A TREATISE

ON

THE STABILITY OF SHIPS.

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1830 - 1906.

BY

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TO

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AND FOR MANY YEARS DIRECTOR OF ADMIRALTY EDUCATION;

WHOSE INFLUENCE UPON THE PROGRESS OF SCIENTIFIC NAVAL EDUCATION GENERALLY

HAS LONG BEEN AS BENEFICIAL AS IT HAS BEEN EXTENSIVE;

WHOSE AUTHORITY UPON THE HIGHER BRANCHES OF THE SCIENCE OF NAVAL

ARCHITECTURE IS AS UNQUESTIONED AS IT IS GREAT;

AND WHOSE FRIENDLY AND CORDIAL INTEREST IN THEM AND THEIR

PROFESSION WILL EVER BE GRATEFULLY REMEMBERED BY NAVAL ARCHITECTS, NOT

A FEW OF WHOM HAVE ATTAINED TO EMINENCE;

This Work

IS RESPECTFULLY DEDICATED

BY

HIS OBLIGED FRIEND,

THE AUTHOR.

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INTRODUCTION.

The Author has been induced to prepare this work by many considerations. In the first place, no general work upon the Stability of Ships exists, so far as he is aware, in our language, although several authors have treated the subject satisfactorily as parts of more comprehensive works. In the next place, both the science and the practice of the subject have recently undergone great developments, at home and abroad, the results being scattered over many and various publications; and it was undoubtedly desirable to bring them together, and place them into due relation to each other. Again, during the present, as during the last century, French investigators have taken a leading part in the extension of this branch of science, and it is essential to the sound education of English naval architects and others, that the results of their labours should be brought within easy reach. Finally, the number of persons who now are required, by their professional avocations in connection with ships, to obtain some knowledge of the doctrines of stability is so great, that this work may fairly be taken as a response, and a somewhat tardy response, to a demand which has long been felt for collected information on the subject, and which, during the last year, has become widespread and urgent.

No degree of justice can be done to the feelings with which one surveys the wide field of this branch of naval science without expressing almost boundless admiration of the genius which has been displayed, and the labour which has been expended upon it, throughout the whole period of its cultivation, by men of science in France. The names of Bouguer and Dupin will probably excite greater and more enduring admiration throughout the world, in connection with this subject, than any other names whatever; and the author cannot but believe that the simple and beautiful manner in which M. Reech (as will be seen hereafter in chapter xiii.) unfolded the system of calculating the stability for various draughts of water and angles of inclination, by means of the co-ordinates of the successive centres of buoyancy, serves to prove that nothing of the mastery displayed by Bouguer and Dupin has been wanting in their successors. This volume will show that the names of Risbec, Leclert, Guyou, and that of the late MM. Dargnies and Ferranty, should not be without honour in this connection; while the very complete and practical system recently worked out by M. V. Daymard, of Marseilles,* has already commanded what it well deserved—the approbation and applause of the entire profession. Of M. Bertin's masterly work in connection with stability, this volume is not without illustrations, although his most important labours have been devoted to the still more difficult and complex study of the movements of ships at sea.

The prime characteristic of the French writers on the doctrines of stability has been their comprehensive grasp of the subject. While on this side of the Channel the tendency has usually been to limit our investigations to the bounds which embrace the actual or expected requirements of those who build ships or command them at sea, on the other side of the Channel, French investigators have usually passed far beyond these bounds, and have explored the whole theory of the stability of floating bodies, in all its geometrical breadth and completeness. Occasionally—as will hereafter be seen in our remarks upon Bouguer's views of the *Métacentrique*, and in some of our observations upon Dupin's work—they have gone a little too fast and too far; but their habit of dealing largely with the subject has secured to them great advantages in recent days, when the construction of ships-of-war of low freeboard, and of merchant steamships of very small initial stability, combined with great variations of stability on the voyage (occasioned by a large consumption of fuel, shifting cargoes, &c.), has made it absolutely necessary to bring all the probable stability conditions of a ship into full view and under calculation. It was only in 1867 or 1868, when proposals for placing low-sided monitors under canvas came officially before the author, at the Admiralty, that the necessity for extending stability calculations to large angles, came strongly under notice, and resulted in the construction of "Curves of Stability." It was later still when the practice of calculating stability at greatly different draughts of water, chiefly in the case of merchant steamers, came into vogue among us. But as long ago as 1863, M. G. Dargnies

* See chapter xvi., commencing on page 311.

was making calculations at Marseilles for numerous angles of inclination, and for four or five draughts of water; and in 1864, M. Reech put forward his admirable system, to which reference has already been made.

Both to Bouguer and to Dupin, and likewise to M. Reech, we have endeavoured to do justice in the course of this volume; but, having just shown that the habit of cultivating large views of the subject has given to France much advantage and honour, we would here commend that habit, at least to younger students of the subject, as combining intellectual pleasure with professional advantage in a high degree. The mind that can clearly follow the movements of the centre of buoyancy of a ship, when inclined continuously in one direction, as it traces out a curve of buoyancy, can almost as readily conceive of the same centre, when the ship is inclined through all possible angles and in all possible directions, tracing out a complete surface of buoyancy. When once this is clearly seen, and the part played by the gravity of the ship, acting through its centre of gravity, is likewise understood, there is opened up a beautiful domain of further study, which will delight the mind of any earnest student of geometrical science. In this domain, Dupin has developed much that is as important as it is attractive. Imagining lines drawn in all directions from the centre of gravity to the surface of buoyancy, "normally," or at right angles to the surface, he shows that when these lines are *minima*, they indicate positions of stable equilibrium, and when *maxima*, unstable equilibrium. This, and other associated doctrines, he demonstrates by the charming device of conceiving spheres, which either just embrace the surface of buoyancy, or just touch its interior, with intermediate spheres situated between them, fulfilling certain conditions. This conception has been improved upon by M. Guyou (see chapter xv.), who imagines a moist sphere of variable diameter expanding within the surface of buoyancy, wetting it at various places as it expands, and thus forming a series of isles and lakelets, which define the limits of stable and unstable equilibrium. It cannot be doubted that where the intelligence is trained to pursue investigations of this nature, in which pure imagination and pure science are brought equally into activity, the advantage to the student is very great indeed. At the same time it is highly satisfactory to know that French investigators, who have achieved so much distinction in the highest parts of the science of stability, have likewise advanced practical systems of calculation in a very remarkable manner.

To M. Risbec is due the credit of promptly and practically reducing M. Reech's system to tabular forms for calculation; the late M. Ferranty expressly directed his labours to a like object; and M. Daymard held back his theoretical results until he was able to accompany them with appropriate practical tables.*

The labours of English investigators, although directed to objects less attractive and brilliant than some of those developed by Dupin and other French writers, have not been deficient either in skill or in value. Atwood, with a simplicity and directness which are perhaps characteristic of most English scientific investigators, turned from Bouguer's somewhat hastily extended metacentric theory, and closely examined the measure of a ship's stability when inclined at a given finite angle. His method of investigation was perfectly sound; the fundamental formulæ which he obtained were no less correct; the abundant illustrations which he worked out were most instructive; and the practical methods of calculation which he employed—both such as were designed to be exact, and such as were avowedly approximate—were deserving of all confidence. Later English investigators have mainly followed in the same path, Mr. F. K. Barnes, of the Admiralty, having distinguished himself more than any other Englishman, by the novel application of graphic and other eminently practical processes to the production of simple and trustworthy methods of calculating stability.

It is a remarkable fact that, notwithstanding the extent to which French processes have anticipated ours in point of time, English systems of calculation suited to the requirements of the present day, and more especially to those of the mercantle marine, appear to have been quite independently developed in this country. Until the present year, no one here, to the best of the author's knowledge, had become acquainted with the real character of the investigations of M. Reech, with the exception of the author, and he only had come into possession of them through the private courtesy of M.V. Daymard. M. Daymard's own admirable *Mémoire* only became known to the author in consequence of, and some time after, the appearance of the report on the *Daphne* accident. But Mr. W. Denny and Mr. John Inglis, of the Clyde, had for some time past been at work upon those extended investigations of stability which the losses of cargo-carrying vessels had probably suggested, and which, as those

* For translations of the tabular forms of MM. Risbec and Daymard, see the end of this volume.

gentlemen were prompt to observe, were really essential to the safety of the mercantile marine. The subject has also for some time past been engaging the attention of Mr. Martell, Mr. White, Mr. John, and several other naval architects, while Professor Elgar, in the course of his practice, was giving close attention to the subject. Mr. White, in 1882, treated it well, and at considerable length in the third chapter of his excellent *Manual of Naval Architecture.* At the meeting of the Royal Society, held on the 6th March, 1884, Professor Elgar produced a valuable Paper on the "Variation of Stability with Draught of Water;"* and a month later, at the meeting of the Institution of Naval Architects, several highly important English Papers upon Stability, all of them exhibiting an equally enlarged method of treatment, were produced simultaneously with the production there of M. Daymard's, and of another by a young German naval architect, Mr. Benjamin.†

In France first, therefore, and later (but, as would appear, quite independently) in this country also, the subject of the Stability of Ships has assumed at length its full importance, and received the comprehensive treatment which it has come practically to require. In the old days, when ships were mainly propelled by means of sails, and when large stability was necessary to enable them to stand up against the force of the wind, it was usual to give them a large measure of righting force, and this enabled their constructors and those who sailed them to dispense with refined calculations. Even in those days, however, insufficient stability was a not uncommon fault, and such contrivances as excessive ballasting, and doubling the plank of the hull at and above the water-line, had not unfrequently to be resorted to. With the abandonment of sail-power in so many ships, and in the belief that smallness of breadth, within reasonable limits, was favourable to speed, it became the general fashion to greatly reduce the stability of new vessels, and it has been placed beyond doubt that many ships have been lost at sea in consequence, including some sailing ships, in which the reduction of the righting force was carried too far. This volume will at least serve to show, it is believed, that there is nothing in the circumstances of either mercantile or war ships to hinder a complete knowledge of their stability, under all probable conditions, being ascertained and formulated.

A highly satisfactory feature of the present condition of the stability question, is the very practical manner in which it is

* See chapter x. hereafter. † See chapter viii. hereafter.

considered by naval constructors, and the efforts they are exerting to make the results of their calculations intelligible and clear to naval officers, both of the Royal and Mercantile Marine. Professor Elgar, who has the highest and most mathematical branches of the subject well within his grasp, has shown a praiseworthy disposition to assist those who cannot be expected at present to comprehend readily either formulæ, or diagrams, or even the technical terms of the naval architects. He has also done good service by bringing into clear view the fact that when we speak of the stability of a ship, we do not usually refer to some intrinsic quality which she possesses of herself, apart from what she carries; but to the stability of the stowed ship, or of the ship and all she carries; and the measure of her stability, therefore, can only be ascertained by taking all the weights on board her into account, both as to their amounts and as to their positions. It is no doubt trite to say that the stability of the ship, thus viewed, varies with every change in the weights on board her, and with every change of position of every weight on board her; but, familiar as the fact may be, its effective force is much too often neglected, and many a ship and many scores of lives have been sacrificed in consequence.

"It often diverts attention from the main cause of loss," says Professor Elgar, "to say that it occurred because the ship was unstable. The fact is, that the ship has frequently so little to do with the matter, and the stowage so much, that it is the latter which should be blamed for the instability, and not the ship herself. When a ship is built for a particular trade, and for the purpose of carrying certain specific cargoes, she may then, of course, be so designed as to be quite stable, in all conditions, while thus employed; but when vessels are built, as they often are, to dimensions fixed by owners, for general trading purposes, it is seldom possible for the designer to provide against instability arising in some possible or conceivable circumstances of loading. The due preservation of stability in such cases requires to be watched and provided for by those who control the loading. It is erroneous to suppose, as appears to be sometimes done, that a cargo-carrying steamer should be so constructed and proportioned as to run no risk of becoming unstable, however she may be laden. If this idea were acted upon, such a mode of preventing instability, however easy and plausible it may at first sight appear to be, would only defeat the desired object of promoting safety at sea, because it would make many vessels dangerously stiff when laden with some classes of cargo. The true and reasonable mode of procedure is not to attempt to construct a ship so that she will be stable however she may be laden, but to see that any tendency she may have towards instability—

if any such exists—is understood by those in charge of her, and that she is always laden with careful reference to it. There are no steamers afloat, whatever tendency they may have towards instability as sometimes laden, that might not be kept perfectly safe if treated with full knowledge of what their stability is, and their stowage regulated accordingly. One great problem that the mercantile naval architect has just now to solve is, how any dangerous features of a ship's stability are to be made clearly known to those in charge of her, and in what manner they can be best taught to regulate the loading in cases where special care may be required."

And Professor Elgar's mode of attempting the solution of this problem is the following:—

"In advising upon how a steamer should be treated and loaded so as to be kept safe in respect of stability, I state, 1st, the quantity of ballast, if any, that is required to enable her to stand up when quite empty, without water in boilers or tanks, coal in bunkers, and with a clean-swept hold, and to be stiff enough for all working requirements in dock or river; 2nd, if she is to be employed in carrying homogeneous cargoes, what proportion of the space in the 'tween-decks it is safe to fill with such cargo, after the holds are full, and what weight of ballast is required in the bottom to enable the vessel to be loaded to her maximum draught with such cargo; 3rd, if required to carry two or more kinds of homogeneous cargo, such as grain and cotton, grain and wool, grain, meat, and wool, &c., the best mode of stowage, and whether or not the space in the 'tween-decks can be filled with the lightest of the cargoes, and in what circumstances ballast, and how much of it will be required; 4th, if not intended for homogeneous cargoes, but for general cargoes, or partly homogeneous and partly general, the average densities of the general goods for various ports is arrived at after a little experience, and the same system adopted. The main point is, to state what space, if any, must be left unfilled in the 'tween-deck cargo spaces, with the different descriptions of cargo, and what ballast, if any, is necessary if the vessel is to be loaded to her maximum draught; 5th, if the consumption of the coal diminishes the stability materially, as is often the case in some classes of steamers, to call prominent attention to this fact, in order that the captain may not be misled by finding his ship appear to be rather stiff on commencing a voyage. The possible consumption of coal is, of course, taken into account in fixing upon the limits that should be imposed upon the stowage in all the conditions named; and, 6th, if there appear to be any circumstances in which a tendency towards instability may arise they are described."*

* Professor Elgar's Paper on "The Use of Stability Calculations in Regulating the Loading of Steamers,"—*Transactions of Institution of Naval Architects* for 1884.

While adducing this as a laudable example of the manner in which the scientific investigator can assist the sailor, the stevedore, the agent, and others, we are not without hope that the time is approaching when the education of the officers of the mercantile marine, will be so far improved as to enable the greater part of them to understand perfectly all the conditions of the stability of their ships when these are reflected in suitable diagrams. Even the smaller classes of merchant steamers are valuable pieces of property, and carry each a few lives that ought not to be thrown away because the man in command is ignorant of some of the conditions of their safety; while, as regards the larger classes of ships, in which many scores, sometimes hundreds, of lives are embarked, it is intolerable that persons who are fully competent to master, in any and every practical form, the conditions of their safety should not be found to command them.

And this consideration leads to an expression of the author's regret that, simple as are the fundamental principles of stability, it is impossible to carry an exposition of them to any great length without the resort to mathematical expressions. Than the primary principles upon which all such expressions and all stability diagrams depend, nothing can be simpler. The whole weight of the ship and all on board tends downwards under the attraction of gravitation, virtually acting through its centre of gravity; the whole buoyancy of the ship acts upwards through its centre of buoyancy. If these two great aggregate forces act in the same vertical line there will be equilibrium; if they act in different vertical lines motion must ensue, and will continue until the two lines come together and coincide; the distance between these two lines, when they do not coincide, is the measure of the leverage with which the ship tends to upright herself or further incline; and, whether she will continue to incline or will return to the upright depends upon the direction in which the two forces tend to turn her, which direction is always pretty obvious: these are really and truly the only essential doctrines of the stability of ships. It is when you come to measure the separation of the two lines aforesaid, for any given position of the ship, that all the difficulty and complication enters, because then you have to take into account the varying form of the ship, which changes more or less from point to point, and is comprised within rounded or curved surfaces, the volumes of which it is difficult to measure.

In writing this work the author has endeavoured to make the earlier chapters intelligible even to those who do not understand mathematics, and in those earlier chapters will be found all that many persons who are concerned with ships require to know. But non-mathematical readers should not be readily deterred from pressing on with their study of the subject by the occasional intrusion of a sign of integration, or other mathematical symbol. The general sense and purpose may often be easily mastered even by those who cannot interpret the mathematical expressions.

I have considered it desirable to do all that was possible in the body of this work to clear up certain ambiguities that have arisen, touching the uses of such terms as "metacentre," "curves of metacentres," "metacentric curves," &c., and have attempted to employ them for single and definite purposes. I trust that my attempt will be supported and furthered by future writers on the subject. It is unnecessary to refer here to more than one ground of ambiguity, to which Professor Osborne Reynolds made serviceable reference at the British Association in 1883. After referring to a proposal previously made to define stability, in a quantitative sense, as measuring "the greatest angular disturbance from which a ship would recover," and to substitute the term "stiffness" to "measure the righting moment in any position," Professor Reynolds said—"My object was to call attention to the importance of such a system. In recent literature on naval architecture the term stability occurs over and over again in the sense of righting moment, and this under circumstances where the context shows the meaning to be incompatible with any meaning that can be given to the word, for stability must refer to some position in which the ship is stable; so that when it is said that a ship has initial stability, and has some stability at a heel of 90 degrees, it would seem that the ship would be stable (*i.e.*, tend to hold its position) in either of the positions; but as this is clearly not what is meant, then it would seem that some stability at 90 degrees means that a ship is stable about the erect position for angular disturbances of 90 degrees. This, however, it appears, is not the sense in which the words are to be understood, some stability meaning that the ship tends to return *towards*, not necessarily *to*, its erect position, and has some positive righting moment." Although there is nothing in this statement of the case which, in any way, corrects or conflicts with the well-understood science of the subject, it deserves careful attention, because it well points out an instance of the looseness with which the word "stability"—like the word "metacentre," as we shall see—has come sometimes to be employed.* In its most general sense, the stability of a floating body is nothing more than its tendency to remain in, or return to, a given position of equilibrium. But wherever this tendency exists, it so exists by virtue of the "righting force," which is called into play upon the disturbance of the body from that given position. Nothing can be more natural, and nothing more convenient, than to identify this righting force with the stability which it produces, and thus to designate the righting force at any angle of inclination (within the range of its operation) the "stability" at that angle. It is of no consequence whether the angle of inclination be small or great, provided the tendency throughout the inclination always is to restore the body to the given original position of equilibrium. Thus far all is clear, and no objection need, we think, be taken to the current use of the word stability to signify the righting force throughout this range. Supposing, however, the inclination of the body to be carried so far that the righting force disappears, and then continued farther still until a new position of stable equilibrium is reached, we shall now have a new righting force coming into play, of the same kind, and acting in the same direction as before; but it is manifest that we can no longer speak of this force as representing the "stability," except on the clear condition that we now refer the word, and the thing, to the second position of equilibrium, and *not* to the first position. Now Professor Osborne Reynolds is perfectly correct in pointing out that this most essential distinction has not been always observed, and that, in speaking of ships, mere *righting force* acting in a given direction, has been spoken of as *stability* without any plain and rigid reference to the position of equilibrium, to which, and to which alone, it has relation. It is easy to see how this has been brought about. The practice of investigating the stability of ships at large angles of inclination has sprung up in quite recent years; and in ships of ordinary type, and in ordinary conditions, the positive righting force, or stability, which has been found to exist, has always had reference to the upright position of the ship, and the word stability has been, therefore, freely used to express the force turning the ship *towards* that upright position. Recent events have, however, brought to light the fact—which had not previously been observed—that actual ships (no less than such prismatic bodies as Atwood and other writers have considered) sometimes, in some

* The following observations are taken mainly from an article contributed by the Author to the *Contemporary Review* for November, 1883.

exceptional conditions, are characterised by the fact that the righting force, tending to return the body "towards" its upright position, either did not exist, or else disappeared at comparatively small angles of inclination; and, after a phase of instability had been passed through, reappeared again while the angle of inclination was still within reasonable limits. In the case of a prismatic body 25 feet square in section, immersed 5 feet, and having its centre of gravity 1 foot above its centre of form, very small instability exists up to about 20 degrees of angle, a series of capsizing forces operating up to that inclination; then a position of stable equilibrium is reached, and there commences (as the body is further inclined) a righting force of small amount, acting, of course, in the opposite direction to those which have been capsizing the body, and, therefore, tending to return the body *towards* the upright position, but only so far towards it as to reach the second position of equilibrium—viz., that at which the body floats inclined at an angle of 20 degrees from the upright; thus it is very easy to see how these later righting forces happen to have been spoken of as so much stability, seeing that they oppose and overcome the further capsizing of the body; but it is equally easy to see that, as Professor Reynolds has pointed out, the stability so spoken of is not stability at all in the sense of restoring the body *to* its erect position, but is so only in the sense of restoring the body *towards* that position. That which is true of the prism spoken of may be, under suitable conditions, equally true of a ship; and, we have now to add, is true (not quantitatively, but characteristically) of very many large and fine ships of modern type; and, being true, gives rise to some grave accidents, and to many more anxieties and apprehensions. Such vessels, when in the condition described, refuse to float upright, but loll over, if allowed, to whatever angle it happens to be at which they find a position of stable equilibrium.

It becomes important to point out that no danger is *necessarily* involved in a ship, under some conditions, having to lie over to even a considerable angle in search of a position of rest. A ship may lie in harbour or in dock just as safely at 12 or 15 or 20 degrees from the upright as in an upright position; nay, if the inclined position be one beyond which the righting forces become great, she may even be safer than some other vessel which has stability in the upright position, but the stability of which may be small in amount or in range. We may even go further than this, and state with perfect confidence, that some ships which have little or no stability in the upright position, but which gather large stability as they incline, and go on increasing it up to very large inclinations, may be safer, very much safer, in storms at sea than some other ships which have considerable stability near the upright, but lose it as the inclination becomes great. On the other hand, it requires no special skill or judgment to see that when ships are, in any given condition of stowage, incapable of standing upright or nearly so, and are liable to loll about with small changes of weight, they are, in fact, exposed to classes of risk from which they would otherwise be free. We had a striking instance of this some time ago, in the case of the *Austral*. In the state of her cargo, stowage, &c., on the night of her sinking, she was exposed to a danger from which a ship endowed with large initial stability under like stowage would have been free. A moderate quantity of coal put on board through her starboard ports sufficed to bring one of her coaling ports under water. The sea poured in, and, further inclining her, brought another and somewhat higher port under the surface; and a comparatively short time sufficed to sink, in this manner, a splendid ship. It is perfectly true that there were many ways of preventing the catastrophe. Water-ballast might have been let in to increase the stability; the coaling ports might have been closed as they came near the water's surface; the coaling-lighter might have been shifted in good time to the opposite side of the ship. But none of these things was done, and the ship was sunk. The owners appear to have been very careful and painstaking in framing their orders, and to have understood their ship quite well. The probability is that had their orders been strictly obeyed the accident would not have happened. But the fact remains that the ship was sunk, and that she was so sunk from those in charge of her either not understanding how to handle her, or not taking all the means necessary for handling her properly. The case is mentioned here only because it seems to illustrate in a remarkable manner the fact that the care in handling which modern ships receive is not equal to their requirements; and that one of two things, probably both, ought to happen—either ships should be built so as to possess greater stability, when discharging and loading, than some of the finest of them now possess, or else the competence of those who have charge of them should be better seen to.

ERRATA.

Page 19. Last line, *for* $G\,Z_2^{\frac{1}{2}}$, *read* $G\,Z_{2\frac{1}{2}}$.
„ 31. Line 4 from top, *for* Fig. 21, *read* Fig. 24.
„ 34. Line 8 from bottom, *for* W L, *read* $w\,l$.
„ 37. In Fig. 28, H′ should be within the triangle L′ O L.
„ 42. Line 3 from bottom, *for* immediately, *read* at once.
„ 72. In the final equation, *for* $l - l'$, *read* $\frac{l - l'}{l}$.
„ 96. Line 11 from top, *for* valves, *read* values.
„ 134. Line 9 from bottom, after tan. γ insert the sign of equality (=).
„ 186. Line 4 from bottom, *for* length *read* lengths.
„ 197. Line 3 from top, *for* Fig. 4, *read* Fig. 133.
„ 219. In equation at bottom of page, *for* $1 \times \beta$, *read* $1 + \beta$.
„ 256. In the equation, *for* $s =$, *read* $\rho =$.
„ 261. Line 18 from top, *for* $B\,d\,\eta$, *read* $B\,d\,z$.
„ 263 and 264. In four places, *for* $f(\theta_1\,V)$, *read* $f(\theta, V)$.
„ 263. Line 3 from bottom, *for* formula, *read* formulæ.
„ 268. Line 5 from bottom, *for* larger, *read* layer.
„ 305. Line 17 from bottom, *for* R, *read* R_0.
„ 306. Line 7 from bottom, *for* m', *read* m.
„ 307. Top line, *for* no, *read* two.
„ 313. First equation, *for* G R, *read* C K.
„ 322. First equation, *for* $V\,d_2$, *read* $V\,d_1$.
„ „ Line 17 from bottom, *for* $k\,\theta$, *read* K_0.
„ „ Line 5 from bottom, *for* V (outside bracket), *read* V_1.
„ „ Equation at bottom, *for* $V - \phi$, *read* $V + \phi$.
„ 345. Last line, *for* 159, *read* 15·9.
„ 349. Line 2 above figure, *for* 36°, *read* 6°.

A TREATISE

ON

THE STABILITY OF SHIPS.

CHAPTER I.

Primary Conditions of Flotation and Stability—Stability of Submarine Vessels—Stable and Unstable Equilibrium—Stability of Partially-Immersed Bodies—Indifferent or Neutral Equilibrium—The Metacentre—Stability of a Homogeneous Cylinder with its Axis Horizontal—Curve of Flotation—Surface of Flotation—Curve of Buoyancy—Surface of Buoyancy—Stability of a Cylinder which is not Homogeneous—Curves of Stability—Double Branch Curves of Stability—Stability of Homogeneous Prisms of Square Section.

ANY body placed freely in still water, or in any other fluid at rest, will sink if its weight exceed the weight of the fluid which it displaces. If its weight be less than that of the fluid which it would displace if wholly immersed, it will float with only a portion of it immersed. If its weight be exactly equal to the weight of the fluid which it displaces when wholly immersed, it will become wholly immersed.

The first case—that of a body which sinks—needs no special consideration here, although it must be acknowledged that it is not wholly devoid of occasional practical interest.*

In the last case—that of a body whose weight and displacement are equal when it is wholly immersed—which has sometimes to be dealt with in connection with submarine vessels, the position which the body will assume, and in which it will remain, will be determined by the relative positions of the body's centre of displacement and its centre of gravity.

If the body in this case be homogeneous throughout, its centre of gravity must of necessity be coincident with its centre of displacement; its weight will act downwards, and its buoyancy (which is equal to, and the effect of the displacement) will act upwards through the same point. These being the only forces acting upon

* There were incidents connected with the foundering of the *Captain*, for example, which required and received some investigation.

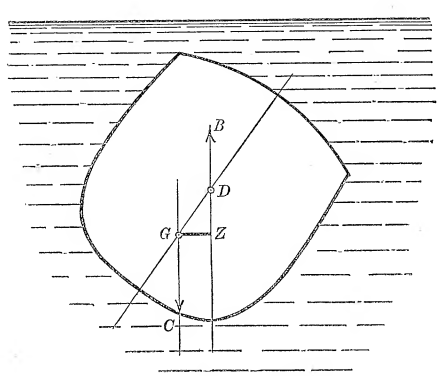
the body, there is obviously no force interfering with its equilibrium, and causing it to revolve about an axis, and it will therefore remain in any position in which it may happen to be placed.

If the body be not homogeneous throughout, it may nevertheless happen to have the weights of its parts so disposed as to bring its centre of gravity into coincidence with its centre of displacement, when the condition of undisturbed equilibrium in any position just described will still hold.

If the body be not homogeneous, and if its centre of gravity be not coincident with its centre of displacement, it will not remain freely at rest, but will turn more or less round, unless and until its centre of gravity comes vertically under its centre of displacement. For in every other position it is obvious that it will be subject to two vertical forces acting in different vertical lines, and these will constitute what is known in mechanics as a "couple," and will turn the body round.

For example, if the accompanying diagram, Fig. 1, represent a submarine vessel equal in total weight to the water which it displaces, D being the centre of its displacement, and G its centre of gravity, and if G Z be drawn at right angles to the vertical lines, D B and G C, along which the buoyancy acts upwards and the gravity downwards respectively, it is manifest that these two forces must tend to turn the body with a leverage proportional to G Z; D will rise or G will descend, or more strictly, both these movements will occur; and as they take place, G Z will shorten until D comes vertically over G, when it (G Z) will disappear.

Fig.1.



The forces of gravity and buoyancy will then neutralise each other and the vessel will remain at rest.

This condition of equilibrium and rest would equally exist if D should be placed vertically beneath G; but on the slightest disturbance of the vessel from its position, the upward tendency of the buoyancy acting through D, and the downward tendency of the gravity acting through G, would be free to take effect, and the vessel would turn round until G had placed itself vertically beneath D. Should a slight temporary disturbance produced by some external cause now occur, the vessel would return as soon as free to this position, with G beneath D. In this position the equilibrium would, therefore, be what is called *stable;* whereas the equilibrium, which, as we have just seen, might exist when G was situated vertically over D, would be *unstable,* the vessel on the slightest disturbance abandoning that latter position, and moving round till the position of stable equilibrium was reached.

It is necessary to include positions of unstable equilibrium within the view of our investigations, because in them the mathematical conditions of equilibrium are fulfilled, and because it is impossible to investigate the stability of many modern ships without giving careful consideration to the amount and range of their instability.

The case of a body less in weight than the fluid which it would displace if wholly immersed, and which, therefore, floats partially immersed, is that which chiefly claims our attention, and which we proceed to consider.

If the body be homogeneous it is obvious that its centre of gravity will be situated higher than its centre of displacement, because the displacement is wholly below the water's surface, while part of the weight is above that surface. But it does not by any means follow in this case (as it did in that of the wholly immersed body), that the equilibrium resulting from the centre of gravity being vertically over the centre of buoyancy will necessarily be unstable; on the contrary, the centre of gravity will usually be higher than the centre of buoyancy in ships of ordinary form and character, and we, therefore, see thus early that vessels wholly immersed, and those but partially immersed, are under very different conditions as regards stability.

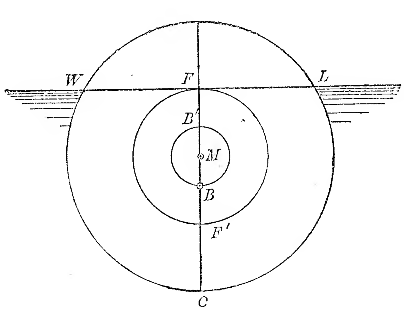
Instead of entering at this stage of our subject upon any general investigation of the stability of a floating body, it may be well to lead up to such an investigation by considering the conditions which hold in the cases of certain bodies of forms simpler than those of ordinary ships; such, for example, as cylinders and prismatic bodies of various sections, observing that for all prismatic bodies their *lengths* may be left out of consideration, as the stability of every unit of length will necessarily be the same, and it is only necessary to attribute to them sufficient length for the purpose when it is desired that they shall be supposed to float with their longitudinal axes horizontal.

And first, let us take what is perhaps the simplest case of all, viz., that of a homogeneous cylinder circular in section. A moment's consideration will show that, whatever be the degree of immersion of this body, every position will be one of equilibrium, because the centre of gravity will always be in the centre of form, and the centre of buoyancy will necessarily be always beneath it, as the immersed section is symmetrical about a vertical line through the centre of form. If moved round from a position of equilibrium through any angle, large or small, into a new position, the centre of buoyancy will still be vertically beneath the centre of gravity, and there will consequently be no force operating either to diminish or to increase the rotation. This state of things introduces us to a third kind of equilibrium, which is known as *indifferent* or *neutral* equilibrium.

Before leaving this simple case of the cylinder, let us take note of the fact that in every position the upward force of buoyancy is directed through the centre of the body, so that successive intersections of such lines of upward force are concentrated in this one point—which we shall afterwards see to be the "metacentre," and the only metacentre of this particular body, the word "metacentre" being here employed in its original and legitimate sense, viz., as signifying the point above which the centre of gravity cannot be raised compatibly with *stable equilibrium*. This point is also in the present instance the centre of gravity, as it is the centre of the homogeneous body.

The water-line at which the body floats may be at any depth less than the diameter, and if the body be caused to make a complete rotation through 360°, its successive water-lines situated indefinitely near to each other, will successively touch a circle described about the centre of the body, with the distance between that centre and the middle point of the water-line as a radius. Let W C L in the accompanying diagram, Fig. 2, represent in section such a floating body, immersed we will suppose, to the water-line, W F L. Let M be the centre of form, and it will also be, as we have said, the metacentre and the centre of gravity likewise,

Fig. 2.



always remaining at a fixed distance below the surface. Describe the circle F F′ about M with the radius M F, and this circle will obviously come to the surface, point by point, as the body rotates. Coming thus to the surface, and always therefore having a point coincident with the water-line, or line of flotation, it is called the "curve of flotation" for this body; and if the length of the body be taken into consideration, the point, of course, corresponds to a straight line, and the curve of flotation to a cylindrical surface (of which the curve is a section), this surface being designated the "surface of flotation." It is manifest that all the water-lines are tangents to the curve of flotation, and all the water-planes are tangents to the surface of flotation. Further, if B be the centre of buoyancy in Fig. 2, let us describe the circle B B′ about M with the radius M B. A moment's reflection will show that as the body is rotated, as before, each point of this circle B B′ must become in succession the centre of buoyancy, because that centre must always in this case lie at one uniform distance below the water's surface; and as M must do the same, the distance of the centre of buoyancy below M must be constant. In other words, the points in B B′, the locus of the centres of buoyancy, must lie in a circle around M as stated. The circle B B′ is therefore called the curve of the centres of buoyancy, or more briefly the "curve of buoyancy;" and, length being again considered, the cylindrical surface of which this is the section is known as the "surface of buoyancy."

Owing to the simplicity of the figures which we are here considering, these "curves" of flotation and buoyancy happen to be circles; but, as we shall see hereafter, these are very special cases only, and generally we shall find them of more complex curvatures.

Fig. 3.

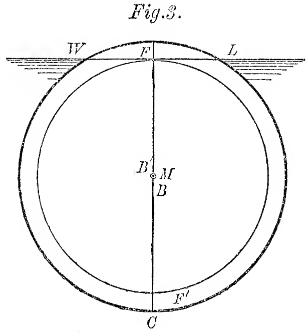
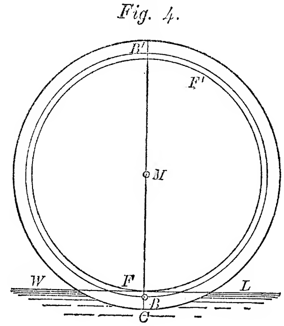


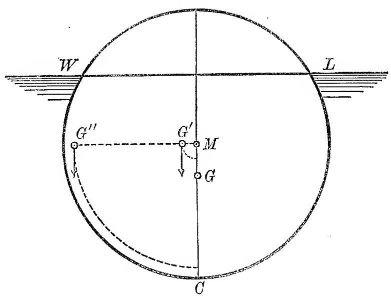
Fig. 4.



It is scarcely necessary to add that in the instance we have been considering, the only effect that would result from increasing or diminishing the draught of water of the cylindrical or spherical body (all the time it remains homogeneous) would be to vary the sizes of the circles B B′ and F F′. As the body becomes more immersed, the circle F F′ is enlarged, and the circle B B′ is reduced in diameter, until, when the body is almost wholly immersed, F F′ lies close to the circumference W C L, and B B′ is almost reduced to the point M, as in Fig. 3. When the body is but very slightly immersed, the circles F F′ and B B′ become almost coincident with W C L—see Fig. 4; and when the body is just half immersed (being one-half the specific gravity of water) the curve or circle of flotation becomes a point coincident with M, the circle B B′ having a radius of $\frac{4d}{3\pi}$* where d is the diameter of the section, $\frac{4d}{3\pi}$ being the distance down of the centre (or centre of gravity) of the immersed semi-circle from the water-line.

Let us next consider the case of a cylinder of circular section which is not homogeneous, and which has its centre of gravity situated in some point other than the centre of form, as in Fig. 5, in which G represents the centre of gravity, and W L the water-line. In this case all those elements which depend solely upon form will be the same as in the last case for corresponding immersions, and therefore M, the centre of form, will be the metacentre as before, and the curves of flotation and buoyancy will also remain as before for any given immersion. But the condition of the body as regards equilibrium and stability will be very different.

Fig. 5.



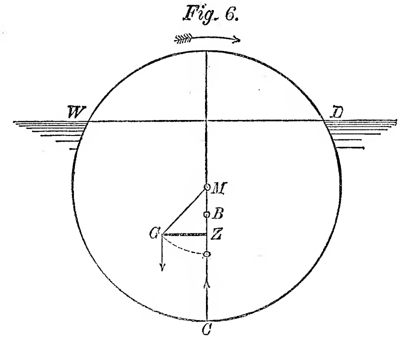
If the body be first floated in the position shown in Fig. 5, with the centre of gravity, G, situated vertically below the metacentre, M, and if an inclining or rotating force be now applied, it obviously cannot take effect and turn the body round, without raising the centre of gravity, G, nearer to the water's surface; if this be done, and if the angular motion be less than 180°, and the body be now left free, the centre of gravity, being left unsupported, will fall back again into the position shown. This, therefore, is a position of

* The Greek letter *pi* (π) is used to denote the ratio of the circumference of a circle to its diameter, which is expressed by the quantity 3·14159..., that being the circumference of a circle whose diameter is 1.

stable equilibrium. If the body be turned round through 180°, and the centre of gravity be thus brought vertically over the metacentre, M, the forces of the buoyancy and gravity will again be in equilibrium, but the equilibrium will now be unstable, and on the slightest disturbance (which could not, in practice, be avoided) G would again fall into the position shown in Fig. 5. There obviously is no other position of equilibrium besides the two just mentioned, and but one of them is a position of stable equilibrium.

This is true wherever the point, G, may be situated between M and the circumference of the body; but the *leverage* with which the body will be urged back to the position of stable equilibrium, after having been moved out of it, will be materially influenced by the position of the centre of gravity. If G be very near to M, that leverage will be proportionately small; but if G be very near the circumference, it will be proportionately great. Let G′ and G″, Fig. 5, illustrate these positions of the centres of gravity respectively when the body has been turned round through 90°. In the former case, the weight of the body, tending to turn it back round M, will act with the small leverage, G′ M, and in the latter case with the large leverage, G″ M; and the righting moment will be proportionate in each case to the leverage. This moment, thus depending only on the weight of the body and the distance, M G, will be the same, whatever the immersion (although immersion and weight are themselves of course practically related), but will vary with every change of angle. It is easy to see, from Fig. 6, what the righting moment must be for any given position of G, and at any given angle of inclination. The buoyancy acts upwards through B and M, the gravity acts downwards through G, and these forces are equal; they therefore constitute a "couple," the arm of which is G Z, G Z being drawn from G at right angles to B C. It will be seen that $GZ = GM \sin. \theta$, if θ be the angle of inclination from the upright position. Starting from nothing, when G is beneath M, the righting moment therefore gradually increases in this instance up to 90°, when it reaches a maximum, and is there equal to the weight of the body multiplied by the distance of the centre of

Fig. 6.

