IMAGE EVALUATION
TEST TARGET (MT-3)

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INVESTIGATION

OF THE

DISTANCE OF THE SUN,

AND OF THE

ELEMENTS WHICH DEPEND UPON IT,

FROM THE

OBSERVATIONS OF MARS,

MADE DURING THE

OPPOSITION OF 1862, AND FROM OTHER SOURCES,

FORMING

[illegible] II TO THE WASHINGTON ASTRONOMICAL OBSERVATORY FOR 1865.

APPENDIX II.

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OBSERVATIONS OF MARS,

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BY SIMON NEWCOMB,
Professor of Mathematics, U. S. Navy.

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APPENDIX II.

INVESTIGATION

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About ten years since, astronomers began to suspect that Encke's value of the Sun's distance, deduced from the transits of Venus in the years 1761 and 1769, was largely in error. The different methods available for its correction all agreed in indicating a diminution of between one twenty-fifth and one-thirtieth of the whole distance. The last doubt of the correctness of the suspicion was removed by the publication of Powalky's paper on the Transit of 1769. In this paper it was shown that, with our more accurate knowledge of the positions of the observing stations, the results of this Transit agreed with those of the modern measures.

The magnitude of the correction being such as seriously to affect the reduction of meridian observations of Mercury, Venus, and Mars, as well as our computations of the mass of the Earth and the parallactic equation of the Moon, it becomes important to determine it with precision, even in advance of the coming transits of Venus. In such a determination the results of all methods which can be relied on, or the precision of which can be estimated, ought, I conceive, to be combined in the final result. Let us, then, glance at the various methods now available.

1. *By Observations of Transits of Venus.*—This method has gone into our school-books as the one superior to all others in the precision of its results. It is true that transits which occurred a century ago, when the art of observation was in its infancy, have furnished the solar parallax which has hitherto been adopted as the standard. It is also possible that, should the civilized world take due interest in the observation of the next two transits, and should circumstances prove favorable, the precision of either result may exceed that of any other one determination. But it is certain that our modern determinations by other methods are more precise than any that can be derived from the past transits of Venus, and opportunities which occur in but one generation of men out of four are too rare to be implicitly relied on in future.

2. *From Observations of Mars when near the Earth.*—Three methods of making these observations have been employed.

a. By nearly simultaneous observations of difference of declination between Mars and a neighboring star, at Observatories situated in different hemispheres of the earth, and by means of Equatorial Telescopes. For the employment of this method the United States Astronomical

Expedition to Chili was organized. It was again proposed by Captain Gilliss, in 1862, and observations in this way were made at Upsala, Leiden, and Washington, in the Northern Hemisphere, and Santiago in the Southern.

β. By similar observations with a Meridian Circle, Mars being compared with a number of pre-selected stars. This is the method proposed by Winnecke, and most extensively carried out in 1862. It was first employed in 1832, between the Observatories of Greenwich, Cambridge, and Altona, in the Northern Hemisphere, and Cape of Good Hope in the Southern. The result was 9″.028,* which it now appears was not only nearer the truth than Encke's value, but was affected with a probable error less than the absolute error of the latter.

As compared with the first, this plan has this advantage: that, comparisons being made with the same stars night after night, there is little danger of observations being lost at one station for want of corresponding ones at another; while, by the other, since the planet must be compared with a different star on every night, they will be lost, unless made on the same night at both stations. The disadvantages are, that the results are affected by the errors arising from erroneous division of the circle, or other causes peculiar to each star, and that the observations cannot be repeated on the same night. The probable magnitude of the first error may be inferred from the results of the investigations of Auwers on the declinations of the fundamental stars, from which it would seem that the probable error arising from these causes is between two and three-tenths of a second. It is, therefore, advisable to compare with as many stars as possible, in order to diminish the chances of error. Inability to repeat the observations will appear a less serious objection, if we reflect that, from some cause or another, micrometric comparisons with an Equatorial do not often exhibit the precision of meridian observations.

On the whole, I conceive that, in a general combination of the principal active Observatories of the world, the micrometric method would be preferable; while, if the number in either hemisphere is limited to one or two, the preference must be given to the Circle observations. The arrangements of 1862 were precisely the reverse of this.

γ. By differences of Right Ascension between Mars and neighboring stars east and west of the Meridian. So far as I am aware, this method was first employed by the Messrs. Bond at the Observatory of Harvard College, during the opposition of 1849–50.† The value then obtained was 8″.605, with a probable error of 0″.4. It was also proposed by the Astronomer Royal, and actually employed at the Royal Observatory, Greenwich, in 1862. The result has not, I believe, been published.

This method has not received the attention it deserves, probably from a general distrust of time observations. If employed at a station of less than forty degrees latitude, with a steady and carefully-adjusted instrument, and if care be taken to eliminate every source of constant personal error, its results might, I conceive, be received with entire confidence. Among the measures necessary to secure a reliable result may be placed the making of the observations on one side of the meridian with an inverting eye-piece, that the apparent direction of motion of the planet may be as nearly as possible the same on both sides of the meridian.

It is possible that observations over the horizontal wires of an Altozimuth might be preferable to that over the right ascension wires of an Equatorial.

3. *From the Observed Parallactic Inequality of the Moon.*—This inequality has the solar parallax as a factor, into which it is multiplied nearly fifteen times. Since astronomers ought to be able to determine the coefficient of this inequality without a probable error of more than a tenth of a second, the solar parallax ought, it would seem, to be determined from it without a probable error exceeding 0″.007, and, therefore, with greater precision than by any other method yet employed. Unfortunately, however, the uncertainty of the observed value of the parallactic inequality still amounts to several tenths of a second, so that there is no hope of attaining this degree of precision.

* Astronomische Nachrichten, No. 262.

† Astronomical Journal, No. 103.

4. *By combining the Lunar Inequality in the Motion of the Earth with the known Mass of the Moon.*—By this method was obtained the value of the solar parallax adopted by Le Verrier in his solar tables. Knowing the parallax and mass of the Moon, we can compute the distance of the centre of the Earth from the common centre of gravity of the Earth and Moon, around which the Earth's centre revolves in a lunar month. Also, from observations of the Sun, Venus, or Mars, we can determine the angle which this same distance subtends when seen from the Sun. A comparison of these two data gives the angle which the radius of the Earth itself subtends, as seen from the Sun, or the solar parallax.

This method is the least precise of all, since it gives the solar parallax as the product of two factors, neither of which are determined with great precision. The observed value of the lunar equation must at present depend on observed right ascensions of the Sun, of which the probable error is very large, and the uncertain factor of this element is about one-third greater than unity. Therefore, supposing the mass of the Moon known, the more logical course would seem to be to determine the lunar inequality from the solar parallax.

5. *From Experimental Determinations of the Velocity of Light, combined with the known value of the Aberration of Light.*—Foucault's beautiful experiments with the revolving mirror are so well known that they need not be described. The theoretical objections to this method do not seem to me to have much force, and I see no insuperable reason why its results should not be as reliable as those of any other method. It is quite true that in experiments so delicate, hidden causes of constant error may defy the scrutiny of the experimentalist. It is also true that Foucalt's operations have not been published with that fullness of detail necessary to satisfy astronomers that his results could not have been vitiated by any such cause. But, to test the reliableness of the results, it is, I think, only necessary that the determination should be repeated with apparatus as different as possible from that used by Foucault. Such a repetition is a desideratum both for Physics and Astronomy. A desirable modification of the apparatus would be, if practicable, placing the fixed reflector at a great distance, say 3,000 or 4,000 metres from the revolving mirror.

§ 2.

When the Great Transit Circle was mounted at the Naval Observatory, the question arose whether, in the reduction of observations of the Sun and Planets, it was possible to employ a value of the parallax so near the truth that there would be little danger of future investigators having to correct our results on account of error in the adopted constant of solar parallax. The most promising source of an accurate parallax seemed to be the observations of Mars, made in 1862 on the plan of Winnecke. They formed a better planned, better executed, and more extended series than was ever before available. In the Southern Hemisphere the Observatories of Williamstown, Cape of Good Hope, and Santiago, worked with remarkable success, securing, altogether, observations of Mars and comparison stars on 143 nights. In the Northern Hemisphere the Observatories of Pulkowa, Petersburg, Helsingfors, Vienna, Berlin, Leiden, Greenwich, Albany, and Washington, are known to have co-operated. So extended a co-operative effort on the part of astronomers all over the civilized world has not, I believe, been seen since the transit of Venus in the last century.

Three partial discussions of these observations have appeared.

1. Winnecke, by a comparison of observations at Pulkowa and the Cape of Good Hope, on thirteen corresponding nights, found the solar parallax 8″.964.*

2. Mr. E J. Stone, of the Royal Observatory, Greenwich, discussed the observations of Greenwich, the Cape, and Williamstown, deducing the parallax 8″.943.†

3. The corresponding observations at Albany, Washington, and Santiago, were discussed by Mr. Ferguson in the Washington Astronomical Observations for 1863, with the following results:

From 12 observations of Washington and Santiago - - - - 8″.834
15 " " Albany - - - - - - - - - - 8″.611

* Astronomische Nachrichten, No. 1409. † Memoirs of the Royal Astronomical Society, Vol. 33, p. 97.

The method adopted by each of these investigators was that of comparison of pairs of corresponding observations made—one in each hemisphere. Many observations at the one station would be lost for want of corresponding observations at the other. Thus, out of a grand total of more than 300 observations, only 26 were employed by Winnecke, 58 by Stone, and 46 by Ferguson. Five of those used by Winnecke and Stone being the same, the sum total used by the three astronomers is only 125. The three results are, therefore, so far from final that a complete discussion is to be desired.

This discussion I have been permitted and enabled to undertake by Rear-Admiral Charles H. Davis, lately Superintendent of the Naval Observatory. At his request, copies of the unpublished observations of Williamstown and the Cape were obtained from Robert J. Ellery, esq., and Sir Thomas Maclear, the directors of those Observatories. The Observatory of Pulkowa, as the originator of the plan, having a prior right to the general discussion, its consent was also obtained through its distinguished director, who communicated the observations of Sawitch and Krenger, made at Petersburg and Helsingfors.

§ 3.

The following considerations may lead to a method of determining the parallax of Mars from observations, more simple and rigorous than that of corresponding pairs of observations: The perturbations in the motions of the Earth and Mars being perfectly known for the period which we consider, every observation of that planet will lead rigorously to an equation of condition between its parallax, the six elements of its orbit, and the six elements of the Earth's orbit. Thirteen or more observations will, when compared with any theory, suffice, formally, to correct the elements of that theory. But, if the observations extend through only a short interval, say one month, the coefficients of the corrections will be so minute that no trustworthy values of the corrections can be deduced. We shall, in fact, find that our equations will only suffice to determine a few functions of the elements, and that the elements themselves, if their values are only chosen so as to satisfy those functions, may all vary widely, without ceasing to satisfy our equations of condition. If, now, we can fix *a priori* on the entire number of functions of this kind, and use them in lieu of the elements of the Earth and Mars, our equations will be practically as rigorous as if we had introduced the entire number of thirteen unknown quantities.

One of these functions, the first one, indeed, will be the error of declination of Mars, since this will be given by a single pair of observations. But, when there are a series of observations, we may take, instead of the declination, the absolute distance of the planet from the plane of the Earth's equator. This distance, or rather its error, may be developed in powers of the time, and the coefficients of this development may be taken in lieu of the elements. That is, we may assume that the error of tabular declination may be expressed in the form

$$\frac{\sec \text{Dec.}}{\Delta}(\alpha+\beta t+\gamma t^2+ \text{etc.})$$

That this assumption is a safe one in the case of Mars, may be shown by taking the observed tabular errors given by Winnecke in his publication, "Beobachtungen des Mars um die zeit der Opposition, 1862," and developing them in this way. Dividing them into five series, and taking the mean of each series as the error corresponding to the mean of the dates, we have the following five tabular errors, and their products by the distance of Mars from the Earth:

Date.	ϵ	$\epsilon\times\Delta$	Obs.
	"	"	
Aug. 31	4.28	1.96	5
Sept. 17	5.28	2.19	11
30	5.19	2.11	7
Oct. 14	4.55	1.94	4
31	3.53	1.73	2

Developing the last column in powers of the time, and retaining only three terms, we shall find that none of the deviations from the formula will amount to 0″.10. The coefficient of the second power of the time being only 0″.00035, it may, for a period of ten days on each side of any epoch, be neglected without danger of error.

