources of error.

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THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS

GEOPHYSICS

1934-1935

REPRINTS FROM

THE BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS, BEING PAPERS PRESENTED BEFORE THE GEOPHYSICS DIVISION OF THE ASSOCIATION IN ANNUAL CONVENTION AT DALLAS, TEXAS, MARCH 24, 1934; FROM THE TRANSACTIONS OF THE AMERICAN GEOPHYSICAL UNION; FROM THE AMERICAN JOURNAL OF SCIENCE; FROM THE PROCEEDINGS OF THE WORLD PETROLEUM CONGRESS; FROM PHYSICS, OF THE AMERICAN PHYSICAL SOCIETY; AND FROM THE PROCEEDINGS OF THE PHYSICAL SOCIETY OF LONDON

TRANSACTIONS

SOCIETY OF PETROLEUM GEOPHYSICISTS

VOL. V

PUBLISHED BY

THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS

TULSA, OKLAHOMA, U.S.A.

PUBLISHED, MARCH, 1935

Composed and Printed by
George Banta Publishing Company
Menasha, Wisconsin, U. S. A.

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Volume 19 Number 1

BULLETIN
of the
AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS

JANUARY, 1935

INFLUENCE OF GEOLOGICAL FACTORS ON LONGITUDINAL SEISMIC VELOCITIES[1]

B. B. WEATHERBY[2] AND L. Y. FAUST[2]
Tulsa, Oklahoma

ABSTRACT

An examination has been made of velocity data obtained from fifty wells located in eight states. There is definite evidence to show that the velocity of shale and sandstone sections increases with the geologic age of the beds. Less reliable evidence points to a similar relationship for limestone velocities.

The velocity of calcareous sandstones and shales is substantially higher than that of a normal sand-shale section.

The effect of depth on the velocity of a given section is investigated. There is a definite increase in velocity with depth in the case of shale and sand sections.

Increase of velocity with age is ascribed to the general increase of the lithification of sediments with age.

INTRODUCTION

The idea that the longitudinal seismic velocity of sediments increases with the age of the sediments has been held rather generally for some time. It was to discover quantitatively what relationship existed that the writers have made an analysis of the velocity measurements available to them in the files of the Geophysical Research Corporation. The data used comprise velocity measurements of good quality taken in fifty wells selected from a total of seventy well velocities available. These wells are located in Mississippi, Louisiana, Texas, New Mexico, Oklahoma, Kansas, Colorado, and Pennsylvania.

[1] Read in preliminary form before the Geophysics Division of the Association at the Dallas meeting, March 24, 1934.

[2] Geophysical Research Corporation. The writers express their thanks to John L. Ferguson, of the Amerada Petroleum Corporation, for his aid in supplying geologic data.

PROCEDURE

The type of data which were analyzed is shown in Figure 1, where velocity-depth curves from three wells are plotted as *AA*, *BB*, and *CC*. These curves have been determined in the usual manner by well shooting. The procedure is to place a charge of dynamite in a hole 20–70 feet deep located about 1,000 feet from a well. A geophone is lowered into the well to some measured depth and this geophone records the time of the first arrival of energy from the exploded charge. The straight-line distance from the shot to the geophone,

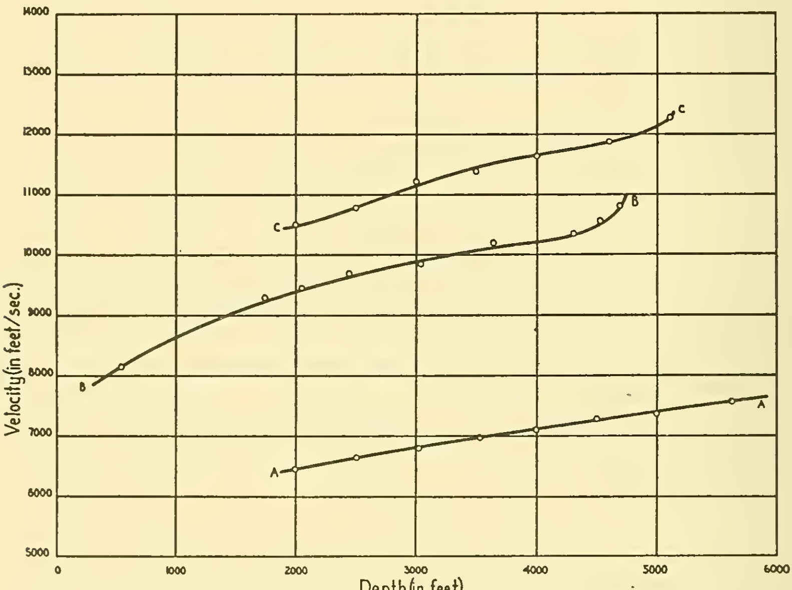

FIG. 1.—Results of velocity determinations in three wells. Average velocity from surface to given depth is plotted against that depth.

divided by this time, gives the average velocity of the section to the depth of the geophone. The velocity so determined is somewhat lower than the average velocity along the actual path of the ray. The actual path of least time is slightly curved and therefore would yield a greater average velocity. The geophone is then lowered to some other depth and another measurement of time is secured and the velocity calculated. It is these velocities plotted against depths of the geophone below the casinghead which are shown in Figure 1.

These curves represent the average velocity of a section from the surface to any depth plotted against that depth. Since the velocity of the first 1,000 feet of material, for example, enters into the velocity obtained for the first 2,000 feet of section, these curves do not indicate clearly the velocity of the section between 1,000 and 2,000 feet deep. This can be obtained for a vertical path by taking the difference in times of arrival of the energy at the 1,000- and the 2,000-foot depths and dividing it into the difference between the two depths. This velocity is termed the differential velocity of that particular section. On the other hand, the average velocity of the section is measured from the surface to any given depth.

It is the differential velocities which are significant in showing the manner in which velocity varies with depth. The results given in this paper are differential velocities and are determined from actual measurements rather than from a smooth curve drawn through the velocities measured. For the most part, these differential velocities represent the velocity of a section about 1,000 feet thick.

To compare velocity in sections of different geologic age, it is necessary that the sections differ from one another as little as possible in all respects except age. Sections of shale and sand were used chiefly for comparison. A section logged as "calcareous" will give velocities considerably higher than a normal shale and sand section. Curve *CC* (Fig. 1), for example, is a curve of average velocities in Permian and Pennsylvanian beds of calcareous sand and shales. The velocities observed are considerably higher than those in the normal Pennsylvanian section represented by *BB*. This example discloses the need for a careful selection of similar sections so that comparisons of velocities may be significant.

As is shown later, the amount of overburden on a given section influences the velocity of that section. This fact requires that comparisons of velocity in sections of different geologic age be made under the same conditions of overburden.

The data available on limestone velocities is scarce and subject to greater errors than the shale velocities. Velocities of limestone at the surface have been measured by short refraction profiles on the limestone outcrop. These results are open to some question, however, since some alteration in velocity is to be expected due to the weathering of the limestone at the surface.

Measurements of limestone velocities in wells are difficult. Most of the limestone sections available for measurement being less than 400 feet thick and the velocities being high, the error in velocity determinations is considerable. It was not possible to secure a comparison of velocities of limestone sections at exactly the same depth below the surface.

RESULTS

SHALE AND SANDSTONE VELOCITIES

The velocities for shale and sandstone from the surface to a depth of 2,000 feet are given in Table I. The velocity given for each age is an average of velocities obtained in several wells. The individual velocities followed the mean closely.

TABLE I

Velocities in Shale and Sandstone to 2,000 Feet

Age	*Velocity (Ft./Sec.)*
Devonian	13,300
Pennsylvanian	9,500
Permian	8,500
Cretaceous	7,400
Eocene	7,100
Pleistocene-to-Oligocene	6,500

Because of the presence near the surface of the weathered layer of rather indefinite extent, some question may exist as to the accuracy of these results. The effect of this surface layer is canceled out in the determination of differential velocities in sections below the surface. Thus there are no weathering effects in the data in Table II, which shows the differential velocities obtained in shale and sand sections for a section extending from 2,000 to 3,000 feet below the surface, and also for a section from 3,000 to 4,000 feet deep. There is also listed the percentage increase in velocity at the greater depth.

TABLE II

Differential Velocities in Shale and Sandstone

Age	*Velocity from 2,000 to 3,000 Feet (Ft./Sec.)*	*Velocity from 3,000 to 4,000 Feet (Ft./Sec.)*	*Percentage Increase*
Devonian	13,400	13,500	0.7
Pennsylvanian	11,200	11,700	4.5
Permian	10,000	—	—
Cretaceous	9,300	10,700	15.0
Eocene	9,000	10,100	12.2
Pleistocene-to-Oligocene	7,200	8,100	11.2

The data shown in Tables I and II are graphically represented in Figure 2 with some additions at the upper ends of the curves.

Figure 2 can not be said to represent the rate of increase of velocity as a function of depth alone. The velocity of the Pennsylvanian sediments shown in Tables I and II, for example, was secured in part by measurements of sections of the Pennsylvanian of different age in the same wells. Consequently, the increase in velocity with depth could be caused partly by the fact that older Pennsylvanian sediments were measured as well as by the fact that they were at the greater depths. The relationship between velocity and depth will be signifi-

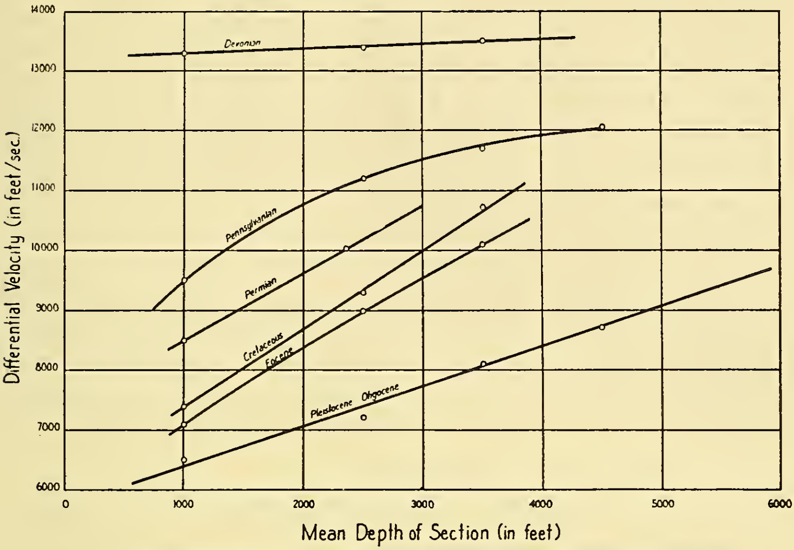

FIG. 2.—Differential velocities plotted against mean depths for sections of different ages.

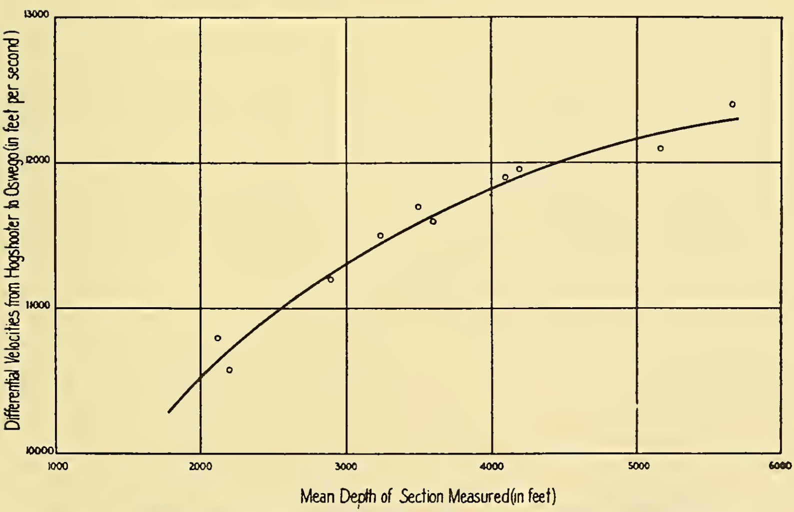

FIG. 3.—Differential velocity in given Pennsylvanian section plotted against mean depth of that section.

cant only when the velocity in a section whose age is kept fixed can be measured while the amount of overburden on that section is varied.

Ideal conditions for determining the velocity-depth function can be approximated by velocity measurements of the section of Pennsylvanian sediments between the Hogshooter and the Oswego. This section is fairly constant in its lithologic character throughout central Oklahoma. The average lime content of the section in the wells where velocity measurements were available was 14 per cent with a maximum variation in any well of 3 per cent from this mean. The average thickness of the section was 750 feet. Figure 3 represents the differential velocity obtained in this section plotted against the mean depth below the surface of the section.

LIMESTONE VELOCITIES

Except for the work of Weatherby, Born, and Harding[3] in the measurement of the longitudinal velocity of Arbuckle limestone, little work has been done on the accurate measurement of surface velocities in limestone. The data available on surface limestone velocities is listed in Table III.

TABLE III

SURFACE LIMESTONE VELOCITIES

Age	*Velocity (Ft./Sec.)*
Cambro-Ordovician (Arbuckle)[3]	17,400
Ordovician (Viola)	16,700
Devonian (Hunton)	4,000
Mississippian (Mayes)	12,500
Pennsylvanian (Belle City)	15,000
Cretaceous (Edwards)	11,000

With the exception of the Pennsylvanian value these velocities follow the age of the limestones in an orderly fashion. The greater velocity of the Pennsylvanian may be due to its having been under a few feet of cover while the rest of these velocities were determined on surface exposures. It should be noted that these velocities may be in rather serious error due to surface weathering and erosion.

TABLE IV

VELOCITIES IN LIMESTONE

Age	*Velocity (Ft./Sec.)*	*Mean Depth of Section Measured (Feet)*
Ordovician	20,000	4,000
Devonian	17,500	4,500
Mississippian	17,000	4,700
Pennsylvanian	15,500	3,000
Permian	15,500	3,900
Cretaceous	13,500	3,300

[3] B. B. Weatherby, W. T. Born, and R. L. Harding, *Bull. Amer. Assoc. Petrol. Geol.*. Vol 18, No. 1 (January, 1934), p. 106.

A list of velocities in limestone at a depth of 3,000–5,000 feet is given in Table IV. Due to the sources of inaccuracy previously mentioned, these velocities may be in error by as much as 1,000 feet per second.

The curve *CC* (Fig. 1) shows a velocity through calcareous Permian sediments to a depth of 2,000 feet of 10,500 feet per second, as against 8,500 feet per second obtained from the average non-calcareous Permian section of this depth. Likewise, the differential velocity on curve *CC* from 2,100 to 3,500 feet is 13,300 feet per second. This is a section of Pennsylvanian beds of 14 per cent limestone and the remainder of the section composed of calcareous sands and shales. A normal section of Pennsylvanian sediments containing 14 per cent limestone and the remainder non-calcareous sand and shale gave a velocity of 11,500 feet per second. A section of 665 feet of these normal sediments plus 735 feet of Pennsylvanian limestone gives the velocity equivalent of the calcareous section.

DISCUSSION

The results obtained show conclusively that the velocity of non-calcareous sections of shale and sandstone increases with the geologic age of the section. That a similar condition holds for velocities in limestone seems to be indicated.

Figure 3 shows the rate of increase of the velocity in a Pennsylvanian section of fixed age with the depth of that section. A comparison of the curve with the Pennsylvanian curve of Figure 2 shows that these two curves agree fairly well. This suggests that the two major causes of variation of velocity in fairly thick sand and shale sections are age and depth. If this is correct, we may consider the group of curves of Figure 2 as being representative of the rate of increase of velocity with depth in shale and sandstone of any given age. It might be expected that the more recent beds, being of lower velocity, would exhibit a greater rate of increase of velocity than the older beds. This appears to be true of the velocities from Cretaceous to Devonian which show some tendency toward approaching the Devonian velocity-depth curve asymptotically. However, the Eocene and more especially the younger sediments show not only a lower rate of increase of velocity with depth but also show a lower percentage increase. A possible explanation for the slower rate of increase of velocity in these younger sediments may be found in the manner of their deposition. In the wells where velocities were measured, these beds were composed mostly of land sediments. They are less tightly cemented than the older sediments which are chiefly marine. It might, therefore, be expected that such sediments would show a different rate of increase under compaction, since the increase of density tending to decrease the velocity might be greatest in uncemented materials.

The increase of velocity with age appears to be linked with the general increase of the lithification of sediments with age. This appears especially reasonable in view of the high velocities observed in sections of calcareous sand and shales.

The formula for longitudinal velocity (v) in a solid is given by the equation

$$v = \sqrt{\frac{k + 4/3n}{\rho}}$$

where k is the bulk modulus, n the rigidity, and ρ the density of the solid. It is, therefore, apparent that any increase of density of a section with depth would tend to decrease the velocity. Since an increase of velocity with depth is observed, it is apparent that the numerator of the equation must be increasing more rapidly than the denominator. That is, either incompressibility or rigidity or both increase with depth more rapidly than the density. Likewise the effect of lithification must be to increase the elastic constants at a greater rate than the density.

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EFFECT OF MOISTURE UPON VELOCITY OF ELASTIC WAVES IN AMHERST SANDSTONE[1]

W. T. BORN AND J. E. OWEN[2]
Tulsa, Oklahoma

ABSTRACT

Laboratory measurements of the velocity of elastic waves in Amherst sandstone show that the bar velocity is dependent on the moisture content, the velocity decreasing as the moisture content is increased. Bar velocities ranging from 7,640 to 4,415 feet per second were observed. It is shown that the change in velocity is caused primarily by a change in the value of Young's modulus of the material. The suggestion is made that this phenomenon may have some bearing on the question of Gulf Coast reflection surveys.

INTRODUCTION

In the course of a program of work being carried on at the laboratory of the Geophysical Research Corporation to determine the velocity of elastic waves in various materials, it was found that one of the samples used showed variations in velocity from day to day of an amount entirely too great to be accounted for by experimental errors. The determinations were made with the sample in an air-dry condition and it was therefore suspected that the velocity variations observed might be caused by a change in the moisture content of the sample due to varying atmospheric conditions. This supposition led to the work here described.

DESCRIPTION OF SANDSTONE

The sample used was a bar of Amherst sandstone approximately 2 inches square and 4 feet long. This rock is fine-textured sandstone of Triassic age, outcropping in the Connecticut Valley. The sample was one of a number obtained from a local stone cutter who, unfortunately, was unable to specify the exact location from which it was obtained. The density of a dry sample of this material was found to be 1.92. To obtain an idea of the total porosity of the stone, a sample

[1] Read before the Geophysics Division of the Association at the Dallas Meeting, March 23, 1934.

[2] Geophysical Research Corporation. The writers wish to extend their thanks to Miss Margeret C. Cobb and Bruce H. Harlton, both of the Amerada Petroleum Corporation, for their study of the rock section, and also to Parker D. Trask and K. E. Lohman of the United States Geological Survey for their kindness in furnishing the photomicrographs.

FIG. 1.—Photomicrograph of Amherst sandstone. Ordinary light; magnification 35×.

Fig. 2.—Photomicrograph of Amherst sandstone. Polarized light; nicols crossed; magnification 35×.

of measured volume was weighed, evacuated, saturated with water and reweighed. It was found that the sample absorbed 23 per cent of water by volume or 12 per cent by weight.

In order to ascertain the structure of the rock, a section was made for microscopic study. Through the courtesy of Parker D. Trask, of the United States Geological Survey, several photomicrographs of the section were obtained and are here reproduced in Figures 1 and 2. The following brief description of the section is the result of examinations made by Miss Margaret C. Cobb and Bruce H. Harlton, of the Amerada Petroleum Corporation.

> The rock is a comparatively pure quartz sandstone. It shows no secondary growth of crystals. The bond is a very fine siliceous iron-bearing silt. Excellent porosity is developed. Feldspar is almost completely absent. Magnetite, hematite, and chloritic alteration products from mica, as well as holite itself, are fairly abundant, and there are the usual heavy residuals.

It is of interest and importance to note that the major constituents of this sandstone are insoluble in water. In order to obtain a rough idea of the amount of water-soluble material present, a 200-gram sample was pulverized and digested with 500 cubic centimeters of distilled water at room temperature for a period of 4 hours. The water was decanted and filtered, the filtrate evaporated to dryness, and the residue weighed. In this way the water-soluble material was estimated to be less than 0.05 per cent by weight.

METHOD AND RESULTS

The method used is a refinement of a method previously[3] described. The apparatus used is shown schematically in Figure 3. The sample bar is held in a horizontal position by a support at its midpoint. Small

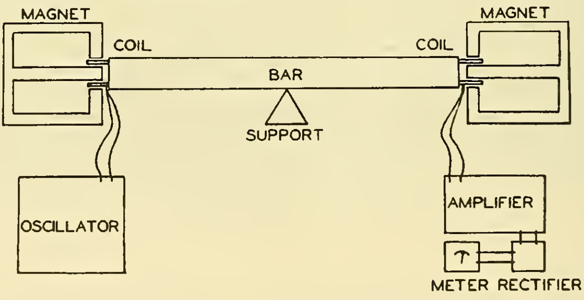

FIG. 3.—Arrangement of apparatus used to determine natural frequency of bar.

[3] B. B. Weatherby, W. T. Born, and R. L. Harding, "Granite and Limestone Determinations in Arbuckle Mountains, Oklahoma," *Bull. Amer. Assoc. Petrol. Geol.*, Vol. 18, No. 1 (January, 1934).

coils of wire are attached rigidly at either end, these coils being placed in a radial magnetic field. The coil at one end of the rod is connected to a calibrated variable-frequency oscillator, while that at the other end is connected through an amplifier to a rectifier and galvanometer. Suitable means are provided to measure and to adjust the current supplied by the oscillator to any desired value. The bar may thus be driven at any desired frequency and the amplitude of the motion at the further end of the bar is indicated by the magnitude of the galvanometer deflection. By holding the driving current constant while varying the oscillator frequency, a resonance curve may be drawn by plotting galvanometer deflection against the frequency of the driving current. In this manner the frequency corresponding with the natural frequency of the bar may be easily and accurately obtained.[4] Although the bar is capable of a number of modes of vibration this arrangement of apparatus allows the desired mode, in this case, the fundamental longitudinal vibration, to be easily distinguished.

Having obtained the natural frequency of the bar, the velocity of elastic waves through it may be calculated at once by means of the relationship[5]

$$v = 2fl$$

where v is the velocity
f is the natural frequency
and l is the length of the bar.

The velocity thus calculated is the so-called "bar velocity" which is in general lower than the "bulk velocity" obtained in field measurements. The ratio of these two velocities is a function of Poisson's ratio for the material. In what follows the term velocity is used to denote the measured bar velocity, unless otherwise stated.

To obtain quantitative data showing the effect of moisture upon the velocity of the sandstone, the percentage of water in the rock was varied, and the natural frequency corresponding to each moisture content was determined. The data obtained are shown graphically in Figure 4, in which the observed natural frequencies are plotted against total weight of the bar. The points on the curve were obtained in the order shown by the numeral opposite each. Point 1 was taken with the sample in an air-dry condition, that is, after it had been

[4] In calculating the natural frequency of the bar from the observed data, a correction is made to take into account the effect of the mass of the coils attached to either end. In the present investigation this correction was negligible.

[5] This relationship is strictly true only if the length of the bar is very large as compared with its other dimensions. In the case of short bars a correction must be applied. This correction is negligible for the bar used in this investigation.

lying in air at room temperature for several months. The observed natural frequency was 901 cycles per second and the weight 6,450 grams. The rod was then placed in an oven at 95°C. for a period of 20 hours, and after it was allowed to cool to room temperature, the weight was found to be 6,440.7 grams and the natural frequency 931 cycles per second (Point 2). The weight of the bar after this baking process was taken to be the "dry" weight, that is, the assumption was made that only a negligible amount of water remained after this baking period. The rod was then moistened and placed in a closed tube at room temperature for a period of 36 hours in order to allow the moisture to distribute itself throughout the bar. The weight and natural

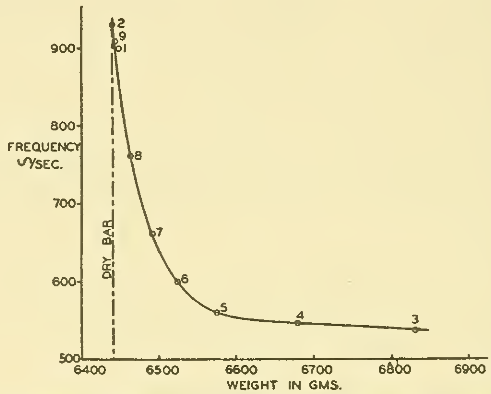

FIG. 4.—Variation in natural frequency of Amherst sandstone bar with weight as water content is changed.

frequency were then determined again, the former being 6,831.3 grams and the latter 538 cycles per second. These values are those corresponding with Point 3 on the curve. The rod was next allowed to dry in air for several hours and then placed in a closed tube for a day to reëstablish moisture equilibrium. Point 4 indicates the weight and natural frequency after this procedure. Subsequent determinations, corresponding to points 5, 6, 7 and 8, were made in similar fashion. To obtain Point 9, the rod was again baked for about 5 hours and the weight and natural frequency determined.

Several features of this curve are of interest. Perhaps the most striking is the extremely rapid reduction of velocity which accompanies the addition of the first increments of moisture and the slower reduction as a greater amount of water is added. A second interesting point is the fact that the curve indicates that the addition and removal of the water caused little, if any, permanent change in the character of the rock. In other words, the "moisture cycle" appears to be reversible. This is evidenced by the fact that points 1 and 2 fall so nearly on the curve determined by the remaining points. This fact must be taken into consideration in seeking for an explanation of the observed phenomena.

Figure 5 is a graph derived from the observed data. In this figure the calculated velocities in feet per second are plotted against the percentage of water added to the dry bar, the weight of the latter being taken as the base. This curve shows most clearly the very small

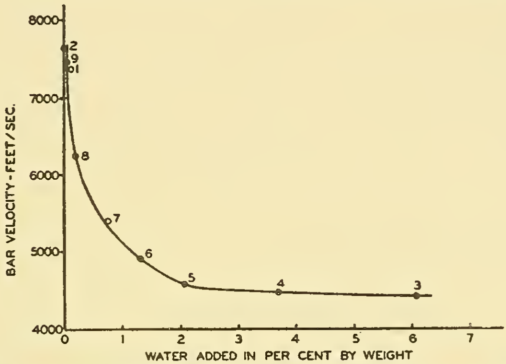

FIG. 5.—Variation of bar velocity of elastic waves in Amherst sandstone as water content is changed.

amount of moisture necessary to cause a great decrease in velocity. The addition of slightly more than 2 per cent of water to the dry bar decreases its velocity from 7,640 feet per second to 4,600 feet per second, which is a reduction of 40 per cent. The addition of 4 per cent more water, however, decreases the velocity to 4,415 feet per second, which is a further reduction of only 2 per cent.

In order to formulate an explanation for the phenomena described, it is first necessary to determine what property of the bar is affected by the moisture. The bar velocity is a function of the value of Young's modulus and the density of the bar, being given by the relationship

$$v = \sqrt{E/\rho}$$

where v is the bar velocity
E is Young's modulus
and ρ is the density of the bar.

A decrease of velocity may therefore be caused by an increase in density, by a decrease in the value of Young's modulus, or by both. The addition of water does, of course, increase the density, but this increase is far too small to account for the large changes in velocity observed.[6] It might therefore seem safe to conclude that the presence of moisture decreases the value of Young's modulus. It is, however, conceivable that the presence of a small amount of water might exert a loading effect upon the vibrations of the bar and consequently reduce the velocity to a much greater extent than would be anticipated. In order to show that water actually changes the value of Young's modulus by an amount approximately sufficient to account for the observed change in velocity, it was necessary to resort to a static method of measuring the change in Young's modulus. This was done in the following fashion. The bar was supported at either end and loaded at its center point. A fiducial point at the center of the bar was observed with the aid of a micrometer microscope in order that the deflection of the center under various loads could be determined. This deflection, to a first approximation, is given by the relation

$$D = K/E$$

where D is the deflection
E is Young's modulus
and K is a constant involving the distance between the supports and the dimensions of the bar.

The data so obtained were found to be in accord with the supposition that water changes the value of Young's modulus rather

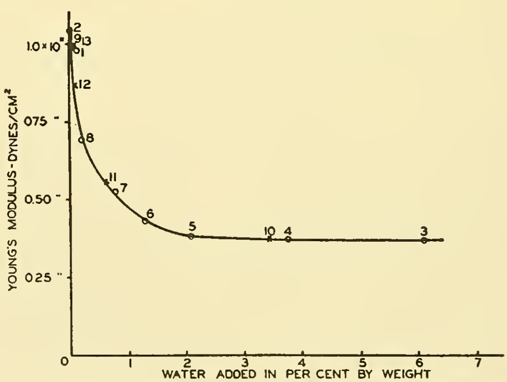

FIG. 6.—Variation in Young's modulus of Amherst sandstone as water content is changed.

[6] The increase in density of the bar produced by the maximum amount of water added was a trifle more than 6 per cent. The maximum decrease in velocity which may be attributed to the change in density is therefore about 3 per cent, whereas changes as great as 42 per cent were observed. In this connection it is of interest to note that the changes in velocity corresponding to that portion of the curve of Figure 5 lying between points 3 and 5 are sufficiently small so that they may be attributed to the change in density.

than the effective density of the bar. This agreement is illustrated by Figure 6. The circled points on the curve are calculated from the observed velocity and measured density of the bar. The points indicated by crosses are those derived from the static measurement. In plotting the latter no effort was made to evaluate accurately the constant K because the cross section of the bar was somewhat irregular. The constant was chosen to make one point fall on the dynamic curve. The others then fell on the curve within the limits of experimental error, which was approximately 2 per cent.

DISCUSSION

The conclusion to be drawn from the work here described is that the observed changes in the velocity of sound in the sample used are primarily due to the observed changes in Young's modulus. In order to formulate an explanation for the phenomena it is therefore necessary to account for the change in Young's modulus as the water content of the sample is varied. Sufficient data are not available at the present time to permit the formulation of a definite explanation of the observed phenomena, but the following discussion is of interest in this connection.

Since the mineral constituents of the rock are highly insoluble in water, it seems safe to conclude that the elastic properties of the individual rock particles themselves are not affected by moisture. The effect may then be localized in the bond holding the particles together. If we picture the rock as made up of particles held together by a bonding material which combines with water as does glue or gelatine, the rock should act somewhat as was observed. A comparatively small amount of a bonding material evenly distributed throughout the sample could, by softening or dissolving, release a large number of bonds and thus cause a large change in the velocity. This explanation fits the observed facts, since the velocity would tend to approach a minimum value determined by the amount and distribution of the bond present. Furthermore, the removal of water would allow the bond to recement the particles and thus restore the rock approximately to its original condition.

An immediate objection to this picture as applied to the sandstone under investigation is that the bond in the sandstone seems to be a siliceous silt which is only slightly soluble. Furthermore the percentage of water-soluble constituents in the bar was found to be extremely small. However, since the absolute amount of bonding material present is minute compared with the total mass of the bar it is not necessary that a large absolute amount of material be dissolved or softened by the water. The process of softening the bond may not be one of ordinary solution. Some hydrolysis of the silica may occur or the bond may form a colloid in the presence of water. It is hoped that further work may make possible a more definite explanation.

PRACTICAL IMPLICATIONS

In considering the practical application of the results obtained, the question at once arises whether a change in the bar velocity of a sample of this sandstone corresponds to a similar change in the bulk velocity of the material as found in the field. It seems almost certain that it should. From an analytical standpoint the bulk velocity must decrease as the bar velocity decreases, unless there takes place simultaneously a compensating change in the value of Poisson's ratio. Furthermore, as the presence of moisture has been shown to affect one of the elastic constants, namely, Young's modulus, there is reason to suppose that it affects the other elastic constants in a similar fashion. While the results of the work described do not permit a quantitative calculation of the effect of moisture upon the bulk velocity it may reasonably be supposed to be of the same order as that found in the case of the bar velocity.

In connection with the practical application of this and other similar work, B. B. Weatherby has called the attention of the writers to the possibility that some of the "geophysical discontinuities" of the Gulf Coast area which reflect seismic energy in spite of the absence of recognized geological discontinuities, may be due to the presence of a moist strata in an otherwise similar section. The suggestion is an interesting one which will require the accumulation of more data before its validity can be tested. There are undoubtedly many factors operating to produce these discontinuities and it seems very probable that the presence or absence of water may prove to be one of them.

BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 19, NO. 1 (JANUARY, 1935), PP. 19-24.

RECENT DEVELOPMENTS IN GRAVITY PROSPECTING ON GULF COAST[1]

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ABSTRACT

The Gulf Coast salt domes found by gravity methods since the discovery of Sugarland are reviewed up to and including the recent discoveries.

Besides the better known torsion-balance method, the growing use of the pendulum and gravity meter characterize the last 2 years of gravity prospecting.

Regional surveys destined to reveal the general gravity grain of the Gulf Coast in relation to the isolated salt masses are steadily growing and emphasize the importance of gravity surveys in geophysical reconnaissance work.

RECENT DEVELOPMENTS IN GRAVITY PROSPECTING

Although several shallow domes were discovered on the Texas Gulf Coast prior to 1927, the main impetus toward increased gravity prospecting was given by the discovery of the Sugarland dome. Since that time domes were found by the torsion balance in rapid succession: Fannett in 1927, Mykawa, Hankamer and Shepherds Mott in 1928, Manvel and Esperson in 1929, Rabbs Ridge and Citrus Grove in 1930, Spurger, Garwood, Livingston, Tomball, and Cleveland in 1933 and Eureka in 1934. In Louisiana the following domes were found: Darrow in 1927, Roanoke in 1928, Iowa and Cameron Meadows in 1929.

Of these, Fannett is the only shallow dome represented by a torsion-balance maximum. All the others are more or less deep-seated and are represented by gravity minima. Many were not discovered earlier because minima were of secondary interest prior to the discovery of Sugarland. Since then the tables have turned completely. Several gravity maxima have been drilled since 1927, but all were rather weak and represented gravel beds or pseudomaxima which are not of a structural nature, but merely a resultant of forces caused by a number of conflicting salt flights. Others were local maximum closures on the regional high trends which are discussed later. The only known oil field on the Gulf Coast producing from a gravity maximum, excepting the shallow cap-rock domes, is the Louise field

[1] Read before the Geophysics Division of the Association at the Dallas meeting, March 23, 1934. Manuscript received, September, 1934.

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in Wharton County, where a maximum extends due south of the producing high wells, where there is a sharp drop from a rather flat subsurface top to the structure. There is no evidence that the Louise is a salt dome. If so, it must be very deep, since wells have been drilled deeper than 7,000 feet without encountering salt.

Undoubtedly the four greatest discoveries by the torsion balance are Sugarland, Rabbs Ridge, Manvel, and Iowa—greatest at least from the economic standpoint.

Full credit for the discovery of Sugarland is given to the North American Exploration Company crew working for H. C. Cockburn. The area had been shot over by refractions several times without being found until the torsion balance showed indications of a minimum. Further refraction seismograph work confirmed the dome.

The Rabbs Ridge dome was found by crews of the Gulf Production Company. Early torsion-balance work gave vague indications, but detailed work in 1929 and 1930 confirmed the presence of a deep-seated salt dome. The picture is not well defined. Lack of permits for work on the southeast necessitated interpolation of isogams over large distances in order to effect the necessary closures. Averaging of gradients was necessarily used to a great extent where irregularities were caused by surface and near-subsurface depositional variations in the Brazos River bottoms. Stations were made very close together. Calculated sections across the gravity anomaly (Δg section) show definitely that Rabbs Ridge is a deep-seated salt dome rather than a part of a regional trough. In that respect, it was also noted that the axis of Rabbs Ridge was not in the direction of the known regional trends.

The Manvel dome was discovered for The Texas Company by the Torsion Balance Exploration Company. Good interpretation was shown in outlining a dome represented by very little closure and hardly any on one side. The actual producing area is south and southeast of the minimum.

Of the other domes listed, Esperson, discovered for the Union Exploration Company by Christian Iden, and Citrus Grove, a Cockburn discovery, are well marked minima.

Hankamer, Chambers County, a Gulf Production Company discovery, stands out fairly well, although obscured on the west side by the strong influence of the shallow Moss Bluff dome.

Garwood, Colorado County, was discovered by the Torsion Balance Exploration Company for the Coyle-Concord Oil Company. Livingston was discovered by the Sloane Prospecting Company and later confirmed. The Shell Petroleum Company discovered Lakeview, where two wells were drilled showing gas and some structure.

Tomball is a discovery by the Vacuum Oil Company and the

Magnolia Oil Company, confirmed by other torsion-balance work and by pendulum work of the Gulf Production Company before being drilled. The Vacuum-Magnolia is also credited with discovering Iowa, Roanoke, and Cameron Meadows in Louisiana.

In this paper, dealing only with developments, no mention is made of the undeveloped prospects or those more or less definitely condemned. It is possible that several prospects will be proved, as the result of further drilling, particularly those that show structurally high wells. In this respect it is important to bear in mind that some deep-seated geophysical dome prospects may be represented as condemned whereas in reality they lack only deep enough tests.

Certain minima undoubtedly represent inceptive or aborted salt domes which either stopped in early Eocene time or were not uplifted later sufficiently to affect horizons within easy reach of the drill and commercial production. Some of these will be drilled when economic conditions and deep-drilling technique have improved, giving some encouragement for commercial success.

RECENT DEVELOPMENTS IN TECHNIQUE AND INSTRUMENTS

In torsion-balance field work great economies have been effected in operation of the crews. Preference has been shown for the smaller instruments because of their easier portability. The introduction of tungsten torsion wires instead of the platinum-iridium wires formerly used, has done away with a great number of fail stations due to temperature effects. This is particularly true where only five points are used of a three-position automatic registration, and there is no check of the drift of the rest position of the instrument due to temperature variations. Recent investigations have shown that a proper adjustment of the torsion head of a balance is essential to obtain reliable readings. This factor was not given all the consideration due it in the past.

Recently a new type of torsion balance was brought to this country by the American Askania Corporation. The beams, instead of being horizontal, or Z-shaped, are hung at an angle on a double suspension attached at the bottom of the torsion wires. This arrangement saves space and the manufacturers claim that 20 minutes is enough to obtain a rest position for this compact little instrument. One hour and 40 minutes is sufficient to complete a station, a great saving of time. This instrument is easily portable and easily adjusted. Experiments have been made in this climate to determine its sensitivity to temperature changes where variations are very pronounced. The best test of its behavior was at a known location where several reliable instruments checked each other within the allowable of one

Eötvös unit or less. There is no abnormal deviation from a mean gradient value for the location, measured with standard Z-type Bamberg instruments.

Pendulum apparatuses were first introduced by the Gulf Production Company in 1931. The company had instruments working in East Texas and the upper Gulf Coast to check a network of torsion-balance stations extending from Rains County in northeast Texas to Orange County and farther west. These Askania pendulums were contracted to the Gulf Production Company by Ludger Mintrop. Pendulum stations were placed at intersections of converging torsion-balance traverses several miles apart where possible. As a rule fairly good checks were obtained with not enough error seriously to influence any possible interesting anomalies. Occasionally, however, one pendulum station would be several millidynes off, although converging torsion-balance traverses from other reliable pendulum stations would check within a millidyne or less, all radiating from fixed points toward that hub of divergence. When repeated, the pendulum station would generally check satisfactorily.

Over long distances (80 to 100 miles), a certain gradually increasing divergence between torsion-balance and pendulum results was observed. The torsion balance showed a larger total number of millidynes over that distance than the pendulum. This divergence increases gradually on departing from the base station, but only in the direction of increase of gravity. It is proportional to the anomaly between the first and last station of an 80-mile line. Along strike, differences were small.

After study of all possible causes of this error: (1) normal values of torsion-balance stations, (2) latitude correction, (3) corrections of elevation, and (4) Bouguer correction on the pendulum, it was decided that none of these was consistent enough to cause such differences. It was finally concluded that the cause was to be found in the then prevailing plan of two day stations and one night station per large Bamberg instrument. The night-station gradients with falling temperature were always smaller in proportion to day-station gradients made with rising temperature. Since there were two of these to one night station, the tendency would naturally be to increase the total anomaly in an area of uniform gravity increase. These results were not confirmed in areas worked with small Bamberg instruments, or where two night and two day stations were made.

In 1932, the Gulf Production Company introduced its own pendulum apparatus. These instruments now number 10 on the Gulf Coast: 6 field instruments and 4 base instruments near the field of work or as far as the radio transmitting sets can be heard by the field parties. These instruments use quartz instead of the invar pendulums previously used. They are sensitive to about 3.10^{-4} except for occasional lapses, when one station will suddenly fail for no apparent reason. The probable error is reduced by two observations with four comparisons with two different base stations. It is then only about 2 or 3.10^{-4}. All direct gravity measuring instruments have the problem of daily and periodical drift and sudden jumps or shifts. Constant comparisons with the base stations are essential daily.

These instruments, if operated to effect a network of stations a mile or two apart, are very valuable for regional gravity prospecting. Larger distances between stations are not advisable because interesting areas may be missed by the larger mesh. It should also be remembered that isolated pendulum stations do not give directional data. Interpolations in contouring are somewhat hazardous except with a close net of reliable stations. The Cleveland dome was found by the pendulum and checked by the torsion balance before the first location was made by the Gulf Production Company.

Gravity meters are based more or less on the basic patent of Kenneth Hartley. They consist of a weight suspended on a delicate spring protected from temperature differences. Variations of the length of the spring due to small differences in gravity are read on a dial controlling a regulating device. The optical system is ingenious and projects two filar images that can be made to overlap and coincide by rotating the dial. There are two or three of these instruments in the field. Their sensitivity is similar to that of the pendulum about 2 or 3.10^{-4} under the best conditions. They can be worked much more rapidly and need no timing device except an ordinary watch to note approximate time for the moon correction. On account of the delicate spring mechanism, the instrument needs great care in handling and must be kept constantly at an even temperature. The clamping device is so constructed as to avoid stresses in the spring while clamping and releasing the weight. Very accurate leveling is of prime importance. These instruments cover a large area in a very short time, but can not always be expected to find domes of very little gravimetric expression, where the total anomaly is about the same as the probable error in reading, or dependent on the additive or neutralizing effect of the probable error in enhancing or wiping out the gravity anomaly.

PROBLEM OF REGIONAL SURVEYS

Since 1927 several of the larger oil companies have been tying in isolated torsion-balance surveys in order to get an idea of the regional grain of the Gulf Coastal country. The regional maxima and minima are all nearly parallel with the coast line, that is, their axes extend northeast and southwest or east and west.

Perhaps the best marked are the parallel maximum and mimimum through Galveston, Brazoria, and Matagorda counties into Calhoun County, Texas. Sections across this system show gravity increases ranging from 10 to 15 millidynes from the minimum to the maximum. In general, there is an increase of gravity out to sea along the coast line.

Salt domes seem to be situated on the flanks of these regional features rather than on the top of the maximum or in the minimum troughs. Little may ever be known as to the meaning of these remarkable gravity features on the Gulf Coast, since these effects are obviously the expression of basement features of great depths beyond reach of the drill. They are unlike anything found in other salt-dome regions, including East Texas, North Louisiana, and Germany.

The effect of these regional gravity maxima and minima is that a number of salt domes, even of the relatively shallow intrusive type, like Danbury and Markham, are not easily distinguished except by deducting a regional gradient. The deeper domes are masked altogether except for a slight bulge in the isogam lines, a slight shortening and lengthening of the gradients respectively against, or following, the regional gravity dip.

A knowledge of the extent and intensity of these regionals over relatively large distances is essential in attempting to isolate a deep gravity anomaly dome from the regional effect. Therefore, local torsion-balance surveys are not of much use without definite knowledge of the regional trend obtained by at least one long traverse into the surrounding territory, except with previous knowledge of general regional conditions in the area under examination.

The torsion balance is ideal for exploration work, because from a distance anomalies can be followed until they are definitely localized. Then they can be checked if so desired by reflection shooting for further geophysical proof, to determine if uplift is present within reach of the drill or if the minimum represents an arrested type salt dome with no uplift in the commercially productive horizons.

As to local gravity elongate closures in the bottom of gravity minimum trends (a good many of which have been drilled, even in recent years), most competent geophysicists will condemn them if they are familiar with regional conditions. Such minimum trends, of course, should not be confused with elongate lines or groups of salt domes at various stages of evolution, such as Lost Lake, Moss Bluff, and Hankamer. Prospects in or near the bottom of regional trends gain in value, if at least on one side a good closure is obtained along the axis of the trend. Discrimination is, therefore, necessary and should be the basis on which the competent geophysicist bases his conclusions.

BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 19, NO. 1 (JANUARY, 1935), PP. 25-36, 2 FIGS.

PREDICTION OF OVERHANG AT BARBERS HILL, CHAMBERS COUNTY, TEXAS: A STUDY IN QUANTITATIVE CALCULATIONS FROM TORSION-BALANCE DATA[1]

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ABSTRACT

A study of the possibility of prediction of the overhang at Barbers Hill was based on quantitative calculations from a diametral torsion-balance profile across the dome and the drilling data in regard to the top of the cap and salt. A series of three alternative positions of the flank of the salt on the left and four on the right were assumed. All combinations of the assumed left and right alternative positions of the edge of the salt were tested. Several variations of the assumed density relations were tested. Several variations of the regional gradient were tested. The gradient profiles which would be produced by the respective assumed forms of the dome were calculated by the writer's graphical method. For the calculated gradient profiles which had the closer fit to the observed gradient profile, the mean-square difference between the calculated and observed profiles was used as the test of the relative closeness of fit. The form of the dome which had considerable overhang on the right and slight overhang on the left consistently had the least mean square. The conclusion therefore is that the overhang at Barbers Hill could be predicted in advance of drilling solely on the basis of torsion-balance data and of whatever data in regard to the cap and salt might be available from drilling.

PURPOSE OF STUDY

This study was made to see whether or not the presence of the overhang at such a dome as Barbers Hill could be determined by calculation from the data of a torsion-balance survey, in advance of determination of the overhang by the drill. The study was based on the assumptions that the form of the top of the cap and the depth to the top of the salt table were known but that there were no drilling data available to show whether or not overhang actually was present. Calculations were then made from the results of the torsion-balance data to see whether or not the presence or absence of overhang could be determined with good, fair, or poor probability of accuracy.

[1] Read before the Geophysics Division of the Association at the Dallas meeting, March 23, 1934. Manuscript received, November 12, 1934. Original data released through the favor of Sidney A. Judson and the permission of the Rio Bravo Oil Company, Sun Oil Company, and Texas Gulf Producing Company.

[2] Humble Oil and Refining Company.

DATA

The data used in the calculations were:

A. Three closely adjacent diametral lines of torsion-balance stations (Fig. 1).

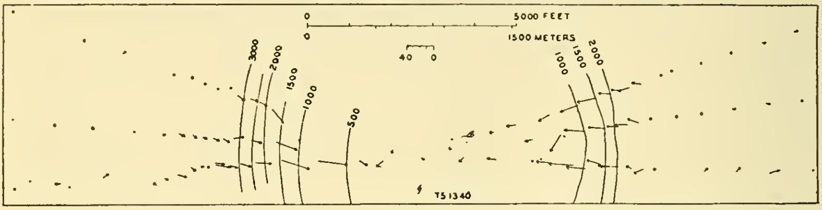

Fig. 1.—Map showing the gradient arrows of the torsion-balance surveys, and the structure contours on top of the cap, from well data, Barbers Hill salt dome.