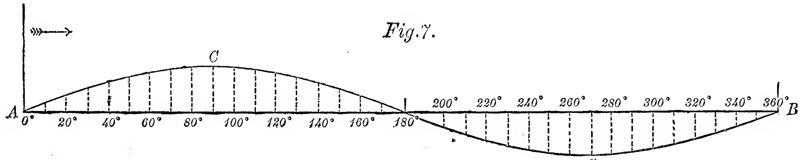


gravity from the centre of the body; it then diminishes with further inclinations, and vanishes at 180°, when G comes vertically over M. From 180° to 360° it will clearly pass through phases the reverse of those previously passed through, for it matters not whether the motion be continued from 180° or reversed. By giving any required values to the weight, to G M, and to the angle of inclination, the righting moment under all possible circumstances can be determined.

Having now before us a case, however simple, of varying stability, and exhibiting variations of righting force under different conditions, it will be well to observe that these variations may be conveniently illustrated graphically in the form of a "Curve of Stability." The instance before us is so elementary, and so easily understood without graphic aid, that it would not be worth while, for the immediate purpose only, to further discuss it; but the principle involved in the construction of curves of stability (or, more correctly, "curves of righting forces") is always the same, and general principles are often most easily and effectively illustrated by elementary examples.

We will, therefore, take as a first example the case shown in Fig. 6, and assume the centre of gravity, G, to be one-half the radius from M, so that M G is equal to one-fourth the diameter of the floating cylinder. Now we can take either of two measures of righting force, as we please, for we can either take the length, G Z, at each angle, which we may call the righting "lever;" or we can take the length of that lever multiplied by the weight of the floating body, which we can call the righting "moment;" and it is obvious that the series of results obtained will differ only in being the one a fixed multiple of the other. Let us leave the weight out of consideration, and deal with the righting levers only. These will be represented at every angle of inclination by the length of G Z at that angle, and we may presume these to be ascertained at a series of positions as numerous as we please; for example, at intervals of 10°. This would give us 36 different lengths of G Z for the full rotation of 360°, or 18 positions for each 180° of inclination. Our object is to exhibit graphically the variations of G Z, and that may best be done by setting them all off from one base-line, A B, Fig. 7, which we

Fig. 7.

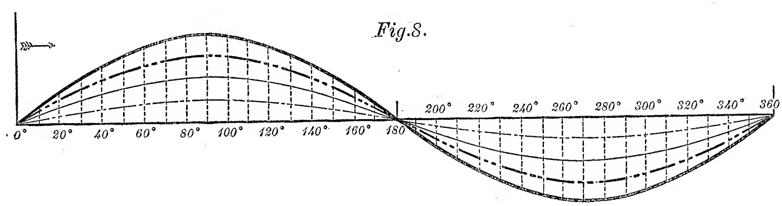


will take to represent by its length the 360° of angle. Dividing this line into 36 parts, we may then set up as an ordinate at each of the points so obtained the length of G Z, corresponding to the inclination, and through the extremities of the ordinates so obtained draw a curved line, C C, Fig. 7. With this curve before us we have the means of ascertaining by simple measurements the length of the righting lever at any angle of inclination whatever. An inspection of the curve shows that regarding the upright position, with G beneath M, as the starting position, indicated by A, as the body is by external force inclined as the arrow indicates, a righting lever comes into play, and increases until an angle of 90° is reached, there attaining a maximum; beyond 90° the righting lever still exists, but it now begins to diminish in amount until an angle of 180° is reached, where the righting ordinate vanishes, the curve there crossing the base-line. At this point G is above M; there is therefore *equilibrium*, but it is *unstable*. Beyond 180° the curve falls below the base-line, and remains there till 360° of inclination is reached, or in other words, till the original upright position is resumed, the ordinates varying in length exactly as they varied above the base-line during the first 180° of inclination. These ordinates being now (from 180° to 360°) always below the base-line, or *negative* in amount, signify that the effect of the inclining lever is now, not to turn the body *back* to its original position by reversing its motion, but to further rotate the body in the direction in which it has been forcibly moved, or in other words to completely capsize it. This curve represents, therefore, the stability (or instability as the case may be) of the vessel, and has come to be called—somewhat loosely, but too generally now to be altered—the "curve of stability" of the body in question.

This curve will always be of the same type as in Fig. 7 (for an unbalanced circular cylinder floating with its axis horizontal), whatever be the position of G in the body. The curve will always reach its maximum at 90°, always cross the base-line at 180°, always reach its *negative maximum*, so to speak, at 270°, and always come to the base-line again at 360°. But the ordinates of the curve will obviously decrease in length as the centre of gravity of the body is placed nearer and nearer the centre of form, and increase in length as it recedes from that centre, since for any given angle whatever G Z of necessity varies directly with G M. The largest righting lever that such a body as this can have is obtained when the centre of gravity is in the circumference (as we may for a moment imagine it to be), and the body is inclined at 90° from the upright. The righting *lever* is then equal to the radius, and this is true whatever be the degree of immersion of the body. The righting *moment* will also be the greatest with the centre of gravity in the circumference, but in this case the weight of the body must also be at a maximum, which it will be when it just, and only just, floats.

We thus see that this simple form of body offers to us the means of constructing a whole series of curves of stability, all following a given law, but differing with every change of position of the centre of gravity. The law by which the ordinates will vary in the respective curves is simple enough, for they will be in each curve directly proportional to G M. As G M gets small all the ordinates of the curve will diminish, and the curve itself will be ultimately merged in the base-line. In Fig. 8 are shown

Fig. 8.

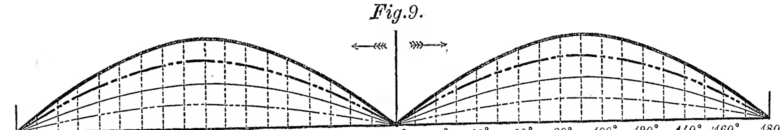


four different curves of stability—which are here curves of uprighting and inclining *levers* — corresponding to the following four positions of the centre of gravity, viz., when G M is equal to one-fourth, one-half, and three-fourths of the radius of the body respectively, and also when it equals the radius itself. The ordinates are of course in these proportions to each other.

The reader who is considering this subject for the first time must be careful to bear in mind that in framing the above curves of righting levers, it has been assumed that the body has been inclined always in one direction, viz., that indicated by the arrow in Fig. 6. It will be clearly seen that had the body been rotated in the opposite direction precisely similar curves would have been obtained. But those who are unaccustomed to such curves often experience a difficulty in understanding them from the fact that, as usually constructed they do not exhibit to the eye that which it seems natural to expect, viz., curves that are symmetrical on either side of the upright position and on either side of other positions of equilibrium. If we take Fig. 7, for example, we know that in the upright positions, at A and B, the stability of the body is the same whether we incline it to the right or to the left, but the curve of stability (if continued far enough), rises on the right of each of these points with positive ordinates, and descends on the left of them with negative ordinates. The explanation is simple enough, viz., that the curve is constructed on the assumption that the rotation takes place in one direction only, and that motion in the opposite direction is provided for by simply changing the sign of the ordinates—making negatives positive and positives negative.

But it will greatly facilitate the ready comprehension of these curves by amateurs and students if we give to them two equal branches—the branch to the right representing the righting levers (or "moments," as the case may be) when the top of the floating body is inclined to the right, and the branch to the left representing the levers when the top of the body is inclined to the left. These two branches will usually be precisely alike for ships, and one of them will, therefore, be superfluous for purposes of calculation; but the second branch will be very far from superfluous to many persons among whom it is desirable to diffuse a sound knowledge of the stability of ships. Applying this method to the curves in Fig. 8, and contenting ourselves with seeing the body capsize (or lose all stability) to right and to left (which happens at 180° in this case), the curves will assume the graphic form of Fig. 9. In referring

Fig. 9.



to curves so constructed, it will always be understood that ordinates above the line will be *righting* ordinates tending to return the body to the position from which it was started, until the curve *first* crosses the base-line, either to the right or to the left. The ordinates, then falling below the base-line, are capsizing ordinates, tending to continue the inclination previously given to the body.

Owing to the special condition of the cylindrical body thus far under consideration, in which positive ordinates do not reappear until the body has made a complete rotation, it is undesirable to pursue this branch of the subject further at present; but hereafter we shall have cases to consider in which positive ordinates reappear at much smaller angles, and then we shall take occasion to show that although their reappearance signifies the reversal of the capsizing forces, it does not signify any tendency on the part of the floating body to return to the upright position.

We have seen that the cylindrical body has but one position of stable equilibrium wherever its centre of gravity may be situated. It is easy to show that this is the case by other than graphic means. It is obvious, for example—in fact we have already seen—that whenever the centre of gravity is not situated in the centre of the body, the moment of the weight about M, at any given angle θ, is $W \times MG \times \sin. \theta$ (see Fig. 6). All the time this moment has a finite value there is a turning force in operation, and the body cannot be in a state of equilibrium. When equilibrium exists this moment must vanish. Now it can only vanish, under the condition supposed, by $\sin. \theta$ vanishing, and this can only be when $\theta = 0$, or $\theta = 180°$. When $\theta = 180°$, the centre of gravity is above the metacentre, and the equilibrium is therefore unstable. The upright position when $\theta = 0$ alone remains, and this, therefore, is the only position of stable equilibrium.

In the cylindrical bodies which we have thus far been considering, the resultant upward pressure of the buoyancy always acts through the centre of the cylindrical section for reasons which are obvious, or which the most elementary knowledge of hydrostatics makes clear. If we now pass to bodies of almost equal simplicity of form—viz., prismatic bodies with parallel sides, and suppose them to be of square section, we shall find that the change from the circular section to the square, simple as it may appear, involves very different conditions of stability.

Fig.10.

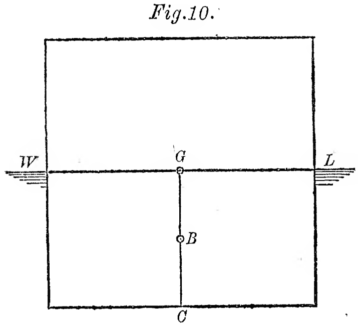
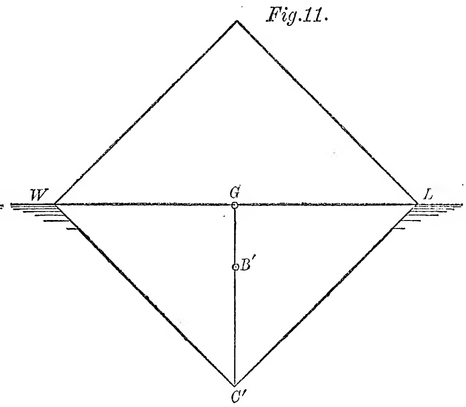
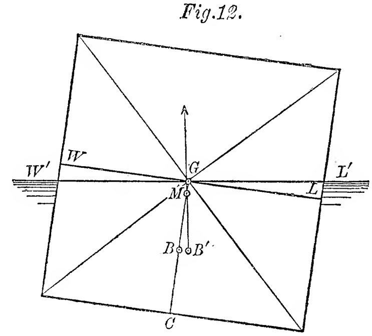


Fig.11.



First, let us consider the case of homogeneous prisms of square section, and let us commence with a body one-half the weight of water, and, therefore, floating with one-half its volume immersed. We will place this body first in the position shown in Fig. 10, with its sides vertical and horizontal. This will be a position of equilibrium, because both its centre of gravity, G, and its centre of buoyancy, B, will lie in the vertical line, G C, and there is consequently no force tending to rotate the body. We will place the body next in the position shown in Fig. 11, with its diagonals horizontal and vertical respectively. This will also be a position of equilibrium, because both the centre of gravity, G, and its centre of buoyancy, B′, will lie in the vertical line, G C′. We will finally place the body, as in Fig. 12, in a position intermediate between Figs. 10 and 11, and consider how it is situated. As compared with its situation in Fig. 10, the following changes will have happened:—Instead of the water-line, W L, it will now (Fig. 12) have a new water-line, W′ L′; and instead of the centre of buoyancy, B, it will have some new centre of buoyancy, B′. B′ will obviously lie to the right of B, because the body has been more immersed on that side by the inclination, the triangle, L′ G L, having been newly immersed on the right side, and the triangle, W G W′, having been taken out of the water on the left side. The upward pressure of buoyancy will act through B′, and, when the angle of inclination is exceedingly small, its line of action will, in this case, intersect G C at a point, M, below G, as may be ascertained either by geometrical construction or by calculation. This point, M, is the metacentre, being, as we see, the intersection of two verticals through two centres of buoyancy lying extremely near to each other, and corresponding, of course, to two positions of the body which differ only by a very slight angle of inclination, one of the two being a position of equilibrium.

Fig. 12.



CHAPTER II.

Fuller Consideration of the Metacentre—Bouguer's Original Use of the Word—Its Precise Signification—Bouguer's "Metacentric" (*Métacentrique*)—His Error regarding it—True Character and Properties of the Metacentric—Pro-Metacentres—The Metacentric a Locus of Metacentres and Pro-Metacentres—The Metacentric an Evolute of which the Curve of Buoyancy is the Involute—The so-called "Shifting Metacentre"—Modern French Usage: "Initial Metacentre" and Metacentre—Rules of M. Reech stated by M. Bertin—Ascending and Descending Metacentrics—Examples of Simultaneous Descent of Metacentric and Increase of Stability—Distinction between "Curve of Metacentres" and "Metacentric Curve."

We have now arrived at a point in our inquiries when it has become necessary to consider somewhat fully the term "metacentre."

The French investigator, Bouguer, nearly a century and a-half ago,* introduced the word metacentre into the nomenclature of naval science. He employed it with specific reference to a ship floating freely in an upright position, and for the specific purpose of indicating that point in the vertical axis of the ship beyond which her centre of gravity could not be raised without inclining the ship.

All the time the centre of gravity (which we presume to be in the vertical axis) is situated below the metacentre, the ship will tend to remain upright, and to return to the upright if slightly disturbed, because as soon as she is inclined a little either to the right side or to the left, the buoyancy moves out towards that side, and the upward vertical thrust of the buoyancy, acting through the new centre of buoyancy, tends to push her back to the upright position.

If the centre of gravity be raised to exactly the same height as the metacentre, and the ship be now slightly inclined either way, the upward thrust of the buoyancy and the downward drag of gravity will both pass through the same point, and no further motion need therefore ensue; the ship will, consequently, be in neutral equilibrium, and if put back to the upright, will remain there. But if the centre of gravity be raised above the metacentre, then, on a

* In his famous *Traité du Navire*, published in 1746.

slight disturbance of the ship, the upward vertical thrust of the buoyancy will not resist the downward drag of the gravity, but will co-operate with it, and further incline the ship. We have here, then, a clear and precise signification of the word "metacentre,"* and we know that it is its original and true meaning.

But Bouguer went a step farther: the metacentre being, as we have seen, the point at which a vertical through a centre of buoyancy, closely adjacent to the original centre of buoyancy, cuts the originally upright axis, Bouguer saw that if the ship were a little further inclined, two verticals through closely-adjacent centres of buoyancy might not, and often would not, intersect at the same point of the originally-upright axis, or in that axis at all, but would intersect at some point lying a little aside from that axis, and, with ships of usual form and condition, would be situated a little higher than the metacentre. If the ship were inclined a little further still, the point of intersection of similar verticals through adjacent centres of buoyancy would usually be a little higher up still, and a little further aside from the axis, and so forth. This is illustrated in Fig. 13 (next page), in which a curved line is drawn through the metacentre, M, and through successive intersections of such verticals, at M_1, M_2, and M_3. This curved line Bouguer called the "metacentric" (*méta-centrique*), and it is manifestly the *locus* of the intersections of successive verticals through adjacent centres of buoyancy, corresponding to successive small inclinations of the ship from the upright.

Bouguer made the mistake of supposing that the rise of the metacentric above the metacentre, or its fall below it, indicated increase or decrease respectively of the righting force or stability; and many writers since have made the same mistake, some of them still repeating it. But it really does nothing of the kind, as Atwood showed at the end of the last century, and as we shall abundantly show hereafter.

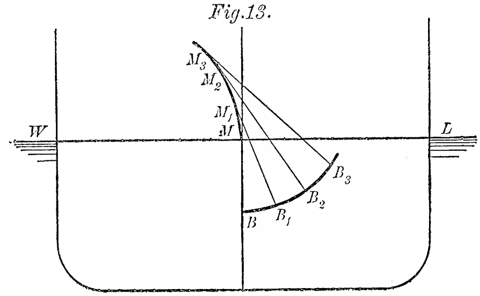
It will easily be seen, if we keep the *ship* still in mind (and with a ship in an upright position Bouguer started), that the points, M_1, M_2, M_3, &c., are not metacentres (although they have lately come to be very frequently spoken of as such) in Bouguer's own

* "The point so found he calls the metacentre, as it appears to some instructed persons, from its being the *meta*, limit or goal beyond which the ship's centre of gravity may not rise. It is to be regretted that Bouguer only hints at the derivation of the word. Another derivation, from the Greek *méta*, signifying change, and *kentron*, centre, does not materially differ from this, provided we understand the centre to be the ship's centre of gravity."—*Naval Science*, vol. iii., p. 439, 1874.

sense of the term, not being in the original upright axis at all, and therefore being incapable, in a ship so loaded as to float upright, of fixing a limit in that axis above which the centre of gravity cannot be raised. Of course if we dismiss the idea of a ship, and have regard only to that of any floating body whatever, and suppose that its centre of gravity can be shifted about just as we please (as *Dupin* afterwards did for his general investigation), we are then at liberty to assume any axis of buoyancy, or line of action of buoyancy, as indicating for the moment the upright axis of the floating body, and in that case any such point as M_1, M_2, &c., may be regarded as the metacentre for the time being. But however useful such artifices as this may be for the purpose of determining general principles, we must be most careful not to regard such points as M_1, M_2, &c., as the metacentres of any ordinary ship, yacht, barge, floating dock, or any other like body with which we may have practically to do,—unless, indeed, it should be made perfectly clear that they have reference solely to some specified inclined position of the ship.

It may be well, however, here to point out that each of these points, of which the metacentric is the locus, has a property of considerable importance, for it is the *centre of curvature* of the curve of buoyancy at the corresponding point of it. We saw in the last chapter that the curve of buoyancy is the locus of the centres of buoyancy, and we have just seen that the metacentric is the locus of the intersections of adjacent verticals through neighbouring centres of buoyancy. Each point in the metacentric will therefore be related to a corresponding point in the curve of buoyancy, and if we join these two corresponding points by a straight line, this line will be at right angles to the curve of buoyancy (*i.e.*, to its tangent) at that point, and will be also its radius of curvature at that point. In Fig. 13, for illustration, MB is the radius of curvature of the curve of buoyancy, B, B_1, B_2, &c., at B; $M_1 B_1$ is the radius of curvature at B_1; $M_2 B_2$ its radius at B_2, and so forth. And this being the case, the curve, M, M_1, M_2, &c.—it may be mentioned in passing—is the *evolute* of the curve, B, B_1, B_2, &c., which is therefore its *in-*

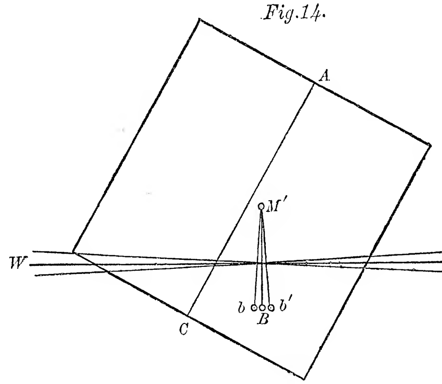
Fig. 13.



volute. If we suppose a string to be led round a rigid curve, M_3, M_2, M_1, and M, and carried down to B, and this string then to be kept stretched and carried outwards from B, the point of the string which touched B will now trace out the curve, B B_1 B_2, &c.

It would be well if we could avail ourselves of this property in giving a new name to such points as M_1, M_2, &c., which we might define, and which have been defined as "centres of curvature of the curve of buoyancy," but this designation is much too cumbrous for ordinary and frequent use. We certainly need a name for them; they are not "metacentres," save in a very strained, misleading, and wholly exceptional sense; and yet we shall have frequently to speak of them. As we cannot find for them, as we should like to do, a designation that is both specific and characteristic, we shall content ourselves with calling them "pro-metacentres," and hereafter, when we speak of a "pro-metacentre," we shall signify by that word a point on the *meta-centric;* or, what is the same thing, a point on the *evolute* of the curve of buoyancy; or, what is still the same thing, a centre of curvature of the curve of buoyancy. An example of an isolated "pro-metacentre" is given in Fig. 14. A floating prism of square section is there shown with the axis, A C, inclined at an angle of about 30° from the upright; B is its centre of buoyancy, when W L is its water-line. By giving the body a *very slight* inclination either way from this position, it will have a new centre of buoyancy given to it. If we incline it one way *b* will indicate this; if we incline it the other way *b′* will indicate it; and for each of these positions there will be a new line of upward action of the buoyancy. These lines of action, together with that through B, will all meet or intersect in one point, M′, and this point will be the pro-metacentre.

Fig.14.



The word "metacentre," qualified by the adjective "shifting," has sometimes been used in this country to signify the intersection of the vertical line of upward action of the buoyancy, when the ship is inclined at any considerable angle with the axis of the ship which was vertical when she floated upright. It can hardly be said, indeed it cannot be said, that the so-called "shifting metacentre" is any metacentre at all in the original sense of the word, because it really has nothing to do with limiting the height to which the centre of gravity can be raised without disturbing the upright position of the ship. But on the other hand it does indicate a point in the ship's axis of symmetry, above which the centre of gravity cannot be raised *without inclining the ship beyond the given angle which determines it*, and this doubtless is why it was designated the "*shifting* metacentre." It is obviously a point that usually would shift as the angle of inclination became altered. But although the term has this measure of justification, its use is not very desirable, and is indeed likely, unless great care is taken, to introduce misconceptions into the subject.*

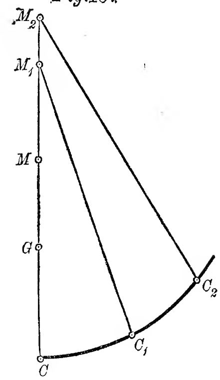
In France, nevertheless, the modern usage is to treat the metacentre proper as the "initial metacentre," and to regard as metacentres the points which we have above seen defined as "shifting metacentres." From a reply which that distinguished investigator, Mons. L. Emile Bertin, of Brest, has been good enough to make to an inquiry of mine on this point, I find that in his opinion, although the diverse acceptations given to the word metacentre at different periods or in different countries have thrown the subject into some confusion, the terminology adopted throughout France for many years past has been such as to leave no room for difficulties of interpretation. "The rules followed," says M. Bertin (whom we translate freely), "which, if they are no older, were certainly employed about 1840 by M. Reech, at the *Ecole du Génie Maritime*, are the following:—

"Let C, Fig. 15, be the centre of buoyancy of the upright ship, C_1 the centre of buoyancy for any inclination whatever, $C M_1 C_1$, equal to θ; then the point, M_1, at which the direction of the upward thrust of buoyancy through C_1 cuts the axis C G, is the metacentre corresponding to the inclination θ.

* "We do not remember having met with the term *shifting metacentre* in any previous publication, but we may observe it is still applied only to points on the axis. This term has never appeared to us a happy one, and we have never regarded it as fixed in the scientific nomenclature of shipbuilding in the same sense as the term metacentre is. It is, we think, open to the very serious objection that it is not a metacentre—*i.e.*, a limiting position of the ship's centre of gravity, in the true sense of the word, as separating stable from unstable or neutral equilibrium. It is very likely to be misunderstood by the unlearned or the sciolist, and seems really to have misled, &c."—*Naval Science*, vol. iii., p. 441.

"In the particular case in which θ is very small, the thrust of the buoyancy through C_1 cuts C G at the point, M, which is the *initial metacentre* of the ship.

Fig.15.



"The variable length, C M, is usually denoted by ρ; the constant distance, C G, by a, and the expression for the arm of the lever of the couple of stability is therefore

$$(\rho - a) \sin. \theta.$$

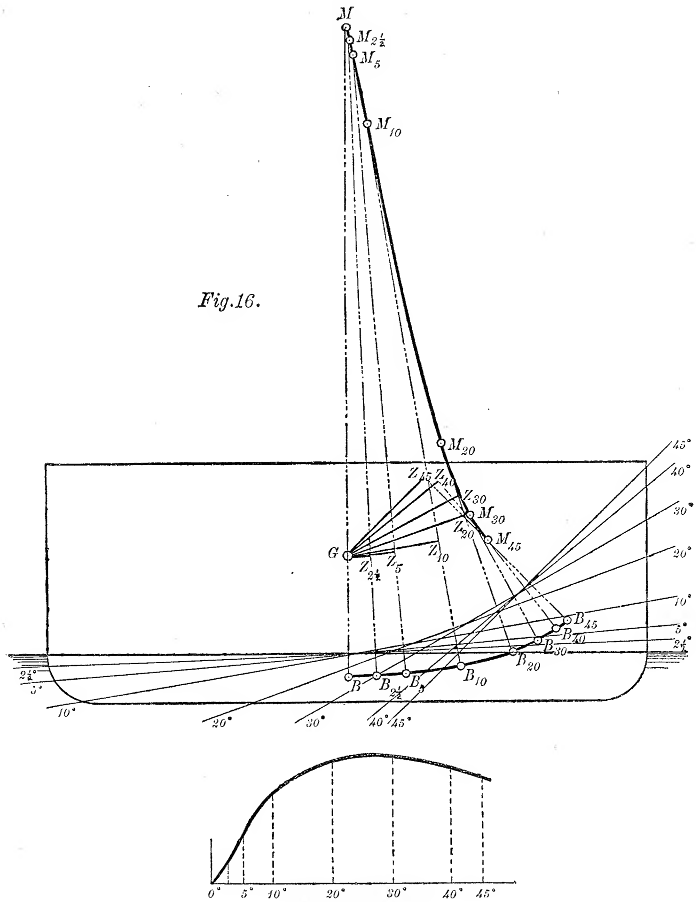
The length, $\rho - a$, or G M, is the metacentric height. The very form in which this is written leaves no room for doubt about the fact that the metacentre M is situated on the line C G.

"When we prolong the directions $C_1 M_1$, $C_2 M_2$, &c., of the successive thrusts of the buoyancy to their own intersections, the point at which two such lines of action which are infinitely near cut each other is named simply the centre of curvature of the curve of centres of buoyancy, or a point of the metacentric evolute. It sometimes happens that this last appellation is abridged to 'metacentric point;' that is a fault, because it may lead to confusion, but it is a simple abbreviation, and an excess of brevity sometimes has its inconveniences."

We have already adverted to the error into which Bouguer fell when he wrote his chapter entitled "On more extended investigation on the metacentres, and on the curved line which these points form when the ship is inclined," and assumed that where the metacentric *ascended* from the metacentre as the vessel was inclined the vessel might be regarded as safe against oversetting, and that when it *descended* she might be regarded as insecure. Instead of demonstrating the inaccuracy of this view—to which we shall have occasion to refer hereafter—we will here give a conclusive and striking example of the contrary. Fig. 16 illustrates the case of a prismatic shallow draught vessel of the section shown; M is its metacentre; $M_{2\frac{1}{2}}$ is its pro-metacentre at $2\frac{1}{2}$ degrees of inclination; and M_5, M_{10}, M_{20}, &c., are its pro-metacentres at inclinations of 5, 10, 20 degrees, &c. Its corresponding centres of buoyancy, B, $B_{2\frac{1}{2}}$, B_5, B_{10}, &c., are similarly indicated.

The pro-metacentres are joined to their corresponding centres of buoyancy by ticked lines (which are radii of curvature), and from the centre of gravity, G, the levers of stability, $GZ_{2\frac{1}{2}}$, GZ_5, GZ_{10}, &c.,

Fig. 16.

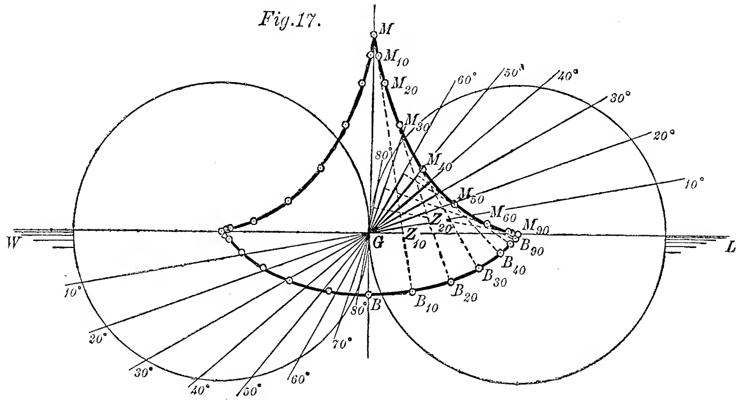


Curve of Stability

are drawn perpendicular upon these respective lines as shown. These levers are employed in forming the curve of stability which is engraved below the figure. Here we manifestly have the metacentric *descending*, and descending very steeply, almost as soon as the inclination commences, whereas the stability (instead of diminishing) increases very rapidly, and continues to increase until the vessel is inclined to about 25 degrees. After that the stability begins to diminish, but it is still very large, and would remain of substantial amount until the vessel was inclined nearly on her beam-ends.

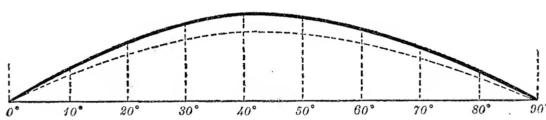
One such example is as good as a thousand, so far as the settlement of the question goes, but we have devised the following example in order to put the simultaneous increase of righting lever of stability and decrease in height of pro-metacentre (*i.e.*, decrease in the radius of curvature of the curve of buoyancy) from the very commencement of the inclination beyond all doubt. We take two homogeneous cylinders placed side by side, Fig. 17, combined into

Fig.17.



one floating body of one-half the specific gravity of water, and therefore floating one-half immersed, and with the centre of gravity, G, in the water-line. On inclining this body through any small angle whatever, it is obvious that the area of its water-line section (which, as the body is prismatic, is sufficiently represented by the breadth of the body measured along the new water-line) must diminish, because whereas the water-line breadth is equal to two *diameters* when the body floats upright, the water-line breadth is only equal to two *chords* of the circular section when the body is inclined, and every such chord of a circle is of necessity less than its diameter. Meantime, the volume immersed will remain the same, the portion of the one cylinder which is emersed by the inclination being equal to the portion of the other which becomes immersed by it, the point G remaining in the water-line, and at its middle at every possible angle of inclination. With the immersed volume remaining the same, and the water-line area thus diminishing from the very beginning of the inclination, and going on diminishing until an angle of 90° from the upright has been reached, it follows (for reasons which will appear hereafter) that the radii of curvature, $B_{10}M_{10}$, $B_{20}M_{20}$, &c. (corresponding to inclinations of 10°, 20°, &c.) must also continually diminish from the very beginning, or, in other words, the pro-metacentre continually descends until 90° of inclination is reached, as shown in the figure. At that angle the water-line will have diminished to nothing, and will be coincident with the point G, the pro-metacentre will have descended to the centre of the immersed circle; the centre of buoyancy also will obviously have travelled to, and have become coincident with, that point. The body will therefore float in equilibrium, but the equilibrium will be unstable, the centre of gravity, G, being poised a half-diameter above the metacentre. But while the pro-metacentre has thus been continually falling, and the radius of curvature thus continually diminishing, there has always been a force of stability at work tending to return the body to its original upright position. In the Fig. 17 the righting levers, GZ_{10}, GZ_{20}, &c., are drawn as perpendiculars upon the successive radii of curvature of the curve of buoyancy B, B_{10}, B_{20}, &c., and a curve of stability, Fig. 18, has been constructed from them.* It will be

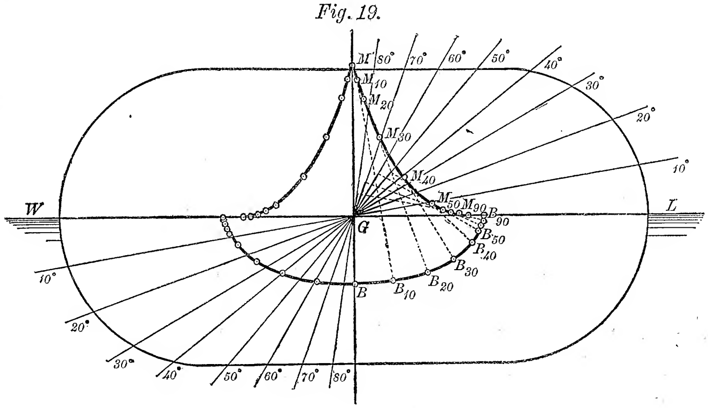
Fig. 18.



seen from both figures that the arm of stability or righting lever increases up to 40° of inclination or more, and then diminishes gradually until it disappears at 90°, so that from the beginning of the inclination up to 40° of angle at least we have a steadily diminishing radius of curvature, and a steadily increasing amount of righting force.

Something like a ship-shape may be given to this body (Fig. 17) by supposing it to be furnished with a flat bottom, and decked over at the top, as indicated in Fig. 19. This will, of course, have

Fig. 19.



the effect of disturbing some of the symmetries and other condi-

*The dotted line in this figure relates to Fig. 19, as will be presently seen.

tions of the previous case, and of somewhat diminishing the stability at every point, as indicated by the light dotted line in Fig. 18, but the same essential characteristic of the pro-metacentre falling while the stability increases will still be preserved.

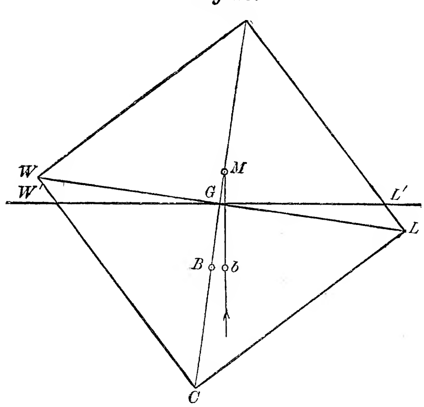
From what has gone before, it will be seen that it is our desire —and for very good reasons—to limit the application of the word "metacentre" to those points which are really metacentres, and are therefore situated in the upright axis of equilibrium, and have relation to small inclinations only. We retain the term "metacentric" as descriptive of the locus of "pro-metacentres," and as the evolute of the curve of buoyancy, but only as such, and not as implying that, for a given draught of water there is, or can be, more than one "metacentre." This metacentric has sometimes been spoken of as a "curve of metacentres," and as a "metacentric curve;" the former it is not; to the latter we can hardly object, because if it be a curve, and be designated the "metacentric," it can hardly be considered wrong, or even irregular, to speak of it as a metacentric curve. It will be well, however, to avoid this term as much as possible in this connection, because, as we shall see hereafter, there is another curve in common use which is, strictly speaking, a "curve of metacentres," and which has also been sometimes called the metacentric curve, but which is really a curve constructed by means of true metacentres, for a series of upright positions at different draughts of water. Although this latter curve, as we shall hereafter see, is artificially constructed, it is a curve of metacentres, and has therefore been called a metacentric curve, although a wholly different thing from the "metacentric." If the last-named term be employed to indicate the locus of the pro-metacentres, and the other curve (which implies different draughts of water) be always called the "curve of metacentres," much confusion may be avoided.

CHAPTER III.

Further Conditions of Stability of Square Prisms—Relation of Loci of Centres of Buoyancy and of Pro-Metacentres to Stability—Relation of Height of Centre of Gravity to Stability—Determination of Position of Metacentres and Pro-Metacentres—Shift of Centre of Gravity—Expression for the Height of Metacentre above Centre of Buoyancy—Expression for the Height of Pro-Metacentre above Centre of Buoyancy—Remarks of Professor Elgar and Mr. W. John—"Surface Stability"—French Method of Treating Stability—Description thereof by Mons. Emile L. Bertin—Notes by Mons. V. Daymard and Mons. Emile L. Bertin.

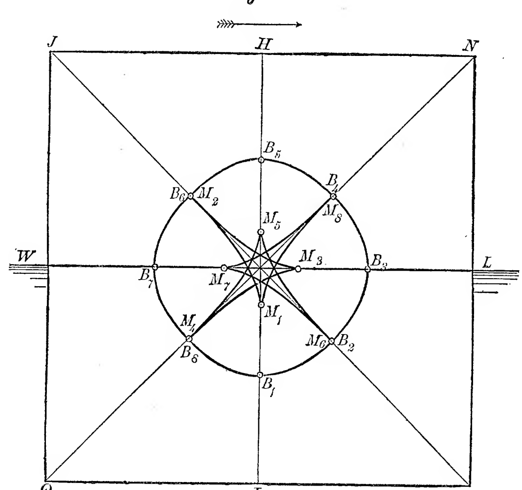
THE square prism, Fig. 12, is in unstable equilibrium when floating with its diagonals inclined. The symmetry of the body suggests that we should place it next with its diagonals nearly upright and horizontal respectively, as in Fig. 20. The centre of buoyancy when G C was upright, was B; now that G C is slightly inclined it is at *b*, and the metacentre M is now situated above G, as shown. The equilibrium is therefore stable in this case.

Fig. 20.



As the body was rotated from the position shown in Fig. 12 to that shown in Fig. 20, with W L for water-line, it passed through an indefinite number of intermediate positions, for each of which there was a corresponding centre of buoyancy and a corresponding pro-metacentre. These would compose in each case a curve or locus, as we have seen. In Fig. 21 are shown the curve of buoyancy, B_1, B_2, B_3, &c., and the metacentric, or curve of pro-metacentres, M_1, M_2, M_3, &c. These curves are not here limited to correspond to an inclination through an angle of 45° only; but are carried on to an extent corresponding to a complete rotation of the prism through 360°. Regarding the body as upright when in the position shown in Fig. 21, with I H central and vertical, B_1 is the corresponding centre of buoyancy, and M_1 the metacentre. As the body inclines in the direction indicated by the arrow, the centre of buoyancy travels along the curve, $B_1 B_2$, arriving, so to speak, at B_2, when the diagonal, J K, becomes upright, by which time the pro-metacentre (starting from M_1) has travelled along its curve, $M_1 M_2$, and has arrived at M_2. Continuing the rotation, the centre of buoyancy and the pro-metacentre travel on, arriving at B_3 and M_3 respectively, when W L has become upright; and so on as the inclination proceeds until the whole rotation has been completed, the centre of buoyancy having then passed through the points B_4, B_5, B_6, B_7 and B_8, and

Fig. 21.



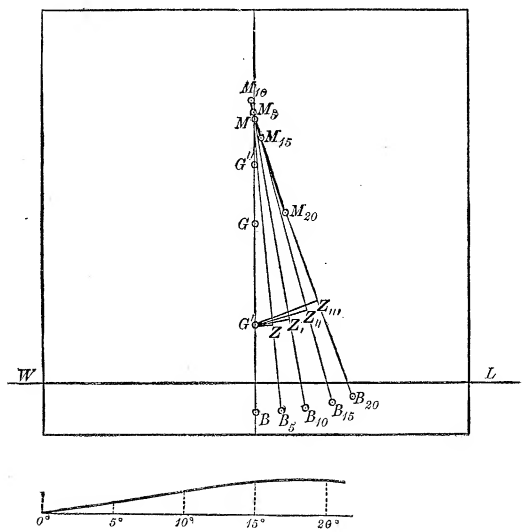
arrived again at B_1, the pro-metacentre having similarly arrived again at M_1. The symmetry of the curves thus traced out indicates what the symmetry of the prism makes certain beforehand, viz.—that the relative positions of the centre of buoyancy and of the pro-metacentre, which held when the body was upright, recur whenever the sides are horizontal; and the relative positions which held when the body floated in stable equilibrium with a diagonal vertical, recur whenever any diagonal becomes vertical.