It is to be remarked that, besides those terms divided by $\varDelta$ in the expression for error of tabular declination, there may be a constant term arising from constant sources of error in the measured polar distances. Putting $\varDelta' = \varDelta \cos$ Dec. and $\delta\pi$ = correction of parallax, each comparison of an observed and computed declination will give an equation of the form

$$\delta \text{ Dec.} = f\delta\pi + D_0 + \frac{\alpha}{\varDelta'} + \frac{\beta t}{\varDelta'}.$$

$\delta\pi$, D_0, α, and β being the unknown quantities to be determined.

§ 4.

The following is the list of observations included in the discussion, with the authorities for them:

NORTHERN OBSERVATORIES.

Pulkowa.—Thirty-one observations. Beobachtungen des Mars von Dr. A. Winnecke.

Helsingfors.—Eighteen observations. Beobachtungen des Mars und der Winneckeschen Vergleichsterne. Herbst, 1862, am Reichenbach-Ertelschen Meridiankreise der Sternwarte zu Helsingfors. (Communicated by M. Struve.)

Leiden.—Twenty-nine observations. Astronomische Nachrichten, Band 62.

Greenwich.—Fourteen observations. Greenwich Observations of 1862.

Albany.—Twenty-six observations. Washington Observations of 1863, p. XLIX.

Washington.—Thirty-six observations. Washington Observations of 1862.

SOUTHERN OBSERVATORIES.

Williamstown.—Fifty-one observations. Results communicated by Robert J. Ellery, esq.

Cape of Good Hope.—Forty-three observations. Observations, with their reductions, communicated by Sir Thomas Maclear.

Santiago.—Forty-nine observations. Observaciones Meridianas i Micrométricas relativas al Planeta Marte al Tiempo de su oposicion en 1862, verificadas en el Observatorio Nacional de Santiago de Chile. Santiago, 1863.

This includes the entire list of those accessible observations the weight of which would be such as to sensibly affect the concluded parallax. The entire number is as follows:

In the Northern Hemisphere - - - - - - - - - -	154
In the Southern Hemisphere - - - - - - - - - -	143
Total - - - - - - - - - - - - - - - - - -	297

In discussing the observations, the first thing to be done is to make them strictly comparable with each other. This is effected by deducing them all differentially from one set of comparison stars. In Winnecke's plan, each observation of Mars can be compared with similar observations of eight stars of comparison. An ephemeris of the positions of these stars being prepared, a comparison of the observed polar distance of any star with the ephemeris gives an apparent correction to the observation. The mean of the eight corrections thus deduced from one night's work, by one observer, is considered a correction applicable to the polar distance of Mars, observed on the same evening.

If every observer observed all eight comparison stars on every night, the adopted mean position of each star would be a matter of entire indifference. But, since a portion of the comparison stars were frequently missed, it becomes important that the different stars should be reduced to the same standard. Rigorously, this standard should be, not that of absolute correctness, but that of each particular instrument as affected with its errors of divisions, and corrected for the constant error of the mean of its positions of all eight stars. This standard being unattainable for want of a sufficient number of observations, we shall be obliged to use one uniform set of star places, differing from absolute truth by amounts less than the accidental errors of division of our instruments. The adopted positions will be derived from the observations of Greenwich, Pulkowa, Albany, and Washington. The adopted standard of declination will be that of Auwers, in his paper on the corrections necessary to reduce the different catalogues to a fundamental system. This reduction will be obtained through the Greenwich Transit Circle, the correction of which, near the Equator is +0''.2. Comparing the observations of Pulkowa, Washington, and Albany with those of Greenwich, thus corrected, we find the following systematic corrections to reduce them to Auwers:

	''
Pulkowa - - - - - - - - -	+1.2
Albany - - - - - - - - - -	+1.6
Washington - - - - - - - -	—1.5

The mean declinations for 1862.0, thus concluded from the observations of each Observatory, with the seconds of concluded north polar distance for the same epoch, are given in Table I. The positions under the names of the Observatories are not corrected for systematic error. The small figures after each result show the number of observations. The weights are not proportional to the number of observations, but on a somewhat arbitrary scale, depending on the probable errors of divisions, and the discordance of individual observations at the several Observatories. In Table II is given an ephemeris of apparent positions, computed with the constants of the American Ephemeris and Nautical Almanac.

TABLE I.

	Pulkowa.				Greenwich.		Washington.		Albany.		Sec. of mean N. P. D for 1862.0.
	°	'	''	n	'	n	''	n	''	n	''
Lalande 47374 . .	— 3	19	45.1	2	43.1	4	43.4	8	. .		44.15
Lalande 261 . . .	+ 0	55	15.6	6	76.8	8	17.9	15	15.8	3	43.21
44 Piscium	+ 1	10	29.4	6	31.0	7	31.9	15	• 29.2	8	29.32
Lalande 670 . . .	+ 4	5	45.5	13	46.8	5	48.3	20	45.1	13	13.24
15 Ceti	— 1	15	47.0	11	47.0	3	44.7	10	48.1	7	46.06
60 Piscium	+ 5	59	12.1	6	12.4	2	14.8	6	11.7	3	46.75
Lalande 1299 . . .	+ 4	31	12.7	9	14.0	5	14.8	10	. .		46.32
δ Piscium	+ 6	49	58.8	10	59.7	6	61.8	15	58.5	18	59.89
20 Ceti	— 1	53	40.6	17	39.5	8	38.7	31	40.9	21	39.53
26 Ceti	+ 0	37	33.7	16	34.9	15	36.4	30	33.6	25	25.00
29 Ceti	+ 1	16	14.2	2	15.2	8	16.8	10	. .		44.64
80 Piscium	+ 4	55	6.3	14	6.8	5	9.1	18	6.0	15	52.48
89 Piscium	+ 2	53	11.0	11	12.2	8	14.0	11	11.0	4	47.67
43 Ceti	— 1	10	21.6	10	21.8	1	19.7	11	22.8	6	20.76
Lalande 2614 . . .	— 1	6	62.3	6	62.5	4	59.7	6	62.6	2	1.32
μ Piscium	+ 5	25	51.6	16	51.9	7	53.9	19	50.2	8	7.50
ν Piscium	+ 4	47	15.2	10	16.2	13	18.4	15	14.8	16	43.45
Lalande 3298 . . .	+ 2	59	42.0	10	43.1	7	45.0	15	41.2	13	16.80
ξ Piscium	+ 2	30	16.6	10	17.7	7	19.6	15	16.3	15	42.08

TABLE II.

Ephemeris of the apparent North Polar distances for transit over the meridian of Washington.

FIRST SERIES.

Date.	δ Pis.	20 Ceti.	26 Ceti.	80 Pis.	μ Pis.	ν Pis.	43298.	ε Pis.
	″	″	″	″	″	″	″	″
Aug. 20	35.25	12.86	59.22	28.06	43.86	20.06	53.18	18.58
30	33.88	11.87	58.13	26.76	42.59	18.83	52.02	17.44
Sept. 9	32.69	11.17	57.28	25.71	41.55	17.83	51.12	16.55
19	31.77	10.74	56.70	24.91	40.73	17.07	50.45	15.94
29	31.06	10.55	56.37	24.33	40.16	16.55	50.05	15.54

SECOND SERIES.

Date.	Lal. 670.	15 Ceti.	60 Pis.	20 Ceti.	89 Pis.	43 Ceti.	Lal. 2614.	μ Pis.
	″	″	″	″	″	″	″	″
Sept. 19	44.43	17.06	18.47	11.06	20.04	52.79	33.50	40.73
29	43.86	16.82	17.80	10.36	19.60	52.60	33.30	40.16
Oct. 9	43.54	16.83	17.38	9.97	19.40	52.61	33.36	39.84

THIRD SERIES.

Date.	Lal. 261.	44 Pis.	Lal. 670.	15 Ceti.	26 Ceti.	80 Pis.	89 Pis.	43 Ceti.
	″	″	″	″	″	″	″	″
Oct. 9	13.52	59.73	43.54	16.83	56.29	23.99	19.40	52.64
19	13.58	59.78	43 43	17.04	56.44	23.85	19.41	52.92
29	13.88	60.08	43.50	17.47	56.74	23.92	19.64	53.42

FOURTH SERIES.

Date.	L. 43734.	Lal. 261.	44 Pis.	Lal. 670.	Lal. 1299.	20 Ceti.	26 Ceti.	29 Ceti.
	″	″	″	″	″	″	″	″
Oct. 19	15.15	13.58	59.78	43.43	17.82	10.89	56.44	16.45
29	15.62	13.82	60.04	43.50	17.95	11.34	56.74	16.76
Nov. 8	16.24	14.22	60.40	43.74	18.49	11.92	57.21	17.20

The observations being all reduced according to one uniform system, no details need be given except for those cases in which a mode of observation different from that Winnecke has adopted, or in which some of the elements of reduction are doubtful or imperfect.

Greenwich.

Only a small number of the comparison stars, seldom more than four, were observed on any one evening. Moreover, the same stars were frequently selected night after night, so that the positions of Mars depend mainly on less than half the entire list of stars. The double wire system was not used, but the single wire was placed alternately tangent to the two limbs of the planet. From the discordance of the measured diameters, this method would appear much less accurate than that of double wires.

ALBANY.

Here, also, a single wire was placed alternately tangent to the two limbs of the planet, several contacts with each limb being made. As the fixed wire was used in these measures, and there was not time to read the microscopes for all the contacts, the readings were referred to the microscopes by an apparatus invented by Mitchell, called a Declinometer. This apparatus is described in the publications of the Dudley Observatory, and the mode of using it in the present case, as well as an abstract of the observations, may be found in the volume of Washington Observations for 1863. Owing to the irregularity of this method, I have hesitated to admit its results. But after a careful scrutiny of the observations, and a personal examination of the apparatus, I am unable to see how any source of constant error could have crept in, and have therefore admitted them with a small weight.

WASHINGTON.

During the first series (until September 24) the star observations were irregular, two of the observers placing the star image between the wires, and the third using wire 1 exclusively. The employment of the former system for a limited number of the observations does not seem to be objectionable, since the constant errors to which it is known to be liable may equally affect bisections of a star by a wire. But when the same wire is used for all the comparison stars, and the mean of the two wires for Mars, any error in the adopted distance of the wires will affect all the observations in the same way. This distance being determined from nadir observations, in the manner described in Washington Observations for 1863, p. XXVII, does not seem reliable. It was therefore redetermined in the following way:

1. Reference was made to the original observations for nadir point in 1862, and every result, both for single and double interval of wires during the entire series of parallax observations, was classified according to the observer, and arranged in the order of time.

These results, being supposed affected only with constant personal errors, gave entirely reliable values of the *changes* of distance at those epochs when the wires were disturbed, though none of the absolute distances are considered reliable. It was thus found that the distance of the wires from September 4 to October 2 was greater by 0 *r*. 0070, or 0″.45, than from October 7 to November.

2. The absolute distance from October 7 to November was derived from observations of couples of stars made on couples of dates, on which the order of use of wires was reversed. Let D_1 and D_2 be the declinations of a star deduced from observations on two dates with wires 1 and 2, respectively; d_2 and d_1 the declinations of another star, deduced on the same dates with wires 2 and 1. Then, if δ be the correction to the adopted distance of wires, we shall have—

$$\delta=\tfrac{1}{2}(D_1-D_2+d_1-d_2).$$

Thus was found, $\delta = -0''.132$.

	″
Adopted distance, October 7 to November - - - - -	13.68
True distance - - - - - - - - - - - - - - - -	13.55
Correction for September - - - - - - - - - - - -	+ 0.45
True distance for September - - - - - - - - - -	14.00
Adopted distance in published reductions - - - - -	14.02
Error of published reductions - - - - - - - - - -	0.02

The correction on account of erroneous half distance, being only a hundredth of a second, has been neglected.

WILLIAMSTOWN.

It is not stated by Mr. Ellery whether a single wire or a pair of wires was used.

