2/14/2013

Mathematica

Mathematical formulae form basic to the advance level.

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I. BASICS

1. $(a + b)^2 = a^2 + b^2 + 2ab$.
2. $(a - b)^2 = a^2 + b^2 - 2ab$.
3. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$.
4. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$.
5. $(a^3 + b^3) = (a + b)(a^2 + b^2 - ab)$.
6. $(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)$.
7. $(a^2 - b^2) = (a + b)(a - b)$.
8. $(a + b + c)^2 = a^2 + b^2 + c^2\ 2ab + 2bc + 2ca$.
9. $(x + a)(x + b) = x + (a + b)x + ab$.
10. $(x + a)(x + b)(x + c) = x^3 + (a + b + c)x2 + (ab + bc + ac)x + abc$.
11. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$.

1. QUADRATIC EQUATION

1. Formulae used to find the value of x in quadratic eqn: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

2. If m & n are the roots of quadratic eqn $ax^2 + bx + c$

Sum of the roots: $m+n = \frac{-b}{a}$

Product of the roots: $mn = \frac{c}{a}$

$x^2-(m+n)x + mn=0$.

2. PROGRESSIONS

1. n^{th} term of arithmetic progression $T_n= a+ (n+1)\, d$ where a is first term & n is no. of terms.
2. Arithmetic series of n^{th} term total $S_n=\frac{n}{2}(a+l)$ and $S_n=\frac{n}{2}[2a+ (n-1)\, d]$ where d is the common difference and l is last term and T_n is n^{th} term of arithmetic series, S_n is arithmetic series.
3. n^{th} term of Geometric progression $T_n= a.r^{n-1}$ where a is first term and r is common ratio.
4. Geometric series of n^{th} term total $S_n= a[\frac{1-r^n}{1-r}]$ where $r< 1$ and

   $S_n= a\,[\frac{r^n-1}{r-1}]$ n= no. of terms.
5. Geometrical mean $G=\sqrt{ab}$, G is geometric mean between a & b.
6. Harmonic progression $T_n=\frac{1}{a+(n-1)d}$ and Harmonic mean (H)

   $H=\frac{2ab}{a+b}$.

3. NUMBERS

We use the following notations to represent the set of numbers mentioned below.

1. Set of Natural numbers N {1, 2, 3, 4, 5 ....}.
2. Set of Whole numbers W {0, 1, 2, 3, 4, 5,}.
3. Set of Integers I or Z {.........-3,-2,-1, 0, 1, 2, 3.......}.

4. Set of Rational numbers Q $\{\frac{p}{q}, q \neq 0$ where p, q are integers}.

5. Set of Irrational numbers Q^{I} $\{\sqrt{p}$: where p is a +ve rational & it is not a perfect square}.

6. Set of Real numbers R $\{1, 2, {}^{3}/_{5}\ \sqrt{5}\}$ R=Q∪Q^{I}.

4. SETS

i. Associative law: A∪B=B∪A, A∩B=B∩A.

ii. Commutative law: (A∪B) ∪C=A ∪(B∪C).

iii. Division law: A ∪(B∩C) = (A∪B) ∩ (A∪C); A∩ (B∪C) = (A∩B) ∪(A∩C).

5. PERMUTATION & COMBINATION

1. $^{n}P_{r}= \frac{\llcorner n}{\llcorner n-r}$ where 'n' is total no. of things and 'r' is selected things.

2. $^{n}C_{r}=\frac{\llcorner n}{\llcorner n-r(\llcorner r)}$. NOTE: '$\llcorner$' REPRESENTS FACTORIAL

6. PROBABILITY

➢ P (A) = $\frac{n(A)}{n(S)} = \frac{\textbf{the total no.of elementary events favourable to A}}{\textbf{the total no.of elementary events}}$.

7. BASICS LAWS OF INDICES

1. $a^m.a^n = a^{m+n}$.
2. $\frac{a^m}{a^n} = a^{m-n}$ where m>n.
3. $(a^m)^n = a^{mn}$.
4. $(\frac{a}{b})^m = \frac{a^m}{b^n}$.

8. FRACTIONS

When any object is divided into a number or equal parts one or more of these equal parts can be expressed as a fraction

TYPES OF FRACTIONS:

1. PROPER FRACTION: - In a proper fraction, the numerator is less than the denominator.

   Ex: $\frac{2}{5}, \frac{4}{6}, \frac{7}{8}$ all proper fractions are less than 1.
2. IMPROPER FRACTION: - In an improper fraction the numerator is greater than denominator.

   Ex: $\frac{4}{3}, \frac{6}{4}, \frac{11}{4}$.
3. MIXED FRACTION: - In a mixed fraction it contains a whole number along with proper fraction. Ex: $1\frac{1}{2}$.

9. GEOMETRY

- TRANSVERSAL: - Any line cutting a set of lines is a transversal.
- ACUTE ANGLE: - Angle which is less than 90°.
- OBTUSE ANGLE: - Angle which is greater than 90°.
- STRIAGHT ANGLE: - Angle which is equal to 180°.
- COMPLEMENTARY ANGLE: - If 2 angles together makes a right angle.
- SUPPLEMENTARY ANGLE: - When 2 angles together form a straight line i.e. 180°.
- CIRCLE: - A circle is set of points equidistant from the Centre. Circumference of the circle; $C= 2\pi r$.

  Area of the circle; $A= \pi r^2$.

10. AREAS

1. Area of square= 2a (a= side).
2. Area of rectangle= l.b (length x breadth).
3. Volume of rectangle = 2[lb + bh + hl].
4. Volume of square = length x breadth x height = l^3.
5. Volume of rectangle = length x breadth x height = lbh.

11. GEOMETRICAL FIGURES

1. Lateral surface area of a pyramid = $\frac{1}{2}$p.l where p= perimeter, l= lateral.
2. Total surface area of a pyramid = B + $\frac{1}{2}$p.l where B= area of a base.
3. Lateral surface area of a prism= p.h where p= perimeter, h= height.
4. Total surface area of a prism= 2B+ph
5. Lateral surface area of a cylinder = $2\pi rh$.
6. Total surface area of a cylinder= $2\pi r(r+h)$.
7. Lateral surface area of a cone= πrl.
8. Total surface area of a cone= $\pi r(r+l)$.
9. Surface area of a sphere= $4\pi r^2$.

12. VOLUMES

1. Volume of prism = Bh.
2. Volume of cylinder = $\pi r^2 h$.
3. Volume of cone = $\frac{1}{3}\pi r^2 h$.
4. Volume of pyramid= $\frac{1}{3}$Bh.
5. Volume of sphere = $\frac{4}{3}\pi r^3$.
6. Volume of cuboid = lbh.

13. COMMERCIAL

1. Simple interest $=\frac{\textbf{principle x time x rate of interest}}{100}$.
2. Amount = principle + interest.
3. Compound interest = p. $(1+\frac{r}{100})^n$.

14. CONICS

1. Parabola = $y^2 = 4ax$ & distance formula= $\sqrt{[(x-a)\ 2+(y-b)\ 2]}$
2. Ellipse = $[\frac{x^2}{a^2}+\frac{y^2}{b^2}] = 1$.
3. Hyperbola = $[\frac{x^2}{a^2}-\frac{y^2}{b^2}] = 1$.

II. HIGHER BASICS

1. PARTIAL FRACTION

1) Rational function: - if f(x) and g(x) are two polynomials in x & g(x) ≠0, then$\frac{f(x)}{g(x)}$.

2) Proper fraction: - a rational fn. $\frac{f(x)}{g(x)}$ is said to be proper fraction if the degree of f(x) is less than the degree of g(x).

Ex:$\frac{3x-5}{2x^2-x-6}$.

3) Improper function: - A rational function $\frac{f(x)}{g(x)}$ is said to be improper fraction if the degree of f(x) is greater than or equal to the degree of g(x).

Ex: $\frac{x^4-1}{x^2(x^2+1)}$

IMPORTANT RESULTS

1. For linear term ax+b we get a contribution of$\frac{A}{ax+b}$.
2. For repeated $(ax+b)^3$ we get a contribution of linear term $\frac{A}{ax+b}+\frac{B}{(ax+b)^2}+\frac{C}{(ax+b)^3}$.
3. For quadratic term ax^2+bx+c we get a contribution =$\frac{Ax+B}{ax^2+bx+c}$.
4. for a repeated quadratic term $(ax^2+bx+c)^2$ we get a contribution of $\frac{Ax+B}{ax^2+bx+c}$+

2. LOGARITHMS

1. $a^x = m \leftrightarrow \log_a m = x$.
2. $\log_a uv = \log_a u + \log_a v$.
3. $\log_a \frac{u}{v} = \log_a u - \log_a v$.
4. $\log_a u^p = p \log_a u$.
5. $\log_b u = \frac{\log_a u}{\log_a b}$.
6. $a \wedge \log_a x = x$.
7. $\log_{a^n} m = \frac{1}{n} \log_a m$.

