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Motion in Two and Three Dimensions

Module Overview

Acknowledgments

This presentation is based on and includes content derived from the following OER resource:

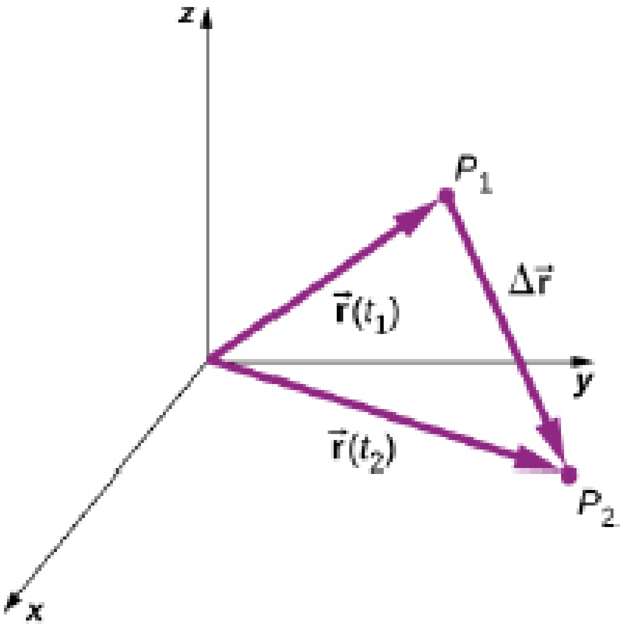
University Physics Volume 1

An OpenStax book used for this course may be downloaded for free at:
https://openstax.org/details/books/university-physics-volume-1

Displacement Vector

In three dimensions, an object's position is usually described by coordinates $x, y,$ and z, which may be functions of time. The three coordinates form a **position vector**, $\vec{\mathbf{r}}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}$.

The **displacement vector**, $\Delta\vec{\mathbf{r}}$, is the difference vector between an object's position vectors at two different instants in time, given by $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}(t_2) - \vec{\mathbf{r}}(t_1)$.



(University Physics Volume 1. OpenStax. Fig. 4.3)

Velocity Vector

In two or three dimensions, the **velocity vector** is the time derivative of the displacement vector. Velocity can be written in terms of components, just like the position vector. Each velocity component is defined as the time derivative of the corresponding position component. The average velocity vector is defined similarly to average velocity in one dimension.

$$\vec{\mathbf{v}}(t) = \frac{d\vec{\mathbf{r}}}{dt}$$

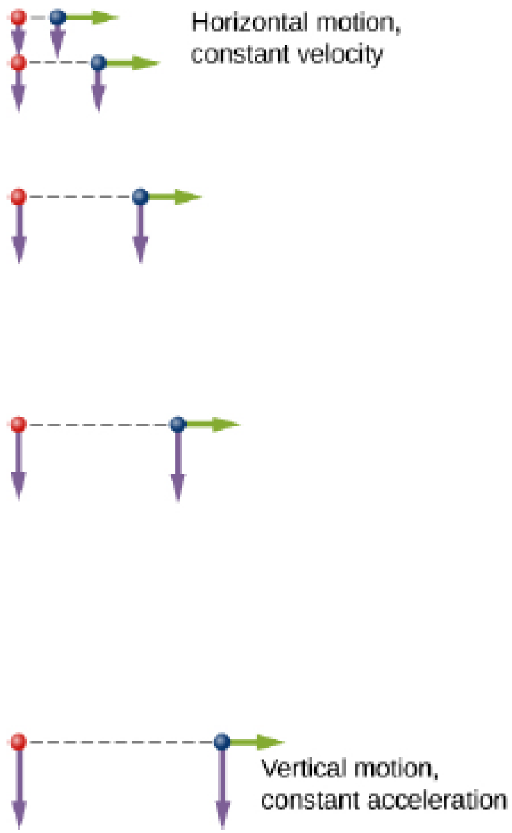
$$\vec{\mathbf{v}}(t) = v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}} + v_z(t)\hat{\mathbf{k}}$$

$$\begin{cases} v_x(t) = \dfrac{dx(t)}{dt} \\ v_y(t) = \dfrac{dy(t)}{dt} \\ v_z(t) = \dfrac{dz(t)}{dt} \end{cases}$$

$$\vec{\mathbf{v}}_{\text{avg}} = \frac{\vec{\mathbf{r}}(t_2) - \vec{\mathbf{r}}(t_1)}{t_2 - t_1}$$

The Independence of Perpendicular Motions

In the kinematic description of motion, we can treat the horizontal and vertical components of motion separately. In many cases, motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.



