QUARTZ OPERATOR'S HAND BOOK.

To
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By his former School mate
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WHEELER & RANDALL.

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QUARTZ OPERATOR'S HAND BOOK.

In the preparation of this Hand Book, the object has been to make it a reliable and practical guide to the Quartz Operator.

No pains have been spared in gathering the material and in rendering the subjects discussed as concise and plain as their nature would admit.

Mindful that science and practice go hand in hand, the authors have confined their investigations to useful and established facts, leaving untouched all that is doubtful and chimerical.

No claim is made to originality, unless it may be in regard to the discussion of the tractory and the grinding effects of differently formed plates — a subject of great importance to every quartz miner.

To what extent the authors have succeeded in their object, is submitted with no little diffidence to the decision of the public.

WHEELER & RANDALL.

SAN FRANCISCO, April 26, 1865.

BLOWPIPE.

The Blowpipe is an instrument used for directing, by a current of air, the flame of a lamp or candle upon a mineral substance to fuse or oxydize it. The flame consists essentially of two parts—*the oxydizing and reducing.*

1st. *The oxydizing* part is the outer and slightly luminous flame.

2nd. *The reducing* part, which is hottest, is the inner blue flame.

The reagents mostly used in making blowpipe tests are charcoal, carbonate of soda, cyanide of potassium and borax.

The charcoal performs the part of a cupel as well as that of a reagent. The best charcoal is made of young hard wood.

The cupel or support consists of a sound piece of coal sawed or broken lengthwise, having a small cavity made in its plain side near the edge to hold the substance to be tested.

The borax is prepared by being melted or vitrified and pulverized.

(*A*) *Blowpipe Assay* of Silver Ores containing sulphur and arsenic.

1st. *Roast the pulverized ore* within a shallow cavity on the coal support. To do this, direct by the blowpipe the oxydizing flame, that is, the extreme point of the outer flame upon the powdered ore; turn the specimen metal. With soda, it forms a magnetic powder of metal on charcoal.

"*Oxyd of Silver* is instantly reduced to metal when brought within the flame. It forms a white opaque glass with borax, and is partly reduced to metal in all instances; with alkaline fluxes it forms metal directly, when brought in the flame.

"*Oxyd of Tellurium* imparts to the flame a green color, fuses and sublimes; on charcoal it is easily reduced to metal. With borax, it melts to a colorless glass in the oxydizing flame; in the reducing flame the glass is gray. With carbonate of soda it acts as with borax but less distinctly.

"*The Oxyds of Tin* are converted into sesqui-oxyd, becoming dirty-yellow in the oxydizing flame; it forms metal after protracted heating on the charcoal support and in the reducing flame. With borax it forms a clear glass, and with alkaline fluxes it is easily reduced to metal on charcoal.

"*Titanic Acid* is not altered in the flame when exposed to it; with borax it melts to a colorless glass, which becomes opaque in cooling. In the reducing flame it becomes first yellow, then amethyst, and darkens in cooling. With carbonate of soda it dissolves with effervesence, forming a faint yellow glass, which becomes gray in cooling. It forms no metal on charcoal.

"*Zinc.*—The oxyd of this metal forms a strong whitish-green flame; it is slightly yellow when hot but turns white in cooling. With borax it forms a transparent glass, which becomes milky by an intermittent flame; in the reducing flame it forms metal which is quickly evaporated. Alkaline fluxes do not alter it in the oxydizing flame; it is reduced on charcoal, and in the reducing flame. The metal burns readily and forms a white floculent oxyd, which is yellow when hot."—*Overman's Treaties on Metallurgy, pages* 154—157.

CHEMICAL TESTS.

Reagents or tests, usually in liquid form, are substances for indicating the presence of other bodies. In the following examples the reagents are arranged at the left hand side of the page, and the precipitates or products at the right. The proper solvents are indicated by the name of the solution holding the substances sought:

TESTS FOR GOLD IN SOLUTION WITH AQUA-REGIA.

Sulphate of Iron gives....	Metallic Gold as a purple powder.
Oxalic Acid gives	Metallic Gold in large flakes
Potash "	Yellow Precipitate.
Soda "	" "

TESTS FOR SILVER IN SOLUTION WITH NITRIC ACID.

Potash gives	Dark-Olive Precipitate.
Soda "	" " "
Plate of Copper gives	Metallic Silver.
Muriatic Acid "	White, Curdy Precipitate.

Common Salt gives .... White Curdy Precipitate.
Tincture of Nut-Gall gives, Brown Precipitate.

TESTS FOR COBALT IN SOLUTION WITH NITRIC ACID.

Potash gives............ Blue Precipitate.
Soda " .... " "
Ferro-Prusiate of Potash
gives .. .. Green "
Carbonate of Potash " Red "

TESTS FOR BISMUTH IN SOLUTION WITH NITRIC ACID.

Pure Water gives....... White Precipitate.
Gallic Acid " .... Greenish Yellow.
Potash " .... White Precipitate.
Soda " .... " "

TESTS FOR LEAD IN SOLUTION WITH NITRIC ACID.

Sulphate of Soda gives... White Precipitate.
Sulphuric Acid " ... " "
Infusion of Nut-Gall gives, " "

TESTS FOR COPPER IN SOLUTION WITH NITRIC ACID.

Plate of Iron gives...... Metallic Copper.
" " Zinc " ...... " "
Potash " ...... Green Precipitate.
Ammonia " ...... Azure-Blue Color.
Infusion of Nut-Gall gives, Brown Precipitate.

TESTS FOR ANTIMONY IN SOLUTION WITH FOUR PARTS OF MURIATIC ACID AND ONE PART OF NITRIC ACID.

Pure Water gives .... White Precipitate.

Plate of Iron gives .... Black Powder of the Metal.

TESTS FOR MERCURY IN SOLUTION WITH NITRIC OR MURIATIC ACID.

Plate of Copper gives.... Metallic Mercury.
" " Iron " .... Dark Powder.
Gallic Acid " .... Orange Yellow.

TESTS FOR IRON IN SOLUTION WITH MURIATIC ACID.

Infusion of Nut-Gall gives, Black Precipitate.
Ferro-Prusiate of Potash gives .... Blue "
Ammonia gives......... Brownish Red Precipitate.

EXPLANATION OF CHEMICAL TERMS.

Aqua Regia.—A fuming liquid composed of nitric acid and muriatic acid, viz: One part of the former and two of the latter. This mixture readily dissolves gold and platinum.

Sulphate of Iron....... / *Proto-Sulphate of Iron* ..	Copperas, Green Vitriol.
Sulphate of Copper	Blue Vitriol, Blue-Stone, Blue Copperas.
Nitrate of Potassa	Nitre, Saltpetre.
Sulphate of Soda	Glauber Salts.
Chloride of Sodium......	Sea Salt, Common Table Salt.
Nitric Acid	Aquafortis.
Sulphuric Acid (Concentrated)	Oil of Vitriol.
Hydrochloric Acid... ...	Muriatic Acid.

Oxalic Acid.—Sorrel Acid.—It is commonly manufactured by the action of nitric acid upon saccharine and farinaceous substances.

Gallic Acid.—An acid obtained from nut-galls or oak-apples. It is also obtained from several other vegetable astringents.

Nut-Galls.—Oak apples, which are excrescences produced by small insects depositing their eggs in the tender shoots of a species of oak.

Potassa.—Pure Potash, when refined by heat, is called pearlash. It is a vegetable fixed alkali.

Aqua-Vitae.—A liquid much used for precipitating hombres, properly called aquamortis, alcohol.

Chloride of Ammonium.—Sal-Ammoniac.

Catechu.—A dry, brown astringent extract obtained by decoction and evaporation from the acacia catechu in India. It contains a large portion of tannin or tannic acid.

ASSAY.

Assays are three kinds—Mechanical, dry and humid.

I. *Mechanical Assays* consist in washing or otherwise freeing, without the aid of chemical agents, the metallic substances from sands and other impurities. "*Panning out*," or separating the gangue (earthy matter) from the metallic substances, by washing in common mining pans, also, "*winnowing*," as practiced on rich, dry sands, are familiar examples of mechanical assays, and require no explanations; the former of which often furnishes safer and more practical data for extensive operations, especially in gold mining, than either the dry or humid way.

II. *The Dry way* of assaying ores, usually requires fluxes for separating the gangue (earthy matters) from the metallic substances.

Assay of Galena.—Fuse in an earthen crucible, at a bright red heat.

Powdered ore..........................	6	parts.
Black Flux,.	.9	"
Iron, in small pieces,.	...2	"

Extract from the slag, and weigh the button of lead thus obtained.

Assay of Iron.—Fuse in a covered crucible, about one hour, a well triturated mixture of

Powdered and roasted ore,........	2	parts
Fluor Spar,......	.1	"
Charcoal, ...	.1	"
Common Salt,..	.4	"

Extract and weigh the button of cast iron thus obtained.

Various other fluxes, as lime, clay, etc., may be employed instead of the above. No general formulæ can be given, as their application depend upon the nature of the ore.

Assay of Copper Ores, containing no other metals besides iron and copper:

Heat gradually, at first, in an earthen crucible, and afterward increase the heat to bright red, which continue fifteen minutes.

Powdered Ore,........................1 part.
Black Flux,............. ....3 "

Extract from the slag, and weigh the button of copper thus obtained.

Assay of Copper Ores containing sulphur, but otherwise similar to the above:

Fuse in an earthen crucible, at a dull red heat, equal parts of the powdered ore and dried borax. Extract from the slag the matte (crude copper) button, which pulverize; roast slowly in an earthen crucible, and stir, in the meantime with a steel rod, till sulphurous acid ceases to be evolved; then increase the temperature to a white heat, which continue for several minutes. Next mix in the same crucible:

The Roasted Matte,.................... 1 part
Black Flux, from... .. ..3 to 4 "

Cover the mixture with a layer of fused borax, and subject it to a cherry heat for twenty minutes, in a wind furnace; then extract and weigh the button of copper.

Assay of Copper Ores containing arsenic and various other metals:

Obtain and pulverize the matte as in the preceding case, then roast with it powdered charcoal, till the garlic odors of arsenic cease to be exhaled.

Reduce the matte thus obtained, as in the last case, with black flux and borax.

Cupel the button in a bone-ash cupel, with pure lead. Throw a little borax glass over the globule when its rotation ceases and brightening occurs; cool and weigh the button of copper.

ASSAY OF GOLD OR SILVER, OR GOLD AND SILVER ORES

Fuse in an earthen crucible:

Powdered Ore,	4 parts.
Litharge,	4 "
Black Flux,	3 "

If the ores contain much oxyd of lead, add only black flux.

If the ores are very rich in pyrites, add litharge and nitre.

If the button obtained be an alloy—for instance, of gold, silver, copper and lead—make additions to it of silver and lead, so that the prepared alloy shall contain as near as may be, of

Gold,	1 part.
Silver,	3 "
Lead,	16 "

First fuse the lead in a bone-ash cupel, within a muffle; then add the gold and silver inclosed in a piece of paper, and continue the heat till the button brightens and becomes tranquil. Cool and weigh the button. To separate the gold from the silver, called "parting of gold," anneal, beat the button into a thin plate, make it into a roll, which is termed a cornet. First heat this plate or cornet in dilute nitric acid as long as the acid acts upon it, then in concentrated nitric acid till all of the silver is dissolved. Thoroughly wash, dry and ignite the cornet. The weight of silver is equal to the weight of the button before "parting," less that of the refined cornet.

HUMID WAY OF ASSAY—ASSAY OF GALENA.

1.—Digest the powdered ore in equal parts of nitric acid and pure water.

2.—Filter and digest the residual several hours with a strong solution of carbonate of soda.

3.—Filter and digest the second residual in dilute nitric acid, and again filter.

4.—Add either a solution of the sulphate of soda, or sulphuric acid to the collected filtrates, as long as any precipitate takes place.

5.—Filter, then wash and dry the residual.

6.—Reduce the residual, with powdered charcoal, in an earthen crucible; cool and weigh the button.

ASSAY OF COPPER ORES.

1.—Digest the powdered ore in dilute nitro-muriatic acid.

2.—Filter the solution.

3.—Add ammonia in excess to the filtrate.

4.—Filter and wash residual in ammonia.

5.—Evaporate the filtrate to dryness.

6.—Dissolve the dried filtrate in muriatic acid.

7.—Add clean iron or zinc plates to the solution much diluted.

8.—Wash dry and weigh the copper precipitate.

ASSAY OF SILVER [ORES.*

1.—Digest the pulverized ore in nitric acid.

**Chloride of Silver*, as found native, is called *horn-silver*; it is completely insoluble in nitric acid. It is readily dissolved by

2.—Add muriatic acid or solution of common salt to the silver solution as long as any precipitate takes place.

3.—Filter and dry the residual.

4.—Reduce the dry residual with carbonate of soda, or black rosin in an earthen crucible, then cool and weigh the button of silver. It also may be reduced with chalk and charcoal.

ASSAY OF GOLD ORES.

1.—Digest the pulverized ores in one part of nitric acid, and four parts hydrochloric acid.

2.—Dilute, filter and evaporate the filtrate to dryness.

3.—Digest the dried filtrate in pure water, then boil the solution with a solution of sulphate of iron, which precipitates the gold as a dark-purple powder.

4.—Filter and heat the residual with hydrochloric acid.

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ammonia, and can be precipitated from this solution by the addition of nitric acid. It is also soluble in a strong, hot solution of common salt, (see Augustin process) from which it may be precipitated in its metallic state by a clean plate of copper.

Quicksilver partially decomposes the chloride of silver forming a silver amalgam; this is attended, however, with a loss of quicksilver, and should be avoided in practical operations. Silver may be revived from its chloride state by being kept from twelve to twenty-four hours in contact with clean iron, copper or zinc plates.

Bromide of Silver, in almost every respect, resembles chloride of silver; it is, however, less soluble in ammonia.

Iodide of Silver, found also native, is readily converted into *chloride of silver* by muriatic acid, and then may be treated as above described.

5.—Filter, wash, dry and weigh the gold powder.

Oxalic acid substituted for the sulphate of iron precipitates the gold in large flakes.

ASSAY OR ANALYSIS OF IRON ORES CONTAINING MANGANESE.

1.—Digest the roasted and pulverized ore in dilute hydrochloric acid.

2.—Filter, wash residual and add washings to the filtrate.

3.—Add muriate of barytes until no farther precipitation takes place.

4.—Filter, wash and add washings to the filtrate.

5.—Evaporate the filtrate nearly to dryness and to it add sufficient nitric acid to transform the sulphate of iron to per-oxyd.

6.—Add solution of caustic ammonia in excess to the solution.

7.—Filter and reduce the iron to the magnetic state by heating the residual with resin in an iron crucible—then cool and weigh.

8.—Precipitate the oxyd of manganese from the filtrate, by expelling the excess of ammonia with heat.

When the ores contain much alumina or silex, flux them with three or four times their weight of caustic potash, then digest in hydrochloric acid and proceed as above.

ASSAY OR ANALYSIS OF ORES CONTAINING GOLD SILVER, COPPER. LEAD, IRON AND SULPHUR.

1.—Digest well the pulverized ore in nitric acid.

2.—Filter, wash residual (1) and add washings to filtrate (1).

3.—Add to filtrate (1) hydro-chloric acid, or a solution of common salt, which precipitates the silver as a chloride.

4.—Filter, and digest residual (2) in hydro-chloric acid.

5.—Filter, wash residual (3) in warm water, and to filtrate (3) with the washings add filtrate (2).

6.—Reduce the chloride of silver with carbonate of soda by fusion, and weigh the button of silver.

7.—Add to filtrate (3), sulphate of soda in solution, which precipitates the lead as a sulphate.

8.—Filter, and add the residual to residual (1).

9.—Evaporate filtrate (4) to any desirable extent.

10.—Add, in excess, to concentrated filtrate (4) ammonia, which precipitates sesqui-oxyd of iron.

11.—Filter, wash residual and add washings to filtrate (5).

12.—Dry and heat the residual in hydrogen gas within a glass tube as long as any vapor of water is disengaged, then weigh the iron. This powder, with glass as a flux at a high heat, becomes a button of iron.

13.—Treat filtrate (5), after evaporating it to dryness with hydro-chloric acid, then add clean iron or zinc plates to the solution diluted. Wash, dry and weigh the copper precipitate.

14.—Treat residual (1), first with a strong solution of carbonate of soda, then with dilute nitric acid; and to the combined filtrates add sulphuric acid, or a solution of the sulphate of soda. Wash, dry and reduce the precipitate with powered charcoal in an earthen crucible; then cool and weigh the button of lead.

15.—Digest the last residual in nitro-muriatic acid; add chloride of sodium in solution, filter, precipitate the gold from its solution by the addition of sulphate of iron in solution; wash, dry and weigh the gold.

16.—If the gold may be alloyed with silver and copper, precipitate the copper from the last filtrate by the addition of iron or zinc plates; wash, dry and add the weight of the precipitate to the copper already obtained.

Heat the residuary ore in a strong solution of chloride of sodium, filter and precipitate the silver with a clean copper plate; wash, ignite and add the silver to that already obtained.

17.—Burn off the sulphur and weigh residuary ore. The sum of the weights of the gold, silver, copper, lead, iron and calcined ore taken from the weight of the original ore, leaves the weight of the sulphur.

RECIPES.

Black Flux.—Black Flux is prepared by introducing gradually in small quantities, into a crucible heated to a very dull redness, a mixture of either two parts of cream of tartar and one of nitre; or equal parts of cream of tartar and nitre. *White Flux* is similarly prepared except that the mixture consists of one part of cream of tartar and two parts of nitre.

Iron Rust Cement.—To one hundred parts of powdered and sifted iron borings, add one part of sal-ammoniac. Moisten the mixture with water to a pasty consistency for use.

Lead Cement.—Red or white lead in oil, four parts; iron borings, two to three parts. Makes a good cement for steam boilers, steam pipes, etc.

Solders, for Lead.—Melt one part of block tin, and when in a state of fusion add two parts of lead. Resin should be used with this solder.

For Tin.—Pewter four parts, tin one part, and bismuth one part; melt them together. Resin is also used with this solder.

For Iron.—Tough brass with a small quantity of borax.