3. SUMMATION

1. $\sum n = \frac{n(n+2)}{2}$.
2. $\sum n^2 = \frac{n(n+2)(2n+1)}{6}$.
3. $\sum n^3 = \frac{n^3(n+1)}{4}$.
4. $\sum n^2 = (\sum n)^2$.

4. BINOMIAL THEOREM

$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n+1} b + {}^nC_2\, a^{n-2} b^2 + \dots\dots\dots + {}^nC_r\, a^{n-r} b^r + \dots\dots\dots + {}^nC_n\, b^n$.

${}^nC_r = \frac{n!}{(n-r)!r!}$.

5. MATRICES & DETERMINANTS

TYPES OF MATRIX:

- Row matrix: A matrix having only one row.

  for Ex: [5 3 2].

- Column matrix: A matrix having only one column.

  For Ex. $\begin{bmatrix} 3 \\ 1 \\ 8 \end{bmatrix}$

- Zero or null matrix: A matrix in which all the elements are zeroes.

  for Ex: $O=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

- Square matrix: A matrix in which the no. of rows is equal to the no. of column.

  For Ex: $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ where a_{11}, a_{22} and a_{33} are the diagonal elements.

- Diagonal matrix: A square matrix in which all the non-diagonal elements are equal to zero.

  For Ex: $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.

- Scalar matrix: A diagonal matrix in which all diagonal elements are equal. For Ex:

  $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

- Unit matrix: A scalar matrix in which all the diagonal elements are equal to 1 i.e. unity and the matrix is denoted by the letter 'I'.

For Ex: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and A-I=I.A=A.

MULTIPLICATION OF MATRICES:

A= $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$, B= $\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}$

A.B= $\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}$

A.B= $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

<u>Transpose of matrix</u>: Let A be a matrix of order mxn. Then the matrix obtained by interchanging the rows & column of A is called the transpose of A & is denoted by A^{I} or A^{T}.

CRAMER'S RULE:

$a_1x+b_1y+c_1z= d_1$

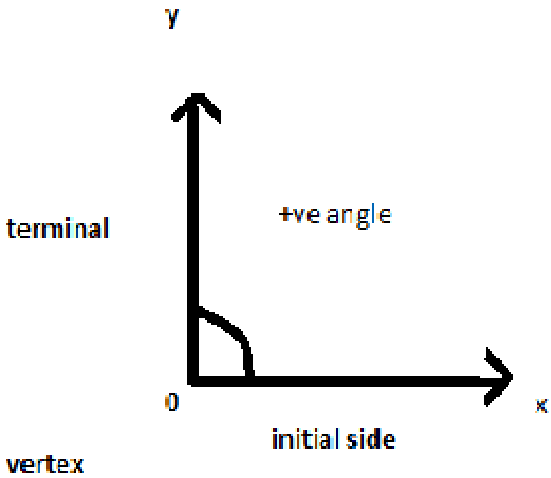
$a_2x+b_2y+c_2z= d_2$ be the set of linear eqn in x, y, z

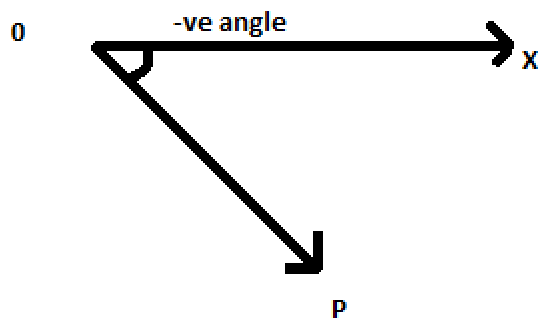
$a_3x+b_3y+c_3z= d_3$

$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$

For 'x' find Δ_1 by replacing first column by constants 'd' same for Δ_2 and Δ_3 to find 'y' & 'z' then x=$\frac{\Delta_1}{\Delta}$, y=$\frac{\Delta_2}{\Delta}$ & $z = \frac{\Delta_3}{\Delta}$.

III. TRIGONOMETRY





If the rotation is clock wise then the angle is –ve & if the rotation is anti-clock wise then the angle is +ve.

In trigonometry angles are measured in terms of: a) degree (sexagesimal system), b) Grade (centesimal system) & c) Radian.

a) Degree: 1 right angle= 90°

$1^\circ = 60'$.

$1' = 60''$.

b) Grade: 1 right angle= 100 grades.

$1^g = 100$ mins.

1 min= 100 secs.

c) Radians: A radian is the angle subtended at the Centre of a circle by an arc whose length is equal to the radius of the circle.

1 radian = $\frac{2}{\pi}$ of a right angle.

$$= \frac{2}{\pi} x\, 90° = \frac{180}{\pi} = 57.296° = 57°17^{I}45^{II}$$

$$1^{c} = \frac{\pi}{180} = 0.017453$$

$\pi^{c} = 180° = 200^{g} = 2$ right ⊾ or $\frac{\pi^{c}}{c} = 90° = 100^{g} = 1$ right⊾.

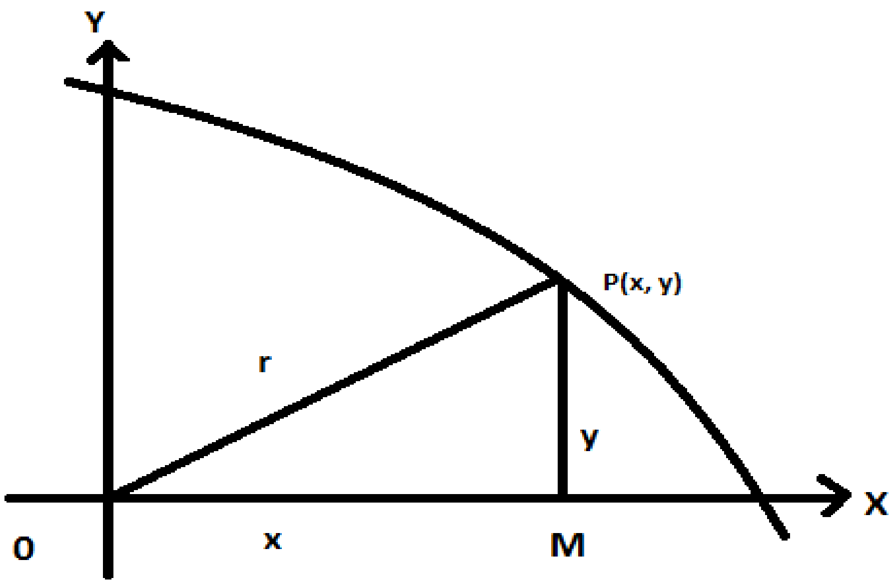
HYPERBOLIC FUNCTION:

1. Sin hx = $\frac{e^{x} - e^{-x}}{2}$
2. Cos hx = $\frac{e^{x} + e^{-x}}{2}$
3. Tan hx = $\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$
4. Cos $h^{2}x$ – sin $h^{2}x$ = 1
5. Sec $h^{2}x$ = 1 - tan $h^{2}x$
6. cosec $h^{2}x$ = cot $h^{2}x$ – 1
7. sin h(x$\pm$y) = sin hx . cos hy $\pm$ cos hx. sin hy
8. cos h(x$\pm$y) = cos hx. cos hy $\pm$ sin hx. sin hy
9. tan h(x$\pm$y)= $\frac{\tan hx \pm \tan hy}{1 \pm \tan hx . \tan hy}$
10. sin h2x = 2. sin hx . cos hx
11. cos h2x= cos $h^{2}x$ + sin $h^{2}x$ = 2 cos $h^{2}x$ – 1.
12. Tan h2x= $\frac{2 \tan hx}{1 + \tan h^{2}x}$

MODULUS FUNCTION:

f(x) = $|x| = \begin{cases} x \text{ where } \geq 0 \\ -x \text{ where } x < 0 \end{cases}$

TRIGONOMETRIC FUNCTION:



r-hypotenuse; y- opposite; x- adjacent

Sin θ= $\frac{\text{opp}}{\text{hyp}} = \frac{y}{r},$

Cos θ= $\frac{\text{adj}}{\text{hyp}} = \frac{x}{r},$

Tan θ= $\frac{\text{opp}}{\text{adj}} = \frac{y}{x},$

Sec θ= $\frac{1}{\cos} = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x},$