(University Physics Volume 1. OpenStax. Fig. 4.8)

Instantaneous Acceleration

The **acceleration vector** is the time derivative of the velocity vector. Acceleration can be written in terms of components, just like the position and velocity vectors. The acceleration vector is also the second derivative of the position vector.

$$\vec{\mathbf{a}}(t) = \frac{d\vec{\mathbf{v}}}{dt}$$

$$\vec{\mathbf{a}}(t) = \frac{dv_x(t)}{dt}\hat{\mathbf{i}} + \frac{dv_y(t)}{dt}\hat{\mathbf{j}} + \frac{dv_z(t)}{dt}\hat{\mathbf{k}}$$

$$\vec{\mathbf{a}}(t) = \frac{d^2x(t)}{dt}\hat{\mathbf{i}} + \frac{d^2y(t)}{dt}\hat{\mathbf{j}} + \frac{d^2z(t)}{dt}\hat{\mathbf{k}}$$

Constant Acceleration

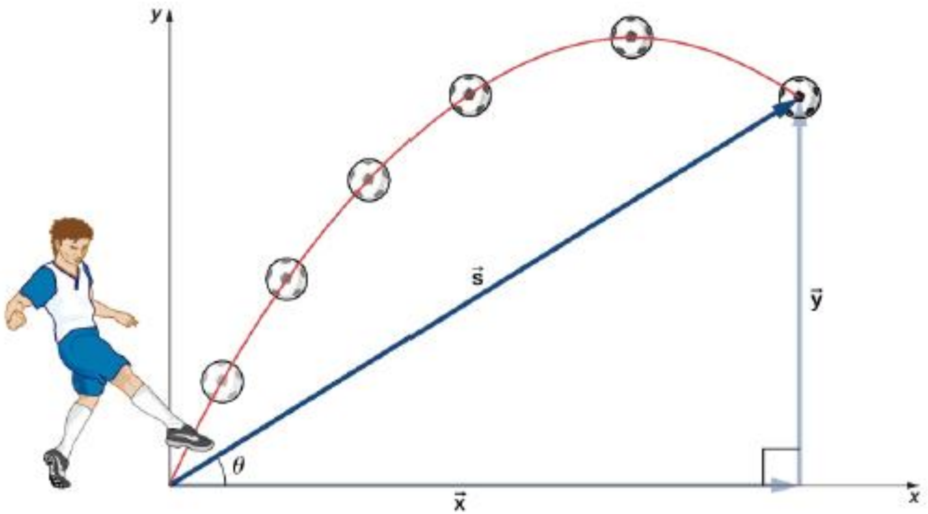
In two or three dimensions, the motion in each direction can be treated independently. Under constant acceleration, the kinematic equations derived earlier apply in each dimension. For example, in two dimensions (ignoring the z-direction), there are eight kinematic equations.

$$x(t) = x_0 + (v_x)_{\text{avg}} t$$
$$v_x(t) = v_{0x} + a_x t$$
$$x(t) = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$
$$v_x^2(t) = v_{0x}^2 + 2a_x(x - x_0)$$
$$y(t) = y_0 + (v_y)_{\text{avg}} t$$
$$v_y(t) = v_{0y} + a_y t$$
$$y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$
$$v_y^2(t) = v_{0y}^2 + 2a_y(y - y_0)$$

Projectile Motion, Part 1

Projectile motion is a special case of two-dimensional motion describing an object thrown into the air, subject only to the acceleration due to gravity. The path of the object is called its **trajectory**.

Each component of the motion can be treated independently. In the horizontal direction, there is no acceleration.



(University Physics Volume 1. OpenStax. Fig. 4.11)

Projectile Motion, Part 2

Since acceleration in the horizontal direction is zero, the equations of motion are reduced from eight to six.