With the locus of centres of buoyancy, $B_1 B_2$, &c., and the locus of pro-metacentres, $M_1 M_2$, &c., Fig. 21, before him, the reader will easily see in what relation these stand to the stability, or righting lever, at any given angle of inclination. Let him join the centre of buoyancy and the pro-metacentre, corresponding to any given angle of inclination, by a straight line (prolonging the line if necessary till it passes the centre of gravity) and observe on which side of this line the centre of gravity lies. If it lies to the right of this line it will turn the upper or unimmersed part of the body to the right; if it lies to the left of this line it will turn the body in the opposite direction.

It is not, as Bouguer hastily assumed, a question of whether the metacentric rises or falls, but whether the force of buoyancy, acting through any given centre of buoyancy, passes the centre of gravity on one side of it or on the other. Whenever the forces of gravity and buoyancy act in different lines they will produce a turning movement, and that will turn the floating body in one way or the other, according as they act on one or other side of each other. When we are dealing with *metacentres* (lying in axes of equilibrium of course) the stability or instability of the equilibrium *is* determined by the relative *heights* of the centres of gravity and the metacentre; but, when we are dealing with *pro-metacentres* which are not metacentres, and which are nothing more than intersections of adjacent verticals through centres of buoyancy, their *heights* relative to the centre of gravity do not usually determine whether stability or instability exists, or in any way measure the amount of it. This we saw with reference to Fig. 16; it appears also from Fig. 22. In this figure, which represents the pro-metacentres, centres of buoyancy, and curve of stability of a square prism immersed $\frac{3}{25}$ths of its depth, M is the metacentre; M_5 the pro-metacentre at an inclination of 5°; M_{10} the pro-metacentre at 10°; and M_{15} and M_{20} those at 15° and 20° respectively. The pro-metacentre rises above the metacentre as the body is inclined up to 10°, but then it falls, and falls rapidly, as the inclination proceeds, lying considerably lower than the metacentre at 15°, and descending to M_{20} at 20°. It is obvious that if the righting force, or arm of stability, were proportionate to the *height* of the pro-metacentre, there would be a great falling off in it from 10° onwards, because of this rapid descent of the pro-metacentres. The curve of stability which is given below the figure shows, however, that if the centre of gravity be situated at the centre of form this is by no means the case, for the stability goes on increasing up to 20°. This increase of stability, as the angle increased, up to 20° would obviously still hold, proportionally, if G were either raised or lowered.

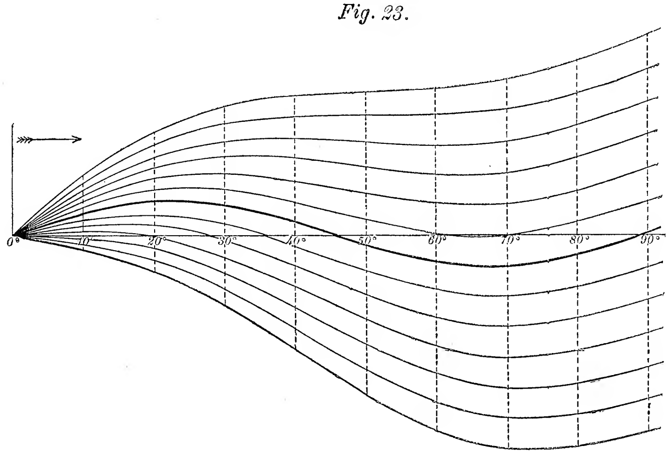
In so far as *height* is concerned, it is the position of the centre of gravity which really does determine the stability at any large

Fig. 22



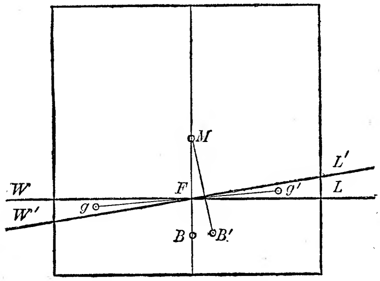
angle. This also will readily be seen from Fig. 22, in which the lines joining the centres of buoyancy with their respective pro-metacentres are shown, the ordinates in the curve of stability being the perpendicular distances of the point, G, from these lines. If we now suppose the centre of gravity to be situated no longer at G but at G′, it is easy to see that we shall get in G′ Z, G′ $Z_{,}$, G′ $Z_{,,}$ and G′ $Z_{,,,}$, a new set of stability ordinates, all *greater* than those obtained from G; and if we suppose the centre of gravity to be raised above G, say to G″, it is clear that we shall obtain another set of ordinates, all *less* than those obtained from G. In fact, the magnitude of the ordinates, such as those shown drawn, G′ Z, G′ $Z_{,}$, &c., will vary directly as the height of G varies, each ordinate being the base of a triangle, the hypothenuse of which is the distance between the point G and the point of the upright axis of equilibrium, at which it is intersected by the line joining the centre of buoyancy and the pro-metacentre (produced if necessary). The ordinate is *positive,* and should be drawn above the base of the curve of stability when this joining line lies upon that side of G toward which the top of the body is turning; and *negative,* and should be drawn below the base-line when it lies on the opposite side of G. Nothing is easier, therefore, than to construct curves of stability for a prismatic body of this form for any given degree of immersion, and for any given position of the centre of gravity, when once the centres of buoyancy and pro-metacentres have been determined for the given draught of water. In Fig. 23 we have shown a series of such curves (or

Fig. 23.



rather half-curves, as they represent inclination one way only), for the 25-feet prism immersed 3 feet, as shown in Fig. 22. In Fig. 23 the dark curve represents the stability (or righting levers) when the centre of gravity is at the centre of the body; the light curves lying above the dark one represents successive lowerings of the centre of gravity, 2 feet each time; and the light curves lying below the dark one represent successive raisings of the centre of gravity, 2 feet each time. The ordinate corresponding to a given angle of inclination increases and decreases by an equal length each time, for the reason before stated—viz., that it varies directly with the height of the centre of gravity measured on the axis of equilibrium.

Fig. 24.



Before proceeding further with the case of square prisms, let us consider how the positions of their pro-metacentres and metacentres may be determined. In Fig. 24, W L and W′ L′ are successive water-lines, making with each other, we will suppose, the very small angle θ. B is the centre of buoyancy corresponding to W L, and B′ the new one; g is the centre of gravity of the triangle W F W′, and g' that of the triangle, L F L′. F will be in the line M B, and the two small triangles just mentioned will be equal. It is easy to find how far B′ is from B, because the travel of the centre of buoyancy has been caused solely by the *emersion* of the small triangle on one side, and *immersion* of the equal triangle on the other side. This is equivalent to saying that a triangle of buoyancy, so to speak, has travelled from the position in which its centre of gravity was at g to one in which its centre of gravity is at g', and from a well-known proposition of mechanics it follows that the area of this triangle multiplied by $g\,g'$ is equal to the whole immersed area of the body multiplied by B B′.*

In other words, if v be the area of the triangle, and V the whole immersed area, then

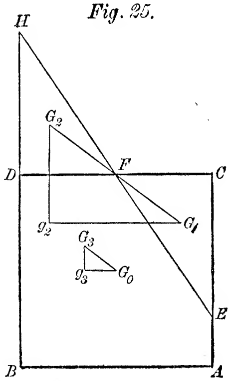
$$\mathrm{V} \times \mathrm{B\,B'} = v \times g\,g';$$

and, therefore,

$$\mathrm{B\,B'} = \frac{v}{\mathrm{V}}\, g\,g'.$$

B B′ is parallel to $g\,g'$ (as we just saw in the foot-note), but we need not dwell on this here, as the angle θ is supposed to be exceedingly small. Later on we shall see that in the case of ships, and for large angles of inclination, we shall have to substitute for $g\,g'$ the

Fig. 25.



* Professor Rankine, in his *Applied Mechanics*, states this principle in so concise and neat a form that we are induced to give it here. He says—"Let A B C D (Fig. 25) be a body of the weight, W_0, whose centre of gravity, G_0, is known. Let the figure of this body be altered by transposing a part, whose weight is W_1, from the position E C F to the position F D H, so that the new figure of the body is A B H E. Let G_1 be the original, and G_2 the new position of the centre of gravity of the transposed part. Then the moment of the body relatively to any axis in a plane perpendicular to $G_1\,G_2$ will be altered by the amount $W_1 . G_1\,G_2$; and the centre of gravity of the whole body will be shifted to G_3, in a direction, $G_0\,G_3$, parallel to $G_1\,G_2$, and through a distance given by the formula

$$\overline{G_0\,G_3} = \overline{G_1\,G_2}\,\frac{W_1}{W_0}.$$

If horizontal lines be drawn from G_1 and G_0 in the figure, and perpendiculars be dropped upon them from G_2 and G_3 respectively, meeting them in g_2 and g_3, then the *horizontal* distance through which the centre of gravity is shifted will be given by the formula

$$\overline{G_0\,g_3} = \overline{G_1\,g_2}\,\frac{W_1}{W_0}.$$

distance apart of their perpendicular projections upon the new water-line.

Now, if we take y to represent the half-breadth of the body (W F or F L), F g and F g' will, by a well-known property of triangles, each equal $\frac{2}{3}y$ (the angle being very small); and, therefore, W L or W′ L′ not differing materially from the whole breadth measured along the line, $g\,g'$,

$$g\,g' = \frac{4}{3}y.$$

The area, v, of one of the triangles will be equal to $\frac{1}{2}$ F L × L L′, but $\frac{\text{L L}'}{\text{F L}} = \tan.\ \theta$, and, therefore, L L′ = F L tan. θ.

Hence,
$$v = \frac{1}{2}\,\text{F L}^2 \tan.\ \theta$$
$$= \frac{1}{2}\,y^2 \tan.\ \theta.$$

Consequently
$$\text{B B}' = \frac{\frac{1}{2}\,y^2 \tan.\ \theta \times \frac{4}{3}\,y}{\text{V}}$$
$$= \frac{\frac{2}{3}\,y^3 \tan.\ \theta}{\text{V}} \quad . \quad . \quad . \quad (1.)$$

But θ being very small, $\frac{\text{B B}'}{\text{B M}}$ may be taken as equal to tan. θ and B B′ = B M tan. θ . . . . (2.)

It follows from equations (1) and (2) that

$$\text{B M} = \frac{2}{3}\,\frac{y^3}{\text{V}}\,.$$

It will be seen later on that there is a general expression for the height of the metacentre above the centre of buoyancy of a ship in the upright position, of which this is a simple form, its simplicity being consequent upon the fact that we are here dealing only with a square prism, and are assuming that its volume is represented by its sectional area.*

* The general expression to which we refer, and which we shall show the reason of afterwards, is

$$\text{B M} = \frac{\frac{2}{3}\int y^3\,d\,x}{\text{V}}.$$

The employment of the sign of integration ($\int$) in this expression and in the next

On looking over the previous demonstration it will be seen that it will hold with perfect accuracy for the height of the intersection of the two consecutive verticals through any two consecutive centres of buoyancy whatever. For example, if the original water-line (Fig. 21) instead of being W L had been W′ L′, and B′ M had been the original vertical, and the second water-line had been taken at a correspondingly greater inclination, the demonstrations would have been unaltered, and only the quantities (y, $g\,g'$, &c.) would have been different. The expression

$$\mathrm{B\,M} = \frac{2}{3}\,\frac{y^3}{\mathrm{V}}$$

would have been precisely of the same form in all such cases, and is, therefore, a general expression for the height of the pro-metacentre above the centre of buoyancy for the square prism in question at all angles of inclination.

It will be further seen that the form of the body below the water-line does not enter into the demonstration, and that the demonstration would have been precisely the same if we had taken not a square prism, but a prism of any section whatever. Consequently, the expression given is true for all prismatic bodies, and the height of the metacentre, or of any pro-metacentre, above the corresponding centre of buoyancy may be found from the fact that it is equal to two-thirds of the cube of the half-breadth of the corresponding water-line divided by the whole immersed area of the section.

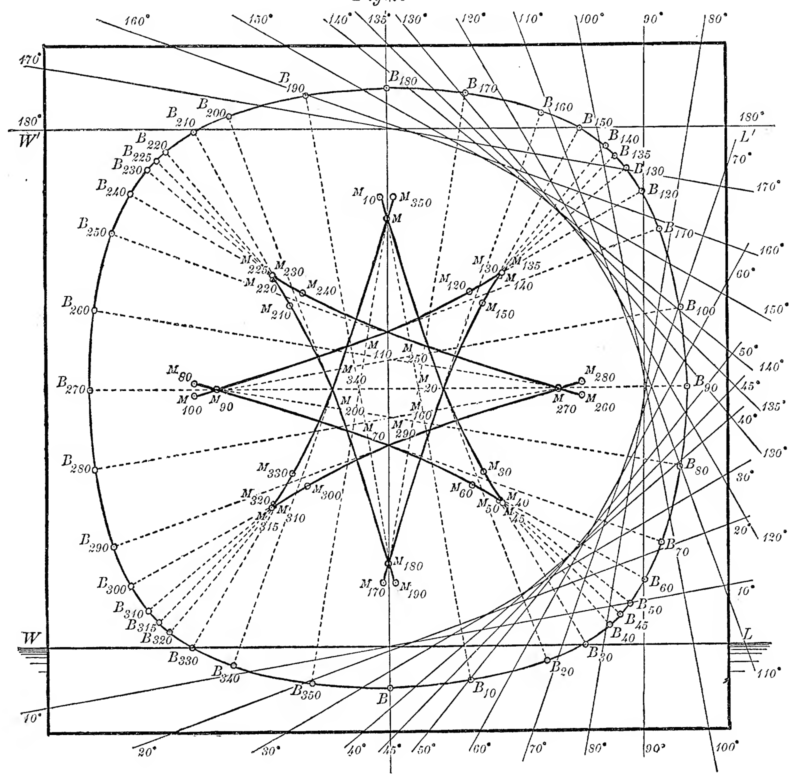
Some interesting consequences follow from these facts. First, it will be seen that, assuming any given specific gravity for the prism, and consequently a given amount of buoyancy (V being then constant), the height of the pro-metacentre above the corresponding centre of buoyancy, at any angle of inclination whatever, will be directly proportional to the cube of the half-breadth of the body measured at the water-line. Hence, for all positions of the body in which the breadth at the water-line is the same, and the displacement the same, the pro-metacentre and the centre of buoyancy will be at equal distances apart. Some readers will find it interesting and instructive to verify this by examining Fig. 26, in which the

chapter, need not trouble those readers who are without knowledge of the Integral Calculus. It is merely a conventional and simple mode of indicating that a number of small elements are to be added together into one sum, as will be more fully explained hereafter.

curves of buoyancy and the curve of pro-metacentres (or the metacentric) are again traced throughout a complete revolution of the body.

It will then be seen that what has just been said is true, not only of the cardinal points, so to speak, of the figure (shown already in Fig. 21), but also of all equal water-lines. For example: at

Fig. 26.



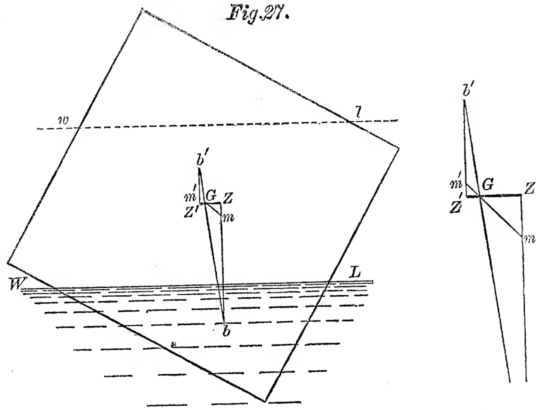
angles of 10°, 80°, 100°, 170°, 190°, 260°, 280°, and 350°, Fig. 26, the water-lines are all equal, being symmetrical with reference to the four sides of the figure; those at 10° and at 350° being evidently in immediate symmetrical relation to the side of the prism that is lowest in the figure; those at 170° and at 190° being in similar relation to the side which is shown at the top, and so on.

The water-lines at 30°, 60°, 120°, 150°, 210°, 240°, 300°, and 330° are all equal, and it will be seen that their pro-metacentres are all at equal distances from their respective centres of buoyancy. Other like symmetries will suggest themselves to the reader.

There are other inferences that may be drawn. For example: with the body floating, as shown in Fig. 26, y, the half-breadth at the water-lines remains the same for all depths of immersion, and therefore in the value of B M $\left(\text{viz.,} \dfrac{\frac{2}{3}y^3}{V}\right)$ the numerator of the fraction remains constant for all depths. B M varies therefore in this particular case, inversely as V, becoming small as V becomes large, and large as V becomes small. But V varies directly as the depth of immersion, and consequently the height of the metacentre above the centre of buoyancy varies inversely as the depth of immersion or draught of water. As the pro-metacentres all follow the same law, it is quite easy in passing from one draught of water to another to determine their positions.

We have already explained that the distances between the centres of buoyancy and the corresponding pro-metacentres must not be taken as measures of the righting forces or stability. But there are symmetrical relations existing between the righting forces arising out of the symmetry of the floating body which we are considering. One of these is very important. Let us suppose in Fig. 26 that the body, instead of being only immersed, as there shown, to W L, is immersed to the line, W′ L′, marked 180°, which is as near to the top of the body as the water-line, W L, shown in the figure is to the bottom of the body. If we now suppose the body to rotate as before through a complete rotation (360°) it is obvious that it will have precisely the same water-lines as before, in regular succession as before, only the parts which were before *emersed* will now be *immersed,* and *vice versa.* They will have, therefore, at every angle of inclination, precisely the same centres as before (but centres of *emersion* and *immersion* interchanged), and a well-known property of the centre of gravity provides that the lines joining the centres of immersion with corresponding centres of emersion must all pass through the centre, G. For the sake of clearness this is illustrated separately and adapted to other draughts of water, in Fig. 27, where W L is the water-line, the body being inclined; $b\,b'$ are the centres of the immersed and non-immersed parts respectively, $m\,m'$ the corresponding pro-metacentres, G Z and G Z′ are perpendiculars drawn from G upon $b\,m$ and $b'\,m'$. Let v and v' be the volumes (represented by the areas, as before) below and above W L respectively, or v the immersed and v' the out-of-water volume. We will still designate the half of W L by y. We know that $b\,m$ and $b'\,m'$ (shown on a larger scale on the side of the figure) are parallel, both being perpendicular to W L, and therefore

Fig. 27.



the angles, G b Z and G b'Z′, are equal. Consequently the right-angled triangles, b G Z and b' G Z′, are similar, and G Z is to G Z′ as G b is to G b'.

$$\text{But} \qquad \mathrm{G}b \times v = \mathrm{G}b' \times v';$$
$$\text{and therefore} \quad \mathrm{GZ} : \mathrm{GZ}' :: \mathrm{G}b : \mathrm{G}b' :: v' : v;$$
$$\text{and} \qquad \mathrm{GZ} \times v = \mathrm{GZ}' \times v'.$$

In other words, the righting *moments*—or rather capsizing moments, as they happen here to be—are exactly the same, whether the body be immersed to W L, as shown in Fig. 24, or be turned upside down, and sunk till it floats at the same water-line—always providing that the body be homogeneous. It follows, of course, that the stability of the prism would remain unaltered if it were simply sunk to the line, W L, provided its centre of gravity still remained in its centre of form. Small immersion and small freeboard therefore are attended in this case by like conditions of stability.

Mr. F. Elgar * drew attention to this class of considerations as follows †:—"Any homogeneous floating body which is symmetrical about the three principal axes at the centre of gravity—such as a rectangular prism or an ellipsoid—will have the same moment of stability at equal angles of inclination, whether floating at a light

* Now Professor Elgar, of Glasgow University.

† In a letter to *The Times*, published September 1, 1883.

draught with a small volume below water, or at a deep draught with a similar volume above water. For instance, if a homogeneous prism of square cross section, the sides of which section are each five feet in length, floats at a draught of water of one foot, it will then have precisely the same moment of stability at equal angles of inclination, and consequently the same curve of stability through out as if it were loaded—without altering the position of the centre of gravity—till it had four feet draught of water and one foot of freeboard. From this it follows that in such elementary forms of floating bodies as these, lightness of draught has the same effect upon stability as lowness of freeboard, and if a low freeboard is unfavourable to stability, so also—and precisely to the same extent—is a correspondingly light draught of water."

Mr. W. John, in following up this statement of the case, said*:—"The proposition laid down is not confined to a body symmetrical about three principal axes at the centre of gravity, but applies to all homogeneous floating bodies of irregular form revolving about a horizontal axis fixed only in direction. This is easily seen in the case of a horizontal prism with any irregular form of vertical section. The line joining the centre of gravity of the immersed section with the centre of gravity of the section above water passes through the centre of gravity of the whole section, and the distances from the latter are inversely as the areas; and the moment of stability being proportional to the immersed area, multiplied by this distance, it will be seen at once geometrically that the moment of stability will be exactly the same if the diagram be turned upside down and the part before out of water be now considered the immersed part."

We have seen this to be correct.

Before advancing further, it will be well to point out that, although there is no such thing as a measurable righting force, and therefore no such thing as stability, apart from a definite weight of body, and a definite position of its centre of gravity, yet the fact of the position and movements of the centre of buoyancy, and the corresponding position and movements of the pro-metacentres, all being dependent upon *form* alone, has induced authors to speak of "stability due to form," or "surface stability," as it has been also called.

Mr. Thearle, in his useful work on *Theoretical Naval Archi-*

* In a letter to *The Times*, published September 5, 1883.

tecture, says:—"By surface stability is meant that tendency of a vessel, when inclined, to return to the upright position, which is due to her form, irrespective of the influence due to her centre of gravity not coinciding with the centre of buoyancy."

Mr. Mackrow, in his *Naval Architects and Shipbuilders' Pocket-Book*, puts the matter more explicitly by saying—"The moment of *statical surface stability* is what the righting moment would be, supposing the centre of gravity of the ship coincident with the centre of buoyancy."

It is sometimes convenient, as will more plainly appear hereafter, thus to separate the actual stability of a ship into two parts, one of which is estimated upon the assumption of the centre of gravity and centre of buoyancy being coincident, the other part being an addition thereto, or subtraction therefrom, according as the centre of gravity is situated below or above the centre of buoyancy. It is perfectly obvious, however, and must never be forgotten that stability measured upon the assumed coincidence of the centre of gravity and centre of buoyancy is not in any true sense stability due to form only, but stability which is just as much due to gravity as any stability is ever due to it. Still it is, as we have said, sometimes convenient to assume the coincidence in question, and calculate the stability then existing, and there is no particular reason why this should not be called "surface stability," or "stability due to form," to distinguish it. It will be seen later on that Mons. V. Daymard, of Marseilles, has turned to account this method of arranging stability calculations with remarkable originality and success.

We will give additional fullness to this chapter, and exhibit the French view of this doctrine of "stability of form," by translating here the remainder of the remarks by Mons. L. Emile Bertin, of Brest, part of which were quoted in the last chapter, adding a note received* from M. Daymard. M. Bertin goes on to say:—

"The points of the metacentric *evolute* (which we here designate pro-metacentres) 'play, in the study of stability, a *rôle* infinitely less important than the metacentres. The only practical advantages which one can discover in considering this evolute appear to be the two following: In the first place, the directions of the upward push of the buoyancy in which the stability is annulled are *normals* to the curve of buoyancy drawn through the centre of gravity, G, and are *tangents* to the metacentric evolute; and through the point

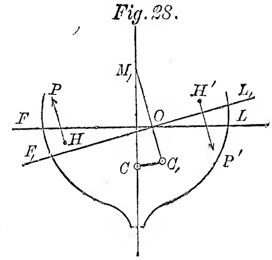
* October, 1883.

G we can draw the *tangents* to the metacentric evolute more easily and more exactly than we can draw the *normals* to the curve of buoyancy. In the second place, the *maxima* values of the arm or lever of stability ($\rho - a$ sin. θ) are the normals drawn through G to the metacentric evolute; the evolute indicates therefore the positions of the maximum couple of stability, which are not indicated by anything in the curve of buoyancy. These two advantages are, however, of little value now that it has become the usage to represent the couples of stability by a curve traced with rectangular co-ordinates, with which we see, still better than on the metacentric evolute, the points where the stability becomes nothing, and those where it reaches a maximum.

"An examination of the two processes employed (in France) for determining the stability of ships at divers inclinations will complete the proof that there can exist no possible doubt respecting the signification of the words *metacentre* and *metacentric height*.

"In the geometrical process we calculate, by the aid of quadratures, the moment $p \times \mathrm{H\,H'}$ of the couple of the two wedges, $\mathrm{OLL}_{,}$ and $\mathrm{OFF}_{,,}$ Fig. 28, the one immerged the other emersed, for a finite inclination, θ, and put

Fig. 28.



$$p \times \mathrm{H\,H'} = \mathrm{P} \times \mathrm{C_{,}M_{,,}} \text{ sin. } \theta,$$

calling M the metacentre, which is the point already defined. In the experimental process, one obtains, by inclining a small model, a series of values of $(\rho - a)$ sin. θ corresponding to divers values of θ, from zero to about 35 degrees; one deduces from the initial value of $(\rho - a)$ and from the initial value of ρ which is known, the value of a for the model; the a of the model is replaced by that of the ship, and thus is obtained the couple of the real ship's stability for divers values of θ. All this supposes the values of ρ and those of the metacentric heights, $\rho - a$, taken on the initial vertical through the centre of buoyancy; this shows, at the same time, the practical utility of the division of the metacentric height into its two terms, ρ and a.

"Often one is not content with distinguishing on the vertical axis, C G′, the two heights, C M and C G, of which the difference is the metacentric height $(\rho - a)$; it is desired further to distinguish, in the couple of stability $\mathrm{P}(\rho - a)$ sin. θ, the two parts

$$P \rho \sin. \theta,$$
$$P \alpha \sin. \theta.$$

The first of these moments is called the couple of *stability of form*, and the second, which would be a capsizing couple, is called the *stability of weight*. There is nothing in this which is inexact in itself, but there may result from it the false impression that there exist two sorts of stability, the one of *form*, the other of *weight* which are not of the same nature. There is a celebrated theorem of David Bernoulli, relative to the equilibrium of a ship on waves, which attributes to these two sorts of stability properties wholly different, and from which Mr. Froude intuitively set the theory of rolling free. Such notions of stability of form and of stability of weight are definitely set aside. If we experience the need of giving particular names to $P\rho \sin. \theta$, and to $\rho \sin. \theta$, we can call them the couple of *geometric stability* and the arm of the lever of geometric stability.

"The rules and usages which I have just described so summarily are doubtless not perfect. We can easily imagine more general and more scientific representations of stability than those furnished by metacentres. But these rules, very little modified since Bouguer, have the advantage of being sanctioned by long practice; they are understood in the same sense by all those who employ them, and in these conditions certainly cannot lead to any error."

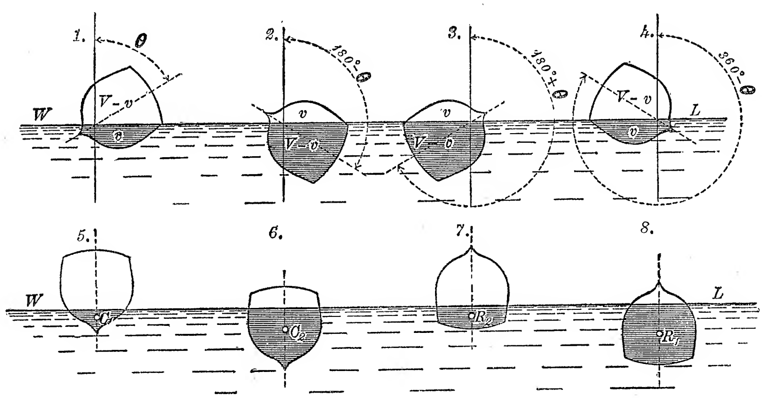
The interesting note from M. Daymard, of Marseilles, bears upon the question of complementary measures of stability, and was suggested by the correspondence previously quoted respecting the relations between the stabilities of a given ship at light draught and with small freeboard, and is to the following effect. M. Daymard says*:—

"There are for any ship whatever, and for every floating body which possesses a longitudinal axis of symmetry, four positions, viz.: Nos. 1, 2, 3, and 4, Fig. 29, inclined at the angles, θ, $180° - \theta$, $180° + \theta$, and $360° - \theta$, in which the areas of flotation are of exactly the same form, and of which the immersed volumes are alternately complementary; v and $V - v$, V being the total volume. To each of these immersed volumes, v and $V - v$, correspond two positions of the ship with its axis vertical, the one upright, the

* The substance of this note has, since this was written, been appended by M. Daymard to his Paper read (April, 1884) at the Institution of Naval Architects.

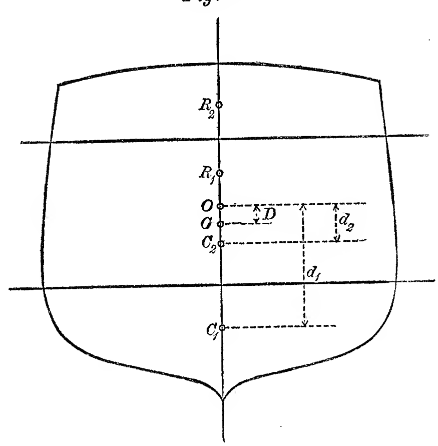
other reversed, or bottom upwards, as Nos. 5, 6, 7, and 8, Fig. 29. There exists a simple relation between the arm of the lever and the moments of stability corresponding to the positions, Nos. 1, 2,

Fig. 29.



3, and 4." M. Daymard proceeds to deduce these relations in his note, but as in doing so he makes reference to the notation employed in his beautiful system of calculating stability, which we shall here-

Fig. 30.



after consider, we will here briefly indicate in our own words his mode of procedure, observing that we may for the present purpose regard the bodies spoken of as of prismatic form. Take, for example, the case illustrated in diagram Nos. 1 and 2, and let d_1, d_2, Fig. 30, be the distances from the centre, O, of the whole volume, V, of the respective centres of buoyancy, C_1, C_2, of the upright displacements, v and $V - v$. Let R_1, R_2, be the corresponding centres of buoyancy when the ship is reversed with like displacements, then we shall have

$$C_1\,O = d_1$$

$$C_2\,O = d_2$$

$$O\,R_1 = d_1 \frac{v}{V - v}$$

$$C_2\,R_1 = d_2 + d_1 \frac{v}{V - v}.$$

Now let l_1 and l_2 be the levers of stability corresponding to the positions, Nos. 1 and 2 in Fig. 29, and we shall have, as may be the more readily seen if Fig. 24 and its descriptive text are referred to,

$$l_2 = \left(d_2 + \frac{d_1\,v}{V - v}\right) \sin.\,\theta - l_1 \frac{v}{V - v}$$

and if we call m_1 and m_2 the corresponding moments, we shall have

$$m_2 = [V\,d_2 + v\,(d_1 - d_2)] \sin.\,\theta - l_1\,v$$

$$= [V\,d_2 + v\,(d_1 - d_2)] \sin.\,\theta - m_1.$$

If in the second case we require the righting lever and the moment for placing the body in the reversed position, as in diagram No. 8, we have, calling these l'_2 and m'_2, respectively,

$$l'_2 = l \frac{v}{V - v}$$

$$m'_2 = l'_1\,v = m_1.$$

In these expressions it has been assumed that the centre of gravity is always coincident with the centre of buoyancy, and consequently variable. If G be a fixed centre of gravity, a, its height above C_1 and a', its height above C_2, we shall have $d_2 - a = d_1 - a = d_2 - a'$ $=$ G O, which we may call D. When the following expressions will hold calling the arms of the righting levers, B_1 and B_2, and the moments, M_1, M_2.

1st case: Volume v, inclination θ, for restoring the body to the upright position, No. 5,

$$B_1 = l_1 - a \sin. \theta$$

$$M_1 = l_1 v - a v \sin. \theta.$$

2nd case: Volume $V - v$, inclination $180° - \theta$, for restoring the body to the upright position, No. 6,

$$\begin{aligned} B_2 &= \left(d_1 \frac{V}{V - v} - a\right) \sin. \theta - l_1 \frac{v}{V - v} \\ M_2 &= (d_1 - a) V \sin. \theta - (l_1 v - a v \sin. \theta) \\ &= V D \sin. \theta - M_1. \end{aligned}$$

If in the second case we require the arm of the lever, and the moment for reaching the reversed position, as in diagram No. 8, calling these B'_2 and M'_2 respectively, we shall have

$$\begin{aligned} B'_2 &= l_1 \frac{v}{V - v} - \left(\frac{d_1 V}{V - v} - a\right)^* \sin. \theta \\ &= - B_2 \\ M'_2 &= l_1 v - a v \sin. \theta - (d_1 - a) V \sin. \theta \\ &= M_1 - V D \sin. \theta \\ &= - M_2. \end{aligned}$$

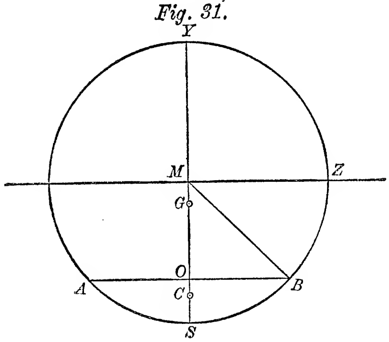
The equalities, $B'_2 = - B_2$; $M'_2 = - M_2$, thus found by calculation are also self-evident.

M. Daymard, in resuming, remarks that the effective moments of stability in the cases, Nos. 1 and 2, with the fixed centre of gravity, are different for the return to the upright positions, but as there exists between them the relation, $M_2 = V D \sin. \theta - M_1$, they are in a certain sense complementary.

* $d_1 \frac{V}{V - v} - a$ is the distance of the centre of gravity, G, from the centre of buoyancy, R_1, of the volume, $V - v$, measured from the top of the body downwards.

M. Bertin has pointed out the fact that the expression (given previously in this chapter) for the height of the metacentre above the centre of buoyancy—which is the radius of curvature of the curve of buoyancy, where that curve cuts the upright axis—can be found by an independent process of reasoning, and in a perfectly general form. The reasoning by which he proceeds to establish this is based upon the following elementary property of the circle, viz., in a circle, the *moment* of the surface of any segment whatever, say A S B, Fig. 31, about its centre, M, is equal to two-thirds of the cube of the half-chord, O B, of the segment. Let R be the radius of the circle; y_0, the half-chord O B; and z_0, the distance M O; then taking M Y and M Z as axes of co-ordinates, and using the word "moment" as the equivalent of "moment of segment, A S B, about the axis M Z," we have,

Fig. 31.



$$\text{moment} = 2\int_{z_0}^{R} y\,z\,d\,z.$$

But we know that

$$y^2 + z^2 = R^2;$$

and therefore

$$\text{moment} = 2\int_{z_0}^{R} \sqrt{R^2 - z^2}\;z\,d\,z,$$

whence

$$\text{moment} = \frac{2}{3}(R^2 - z^2)^{\frac{3}{2}}$$

$$= \frac{2}{3}y_0^3.$$

Applying now this formula to the calculation of stability, if the floating body be a cylinder or part of a cylinder, of which the immersed portion is represented by its section, A S B, and of which G is the centre of gravity, we see immediately that the condition of equilibrium is that G should be immediately below M; and if we put V (= A S B) to represent the immersed volume (and its weight likewise) we shall have as the moment of stability for any inclination, great or small,

$$\text{V} \times \text{M G sin. } \theta.$$

Calling C the centre of buoyancy, and putting for M C, ρ, and for G C, a (as is usual in France) this expression may be written,

$$\text{V} (\rho - a) \text{ sin. } \theta.$$

But the *moment* which we previously calculated being equal to V ρ, it is evident that we have

$$\rho = \frac{2}{3} \frac{y^3}{\text{V}}$$

the value arrived at by the usual investigation for stability.

Passing from the cylindrical floating body to any other having the form of any surface of revolution whatever, of which the axis is at M, it is evident that the upward thrust of the buoyancy at any angle of inclination will pass through the axis, the moment of stability will be calculated as before, and (if we still call ρ the distance between the centre of buoyancy and the axis M) its expression will be

$$\rho = \frac{2}{3} \frac{\Sigma\, y_0^3\, \delta_k}{\Sigma\, s\, \delta_k}$$

the surface of revolution being supposed, for the calculation of the moments, decomposed into a series of slices of the constant thickness, δ_k, in each slice the half-ordinate of the water-line breadth is y_0, and the surface of the transverse section s. The expression for ρ may be written

$$\rho = \frac{2}{3} \frac{\delta_k\, \Sigma\, y_0^3}{\text{V}}$$

observing that $\Sigma s\, \delta_k$ is the immersed volume.

Finally, "if we consider a floating body of any form whatever," says M. Bertin, "and give it infinitely small inclinations, the thrust passes constantly through the centre of curvature of the curve of buoyancy, which performs the same part as the axis, M, in the two preceding cases. The displacements of the centre of buoyancy depend only on the total volume immersed and on the form of the water-line plane, the influence of the form of the transverse sections being neglected, as it may be, for infinitely small angles. The radius of curvature of the curve of buoyancy is therefore equal to the radius of the circle, which would form the curve of buoyancy of a floating body, of the same area of water-line plane and the same immersed volume, which had the form of a solid of revolution. This radius is given by the formula

$$\rho = \frac{2}{3} \; \frac{\delta_k \; \Sigma \; y_0^3}{V}$$

which is therefore of general application.

"To resume," adds M. Bertin, "the metacentre, such as Bouguer defined it, is the axis of the floating body of revolution, to which any other floating body whatever can be assimilated, from the point of view of *initial stability*, and the height of the metacentre above the centre of buoyancy can be directly established by this property."

CHAPTER IV.

General Case: Stability of Body of Irregular Form—Present Treatment thereof purely Statical—Effects of Inclination—Wedges of Immersion and of Emersion—Travel of Centre of Buoyancy—Righting Lever of Stability—Expression for the same—Atwood's Fundamental Formula of Statical Stability—Work to be done in Calculating a Ship's Stability—Construction of Curves of Stability—Example of a Curve of "Moments" of Stability in Foot-tons—Importance of Observing the Scale of Curves of Stability—Curve of B R's—Curve of Sines—Curve of Stability represents the Difference between these Curves—Stability due to Form at Various Draughts of Water—Mons. V. Daymard's Curves—Metacentric Stability—Fundamental Expression for Stability at Evanescent Angle of Inclination—Expression for Height of Metacentre above Centre of Buoyancy—Moment of Inertia of Ship's Plane of Flotation—Atwood's System of Calculation of the "Wedges"—Several Conditions under which Stability remains Unchanged—Atwood's Methods of Equalising the "Wedges"—The late Mr. Scott Russell's Method of Treating Stability.