Cot θ= $\frac{1}{\tan} = \frac{\text{adj}}{\text{opp}} = \frac{x}{y},$

Cosec θ= $\frac{1}{\sin} = \frac{\text{hyp}}{\text{opp}} = \frac{x}{y}.$

BASIC TRIGONOMETRIC IDENTITIES:

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \mathrm{cosec}^2\theta$
4. $\tan\theta + \cos\theta = \sec\theta \, . \, \mathrm{cosec}\theta = \sqrt{(\sec^2\theta + \mathrm{cosec}^2\theta\,)}$
5. $\sin\theta + \cos\theta = \frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$
6. $\sin^3\theta + \cos^3\theta = (\sin\theta + \cos\theta)(1 - \sin\theta.\cos\theta)$

EULERS FORMULAE:

7. $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2}$
8. $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$

TABLE:

	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

- The no. against sine ratio are got as follows;

Write the numbers 0, 1, 2, 3, 4

Divide each by 4 i.e. $^0/_4, ^1/_4, ^2/_4, ^3/_4, ^4/_4.$

Take the square root; $\sqrt{^0/_4}, \sqrt{^1/_4}, \sqrt{^2/_4}, \sqrt{^3/_4}, \sqrt{^4/_4}.$

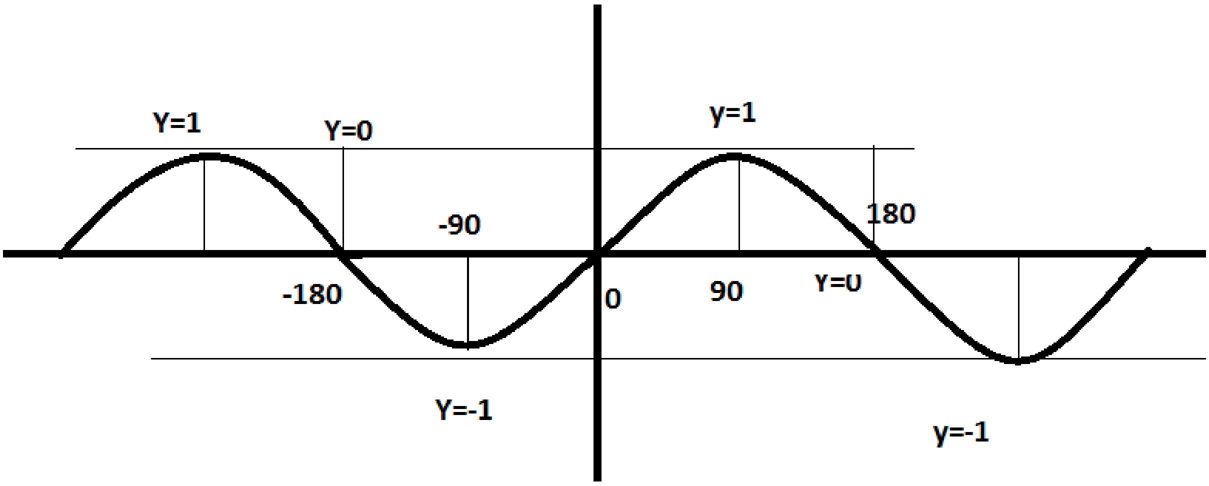
Finally the values are as follows:

$0, 1/2, 1/\sqrt{2}, \quad \sqrt{3}/2, 1$

❖ For the cosine values, the cosine values are exactly the mirror image of the sine values and for tan values sine/cos values.

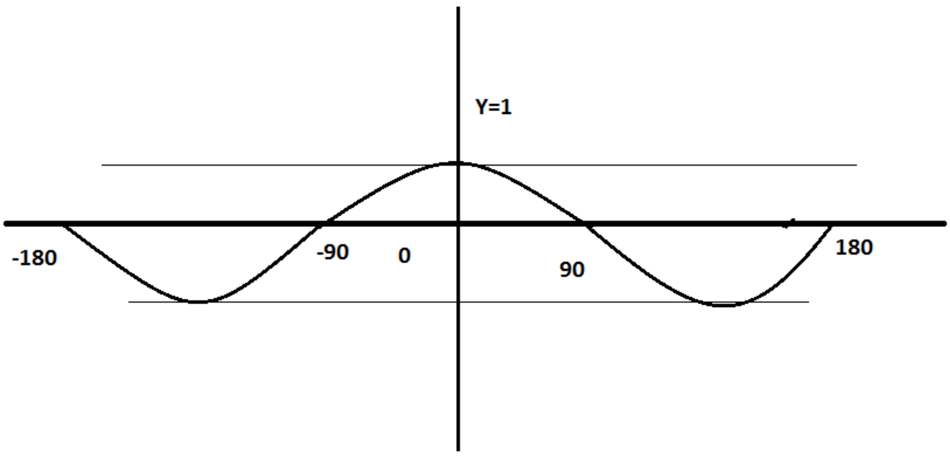
GRAPHS OF TRIGONOMETRIC FUNCTIONS:

1. y= sinθ



It may be noted from the graph that sine curve passes through the origin and the maximum value of sine is +1 and the minimum is -1, hence the value of sine oscillates between +1 & -1.

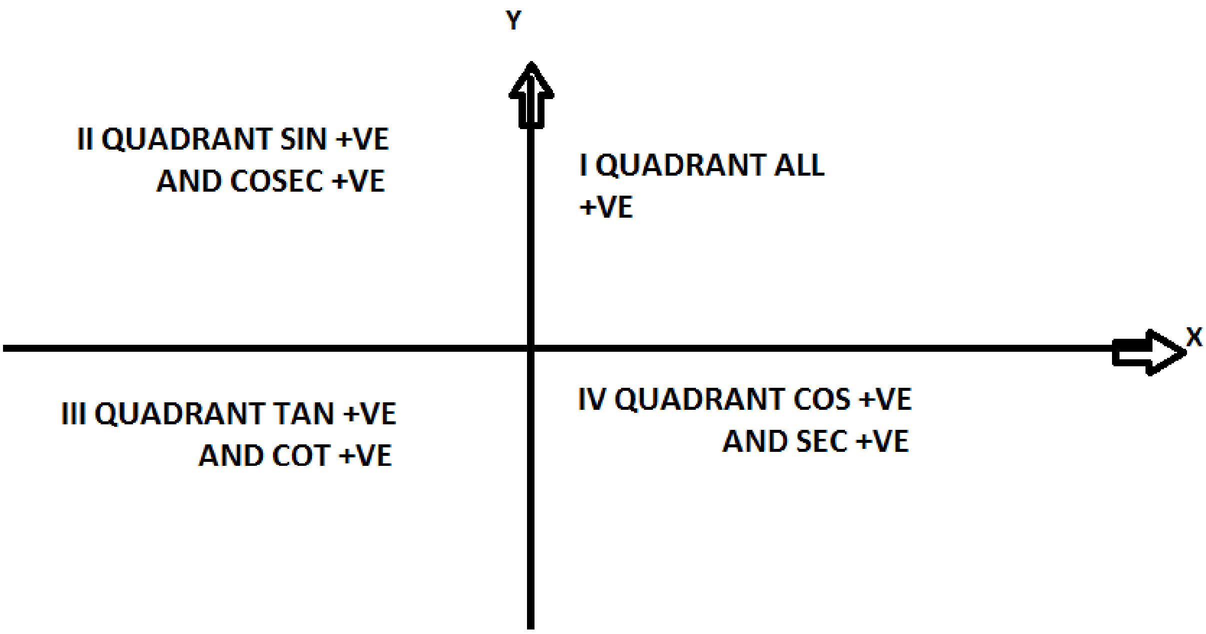
2. y= cosθ.



From the graph, it is observed that the cos curve does not pass through the origin & maximum value of cos is +1 and the minimum is -1 hence the value of cos oscillates between +1 & -1.

