AN

ACCOUNT

OF

SIR ISAAC NEWTON's

Philoſophical Diſcoveries,

IN FOUR BOOKS.

BY

COLIN MACLAURIN, *A. M.*

Late Fellow of the Royal Society, Profeſſor of Mathematics in the Univerſity of EDINBURGH, *and Secretary to the Philoſophical Society there.*

Publiſhed from the Author's Manuſcript Papers,

By *PATRICK MURDOCH,* M. A. and F. R. S.

The SECOND EDITION.

LONDON:

Printed for A. MILLAR, at *Buchanan*'s-Head over againſt *Catharine-ſtreet* in the *Strand.*

M.DCC.L.

IN MEMORY

OF THAT JUST VENERATION

WHICH

THE AUTHOR MY DECEASED HUSBAND

ALWAYS EXPRESSED

FOR HIS ROYAL HIGHNESS

THE DUKE,

THIS ACCOUNT OF SIR ISAAC NEWTON's

PHILOSOPHICAL DISCOVERIES

IS HUMBLY INSCRIBED

TO HIS ROYAL HIGHNESS

BY HIS

MOST OBEDIENT SERVANT,

ANNE MACLAURIN.

a

THE

CONTENTS.

BOOK I.

Of the method of proceeding in natural philosophy, and the various systems of philosophers.

BOOK II.

Of the theory of motion, or rational mechanics.

BOOK III.

Gravity demonstrated by analysis.

BOOK IV.

The effects of the general power of gravity deduced synthetically.

ERRATA.

Page 134, l. 4, r. A×D*d*. Page 360, l. ult. LK, r. LB. Page 361, l. 1, LK, r. LB.

AN

ACCOUNT

OF THE

LIFE *and* WRITINGS *of the* AUTHOR.

COLIN MACLAURIN was descended of an ancient family, which had been long in possession of the island of *Tirrie*, upon the coast of *Argyleshire*. His grand-father, *Daniel*, removing to *Inverara*, greatly contributed to restore that town, after it had been almost entirely ruined in the time of the civil wars; and, by some memoirs which he wrote of his own times, appears to have been a person of worth and superior abilities. *John* the son of *Daniel*, and father of our author, was minister of *Glenderule*; where he not only distinguished himself by all the virtues of a faithful and diligent pastor, but has left, in the register of his provincial synod, lasting monuments of his talents for business, and of his public spirit. He was likewise employed by that synod in completing the version of the psalms into *Irish*, which is still used in those parts of the country where divine service is performed in that language. He married a gentlewoman of the family of *Cameron*, by whom he had three sons; *John*, who is still living, a learned and pious divine, one of the ministers of the city of *Glasgow*; *Daniel*, who died young, after having given proofs of a most extraordinary genius; and *Colin*, born at *Kilmoddan* in the month of *February* 1698.

His father died ſix weeks after; but that loſs was in a good meaſure ſupplied to the orphan family, by the affectionate care of their uncle Mr. *Daniel Maclaurin*, miniſter of *Kilfinnan*, and by the virtue and prudent œconomy of Mrs. *Maclaurin*. After ſome ſtay in *Argyleſhire*, where her ſiſters and ſhe had a ſmall patrimonial eſtate, ſhe removed to *Dumbarton*, for the more convenient education of her children: but dying in 1707, the care of them devolved entirely to their uncle.

In 1709 *Colin* was ſent to the univerſity of *Glaſgow*, and placed under the care of one of the beſt men, and moſt eminent profeſſors, of this age, the learned Mr. *Gerſhom Carmichael*. Here he continued five years, applying himſelf to his ſtudies with that ſucceſs which might be expected from parts like his, cultivated with the moſt indefatigable care and diligence. We find, amongſt his oldeſt manuſcripts, fragments of a diary in which he kept an account of every day, and of almoſt every hour of the day; of the beginning and ſucceſs of every particular ſtudy, enquiry or inveſtigation: of his converſations with learned men, the ſubjects of them, and the arguments on either ſide. Here we read the names of Profeſſor *Carmichael*, the celebrated Mr. *Robert Simſon*, Dr. *Johnſton*, and ſeveral other gentlemen of learning and worth; who all vied who ſhould moſt encourage our young philoſopher, by opening to him their libraries, and admitting him into their moſt intimate ſociety and friendſhip. He could not, afterwards, find time to keep ſo formal a regiſter of his life, but we are aſſured the habit never left him; and that every hour of it was continually filled up with ſomething which he could review with pleaſure.

His genius for mathematical learning diſcovered itſelf ſo early as at twelve years of age, when, having accidentally met with a copy of *Euclid* in a friend's chamber, in a few days he became maſter of the firſt ſix books without any aſſiſtance: and thence, following his natural bent, made ſuch a ſurprizing progreſs, that very ſoon after we find him engaged in the moſt curious and difficult problems. Thus much is certain, that in his ſixteenth year, he had already invented

invented many of the propoſitions afterwards publiſhed under the title of *Geometria Organica*.

In the fifteenth year of his age he took his degree of maſter of arts, with great applauſe; on which occaſion he compoſed and publicly defended a *Theſes* on the power of gravity: and after having ſpent a year in the ſtudy of divinity, he quitted the univerſity, and lived, for the moſt part, in an agreeable country retirement at his uncle's houſe, till near the end of 1717. In this retirement, he purſued his ſtudies with the ſame aſſiduity as he had done at the univerſity; continuing his favourite reſearches in mathematicks and philoſophy, and at other times reading the beſt claſſic authors; for which he naturally had an exceeding good taſte.

In the intervals of his ſtudies, the lofty mountains amidſt which he lived would often invite him abroad, to conſider the numberleſs natural curioſities they contain, and the infinite variety of plants that grow on them; or to climb to their tops, and enjoy the moſt extenſive and moſt diverſified proſpects. And here, his fancy being warmed by the grand ſcenes which preſented themſelves, he would ſometimes break out into a hymn or poetic rhapſody on the beauties of nature, and the perfections of its Author. Of theſe ſome fragments ſtill remain; which, tho' ſo unfiniſhed that it can be only thro' forgetfulneſs they have not been deſtroyed, yet ſhew a genius capable of much greater things in that way. His friends, however, are obliged to the accidents that have preſerved them, together with ſome others of his juvenile performances; for however unfit they may be for the public view, they ſhew the progreſs he had made in the ſeveral parts of learning, at the time they were written: and what can be more delightful, than to obſerve the gradual openings and improvements of a mind like that of Mr. *Maclaurin*?

In the autumn of 1717, he preſented himſelf a candidate for the profeſſorſhip of mathematics in the mariſhal college of *Aberdeen*, which he obtained after a comparative tryal of ten days with a very able competitor: and being fixed in his chair, he ſoon revived the taſte of mathematical

cal learning, and raiſed it higher than it had ever been in that univerſity.

During the vacations of 1719 and 1721, he went to *London* with a view of improving himſelf, and of being introduced to the illuſtrious men there. In his firſt journey, beſides Dr. *Hoadly* then biſhop of *Bangor*, Dr. *Samuel Clarke*, and ſeveral other eminent men, he became acquainted with Sir *Iſaac Newton*; whoſe friendſhip he ever after reckoned the greateſt honour and happineſs of his life. He was admitted a member of the *Royal Society*; two papers of his were inſerted in their tranſactions, and his book intitled *Geometria Organica* was publiſhed with the approbation of their preſident.

In his ſecond journey to *London* in 1721, he became acquainted with *Martin Folkes*, Eſq; now preſident of the royal ſociety; with whom he thence forth cultivated a moſt entire and unreſerved friendſhip, frequently interchanging letters with him, and communicating all his views and improvements in the ſciences.

In 1722, Lord *Polwarth*, Plenipotentiary of the King of *Great Britain* at the congreſs of *Cambray*, engaged Mr. *Maclaurin* to go as tutor and companion to his eldeſt ſon, who was then to ſet out on his travels.

After a ſhort ſtay at *Paris*, and viſiting ſome other towns in *France*, they fixed in *Lorrain*; where, beſides the advantage of a good academy, they had that of the converſation of one of the moſt polite courts in *Europe*. Here Mr. *Maclaurin* gained the eſteem of the moſt diſtinguiſhed perſons of both ſexes, and at the ſame time quickly improved that eaſy genteel behaviour which was natural to him, both from the temper of his mind, and from the advantages of a graceful perſon.

It was here likewiſe that he wrote his piece on the percuſſion of bodies, which gained the prize of the *Royal Academy of Sciences* for 1724; the ſubſtance of this tract is inſerted in his *Treatiſe of Fluxions*, and alſo in *Book* II. *Chap.* 2. of the following work.

Mr.

Mr. *Maclaurin* and his pupil having quitted *Lorrain*, were got as far on their tour as the ſouthern provinces of *France*, when Mr. *Hume* was ſeized with a fever, and died at *Montpelier*. An event ſo ſhocking muſt have affected a heart leſs ſenſible and tender than Mr. *Maclaurin*'s: in ſome letters written on this occaſion, he appears quite inconſolable. His own grief for his pupil, his companion, and friend; and his ſympathy with a family to which he owed great obligations, and which had ſuffered an irreparable loſs in the death of this hopeful young nobleman, rendered him altogether unhappy. Travelling and every thing elſe was become diſtaſteful, ſo he ſet out immediately on his return to his profeſſion at *Aberdeen*.

But being now univerſally diſtinguiſhed as one of the firſt genius's of the age, ſome of the curators of the univerſity of *Edinburgh*, were deſirous of engaging him to ſupply the place of Mr. *James Gregory*, whoſe age and infirmities had rendered him incapable of teaching. Several difficulties retarded this deſign for ſome time; particularly, the competition of a gentleman eminent for mathematical abilities, who had good intereſt with the patrons of the univerſity; and the want of an additional fund for the new profeſſor. But both theſe difficulties were got over, upon the receipt of two letters from Sir *Iſaac Newton*. In one, addreſſed to Mr. *Maclaurin*, with allowance to ſhew it to the patrons of the univerſity, Sir *Iſaac* expreſſes himſelf thus; "I am very glad to hear that you have a "proſpect of being joined to Mr. *James Gregory* in the "profeſſorſhip of the mathematics at *Edinburgh*, not "only becauſe you are my friend, but principally becauſe "of your abilities, you being acquainted as well with the "new improvements of mathematics, as with the former "ſtate of thoſe ſciences; I heartily wiſh you good ſucceſs, and ſhall be very glad of hearing of your being "elected; I am, with all ſincerity, your faithful friend "and moſt humble ſervant."

In a ſecond letter to the then Lord Provoſt of *Edinburgh*, which Mr. *Maclaurin* knew nothing of till ſome years after Sir *Iſaac*'s death, he thus writes, "I am glad

to

"to underſtand that Mr. *Maclaurin* is in good repute "amongſt you for his ſkill in mathematics, for I think he "deſerves it very well; and to ſatisfy you that I do not "flatter him, and alſo to encourage him to accept the "place of aſſiſting Mr. *Gregory*, in order to ſucceed him, "I am ready (if you pleaſe to give me leave) to contri-"bute twenty pounds *per annum* towards a proviſion for "him, till Mr. *Gregory*'s place become void, if I live ſo "long, and I will pay it to his order in *London*."

In *November* 1725, he was introduced into the univerſity: as was at the ſame time his learned collegue and intimate friend, Dr. *Alexander Monro*, profeſſor of anatomy. After this the mathematical claſſes ſoon became very numerous, there being generally upwards of a hundred young gentlemen attending his lectures every year: who being of different ſtandings and proficiency, he was obliged to divide them into four or five claſſes, in each of which he employed a full hour every day, from the firſt of *November* to the firſt of *June*.

In the firſt or loweſt claſs, (ſometimes divided into two) he taught the firſt ſix books of *Euclid*'s Elements, plain trigonometry, practical geometry, the elements of fortification, and an introduction to algebra. The ſecond claſs ſtudied algebra, the 11th and 12th books of *Euclid*, ſpherical trigonometry, conic ſections, and the general principles of aſtronomy. The third claſs went on in aſtronomy and perſpective, read a part of Sir *Iſaac Newton*'s *Principia*, and had a courſe of experiments for illuſtrating them, performed and explained to them. He afterwards read and demonſtrated the elements of fluxions: thoſe in the 4th claſs read a ſyſtem of fluxions, the doctrine of chances, and the reſt of *Newton*'s *Principia*.

All Mr. *Maclaurin*'s lectures on theſe different ſubjects were given with ſuch perſpicuity of method and language, that his demonſtrations ſeldom ſtood in need of repetition: ſuch, however, was his anxiety for the improvement of his ſcholars, that if at any time they ſeemed not fully to comprehend his meaning, or if, upon examining them, he found they could not readily demonſtrate the propoſi-

tions

tions which he had proved, he was apt rather to ſuſpect his own expreſſions to have been obſcure, than their want of genius or attention; and therefore would reſume the demonſtration in ſome other method, to try if, by expoſing it in a different light, he could give them a better view of it.

Beſides the labours of his public profeſſion, he had frequently many other employments and avocations. If an uncommon experiment was ſaid to have been made any where, the curious were deſirous of having it repeated by Mr. *Maclaurin:* if an eclipſe or *comet* was to be obſerved, his teleſcopes were always in readineſs. The ladies too would ſometimes be entertained with his experiments and obſervations; and were ſurprized to find how eaſily and familiarly he could reſolve the queſtions they put to him. His advice and aſſiſtance, eſpecially to the young gentlemen who had been his pupils, was never wanting; nor was admittance refuſed to any, except in his teaching hours, which were kept ſacred. His acquaintance and friendſhip was likewiſe courted by the ingenious of all ranks; who, by their fondneſs for his company, took up a great deal of his time, and left him not maſter of it, even in his country retirements. Notwithſtanding the neceſſary labour and the many interruptions and avocations which he had, he continued to purſue his own ſtudies with the utmoſt aſſiduity, reading whatever was publiſhed, from which he could expect any information or improvement. But to have time for ſo much ſtudy and writing, he was obliged to take from the ordinary hours of ſleep, what he beſtowed on his ſcholars and friends; and by this, no doubt, greatly impaired his health.

Sir *Iſaac Newton* dying in the beginning of the year 1728, his nephew Mr. *Conduitt* propoſed to publiſh an account of his life, and deſired Mr. *Maclaurin*'s aſſiſtance; who, out of gratitude to his great benefactor, chearfully undertook and ſoon finiſhed the hiſtory of the progreſs which philoſophy had made before Sir *Iſaac*'s time. This was the firſt draught of the following work; which was immediately ſent up to *London*, and had the approbation of ſome of the beſt judges. Dr. *Rundle*, in particular, afterwards bishop of *Derry*, was so pleased with the design, that he mentioned it to her late Majesty; who did it the honour of a reading, and expressed a desire to see it published. But Mr. *Conduitt*'s death having prevented the execution of his part of the proposed work, Mr. *Maclaurin*'s manuscript was returned to him. To this he afterwards added the more recent proofs and examples, given by himself or others, on the subjects treated of by Sir *Isaac*, and left it in the state in which it now appears.

Mr. *Maclaurin* had lived a batchelor to the year 1733: but being formed for society as well as for contemplation, and desirous of mixing more delicate and interesting delights with those of philosophy, he married *Anne*, daughter of Mr. *Walter Stewart* sollicitor-general to his late Majesty for *Scotland*; by whom he had seven children, of which, two sons *John* and *Colin*, and three daughters, have survived him.

Dr. *Berkley* bishop of *Cloyne*, having taken occasion from some disputes that had arisen concerning the grounds of the fluxionary method, in a treatise intitled the *Analyst*, published in 1734, to explode the method itself, and, at the same time to charge mathematicians in general with infidelity in religion; Mr. *Maclaurin* found it necessary to vindicate his favourite study, and repel an accusation in which he was most unjustly included. He began an answer to the bishop's book; but as he proceeded, so many discoveries, so many new theories and problems occurred to him, that, instead of a vindicatory pamphlet, his work came out a complete system of fluxions, with their application to the most considerable problems in geometry and natural philosophy.

This work was published at *Edinburgh* in 1742, in two volumes in quarto; in which we are at a loss what most to admire, his solid and unexceptionable demonstrations of the grounds of the method itself, or its application to such a variety of curious and useful problems.

His

His demonſtrations had been, ſeveral years before, communicated to Dr. *Berkley*, and Mr. *Maclaurin* had treated him with the greateſt perſonal reſpect and civility: notwithſtanding which, in his pamphlet on tar-water, he renews the charge, as if nothing had been done; for this excellent reaſon, that different perſons had conceived and expreſſed the ſame thing in different ways.

A ſociety having ſubſiſted ſome years at *Edinburgh* for improving medical knowledge, Mr. *Maclaurin* propoſed to have their plan made more extenſive, ſo as to take in all the parts of phyſics, together with the antiquities of the country. This was readily agreed to; and Mr. *Maclaurin*'s influence engaged ſeveral noblemen and gentlemen of the firſt rank and character, to join themſelves, for that purpoſe, to the members of the former ſociety. The Earl of *Morton* did them the honour to accept of the office of preſident; Dr. *Plummer* profeſſor of chymiſtry, and Mr. *Maclaurin* were appointed ſecretaries; and ſeveral gentlemen of diſtinction, *Engliſh* and foreigners, deſired to be admitted members.

At the monthly meetings of the ſociety, Mr. *Maclaurin* generally read ſome performance or obſervation of his own, or communicated the contents of his letters from foreign parts; by which means the ſociety was informed of every new diſcovery or improvement in the ſciences.

Several of the papers read before this ſociety, are printed in the 5th and 6th volumes of the *Medical Eſſays*. Some of them are likewiſe publiſhed in the *Philoſophical Tranſactions*, and Mr. *Maclaurin* had occaſion to inſert a great many more in his Treatiſe of Fluxions, and in his account of Sir *Iſaac Newton*'s philoſophy. By which means the publication of any volume of the works of the ſociety has been retarded: but we may hope their labours will ſtill be continued with ſucceſs, notwithſtanding the loſs they have ſuſtained by Mr. *Maclaurin*'s death.

He likewise proposed the building an astronomical *observatory*, and a convenient school for experiments in the university; of which he drew an elegant and well contrived plan: and as this work was to be carried on by private contributions, employed all his influence to raise money for that purpose; with so much success, that had not the unhappy disorders of that country intervened, the fabrick might by this time have been far advanced. The Earls of *Morton* and *Hoptoun* shewed their liberality as well as their love of the sciences, upon this occasion; as did the honourable Baron *Clerk*, vice-president of the *philosophical* society: and several noblemen and gentlemen offered to contribute what instruments of value they were possessed of, as soon as the observatory should be ready to receive them.

The Earl of *Morton* being to set out for *Orkney* and *Shetland* in 1739, to visit his estates there, wanted at the same time to settle the geography of these countries, which is very erroneous in all our maps; to examine their natural history, to survey the coasts, and to take the measure of a degree of the meridian: and, for this purpose, desired Mr. *Maclaurin*'s assistance. But his family affairs not permitting him to take such a journey, he could do no more than draw a memorial of what he thought necessary to be observed, furnish the proper instruments, and recommend Mr. *Short*, the famous optician, as a fit operator for managing them.

The account which he received of this voyage, made him still more sensible of the erroneous geography we have of those parts, by which many shipwrecks have been occasioned; and therefore he employed several of his scholars, who were then settled in the northern counties, to survey the coasts.

The reverend Mr. *Bryce* composed from observations a map of the coast of *Caithness* and *Strathnaver*, with remarks on the natural history and rarities of the country, together with directions for sea-faring people. This map was presented to the *Philosophical Society* at *Edinburgh*, and

and publiſhed by their order. The reverend Mr. *Bonnar* drew likewiſe a map of the three moſt northerly iſlands of *Shetland*, which is among Mr. *Maclaurin*'s papers; and we expect ſoon the geography of the *Orkneys* corrected by Mr. *Mackenzie*. It was from obſervations like theſe, made by ſkilful perſons, and with the beſt inſtruments, that Mr. *Maclaurin* expected to ſee a good map of *Scotland*; not from the ſlaviſh copying of map-ſellers, nor from a painful collecting, and patching together of old draughts and ſurveys of little authority; which he thought muſt contribute more to perpetuate than to rectify errors.

Mr. *Maclaurin* had ſtill another ſcheme for the improvement of geography and navigation, of a more extenſive nature. After reading all the accounts he could procure of voyages, both in the ſouth and north ſeas, he imagined the ſea was open all the way from *Greenland* to the ſouth ſea, by the north pole. Of this he was ſo much perſuaded, that he has been heard to ſay, if his ſituation could admit of ſuch adventures, he would undertake the voyage even at his own charges. But when ſchemes, for finding out ſuch a paſſage, were laid before the parliament in 1744, and he was conſulted concerning them by ſeveral perſons of high rank and influence; before he could finiſh the memorials which he propoſed to have ſent, the præmium was limited to the diſcovery of a north-weſt paſſage, and Mr. *Maclaurin* uſed to regret that the word *weſt* was inſerted, becauſe he thought that paſſage, if at all to be found, muſt lie not far from the pole.

Such was the zeal of this worthy perſon for the public good, in every inſtance; the laſt, and moſt remarkable, is that which we are now going to relate.

When it was certainly known, in 1745, that the rebels, after having got between *Edinburgh* and the King's troops, were continuing their march ſouthwards, Mr. *Maclaurin* was among the firſt to rouſe the friends of our happy conſtitution, from the unlucky ſecurity they had hitherto continued in: and tho' he was ſenſible that the city of *Edinburgh*, far from being able to ſtand the attack of a regular army, could not even hold out any conſiderable time againſt the undiſciplined and ill-armed force that was coming againſt it; yet, as he foreſaw of how much advantage it would be to the rebels, to get poſſeſſion of that capital; and, the King's forces under the command of Sir *John Cope* being daily expected; he made plans of the walls, propoſed the ſeveral trenches, barricades, batteries, and ſuch other defences as he thought could be got ready before the arrival of the rebels, and by which, he hoped, the town might be kept till the King's forces ſhould come to its relief. The whole burden, not only of contriving, but alſo of overſeeing the execution, of theſe haſty fortifications fell to Mr. *Maclaurin*'s ſhare; he was employed night and day, in making plans, and running from place to place; and the anxiety, fatigue, and cold to which he was thus expoſed; affecting a conſtitution naturally of weak nerves, laid the foundation of the diſeaſe of which he died.

How this plan came to be neglected, and in what manner the rebels got poſſeſſion of the town, is not a proper enquiry for this place. They got poſſeſſion of it! and, their ſpirits being raiſed by this unaccountable ſucceſs, and by the ſupply of arms and proviſions which it gave them, they ſoon after defeated the King's troops at *Preſton*. The moderation which they had affected before that unhappy battle was now laid aſide, and obedience was to be given to whatever proclamations or orders they thought fit to iſſue, under pain of military execution. Among other deſpotic orders, one was, commanding all who had been volunteers in defence of the town, before a ſtated time, to wait on their ſecretary of ſtate, to ſubſcribe a recantation of what they had done, and a promiſe of ſubmiſſion to their pretended government, under the pain of being deemed and treated as rebels. Mr. *Maclaurin* had been too active and diſtinguiſhed a volunteer, to think he could eſcape the ſevereſt treatment, if he fell into their hands after neglecting to make the ſubmiſſion required; he therefore withdrew privately into *England*, before the laſt day of receiving the ſubmiſſions; but, previous to his eſcape, found means to convey a good teleſcope into the caſtle, and concerted a method of ſupplying the garriſon with proviſions.

As

As ſoon as his Grace, Dr. *Thomas Herring* then Lord Archbiſhop of *York*, was informed that Mr. *Maclaurin* had fled to the north of *England*, he invited him in a moſt friendly and polite manner, to reſide with him during his ſtay in that country. Mr. *Maclaurin* gladly accepted of the invitation, and ſoon after expreſſes himſelf thus in a letter to a friend; " Here (ſays he) I live as happily as a " man can do, who is ignorant of the ſtate of his family, " who ſees the ruin of his country." His Grace, of whoſe merit and goodneſs, Mr. *Maclaurin* ever retained the higheſt ſentiments, afterwards kept a regular correſpondence with him; and when it was ſuſpected that the rebels might once more take poſſeſſion of *Edinburgh*, after their retreat from *England*, invited his former gueſt again to take refuge with him.

At *York* he had been obſerved to be more meagre than ordinary, and with a ſickly look; though not being apprehenſive of any danger at that time, he did not call in the aſſiſtance of a phyſician: but having had a fall from his horſe on his journey ſouthward, and, when the rebel army marched into *England*, having on his return home been expoſed to moſt tempeſtuous cold weather, upon his arrival he complained of being much out of order. In a little time his diſeaſe was diſcovered to be a dropſy of the belly, to remove which, variety of medicines, preſcribed by the moſt eminent phyſicians at *London*, as well as thoſe of *Edinburgh*, and three tappings, were uſed without making a cure.

His behaviour, during this tedious and painful diſtemper, was ſuch as became a philoſopher and a chriſtian; calm, chearful, and reſigned; his ſenſes and judgment remaining in their full vigour, till within a few hours of his death. Then, for the firſt time, his amanuenſis to whom he was dictating the laſt chapter of the following work (in which he proves the wiſdom, the power, goodneſs, and other attributes of the Deity) obſerved ſome heſitation or repetition: no pulſe could then be felt in any part of his body, and his hands and feet were already cold. Notwithſtanding this extremely weak condition, he ſate in his chair, and ſpoke to his friend Dr. *Monro* with his uſual ſerenity and ſtrength of reaſon, deſiring the Doctor to account for a phænomenon which he then obſerved in himſelf: flaſhes of fire ſeeming to dart from his eyes, while in the mean time his ſight was failing, ſo that he ſcarce could diſtinguiſh one object from another. In a little time after this converſation, he deſired to be laid upon his bed; where, on *Saturday* the 14th of *June*, 1746, aged 48 years and 4 months, he had an eaſy paſſage from this world to that ſtate of bliſs, which he had the moſt elevated ideas of, and which he moſt ardently longed to poſſeſs.

The grief for the loſs of this excellent perſon was as general as the eſteem which he had acquired, with all ranks of men: but thoſe of greateſt worth, and who had moſt intimately known him, were the moſt deeply affected. Dr. *Monro*, in an oration ſpoken at the firſt meeting of the univerſity after Mr. *Maclaurin*'s death (from which the ſubſtance of the foregoing account is taken) gives, particularly, a very moving picture of the grief of the late Lord Preſident *Forbes*, on this occaſion. A likeneſs of character, and a perfect harmony of ſentiments and views, had cloſely united them in their lives; in their deaths, they were alas! too little divided: the preſident likewiſe, worn out in the ſervice of his country, was ſoon to be the ſubject of a general mourning.

In the ſame diſcourſe the Doctor ſhews, in a variety of inſtances, that acute parts and extenſive learning were, in Mr. *Maclaurin*, but inferior qualities; that he was ſtill more nobly diſtinguiſhed from the bulk of mankind, by the qualities of the heart; his ſincere love to God and Men, his univerſal benevolence and unaffected piety; together with a warmth and conſtancy in his friendſhips, that was in a manner peculiar to himſelf. He profeſſes likewiſe, that after an intimacy with him for ſo many years, he had but half known his worth; which then only diſcloſed itſelf in its full luſtre, when it came to ſuffer the ſevere teſt of that diſtreſsful ſituation, in which every man muſt at laſt find himſelf; and which only minds prepared like his, armed with virtue and chriſtian hope, can bear with dignity.

But the bounds we are confined to, do not permit us to follow the professor in this delightful track; nor would the modesty of Mr. *Maclaurin*'s surviving friends bear with our being so particular. We must content ourselves to consider him in the character in which he was universally known; by giving a short account of his works, and of the taste and manner in which he cultivated the mathematical sciences; pursuing with such indefatigable pains, studies that seem, to many, rather curious than useful.

His first work, composed in his early youth, was the *Geometria Organica*, in which he treats of the description of curve lines by continued motion. The first and simplest of curves is described by the motion of a right line on a plane, round one of its extremities. Sir *Isaac Newton* had shewn, that the *Conic Sections* might all be described by assuming two centres or poles in a plane, and moving round them two given angles, so as the intersection of two legs be always found in a streight line, given in position in the same plane; for thus the intersection of the other two will trace some conic section. In a similar way, he describes such lines of the third order, as have a double point, that is to say, which returning upon themselves, pass twice through the same point; but the description of the far greater number of those lines, which have no such point, Sir *Isaac* declares to be a problem of much more difficulty. This was reserved for Mr. *Maclaurin*; who not only happily resolved it, but carried the same method of description much higher. By assuming more poles, or by moving the angular points along more lines given in position, or, lastly, by carrying the intersections along curve lines, instead of streight, he has extended, or given hints of extending, the method as far as it can go. And because, by the motion of rulers actually combined, as the case requires, such descriptions may be effected, he calls them by the general name of *Organical*. When he wrote this treatise, the subjects being new and entertaining, his invention in its prime, and the ardor of his curiosity continually urging him on to farther discoveries, he did not take time to finish every demonstration in so elegant a manner as he might have done. His page, we muſt own, is incumbered with algebraical calculations, and theſe have offended the delicate eyes of ſome critics; but, in anſwer to this, we may ſay that what offends them, may be very acceptable to younger ſtudents: nor indeed ſhould we at all have mentioned this blemiſh in ſo great a work, if himſelf had not ſomewhere hinted at it, and, in a letter to one of his friends, expreſſed an intention of reſuming, with his firſt leiſure, that whole theory, and adding to it a *ſupplement*; the greateſt part of which had been printed ſeveral years ago, but whereof we have only an abſtract in the Philoſophical Tranſactions, N°. 439. In the ſame volume, he gives a new theory of the curves which may be derived from any given curve, by conceiving perpendiculars to its tangents to be drawn continually through a given point, whoſe interſections with the tangents will form a new curve; from which laſt a third may be formed in the ſame manner, and ſo on *in infinitum*. This furniſhes many curious theorems: there are likewiſe ſome propoſitions concerning centripetal forces and other ſubjects, which, with the quotations he uſes, ſhew the great progreſs he had already made in every part of mathematical learning, and how well acquainted he was with the writings of the beſt authors.

We ſhall not here repeat what has been ſaid concerning his piece which gained the prize of the *Royal Academy* of *Sciences* in 1724. In the year 1740, the *Academy* adjudged him a prize which did him ſtill more honour, for accounting for the motion of the *Tides*, from the theory of gravity; a queſtion which had been given out the former year, without receiving any ſolution. He happened to have only ten days time to draw up this paper, and could not find leiſure to tranſcribe a fair copy, ſo that the *Paris* edition of it is incorrect; but he afterwards reviſed the whole, and inſerted it in his Treatiſe of Fluxions.

Nor need we mention the occaſions on which ſeveral pieces which he ſent to the Royal Society were written: the following liſt will ſhew their dates, and the ſubjects treated of in them.

1. *Of the construction and measure of curves*, Phil. Transf. N°. 356.

2. *A new method of describing all kinds of curves*, N°. 359.

3. *A Letter to* Martin Folkes, *Esq; on equations with impossible roots*, May, 1726. N°. 394.

4. ——— *Continuation of the same*, March 1729. N°. 408.

5. Decem. 21*st*, 1732. *On the description of curves; with an account of farther improvements, and a paper dated at* Nancy, 27*th* Nov. 1722. N°. 439.

6. *An account of the annular eclipse of the sun, at* Edinburgh, Feb. 18, 1736-7. N°. 447.

7. *An account of the Treatise of Fluxions*, January 27*th*, 1742-3. N°. 467.

8. ——— *The same continued*, March 10*th*, 1742-3. N°. 469.

9. *A rule for finding the meridional parts of a spheroid with the same exactness as of a sphere*, August 1741. N°. 461.

10. *Of the bases of the cells wherein the bees deposit their honey*, Novem. 3, 1743. N°. 471.

But the great work, on which he bestowed the most labour, and which will for ever do him honour, is his *Treatise of Fluxions.*

The occasion of it was related above, namely, the objections of some ingenious men against the doctrine of fluxions, on account of the different modes of explication which had been used by different authors. Nor can it be denied, that the terms *infinite* and *infinitesimal* were become much too familiar to mathematicians, and had been abused both in *arithmetic* and *geometry*: At one time in-

 troducing

troducing and palliating real abfurdities, and, at others, giving thefe fciences an affected myfterious air which does not belong to them. To remedy this growing evil, and for ever take away the handle which it gave to cavilling, Mr. *Maclaurin* found it neceffary, in demonftrating the principles of fluxions, to reject altogether thofe exceptionable terms, and to fuppofe no other than finite determinable quantities, fuch as *Euclid* treats of in his geometry; nor to ufe any other form of demonftration than what the antients had frequently ufed, and which had been allowed as ftrictly conclufive from the firft rife of the fcience: by which means, he has fecured this admirable invention from all future attacks, and at the fame time done juftice to the accuracy of the great inventor. The work coft him infinite pains; but he did not grudge it: he thought that in proportion "as the general me-"thods are valuable, it is important that they be efta-"blifhed above all exception, and fince they fave us fo "much time and labour, we may allow the more for il-"luftrating the methods themfelves *."

To his demonftrations of this doctrine he has added many valuable improvements of it, and has happily applied it to fo many curious and ufeful enquiries, that his work may be called a ftorehoufe of mathematical learning, rather than a treatife on one branch of it. The particulars we need not enumerate, efpecially as there is printed in the Philofophical Tranfactions, N°. 468, 469. a clear and methodical account of them; to which we refer the reader.

Throughout this whole work, though not equally perfect in all its parts, becaufe of the infinite extent of the field into which he was led, there appears a very mafterly genius, and an uncommon addrefs.

An ordinary artift follows the firft, not generally the beft, road that prefents itfelf, and arrives perhaps at the folution of his problem; but it will fcarcely be either elegant or clear; one may fee there is ftill fomething want-

* Introd. to *Fluxions*, at the end.

ing,

ing, the reſult being little more ſcientific than that of an arithmetical operation, where the given numbers and their relations have all diſappeared. This was not the caſe of Mr. *Maclaurin*; he had a quick comprehenſive view, taking in at once all the means of inveſtigation; he could ſelect the fitteſt for his purpoſe, and apply them with exquiſite art and method. This is a faculty not to be acquired by exerciſe only; we ought rather to call it a ſpecies of that taſte, the gift of nature, which in mathematics, as in other things, diſtinguiſhes excellence from mediocrity.

We have in all Mr. *Maclaurin*'s latter works, eſpecially in his treatiſe of fluxions, numberleſs inſtances of this addreſs: We need only inſtance in his reducing ſo many ſolutions which uſed to be managed by the higher orders of fluxions to thoſe of an inferior order, and many of the queſtions concerning the *maxima* and *minima*, even ſome of the moſt difficult, to plane geometry.

Theſe are all the writings which our author lived to publiſh; ſince his deceaſe two volumes more have appeared, his treatiſe of *Algebra*, and this account of Sir *Iſaac Newton*'s philoſophy.

His *Algebra*, tho' it had not the advantage to be finiſhed by his own hand and publiſhed under his eye, is yet allowed to be excellent in its kind; containing, in no large volume, a complete elementary treatiſe of that ſcience, as far as it has hitherto been carried; all the moſt uſeful rules, which lie ſcattered in ſo many authors, being clearly laid down and demonſtrated, and in the order which he had found to be the beſt in a long courſe of methodical teaching. He is more ſparing, it is true, in the practical applications than moſt other writers, but this was deſignedly; he was of opinion that many of thoſe applications deſerve to be treated of apart; and to have taken too much of them into his plan, would have been like disfiguring the elements of *Euclid*, by mixing with them the rules of practical geometry. To this work is ſubjoined, as a proper *appendix*, his Latin tract *De Linearum Geometricarum proprietatibus generalibus*. It is carefully printed from a manu-

manuſcript all written and correĉted by the author's own hand; and we need only add, that as it was among the laſt, ſo it appears to have been, in his own judgment, one of the beſt of his performances.

The account of Sir *Iſaac Newton*'s philoſophy lies now before the reader; who, by caſting his eye on the *table of contents*, may ſee the author's deſign and method; and in peruſing the work itſelf will not, we hope, find himſelf diſappointed.

One queſtion however may be put, which it is proper for us to obviate. Why, in this account, Sir *Iſaac Newton*'s grand diſcoveries concerning light and colours, are but tranſiently and in general touched upon? To this it is anſwered, that our author's main deſign ſeems to have been to explain only thoſe parts of Sir *Iſaac*'s philoſophy that have been, and are ſtill, controverted. But it is known that, ever ſince the experiments, on which his doĉtrine of light and colours is founded, have been repeated with due care, this doĉtrine has ſuffered no conteſtation: Whereas his ſyſtem of the world, his accounting for the celeſtial motions, and the other great appearances of nature, from gravity, is miſunderſtood and even ridiculed to this day: the weak charge of *occult qualities* has been frequently repeated; foreign profeſſors ſtill amuſe themſelves with imaginary triumphs; even the polite and ingenious Cardinal *de Polignac* is ſeduced to lend them the harmony of his numbers.

It was proper therefore that theſe Gentlemen ſhould once more be told (and by Mr. *Maclaurin*) that their objeĉtions are altogether out of ſeaſon; that the ſpeĉtres they are daily combating are a creation of their own, no more related to Sir *Iſaac Newton*'s doĉtrines than obſervation and experience are to occult qualities; that the followers of Sir *Iſaac Newton* will for ever aſſert their right to ſtop where they find they can get no farther upon ſure ground; and to make uſe of a principle firmly eſtabliſhed in experience, adequate to all the purpoſes they apply it to, and in every application uniform and conſiſtent with

itſelf;

itſelf *; although they, perhaps, deſpair of tracing the ulterior cauſe of that principle.

But beſides that Sir *Iſaac Newton*'s treatiſe of optics wanted no defence, it may be ſaid likewiſe, that it ſcarce admits of an explication; it is ſuch an abſolute maſter-piece of philoſophical writing, that it can as little be abridged as enlarged; and we had better take all his experiments, illuſtrations and proofs in the words in which he has delivered them, than riſque the injuring them by a different dreſs. As for the hints which he could not further purſue, and which he propoſes as queries; Mr. *Maclaurin* had too ſound a judgment, and had too thoroughly imbibed the genius and ſpirit of his great Maſter, to run away with them as materials for rearing doubtful theories: He leaves them as he found them, till future diſcoveries can give them another name.

Beſides his printed and more finiſhed works, Mr. *Maclaurin* had by him a number of manuſcript papers, and imperfect eſſays on mathematical and other ſubjects. Theſe the increaſe of his diſtemper did not give him time to put in order, or to leave particular directions how they were to be diſpoſed of: He therefore intruſted them all together to the care of three gentlemen, in whoſe hands he knew they would be perfectly ſafe: his honoured friend *Martin Folkes*, Eſq; preſident of the royal ſociety; *Andrew Mitchell*, Eſq; member of parliament for the ſhire of *Aberdeen*, who, he knew, would ſpare no pains to do juſtice to the memory of a perſon whom he had ſo long, and ſo entirely, loved; and the reverend Mr. *John Hill*, chaplain to his grace the archbiſhop of *Canterbury*, with whom he had for ſome years cultivated a moſt intimate friendſhip. In conſequence of this truſt, theſe Gentlemen immediately ſet about publiſhing what Mr. *Maclaurin* had deſigned and prepared for the preſs; his algebra, and the account of Sir *Iſaac Newton*'s philoſophy: and becauſe

* Of this we ſee a freſh inſtance in a *ſecond admirable* diſcovery of Dr. *Bradley*'s; of a ſmall nutation of the earth's axis, from the motion of the nodes of the lunar orbit.

they

they could not take upon themſelves the immediate care of theſe editions, they appointed, for that purpoſe, a perſon whoſe regard for the author's memory was a ſure pledge of his utmoſt diligence. They likewiſe ſet on foot and ſollicited a ſubſcription for the following work; which the ſituation of Mr. *Maclaurin*'s family made neceſſary. For not to mention, that the thoughts of a philoſopher are not much turned to the ſaving of money, nor is his curioſity to be gratified but at a conſiderable expence, Mr. *Maclaurin*'s liberality was greater than his fortune could well bear: it was not advice and recommendation only that he furniſhed to young men, in whom he could diſcover a promiſing and virtuous diſpoſition; he often ſupplied them with money till his recommendations could take place. This however will not, we hope, upon the whole, be any loſs to his family; as it has been remembred, and rewarded by the generous manner in which many gentlemen of worth have promoted this ſubſcription.

If we now look back upon the numerous writings of our author, and the deep reſearches he had been engaged in, his patience and aſſiduity will be equally aſtoniſhing with his genius. To endeavour to account for it to a perſon who has not himſelf taſted the pleaſures of a contemplative mind, would be a vain attempt. Whoever has devoted himſelf to worldly views, or to the mere joys of ſenſe and imagination, muſt be a ſtranger to the charms of *truth*, naked, unportioned, and unadorned; ſuch as Mr. *Maclaurin* courted her, through his whole life, with a moſt faithful and perſevering paſſion. Call his ſpeculations but a kind of luxury; it is however a higher and more refined luxury than other purſuits can furniſh: an exerciſe, in which the human faculties find themſelves the moſt rationally employed, and the moſt ſenſibly ſtrengthened and improved. At the ſame time, it beſt diſtinguiſhes the limits to which they are confined; inſpiring that humility which belongs to man, and makes a principal part of true wiſdom, *the knowledge of one's ſelf.*

How

How great an example Mr. *Maclaurin* was of this virtue, thoſe who had the happineſs of his acquaintance can teſtify, and his writings abundantly ſhew. The farther he advanced in the knowledge of geometry and of nature, the greater his averſion grew to perfect ſyſtems, hypotheſes, and dogmatizing; without peeviſhly deſpiſing the attainments we can arrive at, or the uſes to which they ſerve, he ſaw there lay infinitely more beyond our reach; and uſed to call our higheſt diſcoveries but a dawn of knowledge, ſuited to our circumſtances and wants in this life; which, however, we ought thankfully to acquieſce in for the preſent, in hopes that it will be improved in a happier and more perfect ſtate.

In weak and unexperienced minds, it is true, the ſtudy of mathematics has often wrought quite different effects: ſometimes an overweening and moſt ridiculous ſelf-conceit, with a contempt of all other ſtudies; at other times, a raſh confounding of the different kinds of evidence, and the different ſubjects to which they can be applied; ſometimes, becauſe demonſtrative evidence is the moſt perfect, it has been taken for granted there is none other; or moral evidence, to bring it to the ſame level, has been diſguiſed in an awkward and diſadvantageous dreſs. But to oppoſe the ſingle example of Mr. *Maclaurin* to ſuch pretenders, will be a ſufficient cenſure of their abſurd conduct; and at the ſame time a ſufficient anſwer to the unjuſt reproaches, which, on occaſion of theſe abuſes, have been thrown out againſt mathematicians.

It was not mental pleaſure and improvement only, that Mr. *Maclaurin* ſought in his favourite ſtudies; he ſaw their great importance in all the arts of civil life, in *aſſiſting* (as my Lord *Bacon* expreſſes it *) *the powers of man, and extending his dominion in nature.* Whoſoever is the leaſt acquainted with the hiſtory or the preſent ſtate of trade and manufactures, is fully apprized that there

* *Nov. Organ.* Lib. I.

is

is nothing great or beautiful, nothing convenient or expeditious, nothing universally beneficial, but wants *their* direction: nor are even the hints which accident throws in our way, to be improved to any tolerable purpose, without the help of *Arithmetic* and *Geometry*.

To this view of general utility, Mr. *Maclaurin* had accommodated all his studies; and we find in many places of his works an application, even of the most abstruse theories, to the perfecting of mechanical arts. He had resolved, for the same purpose, to compose a course of practical mathematics, and to rescue several useful branches of the science, from the bad treatment they often meet with in less skilful hands. But all this his death has deprived us of; unless we would reckon as a part of his intended work, the translation of Dr. *David Gregory*'s practical geometry, which he revised and published, with additions, in the year 1745.

In his life-time, however, he often had the pleasure to serve his friends and country by his superior skill. Whatever difficulty occurred concerning the construction or perfecting of machines, the working of mines, the improvement of manufactures, the conveying of water, or the execution of any other public work, Mr. *Maclaurin* was at hand to resolve it. He was likewise employed to terminate some disputes of consequence, that had arisen at *Glasgow* concerning the gauging of vessels; and for that purpose, presented to the commissioners of excise two elaborate memorials, containing rules by which the officers now act, with their demonstrations.

But what must have given him a higher satisfaction than any thing else of this kind, was the calculations he made, relative to that wise and humane provision, which is now established by law, for the children and widows of the *Scotch* clergy, and of the professors in the universities; entitling them to certain annuities and sums, upon the voluntary annual payment of a certain sum by the incumbent. In contriving and adjusting the scheme, Mr. *Maclaurin* had bestowed great labour; and the gentlemen

men who were appointed to ſollicite the affair at *London*, own that the authority of his name was of great uſe to them, for removing any doubts that were moved concerning the ſufficiency of the propoſed fund, or the due proportion of the ſums and annuities.

To find himſelf thus eminently uſeful, even to late poſterity, muſt have been a delightful enjoyment. But what ſtill more endeared his ſtudies to him, was the uſe they are of in demonſtrating the Being and Attributes of the Almighty Creator, and eſtabliſhing the principles of natural religion on a ſolid foundation; equally ſecure againſt the idle ſophiſtry of *Epicureans*, and the dangerous refinements of *modern metaphyſicians*. He agreed with the great Mr. *Cotes* *, in thinking *that the knowledge of nature will ever be the firmeſt bulwark againſt Atheiſm*, and conſequently the ſureſt foundation of true religion. This knowledge does more than excite mere *wondering*; it inſpires love and adoration of the Creator, our reaſonable *Service*: for it muſt be a ſuperficial view of nature, indeed, that ſuggeſts no *relation*, or *duty*, to Him *in whom we live, move, and have our being*. The argument from final cauſes, from the order and deſign that evidently ſhews itſelf throughout the univerſe, Mr. *Maclaurin* held to be the ſhorteſt and ſimpleſt of all others; and conſequently of moſt general uſe, and the beſt adapted to the human faculties: whereas metaphyſical deductions are to be apprehended but by the few, and are ever liable to be perverted. So that altho' he could uſe them with as much ſubtlety and force as any man living, he choſe rather, in his converſation as well as his writings, to bring the diſpute to a ſhort iſſue in his own way.

He was no leſs ſtrenuous in the defence of revealed religion; which he would warmly undertake as often as it was attacked, either occaſionally in converſation, or in thoſe pernicious books which have brought the name of Free-thinker into diſgrace, and have ſo much contributed to ſpoil our taſte as well as our morals: and how firm his

* In Præfat. ad *Neut.* Principia.

own

own perſuaſion of it was, appeared from the ſupport it afforded him in his laſt hours.

Such was the life of this eminent perſon; ſpent in a courſe of laborious, yet not painful, ſtudy; in continually doing good to the utmoſt of his power: in improving curious and uſeful arts; and propagating truth, virtue, and religion amongſt mankind. He was taken from us at an age when he was capable of doing much more; but has left an example which, we hope, will be long admired and imitated: till the revolution of human affairs puts an end to learning in theſe parts of the world; or the fickleneſs of men, and their ſatiety of the beſt things, have ſubſtituted for this philoſophy ſome empty form of falſe ſcience; and, by the one or the other means, we are brought back to our original ſtate of barbariſm.

AN
ACCOUNT
OF
Sir ISAAC NEWTON's
Philoſophical Diſcoveries.

BOOK I.

Of the method of proceeding in natural philoſophy, and the various ſyſtems of philoſophers.

CHAP. I.

A general view of Sir Iſaac Newton's *method, and of his account of the ſyſtem of the world.*

1. TO deſcribe the *phenomena* of nature, to explain their cauſes, to trace the relations and dependencies of thoſe cauſes, and to enquire into the whole conſtitution of the univerſe, is the buſineſs of natural philoſophy. A ſtrong curioſity has prompted men in all times to ſtudy nature; every uſeful art has ſome connexion with this ſcience; and the unexhauſted beauty and variety of things makes it ever agreeable, new, and ſurpriſing.

But natural philoſophy is ſubſervient to purpoſes of a higher kind, and is chiefly to be valued as it lays a ſure foundation for natural religion and moral philoſophy; by leading us, in a ſatisfactory manner, to the knowledge of the Author and Governor of the univerſe. To ſtudy nature is to ſearch into his workmanſhip: every new diſcovery opens to us a new part of his ſcheme. And while we ſtill meet, in our enquiries, with hints of greater things yet undiſcovered, the mind is kept in a pleaſing expectation of making a further progreſs; acquiring at the ſame time higher conceptions of that great Being, whoſe works are ſo various and hard to be comprehended.

Our views of nature, however imperfect, ferve to reprefent to us, in the moft fenfible manner, that mighty power which prevails throughout, acting with a force and efficacy that appears to fuffer no diminution from the greateft diftances of fpace or intervals of time; and that wifdom which we fee equally difplayed in the exquifite ftructure and juft motions of the greateft and fubtileft parts. Thefe, with perfect *goodnefs*, by which they are evidently directed, conftitute the fupreme object of the fpeculations of a philofopher; who, while he contemplates and admires fo excellent a fyftem, cannot but be himfelf excited and animated to correfpond with the general harmony of nature.

In order to obtain thofe great purpofes, we muft not proceed haftily in our enquiries, but with the utmoft caution. Falfe fchemes of natural philofophy may lead to atheifm, or fuggeft opinions, concerning the Deity and the univerfe, of moft dangerous confequence to mankind; and have been frequently employed to fupport fuch opinions. We have the more reafon to be on our guard, becaufe philofophers have, on many occafions, fhown an unaccountable difpofition to give into extravagant fictions in their accounts of nature. A confiderable party adopted, of old, that monftrous fyftem, which, excluding the influences of a Deity *, attempted to explain the formation of the univerfe from the accidental play of atoms, and derived the ineffable beauty of things, even life and thought itfelf, from a lucky hit in the blind uproar. An horror at the dire effects of fuperftition may have induced them to have recourfe to a doctrine fo oppofite to common fenfe and reafon; but we have not even this

* *Lucret.* de rerum natura, lib. I. v. 63, *&c.*

excuſe to offer in defence of ſome modern philoſophers of great name, who ſeem to have copied too much after thoſe maſters, in their mechanical accounts of the production of the material ſyſtem.

While we guard againſt atheiſm and opinions that approach towards it, we ought likewiſe to beware of liſtening to ſuperſtition; which diſcourages enquiries into nature, leſt, by having our views enlarged, we ſhould eſcape from her bonds, and our diſcoveries ſhould weaken ſome darling tenets. If thoſe tenets are true, they will rather be confirmed by our enquiries; and if they are falſe, ſurely it is better they ſhould be detected. We may purſue truth ſteadily, ſecure that it will be always found conſiſtent with itſelf, and ſtands in no need of the jealouſies and dark ſuſpicions of the ſuperſtitious to ſupport it; in whoſe hands truth itſelf is apt to ſuffer, by the baſe alloy they mix with it, and by the deteſted means which they have too often employed to maintain ſo incongruous an union. The philoſophers who have been devoted to ſo mean views, have never failed to expoſe themſelves to juſt ridicule, without doing ſervice to the cauſe which they eſpouſed. *Coſmas Indopleuſtes* * of old, miſled by an injudicious zeal, compiled a ſyſtem of nature from ſome expreſſions in the ſacred writings; which, againſt the conſtant and univerſal uſe of language, he would needs underſtand in the moſt literal and the very ſtricteſt ſenſe.

The earth therefore, according to him, was not globular, but an immenſe plane of a greater length than breadth, environed by an unpaſſable ocean.

* *Fabrit.* bibliotheca græca, vol. II. p. 609, *&c.* where an account is given from *Photius* and others of this author, with a figure to illuſtrate his ſyſtem.

He placed a huge mountain towards the north, around which the ſun and ſtars performed their diurnal revolutions; and from the conical ſhape which he aſcribed to it, with the oblique motion of the ſun, he accounted for the inequality of the days and the variation of the ſeaſons. The vault of heaven leaned upon the earth extended beyond the ocean, being likewiſe ſupported by two vaſt columns: beneath the arch, angels conducted the ſtars in their various motions. Above it were the celeſtial waters, and above all he placed the ſupreme heavens. However abſurd the conceits of this author, who wrote in darker times, may appear, we have a more inexcuſable inſtance, in the laſt century, of the ſame kind, in what *Kircher* calls his Ecſtatic Voyage to the Planets; who, after many great diſcoveries had been made concerning the celeſtial bodies, produced nothing worthy * of ſo noble a ſubject, or of his own extenſive learning and invention, having determined to make a ſacrifice of both to certain decrees of the church of Rome: he deſcends even ſo low as to adopt the folly or rather impiety, of aſtrologers, in deriving the good or evil that happens to man from the propitious or malignant influences of planets. True religion requires no ſuch ſacrifices; nor are its intereſts advanced by feigning philoſophical ſyſtems purpoſely to favour it: for when we afterwards find theſe to be ill-grounded, we may be in danger of falling into ſcepticiſm.

An entire liberty muſt be allowed in our enquiries, that natural philoſophy may become ſubſervient

* In the planet *Venus*, for example, he finds no other amuſement but to admire the limpid waters and beautiful cryſtals he found there; and to aſk the genie, his companion and guide, whether baptiſm with ſuch water would be valid. The reſt is of a piece with this.

vient to the moſt valuable purpoſes, and acquire all the certainty and perfection of which it is capable: but we ought not to abuſe this liberty by *ſuppoſing* inſtead of *enquiring*, and by imagining ſyſtems, inſtead of learning from obſervation and experience the true conſtitution of things. Speculative men, by the force of genius, may invent ſyſtems that will perhaps be greatly admired for a time; theſe, however, are phantoms which the force of truth will ſooner or later diſpell: and while we are pleaſed with the deceit, true philoſophy, with all the arts and improvements that depend upon it, ſuffers. The real ſtate of things eſcapes our obſervation: or, if it preſents itſelf to us, we are apt either to reject it wholly as fiction, or, by new efforts of a vain ingenuity, to interweave it with our own conceits, and labour to make it tally with our favourite ſchemes. Thus, by blending together parts ſo ill ſuited, the whole comes forth an abſurd compoſition of truth and error.

Of the many difficulties that have ſtood in the way of philoſophy, this vanity perhaps has had the worſt effects. The love of the marvellous, and the prejudices of ſenſe, obſtructed the progreſs of natural knowledge; but experience and reflection ſoon taught men to examine and endeavour to correct theſe. Tho' philoſophers met with great diſcouragements in the dark and ſuperſtitious ages, learning flouriſhed, with liberty, in better times. The diſputes amongſt the ſects, more fond of victory than of truth, produced a talkative ſort of philoſophy, and a vain oſtentation of learning, that prevailed for a long time; but men could not be always diverted from purſuing after more real knowledge. Theſe have not done near ſo much harm, as that pride and ambition, which has led philosophers to think it beneath them, to offer any thing less to the world than a complete and finished system of nature; and, in order to obtain this at once, to take the liberty of inventing certain principles and hypotheses, from which they pretend to explain all her mysteries.

2. Sir *Isaac Newton* saw how extravagant such attempts were, and therefore did not set out with any favourite principle or supposition, never proposing to himself the invention of a system. He saw that it was necessary to consult nature herself, to attend carefully to her manifest operations, and to extort her secrets from her by well chosen and repeated experiments. He would admit no objections against plain experience from metaphysical considerations, which, he saw, had often misled philosophers, and had seldom been of real use in their enquiries. He avoided presumption, he had the necessary patience as well as genius; and having kept steadily to the right path, he therefore succeeded.

Experiments and observations, 'tis true, could not alone have carried him far in tracing the causes from their effects, and explaining the effects from their causes: a sublime geometry was his guide in this nice and difficult enquiry. This is the instrument, by which alone the machinery of a work, made with so much art, could be unfolded; and therefore he sought to carry it to the greatest height. Nor is it easy to discern, whether he has shewed greater skill, and been more successful, in improving and perfecting the instrument, or in applying it to use. He used to call his philosophy *experimental philosophy*, intimating, by the name, the essential difference there is betwixt it and those systems that are the product of genius and invention only. These could not long

long ſubſiſt; but his philoſophy, being founded on experiment and demonſtration, cannot fail till reaſon or the nature of things are changed.

In order to proceed with perfect ſecurity, and to put an end for ever to diſputes, he propoſed that, in our enquiries into nature, the methods of *analyſis* and *ſyntheſis* ſhould be both employed in a proper order; that we ſhould begin with the phænomena, or effects, and from them inveſtigate the powers or cauſes that operate in nature; that, from particular cauſes, we ſhould proceed to the more general ones, till the argument end in the moſt general: this is the method of *analyſis*. Being once poſſeſt of theſe cauſes, we ſhould then deſcend in a contrary order; and from them, as eſtabliſhed principles, explain all the phænomena that are their conſequences, and prove our explications: and this is the *ſyntheſis*. It is evident that, as in mathematics, ſo in natural philoſophy, the inveſtigation of difficult things by the method of *analyſis* ought ever to precede the method of compoſition, or the *ſyntheſis*. For in any other way, we can never be ſure that we aſſume the principles which really obtain in nature; and that our ſyſtem, after we have compoſed it with great labour, is not mere dream and illuſion.

By proceeding according to this method, he demonſtrated from obſervations, analytically, that gravity is a general principle; from which he afterwards explained the ſyſtem of the world. By *analyſis* he diſcovered new and wonderful properties of light, and, from theſe, accounted for many curious phænomena in a *ſynthetic* way. But while he was thus demonſtrating a great number of truths, he could not but meet with hints of many other things, that his ſagacity and diligent obſervation ſuggeſted to him, which

which he was not able to eſtabliſh with equal certainty: and as theſe were not to be neglected, but to be ſeparated with care from the others, he therefore collected them together, and propoſed them under the modeſt title of *queries*.

By diſtinguiſhing theſe ſo carefully from each other, he has done the greateſt ſervice to this part of learning, and has ſecured his philoſophy againſt any hazard of being diſproved or weakened by future diſcoveries. He has taken care to give nothing for demonſtration but what muſt ever be found ſuch; and having ſeparated from this what he owns is not ſo certain, he has opened matter for the enquiries of future ages, which may confirm and enlarge his doctrines, but can never refute them. He knew where to ſtop when experiments were wanting, and when the ſubtilty of nature carried things out of his reach: nor would he abuſe the great authority and reputation he had acquired, by delivering his opinion concerning theſe, otherwiſe than as matter of queſtion. It was long before he could be prevailed on to propoſe his opinion or conjectures concerning the cauſe of gravity; and what he has ſaid of it, and of the other powers that act on the minute particles of matter, is delivered with a modeſty and diffidence ſeldom to be met with amongſt philoſophers of a leſs name. Nor do they act in a conformity with the ſpirit of this philoſophy who ſpeak dogmatically on theſe ſubjects, till a clearer light from new obſervations and experiments brings them from the claſs of queries, and places them on the level of demonſtration.

3. Such was the method of our incomparable philoſopher, whoſe caution and modeſty will ever do him the greateſt honour in the opinion of the unprejudiced.

prejudiced. But this ſtrict method of proceeding was not reliſhed by thoſe who had been accuſtomed to treat philoſophy in a very different way, and who ſaw that, by following it, they muſt give up their favourite ſyſtems. His obſervations and reaſonings were unexceptionable; ſo, finding nothing to object to theſe, they endeavoured to leſſen the character of his philoſophy by general indirect inſinuations, and, ſometimes, by unjuſt calumnies. They pretended to find a reſemblance between his doctrines and the exploded tenets of the ſcholaſtic philoſophy. They triumphed mightily in treating gravity as an occult quality, becauſe he did not pretend to deduce this principle fully from its cauſe. His extending over all the ſyſtem a power which is ſo well known to us on the earth, and explaining by it the motions and influences of the celeſtial bodies, in the moſt ſatisfactory manner; and his determining the meaſures of the various motions that are conſequences of this power, by ſo ſkilful an application of geometry to nature; all theſe had no merit with ſuch philoſophers, becauſe he did not aſſign the mechanical cauſe of gravity. I know not that ever it was made an objection to the circulation of the blood that there is no ſmall difficulty in accounting for it mechanically; for they who firſt extended gravity to air, vapour, and to all bodies round the earth, had their praiſe, though the cauſe of gravity was as obſcure as before; or rather appeared more myſterious, after they had ſhewn that there was no body found near the earth, exempt from gravity, that might be ſuppoſed to be its cauſe. Why then were his admirable diſcoveries, by which this principle was extended over the univerſe, ſo ill reliſhed by ſome philoſophers? The truth is, he had, with great evidence, overthrown the boaſted ſchemes by which they pretended to unravel all the myſteries of nature; and the

the philoſophy he introduced, in place of them, carrying with it a ſincere confeſſion of our being far from a complete and perfect knowledge of it, could not pleaſe thoſe who had been accuſtomed to imagine themſelves poſſeſs'd of the eternal reaſons and primary cauſes of all things.

But to all ſuch as have juſt notions of the great author of the univerſe, and of his admirable workmanſhip, Sir *Iſaac Newton*'s caution and modeſty will recommend his philoſophy; and even the avowed imperfection of ſome parts of it will, to them, rather appear a conſequence of its conformity with nature. To ſuch, all complete and finiſhed ſyſtems muſt appear very ſuſpicious: they will not be ſurprized that refined ſpeculations, or even the labours of a few ages, are not ſufficient to unfold the whole conſtitution of things, and trace every phænomenon through all the chain of cauſes to the firſt cauſe. Is the admirable progreſs which has been made in this arduous purſuit to be deſpiſed or neglected, becauſe more remains behind undiſcovered? Surely we ought rather to rejoice that ſo much is opened to us of the conſummate art by which all things were made; and ought to be afraid to intermix with it our own extravagant conceits.

The proceſſes of nature lie ſo deep, that, after all the pains we can take, much, perhaps, will remain undiſcovered beyond the reach of human art or ſkill. But this is no reaſon why we ſhould give ourſelves up to the belief of fictions, be they ever ſo ingenious, inſtead of hearkening to the unerring voice of nature; for ſhe alone can guide us in her own labyrinths; and it is a conſequence of her real beauty, that the leaſt part of true philoſophy is incomparably more beautiful than the moſt complete ſyſtems which have been the product of invention. This is particularly true of Sir *Isaac Newton*'s philosophy; and we may compare it in this respect with those celebrated pieces of *Apelles*, which, though they never received his last hand, were in greater admiration amongst the ancients, than the most finished pieces of other artists: and we wish posterity may not find cause to say of this philosophy what the ancients said of those pieces,——*Ipsum defectum cessisse in gloriam artificis, nec qui succederet operi ad præscripta lineamenta inventum fuisse.* Plin.

4. It was, however, no new thing that this philosophy should meet with opposition. All the useful discoveries that were made in former times, and particularly in the last century, had to struggle with the prejudices of those who had accustomed themselves not so much as to think but in a certain systematic way; who could not be prevailed on to abandon their favourite schemes, while they were able to imagine the least pretext for continuing the dispute: every art and talent was displayed to support their falling cause; no aid seemed foreign to them that could in any manner annoy their adversary; and such often was their obstinacy, that truth was able to make little progress, till they were succeeded by younger persons who had not so strongly imbibed their prejudices.

Sir *Isaac Newton* had very early experience of this temper of philosophers, and appears to have been discouraged by it. He had a particular aversion to disputes, and was with difficulty induced to enter into any controversy. The warm opposition his admirable discoveries in optics met with, in his youth, deprived the world of a full account of them for many years, till there appeared a greater disposition among

among the learned to receive them; and induced him to retain other important inventions by him, from an apprehenſion of the diſputes in which a publication might involve him. He thus weighed the reaſons of things impartially and coolly, before a publication of them can be ſuſpected to have engaged him in their defence. It is well known how ſlow he was in publiſhing: and we cannot but obſerve that the temper and diſpoſition of mind, as well as the abilities of this great man, fitted him in a particular manner for penetrating far into nature and unfolding her harmony.

Nor did his averſion to diſputes proceed from the love of quiet only. Philoſophy had been in high eſteem of old, but had loſt its antient luſtre from the endleſs idle janglings that had ariſen amongſt the ſects; and could never recover it while a faculty of inventing a ſyſtem readily, and defending it obſtinately, were the admired talents of a philoſopher. While one age or ſect overturned for the moſt part the laborious productions of another, many of the wiſer ſort deſpaired of acquiring certainty in natural knowledge, and choſe rather to content themſelves with the general view of things, open to all men, than attach themſelves to ſchemes which produced no real fruit, and really led them farther from the truth. Our author therefore propoſed that all prejudices ſhould be laid aſide, and the genuine method of treating natural philoſophy, which we have deſcribed from him, ſhould be cloſely followed. By his adhering to it himſelf, we are ſecure that truth and nature are on his ſide; and by following the excellent models which he has given us, we may be able to make farther advances.

Others have pretended to explain the whole conſtitution of things by what they call clear ideas, and by by mere abſtracted ſpeculations. They expreſs a contempt * for that knowledge of cauſes which is derived from the contemplation of their effects, and are unwilling to condeſcend to any other ſcience than that of effects from their cauſes. Therefore they ſet out from the *firſt cauſe*; and from their ideas of him pretend to unfold the whole chain, and to trace a complete ſcheme of his works. This is the philoſophy that ſtands in oppoſition to our author's to this day. It flatters human vanity ſo much, and ſets out in ſo pompous a manner, that they who attend not to the unexhauſtible variety of nature, and conſider not how unequal the human powers are to ſo arduous an undertaking, are deluded by its promiſes. It may be doubted if ſuch a philoſophy lies within the reach of any created being; and it ſeems to be very plain that it far ſurpaſſes the reach of men. But ſince many are devoted to this phantom, and uſe all their art to adorn, and recommend it to more admirers, it will be neceſſary for the ſervice of truth, that, while we proceed, we have in view likewiſe the detection of this impoſture.

5. The view of nature which is the immediate object of ſenſe is very imperfect, and of a ſmall extent; but by the aſſiſtance of art, and the help of our reaſon, is enlarged till it loſes itſelf in an infinity on either hand. The immenſity of things on

* Perſpicuum eſt optimam philoſophandi viam nos ſequuturos, ſi, ex ipſius Dei cognitione, rerum ab eo creatarum explicationem deducere conemur, ut ita ſcientiam perfectiſſimam, quæ eſt effectuum per cauſas, acquiramus. *Cartes Princip.* part. II. § 22. Afterwards, having occaſion to ſpeak of the phænomena, he takes care to tell us, that he would not make uſe of them to prove any thing from them, becauſe he wanted to derive the knowledge of effects from their cauſes, and not reciprocally that of the cauſes from their effects. *Princip.* part III. § 4, &c.

the one ſide, and their minuteneſs on the other, carry them equally out of our reach, and conceal from us the far greater and more noble part of phyſical operations. As magnitude of every ſort, abſtractly conſidered, is capable of being increaſed to infinity, and is alſo diviſible without end; ſo we find that, in nature, the limits of the greateſt and leaſt dimenſions of things are actually placed at an immenſe diſtance from each other. We can perceive no bounds of the vaſt expanſe in which natural cauſes operate, and can fix no border or termination of the univerſe; and we are equally at a loſs when we endeavour to trace things to their elements, and to diſcover the limits which conclude the ſubdiviſions of matter. The objects which we commonly call great vaniſh when we contemplate the vaſt body of the earth; the terraqueous globe itſelf is ſoon loſt in the ſolar ſyſtem: in ſome parts it is ſeen as a diſtant Star. In great part it is unknown, or viſible only at rare times to vigilant obſervers, aſſiſted, perhaps, with an art like to that by which *Galileo* was enabled to diſcover ſo many new parts of the ſyſtem. The ſun itſelf dwindles into a ſtar; *Saturn*'s vaſt orbit, and the orbits of all the comets, croud into a point, when viewed from numberleſs places between the earth and the neareſt fix'd ſtars. Other ſuns kindle light to illuminate other ſyſtems where our ſun's rays are unperceived; but they alſo are ſwallowed up in the vaſt expanſe. Even all the ſyſtems of the ſtars that ſparkle in the cleareſt ſky muſt poſſeſs a ſmall corner only of that ſpace over which ſuch ſyſtems are diſperſed, ſince more ſtars are diſcovered in one conſtellation, by the teleſcope, than the naked eye perceives in the whole heavens *.

* In the conſtellation of *Orion*, 2000 ſtars have been numbered by aſtronomers.

After

After we have risen so high, and left all definite measures so far behind us, we find ourselves no nearer to a term or limit; for all this is nothing to what may be displayed in the infinite expanse, beyond the remotest stars that ever have been discovered.

If we descend in the scale of nature, towards the other limit, we find a like gradation from minute objects to others incomparably more subtile, and are led as far below sensible measures as we were before carried above them, by similar steps that soon become hid to us in equal obscurity. We have ground to believe that these subdivisions of matter have a termination, and that the elementary particles of bodies are solid and uncompounded, so as to undergo no alteration in the various operations of nature or of art. But from microscopical observations that discover animals, thousands of which could scarce form a particle perceptible to the unassisted sense, each of which have their proper vessels, and fluids circulating in those vessels; from the propagation, nourishment and growth of those animals; from the subtilty of the effluvia of bodies retaining their particular properties after so prodigious a rarefaction; from many astonishing experiments of chymists; and especially from the inconceivable minuteness of the particles of light, that find a passage equally in all directions through the pores of transparent bodies, and from the contrary properties of the different sides of the same ray †; it appears, that the subdivisions of the particles of bodies descend by a number of steps or degrees that surpasses all imagination, and that nature is unexhaustible by us on every side. Nor is it in the magnitude of bodies only that this endless gradation is to be observed. Of motions, some are

† *Newton's optics.* Query 26.

performed in moments of time; others are finiſhed in very long periods: ſome are too ſlow, and others too ſwift, to be perceptible by us. The tracing the chain of cauſes is the moſt noble purſuit of philoſophy; but we meet with no cauſe but what is, itſelf, to be conſidered as an effect, and are able to number but few links of the chain. In every kind of magnitude, there is a degree or ſort to which our ſenſe is proportion'd, the preception and knowledge of which is of greateſt uſe to mankind. The ſame is the ground-work of philoſophy *; for tho' all ſorts and degrees are equally the object of philoſophical ſpeculation; yet it is from thoſe which are proportioned to ſenſe that a philoſopher muſt ſet out in his enquiries, aſcending or deſcending afterwards as his purſuits may require. He does well indeed to take

* If we were to examine more particularly the ſituation of man in nature, we ſhould find reaſon to conclude, perhaps, that it is well adapted to one of his faculties and inclinations, for extending his knowledge, in ſuch a manner as might be conſiſtent with other duties incumbent upon him; and that they have not judged rightly who have compared him in this reſpect (*Spinoz.* Epiſt. 15.) with the animalcules in the blood diſcovered by microſcopes. He muſt be allowed to be the firſt being that pertains to this globe, which, for any thing we know, may be as conſiderable (not in magnitude, but in more valuable reſpects) as any in the ſolar ſyſtem, which is itſelf, perhaps, not inferior to any other ſyſtem in theſe parts of the vaſt expanſe. By occupying a lower place in nature, man might have more eaſily ſeen what paſſes amongſt the minute particles of matter, but he would have loſt more than he could have gained by this advantage. He would have been in no condition to inſtitute an analyſis of nature, in that caſe. On the other hand, we doubt not but there are excellent reaſons, why he ſhould not have acceſs to the diſtant parts of the ſyſtem, and muſt be contented at preſent with a very imperfect knowledge of them. The duties incumbent upon him, as a member of ſociety, might have ſuffered by too great an attention to them, or communication with them. Had he been indulged in a correſpondence with the planets, he next would have deſired to pry into the ſtate of the fixed ſtars, and at length to comprehend infinite ſpace.

his views from many points of ſight, and ſupply the defects of ſenſe by a well regulated imagination; nor is he to be confined by any limit in ſpace or time: but as his knowledge of nature is founded on the obſervation of ſenſible things, he muſt begin with theſe, and muſt often return to them, to examine his progreſs by them. Here is his ſecure hold; and as he ſets out from thence, ſo if he likewiſe trace not often his ſteps backwards with caution, he will be in hazard of loſing his way in the labyrinths of nature.

6. From this ſhort view of nature, and of the ſituation of man, conſidered as a ſpectator of its phænomena and as an enquirer into its conſtitution, we may form ſome judgment of the project of thoſe, who, in compoſing their ſyſtems, begin at the ſummit of the ſcale, and then, by clear ideas, pretend to deſcend through all its ſteps with great pomp and facility, ſo as in one view to explain all things. The proceſſes in experimental philoſophy are carried on in a different manner: the beginnings are leſs lofty, but the ſcheme improves as we ariſe from particular obſervations, to more general and more juſt views. It muſt be owned, indeed, that philoſophy would be perfect, if our view of nature, from the common objects of ſenſe, to the limits of the univerſe upwards, and to the elements of things downwards, was complete; and the powers or cauſes that operate in the whole were known. But if we compare the extent of this ſcheme with the powers of mankind, we ſhall be obliged to allow the neceſſity of taking it in parts, and of proceeding with all the caution and care we are capable of, in enquiring into each part. When we perceive ſuch wonders, as naturaliſts have diſcovered, in the minuteſt objects, ſhall we pretend to deſcribe ſo

easily the productions of infinite power in space, that is at the same time infinitely extended and infinitely divisible? Surely we may rather imagine, that in the whole, there will be matter for the enquiries and perpetual admiration of much more perfect beings.

It is not therefore the business of philosophy, in our present situation in the universe, to attempt to take in at once, in one view, the whole scheme of nature; but to extend, with great care and circumspection, our knowledge, by just steps, from sensible things, as far as our observations or reasonings from them will carry us, in our enquiries concerning either the greater motions and operations of nature, or her more subtile and hidden works. In this way Sir Isaac Newton proceeded in his discoveries: he established his account of the system of the world upon the best astronomical observations, on the one hand; and performed, himself, on the other, with the greatest address, the experiments by which he was enabled to pry into the more secret operations of nature, amongst the minute particles of matter. On either side he has extended our views very far, and has left valuable hints and intimations of what yet lies involved in obscurity.

For those purposes he has given us two incomparable treatises, the most perfect in their kind philosophy has to boast of; his mathematical *Principles* of Natural Philosophy, and his Treatise of *Optics*. In the first, he describes the system of the world, and demonstrates the powers which govern the celestial motions, and produce their mutual influences. These are extended from the center of the sun to the utmost altitude of the highest comet, and probably to the farthest limits of the universe. Nor are theſe new or abſtruſe principles, like to thoſe which never had a being but in the imagination of philoſophers, but the ſame which are moſt familiar to mankind, and in common uſe, farther extended and more accurately defined. In the ſecond, he treats of light, which, tho' the moſt potent agent in nature, that is ſenſible to us, acts only at the leaſt diſtances. His admirable diſcoveries, on this ſubject, led him to ſearch into the motions that are amongſt the minute particles of matter, the moſt abſtruſe of all natural phænomena.

In the firſt, he had the obſervations of aſtronomers for many ages to build on, with valuable conſequences that had been derived from them, by the laborious calculations of diligent and ingenious men. The conſtancy and regularity of the celeſtial motions had contributed, with the obſervations of ſome thouſands of years, to render aſtronomy the moſt exact part of the hiſtory of nature; the doctrine of comets only excepted. The vaſt diſtances of the great bodies which compoſe the ſyſtem, from each other, rather favoured a juſt *analyſis* of the powers by which they act on one another; ſince by the greatneſs of the diſtance, theſe muſt be reduced to a few ſimple principles, and be the more eaſily diſcovered. In the ſecond treatiſe, he enquires into more hidden parts of nature, and had moſt of the phænomena themſelves to trace, as well as their cauſes. The ſubject is rather more nice and difficult, becauſe of the inconceivable minuteneſs of the agents, and the ſubtilty and quickneſs of the motions; and the principles combined in producing the phænomena being more various, it could not be expected that they ſhould be ſo eaſily ſubjected to an *analyſis.* Hence it is that what he has delivered in the firſt (tho' ſtill capable of improvement) is more complete and finiſhed in ſeveral reſpects; while his diſcoveries of the ſecond ſort are more aſtoniſhing.

After having eſtabliſhed the principle of the univerſal Gravitation of Matter in the firſt treatiſe, when he is not able to demonſtrate the cauſes of the phænomena deſcribed in the ſecond more evidently, he endeavours to judge of them, by *analogy*, from what he had found in the greater motions of the ſyſtem; a way of reaſoning that is agreeable to the harmony of things, and to the old maxim aſcribed to *Hermes* *, and approved by the obſervation and judgment of the beſt philoſophers, " That what paſſes in the heavens above is ſimilar and analogous to what paſſes on earth below." He had found that all bodies gravitated towards each other, by a power that acts on all their particles equally at equal diſtances, and increaſes according to a ſtated law when the diſtance is diminiſhed. From a like principle, acting at leſs diſtances, with greater vigour, and with more variety, but inſenſibly at larger diſtances, he ſuſpected that the more abſtruſe phænomena of nature proceeded. It was a great matter in philoſophy to be ſecure of one general principle; and one was ſufficient for carrying on the regular motions of the heavenly bodies. A greater variety was neceſſary for conducting the different operations of nature in particular parts; and theſe being involved in ſome obſcurity, till better light ſhould appear, he could find no ſurer ground on which to found a judgment of them, than that principle he had already ſhewn to take place in nature. But be-

* A principle not unlike this is aſcribed to the *Perſian* and *Chaldean* magi, *συμπαθῆ εἶναι τὰ ἄνω τοῖς κατω*. *Pſell.* Declaratio dogmat. Chaldaic. Tho' this, as other maxims, was much abuſed in progreſs of time, when philoſophers degenerated from their firſt ſimplicity.

caufe we often find that phænomena, which, at firft fight, appear of a very different fort, flow nevertheless from the fame caufe, and feveral fuch caufes are often refolved, on farther enquiry, into one more general principle; the whole conftitution of nature (notwithftanding the variety of appearances) manifeftly leading to one fupreme caufe; this great philofopher was hence induced, as well as from feveral obfervations he had made, to think that all thefe powers might proceed from one general inftrument or agent, as various branches from one great ftem, whofe efficacy might be refolved more immediately into the direction or influences of the fovereign caufe that rules the univerfe. But he fpeaks of this in the manner that became a philofopher who had fo much ftudied nature, and knew how obfcure thofe arduous parts of her fcheme muft be to us.

7. As the moft obvious views of the creation fuggeft to all men the perfuafion of the being and government of a Deity; fo every difcovery in natural philofophy enforces it: and with this improvement of his difcoveries, this great man concludes both thofe treatifes. Nor is his philofophy to be thought of little fervice for this purpofe, tho' he has not been able to explain fully the primary caufes themfelves.

The great myfterious Being, who made and governs the whole fyftem, has fet a part of the chain of caufes in our view; but we find that, as he himfelf is too high for our comprehenfion, fo his more immediate inftruments in the univerfe, are alfo involved in an obfcurity that philofophy is not able to diffipate; and thus our veneration for the fupreme author is always increafed, in proportion as we advance in the knowledge of his works. As we arife in philoſophy towards the firſt cauſe, we obtain more extenſive views of the conſtitution of things, and ſee his influences more plainly. We perceive that we are approaching to him, from the ſimplicity and generality of the powers or laws we diſcover; from the difficulty we find to account for them mechanically; from the more and more complete beauty and contrivance, that appears to us in the ſcheme of his works as we advance; and from the hints we obtain of greater things yet out of our reach: but ſtill we find ourſelves at a diſtance from Him, the great ſource of all motion, power and efficacy; who, after all our enquiries, continues removed from us and veiled in darkneſs. He is not the object of ſenſe, his nature and eſſence are unfathomable; the more immediate inſtruments of his power and energy are but obſcurely known to us; the leaſt part of nature, when we endeavour to comprehend it, perplexes us; even *place* and *time*, of which our ideas ſeem to be ſimple and clear, have enough in them to embaraſs thoſe who allow nothing to be beyond the reach of their faculties. Theſe things, however, do not hinder but we may learn to form great and juſt conceptions of him from his ſenſible works, where an art and ſkill is expreſſed that is obvious to the moſt ſuperficial ſpectator, ſurprizes the moſt experienced enquirer, and many times ſurpaſſes the comprehenſion of the profoundeſt philoſopher. From what we are able to underſtand of nature, we may entertain the greater expectations of what will be diſcovered to us, if ever we ſhall be allowed to penetrate to the firſt cauſe himſelf, and ſee the whole ſcheme of his works as they are really derived from him, when our imperfect philoſophy ſhall be compleated.

CHAP. II.

Of the ſyſtems of the ancient philoſophers.

1. THoſe who have not imbibed the prejudices of philoſophers, are eaſily convinced that natural knowledge is to be founded on experiment and obſervation. But there is a philoſophy that intoxicates the mind, while it pretends to elevate and ſatisfy it, which teaches to deſpiſe the plain and ſober way of truth. And it is no eaſy matter to deal with thoſe who have loſt themſelves in the dark ſchemes of an inviolable and univerſal neceſſity, or with thoſe who are ever dreaming themſelves poſſeſt of the eternal reaſons and primary cauſes of things. The leaſt ſhew of an argument in their own viſionary way takes infinitely more with them, than the cleareſt evidence from fact or obſervation; and ſo fond they appear of ſuch airy ſchemes, that they would chuſe rather to go on diſputing for ever, than condeſcend to acquieſce in certainty obtained in a lower way.

To an impartial enquirer, Sir *Iſaac Newton*'s method, deſcribed in the laſt chapter, approves itſelf; and ſome ingenious men have been ſenſible of the neceſſity of following it, in former times. But the general practice of philoſophers has been very different; and ſyſtems founded on abſtracted ſpeculations ſtill ſo much prevail, that it will be neceſſary for our purpoſe to ſhew, by a few obſervations on the hiſtory of learning, how vain and fruitleſs ſuch attempts have always proved.

Theories

Theories of this kind have been invented, and amended again and ágain, with great labour and expence of thought; but ſtill when they came to compare them with nature, how wide has been the difference!—*ibi omnis effuſus labor.* If we look back into the ſtate of philoſophy in the different ages, we ſhall learn from the hiſtory of every period, that as far as philoſophers conſulted nature, and proceeded on obſervation, they made ſome progreſs in true knowledge; but as far as they pretended to carry on their ſchemes without this, they only multiplied diſputes.

The beginnings of learning, as of other things, are uncertain and obſcured with fables: we collect, however, from ſeveral teſtimonies, that the oldeſt and moſt celebrated philoſophers of *Phœnicia* and *Greece* made a *vacuum* and *atoms*, and the *gravity* of atoms, the firſt principles of their philoſophy *; whether theſe were ſuggeſted to them from their early obſervations of nature, before her plain appearances were obſcured by the imaginary ſchemes and the diſputes of ſpeculative men, or were derived from ſome other origin. Afterwards various ſyſtems appeared, but ſome traces of thoſe antient principles are for a long time to be diſcovered amongſt the doctrines of ſucceeding philoſophers, tho' interwoven with their own particular tenets;

* According to *Poſidonius* the ſtoick, as cited by *Strabo* and *Sextus Empiricus*, the doctrine of Atoms was more ancient than the times of the Trojan war, having been taught by *Moſchus* a Phœnician, the ſame probably meant by *Iamblichus*, when he tells us that *Pythagoras* converſed at *Sidon* with the prophets, the ſucceſſors of *Mochus* the phyſiologer. In thoſe early times the characters of lawgiver and philoſopher were united, and this *Mochus* is ſuppoſed by many to have been the ſame with *Moſes* the legiſlator of the *Jews*.

and

and what appears to be moſt uniform in the variety of their opinions ſeems to be derived from this ſource *. The more ancient atomiſts ſeem to have taught that there were living ſubſtances alſo, which pre-exiſted before the union of the ſyſtems of thoſe elementary corpuſcles, and continued to exiſt after their diſſolution. They ſaw the neceſſity of admitting active as well as paſſive principles, life as well as mechaniſm, throughout the world †. But this entire and genuine philoſophy was diſmembered afterwards, and from an affectation of ſimplicity, or for other reaſons, one ſort of permanent ſubſtance was thought ſufficient. One party retained the paſſive and ſluggiſh matter only, and from the fortuitous concourſe of its corpuſcles pretended to explain the formation of the univerſe. Others, more refined, aſcribed reality and permanency to active incorporeal ſubſtances chiefly, or only. And ſo ſimilar were their diviſions and diſputes to thoſe of our own times, that a third ſort ſeem to have rejected the reality of both, while they maintained that there was no ſtability of eſſence or knowledge any where to be found; that all being and knowledge was fantaſtical and relative only; that man was the meaſure of truth to himſelf in all things; and that every opinion or fancy of every one was true ‡. While one ſect thought that nothing was permanent, but that all things were in a continual flux or motion, and

* They taught that nothing was made out of nothing, that no ſubſtance is generated or deſtroyed, that colour and taſte are not in the objects, *&c.* which ſeem to be the genuine doctrines of this atomical philoſophy amongſt the Greeks. See *Ariſtot.* de anima, *Lib.* III. *Cap.* I. who aſcribes ſuch opinions to moſt of the phyſiologers before his time.

† See Dr. *Cudworth*'s intellectual ſyſtem of the univerſe. Book I. Chap. I.

‡ This was the doctrine of *Protagoras* the Abderite. *Plat.* Thætetus, *&c.*

others,

others, that all things conſiſted of one immoveable and infinite eſſence, it is no wonder that their ſucceſſors own themſelves at a loſs to underſtand their meaning ‖.—Oppoſition to each other ſeems to have driven them to extremes, and both aimed at too general and extenſive principles.

As to the particular tenets of *Thales*, and his ſucceſſors of the *Ionic* ſchool, the ſum of what we learn from the imperfect accounts we have of them is, that each overthrew what his predeceſſor had advanced; and met with the ſame treatment himſelf from his ſucceſſor. One of them is ſaid to have made water the principle of all things; another choſe air; a third fire; a fourth preferred earth; and ſome took them all in, and made theſe four the elements or principles of things. So early did the paſſion for ſyſtems begin, and diſputes in conſequence of ſuch precipitancy were unavoidable.

2. In the time of this uncertainty amongſt the phyſiologers (for ſuch all the more antient philoſophers were) *Socrates* appeared in the world. A ſublimity of genius, a ſimplicity of manners, a particular talent of inveſtigating truth and expoſing error, diſtinguiſhed this great man. In his youth he applied himſelf, as his predeceſſors had done, to natural knowledge, and endeavoured to reduce it to a method and principles. But after examining their ſchemes without receiving any ſatisfaction from them, he was too ſincere a lover of truth, and too juſt to mankind, to attempt to invent one of his own, or to diſſemble his ignorance of nature. He ſaw that imaginary knowledge was the greateſt obſtruction to true ſcience, and made thoſe who were puffed up with it very troubleſome to the lovers of ſolid learning.

‖ *Plat. Theætet.*

ing. He therefore took every occafion to expofe it, and had a happy talent in ridiculing the vanity of the fophifts of thofe times, who pretended to know all things. The oracle on a certain occafion had declared him the wifeft of men; and this preference he explained, with his ufual modefty, to be owing to this only, that while others vainly imagined they knew what they were indeed ignorant of, he knew this one thing more than they, "that he knew nothing."