We now come to consider the general case of a body of irregular form, like a ship, and to ascertain what will be the leverages with which the weight of the body and the buoyancy of the water will operate to give motion to the body, presuming it to be inclined from its upright position, and then left free. Like Bouguer, and Atwood, and Dupin, we shall for the present treat the problem as one purely statical, and take account solely of the measures of the righting or capsizing forces which we find at work. We shall presume, as they do, what is of course not usually or really practicable, viz., that the ship or other body, when brought to a given angle of inclination, and into a given position which is not one of equilibrium, is for the moment, while we estimate the forces at work, herself as stationary, and immersed in water that is also as stationary, as if the body were floating upright in perfectly smooth water, and in undisturbed stable equilibrium. It is on this assumption that all the ordinary formulæ of statical stability are obtained. As a matter of fact, when a ship is inclined at an angle at sea, she is usually undergoing more or less oscillation, with her own centre of gravity rising and falling, and has to acquire her stability, whatever its amount, from the pressures of water which is itself undergoing continual movement. There is no fixed position of equilibrium for a ship so circumstanced. This point has been very well stated by Mons. L. E. Bertin, who says,* "rolling

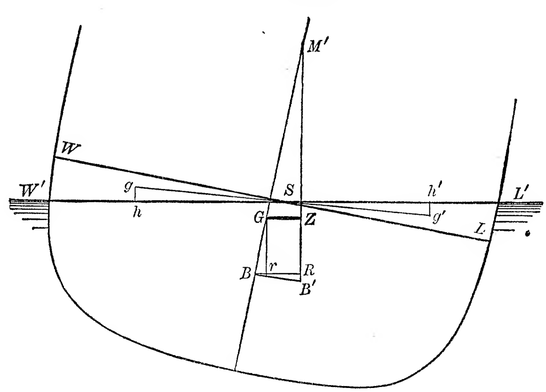
* See *Naval Science*, vol. iii. page 44.

is produced by the alternate variations of the different forces which the water exerts on the immersed hulls of ships. In consequence of these changes the position of equilibrium of the ship, that is to say, the inclination for which the moment of all the external forces is zero, undergoes a periodic motion of the nature of the motion of a pendulum. By virtue of its moment of inertia the ship cannot follow the motion of the position of equilibrium; it assumes round this a proper motion, which would become an ordinary pendulous motion if, at a given moment, the position of equilibrium remained fixed in a certain direction. In order then to understand the phenomenon, we may consider rolling as resulting from the superposition of a *roll of equilibrium* and of a *proper roll*."

All these matters have to be considered, of course, in any exhaustive account of a ship's stability under ordinary conditions at sea; but for the present, as we have said, we shall leave these dynamic questions out of consideration, and presuming the conditions to be statical, and the water in which the body floats to be at rest, shall proceed to examine the problem.

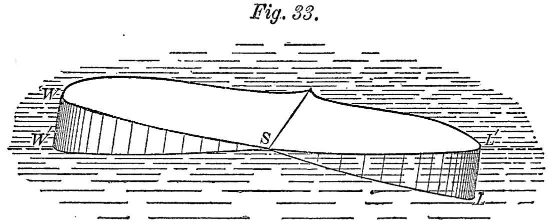
Let Fig. 32 represent a transverse section of a ship, of which

Fig. 32.



W L is the line in which the plane of flotation, when the ship is upright, is cut by the plane of the paper, the centre of gravity of the whole ship being at G, which we will suppose to be either in the plane of the paper, or projected perpendicularly upon it. Let B similarly represent the centre of buoyancy of the whole ship, or its projection. Now, suppose the ship to be inclined through an angle of a few degrees, by some external force that acts horizontally, and therefore does not alter the displacement of the ship, and let W′ L′ be the new water-line, or the line in which the new plane of flotation is cut by the plane of the paper. Let S be the point in which the two water-lines, W L and W′ L′, intersect each other. The point S, may not now be found at the middle point of W L (as it would if the angle of inclination were exceedingly small) because of the irregular form of the ship. The effect of the inclination has obviously been to lift out of the water a wedge-like body enclosed between the two planes of flotation, of which body W S W′ is the section, and to submerge on the opposite side of the ship another somewhat similar wedge-like body, of which the section is L S L′. These wedges, so to speak—commonly known as the wedges of emersion and immersion respectively—will each be bounded on the outside by the outside of the ship, and will therefore usually differ in external form, but they will be precisely equal in volume, for otherwise the whole displacement of the ship could not remain unaltered. They will be of the shape roughly indicated in Fig. 33, their size and form varying, of course, with every variation of size

Fig. 33.



and form in ships. In most ships the inclined water-line will not be symmetrical about a longitudinal axis as the upright water-line is, for the obvious reason that the breadths of ships differ above and below any given water-line, especially near the ends, and more especially near the stern. It is, in fact, as will more clearly appear presently, to the irregular and unsymmetrical form of the wedges of immersion and emersion that most of the labour and trouble of calculating a ship's stability is due. Still, the two planes of flotation must intersect each other in a longitudinal straight line (presuming no change of trim to occur), and of that line we will assume S to be the projection on Figure 32.

During the inclination and the consequent immersion of the wedge whose section is L S L′ and the emersion of the wedge W S W′, the centre of buoyancy was necessarily changed, and we have already seen (in the last chapter, foot-note, page 29) where it is now to be found. It will be situated at a point, B′, Fig. 32, which can be determined by the formula

$$BB' = \frac{v \times gg'}{V}$$

where V is the whole volume of displacement, v the volume of either of the wedges, and gg' the distance between the centres of gravity of the wedges. B B′ will be parallel to gg'. In the inclined position of the ship, the buoyancy will now act vertically upwards through B′ M′, and therefore perpendicularly to the new water-line, W′ L′, while the weight of the ship acts vertically downwards through G, in a line, G r, parallel to B′ M′. From B draw B R, and through G draw G Z, both parallel to W′ L′, or perpendicular to B′ M′.

We are now able to see clearly what will be the righting force acting upon the ship. We have only two forces to deal with, viz., the weight of the ship acting directly downward through G r, and the buoyancy acting directly upward, through B′ M′. Two parallel forces so acting constitute a "couple," and the effect will here be measured by multiplying the weight of the ship into the distance G Z.

At this point let it be observed that if gh and $g'h'$ be drawn perpendicular upon W′ L′, the distance, hh', will be the distance between the centres of gravity of the wedges of immersion and emersion measured along or parallel to the new water-line. Similarly, while B B′ represents the distance travelled by the centre of buoyancy in a direction parallel to gg', B R represents the distance travelled by it parallel to hh' or W′ L′. In the remainder of this investigation, therefore, we need no longer consider the distances gg', or B B′, but only the distances hh' and B R.

If α be the angle between the two planes of flotation, W L and W′ L′, this will also be the angle B G r, and therefore $Br = BG \sin. \alpha$.

We now have all the elements of the case before us.

$$GZ = rR = BR - Br = BR - BG \sin. \alpha.$$

$$\text{But} \quad BR = \frac{v \times hh'}{V}$$

Therefore,

$$G\,Z = \frac{v \times h\,h'}{V} - B\,G \sin. a.$$

This is Atwood's fundamental formula of statical stability. It obviously represents only the *leverage* with which the weight of the ship acts; to get the *moment* of stability, the value of G Z must be multiplied by the weight of the ship.

We saw in a former chapter that French naval architects usually call the height B M′, ρ; and the weight B G, a; and putting θ for the angle of inclination, they write the equation of stability thus:

$$M = P\,(\rho - a) \sin. \theta;$$

where M is the "moment" of stability, and P the upward pressure of the fluid. M is of course equal to W $\times$ G Z, and the above equation is equivalent to Atwood's formula just given, $\rho \sin. \theta$ being the equivalent of $\frac{v \times h\,h'}{V}$.

We now clearly see what has to be done in calculating a ship's stability at a given angle of inclination. We have to ascertain the volume of wedges of immersion and emersion, and take care that they are equal. If they do not come out equal at the first attempt, they must be made so by a process to be described hereafter. Being made equal, the moment of each wedge about the point S has to be ascertained, and the two moments have then to be added together, because, still calling v the volume of the wedge, we have

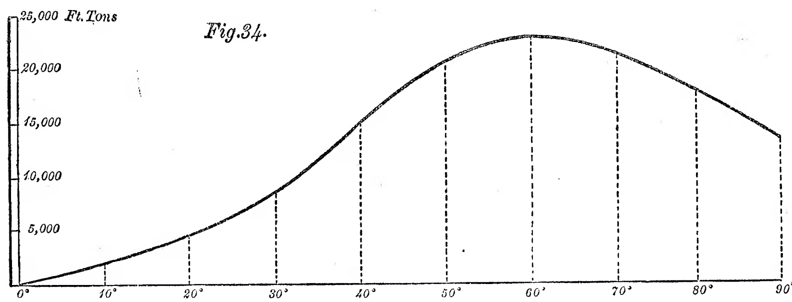
$$v \times h\,h' = v\,(h\,S + S\,h'),$$

and it is convenient to calculate h S and S h' separately. We also require to know the total volume, V, or the displacement, and the positions of the upright centre of buoyancy, B, and of the centre of gravity, G. Without each and all of these particulars, Atwood's formula cannot be applied; with them, the statical stability at any angle can be obtained with certainty and accuracy.

What has thus far been said will not enable a reader to make calculations of stability, but it will enable him to understand in what manner curves of stability are constructed, and what it is that they really represent. The common curve of stability represents nothing more than the lengths of G Z (or of its products when multiplied by the weight of the ship), obtained, by such calculations as have been indicated, or otherwise, at various angles of inclination, and thence inferred for all intermediate angles, by the common process of setting up the calculated G Z's as ordinates along a line representing angles of inclination, and drawing a curve through the extremities of these ordinates. This curve of G Z's will equally well represent the curve of moments of stability, if the *scale* be altered accordingly.

For example, Fig. 34 is the curve of stability of a mail steamer, having at the time for which it was calculated a displacement of

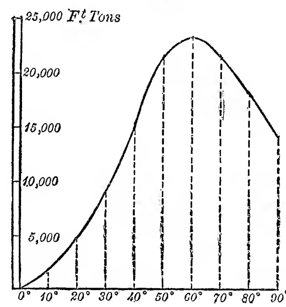
Fig. 34.



about 8,000 tons. It is in this case a curve of *moments* of stability, as will be seen on looking at the scale on the left of the figure which represents foot-tons, but the curve would be precisely the same if it represented leverages, or G Z's, only the ordinates would have to be read on a scale 8,000 times greater than that represented. The scale as shown represents 16,000 foot-tons for each inch of ordinate. If treated as a scale of G Z's each inch of ordinate would therefore represent 2 feet ($\frac{16000}{8000}$) of length of righting lever. In constructing this curve the length of G Z was found for a few specific angles of inclination, and multiplied by 8,000, a point on the curve being thus obtained for each such angle. A sufficient number of these points to fix the form of the curve being determined, the curve was then passed through them, and, that done, the stability at any intermediate angle could be measured by measuring the ordinate to the curve at the corresponding angle.

In examining or employing such curves one must be careful to observe the *scale* upon which the angles of inclination are set off along the base, otherwise very false impressions may be formed. No fixed relation at present exists between the scale adopted for angles along the base, and the scale adopted for either "moments" or "levers" of stability in setting up the ordinates. It may, therefore, happen that curves of stability constructed on scales relatively very different, may come together under consideration, and should the scales of ordinates happen to be alike or nearly alike, while the scales of abscissæ (angles of inclination), differ materially, a hasty view of them may lead to serious misconceptions. To illustrate this we give in Fig. 35 the same curve of stability as is shown in Fig. 34, with the ordinates on the same scale, but with the scale of abscissæ reduced to one-third of what it there is. For all purposes of measurement and careful comparison these curves are precisely the same; but any one looking at them, and not observing their differences of scale, might regard them as signifying very different amounts of stability. There is, however, as has been said, no difference between them.

Fig.35.



If we look again at the fundamental formula—

$$G\,Z = \frac{v \times h\,h'}{V} - B\,G \sin. a,$$

we shall see that its value must ordinarily vary not only with every change of inclination, but with every change in the displacement and draught of water, and with every change in the position of the centre of gravity.

As regards changes of inclination it is obvious that in a given ship v and $h\,h'$ will both vary as the angle a varies, because the magnitude and form of the wedges must change with the angle.

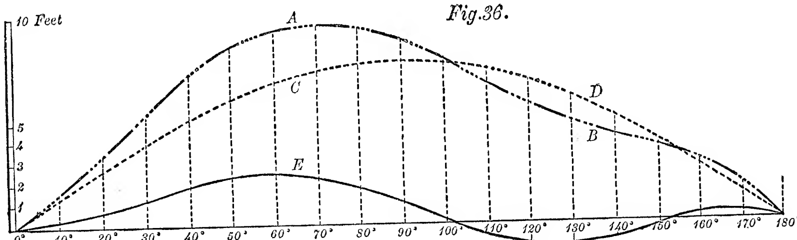
As regards changes of displacement and draught of water, B, and therefore B G, must vary, and v and $h\,h'$ will also vary.

As regards changes in the position of the centre of gravity, G, these affect only the value of the second part of the formula, viz., B G sin. a. This fact has suggested the device referred to in the last chapter, of imagining the centre of gravity, G, to be coincident with the centre of buoyancy, B, and getting out the value of G Z (which then becomes B R) on that supposition, and calling the value so obtained "surface stability," or "stability of form," or "stability due to form." We have already said, and it will now be clearly seen, that although the value of B R is independent of the position of the centre of gravity, it can only be called the *arm* of the *stability* couple, or regarded as the lever of *stability*, on the supposition that G is coincident with B, and therefore *gravity*, and not *form* alone, enters just as much into this measure of stability as into any other. At the same time, it is quite obvious that by the device of assuming G to be situated at B, and treating the length of B R as the measure of "form stability" at a given angle, you obtain a quantity which leaves the position of G out of consideration for the time being, and depends on the geometrical form and dimensions of the ship, and from this you can at once obtain the value of G Z for any and every position of G by subtracting B G sin. a from it, or by adding B G sin. a to it, should G fall below B; for it will be evident from our inspection of Fig. 32 that if G should fall below B, G Z would be greater than B R by the quantity B G sin. a.

One method, therefore, of constructing a curve of stability would be that of ascertaining the values of B R for successive angles of inclination, and constructing a curve with these values for ordinates, then (presuming G to be above B, as it usually is) setting off another curve of which the ordinates are the sines of the corresponding angles multiplied in each case by B G. The differences between the ordinates of these curves would furnish another set of ordinates, which would represent those of the ordinary curve of stability.

Fig. 36 illustrates this mode of procedure, A B being the curve of which each ordinate is the B R of the corresponding angle; C D,

Fig. 36.



the curve of which the ordinates are B G, sin. a at each point; and E F the curve of stability, the ordinates of E F being equal to the difference between the ordinates of A B and C D at each point.

The ordinary curve of stability is applicable only to one given draught of water, and one corresponding displacement for a given ship, and to one definite position only of her centre of gravity; we have now seen that it can have a more extended character given to it by making it a curve of B R's, instead of a curve of G Z's so to speak, because in this latter form it can be made directly available for all conditions of stowage in the ship, *i.e.*, for all possible heights of centre of gravity, care being taken to reduce it from a curve of B R's to a curve of G Z's, when the position of the centre of gravity becomes known, by cutting off from its ordinates at every part a length equal to B G × sine of angle of inclination.

Instead of proceeding, as is indicated in Fig. 36, the process may be conducted as is indicated in Fig. 37, in which A B is the curve

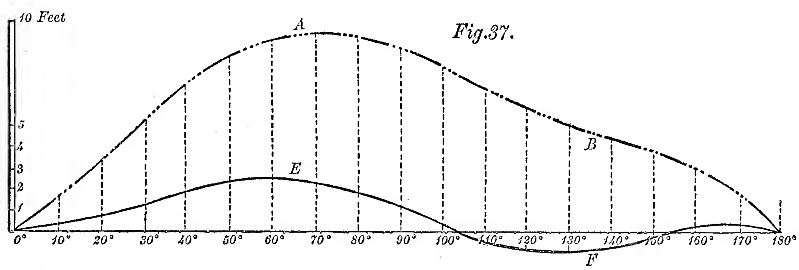


Fig. 37.

of B R's as before, but the ordinates of the curve of sines are set down from this curve of B R's, and a new curve is drawn through the points so obtained, this new curve, E F, being the ordinary curve of stability as in Fig. 36.

However the curve of stability may be obtained, it is obvious that if extended over a sufficient angle, it furnishes an exhaustive record of the stability, under the condition, that all the quantities given in the fundamental formula are known and remain unaltered. It also appears from what has just before been explained, that if the curve of B R's be constructed—which is the so-called curve of "surface stability," or curve of "stability of form"—it may be made available for indicating the limits within which the stability at any given angle of inclination must lie, provided the limits within which the centre of gravity, G, lies are known. For from the curve, A B, Fig. 37, can be set down two sets of points, one set corresponding to B G sin. a, when G is at its highest limit, and the other set corresponding to B G sin. a, when G is at its lowest limit; and if curves be passed through these two sets of points, the lower of the two (that nearest the base) will represent the least stability at every point which the vessel can have at the given immersion, and the upper curve will represent the greatest stability at every point, which she can have at that immersion.

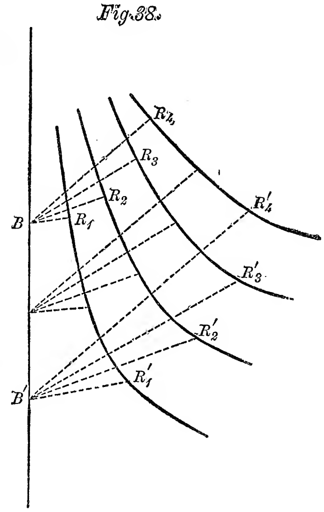
Next, we have to consider cases in which the immersion varies, whether from change of loading, consumption of fuel and stores, or any other cause; we shall here have our fundamental expression,

$$\mathrm{G\,Z} = \frac{v \times h\,h'}{\mathrm{V}} - \mathrm{B\,G}\ \sin.\ a,$$

undergoing changes of value from a cause which thus far we have not much considered, namely, the change of V or of the displacement. With V, v and $h\,h'$ will also usually change, from point to point, and so will the distance B G. For every given immersion, however, curves of stability can be constructed in accordance with either of the methods which have just before been described, and a very complete record of the stability may therefore be obtained for all possible conditions. It is manifest that if we calculate curves of B R's or curves of stability of form, for the greatest immersion contemplated, for the least immersion, and for a few intermediate displacements, and apply the method previously set forth for obtaining the maximum and minimum curves of actual stability at each of these immersions, we shall thus put ourselves into possession of all, or nearly all, the information which can be required concerning the statical stability of a given ship. The stability of a ship which has undergone injury, and become more or less water-logged, is deferred for special consideration hereafter.

A little reflection will show that as the "stability due to form" can be obtained for all degrees of displacement, and for all angles of inclination, without taking the actual position of a ship's centre of gravity into account, something more than we have yet considered may be done in the way of grouping the measures of her stability. For instance, it is perfectly practicable to select two extremes of displacement, one due to the weight at launching, and the other due to the greatest loaded weight, and also a certain number of intermediate displacements, and for each of these to calculate the position of the centre of buoyancy with the ship upright, and the length of B R at the given angles of inclination. Through the centre of buoyancy for any one of the given displacements, a line may be drawn at the requisite inclination to the horizontal, and equal in length to the calculated B R, and by repeating this process the B R's may be obtained for various displacements and inclinations. Curves drawn through the extremities of the lines so obtained, will furnish a ready means of obtaining the lengths of B R for all intermediate draughts of water at corresponding angles. This is illustrated in Fig. 38. B, B′ are centres of buoyancy, corresponding to the load draught and light draught respectively; B R_1, B R_2, B R_3, B R_4, are the B R's as calculated for each given angle of inclination at load draught, and applied as previously described; B′ R'_1, B′ R'_2, B′ R'_3, B′ R'_4, are the respective B R's for light draught. Other points upon $R_1 R'_1$, $R_2 R'_2$, &c., may be similarly obtained for intermediate draughts of water, and the curves, $R_1 R'_1$, $R_2 R'_2$, $R_3 R'_3$, $R_4 R'_4$, be drawn. They will evidently be in each case the locus of the feet of perpendiculars from the upright positions of the centre of buoyancy at different draughts of water upon the verticals through the centre of buoyancy when the ship is inclined at the given angle, and the length of B R for any position of the centre of buoyancy comprised between B and B′, and for either of the given inclinations, may be readily ascertained by drawing a line through the centre of buoyancy at the given inclination until it meets the curve corresponding to that inclination. But this extension of the subject we shall not pursue until we come to consider the more advanced stages of the science of stability, and more especially the system of M. Daymard, who grounds his exhaustive process of calculation upon such curves as those just explained.

Fig. 38.



We must now turn for a time to the question of what is known as "metacentric stability." Referring back to Fig. 32, page 46, let us suppose the angle of inclination, W S W′, and therefore B M′ R to be so very small that W and W′ almost coincide, and M′ becomes the metacentre. The point, S, may then be regarded as situated at the middle point of W L, or of W′ L′. The sectional area of either wedge will be $\frac{\text{S L} \times \text{L L}'}{2}$ and

$$(\text{as L L}' = \text{L S} \times \sin. a)$$

this will equal $\frac{1}{2}$ L S^2 sin. a. $h\,h'$ may be taken as equal to $2 \times \frac{2}{3}$ L S $= \frac{4}{3}$ L S, and, therefore, if v' be the sectional area of the wedge of immersion, we have

$$v' \times h\,h' = \frac{4}{3}\,\mathrm{L\,S} \times \frac{1}{2}\,\mathrm{L\,S}^2 \sin.\,a$$

$$= \frac{2}{3}\,\mathrm{L\,S}^3 \sin.\,a.$$

But obviously the sectional area, v', cannot represent v, for that is the volume of the whole varying wedge of immersion extending right fore and aft the ship. To represent this, we will suppose the fore and aft length of this wedge to be represented by x, and imagine this to be divided into an indefinite number of short pieces each of a length, $d\,x$, and then the quantity above given, if multiplied by $d\,x$ (or $\frac{2}{3}$ L S^3 sin. $a \times d\,x$), will be the value of $v \times h\,h'$ for one of these very short lengths of the wedge; and its value for every other such short length will be represented by a similar expression, in which, however, L S must be supposed to alter as the half-breadth of the ship at the water-line alters. Bringing the integral calculus to our aid, as a convenient means of summing up all these little quantities into one, we shall thus get

$$v \times h\,h' = \frac{2}{3} \int \mathrm{L\,S}^3 \sin.\,a\,d\,x,$$

and it will follow that

$$\mathrm{G\,Z} = \frac{2}{3} \int \frac{\mathrm{L\,S}^3 \sin.\,a}{\mathrm{V}}\,d\,x - \mathrm{B\,G} \sin.\,a.$$

Which is substantially Atwood's fundamental expression for the stability of a ship at an evanescent angle of inclination. Putting $y =$ the half-breadth at the water-line, and θ for the very small angle (instead of a), the above expression takes the well-known form,

$$\mathrm{G\,Z} = \frac{2}{3} \int \frac{y^3\,d\,x}{\mathrm{V}} \sin.\,\theta - \mathrm{B\,G} \sin.\,\theta.$$

If in this we put B G = 0, which is equivalent to making G coincident with B, we shall have

$$GZ = BR = BM \sin.\ \theta = \frac{2}{3} \int \frac{y^3\,dx}{V} \sin.\ \theta,$$

or

$$BM = \frac{2}{3} \int \frac{y^3\,dx}{V},$$

which is the general expression for the height of the metacentre, M, above the centre of buoyancy, B, and is identical with the expression given in the last chapter.

The result here arrived at is clearly independent of the position of the centre of gravity, and expresses the *geometrical* relation between the metacentre and the centre of buoyancy. A demonstration, conducted on the lines of the foregoing, but leaving the position of the centre of gravity altogether out of the question, would bring us to precisely the same expression for the distance of the centre of buoyancy from the corresponding pro-metacentre, or, in other words, for the radius of curvature of the curve of buoyancy at the point, viz.:

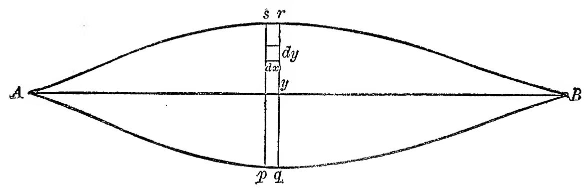
$$BM = \frac{2}{3} \int \frac{y^3\,dx}{V},$$

where y would be the half-breadth of the corresponding water-line.

The height of a ship's metacentre, and of any pro-metacentre, above her corresponding centre of buoyancy is thus seen, as we saw to be the case with prismatic bodies, to depend solely on her water-line breadths, her length, and her volume immersed, or displacement. It will also be seen that as her breadth at every point enters into the expression in its third power, or is multiplied by itself twice over, the breadth of the ship—not her breadth amidships only, but her breadth at each point all along her length—has very much to do with her *metacentric stability*, or stability at and near the upright position.

As the value just given for the height of the metacentre above the centre of buoyancy is in part identical with the *Moment of Inertia* of the water-line area (or area of the plane of flotation), and as the moment of inertia of this area is therefore frequently spoken of as an essential element of a ship's stability, it is desirable here (for the convenience of some of our readers) to briefly explain what is meant. Let Fig. 39 represent the area of a ship's water-line section, and let $p\,q\,r\,s$ be a very small rectangular portion thereof, the length of which $p\,q$ or $r\,s = d\,x$. Let us take an extremely narrow strip of this, parallel to the axis, A B, whose distance from A B is y and whose breadth is $d\,y$. Now the moment of inertia of a body being the sum of the products obtained by multiplying the

Fig. 39.



mass of each of its particles by the square of its distance from the given axis, we shall have (dealing with areas only in lieu of masses and therefore neglecting weight) that the moment of inertia about A B of the strip, whose length is $d\,x$ and whose breadth is $d\,y$, will be

$$d\,x \times d\,y \times y^2,$$

and the expression for the moment of inertia of the whole water-line area becomes

$$\frac{2}{3}\int y^3\,d\,x.$$

This is identical with the numerator of the fraction representing the height, B M, and therefore we may write the equation for the height of the metrecentre thus—

$$\mathrm{B\,M} = \frac{\text{moment of inertia of water-line area}}{\text{volume of displacement}}.$$

When we have to deal with large inclinations we must revert to Atwood's main formula,

$$\mathrm{G\,Z} = \frac{v \times h\,h'}{\mathrm{V}} - \mathrm{B\,G}\ \text{sin.}\ a.$$

In applying this to a ship, the displacement (V) and the positions of the centres of gravity and buoyancy (which are separated by the distance, B G) have to be either known or calculated; and B G sin. a is of course known for any given value of a. The quantities, $h\,h'$ and v are what have to be found in order to complete the known terms of the expression.

The manner in which Atwood deals with this part of the subject is as follows:—The object, of course, is to find by actual measurement and calculation the solid contents of the wedges immersed and emersed on opposite sides of the ship throughout its length, and the distance apart of their centres of gravity. A small portion of these wedges comprised between two transverse planes, W′ W L′ L and $w' \, w \, l' \, l$, is shown in Fig. 40. The planes are only a few feet apart

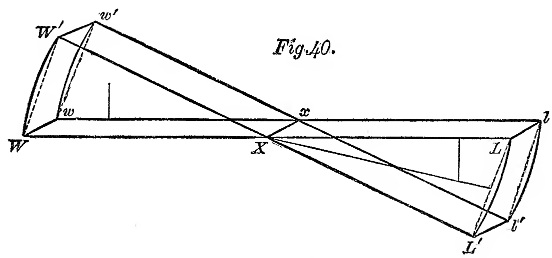


Fig. 40.

represented by $X x = W w = L l$. From the drawings of the ship the areas of X L L′ (embracing the segment bounded by the line representing the curved side of the ship) and of $x \, l \, l'$ can be calculated, and the mean of these two areas multiplied by the thickness, $X x$, will give the solid contents of as much of the depressed or immersed wedge (viz., a portion, $X x$, in length) as is shown in the figure. The aggregate of all such wedges, calculated so as to comprise the whole length of the ship, will be the solid contents of the whole immersed wedge. The solid content of the opposite or emersed wedge must be similarly calculated, and should be equal to that of the immersed wedge. If the two be found unequal, the second water-line must be raised or lowered, and the calculations repeated until an equality between the immersed and emersed wedges is established. The centres of gravity of these wedges are similarly obtained.

It follows from what has gone before that the stability of a ship at any given angle of inclination will remain the same all the time the displacement, the distance apart of the centres of gravity and buoyancy, and the form and magnitude of the wedges of immersion and emersion remain unchanged. The form of the vessel below these wedges may be altered in any way, and to any extent, without changing the value of G Z, subject to the condition that the total displacement and the distance apart of the centres of gravity and buoyancy remain unaltered.

But other variations may follow without changing the stability. It is obvious that the ship's sides, which bound the wedges of immersion and emersion, may undergo any changes of form which are compatible with the equal volumes of the wedges, and equal distances apart of their centres of gravity; in other words, compatible with $h\,h'$ and v remaining unchanged in amount. If we turn again for a time to the case of prismatic bodies (for simplicity's sake), in which we can take the sectional areas to represent the volumes, it will be obvious that $h\,h'$ and v will be the same in two bodies, of which one has the sides spreading outward above the water, but is vertical below the water-line (as in Fig. 41), and the other has them vertical above the water-line, and spreading equally below water, as would be the case if Fig. 41 were turned upside down.

Fig. 41.

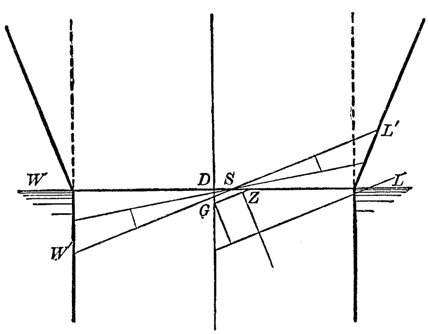
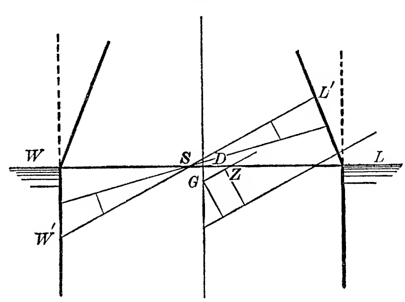


Fig. 42.



The stability of the two bodies will, therefore, be the same if V, B G, and the angle of inclination are the same.

Precisely the same may be said if the sides of the ship, instead of spreading outward above and below the water-line respectively, closed inwards, as in Fig. 42, and in that figure reversed. The stabilities in these two vessels would be alike, other things being equal as before. The *equations* of stability take the same form in all four of the above cases; but with like dimensions the *values* in the cases of Fig. 41 are different from those of case 42.

Again, if the sides of a body, in the region of emersion and immersion, are straight and at equal inclinations throughout, it is of no consequence whether they spread outwards above the water, as in Fig. 43, or spread outwards as in Fig. 44.

Atwood, in his "Royal Society" Papers, gives a demonstration of each of the foregoing equalities; he also shows that the equation of stability is precisely the same for a body with vertical sides, and

Fig. 43.

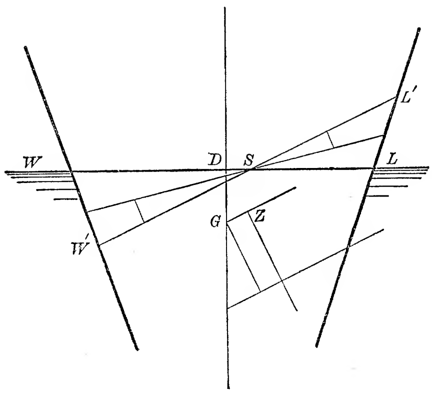
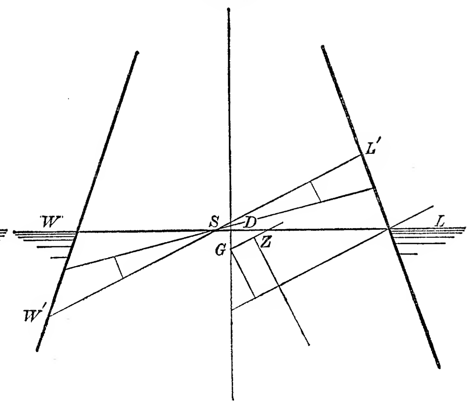


Fig. 44.



for a body whose sides form arcs of a conic parabola. The equation in both cases is

$$\text{G Z} = \frac{b^3 \tan. \theta}{24 \text{ V}} (\cos. \theta + \sec. \theta) - \text{B G} \sin. \theta,$$

where $b =$ the breadth at the water-line, and θ is the angle of inclination.

We have already seen how Atwood proceeded to apply his formula to an actual ship, by equating the volumes of immersion and emersion for the whole length of the ship. But in referring to this matter before, we simply pointed out that if these wedges did not prove to be equal, it was necessary to so shift the new water-line as to make them equal.

He suggests two methods of doing this—assuming for the time being that, although the sections of the body are of irregular form, they are all equal, and the body is prismatic.

It will presently be seen that both these methods rest upon the obvious consideration that, presuming the difference between the volumes of the wedges of immersion and emersion, which are first found, not to be very large—and with usual forms of ships it will not be—then it may be taken for granted that, if we divide the difference of volume by the area of the inclined water-line plane, we shall get the thickness of the slice that must be added to, or deducted from, the whole immersed volume of the ship in order to make the wedges equal. This will readily be seen by aid of what follows.

1*st Method.*—Let W C L (Fig. 45) be the section, W L the water-line with vessel upright; bisect it in D; through D draw W′ D L′ inclined to W L at the given angle of inclination, θ; let the areas of the figures, L D L′ and W D W′ (taken out to the ship's side whatever its form), be found, and suppose that the former comes out greater than the latter by an amount represented by a. From D along D L set off D S, so that

$$\mathrm{D\,S} = \frac{a}{\mathrm{W'\,L'}\sin.\,\theta}.$$

A line, w S l, drawn through S parallel to W′ L′ will cut off the area, L S l, very nearly equal to the area, W S w. Conseqently, $w\,l$ will be the correct water-line.

2nd Method (same figure).—This method consists in first calculating the whole immersed area below W′ L′, and if this is found unequal to the whole immersed area below W L with the vessel upright, and the difference is represented by a, a distance, D S, is set off equal to

$$\frac{a}{\mathrm{W'\,L'}\sin.\,\theta},$$

and the line $w\,l$ drawn as before.

Coming to actual ships, with gradually changing sections, it is no longer possible to simplify the investigations dealing with one section only, even in the determination of the inclined water-line, for the midship wedge sections may be equal and the whole volumes nevertheless unequal, or the midship wedge sections may be unequal and the whole volumes nevertheless equal. It is the equality of the *volumes* that has to be secured.

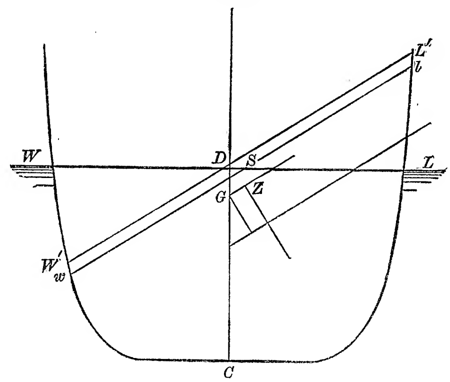
The first object is to fix the point, S, which is, of course, the same for all sections, and Atwood, in dealing with the matter, divides the vessel into a large number of equidistant cross sections, and calculates the areas of the triangles of immersion and emersion at every section with an *approximate* inclined water-line through the middle point of the upright water-line, by the rules for approximating to the areas of surfaces bounded on one side by a curve. From these sectional areas he obtains by similar rules the entire volumes of the wedges of immersion and emersion. If these prove unequal he obtains a point, S (Fig. 45) by means of the equation

$$\mathrm{D\,S} = \frac{\text{difference between the wedge volumes.}}{\text{Area of approximate water section,} \times \sin.\,a,}$$

a being the angle of inclination.

The water-line drawn through S is the corrected one. This being drawn on each cross section of the ship, the areas of the triangles of immersion and emersion are calculated anew, and being summed up as before, will give the volumes of the wedges of

Fig. 45.



immersion and emersion, which will now be equal. The moments of these wedges about the axis through S can also thus be found, and their sum ($v \times h\,h'$) introduced into the equation of stability. The whole process is well described by Atwood, in his 1798 Paper, in which he practically applies it to an actual ship, employing 34 vertical sections, at a common interval of 5 feet.

In his great work, entitled *The Modern System of Naval Architecture,* published in 1865, which may be regarded as a monument of the ability and labour which the late Mr. J. Scott Russell devoted to his profession, and which bears upon every page the impress of his own peculiar methods of treating naval science, the question of stability is discussed with even more than the author's usual originality and abandonment of known and accepted usages. He bases his stability investigations and modes of calculation upon the principle that the portion of the ship which is situated near the water-line may be regarded as the "shoulders" of the ship, tending to keep her upright, while the portion below may be regarded as tending to upset her. The amount of labour and skill devoted by him to the development of this view of stability was enormous, but it cannot be said to have secured for it general approbation and adoption. Recalling all this labour and skill, and cherishing, as we do, so many grateful and pleasant memories of the truly remarkable man who exercised them, it is with regret that we are unable to share the satisfaction felt by him with this mode of treating the stability of floating bodies, which, indeed, appears to us open to many objections.

In the first place, we are unable to regard the distinction between the so-called "shoulders" and the so-called "upsetting" part of a ship as sufficiently well defined, or as sufficiently well definable. Mr. Scott Russell's definition was this: "The shoulder of a ship is that part which, being under water when the ship leans over one way, is then left bare, out of water, when she leans as far over the other way." As an example, he takes the case of a ship leaning over to one side, far enough to *immerse* on that side 2 feet more of her skin than is immersed when she is upright, and then leaning over the other way, far enough to *emerse* 2 feet of her skin which was in the water when she was upright; then "those 4 feet of her skin in each side which lie between these extreme positions are what I call the shoulders of the ship." He goes on to say: "If we take away from the body of the ship the two shoulders, the remainder of the bottom, which never leaves the water, I call the 'under-water body of the ship,' and this under-water body is the part tending to upset her."

It is obvious that the above "definitions" are altogether too indefinite for any practical purpose. To say nothing of the oversight of describing the mere "skin" of the ship as "shoulders," the language employed leaves out of consideration altogether the fact that for every different angle of inclination there is a different volume for the wedges of immersion and emersion, and likewise leaves out of account all changes depending upon differences of draught. It is not surprising, therefore, that later on we find the author giving some extension to the previous definitions by saying that "we may take the shoulders as meaning those portions of a ship which, in heeling contrary ways, rise out of, and sink into, the water," although we here come upon the verbal anomaly of describing as a "shoulder," which is by its buoyancy to sustain weight, the portion of the ship which "rises out of" the water.

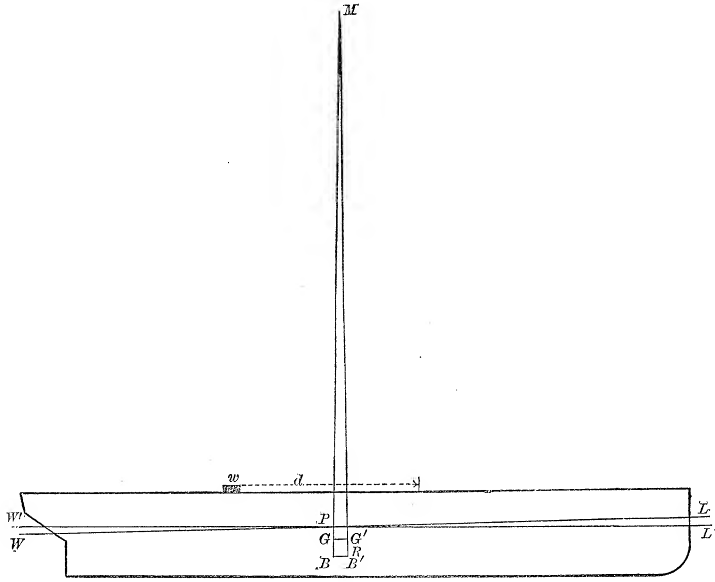
There is no reason to doubt that, notwithstanding this want of clearness in the definitions of what is intended—arising, as it is easy to see, from the peculiar method of treatment adopted—Mr. Scott Russell's system of calculation, carefully carried out, gives the same results as other methods. But with the progress of time the calculations of stability are being so extended, and are now made to comprise so many variations of draught of water and angular inclination, that no practical convenience can result from describing as shoulders the wedges of immersion and emersion, or from employing this somewhat involved and arbitrary mode of viewing the matter. The simpler method of treatment seems to be the usual one of estimating the lateral movement of the common centre of buoyancy, and from this ascertaining the "couple" of stability.