FUNCTION	**X ∈ DOMAIN**	**Y ∈ RANGE**	
SIN	$[\frac{-\pi}{2}, \frac{\pi}{2}]$	[-1, 1]	SIN X= Y
COS	[0, π]	[-1, 1]	COS X= Y
TAN	$[\frac{-\pi}{2}, \frac{\pi}{2}]$	R	TAN X= Y
COT	[0, π]	r	COT X= Y
SEC	$[0, \pi] - \{\frac{\pi}{2}\}$	R- (-1, 1)	SEC X= Y
COSEC	$[\frac{-\pi}{2}, \frac{\pi}{2}] - \{0\}$	R- (-1, 1)	COSEC X= Y

ALLIED ANGLES:



- To express trigonometric function of -θ in terms of +θ:

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$

$\text{cosec}(-\theta) = -\text{cosec}\theta$

- To express trigonometric function of 90-θ in terms of +θ:

$\sin(90 - \theta) = \cos\theta$

$\cos(90 - \theta) = \sin\theta$

$\tan(90 - \theta) = \cot\theta$

$\cot(90 - \theta) = \tan\theta$

$\sec(90 - \theta) = \text{cosec}\,\theta$

$\text{cosec}(90 - \theta) = \sec\theta$

- To express trigonometric function of 90+θ in terms of +θ:

$\sin(90 + \theta) = \cos\theta$

$\cos(90 + \theta) = -\sin\theta$

$\tan(90 + \theta) = -\cot\theta$

$\cot(90 + \theta) = -\tan\theta$

$\sec(90 + \theta) = -\text{cosec}\,\theta$

$\text{cosec}(90 + \theta) = \sec\theta$

❖ To express trigonometric function of 180-θ in terms of +θ:

$\sin(180 - \theta) = \sin\theta$

$\cos(180 - \theta) = -\cos\theta$

$\tan(180 - \theta) = -\tan\theta$

$\cot(180 - \theta) = -\cot\theta$

$\sec(180 - \theta) = -\sec\theta$

$\mathrm{cosec}(180 - \theta) = \mathrm{cosec}\,\theta$

❖ To express trigonometric function of 180+θ in terms of +θ:

$\sin(180 + \theta) = -\sin\theta$

$\cos(180 + \theta) = -\cos\theta$

$\tan(180 + \theta) = \tan\theta$

$\cot(180 + \theta) = \cot\theta$

$\sec(180 + \theta) = -\sec\theta$

$\mathrm{cosec}(180 + \theta) = -\mathrm{cosec}\,\theta$

❖ To express trigonometric function of 270-θ in terms of +θ:

$\sin(270 - \theta) = -\cos\theta$

$\cos(270 - \theta) = -\sin\theta$

$\tan(270 - \theta) = \cot\theta$

❖ To express trigonometric function of 270+θ in terms of +θ:

$\sin(270 + \theta) = -\cos\theta$

$\cos(270 + \theta) = \sin\theta$

$\tan(270 + \theta) = \cot\theta$

❖ To express trigonometric function of $2n\pi \pm \theta$ in terms of $+\theta$:

$\sin(2n\pi + \theta) = \sin\theta$

$\cos(2n\pi + \theta) = \cos\theta$

$\tan(2n\pi + \theta) = \tan\theta$

$\sin(2n\pi - \theta) = -\sin\theta$

$\cos(2n\pi - \theta) = \cos\theta$

$\tan(2n\pi - \theta) = -\tan\theta$

FUNCTIONS OF COMPOUND ANGLES:

1. $\sin(A \pm B) = \sin A.\cos B \pm \cos A.\sin B$
2. $\cos(A \pm B) = \cos A.\cos B \mp \sin A.\sin B$
3. $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A.\tan B}$
4. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
5. $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

MULTIPLE & SUBMULTIPLE ANGLES:

1. $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$
2. $\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2} = 2\cos^2\frac{A}{2} - 1 = 1 - 2\sin^2\frac{A}{2} = \sqrt{\frac{1+\cos 2A}{2}}$
3. $\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}} = \sqrt{\frac{1-\cos 2A}{1+\cos 2A}} = \frac{\sin 2A}{1+\cos 2A} = \frac{\sin A+\sin 2A}{1+\cos A+\cos 2A}$

4. $\sin 2A = \sin A\cos A + \cos A\sin A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A} = \sqrt{\frac{1-\cos 2A}{2}}$

5. $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^4 A - \sin^4 A$

6. $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

7. $\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$

8. $\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$

9. $\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{1-\cos A}{\sin A} = \frac{1+\sin A-\cos A}{1+\sin A+\cos A}$

10. $\text{Sin}3A = 3\sin A - 4\sin^3 A$

11. $\text{Cos}3A = 4\cos^3 A - 3\cos A$

12. $\tan 3A = \frac{3\tan A - \tan 3A}{1-3\tan^2 A}$

13. $\text{Sin}^2 A = \frac{1-\cos 2A}{2}$

14. $\text{Cos}^2 A = \frac{1+\cos 2A}{2}$

15. $\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$

TRANSFORMATION OF PRODUCTS & SUM:

1. $\sin A\cos B = \frac{1}{2}[\sin(A+B)+\sin(A-B)]$

2. $\cos A\sin B = \frac{1}{2}[\sin(A+B)-\sin(A-B)]$

3. $\cos A\cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$

4. $\sin A\sin B = \frac{1}{2}[\cos(A+B)-\cos(A-B)]$

5. $\sin C + \sin D = 2\sin\frac{(C+D)}{2}\cos\frac{(C-D)}{2}$

6. $\text{SinC} - \sin D = 2\cos\frac{(C+D)}{2}\sin\frac{(C-D)}{2}$

7. $\cos C + \cos D = 2\cos\frac{(C+D)}{2}\cos\frac{(C-D)}{2}$

8. $\cos C - \cos D = -2\sin\frac{(C+D)}{2}\sin\frac{(C-D)}{2}$

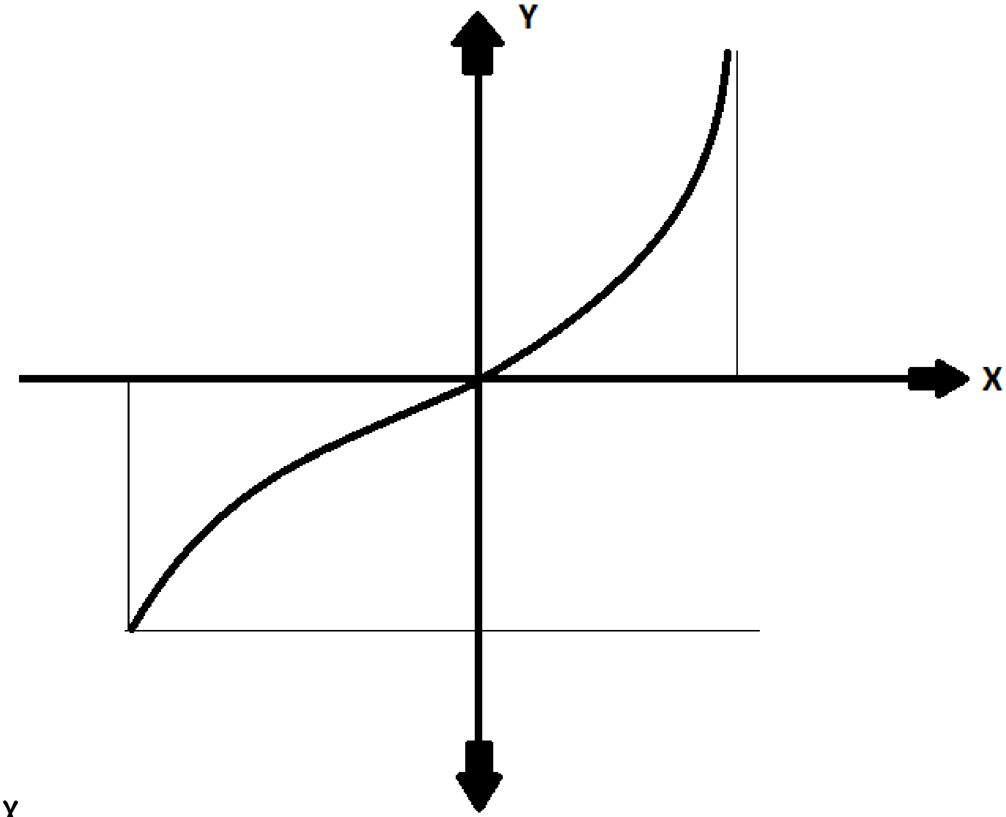
9. $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