After many other fruitlefs attempts he had made in his youth * to fee into the caufes of things, happening to hear that *Anaxagoras* taught that all things were governed by a fupreme mind, and being mightily pleafed with this principle, he had recourfe to his writings; full of expectation to fee the whole fcheme of nature explained from the perfect wifdom of an all-governing mind, and to have all his doubts about the perfection of the univerfe fatisfied. But he was much difappointed, when he found that *Anaxagoras* made no ufe of this fovereign mind in his explications of nature, and referred nothing to the order and perfection of the univerfe as its reafon; but introduced certain aereal, æthereal and aqueous powers, and fuch incredible principles for the caufes of things. Upon the whole, *Socrates* found that this account of nature was no more fatisfactory, than if one who undertook to account for all the actions of *Socrates*, fhould begin with telling that *Socrates* was acted by a principle of thought and defign; and pretending to explain how he came to be fitting in prifon at that time, when he was condemned to die by the unjuft and ungrateful *Athenians*, he fhould acquaint us that the body of *Socrates* confifted of bones and mufcles, that the

* ἐγὼ γάρ νέος ὤν,

bones were ſolid and had their articulations, while the muſcles were capable of being contracted and extended, by which he was enabled to move his body and put himſelf in a ſitting poſture; and after adding an explication of the nature of ſound, and of the organs of his voice, he ſhould boaſt at length that he had thus accounted for *Socrates*'s ſitting and converſing with his friends in priſon; without taking notice of the decree of the *Athenians*, and that he himſelf thought it was more juſt and becoming to wait patiently for the execution of their ſentence, than eſcape to *Megara* or *Thebes*, there to live in exile. "'Tis true, ſays he, that without bones and "nerves I ſhould not be able to perform any action "in life, but it would be an unaccountable way of "ſpeaking to aſſign thoſe for the reaſons of my ac-"tions, while my mind is influenced by the appear-"ance of what is *beſt*."

I have taken notice of this paſſage the rather, becauſe it ſhews how eſſential the greateſt and beſt philoſophers have thought the conſideration of final cauſes to be to true philoſophy; without which it wants the greateſt beauty, perfection and uſe. It gave a particular pleaſure to Sir *Iſaac Newton* to ſee that his philoſophy had contributed to promote an attention to them (as I have heard him obſerve) after *Des Cartes* and others had endeavoured to baniſh them. It is ſurprizing that this author ſhould repreſent it as greater preſumption in us * to aim at the

* *Princip.* Part I. § 28. Nullas unquam rationes circa res naturales a fine, quem Deus aut natura in iis faciendis ſibi propoſuit, deſumemus; quia non tantum debemus nobis arrogare ut ejus conſiliorum participes nos eſſe putemus; ſed ipſum ut cauſam efficientem rerum omnium conſiderantes, videbimus quidnam, ex iis ejus attributis quorum nos nonnullam notitiam voluit habere, circa illos ejus effectus, qui ſenſibus noſtris apparent, lumen naturale quod nobis indidit concludendum eſſe oſtendat.

knowledge of final caufes, than to attempt to derive a complete fyftem of the univerfe from the nature of the Deity, confidered as the fupreme efficient caufe, or, after difcarding mental and final caufality, to refolve all into mechanifm and metaphyfical or material neceffity. Surely this is the fort of caufes that is moft clearly placed in our view; and we cannot comprehend why it fhould be thought arrogant in us, to attend to the defign and contrivance that is fo evidently difplayed in nature, and obvious to all men; to maintain, for inftance, that the eye was made for feeing, tho' we may not be able either to account mechanically for the refraction of light in the coats of the eye, or to explain how the image is propagated from the retina to the mind.

Socrates, finding all dark and uncertain in the various fyftems of his predeceffors, was fatisfied that it was better to reft contented with the general view of nature open to all, than adopt any one of them; and having applied himfelf to promote the practice as well as the theory of moral philofophy amongft his fellow citizens, by his example and precepts, he merited the higheft efteem and admiration of mankind *. *Plato*, however, and his followers, being fenfible of the influence which natural knowledge

* See *Aul. Gellius*, Lib. 6. ch. 10. where an extraordinary inftance of this is given from *Taurus* a Platonic philofopher. The *Athenians*, upon fome difference with the inhabitants of *Megara*, made it capital for any of them to enter *Athens*. *Euclid* of *Megara*, after this edict, ufed to difguife himfelf as a woman, and travel twenty miles in the night to hear *Socrates*. Whence *Taurus* takes occafion to lament how much philofophy was funk in efteem in his time. Now, fays he, we fee philofophers run of their own accord to attend at the gates of the young and rich, and there fit waiting to noon till their difciples have flept out their laft night's debauch. *Diogenes Laertius*, however, fpeaks of a ftranger who came to *Athens* and found fault with *Socrates* in fome things.

must have on the most important truths, returned to it. The beauty of the universe was the favourite subject of the Platonists; and they used to recommend the contemplation and imitation of its regular and constant motions, by the practice of virtue, as the best means to recover their antient conformity with it in a prior state, and to become worthy of returning to the same state again. While a sect of the Atomists resolved all things into the motions and modifications of matter, *Plato* strove to raise the thoughts of men above the objects of sense, and zealously maintained the pre-eminence of active, incorporeal and intellectual beings. These, according to him, are the true substances, the other the shadows; which last only, those gross philosophers could perceive; as he who has his back towards the light sees it not, or the bodies placed betwixt him and it, but the images projected from them only *. He speaks, however, sometimes of the insensible particles of bodies, which can only be perceived by the mind and understanding, ascribing different figures to them in the style of the atomical philosophy †. If he carried his fondness for his ideas too far, we must own, at least, that he erred on the most innocent side of the question, in opposition to the dangerous doctrines of *Democritus* and others. But however laudable the views of this amiable philosopher may have been, surely the unintelligible mystical doctrines of some of his followers ‡ ought to admonish us to be on our guard against excesses, even in a good cause.

* *Plato* de republica, Lib. 7. & 10.

† *Plat.* Timæus.

‡ It were unnecessary to cite here instances of the most profound mysticism from *Plotinus* and other platonists.

3. In

3. In the mean time the followers of *Pythagoras* flouriſhed in *Italy*, and taught a philoſophy that does not appear to have been ſo much the reſult of their own obſervations, as to have been tranſplanted from the eaſt by their great maſter; who ſpent two and twenty years in thoſe parts, and ſcrupled not to comply with the cuſtoms * moſt peculiar to the eaſtern nations, in order to obtain the freer acceſs to their learned men. And as he was a man of extraordinary qualities and at the moſt pains, ſo he ſeems to have been the moſt ſucceſsful of the ancients in getting acquainted with their philoſophy. We find that his followers taught the true account of the planetary motions, particularly that the earth moved daily on its own axis, and revolved annually round the ſun; and gave the ſame account of the comets which is agreeable to modern diſcoveries †. They alſo taught that every ſtar was a world ‡, and that each of them had ſomething correſponding to our earth, air, and water, in the vaſt expanſe. The moon particularly, according to them, was inhabited by larger and more beautiful animals than this globe. We find ſome hints concerning the gravitation of celeſtial bodies, in what is related of the doctrines of *Thales* and his ſucceſſors: but *Pythagoras* ſeems to have been better acquainted with it, and is ſuppoſed to have had a view to it, in what he taught concerning the harmony of the ſpheres §.

* He was circumciſed in *Egypt* after the manner of the prieſts of that country, and is ſaid to have been the moſt graceful perſon of his time. *Clem. Alexandr.* Strom. Lib I.

† *Ariſtot.* Meteorol. Lib. I. cap. 6 *Plutarch.* de placitis philoſoph. Lib. III cap. 2.

‡ Ibid. cap. 13, & 30.

§ *Plin.* Lib. II. cap 22. *Macrob.* in ſomnium *Scip.* Lib. II. cap. 1. See alſo *Plutarch* de animal procreatione, è Timæo. ὅιτε παλαι θεολόγοι, πρεσβύτατοι φιλοσόφων ὄργανα μȣσικα θεῶν, &c. to the end.

A muſical chord gives the ſame notes as one double in length, when the tenſion or force with which the latter is ſtretched is quadruple: and the gravity of a planet is quadruple of the gravity of a planet at a double diſtance. In general, that any muſical chord may become uniſon to a leſſer chord of the ſame kind, its tenſion muſt be increaſed in the ſame proportion as the ſquare of its length is greater; and that the gravity of a planet may become equal to the gravity of another planet nearer to the ſun, it muſt be increaſed in proportion as the ſquare of its diſtance from the ſun is greater. If therefore we ſhould ſuppoſe muſical chords extended from the ſun to each planet, that all theſe chords might become uniſon, it would be requiſite to increaſe or diminiſh their tenſions in the ſame proportions as would be ſufficient to render the gravities of the planets equal. And from the ſimilitude of thoſe proportions, the celebrated doctrine of the harmony of the ſpheres is ſuppoſed to have been derived.

As theſe doctrines of the *Pythagoreans*, concerning the diurnal and annual motions of the earth, the revolutions of the comets, the inhabitants of the moon and ſtars, and the harmony of the ſpheres, are very remote from the ſuggeſtions of ſenſe, and oppoſite to vulgar prejudices; ſo we cannot but ſuppoſe that they who firſt diſcovered them muſt have made a very conſiderable progreſs in aſtronomy and natural philoſophy. It is no eaſy matter to perſuade a perſon unacquainted with the true theory of motion, that the earth, which of all things in nature appears to be moſt fixed and ſtable, is carried on in ſuch a manner, and with ſo much rapidity, in the expanſe. To be ſatisfied of theſe doctrines, ſo as to reckon the earth amongſt the ſtars, and conſider the

the ſtars as ſo many worlds, one muſt have got over many difficulties from ſenſe as well as from the religious prejudices that prevailed in thoſe days When therefore we find the accounts of them given by the *Greeks* to be very imperfect, mixed with errors and miſrepreſentations, it ſeems reaſonable to ſuppoſe that they had ſome hints of them only from ſome more knowing nations who had made greater advances in philoſophy; and that they were able to deſcribe them perhaps not much better than we may imagine an ingenious *Indian*, after paſſing ſome years in *Europe*, and having had ſome acceſs to learned men, would repreſent our ſyſtems to his countrymen after his return. Hence it was that the *Pythagoreans* do not ſeem to have been in a condition to defend their doctrines, tho' true; and *Ariſtotle* refutes them with the appearance of reaſon on his ſide. What he ſays of their ſyſtem ſhews that either it was not deſcribed rightly by them, or that he miſunderſtood them. We are told that they taught that there was an earth oppoſite to our earth, and ſeveral other bodies revolving about the ſun which were concealed from us by the earth, and that from this they explained why there were more eclipſes of the moon than of the ſun *. On this occaſion he urges againſt them a complaint, for which philoſophers have too often given ground, " That inſtead of ſuiting their " philoſophy to nature, they had miſrepreſented the " phænomena, that they might appear conformable " to their own ſuppoſitions." But had he been

* De cœlo, lib. II. cap. 15. We may be the leſs ſurprized that the *Greeks* had ſo imperfect accounts of the eaſtern learning, if it be true that ſome of the moſt noted amongſt their philoſophers, travelled into *Egypt* from a very different view than acquiring their philoſophy. *Plato*'s chief view is ſaid to have been to ſell his oyl.

better acquainted with the phænomena and this ſyſtem, he had formed a better judgment of it.

At this time geometry was in high eſteem. We have reaſon to think that the fondneſs of the *Pythagoreans* and *Platoniſts* for it ſometimes miſled them, by inducing them to derive the myſteries of nature from ſuch analogies of figures and numbers as are not only unintelligible to us, but in ſome caſes ſeem not capable of any juſt explication. The uſe they made of the five regular ſolids in philoſophy is a remarkable inſtance of this, and muſt have been a very important part of their ſcheme, if we may depend upon the antient commentators on *Euclid*; who tell us that he was a platonic philoſopher, and compoſed his excellent elements for the ſake of this doctrine. But as it is a matter of pure ſpeculation, we cannot conceive that there can be any analogy between it and the conſtitution of nature; and they have not been ſucceſsful who have of late endeavoured to explain this analogy; as we ſhall have occaſion to ſhew afterwards, when we come to give ſome account of *Kepler*'s diſcoveries. Nor is this the only inſtance, where a purſuit of analogies and harmonies has led us into error, in philoſophy. Geometry can be of little uſe in it till *data* are collected to build on, and Lord *Verulam* has juſtly obſerved, *Matheſin philoſophiam naturalem terminare debere, non generare aut procreare.*

4. From *Ariſtotle*'s philoſophy we may learn, that the greateſt penetration, without other helps, will ever be of leſs ſervice in enquiries into nature, than in metaphyſics and dialectics; where the force of genius may indeed atchieve wonders. Inſtead of the more antient ſyſtems, he introduced *matter*, *form*, and *privation* as the principles of all things: but it does

does not appear that this doctrine was of great uſe to him in natural philoſophy. He ſurpaſſed all the other philoſophers, in ſtating the diviſions and definitions relating to his ſubjects, with peculiar accuracy; yet ſome of his doctrines are ſo obſcurely expreſſed, according to the confeſſion of his moſt devoted diſciples, that tho' they took the utmoſt pains to diſcover his meaning (and ſome of them, as is reported, in a very extraordinary manner) they were not able to penetrate into it; and it is diſputed to this day what were his ſentiments on ſome of the moſt important ſubjects.

He was enabled by the liberality of his pupil *Alexander* to make vaſt collections relating to the hiſtory of nature, at an immenſe expence, which have been often copied by natural hiſtorians ſince *. But in his general and theoretical writings concerning nature, tho' his reaſonings may appear acute and ſubtle, the concluſions are commonly ſuch as are overthrown by later diſcoveries. How he deſcribed the *Pythagorean* doctrine concerning the two-fold motion of the earth, and endeavoured to refute it, we obſerved above: in one of the treatiſes that are aſcribed to him †, the author pretends to demonſtrate that the matter of the heavens is ungenerated, incorruptible, and ſubject to no alteration; and ſuppoſes the ſtars to be carried round the earth in ſolid orbs. In theſe doctrines he was generally followed, till *Tycho* by his obſervations, and *Galileo* by his arguments, expoſed their fallacy. Some have complained that there is leſs mention of a Deity, in his

* According to *Pliny*, *Ariſtotle* wrote fifty volumes concerning animals, and ſeveral thouſand perſons in *Greece* and *Aſia*, by *Alexander*'s orders, aſſiſted him in his enquiries. The expence is ſaid to have amounted to eighty talents.

† De cœlo.

extensive and various works, than in most of the antient philosophers; that ΠΕΡΙ ΚΟΣΜΟΥ, (or, as some say it ought to be entitled, ΠΕΡΙ ΠΑΝΤΟΣ) excepted; which for this reason has been ascribed to another author. But there are many who judge this admirable piece to be *Aristotle*'s; and *Gassendus* is of opinion that he composed it towards the end of his life, as the result of his most serious thoughts *.

It may be observed in favour of this great philosopher, that perhaps he did not intend his discoveries should be well understood from his public writings; for we are told that when his pupil * complained of his publishing some of his treatises, he insinuated, by his answer, that they would be understood by philosophers only. Had we a more perfect account of his doctrines concerning forms and qualities, possibly they might appear in a better light: perhaps he meant only to assert, in opposition to that branch of the atomists who followed *Democritus*, that the phænomena of nature could not be accounted for from matter and motion only; but that the qualities of bodies arise from hidden powers acting variously on different combinations of the particles of matter, according to the laws established. The conduct of *Callisthenes*, whom he recommended to *Alexander* to accompany him in his *Asiatic* conquests, does great honour to *Aristotle*: A prosecution however, carried on by the *Athenian* priests, obliged him to abandon their city, to avoid the fate of *Socrates*.

Aristotle was for a long time called the prince of philosophers; and possessed the most absolute authority in the schools, not in *Europe* only, but even in

* De physiologia *Epicuri*.

Africa,

Africa, amongſt Mahometans as well as Chriſtians. They had tranſlations of his works in *Perſia* and at *Samarcand*; and no philoſopher ever acquired ſo univerſal or ſo high an eſteem. His opinion was allowed to ſtand on a level with reaſon itſelf; nor was there any appeal from it admitted, the parties, in every diſpute, being obliged to ſhew that their concluſions were no leſs conformable to *Ariſtotle*'s doctrine than to truth. This, however, did not put an end to diſputes, but rather ſerved to multiply them; for neither was it eaſier to aſcertain his meaning than to come at the truth, nor was his doctrine conſiſtent with itſelf. It is not improper to have this ſlaviſh ſubjection of philoſophers in remembrance; becauſe an high eſteem for great men is apt to make us devoted to their opinions even in doubtful matters, and ſometimes in ſuch as are foreign to philoſophy.

5. We have already mentioned the Epicurean ſyſtem, and ſhall have occaſion frequently to make remarks upon it afterwards. Whoever conſiders the extravagant doctrines of this ſect, and of the other Dogmatiſts, of whatever denomination, Peripatetics or Stoics, may admire ſome of them for their morality, and more for their eloquence, it having been their chief buſineſs to diſpute for their ſchemes and declaim upon them; but cannot be greatly ſurprized that, as to what relates to natural knowledge, ſo many joined the ſceptics; and either maintained that it was impoſſible to diſcover truth, with ſome of them; or with others, that men were only in purſuit, not in poſſeſſion of it. The ſects, and ſubdiviſions of ſects, at length became ſo numerous, and their ſyſtems ſo various, that almoſt every perſon of any note addicted himſelf in ſome degree to philoſophy: for none could be at a loſs to find a ſect and doctrine ſuited to his taſte and inclination. But it does not

appear that this great increaſe of philoſophers contributed much to the advancement of the ſcience, or did ſervice to truth: ſuch was their licentiouſneſs, and ſo great the variety of their opinions, that there has hardly appeared any doctrine, in later times, but may be ſupported by the authority of one or other of them. It has been juſtly obſerved that we may learn ſomething from the faults and miſtakes of others, in every art; but we do not find that the errors of one ſect in philoſophy ſerved to put others on their guard. The great maſters we have mentioned had given an unhappy example; and their ſucceſſors exceeded them in grafting one fiction upon another, to ſerve their purpoſes. Thus the Platoniſts became unintelligible myſtics, and the Peripatetics unwearied diſputants; while every ſect had its tale or ſcheme, magnified by the party, but condemned by all the reſt.

When the antients, however, applied themſelves to conſider the heavens, or to collect the hiſtory of nature, they did not loſe their labour; their obſervations, ſometimes, ſuggeſted to them imperfect views of the true cauſes which obtain in the univerſe: and we have reaſon to admire ſome hints of this kind that appear in ſeveral paſſages of their writings, and ſeem to be anticipations of ſome of the moſt valuable modern diſcoveries. But, generally ſpeaking, they indulged themſelves too much in abſtruſe fruitleſs diſquiſitions concerning the hidden eſſences of things, and ſought after a knowledge that was not ſuited to the grounds they had to build on. As to their accounts of the ſyſtem of the world, the *Pythagorean* doctrines were quite forgot, and the opinions of *Ariſtotle* and *Eudoxus* univerſally prevailed. In proceſs of time great liberties were taken with nature, ſolid orbs and epicycles were multiplied,

multiplied, to anſwer every appearance, till the univerſe in their deſcriptions loſt its native beauty, and ſeemed reduced to a chaos again by their unhappy labours.

It is not worth while, nor of uſe for our purpoſe, to trace the hiſtory of learning thro' its various revolutions in the later ages, when philoſophy and philoſophers fell into contempt; when they became more diſtinguiſhed by their extravagant opinions, manners and temper *, than by any real knowledge and merit. How different they were, ſo early as in the times of the *Cæſars*, from the famous *Pythagorean* lawgivers, the incomparable *Socrates*, and others who adorned the firſt ages of philoſophy, we may learn from the picture given of them by *Tacitus*. "*Nero*, ſays this author, uſed to beſtow ſome time "after meals in hearing the reaſonings of different "philoſophers, and while each maintained his own "ſect, and every one expreſly contradicted another, "they all conſpired to expoſe their endleſs variance "and broils, as well as to diſplay their peculiar "and favourite opinions; nay, there were ſome of "thoſe ſolemn maſters of wiſdom, highly fond of "being ſeen with their gloomy aſpect and rigid ac"cent, amidſt the royal exceſſes and recreations "of *Nero* †.

* Sapientiam capillis et habitu jactant, ſays *Lactantius* ſpeaking of them. See alſo the complaint of *Taurus* the philoſopher, cited from *Aul. Gellius* above in the notes on § 2. of this chapter.

† Tacit. annal. lib 14. We have ſaid nothing of the *Chineſe*, for tho' no nation has applied to aſtronomy for ſo long a time, or with ſo much encouragement from the public, they ſeem to have made little progreſs, by the accounts we have of them: this may be aſcribed, in part at leaſt, to their neglect of geometry (without which it is impoſſible to make great advances in aſtronomy) and their having no correſpondence with other nations.

But the ſtate of learning proved ſtill more deplorable in a later period; that ought to be remembered, becauſe it diſcovers to us the moſt cruel enemy to true philoſophy. 'Twas ſometime after the fall of the *Roman empire*, when the majeſty and policy of that people had given way to *Gothic* barbarity, that ſuperſtition reigned uncontrouled, liberty of enquiry was proſcribed, and a ſavage zeal ſought to root out the memory of antient learning, by deſtroying the records of it, the ineſtimable product of the labours of paſt times. The fatal ſcheme proved but too ſucceſsful, for ſoon a thick cloud ſeems to have darkened the underſtandings of men, and to have almoſt extinguiſhed their natural faculties; in ſo much that a part of the ſucceeding times obtained the appellation of the leaden ages, as worſe than the iron age of the poets. Authority for a long time uſurped the place of reaſon, and, under the abuſed pretence of making them more ſubmiſſive to heaven, mankind were enſlaved and degraded. Here and there ſome appeared worthy of better times; but theſe were obliged to conform to the genius of that barbarous age: if they applied to true philoſophy, it was either in a private and myſterious manner, or their abilities and merit ſerved only to provoke ſevere and cruel treatment from their bigotted cotemporaries. This was the fate of the famous *Roger Bacon*, who appears to have made ſurprizing advances in natural knowledge, for thoſe times, and ſeems to have been acquainted with ſome inventions that are moſt commonly ſuppoſed to be of a later date.

Learning, neglected and deſpiſed in *Europe*, found a ſanctuary amongſt the *Saracens*, to whom we are indebted for ſeveral inventions, as well as for the pre-

preſervation of ſome of the works of the antients. They had ſo great a value for theſe, that it was uſual with them to demand copies of them, by particular articles, in their treaties with the *Greek* emperors; tho' they had deſtroyed an ineſtimable treaſure of this kind, at *Alexandria*, in their firſt conqueſts. The caliph *Almaimon* is celebrated for encouraging aſtronomical learning, erecting a great number of obſervatories over his dominions, and providing them with inſtruments of a prodigious ſize. By his order, a degree of the circle of the earth, was, firſt, meaſured with exactneſs, as far as we know. But, at length, their philoſophers ſeem to have devoted themſelves abſolutely to *Ariſtotle*, in no leſs ſlaviſh a manner than the *Europeans*; and to a talkative philoſophy that ſerved only to produce endleſs diſputes.

The cloud was, at length, gradually diſpell'd in *Europe*: the active genius of man could not be enſlaved for ever. The love of knowledge revived, the remains of antient learning, that had eſcaped the wreck of the dark ages, were diligently ſought after; the liberal arts and ſciences were reſtored, and none of them has gained more by this happy revolution than natural philoſophy.

CHAP. III.

Of the modern philoſophers before Des Cartes.

1. THE revolutions of learning were compared, by *Ariſtotle*, to the riſing and ſetting of the ſtars; and *Pliny* ſpeaks of four periods of it that preceded his time, the *Egyptian*, *Aſſyrian*, *Chaldean*, and *Grecian*. Learning, after it was once loſt in thoſe countries, has never revived again; and, of

of the produce of three of thoſe periods, there is little or nothing left. The weſtern parts of *Europe* have been more happy. After a long interval, learning has returned to them; and the period which commenced upon the revolution we have mentioned, has already continued ſome hundred years. It was uſhered in by ſeveral inventions of the greateſt uſe. If we may judge from theſe, from the valuable diſcoveries that have been made in its progreſs, and from thoſe which learned men are ſtill in purſuit of, (which afford matter for their enquiries, and at the ſame time keep up their curioſity and expectation) we may juſtly hope that it will be long ere it comes to an end: and if it ſhould likewiſe have its termination, it cannot, however, but be ever memorable in the hiſtory of learning, in future times; unleſs a general oblivion overwhelm all memory and record.

The invention of convex and concave glaſſes was as old as the thirteenth century, tho' no one thought of putting two of them together to make a teleſcope, till three hundred years later. Upon which it has been juſtly obſerved, that thoſe things which we handle daily may have valuable properties altogether unknown to us, which chance, or future tryals, may diſcover. The polarity of the magnetic needle, which was made uſe of in navigation early in the fourteenth century (if not ſooner) facilitated the correſpondence between diſtinct nations, and conducted *Columbus* to the diſcovery of the new world. It is obvious how advantageous to learning the art of printing has proved, which we owe to the ſame century. Theſe, with ſeveral other new and ſurprizing inventions, produced a great change in the affairs of the world; and a ſpirit of reformation ſoon ſhewed itſelf,

itſelf, in every thing that had any connexion with the arts and ſciences.

2. *Peurbachius*, with his ſcholar *Regiomontanus* and others, revived aſtronomical learning, in the fourteenth century. The celebrated *Copernicus* (who was born at *Thorn* in *Pruſſia* in 1473) ſucceeded them, " a man, ſays *Kepler*, * of a vaſt genius, and " what is of great moment in theſe matters, of a " free mind." When he conſidered the form, diſpoſition and motions of the ſyſtem, as they were then repreſented after *Ptolemy*, he found the whole void of order, ſymmetry and proportion; like a piece (as he expreſſes himſelf) made up of parts copied from different originals, which not fitting each other, ſhould rather repreſent a monſter than a man. He therefore peruſed the writings of the antient philoſophers, to ſee whether any more rational account had ever been propoſed of the motions of the heavens. The firſt hint he had was from *Cicero*, who tells us, in his *academical queſtions* (book 4.) that *Nicetas* a *Syracuſian* had taught that the earth turned round on its axis, which made the whole heavens to appear to a ſpectator on the earth to turn round it daily. Afterwards, from *Plutarch* †, he found that *Philolaus* the Pythagorean had taught that the earth moved annually round the ſun. He immediately perceived that, by allowing theſe two motions, all the perplexity, diſorder and confuſion, he had complained of in the celeſtial motions, vaniſhed, and that, inſtead of theſe, a ſimple regular diſpoſition of the orbits, and a harmony of the motions appeared, worthy of the great author of the world.

* Prefatio ad *Paulum* III. pontif. max.

† De placitis philoſophorum, lib. 3. cap. 13.

'Twas

'Twas ſoon after the year 1500 he began to form this judgment of the ſyſtem, in his own thoughts: but being ſenſible how ill it would be received by the generality of men, and even of the learned of that time, he could not be induced to publiſh his account of the celeſtial motions, for more than thirty years. He had a great inclination, as he tells us, to have followed the manner of the *Pythagoreans*, who would not publiſh their myſteries to the world, but choſe rather to deliver them from hand to hand to poſterity; not that they envied others the knowledge of them, but that the beautiful diſcoveries of great men, the fruit of all their labours, might not become the ſport of the preſumptuous and ignorant. It was not without the greateſt ſollicitations, and much ſtruggling on his part, that at length he gave his papers to his friends, with permiſſion to publiſh them; and he lived only to ſee a copy of his book in 1543, a few hours before his death.

In this treatiſe, he reſtores the antient *Pythagorean* ſyſtem, and deduces the appearances of the celeſtial motions from it. Every age ſince has produced new arguments for it; and, notwithſtanding the oppoſition it met with, from the prejudices of ſenſe againſt the earth's motion, the authority of *Ariſtotle* in the ſchools, the threats of ignorant bigots, and the terror of the inquiſition, it has gradually prevailed. The chief argument that had induced *Ariſtotle*, and his followers, to conſider the earth as the centre of the univerſe, was that all bodies have a tendency towards the centre of the earth. In anſwer to this, *Copernicus* * obſerved, that it was reaſonable to think there

* Equidem exiſtimo gravitatem non aliud eſſe quam appetentiam quandam naturalem, partibus inditam a divina providentia opificis

there was nothing peculiar to the earth in this principle of gravity; that the parts of the ſun, moon, and ſtars, tended likewiſe to each other, and that their ſpherical figure was preſerved in their various motions by this power. Thus every ſtep in true knowledge gives a glimpſe or faint view of what lies next beyond it, tho' yet unrevealed, in the ſcale of nature.

3. The reſtoration of the *Pythagorean* ſyſtem was a ſtep of the utmoſt importance in true philoſophy, and paved the way for greater diſcoveries; but the minds of men were not ſufficiently prepared for it, at that time. A juſt account of the theory of motion was wanting to make them ſenſible of its ſimplicity and beauty, and to enable them to reſolve, in a ſatisfactory manner, the obvious arguments that appeared againſt it. According to *Copernicus*, the earth revolved on its axis, with a rapid motion, from weſt to eaſt. It was objected, that ſuch a motion could not but have ſenſible effects on many occaſions; that a ſtone, for inſtance, drop'd from the ſummit of a tower, ought to ſtrike the ground, not at the foot of the tower, but at a diſtance weſtward, according to this doctrine; the tower being carried, by the diurnal motion, towards the eaſt, while the ſtone was falling. In anſwer to this, the motion of the earth was compared to the uniform progreſſive motion of a ſhip at ſea; and it was affirmed, that a ſtone drop'd from the top of the maſt would ſtrike the deck at the foot of it, tho' the ſhip was under

opificis univerſorum, ut in unitatem integritatemque ſuam ſeſe conferant, in formam globi coeuntes. Quam affectionem credibile eſt etiam ſoli, lunæ, cæteriſq; errantium fulgoribus, ineſſe, ut ejus efficaciâ in eâ qua ſe repreſentant rotunditate permaneant; quæ nihilominus multis modis ſuos efficiunt circuitus. *Nicol. Copernici* revol. lib. 1. cap. 9.

ſail, and advanced at a great rate while the ſtone was falling. This experiment is now beyond a l queſtion: but ſome, who tried it without due care and attention, having reported to *Tycho Brahe* that it had not ſucceeded *, this, with a miſtaken zeal for the ſacred writings, and perhaps an ambition of being the inventor of a new ſyſtem, induced him to reject the doctrine of *Copernicus*, and propoſe a middle ſcheme. *Tycho* was too well acquainted with the planetary motions to ſuppoſe their centre any where elſe than in the ſun; but that the earth might be quieſcent, he ſuppoſed the ſun, with all the planets, to be carried annually around it, while theſe, by their proper motions, revolved about the ſun in their ſeveral periods. Having rejected the diurnal rotation of the earth on its axis, he was obliged to retain the moſt ſhocking part of the *Ptolemaick* ſyſtem, and to ſuppoſe that the whole univerſe, to its fartheſt viſible limits, was carried, by the *primum mobile*, about the axis of the earth every day. In this, however, he was abandoned by ſome of his followers, who choſe rather to ſave this immenſe labour to all the ſpheres, by aſcribing the diurnal motion to the earth, with *Copernicus*; and therefore were called *Semi-Tychonics*.

Tho' this noble *Dane* was not happy in eſtabliſhing a new ſyſtem, he did great ſervice, however, to aſtronomy, by his diligence and exactneſs in making obſervations, for a long ſeries of years. He diſcovered the refraction of the air, and determined the places of a great number of the fixed ſtars, with an accuracy unknown to the aſtronomers of former times. He demonſtrated that the comets were higher than the moon, from their having a very ſmall parallax, againſt the opinion which then pre-

* *Gaſſend.* in vita *Tychonis.*

vailed.

vailed. He difcovered what is called the *variation* in the motion of the moon; and, from his feries of obfervations on the other planets, the theories of their motions were afterwards corrected and improved. For thefe fervices he will be always celebrated by aftronomers.

4. Towards the latter end of the fixteenth century, and about the beginning of the next, *Galileo* and *Kepler* diftinguifhed themfelves in the defence of the *Copernican* fyftem, and by many new difcoveries in the fyftem of the world. The excellent *Galileo* was no lefs happy in his philofophical enquiries, than in the celebrated difcoveries which he made in the heavens, by the telefcope. To the admirable *Kepler* we owe the difcovery of the true figure of the orbits, and the proportions of the motions of the folar fyftem: but the philofophical improvement of thefe phænomena was referved for Sir *Ifaac Newton*.

Kepler had a particular paffion for finding analogies and harmonies in nature, after the manner of the Pythagoreans and Platonifts; and to this difpofition we owe fuch valuable difcoveries as are more than fufficient to excufe his conceits. Three things, he tells us, he anxioufly fought to find the reafon of, from his early youth; why the planets were fix in number, why the dimenfions of their orbits were fuch as *Copernicus* had defcribed from obfervations, and what was the analogy or law of their revolutions. He fought for the reafons of the firft two of thefe in the properties of numbers and plane figures, without fuccefs. But at length reflecting that while the plane regular figures may be infinite in number, the ordinate and regular folids are five only, as *Euclid* had long ago demonftrated; he imagined

 that

that certain myſteries in nature might correſpond with this remarkable limitation inherent in the eſſences of things; the rather that he found the *Pythagoreans* had made great uſe of thoſe five regular ſolids in their philoſophy. He therefore endeavoured to find ſome relation between the dimenſions of thoſe ſolids and the intervals of the planetary ſpheres; and imagining that a cube inſcribed in the ſphere of *Saturn* would touch by its ſix planes the ſphere of *Jupiter*, and that the other four regular ſolids in like manner fitted the intervals that are betwixt the ſpheres of the other planets, he became perſuaded that this was the true reaſon why the primary planets were preciſely ſix in number, and that the Author of the world had determined their diſtances from the ſun, the center of the ſyſtem, from a regard to this analogy. Being thus poſſeſſed, as he thought, of the grand ſecret of the *Pythagoreans*, and being mightily pleaſed with his diſcovery, he publiſhed it in 1596, under the title of *Myſterium Coſmographicum*.

Kepler ſent a copy of this book to *Tycho Brahe*, who did not approve of thoſe abſtracted ſpeculations concerning the ſyſtem of the world, but wrote to *Kepler*, firſt to lay a ſolid foundation in obſervations, and then, by aſcending from them, to ſtrive to come at the cauſes of things. This excellent advice, to which we owe the more ſolid diſcoveries of *Kepler*, deſerves to be copied from his own account of it *. "Argumentum literarum *Brachei* (ſays he) hoc "erat, uti ſuſpenſis ſpeculationibus a priori de-"ſcendentibus, animum potius ad obſervationes, "quas ſimul offerebat, conſiderandas adjicerem. "Inque iis primo gradu facto, poſt demum, ad

* Notæ in editionem ſecundam Myſterii Coſmographici.

"causas ascenderem." In this judgment the great men of different times have frequently conspired, but few have faithfully followed it.

Tycho, however, pleased with his genius, prevailed with *Kepler* to reside with him near *Prague* (where he passed the last years of his life, after having left his native country on some ill usage) and to assist him in his astronomical labours. Soon after this *Tycho* died, but *Kepler* made many important discoveries from his observations: he found that astronomers had erred, from the first rise of the science, in ascribing always circular orbits and uniform motions to the planets; that each of them moves in an ellipsis which has one of its *foci* in the center of the sun; that the motion of each is really unequable; and varies so, that a ray supposed to be always drawn from the planet to the sun describes equal areas in equal times.

It was some years later before he discovered the analogy there is between the distances of the several planets from the sun, and the periods in which they complete their revolutions. He easily saw that the higher planets not only moved in greater circles, but also more slowly than the nearer ones; so that, on a double account, their periodic times were greater; *Saturn*, for example, revolves at a distance from the sun nine times and a half greater than the earth's distance from it; and the circle described by *Saturn* is in the same proportion; and as the earth revolves in one year, so, if their velocities were equal, *Saturn* ought to revolve in nine years and a half; whereas the periodic time of *Saturn* is above twenty nine years. The periodic times of the planets increase, therefore, in a greater proportion than their distances from the sun; but not in so great a proportion as the ſquares of thoſe diſtances; for if that was the law of their motions (the ſquare of $9\frac{1}{2}$ being $90\frac{1}{4}$) the periodic time of *Saturn* ought to be above 90 years. A mean proportion betwixt that of the diſtances of the planets, and that of the ſquares of thoſe diſtances, is the true proportion of the periodic times; as the mean betwixt $9\frac{1}{2}$ and its ſquare $90\frac{1}{4}$ gives the periodic time of *Saturn* in years. *Kepler*, after having committed ſeveral miſtakes in determining this analogy, hit upon it at laſt in 1618, *May* 15th, for he is ſo exact as to mention the preciſe day when he found, that "The ſquares "of the periodic times were always in the ſame "proportion as the cubes of their mean diſtances "from the ſun." This is only a very brief and ſummary account of the fruits of his great labours for many years on the obſervations made by *Tycho* *.

When *Kepler* ſaw that his diſpoſition of the five regular ſolids amongſt the planetary ſpheres was not agreeable to the intervals between their orbits, according to better obſervations, he endeavoured to diſcover other ſchemes of harmony. For this purpoſe, he compared the motions of the ſame planet at its greateſt and leaſt diſtances, and of the different planets in their ſeveral orbits, as they would appear viewed from the ſun; and here he fancied that he found a ſimilitude to the diviſions of the octave in muſic. Theſe were the dreams of this ingenious man, of which he was ſo fond, that, hearing of the diſcovery of four new planets (the ſatellites of *Jupiter*) by *Galileo*, he owns that his firſt reflexions were from a concern how he could ſave his favourite ſcheme, which was threatned

* See his Tabulæ Rudolphinæ, and Comment. de ſtellâ Martis.

by

by this addition to the number of the planets *. The ſame attachment led him into a wrong judgment of the ſphere of the fixed ſtars †: for being obliged, by his doctrine, to allow a vaſt ſuperiority to the ſun in the univerſe, he reſtrains the fixed ſtars within very narrow limits. Nor did he conſider them as ſuns, placed in the centers of their ſeveral ſyſtems, having planets revolving round them; as the other followers of *Copernicus*, from their having light in themſelves, their immenſe diſtances, and from the analogy of nature, have concluded them to be. Not contented with theſe harmonies, which he had learned from the obſervations of *Tycho*, he gave himſelf the liberty to imagine ſeveral other analogies, that have no foundation in nature, and are overthrown by the beſt obſervations. Thus from the opinions of *Kepler*, tho' moſt juſtly admired, we are taught the danger of eſpouſing principles, or hypotheſes, borrowed from abſtracted ſciences, and of applying them, with ſuch liberty, to natural enquiries.

A more recent inſtance of this fondneſs, for diſcovering analogies between matters of abſtracted ſpeculation and the conſtitution of nature, we find in *Huygens*, one of the greateſt geometricians and aſtronomers any age has produced: when he had diſcovered that ſatellite of *Saturn*, which, from him, is ſtill called the *Huygenian* ſatellite, this, with our moon, and the four ſatellites of *Jupiter*, completed the number of ſix ſecondary planets then diſcovered in the ſyſtem: and, becauſe the number of the primary planets is alſo ſix, and this number is called by mathematicians a perfect number, (being equal to

* Diſſert. cum nuncio ſidereo.
† Epitome Aſtronomiæ, lib. 4. part 1.

the ſum of its aliquot parts, 1, 2, and 3, *) *Huygens* was hence induced to believe that the number of the planets was complete, and that it was in vain to look for any more †. We do not mention this to leſſen this great man, who never perhaps reaſoned in ſuch a manner on any other occaſion; but only to ſhew, by another inſtance, how ill-grounded reaſonings of this kind have always proved: for, not long after, the celebrated *Caſſini* diſcovered four more ſatellites about *Saturn*; ſo that the number of ſecondary planets now known in the ſyſtem is ten. The ſame *Caſſini* having found that the analogy, diſcovered by *Kepler*, between the periodic times and the diſtances from the center, takes place in the leſſer ſyſtems of *Jupiter* and *Saturn*, as well as in the great ſolar ſyſtem; his obſervations overturned that groundleſs analogy which had been imagined between the number of the planets, both primary and ſecondary, and the number ſix; but eſtabliſhed, at the ſame time, that harmony in their motions, which will, afterwards, appear to flow from one real principle extended over the univerſe.

5. But to return to *Kepler*, his great ſagacity, and continual meditation on the planetary motions, ſuggeſted to him ſome views of the true principles from which theſe motions flow. In his preface to the commentaries concerning the planet *Mars*, he ſpeaks of gravity as of a power that was mutual betwixt bodies, and tells us that the earth and moon tend towards each other, and would meet in a point ſo many times nearer to the earth than to the moon, as the earth is greater than the moon, if their motions did not hinder it. He adds, that the tides

* Elem. *Euclid.* lib. 7. defin. ult.

† See the dedication of his Syſtema Saturnium.

arise

ariſe from the gravity of the waters towards the moon. But not having juſt enough notions of the laws of motion, he does not ſeem to have been able to make the beſt uſe of theſe thoughts; nor does he appear to have adhered to them ſteadily, ſince in his *epitome* of aſtronomy, publiſhed eleven years after, he propoſes a phyſical account of the planetary motions, derived from different principles.

He ſuppoſes, in that treatiſe, that the motion of the ſun on his axis is preſerved by ſome inherent vital principle; that a certain virtue, or immaterial image of the ſun, is diffuſed with his rays into the ambient ſpaces, and, revolving with the body of the ſun on his axis, takes hold of the planets and carries them along with it in the ſame direction; as a load-ſtone turned round in the neighbourhood of a magnetic needle makes it turn round at the ſame time. The planet, according to him, by its *inertia* endeavours to continue in its place, and the action of the ſun's image and this *inertia* are in a perpetual ſtruggle. He adds, that this action of the ſun, like to his light, decreaſes as the diſtance increaſes; and therefore moves the ſame planet with greater celerity when nearer the ſun, than at a greater diſtance. To account for the planet's approaching towards the ſun as it deſcends from the *aphelium* to the *perihelium*, and receding from the ſun while it aſcends to the *aphelium* again, he ſuppoſes that the ſun attracts one part of each planet, and repells the oppoſite part; and that the part which is attracted is turned towards the ſun in the deſcent, and that the other part is towards the ſun in the aſcent. By ſuppoſitions of this kind, he endeavoured to account for all the other varieties of the celeſtial motions.

Now the laws of motion are better known than in *Kepler*'s time, it is eaſy to ſhew the fallacy of every part of this account of the planetary revolutions. The planet does not endeavour to ſtop in its place in conſequence of its *inertia*, but to perſevere in its motion in a right line. An attractive force makes it deſcend from the *aphelium* to the *perihelium* in a curve concave towards the ſun: but the repelling force, which he ſuppoſed to begin at the *perihelium*, would cauſe it to aſcend in a figure convex towards the ſun. We ſhall have occaſion to ſhew afterwards, from Sir *Iſaac Newton*, how an attraction or gravitation towards the ſun, alone, produces the effects, which, according to *Kepler*, required both an attractive and repelling force; and that the virtue which he aſcribed to the ſun's image, propagated into the planetary regions, is unneceſſary, as it could be of no uſe for this effect tho' it were admitted. For now his own prophecy, with which he concludes his book *, is verified; where he tells us that "the "diſcovery of ſuch things was reſerved for the ſuc-"ceeding age, when the Author of nature would "be pleaſed to reveal thoſe myſteries."

6. In the mean time, *Galileo* made ſurprizing diſcoveries in the heavens by the teleſcope, an inſtrument invented in that time; and, by applying geometry to the doctrine of motion, began to eſtabliſh natural philoſophy on a ſure foundation. He made the evidence of the *Copernican* ſyſtem more ſenſible, when he ſhewed from the phaſes of *Venus*, like to the monthly phaſes of the moon, that *Venus* actually revolves about the ſun. He proved the

* Hæc et cætera hujuſmodi latent in pandectis ævi ſequentis, non antea diſcenda quam librum hunc Deus arbiter ſeculorum recluſerit mortalibus. *Epit.* Aſtron.

revolution

revolution of the ſun on his axis, from his ſpots; and thence the diurnal rotation of the earth became more credible. The four ſatellites that attend *Jupiter* in his revolution about the ſun, repreſented, in *Jupiter*'s leſſer ſyſtem, a juſt image of the great ſolar ſyſtem; and rendered it more eaſy to conceive how the moon might attend the earth, as a ſatellite, in her annual revolution. By diſcovering hills and cavities in the moon, and ſpots in the ſun conſtantly varying, he ſhewed that there was not ſo great a difference between the celeſtial and ſublunary bodies as the philoſophers had vainly imagined *.

He did no leſs ſervice by treating, in a clear and geometrical manner, the doctrine of motion, which has been juſtly called the key of nature. The rational part of mechanics had been ſo much neglected, that there was hardly any improvement made in it, from the time of the incomparable *Archimedes* to that of *Galileo*; but this laſt named author has given us fully the theory of equable motions, and of ſuch as are uniformly accelerated or retarded, and of theſe two compounded together. He, firſt, demonſtrated, that the ſpaces deſcribed by heavy bodies from the beginning of their deſcent are as the ſquares of the times, and that a body, projected in any direction that is not perpendicular to the horizon, deſcribes a parabola. Theſe were the beginnings of the doctrine of the motion of heavy bodies, which

* *Galileo* obſerved ſomething very extraordinary about *Saturn*, which he imagined to be two Satellites almoſt in contact with his body; and *Des Cartes* fancied theſe two Satellites were quieſcent in his vortex, becauſe (as he ſuppoſed) *Saturn* did not turn round on his axis; but *Huygens* ſhewed that this appearance proceeded from a ring that encompaſſes his body, without touching it, and accompanies him in his revolution about the ſun.

has

has been ſince carried to ſo great a height by Sir *Iſaac Newton.*

He alſo diſcovered the gravity of the air, and endeavoured to compare it with that of water; and opened up ſeveral other enquiries in natural philoſophy. He was not eſteem'd and followed by philoſophers only, but was honoured by perſons of the greateſt diſtinction of all nations. *Des Cartes,* indeed, * after commending him for applying geometry to phyſics, complains that he had not examined things in order, but had enquired into the reaſons of particular effects only; adding that, by his paſſing over the primary cauſes of nature, he had built without a foundation. He did not, 'tis true, take ſo high a flight as *Des Cartes,* or attempt ſo univerſal a ſyſtem; but this complaint, I doubt, muſt turn out to *Galileo*'s praiſe; while the cenſure of *Des Cartes* ſhews that he had the weakneſs to be vain of the worſt part of his writings.

But all the merit of this excellent philoſopher and elegant writer could not preſerve him from perſecution in his old age. Some pretended philoſophers, who had imprudently objected againſt his new diſcoveries in the heavens, when they found themſelves worſted and expoſed to ridicule, turned their hatred and reſentment againſt his perſon. He was obliged, by the rancour of the Jeſuits (as 'tis ſaid †) and the weakneſs of his protector, to go to *Rome,* and there ſolemnly renounce the doctrine of the motion of the

* Epiſtol. part 2. epiſt. 91.

† Vir in omni mathematum parte ſummus *Galileus Galilei,* Jeſuitarum in ipſum odio, ac principis Thuſci ſub quo vixit ſocordi metu, coactus ire Romam, ideo quod terram moviſſet, non vetante veſtro *Hortenſio,* durè habitus, ut majus vitaret malum, quaſi ab eccleſia edoctus, ſua ſcita reſcidit. *Hug. Grotius* in epiſtola ad *Voſſium,* Lutét. 17. maii, 1635.

earth,

earth, which he had argued for with ſo much ingenuity and evidence *. After this cruel uſage he was ſilent for ſome time, but not idle; for we have valuable pieces of his of a later date.

7. Sir *Francis Bacon* Lord *Verulam* †, who was cotemporary with *Galileo* and *Kepler*, is juſtly held amongſt the reſtorers of true learning, but more eſpecially the founder of *experimental philoſophy.* When he was but ſixteen years old, he began to diſlike the vulgar phyſics and what was called *Ariſtotle*'s philoſophy. He ſaw there was a neceſſity for a thorough reformation in the way of treating natural knowledge, and that all theory was to be laid aſide that was not founded on experiment. He propoſed his plan in his *inſtauratio magna*, with ſo much ſtrength of argument, and ſo juſt a zeal, as renders that admirable work the delight of all who have a taſte for ſolid learning.

He conſiders natural philoſophy as a vaſt pyramid, that ought to have the hiſtory of nature for its baſis; an account of the powers and principles that operate in nature, which he calls the phyſical part, for its ſecond ſtage; and the metaphyſical part, that treats of the formal and final cauſes of things, for its third ſtage. But as for the ſummit of this pyramid, the ſupreme of nature, *opus quod operatur Deus a principio uſque ad finem*, as he expreſſes it, he doubts if men can ever attain to the full knowledge of it. The philoſophers who ſtrive to erect theſe by the force of abſtract ſpeculation he compares to the

* He was beſides condemned to a years impriſonment in the inquiſition, and the penance of repeating daily ſome penitential pſalms.

† He was born in 1560, *Galileo* in 1564.

giants

giants of old, who, according to the poets, endeavoured to throw mount *Oſſa* upon *Pelion*, and *Olympus* upon *Oſſa*.

An artiſt, ſays this noble author, would expoſe himſelf to the juſteſt ridicule, who, in order to raiſe ſome vaſt obeliſk, ſhould attempt it by the force of his arms, inſtead of employing the proper machines; or if, after finding himſelf unequal to the taſk, he ſhould call for the aid of more workmen in the ſame way. Would he appear leſs ridiculous if he ſhould next ſet about chuſing his men, and examining them carefully, that he might employ the vigorous and robuſt only? or if, after he found this was to no purpoſe, he ſhould then apply himſelf to ſtudy the athletic art, and learn to compoſe curious ointments for ſtrengthening their limbs, or conſult learned phyſicians, who, by proper medicaments, ſhould promote their health and vigour? Nor are they leſs abſurd, in our noble author's judgment, who labour to interpret nature by the force and ſubtlety of genius only, tho' they ſhould aſſume the aid of the acuteſt men in the ſame work, and carry the dialecticks, or the art of reaſoning, to the greateſt height for this purpoſe.

The *empirical* philoſophers, thoſe who have no higher view than to collect the hiſtory of nature, he compares to the ants, who gather the grain and lay it up as they find it (unleſs it be true, as is reported of them, that they firſt take care it ſhould not germinate or become fruitful;) the *Sophiſts* to the ſpiders, who form their webs from their own bowels, to catch unwary inſects in their aerial flights; while the bee that gathers the matter from the flowers of the field, from which with admirable ſkill ſhe makes her honey, is the emblem of the true philoſopher; who

neither

neither trufts wholly to his own underftanding, nor contents himfelf with recording the matter with which he is furnifhed from natural hiftory or mechanical experiments; but, by reafoning fkilfully from them, brings forth truth and fcience, the great and noble production of the human faculties. From the neglect of experiments it arofe, that while nature was infinite, natural knowledge was at a ftand for many ages, and that the various fects wandered in the dark, without kindling any light to guide them, or finding any path to conduct them in her mazes. But, from a happy conjunction of the experimental and rational faculties, Lord *Verulam* conceived the higheft expectations. *Alexander*, he tells us, and *Cæfar* performed exploits that are truly greater than thofe reported of king *Arthur* or *Amadis de Gaul*; tho' they acted by natural means, without the aid of magic or prodigy.

It was with great juftice, and very feafonably, he reprehended thofe * who, "upon a weak conceit of "fobriety, or ill-applied moderation, thought or "maintained that a man can fearch too far, or be "too well ftudied in the book of God's word, or in "the book of God's works. But rather, he adds, "let men awake themfelves, and chearfully endeavour and purfue an endlefs progrefs and proficiency in both; only let them beware left they "apply knowledge to pride, not to charity, to "oftentation, not to ufe." He obferves, that a fuperficial tafte of philofophy may perchance incline the mind to atheifm; but a full draught thereof brings it back again to religion: in the entrance of philofophy, when the fecond caufes moft obvious to the fenfes offer themfelves to the mind, we are apt

* *Bacon*'s Advancement of Learning, lib. 1.

to

to cleave unto them, and dwell too much upon them, ſo as to forget what is ſuperior in nature. But when we paſs further, and behold the dependency, continuation and confederacy of cauſes, and the works of providence, then, according to the allegory of the poets, we eaſily believe that the higheſt link of nature's chain muſt needs be tied to the foot of *Jupiter*'s chair; or perceive "That philoſophy, like "*Jacob*'s viſion, diſcovers to us a ladder, whoſe "top reaches up to the footſtool of the throne "of God."

The *Ariſtotelian* philoſophy appeared unſatisfactory to Lord *Bacon*, not from want of eſteem for its author, whom he always uſed to extol; but becauſe it ſeemed fit for diſputes only, and incapable of producing real fruit. *Ariſtotle*, he ſaid, had ſuited his phyſics to his logic, inſtead of giving ſuch a kind of logic as might be of real uſe in phyſics. To ſupply this defect, he compoſed his *novum organum*; where his chief deſign is to ſhew how to make a good *induction*, as *Ariſtotle*'s was to teach how to make a good *ſyllogiſm*. Had the philoſophers, ſince Lord *Verulam*'s time, adhered more cloſely to his plan, their ſucceſs had been greater; and Sir *Iſaac Newton*'s philoſophy had not found the learned ſo full of prejudices againſt it, in favour of ſome ſyſtems lately invented and mightily extolled by ſpeculative men; that while all admired the ſublime geometry which ſhone throughout his work, few for ſome time appeared to be diſpoſed to hearken to his philoſophy, or in a condition to judge of it impartially.

8. However, Lord *Bacon*'s exhortations and example had a good effect; and experimental philoſophy has been much more cultivated ſince his time than

than in any preceding period. Geometry and philosophy advanced together at a great pace, and gave mutual aid to each other. The evidence of geometry began to take place in philoſophy, while all things were examined by number, weight, and meaſure; and the principles of the theory of motion, being now clearly underſtood, furniſhed excellent illuſtrations of the abſtruſe parts of geometry. *Galileo* had ſcholars worthy of ſo great a maſter, by whom the gravitation of the atmoſphere was eſtabliſhed fully, and its varying preſſure accurately and conveniently meaſured, by the column of quick-ſilver of equal weight ſuſtained by it in the barometrical tube. The elaſticity of the air, by which it perpetually endeavours to expand itſelf, and, while it admits of condenſation, reſiſts in proportion to its denſity, was a phænomenon of a new kind (the common fluids having no ſuch property) and of the utmoſt importance to philoſophy. Theſe principles opened up a vaſt field of new and uſeful knowledge, and explained a great variety of phænomena, which had been accounted for in an abſurd manner before that time. It ſeem'd as if the air, the fluid in which men lived from the beginning, had been then firſt diſcovered. Philoſophers were every where buſy enquiring into its various properties and their effects; and valuable diſcoveries rewarded their induſtry. Of the great number who diſtinguiſhed themſelves on this occaſion, we cannot but mention *Torricelli* in *Italy*, *Paſchal* in *France*, *Otto Guerick* in *Germany*, and *Boyle* in *England*.

The views of philoſophers began now to be mightily enlarged, not by their diſcoveries concerning the air only, but likewiſe by their enquiries into the more potent element fire and its effects, and into the chymical compoſition, reſolution, and changes

of

of bodies. For about this time chymiſts began to ſpeak more intelligibly concerning their art, and to connect it in ſome degree with natural philoſophy, or to conſider it, at leaſt, as not quite foreign to it. This we owe in great meaſure to the honourable Mr. *Boyle*, whoſe favourite ſtudy chymiſtry is ſaid to have been, and who was happy in an eaſy and familiar manner of deſcribing the ſubjects which were treated by him.

It muſt be owned that none ever took ſo great pains to promote natural knowledge, in all its branches, or the beſt improvement that can be made of it, than this excellent perſon. It has been obſerved that he was born the ſame year that Lord *Bacon* died, as if he had been deſtin'd to carry on his plan. He ſpared no labour nor coſt in collecting the hiſtory of nature, and making curious and uſeful experiments of all ſorts. As Lord *Bacon*'s plan comprehended the whole compaſs of nature, ſo the variety of enquiries proſecuted by Mr. *Boyle*, with great care and attention, is very ſurprizing, and perhaps not to be parallel'd. Hydroſtatics, tho' a moſt uſeful branch of mechanical philoſophy, had been but ill underſtood, till he eſtabliſhed its principles, and illuſtrated its paradoxes, by a number of plain experiments, in a ſatisfactory manner. The doctrine of the air afforded him an ample field; and, in all his reſearches, he ſhewed a genius happily turned for experimental philoſophy, with a perfect candour, and a regular condeſcenſion in examining with patience, and refuting, without oſtentation, the errors which philoſophers had been led into from their prejudices, and the many artful ſubterfuges by which they ſtrove to ſupport them. The unexceptionable integrity, extenſive charity, and ſingular piety of this excellent perſon did great honour to philoſophy, and formed an eminent part of his character. The world he conſidered as the temple of God, and * " man (to uſe his own words) as born the prieſt of " nature, ordained (by being qualified) to celebrate " divine ſervice, not only *in* it but *for* it." Not ſatisfied with having promoted the belief of a Deity and the evidence of true religion, to the utmoſt of his power, in the great number of volumes compoſed by him, on every occaſion during the courſe of a laborious life, he has taken care, by his will, to perpetuate a ſucceſſion of advocates for it, who ſhould make the ſame improvement not of his diſcoveries only, or of thoſe of former times, but of what ſhould be produced by future ages. In this deſign, worthy of him, the ſucceſs has been anſwerable to his intentions; and ſurely ſuch a man, we muſt allow, was not an ornament to his own age and country only, but a publick benefit to all times and nations.

We are now arrived at the happy æra of experimental philoſophy; when men, having got into the right path, proſecuted uſeful knowledge; when their views of nature did honour to them, and the arts received daily improvements; when not private men only, but ſocieties of men, with united zeal, ingenuity and induſtry, proſecuted their enquiries into the ſecrets of nature, devoted to no ſect or ſyſtem. But we are obliged to abandon, at preſent, the agreeable taſk of following them in their diſcoveries, in this flouriſhing period of ſcience, to give account of a moſt illuſive ſcheme of ſpeculative philoſophy that prevailed amongſt many at this very time, and, by miſleading ingenious men, corrupted their notions and retarded their progreſs. It ſeems that, however

* *Boyle's* Uſefulneſs of Natural Philoſophy, part 1. eſſay 3.

fertile this period was in new inventions, nature did not unveil herſelf readily enough to ſatisfy the impatience of ſome men, who could not be contented with thoſe views of her which time and induſtry produced to them. Therefore they hearkned again to the vain promiſes of thoſe who pretended to unravel all her myſteries at once, by the force of their abſtracted ſpeculations. The *Carteſian* ſyſtem was the moſt extenſive, and (according to many) the moſt exquiſite in its contrivance, of any that have been imagined. The author of it was a bold philoſopher, and doubtleſs of a ſubtle genius, to indulge which he retired from the world for many years. He valued himſelf on his clear ideas, and is allowed to have contributed to diſſipate the darkneſs of that ſort of ſcience which prevailed in the ſchools. If we may believe ſome accounts, he rejected a *void* from a complaiſance to the taſte which then prevailed, againſt his own firſt ſentiments; and amongſt his familiar friends, uſed to call his ſyſtem his philoſophical romance. It had however great ſucceſs; and his doctrines ſtill prevail ſo much, that it is neceſſary for our purpoſe to give a ſhort account of them.

CHAP. IV.

Of the philoſophical principles of Des Cartes, *the emendations of his followers, and the preſent controverſies in natural philoſophy.*

DES *Cartes* begins his *principia* by ſhewing the neceſſity of doubting firſt of every thing, in order to our obtaining certain knowledge; and recommends to his readers to conſider his reaſons for doubting of all things, not once only, but to employ weeks, or even months, on theſe alone, before he he proceed farther. He firſt eſtabliſhes the certainty of our own exiſtence, and that of our ideas of which we are intimately conſcious to ourſelves; of the exiſtence of which, however, after all he has ſaid, it ſeems impoſſible for us to doubt for a moment. From our having the idea of a Being infinitely perfect and neceſſarily exiſting, he concludes that ſuch a Being actually is; upon whoſe will he makes the certainty of ſelf-evident propoſitions, or axioms *, as well as of all other neceſſary truths, to depend.

From the knowledge of the cauſe eſtabliſhed in this manner, he pretends to deduce a complete knowledge of his effects, by neceſſary ſteps. It is clear, ſays he †, that we ſhall follow the beſt method in philoſophy, if, from our knowledge of the Deity himſelf, we endeavour to deduce an explication of all his works; that ſo we may acquire the moſt perfect kind of ſcience, which is that of effects from their cauſes. As for final cauſes he rejected them from philoſophy, as we obſerved above; and from theſe paſſages, which repreſent the genius of this author's philoſophy, and from the manner in which he ſets out, we may already form ſome judgment how hopeful his project was.

From the veracity of the Deity, he infers the reality of material objects, which are repreſented to us as exiſting without us. He places the eſſence of matter in extenſion; for this alone remains, he ſays,

* According to him, the Deity did not will that the three angles of a triangle ſhould be equal to two right ones, becauſe he knew that it could not be otherwiſe; but, becauſe he would that the three angles of a triangle ſhould neceſſarily be equal to two right ones, therefore this is true and can be no otherwiſe.

† See the paſſages cited above from his Principia, in the notes upon § 4. ch. 1.

when we reject hardneſs, colour, weight, heat and cold, and the other qualities which, we know, a body can be without. Hence he eaſily concludes that there can be no void, or extenſion without matter. He adds, however, immediately afterwards, as properties of matter, that its parts are ſeparable and moveable; tho' theſe ſeem to imply more than mere extenſion.

He defines motion to be the tranſlation of a body from the neighbourhood of other bodies that are in contact with it, and are conſidered as quieſcent, to the neighbourhood of other bodies; and thus makes no diſtinction between abſolute or real, and relative or apparent motions; both of which equally agree to this definition. The reaſon he gives why the ſame quantity of motion muſt be preſerved for ever in the univerſe, without any augmentation or diminution in the whole, muſt appear conciſe, and very extraordinary. It is no other than that God muſt be ſuppoſed to act in the moſt conſtant and immutable manner. From the ſame property of the Deity, he infers that a body muſt continue in its ſtate as to reſt, motion, figure, &c. till ſome external influence produce a change; which is his firſt law of nature: that the direction of motion is naturally rectilinear, or that a body never changes its direction of itſelf; which is his ſecond law: and that a body in motion, when it meets with another moving with a greater force, is reflected without loſing any part of its firſt motion; but when it meets with a body moving with leſs force, it then carries this body along, and loſes as much motion as is transferred to it; and this is his third law of nature. He accounts for the hardneſs of bodies from their parts being quieſcent with reſpect to each other; and for fluidity, from their being moved perpetually in all

all directions. He concludes the ſecond part of his book with telling us, that theſe principles are ſufficient for explaining all the phænomena of nature, and that no other ought to be admitted or even wiſhed for.

He afterwards proceeds to ſhew how the univerſe might have aſſumed its preſent form, and may be for ever preſerved, by mechanical principles. He ſuppoſes the particles of matter to have been angular, ſo as to repleniſh ſpace without leaving any interſtices between them; and to have been in perpetual agitations, by which the angular parts being broke off, the particles themſelves became round, and formed what he calls the matter of the ſecond element. The angular parts, being ground into the moſt ſubtile particles of all, became the matter of his firſt element, and ſerved to fill all the pores of the other. But there being more of this firſt element than was neceſſary for that purpoſe, it became accumulated in the centers of the *vortices*, of which he imagined the univerſe to conſiſt, and formed there the bodies of the ſun and ſtars. The heavens were filled with the matter of the ſecond element, the medium of light. But the planets and comets conſiſted of a third element groſſer than the other two, the generation of which he traces at length through all its ſteps. According to him, the matter of the firſt element muſt have conſtantly flowed out through the interſtices between the ſpherical particles of the ſecond element, where the circular motion is greateſt, and muſt have returned continually at the poles of this motion towards the centre of the vortex; where being apt to cohere together, they at length produced the groſſer particles of the third; and when theſe came to adhere in a conſiderable quantity, they gave riſe to the ſpots on the ſurfaces of the

the ſuns or ſtars. Some being cruſted over with ſuch ſpots became planets or comets; and the force of their rotation becoming languid, their vortices were abſorbed by ſome more potent neighbouring vortex. In this manner the ſolar ſyſtem was formed, the *vortices* of the ſecondary planets having been abſorbed by the vortex of the primary, and all of them by that of the ſun. He contends that the parts of the ſolar vortex increaſe in denſity, but decreaſe in celerity, to a certain diſtance; beyond which he ſuppoſes all the particles to be equal in magnitude, but to increaſe in celerity as they are farther from the ſun. In thoſe upper regions of the vortex he places the comets; in the lower parts he ranges the planets; ſuppoſing thoſe that are more rare to be nearer the ſun, that they may correſpond to the denſity of the vortex where they are carried round.

He accounts for the gravity of terreſtrial bodies from the centrifugal force of the æther revolving round the earth; which, he imagined, muſt impell bodies downwards that have not ſo great a centrifugal force, much in the ſame manner as a fluid impells a body upwards that is immerged in it, and has a leſs ſpecifical gravity than it. He pretended to explain the phænomena of the magnet, and to account for every thing in nature, from the ſame principles.

2. There never was, perhaps, a more extravagant undertaking than ſuch an attempt, to deduce, by neceſſary conſequences, the whole fabric of nature, and a full explication of her phænomena, from any ideas we are able to form of an infinitely perfect Being. Was it not for the high reputation of the author, and of his ſyſtem, it would be hardly excuſable to make any remarks upon ſuch a rhapſody.

Should

Should we allow the principles he builds on, and his method, it muſt be obvious with how weak an evidence the conſequences are connected with each other, in this viſionary chain. How juſt a method he has taken to eſtabliſh the exiſtence and attributes of the Deity we ſhall not enquire, nor how far his making all truth and falſhood dependent on the will of the Deity tends to weaken all ſcience and confound its principles. While he ſuppoſes extenſion to conſtitute the complete eſſence of matter, he neglects ſolidity, and the *inertia* by which it reſiſts any change in its ſtate of motion or reſt; which diſtinguiſh body from ſpace. If extenſion be underſtood to be the eſſence of matter, it is a trifling propoſition to affirm that all ſpace is full of matter, according to this definition. But ſtill the queſtion will remain, whether all ſpace is full of that ſolid, moveable and reſiſting ſubſtance commonly called *body*. And as many parts of ſpace appear to make no ſenſible reſiſtance to motion, while others reſiſt variouſly in proportion to the denſity of the medium diffuſed over them, we thence learn there is ſpace void of what is commonly called *matter*. The comets which move with equal freedom in all directions with very rapid motions, and carry along with them tails of a prodigious ſize, conſiſting of ſome highly rarified matter, ſhew that the heavens are not repleniſhed with denſe fluids that admit no void. For it is evident in experimental philoſophy that the reſiſtance of fluids increaſes, *cæteris paribus*, with their denſity; ſo that all motion would ſoon languiſh in a fluid, which, having no pores, muſt far ſurpaſs quick ſilver, or the heavieſt ſolids, in denſity. Nothing is more evident, than that the force requiſite to move two equal bodies with a given velocity, is double that which would produce the ſame celerity in either of them. When we compound greater bodies from leſſer, or when we reſolve them into their parts, we find that the reſiſtance or *inertia* increaſes or decreaſes in proportion to the quantity of matter. Therefore when the velocity is given, if a body moving in a denſer fluid diſplaces more matter to make way for itſelf, the reſiſtance which it meets with being equal to the motion communicated to the parts of the fluid, it muſt find a reſiſtance proportionally greater.

It is not only from the free motions of the planets and comets that we learn the abſurdity of the doctrine of an univerſal plenitude. The moſt common and plain phænomena of the motion of bodies, at or near the ſurface of the earth, are ſufficient to overthrow it; for we find that they meet with no ſenſible reſiſtance but from the air: whereas ſo denſe a fluid as would repleniſh all ſpace equally would neceſſarily produce a very great reſiſtance.

It is objected *, that by ſuppoſing this denſe fluid which repleniſhes ſpace to penetrate the pores of bodies with the utmoſt freedom, (as light paſſes through tranſparent bodies, and the magnetic and electric *effluvia* through moſt kinds of bodies) its reſiſtance will then be incomparably leſs than in proportion to its denſity; for then the reſiſtance will not be meaſured by the denſity of the fluid, becauſe the much greater part paſſes through the pores of the body in motion, freely without reſiſtance. Suppoſing this to be admitted, it is, however, obvious that, even in this hypotheſis, the reſiſtance of a golden ball in a *plenum* would be ſtill very great. For this ſubtle fluid, how penetrating ſoever it be, muſt reſiſt the

* In a ſmall piece publiſhed on this ſubject, a few years ago, by an ingenious gentleman.

ſolid

ſolid parts of the ball; which cannot move in the fluid without diſplacing its parts, and loſing as much motion as muſt be communicated to thoſe parts; and this reſiſtance depends on the quantity of ſolid parts in the ball: whereas the reſiſtance which the ſame ball meets with in quick ſilver (which we ſuppoſe to have no paſſage through the ball) depends on the quantity of the ſolid parts in an equal bulk of the quick ſilver, which muſt be moved to make way for the ball. And this being leſs than the quantity of ſolid parts in an equal bulk of the golden ball, in proportion as the ſpecific gravity of quick ſilver is leſs than that of gold, it follows that the reſiſtance of a golden ball, moving in ſuch a ſubtile penetrating *plenum*, would ſtill be greater than its reſiſtance in quick ſilver. To illuſtrate this farther, the ſpecific gravity of gold being to that of quick ſilver nearly as 195 to 140, ſuppoſe a golden ball conſiſting of 195 ſolid particles to move in the *plenum* with a given velocity, and to deſcribe a very ſmall ſpace; and then ſuppoſe the ſame ball to move in quick ſilver with the ſame velocity over the ſame ſpace; in the former caſe, the ſolid parts of the ball diſplace a certain quantity of the *plenum*, ſuppoſe a quantity equal to the ball, or 195 parts; in the latter caſe, they diſplace an equal bulk of the quick ſilver, that is, 140 ſolid particles. But becauſe it may be ſaid for thoſe who maintain an univerſal plenitude, that the golden ball meets with a reſiſtance from the ſubtile fluid that repleniſhes ſpace, while it moves in the quick ſilver, as well as from the quick ſilver itſelf; let this likewiſe be allowed, and let us even ſuppoſe it to meet with as much reſiſtance from the *plenum*, while it moves in the quick ſilver, as when it moves in a ſpace free from any groſs fluid; yet it will ſtill appear that the reſiſtance of the golden ball in the *plenum* ought to bear at leaſt as great a proportion

to

to its reſiſtance in quick ſilver, as the denſity of gold is to the ſum of the denſities of gold and quick ſilver, or as 195 to 335, and conſequently ought to be eight times greater than its reſiſtance in water. This is the leaſt reſiſtance ſuch a ball could meet with in a *plenum*, ſhould we allow the ſuppoſitions that are moſt favourable in this doctrine; and this reſiſtance would ſoon put an end to the motions of bodies. But it is evident that we allowed too much in favour of their doctrine, when we ſuppoſed the ball moving in the quick ſilver to meet with a reſiſtance equal to the ſum of the reſiſtances that it would meet with from the *plenum* and quick ſilver ſeparately. For, according to this ſuppoſition, its reſiſtance in quick ſilver would be to its reſiſtance in water, as the ſum of the denſities of gold and quick ſilver to the ſum of the denſities of gold and water, that is, as 335 to 205, or 67 to 41; ſo that the reſiſtance of quick ſilver would not be double of that of water, or even double of that of air; than which nothing can be more contradictory to experiment.

It is of no importance to this argument how rare gold, quick ſilver, or the heavieſt bodies, be ſuppoſed; ſince the reſiſtance of quick ſilver in fact is known to be very great, and is not altered by ſuch ſuppoſitions: neither is the proportion of the denſity of gold to that of quick ſilver (upon which proportion the argument is founded) affected by them. For it will always be found that the reſiſtance of a golden ball in a *plenum* (how freely ſoever it paſs through the pores of the ball, and how large or numerous ſoever theſe pores may be) muſt correſpond to the ſolid matter in the ball; which is greater than the ſolid matter in any equal bulk of any of our fluids; upon which their reſiſtance depends. The ſuppoſing the ſolid matter in the quick

ſilver

ſilver to occupy only the thouſandth or millionth part of its bulk, has no other effect but that it ſuppoſes the *inertia* of a given quantity of ſolid matter to be increaſed in the ſame proportion with the rarity of the quick ſilver, whoſe *inertia* is in fact aſcertained.

The reſiſtance which ariſes from the tenacity or adheſion of the parts of fluids may be diminiſhed; but ſtill the reſiſtance which ariſes from the *inertia* of the matter remains: if this could be taken away, as the matter would have no reſiſtance, ſo it is not eaſy to conceive how it could have any activity or mechanical force to impell bodies, or to produce any of the effects which are attributed to the ſubtile matter of the *Carteſians*. For action and reaction are always equal, and we know of no force in bodies but what ariſes from their reſiſtance to change their ſtate, or their *inertia*. Without this there could be no centrifugal force, the favourite power by which thoſe philoſophers endeavour to explain the phænomena of nature.

They ſuppoſe the particles of thoſe ſubtile fluids to move conſtantly and equally in all directions; and, by the favour of this hypotheſis, they imagine that they may ſuppoſe them to act but not reſiſt. But they have neither made this ſtrange ſuppoſition probable, nor even credible, nor can they ſhew that it would anſwer their purpoſe. A motion of a fluid favours the motion of a body in it, only as far as it is in the ſame direction; and an inteſtine motion of the parts of the fluid, equal in all directions, cannot make the reſiſtance leſs than if there was no motion of the parts. It is ſuppoſed by many that the particles of common fluids, water or air for example, are in a conſtant inteſtine motion; but this does not

not hinder thoſe fluids from reſiſting in proportion to their denſity.

We are told by ſome, that it is impoſſible to conceive a *vacuum*. But this ſurely muſt proceed from their having imbibed *Des Cartes*'s doctrine, that the eſſence of body is conſtituted by extenſion; as it would be contradictory to ſuppoſe ſpace without extenſion. To ſuppoſe that there are fluids penetrating all bodies and repleniſhing ſpace, which neither reſiſt nor act upon bodies, merely in order to avoid the admitting a *vacuum*, is feigning two ſorts of matter, without any neceſſity or foundation; or is tacitly giving up the queſtion. As for Mr. *Leibnitz*'s arguments againſt a *vacuum*, we defer them till we come to conſider the emendations that have been made to this ſyſtem.

The ſame quantity of motion is not always preſerved in the univerſe, as *Des Cartes* raſhly concluded from the immutability of the Deity. The quantity of abſolute motion is continually varying; it is diminiſhed in the compoſition of motion, and, in many caſes, in the colliſions of bodies that have an imperfect elaſticity; and it is increaſed in the reſolution of motion, and, in ſome caſes, in the colliſions of elaſtic bodies. It requires an active principle to account for the hardneſs of bodies; and the particles being at reſt is not ſufficient for this purpoſe; for this would not hinder them to be ſeparated from each other by the leaſt force. There is hardly one article in this ſcheme but what is, in like manner, liable to inſuperable difficulties.

After all, *Des Cartes* ſaw the neceſſity of having recourſe to obſervation, tho' unwillingly; and he appears to be at a loſs how to acknowledge it, after

having

having boaſted ſo much of his principles. He tells us that he found theſe ſo extenſive and fertile *, that many more things followed from them than we find in the viſible world. Other philoſophers have complained that they were able to account for too little of nature: *Des Cartes* finds that his principles were more than ſufficient to account for all her phænomena, and ſeems only to fear leſt he ſhould account for too much. Therefore he has recourſe to the phænomena, not becauſe he would prove any thing from them; for he takes care that we ſhould not have ſo mean an opinion of his philoſophy, as to imagine he would eſtabliſh it on facts; but that he might be able to determine his mind to conſider ſome of thoſe innumerable effects, which he judged might proceed from the ſame cauſes, rather than others. He likewiſe acknowledges †, that the ſame effect might be deduced, from his principles, many different ways; and that nothing perplexed him more than to know which of them obtained in nature. In thoſe paſſages he magnifies his principles, in order to conceal the weakneſs of his ſyſtem, with an affectation that only ſerves to make it more evident, and appear unworthy of ſo great a man.

3. *Des Cartes*, by placing the eſſence of matter in extenſion alone, gave occaſion to others to draw

* He cites the effects, as he tells us, Non quidem ut ipſis tanquam rationibus utamur ad aliquod probandum; cupimus enim rationes effectuum a cauſis, non autem e contrario cauſarum ab effectibus deducere; ſed tantum ut ex innumeris effectibus, quos ab iiſdem cauſis produci poſſe judicamus, ad unos potius quam alios conſiderandos mentem noſtram determinemus.

† Sed confiteri me etiam oportet potentiam naturæ eſſe adeo amplam, ut nullum fere amplius particularem effectum obſervem, quem ſtatim *variis modis* ex iis principiis deduci poſſe non agnoſcam: nihilque ordinario mihi difficilius videri, quam invenire quo ex his modis inde dependet. *De Methodo*, § 6.

con-

consequences, from this doctrine, of a dangerous nature; which undoubtedly he would have disowned, tho' 'tis not easy to see how he could have got rid of them. As we are not able to conceive that space can be annihilated, or that there ever was a time when space or expansion was not; so if we allow that extension alone constitutes the essence of matter, we cannot but ascribe infinity, eternity, and necessary existence to it. In this manner *Spinoza* reasons from the *Cartesian* principles, affirming that matter is not only infinite and necessary, but also that it is one and indivisible *. "This, says he, cannot be denied by those who reject the possibility of a *vacuum*; "for if matter could be so divided that its parts "should be really distinct, why might not one part "be annihilated, the rest remaining connected with "each other as before? since of things which are "really distinct from each other, the one can exist "and remain in its state without the other." In another place, he tells us, that if any one part of matter was annihilated, all extension would vanish with it †. This author appears to have been very con-

* Nam si substantia corporea ita posset dividi ut ejus partes realiter distinctæ essent, cur ergo una pars non posset annihilari manentibus reliquis, ut ante, inter se connexis? Et cur omnes ita aptari debent ne detur vacuum? Sane, rerum quæ realiter ab invicem distinctæ sunt, una sine aliâ esse & in suo statu manere potest. Cum igitur vacuum in naturâ non detur, sed omnes partes ita concurrere debent ut detur vacuum, sequitur hinc etiam easdem non posse realiter distingui; hoc est, substantiam corpoream, quatenus substantia est, non posse dividi. *Spinoz. Ethic.* part 1. prop. 15. schol.

† Si una pars materiæ annihilaretur, simul etiam tota extensio evanesceret. *Epist.* 4. *ad Henr. Oldenb.*

From these and other passages it appears, that this author was unhappily misled by the doctrine of *Des Cartes*, that the essence of matter is constituted by extension. It must be owned, however, that many of the *Cartesians* endeavoured to wrangle away the dreadful conclusion: but they had shortned their work, and converſant in the writings of *Des Cartes* ‡, the two firſt parts of whoſe *principia* he reduced into the geometrical form. Mr. *Leibnitz* himſelf calls ſpinoziſm *un Carteſianiſme outré*; and it is apparent that his method, and many of his doctrines, were derived from this ſource.

As *Des Cartes* had concluded, from the idea of an infinitely perfect neceſſarily-exiſting Being, that ſuch a Being muſt exiſt; ſo *Spinoza*, from our having a true idea (that is a clear and diſtinct idea, as he himſelf explains it) of a ſubſtance, infers that it muſt neceſſarily exiſt *; or, to uſe his own words, that its exiſtence as well as its eſſence muſt be an eternal truth. As *Des Cartes* pretended to deduce all the phænomena of nature from the nature and properties of the firſt cauſe; ſo *Spinoza* pretends, that all our knowledge is to be derived from true ideas (as he always calls them) and that thoſe true ideas ought

and had proceeded on better grounds, if they had rejected the principle. Yet *Spinoza*, in his ſeventy-third letter, pretends to find fault with *Des Cartes* for defining matter by extenſion, which, according to him, ought to have been explained by an attribute that ſhould expreſs an *eſſential and infinite eſſence.*

‡ Quum ille ſummo ſciendi amore arderet, quid in his ingenii vires valerent experiri decrevit. Ad hoc propoſitum urgendum ſcripta philoſophica nobiliſſimi & ſummi philoſophi *Renati Des Cartes* magno ei fuerunt adjumento. *Spinoz.* oper. poſth. præfat.

* Si quis dicerit ſe claram & diſtinctam, hoc eſt veram, ideam ſubſtantiæ habere, & nihilominus dubitare num talis ſubſtantia exiſtat, idem hercle eſſet ac ſi diceret ſe veram habere ideam, & nihilominus dubitare num falſa ſit (ut ſatis attendenti ſit manifeſtum :) vel ſi quis ſtatuat ſubſtantiam creari, ſimul ſtatuit ideam falſam factam eſſe veram; quo ſane nihil abſurdius concipi poteſt: adeoque fatendum neceſſario eſt, ſubſtantiæ exiſtentiam ſicut ejus eſſentiam æternam eſſe veritatem. *Ethic.* part 1. prop. 8. ſchol. 2.

to be produced by the mind †, from that idea which reprefents the moft perfect Being, the origin and fountain of nature. *Des Cartes* rejected the confideration of final caufes from philofophy; and *Spinoza* tells us they are nothing but human fiction ‡, and laughs at thofe who imagine that the eyes were defigned for feeing, or the fun for giving light. He derives our notions of good and evil, order and confufion, beauty and deformity, from the fame fource. As *Des Cartes* reprefented the univerfe as a machine that might have been produced at firft, and may continue to exift for ever, by mechanical laws only, the fame quantity of motion remaining always in it unalterable; fo *Spinoza* reprefented it as infinite and neceffary, endowed always with the fame quantity of motion, or (to ufe his inaccurate expreffion *) having always the fame proportion of motion to reft in it, and proceeding by an abfolute natural neceffity; without any felf-mover or principle of liberty.

In all thefe, *Spinoza* has added largely, from his own imagination, to what he had learned from

† Ut mens noftra omnino referat naturæ exemplar, debet omnes fuas ideas producere ab eâ quæ refert originem & fontem totius naturæ, ut ipfa etiam fit fons ceterarum idearum. *Spinoz.* de emendatione intellect.

‡ Ut jam oftendam naturam nullum fibi finem præfixum habere, & omnes caufas finales nihil nifi humana effe figmenta, non opus eft multis, &c. Hoc adhuc addam, nempe hanc de fine doctrinam naturam omnino evertere. *Append.* prop. 36. part 1. *Ethic.*

* Omnia corpora ab aliis circumcinguntur, & ab invicem determinantur ad exiftendum & operandum, certâ ac determinatâ ratione, fervatâ femper in omnibus fimul, hoc eft in toto univerfo, eâdem ratione motus ad quietem, *Epift.* 15——Corpus motum vel quiefcens ad motum vel quietem determinari debuit ab alio corpore, quod etiam ad motum vel quietem determinatum fuit ab alio, & illud iterum ab alio, & fic in infinitum. *Ethic.* part 2. prop. 13. lem. 3.

Des Cartes. But from a comparifon of their method and principles, we may beware of the danger of fetting out in philofophy in fo high and prefumptuous a manner; while both pretend to deduce compleat fyftems from the clear or true ideas, which they imagined they had, of eternal effences and neceffary caufes. If we attend to the confequences of fuch principles, we fhall the more willingly fubmit to experimental philofophy, as the only fort that is fuited to our faculties. It were unreafonable to charge upon *Des Cartes* the impious confequences which *Spinoza* may have been led into from his principles: but we cannot but obferve, to the honour of Sir *Ifaac Newton*'s philofophy, that it altogether overthrows the foundation of *Spinoza*'s doctrine, by fhewing that not only there may be, but that there actually is a *vacuum*; and that, inftead of an infinite, neceffary, and indivifible, plenitude, matter appears to occupy but a very fmall portion of fpace, and to have its parts actually divided and feparated from each other.

It would be of no ufe to give a more particular account of the fyftem of *Spinoza*; nor is it poffible to defcribe fully, in an intelligible manner, fo abfurd a doctrine. It is allowed even by thofe who, on other occafions, have fhewn a difpofition towards fcepticifm, in relation to the foundations of natural religion, to be the moft monftrous that can be imagined; and to be fo oppofite to the moft evident notions we are able to form *, that no perfon of a right

* Thefe are the words of Mr. *Bayle* in the article of *Spinoza*; where he expofes the abfurdities of this fyftem very clearly, and affirms that the weakeft of its adverfaries was able to have overturned it. Our view in giving fome account of it, was not only to fhew the abfurd confequences to which *Des Cartes*'s fyftem leads,

right mind can be in hazard of giving into it. He pretends, indeed, to proceed in the geometrical method and ſtyle; but while he aſſumes a definition of ſubſtance and of its attributes at his pleaſure, and paſſes from his definitions as true ideas (as he calls them) to the neceſſary exiſtence of the thing defined, by a pretended immediate conſequence, which he will not allow to be diſputed, his whole ſuperſtructure appears a mere *petitio principii* or fiction. By his way of proceeding, any ſyſtem whatſoever might be eſtabliſhed. But it does not appear poſſible to invent another ſo abſurd, while he maintains that there is but one ſubſtance in the univerſe, endowed with infinite attributes, (particularly, infinite extenſion and cogitation) that produces all other things, in itſelf, neceſſarily, as its own modifications; which alone is, in all things, cauſe and effect, agent and patient, in all reſpects phyſical and moral.

The *Carteſian* doctrine has been often altered, and variouſly mended, ſince it was firſt propoſed by its author; and, for a hundred years together, many ingenious men have been making their utmoſt efforts to patch it up, and ſupport its credit, by reforming firſt one part, and then new-modeling another part of this extenſive ſyſtem. But the foundation is ſo faulty, and the whole ſuperſtructure ſo erroneous, that it were much better to abandon the fabrick, and ſuffer the ruins to remain a memorial, in all time to come, of the folly of philoſophical preſumption and pride.

leads, but likewiſe to trace *Spinoza*'s doctrine to its ſource (for the ſake of ſome who may have been miſled into a favourable opinion of it), which is no other than the *Carteſian* fable; of which almoſt every article has been diſproved by Sir *Iſaac Newton*, or others.

Mr.

Mr. *Leibnitz* retained the *Cartesian* subtile matter, with the universal plenitude and vortices; and represented the universe as a machine that should proceed for ever, by the laws of mechanism, in the most perfect state, by an absolute inviolable necessity; tho' in some things he differs from *Des Cartes*. After Sir *Isaac Newton*'s philosophy was published (in 1687), he printed an essay on the celestial motions (*Act. Erudit.* 1689) where he admits of the circulation of the ether with *Des Cartes*, and of gravity with Sir *Isaac Newton*; but he never explained how these could be reconciled, and adjusted together, so as to account for the planetary revolutions; or how gravity arose from the impulse of this ether. Nor did he shew how his harmonical circulation of the ether could be reconciled with the law of the motions of the several planets, in their respective orbits; which is very different from the law of the motions of the same planet, at its various distances from the sun. The angular velocity of any one planet, decreases from the *perihelium* to the *aphelium*, in the same proportion as its distance from the sun increases, and this is what he calls the harmonical circulation. If this law took place likewise in the motions of the different planets compared together, throughout the system, this hypothesis, of their being carried along with a circulating ether, might appear more tolerable: but the velocities of the planets, at their mean distances, decrease in the same proportion as the square-roots of the numbers which express those distances from the sun. Neither did he shew how to reconcile this circulating motion of the ether with the free motions of the comets in all directions, or with the obliquity of the planes in which the planets revolve to the equator of the sun and to one ano-

 ther;

ther; or resolve the other objections to which this hypothesis of a *plenum* and *vortices* is liable.

Afterwards however, on occasion of some disputes that had arisen concerning his title to the invention of the calculus of infinitesimals, or method of fluxions, he appeared with great warmth against Sir *Isaac Newton*'s philosophy, and placed himself at the head of its opposers. It is needless to insist here on the passion and prejudices that his followers have expressed against it, and against those that have appeared in its defence. It is better to forget these, and to confine a philosophical dispute to philosophical matters.