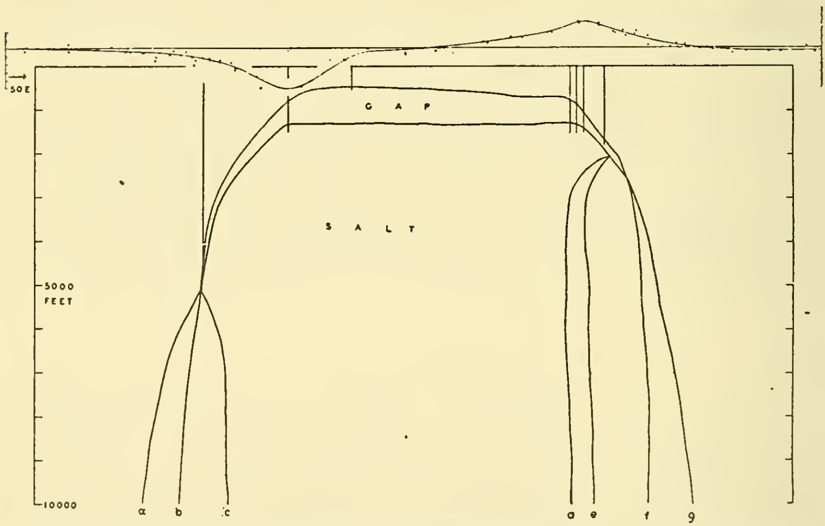

Fig. 2.—Section and gradient profile, Barbers Hill salt dome. Above: gradient profile. Below: structure section showing the known data of the cap and the assumed positions (*a* to *g*) of the flank of the salt core.

The torsion-balance lines were part of a survey by the Torsion Balance Exploration Company for the Humphreys Corporation several years ago and were obtained through the courtesy of Sidney A. Judson of the Texas Gulf Producing Company.

The stations presumably are good normal stations, as the survey was made by the Torsion Balance Exploration Company. For quantitative calculations of this type, care beyond that used normally in the preparation of stations and in the taking of the stations should be used. But it is not known whether precautions beyond that company's usual good care were used on this survey.

The stations were not placed most advantageously for the purposes of the calculations. It would have been strongly advantageous for this study, if the torsion-balance lines had been extended much farther out from the edge of the dome. Preferably also the stations of two of the lines should have been all on the same line; and the stations of the third line on the ends of that line.

B. The known drilling data in regard to the top of the cap and the top of the salt table. A host of wells have been drilled through the edge of the cap, but over the top of the dome wells are rather sparsely scattered.

C. Past experiences in regard to the probable specific gravities of the cap, salt, and sediments.[3]

METHODS OF CALCULATION

The writer's graphical method of calculating was used.[4]

A chart was constructed specially for use on this diameter of the Barbers Hill dome. These charts are composed of assumed prisms at right angles to the vertical plane of the section. The length of the chart prisms was chosen to fit the northwest-southeast conformation of the cap through the central half of the dome. Corrections were then calculated to make the chart fit the northwest-southeast conformation of the cap in the outer quarters of the northeast-southwest cross section. This chart was used for the cap and salt above a depth of 4,000 feet. A similar chart with slightly different specifications was used to calculate the effects from 4,000 feet down to 12,000 feet. The effect of the salt below 12,000 feet was neglected.

The observed gradient profile had first to be smoothed. Seriously aberrant values were discarded. The smoothing of the others was studied by two methods:

A. By replacing the dot for each observed gradient value by a dot at the center of gravity of the triangle composed of the dot and the adjacent dot on each side, and

B. By the formula:

$$C' = \frac{a + 2b + 3c + 2d + e}{9}.$$

[3] D. C. Barton, "Belle Isle Torsion-Balance Survey, St. Mary Parish, Louisiana," *Bull. Amer. Assoc. Petrol. Geol.*, Vol. 15, No. 11 (November, 1931), pp. 1341–42.

[4] ———, "Calculations in the Interpretation of Observations with the Eötvös Torsion Balance," *Geophysical Prospecting, 1929* (Amer. Inst. Min. Met. Eng., 1930), pp. 416–80.

The effect of the cap rock was then calculated. These calculations would have been simple, if the actual form and density of the cap rock had been known. But as a matter of fact, the form of the cap-rock mass along the southwest-northeast diameter is known only approximately, and it is necessary to go through the whole tedious series of trial and error calculations to determine the most probable gradient profile of the cap-rock effect. The observed gradient profile is predominantly the effect of the cap rock. The form of the cap rock was altered within the limits which were imposed by the drilling data; and the gradient profile for each form was calculated. The calculated gradient profile which most nearly fitted the observed gradient profile was assumed to be the actual effect of the cap-rock mass.

Detailed calculations of the salt effect were then made. It was assumed that the edge of the salt core might have any of the forms, *a*, *b* *c*, *d*, *e*, *f*, *g*, of Figure 2. The form might take that of any combination of (*a*, *b*, *c*) with (*d*, *e*, *f*, *g*).

Seventy-eight calculation points, 200 feet apart, were taken, covering the whole length of the torsion-balance profile. The gradient effect of the cap and the salt was calculated for every fifth station. The values for the intermediate stations were interpolated. The difference between the calculated value at each station and the corresponding value of the smoothed observed curve was then recorded. The respective differences were then squared, the squares were added, and the mean square found.

The mean square is taken to be the measure of probability of the particular set of assumptions used in the respective calculations. The less the mean square, the more probable is that set of assumptions. The set of assumptions which has the least mean square is assumed to be the most probable.

Three unknowns have to be evaluated by the trial and error calculation of the least mean square: deviation of the calculated from the smoothed, observed gradient profile:

A. The form of the salt and cap;

B. The relative density of the salt and cap in reference to the sediments; and

C. The regional gradient.

Any one set of assumptions for a given calculation comprises:

A. One of the forms $(a-e)$, $(a-f)$, et cetera.

B. A net of assumptions regarding the density of the cap, the density of the salt, the respective density of the sediments at various depths.

C. Some assumption in regard to the regional gradient.

A long series of combinations of these geologically probable assumptions are possible and have to be calculated directly or indirectly.

For many such sets of assumptions, it was unnecessary to carry the calculations through to the point of finding the mean square. If the unsquared differences are plainly large, the mean square necessarily is large. The calculation of any set of assumptions was dropped as soon as it was very evident that the unsquared differences were large.

The work of the construction of the charts and of the calculation was done by the writer's then assistant, Maude Hickey.

Approximately 1 month was spent in calculation in connection with the effect of the cap rock and approximately 3 months on the effect of the salt. The methods evolved considerably during the calculations. Much of the time was spent in refinement of the technique and in recalculating calculations which had been done earlier by less refined technique. It is probable that a similar calculation of a diametral profile could be made by an experienced calculator in 2 months, although if difficulties came up, a slightly longer time might be required. The time required for satisfactory results would vary inversely with the amount of geologic information that would limit the range of variations in our fundamental assumptions.

RESULTS

The calculations indicate that the overhang at Barbers Hill can be detected by calculations based on torsion-balance data, for the results of the calculations indicate that the series of assumed forms rank in the following order of decreasing probability.

		Relative Rating
cd	Maximum overhang	1.00
ce	Overhang both sides but on right not so much as in preceding	0.80
cf	Overhang left; no overhang right	0.44
be	Overhang right; no overhang left	0.43
bf	No overhang	0.22

Preliminary calculations showed clearly that all forms including *a* or *g* would produce calculated gradient profiles which fit the smoothed gradient profile very poorly. Those forms were dropped from the further calculations.

A summary of the results of the calculations for the forms (*b*,*c*) and (*d*, *e*, *f*) are shown in Table I. Many trials which were started are not reported on that sheet, for trials of any set of assumptions were stopped as soon as the unsquared differences were seen to be

TABLE I

MEAN SQUARES OF DEVIATION

			ad	ae	af	ag	bd	be	bf	cd	ce	cf	bg cg
Density Assumption A													
Varying Regional Gradient		Mean						3.50	3.87	3.00	2.80	3.57	
I-*A*	4 N.E. Stations Omitted	Median						2.03	2.73	1.50	1.44	2.03	
		Sum						5.33	6.60	4.50	4.24	5.60	
II-*B*	All Stations		LARGE	LARGE	LARGE	LARGE	*bd* = very close to *be*—not calculated	3.46	3.69	2.85	2.80	3.40	LARGE
								2.18	2.40	1.38	1.50	1.94	
								5.64	6.09	4.23	4.30	5.34	
III-*C*	15 Middle and 4 N.E. Stations Omitted							2.39	2.71	1.65	1.63	2.24	
								1.92	1.90	1.21	1.17	1.59	
								4.31	4.61	2.86	2.80	3.83	
IV-*D*	15 Middle and 4 N.E. Weight 1 All Others Weight 2							2.99	3.26	2.42	2.29	2.89	
								2.13	2.25	1.30	1.44	1.69	
								5.12	5.51	3.72	3.73	4.58	
No Regional Gradient													
V-*A*					ditto			4.16	5.89	3.53	4.03	6.49	LARGE
								3.15	4.85	2.17	3.24	5.53	
								7.31	10.74	5.70	7.27	12.02	
VI-*B*					ditto			4.30	5.72	3.94	4.23	6.31	
								3.43	4.42	2.57	3.50	5.29	
								7.83	10.14	6.51	7.73	11.60	
VII-*C*					ditto			3.37	4.78	2.61	2.99	4.86	
								2.13	3.69	1.69	2.70	4.81	
								5.50	8.47	4.30	5.69	9.67	
VIII-*D*					ditto			3.90	5.31	3.36	3.69	5.68	
								2.69	4.01	2.02	3.24	5.29	
								6.59	9.31	5.38	6.93	10.97	
Density Assumption B													
IX-*A*					ditto				8.10	ALL MEAN SQUARES			
									6.90				
									15.00				
X-*B*					ditto				7.80				
									6.26				
									14.06				
XI-*C*					ditto				7.60				
									6.16				
									13.76				
XII-*D*									7.71				
									6.15				
									13.86				
Density Assumption C									>20				ditto
Density Assumption D—No Regional													
XIII-*A*									4.14		4.49		
									3.24		1.63		
									7.38		6.12		
XIV-*B*									5.66		5.09		
									3.43		1.76		
									9.09		6.85		
XV-*C*									5.41		3.52		
									2.50		1.17		
									7.91		4.69		
XVI-*D*									5.00		4.41		
									3.24		1.44		
									8.24		5.85		
Regional Gradient >1 <1.5*E*													
XVII-*A*									4.2		3.8		
									1.6		1.1		
									5.8		4.9		
XVIII-*B*									4.2		4.0		
									1.8		1.6		
									6.0		5.6		

TABLE I (*Continued*)
MEAN SQUARES OF DEVIATION

First	*Second*	*Third*	*Fourth*	*Fifth*	*Sixth*	*Seventh and Poorer*	
ce	*cd*	(*bd, be, cf*)			*bf*	(All a's, All g's)	Cap 2.59–2.61 Salt 2.20–2.21
ce	*cd*	ditto			*bf*		
ce	*cd*	ditto			*bf*		Sediments 2.12 to 2.37 at 12,000 feet
ce	*cd*	(*cf, be, bd*)			*bf*		
cd	*ce*	ditto			*bf*	ditto	
cd	*ce*	ditto			*bf*		
ce	*cd*	*cf*	*be*		*bf*		Regional Gradient >0, <1.5 *E*
ce	*cd*	*cf*	and		*bf*	ditto	
ce	*cd*	*cf*	*bd*		*bf*		
ce	*cd*	*cf*	*be*		*bf*		
ce	*cd*	*cf*	and		*bf*	ditto	
ce	*cd*	*cf*	*bd*		*bf*		
cd	*ce*	(*be, bd*)		*bf*	*cf*		
cd	*be*	(*ce, bd*)		*bf*	*cf*	ditto	
cd	*ce*	(*be, bd*)		*bf*	*cf*		
cd	*ce*	(*be, bd*)		*bf*	*cf*		No Regional Gradient
cd	*be*	(*ce, bd*)		*bf*	*cf*	ditto	
cd	*ce*	(*be, bd*)		*bf*	*cf*		
cd	*ce*	(*be, bd*)		*bf*	*cf*		
cd	*be*	(*ce, bd*)		*bf*	*cf*	ditto	
cd	*be*	(*ce, bd*)		*bf*	*cf*		
cd	*ce*	(*be, bd*)		*bf*	*cf*		
cd	*be*	(*ce, bd*)		*bf*	*cf*	ditto	
cd	*be*	(*ce, bd*)		*bf*	*cf*		
LARGE						ditto	Cap 2.60–2.63 Salt 2.21 Sediments 2.13 to 2.38 at 12,000 feet
							Cap 2.70–2.74 also 2.66–2.70 Salt 2.21 also 2.21 Sediments 2.15–2.39 also 2.15–2.39
							Cap 2.57 Salt 221 Sediments 2.12 to 2.36 at 12,000 feet
							XIII–XVIII are based on cruder calculations.

running consistently large. The table gives merely a synopsis of the more important mean squares.

The mean squares are given in 3 forms: the mean, the median, and the sum of the mean and median. The median which is used actually is the mean of the median 4 or 5 squared differences. The mean square differences are given in 4 sets: *A*, *B*, *C*, *D*. In set *B*, all 82 calculation stations were used. In set *A*, 4 slightly dubious stations on the right end of the observed profile were neglected. The central part of the observed profile was plainly irregular and reflecting the effects of shallow features not connected with the cap or salt. In *C*, therefore, the 15 central and the 4 northeast calculation stations at the right end were neglected. In *D*, the 15 central and the 4 calculation stations at the right end were weighted one; and the other 62 were weighted two.

The results I–IV are really the condensed final results; and V–VIII are the condensed next-to-the-last results.

A single line under I–VIII does not give a simultaneous series of mean squares respectively for $(b-e)$ et cetera. But if, for example, the line I-*A*-mean is referred to, then 2.80 was the least mean square which could be obtained for $(c-e)$ by varying the regional gradient within certain plausible limits. But for the regional gradient under which *ce* had a mean square of 2.80, the mean square of *cd* is greater than 3.00, of *cf* is greater than 3.57 et cetera. By varying the regional gradient, the mean square of *cd* could be brought down to 3.00, but no regional gradient was found by which it was possible to bring it down to 2.80, and so on for the other forms, *be*, *bf*, and *cf*.

The use of a small regional gradient toward the right gave distinctly better results than the use of no regional gradient.

Density assumption *A* gave the closest fit of calculated to observed gradient profile. The very slight change of the density assumptions to those of *B* or *C* greatly increased the size of the mean square for all combinations of $(b, c)-(d, e, f)$. Density assumption *D* gave some small mean squares for the median but a large discrepancy between the median and the mean. The whole series of results under density assumption *D* showed the same discrepancy and also a combination of very small and very large differences between the observed and calculated profiles.

The two forms, *cd* and *ce*, were consistently the best two in all the calculations. Either *cd* or *ce* was first in all calculations made, and in the final series I–IV, these two forms held first and second places. In some of the earlier calculations, one of them, more commonly *ce*, was displaced from second to third place by *be*. The difference in those cases, however, was small between *ce* and *be*, 3.15 against 3.24; 3.43 against 3.50; 2.13 against 2.70; 2.69 against 3.24.

The relative standing of the various forms in reference to the order of occurrence of their respective mean squares is given in Tables II and III. In Table II, the "mean" mean squares and the median mean squares of the series, I–VIII, are given in order of their

TABLE II

Weight	Means	Weight	Medians
25	1.63 *ce*	25	1.17 *ce*
	1.65 *cd*		1.21 *cd*
	2.24 *cf*		2.30 *cd*
	2.29 *ce*		1.38 *cd*
	2.39 *be*		1.44 *ce*
20	2.42 *cd*	20	1.44 *ce*
	2.61 *cd*		1.50 *ce*
	2.71 *bf*		1.50 *cd*
	2.80 *ce*		1.59 *cf*
	2.80 *ce*		1.69 *cd*
15	2.85 *cd*	15	1.69 *cf*
	2.89 *cf*		1.90 *bf*
	2.99 *ce*		1.92 *be*
	2.99 *be*		1.94 *cf*
	3.00 *cd*		2.02 *cd*
10	3.26 *bf*	10	2.03 *be*
	3.36 *cd*		2.03 *cf*
	3.37 *be*		2.13 *be*
	3.40 *cf*		2.13 *be*
	3.46 *be*		2.17 *cd*
5	3.50 *be*	5	2.18 *be*
	3.53 *cd*		2.25 *bf*
	3.57 *cf*		2.40 *bf*
	3.69 *ce*		2.57 *cd*
	3.69 *bf*		2.69 *be*
0	3.87	0	2.70 *ce*
	3.90		2.73 *bf*
	3.94		3.24 *ce*
			3.24 *ce*
			3.50 *ce*

magnitude for the mean squares, less than 4.00 for the means, and 3.50 for the medians. The relative position of each form in regard to the magnitude of its mean square is given in Table II for the first 25 in each list. First place is ranked 25 and last place, 1.

Although strictly the least mean square should indicate the most probable set of assumptions, there is considerable probable error in our calculations and we can not be certain that the least mean square of our calculations actually is significantly less than some slightly larger mean square. Tables II and III give a study of the distribution

TABLE III

Means

be	*bf*	*cd*	*ce*	*cf*
21	18	24	25	23
12	10	20	22	14
8	1	19	17	7
6		15	16	3
5		11	13	
		9	2	
52	29	102	95	47

Medians

be	*bf*	*cd*	*ce*	*cf*
13	14	24	25	17
10	4	23	21	15
8	3	22	20	12
7		18	19	9
5		16		
1		11		
		6		
		2		
44	21	122	85	53

of the mean squares. And from them, conclusions can be drawn in regard to the relative probability of the five forms, *be*, *bf*, *cd*, *ce*, *cf*.

In absolute value of the mean square, *ce* takes first place over *cd* both in the means and the medians, but by so small a difference, 1.63 against 1.65, and 1.17 against 1.21, that it is presumably of no significance. The least mean square of any of the other three forms is 2.24 in the means and 1.59 in the medians. The difference seems significant.

In probability from frequency of placement and position of placement among the 25 highest mean squares of I–VIII, *cd* seems slightly more probable than *ce;* and *cd* and *ce* together are decisively more probable than *be*, *bf*, and *cf*. The form *cd* was placed 7 times against *ce*'s 6 times in the first 25 means, and 8 times against *ce*'s 4 times in the medians. The forms *be* and *cf* each held two places in the first 10 of the means, but otherwise *be*, *bf*, and *cf* were in the last 15; and in the medians of these three, only *cf* won a single place in the first 10. The sum of the positions of placement of each form gives a measure of its probable relative position in reference to the others. The form *cd*, therefore, would seem to be most probably first with *ce* a fairly close second. The forms *bc* and *cf* are almost tied for a poor third and fourth, with *cf* slightly in the lead and *bf* a poor fifth.

The value of *bd* runs close to that of *be*. It does not give any important information beyond that given by *be*. It was dropped from the calculations, therefore, in order to save time.

The relative order (*cd* and *ce*), (*be* and *cf*), *bf* is to be expected if there is overhang on both sides; for then

	Assumptions in regard to	
	Southwest Flank	*Northeast Flank*
cd and *ce*=	Approximately right	Approximately right
be=	Wrong	Approximately right
cf=	Approximately right	Wrong
bf=	Wrong	Wrong

CONCLUSIONS

The calculations from the torsion-balance survey at Barbers Hill indicate the presence of considerable overhang on the right and a smaller overhang on the left. The degree of probability of the indication is "good" for commerical purposes and fair for purely scientific purposes. If some important fundamental physical constant were being determined, and if the determination were to have to stand for a long time, the technique of the calculations should be refined yet further; and further tests should be made to see if some geologically possible set of assumptions would not give a lesser mean square than *cd* or *ce*. But in terms of the accuracy of commercial geological and geophysical work on which oil companies base leasing and drilling operations, the accuracy of the indication from these calculations is good.

The results of the calculations also give a fair indication that there is much more overhang on the northeast than on the southwest. From the small differences between the mean squares of *ce* and *cd* and the much larger difference between those mean squares and the mean squares of *cf*, it would seem probable that the actual position of the edge of the salt would lie between *d* and *e*. From the closeness of the mean squares of *be* and *cd* (and *ce*), it seems probable that the actual position of the edge of the salt lies considerably nearer *c* than *b* and that it might lie even slightly inside of *c*, although it looks improbable that the edge of the salt could lie far inside of *c*.

How much further the accuracy of this type of calculation and prediction can be carried by more refined technique is an open question. It should be possible to obtain a little higher accuracy than that of the calculations of this study. The calculations another time could be done in a way to reduce the probable error of calculating. Added corrections could be made, as, for example, for the aureole of upturned beds. If the field stations were laid out better for the purpose and the observations taken with extraordinary care, the basic original data would be more accurate. But in attempting to obtain better quantitative accuracy, the method would be forced well toward the limit of its accuracy.

A limiting factor comes in to affect quantitative calculations. The salt effect is small. The effect of deficiencies of salt (for example, in an overhang) is yet smaller. Within certain limits, it is impossible for us to determine in our calculations whether the deficiency of salt (which forms the overhang) is spread vertically up and down the edge of the salt, or whether it is mainly high or mainly low. The writer therefore would hesitate about attempting to push the quantitative predictions much further. Yet in some cases, for commercial purposes, it might be worth while to know whether there was a 1-to-4 chance of greater width of the overhang than was commonly believed.

The degree of success of the calculations at Barbers Hill indicates that this type of calculation might be successful in the determination of overhang on other domes. Barbers Hill is a fairly good dome for such calculations as the terrane is good for torsion-balance observations; the dome is far enough inland from the coast so that the density of the sediments is not too low, and a fair amount is known about the form of the cap rock. The calculations will become tedious and more inaccurate for domes on which there are only a few wells drilled into the cap and salt. The accuracy, also, will probably not be as good for domes nearer the coast as for those farther inland, for there is a thicker section of light sediments near the coast.

An advantage of this torsion-balance method over the suggested seismic method of determination of overhang is that the torsion-balance method is independent of any flank wells. The calculations could be made if there were only one well on the dome, if that well went through the cap into the salt. It would require much less time and be more satisfactory, however, to have more data in regard to the top of the cap. But flank wells are in no way necessary to the torsion-balance calculations.

BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 19, NO. 1 (JANUARY, 1935), PP. 37-57, 7 FIGS.

EFFECT OF ANISOTROPY ON APPARENT RESISTIVITY CURVES[1]

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ABSTRACT

The anisotropic character of the sedimentary formations is a fact which has been so far generally overlooked in the interpretation of resistivity curves. The purpose of the present article is to indicate the normal distortion of the resistivity-depth curves for non-homogeneous horizontal stratified conditions. Two cases are investigated, namely where the anisotropies of the individual layers are the same and where they assume different values. The amount of error made in the accepted methods of interpretation is also indicated. The writer proposes a method of interpreting three-layer resistivity curves when due allowance is made for the anisotropic conditions of the ground.

A practical case illustrates the method.

INTRODUCTION

The anisotropic character of the sedimentary formations is a fact which has been, so far, generally overlooked in the interpretation of resistivity curves. The distortion of the electrode spacing-apparent resistivity curves by non-homogeneous conditions of the individual layers accounts for the numerous empirical methods of interpretation. Gish and Rooney (1)[3] proposed, as a first approximation, to interpret the depth to horizontal discontinuities as being equal to the electrode separation at which a point of maximum curvature is obtained in the resistivity curve. Schlumberger (2) estimates the depth as being three-fourths of the same distance. Lancaster-Jones (3) proposed a closer approximation for the depth by taking two-thirds of the electrode spacing for which an inflection point occurs in the resistivity curve. However, by virtue of the nature of such a point which lies on a practically straight part of the curve, the depth determination may be erroneous. The closest approximation in depth determination was obtained by Tagg (4) who really advocated a mathematical solution of the resistivity curve by making use of nomographic charts accurately determined from the formula of the apparent resistivity

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[3] Numbers in parentheses indicate references at end of this article.

in the case of two plane boundaries. Tagg, however, stressed the point that before his method could be applied, the ground under investigation must satisfy the hypothesis at the base of the establishment of the charts; mainly, the individual layers must be homogeneous and isotropic. A misleading extension of Tagg's method to the interpretation of the three-layer resistivity curves has been proposed by Manhart (5) and Tattam (6), since they actually overlook the ground condition in their method. The author of the present article proposes here a "successive approximation method" for the extension of Tagg's two-layer method to the case of three horizontal layers. However, the ground under investigation must be homogeneous and isotropic or at least the individual layers must have the same coefficient of anisotropy.

Consequently, the method is best applicable to the determination of depth to bed rock, igneous or vertically stratified, under a cover of placer or gravel beds. The method can also be advantageously applied to the determination of thickness of limestone beds, lava flows, et cetera. An example of application of the writer's method is indicated here, together with the indication of the correction to be applied in order to compensate for the anisotropy of the ground.

ANISOTROPY OF SEDIMENTARY LAYERS

The effect of anisotropy on the apparent resistivity has been mentioned only a few times in the geophysics literature by Maillet and Doll (7), Slichter (8), and Müller (9).

It is generally accepted that in stratified media the conductivity is larger parallel with the bedding plane than perpendicular to the stratification, the mobility of the ions being larger parallel with the schistosity than at right angles to it. However, if the apparent resistivity ρa of such a stratified ground be measured with the aid of four equidistant contacting electrodes, the outside ones being the current electrodes, and the inside ones the potential electrodes (Wenner-Gish-Rooney set-up), the formula,

$$\rho_a = 2\pi a \frac{V}{I},$$

where

a = electrode spacing
V = potential at the inside electrodes
I = current at the outside electrodes,

gives larger apparent resistivities ρa parallel with the bedding surfaces than perpendicular to them. The explanation of this paradox is beyond the scope of the present paper, but a few practical results are given.

Measurements were made on sedimentary beds *in situ* using one-foot electrode separation.

An outcrop of hard broken clay (10 feet thick), slightly stratified, of the Laramie formation (Tertiary) in Golden, Colorado, has given the following results.

Resistivity parallel with the bedding surface: 825 ohm feet
Resistivity perpendicular to the bedding surface: 388 ohm feet

A hard laminated shale interbedded with sandstone of the Benton formation (Tertiary) in Golden gave the following results.

Resistivity parallel with the bedding surface:
Electrodes in sandstone seam: 3,000 ohm feet
Electrodes in shaly seam: 1,760 ohm feet
Resistivity perpendicular to the bedding plane:
204 ohm feet

Another series of measurements was made at the same location on a laminated fine white sandstone (Benton formation) and the following results were obtained.

Resistivity parallel with the bedding plane: 1,770 ohm feet
Resistivity perpendicular to the bedding plane: 970 ohm feet

From the preceding results, one should not generalize that the large axis of the anisotropy ellipse will always be in the stratification plane; actually, jointing in shales may produce an anomalous anisotropy. It is thus possible for the conductivity to be larger perpendicular to, than parallel with, the stratification. The anisotropy coefficient α of sedimentary formations which is defined as the ratio of the apparent parallel resistivity to the apparent normal resistivity may then assume values ranging from close to zero to values as high as ten. The isotropic conditions of a layer are represented by $\alpha = 1$.

ANISOTROPY REDUCTION THEOREM

The resistivity method of prospecting being a surface potential method, it is necessary to investigate the distribution of potential in an anisotropic medium in order to calculate the apparent resistivity. Let us consider the case of two superimposed layers designated by the indexes 1 and 2 and of anisotropy coefficients $\alpha 1$ and $\alpha 2$. The resistivities perpendicular to the stratification are designated by ρ_{1v} and ρ_{2v} and the resistivities parallel with the bedding planes by ρ_{1h} and ρ_{2h} which latter resistivities may generally be assumed to be constant in any horizontal direction due to the continuity of character of sedimentary formations parallel with the bedding surfaces. Thus we have:

$$\frac{\rho_{1h}}{\rho_{1v}} = \alpha_1 \quad \text{and} \quad \frac{\rho_{2h}}{\rho_{2v}} = \alpha_2$$

Suppose that the current is introduced at the surface at two points C_1 and C_2 (Fig. 1). The densities of current across surfaces

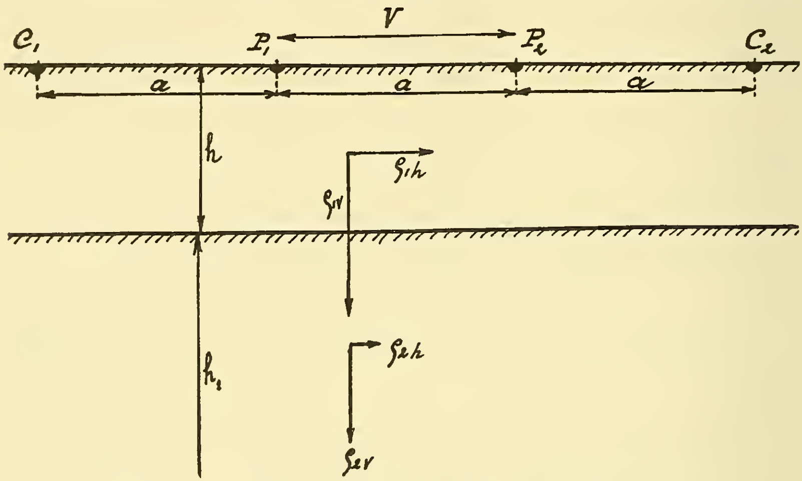

FIG. 1.—Gish-Rooney electrode set-up and resistivity vectors.

perpendicular to the axes OX, OY, and OZ are respectively:

$$i_x = -\frac{1}{\rho_{1h}} \times \frac{\partial V_1}{\partial x}$$

$$i_y = -\frac{1}{\rho_{1h}} \times \frac{\partial V_1}{\partial y}$$

$$i_z = -\frac{1}{\rho_{1v}} \times \frac{\partial V_1}{\partial z},$$

when the axis OZ is chosen perpendicular to the bedding plane XOY in which the axes ox and oy may have any particular position. V_1 is the electrical potential at any point in medium 1 and i_x, i_y and i_z the densities of current perpendicular to the directions x, y, and z at that point.

The total current at the point considered is then given by

$$I = ii_x + ji_y + ki_z$$

where i, j and k are unit vectors taken along the axes ox, oy, and oz. Since we must have conservation of the vector current, we find that the divergence of I is zero:

$$\nabla \cdot I = -\left(\frac{1}{\rho_{1h}}\frac{\partial^2 V_1}{\partial x^2} + \frac{1}{\rho_{1h}}\frac{\partial^2 V_1}{\partial y^2} + \frac{1}{\rho_{1v}}\frac{\partial^2 V_1}{\partial z^2}\right) = 0$$

Thus Laplace's equation in the anisotropic medium considered becomes:

$$\frac{1}{\rho_{1h}}\left(\frac{\partial^2 V_1}{\partial x^2} + \frac{\partial^2 V_1}{\partial y^2}\right) + \frac{1}{\rho_{1v}}\frac{\partial^2 V_1}{\partial z^2} = 0$$

If we substitute in this equation

$$x = \xi\sqrt{\frac{1}{\rho_{1h}}}$$
$$y = \eta\sqrt{\frac{1}{\rho_{1h}}}$$
$$z = \zeta\sqrt{\frac{1}{\rho_{1v}}}$$

we obtain the equation:

$$\frac{\partial^2 V_1}{\partial \xi^2} + \frac{\partial^2 V_1}{\partial \eta^2} + \frac{\partial^2 V_1}{\partial \zeta^2} = 0,$$

which is the ordinary Laplace's equation. We have thus reduced the space xyz into a space ξ, η, ζ in which Laplace's equation is satisfied but where the dimensions are expanded by the factor $\sqrt{\rho_{1h}}$ in the horizontal directions and $\sqrt{\rho_{1v}}$ in the vertical direction.

If we leave the vertical dimensions unchanged, the horizontal dimensions of the space x, y, z, are then expanded by a factor

$$\sqrt{\frac{\rho_{1h}}{\rho_{1v}}} = \sqrt{\alpha_1}$$

in order to obtain a medium ξ, η, ζ in which Laplace's equation is satisfied. Thus we have the transformation:

I.
$$\xi = x\sqrt{\alpha_1}$$
$$\eta = y\sqrt{\alpha_1}$$
$$\zeta = z.$$

The medium ξ, η, ζ obtained by the preceding transformation of coördinates will be rendered homogeneous by making the following substitutions for the horizontal and vertical resistivities.

$$\rho'_{1h} = \rho_{1h} \frac{1}{\sqrt{\alpha_1}}$$

$$\rho'_{1v} = \rho_{1v} \sqrt{\alpha_1}$$

where ρ'_{1h} and ρ'_{1v} are the resistivities parallel with, and perpendicular to, the bedding surface in the medium ξ, η, ζ.

Similarly the medium 2 will be reduced to homogeneous conditions by the following transformation:

II.

$$\xi' = x\sqrt{\alpha_2}$$

$$\eta' = y\sqrt{\alpha_2}$$

$$\zeta' = z$$

$$\rho''_{2h} = \rho_{2h} \frac{1}{\sqrt{\alpha_2}}$$

$$\rho''_{2v} = \rho_{2v} \sqrt{\alpha_2}$$

In order that the points at infinity in medium 2 shall correspond with the points at infinity in medium 1, it is necessary to expand the medium ξ', η', ζ' by the factor $\sqrt{\frac{\alpha_1}{\alpha_2}}$, horizontal and vertical dimensions as well. The transformation II becomes:

III.

$$\xi = x\sqrt{\alpha_1}$$

$$\eta = y\sqrt{\alpha_1}$$

$$\zeta = z\sqrt{\frac{\alpha_1}{\alpha_2}}$$

$$\rho'_{2h} = \rho_{2h} \frac{\sqrt{\alpha_1}}{\alpha_2}$$

$$\rho'_{2v} = \rho_{2v} \sqrt{\alpha_1}$$

We have thus reduced media 1 and 2 to a single system of coordinates ξ, η, ζ in which we have homogeneous and isotropic con-

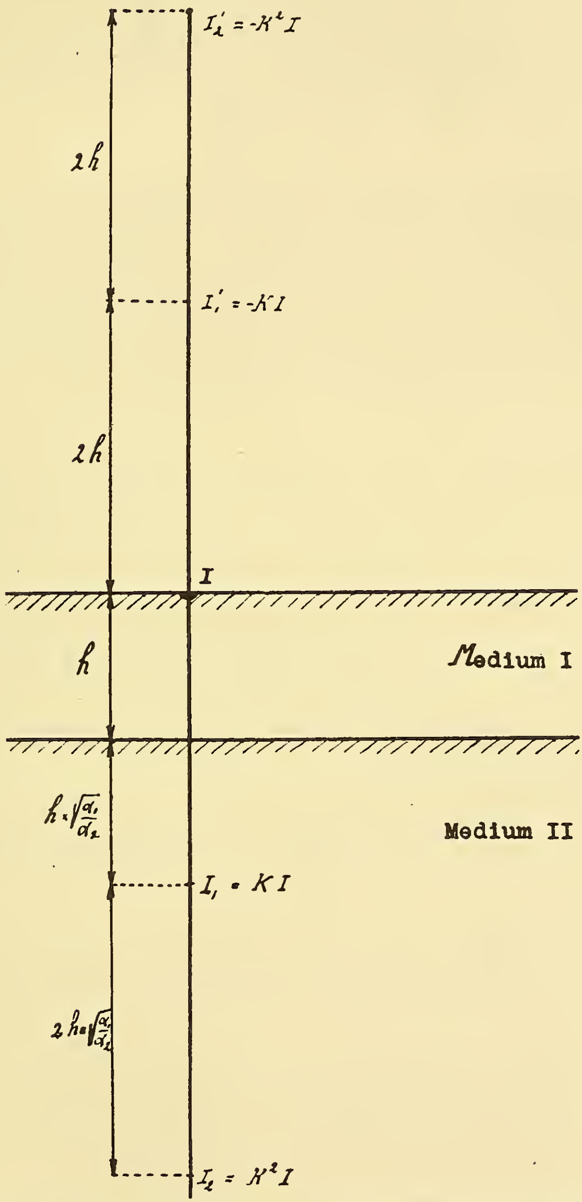

FIG. 2.—Electrical images for two media of different anisotropies.

ditions. Consequently the formulae of the apparent resistivity which have been established by Hummel (10) for homogeneous and isotropic layers can be easily extended to the anisotropic media considered here. We notice first that the reflection coefficient or resistivity factor

$$K = \frac{\rho'_{2v} - \rho'_{1v}}{\rho'_{2v} - \rho'_{1v}}$$

which governs the intensities of the electrical images has not changed. We will then calculate the potential at a point of coördinates ξ, η, o on the surface of the ground (Fig. 2) when a current I is introduced into the ground at the origin of the system of coördinates. In order to facilitate the calculations, the dimensions in medium 2 are expressed as functions of the coördinate system ξ', η', ζ'.

Thus the electrical potential due to I at the point (ξ, η, o) is given by:

$$V = \frac{I\rho'_{1v}}{2\pi}\left[\frac{1}{\sqrt{\xi^2+\eta^2}} + \sum_{n=1}^{n=\infty} K^n \frac{1}{\sqrt{\xi^2+\eta^2+(2nh)^2}} + \sum_{n=1}^{n=\infty} \frac{K^n}{\sqrt{\xi^2+\eta^2+\left\{h\left[1+(2n-1)\sqrt{\frac{\alpha_1}{\alpha_2}}\right]\right\}^2}}\right]$$

Expressing now ξ and η as functions of the original coördinates x and y, we obtain:

$$V = \frac{I\rho'_{1v}}{2\pi}\left[\frac{1}{\sqrt{(x^2+y^2)\alpha_1}} + \sum_{n=1}^{n=\infty} \frac{K^n}{\sqrt{(x^2+y^2)\alpha_1+(2nh)^2}} + \sum_{n=1}^{n=\infty} \frac{K^n}{\sqrt{(x^2+y^2)\alpha_1+\left[1+(2n-1)\sqrt{\frac{\alpha_1}{\alpha_2}}\right]^2 h^2}}\right]$$

If we consider now the Wenner-Gish-Rooney set-up of electrodes as given in Figure 1 and choose the x axis as the electrode line, the difference of potential at the inside electrodes due to the current I flowing through the outside electrodes is given by:

$$V_{P_1} - V_{P_2} = \frac{I'\rho_{1v}}{2\pi}\left[\frac{1}{a\sqrt{\alpha_1}} + 2\sum_{n=1}^{n=\infty} K^n \left\{\frac{1}{\sqrt{a^2\alpha_1+(2nh)^2}}\right.\right.$$

$$- \frac{1}{\sqrt{4a^2\alpha_1 + (2nh)^2}} \Bigg\}$$

$$+ 2 \sum_{n=1}^{n=\infty} K^n \left\{ \frac{1}{\sqrt{a^2\alpha_1 + \left[1 + (2n-1)\sqrt{\frac{\alpha_1}{\alpha_2}}\right]^2 h^2}} \right.$$

$$\left. - \frac{1}{\sqrt{4a^2\alpha_1 + \left[1 + (2n-1)\sqrt{\frac{\alpha_1}{\alpha_2}}\right]^2 h^2}} \right\} \Bigg]$$

The formula for the apparent resistivity in this case is given by:

$$\rho_a = 2\pi a\sqrt{\alpha_1} \frac{V_{P_1} - V_{P_2}}{I}$$

in the ξ, η, ζ system as a function of the distance a measured in the xyz system.

Thus the relative apparent resistivity $\frac{\rho_a}{\rho'_{1v}}$ for the case of two non-homogeneous layers is given by:

IV.

$$\frac{\rho_a}{\rho'_{1v}} = 1 + 2 \sum_{n=1}^{n=\infty} K^n \left\{ \frac{1}{\sqrt{1 + \left(2n \frac{h}{a\sqrt{\alpha_1}}\right)^2}} - \frac{1}{\sqrt{4 + \left(2n \frac{h}{a\sqrt{\alpha_1}}\right)^2}} \right\}$$

$$+ 2 \sum_{n=1}^{n=\infty} K^n \left\{ \frac{1}{\sqrt{1 + \left[1 + (2n-1)\sqrt{\frac{\alpha_1}{\alpha_2}}\right]^2 \left(\frac{h}{a\sqrt{\alpha_1}}\right)^2}} - \frac{1}{\sqrt{4 + \left[1 + (2n-1)\sqrt{\frac{\alpha_1}{\alpha_2}}\right]^2 \left(\frac{h}{a\sqrt{\alpha_1}}\right)^2}} \right\}$$

It is well to stress the point that ρ'_{1v} represents the resistivity of the homogeneous top layer corresponding with the anisotropic case under investigation and that in order to obtain comparable resistivity curves the ratio $\frac{\rho_a}{\rho'_{1v}}$ or the relative apparent resistivity will be studied as a function of the relative depth $\frac{h}{a}$.

A few applications of the theorem of the reduction of anisotropic conditions to homogeneous conditions are considered here.

APPLICATIONS

I. TWO STRATA OF THE SAME ANISOTROPY

If we suppose that the anisotropy coefficients α_1 and α_2 are equal, formula IV reduces to the simpler form:

$$\text{V.} \quad \frac{\rho_a}{\rho'_{1v}} = 1 + 4\sum_{n=1}^{n=\infty} K^n \left\{ \frac{1}{\sqrt{1 + \left(2n \frac{h}{a\sqrt{\alpha_1}}\right)^2}} - \frac{1}{\sqrt{4 + \left(2n \frac{h}{a\sqrt{\alpha_1}}\right)^2}} \right\}$$

From this formula, we calculated the relative apparent resistivity $\frac{\rho_a}{\rho'_{1v}}$ for the following cases.

1) $\alpha_1 = 1$ Isotropic conditions
2) $\alpha_1 = 2$ Condition which is often obtained for sedimentary beds
3) $\alpha_1 = 4$ Condition obtained for intense compaction and dynamometamorphism

We further assumed the following resistivities and thicknesses.

$$\rho_{1v} = 2 \qquad h = 4$$

$$\rho_{2v} = 0 \qquad h_1 = \infty$$

By carrying out the calculations indicated by formula V, the resistivity curves *a*, *b*, and *c* in Figure 3 are obtained using as coördinates the relative resistivity ρ_a/ρ'_{1v} and the relative electrode spacing h/a. The curves obtained for $\alpha_1 = 0.5$ and $\alpha_1 = 0.25$ are also indicated in Figure 3, although such conditions are seldom encountered in field practice.

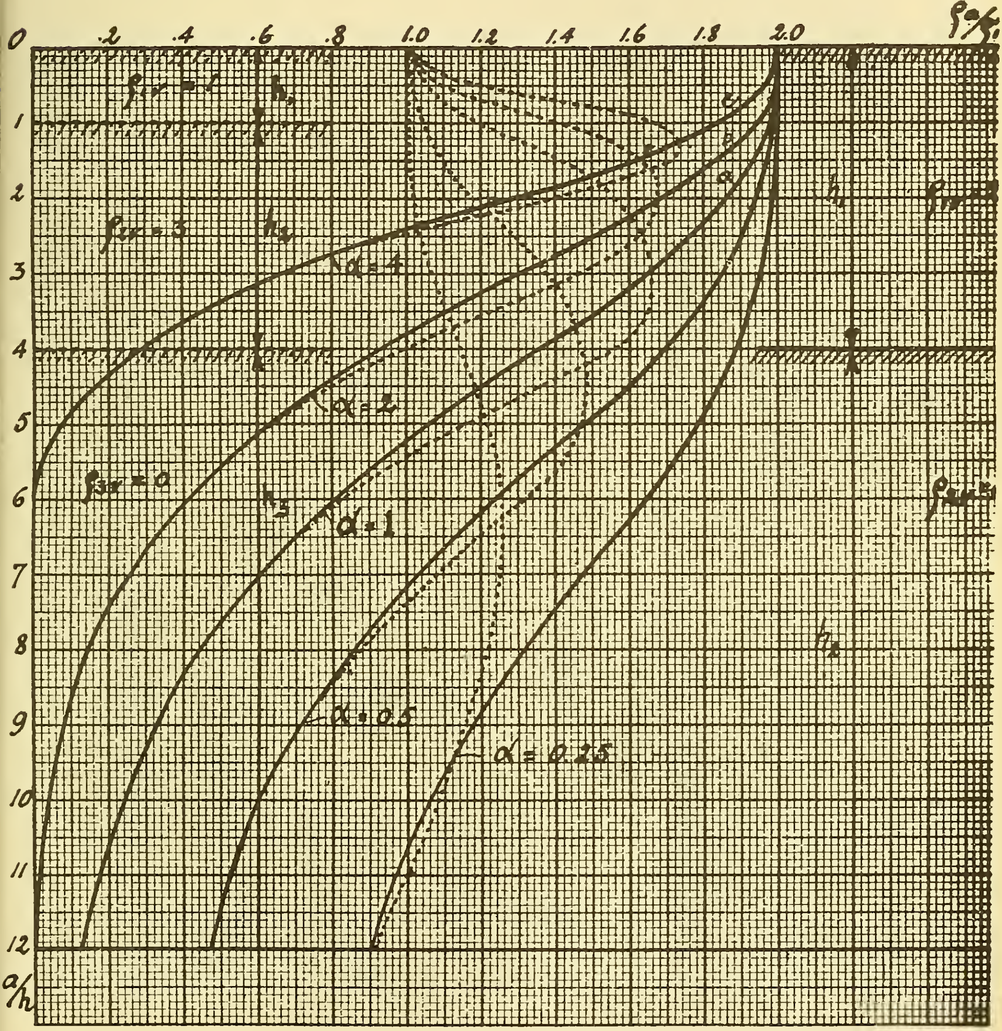

FIG. 3.—Theoretical resistivity curves. Right: two-layer case. Left: three-layer case.

2. THREE STRATA OF SAME ANISOTROPY

The theory previously given for a two-layer case can be easily extended to the three-layer case when the individual layers present the same anisotropy. It can be easily proved that the expression of the apparent resistivity in such a case is given by:

$$
\begin{aligned}
\frac{\rho_a}{\rho'_{1v}} = 1 + 4\sum_{n=0}^{n=\infty} K^{n+1} & \left\{ \frac{1}{\sqrt{1 + \left[2(n+1)\dfrac{h}{a\sqrt{\alpha}} \right]^2}} - \frac{1}{\sqrt{4 + \left[2(n+1)\dfrac{h}{a\sqrt{\alpha}} \right]^2}} \right\} \\
& + 4(1 - K^2)K_1 \sum_{n=0}^{n=\infty} (n+1)K^n \\
& \left\{ \frac{1}{\sqrt{1 + \left[2\dfrac{h_1}{a\sqrt{\alpha}} + \left\{ 2(n+1)\dfrac{h}{a\sqrt{\alpha}} \right\} \right]^2}} - \frac{1}{\sqrt{4 + \left[2\dfrac{h_1}{a\sqrt{\alpha}} + \left\{ 2(n+1)\dfrac{h}{a\sqrt{\alpha}} \right\} \right]^2}} \right\} \\
& + 4(1 - K^2)K_1^2 \sum_{n=0}^{n=\infty} \left[\frac{n(n+1)(1-K^2)}{2} - (n+1)K^2 \right] K^{n-1} \\
& \left\{ \frac{1}{\sqrt{1 + \left[4\dfrac{h_1}{a\sqrt{\alpha}} + 2(n+1)\dfrac{h}{a\sqrt{\alpha}} \right]^2}} - \frac{1}{\sqrt{4 + \left[4\dfrac{h_1}{a\sqrt{\alpha}} + 2(n+1)\dfrac{h}{a\sqrt{\alpha}} \right]^2}} \right\} \\
& + 4(1 - K^2)K_1^3 \sum_{n=0}^{n=\infty} \left[\sum_{m=0}^{n} \frac{(m-1)m}{2}(1-K^2)^2 \right. \\
& \left. - n(n+1)(1-K^2)K^2 + (n+1)K^4 \right] K^{n-2}
\end{aligned}
$$

$$\left\{\frac{1}{\sqrt{1+\left[6\,\frac{h_1}{a\sqrt{\alpha}}+2(n+1)\,\frac{h}{a\sqrt{\alpha}}\right]^2}}\right.$$

$$\left.-\frac{1}{\sqrt{4+\left[6\,\frac{h_1}{a\sqrt{\alpha}}+2(n+1)\,\frac{h}{a\sqrt{\alpha}}\right]^2}}\right\}$$

$$+\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

where $K=\dfrac{\rho_{2v}-\rho_{1v}}{\rho_{2v}+\rho_{1v}}$ and $K_1=\dfrac{\rho_{3v}-\rho_{2v}}{\rho_{3v}+\rho_{2v}}$

h and h_1 are respectively the thicknesses of the top and intermediate layer, the third layer having an infinite thickness.