10. $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

11. $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

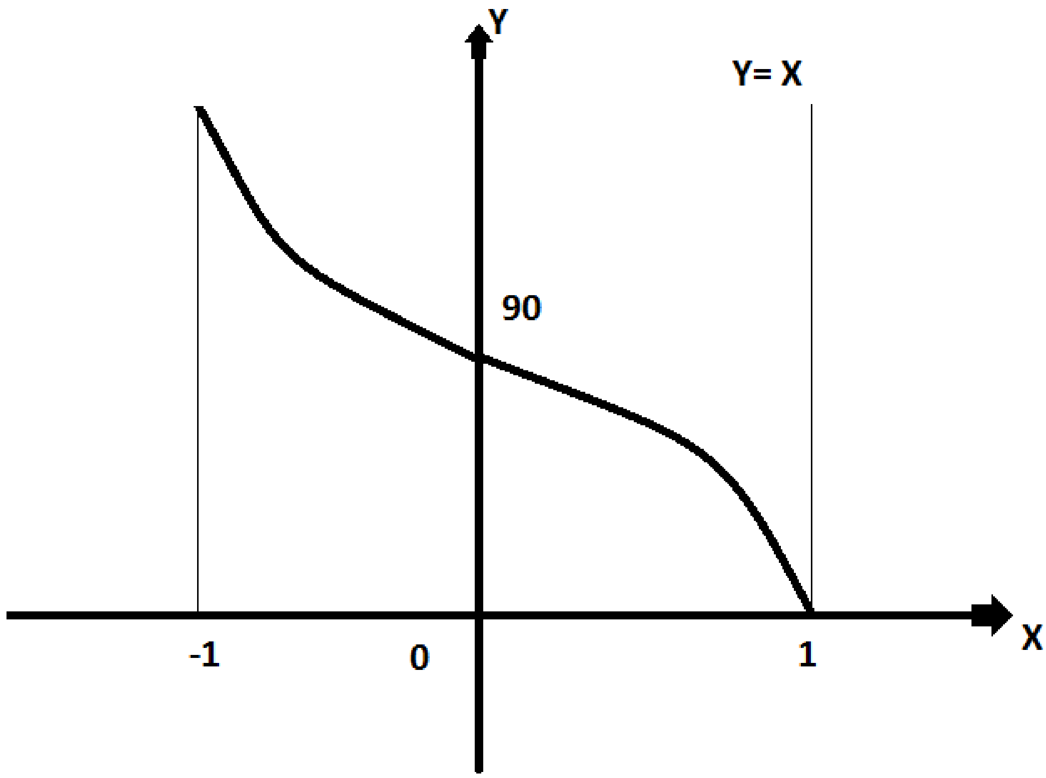
12. $-2\sin A\sin B = \cos(A+B) - \cos(A-B)$

INVERSE TRIGONOMETRIC FUNCTION:

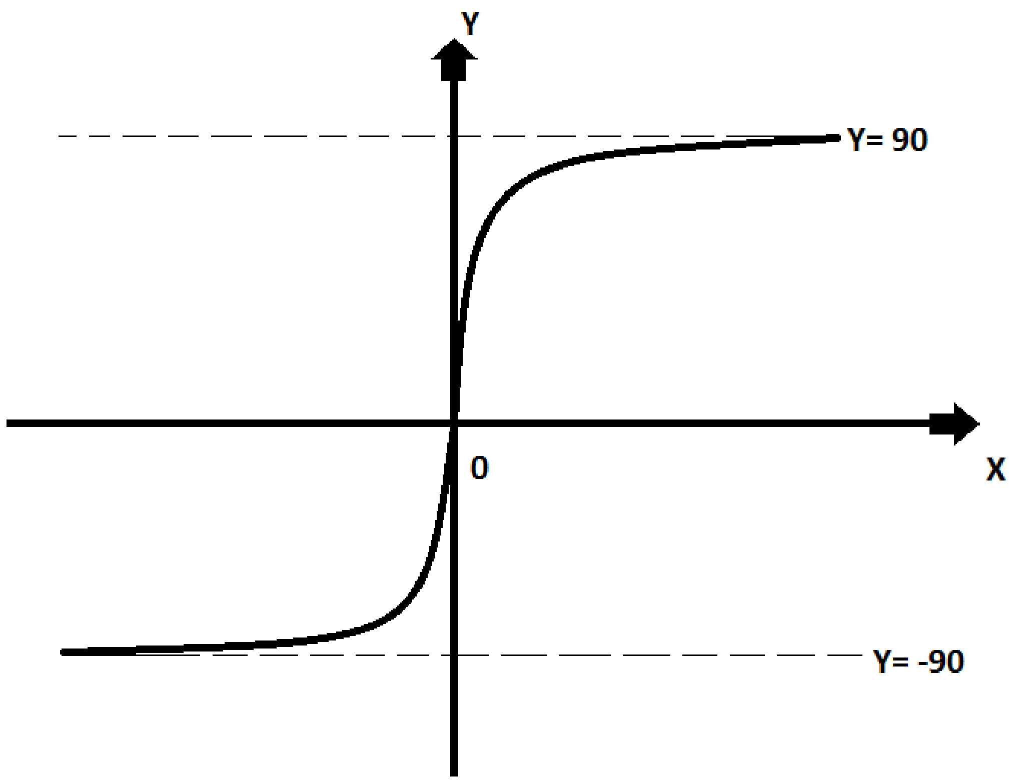


$Y = \text{SIN}^{-1}X$

$Y = COS^{-1}X$



$Y = TAN^{-1}X$



1. $Sin^{-1}(\sin y) = y$; where $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$.

2. $Cos^{-1}(\cos y) = y$; where $0 \leq y \leq \pi$.

3. $\tan^{-1}(\tan y) = y$; where $\frac{-\pi}{2} < y < \frac{\pi}{2}$.

4. $Sin^{-1}(-x) = -\sin^{-1}x$; where $-1 \leq x \leq 1$

5. $Cos^{-1}(-x) = \pi - \cos^{-1}x$; where $-1 \leq x \leq 1$

6. $\tan^{-1}(-x) = -\tan^{-1}x$; where $-\infty < x < \infty$

7. $Sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$; where $-1 \leq x \leq 1$

8. $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$; where $-\infty < x < \infty$

9. $Sec^{-1}x + cosec^{-1}x = \frac{\pi}{2}$, where $|x| \geq 1$

10. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$; where $x>0; y>0$ & $xy<1$

11. $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$; where $x>0; y>0$ & $xy>1$

12. $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$; where $x>0; y>0$

13. $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$; $\forall$ x such that $x^2 < 1$

14. $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-(xy+yz+zx)}\right)$

15. $Sin^{-1}x \pm \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\}$

16. $\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\{xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\}$

IV. COMPLEX NUMBERS

- If z= x+ iy then $\bar{z}$= x- iy
- e^{ix}= cosx + $i$sinx is called eulers formula

z= x + iy= r(cosθ + $i$sinθ); where r= $\sqrt{x^2 + y^2}$ & θ= $\tan^{-1}\frac{y}{x}$

MODULUS OF A COMPLEX NUMBER:

If z= x+ iy is a complex number then $\sqrt{x^2 + y^2}$ is called the modulus or absolute value of z & is denoted by $|z|$

$|z|= \sqrt{x^2 + y^2}$

$|z_1 . z_2|= |z_1| . |z_2|$

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$|z_1 + z_2| \leq |z_1| + |z_2|$

$|z_1 - z_2| \geq |z_1| - |z_2|$

Cosθ + is specifically denoted by cisθ.

DEMOIVRE'S THEOREM:

$$\cos n\theta + i \sin n\theta = (\cos\theta + i\sin\theta)^n = e^{in\theta}$$

V. ADVANCED

CALCULUS:

1. $\lim_{x\to a}\{f(x)+g(x)\} = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a}\{f(x)\,.\,g(x)\} = \lim_{x\to a} f(x)\,.\lim_{x\to a} g(x)$
3. $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$
4. $\lim_{x\to a} k\,f(x) = k\lim_{x\to a} f(x)$; k is independent of x
5. $\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$; provided $\lim_{x\to a} f(x)$ is +ve
6. $\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right) = n\,a^{n-1}$
7. $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$
8. $\lim_{\theta\to 0}\frac{\tan\theta}{\theta} = 1$
9. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$
10. $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$

1. DIFFERENTIAL CALCULUS

- The derivative of a function at a is defined by $f'(a)= \lim_{h\to 0}\left[\frac{f(a+h)-f(a)}{h}\right]$
- Derivative of a function $\lim_{\delta x\to 0}\left[\frac{\delta x}{\delta y}\right]= \lim_{\delta x\to 0}\left[\frac{f(x+\delta x)-f(x)}{\delta x}\right]$
- $\frac{dy}{dx}= \lim_{\delta x\to 0}\left[\frac{\delta x}{\delta y}\right]$

Y	$\frac{dy}{dx}$ or y'
1. K	0
2. X	1
3. x^n	nx^{n-1}
4. $\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
5. e^{ax}	$a\,e^{ax}$
6. e^{-x}	$-e^{-x}$
7. e^x	e^x
8. sin ax	a. cos ax
9. sin x	cos x
10. cos ax	-a sin ax
11. cos x	-sin x
12. tan ax	$a\sec^2 ax$
13. tan x	$\sec^2 x$