For Iron, Copper and Brass.—Spelter, that is an alloy of zinc and copper in nearly equal parts, is used.

Quicksilvering of Copper Plate.—First thoroughly cleanse the surface of the plate, and rub it over with quicksilver, or with the nitrate of mercury.—The surface is sometimes cleansed by simply scouring it with wood ashes, brick dust, or fine sand; and sometimes by washing it with dilute acid or strong alkali. When acid is employed, its corrosive qualities should be neutralized before the application of the quicksilver. Nitrate of mercury, when crystalized, is readily converted to a liquid by heat, in which state it may be applied as a wash to the plate.

ROASTING.

Roasting is employed to dissipate the volatile parts of ore by heat, and is effected in heaps or furnaces.

In Heaps.—Alternate layers of fuel and ore, usually as it comes from the mine, are heaped up to the depth of several feet. The lowest or ground layer is of wood, arranged by cross-piling so as to afford a free circulation of air. The upper layers may be of wood or coal.

The ratio of fuel in volume to that of ore varies from 1 to 6 to 1 to 18. Fine ores and those rich in sulphur require less than coarse ores and poor in sulphur. The fire is kindled through vertical openings or chimnies which extend to the ground layer. These openings are closed when the fuel has well taken fire. The roasting should be slow and uniform in all parts of the heap. The heat may be regulated by opening or closing the draft holes and chimnies. Several days and even months, sometimes, are required for roasting one heap. Ores similarly piled with fuel are sometimes roasted in walled inclosures provided with side openings.

Furnaces.—There are a great variety of furnaces. Those mostly approved for the *roasting* of ores embracing also *calcining* and *chloridizing* are the reverberatory. The interior walls of the furnace should be of the best fire brick laid edgewise; the outer walls may be of common building brick or stone. The furnace must be well tied with iron rods, and carefully dried before being used.

The Reverbreatory Furnace is constructed sometimes with one and sometimes with two hearths or soles one above the other. In the *double hearth* furnace, for instance, in the treatment of silver ores the *roasting* and *sulphatization* are effected on the upper sole, and the *calcining* and chloridizing on the lower. The ore pulverized fine, is charged upon the upper sole to the depth of from two to four inches, and is kept well stirred during the roasting. The heat should be at a low temperature, not exceeding brown or dull red. The access of air should be free. A small jet of steam into the furnace assists in regulating the temperature and also facilitates oxydation. The addition of powdered charcoal in small quantities may be made to advantage when the ores contain arsenic. If the ores are poor in sulphur add from two to three per cent. of the sulphate of iron. The first operation of *roasting* and *sulphatizing* is accomplished in four or five hours. Then through an opening in the upper hearth the ore is let fall upon the lower, where it is heated for some time at a temperature not much higher than that above. The heat is then gradually increased to cherry red, at which it is kept during the time required for calcining and chloridizing. The heat should never exceed bright red. The ore is frequently stirred. When calcination is complete a mixture of common salt melted and pulverized and seven parts of cold calcined ore are added to the hot ore, estimated at fifteen parts, and quickly and thoroughly mixed with it by stirring. Calcination is usually effected in four or five hours, and chlorination in fifteen or twenty minutes.

PURIFICATION OF MERCURY.

Mercury for the purposes of amalgamation should be pure. Any foreign substance such as lead, tin, zinc, or bismuth diminishes its properties of combining with gold and silver. To free from these and other impurities,

1st. Distil the impure mercury. A retort for this process may readily be made of a common quicksilver flask and iron pipe of syphon form. The short leg of the pipe, a few inches long, is attached to the flask in the place of the removed stopper.

The long leg, three or four feet in length, inclines downward from the bend. The retort should not be over two thirds filled with mercury. The heat ought first to be applied to the short leg of the pipe and upper part of the retort, then to all parts of the flask alike. The long leg of the pipe must be kept cold. This may be effected by wrapping it with cloths and pouring on cold water. The discharge end may also be immersed in cold water, kept in the receiver. The heat should be uniform, and the distillation slow. The common covered retort is far preferable to the one described.

2. Heat and frequently agitate the distilled mercury in thin sheets, with one part of nitric acid and two parts

of pure water. The heat should be kept at 120 degrees Fahrenheit, for several hours. Repeat these operations until satisfactory results are obtained. Then pour off the mercury for use.

3. Digest the crust (nitrate of mercury and impurities) in nitric acid. Then dilute the solution, filter, precipitate the mercury by metallic copper, and add it to the mercury already obtained. Or the nitrate of mercury may be converted to a liquid, simply by heat, and the metal then precipitated by copper plate.

EXTRACTION OF GOLD BY THE PAN PROCESS.

1. The rock, as it comes from the mines, is usually crushed wet by stamps, to a fine granular state, and run into large tanks.

2. Charges of the reduced ore, with sufficient water to form a thin paste, are thoroughly ground in iron pans. As gold found in rock exists almost without exception in a metalic state, friction alone is required to fit it for amalgamation.

3. Quicksilver is ordinarily added to the pulp, as the pans commence running. To avoid grinding the quicksilver excessively, the addition is sometimes made with the muller slightly raised, after the reduction of the ores

4. The charge is then drawn off and washed, leaving the amalgam in the separators.

5. The proportions usually observed, for instance, in the Wheeler & Randall grinders and amalgamators, are

Ore to the charge,.............. 1,200 pounds.
Quicksilver to the charge of ore,.... 75 "

Revolutions of muller,............	60 to 75
Time of reducing,................	2 to 3 hours.

As gold-bearing rock is seldom found sufficiently rich to render it advisable to treat the entire mass in pans, the above method is subject to various modifications, of which the following are a few:

1. The heavier and richer portions of the rock, as crushed, are concentrated by revolving-blankets, buddles or other machinery, and then pulverized and amalgamated in pans.

2. Amalgamation is commenced in the batteries during the crushing operation, and is carried on through a series of shaking tables, riffles, and copper plates. The richer portions of the tailings are then concentrated and treated in pans.

3. Grinding and amalgamating are effected in pans while the reduced ores are flowing continuously through them.

4. The sulphurets or concentrated tailings are sometimes roasted in a reverberating furnace, before being ground and amalgamated.

5. Thin layers of the concentrated sulphurets or tailings are spread in inclosures open to the sky, and allowed to remain a long time, for instance, a year. The tailings are occasionally turned with shovels and the lumps broken, so as to expose as much surface as possible to the action of the air. Common salt mixed with the tailings assists in their oxydation. When quite

thoroughly oxydized, they are treated in pans. This is very economical and effectual, and by it the yield of gold, (especially if very fine) to the ton is frequently much greater than was obtained at first from the same ores.

EXTRACTION OF GOLD BY CHLORINATION.

1. *Pulverized Ores*, containing gold, having been well roasted, cooled and moistened with water, are put into closely covered wooden cisterns, whose bottoms are so constructed that chlorine gas can permeate the mass from underneath.

2. *Chlorine gas* produced by heating sulphuric acid, per oxyd of manganese and common salt, in a suitable generator, is caused to enter the cisterns at the bottom, through leaden pipes. The effect of the chlorine on the gold, is to produce terchloride of gold.

3. *Pure water*, after the chloride has done its duty, which takes from ten to fifteen hours, the covers being removed, is added sufficient to keep the cisterns even with the mass. The effect of the water is to dissolve the terchloride of gold. The solution is then drawn off into glass vessels.

4. *Sulphate of iron*, in solution, is used to precipitate the gold, which may then be gathered as a powder.

EXTRACTION OF SILVER BY THE PATIO PROCESS.

1. *Patio* signifies a yard. For amalgamating purposes, the floor of the yard is made level, paved with brick or granite blocks, surrounded by high walls, and

usually left open to the sky. On this floor circular batches of silver ore, reduced to an inpalpable paste by stamps and arastras, or other machinery, are spread to the depth of seven to twelve inches, and inclosed by low close curbs.

2. *Salt*, varying in quantity according to its quality and the richness of the ore, is well mixed with the pulp by treading it with horses, mules, or oxen, and turning it with shovels. The effect of the salt is to desulphurize the sulphurets, and produce chloride of silver. The batch is then left one entire day.

3. *Magistral*, that is, roasted and pulverized copper pyrites, varying in quantity with its quality, the richness of the ores and season, is well mixed with the pulp after it has been subjected to the treading and turning operation one hour. The ultimate effect of the magistral is to revive the silver by depriving it of its chlorine.

4. *Quicksilver* is added, usually in three charges to the mass, by being sprinkled in minute particles through cloth or other porous substance. After the addition of the first charge of quicksilver, the batch is thoroughly mixed, thrown into heaps of about one ton each, smoothed and left at rest one whole day. The treading, turning and heaping operation is performed every other day, occupying five or six hours, and is found much more effective in a morning than an evening. The second charge of quicksilver is added and similarly treated when it is ascertained by washing a small quantity of the mixture, that the first has been well incorporated.

After the second charge has performed its work, the third charge is added to take up any stray particles of silver, and to fit the amalgam better for separation.

5. *Lime* is added to cool, and magistral to heat the mass, according as it may be too hot or too cold. Too much heat is indicated by the quicksilver becoming extremely divided, and of a dark color, with occasional brown spots upon its surface. Too little heat is indicated by the quicksilver retaining its natural color and fluidity. A proper degree of heat is indicated by the amalgam's being of a greyish-white color, and yielding readily to a slight pressure.

6. *The proportions* to the ton of ore, valued at fifty dollars, are:

Sea Salt, of good quality,		80	pounds.
Magistral.—When containing ten per cent., of the sulphate of copper,			
	in summer,	20	"
	in winter,	10	"
Quicksilver—	First charge,	14	"
	Second charge,	5	"
	Third charge,	7	"
Lime.—More or less, see section 5th,		15	"

An excess of *magistral*, *quicksilver*, or *lime* is injurious. An excess of salt causes a loss of quicksilver but is not otherwise injurious.

The time employed in treating a batch of ore varies from twelve to sixty days. Light and good weather greatly facilitate operations.

7. *The separation* is accomplished by agitating the

pulp or mixture with abundance of water, in a large, deep, circular vessel, and causing the lighter portions of the mass to flow slowly off, until the amalgam is gathered by itself.

EXTRACTION OF SILVER BY THE FREYBERG PROCESS.

1. *This process* takes its name from Freyberg, a place in Germany, where it was first practiced. The ores, if possible, are assorted so as to contain not less than twenty-five per cent. of sulphurets. When they contain less, the sulphate of iron is added to make up the deficiency. When more, then a sufficient quantity of the richest in sulphurets is roasted without sea-salt to make good the ratio : the ores are crushed dry.

2. *Sea-Salt* and crushed ores are thoroughly mixed together, roasted in a reverberatory furnace, and then reduced to an impalpable powder in a suitable mill. The salt and heat transform the sulphurets of silver to chloride of silver.

3. *Wrought Iron*, in small pieces, with a pasty mixture of the reduced ores and water are put into German barrels, which, making twenty revolutions a minute, are run two hours. The effect of the iron is to revive the silver to its metallic state.

4. *Quicksilver* is then poured into the barrels, after which they are run sixteen hours continuously, except the time taken to regulate the consistency of the pulp, by the addition of ore or water. At the end of the time run, the casks are filled with water and revolved quite

slowly for one or two hours, when the mass is discharged into large vats and the amalgam separated by washing.

5. *The Proportions* to the ton of ore valued at $75.00 per ton, are

Sea-Salt, added before the roasting process, 200 lbs.
Wrought Iron, added to the ton of roasted ore.................. .. .. 200 "
Quicksilver, added to the ton of roasted ore.......... .... 1000 "

EXTRACTION OF SILVER BY THE VEATCH PROCESS.

The only essential difference between this and the Freyberg process consists in the employment of tubs instead of barrels, and the use of steam directly in the pulp. Vertical plates of iron or copper, for reviving the silver from its chloride state, are fastened to the muller arms, so as to revolve edgewise through the pulp or mass. The operations are greatly hastened by the application of steam, so that not more than five or six hours are required for the treatment of a charge of ore.

EXTRACTION OF SILVER BY THE PAN PROCESS.

1. *The Ores*, as they come from the mines, are usually crushed wet to a granular state by stamps, and run into a series of large settling tanks. To crush wet, and at the same time fine, is very objectionable, as much silver thereby is carried off by the water.

2. *Charges* of the reduced ores, with sufficient water to form a soft, pasty mass, are put into iron pans con-

structed as grinders, which are run from two to six hours, according to their reducing properties. Water is occasionally added during the grinding process, as the condition of the pulp may require.

3. *Quicksilver* is ordinarily poured into the pans as they commence running. Sometimes, to avoid grinding it excessively, the muller is slightly raised and the addition made after the reduction of the ores.

4. *Chemicals*, differing in kind and proportions, to almost an indefinite extent are employed. As to their practical value, a diversity of opinion prevails among the most experienced and intelligent amalgamators and mill-men. In pans of slow motion and of little grinding capacity, certain chemicals, in the treatment of some ores, have been used to advantage. Their employment and proportions, in all cases, depend upon the composition and character of the ores. Experience thus far, chiefly goes to show that the chemicals in pans, which grind rapidly, are not only valueless but in many instances injurious to amalgamation. In pans of this character, the sulphurets of silver ores become not only mechanically divided, but chemically decomposed. The heat of the steam contributes to the attainment of this desirable object; the iron of the pans serves also to revive any silver existing as a chloride.

The proportions usually observed in operating the Wheeler & Randall grinders and amalgamators, are:

REC. V.

Sulphate of Iron,.......	1.5	"
Nitric Acid,....	1.5	"
Common Salt,......................	15.0	"

REC. VI.

Muriatic Acid,...............	30	ounces.
Peroxyd of Manganese,..	8	"
Sulphate of Copper,...........	10	"
Sulphate of Iron,...................	10	"

The salt is applied half an hour before the other chemicals.

SEPARATION OF SILVER FROM LEAD BY THE PATTINSON PROCESS.

1. This process is founded on these facts: If a melted alloy of silver and lead is stirred while cooling slowly, crystals of lead form and sink, which may be removed with a drainer. A large portion of the lead may thus be separated from the silver.

2. Cast-iron pans, capable of holding about five tons each, and provided with fire places, are arranged in a series, as A, B, C, D, E, F, G, in a straight line.

3. The metal of ores containing silver and lead as it comes from ordinary smelting works, is melted, for instance, in pan D, and then allowed to cool very slowly. The metal while cooling is stirred, especially near the edges of the pan, with an iron bar. As soon as crystals form and sink to the bottom, they are taken out with an iron drainer raised to a temperature somewhat higher

than that of the metal bath. From one half to two thirds of the charge is thus removed to pan E, and the balance taken to pan C. Other charges of D, are similarly treated and disposed of. The charges of C and E are treated and disposed of in like manner, except that the crystals of E go to F, and the balance to D, and the crystals of C go to D, and the balance to B. Thus, after successive meltings and drainings, the alloys rich in silver pass to A, while the lead, almost entirely deprived of silver, goes to G. The alloys obtained in pan A are then subjected to cupellation. An alloy containing over six hundred dollars of silver to the ton should not be treated by this process.

SEPARATION OF SILVER FROM COPPER BY THE LIQUATION PROCESS.

1. This process is founded on these facts: Lead and copper fused together form an alloy, which, if rapidly cooled, maintains an intimate admixture, but if slowly cooled, separates. An alloy of lead and copper slowly heated to near its point of fusion, also separates. Silver, if contained in the alloy, goes with the lead.

2. Either an alloy of copper or silver, or matte (crude black copper reduced, but not refined from sulphur and other impurities), containing silver, as it comes from the smelting furnace, is melted with lead of about four times its weight, in a cupola furnace, and cast into plain circular plates which are suddenly cooled. These plates, called "liquation cakes," are arranged on their edges,

with alternate layers of charcoal, in a liquation furnace. The charcoal is then ignited, and a degree of heat produced somewhat below that of the fusing point of copper. The lead and silver melt and flow into a receiver, while the copper, in a porous state, retains the forms of the original cakes. If the separation may have been imperfect, the cakes are farther treated by being raised to a higher degree of heat in the "sweating furnace." The silver is then separated from the lead by cupellation.

SEPARATION OF SILVER FROM LEAD BY THE PARKE PROCESS.

1. *Lead, containing silver*, is fused in large cast-iron pots. Melted zinc is added and well stirred in the alloy. The fire being withdrawn from under the pot, the whole is left at rest for a short time.

2. *The silver and zinc* separating from the lead, form an independent alloy, which is skimmed from the surface of the metal bath, as long as it rises.

3. *This scum alloy*, containing some lead, is heated in a liquation retort. The silver and lead fuse, and, to a great extent, flow into prepared moulds. The alloy thus run off is then cupelled; the alloy of zinc and silver remaining in the retort, are partially separated by distillation. The silver thus obtained is freed of its impurities by cupellation.

4. *The proportions* are:

Charge of argentiferous lead to the pot
usually from ....6 to 7 tons.
Charge of zinc to the ounce, of silver
by estimation,....... ....1.5 to 2 pounds
Quantity of silver to the ton of lead..10 to 15 ounces
Time of stirring alloy, after the addition
of zinc, from.................10 to 15 hours.

The alloy prepared for cupellation contains, of silver, to the ton, about 1,000 ounces.

SEPARATION OF SILVER FROM LEAD BY CUPELLATION.

1. *The Alloy* of silver and lead is melted in a circular reverberatory furnace provided with openings through its sides for the admission of metal, heat, currents of air, and for the escape of vapors or litharge. The escape is opposite the blast opening. The roof or top of the furnace is of dome-form and movable. At each cupellation, the hearth, usually of hollow form, is broken up and replaced by one made of clay, sand and carbonate of lime.

2. *Blasts*, or currents of air, are blown continually during the operation upon the surface of the fused alloy, promoting oxydation of the lead and causing the litharge to pass out through the escape opening. The gate-way of this opening is kept level with the surface of the metal withiu. The silver thus separated from the lead remains on the hearth of the furnace in nearly a pure state. It is deprived of what lead it may contain by the humid way of assay.

EXTRACTION OF SILVER BY THE AUGUSTIN PROCESS.

1. *This Process*, employed thus far chiefly in the treatment of matte, (impure copper) containing silver, is founded on the solubility of chloride of silver in a hot, concentrated solution of common salt.