The resistivity curves have been calculated for the following conditions:

$\rho_{1v}=1$ $\qquad h=1$

$\rho_{2v}=3$ $\qquad h_1=3$

$\rho_{3v}=0$ $\qquad h_2=\infty$

and for the following anisotropy coefficients:

$$\alpha=4,\ 2,\ 1,\ 0.5 \text{ and } 0.25$$

For large electrode spacings, the average resistivity of the two top layers can be calculated by Kirchhoff's law for two resistances connected in parallel. This average resistivity is precisely 4 here, and for large electrode spacings the three-layer case is reduced to the two-layer case previously investigated. This facilitates considerably the numerical computations since for increasing relative electrode separation the three-layer curves and the two-layer curves are tangent and finally superimposed. The results obtained are plotted in Figure 3.

By considering the curves in Figure 3, the following conclusions for a three-layer curve can be arrived at.

1. Anisotropies of sedimentary beds larger than one, shift the points of maximum curvature toward smaller electrode spacings and the greater the anisotropy the larger the amount of displacement.

2. Anisotropic ground conditions ($\alpha<1$) reduce the anomaly in the resistivity curve and the smaller the anisotropy coefficient, the larger the reduction. For anisotropy coefficient α larger than 1, the anomaly is increased and the curves are flattened. This is a fairly general condition encountered in field practice.

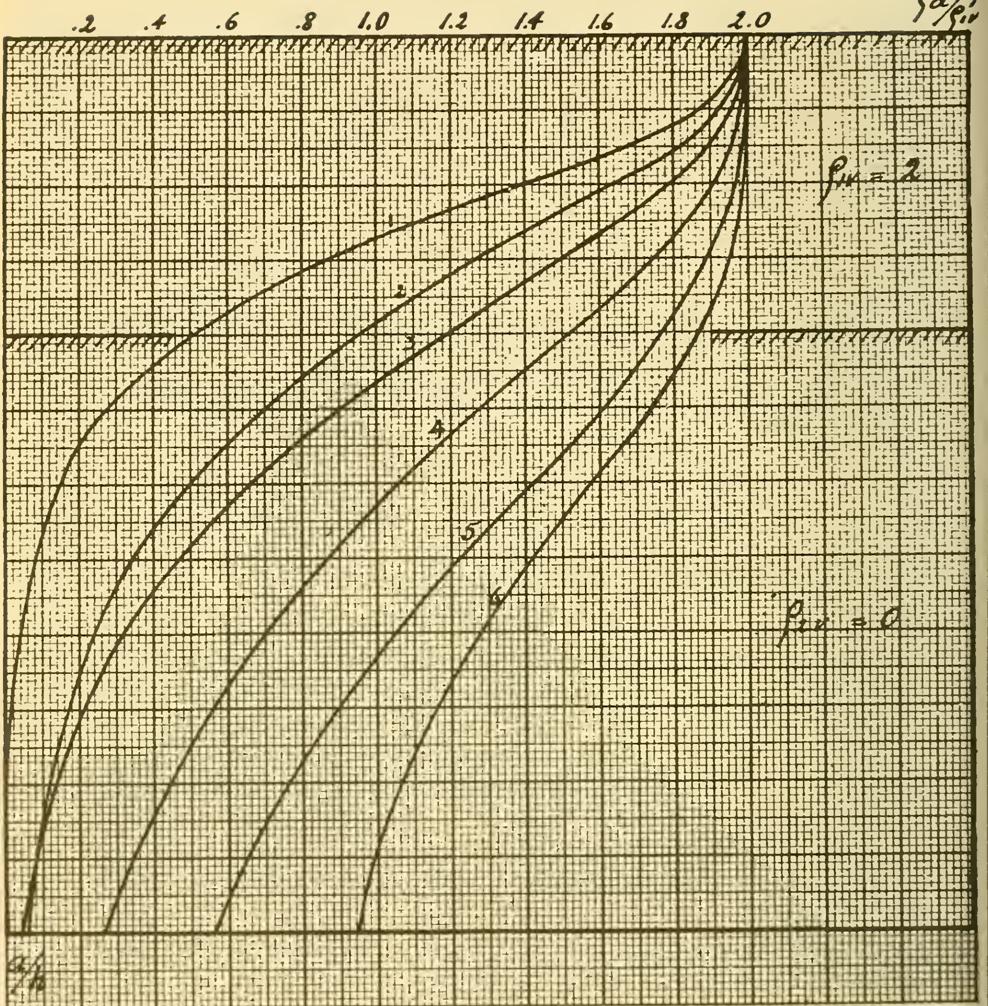

FIG. 4.—Theoretical resistivity curves for different individual anisotropies.

1. $\alpha_1 = 4, \alpha_2 = 2$
2. $\alpha_1 = 2, \alpha_2 = 4$
3. $\alpha_1 = 2, \alpha_2 = .5$
4. $\alpha_1 = 1 \quad , \alpha_2 = .25$
5. $\alpha_1 = 5 \quad , \alpha_2 = .25$
6. $\alpha_1 = \ .25, \alpha_2 = .5$

3. The application of Tagg's method of interpretation to depth determination gives a solution which is multiplied by a factor equal to the square root of the anisotropy coefficient.

4. The Gish-Rooney rule of depth determination is approximately verified when the anisotropy coefficient α equals 1/2.

3. TWO STRATA OF DIFFERENT ANISOTROPIES

This case has been investigated previously and formula IV gives the value of the relative apparent resistivity. In order to obtain an idea of the amount of deviation in the resistivity curve produced by the presence of anisotropy we have computed the apparent resistivity curves for the following two-layer case.

$$h = 4 \qquad \rho_{1v} = 4$$
$$h_1 = \infty \qquad \rho_{2v} = 0$$

and for different combinations of the anisotropy coefficients α_1 and α_2 as shown in Figure 4.

Comparative studies of the two-layer curves in Figures 3 and 4 indicate that the anisotropy of the surface layer governs the general outline of the apparent resistivity curve; however, it appears rather impractical to compute a correcting factor for the depth determination. Considering the theory and the results obtained in the case of different anisotropies for a horizontally stratified ground, it appears that a determination of the depth to horizontal discontinuities can not be obtained by application of Tagg's method of interpretation.

Furthermore, the extension of the method to the solution of three-layer problems as proposed by Manhart and Tattam may lead to considerable error since in many cases it is not less than imposing a solution on the problem.

In order to minimize the chances of errors in interpreting the three-layer resistivity curves, the writer proposes the method outlined in the following paragraphs for the interpretation of homogeneous or equally anisotropic layers.

INTERPRETATION OF THREE-LAYER CURVES

For the interpretation of resistivity curves when Tagg's charts are applied to the solution of a three-layer case, the writer of the present article proposes the following "Successive Approximation method" for which the steps are as follows.

1. Average the resistivities of the surface for electrode spacings from 5 to 20 feet, which will give a value of the resistivity of the top layer ρ_1. Very careful measurements must be made in the field and

precautions should be taken in order that the depth of penetration of the electrode stakes shall not exceed 1/5 of the electrode spacings. It is recommended that measurements be made for different values of the current, and of the commutating frequency. Since the value of the method depends greatly on the accuracy with which the resistivity of the top layer is known, too much precaution can not be taken in this determination.

2. ρ_1 is plotted to scale on the resistivity axis and a line drawn from ρ_1 to meet tangentially the first part of the curve.

3. Resistivities are read for several electrode spacings on the curve just drawn and Tagg's method is applied to that first part of the curve. The process yields the resistivity of the second layer ρ_2, the thickness of the first layer h_1, and the resistivity factor $K_1 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$.

4. The depth to the third layer is estimated by Lancaster-Jones' method. Thus $h_1 + h_2 = \frac{2}{3}d$ where d is the distance to the inflection point comprised between the two lower maximum curvature points.

5. The apparent or average resistivity ρ'_1 of the two layers of resistivities ρ_1 and ρ_2 in parallel is calculated by the formula:

$$\frac{h_1 + h_2}{\rho'_1} = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_2}$$

6. Tagg's method is then applied to the bottom part of the curve. A more accurate depth h'_2 than the one previously estimated is then obtained by tracing the curve's depth-resistivity factor. When they converge in a small area, the depth $h_1 + h'_2$ and the resistivity factor $K_2 = \frac{\rho_3 - \rho_2}{\rho_3 + \rho_2}$ are known with a fair degree of accuracy. From this expression of K_2, the value of the resistivity ρ_3 of the lower medium supposedly of infinite vertical extent can be calculated.

7. If the degree of accuracy with which $h_1 + h'_2$ is known is not sufficient, the process may be recalculated once more by using $h_1 + h'_2$ instead of $\frac{2}{3}d$ in the calculation of the average resistivities of the two top layers. A value ρ''_1 will be obtained and a more accurate value $h_1 + h''_2$ will be found for the depth to the third layer. In other words, the mathematical process known as the "Successive Approximation method" is proposed.

A practical example worked from an actual resistivity curve is given here.

The problem is one of water-table determination which was worked by Tattam (curve 26) in the Newman district (New Mexico).

It is a typical three-layer curve (Fig. 5) with some irregularities of the order of the errors of observation expected with the equipment in use. They are then smoothed out before calculations. The determinations were made in an area underlain by valley fill and at about 2 miles from two wells which encountered water at 271 feet. Consequently the cone of exhaustion did not reach the region where the measurements were taken.

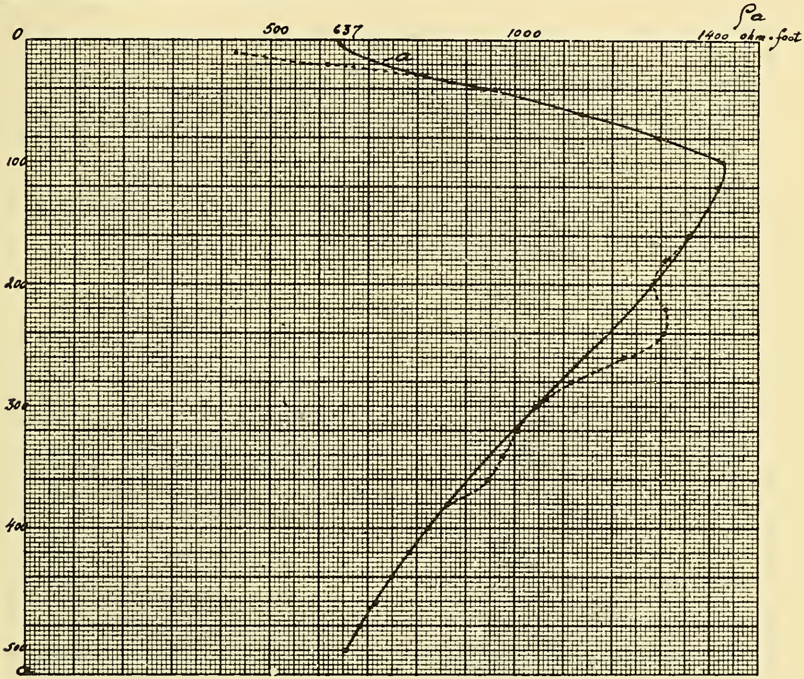

FIG. 5.—Apparent resistivity-electrode spacing curve.
– – – – Actual data.
——— Graphic approximation curve.

According to the method developed in the present article the three top values of the resistivities are averaged, which gives $1/3(428+611+932)=\rho'_1=637$ ohm feet for the resistivity of the top layer. The graphic approximation curve *a* is then traced, which meets the original curve tangentially. This curve is used to determine the thickness of the upper layer by the application of Tagg's method. The tabulation then obtained is shown in Table I.

The curve's depth-resistivity factors plotted from Table I on Figure 6 converge in a rather small area and consequently the depth

TABLE I

a	10	20		40		60		80		100	
$\rho'_1/\rho a$	.982	.920		.718		.582		.505		.463	
$\downarrow K_1$		h_1/a	h_1	h_1/a	h_1	h_1/a	h_1	h_1/a	h_1	h_1/a	h_1
0.1	No Intersection	.576	15.2	—	—	—	—	—	—	—	—
0.2		.920	18.4	.228	9.5	—	—	—	—	—	—
0.3		1.17	23.4	.45	18.0	.156	9.3	—	—	—	—
0.4		1.33	26.6	.588	23.5	.331	19.9	.190	15.2	.088	8.8
0.5		1.51	30.2	.709	28.3	.436	26.1	.323	25.8	.253	25.3
0.6		1.625	32.5	.799	31.9	.540	32.4	.422	33.7	.357	35.7
0.7		1.765	35.5	.889	35.6	.621	36.6	.497	39.8	.433	43.3
0.8		1.835	36.7	.969	38.7	.700	42.0	.579	46.3	.510	51.0
0.9		1.095	38.1	1.039	41.5	.770	46.0	.648	51.8	.578	57.8
1.0		2.00	40.0	1.118	44.8	.834	50.0	.707	56.4	.640	64.0

determination to the second layer may be considered as very good. We have: $h_1 = 33$ feet and $K_1 = 0.6$. Thus the value of the resistivity of the second layer is $\rho_2 = \rho'_1 \left(\frac{1+K_1}{1-K_1}\right) = 2{,}630$ ohm feet. By mak-making use of the inflection point on the lower part of the curve situated approximately at the electrode spacing 300 feet, the total thickness of the two top layers is estimated at $h_1 + h_2 = \frac{2}{3}\ 300 = 200$

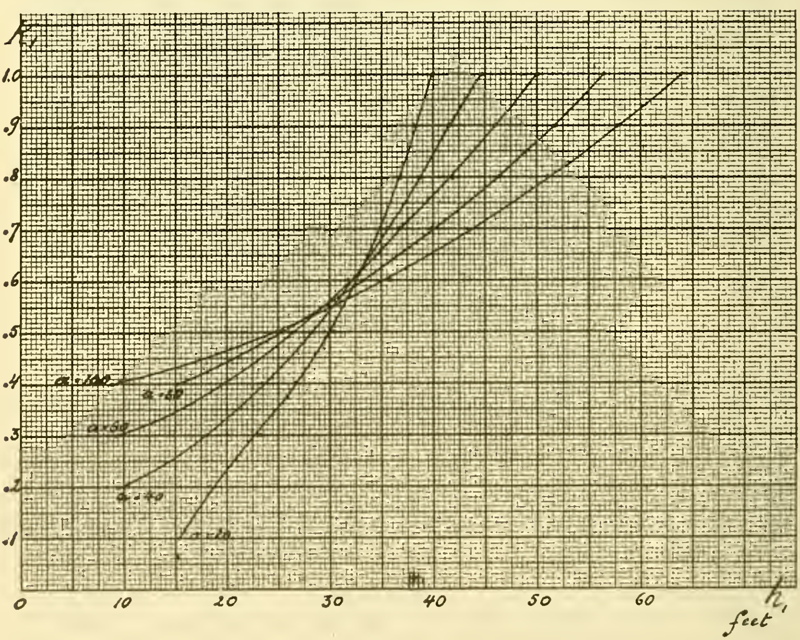

FIG. 6.—Depth-resistivity factor curves.

TABLE II

a	240		260		280		300		320		340		360		380	
$K_2 \downarrow$ $\rho a/\rho'_1 \rightarrow$	.712		.663		.624		.595		.566		.541		.517		.492	
0.1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.2	.251	60.5	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.3	.472	113	.395	106	.323	90	.265	80	.184	59	.084	28.6	—	—	—	—
0.4	.616	147	.532	138	.475	133	.440	132	.399	128	.362	123	.325	117	.284	108
0.5	.719	172	.637	166	.580	160	.544	163	.507	162	.474	161	.446	161	.414	157
0.6	.811	195	.730	190	.673	189	.635	191	.596	191	.566	192	.539	190	.508	193
0.7	.882	212	.800	208	.742	208	.704	211	.664	212	.630	214	.600	216	.569	216
0.8	.939	226	.856	223	.799	224	.763	229	.724	232	.695	236	.670	241	.640	243
0.9	1.0	240	.910	238	.850	238	.812	244	.774	248	.746	254	.720	259	.691	262
1.0	1.05	252	.960	250	.903	252	.869	261	.832	266	.800	272	.776	279	.746	283

feet. Thus $h_2 = 200 - 33 = 167$ feet. The average resistivity of the two upper layers in parallel is then given by $\frac{200}{\rho'_1} = \frac{33}{657} + \frac{167}{2630}$ and $\rho'_1 = 1{,}755$ ohm feet. By the application of Tagg's charts to the lower part of the curve, Table II is obtained.

The depth-resistivity factor curves computed from Table II give a good area of convergence (Fig. 7) for which the coördinates are $h_1 + h'_2 = 187$ feet and $K_2 = 0.58$. The value of the resistivity of the lower medium is obtained by

$$\rho_3 = 1755 \frac{.42}{1.58} = 467 \text{ ohm feet}$$

If a better approximation for the depth to the third layer is desired, the process may be reworked using 187 feet instead of 200 for the

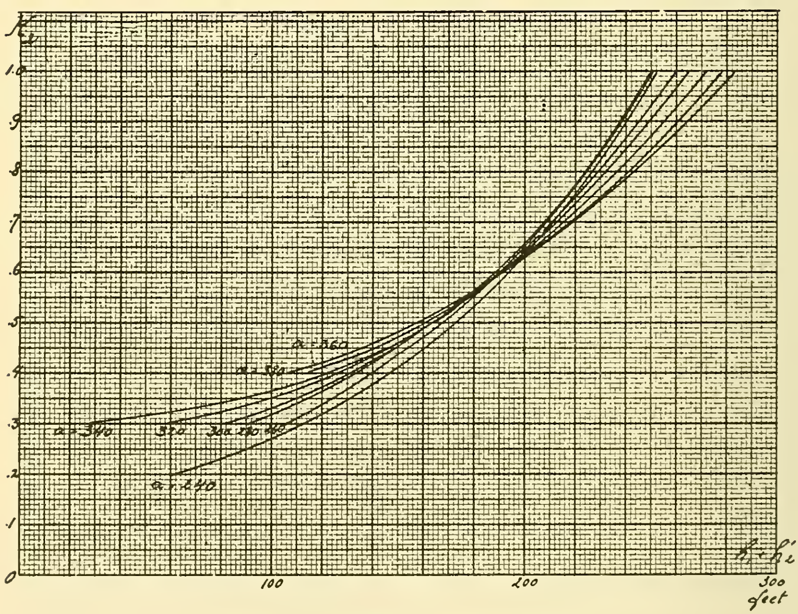

FIG. 7.—Depth-resistivity factor curves.

calculation of the average resistivity of the two upper layers. In general, however, one determination should give a close enough approximation.

If one takes pains to consider the curves given in Figure 3 and compares them with the outline of the curve in Figure 5, one notices that the resistivity anisotropy in the present case can be estimated at approximately 2; consequently the depth to the water table can be estimated to be approximately $187 \times \sqrt{2} = 264$ feet, whereas Tattam and Manhart's method places the water table at 190 feet.

CONCLUSIONS

The present method of interpretation offers a means of solving completely a three-layer resistivity curve, the resistivities and thicknesses of the individual layers being obtained. The method permits the obtaining of results with an acceptable probable error of 5–10 per cent when the field and ground conditions fulfill the requirements of the theory of apparent resistivity for a horizontally stratified medium. The isotropy or equal anisotropy of the individual layers is a necessary condition for the application of this method. If no converging point is obtained in steps 3 and 6 of the present method, it is an indication that the anisotropies of the three layers are different, and the method can not be applied.

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BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 19, NO. 1 (JANUARY, 1935), PP. 58-63, 3 FIGS.

STUDY OF STRUCTURE OF SUWA BASIN NEAR KYOTO, JAPAN, BY TORSION BALANCE[1]

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ABSTRACT

The origin of the Suwa Basin near Kyoto, Japan, was sought from the geophysical standpoint by use of the torsion balance. An isogam map suggests Lake Suwa as a crater lake, but this conclusion is not reasonable from a geological standpoint. Therefore, the origin of the lake seems to be in subsidence following the eruption of Yatsugadake Mountain, near by.

The gravity field of the basin is negative while that of the mountains which surround the basin is positive. Hence, from the standpoint of isostasy, the deficiency of the subterranean mass is clear, and shows that the basin is a result of faulting or subsidence.

PURPOSE OF RESEARCH

From the geological standpoint, the origin of the Suwa Basin has been studied and the results of the study have been published and discussed by geologists. To examine the opinion of the geologists, this research was carried out from the geophysical standpoint so as to determine the reasons for the occurrence of the Suwa Basin and the geological history of Lake Suwa.

FIELD WORK

The field work was performed in April, 1927, and again in May and June, 1928. The Eötvös original type torsion balance was used on these two occasions. The number of observation points was 13 in the field work of 1927, and 32 in the field work of 1928, making a total of 45 stations.

COMPUTATION OF RESULTS

The topographic (including terrane, topographic, and cartographic corrections) and normal corrections were applied to the observed values. There are a number of formulas for the computation of the topographic correction, but none of them is absolute. The cor-

[1] Manuscript received, November 13, 1934.

[2] Kyoto Imperial University.

rection formula derived by C. A. Heiland[3] seems to be the most reasonable at present, and was used for the correction.

The results of density determination on a large number of samples of rocks and soils are shown in Table I.

TABLE I
DENSITY OF ROCKS AND SOILS

Names of Rocks		*Specific Gravity*	*Samples are Taken from*
Contact metamorphosed (rock)		3.05	
Andesite		2.75	
Granite		2.72	
Diabase		2.70	
Conglomerate		2.66	
Diabasic tuff		2.65	
Gabbro?		2.64	
Tuff		2.44	
Andesite		2.43	Kosaka Pass
	Mean	2.71	
Soil		1.71	Road
Soil		1.52	House lot or garden
Soil		1.48	Field
	Mean	1.53	

The densities of rocks surrounding the Suwa Basin differ, but from the nature of rock distribution, it was confirmed that by taking the mean density of 2.71, an error of more than 5 per cent would not arise. Hence, the density of soil was taken as 1.53 and that of rock was taken as 2.71 in the computation.

The topographic corrections were made in three stages as shown in Table II.

TABLE II
AREA OF TOPOGRAPHIC CORRECTION

Stage	*Radius in Meters with Observation Station as Center; Corrected Area*	*Maps Used for Drawing Profiles*
1	40	Surveyed by plane table
2	5,000	Topographic map Scale: 1/50,000
3	20,000	Topographic map Scale: 1/200,000

The corrected value from the three stages (Table II) was maximum in the second stage and was minimum in the first stage, due to the attention given in selecting the stations. The normal correction is the correction for the longitude of the station and has nothing to do with the latitude. The difference in longitudes between the extreme northern and southern observation stations was only 5.6 minutes.

[3] C. A. Heiland, "A New Graphical Method for Torsion Balance-Topographic Corrections and Interpretations," *Bull. Amer. Assoc. Petrol. Geol.*, Vol. 13, No. 1 (January, 1929), pp. 39–74.

Hence the results obtained by taking the average value of 36° 20′ and calculating the influence of it was as follows:

$$B - A = \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} = +\,6.8 \qquad H = \frac{\partial^2 U}{\partial x \partial y} = 0$$

$$G = \frac{\partial^2 u}{\partial x \partial z} = +\,7.8 \qquad F = \frac{\partial^2 u}{\partial y \partial z} = 0$$

The results of correcting the observed values by the foregoing are shown in Figure 1.

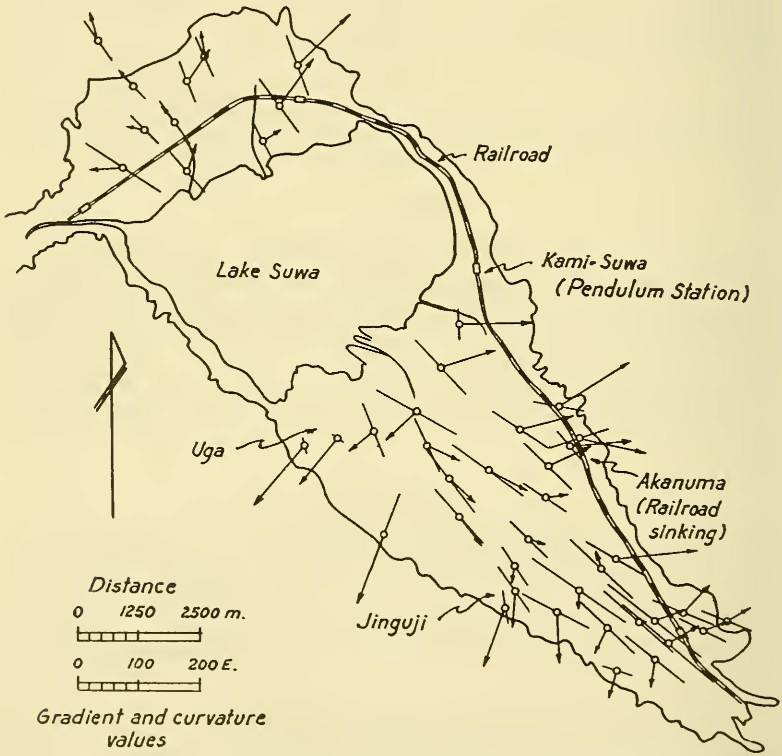

FIG. 1.—Diagram showing gravity gradients and curvature values at Suwa Basin. (To conform to American practice in plotting, R-lines should be rotated through 90°.)

The determination of gravity by means of a pendulum at Kami-Suwa, conducted by the Bureau of Geodesy of Japan, gave the value of 979.629 dynes. Therefore, by taking this value as zero, the isogam lines calculated gave the results shown in Figure 2.

VARIATION OF GRAVITY FIELD

In looking at Figure 1, it is seen that the gradients point radially away from Lake Suwa, but a close examination reveals that the gradients increase in value northeast and southwest from the major axis of the Suwa Basin; near the margin of the basin they have their

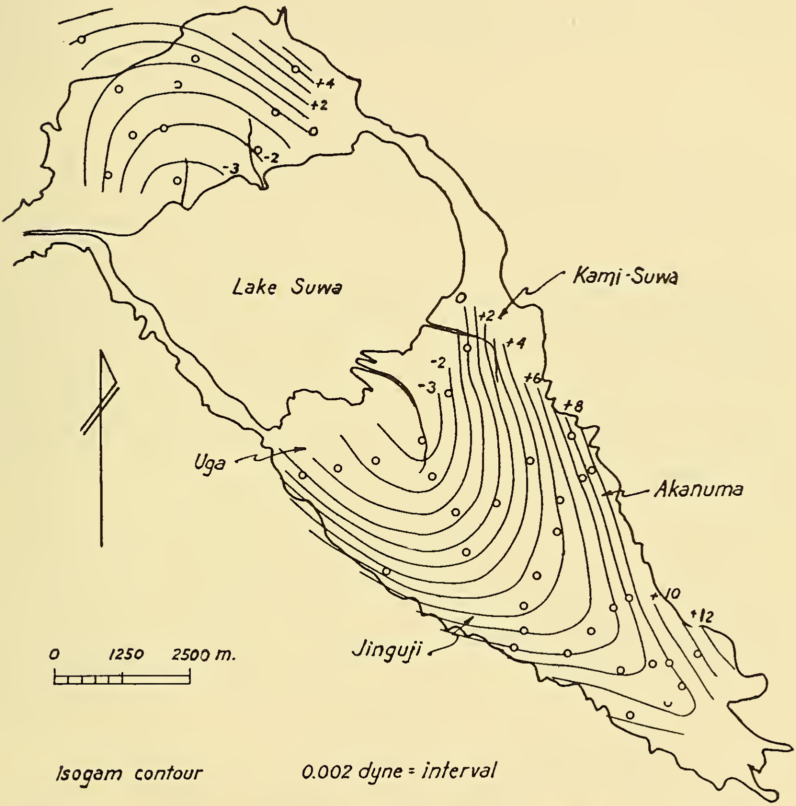

FIG. 2.—Isogam contours in Suwa Basin. (Igneous and metamorphic rocks surround Suwa Basin.)

maximum value. Along the major axis, the direction of the gradients is northwest on the northern side and southwest on the southern side of the lake. The curvature value is maximum at the center of the basin and decreases as the distance from the center increases. Furthermore, the direction of maximum curvature is parallel with the major axis at the center of the basin and is nearly perpendicular at the foot of the mountain.

DEPTH OF BED ROCK

Gradients were calculated on the assumption that a fault, the throw of which was large, is present along the east side of the margin of the basin. The variation in the observed gradients was found to be far too small. Therefore, the assumption was incorrect.

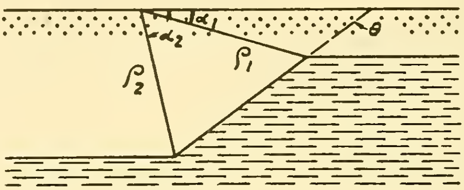

FIG. 3.—Assumed structure in Suwa Basin.

From the topography and the distribution of the corrected values of gradient and curvature, it seemed that the subterranean structure extends in the direction of the major axis of the basin. An assumed cross section is shown in Figure 3. By comparing the anomaly that arises from this assumed structure, which was computed from the following formula, the corrected observation values are obtained and the results are shown in Table III.

$$g\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = -\ 2G\sigma\left\{\sin\theta\cos\theta\log\frac{\rho_2}{\rho_1} + \sin^2\theta(\alpha_2 - \alpha_1)\right\}$$

$$\frac{\partial g}{\partial s} = +\ 2G\sigma\left\{\sin^2\theta\log\frac{\rho_2}{\rho_1} - \sin\theta\cos\theta(\alpha_2 - \alpha_1)\right\}$$

TABLE III

GRAVITY ANOMALIES DUE TO STRUCTURE SHOWN IN FIGURE 3

Place	*No. of Sta.*	*Assumed Value* *Inclination*	*Depth (Meters)*	*Horizontal Distance (Meters)*	*Observed Value*	*Calculated Value*
	11			275	132.4	108.5
Akanuma	15	12°44′	313	447	92.0	101.2
	16			1,010	77.6	76.5
	17			1,327	36.0	46.7
	21			192	102.0	103.1
Jingugi	20	12°44′	173	568	50.4	72.5
	19			983	27.8	12.3
	6			308	123.2	109.0
Uga	4	12°34′	450	900	94.0	94.1
	5			1,519	51.0	73.9

In the vicinity of observation station No. 14 the presence of an active fault is being considered because of the subsidence of the railroad track and because of the geological features. If the active fault is slipping westward, an explanation of the discrepancy between the calculated and the corrected observation values can be made.

If the depth and the dip of the bed rock are assumed, as shown in Figure 3, the calculated results satisfactorily coincide with the observed values. The depth of the bed rock increases as the center of the lake is approached.

CONCLUSION

The maximum computed depth to bed rock is 450 meters; therefore, the basin can not be considered as formed by weathering only, when located in Japan. Its origin should be considered as from sinking or from volcanic eruption.

The isogams of Figure II lead us to believe that Lake Suwa is a crater lake. This theory is rejected, however, as unreasonable from the geologic standpoint. Therefore, the origin of the lake should be regarded as due to a sinking or subsidence following the eruption of the Yatsugadake Mountain, near by. If the origin is sinking, the occurrence of a fault at the margin of the basin is a matter of course. The volcanic lava and debris from the Yatsugadake Mountain were carried down into the sunken zone and deposited in the basin, forming an alluvial fan. The thickness of the deposition is much greater on the southern side than on the northern side of the lake.

The gravity field of the basin is negative, while that of the mountains which surround the basin is positive. Hence, from the standpoint of isostasy, the deficit of the subterranean mass is clear, and shows that the basin is a result of faulting or subsidence.

BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 19, NO. 1 (JANUARY, 1935), PP. 64-77, 7 FIGS.

DEEP ELECTRICAL PROSPECTING[1]

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Dallas, Texas

ABSTRACT

By deep electrical prospecting is meant depths of a half mile or more. Most data so far published refer to depths appreciably shallower than this. A continuous current field is most satisfactory together with electrode spacing comparable with the depth of the insulating media. Electrical prospecting is here regarded in the light of its ability to determine the presence of insulating rather than conducting media. Use is made of resistivity determinations at various electrode spacings together with inductance and resistivity slope functions. Inductance is determined by measuring the time constant of the current decay. Three areas are considered. One, the Hugoton gas area, Stevens County, Kansas, is an ideal case for the electrical method. In the other two cases considered, the Hebbronville area, Jim Hogg County, Texas, and the Anderson County, Kansas, area, the electrical method is quite ineffectual.

Most of the published data on the electrical method of geophysical prospecting have been for rather short electrode spreads and consequently afforded information for relatively shallow depths. A further limitation as regards depth has been the use of an alternating current source to establish a field in the earth. In the following examples a continuous current was employed and interrupted at regular intervals of about 10 seconds. With the longer electrode spacings time constants of the order of one-tenth second were not unusual. It is therefore obvious that for deep penetration it is undesirable to employ alternating current of frequencies higher than a few cycles per second. In view of the difficulty of obtaining large currents of such low frequency, the advantages of a continuous current are apparent.

A current spread of one-half mile (distance between current electrodes) was used throughout. A pair of potential electrodes was placed at varying distances in line with the current electrodes and outside the current electrodes, as indicated in Figure 1. The distance from the inside current electrode to the inside potential electrode is the distance between current and potential electrodes referred to in what follows. The separation of potential electrodes was made as large as necessary to obtain a satisfactory value of voltage. This

[1] Read before the Geophysics Division of the Association at the Dallas meeting, March 23, 1934. Manuscript received, December 4, 1934.

[2] Geophysical Service, Inc., Republic Bank Building.

voltage was amplified by means of a calibrated D.C. amplifier and applied to a galvanometer, the motion of which was photographed. This gave a permanent record and permitted the determination of the voltage and time constant. The disposition of electrodes is designated by the current electrode separation, the distance between the nearest current and potential electrodes, and the potential electrode spacing. For instance, an electrode disposition represented by .5–.4–500′ means that the current electrode separation is one-half mile, the distance between the nearest current and potential electrodes is four-tenths mile and the potential electrode separation is 500 feet. A storage battery of several hundred volts employed as a source delivered 10–20

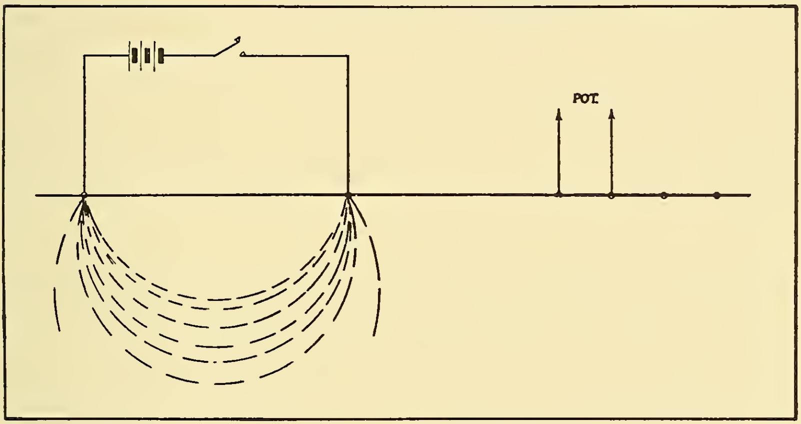

FIG. 1.—Disposition of electrodes.

amperes to the current electrodes which were a multiplicity of copper rods. The usual non-polarizing potential pots were used for the potential electrodes.

Electrical prospecting is here regarded in the light of its ability to determine the presence of insulating rather than conducting media. It would seem that if the electrical method is to be of any considerable value to the petroleum geologist it must prove itself of value in this respect. As rocks filled with oil and gas exhibit much higher resistivities than when filled with mineralized water in the usual case, this would seem the most logical and desirable use of the electrical method. It is well known that the resistivity of dry rock is very high. The comparatively low resistivities observed in electrical prospecting are due to the mineralized water in the rock pores acting as an electrolyte. It is apparent, therefore, that when the pore space is small or when this space is filled with fresh water, gas, or oil, the apparent resistivity of the rock as observed by the ordinary methods of electrical prospecting will be large. Non-porous limestone, therefore, and porous sands where the mineral content of the water is small will indicate high resistivity. This, together with the fact that the areal extent of oil- or gas-filled media is in general small compared with the depth of such media below the surface, suggests at the outset that there are real limitations to the use of the electrical method as far as its use for the direct detection of reasonably deep oil- and gas-saturated media is concerned. Likewise the use of the electrical method for exploring the subsurface structurally at any depth likely to be of value will find its limitations in the lateral variation of resistivity due to changing porosity of the various media and mineral content of subsurface waters, the first mentioned probably offering the greater obstacle. Granite, due to its small porosity, exhibits a very high resistivity. It should, therefore, be possible to determine subsurface structure in the granite provided the lateral extent of the uplift and the amount of relief are sufficiently large in comparison with the depth of the surface of the granite.

If it is assumed that the electrical method is to be used to discover the presence of electrically resistant media, such as oil and gas sands, it is safe to generalize to the extent of saying that to determine the presence of such media with a reasonable degree of certainty *the smallest lateral dimension of the high-resistance medium must be at least several times the depth of the medium below the surface.* The Hugoton gas field in Stevens County, Kansas, as here demonstrated, definitely comes under this category. Without much doubt the East Texas field also should be included, although this is now impossible to determine due to the large number of cased wells in the field. The electrical method is of no value if cased wells are in the vicinity of any of the electrodes.

The resistivity was determined from the following relation:

$$\Delta V = \frac{I\rho A}{\pi(x_1^2 - A^2)} - \frac{I\rho A}{\pi(X_2^2 - A^2)}$$

$$\rho = \frac{\pi \Delta V}{IA} \times \frac{1}{\dfrac{1}{X_1^2 - A^2} - \dfrac{1}{X_2^2 - A^2}}$$

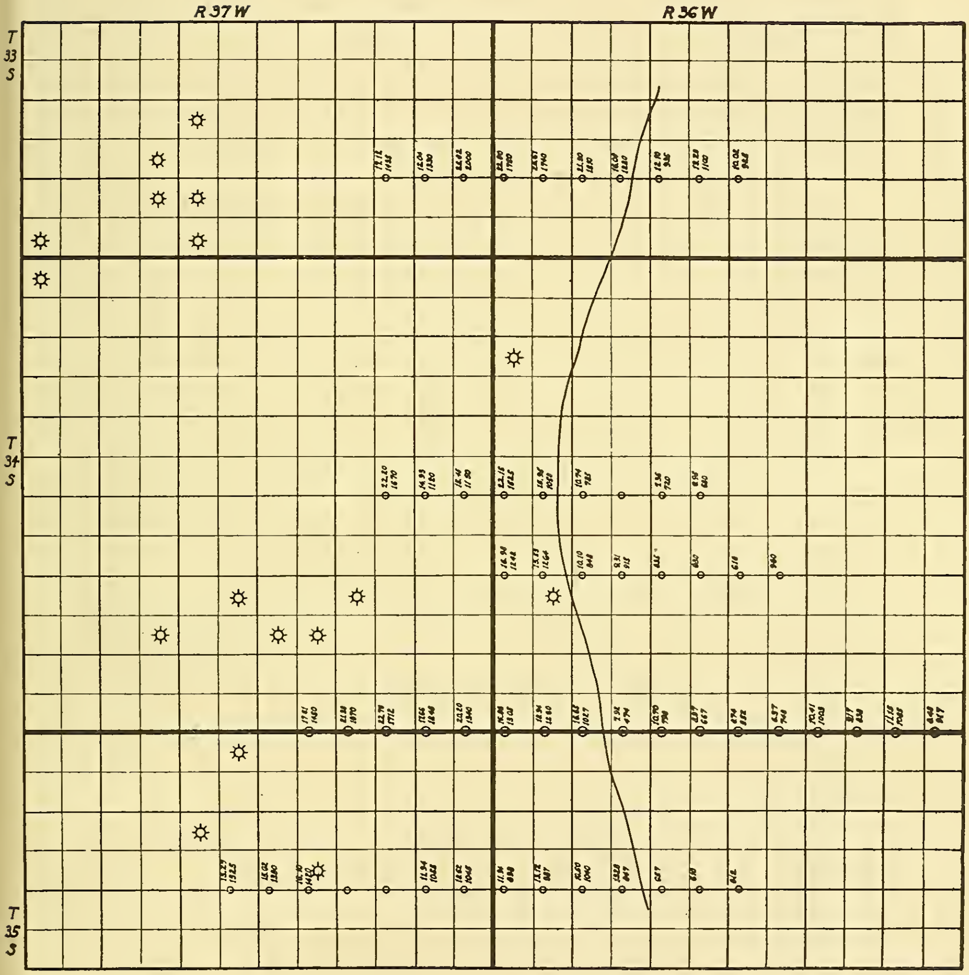

FIG. 2.—Hugoton gas area.

where ΔV is the potential difference between potential electrodes in volts, I is the current flowing from the current electrodes in amperes, ρ is the resistivity in ohms per centimeter cube, A is the half current spread in centimeters, X_1 and X_2 are the distances from the center of the current spread to the potential electrodes. It is obvious that if the medium is homogeneous the value of ρ will be independent of the position of the potential electrodes (X_1 and X_2). As the ground is non-homogeneous the value of ρ will depend on the position of the potential electrodes. The variations from normal as the potential electrodes are moved permits an approximation of subsurface resistivities.

As the current does not reach its maximum immediately upon application of a voltage on the current electrodes, or does not immediately return to zero on removal of the voltage, the path of the current through the ground has an effective inductance. The current rises and decays logarithmically in point of time, exactly the characteristic of an electrical circuit comprising an inductance and resistance in series. Now L/R is the time constant of the circuit and represents the time required for the current to decay to $1/\epsilon$ of its initial value. This may easily be measured from the photographic record of the voltage and R may be approximated from the resistivity measurements. Thus we arrive at a quantity which is a functoin of the inductance and is here designated the inductance function.

Although the shorter spreads are affected principally by the near-surface conditions, an abnormal surface resistance will have some residual effect on the longer spreads. A quantity, therefore, which is a function of the resistivity as determined by the longer spread and the slope of the resistivity profile will further eliminate the near-surface effects. This we will call the resistivity-slope function. The slope is measured by the angle the resistivity profile makes with the vertical and increases in a clockwise direction.

Electrical surveys of three areas are here described. In one area the Hugoton gas field, resistivities, inductance and resistivity-slope functions are all determined and compared. In the other two areas, the Hebbronville area in Jim Hogg County, Texas, and the Anderson County area in Kansas, only the resistivities are used. J. V. Polk, of Geophysical Service, Inc., deserves credit for the field work, most of which was conducted under his direction during 1932.

HUGOTON GAS AREA

The Hugoton gas field is one of the largest gas fields known, covering the western part of Stevens County, Kansas, and extending into several adjacent counties. The gas sand, the thickness of which is not

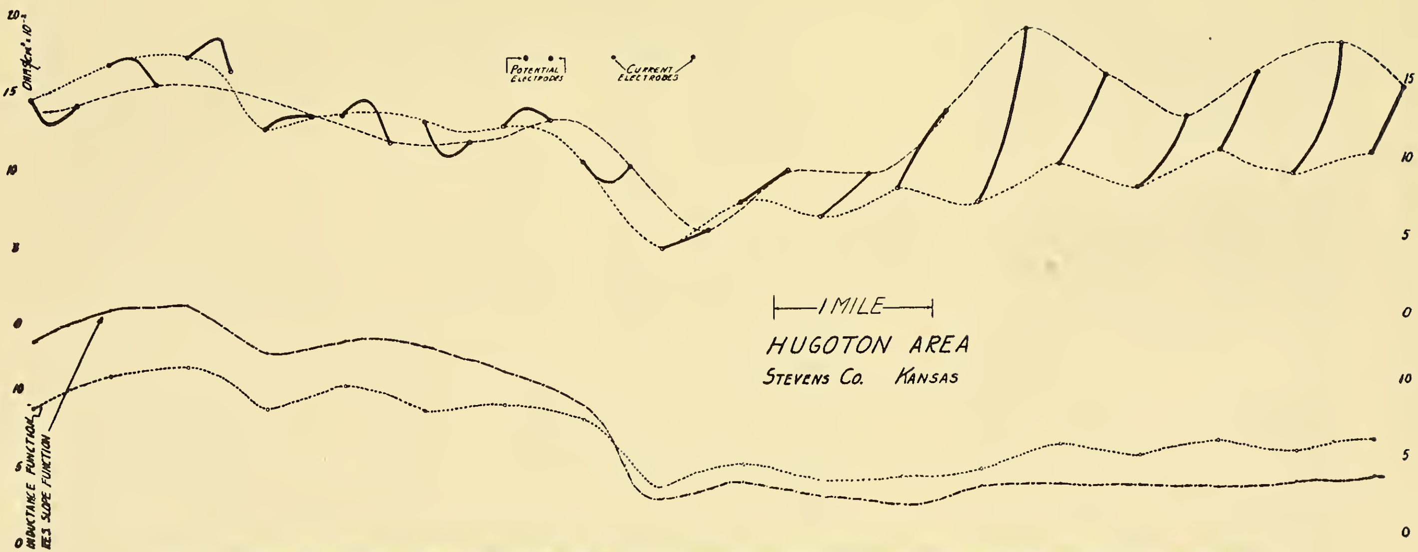

FIG. 3.—Electrical profile across east edge of Hugoton gas area, showing values of resistivity, also inductance function for longest spread.

accurately known, lies at a depth somewhat greater than one-half mile below the surface. A number of lines of electrical profiles were run in east and west directions, traversing the east edge of the field. This particular portion of the field is shown in Figure 2, together with the resistivity values and the inductance function values for the longest electrode spread on each profile. A profile consists of four potential electrode positions, namely, .4, .5, .6, and .7 mile from the inside potential electrode to the inside current electrode. The separation of the two potential electrodes ranged from 500 feet to 1,000 feet, the minimum value necessary to secure a good record being chosen in each case. This choice is not critical. The distance between current electrodes was .5 mile. These, according to a previous statement of nomenclature, are designated as:

.5–.4– 500′
.5–.5– 500′
.5–.6–1,000′
.5–.7–1,000′

The north-south trending line through R. 36 W. indicates the electrically determined east edge of the gas field.

In the upper portion of Figure 3 are shown the resistivity determinations for the line of profiles indicated in Figure 2 along the township line. Although only the two end values are indicated on the profiles, the profiles were determined from the four values specified above. The profiles run from right to left, the shorter electrode spacing being on the right. The relative disposition of the electrodes is indicated for one profile.