14.	cosec ax	-cot ax. Cosec ax
15.	sec ax	a sec ax . tan ax
16.	cot ax	-a $cosec^2 ax$
17.	a^x	$a^x \log_e a$
18.	$\log_e x$	$\frac{1}{x}$
19.	$Sin^2 x$	2 sinx cosx
20.	$Cos^2 x$	-2 cosx sinx
21.	$\sqrt{\cos x}$	$\frac{-\sin x}{2\sqrt{\cos x}}$
22.	$\sqrt{\sin x}$	$\frac{\cos x}{2\sqrt{\sin x}}$
23.	$Sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
24.	$Cos^{-1}x$	$\frac{-1}{\sqrt{1-x^2}}$
25.	$tan^1 x$	$\frac{1}{1+x^2}$
26.	$cot^{-1}x$	$\frac{-1}{1+x^2}$
27.	$sec^{-1}x$	$\frac{1}{x\sqrt{x^2-1}}$
28.	$cosec^{-1}x$	$\frac{-1}{x\sqrt{x^2-1}}$
29.	$sinh^{-1}x$	$\frac{1}{\sqrt{1+x^2}}$
30.	$cosh^{-1}x$	$\frac{1}{\sqrt{x^2-1}}$
31.	$tanh^{-1}x$	$\frac{1}{1-x^2}$
32.	$coth^{-1}x$	$\frac{-1}{x^2-1}$
33.	$sech^{-1}x$	$\frac{-1}{x\sqrt{1-x^2}}$

34.	$\text{cosech}^{-1}x$	$\frac{-1}{x\sqrt{1+x^2}}$
35.	sinhx	coshx
36.	coshx	sinhx
37.	tanhx	sech^2x
38.	cothx	cosech^2x
39.	sechx	- sechx tanhx
40.	cosechx	- cosechx cothx

RULES OF DIFFERENTIATION:

1. If 'k' is a constant & f(x) is any function of x $\frac{d}{dx}[k \,.\, f(x)] = k \,.\, \frac{d}{dx}[f(x)]$
2. If f(x), g(x), h(x)....... are the function of x

   $\frac{d}{dx}[f(x) \pm g(x) \pm h(x) \pm \ldots] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)] \pm \frac{d}{dx}[h(x)] \ldots\ldots$
3. Product rule; if y= uv where u & v are the function of x then $\frac{d}{dx}(uv) = u\frac{dv}{dx}$

   $+ v\frac{du}{dx}$
4. Quotient rule; if $y = \frac{u}{v}$, where u & v are the function of x then $\frac{dy}{dx} =$

   $\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{x^2}$
5. Extension of product rule; if y= uvw, where u, v & w are the function of x then

   $\frac{d}{dx}(uvw) = (vw)\frac{du}{dx} + (wu)\frac{dv}{dx} + (uv)\frac{dw}{dx}$

2. COMPOSITE FUNCTION OR CHAIN RULE

1. If y= f(u) & u= g(x) then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
2. If y= f(u), u= g(v) and v= h(x) then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$
3. $\frac{d}{dx}(u^n) = n\,u^{n-1}\left(\frac{du}{dx}\right)$
4. $\frac{d}{dx}\left(\frac{1}{u^2}\right) = \left(\frac{-2}{u^3}\right)\left(\frac{du}{dx}\right)$
5. $\frac{d}{dv}\sqrt{u} = \left(\frac{1}{2\sqrt{u}}\right)\left(\frac{du}{dx}\right)$
6. $\frac{d}{dx}(\log u) = \frac{1}{u}\left(\frac{du}{dx}\right)$

3. IMPORTANT STANDARD RESULTS:

Y	$\left(\frac{dy}{dx}\right)$ or y^1
1. $Log\left[\frac{1-\cos x}{1+\cos x}\right]$	2 cosec x
2. $Log\left[\frac{1+\sin x}{1-\sin x}\right]$	2 sec x
3. $Log\left[\frac{\sqrt{1-\cos x}}{\sqrt{1+\cos x}}\right]$	cosec x
4. $Log\left[\frac{\sqrt{1+\sin x}}{\sqrt{1-\sin x}}\right]$	sec x
5. $Log\left[\frac{a+b\tan x}{a-b\tan x}\right]$	$\frac{2ab}{a^2\cos^2 x - b^2\sin^2 x}$
6. $Log\left[\frac{1+\tan x}{1-\tan x}\right]$	2 sec2x
7. $Sin^{-1}[f(x)] + cos^{-1}[f(x)]$	0

8. $\text{Sin}^{-1}[f(x)] + \sec^{-1}\left[\frac{1}{f(x)}\right]$ 0

9. $\tan^{-1}[f(x)] + \tan^{-1}\left[\frac{1}{f(x)}\right]$ 0

4. SUBSTITUTION METHOD

Expression	substitution
1. $\sqrt{a^2 - x^2}$	$x = a \sin\theta$
2. $\sqrt{x^2 - a^2}$	$x = \frac{a \sec\theta}{a \cos\theta}$
3. $\sqrt{a^2 + x^2}$	$x = a \tan\theta$
4. $\sqrt{a - x}$ or $\sqrt{a + x}$	$x = a \cos 2\theta$ or $x = a \cos\theta$
5. $\sqrt{a - x^n}$ & $\sqrt{a + x^n}$	$x^n = a \cos 2\theta$ or $a \cos\theta$

STANDARD EXAMPLES

Y	y^1
• $\tan^{-1}\frac{1-x}{1+x}$	$\frac{-1}{1+x^2}$
• $\tan^{-1}\frac{x}{1+x}$	$\frac{1}{2x^2+2x+1}$
• $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$	$\frac{-1}{2\sqrt{1-x^2}}$
• $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$	$\frac{2}{1+x^2}$

- $\tan^{-1}\left(\frac{bx-a}{ax+b}\right)$ $\frac{1}{1+x^2}$
- $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ $\frac{x}{\sqrt{1-x^4}}$
- $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ $\left(\frac{2}{1+x^2}\right)$
- $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ $\left(\frac{-2}{1+x^2}\right)$
- $\tan^{-1}\left[\frac{\sqrt{1+x^3}-\sqrt{1-x^3}}{\sqrt{1+x^3}+\sqrt{1-x^3}}\right]$ $\frac{3x^2}{2\sqrt{1-x^6}}$
- $_x x^{x\cdots}$ $\frac{y^2}{x(1-y\log x)}$
- $\log_a x$ $\frac{1}{\log_a} \times \frac{1}{x}$

5. APPLICATION OF DERIVATIVES

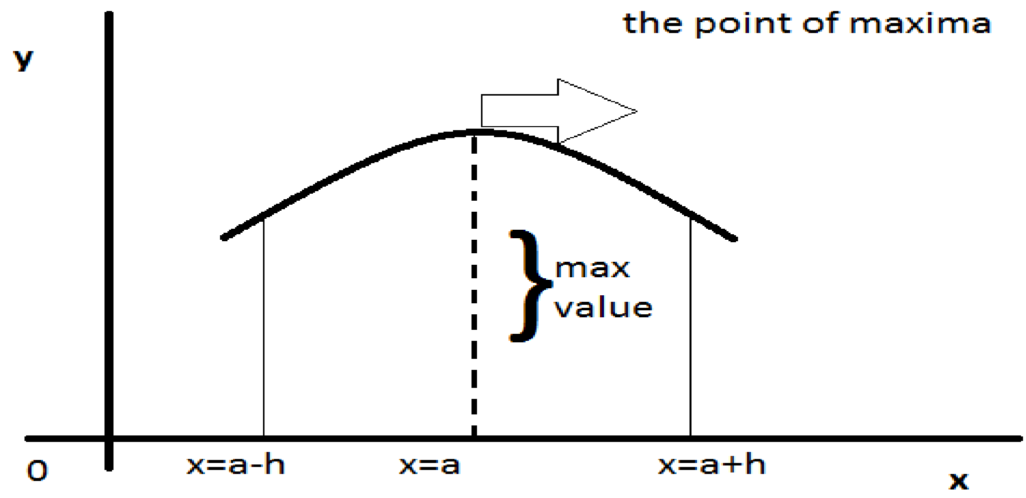
1. Slope of a line: - If a line 'L' makes an angle 'θ' w.r.t x-axis in +ve direction than tan θ is called slope of that line & usually denoted by m; m= slope of line= tan θ
2. Angle between two curves; tan $\theta = \frac{m_1-m_2}{1+m_1m_2}$
3. Subtangent ; $\frac{y_1}{(y_1)p}$
4. Subnormal; $(y_1)p\ y_1$