The North Polar Distances, as forwarded in manuscript, were not corrected for errors of division. They were, however, accompanied by a table, giving the errors of division for every degree of zenith distance. Unfortunately, it was not stated either in what direction the zenith distance was counted, or whether the correction was to be applied to the circle reading, which might increase toward the north, or the polar distance, which increased toward the south. In this difficulty, recourse was had to the paper of Mr. Stone, who might be supposed acquainted with the facts, and who published a similar table for every 10°. It was found that he counted the zenith distance toward the north, and applied the corrections in the same direction. To ascertain whether the corrections thus applied were real, the corrections to the adopted places of the stars were deduced from all the Williamstown observations, without applying errors of division, and then the corrections were corrected for errors of division, in order to see whether they were thus diminished. The results were:

Series.	I.	II.	III.	IV.	Sum.
	"	"	"	"	"
Sum of corrections uncorrected	2.9	3.7	3.9	3.2	13.7
Sum of corrections corrected	1.1	2.5	1.8	2.9	8.3
Sum of squares uncorrected	1.83	2.41	2.33	1.98	8.55
Sum of squares corrected	0.27	0.95	0.70	1.37	3.29

The improvement is so well marked, the probable error being reduced from 0″.39 to 0″.24, that the correctness of the assumption can hardly be doubted. The errors of division have therefore been thus applied.

Santiago.

The observations appear to have been carefully made throughout. They do not, however, impress one with a high sense of the excellence of the Meridian Circle, or, at least, of the precision with which its microscopes can be read. There is also a weak point in one of the important elements of reduction, namely, the inclination of the declination wires. There are two of these wires, one fixed, the other movable by a micrometer screw. In the Mars observations, the latter was set over or under the fixed wire, at a distance somewhat less than the diameter of Mars, and the observations were then made in strict accordance with Winnecke's programme. The only information respecting the inclination of the wires is in the following words:

"En el campo de vision del anteojo del Circulo Meridiano està estendido un hijo fijo paralelo al camino que recorra una estrella ecuatorial. * * * Por fin, es de advertir que el hilo mòvil no es exactamenta paralelo al hilo fijo. Estando el anteojo dirijido al Norte, la estremidad occidental del hilo mòvil queda encima del hilo fijo, i la inclination de los dos hilas ascienda a 0°7′."

From this it is deducible that the correction to the observed north polar distance, on account of inclination of movable wire, is negative when the observation is made before meridian passage, and positive afterward, and that the amount of the correction is as follows:

Interval from meridian.	Correction.
s.	"
10	0.31
20	0.61
30	0.92
40	1.22
50	1.53
60	1.84
70	2.14

Either the clock time of transit or the vertical wire being given for all extra meridian observations, the polar distances observed with the movable wire were corrected for inclination according to the above table. It was soon seen that the effect of this correction was to produce large discordances in the results for polar point deduced from the several stars, and this effect was so uniform and well marked as to leave little doubt that the correction had no existence in fact.

It was then found that some correspondence had passed between the Superintendent of the Naval Observatory and Mr. Moesta on this very subject, in 1864, the effect of which was to throw darkness on the nature of the inclination correction of the Santiago Circle during this critical period.

I next attempted to determine the inclination from observations of the same star, made on different sides of the meridian. To effect this, a table was drawn up, showing the hour angles of Mars and each of the eight stars, for each of the Santiago observations. It was found that the same star was nearly always observed with great regularity on the same side of the meridian. It was not possible, therefore, to determine the inclination of wires.

The following plan was adopted: In each determination of the polar point, the inclination of the mean of the wires was included as an unknown quantity multiplied by a numerical coefficient, equal to the mean of the hour angles. The correction to the polar distance of Mars was then equal to this unknown quantity, multiplied by the difference of hour angle between Mars and the rest of the stars. The stars whose hour angles were farthest from that of Mars were then rejected in such number that the sum of the coefficients of the inclination should be quite small for each series, and, as nearly as practicable, vanish entirely for the mean of all the observations.

§ 5.

The North Polar Distances of Mars thus deduced from observations, with the resulting equations of condition, are given in Table III.

The first column of this table gives the mean solar date, the day changing between the meridians of Washington and Williamstown.

The third column gives the seconds of North Polar distance of Mars, derived in the way set forth in the last section.

Following these are the multipliers of the equations of condition to reduce them to the same probable error. These multipliers are inversely as the concluded probable errors of the positions of Mars. They are on a scale of 1 to 3 only, as it did not seem worth while to attempt dividing the observations into more than three classes with respect to excellence. The multipliers are assigned by the following considerations:

1. The mean error of an observation at any one Observatory, as deduced from discordance of results for polar points derived from the several stars observed on any one evening.

2. The number of stars on which the polar point depends. The omission of several stars does not, however, seriously diminish the multiplier, unless the same stars are missed night after night, so that the final result of the work with any one instrument will depend on too few stars.

3. The notes of the observers with respect to the quality of the images. The precision of the observations, however, appears to be much less affected by this cause than might be supposed. The greatest extremes of description occur in the Cape observations. Here it is found that the mean discordance on six good nights is 0″.37; while, on six nights noted as very bad, or deplorably bad, it is 0″.35. Here, at least, the effects of the cause in question would appear to be entirely masked by those of other causes.

4. The number of observers. There being always a possibility of personal differences in the measures, greater weight should be given to a series made by several than to a similar series made by one observer.

The first consideration was the fundamental one. The mean errors for the different observations were found to be as follows:

	″
Pulkowa	0.31
Helsingfors	0.73
Leiden	0.33
Greenwich	0.53
Albany	0.64
Washington	0.56
Williamstown	0.37
Cape	0.36
Santiago	0.62

The adopted multiplier is supposed to carry the mean error of each equation to about 0.″9.

The next column gives the computed parallax in polar distance for the observation, the adopted constant of Sun's equatorial horizontal parallax being

8″.90.

The following are the adopted co-ordinates of the different observations:

Observatory.	Geocentric latitude.	Log ρ.	Longitude from Pulkowa.	The same in parts of a day.
			h. m. s.	d.
Williamstown	− 37 41.0	9.99946	− 7 38 36	− .31830
Pulkowa	59 36.3	9.99890	0 0 0	.00000
Helsingfors	60 0.0	9.99890	+ 0 21 28	+ .01490
Cape	− 33 45.4	9.99955	0 47 23	.03291
Leiden	51 58.3	9.99910	1 43 21	.07177
Greenwich	51 17.4	9.99912	2 1 19	.08425
Santiago	− 33 16.1	9.99956	6 43 51	.28056
Albany	42 28.4	9.99934	6 56 18	.28910
Washington	38 42.5	9.99930	7 9 30	.29827

Next, we have the polar distances of Mars given by the ephemeris. In the adopted method of discussion it is essential that the differences between the ephemeris and the actual position of the planet should vary regularly during each period of twenty days. To insure this, heliocentric ephemerides of the Earth and Mars were computed by Mr. Charles Thirion, aid at the Naval Observatory, for every other day during the parallax observations, using Le Verrier's tables. In the ephemeris of the Earth the planetary perturbations were smoothed off by differences, while the lunar perturbation, instead of being taken from the tables, was rigorously computed from the co-ordinates of the Moon. In the Mars ephemeris the perturbations of each separate planet were mainly developed in powers of the time, to reduce the accidental errors in the last place of decimals, produced by adding so many terms.

Comparing these positions with those of Winnecke's ephemeris, the variations of the differences from the desired law were found to be altogether insignificant, seldom amounting to 0″.02 in longitude, and still less in latitude. I still feared that, as Winnecke used but seven decimals in computing his geocentric plans, the imperfections of the last decimal might have affected his declinations. Differencing his two-day ephemeris, the accidental errors were found not to exceed, on the average, 0″.02, so that their influence would be altogether insensible. His ephemeris was therefore adopted as a basis. From the right ascension observations made at Pulkowa, it appeared that the tabular heliocentric longitude of the planet was too great by about 2″.40.

Supposing, therefore, that the heliocentric longitude required the constant correction −2″.4, the effect of this upon the geocentric north polar distance was computed for every ten days, with the following result:

1862. August	18,	″ +2.32;
	28,	2.62;
September	7,	2.92;
	17,	3.15;
	27,	3.24;
October	7,	3.16;
	17,	2.93;
	27,	2.60;
November	6,	+2.28.

These corrections being interpolated to every day, and applied to Winnecke's Ephemeris, we have the following ephemeris of the theoretical north polar distances of Mars at transit over the meridian of Pulkowa:

Date.	Apparent north polar distance.	First difference.	Second differ'ce.	Log first difference.
Aug. 20	87 9 48.02	−2 51.58	+15.08	2.21199
21	87 6 53.41	2 39.50	15.10	.20276
22	87 4 13.91	2 21.40	15.15	2.15956
23	87 1 49.54	2 9.25	15.15	.11143
24	86 59 40.29	1 54.10	15.13	2.05729
25	86 57 46.19	1 38.97	15.12	1.99550
26	86 56 7.22	1 23.85	15.03	1.92350
27	86 54 43.37	1 8.82	14.96	1.83771
28	86 53 34.55	0 53.86	14.83	1.73127
29	86 52 40.69	0 39.03	14.71	1.59140
30	86 52 1.66	0 24.32	14.58	1.38596
31	86 51 37.34	−0 9.74	14.49	−0.98856
Sept. 1	86 51 27.60	+0 4.75	14.31	+0.67669
2	86 51 32.35	0 19.06	14.18	1.28012
3	86 51 51.41	0 33.24	14.03	1.52166
4	86 52 24.65	0 47.27	13.90	1.67459
5	86 53 11.92	1 1.17	13.67	1.78651
6	86 54 13.09	1 14.84	13.46	1.87413
7	86 55 27.93	1 28.30	13.18	1.94596
8	86 56 56.23	1 41.48	12.93	2.00638
9	86 58 37.71	1 54.41	12.60	2.05846
10	87 0 32.12	2 7.01	12.25	2.10384
11	87 2 39.13	2 19.26	11.82	2.14383
12	87 4 58.39	2 31.08	10.39	2.17921
13	87 7 29.47	2 42.47	10.88	2.21077
14	87 10 11.94	2 53.35	10.38	2.23892
15	87 13 5.29	3 3.73	9.73	2.26418
16	87 16 9.02	3 13.46	9.07	2.28659
17	87 19 22.48	3 22.53	8.38	2.30649
18	87 22 45.01	3 30.91	7.66	2.32410
19	87 26 15.92	3 38.57	6.85	2.33959
20	87 29 54.49	3 45.42	6.05	2.35299
21	87 33 39.91	3 51.47	5.15	2.36149
22	87 37 31.38	3 56.62	4.21	2.37405
23	87 41 28.00	4 0.83	3.28	2.38171
24	87 45 28.83	4 4.11	2.26	2.38759
25	87 49 32.94	4 6.37	1.36	2.39159
26	87 53 39.31	4 7.73	+ 0.39	2.39398
27	87 57 47.04	+4 8.12	− 0.56	+2.39466
Sept. 28	88 1 55.16	+4 7.56	− 1.51	+ .39368
29	88 6 2.72	4 6.05	2.41	.39102
30	88 10 8.77	4 3.64	3.28	.38675
Oct. 1	88 14 12.41	4 0.36	4.17	.38086
2	88 18 12.77	3 56.19	5.05	.37326
3	88 22 8.96	3 51.14	5.87	.36388
4	88 26 0.10	3 45.27	6.64	.35270
5	88 29 45.37	3 38.63	7.47	.33971
6	88 33 24.00	3 31.16	8.24	.32461
7	88 36 55.16	3 22.92	8.99	.30733
8	88 40 18.08	3 13.93	9.76	.28764
9	88 43 32.01	3 4.17	10.41	.26522
10	88 46 36.18	2 53.74	11.10	.23990
11	88 49 29.92	2 42.64	11.77	.21123
12	88 52 12.56	2 30.87	12.43	.17860
13	88 54 43.43	2 18.44	12.98	.14126
14	88 57 1.87	2 5.46	13.57	2.09851
15	88 59 7.33	1 51.89	14.09	2.04879
16	89 0 59.22	1 37.80	14.57	1.99034
17	89 2 37.02	1 23.23	15.03	1.92028
18	89 4 0.25	1 8.20	15.45	1.83378
19	89 5 8.45	0 52.75	15.84	1.72222
20	89 6 1.20	0 36.91	16.21	1.56714
21	89 6 38.11	0 20.70	16.49	1.31597
22	89 6 58.81	+0 4.21	16.72	0.62428
23	89 7 3.02	−0 12.51	16.91	1.09726
24	89 6 50.51	0 29.42	17.03	1.46861
25	89 6 21.09	0 46.45	17.09	1.66699
26	89 5 34.64	1 3.54	17.13	1.80305
27	89 4 31.10	1 20.67	17.08	1.90671
28	89 3 10.43	1 37.75	16.98	1.99012
29	89 1 32.68	1 54.73	16.83	2.05968
30	88 59 37.95	2 11.56	16.65	2.11912
31	88 57 26.39	2 28.21	16.56	2.17088
Nov. 1	88 54 58.18	2 44.77	16.41	2.21688
2	88 52 13.41	3 1.18	−16.21	2.25811
3	88 49 12.23	−3 17.39	. .	+2.29533
4	88 45 54.84			

From the above ephemeris the positions in the sixth column were computed by interpolation, using the adopted longitudes of the Observatories already given.