Attention is called to the fact that resistivity values for the .5–.4–500′ set-up are on the average equally high, while some are even higher, east of the gas field than over the gas sand. On the other hand, the .5–.7–1,000′ resistivity values are on the average decidedly higher over the gas sand than outside the bounds of the field. As a result of this, the shape of the profile curve is markedly different in the two cases. One of the curves in the lower portion of the figure represents the inductance function. This follows the same general pattern as that of the .5–.7–1,000′ resistivity line above, but the change across the field border is of greater magnitude. In this particular case all indices, the longer spread resistivities, inductance function and resistivity-slope function clearly indicate the presence of the insulating gas sand with a remarkable degree of certainty. This field amply satisfied the condition postulated, namely that the minimum dimensions of the insulating layer should be at least several times the depth. It is evident also that, in such a case as this, further manipulation of the data

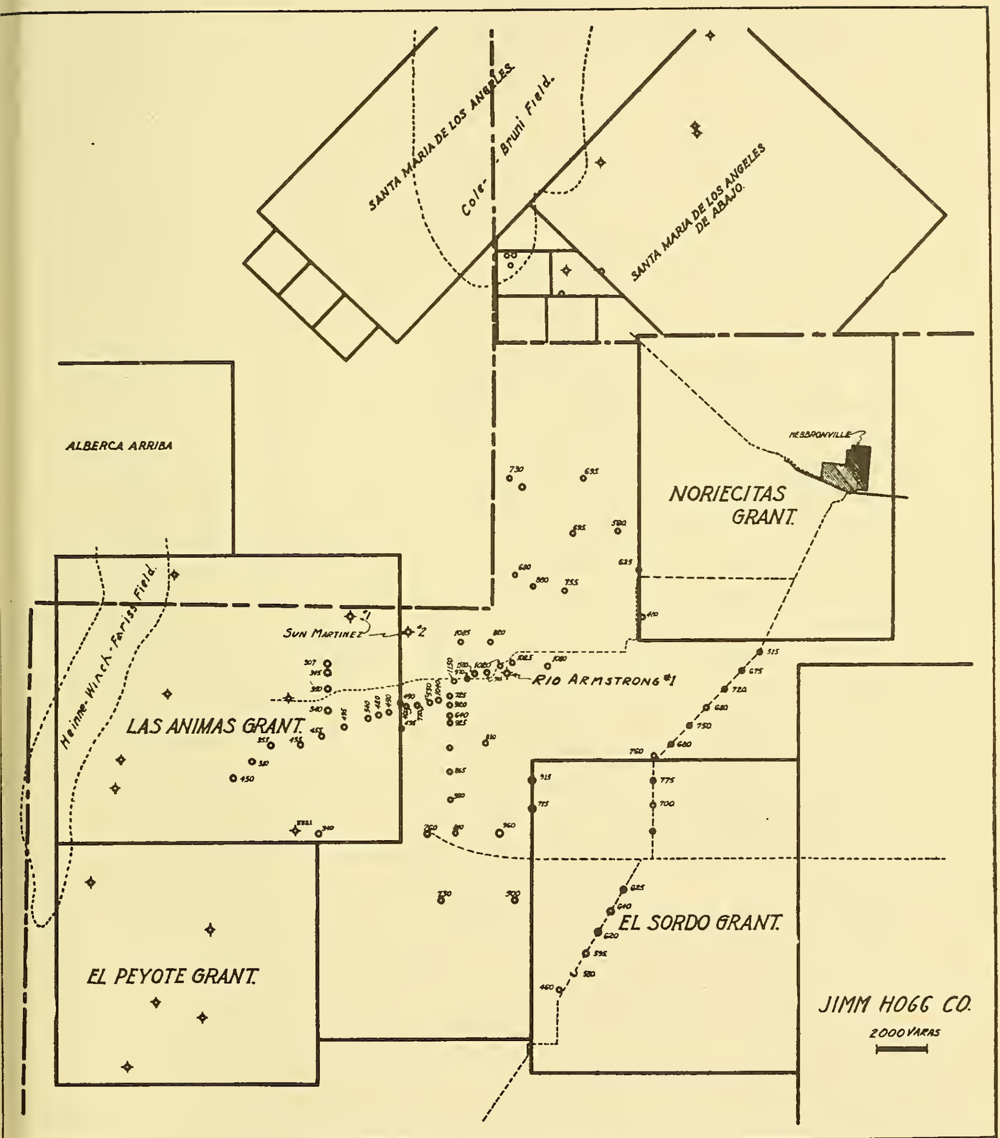

FIG. 4.—Hebbronville area.

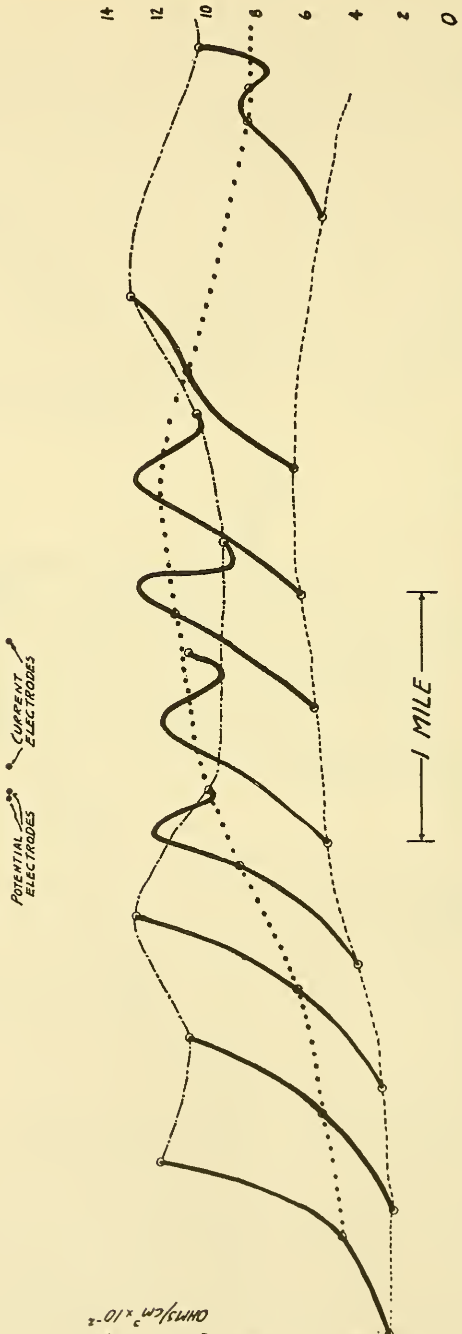

Fig. 5.—Electrical profile across Hebbronville area.

beyond the original resistivity determination adds but little to the value of the data. It would seem therefore that the limitations of the electrical method are apt to be more fundamental than the choice of instrument design or electrode arrangement. The following two cases support this view.

HEBBRONVILLE AREA

The center of the area surveyed lies about 10 miles southwest of the town of Hebbronville in Jim Hogg County, Texas. The electrical work in this area was completed previous to the drilling of any wells. The resistivity measurements were made in a manner similar to that in the foregoing illustrations. The location of resistivity points is indicated in Figure 4. The effective resistivity point is assumed to be half way between the inside potential electrode and the inside current electrode. The potential electrodes were placed at distances of .1, .2, .3, .4, .6, and .8 mile from the nearest current electrode. For the sake of clarity, not all of these positions are indicated in Figure 5, which represents a line of profiles east and west across the center of the area, east being at the right. The Rio Oil Corporation Armstrong No. 1 was drilled slightly east of the center of the profile. The broken connecting lines shown join corresponding points on the different profiles, namely, the .1, .4, and .8 mile potential-electrode points. The .1 mile point indicates a resistivity syncline where the .4 and .8 mile potential points evidence a resistance anticline, that is, high-resistance values. The .2 potential point not shown also demonstrates a resistivity syncline. It is therefore evident that the resistivity "high" indicated by the longer spreads is not the reflection of a surface condition but must be attributed to a subsurface effect at a depth of the order of the electrode separation, namely, .4 mile. This is assumed to be the case because the short electrode separations are affected oppositely and the longest separation, namely .8 mile, indicates a smaller resistivity "high" than the .4 mile position. This is the effect expected in the case of a poor insulator. The contour maps of Figure 6 are the result of contouring all resistivity values in the one case for the .5–.4–500′ set-up and in the other for the .5–.8–1,000′ set-up.

The Rio Oil Corporation well was drilled at the center of the "high" as shown in Figure 6. This well had no showing of gas or oil and upon correlation with the two Sun Oil Company Martinez wells later drilled at the west, was found to be structurally much lower than either. The following datum values on the Vicksburg were kindly furnished by the Sun Oil Company.

Sun Oil Company	Martinez No. 1	− 710
Sun Oil Company	Martinez No. 2	− 930
Rio Oil Corporation	Armstrong No. 1	−1390

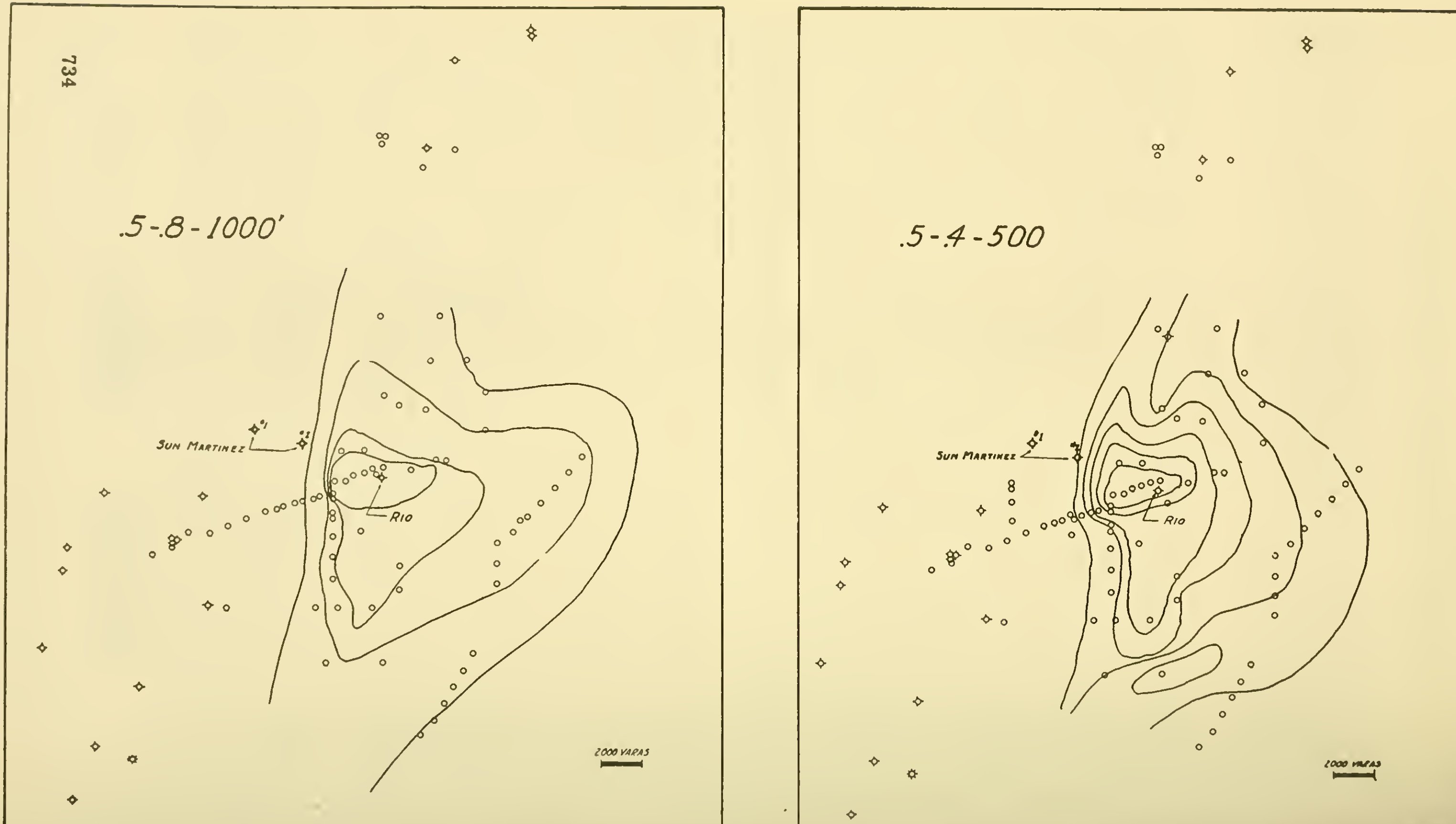

FIG. 6.—Resistivity contours, Hebbronville area.

Obviously there is no agreement whatever between the electrical and well data, either as regards the presence of an oil or gas sand or the structural condition. Therefore, it must be concluded that the resistivity "high" was caused by a lateral change in porosity or mineral content of subsurface water at a depth of approximately .4 mile. It was reported that the well made considerable fresh water between

TABLE I

Rio Oil Corporation's E. L. Armstrong No. 1

Jim Hogg County, Texas

Formation	*Depth in Feet*
Caliche	20
Sand and caliche	100
Sand	125
Hard shale	150
Gravel	160
Hard shale	200
Sand	205
Shale	275
Sand and gravel	290
Hard shale	380
Hard sand	402
Hard cemented gravel	442
Hard sand gravel	505
Hard cemented gravel	597
Hard sand gravel	652
Hard sand	662
Hard gravel	682
Hard shale	700
Hard sand gravel	732
Shale	1051
Sticky shale	1325
Sandy shale	1425
Shale	1576
Sticky shale	1650
Shale	1700
Sticky shale	1776
Sandy shale (cored)	1781
Shale	1941
Sticky shale (cored 1,935–55)	1975
Sticky shale	2016
Shale (cored 2,016–36)	2110
Sticky shale	2186
Sand (cored)	2223
Hard sand	2240
Shale	2269
Hard sand (cored 2,270–75)	2275
Sand	2290
Broken sand	2320
Shale	2369
Hard sand rock (cored, 2,369–71)	2375
Sand (cored)	2382
Hard sand rock	2384
Hard sand	2435
Hard sand rock	4440
Shale	2460
Shale and lime (cored)	2468
Shale	2505

1,600 and 2,000 feet. If so, this fact may well account for the resistivity "high" observed. This case illustrates very well some of the limitations of the electrical method as previously discussed. A driller's log of the well is given in Table I.

ANDERSON COUNTY, KANSAS, AREA

The area under consideration is located in Sec. 10, T. 23 S., R. 20 E., Anderson County, Kansas. The procedure was exactly the same as followed in the two foregoing cases. The following spreads were employed: .5–.1–10′, .5–.2–20′, .5–.3–150′, and .5–.4–500′. The contour maps shown in Figure 8 for the four different spreads suggest the presence of an insulator at a depth of somewhat less than 500 feet. As the spread is increased, the effect of this disappears as shown

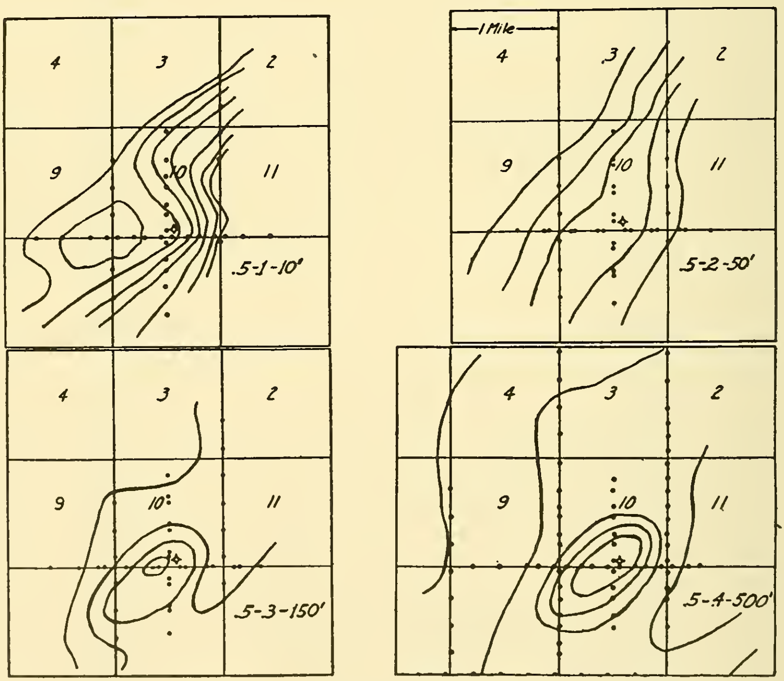

FIG. 7.—Resistivity contours, Anderson County, Kansas, area.

on the contour map for the .5–.2–50′ set-up. Upon further lengthening the spread, the presence of another insulator is indicated. A well drilled on the south side of Section 10, with the exception of a very slight showing of gas at 550 feet, was abandoned at a total depth of

1,056 feet as a dry hole. The log of this well (Table II) shows considerable limestone. The only conclusion that may be drawn is that

TABLE II

WREN WELL NO. 1, SEC. 10, T. 23 S., R. 20 E.,

ANDERSON COUNTY, KANSAS

Formation	*Depth in Feet*
Shale	10
Lime	12
Shale	115
Lime	150
Blue shale	185
Lime	188
Shale, dark	216
Lime	220
Shale	225
Lime	256
Shale	258
Lime	281
Dark shale	286
Lime	306
Shale	311
Lime	335
Shale	510
Lime	512
Shale	525
Lime	531
Shale	602
Lime	617
Blue shale	637
Lime	652
Blue shale	663
Lime	678
Blue shale	686
Lime	692
Shale	765
Lime	768
Dark shale	788
Lime	790
Blue shale	805
Light shale	815
Dark shale	837
Light shale	840
Dark shale	930
Sand	941
Blue shale	967
Sand	982
Broken sand	990
Dark shale	1049
Broken lime	1054
Lime	1056

this case falls into the same category as the preceding one of the Hebbronville area.

BULLETIN OF THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
VOL. 19, NO. 1 (JANUARY, 1935), PP. 78-115, 2 FIGS.

NORMAL GEOTHERMAL GRADIENT IN UNITED STATES[1]

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Washington, D.C.

ABSTRACT

The objects in preparing this paper have been, first, to prepare a brief summary of the gradients deduced from recent geothermal surveys in the United States; and second, to discuss the data thus summarized from the standpoint of a normal geothermal gradient.

INTRODUCTION

In 1920, N. H. Darton (1)[3] published a summary of geothermal data in the United States. With but few exceptions, the data for the individual states consisted of temperatures recorded in flowing wells. Since the appearance of Darton's publication, the United States Geological Survey and the American Petroleum Institute (2) have added to the list the records of about 700 non-flowing wells located chiefly in the oil-producing states. These observations, including Darton's records, have never been discussed from the standpoint of a normal geothermal gradient; but, in view of the constant use of the term in geological literature, it is necessary that the term be properly defined and that the limitations of the definition be determined, as nearly as possible, from existing data.

HORIZONTAL AND VERTICAL VARIATION OF GRADIENTS

Recent geothermal surveys have shown without question that geothermal gradients vary uniformly in both the horizontal and the vertical. Any attempt, therefore, to approximate to a normal gradient must take these two important variations into account.

Variation in the horizontal is illustrated by reference to Figure 1, which represents the cross section of a typical oil field anticline. Observations show that in a majority of cases the isogeotherms ($a''a'a$,

[1] Published with the permission of the director, United States Geological Survey.

[2] Geophysicist, United States Geological Survey. Manuscript received, June 28, 1934. To the writer's assistant, H. Cecil Spicer, special credit is due for very valuable and efficient service rendered in carrying out the extensive calculations involved in the preparation of this and other recent papers on geothermal gradients.

[3] This and the following references are in the bibliography at the end of this article.

$c''c'c$) rise in passing over the anticlines. Expressed in terms of gradients, this is the equivalent of saying that the gradients are a maximum, or the reciprocal gradients are a minimum, along the crests of the anticlines.

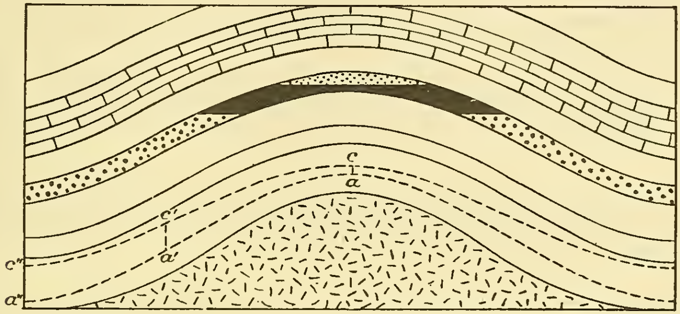

FIG. 1.—Cross section of typical anticline.

Variations in the vertical are illustrated by the concave and convex types of depth-temperature curves (AA' and BB') shown in Figure 2. Each of the lines *cc* and *ee* represents the least square adjustment of a straight line through all of the observed points from

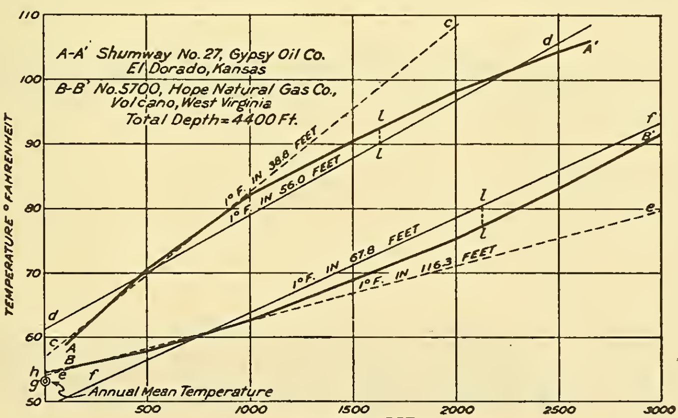

FIG. 2.—Concave and convex types of depth-temperature curves.

100 to 1,000 feet inclusive on the respective curves. Similarly, the lines *dd* and *ff* were obtained by adjusting, in each case, a straight line through all of the points from 100 feet to the greatest depth, inclusive, at which a reading was taken. In curves of the concave

TABLE I

TEMPERATURE GRADIENTS. ISOLATED LOCATIONS AND FLANKS OF STRUCTURES

Town	State	No. of Wells	$b \times 10^5$. Degrees F. per Foot 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet	Total Depth Feet	$b \times 10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b \times 10^5$ 0–Total Depth
Albany	Alabama	1	259	398	—	—	2,000	398	251.1	355
Birmingham	Alabama	7	914	955	—	—	2,000	955	104.7	750
Fayette	Alabama	1	780	—	—	—	1,250	832	120.1	632
El Dorado	Arkansas	7	1,862	2,329	—	—	2,000	2,329	42.9	2,240
Bakersfield	California	5	1,198	1,236	1,338	1,464	4,250	1,569	63.7	1,911
Coalinga	California	51	1,413	—	—	—	1,370	1,351	74.0	1,956
Fullerton	California	1	2,009	2,070	1,983	1,932	4,270	1,919	52.1	2,133
Grass Valley	California	1	593	569	537	—	3,400	527	189.8	497
Huntington Beach	California	4	2,225	2,456	2,624	—	3,500	2,643	37.8	2,786
Kettleman Hills	California	4	1,612	—	—	—	1,350	1,671	59.9	2,244
Long Beach	California	40	1,607	1,681	—	—	2,500	1,642	60.9	2,204
Santa Fe Springs	California	31	1,535	1,819	1,895	—	4,250	1,904	52.5	1,995
Seal Beach	California	1	2,949	2,398	2,277	—	3,500	2,256	44.3	2,346
Whittier	California	4	1,670	1,809	1,885	—	3,750	1,874	53.4	2,136
Calhan	Colorado	1	1,575	1,570	1,610	1,712	4,000	1,712	58.4	1,863
Florence	Colorado	3	1,532	1,848	—	—	2,900	2,020	49.5	2,090
Fort Collins	Colorado	2	1,667	1,755	1,858	1,956	4,000	1,956	51.1	2,050
Longmont	Colorado	1	1,775	2,018	2,152	2,251	6,500	2,399	41.7	2,489
Bridgeport	Illinois	4	1,456	—	—	—	1,250	1,526	65.5	1,704
Ames	Iowa	1	695	731	—	—	2,100	710	140.9	686
Augusta	Kansas	2	1,945	2,009	—	—	2,250	1,984	50.4	2,013
El Dorado	Kansas	24	1,777	1,827	—	—	2,250	1,799	55.6	1,689
Florence	Kansas	4	1,361	—	—	—	1,750	1,590	62.9	1,686
Haverhill	Kansas	3	1,720	1,819	—	—	2,750	1,783	56.1	1,811
Syracuse	Kansas	1	1,219	1,277	1,338	—	4,600	1,428	70.0	1,485

TABLE I (*Continued*)

Town	State	No. of Wells	$b \times 10^5$. Degrees F. per Foot 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet	Total Depth Feet	$b \times 10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b \times 10^5$ 0–Total Depth
Prater Creek	Kentucky	1	820	—	—	—	1,500	637	157.0	620
Rockcastle Creek	Kentucky	1	844	—	—	—	1,500	764	130.9	600
Blue Lake	Louisiana	2	1,786	—	—	—	1,500	1,993	50.2	2,180
Caddo Lake	Louisiana	1	2,653	2,423	—	—	2,000	2,423	41.3	2,325
Caddo Lake (near)	Louisiana	3	2,674	2,413	—	—	2,250	2,391	41.8	2,333
Haynesville	Louisiana	1	2,075	2,273	—	—	2,750	2,245	44.5	2,160
Homer	Louisiana	8	2,518	—	—	—	1,150	2,646	37.8	2,357
Many	Louisiana	6	1,883	—	—	—	1,250	1,926	51.9	1,832
Pine Island	Louisiana	1	2,845	2,451	2,269	—	3,500	2,218	45.1	2,234
Sligo	Louisiana	2	2,588	—	—	—	1,650	2,318	43.1	2,218
Zwolle	Louisiana	3	1,751	—	—	—	1,250	1,771	56.5	1,792
Houghton	Michigan	4	771	—	—	—	6,250	922	108.4	845
Lake Linden	Michigan	1	624	—	—	—	1,435	757	132.2	983
Surrey Twp.	Michigan	1	1,186	1,378	1,748	—	3,700	1,887	53.0	1,754
Jackson	Mississippi	1	2,906	2,861	—	—	2,250	2,832	35.3	2,676
Anaconda	Montana	1	2,635	—	—	—	1,600	2,111	47.4	1,969
Kevin-Sunburst	Montana	10	930	978	987	—	3,000	987	101.4	1,123
Virginia City	Nevada	1	3,396	3,129	—	—	2,300	3,105	32.2	3,061
Franklin-Furnace	New Jersey	1	362	—	—	—	2,500	585	170.8	640
Artesia	New Mexico	3	560	478	—	—	2,892	439	227.7	598
Carlsbad	New Mexico	1	680	583	—	—	2,874	578	173.1	703
Lovington	New Mexico	1	1,252	941	733	594	4,500	554	180.5	640
Roswell	New Mexico	2	439	540	576	—	2,950	576	173.5	773
Lonetree	North Dakota	1	1,578	1,654	1,806	—	3,750	1,998	50.0	2,157
Ardmore	Oklahoma	4	657	—	—	—	1,500	678	147.5	667

TABLE I (*Continued*)

Town	State	No. of Wells	$b\times10^5$. Degrees F. per Foot: 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–3,000 Feet	Total Depth Feet	$b\times10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b\times10^5$ 0–Total Depth
Ardmore-Healdton	Oklahoma	9	616	—	—	—	1,500	657	152.2	593
Ardmore-Hewett	Oklahoma	8	808	—	—	—	1,750	950	105.3	800
Bessie	Oklahoma	1	707	—	—	—	1,500	828	120.7	920
Billings	Oklahoma	1	1,891	1,910	1,895	—	3,500	1,911	52.3	1,937
Blackwell	Oklahoma	2	2,038	2,027	—	—	2,000	2,027	49.3	1,900
Braman	Oklahoma	3	1,758	—	—	—	1,410	1,829	54.7	1,787
Burbank	Oklahoma	3	1,893	2,093	—	—	2,850	2,188	45.7	2,172
Covington	Oklahoma	1	—	—	—	—	1,600	1,933	51.7	1,919
Cromwell	Oklahoma	9	1,886	—	—	—	1,200	1,918	52.1	1,775
Davenport	Oklahoma	1	882	988	1,101	—	3,375	1,162	86.0	1,132
Dilworth	Oklahoma	11	1,743	1,848	1,874	—	3,000	1,874	53.4	1,953
Drumright	Oklahoma	28	880	1,381	—	—	2,210	1,467	68.2	1,416
Glenn Pool	Oklahoma	18	2,284	—	—	—	1,500	2,383	42.0	2,187
Holdenville	Oklahoma	1	—	—	1,891	1,925	5,200	2,002	49.9	1,996
Hubbard	Oklahoma	1	2,066	2,054	—	—	2,870	2,103	47.6	2,136
Morris	Oklahoma	1	2,551	—	—	—	1,610	2,454	40.7	2,410
Newkirk	Oklahoma	1	2,264	2,202	2,224	—	3,000	2,224	45.0	2,270
Okemah	Oklahoma	4	2,341	2,240	—	—	2,500	2,241	44.6	2,444
Oklahoma City	Oklahoma	3	659	765	841	904	4,000	904	110.7	1,023
Papoose	Oklahoma	4	1,903	1,945	—	—	2,500	2,014	49.7	2,168
Perry	Oklahoma	1	1,820	1,677	1,717	—	3,000	1,717	58.2	1,670
Sasakwa	Oklahoma	2	1,326	1,480	—	—	2,620	1,518	65.9	1,443
Seminole	Oklahoma	19	1,019	1,314	—	—	2,750	1,340	74.6	1,284
Tonkawa	Oklahoma	15	1,569	1,831	—	—	2,750	1,840	54.3	1,876
Walters	Oklahoma	10	750	—	—	—	2,150	864	115.7	902

TABLE I (*Continued*)

Town	State	No. of Wells	$b \times 10^5$. Degrees F. per Foot				Total Depth Feet	$b \times 10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b \times 10^5$
			100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet				0–Total Depth
Wewoka	Oklahoma	5	1,951	2,063	2,107	—	3,000	2,107	47.5	2,087
Astoria	Oregon	1	1,503	1,615	1,656	—	3,780	1,670	59.9	1,659
Burns	Oregon	1	—	—	—	—	300	5,850*	17.1	8,100*
Klamath Falls	Oregon	2	—	—	—	—	625	4,909*	20.4	5,840*
Klamath Falls	Oregon	1	—	—	—	—	500	730	137.0	900
Medford	Oregon	2	—	—	—	—	899	1,320	75.7	1,846
Merrill	Oregon	1	—	—	—	—	660	2,471	40.5	2,924
Vale	Oregon	1	5,110*	—	—	—	1,295	4,575*	21.9	5,042*
Gaines Junction	Pennsylvania	1	1,060	1,085	1,218	1,413	5,500	1,619	61.8	1,569
Johnstown	Pennsylvania	1	919	1,121	1,171	1,157	6,500	1,313	76.2	1,358
Long Bridge	Pennsylvania	3	1,370	1,473	1,480	1,482	6,500	1,674	59.7	1,634
McDonald	Pennsylvania	1	764	903	1,045	1,190	6,975	1,444	69.2	1,325
McKeesport	Pennsylvania	1	—	—	1,249	—	3,250	1,268	78.9	1,243
Bastrop	Texas	1	2,073	2,110	—	—	2,000	2,110	47.4	2,430
Big Lake	Texas	16	672	—	—	—	1,500	636	157.3	873
Blue Ridge	Texas	7	—	1,706	—	—	2,900	1,800	55.6	1,828
Boggy Creek	Texas	3	—	2,499	2,820	—	3,703	2,867	34.9	2,768
Bonham	Texas	1	2,276	—	—	—	1,140	2,186	45.7	2,237
Callisburg	Texas	1	971	918	949	—	3,680	982	101.8	978
Canadian	Texas	1	794	806	918	939	4,758	1,001	99.9	1,051
Dale	Texas	1	2,144	2,298	—	—	2,250	2,299	43.5	2,262
Damon	Texas	1	—	1,434	1,512	1,619	4,000	1,619	61.8	1,715
Del Rio	Texas	2	1,262	1,432	1,372	—	3,686	1,311	76.3	1,373
De Witt County	Texas	1	1,753	1,902	—	—	1,970	1,902	52.6	1,949
Dickens	Texas	1	1,024	871	—	—	2,400	877	114.0	1,017

TABLE I (*Continued*)

Town	State	No. of Wells	$b\times10^5$. Degrees F. per Foot: 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet	Total Depth Feet	$b\times10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b\times10^5$ 0–Total Depth
Eastland	Texas	1	1,760	1,857	1,885	—	3,000	1,885	53.0	1,973
Edwards	Texas	1	1,251	—	1,267	1,365	6,600	1,666	60.0	1,691
Electra	Texas	1	1,159	—	—	—	1,830	1,224	81.7	1,213
Ennis	Texas	1	1,716	—	—	—	1,325	1,856	53.9	2,181
Ferris	Texas	1	2,034	—	—	—	1,250	2,024	49.4	2,032
Graford	Texas	1	1,736	1,734	1,856	—	3,000	1,856	53.9	1,933
Graham	Texas	2	1,502	1,630	1,638	—	3,000	1,638	61.0	1,830
Grand Saline	Texas	4	—	—	—	—	875	1,606	62.2	1,977
Greenville	Texas	1	1,952	—	—	—	1,750	1,855	53.9	1,903
Guthrie	Texas	1	1,068	841	898	—	2,950	898	111.3	1,108
Humble	Texas	28	1,458	1,677	—	—	2,000	1,677	59.6	1,560
Leonard	Texas	1	1,799	1,586	—	—	2,500	1,487	67.3	1,532
Long Point	Texas	8	1,789	1,831	1,777	—	3,300	1,765	56.7	1,794
Luling	Texas	4	1,489	1,832	—	—	2,500	1,886	53.0	2,184
Lytton Springs	Texas	7	2,464	—	—	—	1,350	2,297	43.5	2,356
Mexia	Texas	6	1,820	2,072	—	—	2,900	2,099	47.6	2,369
Muenster	Texas	1	1,360	—	—	—	1,500	1,454	68.8	1,173
Odessa	Texas	1	693	717	—	—	1,985	717	139.4	912
Ozona	Texas	1	1,030	1,135	1,369	1,572	5,500	1,624	61.6	1,556
Panhandle	Texas	6	640	710	—	—	2,785	597	167.6	944
Pierce Junction	Texas	3	—	1,663	—	—	4,303	1,486	67.3	1,769
Powell	Texas	14	1,147	1,347	1,460	—	2,950	1,460	68.5	2,254
Ranger	Texas	5	2,027	2,015	2,067	—	3,000	2,067	48.4	2,150
Rosewood	Texas	1	1,950	—	—	—	1,500	2,027	49.3	2,320
Sherman	Texas	2	1,460	—	—	—	1,450	1,388	72.0	1,379

TABLE I (*Continued*)

Town	State	No. of Wells	$b \times 10^5$. Degrees F. per Foot 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet	Total Depth Feet	$b \times 10^5$ 100–Total Depth	1/b 100–Total Depth Ft. °F.	$b \times 10^5$ 0–Total Depth
St. Jo	Texas	1	1,160	—	—	—	1,729	1,190	84.1	1,029
Thrifty	Texas	1	1,640	—	—	—	1,250	1,624	61.6	1,872
Waco	Texas	2	1,964	1,734	—	—	1,990	1,734	57.7	2,000
Wolfe City	Texas	2	1,823	—	—	—	1,700	1,973	50.7	1,982
Wortham	Texas	2	1,993	1,974	—	—	2,900	1,954	51.2	2,303
Benton City	Washington	1	2,265	2,085	—	—	2,200	2,064	48.5	2,282
Moclips	Washington	1	985	1,239	1,342	—	3,500	1,378	72.6	1,243
Seattle	Washington	1	929	971	—	—	2,500	949	105.4	820
Bridgeport	West Virginia	1	1,167	1,007	1,047	1,100	7,310	1,401	71.4	1,456
Fairmont	West Virginia	1	1,243	1,232	1,290	1,285	7,500	1,521	65.8	1,544
Grantsville	West Virginia	1	1,152	1,110	1,130	1,165	4,500	1,204	83.1	1,258
Mannington	West Virginia	1	1,371	1,140	1,105	—	3,225	1,096	91.2	1,101
Spencer	West Virginia	2	980	922	999	—	6,500	1,293	77.3	1,246
Volcano Junction	West Virginia	1	882	1,076	1,259	1,389	4,250	1,437	69.6	1,393
Volcano	West Virginia	1	860	1,081	1,267	1,411	4,400	1,476	67.8	1,448
Wheeling	West Virginia	1	—	1,180	1,186	—	4,462	1,349	74.2	1,268
Wyatt	West Virginia	1	1,085	1,018	—	—	2,000	1,018	98.2	1,065
Big Muddy	Wyoming	4	1,416	1,475	1,707	—	3,000	1,707	58.6	1,987
Cody	Wyoming	2	1,607	1,726	1,743	1,707	4,250	1,698	58.9	1,854
Grass Creek	Wyoming	3	—	—	—	—	800	1,934	51.7	1,988
Lance Creek	Wyoming	3	2,341	2,643	2,718	—	3,000	2,718	36.8	2,797
Lost Soldier	Wyoming	3	3,176	2,916	2,638	—	3,400	2,569	38.9	2,635
Pine Mountain	Wyoming	1	2,555	2,207	--	—	2,240	2,057	48.6	1,911
Rawlins	Wyoming	4	1,378	1,708	—	—	2,000	1,708	58.6	1,760
Rock River	Wyoming	3	1,343	—	1,431	—	3,000	1,431	69.9	1,540

TABLE I (*Continued*)

Town	State		No. of Wells	$b\times10^5$. Degrees F. per Foot: 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet	Total Depth Feet	$b\times10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b\times10^5$ 0–Total Depth
Salt Creek	Wyoming		21	1,606	1,952	—	—	2,650	2,079	48.1	2,102
Tea Pot	Wyoming		1	2,232	2,551	—	—	2,736	2,744	36.4	2,727
Thermopolis	Wyoming		9	—	—	—	—	900	4,206	23.8	4,711*
Thermopolis (Gebo)	Wyoming		1	1,772	—	—	—	1,407	1,945	51.4	2,395
Thermopolis (Little Sand Draw)	Wyoming		1	1,457	1,506	1,612	—	3,000	1,612	62.9	1,723
A. Arithmetic mean. All values included. Formula 7		n	658	139	102	60	22	155	155		155
		b		.01559	.01606	.01556	.01433		.01720		.01811
		r		.00474	.00400	.00353	.00264		.00524		.00619
		r_0		.00040	.00040	.00046	.00056		.00042		.00050
		$1/b$		64.13	62.28	64.28	69.77		58.14		55.23
		m						2,781			
B. Arithmetic mean. Values for which $v \geqq 5r$ have been rejected. Formula 7		n	645	138	102	60	22		151		151
		b		.01534	.01606	.01556	.01433		.01636		.01702
		r		.00429	.00400	.00353	.00264		.00391		.00402
		r_0		.00036	.00040	.00046	.00056		.00032		.00033
		$1/b$		65.21	62.28	64.28	69.77		61.12		58.77
C. Arithmetic mean. Values for which $b \geqq 0.02470$ have been rejected. $1/b \leqq 40.49$. Formula 7		n	627	126	96	56	22		142		142
		b		.01414	.01533	.01474	.01433		.01567		.01634
		r		.00351	.00358	.00295	.00264		.00358		.00369
		r_0		.00031	.00036	.00039	.00056		.00030		.00031
		$1/b$		70.71	65.23	67.84	69.77		63.83		61.18
D. Weighted means of *A*-values. Formula 8		n	658						155		155
		b							.01645		.01707
		$1/b$							60.80		58.59

TABLE I (*Continued*)

Town	State	No. of Wells	$b \times 10^5$. Degrees F. per Foot: 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet	Total Depth Feet	$b \times 10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b \times 10^5$ 0–Total Depth
E. Weighted mean of *B*-values. Formula 8	*n*	645						151		151
	b							.01623		.01680
	1/*b*							61.63		59.53
F. Weighted mean of *C*-values. Formula 8	*n*	627						142		142
	b							.01549		.01608
	1/*b*							64.56		62.20
G. Equal weights. *A*-values. Formula 9	*n*	658								155
	b									.01654
	1/*b*									60.46
H. Equal weights. *B*-values. Formula 9	*n*	645							*	151
	b									.01647
	1/*b*									60.72
I. Weighted mean of *A*-values. Weights determined from Formula 12	*n*	658	139	102	60	22		155		
	b		.01560	.01594	.01514	.01392		.01583		
	1/*b*		64.11	62.72	66.04	71.84		63.19		
J. Weighted mean of *A*-values. Weights vary inversely as the squares of the probable errors	*n*	658	139	102	60	22		155		
	b		.01255	.01302	.01293	.01341		.01302		
	1/*b*		79.66	76.79	77.36	74.56		76.81		

* Theoretically rejected observations.

type, the reciprocal gradients increase with depth, whereas in curves of the convex type just the reverse condition exists. Thus in the latter case, the reciprocal gradients diminish from 116.3 to 67.8; in the former, they increase from 38.8 to 56.0 feet per degree Fahrenheit. In some fields, the predominating curve is a combination of curves of the concave and convex type.

TABULATIONS OF DATA

The gradients (b) in Table I are tabulated for different groups of depths, namely, 100 to 1,000 feet; 100 to 2,000 feet; and so on. This means that the constants in the straight line equation, $y=a+bx$, have been computed for each group of observations. In the first group, only those curves are included which reach a depth of 1,000 feet or more. Curves of depth less than 1,000 feet are included in the column designated "*100—Total Depth.*" The columns marked "*100–2,000* feet" include all curves that reach a depth of 2,000 feet or more, and so on for the 3,000- and 4,000-foot columns.

The gradients tabulated in the column, "*100—Total Depth*," are obtained by making a least-square adjustment of a straight line (*dd* or *ff*) through all of the points on the curve. In the following column, the values of $1/b$, the reciprocal gradient, are given in feet per degree Fahrenheit. In the last column, the gradients have been computed from the annual mean temperature of the air (g, Fig. 2) and the single observation at the greatest depth in the well. As the annual mean temperature of the soil just beneath the surface of the ground must exceed the annual mean temperature of the air just above the surface of the ground, the constant value, 1°F., was added to the annual mean temperature of the air as interpolated from the volume on *Climatological Data of the United States Weather Bureau.* This value of the excess of soil temperature over air temperature was adopted by the Committee on Underground Temperatures of the British Association for the Advancement of Science in its report for the year 1882. The excess varies somewhat from place to place. An average value of 1.54°F. has been deduced from 514 tests in the United States (3). The values of the gradient in the last column are subject therefore to revision when the values of the excess of soil temperature over air temperature and the annual mean temperature of the air at the station are known more accurately; but the remaining tabulations of the gradients, being obtained from least-square adjustments of the original observations, are not subject to revision.

ERRORS IN GRADIENTS

A word of caution should be spoken in regard to errors in the observations. For example, the curves in Figure 2 are excellent examples of precise measurement; nevertheless, the resulting gradients may be incorrect. The flattening of the Shumway curve as it enters the granite at a depth of about 2,700 feet is to be expected for the reason that granite is a better conductor of heat than sedimentary deposits. The extreme curvature, however, may be the result of swabbing oil from the well a few hours before the test was made. The extreme convexity of the Volcano well is due in part to surface topography. Unstable temperature equilibrium is a disturbing factor in a large number of the wells. As it is impossible to eliminate these errors and irregularities, it is necessary to bear in mind that our final determinations are to be regarded as rather rough approximations to the true values. Definitive determinations can not be obtained from existing data.

FORMULAS FOR AVERAGING GRADIENTS

It is impossible to correctly interpret the numerous averages of gradients that can be obtained without the assistance of mathematical analysis. In order, therefore, that we may properly interpret our results, let us consider the significance of the formulas by means of which the averages have been obtained.

Let us put,

y_1 = rise in temperature from a point just beneath surface of ground to depth x_1 in well A_1

y_2 = same for depth x_2 in well A_2

. . . .

y_n = same for depth x_n in well A_n

Also, let us put for the residuals (v),

$$v_1 = y_1 - \text{computed value of } y_1$$
$$v_2 = y_2 - \text{computed value of } y_2$$
$$\cdots\cdots\cdots$$
$$v_n = y_n - \text{computed value of } y_n.$$

Then we have by definition:

$$\begin{aligned} b &= y_1/x_1 = \text{gradient in well } A_1 \\ b &= y_2/x_2 = \text{gradient in well } A_2 \\ &\cdots\cdots\cdots \\ b &= y_n/x_n = \text{gradient in well } A_n \end{aligned} \qquad (1)$$

The arithmetic mean of the gradients is

$$b = \frac{y_1/x_1 + y_2/x_2 + \cdots y_n/x_n}{n} \tag{2}$$

and assuming that the weight of each value of b is proportional to the depth, we have from 1,

$$x_1 b = y_1 \qquad x_2 b = y_2 \cdots x_n b = y_n$$

which gives for the weighted mean,

$$b = \frac{y_1 + y_2 + \cdots y_n}{x_1 + x_2 + \cdots x_n}. \tag{3}$$

Equations 2 and 3 are the ones in general use. In order to determine the real significance of these equations, let us consider the adjustment of a straight line through the origin. Let it be assumed that we have n observation equations:

$$\begin{aligned} x_1 b &= y_1 \text{ weight } p_1 \\ x_2 b &= y_2 \text{ weight } p_2 \\ &\cdots\cdots\cdots \\ x_n b &= y_n \text{ weight } p_n \end{aligned} \tag{4}$$

the solution of which by the method of least squares gives

$$b = \frac{p_1 x_1 y_1 + p_2 x_2 y_2 + \cdots p_n x_n y_n}{p_1 x_1^2 + p_2 x_2^2 + \cdots p_n x_n^2}, \tag{5}$$

the residuals (v_1, v_2, . . . v_n) being subject to the condition

$$p_1 x_1 v_1 + p_2 x_2 v_2 + \cdots p_n x_n v_n = 0 \tag{6}$$

Following are important special cases of 5:

$$b = \frac{y_1/x_1 + y_2/x_2 + \cdots y_n/x_n}{n} \qquad p_1 = \frac{1}{x_1^2} \quad p_2 = \frac{1}{x_2^2} \cdots p_n = \frac{1}{x_n^2} \tag{7}$$

$$b = \frac{y_1 + y_2 + \cdots y_n}{x_1 + x_2 + \cdots x_n} \qquad p_1 = \frac{1}{x_1} \quad p_2 = \frac{1}{x_2} \cdots p_n = \frac{1}{x_n} \tag{8}$$

$$b = \frac{x_1 y_1 + x_2 y_2 + \cdots x_n y_n}{x_1^2 + x_2^2 + \cdots x_n^2} \qquad p_1 = p_2 = \cdots p_n \tag{9}$$

$$b = \frac{x_1^2 y_1 + x_2^2 y_2 + \cdots x_n^2 y_n}{x_1^3 + x_2^3 + \cdots x_n^3} \qquad p_1 = x_1 \quad p_2 = x_2 \cdots p_n = x_n \tag{10}$$

Equations 7 and 8 are the same as equations 2 and 3. Since the weights vary inversely as the squares of the probable errors, it follows that in equations 2 and 7, the errors of the observed temperatures are proportional to the depths, whereas in equations 3 and 8 the errors are proportional to the square roots of the depths. Equation 9 amounts to assuming that the errors in the temperature readings at the various depths are the same. In 10, the errors are assumed to vary inversely as the square root of the depth, that is, the errors of the observed temperatures are supposed to diminish with the depth. There is no justification for this assumption, but equations 7, 8, 9, in which the errors are supposed to be independent of the depth 9, or to increase linearly with the depth 7, or the square root of the depth 8, rest on a sufficiently sound basis to justify their use. Equation 6 is a partial check on the calculations.