2. *The Matte*, as it comes from the cupola or high furnace, is crushed dry by stamps, pulverized in suitable mills and bolted. The coarser portions thus obtained are taken to the copper works.

3. *The Roasting* of the powdered matte in a reverberatory furnace is commenced at a low temperature with a free access of air. By careful, uniform roasting, at a dull red heat, the sulphurets of silver, iron and copper are produced. The heat is then increased to cherry-red which decomposes the sulphates of iron and copper, but not the sulphate of silver.

4. *Salt*, previously melted, pulverized and mixed with cold calcined matte, is added to the hot matte in the furnace and thoroughly mixed with it by stirring. The sulphate of silver is thus transformed to chloride of silver.

5. *The Apparatus* for the humid operations consist of a large heating reservoir, a series of dissolving tubs, two large settling cisterns, four precipitating tubs to each one of the dissolving tubs, and two large receptacles, arranged in the order here given on descending steps. The dissolving and precipitating tubs are nearly cylindrical. They are provided with filters made of small

sticks and straw, covered with cloth; a vertical partition, resting on the filter, divides each tub into two unequal compartments.

6. *The Chloridized Matte* being put into the larger compartments of the dissolving tubs, sufficient of the hot salt solution from the heating reservoir above to completely immerse the matte, is let into the tubs; they are then left at rest one hour. The discharge cocks of the heating reservoir and tubs then being opened, the hot salt solution is filtered through the contents of the tubs, and run off from the smaller compartments, at openings at first above the level of the matte, afterwards at openings near the bottoms of the tubs, into the settling cisterns, until a test with clean copper plate shows no trace of silver in the filtered solution.

7. *Copper* (copper cement) is put into each of the upper two precipitating tubs in the several series of four, and *iron* (wrought scrap iron) into each of the lower two. The chloride solution from the settling cisterns is then slowly filtered through the several series of precipitating tubs, and the filtered solution run into the large *receptacles* below. The silver is precipitated by the copper in the upper tubs, and the copper in solution is precipitated by the iron in the lower tubs. The silver is taken twice a week from the precipitating tubs and refined. The copper precipitated in the lower tubs is transferred to the upper tubs. The filtered matte is washed and taken to the copper works. The filtered

solution, in the receptacles, is pumped into the heating reservoir and used again.

8. *The Proportions* usually observed are:

Matte, before roasting, should contain of sulphur, not less than,	20 per ct.
Charge of Matte, to the furnace, for roasting and calcining,..	500 pounds.
Charge.—Melted Salt,.........	35 "
Roasted Matte,...........	220 "
Add same for chloridizing.	
Time.—Roasting on the upper sole of Furnace........... ..	4 to 4½ hs.
Calcining on lower sole of Furnace................	4 to 4½ "
From...............	8 to 9 "
Time of chloridizing, from......	15 to 20 minutes.
Charge of chloridized matte to the tub, from.......	1000 to 1200 lbs.
Salt—In the solution, in reference to the water, from...... ..	20 to 25 per ct.
Degrees of Heat of salt solution..	131° Fahr.
Time of dissolving and precipitating, from	20 to 24 hours.
Solution of Salt, run through each tub to 1000 pounds of matte, from	200 to 250 cubic ft.
Depth of Copper in precipitating tubs, about....	6 inches.
Depth of Iron in precipitating tubs, about...	6 "

EXTRACTION OF SILVER BY THE ZIERVOGEL PORCESS.

1. *This Process*, employed thus far chiefly in the treatment of matte (impure copper) containing silver, is founded on the solubility of sulphate of silver in *hot water.*

2. *The Matte*, as in the Augustin process, having been thoroughly pulverized, is carefully roasted and calcined till the sulphates of iron and copper are completely decomposed, but none of the sulphate of silver. When small quantities of the roasted matte, thrown hot into water, give only a very slight blue color, the calcination is regarded complete.

3. *The Sulphatized Matte* is then treated, in all respects, the same as the chloridized matte, (see sec. 6, page [redacted]) in the Augustin process, except that *pure water* is employed instead of solution of salt.

4. *The Proportions* usually observed are:

Matte, before roasting, should contain of sulphur, not less than,	20 per ct.
Charge of Matte to the Furnace..	500 lbs.
Time.—Roasting on upper sole of Furnace,.	4 to 4½ hours.
Calcining on lower sole of Furnace,..	4 to 4½ hours.
From	8 to 9 "
Charge of sulphatized matte to the tub, from	1000 to 1200 lbs.
Degrees of Heat of the water for	

dissolving................	149° Fahr.
Time of dissolving and precipitating, from	20 to 24 hours.
Hot Water run through each tub, from	200 to 250 cubic ft.
Depth of Copper in upper precipitating tubs,...............	6 inches.
Depth of Iron in lower precipitating tubs,................	" "

EXTRACTION OF SILVER BY THE PATERA PROCESS.

1. *In this Process* the ores are thoroughly pulverized and chloridized by roasting with common salt.

2. *Hot Water* to dissolve the chlorides of various base metals is filtered through the chloridized ores put in tubs similar to the dissolving tubs in the Augustin process. The ores are then cooled and transferred to similar, but smaller tubs.

3. *Hyposulphite of Soda*, in cold solution, is then filtered through the ores and run into precipitating tubs until all the chloride of silver is completely dissolved.

4. *Polysulphide of Sodium*, sufficient to produce a neutral liquor, is then added, which precipitates the silver as a sulphide in sacks fitted to the inside of the tubs. This neutral liquor is preserved for lixiviating purposes.

5. *The Sulphide of Silver* thus obtained, after being washed in warm water, pressed and dried, is heated under muffles with free access of air till nearly all the

sulphur is expelled. The metallic silver is then re-refined.

MECHANICS.

Force.—That which produces or tends to produce motion, or change of motion, is termed force.

Work.—The product of force and the distance through which it is exerted, is termed *work—mechanical work.*

Units.—The units of *force*, *distance*, and *time* are respectively one (1) pound, one (1) foot, and one (1) minute.

Horse Power.—Thirty three thousand (33,000) units of work constitute one (1) "horse power," that is, thirty-three thousand pounds raised vertically one (1) foot in one (1) minute, or its equivalent.

TO FIND THE HORSES' POWER IN A GIVEN TIME.

Rule.—Divide the product of the weight in pounds and the vertical distance in feet through which the weight is to be raised, by the product of the time in minutes and thirty-three thousand. (33,000.)

Ex. 1.—Required the *horses' power* necessary to drive forty-five (45) stamps, each stamp weighing six hundred and forty (640) pounds, falling ten inches, and making seventy-seven (77) drops per minute, allowing twenty-five per cent. for friction.

Cal. $45 \times 640 \times 77 \times 10 \div 12 = 1848000.$

$1848000 \times 1.25 = 2310000.$

$2310000 \div 33000 = 70$ horses' power. Ans.

Ex. 2.—How many horses' power are required to raise water three hundred (300) feet by a single acting pump seven (7) inches diameter, thirty (30) inches stroke, making fifteen (15) lifting strokes per minute, allowing thirty-five (35) per cent. for friction?

Cal. 15×30=450 inches, column of water raised.
450÷12=37.5 feet, column of water raised.

7×7×,7854÷144=,2672 area of end of column of water.

,2672×37.5=10.02 solid inches in column of water.
10.02×62. 5=626.25 pounds in column of water.
626.25×1.35=845.4375 pounds with friction added.
845.4375×300÷330000=7.68 horses' power. Ans.

Ex. 8.—The slant depth of the hoisting shaft of the Eureka Mine, Sutter Creek, is one thousand (1000) feet; the dip of the lode being sixty (60°) degrees; how many horses' power are required to raise one ton (2,000 pounds) of rock to the surface in five minutes, allowing fifty per cent. for friction?

90°—60°=30° complement.

1000÷2=500 feet side of right angled triangle opposite 30° 1000×1000=100000; 500×500=250000; 1000000—250000=750000.

$\sqrt{750000}$=866.0254 perpendicular depth of mine.
866.0254×1.50=1299.0376, with friction added.
1299.0376×2000=2598075; 33000×5=165000.
2598085÷165000=15.75 horses' power. Ans.

Remarks.—1. It is found by experience that it requires to reduce by stamps hard quartz rock (frequently called by miners, live rock,) from the size usually fed into batteries to the ordinary granular size coming from the same, about one horse power to the ton in twenty-four hours; and to farther reduce it from the granular state by the common pan-grinders and amalgamators to a "slum" or slime of economical fineness, about one horse power in the same length of time.

2. That the Wheeler & Randall Grinder and Amalgamator, four feet diameter, making sixty-five revolutions per minute, will reduce from the granular to the slime condition, five tons of rock per twenty-four hours.

Note.—It is estimated by those using the Wheeler and Randall Grinder and Amalgamator, that it requires to run each four foot machine sixty-five revolutions per minute, three (3) horse power. This is somewhat less than the inventors were ready to believe, on account of the extraordinary reducing properties of their invention.

3. That it requires to run a Separator seven feet diameter, (not grinding,) about one half of one horse power.

VARIED MOTION.

Laws of Uniformly Varied Motion.—1. In uniformly varied motion, the path described at the end of any time is half that which the body would describe in the same time if it were to move uniformly with the velocity acquired during this time.

2. In uniformly accelerated motion, the paths described at the end of any two times, are to each other as the squares of these times.

3. That these paths are to each other as the square

of the velocities acquired at the end of the corresponding times.

Let one second be a unit of time; then will the times of a falling body, *in vacuo*, be

1, 2, 3, 4, 5, 6, 7, 8, 9, etc., etc.

The corresponding fall during each second, will be

1, 3, 5, 7, 9, 11, 13, 15, 17, etc., etc.

The fall during any number of seconds, will be

1, 4, 9, 16, 25, 36, 49, 64, 81, etc., etc.

And the velocities acquired at the end of each second, will be

2, 4, 6, 8, 10, 12, 14, 16, 18, etc., etc.

Now, a body at the equator falls, *in vacuo*, as determined by experiments, 16.0904 feet in one second of time. The resistance of the atmosphere does not much retard the velocity of heavy falling bodies. Let then, sixteen feet represent the distance which a falling body will describe in one second of time, when impressed by gravity alone. The velocity acquired at the end of the first second of time, is called the "*initial velocity*," and is found to be 32.1808 feet; that is, twice 16.0904 feet. This velocity, due to the force of gravity, is usually denoted by g; that is, $g=32.1808$, which, for most practical purposes, may be taken at 32 feet.

TO FIND THE DISTANCE A BODY WILL FALL IN TERMS OF THE VELOCITY.

Rule 1.—Divide the square of the velocity by sixty-four (64).

Example.—The velocity is 256 feet, what distance has the body fallen?

Calculation. $256 \times 256 \div 64 = 1024$ feet. Ans.

TO FIND WHAT DISTANCE IN FEET A BODY WILL FALL IN A GIVEN TIME.

Rule 2.—Multiply the square of the time in seconds by si teen (16).

Example.—What distance will a body fall in one minute?

Calculation. $60 \times 60 \times 16 = 57600$ feet. Ans.

TO FIND THE VELOCITY IN FEET, IN TERMS OF THE TIME.

Rule 3.—Multiply the time in seconds by thirty-two.

Example.—What velocity does a falling body acquire in seven seconds (7)?

Calculation. $32 \times 7 = 224$ feet. Ans.

TO FIND THE VELOCITY IN TERMS OF THE DISTANCE.

Rule 4.—Multiply the square root of the distance by eight (8).

Example.—What velocity will a body acquire by falling one hundred and ninety-six feet (196)?

Calculation. $8\sqrt{196} = 112$ feet. Ans.

TO FIND THE TIME FALLEN, THE VELOCITY BEING GIVEN.

Rule 5.—Divide the velocity by thirty-two (32).

Example.—The velocity is 1920 feet, what time has it fallen?

Calculation. $1920 \div 32 = 60$ seconds. Ans.

TO FIND THE TIME A BODY HAS FALLEN, THE DISTANCE BEING GIVEN.

Rule 6.—Divide the square root of the distance fallen by four (4).

Example.—How long will it take a body to fall one hundred and forty-four feet (144)?

Calculation. $\sqrt{144} \div 4 = 3$ seconds. Ans.

WATER POWER.

The theoretical velocity with which a liquid issues from an orifice in the bottom or side of a vessel that is kept full, is equal to that which a heavy body would acquire by falling from the level of the surface to the level of the orifice. The rules, therefore, under the head of "Varied Motion," apply equally well to falling bodies and to hydraulics.

The practical velocity estimated for the entire opening, is considerably less than the theoretical velocity owing to oblique currents and to friction. These oblique currents produce a contraction in the vein or stream. The minimum transverse section of the contracted vein, is the plane at which the velocity is nearly equal to the theoretical velocity. The quantity of water which will be discharged in a certain time, depends upon the form of the opening, as well as upon the head. Thus, by means of a conical tube of the form of the contracted vein, the velocity at the opening or smaller end of the tube, is nearly equal to the theoretical velocity. The theoretical velocity per second (rule 4, varied motion), is eight times the square root of the head in feet. The actual velocity estimated for the entire opening, as ordinarily constructed, is five and four-tenths the square root of the head.

Water, as a power or force, is exerted on water wheels by its weight and by its impulse. Weight and impulse are combined on the overshot and breast wheels.

The theoretical work accomplished by weight is the product of its force and the vertical distance through which it is exerted. The theoretical work accomplished by impulse, is the product of the force produced by weight of the flow of water, and the vertical height or head necessary to produce the velocity with which the weight moves.

The available work depends not only upon the magnitude of the force exerted, but upon the direction of that force in reference to the direction given to the resistance; also upon the form of the floats or buckets of the wheel, friction, losses by leakage, etc.

The average efficiency of various water wheels, running under favorable circumstances, as found by experience, is as follows, to wit:

Undershot, having flat, radial floats 33
Poncelet, improved undershot, ...60
Turbine, for example, the "Jouval,".........68
Reaction, for example, the Scotch Turbine,... 66
Overshot and Breast, the efficiency of that part of the fall acting by weight, is about.. ...78

And of that part acting by impulse,.... 40

The best velocities of the various water wheels, as compared with the supply velocities, are as follows:

Undershot and Low Breast, at circumference,..50
Turbines, at the middle of ring of buckets,.....65
Reaction, at circumference,..................97
Overshot, at circumference, ...50

The velocity of the overshot wheel at its circumference, should be about six feet; which is due a head of 2.25 feet.

Let the vertical distance from the centre of the opening in the gate, to the surface of the water in the flume or reservoir, be termed the *head*, and the vertical distance from the centre of the opening in the gate to the lower edge of the wheel, the *fall*.

TABLE OF COEFFICIENTS FOR ESTIMATING THE HORSES' POWER OF WATER WHEELS.

Head. ft. in.	coefficient.	Head. ft. in.	coefficient	Head. ft. in.	coefficient.	Head. ft. in.	coefficient.
1	12	1 7	54	3 2	76	9.	128
2	17	1 8	55	3 4	78	10.	135
3	21	1 9	56	3 6	80	12.	148
4	25	1.10	58	3 8	82	14.	160
5	28	1.11	59	3 10	84	16.	171
6	30	2 0	60	4 0	85	20.	191
7	33	2 1	62	4 3	88	25.	213
8	35	2 2	63	4 6	90	30.	233
9	37	2 3	64	4 9	93	36.	256
10	39	2 4	65	5 0	95	49.	298
11	41	2 5	66	5 4	98	64.	341
1 0	43	2 6	67	5 8	101	81-	384
1 1	44	2 7	69	6 0	104	100.	426
1 2	46	2 8	70	6 6	109	121.	469
1 3	48	2 9	71	7 0	113	144.	511
1 4	49	2.10	72	7 6	117	169.	554
1 5	51	2.11	73	8 0	121	196.	597
1 6	52	3 0	74	8 6	124	225.	639

TO FIND THE HORSES' POWER FOR VARIOUS WATER WHEELS.

Rule.—Multiply the product of the tabular coefficient opposite the given head, the area of the opening in the gate in square inches, the entire head in feet (head and fall in case of overshot and breast wheels), by the efficiency of the class of wheel, pointing off six figures as decimals.

Example 1.—The dimensions of a stream are two inches by two hundred inches. What is its horses' power, applied to a breast wheel affording a fall of ten feet?

Calculation. 2 inches by 200 inches=400 square inches opening.

Tabular coefficient opposite 2 feet 3 inches,.... 64
Efficiency of wheel, arising from impulse...... 40
Efficiency of wheel arising from weight,....... 78
Head, 2 feet 3 inches =2.25

2.25×40=90, product of efficiency and head.

10×78=780 product of efficiency and fall.

90+780=870 sum of products.

870×400×64=22.27 horses' power. Ans.

Example 2.—The dimensions of the stream are ten inches square, the head twenty-five feet, what is its horses' power applied to a good turbine?

Calculation. 10×10=100 square inches opening.

Tabular coefficient opposite head of twenty-five feet,......... .... =213

Efficiency of turbine =68.

100×213×68×25=36.21 horses' power. Ans.

STEAM POWER.

Steam, as a force, acts by elastic pressure. The law "that in compressing the same quantity of air, or of a perfect gas into smaller spaces, the volumes occupied by it are inversely proportioned to the pressures," does not hold good in relation to saturated steam. In the following table, P denotes the total pressure in pounds per square inch; T the corresponding temperature, and V the volume of the steam compared to the volume of the water that has produced it.

TABLE OF PRESSURES, TEMPERATURES AND VOLUMES.

P.	T.	V.	P.	T.	V.	P.	T.	V.
1	102.1°	20582	60	292.7°	437	180	372.9°	155
5	162.3	3813	65	298.0	405	190	377.5	148
10	193.3	2358	70	302.9	378	200	381.7	141
14.7	212.0	1642	75	307.5	353	210	386.0	135
15	213.1	1610	80	312.0	333	220	389.9	129
20	228.0	1229	90	320.2	298	230	393.8	123
25	240.1	996	100	327.9	270	240	397.5	119
30	250.4	838	110	334.6	247	250	401.1	114
35	259.3	726	120	341.1	227	260	404.5	110
40	267.3	640	135	350.1	203	270	407.9	106
45	274.4	572	150	358.3	184	280	411.2	102
50	281.0	518	165	366.0	169	300	417.5	96

TABLE FOR ESTIMATING THE MEAN PRESSURE OF STEAM FOR A GIVEN CUT-OFF OF STROKE.