CHAPTER V.

Longitudinal Metacentre—General Expression for B M—Change of Trim—Effect on Stability of Admitting Water into Central Water-tight Compartments—Consequent Change in Height of Metacentre—Table of Heights of Metacentres—Inferences therefrom—Effect when Compartments are not Central—Table of Metacentric Heights under this condition—Inferences therefrom—Effect of Water-tight Decks.

HITHERTO we have only dealt with the transverse metacentre; we will now give a short account of the longitudinal metacentre, observing that the scientific principles underlying both cases are precisely the same. When a vessel is inclined longitudinally, the vertical line through the centre of buoyancy in the inclined position intersects the vertical through the centre of buoyancy in the upright or initial position in a certain point. In the limiting position, when the angle of inclination is very small, this point is called the longitudinal metacentre. We can, therefore, see that a determination of this point is of great service in determining changes of trim, caused by shifting the weights already on board a vessel in a fore and aft direction, or by putting moderate weights into her, or taking them out of her. Referring to Fig. 46, let W L represent the water-line of a vessel, B and G her centres of buoyancy and gravity respectively, B G M the vertical through these points. Now, suppose a weight, w, on board the vessel to be moved forward through a distance, d, the water-line now becoming W′ L′. Let B′ and G′ be the altered positions of the centres of buoyancy and gravity respectively. A vertical through these points will intersect the original vertical through B and G in a point, M, which, when the angle of inclination is indefinitely small, is the longitudinal metacentre for the water-line, W L. Through B draw B R perpendicular to B M. It is evident that B R = B M tan. θ, where θ is the angle of inclination of the vessel, also that the wedges, L′ P L and W P W′ are equal, and B R the distance moved through by the centre of buoyancy parallel to W L is equal to the horizontal

Fig. 46.



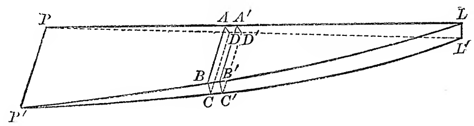
distance between the centre of buoyancy of the wedges, multiplied by $\frac{V}{D}$, where V = the volume of either of the wedges, and D = the volume of the total displacement, or expressed otherwise—

$$B\,R = \frac{\text{moment of wedges about P}}{\text{displacement}}.$$

Therefore, $$B\,M \tan.\,\theta = \frac{\text{moment of wedges about P}}{\text{displacement}}.$$

To determine the moment of the wedges about P, which axis evidently contains the centre of gravity of the water-plane repre-

Fig. 47.



sented by its trace, W L, referring to Fig. 47, let A B C D and A′ B′ C′ D′ represent sections of one of the wedges made by planes, perpendicular to the water-plane, W L, and to the longitudinal vertical plane of the vessel, at a horizontal distance, $d\,x$, apart; the vertical distance apart of the two planes at that place being x tan. θ; then the volume of the prism intervening between the two planes $=\mathrm{A\,D}\times\mathrm{A\,A'}\times\mathrm{A\,B}=x\,\text{tan.}\,\theta\times dx\times y=x\,y\,dx\,\text{tan.}\,\theta$. But $x\,y\,dx$ = the moment of the section about the axis, P P′. Supposing the wedges to be divided into an infinitely large number of such prisms; then the volume of the wedges would equal the moment of the water-line area about the axis, P P′, multiplied by tan. θ, and in the limiting position, therefore, the volume of the wedges is equal to the moment of the water-line area about the axis, P P′. Again, the moment of each of the small prisms = its volume ($x\,y\,dx$ tan. θ) $\times x = x^2\,y\,dx$ tan. θ; and, therefore, the moment of the wedges about the axis = the moment of inertia of the water-line area, W L, about the same axis multiplied by tan. θ.

$$\therefore \mathrm{B\,M}\ \text{tan.}\ \theta = \frac{\text{moment of inertia of water-line area} \times \text{tan.}\ \theta}{\text{volume of displacement.}}$$

$$\therefore \mathrm{B\,M} = \frac{\text{moment of inertia of water-line area}}{\text{volume of displacement.}}$$

In practice, it is usual to obtain the moment of inertia of the water-plane with reference to an axis corresponding to the middle ordinate, and having obtained this, the necessary modification in order to determine the moment of inertia about a parallel axis passing through the centre of gravity of the water section is readily obtained by deducting from the result the area of the water-plane multiplied by the square of the distance of its centre of gravity from the axis taken, this process depending upon a well-known property of the moment of inertia.

Having thus determined the height of the longitudinal metacentre above the centre of buoyancy, we may now call attention to longitudinal inclinations or changes of trim. In the first place, difference of trim signifies the difference of the draughts of water at the extremities of a vessel, and the vessel is said to trim by the head or stern respectively, as the draught of water there is the greatest. Suppose now the trim is changed, by moving a weight on board forward or aft through a certain distance, or by other means, then the change of trim is the sum of the increase in the draught of water at one extremity, and the decrease in the draught of water at the other extremity.

In Fig. 46 let θ be the longitudinal inclination as before-mentioned due to change of trim. The change of trim evidently equals $l \times \tan. \theta$, where l is the length of the water-plane; and the shift of the centre of gravity caused by moving the weight, w, through a distance, d, evidently equals $\frac{w\,d}{D}$,

$$\text{consequently, } G\,G' = \frac{w\,d}{D}$$

but $G\,G' = G\,M \tan. \theta$. Therefore, $\tan. \theta = \frac{w\,d}{D \times GM}$;

and change of trim $= l \tan. \theta = \frac{l \times w \times d}{D \times GM}$.

Supposing we wish to know the moment required to change trim 1 inch at the water-plane, we have

$$\frac{1}{12} = \frac{l \times w \times d}{D \times GM};$$

$$\text{therefore, } w\,d = \frac{D \times GM}{l \times 12};$$

and this may be expressed in words as follows:—The moment in foot-tons required to change the trim 1 inch at the water-line is equal to the displacement in tons, multiplied by the height of the longitudinal metacentre above the centre of gravity, in feet, divided by twelve times the length of the water-line.

When adding moderate weights to a vessel the change of trim is determined in the following manner:—The weight in the first instance is supposed to be placed in the vessel directly over the centre of gravity of the load-water-plane, which will cause her to retain the same trim, but to displace more water, dependent of course upon the amount of the weight. The weight may be now moved to its required position, and the question simply resolves itself into the change of trim when a given weight on board is shifted through a certain known distance in a fore or aft direction, taking into account, of course, the increased displacement due to the weight added.

When taking moderate weights out of a vessel an operation the reverse of that just described is performed, in order to determine the change of trim due to such readjustment.*

* The object of the longitudinal metacentre, and of its use in determining changes of trim, was discussed substantially as in the text, and with somewhat greater fullness, in a paper read by Mr. F. K. Barnes, of the Admiralty, at the Institute of Naval Architects in the year 1864.

The stability of a vessel fitted with water-tight compartments, and having water admitted to one or more of them by means of collision or otherwise, deserves consideration. It will only be necessary, however (after what has already been said about longitudinal trim), to consider the matter here in its relation to transverse stability. There may be several distinct conditions set up:—

1. A compartment may be totally filled with water which it completely encloses.

2. A compartment may be partially filled by water which it completely encloses.

3. A compartment may have water in it in free communication with the sea, and at the sea-level for all inclinations.

It is evident that in the first of these cases the stability will be affected in much the same manner as it would be were the enclosed water replaced by a solid body equal in weight, and having its centre of gravity in the same place. This is the case of a water-ballast compartment being wholly filled with water. If the surface of the enclosed water be such as to remain always below the surface of the sea during the rolling of the vessel, it is of little consequence whether it be wholly enclosed or in communication with the sea, the result being equivalent in each case to a corresponding loss of displacement. The admission of this water would, however, as will be obvious, bring about a change of position in the water-line, the centre of buoyancy, and the metacentre, which must be newly calculated, if it is desired to ascertain their new positions.

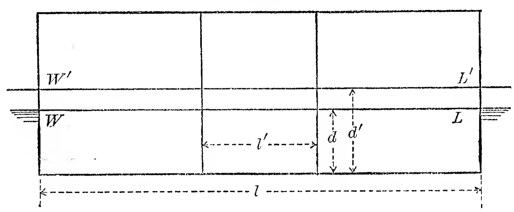
When, however, as in our 2nd case, a compartment is not full, and the volume of water within it, although completely enclosed, is free to alter its form and position as the vessel rolls, we have a wholly different state of things, something more even than a statical investigation of the stability at given angles being now necessary. This is the case of a vessel with a water-ballast compartment partly filled only; or of a vessel carrying liquid at large in a tank imperfectly filled; and may be approximated to in some cases by loose cargoes of quasi-fluid, grain, &c., badly stowed in bulk.

But although the complete determination of the change of stability induced by a case of this kind involves dynamical considerations, an indication of its amount, sufficient for most practical purposes, may be obtained from statical investigations, which take into account the form and position of the free water at various angles of inclination of the ship.

Mr. F. K. Barnes, of the Admiralty, of whose contributions to the science of stability we have so frequently had occasion to speak with praise, has dealt with this branch of the question also, and we need do little more than go over the ground which he long since laid out.* We may, however, deal somewhat differently from him with the details of the investigation.

Let us consider with him, in the first place, the case of a central compartment being laid open to the sea and filled with water, and let us, for simplicity's sake, presume the vessel to be prismatic, and of rectangular section. Fig. 48 represents its elevation, W L being

Fig. 48.



its water-line before the vessel is injured, and W′ L′ its water-line afterwards. As the *weight* of the body proper is not altered by the admission of the water, the displacement must be the same before and after the injury; from which it follows that the displacement of the two end compartments below the line, W′ L′, must be equal to the whole displacement below W L, and consequently (as the breadth is everywhere the same), if l be the whole length, l' the length of the inside compartment, d the depth below W L, and d' the depth below W′ L′, we shall have

$$d' \, (l - l') = d \, l$$

and

$$d' = d \frac{l}{l - l'} \, .$$

This, therefore, is the new draught of water.

The centre of buoyancy before injury is, of course, at one-half

* See a paper "On Water-Tight Compartments in Ships as Affording Security against Foundering," in the *Transactions of the Institute of Naval Architects*, for 1867, vol. viii.

the draft $\left(\frac{d}{2}\right)$ from the bottom; after injury it is $\frac{d'}{2}$, and the value of d' in terms of d has just been seen.

If b equals the breadth, then before the injury the height of the metacentre above the centre of buoyancy is

$$\frac{\frac{2}{3}\,l\left(\frac{b}{2}\right)^3}{l\,b\,d}=\frac{1}{12}\,\frac{b^2}{d}\,;$$

after the injury this height of metacentre becomes

$$\frac{\frac{2}{3}(l-l')\left(\frac{b}{2}\right)^3}{l\,b\,d}=\frac{1}{12}\,\frac{b^2}{d}\,(l-l')\,.$$

If we desire to know the height of these metacentres above the bottom of the vessel, we must in each case add the half-draught, or $\frac{d}{2}$, in the first case, and $\frac{d'}{2}$ in the second case.

Mr. Barnes prepared a table of corresponding heights of metacentres which is worth reproduction. In order to adapt it to a variety of cases, he assumed the breadth to vary from d, the original immersed depth, or draught of water, up to four times that amount; and he assumed the length of the central or injured compartment to vary from one-hundredth of the whole length to one-half of that length, l' equalling in succession $\frac{l}{100}$, $\frac{l}{10}$, $\frac{l}{4}$, and $\frac{l}{2}$. The following is the table slightly modified to suit our nomenclature, and omitting the cases in which l' is supposed to be $\frac{1}{100}$th part of l, leaving the remaining three cases:—

Value of b.	Value of l'.	Metacentre above Centre of Buoyancy.		Metacentre above Keel.	
		Before the Compartment Fills.	After the Compartment Fills.	Before the Compartment Fills.	After the Compartment Fills.
d	$\frac{l}{10}$	$\cdot 08\dot{3}\,d$	$\cdot 075\,d$	$\cdot 583\,d$	$\cdot 6305\,d$
,,	$\frac{l}{4}$	,,	$\cdot 0625\,d$	,,	$\cdot 7292\,d$
,,	$\frac{l}{2}$	,,	$\cdot 0416\,d$	,,	$1\cdot 0416\,d$
$2\,d$	$\frac{l}{10}$	$\cdot\dot{3}\,d$	$\cdot 3300\,d$	$\cdot 8\dot{3}\,d$	$\cdot 8\dot{5}\,d$
,,	$\frac{l}{4}$	,,	$\cdot 250\,d$	,,	$\cdot 91\dot{6}\,d$
,,	$\frac{l}{2}$	,,	$\cdot 16\dot{6}\,d$	,,	$1\cdot 16\dot{6}\,d$
$3\,d$	$\frac{l}{10}$	$\cdot 75\,d$	$\cdot 675\,d$	$1\cdot 25\,d$	$1\cdot 230\,d$
,,	$\frac{l}{4}$	,,	$\cdot 5625\,d$	,,	$1\cdot 2291\,d$
,,	$\frac{l}{2}$	,,	$\cdot 375\,d$	,,	$1\cdot 375\,d$
$4\,d$	$\frac{l}{10}$	$1\cdot\dot{3}\,d$	$1\cdot 200\,d$	$1\cdot 8\dot{3}\,d$	$1\cdot 7\dot{5}\,d$
,,	$\frac{l}{4}$	,,	$1\cdot 000\,d$	,,	$1\cdot\dot{6}\,d$
,,	$\frac{l}{2}$	,,	$\cdot\dot{6}\,d$	,,	$1\cdot\dot{6}\,d$

Mr. Barnes infers from this table that in all cases "when the breadth is equal to the depth, and to twice the depth, the height of the metacentre above the lower edge of the keel is greater after the compartment is injured than it was before; and, as already stated, we assume that the volume of the iron forming the sides of the compartment is equal to zero, and that when the compartment is empty the centre of gravity of the ship remains unaltered; consequently, also, the stability of the ship is in all these cases greater after the compartment is injured than it was before. It follows, therefore, that if sufficient freeboard be given to such ships, to admit of their immersion being increased to the extent due to the volume of any one or more of its compartments, they will be quite safe when the said compartments are injured. It also follows that ships of the above forms and relative proportions would be lost by going down bodily in the water and losing their freeboard, and not from losing their stability and turning over. The same remark is practically applicable to the case in which the breadth is equal to three times the depth.

"Where the breadth is equal to four times the depth, the metacentre falls slightly between the limits taken; but it manifestly rises again as the bulkheads are placed nearer the extremities of the ship.

"As the breadth increases above this in proportion to the depth, the relative depression of the metacentre by injury to the compartments will be increased; but it must be borne in mind that in such cases the metacentre, before injury to the compartments, would be exceedingly high."

The foregoing investigation and remarks assume that the injured compartment is exactly central, and that the ship, therefore, becomes additionally immersed without change of trim. If this assumption be not approximately correct, and if a strict investigation be needed, the same general consideration will apply, but the change of trim consequent on the admission of the water must be calculated, and the resulting change in the area of the water-line must be taken into account in the expression for the height of metacentre. In the case of the prismatic vessel of rectangular section, it will be obvious that any change of longitudinal trim, which does not immerse any portion of the top or deck of the vessel, nor emerge any part of the bottom, must, with any given displacement, give an increase of metacentric height, whether the vessel be uninjured or injured, because it must increase the length, and (in this case), therefore, the area of the load water-plane.

The case of a vessel divided into longitudinal water-tight compartments is also considered by Mr. Barnes, who assumes her to possess two longitudinal water-tight bulkheads equidistant from the sides, say one at a distance, b', from each side; and, as provision is always made for letting water into such compartments, if necessary, he assumes that the sea is let into both sides of the ship at once, the central space between the bulkheads being kept free of water. Using the same notation as before, and observing that in this case l remains always unaltered, and b only undergoes diminution, we shall have

$$d' (b - 2 b') = d b$$

and

$$d' = d \frac{b}{b - 2 b'}.$$

The centre of buoyancy, which before injury is $\frac{d}{2}$ above the bottom, is after injury $\frac{d'}{2}$, and the value of d' we now know.

The height of the metacentre above the centre of buoyancy, which, as we previously saw, is $\frac{1}{12}\,\frac{b^2}{d}$ before injury, becomes after injury

$$\frac{\frac{2}{3}\,l\left(\frac{b}{2} - b'\right)^3}{l\,b\,d}$$

Putting for b', $\frac{b}{n}$, this expression becomes

$$\frac{\frac{2}{3}\,l\,b^3\left(\frac{1}{2} - \frac{1}{n}\right)^3}{l\,b\,d}$$
$$= \frac{2}{3}\,\frac{b^2}{d}\left(\frac{1}{2} - \frac{1}{n}\right)^3$$

For the height of the metacentre above the bottom of the vessel, we must add the half-draught in each case.

In this case also Mr. Barnes has tabulated the heights of the metacentres for the cases in which b equals d, $2\,d$, and $4\,d$ respectively, and n equals 100, 10, 4, and 2 respectively. We omit as before the case of $n = 100$, and give the others :—

Values of b.	Values of n.	Metacentre above Centre of Buoyancy.		Metacentre above Keel.	
		Before the Compartments Fill.	After the Compartments Fill.	Before the Compartments Fill.	After the Compartments Fill.
d	10	$\cdot 08\dot{3}\,d$	$\cdot 042\dot{6}\,d$	$\cdot 58\dot{3}\,d$	$\cdot 6676\,d$
,,	4	,,	$\cdot 0104\,d$	,,	$1\cdot 0104\,d$
,,	2	,,	$\cdot 0000$	,,	∞
$2\,d$	10	$\cdot\dot{3}\,d$	$\cdot 170\dot{6}\,d$	$\cdot 8\dot{3}\,d$	$\cdot 7956\,d$
,,	4	,,		,,	$1\cdot 0416\,d$
,,	2	,,		,,	∞
$4\,d$	10	$1\cdot\dot{3}\,d$	$\cdot 6826\,d$	$1\cdot 8\dot{3}\,d$	$1\cdot 3076\,d$
,,	4	,,	$1\cdot 66\dot{6}\,d$	,,	$1\cdot 16\dot{6}\,d$
,,	2	,,	$\cdot 0000$	,,	∞

In view of this table Mr. Barnes makes the following remarks:—

"Where the breadth is equal to the depth, the height of the metacentre above the bottom of the vessel is greater after the compartments are filled than it was before; and, since from the suppositions we have made, the centre of gravity of the ship is unaltered, the ship, if stable before the compartments are filled, will be more stable after they are filled.

"When the breadth is equal to twice the draught of water, the metacentre descends when the vertical longitudinal bulkhead is very close to the ship's side, and it reaches its lowest position when the bulkheads are fixed somewhere between one-fourth and one-tenth of the breadth from the ship's side. From this position, as the bulkheads are placed nearer to the middle line, the metacentre continually rises as they approach the middle line. The same remarks apply when the breadth is equal to four times the draught of water; but the lowest position of the metacentre will not be reached until the bulkheads are relatively much nearer to the middle line than when the breadth of the ship is equal to twice the draught of water."

In lieu of, or in addition to, water-tight bulkheads, water-tight decks may be employed for dividing a ship into compartments, and this case needs some remark. If a compartment so formed be completely filled, without being laid freely open to the sea, the water so admitted is approximately equivalent to any equal weight in the form of a solid being introduced into the ship, with its centre of gravity in the position occupied by the centre of gravity of the volume of 'tween-deck water admitted. Presuming that the addition of this weight to the ship does not materially change the area or form of the water-line plane, then, whether the stability will be diminished, unaltered, or increased, depends upon whether the centre of gravity of the added weight be above, coincident with, or below that of the added displacement.* In the case of water admitted

* The above proposition is demonstrated in *Shipbuilding: Theoretical and Practical*, of which Mr. Barnes was one of the authors. The substance of the demonstration, which was doubtless due to him, was as follows:—Taking W as the original weight, w the added weight, a the distance of the centre of gravity of the latter above the original centre of buoyancy, and c the centre of gravity of the added displacement above the centre of buoyancy: then, θ being the angle of inclination, and the usual notation adopted, we shall have for the original value of the righting lever, G Z,

$$W \cdot BM \cdot \sin.\theta - W \cdot BG \cdot \sin.\theta;$$

and for the new value, after the weight, w, is on board,

$$(W + w)\, B_1 M_1 \sin.\theta - (W \times w)\, B_1 G_1 \sin.\theta. \qquad (1.)$$

from the sea, its centre of gravity never can be situated above that of the added displacement; and can only be near to it when it happens to be admitted above a water-tight deck, just beneath the original water-line of the ship. In all cases, therefore, the admission of water sufficient to fill such a 'tween-deck compartment must add to the stability, and usually must add considerably to it, the amount added being proportional to the size of the compartment and to its depth below the water's surface.

Now, if A be the volume of either of the wedges of immersion and emersion in the first case, and b the distance between their centres of gravity, and A_1 and b_1 be the corresponding volume and distance for the new water-line; then

$$W \times B\,M \sin.\ \theta = b\,A\,;$$

and

$$(W + w)\,B_1 M_1 \sin.\ \theta = b_1\,A_1.$$

Again,

$$B_1 G_1 = B\,G + G\,G_1 - B\,B_1\,;$$

(any reader who follows the argument can make for himself the diagram, putting B_1 a little above B, and G_1 a little above G); and therefore,

$$(W + w)\,B_1 G = W \cdot B\,G + w\,B\,G^* + (W + w)\,G\,G_1 - (W + w)\,B\,B_1.$$

But w B G + (W + w) G G is the moment of the weights added (w) about the original centre of buoyancy $= w\,a$; and $(W + w)\,B\,B_1$ is the moment of the additional displacement about the original centre of buoyancy $= w\,c$.

Substituting in equation (1.), we have the stability of the ship at the new water-line,

$$= b_1\,A_1 - W \cdot B\,G \sin.\ \theta - w\,(a - c) \sin.\ \theta. \qquad (2.)$$

The stability at the first water-line was

$$= b\,A - W\,B\,G \sin.\ \theta. \qquad (3.)$$

Subtracting (3.) from (2.), we have the difference of stability in the two cases—

$$= b_1\,A_1 - b\,A - w\,(a - c) \sin.\ \theta. \qquad (4.)$$

If we assume that the weight, w, which has been added, is moderate, and, therefore, that the form of the water-line area has not materially changed, and that the same may be said of the wedges of immersion and emersion, then $b_1\,A_1$ is practically equal to b A; and the difference of stability at the two draughts becomes simply

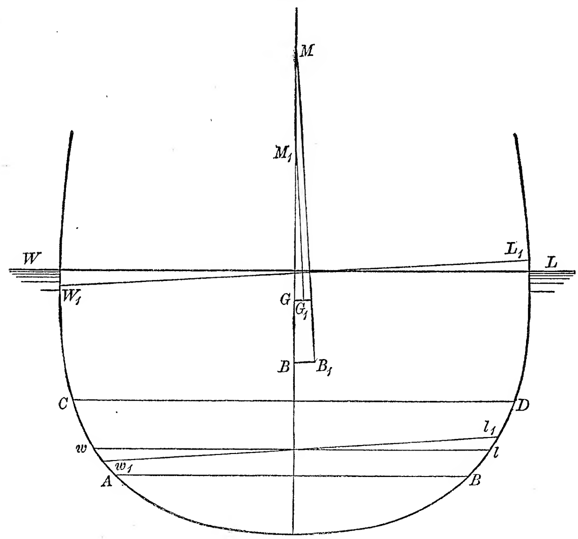
$$= -\,w\,(a - c) \sin.\ \theta\,;$$

and if $a = c$, this quantity becomes nothing, and there is no change in the stability consequent on the introduction of the weight, w. This is the case when the centre of gravity of the weight introduced coincides with the centre of gravity of the displacement added. If it be above it, a will be greater than c, and the stability will be diminished [by $w\,(a - c)$ sin. θ], and *vice versa*.

*There is an error at this point in *Shipbuilding: Theoretical and Practical*—probably a misprint, W being put for w.

To the case of a 'tween-deck compartment being slightly injured, and the water admitted to it being kept down to a small amount, Mr. Barnes gives special consideration, substantially as follows:—In Fig. 49 let W L represent the upright water-line of the vessel, and

Fig. 49.



$w\,l$ the surface of the free water in the compartment. Let G and B be the centres of gravity and buoyancy, and M the metacentre of the whole ship when upright with the free water on board. Now, let the ship have a very small inclination, θ, given to her, and let $W_1\,L_1$, $w_1\,l_1$, be the new water-planes corresponding to W L and $w\,l$, and B_1 the new centre of buoyancy. The shift of the free water from the upright to the inclined position occasions a transfer of the common centre of gravity of the ship and free water to the point G_1, along a line nearly parallel to $B\,B_1$.

Let M_1 be the intersection of the vertical through the point, G_1, with the original vertical, B M. Now, since the angle of inclination is very small,

$$BM = \frac{\frac{2}{3}\int y^3\,dx}{W},$$

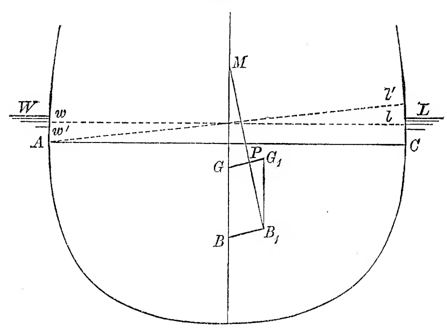
where y is the half-breadth of the water-line and D is the displacement. If the process of investigation which led to this expression be applied to the case of the free water it will be found that

$$G M_1 = \frac{\frac{2}{3}\int y_1^3 \, d x}{W},$$

where y_1 is the half-breadth of the free water surface; and it is evident that when $G M_1$ is less than $G M$ (or when $\frac{2}{3}\int \frac{y_1^3 \, d x}{W}$ is less than $\frac{2}{3}\int \frac{y^3 \, d x}{W} - B G$), the ship will float safely; if $G M_1$ is greater than $G M$ the ship is unstable; and when $G M_1$ is equal to $G M$ the equilibrium is indifferent.

Taking now the case, already mentioned, of free water in a compartment above a water-tight flat, $A C$, situated a very short distance below the water-line of the vessel, in Fig. 50, let $W L$ represent the

Fig. 50.



water-line of a vessel, $w \, l$ the surface of the free water in the compartment under consideration. Let G and B be the centres of gravity and buoyancy respectively, and M the metacentre. Let θ be the inclination, and B_1 the new position of the centre of buoyancy. It is evident that if from G a line, $G G_1$, be drawn equal and parallel to $B B_1$, that G_1 is the new position of the centre of gravity of the vessel, because the wedge of water within the vessel corresponds exactly with what is known as the wedge of immersion of the vessel itself, and the distance between G_1 and the line, $B_1 M$,—in other words, the length, $G_1 P$,—multiplied by the displacement of the ship, is the moment of the upsetting couple, and

$$= W \times B_1 G_1 \sin. \theta = W \times B G \sin. \theta.$$

The maximum upsetting force, up to an inclination of 90°, due to such water is evidently when the surface of the free water meets the water-tight flat at the vessel's side, for in that case the moment of inertia of the plane of flotation is the same as that of the free water surface; for any further inclination, the moment of inertia of the plane of flotation exceeds that of the surface of the free water, and the capsizing force therefore diminishes. The position of equilibrium will only be reached after much further inclination.

From the preceding remarks it will be clear that in such a condition as that just described a vessel will not float upright, as we have shown that the position is one of instability.

When a compartment is in free communication with the sea, and the surface of the water in it is at the sea level for any inclination, this condition evidently has the same effect as if that portion of the vessel so occupied with water were, so to speak, not part of the vessel's volume, and in this case the vessel would have an increased immersion in volume equal to the volume of water in the damaged compartment, and the centre of buoyancy and metacentre will therefore have a new position.

We may, however, further remark that when the moment of inertia of the surface of the free water on any water-tight flat about a longitudinal axis is equal to, or greater than, the moment of inertia of the water-plane about its longitudinal axis, provided also that the centre of gravity of the vessel is above its centre of buoyancy, the upright position of the vessel will be one of instability, and she will loll over to an inclination, dependent of course on the quantity of water in the compartment, until she arrives at a position of stable equilibrium. It must, however, be clearly borne in mind that this condition is based on the assumption that the centre of gravity is above the centre of buoyancy; if the centre of buoyancy were situated above the centre of gravity (which is very unusual in a ship), the upright position would be one of stable equilibrium.

CHAPTER VI.

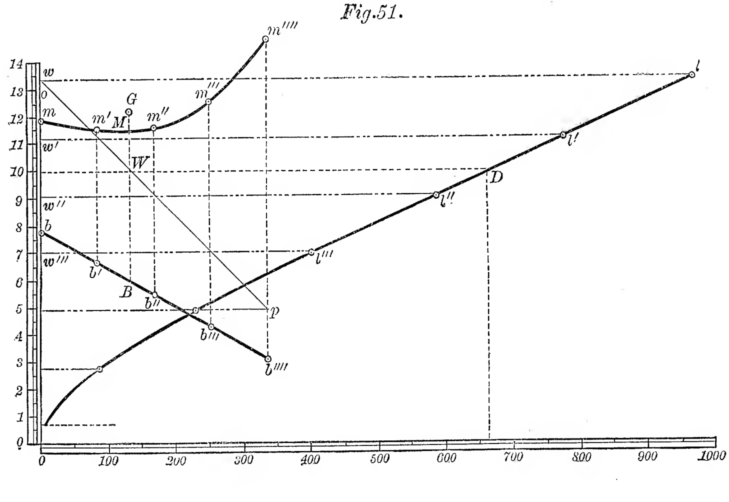
Purpose of "Metacentric Diagrams"—Their Construction—Association of Metacentric Diagram with Displacement Curve—Also with Midship Section of Vessel—Distinction between Curve of Metacentres and Metacentric—Form of Curve of Buoyancy Theoretically Considered—Variety in Form of Curves of Metacentres—Curves of Metacentres and of Buoyancy of some Prismatic Bodies—Examples of such Curves for Ships—Capacity and Stowage Diagram—Stability of the *Captain*—Stability of a Transatlantic Passenger Steamer—Stability of the *Austral*—Stability of a Raised Quarter-Deck Steamer—Other Examples of Stability Curves—Effect on Stability of Decreased Breadth and of Increased Freeboard—Relation of Beam to Stability Illustrated by Case of Prism—Further Examples of Stability Curves.

We have now seen that the height of the metacentre of a given ship above her centre of buoyancy, at a given draught of water, can be obtained by dividing the moment of inertia of the water-line area about the longitudinal middle line by the volume of displacement. In order to ascertain, and to describe graphically, the variations which the initial stability of a ship undergoes when her draught of water varies, it has become usual to calculate this height of the metacentre above the centre of buoyancy for several different draughts of water, and thus to get a series of such heights for the corresponding water-lines and displacements. Having obtained these, both the centres of buoyancy and the metacentres are set off on a diagram, and a curve is passed through each set of points. It is then assumed (and correctly assumed for ordinary forms of ships) that the height of the metacentre above the centre of buoyancy may be ascertained at any intermediate draught of water, lying within the limits of the calculated points of these curves, by simply measuring the vertical distance between the two curves. The positions of the centres of gravity, when known, can also be indicated in their correct relation to the metacentres and centres of buoyancy.

Such a diagram, known as a "Metacentric Diagram" (and first constructed, employed, and made public by that able naval architect and calculator, Mr. F. K. Barnes, of the Admiralty), is usually arranged as shown in Fig. 51. A series of horizontal lines, $w\,l$, $w'l'$, $w''l''$, &c., are drawn at heights representing on some convenient scale the various mean draughts of water at which the positions of the metacentres and centres of buoyancy have been calculated. An oblique line, $o\,p$, is drawn across these horizontal water-lines, inclined to them at an angle of 45 degrees; and from the points at which this line intersects the respective water-lines are drawn vertical lines, upon which are set off, on the same scale as before, the distances down of the centres of buoyancy, b, b', b'', &c., below the corresponding water-lines, and from these centres of buoyancy are set up the corresponding metacentres, m, m', m'', &c. A fair curve passed through all the metacentres so obtained, and another passed through all the centres of buoyancy, will respectively be a curve or locus of metacentres, and a curve or locus of centres of buoyancy.

For a "metacentric diagram" alone what has been described is all that is required; but it is often found convenient to have the scale of displacement represented on the same diagram. For this purpose a vertical line is drawn through the intersection of the oblique line before-mentioned with the water-line corresponding to the load-draught of the ship. From this vertical line are set off, on any convenient scale of tons, in a horizontal direction the calculated displacements at the draughts represented by the water-lines before used; a fair curve passed through all the points thus obtained is the

Fig. 51.



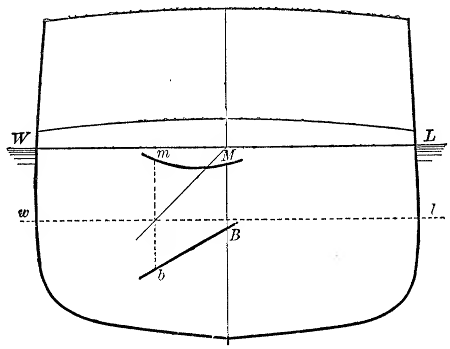
curve of displacement, as shown in Fig. 51. From this complete diagram the position of the metacentre can be obtained for any given displacement, draught of water, or position of centre of buoyancy within the given limits; and in like manner, for any given value of either of the latter, the other corresponding positions can be obtained.

For example, suppose it is desired to know the metacentric height corresponding to a draught of 10 feet, for the vessel whose various conditions are represented by Fig. 51:—A line drawn parallel to either of the water-lines, from the 10 feet mark on the vertical scale will cut the oblique line at the point, W. A vertical line drawn through that point will cut the curve of metacentres at M, which will be the required position of metacentre; the same vertical will cut the curve of centres of buoyancy at B, which will be the position of the centre of buoyancy corresponding to the 10 feet draught, and B M is of course the height of the metacentre above that centre of buoyancy. If the 10 feet water-line be continued, it will cut the curve of displacement at D, and a perpendicular dropped from that point on to the scale of tons will give the displacement corresponding to that draught of water.

Although the method just described is the one usually employed for arranging metacentric diagrams, there are many others that may be adopted according to the object in view when constructing them; but, whatever the method employed, the curves recorded are essentially the same. For example, it is sometimes found convenient, instead of placing the curve of displacements and the metacentric diagram in the relative positions shown in Fig. 51, to apply them in positions respectively at right angles thereto, observing that by constructing the curves, as we have seen, about an oblique line inclined at 45° to the water-lines and verticals, the same scale of linear measurements can be employed for measurements at right angles to each other. It is a mere matter of convenience, therefore, to decide in what relation to each other the displacement and metacentric diagrams shall be placed.

Again, for certain purposes it is convenient to have the curve of metacentres shown upon the midship section of the vessel, so that her metacentric stability under various conditions of draught may be at once clearly seen. This is illustrated by Fig. 52, which shows the metacentric curve and that of the centres of buoyancy, the lines used in their construction being suppressed, excepting the oblique line, which is necessary for defining the positions of the centres of buoyancy and metacentres at any given draught of water, as indicated by dotted lines. This figure represents the section and curves of metacentres and buoyancy for an actual ship about 370 feet long, and 46 feet broad. W L is her water-line, when fully equipped for sea, with cargo, coals, and water-ballast on board. M and B are respectively the position of the metacentre and centre of buoyancy corresponding to this condition; *w l* is her water-line,

Fig.52.



when ready for sea, but without cargo, coals, or ballast on board; and *m* and *b* are the corresponding metacentre and centre of buoyancy respectively.

In considering such curves it must be most carefully borne in mind that the locus of metacentres thus recorded is a wholly different locus from that which, following Bouguer, we call the *metacentric*, that being the locus of *pro-metacentres*, all pertaining to one given displacement, and this (Figs. 51 and 52) indicating merely the *rise and fall* of the *metacentre* as the ship's draught is changed, she always remaining in the upright position. We here see that much confusion is avoided by the introduction of the word "pro-metacentre;" it is no longer necessary to speak of the *metacentric* as a locus of metacentres, or a metacentric curve, but as a locus of pro-metacentres, the designations, "locus of metacentres" and "metacentric curve" being now strictly confined to such curves as that shown in Fig. 51, which are necessarily associated with varying displacements.

The diagram just mentioned presents to us at once the limit to which it is possible, at every draught of water within its range, to raise the centre of gravity of the ship and its cargo or other load, without sacrificing all metacentric stability. The metacentric curve furnishes this limit. Let any draught of water whatever, comprised within the diagram, Fig. 51, be taken, say that already used as an illustration, viz., 10 feet. All the time the centre of gravity is situated below the point M, the ship will possess metacentric stability, and will float in the upright position in smooth water. If the centre of gravity should be situated at some point, G, above the metacentre, M, then the ship, even in still water, will roll either to port or starboard, in search of a position of stable equilibrium. When and where she will find one can only be ascertained by bringing into use, in some form or other, the fundamental formula of stability for larger angles of inclination, viz:

$$\mathrm{G\,Z} = \frac{v \times h\,h'}{\mathrm{V}} - \mathrm{B\,G} \sin. \alpha.$$

In constructing such curves, although their use is limited within certain fixed light and load draughts of water, it is usually desirable to calculate points in the curves of buoyancy and metacentres lying somewhat beyond these limits, in order to ensure correctness in the curves within the required limits; otherwise, unless the number of points calculated is unusually large, there is a risk of inaccuracy in drawing in the curves through the points obtained.

The curve of buoyancy for ordinary ships does not depart very materially from a straight line, especially between the limits of load and light draught. When it does depart, it is usually concave to the base-line, but with special forms of vessels it is sometimes convex to that line, or concave upwards. It is easy to see why all this should be so. For any body of regular form, say a prismatic body of rectangular, or triangular, or parabolic section,* the centre of buoyancy at any given draught of water will be at a fixed proportion of the draught of water below the water's surface. With a rectangular section it will always be at one-half the depth, of course; with a triangular, at one-third; with a parabolic, at two-fifths, and so forth. The respective loci of all such centres of buoyancy, therefore, constructed as we have described, must of necessity, in each case, be a straight line. It is also easy to see that the straight line, representing the locus of centres of buoyancy for a rectangular section, must lie at a less inclination to the base (*i.e.*, to the horizontal) than a line representing the locus of such centres for a parabolic section, and this again at a less inclination than a line representing the locus of such centres for a triangular section, because the angle of inclination is in these cases that of

* Although ships are not of prismatic or parallel form, we may calculate for them a *mean section*, and then assume them to be prismatic, for the purposes of such general investigations as the present.

which the tangent is $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{2}{3}$, respectively. A ship of irregular but ordinary form would approximate to a compound of, or a compromise between these three figures, and as at the region of the water-line the form tends to the rectangular, the curve will usually be flattest, or at the least inclination to the horizontal near the top, or when the ship is most immersed, becoming somewhat steeper as the water falls and leaves the more rectangular parts of the section, and as the parts of the section remaining in the water approximate more and more towards the parabolic or triangular form.