After solving for the position and velocity components, the components can be used to write the displacement vector, $\vec{\mathbf{s}}$, and the velocity vector, $\vec{\mathbf{v}}$.

Horizontal motion

$$x = x_0 + v_x t$$
$$v_x = v_{0x}$$

Vertical motion

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$
$$v_y = v_{0y} - gt$$
$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$
$$v_y^2(t) = v_{0y}^2 - 2g(y - y_0)$$

Time of Flight, Trajectory, and Range

The **time of flight** of an object can be calculated by noting that the displacement in y is equal to zero. The **trajectory** of the object, or its y position as a function of x, can be calculated by eliminating t from the kinematic equations. The **range** of the object, or the horizontal distance traveled, is found by noting that y is equal to zero in the equation for trajectory.

$$T_{\text{tof}} = \frac{2v_0 \sin\theta_0}{g}$$

$$y = \tan\theta_0 x - \left[\frac{g}{2v_0\cos\theta_0}\right]x^2$$

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

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Uniform Circular Motion

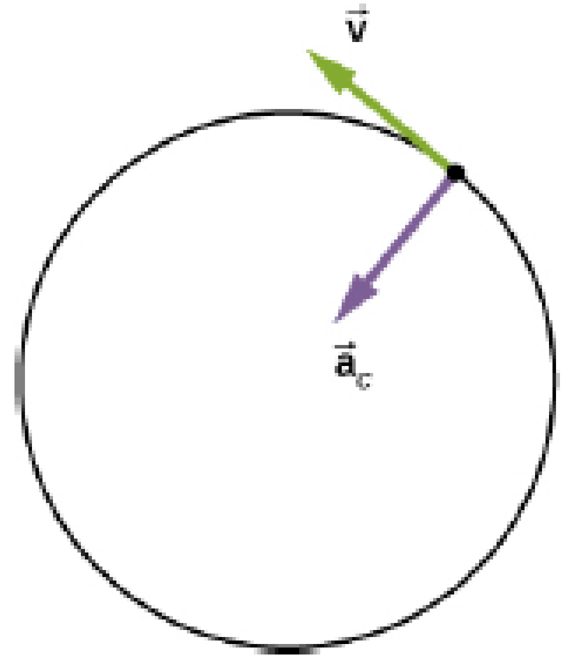
Uniform circular motion is a specific type of motion in which an object travels in a circle with a constant speed.

Examples include any point on an airplane's propeller spinning at a constant rate and the second, minute, and hour hands of a watch.

Even though the rotation rate is constant, points on these rotating objects are actually accelerating.

Centripetal Acceleration

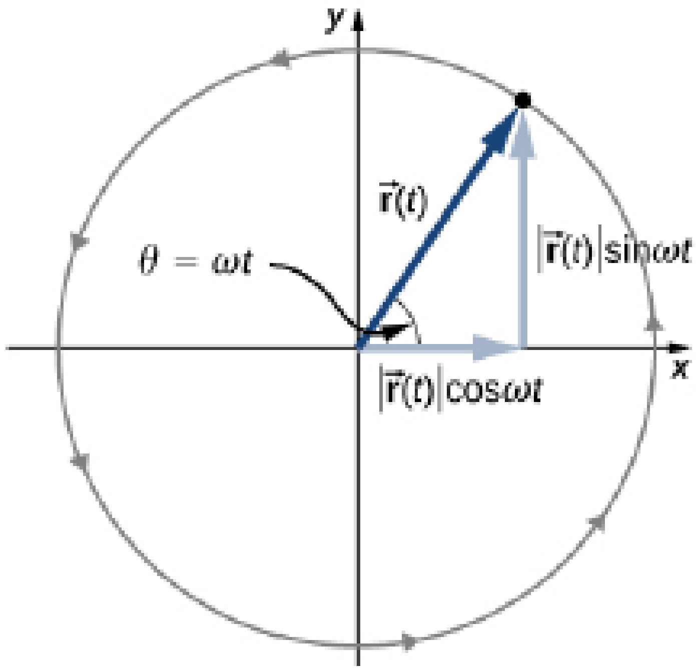
When both acceleration and speed are constant, the resulting motion is circular. The magnitude of the acceleration is related to the radius and speed of the motion by $a_C = \frac{v^2}{r}$. The acceleration is called **centripetal acceleration**, and it points toward the center of the motion.