UNJACKETED CYLINDER.			JACKETED CYLINDER.		
cut-off	coefficient.	correction.	cut-off.	coefficient.	correction.
1/20	.177	12.098	1/20	.186	11.966
3/40	.244	11.113	3/40	.254	10.966
1/10	.303	10.246	1/10	.314	10.084
1/8	.356	9.467	1/8	.370	9.261
3/20	.407	8.717	3/20	.417	8.570
1/5	.496	7.409	1/5	.505	7.297
1/4	.572	6.290	1/4	.582	6.145
3/10	.639	5.307	3/10	.648	5.174
7/20	.697	4.454	7/20	.707	4.307
2/5	.748	3.704	2/5	.756	3.587
9/20	.797	2.984	9/20	.800	2.940
1/2	.833	2.455	1/2	.840	2.352
11/20	.869	1.926	11/20	.874	1.852
3/5	.894	1.558	3/5	.900	1.470
13/20	.923	1.132	13/20	.929	1.044
7/10	.945	0.808	7/10	.945	0.808
3/4	.960	0.588	3/4	.960	0.588
4/5	.976	0.353	4/5	.976	0.353
17/20	.986	0.206	17/20	.986	0.206
9/10	·997	0.044	9/10	.997	0.044

By the table of pressures, temperature and volumes, it will be seen that the volume of steam under a pressure of thirty pounds to the square inch produced from a cubic inch of ice-cold water is 838 cubic inches; while under a pressure of ninety pounds to the square inch, the volume is 298 cubic inches. Thus the ratio of the two pressures is as 30 to 90, or as 1 to 3, while the inverse ratio of the respective volumes of steam is as 1 to 2.81. The mechanical effect deduced from the above data is as follows: $838 \times 30 \div 12 = 2095$, and $298 \times 90 \div 12 = 2235$. Then $2235 - 2095 = 140$ difference of mechanical effects, and $140 \div 2095 = .0668$; showing an advantage of nearly seven per cent. in favor of using steam at the higher pressure.

By the table for estimating the mean pressure of steam for a given cut-off of stroke, the coefficients for one-fourth ($\frac{1}{4}$) cut-off are 572 in the unjacketed cylinder, and 582 in the jacketed cylinder.

$$\text{Then } 582 - 572 = 10,$$
$$\text{and } 10 \div 572 = .0175;$$

showing an advantage of one and three-fourths of one per cent. in favor of the jacketed cylinder for the given cut-off of stroke.

The back pressure of steam in the cylinder of an engine of ordinary structure is found, by experience, to be about four pounds to the square inch above the atmospheric pressure; the velocity of piston being three hundred (300) feet per minute. It is also found by experience that the excess of the back pressure above the atmospheric pressure varies nearly as the square of the velocity of the piston.

Thus, if the velocity of the piston be four hundred (400) feet per minute, the back pressure will be 7.11 pounds.

Calculation. 300×300=90000; 400×400=160000; 4×160000÷90000=7.11 pounds.

TO FIND THE MEAN PRESSURE OF STEAM FOR A GIVEN CUT-OFF OF STROKE.

Rule.—Multiply the excess of the pressure of steam above the atmospheric pressure per square inch as it enters the cylinder by the tabular coefficient opposite the given cut-off, pointing off three figures as decimals, and deduct from the product the tabular correction for the same cut-off.

Example.—What is the mean pressure of steam entering the cylinder at a pressure of ninety pounds to the square inch, and cut-off at three-tenths stroke?

Calculation.—Tabular coefficient, unjacketed
cylinder, for,................$\frac{3}{10}$ stroke=,639
Correction for same,..................=5.307
Then, ,639×90=57.510
57.510—5.307=52.203 pounds. Ans.
And, jacketed cylinder,
,648×90=58.320
58.320—5.174=53.146.

TO FIND THE EFFECTIVE HORSES' POWER OF A NON-CONDENSING STEAM ENGINE.

Rule.—Multiply four times the square of the diameter of the piston in inches by the product of the number of revolutions, length of stroke in feet, and the difference between the average forward pressure and the back pressure of steam in pounds per square inch, pointing off five figures as decimals.

Example.—What is the effective horses' power of an engine, the diameter of the piston being sixteen inches, the length of stroke three feet, the number of revolutions fifty per minute, and the average forward pressure, above the atmospheric pressure, seventy-five pounds per square inch?

Cal.—The back pressure =4 pounds.

75—4=71 pounds.

Then,

16×16×4×50×3×71=109.05 horses' power. Ans.

Ex. 2.—What is the effective horses' power of an engine, the diameter of the piston being twelve inches, the length of stroke two feet, the pressure of steam, as it enters the cylinder, sixty pounds in excess of atmospheric pressure, cut-off at one-half stroke, and making seventy-five revolutions per minute?

Cal.—Back pressure 4 pounds.

Tabular coefficient for ½ stroke=,833.

" correction for same =2.455.

833×60—2.455—4=43.525 effective pressure.

Then, $12 \times 12 \times 4 \times 43.525 \times 2 \times 75 = 37.60$ horses' power. Ans.

MECHANICAL POWERS.

There are three classes of mechanical powers, viz :—the lever, pulley and inclined plane.

The wheel and axle belong to the first class, and are sometimes termed the *perpetual lever.*

The wedge and screw belong to the third class.

Weight signifies the resistance to be overcome, and power the force which overcomes or tends to overcome the resistance.

THE LEVER.

The arms of a lever are the portions of it which are intercepted between the power and fulcrum, and between the weight and fulcrum.

There are three kinds of levers, in which :—

1. The fulcrum is between the power and weight.
2. The weight is between the power and fulcrum.
3. The power is between the fulcrum and weight.

Assuming the lever itself to have no weight, and no friction, the condition of equilibrium or balance, is as follows:

The product of the power and the length of arm to which it is applied, is equal to the product of the weight and the length of arm to which it is applied.

Ex. 1. If one arm of a lever be ten feet, and the other two feet, what power must be applied to the longer arm to balance a weight of 1000 pounds?

Cal. $1000 \times 2 \div 10 = 200$ pounds. Ans.

Ex. 2. If the radius of the axle, of "the wheel and axle," be six inches, and the radius of the wheel forty-eight inches, what power must be applied to the circumference of the wheel to balance 2400 pounds at the circumference of the axle?

Cal. $2400 \times 6 \div 48 = 300$ pounds. Ans.

THE PULLEY.

Assuming the pulley (as commonly arranged) to be without weight and free from friction and stiffness of cordage, the condition of equilibrium or balance is, that the weight is equal to the product of the power and the number of cords at the movable block.

Ex.—What weight will be balanced by a power of one hundred pounds, there being three movable pulleys, or, in other words, six cords at the movable block?

Cal. $100 \times 6 = 600$ pounds. Ans.

THE INCLINED PLANE.

Assuming that there is no friction, and that the direction of the power applied is parallel to the plane, the conditions of equilibrium of a body sustained by any force on an inclined plane, is as follows:

The product of the power and the length of the plane is equal to the product of the weight and the height of the plane.

Ex.—What power would be necessary to sustain a rolling weight of 1,200 pounds upon an inclined plane of 10 feet length and 6 feet perpendicular height?

Cal.—$1200 \times 6 \div 10 = 720$ pounds. Ans.

If the power acts parallel to the base of the plane, then the product of the power and the length of the base is equal to the product of the weight and the height of the plane.

Ex.—What power would be necessary to sustain a rolling weight of 1200 pounds upon an inclined plane whose base is 8 feet and height 6 feet?

Cal. $1200 \times 6 \div 8 = 900$ pounds. Ans.

Ex. 3d.—Omitting the consideration of friction, what power applied to the back of a wedge of the form of either a single or double inclined plane, and in the direction of the base of the inclined plane, will raise a weight of 2400 pounds, the back of the wedge being 3 inches thick, and the base being 48 inches long?

Cal. $2400 \times 3 \div 48 = 150$ pounds. Ans.

Ex. 4.—Omitting the consideration of friction, if the threads of a screw be 2 inches apart, and a power of 500 pounds be exerted at the end of a lever 84 inches long, what weight or force will be produced at the end of the screw?

Cal. $84 \times 2 \times \frac{22}{7} = 528$ inches, base of inclined plane. Distance of threads apart$=2$ inches, height of inclined plane, $528 \times 500 \div 2 = 132000$ pounds. Ans.

THIN CYLINDERS.

To determine the thickness of a thin hollow cylinder;

the internal radius, pressure, and the tenacity of the material being given.

Rule. Multiply the internal radius in inches by the fluid pressure in pounds, and divide the product by the tenacity per square inch of the material.

Ex.—The internal radius of a cylinder being 30 inches, the fluid pressure 250 pounds to the square inch, and the tenacity of the material of the cylinder twelve thousand pounds per square inch, what is the thickness of the cylinder?

Cal. $250 \times 30 = 7500$.

$7500 \div 12000 = \frac{5}{8}$ inches. Ans.

To determine the fluid pressure, the internal radius, thickness of cylinder, and tenacity of material being given.

Rule.—Divide the product of the thickness of the cylinder and tenacity of the material, by the internal radius.

Ex.—The thickness of the cylinder being one-fourth of an inch, the tenacity eighteen thousand pounds, and the radius six inches, what fluid pressure will the cylinder withstand per square inch?

Cal. $18000 \times \frac{1}{4} \div 6 = 750$ pounds. Ans.

THICK HOLLOW CYLINDERS.

To determine the thickness of thick, hollow cylinders, the internal radius, the fluid pressure and the tenacity of the material of the cylinder being given.

Rule.—Subtract one from the square root of the quotient of the sum and difference of the tenacity per square inch of the material of the cylinder, and the fluid pressure per square inch, and multiply this difference by the internal radius.

Ex. 1.—The internal radius of a thick, hollow cylinder being nine inches, the tenacity of the material of the cylinder ten thousand pounds per square inch, what is the requisite thickness of the cylinder?

Cal. Sum of tenacity and fluid pressure,	10000÷8000=18000
Difference of tenacity and fluid pressure,	10000--8000=2000
Quotient of sum and difference,	18000÷2000=9
Square root of quotients,.	$\sqrt{9}=3$
Difference between root and one,	3—1=2
Product of radius and this difference	9×2=18 inches. Ans.

To determine the fluid pressure per square inch, which a thick, hollow cylinder will withstand, the internal and external radii, and the tenacity of the material of the cylinder being given.

Rule.—Divide the difference of the squares of the radii, and multiply the quotient by the tenacity per square inch of the material of the cylinder.

Ex. 2.—The internal and external radii of a thick, hollow cylinder being respectively nine inches and twenty-seven inches, and the tenacity per square inch of the material of the cylinder ten thousand pounds, what fluid pressure per square inch will it withstand?

Cal. Square of external radius,	27×27=729
Square of internal radius,	9×9=81
Difference of squares,	729—81=648
Sum of squares,	729+81=810
Then 10000×648÷810=8000 pounds. Ans.	

SUSPENSION RODS OF UNIFORM STRENGTH.

To determine the transverse section at any point of a suspension rod of uniform strength:

Rule 1.—Divide the constant weight to be raised by the uniform tension per square inch, due the tenacity of the material in the rod, and multiply the quotient by 2,71828, raised to a power equal to the product of the length of the rod in inches, and the weight of one cubic inch of the rod, divided by the intensity of the tension per square inch.

Ex.—The weight to be raised being 27000 pounds, the intensity per square inch of the working tension 3,000, the weight of rod per cubic inch $\frac{5}{18}$ of a pound, and the length of rod 600 feet. Required the transverse section at the upper end of rod?

Cal.—Weight divided by intensity of tension,

27000÷3000=9

Product of length in inches and

weight of cubic inch,.........7200×$\frac{5}{18}$=2000

This product divided by intensity

of tension,..................2000÷3000=$\frac{2}{3}$

Product of 434295 and last quotient, ,434295$\times\frac{2}{3}$= ,289530.

Number corresponding to logarithm,.. ....289530=1,94887

And 1,94887$\times$9=17,5983 inches. Ans.

TO DETERMINE THE WEIGHT OF THE ROD THE SAME DATA BEING GIVEN AS IN EXAMPLE UNDER RULE ONE.

Rule 2.—Multiply the transverse section by the intensity of the tension, and subtract from the product the constant weight to be raised.

Ex.—The transverse section being 17,5983 square inches, as determined by solution of example one, the intensity of tension being 3,000 pounds per square inch, and the constant weight to be raised 27,000 pounds, what is the weight of the rod?

Cal. 17,5983$\times$3000=52794.9
52794.9—27000.0=25794.9 pounds. Ans.

WATER PIPES.

To determine the velocity of water per second, flowing through long pipes, the head or hight of reservoir above the point of delivery, the length and diameter of the pipe being given.

Rule.—Multiply the product of the head and diameter of the pipe in feet by twenty-three hundred. Divide this product by the sum of once the length and fifty-two times the diameter of the pipe, and extraet the square root of the quotient.

Ex.—The head is six hundred feet; the diameter of pipe nine inches; the length of pipe six thousand feet; what is the velocity of the water per minute?

Cal.—Diameter of pipe =9 inches=,75 feet.

Produet of head, diameter, etc., 600×2300×,75=1035000,00.

Sum of length and product, 6000+52×,75=6039

Quotient, 1035000÷6039=171,386.

Square root $\sqrt{171,386}$=13,09 feet, velocity per sec.

Per minute, 13,09×60=785,4 feet. Ans.

Plane Circular Plates.—To determine the grinding effects by one revolution of a plane circular plate of the usual ring form, of uniform hardness, about its axis perpendieular to the grinding plane—the greater and less diameters and pressure per square inch being given.

Rule.—Multiply the difference of the fourth powers of the radii by the product of the pressure per square inch, and the square of the ratio between the radius and circumference of a circle, and divide the product by the greater radius. Deduced from formulas 4_{6} and 5_{6} Discussion of Tractory, etc.

Ex.—The greater diameter of a plane circular plate (muller) of the usual ring form, and of uniform hardness, being forty inches; the less diameter sixteen inches, and the pressure per square inch five pounds, what is the grinding effect by one revolution?

Cal. $48 \div 2 = 24$ greater radius.

$16 \div 2 = 8$ less radius.

$24 \times 24 \times 24 \times 24 \times = 331776$ fourth power of greater radius.

$8 \times 8 \times 8 \times 8 = 4096$ fourth power of less radius.

$331776 - 4096 = 327680$ differenee of fourth powers.

$327680 \times \frac{22}{7} \times \frac{22}{7} \times 5 \div 24 = 674307.47.$* Ans.

Conical Plates.—To determine the grinding effeet by one revolution of a conieal plate (muller) of the usual ring form, of uniform hardness, about its axis perpendicular to the plane of the base, the greater and less diameters of the frustum, the height of the cone and the pressure per square inch, parallel with the axis of revolution being given.

Rule.—Multiply the square root of the sum of the squares of the greater radius, and height of the cone by the difference of the fourth powers of the radii, and this produet by the product of the pressure per square ineh, and the square of the ratio between the diameter and circumference of a circle, divided by the square of the greater radius. Dedueed from formulas 9_7 and 10_7.

Ex.—The greater diameter of a eonieal plate (muller) of the usual ring or frustum form being forty-eight

* Remark.—The unit of grinding effect here taken is the result of a body impressing one square inch of another body, with on pound pressure, and moving, under these circumstances, on inch. See Laws of Grinding, Discussion of Tractory, etc.

inches, the less diameter sixteen inches, the height of the cone twelve inches, the pressure per square inch, parallel with the axis of revolution, five pounds, what is the grinding effect by one revolution?

Cal. $48 \div 2 = 24$ greater radius.

$16 \div 2 = 8$ less radius.

$24 \times 24 = 576$ square of greater radius.

$12 \times 12 = 144$ square of hight of cone.

$\sqrt{576+144} = 26.8328$ square root of sum.

$24 \times 24 \times 24 \times 24 = 331776$ fourth power of greater radius.

$8 \times 8 \times 8 \times 8 = 4096$ fourth power of less radius.

$331776 - 4096 = 327680$ difference of fourth powers.

$327680 \times 26.8328 \times \frac{22}{7} \times \frac{22}{7} \times 5 \div 576 = 753875.77$.

Ans.

Tractory Conoidal Plates.—To determine the grinding effect by one revolution of a tractory conoidal plate (muller) of frustum form, of uniform hardness, about its axis perpendicular to the plane of its base, the greater and less diameters of the frustum, and the pressure per square inch parallel with the axis of revolution being given.

Rule.—Multiply the difference of the squares of the radii of the frustum by twice the product of the greater radius, pressure per square inch, and the square of the ratio between the diameter and circumference of a circle. Deduced from formula 5.

Ex.—The greater diameter of a tractory conoidal plate (muller) being forty-eight inches, the less diameters sixteen inches, and the pressure, per square inch, five pounds; what is the grinding effect by one revolution.

Cal. $48 \div 2 = 24$ greater radius.

$16 \div 2 = 8$ less radius.

$24 \times 24 = 576$ square of greater radius.

$8 \times 8 = 64$ square of less radius.

$576 - 64 = 512$ difference of squares.

$512 \times 24 \times 5 \times \frac{22}{7} \times \frac{22}{7} \times 2 = 1213753.47$. Ans.

Remark.—Since, in this style of plate, the wear is perfectly uniform, the grinding effect will be the same, whether the central opening be large or small, providing the weight of the muller be the same in each case.

MENSURATION.

Prop. 1.—The square root of the sum of the squares of the base, and perpendicular of a right-angled triangle is equal to the hypothenuse.

Ex.—The base of a right-angled triangle is eight feet, the perpendicular fifteen feet, what is the hypothenuse?

Cal. $8 \times 8 = 64$ $15 \times 15 = 225$

$64 + 225 = 289$ $\sqrt{289} = 17$ feet. Ans.

Prop. 2.—The square root of the difference of the squares of the hypothenuse, and one of the sides of a right angled triangle is equal to the other side.

Ex. 1.—The hypothenuse is thirteen feet, the perpendicular twelve feet; what is the base?