In forming the equations of condition, the observations are divided into five series. The first two series comprise the observations made with the first group of Winnecke's comparison stars, and the next three those made with the three following groups.

In forming the equations of condition, the errors of the north polar distance of the ephemeris proved to be so minute that the simple error of geocentric declination was introduced into the equations instead of the error of the linear co-ordinate z, which it was intended to use. The three unknown quantities in the equations are as follows:

α, the error of north polar distance at the middle date of each series.

β, the change of α in ten days, supposed constant throughout each series.

π', the error of the Sun's mean equatorial horizontal parallax divided by 0.89.

The general form of the equations of condition is

$$0 = \mathrm{P}\left\{ \alpha + \frac{t}{10}\beta + \frac{0.89 \sin z'}{\Delta}\pi' + \Delta\mathrm{N.P.D.} \right\}$$

where

P = measure of precision, (column 4.)

t = time in days from the middle of each series.

z' = planet's apparent geocentric zenith distance south.

Δ = planet's distance from the earth.

ΔN.P.D. = the computed, minus the observed, geocentric north polar distance.

Comparison of North Polar distances of Mars derived from observation with those given by the Ephemeris, and Equations of condition given by the comparison.

FIRST SERIES.

Date.	Observatory.	Second of observed north polar distance.	Precision.	Comp. parallax.	Sec. of computed geocentric north polar distance.	Equations of condition given by the observations.			
1862.									″
Aug. 21	Cape	36.82	3	+ 10.71	47.95	$0=3a_1$	$-3.3\delta_1$	$-3.2\pi'$	+1.3
	Santiago	56.3	1	+ 10.6	7.17	1	−1.4	−1.1	+0.3
	Albany	16.47	1	− 11.39	5.78	1	−1.1	+1.1	+0.7
22	Williamstown	51.22	2	+ 11.73	3.09	2	−2.1	−2.3	+0.3
	Cape	57.92	3	+ 10.81	8.95	3	−3.0	−3.2	+0.7
	Santiago	21.3	2	+ 10.7	30.90	2	−2.0	−2.1	−2.2
23	Williamstown	22.75	2	+ 11.84	33.87	2	−1.9	−2.4	−1.4
	Pulkowa	4.63	3	− 15.21	49.54	3	−2.7	+4.6	+0.4
	Cape	34.10	3	+ 10.91	45.05	3	−2.7	−3.3	+0.1
	Santiago	1.4	2	+ 10.8	11.75	2	−1.8	−2.2	−0.9
24	Williamstown	7.53	2	+ 11.95	19.80	2	−1.7	−2.4	+0.6
	Cape	21.97	3	+ 11.01	36.29	3	−2.4	−3.3	+0.9
	Santiago	55.8	1	+ 10.9	6.75	1	−0.8	−1.1	+0.0
	Albany	45.82	1	− 11.71	5.75	1	−0.8	+1.2	+1.6
25	Williamstown	7.10	2	+ 12.05	20.85	2	−1.5	−2.4	+3.4
	Cape	31.29	3	+ 11.10	42.69	3	−2.1	−3.3	+0.9
	Greenwich	50.33	1	− 13.82	37.27	1	−0.7	+1.4	+0.8
	Santiago	6.2	2	+ 11.0	16.90	2	−1.4	−2.2	−0.6
	Washington	26.08	1	− 10.86	15.09	1	−0.7	+1.1	−0.1
26	Cape	53.12	2	+ 11.20	4.22	2	−1.2	−2.2	−0.2
	Santiago	32.1	2	+ 11.1	42.18	2	−1.2	−2.2	−2.0
27	Williamstown	55.34	2	+ 12.27	8.43	2	−1.1	−2.5	+1.6
	Greenwich	52.25	1	− 14.01	36.99	1	−0.5	+1.4	−1.2
	Santiago	12.3	2	+ 11.2	22.55	2	−1.0	−2.2	−1.9
28	Williamstown	43.20	2	+ 12.37	51.83	2	−0.9	−2.5	−1.5
	Greenwich	45.14	1	− 14.14	29.41	1	−0.4	+1.4	−1.6
	Santiago	5.7	1	+ 11.3	17.94	1	−0.4	−1.1	+0.9
	Washington	26.79	1	− 11.07	16.92	1	−0.4	+1.1	+1.2
29	Williamstown	43.60	2	+ 12.46	56.22	2	−0.7	−2.5	+0.3
	Cape	27.67	2	+ 11.48	39.17	2	−0.6	−2.3	+0.0
	Santiago	18.1	1	+ 11.3	28.25	1	−0.3	−1.1	−1.2
	Albany	39.09	1	− 12.14	27.89	1	−0.3	+1.2	+0.9
30	Williamstown	59.89	2	+ 12.56	12.50	2	−0.5	−2.5	+0.1
31	Williamstown	30.11	1	+ 12.66	43.51	1	−0.1	−1.3	+0.7
	Cape	21.56	2	+ 11.66	36.79	2	−0.2	−2.3	+1.1
	Santiago	21.1	2	+ 11.5	33.14	2	−0.2	−2.3	+1.1
Sept. 1	Pulkowa	43.72	3	− 16.29	27.60	3	0.0	+4.9	+0.5
	Cape	15.66	3	+ 11.75	27.53	3	0.0	−3.5	+0.4
2	Pulkowa	48.83	3	− 16.40	32.35	3	+0.3	+1.9	−0.2
	Cape	21.31	3	+ 11.81	32.75	3	+0.3	−3.6	−1.3
	Albany	48.26	2	− 12.50	36.40	2	+0.2	+2.5	+1.3
	Washington	48.65	2	− 11.48	36.51	2	+0.2	+2.3	−1.3
3	Pulkowa	7.33	3	− 16.51	51.11	3	+0.6	+5.0	+1.8
	Cape	39.93	1	+ 11.92	52.27	1	+0.2	−1.2	+0.4
	Greenwich	8.45	2	− 11.76	53.66	2	+0.4	+3.0	−0.1
	Santiago	47.3	1	+ 11.8	59.32	1	+0.2	−1.2	+0.2
	Albany	11.64	1	− 12.59	59.57	1	+0.2	+1.3	+0.5
	Washington	11.25	2	− 11.56	59.84	2	+0.4	+2.3	+0.3
4	Williamstown	58.28	1	+ 13.03	12.55	1	+0.3	−1.3	+1.2
	Pulkowa	41.30	3	− 16.62	24.65	3	+0.9	+5.0	−0.1
	Cape	13.77	3	+ 12.00	25.98	3	+0.9	−3.6	+0.6
	Santiago	25.7	1	+ 11.9	37.50	1	+0.3	−1.2	−0.1
	Albany	49.09	1	− 12.68	36.88	1	+0.3	+1.3	+0.5
	Washington	47.83	2	− 11.64	37.29	2	+0.6	+2.3	+2.2
5	Williamstown	42.67	1	+ 13.11	55.37	1	+0.4	−1.3	−0.4
	Cape	1.23	2	+ 12.07	13.71	2	+0.8	−2.4	+0.8
	Santiago	16.9	2	+ 11.9	27.69	2	+0.8	−2.4	−2.2
	Albany	40.37	1	− 12.77	28.18	1	+0.4	+1.3	+0.6
	Washington	40.52	2	− 11.72	28.73	2	+0.8	+2.3	−0.1
6	Williamstown	37.37	1	+ 13.19	52.14	1	+0.5	−1.3	+1.6
	Washington	45.28	2	− 11.80	34.01	2	+1.0	+2.4	+1.1
7	Santiago	39.4	2	+ 12.1	51.33	2	+1.2	−2.4	−0.3
	Albany	3.88	1	− 12.94	52.09	$0=1a_1$	$+0.6\delta_1$	$+1.3\pi'$	+1.1

FIRST SERIES—Continued.

Date.	Observatory.	Second of observed north polar distance.	Precision.	Comp. parallax.	Sec. of computed geocentric north polar distance.	Equations of condition given by the observations.			
1862.									
Sept. 8	Williamstown	11.61	1	+ 13.31	26.68	$0=1a_1$	$+0.7\beta_1$	$-1.3\pi'$	+1.7″
	Cape	46.66	2	+ 12.28	59.35	2	+1.1	−2.5	+0.8
	Santiago	11.1	1	+ 12.2	23.38	1	+0.7	1.2	+0.1
	Albany	36.22	1	− 13.02	21.22	1	+0.7	+1.3	+1.0
	Washington	36.17	2	− 11.97	25.11	2	+1.1	+2.1	+1.9
9	Albany	20.05	1	− 13.11	9.17	1	+0.8	+1.3	+2.5
10	Williamstown	40.30	2	+ 13.48	51.31	2	+1.7	−2.7	+1.1
	Pulkowa	49.39	3	− 17.29	32.12	3	+2.7	+5.1	+0.1
	Leiden	55.70	2	− 15.63	40.82	2	+1.8	+3.1	+1.5
	Greenwich	57.65	1	− 15.46	42.35	1	+0.9	+1.5	+0.2
	Albany	20.61	1	− 13.20	7.56	1	+0.9	+1.3	+0.1
11	Williamstown	44.21	2	+ 13.56	57.36	2	+1.9	−2.7	−0.8
	Leiden	3.47	2	− 15.72	48.72	2	+2.0	+3.1	+2.5
	Greenwich	5.46	1	− 15.56	50.40	1	+1.0	+1.6	+0.5
12	Pulkowa	16.15	2	− 17.49	58.39	2	+2.2	+3.5	−0.5
	Helsingfors	17.92	1	− 17.57	0.56	1	+1.1	+1.8	+0.2
	Cape	50.81	3	+ 12.52	3.17	3	+3.3	−3.8	−0.5
	Leiden	21.19	3	− 15.81	8.81	$0=3a_1$	$+3.3\beta_1$	$+1.7\pi'$	+1.4

SECOND SERIES.