(University Physics Volume 1. OpenStax. Fig. 4.19)

Equations of Motion: Uniform Circular Motion, Part 1

The position vector of an object in circular motion has constant magnitude, A, and completes a full revolution in time, T, the period of the motion. The **angular frequency** of the motion, ω, describes the rate at which the object traverses the circular path. It is defined as $\omega = \frac{2\pi}{T}$, and has units of radians per second.



(University Physics Volume 1. OpenStax. Fig. 4.20)

Equations of Motion: Uniform Circular Motion, Part 2

The position vector can be fully described in terms of the magnitude and angular frequency, $\vec{\mathbf{r}}(t) = A\cos\omega t\,\hat{\mathbf{i}} + A\sin\omega t\hat{\mathbf{j}}$.

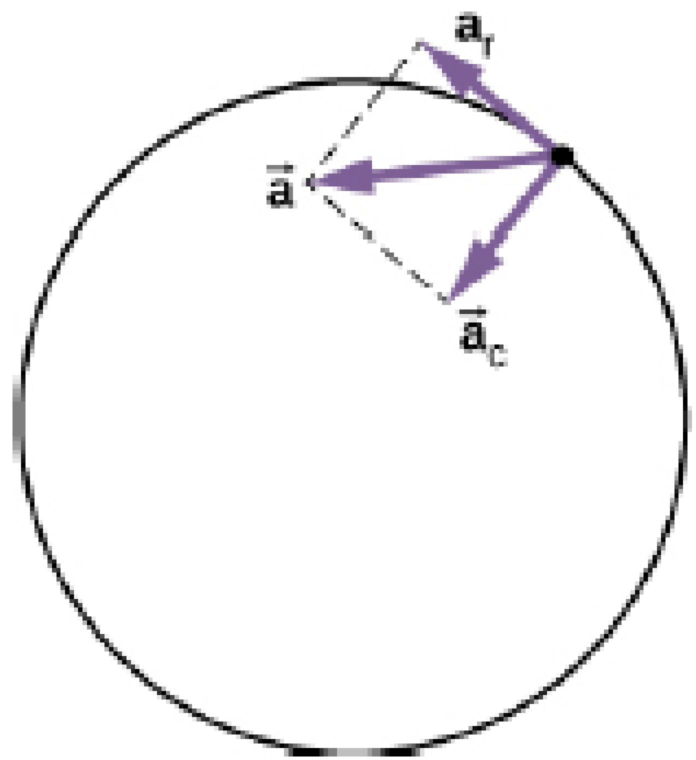
The velocity vector is given by the time derivative of the position vector, $\vec{\mathbf{v}}(t) = -A\omega\sin\omega t\,\hat{\mathbf{i}} + A\omega\cos\omega t\,\hat{\mathbf{j}}$.

The acceleration vector is defined as the time derivative of the velocity vector, $\vec{\mathbf{a}}(t) = -A\omega^2\cos\omega t\,\hat{\mathbf{i}} - A\omega^2\sin\omega t\;\hat{\mathbf{j}}$. The acceleration vector has magnitude $A\omega^2$ and is directed opposite the position vector, toward the origin.

Nonuniform Circular Motion

An object in circular motion may undergo **tangential acceleration** that changes its speed, $a_{\mathrm{T}} = \frac{d|\vec{\mathbf{v}}|}{dt}$.

Centripetal acceleration causes circular motion, while tangential acceleration causes the radius of the object's path to change. The object's **total acceleration** is the vector sum of the two orthogonal accelerations, $\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\mathrm{C}} + \vec{\mathbf{a}}_{\mathrm{T}}$.