Cal. $13\times13=169$ $\quad 12\times12=144$

$169-144=25$ $\quad \sqrt{25}=5$ feet. Ans.

Ex. 2.—The hypothenuse is thirty-seven feet, the base twelve feet; what is the perpendicular?

Cal. $37\times37=1369$ $\quad 12\times12=144$

$1369-144=1225$ $\quad \sqrt{1225}=35$ feet. Ans.

Prop. 3.—The circumference of a circle is equal to twenty-two times the diameter, divided by seven, (nearly).

Remark.—The true circumference of a circle lies between $3\frac{10}{70}$ and $3\frac{10}{71}$ times the diameter, which is nearly 3.1416. This ratio is usually represented in formulas by the character Π.

Ex.—The diameter of a circle being ten feet, what is the circumference?

Cal. $10\times22\div7=31.428$, or $10\times3.1416=31.416$. Ans.

Prop. 4.—The length of an arc of a circle containing any number of degrees is equal to the product of the number of degrees in the arc, diameter of the circle and 3.1416 divided by three hundred and sixty.

Ex.—What is the length of an arc containing seventy-five degrees, the diameter of whose circle is twenty-one.

Cal. $75\times21\times3.1416\div360=13.74$ feet. Ans.

Prop. 5—The length of an arc of a circle is equal to one-third of the difference between eight times the chord of half the arc, and once the chord of the arc.

Ex.—Required the length of an arc whose chord is ten feet, and the chord of one-half the arc is 5.177 feet?

Cal. $5.177 \times 8 - 10 = 31.416$. $31.416 \div 3 = 10.472$ feet. Ans.

Prop. 6—The diameter of a circle is equal to the sum of the hight of any segment of the circle, and one third the square of half the base of that segment.

Ex.—The base of a segment of a wheel is fifty-four inches; the hight of the segment three inches; what is the diameter of the wheel.

Cal. $54 \div 2 = 27$ $27 \times 27 \div 3 = 243$
$243 + 3 = 246$ inches. Ans.

Prop. 7.—The length of the *common cycloid* is equal to four times the diameter of the generating circle.

Ex.—What is the length of a common cycloid, the length of whose generating circle is five feet?

Cal. $5 \times 4 = 20$ feet. Ans.

Prop. 8.—The hight of a *common parabola* is equal to the square of half the base, or the square of the ordinate divided by twice the parameter.

Ex.—The base of a common parabola is eight feet; the perameter four thirds of a foot; what is its hight?

Cal. $8 \div 2 = 4$ $4 \times 4 = 16$
$4 \times 2 \div 3 = 8 \div 3$ $16 \times 3 \div 8 = 6$ feet. Ans.

Prop. 9.—The length of an arc of the common parabola, measured from its vertex, is as follows:

$$\text{Length of arc.} \quad = \frac{y\sqrt{p^2+y^2}}{2p} + \frac{p}{2}\log\left(\frac{y+\sqrt{p^2+y^2}}{p}\right).$$

Ex.—The half base or coordinate (y) being twelve feet, and the half parameter (p) being five feet, what is the length of the arc of the parabola, measured from the vertex?

Cal. $12\times12=144 \quad 5\times5=25$

$144+25=169 \quad \sqrt{169}=13$

$13\times12\div10=15.6 \quad 13+12=25$

$25\div5=5$ Naperian log, $5=1.61$

$1{,}61\times5\div2=4.02 \quad 15.6+4.02=$

Prop. 10.—An ordinate of an ellipse is equal to the product of the minor axis, and the square root of the difference of the squares of the major axis, and the coordinate divided by the major axis.

Ex.—The axes of an ellipse are seven and ten feet An ordinate is six feet; what is the coordinate.?

Cal. $10\times10=100 \quad 6\times6=36$

$100-36=64 \quad \sqrt{64}=8$

$8\times7\div10=5.6$ feet. Ans.

Prop. 11.—The circumference of an ellipse is as follows:

$$\text{Circumference} = 2\pi A\left(1-\frac{E^2}{4}-\frac{3E^4}{64}-\frac{45E^6}{2304}-\text{etc}\right).$$

Remark.—In the above formula, A denotes the semi-major axis; E the eccentricity; that is, the distance between the centre and one of the foci of the ellipse, divided by the semi-major axis.

Ex. $E^2 \div 4 = ,01.$ $E^4 \times 3 \div 64 = ,000075.$
$E^6 \times 45 \div 2304 = ,000001.$

$.01 + 000075 + ,000001 = .010076$

$1 - .010076 = ,989924$

$.989924 \times 2 \times 20 \div 2 \times 3.1416 = 62.1989$ feet. Ans.

Prop. 12.—An ordinate of an *hyperbola* is equal to the product of the conjugate axis, and the square root of the difference of the squares of the co-ordinate and transverse axis, divided by the transverse axis.

Ex.—The transverse axis of an hyperbola is four feet; the conjugate axis three feet, and an ordinate on the transverse axis and hight of the hyperbola ten feet; what is the co-ordinate?

Cal. $10 \times 10 = 100$ $4 \times 4 = 16$ $100 - 16 = 84$
$\sqrt{(84)} = 9.165$ $9.165 \times 3 \div 4 = 6.874$ feet. Ans.

Prop 13.—The length of an hyperbola is as follows, to wit:

$$= \pi A\left[1 - \tfrac{1}{4}(2 - E^2) - \tfrac{3}{64}(2 - E^2)^2 - \tfrac{45}{2304}(2 - E^2)^3\right]$$

Remark.—In the above formula, A represents the semi-transverse axis; E the eccentricity; that is, the distance of the center from one of the foci divided by the semi-transverse axis.

Ex.—The difference of the distances of a point in the curve, from the foci, being twenty feet, and the distance from the center from one of the foci, fifteen feet, what is the length of the hyperbola?

Cal. $20 \div 2 = 10.$ $15 \div 10 = E.$ Eccentricity.

$-(2-E^2) \div 4 = +,1875$; $3(2-E^2)^2 \div 64 = -,0264$

$-45\,(2-E^2)^3 \div 2304 = +.0082$ $1.+.1875+.0082$
$-,0264 = 1.1693$

$1.1693 \times 10 \times 3.1416 = 36.7347$ feet. Ans.

Prop. 14.—The length of the spiral of Archimedes is as follows:

$$\text{Length.} = \frac{A}{2} \left\{ t \sqrt{(1+t^2)} + \log \left[\sqrt{(1+t^2)} + t \right] \right\}$$

Remark.—In the above formula, (t) represents the measuring arc, and (A) the relation between the radius vector and measuring arc.

Ex.—The measuring arc being the entire circle, and the radius vector unity, what is the length of the spiral or spire?

Cal. $t = \Pi 2 = 6,2832.$ $a = \frac{1}{2\Pi} = \frac{1}{6,2832}$

$$\tfrac{1}{2}\sqrt{1+4\times(3,1416)^2} = 3,1811$$

$$\frac{1}{4\times 3,1416} \log \left[\sqrt{1+4\times(3.1416)2} + 2\times 3.1416 \right] = .2019$$

$3.1811 + ,2019 = 3.383.$ Ans.

Ex. 2.—The measuring arc being twice the circumference of a circle whose radius is unity, what is the length of the spiral, and what is the length of the second spire?

Cal. $t=4\pi$; $a=\frac{1}{2\pi}$ a constant quantity.

$\sqrt{1+16\times(3.1416)^2}=12.6060$

$\frac{1}{4\times 3,1416}\log\left[\sqrt{1+16\times(3.1416)^2}+4\times 3.1416\right]=,2567$

12,6060 +, 2567=12,8627 length of spiral. Ans.

The first spire, as found, for example one, is 3.383.

Hence, 12.8627—3.383=9,4797 length of second spire. Ans.

Prop. 15.—The area of a square, a rectangle, or a parallelogram, is equal to the product of its base and altitude.

Ex.—What is the area of a floor forty-five feet by one hundred and twenty-five feet?

Cal. 125×45=5625 square feet. Ans.

Prop. 16.—The area of a triangle is equal to half the product of the base and perpendicular.

Ex.=What is the area of a triangle whose base is twelve feet, and altitude eighteen feet?

Cal. 12×18÷2=108 square feet. Ans.

Prop. 17.—The area of a trapezoid is equal to half the product of the sum of the two parallel sides and the altitude.

Ex.—The parallel sides being thirteen feet and seventeen feet, and their distance apart seven feet, what is the area of the trapezoid?

Cal. 13+17=30. 30×7÷2=105 square feet. Ans.

Prop. 18.—The area of an irregular polygon is equal to the sum of the areas of the separate triangles composing it.

TABLE.

NAMES.	SIDES.	AREAS.	NAMES.	SIDES.	AREAS.
Triangle,	3	,4330127	Octagon	8	4,8284271
Square	4	1,0000000	Nonagon	9	6,1818242
Pentagon,	5	1,7204774	Decagon,	10	7,6942088
Hexagon,	6	2,5980762	Undecagon	11	9,3656399
Heptagon,	7	3,6339124	Dodecagon	12	11,1961524

Prop. 19.—The area of a regular polygon is equal to the square of one of its sides multiplied by the area of a polygon of the same number of sides, and whose side are unity.

Ex.—What is the area of a nonagon whose side is ten feet?

Cal. $10\times10=100$. Tabular area $=6,1818242$
$6,1818242\times100=618,18242$ square feet. Ans.

Prop. 20.—The area of a circle is equal to one fourth the product of the diameter and circumference; also equal to the product of the square of the diameter and ,7854; and also equal to the square of the diameter and eleven divided by fourteen.

Ex.—What is the area of a circle whose diameter is forty feet?

Cal. $40\times40\times11\div14=1257,14$ square feet. Ans.

Prop. 21.—The area of a sector of a circle is equal to half the arc of the sector multiplied by half the diameter of the circle.

Ex.—What is the area of a sector whose arc is forty degrees, and the diameter of the circle thirty feet.

Cal. 3.1416×40÷360×30=10.472

10.472÷2×15=78.54 square feet. Ans.

Prop. 22.—The area of a circular ring is equal to the difference of the squares of the diameters multiplied by ,7854.

Ex.—What is the area of a circular ring whose greater diameter is forty-two feet, and less diameter seven feet?

Cal. 42×42=1764. 7×7=49.

1764--49=1715. 1715×,7854=1346,96 square feet. Ans.

Prop. 23.—The area of the common cycloid is equal to three times the area of the generating circle.

Ex.—What is the area of a common cycloid, the diameter of whose generating circle is seven feet?

Cal. 7×7×11÷14×3=115.5 square feet. Ans.

Prop. 24.—The area of the common parabola is equal to two-thirds the product of the base and altitude.

Ex.—The base of a common parabola is twelve feet, and the hight fifteen feet, what is the area?

Cal. 12×15×2÷3= 120 square feet. Ans.

Prop. 25.—The area of an ellipse is equal to the product of the semi-diameters multiplied by 3.1416.

Ex.—What is the area of an ellipse, the semi-diameters being ten feet and eight feet?

Cal. 10×8×3.1416=251.328 square feet. Ans.

Prop. 26.—The area of an hyperbola is as follows:

$$\text{Area} = xy - A \times B \log \left(\frac{x}{A} + \frac{y}{B}\right)$$

Remark.—In this formula, A represents the semi-transverse axis, B the conjugate axis, x and y the general co-ordinates.

Ex.—Given the base, 13,748 feet, the hight six feet, transverse axis eight feet, and the conjugate axis six feet, what is the area of the hyperbola?

Cal. A=8÷2=4 semi-transverse axis.
B=6÷2=3 semi-conjugate axis.
x=6+4=10 co-ordinate.
y=13.748÷2=6.874 co-ordinate.

Then area=68.74—12 log 4.791=49.88 square feet. Ans.

Prop. 27.—The area of the equable spiral or spiral of Archimedes, is equal to one-third the difference between the cube of the number of revolutions and the cube of a number one less than that of the revolutions, multiplied by 3.1416.

Ex.—What is the area of an equable spiral, whose radius vector is seven feet, described by seven revolutions?

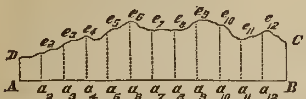
Cal. 7×7×7=343 cube of revolutions $=n^3$
6×6×6=216 cube of number one less $=(n-1)^3$

Then 343=216=127 difference $=n^3-(n-1)^3$
127×3,14.16÷3=132.99 square feet. Ans.

Prop. 28.—The area of an irregular plane surface is equal to one-third the distance between any two consecutive ordinates multiplied by the sum of the extreme ordinates, increased by four times that of the even ordinates, and twice that of the uneven ordinates.

Remark.—The entire number of ordinates or perpendiculars is to be uneven, and at equal distances apart.

Fig. 1.



Ex. Having given as in figure 1, the base A B, equal to 540 feet, and having found the perpendiculars A D = 50, $a_2\,e_2 = 55$, $a_3\,e_3 = 80$, $a_4\,e_4 = 100$, $a_5\,e_5 = 120$, $a_6\,e_6 = 110$, $a_7\,e_7 = 115$, $a_8 e_8 = 130$, $a_9\,e_9 = 128$, $a_{10}\,e_{10} = 90$, $a_{11}\,e_{11} = 110$, $a_{12}\,e_{12} = 108$, $a_{13}\,e_{13} = 75$ feet, what is the area of ABCD?

Cal. 540÷12=45 distance apart of perpendiculars.

45÷3=15 one-third distance between any two perpendiculars.

50+75=125 sum of extremes.

55+100+110+130+90+108=593 sum of even ordinates or perpendiculars.

80+120+115+128+110=553 sum of uneven ordinates or perpendiculars.

593×4=2372 ; 553×2=1106.

125+2372+1106=3603.

3603×15=Area of ABCD,=54045 square feet. Ans.

Prop. 29.—The area or convex surface of a right prism, or of a cylinder, is equal to the primeter of the base multiplied by the altitude.

Ex.—What is the convex surface of a cylinder whose diameter is seven feet and length ten feet?

Cal. $7\times22\div7=22$; $22\times10=220$ square feet. Ans.

Prop. 30.—The area or convex surface of a right pyramid or cone is equal to the perimeter of the base multiplied by one half of the slant height.

Ex.—What is the area or convex surface of a cone whose slant height is twenty feet, and the diameter of whose base is seven feet?

Cal. $7\times22\div7=22$; $22\times20\div2=220$ square feet. Ans.

Prop. 31.—The area or convex surface of a spherical zone is equal to the altitude of the zone multiplied by the circumference of a great circle of the sphere.

Ex.—The height of a zone is eight feet, the diameter of the sphere twenty-five feet. What is the area of the zone?

Cal. $8\times25\times3.1416=628.32$ square feet. Ans.

Prop. 32.—The area of a sphere is equal to the product of the diameter and circumference.

Ex.—What is the area of a ball ten inches in diameter?

Cal. $10\times3.1416=31.416$; $31.416\times10=314.16$ square feet. Ans.

Prop. 33.—The area or surface described by revolving a cycloid about its base is sixty-four thirds of the generating circle.

Ex.—What is the area or surface described by the revolution of a cycloid about its base, the diameter of the generating circle being seven feet?

Cal. $7\times22\div4=38.5$; $38.5\times64\div3=821.33$ square feet. Ans.

Prop. 34.—The area or convex surface of a paraboloid is as follows:

$$\text{Area}=\frac{4bh\pi}{3}\left\{\left(1+\frac{b^2}{4h^2}\right)^{\frac{3}{2}}-\frac{b^3}{8h^2}\right\}$$

Remark.—In this formula b represents one-half the diameter of the base, h the height of the paraboloid, and $\pi=3.1416$.

Ex.—The diameter of the base of a paraboloid being six feet, and the height two feet, what is the surface of revolution?

Cal. $4\times6\div2\times2\times3.1416\div3=25.1328$.

$$\left(1+\frac{9}{16}\right)^{\frac{3}{2}}=\frac{125}{64};\quad \frac{27}{8\times8}=\frac{27}{64};\quad \frac{125}{64}-\frac{27}{64}=\frac{98}{64};$$

$25.1328\times98\div64=38.48$ square feet. Ans.

Prop. 35.—The area of an ellipsoid described by revolving an ellipse about its major axis, is as follows:

$$\text{Area}=4\pi AB\left(1-\frac{e^2}{6}-\frac{e^4}{40}-\frac{e^6}{112}-\&c.\right)$$

Ex.—The major axis being twenty feet, and the eccentricity four-tenths of a foot, what is the area of the ellipsoid?

Cal. $B=\sqrt{(100-16)}=9.165$ semi-conjugate axis, $4\times3.1416\times10\times9.165\times(1-.0167-.0064-\&c.)=$ 1113.59 square feet. Ans.

Prop. 36.—The convex surface of a hyperboloid is as follows:

$$\text{Area}=\frac{\pi B}{a}\left\{x\sqrt{(x^2-a^2)}-A\sqrt{(A^2-a^2)}\right.$$

$$\left.+a^2 log\ \left[\frac{A+\sqrt{(A^2-a^2)}}{x^2+\sqrt{(x^2-a^2)}}\right]\right\}$$

Remark.—In the above formula, (A) represents the transverse axis; (B) the conjugate axis; (x) an ordinate on the axis of revolution; and, (a) equal to the square root of $\frac{A^4}{A^2+B^2}$.

Ex.—The transverse axis (A) is four feet, the conjugate axis (B) three feet, and the hight six feet; what is the area of the hyperboloid?

Cal. $x=4+6=10$. $a=\frac{16}{5}$. $a^2=\frac{256}{25}$.

$\frac{15\times3.1416}{16}(94.742-9.6-11.397)=217.31$ square feet. Ans.

Prop. 37.—The solid contents of a prism, or of a cylinder, are equal to the area of the base multiplied by the altitude.

Ex.—The diameter of a cylinder is twenty-one inches and its length forty inches; what are the solid contents?

Cal. $21 \div 7 \times 22 = 66$; $66 \times 21 \div 4 = 346.5$
$346.5 \times 40 = 13860$ solid inches. Ans.

Prop. 38.—The solid contents of a pyramid, or of a cone, are equal to the base multiplied by one-third of the altitude.

Ex.—The square inches in the base of a cone are 346.5, and the hight forty inches; what are the solid contents?

Cal. $346.5 \times 40 \div 3 = 4620$ solid inches. Ans.