Date.	Observatory.	Second of observed north polar distance.	Precision.	Comp. parallax.	Sec. of computed geocentric north polar distance.	Equations of condition given by the observations.			
Sept. 13	Williamstown	25.97	2	+ 13.68	10.11	$0=2a_2$	$-1.1\beta_2$	$-2.7\pi'$	+1.0″
	Pulkowa	47.26	3	− 17.59	29.47	3	−1.5	+5.3	−0.6
	Helsingfors	48.85	2	− 17.67	31.82	2	−1.0	+3.5	+1.3
	Cape	21.02	3	+ 12.57	34.61	3	1.5	−3.8	+3.2
	Leiden	56.85	3	− 15.90	40.75	3	−1.5	+1.8	−0.6
	Santiago	1.6	1	+ 12.4	13.93	1	−0.5	−1.2	0.1
	Washington	28.87	2	− 12.31	16.76	2	−1.0	+2.5	+0.5
14	Williamstown	5.38	3	+ 13.71	18.91	3	−1.3	−1.1	−0.6
	Pulkowa	29.96	3	− 17.68	11.91	3	−1.2	+5.3	−1.0
	Leiden	39.78	2	− 15.99	24.02	2	−0.8	+3.2	+0.5
	Santiago	47.0	1	+ 12.5	59.50	1	−0.4	−1.2	+0.0
15	Williamstown	55.20	2	+ 13.79	8.96	2	−0.7	−2.8	−0.1
	Pulkowa	23.36	2	− 17.77	5.29	2	−0.6	+3.5	−0.6
	Leiden	33.97	3	− 16.08	18.11	3	−0.9	+1.8	+0.8
	Washington	12.19	2	− 12.48	59.01	2	−0.6	+2.5	−1.3
16	Williamstown	55.73	3	+ 13.83	9.45	3	−0.7	−1.1	−0.3
	Pulkowa	27.08	4	− 17.86	9.02	3	−0.6	+5.1	−0.6
	Leiden	38.46	3	− 16.16	22.59	3	−0.6	+1.8	+0.9
	Greenwich	40.63	1	− 16.00	24.99	1	−0.2	+1.6	+0.4
	Santiago	50.0	1	+ 12.6	2.35	1	−0.2	−1.3	−0.2
17	Williamstown	6.06	3	+ 13.87	19.88	3	−0.4	−1.1	−0.2
	Pulkowa	40.28	3	− 17.91	22.48	3	−0.3	+5.4	+0.1
	Helsingfors	44.58	1	− 17.98	25.44	1	−0.1	+1.8	−1.2
	Leiden	52.22	1	− 16.21	36.73	1	−0.1	+1.6	+0.8
	Greenwich	55.52	1	− 16.07	39.23	1	−0.1	+1.6	−0.2
	Santiago	5.5	1	+ 12.6	18.42	1	−0.1	−1.3	+0.3
18	Pulkowa	2.44	3	− 18.02	45.01	3	0.0	+5.1	+1.8
	Helsingfors	5.95	1	− 18.03	48.10	1	0.0	+1.8	+0.2
	Cape	39.03	3	+ 12.76	51.82	3	0.0	−3.8	+0.1
	Leiden	16.48	3	− 16.31	59.89	3	0.0	+1.9	−0.8
	Washington	0.30	2	− 12.69	47.08	2	0.0	+2.5	−1.1
19	Williamstown	53.63	3	+ 13.95	7.93	3	+0.2	−1.2	+1.0
	Pulkowa	34.97	3	− 18.09	15.92	3	+0.3	+5.4	−2.9
	Cape	10.16	3	+ 12.78	22.99	3	+0.3	−3.8	+0.2
	Leiden	48.75	1	− 16.38	31.36	1	+0.1	+1.6	−1.0
	Santiago	5.3	1	+ 12.6	16.51	1	+0.1	−1.3	−1.1
20	Leiden	26.66	3	− 16.41	10.46	3	+0.6	+1.9	+0.7
	Santiago	46.1	2	+ 12.7	57.08	2	+0.4	−2.5	−3.1
21	Williamstown	13.94	2	+ 14.00	27.47	2	+0.5	−2.8	−0.9
	Pulkowa	58.17	3	− 18.22	39.91	3	+0.9	+5.5	−0.1
	Albany	58.56	1	− 13.97	46.25	1	+0.3	+1.4	+1.7
	Washington	2.42	1	− 12.85	48.35	1	+0.3	+1.3	−1.2
22	Pulkowa	50.43	3	− 18.27	31.38	$0=3a_2$	$+1.2\beta_2$	$+5.5\pi'$	−2.3

SECOND SERIES—Continued.

Date.	Observatory.	Second of observed north polar distance.	Precision.	Comp. parallax.	Sec. of computed geocentric north polar distance.	Equations of condition given by the observations			
1862.									″
Sept. 22	Helsingfors	51.48	1	− 18.36	31.88	$0=1a_2$	$+0.4\beta_2$	$+1.8\pi'$	+1.8
	Cape	25.86	3	+ 12.83	39.40	3	+1.2	−3.8	+1.2
	Leiden	5.55	1	− 16.56	48.21	1	+0.4	+1.7	−0.8
	Greenwich	6.95	1	− 16.39	51.13	1	+0.4	+1.6	+0.6
	Santiago	25.1	1	+ 12.7	37.30	1	+0.4	−1.3	−0.5
	Albany	52.12	1	− 14.02	39.32	1	+0.4	+1.4	+1.2
	Washington	54.16	2	− 12.90	41.47	2	+0.8	+2.6	+0.4
23	Williamstown	57.86	3	+ 14.02	12.17	3	+1.4	−4.2	+0.9
	Pulkowa	47.42	2	− 18.32	28.00	2	+1.0	+3.7	−2.2
	Cape	23.32	3	+ 12.84	35.86	3	+1.5	−3.8	−0.9
	Leiden	0.94	1	− 16.61	45.16	1	+0.5	+1.7	+0.8
	Greenwich	4.41	1	− 16.44	48.16	1	+0.5	+1.6	+0.2
	Santiago	22.8	1	+ 12.7	35.18	1	+0.5	−1.3	−0.3
	Albany	50.66	1	− 14.07	37.25	1	+0.5	+1.4	+0.7
	Washington	51.65	2	− 12.91	39.41	2	+1.0	+2.6	+1.5
24	Williamstown	57.52	2	+ 14.02	11.77	2	+1.1	−2.8	+0.5
	Helsingfors	51.42	1	− 18.46	32.45	1	+0.6	+1.8	−0.5
	Cape	23.73	3	+ 12.84	36.82	3	+1.8	−3.8	+0.8
	Leiden	2.43	2	− 16.66	46.26	2	+1.2	+3.3	+1.0
	Santiago	24.3	2	+ 12.7	37.01	2	+1.2	−2.5	+0.1
	Albany	52.71	1	− 14.12	39.12	1	+0.6	+1.4	+0.5
	Washington	53.61	2	− 12.99	41.35	$0=2a_2$	$+1.2\beta_2$	$+2.6\pi'$	+1.5

THIRD SERIES.

Date.	Observatory.	Second of observed north polar distance.	Precision.	Comp. parallax.	Sec. of computed geocentric north polar distance.	Equations of condition given by the observations			
Sept. 25	Williamstown	1.05	2	+ 14.02	14.93	$0=2a_3$	$-1.1\beta_3$	$-2.8\pi'$	−0.″3
	Helsingfors	54.48	1	− 18.50	36.60	1	−0.5	+1.8	+0.6
	Cape	27.96	2	+ 12.83	41.02	2	−1.0	−2.6	+0.5
	Greenwich	9.65	2	− 16.51	53.62	2	−1.0	+3.3	+1.0
	Santiago	28.2	2	+ 12.7	41.88	2	−1.0	−2.5	+2.0
	Albany	56.47	1	− 14.16	43.98	1	−0.5	+1.4	+2.0
	Washington	58.19	2	− 13.03	46.23	2	−0.5	+2.6	+2.2
26	Williamstown	7.12	3	+ 14.01	20.68	3	−1.3	−4.2	−1.4
	Pulkowa	58.23	3	− 18.44	39.31	3	−1.2	+5.5	−1.4
	Leiden	13.47	3	− 16.72	57.06	3	−1.2	+5.0	+0.9
	Santiago	36.8	2	+ 12.7	48.72	2	−0.8	−2.5	−1.6
	Albany	3.45	1	− 14.19	50.83	1	−0.4	+1.4	+1.6
	Washington	6.41	2	− 13.06	53.11	2	−0.8	+2.6	−0.5
27	Williamstown	13.53	2	+ 14.00	28.09	2	−0.7	−2.8	+1.1
	Pulkowa	5.45	2	− 18.46	47.04	2	−0.6	+3.7	+0.1
	Helsingfors	10.22	2	− 18.56	50.74	2	−0.6	+3.7	−1.8
	Leiden	21.28	3	− 16.75	4.85	3	−0.9	+5.0	+1.0
	Santiago	43.3	1	+ 12.7	56.66	1	−0.3	−1.3	+0.7
	Albany	11.67	1	− 14.21	58.78	1	−0.3	+1.4	+1.3
28	Williamstown	21.39	2	+ 13.98	36.20	2	−0.5	−2.8	+1.7
	Pulkowa	13.51	2	− 18.48	55.16	2	−0.4	+3.7	+0.2
	Helsingfors	14.34	2	− 18.58	58.86	2	−0.4	+3.7	(+6.2)
	Santiago	51.6	2	+ 12.6	4.71	2	−0.4	−2.5	+1.0
	Washington	22.91	1	− 13.11	9.11	1	−0.2	+1.3	−0.7
29	Williamstown	29.53	2	+ 13.96	44.04	2	−0.2	−2.8	+1.1
	Pulkowa	21.85	3	− 18.49	2.72	3	−0.3	+5.5	−1.9
	Helsingfors	23.30	2	− 18.59	6.40	2	−0.2	+3.7	+3.4
	Leiden	37.03	2	− 16.79	20.45	2	−0.2	+3.4	+0.4
	Washington	30.12	1	− 13.14	16.31	1	−0.1	+1.3	−0.7
30	Williamstown	37.15	2	+ 13.93	50.67	2	0.0	−2.8	−0.8
	Pulkowa	27.77	2	− 18.49	8.77	2	0.0	+3.7	−1.0
	Santiago	5.1	1	+ 12.6	17.42	1	0.0	−1.3	−0.3
Oct. 1	Williamstown	41.17	2	+ 13.90	55.17	2	+0.2	−2.8	+0.2
	Pulkowa	30.60	3	− 18.49	12.41	3	+0.3	+5.5	+0.9
	Leiden	47.41	2	− 16.80	29.78	2	+0.2	+3.4	−1.7
2	Williamstown	43.44	3	+ 13.87	56.67	3	+0.6	−4.1	[illegible]
	Cape	8.19	3	+ 12.68	20.61	3	+0.6	−3.8	−0.8
	Santiago	6.9	1	+ 12.5	19.49	1	+0.2	−1.2	+0.1
	Washington	36.57	2	− 13.15	23.70	$0=2a_3$	$+0.4\beta_3$	$+2.6\pi'$	+0.6

THIRD SERIES—Continued

Date.	Observatory.	Second of observed north polar distance.	Precision.	Comp. parallax.	Sec. of computed geocentric north polar distance.	Equations of condition given by the observations.			
1862.									''
Oct. 3	Williamstown	40.11	2	+ 13.83	54.28	$0=2a_3$	$+0.5\beta_3$	$-2.8\pi'$	+0.7
	Cape	4.04	2	+ 12.61	16.64	2	+0.6	−2.5	−0.1
4	Williamstown	32.92	2	+ 13.78	47.12	2	+0.8	−2.8	+0.8
	Pulkowa	18.67	3	− 18.44	0.10	3	+1.2	+5.5	−0.1
	Helsingfors	19.88	1	− 14.54	3.50	1	+0.4	+1.5	−1.8
5	Williamstown	21.41	3	+ 13.73	34.35	3	+1.4	1.1	1.5
	Cape	41.10	3	+ 12.53	52.67	3	+1.5	−3.8	−2.9
	Leiden	18.62	1	− 16.73	1.29	1	+0.5	+1.7	−0.6
	Santiago	34.6	1	+ 12.4	47.42	1	+0.5	−1.2	+0.1
6	Pulkowa	42.43	3	− 18.37	24.00	3	+1.8	+5.5	−0.2
	Helsingfors	45.09	1	− 18.47	27.21	1	+0.6	+1.8	+0.6
	Cape	17.66	3	+ 12.48	31.07	3	+1.8	−3.7	+2.8
	Leiden	55.77	3	− 16.70	39.42	3	+1.8	+5.0	+1.0
	Santiago	12.0	1	+ 12.3	24.03	1	+0.6	−1.2	−0.3
	Washington	41.70	1	− 13.10	27.77	$0=1a_3$	$+0.6\beta_3$	$+1.3\pi'$	−0.8

FOURTH SERIES.