6. MAXIMA AND MINIMA OF A FUNCTION

Defination; At a function y= f(x) is said to be maxima at x= a if f(a) is greater than any other values of a function

$$i.e, f(a) > f(a-h)$$

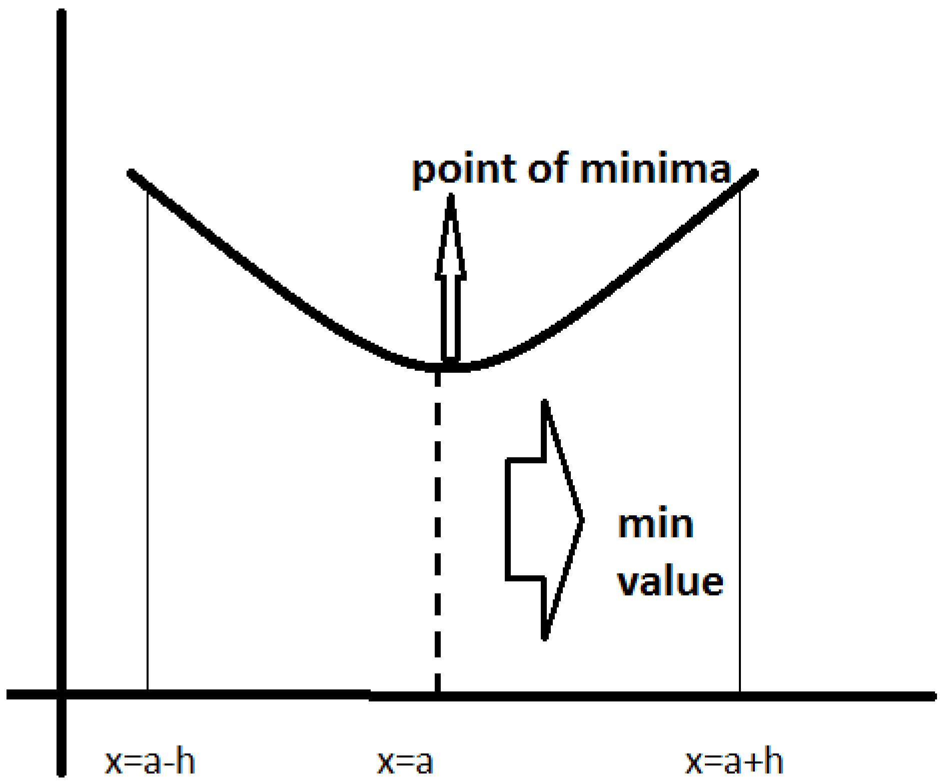
f(a) > f(a+h)



Defination; At a function y= f(x) is said to be minima at x= a if f(a) is lesser than any other values of a function

i.e, f(a) < f(a-h)

f(a) < f(a+h); where h is small +ve real number



7. INTEGRAL CALCULUS

$$\int f(x)\, dx = \emptyset(x) + c$$

RULES OF INTEGRATION:

1. Integration of $\int [f(x) \pm g(x) \pm h(x)]dx = \int f(x)\, dx \pm \int g(x)dx \pm \int h(x)dx.$
2. Integration of constant $\int kf(x)\, dx$= k$\int f(x)dx$
3. Integration of some function $\int f(ax)dx$=$\frac{1}{a}\phi$(ax) +c
4. Integration of $\int y^1\, dx$= $\int \frac{dy}{dx} dx$= y

STANDARD INTEGRATION FORMULAE: -

C to be added ∀ answers

1. $\int 0\ \ dx$ = c
2. $\int 1\, dx$ = x
3. $\int \sin x\ \ dx$= - cos x
4. $\int \cos x\ \ dx$ = sin x
5. $\int sec^2 x\ \ dx$= tan x
6. $\int cosec^2 x\ dx$= - cot x
7. $\int (secx\ .\ tanx)\ dx$= sec x
8. $\int (cosecx\ .\ cotx)\ dx$ = - cosec x
9. $\int \tan x\ dx$= log(sec x)

10. $\int \sec x\ dx$= log(secx + tanx)

11. $\int cosec\ x\ dx$= log(cosecx - cotx)

12. $\int coshx\ \ dx$= sinhx

13. $\int \sin hx\ \ dx$ = coshx

14. $\int sec^2 x\ \ dx$= tanhx

15. $\int (sechx\ .\ tanhx)\ dx$= - sechx

16. $\int (cosechx\ .\ cothx)\ dx$= - cosechx

17. $\int cosech^2 x\ dx$= - cothx

18. $\int tanhx\ dx$= -log(sechx)

19. $\int cothx\ dx$= log(sinhx)

20. $\int cotx\ dx$= log(sinx)

21. $\int e^x\ dx = e^x$

22. $\int a^x\ dx = \frac{a^x}{\log a}$

23. $\int \frac{1}{1+x^2}\ dx$= $\tan^{-1}x$

24. $\int \frac{1}{\sqrt{1-x^2}}\ dx$= $\sin^{-1}x$

25. $\int \frac{1}{\sqrt{x^2-1}} dx$= - $\cosh^{-1}x$

26. $\int \frac{1}{x\sqrt{x^2-1}}\ dx$= $\sec^{-1}x$

27. $\int \frac{1}{x\sqrt{1-x^2}}\ dx$= - $\text{sech}^{-1}x$

28. $\int \frac{1}{x\sqrt{1+x^2}}\ dx$= - $\text{cosech}^{-1}x$

29. $\int \frac{1}{1-x^2}\ dx$= $\tanh^{-1}x$

30. $\int \frac{1}{x}\ dx$= logx

31. $\int x^n\ dx = \frac{x^{n+1}}{n+1}$

32. $\int \frac{1}{\sqrt{x}}\ dx = 2\sqrt{x}$

33. $\int \sqrt{x}\ dx = \frac{2}{3} x^{\frac{3}{2}}$

34. $\int \frac{1}{\sqrt{1+x^2}}\ dx = \sinh^{-1} x$

35. $\int \frac{f^1(x)}{f(x)} dx = \log[f(x)]$

IMPORTANT STANDARD RESULTS

36. $\int \frac{1}{a^2+x^2}\ dx = \frac{1}{a}\ tan^{-1}(x/a)$

37. $\int \frac{1}{a^2+(x\pm b)^2}\ dx = \frac{1}{a}\ tan^{-1}\left(x \pm b/a\right)$

38. $\int \frac{1}{\sqrt{a^2-x^2}}\ dx = \sin^{-1}(x/a)$

39. $\int \frac{1}{\sqrt{a^2-(x\pm b)^2}}\ dx = \sin^{-1}\frac{x\pm b}{a}$

40. $\int \frac{1}{\sqrt{a^2+x^2}}\ dx = \sinh^{-1}(x/a) = \log(x+\sqrt{a^2+x^2})$

41. $\int \frac{1}{\sqrt{x^2-a^2}}\ dx = \cosh^{-1}(x/a) = \log(x+\sqrt{x^2-a^2})$

42. $\int \frac{1}{a^2-x^2}\ dx = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)$

43. $\int \frac{1}{x^2-a^2}\ dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)$

44. $\int \sqrt{a^2-x^2}\ dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}(x/a)$

45. $\int \sqrt{x^2-a^2}\ dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}(x/a)$

46. $\int \sqrt{x^2+a^2}\ dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\sinh^{-1}(x/a)$

47. If $\int \frac{1}{a.cosx+b.sinx+c} dx$

Put t= tan$\left({}^{x}/_{2}\right)$, sinx=$\left({}^{2t}/_{1+t^2}\right)$, cosx=$\left({}^{1-t^2}/_{1+t^2}\right)$,

dx=$\left({}^{2}/_{1+t^2}\right)$

48. If $\int \frac{px+q}{ax^2+bx+c}dx$ & $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$; p, q, a, b, c are real numbers. These kind of integrals are evaluated by following method.

i. Express the numerator expression in the form of; Numerator= constant. $\frac{d}{dx}$(quadratic eqn) + Another constant.