Hence the usual concavity of the curve towards the base-line. On the other hand, great flare in an immersed section near the load water-line tends to lift the upper part of the curve of centres of buoyancy, and, therefore, tends to give the curve convexity downwards, but this convexity is very unusual. In his work on the *Modern System of Naval Architecture*, the late Mr. J. Scott Russell gives twenty-five different forms of sections, and tabulates their particulars; and out of these there are but five with curves of buoyancy which have their convexity towards the base, whilst three give straight lines (within the limits of the three draughts of water which alone are given), and eighteen exhibit downward concavity, the concavity in most cases being very small.

The "curve of metacentres" is susceptible of a great variety of forms. Being set up, at each calculated position, from the centre of buoyancy, and the curve of buoyancy, as we have just seen, approximating to a straight line, the form of the curve of metacentres reflects directly and somewhat closely the varying actual heights of the metacentres at different draughts of water.

Let us consider the cases of a few prismatic bodies of simple section. Taking, first, a body of rectangular section, floating with two sides horizontal and two upright, let us observe what form its curve of metacentres will take. The formula for the height of the metacentre above the centre of buoyancy is, as we have seen,

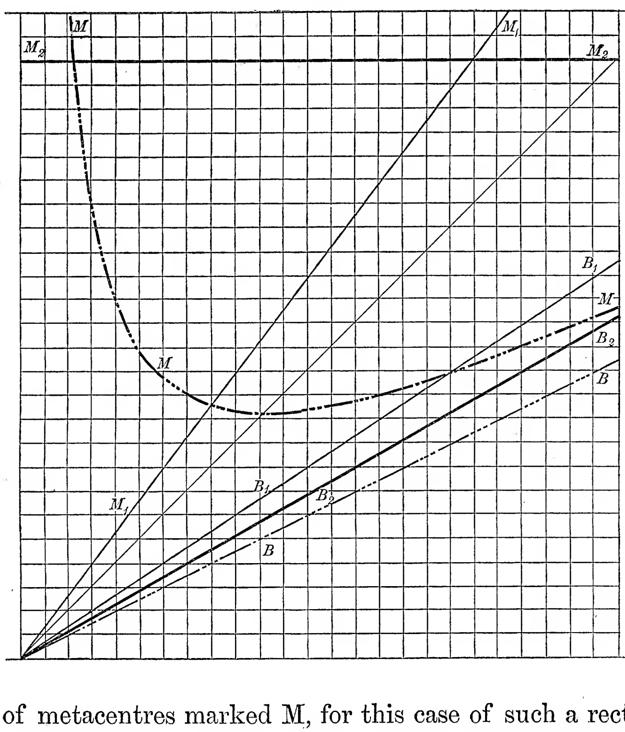
$$\mathrm{B\,M} = \frac{\text{moment of inertia of water-line area}}{\text{displacement.}}$$

Let us put b for the breadth of the body, and h for the immersed depth. Then, as the body is prismatic, we may take b to represent the *area* of the water-line, and $b \times h$ to represent the *volume* of the displacement; and we shall have

$$\mathrm{B\,M} = \frac{\frac{b^3}{12}}{b \times h} = \frac{b^2}{12\,h}.$$

This is the height of the metacentre above the centre of buoyancy for each draught of water. If we now give successive values to h (which means successive draughts of water), say $h = 1, 2, 4, 8, 16$ successively, we shall have $\frac{b^2}{12}, \frac{b^2}{24}, \frac{b^2}{48}, \frac{b^2}{96}$ and $\frac{b^2}{192}$ respectively, for the heights required. The height, B M, will obviously become less and less as the depth to which the rectangular body is immersed becomes greater and greater. This state of things is represented in Fig. 53, which shows the curves of centres of buoyancy marked

Fig.53.



B, and of metacentres marked M, for this case of such a rectangular prism. The height up of the centre of buoyancy above the base-line will of course be $\frac{h}{2}$ for every draught of water. The fraction $\frac{b^2}{12\,h}$, diminishing in value as the height, h, increases (the breadth, b, remaining unaltered), it is clear that the curve of metacentres will approach continually nearer to the curve of centres of buoyancy as the immersion is increased. On the other hand, as the height, h, diminishes, the value of the fraction $\frac{b^2}{h}$ increases, and the curve of metacentres rises continually higher above the curve of centres of buoyancy, and of course springs to immense heights as the draught of water approaches zero. If, for example, we assume the figures with which we have been dealing to be feet, and put the breadth $b = 25$ feet, the height, B M, will for a draught of 5 feet, be slightly over 10 feet, while for a draught of 1 foot, it will be 52 feet, for a draught of 6 inches, 104 feet, and for a draught of 1 inch, 625 feet, the *limit* being infinitely great.

If, instead of a rectangular prismatic body, we consider a triangular one with two equal sides floating apex downwards, calling b the breadth at the water-line (so that b will now vary with the draft of water), and h the immersed depth as before, we shall have

$$\mathrm{B\,M} = \frac{\frac{b^3}{12}}{\frac{b\,h}{2}} = \frac{b^2}{6\,h}$$

the displacement now obviously being one-half of what it was before. The form of the locus of metacentres, therefore, it is at once seen, will be very different from that in the last case (with rectangular section), because of the variation of b, the breadth, which will now take place.

If we call the angle of the immersed apex, $2\,\theta$, we shall then establish a fixed relation between b and h, because we shall have $\frac{b}{2\,h} = \tan.\,\theta$, and $b = 2\,h\tan.\,\theta$, and we can then write

$$\mathrm{B\,M} = \frac{4\,h^2 \tan.^2\,\theta}{6\,h} = \frac{2}{3}\,h\tan.^2\,\theta.$$

As the tangent of an angle increases from zero up to infinity, as the angle increases from 0 to 90° (being 1 at 45°), and as we have here the square of the tangent entering into the expression for B M, it is easy to see that the height of the metacentre above the centre of buoyancy increases largely with the increase of the angle immersed, and therefore we shall have a different locus of metacentres for every change of this angle. But, presuming the apex angle to be fixed, and called $2\,a$, we shall then have

$$B M = \frac{2}{3} h \tan.^2 a.$$

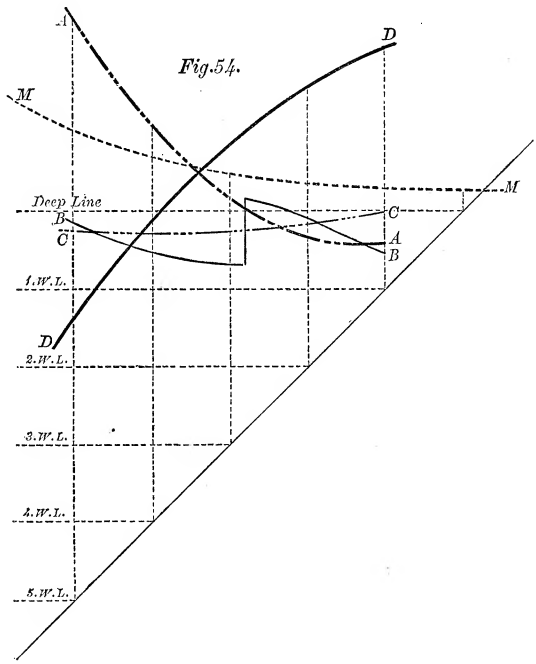
Tan.^{2} a will now be a fixed quantity, and this multiplied by $\frac{2}{3}$rds the immersed depth of the triangle, will be the height of its metacentre above its centre of buoyancy. That height will therefore vary directly with the immersed depth of the triangle, increasing as the depth increases, and diminishing as it diminishes. This locus of metacentres must therefore be a straight line, as illustrated in Fig. 53, for the triangle there shown by the line marked M_1, the corresponding locus of centres of buoyancy being marked B_1. It will be always a straight line for a triangular prism, whatever be the apex angle.

Loci of centres of buoyancy and metacentres for a prismatic vessel of circular section will both be straight lines, the locus of centres of buoyancy being an inclined line, and the locus of metacentres a horizontal line, the metacentre for every draught of water being at the centre of the section. This is shown in Fig. 53, where the line, B_2, is the locus of the centres of buoyancy, and the line, M_2, that of the metacentres. The locus of centres of buoyancy and that of the metacentres in this case meet when the whole section is immersed, the centre of buoyancy and the metacentre being then coincident.* In the figure they are only carried to the immersion of the lower semicircle.

It has been said that the curve of metacentres for actual ships is always convex to the base of the diagram; but this is not correct, as any one who carefully considers the expression for M B, and remembers what various forms ships assume, will readily discern. In his *Theoretical Naval Architecture,* Mr. Thearle gives a diagram of metacentric curves such as we produce in Fig. 54, in which curve A A represents certain of H.M. gun-boats; B B ships with a projecting armour shelf like that of H.M.S. *Devastation;* C C an ordinary broadside iron-clad; and D D, which is *concave* to the base-line, a man-of-war brig with a rising floor. M M he gives as representing curves of the character ordinarily met with, the value of B M usually increasing very rapidly as the draught diminishes. "This is especially the case," he correctly says, "in vessels having a very flat floor; as the moment of inertia of the water-plane

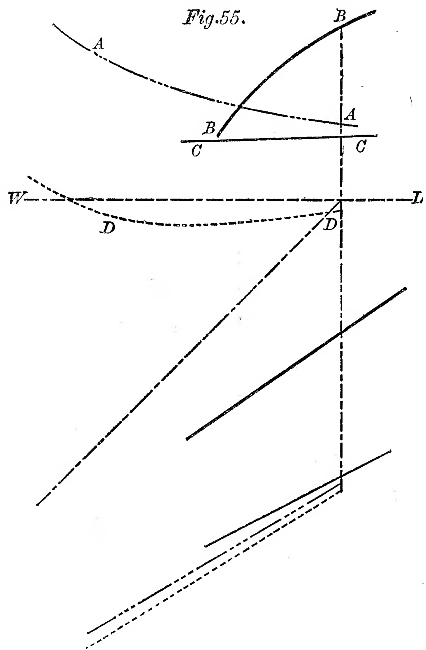
* In the paper, "On Curves of Buoyancy and the Metacentre for Vertical displacements," by Mr. Stanbury, in the *Annual* of the Royal School of Naval Architecture and Marine Engineering, for 1872, several equations for such curves will be found.

remains very considerable, while the displacement becomes almost zero. If, however, the vessel has a very rising or a hollow floor,



the curve of metacentres is flatter, being, indeed, in some cases slightly concave with regard to the water-lines in the diagram." In a "Note on the Geometry of Metacentric Diagrams," read at the Institution of Naval Architects in 1878, Mr. W. H. White gives corroborative diagrams calculated at the Admiralty, which we have reproduced in Fig. 55. We have brought the several curves into a single figure, making the horizontal dotted line the common load water-line of all the cases. A A is given as a common case for warships of ordinary form. B B illustrates the case of a "Symondite" or "peg-top" vessel, in which again the curve of metacentres is concave to the base-line, and the height of the metacentre above the curve of buoyancy decreases as we pass from load to light draught. C C is given as representing such a ship as the *Inconstant,* the curve of metacentres being nearly horizontal; and D D exhibits a case in which this curve drops slightly as it passes from the load draught to a lighter draught, then becomes horizontal, and afterwards rises as it passes to a still lighter draught. In speaking of a similar curve in his *Manual of Naval Architecture,* Mr. White says the condition of things described in our last sentence "frequently occurs in merchant-ships of deep draught (in proportion to their beam) when fully laden, and with approximately vertical sides in the region between the load and light lines." . . . "The highest position of the metacentre on these ships," he adds, "usually corresponds to the light line, and the lowest to a draught intermediate between the load and light lines. Very frequently the heights at the load and light lines are nearly equal, and the metacentric locus lies wholly below the load line. In war-ships, on the contrary, that locus usually lies wholly above the load line, the ratio of breadth to load draught being greater than the corresponding ratio for merchant-ships — the range of draught from the load to the light condition being much less for war-ships than for merchant-ships."

Fig. 55.



The principal value of these diagrams of metacentres at various draughts of water lies, of course, in the facility they give for indicating the stability of the ships at those various draughts when the corresponding positions of the centres of gravity are known. Mr. John Inglis, shipbuilder, of Pointhouse Shipyard, Glasgow, has taken a leading part in the development of this very important matter. Fig. 56 is a reduced copy of a diagram with which he has favoured the author, and which exhibits the system that he pursues.* The horizontal scales at the bottom of the figure are two in number, the one being a scale of feet for showing the height above the floor and ceiling available for the cargo, the zero being at the top of keel, and the cargo space commencing somewhat more than 2 feet above it; the other being a scale (also in feet) of draught of water, and the stowage of cargo, on the assumption that the cargo is of such a specific gravity as to bring the ship (when filled with it)

* See also White's *Manual of Naval Architecture*, second edition, p. 94.

to her designed load draught, and is poured in, so to speak, in such manner as to keep its surface always level. The upright scales are also two in number, the one being a scale of cargo capacity in cubic feet, from which may be read off the quantity of the homogeneous cargo on board at any time by means of the "curve of capacity" to be presently mentioned; and the other being a scale of feet, set off

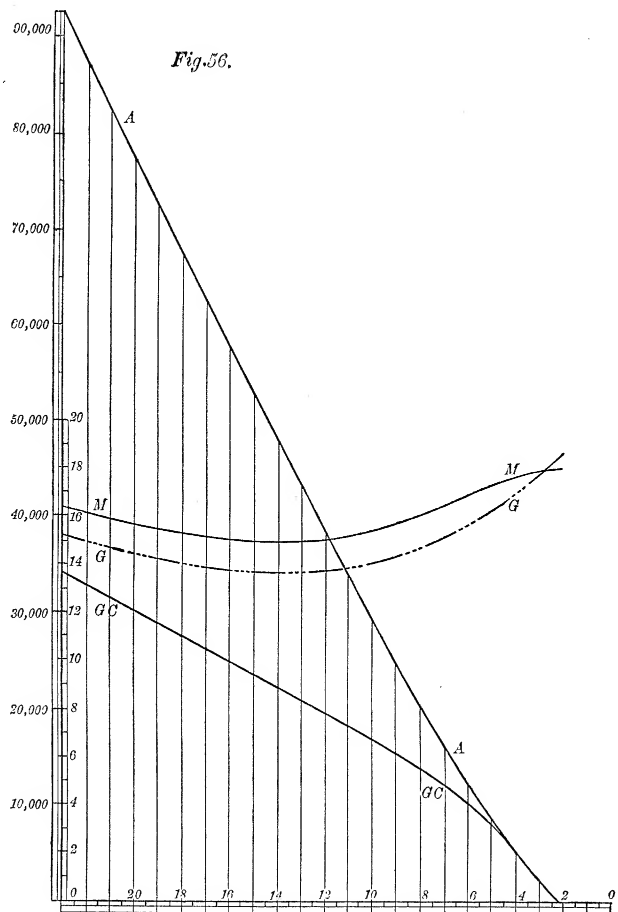


Fig. 56.

above the top of the keel, serving as a scale of heights for centres of gravity and metacentres. The curve, A A, is the curve of capacity before-mentioned; by taking any point upon this curve, and projecting it horizontally upon the vertical scale of capacity, the number of cubic feet on board (from which the number of tons which it weighs may be inferred, there being allowed in this instance 58·5 cubic feet per ton of dead weight, or a specific gravity of ·615) can at once be seen; while projecting the same point vertically downwards upon the horizontal scales, the corresponding depth of cargo in the hold, and the corresponding draught of water of the vessel, can be read off. The curve marked G C, exhibits from point to point the heights of the centre of gravity of the homogeneous cargo, these heights being read off from the vertical scale of feet at the side of the figure; the curve, G G, represents the heights of the common centre of gravity of both the ship and the cargo; and the curve, M M, represents the heights of the metacentres (within the requisite limits), both of these latter sets of heights being read off from the same vertical scale at the side of the figure. A comparison of the curves, G G and M M, at any point exhibits the measure of metacentric stability which the ship possesses, with the corresponding quantity of homogeneous cargo of the given specific gravity on board.

In arranging this diagram, and making the assumptions as to specific gravity of cargo and stowage on which it rests, it is presumed that the worst case which need arise is provided for, because the cargo is the lightest possible compatibly with its being homogeneous, and yet bringing the ship down to her load draught, observing that its assumed specific gravity (·615) is less than one-half that of coal. If any part of the cargo be heavier than the homogeneous cargo here considered, it may be inferred that this heavier part may be placed low, so as to bring down the centre of gravity and add to the stability—add to it, both by being itself placed low, and by displacing, so to speak, part of the lighter homogeneous cargo. It may be feared, however, that the exigencies of trade under which ships are loaded do not always admit of the heaviest portion of the cargo being placed low down in the hold; in fact, cases have often come to our knowledge, and must have come to the knowledge of many, in which parts of machinery, armour-plates,* and other heavy materials have come to the wharf for shipment after most of the cargo was on board, and doubtless have often brought the centre of gravity so near to the metacentre as to leave much too small a margin of stability for sea conditions.

* We remember an instance of two ships, sailing from the Humber on successive Saturdays, being sunk on the Dogger Bank in consequence of armour-plates delivered late for shipment, and stowed high, getting adrift, and breaking their way out through the bottom.

But the greater the tendency that exists in trade conditions to disturb the conditions of stowage which science would suggest, and which the shipbuilder would fain provide, the greater is the necessity for all those who have to do with ships being thoroughly informed respecting the elements essential to safety; and this consideration it is which adds so much to the merit of Mr. Inglis, and of all those who contribute to formulate, and exhibit in diagrams, the actual conditions of merchant ships.

An examination of the diagram, Fig. 56, reveals the fact that when all the cargo is out of the ship, the curve of centres of gravity has crossed the curve of metacentres, and the metacentre has fallen below the centre of gravity. In the light condition, therefore, the ship is unstable in the upright position, and needs ballast to enable her to float in that position. How much ballast was essential to safety this diagram does not show, because that depends upon the greater or less rapidity with which the ship acquires stability as she inclines from the upright, and upon the magnitude of the angle of inclination, through which the continued acquisition of stability proceeds. This can only be shown by the ship's "curve of stability," or by some equivalent means—a fact which should put the reader on his guard against attributing to these "curves of metacentres" and of centres of gravity any more value than they possess as indications of the stability of vessels, *in the upright or nearly upright position* in various conditions of lading. They cannot of themselves furnish any complete or satisfactory account of a given ship's stability. Unless more is known of it they must not be relied upon, even when indicating a good "metacentric height." * When curves of stability at various draughts of water have once been worked out, or when similar facts are known for ships of closely similar forms, then the relation between the metacentric curve and the curve of common centres of gravity of ship and cargo may be sufficient; but the "metacentric stability" of cargo-carrying ships cannot alone be regarded as a sufficient indication of safety at sea, even for a steamship, and still less for a sailing ship.

A signal instance of the impropriety of taking the curve of metacentres to furnish a complete account of a ship's stability is illustrated in Figs. 57 and 58. Fig. 57 shows the relative positions

* The "metacentric height" is the height of the *metacentre* above the *centre of gravity*.

of the metacentre, centre of gravity, and centre of buoyancy under various conditions for H. M. late S. *Captain*, and Fig. 58 shows the curve of stability of the same ship calculated to a mean draught of 25 feet 4 inches, or corresponding to a displacement of 7,907 tons, and with a metacentric height of 2·66 feet. Referring in the first case to Fig. 57, M M is there the curve of metacentres, and B B the curve of centres of buoyancy at corresponding draughts of water. G_1 the centre of gravity for a displacement of 7,907 tons with 617 tons of coal, and fully equipped, giving a metacentric height of 2·66 feet, at a mean draught of water of 25 feet 4 inches; G_2 is the centre of gravity corresponding to a displacement of 7,790 tons with 500 tons of coal, and fully equipped, giving a metacentric height of 2·6 feet at a mean draught of 25 feet $\frac{1}{2}$ inch. Similarly, G_3, G_4, G_5, are centres of gravity corresponding to different conditions of the ship with varying displacement, but in each case the metacentric height is less than that due to a displacement of 7,907 tons, including 617 tons of coal, and the ship fully equipped at mean draught of 25 feet 4 inches, viz.:—2·66 feet. Now, referring to Fig. 58, which is the curve of stability calculated to those conditions of the ship which give the greatest metacentric height, we see that the curve reveals the fact that the range of stability is only $54\frac{1}{2}°$, and the maximum stability is reached at 21°, where it is only 7,100 foot-tons; or the arm of the righting couple is less than 1 foot. These features of danger would in many cases be overlooked should the curve of metacentres alone be held to afford sufficient information concerning a ship's stability.

Fig. 57.

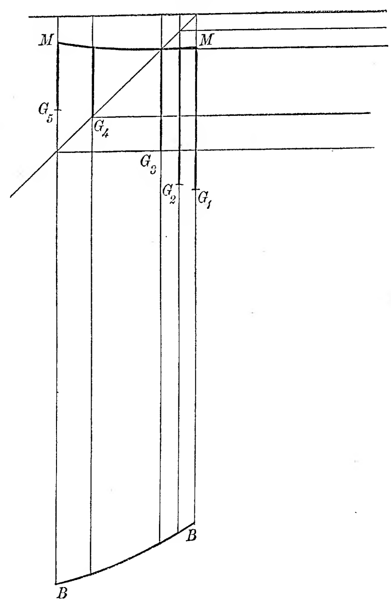
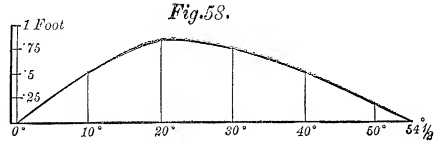


Fig. 58.



It may be added that Mr. White, in his work before quoted, gives an example of a ship in which the metacentre is below the centre of gravity, not only when the ship is light (as in Fig. 56), but also when she is loaded, and he says with reference to it, "this vessel represents a class which is successfully employed in certain trades, with the frequent use of water-ballast when homogeneous cargoes are carried."

In Figs. 59 and 60 we give the case of a transatlantic passenger steamship, with tracings and full particulars of which we have been favoured by Messrs. J. & G. Thomson, of Glasgow. The curve of metacentres, Fig. 59, shows the relative positions of the metacentre and centre of gravity of the ship in several conditions. The following are some of the valves of the "metacentric height":—

A draught of water of 25 feet corresponds to the conditions of 2,500 tons of cargo, and 900 tons of coal being on board. G_1 is the centre of gravity in this case, when the cargo is all below the lower deck, and the coal-bunkers are full. The metacentric height is then 2·6 feet. If the cargo be supposed homogeneous and to occupy the holds and 'tween-decks up to the main-deck, the centre of gravity is then raised 1·5 feet, leaving a metacentric height of 1·1 feet.

A draught of 23 feet corresponds to the condition of the cargo on board being 2,400 tons, with coals out, but ballast-tanks full. G_2 is the centre of gravity in this case, when the cargo is all below the lower deck. The metacentric height is then 2·9 feet. If the cargo be supposed homogeneous, and to fill the ship up to the main-deck, the centre of gravity is then raised 1·9 feet, leaving a metacentric height of 1 foot.

A draught of 22 feet corresponds to the condition of 2,300 tons of cargo being on board, the ship being without coals or water ballast. G_3 is the centre of gravity in this case, when the cargo is all below the lower deck. The metacentric height is then 1·7 feet. If the cargo be homogeneous, and extend up to main-deck, the centre of gravity is then raised 2·0 feet, leaving no metacentric height, the centre of gravity in this case being ·3 foot *above* the metacentre. If now 900 tons of cargo be taken out and 900 tons of coals (the full supply) be put into the bunkers, the draught of water will of course remain the same, but, presuming the remaining 1,500 tons of cargo be stowed away below the lower deck, G_4 will be the new centre of gravity, giving a metacentric height of 1·2 feet. But if the 1,500 tons of cargo be homogeneously disposed up to the main-deck, the centre of gravity will be raised 1·1 feet above G_4, leaving a metacentric height of ·1 foot.

With coal-bunkers and water-ballast tanks full, but with no cargo on board, the ship draws 18·7 feet, G_5 is her centre of gravity, and her metacentric height is ·5 foot.

Fig. 59.

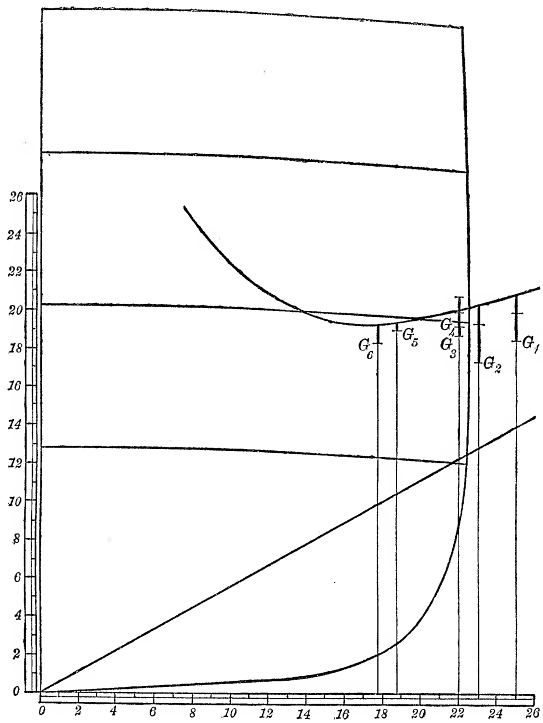
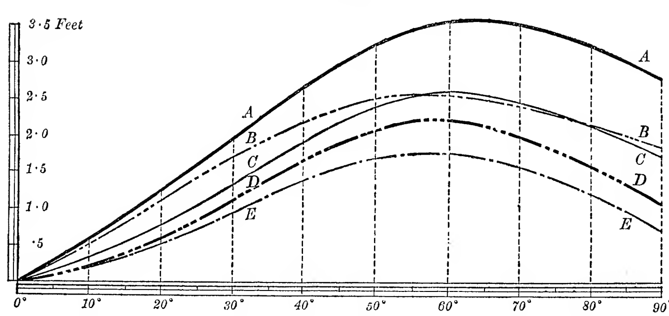


Fig. 60.



With 550 tons in coal-bunkers and water-tanks all full, the ship draws 17·7 feet, and her metacentric height is **1** foot, G_6 being then her centre of gravity.

With coal-bunkers full and no water-ballast or cargo on board, the draught is an inch or two less, but the centre of gravity is then

·9 foot above the metacentre. The necessity of the water-ballast is in this case manifest.

In remarking upon this case, Messrs. Thomson say:—"If we assume that the cargo is homogeneously stowed to the main-deck instead of to the lower, then, instead of 2·6 feet, 1·7 feet, and 2·9 feet, as given above, we shall have 1·1 feet, ·3 foot, and 1·0 foot. These last conditions are neither of them likely to exist, as in most cargoes there is sufficient variation in density to allow of the centre of gravity of the cargo being very much lower than it would be if homogeneous, by merely putting the heavier parts below. From this, however, it will be seen that the worst possible condition the ship can get in, when water-ballast is available, is with a metacentric height of 1 foot. The condition of no cargo in, but all bunkers and ballast-tanks full, is shown. The metacentric height in this condition is ·5 foot. This is scarcely sufficient to enable her to be worked easily in port. But with her bunkers filled to the lower-deck, and ballast-tanks full, she would have a metacentric height of ·9 foot."

Fig. 60 shows the stability of this ship under various conditions, at all angles of keel from the upright to beam-ends position. The following table gives the principal particulars in a convenient form:—

	To Lower-Deck.				To Main-Deck.			
Draught.	22 ft.	22 ft.	23 ft.	25 ft.	22 ft.	22 ft.	23 ft.	25 ft.
Condition.	1500 Tons Cargo, Coals in.	2400 Tons Cargo, Coals out.	2600 Tons Cargo, Coals out, Ballast-tanks full.	2500 Tons Cargo, Coals and Stores in.	1500 Tons Cargo, Coals in.	2400 Tons Cargo, Coals out.	2600 Tons Cargo, Coals out, Ballast-tanks full.	2500 Tons Cargo, Coals and Stores in.
GM, ...	1·205	1·7	2·9	2·6	·17	− ·31	1·0	1·1
BG, ...	6·78	6·28	4·70	4·55	7·8	8·26	6·61	6·12
Displacement, ...	6,450	6,450	6,852	7,575	6,450	6,450	6,852	7,575
Righting moment when deck edge is immersed, . . .	6,654	8,286	13,656	12,975	3,310	1,805	6,136	6,423
Maximum righting moment, ...	13,867	13,384	24,701	19,316	8,385	5,830	11,785	9,809
Righting moment at 90°,	7,160	11,255	19,528	13,783	806	0	4,933	2,878
Angle of maximum stability, ...	59·25°	60°	64°	60°	56°	51°	59°	50°

The curve of greatest stability, A A, Fig. 60, corresponds to the condition of 23 feet draught, with 2,600 tons of cargo stowed below lower-deck, with all the coals consumed, but with her water-ballast tanks full. This would roughly correspond to the state in which such a ship, having left Europe with a full cargo of European exports (which are usually much heavier, bulk for bulk, than a homogeneous cargo filling the ship up to her main-deck at the same draught) would, after a protracted passage, arrive at New York.

The next best condition as to amount of stability is that illustrated by curve, B B. The ship is still stowed as just stated, as to cargo, but now has her bunkers full of coal in lieu of water-ballast. She then draws 25 feet of water.

The curve of stability next in order of magnitude is C C, corresponding to the same amount and stowage of cargo as before, but with coals and stores consumed, and the water-ballast tanks empty, the draught now being 22 feet.

On comparing the curves, B B and C C, it will be seen that the lever or arm of stability at moderate angles of inclination is considerably reduced as the coal and stores are consumed; but at the large angle of 56 degrees the two curves cross, and the lengths of the righting levers become equal in amount; while from that inclination onward they do not greatly differ. The *amount* of the stabilities, of course, differ in proportion to the differences of displacement.

Curve D D illustrates the state of the stability (as to righting lever) at a draught of 22 feet with coals in, and 1,500 tons of cargo below the lower-deck.

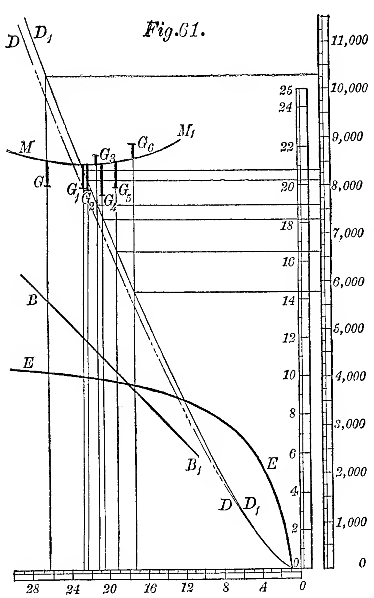
In all the above cases it will be seen that the righting lever increases up to angles of nearly 60 degrees, and the lowest of the curves indicates a maximum length of lever of over 2 feet.

The worst condition in which a ship of this description need find herself, as regards stability, is that of having on board at sea homogeneous cargo stowed up to the main-deck, and with all her coals and stores consumed. In this remark we take no account of the ship's condition when without cargo, because, however inconvenient it may be to have ships (as so many recent ones are) devoid of initial stability in port, or possessing extremely small stability there, their condition *ought* to be understood, and danger *ought* to be avoided. But a ship at sea is liable to various displacements and to protracted voyages, and may unavoidably find herself with coal and stores gone, but cargo remaining, and in this state exposed to stormy weather. This vessel would then have no metacentric stability without water-ballast, but, with water-ballast in, she would possess the amount of stability indicated by curve E E, with a maximum righting lever of $1\frac{3}{4}$ feet.

It is true that such a ship as this would have much less stability than the last-mentioned at large angles of inclination, with her coal on board, and a homogeneous cargo to the main-deck, then drawing 25 feet of water, and therefore too deep to use water-ballast freely; but she would still possess a very large range of stability, and her stability could be continually increased by the substitution, weight for weight, of water-ballast for coal consumed, always approaching the condition indicated by curve E E.

Fig. 61 represents the curve of metacentres of the Orient Line steamship *Austral*, as produced at the time of the inquiry into her loss. BB_1 is the curve of buoyancy, and MM_1 the curve of metacentres. Two other curves, D D and D_1 D_1, are curves of displacement, the former for fresh-water and the latter for salt-water; E E is a curve representing the tons per inch of immersion at various draughts of water. The vertical lines in this diagram indicate the draughts of water measured upon the scale at the bottom of the diagram. The heights of the metacentres and centres of buoyancy are measured upon the vertical scale of feet on the right-hand side of the diagram. The centres of gravity are shown upon the same scale, being joined with the metacentres by a dark line in each case.

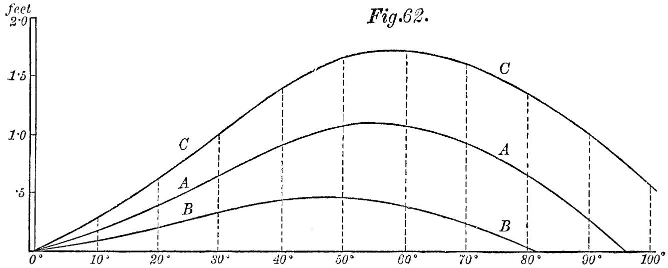
Fig.61.



Centre of gravity, G (with its corresponding metacentre and centre of buoyancy) is for the load-draught of the ship (26 feet 6 inches) laden with a homogeneous cargo (measuring 100 cubic feet per ton) and with bunkers, tanks, &c., full. G_1 is the centre of gravity laden with the same cargo as before, but with coals, water, and stores consumed, and ballast-tanks full. The position, G_2, was given as the centre of gravity at the time of the accident which occurred to this ship in Sydney Harbour, 11th November, 1882. It will be seen that in all the above cases the centre of gravity is given as a little (about 1 foot 3 inches) below the metacentre. G_3 is the centre of gravity when the ship is laden with a homogeneous cargo, but with all coals, water, and stores consumed, and shows the vessel to have, when in this condition, a negative metacentric height, when upright, of about four-tenths of a foot, which would cause her to loll over until a position of rest was found. G_4 represents the position of the centre of gravity when the ship was inclined at Glasgow on the 6th August, 1883, after her return from the scene of the accident, and indicates that, under the conditions of the inclining experiment, she possessed a metacentric height of about 1·6 feet. G_5 is the centre of gravity of the hull and machinery of the ship without coals, cargo, or stores, but with the water-ballast tanks filled, the draught of water being 19 feet 3 inches. G_6 is the centre of gravity of the hull and machinery under similar conditions, but with the important exception of the water-ballast tanks being empty, and shows the vessel to have, when in this condition, a negative metacentric height of eight-tenths of a foot, and represents the condition under which she would leave the hands of the builders, with water-tanks empty. It will be observed, however, from the position of G_5 and G_6 in this diagram that the metacentric height of the ship, when light—that is without cargo, coals, or stores—may be increased by over 2 feet by simply filling the water-ballast tanks with water. The vertical scale of tons on the right-hand side of this diagram represents the displacement of the ship at various draughts.

Fig. 62 represents curves of stability prepared by Mr. Thomas

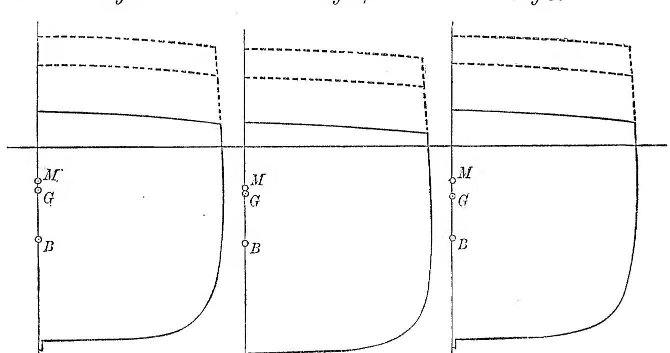
Fig. 62.



Phillips, Lloyd's Surveyor, for a raised quarter-decked screw-steamer of the well-decked type. The dimensions of this vessel are—Length, 267·5 feet; breadth, 35·5 feet; depth, 19·7 feet; tonnage under deck, 1,509 tons; gross tonnage, 1,866 tons; net register tonnage, 1,009 tons. Curve A represents the ship laden with a homogeneous cargo, which entirely fills the cargo holds, the bunkers being assumed as quite full of coal, the boilers filled with water, all stores on board, but no water in ballast-tanks, the vessel otherwise being in sea-going condition. The displacement of the ship under the above conditions is 3,870 tons, the mean draught of water 19 feet $4\frac{1}{2}$ inches, the freeboard 2 feet $4\frac{1}{2}$ inches, and the metacentric height ·85 foot. Fig. 63 shows a transverse half-section of the ship in the above condition, denoting the positions of the centre of buoyancy, centre of gravity of ship and cargo, the metacentre, and the height of the respective decks. It will be observed from the curve (Fig. 62) that the angle of maximum stability is $55\frac{1}{4}$ degrees, and the righting moment in foot-tons at this angle is 4,218; the angle of vanishing stability is $96\frac{1}{2}$ degrees; and the angles at which the edges of the exposed main-deck, raised quarter-deck, and bridge-deck become immersed are 10 degrees, 22 degrees, and 30 degrees respectively.

Curve B, Fig. 62, represents the vessel under all the conditions as described for curve A, but assuming the forewell to be filled with water, amounting to 267 tons. The displacement is 4,137 tons, the mean draught 20 feet $6\frac{1}{2}$ inches, the freeboard 1 foot $2\frac{1}{2}$ inches, and the metacentric height 4 feet. Fig. 64 is a transverse half-section showing the position of the centre of buoyancy and metacentre at this draught, and also the position of the common centre of gravity of the ship, cargo, and water in well; and the height of the various decks at this immersion. The angle of maximum stability is reached at $46\frac{1}{2}$ degrees, the righting moment at this angle being 1,820 foot-tons. The angles at which the respective deck edges

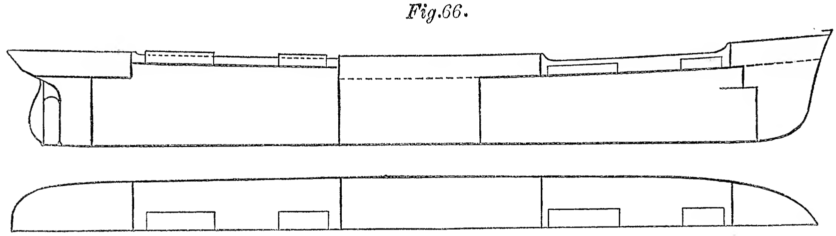
Fig. 63. Fig. 64. Fig. 65.



become immersed are 6 degrees, 18 degrees, and 27 degrees, and the angle of vanishing stability is $80\frac{1}{2}$ degrees. Curve C, Fig. 62, represents the vessel when laden with a heavier cargo than in curve A, the holds not being full, as in the case of the heaviest description of coals. The displacement and draught are the same as in curve A, but the metacentric height is increased to 1·6 feet. Fig. 65 is a transverse half-section showing the respective positions of the centre of buoyancy, common centre of gravity of ship and cargo, metacentre, and decks when in the above condition. The angle of maximum stability is 60 degrees, and the righting moment at this point is 6,656 foot-tons. The angles at which the edges of the various decks become immersed are the same as those given in the description of curve A, the draught of the vessel being the same. The range of stability is much increased, the vanishing point being at $112\frac{1}{2}$ degrees.

Fig. 66 shows the profile and plan of the vessel referred to

Fig. 66.