Date.	Observatory.	Second of observed north polar distance.	Precision.	Comp. parallax.	Sec. of computed geocentric north polar distance.	Equations of condition given by the observations.			
Oct. 7	Leiden	26.37	3	− 16.66	10.03	$0=3a_4$	$-1.8\beta_4$	$+5.0\pi'$	+0.''8
	Santiago	41.1	1	+ 12.3	52.97	1	−0.6	−1.2	−0.4
	Albany	7.12	1	− 14.17	54.71	1	−0.6	+1.4	+1.8
	Washington	9.80	1	− 13.08	56.58	1	−0.6	+1.3	−0.1
8	Williamstown	59.65	3	+ 13.56	13.41	3	−1.6	−4.1	+0.6
	Leiden	48.36	3	− 16.62	32.34	3	−1.5	+5.0	+1.8
	Santiago	0.9	1	+ 12.2	13.43	1	−0.5	−1.2	+0.3
	Albany	28.37	1	− 14.13	15.11	1	−0.5	+1.4	+0.9
	Washington	27.76	1	− 13.05	16.91	1	−0.5	+1.3	+2.2
9	Williamstown	18.37	3	+ 13.49	31.28	3	−1.3	−4.0	−1.7
	Helsingfors	53.92	1	− 18.31	34.83	1	−0.4	+1.8	−0.8
	Leiden	2.10	3	− 16.56	45.60	3	−1.2	+5.0	+0.2
	Washington	40.35	2	− 13.02	27.99	2	−0.8	+2.6	+1.3
10	Williamstown	24.28	2	+ 13.42	38.62	2	−0.7	−2.7	+1.8
	Cape	29.95	3	+ 12.22	42.08	3	−0.9	−3.7	−0.3
	Leiden	4.87	3	− 16.50	49.01	3	−0.9	+5.0	+1.9
	Santiago	13.5	2	+ 12.0	26.00	2	−0.6	−2.4	+1.0
11	Williamstown	22.15	2	+ 13.35	35.74	2	−0.5	−2.7	+0.5
	Pulkowa	47.39	3	− 18.07	29.92	3	−0.6	+5.4	+1.8
	Helsingfors	49.73	2	− 18.18	32.42	2	−0.4	+3.6	+1.7
12	Williamstown	8.31	2	+ 13.27	21.98	2	−0.3	−2.7	+0.8
	Pulkowa	30.66	3	− 17.99	12.56	3	−0.3	+5.4	−0.3
	Cape	5.99	3	+ 12.08	17.71	3	−0.3	−3.6	−1.1
	Santiago	44.4	1	+ 11.9	56.00	1	−0.1	−1.2	−0.3
13	Williamstown	42.71	2	+ 13.19	56.71	2	0.0	−2.6	+1.6
	Helsingfors	4.45	1	− 18.01	45.58	1	0.0	+1.8	−0.9
	Cape	35.90	3	+ 12.00	48.19	3	0.0	−3.6	+0.9
14	Leiden	26.98	2	− 16.21	11.31	2	+0.2	+3.2	+1.1
	Albany	52.23	1	− 13.80	39.51	1	+0.1	+1.4	+1.1
15	Pulkowa	24.71	3	− 17.71	7.33	3	+0.6	+5.3	+1.0
	Cape	59.26	2	+ 11.85	11.23	2	+0.4	−2.4	+0.2
	Greenwich	33.88	2	− 15.95	17.29	2	+0.4	+3.2	−1.3
	Santiago	27.5	2	+ 11.7	40.11	2	+0.4	−2.3	+1.8
16	Pulkowa	16.55	3	− 17.60	59.22	3	+0.9	+5.3	+0.8
	Leiden	22.51	2	− 16.03	6.72	2	+0.6	+3.2	+0.5
17	Washington	15.92	1	− 12.53	3.39	1	+0.4	+1.3	+0.0
18	Williamstown	22.02	2	+ 12.74	35.35	2	+1.0	−2.5	+1.2
	Washington	35.23	2	− 12.45	22.18	2	+1.0	+2.5	−1.2
19	Cape	59.61	2	+ 11.55	10.44	2	+1.2	−2.3	−1.4
	Santiago	13.0	2	+ 11.3	24.83	$0=2a_4$	$+1.2\beta_4$	$-2.3\pi'$	+1.1

FIFTH SERIES.

Date.	Observatory.	Second of observed north polar distance.	Precision.	Comp. parallax.	Sec. of computed geocentric north polar distance.	Equations of condition given by the observations.			
1862.									″
Oct. 20	Santiago	2,1	2	+ 11,3	13,18	$0=2a_5$	$-1,4\beta_5$	$-2,3\pi'$	−1,0
	Washington	26,19	2	− 12,28	13,89	2	−1,4	+2,5	+0,0
21	Cape	27,32	3	+ 11,32	39,05	3	−1,8	−3,4	+1,2
22	Williamstown	10,56	2	+ 12,35	53,99	2	−1,1	−2,5	+2,2
	Cape	47,92	3	+ 11,23	59,21	3	−1,5	−3,4	+0,2
	Washington	12,58	2	− 12,09	1,81	2	−1,0	+2,4	+2,6
23	Williamstown	51,35	2	+ 12,25	3,47	2	−0,9	−2,4	−0,3
	Cape	52,01	3	+ 11,13	2,88	3	−1,2	−3,3	−0,8
	Santiago	50,2	2	+ 11,0	1,21	2	−0,8	−2,2	+0,0
	Albany	12,75	1	− 12,96	1,15	1	−0,4	+1,3	+1,4
	Washington	12,09	1	− 11,99	1,05	1	−0,4	+1,2	+1,0
24	Williamstown	41,55	2	+ 12,15	56,31	2	−0,7	−2,4	−0,8
	Cape	38,37	3	+ 11,01	49,81	3	−0,9	−3,3	+1,2
	Leiden	3,29	2	− 15,11	48,97	2	−0,6	+3,0	+1,6
	Santiago	32,7	1	+ 10,9	43,98	1	−0,3	−1,1	+0,4
	Washington	51,31	1	− 11,88	43,52	1	−0,3	+1,2	+1,1
25	Cape	8,73	3	+ 10,95	19,83	3	−0,6	−3,3	+0,4
	Santiago	57,9	1	+ 10,8	9,78	1	−0,2	−1,1	+1,1
26	Williamstown	38,81	3	+ 11,91	51,27	3	−0,4	−3,6	+1,6
	Leiden	45,05	2	− 14,83	30,65	2	−0,2	+3,0	+0,9
27	Cape	17,89	3	+ 10,75	28,72	3	0,0	−3,2	+0,2
	Santiago	58,5	2	+ 10,6	10,19	2	0,0	−2,1	+2,2
	Albany	21,67	1	− 12,51	9,54	1	0,0	+1,3	+0,4
28	Cape	57,40	3	+ 10,65	7,48	3	+0,3	−3,2	−1,7
	Albany	55,06	2	− 12,40	43,92	2	+0,2	+2,5	+2,5
	Washington	53,98	2	− 11,11	43,06	2	+0,2	+2,3	+1,0
29	Williamstown	51,25	3	+ 11,63	5,61	3	+0,5	−3,5	−0,7
	Greenwich	38,04	2	− 14,26	23,66	2	+0,4	+2,9	−0,2
	Santiago	51,7	1	+ 10,4	2,20	1	+0,2	−1,0	+0,1
	Washington	10,28	1	− 11,33	0,23	1	+0,2	+1,1	+1,3
30	Williamstown	4,31	3	+ 11,53	16,30	3	+0,8	−3,5	+1,4
	Cape	23,20	3	+ 10,47	33,89	3	+0,9	−3,1	+0,7
	Santiago	53,8	1	+ 10,3	2,73	1	+0,3	−1,0	−1,4
	Albany	12,09	1	− 12,11	1,63	1	+0,3	+1,2	+1,7
	Washington	11,19	2	− 11,21	0,47	2	+0,6	+2,2	+1,0
31	Williamstown	58,29	3	+ 11,42	10,08	3	+0,1	−3,4	+1,1
	Pulkowa	41,29	3	− 15,50	26,39	3	+1,2	+4,6	+1,8
	Helsingfors	37,65	1	− 15,61	24,29	1	+0,4	+1,6	+2,2
	Cape	10,78	3	+ 10,38	21,78	3	+1,2	−3,1	+1,8
	Leiden	29,67	3	− 14,11	16,30	3	+1,2	+4,2	+2,2
	Santiago	36,5	1	+ 10,2	46,49	1	+0,4	−1,0	−0,2
	Washington	51,48	2	− 11,09	43,92	2	+0,8	+2,2	+1,1
Nov. 1	Williamstown	35,20	3	+ 11,32	47,15	3	+1,4	−3,4	+1,9
	Pulkowa	12,96	3	− 15,31	58,18	3	+1,5	+4,6	+1,7
	Cape	43,02	3	+ 10,29	53,01	3	+1,5	−3,1	−0,9
	Santiago	3,6	1	+ 10,2	13,62	1	+0,5	+1,0	−0,2
	Albany	22,30	1	− 11,88	12,25	1	+0,5	+1,2	+1,8
2	Helsingfors	27,02	2	− 15,29	10,82	2	+1,2	+3,1	−1,8
	Cape	57,10	2	+ 10,20	7,71	2	+1,2	−2,0	+0,8
	Santiago	13,7	1	+ 10,1	24,26	1	+0,6	−1,0	+0,5
	Washington	31,36	2	− 10,85	21,08	2	+1,2	+2,2	+1,1
3	Williamstown	0,09	2	+ 11,11	11,66	2	+1,3	−2,2	+0,9
	Cape	57,23	2	+ 10,12	5,99	2	+1,4	−2,0	−2,7
	Greenwich	8,96	1	− 13,97	56,22	1	+0,7	+1,4	+1,2
	Santiago	9,0	1	+ 10,0	18,46	1	+0,7	−1,0	−0,5
	Washington	25,26	1	− 10,73	15,06	1	+0,7	+1,1	+0,5
3	Helsingfors	6,32	2	− 14,96	51,77	$0=2a_5$	$+1,6\beta_5$	$+3,0\pi'$	+0,8

Treating these equations by the method of least squares, we have the following normal equations:

First series.

$$311\ a_1 - 11.5\beta_1 - 32.4\pi' + 18''.5 = 0,$$
$$-11.5a_1 + 145.8\beta_1 + 114.0\pi' + 11\ .0 = 0,$$
$$-32.4a_1 + 114.0\beta_1 + 533.9\pi_1 + 41\ .8 = 0.$$

Second series.

$$308\ a_2 + 6.2\beta_2 + 122\ 7\pi' + 2''.5 = 0,$$
$$6.2a_2 + 41.1\beta_2 - 19.9\pi' - 1\ .3 = 0,$$
$$122.7a_2 - 19.9\beta_2 + 719.6\pi' - 25\ .3 = 0.$$

Third series.

$$237\ a_3 + 4.8\beta_3 + 67.9\pi' + 3''.9 = 0,$$
$$4.8a_3 + 33.6\beta_3 - 12.4\pi' - 7\ .1 = 0,$$
$$67.9a_3 - 12.4\beta_3 + 567.1\pi' + 13\ .0 = 0.$$

Fourth series.

$$192\ a_4 - 23.8\beta_4 + 62.1\pi' + 41''.4 = 0,$$
$$-23.8a_4 + 24\ 7\beta_4 - 11.0\pi' - 9\ .7 = 0,$$
$$62.1a_4 - 11.0\beta_4 + 427.9\pi' + 38\ .4 = 0.$$

Fifth series.

$$264\ a_5 + 23.5\beta_5 - 83.2\pi' + 75''.1 = 0,$$
$$23.5a_5 + 45.4\beta_5 + 26.4\pi' + 7\ .5 = 0,$$
$$-83.2a_5 + 26.4\beta_5 + 378.2\pi' + 38\ .6 = 0.$$

The separate solution of each series of equations gives the following results:

First series - - - -	$a_1 = -0''.167$;	$\beta_1 = -0''.053$;	$\pi' = -0''.077$.
Second series - - -	$a_2 = -0.020$;	$\beta_2 = +0.024$;	$\pi' = +0.039$.
Third series - - -	$a_3 = -0.016$;	$\beta_3 = +0.210$;	$\pi' = -0.016$.
Fourth series - - -	$a_4 = -0.188$;	$\beta_4 = +0.187$;	$\pi' = -0.057$.
Fifth series - - - -	$a_5 = -0.354$;	$\beta_5 = +0.119$;	$\pi' = -0.188$.

These are merely first approximations to the values of the unknown quantities. The rigorous solution would require us to take the last equation of each series and add them together to form a single one, and then find the values of the eleven unknown quantities from the eleven equations to which the fifteen would thus be reduced. This we shall do by successive approximations.

There is another consideration which will modify their treatment. It will be remembered that β is simply the change in a during ten days, that change being supposed uniform. Now, having a series of values of a at intervals of twelve or fifteen days, we could, if they were strictly comparable, deduce from their differences the values of β. In fact, only the first two are strictly comparable, different stars being used in each of the following series, the adopted positions of which may not strictly correspond to those of the first series. The probable differences between the means of eight stars, several of which are common, is, however, so small that the values of β, deduced by differences, can hardly be appreciably in error from this cause.

The comparison of the five successive values of a give the following values of β, alongside of which we place, for comparison, the values which have just been derived from the equations.

Value of β_1	from differences is	$+0''.09$;	from equations,	$-0''.05$.
" β_2	" "	$+0.05$;	" "	$+0.02$.
" β_3	" "	-0.05;	" "	$+0.21$.
" β_4	" "	-0.12;	" "	$+0.19$.
" β_5	" "	-0.12;	" "	$+0.12$.