Mr. *Leibnitz*'s system has been the more acceptable to many, because, from the wisdom and goodness of the Deity, he concluded the universe, upon the whole, to be a perfect work, or the best that could possibly have been made. This doctrine was very agreeable in all times to the philosophers who acknowledged a supreme beneficent governor; but the origin of evil perplexed them. The solution of this was what *Socrates* expected from the writings of *Anaxagoras*, but was disappointed. The supreme Being, according to *Timæus Locrus*, was δαιμεργος τω βελτίονος. *Plato* taught that the supreme governor has disposed and complicated all things for the happiness and virtue of the whole, and that our complaints are groundless, arising from our narrow views of things. *Chrysippus* was of opinion * that it could

* Existimat *Chrysippus* non hoc fuisse naturæ principale consilium ut faceret homines morbis obnoxios, nunquam enim hoc convenisse naturæ auctori, parentique rerum omnium bonarum; sed quum multa atque magna gigneret, pareretque aptissima & utilissima, alia quoque simul agnata sunt incommoda iis ipsis quæ faciebat

could never have been the aim or firſt intention of the Author of nature, and parent of all good, to make men obnoxious to diſeaſes; but that while he was producing many excellent things, and forming his work in the beſt manner, other things alſo aroſe, connected with them, that were incommodious; which were not made for their own ſakes, but permitted as neceſſary conſequences of what was beſt. Mr. *Leibnitz* has wrote at great length in defence of this doctrine, and has endeavoured to anſwer the objections that have been made againſt the perfection of the univerſe.

But this learned author's ſpeculations, tho' they may perplex a cautious reader, cannot ſatisfy him. He propoſes two principles as the foundation of all our knowledge; the firſt, that it is impoſſible for a thing to be, and not to be at the ſame time, which, he ſays, is the foundation of ſpeculative truth. The other is, that nothing is without a *ſufficient reaſon* why it ſhould be ſo rather than otherwiſe; and by this principle, according to him, we make a tranſition from abſtracted truths to natural philoſophy. From this principle he concludes, that the mind is naturally determined, in its volitions or elections, by the greateſt apparent good; and that it is impoſſible to make a choice between things perfectly like, which he calls *indiſcernibles*; from whence he infers, that two things perfectly like could not have been produced even by the Deity. For this reaſon, and other metaphyſical conſiderations, he rejects a *vacuum*, the parts of which muſt be ſuppoſed perfectly like to each other. For the ſame reaſon he alſo rejects atoms, and all ſimilar particles of matter; to

faciebat cohærentia; eaque non per naturam, ſed per ſequelas quaſdam neceſſarias, facta dicit, quod ipſe appellat κατὰ παρακολούθησιν. *Aul. Gell.* lib. 6. cap. 1.

each of which, tho' divisible *in infinitum*, he ascribes * a *monad*, or active kind of principle, in which, says he, are as it were perception and appetites. The essence of substance he places in action or activity, or rather (as he expresses it) in something that is between acting and the faculty of acting. He affirms absolute rest to be impossible, and holds motion, or a sort of *nisus*, to be essential to all material substances. Each *monad* he describes as representative of the whole universe from its point of sight; and, after all, in one of his letters tells us, that matter is not a substance, but a *substantiatum*, or *phenomene bien fondé*.

Such are the doctrines and expressions of a philosopher who valued himself upon his clear and adequate ideas, and ridiculed the metaphysics of the *English*, as narrow, and founded on unadequate notions. The *criterion* of truth is usually placed in clear and evident perception; but some philosophers seem to value doctrines in proportion as they are obscure. Who would imagine that, in natural philosophy, such arguments should be preferred to the plainest facts and experiments for determining the question concerning a *vacuum*? Let any man reflect on his own thoughts, from which only any notions we have of liberty (and consequently of the divine liberty) can be derived; and if he is satisfied that he could chuse between two desirable things that appear equally good, rather than want both, such arguments can have no force upon him. His difficulty seems still to remain against the particles of matter, after all the pains he had taken to distinguish them by his *monads*; for how shall we distinguish the *monads* themselves? or if that may be practicable,

* Acta Lipsiæ, 1698, p. 435.

how

how ſhall we diſtinguiſh the ſame *monad* from itſelf, in all the moments of its exiſtence? If two things perfectly like to each other can exiſt in different times, ſurely they may exiſt in different places at the ſame time. This learned author appeared very averſe to thoſe doctrines which he imagined had a tendency to reſtore the exploded tenets of the ſcholaſtic philoſophy; yet theſe *monads*, as far as he has condeſcended to deſcribe them, appear to be as incomprehenſible as their ſubſtantial forms, *entelecheia*, or moſt occult qualities.

He makes great uſe of a compariſon between the effects of oppoſite motives on the mind, and of weights placed in the ſcales of a ballance, or of powers acting upon the ſame body with contrary directions. His learned antagoniſt denies that there is a ſimilitude between a ballance moved by weights, and a mind acting upon the view of certain motives; becauſe the one is entirely paſſive, and the other not only is acted upon, but acts alſo. The mind, he owns, is purely paſſive in receiving the impreſſion of the motive, which is only a perception, and is not to be confounded with the power of acting after, or in conſequence of, that perception. The difference between a man and a machine does not conſiſt only in ſenſation, and intelligence; but in this power of acting alſo. The ballance for want of this power cannot move at all, when the weights are equal: but a free agent, ſays he, when there appear two perfectly alike reaſonable ways of acting, has ſtill within itſelf a power of chuſing; and it may have ſtrong and very good reaſons not to forbear. It is evident that as it is from internal conſciouſneſs I know any thing of liberty, ſo no aſſertion contrary to what I am conſcious of concerning it can be admitted; and it were better perhaps to treat of this abſtruſe ſubject after the manner of experimental philoſophy, than to fill a thouſand pages with metaphyſical diſcuſſions concerning it. But to leave this ſubject, the doctrine of liberty is ſo foreign to the queſtions concerning a *vacuum* and atoms, that it muſt appear a far-fetched uncommon ſtretch of metaphyſics to pretend to determine them by it; and very unaccountable to refuſe the Deity the power of producing, by one act of his will, all the matter in the univerſe at once, tho' it ſhould be ſuppoſed perfectly ſimilar and uniform.

5. From the ſame principle, Mr. *Leibnitz* concluded, that the material ſyſtem is a machine abſolutely perfect, that can never fall into diſorder, or require to be ſet right; and that to imagine that God interpoſes in it, is to leſſen the ſkill of the Author, and the perfection of his work.

But this is more than his own principles require. For tho' it ſhould be allowed that nothing is limited without a ſufficient reaſon; yet, upon the whole, it may be better that the Author of the world ſhould act immediately in it, cheriſhing and governing his work, and ſometimes changing or renewing it. Can the beauty and perfection of the univerſe be the worſe for His acting in it, who muſt be ſuppoſed to act always with perfect wiſdom? It was fit that there ſhould be, in general, a regularity and conſtancy in the courſe of nature; not only for the ſake of its greater beauty, but alſo for the ſake of intelligent agents, who without this could have had no foreſight, or occaſion for choice and wiſdom in judging of things by their conſequences, and no proper exerciſe for their other faculties. But tho' the courſe of nature was to be regular, it was not neceſſary that it ſhould be governed by thoſe princi-

ples

ples only which ariſe from the various motions and modifications of unactive matter, by mechanical laws; and it had been incomparably inferior to what it is, in beauty and perfection, if it had been left to them only.

Sir *Iſaac Newton* was of opinion that the fabrick of the univerſe, and courſe of nature, could not continue for ever in its preſent ſtate, but would require, in proceſs of time, to be re-eſtabliſhed or renewed by the ſame hand that formed it. Yet this philoſophy was condemned by Mr. *Leibnitz* as leading to impiety; and, which is very ſurprizing, this particular doctrine was excepted againſt as having ſuch a tendency. He objected, that as a good artiſt made his workmanſhip as perfect as poſſible, ſo it argued a want of power or ſkill in the Author of the world, if it ſhould ever require to be reformed or wound up again. But Sir *Iſaac Newton* thought it altogether conſiſtent with the notion of a moſt perfect Being, and even more agreeable to it, to ſuppoſe that he ſhould form his work dependent upon himſelf, ſo as after proper periods to model it anew, according to his infinite wiſdom. To exclude the Deity from acting in the univerſe, and governing it, is to exclude from it what is moſt perfect and beſt, the abſence of which no mechaniſm can ſupply. Such a doctrine could not have been propoſed by one of Mr. *Leibnitz*'s ſentiments concerning the perfection of the univerſe, if he had not been miſled by an exceſſive fondneſs for neceſſity and mechaniſm.

The capital doctrine of this philoſophy that repreſents the univerſe as a perfect machine, ſuch as may continue for ever by mechanical laws in its preſent ſtate, is, that the ſame quantity of force and vigour remains

remains always in it, and paſſes from one portion of matter to another, without undergoing any change in the whole. *Des Cartes* maintained that the ſame quantity of motion is always preſerved in the univerſe. *Spinoza* called it the ſame proportion of motion to reſt. Mr. *Leibnitz* diſtinguiſhed between the quantity of motion, and the force of bodies; he owns that the former varies, but maintains that the quantity of force is for ever the ſame in the univerſe: and yet there is no doctrine more repugnant to perpetual experience and common obſervation than this is, even tho' we ſhould meaſure the forces of bodies by the ſquares of the velocities, according to his doctrine. If all bodies in the world had a perfect elaſticity, there might be ſome pretence for maintaining this principle. But there never has been diſcovered as yet any one body, whoſe elaſticity is perfect; and when any two bodies meet with equal motions, they rebound with leſs motions, and there is always force loſt by their colliſion; and if the bodies are ſoft, they both ſtop, becauſe of the impenetrability of their parts; or, to ſpeak in this author's favourite ſtyle, becauſe there can be no ſufficient reaſon why one of them ſhould prevail, rather than the other. In this caſe, their whole motion is loſt; and the motion of the one being deſtroyed by the oppoſite motion of the other, it is without ground, and merely to ſave an hypotheſis, that a fluid is imagined, which they feign to receive and retain the forces of thoſe bodies. When liberty is taken to ſupport one fiction by another, this by a third, and ſo on, any ſyſtem may be maintained. According to our firſt views of matter and motion, from the plaineſt experiments, matter appears to be an unactive ſubſtance of no elaſticity; yet they aſcribe a perfect elaſticity to all their ſubtile matter; and laws of motion are propoſed by them as general, which can hold of perfectly elastic bodies only, that is, of bodies not one of which has hitherto been found in nature. They have never been able to explain how this perfect elasticity arises from the laws of mechanism; yet, according to them, the world is a mechanical perpetual movement.

The genius of this kind of philosophy appears on no occasion so evidently, as from the arts which have been used to get rid of the insuperable objections against the *vortices*. To remove the difficulty a step farther, or to involve the question in obscurity, new *vortices* are introduced in every infinitely small particle of matter. From these, if there be occasion, they will descend into another order infinitely less; and so on; for they expresly pretend to take the same benefit from the infinite orders of infinitesimals, in philosophy *, that is claimed by some late geometricians in the resolution of their problems. Thus (as we observed elsewhere †) an absurd philosophy is the natural product of a vitiated geometry. For tho' it follows from our notion of magnitude, that it always consists of parts, and is divisible without end, yet an actual division *in infinitum* is absurd, and an infinitely little quantity (even in Mr. *Leibnitz*'s judgment ‡) is a mere fiction. Philosophers may allow themselves to imagine likewise infinite orders of infinitely small particles of matter, and suffer themselves to be transported with the idea; but these illusions are not supported by sound geometry, nor agreeable to common sense. After all that has been said for the *vortices*, there is not one experiment to favour them; and some of the most

* Mem. de l'Academie Royale des Sciences, 1729.
† Treatise of *Fluxions* Introd. p. 47.
‡ Essay de *Theodicée*, § 70.

common

common and ſimple are againſt admitting ſuch fluids and their motions.

We have another inſtance of the art by which they ſupport their ſchemes, in the pretended demonſtration they give againſt the poſſibility of atoms, or of any perfectly hard and inflexible bodies. According to what they call the law of *continuity*, all changes in nature are produced by inſenſible and infinitely ſmall degrees; ſo that no body can, in any caſe, paſs from motion to reſt, or from reſt to motion, without paſſing through all poſſible intermediate degrees of motion; from which they conclude that atoms, or any perfectly hard bodies, are impoſſible; becauſe if two of them ſhould meet with equal motions, in contrary directions, they would neceſſarily ſtop at once, in violation of the law of *continuity* *. But upon what grounds have they made this an univerſal law of nature? Tho' in common bodies (which are looſely compounded of particles that are themſelves compounded of others of a lower order, and ſo on; ſo that we cannot arrive at the elements, or atoms, till after we know not how many reſolutions) the parts yield in their colliſions, we cannot affirm this of the atoms or ultimate elements themſelves. This yielding is a conſequence of the contexture of bodies, which have always much more of void interſtices than of ſolid matter, and conſiſt of particles that muſt be ſuppoſed to adhere to one another with a force incomparably leſs than that by which the matter of the elementary particles themſelves holds together †. The

* Diſcours ſur le Mouvement, *Paris* 1726.

† The author of the above cited diſcourſe on motion, tells us, that if nature could paſs from a ſtate of motion to a ſtate of reſt at once, without paſſing through the intermediate degrees of motion,

The truth is, they found it neceſſary to reject bodies of a perfect hardneſs; becauſe it was impoſſible to explain the effects of their colliſions, in a manner conſiſtent with the preſervation of the ſame quantity of force in the univerſe, or with their new doctrine, *That the forces of bodies are as the ſquares of the velocities*; and therefore they had recourſe to this new law of *continuity* to proſcribe them. If ſuch a body ſhould ſtrike another equal quieſcent body, of the ſame kind, the velocity of the firſt would be equally divided by the ſtroke between them; but if we meaſure the force by the ſquare of the velocity, each of them would have but one fourth part of the force of the firſt body; and both together would have but one half of its force; ſo that the other half would be neceſſarily loſt, without producing any ſort of effect. In order to get rid of objections of this kind, ſome of the favourers of the new doctrine, concerning the menſuration of the force of bodies, content themſelves with obſerving, that no bodies of a perfect hardneſs have been found in nature; tho' there is the ſame objection againſt admitting and treating of bodies of a perfect elaſticity. But others boldly reject ſuch hard bodies as impoſſible, from thoſe far-fetched metaphyſical conſiderations we have deſcribed. How much they have endeavoured to perplex the theory of motion, in its plaineſt parts, from a zeal for the ſame doctrine, will appear afterwards.

motion, then one ſtate would be deſtroyed before nature could know what new ſtate ſhe ought to determine herſelf to; and asks how ſhe could then determine herſelf to any one ſtate rather than another? In anſwer, we need only obſerve, that to ceaſe to move is the ſame as to be at reſt, and that when the equal atoms ſtop each other at once, there is no interval between the ſtate of motion and that of reſt; and that when motion is deſtroyed, reſt neceſſarily enſues.

The

The power of mechaniſm was never more magnified than by Mr. *Leibnitz*'s famous doctrine of a *pre-eſtabliſhed harmony*, as he calls it. According to *Des Cartes*, the brutes were mere machines; and this doctrine, to many, appeared incredible. But this is nothing in compariſon to what Mr. *Leibnitz* would have us believe, when he tells us that the ſoul does not act on the body, nor the body on the ſoul; that both proceed by neceſſary laws, the ſoul in its perceptions and volitions, and the body in its motions, without affecting each other; but that each is to be conſidered as a ſeparate independent machine. The volitions of the mind are followed inſtantly by the deſired motions of the body, not in conſequence of thoſe volitions in the leaſt, but of the nice and well adjuſted machinery of the body. The impreſſions produced in the ſenſory have no effect on the mind, but the correſponding idea ariſes, at that preciſe time, in conſequence of a chain of cauſes of a different kind. Thus all that men do or ſay, is no more than the effect of exquiſite machinery, according to him. But it is time for us to leave thoſe fictions, leſt the reader ſhould be tempted to think that all philoſophy is illuſion.

CHAP. V.

Concluſions from the foregoing obſervations.

1. THE ſum of what we have obſerved is, that tho' theſe learned men may have ſhewn abundance of genius and invention in their writings; yet they, and all others who have followed a like method, have begun at the wrong end, in tracing the chain of cauſes, and have attempted to form a ſcheme of philoſophy that far ſurpaſſes the human facul-

faculties. The eternal reaſons and primary cauſes of things, which they imagine they poſſeſs, riſe infinitely above them; while certain obſervation, and plain facts, perpetually appear in contradiction to their boaſted ſpeculations.

We are to endeavour to riſe, from the effects thro' the intermediate cauſes, to the ſupreme cauſe. We are, from his works, to ſeek to know God, and not to pretend to mark out the ſcheme of his conduct, in nature, from the very deficient ideas we are able to form of that great myſterious Being. Thus natural philoſophy may become a ſure baſis to natural religion, but it is very prepoſterous to deduce natural philoſophy from any hypotheſis, tho' invented to make us imagine ourſelves poſſeſt of a more complete ſyſtem of metaphyſics, or contrived perhaps with a view to obviate more eaſily ſome difficulties in natural theology. We may, at length, reſt ſatisfied, that in natural philoſophy, truth is to be diſcovered by experiment and obſervation, with the aid of geometry, only; and that it is neceſſary firſt to proceed by the method of *analyſis*, before we preſume to deliver any ſyſtem *ſynthetically*.

We may alſo learn at length, from the bad ſucceſs of ſo many fruitleſs attempts, to be leſs fond of perfect and finiſhed ſchemes of natural philoſophy; to be willing to ſtop when we find we are not in a condition to proceed farther; and to leave to poſterity to make greater advances, as time and obſervation ſhall enable them. For we cannot doubt but that nature has diſcoveries in ſtore for future times alſo, which may be retarded by our raſh and ill-grounded anticipations. By proceeding with due care, every age will add to the common ſtock of knowledge; the myſteries that ſtill lie concealed in nature

nature may be gradually opened, arts will flouriſh and increaſe, mankind will improve, and appear more worthy of their ſituation in the univerſe, as they approach more towards a perfect knowledge of nature.

2. 'Twas thus the ſpeculative parts of the mathematics gradually aroſe, from ſmall beginnings, by the conſpiring labours of great men, in the diſtant ages of the world. The *Egyptians* began this ſcience, the *Greeks* purſued it, the *Arabians* preſerved it, when it was loſt in *Europe*, and ſet a high value upon it while their empire flouriſhed; and ſince the late memorable reſtoration of letters in *Europe*, its great progreſs has been the boaſt of modern learning.

The inundations of the *Nile* made it neceſſary for the *Egyptians* to invent ſome art by which they ſhould be able to meaſure their land, and to this, we are told, geometry owes its origin and name. The prieſts of that country, abounding in leiſure and genius, improved it into a ſcience; and their kings wrote treatiſes upon it. *Thales* brought the principles of it into *Greece*, where it was ſo diligently cultivated that the elementary part was ſoon compleated, and was ſo highly eſteemed as to have the appellation of the *mathemata* in a manner appropriated to it. An oracle appointing the cubical altar of *Apollo* to be doubled was, we preſume, of greater advantage to geometry than to the *Athenians* then afflicted with the plague; as it gave occaſion to *Plato* to conſider the famous problem of the duplication of the cube, and produced the *ſolid* geometry. It afterwards received great improvements from the incomparable *Archimedes*, who ſquared the area of the *parabola*, made ſome progreſs in the menſuration

tion of the circle, and enriched this ſcience with many diſcoveries worthy of ſo excellent a genius.

It appears that it advanced but by degrees, and ſometimes by very ſlow ſteps: one, we are told, diſcovered that the three angles of an equilateral triangle were equal to two right ones; another went farther, and ſhewed the ſame thing of thoſe that have two ſides equal and are called *iſóſceles* triangles; and it was a third who found that the theorem was general, and extended it to triangles of all ſorts *. In like manner, when the ſcience was farther advanced, and they came to treat of the conic ſections, the plane of the ſection was always ſuppoſed perpendicular to the ſide of the cone; the *parabola* was the only ſection that was conſidered in the right-angled cone, the *ellipſe* in the acute-angled cone, and the *hyperbola* in the obtuſe-angled. From theſe three ſorts of cones, the figures of the ſections had their names, for a conſiderable time; till at length *Apollonius* ſhewed how they might be all cut out of any one cone, and by this diſcovery merited in thoſe days the appellation of the *great geometrician*.

By ſuch ſteps this ſcience roſe, in proceſs of time, to that vaſt height for which it is admired. Problems that appeared of an inſuperable difficulty in one age were reſolved in another, and, in a third, were in a manner deſpiſed as too ſimple and eaſy; particular theorems were firſt inveſtigated that led to more extenſive diſcoveries; laborious methods were followed, till others were found that were more ſimple and general; but the greateſt care was always taken of the certainty and evidence of the ſcience, as it was carried on. There was indeed a long

* *Procli* Comment. in *Euclidem*.

interval of many ages, between the period when it flouriſhed in *Greece*, and revived in *Europe*: but the antients, having founded it on unexceptionable grounds, and carried it on with the utmoſt accuracy, when learning was reſtored, their works ſerved for a baſis, as well as for models, to the modern inventors. Thus the gradual progreſs of mankind in this ſcience appears ſimilar, in ſome reſpects, to the advances of a man in vigour and knowledge. They firſt made eſſays of a weak and unexperienced ſtrength, which by degrees acquired more and more force, till at length, after the ſucceſsful labours of ſeveral ages, nothing ſeem'd too high for them.

3. From what we have obſerved concerning the hiſtory of natural philoſophy, it may eaſily be underſtood why its progreſs has been ſo different; and whence it proceeds that we ſeldom have found in it, as in geometry, that pleaſing gradual riſe from ſmall beginnings to greater heights. Inſtead of ſearching into nature, men retired to contemplate their own thoughts; inſtead of tracing her operations, they gave their imaginations full play: where they ought to have heſitated, they decided; and where there was no difficulty, they doubted. What was ſimple they divided, and defined what was plain; but in what was more intricate, the ſubterfuges of art were ſet up in oppoſition to nature, and captious ſcience againſt common reaſon; while one ill-grounded maxim was imagined, to ſupport another, and fiction was grafted on fiction. Hypotheſes were invented, not for reducing facts or obſervations of a complicated nature to rules and order, (for which purpoſe they may be of ſervice) but as principles of ſcience. Theſe were of ſo great authority as not to be overturned by contradictory obſervations, or by the extravagant conſequences that aroſe from them; but the

the author, charm'd with his rhapſody, proceeded, without minding theſe, to the concluſion of his fable.

Thus one age or ſect could not but deſtroy, for the moſt part, the labours of another. Sometimes the *numbers* and *harmony* of the *Pythagoreans* ſerved for explaining what was moſt myſterious in nature; the *ideas* of *Plato*, the *matter* and *form* of *Ariſtotle* prevailed in their turn; but theſe were of uſe only to veil the ignorance of men. *Epicurus* employed his philoſophy to overthrow the plain and evident dictates of ſenſe and reaſon; yet diſciples were not wanting to ſupport and adorn ſo abſurd a ſcheme. The *Sceptics* went into the oppoſite extreme, and became ſo fond of darkneſs that they would not ſee the light tho' never ſo clear; and ſome of them choſe rather to doubt that they doubted, than to acknowledge any thing; yet they too had numerous followers. Afterwards philoſophy was in no eſteem but as far as it ſerved, by a perplexed and falſe gloſs, to promote the ends of ſuperſtition. Of late, the pretended clear ideas of *Des Cartes*, and metaphyſical ſpeculations of Mr. *Leibnitz*, have been received by many for true philoſophy; not to mention the extravagancies of *Spinoza*, and a thouſand crude notions that deſerve no memory.

We have ſeen, in the foregoing account of the ſtate of philoſophy in different periods, that they who have indulged themſelves in inventing ſyſtems and compleating them, tho' they have ſometimes ſet out in a manner that has appeared plauſible, yet, in purſuing thoſe ſchemes, ſuch conſequences have ariſen as could not fail to diſguſt all but ſuch as were intoxicated with the deceit. Some, from their fondneſs to explain all things by mechaniſm, have been led to exclude every thing but matter and motion out of the univerſe: others, from a contrary diſpoſition, admit nothing but perceptions, and things which perceive; and ſome have purſued this way of reaſoning, till they have admitted nothing but their own perceptions. Others, while they overlook the intermediate links in the chain of cauſes, and haſtily reſolve every principle into the immediate influence of the firſt cauſe, impair the beauty of nature, put an end to our enquiries into the moſt ſublime part of philoſophy, and hurt thoſe very intereſts which they would promote. In framing thoſe ſyſtems, he who has proſecuted each of them fartheſt has done this valuable ſervice, that, while he vainly imagined he improved or compleated it, he really opened up the fallacy, and reduced it to an abſurdity. Many who ſuffered themſelves to be pleaſed with *Des Cartes*'s fable, were put to a ſtand by *Spinoza*'s impieties. Many went along with Mr. *Leibnitz*'s ſcheme of abſolute neceſſity, but demurred at his *monads* and *pre-eſtabliſhed harmony*. And ſome, willing to give up the reality of matter, could not think of giving up their own and other minds.

The variety of opinions and perpetual diſputes amongſt philoſophers has induced not a few, of late as well as in former times, to think that it was vain labour to endeavour to acquire certainty in natural knowledge, and to aſcribe this to ſome unavoidable defect in the principles of the ſcience. But it has appeared ſufficiently, from the diſcoveries of thoſe who have conſulted nature and not their own imaginations, and particularly from what we learn from Sir *Iſaac Newton*, that the fault has lain in the philoſophers themſelves, and not in philoſophy. A com-

compleat ſyſtem indeed was not to be expected from one man, or one age, or perhaps from the greateſt number of ages; could we have expected it from the abilities of any one man, we ſurely ſhould have had it from Sir *Iſaac Newton*: but he ſaw too far into nature to attempt it. How far he has carried this work, and what are the moſt important of his diſcoveries, we now proceed to conſider.

BOOK II.

Of the theory of motion, or rational mechanics.

CHAP. I.

Of ſpace, time, matter, and motion.

1. AS we are certain of our own exiſtence, and of that of our ideas, by internal conſciouſneſs; ſo we are ſatisfied, by the ſame conſciouſneſs, that there are objects, powers, or cauſes without us, and that act upon us. For in many of our ideas, particularly thoſe that are accompanied with pain, the mind muſt be paſſive, and receive the impreſſions (which are involuntary) from external cauſes or inſtruments, that depend not upon us. We eaſily diſtinguiſh theſe objects into two general claſſes. The firſt is of thoſe which we perceive to have a ſpontaneity, or ſelf-moving power, and ſeveral properties and affections ſimilar to thoſe of our own minds, ſuch as reaſoning, judging, willing, loving, hating, &c. The ſecond general claſs is of thoſe in which no ſuch affections appear, but which are ſo far of a paſſive nature, that they never move of themſelves, neither, when they are in motion, do they ever ſtop without ſome external influence. If one of theſe move out of its place, without the appearance of a mover, we immediately conclude that this is owing to ſome inviſible agent; ſo much are we perſuaded of its own *inertia*. If we lay up one of them in any place, we expect to find it there at any

any diſtance of time, if no other powers have had acceſs to it. This paſſive nature, or *inertia*, is what chiefly diſtinguiſhes the ſecond claſs of external objects, which is called *body* or *matter*; as the former is called *mind* or *ſpirit*.

2. How external objects, of either claſs, act upon the mind, by producing ſo great a variety of impreſſions or ideas, is not our buſineſs at preſent to enquire: neither is it neceſſary for us to determine how exact or perfect the reſemblance may be between our ideas and the objects or ſubſtances they repreſent. In our ideas which are repetitions of other ideas, we find very different degrees of reſemblance between them and thoſe of which they are repetitions. The idea we form in our imagination of a perſon, place, or figure which we have often ſeen, has a much more perfect reſemblance to the impreſſion we receive from ſenſe, than the idea we are able in our imagination to form of pain, as to the ſenſation we have felt of it. And as it is no objection againſt the exiſtence of the ſouls of other men, that they may be very different from the notion or conception we may have formed of them; ſo it is no juſt reaſon againſt admitting the exiſtence of body, that its inward eſſence, or *ſubſtratum*, may be very different from any thing we know of it. It is, however, rating our ideas of external objects by much too low, to compare them to words or arbitrary ſigns, ſerving only to diſtinguiſh them from each other. For it is from our ideas of them that we learn their properties, relations, and their influences upon each other, and upon our minds and thoſe of others, and acquire uſeful knowledge concerning them and ourſelves. For example, by comparing and examining our ideas, we judge of order and confuſion, beauty and deformity, fitneſs and

 unfit-

unfitneſs, in things. The ideas of number and proportion, upon which ſo uſeful and extenſive ſciences are founded, have the ſame origin.

3. The mind is intimately conſcious of its own activity in reflecting upon its ideas, in examining and ranging them, in forming ſuch as are complex from the more ſimple, in reaſoning from them, and in its elections and determinations. From this, as well as from the influence of external objects upon the mind, and from the courſe of nature, it eaſily acquires the ideas of cauſe and effect. When a figure deſcribed upon a board produces a ſimilar idea or impreſſion on all thoſe who ſee it, it is as natural to aſcribe this to one cauſe, as, when we ſpeak to a numerous audience, the effect of the diſcourſe is to be aſcribed to us; tho' we may be unable to explain how the impreſſion of the figure is communicated to the ſeveral ſpectators, or the diſcourſe to the hearers. It were eaſy to make many more remarks on the philoſophy of thoſe whoſe principles would lead them to maintain, that external objects vary with our preceptions, and that the object is always different when perceived by different minds, or by the ſame perſon at different times, or in different circumſtances. It will not be expected from us that we ſhould enter farther, in a treatiſe of this kind, into the examination of doctrines as fruitleſs as they are extravagant.

4. Body not only never changes its ſtate of itſelf, in conſequence of its paſſive nature or *inertia*, but it alſo reſiſts when any ſuch change is produced: when at reſt, it is not put in motion without difficulty; and when in motion, it requires a certain force to ſtop it. This force with which it endeavours to perſevere in its ſtate, and reſiſts any change, is called its *vis inertiæ*; and ariſes from the *inertia* of

of its parts, being always proportional to the quantity of matter in the body; insomuch that it is by this *inertia* only we are able to judge of the quantity of matter. And this judgment is well founded, because we constantly find that when we double or triple a body, or increase or diminish it in any proportion, we must double or triple the force that is requisite to move it with the same celerity, or increase or diminish it in the same proportion with the body. If the solid, uncompounded particles void of pores, of equal bulk, have their *inertia* equal, then this must be accurately true: but if matter be of kinds so different from each other, that the solid elementary particles of the one have a greater *inertia* than equal solid elementary particles of the other kind, then it is only when we compare those of the same kind, that we can affirm the *inertia* to be proportional to the quantity of matter. Such different kinds of matter may exist for ought we know; but it is by diminishing or increasing the number or dimensions of the pores of bodies that they are condensed or rarified, according to our experience, and thereby the *inertia* of a given bulk increased or diminished.

5. Space is extended without limits, immoveable, uniform and similar in all its parts, and void of all resistance. It consists indeed of parts which may be distinguished into other parts, less and less, without end, but cannot be separated from each other, and have their situation and distances changed.

6. Body is extended in space, moveable, bounded by figure, solid, and impenetrable, resisting by its *inertia*, divisible into parts, less and less, without end, that may be separated from each other and have their situation or distances changed in any manner.

7. From the ſucceſſion of our own ideas, and from the ſucceſſive variations of external objects in the courſe of nature, we eaſily acquire the ideas of *duration* and *time*, and of their meaſures. We conceive true or abſolute time, to flow uniformly in an unchangeable courſe, which alone ſerves to meaſure with exactneſs the changes of all other things. For unleſs we correct the vulgar meaſures of time, which are groſs and inaccurate, by proper equations, (as in predicting the eclipſes of the ſatellites of *Jupiter*, and moſt other aſtronomical phænomena) the concluſions are always found inaccurate and erroneous: and however various the flux of time may appear to different intellectual beings, it cannot, at leaſt, be thought to depend upon the ideas of any created being. Time may be conceived to be divided into ſucceſſive parts that may be leſs and leſs without end; tho', with reſpect to any one particular being, there may be a leaſt ſenſible time, as well as a *minimum ſenſibile* in other magnitudes.

8. Motion is the change of place; that is, of the part of ſpace which the body occupies, or in which it is extended. The motion is *real* or *abſolute*, when the body changes its place in abſolute ſpace. It is called *relative*, when the body changes its place with relation only to ambient bodies; and it is *apparent* motion, when the body changes its ſituation with reſpect to other bodies that appear to us to be at reſt. The parts of abſolute ſpace not being the objects of our ſenſes, it is one of the great difficulties in philoſophy to diſtinguiſh which motions are true and real, and which are apparent only. However, philoſophers by proper care are often able to effect this, by arguing juſtly from the cauſes of the motion when known, or from their properties and effects. A real

circu-

circular motion, for example, is always accompanied with a centrifugal force, arifing from the tendency which a body always has to proceed in a right line. Thus, from the centrifugal force which, at the æquator, diminifhes the gravity and retards the motion of the pendulum, fo that it moves more flowly there than towards either pole, we have a proof of the earth's diurnal rotation on its axis. At the fame time, the diurnal revolution of the heavenly bodies about the earth muft be apparent only; fince if it was real, an immenfe centrifugal force would thence arife, which could not but difcover itfelf; becaufe they move in free fpaces, and the folid orbs have been exploded upon the moft evident grounds.

9. I know that fome metaphyficians of great character condemn the notion of abfolute fpace, and accufe mathematicians in this of realizing too much their ideas: but if thofe philofophers would give due attention to the phænomena of motion, they would fee how ill grounded their complaint is. From the obfervation of nature, we all know that there is motion; that a body in motion perfeveres in that ftate, till by the action or influence of fome power it be neceffitated to change it; that it is not in relative or apparent motion in which it perfeveres, in confequence of its *inertia*, but in real and abfolute motion. Thus the apparent diurnal motion of the ftars would ceafe, without the leaft power or force acting upon them, if the motion of the earth was ftopt; and if the apparent motion of any ftar was deftroyed by a contrary motion impreffed upon it, the other celeftial bodies would ftill appear to perfevere in their courfe, the centrifugal force at the æquator would ftill fubfift, with the fpheroidical figure of the fluid ocean; the confequences of the real motion of the earth

upon

upon its axis. They who are not well acquainted with the theory of motion, more eaſily allow that a body at reſt continues at reſt, in conſequence of its paſſive nature or *inertia*, than that when in motion it continues in motion: but this perſeverance of a body in a ſtate of reſt can only take place with relation to abſolute ſpace, and can only be intelligible by admitting it. When a topp turns upon a ſmall pivot, its circular motion will continue ſmooth for a long time, but any body placed upon its ſurface does not continue in that place, but immediately flies off. When a ſhip moves ſteadily, any body placed in the cabin continues in its place, as if the whole was at reſt; but when the motion of the ſhip is ſtopt, the body flies off in the direction of its former motion; for, in conſequence of its *inertia*, it endeavours to perſevere, not in its ſtate of reſt in the ſhip, but in its ſtate of motion or reſt with regard to abſolute ſpace. It were eaſy to enlarge on this ſubject, and to ſhew that there is no explaining the phænomena of nature without allowing a real diſtinction between true, or real, and apparent motion, and between abſolute and relative ſpace. Whatever thoſe philoſophers may pretend, we have no clearer idea than of ſpace; and tho' ſome puzzling diſputes may ariſe in ſome of our enquiries concerning it, this is what we meet with in all our enquiries into nature; our knowledge of which we ought to take care to have as clear and well founded as poſſible, tho' it is in vain to pretend to make it complete and perfect; as we obſerved in the firſt book.

10. Body being diſtinguiſhed from ſpace by its *vis inertiæ* or reſiſtance, it is an obvious ſuggeſtion of common ſenſe that all ſpace is not equally full of matter; and it is the reſult of philoſophical enquiries, that the ſolid matter in the denſeſt bodies bears

a ſmall proportion to their whole bulk. The rays of light find a paſſage through a glaſs globe in all directions, which argues the great rarity of the globe, as well as the ſubtility of light. The ſame is to be ſaid of the magnetic and electric *effluvia*, and of the ſubtile matter that pervades the pores of bodies with great freedom in chymical experiments. As for thoſe fluids which philoſophers have invented, in order to repleniſh the pores of bodies, ſo as to exclude a void out of the univerſe, we made ſome obſervations upon them in the firſt book; and we may have occaſion afterwards to ſhew how improper they are for accounting for the phænomena which have been aſcribed to them.

11. Space and time ſerve to meaſure each other, reciprocally, by motion: time is in a perpetual flux and periſhing; but a repreſentation of it is preſerved in the ſpace deſcribed by the motion. When the ſpace flows as the time, that is, when equal parts of ſpace are deſcribed in any equal parts of the time, then the motion is uniform, and the velocity is conſtant or unvaried during the motion. When the parts of ſpace, deſcribed in any equal ſucceſſive parts of the time, continually increaſe, the motion is accelerated; and when thoſe parts of ſpace continually decreaſe, the motion is retarded. In general, the velocity of motion is always meaſured by the ſpace that would be deſcribed by that motion continued uniformly for a given time. It is obvious that the ſpace, deſcribed by an uniform motion, is in the compound proportion of the time and velocity of the motion: but in general, let A B, (*Fig.* 1.) the baſe of a figure, repreſent the time of a motion, and the ordinate or perpendicular P M, at any point P of the baſe, meaſure the velocity at the correſponding term of time, (that is, the ſpace which would be de-

defcribed by the motion continued uniformly from that term for a given time) then the area of the figure ABD fo formed will meafure the fpace defcribed by the motion, in the time reprefented by the bafe AB. Thus a rectangular parallelogram ferves to meafure the fpace defcribed by an uniform motion, the time being reprefented by the bafe, and the conftant velocity of the motion by the perpendicular. The fpace defcribed by a motion which is uniformly accelerated (the velocity of which increafes uniformly as the time, that is, receives equal augments in any equal fucceffive parts of time) is reprefented by a triangle; the time being reprefented by the bafe, and the increafing velocity by the perpendicular, which increafes in the fame proportion as the bafe. Becaufe the triangle is the half of a parallelogram of the fame bafe and altitude, the fpace defcribed by a motion uniformly accelerated, during any time, from the beginning of the motion, is one half of what would have been defcribed if the motion had been uniform, and the velocity had been the fame as is acquired at the end of that time. Becaufe fimilar triangles are as the fquares of their analogous fides, the fpaces defcribed by a motion uniformly accelerated, being meafured by fuch triangles, are as the fquares of the times from the beginning of the motion; or as the fquares of the velocities acquired at the end of thofe times. The fpaces defcribed by motions uniformly retarded are meafured in the fame manner; only the times and velocities are to be taken in a contrary order, till the extinction of the motion. In other cafes, the fpaces are meafured by curvilinear areas. And becaufe there are areas whofe ordinates decreafe in fuch a manner, that tho' the figure be produced indefinitely, the area never amounts to a certain finite fpace; it appears that the velocities of a retarded motion may

may decreaſe in ſuch a manner, that, tho' the motion was continued ever ſo long, yet the ſpace deſcribed by it ſhould not exceed any certain given line. For example, if the velocity during the firſt hour be double of what it is in the ſecond hour, and this be reduced to its half in the third hour, and ſo on for ever, then the ſpace deſcribed by this motion, tho' it was to continue for the greateſt number of ages, will never amount to the double of the line deſcribed in the firſt hour.

12. The quantity of motion in a body being the ſum of the motions of its parts, is in the compounded ratio of its quantity of matter and of the velocity of the motion. If the body A, of a quantity of matter repreſented by 2, moves with a velocity repreſented by 5, and the body B, repreſented by 3, moves with a velocity repreſented by 4; then the quantity of motion of A, ſhall be to the quantity of motion of B, in the compounded ratio of 2 to 3 and of 5 to 4, that is as 2 × 5 to 3 × 4, or as 10 to 12. There appears to be no ground for making a diſtinction between the *quantity of motion* and the *force* of a body in motion; as all the power or activity of body ariſes from and depends upon its motion. We are not, however, to expect that all the effects of the motion of bodies ſhould be proportional to the quantity of motion, unleſs a due regard be had to the time of the motion, and to the direction in which it acts, according to the true principles of mechanics. A body, in conſequence of its uniform motion, deſcribes a certain ſpace in a certain time; but there is no ſpace ſo great that may not be deſcribed by it, if the time be not limited. When a body acts upon another body, the effect is very different according to the direction in which it acts. How neceſſary it is to have regard to theſe,

in

in determining the effects of the motions and actions of bodies, will appear more fully in the next chapter.

13. When a body tends to move, but is hindered by some obstacle, this tendency is called *pressure*. It is not to be compared with the force of a body in motion, no more than a line is to be compared with the rectangle that is generated by it. Of this kind is the gravity of a body that rests and presses upon a table, or of water upon the bottom of a vessel, or of air upon the sails of a ship. When the obstacle is removed, the continual action of the pressure generates motion in the body, in any finite time. Thus gravity accelerates the motion of falling bodies, by acting incessantly upon them. When an orifice is opened in the bottom of a vessel, the pressure of the fluid accelerates the motion of the issuing water, and, in an exceeding little time, brings its velocity to a height. When the wind acts upon the sails of a ship, it accelerates her motion for some time, till the resistance of the water (which increases with the increasing velocity of the ship) ballances the action of the wind; after which her motion becomes uniform. In these, and all such other instances, the motion begins from nothing; and it is in consequence of the continual incessant action of the power or pressure, that the velocity, generated in any finite time, is finite. If we were to suppose that each action of the power produced a finite augmentation of velocity, the motion acquired in the least finite time would be infinite, or surpass any assignable velocity; as we have demonstrated elsewhere *.

* See the Treatise of *Fluxions*, § 44.

14. *Gravity* is the beſt known to us of all thoſe powers or preſſures. Becauſe all bodies deſcend with equal velocity in a void, the gravity of bodies muſt be proportional to their quantity of matter; and depends not upon the figure or texture of the parts, but upon their ſolid matter only. This is evident by experiments of the motion of pendulums, made with the greateſt exactneſs. For when the lengths of the pendulums are equal, bodies of very different bulks, and different internal and external texture, perform their vibrations in times exactly equal in equal arcs, keeping always pace together, and acquiring always equal velocities at the correſponding points of thoſe arcs, unleſs ſo far as the reſiſtance of the air acts upon them unequally. In the common buſineſs of life, the quantity of matter of bodies has been always meaſured out by their weight; tho' the influence of the air is various in its different ſtates, and renders this menſuration ſomewhat unaccurate in things of great value. Tho' the gravity of bodies really ariſes from their gravitation towards the ſeveral parts of the earth (as will appear afterwards) yet, becauſe this power acts around in all parts, and its direction is nearly towards the centre of the earth, it is therefore called a centripetal force. We ſhall, afterwards, ſhew that ſimilar centripetal forces tend to the ſun and planets. Theſe forces are of three kinds: the *abſolute* force is meaſured by the motion that would be produced by it in a given body, at a given diſtance. For example, the abſolute centripetal force tending towards the ſun is to that which tends toward the earth, as the motion which would be produced by the force tending toward the ſun in a given body, at a given diſtance without the ſun's body, is to the motion which would be produced by the force tending towards the earth in an equal body, at an equal diſtance from it. As when we compare

 the

the forces of two magnets, we must compare their effects at equal distances; so when we compare the absolute forces which tend to the central bodies, the comparison cannot be just unless it be from effects produced when the circumstances are alike.

The second sort of centripetal force is the *accelerating* force, which is measured by the velocity generated by it in a given time, and is different at different distances from the same central body, but depends not on the quantity of matter of the body that gravitates, being equal in all sorts of bodies at equal distances from the centre. The third sort is the weight, or the *vis motrix*, and is measured by the quantity of motion that is generated in a heavy body in a given time; and differs from the accelerating force in the same manner as motion differs from velocity.

15. Because the power of gravity is so well known to us, when we enquire into other powers, we endeavour to compare them with that of gravity, and to determine their proportion. We find a great variety of powers analogous to it in nature; such as that by which the particles of fluids form themselves into drops; that by which the parts of hard bodies cohere together; that by which the rays of light, in entering into water or glass, or into any medium of a greater refractive power, are constantly bent towards the perpendicular, and when they are incident upon the farther surface of the glass, with a sufficient obliquity, are all turned back into the glass, though there be no sensible medium behind the glass to reflect it; in the same manner as a heavy body projected obliquely upwards is bent into a curve, and brought back to the earth again by its gravity. These, and many other powers in nature, have an analogy

analogy to gravity, but extend to leſs diſtances, and obſerve laws ſomewhat different. It has been found very difficult to account for them mechanically. For this purpoſe, ſome have imagined certain *effluvia* to proceed from bodies, or atmoſpheres environing them; others have invented *vortices*; but all their attempts have hitherto proved unſatisfactory. That ſuch powers take place in nature, and contribute to produce its chief phænomena, is moſt evident; but their cauſes are very obſcure, and hardly acceſſible by us. In all the caſes when bodies ſeem to act upon each other at a diſtance, and tend towards one another without any apparent cauſe impelling them, this force has been commonly called *attraction*; and this term is frequently uſed by Sir *Iſaac Newton*. But he gives repeated cautions that he pretends not, by the uſe of this term, to define the nature of the power, or the manner in which it acts. Nor does he ever affirm, or inſinuate, that a body can act upon another at a diſtance, but by the intervention of other bodies. It is of the utmoſt importance in philoſophy to eſtabliſh a few general powers in nature, upon unqueſtionable evidence, to determine their laws, and trace their conſequences, however obſcure the cauſes of thoſe powers may be; and this he has done with great ſucceſs.

16. But however commodious the term *attraction* may be, to avoid an uſeleſs and tedious circumlocution, yet becauſe it was uſed by the ſchool-men to cover their ignorance, the adverſaries of Sir *Iſaac Newton*'s philoſophy have taken an unjuſt handle from his uſe of this term, after all his precautions, to depreciate and even ridicule his doctrines; by which they only convince us that they neither underſtand them, nor have impartially and duely conſidered them. Mr. *Leibnitz* made uſe of this ſame

term, in the ſame ſenſe with Sir *Iſaac Newton*, before he ſet up in oppoſition to him; and it is often to be met with in the writings of the moſt accurate philoſophers, who have uſed it without always guarding againſt the abuſe of it, as he has done. A term of art has been often employed by crafty men, with too much ſucceſs, to raiſe a diſlike againſt their opponents, and miſlead the unwary, and to diſguſt them from enquiring into the truth; but ſuch diſingenuity is unworthy of philoſophers. No writer hath appeared againſt Sir *Iſaac Newton*, of late, by whom this argument, tho' altogether groundleſs, is not inſiſted on at great length; and ſometimes adorned with the embelliſhments of wit and humour; but if the reader will take the trouble to compare their deſcriptions with Sir *Iſaac Newton*'s own account, he will eaſily perceive how little it was minded by them; and that the ſum of all their art and ſkill amounts to this only, that they were able to expoſe a creature of their own imagination. Poſſibly ſome unſkilful men may have fancied that bodies might attract each other by ſome charm or unknown virtue, without being impelled or acted upon by other bodies, or by any other powers of whatever kind; and ſome may have imagined that a mutual tendency may be eſſential to matter, tho' this is directly contrary to the *inertia* of body deſcribed above; but ſurely Sir *Iſaac Newton* has given no ground for charging him with either of theſe opinions: he has plainly ſignified that he thought that thoſe powers aroſe from the impulſes of a ſubtile ætherial *medium* that is diffuſed over the univerſe, and penetrates the pores of groſſer bodies. It appears from his letters to Mr. *Boyle* *, that this was his opinion early; and if

* See the life of Mr. *Boyle* premiſed to the late complete edition of his works.

he

he did not publiſh it ſooner, it proceeded from hence only, that he found he was not able, from experiment and obſervation, to give a ſatisfactory account of this medium, and the manner of its operation, in producing the chief phænomena of nature. They who imagine that he has only introduced a new phraſe or two into philoſophy, without any real benefit, may be eaſily ſatisfied of their miſtake, if they will but conſider with what evidence he has reſolved the chief phænomena of the ſyſtem of the world from thoſe powers; how he has computed the quantity of matter and denſity of the ſun, and of ſeveral of the planets, from them; how nearly he has determined the motion of the *nodes* of the moon, from its cauſe; and explained many of her irregularities, and the other motions of the ſyſtem. But we have inſiſted upon this perhaps at too great length; for as no philoſopher ſcruples to ſay that the magnet attracts iron, and that electric bodies, when their virtue is raiſed by friction, attract light ſubſtances; it muſt be allowed to be at leaſt as juſtifiable an expreſſion, or even more unexceptionable, to ſay that the earth attracts heavy bodies towards it; ſince all of them deſcend towards it with forces proportional to their quantity of matter, at equal diſtances from it; and this power extends to all diſtances, varying according to a certain known law.

CHAP. II.

Of the laws of motion, and their general corollaries.

1. THE first law of motion is, "That a body "always perseveres in its state of rest, or "of uniform motion in a right line, till by some "external influence it be made to change its state." That a body, of itself, perseveres in its state of rest, is matter of most common and general observation, and is what suggests to us the passive nature of body: but that it likewise, of itself, perseveres in its state of motion, as well as of rest, is not altogether so obvious, and was not understood, for some time, by philosophers themselves, when they demanded the cause of the continuation of motion. It is easy, however, to see that this last is as general and constant a law of nature as the first. Any motions we produce, here on the earth, soon languish and at length vanish; whence it is a vulgar notion that, in general, motion diminishes and tends always toward rest. But this is owing to the various resistances which bodies here meet with in their motion, especially from friction, or their rubbing upon other bodies in their progress, by which their motion is chiefly consumed. For when, by any contrivance, this friction is much diminished, we always find that the motion continues for a long time. Thus, when the friction of the axis is lessened by friction wheels applied to it, and turning round with it, the great wheel will sometimes continue to revolve for half an hour. And when a brass topp moves on a very small pivot on a glass plane, it will continue in motion very smoothly for a great number of minutes.

A pendulum, ſuſpended in an advantageous manner, will vibrate for a great while, nótwithſtanding the reſiſtance of the air. Upon the whole, it appears, that, if the friction and other reſiſtances could be taken quite away, the motions would be perpetual. But what ſets this in the cleareſt light, is, that a body placed on the deck, or in the cabin, of a ſhip, continues there at reſt while the motion of the ſhip remains uniform and ſteady; and the ſame holds of a body that is carried along in any ſpace that has, itſelf, an uniform motion in a right line. For if a body in motion tended to reſt, that which is in the cabin of a ſhip ought to fall back towards the ſtern, which would appear as ſurprizing, when the motion of the ſhip is uniform and ſteady, as if the body ſhould, of itſelf, move towards the ſtern when the ſhip is at reſt. It is for this reaſon that the uniform motion of the earth upon its axis has no effect on the motion of bodies at the ſurface; that the motion of a ſhip carried away with a current is inſenſible to thoſe in the ſhip, unleſs they have an opportunity to diſcover it by objects which they know to be fixed, as the ſhores, and the bottom of the ſea, or by aſtronomical obſervations; and that the motions of the planets and comets, in the free celeſtial ſpaces, require no new impulſes to perpetuate them.

2. It is a part of the ſame law, that a body never changes the direction of its motion, of itſelf, but by ſome external influence only; and it is as natural a conſequence of the paſſive nature of body, as that it never changes its velocity of itſelf. As body has no ſelf-motive power, or ſpontaneity, if it was to change its direction, how could it determine itſelf to any one direction rather than to another? This part of the law is likewiſe confirmed by conſtant experience. If upon any ſmooth plane a globe of an

uniform texture be projected, it proceeds always in a right line, without turning to either ſide, till its motion be extinguiſhed by the friction of the plane and reſiſtance of the air. It is true, that, in certain caſes, a ball proceeds upon a billiard table firſt in a right line, and, afterwards, returns of itſelf a little way in the ſame right line; but this ariſes from the ball's having a motion upon its axis, with a direction contrary to that of its progreſſive motion on the table; which, when the progreſſive motion is deſtroyed by the friction, brings the ball back again, till this motion is likewiſe deſtroyed by the ſame friction. When a ball is projected in the air, its gravity indeed bends its motion into a curve, but it continues to move in the plane of its firſt projection perpendicular to the horizon, without turning to either ſide of that plane; unleſs in ſome caſes, when, becauſe of its motion upon its axis, the reaction of the air makes it deviate ſomewhat from it. If bodies changed the direction of their motion of themſelves, they could not continue at reſt in a ſpace that is carried uniformly forward in a right line; as they are always found to do. As body, therefore, is paſſive in receiving its motion and the direction of its motion, ſo it retains them or perſeveres in them, without any change, till it be acted upon by ſomething external. This law is now generally received upon the beſt evidence, but was not clearly underſtood even ſo lately as in *Kepler*'s time, as appears by the account we gave of his doctrines in the firſt book. From this law it appears, why we enquire not, in philoſophy, concerning the cauſe of the continuation of the reſt of bodies, or of their uniform motion in a right line. But if a motion begin, or if a motion already produced is either accelerated or retarded, or if the direction of the motion is altered, an enquiry into the power or cauſe that produces this

this change is a proper ſubject of philoſophy: the chief buſineſs of which (as Sir *Iſaac Newton* obſerves) is to diſcover the powers that produce any given motions; or, when the powers are given, to trace the motions that are produced by them.

3. The ſecond general law of motion is "that "the change of motion is proportional to the force "impreſſed, and is produced in the right line in "which that force acts." Thus when a motion is accelerated, as that of a heavy body deſcending in the vertical line, the acceleration is proportional to the power that acts upon the body. If a body deſcend along an inclined plane, the acceleration of the motion along the plane is proportional not to the total force of gravity, but to that part only which acts in the direction of the plane, as will better appear when we come to treat of the reſolution of motion. When a fluid acts upon a body, as water or air upon the vanes of a mill, or wind upon the ſails of a ſhip, the acceleration of the motion is not proportional to the whole force of thoſe fluids, but to that part only which is impreſſed upon the vanes or ſails, which depends upon the exceſs of the velocity of the fluid above the velocity which the vane or ſail has already acquired: for if the velocity of the fluid be only equal to the velocity of the vane or ſail, it juſt keeps up with it, but has no effect either to advance or retard its motion.

It is, at the ſame time, of the utmoſt importance to have regard to the direction in which the force is impreſſed, in order to determine the change of motion produced by it. It would be very erroneous to ſuppoſe that the acceleration of the motion of the ſhip, in the direction in which ſhe ſails, is proportional to the force impreſſed when it acts obliquely upon

upon the ſail, or when the poſition of the ſail is oblique to the direction in which the ſhip moves. The change of her motion is firſt to be eſtimated in the direction of the force impreſſed, and thence, by a proper application of mechanical and geometrical principles, the change of the motion of the ſhip in her own direction is to be derived. When gravity or any centripetal force, acts upon a body moving with a direction oblique to the right line drawn from it to the center, the change of its motion is not proportional to the whole centripetal force which acts upon it, but to that part only, which, after a juſt reſolution of the force, is found to act in the direction of its motion. It appears from theſe inſtances, of how extenſive an uſe theſe general laws are in the doctrine of motion.

4. The third general law of motion is, "that "*action* and *reaction* are equal with oppoſite directions, and are to be eſtimated always in the ſame "right line." Body not only never changes its ſtate of itſelf, but reſiſts, by its *inertia*, againſt every action that produces a change in its motion. When two bodies meet, each endeavours to perſevere in its ſtate and reſiſts any change; and, becauſe the change which is produced in either may be equally meaſured by the action which it exerts upon the other, or by the reſiſtance which it meets with from it, it follows that the changes produced in the motions of each are equal, but are made in contrary directions. The one acquires no new force but what the other loſes in the ſame direction; nor does this laſt loſe any force but what the other acquires; and, hence, tho' by their colliſions, motion paſſes from the one to the other, yet the ſum of their motions, eſtimated in a given direction, is preſerved the ſame, and is unalterable by their mutual actions upon each other. In

col-

collecting this ſum, motions that have contrary directions are to be affected with contrary ſigns; a motion eaſtward is contrary to a motion weſtward; ſo that if the motions are ſummed up as having a weſtern direction, a motion eaſtward is to be conſidered as negative, or to be ſubducted from the reſt. In this manner, this law ſerves to render the firſt law more general, and to extend it to any number of bodies; for as, by the firſt law, a body perſeveres in its ſtate of reſt, or of uniform rectilinear motion, till ſome external influence affect it; ſo it follows from this law, "that the ſum of the motions of "any number of bodies, eſtimated in a given di-"rection, perſeveres the ſame in their mutual ac-"tions and colliſions, till ſome external influence "diſturb them."

5. The truth of this third law appears from manifold experiments, in the colliſions of bodies of all kinds. But the meaning of it ſeems to have been miſtaken, in ſeveral inſtances, by ingenious men; which it is neceſſary for us to guard againſt. They who maintain the new opinion concerning the forces of bodies, meaſuring them by the compounded proportion of the quantity of matter and the ſquare of the velocity, found it impoſſible to explain the actions and colliſions of bodies of a perfect hardneſs, void of all elaſticity, conſiſtently with this doctrine. Therefore, in order to get rid of them, ſome pretended that it is abſolutely impoſſible ſuch bodies ſhould exiſt, upon grounds the weakneſs of which was ſhewn in the firſt book; while others contented themſelves with obſerving that they knew of no ſuch bodies in nature, and thought this a ſufficient excuſe for giving no account of their colliſions; tho' at the ſame time they treated largely of bodies of a perfect elaſticity, none of which are to be met with

with in nature; and we have much better reaſon to conclude that there are bodies of a perfect hardneſs, than of a perfect elaſticity; becauſe we cannot but ſuppoſe the ultimate elementary particles of bodies that are void of all pores, or atoms, to be perfectly hard or inflexible, ſo as not to yield in the ordinary actions and colliſions of bodies. But after all this art in ſcreening their favourite opinion, the difficulty ſtill recurred in explaining the colliſions of ſoft bodies; and ſome farther new invention was requiſite to reconcile the phænomena with their doctrine. For if a ſoft body, with the velocity *u*, ſtrikes another equal quieſcent ſoft body, they will proceed as in one maſs with the velocity $\frac{1}{2}u$, dividing the motion of the firſt body equally between them, in conſequence of the third general law of motion. According to the new opinion, the force of the firſt body before the ſtroke was *uu*, the force of each of them after the ſtroke is $\frac{1}{2}u \times \frac{1}{2}u$ or $\frac{1}{4}uu$; and the ſum of their forces after the ſtroke is $\frac{1}{2}uu$; ſo that the ſum of the forces, after the ſtroke, is only one half of what it was before the ſtroke, while the quantity of motion is preſerved the ſame as it was, without any change. Now the difficulty was, how to account for the loſs of one half of the force of the firſt body in the ſtroke: for this purpoſe, they advanced, without any other proof, this new doctrine, that when the parts of ſoft bodies yield without reſtoring themſelves, being void of elaſticity, a certain quantity of force is loſt in the compreſſion of their parts by the colliſion; whereas we know no way by which force is loſt in one body, but by its being communicated to another. The parts of ſoft bodies are indeed moved out of their places, in the colliſion, and ſome motion is loſt in the firſt body by being communicated, in this manner, to the parts of the ſecond; but theſe parts cannot loſe this motion

tion otherwiſe than by communicating it to other parts, or by its accruing to the whole body; ſo that there is no juſt reaſon for ſuppoſing that any motion or force is loſt in flattening or hollowing of ſoft bodies, in their colliſions; and this new tenet is invented merely to ſerve a particular purpoſe.

6. The moſt learned and ſkilful advocate for this new doctrine appears to have greatly miſtaken this third law of motion, when he tells us that the preſervation of the ſum of the abſolute motions of bodies, in their colliſions, is ſo immediate a conſequence of the equality of *action* and *reaction*, that to endeavour to prove it would only render it more obſcure, the augmentation or diminution of the force of the one (ſays he) being the neceſſary conſequence of the diminution or augmentation of the force of the other. Now it is plain that this third law of motion is general, extending to bodies of all kinds; and it is well known that when ſoft bodies meet in oppoſite directions, the ſum of their abſolute motions or forces is diminiſhed; and when the bodies are equal, and their velocities likewiſe equal, it is totally deſtroyed by their colliſion. It is not the ſum of the abſolute motions or forces of bodies, but this ſum eſtimated in a given direction, that is preſerved unaltered in their colliſions, in conſequence of this third law of motion: nor can the preſervation of the ſum of the abſolute forces of any ſort of bodies be conſidered as an immediate conſequence of it. On the contrary, the ſum of the abſolute motions of even perfectly elaſtic bodies is ſometimes increaſed, and in ſome caſes diminiſhed, by their colliſions; ſo that a proof was neceſſary that the ſum of their abſolute forces (in whatever manner thoſe forces are meaſured) is preſerved unalterable, in their colliſion; eſpecially ſince this ſum, according to his

own

own doctrine, undergoes an infinite variety of changes, during the ſmall time in which the bodies act upon each other, while the parts firſt yield and then reſtore themſelves to their former ſituations.

7. The ſame philoſophers miſtake this third law, or a moſt eſſential part of it, when they meaſure action and reaction on different right lines. In a celebrated argument which they advance for their new doctrine concerning the forces of bodies, and which is much applauded by thoſe who favour it, they ſhew that a body with a velocity as 2, is able to bend and overcome the reſiſtance of four ſprings, one of which alone is equivalent to the force of the ſame body moving with a velocity as 1; from which they infer that, in the former caſe, the force is quadruple, tho' the velocity be only double of what it is in the latter caſe. In like manner, becauſe a body moving with a velocity proportional to the diagonal of the rectangle is able to ballance the reſiſtance of two ſprings proportional to the ſides of the ſame rectangle, they thence infer that the force of a body moving with a velocity as the diagonal is equal to the ſum of the forces of two bodies moving with velocities proportional to the ſides of the rectangle; and, becauſe the ſquare of the diagonal is equal to the ſum of the ſquares of the two ſides, they thence infer that the forces of equal bodies are as the ſquares of their velocities. But in all theſe arguments (which are the moſt plauſible of any that have been offered for their new doctrine, and are moſt apt to miſlead their readers) they do not conſider that the force which one body loſes, in acting upon another, is not equal to that which it produces or deſtroys in the other, eſtimated in any direction at pleaſure, but in that only in which the firſt body acts; and that body, in conſequence of its *inertia*, not only reſiſts

any

any change in its quantity of motion, but likewiſe any change in the direction of its motion. If any planet revolves in a circle, the gravity of it towards the centre is employed, during the whole revolution, in changing the direction of its motion only, without producing the leaſt augmentation or diminution of the motion itſelf. But theſe things will more eaſily appear after we have treated of the compoſition and reſolution of motion: we only obſerve here, that, in order to ſupport their favourite doctrine, they embarraſs the plain, ſimple and beautiful theory of motion, in ſome caſes by neglecting the time, and in others by confounding the directions in which bodies act upon each other, or upon ſprings; while all the valuable conſequences which they pretend to draw from this doctrine follow more naturally, and in a ſatisfactory manner only, from the laws of motion rightly underſtood and applied.

8. Our author's firſt corollary, from the laws of motion, is, that when a body is acted upon by two forces at the ſame time, it will deſcribe the diagonal, by the motion reſulting from their compoſition, in the ſame time that it would deſcribe the ſides of the parallelogram by thoſe forces acting ſeparately. Let the body A (*Fig.* 2.) have a motion in the direction A B, repreſented by the right line A B, at the ſame time let another motion be communicated to it in the direction A D, repreſented by the right line A D; complete the parallelogram A B C D; and the body will proceed in the diagonal A C, and deſcribe it in the ſame time that it would have deſcribed the ſide A B by the firſt motion, or the ſide A D by the ſecond. To underſtand our author's demonſtration of this corollary, we muſt premiſe this obvious principle, that when a body is acted upon by a motion or power parallel to a right line given in poſition,

this power or motion has no effect to cause the body to approach towards that right line or recede from it, but to move in a line parallel to that right line only; as appears from the second law of motion. Therefore A D being parallel to B C, the motion in the direction A D has no effect in promoting or retarding the approach of the body A towards the line B C; consequently it will arrive at this line B C in the same time as if the first motion A B only had been imprest upon it. In like manner, because A B is parallel to D C, the motion A B has no effect in promoting or retarding the approach of the body A towards the line D C; consequently it will arrive at the line D C in the same time as if the motion A D only had been impressed upon it. Therefore the body A will arrive at both the lines B C and D C in the same time, that, by the first motion alone, it would have described A B, or, by the second alone, it would have described A D. But it can arrive at both the lines B C and D C no other way than by coming to their intersection C: therefore, when the two motions A B and A D are imprest upon it at once, it moves from A to C, and describes the diagonal A C, in the same time that, by these motions acting separately, it would have described the sides A B and A D.

9. Because this corollary is of very extensive use, it may be worth while to illustrate it farther. Suppose (*Fig.* 3.) the space E F G H to be carried uniformly forward in the direction A B, and with a velocity represented by A B. Let a motion in the direction A D, and measured by the right line A D, be imprest upon the body A in the space E F G H. To those who are in this space, the body A will appear to move in the right line A D; but its real or absolute motion will be in the diagonal A C of the paral-

parallelogram A B C D; and it will defcribe A C in the fame time that the fpace by its uniform motion, or any point of it, is carried over a right line equal to A B, or that the body A, by its motion acrofs the fpace, defcribes A D. For it is manifeft that the line A D, in confequence of the motion of the fpace, is carried into the fituation B C, and the point D to C; fo that the body A really moves in the diagonal A C.

10. The converfe of this corollary is, that the motion in the diagonal A C may be refolved into the motions in the fides of the parallelogram A B and A D. For it is manifeft that if (*Fig.* 4.) A K be taken equal to A D with an oppofite direction, and the parallelogram A K B C be compleated, the right line A B fhall be the diagonal of this parallelogram; confequently, by the two laft articles, the motion A C compounded with the motion A K equal and oppofite to the motion A D, produces the motion A B; that is, if from the motion A C, in the diagonal, you fubduct the motion A D in one of the fides, there will remain the motion A B in the other fide of the parallelogram A B C D.

11. This doctrine will receive farther illuftration by refolving each of the motions A B and A D into two motions, one in the direction of the diagonal A C and the other in the direction perpendicular to it; that is, by refolving (*Fig.* 5.) the motion A B into the motions A M and A N, and the motion A D into the motions A K and A L. For the triangles A D K and B C M being equal and fimilar, D K is equal to B M, or A L to A N; fo that the motions A L and A N, being equal and oppofite, they deftroy each others effect: and it being an obvious and general principle, that the motion of a body in a right line is no way

affected by any two equal powers or motions that act in directions perpendicular to that line, and opposite to each other, it thus appears how the body A is determined to move in the diagonal A C; and, because A K is equal to M C, it appears how the remaining motions A M and A K are accumulated in the direction A C, so as to produce a motion measured by A C. It appears likewise how absolute motion is lost in the composition of motion; for the parts of the motions A B and A D that are represented by A N and A L, being equal and opposite, destroy each others effect, and the other parts A M and A K, only, remain in the direction of the compounded motion A C: while, on the contrary, in the resolution of motion, the quantity of absolute motion is increased, the sum of the motions A B and A D, or B C, being greater than the motion A C. But the sum of the motions, estimated in a given direction, is no way affected by the composition or resolution of motion, or indeed by any actions or influences of bodies upon each other, that are equal and mutual and have opposite directions.

For suppose that (*Fig.* 6.) the motions are to be estimated in the direction A P; let C P, B R, D Q, be perpendicular to this direction in the points P, R and Q; then the motions A C, A B, A D, reduced to the direction A P, are to be estimated by A P, A R and A Q respectively, the parts which are perpendicular to A P having no effect in that direction. Let A P meet B C in S; then because R P is to S P, as B C (or A D) to C S, that is, as A Q to S P, it follows that A Q is equal to R P, and that A R + A Q is equal to A P; that is, that the sum of the motions A B and A D, reduced to any given direction A P, is equal to the compounded motion A C reduced to the same direction. From which it is obvious, that, in general,

ral, when any number of motions are compounded together, or are reſolved according to this general corollary, the ſum of their motions continues invariably the ſame, till ſome foreign influence affects them.

12. The uſefulneſs of the ſame corollary has induced authors to invent other demonſtrations for the farther illuſtration of it. We ſhall only add a proof of the ſimpleſt caſe, when the motions A B and A D are equal, and the angle B A D is a right one; in this caſe A B C D (*Fig.* 7, 8.) is a ſquare, and the diagonal A C biſects the angle B A D; and, becauſe the powers and motions of A D and A B are equal, and there can be no reaſon why the direction of the compounded power or motion ſhould incline to one of theſe more than to the other, it is evident that its direction muſt be in the diagonal A C; and that the compounded power or motion is meaſured by A C appears in the following manner. If it is not meaſured by A C, firſt let it be meaſured by any right line A E leſs than A C; join B D interſecting A C in K, upon A C take A M greater than A K, in the ſame proportion that A C is greater than A E; thro' the point M draw the right line F G parallel to B D, meeting A D in G and A B in F; compleat the parallelograms A M G H and A M F N: then becauſe theſe parallelograms are ſquares as well as A B C D, and A D is to A G, as A K to A M, that is as A E to A C; and A B to A F in the ſame proportion; and becauſe A E is ſuppoſed to be the power or motion compounded from A B and A D, it follows that the power or motion A D may be ſuppoſed to be compounded from the powers or motions A M and A H, and A B from A M and A N. But A H and A N, acting equally with oppoſite directions, deſtroy each others effect; ſo that it would follow that the re-

maining powers or motions AM + AM (*i. e.* 2 AM) which are accumulated in the direction of the diagonal AC, ought to be equal to AE; which is absurd, for AM is greater than AK by the construction, and 2 AM greater than 2 AK or AC, which is supposed to be greater than AE. In like manner, it is shewn (*Fig.* 8.) that the compounded power or motion, in the diagonal AC, is not measured by a right line greater than AC; and therefore it is measured precisely by the diagonal AC itself.

13. The state of any system of bodies, as to motion or rest, is judged by that of their centre of gravity, in the most simple and convenient manner. In a regular body of a homogeneous texture, the centre of gravity is the same with the centre of magnitude; and, in general, it is that point of an heavy body, which being sustained the body is in consequence itself sustained. In two equal bodies it is in a right line joining their centres, at equal distances from both: when the bodies are unequal, it is nearer to the greater body, in proportion as it is greater than the other; or its distances from their centres are inversely as the bodies. Let A (*Fig.* 9.) be greater than B, join AB, upon which take the point C, so that CA may be to CB, as the body B is to the body A, or that A × CA may be equal to B × CB, then is C the centre of gravity of the bodies A and B; and we shall afterwards shew, that if A and B be joined by an inflexible rod AB void of gravity, and the point C be sustained, then the bodies A and B shall be in *æquilibrio*. If the centre of gravity of three bodies be required, first find C the centre of gravity of A and B, and supposing a body to be placed there equal to the sum of A and B, find G the centre of gravity of it and D; then shall G be the centre of gravity of the three bodies A, B, and

and D: in like manner, the centre of gravity of any number of bodies is determined.

14. The ſum of the products that ariſe by multiplying the bodies by their reſpective diſtances from a right line, or plane, given in poſition, is equal to the product of the ſum of the bodies multiplied by the diſtance of their centre of gravity from the ſame right line or plane, when all the bodies are on the ſame ſide of it: but when ſome of them are on the oppoſite ſide, their products when multiplied by their reſpective diſtances from it are to be conſidered as negative, or to be ſubducted. Let IL (*Fig.* 10.) be the right line given in poſition, C the centre of gravity of the bodies A and B, A*a*, B*b*, C*c* perpendiculars to IL in the points *a*, *b*, *c*; then if the bodies A and B be on the ſame ſide of IL, we ſhall find $A \times Aa + B \times Bb = \overline{A + B} \times Cc$. For drawing thro' C the right line MN parallel to IL, meeting A*a* in M, and B*b* in N, we have A to B, as BC to AC, by the property of the centre of gravity; and conſequently A to B, as BN to AM, or $A \times AM = B \times BN$; but $A \times Aa + B \times Bb = A \times Cc + A \times AM + B \times Cc - B \times BN = A \times Cc + B \times Cc = \overline{A + B} \times Cc$.

When (*Fig.* 11.) B is on the other ſide of the right line IL, and C on the ſame ſide with A, then $A \times Aa - B \times Bb = A \times Cc + A \times AM - B \times BN + B \times Cc = \overline{A + B} \times Cc$: and when the ſum of the products of the bodies on one ſide of IL multiplied by their diſtances from it, is equal to the ſum of the products of the bodies multiplied by their diſtances on the other ſide of IL, then C*c* vaniſhes, or the common centre of gravity of all the bodies falls on this right line IL.

15. Suppoſe now the bodies A and B to proceed in the right lines AD and BE, (*Fig.* 12.) and when

they come to D and E their common centre of gravity to be found in G: let D d, E e, G g be perpendiculars to IL, in d, e, g; let DM, EN, GK, parallel to IL, meet A a, B b, C c, reſpectively, in the points M, N, K. By the laſt article, $\times Dd + B \times Ee = \overline{A+B} \times Gg$; and, ſubducting this from the equation in the preceding article, *viz.* $A \times Aa + B \times Bb = \overline{A+B} \times Cc$, then $A \times AM + B \times BN = \overline{A+B} \times CK$. By proceeding in the ſame manner it will appear that $A \times DM + B \times EN = \overline{A+B} \times GK$. The motions of A and B being ſuppoſed uniform, the right lines AM and BN will increaſe uniformly; ſo as to become double in double the time; conſequently CK will alſo increaſe uniformly, or in the ſame proportion as the time. And becauſe DM, EN, increaſe uniformly, it follows that GK alſo increaſes uniformly; and that CK is to KG in the conſtant ratio of $A \times AM + B \times BN$ to $A \times DM + B \times EN$. Hence it appears, that when any number of bodies move in right lines with uniform motions, their common centre of gravity moves likewiſe in a right line with an uniform motion; and that the ſum of their motions, eſtimated in any given direction, is preciſely the ſame as if all the bodies, in one maſs, were carried on with the direction and motion of their common centre of gravity. Becauſe the ſum of the motions of the bodies, eſtimated in any given direction, is preſerved invariably the ſame in their colliſions, without being affected by their actions upon each other, that are equal and mutual and have contrary directions; it follows, that the ſtate of their centre of gravity is no way affected by their colliſions or any ſuch actions; and that it perſeveres in its ſtate of reſt or uniform motion, in the ſame manner as by the firſt law of motion any one body perſeveres in its ſtate, till ſome external influence diſturb it. Theſe propoſitions repreſent to us the theory of

motion

motion in a plain and beautiful light; and enable us to judge the motions of a ſyſtem of bodies, with almoſt the ſame facility as of thoſe of one body.

16. The motions and actions of bodies upon each other, in a ſpace that is carried uniformly forward, are the ſame as if that ſpace was at reſt; and any powers or motions that act upon all the bodies, ſo as to produce equal velocities in them in the ſame or in parallel right lines, have no effect on their mutual actions or relative motions. Thus the motion of bodies aboard a ſhip, that is carried ſteadily and uniformly forward, are performed in the ſame manner as if the ſhip was at reſt. When a fleet of ſhips is carried away by an uniform current, their relative motions are no way affected by the current, but are the ſame as if the ſea was at reſt. The motion of the earth and air round its axis has no effect on the actions of bodies and agents at its ſurface, but ſo far as it is not uniform and rectilineal. In general, the actions of bodies upon each other depend not upon their *abſolute* but *relative* motion; which is the difference of their *abſolute* motions when they have the ſame direction, but their ſum when they are moved in oppoſite directions.

17. No principle being more univerſally allowed than this, or more evidently eſtabliſhed upon common experience, we deduced the following argument from it againſt the new doctrine concerning the forces of bodies in motion, in a piece that obtained the prize of the *royal academy* of ſciences at *Paris*, in 1724; which, becauſe of its plainneſs and ſimplicity, we ſhall deſcribe here again. Let A and B (*Fig.* 13.) be two equal bodies that are ſeparated from each other by ſprings interpoſed between them

(or in any other equivalent manner) in a ſpace E F G H, which in the mean time proceeds uniformly in the direction B A (in which line the ſprings act) with a velocity as 1; and ſuppoſe that the ſprings impreſs on the equal bodies A and B equal velocities, in oppoſite directions, that are each as 1. Then the abſolute velocity of A (which was as 1) will be now as 2; and according to the new doctrine its force as 4: whereas the abſolute velocity and the force of B (which was as 1) will be now deſtroyed; ſo that the action of the ſprings adds to A a force as 3, and ſubducts from the equal body B a force as 1 only; and yet it ſeems manifeſt, that the actions of the ſprings, on theſe equal bodies, ought to be equal; (and Mr. *Bernoulli* expreſsly owns them to be ſo): that is, equal actions of the ſame ſprings upon equal bodies would produce very unequal effects, the one being triple of the other according to the new doctrine; than which hardly any thing more abſurd can be advanced in philoſophy or mechanics. In general, if m repreſent the velocity of the ſpace E F G H in the direction B A, n the velocity added to that of A and ſubducted from that of B, by the action of the ſprings, then the abſolute velocities of A and B will be repreſented by $m+n$ and $m-n$ reſpectively, the force added to A by the ſprings will be $2mn+nn$, and the force taken from B will be $2mn-nn$, which differ by $2nn$. Farther, it is allowed that the actions of bodies upon one another are the ſame in a ſpace that proceeds with an uniform motion as if the ſpace was at reſt: but if the ſpace E F G H was at reſt, it is allowed that the forces communicated by the ſprings to A and B had been equal; and, according to the new doctrine, the force of each had been repreſented by nn; whereas the force communicated to A by the ſprings in the ſpace E F G H is repreſented by $2mn+nn$, and the force taken

taken from B will be 2 $mn-nn$. Theſe arguments are ſimple and obvious, and ſeem, on that account, to be the more proper in treating of this queſtion. They who maintain the new doctrine may define force in ſuch a manner, as to make the diſpute appear to relate merely to words; but, as the terms *action* and *force* ſeem to be very nearly allied to each other, it ſurely tends to confound our notions and language, to maintain that equal actions generate or produce unequal forces in the ſame time. But what evidently ſhews that the authors on the ſide of this new opinion did not underſtand what they taught, is, their telling us, that the quantity of abſolute force is unalterable by the colliſions of bodies, and that this follows ſo evidently from the equality of *action and reaction*, that to endeavour to demonſtrate it would only render it more obſcure. For hence it appears, that they underſtood equal changes to be produced in the forces of bodies in conſequence of the equality of *action* and *reaction*; and yet it is evident from what we have ſhewn, that the changes produced in the forces of bodies muſt be very unequal, according to this new doctrine, tho' the *action* and *reaction* by which they are produced be equal. It ſeems to have been by a miſtake, that Mr. *Leibnitz* firſt found himſelf engaged to maintain this new doctrine, in 1686; and in like manner, ſome of his diſciples ſeem to have raſhly adopted the ſame, without having attended to the conſequences.

18. In the theory of motion, rightly underſtood, the ſame laws that ſerve for comparing, compounding, or reſolving motions, are obſerved likewiſe by preſſures; that is, the powers that generate motion, or tend to produce it: for forces are nothing elſe but the ſums of ſuch preſſures accumulated in the body,

body, in consequence of the continued action of the powers for a finite time; and pressures are considered as infinitely small forces, or as the elements from which the forces are produced: and it adds no small beauty and evidence to this theory of motion, that both observe the same laws. When a force is generated in any body, by the accumulation of other forces or impulses, that which is generated, in any direction, must be equal to the sum of those which are all employed and consumed, in that direction, in producing it; and if the force is produced by a continual successive action, the motion generated must be equal to the sum of the pressures that are exerted in producing it. In like manner, if motion is destroyed by the resistance of any opposite power, it must be equal to the sum of all the actions by which it is totally destroyed. On the other hand, the intensity of the power that generates motion in any body, is proportional to the augment of force which it generates in a given time, and the intensity of the power that resists or destroys motion, is measured by the decrement of force produced in a given time; since the augment in the first case, and decrement of motion in the second case, are the adequate effects of the power; which is supposed to be of such a nature as to be renewed every moment, and exert all its influence at once. In general, the intensity of any power that generates or destroys motion is the greater, in proportion as the change of velocity produced by it in the direction of that power is greater, and the less the time is in which that change is produced, if the intensity of the power continues uniform during that time: but if the power varies, its intensity, at any given term of the time, is to be measured by the change of velocity which would have been produced, in a given time, by the power continued uniformly for that time.

19. The preſſure or power that generates motion in a body is in the compounded ratio of the quantity of matter in the body, and of the velocity which it would generate in it in a given time, if it was continued uniform for that time; and thoſe preſſures are equal in any two bodies, when their quantities of matter are reciprocally as thoſe velocities, that is, when the intenſity of the power that acts upon the greater body A, is leſs than the intenſity of that which acts upon the leſſer body B, in the ſame proportion as B is leſs than A. If two bodies that are acted upon by ſuch powers, with oppoſite directions, be in contact, neither of the powers will prevail, and no motion will be produced. In the ſame manner, if two bodies, moving with velocities inverſely proportional to their quantities of matter, meet with oppoſite directions, their motions will deſtroy each other, if they are ſoft bodies; or if they are ſo perfectly hard as that their parts are quite inflexible, they will both ſtop after the ſtroke: but if they have any elaſticity, they will be reflected after the ſtroke with equal motions. Thus there is a perfect harmony between the laws of preſſures, or powers, and the laws of motions or forces produced by thoſe powers; as, in general, there muſt be an analogy between the powers that generate or produce any effect, and the effects themſelves which are generated. But this harmony is quite loſt, as to the forces of bodies, according to the new opinion concerning their menſuration; for, according to this opinion, when the velocity is finite, how ſmall ſoever it may be, the force is meaſured by the ſquare of the velocity; but when the velocity is infinitely little (as it is, according to the favourers of the new opinion) in conſequence of the firſt impulſe of the power that generates the motion, the force is

ſimply

ſimply as the velocity; and we cannot but obſerve, that this ſudden change of the law does not appear to be conſiſtent with the favourite principle of *continuity*, ſo zealouſly maintained by the ſame philoſophers. According to the ſame opinion, forces that ſuſtain each other, with oppoſite directions, and deſtroy each others effect, may be unequal in any given ratio; and when bodies meet with equal forces in oppoſite directions, they do not therefore ſuſtain each other, but that which has the greater velocity carries it againſt the other. Let V denote the velocity of A, and v the velocity of B; then $A \times V$ will denote the motion or force of A, and $B \times v$ the motion or force of B; ſo that theſe motions are equal when $A \times V = B \times v$, that is, when V is to v, as B is to A: and this is the caſe wherein conſtant experience teaches us that the motions ſuſtain each other, provided their directions be oppoſite. But, according to the new opinion, the force of A is meaſured by $A \times VV$, and the force of B by $B \times vv$, which are to each other in the ſame proportion as V to v, in the preſent caſe, becauſe we ſuppoſe $A \times V = B \times v$. Theſe forces, therefore, according to the new opinion, are ſo far from being equal, that the force of A is leſs than the force of B, in proportion as V is leſs than v, or B leſs than A; ſo that, according to this doctrine, a force might ſuſtain, or even overcome, a force 1000 times greater than itſelf, or greater than itſelf in any aſſignable proportion. According to the ſame doctrine, the forces of A and B are equal, when $A \times VV = B \times vv$, that is, for example, when A being quadruple of B, the velocity of B is double of the velocity of A; in which caſe the quantity of motion, or *momentum*, of A is double of that of B; and the motion of A appears, from experience, to be more than ſufficient to ſuſtain the motion of B. It has coſt the favourers of the new opinion

opinion a great deal of pains to compose their accounts, by which they endeavoured to reconcile their theory with experience; and how unsatisfactory their accounts have proved, will easily appear to the reader who will take the trouble to examine them.

20. Let the bodies A and B (*Fig.* 14.) by moving towards each other, compress equal and similar springs placed between them, till by the reaction of those springs their motions be destroyed. Mr. *Bernovilli* expressly owns, that the actions of the springs on those bodies are constantly equal to each other, and yet maintains that they destroy a force in B greater than the force of A, in the same proportion as the body A is greater than B, or (C being the centre of gravity of A and B) as C B is greater than C A. He therefore maintains, that equal pressures or actions of springs generate, in the same time, forces that may be unequal in any assignable ratio; which is repugnant to the plainest notions we are able to form of action and force, and serves only to introduce mysterious and obscure conceptions into the theory of motion, without any necessity. If we suppose the body A to compress the springs from A to C, then the body B will compress all the springs from B to C, in the same degree, and in the same time; and thence he infers, that the force of A is to the force of B, in the same proportion as the number of springs from C to A, to the number of springs from C to B. But since the motion, force, or effect of any kind, produced or destroyed in A or B, depends upon the immediate action which produces the effect, and upon it only; and since, in this case, the actions of the springs upon the bodies A and B are those which destroy their motions; and since it is allowed by him that the actions of the springs upon these bodies are equal, is it not evident that the forces

forces deſtroyed by them in the ſame time muſt be equal? And is it not manifeſt, that the forces which are produced or deſtroyed in bodies, are to be meaſured by the efforts which the ſprings exert upon the bodies in producing this effect, and not by the number of ſprings? It is the laſt ſpring only, which is in contact with the body, that acts upon it, the reſt ſerving only for ſuſtaining it in its action; ſo that any change produced in the body, by whatever name it be called, ought to be determined from the action of this laſt ſpring only, and in juſt reaſoning ought to be computed from it alone. Had he defined force by the number of equal and ſimilar ſprings, that, by a given degree of expanſion or compreſſion, produce or deſtroy it, juſt exceptions might have been made againſt the propriety and convenience of ſuch new and unneceſſary expreſſions, as tending to perplex and darken this moſt uſeful theory of motion, which was before very clear and evident: but then this controverſy would have appeared to relate chiefly to words and terms of art, and there would not have been ſo much danger of miſtakes ariſing from their doctrines. But he does not give this for the definition of force.

21. When a body deſcends by its gravity, the motion generated may be conſidered as the ſum of the uniform and continual impulſes accumulated in the body, during the time of its falling. And when a body is projected perpendicularly upwards, its motion may be conſidered as equivalent to the ſum of the impulſes of the ſame power till they extinguiſh it. When the body is projected upwards with a double velocity, theſe uniform impulſes muſt be continued for a double time, to be able to deſtroy the motion of the body; and hence it ariſes, that the body, by ſetting out with a double velocity, and aſcend-

aſcending for a double time, muſt ariſe to a quadruple height, before its motion is exhauſted. But this proves that a body with a double velocity moves with a double force, ſince it is produced or deſtroyed by the ſame uniform power continued for a double time, and not with a quadruple force, tho' it ariſe to a quadruple height. This, however, was the argument upon which Mr. *Leibnitz* firſt built this doctrine; and thoſe which have been ſince derived from the indentings or hollows produced in ſoft bodies by others falling into them, are much of the ſame kind and force. Cauſes are not to be meaſured by any effects produced by them, taken without any choice, or judgment, or regard to their circumſtances. Motions and forces are not to be meaſured by the effects produced, without regard to the times and directions of the motions, according to the principles of geometry and mechanics. In geometry, we judge of wholes by comparing their parts, or the elements from which they are generated; and, in mechanics, we can have no better method of judging of motions, or forces, than from the powers that produce them. The motion, or force, of a body has a much more ſimple and plain analogy to the power that produces it, than to the ſpace deſcribed by it in ſoft clay or any other reſiſting medium.