Prop. 39.—The solid contents of a frustum of a pyramid, or a cone, are equal to one-third of the altitude of the frustum, multiplied by the sum of the two bases, increased by a mean proportional between them.

Ex.—What is the solidity of the frustum of a cone whose lower diameter is ten inches, upper diameter eight inches, and hight twenty-four inches?

Cal. $10 \times 10 = 100$ $8 \times 8 = 64$ $10 \times 8 = 80$
$100 + 64 + 80 = 244$; $244 \times ,7584 \times 24 \div 3 = 1533.10$ solid inches. Ans.

Prop. 40.—The solidity of a sphere is equal to one-third of the area of a great circle, multiplied by the radius; or, it is equal to the cube of the diameter, multiplied by ,5236.

Ex.—What is the solidity of a ball fourteen inches in diameter?

Cal. 14×22÷7=44; 14×44×14÷6=1437.33 cubic inches. Ans.

Prop. 41.—The solidity of a spherical segment is equal to one-half the hight of the segment multiplied by the sum of the areas of the two basis; and this product increased by the solid contents of a sphere whose diameter is equal to the hight of the segment.

Remark.—If the segment has but one base, the other is to be regarded equal to 0 (zero).

Ex.—What is the solidity of a spherical segment, the diameter of the sphere being forty inches, and the distances from the center to the bases sixteen inches, and ten inches?

Cal. 20×20=400; 16×16=256; 10×10=100; 400—256=144; 400—100=300; 144+300=444; 444×4=1776; 1776×,7854=1394.8704; 1394,8704 ×3=4184.6112; 6×6×6×,5236=113.0976; 4184 ,6112+113.0976=4297.7088 solid inches. Ans,

Prop. 42.—The solidity of a regular polyedron is equal to the cube of one of its edges multiplied by the solidity of a similar polyedron whose edge is one.

Table of Regular Polyedrons whose edges are one.

Names.	No. of Faces.	Surface.	Solidity.
Tetraedron. . .	4	1.7320508	0.1178513
Hexaedron. . .	6	6.0000000	1.0000000
Octaedron. . .	8	3.4641016	0.4714045
Dodecaedron..	12	20.6457288	7.6631189
Icosaedron....	20	8.6602540	2.1816950

Ex.—What is the solidity of an icosaedron whose edge is twenty inches?

Cal. $20\times20\times20\times2.1816950=17453.56$ solid inches. Ans.

Prop. 43.—The solidity of a solid, generated by the revolution of the cycloid about its base, is equal to five-eighths of a cylinder whose length is equal to the base of the cycloid, and whose diameter is twice that of the generating circle.

Ex.—What is the solidity of a solid, generated by the revolution of a cycloid about its base, the diameter of whose generating circle is seven feet?

Cal. $7\times22\div7=22$ length of solid, $7\times2=14$ diameter, $7\times22=154$; $154\times22\times5\div8=2117.5$ solid feet. Ans.

Prop. 44.—The solidity of a paraboloid is equal to one-half of the solid contents of a cylinder of the same hight, and same base as the paraboloid.

Ex.—What are the solid contents of a paraboloid, the diameter of whose base is twenty-eight inches, and height forty inches?

Cal. $28\times22\div7=88$; $88\times28\div4=616$; $616\times40\div2=12320$ solid inches. Ans.

Prop. 45.—The solidity of a spheroid is equal to two-thirds the solidity of a circumscribing cylinder.

Ex. 1.—What is the solidity of an oblate spheroid whose major diameter is twenty-eight inches, and minor diameter twenty-one inches?

Cal. $(28\times88\div4)\times(21\times2\div3)=8624$ solid inches. Ans.

Ex. 2.—What is the solidity of a prolate spheroid, the diameters being as in example 1.

Cal. $(21\times66\div4)\times(28\times2\div3)=6468$ solid inches. Ans.

Prop. 46.—The solidity of an hyperboloid is as follows:

$$\text{Solidity.}=\frac{11B^2}{A^2}\left(\frac{x^3+2A^3}{3}-A^2x\right)$$

Remark.—In this formula the transverse and conjugate axes are respectively represented by (A) and (B), and the ordinate on the axis of revolution by (x).

Ex.—What is the solidity of an hyperboloid whose transverse axis is four feet, conjugate axis three feet, and hight six feet?

Cal. $6+4=10$; $3.1416\times9\div16=1.76715$; $1000+128=1128$; $1128\div3=376$; $376-160=216$; $1.76715\times216=381.7$ solid feet. Ans.

Remark.—The relations of the co-ordinates to each other in the tractory curve, the length of the curve, the quadrature of the meridian plane coinciding with the axis, the surface of revolution of the tractory conoid; and the solid contents of that solid will be found under the heading "Discussion of the Tractory and differently formed Grinding Plates."

INVOLUTION.

Involution is the raising of quantities to any proposed power.

The power of any quantity is that quantity multiplied any number of times by itself.

Thus, $3\times3=9$ is the second power of 3;
$5\times5\times5=125$ is the third power of 5.

The power is sometimes expressed by writing the quantity with the number of the power a little above it, and at the right hand.

Thus, to express the power, write

$3^2=3\times3=9$ square of 3.

$5^3=5\times5\times5=125$ cube of 5.

$2^7=2\times2\times2\times2\times2\times2\times2=128$ seventh power of 2.

The number of the power, as 2, 3, 7 above, is termed the exponent or index.

EVOLUTION.

Evolution is the extracting of roots.

The root of any quantity is a quantity which, if multiplied by itself a certain number of times, produces the original quantity; and is called the second or square root, the third or cube root, etc., according to the number of multiplications.

Thus the second or square root of 9 is a quantity whose square or second power produces 9—that is 3. And the seventh root of 128 is a quantity whose seventh power produces 128—that is 2.

Roots are sometimes represented by the symbol $\sqrt{}$, with the number of the root written within the angle of the symbol, and sometimes by exponents or indices.

Thus the square root is represented

$\sqrt[2]{9}=9^{\frac{1}{2}}=3$ and is read square root or half power of

$\sqrt[3]{125}=125^{\frac{1}{3}}=5$ and is read cube root or one-third power of 125.

$\sqrt[7]{128}=128^{\frac{1}{7}}=2$ and is read seventh root or one-seventh power of 128.

The number 2 within the angle of the symbol in expressing the square root, is usually omitted.

Both *involution* and *evolution* are sometimes expressed by fractional exponents or indices.

Thus $32^{\frac{3}{5}}$ ·The numerator shows that the quantity is to be raised to the third power, $32\times32\times32=32768$, and the denominator shows that the fifth root of that power is to be extracted—$\sqrt[5]{32768}=32768^{\frac{1}{5}}=8$.

Or the denominator shows that the fifth root of the quantity is to be extracted $\sqrt[5]{32}=32^{\frac{1}{5}}=2$.

And the numerator shows that the root thus obtained is to be cubed or raised to the third power, $2\times2\times2=8$.

TO EXTRACT ANY ROOT OF A POWER OR QUALITY.

Rule.—1st. Point off the given power or quantity into periods, containing each, except the left-hand period, as many figures as the required root indicates, beginning at the units place and pointing to the left in integers and to the right in decimals.

2d. Find by trial the first figure of the root and set it to the right of the quantity or power in the quotient's or root's place. Also place its first power at the head of a column on the extreme left, and its successive higher powers (regularly increasing the exponents by one) at the heads of columns following in order toward the right; thus forming as many columns as there are units in the exponent of the power whose root is sought. The highest power of the first root figure falls under the left-hand period of the quantity, and is to be subtracted therefrom, and with the remainder the next period is to be brought down.

3d. Add the root figure to the first column, and multiply this sum by the root figure, placing the product in the next right hand column and adding it thereto. Multiply this sum by the root figure, placing the product in the next right hand column, which add thereto, and thus proceed adding and multiplying until the number of additions shall be one less than the exponent of the number whose root is sought. The number thus found is termed the trial divisor. The root figure is to be added to the first column (the successive multiplications and additions following as above, except that one multiplication less is made each time) as many times as there are units in the exponent of the number whose root is sought.

4th. Find how many times the trial divisor is contained in the remainder, with the first left hand figure of the next period brought down, and place the quotient as a second root figure. Also add this root figure, removed one place to the right, to the first column. Multiply the sum thus obtained by this root figure, adding the product, removed two places to the right, to the second column. Thus continue to multiply and add to the successive columns, removing the product at each addition one figure further to the right, until the product, falling under the quantity whose root is sought, is to be subtracted therefrom, and the remainder with the next period brought down as before.

5th. In a similar manner the successive figures of the root are determined.

Example 1*st.* Required the side of a square inclosure containing 55225 square rods. Ans. 235 rods.

		′ ′ ′
Cal.		55225 [235
	2	4
	2	——
	—	152
	43	129
	3	——
	——	2325
	465	2325

Ex. 2*d.* Required the side of a cubical reservoir containing 9663597 solid inches. Ans. 213 inches.

		′ ′ ′
Cal.		9663597 [213
2	4	8
2	8	——
–	——	1663
4	1261	1261
2	62	——
—	——	402597
61	1323	402597
1	1899	
—	——	
62	134199	
1		
——		
633		

Ex. 3d. Required the fifth root of 6436343. Ans. 23.

Cal.				6436343 [23
2	4	8	16	32
2	8	24	64	
—	—	—	—	———
4	12	32	80	3236343
2	12	48	278781	3236343
—	—	—	———	
6	24	80	1078781	
2	16	12 927		
—	—	———		
8	40	92927		
2	309			
——	——			
10 3	4309			

DISCUSSION OF THE TRACTORY AND DIFFERENTLY FORMED GRINDING PLATES.

The discovery of the *Tractory Curve* was long attributed to Huygens, an eminent mathematician of the seventeenth century. That the curve, however, was known long prior to the discovery of Huygens, is proven by recent excavations in the ruins of Pompeii, where has been found a mill whose grinding surfaces are of tractory conoidal form.

What properties of the curve Huygens may have investigated does not appear. D'Alembert says that the evolute of the curve is the common catenary. Dr. Peacock says that the *Tractory* is an inverted semicycloid. As the term cycloid here used is not qualified, it must be inferred that the common is meant. Now, D'Alembert and Dr. Peacock cannot both be right; for the involute of the common cycloid is similar and equal to the evolute, that is, the cycloid itself. But the cycloid is essentially dissimilar and unequal to the catenary. Therefore, the cycloid and the involute of the catenary cannot both coincide with, or be the *Tractory.*

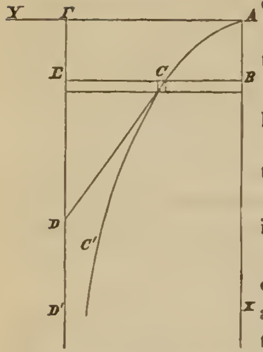
Again, as the equations of the *Tractory*, *Catenary* and *cycloid*, are essentially different, it is evident that neither D'Alembert, nor Dr. Peacock has correctly demonstrated and set forth the properties of the *Ttractory*.

The discrepancy of the above named authors, and the unsatisfactory and unreliable manner of laying down the curve mechanically, as adopted by C. Schiele, of Oldham, have led to the following discussion, the correctness of the result of which is not only certified to by several distinguished mathematicians, but is farther confirmed by numerous and carefully made experiments.

A tractory is defined to be "A curve whose tangent is always equal to a given line."

The directrix of the tractory here investigated is parallel with one of the axes, and the point of origin of the curve is in a line at right angles to the directrix.

Fig.1.



Let A Fig. 1 be the point of origin.

Lay off on the axis AY the constant tangent $AF=a$.

Draw the directrix FD′ parallel with the axis AX.

Draw $CD=a$ tangent to the curve ACC′ at the point C

Produce BC in a right line intersecting FD in E.

Let x and y be co-ordinates of the curve at any point C, and let z denote the length of the curve AC.

Then $CE=BE-BC=a-y$, and $ED=(\overline{CD}^2-\overline{CE}^2)^{\frac{1}{2}}$

$$=(2ay-y^2)^{\frac{1}{2}}.$$

1. By similar triangles $\quad dx=\dfrac{(2ay-y^2)^{\frac{1}{2}}}{a-y}dy$

2. By similar triangles $\quad dz=\dfrac{ady}{a-y}$

1_1. Putting $\quad u=(2ay-y^2)^{\frac{1}{2}}$

2_1. Squaring Eq. 1_1, $\quad u^2=\ 2ay-y^2$

3_1. Differentiating Eq. 2_1, $\quad 2udu=2ady-2ydy$

4_1. Dividing Eq. 3_1, by $2(a-y)$ $\quad dy=\dfrac{udu}{a-y}$

5_1. Transposing Eq. 2_1, $\quad y^2-2ay=-u^2$

6_1. Completing square to Eq. 5_1, $y^2-2ay+a^2=a^2-u^2$

7_1. Extracting sq. root Eq. 6_1, $y-a=\pm\sqrt{(a^2-u^2)}$

8_1. Or, $a-y=\pm\sqrt{(a^2-u^2)}$

3. Transposing Eq. 7_1, $y=a\pm\sqrt{(a^2-u^2)}$

9_1. Substituting value of $(a-y)$ Eq. 8_1, in Eq. 4_1,

$$dy=\frac{udu}{\pm\sqrt{(a^2-u^2)}}$$

4. Substituting values of $\sqrt{(2ay-y^2)}$ Eq. 1_1, $(a-y)$ Eq. 8_1, and dy Eq. 4_1, in Eq. 1,

$$dx=\frac{u^2du}{a^2-u^2}$$

5. Integrating Eq. 4, $x=\int\frac{u^2du}{a^2-u^2}$

1_2. By division $\frac{u^2}{a^2-u^2}=-1+\frac{a^2}{a^2-u^2}$

2_2. Decomposing $\frac{a^2}{a^2-u^2}=\frac{A}{a-u}+\frac{B}{a+u}$

3_2. Reducing to common denominator

$$\frac{a^2}{a^2-u^2}=\frac{Aa+Au+Ba-Bu}{a^2-u^2}$$

4_2. Clearing Eq. 3_2 of fractions

$$a^2=(A+B)a+(A-B)u$$

5_2. Transposing Eq. 4_2 $0=(A+B)a+(A-B)u-a^2$

6_2. Eq. 4_2 being true for all values of u, is true when $u=0$; hence $A-B=0$;

7_2. And $A+B=a$

8_2. Adding Eqs. 6_2 and 7_2; and dividing by 2, $A=\frac{a}{2}$

9_2. Subtraeting Eq. 6_2 from Eq. 7_2; dividing by 2, $B=\frac{a}{2}$

10_2. Substituting values of A and B

in Eq. 2_2 $\frac{a^2}{a_2-u_2}=\frac{a}{2(a-u)}+\frac{a}{2(a+u)}$

11_2. Substituting value of

$\frac{a^2}{a^2-u^2}$ in Eq. 1_2 $\frac{u^2}{a^2-u^2}=-1+\frac{a}{2(a-u)}+\frac{a}{2(a+u)}$

6. Substituting value of

$\frac{u^2}{a^2-u^2}$ in Eq. 5; $x=\int -du+\int\frac{adu}{2(a-u)}+\int\frac{adu}{2(a+u)}$

$$=-u-\frac{a}{2}l(a-u)+\frac{a}{2}l(a+u)+C_1$$

When $x=0$, $u=0$ also $C=0$;

7. henee, $x=-u-\frac{a}{2}l(a-u)+\frac{a}{2}l(a+u)$

8. whieh may be rendered $x=-u+\frac{a}{2}l\left(\frac{a+u}{a-u}\right)$

9. Integrating Eq. 2, $z=\int\frac{ady}{a-y}=-al(a-y)+C$

When $y=0$, $z=0$ and $C=ala$

10. hence, $z = ala - al(a - y)$

11. which may be rendered $z = al\left(\frac{a}{a-y}\right)$

l, in the above calculations denotes the Naperian logarithm.

TO FIND THE AREA OF THE MERIDIAN PLANE COINCIDING WITH THE AXIS OF REVOLUTION OF THE TRACTORY CONOID.

$(b)_0$. Differential Equation, $d(\text{area}) = ydx$

$(b)_1$. Substituting value (dx) Eq. 1; $ydx = \sqrt{(2ay - y^2)}dy$

$(b)_2$. Putting $y = a - n$; $ydx = -\sqrt{(a^2 - n^2)}dn$

$(b)_3$. Decomposing and integrating,

$$\int ydx = -\int a^2(a^2 - n^2)^{-\frac{1}{2}}dn + \int n^2(a^2 - n^2)^{-\frac{1}{2}}dn$$

$(b)_4$. Second term of Eq. $(b)_3$;

$$\int n^2(a^2 - n^2)^{-\frac{1}{2}}dn = -n\sqrt{(a^2 - n^2)} + \int dn\sqrt{(a^2 - n^2)}$$

$(b)_5$. Substituting in Eq. $(b)^3$, $-\int(a^2 - n^2)^{-\frac{1}{2}}dn =$

$$-\int a^2(a^2 - n^2)^{-\frac{1}{2}}dn - n(a^2 - n^2)^{\frac{1}{2}} + \int(a^2 - n^2)^{\frac{1}{2}}dn$$

$(b)_6$. Transposing,

$$-\int(a^2 - n^2)^{-\frac{1}{2}}dn = \frac{a^2}{2}\int\frac{-dn}{\sqrt{(a^2 - n^2)}} - \frac{n}{2}\sqrt{(a^2 - n^2)}$$

$(b)_7$. But, $\frac{a^2}{2}\int\frac{-dn}{\sqrt{(a^2-n^2)}}=\frac{a^2}{2}\cos^{-1}\left(\frac{n}{a}\right)$

$(b)_8$. Hence,

$$-\int(a^2-n^2)^{\frac{1}{2}}dn=\frac{a^2}{2}\cos^{-1}\left(\frac{n}{a}\right)-\frac{n}{2}\sqrt{(a^2-n^2)}+C$$

$(b)_9$. Restoring value of n, $\int\sqrt{(2ay-y^2)}dy=$

$$\frac{a^2}{2}\cos^{-1}\left(\frac{a-y}{a}\right)-\frac{a-y}{2}\sqrt{(2ay-y^2)}+C$$

Making $y=0$; Then $C=0$.