When the observations in the different wells are made at the same depth (x_1), equations 7, 8, 9, 10 reduce to

$$b = \frac{y_1/x_1 + y_2/x_2 + \cdots y_n/x_n}{n},$$

the arithmetic mean of the gradients. Equation 6 shows that the sum of the residuals vanishes for these special cases. That is,

$$v_1 + v_2 + \cdots v_n = 0$$

Formulas 7, 8, 9 are applicable to the data in the last column of Table I. In the remaining tabulations of b in Table I, the values have been determined from a series of n' observed temperatures in the same well by making a least-square solution of the n' observation equations:

$$\begin{aligned} a + x_1 b &= y_1 \\ a + x_2 b &= y_2 \\ &\cdots\cdots \\ a + x_n' b &= y_n' \end{aligned} \qquad (11)$$

where y is now the observed temperature at depth x; a is the intercept of the straight line on the y-axis; and b is the computed gradient.

The least-square solution of equations 11 gives the normal equations:

$$\begin{aligned} n'a + (\Sigma x)b &= \Sigma y \\ (\Sigma x)a + (\Sigma x)^2 b &= \Sigma xy \end{aligned}$$

from which the values of a and b can be obtained by algebraic methods. The weights of a and b are respectively:

$$p_a = n' - \frac{(\Sigma x)^2}{\Sigma x^2} \qquad p_b = \Sigma x^2 - \frac{(\Sigma x)^2}{n'}. \tag{12}$$

In these equations,

$$\Sigma x = x_1 + x_2 + \cdots x_n'$$
$$\Sigma y = y_1 + y_2 + \cdots y_n'$$
$$\Sigma xy = x_1y_1 + x_2y_2 \cdots x_n'y_n'$$
$$\Sigma x^2 = x_1^2 + x_2^2 + \cdots x_n'^2$$

Equation 12 shows that the weight of b is dependent on n', the number of observed points in the well, and also on the distribution of these points in the well.

AVERAGES DEDUCED FROM TABLE I

The summary at the end of Table I shows that it is based on a total of 658 wells. The total number of fields and single locations represented is 155. Out of this total of 155 wells, 139 wells reached a depth of 1,000 feet; 102 wells reached a depth of 2,000 feet; and so on for the other depths. The average depth (m) of the 155 wells is 2,781 feet. In fields in which there are more than one well, that well has been selected in which the gradient is a minimum or the reciprocal gradient is a maximum. When there is precise correlation of temperature with structure, the well thus selected is, of course, the lowest on the structure.

Comparison of the values of $1/b$ in the last column of the summary with the tabulations in the third from the last column shows without exception that the smaller values are always found in the last column. The reciprocal of the mean of 6 average values of the gradient in the second from the last column is 61.60; in the last, 59.16, leaving a difference of 2.44 feet per degree Fahrenheit. This difference is readily explained. Referring to Figure 2, it will be seen that a straight line drawn from g, or, more accurately, from a point 1°F. above g, to the 4,400-foot point on curve BB', makes a greater angle with the depth-axis than the straight line ff which is adjusted by the method of least squares to all of the points on the curve BB'. The reverse condition exists for the concave curves AA', but the convex curves outnumber the concave curves about 2 to 1; consequently, the average for a number of curves reflects the more rapid rise of the convex curves.

However, the mean difference of only 2.44 feet per degree Fahrenheit is small and indicates that the total error introduced into the mean by the use of either method is not particularly serious.

The calculations carried out to obtain the results in the last column of the table are the same as those by means of which the Committee on Underground Temperature of the British Association for the Advancement of Science (Report 1882, page 88) obtained the value of 64 feet per degree Fahrenheit. The calculations (Formula 8) were based on 36 observations in Europe, chiefly in mines and artesian wells. As a result of correcting the gradients in Mont Cenis and St. Gothard tunnels, the Report of the Committee for the year 1883 (page 49) contains the value of 1°F. in 60 feet. The fact that two corrections in the British data change the rate of temperature increase by 4 feet per degree is an indication that averages taken in this way are not altogether reliable. The defect in this method of averaging is due to the fact that the observations are not sufficiently representative of the different geological conditions. With a few observations on each of a very great number of different geological units distributed over a large area, the values would tend to be consistent. This definition of a normal gradient corresponds to the mathematical definition in which the number of gradients in a large area is infinite, or, from a practical standpoint, very large.

Strict compliance with the mathematical requirements is impossible. The next best method of procedure is to obtain as much information as possible from a limited number of observations by means of the mathematical theory of probability and least-square adjustments of the data of observation.

The curvature of the depth-temperature curves greatly complicates the problem of evaluating a theoretically correct average or normal gradient. If the depth-temperature curves were straight lines, the weight of b would be given correctly by the well known relation that the weights are inversely proportional to the squares of the probable errors, the latter being obtained rigorously from the usual least-square adjustment of the straight line, $y=a+bx$, to the observed temperatures at the given series of depths. The curves in Figure 2 show that the residuals ($v=ll$) are very large in comparison with the errors of observation. This method, therefore, assigns minimum weights to curves of maximum curvature, disregarding, for the most part, the accuracy of the observations. Furthermore, the value of the gradient itself varies rapidly with the depth. When the range of depths is large, as in the last two columns of tabular values of b, the resulting average gradient is difficult of correct interpretation. For this reason, no further consideration will be given the tabulations in these two columns in attempting to deduce an average gradient from the data of observation.

In group *J*, where the weights are inversely proportional to the squares of the probable errors, the resulting values are not in agreement with those obtained by means of formulas 7, 8, 9. From what has just been said in regard to the effect of the curvature of the depth-temperature curves on the weights of the gradients, it follows that the results summarized in the *J*-groups are dependent chiefly on those depth-temperature curves which approach a straight line.

In the last column are shown the different results obtained by the use of formulas 7, 8, 9. The mean values are not very consistent and differ widely from those given in the *J*-group. The inconsistencies emphasize the importance of assigning theoretically correct weights to the individual values. Formula 9 is the correct formula to use when true rock temperatures are recorded at a single point in each well.

In group *I*, the weights have been computed from Formula 12. Since the observations have generally been made at practically the same points, 100, 250, 500, . . . feet, the values of the weights deviate only slightly from constancy, except for the column of total depth. It is to be expected, therefore, that the mean values in the first four columns of the *I*-group will approximate closely to those found in the *A*-group. Comparison of the results shows a sufficiently close agreement with theory. In comparison with the *J*-group, it amounts to assuming that r, the probable error of an observation of weight unity in the *J*-group, is equal to a constant.

In the *B*-group, those values of the gradient for which v, the deviation from the mean, equals or exceeds $5r$ have been rejected. This procedure is based on the statistical deduction that there is only 1 chance in about 1,000 that any individual residual in a statistical system should equal or exceed $5r$. The rule is a very useful guide. Like other statistical deductions, it is not absolute. The summary shows that one value in the first column and four in the last two columns were rejected. The rejected values are indicated by an asterisk in the main part of the table.

In group *C* all values of b for which the reciprocal gradient in the *B*-group exceeds 1°F. in about 40 feet have been arbitrarily rejected. The object of this procedure is to eliminate the abnormally high values of the gradients which are frequently found in the oil fields. The table shows that the reciprocal gradient for the first 1,000 feet is changed by this operation from 65.21 to 70.71 feet per degree Fahrenheit.

With the exception of the *C*-group and the *J*-group, the results for the range, 100–1,000 feet, are quite consistent. The means for the other three ranges of depth show irregularities that increase as the number of wells decreases. That is, these means show the effects of the areal distribution of the wells. The effect appears also in the range, 100–1,000 feet, but to a lesser degree on account of the greater number of wells.

AVERAGES DEDUCED FROM TABLES II AND III

Tables II and III have been constructed on the same general plan as Table I. The object of these tables is to take into account the variation of temperature with structure. Hence, in Table II, only those wells are considered that are highest on the structure. In Table III, the wells that are supposed to be lowest on the structure have been included. On account of irregularities in the observations, cases can be found in which the most rapid rate of temperature increase is not found on the crest of the structure; in fact, in a few instances, just the reverse of these conditions exists. In general, however, variation of temperature across a structure is much more probable than no variation, and in the observed exceptions to the rule, it is highly probable that a variation exists, so that the assumption of a variation regardless of the positions of the wells is a closer approximation to the facts than is the assumption that geothermal synclines stand in correlation with geological anticlines. Furthermore, exceptions in these cases are offset by numerous other cases, particularly in Wyoming and the salt-dome area of the Gulf Coast, where the lowest recorded gradient is high on the structure. For example, the highest reciprocal gradient recorded at Lance Creek, Wyoming, is 42.7 feet per degree Fahrenheit. As this well is near the top of the structure, it is not unreasonable to assume that the reciprocal gradients in the area surrounding Lance Creek are equal to or greater than 60 or 70 feet per degree. Hence, it is believed that the averages here obtained are approximately correct representations of the actual conditions to be expected on anticlines and domes.

The tables show that the mean depth of 63 wells on the crest of the structure is 2,316 feet; the same for 63 wells on the flank, is 2,489 feet. This shows that a majority of the wells in which there is a rapid rate of temperature increase are located on or near the crests of the structures.

Comparison of the last two columns of tabulations shows that Table III agrees with Table I in that minimum values of $1/b$ are found in the last column. The predominating curve represented in

TABLE II
GRADIENTS ON CREST OF STRUCTURE

Town	State	No. of Wells	$b\times10^5$. Degrees F. per Foot 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet	Total Depth Feet	$b\times10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b\times10^5$ 0–Total Depth
El Dorado	Arkansas	7	2,055	1,936	—	—	2,120	1,891	52.9	1,755
Bakersfield	California	5	1,532	1,624	—	—	2,850	1,666	60.0	1,884
Coalinga	California	51	2,289	1,792	—	—	1,954	1,792	55.8	2,201
Huntington Beach	California	4	2,503	—	—	—	1,000	2,503	40.1	3,060
Kettleman Hills	California	4	2,125	—	—	—	1,190	2,052	48.7	2,487
Long Beach	California	40	2,183	2,024	1,979	—	3,000	1,979	50.5	2,280
Santa Fe Springs	California	31	2,455	2,098	1,993	—	3,000	1,993	50.2	2,213
Whittier	California	4	1,774	1,824	1,835	—	3,500	1,842	54.3	2,117
Florence	Colorado	3	2,220	2,127	—	—	2,400	2,074	48.2	2,225
Fort Collins	Colorado	2	1,844	1,795	1,861	—	3,000	1,861	53.7	2,023
Bridgeport	Illinois	4	1,827	—	—	—	1,800	1,725	58.0	1,672
Augusta	Kansas	2	2,028	1,944	—	—	2,250	1,908	52.4	1,924
El Dorado	Kansas	24	2,167	2,046	—	—	2,250	2,017	49.6	1,920
Florence	Kansas	4	1,731	1,673	—	—	2,150	1,647	60.7	1,619
Haverhill	Kansas	3	1,967	1,906	—	—	2,250	1,936	51.6	1,973
Blue Lake	Louisiana	2	1,800	2,250	—	—	2,000	2,250	44.4	2,390
Caddo Lake (Pine Island)	Louisiana	5	2,845	2,451	2,269	—	3,500	2,218	45.1	2,237
Homer	Louisiana	8	3,023	—	—	—	1,300	3,006	33.3	2,692
Many	Louisiana	6	2,019	2,195	—	—	2,470	2,402	41.6	2,397
Zwolle	Louisiana	3	1,980	2,249	2,186	—	3,380	2,191	45.7	2,290
Kevin-Sunburst	Montana	10	1,399	—	—	—	1,250	1,442	69.4	1,584
Artesia	New Mexico	3	772	—	—	—	1,500	611	163.6	793
Ardmore	Oklahoma	4	831	881	978	—	3,000	978	102.3	1,010
Ardmore-Healdton	Oklahoma	9	1,265	—	—	—	1,000	1,265	79.0	1,240
Ardmore-Hewett	Oklahoma	8	1,152	—	—	—	1,000	1,152	86.8	820

TABLE II (*Continued*)

Town	State	No. of Wells	$b \times 10^5$. Degrees F. per Foot: 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet	Total Depth Feet	$b \times 10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b \times 10^5$ 0–Total Depth
Blackwell	Oklahoma	2	2,162	2,111	—	—	1,835	2,111	47.4	1,896
Braman	Oklahoma	3	2,462	—	—	—	1,375	2,313	43.2	1,891
Burbank	Oklahoma	3	2,363	2,114	—	—	2,750	2,144	46.6	2,236
Cromwell	Oklahoma	9	2,037	2,123	2,213	—	3,000	2,213	45.2	2,113
Dilworth	Oklahoma	11	2,045	—	—	—	1,766	2,082	48.0	2,027
Drumright	Oklahoma	28	1,826	—	—	—	1,315	1,940	51.5	1,947
Glenn Pool	Oklahoma	18	2,687	—	—	—	1,260	2,607	38.4	2,278
Okemah	Oklahoma	4	2,572	2,595	—	—	2,500	2,549	39.2	2,424
Oklahoma City	Oklahoma	3	784	829	885	933	4,000	933	107.1	953
Papoose	Oklahoma	4	2,136	2,240	—	—	2,750	2,367	42.3	2,222
Sasakwa	Oklahoma	2	1,646	1,769	1,842	—	3,500	1,878	53.3	1,869
Seminole	Oklahoma	19	1,530	1,734	—	—	2,500	1,789	55.9	1,672
Tonkawa	Oklahoma	15	1,944	1,866	—	—	2,820	1,910	52.4	1,947
Walters	Oklahoma	10	916	898	—	—	2,000	898	111.4	890
Wewoka	Oklahoma	5	2,255	2,167	2,066	—	3,000	2,066	48.4	2,067
Long Bridge	Pennsylvania	3	1,466	1,481	1,500	1,498	6,000	1,645	60.8	1,612
Big Lake	Texas	16	1,000	838	—	—	3,175	699	143.1	699
Blue Ridge	Texas	7	—	—	—	—	700	2,677	37.4	2,543
Boggy Creek	Texas	3	2,935	2,623	2,495	—	3,590	2,333	42.9	2,423
Graham	Texas	2	1,804	1,643	1,632	1,691	4,334	1,732	57.7	1,797
Grand Saline	Texas	4	—	—	—	—	200	4,000	25.0	3,850
Humble	Texas	28	4,613*	—	—	—	1,080	4,517	22.1	4,028
Long Point	Texas	8	—	—	—	—	857	3,715	26.9	3,617
Luling	Texas	4	2,305	2,238	—	—	2,000	2,238	44.7	2,265
Lytton Springs	Texas	7	2,636	—	—	—	1,175	2,559	39.1	2,536

TABLE II (*Continued*)

Town	State	No. of Wells	$b \times 10^5$. Degrees F. per Foot: 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet	Total Depth Feet	$b \times 10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b \times 10^5$ 0–Total Depth
Mexia	Texas	6	2,568	2,419	2,393	—	2,985	2,393	41.8	2,335
Pierce Junction	Texas	3	1,692	1,817	1,762	—	3,300	1,753	57.0	1,812
Powell	Texas	14	2,584	2,301	—	—	2,750	2,237	44.7	2,269
Ranger	Texas	5	2,347	2,216	2,230	—	3,000	2,230	44.8	2,273
Wortham	Texas	2	2,145	2,127	—	—	2,800	2,069	48.3	2,361
Big Muddy	Wyoming	4	1,886	1,792	1,880	—	3,250	1,884	53.1	1,954
Grass Creek	Wyoming	3	1,998	—	—	—	1,090	1,953	51.2	2,037
Lance Creek	Wyoming	3	2,469	2,718	2,728	—	3,500	2,720	36.8	2,843
Lost Soldier	Wyoming	3	5,300*	4,895	—	—	1,925	4,895	20.4	5,210
Rawlins	Wyoming	4	2,324	—	—	—	1,500	2,034	49.2	2,100
Rock River	Wyoming	3	1,802	—	—	—	2,700	1,918	52.1	1,926
Salt Creek	Wyoming	21	3,440	—	—	—	1,500	3,483	28.7	3,493
Thermopolis	Wyoming	9	4,784*	—	—	—	1,000	4,784	20.9	5,310
A. Arithmetic mean. All values included. Formula 7	n	544	60	41	19		3	63	63	63
	b		.02154	.02009	.01933		.01374		.02169	.02199
	r		.00567	.00436	.00313		.00266		.00561	.00577
	r_0		.00073	.00068	.00072		.00153		.00071	.00073
	$1/b$		46.42	49.78	51.73		72.78		46.10	45.48
	m							2,316		
B. Arithmetic mean. Values for which $v \gtreqless 5r$ have been rejected from *A*. Formula 7	n	504	57	40	19	3		63		61
	b		.02010	.01937	.01933	.01374		.02169		.02099
	r		.00379	.00309	.00313	.00266		.00561		.00444
	r_0		.00050	.00049	.00072	.00153		.00071		.00059
	$1/b$		49.76	51.63	51.73	72.78		46.10		47.65

TABLE II (*Continued*)

Town	State	No. of Wells	$b \times 10^5$. Degrees F. per Foot: 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100 4,000 Feet	Total Depth Feet	$b \times 10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b \times 10^5$ 0–Total Depth
C. Arithmetic mean. Fields in	n	490	50	33	16	2		56		54
which there are only 2 wells	b		.02022	.01934	.01962	.01216		.02194		.02107
have been omitted from *B*.	r		.00402	.00335	.00337	.00270		.00596		.00459
Formula 7	r_0		.00057	.00058	.00084	.00191		.00080		.00062
	$1/b$		49.44	51.70	50.97	82.27		45.58		47.47
D. Weighted mean of *A*-values.	n	544						63		63
Formula 8	b							.02035		.02077
	$1/b$							49.15		48.15
E. Weighted mean of *B*-values.	n	504						63		61
Formula 8	b							.02035		.02012
	$1/b$							49.15		49.70
F. Weighted mean of *C*-values.	n	490						56		54
Formula 8	b							.02051		.02013
	$1/b$							48.77		49.68
G. Equal weights. *A*-values.	n	544								63
Formula 9	b									.02001
	$1/b$									49.99
H. Equal weights. *B*-values.	n	504								61
Formula 9	b									.01963
	$1/b$									50.95
I. Weighted mean of *A*-values.	n	544	60	41	19	3		63		
Weights determined from	b		.02179	.02039	.01946	.01366		.01941		
Formula 12	$1/b$		45.90	49.03	51.40	73.18		51.52		
J. Weighted mean of *A*-values.	n	544	60	41	19	3		63		
Weights vary inversely as	b		.02098	.01740	.01793	.01426		.02295		
the square of the probable errors	$1/b$		47.67	57.47	55.77	70.10		43.57		

* Theoretically rejected observations.

TABLE III
GRADIENTS ON FLANK OF STRUCTURE

Town	State	No. of Wells	$b \times 10^5$. Degrees F. per Foot 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet	Total Depth Feet	$b \times 10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b \times 10^5$ 0–Total Depth
El Dorado	Arkansas	7	1,862	2,329	—	—	2,000	2,329	42.9	2,240
Bakersfield	California	5	1,198	1,236	1,338	1,464	4,250	1,569	63.7	1,911
Coalinga	California	51	1,413	—	—	—	1,370	1,351	74.0	1,956
Huntington Beach	California	4	2,225	2,456	2,624	—	3,500	2,643	37.8	2,786
Kettleman Hills	California	4	1,612	—	—	—	1,350	1,671	59.9	2,244
Long Beach	California	40	1,607	1,681	—	—	2,500	1,642	60.9	2,204
Santa Fe Springs	California	31	1,535	1,819	1,895	—	4,250	1,904	52.5	1,995
Whittier	California	4	1,670	1,809	1,885	—	3,750	1,874	53.4	2,136
Florence	Colorado	3	1,532	1,848	—	—	2,900	2,020	49.5	2,090
Fort Collins	Colorado	2	1,667	1,755	1,858	1,956	4,000	1,956	51.1	2,050
Bridgeport	Illinois	4	1,456	—	—	—	1,250	1,526	65.5	1,704
Augusta	Kansas	2	1,945	2,009	—	—	2,250	1,984	50.4	2,013
El Dorado	Kansas	24	1,777	1,827	—	—	2,250	1,799	55.6	1,689
Florence	Kansas	4	1,361	—	—	—	1,750	1,590	62.9	1,686
Haverhill	Kansas	3	1,720	1,819	—	—	2,750	1,783	56.1	1,811
Blue Lake	Louisiana	2	1,786	—	—	—	1,500	1,993	50.2	2,180
Caddo Lake	Louisiana	5	2,653	2,423	—	—	2,000	2,423	41.3	2,325
Homer	Louisiana	8	2,518	—	—	—	1,150	2,646	37.8	2,357
Many	Louisiana	6	1,883	—	—	—	1,250	1,926	51.9	1,832
Zwolle	Louisiana	3	1,751	—	—	—	1,250	1,771	56.5	1,792
Kevin-Sunburst	Montana	10	930	978	987	—	3,000	987	101.4	1,123
Artesia	New Mexico	3	560	478	—	—	2,892	439	227.7	598
Ardmore	Oklahoma	4	657	—	—	—	1,500	678	147.5	667
Ardmore-Healdton	Oklahoma	9	616	—	—	—	1,500	657	152.2	593
Ardmore-Hewett	Oklahoma	8	808	—	—	—	1,750	950	105.3	800

TABLE III (Continued)

Town	State	No. of Wells	$b \times 10^5$. Degrees F. per Foot: 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet	Total Depth Feet	$b \times 10^5$ 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b \times 10^5$ 0–Total Depth
Blackwell	Oklahoma	2	2,038	2,027	—	—	2,000	2,027	49.3	1,900
Braman	Oklahoma	3	1,758	—	—	—	1,410	1,829	54.7	1,787
Burbank	Oklahoma	3	1,893	2,093	—	—	2,850	2,188	45.7	2,172
Cromwell	Oklahoma	9	1,886	—	—	—	1,200	1,918	52.1	1,775
Dilworth	Oklahoma	11	1,743	1,848	1,874	—	3,000	1,874	53.4	1,953
Drumright	Oklahoma	28	880	1,381	—	—	2,210	1,467	68.2	1,416
Glenn Pool	Oklahoma	18	2,284	—	—	—	1,500	2,383	42.0	2,187
Okemah	Oklahoma	4	2,341	2,240	—	—	2,500	2,241	44.6	2,444
Oklahoma City	Oklahoma	3	659	765	841	904	4,000	904	110.7	1,023
Papoose	Oklahoma	4	1,903	1,945	—	—	2,500	2,014	49.7	2,168
Sasakwa	Oklahoma	2	1,326	1,480	—	—	2,620	1,518	65.9	1,443
Seminole	Oklahoma	19	1,019	1,314	—	—	2,750	1,340	74.6	1,284
Tonkawa	Oklahoma	15	1,569	1,831	—	—	2,750	1,840	54.3	1,876
Walters	Oklahoma	10	750	—	—	—	2,150	864	115.7	902
Wewoka	Oklahoma	5	1,951	2,063	2,107	—	3,000	2,107	47.5	2,087
Long Bridge	Pennsylvania	3	1,370	1,473	1,480	1,482	6,500	1,674	59.7	1,634
Big Lake	Texas	16	672	—	—	—	1,500	636	157.3	873
Blue Ridge	Texas	7	—	1,706	—	—	2,900	1,800	55.6	1,828
Boggy Creek	Texas	3	—	2,499	2,820	—	3,703	2,867	34.9	2,768
Graham	Texas	2	1,502	1,630	1,638	—	3,000	1,638	61.0	1,830
Grand Saline	Texas	4	—	—	—	—	875	1,606	62.2	1,977
Humble	Texas	28	1,458	1,677	—	—	2,000	1,677	59.6	1,560
Long Point	Texas	8	1,789	1,831	1,777	—	3,300	1,765	56.7	1,794
Luling	Texas	4	1,489	1,832	—	—	2,500	1,886	53.0	2,184
Lytton Springs	Texas	7	2,464	—	—	—	1,350	2,297	43.5	2,356

TABLE III (*Continued*)

Town	State		No. of Wells	$b\times 10^5$. Degrees F. per Foot: 100–1,000 Feet	100–2,000 Feet	100–3,000 Feet	100–4,000 Feet	Total Depth Feet	$b\times 10^5$: 100–Total Depth	$1/b$ 100–Total Depth Ft. °F.	$b\times 10^5$: 0–Total Depth
Mexia	Texas		6	1,820	2,072	—	—	2,900	2,099	47.6	2,369
Pierce Junction	Texas		3	—	1,663	—	—	4,303	1,486	67.3	1,769
Powell	Texas		14	1,147	1,347	1,460	—	2,950	1,460	68.5	2,254
Ranger	Texas		5	2,027	2,015	2,067	—	3,000	2,067	48.4	2,150
Wortham	Texas		2	1,993	1,974	—	—	2,900	1,954	51.2	2,303
Big Muddy	Wyoming		4	1,416	1,475	1,707	—	3,000	1,707	58.6	1,987
Grass Creek	Wyoming		3	—	—	—	—	800	1,934	51.7	1,988
Lance Creek	Wyoming		3	2,341	2,643	2,718	—	3,000	2,718	36.8	2,797
Lost Soldier	Wyoming		3	3,176	2,916	2,638	—	3,400	2,569	38.9	2,635
Rawlins	Wyoming		4	1,378	1,708	—	—	2,000	1,708	58.6	1,760
Rock River	Wyoming		3	1,343	—	1,431	—	3,000	1,431	69.9	1,540
Salt Creek	Wyoming		21	1,606	1,952	—	—	2,650	2,079	48.1	2,102
Thermopolis	Wyoming		9	—	—	—	—	900	4,206	23.8	4,711*
A. Arithmetic mean. All values included. Formula 7		n	544	57	42	19	4	63	63		63
		b		.01609	.01806	.01850	.01452		.01808		.01911
		r		.00368	.00317	.00376	.00290		.00403		.00421
		r_0		.00049	.00049	.00086	.00086		.00051		.00053
		$1/b$		62.14	55.36	54.06	68.89		55.32		52.34
		m						2489			
B. Arithmetic mean. Values for which $v \gtreqless 5r$ have been rejected from *A*. Formula 7		n	535	57	42	19	4		62		62
		b		.01609	.01806	.01850	.01452		.01769		.01865
		r		.00368	.00317	.00376	.00290		.00349		.00347
		r_0		.00049	.00049	.00086	.00086		.00044		.00044
		$1/b$		62.14	55.36	54.06	68.89		56.53		53.61
C. Arithmetic mean. (Fields in which there are only 2 wells have been omitted from *B*.) Formula 7		n	521	50	36	17	3		55		55
		b		.01590	.01805	.01862	.01283		.01757		.01853
		r		.00387	.00338	.00397	.00222		.00367		.00363
		r_0		.00055	.00056	.00096	.00128		.00050		.00049
		$1/b$		62.91	55.39	53.71	77.92		56.93		53.95

TABLE III (*Continued*)

Town	State	No. of Wells	$b \times 10^5$. Degrees F. per Foot: 100–1,000 Feet	$b \times 10^5$. Degrees F. per Foot: 100–2,000 Feet	$b \times 10^5$. Degrees F. per Foot: 100–3,000 Feet	$b \times 10^5$. Degrees F. per Foot: 100–4,000 Feet	Total Depth Feet	$b \times 10^5$: 100–Total Depth	$1/b$: 100–Total Depth Ft. °F.	$b \times 10^5$: 0–Total Depth
D. Weighted mean of *A*-values.	n	544						63		63
Formula 8	b							.01799		.01907
	$1/b$							55.59		52.44
E. Weighted mean of *B*-values.	n	535						62		62
Formula 8	b							.01785		.01891
	$1/b$							56.02		52.89
F. Weighted mean of *C*-values.	n	521						55		55
Formula 8	b							.01776		.01882
	$1/b$							56.31		53.14
G. Equal weights. *A*-values.	n	544								63
Formula 9	b									.01907
	$1/b$									52.44
H. Equal weights. *B*-values.	n	535								62
Formula 9	b									.01902
	$1/b$									52.58
I. Weighted mean of *A*-values.	n	544	57	42	19	4		63		
Weights determined from	b		.01618	.01824	.01856	.01463		.01790		
Formula 12	$1/b$		61.79	54.82	53.88	68.35		55.87		
J. Weighted mean of *A*-values.	n	544	57	42	19	4		63		
Weights vary inversely as	b		.01322	.01559	.01627	.01454		.01436		
the square of the probable errors	$1/b$		75.63	64.13	61.46	68.76		69.64		

* Theoretically rejected observations.

these columns is therefore of the convex type. In Table II, the reciprocal gradients tabulated in the last column of averages are generally larger than those of the next adjacent column. This implies a predominance of straight lines and concave curves, as might be expected from the shallower depths and the more rapid approach of the wells to granite on the crests of the structures.

The results assembled in Table II are quite consistent; even the *J*-group which is abnormal in the other two tables is here in fair agreement with the other means. The agreement is not a coincidence. It is due to the fact, as previously stated, that the weights in the *J*-group are more and more in agreement with theory as the depth-temperature curves approach straight lines.

The arbitrary rejection of a total of 7 fields in each of which there are only 2 wells, group *C*, does not seriously affect the results obtained on either the crests or the flanks of the structures.

REJECTED OBSERVATIONS

Theoretically rejected observations in all of the tables have been indicated by an asterisk. Rejections occur in Oregon, Texas, and Wyoming. The Oregon data are based on temperature measurements that are sufficiently precise, but on account of the irregularly shaped depth-temperature curves, it is difficult to determine a representative gradient from them. The one gradient in the entire list that is most likely to be incorrect is the one at Albany, Alabama. Here the measurements were made in an uncased well which was 12.5 inches in diameter at the top and 8 inches in diameter at 2,000 feet. The rise in the depth-temperature curve to a depth of 750 feet is almost imperceptible. The results suggest a circulation of water either by convection or by the movement of water from one bed to another. Theoretical calculations do not indicate that this well should be rejected. The other wells that appear in the list of rejections represent extreme geological conditions. They can not be rejected on the basis of errors of observation.

INTERPRETATION OF TABLES I, II, AND III

Excluding from Table I the Albany well and the wells that are known to be in proved or prospective oil fields, we have the following series of gradients from the first three columns of tabulations:

	100–1,000 Feet	100–2,000 Feet	100–3,000 Feet
Birmingham	0.00914	0.00955	—
Grass Valley	0.00593	0.00569	0.00537
Ames	0.00695	0.00731	—
Houghton	0.00771	—	—
Franklin Furnace	0.00362	—	—
Lonetree	0.01578	0.01654	0.01806
Astoria	0.01503	0.01615	0.01656
Moclips	0.00985	0.01239	0.01342
Seattle	0.00929	0.00971	—
b	0.00926	0.01105	0.01335
$1/b$	108.04	90.51	74.89

Comparison of these values with the corresponding values in Tables I, II, and III, shows that the mean gradients recorded in the tables are determined almost entirely by the much larger values of the gradients found in most of the oil-bearing areas.

TABLE IV

RATIOS OF GRADIENTS

	2,000/1,000	3,000/1,000	4,000/1,000	Remarks
n	97	55	20	
m	1.0523	1.0928	1.1928	Table I
r	0.0927	0.1197	0.1829	
r_0	0.0094	0.0161	0.0409	
b	0.01614	0.01676	0.01830	
$1/b$	61.95	59.65	54.65	
n	95			Table II
m	1.0417			
r	0.0793			
r_0	0.0081			
b	0.01598			
$1/b$	62.58			
n	41	19	3	Table III
m	0.9838	0.9944	1.0497	
r	0.0609	0.0753	0.0869	
r_0	0.0095	0.0173	0.0502	
b	0.01977	0.01999	0.02110	
$1/b$	50.57	50.03	47.40	
n	39	18	4	Table III
m	1.0974	1.1104	1.2122	
r	0.0842	0.0721	0.0818	
r_0	0.0135	0.0170	0.0409	
b	0.01766	0.01787	0.01950	
$1/b$	56.63	55.97	51.27	
n	38			Table III
m	1.0850			
r	0.0669			
r_0	0.0108			
b	0.01746			
$1/b$	57.28			

In the preceding summary, the values of $1/b$ diminish uniformly with the depth. This is not the case in Tables I, II, and III. The discrepancies are the results of areal distribution of the wells. Table IV shows the results of attempting to eliminate these inconsistencies. In this table, the mean (m) of the ratios of the gradients for the different depths to the gradient computed for the first 1,000 feet has been tabulated.

The first part of the table shows that the mean of 97 ratios of the gradients from 100 to 2,000 feet to the gradients from 100 to 1,000 feet is 1.0523, the probable errors being represented as heretofore by r and r_0, the latter being the probable error of the mean. Multiplying the mean ratio, 1.0523, by 0.01534, the value of b in the B group, Table I, we have for the mean gradient from 100 to 2,000 feet, the value, 0.01614, corresponding with the reciprocal gradient 1°F. in 61.95 feet. The numbers in the remaining columns are computed in a similar manner. Two rejections occur in the first column of tabular results. The final sequence in the values of the reciprocal gradients, namely, 65.21, 62.58, 59.65, 54.65, shows that the normal curve is convex to the depth axis. The summary for Table III—62.14, 57.28, 55.97, 51.27—shows a similar sequence, but the numerical values of the reciprocal gradients are greatly diminished, again showing the influence of the higher temperatures in the oil fields. The sequence for Table II—49.76, 50.57, 50.03, 47.40—shows that an average of the depth-temperature curves on the crests of anticlines tends to approximate rather closely to a straight line.

Summarizing the results of Table IV with reference to Tables II and III, we can say that the convexity of the depth-temperature curves increases as we pass from the crest to the flank of the structure. This experimental result is partly accidental, but it follows, also, from the geometry of the isogeotherms. Thus, for convenience, let it be assumed that $a''a'a$ and $c''c'c$ (Fig. 1) are isogeotherms. Then the gradients are determined by the constant difference in temperature between the two isogeotherms divided by the lengths ac, $a'c'$, $a''c''$. Since $a'c'$ is greater than ac, it follows that the gradient at $a'c'$ is less than at ac; but, since the temperatures must be the same at the depth at which the isogeotherms become parallel to a horizontal surface, it follows that the temperatures must increase much more rapidly along $a'c'$ extended than along ac extended. Hence the convexity of the depth-temperature curve $a'c'$ is greater than that of ac.

Comparison of the B groups in Tables II and III shows that the reciprocals of the mean gradients vary from 49.76 on the crest to 62.14 on the flank of the structure. As the tests were made mostly in producing wells, it is reasonable to infer that the point for which a rate of 62.14 feet per degree Fahrenheit was obtained is not far from edge water, and, in view of the relative effects of oil and salt water on the transmission of heat, it seems reasonable to expect that an equal increment will accrue as we pass from edge water to $a''c''$ at the base of the structure. Assuming such an increase, we have from Table IV for the sequence of reciprocal gradients at a'' c'', the values 82.8, 66.0, 63.5, 55.9, for the successive series of depths. These values are merely plausible estimates of the reciprocals of the mean gradients between domes and anticlines. The geometry of the isogeotherms (Fig. 1) leads one to suspect that the average depth-temperature curve in these areas approximates more closely to a straight line than is indicated by the preceding series of numbers.

Conclusive evidence on the diminution of temperature in the area immediately surrounding an oil dome was found by E.M. Hawtof (4) at Big Lake, Texas. He reports a rate of 111.2, on top of the dome; 137.9, on the north edge; 133.6, on the south edge; 133 on the west edge; and 148.2 feet per degree Fahrenheit at a point about 12 miles northwest of the dome. An important paper by Strong (5), just received, contains graphs showing a close correlation of the iso-geothermal surfaces with the anticlines and synclines in some of the oil fields in Persia. Tests between domes and anticlines should be of great value in establishing the validity of the hypothesis of a variation of temperature with structure and it would enable us also to make a much more accurate estimate of an average gradient in sedimentary areas.

OTHER OBSERVATIONS

In Table V are tabulated the records from 25 non-flowing wells taken chiefly from the bulletin by N. H. Darton (1). The accuracy of many of these observations is probably questionable and the range of depths over which the temperatures have been taken are so irregular that it is difficult to properly interpret the averages with reference to the summaries contained in this paper. Excluding Lake Tahoe, for which the gradient is negative, an average value for 24 stations is 63.7 feet per degree Fahrenheit.

The reciprocal of the mean gradient determined from the records of 3,011 overflowing wells, contained chiefly in Darton's (1) bulletin, is 1°F. in 48.92 feet. Rejecting 164 observations for which the residual equals or exceeds $5r$, the reciprocal of the mean gradient is 1°F. in 47.82 feet. No further rejections are indicated by the calculations. Rejection of 164 observations changed the value of the mean depth

TABLE V

TEMPERATURE GRADIENTS. OLD OBSERVATIONS

Town	*State*	*Range*	$b\times 10^5$	*Range Feet*	$b\times 10^5$	$1/b$	$b\times 10^5$
Phoenix	Arizona	—	—	100– 835	836	119.7	1,473
Prescott	Arizona	—	—	205–1,285	1,124	89.0	2,202
St. Augustine	Florida	170–1,110	606	170–1,340	919	108.8	1,142
Des Moines	Iowa	250–1,000	1,400	250–2,250	749	133.5	867
Belle Island	Louisiana	—	—	427– 975	1,269	78.8	1,179
Bay City	Michigan	—	—	1,304–3,455	1,535	65.2	1,427
Cheboygan	Michigan	100–2,000	810	100–2,700	861	116.1	1,085
Grayling	Michigan	500–2,150	2,301	500–2,600	2,222	45.0	2,023
Alma	Michigan	—	—	157–2,863	1,766	56.6	1,788
Keweenaw Point	Michigan	60–1,080	1,009	60–5,367	852	117.4	919
Freeda	Michigan	—	—	100– 950	952	105.0	1,505
Muskegon	Michigan	—	—	240–1,150	1,544	64.8	1,704
Port Huron	Michigan	—	—	300– 838	603	165.9	1,253
Lake Tahoe	Nevada	—	—	50–1,506	1,779*	56.1	2,656*
Tonopah	Nevada	—	—	100– 780	2,019	49.5	2,833
Tonopah	Nevada	—	—	317– 600	2,299	43.5	3,183
Tonopah	Nevada	—	—	100– 766	2,586	38.7	3,473
Virginia City	Nevada	—	—	1,482–2,128	2,542	39.3	3,289
Virginia City	Nevada	—	—	845–2,017	3,378	29.6	3,342
Isleta	New Mexico	—	—	443– 843	1,740	57.5	2,693
Findlay	Ohio	—	—	1,165–2,980	801	124.9	1,050
Plymouth	Ohio	—	—	1,400–3,000	1,363	73.4	1,390
Homewood	Pennsylvania	—	—	3,600–4,295	3,220	31.1	1,402
West Elizabeth	Pennsylvania	525–2,397	945	525–5,380	1,486	67.3	1,362
Mannington	West Virginia	100–1,000	956	100–2,500	996	100.4	1,044
A. Arithmetic mean. Formula 7		n	7		24		24
		b	.01147		.01569		.01818
		r	.00379		.00534		.00574
		r_0	.00143		.00109		.00117
		$1/b$	87.2		63.7		55.0
B. Weighted mean. Formula 8		m		2,162			
		b			.01579		.01587
		$1/b$			63.3		63.0
C. Arithmetic mean. Formula 7. $1/b \gtrless 40.49$ rejected		n			20		20
		b			.01297		.01606
		$1/b$			77.1		62.3

* Theoretically rejected observations.

of the wells from 445 to 400 feet. As it is hardly to be expected that reliable information can be obtained from wells of such shallow depths, no further consideration will be given these observations in this paper.

From DeGolyer's (6) observations in Tuxpam, Mexico, there results the following reciprocal gradients by using all of the observations, inclusive, between the indicated depths.

	Feet	*Feet per °F.*
Llano Grande	100–2,683	32.7
Tanhuijo	1,060–3,558	32.0
Tlacolula	75–4,026	42.6
Tlacolula	75–4,080	40.3

Roy O. Armstrong has kindly communicated to the writer the results of two tests in the oil fields of Canada. Averaging the observations by the usual least-square method, from a depth of 250 to 1,500 feet, inclusive, in a well located in Sec. 17, T. 19, R. 2W, 5th, Alberta, the resulting reciprocal gradient is 71.1 feet per degree Fahrenheit. From a well in the southern part of the field in Turner Valley, Alberta, the value 44.8 feet was obtained from a range of depths of 250 to 1,267 feet.

In marked contrast with these high temperatures in the oil fields of Mexico and Canada, we have the very interesting record of low temperatures obtained by Ralph H. Cleland (7) in the mining districts of Northern Ontario.

			Feet	*Feet per °F.*
Dome	Mine	Porcupine Dist.	248–3,065	164.0
Hollinger	"	"	235–3,892	223.4
McIntyre	"	"	181–3,865	233.2
Kirkland Lake	"	Kirkland Lake	2,490–4,905	130.7
Teck-Hughes	"	"	1,110–4,225	137.9
Lake Shore	"	"	391–3,575	167.1
Wright-Hargreaves	"	"	587–3,003	118.4
Sylvanite	"	"	495–2,993	142.2
Frood	"	Sudbury	400–3,100	155.2

The preceding averages were obtained by making least-square adjustments of all the observations recorded in each mine.

GENERAL CONSIDERATIONS

If our records represented true rock temperatures, it would be possible to draw tangents to the depth-temperature curves at desired depths. With smooth curves like those shown in Figure 2, it would be easy to make these determinations, and the results thus obtained would necessarily possess a physical significance not possessed by the successive straight lines used in this paper. However, imperfections in the present records preclude the possibility of using this method of evaluating the constants for very many of the curves.

Another method consists in computing the derivatives from a truncated power series that has been properly adjusted. In some of the fields, Salt Creek, for example, the depth-temperature curves possess a double curvature somewhat resembling a letter S. In these fields, the computation of the gradients from a truncated power series would lead to a misrepresentation of the facts.

Scientists use the term "normal gradient" without realizing, perhaps, that the term can be defined in a great number of different ways, and that some of the mathematical definitions can not be realized experimentally. The definitions in common use, equations 7 and 8, imply an average of a great number of gradients distributed uniformly or at random throughout a large area. This definition is theoretically correct but impossible of practical realization because of the very great number of observations required to determine an average that approaches a constant value. This is the method used by the Committee on Underground Temperatures of the British Association for the Advancement of Science in obtaining the commonly accepted value of 1°F. in 60 feet.

In the absence of complete evidence in the areas surrounding domes and anticlines, it is impossible to do other than to infer possibilities. About the only approach to a definite conclusion that can be made at present in regard to these areas is the possibility that the values of $1/b$ in Table II should be multiplied by factors which vary uniformly from a little more than unity for $1/b=150$ feet per degree Fahrenheit to 3 or 4 times the tabular values as the lower limit of about 20 feet per degree Fahrenheit is reached. A closely related statement is the following: in oil-bearing areas, free from intrusives, the probability that a location is on a dome or the crest of an anticline increases from about 0.5 to practically a certainty (1.0) as the values of the reciprocal gradients decrease from about 50 to approximately 20 feet per degree Fahrenheit.

Koenigsberger (8) has grouped geothermal data for different geologic and topographic features into 10 groups and concludes that the averages for the different groups differ from one another—that for the bituminous group, particularly petroleum, being higher than the others, except for areas of recent intrusives.

Classification of the gradients in accordance with the sequence of geological epochs may provide a means of establishing a normal gradient for each epoch. The last gradient in the series would represent a normal gradient for an undisturbed earth.

An entirely different approach to the solution of the problem was proposed by the late G. K. Gilbert of the United States Geological Survey. In the first and third Year Books of the Carnegie Institution of Washington (1902 and 1904) he proposed the drilling of a deep well in plutonic rocks. The importance of this suggestion has been again emphasized by the recent observations of R. H. Cleland (7) in northern Ontario, Canada, which show that the reciprocal of the mean gradient in undisturbed areas of crystalline rocks may equal or exceed 200 feet per °F. (109.7 meters per °C.).

Jeffreys (9) estimates that subsidence to a depth of 10 kilometers during a geologic period of 130 million years causes a rise in temperature of about 250° C. at the plane of contact of the sediments and the crystalline rocks. As the heat of compression is less than 1° C., the rise in temperature is due almost entirely to the flow of radioactive heat into the sediments. With these facts in mind, let us consider the changes in the gradient at the surface of a sedimentary area. First, depression of isogeotherms immediately beneath ocean floors causes an increase in the flow of heat beneath oceans as compared with undisturbed land areas; and likewise, the sinking of the base of the sedimentary column into rocks of high temperature tends to increase the gradient during subsidence. Second, inasmuch as the total height of material eroded from a mountain range during the process of base leveling the range is several times the original height of the range (10), it follows that large quantities of heat in these areas are brought nearer to the surface of the earth by mass displacement. The net result of these activities is that in areas of sediments which have for one or more times been subject to uplift and subsidence, the total quantity of heat remaining in the rocks down to the level of concentric isogeotherms is probably at a minimum: the observed gradients at the surface, however, because of erosion and mass displacement outward in the vertical, are at a maximum. Thus, the absorption of heat during subsidence, and the subsequent erosion and displacement of rocks toward the surface, may account, in part, for the relatively high gradients found in sedimentary areas. Even thin sediments which are the remnants of extensive erosion, as at El Dorado, Kansas, may retain some of the absorbed heat. In undisturbed areas, however, the gradients are probably at a minimum and represent a normal gradient in the sense that such a gradient is the result of an undisturbed flow of heat that began immediately following the solidification of the crust. As suggested by J. S. De Lury (11), a normal gradient as thus defined may equal or exceed 200 feet per degree Fahrenheit. Observations at Grass Valley, California, Franklin Furnace, New Jersey, and the upper peninsula, Michigan (Table I), tend to support this conclusion. Core-drill holes in areas of undisturbed igneous rocks would give us valuable information on this important question.

RECAPITULATION

In order to give a general idea of the areal distribution of gradients, the maximum and minimum reciprocal gradients have been tabulated in those areas in which there are a considerable number of observations. We have for the first 1,000 feet:

	Feet per °F.	*Meters per °C.*
Appalachian Mountains	68–131	37–72
Wyoming	19–74	10–41
Southern California	40–83	22–46
Louisiana	33–57	18–31
Kansas	46–82	25–45
Oklahoma	37–162	20–89
Texas	22–156	12–86
New Mexico	130–228	71–125

The well at Albany, Alabama, is not included in the preceding summary. The lowest temperatures are found in the Permian basin (12)—that is, southeastern New Mexico, western Kansas, western Oklahoma, and western Texas. The highest oil-field temperatures occur on the salt domes of the Gulf Coast, including northern Louisiana, and on structures of large closure in Wyoming.

The computed averages are as follows.