22. The principle, "that the cauſe is to be meaſured by its effect," is one of thoſe that will be very apt to lead us into error, both in metaphyſics and natural philoſophy, if applied in a vague and indiſtinct manner, without ſufficient precautions. Force is defined to be that power of acting in a body which muſt be meaſured by its whole effect till its motion be deſtroyed, by thoſe who favour the new opinion, or ſome of them at leaſt, and by ſome

who

who would repreſent this diſpute as merely about words. But the ſame authors tell us likewiſe, that force is proportional to the number of ſprings which it can bend before it be deſtroyed; and this they propoſe, without any proof, as a definition or axiom. Did they content themſelves with the latter of theſe only, we ſhould allow the diſpute to be of very little moment, farther than as ſuch liberties tend to confound our notions of the action and motion of bodies, as we obſerved above. But while they pretend that force, defined by them at their pleaſure, is to be conſidered as the cauſe of the effects produced by motion, and is to be meaſured by thoſe effects, the diſpute appears no longer to be about words only. Sir *Iſaac Newton*, in his ſecond law of motion, points out to us that the impreſſed force being conſidered as the cauſe, the change of motion produced by it is the effect that meaſures the cauſe; and not the ſpace deſcribed by it againſt the action of an uniform gravity, nor the hollows produced by the body falling into clay. This law of motion is the ſureſt guide we can follow, in determining effects from their cauſes, or converſely the cauſes from their effects.

23. The harmony between the laws of preſſures, or powers, that generate motion, and the laws of theſe motions themſelves, appears in a fuller light when we attend to their compoſition and reſolution. Powers acting in the directions AB and AD, (*Fig.* 4.) proportional to thoſe right lines, compound a power that acts in the direction of the diagonal AC, and is meaſured by AC. Becauſe AC is leſs than AB+AD, the power compounded from AB and AD is always leſs than thoſe powers themſelves; and this is fully accounted for by reſolving the power AB into AM and AN, (*Fig.* 5.) and the power AD into AK and

AL; of which AN and AL are oppoſite and equal and deſtroy each others effect, ſo that there remains AM+AK, or AC, the meaſure of the compounded power. The favourers of the new opinion agree with us in arguing in this manner, concerning powers and preſſures; but in a manner quite inconſiſtent with this, in the compoſition and reſolution of forces. When the angle BAD is right, the compounded force is equal to the ſum of the forces AB and AD, according to them; and no force is loſt, notwithſtanding the oppoſite directions of the forces AL and AN; tho' it is not eaſy to conceive how this ſhould not have an effect in the compoſition of forces, as well as of powers and preſſures. When the angle BAD (*Fig.* 15.) is acute, the ſquare of the diagonal AC exceeding the ſum of the ſquares of AD and DC, (*Euclid*, 12. 2.) or of AD and AB, the two forces in the directions AD and AB muſt, according to the new doctrine, compound a force AC greater than their ſum. Now this appears directly contradictory to the metaphyſical principle ſo much inſiſted on by them, that the effect is proportional to the cauſe which produces it; for, in this caſe, the effect is greater than the cauſe; and this ſeems to be as abſurd, in mechanics, as that two quantities collected together ſhould produce a greater quantity than their ſum, in geometry. When this was objected, the anſwer * given to it deſerves to be copied, for a ſpecimen of their way of getting over difficulties: it is no more but that " no abſurdity follows from the " new opinion, which by meaſuring forces, not by " *momenta*, but, by the ſquare of the velocities, " concludes that on account of the angle DAB its " being acute, the ſquare of AC (which is the force

* See *Deſagulier's* courſe of experimental philoſophy, vol. 2. in the note at the bottom of page 72.

"compounded) is greater than the ſquares of A B
"and A D, the ſum of what they call the com-
"pounding forces."

24. To illuſtrate this farther, ſuppoſe that the elaſtic body A (*Fig.* 16.) receives its force, in the direction A B, from the equal elaſtic body H, and its force, in the direction A D, from the equal elaſtic body G, at the ſame time. According to the patrons of the new doctrine, the forces of H and G are communicated to A by infinitely ſmall degrees, or by an uninterrupted ſucceſſion of preſſures, and the whole force communicated to A is the ſum of the effects of theſe preſſures. Now in every inſtant the preſſure, or infinitely ſmall force impreſſed on A, is leſs than the ſum of the preſſures exerted in that inſtant by H and G, in proportion as AC is leſs than A B+A D, as is allowed on all ſides. Therefore the ſum of all the preſſures, or the force impreſs'd on A, muſt be leſs than the ſum of all the preſſures, or the ſum of the forces exerted by H and G, in the ſame proportion of AC to A B+A D; that is, the forces of A, H, and G, muſt be as the lines AC, A B and A D, and not as their ſquares. It is not poſſible to conceive that while the force in A ariſes from the accumulation of the preſſures, or infinitely ſmall forces, which it receives every moment from the actions of H and G, and each of theſe preſſures, or infinitely ſmall forces, is leſs than the ſum of the actions of H and G that produce them; yet the whole force of A ſhould nevertheleſs exceed the ſum of the whole actions or forces of H and G. I ſpeak here of infinitely ſmall forces, to comply as much as poſſible with the ſtile of the favourers of this new opinion. To * this they gave no other anſwer than that what we call

* Ibid. p. 73, in the laſt note.

forces

forces here ought to be called *momenta*. But they pretend not to explain how the infinitely ſmall forces impreſſed upon A, in the direction AC, come to produce a finite force far greater than their ſum total; or how the effect ſhould be ſo far from correſponding to the cauſe; the metaphyſical principle which they ſeem to uſe, or reject, juſt as it ſerves their turn. If we ſuppoſe the angle BAD to be infinitely acute, the ſame forces (according to the new opinion generate a force in A which exceeds their ſum as much as the ſquare of AB+AD exceeds the ſum of the ſquares of AB and AD; ſo that if AD be equal to AB, they will in that caſe generate at A a force double of their ſum, for then the ſquare of AB+AD will be equal to the ſquare of 2 AB, that is to 4 AB^2; tho' the two equal forces which are ſuppoſed to produce this, taken together, amount only to 2 AB^2, according to their own computation; ſo that, in this caſe, a cauſe produces an effect of the ſame kind double of itſelf. To this it has been † anſwered, that, according to the new opinion, a double *momentum* may produce a quadruple effect, if the velocity is double. But ſurely the author who gave this anſwer did not attend to the objection; for what we have proved, is not that a double *momentum* produces a quadruple effect, but that a double force, according to their own notion and computation, produces a quadruple force, according to the ſame notion and computation. And indeed the ſum of the anſwers they have made to the abſurdities which have been deduced from their favourite opinion amounts to this, *viz.* that they are no abſurdities becauſe their new opinion obliges them to admit them.

† Ibid. p. 74, in the notes.

25. The resolution of powers, or pressures, is a necessary consequence of their composition. As motion is lost in the composition, so it is necessarily gained in the resolution of motion; and as this is allowed of motions, and of the powers that generate motion, there can be no good reason given why it ought not to be allowed of the effects of those powers, or of the force of bodies. The same reasons that argue for an increase in the one case, prove, with the same evidence, that an increase of the other ought likewise to be allowed. Let the body C (*Fig.* 17.) moving in the direction DC, the diagonal of the parallelogram CLDK, strike the equal body A obliquely, so as to impell it in the direction CA the continuation of CK, and at the same time the equal body B, in the direction CB the continuation of CL; the body A will proceed in the right line CA, and the body B will proceed in the direction CB the continuation of CL, and C having communicated all its force to them will stop. It will not appear strange that the motions and forces of A and B exceed the motion or force of C, if we consider that C communicates the whole motion or force CK to A, and the whole motion or force CL to B, that the resistance or *inertia* of A reacting upon C, not in the direction of its motion CD, but in the direction CK oblique to it, the absolute motion or force of C, in the direction DC, is not so much diminished by this reaction as if it was directly opposite to the motion of C; for no power, or resistance, can produce so great an effect in any direction as in that wherein it acts. In like manner the reaction of B destroys the motion or force LC in the body C, in the direction in which B reacts; but not so great a motion or force in the direction DC to which it is oblique; and thus it appears, that the motion or force of C, in the direction

DC, muſt neceſſarily be leſs than the ſum of the motions or forces of the bodies A and B in their reſpective directions. If it be objected, that, in this caſe, the motion of C, in the direction DC, is the cauſe of the motions of A and B, in the directions CA and CB; ſo that a cauſe produces effects whoſe ſum is greater than itſelf; in anſwer to this, we have already obſerved, that as this is allowed on all hands of motions and preſſures, it cannot be abſurd to extend it to forces, but muſt obtain in them for the ſame reaſons. But farther, we are to obſerve, that, in conſequence of the *inertia* of body, it not only reſiſts any change of its motion, but likewiſe any change in the direction of its motion; and that when the action of bodies upon each other is not in a right line, both theſe are to be taken into the account. Suppoſe the body C firſt to ſtrike upon A, then the reaction of A has a twofold effect; it ſubducts ſomewhat from the motion or force of C, and at the ſame time it produces a change in the direction of C; and the reaction of A (to which the motion or force produced in it is equal) is not to be eſtimated by one of thoſe effects only, but by both conjointly. After the body C has ſtruck A, it proceeds in the right line CB with a motion or force as CL, and, impinging upon B directly, it communicates its whole motion or force to B which reacts directly againſt it. We have ſuppoſed the bodies C, A, and B to be perfectly elaſtic, in conformity to the ſuppoſitions of our opponents, ſome of whom confine themſelves in their enquiries to theſe only.

26. If we ſubſtitute ſprings in place of the bodies A and B, and their reſiſtances be meaſured by CK and CL, it will appear, in the ſame manner, that the reſiſtances of thoſe ſprings are not the proper meaſures of the force of the body C, but that taken

together they muſt exceed it; for the ſpring A acts at a diſadvantage againſt the motion or force of C. It has its whole effect in the direction CK in which it reſiſts; but not ſo great an effect in the direction CD, which is oblique to that in which it acts. If the ſpring A acted with the ſame advantage as B, they would together produce a greater effect than in the ſituation they have in the figure; and therefore the greateſt reſiſtances which they are able to exert taken together, muſt exceed the force of the body C. Thus it appears that this argument, inſtead of overthrowing our doctrine, confirms it, and that they who advanced it ſuppoſed thoſe forces to be equal, which, according to the known principles of mechanics, are unequal. If it is aſked what becomes of the exceſs of the force of the ſpring A, above what is ſubducted from the force of C? It may be anſwered, that it is not without its effect: for the direction of the body is changed from the line DC into the right line CB; and no principle, either in metaphyſics or mechanics, teaches us that this effect is to be neglected, in comparing the cauſe and effects together on this occaſion. On the contrary, many inſtances might be given where a force is employed in producing a change in the direction of a motion of a body only, without either accelerating or retarding it. The force that is ſufficient to carry a body upwards in the perpendicular to the horizon, to a double diſtance from the centre of the earth, is equal to that which, impreſſed in a horizontal direction, would carry it in a circle about the earth for ever, abſtracting from the reſiſtance of the air; as appears from the theory of gravity: and yet the firſt would overcome the reſiſtance ariſing from the gravity of the body for a certain time only; whereas the other would overcome that reſiſtance for ever, without any diminution of motion. In the firſt

caſe,

caſe, the gravity of the body would act directly againſt its force; in the ſecond, it would act in a line perpendicular to the direction of its motion: in the firſt caſe, the action of gravity is entirely employed in conſuming the force of the body; in the other, in changing its direction only. The arguments drawn in favour of the new opinion from the reſolution of motion, ſeem, at firſt ſight, the moſt plauſible of any that have been offered for it; but, from the conſiderations which we have ſuggeſted, it may appear to an impartial reader, that inſtead of overthrowing the common doctrine, they rather confirm it. As, in other inſtances, Mr. *Leibnitz*'s followers neglect the conſideration of time, in reaſoning concerning the forces of bodies; ſo here we find that they have not due regard to the directions of motions and forces, in eſtimating and comparing their effects; which, however, in mechanical enquiries, are of no leſs importance than the motions or forces themſelves.

27. We have inſiſted on theſe obſervations, becauſe they ſet the theory of motion in a plain and juſt light. We often obtain this advantage from diſputes concerning the elementary propoſitions of any ſcience, that they are the more carefully enquired into, and when found juſt, are illuſtrated and the better underſtood for having been diſputed. We cannot, however, leave this ſubject without mentioning an experiment, made by the ingenious and accurate Mr. *Graham*, to whom the mechanical ſciences are ſo much indebted. He prepared a pendulous body with a cavity in it capable to receive another body of an equal weight, at the loweſt point of its vibration; and when the body was drop'd into it, he found, by the ſubſequent vibration, that the velocity of the double maſs was preciſely one half of

what the velocity of the pendulum was before; from which it appears, that the same force produces in a double quantity of matter one half of the velocity only; which is agreeable to the common doctrine, but directly repugnant to the new one, concerning the forces of bodies in motion. Many ingenious pieces have been writ against this new doctrine by learned men, to which we refer the reader who desires to see more on the subject *. It is pretended, that by this new doctrine we are enabled to resolve problems in an easy manner, which are otherwise of great difficulty; but by the rejecting hard and inflexible bodies, there is more lost than gained in this respect, as we have shewn elsewhere, and as will appear afterwards, when we come to determine more particularly the effects of the collisions of bodies.

28. It is because *action* and *reaction* are always equal, that the mutual actions of bodies upon one another have no effect upon the motion of the common centre of gravity of the system to which they appertain. If there was any *action* in the system that had not a contrary and equal *reaction* always corresponding to it, it would affect the state of the centre of gravity of the system, and disturb its motion: and, conversely, if it be allowed that the state of the centre of gravity of a system is not disturbed by the actions of bodies upon one another that are its parts, we may conclude that their actions are mutual, equal, and have contrary directions. It will therefore be found agreeable to the course of things, and to perpetual experience, that the third law of motion be extended generally to all sorts of

* As a piece of Mr. *de Mairan*, in the memoires de l'academie royale des sciences 1728. Several pieces of Dr. *Jurin*, philosophical transactions, *&c.*

powers

powers that take place in nature, thoſe of *attraction* and *repulſion* as well as others, (and not to be a ſuppoſition arbitrarily introduced by Sir *Iſaac Newton*;) when thoſe powers are found to depend upon the bodies that are ſaid to *attract* or *repell*, as well as upon thoſe that are *attracted* or *repelled*. We find the loadſtone attracts iron, and that iron attracts the loadſtone with equal force; and becauſe they attract each other equally, they remain at reſt when they come into contact. If a mountain by its gravity preſſed upon the earth, and the earth did not react equally on the mountain; then the mountain would neceſſarily carry the earth before it, by its preſſure, with a motion accelerated *in infinitum*. The ſame is to be ſaid of a ſtone, or the leaſt part of the earth, as well as of a mountain. Bodies act upon light in proportion to their denſity, *cæteris paribus*, by refracting it when it enters into them; and converſely, light acts upon bodies by heating them and putting their parts in motion. This equality of *action* and *reaction* obtains ſo generally, that when any new motion is produced by any power or agent in nature, there is always a correſponding equal and oppoſite motion produced by its *reaction* at the ſame time, or ſome equal motion in the ſame direction deſtroyed. When from an engine a weight is thrown, the engine reacts with an equal force on the earth or air. If it was not for this law, the ſtate of the centre of gravity of the earth would be affected by every action or impulſe of every power or agent upon it. But by virtue of this law, the ſtate of the centre of gravity of the earth, and the general courſe of things, is preſerved, independent of any motions that can be produced at or near its ſurface, or within its bowels. By the ſame law, the ſtate of the leſſer ſyſtems of the planets, and the

the repoſe of the general ſyſtem, is preſerved, without any diſturbance from the actions of whatever agents there may be in them. We muſt therefore allow, that in the *attracting* and *repelling* powers which obtain in nature, from whatever ſort of cauſe they may ariſe, *action* and *reaction* are always equal; and ſince this law obtains in all ſorts of motions that ariſe from impulſe, we may be the more ſurprized if we ſhould find the philoſophers that explain thoſe powers from impulſe call it in queſtion. Even in the motions produced by voluntary and intelligent agents, we find the ſame law take place; for tho' the principle of motion, in them, be above mechaniſm, yet the inſtruments which they are obliged to employ in their actions are ſo far ſubject to it as this law requires. When a perſon throws a ſtone, for example, in the air, he at the ſame time reacts upon the earth with an equal force; by which means the centre of gravity of the earth and ſtone perſeveres in the ſame ſtate as before. And the neceſſity of this law, for preſerving the regularity and uniformity of nature, well deſerved the attention of thoſe who have wrote ſo fully and uſefully of *final cauſes*, if they had attended to it.

CHAP. III.

Of the mechanical powers.

1. THE knowledge of mechanics is one of thoſe things that contribute moſt to diſtinguiſh civilized nations from barbarians: the works of art derive their chief beauty and value from it; and without it we can make very little progreſs in the knowledge of the works of nature. It is by this ſcience that the utmoſt improvement is made of every

every power and force in nature, and the motions of the elements, water, air, and fire, are made ſubſervient to the purpoſes of life, when induſtry, with materials for the neceſſary inſtruments, are not wanting. However weak the force of man appears to be, when unaſſiſted by this art, yet with its aid, there is hardly any thing above his reach. It is a ſcience that admits of the ſtricteſt evidence; and certainly it is worth while to eſtabliſh it on its juſt principles, and to cultivate it with the greateſt diligence.

It is diſtinguiſhed by Sir *Iſaac Newton* into *practical* and *rational* mechanics; the former treats of the mechanical powers, *viz.* the *lever*, the *axis* and *wheel*, the *pulley*, the *wedge*, and the *ſcrew*, to which the *inclined plane* is to be added; and of their various combinations together. Rational mechanics comprehends the whole theory of motion; and ſhews, when the powers or forces are given, how to determine the motions that are produced by them; and, converſely, when the phænomena of the motions are given, how to trace the powers or forces from which they ariſe. Thus it appears that the whole of natural philoſophy, beſides the deſcribing the phænomena of nature, is little more than the proper application of rational mechanics to thoſe phænomena; in tracing the powers that operate in nature from the phænomena, we proceed by *analyſis*; and in deducing the phænomena from the powers or cauſes that produce them, we proceed by *ſyntheſis*. But in either caſe, in order to proceed with certainty, and make the greateſt advances, it is neceſſary that the principles of this art ſhould be premiſed and clearly eſtabliſhed, being the grounds of our whole work. We have already conſidered the *inertia* or paſſive nature of body, according to which it perſeveres in its ſtate of motion or reſt, receives motion in in proportion to the force impreſt, and reſiſts as much as it is reſiſted; which is the ſum of the three general laws of motion: from which, and their general corollaries, demonſtrated in the laſt chapter, we are now to deduce the principles of mechanics. As theſe laws and their corollaries take place, tho' the cauſes of the motions, the nature of the impreſt force, or of the reſiſtance, be unknown or obſcurely underſtood; ſo the obſcurity of the nature and cauſe of the power that produces the motions, does not hinder us from tracing its effects in mechanics with ſufficient evidence, provided we can ſubject its action to a juſt menſuration: and, in fact, we know that excellent contrivances have been invented for raiſing weights, and overcoming their reſiſtances, by ſuch as gave themſelves no trouble to enquire into the cauſe of gravity.

2. In treating of the mechanical engines, we always conſider a *weight* that is to be raiſed, the *power* by which it is to be raiſed, and the *inſtrument* or engine by which this effect is to be produced. There are two principal problems that ought to be reſolved in treating of each of them. The firſt is, "to determine the proportion which the power and "weight ought to have to each other, that they "may juſt ſuſtain one another, or be in *æquilibrio*." The ſecond is, "to determine what "ought to be the proportion of the power and "weight to each other, in a given engine, that it "may produce *the greateſt effect poſſible, in a given* "*time*." All the writers on mechanics treat of the firſt of theſe problems, but few have conſidered the ſecond; tho' in practice it be equally uſeful as the other. As to the firſt, there is a general uniform rule that holds in all the powers, is founded on the laws of motion, and is another inſtance of the beauty

beauty and harmony that reſults from the ſimplicity of the theory of motion deſcribed in the laſt chapter. Suppoſe the engine to move, and reduce the velocities of the power and weight to their reſpective directions in which they act; find the proportion of thoſe velocities; then if the power be to the weight, as the velocity of the weight is to the velocity of the power, or, (which amounts to the ſame thing) if the power multiplied by its velocity give the ſame product as the weight multiplied by its velocity, this is the caſe wherein the power and weight ſuſtain each other and are in *æquilibrio:* ſo that in this caſe, the one would not prevail over the other, if the engine was at reſt; and, if it is in motion, it would continue to proceed uniformly, if it was not for the friction of its parts, and other reſiſtances. This principle has a plain analogy to that by which the equality of the motions, or forces, of bodies was determined in general, in chap. 2. § 19. For, as the motion of bodies are equal, and deſtroy each others effect, if their directions are contrary, when the firſt is to the ſecond, as the velocity of the ſecond is to the velocity of the firſt, the greater velocity of the leſſer body juſt compenſating its deficiency in quantity of matter; ſo the actions of the power and weight are equal, and deſtroy each others effect upon the engine, when the power is to the weight, as the velocity of the weight is to the velocity of the power. But tho' it is uſeful and agreeable, to obſerve how uniformly this principle prevails in engines of every ſort, throughout the whole mechanics, in all caſes where an *æquilibrium* takes place; yet it would not be right to reſt the evidence of ſo important a doctrine upon a proof of this kind only. Therefore we ſhall demonſtrate the law of the *æquilibrium* in the *lever* or *vectis* (which is the foundation of all the other propoſitions of this kind in mechanics)

nics) by a new method, that ſeems to us to be founded on the plaineſt and moſt evident principles; to which we ſhall ſubjoin the demonſtration given by Sir *Iſaac Newton* of the ſame law, and that which is aſcribed to *Archimedes*.

3. In the *firſt* place it is evident, that if equal powers act at equal diſtances on different ſides of the prop, or centre of motion, with directions oppoſite and parallel to each other, they will have the ſame effect. Thus, A B (*Fig.* 18.) being biſected in C, if a power A act upon the lever in the direction A F, and an equal power B act upon it with an oppoſite and parallel direction B E, then the effects of thoſe powers, to move the lever about the centre C, will be preciſely equal; ſo that the one may be always ſubſtituted for the other. A *ſecond* principle is, that, gravity being ſuppoſed to act in parallel lines, if the prop C (*Fig.* 19. *n.* 1.) be between the bodies A and B, it muſt bear the ſum of their weights; becauſe the lever being loaded with thoſe weights, it muſt give way if the prop does not ſuſtain their ſum; but that when the powers A and B are on the ſame ſide of the prop or fulcrum C, (*Fig.* 19. *n.* 2.) in which caſe one of them, as A, muſt pull upwards, while the other B pulls downwards, that there may be an *æquilibrium*, it is then only loaded with the difference of the powers A and B. The one of thoſe caſes always follows from the other, if we conſider, that in the caſe of the *æquilibrium*, any one of the three powers that act at A, B, and C, may be conſidered as that of the prop, and the other two as endeavouring to turn the lever about it. From theſe principles we deduce the law of the *æquilibrium* in the lever, in the following manner.

4. Sup-

4. Suppoſing firſt two equal powers, A and B (*Fig.* 20.) acting in the directions A F, B H, to carry a body C, upon the lever A B, placed at C at equal diſtances from them; it is evident, that, in this caſe, each of the powers A and B ſuſtains one half of the weight C, by dividing it equally between them. Imagine now that the power A is taken away, and that inſtead of reſting upon it, the end A of the lever reſts upon a prop at A; it is manifeſt that the power B, and the prop at A, ſuſtain, as before, each one half of the weight C; the prop now acting, in every reſpect, as the power at A before; and, the *æquilibrium* continuing, it appears, that, in this caſe, a power B equal to one half of the weight C ſuſtains and ballances it, when the diſtance of C from the prop A is one half of the diſtance of B from the ſame; that is, when B is to C, as C A to B A, or $B \times BA = C \times CA$. From this ſimple inſtance we ſee, that powers act upon a lever not by their abſolute force only, but that their effect neceſſarily depends upon the diſtance of the point where they act from the prop, or centre of motion; and particularly, that a power ballances a double power which acts at half its diſtance from the prop, on the ſame ſide of it, with an oppoſite direction.

The caſe when the two powers act on different ſides of the prop, follows from this, by the principles laid down in the laſt article. For let B H and C G (*Fig.* 21.) repreſent the directions and forces with which the powers B and C act upon the lever; upon B A produced take A E equal to A C, or $\frac{1}{2}$ A B, and in place of the power C G ſubſtitute an equal power E K at E, with an oppoſite direction; and, by the firſt of thoſe principles, this power E K will have the ſame effect as C G, only the prop, or centre of mo-

motion, A will now ſuſtain the ſum of the forces EK and BH, by the ſecond principle in the laſt article. But the *æquilibrium* between the powers BH and EK will continue as it was before between BH and CG; ſo that the powers BH and EK will be *in æquilibrio*, when the power BH is one half of EK, and the diſtance of EK from the prop A is one half of the diſtance of BH from the ſame; that is, when the power at B is to the power at E, as AE to AB, or B×BA=E×EA. In this caſe, the prop A being loaded with both the powers B and E which act with the ſame direction, its reaction muſt be equal to their ſum EK+BH=3 BH, and muſt be in the oppoſite direction AF. In place of this reaction let us now (*Fig.* 22.) ſubſtitute a power AF at A, equal to thrice BH; and in place of the power EK, let us ſubſtitute a prop at E, ſuſtaining that end of the lever BE; and ſince the *æquilibrium* continues as before, it follows that the prop, or centre of motion, being at E, the power BH ſuſtains the power AF which is triple of BH, when the diſtance of BH from the prop E is triple of the diſtance of the power AF from the ſame, that is, when BH×BE =AF×AE.

If we ſuppoſe the power EK to remain (*Fig.* 23.) but the end B of the lever EB to reſt upon a prop, then the powers AF and EK will ſuſtain and ballance each other, the prop at B now coming in place of the power BH; in which, AF=3 BH, and EK=2 BH; ſo that AF is to EK as 3 to 2; and the diſtances EB and AB being in the ſame proportion, it appears that when two powers in the proportion of 3 to 2 act upon a lever on the ſame ſide of the prop, or centre of motion, with oppoſite directions, at diſtances in the proportion of 2 to 3, they then ſuſtain each other. We have demonſtrated therefore,

fore, that when the powers are in the proportion either of 2 to 1, or of 3 to 1, or of 3 to 2, and the diſtances of their application from the centre of motion are in the inverſe proportion, then thoſe powers ballance each other, or are in *æquilibrio.*

5. Upon BE produced (*Fig.* 25. *n.* 1.) take EL=EA; and in place of the power AF ſubſtitute a power LM=AF, but with a contrary direction; this power LM will have the ſame effect to turn the lever round the centre of motion E as AF had, by the firſt principle in § 3; conſequently it will be in *æquilibrio* with the power BH, as AF was. Therefore when two powers LM and BH, in the proportion of 3 to 1, act upon a lever with the ſame direction, they are in *æquilibrio*, if their diſtances from the centre of motion LE and EB be in the ratio of 1 to 3; that is, when LM×LE=BH×BE. In this caſe, the powers LM and BH acting with the ſame direction, the prop E muſt ſuſtain their ſum LM+BH=4BH, by the ſecond principle of § 3. Therefore a power at L as 3, and a power acting at B with the ſame direction as 1, are ſuſtained by a power acting at E, with a contrary direction, as 4. From which it follows, by ſubſtituting in the place of the power LM a prop at L, that a power at B as 1 ſuſtains a power at E as 4, acting with a contrary direction, when BL is to EL as 4 to 1; that is, when the powers are inverſely as their diſtances from the prop, or centre of motion. By ſubſtituting the prop at B in the place of the power BH, it appears that a power LM at L, as 3, ſuſtains a power, acting with an oppoſite direction, at E, as 4, when their diſtances LB and EB from the prop B, are to each other as 4 to 3, or when LM×LB=EK×EB. By taking upon LB produced B*e*=BE, (*Fig.* 24. *n.* 2.) and in place of the power at E, ſubſtituting an equal power at *e* with a

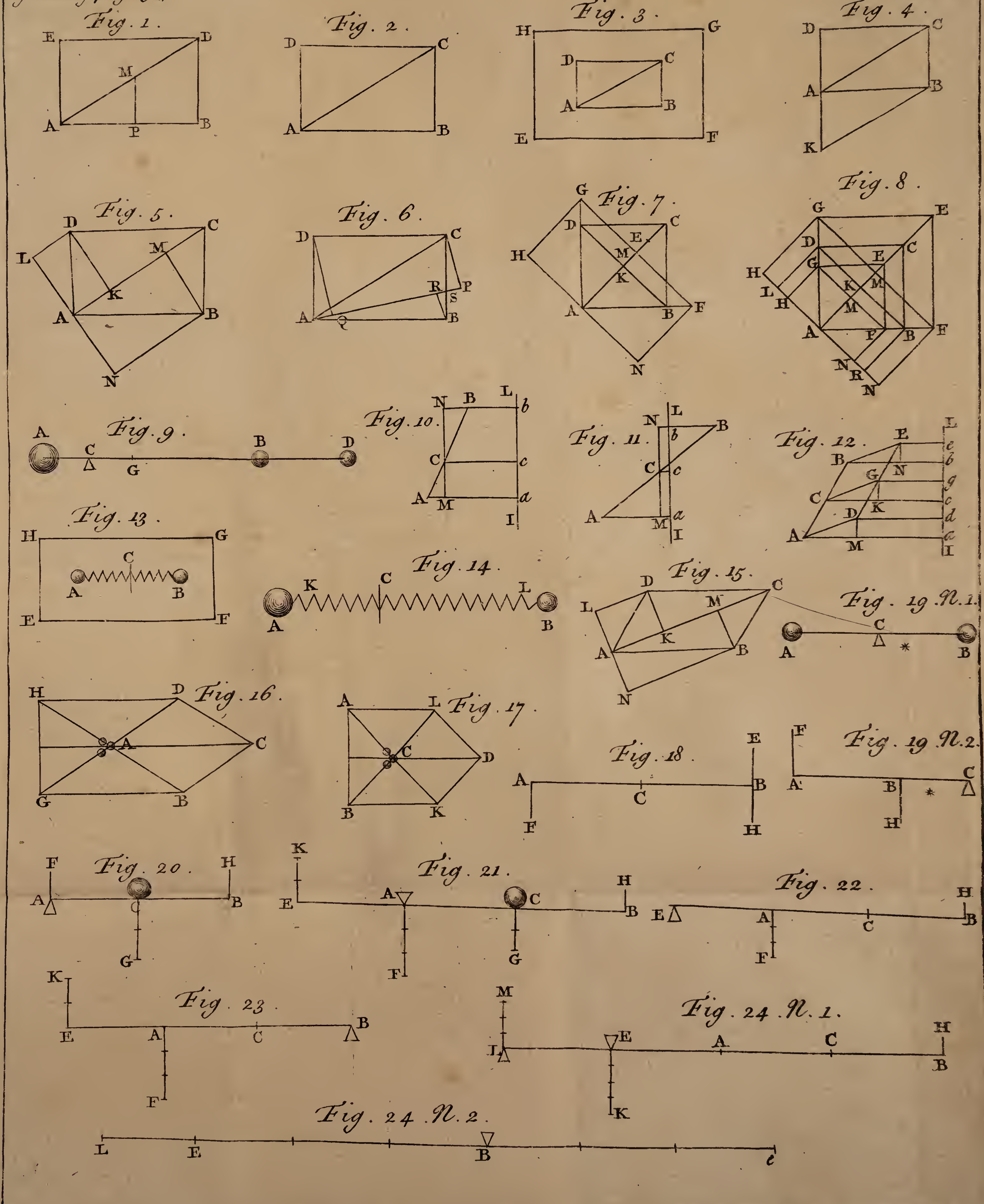
contrary direction, it appears, by the firſt principle in § 3. that a power at L as 3 ſuſtains a power acting at *e*, with the ſame direction, as 4, when the diſtance LB is to the diſtance *e* B, as 4 to 3. In this caſe, the prop at B ſuſtains the ſum of the powers acting at L and *e*, that is, a power equal to ſeven times BH. From which it follows, by ſubſtituting a prop at L, or *e*, in place of the powers that act there, that a power at *e* as 4 ſuſtains a power at B as 7, about the centre of motion L, when their diſtances from it *e* L, BL are to each other as 7 to 4: and that a power at L as 3 ſuſtains the power at B as 7, about the centre of motion *e*, when their diſtances from it, L *e* and B *e*, are to each other as 7 to 3.

6. By proceeding in this manner it appears, that when the powers are to each other as number to number, and when their diſtances from the centre of motion are in the inverſe ratio of the ſame numbers, then the powers ſuſtain each other, or are in *æquilibrio*. From which it is eaſy to ſhew, in general, that when the powers are to each other in any ratio, tho' incommenſurable, and the diſtances of their application from the centre of motion in the ſame inverſe ratio, then they are in *æquilibrio*; becauſe the ratio of incommenſurable quantities may be always limited, to any degree of exactneſs at pleaſure, between a greater and a leſſer ratio of number to number. And this I take to be the moſt direct and natural proof of the law of *æquilibrium* in the lever, the fundamental propoſition of mechanics.

7. When the centre of motion C is between the bodies A and B, it is the ſame point which was called their centre of gravity, chap. 2. § 13. And hence it appears, that when the two bodies are ſuppoſed

fronting pag. 162.

TAB. I.



to be joined by an inflexible rod void of gravity, if the centre of gravity be ſuſtained, then the bodies ſhall be ſuſtained.

If two powers or weights, B and D, (*Fig.* 25.) act upon a lever at the diſtances B C and D C from the centre of motion, the forces with which they act upon the lever ſhall be in the ſame proportion of B×B C to D×D C; that is, in the ratio compounded of the ratio of the powers, or weights, and that of their diſtances from the centre of motion. For the effort of B is ſuſtained by A, if A×A C be equal to B×BC; and the effort of the power D is ſuſtained by K applied at the diſtance C A, if K×AC=D×DC. But the efforts of the powers, or weights, B and D, upon the lever, are in the ſame proportion to each other as the powers A and K, which, applied at the ſame diſtance C A from the centre of motion, ſuſtain them, or as A×AC to K×AC, and therefore as B×BC to D×DC. From this it appears, that when any number of powers act upon a lever, if the ſum of the products that ariſe by multiplying each power by its reſpective diſtance from the centre of motion, on one ſide of it, be equal to the ſum of the products that ariſe by multiplying each power on the other ſide of the centre of motion by its reſpective diſtance from it, then theſe powers ſuſtain each other, and the lever is in *æquilibrio.* But by what was ſhewn in § 13. chap. 2. the centre of motion coincides, in this caſe, with what was there called the centre of gravity. Therefore if any number of powers or weights act upon a lever, and, their centre of gravity being determined by the conſtruction in that article, if the prop or *fulcrum* be applied at this centre, the lever ſhall be in *æquilibrio.* In the ſame manner, if any number of powers or weights be applied upon a plane that reſts upon a given right

 line

line IL, (*Fig.* 26.) and the centre of gravity of all the powers or weights fall upon that line, the plane ſhall be in *æquilibrio:* for, by that article, the ſums of the products that ariſe by multiplying each power by its reſpective diſtance from the axis of motion, being equal on the different ſides of this axis, their efforts to move the plane muſt be equal and contrary, and deſtroy each others effect. Therefore as the ſtate of any ſyſtem of bodies, as to motion or reſt, depends on the motion or reſt of the point called the centre of gravity, by what was ſhewn above in the laſt chapter; ſo it is another notable property of this point, that if the bodies be joined together, and to it, by inflexible lines void of gravity, and this point be ſuſtained, the whole ſyſtem ſhall be ſuſtained and remain in *æquilibrio.*

8. When any powers B and D (*Fig.* 25, 26.) act upon a lever, endeavouring to turn it about the centre of motion C, or when they act upon a plane, endeavouring to turn it about the axis of motion IL, their effect is the ſame as if a power or weight equal to their ſum was ſubſtituted in place of them at their common centre of gravity N. For, by § 14. chap. 2, $B\times BC+D\times DC=\overline{B+D}\times NC$; or if Bb, Dd, Nn be perpendicular to IL in the points b, d, n, then, by the ſame article, $B\times Bb+D\times Dd=\overline{B+D}\times Nn$. If G, the centre of gravity of all the powers, or weights, that act upon the lever, fall on one ſide of C the centre of motion; or the centre of gravity of all the powers that act upon the plane, is on one ſide of the axis IL; then the preponderancy will be on that ſide, and will be the ſame as if, in place of all thoſe powers, one power equal to their ſum was ſubſtituted at their common centre of gravity. For it was ſhewn that $B\times BC+D\times DC-A\times AC=\overline{A+B+D}\times$ GC,

GC, when the power A acts on one ſide and the powers B and D on the other. Therefore, as when the centre of gravity of the powers reſts upon the centre of motion, the whole is in *æquilibrio*, and the prop C ſuſtains a force equal to their ſum; ſo when the centre of gravity is not ſuſtained by the prop, but falls on one ſide of it, the preponderancy is on that ſide, and is the ſame as if all the powers or weights were collected together at that centre. The analogy between theſe ſtatical theorems, and thoſe in the theory of motion relating to this centre, deſcribed in the laſt chapter, deſerve our attention; and farther illuſtrate the ſimplicity of this doctrine and the harmony of all its parts.

9. Sir *Iſaac Newton* demonſtrates the fundamental propoſition concerning the lever, from the reſolution of motion. Let C (*Fig.* 27.) be the centre of motion in the lever KL; let A and B be any two powers, applied to it at K and L, acting in the directions KA and LB. From the centre of motion C, let CM and CN be perpendicular to thoſe directions in M and N; ſuppoſe CM to be leſs than CN, and from the centre C, at the diſtance CN, deſcribe the circle NHD, meeting KA in D. Let the power A be repreſented by DA, and let it be reſolved into the power DG acting in the direction CD, and the power DF perpendicular to CD, by compleating the parallelogram AFDG. The power DG, acting in the direction CD from the centre of the circle, or wheel, DHN towards its circumference, has no effect in turning it round the centre, from D towards H, and tends only to carry it off from that centre. It is the part DF only that endeavours to move the wheel from D towards H and N, and is totally employed in this effort. The power B may be conceived to be applied at N as well as at L, and to be wholly employed

in endeavouring to turn the wheel the contrary way, from N towards H and D. If therefore the power B be equal to that part of A which is repreſented by D F, theſe efforts, being equal and oppoſite, muſt deſtroy each others effect; that is, when the power B is to the power A, as DF to DA, or, (becauſe of the ſimilarity of the triangles A F D, D M C) as C M to C D, or as CM to C N, then the powers muſt be in *æquilibrio*; and thoſe powers always ſuſtain each other that are in the inverſe proportion of the diſtances of their directions from the centre of motion; or, when the product of the one power multiplied by the diſtance of its direction from the centre, is equal to the product of the power on the other ſide multiplied by the like diſtance from it.

10. The demonſtration commonly aſcribed to *Archimedes* is founded upon this principle, that when any cylindric or priſmatic body is applied upon a lever, it has the ſame effect as if its whole weight was united and applied at the middle point of its axis. Let A B (*Fig.* 28.) be a cylinder of an uniform texture, C its middle point; and it is manifeſt, that if the point C be ſupported, the equal halves of the cylinder, C A and CB, will ballance each other about the point C, and the body will remain in *æquilibrio*. Let the cylinder A B be diſtinguiſhed into any unequal parts, AD and D B; biſect A D in E, and D B in F; then a power applied at E, equal to the weight of the part A D, with a contrary direction, will ſuſtain it; and a power applied at F, equal to the weight of the part D B, with a contrary direction, will ſuſtain that part; ſo that theſe two powers acting at E and F, reſpectively equal to the weights of A D and D B, have preciſely the ſame effect as a prop at C, ſuſtaining the whole cylinder A B, and may be conſidered as in *æquilibrio* with a power, acting at C, equal

equal to the whole weight of the cylinder. But the diſtance CE = CA — AE = $\frac{1}{2}$ AB — $\frac{1}{2}$ AD = $\frac{1}{2}$ DB; and, in like manner, the diſtance CF = CB — BF = $\frac{1}{2}$ AB — $\frac{1}{2}$ DB = $\frac{1}{2}$ AD; conſequently CE is to CF, as DB to AD; that is, as the power applied at F to the power applied at E, theſe being in *æquilibrio* with the weight of the whole cylinder applied at C. From which it appears, that powers applied at E and F, which are to each other in the proportion of CF to CE, ſuſtain one another about the centre C.

11. Suppoſe the lever AB (*Fig.* 29.) with the weights A and B, to turn round the centre C; the bodies A and B will deſcribe ſimilar arcs A*a* and B*b*; and A*a* will be to B*b*, as CA to CB, or as B to A; conſequently A × A*a* = B × B*b*; that is, the *momenta*, or quantities of motion, of A and B will be equal; and conſidering one of them as the power and the other as the weight, the power will be to the weight, as the velocity of the weight to the velocity of the power. Therefore in this, as in all mechanical engines, when a ſmall power raiſes a great weight, the velocity of the power is much greater than the velocity of the weight; and what is gained in force is therefore ſaid to be loſt in time. In like manner, when a number of powers are ſuppoſed to act upon the lever, and it is turned round about their common centre of gravity C, the ſums of the *momenta* on the different ſides of C are equal.

12. The lever, or *vectis*, is commonly diſtinguiſhed into *three* kinds. In the *firſt*, the centre of motion is between the power and weight. In the *ſecond*, the weight is on the ſame ſide of the centre of motion with the power, but applied between them. In the *third*, the power is applied between the weight and centre of motion. In this laſt, the

power muſt exceed the weight, in proportion as its diſtance from the centre of motion is leſs than the diſtance of the centre from the weight. But as the firſt two ſerve for producing a ſlow motion by a ſwift one; ſo the laſt ſerves for producing a ſwift motion of the weight by a ſlow motion of the power. It is by this kind of levers that the muſcular motions of animals are performed; the muſcles being inſerted much nearer to the centre of motion than the point where the centre of gravity of the weight to be raiſed is applied; ſo that the power of the muſcle is many times greater than the weight which it is able to ſuſtain. Tho' this may appear at firſt ſight a diſadvantage to animals, becauſe it makes their ſtrength leſs; it is, however, the effect of excellent contrivance: for if the power was, in this caſe, applied at a greater diſtance than the weight, the figure of animals would not only be awkward and ugly, but altogether unfit for motion; as *Borelli* has ſhewn in his treatiſe *de motu animalium*.

13. When the two arms of a lever are not in a right line, but contain any invariable angle at C, (*Fig.* 30.) the law of the *æquilibrium* is the ſame as in the former caſe; that is, if the power P be applied at B to the arm CB, and the weight W act, by means of a pulley M, in the direction AM perpendicular to the arm CA, the power and weight will ſuſtain each other if P be to W, as CA to CB, or $P \times CB = W \times CA$. If ſeveral powers act upon the arm CA, find their centre of gravity A, on the arm CA, by § 13. chap. 2. ſuppoſe all the powers to be united there; and if the power P be to their ſum, as CA to CB, it will ſuſtain them. The ſum of the powers being ſuppoſed given, it is manifeſt that the farther their centre of gravity A is removed from the centre of motion C, the greater reſiſtance they will oppoſe againſt

againſt the power P, and it will require the greater force in the power to overcome them. From this *Galileo* juſtly concludes, that the bones of animals are the ſtronger for their being hollow, their weight being given; or, if the arm C B F repreſent their length, the circle C H D a ſection perpendicular to the length, P any power applied along their length, tending to break them; then the ſtrength or force of all their longitudinal fibres, by which the adheſion of the parts is preſerved, may be conceived to be united in A the centre of the circle C H D, which is the common centre of gravity of thoſe forces, whether the ſection be a circle or *annulus*. But it is plain that when the area of the ſection, or the number of ſuch fibres, is given, the diſtance C A is greater when the ſection is an annulus, than when it is a circle without any cavity; conſequently the power with which the parts adhere, and which reſiſts againſt P which endeavours to ſeparate them, is greater in the ſame proportion. For the ſame reaſon, the ſtalks of corn, the feathers of fowls, and hollow ſpears, are leſs liable to accidents that tend to break them, than if they were of the ſame weight and length, but ſolid without any cavity. In this inſtance, therefore, art only imitates the wiſdom of nature.

14. The ſame excellent author obſerves, that in ſimilar bodies, engines, or animals, the greater are more liable to accidents than the leſſer, and have a leſs relative ſtrength; that is, the greater have not a ſtrength in proportion to their magnitude. A greater column, for example, is in much more danger of being broke by a fall than a ſimilar ſmall one; a man is in greater danger from accidents of this kind than a child; an inſect can bear a weight many times greater than itſelf, whereas a large animal, as a horſe, can hardly bear a burthen equal to his own weight.

To

To account for this, it will be sufficient to shew, that, in similar bodies of the same texture, the force which tends to break them, or to make them liable to hurtful accidents, increases in the greater bodies in a higher proportion than the force which tends to preserve them entire, or secure against such accidents. Suppose the similar beams ABDE, FGHK, (*Fig.* 31.) of a cylindric or prismatic figure, to be fixed in the immoveable wall IL; and let us at present abstract from any other force that may tend to break them, besides their own weight. Bisect AB in C, and FG in M; and their weights may be conceived to be accumulated at the points C and M, which are directly under their centres of gravity. For the greater facility of the computation, suppose AB=2 FG, and consequently the weight of the beam ABDE will be eight times greater than the weight of the similar beam FGHK; and the weight of the former being conceived to be accumulated in C, and that of the latter in M, and AC being double the distance FM, it follows, that the force which tends to break the former at A, being eight times greater than that which tends to break the latter at F, and at the same time acting at a double distance, on both these accounts its effort must be sixteen times greater than that of the latter. Now, to compare the forces which tend to preserve those beams entire and fixed in the wall, let ARE be a section of the greater beam, and FSK a section of the latter, perpendicular to their lengths at the points A and F; bisect AE in *p*, and FK in *q*; then the number of longitudinal fibres, whose adhesion tends to preserve the beams entire, or rather the quantity of this adhesion, in the greater beam, will be to the quantity of adhesion in the lesser beam, as the area of the section ARE to the area of the section FSK, that is, in the present case (because of the similarity of the figures) as the squares of of A E to the ſquare of F K, or as 4 to 1. But the adheſion of the parts that are in contact with each other in the ſection A R E may be conceived to be accumulated at p their centre of gravity; and the adheſion of the parts in contact with each other in the ſection F S K is to be conceived as accumulated in q, for the ſame reaſon. The adheſion, therefore, which tends to preſerve the greater beam entire is quadruple of that which tends to preſerve the leſſer beam entire, and at the ſame time is to be conceived as acting at a double diſtance from the centre of motion, becauſe $Ap = 2Fq$; ſo that the effort which tends to preſerve the greater beam from breaking, is eight times greater than that which tends to preſerve the leſſer beam entire. We have found, therefore, that the effort which tends to break the greater beam at A, is ſixteen times greater than that which tends to break the leſſer beam at F; but that the effort, which, on the other hand, endeavours to preſerve the adheſion of the greater beam entire, is only eight times greater than that which tends to preſerve the adheſion in the leſſer beam. In general, it will eaſily appear, in the ſame manner, that the efforts tending to deſtroy the adheſion of the beams, ariſing from their own gravity only, increaſe in the quadruplicate ratio of their lengths; but that the oppoſite efforts, tending to preſerve their adheſion, increaſe only in the triplicate ratio of the ſame lengths. From which it follows, that the greater beams muſt be in greater danger of breaking than the leſſer ſimilar ones; and that, tho' a leſſer beam may be firm and ſecure, yet a greater ſimilar one may be made ſo long, as neceſſarily to break by its own weight. Hence *Galileo* juſtly concludes, that what appears very firm, and ſucceeds well, in models, may be very weak and infirm, or even fall to pieces by by its weight, when it comes to be executed in large dimenſions according to the model.

15. From the ſame principles he argues, that there are neceſſarily limits in the works of nature and art, which they cannot ſurpaſs in magnitude. Were trees of a very enormous ſize, their branches would fall by their own weight. Large animals have not ſtrength in proportion to their ſize; and if there were any land-animals much larger than thoſe we know, they could hardly move, and would be perpetually ſubjected to moſt dangerous accidents. As to the animals of the ſea, indeed, the caſe is different, as the gravity of the water ſuſtains thoſe animals in great meaſure, and in fact theſe are known to be ſometimes vaſtly larger than the greateſt land-animals. Nor does it avail againſt this doctrine to tell us, that bones have been found which were ſuppoſed to have belonged to giants of an immenſe ſize, ſuch as the ſkeletons mentioned by *Strabo* and *Pliny*; the former of which was 60 cubits high, and the latter 46; for the naturaliſts have concluded, on juſt grounds, that in ſome caſes thoſe bones had belonged to elephants; and that the larger ones were bones of whales, which had been brought to the places where they were found, by the revolutions of nature that have happened in paſt times. Tho' it muſt be owned, that there appears no reaſon why there may not have been men that have exceeded, by ſome feet in height, the talleſt we have ſeen. The reader will find a curious and uſeful diſſertation on this ſubject, by the celebrated Sir *Hans Sloane*, in the *Philoſophical Tranſactions*, or in the *Memoires de l' Academie Royale des Sciences*, 1727. If, in the other planets, the ſame law of coheſion and other attractions takes place as in the earth, it may be of uſe that the gravity near their ſurfaces ſhould not be vaſtly

vaſtly different from what it is near the ſurface of the earth; it was perhaps with ſome view to this, that Sir *Iſaac Newton* inſinuates, that it was not without deſign and contrivance that the gravities at the ſurfaces of the planets ſhould differ ſo much leſs from each other, than, at firſt ſight, might be expected from the attractions of bodies of ſo unequal magnitude.

16. It follows, from § 14th, that in order to make bodies, engines, or animals, of equal relative ſtrength, the greater ones muſt have groſſer proportions. Thus in order that the greater cylinder ABDE may be as firm and ſecure againſt accidents as the leſſer cylinder FGHK, the ſection ARE and its diameter AE muſt be increaſed, till the effort ariſing from the adheſion of the parts bear as great a proportion to the effort that tends to overcome this adheſion, in the greater, as in the leſſer cylinder. And this ſentiment being ſuggeſted to us by perpetual experience, we naturally join the idea of greater ſtrength and force with the groſſer proportions, and the idea of agility with the more delicate ones. In architecture, where the appearance of ſolidity is no leſs regarded than real firmneſs and ſtrength, this is particularly conſidered, in order to ſatisfy a judicious eye and taſte; the various orders of the columns ſerving to ſuggeſt different degrees of ſtrength. But, by the ſame principle, if we ſhould ſuppoſe animals vaſtly large, from the groſs proportions, a heavineſs and unwieldineſs would neceſſarily ariſe, which would make them uſeleſs themſelves, and diſagreeable to the eye. In this, as indeed in all other caſes, whatever generally pleaſes taſtes not vitiated by education, or by fabulous and marvellous relations, may be traced till it appear to have

have a juſt foundation in nature; tho' the force of habits is ſo ſtrong, and their effects upon our ſentiments ſo quick and ſudden, that it is often no eaſy matter to trace, by reflexion, the grounds of what pleaſes us.

17. We have inſiſted at ſo great length on the lever, that we may be brief in treating of the other mechanical powers. The common *ballance* is a lever that has equal arms A G and G B, (*Fig.* 32.) with the centre of motion C commonly placed directly over G. If the centre of motion was in G, equal weights, ſuſpended from A and B, would ſuſtain each other, in any poſition of the lever A B; but when the centre of motion is above G, they ſuſtain each other when the lever A B is level only; and when the weight at A is but a little greater than the weight at B, the ends A and B deſcend and aſcend by turns, till their common centre of gravity *g* ſettles in the vertical line C G; where they ſuſtain each other, becauſe their centre of gravity is ſuſtained by C. The ballance is falſe when the arms A G and G B are unequal: and the exactneſs of this inſtrument chiefly depends upon making the friction, at the centre of motion C, as ſmall as poſſible.

18. The *axis and wheel* has a near analogy to the lever; the power is applied at the circumference of the wheel, and the weight is raiſed by a rope that is gathered up (while the machine turns round) on the axis. The power may be conceived as applied at the extremity of the arm of a lever equal to the radius of the wheel, and the weight as applied at the extremity of a lever equal to the radius of the axis; only thoſe arms do not meet at one centre of motion as in the lever, but in place of this centre, we have an axis of motion, *viz.* the axis of the

3 whole

whole engine. But as this can produce no difference, it follows that the power and weight are in *æquilibrio* when they are to each other inversely as the distances of their directions from the axis of the engine; or when the power is to the weight, as the radius of the roller to the radius of the wheel, the power being supposed to act in a perpendicular to this radius; but if the power act obliquely to the radius, substitute a perpendicular from the axis on the direction of the power, in the place of the radius. Thus if A B D E (*Fig.* 33.) represent the cylindric roller, H P N the wheel, L M the axis or right line upon which the whole engine turns, Q the point of the surface of the roller where the weight W is applied, P the point where the power is applied, K Q the radius of the roller, C P the radius of the wheel; then if the power P act with a direction perpendicular to C P, the power and weight will sustain each other when P is to W, as K Q to C P or C H: but if the power act in any other direction P R, let C R be perpendicular from C, the centre of the wheel, on that direction; then P and W will sustain each other when P is to W, as K Q to C R; because, in this case, a power P has the same effect as if it was applied at the point R of its direction, acting in a right line perpendicular to C R.

19. The *simple pulley* serves only to change the direction of the power, or motion, without any mechanical advantage, or any disadvantage but what arises from the friction. Let M (*Fig.* 34.) represent a simple pulley, P N W the rope that goes over the pulley from the power P to the weight W: and it is manifest, that if P and W be equal, they will sustain each other as if suspended at equal distances, M A and M B, from the centre of the lever A B. But, if besides the fixed pulley M, there be (*Fig.* 35.) another

another moveable pulley L, to which the weight W is fixed, and the rope that goes from the power P, over the fixed pulley M, and under the moveable pulley L, be fixed above at E, then it is manifeſt that the power P ſuſtains only one half of W, becauſe the rope KN ſuſtains only one half of it, the other half being ſuſtained by the rope KE.

There is an obvious analogy between this caſe of pullies, and that wherein a power ſuſtains a double weight at half its diſtance from the centre of motion, on the ſame ſide. For if AB be the diameter of the pulley L, at whoſe extremities the parallel ropes, AE and BN, touch it, the power P may be conceived to be applied at B, the weight W at L, and the centre of motion to be at A. If we ſuppoſe the power P and weight W to move, as P is equal to one half of W, ſo the velocity of W is one half of the velocity of P, or P multiplied by its velocity gives a product equal to W multiplied by its velocity; for, that the weight W may be elevated one inch, each of the parts of the rope EK and KN muſt be ſhortned by one inch; and the power P that draws the whole rope from E by K and N, muſt deſcend two inches. A ſimilar reaſoning may be applied to all the combinations of pullies.

20. When a weight W (*Fig.* 36.) deſcends along an *inclined plane* AC, a part of its gravity is ſuſtained by the reaction of the plane, and the remaining part produces its motion along the plane. Let AB be the height of the plane, BC the baſe, and the gravity of the weight W being repreſented by the vertical line WM, let this power be reſolved into the power WN perpendicular to the plane, and WQ parallel to it. The former WN is deſtroyed by the reaction of the plane, and the latter WQ is that

which produces the motion of the body along the plane. Becauſe the triangles W Q M and A B C are ſimilar, W Q is to W M, as A B to A C; and the force with which a body deſcends along the plane is to its gravity, as the height of the plane to its length; conſequently a force acting upon the body W, with the direction Q W parallel to the plane A C, will ſuſtain it, if it be to the whole weight of the body, as A B to A C.

21. Let A B C (*Fig.* 37.) repreſent a *wedge* driven into the cleft E D F, of which D E and D F are the ſides; and if we ſuppoſe thoſe ſides D E and D F to re-act upon the wedge with directions perpendicular to D E and D F, let the horizontal line E F meet D F in F; then when the force impelling the wedge, ſuppoſed perpendicular to the horizon, is in *æquilibrio* with the reſiſtances of the ſides of the cleft D E and D F, theſe three powers are in the ſame proportion as the three right lines E F, D E and D F. For it follows from the compoſition of motion, that when three powers are in *æquilibrio* with each other, they are in the ſame proportion as the three ſides of a triangle parallel to their reſpective directions, and, conſequently, as the three ſides of a triangle perpendicular to their directions; ſuch a triangle being evidently ſimilar to the former. But E F is perpendicular to the direction in which the weight of the wedge, or the power that impells it, is ſuppoſed to act; and D E, D F are perpendicular to the directions in which their reſiſtances are ſuppoſed to act, conſequently the power that impells the wedge and thoſe reſiſtances are in the ſame proportion as E F, D E and D F. If other ſuppoſitions are made concerning the reſiſtances of the ſides of the cleft D E and D F, the proportions of the powers may be determined, from the ſame principles.

22. When a point moves along the ſide of a cylinder, with an uniform motion, upon its curve ſurface, while this ſide is itſelf carried with an uniform motion about the axis of the cylinder, the line traced, by this compounded motion, upon the curve ſurface of the cylinder, is called a *ſpiral*. When this line is raiſed upon the external ſurface of the cylinder, it is called the *external ſcrew*; but if it is carried on in the internal ſurface, it is called the *internal ſcrew*. While one of theſe is converted about the other, one of them ought to be fixed; and they form a machine of great force for ſqueezing or moving bodies. If a power P (*Fig.* 38.) turn either of the ſcrews with a direction parallel to the baſe, it will ſuſtain the weight W which is to be raiſed, if it be to W in the ſame proportion as the diſtance between the two neareſt ſpirals is to the circumference of the circle deſcribed by the power P; becauſe while the power makes a compleat revolution, the ſcrew advances by the diſtance of the two neareſt ſpirals, and the velocity of the power is to the velocity of the weight, as the circumference deſcribed by P to that diſtance. The ſame will appear by conſidering the ſcrew as an inclined plane involved about a cylinder. In this engine the friction is very great.

23. From theſe ſimple machines, compounded ones are formed by various combinations, and ſerve for different purpoſes; in which the ſame general laws take place, particularly that which was deſcribed in § 3. That the power and weight ſuſtain each other when they are in the inverſe proportion of the velocities which they would have in the directions wherein they act, if they were put in motion. By theſe the famous problem is reſolved, of moving any given weight by any given power, provided the reſiſt-

resiſtance ariſing from the friction can be overcome. It being of great importance to diminiſh this friction, ſeveral contrivances have been invented for that purpoſe. In wheel-carriages the friction is transferr'd from the circumference of the wheel (where it would act if the wheel did not turn round) to the circumference of the axis; and, conſequently, is diminiſhed in the proportion of the radius of the axis to the radius of the wheel. In theſe, therefore, the friction is always diminiſhed by diminiſhing the diameter of the axis, or by increaſing the diameter of the wheel. The friction is likewiſe diminiſhed by making the axis of an engine to reſt upon the circumferences of wheels that turn round with it, inſtead of reſting in fixed grooves that rub upon it; for by this contrivance, the friction is transferred from the circumferences of thoſe wheels to their pivots; and the friction may be ſtill diminiſhed farther by making the axles of thoſe wheels reſt upon other friction-wheels that turn round with them. It is hardly poſſible to give general and exact rules concerning friction, ſince it depends upon the ſtructure of bodies, the form of their prominent parts and cavities, and upon their rigidity, elaſticity, their coherence, and other circumſtances. Some authors have made the friction upon a horizontal plane equal to one third of the weight; but others have found that it was only one fourth of it, and ſometimes only $\frac{1}{6}$ or $\frac{1}{7}$ of it. Of late, authors have told us that the friction depends not on the ſurface of the body, but its weight only; but neither is this found to be accurately true. In leſſer velocities, the friction is nearly in the ſame ratio as the velocities; but in greater velocities, the friction increaſes in a higher proportion, whether the bodies are dry or oil'd.

24. The ſecond general problem in mechanics, mentioned above, is, to determine the proportion which the power and weight ought to bear to each other, that, when the power prevails, and the engine is in motion, the greateſt effect poſſible may be produced by it in a given time. It is manifeſt that this is an enquiry of the greateſt importance, tho' few have treated of it. When the power is only a little greater than that which is ſufficient to ſuſtain the weight, the motion is too ſlow; and tho' a greater weight is raiſed in this caſe, it is not ſufficient to compenſate the loſs of time. When the weight is much leſs than that which the power is able to ſuſtain, it is raiſed in leſs time; and this may happen not to be ſufficient to compenſate the loſs ariſing from the ſmallneſs of the load. It ought, therefore, to be determined when the product of the weight multiplied by its velocity is the *greateſt* poſſible; for this meaſures the effect of the engine in a given time, which is always the greater in proportion as the weight that is raiſed is greater, and as the velocity with which it is raiſed is greater. We ſhall, therefore, ſubjoin ſome inſtances of this kind that may be demonſtrated from the common elementary geometry; wiſhing that farther improvements may be made in this moſt uſeful part of mechanics.

25. When the power prevails, and the engine begins to move, the motion of the weight is at firſt gradually accelerated. The action of the power being ſuppoſed invariable, its influence in accelerating the motion of the weight decreaſes while the velocity of the weight increaſes. Thus the action of a ſtream of water, or air, upon a wheel is to be eſtimated only from the exceſs of the velocity of the fluid above the velocity already acquired by the part of the

the engine which it ſtrikes, or from their relative velocity. On the other hand, the weight of the load that is to be elevated, and the friction, tend to retard the motion of the engine; and when theſe forces, *viz.* thoſe that tend to accelerate it, and thoſe that tend to retard it, become equal, the engine then proceeds with the uniform motion it has acquired.

Let AB (*Fig.* 39.) repreſent the velocity of the ſtream, AC the velocity of the part of the engine which it ſtrikes, when the motion of the machine becomes uniform; and CB will repreſent their relative velocity, upon which the effect of the engine depends. It is known that the action of a fluid, upon a given plane, is as the ſquare of this relative velocity; conſequently, the weight raiſed by the engine, when its motion becomes uniform, being equal to this action, it is likewiſe as the ſquare of CB. Let this be multiplied by AC, the velocity of the part of the engine impell'd by the fluid; and the effect of the engine in a given time will be proportional to $AC \times CB^2 =$ (ſuppoſing CB to be biſected in D) $AC \times 2\,CD \times 2\,DB = 4\,AC \times CD \times DB$; conſequently, the effect of the engine is greateſt when the product of AC, CD, and DB is greateſt. But it is eaſy to ſee, that this product is greateſt when the parts AC, CD and DB are equal; for, if you deſcribe a ſemicircle upon AD, and the perpendicular CE meet the circle in E, then $AC \times CD = CE^2$, and is greateſt when C is the centre of the circle; ſo that in order that $AC \times CD \times DB$ may be the greateſt poſſible, AD muſt be biſected in C; and CB having been biſected in B, it follows that AC, CD, DB muſt be equal; or that AC, the velocity of the part of the engine impelled by the ſtream, ought to be but one third of AB the velocity of the ſtream. In this caſe, when (abſtracting from friction) the engine acts with the utmost advantage, the weight raised by it is to the weight that would just sustain the force of the stream, as the square of CB, the relative velocity of the engine and stream, to the square of AB, which would be the relative velocity if the engine was quiescent; that is, as 2×2 to 3×3 or 4 to 9. Therefore, that the engine may have the greatest effect possible, it ought to be loaded with no more than $\frac{4}{9}$ of the weight which is just able to sustain the efforts of the stream. Of this the reader will find more in my *Treatise of Fluxions*, § 908.

26. For another example, suppose that a given weight P, (*Fig.* 40.) descending by its gravity in the vertical line, raises a greater weight W likewise given, by the rope PMW (that passes over the fixed pulley M) along the inclined plane BD, the height of which BA is given; and let it be required to find the position of this plane, along which W will be raised in the least time, from the horizontal line AD to B. Let BC be the plane upon which if W was placed, it would be exactly sustained by P, and, by § 20. of this chapter, P shall be to W, as AB to BC; but W is to the force with which it tends to descend along the plane BD, as BD to AB, by the same article; consequently the weight P is to that force, as BD to BC. Therefore the excess of P above that force (which excess is the power that accelerates the motions of P and W) is to P, as BD—BC to BD; or, taking BH upon BC equal to BD, as CH to BD. But it is known that the spaces described by motions uniformly accelerated are in the compound ratio of the forces which produce them and the squares of the times; or, that the square of the time is directly as the space described in that time, and inversely as the force; consequently, the square of the time, in which BD is

described by W, will be directly as BD and inversely as $\frac{CH}{BD}$, and will be least when $\frac{BD^2}{CH}$ is a *minimum*; that is, when $\frac{BC^2}{CH}+CH+2BC$, or (because $2BC$ is invariable) when $\frac{BC^2}{CH}+CH$ is a *minimum*. Now as, when the sum of two quantities is given, their product is a *maximum* when they are equal to each other; so it is manifest, that, when their product is given, their sum must be a *minimum* when they are equal. Thus it is evident, that, as in the last section the rectangle or product of the equal parts AC and CD was CE^2; so the rectangle or product of any two unequal parts, into which AD may be divided, is less than CE^2, and AD is the least sum of any two quantities the product of which is equal to CE^2. But the product of $\frac{BC^2}{CH}$ and CH is BC^2, and consequently given; therefore the sum of $\frac{BC^2}{CH}$ and CH is least when these parts are equal, that is, when CH is equal to BC, or BD equal to 2 BC. It appears, therefore, that when the power P and weight W are given, and W is to be raised by an inclined plane, from the level of a given point A to the given point B, in the least time possible, we are first to find the plane BC upon which W would be sustained by P, and to take the plane BD double in length of the plane BC; or, we are to make use of the plane BD upon which a weight that is double of W could be sustained by the power P.

27. Let a fluid, moving with the velocity and direction AC (*Fig.* 41.) strike the plane CE, and suppose that this plane moves parallel to itself in the direction CB, perpendicular to CA, or that it cannot move in any other direction; then let it be required to find the most advantageous position of the plane

CE, that it may receive the greateſt impulſe from the action of the fluid. Let AP be perpendicular to CE in P, draw AK parallel to CB, and let PK be perpendicular upon it in K; and AK will meaſure the force with which any particle of the fluid impells the plane EC, in the direction CB. For the force of any ſuch particle being repreſented by AC, let this force be reſolved into AQ parallel to EC, and AP perpendicular to it; and it is manifeſt, that the latter AP only has any effect upon the plane CE. Let this force AP be reſolved into the force AL perpendicular to CB, and the force AK parallel to it; then it is manifeſt, that the former, AL, has no effect in promoting the motion of the plane in the direction CB; ſo that the latter AK, only, meaſures the effort by which the particle promotes the motion of the plane CE, in the direction CB. Let EM and EN be perpendicular to CA and CB, in M and N; and the number of particles, moving with directions parallel to AC, incident upon the plane CE, will be as EM. Therefore the effort of the fluid upon CE, being as the force of each particle and the number of particles together, it will be as $AK \times EM$; or, becauſe AK is to AP ($=EM$,) as EN to CE, as $\frac{EM2EM2\times EN}{CE}$; ſo that CE being given, the problem is reduced to this, to find when $EM^2 \times EN$ is the greateſt poſſible, or a *maximum*. But becauſe the ſum of EM^2 and of EN^2 ($=CM^2$) is given, being always equal to CE^2, it follows that $EN^2 \times EM^4$ is greateſt when $EN^2 = \frac{1}{3} CE^2$; in the ſame manner as it was demonſtrated in § 25. that when the ſum of AC and CB was given, $AC \times CB^2$ was greateſt when $AC = \frac{1}{3} AB$. But when $EN^2 \times EM^4$ is greateſt, its ſquare-root $EN \times EM^2$ is of neceſſity at the ſame time greateſt. Therefore the action of the fluid upon the plane CE in the direction CB is greateſt

eſt when $EN^2 = \frac{1}{3}CE^2$, and conſequently $EM^2 = \frac{2}{3}CE^2$; that is, when EM the ſine of the angle ACE in which the ſtream ſtrikes the plane is to the radius, as $\sqrt{2}$ to $\sqrt{3}$; in which caſe it eaſily appears, from the trigonometrical tables, that this angle is of 54°. 44′.

28. Several uſeful problems in mechanics may be reſolved by what was ſhewn in the laſt article. If we repreſent the velocity of the wind by AC, a ſection of the ſail of a wind-mill perpendicular to its length by CE, as it follows from the nature of the engine, that its axis ought to be turned directly towards the wind, and the ſail can only move in a direction perpendicular to the axis, it appears, that, when the motion begins, the wind will have the greateſt effect to produce this motion, when the angle ACE in which the wind ſtrikes the ſail is of 54°. 44′. In the ſame manner, if CB repreſent the direction of the motion of a ſhip, or the poſition of her keel, abſtracting from her lee-way, and AC be the direction of the wind, perpendicular to her way, then the moſt advantageous poſition of the ſail CE, to promote her motion in the direction CB, is when the angle ACE, in which the wind ſtrikes the ſail, is of 54°. 44′. The beſt poſition of the rudder, where it may have the greateſt effect in turning round the ſhip, is determined in like manner. And how this ſame angle enters into the determination of the figure of the rhombus's that form the baſes of the cells in which the bees depoſite their honey, in the moſt frugal manner, I have ſhewn in a letter to the learned and worthy *Martin Folkes*, Eſq; preſident of the royal ſociety. *Philoſophical Tranſactions*, N°. 471.

29. But it is to be carefully obſerved, that when the ſine of the angle ACE is to the radius as $\sqrt{2}$ to $\sqrt{3}$, or (which is the ſame thing) when its tangent is to the radius, as the diagonal of a ſquare to its ſide, this is the moſt advantageous angle only at the beginning of the motion of the engine; ſo that the ſails of a common wind-mill ought to be ſo ſituated, that the wind may indeed ſtrike them in a greater angle than that of 54°. 44′. For we have demonſtrated elſewhere, that when any part of the engine has acquired the velocity c, the effort of the wind upon that part will be greateſt, when the tangent of the angle in which the wind ſtrikes it is to the radius, not as the $\sqrt{2}$ to 1, but as $\sqrt{2 + \frac{9cc}{4aa}} + \frac{3c}{2a}$ to 1, the velocity of the wind being repreſented by a. If for example $c = \frac{1}{3}a$ then the tangent of the angle ACE ought to be double of the radius, that is, the angle ACE ought to be of 63°. 26′. If $c = a$ then ACE ought to be of 74°. 19′. This obſervation is of the more importance, becauſe, in this engine, the velocity of the parts of the ſail remote from the axis, bear a conſiderable proportion to the velocity of the wind, and perhaps ſometimes are equal to it; and becauſe a learned author, Mr. *Daniel Bernouilli*, has drawn an oppoſite concluſion from his computations in his *hydrodynamics*, by miſtaking a *minimum* for a *maximum*; where he infers, that the angle in which the wind ſtrikes the ſail ought to decreaſe as the diſtance from the axis of motion increaſes, that if $c = a$ the wind ought to ſtrike the ſail in an angle of 45°, and that, if the ſail be in one plane, it ought to be inclined to the wind, at a *medium*, in an angle of about 50°. How he fell into theſe miſtakes, we have explained elſewhere *. In like manner, tho' the

* *Treatiſe of Fluxions*, § 914.

the angle ACE of 54°. 44′. be the moſt advantageous at the beginning of the motion, when a ſhip ſails with a ſide wind, yet it ought to be enlarged afterwards as the motion increaſes. In general, let A*a*, parallel to CB, be to AC, as the velocity which the engine has already acquired in the direction CB, to that of the ſtream; upon AC produced take AD to AC as 4 to 3, draw DG parallel to CB, and let a circle deſcribed from the centre C with the radius C*a* meet DG in *g*; and the plane CE ſhall be in the moſt advantageous ſituation for promoting the motion of the engine, when it biſects the angle *a*C*g*. It is generally ſuppoſed, that a direct wind always promotes the motion of a ſhip, the ſail being perpendicular to the wind, more than any ſide-wind; and this has been affirmed in ſeveral late ingenious treatiſes; but, to prevent miſtakes, we are obliged to obſerve, that the contrary has been demonſtrated in our treatiſe of *fluxions*, § 919; where other inſtances of this ſecond general problem in mechanics are given, to which we refer.

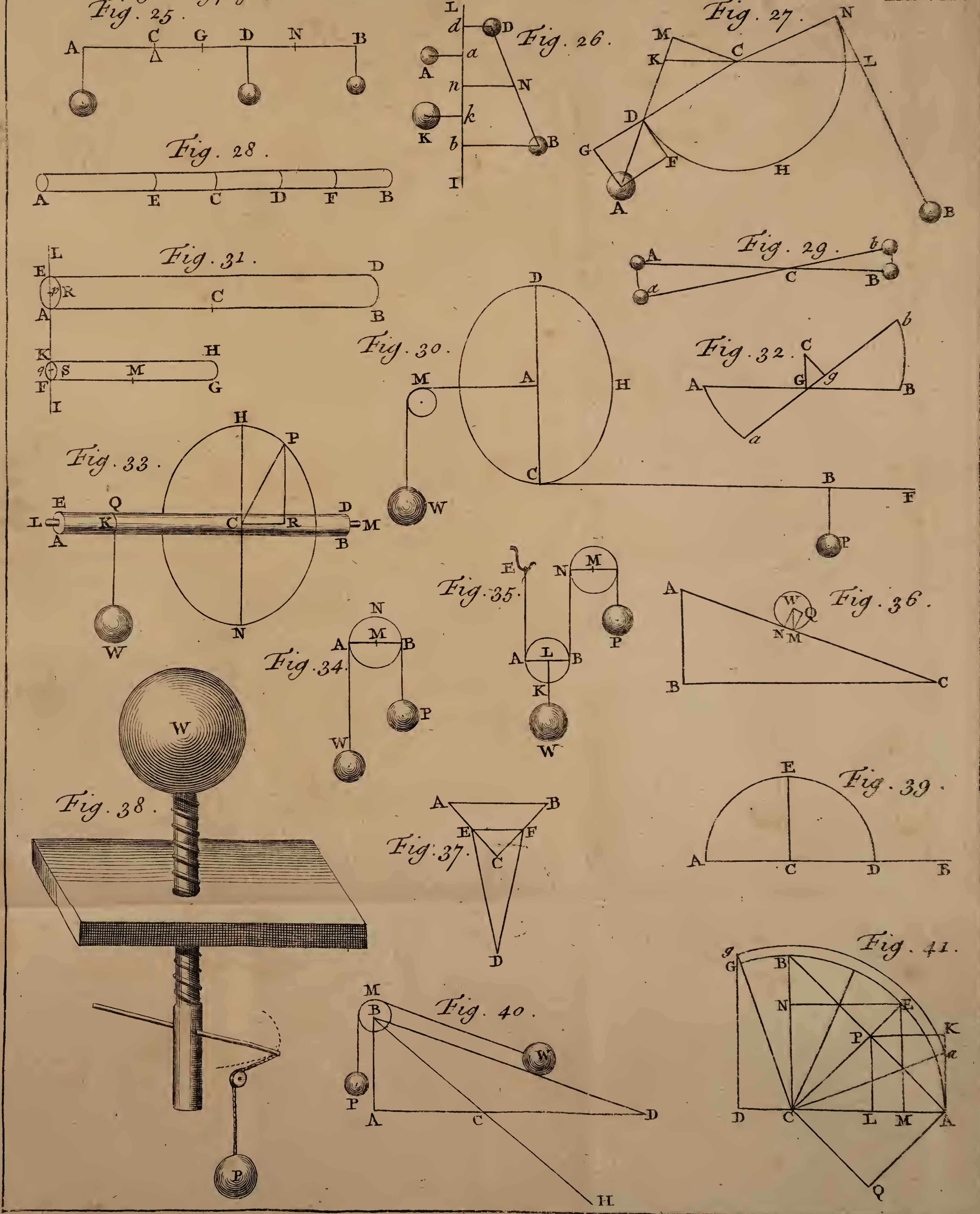
30. The mechanical powers, according to their different ſtructure, ſerve for different purpoſes; and it is the buſineſs of the ſkilful mechanic to chuſe them, or combine them, in the manner that may be beſt adapted to produce the effect required, by the power which he is poſſeſſed of, and at the leaſt expence. The lever can be employed to raiſe weights a little way only, unleſs the engine itſelf be moved, as, for example, to raiſe ſtones out of their beds in quarries. But the axis and wheel may ſerve for raiſing weights from the greateſt depths. The pullies being eaſily portable aboard ſhips, are therefore much employed in them. The wedge is excellent for ſeparating the parts of bodies; and the ſcrew, for compreſſing or ſqueezing them together; and its great

great friction is even sometimes of use, to preserve the effect already produced by it. The strength of the engine, and of its parts, must be proportioned to the effects which are to be produced by it. As we found, that, when the centre of motion is placed between the power and weight, it must sustain the sum of their efforts; a small ballance ought not to be employed for weighing great weights; for these disorder its structure, and render it unfit for serving that purpose with accuracy. Neither are great engines proper for producing small effects: the detail of which things must be left to the skilful and experienced mechanic.

31. But, besides the raising of weights and overcoming resistances, in mechanics we have often other objects in view. To make a regular movement, that may serve to measure the time as exactly as possible, is one of the most valuable problems in this science; and has been most successfully effected, hitherto, by adapting pendulums to clocks; tho' many ingenious contrivances have been invented to correct the irregularities of those movements that go by springs. Some have endeavoured to find a perpetual movement, but without success: and there is ground to think, from the principles of mechanics, that such a movement is impossible. In many cases, when bodies act upon each other, there is a gain of absolute motion; but this gain is always equal in opposite directions, and the quantity of direct motion is never increased. To make a perpetual movement, it appears necessary that a certain system of bodies, of a determined number and quantity, should move in a certain space for ever, and in a certain way and manner; and for this, there must be a series of actions returning in a circle, to make the movement continual; so that any action by which the absolute

fronting pag. 188.

TAB. II.



ſolute quantity of force is increaſed, of which there are ſeveral ſorts, muſt have its correſponding counter-action, by which that gain of force is deſtroyed, and the quantity of force reſtored to its firſt ſtate. Thus, by theſe actions, there will never be any gain of direct force, to overcome the friction and the reſiſtance of the *medium*. But every motion will be abated, by theſe reſiſtances, of its juſt quantity; and the motions of all muſt, at length, languiſh and ceaſe.

32. To illuſtrate this, it is allowed, that, by the reſolution of force, there is a gain or increaſe of the abſolute quantity of force; as the two forces AB and AD (*Fig.* 2.) taken together, exceed the force AC which is reſolved into them. But you cannot proceed reſolving motion *in infinitum*, by any machine whatſoever; but thoſe you have reſolved muſt be again compounded, in order to make a continual movement, and the gain obtained by the reſolution will be loſt again by the compoſition. In like manner, if you ſuppoſe A and B (*Fig.* 42.) to be perfectly elaſtic, and that the leſſer body A ſtrikes B quieſcent, there will be an increaſe of the abſolute quantity of force, becauſe A will be reflected; but if you ſuppoſe them both to turn round any centre C, after the ſtroke, ſo as to meet again in *a* and *b*, this increaſe of force will be loſt, and their motion will be reduced to its firſt quantity. Such a gain, therefore, of force as muſt be afterwards loſt in the actions of the bodies can never produce a perpetual movement. There are various ways, beſides theſe, by which abſolute force may be gained; but ſince there is always an equal gain in oppoſite directions, and no increaſe obtained in the ſame direction; in the circle of actions neceſſary to make a perpetual movement, this gain muſt be preſently loſt, and

will

will not ſerve for the neceſſary expence of force em-ployed in overcoming friction and the reſiſtance of the *medium*.

33. We are to obſerve, therefore, that tho' it could be ſhewn that in an infinite number of bodies, or in an infinite machine, there could be a gain of force for ever, and a motion continued to infinity; it does not therefore follow that a perpetual move-ment can be made. That which was propoſed by Mr. *Leibnitz*, in *Auguſt* 1690, in the *Leipſick* acts, as a conſequence of the common eſtimation of the forces of bodies in motion, is of this kind; and, for this and other reaſons, ought to be rejected. It is, however, neceſſary to add, that tho' on many ac-counts, it appear preferable to meaſure the forces as well as motions of bodies by their velocities, and not by the ſquares of their velocities; yet, in order to produce a greater velocity in a body, the power or cauſe that is to generate it muſt be greater in a higher proportion than that velocity; becauſe the action of the power upon the body depends upon their relative motion only; ſo that the whole action of the power is not employed in producing motion in the body, but a conſiderable part of it in ſuſtaining the power, ſo as to enable it to act upon the body and keep up with it. Thus the whole action of the wind is not employed in accelerating the motion of the ſhip, but only the exceſs of its velocity above that of the ſail on which it acts, both being reduced to the ſame direction. When motion is produced in a body by ſprings, it is the laſt ſpring only which acts upon the body by contact, and the reſt ſerve only to ſuſtain it in its action; and hence a greater number of ſprings is requiſite to produce a greater velocity in a given body, than in proportion to that velocity. A double power, like that of gravity, will produce a double

motion

motion in the ſame time; and a double motion in an elaſtic body may produce a double motion in another of the ſame kind. But two equal ſucceſſive impulſes, acting on the ſame body, will not produce a motion in it double of what would be generated by the firſt impulſe; becauſe the ſecond impulſe has neceſſarily a leſs effect upon the body, which is already in motion, than the firſt impulſe which acted upon it while at reſt. In like manner, if there is a third and fourth impulſe, the third will have leſs effect than the ſecond, and the fourth leſs than the third. From this it appears what anſwer we are to make to a ſpecious argument that is adduced to ſhew the poſſibility of a perpetual motion. Let the height AB (*Fig.* 43.) be divided into four equal parts AC, CD, DE, EB: ſuppoſe the body A to acquire, by the deſcent AC, a velocity as 1, and this motion by any contrivance to be tranſmitted to an equal body B; then let the body A, by an equal deſcent CD, acquire another motion as 1, to be tranſmitted likewiſe to the ſame body B, which in this manner is ſuppoſed to acquire a motion as 2, that is ſufficient to carry it upwards from B to A; and becauſe there yet remain the motions which A acquires by the deſcents DE and EB, that may be ſufficient to keep an engine in motion, while B and A aſcend and deſcend by turns, it is hence concluded that a ſufficient gain of force may be obtained in this manner, ſo as to produce a perpetual movement. But it appears from what has been ſhewn, that a motion as 2 cannot be produced in B, by the two ſucceſſive impulſes tranſmitted from A, each of which is as 1.

Some authors have propoſed projects for producing a perpetual movement, with a deſign to refute them; but, by miſtaking the proper anſwer, have rather confirmed the unſkilful in their ground-

leſs

leſs expectations. An inſtance of this we have in Dr. *Wilkin's Mathematical Magick*, book 2. chap. 13. A load-ſtone at A (*Fig.* 44.) is ſuppoſed to have a ſufficient force to bring up a heavy body along the plane FA, from F to B; whence the body is ſuppoſed to deſcend by its gravity, along the curve BEF, till it return to its firſt place F; and thus to riſe, along the plane FA, and deſcend, along the curve BEF, continually. But ſuppoſing BZE to be the ſurface upon which if a body was placed, the attraction of the load-ſtone and the gravity of the body would ballance each other, this ſurface ſhall meet BEF at ſome point E between A and F, and the body muſt ſtop in deſcending along AEF at the point E.

CHAP. IV.

Of the colliſion of bodies.

1. THO' the laws of motion and principles of mechanics are ſufficiently explained and eſtabliſhed in the preceding chapters, it will be of uſe, before we proceed to apply them to ſubjects of a higher nature, to conſider the moſt ſimple and obvious motions and phænomena that are derived from them; by which they may be farther tried and examined, and our methods of reaſoning from them juſtified: and theſe are the motions which are produced by bodies impinging upon one another, which fall frequently under our obſervation, and can be repeated by us in experiments. It is always from the moſt ſimple kind of phænomena that we can trace with the greateſt certainty the analyſis of the laws of nature; from which we afterwards may proceed to ſuch as are more complicated and abſtruſe: but it would be contrary to the rules of good method to begin

begin with the latter. It would be very prepofterous, for example, in defining or afcertaining the true notion of the *inertia* of body, to begin with chymical experiments concerning fermentation, the folutions of bodies by menftruums, the phænomena of generation and corruption, or others of that complicated kind. If we fhould begin with fixing our attention on thefe, we fhould be apt to afcribe to body an activity which is really repugnant to its nature. It is from obfervations and experiments concerning the fenfible and grofs bodies, that we muft acquire our knowledge of the firft principles of this fcience. The doctrine of the collifion of bodies was very plain and clear, and deduced in a fatisfactory manner from the laws of motion, before fome late authors endeavoured to cloud it, by introducing abftrufe notions into it, in favour of their new doctrine concerning the eftimation of the forces of bodies in motion. But we fhall have no regard to thefe; and fhall endeavour to deduce it, in a plain and fatisfactory manner, from the principles eftablifhed and illuftrated in the fecond chapter.

2. Bodies have been commonly diftinguifhed into three forts. Thofe are called perfectly *hard* whofe parts yield not at all in their collifions, but are abfolutely inflexible; and fuch the laft elements of bodies, or atoms, are fuppofed to be. Thofe are called *foft* whofe parts yield in their collifions, but reftore not themfelves again towards their firft pofitions. Thofe are faid to be *elaftic* which yield in their collifions, but reftore themfelves fo as to recover their firft fituation; and they are faid to be perfectly elaftic, when they reftore themfelves with the fame force with which they are compreffed. The actions of perfectly hard or inflexible bodies on one another are confummated in a moment: and, as

there is no ſpring, nor any force, to ſeparate them, they muſt go on together after their colliſion as if they formed one body. But when an elaſtic body is acted on by any force or power, its parts yield at firſt, and afterwards reſtore themſelves by degrees to their firſt ſituations. There is a time required for this, which may be diſtinguiſhed into two portions; the firſt is the time during which the parts yield and become more and more compreſſed; the other is the time during which they reſtore themſelves to their firſt ſituations. When two ſpherical elaſtic bodies meet, at firſt they touch one another in a point, but their contact gradually increaſes, as the parts that touch and preſs on one another yield, till their greateſt compreſſion: and afterwards theſe parts recover by the ſame ſteps, tho' in a contrary order, their firſt ſituations. The actions of elaſtic bodies may be explained by imagining ſprings KL placed betwixt hard bodies A and B (*Fig.* 14.); for the ſprings muſt have the ſame effect in this caſe, as the elaſticity of the parts of the bodies in the other caſe. If A move towards B and compreſs the ſprings, and, by their mediation, act on B, the ſprings will become more and more compreſſed, till the two bodies have equal velocities in the ſame direction; and then, no force acting on the ſprings, they will have liberty to begin to expand themſelves; which they will do by the ſame degrees as they were compreſſed, in a contrary order: and this is the ſecond period of the action of the bodies on one another. In the firſt period of the action of elaſtic bodies, or of bodies acting by the intervention of ſprings, the ſame effects are produced as if the bodies were perfectly hard. At the end of this period the reſpective velocity of the bodies is deſtroyed, and in the inſtant when it ceaſes the ſecond begins, the velocities of the bodies in the ſame direction being now equal. In this ſecond period of the

the action of the bodies, if the elasticity is perfect, the springs expanding themselves by the same force with which they were compressed, the bodies must be separated with a respective velocity equal to that they had before their collision; and whatever motion was added to, or subducted from, either body, in the first period, as much will be added to, or subducted from it, in the same direction, in the second; so that there will be twice as much force lost, or twice as much gained, by either, as if the bodies had been perfectly hard.

3. The effects produced in the first period of the action of bodies that have an imperfect elasticity are the same as when the bodies are perfectly elastic; but, because their parts recover their first situations with less force than that whereby they were displaced from them, there is less force lost or gained in the second than in the first period. There is, however, a constant proportion observed between what is lost or gained in these two periods, in the same sort of bodies; so that there is a constant proportion between their respective velocities before and after their collision. In glass, for example, this proportion is observed to be that of 16 to 15.