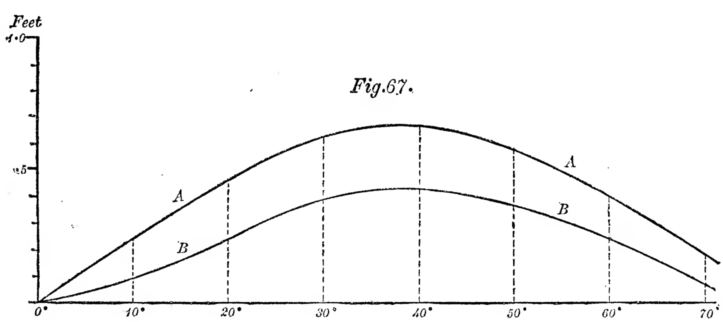


in the preceding curves of stability, and exhibits the respective lengths of the raised quarter-deck, poop, bridge, and forecastle.

Taking the load displacement, as in curves A and C, namely 3,870 tons, the surplus buoyancy due to the parts of the ship above the water-line is 1,967 tons, or 33·7 per cent. of what would be the total displacement if it were wholly submerged. The portion of the ship between the load-line and the main-deck gives 560 tons displacement, or 12·6 per cent. surplus; that due to the sheer of the vessel is 301 tons, which, added to 560 tons, gives 18·2 per cent; that due to the quarter-deck is 307 tons, which, added to 861 tons, gives 23·2 per cent.; that due to the poop is 196 tons, which, added to 1,168 tons, gives 26·1 per cent.; that due to the bridge is 465 tons, which, added to 1,364 tons, gives 32·1 per cent.; that due to the forecastle is 138 tons, which, added to 1,829 tons, gives the total 33·7 per cent. surplus buoyancy.

In some remarks* upon the stability of Well-deck Steamers, based upon curves of stability supplied by Mr. Martell (of Lloyd's Register Office), Professor Elgar has considered the case of such a vessel, 257 feet by 35½ feet by 18½ feet, with a well 60 feet in length, bulwarks 5 feet high, and has assumed that there is no other outlet for the volume of water filling the well than that which it finds by pouring itself out over the bulwarks as the vessel inclines. The diagram which he gives shows that although the volume of water which the well holds (186 tons), reduces the initial stability to nothing, and keeps the ship unstable up to 10° of inclination, the stability becomes positive at that angle, when only 98 tons of water remain in the well. At 20° this water is reduced to 28 tons, and at 30° the stability becomes the same as if the well did not exist, and remains the same for all larger angles of inclination. His conclusion is that, "so far as stability is concerned, the well cannot be regarded as a serious element of danger."

Other examples of cargo steamers worked out in Lloyd's Register Office in London, and presented by Mr. Martell to the Load-Line Committee, are given in Figs. 67 to 71 inclusive. Curve A, Fig. 67, represents the curve of stability of a cargo-carrying



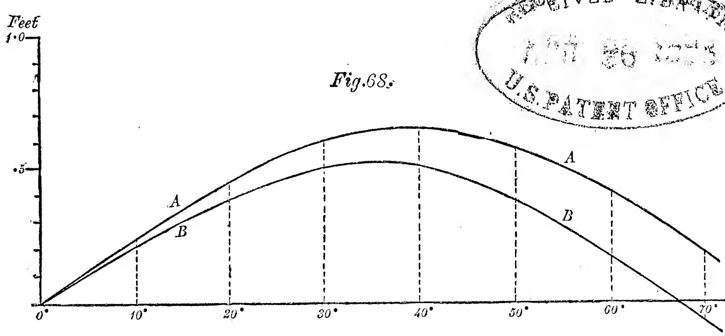
steamer of the following registered dimensions: length 289·5 feet, breadth 32·1 feet, depth moulded 23·1 feet. The centre of gravity of the vessel was ascertained by an inclining experiment, no cargo being on board at the time, and the boilers empty, but with 60 tons of coal in the bunkers, it was found to be ·98 foot *below* the metacentre. Two hundred and forty tons of additional coal were then assumed to be placed in the bunkers, and the cargo holds and

* See a "Paper on the Use of Stability Calculations in Regulating the Loading of Steamers," read at the Institution of Naval Architects, April, 1884.

'tween-decks completely filled with a homogeneous cargo, which occupied 61·2 cubic feet to the ton, the vessel having, when so laden, 4 feet 7 inches freeboard, which is that required by Lloyd's Tables of Freeboard. It will be observed that the angle of maximum stability is reached at about 40°, the length of the righting lever at this point being ·68 foot, and the stability vanishes at 77°.

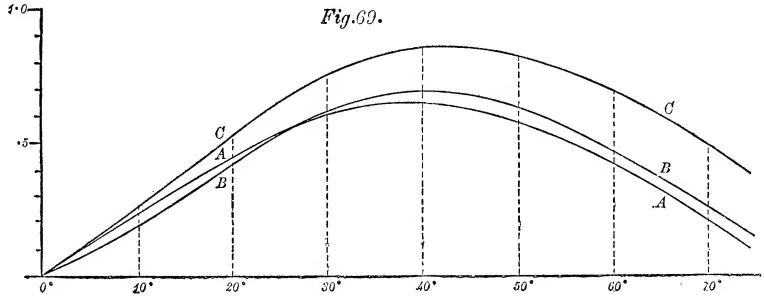
The effect on the stability by decreasing the breadth of this vessel by 2 feet is well illustrated by curve B, in the above Fig. 67, the length, depth, freeboard, and the assumed conditions as to the nature and amount of the cargo remaining the same. The curve is much reduced, the maximum length of the righting lever being only ·43 foot. Assuming this vessel to be fitted with water-ballast tanks, 2 feet high above the floors of the fore and after holds, thus raising the position of the centre of gravity of the cargo, which is taken to be similar in all respects to that previously described, the vessel's stability is reduced from curve A to curve B, as shown in Fig. 68. In Fig. 69 curves are shown illustrating the effect upon

Fig. 68.



the stability for the same vessel by an increase in the freeboard, the cargo being in each case supposed to fill the vessel; and also

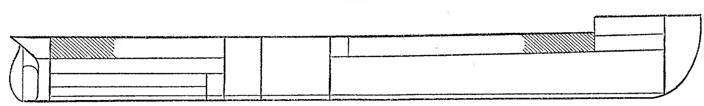
Fig. 69.



the effect of variations in the density of the cargo. Curves A and B show the difference in the stability, with 6 inches additional freeboard, the homogeneous cargo with which she is filled being proportionately lighter than that which fills her with 4 feet 7 inches freeboard. There is no material change in the stability under these conditions, other than that a part of the area of curve A at small angles of heel is transferred to larger angles of heel, and somewhat increases the stability at these angles. Curve C represents the stability when the vessel is laden with cargo of the same density as in curve A, but, having a freeboard of 5 feet 1 inch, the spaces in the 'tween-decks at the ends of the vessel being left empty. This curve exhibits a marked improvement in the stability of the ship, from which it will be seen that, in cases where vessels have insufficient stability when laden with homogeneous cargo which practically fills them, it will generally be effective to restrict the amount of cargo stowed between decks.

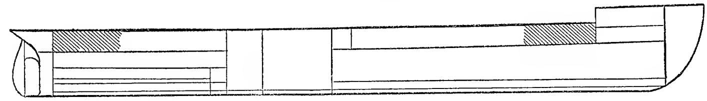
Fig. 70 is a longitudinal section of this vessel without water-ballast tanks, filled with a homogeneous cargo which gives her

Fig. 70.



5 feet 1 inch freeboard, but with the spaces shaded in the 'tween-decks left empty. This vessel has the stability represented by curve C, Fig 69. The space available for cargo with this arrangement is 102,600 cubic feet, and the spaces in the 'tween-decks which are left empty contain 6,700 cubic feet. Fig. 71 shows the

Fig. 71.



vessel fitted with water-ballast tanks in the fore and after holds, which reduce the space available for cargo by 6,000 cubic feet. Assuming her under these circumstances to be filled with a homogeneous cargo which gives her 4 feet 7 inches freeboard, her stability would be reduced to curve A, Fig. 68, that is, if the spaces shaded in the 'tween-decks were left empty. This shows that she could be safely laden to 4 feet 7 inches freeboard, with all cargoes which do not exceed the density of 53·4 cubic feet per ton. If 90 tons of ballast consisting of kentledge, or an equal weight of water in the water-ballast tanks be placed in the bottom of the vessel, she could load to the same freeboard with a full cargo of 61 cubic feet to the ton, and still have the same stability as is represented by this curve, but for all lighter cargoes both the freeboard and stability would be increased.

We have shown in the preceding illustration the effect on the stability of a vessel of reducing her beam by 2 feet, and there have been, without doubt, many vessels built of late years in which the breadth is so reduced, relatively to the depth, that their margin of stability is insufficient for safety when filled with homogeneous cargoes.

Perhaps the relation of beam to stability can be better illustrated by taking a prism of rectangular section $50\frac{1}{2}$ feet broad, and immersing it 21 feet in the water, leaving $6\frac{1}{2}$ feet freeboard, and assuming the centre of gravity to be 3 feet below the water-line, and constructing for this floating body a curve of stability marked A, Fig. 72; it will be seen that the angle of maximum stability is 20 degrees,

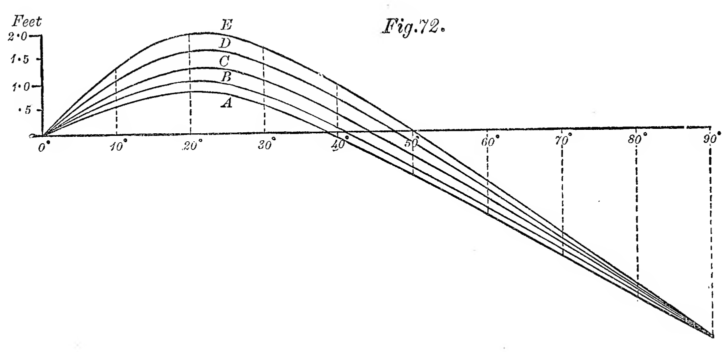


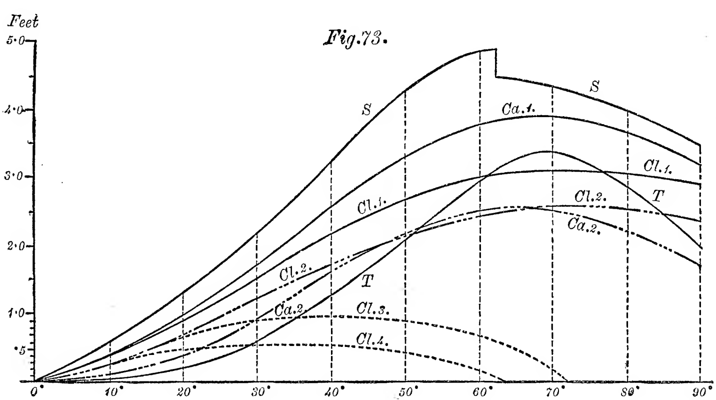
Fig. 72.

and the curve has a range of $38\frac{3}{4}$ degrees. By increasing the beam of this floating prism by $2\frac{1}{2}$ feet, the curve of stability at the same draught is represented by B in the figure, the angle of maximum stability being the same as before, namely, 20 degrees, but the range is extended to $41\frac{1}{4}$ degrees. By adding successive increments of $2\frac{1}{2}$ feet to the beam up to 60 feet, and retaining the same amount of freeboard, $6\frac{1}{2}$ feet, the curves, C, D, E, would indicate respectively the stability due to these additions, and assuming the position of the centre of gravity to remain unaltered, the position of the angle of maximum stability will remain unaltered also, although the *amount* of stability will be more than doubled, and it would absorb more than twice the amount of applied force, to heel the broad prism to an angle of, say 10 degrees, than it would to heel the narrow prism to the same angle. It will be observed that the curves, A, B, C, D, E, Fig. 72, produced by varying the beam in this manner, rapidly leave each other at starting, and converge again at large angles of inclination, finally meeting in a point at 90 degrees, or when the prism is on its side and the top and bottom become vertical, at which point the amount of instability in all cases is the same.*

Useful particulars relating to the stability of merchant-ships have at various times been placed before the Institution of Naval Architects at its annual meetings, and we will add here a few of the most interesting of them :—

In March, 1882, Mr. J. H. Biles read a paper on "Curves of Stability of some Mail Steamers," his illustrations being taken exclusively from ships built by Messrs. J. & G. Thomson. We reproduce these curves in Fig. 73, and have put the particulars

Fig. 73.



relating to them in tabular form further on. The curve, S, is that of the Cunard Royal Mail S.S. *Servia*, when she has 1,700 tons of coal and 3,000 tons of cargo on board, assumed to be stowed homogeneously. Her deck edge becomes immersed at an angle of 34°,

* For this and other illustrations see an able "Paper on the Relative Influence of Breadth of Beam and Height of Freeboard in Lengthening out Curves of Stability," by Mr. N. Barnaby, C.B., Director of Naval Construction, Admiralty, in vol. xii. of the *Transactions of the Institution of Naval Architects.*

and her stability reaches its maximum at 62°, the value of the righting moments at these angles being given in the table. The break in the curve at 63½° is due to the assumed admission of water into the forecastle by the forecastle door, which begins to be immersed at that angle.

The curve, *Ca*. 1. is that of another Cunard Liner, the *Catalonia*, when she has 970 tons of coal and 2,950 tons of cargo on board, assumed to be stowed homogeneously up to the lower-deck. The curve, *Ca*. 2. is for the same ship and same weights, but with the cargo assumed to be stowed up to the main-deck. Her deck edge becomes immersed at an inclination of 32½°, and her stability attains its maximum at 67° in the first case, and at 65° in the second.

The curve, T, is that of the *Thames*, a fine-lined passenger steamer belonging to the Peninsular and Oriental Company, assumed to be loaded with 600 tons of coal and 2,700 tons of cargo stowed homogeneously to the main-deck. Under these conditions her deck edge becomes immersed at an inclination of 30°, and her maximum stability is reached at 67½°.

The four curves, C*l* (with suffixes 1, 2, 3 and 4), are those of the S.S. *Claymore*, a vessel running between Glasgow and the North of Scotland. She is an awning-decked vessel, but has an opening the full breadth of the ship in the awning-deck forward at the fore-hatch, so that if steadily heeled until the deck edge became immersed, she would begin to take in water at this opening. The curve, C*l*. 1., was calculated on the assumption that no cargo is carried above the main-deck, and that no water gets into the opening forward. The curve, C*l*. 2., assumes the cargo to be carried homogeneously to the upper or awning-deck. In calculating the curve, C*l*. 3., it was assumed that the cargo is so arranged that its centre of gravity is at the same height as in curve, C*l*. 1., but that none of the space between the upper and main decks, except the poop, excludes water. For the curve, C*l*. 4., the centre of gravity was assumed to be at the same height, and the ship under the same conditions as to water-tightness as for curve C*l*. 3. The stability of the ship would probably never be better than in curve C*l*. 1., and it need not be worse than in curve C*l*. 4.; generally it would be something between those, depending on the stowage of cargo.

The table referred to on the preceding page is as follows:—

Name of Vessel.	Length between perpen-diculars	Breadth	Mean Draught	Dis-place-ment.	Meta-centric Height above C.G.	Righting Moments. When Deck Edge is Im-mersed.	Righting Moments. Maxi-mum.	Righting Moments. At 90°.	Maxi-mum Right-ing Lever.
	ft. in.	ft. in.	ft. in.	tons.	feet.	ft.-tons.	ft.-tons.	ft.-tons.	feet.
Servia, . . .	515 0	52 0	26 0	12,360	3·6	32,050	59,500	42,700	4·81
Catalonia, . (1)	430 0	43 0	24 0	8,285	2·46	15,450	32,750	25,900	3·95
,, . (2)	,,	,,	,,	,,	1·0	8,500	21,100	13,800	2·5
Thames, . .	392 0	42 0	24 0	7,348	·5	5,000	24,100	15,200	3·28
Claymore, . (1)	220 0	29 6	14 0	1,445	2·4	2,730	4,565	4,170	3·16
,, . (2)	,,	,,	,,	,,	1·75	2,200	3,740	3,735	2·59
,, . (3)	,,	,,	,,	,,	2·4	585	1,415	...	·98
,, . (4)	,,	,,	,,	,,	1·75	440	830	...	·58

Diagrams, Fig. 74 to Fig. 77, are also derived from the *Transactions of the Institution of Naval Architects,* occurring in a paper

Fig.74.

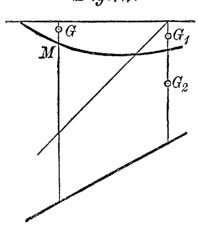
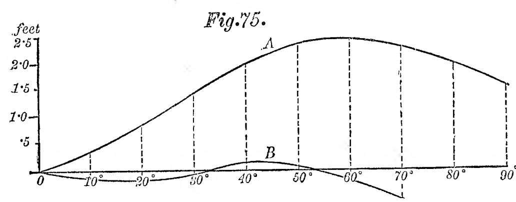


Fig.75.



read by Mr. W. H. White in 1881. In Fig. 74 are shown the curves of metacentres and centres of buoyancy for a cargo-carrying screw-steamer, whose dimensions are given in the table which follows. She has a deep water-ballast tank above her floors. When this ship is in the light condition, with no water in the ballast tank, but with her boilers full, the common centre of gravity is at G, about 1·25 feet above the metacentre, M. If the whole of the cargo spaces were filled with homogeneous cargo, G_1 would be the position of the centre of gravity, about ·8 foot above the metacentre, her draught in this condition being 18 feet 3 inches. Fig. 75 shows the curve of stability, B, given by this position of the centre of gravity. Assuming the cargo to be so stowed as to give a metacentric height of 2·25 feet with the same draught of water, the centre of gravity would be at G_2, Fig. 74, and A, Fig. 75, is the corresponding curve of stability.

Fig. 76 shows the curves of metacentres and centres of buoyancy for a cargo and passenger steamer of the dimensions given in the table below. G is the position of the centre of gravity when the ship is light and without water-ballast, the metacentre, M, being then

Fig. 76.

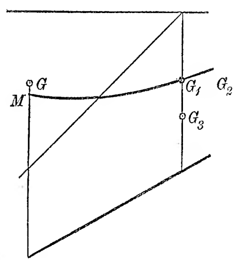
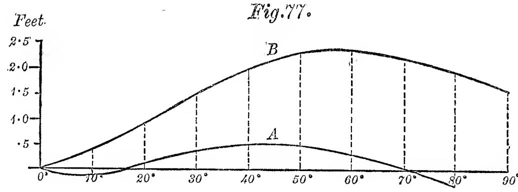


Fig. 77.

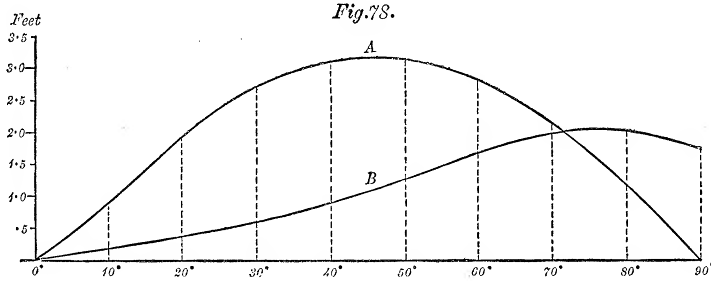


·7 foot below the centre of gravity. Assuming the ship to be filled with a homogeneous cargo giving her a draught of 23 feet, G_1 is the centre of gravity, coinciding, in this case, with the metacentre. The curve of stability corresponding to this assumed condition is shown by the curve A in Fig. 77. If the cargo were so stowed as to give, with the same draught of water (23 feet) a metacentric height of 2·25 feet, the centre of gravity would be at G_3, Fig. 76, and the corresponding curve of stability is shown by the curve B in Fig. 77. G_2 is the position of the centre of gravity, assuming the ship to be laden with a homogeneous cargo, which would give her a draught of 25 feet; the metacentre would then be about 6 feet above the centre of gravity.

Type of Ship.	Length.	Breadth.	Moulded Depth.	Light Draught.	Light Displacement.	Load Draught.	Load Displacement.
	ft. in.	ft. in.	ft. in.	ft. in.	tons.	ft. in.	tons.
Cargo-carrying Steamer,	320 0	34 0	22 3	11 4	1,880	18 3	3,570
Cargo and Passenger Steamer, . .	390 0	39 0	31 0	13 6	3,200	23 0	6,330

In Fig. 78 we give two curves of stability which are of special interest, being those of two steamers, in the launching condition, which were successfully launched upon the Clyde. The ship whose curve is marked A has a length of 275 feet; she is 35 feet broad and 21 feet 6 inches deep; her mean draught at launching was 6 feet 8 inches. The ship of which B is the curve is 430 feet long, has a

Fig. 78.



breadth of 43 feet, and a depth of 35 feet 1 inch, her draught at launching being nearly 11 feet 8 inches. Curve A may be taken as illustrating the large amount of stability up to comparatively large angles that a ship may have in the launching condition, while curve B shows the small amount that may be made sufficient when careful precautions are taken. The striking contrast between the two curves illustrates in a very marked manner the great difference that may exist in different types of ship under approximately the same conditions. In curve A the stability increases very rapidly at small angles, attains its maximum at an angle of about 46 degrees; and then decreases as rapidly, being lost completely at 90 degrees. In curve B, on the other hand, the stability increases very gradually up to an inclination of 50 degrees, then a little more rapidly until its maximum is reached at about 75 degrees; thence it decreases very gradually, being comparatively little less than the maximum at 90 degrees.

CHAPTER VII.

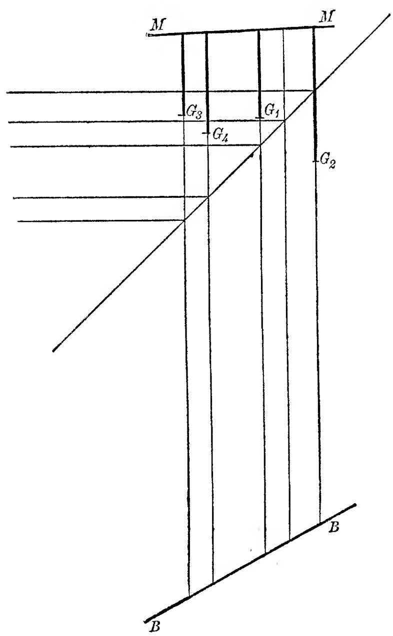
Stability of Certain War-Ships—Stability of the Frigates *Inconstant, Invincible, Iron Duke, &c.*—Stability of War-Vessels Comparatively easy to Ascertain—List of War-Ships whose Curves of Stability are Given—Admiralty Form for Metacentric Heights and Stability—Serious Diminution of Stability Caused by Low Freeboard—*Monarch's* and *Captain's* Compared—Case of the *Atalanta*—Effects upon Stability of Differences of Form and Relative Proportions—Tabulated Examples—Amount of Stability Requisite in Merchant-Ships—Stability at Large Angles of Heel—The Torpedo-ram *Polyphemus*—Comparison of her Section with Wall-sided Section—Investigations in the Geometry of Curves of Metacentres.

In the last chapter we considered the stability at various draughts of water of several merchant-ships, taking into consideration conditions of stowage both favourable and unfavourable. It will be both instructive and interesting to pursue this branch of the subject, and in the next place to bring into comparison with those cases the stability of some of H.M. ships of war, and more especially such of them as have had their stability more or less called in question, observing that the height of the centre of gravity undergoes less variation in a war-ship than in a merchant-ship, owing to most of the weights of the war-ship being fixed in amount and in position.

In 1871 a large committee of scientific and nautical gentlemen, presided over by Lord Dufferin, reported to the Admiralty, at their Lordships' request, upon the qualities of several ships of the Royal Navy. Among other things they said—" Naval architects have been induced" (in order to secure accuracy in the fire of their guns) "to seek steadiness of platform by diminishing, as far as safety would allow, the statical stability and stiffness of the ship. In some recent instances (*e.g.*, the *Inconstant* and the *Invincible*) this was carried to a degree which, together with an alteration in the distribution of weights during construction, has led to a considerable weight of ballast being placed on board these ships in order to correct the crankiness so caused." It may be well to state at once that the displacement of the *Inconstant* was 5,782 tons, and the amount of ballast put into her was 180 tons, or slightly over 3 per cent.; while the displacement of the *Invincible* was nearly 6,000 tons, and the amount of ballast put into her 135 tons, or a little over 2 per cent. It will at once be seen from these figures how little "crankiness" there was to correct!

The initial stability of the *Inconstant* in different conditions as to weights on board is illustrated in Fig. 79, which represents the relative positions of the metacentre, centre of gravity, and centre of buoyancy under such conditions. M M is the curve of metacentres and BB the curve of the centres of buoyancy at corresponding draughts of water.

Fig. 79.



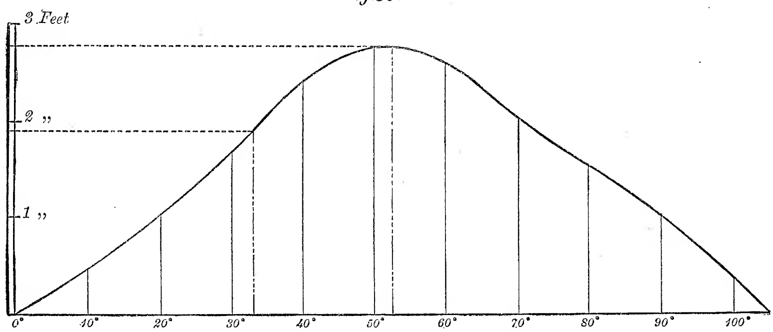
The height of the centre of gravity, G_1, which was ascertained by experiment, was found to be 1·8 feet below the metacentre, 90 tons of ballast being on board at the time, the draught of water being 22 feet 9½ inches. A further 90 tons of ballast was placed in the ship, which, with the other weights put on board had the effect of increasing the draught of water to 23 feet 10½ inches, and the metacentric height to 2·8 feet, the centre of gravity of the ship when in this condition being indicated on the diagram by G_2.

When the vessel was at a lighter draught, viz., 21 feet 2 inches with the 180 tons of ballast on board, but with the boilers and condensers empty—a condition of things which need never take place at sea—the ship still had a metacentric height of 1·66 feet, G_3 being the position of the centre of gravity. When all coal, provisions, and water were consumed, the boilers and condensers being filled—a condition of things which might possibly happen at sea—the metacentric height was increased to 2·05 feet, the draught of water being 21 feet 8½ inches. G_4 on the diagram represents the position of the centre of gravity in this case.

Fig. 80 is the curve of statical stability of the *Inconstant* when the ship is floating at a mean load-draught of water of 23 feet 10½ inches, the displacement at that draught being 5,782 tons;

the centre of gravity below the metacentre, or the metacentric height, at this draught being 2·8 feet, as illustrated in the preceding figure. The height of the edge of the upper-deck above water, when floating at this draught, is 15 feet 3½ inches, and from the curve it will be observed that the vessel heels over to an angle of

Fig.80.



33° before the edge of the deck becomes immersed; the righting moment at this point being 11,362 foot-tons. The angle of maximum stability is 52½°, and the righting moment at this angle is 16,276 foot-tons, the length of the arm of the righting lever, G Z, being 2·82 feet.

When this vessel has been heeled over to an angle of 72½°, it will be observed that she possesses the same amount of righting force to return her to the upright position as she had at the moment the edge of the upper-deck became immersed, the length of the righting lever being about 2 feet. The range of stability of the *Inconstant* when in the loaded condition is very large, the stability not vanishing even with the ship on her beam-ends.

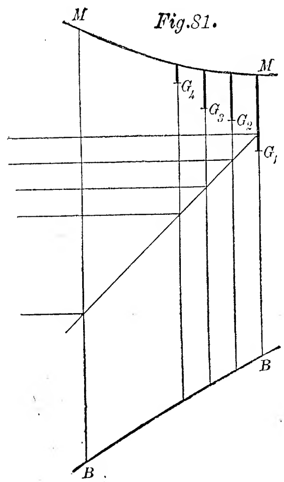
The following are the principal particulars illustrated by this curve of stability :—

CURVE OF STABILITY OF THE *INCONSTANT.*

Angle at which edge of deck is just immersed, .	33°
„ of maximum stability, ...	52½°
„ „ no stability, ...	105½°
„ where stability is the same as at 33°, . .	72½°
Stability with deck just immersed, ...	11,362 foot-tons.
Maximum stability, ...	16,276 „
Displacement in tons, ...	5,782 „
Mean draught of water, ...	23 ft. 10½ ins.

Fig. 81 shows the relative positions of the metacentre, centre of gravity, and centre of buoyancy of the *Iron Duke,* which is a sister-ship to the *Invincible,* under various conditions of draught of water and weights on board. M M is the curve of metacentres, and B B the curve of centres of buoyancy. G_1 is the centre of gravity when the vessel is floating at the constructed load-draught of 22 feet. Owing to the disposition of weights on board, it will be observed that the vessel has a metacentric height of 3 feet when loaded for sea. G_2 is the centre of gravity of the ship when floating at a draught of water of 20 feet 11 inches, and shows the vessel to have a metacentric height of 1·88 feet. The centre of gravity of the *Iron Duke* was ascertained by an inclining experiment, and its position is indicated on the diagram by G_3, the draught of water at the time being 19 feet 10 inches, and the metacentric height about $1\frac{1}{2}$ feet. G_4 is the centre of gravity of the vessel when all coals, provisions, and water are consumed, and with the boilers and engine condensers empty. The metacentric height when in this condition is ·65 foot, and the draught of water 18 feet 9 inches. This is a condition in which the vessel should never be placed, even although the whole of the consumable stores are exhausted. There is no reason why the boilers and engine condensers should remain empty, and thus unnecessarily reduce the metacentric height.

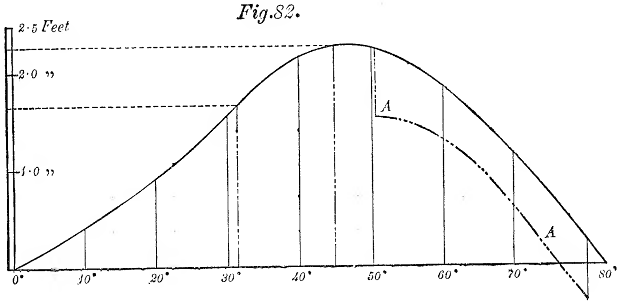
Fig. 81.



It will be noted that, in order to construct the metacentric diagram of the *Iron Duke* more accurately, an offset has been calculated at the comparatively shallow draught of water of 14 feet 8 inches, at which draught the centre of buoyancy has been found, and the point set off on its corresponding vertical, the curve of centres of buoyancy being extended and made to pass through this point. The corresponding position of the metacentre has also been ascertained, and set off on the vertical above this centre of buoyancy, which enables the direction of the curve of metacentres to be continued accurately beyond the points previously obtained as described above.

Fig. 82 is the curve of stability of the *Iron Duke* when the ship is floating at a mean load-draught of 22 feet. This curve has been constructed in disregard of the increase of stability derived, at large angles of inclination, from the continuous bulwarks on the upper-deck, and the ship is treated as if she had no bulwarks or forecastle, the

Fig.82.



space enclosed by the central armour-plated battery only being taken into consideration. It will be seen from the curve that the angle at which the edge of the upper deck becomes immersed is $31\frac{1}{2}°$, the length of the arm of the righting lever being 1·7 feet, the amount of righting force at this point being about 10,000 foot-tons. The angle of maximum stability is about 45°, the length of righting lever 2·4 feet, and the maximum righting moment 14,000 foot-tons. When the vessel has been heeled over to the large angle of 63°, the righting moment would be precisely the same as when she was so inclined that the edge of the upper-deck became just immersed.

The angle at which the stability of the *Iron Duke* vanishes is 84°, that is, supposing the space enclosed by the central armour-plated battery to remain water-tight, and this space has been considered in constructing this curve as contributing, as it manifestly must, to the stability of the vessel at large angles of inclination, and more especially as opposing sudden inclining forces. In the event of the space enclosed by the battery not remaining wholly water-tight, and water finding access from any cause to the interior of the battery, such, for instance, as the doors being open which lead into the battery, then the stability of the ship would be proportionately diminished, the curve of stability under these conditions being marked A A on the diagram. Supposing these doors to be open in the battery after the vessel had heeled to about 50°, the water would flow into the enclosed armoured space and reduce the range of stability of the ship to about 76°, as illustrated in the diagram.

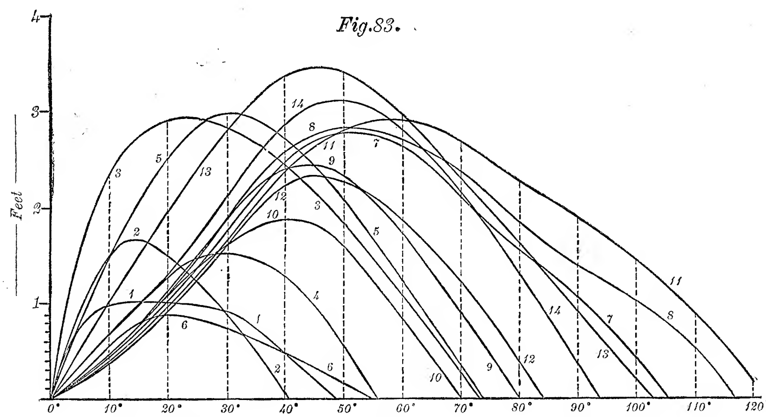
The following are the particulars relating to the curves of stability of the *Iron Duke*:—

CURVES OF STABILITY OF THE *IRON DUKE*.

Angle at which edge of deck is just immersed, .	$31\frac{1}{2}°$
Angle of maximum stability, ...	$44\frac{3}{4}°$
„ „ no stability, ...	84°
„ where stability is the same as at 31°, . .	63°
Stability with deck just immersed, . . .	10,021·5 foot-tons.
Maximum stability, ...	14,000·6 „
Displacement in tons, ...	5,895
Mean draught of water, ...	22 feet.

In Fig. 83 we have brought together the curves of statical stability of several of H.M. ships, a careful study of which, in

Fig. 83.



1. *Glatton.* 6. *Captain.* 11. *Custozza.*
2. *Mjölner.* 7. *Inconstant.* 12. *Iron Duke.*
3. *Miantonomoh.* 8. *Radetzky.* 13. *Kaiser.*
4. *Devastation.* 9. *Achilles.* 14. *Serapis.*
5. *Peter the Great.* 10. *Monarch.*

conjunction with the table of particulars which follows, will be found interesting and instructive. To add to its interest we give also, in the same figure, some curves of ships belonging to foreign powers, the particulars of which will also be found below. These curves all refer to the fully laden condition. The low freeboard ships in our own Navy, such as the *Devastation, Clycops*, &c., differ considerably from the simple monitor, such, for instance, as the *Mjölner* of the Norwegian Navy, by having a central breastwork of great volume, which adds materially to the effective height

Name of Vessel.	Length between Perpendiculars.		Breadth, Extreme.		Draught, Mean.		Displacement in Tons.	Height of Metacentre above Centre of Gravity.	
	ft.	in.	ft.	in.	ft.	in.		ft.	
Low-Sided Ships.									
Glatton, . . .	245	0	54	0	19	0	4,910	...	Pure Monitors.
Mjölner, . . .	187	6	45	6	11	4	1,566	13·8	
Miantonomoh, .	250	0	55	2	14	3	3,842	15·8	
Devastation, . .	285	0	62	3	26	$1\frac{1}{2}$	9,062	3·85	Breast-work Monitors.
Peter the Great, .	321	0	64	0	23	9	9,662	8·64	
Captain, . . .	320	0	53	3	25	4	7,916	2·6	...
High-Sided Ships.									
Inconstant, . .	337	4	50	3	23	$10\frac{1}{2}$	5,782	2·8	...
Radetzky, . . .	253	0	47	0	23	2	4,033	3·32	...
Achilles, . . .	380	0	58	$3\frac{1}{2}$	26	5	9,484	...	...
Monarch, . . .	330	0	57	6	26	0	8,306	2·37	...
Custozza, . . .	287	0	71	$1\frac{1}{2}$	24	10	7,390	...	...
Iron Duke, . . .	280	0	54	0	22	0	6,000	3·0	...
Kaiser, ...	285	0	62	0	24	6	7,600	...	...
Serapis, ...	360	0	49	0	19	5	5,976	...	...

of freeboard, and, beyond the angle at which the water-line meets the junction of the breast-work with the deck, entirely changes the character of the curve from that of a simple monitor. The curve is very much modified beyond this point by an increase in the length of the ordinates, and by the greater angle to which the ship may incline before losing her stability. The *Mjölner* is a pure monitor, or vessel with a nearly flat deck a little raised above the water, upon the centre of which deck a turret is supported. She is without masts, designed purely for defensive purposes; the amount and range of her stability are obtained by means of a very great metacentric height; this is as much as 13·8 feet, as may be seen by reference to the table.

The *Miantonomoh,* of the United States Navy, is a low freeboard monitor of the type originated by Ericsson, with a freeboard of only

2 feet 6 inches amidships. In fact she is, in many respects, a reproduction of the old *Miantonomoh*, being of the same type and arrangements, and nearly of the same dimensions. The *Peter the Great*, of the Imperial Russian Navy, is a breast-work monitor similar to our *Devastation*, but larger and more powerful, having a breast-work which extends to the full width of the deck, with a forecastle that extends to the breast-work, and a flying deck over all: her large amount and range of stability are likewise obtained by a great metacentric height, which is 8·64 feet, or nearly double that of the *Devastation*.

The *Radetzky*, of the Imperial Austrian Navy, is a fast unarmoured steam frigate with high sides, owing to which she possesses a large range of stability, and her power to resist inclination compares favourably with that of other vessels of known good sailing qualities. The *Custozza*, also of the Imperial Austrian Navy, is a powerful ironclad with projecting battery, and possessing, as will be seen from the diagram, a very large range of stability.

The *Kaiser*, of the Imperial German Navy, a sister-ship to the *Deutschland*, was built from the designs of the author, has a central battery, and a large amount and range of stability.

The vessels belonging to our own Navy, the curves of which are given, are too well-known to require any description in this place beyond what is to be found in the table.

The complete knowledge of the stability of a war-vessel in the various conditions of load- and light-draught can be much more readily arrived at, and more accurately determined than that of a merchant-ship, because in the latter the variations of the distribution of weight due to the differences in kind, and consequently in the stowage of the cargoes she may have to carry, make it necessary to *assume* conditions of stowage for the purposes of calculation, to which the actual conditions in practice may approximate only in a greater or less degree; whereas, when the distribution of weight for a war-vessel has once been arranged in a design it is practically fixed (as already intimated), and the deviations from that arrangement for different conditions of draught due to the consumption of stores, coal, &c., can be made with any required degree of accuracy.

To this is doubtless due in large measure the fact that, whereas, no information relating to the stability of their ships has, until recently, been given to the commanders of merchantmen, it has for some time past been the practice with the Admiralty to include a "statement of stability" in the information supplied for each of

H. M. ships. This "statement" gives the metacentric height when fully equipped, and also when light, what is considered the light condition being clearly specified. The angle of heel at which the ship reaches her maximum stability when in the load condition is also given, as well as the angle at which her stability completely vanishes in that condition. These angles are not given for the light condition, because the "reduction in metacentric height generally lessens these angles more than greater freeboard increases them." The form in which this information is supplied is as follows:—

H.M.S.________________

STATEMENT OF METACENTRIC HEIGHTS AND STABILITY.