	Feet per °F.	*Meters per °C.*
Crests of 57 structures	49.8± 1.2	27.3±0.7
Flanks of 57 structures	62.1± 1.9	34.1±1.0
138 locations, including flanks of oil structures	65.2± 1.6	35.8±0.9
9 locations, excluding flanks of oil structures	108.0±10.6	59.3±5.8
4 locations in oil fields, Tuxpam, Mexico	36.3± 1.8	19.9±1.0
2 locations in oil fields, Alberta, Canada	52.5	28.8
9 mines, northern Ontario, Canada	156.0± 7.3	85.6±4.0

The preceding results, with the exception of the oil fields at Tuxpam and the mines in northern Ontario, are the averages for the first 1,000 feet.

From Table IV and the preceding tabulations, we have the following averages for the indicated successive ranges of depths.

	100 to 1,000 Feet	*100 to 2,000 Feet*	*100 to 3,000 Feet*	*100 to 4,000 Feet*
Crests of 57 structures	49.8	50.6	50.0	47.4
Flanks	62.1	57.3	56.0	51.3
138 locations, including flanks of oil structures	65.2	62.6	59.6	54.6
9 locations, excluding flanks of oil structures	108.0	101.0	95.7	—

On account of the rapid diminution of the number of wells with depth (Table IV) the accuracy of the preceding sequences of numbers diminishes rapidly as the depth increases.

No attempt has been made in this paper to extend an average depth-temperature curve beyond a depth of 4,000 feet. Such an extension probably involves the average thickness of the sediments and other factors that have not been investigated.

CONCLUSIONS

Practically nothing is known in regard to the positions of the isogeotherms over metalliferous deposits, such as are found in northern Ontario, Canada, and elsewhere throughout the world. As a result of ascending waters that have long since become extinct, it is reasonable to infer that the isogeotherms in many of these areas are depressed relative to those in the immediately surrounding zone. However, granting this possibility, it seems rather improbable that the reciprocal gradients in the extended zone surrounding these areas of very low temperatures should be less than 100 feet per °F. (54.9 meters per °C.); nor is it altogether improbable that the rate may not exceed 200 feet per °F. (109.7 meters per °C.).

Comparison of this low rate of 1° F. in 200 feet in the slightly disturbed rocks of the Canadian shield with the preceding rates (49.8, 62.1, 65.2) shows, as suggested on page 111, that there are at least two distinct types of normal gradient—one for sediments and one for exposed basement rocks that have remained static, or almost static, since their solidification.

Apart from differences in the thermal constants of the rocks and the variation of the annual mean temperature of the air with elevation, latitude, and geologic climate, the average of even a comparatively small number of gradients in the undisturbed crystalline rocks must be nearly constant over the surface of the earth.

In the sedimentary areas, however, uplift, subsidence, erosion, chemical reactions, and many other factors produce variations in the rock temperatures. Hence, a normal gradient determined from such areas depends on the number and distribution of the wells and the results of endless geological changes. For purposes of theoretical definition, the number of wells can be assumed to be so great that the average approaches a definite value, but, for practical purposes, it can not be assumed that the average obtained from a limited number of wells approaches a definite value. To illustrate, the reciprocal of the mean gradient, from 10 wells in western Texas is, say, 150; in Wyoming, again using 10 wells, the reciprocal is perhaps 40 feet per degree. The reciprocal of the mean of the corresponding gradients is 63.2, but, by varying the number of wells selected from the respective states, it is possible to obtain reciprocals of the mean gradient that vary from a little more than 40 to a little less than 150 feet per degree. This result suggests that the average for each geologic feature, the Permian Basin, for example, should be considered as a unit in determining the average. The distribution of these averages about a mean should be definitely related to the geological facts.

Summarizing the evidence, it appears that the normal gradient in the first 1,000 feet of sediments is probably greater than 65.2 feet per °F. (35.8 meters per °C.). The determination of an upper limit is not easily made. Assuming an arbitrary value of 82.8 as an average, and a minimum of 62.14, we have for the maximum reciprocal gradient, 103.5 feet. The mean from 9 locations exclusive of the oil fields is 108.0. It seems reasonable, therefore, to assume that the average of a large number of gradients covering the entire area of the United States, that is, the reciprocal of the normal gradient corrected for surface topography, is certainly greater than 60 feet per °F. (32.9 meters per °C.) and probably less than 110 feet per °F. (60.4 meters per °C.).

The wide interval between the preceding limits may seem unjustifiable, but when it is recalled that we have practically no observations in the large areas surrounding domes and anticlines, where the temperatures are presumably low, it is evident that a large element of uncertainty enters into our calculations. For depths exceeding 1,000 feet, the preceding numbers should probably be replaced by numbers of a lesser numerical magnitude. Existing observations do not give us very much information on this point.

The corresponding estimates for undisturbed basement rocks are: 100 feet per °F. (54.9 meters per °C.) for the minimum and probably not less than 200 feet per °F. (109.7 meters per °C.) for the maximum. These estimates are based on the data from northern Ontario, Canada (118.4 to 233.2).

In conclusion, it may be of interest to call attention to the fact that a value of the reciprocal of the normal gradient equal to 200 feet per °F. implies an age of a nonradioactive earth of 1,000 or 1,500 million years. Thermal data from undisturbed basement rocks would be of great value, therefore, in estimating the age of a nonradioactive earth, and incidentally, it would enable us, also, to estimate the amount of heat which is being supplied to the earth from radioactive sources.

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THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
TRANS. SOCIETY PETROLEUM GEOPHYSICISTS, VOL. V (MARCH, 1935), PP. 117-120, 1 FIG.

A FORMULA FOR WEATHERING CORRECTION[1]

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It is a well known fact that practically everywhere the outer portion of the Earth's crust is composed of weathered material. The thickness of this weathered zone may vary from zero to 50, or more, feet within short distances. The velocity of seismic waves in this material is not uniform but varies in a very marked manner from top to bottom, as is readily seen on the weathered-zone time-distance chart. As a consequence, it has been found necessary in seismic reflection-work to base all calculations from the bottom of the weathered zone. An average velocity can be used below the weathered zone, since the velocity becomes more uniform in this part of the section.

It is common practice, however, to calibrate an average velocity for the weathered zone itself due, perhaps, to the fact that no better method has been presented. It is at once apparent, however, that such a calibrated weathered-zone velocity is not actually valid and may lead to serious errors in determining the amount of time to be subtracted for the weathered zone, as well as its thickness. It is the purpose of this paper to present a formula whereby the thickness of the weathered zone may be calculated, and thus the proper time-correction applied. The usual method for the calculation of depths by the seismic reflection-method has been given by the author in a previous paper.[3]

It is necessary, in order to calculate the thickness of the weathered zone, to know in what manner the velocity of propagation varies with the depth of penetration. A method for doing this was first presented by Ewing.[4] Ewing found that the penetration is given by the formula,

$$P(D) = (1/\chi) \int_0^D [\cosh^{-1} V(D)/V(x)]dx.$$

[1] Reprinted from The National Research Council of The National Academy of Sciences, *Trans. Amer. Geophysical Union* (June, 1934), pp. 78–80, with the permission of the general secretary of The American Geophysical Union and the author.

[2] Seismograph Station, University of Pittsburgh.

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[4] M. Ewing and L. D. Leet, "Comparison of Two Methods for Interpretation of Seismic Time-Distance Graphs Which Are Smooth Curves," Trans. Amer. Inst. Min. Eng., *Geophysical Prospecting, 1932*, pp. 263–270.

Here $P(D)$ represents the penetration and $V(D)$ is the velocity at the mid-point of the path of a ray with shot-length D.

Now, it has been found that the time-distance data for the weathered zone fits a function of the form

$$X = at^2 + bt.$$

An example of this is shown later. If the above function be substituted in Ewing's formula for penetration, the result is given by

$$P(D) = (1/4a\chi)\left[V\sqrt{V^2 - b^2} - b^2 ln(V/b + \sqrt{V^2/b^2 - 1})\right].$$

The above expression is rather involved but fortunately it can, in the case of the weathered-zone data, be represented by an approximation, or

$$V = a\sqrt{y} + b$$

where V is the velocity, y is the penetration below the surface, and a and b are constants.

The author has shown that the time-distance relationships for the case of a high-velocity bed whose overburden consists of material in which the velocity is a steadily increasing function of the depth is given by[5]

$$t = 2\int_0^Z V dy/\left[V(y)\sqrt{V^2 - V^2(y)}\right]$$
$$- (2/V)\int_0^Z V(y)dy/\sqrt{V^2 - V^2(y)} + D/V$$

where Z is the depth of the high-speed bed below the surface, or the thickness of the overburden, V is the velocity of the high-speed bed, $V(y)$ is the velocity-depth function, and t and D are the time and horizontal distance on the surface for any ray which has been refracted over the high-speed bed. We now substitute for $V(y)$ in the above expression its approximation, that is

$$V(y) = a\sqrt{y} + b$$

and get

$$t = 2\int_0^Z V dy/(a\sqrt{y} + b)\sqrt{V^2 - (a\sqrt{y} + b)^2}$$
$$- 2/V\int_0^Z (a\sqrt{y} + b)dy/\sqrt{V^2 - (a\sqrt{y} + b)^2} + D/V.$$

[5] H. M. Rutherford, *Trans. Amer. Geophys. Union*, fourteenth annual meeting, 1933, pp. 289–303.

The various integrals can be evaluated by making the substitution $u=(a\sqrt{y}+b)$. The final result is

$$t = (4V/a^2)\{\sin^{-1}[(a\sqrt{Z}+b)/V] - \sin^{-1}(b/V)$$

$$+ (b/V)ln[(V+\sqrt{V^2-(a\sqrt{y}+b)^2}]/(V+\sqrt{V^2-b^2})\}$$

$$- (4/a^2V)\{[(V^2/2)\sin^{-1}[(a\sqrt{Z}+b)/V]$$

$$+ [(b-a\sqrt{Z})/2]\sqrt{V^2-(a\sqrt{Z}+b)^2}]$$

$$- [(V^2/2)\sin^{-1}(b/V) + (b/V)\sqrt{V^2-b^2}]\} + D/V.$$

The above formula is general and holds for depths for any high-speed bed if the overburden has a velocity-depth function of the form given. The formula for the case of reflections is given in its parametric form by merely substituting for V the parameter K, and writing the corresponding formulas for time and distance.

AN EXAMPLE

The following formula was found to hold for some weathering data in Arkansas

$$X = 29.722t^2 + 0.039t \quad \pm 0.002 \text{ second}$$

and for the high-speed bed

$$X = 6.3t - 0.315 \quad \pm 0.0009 \text{ second.}$$

In the above formulas X is given in units of 1,000 feet and t in seconds. The velocity-depth function was found to be approximated by

$$V = 16.917\sqrt{y} - 0.0035 \quad \pm 0.0018.$$

The values represented in this equation are in units of 1,000 feet. It will be noticed that in the above the fits are exceedingly good and well within experimental error. The depth computed by use of the above formula gives $Z=50$ feet. Computation of the depth by rectilinear propagation-paths leads to a figure of $Z=40$ feet, though this figure varied depending on the manner the lines were drawn on the graph. The accompanying figure shows the data and the computations.

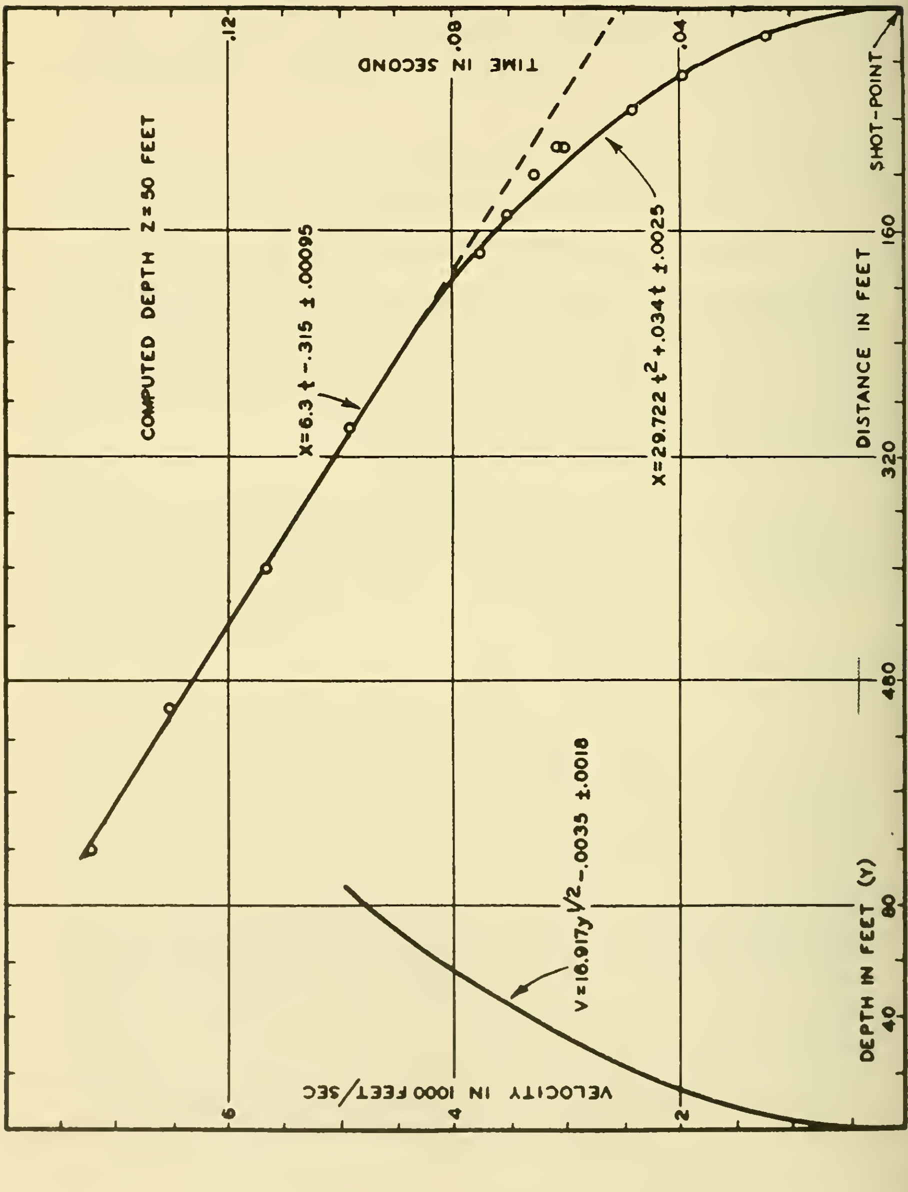

FIG. 1.—A typical profile which penetrates the weathered zone (Arkansas).

THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
TRANS. SOCIETY PETROLEUM GEOPHYSICISTS, VOL. V (MARCH, 1935), PP. 121-131, 4 FIGS.

A SEMI-GRAPHICAL METHOD OF DETERMINING DEPTHS OF MULTI-LAYER, DIPPING STRATA FROM SEISMIC TIME-TRAVEL CURVES[1]

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Although refraction *shooting* has been largely replaced by reflection *shooting* in the commercial field, it appears that theories and problems relating to the former type of seismic exploration are still of general interest. This fact is evidenced by recent papers which have been published on refraction *shooting*.[3,4]

The object of this paper is to describe a method of interpreting depths from seismic time-travel curves used by the writer on actual

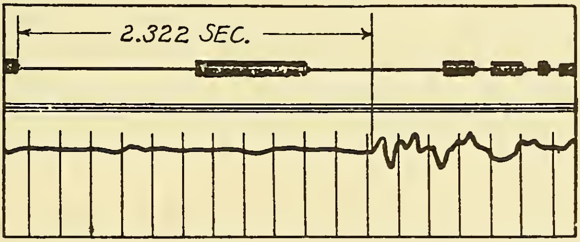

Fig. 1.—A seismogram obtained by an electromagnetic type seismograph.

field data. No theoretical considerations are taken up and no new mathematical formulas are developed. However, the graphical method of solution is original, and has not been described in the literature to date. It was thought that a purely practical interpretation of seismic results would be of interest to the general reader.

A section of a seismogram obtained from an electromagnetic seismograph used by the Humble Oil and Refining Company is shown in Figure 1. The top line is recorded to indicate the time of firing of the

[1] Published by permission of the Humble Oil and Refining Company. Reprinted from *Amer. Jour. Sci.*, 5th Ser., Vol. XXVII, No. 157 (January, 1934), pp. 13–23, with the permission of the editor of the *American Journal of Science* and the author.

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[3] L. B. Slichter, *Physics*, 3, 273, 1932.

[4] M. Muskat, *Physics*, 4, 14, 1933.

charge of dynamite used to produce the shock waves. The buzzer break at the extreme left end of this line indicates the exact instant the charge was fired. The second line from the top has no significance, but the third line is the actual seismogram; i.e., it is a record of the velocity of the motion of the earth's surface in the vertical direction. The arrival of the first shock is plainly indicated. The vertical marks are time marks put on the record by a chronometer. The interval between marks is one-fifth of a second. By careful interpolation between these marks it is possible to read the time interval between the firing of the charge and the arrival of the first shock at the seismograph to .005 of a second. This time interval for the particular record shown in Figure 1 is indicated on the record as 2.322 seconds; the last figure, of course, may be anywhere between 0 and 5.

The usual number of seismographs employed by a *party* is four. These instruments are placed at various distances from the *shot point* (the point where the dynamite is discharged), and the charge is fired; then the instruments are moved to new locations and another charge is fired. This process is continued until records are finally obtained with the instruments as far as six kilometers from the *shot point*. Cases are known where instruments were placed as far as 15 kilometers from the *shot point* and charges of 2,000 pounds of dynamite have been used. In one case, a charge of this magnitude was fired in the Gulf of Mexico, and the resulting shocks were recorded by instruments 15 kilometers away on the shore.

The records obtained are usually numbered A_1, A_2, A_3, A_4, for the first shot; B_1, B_2, B_3, B_4, for the second shot, and so on. For the first shot, the instruments are usually placed 50 meters apart and a 5-pound charge of dynamite is exploded; thereafter the stations are 500 meters apart and successively increasing charges of dynamite are used. Figure 2 illustrates a typical profile obtained from seismic results shot over parallel, horizontal strata of successively increasing velocities of transmission of shock waves. The lower part of the figure is the profile of the stratification, while the upper part is the plot of the time travel curves obtained from the seismic data. The circles at each end of the surface represent the *shot points* Nos. 10 and 11, and the little triangles on the surface represent the various locations of the seismographs. The distances of these stations from their respective *shot points* are very accurately determined, and the time-travel curves are obtained by plotting the time required for the first shock to arrive at each station against the distance of this station from the *shot point*. It will be noticed that the curves obtained are a series of straight lines of different slopes. If the laws of optics are assumed to hold, it is easy to show that the reciprocals of these slopes are the velocities of transmission of the shock waves along the respective strata. The solid lines through the strata represent the *Mintrop shortest time paths* of these shock waves from *shot point* 10 to the respective stations, and the dotted lines represent these same paths for the shock waves from *shot point* 11 to the respective stations. In order to obtain the true velocities of the beds in the case of tilting, it is necessary to shoot in both

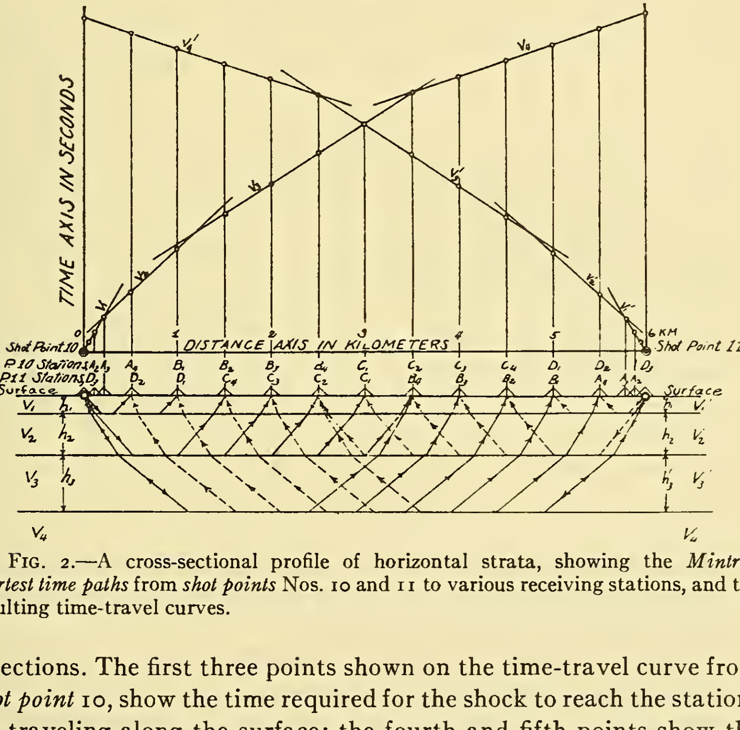

FIG. 2.—A cross-sectional profile of horizontal strata, showing the *Mintrop shortest time paths* from *shot points* Nos. 10 and 11 to various receiving stations, and the resulting time-travel curves.

directions. The first three points shown on the time-travel curve from *shot point* 10, show the time required for the shock to reach the stations by traveling along the surface; the fourth and fifth points show the time required for the shock to reach these stations by traveling through the first layer and along the surface of the second layer. For these last two stations it will be seen that less time is required for the transmission of the waves along the latter path than for the transmission of the waves along the surface. The sixth, seventh, eighth, ninth, and tenth points show the time required for the transmission of the impulses through the first and second layers, and along the surface of the third layer to the respective stations. It is seen that the time of arrival of the shocks refracted along the surface of the third and fourth beds is the same for the tenth point. The thicknesses and depths of the beds can be determined from the time-travel curves by substituting the proper values of X, T, and V in the following relations:

$$h_1 = 1/2 \cdot \tan \alpha_4 (T_2 V_2 - X_2) \tag{1}$$

$$h_2 = 1/2 \cdot \tan \beta_3 \left(T_3 V_3 - \frac{2h_1}{\tan \alpha_3} - X_3 \right) \tag{2}$$

$$h_3 = 1/2 \cdot \tan \gamma_2 \left(T_4 V_4 - \frac{2h_1}{\tan \alpha_2} - \frac{2h_2}{\tan \beta_2} - X_4 \right) \tag{3}$$

$$h_4 = 1/2 \cdot \tan \psi_1 \left(T_5 V_5 - \frac{2h_1}{\tan \alpha_1} - \frac{2h_2}{\tan \beta_1} - \frac{2h_3}{\tan \gamma_1} - X_5 \right) \tag{4}$$

where the values T_2, T_3, T_4, and T_5 are the times of arrival of the first shocks at stations so situated at distances X_2, X_3, X_4, and X_5 from the *shot point* that these stations fall within the sections of the time-travel curves whose slopes are given by $1/V_2$, $1/V_3$, $1/V_4$, and $1/V_5$, respectively. The values of the angles used in the above relations are obtained from the following expressions:

$$\sin \alpha_4 = V_1/V_2 \tag{5}$$

$$\sin \alpha_3 = V_1/V_3 \tag{6}$$

$$\sin \beta_3 = V_2/V_3 \tag{7}$$

$$\sin \alpha_2 = V_1/V_4 \tag{8}$$

$$\sin \beta_2 = V_2/V_4 \tag{9}$$

$$\sin \gamma_2 = V_3/V_4 \tag{10}$$

$$\sin \alpha_1 = V_1/V_5 \tag{11}$$

$$\sin \beta_1 = V_2/V_5 \tag{12}$$

$$\sin \gamma_1 = V_3/V_5 \tag{13}$$

$$\sin \psi_1 = V_4/V_5. \tag{14}$$

If a difference in elevation between receiving stations and *shot point* occurs, it is necessary to correct the indicated times of arrivals of the shocks for these differences in elevation.

Figure 3 illustrates the case of a single inclined layer. In the case of an inclined layer, it is necessary to shoot a reversed profile; i.e., two profiles are shot over the same line: one up dip, and the other down dip. The depths, h and k, under the two *shot points* and the angle of dip θ are determined from the following relations:

$$h = \frac{T_2'' V_1 - X_2'' \sin (B)}{2 \cos \alpha \cos \theta} \tag{15}$$

$$k = \frac{T_2' V_1 - X_2' \sin (A)}{2 \cos \alpha \cos \theta} \tag{16}$$

$$\sin (A) = \sin (\alpha - \theta) = V_1/V_2' \tag{17}$$

$$\sin (B) = \sin (\alpha + \theta) = V_1/V_2'' \tag{18}$$

$$\theta = \frac{B - A}{2} \tag{19}$$

where V_2' is the apparent velocity obtained from the slope of the time-travel curve shot up dip, V_2'' is the apparent velocity obtained from the slope of the time-travel curve shot down dip, T_2' and X_2' are coordinates of a point on the time-travel curve shot up dip, and T_2''

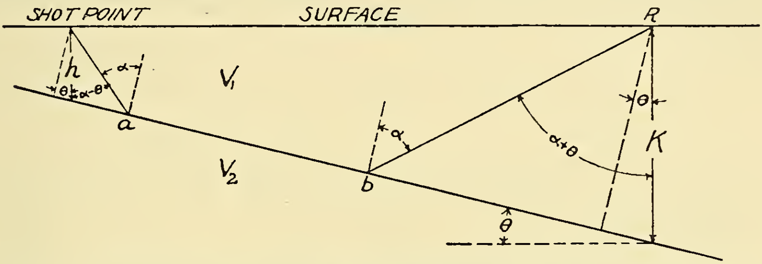

FIG. 3.—A cross-sectional profile of a single dipping stratum with the *Mintrop shortest time* path from the shot point to the receiving station *R*.

and X_2'' are the coordinates of a point on the time-travel curve shot down dip. The true velocity V_2 of the inclined layer is obtained from the following relations:

$$\alpha = \frac{B + A}{2} \tag{20}$$

$$\overline{V_2} = V_1/\sin \alpha . \tag{21}$$

The equations (1)–(21) apply only where the velocities of the layers are all in an ascending order of magnitude from V_1 to V_n, and these equations are essentially the ones given by D. C. Barton in a paper entitled "The Seismic Method of Mapping Geologic Structure."[5] However, if the area under consideration yields strata of several subsequent dips of different angles, the calculations become very complicated and it is quite laborious to determine the various dips, true velocities, and thicknesses of the beds from enlarging on the expressions (1)–(21). To simplify this problem, the following semi-graphical method was developed.

In most cases the thickness of the first bed can be calculated by the straightforward formula for parallel beds. The result, of course, gives the depth of the surface of the second bed only in the immediate vicinity of the *shot point* at each end of the profile. In general it may be assumed that the surface of this bed extends across the profile in a linear variation from the two *shot points*, and that this bed has the

[5] D. C. Barton, *Geophysical Prospecting for 1929*, p. 572.

seismic velocities at each end of the profile indicated by the time-travel curves. It must be remembered that the dip formulas can be used only where either the true velocity of the dipping bed is known or where the reversed profiles overlap. Suppose the surface of the third bed indicates a dip of θ_1 degrees with the horizontal surface. As the true velocities of the first two beds are known, and as the surface of the second bed is parallel to the surface of the ground, the true velocity and dip of the third bed may be determined from the following relations:

$$\theta_1 = \frac{B - A}{2}, \text{ where } \sin B = V_2/V_3'' \qquad (22)$$
$$\sin A = V_2/V_3'$$

where V_3'' is the apparent velocity down dip, and V_3' is the apparent velocity up dip.

$$\overline{V_3} = \text{true velocity} = V_2/\sin(A + \theta_1). \qquad (23)$$

It now remains only to find the thickness of the second bed under each *shot point*. First, calculate the angles of refraction for the *Mintrop path* refracted along the surface of the third bed by using relations (6) and (7), and lay off the path accordingly for a station on the V_3'' and V_3' curves. Then, measure the path length through the first layer and determine the time required to traverse this layer by dividing the path length by the velocity V_1. Subtract this time interval from the time coordinates of the chosen points on the V_3'' and V_3' curves and use the remainders as the values of T in relations (15) and (16). It must be remembered that the values of X used in the calculations of h and k are the distances of the stations chosen from the respective *shot points* minus the mirage distances on the surface of the V_2 layer. The mirage distance is illustrated by the distances ab and pq in Figure 4.

Suppose the surface of the fourth bed has a dip. On account of the dip of the third layer, the slopes of the time-travel curves of the fourth bed will be affected whether the fourth bed has a dip or not. In order to remove this effect of the third bed the following procedure is taken. Assume a true velocity for the fourth bed; a mean value between the velocities obtained from the up and down dip time-travel curves is usually a fair approximation. Calculate the angles of diffraction of the *Mintrop path* diffracted from the surface of the fourth bed, using the assumed velocity for V_4 in the relations (8)–(10), and lay off the parts of the path through the first and second layers for at least two points on the up dip profile and the same number on the down dip profile. Calculate the time required in traversing the first and second beds for each of these points and subtract the corresponding time interval for each point. Then draw the two new time-travel curves through these points. The slopes of these two new curves will give the dip of the surface of the V_4 bed relative to the surface of the V_3 bed. The above action will, of course, rotate the plot of the time-travel curves through an angle θ so that the abscissa, or distance axis, will be parallel to the secondary surface, which is the surface of the V_3 layer, and the coordinates of the time-travel curves must be read ac-

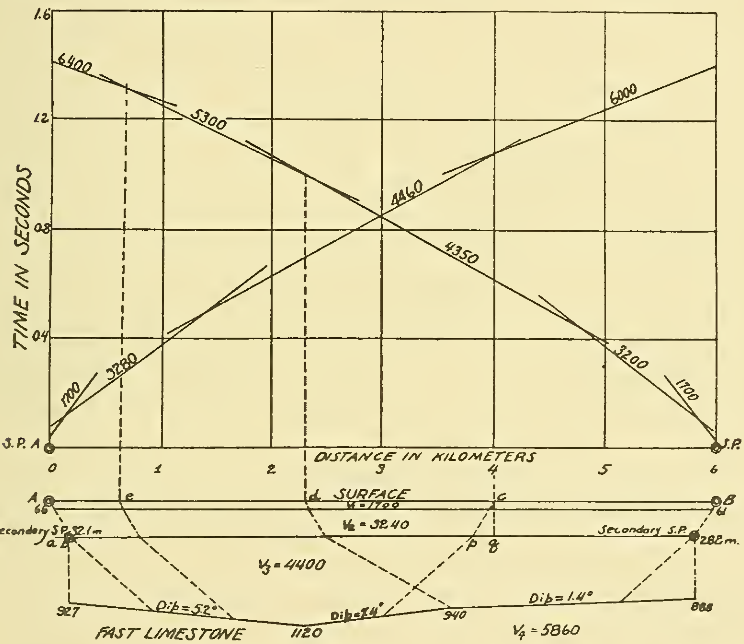

FIG. 4.—The time-travel curves obtained from actual data taken from records shot in an area in West Texas, and the resulting profile calculated according to the method described in this paper.

cordingly. With this action accomplished, the procedure is the same as for the case of a single dipping layer. The dip angle calculated in the usual manner will be the angle of dip between the surfaces of the V_3 and V_4 beds, and the values of h and k will be the thickness of the V_3 bed under the secondarys *hot points* on the surface of this bed. The true velocity of V_4 can be calculated in the same manner as $\overline{V}_3$ was previously determined; and if the result is not in good agreement with the assumed value of V_4, a new value for this quantity can be assumed and the entire procedure repeated. A high degree of accuracy can be obtained by repeating this procedure a few times. The writer has obtained numerous checks which were well within the experimental error on the first round.

A certain area yielded the time-travel curves illustrated in Figure 4. The velocities in meters per second are as indicated on the curves. The scale of plotting is, of course, greatly reduced from the work sheet. From the curves, it is apparent that there are four beds represented; the velocities of the fourth bed indicated from *shot point B* show definitely a hole in the surface of the high speed bed. It should be stated that before any depth determinations were made from this profile, a careful consideration was made of the characteristics of the beds throughout the entire region under investigation. A total of six profiles were shot on this particular region. It would be very hard to determine the true velocity of the high speed bed from the profile illustrated in Figure 4, but from another profile shot in the same vicinity which was more regular, a normal of 5,860 m./sec. was fairly well defined for the lower bed. In the calculations of the dips of this bed the 5,860 m./sec. true velocity was used. The fact that the V_1 curves in Figure 4 do not pass through the origins at each *shot point*, is explained by the fact that the shots were fired from the bottoms of small wells 50 feet deep. This discrepancy may be corrected by pushing up the distance axis so that the V_1 curves will pass through the origins at the two *shot points*.

The thickness of the first bed under each shot was determined by using the values of V_1 and V_2 written on the time-travel curves at each end of the profile in equation (5), and by substituting the proper coördinates of points obtained from the V_2 curves in equation (1). The results are written on the subsurface chart shown in Figure 4. The surface of the third bed showed a dip down towards *shot point A*. It was within the limit of accuracy of the data to take a mean between the values of V_2 given at each end of the profile as the true velocity of the second bed, i.e., $\overline{V}_2 = 3{,}240$ m./sec. The angle of dip of the surface and the true velocity of the third bed were determined by using relations (22) and (23). The thickness of the second bed under each *shot point* was determined by the method previously described.

The surface of the fourth bed, which was of principal interest in this case, was obtained in the following manner. As the true velocity of this bed was known from additional data, no true velocity determination was necessary. From the time-travel curves refracted from the surface of the bed under consideration, it is apparent that there were three distinct dips. The dip angle of the surface of V_4 with the surface of V_3 under *shot point A* was determined as follows:

$$\sin A = 4{,}400/6{,}400 = .687, \ A = 43.4°$$

$$\sin (A + \theta) = 4{,}400/5{,}860 = .750, \ A + \theta = 48.6°$$

$$\theta = \text{dip angle} = 5.2° \text{ down from } \textit{shot point } A .$$

The dip down from *shot point B* was determined in a similar manner and was found to be 1.4°. It is seen from the time-travel curve that a dip downward on the *shot point B* profile occurs between stations *d* and *e* (Fig. 4). This dip was calculated to be 7.4° by the same method as the other dips were determined. The thickness of the V_3 bed is unusually difficult to compute in this case as the surface of the V_4 bed is not on a steady dip between the two *shot points*. The method used here is essentially a cut and try method, but it must be remembered that all geophysical interpretations are largely accomplished by setting up conditions which will give the observed results. By using 5,860 as the value of V_4 in relations (8)–(10) the angles of refraction of the *Mintrop* paths were determined and the paths laid off as shown in dotted lines in Figure 4. From other profiles of more regular character, the depth of the V_4 surface at each end of the profile in question was found to be nearly the same. The time required to traverse the V_1 and V_2 beds along a path through the V_4 bed from *shot point A* to *shot point B* was determined by measuring the sections of this path through V_1 and V_2, dividing the results by the respective velocities, and the result was found to be 0.254 seconds. The correction for X, which was the sum of the distances from the actual *shot points* to the respective secondary *shot points* on the surface of the V_3 bed was found to be 350 meters. The value of T was determined by subtracting 0.254 sec. from the time coördinate of the point at the end of each profile.

This time coördinate was, of course, the same for both end points. The thickness of the V_3 bed under each *shot point* was then determined by substituting the corrected values of X and T in relation (1) with the values of V_4, T_4, and X_4 being used in place of the values of V_2, T_2, and X_2. The thickness of the third bed under each *shot point* came out to be 606 meters. The depth of the V_4 surface in the immediate vicinity of each *shot point* was thus determined and plotted on the subsurface chart as shown in Figure 4. By drawing lines through the depth points at angles previously determined, the surface of the V_4 bed was obtained at each end of the profile. The slope of the V_4 surface which connects the dips at each end of the line was previously determined to be 7.4°. From the seismic times of the stations marked *d* and *e* on the surface, it is apparent that the sharp dip occurs between *d* and *e*. Hence, by drawing in the 7.4° dip from the surface of the V_4 bed at the point where the path through *d* intersects the line drawn from *shot point B*, and extending this line to intersect the 5.2° dip line drawn from *shot point A*, the surface of the high speed bed was completed.

A good method of checking the work is to select some point which lies on one of the V_4 curves and trace its path from the station to the *shot point.* By measuring the sections of this path and dividing them by their respective velocities, the time of arrival of the first shock at the station located on this path is computed. This calculated time should check the seismic time on the time-travel curve. Several checks of this sort were made when the profile illustrated in Figure 4 was first computed. All the checks came within .005 of a second of checking the data. This accuracy was within the limits of error of the instruments, which were Petty[6] condenser type seismographs equipped with a string galvanometer recording system. Chronometer marks were made on the seismograms which enabled the time of arrival of the first shock to be read to .005 second.

In summary, it will be seen that in order to arrive at any satisfactory conclusions as to the depths of consolidated deposits from refraction shooting, it is necessary to shoot extensively over the area under consideration and then plot the entire data and arrive at approximate true velocities of the deeper beds, before attempting computations of depths. Then each profile must be taken up separately and a rough contour of the subsurface bed of most interest (usually the deepest one) sketched from the time-travel curves before actual calculations are started. This procedure gives the computer an outline to work from, and is of great aid in keeping the results within reason. In the case of the example illustrated in this paper, all six profiles were worked out and a subsurface contour map made of the limestone surface. The contour map showed a decided *high* in the surface of the limestone. This *high* was later drilled into and the difference between the well log and the calculated depth of the *high* was only 4% of the calculated depth.

The fact that McCollum and Snell[7] found that the velocity of the *sound waves* is not the same in all directions in consolidated deposits introduces an error in the method described in this paper. However, if the velocity across the beds is determined from other methods, these values can be used in determining the time required to cross the beds. It might be said that in regions similar to the one illustrated in

[6] Made by the Petty Geophysical Corporation, San Antonio, Texas.

[7] B. McCollum, and F. A. Snell, *Physics*, 2, 174, 1932.

this paper, the method described here was found to check admirably with later determined well logs, and it is doubtful if the difference in the velocity of the shock waves across and along the beds in these cases was very pronounced, or that they introduced a serious error in the results.

In support of the method just described, it may be said that it facilitates mathematical work to a high degree, enables the computer to break up the problem into sections which aid him in placing the observed time differences of the records in their proper places in the subsurface chart, and checks to a close margin calculations made from much more lengthy derivations.

I wish to express my appreciation to Mr. J. E. LaRue of the Humble Oil and Refining Company for permission to publish this article.

THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
TRANS. SOCIETY PETROLEUM GEOPHYSICISTS, VOL. V (MARCH, 1935), PP. 132-153, 12 FIGS.

MAPPING OF GEOLOGICAL STRUCTURE BY THE REFLEXION OF ELASTIC WAVES[1]

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SUMMARY

The utilisation of reflected elastic waves in structure mapping is an adaptation of the older technique developed for depth-sounding in water. The geological problem is more complex due to the inferior qualities of rock contacts as reflecting planes and the existence of transverse waves which are not present in a fluid. The principal waves generated by a disturbance are the longitudinal, transverse, and surface waves; time-distance curves for these waves in the two layer problem are presented. Refraction seismic prospecting utilizes the first wave front to reach any point remote from the disturbance by any path, whilst reflexion seismic prospecting utilizes the reflected longitudinal waves which must arrive later than the same wave fronts propagated directly along the surface. The energy contained in the wave train reflected from a slate-granite contact has been computed as being less than 4% of the incident wave for angles of incidence below the critical angle. A reflexion unit must be capable of recording clearly these weak wave trains which arrive superimposed on other disturbances; this can be achieved by selective amplification of the dominant frequency of the reflected wave train, which is usually between 30 and 60 cycles per second. The component elements of a reflexion unit are described. A typical reflexion seismogram is reproduced showing the normal succession of events, namely, the instant of explosion, arrival of the surface longitudinal wave, arrival of a reflected event and arrival of the sound wave through the atmosphere. The usual field procedure is discussed. Two radically different methods of interpretation are possible: the first is by correlation which requires that a particular burst of reflected energy can be identified over large areas; the second, by dip, requires that an event can be identified over only a short distance. Examples of typical seismograms permitting mapping by correlation are reproduced. The three possible sources of error inherent in this method are the correlation of events between seismograms obtained at adjacent locations, in the measurement of time intervals and in the computation of depths to reflecting planes; the first of these, which may lead to major errors, is governed by geological conditions and instrument design, the second can be eliminated by sufficiently careful technique and the last largely reduced by accurate determination of surface corrections. The results of such a survey in Oklahoma are reproduced. Examples of typical seismograms from which interpretation must be derived by the dip method are reproduced, and a graphical method of obtaining the reflecting plane explained. The principal source of error in this method is in deriving the correction for the low velocity surface zone. An example of a profile across a deep-seated salt dome derived by this method is reproduced. The conclusion is drawn that many structural problems confronting the petroleum geologist can be solved by the two methods of reflexion shooting outlined.

Seismology, in its application to geological problems, has as its tools two measurable quantities, time and distance, and one physical property, velocity. All methods involve the measurement of time intervals and can be broadly grouped into (*a*) those in which distance

[1] Reprinted from *Proc. World Petroleum Congress* (London), Vol. I (1934), pp. 155–62, with the permission of the editors and the author.

[2] Shell Petroleum Corporation.

is the other known factor and conclusions are drawn from differences in velocity, and (*b*) those in which velocities are known or assumed and depths derived. An example of the first group is the method of refraction shooting in which a disturbance is generated, usually by explosion of a charge of dynamite, and at substantially equidistant surrounding points the times of arrival of the resulting elastic waves are recorded. Knowing the quantities, time, and distance, we can derive the average velocity of propagation of the initial impulse which reaches these encircling points. Velocity of propagation being a characteristic property whose value is now known for many minerals, we have therefore a direct method of approximating the character of material traversed by the impulse. This method met with outstanding success in discovery of intruded salt plugs piercing the Oligocene and younger strata of south Texas and Louisiana. The latter group includes the reflexion method in which a disturbance is generated at the surface and the time taken for the resulting wave front to travel from the origin down to a selected reflecting plane and back to the surface is measured; knowing the average velocity of propagation of the leading wave front it is obviously a simple matter to determine the depth to the reflecting plane. The best example of the brilliant application of this principle is in the method of determining the depth of water known as echo depth-sounding. In the simplest application of this method a disturbance is generated on one side of the hull of a ship and the travel time for an impetus to be transmitted downward through the water to the ocean bed and back to a detector on the other side of the hull is measured; a knowledge of the velocity of propagation of elastic waves in water permits the depth to be computed. This practical application antedated the advent of the method as a geological tool and must be regarded as the prior art which led to such application.

Two great simplifications are present in mapping a water-rock contact by this method which do not pertain to the geological problem. In the first place, the contact between two media such as water and rock is an excellent mirror for sonic waves, whereas the contacts of geological strata normally are but indifferent reflecting planes. Secondly, water being a fluid it transmits only longitudinal and surface waves, whereas any disturbance in a solid will set up a complete suite of longitudinal, transverse, and surface waves.

VARIOUS WAVES RECORDED

Let us now investigate the many impetii which strike a series of seismographs placed on the surface at varying distances from a source of disturbance. These are best represented by a time-distance graph such as Figure 1, which depicts the relation between the distance from the origin and the time of arrival of the various wave fronts resulting from two strata, the upper one, in which the disturbance is generated, having the lower velocity of propagation, V_1. The disturbance, let us say due to a dynamite explosion, set up at shot point A gives rise to at least three trains of elastic waves, namely, a longitudinal wave, a transverse wave, and a surface, or Rayleigh, wave; the velocities of propagation are here arranged in descending magnitude. Further, the longitudinal wave on striking the interface between V_1 and V_2 will be in part reflected as a longitudinal wave and in part as a transverse wave, and will set up a new series of surface waves on the interface; likewise the transverse wave on striking this interface will suffer reflexion as both longitudinal and transverse waves. Waves striking the interface at their critical angles will be refracted along the surface of V_2 with velocities characteristic of the particular wave in that stratum and will be radiated thence back to the surface. Only selected impetii have been chosen for plotting on the time-distance graph to avoid too great complication; however, the principal longitudinal wave front impetii are all shown. Refraction seismic prospecting concerns itself with the first longitudinal wave front impetus to reach any point by any path, whereas reflexion seismic prospecting as applied at the present time confines itself to the longitudinal wave front impetii reaching the seismograph after reflexion from an underlying discontinuity and consequently arriving later than the same wave travelling directly along the surface of the ground. Consider the various wave fronts striking a seismograph placed at point B; this ordinate erected to the time-distance graph shows that the first wave front to arrive will be the longitudinal wave in V_1, the second will be the transverse wave in V_1, the third will be the Rayleigh wave along the surface, the fourth will be the sound wave in air, and lastly the longitudinal wave reflected from the interface of V_1 and V_2. The relative intensities of these various waves can only be learned by experience in any particular area, since they do not seem to be amenable to mathematical analysis. Professor C. G. Knott, in his magnificent paper on the quantitative aspect of the reflexion and refraction of elastic waves at a rock interface,[3] states that the energy carried by the various waves can be computed when complete physical data are available. In the particular case of the reflected longitudinal wave at an interface between slate and granite, Knott computes the energy contained in the longitudinal wave reflected at perpendicular incidence to be some

[3] *Phil. Mag.*, 1899, 48.

4% of the incident wave. This percentage rapidly decreases almost to zero until the critical angle is reached, and at greater angles of incidence 100% of the energy is reflected as a longitudinal wave. These

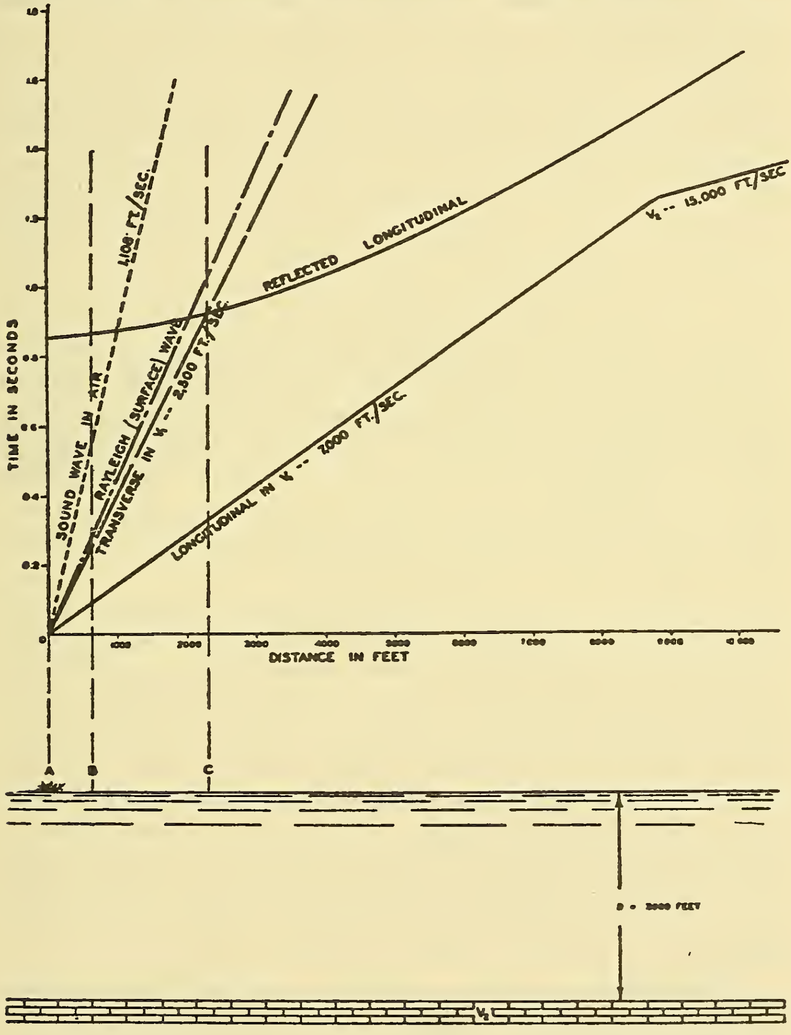

Fig. 1.—Time-distance graph of wave fronts resulting from two layers.

figures do not seem to be borne out in practical cases, as for angles of incidence ranging from perpendicular to 30° I have not observed any substantial reduction in the amplitude of reflected impetii. Consider now a seismograph placed at a point *C* and again erect an ordinate to the time-distance graph; the first wave front to arrive is still the longitudinal wave propagated in V_1, but now the longitudinal reflected wave and the transverse wave arrive simultaneously. At any point more remote than C from the origin of the disturbance, the reflected longitudinal wave arrives ahead of any transverse wave.