4. In soft bodies, whose parts yield so as not to restore themselves at all to their first situations, the action must be the same as in the first period of perfectly elastic bodies, and the same as in perfectly hard bodies. By their collision their respective velocity is destroyed, the *inertia*, or resistance of the parts, having the same effect in this case, as their spring in the other. After the collision they go on together as one mass, there being no spring to separate them. Because the parts yield, in their collisions, certain philosophers have imagined that some force must be

loſt in producing this effect: but there is no motion communicated to any one part that it can loſe without communicating it to others; a body moving in a fluid loſes no force but what it communicates to the parts of the fluid; and a body acting upon a ſoft body can loſe no force but what muſt be communicated to the parts of that body, which therefore muſt be accumulated to the force of the whole. The parts are indeed moved out of their firſt places, but this can produce no loſs of force; for it is manifeſt, that if A move and ſtrike B, (*Fig.* 45.) and make it go into the place *b*, and there ſtrike C, ſo that it remain itſelf in the place *b*, all the force which A had at firſt muſt be ſtill found in A or C, and there can be none loſt or conſumed in carrying B from its firſt place B, to its laſt place *b*, ſince A loſt none but what it gave to B, and B could loſe none, but what is communicated to C. There can be no force loſt in this caſe more than if B had ſtruck C in its firſt place B, nor would there be more force loſt in B moved twice or thrice as far before it ſtruck C. In like manner, when a body acts upon a ſoft body and moves its parts out of their places, the force which the firſt body loſes is employed in moving thoſe parts indeed, by which they acquire whatever is loſt by it, and loſe none of what they thus acquire, but by communicating to other particles; nor is it of moment how far they are moved from their places, but what force is communicated to them, which it is not poſſible to conceive they can loſe by merely moving out of their places, but by acting on other particles.

5. This will ſtill be found true, tho' you ſuppoſe the particles of the ſoft body to cohere with ſome certain degree of force. That caſe may be explained by ſuppoſing particles, B, C and D, (*Fig.* 46.) cohering by a ſtring of a certain degree of ſtrength, and

and that A impelling C changes the ſituation of the particles with reſpect to one another. In this caſe, A will loſe no force which will not be all communicated to C, but ſome part, by mediation of the ſtring, muſt be imprinted on B and D, and all that A loſes and is not given to C, muſt be communicated to B and D, if we ſuppoſe the ſtring infinitely fine, or abſtract from its *inertia*, and reckon all the force in the ſame direction. It is true the ſtring will be ſtretched by the force which is at firſt imprinted on C, but as C can loſe none but what B and D receive, there can be no force loſt from that cauſe; and, if the ſtring ſhould break, the only conſequence can be, that there will be no more force communicated from C to B and D, after that happens. From the equality of action and reaction it follows, that the ſtring acts equally on C and B, and on C and D; ſo that it adds as much force to B and D as it takes from C; and, as this is always true, it muſt hold in the inſtant when the ſtring breaks, as well as before: the coheſion of the particles, therefore, can be the occaſion of no loſs of force, taking in all that are affected in the colliſion, and there appears no ground for ſuppoſing that any force is conſumed, in making the parts of ſoft bodies yield, but what is accumulated to the whole maſs of body, while its parts continue all together.

6. Theſe things being premiſed, firſt let the bodies A and B (*Fig.* 47.) be ſuppoſed void of elaſticity, let C be their centre of gravity, and let A D and B D repreſent their velocities before the ſtroke. Then ſuppoſing the ſtroke to be direct, after it they will proceed together as forming one maſs, and their centre of gravity being carried along with them, their common velocity will be the ſame as the velocity of that centre, which (by § 15. chap. 2.) is the ſame

after the ſtroke as before it. But while the bodies deſcribed A D and B D before the ſtroke, their centre of gravity moves from C to D, the place where they meet, or the one overtakes the other; therefore the common velocity of A and B after the ſtroke is meaſured by C D, their velocities before the ſtroke being repreſented by A D and B D reſpectively. The right line C D ſhews the direction as well as the velocity of their motions after the ſtroke; for it is always in the direction from C to D. If D fall upon C, then C D vaniſhes, and their motions are deſtroyed by the ſtroke. This propoſition ſerves for determining the caſes when the bodies are either perfectly hard, or perfectly ſoft.

7. But if the bodies are perfectly elaſtic, take C E equal to C D in an oppoſite direction; and the velocities of A and B after the ſtroke, with their directions, will be repreſented by E A and E B reſpectively. For the change produced in their motions by the ſtroke, being, in this caſe, double of what it was in the former, by § 2; and the difference of A D and C D (the change produced in the velocity of A in the former caſe) being equal to the difference of C D, or C E, and E A, it follows that the velocity of A after the ſtroke is meaſured by E A; and the difference of E B and C D, or C E, *viz.* C B, being equal to the difference of C D and B D, it follows, that E B is the velocity of B after the ſtroke. If B have no motion before the ſtroke, let A B repreſent the velocity of A, take C E equal and oppoſite to C B, and E A, E B, will repreſent the velocities of A and B after the ſtroke: in which caſe, the velocity of A before the ſtroke is to the velocity of B after it, as A B to E B, or 2 C B; that is, as one half A B to C B, and therefore (by the property of the centre of gravity) as half the ſum of the bodies A and B to A.

2 From

From this theorem, all the cafes relating to the motion of bodies that have a perfect elafticity may be immediately deduced. For example, if the bodies A and B be equal, then CA=CB, and fince CE=CD, it follows that EA=BD, and EB=AD; that is, the bodies exchange their velocities by the ftroke.

8. But if the elafticity of the bodies is imperfect, take CE (*Fig.* 48. *n.* 1.) equal and oppofite to CD, but C*a* is lefs than CA, and C*b* lefs than CB, in the fame proportion as their elafticity is lefs than a perfect elafticity; and the right lines E*a* and E*b* will reprefent their velocities after the ftroke, by § 3: becaufe if we diftinguifh the time in which the bodies act upon each other into two periods, as in that article, the effect produced in the fecond period will be lefs than the effect produced in the firft period, in that ratio. In this cafe their refpective velocity after the ftroke is reprefented by *ab*, and is to their refpective velocity before the ftroke, as *ab* to AB. In glafs, Sir *Ifaac Newton* found this ratio to be that of 15 to 16, as was obferved above; confequently in determining the effect of their collifions, we are to take C*a*$=\frac{15}{16}$CA, and C*b*$=\frac{15}{16}$CB.

9. If motion be communicated, in this manner, from a body A to a feries of bodies in a geometrical progreffion, then the velocity fucceffively communicated to thofe bodies will be likewife in a geometrical progreffion; and if A and B be the two firft bodies, the common ratio of the velocities will be that of half the fum of A and B to A; that is, if the bodies A, B, be reprefented by the right lines *oa* and *ob*, (*Fig.* 48. *n.* 2.) and *ab* be bifected in *e*, the common ratio of any two fubfequent velocities in the progreffion will be that of *oe* to *oa*; and if *n*

represent the number of bodies without including the first A, the velocity of the last will be to the velocity of the first, as the power of *o a* whose exponent is *n* to the same power of *o e*.

10. Any three bodies being represented by *o a*, *o b*, and *o d*, take *o f* to *o d*, as *o a* is to *o b*; then supposing the motion to begin from the first *o a* (which was supposed to strike *o b* quiescent, and *o b* afterwards to strike *o d* quiescent) the velocity communicated, in this manner, to the third shall be to the velocity of the first, as *o a* is to one fourth part of the sum of *o a*, *o b*, *o f* and *o d*. For the velocity of the first *o a* is to the velocity of the second *o b*, as the sum of *o a* and *o b* to 2 *o a*; the velocity of *o b* is to that of *o d*, as the sum of *o b* and *o d* to 2 *o b*; consequently the velocity of the first *o a* is to the velocity of the third *o d*, in the compound ratio of *oa*+*o b* to 2 *o a* and of *ob*+*o d* to 2 *o b*, that is, (since *o a*, *ob*, *of*, *o d*, are proportional, so that *o a* is to *o b*, as *o a*+*o f* to *o b*+*o d*, and *o a*+*o b* to *o b*, as the sum of *o a*, *o b*, *o f* and *o d* to *o b*+*o d*) as the sum of *o a*, *ob*, *of* and *o d* is to 4 *o a*. Hence the velocity of *o a* being given, the velocity communicated to *o d* is inversely as the sum of *o a*, *o b*, *o f* and *o d*, and is greatest when this sum is least; that is, if *o a* and *od* be given, when *ob* and *o f* coincide with each other and with *o k* the mean proportional between *o a* and *o d*. Therefore the velocity communicated to *o d* is greatest when *o b*, the body interposed between *o a* and *o d*, is a mean proportional between them. This is one of Mr. *Huygens*'s theorems; from which it follows, that the more such geometrical mean proportionals are interposed between *o a* and *o d*, the greater is the velocity communicated to *o d*. There is, however, a limit which the velocity communicated to *o d* never amounts to, (the bodies *o a*, *o d*, and

and the velocity of *o a* before the ſtroke, being given) to which it approaches continually, while the number of ſuch bodies interpoſed between *o a* and *o d* is always increaſed. And this limit is a velocity which is to the velocity of the firſt *o a* before the ſtroke, in the ſubduplicate ratio of *o a* to *o d*; as we have demonſtrated in our *fluxions*, § 514.

11. The ſame principles will ſerve for determining the effects of the colliſions, when a body ſtrikes any number of bodies at once, in any directions whatever. Let the bodies firſt be perfectly hard and void of elaſticity, and the body C (*Fig.* 49.) moving in the direction C D with a velocity repreſented by CD, ſtrike at once the bodies A, B, E, &c. that are ſuppoſed at reſt before the ſtroke, in the directions C F, C H, C K, &c. in the ſame plane with CD, and let D *a*, D *b*, D *e*, be perpendicular to C F, C H, C K, in *a*, *b*, and *e*, reſpectively. Determine the point P where the common centre of gravity of the bodies C, A, B, E, &c. would be found, if their centres were placed at the points C, *a*, *b*, *e*, &c. reſpectively, (by § 13. chap. 2.); join D P, and C L parallel to D P ſhall be the direction of the body C after the ſtroke. Let P R, perpendicular to D P, meet C D in R, and D L, perpendicular to C D, meet C L in L; then if C L be divided in G, ſo that C G be to C L in the ratio compounded of that of C D to C R, and that of the body C to the ſum of all the bodies, the velocity of C after the ſtroke will be repreſented by C G; that is, the velocity of C after the ſtroke will be to its velocity before it, as C G is to C D. Let G *f*, G *h*, and G *k* be reſpectively perpendicular to C F, C H, and C K, in *f*, *h*, and *k*; and the velocities of A, B, and E, after the ſtroke, will be repreſented by C *f*, C *h* and C *k*.

But

But if we now ſuppoſe the bodies to be perfectly elaſtic, or the relative velocities, before and after the ſtroke, to be always equal when meaſured on the ſame right line; produce D G till D *g* be equal to 2 D G, join C *g*, and the body C will deſcribe C *g* after the ſtroke, in the ſame time that it would have deſcribed a right line equal to C D, before the ſtroke. And, in like manner, the motions are determined when the elaſticity is imperfect, if the relative velocity after the ſtroke is always in a given ratio to the relative velocity before it in the ſame right line. Mr. *Bernouilli* has reſolved only a very limited caſe of this problem, in his *Eſſay* on motion, *Paris* 1726; for he ſuppoſes the bodies to be perfectly elaſtic, and that, for each body on one ſide of the line of direction C D, there is always an equal body on the other ſide, that is impell'd in a right line forming an equal angle with C D; ſo that the body C moves with the ſame direction after the ſtroke as before. The ſolution of this particular caſe, (which he repreſents as a matter of uncommon difficulty, and magnifies as the fruit of the new doctrine concerning the forces of bodies) he derives from this principle, "that the ſum of the bodies multiplied by the ſquares of their velocities is the ſame before and after the ſtroke;" which principle, however, had never been demonſtrated by him; for it cannot be conſidered as an immediate conſequence of the equality of action and re-action, as he too haſtily concluded, by what was ſhewn above. But the ſolution of theſe and other problems of this kind is derived, in a natural eaſy and general manner, from the laws concerning the ſum of the motions of a ſyſtem of bodies eſtimated in a given direction, and concerning the motion of their centre of gravity, which is never affected by their colliſions.

12.

12. The ſame things being ſuppoſed as in § 7. becauſe CE=CD, (*Fig.* 47.) it follows that $AD^2-AE^2=4CE\times CA$; and that $EB^2-BD^2=4CE\times CB$. But $A\times 4CE\times CA=B\times 4CE\times CB$, by the property of the centre of gravity C: therefore $A\times AD^2-A\times AE^2=B\times EB^2-B\times BD^2$, or $A\times AD^2+B\times BD^2=A\times AE^2+B\times EB^2$; that is, when the bodies are perfectly elaſtic, the ſum ariſing when each is multiplied by the ſquare of its velocity, is the ſame after the ſtroke as before it. The ſame things being now ſuppoſed as in the laſt article, let DQ, gq, fm, hn, kr, be perpendiculars to CG, in Q, q, m, n, and r; then the rectangles contained by Cm and CG, Cn and CG, Cr and CG, will be reſpectively equal to the ſquares of Cf, Ch, and Ck. If the bodies C, A, B, E, be ſuppoſed to have no elaſticity, their velocities after the ſtroke will be repreſented by CG, Cf, Ch and Ck, the velocity of C before the ſtroke being repreſented by CD, becauſe, in this caſe, no relative velocity is generated by the ſtroke in their reſpective directions; and the ſum of $A\times Cm$, $B\times Cn$, $E\times Cr$ is equal to $C\times GQ$, becauſe the ſum of the motions which would be communicated to A, B and E, in the direction CG, is equal to the motion which C would loſe in the ſame direction, by § 4. chap. 2. Therefore the ſum of $A\times Cf^2$, $B\times Ch^2$, $E\times Ck^2$ is equal to $C\times CG\times GQ$; and to theſe if we add $C\times CG^2$, the ſum of all the bodies multiplied by the ſquares of their velocities in this caſe would be $C\times CG\times CQ$. But when the bodies are ſuppoſed to be perfectly elaſtic, the velocities of A, B and E, are to be repreſented by 2Cf, 2Ch, and 2Ck, reſpectively; the ſum of $A\times 4Cf^2$, $B\times 4Ch^2$ and $E\times 4Ck^2$, is equal to $C\times 4CG\times GQ$ or (*Elem.* 8. 2.) $C\times CQ^2-C\times Cq^2$; to which if we add $C\times Cg^2$ (or $C\times Cq^2+C\times DQ^2$) the whole ſum of the products, when each body is multiplied

tiplied by the ſquare of its velocity, is equal to C × CD²; and conſequently the ſame after the ſtroke as it was before the ſtroke. When therefore the bodies are void of elaſticity, this ſum is leſs after the ſtroke than before it, in the ratio of CG×CQ to CD², or of CG to CL, L being the point where LD perpendicular to CD meets CG. And when the bodies A, B, E, move, before the ſtroke, in directions different from thoſe in which C acts upon them, the propoſition will appear by reſolving their motions into ſuch as are in thoſe directions (which alone are affected by the ſtroke,) and ſuch as are in perpendiculars to thoſe directions, from *Elem.* 47. 1. This propoſition likewiſe holds when bodies of a perfect elaſticity ſtrike any immoveable obſtacle as well as when they ſtrike one another, or when they are conſtrained, by any power or reſiſtance, to move in directions different from thoſe in which they impell one another. But it is manifeſt, that it is not to be held a general principle or law of motion, ſince it can take place in the colliſions of one ſort of bodies only. The ſolutions of ſome problems which have been deduced from it may be obtained, in a general and direct manner, from plain principles that are univerſally allowed, by determining firſt the motions of hard bodies, which are ſuppoſed to have no elaſticity, and thence deducing the ſolutions of other caſes, when the relative velocities before and after the ſtroke are equal, or in any given ratio.

13. From what was ſhewn in the laſt article, we are led to the principle, which, by Mr. *Huygens*, was called the *conſervatio vis aſcendentis.* It is well known, and was proved in § 11. chap. 1: that the heights to which bodies will riſe againſt the direct reſiſtance of an uniform gravity are as the ſquares of the velocities with which they ſet out. In the laſt

article we found that the ſum of the products, when the bodies are multiplied by the ſquares of their velocities, is the ſame after as before the ſtroke; provided the bodies be perfectly elaſtic. If, therefore, we ſuppoſe the motion of the bodies to be turned upwards in vertical lines, the ſum of the products when each body is multiplied by the height to which it would ariſe is the ſame after as before the ſtroke. But by the property of the centre of gravity, in § 15. chap. 2. the ſum of the products of the bodies multiplied by thoſe heights is equal to the product of the ſum of the bodies multiplied by the height to which their centre of gravity would ariſe. Therefore when the motions of bodies are ſuppoſed to be converted upwards in vertical lines, before or after their colliſions, their common centre of gravity will always ariſe to the ſame height; and that is what is meant by Mr. *Huygens* when he tells us the *vis aſcendens* of any ſyſtem of bodies is not affected by their colliſions or mutual actions, provided they be perfectly elaſtic; for if they are ſoft bodies, or have an imperfect elaſticity (which indeed is the caſe of all bodies we have acceſs to examine,) then it is obvious that by their colliſions their motions are often diminiſhed, and ſometimes totally deſtroyed; ſo that the centre of gravity will neceſſarily ariſe to a leſs height after their colliſion than before it, if the motions of the bodies be ſuppoſed to be converted upwards in vertical lines.

14. When bodies are moved by their gravity, and at the ſame time act upon each other, it will ſtill be found, that the ſum of the products that ariſe when each body is multiplied by the ſquare of the velocity acquired by it, is equal to the difference of the ſum of the products of thoſe that deſcend multiplied by the

the ſquares of the velocities that would have been acquired by the ſame deſcents, if the bodies had fallen freely without acting upon each other, and of the ſum of the products of the bodies that aſcend multiplied by the ſquares of their reſpective velocities that would be acquired by falling freely along the reſpective altitudes to which they have ariſen; provided that the elaſticity of the bodies be perfect; or if it be imperfect, that there be no colliſion, or ſudden communication of motion from one body to another. For if the relative velocities in their reſpective directions be leſs immediately after that action than before it; in thoſe caſes, the ſum of the products of the bodies multiplied by the ſquares of their velocities will be leſs than it would have been if the bodies had deſcended freely from the ſame reſpective altitudes; and if the bodies be ſuppoſed to aſcend with their reſpective velocities at any time, and their motions be retarded by their gravity only, the common centre of gravity will not aſcend to the ſame level from which it deſcended; as we have ſhewn at length in our *Treatiſe of Fluxions*, from § 521 to 533.

15. The true general principle on this ſubject, is, that when any number of bodies, moved by their gravity, are connected together in any manner ſo as to act upon each other while they move, the aſcent of their common centre of gravity, in their vibrations or revolutions, will be always found to be either equal to its deſcent, or leſs than it, but never to exceed it. And, from this principle, the impoſſibility of a perpetual motion is juſtly derived. For it appears, that, in ſuch vibrations and revolutions, the ſucceſſive aſcents of the centre of gravity muſt continually diminiſh, in conſequence of the attrition

tion of the parts of bodies, and the reſiſtance of the medium; ſince the aſcent of the centre of gravity being never greater than the deſcent (tho' often leſs than it,) there can be no gain of force to overcome thoſe reſiſtances. All motion, therefore, muſt be abated and gradually languiſh in our mechanical engines, unleſs they be ſupplied by new and repeated influences of the power.

16. It is very well known, that, when allowance is made for the defect of elaſticity in bodies, for attrition, and the reſiſtance of the medium, theſe concluſions are perfectly agreeable to experience; and therefore ſerve to confirm the general laws of motion with their corollaries, and our methods of reaſoning from them.

CHAP. V.

Of the motion of projectiles in vacuo; *of the cycloid, and the motion of a pendulum in it* *.

LEMMA I.

Suppoſe the motion of a body to be uniformly accelerated; let the time be repreſented by the right line AM, (*Plate* IV. *Fig.* I.) and any part of it by AK, draw MN, KL perpendiculars to AM in M and K, and AN interſecting them in N and L: then the velocities acquired in the times AM, AK, reckoned from the beginning of the motion, will be as the

* To render the ſecond book more complete, we have added this *ſupplement*, from two pieces which the author uſed to give his ſcholars. The ſubſtance of them is taken from the learned Mr. *Cotes*'s tracts, printed at the end of his *Harmonia Menſurarum*.

per-

perpendiculars M N, K L, but the ſpaces deſcribed in theſe times will be as the areas A M N, A K L.

This propoſition has been demonſtrated elſewhere; but we ſhall here add the proof that is commonly given of it, by the method of *indiviſibles.*

Since the motion of the body is ſuppoſed to be uniformly accelerated, that is, to receive equal increments of velocity in equal times, the velocities acquired will be always proportional to the times: ſo that if M N repreſent the velocity acquired in the time A M, it follows, becauſe A M : A K : : M N : K L, that K L will repreſent the velocity acquired in the time A K. After the ſame manner, the velocities acquired in the times A B, A C, A D, *&c.* will be repreſented by the perpendiculars B E, C F, D G, *&c.* reſpectively.

The ſpace deſcribed by any uniform motion is as the rectangle contained by the right lines that repreſent the velocity and the time: therefore the ſpaces deſcribed in the times A B, B C, C D, D H, *&c.* with the velocities B E, C F, D G, H I, *&c.* are as the rectangles A E, B F, C G, D I, *&c.* and the ſpaces deſcribed in the whole time A K as the ſum of theſe rectangles. That the motion may be uniformly and continually accelerated, ſuppoſe the number of the parts A B, B C, C D, *&c.* into which the line A K is divided, to be increaſed *in infinitum*, and the ſum of the rectangles A E, B F, C G, *&c.* will become equal to the triangle A K L. Therefore, in a motion uniformly accelerated, the ſpaces deſcribed in any times A K, A M, from the beginning of the motion, are as the areas A K L, A M N.

Corol.

Corol. 1. The ſpace deſcribed by a motion uniformly accelerated, in any time, is half the ſpace that would be deſcribed, in the ſame time, by an uniform motion with the velocity acquired at the end of that time.

The ſpace deſcribed by a motion uniformly accelerated, in the time A K, is repreſented by the triangle A K L; the ſpace that would be deſcribed by an uniform motion, in the ſame time, with the velocity K L, is repreſented by the rectangle contained by A K and K L, but the triangle A K L is half of that rectangle; and the propoſition is manifeſt.

Corol. 2. The ſpaces deſcribed by a motion uniformly accelerated, are as the ſquares of the times from the beginning of the motion; for thoſe ſpaces are as the ſimilar triangles A K L, A M N, whoſe homologous ſides A K, A M, repreſent the times. For the ſame reaſon, the ſpaces are alſo as the ſquares of (K L, M N.) the velocities acquired at the end of thoſe ſpaces.

Corol. 3. If the accelerating force is ſuppoſed to be greater or leſſer in any given ratio, the velocities generated by it, in a given time, will be increaſed or diminiſhed in the ſame ratio. And in any times, the velocity generated by this force, will be to that generated by the former, in the compounded ratio of the forces and of the times.

Corol. 4. The fall of heavy bodies, either perpendicular or along inclined planes, being a motion uniformly accelerated, the preceding *Lemma* and its corollaries may be applied to them.

LEMMA II. *Fig.* II.

If two heavy bodies fall from reſt at C to the horizontal line A B, one in the vertical C B, and the other along the inclined plane C A; the time of deſcent from C to B, will be to the time of deſcent from C to A, as C B to C A; and the velocities acquired at B and A will be equal.

For let the force of gravity by which the body deſcends in the vertical C B, be repreſented by C B, and reſolved into the forces B D perpendicular to C A, and C D; the other body is urged along the inclined plane by C D only. Therefore the accelerating forces by which the bodies deſcend in the vertical C B and along the inclined plane C A, are repreſented by C B and C D. The ſpaces deſcribed in equal times, by the uniform continued action of any forces, are in the ſame ratio as thoſe forces: therefore the bodies will fall from C to B, and from C to D, in equal times. But the time of deſcent from C to D is to the time of deſcent from C to A (by *Corol.* 2. and 4. *Lem.* 1.) in the ſubduplicate ratio of C D to C A, that is, (becauſe C D, C B, C A, are in continued proportion) in the ratio of C D to C B, or of C B to C A.

Again, the velocities generated in the falls are in the compound ratio of the generating forces, and of the times of their generation (*Corol.* 3. *Lem.* 1.) that is, in the preſent caſe, in the compound ratio of C A to C B, and of C B to C A; which compound ratio is that of equality.

LEM-

LEMMA III.

Upon the ſame horizontal plane, let there be raiſed another plane *c a*, whoſe elevation is *c* B; from C draw C I parallel to *c a*, meeting B A in I, and from B the line B *d* perpendicular to C I. Then CB repreſenting, as before, the conſtant force of gravity, CD, C *d* will repreſent the accelerating forces along the planes C A and (C I or) *c a*; and their ratio being compounded of thoſe of CD to C B, and of C B to C *d*, that is, of CB to CA, and (C I to CB or) *c a* to *c* B; it follows that thoſe accelerating forces are directly as the elevations of the planes, C B, *c* B, and inverſely as their lengths C A, *c a*.

Corol. 1. Compound now theſe three ratios; that of C A to C B, of $\surd$ C B to $\surd$ *c* B, and of *c* B to *c a*, their ſum gives the ratio of the times of falling thro' CA and *c a*, being the direct ratio of the lengths CA, *c a*, and the inverſe ſubduplicate of the elevations CB, or *c* B.

Corol. 2. The velocities acquired being as the accelerating forces and the times in which they act; compound the ratio of theſe found in the preceding *Lemma* and *Corollary*, and there will reſult that of the velocities, *viz.* the direct ſubduplicate of the elevations C B, *c* B.

Corol. 3. Hence likewiſe it is inferred, that if (*Fig.* III.) a body fall from reſt at C, to A in the horizontal line A B, along any number of planes C D, D E, E A, inclined to each other any how, as at D and E, the velocity at A will be the ſame as if the body had fallen in the vertical C B; abſtracting however

 from

from the loſs of velocity that happens by its impulſes at D and E, upon the contiguous planes.

That multiplying the number of planes from C to A, till the path of the body becomes curvilinear, the velocity at A will be accurately the ſame as in the perpendicular fall C B.

And laſtly, that if a ſeries of planes, *c d*, *d e*, &c. ſimilar and ſimilarly ſituated to the former, or two ſimilar and ſimilarly ſituated arcs of a curve, be the path of the body; the velocities will be as the lengths of the paths; and the times in the ſubduplicate ratio of thoſe lengths, of the heights C B, *c b*, or of any two homologous lines belonging to the figures.

Corol. 4. Let A D (*Fig.* IV.) be the diameter of a circle touching the horizontal line in A; C A, *c a*, any two chords drawn to A. Then, if bodies deſcend by the force of gravity along theſe chords, the times of deſcent will be equal; and the velocities will be proportional to the chords C A, *c* A.

For, joining D C, D *c*, and making C E, *c e*, perpendiculars to the diameter; becauſe the triangles D C A, E C A are ſimilar, as alſo D *c* A, *e c* A; it is eaſily ſhewn, that C A is to *c* A in the ſubduplicate ratio of the elevations A E, A *e*: and this compounded with the ſame ratio inverted, gives the ratio of equality; which, by *Corol.* 1. is that of the times.

And, by *Corol.* 2. the velocities are in the ſubduplicate ratio of AE to A *e*, or that of C A to *c* A.

I. *Of*

I. *Of the motion of projectiles.*

PROPOSITION I. *Fig.* V.

The line described by a heavy body, thrown in any direction not perpendicular to the horizon, is a parabola.

Suppose a body projected in the direction A D, with the velocity it would have acquired by falling from B to A, the body, by that force alone acting upon it, would uniformly describe the right line A D; and any part of the line of direction, as A H, represents the time in which it would be described.

Suppose that the force of gravity, acting alone, would have, in the same time, carried the body from A to P; compleat the parallelogram A P M H, and, at the end of the time represented by A H, the body will actually be found in M. Since, by the first *Corollary* of the first *Lemma*, the time in which the body falls from B to A is the same in which it would describe 2 A B by an uniform motion, with a velocity equal to that acquired at A, therefore that time will be represented by 2 A B. But the time in which the body would fall from A to P being represented by A H, it follows, from the second *Corollary* of the same *Lemma*, that $AP : AB :: AH^2 : 4AB^2$, and $4AB \times AP = AH^2 = PM^2$: from which it appears that the point M is a point in the *Parabola* whose diameter is A P and vertex A, having the parameter of that diameter equal to 4 A B.

Corol. 1. It is evident that the line A H is a tangent to the *Parabola* in A, because it is parallel to the ordinate P M.

Corol. 2. Since 4 A B is the parameter of the diameter A P, it follows that the parameters belonging to the vertex A of the diameter A P are always in the duplicate ratio of the velocities of the projection, the ſpace A B being always as the ſquare of the velocity acquired by falling from B to A. It follows alſo that the parameter of A P is the ſame when the velocity of the projection is the ſame, whatever the direction A H of the projectile be.

Corol. 3. If from A as centre you deſcribe the ſemicircle B Q L, its circumference ſhall be the *locus* of all the foci of the parabolas that can be deſcribed by a projectile thrown from A, with the velocity it could acquire falling from B to A: for, by a known property of the parabola, the diſtance of the focus from A is always equal to $\frac{1}{4}$ of the parameter of the diameter that paſſes thro' A: that is, to $\frac{1}{4}$ of 4 A B or to A B itſelf; all the foci muſt therefore be found in the ſemicircle B Q L.

Corol. 4. Hence it is eaſy to determine the parabola deſcribed when the direction of the projectile is given; for you need only draw A F ſo as to make the angle F A D equal to the given one D A B, which the direction A D makes with the perpendicular A B, and the point F where A F cuts the ſemicircle B Q L ſhall be the focus required; and, if you draw thro' F the line F N parallel to A B cutting the *directrix* B E in N, it ſhall be the axis, and I, the middle point betwixt F and N, ſhall be the vertex of the parabola, 4 F I being the parameter of the axis.

Corol. 5. If you draw a line thro' the vertex I parallel to the *directrix*, meeting A B in C it muſt be biſected by the line of direction in D; and if you draw

draw a line from the focus F, to D, it will be perpendicular to the tangent, and will paſs thro' B if produced, as appears from the properties of the parabola: and therefore a ſemicircle deſcribed upon A B as diameter will always paſs thro' the point D, where the line of direction cuts C I the tangent to the vertex of the parabola.

Definition. If you draw a line thro' the point A, parallel to the horizon, cutting the axis in O and the parabola in K, then A K is called the *amplitude* of the parabola.

PROPOSITION II.

The amplitude of any parabola is always equal to four times the ſine of double the angle which the line of direction makes with the vertical, taking the half of A B *for radius.*

For A K = 2 A O = 2 C I = 4 C D; but A K is the amplitude of the parabola, and C D is the ſine of the angle D G B, which is the double of B A D, if you take G B ($=\frac{1}{2}$ A B) for radius.

Therefore the amplitude is equal to 4 times the ſine of double the angle B A D, which the vertical makes with the line of direction.

Corol. 1. The velocity of projection being given, the amplitudes are to one another as the ſines of double the angles of inclination.

Corol. 2. If the angle B A D does not exceed 45°, then it is plain that the more acute that angle is, the amplitude A K muſt be the leſs; ſince the ſine of double that angle muſt become leſs, and the amplitude is equal to four times the ſine.

When the angle B A D vaniſhes, then the parabola A I K coincides with the ſtreight line A B; and the projectile, inſtead of deſcribing a curve, will only riſe to B and fall again to A.

On the other hand, the more the angle B A D approaches to 45°, the line C D, which is the ſine of double that angle, becomes the greater: and therefore the amplitude A K, which is quadruple of that ſine, muſt alſo become the greater.

Corol. 3. When the angle B A D becomes 45°, the points F and O ſhall fall on the point Q, where the ſemicircle B Q L cuts the horizontal line A K; the ſine C D of double B A D becomes now the ſine of 90°, and therefore is equal to the radius G A.

But ſince the radius is the greateſt ſine, it is plain that now the amplitude A K is the greateſt that can be deſcribed by any projectile thrown from A with a velocity which it would have acquired by falling from B to A: and this greateſt amplitude is always double of B A; for A K in this caſe is equal to 4 A G = 2 A B. Hence it appears, that if you throw a body in a direction that makes an angle of 45° with the horizon, it will be carried farther on the horizontal line, than if you threw it with the ſame force in any other direction.

Corol. 4. When the angle B A D is greater than 45°, then according as it approaches to a right angle, the parabola becomes more and more open, but the amplitudes A K decreaſe as the angle B A D increaſes; for

for A K=4 C D, and C D muſt, in this caſe, decreaſe according as B A D increaſes.

If of two directions A D and A *d*, the elevation of the one exceeds that of 45° as much as the elevation of the other wants of it, their amplitudes will be equal; for the ſines of double theſe angles muſt be equal, becauſe they are ſupplements to two right angles, to one another: but the amplitudes of the parabola are always quadruple of theſe ſines, and therefore they muſt alſo be equal to one another. That the doubles of theſe angles are ſupplements to one another appears thus: let their difference from 45° be called A, and the greater ſhall be 45°+A, the leſſer 45°—A, their doubles ſhall be 90°+2 A and 90°—2 A, which are ſupplements to each other becauſe together they make up 180°.

Corol. 5. When the angle B A D becomes a right angle, then A B becomes the axis, and A the vertex of the parabola, C D vaniſhes, and A K becomes =0.

Corol. 6. When the angle B A D becomes greater than a right one, then the curve deſcribed ſhall be only a portion of the parabola that we have conſidered in the preceding corollaries, lying on the other ſide of A.

Corol. 7. If there is given the *impetus* or velocity wherewith the projectile is thrown, and the angle of elevation, or its complement B A D, you may find the amplitude A K, and the altitude of the parabola deſcribed by this projection. For ſeeing the amplitude of 45° is 2 A B (which is the line that always expreſſes the velocity, ſince by falling thro' it the velocity is acquired) you may ſay as the radius (or ſine

ſine of 90°) is to the ſine of double the angle BAD, ſo is 2 AB to AK the amplitude ſought, (by *Cor.* 1.): the amplitude being found, you may find the altitude by ſaying, as the radius is to the tangent of the angle of elevation, ſo is CD ($=\frac{1}{4}$AK) to AC the altitude ſought.

Corol. 8. If you have given, the amplitude AK, and the angle of elevation DAK, you may find the impetus neceſſary to deſcribe a parabola that ſhall have that amplitude, by this proportion; as the ſine of double the angle of elevation, is to the radius, ſo is one half of the given amplitude to AB, the ſpace thro' which a body muſt fall to acquire the neceſſary impetus.

Corol. 9. If the impetus and amplitude be given, the direction may be found by this rule. Firſt find AB, by falling thro' which the given impetus may be acquired; then ſay, as the double of this line to the given amplitude, ſo is the radius, to the ſine of double the angle of elevation, and this angle or its complement will ſatisfy the problem.

PROPOSITION III. *Fig.* VI.

A projectile thrown in the direction AE, *with the velocity it would acquire by falling from* B *to* A, *will ſtrike any line* AN *in* K, *ſo that* AK *ſhall be equal to* 4CD: *ſuppoſing* AG *perpendicular to the line* AN, *the angle* GBA=GAB, *and that the circle deſcribed from* G *as centre, with the radius* GA, *cuts the direction* AE *in* D, *and that* DC *is parallel to* AN, *meeting* AB *in* C.

For it is plain that the angle ADC (=DAK) =DBA, by *Eucl.* 32. 3. and that conſequently the tri-

triangles A D C, A D B are ſimilar, having the angle at A common, and the angle A D C = A B D; therefore A C : A D :: A D : A B :: (becauſe of the ſimilar triangles A C D, P A K) A P : P K :: (by the property of the parabola) P K : 4 A B, therefore A D = $\frac{1}{4}$ P K, and conſequently C D = $\frac{1}{4}$ A K, or A K = 4 C D.

Corol. 1. Draw thro' D a parallel to A B meeting the circle in *d*, and draw A *d*; then will the projectile thrown in the direction A *d* ſtrike the line A N in the ſame point K; for C D = *c d*.

Corol. 2. Let H L, parallel to A B, touch the circle in H, then ſhall A H be the direction which will carry the projectile fartheſt on the line A N; becauſe when D comes to H, then C D is the greateſt it can poſſibly be, and conſequently A K (= 4 C D) is then the greateſt diſtance the projectile can be carried to, on the line A N, by the velocity acquired by falling from B to A. But it is plain that the angle H A N = H B A = H A B, therefore the direction A H biſects the angle B A N which the line A N makes with the vertical A B.

Corol. 3. The lines A D, A *d*, make equal angles with A H, alſo the angle D A N = *d* A B; and when theſe angles are equal the diſtance A K is the ſame.

Corol. 4. When A K is given and the direction is required, take A R = $\frac{1}{4}$ of A K, and thro' R draw R D parallel to A B, meeting the circle in D and *d*; then draw A D, A *d*; and theſe will be the directions *.

* See more on this ſubject in Mr. *Gray*'s treatiſe of *Gunnery*, *London* 1731.

II. *Of the cycloid; and the motion of a pendulum in it.*

Definitions. If the circle C D H (*Fig.* VII.) roll on the given ſtreight line AB, ſo that all the parts of the circumference be applied to it one after another, the point C that touched the line A B in A, by a motion thus compounded of a circular and rectilineal motion, will deſcribe the curve line A C E B which is called the *cycloid.* The ſtreight line A B is called the *baſe*; the line E F perpendicular to A B, biſecting it in F, the *axis*; and the point E the *vertex* of the cycloid. The circle by whoſe revolution the curve line is deſcribed, is called the generating circle. The line CK parallel to the baſe A B, meeting the circle in C and the axis in K, is called an *ordinate* to the axis; and a line meeting the curve in one point, that produced does not fall within the curve, is called a *tangent* to the curve in that point.

PROPOSITION I.

On the axis E F *deſcribe the generating circle* E G F, *meeting the ordinate* C K *in* G; *and the ordinate will be equal to the ſum of the arc* E G *and its right ſine* G K; *I ſay,* CK=EG+GK.

It is plain, from the definition, that the line A B is equal to the whole circumference of the generating circle, and therefore A F muſt be equal to the ſemicircumference E G F. It is alſo obvious, from the deſcription of the curve, that the arc C D is equal to the line A D, and conſequently the arc C H equal to D F or I K or C G; but the arc CH is equal to the arc E G; therefore C G is equal to the arc E G, and the

the ordinate CK (=CG+GK) muſt be equal to the ſum of the arc EG and the right line GK.

PROPOSITION II.

The line CH, *parallel to the chord* EG, *is a tangent to the cycloid in* C.

Draw an ordinate ck very near CK, meeting the curve in c, the circle in g, and the axis in k: let Cu and Gn, parallel to the axis, meet the ordinate ck in u and n; and from O the centre of the circle EGF, draw the radius OG. Since ck=Eg+gk, therefore cu=Gg+gn; and if you ſuppoſe the ordinate ck to approach to the ordinate CK, and at length to coincide with it, as Gg and Gn vaniſh, the triangles Ggn and GOK become ſimilar, whence Gg:gn:: OG:OK, and Gg+gn:gn::OG+OK(=FK):OK; but Gn:gn::GK:OK, therefore Gg+gn:Gn:: FK:GK::GK:EK; and conſequently cu:Cu:: GK:EK; and if you draw the chord Cc, the triangles Cuc, EKG will be ſimilar; ſo that the chord Cc, as the points C and c coincide, becomes parallel to EG: therefore the tangent of the cycloid at C is parallel to EG.

PROPOSITION III.

The arc of the cycloid EL *is double of the chord* EM *of the correſponding arc of the generating circle* EMF.

Let KL and kS be two very near ordinates of the cycloid, meeting the generating circle in M and Q; produce the chord EM till it meet the ordinate kS in P; let Qo be the perpendicular from Q on MP; then draw the lines EN and MN, touching the circle in E and M.

Becauſe

Because the triangles E N M, P Q M are similar, and EN=NM, therefore P Q is equal to Q M; and the triangle P Q M being isosceles, the perpendicular Q *o* bisects the base PM; so that M P is double of M *o*: but, by the last proposition, L S is parallel, and consequently equal, to M P, and L S is equal to 2 M *o*. The line L S is the increment of the curve E L, generated in the same time that the chord E M increases by M *o*, since E Q is equal to E *o*, when the points Q and M come together: Therefore the curve increases with double the velocity that the chord increases; and since they begin, at E, to increase together, the arc of the cycloid E L will be always double of the chord E M.

Corol. The semi-cycloid E L B is equal to twice the diameter of the generating circle, E F; and the whole cycloid A C E B is quadruple of the diameter E F.

PROPOSITION IV.

Let E R *be parallel to the base* A B, *and* C R *parallel to the axis of the cycloid; and the space* E C R, *bounded by the arc of the cycloid* E C *and the lines* E R *and* R C, *shall be equal to the circular area* E G K.

Draw *c r* parallel to CR; and since *c u* : C *u* : : G K : E K; therefore E K × *c u* = G K × C *u*, and consequently R *r* × C R = G K × K *k*: therefore the little space C R *r c* = G K *kg*. So that the areas E C R, E G K increase by equal increments; and since they begin to flow together, therefore they must be equal.

Corol.

Corol. 1. Let A T, perpendicular to the baſe A B, meet E R in T, and the ſpace E T A C E will be equal to the ſemicircle E G F.

Corol. 2. Since A F is equal to the ſemicircumference E G F, the rectangle E F A T, being the rectangle of the diameter and ſemicircumference, will be equal to four times the ſemicircle E G F: and therefore the area E C A F E will be equal to three times the area of the generating ſemicircle E G F.

Corol. 3. If you draw the line E A, the area intercepted betwixt the cycloid E C A and the ſtreight line E A will be equal to the ſemicircle E G F; for the area E C A F E is equal to three times E G F, and the triangle E A F $=$ A F $\times \frac{1}{2}$ E F the rectangle of the ſemicircle and radius, and conſequently equal to 2 E G F; therefore their difference, the area E C A E, is equal to E G F.

PROPOSITION V.

Take E b $=$ O K, *draw* b Z *parallel to the baſe, meeting the generating circle in* X, *and the cycloid in* Z, *and join* C Z, F X: *then ſhall the area* C Z E C *be equal to the ſum of the triangles* G F K and b F X.

Draw Z d parallel to the axis E F, meeting E T produced in d, and the trapezium R C Z d will be equal to $\overline{\frac{1}{2}\,CR + \frac{1}{2}\,Zd} \times Rd =$ (becauſe Z d $=$ E b $=$ O K) $\frac{1}{2}$ O E $\times$ R d. But $Rd = RE + Ed = CK + bZ = EG + GK + EX + bX$; therefore the trapezium R C Z d is equal to the ſum of the rectangles of half the radius and the arcs E G, E X, added to their ſines G K, and b X. But the area E G F, *i. e.* the triangle E G F and the ſegment

ment cut off by the chord EG, is equal to the rectangle contained by half the radius and the ſum of the arc EG and its right ſine GK; and the area EXF conſiſting of the ſector EOX and the triangle XOF is equal to the rectangle of half the radius and the ſum of the arc EX and its right ſine *b* X; therefore the trapezium RCZ*d* is equal to the ſum of the areas EGF and EXF. By the laſt propoſition, the area ECR is equal to EGK, and EZ*d*=E*b*X; from the trapezium RCZ*d* ſubtract the areas ECR, EZ*d*, and from the areas EGF, EXF, ſubtract the areas EGK, E*b*X, and there will remain the area CZEC equal to the ſum of the triangles, GFK, *b*FX.

Corol. 1. Hence, an infinite number of ſegments of the cycloid may be aſſigned that are perfectly quadrable. For example, if the ordinate CK be ſuppoſed to cut the axis in the middle of the radius OE, then K and *b* coincide; and the area ECK becomes in that caſe equal to the triangle GKF, and E*b*Z becomes equal to F*b*X; and theſe triangles themſelves become equal.

Corol. 2. Suppoſe now that K comes to the centre O, and C comes to *i*; then becauſe OK vaniſhes, therefore E*b* vaniſhes, and the ſpace CZEC becomes in this caſe EC*i*E, which is equal to $\frac{1}{2}$ OE^2; for the triangle *b*FX in this caſe vaniſhes.

But to return from this digreſſion;

PROPOSITION VI. *Fig.* VIII.

Let ATC *be a ſemi-cycloid having its baſe* EC *parallel to the horizon, and its vertex* A *downwards: ſuppoſe a ſtring, with a pendulum, of the length of the ſemi-*

ſemi-cycloid, ſuſpended at C, *and applied to the ſemi-cycloid* CTA; *the body* P, *by its gravity, will gradually ſeparate the ſtring from the ſemi-cycloid* CTA, *and will deſcribe an equal ſemi-cycloid* APV, *having its vertex in* V, *and its axis perpendicular to the horizon.*

On the axis AE deſcribe the generating ſemicircle AGE, draw AB cutting the vertical line CV in D, and on DV, taken equal to AE, deſcribe the ſemi-circle DHV. Then, ſince the ſemi-cycloid CTA is equal to 2AE or CV, (by *Cor. Prop.* III.) therefore the body P will come to V, when the ſtring CTP comes to a vertical ſituation. Thro' T and P draw TG and PH parallel to AD, meeting the ſemicircles in G and H; and ſince the ſtreight part of the ſtring TP is equal to the curve TA to which it was applied, therefore TP=2AG=2TK, and conſequently TK and KP are equal, and the points G and H muſt be equally diſtant from the line AD: and therefore the arc AG will be equal to DH, and conſequently the angle GAD=ADH; and the chords GA, DH, are parallel. But TP, being a tangent to the cycloid in T, is parallel to GA; therefore DKPH is a parallelogram, and DK is equal to PH. But the arc AG is equal to GT, by *Prop.* I. and therefore the arc AG =AK; and ſince AD=AGE, it follows that DK or PH=GE or HV: and if PH be produced till it meet the axis in R, then ſhall the ordinate PR be equal to the ſum of the arc HV and its right ſine HR, and therefore the point P, by *Prop.* I. muſt be in a ſemi-cycloid, whoſe generating circle is DHV, its axis DV, and vertex V.

Corol. If another ſemi-cycloid equal to CTA, as C*t*B, be placed in a contrary ſituation, it is plain, that, by means of theſe ſemi-cycloids, a pendulum may be made to deſcribe the cycloid AVB in its oſcillations.

PROPOSITION VII.

Let VL, *perpendicular to* DV, *be equal to any arc of the cycloid* VML; *deſcribe with the radius* VL *the ſemicircle* LZ*l*; *and ſuppoſing the pendulum to begin an oſcillation from* L, *the velocity acquired at* M, *in the cycloid, will be as* MX *the ordinate of the circle at the correſponding point* M *in the ſtreight line* VL: *and the force by which the motion of the pendulum is accelerated in* M, *is as the arc of the cycloid* VM *that remains to be deſcribed.*

Let LR, MS be perpendiculars to the axis DV, meeting the generating circle in O and Q, and draw the chords VO, VQ: then by *Cor.* 3. *Lemma* 3. the velocity of the pendulum at M, will be the ſame as would have been acquired by a body directly falling from R to S, and the velocity acquired at V will be the ſame as would have been acquired by a body directly falling from R to V; but theſe velocities are to one another as $\sqrt[2]{RS}$ to $\sqrt[2]{RV}$, by *Cor.* 2. *Lemma* 1. and ſince $RV:SV::VO^2:VQ^2$, and $RV:RV-SV\ (=RS)::VO^2:VO^2-VQ^2::VL^2:VL^2-VM^2$ (becauſe $VL=2VO$ and $VM=2VQ$), it follows that the velocity of the pendulum acquired in M is to the velocity acquired in V, as $\sqrt[2]{VL^2-VM^2}$ to $\sqrt[2]{VL^2}$, or as MX to VZ.

The force of gravity that is ſuppoſed invariable, acting in the direction of the diameter DV, may be repreſented by DV; and may be reſolved into the two forces DQ and VQ, whereof the firſt DQ, parallel

rallel to *t* M the ſtring, ſerves only to ſtretch the ſtring, and does not at all contribute to accelerate the motion of the pendulum; it is only the force repreſented by the chord V Q that accelerates the motion of it along the curve M *m*, and is all employed to produce that effect, the direction V Q being parallel to the tangent of the cycloid at M, by *Prop.* II. But V M = 2 V Q, by *Prop.* III; therefore the force that accelerates the pendulum at M, is as the arc of the curve V M.

Corol. It is obvious from the demonſtration, that the part of the gravity which the ſtring ſuſtains in any point M, is to the whole weight of the pendulum, as the chord D Q to the diameter.

PROPOSITION VIII.

Suppoſe that the circle L Z *l* *is deſcribed by the body* X *with an uniform motion, by the velocity acquired by the pendulum in* V; *and any arc of the cycloid, as* M N, *will be deſcribed by the pendulum, in the ſame time as the arc of the circle* X Y *by that uniform motion: taking* V N, *on the ſtreight line* V L, *equal to* V N *in the cycloid, and drawing* N Y *parallel to* V Z, *meeting the circle in* Y.

Let *x m* be an ordinate very near to X M, and draw X *r* parallel to the diameter L *l*, meeting *x m* in *r*; then, ſince the triangles X *r x* and V X M are ſimilar, it follows that X *x* : M *m* (= X *r*) : : V X : M X, that is, as the velocity of the body X to that of the body M: and conſequently the ſpaces X *x* and M *m* will be deſcribed in the ſame time by theſe bodies, the times being always equal when the ſpaces are taken in the ſame ratio as the velocities. After the ſame manner, the other correſponding parts of the lines M N and x y will be deſcribed in the ſame time; and therefore the whole ſpace m n will be deſcribed in the ſame time as the arc x y.

Cor. Therefore the pendulum will oſcillate from l to v, in the ſame time as the body x will deſcribe the quadrant l z.

PROPOSITION IX.

The time of a complete oſcillation in the cycloid is to the time in which a body would fall thro' the axis of the cycloid d v, *as the circumference of a circle to its diameter.*

The time in which the ſemi-circumference l z *l* is deſcribed by the body x, is to the time in which the radius l v could be deſcribed with the ſame velocity; as the circumference of a circle, to its diameter. But the ſame time, in which the ſemi-circumference l z *l* is deſcribed by the body x, is equal to the time of the complete oſcillation l v p in the cycloid, by the *Corollary* of the laſt propoſition. The time in which a body falls from o to v, along the chord o v, is equal to the time in which l v (=2 o v) could be deſcribed by the velocity acquired at the point v, by *Cor.* 1. *Lem.* 1. and *Cor.* 3. *Lem.* 3. and the time of the fall thro' the chord o v is equal to the time of the fall thro' the diameter d v, by *Cor.* 4. *Lem.* 3. conſequently the time in which l v could be deſcribed by a velocity equal to that of the body x, is equal to the time of a fall thro' the diameter d v. It follows therefore that the time of the entire oſcillation l v p, is to the time of a fall thro' the diameter d v; as the circumference of a circle, to its diameter.

Corol.

Corol. 1. Hence the oſcillations in the cycloid are all performed in equal times; for they are all in the ſame ratio to the time in which a body falls thro' the diameter D V. If therefore a pendulum oſcillates in a cycloid, the time of the oſcillation in any arc is equal to the time of the oſcillation in the greateſt arc B V A, and the time in the leaſt arc is equal to the time in the greateſt.

Corol. 2. The cycloid may be conſidered as coinciding, in V, with any ſmall arc of a circle deſcribed from the centre C; paſſing thro' V; and the time in a ſmall arc of ſuch a circle will be equal to the time in the cycloid; and hence is underſtood why the times in very little arcs are equal, becauſe theſe little arcs may be conſidered as portions of the cycloid as well as of the circle.

Corol. 3. The time of a complete oſcillation in any little arc of a circle, is to the time in which a body would fall thro' half the radius; as the circumference of a circle, to its diameter: and ſince the latter time is half the time in which a body would fall thro' the whole diameter, or any chord, it follows that the time of an oſcillation in any little arc, is to the time in which a body would fall thro' its chord; as the ſemicircle, to the diameter.

Suppoſe N V a ſmall arc of the circle deſcribed from the centre C; then the time in the arc N V is ſo far from being equal to the time in the chord N V, even when they are ſuppoſed to be evaneſcent, that the laſt ratio of theſe times is that of the circumference of a circle to four times the diameter: and hence an error in ſeveral mechanical writers is to be corrected, who, from the equality of the evaneſcent arcs and their chords, too raſhly conclude the time of a fall of a body in any of theſe arcs equal to the time of the fall of a body in their chords.

Corol. 4. The times of the oſcillations in cycloids, or in ſmall arcs of circles, are in a ſubduplicate ratio of the lengths of the pendulums. For the time of the oſcillation in the arc L V P is in a given ratio to the time of the fall thro' D V, which time is in the ſubduplicate ratio of the ſpace D V, or of its double C V the length of the pendulum.

Corol. 5. But if the bodies that oſcillate be acted on by unequal accelerating forces, then the oſcillations will be performed in times that are to one another in the ratio compounded of the direct ſubduplicate ratio of the lengths of the pendulums, and inverſe ſubduplicate ratio of the accelerating forces: becauſe the time of the fall thro' D V is in the ſubduplicate ratio of the ſpace D V directly and of the force of gravity inverſely; and the time of the oſcillations is in a given ratio to that time. Hence it appears, that if oſcillations of unequal pendulums are performed in the ſame time, the accelerating gravities of theſe pendulums muſt be as their lengths; and thus we conclude that the force of gravity decreaſes as you go towards the equator; ſince we find that the lengths of pendulums that vibrate ſeconds are always leſs at a leſs diſtance from the equator.

Corol. 6. From this propoſition we learn how to know exactly what ſpace a falling body deſcribes in any given time: for finding, by experiment, what pendulum oſcillates in that time, the half of the length of the pendulum will be to the ſpace required, in the duplicate ratio of the diameter to the circumference; becauſe ſpaces deſcribed by a falling body, from

from the beginning of its motion, are as the squares of the times in which they are described; and the ratio of the times, in which these spaces are described, is that of the diameter to the circumference: and thus Mr. *Huygens* demonstrates that falling bodies, by their gravity only, describe 15 *Parisian* feet and 1 inch in a second of time.

Schol. That it may be understood how the time in a small arc is not the same with that in its chord, tho' the evanescent arc is equal to its chord, we may here demonstrate, that if V *k* and N *k* be two planes touching the arc N V in V and N. Tho' the evanescent chord N V be equal to the sum of these tangents V *k* and N *k*, yet the time in the chord is to the time in these tangents as 4 to 3.

By *Cor.* 1. *Lem.* 3. the time in N *k* is to the time in N V as N *k* to N V, or as 1 to 2; but *k* V being horizontal, the motion in *k* V must be uniform, and it will be described by that uniform motion in half the time the body falls from N to *k*: therefore if the time in which *k* V is described uniformly be called T, the time in which N *k* is described will be 2 T, and the time in which the chord N V will be described will be 4 T: and consequently the time in which a body would fall along the two tangents, is to the time in which it would describe the chord, as 3 to 4.

BOOK III.

Gravity demonstrated by analysis.

CHAP. I.

Of the theory of gravity as far as it appears to have been known before Sir Isaac Newton.

1. FROM experiments and observation alone, we are enabled to collect the history of nature, or describe her phænomena. By the principles of geometry and mechanics, we are enabled to carry on the *analysis* from the phænomena to the powers or causes that produce them; and, by proceeding with caution, we may be satisfied that our foundations are well laid, and that the superstructure raised upon them is secure. The first views which philosophers had of nature were no better than those of the vulgar, being the immediate suggestions of sense. But by comparing these together, examining the nature of the senses themselves, correcting and assisting them; and by a just application of geometrical and mechanical principles, the scheme of nature soon appears very different to a philosopher from that which is presented to a vulgar eye. At first sight, the surface of the earth appears of an unbounded extent, and of a most irregular form; while all the rest of the universe, the clouds, meteors, moon, sun, and stars of all sorts, appear in one concave surface bent towards the earth. This was the opinion concerning the system that most com-

commonly prevailed at firſt, while their imagination, influenced by ſuch prejudices, made men fancy that they ſaw and heard things impoſſible. Thus the *Roman* poet repreſents their army when in *Portugal* (the weſtern boundary of the great continent) as hearing the ſun enter with a hiſſing noiſe into the ocean,

Audiit herculeo ſtridentem gurgite ſolem.

LUCAN.

while other travellers have talked of a vaſt cavity in the moſt remote parts of the eaſt, from whence the ſun was heard to iſſue every morning with an unſufferable noiſe. But philoſophers ſoon diſcovered that the earth was not of an unbounded extent, but of a globular form; and that the meteors, planets, and ſtars, were not confined to one concave ſurface, but diſperſed in *ſpace* at very different diſtances; that their real magnitudes and motions are very different from their apparent ones, and are not to be deduced from the appearances in any one place, but from views taken from divers points of ſight, compared together by geometrical principles.

2. As our *analyſis* of the ſyſtem muſt be founded upon the real figures, magnitudes and motions, of the bodies of which it is compoſed; ſo we ſhall have an excellent inſtance of the method of proceeding by *analyſis* and *ſyntheſis* if we deſcribe in what manner we are enabled, from the apparent phænomena, to deduce an account of the real; without the knowledge of which our enquiries into the powers or cauſes that operate in nature muſt be doubtful or erroneous. The knowledge of the diſpoſition and motions of the celeſtial bodies muſt precede a juſt enquiry into their cauſes. The former is more ſimple, the latter more arduous; and the former will pre-

prepare the way for the latter, and ſerve to make the reader acquainted with this method (the only one by which certainty can be acquired in this ſcience) in eaſy caſes, before he proceed to thoſe of a more complicated nature. We ſhall therefore begin with the plaineſt and moſt ſimple inſtance of this kind, by ſhewing briefly how, from the phænomena, the true figure, magnitude and motions of the earth are derived; and how, theſe being eſtabliſhed, innumerable phænomena are deduced by *ſyntheſis.*

3. It is to *ſight* that our knowledge of the diſtant parts of the ſyſtem is owing, thoſe objects that are very near us falling under the obſervation of the other ſenſes only: but this ſenſe, however admirable, has its imperfections. Viſion depends upon the picture of external objects formed on the *retina*, together with a judgment of the underſtanding, acquired by habit and experience; which is ſo immediately connected with the ſenſe, that it is impoſſible, by an act of reflection, to trace it, or, when it is erroneous, ſuddenly to correct it. If viſion depended upon the picture only, then equal pictures upon the *retina* would ſuggeſt ideas of equal magnitudes of the objects; and if the ſmalleſt fly was ſo near that it could cover a diſtant mountain from it, the fly ought to appear to us to be equal to the mountain. But we have, by habit, acquired a faculty of compounding the opinion, or prejudice, formed concerning the diſtance with the apparent magnitude or bulk of the image formed on the *retina*; and this with an inconceivable quickneſs of thought, ſo that the idea or image we form to ourſelves of its magnitude is the reſult of both; an allowance being made for the greater diſtance, agreeable to the notion we have conceived of it. Hence it is eaſy to ſee how many fallacies in viſion muſt ariſe: for as

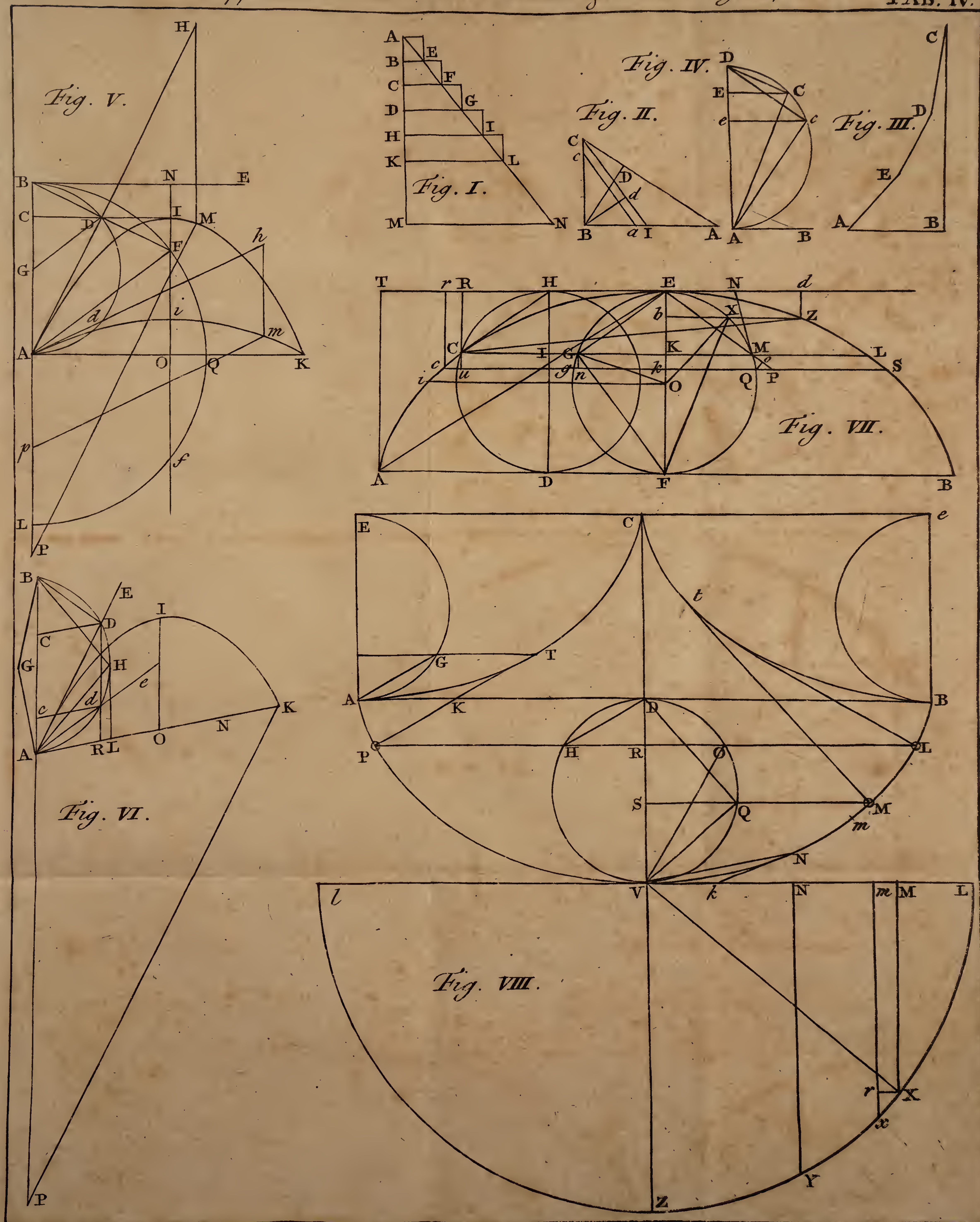


Fig. I.
Fig. II.
Fig. III.
Fig. IV.
Fig. V.
Fig. VI.
Fig. VII.
Fig. VIII.

we may be often miſtaken in our notion of diſtance, ſo every ſuch miſtake muſt produce a correſponding error in our idea of the magnitude of the object. Beſides, in many caſes, this notion of diſtance ariſes without reflexion, from the force of habit; and we find the effect of it takes place even after the underſtanding is better informed, and the judgment corrected. Thus the moon continues to appear bigger to us at the horizon than at the meridian, even after it has been demonſtrated to us that her diſtance is then greater, ſo that ſhe ought really to appear leſs. Becauſe (according to *Kepler*'s obſervation) the heavens appear to us, not in an hemiſpherical dome, but as a ſegment of a ſphere leſs than the hemiſphere, we have been accuſtomed to aſcribe a greater real magnitude to objects ſeen at a great diſtance along the horizon, than to thoſe of an equal apparent magnitude (or that have equal images on the *retina*) ſeen at a conſiderable elevation about it; and hence he ingeniouſly accounts for the moon's appearing bigger to us at the horizon than at the meridian. But after we are better informed, and know that the apparent magnitude of the moon is leſs at the horizon in the ſame proportion as the diſtance is greater, we continue to make an allowance not on this account only, but a much greater than this requires, from the great influence of habit and cuſtom *; the effect of which on the mind and its operations is a ſubject that well deſerves the particular attention of philoſophers,

* Perhaps the concave ſurface of the heavens appears to us as a portion leſs than a hemiſphere, becauſe we have been always accuſtomed to ſee greater diſtances along the horizon than in the vertical line towards the zenith. But whatever the reaſon of this appearance (ſuppoſing it true) may be, it would ſeem that an habitual way of thinking to the contrary ought to have ſome effect; and ſome obſerve that the moon never appears to them ſo large at the horizon, as it did formerly when they were young and unacquainted with her motions.

but

but is improper to be inſiſted on in this place, leſt we ſhould ſeem to mix, without neceſſity, what is obſcure and uncertain with what is clear and ſatiſfactory. For the *analyſis* we are to deſcribe, depends not on any diſputed principles, but on thoſe of practical geometry applied to the heavens.

4. Experience has taught us ſeveral ways of forming a judgment concerning the diſtances of objects, when they are not very remote from us; as by the different diſpoſition of our eyes when we look at a near object with both; it being manifeſt that when the object is near, the eyes muſt be turned more towards each other, in order that they may be directed towards the ſame point of it, than when it is at a greater diſtance. We ſoon learn from experience, likewiſe, that when the object is very near, the image is obſcure and confuſed, and we are obliged to ſtrain the eye to render it tolerably diſtinct. The image is alſo found to be more luminous and bright when the object is near than when it is remote. But the moſt uſual way of eſtimating the diſtance is from the intervening objects; or, when the object itſelf is of a kind with which we are well acquainted, by the bulk its image bears in the picture upon the *retina*. By theſe, and perhaps other methods, we are enabled to form ſome judgment of the diſtance of near objects *. But when they are very remote, and no objects

* A learned author, of a diſtinguiſhed character, begins an ingenious treatiſe upon this ſubject, by obſerving, "it is, I "think, agreed by all, that diſtance, of itſelf and immediately, "cannot be ſeen. For diſtance being a line directed endwiſe "to the eye, it projects only one point in the fund of the eye, "which point remains invariably the ſame, whether the diſ-"tance be longer or ſhorter." The diſtance here ſpoken of, is diſtance from the eye; and what is ſaid of it is not to be applied to diſtance in general. The apparent diſtance of two ſtars is

objects intervene, as is the caſe of the celeſtial bodies, theſe methods fail us, the ſenſe is at a loſs in comparing their diſtances together, and is unable to determine which are greater or leſs, without the aid of geometry, or ſome equivalent art. In ſuch caſes, therefore, the objects are all referred by the ſenſe to one concave ſurface. Thus the clouds, meteors, planets, and ſtars of all kinds, appear to the ſenſe in one concave ſurface of heaven, tho' there be the greateſt variety in their real diſtances. It is in theſe caſes that practical geometry brings us its neceſſary and ſure aid. By it we ſoon find that the clouds are not only nearer us than the celeſtial bodies, which they often cover from us, but that their diſtance is only of a few miles; a ſmall change of the place producing a great change in their poſition with reſpect to us, while thoſe that are ſeen by us at one place are different in poſition from thoſe that are ſeen at the ſame time in places remote from it. We ſoon perceive that the moon is at a vaſtly greater diſtance; becauſe ſhe is ſeen over one half of the earth at once, and nearly in the ſame direction, or in the ſame ſituation among the fixed ſtars. We eaſily learn that the moon is at a leſs diſtance from us than the ſun, becauſe by coming between us and the ſun ſhe produces the ſolar eclipſes; and that *Venus* and

is capable of the ſame varieties as any other quantity or magnitude. Viſible magnitudes conſiſt of parts into which they may be reſolved as well as tangible magnitudes, and the proportions of the former may be aſſigned as well as of the latter; ſo that this author goes too far, when he tells us that viſible magnitudes are to be no more accounted the object of geometry than words; and when he concludes of diſtance in general, what had only been ſhewn of diſtance directed " end-wiſe to the eye;" and pretends " to demonſtrate that the ideas of ſpace, outneſs, and " things placed at a diſtance, are not, ſtrictly ſpeaking, the object of ſight; and are not otherwiſe perceived by the eye than " by the ear."

Mercury are nearer to us in their inferior conjunctions than the ſun, becauſe they are then ſeen as dark ſpots upon his diſk. If our inſtruments were abſolutely perfect, and our obſervations could be made with the utmoſt accuracy, then each celeſtial body might have its diſtance preciſely aſcertained, and the whole diſpoſition of the ſyſtem might be exactly known. But this ſubject being of the utmoſt importance in our preſent *analyſis*, it deſerves ſome farther illuſtration.

5. Let A and C (*Plate* III. *Fig*. 50.) repreſent two ſpectators, or two different ſtations of the ſame ſpectator, D the object or phænomenon whoſe diſtance is required. This object appears to the ſpectator at A in the right line A D F, and to the ſpectator at C in the right line C D E; the angle contained by which, A D C, ſhews how much the poſition of the object D varies with reſpect to the two ſpectators. When this angle is great, the diſtance A D bears not a great proportion to A C; but when this angle is very ſmall, as when the object is removed from D to H, then its diſtance from A muſt be much greater than A C the diſtance of the two ſpectators or ſtations; becauſe A C is always to A D, as the ſine of the angle A D C to the ſine of A C D, by common trigonometry. Thus when A C conſiſts of ſome miles, and D repreſents a cloud, the angle A D C is found to be conſiderable; and thence we learn that its diſtance is not very great. If E D C repreſent the right line in which the ſun ſhines, then C will repreſent the ſhadow of the cloud upon the plane A C; and the proportion of A D to A C may be determined by obſervations taken from one ſtation A. But tho' the right line A C conſiſt of hundreds of miles, if H repreſent the moon, it is found that the angle A H C is exceeding ſmall; and thence we conclude, that the diſtance of the moon

moon is not to be expreſſed but by a great number of miles.

6. Let C (*Fig.* 51.) repreſent the centre of the earth, A a place upon its ſurface, C A *e* the vertical line of this place, *d* any object or phænomenon in the zenith; A D F a tangent to the ſurface of the earth at A, the ſenſible horizon at that place. Then the object *d* being ſuppoſed to project upon the fixed ſtar *e*, when in the vertical line, to a ſpectator at A as well as at C, it will be otherwiſe when the object *d* comes to the horizon at D. For tho' the centre C, the object D and the ſtar E (abſtracting from their proper motions) be ſtill in a ſtrait line, yet D and E are no longer in a right line with A the place of the ſpectator; but while D appears to be ſet at F, the ſtar appears ſtill elevated above the horizon by the arc E F, which meaſures the angle E D F, or A D C; the ſine of which is to the radius, as C A the ſemidiameter of the earth is to C D the diſtance of the object from the centre of the earth. This angle A D C is what is called the *horizontal parallax* of the object or phænomenon, and ſhews under what angle the ſemidiameter of the earth C A would appear if viewed at the diſtance of the object C D. And to find this horizontal parallax of any object, is no more than to determine how great (or under how many *minutes* and *ſeconds*) the ſemidiameter of the earth would appear viewed at that object. Suppoſe any number of objects in the right line A F, as D, G, H; and ſpectators at each of theſe viewing the ſemidiameter of the earth C A; it will appear to them under the reſpective angles C D A, C G A, C H A, which are the reſpective parallaxes of thoſe objects, and which gradually decreaſe as their diſtances increaſe. We diſcover therefore the diſtances of thoſe objects by determining what appearance, as to bulk or apparent

parent magnitude, the earth's ſemidiameter makes at thoſe objects: and it is obvious that this method is well founded, it being manifeſt, that the diſtances at which the earth appears great to a ſpectator muſt be leſs, and that thoſe diſtances at which the earth appears ſmall to him muſt be greater. Thus to a ſpectator carried to a few hundred miles diſtance only, the earth would appear very large; to a ſpectator at the moon, the ſemidiameter of it would appear under an angle leſs than a degree; to a ſpectator at *Venus*, of much about the ſame bigneſs as *Venus* appears to us; and to a ſpectator as remote as *Jupiter* or *Saturn* it would hardly be viſible at all, unleſs his ſenſe was more acute than ours, or aſſiſted by art. And as, when the proportion of the diſtance of the ſpectator from the centre of the earth to its ſemidiameter is known, it is eaſily aſcertained how great an appearance the earth will make to that ſpectator; ſo converſely, when this appearance is determined, it is eaſy to aſſign the ſpectator's diſtance from it.

7. In this manner, menſuration is carried from the earth to the heavens; and the diſtances of the celeſtial bodies compared with ſemidiameters of the earth, and with one another. For the further illuſtration of what is of ſuch importance in aſtronomy, a ſcience that affords us ſo noble and extenſive views of nature, let us imagine a ſpectator at A viewing the immenſe expanſe around him, while a right line D L, perpendicular to A D and equal to the ſemidiameter of the earth, moves off on the right line A F from the leaſt to the greateſt diſtances; then the parallax belonging to any diſtance is nothing elſe than the angle which the ſemidiameter of the earth at that diſtance ſubtends to the ſpectator at A. Thus the parallaxes belonging to the ſeveral diſtances A D, A G,

A H,

A H, &c. are the reſpective angles D A L, G A M, H A N, &c; which meaſure the apparent magnitude of the ſemidiameter of the earth viewed, at thoſe diſtances, by a ſpectator at A. While we ſuppoſe this ſemidiameter to be carried off *in infinitum*, theſe apparent magnitudes gradually decreaſe, nearly in the ſame proportion as the diſtance increaſes. The parallaxes decreaſe in the ſame manner; and a ſcale of the one affords us a ſcale of the other. It is obvious, that, from the moment any object departs from the vertical line, it appears to a ſpectator at A depreſſed towards the horizon, and is the more depreſſed in proportion as it is nearer to him. The true place of the object D is at E, where it would be ſeen from the centre C; but its apparent place to a ſpectator at A is at F, and its depreſſion or parallax is meaſured by the arc E F, or by the angle E D F equal to A D C. Now in order to find this depreſſion, it is ſufficient to make uſe of the fixed ſtar E, which has no ſenſible parallax, and was ſuppoſed to be in conjunction with the object in the vertical line A *d e*; for the depreſſion of the object D below the ſtar E, viewed from A, gives the parallax. By proceſſes of this kind, it is found, from aſtronomical obſervations, that the mean diſtance of the moon from the centre of the earth, is about 60½ ſemidiameters of the earth.

8. The figure of a body is more eaſily known when we are able to view it from great diſtances than from very ſmall ones; becauſe when it is at a great diſtance, the eye takes in a conſiderable portion of it in one view, from which the figure of the whole is more eaſily collected: whereas when it is viewed at a ſmall diſtance, ſmall irregularities on its ſurface have too great an effect upon the ſenſe, and are apt to miſlead us in our judgment concerning the whole.

It is very eaſy to ſee, for example, that the ſun and moon are globular, becauſe in all poſitions they conſtantly appear to us as bounded by a circle, a property which belongs to the ſphere or globe alone. But the figure of the earth is not ſo eaſily diſcovered by us, becauſe the largeſt views we are able to take of it, from the tops of the higheſt mountains, bear a ſmall proportion to the whole ſurface; and the curvature or ſphericity is hardly ſenſible in thoſe proſpects of it. However, we have undoubted proofs that the earth is globular, tho' not exactly ſpherical. We are aſſured that the meridian ſections of the earth, or ſections thro' its poles, are circular, becauſe as we go ſouthwards the northern ſtars are depreſſed, and the ſouthern ſtars elevated, nearly in a regular courſe; ſo that a degree of depreſſion of the former, or elevation of the latter, always correſponds to 60 *Italian* or geophraphical miles on the meridian; whence we conclude, that a meridian ſection of the earth is a circle, a degree of which is 60 ſuch miles, and the whole circumference is 60× 360, or 21600, of the ſame miles. At the equator, both the poles are in the horizon; as we remove northwards, the northern pole riſes till we come to the pole of the earth, where the celeſtial pole is in the zenith; and, in general, the elevation of the pole increaſes gradually and regularly with the diſtance from the equator. The equator and its parallels appear to be circular from the regular daily progreſs of light, from eaſt to weſt, along their ſurface. The ſun arrives at the meridian of places that are more eaſterly, ſooner than to the meridian of thoſe that are towards the weſt, in proportion to the diſtance of the meridians meaſured upon the equator. The ſpherical figure of the earth appears likewiſe from *levelling*, where it is found neceſſary to make an allowance for the difference between the apparent and

and the true level; the former being a plane that touches the earth's ſurface, the latter the globular ſurface itſelf, which falls below the tangent plane.

9. But we have the plaineſt and moſt ſimple proof of the globular figure of the earth, from that of its ſhadow projected on the moon in a lunar eclipſe. For this ſhadow being always bounded by an arc of a circle, it follows that the earth which projects it is of a ſpherical figure. If there was any remarkable angle, or very conſiderable irregular protuberance, on the earth, it would, on ſome occaſion or other, appear by the ſhadow. The mountains, indeed, are irregularities on the ſurface of the earth; but they bear ſo ſmall a proportion to its vaſt bulk, that they make no appearance upon its ſhadow. There is likewiſe a gradual riſing from the ſea ſhore towards the inland parts of the great continents; as in *Europe* from the ſhores of the ocean, the *Mediterranean*, and the *Euxine* ſea, towards *Switzerland*; but this gradual riſing is ſmall, and has little effect on the figure of the earth. If it was conſiderable, it would carry the inland parts too high in the atmoſphere; but it is ſufficient for giving a courſe to the rivers, and preſerving the beautiful circulation of water, ſo neceſſary to the good condition of this globe; and the extent of the continents has been probably contrived with a view to this great purpoſe. Upon the whole, the earth is evidently globular tho' not an exact ſphere, and if ſeen at a diſtance would appear to us as the ſun or moon; that is, always terminated by a circular figure, unleſs this diſtance was ſo great as to make it appear like *Venus* or *Mars*; when, in conſequence of the contraction of the apparent diameter, the whole ſurface would appear to be crouded in one point, and the *Alps*, *Pyrenees*, and even the diſtant *Cordelleras*, would reflect undiſtin-

guiſhed rays. At ſuch diſtances its figure could not be diſcerned by ſenſe, unleſs it was aſſiſted by a teleſcope or ſome equivalent inſtrument.

10. The ocean, which covers a great part of the ſurface of the earth, is more accurately globular than the ſolid parts; and it is manifeſt that this ariſes from the gravitation of its parts towards the earth, acting in right lines perpendicular to its ſurface. For if its direction formed an acute angle with the ſurface, the fluid water would neceſſarily move towards that ſide, and could not be in *æquilibrio* till the direction o gravity became perpendicular to the ſurface every where, ſo as to give no inclination to the fluid to move towards either ſide. The perpendiculars to a ſpherical ſurface meet all in the centre of the ſphere. Therefore, ſince the earth is nearly a ſphere, the di rection of the gravity is nearly towards it centre not as if there was really any virtue or charm in th point called the centre, by which it attracted bodies but becauſe this is the reſult of the gravitation o bodies towards all the parts of which the earth con ſiſts; as will appear more fully afterwards. Th direction of gravity is not any one fixed or deter mined one, as the vulgar are apt to imagine; nor i there any occaſion for pillars or inſtruments of an kind to ſupport the earth; that direction being a ways downwards which is towards the centre, or (t ſpeak more accurately) which is perpendicular to th fluid ſurface or level, on the concave ſide; and tha direction being upwards which lies in a perpendicula to the ſurface on the convex ſide. Was the eart all fluid, all the ſurface would be on one level, an no one part would have a pre-eminence above th reſt in this reſpect; and bodies would be ſuſtaine by the earth equally round all its ſurface with equa firmneſs and ſecurity. Thus there is no difficult

i

in conceiving that there are *Antipodes* ; and it appears equally abſurd that bodies ſhould fall off from any other part of the earth, as that they ſhould riſe here into the air.

11. This principle of gravity extends to all bodies around the earth. For the gravity of the air being eſtabliſhed beyond all diſpute, by the celebrated experiments of *Galileo* and *Torricelli*, and many others of the ſame kind, it eaſily appears that all terreſtrial bodies whatſoever are heavy, or gravitate towards the earth ; and that the apparent levity of ſome of them proceeds only from the greater gravity of the ambient air, which makes them riſe upwards, for the ſame reaſon that cork riſes in water, and lead in quick-ſilver ; or from their being carried off by ſome medium entangled in its parts. The gravity of terreſtrial bodies muſt the rather be allowed to be univerſal, becauſe, by the moſt accurate experiments, it is always found to obſerve the ſame proportion as their quantities of matter ; and not to depend on the figure or bulk of bodies, or the contexture of their parts, but always to meaſure their quantity of matter, and to be meaſured by it only, abſtracting from the influence of the medium in which they ſwim. For gravity always generates the ſame velocity, in bodies of all ſorts, in the ſame time ; and therefore muſt act equally on equal portions of matter, and on a greater portion with a force proportionally greater. The direction of this power is nearly towards the centre of the earth ; for, at preſent, we abſtract from the variation of its figure from that of a perfect ſphere, ariſing from its motion on its axis. The force of this power is ſuch, that it carries all bodies downwards about $15\frac{1}{12}$ feet, of *Paris* meaſure, in a ſecond of time. This is the reſult of accurate experiments ; every body would fall juſt ſo

much if it defcended freely in the plump-line, or perpendicular to the horizon, and met with no refiftance from the air or ambient medium. When a body is projected in a right line that is not perpendicular to the horizon, it moves in a curve, but fo as to fall always below the point in the line of projection which is directly over it, as much as it would have fallen by defcending freely in the perpendicular in the fame time; provided we fuppofe gravity to act in parallel lines, as was ufual before Sir *Ifaac Newton* found it neceffary to confider this fubject more accurately, and which may be admitted, without any fenfible error, in fuch motions as our engines are able to produce.

12. The globular figure of the earth, with the direction and force of gravity, being difcovered by this *analyfis*, a great variety of phænomena may be thence deduced by the *Synthetic* method. The whole doctrine of the fphere may be explained from the figure of the earth, either in the *Pythagorean* or *Ptolemaic* fyftem. As the fun appears to go round the whole circle of 360 degrees in 24 hours, fo in one hour he appears to defcribe 15 degrees, and one degree in 4 minutes of time, on the equator or its parallels. Hence the diftance of meridians at two places, meafured upon the equator, or their difference of longitude, being known, it is eafy to compute how much the hours at one place precede the fame hours at the other, by allowing 4 minutes of time for each degree of that diftance; and converfely, the difference of time being given, the difference of longitude is computed by allowing one degree for each 4 minutes of time, and proportionally in greater or leffer differences. And it is obvious that the hours of the day, which are fucceffive in any one place, are co-exiftent when you take in the whole globe;

globe; ſo that no hour of the day can be aſſigned, but a meridian can be likewiſe aſſigned where it is that hour at this preſent time. The *ſenſible horizon* of any place is a plane perpendicular to the plumb-line at that place, and tangent to the earth's ſurface there. The *rational horizon* is a plane thro' the earth's centre parallel to this, whoſe poles are the *zenith* and *nadir*, in the ſame manner as the north and ſouth poles of the world are the poles of the equator. The particular phænomena of places depend upon the poſition of their horizon with reſpect to the circles of the apparent diurnal motion of the ſun and ſtars. The horizon of a place at the equator paſſes thro' the poles, and divides equally the equator and its parallels. Hence the days and nights are always equal in ſuch places, and each of the ſtars performs one half of its revolution above their horizon, and the other half under it. The circles of diurnal motion are all perpendicular to their horizon, and therefore they are ſaid to be in a *right ſphere*. When the ſun moves in the equator, he riſes directly from their horizon to their zenith, and then deſcends directly to their horizon again; in other caſes, after riſing perpendicularly, he ſlopes away in his parallel towards the north or ſouth ſide of their zenith, according to the ſeaſon of the year; which muſt be a conſiderable relief to them, as the heat muſt thereby be abated. At the poles, their horizon coincides with the equator; ſo that the northern celeſtial hemiſphere muſt be always in view of the northern pole, being above their horizon, while no part of the ſouthern hemiſphere is viſible to them, being always beneath it. The circles of the diurnal motion being parallel to the æquator, and conſequently to their horizon, the ſun and ſtars appear to them to move in parallels to their horizon; the fixed ſtars never riſe nor ſet, and the ſun riſes at the vernal equinox and ſets at the autumnal; ſo that they have day for one half year and night for the other. They are ſaid to be under a *parallel ſphere.* In intermediate places, the circles of the diurnal motion are oblique to their horizon; one pole is always elevated above it by an arc equal to the latitude of the place, and the other pole is depreſſed under it by an equal arc. All the ſtars whoſe diſtance from the elevated pole exceeds not the latitude of the place are conſtantly above their horizon; and thoſe within the ſame diſtance of the other pole are depreſſed under it, and are never viſible to them. The equator and horizon being great circles divide each other equally, whence the days and nights are equal every where when the ſun deſcribes the celeſtial equator. But when the ſun is on the ſame ſide with the elevated pole, a greater portion of his parallel is above the horizon than under it, and therefore the days are longer than the nights: and when the ſun is on the other ſide of the equator, a greater portion of his diurnal parallel is below the horizon than above it; and conſequently the nights are longer than the days. Theſe are ſaid to be under an *oblique ſphere.* In all thoſe different places, the time in which they have day (that is, when the centre of the ſun is above the horizon) is equal to the time in which they have night, or when the centre of the ſun is beneath their horizon, taking the whole year together; abſtracting from the effects of refraction and the elliptic figure of the earth's orbit, which are not conſidered in the doctrine of the ſphere. But theſe equal times are diſtributed with a good deal of variety. At the equator they have 12 hours day and 12 hours night, perpetually ſucceeding each other. At the poles they have their day all at once and their night at once, each of half a year. In intermediate places, the

the length of their days at one ſeaſon is compenſated by the length of the nights at another. Within the polar circles, they have the ſun continually for ſome days, or weeks, circulating above their horizon; but, in the oppoſite ſeaſon of the year, he continues as long beneath their horizon; and thus the equality of the times of day and night is preſerved, when we abſtract from the ſun's having a ſenſible diameter, from the effects of refraction and twilight, and the elliptic figure of the earth's orbit; but, in conſequence of theſe, the time in which they have day conſiderably exceeds what is commonly called night, particularly in the northern hemiſphere. The *amplitude* of the ſun, or his range upon the horizon, has likewiſe great varieties, which are eaſily deduced from the ſame principles. It is leaſt at the equator, amounting there to 23° 29′ on each ſide, towards the north and ſouth of the eaſt and weſt points. In the latitude of 56° it amounts to above 45°, on each ſide of the ſame points; and the arc between the moſt northern and ſouthern points where he riſes, and ſets, is above a quadrant. At the polar circles, his range on the horizon is the whole ſemicircle from north to ſouth. A circle perpendicular to the meridian and horizon is called the *prime vertical*, and, being a great circle, it cuts the equator equally, and all places that are under it bear due eaſt or weſt from us; whence many of the geographical paradoxes are explained. The art of *dialling* is deduced from the ſame principles. The moſt ſimple kind of dial is an equinoctial one, where the ſhadow is received upon a plane parallel to the circles of the ſun's diurnal motion, and is projected by a *ſtylus*, or right line, perpendicular to thoſe planes. Becauſe the ſun moves over equal arcs on its parallel in equal times, the motion of the ſhadow in this dial muſt likewiſe be uniform, ſo that the intervals between the hours

muſt

muſt be equal; which is therefore made by dividing a circle into 24 equal parts. The conſtruction of other dials is eaſily deduced from this: but our deſign obliges us to mention theſe things very briefly. We have a remarkable inſtance of the beauty of truth when we obſerve what a variety of phænomena ariſe from ſo few ſimple principles as the ſpherical figure of the earth, its diurnal motion, and the obliquity of its axis, as we take a ſurvey of the earth from the torrid to the frigid zone, or from the equator to the poles, and attend to the phænomena of heat and cold, as well as of thoſe of day and night, and of the apparent motions of the ſtars. A diverſity of phænomena ſo very great, ariſing from two principles of ſo ſimple a nature, affords a curious ſpeculation to the underſtanding, as well as a pleaſing entertainment to the imagination, and ſerves to ſuggeſt the admirable fertility of which nature is capable in its productions; inſomuch that upon one globe we have ſome image or repreſentation, in the climates from the equator to the poles, of that great variety that we may ſuppoſe to take place in the ſolar ſyſtem, from *Mercury*, the neareſt and hotteſt, to *Saturn*, the remoteſt and coldeſt of all the planets.