$(b)_{10}$. Hence, $\int\sqrt{(2ay-y^2)}dy=$

$$\frac{a^2}{2}\cos^{-1}\left(\frac{a-y}{a}\right)-\frac{a-y}{a}\sqrt{(2ay-y^2)}$$

$(b)_{11}$. Making $y=a$. Area $=\frac{a^2\cos^{-1}}{2}(0)=\frac{a^2\times 2\pi}{2\times 4}=\frac{a^2\pi}{4}$

$(b)_{12}$. Making $y=2a$. Area $=\frac{a^2}{2}\cos^{-1}(-1)=\frac{a^2}{2}\times\frac{2\pi}{2}=\frac{a^2\pi}{2}$

Making $y=\frac{4a}{10}$, as adopted in one of the inventions of Wheeler & Randall, viz: in their grinder and amalgamator having the greater base of its muller upward. Making $a=1$, or unity.

Area $=\frac{1}{2}\cos^{-1}(,6)-\frac{6}{2}(,8)=,2232$ half plane of muller.

Hence, area $=,4464$ entire plane of muller.

TO FIND THE SURFACE OF REVOLUTION OF THE TRACTORY CONOID.

$(c)_1$. Differential Equation, $d(\text{surface}) = (a-y)dz$

$(c)_2$. Substituting value of dz Eq. 2, $ds = 2\pi a dy$

$(c)_3$. Integrating $s = 2\pi a y$

$(c)_4$. Making $y = a$ $s = 2\pi a^2$

Making $y=,4$, as adopted in that grinder and amalgamator of Wheeler & Randall, having its greater base upward.

$(c)_5$. Surface of revolution $= s = 2,5028$

TO FIND THE SOLID CONTENTS OF THE TRACTORY CONOID.

e_1. Differential Equation, d (solid contents) $= \pi(a-y)^2 dx$

e_2. Substituting val. dx. eq. 1,

$$dC_0 = \pi(a-y)\sqrt{(2ay-y^2)}dy$$

e_3. Put $a-y=,n$ then $y=a-n$

e_4. $dC_0 = -\pi a\sqrt{(a^2-n^2)}dn$

e_5. Put $s=a^2-n^2$, then $ds=-2ndn$, and $\frac{ds}{2} = -ndn$

e_6. Substituting in Eq. e_4 $dC_0 = \frac{\pi s^{\frac{1}{2}} ds}{2}$

e_7. Integrating Eq. e_6 $C_0 = \frac{\pi s^{\frac{3}{2}}}{3}$

e_8. Restoring (a^2-n^2) for s, $C_0=\frac{11(a^2-n^2)^{\frac{3}{2}}}{3}$

e_9. Restoring $(a-y)$ for n $C_0=\frac{11(2ay-y^2)^{\frac{3}{2}}}{3}$

e_{10}. Making $y=a$ $C_0=\frac{11a^3}{3}$

e_{11}. Making $y=\frac{4a}{10}=$,4 when $a=$unity $C_0=$,5362

The following table is computed by making the tangent $a=1$, and assigning different values to u, not greater than a as shown in column marked u. That is, these values are substituted in equations 8, 3 and 11, and the co-ordinate values of x and y, and the length of the tractory curve z, thus determined in terms of the tangent or unity :

TRACTORY TABLE.

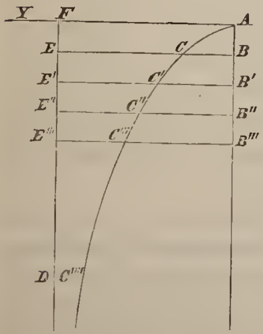
$a=1$									
No	u	x	y	z	No	u	x	y	z
1	,100	,000336	,005014	005027	14	,725	193102	311251	368505
2	,150	,001136	,011316	011379	15	,750	222956	338562	413293
3	,200	,002732	,020205	029412	16	775	257730	368039	458928
4	,250	,005415	,031756	032260	17	,800	298611	400000	510827
5	,300	,009514	,046063	047157	18	,825	347276	434867	570796
6	,350	,015454	,063252	065571	19	,850	406153	473297	641118
7	,400	,023648	,083485	087178	20	,875	479027	515878	725418
8	,450	,034699	,106972	113137	21	,900	572220	564109	830367
9	,500	,049304	,133976	143842	22	,925	697603	620033	967651
10	550	,068381	,164835	180127	23	,950	881785	687751	1,161903
11	,600	,093146	,200000	223144	24	,975	1,209723	777795	1,504152
12	,650	,125296	,240065	274523	25	,990	1,656677	858933	1,958518
13	,700	,167304	,285857	336672	26	,999	2,801193	955289	3,107543

TO CONSTRUCT THE TRACTORY BY THE PRECEDING TABLE.

Draw the rectangular co-ordinate axes AX, AY, Figure 2.

On AY, lay off AF, equal to the given tangent, and draw the directrix FD, parallel with the axis AX, through the point F.

Fig. 2.



Multiply any tabular quantity in column x by the given tangent, and lay off the product as AB or AX. Draw BE parallel with AF.

Multiply the quantity in column y, horizontal with the quantity taken in column x, by the given tangent, and lay off the product as BC on the line BE. The point C is in the curve.

In a similar manner determine the points C′, C″, C‴, C⁗, etc., etc., in the curve.

Multiply the quantity in column z horizontal with the quantities thus taken in column x and y, and the product will be the length of the curve A C, A C C′, etc., etc.

When the points A, C, C', C'', C''', etc., are laid down, the arc of a circle drawn through any three of them taken consecutively, approximates the true curve sufficiently near for most practical purposes.

Ex.—Let it be required to find the values of x, y, and z, when the tangent $a = 24$ for the point C'.

Now since C' may be any point of the curve, let the coordinate x be taken in the table opposite No. 17.

Then, $24\,x = AB' = 24 \times .2[illegible] = [illegible]$

$24\,y = B'C' = 24 \times [illegible] = [illegible]$

$24\,z = ACC' = 24 \times [illegible] = 12.2[illegible]$

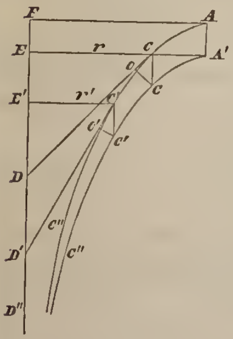
Laws of Grinding.—1. The measure of a unit of grinding effect is the result of a body impressing a unit surface of another body with a unit pressure, and moving over a unit space.

2. A body impressing a unit surface of another body with a unit pressure, and moving under these circumstances over a unit space, producing thereby a unit of grinding effect, (the pressure being uniform and the square of the surfaces of contact being the same) produces as many units of effect as there are units in the distance passed over.

3. The wear or abrasion in the moving body perpendicular to the surfaces of contact, bears a constant ratio to the distance passed over.

To prove that to grinding plates coinciding, of uniform hardness, and of tractory conoidal form, the wear parallel with the axis of revolution is the same at all points of the grinding surface.

Fig. 3.



Let A F D″ C″ represent one half of a meridian plane of a tractory conoid taken through the axis or directrix F D″.

Draw C D $= a$, and C′ D′ $= a$ each tangent to the curve at any points C, C′; also draw the radii C E $= r$ and C′ E′ $= r'$ parallel with A F.

Let $w = o\,c$ denote the wear perpendicular to C D, and $w' = o'\,c'$ the wear perpendicular to C′ D′.

Let P $=$ C c and P′ $=$ C′ c' denote the wear respectively at the points C, C′ and each parallel with the axis F D″.

1_3. Then by law 2d, $$w' = \frac{w\,r'}{r}$$

2_3. By similar triangles, $$P = \frac{a\,w}{r}$$

3_3. By similar triangles, $$P' = \frac{a\,w'}{r'}$$

4, Substituting value of w' Eq. 1, in Eq. 3, $P = \frac{a\,w\,r'}{r\,r'}$.

5, Reducing 2d member of Eq. 4, $P' = \frac{a\,w}{r}$

6, Substituting value of $\frac{a\,w}{r}$ in Eq. 2, $P' = P$

7, That is, $C'\,c' = C\,c$

(*a*) Hence, as the wear is the same at any two points as shown by Eqs. 6, and 7, it is the same at all points parallel with the axis, of revolution.

(*b*) And hence the curve $A'\,c\,c'\,c''$ is similar and equal to the tractory $A\ C\ C'\ C''$.

Again, plates whose grinding surfaces are of tractory conoidal form, alone passes the property of uniform wear parallel with the axis of revolution.

To prove this proposition
Put (Fig. 3) $a' = C'\,D'$

8, Then Eq. 5, becomes $P' = \frac{a'\,w}{r}$

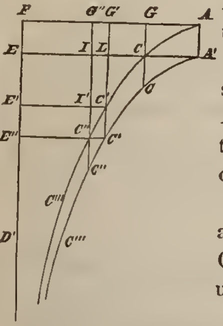
9, From Eq. 2, and 8, we have $P : P' :: \frac{a\,w}{r} : \frac{a'\,w}{r}$

10, Reducing 2d couplet $P : P' :: a : a'$

Proportion 10, shows that the wear P is equal to P' only when tangent a is equal to a'; but when a is equal to a' the line or curve is a tractory. Q. E. D.

TO DETERMINE THE SOLIDITY OF A HOLLOW TRACTORY CONOID.

Fig. 4.



Let $AFD'C'''$ represent one-half of a meridian plane taken through the axis $F\,D'$.

Draw the curve $A'\,c\,c'\,c''\,c'''$ similar and equal to the tractory $A\,C\,C'\,C''\,C'''$. and at the distance $A\,A'$, $C''\,c''$ from it. (See conclusion (*b*).

Draw $A'\,E$, $C'\,E'$, $c'\,E''$, parallel with $A\,F$, also $G\,c$ through C, $G'\,c'$, through C', $G''\,c''$, through C'' parallel with $F\,D'$.

Since the curve $c'\,c''$ is similar and equal to $C'\,C''$, and the line $C'\,c' = C''\,c''$, and $C'\,I' = c'\,C''$, and the angle $c'\,C''\,c'' =$ the angle $C'\,I'\,C''$; it follows that the triangle $c''\,c'\,C'' = C''\,C'\,I'$, and also similar to it.

And since the conoidal triangle $C''\,c'\,C'$ is common to the parallelogram $C''\,c'\,C'\,I'$, and to the conoidal section $c''\,c'\,C'\,C''$, it follows that the conoidal section $c''\,c'\,C'\,C''$ is equal to the parallelogram $C''\,c'\,C'\,I'$.

Again, since $c'\,C' = A'\,A = L\,G'$, and $C''\,c = I\,L$, it follows that the parallelogram $c'\,I' =$ the parallelogram $L\,G''$. Hence the conoidal section $c''\,c'\,C'\,C'' =$ the parallelogram $L\,G''$; and since its position, in all respects, is at the same distance from the axis $F\,D'$, it is evident, if it be revolved about F D′ as an axis, it will generate a conoidal ring of the same magnitude as a ring generated by revolving L G″ about the same axis.

In the same manner may it be shown that any conoidal ring, under similar circumstances, is equal to the ring generated by revolving the corresponding parallelogram taken in A′ A F E around the axis F D′.

Hence the solidity of a hollow tractory conoidal is equal to the solidity of a cylinder having the same base and same height as A′ A, c'' C″, measured parallel with the axis of revolution.

Tractory Plates.—To determine the *grinding effect* of hollow tractory conoidal plates of uniform hardness.

Let, as in Figs. 1 and 2, the tangent or greater radius $= a$.

Let x and y be co-ordinates of any point, C, etc.

Let z denote the length of curve A C, etc.

Let P denote the wear parallel with the axis of revolution, by one revolution under a unit pressure to a unit surface.

Let S denote the grinding surface.

Let π denote the ratio of the diameter to the circumference of a circle.

1_4. Then, $ds = 2\pi(a-y)dz$

2_4. Substituting value of dz of Eq. 2,

in Eq. 1_4, $ds = 2\pi a dy$

3_4. Then, by one revolution,

$$2\pi P(a-y)ds = 4\pi^2 Pa^2dy - 4\pi^2 Paydy$$

4_4. Integrating Eq. 3_4,

$$\int 2\pi P(a-y)ds = 4\pi^2 Pay - 2\pi^2 Pay^2$$

5_4. Resolving 2d member of Eq. 4_4 into factors;

$$\int 2\pi P(a-y)ds = 2\pi^2 Pa(2ay - y^2)$$

6_4. By making $y=a$ in Eq. 5_4;

$$\text{Grinding effect} = 2\pi^2 Pa^3$$

Plane Plates.—To determine the grinding effect of plane circular plates, increasing in hardness from the center to the circumference in the ratio of the increase of the radius.

Let $a =$ the radius of the plates.

Let $y =$ any radius of the plates less than a.

Let $P =$ the wear at the circumference, perpendicular to the grinding surface, under a unit pressure to the unit surface, by one revolution.

Let $\pi =$ ratio of diameter to the circumference of a circle.

1_5. Then $ds = 2\pi ydy$

2_5. By one revolution, $2\pi Pyds = 4\pi^2 Py^2dy$

3_5. Integrating Eq. 2_5, $\int 2\pi Pyds = \frac{4}{3}\pi^2 Py^3$

4_5. By making $y=a$ in Eq. 3_5;

$$\text{Grinding effect} = \frac{4}{3}\pi^2 Pa^3$$

Plane Plates.—To determine the grinding effect of plane circular plates of uniform hardness.

Let $a =$ the radius of the plates.

Let $y =$ any radius less than a.

Let $P =$ the tendency to wear at the circumference, perpendicular to the grinding surfaces.

Let $S =$ the surface of which y is the radius.

Let $\pi =$ ratio, etc.

1_6. Then $$ds = 2\,\pi\,y\,dy$$

Now, it is evident that when the tendency to wear at the circumference and at the distance a is P, that the tendency to wear at the distance y from the centre will be $\frac{Py}{a}$

2_6. Then by one revolution $\frac{2\pi^2 P}{a} y ds = \frac{4\pi^2 P y^2 dy}{a}$

3_6. Integrating Eq. 2_6 $\int \frac{2\pi^2 P}{a} y ds = \frac{4\pi^2 P y^4}{4a}$

4_6. Reducing 2d member of Eq. 3_6 —;

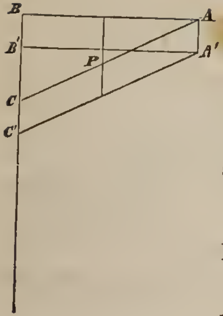
$$\int \frac{2\pi^2 P}{a} y ds = \frac{\pi^2 P y^4}{a}$$

5_6. By making $y = a$ in Eq. 4_6,

$$\text{Grinding effect} = \pi^2 P a^3$$

Conical Plates.—To determine the grinding effect of hollow conical plates of uniform hardness.

Fig. 5.



Let, in Fig. 5, the right angled triangles A B C and A′ B′ C′, similar and equal, and at the distance A A′ = C C′ apart, be revolved about the axis B C′.

Then will the solid generated by A A′ B B′ be equal to the solid generated by C′ A′ A C.

Let a = the radius B A.

Let h = height of cone B C = B′ C′.

Let P = tendency to wear at A, parallel with the axis of revolution, by one revolution of the plate under a unit pressure to a unit surface.

Let x and y be co-ordinates of any point P.

Let z = C P the slant height of that cone the radius of whose base is y.

Let S = the surface whose slant height is z.

Let Π = ratio, etc.

1_7. Then by similar triangles $x = \frac{h\,y}{a}$

2_7. And (Euclid, Book 1, Prop. 47) $x^2 + y^2 = z^2$

3_7. Squaring Eq. 1_7 $x^2 = \frac{h^2\,y^2}{a}$

4_7. Substituting value of x^2 of Eq. 3_7 in Eq. 2_7

$$z^2 = \left(\frac{h^2}{a^2} + 1\right) y^2$$

5_7. Extracting square root of Eq. 4_7

$$z = \sqrt{\left(\frac{h^2}{a^2} + 1\right)}\, y$$

6_7. Surface of cone whose slant height is z

$$S = \pi \sqrt{\left(\frac{h^2}{a^2} + 1\right)}\, y^2$$

7_7. Differentiating Eq. 6_{10} $ds = 2\pi \sqrt{\left(\frac{h^2}{a^2} + 1\right)}\, y\, dy$

When the tendency to wear at A is P, it is evident that the tendency to wear at P is $\frac{P\,y}{a}$, parallel with the axis.

8_7. Hence differential of effect

$$\frac{2\pi P y^2}{a}\, ds = \frac{4\pi^2 P \sqrt{}}{a}\left(\frac{h^2}{a^2} + 1\right) y^2\, dy$$

9_7. Integrating Eq. 8_7 $\int \frac{2\pi P y^2}{a}\, ds = \frac{\pi^2 P}{a}\left(\frac{h^2}{a^3} + 1\right) y^4$

10_7. By making $y = a$ in Eq. 9_7.

$$\text{Grinding effect} = \pi^2 P \sqrt{\left(\frac{h^2}{a^2} + 1\right)}\, a^3$$

11_7. By making $h = \frac{a}{2}$ in Eq. 10_7.

$$\text{Grinding effect} = \frac{\pi^2 P a^3}{2} \sqrt{5}$$

12_7. Removing surd sign from 2d member of Eq. 11_7. Grinding effect $= 1.118\, \pi^2 P a^3$

Comparison.—Let a comparison now be instituted between the grinding effects of *plane circular plates*, increasing in hardness from the center to the circumference in the ratio of the increase of the radius, plane circular plates of uniform hardness, "Randall's Patent Grinding Plates," conical plates of uniform hardness, and tractory conoidal plates, and all of the usual ring, form, same diameter, same weight, and running at the same velocity.

1st. Plane circular plates, increasing in hardness from the center to the circumference in the ratio of the increase of the diameter.

Making y less than a. For example, $y = \frac{a}{3}$, and substituting this value for y in Eq. 3_5, and we have

1_8. Grinding effect (the radius being $\frac{a}{3}$) $= \frac{4}{27} \Pi^2 P a^3$

2_8. Subtracting Eq. 1_8 from Eq. 4_5.

$$\text{Grinding effect (ring)} = \frac{32}{27} \Pi^2 P a^3$$

3_8. Or expressing decimally.

$$\text{Grinding effect (ring)} = 1{,}1852\ \Pi^2 P a^3$$

2d. *Plane Circular Plates* of uniform hardness.

