H. Zhao's **MindMap** of **Galaxy** & **Accretion** Physics **Common Equations** & **Concepts/Examples**

I. Poisson Eq.: $\frac{\nabla\cdot[\nabla\Phi(t,\mathbf{X})]}{4\pi G} = \overbrace{\rho(t,\mathbf{X}) = m^{DM}n^{DM} + \rho^{gas}(t,\mathbf{X}) + ...+}^{grav.density.=\sum_{p=gas,*}^{DM}\rho^p(t,\mathbf{X})=\sum_p m^p n_p} \quad \int_\infty d\mathbf{v}_*^3 \quad \underbrace{\overbrace{[m_* f_*(t,\mathbf{X},\mathbf{v}_*)]}^{O(10^{10}M_\odot)/[10kpc.100km/s]^3}}_{\to 0\ at\ big(X,v_*)}$,

$$\textbf{E.g.,}\qquad \overbrace{\ddot{\mathbf{x}}^* = \underbrace{-\nabla_{\mathbf{x}^*}\Phi(t,\mathbf{x}_*)}_{\mathbf{g}}}^{Stellar.Eq.of.Motion} = \nabla_{\mathbf{x}_*}\int_\infty \frac{\overbrace{[d\mathbf{X}^3\rho(t,\mathbf{X})]}^{dM(\mathbf{X})}G}{|\mathbf{x}_* - \mathbf{X}|},\ \overbrace{\frac{v_{cir}^2}{r}}^{shell.accel.} = |\mathbf{g}| = \frac{d}{dr}\overbrace{\Phi(r)}^{-v_{esc}^2/2} = \frac{4\pi G}{(4\pi r^2)}\overbrace{\int_0^r \rho(r_1)4\pi r_1^2 dr_1}^{enclosed.\sum_p M^p(r)}.$$

E.g.: No relaxation of Sun's angular momentum in 10^9 stress-free harmonic **periods** $\frac{2\pi}{\kappa} \sim \frac{2\pi}{\nu} \sim \frac{2\pi(R_0\sim 10kpc)}{v_{cir}\sim 200km/s}$.

$$\overbrace{[^{\ddot{R}/(R_0-R)}_{\ddot{Z}/(0-Z)}]_{Z\to 0}^{R\to R_0} = [^{\kappa^2}_{\nu^2}]}^{restoring.freq.^2\sim O(G\rho)} \equiv [^{\partial_R^2}_{\partial_Z^2}]\overbrace{[\Phi(R,Z) + (R\dot{\psi})^2/2]}^{eff.pot.},\quad \overbrace{J_z m_p = R(\dot{\psi}R)M_\odot = Rv_\psi M_\odot}^{looporbit}$$

E.g.:Meaning of BH tide beats centrifugal, or luminosity-driven force

$$1 \geq Q^{centrf} \equiv \frac{J_z^2/R^3}{G[\frac{M_\bullet}{R^2}]} \equiv \frac{R_{centrf}}{R},\quad 1 \geq Q_{tide}^{centrf} = \frac{(GM_\odot R_\odot)/R_\odot^3}{\frac{GM_\bullet}{R^2} - \frac{GM_\bullet}{(R+R_\odot)^2}} \equiv \frac{R^3}{R_{tide}^3}.$$

E.g.: Meaning of Virial$_{jj}$ tensor

$$\overbrace{M_*\sum_{j=1,3}\overline{\mathbf{v_j^2}}^*}^{2T} = \overbrace{M_*\overline{\mathbf{x}\cdot\nabla_\mathbf{x}\mathbf{\Phi}}^*}^{M_*\overline{\mathbf{v^2_{cir}}}^*} \overbrace{=}^{selfgravity} = \overbrace{\underbrace{\overbrace{M}}_{\approx M_*}[-\Phi/2]}^{W} = \frac{1}{2}\iint_\infty \overbrace{[\rho(\mathbf{x})d^3\mathbf{x}]}^{dM_*(\mathbf{x})}[\overbrace{\rho(\mathbf{X})}^{\approx m_* n_*}d^3\mathbf{X}]/|\mathbf{x}-\mathbf{X}|$$

II. Mass Conservation Eq.: Viscous flow onto a particle m^p (of Bondi size $2B \equiv \frac{2Gm^p}{\bar{\mathbf{v}}^2+\sigma^2}$):

$$\overbrace{cst\ accretion\ rate}^{steady} = \dot{M}_p = \frac{M_p}{t_{visc}^{dyf,rlx}} = -\oint \rho_p\langle\mathbf{v}_p\rangle\cdot\overbrace{d\mathbf{A}}^{O(4\pi B^2)} = \overbrace{\frac{\partial}{\partial t}\int_{\in\mathbf{A}}\rho_p d^3\mathbf{x}}^{\dot{M}_p} \approx \overbrace{(2\pi B\sqrt{\langle\mathbf{v}\rangle_p^2+\sigma^2})}^{viscosity}\overbrace{\int_{-B}^{B}dZ\rho_{gas}}^{surf.dens.}$$