13. Tho' the doctrine of the ſphere may be explained from the *Ptolemaic*, as well as from the *Pythagorean* or *Copernican* ſyſtem, by ſuppoſing the *primum mobile* to penetrate the whole univerſe (the earth and its appendicles only excepted) and to carry every thing round the earth's axis every day; yet this hypotheſis, to every thinking perſon who has not devoted his judgment entirely to the prejudices of ſenſe or dictates of ſuperſtition, appears ſo very abſurd, that it is now almoſt univerſally exploded. The motions of the *comets*, performed with ſo much

freedom in the celeſtial ſpaces, ſhew us that the ſolid orbs are imaginary, and that there can be no ſuch univerſal mover that carries all the univerſe along with it: nor is there any axis upon which this immenſe machine can be ſuppoſed to turn. The prodigious velocity, which, according to this doctrine, muſt be aſcribed to the remote fixed ſtars, cannot but ſhock thoſe that have any juſt notion of the vaſt extent of the univerſe. The aſcribing ſo extraordinary a pre-eminence to the earth, to which it appears to have no title, argues a partiality unworthy of philoſophers; eſpecially ſince we ſee that moſt of the other bodies of the ſyſtem, even the ſun himſelf, turn round upon their axes, which would induce us, if we were upon the ſurface of any of them, to aſcribe the ſame pre-eminence to that one, and to place it in the centre of the whole. But beſides theſe and other conſiderations, the retardation of pendulums carried to the equator, with the increaſe of the degrees of the meridian from thence to the poles, are obſervations that demonſtrate a centrifugal force, greateſt at the equator, and gradually diminiſhing towards either pole, where it vaniſhes. Now this centrifugal force is an evident proof of the diurnal rotation of the earth upon its axis; therefore, in treating of the celeſtial motions, we ſhall entirely abſtract from the apparent diurnal motions of the planets, as pertaining to the earth only: and thus our *analyſis* of the cauſes that produce the celeſtial motions is founded on the real ſtate of things, and not on fallacious appearances.

14. The doctrine of the ſphere is eaſily deduced from theſe true motions. One half of the earth is illuminated by the ſun at all times, and the other half always deprived of his light. The boundary of light and darkneſs is a great circle of the earth.

It

It is day at any place while it revolves in the illuminated part, but night while it moves in the part that is hid from the sun's rays. The diurnal motion is from west to east, and the sun rises to any place when it arrives at the boundary of light and darkness on the west side, and sets when it arrives at the same boundary on the east. The point where a right line joining the centres of the sun and earth cuts the surface of the earth, is that which has the sun in the vertex or zenith, and is the pole or middle point of the illuminated disk. The circle described by the earth's annual motion, or the sun's apparent motion, is the *ecliptic*; and, because the axis of the earth is oblique to the plane of this circle, it cuts the equator (in an angle of 23° 29′), and the two points of intersection are called the equinoctial points; in which the sun appears when the axis of the earth is perpendicular to the right line drawn from its centre to the centre of the sun. Those are called the solstitial points which are at 90° distance from the former, and where the sun appears when he declines most towards the poles. The equator being a great circle, so as to be equally divided by the boundary of light and darkness, the day therefore at the equator is always equal to the night. It is obvious that when the sun appears on the north side of the equator, the northern pole must be in the illumined hemisphere; so that it must be day there from the vernal to the autumnal equinox, but that they must be deprived of the sun's light from the autumnal to the vernal equinox; and that it is the contrary at the south pole. In any place that is on the same side of the equator with that which has the sun in the zenith, a greater part of the parallel to the equator described by that place must be in the illuminated hemisphere than in the other; so that the day must be longer than the night: but it is the contrary when the place

is

is on the oppoſite ſide of the equator, and then the night muſt be longer than the day. In the ſame manner, all the other phænomena of the doctrine of the ſphere may be deduced from the true motions in the ſyſtem.

15. We have given a ſummary account of what was known concerning the gravity of terreſtrial bodies, before Sir *Iſaac Newton*. As the figure of the earth is owing to this principle; ſo, as *Copernicus* very juſtly obſerved *, it is highly reaſonable to ſuppoſe that by a like principle, diffuſed from the ſun and planets, their figures are preſerved in their various motions. Various attempts and ſchemes have been propoſed, for explaining the nature of this power and its cauſe; but all have proved unſucceſsful. *Des Cartes* deduced it from the centrifugal force of his ſubtile matter revolving on the axis of the earth; but this account has been already refuted †. Others conſidered it as a ſort of magnetiſm; but the powers of gravity and magnetiſm differ widely in moſt eſſential circumſtances. Others derived it from the preſſure of the atmoſphere; altho' the air is ſo far from producing gravity, that it conſtantly ſubducts from the weight of bodies. But all we want to conclude here, is, that this power extends univerſally to all ſorts of ſenſible bodies, at or near the earth's ſurface; and that it has theſe two remarkable properties; firſt, that it is proportional to the quantity of matter in bodies; ſecondly, that it acts inceſſantly or continually, and with the ſame force upon a body that is already in motion as upon a body that is at reſt. The laſt property appears from hence, that it produces equal accelerations in

* See Book I. Chap. 3. § 2.
† See Book I. Chap. 4. § 4.

falling

falling bodies in equal times. Both these properties distinguish it from such causes as are wholly mechanical; which either act in proportion to the surface or to the bulk of bodies, and produce a less acceleration in a body that is already in motion, in the direction in which the cause acts, than upon a body at rest, in the same time. We here observe these things concerning gravity, not with a view to determine any thing concerning its cause, but only to pave the way for what follows concerning the universality of this principle.

CHAP. II.

The moon is a heavy body, and gravitates towards the earth in the same manner as terrestrial bodies.

1. SIR *Isaac Newton* considering that the power of gravity acts equally on all matter on the surface of the earth or near it, that it is not sensibly less on the tops of the highest mountains, that it affects the air and reaches upward to the utmost limits of the atmosphere, and that it cannot be owing to the influence of any sensible terrestrial matter; he could not believe that it broke off abruptly, but was induced, on these grounds, to think it might be a more general principle, and extend to the heavens; so as to affect the moon at least, which is by much the nearest to us of all the bodies in the system. The absurdity of those who had taught that the heavenly bodies were made of some inexplicable substance, essentially different from that of our earth, had sufficiently appeared from modern discoveries: the philosophers no longer made that distinction, which had been founded on superstition and vulgar prejudices only. The earth was allowed to be of the number

number of the planets, and the planets were confidered as like our earth. To complete this refemblance, our author has fhewn that they confift of the fame heavy gravitating fubftance of which the earth is formed.

2. The effects of the power of gravity upon terreftrial bodies may be reduced to three claffes: *Firft*, in confequence of it, a body at reft, fupported by the ground, or fufpended by a ftring or line of any kind, or that is any way kept from falling, endeavours, however, always to move; and in fuch cafes, its gravity is meafured by the preffure of the quiefcent body upon the obftacle that hinders its motion. *Secondly*, when a body defcends in the vertical or plumb-line, its motion is continually accelerated, in confequence of the power of gravity's acting inceffantly upon it; or if it be projected upwards in the fame right line, its motion is continually retarded, in confequence of the fame power's acting inceffantly upon it with a contrary direction: and, in fuch cafes, the force of gravity is meafured by the acceleration or retardation of the motion produced in a given time, by the power continued uniformly for that time: but if the body defcend or afcend along an inclined plane, or move in a refifting medium, then, in meafuring this power, due regard muft be had to the principles of mechanics defcribed in the preceding book. *Thirdly*, when a body is projected in any direction different from the vertical line, the direction of its motion is continually varied, and a curve line is defcribed, in confequence of the inceffant action of the power of gravity, which in fuch cafes is meafured by the flexure or curvature of the line defcribed by it; for the power is always the greater, *cæteris paribus*, the more it bends the way or courfe of the body from the tangent or direction

in

in which it was projected. Effects of the power of gravity, of each kind, fall under our conſtant obſervation, near the ſurface of the earth; for the ſame power which renders bodies heavy while they are at reſt, accelerates them when they deſcend perpendicularly, and bends their motion into a curve line when they are projected in any other direction than that of their gravity. But we have acceſs to judge of the powers that act on the celeſtial bodies by the effects of the laſt kind only: we ſee bodies near the earth falling towards it; but this is a proof of the moon's gravity that cannot be had, till the preſent ſtate of things comes to its diſſolution. When a body is projected in the air, we do not ſee it fall in the perpendicular towards the earth, but we ſee it falling every moment from the tangent to the curve, that is, from the direction in which it would have moved if its gravity had not acted for that moment. And this proof we have of the moon's gravity: for tho' we do not ſee her falling directly towards the earth in a right line, yet we obſerve her deſcending every moment towards the earth from the right line which was the direction of her motion at the beginning of that moment; and this is no leſs evidently a proof of her being acted upon by gravity, or ſome power like to it, than her rectilineal deſcent would be was ſhe allowed to fall freely towards the earth.

3. If we had engines of a ſufficient force, bodies might be projected from them ſo as not only to be carried a vaſt way without falling to the earth, but ſo as to move over a quarter of a great circle of it, or (abſtracting from the effects of the air's reſiſtance) ſo as to move round the whole earth without touching it, and, after returning to their firſt place, commence a new revolution with the ſame force they firſt received from the engine, and after that a third,

and thus revolve as a moon or ſatellite round the earth for ever. If this could be effected near the earth's ſurface it might be done higher in the air or even as high as the moon, could the engine, or an equivalent power, be carried up and made to act there. By increaſing the force of the power, a body proportionally larger might be thus projected: and, by a power ſufficiently great, a heavy body not inferior to the moon might be put in motion at firſt; which, being perpetually reſtrained by its gravity from going off in a right line, might revolve for ever about the earth. Thus Sir *Iſaac Newton* ſaw that the curvilineal motion of the moon in her orbit, and of any projectile at the ſurface of the earth, were phænomena of the ſame kind, and might be explained from the ſame principle extended from the earth ſo as to reach the moon; and that the moon was only a greater projectile that received its motion, in the beginning of things, from the Almighty Author of the univerſe.

4. But, to make this perfectly evident, it was neceſſary to ſhew that the powers which act on the moon, and on projectiles near the earth, and bend their motions into a curve line, were directed to the ſame centre, and agreed in the quantity of their force as well as in their direction. All we know of force relates to its direction or quantity, and a conſtant coincidence and agreement in theſe two reſpects is ſufficent ground to conclude them to be the ſame, or ſimilar, phænomena derived from the ſame, or from like cauſes. It was ſhewn in the laſt chapter, that the gravity of heavy bodies is directed towards the centre of the earth; and it appears from the obſervations of aſtronomers, that the power which acts on the moon, inceſſantly bending her motion into a curve, is directed towards the ſame centre: for they

 find

find that the moon does not deſcribe an exact circle about the earth; but an *ellipſe* or oval; and that ſhe approaches to the earth, and then recedes from it, in every revolution, but ſo as to have her motion accelerated while ſhe approaches to the centre of the earth, and retarded as ſhe recedes from it; which is an indication that ſhe is acted on by a power directed, accurately or nearly, towards the centre.

5. That this may appear more fully, let us ſuppoſe that a body is projected in any right line, and, if no new force act upon it, then muſt it proceed in that line, deſcribing equal ſpaces in equal times, by the firſt law of motion; and if you imagine a ray drawn always from the body to ſome fixed point, that is not in the line of its motion, while the body moves over equal ſpaces in equal times, that ray will deſcribe equal triangular ſpaces * in equal times;

* All the reaſoning here ſuppoſes only one propoſition very generally known, that "triangles on the ſame baſe, or on equal baſes, that have the ſame height, are equal to each other;" from which it eaſily follows, 1. That while a body by an uniform motion deſcribes the line A F, (*Fig.* 52.) and moves over the equal parts A B, B C, in equal times, the triangles deſcribed by a ray drawn always from the body to the given point S, *viz.* ASB, BSC, muſt be equal, becauſe their baſes AB, BC are equal, and they have their common vertex in S. 2. Suppoſe a force to act on the body in B, directed toward S, that would carry it to E, if it acted alone upon the body, in the ſame time in which the body by its uniform motion would deſcribe B C, and the body will now deſcribe B D the diagonal of the parallelogram B E D C in the ſame time, and the ray drawn from the body to S will deſcribe the triangle B S D equal to B S C becauſe they are on the ſame baſe B S and between the parallels BS, CD; that is, the ſpace deſcribed now by the ray is equal to the ſpace that would have been deſcribed by it if no new force had acted on the body B: from which it appears, that the ſpace deſcribed by the ray is not increaſed or diminiſhed by any action of the body directed towards S, and therefore the ray drawn from the body to S will ſtill continue to deſcribe equal ſpaces in equal times, if no new force act upon it but what is directed towards S.

becauſe

becauſe theſe triangles, deſcribed by the ray in equal times, will have equal baſes on the line of projection, and one common vertex in that fixed point. Suppoſe next that a force, directed to the ſame fixed point, acts upon the body, and it will now be carried out of the firſt line of its motion into a new direction, but the area or ſpace deſcribed by the ray, drawn always from the body to that fixed point, will be equal to the ſpace that would have been deſcribed by the ray in the ſame time if no ſuch force had acted upon the body; for theſe ſpaces are triangles ſtanding on the ſame baſe (*viz.* the firſt diſtance of the body from that fixed point) and between the ſame parallel lines. The power, therefore, directed towards the given point has no effect on the magnitude of the area or ſpace deſcribed by the ray that is ſuppoſed to be drawn always from the body to that point; it may accelerate or retard the motion of the body, but affects not the area. Therefore the ray muſt ſtill continue to deſcribe the ſame ſpaces in equal times about the given point, as it would have done if no new force had acted on the body, but it had been permitted to proceed uniformly in the line of projection.

6. As one impulſe towards the given point has no effect on the area, or ſpace, deſcribed by the ray tending always from the body to that point, ſo any number of ſucceſſive impulſes directed to the ſame point can have no effect on that area, ſo as to accelerate or retard its deſcription; and, if you ſuppoſe the power directed to that point to act continually, it will bend the way of the body's motion into a curve; and may accelerate or retard its velocity, but can never affect the area deſcribed in a given time by the ray ſuppoſed to be drawn always from the body to the given point; which therefore will

be always of an invariable quantity, equal to that which would have been deſcribed in the ſame time, if the body had proceeded uniformly in a right line, from the beginning of the motion.

7. The converſe of this theorem ſhews, that the equable increaſe of the areas deſcribed by a ray, drawn always from a body to a given point, is an indication that the direction of the power that acts upon the body, and bends its way into a curve, is directed to that point. It is eaſy to ſee, that if that power was directed to either ſide of the point *, it would increaſe or diminiſh the area deſcribed by the ray drawn from the body to the point; ſo that if equal areas continue to be deſcribed about it in equal times, we may be aſſured that the power is directed to that point. If a body deſcribe a circle with an equable motion, ſo as to move over equal arcs in equal times, the areas deſcribed in equal times by a ray drawn from the body to the centre of the circle will be equal, and it is plain that the force which bends the body into the curve muſt tend to that centre; for if it was directed to any other point, the body would be accelerated in its motion as it approached to that point, and retarded as it removed to a greater diſtance from it. We have explained this propoſition at ſome length, becauſe it is of the greateſt conſequence in this philoſophy. From it we learn, that the force which retains the moon in her orbit is directed to the centre of the earth, becauſe ſhe deſcribes, by a ray drawn to the centre of the earth, equal ſpaces in equal times, being accelerated

* If a new force acted upon the body at B, that was directed to either ſide of S, the body, inſtead of being found in the line CD, would, in the ſame time, either paſs that line or fall ſhort of it, and the area deſcribed by the ray drawn from the body would either be greater or leſs than BSC.

in her motion as ſhe approaches to the earth, and retarded as ſhe recedes from it. We ſhall, afterwards, ſee that a ſmall inequality in theſe ſpaces only ſerves to confirm our author's philoſophy.

8. There is, therefore, a power which acts on the moon, like to gravity, directed to the centre of the earth; and as this power makes her fall from the direction of her motion every moment towards the earth; ſo, if her projectile motion was deſtroyed, the ſame power would make her fall to the earth, in a direct line: and becauſe this power acts inceſſantly, bending, every moment, her way into a curve, it therefore would make her deſcend to the earth with an accelerated motion, like that of heavy bodies in their fall. It remains only to ſhew, that the power which acts on the moon agrees with gravity in the quantity of its force, as well as in all other reſpects. But, before we compare them in this particular, we are to obſerve, that the power which acts upon the moon is not the ſame at all diſtances from the earth, but is always greater when ſhe is nearer to the earth. To be ſatisfied of this, it is only neceſſary to ſee that to bend the motion of a body into a curve, when it moves with a greater velocity, requires the action of a greater power than when it deſcribes the ſame curve with a leſs velocity. This is obvious enough, but may appear more fully thus: imagine a tangent (*Fig.* 53.) drawn at the beginning of a ſmall arc deſcribed by the body, and as this is the line which the body would have followed if no new power had acted upon it, the effect of that power is eſtimated by the depreſſion of the other extremity of the arc under that tangent: now it is plain, that in arcs of the ſame curvature or flexure, the greater the arc is, the farther muſt one extremity of it fall below the tangent drawn at the other extremity;

tremity; and consequently when a body describes a greater arc, it must be acted on by a greater power than when it describes a lesser arc in the same time. Now as the moon approaches to the earth, her motion is accelerated, is swiftest at her least distance, and slowest at her greatest distance, and the arcs which she describes at her greatest and least distance have the same curvature, therefore the force which acts upon her at her least distance, when her motion is swifter, must be the greater force.

9. It will not be difficult to see according to what law this power varies, at her greatest and least distances from the earth. That it may appear more easily, let us assume a simple case, and suppose that her least distance is the half of her greatest distance. If this was true, the moon would move with a double velocity in her least distance, that the area described there by a ray from her to the earth might be equal to the area described by such a ray, in the same time, at her greatest distance; so that she would describe at her least distance an arc, in one minute, equal to the arc she would describe in two minutes at her greatest distance; and would fall as much below the tangent at the beginning of the arc, in one minute in the lower part of her orbit, or the *perigaeum*, as in two minutes in the higher part of it, or her *apogaeum*. If therefore her projectile motion was destroyed at her least distance, she would fall towards the earth as much in one minute, as in two minutes if her projectile motion was destroyed at her greatest distance. But the spaces described by a heavy body in its descent are as the squares of the times, by *Book* II. *Chap.* 1. § 11; and such a body descends thro' a quadruple space in a double time; so that the moon descending freely at her greatest distance, would necessarily fall four times as far in two minutes

nutes as in one minute. Therefore ſhe would fall thro' four times as much ſpace, in one minute, at her leaſt diſtance, as at her greateſt diſtance in the ſame time. But the forces with which heavy bodies deſcend, are in the ſame proportion as the ſpaces deſcribed, in conſequence of thoſe forces, in equal ſmall parts of time; conſequently the power that acts at the leaſt diſtance is quadruple of that which acts at the greater diſtance, when the latter is ſuppoſed to be double of the former; or the forces are as 4 to 1, when the diſtances are as 1 to 2. We find, therefore, that the force which acts upon the moon, and bends her courſe into a curvilinear orbit, increaſes as the diſtance from the centre of the earth decreaſes, ſo as to be quadruple at half the diſtance. In the ſame manner it is ſhewn, that if her leaſt diſtance was the third part only of her greateſt diſtance, her velocity would be triple at the leaſt diſtance, to preſerve the equability of the areas deſcribed by a ray drawn from her to the centre of the earth; and that ſhe would be acted upon by a power which would have the ſame effect there in one minute, as in three minutes at her greateſt diſtance; ſo that if ſhe was allowed to deſcend freely from each diſtance, ſhe would fall nine times as far from the leaſt diſtance as from the greateſt, in the ſame time; conſequently, the power itſelf which cauſes her deſcent would be nine times greater at the third part of the diſtance; or the diſtances being as 1 to 3, the force of gravity at thoſe diſtances would be as 9 to 1, that is, inverſely as the ſquares of the diſtances. In the ſame manner, it appears that when the greateſt and leaſt diſtances are ſuppoſed to be in any proportion of a greater to a leſſer number, the velocities of the revolving planet are in the inverſe ratio of the ſame numbers; and that the powers, which bend its motion into a curve, are in the inverse ratio of the squares of those numbers.

10. In general, let T (*Fig.* 53.) represent the centre of the earth, ALP the moon's elliptical orbit, A the *apogaeum*, P the *perigaeum*, AH and PK the tangents at those points, AM and PN any small arcs described by the moon in equal times, at those distances; MH, NK, the subtenses of the angles of contact, terminated by the tangents in H and K: then MH and NK will be equal to the spaces which would be described by the moon, if allowed to fall freely from the respective places A and P, in equal times; and will be in the same proportion to each other, as the powers which act upon the moon, and inflect her course, at those places. Let A *m* be taken equal to PN, and *m h*, parallel to AP, meet the tangent at A in *h*; then, because the curvature of the ellipse is the same at A as at P, *m h* is equal to KN; and, if the moon was to fall freely, from the places P and A, towards the earth, her gravity would have a greater effect at P than at A, in equal times, in proportion as *m h* is greater than MH. But *m h* is the space which the moon would describe freely by her gravity at A, in the time in which A *h* would be described by her projectile motion at A; and MH is the space thro' which she would descend freely by her gravity at A, in the time in which AH would be described by her projectile motion; and those spaces being as the squares of the times, it follows that *m h* is to MH, as the square of A *h* to the square of AH, or (because of the equality of the areas TAH, TPK) as the square of TP to the square of TA. Therefore the gravity at P is to the gravity at A, as the square of TA to the square of TP; that is, the gravity of the moon towards the earth increases in the same proportion as the square of the distance from

the

the centre of the earth decreaſes. Sir *Iſaac Newton* ſhews the univerſality of this law, in all her diſtances, from the direction of the power that acts upon her, and from the nature of the *ellipſis*, the line which ſhe deſcribes in her revolution; and it follows from the properties of this curve, that, if you take ſmall arcs deſcribed by the moon in equal times, the ſpace by which the extremity of any arc deſcends towards the earth below its tangent at the other extremity, is always greater in proportion as the ſquare of the diſtance from the focus is leſs: from which it follows that the power which is proportional to this ſpace obſerves the ſame proportion.

11. The moon's orbit, according to the obſervations of aſtronomers, differs not much from a circle of a radius equal to ſixty times the ſemi-diameter of the earth; and the circumference of her orbit, is, therefore, about ſixty times the circumference of a great circle of the earth; which, by the *French* mathematicians, was found to be 123249600 *Pariſian* feet. The circumference of the moon's orbit is eaſily computed from this; and, ſince ſhe finiſhes her revolution in 27 days, 7 hours and 43 minutes, it is eaſy to calculate what arc ſhe deſcribes in one minute. Now, to compute by what ſpace one end of this arc falls below a tangent drawn at the other end, we learn from geometry that this ſpace is nearly a third proportional to the diameter of her orbit and the arc ſhe deſcribes in a minute; and by an eaſy calculation this ſpace is found to be $15\frac{1}{12}$ *Pariſian* feet. This ſpace is deſcribed in conſequence of her gravity towards the earth, which, therefore, is a power, that, at the diſtance of ſixty ſemi-diameters of the earth, is able to make her deſcend in one minute

nute through $15\frac{1}{12}$ *Parisian* feet. This power increases as she approaches to the earth: in order to see what its force would be at the surface of the earth, let us suppose her to descend so low in her orbit as, at her least distance, to pass by the surface of the earth. She would then come sixty times nearer to the centre of the earth, and move with a velocity sixty times greater, that the areas, described by a line drawn from her to that centre in equal times, might still continue equal. The moon therefore passing by the surface of the earth, at her lowest distance, would describe an arc in one second of time (which is the sixtieth part of a minute) equal to that which she describes in a minute at her present mean distance, and would fall as much below the tangent at the beginning of that arc in a second, as she falls from the tangent at her mean distance in a minute; that is, she would fall near the surface of the earth $15\frac{1}{12}$ *Parisian* feet in one second of time. Now this is exactly the same space through which all heavy bodies are found by experience to descend by their gravity, near the surface of the earth, as we observed above. The moon, therefore, would descend at the surface of the earth with the same velocity, and every way in the same manner, as heavy bodies fall towards the earth; and the power which acts upon the moon, agreeing in direction and force with the gravity of heavy bodies, and acting incessantly every moment, as their gravity does, they must be of the same kind, and proceed from the same cause.

12. The computation may be made also after this manner: the mean distance of the moon from the earth being sixty times the distance of heavy bodies at the surface from its centre, and her gravity increasing in proportion as the square of her distance from

from the centre of the earth decreaſes, her gravity would be 60×60 times greater near the ſurface of the earth than at her preſent mean diſtance, and therefore would carry her through $60 \times 60 \times 15\frac{1}{12}$ *Pariſian* feet in a minute near the ſurface: but the ſame power would carry her through 60×60 times leſs ſpace in a ſecond than in a minute, by what has been often obſerved of the deſcent of heavy bodies; and, therefore, the moon in a ſecond of time would fall by her gravity near the ſurface of the earth $15\frac{1}{12}$ *Pariſian* feet; which therefore is the ſame with the gravity of terreſtrial bodies.

13. Thus Sir *Iſaac Newton* ſhewed that the power of gravity is extended to the moon; that ſhe is heavy, as all bodies belonging to the earth are found by perpetual experience to be; and that the moon is retained in her orbit from the ſame cauſe, in conſequence of which a ſtone, bullet, or any other projectile, deſcribes a curve in the air. If the moon, or any part of her, was brought down to the earth, and projected in the ſame line and with the ſame velocity as a terreſtrial body, it would move in the ſame curve; and if any body was carried from our earth to the diſtance of the moon, and was projected in the ſame direction and with the ſame velocity with which the moon is moved, it would proceed in the ſame orbit which the moon deſcribes, with the ſame velocity. Thus the moon is a projectile, and the motion of every projectile gives an image of the motion of a ſatellite or moon. Theſe phænomena are ſo coincident, that it is manifeſt they muſt flow from the ſame cauſe.

CHAP.

CHAP. III.

Of the ſolar ſyſtem: and the parallaxes of the planets and fixed ſtars.

1. HAVING ſhewed that gravity is extended from the ſurface of the earth to the moon, and to all diſtances upwards, decreaſing in a regular courſe as the ſquares of thoſe diſtances increaſe, our author did not ſtop here: as any conſiderable diſcovery in nature generally opens a new ſcene, ſo valuable a one as this could not be barren in Sir *Iſaac Newton*'s hands. The gravity of the moon ſuggeſted to him the univerſal gravitation of matter; and ſo ſucceſsful an account of her motion led him to explain all the curvilinear motions in the ſolar ſyſtem, from the ſame principle. The earth cannot be conſidered as the centre of the motions of any body in the ſyſtem but of the moon only, with which ſhe forms one of thoſe leſſer ſyſtems of which the vaſt ſolar ſyſtem conſiſts. The inferior planets, *Mercury* and *Venus*, do not ſo much as include the earth within their orbits, but manifeſtly revolve round the ſun; for ſometimes they are farther diſtant from us than the ſun, and at other times paſs between him and us, but never are ſeen oppoſite to the ſun, or appear removed from him beyond a certain arc, which is called their *greateſt elongation*. The higher planets, *Mars*, *Jupiter* and *Saturn*, move in orbits which include the earth indeed; but it appears from their motions, which viewed from the earth are ſubject to many irregularities, that the earth is not to be conſidered as the centre of their orbits. Sometimes they appear to *proceed* in theſe orbits from weſt to eaſt, ſometimes they ſeem *ſtationary*

or

or without motion, and at other times they appear *retrograde*, or to go backwards from eaſt to weſt: and theſe irregularities, tho' different in the different planets, are exactly ſuch, in all of them as ſhould appear to us in conſequence of the motion of the earth in her orbit.

2. The motions of all the planets about the ſun are conſtant and regular. They all move round him from weſt to eaſt, almoſt in the ſame plane, in elliptic orbits that have the ſun in one of the *foci*, but of which ſome approach very near to circles. *Mercury* poſſeſſes the loweſt place; where moving with the greateſt velocity of them all, and in the leaſt orbit, he finiſhes his revolution in two months and 28 days. The planet *Venus*, which is called by us ſometimes the evening ſtar, ſometimes the morning ſtar, according as it appears to us eaſtward or weſtward from the ſun, and conſequently ſets later or riſes earlier, is next to *Mercury* in the ſyſtem, and revolves in about ſeven months and 15 days. Above theſe next in order revolves the *earth*, with her ſatellite the *moon*, in the ſpace of a year. *Mars* is above the earth, and is the firſt which includes the earth, as well as the ſun, in his orbit; which he deſcribes in one year, ten months and 22 days. Higher in the ſyſtem and at a great diſtance *Jupiter* revolves, with his four ſatellites, in eleven years, ten months and 15 days. Laſt of all, *Saturn*, with five ſatellites, and a ring peculiar to him, moves in a vaſt orb with the ſloweſt motion, and finiſhes his period in twenty-nine years, five months and 27 days.

3. Suppoſe the earth's mean diſtance from the ſun to be divided into 100 equal parts, then the mean diſtances of *Mercury*, *Venus*, *Mars*, *Jupiter* and *Saturn*, from the ſun, ſhall conſiſt of nearly 38,

72, 152, 520 and 954 ſuch parts, reſpectively. Or if they be required with greater exactneſs, let the earth's mean diſtance be repreſented by 100000, and the diſtances of thoſe ſeveral planets ſhall be repreſented by the numbers 38710, 72333, 152369, 520096, 954006, reſpectively.

The diſtances of *Mercury* and *Venus* are determined by their greateſt elongations from the ſun. Let S (*Fig*. 54.) repreſent the ſun, T the earth, and ſuppoſing AVB the orbit of *Venus* to be perfectly circular, draw TV a tangent; then ſhall V repreſent the place of *Venus* where her elongation from the ſun is greateſt, and the triangle SVT being right angled at V, it follows that ST, the diſtance of the earth from the ſun, is to SV, the diſtance of *Venus* from the ſun, as the radius to the ſine of the angle STV the greateſt elongation of *Venus* from the ſun. In this manner, the diſtances of the inferior planets are compared with the diſtance of the earth from the ſun. The diſtances of the ſuperior planets are determined from their retrogradations, and, in ſuch as have ſatellites, by the eclipſes of thoſe ſatellites. For example, let I (*Fig*. 55.) repreſent the planet *Jupiter*, and if the right line SI, joining the centres of the ſun and *Jupiter*, be produced to M, then ſhall IM be the axis of his ſhadow, the poſition of which is determined by the eclipſes of the ſatellites, and ſhews the *heliocentric* place of *Jupiter*, *i. e.* his place viewed from the ſun. Produce the line TI, which joins the centres of the *Earth* and *Jupiter*, to N, and N ſhall repreſent the *geocentric* place of *Jupiter*, *i. e.* his place when viewed from the earth. The difference of thoſe places gives the angle NIM or TIS; the angle ITS, the elongation of *Jupiter* from the ſun as ſeen from the earth at T, is eaſily found by obſervation; conſequently all the angles of the

the triangle T I S are known, with the proportion of its ſides, which is the ſame as of the ſines of thoſe angles; and thus the proportion of S I, the diſtance of *Jupiter* from the ſun, to S T, the diſtance of the earth from the ſun is diſcovered. The angle T I S is that under which S T the ſemi-diameter of the earth's orbit would appear if viewed from I, or the elongation of the earth from the ſun as it would appear to a ſpectator at *Jupiter*.

4. In the firſt chapter of this book, we explained at length how the diſtances of the celeſtial bodies are diſcovered by what is called the diurnal parallax, that is, the angle under which the ſemi-diameter of the earth would appear at thoſe diſtances. By this method the diſtance of the moon from the earth is compared with its ſemi-diameter. When *Venus* and *Mars* are at their leaſt diſtances from the earth, it is of uſe likewiſe for eſtimating thoſe diſtances. But in moſt other caſes, the diſtances of the celeſtial bodies are ſo great, and the ſemi-diameter of the earth bears ſo ſmall a proportion to them, that the angle under which it would appear, viewed at ſo great diſtances, cannot be diſcovered by our inſtruments, with any tolerable accuracy. Therefore aſtronomers have been obliged to have recourſe to other inventions. The method propoſed by *Ariſtarchus* for determining the diſtance of the ſun, by obſerving the time when the moon's diſk appears to be half illuminated by the ſun, may be conſidered as an attempt to ſubſtitute the ſemi-diameter of the moon's orbit in place of the ſemi-diameter of the earth. Let S and T (*Fig.* 56.) repreſent the ſun and earth, L the moon's place when T L is perpendicular to S L, at which time her diſk ought to appear to us to be biſected by the boundary of light and darkneſs upon her ſurface; and it is manifeſt that T S, the diſtance of

of the earth from the ſun, is then to T L, the diſtance of the moon from the earth, as the radius to the ſine of the angle L S T, the complement of the angle S T L the elongation of the moon from the ſun at that time. But this method, tho' very ingenious, has proved unſucceſsful; aſtronomers finding it impracticable to determine the time of this biſection of the lunar diſk with ſufficient exactneſs for this purpoſe. We learn from it, however, that the diſtance of the ſun is vaſtly greater than that of the moon; for it is obvious that the nearer the angle S T L approaches to a right one, the greater muſt the diſtance S T be in proportion to T L; and that if this diſtance S T was infinite, then S T L would be a right angle. Now aſtronomers find it very difficult to diſcover any difference between the angle S T L and a right angle, or between the time when the lunar diſk appears to be biſected and the quadrature; from which it follows that S T is vaſtly greater than T L.

5. Aſtronomers finding the diurnal parallax of no uſe for determining or comparing the greater diſtances in the celeſtial ſpaces, the ſemi-diameter of the earth being too ſmall a baſe for this purpoſe, have had recourſe to what they call the annual parallax. In place, therefore, of the ſemi-diameter of the earth, they ſubſtituted the ſemi-diameter of the orbit deſcribed by the earth annually about the ſun; or, in place of two ſtations or ſpectators, one of which was ſuppoſed to be at the ſurface and the other at the centre of the earth, they ſubſtituted one at the earth and another at the ſun. In this manner they obtained a baſe that bears a conſiderable proportion to any diſtances within the ſolar ſyſtem, and with which they were able to compare them by accurate obſervations. As, in the former caſe, they compared the diſtances in the heavens with the ſemi-diameter of the

the earth, by finding under what angle it would appear at thoſe diſtances; ſo, in this caſe, they compare the vaſt diſtances of the planets from the ſun with the ſemi-diameter of the earth's orbit, by finding under what angle this ſemi-diameter appears at thoſe diſtances. This angle is greater at the diſtance of *Mars* than at that of *Jupiter*, and is greater there than at the diſtance of *Saturn*; decreaſing always with the diſtance; till at length it become too ſmall to be diſcernible by the exacteſt inſtruments we have. Let I (*Fig.* 55.) repreſent any remote object in the ſyſtem, A the point where the earth paſſes betwixt the ſun S and that object I, I T a tangent from the point I to the earth's orbit, ſuppoſed to be circular: and when the earth is at A, the object I will appear in the ſame place to the earth and ſun; but when the earth comes to T, if we ſuppoſe I to have no motion, it will appear to the earth in the right line T I, and will appear to have gone backward by the arc that meaſures the angle T I S, the ſame which the ſemi-diameter of the earth's orbit S T ſubtends at I; and this angle being determined by obſervation, its ſine will be to the radius, as S T to S I; that is, as the diſtance of the earth from the ſun to the diſtance of the object I from the ſun; which proportion, therefore, is eaſily computed by trigonometry. When the object I has a proper motion, an allowance muſt be made for this motion, after it is determined by obſervation.

The appearances, in this caſe, may be explained in the following manner. Let S I produced meet the ſphere in which the fixed ſtars are apparently diſpoſed in M, let the two tangents T I and *t* I meet the ſame in N and *n*, and ſuppoſing the object I to vibrate continually between N and *n* like a pendulum, imagine this arc N *n* itſelf to be carried along

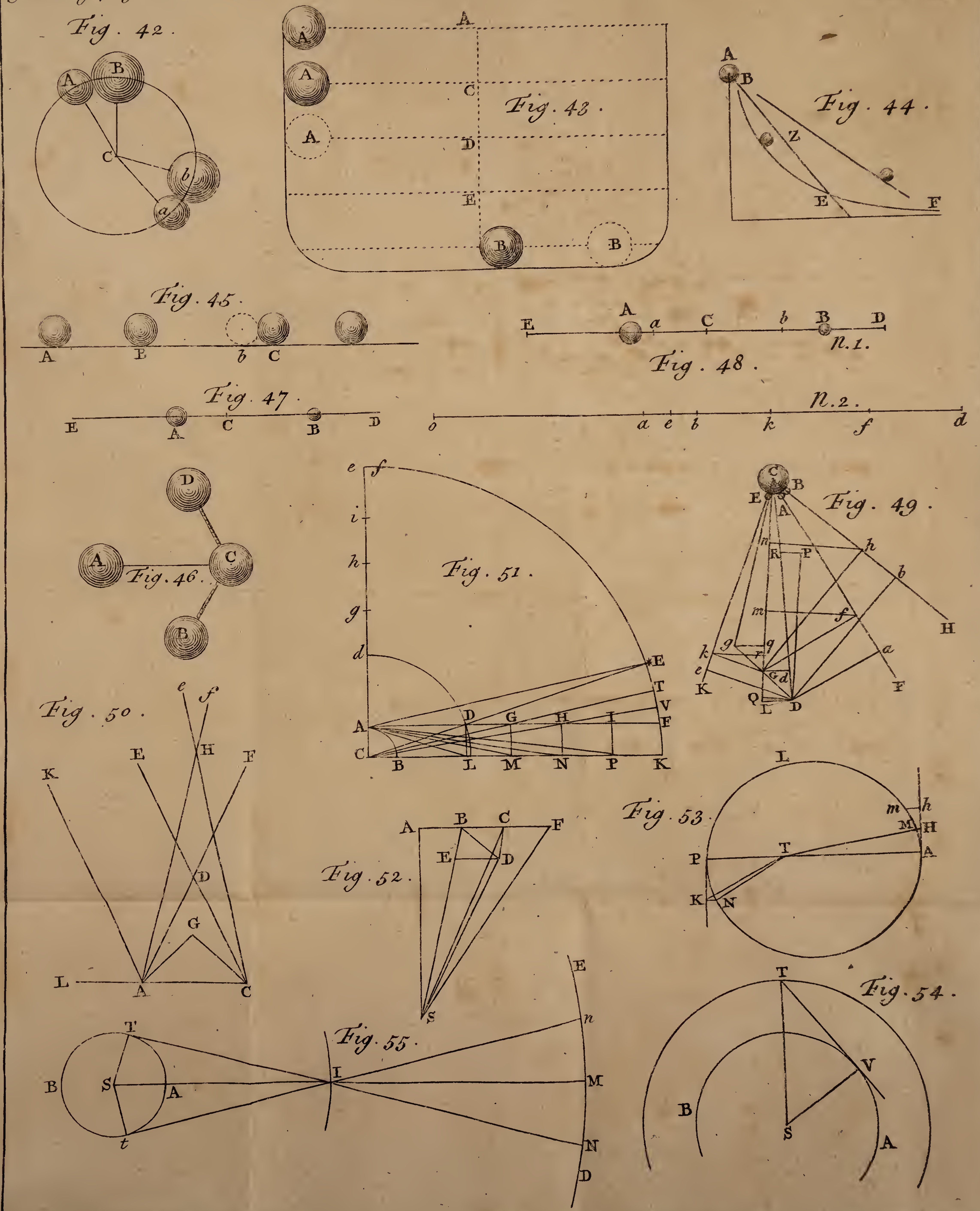
T the

the arc DME with the proper motion and direction of the object I. If I represent a planet, the arc N*n* which measures the angle NI*n* or TI*t*, will shew how much the planet is retrogade, the half of which angle is SIT; which being known, the proportion of SI to ST is computed as above.

6. We ascribe the annual motion to the earth and not to the sun, according to the *Pythagorean* system revived by *Copernicus*, for many reasons; some of which were briefly mentioned in § 1. and 2. By comparing the periodic times of the primary planets and their distances from the sun, and by comparing the periodic times of the satellites that revolve about *Jupiter* and *Saturn* with their respective distances from their primary planets, it appears to be a general law in the solar system, that when several bodies revolve about one centre, the squares of the periodic times increase in the same proportion as the cubes of the distances from that centre; that is, the periodic times increase in a higher proportion than the distances, and not in so high a proportion as the squares of those distances, but accurately as the power of the distance whose exponent is $1\frac{1}{2}$, or as the number which is a mean proportional between those numbers that represent the distance and its square. The earth is the centre of the motion of the moon, in all the systems. If the sun likewise revolved round the earth, we should expect that the same general law would take place in their periodic times and distances compared together; or that the square of 27 days, 7^h, $43'$ would be to the square of 365 days, 6^h, $9'$, as the cube of the moon's distance from the earth to the cube of the sun's distance from the same: from which it is easy to compute that the sun's distance ought to be little more than $5\frac{3}{5}$ times greater than the moon's distance; whereas it is evident, from

fronting pag. 274.

TAB. III.



from the minuteneſs of the ſun's diurnal parallax, that the ſun's diſtance is ſome hundred times greater than the moon's diſtance from the earth. But if, with *Copernicus*, we ſuppoſe the *earth* to revolve about the ſun, in an orbit placed betwixt thoſe of *Venus* and *Mars*, this law will be found to obtain between the periodic times and diſtances of the earth and any of the planets from the ſun compared together; and the harmony of the ſyſtem will appear complete. The retrogradations and ſtations of the planets, and the many apparent irregularities in their motions and diſtances from the earth, furniſh us with ſo many arguments againſt the *Ptolemaic* ſyſtem, according to which thoſe appearances are explained by a number of perplexed ſolid orbs and epicycles, in a manner unworthy of the noble ſimplicity and beauty of nature. It is likewiſe to be remarked, that thoſe inequalities are different in the different planets, but in each of them are ſuch as ought to ariſe from the annual motion of the earth. The arguments derived from the magnitude of the ſun, and its great uſefulneſs to all the bodies in the ſyſtem, which ſeem to entitle it to the moſt centric place, are too obvious to require our inſiſting on them. The earth and planets revolve about the ſun, in order to enjoy the benefits of his light and heat; but no reaſon appears why the ſun and planets ſhould revolve around the earth.

7. There is but one argument againſt the annual motion of the earth that deſerves any notice, *viz.* The want of an annual parallax in the fixed ſtars. Let T A *t* (*Fig.* 57.) repreſent the earth's orbit about the ſun S, T X the axis of the earth, and *t x*, parallel to T X, ſhall repreſent the poſition of the ſame axis at the oppoſite point *t*. Suppoſe T X to be di-

rected towards the ſtar P; and it is manifeſt that the axis of the earth will not be directed to the ſame ſtar when it comes to the ſituation *t x*, but will contain an angle *x t* P with the line *t* P joining the earth and ſtar, equal to the angle *t* P T, under which the diameter T *t* of the earth's orbit appears to a ſpectator, viewed from the ſtar P. It might be expected, therefore, that by obſerving the fixed ſtar P from the different parts of the earth's orbit T, *t*, (which may be conſidered as two ſtations in this problem, the moſt ſublime of all that can be brought into practical geometry,) we ought to be able to judge, from its different appearances at thoſe ſtations, of the angle T P *t*, and conſequently of the proportion of T P, the diſtance of the ſtar, to T *t*, the diameter of the earth's orbit, or double diſtance of the ſun. Yet it is certain that aſtronomers, hitherto, have not been able to diſcover any difference in the apparent ſituations of the fixed ſtars, with reſpect to the axis of the earth or to one another, that can ariſe from the motion of the earth; tho', ſince the reſtoration of the *Pythagorean* doctrine, they have taken great pains to examine this matter. In anſwer to this objection, it is obſerved, that the diſtance of the fixed ſtars is ſo very great, that the diameter of the earth's orbit bears no ſenſible proportion to it; ſo that the angle T P *t* is not to be diſcovered by our exacteſt inſtruments. Nor is this immenſe diſtance of the fixed ſtars advanced by the *Copernicans* as an hypotheſis, merely for the ſake of ſolving this objection; for, as they had reaſon to ſuppoſe the fixed ſtars like to our ſun, they had ground to conclude their diſtance to be vaſtly great, ſince they appear to us with ſo faint a light, and of no ſenſible diameter, even in the largeſt teleſcopes. If we ſhould ſuppoſe the diſtance between us and a fixed ſtar to be divided into 300 equal parts, and a ſpectator, after paſſing

over

over 299 of thoſe parts, ſhould view it from the laſt diviſion, or at $\frac{1}{300}$th part of the whole diſtance, the ſtar, indeed, would appear brighter to him, but not ſenſibly magnified in diameter; becauſe it would appear of the ſame magnitude to him at that diſtance, as it was in a teleſcope that magnified 300 times. The immenſe diſtance of the fixed ſtars likewiſe appears from hence, that when the moon or any other planet covers them from us, this is done in an inſtant; they diſappear at once, and not gradually as the more remote planets when covered by the nearer ones. If we join theſe obſervations together, they will rather appear to confirm one another and the motion of the earth, than to make againſt it. The immenſe diſtance of the fixed ſtars, that ariſes from them jointly, rather ſtrengthens the evidence of the *Copernican* ſyſtem; becauſe the more remote the ſtars are, the more abſurd it muſt appear to ſuppoſe ſo immenſe a ſpace to revolve about our earth, ſo inconſiderable a point! that to our neighbouring planets it is ſeen but as a ſmall ſpark of light; to others of them is hardly known; and to ſome of the fixed ſtars, neither it nor the whole ſolar ſyſtem to which it belongs is viſible. How can it be imagined that thoſe immenſe bodies, ſunk ſo deep in the abyſs of ſpace, deſcribe daily ſuch vaſt rounds about ſo mean a centre; eſpecially if it be conſidered that it is highly probable ſome of the fixed ſtars are immenſely farther diſtant than others, and that all the ſyſtem of the fixed ſtars, viſible to the naked eye in a clear night, form but a ſmall corner of the univerſal ſyſtem?

8. But this is not all we learn from the diligence and accuracy of late aſtronomers, in confirmation of the motion of the earth about the ſun, and that ſerves to reſolve this the only material objection

againſt it. An inſtrument was contrived by the famous Mr. *Graham* (for a deſcription of which we refer the reader to Dr. *Smith*'s excellent treatiſe of *optics*) and executed with ſurpriſing exactneſs, which being placed in the vertical line, a ſtar in the conſtellation *Draco* that paſſed near the zenith was obſerved by this inſtrument for a number of years, with a view to diſcover its parallax, by Mrs. *Molyneux*, *Bradley* and *Graham*. They ſoon diſcovered that the ſtar did not appear always in the ſame place in the inſtrument, but that its diſtance from the zenith varied, and that the difference of its apparent places amounted to 21 or 22 ſeconds. This ſtar is near the pole of the ecliptic. They made ſimilar obſervations on other ſtars, and found a like apparent motion in them, proportional to the latitude of the ſtar. This motion was by no means ſuch as was to have been expected as the effect of a parallax; and it was ſome time before they diſcovered any way of accounting for this new phænomenon: but at length Mr. *Bradley* reſolved all its variety in a ſatisfactory manner, by the motion of light and the motion of the earth compounded together.

Let AD (*Fig.* 58.) repreſent a ſmall portion of the earth's orbit, CD a ray of light moving from the ſtar with the direction CD; and if the earth was at reſt, the teleſcope would be directed to the ſtar, by placing it in the right line AE parallel to DC. Let AD be to DC, as the velocity of the earth in its orbit to the velocity of light, and it is manifeſt that the teleſcope muſt now be placed in the ſituation AC, that the ray of light may run along its axis, and, after entering the middle of the object glaſs at C, may iſſue at the middle of the eye glaſs at A; becauſe, while the ray deſcribes the right line CD, the point A is carried forwards to D, and the teleſcope

by

by moving parallel to itſelf is carried into the ſituation D c. But the apparent place of the ſtar is determined by the poſition of the teleſcope, and conſequently the ſtar will appear in the right line A C, and not in its true ſituation A E. Thus a ſtar in the pole of the ecliptic will appear to have its latitude diminiſhed by the angle E A C or A C D; which will be found to exceed 20 ſeconds, if the velocity of light be to the velocity of the earth as 8000 to 1: and this ſtar will in appearance deſcribe a ſmall circle round the pole of the ecliptic at a diſtance from it of about 20″. In other caſes, the ſtar will appear to deſcribe a ſmall ellipſis having its centre in the true place of the ſtar, (*i. e.* the place where it would appear if the earth was at reſt) its tranſverſe axis parallel to the ecliptic, and its ſecond axis perpendicular to it; the former of which gives its greateſt aberration in longitude, and the latter its greateſt aberration in latitude. If the ſtar be in the plane of the ecliptic, the aberration then is only in longitude. In this caſe, if the rays from the ſtar touch the earth's orbit in G and H, and be perpendicular to it in A and B, the motion of the earth, at G and H, being in the direction of the ray, the ſtar will appear in its true place, and there will be no aberration at thoſe points; but the aberration in longitude will be greateſt at A and B. He has explained all the appearances of the ſtars obſerved by Mr. *Molyneux* and himſelf, in this manner; and tho' he has not diſcovered any parallax by theſe obſervations, he has produced from them a new argument for the motion of the earth, by a ſeries of obſervations made on different ſtars in different places. He finds ground to conclude from theſe, that the parallax of the fixed ſtars can hardly exceed one ſecond; from which their diſtance ought to be 400,000 greater than the diſtance of the ſun.

The true motions in the ſyſtem being eſtabliſhed, we may now proceed ſafely with our *analyſis.*

9. Each of the primary planets bend their way about the centre of the ſun, and are accelerated in their motion as they approach to him, and retarded as they recede from him; ſo that a ray drawn from any one of them to the ſun always deſcribes equal ſpaces, or areas, in equal times: from which it follows, as in *Chap.* 2. § 5, 6, 7. that the power which bends their way into a curve line muſt be directed to the ſun. This power always varies in the ſame manner as the gravity of the moon towards the earth. The ſame reaſoning by which the gravity of the moon towards the earth at her greateſt and leaſt diſtances were compared together, in *Chap.* 2. § 8, 9, 10. may be applied in comparing the powers which act on any primary planet, at its greateſt and leaſt diſtances from the ſun; and it will appear, that theſe powers increaſe as the ſquare of the diſtance from the ſun decreaſes. Our author ſhews this generally, from the nature of the elliptic curve in which each planet moves.

10. But the univerſality of this law, and the uniformity of nature, ſtill farther appears by comparing the motions of the different planets. The power which acts on a planet that is nearer the ſun is manifeſtly greater than that which acts on a planet more remote; both becauſe it moves with more velocity, and becauſe it moves in a leſſer orbit, which has more curvature, and ſeparates farther from its tangent, in arcs of the ſame length, than a greater orbit. By comparing the motions of the planets, it is found that the velocity of a nearer planet is greater than the velocity of one more remote, in proportion as the ſquare root of the number which expreſſes the greater

greater diſtance to the ſquare root of that which expreſſes the leſſer diſtance; ſo that if one planet was four times farther from the ſun than another planet, the velocity of the firſt would be half the velocity of the latter, and the nearer planet would deſcribe an arc in one minute, equal to the arc deſcribed by the higher planet in two minutes: and tho' the curvature of the orbits was the ſame, the nearer planet would fall by its gravity as much in one minute as the other would fall in two, and therefore the nearer planet would deſcribe by its gravity four times as much ſpace as the other would deſcribe in the ſame time, by the law of motion of falling bodies ſo often mentioned; the gravity of the nearer planet would therefore appear to be quadruple, from the conſideration of its greater velocity only. But beſides, as the radius of the leſſer orbit is ſuppoſed to be four times leſs than the radius of the other, the leſſer orbit muſt be four times more curve, and the extremity of a ſmall arc of the ſame length will be four times farther below the tangent drawn at the other extremity in the leſſer orbit than in the greater; ſo that, tho' the velocities were equal, the gravity of the nearer planet would, on this account only, be found to be quadruple. On both theſe accounts together, the greater velocity of the nearer planet, and the greater curvature of its orbit, its gravity towards the ſun muſt be ſuppoſed ſixteen times greater, tho' its diſtance from the ſun is only four times leſs than that of the other; that is, when the diſtances are as 1 to 4, the gravities are reciprocally as the ſquares of theſe numbers or as 16 to 1. In the ſame manner, by comparing the motions of all the planets, it is found that their gravities decreaſe as the ſquares of their diſtances from the ſun increaſe.

11. Thus, by comparing the motions of any one planet in the different parts of its elliptic orbit, and the motions of the different planets in their different orbits, it appears that there is a power like the gravity of heavy bodies ſo well known to us on the earth, extending from the ſun to all diſtances, and conſtantly decreaſing as the ſquares of theſe diſtances increaſe. If any one planet deſcended to the diſtance of another, it would be acted on in the ſame manner, and by the ſame power, as that other: and as gravity preſerves the ſubſtance of the earth together, and hinders its looſer parts from being diſſipated by its various motions; ſo a like power, acting at the ſurface of the ſun, and within its body, keeps its parts together and preſerves its figure, notwithſtanding its rotation on its axis.

12. In the ſame manner as this principle governs the motions of the planets in the great ſolar ſyſtem, it governs alſo the motions of the ſatellites in the leſſer ſyſtems of which the greater is compoſed. There is the ſame harmony in their motions compared with their diſtances, as in the great ſyſtem: we ſee *Jupiter*'s ſatellites bending their way round him, and falling every moment from the lines that are the directions of their motions, or the tangents of their orbits, towards him; each deſcribing equal areas in equal times by a ray drawn to his centre, to which their gravity is therefore directed. The nearer ſatellites move with greater celerity, in the ſame proportion as the nearer primary planets move more ſwiftly round the ſun, and their gravity, therefore, varies according to the ſame law. The ſame is to be ſaid of *Saturn*'s ſatellites. There is, therefore, a power that preſerves the ſubſtance of theſe planets in their various motions, acts at their ſurfaces,

faces, and is extended around them, decreaſing in the ſame manner as that which is extended from the earth and ſun to all diſtances.

13. Theſe ſecondary planets muſt alſo gravitate towards the ſun. It is impoſſible they ſhould move ſo regularly round their reſpective primaries, if they were not acted on by the ſame powers. If we ſuppoſe them to be acted on by the ſame accelerating power in parallel lines, there will no diſorder or perplexity ariſe from thence; for they will then accompany their primary planets in their motions round the ſun, and move about them at the ſame time, with the ſame regularity as if their primary planets were at reſt. It will be as in a ſhip, or in any ſpace carried uniformly forward: in which the mutual actions of bodies are the ſame as if the ſpace was at reſt, being no way affected by that motion which is common to all the bodies. As every projectile, while it moves in the air, gravitates towards the ſun, and is carried along with the earth about the ſun, while its own motion in its curve is as regular as if the earth was at reſt; ſo the moon, which we have ſhewed to be only a greater projectile, muſt gravitate toward the ſun, and, while it is carried along with the earth about the ſun, is not hindered by that motion from performing its monthly revolutions round the earth. *Jupiter*'s ſatellites gravitate toward the ſun as every part of *Jupiter*'s body, and *Saturn*'s ſatellites gravitate toward the ſun as if they were parts of *Saturn*. Thus the motions in the great ſolar ſyſtem, and in the leſſer particular ſyſtems of each planet, are conſiſtent with each other, and are carried on with a regular harmony without any confuſion, or mutually interfering with one another, but what neceſſarily ariſes from ſmall inequalities in the gravities of primary and ſecondary planets, and the

the want of exact parallelism in the directions of those gravities; of which we are to treat afterwards.

14. Nor is there any body that comes, tho' rarely and as a stranger, into the lower parts of our system, exempted from this universal gravitation toward the sun. When a *comet* appears, we see the effect of the same power acting on it; since it descends with an accelerated motion as it approaches the sun, and ascends with a retarded motion, bending its way about the sun, and describing equal areas in equal times by a ray drawn from it to his centre. This power that acts on the comets varies according to the same law as the gravity of the planets, as appears from their describing either *parabolas* *, or very eccentric *ellipses* having one of their *foci* in the centre of the sun: our author having demonstrated, that the power which makes a body describe a parabola about its focus, must likewise vary according to the law so often mentioned. If a body was projected from our earth in a line perpendicular to the horizon, with a certain force, (*viz.* that which would carry it over about 420 miles with an uniform motion in a minute), it would rise in that line for ever and return to the earth no more. Its gravity would, indeed, retard its motion continually, but never be able to exhaust it, the force of gravity upon it decreasing as it rises to a greater height. If the body was projected with the same force in any other direction, it would go off in a *parabola* having its *focus* in the centre of the earth, and never return to the earth again. A force a little less would make it move in a very eccentric *ellipsis*, in which it would return after a long period to its first place; if it was

* *Princip.* Lib. III. Prop. 40.

not

not diverted in its courſe by approaching too near to ſome celeſtial body. In the ſame manner, a planet projected with a certain force would go off for ever in a parabolic curve having the ſun in its focus; and if it was projected with a force a little leſs would revolve in a very eccentric ellipſis having its focus in the ſun. All theſe motions, therefore, proceed from the ſame principle, acting in a various but moſt regular manner in different circumſtances, and are all analogous to the motions of heavy bodies projected from our earth. Effects ſo ſimilar are to be reſolved into the ſame cauſe, and there is hardly more evidence for ſuppoſing that it is the ſame power of gravity that acts upon terreſtrial bodies in *Europe* and in *America*, at the equator and at the poles, than that it is the ſame principle which acts over the whole ſyſtem, from the centre of the ſun to the remote orb of *Saturn*, or to the utmoſt altitude of the moſt eccentric comet.

15. From ſeveral phænomena we have reaſon to conclude, that there is an atmoſphere environing the ſun and extended from it to a conſiderable diſtance. The ring of light obſerved around the moon, in a total eclipſe of the ſun, in 1605, mentioned by *Kepler*, and of late in 1706 and 1724, when it was obſerved to extend to 9 or 10 degrees diſtance from the moon, ſeems rather to have proceeded from the reflexion of that atmoſphere, while the ſolar direct rays were intercepted by the moon, than from the refraction of any atmoſphere about the moon. The matter of this atmoſphere appears to gravitate towards the ſun, from the effect it has upon the vapour which ariſes in the tails of comets from their *Nucleus* and atmoſphere, with a direction oppoſite to that of their gravity towards the ſun. For this vapour,

vapour, being highly rarified, ſeems to ariſe with this direction in conſequence of the greater gravity of the ſolar atmoſphere towards the ſun; in the ſame manner as a column of vapour riſes in the air, in conſequence of the air's greater gravity towards the earth; the rather that this vapour riſes with more rapidity, as well as in greater plenty, in proportion as the comet is nearer the ſun. Thus there is no ſort of matter in the ſolar ſyſtem but what we have ground to conclude gravitates towards the ſun.

16. As to the fixed ſtars, they are removed to ſuch an immenſe diſtance, that their gravity toward the ſun can have no ſenſible effect upon them in many ages, and cannot appear to us by the phænomena. The power of gravity decreaſes in proportion as the ſquare of the diſtance increaſes; the neareſt fixed ſtar ſeems to be ſeveral hundred thouſand times farther diſtant from us than the earth is from the ſun; and therefore their gravity muſt be ſome 100000×100000 times leſs than the gravity of the earth toward the ſun. It is not therefore from phænomena, but from analogy only, that we can extend the power of gravity to the fixed ſtars. There is no influence but their light only which is able to traverſe that vaſt abyſs of ſpace that is between us and them, ſo as to have any ſenſible effect. However, as their light is every way the ſame as that of our ſun, our author thinks the argument from analogy may have its weight in this caſe. If they alſo gravitate toward the ſun, and toward each other, then we may ſuppoſe that the unfathomable void that intervenes between the ſyſtems of which they are probably the centres, as the ſun is of our ſyſtem, may ſerve to hinder them from diſturbing each others motions, and from coming together into

one

one vaſt unformed maſs of matter. It will not ſeem ſtrange that where the ſun itſelf is ſcarcely viſible, the gravity toward it ſhould be inſenſible; and that we ſhould here find no effects of any gravitation toward the fixed ſtars.

17. As *action* and *reaction* are always equal and in oppoſite directions, ſo that the earth, for example, gravitates toward every mountain as well as every mountain toward the earth, and gravitates toward every projectile while it is moving in the air, as well as the projectile gravitates towards it; and without this law nothing would be ſteady or conſtant in nature: hence it follows, that the ſun gravitates toward all the bodies in the ſyſtem, and that the primary planets gravitate toward their ſatellites. The primary planets alſo gravitate toward one another; ſome minute irregularities in their motions, eſpecially in thoſe of *Jupiter* and *Saturn*, the two greateſt planets, when they are in conjunction and come neareſt to each other, are evidences of this. The motions of the ſatellites of *Jupiter* and *Saturn* are alſo ſaid to be ſubject to irregularities that proceed from their mutual actions. From ſo many indications we may at length conclude, that all the bodies in the ſolar ſyſtem gravitate toward each other; and tho' we cannot conſider gravitation as eſſential to matter, we muſt allow that we have as much evidence, from the phænomena, for its univerſality, as for that of any other affection of bodies whatſoever.

CHAP.

CHAP. IV.

Of the general gravitation of matter.

1. HItherto we have considered only the accelerating force of gravity at different distances, to which the velocity generated by it, in a given time, is always proportional. It remains to shew that the motion produced by this power, at equal distances from a given centre, is always proportional to the quantity of matter in the heavy body; that the gravity of bodies arises from the mutual gravitation of their parts; and to ascertain the law of the gravitation of the particles of bodies. It is allowed as to terrestrial bodies, and was confirmed from many accurate experiments by Sir *Isaac Newton*, that bodies of the same bulk and figure, tho' of very different kinds, suspended by lines of the same length, performed their vibrations, when moving as pendulums, exactly in the same time; from which it follows, that the force of their gravity is exactly proportional to their quantity of matter: nor would there be any difference in the times of their vibrations tho' their figure and bulk were different, the distances between their centres of suspension and of oscillation being equal, if it was not for the resistance of the air. It has been already shewed, that the moon would fall toward the earth with the same velocity as any other heavy body, if she was at the same distance from its centre; and it is plain that the forces of bodies moved with equal velocities are as their quantities of matter: so that the weight of the moon would be to the weight of any heavy body at the same distance from the centre of the earth, in the same proportion as the matter of the moon is to

the matter of that heavy body. The primary planets are acted on variouſly in their different diſtances, but according to the law which ſhews that if they were at equal diſtances they would deſcend with equal velocities toward the ſun, ſo that their motion would be proportional to their quantity of matter. In the ſame manner it appears, that if the ſatellites of *Jupiter* and *Saturn* were at equal diſtances from the centres of their reſpective primary planets, they would deſcend towards them with equal velocities. The earth and moon, at equal diſtances from the ſun, are acted upon by equal accelerating forces, and would deſcend with equal velocities toward it. *Jupiter* and his ſatellites would deſcend with the ſame velocity toward the ſun, if their projectile motions were deſtroyed. The ſame is to be ſaid of *Saturn* and his ſatellites. A very ſmall inequality in the accelerating forces that act upon the primary planet and its ſatellites would produce very great irregularities in their motion. In all theſe caſes, equal velocities being generated in equal times, the motions of the bodies, and conſequently the gravities that produce theſe motions, muſt be proportional to the quantities of matter in the bodies; from which it follows, that all equal portions of matter, at equal diſtances from the centre of gravitation, are equally heavy; without regard to figure, bulk, or the texture of their parts: and that the gravitation of bodies ariſes from the gravitation of the particles of which they are compoſed.

2. Becauſe *action* is always equal to *reaction*, if you ſtill ſuppoſe the planets at equal diſtances from the ſun, and therefore gravitating toward the ſun with forces proportional to their quantities of matter, the ſun will gravitate towards each of the planets with forces in the ſame proportion. In general,

the ſame body gravitates towards any other bodies, at equal diſtances from them, with forces proportional to their quantities of matter; becauſe it gravitates toward them with the ſame forces with which they gravitate towards it, which are as their quantities of matter. The power, therefore, that is extended from the centre of the ſun and of each of the planets, to all diſtances around them, is, at equal diſtances from their centres, proportional to their quantities of matter: and, in general, it appears that the weight or gravity of a body is the greater, in proportion as its quantity of matter is greater, as the quantity of matter in the body to which it gravitates is greater, and as the ſquare of the diſtance from it is leſs. By compounding theſe three proportions together, the weight, and motion, of bodies, ariſing from their gravitation, may always be determined.

3. Gravity being found, by ſo many experiments and obſervations, to affect all the matter of bodies equally, we have hence more reaſon ſtill to conclude its univerſality; ſince it appears to be a power that acts not only at the ſurfaces of bodies, and on ſuch bodies as are removed at a diſtance from them, but to penetrate into their ſubſtance, and into that of all other bodies, even to their centres; to affect their internal parts with the ſame force as the external, to be obſtructed in its action by no intervening body or obſtacle; and to admit of no kind of variation in the ſame matter, but from its different diſtances only from that to which it gravitates.

4. The action of gravity on bodies ariſes from its action on their parts, and is the aggregate of theſe actions; ſo that the gravitation of bodies muſt ariſe from the gravity of all their particles towards each other.

other. The weight of a body toward the earth arifes from the gravity of the parts of the body: the gravity of a mountain toward the earth arifes from the gravitation of all the parts of the mountain towards it. The gravitation of the northern hemifphere toward the fouthern arifes from the gravitation of all its parts towards it; and if we fuppofe the earth divided into two unequal fegments, the gravitation of the greater toward the leffer arifes from the gravitation of all the parts of the greater toward the leffer. In the fame manner, the gravity of the whole earth, one particle being excepted, toward that particle, muft arife from the quantity of gravitation of all the other particles of the earth toward that particle. Every particle, therefore, of the earth gravitates toward every other particle of it; and, for the fame reafon, every particle of matter in the folar fyftem gravitates toward every other particle in it.

5. We now proceed to an important part of this doctrine, to determine the law according to which the particles of bodies gravitate towards each other; after having difcovered the law which is obferved by bodies compofed of thofe particles. To a fuperficial enquirer, at firft fight, the former might poffibly appear to be neceffarily the fame with the latter: but it is eafily fhewn, that the law which is obferved in the attractions of the minute particles of matter is often very different from that which is obferved by fpheres compofed of fuch particles. If, for example, the gravitation of the particles decreafe in the fame proportion as the cubes of their diftances increafe, or in any higher proportion, the fpheres compofed of fuch particles will not gravitate towards each other with forces that decreafe in the fame proportion as the cubes of the diftances of their centres increafe, or in that higher proportion; for fpheres

in contact ſhall attract each other, in thoſe caſes, with a force infinitely greater than when they are removed to the leaſt diſtance from contact, tho' there be very little difference betwixt the diſtances of their centres in thoſe two caſes. This made it neceſſary for Sir *Iſaac Newton* to treat of this ſubject fully; and as it is a very uſeful part of the theory of gravity, but not to be underſtood, as he has delivered it, without a profound ſkill in geometry and prolix computations, we ſhall endeavour to deſcribe it in a more eaſy manner, by chuſing (as on other occaſions) the moſt ſimple caſes. Suppoſe, firſt, that the gravitation towards any particle decreaſes in the ſame proportion that the ſquare of the diſtance from it increaſes, let P A E *a*, P B F *b* (*Fig.* 59.) be ſimilar cones conſiſting of ſuch particles, terminated by ſpherical baſes A E *a*, B F *b* that have their centre in P; and the gravitation at P toward the ſolid P A E *a*, will be to the gravitation at P towards P B F *b*, as P A to P B, or in the ſame ratio as any homologous ſides of theſe ſimilar ſolids. For let M N *m* be any ſurface ſimilar to A E *a*, having its centre likewiſe in P; and the gravitation towards the ſurface A E *a* will be to that towards M N *m*, in the ratio compounded of the direct ratio of the ſurface A E *a* to M N *m* (or PA^2 to PM^2) and of the inverſe ratio of PA^2 to PM^2, that is, in the ratio of equality; conſequently, the gravitation towards the ſurface A E *a* A being repreſented by A, the gravitation towards the ſolid P A E *a* will be repreſented by $A \times PA$, and that towards the ſimilar ſolid P B F *b* by $A \times PB$, which are in the ratio of P A to P B. In the ſame manner, the gravitation towards the fruſtum that is bounded by the ſurfaces A E *a*, M N *m*, is repreſented by $A \times AM$. It is evident, likewiſe, that tho' the ſurfaces A E *a* and M N *m* be of any other form, yet the ultimate ratio of the gravitations at P towards the conical or pyramidical ſolids

ſolids PAE*a*, PMN*m*, is that of PA to PM; and that if AQ and M*q* be perpendicular to PH in Q and *q*, theſe forces reduced to the direction PH will be ultimately in the ratio of PQ to P*q*. Whence it appears, that, if PB be equal to BA, the attraction of the particle P by the cone PB*b*, with which the particle is in contact, will be equal to the attraction of the fruſtum of the cone terminated by the ſurfaces AE*a*, BF*b*, when the attraction of the particles is ſuppoſed to increaſe as the ſquare of the diſtance decreaſes; and that, in this caſe, the attraction of a portion of matter is not much greater when it is in contact with the particle attracted, than when it is removed to a ſmall diſtance from it.

6. But it is otherwiſe when we ſuppoſe the attraction of the particles to decreaſe as the cubes of their diſtances increaſe. For, in this caſe, the particle P will tend to the ſurface MN*m* with a force that is as the ſurface, or the ſquare of PM directly, and the cube of PM inverſely; that is, with a force which is as PM inverſely, or directly as MV the ordinate of the æquilateral hyperbola KVI, deſcribed between the aſſymptotes PA and PH. Therefore the attraction of the fruſtum MN*m*AE*a* will be meaſured by the hyperbolic area MVIA bounded by the ordinates at A and M; and the attraction of the cone PMN*m*, by the infinite hyperbolic area that is conceived to be formed betwixt the ordinate MV and the aſſymptote PH. It follows then, that, if ſuch a law could take place, the particle P would tend towards the leaſt portion of matter in contact with it, with a greater force than towards the greateſt body at any diſtance, how ſmall ſoever, from it. The ſame is eaſily ſhewn when the attraction of the particles decreaſes as any powers of the diſtances, higher than their cubes, increaſe. It appears, therefore, that the at-

traction of a particle in contact with a body is not sensibly increased by the addition or diminution of new matter, at any distance, how small soever, from the contact; whether this addition or diminution be made to the body or particle; and, in such cases, the less the particle is, the motions produced in it at infinitely small distances, by such attractions, must be the more violent; because the same force acting on a particle generates a velocity in it that is always greater in proportion as the particle itself is less.

7. The same things may be demonstrated without having recourse to the property of the hyperbolic area. Let PA (*Fig.* 60.) be to PB, as PB to PD; let AB and BD be conceived to be divided into an infinite number of similar equal parts A*k*, *k l*, &c. and B*m*, *m n*, &c; then A*k* will be to B*m* as AB to BD, and the matter between the surfaces whose radii are PA and P*k*, shall be to the matter between the surfaces whose radii are PB and P*m*, as $PA^2 \times Ak$ to $PB^2 \times Bm$; that is, as PA^3 to PB^3. The attractive powers of equal particles placed betwixt the surfaces of the radii PA and P*k*, and the surfaces of the radii PB and P*m*, are in the inverse proportion, or as PB^3 to PA^3, by the supposition; and these two proportions compounded together give a ratio of equality. Therefore, because the attractive powers of the matter bounded by two such surfaces are in the compound ratio of the attractions of equal particles, and of the number of particles, it follows that the attraction of the matter contained by the surfaces of the radii PA and P*k* must be equal to the attraction of the matter contained by the surfaces of the radii PB and P*m*. In the same manner the attraction of the matter contained by the surfaces whose radii are P*k* and P*l*, is equal to the attraction of the matter between the surfaces whose radii are P*m* and P*n*;

and

and the attraction of the frustum A E *a* B F *b* is equal to the attraction of the frustum B F *b* D G *d*. In the same manner, if P B be to P D, as P D to P H, the attraction of the frustum D G *d* H R *h* appears to be equal to the attraction of the frustum A E *a* B F *b*; and if this series of decreasing geometrical proportionals be continued, the attraction of the frustum contained by surfaces whose radii are any two subsequent terms of the progression, must be equal to the attraction of the first frustum A E *a* B F *b*. But in this decreasing progression continued from P B the number of terms is infinite; and in the solid P B F *b* there is an infinite number of frustums, the attraction of each of which is equal to the attraction of the first frustum terminated by the surfaces A E *a*, B F *b*; therefore the attraction of the solid B F *b*, which is in contact with the particle P, is infinitely greater than the attraction of the frustum bounded by the surfaces A E *a*, B F *b*, which is the greater solid, but is removed from the contact of the particle P. We have taken this opportunity to illustrate and demonstrate this theorem here, because it will be of use to us afterwards, and serves to shew the advantages of the law of gravity which takes place in the solar system above other laws; tho' these, on other occasions, may be preferable.

8. The gravitation of the particles being supposed to decrease as the squares of their distances increase, the forces with which particles, similarly situated with respect to similar homogeneous solids, gravitate towards these solids, are as their distances from any points similarly situated in the solids, or as any of their homologous sides. For such solids may be conceived to be resolved into similar cones, or frustums of cones, that have always their vertex in the particles, and the gravitation towards these cones, or

frustums, will be always in the same ratio by § 5. But if the gravitation of the particles decrease as the cubes of the distance increase, the forces, with which particles, similarly situated with respect to similar homogeneous solids, tend toward those solids, shall be equal. For such solids being resolved into similar frustums of cones that have always their vertex in the particles, and are similarly situated with respect to them, the gravitation towards these frustrums will be always equal, by what was shewn in the last article; in the same manner as the forces with which the particle P tends toward similar frustums A E *a* B F *b*, D G *d* H R *b* were demonstrated to be equal.

9. The gravitation of the particles being supposed to decrease as the squares of their distances from each other increase, if a particle be placed within the hollow solid generated by the annular space terminated by two concentric circles, or similar concentric ellipses, A D B E and *a d b e*, (*Fig.* 61.) revolving about the axis A B, it shall have no gravity towards this solid. For let *p* be any such particle, *p r* any right line from *p* that meets the internal circle or ellipse in any points *f* and *q*, and the external figure in *x* and *r*; then if *x r* be bisected in *z*, *f q* will be likewise bisected in *z*, because the figures are similar and similarly situated; consequently *f x* is equal to *q r*; and the gravitations of *p* towards opposite frustums of the solid that have their vertex in *p*, and are terminated by the same right lines produced from *p*, with opposite directions, will be always equal, by § 5. and mutually destroy each others effect. It follows from this, that the gravity of any point Q in the semi-diameter C P, towards the sphere or spheroid, is to the gravity at P, as C Q to P C, supposing the point Q to be within the solid; because the gravitation

tion towards the ſolid generated by the annular ſpace, which is included between APB and aQb, has no effect upon a particle at Q; ſo that the gravity at Q towards the whole ſolid ADBE is the ſame as the gravity at Q towards the ſolid $adbe$, which is to the gravity at P towards the ſolid ADBE as CQ to CP, by the laſt article. It appears, therefore, that when a ſphere or ſpheroid, of an uniform denſity, conſiſts of particles that attract with a force decreaſing as the ſquare of their diſtance increaſes, the gravitation towards the ſolid decreaſes from the ſurface to the centre, in any given ſemidiameter, in the ſame proportion that the diſtance from the centre decreaſes.

10. Suppoſe now the particle P (*Fig.* 62.) to be placed without the ſphere ADBE, at the diſtance PC from the centre C; and this particle ſhall be attracted towards the ſphere with a force that decreaſes as the ſquare of the diſtance PC increaſes. For let PNM be any right line from P meeting the generating ſemicircle ADB in N and M, and the arc CH, deſcribed from the centre P with the radius PC, in L; let Pnm be another ſuch right line from P, conſtituting an infinitely ſmall angle with PM, meeting the ſemicircle in n, m, and the arc CH in l; draw LR, lr, perpendicular to PC in R and r, and CV perpendicular to PM in V. Suppoſe another circle AdBe to interſect the circle ADBE in the axis AB, and to conſtitute with it an infinitely ſmall angle; and let Lu and lx, perpendicular to the plane ADB, meet AdB in u and x. Then the gravitation of the particle P, towards the matter in the phyſical ſurface Luxl, ſhall be meaſured by $\frac{\text{L}l \times \text{L}u}{\text{PL}^2}$ or $\frac{\text{L}l \times \text{L}u}{\text{PC}^2}$; conſequently the gravitation of P towards the pyramidical fruſtum, terminated by the circular planes ADB and AdB, and by planes per-

pendicular

pendicular to ADB in NM and nm, ſhall be meaſured by $\frac{Ll \times Lu}{PC^2} \times NM$, by § 5. of this chapter. But, the angle contained by the planes ADB, AdB, being given, Lu is to LR, as Dd, the arc intercepted by theſe circular planes at the diſtance CD, to CD (or CA;) and, Ll being to Rr, as PL, or PC, to LR, ſo that $Ll \times LR$ is equal to $PC \times Rr$; it follows that the gravitation of P towards that fruſtum ſhall be meaſured by $\frac{Ll \times LR \times 2VM \times Dd}{PC^2 \times CD}$ or $\frac{Rr \times 2VM \times Dd}{PC \times CA}$. This gravitation is reduced to the direction PC by diminiſhing it in the ratio of PV, or PR, to PC; and is then meaſured by $\frac{Dd \times Rr \times PR}{CA \times PC^2} \times 2VM$; or (the ſimultaneous increment of VM being repreſented by Vo, and PR^2, or PV^2, being equal to $VM^2 +$ NPM, by *Eucl.* 2. 6. or to $VM^2 +$ APB, ſo that APB being conſtant, the increments of PR^2 and VM^2 muſt be equal, and $Rr \times PR$ equal to $Vo \times VM$) by $\frac{Dd \times 2VM^2 \times VO}{CA \times PC^2}$; which is the ſimultaneous increment of $\frac{Dd \times 2VM^3}{CA \times 3PC^2}$, in the ſame manner as the increment of VM^3, while VM acquires the infinitely ſmall augment Vo, is $3VM^2 \times Vo$. Therefore the attraction of the part of the ſlice of the ſphere terminated by the circular planes ADB, AdB, which is cut off by a plane perpendicular to ADB in the right line NM, is as $\frac{Dd}{CA} \times \frac{2VM^3}{3PC^2}$; and the attraction of the portion of the ſphere which is generated by the revolution of the ſegment MDN about the axis AB bearing the ſame proportion to the attraction of that ſlice, as the circumference of the whole circle to the arc Dd, it is meaſured by $\frac{c}{r} \times \frac{2VM^3}{3PC^2}$, where $\frac{c}{r}$ expreſſes the ratio of the circumference of a circle to the

the radius; and consequently is directly as the cube of the chord MN, and inversely as the square of PC, the distance of the particle P from the centre of the sphere. Hence the gravity at P towards the whole sphere is as the cube of its diameter, or its quantity of matter (the density being given) directly, and the square of the distance PC inversely, the chord MN coinciding with the diameter AB, when the attraction of the whole sphere is considered; so that this attraction is measured by $\frac{c}{r} \times \frac{2\,CA^3}{3\,PC^2}$.

11. It appears from what has been shewn, that any particle P, without the sphere, is attracted by it with the same force as if the whole matter of the sphere was collected in the centre, and attracted as one particle from that centre. For the circumference of the circle ADBE is expressed by $\frac{c}{r} \times CA$, its area by $\frac{c}{r} \times \frac{CA^2}{2}$, the surface of the sphere by $\frac{c}{r} \times 2\,CA^2$, and its solid content by $\frac{c}{r} \times \frac{2\,CA^3}{3}$; so that the attraction of this solid content acting from the centre C, at the distance PC, is measured by $\frac{c}{r} \times \frac{2\,CA^3}{3\,PC^2}$, the very same which measures the attraction of the sphere at that distance, by the last article. The same is to be said of the gravity towards the aggregate of any number of such spheres that have a common centre; from which it follows, that however variable the density of a sphere may be at different distances from the centre, provided the density be always the same at the same distance from it, the gravity of a particle (that is not within the sphere) towards it will be as the quantity of matter in the sphere directly, and the square of the distance of the parti-

particle from its centre inverſely. If the attraction of the particles increaſed or decreaſed in the ſame proportion as their diſtances increaſe or decreaſe, the ſphere would act, in this caſe likewiſe, in the ſame manner as if all its matter was lodged in the centre as one particle; but the caſe is different when the attraction of the particles obſerves other laws. Suppoſe that the attraction of the particles is inverſely as the power of the diſtance of any exponent n leſs than 3, and the attraction of a ſphere conſiſting of ſuch particles, at its ſurface, will be to the force with which the whole matter of the ſphere collected in its centre would attract at the ſame diſtance, as $3 \times 2^{2-n}$ to $\overline{3-n} \times \overline{5-n}$. If, for example, the attraction of the particles be the ſame at all diſtances (in which caſe we ſuppoſe $n=0$) this ratio is that of 4 to 5; and if the attraction of the particles be inverſely as their diſtance, it is that of 3 to 4; as we have ſhewn elſewhere *.

12. Having ſhewn that when the particles gravitate towards each other with forces that are inverſely as the ſquares of their diſtances, the action of a ſphere upon a particle placed without it obſerves the ſame law as that of the particles themſelves, and decreaſes in the ſame proportion as the ſquare of the diſtance of the particle from the centre of the ſphere increaſes; it follows, becauſe *action* and *reaction* are equal, that the particle will attract the ſphere by a force varying in the ſame proportion; and if, in place of the particle, a ſecond ſphere be ſubſtituted conſiſting of ſuch particles, ſince the total action of this ſecond ſphere will be the ſame as if all its matter was lodged in its centre, therefore the two ſpheres

* Treatiſe of Fluxions, § 902.

muſt obſerve the ſame law, in acting upon each other, as two particles placed in their centres; that is, their attraction muſt decreaſe in proportion as the ſquare of the diſtance betwixt their centres increaſes.

13. The gravitation of bodies having been reſolved by Sir *Iſaac Newton* into the gravitation of their particles, and the law which is obſerved by the gravity of bodies having been diſcovered from the phænomena deſcribed at length above; it appears from the preceding concluſions, that the gravity of the particles of which the bodies are compounded obſerves the very ſame law. He was likewiſe enabled, by the ſame ſteps, to determine the progreſs of gravity from the centre of any ſphere to the greateſt diſtance from it. At the centre a particle can have no gravity at all, being equally attracted every way by the matter of the ſphere about it. If it is placed within the ſphere at ſome diſtance from the centre, its gravity will be the greater, the greater this diſtance is, by § 9; for theſe parts of the ſphere only having an effect upon it that are at a leſs diſtance from the centre than itſelf, and its gravity being as the attracting matter directly and the ſquare of the diſtance from the centre reciprocally, ſince the matter is as the cube of the ſame diſtance, the gravity muſt be as the diſtance itſelf. From the centre to the ſurface, its gravity increaſes in proportion as its diſtance from the centre increaſes; at the ſurface, its gravity is greateſt; and from the ſurface upwards, its gravity decreaſes in proportion as the ſquare of its diſtance from the centre increaſes; regularly obſerving this law to the utmoſt limits of ſpace. Here we ſpeak of the accelerating power of gravity, which is proportional to the velocity that it is able to generate in any given ſmall moment of time; and ſince it

it generates the ſame velocity in the ſame time in all bodies whatſoever at the ſame diſtance, it follows that their weight or motion ariſing from it, muſt be proportional to their quantities of matter. In general, to eſtimate the weight or motion of any ſphere that is attracted by another whoſe parts are equally denſe at equal diſtances from its centre, we are to meaſure it by compounding three proportions; that of the matter in the heavy bodies that gravitate; that of the matter in the attracting ſpheres to which they gravitate, and the reciprocal proportion of the ſquares of the reſpective diſtances betwixt the centres of the ſpheres that tend towards each other; and this is the law which we found from the phænomena to take place in the ſyſtem. See *art.* 2. of this chapter.

14. Thus Sir *Iſaac Newton* diſcovered and fully deſcribed, from undiſputed obſervations and unexceptionable calculations, this ſimple principle of the gravitation of the particles of matter towards each other; which being extended over the ſyſtem to all diſtances, and diffuſed from the centre of every globe, is the chain that keeps the parts of each together, and preſerves them in their regular motions about their proper centres. The ſame gravity, which is ſo well known to us on the earth, affects them all; the whole maſs of the ſyſtem is, in this reſpect, of a piece; and this one principle, ſo regularly diffuſed over the whole, ſhews one general influence and conduct, flowing from *one cauſe* equally active and potent every where. Several obſervations have been made of late that greatly confirm his doctrine, and particularly ſerve to ſhew that the gravitation towards bodies ariſes from the gravitation towards their particles. Of this kind are the meaſures of a degree on the meridian made lately, with great

great accuracy, by the *French* mathematicians; and the declination of the plumb-line from the true vertical, in consequence of the attraction of a great mountain in the neighbourhood.

CHAP. V.

Of the quantity of matter, and density, of the sun and planets.

1. THUS far our author ascends by way of *analysis*, tracing the causes from their effects, and from the coincidence, or perfect similarity, of many effects, shewing the cause to be more general. But in order to descend by the *synthesis*, and to determine the effects from the cause now known, it was not sufficient to establish the general gravitation of the particles of matter; it was requisite to determine, as far as possible, the quantities of the powers which act in the system. We have seen that there is a gravity extending from each body in the system on all sides, at equal distances from their centres proportional to their quantities of matter. We know, from experience, the force of this power at the surface of our own earth, and have seen how to estimate its efficacy at any other distance. In order to be able to estimate all the powers in the system directed to their different bodies, it is necessary to determine the proportion of their quantities of matter to that of our earth. If this is once obtained, all the powers that operate in the system being known, it will require no more but a skilful application of geometry and mechanics to determine the motions and phænomena of the celestial bodies, which all flow from them.

2.

2. To meaſure the matter in the ſun and planets was an arduous problem, and, at firſt ſight, ſeemed above the reach of human art. But the principles of this philoſophy afforded a natural and eaſy ſolution of it in the moſt important caſes, and Sir *Iſaac Newton* has determined the proportions of the matter that is in the *Sun*, *Jupiter*, *Saturn*, and the *Moon*, to that in our *Earth*; that is, he has ſhewed how many earths might form a *Sun*, a *Jupiter*, or a *Saturn*. To underſtand how he was able to diſcover this, we are to recollect that the matter in each of theſe is in the ſame proportion as the force of gravity toward them, at equal diſtances from their centres. We know the force of gravity towards our earth from the deſcent of heavy bodies, and alſo by calculating how much the moon falls below the tangent of her orbit in any given time. We have no experience of any rectilineal deſcent of heavy bodies toward the *Sun*, *Jupiter*, or *Saturn*; but as the primary planets revolve about the ſun, and their ſatellites revolve about *Jupiter* and *Saturn*, by computing from their motions how much a primary planet falls below its tangent in a given time, and how much any of *Jupiter*'s and *Saturn*'s ſatellites fall below their tangents in the ſame time, we are able to determine the proportion which the gravity of a primary planet to the ſun, and of a ſatellite towards its primary, bears to the gravity of the moon towards the earth, in their reſpective diſtances: then from the general law of the variation of gravity, the forces that would act upon them at equal diſtances from the *Sun*, *Jupiter*, *Saturn*, and the *Earth* are computed; which give the proportion of the matter contained in theſe different bodies.

3.