The contrary progression and contrary signs of the two systems of values are a little singular. I can attribute them only to accidental errors. From the two series are deduced the following, as the most probable values of β:

$$\beta_1=+0''.04,$$
$$\beta_2=+0.04,$$
$$\beta_3=\ \ 0.00,$$
$$\beta_4=-0.03,$$
$$\beta_5=-0.03.$$

A second approximation to the value of π' gives

$$\pi'=-0''.05.$$

Substituting these values of β and π' in the first equation of each series we have the following values of a:

$$a_1=-0''.160,$$
$$a_2=+0.011,$$
$$a_3=-0.002,$$
$$a_4=-0.219,$$
$$a_5=-0.295.$$

These values of a, and the above of β, being substituted in the last equation of each series, these equations assume the following form, and give the following values of the solar parallax:

First series - - - -	$533.9\pi'=-\ 51''.5$;	$\pi'=-0''.096$;	$\pi=8''.815$.
Second series - - -	$719.6\pi'=+\ 24.8$;	$+0.034$;	8.930.
Third series - - - -	$567.1\pi'=-\ 12.9$;	-0.023;	8.880.
Fourth series - - -	$427.9\pi'=-\ 25.1$;	-0.059;	8.847.
Fifth series - - - -	$378.2\pi'=-\ 66.0$;	$=0.175$;	8.744.
Total - - - - -	$2626.7\pi'=-130.7$;	$\pi'=-0.050$;	$\pi=8.855$.

The probable error of each equation is about $0''.82$; the probable error of the concluded value of π' is approximately equal to the quotient of this quantity by the square root of the coefficient of π' in the final equation, or $0''.016$; the probable error of π itself, therefore, is by the usual method, $0''.014$. But this method presupposes that the errors of all the separate equations are entirely independent—an unsafe hypothesis until we ascertain whether the observations made at each Observatory may not be affected with errors peculiar to the observer. This we do by substituting in each equation of condition the concluded values of the unknown quantities, taking the algebraic sums of the residuals of the equations belonging to each Observatory, and dividing by the sum of the numbers by which the errors of observation have been multiplied, which is the same as the sum of the coefficients of a. The following are the separate sums of residuals and multipliers for each series of equations, with the final mean residual :

	Williamstown.		Pulkowa.		Helsingfors.	
	"		"		"	
First series . . .	+ 6.0	25	− 2.9	20	−0.1	1
Second series . .	+ 2.6	23	− 8.3	28	+0.9	6
Third series . . .	+ 2.0	25	− 5.5	18	+0.3	7
Fourth series . .	+ 2.5	16	− 0.5	22	−1.3	4
Fifth series . . .	+ 2.1	23	+ 1.1	6	−0.8	5
Sum	+15.2	112	−16.1	94	−1.0	23

	Cape.		Leiden.		Greenwich.	
	"		"		"	
First series . . .	+ 2.5	38	+ 3.4	7	−3.1	6
Second series . .	+ 5.5	18	− 0.4	23	+0.6	4
Third series . . .	+ 0.4	13	− 0.3	14	+0.8	2
Fourth series . .	− 3.4	11	+ 1.3	16	−1.9	2
Fifth series . . .	− 7.6	34	+ 2.1	7	−0.1	3
Sum	− 2.6	114	+ 6.4	67	−3.7	17

	Santiago.		Albany.		Washington.	
	"		"		"	
First series . . .	−11.4	23	+ 8.1	12	+2.2	14
Second series . .	− 4.9	11	+ 3.9	4	−0.8	13
Third series . . .	+ 2.8	11	+ 4.7	3	−0.5	9
Fourth series . .	+ 2.1	9	+ 2.9	3	+0.2	7
Fifth series . . .	− 3.4	14	+ 5.6	6	+5.0	16
Sum	−14.8	68	+25.2	28	+6.1	59

The probable algebraic sum of the residuals on the hypothesis of no constant errors peculiar to each Observatory will be $0''.82\sqrt{N}$; N being the number of observations, and the mean value will be $0''.97\sqrt{N}$.

The following table exhibits a comparison of the actual and probable sums, and the actual and probable mean residuals of the individual observations in the entire number made at each Observatory:

Observatory.	Probable sum.	Actual sum.	Probable mean.	Actual mean.
	"	"	"	"
Williamstown - -	±5.8	+15.2	±0.05	+0.14
Pulkowa - - - -	±4.5	−16.1	±0.05	−0.17
Helsingfors - - -	±3.6	− 0.1	±0.16	−0.04
Cape - - - - -	±5.4	− 2.6	±0.05	−0.02
Leiden - - - -	±4.4	+ 6.4	±0.07	+0.10
Greenwich - - -	±3.0	− 3.7	±0.18	−0.22
Santiago - - - -	±5.7	−14.8	±0.08	−0.22
Albany - - - -	±4.3	+25.2	±0.15	+0.90
Washington - - -	±4.9	+ 6.1	±0.08	+0.10

It will be seen that the actual exceeds the probable residual in seven cases out of the nine, so that the probability in favor of systematic differences is very great. In the case of Albany the evidence in favor of extraordinary systematic difference is indisputable, the observed polar differences being nine-tenths of a second less than those of the other northern Observatories throughout the entire series. This great discrepancy gives rise to the question whether the observations exhibiting them ought not to be considered as affected with some abnormal source of error, and rejected entirely; or, in other words, whether that standard to which the three southern Observatories is comparable, is more likely to be the mean of all the northern Observatories, including Albany, or only the mean of those five which agree well between themselves. Rejecting Albany altogether, the final equation in π' would be, approximately,

$$2574\pi' = -97''.$$

The resulting parallax would, therefore, be 8''.866, the Albany observations entering into the final result for parallax with a weight of only one-fiftieth that of all the others. I think we may consider them entitled to this weight notwithstanding their discordance, and shall, therefore, consider the parallax already deduced the most probable result of the meridian observations. Owing, however to the evidence of constant errors, the probable error of the result must be increased to 0''.020, giving, as the parallax from meridian observations of Mars, made in 1862, according to Winnecke's plan,

$$8''.855 \pm 0''.020.$$

§ 6.

Micrometric Observations of Mars, 1862.—These observations are discussed by Professor Hall, in the Introduction to the Washington Observations for 1863, in a manner which, so far as I see, leaves nothing to be desired. I shall, therefore, accept his result, which is

$$8''.842 \pm 0''.04,$$

the probable error being a rough estimate from the discordance of the results, and the probable systematic errors of the observers.

§ 7.

Solar Parallax from Observed Parallactic Inequality of the Moon.—The observations of the Moon, especially the older ones, do not present values of the parallactic inequality as accordant as we might expect from their number. In his second memoir on the corrections of the elements of the Moon's orbit, the Astronomer Royal finds, from all the Greenwich meridian observations of the Moon, from 1750 to 1851, the value 122''.79,* while the Altazimuth observations alone give the value 125''.50. When the observations previous to 1811 are rejected, owing to uncertainty what value of the semi-diameter should be used, the result is increased to 124''.37. Finally, it is concluded that the real value of the coefficient cannot be far from 124''.7.

Hansen's discussion of the Greenwich observations appears, however, to have led to a materially different result. In calculating the coefficients of the lunar perturbations, he found, from an assumed solar parallax, the value 121''.368.† By comparison with observations, however, it results that this value of the inequality must be multiplied by the factor 1.03573, in order to satisfy the observations‡ This gives for the true value of the coefficient, 125''.70.

The comparison of these publications of Professor Hansen shows that these coefficients are those of the development of *his disturbed mean anomaly*, while the usual development is that of the *true longitude*. They cannot be compared with other values until they are reduced to the latter development. If we represent by e_1, e_2, etc., the coefficients of sine mean anomaly, sine 2× mean anomaly, etc., in the development of the true anomaly, we find the following value of the perturbations of the latter from the formulæ on page 3 of Hansen's tables:

$$\begin{aligned} \delta f = \quad & n\delta z\,[1 + e_1 \cos g + 2e_2 \cos 2g + \text{etc}]. \\ & + (n\delta z)^2[\;-\tfrac{1}{2}e_1 \sin g - 2e_2 \sin 2g - \text{etc}]. \end{aligned}$$

* Memoirs of the Royal Astronomical Society, vol. XXIX, p. 16.
† Monthly Notices R. A. S., vol. XXIII, p. 242; Tables de la Lune, p. 8.
‡ Monthly Notices, XV, 9; XXIII, 242; XXIV, 10; Tables de la Lune, p. 16.

If we represent by $a \sin N$ any term in $n\delta z$, there will result in δf, in virtue of the first term of this equation, the terms

$$a \sin N + \tfrac{1}{2}e_1 \sin (N+g) + \tfrac{1}{2}e_1 \sin (N-g) + e_2 \sin (N+2g) + \text{etc.}$$

The powers of $n\delta z$ are to be developed in like manner.

In developing the square of $n\delta z$, I find no terms which will sensibly affect the parallactic inequality. The latter will therefore depend altogether in the following terms in Hansen's $n\delta z$:

$-\ \ 11''.692 \sin (-g'+\omega-\omega')$	producing the coefficient	$0''.641$,
$-121''.368 \sin (g-g'+\omega-\omega')$	" "	$121''.368$,
$-\ \ 1''.614 \sin (2g-g'+\omega-\omega')$		$0''.089$.

The total value of the theoretical coefficients is therefore 122''.098, which, being multiplied by 1.03573, gives

$$126''.46$$

for the actual value of the parallactic inequality deduced by Hansen from the observations of Greenwich and Dorpat, and adopted in his tables.

The Monthly Notices of the Royal Astronomical Society for May, 1867, contain a short abstract of a paper by Mr. Stone, in which he deduces from 2,075 Greenwich observations the value

$$125''.36.$$

This result I shall accept as the definitive result of the Greenwich observations.

The Washington observations of the Moon, from 1862 to 1865, inclusive, are regularly compared with Hansen's tables. I have discussed those made within two days of the time of maximum and minimum parallactic inequality, on the supposition that the effect of errors in the other inequalities will destroy each other in the course of the four years. Thus, the following corrections to Hansen's parallactic inequality are obtained for the several years:

	''
1862,	−2.2;
1863,	−2.2;
1864,	−2.0;
1865,	−2.0.

These results are still subject to correction for adopted semi-diameter of Moon. Seven transits of both limbs of the nearly full Moon were observed during the above period. The mean correction to Hansen's semi-diameter was zero. If, then, we suppose this same semi-diameter applicable to the Moon at her first and last quarters, the coefficient of parallactic inequality will be

$$126''.46-2''.10=124''.36.$$

But the same semi-diameter will not be applicable, because one-half the observations for parallactic inequality are made while the Sun is above the horizon, and a considerable fraction of the remaining half are made during twilight, while those on which the semi-diameter depends are made at midnight, when the brilliancy of the Moon is such as to excite the eye to a disagreeable extent. From the experiments of Dr. Robinson,* and the researches of Mr. Breen,† and other data, it seems that the effect of this brilliancy is to increase the apparent semi-diameter of the Moon by about 2''. About one-half of the observations being thus affected, the correction to the parallactic inequality from this cause ought to be about +1''.0.

* Memoirs Royal Astronomical Society, vol. v.

† Greenwich Observations for 1864, Appendix.

To obtain an independent determination of this correction, I have made a general comparison of the apparent errors of Hansen's tables in right ascension, when the observations were made during daylight with the corresponding errors when they were made at night. The selected night hours were, on the average, a very little nearer to midnight than the day hours were to noon. The results were for the apparent errors of the tables in right ascension:

	s
Before sunset	−0.154
After bright daylight in the evening	−0.093
Before bright daylight in the morning	+0.091
After sunrise	+0.153

From this investigation, the real enlargement would appear to be 0″.92, and the correction to the parallactic inequality 0″.5. But this correction is so affected by the correction of the coefficient of variation that it cannot be relied on.

There is still another cause of smaller apparent diameter about sunrise and sunset. At those times the Moon's disk is generally very sharply defined, while at midnight there is generally more or less spurious enlargement, called "blurring."

Finally, the following are adopted as the most probable corrections to the semi-diameter at midnight:

	″
On account of irradiation	−0.9
" " spurious enlargement	−0.2
Total	−1.1

The effect of this correction will be to increase the parallactic inequality derived from the Washington observations to

125″.46.

The different results will be combined by giving this the weight 4, Stone's the weight 8, and Hansen's the weight 1; the latter being derived from the Dorpat as well as the Greenwich observations. This gives

125″.49

as the most probable value of the parallactic inequality derived from observations.

Owing to the uncertainty respecting the proper semi-diameter of the Moon to be adopted, and to the fact that owing to the libration of the Moon's disk the points of the Moon's surface observed at quadratures may be systematically different from those observed at full Moon, I estimate the probable error of the above result at

0″.35.