III. Momentum (Jeans) Eqs. of a ^{p}opulation from integrated **6D CBE**: $\frac{1}{\rho_p}\int\{\mathbf{v}_p\frac{d[f_p m_p]}{dt} - \bar{\mathbf{v}}_p\frac{d[f_p m_p]}{dt}\}d^3\mathbf{v} = 0$.

$$\overbrace{(\frac{\partial}{\partial t} + \sum_{j=1}^{3}\langle v_j^p\rangle\frac{\partial}{\partial x_j})\langle v_i^p\rangle}^{flow.accel.\langle\dot{v}_i^p\rangle\sim O(\overline{\dot{\psi}^2}R)} \underbrace{=}_{EoM} \overbrace{\frac{-\partial\Phi(t,\mathbf{x})}{\partial x_i}}^{g_i\sim O(-GM/R^2)} \underbrace{\overset{pressure}{-}}_{balance} \sum_{j=1}^{3}\frac{\partial}{\rho^p\partial x_j}\overbrace{[\underbrace{\rho^p(t,\mathbf{x})}_{\int_\infty m_p f_p d^3\mathbf{v}}\ \underbrace{\sigma_{ji}^p(t,\mathbf{x})}_{O(c_s^2)}]}^{\int_\infty d\mathbf{v}^3(\mathbf{v}_j-\langle\mathbf{v}\rangle_j^p)(\mathbf{v}_i-\langle\mathbf{v}\rangle_i^p)m_p f_p} - \overbrace{\frac{\langle v_i^p\rangle}{t|_{visc\ m_p=M_{gas}}^{dyn.fric,relax.}}}^{snow.plough},$$

E.g.: Jeans Eq. for static spherical system is $0 = -\partial_r\Phi + \frac{\sigma_\theta^2+\sigma_\psi^2-2\sigma_r^2}{r} - \frac{\partial_r(\rho\sigma_r^2)}{\rho}$, where $\langle\mathbf{v}\rangle^p \equiv \frac{\int\mathbf{v}f_p dv_r dv_\theta dv_\psi}{\int f_p dv_r dv_\theta dv_\psi} = 0$ everywhere. If $\sigma_\theta^2 = \sigma_\psi^2 = \sigma_r^2$ then the anisotropy term cancels.

E.g.: This Jeans eq. is satisfied by a BH cluster of size $B = 1kpc$, $M = N_p m_p = 10^3\times 10^6 M_\odot = \int m_p f_p d\mathbf{x}^3 d\mathbf{v}^3$ with $f_p(\mathbf{x},\mathbf{v}) \propto |1 + \frac{E-\frac{J^2}{2B^2}\to 0.5|v_r^2+(1-\frac{r^2}{B^2})(v_\theta^2+v_\psi^2)|+\Phi(|\mathbf{x}|)}{[GM/B]\sim\mathbf{Virial}\sim[c_s\sim\sigma\sim v_{esc}\sim 60kms^{-1}\sim v_{cir}]^2}|^{-0.5}$ is static, uniform $\rho_p(|\mathbf{x}|) = \frac{N_p m_p}{(4\pi B^3/3)} = \frac{d}{r^2 dr}[\frac{r^2 d\Phi}{4\pi G dr}]$. It has **anisotropic dispersion** $\sigma_\theta^2 = \sigma_\psi^2 = \frac{GM}{2B}, \sigma_r^2 \equiv \frac{GM}{2B}(1-\frac{r^2}{B^2}) = -\Phi(r) - \frac{GM}{B}$. Its **Virial** $\overline{(0.5v_{esc})^2} = \overline{|0.5\Phi|} = \overline{v_{cir}^2} = \overline{\sigma_\theta^2+\sigma_\psi^2+\sigma_r^2} = \frac{3GM}{5B}$. **Bondi radius** $B(t)$ doubles once BHs relax into $500\times(2\times 10^6 M_\odot)$ pairs of size $2B = \frac{2Gm_p}{2\sigma^2} \sim 2pc$ by wake-dragging/accretion each $t_{rlx}^{dyf} \sim \frac{10^3\times B}{c_s} \sim \frac{10^3}{\sqrt{G\rho_p}} \sim$ Hubble Time, $m_p \sim 10^6 M_\odot e^{\frac{t}{10Gyr}}$.