3. That the quantity of matter in *Jupiter* is greater than the quantity of matter contained in the earth, we may eaſily learn from the motion of his ſatellites; all of which revolve about his centre in leſs time than the moon revolves about the earth, and are all, excepting the firſt, at a greater diſtance from his centre than the moon is from the earth. The ſecond ſatellite is farther diſtant from *Jupiter* than the moon is from the earth in the proportion of 3 to 2 nearly; and moves in an orbit greater in the ſame proportion. But this ſatellite finiſhes its revolution in 3 days, 13 hours, which is leſs than a ſeventh part of the moon's periodic time about the earth; conſequently its motion muſt be much more ſwift than that of the moon. A ſatellite nearer *Jupiter* would move ſtill more ſwiftly than this ſatellite: ſo that if a ſatellite revolved about *Jupiter* at a diſtance from his centre equal to the diſtance of the moon from the earth, it would move much more ſwiftly than the moon moves about the earth, and therefore would be acted on by a much greater centripetal force; for it requires always a greater force to bend into the ſame orbit a body that moves with a greater velocity. But the quantities of matter in the central bodies are proportional to their attractive powers at equal diſtances, and therefore the matter in *Jupiter* muſt very much exceed the matter in the earth. In like manner, we may eaſily obſerve that *Mercury* revolves about the ſun in very little more than thrice the time in which the moon revolves about the earth, and yet moves in an orbit about 140 times greater, being ſo many times farther diſtant from the centre of his motion; from which it is eaſy to ſee that if a ſatellite revolved about the earth as far diſtant from it as *Mercury* is from the ſun, this ſatellite would move vaſtly ſlower than *Mercury*:

whence it follows that the attractive power of the ſun muſt be vaſtly ſuperior to that of the earth, and therefore that the ſun muſt contain vaſtly more matter than the earth. The matter in *Saturn* is alſo found to be greater than that in the earth. From our author's calculations, founded on theſe principles, it follows that the quantities of matter in the *Sun*, *Jupiter*, *Saturn* and the *Earth* are to each other as the numbers 1, $\frac{1}{1067}$, $\frac{1}{3021}$, $\frac{1}{169282}$.

4. The quantities of matter in theſe bodies being thus determined, and their bulk being known from aſtronomical obſervations, it is eaſy to compute what matter each of them contains in the ſame bulk; which gives the proportion of their denſities. Thus our author finds the denſities of the *Sun*, *Jupiter*, *Saturn* and the *Earth*, to be as the numbers 100, $94\frac{1}{2}$, 67 and 400.

From which it appears that the earth is more denſe than *Jupiter*, and *Jupiter* more denſe than *Saturn*; that is, thoſe planets which are nearer the ſun are found to be more denſe, by which they are enabled to bear the greater heat of the ſun. This is the reſult of our moſt ſubtile enquiries into nature, that all things are in the beſt ſituations, and diſpoſed by perfect wiſdom. If our earth was carried down into the orb of *Mercury*, our ocean would boil and ſoon be diſſipated into vapour, and the dry land would become uninhabitable. If the earth was carried to the orb of *Saturn*, the ocean would freeze at ſo great a diſtance from the ſun, and the cold would ſoon put a period to the life of plants and animals. A much leſs variation of the earth's diſtance from the ſun than this would depopulate the torrid zone if the earth came nearer the ſun, and the temperate zones, if it was carried from the ſun. A leſs heat

at

at *Jupiter*'s diſtance is adapted to the greater rarity of his ſubſtance: the conſequences might be as fatal in *Jupiter*, if he was carried into the orb of the earth, as it would be to us to be carried into the orb of *Mercury*. The ſtill greater rarity of *Saturn* is fitted to his more remote orb; ſo that tho' he is the laſt of the planets, and receives 90 times leſs light and heat from the ſun than we do, he may nevertheleſs be in the beſt ſituation that could poſſibly be aſſigned him in the ſyſtem; and there the ſituation of *Jupiter*, and of all the lower planets, may appear as terrible as that of *Mercury* does to us. *Saturn* terminates the planetary revolutions; and, as if the heat of the ſun was too weak in the higher orbs, we find no bodies revolving higher, but ſuch as deſcend in ſome part of their orbit nearer to this great centre of light and heat. Upon the whole we have reaſon to conclude, that they are all diſpoſed in ſuch order, and in ſuch ſituations, from which any conſiderable variation would produce fatal effects. The hypotheſis of *Des Cartes* led him to place the more denſe planets at a greater diſtance from the ſun; but a philoſophy founded on the obſervation of nature correſponds better with the final cauſes of things, and proves, on every occaſion, the wiſdom of the author.

5. As aſtronomers have found no ſatellites revolving about *Mercury*, *Venus*, or *Mars*, we are deprived of the like opportunities of comparing their attractive powers and proportional quantities of matter. But it is highly probable, from what we have ſaid of the *Earth*, *Jupiter* and *Saturn*, that the denſities of the other planets correſpond to their diſtances from the ſun, and are greater in the nearer planets. Our author has alſo computed the proportion of the attractive powers of the *Sun*, *Jupiter*, *Saturn*, and

the *Earth*, at their reſpective ſurfaces, and finds them to be in proportion as theſe numbers, 10000, 943, 529, 435, reſpectively. From which it appears, that the force of gravity towards theſe very unequal bodies approaches ſurpriſingly to an equality at their ſurfaces; ſo that tho' *Jupiter* be ſeveral hundred times greater than the earth, the force of gravity at his ſurface is very little more than double what it is at the ſurface of the earth; and the force of gravity at the ſurface of *Saturn* is but about $\frac{1}{4}$ greater than that of terreſtrial bodies.

6. The moſt conſiderable powers that act in the ſyſtem being thus determined; before we proceed to conſider their effects, it is neceſſary, firſt, to enquire whether they act in a *void*, or if there is any medium that reſiſts the motions produced by them. We find that the air makes a conſiderable reſiſtance to the motion of projectiles near the earth; which, if it extended unto the planetary regions, would alſo very conſiderably affect their motions. But experiments ſhew that the denſity of the air is proportional to the force that compreſſes it, and that the weight of the ſuperincumbent atmoſphere is the force which compreſſes the air in every altitude; ſo that the higher any portion of air is, having a leſs weight of air above it to compreſs it, it muſt have leſs denſity in the ſame proportion: and from this it follows, that if we abſtract from the diminution of gravity, and the altitudes from the ſurface of the earth be taken in arithmetical progreſſion, the denſities of the air at theſe altitudes will decreaſe in geometrical progreſſion *. Since, therefore, it appears from ſeveral experiments, made in *France* and *England*, that the

* See Dr. *Halley* in *Phil. Tranſ.* N° 181. and *Schol.* Prop. 22. Lib. II. Princip.

density of the air decreases in such a manner, that at the height of seven perpendicular miles it is about $\frac{1}{4}$ of the density it has at the level of the sea, at 14 miles it must be $\frac{1}{16}$ of it, at 21 miles $\frac{1}{64}$, at 28 miles $\frac{1}{256}$, at 35 miles $\frac{1}{1024}$, at 42 miles $\frac{1}{4096}$, at the height of 49 miles $\frac{1}{16384}$ part of it, and at the height of a semidiameter of the earth altogether insensible. It appears from the laws of motion, and from many accurate experiments, that the resistance of fluids, arising from the *inertia* of their matter, is proportional to their density; and therefore the resistance of the air, tho' sensible at the surface of the earth, would be 16384 times less at the height of 49 miles, and could not be sensible in the greatest number of ages at the height of a semidiameter of the earth: it must be still less at the distance of the moon, which therefore, meeting with no resistance, continues to revolve for ever in her orbit, without any impediment or diminution of motion. As for a more subtile medium than the air, no experiments nor observations shew that there is any here, or in the celestial spaces, from which any sensible resistance can arise.

BOOK IV.

The effects of the general power of gravity deduced synthetically.

CHAP. I.

Of the centre of the solar system.

1. SIR *Isaac Newton* having established the general principle of the gravitation of the particles of matter, and having determined the chief powers that act in the system, *viz.* those which tend to the *Sun*, *Jupiter*, *Saturn*, and the *Earth*; and having found that the celestial motions are performed in free spaces, where the resistance is insensible; he has now prepared the way for proceeding *synthetically* in his account of the system of the world, and enquiring into the various effects that arise from a power so evidently established. Any general principle ascertained in nature is a great acquisition to philosophy, especially when the variations of this power, with its direction and force, are clearly determined; and the fertility of this principle will appear from the various phænomena resolved by it *synthetically*, of which we are now to treat. Sir *Isaac Newton* begins with enquiring into the centre of the system. The *Pythagoreans* ascribed this place to the centre of the sun, the followers of *Aristotle* and *Ptolemy* to the earth. But Sir *Isaac*, having found that these gravitate towards each other and towards all the other bodies in the system, neither

of them, nor indeed any body in the ſyſtem, can be ſuppoſed to be void of all motion.

2. It is the centre of gravity of the whole ſyſtem that is the only point which can be ſuppoſed quieſcent in it; the ſame point about which all the matter of the ſyſtem would ſoon be accumulated, if the progreſſive motions of the bodies in it were deſtroyed, and their gravity was permitted to bring them together. The mutual actions of bodies on each other never affect the ſtate of this centre; their attracting or repelling each other produces no effect upon it; and it muſt either be quieſcent, or proceed uniformly in a right line. All ſeem agreed that the centre of the ſyſtem is at reſt, and no reaſon or obſervation argues for our aſcribing any motion to it. The centre of gravity of the ſyſtem is, therefore, the only immoveable point, while all the bodies in the ſyſtem move round it with various motions.

3. As we have our knowledge of gravity, and the laws of nature, from what paſſes on the ſurface of the earth, we cannot illuſtrate the motions of the bodies of the ſolar ſyſtem, ariſing from their mutual gravity, better than by ſome images we find of them on the earth, after having ſhewn ſo fully the ſimilarity of the powers that act on the parts of the earth and on the celeſtial bodies. We know that when, by any power or machine, a body is projected in the air, the power reacts on the earth with an equal force, and that if the power was ſufficient to project a mountain or a much larger part of the earth, it would act on the remainder of the earth with an equal force, in an oppoſite direction; ſo that while the projected part began to move in its curve, the remainder of the earth would begin at the ſame time to move in an oppoſite direction, with

an equal quantity of motion, but with a velocity ſo much leſs as the matter in it is greater than in the projected part; and both would revolve in certain orbits about the common centre of gravity, which would continue in the ſame ſtate as before the projection. If, by the reſiſtance of the medium, the motions of theſe parts of the earth came to be deſtroyed, they would come together again and be accumulated in one maſs about the ſame centre. If there were more ſuch parts of the earth projected, the centre of gravity of all would be no way affected by ſuch projections, but they would move round it, ſo that the ſum of the motions on one ſide of it ſhould be equal to the ſum of the motions on the other ſide: and this obtains even in thoſe ſmall motions that are every day produced by powers and agents on the earth.

4. The motions of the great bodies in the ſolar ſyſtem are analogous to theſe: the different parts of the ſolar ſyſtem gravitate to each other, as the parts of the earth gravitate towards one another; and the different parts of the ſyſtem move in the ſame manner about their common centre of gravity, as the parts into which we ſuppoſed the earth to be divided, if projected in any direction, would all move about their common centre of gravity; or as the earth, and all the bodies that are actually projected every day on its ſurface, revolve about the common centre of gravity of the earth and theſe projectiles. Only there is this difference, that the bodies of the great ſyſtem were projected at great diſtances from each other, and in ſuch a manner that the planets revolve in orbits almoſt circular, ſo as not to come too near to the ſun, or to be carried too far from him, in their revolutions. The creator of the world had in vain made them of denſities adapted to certain diſtances,
if

if he had not projected them with the forces that were requiſite to preſerve them revolving at thoſe diſtances, or near to them; and as the greatneſs of the force impreſſed on thoſe vaſt bodies, ſome of which are many times greater than our earth, ſhews the power, its juſt quantity, varied regularly in the different diſtances of the planets, and its proper direction, ſhew the ſkill of the *firſt mover.*

5. We may ſuppoſe that all the matter of which the ſyſtem conſiſts was formed firſt in one maſs, where now the centre of gravity of the whole ſyſtem is found; that of this maſs various bodies were formed, and ſeparated from each other to proper diſtances, where they received their projectile motions; and that the powers which ſeparated and moved them obſerved the law of nature that requires an equality between action and reaction, and is obſerved in all the actions of powers at preſent: and thus theſe motions would begin, and continue for ever, without producing any motion in the centre of gravity of the ſyſtem.

6. When the bodies were thus moved in their juſt orbits, we may conceive ſome of them to have been ſubdivided again, by actions obſerving the ſame laws, into ſeveral other bodies, which in like manner were formed into leſſer ſyſtems; as that of the earth and moon, thoſe of *Jupiter* and *Saturn* and their ſatellites. There is not any of theſe quieſcent in its particular ſyſtem; the earth and moon move about their common centre of gravity, while it is carried with a regular motion round the centre of gravity of the whole ſyſtem. The ſame is to be ſaid of *Jupiter* and *Saturn* and their ſatellites; and it is certain from the laws of nature, that the motions in any leſſer ſyſtem about its centre of gravity, and the motion of

of that centre about the centre of gravity of the whole ſyſtem, interfere not with each other. A leſſer ſyſtem being thus formed, one of the bodies that compoſe it might be ſubdivided into leſſer bodies that might form a ſyſtem of an inferior order. But we do not find that nature carries this ſubordination ſo far, unleſs we would conſider the motion of projectiles, near the ſurfaces of the ſecondary planets, as an example of this kind.

7. It is next to be conſidered, where this point of reſt of the common centre of gravity of the ſyſtem is to be found; and it is plain from what we have already ſeen, that it can never be far removed from the ſun, becauſe the matter in the ſun vaſtly exceeds the matter in all the planets taken together: and, from what we ſaid of the centre of gravity above, it appears that it is always nearer the greater body in proportion as it is greater. *Jupiter* is the largeſt of the planets, and yet is but $\frac{1}{1067}$ of the ſun, ſo that their centre of gravity muſt be 1067 times nearer the ſun than *Jupiter*; and as the diſtance of *Jupiter* is little more than 1067 ſemidiameters of the ſun, it follows that the centre of gravity of the ſun and *Jupiter* cannot be much above the ſurface of the ſun. *Saturn* is leſs than *Jupiter* both in bulk and denſity, and the centre of gravity of the *Sun* and *Saturn* falls within the body of the ſun: and thus it eaſily appears, that tho' all the planets were on one ſide of the ſun in one line, the centre of gravity of the ſun and them all could ſcarcely be above a ſemi-diameter of the ſun from his ſurface: and this is the fartheſt that the ſun is ever removed from that centre. It appears, therefore, that tho' the ſun is in perpetual agitation about this centre, yet, being always ſo near it, he may very well be conſidered by aſtronomers as the centre of the ſolar ſyſtem. Thus, tho' the ter-

terraqueous globe receives an impreſſion from every power that move projectiles in the air, and is, to ſpeak accurately, agitated a little by theſe powers with a very complex motion, yet we conſider it as at reſt, neglecting ſuch exceeding minute actions and their effects.

CHAP. II.

Shewing how gravity produces ſome ſmall irregularities in the motions of the planets.

1. IF the planets were acted on by a power directed to the centre of the ſun only, varying according to the general law of gravity, and that centre was quieſcent, their motion about it would be perfectly regular. But we found that each of the planets was acted on by a power directed to every body in the ſyſtem. In order to judge of the effects of theſe actions, our author firſt ſuppoſes two bodies equally gravitating towards each other, and revolving about their common centre of gravity: and, ſince the direction of their mutual gravitation paſſes always from the one to the other through their centre of gravity, and their diſtances from it vary always in the ſame proportion as their diſtances from each other: it follows, that they muſt deſcribe equal areas in equal times about that centre, and about each other, and deſcribe ſimilar figures about that point and about each other *. So that in the motions of two bodies no irregularities ariſe in their motions about each other from their mutual attractions; whatever the law of their gravity be ſuppoſed to be: only they will finiſh their revolutions about

* *Princip.* Lib. I. Prop. 58.

the

the centre of gravity in leſs time than if the one was to revolve about the other quieſcent, at the ſame diſtance, and with the ſame centripetal force; becauſe the orbit deſcribed about the centre of gravity being leſs than that which is deſcribed by any one of them about the other quieſcent (their diſtance from each other being equal in both caſes) and being alſo ſimilar to it, it muſt be deſcribed in leſs time.

2. If three or more bodies mutually attract each other, the gravitation of any one, ariſing from the actions of the reſt, may be determined by the rule for the compoſition of motion; and if the law of gravity be ſuch as we find to obtain in the ſolar ſyſtem, its gravitation will not be always directed to the centre of gravity of the other bodies, or indeed to any fixed point, but ſometimes to one ſide of that centre and ſometimes to the other; and therefore, equal areas will not be deſcribed in equal times about any point in the ſyſtem, and ſeveral irregularities will neceſſarily ariſe in the motions of the bodies. But if you ſuppoſe one of theſe bodies to be vaſtly greater than the reſt, ſo that the actions of the other bodies may be neglected if compared with its action, and the centre of gravity of the ſyſtem be always found near it, then the irregularities in the motions in ſuch a ſyſtem will be very ſmall. The areas deſcribed in equal times, about the centre of that great body, will be nearly equal, and the orbits deſcribed will be nearly elliptic, having that centre in their focus. That this is the caſe of the ſun and planets, appears from what we have ſhewn concerning their quantities of matter: and thus we ſee that not only the regular motions of the planets are to be derived from the principle of gravity, but alſo how their minute errors and irregularities are accounted for from it. The ſame is the caſe of *Jupiter* and *Saturn*

their *ſatellites.* As for the *Earth* and the *Moon*, tho' there be a leſs diſproportion in their magnitudes, and their common centre of gravity be ſenſibly removed from the earth, yet as there are only two in their ſyſtem, no irregularities ariſe from their mutual actions in their motions about their common centre of gravity, or they are eaſily determined when the poſition of their centre of gravity is known. Theſe leſſer ſyſtems of the *Earth*, *Jupiter*, and *Saturn*, are carried about the centre of gravity of the general ſolar ſyſtem, without receiving any diſturbance from any action of the ſun or planets, which is equal on all their parts and in the ſame direction. When a fleet of ſhips is carried away by a current that affects them equally, it has no effect on their particular motions amongſt themſelves, nor is the motion proceeding from the current diſcovered by them, if they have no body in ſight that is not affected by it. In the ſame manner, if the gravity towards the ſun acted equally, and in the ſame direction, on the parts of theſe leſſer ſyſtems, it would have no effect on their motions amongſt one another, and could only be diſcovered by comparing their motions with the fixed ſtars, or with ſome body foreign to that leſſer ſyſtem, which is acted on in a different manner by the ſun. But as there is ſome variation in the actions of the ſun upon the parts of theſe ſyſtems, and in the directions of theſe actions, from hence ſome irregularities neceſſarily ariſe.

3. Tho' the actions of the ſun and of the inferior planets, compounded together, do not always produce in a ſuperior planet a gravitation exactly directed towards their centre of gravity; yet, as upon the whole it is more nearly directed to that point than to any other, the motions of a ſuperior planet will be found more regular by ſuppoſing that point

to

to be the centre of its attraction, rather than any other, and its ellipſe will be juſt by placing its *lower focus* there. A planet that is higher than this will, by its attraction, have ſome effect on the motion in this ellipſe, but as it alſo acts on the inferior planets at the ſame time, there will no irregularity ariſe from that part of its action which is equal and in the ſame direction on them all, but from the differences of its actions only; which being exceedingly minute, and having contrary effects in the oppoſite ſituations of that higher planet, can produce effects ſcarcely ſenſible in many revolutions.

4. The action of *Jupiter* on *Saturn* when greateſt (that is in their conjunction when their diſtance is leaſt) is found to be $\frac{1}{204}$ of the action of the *Sun* upon *Saturn*, by comparing the matter of *Jupiter* with the matter in the *Sun*, and the ſquare of the diſtance of the *Sun* from *Saturn*, with the ſquare of the diſtance of *Jupiter* from *Saturn*. The effect of this action on *Saturn* is not altogether inſenſible. But the elliptic orb of *Saturn* will be found to be more juſt, if you ſuppoſe its focus not to be in the centre of the *Sun*, but in the centre of gravity of the *Sun* and *Jupiter*, or rather in the centre of gravity of the *Sun* and of all the planets below *Saturn*. In the ſame manner, the elliptic orb of any other planet will be found more accurate, by ſuppoſing its focus to be in the centre of gravity of the *Sun* and all the planets that are below it.

5. The whole action of *Jupiter* diſturbs the motion of *Saturn* in their conjunction, becauſe *Jupiter* acts upon *Saturn* and upon the *Sun* with oppoſite directions, at that time. But, becauſe *Saturn* acts then in the ſame direction on *Jupiter* and on the *Sun*, if it acted alſo with the ſame force on both, it would

would have no effect on the motion of *Jupiter* about the *Sun*, and it is by the excefs of its action on *Jupiter* above its action on the *Sun* that it difturbs the motion of *Jupiter*. This excefs is found to be $\frac{1}{1923}$ of the action of the *Sun* on *Jupiter*, and therefore is much lefs than the force with which *Jupiter* difturbs the motion of *Saturn*. The actions of the other planets on each other are incomparably lefs than thefe, and the irregularities proceeding from thofe actions are always lefs in any planet as it is nearer the fun. Only the orbit of the earth may appear a little more irregular than that of its neighbouring planets, becaufe it revolves about the centre of gravity of the earth and moon, while that centre annually revolves about the fun.

6. If the planets were attracted by the fun and by one another, but the fun was not reciprocally attracted by them, the centre of gravity of the fyftem, becaufe of the deficiency of this reaction, would neceffarily be in motion; and this would be a new fource of errors and irregularities. If the primary planets were not attracted by their fatellites, as well as the fatellites by their primary planets, other irregularities would neceffarily arife. If the great planets, *Jupiter* and *Saturn*, had moved in the lower fpheres, their influences would have had much more effect to difturb the planetary motions. But while they revolve at fo great diftances from the reft, they act almoft equally on the fun and on the inferior planets, and have the lefs effect on their motions about the fun, and the motions of their fatellites are at the fame time lefs difturbed by the action of the fun. The earth and moon move in a lower fphere, but their motions are the lefs irregular becaufe there are only two in their fyftem. We fhall aftewards fee that the comets continue for a very fmall time

among the planetary ſpheres, and that in the far greater part of their revolutions they are carried to ſuch vaſt diſtances that their actions can have very little effect on the motions of the planets. Such is the law of gravity, and the manner of its operation, and ſuch is the diſpoſition of the bodies in the ſyſtem, as ſeems well adapted for preſerving their motions with great regularity; but this will appear ſtill more fully from the following chapter.

CHAP. III.

Of the approach and receſs of the planets to and from the ſun, in every revolution.

1. THUS far we have conſidered the powers that act in the ſyſtem of the ſun, and have found that thoſe which produce the regular motions of the planets vaſtly exceed thoſe that diſturb them. We are next to conſider how the motions in their orbits proceed from the action of thoſe powers; and how the planet is made to aſcend and deſcend by turns, at the ſame time that it revolves about the centre of its gravitation. This requires an illuſtration, the rather becauſe we have nothing ſimilar to it in the motion of heavy bodies at the earth's ſurface; for theſe are always made to fall to the earth by their gravity: in whatever direction they are projected, upwards, perpendicularly, or obliquely, their gravity ſoon brings them down to the earth again. Hence many find it hard to conceive how a planet after approaching to the ſun can recede from it again, eſpecially ſince its gravity is increaſed as its diſtance decreaſes. They imagine that it ought to continue to approach to the ſun, and at length fall upon his body, as heavy bodies fall to the earth.

2. But we are to remember, that the force with which heavy bodies are projected, from our moſt powerful engines, is inconſiderable, compared with the motions which their gravity could generate in them in a few minutes; and they move over ſuch ſmall ſpaces, when compared with their diſtance from the centre of the earth, that their gravity is conſidered as acting in parallel lines, without any ſenſible error, ſo that the centrifugal force ariſing from the rotation about that centre is altogether neglected. But when we examine the motion of a projectile in larger ſpaces, and trace it in its orbit, we muſt conſider the action of gravity as directed to a centre, and take in the centrifugal force ariſing from its motion of rotation about that centre; and it will appear, that there are indeed ſome laws of gravity which would make the body approach to the centre continually, till it fall into it, but that there are other laws which make bodies approach to the centre, and ſuffer them to recede from it, by turns. How to diſtinguiſh theſe we ſhall now conſider.

In the firſt place, it will be eaſily underſtood that if S (*Fig.* 63.) be the centre of attraction, and a body is projected with a certain force in the line AE, perpendicular to AS, it will deſcribe the circle AL*a* with an equable motion, and after a complete revolution return to its firſt place A, with its firſt motion. The ſame gravity that acted at A upon it, and carried it below the tangent AE, acts upon it at any other point L, at an equal diſtance from the centre S, and brings it from the tangent at L thro' the ſame length in the ſame time. The centrifugal force, ariſing from its rotation, being equal to its gravity, neither of them prevails, and the body therefore neither approaches to the centre nor recedes from it.

If you ſuppoſe the motion of projection at A to be increaſed, the gravity neceſſary to keep it in the ſame circle muſt be increaſed alſo; ſo that if the velocity of the projection be double, the gravity requiſite to retain the body in the ſame circle muſt be quadruple; becauſe A K being double of A L, the point K falls four times farther below the tangent than the point L, as we ſhewed above: in general, the gravity neceſſary to retain a body in the ſame circle is in the duplicate proportion of the motion of projection; and the velocity, therefore, in the ſubduplicate proportion of the gravity; ſo that when the gravities are as 1 to 4, the velocities are as 1 to 2.

3. If the body is projected at a leſs diſtance from the centre of attraction, as at D, with the ſame velocity, the gravity muſt be greater to retain it in a circle; becauſe the curvature being greater, the extremity P of the arc D P, equal to A L, falls farther below the tangent at D, than L falls below the tangent at A, in proportion as the arc D P is more curve, that is, in proportion as the diſtance S D is leſs than S A. If the velocity of projection is increaſed at D, ſo that the body deſcribe a greater arc D Q in the ſame time, then the force of gravity, neceſſary to retain the body in a circle there, muſt be increaſed in a duplicate proportion; becauſe Q T is to P R in the duplicate proportion of D Q to D P. If the velocity at D, for example, is greater than that at A in proportion as S A is greater than S D, then Q T will be to P R as the ſquare of S A is to the ſquare of S D, and Q T will be to L M as the cube of S A is to the cube of S D; that is, the force requiſite to retain bodies in circles muſt be reciprocally as the cubes of the ſemidiameters, when the velocities in theſe circles are reciprocally as the ſemidiameters themſelves; and *converſely*,

verſely, if the gravities increaſe as the cubes of the diſtances from the centre decreaſe, the velocities neceſſary to carry bodies in circles, at different diſtances from the centre of attraction, muſt increaſe in proportion as the diſtances decreaſe.

4. In general, as the gravities of bodies that deſcribe circles about the ſame centre increaſe in proportion as the ſquares of the velocities increaſe, and as the diſtances decreaſe; it follows converſely, that, in order to compare the velocities of projection that are neceſſary to carry bodies in circles at theſe different diſtances, we muſt compound the proportion of the gravities and the proportion of theſe diſtances together, for this compounded proportion will give that of the ſquares of the requiſite velocities. So in the ſolar ſyſtem, if the diſtances of two planets were as 1 to 4, the gravities being as 16 to 1, theſe proportions compounded give that of 16 to 4, or of 4 to 1, which is that of the ſquares of the velocities, and therefore the velocities themſelves are as 2 to 1. In like manner we can determine the law according to which the velocities, neceſſary to carry bodies in circles about S, vary at any diſtances, in any given law of gravity.

5. If a body is projected at A (*Fig.* 64.) with a velocity leſs than that which is neceſſary to carry it in a circle there, it muſt fall within the circle, the centrifugal force, ariſing from the motion of rotation about S, is leſs than that which it would have in the circle AL, in proportion as the ſquare of its velocity is leſs, and is therefore leſs than its gravity in the ſame proportion: the body, therefore, by the exceſs of its gravity above its centrifugal force, is made to approach to the centre. The motion of the body, as it deſcends in the orbit AMB, muſt be

accelerated ſo as to deſcribe equal areas in equal times about s, and the velocity of its motion at M muſt be greater than its velocity at A, in proportion as SA is greater than SP, the perpendicular from S on the tangent to its orbit at M; becauſe if the arcs AK, MN, be deſcribed in the ſame time, the triangular ſpaces ASK, MSN, being equal, the baſes AK, MN muſt be reciprocally as their altitudes SA, SP, and the velocities are as the arcs AK, MN, deſcribed in the ſame time, and therefore reciprocally as SA, SP. The velocity, therefore, in the orbit from A to M, increaſes in a higher proportion than that in which the diſtances SA, SM decreaſe, becauſe SA is to SP in a higher proportion than SA is to SM: only if the direction of the body ever become perpendicular again to the ray drawn from S, at any point, as B, there SM and SP will coincide, and the proportion of the velocities will be the ſame as the reciprocal of the diſtances SA, SB.

6. If a body is projected at B in a direction perpendicular to SB, with a velocity greater than that which is neceſſary to carry it in the circle BGH about the centre of attraction, at the diſtance SB, it muſt be carried without that circle, and recede from the centre S. The centrifugal force, in this caſe, ariſing from its motion of rotation, is greater than that which would ariſe from its motion in the circle BGH, and therefore greater than its gravity; and by the exceſs of its centrifugal force above its gravity, it recedes from S the centre of attraction. The motion of the body decreaſes as it riſes, being retarded by the action of its gravity, ſo that the velocity is always leſs than the velocity at B, in proportion as SB is leſs than S*p*, the perpendicular from S on the direction of its motion.

7.

7. A planet deſcends from A, which is called its higher *apſis*, to B, which is called its lower *apſis*, and reaſcends again from B to A. It deſcends from A, approaching to the centre of attraction, becauſe its velocity at A is leſs than that which would be able to carry it in a circle about S, at the diſtance SA. As it deſcends to leſſer diſtances, its velocity in its orbit increaſes in a higher proportion than the velocities, which would be ſufficient to carry bodies in circles at theſe diſtances, increaſe. For the velocity in the orbit at B is greater than that at A, in proportion as SA is greater than SB; whereas the velocity in a circle at B is greater than the velocity in a circle at A, as $\sqrt{SA}$ is greater than $\sqrt{SB}$. If SA were to SB as 4 to 1, the firſt proportion would be that of 4 to 1, but the ſecond that of 2 to 1 only. Hence it appears how the velocity in the orbit at B, exceeds that in a circle at the ſame diſtance, tho' the velocity in the orbit at A was exceeded by the velocity that was able to carry it in a circle at the diſtance SA. In the higher part of the orbit, the velocity of the body is leſs than that which would carry it in a circle there about S; but the velocity in the orbit increaſes more, by the approach of the body to the centre of attraction, than the velocities requiſite for carrying bodies in circles do, and ſo gets the better of them in the lower part of the orbit. Of theſe two each prevails over the other by turns, in the two apſides; the velocity in the circle in the higher apſis, and the velocity in the orbit in the lower apſis. After the body is carried off at B by its ſuperior velocity, the velocity in a circle afterwards gets the better, becauſe it does not decreaſe ſo quickly as the velocity in the orbit, and the body is made to move, in its aſcent, in a ſemi-ellipſe equal and ſimilarly ſituated to that which it deſcribed in its deſcent.

8. The gravity indeed at B is greater than the gravity at A, in proportion as the ſquare of the diſtance is leſs. But the centrifugal force ariſing from the circular motion about S increaſes in a higher proportion, *viz.* as the cubes of the diſtances decreaſe; for theſe centrifugal forces are in the direct proportion of the ſquares of the velocities and their inverſe proportion of the diſtances, compounded together: the firſt of theſe is the inverſe proportion of the ſquares of the diſtances, and the two together compound the inverſe proportion of the cubes of the diſtances. The centrifugal forces, therefore, increaſe more quickly than the gravities; and tho' the gravities prevail in the higher part of the orbit, the centrifugal forces get the better in the lower part of it. The gravity prevailing in the higher apſis makes the body approach to S, the centrifugal force prevailing in the lower apſis makes the body recede from it; and, by their actions, the body for ever revolves from the one to the other.

9. It is eaſy to ſee from what we have ſaid, that the body can deſcend from the higher apſis to the lower, and aſcend again from the lower apſis to the higher, when the velocities neceſſary to carry bodies in circles about the centre of attraction increaſe, in approaching to that centre, in a leſs proportion than the velocity of a body moving in an orbit AMB increaſes. For tho' the velocity in a circle in the greater diſtances exceed the velocity in the orbit, this latter, by increaſing more quickly as the diſtance decreaſes, gets the better of the other in the lower part of the orbit, and carries the body off again. But if the velocities by which circles can be deſcribed about the centre of attraction increaſe, in approaching to that centre, in a higher proportion, or in the

ſame

ſame proportion, as the velocity in the orbit increaſes, then this latter having been ſuppoſed at A leſs than the former, it muſt always continue leſs than it, and never get the better of it, ſo as to be able to carry off the body; and therefore, in all ſuch caſes, the body can never recede from the centre after it has once begun to approach to it, but muſt deſcend to diſtances leſs and leſs, till it fall into the centre. It approaches at A, becauſe its velocity is leſs than that which is requiſite to carry it in a circle there: its velocity indeed increaſes as it deſcends to leſſer diſtances, but the velocities which would carry bodies in circles at theſe diſtances about S, increaſing alſo in as great a proportion, the velocity in the orbit muſt ſtill continue to be leſs than in theſe circles, and the body muſt ſtill continue to approach to the centre.

10. To fix the limit of theſe two caſes, we are to conſider, that the velocities in an orbit, at A and B, are in the inverted proportion of the diſtances there from the centre of gravitation; and that, if the gravity increaſe as the cubes of the diſtances decreaſe, the velocities neceſſary to deſcribe circles at A and B are in the ſame inverted proportion of the diſtances at A and B from S. In this caſe, therefore, the velocities in circles, and in the orbit at A and B, vary in the ſame proportion, and the ſame which exceeds at the one diſtance muſt exceed at the other; ſo that, for the ſame reaſon for which the body approached to S at A, it would approach to it at B, and if it receded from it at B, it muſt recede from it at A; that is, if it once begin to approach, it muſt always approach to S, and if it once begin to recede, it muſt always recede from it. This alſo appears from what we ſaid of the centrifugal force, which, in the ſame orbit, increaſes as the cube of the diſtance decreaſes; and conſequently in the ſame proportion

in which the gravity is ſuppoſed to increaſe in this caſe; ſo that, of theſe two, which ever is ſuppoſed to prevail in any one apſid, the ſame muſt prevail in any other apſid, if ſuch could be aſſigned; and the body muſt either deſcend continually to the centre, or riſe from it for ever.

11. If the gravity increaſe in a higher proportion than as the cubes of the diſtances from the centre of attraction decreaſe, then the velocities neceſſary to carry bodies in circles about that centre, in approaching to it, will increaſe in a higher proportion than the diſtances decreaſe; that is, in a higher proportion than the velocity in an orbit increaſes from A to B; ſo that as the velocity in a circle at A exceeded the velocity in the orbit there, it will much more exceed it at B, and therefore the body, acted on by a gravity varying in ſuch a manner, muſt approach to the centre till it fall into it, if it once begin to approach to it at A; and if it once begin to recede from it, it muſt continue to recede from it for ever. The higher the power of the diſtance is to which the gravity is reciprocally proportional, the body will deſcend in a leſs number of revolutions to the centre, in like circumſtances. If the gravity is reciprocally proportional to the cubes of the diſtances, the body will deſcend after an infinite number of revolutions. If the gravity increaſe as the 4th power of the diſtance decreaſes, and the body is projected at A with a velocity leſs than that which would carry it in a circle about S in proportion as $\sqrt{2}$ is leſs than $\sqrt{3}$, the body will deſcribe a certain *epicycloid* about S, and fall into it after half a revolution. If the gravity increaſe as the 5th power of the diſtance decreaſes, and the velocity of the projection be to that which would carry it in a circle about the centre S as 1 is to $\sqrt{2}$, it will deſcend in a ſemicircle deſcribed on the

the diameter S A, and fall into the centre in a quarter of a revolution. If the gravity increaſe as the 7th power of the diſtance decreaſes, and theſe velocities be as 1 to $\sqrt{3}$, it will fall into the centre in $\frac{1}{8}$ of a revolution. In general, if the gravity increaſe as the $n+3$ power of the diſtance decreaſes, and the velocity of projection at A be to the velocity which would carry the body in a circle there, about S, as 1 to $\sqrt{1+\frac{n}{2}}$, it will fall into the centre in the $\frac{1}{2n}$ part of a revolution. If the gravity increaſe as the $3\frac{1}{100}$ power of the diſtance decreaſes, and the velocities be as 1 to $\sqrt{1+\frac{1}{200}}$, the body muſt fall into the centre after 50 revolutions. We cannot pretend to demonſtrate theſe things here, and have mentioned them only to illuſtrate this theory *.

12. If the gravity increaſe in a leſs proportion than that in which the cubes of the diſtances decreaſe, the velocities, neceſſary to carry bodies in circles about the centre S, will increaſe, in approaching to it, in a leſs proportion than the ſimple proportion in which the diſtances decreaſe, and therefore in a leſs proportion than the velocity in the orbit from A to B; ſo that, tho' the former exceed in the greater diſtances, the latter may exceed in the leſſer diſtances, and the body may conſequently deſcend from the higher apſis to the lower, and aſcend from the lower apſis to the higher by turns. The gravity may prevail over the centrifugal force in the higher parts of the orbit, but, increaſing more ſlowly in deſcending to the leſſer diſtances than the centrifugal force, it is overcome by it in the lower parts of the orbit, and the body is made to recede again to its firſt diſtance. If the gravity increaſe as the cubes of the diſtances decreaſe, the body never can arrive at the

* See *Treatiſe of Fluxions. Art.* 437.

lower

lower apsis B. If the gravity increase as the squares of the distances decrease, the body will descend in a semi-ellipse from the higher to the lower apsis in half a revolution.

13. If the gravity increase in the reciprocal proportion of some power of the distance betwixt the square and cube, the body will take more than half a revolution to descend from the higher to the lower apsis, the more the increase of gravity approaches to the reciprocal proportion of the cubes of the distances; for the velocity in the orbit will find the more difficulty to get the better of the motion that would carry the body in a circle, or the centrifugal force will with more difficulty get the better of the gravity. But if the gravity increase in proportion as some power of the distance less than the square decreases, the velocities in circles increasing less in approaching to the centre, the velocity in the orbit will the more easily prevail, and the centrifugal force will sooner exceed the gravity; and therefore the body will descend to the lower apsis in less than half a revolution, and return to the higher apsis in less than a complete revolution. From which it appears, that as the apsides are fixed in the regular course of gravity, that is, while it increases as the squares of the distances decrease, they must be carried forwards, in the direction of motion of the body, when gravity varies in a higher proportion than that, and must be carried backwards with a contrary motion when gravity varies less than in that proportion. As a change from the proportion of the squares to that of cubes gives an infinite motion to the apsides, so that the body never arrives at either of them again; a very small change in the course of gravity will produce a sensible motion in the apsides, and the least change from the regular

course

courſe of gravity muſt become very ſenſible, in a great many revolutions, by the motion of the apſides. From which we learn, that ſince the apſides of the planets have ſo ſmall a motion that ſome aſtronomers neglect it altogether, and doubt if there is indeed any ſuch motion at all, we may conclude that their gravity muſt obſerve very accurately, in its variations, the law of the ſquares of the diſtances.

14. Our author, to reduce to a computation the motion of the apſides ariſing from a variation from the regular courſe of gravity, ſuppoſes, with aſtronomers, that the body moves in an ellipſe that is carried at the ſame time with a regular motion about S, which, in an entire revolution, gives the motion of the apſides. In a quieſcent ellipſis, (*Fig.* 65.) the curvature at A and B being the ſame, the centripetal forces there were found, above, to follow the inverſe proportion of the ſquares of the diſtances SA, SB. Suppoſing that the body moves in the ellipſe *alb*, while this ellipſe itſelf is carried about S with an angular motion, ſo that, S*l* in the moveable orbit being equal to SL in the fixed orbit, the angle AS*l* may be to ASL in a conſtant invariable proportion, ſuppoſe that of G to F; then the increments of theſe angles, while SL and S*l* decreaſe equally, will obſerve the ſame conſtant proportion; and the angular motions about S of two bodies *l* and L, revolving in the ſame time in theſe orbits, will be in the ſame proportion, as alſo the areas deſcribed by rays drawn from theſe bodies to S: ſo that if the bodies be projected together at A with velocities in the ſame proportion, and are acted on by the neceſſary centripetal forces, they will move in theſe orbits, and approach equally towards S, and arrive at *l* and L in the ſame time. The motion of approach to the centre being the ſame at equal diſtances from it, and

and this motion being caufed by the excefs of their gravities above the centrifugal forces arifing from their circular motions about S; the gravity will exceed the centrifugal force in the one orbit by the fame excefs as in the other, and therefore the difference of the centrifugal forces muft be the fame as the difference of their gravities; fo that, to find the gravity in the moveable orbit, we are to add to the gravity in the fixed orbit, at the fame diftance, the excefs of the centrifugal force in the moveable orbit there, above the centrifugal force in the fixed orbit at the fame diftance. Thefe centrifugal forces are in a given proportion to each other, *viz.* in that of the fquares of the angular motions, or in the proportion of G^2 to F^2, and their difference muft be in a given proportion to either; the fame centrifugal forces, at different diftances, are reciprocally as the cubes of the diftances, as we fhewed above, and their differences muft vary in the fame proportion: fo that the difference of the gravities in the moveable and immoveable, muft vary in the reciprocal proportion of the cubes of the diftances.

15. If the ellipfe is carried about S with a progreffive motion, that is in the direction of the motion of the body, the angular motion of the body in the moveable orbit is greater than in the fixed orbit, and the centrifugal force, and confequently the gravity, is greater. But if the ellipfe is carried about S with a retrograde motion, the angular motion in the moveable orbit, and confequently the gravity, is leffer. In the firft cafe, the difference of the centrifugal forces is to be added to the gravity in the fixed orbit, to find the gravity in the revolving orbit at the fame diftance from S. In the latter cafe, the difference of the centrifugal forces is to be fubtracted from the gravity in the fixed orbit, to find the

gravity in the revolving orbit at the ſame diſtance from s.

16. The force in the fixed ellipſe increaſes as the ſquare of the diſtance decreaſes; add to this a force that increaſes as the cube of the diſtance decreaſes, and the ſum muſt increaſe in a higher proportion than that of the ſquares of the diſtances, but never in ſo high a proportion as their cubes. A body therefore that moves in an ellipſe that has itſelf a progreſſive motion about s, muſt be acted on by a force that varies according to ſome power of the diſtance higher than the ſquare, but leſs than the cube. The greater this motion of the ellipſe is, the greater is the exceſs of the centrifugal force in the moveable ellipſe above that in the fixed ellipſe, at the ſame diſtance from s; and the greater is the quantity that varies as the cube of the diſtance in the aggregate, in proportion to that which varies in it as the ſquare of the diſtance only; and the more does the proportion of the aggregate vary from that of the ſquares towards that of the cubes of the diſtances. In ſuch a moveable ellipſe, the gravity, which is as the aggregate, cannot be ſaid to vary in the proportion of any one power of the diſtance accurately; but if the ellipſe is very near to a circle, the proportion of the aggregate will be found to vary very nearly as a certain power of the diſtance, and the motion of the ellipſe may be adjuſted ſo as that the aggregate may vary, very nearly, as any power of the diſtance that can be aſſigned betwixt the ſquares and the cubes.

17. If from a force that increaſes as the ſquare of the diſtance decreaſes, you ſubduct a force that increaſes in a higher proportion, *viz.* as the cube of the diſtance decreaſes, the remainder muſt increaſe in

in a leſs proportion than that in which the ſquare of the diſtance decreaſes. A body, therefore, that moves in an ellipſe which revolves itſelf at the ſame time with a retrograde motion about s, muſt be acted on by a gravity that varies in a leſs proportion than the ſquare of the diſtance; and the greater the motion of the ellipſe is, the gravity will vary in a leſs proportion, ſo that if the motion of the ellipſe be ſufficiently great, the gravity may decreaſe inſtead of increaſing as the diſtance decreaſes. By ſuppoſing the orbit near to a circle, the motion of the ellipſe may be adjuſted, that the remainder may vary according to any proportion leſs than that of the ſquares of the diſtances.

18. Our author has made an improvement of this, to judge of the motion of the apſides in any law of gravity: for, by ſuppoſing the gravity in the moveable ellipſe, when near to a circle, computed from the foreſaid principles, to vary according to any given law, he determines what muſt be the motion of the ellipſe, or of the apſides, in conſequence of this ſuppoſition; or, the motion of the ellipſe being given, he determines what is the power of the diſtance according to which the gravity varies, nearly, when the ellipſe revolves with that given motion *.

19. We have ſaid as much as our deſign will allow us, of the motions ariſing from gravity, that are performed in regular revolutions from the one apſis to the other; where the diſtance from the centre of gravitation varies indeed, but ſo as to keep within certain limits, betwixt which the body conſtantly revolves; and we have ſhewn that the motion

* See *Princip.* Lib. I. Sect. 9.

of

of the body may be of this kind, if the gravity decreaſe in a leſs proportion than that in which the cubes of the diſtances from the centre increaſe. But the motion of the body is not always of this kind, in theſe caſes; for if the velocity of projection at B is ſufficiently great, the body will, in ſome of theſe caſes, recede for ever from the centre of gravitation, and never arrive at the higher apſis A. We have already ſhewed that if the gravity decreaſe as the cubes, or any higher powers, of the diſtance increaſe, and the velocity at B exceed, in the leaſt, that which would carry the body in a circle there, about the centre of gravitation, it will recede from S for ever. If the gravity decreaſe in a leſs proportion than that of the cubes of the increaſing diſtances, it may be projected at B with a motion which will ſtill carry it for ever from the centre, provided the gravity decreaſe in a proportion greater than that in which the diſtances increaſe: for the limit here is the inverſe ſimple proportion of the diſtances. If gravity vary more, the body may be carried off for ever from the centre by a finite motion of projection; but if the gravity varies in that proportion, or in any leſs proportion, then no finite force will be able to make the body move in ſuch a manner, as to recede from the centre S for ever: but the body in theſe caſes muſt always revolve betwixt the two *apſides*.

20. In order to ſee this, we may firſt ſuppoſe a body to be projected perpendicularly to the horizon, that is acted on by a gravity decreaſing in a higher proportion than that of the increaſing diſtances; and if the force of projection be ſufficiently great, it will riſe for ever with a motion continually retarded by the action of its gravity, but that ſhall never be altogether deſtroyed by theſe actions; becauſe they

decreafe in fuch a manner that the fum of an infinite number of them amounts to a finite quantity.

21. The fame law of gravity is the limit betwixt the cafes of infinite afcents, in curvilineal motions and in rectilineal: for our author has fhewn, that if one body move in a curve, and another afcend or defcend in a right line, acted on by the fame gravity, and their velocities be equal in any equal altitudes, they will be equal in all other equal altitudes*: and fince the gravity of the body projected upwards in a vertical line, with a certain affignable force, is not able to bring it back again; it will not be able to make it return, if it was projected with the fame force obliquely upwards, fo as to move in a curve. For the centrifugal force, arifing from the motion of rotation about S, leffens the effect of the gravity, and makes it lefs capable to deftroy the motion of

* Suppofe the velocities of bodies L and P (*Fig.* 66.) to be equal at L and P, at equal diftances SL, SP; and let them defcribe the very fmall lines L *l*, P *p*, fo that S *p* being equal to S *l*, and *p* N *l* a circular arc defcribed from the centre S meeting S L in N, L N muft be equal to P *p*. The gravity of L toward S may be refolved into two forces, one of which may be reprefented by L R, and acts in the direction of the tangent L R, the other in a direction R S perpendicular to the tangent or the direction of the body's motion. The latter has no effect in accelerating its motion, being perpendicular to it, and the former is to the gravity, as L R is to S L, or as L N is to L *l*. The motion of the body P is accelerated by its whole gravity, fo that the forces which accelerate the bodies L and P are to each other, as L N (or P *p*) to L *l*; but the velocities at L and P having been equal, the times in which L *l* and P *p* are defcribed are in the proportion of the fpaces L *l* and P *p*; fo that tho' the body L is accelerated by a lefs force in defcending to *l*, the time of its acceleration is greater in the fame proportion: from which it appears that their accelerations are equal in defcribing thefe fpaces, and their velocities confequently equal at *l* and *p*. The velocities therefore of thefe bodies muft be equal in all equal altitudes. See *Princip. Math.* Lib. I. Prop. 40.

afcent

aſcent in this caſe, than in the caſe of a perpendicular aſcent. Therefore if gravity varies in the reciprocal proportion of ſome power of the diſtance higher than unit, a body may run out to infinity in its orbit, if it be projected with a certain force.

22. If this force is the ſame which it would acquire by falling from an infinite height, it will go off in a curve of the *parabolic* kind. But if it is projected with a greater force than that which would be acquired even by an infinite deſcent, the curve will be of the *hyperbolic* kind. If it is projected with the ſame velocity which it would acquire by falling from an infinite height (aſſuming different laws of gravity, but other circumſtances ſimilar) it will go off to infinity after a greater or leſs part of a revolution, or after a greater or ſmaller number of revolutions, according as the power of the diſtance, which is reciprocally proportional to the gravity, is greater or leſs. The limit here is a quarter of a revolution from the apſis, or the place where the direction of the body's motion is perpendicular to the line drawn to the centre; for it muſt always take more than that to get off from the apſis to an infinite diſtance. If gravity obſerve the reciprocal ſeſquiplicate proportion of the diſtance, then the body will go off in $\frac{1}{3}$ of a revolution. If it obſerve the reciprocal duplicate, it will go off for ever in a parabola, in half a revolution. If it obſerves the reciprocal $\frac{5}{2}$ power of the diſtance, it will go off in a complete revolution. But if gravity obſerve the reciprocal triplicate proportion of the diſtance, and the body be projected oblique to the radius, it will go off in an infinite number of revolutions *.

23.

* In general, if gravity vary as the *m* power of the diſtance reciprocally, and the body is projected obliquely upwards with

23. If gravity decreaſe in a leſs proportion than the reciprocal ſimple proportion of the diſtances, and a body is projected from the apſis with any finite force whatſoever, it cannot riſe for ever; but will have the ſame velocity at any diſtance, as it would have had at the ſame diſtance, ſuppoſing it had been projected at A directly upwards with the ſame force of projection: and ſince any finite force would have been deſtroyed in the perpendicular, if the body move in a curve it muſt return again, and after paſſing the higher apſis, deſcend again to the lower apſis, tho' that apſis be not in the ſame place as before. If gravity increaſe as the diſtance increaſes, *a fortiori* the body will never be able to aſcend to an infinite diſtance. Theſe obſervations ſhew the limits of the various ſorts of motions, that can proceed from various laws of gravity.

CHAP. IV.

Of the motion of the moon.

1. WE have explained the motions of the bodies in the ſolar ſyſtem, from gravity, and have taken notice of ſome inequalities or errors in their motions, that ariſe from the ſame principle.

a force that is to that which would carry it in a circle as 1 to $\sqrt{\frac{m-1}{2}}$, it will riſe for ever from the centre, and go off in the $\frac{1}{6-2m}$ part of a revolution, or in the $\frac{1}{2}n$ part of the revolution. Suppoſing $\frac{1}{n}$ to be the exceſs of 3 above the number m. If the gravity follow the reciprocal proportion of the $2\frac{99}{100}$ power of the diſtance, the body will go off in 50 revolutions. See *Fluxions*, § 416, & *ſeq.*

But

But the manifold irregularities that are produced by it in the motion of the moon deſerve particularly to be conſidered, as ſhe is the neareſt to us in the ſyſtem, and as great advantages might be deduced from her motions, if they could be ſubjected to exact computation. Formerly, they who built ſyſtems had great difficulties to reconcile their principles with the phænomena: our author anticipates obſervations, and the more perfect our knowledge of the motions in the ſyſtem ſhall become, the more will this philoſophy be eſteemed. Poſterity will ſee its excellence yet more fully than we do, when the celeſtial motions ſhall be determined more accurately, by a ſeries of long-continued exact obſervations.

2. To give the principles of our author's computations on this perplexed ſubject, in as plain a manner as poſſible, we muſt recollect what has been already obſerved; that if the ſun acted equally on the earth and moon, and always in parallel lines, this action would ſerve only to reſtrain them in their annual motions round the ſun, and no way affect their actions on each other, or their motions about their common centre of gravity. In that caſe, if they were both allowed to fall directly towards the ſun, they would fall equally, and their reſpective ſituations would be no way affected by their deſcending equally towards it. We might then conceive them as in a plane, every part of which being equally acted on by the ſun, the whole plane would deſcend towards the ſun, but the reſpective motions of the earth and moon would be the ſame in this plane as if it was quieſcent. Suppoſing then this plane, and all in it, to have the annual motion imprinted on it, it would move regularly round the ſun, while the earth and moon would move in it, with reſpect to each other, as if the plane was at reſt, without any

irregularities. But becauſe the moon is nearer the ſun in one half of her orbit than the earth is, and in the other half of her orbit is at a greater diſtance than the earth from the ſun, and the power of gravity is always greater at a leſs diſtance; it follows, that in one half of her orbit the moon is more attracted than the earth towards the ſun, and in the other half leſs attracted than the earth; and hence irregularities neceſſarily ariſe in the motions of the moon, the exceſs, in the firſt caſe, and the defect, in the ſecond, of the attraction, becoming a force that diſturbs her motion: add to this, that the action of the ſun on the earth and moon is not directed in parallel lines, but in lines that meet in the centre of the ſun.

3. To ſee the effects of theſe powers, let us ſuppoſe that the projectile motions of the earth and moon were deſtroyed, and that they were allowed to fall freely towards the ſun. If the moon was in conjunction with the ſun, or in that part of her orbit which is neareſt to him, the moon would be more attracted than the earth, and fall with greater velocity towards the ſun; ſo that the diſtance of the moon from the earth would be increaſed in the fall. If the moon was in oppoſition, or in the part of her orbit which is fartheſt from the ſun, ſhe would be leſs attracted than the earth by the ſun, and would fall with a leſs velocity towards the ſun than the earth, and the moon would be left behind by the earth; ſo that the diſtance of the moon from the earth would be increaſed, in this caſe alſo. If the moon was in one of the quarters, then the earth and moon being both attracted towards the centre of the ſun, they would both directly deſcend towards that centre, and by approaching to the ſame centre, they would neceſſarily approach at the ſame time to each

each other, and their diſtance from one another would be diminiſhed, in this caſe. Now, wherever the action of the ſun would increaſe their diſtance, if they were allowed to fall towards the ſun, there we may be ſure the ſun's action, by endeavouring to ſeparate them, diminiſhes their gravity to each other; wherever the action of the ſun would diminiſh their diſtance, there the ſun's action, by endeavouring to make them approach to one another, increaſes their gravity to each other: that is, in the conjunction and oppoſition, their gravity towards each other is diminiſhed by the action of the ſun; but in the two quarters it is increaſed by the action of the ſun. To prevent miſtaking this matter, it muſt be remembred, it is not the total action of the ſun on them that diſturbs their motions, it is only that part of its action by which it tends to ſeparate them, in the firſt caſe, to a greater diſtance from each other; and that part of its action by which it tends to bring them nearer to each other, in the ſecond caſe, that has any effect on their motions with reſpect to each other. The other, and the far more conſiderable, part has no other effect but to retain them in their annual courſe, which they perform together about the ſun.

4. In conſidering, therefore, the effects of the ſun's action on the motions of the earth and moon with reſpect to each other, we need only attend to the exceſs of its action on the moon above its action on the earth, in their conjunction; and we muſt conſider this exceſs as drawing the moon from the earth towards the ſun in that place. In the oppoſition, we need only conſider the exceſs of the action of the ſun on the earth above its action on the moon, and we muſt conſider this exceſs as drawing the moon from the earth, in this place, in a direction

oppoſite to the former, that is, towards the place oppoſite to where the ſun is; becauſe we conſider the earth as quieſcent, and refer the motion, and all its irregularities, to the moon. In the quarters, we conſider the action of the ſun as adding ſomething to the gravity of the moon towards the earth.

5. Suppoſe the moon ſetting out from the quarter that precedes the conjunction, with a velocity that would make her deſcribe an exact circle round the earth, if the ſun's action had no effect on her; and becauſe her gravity is increaſed by that action, ſhe muſt deſcend towards the earth, and move within that circle: her orbit, there, will be more curve than otherwiſe it would have been; becauſe this addition to her gravity will make her fall farther at the end of an arc below the tangent drawn at the other end of it; her motion will be accelerated by it, and will continue to be accelerated till ſhe arrives at the enſuing conjunction; becauſe the direction of the action of the ſun upon her, during that time, makes an acute angle with the direction of her motion. At the conjunction, her gravity towards the earth being diminiſhed by the action of the ſun, her orbit will be leſs curve there, for that reaſon; and ſhe will be carried farther from the earth as ſhe moves to the next quarter; and, becauſe the action of the ſun makes then an obtuſe angle with the direction of her motion, ſhe will be retarded by the ſame degrees by which ſhe was accelerated before.

6. Thus ſhe will deſcend a little towards the earth, as ſhe moves from the firſt quarter towards the conjunction, and aſcend from it, as ſhe moves from the conjunction to the next quarter. The action which diſturbs her motion will have a like, and almoſt equal, effect upon her, while ſhe moves

in

in the other half of her orbit, I mean, that half of it which is fartheſt from the ſun: ſhe will proceed from the quarter that follows the conjunction with an accelerated motion to the oppoſition, approaching a little towards the earth, becauſe of the addition made to her gravity, at that quarter, from the action of the ſun; and receding from it again, as ſhe goes on from the oppoſition to the quarter from which we ſuppoſed her to ſet out. The areas deſcribed in equal times by a ray drawn from the moon to the earth will not be equal, but will be accelerated by the conſpiring action of the ſun, as ſhe moves towards the conjunction or oppoſition from the quarters that precede them; and will be retarded by the ſame action, as ſhe moves from the conjunction or oppoſition to the quarters that ſucceed them.

7. Our author has computed the quantities of theſe irregularities from their cauſes. He finds, that the force added to the gravity of the moon in her quarters, is to the gravity with which ſhe would revolve in a circle about the earth, at her preſent mean diſtance, if the ſun had no effect on her, as 1 to $178\frac{29}{40}$. He finds the force ſubducted from her gravity, in the conjunctions and oppoſitions, to be double of this quantity, and the area deſcribed in a given time in the quarters, to be to the area deſcribed in the ſame time in the conjunctions and oppoſitions, as 10973 to 11073. He finds, that, in ſuch an orbit, her diſtance from the earth in her quarters, would be to her diſtance in the conjunctions and oppoſitions, as 70 to 69.

8. The moon does not move in the ſame plane, round the earth, in which the earth moves round the ſun, but in a plane that is inclined to it in an angle

of about 5 degrees: and hence it is that the centre of the moon appears to us to trace a different circle from the *ecliptic*, the circle which the centre of the ſun appears to deſcribe in the heavens. Theſe circles cut each other in two oppoſite points, that are called by aſtronomers the *nodes* of the moon; at the greateſt diſtance from the nodes, theſe circles are ſeparated from each other by about five degrees. The eclipſes of the ſun and moon depend on their diſtances from theſe nodes, at the time of the new and full moon; for, if the change of the moon happen when ſhe is near one of the nodes, ſhe eclipſes the ſun; and, if the moon is full, near one of the nodes, ſhe muſt fall into the ſhadow of the earth, and there become eclipſed. Aſtronomers have at all times been very attentive to the ſituation of the nodes, in order to calculate theſe eclipſes, which have been always a phænomenon much conſidered by them. The nodes are not fixed in the ſame part of the heavens, but are found to move over all the ſigns in the ecliptic, with a retrograde motion, in about eighteen or nineteen years.

9. Sir *Iſaac Newton* has not only ſhewed that this motion ariſes from the action of the ſun, but has calculated, with great ſkill, all the elements and varieties in this motion, from its cauſe. We called theſe points the moon's nodes, in which her orbit cuts the plane in which the earth revolves about the ſun, and the line that joins the points we call the line of the nodes. We ſay the motion of the nodes is direct when they proceed in the ſame way as the moon moves in her orbit, *viz.* from weſt to eaſt, according to the order of the ſigns *Aries*, *Taurus*, &c. in the ecliptic; and we ſay their motion is retrograde, when they move with a motion contrary to that of the moon, or from eaſt to weſt, contrary

to

to the order of the ſigns. We conceive the plane of the moon's motion to paſs always through the centre of the earth and the centre of the moon, and to be a plane in which the right line joining their centres, and the right line that is the direction of the moon's motion, or the tangent of her orbit, are always found. It is certain, that if the earth and moon were always acted on equally by the ſun, they would deſcend equally toward the ſun; the plane determined always by theſe two lines, would deſcend with them, keeping always parallel to itſelf, ſo that the moon would appear to us to revolve in the ſame plane conſtantly, with reſpect to the earth. But the inequalities in the action of the ſun, deſcribed above, will bring the moon out of this plane, to that ſide of the plane on which the ſun is, in the half of her orbit that is neareſt the ſun, and toward the other ſide, in the half of her orbit that is fartheſt from the ſun.

10. From which we have this general rule for judging of the effect of the ſun on the nodes; that while the moon is in the half of her orbit that is neareſt the ſun, the node towards which ſhe is moving is made to move towards the conjunction with the ſun; and while the moon is in the half of her orbit which is fartheſt from the ſun, the node towards which ſhe is moving is made to move towards the oppoſition: but when the nodes are in conjunction with the ſun, its action has no effect upon them. In the firſt caſe, the moon is brought into a direction which is on the ſame ſide, as the ſun is, of that direction which ſhe would follow of herſelf: and the interſection of a plane paſſing through this direction, and through the centre of the earth, will cut the ecliptic, on that ſide towards which the moon moves, in a point nearer the con-

junction, than if there was no action of the sun to disturb her motion. In the other case, the action of the sun has a contrary direction, and for the same reason, makes the ensuing node move towards the opposition. When the line of the nodes produced passes through the sun, then the sun, being in the plane of the moon's motion, has no effect to bring her to either side; and therefore, in that case, the nodes have no motion at all.

11. From this general rule, it appears, that if you suppose the nodes to be in the quarters A and C, (*Fig.* 67.) after the moon sets out from the node A, that is, in the quarter preceding the conjunction B, the ensuing node C moves towards the conjunction B, and is therefore retrograde; because it moves in a direction opposite to that in which the moon moves; and, in all this revolution of the moon, the nodes are manifestly retrograde; for, after the moon passes the quarter C that succeeds the conjunction, then the ensuing node A moves towards the opposition D, so that the nodes are, in that half of her orbit also, retrograde.

12. Suppose the nodes in the situation N *n*, so as one of them may be between the quarter A and the ensuing conjunction B, while the other node *n* falls on the opposite point of the moon's circle, between the subsequent quarter C and the opposition D. In this case, while the moon moves from A to N, the node N moves towards the conjunction B (by the general principle in § 10.) and therefore its motion is direct. While the moon moves from N to C, the ensuing node *n* moves towards the conjunction B, and therefore is retrograde; and because the arc N C exceeds A N, the retrograde motion exceeds the

direct

direct motion. While the moon moves from C to *n*, the enſuing node *n* moves towards the oppoſition D, and the motion of the nodes is then direct. But while the moon moves from *n* to A, the enſuing node N moves towards the oppoſition D, and then the motion of the nodes being contrary to the motion of the moon, their motion is retrograde; and becauſe the arc *n* A exceeds *n* C, it is apparent that the motion is more retrograde than direct.

13. When (*Fig.* 68.) one node N is between the conjunction B and the enſuing quarter C, while the moon moves from A to N, the enſuing node N moves towards the conjunction B, and therefore is retrograde: while the moon moves from N to C, the enſuing node *n* moves towards the conjunction, and is direct. But as the arc A N exceeds N C, the retrograde motion of the nodes muſt exceed the direct motion. While the moon moves from C to *n*, the motion of the enſuing node is towards the oppoſition D, and is therefore retrograde. While the moon moves from *n* to A, the enſuing node N moves towards the oppoſition D, and therefore is direct. But, as the arc C *n* exceeds A *n*, it follows that the retrograde motion exceeds the direct motion.

It appears, therefore, that in every revolution of the moon, the retrograde motion of the nodes exceeds the direct motion, excepting only when the line of the nodes paſſes through the ſun, in which caſe there is no motion of the nodes at all. We ſee then, how, from the principle of gravity, the nodes of the moon are made to recede every year. Our author has determined the quantity * of this retrograde motion in every revolution of the moon, and

* *Princip.* Lib. III. Prop. 32.

in every year; and it is no ſmall pleaſure to ſee how exactly the theory of theſe motions, drawn from their cauſes, agrees with the obſervations of aſtronomers. He finds, from the theory of gravity, that the nodes ought to move backward about 19° 18′ 1″ in the ſpace of a year, and the aſtronomical tables make this motion 19° 21′ 21″; whoſe difference is not $\frac{1}{300}$ of the whole motion of the nodes in a year. By a more correct computation of this motion from its cauſe, the theory and obſervation agree within a few ſeconds.

14. The inclination of the moon's orbit to the ecliptic, is alſo ſubject to many variations. When the nodes are in the quarters A and C, while the moon moves from the quarter A to the conjunction B, the action of the ſun diminiſhes the inclination of the plane of her orbit; the inclination of this plane is leaſt of all when the moon is in the conjunction B: it increaſes again as ſhe moves from the conjunction B to the next quarter at C, and is there reſtored to its firſt quantity nearly. When the nodes of the moon are in B and D, ſo that the line of the nodes paſſes through the ſun, the inclination of the moon's orbit is not affected by the action of the ſun; becauſe, in that caſe, the plane of her orbit produced paſſes through the ſun: and therefore the action of the ſun can have no effect to bring the moon out of this plane to either ſide. It is in this laſt caſe that the inclination of the moon's orbit is greateſt; it decreaſes as the nodes move towards the quarters; and it is leaſt of all when the nodes are in the quarters, and the moon either in the conjunction or oppoſition. Our author calculates theſe irregularities from their cauſes, and finds his concluſions agree

TAB. V.

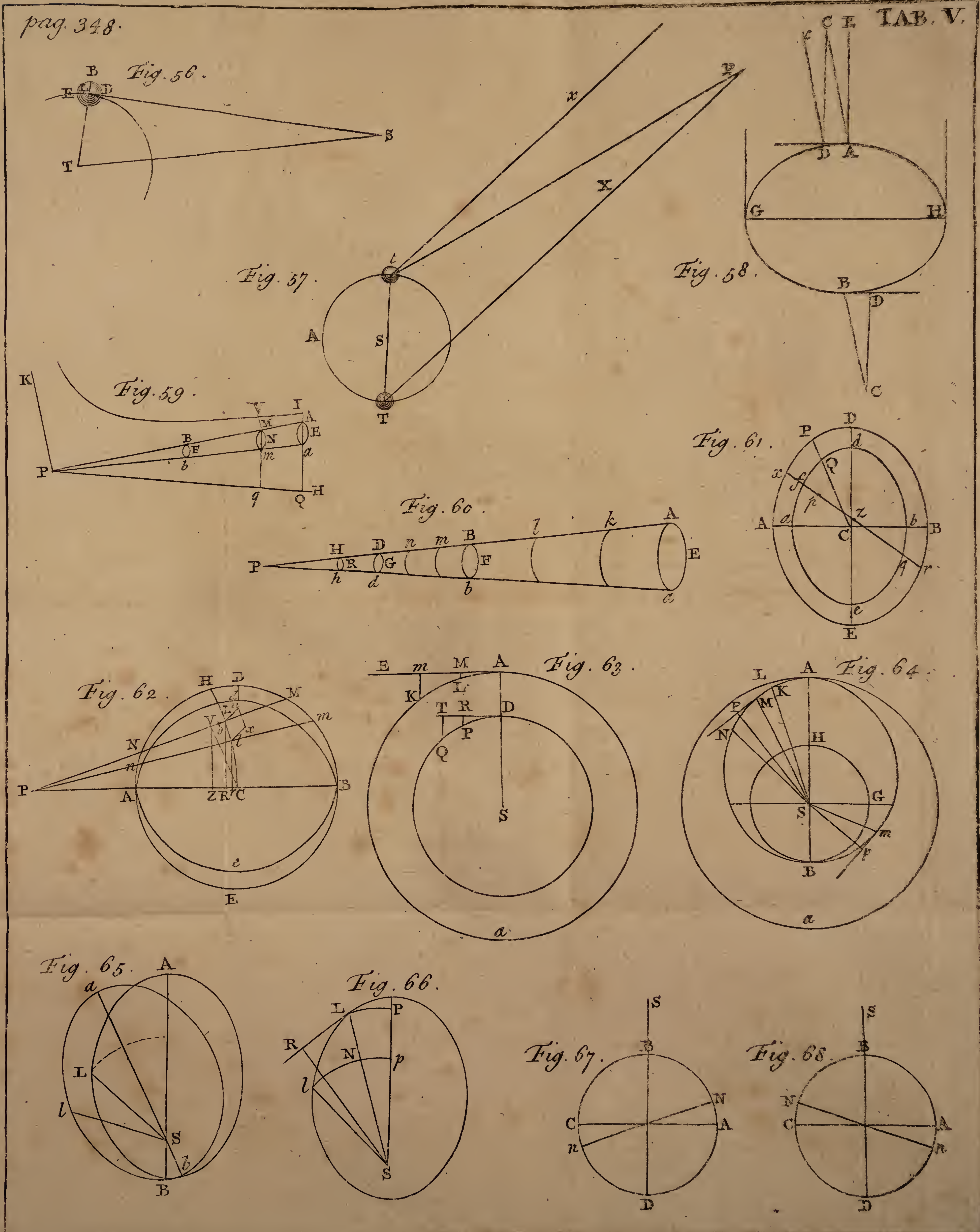


Fig. 56.
B E D S T
Fig. 57.
t A S T x X P
Fig. 58.
e C E D A G H B D C
Fig. 59.
K V I A M E B F N b m a P q Q H
Fig. 60.
P H R h D G d n m B F b l k A E a
Fig. 61.
P D d Q x f p z A a C b B q r e E
Fig. 62.
H D d M m N n P A Z R C B e E
Fig. 63.
E m M A K L T R D P Q S a
Fig. 64.
L A K M P N H G S m p B a
Fig. 65.
A a L l S b B
Fig. 66.
L P R N p l S
Fig. 67.
S B N C A n D
Fig. 68.
S B N C A n D

agree very well with the obſervations of aſtronomers *.

15. The action of the ſun diminiſhes the gravity of the moon towards the earth, in the conjunctions and oppoſitions, more than it adds to it in the quarters, and, by diminiſhing the force which retains the moon in her orbit, it increaſes her diſtance from the earth and her periodic time: and becauſe the earth and moon are nearer the ſun in their perihelium than in their aphelium, and the ſun acts with a greater force there, ſo as to ſubduct more from the moon's gravity towards the earth; it follows, that the moon muſt revolve at a greater diſtance, and take a longer time to finiſh her revolution in the perihelium of the earth, than in the aphelium; and this alſo is conformable to obſervation.

16. There is another remarkable irregularity in the moon's motion, that alſo ariſes from the action

* To make the foregoing account of the motion of the moon's nodes ſtill clearer, we have added *Fig.* 69, (*Plate* VI.) in which, the plane of the ſcheme repreſenting that of the ecliptic, S is the ſun, T the centre of the earth, L the moon in her orbit D N *d n*; N *n* is the line of the nodes paſſing between the quadrature Q and the moon's place L, in her laſt quarter. Let now L P, any part of L S, repreſent the exceſs of the ſun's action at L, above his action at T, and this being reſolved into the force L R, perpendicular to the plane of the moon's orbit, and P R parallel to it, 'tis the former only that has any effect to alter the poſition of the orbit, and in this it is wholly exerted. Its effect is twofold; (1.) It diminiſhes its inclination, by a motion which we may conceive as performed round the diameter D *d*, to which L T is perpendicular. (2.) Being compounded with the moon's tangential motion at L, it gives it an intermediate direction L *t*; thro' which, and the centre of the earth, a plane being drawn muſt meet the ecliptic nearer the conjunction C than before: and in the ſame manner, the other caſes are explained.

of

of the ſun: I mean, the progreſſive motion of the apſides. The moon deſcribes an ellipſe about the centre of the earth, having one of the foci in that centre. Her greateſt and leaſt diſtances from the earth are in the apſides, or extremities of the longer axis of the ellipſe. This is not found to point always to the ſame place in the heavens, but to move with a progreſſive motion forwards, ſo as to finiſh a revolution round the earth's centre in about nine years.

To underſtand the reaſon of this motion of the apſides, we muſt recollect what was ſhewed above, that, if the gravity of a body decreaſed leſs as the diſtance increaſes, than according to the regular courſe of gravity, the body would deſcend ſooner from the higher to the lower apſis, than in half a revolution; and therefore the apſis would recede in that caſe, for it would move in a contrary direction to the motion of the body, meeting it in its motion. But if the gravity of the body ſhould decreaſe more as the diſtance increaſes than according to the regular courſe of gravity, that is, in a higher proportion than as the ſquare of the diſtance increaſes, the body would take more than half a revolution to move from the higher to the lower apſis; and therefore, in that caſe, the apſides would have a progreſſive motion in the ſame direction as the body.

In the quarters, the ſun's action adds to the gravity of the moon, and the force it adds is greater, as the diſtance of the moon from the earth is greater; ſo that the action of the ſun hinders her gravity towards the earth, from decreaſing as much while the diſtance increaſes, as it ought to do according to the regular courſe of gravity; and therefore, while the moon is in the quarters, her apſides muſt recede.

In

In the conjunction and oppofition, the action of the fun fubducts from the gravity of the moon towards the earth, and fubducts the more the greater her diftance from the earth is, fo as to make her gravity decreafe more as her diftance increafes, than according to the regular courfe of gravity; and therefore, in this cafe, the apfides are in a progreffive motion. Becaufe the action of the fun fubducts more in the conjunctions and oppofitions from her gravity, than it adds to it in the quarters, and, in general, diminifhes more than it augments her gravity; hence it is that the progreffive motion of the apfides exceeds the retrograde motion; and therefore, the apfides are carried round according to the order of the figns.

17. Thus the various irregularities of the moon's motion are explained from gravity: and from this theory, with the affiftance of a long feries of accurate obfervations, her motion may be at length reduced fo exactly to computation, and her appulfes to the fixed ftars, over which fhe paffes in her courfe, may be predicted with fo much accuracy, as to afford, on many occafions, an opportunity to navigators, to difcover their longitude at fea.

CHAP.

CHAP. V.

Of the path of a secondary planet upon an immoveable plane; with an illustration of Sir Isaac Newton's *account of the motions of the satellites, from the theory of gravity* *.

IN describing the motions of the solar system, it is usual to consider the primary planets, as revolving in immoveable planes, but to refer the motions of the satellites to planes that are carried along with their primaries about the sun. Sir *Isaac Newton* follows the same method, in accounting for their motions from the theory of gravity: by this analysis, the explication of the motions themselves, and of the powers that produce them, is rendered more simple and easy, than if we should refer the motion of the satellite to an immoveable plane, and contemplate only the path described by it, in consequence of so compounded a motion, in absolute space.

The properties, however, of this path are more simple than perhaps will be expected on a superficial consideration of it; and the referring of the motion of the satellite to it, may be of use on some occasions, particularly for resolving the difficulties some have found to understand Sir *Isaac Newton*'s account of the motions of the satellites, from gravity. This path is, in some cases, concave towards the sun throughout; in other cases, the part of it nearest

* The following chapter, as belonging properly to this place, is inserted from a letter of the author, to his learned friend Dr. *Benjamin Hoadley*, physician to his Majesty's household.

the

the ſun is convex towards the ſun, and the reſt is concave. An inſtance of the former we have in the moon, of the latter in the ſatellites of the ſuperior planets.

The force that bends the courſe of the ſatellite into a curve, when the motion is referred to an immoveable plane, is, at the conjunction, the difference of its gravity towards the ſun, and of its gravity towards the primary. When the former prevails over the latter, the force that bends the courſe of the ſatellite tends towards the ſun; conſequently, the concavity of the path is towards the ſun: and this is the caſe of the *Moon*, as will appear afterwards. When the gravity towards the primary exceeds the gravity towards the ſun, at the conjunction, then the force that bends the courſe of the ſatellite tends towards the primary, and therefore towards the oppoſition of the ſun; conſequently the path is there convex towards the ſun: and this is the caſe of the ſatellites of *Jupiter*. When theſe two forces are equal, the path has, at the conjunction, what mathematicians call a point of *rectitude*; in which caſe, however, the path is concave towards the ſun throughout.

Becauſe the gravity of the moon towards the ſun is found to be greater, at the conjunction, than her gravity towards the earth, ſo that the point of equal attraction, where thoſe two powers would ſuſtain each other, falls then between the moon and earth, ſome * have apprehended that either the parallax of the ſun is very different from that which is aſſigned by aſtronomers, or that the moon ought neceſſarily to abandon the earth. This apprehenſion may be

* See *Coſmotheoria* puerilis.

 eaſily

eaſily removed, by attending to what has been ſhewn by Sir *Iſaac Newton*, and is illuſtrated by vulgar experiments, concerning the motions of bodies about one another, that are all acted upon by a third force in the ſame direction. Their relative motions, not being in the leaſt diſturbed by this third force, if it act equally upon them in parallel lines; as the relative motions of the ſhips in a fleet, carried away by a current, are no way affected by it, if it act equally upon them; or as the rotation of a bullet, or bomb, about its axis, while it is projected in the air, or the figure of a drop of falling rain, are not at all affected by the gravity of the particles of which they are made up, towards the earth. It is to the inequality of the actions of the ſun upon the earth and moon, and the want of paralleliſm in the directions of theſe actions, only, that we are to aſcribe the irregularities in the motion of the moon.

But it may contribute towards removing this difficulty, to obſerve, that if the abſolute velocity of the moon, at the conjunction, was leſs than that which is requiſite to carry a body in a circle there around the ſun, ſuppoſing this body to be acted on by the ſame force which acts there on the moon, (*i. e.* by the exceſs of her gravity towards the ſun, above her gravity towards the earth,) then the moon would, indeed, abandon the earth. For, in that caſe, the moon having leſs velocity than would be neceſſary to prevent her from deſcending within that circle, ſhe would approach to the ſun, and recede from the earth. But tho' the abſolute velocity of the moon, at the conjunction, be leſs than the velocity of the earth in the annual orbit, yet her gravity towards the ſun is ſo much diminiſhed by her gravity towards the earth, that her abſolute velocity is ſtill much ſuperior to that which is requiſite to carry a body in a

circle,

circle, there, about the ſun, that is acted on by the remaining force only. Therefore, from the moment of the conjunction, the moon is carried without ſuch a circle, receding continually from the ſun to greater and greater diſtances, till ſhe arrive at the oppoſition; where, being acted on by the ſum of thoſe two gravities, and her velocity being now leſs than what is requiſite to carry a body in a circle, there, about the ſun, that is acted on by a force equal to that ſum, the moon thence begins to approach to the ſun again. Thus ſhe recedes from the ſun and approaches to it by turns, and in every month her path has two apſides, a *perihelium* at the conjunction, and an *aphelium* at the oppoſition; between which ſhe is always carried, in a manner ſimilar to that in which the primary planets revolve between their apſides. The planet recedes from the ſun at the perihelium, becauſe its velocity, there, is greater than that with which a circle could be deſcribed about the ſun, at the ſame diſtance, by the ſame centripetal force; and approaches towards the ſun from the aphelium, becauſe its velocity there, is leſs than is requiſite, to carry it in a circle, at that diſtance, about the ſun. See my *Treatiſe of Fluxions*, Art. 447.

Tho' the path of the moon be concave towards the ſun throughout, its curvature is very unequal: it is leaſt at each lower apſide or conjunction, and greateſt at each higher apſide or oppoſition. The path of a ſatellite of *Jupiter* has likewiſe two apſides, in the part which is deſcribed every ſynodic revolution; but in the lower apſide, the convexity is towards the ſun; and it has likewiſe two points of contrary flexure in every ſuch part *.

* See the note to *Corol.* 1. Prop. II. below.

By confidering this path, we fhall arrive at the fame conclufions which Sir *Ifaac Newton* derived, more briefly, from the laws of motion; that if the folar action was the fame on the fatellite and on the primary, and in the fame direction, the motion of the fatellite around the primary, would be the fame as if the fun was away. This will appear from the following propofitions, where we fuppofe the orbits of the primary about the fun, and of the fatellite about the primary, to be both circular, and the motions in thefe orbits to be uniform and in the fame plane.

PROPOSITION I. *Fig.* 70. *Pl.* VI.

The path of the fatellite, on an immoveable plane, is the epicycloid *that is defcribed by a given point in the plane of a circle, which revolves on a circular bafe, having its centre in the centre of the fun, and its diameter in the fame proportion to the diameter of the revolving circle, as the* periodic *time of the primary about the fun, to the time of the* fynodic *revolution of the fatellite about the primary: the tangent of the path is perpendicular to the right line that joins the fatellite to the contact of the two circles: and the abfolute velocity of the fatellite is always as its diftance from that contact.*

Let T denote the periodic time of the primary about the fun, *t* the periodic time of the fatellite about the primary. Let S reprefent the fun, A *a* the orbit of the primary; upon the radius A S, take A E to A S as *t* is to T. From the centre S defcribe the circle E *e* Z, and from the centre A the circle E M F. Let this circle E M F revolve on the other E *e* Z, as its bafe: then a point L, taken on the plane of of the circle E M F, at the diſtance A L, equal to the diſtance of the ſatellite from the primary, ſhall deſcribe the path of the ſatellite.

For ſuppoſe the circle E M F to move into the ſituation *emf*, the point A to *a*, L to *l*, and let A L and *a l*, produced, meet E M F and *emf*, in M and *m*. Upon the arc *em* take *er*=E M, then the angle *ear*=E A M. Let *ar* meet the circle *c l d*, deſcribed from the centre *a* with the diſtance *a l*, in *q*; and becauſe *eaq*=E A L, the angle *eaq* repreſents the elongation of the ſatellite from the ſun at its firſt place L. Becauſe *em* (=*er*+*rm*) =*e* E+E M and *er*=E M, it follows that *rm*=*e* E; conſequently the angle *ram* : *e*S E :: S E : A E :: T—*t* : *t*; or, as the angular velocity of the ſatellite from the ſun, to the angular velocity of the primary about the ſun. But E S *e* is the angle deſcribed by the primary about the ſun, conſequently *ram*, or *qal*, is the ſimultaneous increment of the elongation of the ſatellite from the ſun; *l* is its place when the primary comes to *a*; and the epicycloid deſcribed by *l* is the path of the ſatellite.

Becauſe the circle EMF moves on the point E, the direction of the motion of any point L is perpendicular to E L; or the tangent of the path at any point L is perpendicular to E L. The velocity of any point L is as its diſtance E L; and, the motion of the primary A being ſuppoſed uniform and repreſented by E A, the velocity of the ſatellite ſhall be repreſented by E L.

PROPOSITION II.

Upon A S *take* A B : A S :: tt : T T (*or* A B : A E :: A E : A S); *upon the diameter* E B *deſcribe the circle*

E K B *meeting* E L *in* K, *take* L O *a third proportional to* L K *and* L E, *on the same side of* L *with* L K; *and* O *shall be the centre of the curvature at* L *of the path, and* L O *the ray of curvature.*

Because E L and *e l* are perpendicular to the path at the points L and *l*, let them be produced, and their ultimate intersection O shall be the centre of curvature at L. Produce *q e* till it meet L E in V, join S V, and the angle S *e* V = *q e a* = L E A = S E V; consequently the angle *e* V E = *e* S E, the angle E V S = *e* S E, and the angle E V S = E *e* S, and S V is ultimately perpendicular to E O. Now the angle E O *e* is ultimately to E V *e* (= E S *e*) as E V to E O, that is (because E V : E K :: E S : E B :: A S : A E) as E K × A S to E O × A E. But the angular motion of E L being equal to the angular motion of E A, while the circle E M F turns on the point E, L E *l* is therefore ultimately equal to A E *a*, which is to E S *e* as S A to A E; and E O *e* being to L E *l* as E L to L O, it follows that E O *e* : E S *e* :: S A × E L : A E × L O :: E K × S A : E O × A E. Therefore E L : L O :: E K : E O, and E L : L K :: L O : E L, or L K, L E and L O are in continued proportion. This theorem serves for determining the ray of curvature of *epicycloids* and *cycloids* of all sorts; only when the base E *e* is a right line, A B vanishes, and E B becomes equal to E A.

Corol. I. When A L or A C is less than A B, then (because L O is always on the same side of the point L with L K) the path is concave towards S throughout. When A C = A B, the curvature at the conjunction vanishes, or the path has there a point of *rectitude*. When A C is greater than A B (or A S × $\frac{tt}{TT}$), a portion of the path near the conjunction is convex towards S, because a part of the circle C L D

falls

falls within the circle B K E; and when L comes to either of the interſections of theſe two circles, the path has a point of contrary flexure *.

Corol. 2. In the caſe of the moon, tt : T T :: 1 : 178, and A B $= \frac{1}{178} \times$ A S; but A C is about $\frac{1}{337} \times$ A S; conſequently A C is leſs than A B, and the path of the moon is concave towards the ſun throughout.

PROPOSITION III.

Let A B : A S :: t t : T T, *and the force by which the path of the ſatellite can be deſcribed on an immoveable plane, is always directed to the point* B *upon the ray* A S; *and is always meaſured by* B L *the diſtance of the ſatellite from the point* B, *the gravity of the primary towards the ſun being repreſented by* B A.

We conceive the force by which this path could be deſcribed, on an immoveable plane, to be reſolved into a force that acts in the direction L O, perpendicular to the path, and bends the path, but has no effect on the velocity of the ſatellite; and a force perpendicular to L O that accelerates or retards the motion of the ſatellite. The former of theſe is meaſured by L K, the latter by B K, the gravity of the primary towards the ſun being meaſured by A B. For the former is to the gravity of the primary towards S, as $\frac{EL^2}{LO}$ to $\frac{EA^2}{AS}$ (thoſe forces being directly as the ſquares of the velocities, and inverſely as the

* If A C = A E, theſe points meet again, and form a *cuſp*: and if A C is greater than A E, the path has a *nodus*: which laſt is the caſe of the innermoſt of the ſatellites of *Jupiter* and *Saturn*.

rays of the curvature) that is, as L K to A B, by *Prop.* II. Therefore the gravity of the primary being reprefented by A B, the former force will be meafured by L K.

The fecond force that acts on the fatellite in the direction of the tangent of its path, and accelerates or retards its motion, is as the fluxion of the velocity E L directly, and the fluxion of the time inverfely. The fluxion of the time is meafured by $\frac{\text{A}a}{\text{EA}}$ (A a being the arc defcribed by the primary, and E A the velocity with which it is defcribed) $= \frac{\text{E}e}{\text{EB}} = \frac{rm}{\text{EB}} = \frac{lq \times \text{AE}}{\text{EB} \times \text{AC}} =$ (fuppofing an and qu to be perpendiculars to el in n and u, becaufe $lq : lu :: ac : an$, or AC : AN) $\frac{\text{AE} \times lu}{\text{EB} \times \text{AN}} = \frac{lu}{\text{BK}}$. Therefore the force which is meafured by lu, the fluxion of the velocity E l, or E L, divided by the fluxion of the time or $\frac{lu}{\text{BK}}$, is meafured by B K. The force, therefore, in the direction L E being meafured by L K, and the force in the perpendicular direction K B by K B, the compounded force is meafured by L B, and is directed from L to B.