To deduce the solar parallax from this value of the parallactic inequality, the formulæ of Delaunay and Plana will be adopted.* They give, for the parallactic inequality in terms of the solar parallax,

$$F.\frac{1-\mu}{1+\mu}.\frac{\sin\pi}{\sin P\left(1-\frac{m^2}{6}\right)}.$$

* Theorie du mouvement de la Lune, tome II, p. 847. Mr. Delaunay was good enough to communicate the formulæ for F in advance of the volume.

Where

π = constant of solar parallax.

μ = mass of the Moon. Adopted value, $\frac{1}{[illegible]}$.

P = constant of lunar parallax, = 3422″.7.

m = ratio of mean motions of Sun and Moon.

F, a factor whose value, according to Delaunay's theory, is formed as follows:

Terms multiplied by m	0.13865
" " m^2	.06500
" " m^3	.02262
" " m^4	.00885
" " m^5	.00382
" " m^6	.00136
" " m^7	.00064
Higher terms, (by induction)	.00029
Total value of F	.24123

Whence, solar parallax from parallactic inequality of the Moon =

8″.838 ± 0″.025.

As a test of the theory, this result may be compared with that of Hansen, in the Monthly Notices, vol. 24. From a value of the Moon's mass $\frac{1}{80}$, and the parallactic inequality of his tables, (126″.46,) he finds for the solar 8″.916. Altering the result to correspond to the data of the present paper, it will be

8″.844,

agreeing satisfactorily with the theory of Delaunay.

§ 8.

Solar Parallax from the Observed Lunar Equation of the Earth combined with the Mass of the Moon.—In constructing his tables of the Sun, Le Verrier investigated the lunar inequality of the earth from 35 years of Greenwich, 42 of Paris, and 17 of Konigsberg observations, with the result*

6″.50,

and a probable error of about 0″.03.

To complete the investigation, I have added the results of 14 years of Greenwich and 5 years of Washington observations. The results for the separate years are as follows:

Greenwich Observations.

	Cor.	ineq.		Cor.	ineq.
1851.	= +0″.33 ;	= 6″.87.	1858.	= +0″.01 ;	= 6″.45.
1852.	−0.13 ;	6.41.	1859.	+0.21 ;	6.65.
1853.	+0.25 ;	6.79.	1860.	+0.16 ;	6.60.
1854.	−0.11 ;	6.33.	1861.	−0.35 ;	6.19.
1855.	+0.03 ;	6.47.	1862.	+0.22 ;	6.76.
1856.	+0.39 ;	6.83.	1863.	+0.11 ;	6.65.
1857.	−0.10 ;	6.34.	1864.	−0.02 ;	6.48.

Resulting value of the lunar inequality,

6″.56 ± 0″.04.

* Annales de l'Observatoire Impérial de Paris, Memoires, tome iv, p. 100.

Washington Observations.

1861–'2.	Cor. =	$-0''.01$;	ineq. = $6''.37$.	1864.	Cor. =	$-0''.01$;	ineq. = $6''.40$.
1863.		$+0.27$;	6.68.	1865.		$+0.18$;	6.59.

Resulting value of the lunar inequality,

$$6''.51 \pm 0''.07.$$

It will be seen that the following values of the tabular coefficient have been used in obtaining the inequality from the correction given by the observations:

Greenwich,	1851–1853	$6''.54$
	1854–1860	6.44
	1861–1863	6.54
	1864	6.50
Washington		6.44

It is necessary to explain how these values have been obtained.

The above corrections, in the case of Greenwich, were deduced from the "Apparent Error of the Tables in R. A.," given each year in the Greenwich Observations in connection with the observed positions of the Sun, by a comparison of the "Apparent Errors" within three days of the maxima and minima values of lunar inequality. The next step is to find the value of the inequality actually contained in the ephemeris. The latter is, until 1863, that deduced from Carlini's Tables and published each year in the British Nautical Almanac. By induction from Carlini's Table V, it appeared that his value of the inequality was 6''.54. Afterward, I found that in a preceding volume of the "Effemerides" he had deduced the value 6''.537 from theory, and probably the table was constructed from this value. If, then, the ephemeris corresponded exactly with the tables, this would be the value to which the corrections correspond.

But on page V of each volume of the Nautical Almanac from 1854 to 1860, inclusive, it is stated that "The Longitude and Radius-Vector have been computed accurately from the Tables for the Mean Noon of every 6th day of the year, and interpolated with fourth differences for each day." Now, since the lunar inequality goes through its period in a month, its successive orders of differences for each sixth day will be divergent, and interpolation with fourth differences will result in the interpolated inequality being generally too small numerically. To find how much too small, actual trial was resorted to. A number of six-day series of values of the inequality was taken from Carlini's Table V, interpolated to days near the maxima and minima, and compared with the corresponding tabular values. The result showed that the interpolated values were, on the average, numerically too small by 0''.105. Since one-sixth of the values would be accurate, the actual diminution of Carlini's inequality would be 0''.087, reducing it to 6''.45.

From 1858 forward, the Sun's positions given in the American Ephemeris are deduced from Hansen's Tables, in which the value of the lunar inequality is 8''.44. Comparing these positions with the corresponding ones of the Nautical Almanac, the following differences were found:

[illegible]	$+0''.02$;
[illegible]	$+0.04$;
[illegible]	$+0.01$;
Mean,	$+0.02$;

giving, for the value of the inequality actually contained in the Nautical Almanac Ephemeris, 6''.43. The mean of this and the former result is 6''.44, which was considered the most probable value of the quantity in question.

Without the limits of the seven years, 1854–'60, the computations of the Nautical Almanac are probably at sufficiently short intervals to avoid the error of interpolation.

For 1864, LeVerrier's Solar Tables were adopted, the value of the lunar equation in which is 6″.50. We have, then, the following three values of the quantity sought for, deduced from observations:

	″	″	
From Greenwich, Paris, and Konigsberg observations,	6.50	±0.03;	wt. = 11.
Greenwich, 1851 to 1864 - - - - - - - -	6.56	±0.04;	6.
Washington, 1861 to 1865 - - - - - - - -	6.51	±0.07;	2.
Mean, by weights - - - - - - - - - - - - -	6.520	±0.023.	

Although the accidental errors of the observations on which this result depends are quite large, the observations have this invaluable characteristic, that they seem to be perfectly free from any cause of systematic error. Among all the constant sources of error to which observations of the Sun are liable, I can think of none which can systematically change with the first and last quarters of the Moon. If there are none, the precision of the determination of the lunar equation will go on increasing indefinitely with the number of observations.

The next step is to determine the mass of the Moon. The most precise determination is obtained by a comparison of the constants of precession and nutation, which gives the ratio of the disturbing forces of the Sun and Moon in changing the direction of the earth's axis of rotation. The value of this ratio will be deduced from the exhaustive memoir of Serret*, after reconstructing his expression for Ω so as to include the terms of the third order with respect to the inclination and eccentricity of the Moon's orbit, which he has neglected. This is effected by substituting the expression

$$(1+\tfrac{3}{2}e'^2)\sin e\cos e$$

for e in his value of Ω.

Let us put

μ = mass of the Moon, that of the Earth being unity.
p = sine of its parallax, in seconds.
M = mass of the Sun.
ε = ratio of disturbing forces of Sun and Moon.
$\varkappa$ = disturbing force of the Sun.
a = luni-solar precession for 1850.
N = constant of nutation.
P = coefficient of lunar equation of Earth.

Then the observed length of the seconds pendulum compared with the siderial year gives

$$\log M\pi^3 = 8.35488.$$

Whence

$$\varepsilon = \frac{\mu p^3}{M\pi^3} = [2.24812]\mu.$$

The formulæ of precession and nutation give

$$N = [9.38669]\varkappa\varepsilon,$$
$$a = [9.96272]\varkappa + [9.95922]\varkappa\varepsilon.$$

Peters's concluded value of the constant of nutation is

$$N = 9''.223.$$

*Annales de l'Observatoire Impérial de Paris, vol. V, p. 323.

The value of the luni-solar precession derived from Struve's general precession, with the mass of Venus concluded by LeVerrier, from his investigations of the motion of that planet, is

$$a = 50''.378$$

Substituting these values of a and N in the above equations, we find

$$\log \kappa\epsilon = 1.57818,$$
$$\log \kappa = 1.23898,$$
$$\log \epsilon = 0.33920,$$
$$\mu = \frac{1}{81.08}.$$

Developing the longitude and parallax of the Moon so as to include the variation, and the corresponding term in the parallax, we find

$$P = 1.0080 \frac{1}{1+\mu} \frac{\pi}{p} = [1.78351]\, \pi \frac{\mu}{1+\mu},$$

or,

$$\pi = .016461\, P \left(1 + \frac{1}{\mu} \right).$$

Substituting the value of P, already found from observation, we have

$$\pi = 8''.809.$$

The most uncertain data which enter into this result are the constant of nutation, with the resulting mass of the Moon, and the lunar equation of the earth. The probable error of the nutation constant is perhaps $\frac{1}{600}$ of its whole amount, which would involve an error of $\frac{1}{200}$ in the resulting mass of the Moon and solar parallax, or, of $0''.044$ in the latter. The uncertainty of the other factor involves a probable error of $0''.031$, so that the total probable error of the result is $0''.054$.

§ 9.

Transit of Venus in 1769.—The results of Powalky's discussion* will be accepted. He finds $\pi = 8''.832 \pm 0''.021$. But considering that the longitude of the observing station at San José is uncertain, he arbitrarily changes it by 10s., which increases the parallax to $8''.86$, which he considers the most probable value.

That so small a change in the longitude of a single station should change the parallax so largely, shows that the probable error $0''.021$ must be illusory. I think $0''.04$ a more likely value of this element.

§ 10.

Concluded Parallax and Distance of the Sun.—The separate results for the solar parallax with their probable errors, and their consequent weights, are as follows:

	″	″	
From meridian observations of Mars, 1862 - -	8.855 ±	.020 ;	wt. = 25.
From micrometric observations of Mars, 1862 -	8.842 ±	.040 ;	6
From parallactic inequality of the Moon - - -	8.838 ±	.028 ;	16 .
From the lunar equation of the Earth - - - - -	8.809 ±	.054 ;	3.
From the transit of Venus in 1769 - - - - - -	8.860 ±	.040 ;	6.
From Foucalt's experiments on light - - - - -	8.860	?	?

* Additions à la Connaissance des Temps, 1867, p. 22.

The last may not be considered a strictly astronomical result, and it is difficult to assign its probable error. The mean by weights of the other results is 8″.847. From a consideration of all the results, it is concluded that in the present state of astronomical science the most probable value of the mean equatorial horizontal parallax of the Sun is

8″.848,

with a probable error of

±0″.013,

corresponding to a mean distance of

92,380,000 statute miles.

For astronomical purposes the value of π,

8″.85

may be taken as a round number of hundredths having equal weight with the above concluded value.

§ 11.

Conclusions respecting the different Elements which depend on the parallax of the Sun.—From the equation of § 7, which gives the mass of the Sun in terms of its parallax, we find for the value of that mass

$$326800 \pm 1360,$$

taking the mass of the Earth as unity.

The value of the lunar equation of the Earth derived in the same section gives for the mass of the Moon

$$\frac{1}{81.44 \pm 0.33}.$$

Taking the mass of the Sun as unity, the combined masses of the Earth and Moon will therefore be

$$\frac{1}{322800}.$$

With the above value of the mass of the Moon we find from the equations of § 7,

$$\varepsilon = 2.174,$$

$$N = 9''.210 \pm 0''.011,$$

a value of the constant of nutation rather more probable, and more easily obtained than any derived from direct observation. The advantage of the theoretical mode of deriving this constant arises from the fact that an error in the adopted mass of the Moon produces an error of less than one-third its proportionate amount in the resulting constant of nutation.

The theory does not appear to be subject to any objection arising from our ignorance of the physical constitution of the interior of the Earth.

From the data of § 6, Delaunay's theory gives for the parallactic inequality of the Moon

$$125''.63 \pm 0''.19.$$

Taking the constant of aberration as 20″.4451, we have for the velocity of light

185,600 miles per second.

This is slightly greater than the result of Foucault's experiments with the revolving mirror. Adopting that determination, the constant of aberration would be increased about 0″.03. But the distance of the Sun and the terrestrial determination of the velocity of light are both uncertain to an amount greater than this increase, which is therefore altogether unreliable. The constant of aberration must be found by direct observation.