Applying these considerations to a practical case it is obvious that reflected events always appear on a seismogram subsequent to the arrival of at least one other wave train, and we must therefore have instruments so designed that this first impetus will be suppressed sufficiently to permit the recognition of a subsequent burst of energy even though the latter be of much less intensity. It is possible to locate a seismograph sufficiently far from the origin of disturbance to insure that the reflected longitudinal wave from a particular depth will be the second event to occur on the seismogram. For instance under the conditions of Figure 1 this distance will be about 2,300 feet. When investigating areas where reflexions are obtained from great depths it is frequently not desirable to use such large intervals between explosion points and recording station as would be imposed by this condition, and we consequently must be prepared to identify reflected events which arrive simultaneously with or subsequent to the transverse wave propagated in the upper stratum and the Rayleigh wave travelling along the surface. I might mention that the velocity of propagation of these two waves differs by very little and we are not able to differentiate between them; however, when a hard formation exists at the surface a wave having a velocity about in agreement with what might be expected from either of these forms is nearly always present and is extraordinarily persistent, the rate of decay in amplitude with distance being rather small. This suggests that propagation is taking place in two dimensions rather than in three dimensions and would bias one in favour of ascribing this disturbance to a Rayleigh wave. These waves constitute the phenomenon usually referred to as "ground roll." It was on the stumbling-block of eliminating these extraneous wave trains that early efforts to apply this method to economic problems failed.

Experience has pointed the way to elimination of most of these unwanted wave trains. The "ground roll" has a frequency of about 20 cycles per second, which is much removed from that usually observed in longitudinal waves and can be eliminated when electrical seismographs are used by means of a wave filter or by the use of transformers in the amplifier circuit, which are very inefficient in amplifying the frequency band in which these waves fall; alternatively, a seismograph can be designed having a natural vibration frequency which is identical with the reflected longitudinal wave and so will amplify that wave to a much greater degree than waves of lower frequency. The longitudinal wave propagated through surface beds or refracted from deeper beds is found to be very quickly dissipated, and when recording reflected events from appreciable depths is not a serious factor. The sound wave in air can be eliminated to a great extent by burying the seismographs at shallow depths. Micro-seismic unrest and audible frequency disturbances such as telephone wire hum and rustling of trees is usually of sufficiently high frequency to permit of its being by-passed to earth through condensers in the amplifier or of such low frequency as not to affect the unit.

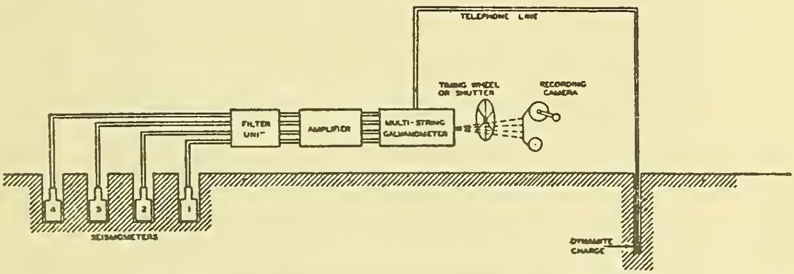

FIG. 2.—Elements of reflexion seismograph unit.

INSTRUMENTS

Instrumental design has followed widely separated paths, but there is evidence of convergence of these paths as the objective sought is more clearly visualised.

Electrical detectors, usually referred to as seismometers, are universally used, some employing moving coils in electromagnetic or permanent magnetic fields, some the carbon button microphone, and others the piezo-electric type of detector. Amplification of the weak electrical image can be obtained by audio-frequency vacuum-tube amplifier circuits employing from one to many stages. Recording of the image is through either oscillographs or string galvanometers. In average commercial work timing of the interval between the explosion and the arrival of an event at a seismometer must be accurate within limits of 0.1%. This accuracy is readily obtainable by means of an electrically maintained tuning-fork which may be arranged either to drive a synchronous motor or a shutter which periodically interrupts the light beam of the galvanometer. Experience has shown that the greatest energy is carried by the component of the reflected longitudinal wave having a frequency ranging from 30 to 60 cycles per second, and it is standard practice to peak the frequency amplification

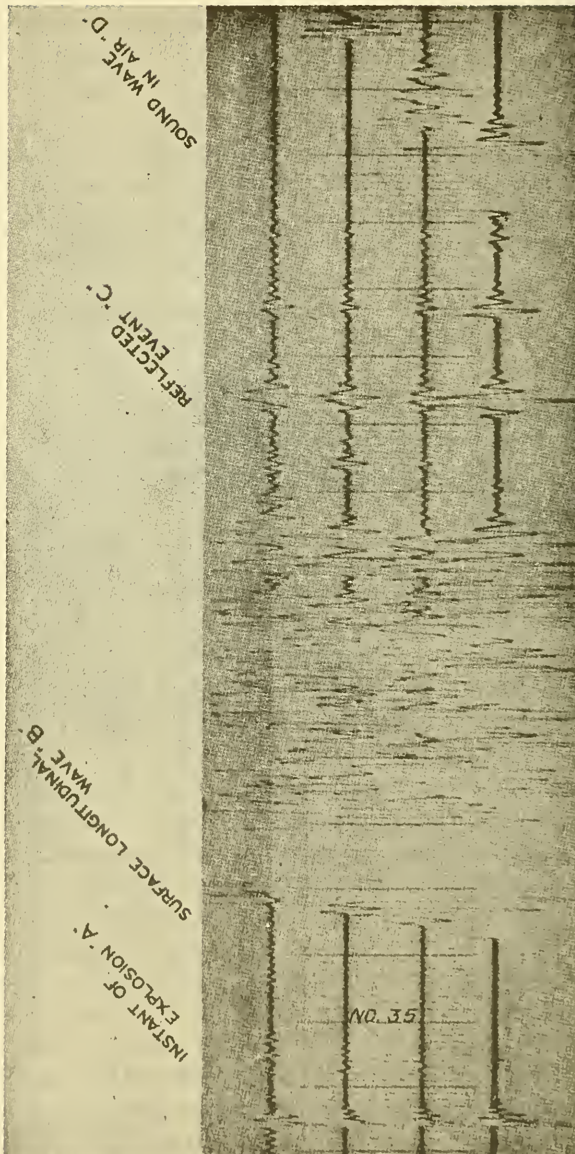

Fig. 3.—Typical seismogram recorded by reflexion seismograph unit.

curve between these values. I personally prefer a unit in which this is done without resorting to resonance and I believe this has become more and more generally accepted. Since the events we are attempting to record are transient, it is very desirable that all components of a reflexion unit should be aperiodically damped. The elements of a reflexion seismograph unit are shown diagrammatically in Figure 2. Let us now examine a record obtained from such a unit; Figure 3 is a reproduction of a seismogram obtained by a unit designed and constructed by the McCollum Exploration Company, who were one of the pioneers in this field. The four traces are produced by four seismometers at intervals of about 150 feet apart, the first seismometer being approximately 1,400 feet from the shot point. A total time of 1.8 seconds is covered by the portion of the seismogram reproduced. The instant of explosion was transmitted by telephone line and is recorded as the first event of the seismogram (A). The arrival of the longitudinal wave through the surface beds is next recorded (B), and after these have subsided it is obvious that much of the energy being recorded is reflected from considerable depths, since the apparent velocity of propagation is exceedingly high, indicating that the wave front is arriving almost parallel to the surface. This is the best criterion for identifying reflected events; consider the burst of energy at C on Figure 3 and compare its apparent velocity with the velocity of propagation of the sound wave in air at D; it will be found to exhibit a velocity some sixty times as great, or about 66,000 feet per second, which is much higher than the velocity of propagation of any elastic wave in rock minerals. This burst of energy must therefore have arrived at the surface subsequent to reflexion from a subsurface horizon, and if we know the average velocity of propagation of the wave in the material traversed we can compute the depth to the reflecting horizon.

FIELD PROCEDURE

The technique employed in the field varies with the conditions of the problem. In Oklahoma where this method was first developed to commercial success, the objective is usually to contour the top of Mississippian or Ordovician limestones in contact with shale members at depths of from 3,000 feet to 8,000 feet. It has been found possible to obtain satisfactory seismograms when seismometers are located at distances from the shot point ranging from a few feet to over 3,000 feet; between 1,000 feet and 2,000 feet is a common range, with some operators preferring to use almost vertical reflexions requiring that seismometers be located very near the shot point. This latter procedure is only possible if the "ground roll" is small or if it can be more or less completely eliminated by such means as a wave filter. Common practice is to use from four to six seismometers spaced from 50 to 200 feet apart. Shot points are normally located at intervals of about half a mile, but this spacing is determined by the complexity of the structure problem and also by the quality of the records obtained. The dynamite charges range from less than a pound to over 50 pounds, about one pound of 60% gelatin dynamite being a normal charge. The dynamite is usually loaded in machine-dug holes 20 to 100 feet or more in depth, the depth being governed by experience in a particular area.

METHODS OF INTERPRETATION

The method of interpretation used in Oklahoma is that of "correlation," which implies that the same events can be recognised on all seismograms and correlated one with another over considerable dis-

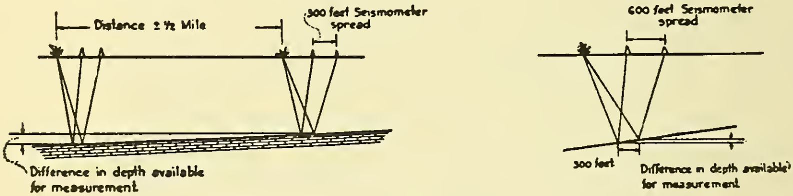

FIG. 4.—"Correlation" and "dip" methods of procedure.

tances. In areas where limestone members are continuous and where correlation of well logs by lithology is possible this method of interpretation is also possible. In fact, we may look upon a reflexion seismogram as a derivative of the lithological log of the stratigraphical column being penetrated by the reflected waves. In areas where lithological correlation cannot be carried over complete structures the same limitations will pertain to the interpretation of reflexion shooting data. For such areas the technique of "dip" shooting has been developed. This involves the very accurate determination of the running time of events reflected from adjacent points on a reflecting surface. Figure 4 will assist in visualising the differences of these methods, and to those familiar with core drilling the analogy between core-drill mapping where a recognisable marker is available and mapping by dips measured on core samples gives a fair representation of the relative power of the two methods of attack.

MAPPING BY CORRELATION

Experience has shown that where events on seismograms can be correlated with certainty, all other processes of the method can be controlled to reduce the probable error in relative depth measurement over short distances to between 10 and 25 feet even when contouring a horizon at 5,000 or 6,000 feet in depth. Our first interest, therefore, is in the quality of seismogram which can be obtained and the certainty of correlations made between adjacent points. Figure 5 (*a*) to (*d*) are reproductions of typical seismograms from various areas.

Figure 5 (*a*) is a seismogram made by the McCollum Exploration Company in central East Texas. The prominent reflexion occurring at about 0.93 second is from the Austin chalk, a limestone member of the Upper Cretaceous which can be identified on reflexion seismograms throughout this area.

Figure 5 (*b*) shows a Nacatoch reflexion obtained by a Geophysical Service Inc. unit in the extreme north-east corner of the East Texas basin. The Nacatoch is a bed in the Navarro formation of Upper Cretaceous age consisting of sands and shales with a limestone member which is no doubt responsible for the reflexion.

Figure 5 (*c*) is a typical Oklahoma seismogram recorded by a Geophysical Service Inc. unit in central Oklahoma. The Oswego and Mississippi limestone reflexions occur at 0.72 and 0.87 second respectively. These beds, or their equivalents can be followed throughout central Oklahoma and parts of Kansas; they are responsible for the outstanding success of this method in the Mid-Continent area.

Figure 5 (*d*) reproduces a series of seismograms made by a Seismograph Service Corp. unit in northern Oklahoma. The event occurring on all these records at about 1.20 seconds is the Mississippi limestone reflexion on which correlation was based. This series of records demonstrates the degree of certainty normally achieved in correlation of seismic events in this area.

From a study of these seismograms I think it must be admitted that under a variety of conditions in widely scattered areas we are able to comply with the mechanical requirements of the method. We must now question the accuracy of depth determinations made from these seismograms.

ERRORS IN DERIVED DATA

Sources of error fall in three categories: (*a*) errors in correlation, (*b*) errors in time measurement, and (*c*) errors in computation of depths.

Errors in correlation may be of two kinds: the first would arise if two bursts of energy were present in an area as in the case of the seismogram reproduced in Figure 3, and at adjacent points the deeper

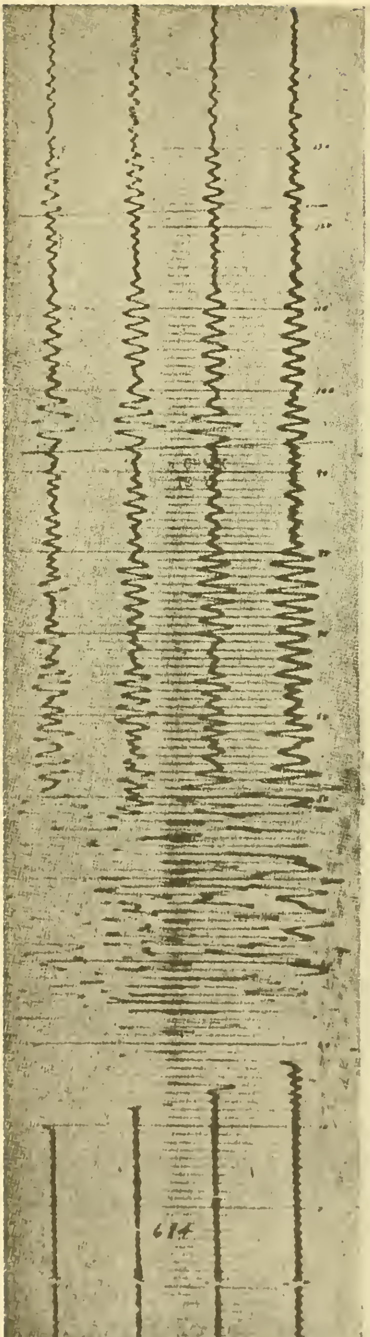

FIG. V (a) AUSTIN CHALK REFLEXIONS IN EAST TEXAS.

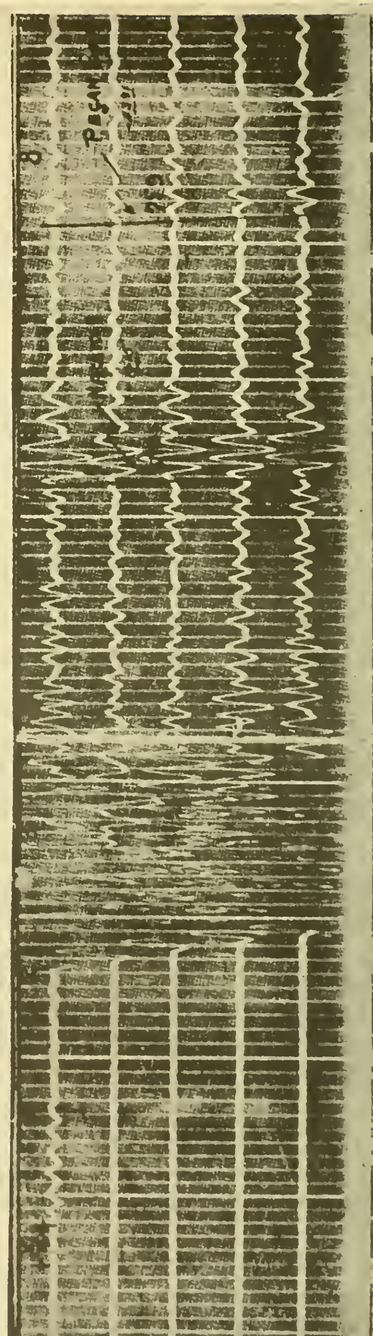

FIG. V (b) NACATOCH AND PECAN GAP REFLEXIONS IN EAST TEXAS.

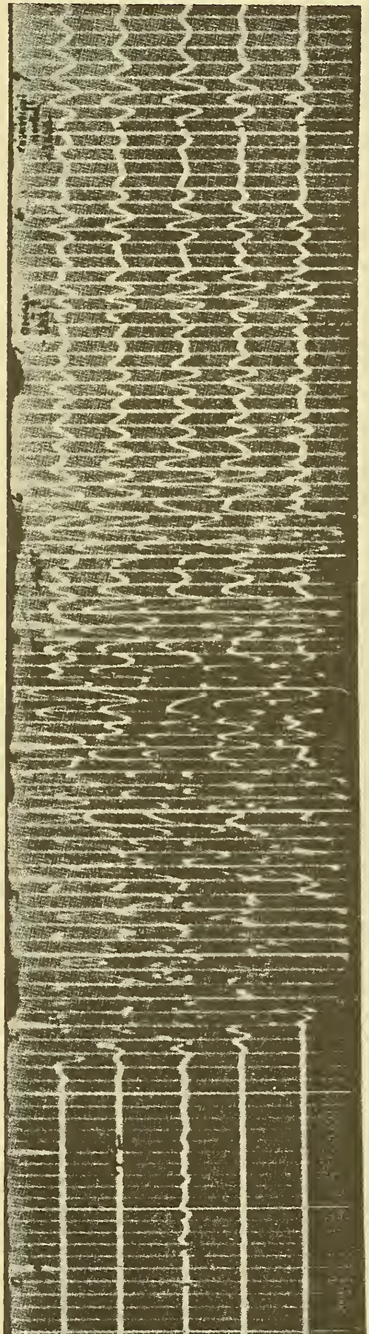

FIG. V (c) OSWEGO AND MISSISSIPPI LIMESTONE REFLEXIONS IN LINCOLN CO., OKLAHOMA.

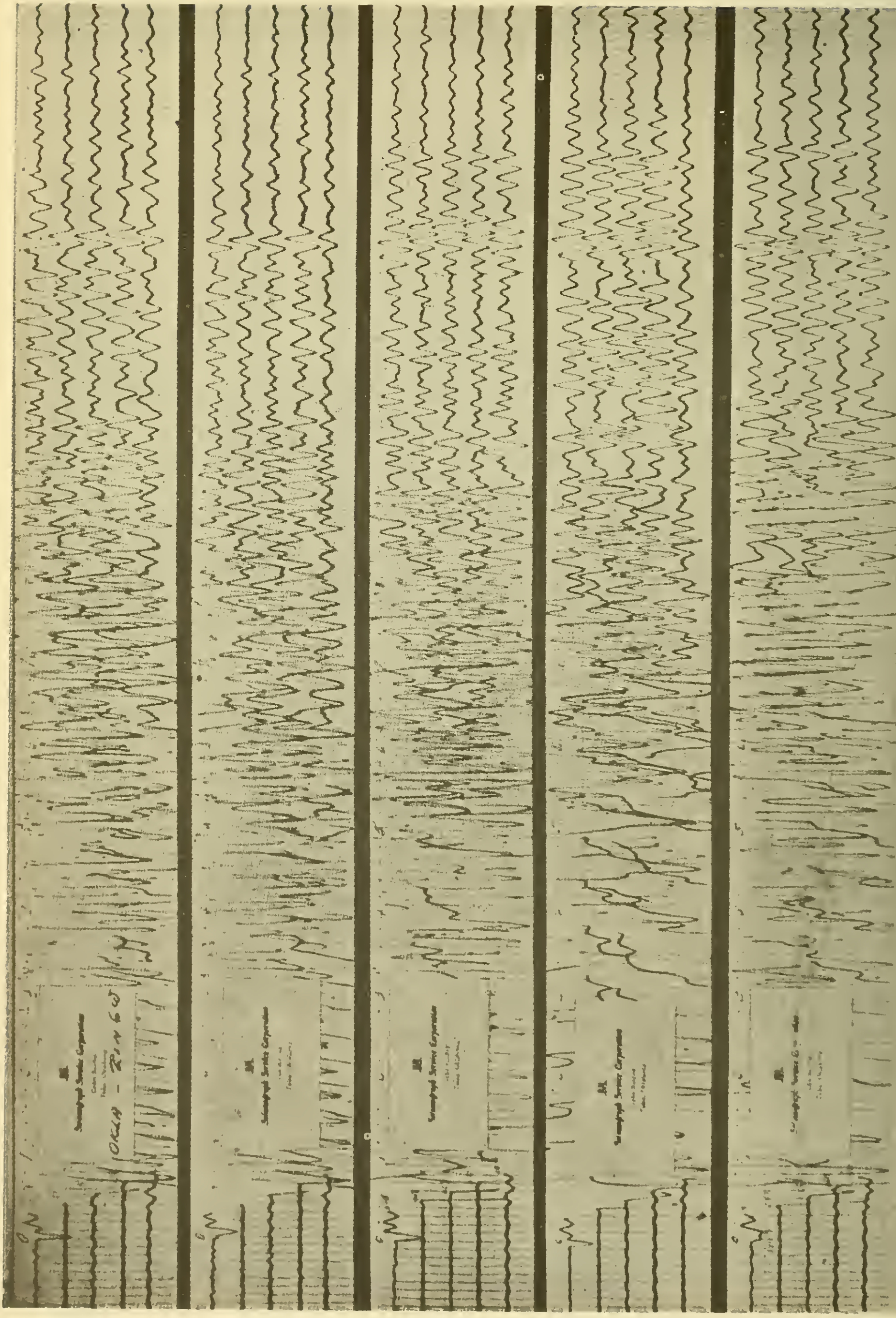

FIG. V (d) CORRELATION OF SEISMOGRAMS

event on one seismogram was correlated with the shallower event on the other; such errors may be of any magnitude. Errors of this nature are usually detected by the unreasonable structural interpretation resulting. It is, however, possible to correlate bursts of energy correctly between two seismograms and yet to choose different points as representing the commencement of the event in the two cases, resulting in the correlation being most probably in error by one complete cycle or wave length which may represent an error in computed depth of from 50 to 150 feet, depending on the average velocity of propagation and other factors. This is a common error and can best be eliminated by striving to obtain good quality seismograms in which the beginning of a burst of energy is readily identified. Where several horizons are being followed simultaneously and local geology leads one to believe their interval will remain uniform, this error is obviously under much better control, since it would disclose itself as apparent changing of interval between horizons being followed.

Errors in time measurement are entirely a matter of instrumental design and can readily be reduced to 0.1%, which rarely represents an error in computed depth of more than 10 feet. The most serious factor is usually the method used in recording the instant of explosion of the dynamite charge, as electric blasting caps frequently exhibit a disconcerting irregularity on time of explosion under differing conditions.

Errors in computation of depths mainly arise from incorrect assumed velocities of propagation for the various strata traversed by the wave front. Figure 6 represents a typical stratigraphic column in Oklahoma and also shows the velocity of propagation of the longitudinal wave in the various members. Tracing the velocity zones upwards from the Viola limestone we first encounter a series of limestones interspersed with shales, which zone has a rather high average velocity of propagation, being approximately 15,000 feet per second. The thick overlying series of Pennsylvanian shales and thin limestone beds is found to exhibit a velocity which decreases with decreasing depth, the range being about 13,000 to 9,000 feet per second. A zone some 100 feet thick which may possibly be caused by the ground water table is found to have a rather lower velocity of about 7,000 feet per second, and finally we have the surface zone, which is, I believe erroneously, sometimes referred to by seismologists as the "weathered" surface layer. This superficial zone in Oklahoma has a velocity of propagation of some 2,000 feet per second. My objection to the description "weathered" layer is that where the surface is cut by a ditch adjacent to a shot point I have many times observed that the surface weathering as understood by geologists is only a fraction of the thickness of this low velocity zone. We sometimes have recorded a thin surface layer which has a velocity of propagation lower than air and presumably is the actual weathered zone. The determination of the velocities of all but the upper 100 feet is best made by correlation with subsurface data obtained by drilling, but it is possible to derive an approximation by direct observation if a very long reflexion profile is shot; the slopes of the t^2-d^2 curves plotted for reflexions from

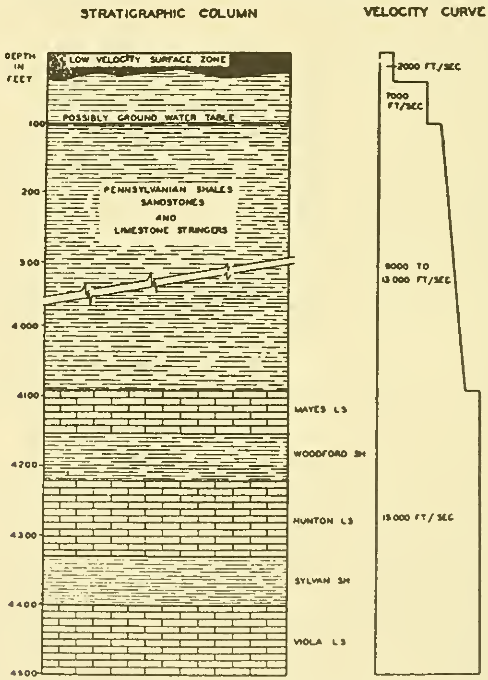

FIG. 6.—Typical stratigraphic column and velocity curve in Oklahoma.

various depths give a first approximation to this velocity-depth relationship. Velocities determined by refraction shooting will naturally almost invariably be too high.

Since it is obvious that an error in the assumed thickness of the surface zone will reproduce itself in the computed depth to a horizon below the Pennsylvanian multiplied by a factor of over six, determination of the thickness of this low velocity surface layer is a critical factor. This is best done by shooting a small charge close to the last seismometer on a spread and computing this zone from the resulting observed time-distance curve. Figure 7 demonstrates such a case;

care must be taken that the shot is sufficiently close to the first seismometer to give the true velocity of the surface zone. Corrections made in this manner should not leave a residual error in the computed Viola datum in central Oklahoma of more than 15 feet.

A typical example of the successful application of this method in Oklahoma is furnished by results obtained at Lucien, in Noblee County, reproduced in Figure 8; field-work was carried out by the Shell Petroleum Corporation. The surface evidence for this structure is very meagre and it can justly be claimed as a reflexion seismic discovery. Drilling carried out subsequent to completion of the seismic

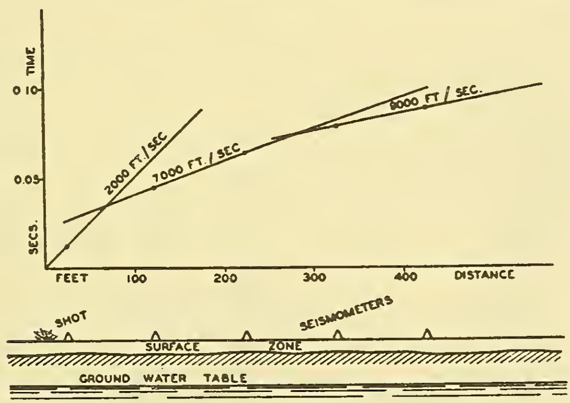

FIG. 7.—Showing method of correcting for surface zones.

survey has amply confirmed the general accuracy of the predictions. In deriving the contouring depicted in Figure 8 a group of reflexions from the Mississippi and Viola limestones were followed; over the center of the structure the upper of these limestones, the Mississippi, is missing and the chance of miscorrelating reflexions is therefore very great. It seems, however, that in this instance the pitfalls were avoided and the results achieved by the method can justly be described as brilliant.

MAPPING BY "DIP"

In many localities we are confronted with the problem of mapping structure in sediments which do not contain limestones or other reflecting horizons continuous over appreciable areas. Such a condition as is represented in Figure 9 can obviously not be investigated by a method involving correlation over more than a very limited distance. Experience has shown that in many such localities we do obtain reflexions from lenticular beds, and in the particular case of the Gulf Coastal Plain of Texas and Louisiana these reflexions frequently persist and are correlatable over distances in excess of 1,000 feet.

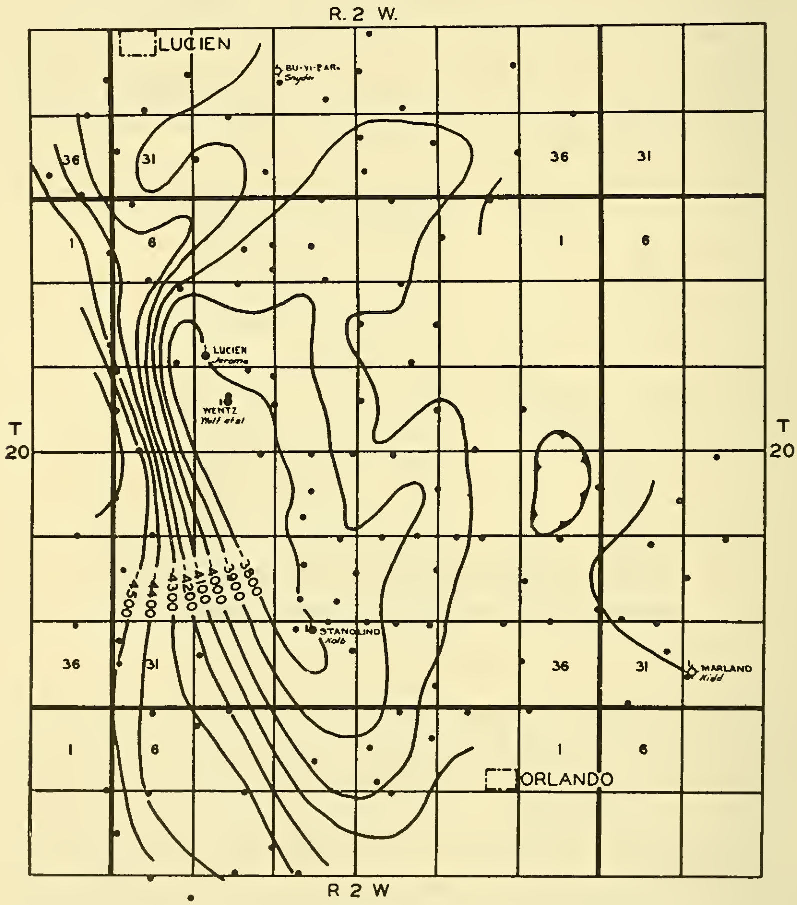

FIG. 8.—Lucien area, Noble County, Oklahoma. Contoured on Viola limestone data obtained by reflexion shooting.

Figure 10 (*a*) to (*c*) shows examples of typical seismograms obtained under such conditions.

Figure 10 (*a*). The shot point in this instance is located down dip from the seismometers as evidenced by the fact that the reflected energy is arriving almost parallel to the surface; in fact the last event indicated with an arrow on the record reaches the most remote seismometer ahead of the nearer seismometers. In this instance the shot point was 500 feet distant from the nearest seismometer and 1,100 feet from the most remote.

Figure 10 (*b*). The disposition of the shot point and seismometer spread is exactly as in Figure 10 (*a*), but now the shot point is located up dip from the recording unit, resulting in reflected events displaying a much slower apparent velocity of propagation.

Figure 10 (*c*). Here the shot point is located midway between the outside seismometers. It is obvious that under these conditions events reflected from horizontal strata will arrive simultaneously at each

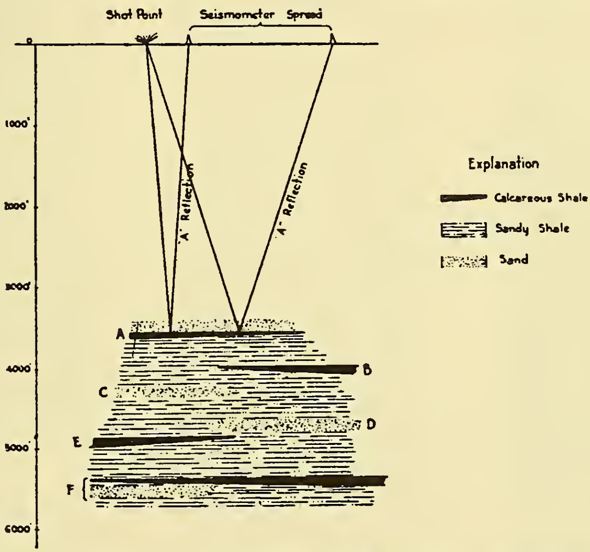

FIG. 9.—Sedimentary conditions necessitating "dip" shooting.

equidistant seismometer; they fail to do so on the seismogram reproduced, indicating that reflecting plans are inclined. The shot point was 300 feet distant from each of the outside seismometers.

The only evidence available on which to determine the dip is the time interval between arrival of a reflected event at the first and last seismometer. This interval is a function of the average velocity of propagation, distance from shot point to seismometer spread, distance between outside seismometers and the inclination of the reflecting plane. If the first three quantities are known the last can be computed. Figure 11 demonstrates a simple graphical method of deriving the dip of a reflecting surface from the running times of a reflected event to two points on a radial line from the shot point; it is assumed that the profile is laid out perpendicular to the strike.

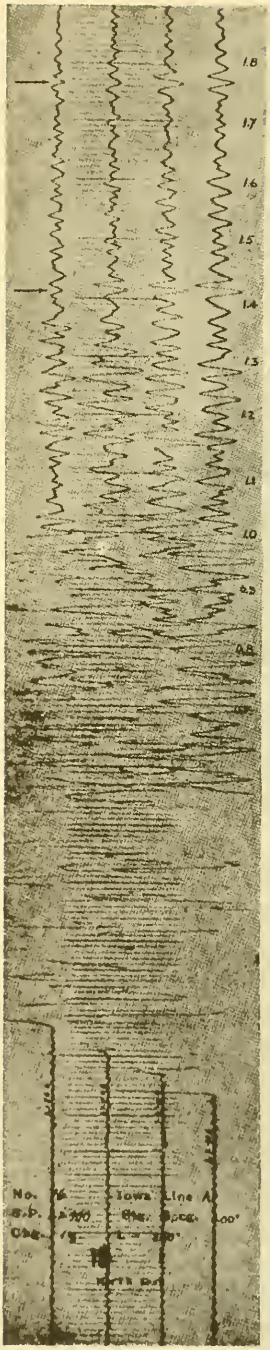

FIG. X (a) SHOT POINT LOCATED DOWN DIP.

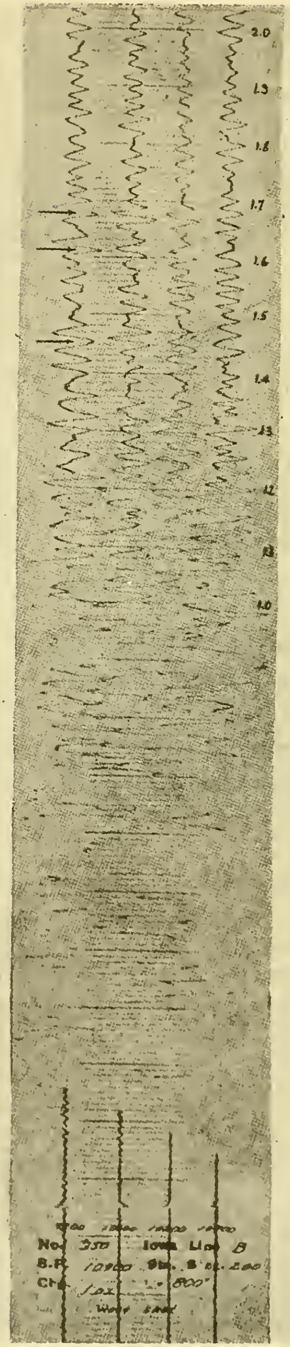

FIG. X (b) SHOT POINT LOCATED UP DIP.

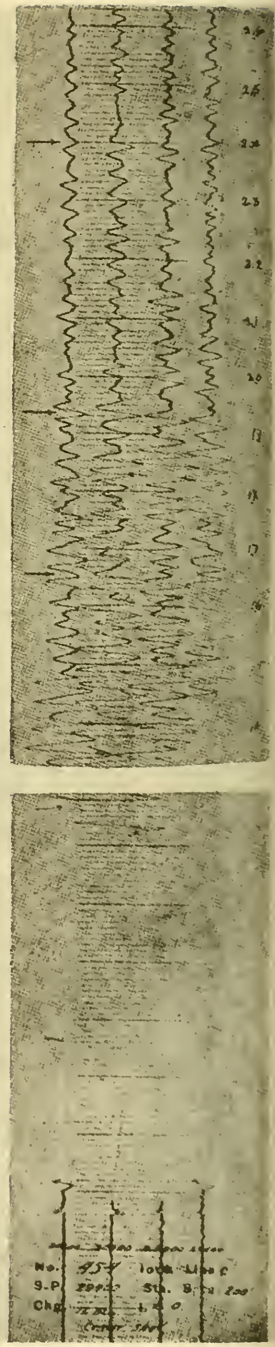

FIG. X (c) SHOT POINT LOCATED IN MIDDLE OF SPREAD

Since a relatively steep dip results in only a very small change in the time interval being measured, it is necessary that the correction for surface effects be either negligible or very carefully determined. To realise the importance of this I quote an actual example from the Gulf Coast of Texas. In measuring dips on a surface at about 6,000 feet in depth and using a seismometer spread of 600 feet, a dip of 5 degrees will produce a variation from the normal running time of 0.01 second. It is obvious that a similar error in determining the difference between corrections for surface layers at the extremities of the seismometer spread will produce an error of like amount in the computed dip. It is not the purpose of this paper to discuss possible errors in great detail, but a warning that many insidious possibilities of error exist must be given. The problem is strictly a three-dimensional one, and yet we usually derive our interpretation assuming only two dimensions. Propagation paths are curved, and yet our limited knowledge usually constrains us to assume straight line propagation.

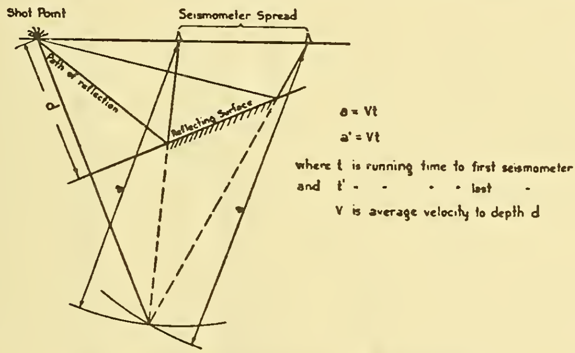

FIG. 11.—Graphical method of computing dip.

Nevertheless, this method also has achieved brilliant results. Figure 12 shows a profile obtained by this method across a deeply buried salt dome in southern Louisiana. This salt dome gives no surface indication of its presence, and yet by this method we can define the probable location of the crest of the associated uplift in the sediments to within limits of error of 1,500 feet.

POSSIBLE APPLICATIONS

The ideal conditions under which to apply reflexion shooting exist when a limestone or series of limestones are present in contact with shale or sand at depths of from 2,000 feet to 7,000 feet or even more; this condition guarantees the reflexion of energy. The best sur-

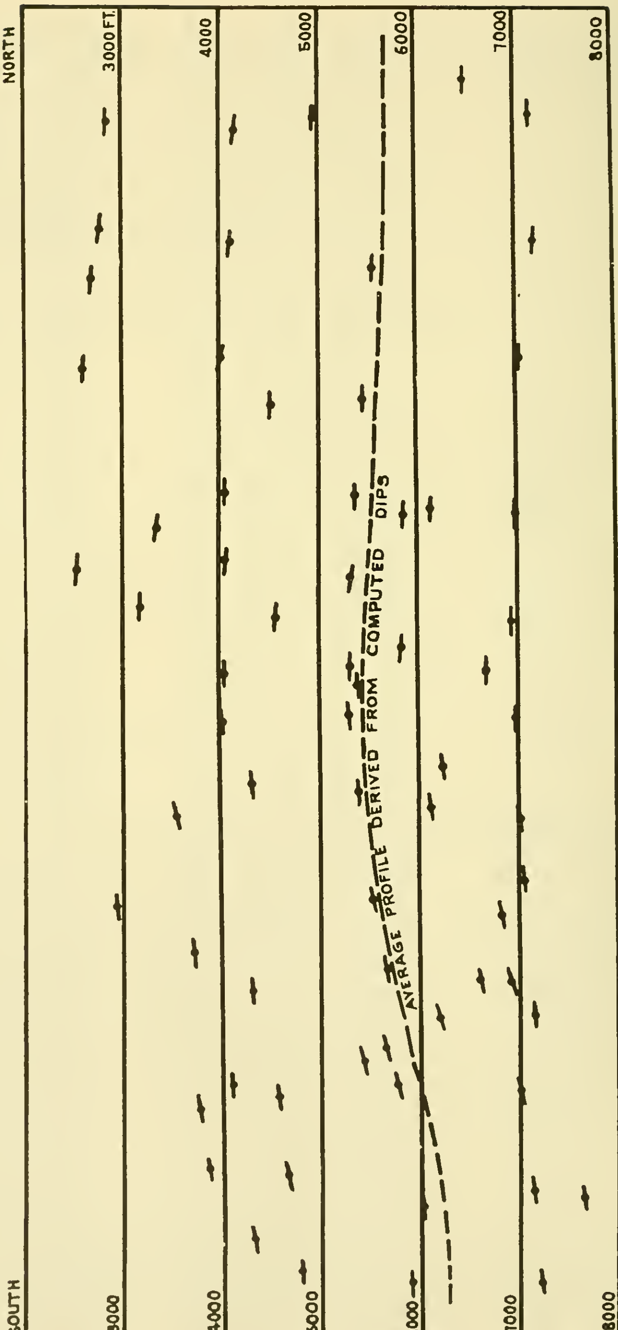

FIG. 12.—Typical dip profile obtained by reflexion shooting across deep seated salt dome in southern Louisiana.

face outcrop for our purpose is clay, and we have found that where the surface consists of soft sands or hard limestones our chances of success are much reduced. Special technique devised to overcome local difficulties has resulted in a measure of success being attained in nearly every area so far investigated, and I feel that the two methods of attack outlined in this paper should be competent to furnish results in most gently folded areas which have failed to yield surface evidence of sub-surface structure.

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STUDY OF EMERGENCE ANGLE AND PROPAGATION PATHS OF SEISMIC WAVES[1]

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SUMMARY

Travel-time curves are given for elastic waves through the earth in a region in which the velocity increases continuously with depth. These time-distance curves have been approximated by an equation of the form $X=aT^2+bT$ from which the velocity depth relation has been deduced. A new method for measurement of the emergence angle has been used and the values obtained agree reasonably well with those deduced from theoretical treatment of the time-distance curve. Formulas for the travel-time between any two points in the medium under consideration are derived. These formulas agree closely with two sets of direct measurements. In one of these the seismograph was placed at various distances directly beneath the explosion. In the second case it was placed at a fixed depth beneath the surface and the explosion was located on the surface at various distances. The approximate depth to bed rock is obtained from the time-distance curves by use of the formulas mentioned above.

The theory of the propagation of elastic waves through the earth in a region in which the velocity increases with depth in such a fashion that the observed time-distance curve may be represented by the equation

$$X = aT^2 + bT \qquad (1)$$

has been treated by Ewing and Leet.[3] The present paper includes an extension of the theory and an experimental investigation of many points in connection therewith. The data used were taken near Green Pond, Northampton County, Pennsylvania. The material at the surface was a fine grained alluvial deposit, which was underlain by limestone. The surface of the limestone was probably quite irregular.

TIME-DISTANCE DATA AND VELOCITY-DEPTH DETERMINATION

The time-distance data are shown in Figure 1. The curved lines drawn represent Eq. (1) with

$$a = 29,690 \text{ ft./sec.}^2, \qquad b = 752.9 \text{ ft./sec.}$$

[1] Reprinted from *Physics*, Vol. 5, No. 10 (October, 1934), pp. 317–20, with the permission of the editor of *The American Physical Society* and the authors. Manuscript received by *Physics*, May 24, 1934.

[2] Lehigh University.

The values of these constants were obtained by a least square solution which included all points representing wave paths lying entirely in the upper laver. By use of the following equations

$$V = 2aT + b, \tag{2}$$

$$b/\sin\theta_0 = v/\sin\theta \doteq V, \tag{3}$$

$$P = b^2/8\pi a(\sinh q - q),^3 \tag{4}$$

$$q = 2 \text{ arc cosh } V/b$$

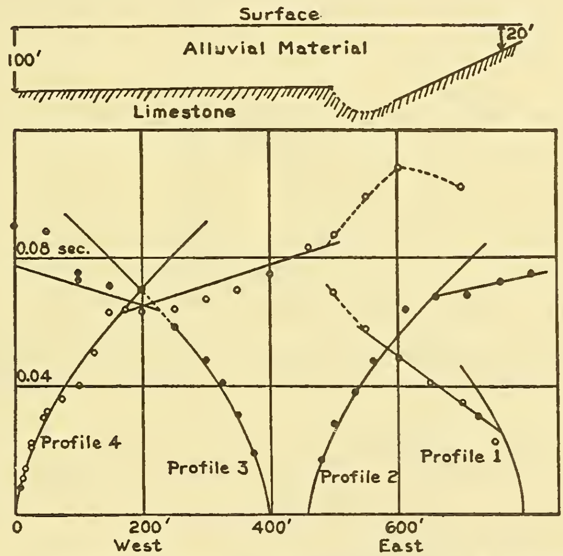


FIG. 1.—Time-distance curves and approximate section at Green Pond.

the velocity depth relation is determined. Here V is the velocity at the midpoint of the path of length X and maximum penetration P,

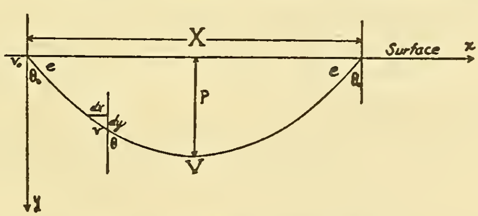


FIG. 2.—Diagram of wave path.

and θ_0 is the emergence angle for this path. Figure 2 illustrates the meaning of each of these symbols. The velocity-depth curve thus obtained is shown in Figure 3.