It appears, from what has been demonftrated, that the path may be defcribed by a force directed towards the point B, (which is given upon the ray A S, but revolves along with this ray about S) or by any forces which, compounded together, generate a force tending to B, and always proportional to L B, the diftance of the fatellite from B. Let L H be equal and parallel to A B, and A B H L fhall be a parallellogram, and the force L K may be compounded of of L H and L A; that is, the force L K may be the reſult of a force L H acting on the ſatellite, equal and parallel to A B, the gravity of the primary towards the ſun, and of a force L A tending to the primary, and equal to the gravity by which the ſatellite would deſcribe the circle C L D about the primary, in the ſame periodic time t, if the ſun was away; becauſe ſuch a force is to the gravity of the primary towards the ſun, (repreſented by A B) as $\frac{AL}{tt}$ to $\frac{AS}{TT}$ or as A L to $AS \times \frac{tt}{TT} = AB$.

Thus we arrive at the ſame concluſion which Sir *Iſaac Newton*, more briefly, derived from an analyſis of the motions of the ſatellite; that while the ſatellite gravitates towards the primary, if, at the ſame time, it be acted on by the ſame ſolar force as the primary, and with a parallel direction, it will revolve about the primary, in the ſame manner as if this laſt was at reſt, and there was no ſolar action. Theſe two forces, the gravitation towards the primary, and a force equal and parallel to the gravitation of the primary towards the ſun, are exactly ſufficient to account for the compounded motion of the ſatellite in its path, however complex a curved-line it may appear to be. Nor is there any perturbation of the ſatellite's motion, but what ariſes from the inequality of the gravity of the ſatellite, and of the primary towards the ſun, or from their not acting in parallel lines. If we ſhould ſuppoſe them to move about their common centre of gravity, while this is carried round the ſun, or if we ſuppoſe the orbits to be elliptical, the concluſions will ſtill be found conſonant to what was more briefly deduced by this great author.

CHAP. VI.

Of the figure of the earth, and the precession of the equinoxes.

1. IF the earth was fluid, and had no motion on its axis, the equal gravitation of its parts towards each other would give it a figure exactly spherical, the columns from the surface to the centre mutually sustaining each other at equal heights from it. But, because of the diurnal rotation of the earth on its axis, the gravity of the parts at the equator is diminished by the centrifugal force arising from this rotation; the gravity of the parts on either side of the equator is diminished less, as the velocity of rotation is less, and the centrifugal force, arising from it acts less directly against the gravity of the parts; while the gravity at the poles is not at all affected by the rotation. The equilibrium that was supposed to be amongst the parts will not, therefore, now subsist in a spherical figure, but will be destroyed by the inequality of their gravitation, till the water rise at the equator and sink at the poles, so as, by a greater height at the equator, to compensate the greater gravity at the poles; and till, by assuming an intermediate height in the intermediate places, the whole earth become of an oblate spheroidal form, whose diameter at the equator will be the greatest, and the axis the least, of all the lines that can pass through the centre.

2. If the gravity of a body at the equator was destroyed, the motion of rotation would there make it go off in a tangent to the earth; and by moving in the tangent it would rise, in a second of time,

from the ſpherical body of the earth, as much as one extremity of the arc which bodies deſcribe there, in a ſecond, falls below the tangent drawn at the other extremity: and this is found to be a ſpace of about 7,54 lines, *French* meaſure. The effect of the centrifugal force of bodies at the equator, in a ſecond of time, is proportional to this ſpace. The effect of the centrifugal force at any place at a diſtance from the equator, for example, at *Paris*, is leſs for the reaſons above mentioned; and, there, it is found, by calculation, that it could only produce a motion of 3,267 lines in a ſecond. Add this to what, by experiments, bodies are found to deſcribe by their gravity at *Paris*, *viz.* 15 feet, 1 inch and 2 lines, and the ſum 2177,267 lines will ſhew the ſpace which bodies would deſcribe by their gravity, in a ſecond of time, if there was no centrifugal force there. By comparing this with the effect of the centrifugal force at the equator, in the ſame time, we ſhall find that the centrifugal force, there, is the $\frac{1}{289}$ part of the power of gravity, becauſe 7,54 is to 2177,267 as 1 to 289.

3. From this it follows, that a body at the equator loſes $\frac{1}{289}$, at leaſt, of its gravity; and the equator muſt be, at leaſt, $\frac{1}{289}$ higher than the poles from the centre of the earth. But as the parts of the equator loſe ſtill of their gravity as they riſe from the centre of the earth, and the regular courſe of gravity is altered by the change of figure, this is not the true proportion of the height of the earth at the equator, to its height at the poles.

Our author, who was never at a loſs to find ſome expedient by which he might determine, accurately or near the truth, what he wanted; in order to take in theſe perplexed conſiderations, aſſumes, as an *hypo-*

hypothesis, that the axis of the earth is to the diameter of the equator, as 100 to 101; he thence determines what must be the centrifugal force at the equator, that the earth might take such a form, and finds it must be $\frac{4}{505}$ of gravity, and therefore would exceed the present centrifugal force there, which is only $\frac{1}{289}$ of gravity. By the rule of proportion, he says, that if a centrifugal force equal to $\frac{4}{505}$ of gravity, would make the earth higher at the equator than at the poles, by $\frac{1}{100}$ of the whole height at the poles, a centrifugal force that is the $\frac{1}{289}$ of gravity, will make it higher by a proportional excess, which is found by calculation to be $\frac{1}{229}$ of the height at the poles; and thus our author discovers that the diameter at the equator is to the diameter at the poles, or the axis, as 230 to 229.

4. This computation supposes the earth to be of an uniform density every where: but if the earth is more dense near the centre, then bodies at the poles will be more attracted by this additional matter being nearer to it; and, for this reason, the excess of the semidiameter of the equator above the semi-axis will be different. What we have said of a fluid earth must hold of the earth in its present state; for if it had not this figure in its solid parts, but a spherical figure, the ocean would overflow all the equatorial regions, and leave the polar regions elevated many miles above the level of the sea; whereas we find the one is no more elevated above the level of the ocean, than the other.

5. The planet *Jupiter* revolves on his axis with much more rapidity than our earth, and finishes his diurnal rotation in less than ten hours. The density of that planet is also less; and therefore his figure is more different from a sphere than the figure of the

the earth, and his equatorial diameter exceeds his axis in a greater proportion. Their difference is ſo ſenſible, that they are found, by the obſervations of aſtronomers, to be to one another as 13 is to 12.

6. The decreaſe of gravity from the poles towards the equator, is very manifeſt from the motion of pendulums. A pendulum that vibrates, in a ſecond, in the northern regions, when carried to the equator, is always found to move too ſlow, and requires to be made ſhorter to vibrate truly in a ſecond. This ſhews the gravity is leſs there: and this obſervation confirms the diurnal motion of the earth, and its oblate ſpheroidal figure at the ſame time. It is alſo a conſequence of this figure of the earth, that the degrees in a meridian muſt increaſe from the equator to the poles; but the difference is ſo ſmall that it cannot be diſcovered, from obſervation, but in latitudes that differ conſiderably from each other; and the variation of the degrees, that are near one another, appears, by our author's computations, to be incomparably leſs adapted for judging of the figure of the earth, than the motion of pendulums, in which the leaſt variation becomes very ſenſible, in a great number of vibrations.

7. Some have imagined the ſlowneſs of the pendulums, at the equator, may have proceeded, from the rod of the pendulum being extended to a greater length, by the heat: but our author has ſhewed, that this could produce but a very ſmall part of the effect. Mr. *Richer*, who was very careful in making his obſervations, found, that a pendulum vibrating in a ſecond of time, in the iſland of *Cayenne*, was ſhorter than one that vibrated, in the ſame time, at *Paris*, by one line and a fourth part of a line. Our author, with good reaſon, thinks that a difference

of

of one ſixth part of a line, may be allowed as the effect of the heat; and, ſubducting this from the difference obſerved by Mr. *Richer*, the remainder, 1 line and $\frac{1}{12}$ of a line, is the difference owing to the decreaſe of gravity, and is very conſonant to what our author draws from his theory. This obſervation and our author's theory agree, in allowing ſeventeen miles for the exceſs of the height of the earth at the equator, above its height at the poles.

8. From the oblate figure of the earth, our author has accounted for the *preceſſion* of the equinoxes. We commonly ſuppoſe, that, while the earth moves in her orbit round the ſun, her axis continues always parallel to itſelf, ſo as to form an invariable angle with the ecliptic of about $66\frac{1}{2}°$: from hence it is, that the plane of the equator is inclined to the ecliptic, in an angle of $23\frac{1}{2}°$, and produced paſſes through the centre of the ſun, twice only in every revolution. The points of the heavens, where the centre of the ſun appears to be, in theſe two caſes, are called the equinoctial points. In any other parts of the earth's orbit, the ſun is on one ſide of the plane of the equator; being to the north of it in the ſummer half of the year, and to the ſouth of it in the winter half. Theſe equinoctial points are not fixed in the heavens, but have a ſlow motion, from eaſt to weſt, among the ſtars, of about 50″ in a year; and hence it is, that the interval of time betwixt any equinox and that ſame equinox, in the following revolution of the earth (which aſtronomers call the *tropical* year), is ſome minutes ſhorter than the *ſidereal* year, or the period wherein the earth revolves from one point of her orbit, to the ſame point again: and, becauſe the retrograde motion of the equinoctial points thus advances the time of every equinox a little ſooner than it would otherwiſe have hap-

happened, this phænomenon is called the preceſſion of the equinoxes. The philoſophers who maintained the *Ptolemaic* ſyſtem aſcribed this motion to the fixed ſtars; and, in their ordinary way, made no ſcruple to contrive a ſphere for this purpoſe, which they ſuppoſed to revolve with a very ſlow motion on the poles of the ecliptic, and to carry all the fixed ſtars along with it; whereas this phænomenon is accounted for by a retrograde motion of the nodes of the equator and ecliptic, ſimilar to the motion of the nodes of the moon's orbit.

It was ſhewn above, how the action of the ſun produces the retrograde motion of the nodes of the moon; and it follows, from the ſame principles, that if a planet revolved about the earth near to its ſurface in the plane of the equator, its nodes would alſo go backward, tho' with a ſlower motion than thoſe of the moon, in proportion as its diſtance from the earth's centre was leſs than that of the moon. Suppoſe the number of ſuch planets to be increaſed till they touch each other, and form a ring in the equator, and the nodes of this ring would go backward in the ſame manner as the nodes of the orbit of any one planet revolving there. Suppoſe then this ring to adhere to the earth; and its nodes would ſtill go backward, but with a much ſlower motion, becauſe the ring muſt move the whole earth, to which it is ſuppoſed to adhere. The elevation of the equatorial parts of the earth has the ſame effect as ſuch a ring would have; only the motion of the nodes of the equator, or of the equinoctial points, is ſlower, becauſe the accumulated parts of the earth, above a ſpherical figure, are diffuſed over its ſurface, and have a leſs effect than if they were all collected in the place of the equator, in the form of a ring. The moon has a greater force on this ring than the ſun,

ſun, becauſe of her leſs diſtance from the earth; and they both contribute to produce the retrograde motion of the equinoctial points: the motion, however, produced by both is ſo ſlow, that thoſe points will not finiſh a revolution in leſs than 25000 years. Our author has determined the quantity of this motion, from its cauſes, and finds it, from the theory, to be perfectly conſonant with the obſervations of aſtronomers.

There is another effect of the action of the ſun and moon on this ring, which is too ſmall to be ſenſible in aſtronomical obſervations: their action on the ring, makes its inclination to the ecliptic to decreaſe and increaſe, by turns, twice every year.

CHAP. VII.

Of the ebbing and flowing of the ſea.

IT is not in the motions of the celeſtial bodies only, that the effects of their mutual gravitation are viſible, for we are now to ſhew, that a phænomenon which paſſes on our earth, and is known to every body, proceeds from the ſame principle; I mean, the ebbing and flowing of the ſea, the ſolution of which, from the bad ſucceſs of thoſe who attempted it before our author, had become a reproach to philoſophy. But he has very plainly and fully accounted for it, from the unequal gravitations of the parts of the earth towards the ſun and moon. It will be worth while, becauſe it is a very celebrated queſtion, to be the more particular in explaining his ſolution of it.

It is obvious, that, if the earth was entirely fluid, and quiescent, its particles, by their mutual gravity towards each other, would form themselves into the figure of an exact sphere. Suppose now, that some power acts on all the particles of this earth, with an equal force, and in parallel directions, the whole mass will be moved by such a power, but its figure will suffer no alteration by it; because all the particles being equally moved by this power, in parallel lines, they will still keep the same situation with respect to each other, and still form a sphere, whose centre will have the same motion as each particle. For, as a drop of water, while it falls towards the earth, retains its spherical figure; and, as the situation of bodies in a ship, that moves with an uniform motion forward, is no way affected by the motion which is common to the ship and all the bodies in it; so the situation of the parts of the earth, with respect to each other, can be no way affected by any power that acts with the same force, and in the same direction, on every part, and promotes each equally.

We have already shewed, that the particles of the earth gravitate towards the moon, and if the gravitation of the particles was every where the same, and acted in the same direction, it would have no effect on the figure of the earth; so that, if the motion of the earth round the common centre of gravity of the earth and moon was destroyed, and the earth was left to the influence of its gravitation towards the moon, the earth falling towards the moon would retain its spherical figure, all the parts being equally carried on, and retaining, therefore, the same situation with respect to each other.

But the actions of the moon, on different parts of the earth, are unequal; thoſe parts, by the general law, being moſt attracted which are neareſt the moon, and thoſe being leaſt attracted which are fartheſt from the moon; while the parts that are at a middle diſtance, are attracted by a mean degree of force: nor are all the parts acted on in parallel lines, but in lines directed towards the centre of the moon: and, on theſe accounts, the ſpherical figure of the earth muſt ſuffer ſome change from the moon's action.

Suppoſe the earth to fall towards the moon, as before, and let us abſtract from the mutual gravitation of its parts towards each other, as alſo from their coheſion; and it will eaſily appear, that the parts neareſt the moon would fall with the ſwifteſt motion, being moſt attracted, and that they would leave the centre or greater bulk of the earth behind them in their fall; while the more remote parts would fall with the ſloweſt motion, being leſs attracted than the reſt, and be left a little behind the bulk of the earth, ſo as to be found at a greater diſtance from the centre of the earth than at the beginning of the motion. From which it is manifeſt, that the earth would ſoon loſe its ſpherical figure, and form itſelf into an oblong ſpheroid, whoſe longeſt diameter would point at the centre of the moon. If the particles of the earth did not gravitate towards each other, but towards the moon only, the diſtances betwixt the parts of the earth that are ſuppoſed to be neareſt to the moon, and the central parts, would continually increaſe, becauſe of their greater celerity in falling; and the diſtance betwixt the central parts, and the parts that are fartheſt from the moon, would increaſe continually at the ſame time; theſe being

 left

left behind by the central parts, which they would follow, but with a leſs velocity. Thus the figure of the earth would become more and more oblong, that diameter of it which pointed towards the moon continually increaſing.

But this is not the only reaſon why the earth would ſoon aſſume an oblong ſpheroidal form, if its parts were allowed to fall freely by their gravity towards the moon's centre. The lateral parts of the earth (that is, thoſe which are at the diſtance of a quarter of a circle from the point which is directly below the moon) and the central parts deſcending with equal velocities, towards the ſame point, *viz.* the centre of the moon, in approaching to it, would manifeſtly approach, at the ſame time, to each other, and, their diſtance growing leſs, the diameters of the earth paſſing through them would become leſs; ſo that the diameter of the earth that points towards the moon would increaſe, and thoſe diameters of the earth that are perpendicular to the line joining the centres of the earth and moon, would decreaſe at the ſame time, and render the figure of the earth ſtill more oblong for this reaſon.

Let us now allow the parts of the earth to gravitate towards its centre; and, as this gravitation far exceeds the action of the moon, and much more exceeds the differences of her actions on the different parts of the earth, the effect that will reſult from the inequalities of theſe actions of the moon, will be only a ſmall diminution of the gravity of thoſe parts of the earth which it endeavoured, in our former ſuppoſition, to ſeparate from its centre, and a ſmall addition to the gravity of theſe parts which it endeavoured to bring nearer to its centre; that is, thoſe parts of the earth which are neareſt to the moon, and thoſe which are fartheſt from her, will have their gravity towards the earth ſomewhat abated; whereas the lateral parts will have their gravity increaſed: ſo that, if the earth be ſuppoſed fluid, the columns from the centre to the neareſt and to the fartheſt parts muſt riſe, till, by their greater height, they be able to ballance the other columns, whoſe gravity is either not ſo much diminiſhed, or is increaſed by the inequalities of the action of the moon: and thus the figure of the fluid earth muſt be ſtill an oblong ſpheroid.

We have hitherto ſuppoſed the earth to fall towards the moon by its gravity. Let us now conſider the earth as projected in any direction, ſo as to move round the centre of gravity of the earth and moon: it is manifeſt, that the gravity of each particle towards the moon will endeavour to bring it as far from the tangent, in any ſmall moment of time, as if the earth was allowed to fall freely towards the moon; in the ſame manner as any projectile at our earth, falls from the line of projection as far as it would fall by its gravity in the perpendicular, in the ſame time. Therefore the parts of the earth neareſt to the moon, will endeavour to fall fartheſt from the tangent, and thoſe fartheſt from the moon will endeavour to fall leaſt from the tangent, of all the parts of the earth; and the figure of the earth, therefore, will be the ſame as if the earth fell freely towards the moon: that is, the earth will ſtill affect a ſpheroidal form, having its longeſt diameter directed towards the moon.

What muſt be carefully attended to here, is, that it is not the action of the moon, but the inequalities in that action, that produce any variation from the ſpherical figure; and that if this action was the ſame in all the particles as in the central parts, and acted in the ſame direction, no ſuch change would enſue. Our author, therefore, to account for this matter, conceives firſt the attraction of the central parts to be diffuſed with an equal force over all the parts, in the ſame direction, and then conceives the inequalities as ariſing from a power ſuperadded, and directed towards the moon where there is an exceſs, and directed in the oppoſite line where there is a defect, in the attraction of the parts, compared with the attraction of the central parts: for thus the ſum of theſe forces, in the firſt caſe, will account for the attraction where it exceeds, and their difference will account for the attraction where it is leſs than in the central parts. And when the effects of theſe powers are conſidered as they affect the figure of the earth, it is manifeſt that they muſt produce ſuch an oblong ſpheroid as we have deſcribed; the ſuperadded force drawing the parts neareſt the moon towards her, and therefore from the earth's centre, while it draws the parts fartheſt from the moon in an oppoſite direction; and therefore ſtill draws from the centre of the earth alſo.

The action of the moon on the lateral parts is reſolved into two, one equal and parallel in its direction to her action on the central parts, and another directed from thoſe lateral parts towards the centre of the earth; the firſt of theſe can have no effect upon the figure of the earth, being conſidered as common to all the particles, and therefore to be neglected in this enquiry: it is the other that adds to the gravity of the lateral parts towards the centre of the earth, and, by adding to the weight of the lateral columns, it makes them ſuſtain the other columns, whoſe gravity is diminiſhed by the action of the moon, to a greater height; and the power which alters the ſpherical figure is to be eſtimated as the ſum of two powers, that which is added to the gravity of the one, and is ſubducted from the gravity of the other.

Hitherto we have abſtracted from the motion of the earth on its axis: but this muſt alſo be conſidered in order to know the real effect of the moon's actions on the ſea. Was it not for this motion, the longeſt diameter of the ſpheroidal figure, which the fluid earth would aſſume, would point at the moon's centre; but, becauſe of the motion of the whole maſs of the earth on its axis from weſt to eaſt, the moſt elevated part of the water no longer anſwers preciſely to the moon, but is carried beyond the moon towards the eaſt in the direction of the rotation.

The water continues to riſe after it has paſſed directly under the moon, tho' the immediate action of the moon there begins to decreaſe, and comes not to its greateſt elevation till it has got half a quadrant further. It continues to deſcend after it has paſſed at 90 degrees diſtance from the point below the moon, tho' the force which the moon adds to its gravity begins to decreaſe there. For ſtill the action of the moon adds to its gravity, and makes it deſcend till it has got half a quadrant farther: the greateſt elevation, therefore, is not in the points which are in a line with the centres of the earth and moon, but about half a quadrant to the eaſt of theſe points in the direction of the motion of rotation.

Thus it appears that the ſpheroidal form, which the fluid earth would affect, will be ſo ſituated that the longeſt diameter of that figure will point to the eaſt of the moon, or that the moon will always be to the weſt of the meridian of the parts of greateſt elevation. Suppoſe now an iſland in this fluid earth, and it will approach in every revolution to each elevated part of this ſpheroid, and the water on the ſhore of this iſland will neceſſarily riſe twice every lunar day; and the time of high water will be when it approaches to theſe elevated parts, that is, when it has paſſed to the eaſt of the moon, or when the moon is at ſome diſtance to the weſt of the meridian.

We have hitherto taken notice of the action of the moon only: but it is manifeſt, that, for the ſame reaſons, the inequality of the ſun's action on the different parts of the earth would produce a like effect, and that theſe alone would produce a like variation from the exact ſpherical figure of a fluid earth. Indeed the effect of the ſun, becauſe of his immenſe diſtance, muſt be conſiderably leſs, tho' the gravity towards the ſun be vaſtly greater. For it is not their actions, but the inequalities in the actions of each, which have any effect; as we have often obſerved. The ſun's diſtance is ſo great, that the diameter of the earth is a point compared to it, and the difference between the actions of the ſun on the neareſt and fartheſt parts becomes, on this account, vaſtly leſs than it would be if the ſun was as near as the moon, whoſe diſtance from us is about 30 diameters of the earth. Thus the inequality of the action of the earth on the parts of a drop of water is altogether inſenſible, becauſe the diameter of the drop is an inſenſible quantity compared with its diſtance from the centre of the earth.

However, the immenſe bulk of the ſun makes the effect ſtill ſenſible at ſo vaſt a diſtance; and therefore, tho' the action of the moon has the greateſt ſhare in producing the tides, the action of the ſun adds ſenſibly to it when they conſpire together, as in the change and full of the moon, when they are nearly in the ſame line with the centre of the earth, and therefore unite their forces; ſo that then the tides are greateſt, and are what we call the *ſpring tides*. The action of the ſun diminiſhes the effect of the moon's action in the quarters, becauſe the one raiſes the water in that caſe where the other depreſſes it; and therefore the tides then are leaſt; and theſe we call the *neap tides*. Tho', to ſpeak accurately, the ſpring and neap tides muſt be ſome time after; becauſe, as in other caſes, ſo in this, the effect is not greateſt or leaſt when the immediate influence of the cauſe is greateſt or leaſt. As the greateſt heat, for example, is not on the ſolſtitial day, when the immediate action of the ſun is greateſt, but ſome time after.

That this may be more clearly underſtood, let it be conſidered, that, tho' the actions of the ſun and moon were to ceaſe this moment, yet the tides would continue to have their courſe for ſome time: For the water where it is now higheſt would ſubſide and flow down on the parts that are lower, till, by the motion of deſcent, being there accumulated to too great a height, it would neceſſarily return again to its firſt place, tho' in a leſs meaſure, being retarded by the reſiſtance ariſing from the attrition of its parts. Thus it would for ſome time continue in an agitation like to that in which it is at preſent. The waves of the ſea that continue after a ſtorm ceaſes, and every motion almoſt of a fluid, may illuſtrate this.

The high water does not always anſwer to the ſame ſituation of the moon, but happens ſometimes ſooner,

ſooner, and ſometimes later, than if the moon alone acted on the ſea. This proceeds from the action of the ſun, which brings on high water ſooner when the ſun alone would produce a tide earlier than the moon, as the ſun manifeſtly would in the firſt and third quarter; and retards the time of high water a little, when the ſun alone would produce a tide later than the moon, as in the ſecond and laſt quarters. The different diſtances of the moon from the earth, produce likewiſe a ſenſible variation in the tides. When the moon approaches the earth, her action on every part increaſes, and the differences of that action, on which the tides depend, increaſe. For her action increaſes as the ſquares of the diſtances decreaſe; and tho' the differences of the diſtances themſelves be equal, yet there is a greater diſproportion betwixt the ſquares of leſs than the ſquares of greater quantities. As for example 3 exceeds 2 as much as 2 exceeds 1, but the ſquare of 2 is quadruple of the ſquare of 1, while the ſquare of 3 (*viz.* 9.) is little more than double the ſquare of 2 (*viz.* 4.) Thus it appears, that, by the moon's approach, her action on the neareſt parts increaſes more quickly than her action on the remote parts, and the tides, therefore, increaſe in a higher proportion as the diſtances of the moon decreaſe. Our author ſhews that the tides increaſe in proportion as the cubes of the diſtances decreaſe, ſo that the moon at half her preſent diſtance would produce a tide eight times greater. The moon deſcribes an ellipſe about the earth, and in her neareſt diſtance produces a tide ſenſibly greater than at her greateſt diſtance from the earth: and hence it is that two great ſpring tides never ſucceed each other immediately; for, if the moon be at her neareſt diſtance from the earth at the change, ſhe muſt be at her greateſt diſtance at the full, having, in the intervening time, finiſhed half a revolution; and there-

therefore the ſpring tide then will be much leſs than the tide at the change was: and for the ſame reaſon, if a great ſpring tide happen at the time of full moon, the tide at the enſuing change will be leſs.

It is manifeſt, that if either the ſun or moon was in the pole, they could have no effect on the tides; for their action would raiſe all the water at the equator to the ſame height; and any place of the earth, in deſcribing its parallel to the equator, would not meet, in its courſe, with any part of the water more elevated than another; ſo that there could be no tide in any place. The effect of the ſun or moon is greateſt when in the equator: for then the axis of the ſpheroidal figure, ariſing from their action, moves in the greateſt circle, and the water is put into the greateſt agitation; and hence it is, that the ſpring tides produced when the ſun and moon are both in the equator, are the greateſt of any, and the neap tides are the leaſt of any, about that time. But the tides produced when the ſun is in either of the tropics, and the moon in either of her quarters, are greater than thoſe produced when the ſun is in the equator, and the moon in her quarters; becauſe, in the firſt caſe, the moon is in the equator; and, in the latter caſe, the moon is in one of the tropics: and the tide depends more on the action of the moon than that of the ſun, and is therefore greateſt when the moon's action is greateſt. However, becauſe the ſun is nearer the earth in winter than in ſummer, hence it is, that the greateſt ſpring tides are after the autumnal and before the vernal equinox.

When the moon declines from the equator towards either pole, one of the greateſt elevations of the water follows the moon, and deſcribes nearly the parallel on the earth's ſurface which is under that which

which the moon, becauſe of the diurnal motion, ſeems to deſcribe: and the oppoſite greateſt elevation, being *Antipodes* to that, muſt deſcribe a parallel as far on the other ſide of the equator: ſo that while the one moves on the north ſide of the equator, the other moves on the ſouth ſide of it, at the ſame diſtance. Now the greateſt elevation which moves on the ſame ſide of the equator, with any place, will come nearer to it than the oppoſite elevation, which moves in a parallel on the other ſide of the equator; and therefore, if a place is on the ſame ſide of the equator with the moon, the day tide, or that which is produced while the moon is above the horizon of the place, will exceed the night tide, or that which is produced while the moon is under the horizon of the place. It is the contrary if the moon is on one ſide and the place on the other ſide of the equator; for then the elevation which is oppoſite to the moon, moves on the ſame ſide of the equator with the place, and therefore will come nearer to it than the other elevation. This difference will be greateſt when the ſun and moon both deſcribe the tropics; becauſe the two elevations in that caſe deſcribe the oppoſite tropics, which are the fartheſt from each other of any two parallel circles they can deſcribe. Thus it is found, by obſervation, that the evening tides in the ſummer exceed the morning tides, and the morning tides in winter exceed the evening tides. The difference is found at *Briſtol* to amount to fifteen inches, at *Plymouth* to one foot. It would be ſtill greater, but that a fluid always retains an impreſſed motion for ſome time; ſo that the preceding tides affect always thoſe that follow them *.

The

* See *Fig.* 71. (from Sir *Iſaac Newton)* in which the ſpheroid P A *p* E repreſents the earth, P, *p*, the poles, A E the equator,

The phænomena of particular places agree with theſe general obſervations, if the ſituation and extent of the ſeas and ſhores, in which they are ſituated, are conſidered. It has been always known that the tides follow the motion of the moon, riſing twice in one revolution of the moon to the meridian of any place; which exceeds a ſolar day by above $\frac{3}{4}$ of an hour, becauſe the proper motion of the moon retards ſo much her appulſe to the meridian of the place. All the effects of the ſun's action, ſometimes promoting, ſometimes abating the effects of the action of the moon, as before mentioned, are alſo conformable to perpetual obſervation: and the tides in places that lie on a deep and open ocean, where the water can eaſily follow the influences of the ſun and moon, are agreeable to this theory.

That the tides may have their full motion, the ocean in which they are produced ought to be extended from eaſt to weſt 90°, or a quarter of a circle of the earth, at leaſt. Becauſe the places,

tor, F any place not in the equator, F *f* its parallel, D *d* a parallel on the other ſide of the equator, L the moon's place three hours before, H the place of the earth to which L is vertical, and *h* the oppoſite place, K, *k*, places 90° diſtant from theſe. Then will C H, C *h*, meaſure the greateſt elevations of the water, and CK, C *k*, the leaſt. C F, C *f*, C D, C *d*, will be the elevations at F, *f*, D, *d*. And if N M is a circle of the ſpheroid, meeting the equator and theſe parallels in S, R, T, C N will be the elevation of the water at S, R, T, or any other places in the circle N M. The higheſt tides at any place F, happen at F and *f*, three hours after the moon's paſſing the meridian, above or below the horizon; and the loweſt at Q three hours after her ſetting or riſing. And if F and L are on the ſame ſide of the equator, the day tide will riſe higher than the night tide, C F being greater than C *f*. 'Tis the contrary, when the moon's declination and the latitude of a place D are of oppoſite denominations, the one north and the other ſouth; becauſe then C D is greater than C *d*.

where

fronting pag. 389.

Fig. 70.

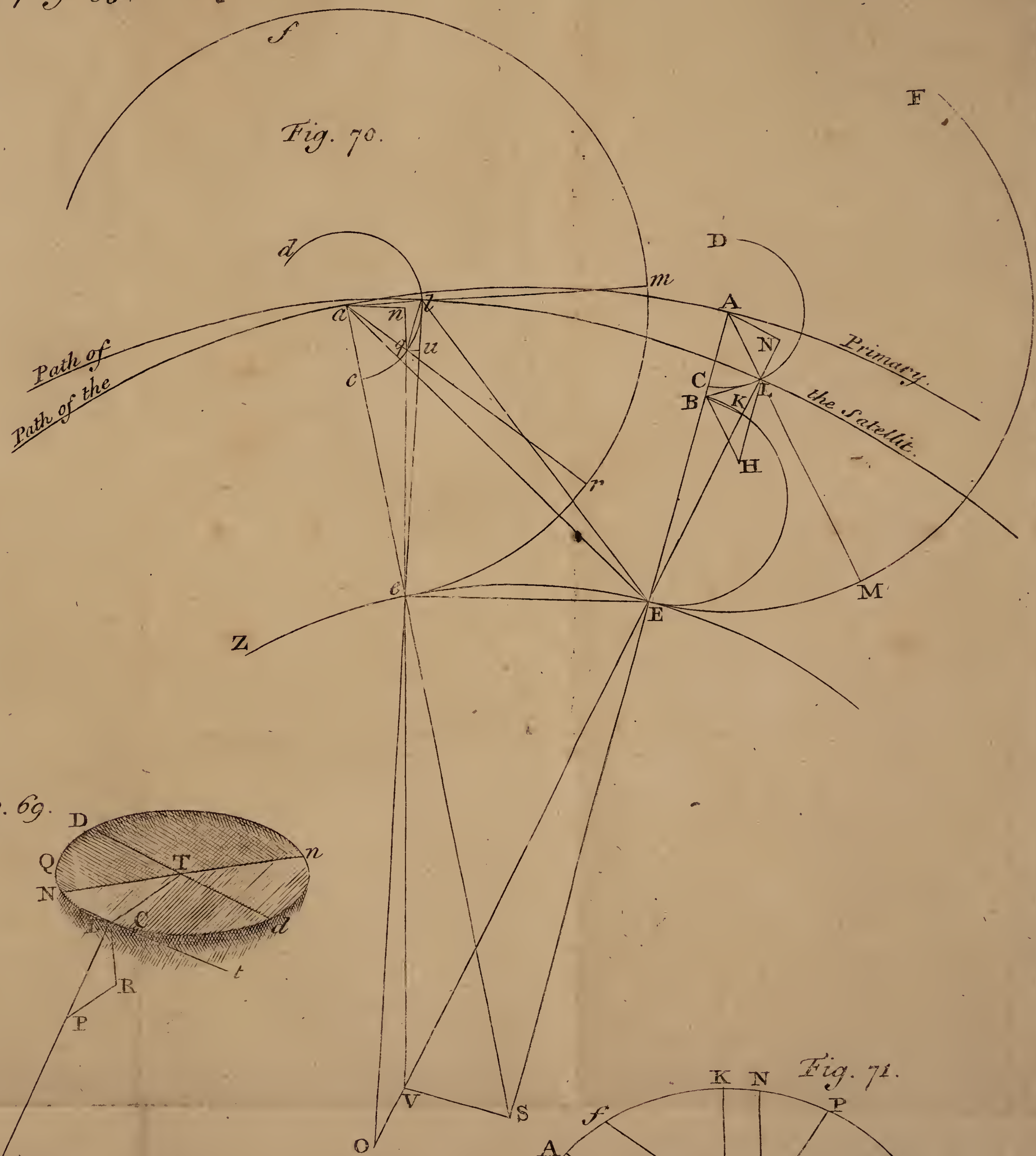


Fig. 69.

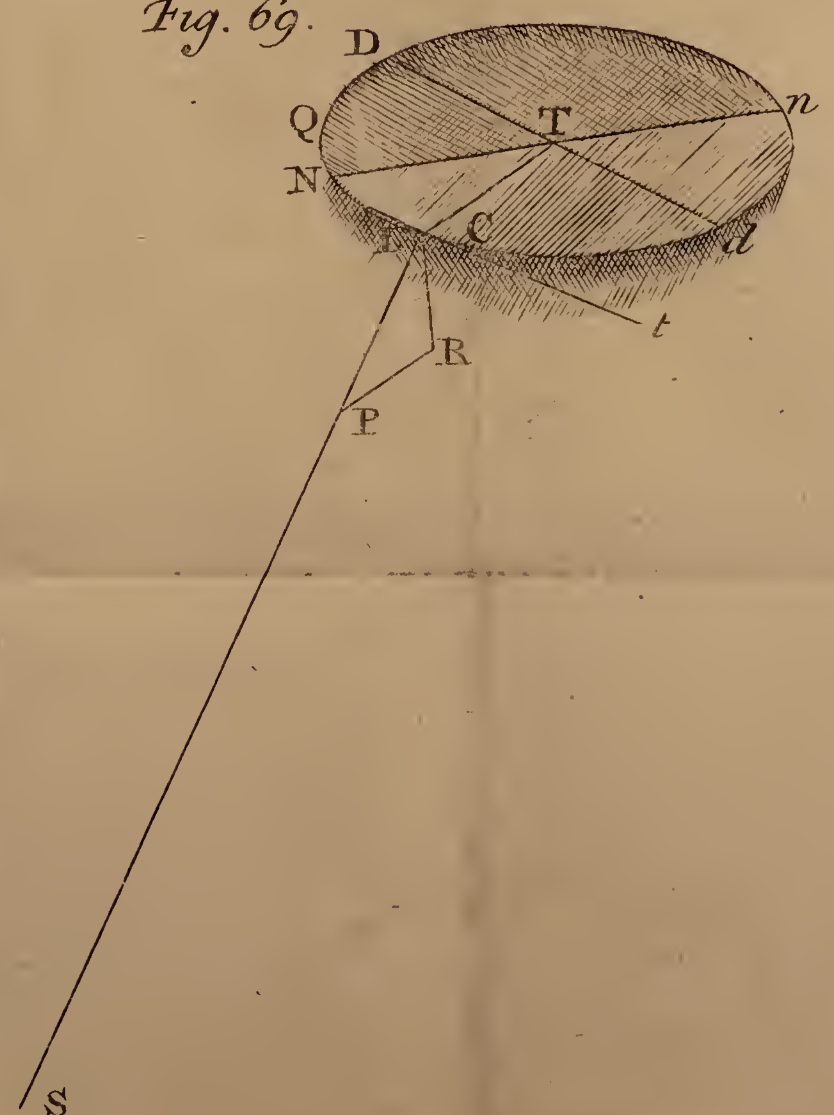
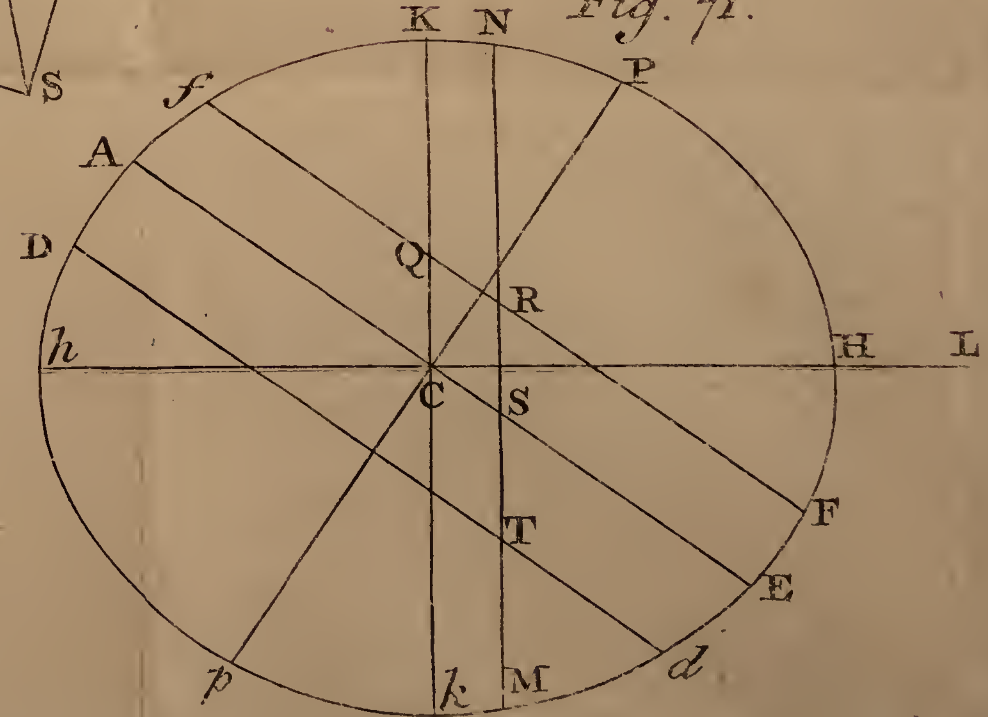


Fig. 71.



where the moon raiſes moſt, and moſt depreſſes, the water, are at that diſtance from each other. Hence it appears, that it is only in the great oceans that ſuch tides can be produced; and why in the larger *pacific* ocean they exceed thoſe in the *Atlantic* ocean. Hence alſo, it is obvious why the tides are not ſo great in the torrid zone, between *Africa* and *America*, where the ocean is narrower, as in the temperate zones on either ſide; and, from this alſo, we may underſtand why the tides are ſo ſmall in iſlands that are very far diſtant from the ſhores. It is manifeſt, that, in the *Atlantic* ocean, the water cannot riſe on one ſhore but by deſcending on the other; ſo that, at the intermediate diſtant iſlands, it muſt continue at about a mean height betwixt its elevation on the one and on the other ſhore.

As the tides paſs over ſhoals, and run through ſtraits into bays of the ſea, their motion becomes more various, and their height depends on a great many circumſtances. The tide that is produced on the weſtern coaſts of *Europe*, in the *Atlantic*, correſponds to the ſituation of the moon we deſcribed above. Thus it is high water on the coaſts of *Spain*, *Portugal*, and the weſt of *Ireland*, about the third hour after the moon has paſſed the meridian. From thence it flows into the adjacent channels, as it finds the eaſieſt paſſage. One current from it, for example, runs up by the ſouth of *England*, another comes in by the north of *Scotland*: they take a conſiderable time to move all this way, and it is high water ſooner in the places to which they firſt come; and it begins to fall at thoſe places, while they are yet going on to others that are farther in their courſe. As they return, they are not able to raiſe the tide, becauſe the water runs faſter off than it returns, till, by a new tide propagated from the open ocean, the return of the current is ſtop'd, and the water begins to riſe again. The tide takes twelve hours to come from the ocean to *London-bridge*, ſo that, when it is high water there, a new tide is already come to its height in the ocean; and, in ſome intermediate place, it muſt be low water at the ſame time. In channels, therefore, and narrow ſeas, the progreſs of the tides may be, in ſome reſpects, compared to the motion of the waves of the ſea. Our author alſo obſerves, that when the tide runs over ſhoals, and flows upon flat ſhores, the water is raiſed to a greater height than in the open and deep oceans that have ſteep banks; becauſe the force of its motion cannot be broke, upon theſe level ſhores, till the water riſes to a greater height.

If a place communicates with two oceans, (or two different ways with the ſame ocean, one of which is a readier and eaſier paſſage) two tides may arrive at that place in different times, which, interfering with each other, may produce a great variety of phænomena. An extraordinary inſtance of this kind is mentioned by our author at *Batſha*, a port in the kingdom of *Tunquin* in the *Eaſt Indies*, of northern latitude 20° 50′. The day in which the moon paſſes the equator, the water ſtagnates there without any motion: as the moon removes from the equator, the water begins to riſe and fall once a day; and it is high water at the ſetting of the moon, and low water at her riſing. This daily tide increaſes for about ſeven or eight days, and then decreaſes for as many days by the ſame degrees, till this motion ceaſes when the moon has returned to the equator. When ſhe has paſſed the equator, and declines towards the ſouth pole, the water riſes and falls again, as before; but 'tis high water now at the riſing, and low water at the ſetting, of the moon.

Our

Our author, to account for this extraordinary tide, confiders that there are two inlets to this port of *Batſha*, one from the *Chineſe* ocean betwixt the continent and the *Manillas*, the other from the *Indian* ocean betwixt the continent and *Borneo*. This leads him to propoſe, as a ſolution of the phænomenon, that a tide may arrive at *Batſha*, through one of theſe inlets, at the third hour of the moon, and another through the other inlet ſix hours after, at the ninth hour of the moon. For, while theſe tides are equal, the one flowing in as the other ebbs out, the water muſt ſtagnate: now they are equal when the moon is in the equator; but as ſoon as the moon begins to decline on the ſame ſide of the equator with *Batſha*, we have ſhewed that the diurnal tide muſt exceed the nocturnal, ſo that two greater and two leſſer tides muſt arrive at *Batſha* by turns. The difference of theſe will produce an agitation of the water, which will riſe to its greateſt height at the mean time betwixt the two greateſt tides, and fall loweſt at a mean time betwixt the two leaſt tides; ſo that it will be high water about the ſixth hour at the ſetting of the moon, and low water at her riſing. When the moon has got to the other ſide of the equator, the nocturnal tide will exceed the diurnal; and therefore, the high water will be at the riſing, and low water at the ſetting, of the moon. The ſame principles will ſerve to account for other extraordinary tides, which, we are told, are obſerved in places whoſe ſituation expoſes them to ſuch irregularities.

Our author does not content himſelf with theſe general obſervations, but calculates the effects of the ſun and moon upon the tides, from their attractive powers. The augmentation of the gravity of the lateral parts of the earth, produced by the action of of the ſun, is a ſimilar effect to an augmentation, eſtimated by him before, that is made to the gravity of the moon towards the earth by the ſame action, when the moon is in the quarters; only the addition made to the gravity of the lateral parts is * about $60\frac{1}{2}$ times leſs, becauſe their diſtance from the earth's centre is ſo many times leſs than the diſtance of the moon from it. The gravity of thoſe parts of the earth that are directly beneath the ſun, and of thoſe oppoſite to it, is diminiſhed by a double quantity of what is added to the lateral parts; and as the diminution of gravity of the one, and augmentation of the gravity of the other, conſpire together in raiſing the water under the ſun, and the parts oppoſite to it, above its height in the lateral parts; the whole force that produces this effect is to be conſidered as triple of what is added to the gravity of the lateral parts: and is thence found to be to the gravity of the particles as 1 to 12868200, and to the centrifugal force at the equator as 1 to 44527. The elevation of the waters, by this force, is conſidered by our author as an effect ſimilar to the elevation of the equatorial parts above the polar parts of the earth, ariſing from the centrifugal force at the equator; and, being 44527 times leſs, is found to be 1 foot and $11\frac{1}{30}$ inches, *Paris* meaſure. This is the elevation ariſing from the action of the ſun upon the water.

In order to find the force of the moon upon the water, he compares the ſpring tides at the mouth of the river *Avon* below *Briſtol* (which are the effect of the ſum of the forces of the ſun and moon when their actions almoſt conſpire together,) with the neap tides there (which are the effect of the difference of theſe forces when they act almoſt againſt one ano-

* *Princip.* Lib. III. Prop. 36.

ther,)

ther,) and finds their proportion to be that of 9 to 5; from which, after several necessary corrections, he concludes that the force of the moon is to the force of the sun, in raising the waters of the ocean, as 4,4815 to 1; so that the force of the moon is able, of itself, to produce an elevation of 8 feet and $7\frac{5}{22}$ inches, and the sun and moon together may produce an elevation of about $10\frac{1}{2}$ feet, in their mean distances from the earth, and an elevation of about 12 feet when the moon is nearest the earth. The height to which the water is found to rise, upon the coasts of the open and deep ocean, is agreeable enough to this computation.

It is from this last calculation that he is able to make an estimate of the density and quantity of matter in the moon. Her influence on the tides is the only effect of the moon's attracting power which we have access to measure, and it enables us to estimate her density compared with that of the sun, which we find it exceeds in the proportion of 4891 to 1000; and since the density of the earth is to that of the sun as 4000 to 1000, it follows that the moon must be more dense than the earth in the proportion of 4891 to 4000, or of 11 to 9 nearly. The proportion of the diameter of the earth to that of the moon is known, from astronomical observations, to be that of 365 to 100; and from these two proportions it easily follows, that the quantity of matter in the moon is to the matter in the earth as 1 to 39,788; and the centre of gravity of the earth and moon must be, therefore, almost 40 times nearer to the earth than to the moon; and, the situation of their centre of gravity being known, the motions in their system may be determined with great preciseness.

 Our

Our author enquires into the figure of the moon: and, becauſe the earth contains near 40 times more matter than the moon, the elevation produced by the action of the earth in the parts of the moon that are neareſt to it, and in the parts oppoſite to theſe, would be near 40 times greater than that which the moon produces in our ſeas, if this elevation was not to be diminiſhed in proportion as the ſemidiameter of the moon is leſs than the ſemidiameter of the earth, that is, in the proportion of 100 to 365. By compounding theſe proportions he finds, that the diameter of the moon that paſſes through the centre of the earth, muſt exceed thoſe that are perpendicular to it, by about 186 feet. He thinks the ſolid parts of the moon muſt have been formed into ſuch a ſpheroidal figure, having its longeſt diameter directed towards the earth; and this may be the reaſon why the moon always turns the ſame ſide towards the earth. If there were great ſeas in the moon, and if ſhe revolved on her axis ſo as to turn different ſides towards the earth, there would have been very great tides produced in them, ſuch as would exceed our tides ten times; but, by her keeping one ſide always towards the earth, there are no tides produced in her ſeas, but what proceed from the differences of their diſtances from the earth, and from the moon's librations; for the action of the ſun can have very little effect upon them.

CHAP.

CHAP. VIII.

Of the comets.

1. HItherto we have treated of the planets: but, besides these, we find in the expanse of heaven many other bodies belonging to the system of the sun, that seem to have much more irregular motions. These are the *comets*, which, descending from the far distant parts of the system with great rapidity, surprize us with the singular appearance of a train, or tail, which accompanies them; become visible to us in the lower parts of their orbits, and, after a short stay, are carried off again to vast distances, and disappear. Tho' some of the ancients had more just notions of them, yet the opinion having prevailed, that they were only meteors generated in the air, like to those we see in it every night, and in a few moments vanishing, no care was taken to observe or record their phænomena accurately, till of late. Hence this part of astronomy is very imperfect. The number of the comets is far from being known: many have been noted by historians formerly, and not a few of late observed by astronomers; and some have been discovered accidentally by telescopes, passing by us, that never became visible to the naked eye: so that we may conclude their number to be very great. Their periods, magnitudes, and the dimensions of their orbits, are also uncertain. This is a part of science, the perfection of which may be reserved for some distant age, when these numerous bodies, and their vast orbits, by long and accurate observation, may be added to the known parts of the solar system. Astronomy will appear as a new science, after all the discoveries we

now boaſt of: but then it will be remembred, even in thoſe flouriſhing days of aſtronomy, that it was Sir *Iſaac Newton* who diſcovered and demonſtrated the principles by which alone ſuch great improvements could be made; and that he begun and carried this work ſo far, that he left to poſterity little more to do, but to obſerve the heavens, and compute after his models.

Having this part of aſtronomy to deduce almoſt from its elements, he begins with ſhewing, againſt the ſcholaſtic philoſophers, that the comets are above the moon; becauſe they participate of the apparent diurnal motion, riſing and ſetting daily, as all things that are not appendant to the earth do, and that without any ſenſible diurnal parallax. But, as they are all affected by the annual motion of the earth, appearing, like the planets, ſometimes direct, ſometimes retrograde, he concludes that, when they become viſible to us, they muſt be in the regions of the planets. As they are all affected by the motion of the earth, and it is impoſſible to bring their motions to any regularity without allowing that motion; and it, alone, ſuffices for explaining the irregularities of every comet, as well as of every planet; we obtain from this a new confirmation of the motion of the earth, and find all the parts of this philoſophy perfectly conſiſtent.

Our author having ſhewed that the comets deſcend into the planetary regions when they are viſible to us, againſt the opinion of *Des Cartes*, he proceeds to trace them in their courſes. It follows, from the general law of gravity already eſtabliſhed, that they muſt move either in *parabolic*, or very excentric *elliptical*, orbits, that have one focus in the centre of the ſun. He then enquires, with his uſual ſkill

ſkill and a great deal of labour, how a motion in a parabola may agree with the obſervations that have been made upon the comets; and, for this end, ſhews how, from three obſervations, the parabolic trajectory which a comet deſcribes may be determined: and, from ſeveral examples which he has given, there appears ſo perfect a harmony between his theory and the obſervations, as adds a new evidence to it, and ſhews its uſe in carrying on the knowledge of our ſyſtem.

He inſiſts particularly on the celebrated comet that appeared near the end of the year 1680, and in the beginning of 1681. He determines its trajectory, or curve, from three obſervations made by Mr. *Flamſteed*; and then compares all the obſervations, that were made by himſelf or others, with the motion of a body in that curve, and finds the differences betwixt the obſerved places of this comet and thoſe computed for it in the curve, for the ſame time, to be very ſmall. It was the ſame comet that was ſeen in *November* 1680, and in *December*, *January*, *February* and *March* following, tho' they had been generally eſteemed two different comets. In *November* it was deſcending towards the ſun; it paſſed very near the ſun on the 12th day of *December*; where, having been heated to a prodigious degree, tho' the light of the head or *nucleus* was duller, yet, while it aſcended in the other half of its orbit, its tail was vaſtly greater than before, extending ſometimes 70° in length, and continuing viſible even after the head or nucleus was carried out of ſight.

Dr. *Halley*, to whom every part of aſtronomy, but this in a particular manner, is highly indebted, has joined his labours to our author's on this ſubject; nor is it neceſſary for us to ſeparate them. Finding three obſervations of comets recorded in hiſtory, agreeing with this in remarkable circumſtances, and returning at the diſtance of 575 years from each other, he ſuſpected that theſe might be one and the ſame comet, revolving in that period about the ſun. He therefore ſuppoſed the *parabola* to be changed into ſuch an excentric *ellipſe* as the comet might deſcribe in 575 years, and as ſhould nearly coincide with the parabola in its loweſt part; and, having computed the places of the comet in this elliptic orbit, he found them to agree ſo well with thoſe in which the comet was obſerved to paſs, that the variations did not exceed the differences which are found betwixt the computed and the obſerved places of the planets, whoſe motions had been the ſubject of aſtronomical calculation for ſome thouſand years. This comet may, therefore, be expected again after finiſhing the ſame period, about the year 2255. If it then return, it will give a new luſtre and evidence to Sir *Iſaac Newton*'s philoſophy, in that diſtant age. And ſhould the inconſtancy of human affairs, and the perpetual revolutions to which they are ſubject, occaſion any neglect of our philoſophy in the intervening ages; this comet will revive it, and fill every mouth again with this great man's name. Nor need this be eſteemed a vain prediction; for we cannot but ſuppoſe that the attention of the aſtronomers of thoſe days, to this comet muſt be raiſed to a great pitch, becauſe in one part of its orbit it approaches very near to the orbit of our earth; ſo that, in ſome revolutions, it may approach near enough to have very conſiderable, if not fatal, effects upon it. Nor is it to be doubted but that, while ſo many comets paſs among the orbits of the planets, and carry ſuch immenſe tails along with them, we ſhould have been called, by very extraordinary conſequences, to attend to theſe bodies long ago, if the motions in the uni-

verſe

verſe had not been at firſt deſigned, and produced, by a Being of ſufficient ſkill to foreſee their moſt diſtant conſequences. Our earth was out of the way when this comet laſt paſſed near her orbit; but it requires a perfect knowledge of the motion of the comet, to be able to judge if it will always paſs by us with ſo little effect. We may here obſerve, that theſe great periods, and diſtant depending obſervations, promiſe this good effect, that they muſt contribute to preſerve the reliſh for learning from the revolutions it has been formerly ſubject to. By them, diſtant ages are connected together, and perpetual matter for reviving the curioſity of men is provided, from time to time.

But we are not to wait for the return of this diſtant comet to have our author's theory verified, and to ſee predictions of this kind begin to take place. By comparing together the orbits of the comets that appeared in 1607 and 1682, they are found ſo coincident, that we cannot but ſuppoſe them to be one and the ſame comet, revolving in 75 years about the ſun. If this comet, according to this period, return in 1758, aſtronomy will then have ſomething new to boaſt of. It ſeems to be of thoſe that riſe to the leaſt height from the ſun, its greateſt diſtance being only 35 times greater than the diſtance of the earth from the ſun; ſo that, at the fartheſt, it does not run out four times farther from us than *Saturn*. It will probably be the firſt that will be added to the number of the revolving planets, and eſtabliſh this part of our author's theory.

Beſides theſe comets we have mentioned, our author has conſidered the motions of ſeveral others, and finds his theory always conſonant with obſervation. He particularly computes the places of a remarkable comet that appeared in 1664 and 1665. It moved over 20° in one day, and defcribed almoft fix figns in the heavens before it difappeared; its courfe deviated from a great circle, towards the north, and its motion, that had been before retrograde, became direct towards the end: and, notwithftanding fo unufual a courfe, its places, computed from our author's theory, agree with the obferved places, as well as thofe of the planets agree with their theory.

The phænomena of all the comets, but efpecially of the comet of 1680, fhew them to be folid, fixed and durable bodies. This comet was, in its *perihelium*, 166 times nearer to the fun than our earth is: and, from this, our author computes that it muft have conceived a heat 2000 times greater than that of iron almoft going into fufion, and that, if it was equal to our earth, and cooled in the fame manner as terreftrial bodies, it would take 50,000 years to cool: to bear fo prodigious a heat, it muft furely be a very folid and fixed body.

There is a phænomenon that attends each comet, and is peculiar to them, called its *tail*: fome have imputed this appearance to the refraction of the fun-beams, paffing through the nucleus or head, which they fuppofed to be tranfparent: others, to the refraction of the beams reflected from the head, as they pafs through the intermediate fpaces to us. Our author refutes both thefe opinions, and fhews that the tail confifts of a vapour arifing continually from the body of the comet, towards thofe parts that are oppofite to the fun, for a like reafon that vapour or fmoke rifes in the atmofphere of the earth. Becaufe of the motion of the body of the comet, the tail is bent a little towards thofe parts which the comet leaves in its motion. Theſe tails are found greateſt after it has paſſed its perihelium, or leaſt diſtance from the ſun, where its heat is greateſt, and the atmoſphere of the ſun is moſt denſe. The head appears, after this, obſcured by the thick vapour that riſes plentifully from it. The tail of the comet of 1680 was of a prodigious ſize: it was extended from the head to a diſtance ſcarcely inferior to the vaſt diſtance of the ſun from the earth. As the matter of the tail participates of the motion of the comet, it is thereby carried along with the comet in its motion, and ſome part of it returns again with it: and as the matter in the tail riſes, it becomes more and more rarified; as appears from the tail's increaſing in breadth upwards. By this rarefaction a great part of the tail muſt be dilated and diffuſed over the ſyſtem; ſome of this, by its gravity, may fall towards the planets, mix with their atmoſpheres, and ſupply the fluids, which, in natural operations, are conſumed; and may, perhaps, ſupply that ſubtile ſpirit in our air, which is neceſſary for the life of animals, and for other natural operations.

We are not to expect that the motions of the comets can be ſo exact, and the periods of their revolutions ſo equal, as thoſe of the planets; conſidering their great number, and their great diſtance from the ſun in their aphelia, where their actions upon each other muſt have ſome effect to diſturb their motions. The reſiſtance which they meet with in the atmoſphere of the ſun, when they deſcend into the lower parts of their orbits, will alſo affect them. By the retardation of their motion in theſe lower parts, their gravity will be enabled to bring them nearer the ſun in every revolution, till at length they fall into him, and ſupply fewel to that immenſe body of fire. The comet of 1680 paſſed at a diſtance from

from the ſurface of the ſun, no greater than the 6th part of his diameter; it will approach ſtill nearer in the next revolution, and fall into his body at length. The fixed ſtars may receive ſupplies, in the ſame manner, by comets falling into them; and ſome of them, whoſe light and heat are almoſt exhauſted, may receive new fewel in this way. Of this kind thoſe ſtars ſeem to be, which have been obſerved to break out at once with great ſplendor, and to vaniſh gradually afterwards. Such was the ſtar in *Caſſiopeia*, that was not viſible on the 8th of *November* 1572, but ſhone the following night with a brightneſs almoſt equal to that of the planet *Venus*, and decreaſed continually afterwards, till in 16 months time it vaniſhed. Another of the ſame kind appeared to *Kepler*'s ſcholars in the right foot of *Serpentarius*, on the 30th of *September* 1604, brighter than *Jupiter*, tho' it was not viſible the preceding night; which alſo decreaſed gradually, and vaniſhed in fifteen or ſixteen months. By ſuch a new ſtar appearing with an extraordinary brightneſs in the heavens, *Hipparchus* is ſaid to have been induced to make his catalogue of the fixed ſtars. But thoſe ſtars which appear and diſappear, gradually increaſing and decreaſing by turns, ſeem to be of a different kind; and to have a luminous and an obſcure ſide, which, by their rotation on their axis, they turn towards us alternately.

The argument againſt the eternity of the univerſe, drawn from the decay of the ſun, ſtill ſubſiſts; and even acquires a new force from this theory of the comets: ſince the ſupply which they afford muſt have been long ago exhauſted, if the world had exiſted from eternity. The matter in the comets themſelves, that ſupplies the vapour which riſes from them in every revolution to the perihelium, and

forms

forms their tails, muſt alſo have been exhauſted long ere now. In general, all quantities that muſt be ſuppoſed to decreaſe or increaſe continually, are repugnant to the eternity of the world; ſince the firſt had been exhauſted, and the laſt had grown into an infinite magnitude, at this time, if the world had been from eternity: and of both kinds there ſeem to be ſeveral ſorts of quantities in the univerſe.

The deſcent of the comets into the planetary regions ſhews that the ſolid orbs, in which the planets were ſuppoſed, by the ſchoolmen, to move, are imaginary. And the regularity of their motions, while they are carried in very excentric orbs, in all directions, into all parts of the heavens, conſpire with many other arguments to overthrow the *Carteſian* vortices.

Sir *Iſaac Newton* further obſerves, that while the comets move in all parts of the heavens, with different directions, and in very excentric orbits, whoſe planes are inclined to one another in large angles; it cannot be attributed to blind fate that the planets move round the ſun, and the ſatellites round their reſpective primaries, all with one direction, in orbits nearly circular, and almoſt in the ſame plane. The comets, by moving in very excentric orbits, deſcend with a vaſt velocity, and are carried quickly thro' the planetary regions, where they approach the neareſt to each other, and to the planets, ſo as to have as little time as poſſible to diſturb their own motions, or thoſe of the planets. By their moving in very different planes, they are carried to a vaſt diſtance from each other in the higheſt parts of their orbits, or aphelia; where, becauſe of the ſlowneſs of their motions, and the weakneſs of the ſun's action at ſo great diſtances, their mutual actions, but for this

precau-

precaution, would produce the greateſt diſorders. Thus we always find, that what has, at firſt ſight, the appearance of irregularity and confuſion in nature, is diſcovered, on further enquiry, to be the beſt contrivance and the moſt wiſe conduct.

Sir *Iſaac Newton* proceeds to make ſome reflections on the nature of the ſupreme *cauſe*, and infers, from the ſtructure of the viſible world, that it is governed by *One Almighty*, and *All-wiſe Being*, who rules the world, not as its *Soul* but as its *Lord*, exerciſing an abſolute ſovereignty over the univerſe, not as over his *own body* but as over his *work*; and acting in it according to his pleaſure, without ſuffering any thing from it. What he has delivered concerning the Deity will be further explained in the next chapter.

CHAP. IX.

Of the Supreme Author and Governor of the univerſe, the True and Living God.

1. ARISTOTLE concludes his treatiſe *de mundo*, with obſerving, that "to treat of the world without ſaying any thing of its Author would be impious;" as there is nothing we meet with more frequently and conſtantly in nature, than the traces of an All-governing Deity. And the philoſopher who overlooks theſe, contenting himſelf with the appearances of the material univerſe only, and the mechanical laws of motion, neglects what is moſt excellent; and prefers what is imperfect to what is ſupremely perfect, finitude to infinity, what is narrow and weak to what is unlimited and almighty, and what is periſhing to what endures for ever. Such who

who attend not to ſo manifeſt indications of ſupreme wiſdom and goodneſs, perpetually appearing before them wherever they turn their views or enquiries, too much reſemble thoſe antient philoſophers who made night, matter, and chaos, the original of all things.

2. As we have neither ideas nor words ſufficient to deſcribe the firſt cauſe, ſo *Ariſtotle*, in the concluſion of the above mentioned treatiſe, is obliged to content himſelf with comparing him with what is chief and moſt excellent, in every kind *. Thus we ſay he is the king or lord of all things, the parent of all his creatures, the ſoul of the world, or great ſpirit that animates the whole. Such expreſſions, tho' well meant at firſt, were ſometimes abuſed afterwards; particularly, that of his being the *anima mundi*, which was apt to repreſent him not only as the active and ſelf-moving principle, but likewiſe as paſſive and ſuffering from the actions and motions of bodies. The abſtruſe nature of the ſubject gave occaſion to the later *Platoniſts*, particularly to *Plotinus*, to introduce the moſt myſtical and unintelligible notions concerning the Deity and the worſhip we owe to him; as when he tells us that intellect or underſtanding is not to be aſcribed to the Deity, and that our moſt perfect worſhip of him conſiſts, not in acts of veneration, reverence, gratitude or love; but in a certain myſterious ſelf-annihilation, or total extinction of all our faculties. Theſe doctrines, however abſurd, have had follow-

* Καθόλου δὲ, ὅπερ ἐν νηῒ κυβερνήτης, ἐν ἅρματι δὲ ἡνίοχος, ἐν χορῷ δὲ κορυφαῖος, ἐν πολει δὲ νόμος, ἐν στρατοπέδῳ δὲ ἡγεμών· τοῦτο θεὸς ἐν κόσμῳ· πλὴν καθ᾽ ὅσον, τοῖς μεν καματηρὸν τὸ ἄρχειν, πολυκίνητόν τε καὶ πολυμέριμνον· τῷ δὲ, ἄλυπον, απονόν τε καὶ πάσης κεχωρισμένον σωματικῆς ασθενείας· ἐν ἀκινήτῳ γὰρ ἱδρυμένος πάντα κινεῖ, καὶ περιάγει ὅπου βούλεται, καὶ ὅπως, ἐν διαφόροις ἰδέαις τε καὶ φύσεσιν. Cap. 6.

ers,

ers, who, in this as in other cafes, by aiming too high, far beyond their reach, overftrain their faculties, and fall into folly or madnefs; contributing, as much as lies in them, to bring true piety and devotion into contempt.

3. Neither are they to be commended, who, under the pretence of magnifying the effential power of the fupreme caufe, make truth and falfhood entirely to depend on his will; as we obferved of *Des Cartes*, Book I. Chap. 4. Such tenets have a direct tendency to introduce the abfurd opinion, that intellectual faculties may be fo made, as clearly and diftinctly to perceive that to be true, which is really falfe. They judge much better, who, without fcruple, meafure the divine omnipotence itfelf, and the poffibility of things, by their own clear ideas concerning them; affirming that God himfelf cannot make contradictions to be true at the fame time; and reprefent the certain part of our knowledge, in fome degree, as the knowledge and wifdom of the Deity imparted to us, in the views of nature which he has laid before us.

4. The fublimity of the fubject is apt to exalt and tranfport the minds of men, beyond what their faculties can always bear: therefore, to fupport them, allegorical and enigmatical reprefentations have been invented, which in procefs of time have produced the greateft abufes. When metaphorical figures and names came to be confidered as realities, in place of the true God, falfe deities were fubftituted without number, and, under the pretence of devotion, a worfhip was paid to the moft deteftable characters, that tended to extinguifh the notions of true worth and virtue amongft men.

5. As there are no enquiries of a more arduous nature than thoſe that relate to the Deity, or of near ſo great importance to intellectual beings, that diſcern betwixt truth and falſhood, betwixt right and wrong; ſo it is manifeſt, that there are none in which the utmoſt caution and ſoberneſs of thought is more requiſite. Hence it is a very unpleaſant proſpect to obſerve with how great freedom, or rather licentiouſneſs, philoſophers have advanced their raſh and crude notions concerning his nature and eſſence, his liberty and other attributes. What freedoms were taken by *Des Cartes* in deſcribing the formation of the univerſe without his interpoſition, and in pretending to deduce from his attributes conſequences that are now known to be falſe, we explained in the firſt book, almoſt in his own words. A manner of proceeding ſo unjuſtifiable, in ſo ſerious and important a ſubject, ought, one would think, to have diſguſted the ſober and wiſe part of mankind. *Spinoza*, while he carried the doctrine of abſolute neceſſity to the moſt monſtrous height, and ſurpaſſed all others in the weakneſs of his proofs as well as the impiety of his doctrines, yet affects to ſpeak, on ſeveral occaſions, in the higheſt terms of veneration for the Deity. Mr. *Leibnitz* and many of his diſciples have likewiſe maintained the ſame doctrine of abſolute neceſſity, extending it to the Deity himſelf, of whom our ideas are ſo inadequate, and whom it ſo much concerns us not to miſrepreſent. But Sir *Iſaac Newton* was eminently diſtinguiſhed for his caution and circumſpection, in ſpeaking or treating of this ſubject, in diſcourſe as well as in his writings; tho' he has not eſcaped the reproaches of his adverſaries even in this reſpect. As the Deity is the ſupreme and firſt cauſe, from whom all other cauſes derive their whole force and energy, ſo he thought

thought it moſt unaccountable to exclude *Him only* out of the univerſe. It appeared to him much more juſt and reaſonable, to ſuppoſe that the whole chain of cauſes, or the ſeveral *ſeries* of them, ſhould centre in him as their ſource and fountain; and the whole ſyſtem appear depending upon him the only independent cauſe.

6. The plain argument for the exiſtence of the Deity, obvious to all and carrying irreſiſtible conviction with it, is from the evident contrivance and fitneſs of things for one another, which we meet with throughout all parts of the univerſe. There is no need of nice or ſubtle reaſonings in this matter: a manifeſt contrivance immediately ſuggeſts a contriver. It ſtrikes us like a ſenſation; and artful reaſonings againſt it may puzzle us, but it is without ſhaking our belief. No perſon, for example, that knows the principles of optics and the ſtructure of the eye, can believe that it was formed without ſkill in that ſcience; or that the ear was formed without the knowledge of ſounds; or that the male and female in animals were not formed for each other, and for continuing the ſpecies. All our accounts of nature are full of inſtances of this kind. The admirable and beautiful ſtructure of things for final cauſes, exalt our idea of the *Contriver*: the unity of deſign ſhews him to be *One*. The great motions in the ſyſtem, performed with the ſame facility as the leaſt, ſuggeſt his *Almighty Power*, which gave motion to the earth and the celeſtial bodies, with equal eaſe as to the minuteſt particles. The ſubtility of the motions and actions in the internal parts of bodies, ſhews that his influence penetrates the inmoſt receſſes of things, and that He is equally *active* and *preſent* every where. The ſimplicity of the laws that prevail in the world, the excellent diſpoſition of

of things, in order to obtain the beſt ends, and the beauty which adorns the works of nature, far ſuperior to any thing in art, ſuggeſt his conſummate *Wiſdom*. The uſefulneſs of the whole ſcheme, ſo well contrived for the intelligent beings that enjoy it, with the internal diſpoſition and moral ſtructure of thoſe beings themſelves, ſhew his unbounded *Goodneſs*. Theſe are the arguments which are ſufficiently open to the views and capacities of the unlearned, while at the ſame time they acquire new ſtrength and luſtre from the diſcoveries of the learned. The Deity's acting and interpoſing in the univerſe, ſhew that he *governs* it as well as formed it, and the depth of his counſels, even in conducting the material univerſe, of which a great part ſurpaſſes our knowledge, keep up an inward veneration and awe of this great Being, and diſpoſe us to receive what may be otherwiſe revealed to us concerning him. It has been juſtly obſerved, that ſome of the laws of nature, now known to us, muſt have eſcaped us if we had wanted the ſenſe of ſeeing. It may be in his power to beſtow upon us other ſenſes of which we have at preſent no idea; without which it may be impoſſible for us to know all his works, or to have more adequate ideas of himſelf. In our preſent ſtate, we know enough to be ſatisfied of our dependency upon him, and of the duty we owe to him the lord and diſpoſer of all things. He is not the object of ſenſe; his eſſence, and indeed that of all other ſubſtances, is beyond the reach of all our diſcoveries; but his attributes clearly appear in his admirable works. We know that the higheſt conceptions we are able to form of them are ſtill beneath his real perfections; but his power and dominion over us, and our duty towards him, are manifeſt.

7. Sir *Iſaac Newton* is particularly careful, always to repreſent him as a free agent; being juſtly appre-

henfive of the dangerous confequences of that doctrine which introduces a fatal or abfolute neceffity prefiding over all things. He made the world, not from any neceffity determining him, but when he thought fit: matter is not infinite or neceffary, but he created as much of it as he thought proper: he placed the fyftems of the fixed ftars at various diftances from each other, at his pleafure: in the folar fyftem, he formed the planets of fuch a number, and difpofed them at various diftances from the fun, as he pleafed: he has made them all move from weft to eaft, tho' it is evident from the motions of the comets, that he might have made them move from eaft to weft. In thefe and other inftances, we plainly perceive the veftiges of a wife agent, but acting freely and with perfect liberty.

As caution was a diftinguifhing part of Sir *Ifaac Newton*'s character, but no way derogatory from his penetration and the acutenefs and fublimity of his genius; fo we have particular reafon on this occafion to applaud it, and to own that his philofophy has proved always fubfervient to the moft valuable purpofes, without ever tending to hurt them.

8. As in treating of this unfathomable fubject we are at a lofs for ideas and words, in any tolerable degree, adequate to it, and, in order to convey our notions with any ftrength, are obliged to have recourfe to figurative expreffions, as was obferved already; fo it is hardly poffible for the moft cautious to make ufe of fuch as may not be liable to exceptions, from angry and captious men. Sir *Ifaac Newton*, to exprefs his idea of the divine *Omniprefence*, had faid that the Deity perceived whatever paffed in fpace fully and intimately, as it were in his *Senforium*. A clamour was raifed by his adverfaries, as if he meant that

that ſpace was to the deity what the *Senſorium* is to our minds. But whoever conſiders this expreſſion without prejudice, will allow that it conveys a very ſtrong idea of the intimate preſence of the Deity every where, and of his perceiving whatever happens in the completeſt manner, without the uſe of any intermediate agents or inſtruments, and that Sir *Iſaac* made uſe of it with this view only; for he very carefully guards againſt our imagining that external objects act upon the Deity, or that he ſuffers any paſſion or reaction from them. It is commonly ſuppoſed that the mind is intimately conſcious of the impreſſions upon the ſenſorium, and that it is immediately preſent there, and there only; and as we muſt derive our ideas of the attributes of God from what we know of our minds, or of thoſe of others, in the beſt manner we can, by leaving out all imperfection and limitation; ſo it was hardly poſſible to have repreſented to us the divine *Omnipreſence* and *Omniſcience* in a ſtronger light, than by this compariſon. But the fondneſs of philoſophers for their favourite ſyſtems, often irritates them againſt thoſe, who, in the purſuit of truth, innocently overturn their doctrines; and provokes them to catch at any occaſion of finding fault.

9. But the greateſt clamour has been raiſed againſt Sir *Iſaac Newton*, by thoſe who have imagined that he repreſented *infinite ſpace* as an attribute of the Deity, and that He is preſent in all parts of ſpace by diffuſion. The truth is, no ſuch expreſſions appear in his writings: he always thought and ſpoke with more veneration of the divinity than to allow himſelf ſuch liberties. On the contrary, he tells * us that "the

* Æternus eſt & infinitus, omnipotens & omniſciens, id eſt, durat ab æterno in æternum, & adeſt ab infinito in infinitum: omnia

"the Deity endures from eternity to eternity, and is preſent from infinity to infinity; but that he is not eternity or infinity, ſpace or duration." He adds indeed, that as the Deity exiſts neceſſarily, and by the ſame neceſſity exiſts every where and always, he conſtitutes ſpace and duration: but it does not appear that this expreſſion can give any juſt ground of complaint; for it is ſaying no more than that ſince he is eſſentially and neceſſarily preſent in all parts of ſpace and duration, theſe of conſequence, muſt alſo neceſſarily exiſt.

10. This idea is ſo far from giving any juſt ground of complaint, that it accounts for the neceſſary exiſtence of ſpace, in a way worthy of the Deity, and ſuggeſts the noble improvement we may make of this doctrine, which lies ſo plain and open before us. Sir *Iſaac Newton* is ſo far from repreſenting the Deity as preſent in ſpace by diffuſion (as ſome have advanced very unjuſtly) that he expreſly tells us * there are ſucceſſive parts in duration, and co-exiſtent parts in ſpace. But that neither are found in the ſoul or principle of thought which is in man; and that far leſs can they be found in the divine ſubſtance. As man is one and the ſame in all the periods of his life, and thro' all the variety of ſenſations and paſſions to which he is ſubject; much more muſt we allow the ſupreme Deity to be one and the ſame in all time, and

nia regit, & omnia cognoſcit, quæ fiunt aut fieri poſſunt. Non eſt æternitas & infinitas, ſed æternus & infinitus; non eſt duratio & ſpatium, ſed durat & adeſt. Durat ſemper, & adeſt ubique, & exiſtendo ſemper & ubique, durationem & ſpatium conſtituit. *Neut. Princip. Scholium Generale*, pag. 528.

* Partes dantur ſucceſſivæ in duratione, coexiſtentes in ſpatio, neutræ in perſona hominis ſeu principio ejus cogitante; & multo minus in ſubſtantia cogitante Dei. Omnis homo quatenus res ſentiens, eſt unus & idem homo durante vitâ ſuâ in omnibus & ſingulis ſenſuum organis. Deus eſt unus & idem Deus ſemper & ubique, *ibid.*

in all ſpace, free from change and external influence. He adds, that the Deity is preſent every where, *non per virtutem ſolam ſed etiam per ſubſtantiam, ſed modo prorſus incorporeo, modo nobis penitus ignoto.* It is plain, therefore, that he was far from meaning that the Deity was preſent every where by the diffuſion of his ſubſtance, as a body is preſent in ſpace by having its parts diffuſed in it. Nor is it ſurprizing that we ſhould be at a loſs to give a ſatisfactory account of the manner of God's omnipreſence. Our knowledge of things penetrates not into their ſubſtance: we perceive only their figure, colour, external ſurface, and the effects they have upon us, but no ſenſe, or act of reflection, diſcovers to us their ſubſtance; and much leſs is the divine ſubſtance known to us. As a blind man knows not colours, and has no idea of the ſenſation of thoſe who ſee, ſo we have no notion how the Deity knows and acts.

11. His exiſtence and his attributes are, in a ſenſible and ſatisfactory manner, diſplayed to us in his works; but his eſſence is unfathomable. From our exiſtence, and that of other contingent beings around us, we conclude that there is a *firſt cauſe*, whoſe exiſtence muſt be neceſſary, and independent of any other being; but it is only *a poſteriori* that we thus infer the neceſſity of his exiſtence, and not in the ſame manner that we deduce the neceſſity of an eternal truth in geometry, or the property of a figure from its eſſence: nor is it even with that direct ſelf-evidence which we have for the neceſſary exiſtence of ſpace. We mention this only to do juſtice to Sir *Iſaac Newton*'s notion, when he ſuggeſts that the neceſſary exiſtence of ſpace is relative to the neceſſary exiſtence of the Deity. Philoſophers have had always diſputes about infinite ſpace and duration; and probably their conteſts on theſe ſubjects will never

have an end: all we want to represent is only, that what is so briefly and modestly advanced by this great man on those subjects, is, at least, as rational and worthy of the Deity, and as well founded in true philosophy, as any of their schemes; tho' it must be expected that the best account we can form of matters of so arduous a nature, will be liable to difficulties and objections. As for those who will not allow space to be any thing real, we observed above that the reality of motion, which is known by experience, argues the reality of absolute space; without admitting which, we should have nothing but confusion and contradictions in natural philosophy. Many other arguments, particularly those drawn from the axiom, *non entis nulla sunt attributa*, for the reality of space, whose parts are subject to mensuration and various relations, have been treated of largely by others.

12. We observed above, that as the Deity is the first and supreme cause of all things, so it is most unaccountable to exclude him out of nature, and represent him as an *intelligentia extramundana*. On the contrary, it is most natural to suppose him to be the chief mover throughout the whole universe, and that all other causes are dependent upon him; and conformable to this is the result of all our enquiries into nature; where we are always meeting with powers that surpass mere mechanism, or the effects of matter and motion. The laws of nature are constant and regular, and, for ought we know, all of them may be resolved into one general and extensive power; but this power itself derives its properties and efficacy, not from mechanism, but, in a great measure, from the immediate influences of the first mover. It appears, however, not to have been his intention, that the present state of things should continue for ever without alteration; not only from what

what paſſes in the moral world, but from the phænomena of the material world likewiſe; as it is evident that it could not have continued in its preſent ſtate from eternity.

13. The power of gravity, by which the celeſtial bodies perſevere in their revolutions, penetrates to the centres of the ſun and planets without any diminution of virtue, and is extended to immenſe diſtances, decreaſing in a regular courſe. Its action is proportional to the quantity of ſolid matter in bodies, and not to their ſurfaces, as is uſual in mechanical cauſes: this power, therefore, ſeems to ſurpaſs mere mechaniſm. But, whatever we ſay of this power, it could not poſſibly have produced, at the beginning, the regular ſituation of the orbs and the preſent diſpoſition of things. Gravity could not have determined the planets to move from weſt to eaſt in orbits nearly circular, almoſt in the ſame plane; nor could this power have projected the comets with all variety of directions. If we ſuppoſe the matter of the ſyſtem to be accumulated in the centre by its gravity, no mechanical principles, with the aſſiſtance of this power of gravity, could ſeparate the vaſt maſs into ſuch parts as the ſun and planets, and, after carrying them into their different diſtances, project them in their ſeveral directions, preſerving ſtill the equality of action and reaction, or the ſtate of the centre of gravity of the ſyſtem. Such an exquiſite ſtructure of things could only ariſe from the contrivance and powerful influences of an intelligent, free, and moſt potent agent. The ſame powers, therefore, which at preſent govern the material univerſe, and conduct its various motions, are very different from thoſe which were neceſſary to have produced it from nothing, or to have diſpoſed it in the admirable form in which it now proceeds.

14. As we cannot but conceive the univerſe, as depending on the firſt cauſe and chief mover, whom it would be abſurd, not to ſay impious, to exclude from acting in it; ſo we have ſome hints of the manner in which he operates in nature, from the laws which we find eſtabliſhed in it. Tho' he is the ſource of all efficacy, yet we find that place is left for ſecond cauſes to act in ſubordination to him; and mechaniſm has its ſhare in carrying on the great ſcheme of nature *. The eſtabliſhing the equality of action and reaction, even in thoſe powers which ſeem to ſurpaſs mechaniſm, and to be more immediately derived from him, ſeems to be an indication that thoſe powers, while they derive their efficacy from him, are however, in a certain degree, circumſcribed and regulated in their operations by mechanical principles; and that they are not to be conſidered as mere immediate volitions of his (as they are often repreſented) but rather as inſtruments made by him, to perform the purpoſes for which he intended them. If, for example, the moſt noble phænomena in nature be produced by a rare elaſtic *ætherial medium*, as Sir *Iſaac Newton* conjectured, the whole efficacy of this medium muſt be reſolved into his power and will, who is the ſupreme cauſe. This, however, does not hinder, but that the ſame medium may be ſubject to the like laws as other elaſtic fluids, in its actions and vibrations; and that, if its nature was better known to us, we might make curious and uſeful diſcoveries concerning its effects, from thoſe laws. It is eaſy to ſee that this conjecture no way derogates from the government and influences of

* Ἀλλὰ τοῦτο ἦν τὸ θειότατον, τὸ μετὰ ῥαστώνης καὶ ἁπλῆς κινήσεως παντοδαπὰς ἀποτελεῖν ἰδέας, ὥσπερ ἀμέλει δρῶσιν οἱ μηχανοποιοί διὰ μιᾶς ὀργάνου σχαστηρίας, πολλὰς καὶ ποικίλας ἐνεργείας ἀποτελοῦντες. *Ariſtot.* ubi ſupra.

the

the Deity; while it leaves us at liberty to purſue our enquiries concerning the nature and operations of ſuch a medium. Whereas they who haſtily reſolve thoſe powers into immediate volitions of the ſupreme cauſe, without admitting any intermediate inſtruments, put an end to our enquiries at once; and deprive us of what is probably the moſt ſublime part of philoſophy, by repreſenting it as imaginary and fictitious: by which means, as we obſerved above *, they hurt thoſe very intereſts which they appear ſo ſanguine to promote; for the higher we riſe in the ſcale of nature, towards the ſupreme cauſe, the views we have from philoſophy appear more beautiful and extenſive. Nor is there any thing extraordinary in what is here repreſented concerning the manner in which the Supreme Cauſe acts in the univerſe, by employing ſubordinate inſtruments and agents, which are allowed to have their proper force and efficacy; for this we know is the caſe in the common courſe of nature; where we find gravity, attraction, repulſion, *&c.* conſtantly combined and compounded with the principles of mechaniſm: and we ſee no reaſon why it ſhould not likewiſe take place in the more ſubtile and abſtruſe phænomena and motions of the ſyſtem.

15. It has been demonſtrated by ingenious men, that great revolutions have happened in former times on the ſurface of the earth, particularly from the phænomena of the *Strata*; which ſometimes are found to lie in a very regular manner, and ſometimes to be broken and ſeparated from each other to very conſiderable diſtances, where they are found again in the ſame order; from the impreſſions of plants left upon the hardeſt bodies dug deep out of

* Book I. Chap. 5. § 3.

the earth, and in places where ſuch plants are not now found to grow; and from bones of animals both of the land and ſea, diſcovered ſome hundreds of yards beneath the preſent ſurface of the earth, and at very great diſtances from the ſea. Some philoſophers explain theſe changes by the revolutions of comets, or other natural means: but as the Deity has formed the univerſe dependent upon himſelf, ſo as to require to be altered by him, tho' at very diſtant periods of time; it does not appear to be a very important queſtion to enquire whether theſe great changes are produced by the intervention of inſtruments, or by the ſame immediate influences which firſt gave things their form.

16. We cannot but take notice of one thing, that appears to have been deſigned by the author of nature: he has made it impoſſible for us to have any communication from this earth with the other great bodies of the univerſe, in our preſent ſtate; and it is highly probable, that he has likewiſe cut off all communication betwixt the other planets, and betwixt the different ſyſtems. We are able, by teleſcopes, to diſcover very plainly mountains, precipices and cavities in the moon: but who tread thoſe precipices, or for what purpoſes thoſe great cavities (many of which have a little elevation in the middle) ſerve, we know not; and are at a loſs to conceive how this planet, without any atmoſphere, vapours, or ſeas, (as is now the common opinion of aſtronomers) can ſerve for like purpoſes as our earth. We obſerve ſudden and ſurprizing revolutions on the ſurface of the great planet *Jupiter*, which would be fatal to the inhabitants of the earth. We obſerve, in them all, enough to raiſe our curioſity, but not to ſatisfy it. From hence, as well as from the ſtate of the moral world, and many other conſi-

confiderations, we are induced to believe, that our prefent ftate would be very imperfect without a fubfequent one; wherein our views of nature, and of its great author, may be more clear and fatisfactory. It does not appear to be fuitable to the wifdom that fhines throughout all nature, to fuppofe that we fhould fee fo far, and have our curiofity fo much raifed concerning the works of God, only to be difappointed at the end. As man is undoubtedly the chief being upon this globe, and this globe may be no lefs confiderable, in the moft valuable refpects, than any other in the folar fyftem, and this fyftem, for ought we know, not inferior to any in the univerfal fyftem; fo, if we fhould fuppofe man to perifh, without ever arriving at a more complete knowledge of nature, than the very imperfect one he attains in his prefent ftate; by analogy, or parity of reafon, we might conclude, that the like defires would be fruftrated in the inhabitants of all the other planets and fyftems; and that the beautiful fcheme of nature would never be unfolded, but in an exceedingly imperfect manner, to any of them. This, therefore, naturally leads us to confider our prefent ftate as only the dawn or beginning of our exiftence, and as a ftate of preparation or probation for farther advancement: which appears to have been the opinion of the moft judicious philofophers of old. And whoever attentively confiders the conftitution of human nature, particularly the defires and paffions of men, which appear greatly fuperior to their prefent objects, will eafily be perfuaded that man was defigned for higher views than of this life. Thefe the author of nature may have in referve to be opened up to us, at proper periods of time, and after due preparation. Surely it is in his power to grant us a far greater improvement of the faculties we already poffefs, or even to endow us with new facul-

faculties, of which, at this time, we have no idea, for penetrating farther into the ſcheme of nature, and approaching nearer to himſelf, the firſt and ſupreme cauſe. We know not how far it was proper or neceſſary that we ſhould not be let into knowledge at once, but ſhould advance gradually, that, by comparing new objects, or new diſcoveries, with what was known to us before, our improvements might be more complete and regular; or how far it may be neceſſary or advantageous, that intelligent beings ſhould paſs through a kind of infancy of knowledge. For new knowledge does not conſiſt ſo much in our having acceſs to a new object, as in comparing it with others already known, obſerving its relations to them, or diſcerning what it has in common with them, and wherein their diſparity conſiſts. Thus our knowledge is vaſtly greater than the ſum of what all its objects ſeparately could afford; and when a new object comes within our reach, the addition to our knowledge is the greater, the more we already know; ſo that it increaſes not as the new objects increaſe, but in a much higher proportion. * * *

F I N I S.

f.