Px+q= l$\frac{d}{dx}$ (ax^2+bx+c) + m where l and m are the constant to be determined

Px+q= l . (2ax+b)+m

On comparing the coeffiecents of x and constant terms we can find the values of l & m.

ii. On substituting the values of px+q (numerator value) in the given integral, splitting the term along with the integrals, given integral becomes in the form;

$\int \frac{px+q}{ax^2+bx+c}dx$ = l . $\int \frac{f^1(x)}{f(x)}dx$ + m$\int \frac{1}{ax^2+bx+c}dx$

49. If $\int \frac{p\,cosx+qsinx}{a\,cosx+b\,sinx}dx$=

i. Numerator= l (denominator) + m. $\frac{d}{dx}$(denominator)

ii. $p\,cosx + qsinx$= l($a\,cosx + b\,sinx$)+ m.[-a sinx +b cosx]

iii. $comparing\ sinx\ \&\ cosx$ coefficient we get l & m

iv. With the value of m & n using it on the numerator and separating the terms along with integrals becomes i=

$\int l\, dx + \int m \frac{-a \text{ sinx } + b \text{ cosx}}{a\, cosx + b\, sinx}\, dx$= lx+mlog(acosx+bsinx)+c

50. $\int e^x[f(x) + f^1(x)]\; dx$= e^xf(x)

8. INTEGRATION BY PARTS

51. $\int u.v\, dx = u \int v\, dx - \int [\frac{d}{dx}(u)\; \int v\, dx]\, dx$

52. $\int 1.2\, dx = 1 \int 2\, dx - \int \left[\frac{d}{dx}(1)\; \int 2\, dx\right] dx$

Choose the first function and second function of the order ILATE

I-INVERSE TRIGONOMETRIC FUNCTION

L-LOGARITHMIC FUNCTION

A-ALGEBRAIC

T-TRIGONOMETRIC FUNCTION

E-EXPONENTIAL

53. $\int x\, cosx\;\; dx$= x sinx+ cosx

54. $\int logx\, dx$= x(logx - 1)

55. $\int \sin^{-1} x\;\; dx$= x $\sin^{-1}$ x + $\sqrt{1 - x^2}$

56. $\int e^x\, sinx\;\; dx$= $\frac{e^x}{2}$(sinx - cosx)

57. $\int x\, e^x\, dx$= (x-1)e^x

58. $\int x\, sinx\, dx$= -x cosx + sinx

59. $\int \tan^{-1} x\;\; dx$= x $\tan^{-1} x$ - $\frac{1}{2}$log(1+x^2)

60. $\int x \cos^{-1} x \, dx = \frac{x^2}{2} \cos^{-1} x + \frac{1}{2} \sqrt{1-x^2}$

61. $\int \tanh^{-1} x \, dx = x \tanh^{-1} x - \frac{1}{2} \log(1-x^2)$

62. $\int x \tanh^{-1} x \, dx = \frac{1}{2}(x^2+1) \tanh^{-1} x - \frac{x}{2}$

63. $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \cosh x + b \sinh x)$

9. DEFINATE INTEGRAL

$$\int_{x=a}^{x=b} f(x) \, dx = [\phi(x) + c]_{x=a}^{x=b}$$

$$= [\phi(b) + c] - [\phi(a) + c]$$

$$= \phi(b) - \phi(a) = \{\text{upper limit} - \text{lower limit}\}$$

- $\int_0^{\frac{\pi}{2}} \sin x^n \, dx = \int_0^{\frac{\pi}{2}} \cos x^n =$

$$\begin{cases} \frac{(n-1)}{n} \cdot \frac{(n-3)}{(n-2)} \cdot \frac{(n-5)}{(n-4)} \ldots \ldots \frac{2}{3} & n = odd\ integers \\ \frac{(n-1)}{n} \cdot \frac{(n-3)}{n} \cdot \frac{(n-5)}{(n-4)} \ldots \ldots \frac{1}{2} \cdot \frac{\pi}{2} & n = even\ integer \end{cases}$$

PROPERTIES OF DEFINATE INTEGRALS

1. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
2. $\int_a^a f(x)dx = 0$
3. $\int_a^b 0 \, dx = 0$
4. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
5. $\int_a^b f(x)dx = \int_{-a}^b f(a+b-x)dx$

6. $\int_{-a}^{a} f(x)dx = \begin{cases} 0 & \rightarrow odd\ finite \\ 2\int_{0}^{a} f(x)dx & \rightarrow even\ finite \end{cases}$

7. $\int_{0}^{a} f(x)dx = \begin{cases} 0 & if\ f(a-x) = -f(x) \\ 2\int_{0}^{\frac{a}{2}} f(x)dx & if\ f(a-x) = f(x) \end{cases}$

8. $\int_{0}^{2a} f(x)dx = \begin{cases} 0 & if\ f(2a-x) = -f(x) \\ 2\int_{0}^{a} f(x)dx & if\ f(2a-x) = f(x) \end{cases}$

10. DIFFERENTIAL EQN

1. Separable eqn: - $\frac{dy}{dx}$= g(x). f(y)

   I. Separable x's & y's on different sides $\frac{1}{f(y)}$dy= g(x)dx

   II. integrating both sides $\int \frac{1}{f(y)}\ \mathrm{dy} = \int \mathrm{g(x)\ dx} + c$

   III. Express y in terms of x where if $|y|$= h(x) then y= $\pm$h(x) and if y= $\pm$ e^ch(x), then y= A h(x) where A is real number.

   IV. Check that constant solution y= c where f(c)= 0 are not missed

2. First order linear equation; - y^1+ P(x)y= Q(x)

   I. Find integrating factor I(x) = $e^{\int p(x)dx}$

   II. Write differential equation as$(I(x)y)^1$= I(x) Q(x)

   III. Integrating both sides I(x)y= $\int\{\mathrm{I(x)\ Q(x)}\}$dx+c

   IV. Divide both sides I(x)

   y= $\frac{1}{I(x)}(\int\{\mathrm{I(x)\ Q(x)}\}dx+c)$

If $y^1 + \frac{1}{x}y = 1$ the integrating factor is I(x) = x

$y^1 - \frac{1}{x}y = 1$; $I(x) = \frac{1}{x}$.

3. Second order linear homogenous eqn ($a\,y^{11} + b\,y^1 + cy) = 0$
   I. Write down the auxiliary equation $a.r^2 + b.r + c = 0$
   II. Solve the auxiliary equation $r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
   III. Depending on the roots
      A. b^2-4ac > 0. two real roots r_1, r_2; $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
      B. b^2-4ac = 0. One real root $r = r_1 = r_2$ $y = c_1 e^{rx} + c_2 x e^{rx}$
      C. b^2-4ac < 0. Two complex roots $\alpha \pm i\beta$ $y = c_1 e^{\alpha x} \cos\beta x + c_2 x e^{\alpha x} \sin\beta x$
4. Second order linear non-homogenous eqn ($a\,y^{11} + b\,y^1 + cy) = G(x)$
   A. Solve the complementary equation

      $a\,y_c^{11} + b\,y_c^1 + cy_c = 0$

      $Y_c = c_1 y_1(x) + c_2 y_2(x)$
   B. Write down trial solution
      i. $G(x) = P(x)$ $y_p = Q(x)$
      ii. $G(x) = P(x)\,e^{kx}$ $y_p = Q(x)\,e^{kx}$
      iii. $G(x) = P(x)\,e^{kx}\cos mx$ (or) $P(x)\,e^{kx}\sin mx$

         $y_p = Q(x)e^{kx}\cos mx + R(x)e^{kx}\sin mx$; here P(x), Q(x), R(x) are polynomials of same degree multiply y_p by x (or) x^2 if one of the term in the sum is $y_1(x)$ or $y_2(x)$
   C. Substitute y_p into the differential equation ($a\,y^{11} + b\,y^1 + cy) = G(x)$ group terms of the same form together. Ex; $x^n e^{kx} \cos mx$, $x^n e^{kx} \sin mx$ & solve for the unknown coefficients.

D. Write down the general solution

$y(x) = y_c(x) + y_p(x)$

5. Method of variation of parameters

i. Solve the complementary eqn

$a\, y_C^{11} + b\, y_C^{1} + c y_c = 0$

$y_c = c_1 y_1 + c_2 y_2$

ii. The particular solution has the form $y_p = u_1 y_1 + u_2 y_2$ write down the two condition

$U_1^1 y_1 + U_2^1 y_2 = 0$

$U_1^1 y_1 + U_2^1 y_2 = \frac{G(x)}{a}$

iii. Solve the conditions for U_1^1 & U_2^1

$U_1^1 = \frac{G(x) y_2}{a(y_1^1 y_2 - y_2^1 y_1)}$; $U_2^1 = \frac{G(x) y_1}{a(y_2^1 y_1 - y_1^1 y_2)}$

iv. Integrate U_1^1, U_2^1 to get u_1, u_2

$U_1 = \int u_1^1 dx + c_1$, $U_2 = \int u_2^1 dx + c_2$

v. Write down the general solution

$Y = (\int u_1^1 dx + c_1) y1 + (\int u_2^1 dx + c_2) y2$