Making y less than a. For example, $y = \frac{a}{3}$, and substituting this value for y in Eq. 4_6, and we have

1_9. grinding effect (radius being $\frac{a}{3}$) $= \frac{\Pi^2 P a^3}{81}$

2_v Subtracting Eq. 1_s from Eq. 5_o.

$$\text{Grinding effect (ring)} = \frac{80\,\pi^2 P a^3}{81}$$

3_v Or expressing decimally.

$$\text{Grinding effect (ring)} = .9877\,\pi^2 P a^3$$

3d. *Randall's Patent Grinding Plates.*—These plates consist of two or more concentric rings of different hardness. The softer plates are arranged nearer the center where there is the less wear, and the harder plates more remote where the wear is greater. This arrangement remedies in a great measure the otherwise fatal defects of plane circular and conical plates.

For Example.—Let $a =$ the greater radius of the plates.

Let $\frac{a}{3} =$ the radius of the opening.

Let $\frac{2a}{3}$ the greater radius of the inner ring.

1st. Making $y = \frac{a}{3}$, and substituting this value for y in Eq. 4_o, and we have

1_w Grinding effect $\left(\text{radius } \frac{a}{3}\right) = \frac{\pi^2 P a^3}{81}$

2d. Making $y = \frac{2a}{3}$, and substituting this value for y in Eq. 4_o, and we have

2_w Grinding effect $\left(\text{radius } \frac{2a}{8}\right) = \frac{16\pi^2 P a^3}{81}$

3_{10}. Subtracting Eq. 1_{10} from Eq. 5_6;

$$\text{Grinding effect (ring)} = \frac{80}{81}\Pi^2 Pa^3$$

4_{10}. Subtracting Eq. 2_{10} from Eq. 5_6

$$\text{Grinding effect (ring)} = \frac{65}{81}\Pi^2 Pa^3$$

5_{10}. Comparative weight of the outer ring,

$$= a^2 - \frac{4a^2}{9} = \frac{5a^2}{9}$$

6_{10}. Comparative weight of entire plane, less the opening,

$$= a^2 - \frac{a^2}{9} = \frac{8a^2}{9}$$

7_{10}. Then $$\frac{5a^2}{9} : \frac{8a^2}{9} :: \frac{65\Pi^2 Pa^3}{81} : \frac{104\Pi^2 Pa^3}{81}$$

8_{10}. Or grinding effect of outer ring, $= \frac{104}{81}\Pi^2 Pa^3$

9_{10}. The loss sustained by plates of uniform hardness,

$$= \frac{104}{81}\Pi^2 Pa^3 - \frac{80}{81}\Pi^2 Pa^3 = \frac{24}{81}\Pi^2 Pa^2$$

Let it now be assumed that the inner plates or rings are one-third as hard as the outer plates.

10_{10}. See Eq. 9_{10}. Loss sustained,

$$= \frac{24}{81}\Pi^2 Pa^3 \div 3 = \frac{8\Pi^2 Pa^3}{81}$$

Subtracting Eq. 10_{10} from Eq. 8_{10}, gives for "*Randall's Patent Grinding Plates*, viz.:

$$\text{Grinding effect} = \frac{96\pi^2 Pa^3}{81}$$

12_{10}. Expressing decimally;

$$\text{Grinding effect} = 1.1852\pi^2 Pa^3.$$

Subtracting Eq. 2_9 from Eq. 11_{10}, and dividing the remainder by Eq. 2_9, gives the per centage of "Randall's Patent Grinding Plates," both over plane circular, and conical plates of uniform hardness, as follows:

13_{10}. Per centage in favor of "Randall's Patent Grinding Plates," $\left(\frac{96\pi^2 Pa^3}{81} - \frac{80\pi^2 Pa^3}{81}\right) \div \frac{81}{80\pi^2 Pa^3} = ,20$

4th. *Conical Plates* of uniform hardness and of ring form.

1_{11}. Making y less than a, for example $y = \frac{a}{3}$, and substituting this value for y in Eq. 9_7, and we have the grinding effect $\left(\text{the radius } \frac{a}{3}\right) = \frac{\pi^2 Pa^3}{81} \sqrt{\left(\frac{h^2}{a^2} + 1\right)}$

2_{11}. Subtracting Eq. 1_{11} from Eq. 10_7.

$$\text{Grinding effect (ring)} = \frac{80\pi^2 Pa^3}{81} \sqrt{\left(\frac{h^2}{a^2} + 1\right)}$$

3_{11}. Making $h = \frac{a}{2}$ in Eq. 2_{11}.

$$\text{Grinding effect (ring)} = \frac{80}{81}\pi^2 Pa^3 \sqrt{\left(\frac{5}{4}\right)}$$

4_{11} Expressing decimally.

Grinding effect (ring) $= 1{,}1042\ \text{II}^2\ P\ a^3$

5th. *Tractory Conoidal Plates* of uniform hardness.

Making y less than a. For example $y = \frac{2a}{5}$, as is the case in the Wheeler & Randall muller or grinding plates.

1_{12}. Substituting $\frac{2\,a}{5}$ for y in Eq. 5_4.

$$\text{Ring effect} = \frac{32\ \text{II}^2\ P\ a^3}{25}.$$

2_{12}. Making the tractory conoidal plate of the same weight or solidity as the plane circular or conical plates, and Eq. 1_{12} becomes Grinding effect (ring) $= \frac{16}{9}\ \text{II}^2\ P\ a^3$

3_{12}. Expressing decimally.

Grinding effect $= 1{,}7778\ \text{II}^2\ P\ a^3$

Recapitulation.—To express the relative grinding effects of the differently formed plates now considered, the literal factors $\text{II}^2 P\ a^3$, common to all their formulas, may be omitted.

Omitting the literal factors, and the relative grinding effects of differently formed plates become as follows, to wit:

1_{13}. Eq. 3_9.—Plane circular plates of uniform hardness $=$,9877.

2_{13}. Eq. 12_{10}.—Randall's Patent Grinding Plates = 1,1752.

3_{13}. Eq. 4_{11}.—Conical plates of uniform hardness = 1,1042.

4_{13}. Eq. 3_{12}.—Tractory conoidal plates of uniform hardness = 1,7778.

Hence the conclusion that *tractory conoidal plates* not only differ materially in form from *plane circular* and from *conical plates*, but also differ essentially from, and are greatly superior to them in their grinding properties.

PRORERTIES OF BODIES.

NAME.	Specific gravity at 32° fh.	Melt'ng points at fahr.	Rates of h'rd-ness.	Tenacity in lbs. per sqr. inch.	Crush'ng force in lbs. sq. in	Volatile at
Platinum	20.336	3280°	.....	....	......	very high ht.
do drawn wire	21.042	.....	.....	34000	......	very high ht.
do plates....	22.069	.....	.....	....	......	very hinh ht.
Gold, cast.....	19.258	2016	1.8	18000	......	moderate ht.
do hammered.	19.361	.....	.....	....	......	....
do wire......	......	.....	.....	28000	......	....
Silver, cast....	10.474	1873	2.4	36000	......	high heat.
do hammered	10.510	.....	.....	....	......	high heat.
do wire......	......	.....	.....	34000	......	high heat.
Copper, cast...	8.788	1996	2.8	17000	117000	high heat.
do hammered	8.910	.....	.....	30000	92000	high heat.
do sheets ...	......	.....	.....	42000	103000	
do wire	......	.....	.....	55000	......	high heat.
Tin, cast......	7.291	442	1.2	4000	15500	....
do hardened	7.291	.....	.....	....	......	....
do wire.....	......	.....	.....	6000	......	....
Zinc, cast.....	6.200	773	1.6	....	......	white heat.
do sheets....	7.191	.....	.....	....	......	....
Lead, cast.....	11.352	.....	.....	1600	7730	white heat.
do milled....	......	.....	.....	3000	......	....
do wire.....	......	.....	.....	2200	......	....
Mercury	13.598	—39	.....	....	......	680°
Iron, cast, gray.	7.248	2800	any	....	......	very high ht.
do do white.	7.500	.....	deg.	....	......	....
do forged....	7.788	.....	4.7	....	......	....
do wire, 1-20 to 1-30 in. diam	....	.....	...	120000 182000	......	....
Iron, wire, 1-10 inch diameter..	....	.....	...	72000 86000	......	....
Iron bars, Russian, mean....	....	.....	.....	54000	......	....
Iron, American gun metal.....	....	.....	....	....	147803	....
Iron, American gun metal, mean	....	.....	....	....	129000	....
Iron, English stirling	....	.....	....	....	119550	....
Iron, English stirling, mean..	....	.....	....	....	90833	....

PROPRTIES OF BODIES—CONTINUED.

NAME.	Specific gravity at 32° fh.	Melt'ng points at fahr.	Rates of hr'd-ness.	Tenacity in lbs. per sqr. inch.	Crush'ng force in lbs. sq in	Volatile at
Mica	2.650					
Quartz	2.624 3.750					
Serpentine	2.264					
Cinnabar	6 902					
Felspar	2.438					
Flint	2.582					
Ash	.845				8663	
Oak, American					4100	
do Canadian					5982	
do English	.920				6484	
Pine, Yellow					5375	
Walnut					6645	
Cedar					5768	
Elm	.671			12000	10331	
Fir	.600			10000		
Box	.862			18000		
Teak					12100	

MISCELLANEOUS.

5760 grains = 1 pound troy = 1 pound apothecary.

480 grains = 1 ounce troy = 1 ounce apothecary.

12 ounces = 1 pound troy = 1 pound apothecary.

7000 grains = 1 pound avoirdupois.

437,5 grains = 1 ounce avoirdupois.

16 ounces avoirdupois = 1 pound avoirdupois.

252,458 grains = 1 cubic inch distilled water, English standard 62° Fahr., Barometer at 30 inches.

252,693 grains = 1 cubie inch distilled water, U. S. standard 30.83° Fahr., Barometer at 30 inehes.

27,7015 cubic inches distilled water = 1 pound avoirdupois.

1 cubic foot distilled = 62,37929 pounds avoirdupois.

321 eubic inches = 8,3388822 pounds avoirdupois = 1 gallon U. S.

277,274 cubic inches = 10 pounds avoirdupois = 1 gallon Imperial.

2150.42 cubic inches = 77,627413 pounds avoirdupois = 1 bushel.

1 grain Gold, 1000 fine = $,0430663 Mint value.

1 grain Silver, 1000 fine = ,0026936 Mint value.

1 grain Copper, 1000 fine = ,0000595 Mint value.

1 ounce Gold, 1000 fine = 20,671791 Mint valuc.

1 ounce Silver, 1000 fine = 1,292929 Mint value.

1 ounce Copper, 1000 fine = ,028571 Mint value.

23.22 grains Gold, 1000 fine + 2.58 grains alloy = 25.8 grains = $1.00.

371.25 grains Silver, 1000 fine + 41.25 grains alloy = 412.5 grains = $1.00.

16800 grains Copper, 1000 fine = $1.00.

1 cubic inch Gold, 1000 fine = 10,12883 ounces troy = $209.38.

1 cubic inch Silver, 1000 fine = 5,50885 ounces troy = $7.13.

1 cubic inch Copper, 1000 fine = 4,62209 ounces troy = $0.133.

Gold and Silver, when pure, are said to be 1000 fine; or, by the old method, 24 carats fine.

The standard fineness of the United States Coin is 900; or, by the old method, 24 × ,900 = 21.6 carats fine.

GUNTER'S CHAIN.

7.92 inches = 1 link.

100 links = 4 rods = 1 chain.

5280 feet = 320 rods = 80 chains = 1 mile.

69.77 statute miles = 1 degree of a great circle of the earth.

160 square rods = 10 square chains = 1 acre.

640 acres = 1 square mile.

FRENCH WEIGHTS AND MEASURES.

1 Metre = 39.371 inches.

1 Are = 3.953 square rods.

1 Litre = 61.028 cubic inches.

1 Stere = 35,31714 cubic feet.

1 Gramme = 15,434 grains troy.

The Greek prefixes *Deca*, *Hecto*, *Chilo*, and *Myria*, respectively, signify 10 times, 100 times, 1000 times, and 10000 times.

And the Latin prefexes *Deci*, *Centi*, and *Milli*, respectively, 10th part, 100th part, and 1000th part.

Thus, 1 Deca-metre = 10 metres, and 1 metre = 10 Deci-meters.

Thus, 1 Chilo-gramme = 1000 grammes, etc.

1 Arroba (Mexican) = 25 pounds avoirdupois.

1 Fanega = 1,599 bushels, U. S., = 3438.52 cubic inches.

1 Marc or Marco = 8 ounces troy.

1 Vara = 33,384 inches.

25 cubic feet of sand = 1 ton.

18 cubic feet of earth = 1 ton.

17 cubic feet of clay = 1 ton.

13 cubic feet of quartz, unbroken in lode, = 1 ton.

18 cubic feet of gravel or earth, before digging, = 27 cubic feet when dug.

20 cubic feet of quartz, broken, (of ordinary fineness) = 1 ton, contract measurement.

OAKLAND, August 27, 1864.

Mr. Randall—SIR :—I have carefully examined your demonstration of the Tractory Curve, and of the grinding effects of differently formed plates, and find your calculations correct.

Yours, etc., FRANCIS D. HODGSON,

Prof. Math. College of California.

SAN FRANCISCO, September 7, 1864.

Mr. M. P. Randall—DEAR SIR :—I have to thank you for the opportunity of inspecting the drawing and model of your and Mr. Wheeler's new form of grinding and amalgamating apparatus, in which you have adopted the Tractory Conoid as the form of the grinding surfaces.

Your mathematical demonstration of the mechanical properties of this curve is, so far as I am informed, original and very interesting, and satisfies perfectly the practical requirements of the problem. The Tractory Conoid is a solid the nature of whose curve is as different from that of the surface of a cone as is a cycloid from an inclined plane.

Your mathematical analysis of the problem of uniform grinding, by tractoroidal surfaces, is extremely interesting, and furnishes a fine illustration of the value of this method of discussion applied to a case which at first sight would seem to be completely beyond the reach of such subtle and exact tests. The practical value of this discussion and of the results which it appears to sustain, are such as commend it to the serious attention of all who are interested in the development of the resources of the Pacific coast in the precious metals.

Yours, respectfully,

B. SILLIMAN, JR.

SAN FRANCISCO, May 4, 1865.

GENTLEMEN:—Having made a careful and critical examination of your "Quartz Operator's Hand Book," it is with extreme pleasure that I certify to the correctness of your statements and deductions. It bears the impress of extensive research and thorough investigation. Your discussions of all the various subjects are remarkably clear, concise and rigidly exact; but permit me more especially to congratulate you upon your masterly discussion of the *Tractory and the grinding effects of differently formed plates*—a subject practically of the highest importance to every quartz miner.

With sentiments of high regard, I remain,

Yours, truly,

W. R. ECKART, JR.,

Engineer (late of) U. S. N.

To Messrs. Wheeler & Randall.

San Francisco, May 29, 1865.

Messrs. Wheeler & Randall:—Having examined your "Quartz Operator's Hand Book," I take pleasure in recommending it to miners and millmen, as a work likely to be of great use in properly understanding the nature of their ores, and consequently the treatment necessary to produce favorable results.

Respectfully, yours, etc.,

W. M. Belshaw,

Assayer, and Sup't. of the S. T. M. Co.

INDEX.

F4

ERRATA.

17th Page, 26th line.—After the word "acid," add *dilute and filter.*

28th page, 17th line.—For "chloride" read *chlorine.*

28th	"	19th	"	Add the word *filled* after "cisterns."
50th	"	25th	"	For "Jouval" read *Jonval.*
53d	"	2d	"	After the word "feet" add *the head being 2 feet 3 inches.*
89th	"	5th	"	For "Quality," read *Quantity.*
93d	"	13th	"	For "Catenary," read *Involute of the Catenary.*
101st	"	11th	"	For "or AX," read *on AX.*
102d	"	23d	"	For "distruction," read *destruction.*
102d	"	10th	"	For "uniie," read *unit.*
104th	"	12th	"	For "passes," read *possess.*
106th	"	9th	"	For "Conoidal," read *Conoid.*

F6


M
N
A
H
K
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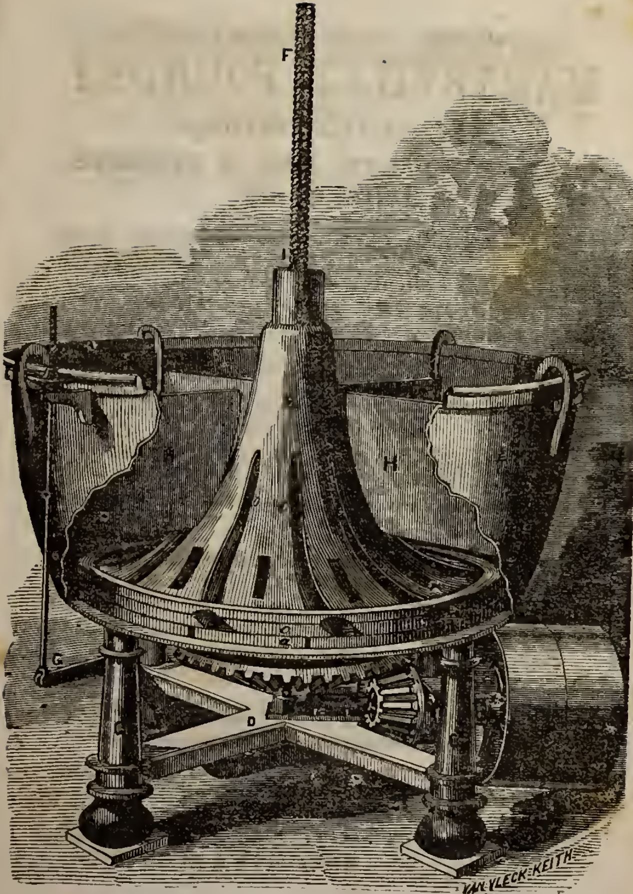
B
VAN VLECK-KEITH SC.


F
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VAN VLECK-KEITH