[3] Ewing and Leet, *Trans. A.I.M.E., Geophysical Prospecting, 1932.*

EMERGENCE ANGLE

A new method was used to measure the emergence angle θ_0 of the waves. The seismograph was of the type technically known as a

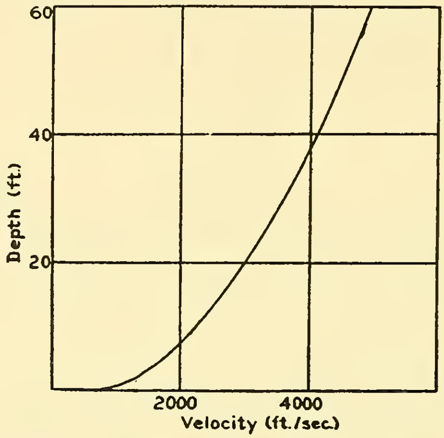


FIG. 3.—Velocity-depth relation (Green Pond).

geophone. It was originally designed to measure only vertical motions but after an alteration of the spring suspension was made it would

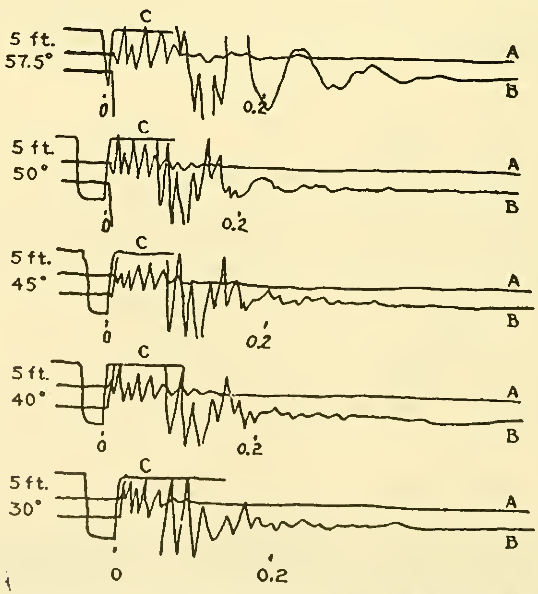


FIG. 4.—Seismograms illustrating method used to determine emergence angle. Angles given represent tilt of seismograph from vertical position. *A*, low sensitivity; *B*, high sensitivity; *C*, timing line.

operate when tilted at any angle between the vertical and the horizontal. The tilting produced some change in the constants of the instrument, but this change was of no importance in the investigation. The idea of the experiment was to tilt the geophone toward the shot until its axis became perpendicular to the direction of the first motion produced by the shot. Successive tests were made to determine the angle of tilt at which the direction of first motion on the seismogram showed a reversal. (This angle could easily be determined to one or two degrees in the range of distances studied.) Figure 4 shows

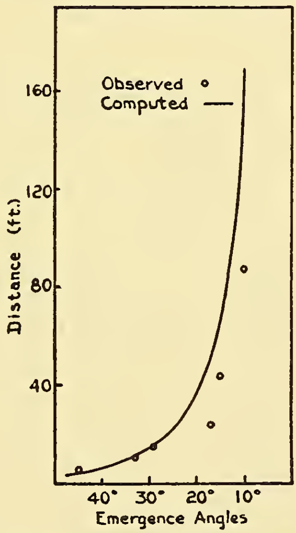


FIG. 5.—Relation of emergence angle to distance traversed.

the set of observations taken at 5 feet. In Figure 5 emergence angle θ_0 is plotted against the distance traversed by the wave along the surface. The curve drawn represents the theoretical values of the angle as determined from Eq. (3) using the values of a and b obtained from the time-distance curves. The observed values, indicated by the points, agree with the calculated values within the limits of experimental error. It was not considered necessary to correct for the difference between true and apparent emergence angles.[4]

TRAVEL-TIME BETWEEN ANY TWO POINTS ON A PATH

From Eq. (4) the relation

$$dy/dv = (v^2 - b^2)^{1/2} 2\pi a, \tag{5}$$

[4] E. Wiechert, *Gött. Nachr.* (1907), No. 1.

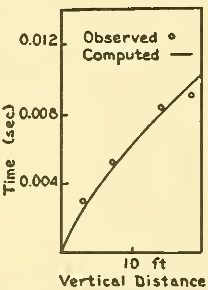
where v is the velocity at the depth y, is obtained. Eq. (2) yields

$$dy/dx = \pm\, (b^2 \csc^2 \theta_0 - v^2)^{1/2}/v, \tag{6}$$

which may be combined with (5) to give the relation:

$$x = \pm\, (1/2\pi a) \int_b^v v\left[(v^2 - b^2)/(b^2 \csc^2 \theta_0 - v^2)\right]^{1/2} dv$$
$$= (b^2 \cot^2 \theta_0/4\pi a)\left[\alpha - \sin\alpha \cos\alpha\right], \tag{7}$$

where $\sin\alpha = \pm \tan\theta_0(v^2/b^2 - 1)^{1/2}$. The positive sign should be taken if the wave has not reached its maximum depth.



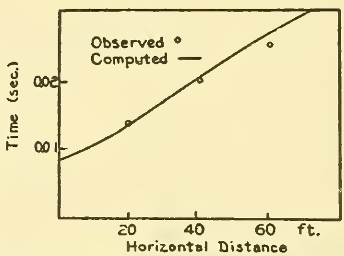


FIG. 6.—Travel-time curve for waves travelling vertically.
FIG. 7.—Travel-time curve for waves travelling from surface to depth of 18 feet 6 inches

The travel-time $t = \int ds/v$ becomes, upon substitution from (5) and (6)

$$t = \pm\, (b/2\pi a \sin\theta_0) \int_b^v (1/v) \times \left[(v^2 - b^2)/(b^2 \csc^2 \theta_0 - v^2)\right]^{1/2} dv$$
$$= (b/2\pi a \sin\theta_0) \times \left[\alpha - \sin\theta_0 \tan^{-1}(\tan\alpha/\sin\theta_0)\right]. \tag{8}$$

For the limiting case, $\theta_0 = 0$, the path is vertical and (7) and (8) become

$$x = 0 \tag{7'}$$
$$t = (b/2\pi a)\left[(v^2/b^2 - 1)^{1/2} - \tan^{-1}(v^2/b^2 - 1)^{1/2}\right]. \tag{8'}$$

Formulas (7) and (8) make it possible to calculate the time required to traverse any part of any path. Two sets of data were taken to which these formulae may be applied. In the first case records were taken with the geophone buried at a series of depths up to about 20 ft., the shot in each case being at the surface directly above. In the second case the geophone remained at a depth of 18.5 ft. while shots were fired on the surface at various distances from the mouth of the hole. Figs. 6 and 7 compare the travel-time obtained from these observations with those computed from the above equations using the values of a and b as computed from the time-distance curves.

WAVES REFLECTED OR REFRACTED AT DISCONTINUITIES

By use of the formulas developed in the preceding section it is possible to calculate time-distance curves for waves reflected or refracted at the surface of a layer of rocks covered by material in which the velocity increases with depth in the manner discussed above. For a given depth the velocity v is given by Eq. (4). From Eqs. (7) and (8) it is possible to calculate time-distance curves for waves reflected at this depth by assuming a series of values for θ_0. If the reflecting surface is horizontal the time and the distance will be just double that calculated from these equations. If the reflecting surface is sloping it is necessary to make separate calculations for the incident and the reflected waves.

In the case of waves which are refracted along the surface of a buried rock the angle of emergence is known from the velocity of the refracting layer. If, then, a depth is assumed the horizontal distance and the time of the wave in the upper layer can be computed from the value of the emergence angle and the velocity in the upper layer corresponding to the depth assumed, as in the case of reflected waves. The remainder of the distance traversed is in the higher speed material and the total time is the sum of the time spent there and the time of the incident and emergent rays in the upper layer.[5] For waves refracted along a sloping layer, the computations become increasingly more difficult as the emergence angle depends not only on the velocity of the high speed layer but also upon its depth.

Although the primary purpose of the present work was to study waves propagated in the upper layer, the data represented in Figure 1 afford a good illustration of refracted waves. The straight lines drawn in Profiles 3 and 4 represent an assumed horizontal layer which is 100 ft. beneath the surface and has a velocity of 17,000 ft./sec. Because of irregularities in the surface of the rock the observed points deviate considerably from the lines, but the depth assumed is certainly accurate to 5 or 10 per cent. Profiles 1 and 2 indicate a limestone surface sloping up sharply to the east. By the aid of Figure 6 the depth at the east end may be estimated at approximately 20 ft. In the region between the 500 and 600 ft. stations there are systematic deviations marked by dotted lines on the time-distance curve. These indicate a fairly sharp depression in the surface of the limestone between these stations. The deviations of the points in general are an indication of the roughness of the limestone surface. An approximate cross section of this region is shown in Figure 1.

[5] Rutherford, *Amer. Geophys. Union. Trans.* (1933), p. 292.

Professor B. L. Miller of the Department of Geology, of Lehigh University, who recommended the location which was chosen for these experiments, has suggested that the structure of the limestone revealed by these results represents the buried valley of a pre-glacial stream.

We are indebted to the Geophysical Research Corporation for the seismic instruments, to the Trojan Powder Company for the explosives, to Mr. H. S. Snyder upon whose property the experiments were carried out, and to Mr. A. M. Thorne, Jr., for assistance with the experimental observations.

THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
TRANS. SOCIETY PETROLEUM GEOPHYSICISTS, VOL. V (MARCH, 1935), PP. 161-179, 7 FIGS.

THE RAPID ADJUSTMENT OF OBSERVATIONS IN A NETWORK OF GEOPHYSICAL STATIONS BY THE METHOD OF LEAST SQUARES[1]

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ABSTRACT

A frequent problem in geodesy and geophysics concerns the adjustment of a set of values for a physical magnitude at a number of stations. The values are obtained from other observed magnitudes by computations involving the geometrical links of the station network, and there is ambiguity on account of the multiplicity of connections. Since the magnitude desired is single-valued at each station, the values obtained are usually adjusted by the method of least squares. Where the number of stations is large and the network is complex, the normal equations for least-square adjustment are very numerous and their solution by standard procedure is tedious. The paper develops a method of establishing the normal equations, and solving them by successive approximations, which is simple, rapid and satisfactory in practice, and is applicable to any network, however large and complex. Although particular attention is focused on the problem of obtaining isogams from observations of gravity gradients in applied geophysics, the method has obvious applications in other fields where the mathematical conditions are similar.

§1. INTRODUCTION

In many branches of geodesy and geophysics, observations of physical magnitudes are made at a series of stations which form a network of triangles or polygons. From these measured magnitudes it is frequently desired to compute the values of other magnitudes at the stations concerned. By reason of the relationship between the observed and computed magnitudes, and the geometry of the network, it usually happens that there is ambiguity in the final values attained, and it is customary to apply the method of least squares to resolve the ambiguity.

A particular example occurs in applied geophysics in the computation of isogams from observations of the gradients of gravity at stations of the network. The difference in gravity $g_B - g_A$ at two stations A, B of the network may be obtained by taking any path formed of straight lines joining adjacent stations between A and B, assuming that the average gradient of gravity along the rectilinear

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[1] From *Proc. Phys. Soc. London*, Vol. 45 (November 1, 1933), pp. 792–807. Reprinted with the permission of *The Physical Society of London* and the author. Communicated by Professor A. O. Rankine, July 28, 1933.

sections of the path is equal to the mean of the values at the two stations connected by it, and taking the sum of the sectional differences of gravity thus computed. Thus, if P and Q are any two consecutive stations on the path chosen, and if their coordinates are (x_1, y_1) and (x_2, y_2) whilst their component gravity-gradients are (X_1, Y_1) and (X_2, Y_2) respectively, we have

$$2(g_B - g_A) = \sum_A^B [(X_1 + X_2)(x_2 - x_1) + (Y_1 + Y_2)(y_2 - y_1)].$$

If, however, we take another path connecting A to B and involving other intervening stations, we shall usually obtain a different value for $g_B - g_A$.

This is a particular case of a more general problem concerned with single-valued functions, and it is with this general class that we are concerned.

§2. THE PROBLEM

Given a network of stations $A, B, C, \cdots$ and the observed or computed values of the increments $(ab), (bc), (ca), \cdots$ between adjacent stations of a single-valued function U, we have to find the best possible values of U at $A, B, C, \cdots$.

The problem resolves itself into one which may be expressed thus: To find the adjusted values $(AB), (BC), (CA), \cdots$ of the increments $(ab), (bc), (ca), \cdots$, respectively, where, around any closed polygonal path $ABC \cdots PA$, we have the condition that

$$(AB) + (BC) + (CD) + \cdots + (PA) = 0,$$

whereas, in general

$$(ab) + (bc) + (cd) + \cdots + (pa) \neq 0 \text{ but } = \text{some value } d.$$

We may call d the "excess" around the path $ABC \cdots PA$.

In the particular cases of gravitational and magnetic observations in applied geophysics, this problem and its solution by the method of least squares have been considered by I. Roman.[2] Roman states the problem in a different but equivalent manner and obtains rather more complex normal equations, which he solves by the usual reduction method of Gauss.

The present writer believes that the treatment given below is particularly adapted to this problem. The normal equations are simple and can be written down by inspection. Methods of resolution of these equations are suggested which are much less laborious than the Gaussian reduction methods, and permit the labour of reduction to

[2] *Trans. Amer. Inst. Min. Metall. Engrs., Geophysical Prospecting,* 1932, p. 460.

be accommodated to the particular degree of accuracy in adjustment which is considered worth while. Most practical geophysicists are agreed that an elaborate, ultra-precise least-square adjustment of their observations is an unwarranted expenditure of time and labour. On the other hand, some adjustment is obviously necessary.

§3. THE NORMAL EQUATIONS

In the terms used in treatises on the method of least squares[3] we have given a series of "conditional" equations

$$(AB) = (ab),$$
$$(BC) = (bc),$$
$$(CA) = (ca),$$
$$\cdots\cdots$$

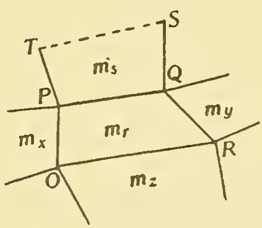

FIG. 1

together with a series of "rigorous" equations

$$(AB) + (BC) + (CA) = 0,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots.$$

There is one conditional equation for every link in the network, and one rigorous equation for every *independent* polygon in the network. We shall assume that all the observations and link-increments such as (ab) have equal weight.

For such a system of conditional and rigorous equations, the normal equations for a least-square adjustment are obtained as follows. For each rigorous equation, provide an unknown multiplier or correlate m. The unknown increment (PQ), which occurs in one only of the conditional equations, namely $(PQ)-(pq)=0$, occurs also in at most two of the rigorous equations. For normally it will be a common section of the boundary of two adjacent polygons, e.g., $OPQR$, $SQPT$, Figure 1, and at the extreme boundaries of the network one of these

[3] E. T. Whittaker, and G. Robinson, *The Calculus of Observations*, chap. IX, and particularly §129, pp. 252–254 (1924).

polygons will be absent. The increments (pq), (PQ) are of course algebraical quantities, so that $(pq) = -(qp)$, $(PQ) = -(QP)$.

If we assume that the polygonal circuits such as $OPQR$ are always traversed in the same direction, e.g., clockwise, then (PQ) occurs positively in $OPQR$, but negatively in $SQPT$.

For such a link as (PQ) there will result a normal equation

$$(PQ) = (pq) - m_r + m_s, \tag{1}$$

where m_r and m_s are the multiplier correlates attached to the polygons $OPQR$ and $SQPT$ respectively.

Similarly for every other link, except on the boundary of the network where one of the m's will be absent.

Since the rigorous equations continue to be satisfied, we can replace every term such as (PQ) in them by its equivalent in the above equation. We thus get a series of equations, one for each independent polygon, of the form (see Figure 1).

$$(op) - m_r + m_x + (pq) - m_r + m_s + (qr)$$
$$- m_r + m_y + (ro) - m_r + m_z = 0,$$

where the polygons x, y and z as well as s also touch polygon r at OP, QR and RO respectively.

Thus, for a four-sided polygon r, having contacts with four others x, s, y and z, we have an equation

$$4m_r - m_x - m_s - m_y - m_z = (op) + (pq) + (qr) + (ro) = d_r,$$

where d_r is the observed or computed excess around the polygon. Similarly, for an n_r-sided polygon r, which has contacts with adjacent polygons x, s, y, $\cdots$, we have an equation

$$n_r m_r - m_x - m_s - m_y - \cdots = d_r. \tag{2}$$

There is one such equation for each independent polygon of the network, so that, for the set of unknown multipliers or correlates m, we have an equal number of linear equations such as (2). From these we can obtain uniquely each of the correlates m_1, m_2, $\cdots$, and by using equations (1) we can then obtain quite simply each adjusted link increment (PQ). The whole procedure thus hinges on the solution of the set of correlate equations (2). As has been stated and demonstrated above, these can be written down for any network by inspection.

§4. SOLUTION OF THE CORRELATE EQUATIONS. APPROXIMATIONS

The series of correlate equations of the type

$$n_r m_r - \Sigma m_x = d_r$$

can be solved by any of the methods appropriate to linear equations, including that of Gauss. This series is, however, obviously adapted to solution by gradual approximation. Every coefficient is unity except those for the key correlates such as m_r, and these are integers not less than 3.

As to the first approximation, we can put

$$m_r = m_r' + \Delta m_r,$$

where

$$m_r' = d_r/n_r.$$

Similarly

$$\left.\begin{aligned} m_x' &= d_x/n_x \\ m_y' &= d_y/n_y, \\ &\cdots\cdots \end{aligned}\right\} \qquad (3)$$

whence

$$\Delta m_r = \frac{1}{n_r}\Sigma m_x = \frac{1}{n_r}\Sigma d_x/n_x + \frac{1}{n_r}\Sigma\Delta m_x.$$

Writing

$$m_r'' = \frac{1}{n_r}\Sigma d_x/n_x, \qquad (4)$$

we have $m_r'+m_r''$ as a second approximation to m_r.

Similarly we can find the values m_x'', m_s'', $\cdots$ and obtain a third approximation $m_r'+m_r''+m_r'''$ to m_r by writing

$$m_r''' = \frac{1}{n_r}\Sigma m_x''. \qquad (5)$$

The procedure can be continued as far as desired, and can be terminated at any stage when the added increments to m_r', m_r'', m_r''', become sufficiently small. It is simple and lends itself to routine execution.

Although exact solutions for particular networks, and methods of more rapid approximation, will be discussed later, we may test this particular method for the case discussed by Roman in the paper previously cited.

Figure 2 illustrates the network solved by Roman. There are seven station points and six triangles. In Table I, column d gives the original excesses for each triangle. The successive approximations to the m's are calculated from these d's and the previously obtained approximations for the adjacent triangles given in the third column. The residuals do not disappear very rapidly, but the procedure is easy and rapid.

TABLE I

Δ	d	Adjacent Δs	m'	$3m''$	m''	$3m'''$	m'''	$3m^{iv}$	m^{iv} . . .
1	+ 5	2+5	1.667	5	1.667	−3.333	−1.111	0.741	0.247 . .
2	+10	1+3	3.333	−1.666	−0.556	0.556	0.185	−1.481	−0.494 . . .
3	−10	2+4	−3.333	−3.334	−1.111	−0.110	−0.370	−1.111	−0.370 . . .
4	−20	3+5	−6.667	−1.666	−0.556	−3.889	−1.296	0.186	0.062 . . .
5	+ 5	1+4+6	1.667	−8.333	−2.778	1.667	0.556	−3.333	−1.111 . . .
6	−10	5	−3.333	1.667	0.556	−2.778	−0.926	0.556	0.185 . . .

The values for m obtained by summing the first four elements m' to m^{iv} are within 1.0 of the correct values, and would be quite near enough in practice for obtaining the values of gravity at the stations and drawing up the isogam chart. The original link-increments from

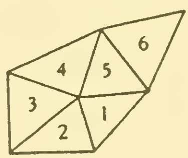

FIG. 2.

which the d's were calculated were of the order of 50 units, so that the final values adjusted on the basis of this fourth approximation would be correct within 2 per cent, which is the limit for gravity work of this type. The final values, obtained by putting m equal to $m'+m''+m'''+m^{iv}$ are $m_1=2.47$, $m_2=2.47$, $m_3=-5.19$, $m_4=-8.46$, $m_5=-1.67$, $m_6=-3.52$, which should be compared with the values obtained in the next section.

A more rapid approximation in the solution of the set of correlate equations

$$n_r m_r - \Sigma m_x = d_r$$

can be obtained by solving partially for those which involve m_r, as follows.

We have, say,

$$n_r m_r - m_x - m_s - m_y - m_z = d_r,$$

$$n_x m_x - m_r - m_a - m_b - \cdots = d_x,$$

$$n_s m_s - m_r - m_e - m_f - \cdots = d_s,$$

$$n_y m_y - m_r - m_h - m_k - \cdots = d_y,$$

$$n_z m_z - m_r - m_p - m_q - \cdots = d_z.$$

From which we can obtain, by dividing the second by n_x, third by n_s, · · · and adding

$$m_r\left(n_r - \Sigma\frac{1}{n_x}\right) - \Sigma\frac{1}{n_x}(m_a + m_b + \cdots) = d_r + \Sigma\frac{d_x}{n_x}.$$

For a first approximation m_r' we can put

$$m_r' = \left(d_r + \Sigma\frac{d_x}{n_x}\right) \Big/ \left(n_r - \Sigma\frac{1}{n_x}\right), \tag{6}$$

and subsequent increments to this are given by

$$m_r'' = \left(\Sigma\frac{1}{n_x}(m_a' + m_b' + \cdots)\right) \Big/ \left(n_r - \Sigma\frac{1}{n_x}\right), \tag{7}$$

$$m_r''' = \left(\Sigma\frac{1}{n_x}(m_a'' + m_b'' + \cdots)\right) \Big/ \left(n_r - \Sigma\frac{1}{n_x}\right), \tag{8}$$

. . . .

When all the polygons concerned have the same number n of sides, the formulae become

$$m_r' = (nd_r + \Sigma d_x)/(n^2 - n_r'), \tag{9}$$

where n_r' is the number of polygons adjacent to, or at "first remove" from, the key polygon r.

Also

$$\left.\begin{aligned} m_r'' &= \Sigma m_a'/(n^2 - n_r') \\ m_r''' &= \Sigma m_a''/(n^2 - n_r') \\ &\cdots\cdots \end{aligned}\right\}. \tag{10}$$

Here Σ denotes the series of polygons excluding r which are at second remove from r, i.e., are adjacent to these at first remove from r.

We can again illustrate this system of approximation with reference to the network of six triangles previously considered, Figure 2. Table II defines the constants concerned in each triangle, n being equal to 3 throughout.

TABLE II

Δ	d	1st-remove Δs	2nd-remove Δs	$n^2 - n_r'$	$nd_r + \Sigma d_x$
1	+5	2, 5	3, 4, 6	7	30
2	+10	1, 3	4, 5	7	25
3	−10	2, 4	1, 5	7	−40
4	−20	3, 5	1, 2, 6	7	−65
5	+5	1, 4, 6	2, 3	6	−10
6	−10	5	1, 4	8	−25

In Table III, where $N_r = n^2 - n_r'$, the approximations are shown worked out to m^{v}, at which stage the residues are well below 0.1 and the values obtained for the m's are within 0.02 of the correct values. This would be an over-elaborate refinement in practice, but illustrates the rapidity of approximation obtainable by the method.

TABLE III

N	Δ	m'	Δ''	$+m''$	$m''-$	$+m'''$	$m'''-$	$+m^{iv}$	$m^{iv}-$	$+m^{v}-$	m
7	1		3		5.714	0.374			0.421	0.020	4.347
			4		9.286	0.676			0.683	0.019	−2.798
			6		3.125		0.625		0.239	0.078	
					18.125	0.425			1.343	0.117	
		4.286	m		−2.589	0.061			−0.192	−0.017	1.549
7	2		4		9.286	0.676			0.683	0.019	3.617
			5		1.667		0.357		0.199	0.063	−1.703
					10.953	0.319			0.882	0.082	
		3.571	m		−1.565	0.046			−0.126	−0.012	1.914
7	3		1	4.286			2.589	0.061		0.192	0.374
			5		1.667		0.357		c.199	0.c63	−6.191
				2.619			2.946		0.138	0.255	
		−5.714	m	0.374			−0.421		−0.020	−0.036	−5.817
7	4		1	4.286			2.589	0.061		0.192	
			2	3.571			1.565	0.046		0.126	0.676
			6		3.125		0.625		0.239	0.078	−10.045
				4.732			4.779		0.132	0.396	
		−9.286	m	0.676			−0.683		−0.019	−0.059	−9.369
6	5		2	3.571			1.565	0.046		0.126	
			3		5.714	0.374			0.421	0.020	
					2.143		1.191		0.375	0.146	
		−1.667	m		−0.357		−0.199		−0.063	−0.024	−2.310
8	6		1	4.286			2.589	0.061		0.192	
			4		9.286	0.676			0.683	0.019	
					5.000		1.913		0.622	0.211	
		−3.125	m		−0.625		−0.239		−0.078	−0.026	−4.093

The final values obtained are

$$m_1 = 1.55,\ m_2 = 1.91,\quad m_3 = -5.82,$$

$$m_4 = -9.37,\ m_5 = -2.31,\ m_6 = -4.09.$$

§5. RECTANGULAR NETWORKS

The case of a rectangular network has a particular importance, since it corresponds to an ideal arrangement of stations often used in practical geophysical surveys on suitable terrain.

Referring to equations (9) and (10) of §4, we have

$$n = 4 \text{ and } n_r' = 1, 2, 3 \text{ or } 4 \text{ generally.}$$

$$\therefore N_r = n^2 - n_r' = 12, 13, 14 \text{ or } 15,$$

and we can solve by successive approximation, as in the case of triangles, using the formulae

$$\left.\begin{aligned} m_r' &= (4d_r + \Sigma d_x)/N_r \\ m_r'' &= \Sigma m_a'/N_r \\ m_r''' &= \Sigma m_a''/N_r \\ &\cdots\cdots\cdots\cdots \end{aligned}\right\}, \qquad (11)$$

which give approximations approaching the complete solution m_r with rapidity rather greater than in the case of triangles.

It should be carefully noted that, throughout the above reasoning, we have assumed that each first-remove and second-remove polygon is independent. If any polygon occurs as a common second-remove to two first-remove polygons, it must be counted twice in the sums $\Sigma m_a', \Sigma m_a'', \cdots$.

§6. SOLUTIONS OF INCREASING PRECISION. RECTANGULAR NETWORKS

In networks which have a regular pattern, whether of triangles or rectangles, formulae can be developed for expressing the correlate of any particular figure (polygon) in terms of the excesses of its surrounding figures, to any degree of remoteness.

Previously we have only considered the elimination of the first-remove polygons, leaving the second-remove ones as first residuals. We next consider how to eliminate polygons of second and greater remoteness.

Figure 3 shows a rectangular network in which the rectangles are numbered from one corner 00 in rows and columns.

00	01	02	03	04
10	11	12	13	14
20	21	22	23	24
30	31	32	33	34

FIG. 3

Any rectangle has a number of first-remove rectangles varying from 2 to 4 according to its position at a corner of the complete network, on the outer run or in the interior of the network.

From the geometry, each first-remove rectangle may have contact with a second-remove rectangle which is common to another first remove. For instance, 11 is common to 01 and 10, both first removes of 00.

We shall first find expressions for the correlate of any rectangle, eliminating both the first-remove and the common second-remove correlates. The corner rectangle 00 has two first removes and one common second remove, and the correlate equations are

$$\begin{aligned} 4m_{00} - m_{01} - m_{10} \qquad\qquad\qquad\qquad &= d_{00}, \\ -m_{00} + 4m_{01} \qquad - m_{11} - m_{02} \qquad &= d_{01}, \\ -m_{00} \qquad + 4m_{10} - m_{11} \qquad - m_{20} &= d_{10}, \\ - m_{01} - m_{10} + 4m_{11} - m_{12} - m_{21} &= d_{11}. \end{aligned}$$

Multiplying the equations by 7, 2, 2 and 1 respectively and adding, we eliminate m_{01}, m_{10} and m_{11} and obtain

$$\begin{aligned} 24m_{00} - 2(m_{02}+m_{20}) - (m_{12}+m_{21}) &= 7d_{00} + 2(d_{01}+d_{10}) + d_{11}, \qquad (12) \\ &= 24D_{00}, \text{ say,} \end{aligned}$$

and $m_{00}' = D_{00}$ gives a good approximation to m_{00}, which can be continued by writing

$$m_{00} = m_{00}' + m_{00}'' + m_{00}''' + \cdots ,$$

where

$$\begin{aligned} 24m_{00}'' &= 2(m_{02}' + m_{20}') + m_{12}' + m_{21}', \\ 24m_{00}''' &= 2(m_{02}'' + m_{20}'') + m_{12}'' + m_{21}'', \\ &\cdots\cdots\cdots\cdots\cdots \end{aligned}$$

and these m_{02}', m_{20}', $\cdots$ are approximations similarly obtained. The rectangles such as 01, 02, $\cdots$, 10, 20, $\cdots$, which are on the boundary but not in corners, have each three first-remove and two common second-remove rectangles. Two of the first removes are on the boundary and one is in the interior; this third interior rectangle touches both the common second removes.

For 01, for instance, the correlate equations are expressed as shown in Table IV, by means of the coefficients only, all applicable to any one m being in the same column while all applicable to any one rectangle are in the same row as its excess d.

TABLE IV

d	m_{01}	m_{00}	m_{11}	m_{02}	m_{10}	m_{12}	Residues
01	4	-1	-1	-1			
00	-1	4			-1		
11	-1		4		-1	-1	$-m_{21}$
02	-1			4		-1	$-m_{03}$
10		-1	-1		4		$-m_{20}$
12			-1	-1		4	$-m_{13}-m_{22}$

Multiplying by 52, 15, 17, 15, 8 and 8 respectively and adding, we get

$$161m_{01} - 17m_{21} - 15m_{03} - 8(m_{20} + m_{13} + m_{22}) = 161D_{01}, \quad (13)$$

where

$$161D_{01} = 52d_{01} + 17d_{11} + 15(d_{00} + d_{02}) + 8(d_{10} + d_{12})$$

and $m_{01}' = D_{01}$ gives a first approximation to m_{01}. It should be noted that all the rectangles whose correlates are thus eliminated are those which are laterally and diagonally once-removed from the key rectangle 01, whilst the residual correlates m_{20}, m_{21}, m_{22}, m_{03} and m_{13} are the ones adjacent externally to those eliminated.

Similarly for 02 we should get

$$161m_{02} - 17m_{22} - 15(m_{00} + m_{04}) - 8(m_{10} + m_{21} + m_{23} + m_{14})$$
$$= 52d_{02} + 17d_{12} + 15(d_{01} + d_{03}) + 8(d_{11} + d_{13}),$$

and so on for any other boundary rectangle not a corner.

The drawback to the above series for m_{01}, m_{02}, $\cdots$ is that the coefficients involved are not so simple as in the case of equations (11) and (12). There is some advantage in using for these rectangles the same multipliers, namely 7, 2 and 1 for the key equation, for all first-remove equations and for common second-remove equations, as were used in equation (12).

These give the equation

$$22m_{01} - m_{11} - 2(m_{21} + m_{03}) - (m_{20} + m_{13} + m_{22})$$
$$= 7d_{01} + 2(d_{00} + d_{11} + d_{02}) + d_{10} + d_{12} = 22D_{01}', \quad (14)$$

in which m_{11} is left as an additional residual to the previous series; but the series of residuals left for m_{01}'', after we have written

$$m_{01} = m_{01}' + m_{01}'' + \cdots \text{ and } m_{01}' = D_{01}',$$

is very easy to use for further approximation.

For the inner rectangles, such as 11, 12, 21, 22, $\cdots$, which have four first-remove and four common second-remove rectangles, together forming a rectangle of three by three cells, we use multipliers 6, 2 and 1 respectively for each order of remoteness and obtain

$$20m_{11} - 2(m_{13} + m_{31}) - (m_{03} + m_{23} + m_{32} + m_{30})$$
$$= 6d_{11} + 2(d_{01} + d_{12} + d_{21} + d_{10}) + (d_{00} + d_{02} + d_{22} + d_{20}),$$
$$20m_{12} - 2(m_{10} + m_{32} + m_{14}) - (m_{00} + m_{04} + m_{24} + m_{33} + m_{31} + m_{20})$$
$$= 6d_{12} + 2(d_{11} + d_{02} + d_{13} + d_{22}) + (d_{01} + d_{03} + d_{23} + d_{21}),$$
$$\cdots\cdots\cdots\cdots \quad (15)$$

in which the residual m's multiplied by the coefficient 2 belong to all those rectangles in external contact with the first-remove rectangles, and those multiplied by the coefficient unity belong to those rectangles in contact with the common second-remove rectangles. There may be as many as four of the former and eight of the latter.

By a similar procedure of selecting suitable multipliers and summing the equations, it is possible to obtain for any correlate formulae which involve the elimination of all or most of the correlates of cells which are adjacent to the key cell, and therefore give a better first approximation than the preceding equations (11), (12), (14) and (15). This advantage is, however, more than counterbalanced by the increasing complexity of the residuals and series of excess terms, and for practical purposes either equations (11) or equations (12), (14) and (15) are recommended for use with a rectangular network.

Single chain of rectangles. In the particular case of a single chain of rectangles, as in Figure 4

$S-4$	$S-3$	$S-2$	$S-1$	S	$S+1$	$S+2$	$S+3$	$S+4$

FIG. 4

there is some advantage in forming the equation for any correlate m of cell S, by eliminating pairs of cells on either side of it as far as symmetry permits, say, to the cells $S-3$ and $S+3$. We have the correlate equation coefficients shown in Table V.

TABLE V

m									d	$\times by$
$S-4$	$S-3$	$S-2$	$S-1$	S	$S+1$	$S+2$	$S+3$	$S+4$		
			-1	4	-1				$=S$	56
		-1	4	-1					$=S-1$	15
				-1	4	-1			$=S+1$	15
	-1	4	-1						$=S-2$	4
					-1	4	-1		$=S+2$	4
-1	4	-1							$=S-3$	1
						-1	4	-1	$=S+3$	1

And using the multipliers 1 for $S-3$ and $S+3$, 4 for $S-2$ and $S+2$, $(4\times 4-1)$ or 15 for $S-1$ and $S+1$, and $(4\times 15-4)$ or 56 for S, and adding, we get

$$194m_S - m_{S-4} - m_{S+4} = 56d_S + 15(d_{S-1} + d_{S+1})$$
$$+ 4(d_{S-2} + d_{S+2}) + (d_{S-3} + d_{S+3}), \qquad (16)$$

which gives a very rapid approximation for any central cell S. For cells near and at the ends of the chain the same multipliers are valid, but the residual m's have slightly different coefficients. For example, if the chain terminates in $S+2$, the above equation still holds for m_S if the term $-m_{S+4}$ on the left is replaced by $+m_{S+2}$ and d_{S+3} is omitted on the right.

The series of multipliers available for such a single chain of rectangular cells is the series

$$1, 4, 15, 56, \cdots p_r,$$

where

$$p_r = 4p_{r-1} - p_{r-2}, \qquad (17)$$

for any positive integral value of r, and

$$p_1 = 1, \quad p_2 = 4.$$

§7. TRIANGULAR NETWORKS. MORE RAPID APPROXIMATIONS

As in the case of the rectangular networks just considered, so in triangular networks can more rapid approximations for the correlate of any cell be obtained by eliminating the correlates for second- as well as first-remove triangles.

As the triangles will normally be restricted to those in which no angle is less than 45° and none greater than 90°, we can assume that only rare instances will occur of common second-remove cells. The normal case will be one in which each triangular cell has from one to three first-remove triangles touching it, and each of these has one or two independent second-remove triangles in external contact with it.

Single chain of triangles. Consider the case of Figure 5, which relates to triangular cells in the same way that Figure 4 relates to rectangular cells.

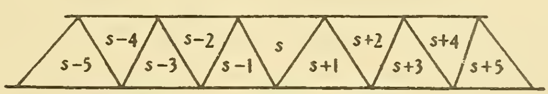

FIG. 5

The table of correlate equations with cell 5 as center is shown in Table VI.

TABLE VI

m											d
$S-5$	$S-4$	$S-3$	$S-2$	$S-1$	S	$S+1$	$S+2$	$S+3$	$S+4$	$S+5$	
				−1	3	−1					S
			−1	3	−1						$S-1$
					−1	3	−1				$S+1$
		−1	3	−1							$S-2$
						−1	3	−1			$S+2$
	−1	3	−1								$S-3$
							−1	3	−1		$S+3$
−1	3	−1									$S-4$
								−1	3	−1	$S+4$

As before, we can find a series of multipliers, namely,

$$1, 3, 8, 21, 55, \cdots p_{r-2}, p_{r-1}, p, \cdots ,$$

where

$$p_r = 3p_{r-1} - p_{r-2}, \qquad (18)$$

$$p_2 = 3,$$

$$p_1 = 1,$$

which will eliminate all the correlates of intermediate pairs of triangles down to any desired residue, e.g., $S \pm 5$. For example, to eliminate from $S \pm 1$ to $S \pm 4$ inclusive, we use the series to the 5th term 55 and get

$$123m_S - m_{S-5} - m_{S+5} = 55d_S + 21(d_{S-1} + d_{S+1}) + 8(d_{S-2} + d_{S+2}) + 3(d_{S-3} + d_{S+3}) + d_{S-4} + d_{S+4}. \qquad (19)$$

For cells near or at a terminus of the chain, we use the same series of multipliers.

For triangles with two second-remove cells to each first-remove cell we use the multiplier series 1, 3, 7; for instance, in the network of figure 6*a*, cell 0 has one first-remove cell 11 and this has two second-remove cells, 21 and 22.

The correlate table for m_0 is Table VII.

TABLE VII

0	11	21	22	d	p
3	−1			0	7
−1	3	−1	−1	11	3
	−1	−3		21	1
	−1		3	22	1

Whence, multiplying (see column p) by 7, 3, 1 and 1 respectively,

$$18m_0 = 7d_0 + 3d_{11} + d_{21} + d_{22}. \qquad (20)$$

Similarly, in the network of figure 6*b*, we should get

$$12m_0 = 7d_0 + 3(d_{11} + d_{12} + d_{13}) + d_{21} + d_{22} + d_{23} + d_{24} + d_{25} + d_{26}. \quad (21)$$

The coefficient of m_0 is always $21-3n_1$, where n_1 is the number of first-remove cells.

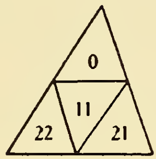

FIG. 6*a*

If in the network of triangles actually under consideration there are third-remove triangles, or triangles of still greater remoteness from the key triangle for which approximation is desired, there will be residual correlates, m_{31}, m_{32}, $\cdots$, introduced into equations of type (19) and (20). There is no difficulty in finding series of multipliers to carry the elimination to further stages, especially if the cells are

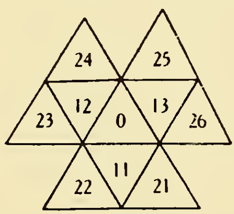

FIG. 6*b*

arranged symmetrically about the key cell. For example, the network of Figure 6*b* may be completed by the insertion of three triangles 31, 32 and 33 between 22 and 23, and 24 and 25, and 26 and 21 respectively, giving three extra common third-removed triangles. The desired series of multipliers is then 2, 3, 7 and 15 and we get

$$24m_0 = 15d_0 + 7(d_{11} + d_{12} + d_{13}) + 3(d_{21} + \cdots + d_{26}) + 2(d_{31} + d_{32} + d_{33}). \quad (22)$$

§8. GENERAL NETWORK. PROCEDURE FOR ADJUSTMENT

The foregoing analysis will have made quite clear the limitless possibilities of mathematical solution by means of correlate equations and multipliers suited to any cell of a network and its immediate surroundings. Interesting as such analysis is, its application to a practical problem of least-square adjustment must be governed by considerations of the accuracy justified in the particular survey, and the most economical method of executing the calculations involved. Adjustment involving elimination of the correlates of the first-remove cells on the lines of §§4 and 5, using equations (6) to (11), is a simple, reasonably rapid process, readily applicable to any cell in any network however complicated. When the network is homogeneous, i.e., when all the cells are polygons of the same species and regular in arrangement, the more rapid approximations discussed in §§6 and 7 may be used with advantage. Also when there are outstanding runs or chains of cells equations (16) to (19) are easy to apply and give very rapid approximations.

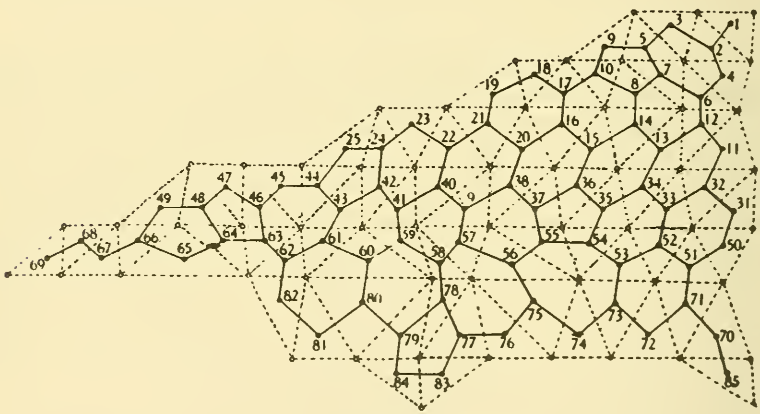

FIG. 7

For general purposes, the procedure of §§4 and 5 is recommended as being simple, universally applicable and almost as rapid as any of the subsequent processes when account is taken of all the stages of the operation. As a test of this procedure, an adjustment made for a network of 56 Eötvös gravity stations may be cited. The observations were made in the course of a practical survey in Cumberland. The stations are shown as open circles in Figure 7, where they are connected by broken-line links to form 80 triangles numbered 1 to 85, numbers 26 to 30 being omitted. It is evident that the network thus formed is typical of a large regional survey and that the procedure of linking could be adapted to any group of stations, however numerous. From the measured data of the gravity survey the increments in g were calculated for every link, and from these the excesses were derived for each triangle of the network. The excesses for the first ten triangles are tabulated in Table VIII in convenient units which were subsequently transformed into decimal fractions of a dyne. The excesses varied in sign and size, many exceeding 100 units and the largest being 346. Many of the actual link-increments exceeded 1,000 units, and as these depend on measured gravity-gradients of which the accuracy rarely exceeds 2 to 3 per cent in practice, it is considered meticulous to obtain the adjusted values of the link-increments to nearer than 10 units. The procedure thus resolves itself into one of obtaining from the excesses the corresponding correlates to the nearest 10 units.

TABLE VIII

Δ	d'	$\sum_1^{iv} m$	d^{iv}
1	−13	−14	−5
2	5	−34	1
3	3	−46	−4
4	37	−46	−2
5	−121	−111	−4
6	−130	−143	−3
7	−290	−205	−1
8	−136	−72	−4
9	180	35	4
10	45	40	2

To facilitate setting down the related first- and second-remove correlates corresponding to each key correlate, it is convenient to replace every triangle in Figure 7 by a correspondingly numbered dot, situated roughly at its center of position, and to join these dots by lines representing contacts of adjacent triangles. A triangle is thus represented by a point from which radiates a number (one, two or three) of lines to each of its first-remove points, from which again outwards radiate lines to the second-remove points. Commencing at any one point it is easy for the eye to pick up the first- and second-remove points in turn. These are tabulated on the working sheets, of which Table IX shows a typical example used for the first five points in order. The first column, headed N, is used for the common divisors obtained from formulae (9) and (10) by simply subtracting the number of first-remove points from 9 in each case, since in this network $n=3$ and $n^2=9$ throughout.

The second column marked Δ contains the key-point number, followed after an interval of one line by the numbers of its first-remove points in order. The third column, marked $+m'-$, is used for computing m' from the excesses given in Table VIII and equation (9). The excess for the key point is multiplied by 3 and set down opposite that point; the excess for each first-remove point is set down directly; the $+$ and $-$ columns are totalled and the aggregate sum divided by

TABLE IX

N	Δ	$+$ m'	$-$	Δ''	$+$ m''	$-$	$+$ m'''	$-$	$+$ m^{iv}	$-$
8	1		39	3		15		21		6
				4		2		37		1
	2	5								
			34			17		58		7
		−4			−2		−7		−1	
6	2	15		5		78		23		7
				6		115		14		10
	1		13							
	3	3								
	4	37								
		42				193		37		17
		+7			−32		−6		−3	
7	3	9		1		4		2		7
				4		2		37		1
	2	5		7		210	27			22
	5		121	9	66			31	5	
			107			150		43		25
		−15			−21		−6		−4	
7	4	111		1		4		2		7
				3		15		21		6
	2	5		7		210	27			22
	6		130	12		28		12		8
			14			257		8		43
		−2			−37		−1		−6	
6	5		363	2	7			32		6
				6		115		14		10
	3	3		8		83	20			11
	7		290	10	51			16	7	
	9	180								
			470			140		42		20
		−78			−23		−7		−3	

N from column 1. The result is m' for this key point. In the fourth column headed Δ'' are set down the numbers of the second-remove points in order. These are used to obtain m'', m''', m^{iv} in succession according to equation (10) by means of the subsequent columns headed $+m''-$, $+m'''-$, $\cdots$, and the values previously obtained in column m' for each second-remove point.

The procedure is thus to set down the key, first-remove and second-remove points in columns 2 and 4, then the values N in column 1; compute m' for every key point and use the values so obtained in column 5 to get m'' for each point; use these latter values to get m''', and these to get m^{iv}, and so on. The values actually obtained are shown under the respective columns in Table VIII. The approximations to m obtained by summation $m' + m'' + m''' + m^{iv}$ are next given. The whole process for 80 points took 15 hours, an average of just over a quarter of an hour for each of the 56 gravity stations.

The adjusted link-increment values are easily obtained from the calculated m's by using equation (1), and the adjusted gravity value of each station relative to any one taken as datum can then easily be derived, and the isogams can be drawn.

A check on the accuracy of reduction is obtained by computing the residual excess for each triangle. From equations (1) and (2) it is easy to see that the residual excess d' is given by the formula

$$d_r' = d_r + \Sigma m_x - 3m_r,$$

which ought to be zero if the values m are accurately computed. The actual residual excesses which remained when the calculation was stopped at m^{iv} are given in Table VIII, column d', and are seen to be below the agreed limit of error of 10 units.

THE AMERICAN ASSOCIATION OF PETROLEUM GEOLOGISTS
TRANS. SOCIETY PETROLEUM GEOPHYSICISTS, VOL. V (MARCH, 1935), PP. 180-183

THE SOCIETY OF PETROLEUM GEOPHYSICISTS, 1934–1935

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THE SOCIETY OF PETROLEUM GEOPHYSICISTS FINANCIAL STATEMENT, 1934

MR. E. E. ROSAIRE, PRESIDENT
SOCIETY OF PETROLEUM GEOPHYSICISTS
TULSA, OKLAHOMA

Dear Sir:

Pursuant to instructions received, we have made an audit of the financial transactions of B. HUBBARD, TREASURER, SOCIETY OF PETROLEUM GEOPHYSICISTS, for the period from March 24, 1934, to December 31, 1934, inclusive.

Our examination did not embrace a confirmation by correspondence of dues from members and delinquents in respect thereto.

Exhibit *A* [not included in this printing] is a statement of dues receivable from March 24, 1934, to December 31, 1934, inclusive. The list of members and "back dues" are taken from the records of the Treasurer. Upon inquiry, we were advised there was no former report or list of delinquents with which these records could be reconciled.

Summaries of the "Dues Receivable" account and cash transactions are presented below:

Dues Receivable:		
Back dues		$ 220.00
Current dues	$870.00	
Less—paid to former Treasurer	5.00	865.00
Total dues receivable		$1,085.00
Collections—back dues	$100.00	
Collections—current dues	635.00	
Total collections		735.00
Delinquent		$ 350.00
Cash Transactions:		
Balance received from former Treasurer	$ 70.45	
Collections	735.00	$ 805.45
Disbursements:		
To A.A.P.G.—bulletin cards	$ 30.00	
Secretarial work (May to December)	80.00	
Application blanks	17.02	
Membership cards	4.55	
Postage	15.00	
Stationery and miscellaneous	28.42	174.99
Balance December 31, 1934		$ 630.46

Yours very truly,
(Signed) FRAZER AND TORBET
Certified Public Accountants

TULSA, OKLAHOMA
January 10, 1935