(University Physics Volume 1. OpenStax. Fig. 4.22)

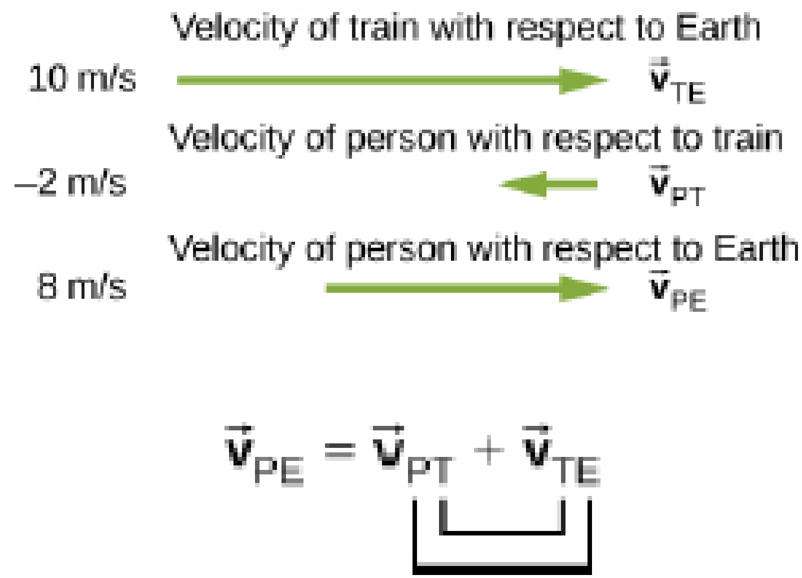
Reference Frames

The velocity of an object is always defined in some **reference frame**. In most cases previously considered, the reference frame is the Earth. For example, when we say a person in a car is traveling at 30 mph, we mean that the person is moving at that speed with respect to the Earth.

When considering relative motion of two or more objects, a good choice of reference frame may significantly simplify the mathematics of a problem, while a poor choice of reference frame may make a problem very difficult. An example of a bad reference frame is considering the motion of a ball on Earth with reference to the Sun. The ball undergoes parabolic motion while Earth rotates while orbiting the Sun, making the mathematics very difficult.

Relative Motion in One Dimension

Consider the motion of a person walking on a moving train. The train travels with a velocity with respect to the Earth, $\vec{\mathbf{v}}_{\mathrm{TE}}$. The person walks on the train with a velocity with respect to the train, $\vec{\mathbf{v}}_{\mathrm{PT}}$. The net velocity of the person with respect to the Earth is given by the vector sum, $\vec{\mathbf{v}}_{\mathrm{PE}} = \vec{\mathbf{v}}_{\mathrm{PT}} + \vec{\mathbf{v}}_{\mathrm{TE}}$.



(University Physics Volume 1. OpenStax. Fig.s 4.24, 4.25)

Relative Velocity in Two Dimensions

Consider a particle P and reference frames S and S'. The particle's position in frame S, $\vec{\mathbf{r}}_{PS}$, and its position in frame S', $\vec{\mathbf{r}}_{PS'}$, are related by the position of frame S in frame S'. The velocities and accelerations are related in the same way. These relationships can be extended to an arbitrary number of reference frames.

$$\vec{\mathbf{r}}_{\mathrm{PS}} = \vec{\mathbf{r}}_{\mathrm{PS'}} + \vec{\mathbf{r}}_{\mathrm{S'S}}$$
$$\vec{\mathbf{v}}_{\mathrm{PS}} = \vec{\mathbf{v}}_{\mathrm{PS'}} + \vec{\mathbf{v}}_{\mathrm{S'S}}$$
$$\vec{\mathbf{a}}_{\mathrm{PS}} = \vec{\mathbf{a}}_{\mathrm{PS'}} + \vec{\mathbf{a}}_{\mathrm{S'S}}$$

$$\vec{\mathbf{v}}_{\mathrm{PC}} = \vec{\mathbf{v}}_{\mathrm{PA}} + \vec{\mathbf{v}}_{\mathrm{AB}} + \vec{\mathbf{v}}_{\mathrm{BC}}$$

How to Study this Module

- Read the syllabus or schedule of assignments regularly.
- Understand key terms; look up and define all unfamiliar words and terms.
- Take notes on your readings, assigned media, and lectures.
- As appropriate, work all questions and/or problems assigned and as many additional questions and/or problems as possible.
- Discuss topics with classmates.
- Frequently review your notes. Make flow charts and outlines from your notes to help you study for assessments.
- Complete all course assessments.

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