CONDITIONS	FEET	REMARKS
The ship when fully equipped at a mean draught of______feet______inches has a metacentric height of		
When lightened to a mean draught of______feet ______inches, or when boilers are__________ engine condensers empty, and all coals, water, provisions, and one-half Warrant Officers' and Engineers' stores consumed, the metacentric height is		
	DEGREES	
* The angle at which the ship reaches her maximum stability in the load condition, and beyond which the righting force diminishes, is . .		
* The angle at which her stability entirely vanishes in the load condition, is		

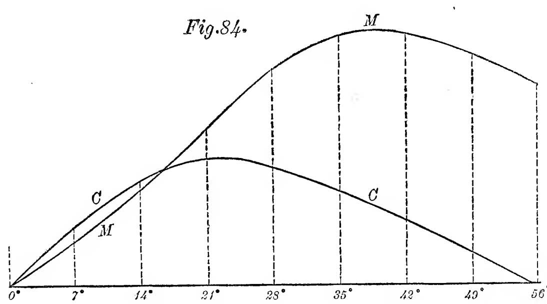
*Admiralty,*____________________188_____.

* These are not calculated for the light condition. Reduction in metacentric height generally lessens these angles more than greater freeboard increases them.

Mr. W. H. White says,* in connection with this subject, and speaking of the Royal Navy, "in cases where special precautions are needed, special standing orders are given. For instance, in some low freeboard ships it is stringently ordered that a certain maximum load-draft shall not be exceeded, because any diminution of the corresponding freeboard would cause an objectionable decrease in the range and area of the curve of stability. Again, in some vessels, as coals and stores are consumed, the stability is considerably diminished, and then orders are given that the ship shall not be lightened beyond a certain minimum draught, that draught being maintained, if necessary, by the admission of water-ballast. All these regulations are based upon careful experiment and detailed calculations."

The very serious diminution in the stability of a ship at considerable angles of inclination that may be caused by low freeboard, when this is not compensated for in some way (by a lowering of the centre of gravity, for instance, or by increased beam) is strikingly illustrated by the case of the *Captain*. For purposes of comparison we will take the *Monarch* as an example of a ship with high freeboard, and will consider both ships in their fully equipped condition, ready for sea. Fig. 84 represents upon the

Fig. 84.



same scale the curves of statical stability for the two vessels, C C being that of the *Captain* and M M that of the *Monarch*.† There are three things to be noted in examining these curves: First, their very small difference up to an inclination of 16°, the *Captain* having slightly the advantage of the *Monarch*, her

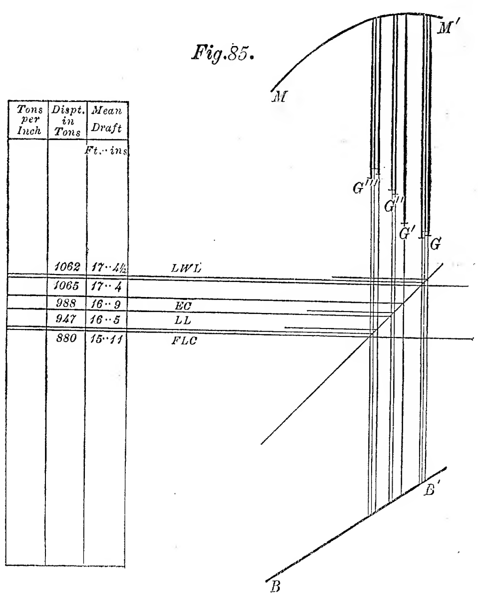
* In *Nature*, dated 15th November, 1883.

† The curve of stability of the *Captain* was given in a previous chapter, to illustrate a somewhat different point; it is convenient to reproduce it here.

righting levers being somewhat longer; second, the great difference in amount of the maximum stability of the two vessels, and in the angles of inclination at which the maximum occurs—that of the *Captain* being reached at 21°, and amounting to a righting lever of only $10\frac{3}{4}$ inches, whereas, when the *Captain's* stability has thus alarmingly decreased, the *Monarch's* has not ceased to grow with every increase of inclination, attaining its maximum at 40°, where the length of righting lever is $21\frac{3}{4}$ inches, or double the maximum of the *Captain's;* third, the very great righting power still possessed by the *Monarch* when the *Captain* has reached her extreme limit of stability at $54\frac{1}{2}$°.

This enormous difference between the statical stability of the two ships arises solely from the one ship having a low freeboard, and the other a high freeboard. As we have seen, the stability of the low freeboard ship compares favourably with that of the high freeboard ship up to an inclination of 16 degrees; but it is precisely at that inclination that the edge of the *Captain's* deck begins to be immersed, and to this fact is due her decrease of stability compared with the

Fig. 85.



Monarch, as the inclination increases. If, all other conditions of the *Captain* remaining unaltered, her side had been as high as the *Monarch's,* she would have retained an advantage over the latter at all angles of inclination.

Fig. 85 is a diagram showing the curves of metacentres, curve of centres of buoyancy, and positions of centres of gravity of the late training-ship, *Atalanta* (which disappeared at sea*) as calculated in the Admiralty Office. B B′ is the curve of buoyancy, and M M′ is the curve of metacentres. The centres of gravity at the draughts of water shown, and under the conditions hereafter stated, are situated in each case respectively at the lower ends of the dark portions of the vertical lines drawn downward from the respective metacentres. The upper group of three water-lines, L W L (one of which is short) represent the load conditions of the ship, with guns removed, but with small variations of load, the highest line (the short one) of the three answering to an extra 10 tons of ballast, the centre of gravity is at G. The line next below this group (at draught 16 feet 9 inches), marked E C for "Experimental Condition," corresponds to the condition of the ship when the experiment for ascertaining the position of the ship's centre of gravity was made at Pembroke. The centre of gravity for this condition is marked G. The next line, marked LL, answers to the usual light condition of the ship, but with half of her hold water-tank full. The centre of gravity is at G″. The lowest group of lines, marked F L C, indicates the final light condition, with all the hold water-tanks (109 tons) empty, and provisions and culinary coals consumed. The centre of gravity is at G‴. The short lines in each case indicate the extra immersion due to the extra 10 tons of ballast, &c. Some further consideration will be given to this case hereafter.

TABLE I.

Reference Letter.	Registered Dimensions.			Tonnage under Deck.	Load-Draught.	Load Displacement.	Light-Draught.	Light Displacement.	Proportions.		Coefficients of Fineness.
	Length. L	Breadth B	Depth. D						$\frac{D}{B}$	$\frac{L}{B}$	
	Feet.	Feet.	Feet.	Tons.	Ft. Ins.	Tons.	Ft. Ins.	Tons.			
A	273·0	43·1	23·6	1,931	19 11	3,980	9 7½	1,440	·54	6·3	·59
B	267·0	39·3	23·3	1,571	20 1½	3,407	9 11	1,235	·59	6·8	·56
C	230·3	37·9	23·1	1,280	19 9	2,910	10 3	1,180	·60	6·0	·59
D	197·5	32·2	19·9	799	17 3	1,775	8 6½	643	·61	6·1	·56
E	197·4	33·9	21·0	853	19 6½	1,990	10 8½	767	·61	5·8	·53
F	148·0	26·9	14·1	340	12 7½	787	6 5	290	·52	5·5	·54

* The *Atalanta* was originally the beautiful *Symondite*, 28 guns, sailing frigate *Juno*.

We next proceed to consider the curves of metacentres of several ships, in order to further show the effects upon stability of differences of form and relative proportions. We give their principal dimensions and other particulars in Table I. above, assigning to each ship a distinguishing letter for the purpose of future reference.

* In Table II. we give the heights of the metacentres and centres of buoyancy above the bottom of keel, for the light- and load-draughts respectively, and also the heights of the centre of gravity of the ship when light, and of the homogeneous cargo, as well as the height of the common centre of gravity of the hull and cargo—all these heights being taken above the bottom of keel. We give besides the position of the centre of gravity relative to the metacentre for the light and load conditions respectively:—

TABLE II.

Reference Letter.	Light-Draught.				Height of Centre of Gravity of Homogeneous Cargo.	Load-Draught.			
	Height of Centre of Buoyancy.	Height of Metacentre.	Centre of Gravity Light, above or below Metacentre.	Height of Centre of Gravity Light.		Height of Centre of Buoyancy.	Height of Metacentre.	Height of Centre of Gravity of Hull and Cargo.	Centre of Gravity of Hull and Cargo below Metacentre.
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
A	6·0	23·6	2·7 below.	20·9	16·4	11·5	19·9	18·0	1·9
B	6·2	20·6	1·3 above.	21·9	16·0	11·9	18·7	18·0	·7
C	6·4	19·9	0·9 ,,	20·8	15·7	11·6	18·4	17·7	·7
D	5·4	17·0	0·8 below.	16·2	13·5	10·2	15·45	14·5	·9
E	7·0	17·7	0·8 ,,	16·9	14·0	12·1	17·9	15·1	2·8
F	4·2	14·8	2·0 ,,	12·8	9·2	7·5	12·9	10·5	2·4

In examining the fourth column of Table II. it will be seen that two of the vessels, viz., B and C, have their centre of gravity above the metacentre, in the light condition, and would be therefore unable to remain upright without the aid of ballast. Ships D and E have the centre of gravity a little below the metacentre, while A and F possess sufficient metacentric height in the light condition to enable them to be shifted about in harbour with perfect safety. The

*Tables I. and II. are abridged from the *Report on Masting*, principally written by Mr. W. John for the Committee of Lloyd's Register.

metacentric heights given in the last column of Table II. are those obtained by assuming that each ship is stowed with a homogeneous cargo of such a density as would bring her down to her assigned load-line. These metacentric heights are in every case less than should be possessed by a sailing-ship about to proceed to sea, though there are evidently considerable differences in their approach to what would be considered a sufficient metacentric height. For instance, assuming the requirement to be a height of $3\frac{1}{2}$ feet, and considering the relative amounts of dead weight the ships would require with similar cargoes to bring them into this condition, we can see clearly that ships, A, E, and F, whose metacentric heights are already considerable, would require very much less than B, C, D, whose metacentric heights are so small. Another interesting comparison to be made from Table II. is that of the differences between the metacentric heights of the several ships in their light and load conditions. For instance, ship A has a metacentric height of 2·7 feet when light, which when loaded is reduced to 1·9 feet. Ship F has nearly the same metacentric height (slightly more) when loaded as when light. Whereas, ship E, whose metacentric height is only ·8 foot when light, has, when loaded, the very greatly increased one of 2·8 feet. These differences can to a certain extent be explained by a reference to the 3rd and 8th columns of Table II., giving the heights of the metacentre above the keel at the light- and load-draughts respectively. We find that for ship A the metacentre at the load-line is 3·7 feet lower than at the light-line; for ship F it is 1·9 feet lower at the load- than at the light-line; and for ship E it is ·2 foot higher at the load- than at the light-line. "By filling ship A, therefore, with a homogeneous cargo having a centre of gravity 6·5 feet below the light centre of gravity, and 7·2 feet below the light metacentre, the common centre of gravity is lowered 2·9 feet; but, owing to the metacentre going down 3·7 feet, the metacentric height, when so loaded, is actually less than it was when the ship was light by ·8 foot. By filling ship E, on the other hand, with a homogeneous cargo having its centre of gravity 2·9 feet below that of the ship when light, the common centre of gravity is lowered 1·8 feet, and owing, in this case, to the metacentre rising ·2 foot, the metacentric height, when loaded, is increased 2.0 feet, making 2·8 feet loaded, against ·8 foot when light."—*Report on "Masting."*

These variations will be still more clearly seen and readily understood by a study of the curves of metacentres for these ships, Fig. 86 to Fig. 91 inclusive, taken from the Report just quoted.

In each figure the curve of metacentres, curve of centres of buoyancy, and the light- and load-lines are shown, and on the latter are

Fig.86.

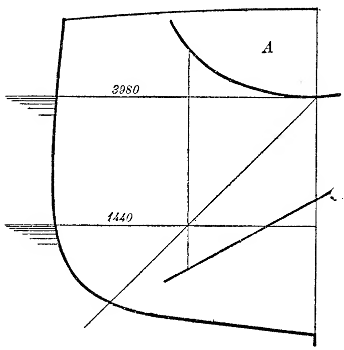


Fig.87.

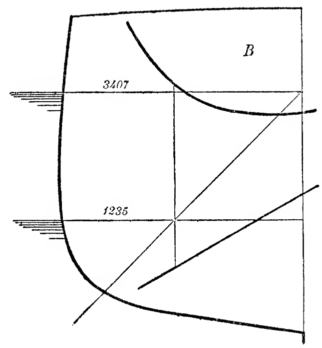


Fig.88.

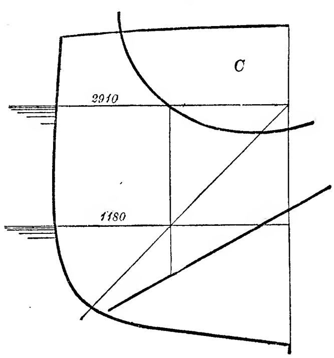
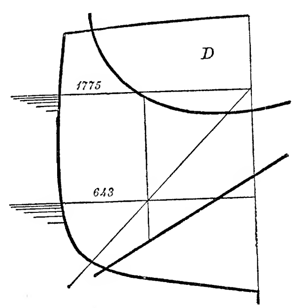


Fig.89.



marked the corresponding displacements in tons. The difference in form of some of the curves is very noticeable. For instance, that in Fig. 90 for ship E, with those in Figs. 86 and 89 for ships A

Fig.90.

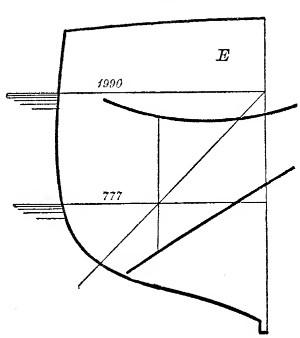
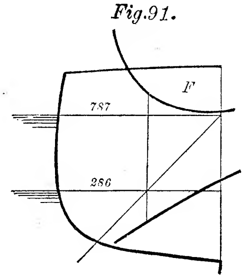


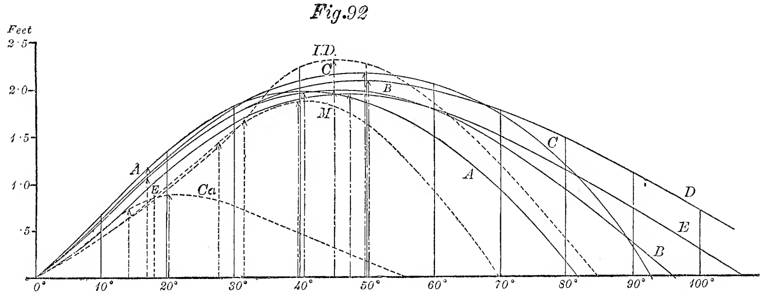
Fig.91.



and D, may be compared respectively. The metacentric curve in Fig. 90, starting from the load-line, drops as the ship lightens, and continues to do so until it reaches its lowest point at about 14 feet draught, when it commences to rise, and continues rising slowly as the draught decreases, until at the light-draught it is ·2 foot lower than at the load-draught. The curve in Fig. 89, on the other hand, drops but very little for a short distance, and then commences to rise slowly at first, but more rapidly as the ship reaches her light-draught, where it is 1·5 feet higher than at the load-draught. The curve in Fig. 91 is almost level for some distance, and then commences to rise rapidly as the ship lightens, reaching a position 3·7 feet higher at the light-line than that it occupied at the load-line.

In order to complete the comparison of the stability of the several ships we have been considering, we give in Fig. 92 curves

Fig. 92

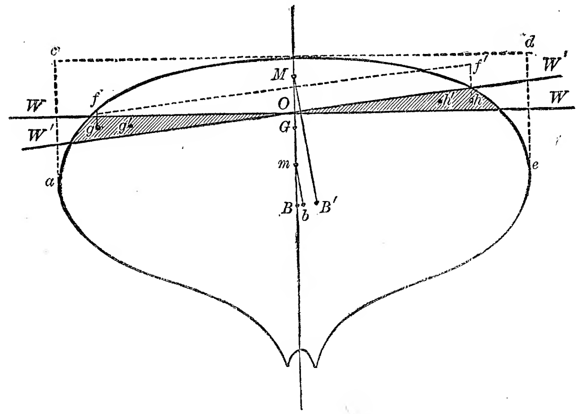


showing their stability at large angles of keel. These curves were all obtained by assuming a metacentric height of $3\frac{1}{2}$ feet in each case. The angle at which the deck edge becomes immersed is indicated by an ordinary dotted line in each case, and that at which the maximum stability is reached is indicated by a different kind of dotted line. The curves of the sailing-ships are marked with the reference letter used in the preceding tables. Those of H.M. ships are drawn with dotted lines; that marked I D being for the *Iron Duke*, that marked M for the *Monarch*, and that marked Ca for the *Captain*. It will be noticed that the curves of four out of the five sailing-ships have a greater range than those of either of H.M. ships, the angle at which the stability vanishes being very large. This is, of course, due to the fact that, although the ships of the Royal Navy possess much greater proportionate beam than merchant-ships, and have greater metacentric height, the heavy weights of armour and guns which they have to carry above the water raise their centres of gravity very high, and, therefore, at large angles of heel they stand at a disadvantage as compared with merchant-ships. The angle of maximum stability (Fig. 92) varies from about 40° to 50°. The ship whose curve is marked A was struck by a heavy squall in the Indian Ocean, and capsized when on her way home with a cargo of cotton; she had a metacentric height of about 3 or $3\frac{1}{2}$ feet. A very large amount of canvas was spread, and she was doubtless in a very critical position relative to the wave-surface when struck by the squall, considering the large resistance to capsizing which her curve shows her to have possessed.

The torpedo-ram, *Polyphemus*, presents an unusual and interesting example, as regards the stability of special types of ships. As first described, and as illustrated in Mr. King's *War-ships and Navies of the World*, she is without the superstructures with which she was ultimately built, and it is in her original condition that she presents the most interesting features. The vessel is essentially a torpedo-ram, her means of attack being the ram and the torpedo, and these only. She carries no guns, or only such as will suffice to protect her against small torpedo-boats—viz., Nordenfelt guns. The object aimed at apparently was to produce a vessel as low down in the water as is consistent with safety, so as to be as little liable to be struck by shot as possible, having great speed and great manœuvring power, these latter elements being pre-eminently essential to her intended mode of attack. She is 240 feet long, 40 feet wide, draws 20 feet of water, and has a displacement of 2,640 tons. The total depth of her hull (exclusive of certain superstructures) is 24 feet, so that the height above water of the hull proper is only 4 feet. The transverse section of the vessel is very similar to the section of a peg-top, and is indicated on the accompanying Fig. 93 by the drawn line. The line, W W, is the load water-line. It will be seen that the vessel had originally no sides above water such as are possessed by an ordinary ship. The upper rounded surface of the hull formed the upper-deck, or turtle-back deck, and this sloped away obliquely into the water, only extending 4 feet above the water along its middle. In the side view, or elevation of the hull proper, the turtle-back deck rounded downward at each end to a point several feet below water. The fine portions of the hull, forward and aft, below which would constitute the *fore-foot* and *heel*, were cut away to improve the manœuvring power.

A novel and interesting feature in this ship is the carrying of 300 tons of cast-iron ballast in large blocks, in a recess or groove along the bottom of the ship, at the keel, so attached that they can be detached from the ship when required and allowed to sink. Their detachment may be regarded as a means of acquiring so much

Fig. 93.



additional buoyancy available in case of necessity, and as compensating, in some measure at least, for the smallness of the capacity of the ship above water. If an ordinary war-vessel be badly hit in action and water enters the hull, she will sink in the water and draw upon her spare buoyancy, until the additional volume of displacement is equal to the volume of the water which has entered the vessel. This process will go on if compartment after compartment be flooded —supposing the ship to remain stable—until the whole of the spare buoyancy has disappeared, when the ship will sink. In such a ship the displacement per inch of additional immersion (or the weight necessary to sink the ship 1 inch at that draught) would remain nearly constant for several feet of immersion; whereas, in the *Polyphemus,* the tons per inch of additional immersion would rapidly decrease with every inch of additional immersion, and after a comparatively small increase of immersion would disappear for all useful purposes. The 300 tons of ballast may be regarded as representing about 20 inches of spare immersion, which would be at once used in the event of serious damage.

The peculiar form of section adopted in the *Polyphemus* was designed to give her a high centre of buoyancy and a low centre of gravity, since the stability she can derive from her form must be very small indeed. She is worse off in this respect than the low-sided monitors, as these vessels derive great stability from their form until their edge of deck enters the water, when it rapidly decreases. But the *Polyphemus* (as shown by Mr. King) has, strictly speaking, no height of side above water, or, at any rate, what may be called her sides slope inwards so much that, as regards stability of form, the vessel has not very much more stability than the monitor class, when their deck has entered the water. The reason of this will be at once seen. Compare the condition of the ship with that of a ship having the same bottom, and the same height above water, but with vertical sides, as indicated by the dotted lines, *a c d e*, Fig. 93. Let B be the centre of buoyancy of this vessel, and suppose G her centre of gravity. The vertical through B must also pass through G, and in order that the ship may have stability initially when she is heeled through a small angle, the vertical through the new centre of buoyancy must cross the original vertical through B and G at a point situated above G. Suppose the vessel heeled so as to float at the water-line, W′ W′. Let B′ be the position of the new centre of buoyancy, and let the vertical through it intersect the original vertical through B and G in the point M.

We see that in this case the wedge of displacement, which travels from one side to the other and is lightly shaded in the figure, is very much smaller than that (W O W′) for the wall-sided ship, neither does it travel over through such a great distance; the centres of gravity of the two wedges of hull displacing the water, would be situated about as indicated, viz., g' and h' for the *Polyphemus*, and g and h for the upright side. Hence, the centre of buoyancy will move through a very much shorter distance. Its new position for this case is indicated by b, and the corresponding metacentre by m, and we see that if G were in the same position as that chosen for the wall-sided ship, the force of buoyancy would pass on the left of the centre of gravity, so that the couple formed by this force, and the weight of the ship would tend to heel her further from the upright, and the vessel would turn over. The centre of gravity must therefore be much lower, and is probably situated at or near the centre of buoyancy. As the wall-sided vessel heels over, the distance through which the wedge of displacement is shifted increases until the edge of deck passes into the water, when it begins to decrease very rapidly; but in the case of the *Polyphemus*, this distance begins to decrease at once as the water passes over the round top of the deck; hence, as we have stated, the stability due to form in the *Polyphemus* is approximately the same as in the monitor class when the water reaches the edge of the deck.

Mr. W. H. White has worked out some interesting considerations connected with the geometry of curves of metacentres,* which may be briefly noticed here. He first investigated the conditions under which the curve becomes *horizontal,* which corresponds, of course, to the circumstances that, notwithstanding differences of draught of water, the metacentre remains at the same height on the axis of the ship throughout the range of the curve's horizontality. We have seen that in the case of a prism of circular section, and of a sphere, the curve of metacentres is throughout horizontal; and we have also had before us repeated instances of curves of metacentres for actual ships in which the curves were horizontal for a greater or less distance. Mr. White shows that one geometrical condition corresponding to this state of things is, that the principal transverse sections of the surfaces of flotation and buoyancy must be concentric; in other words, that the centre of curvature of the curve of flotation must be coincident with the metacentre, a condition which is obviously fulfilled in the case of a spherical vessel, and of a cylindrical vessel floating with its axis horizontal. He proves the propositions generally by showing that the height of the metacentre above the water-line has for its value, under such circumstances, the expression, $\frac{d\,\mathrm{I}}{d\,\mathrm{V}}$, or the differential of the moment of inertia, divided by the differential of the volume displaced, presuming the draught of water to be slightly varied, and this expression, $\frac{d\,\mathrm{I}}{d\,\mathrm{V}}$ has been shown by M. Leclert (as we shall see hereafter), to be also the value of the radius of curvature of the curve of flotation. Another particular case, says Mr. White, is that where a ship is "wall-sided" in the neighbourhood of the given water-line. The centre of curvature then lies in the water-line ($d\,\mathrm{I}$ being zero), and if the curve of metacentres has a horizontal tangent at that water-line, the metacentre must lie on that line, that being,

* See *Transactions of Institution of Naval Architects,* vol. xix., 1878. Mr. White designates them "metacentric curves," a designation which, as previously stated, we desire to reserve for curves known as "metacentrics" (*les métacentriques* of Bouguer).

in fact, the horizontal tangent. This, of course, presumes the wall-sidedness to exist from stem to stern—a very unusual case.

It will be seen hereafter that Professor Leclert suggests the construction of a curve, of which values of V, the volume of displacement, shall form the abscissæ, and values of $r\ \frac{d\,\mathrm{I}}{d\,\mathrm{V}}$, the radius of curvature of the surface of flotation at different draughts, should form the ordinates. Such curves (but having moments of inertia for ordinates) were employed by Mr. White (when teaching Naval Architecture at the Royal School at Greenwich) to illustrate the fact that a curve of metacentres may become horizontal at more places than one, and the following are two examples given by him as having been constructed by pupils of his:—Fig. 94 represents a curve of metacentres obtained from an actual ship, and it is horizontal both at A′ and at C′. Fig. 95 is a companion curve, con-

Fig. 94.

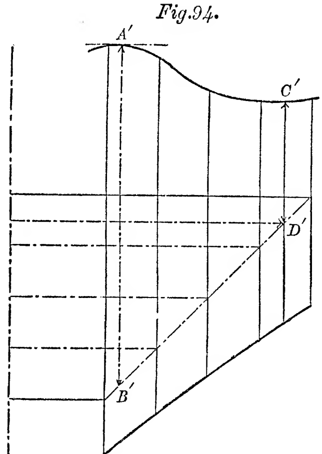
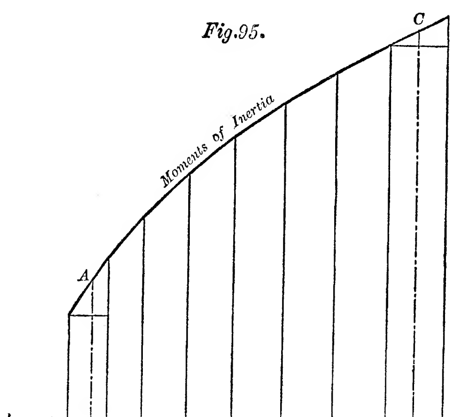


Fig. 95.



structed as just described, with volumes of displacement set off along B D, and moments of inertia (I) of the water-line areas set up as ordinates of the curve, A C. A tangent to the curve at any point represents the ratio, $\frac{d\,\mathrm{I}}{d\,\mathrm{V}}$, and consequently expresses the value of the corresponding radius of curvature of the plane of flotation. To the point, A′, on the diagram, Fig. 94, corresponds the ordinate, A B, on Fig. 95. Hence, according to the general principle above stated, the ratio $\frac{\mathrm{I}}{\mathrm{V}}$ at the point, A, should be equal to the height of A′ above the corresponding water-line; this height is indicated by the length of A′ B′ in Fig. 94, and is 14·8 feet, which exactly equals the ratio of $\frac{d\,\mathrm{I}}{d\,\mathrm{V}}$ given by Fig. 95 at the point A. Similarly it will be seen that at the point, C, in Fig. 95, the ratio, $\frac{d\,\mathrm{I}}{d\,\mathrm{V}} = 5{\cdot}5$, which again is exactly equal to the height, C′ D′ of the metacentre above the corresponding water-line when the tangent (at C′) to the curve of metacentres is horizontal. Figs. 96 and 97

Fig. 96.

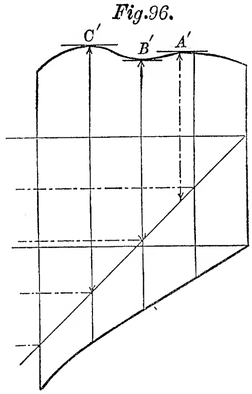
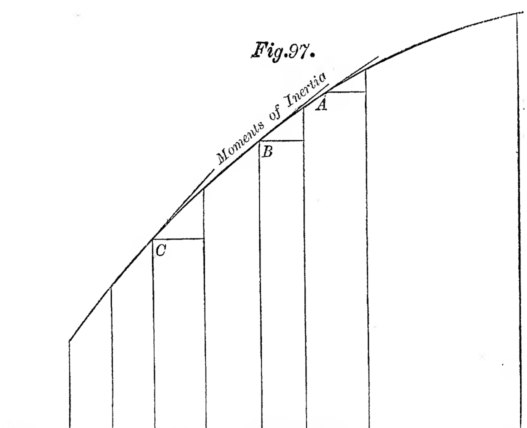


Fig. 97.



illustrate the corresponding conditions for another ship where the points, A′, B′, C′ on Fig. 96, at which the curve of metacentres has horizontal tangents, correspond to A, B, and C respectively, on Fig. 97. In each case the ratio $\frac{d\,\mathrm{I}}{d\,\mathrm{V}}$ equals the height of the metacentre above its appropriate water-line. Calling the tangential angles at A, B, and C respectively, α, β and γ, it was found that tan. $\alpha = \frac{4{\cdot}25 \times 10}{6{\cdot}4} = 6{\cdot}65$; tan. $\beta = \frac{4{\cdot}8 \times 10}{6} = 8$; tan. $\gamma \; \frac{6{\cdot}2 \times 10}{5{\cdot}8} = 10{\cdot}7$.

Having had occasion, in this and the preceding chapter, to bring into view curves of stability constructed for different draughts of water in connection with curves of metacentres and of centres of gravity, it is fitting that we should here describe the ordinary English method by which a curve of stability for a vessel at a given draught is deduced from a known curve of stability for another draught. This method was worked out by the Admiralty draughtsmen, and is now generally used. It was first publicly described in a paper by Mr. G. F. Stanbury, M.I.N.A., in the *Annual* of the Royal School of Naval Architecture, of which he is a distinguished Fellow.

"Most of the readers of this *Annual*," remarks Mr. Stanbury, "will be acquainted with the fact that the range—or position of the angle of vanishing stability—of the curve of stability is considerably less for ordinary ships when all the coals, water, provisions, stores, &c., are consumed, than when these things are on board. This arises from the fact that, in all ordinary cases, the centre of gravity of a vessel ascends as the coals, stores, &c., are consumed, to such an extent as to more than counterbalance the contrary effect of the additional freeboard in lengthening out the curve of stability. In the case of a comparatively low freeboard vessel, an increase of freeboard due to the stores, coals, &c., being consumed, has, of course, a greater proportionate effect on the form and range of the curve than an equal increase would have in the case of a high freeboard vessel. Hence, in practice we generally find that the differences in the ranges of the load and light curves of stability is less with a low freeboard vessel than it is with a high freeboard. The foregoing remarks, of course, apply only to vessels properly stowed or equipped, or, in other words, it is assumed that the centre of gravity of the total weight of consumable stores, coals, &c., is below the centre of gravity of the whole vessel fully equipped. Heavy deck cargoes or bad stowage might, and no doubt often do, seriously modify the above statements. In such cases the curve for the fully laden condition might have a shorter range than the light curve.

"Taking for granted the importance of calculating the curve of stability for the light condition of lading—especially for ocean-going vessels carrying a large spread of canvas—let us enquire what the worst conditions as regards the positions of weights on board would be. This is generally taken to be the case when—(1) The boilers are empty; (2) the tanks are empty; (3) all provisions consumed; (4) no water in engine condensers; (5) all coals consumed; (6) all consumable stores used. Some persons may consider this to be a very improbable condition for any captain to allow his vessel to get into. While admitting this, it must be regarded as a possible condition; and the curve calculated for such a case must be considered as indicating the lowest limit to a vessel's statical stability; while the curve for the fully-equipped condition shows the highest limit.

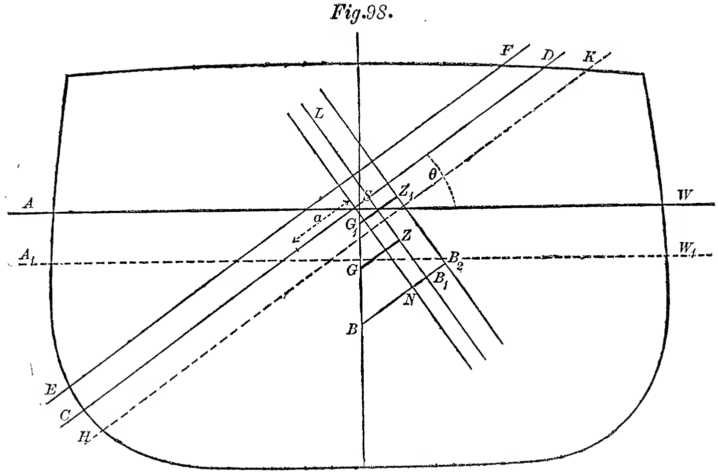
"The weight of coals, stores, &c., consumed, and the vertical position of the centre of gravity of the vessel in her light condition being calculated, let us now proceed to deduce the curve of stability for this condition from the calculations for the curve at deep draught. The corrections to be made on the known curve are—(1) That due to alteration in mean draught; (2) that due to the vertical rise of the centre of gravity. Any small alteration of trim is not taken into account in making these calculations, because it has been shown, in the paper already referred to, that the curve is not affected to any appreciable extent by it."

The following is the Admiralty method of proceeding as described by Mr. Stanbury: Let d represent the total weight of the stores, water, coal, &c., consumed, or the layer of displacement between the light and load water-lines. If D be the displacement of the fully-equipped vessel and D^1 the displacement when light, then

$$D^1 + d = D.$$

In Fig. 98,* let AW be the load water-line, A_1W_1 the light line, the displacement between AW and A_1W_1 being equal to d; B the

Fig. 98.



centre of buoyancy for the upright position at deep draught, G the centre of gravity at that draught, and G_1 the centre of gravity at light draught. Then GG_1 is the rise of the centre of gravity. Now, suppose the vessel heeled to an angle, θ, for which the righting couple, GZ, has been calculated in the fully-equipped case, the load displacement remaining constant for all angles of heel, EF can

* This figure has been so drawn as to show all the points of the process, and not to represent any actual case.

be drawn in its true position from the results of the calculations just referred to. Find by the usual process the position of the water-line, H K, cutting off a layer of displacement of thickness, t, equal to d; also the centre of gravity of the layer. Let a be the horizontal distance between the centre of gravity of the layer and the line, S N. Then $\mathrm{B\,B_1} - \mathrm{B\,S}$ sin. θ is the distance between the perpendicular through the centre of buoyancy, $\mathrm{B_1}$ (in fully-equipped condition), and S N. Again, $a + (\mathrm{B\,B_1} - \mathrm{B\,S}\ \text{sin.}\ \theta)$ equals the distance of the centre gravity of the layer from $\mathrm{B_1\,L}$. Taking moments about $\mathrm{B_1\,L}$, we have

$$d\left\{a + (\mathrm{B\,B_1} - \mathrm{B\,S}\ \text{sin.}\ \theta)\right\} = \mathrm{D_1}\ (\mathrm{B_1\,B_2})\ ;$$

or,

$$\mathrm{B_1\,B_2} = \frac{d\left\{a + (\mathrm{B\,B_1} - \mathrm{B\,S}\ \text{sin.}\ \theta)\right\}^{*}}{\mathrm{D_1}}\ .$$

The arm of the righting couple for the angle, θ, may now be found from the equation

$$\mathrm{G_1\,Z_1} = \mathrm{B\,B_2} - \mathrm{B\,G_1}\ \text{sin.}\ \theta.$$

The values of $\mathrm{B\,B_2}$ and $\mathrm{G_1\,Z_1}$ should be calculated for each of the angles at which the length, G Z, has been calculated.

We may extend this chapter by giving a place to the following ingenious and instructive note on a case of a floating body in which the metacentre is below the centre of buoyancy, which appeared in the *Annual* just referred to, bearing the well-known initials of Mr. Crossland, late of the Admiralty:—

"In all ordinary cases of floating bodies the metacentre is necessarily above the centre of buoyancy. There is, however, a case in which the metacentre is below the centre of buoyancy. Suppose a box without a lid placed in the water with its mouth

* "It often happens," says Mr. Stanbury, "that the line, $\mathrm{B_1\,L}$, and sometimes that the centre of the layer, are on different sides of S N for different angles of heel. The signs in the expression $\left\{a + (\mathrm{B\,B_1} - \mathrm{B\,S}\ \text{sin.}\ \theta)\right\}$ should therefore be carefully considered for each angle. In the case of a vessel with high freeboard, the centre of gravity of the layer at the angles of inclination just before the edge of the deck is immersed, is on the immersed side of S N, then, of course, $\mathrm{B_2}$ falls on the opposite side of $\mathrm{B_1\,L}$, to that shown in Fig. 98. These simple points are mentioned merely to call to them the attention of those readers who may not be conversant with the principles upon which the stability calculations are based. If $\mathrm{B_2}$ is on the immersed side of $\mathrm{B_1\,L}$ (as in the figure), then $\mathrm{B_1\,B_2}$ should be added to $\mathrm{B\,B_1}$; if on the emersed side, it should be subtracted."

downwards and bottom upwards. Suppose it to be of such a weight as to float, the water being kept down by the air inside, as in the case of a diving-bell. Let A B, Fig. 99, be the water-line outside the box, and C D the water-line inside the box. If the thickness of the box be neglected, then A B D C is the displacement. If we suppose a small angular displacement to take place, the two water-lines become *a b* and *c d*. It will be seen that the variation of the water-line, A B, has the effect of moving the centre of

Fig. 99.

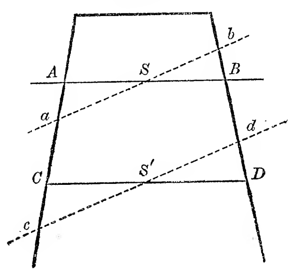
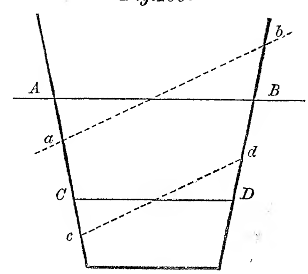


Fig. 100.



buoyancy to the right, while the variation in the water-line, C D, has the effect of moving the centre of buoyancy to the left. And if y and y' be put for S B and S′ D, it will be readily seen that the height of metacentre above the centre of buoyancy is

$$\frac{2}{3\,\mathrm{D}}\left\{\int y^3\,dx - \int y'^3\,dx\right\}$$

or $$\frac{2}{3\,\mathrm{D}}\int_x\left\{y^3 - y'^3\right\}.$$

And it is obvious that in the case of the box, if A B be less than C D, the metacentre will then be below the centre of buoyancy. I have not here investigated the positions of the two water-lines with reference to the elasticity of the air, &c., as I conclude that it would not affect the truth of this proposition as to the mere metacentric stability.

"If we turn the box up the other way, and let it float as in Fig. 100, with its open side uppermost, and if we suppose it to be partially filled with water, then the same principle applies if we assume, as we may, that the displacement is A B D C, A B being the water-line outside the box, and C D the water-line inside the box. If y and y' have the same meaning, we get the same expression for the height of metacentre above the centre of buoyancy. From this it is obvious, that if such a box have any tumble home, the metacentre would be below the centre of buoyancy. These considerations show the great importance, with regard to stability, of water in the hold of the ship if capable of washing from side to side."

It is obvious that a curve of metacentres might be perfectly well constructed for a case like this, should the necessity for it arise.

Since the previous part of this chapter was written, Mr. P. Jenkins, M.I.N.A. (a Surveyor to Lloyd's Register), has contributed to the Institution of Naval Architects* some further considerations in connection with the construction of metacentric diagrams and the initial stability of vessels, which exhibit much ability, and probably give practical completeness to the geometry of metacentric diagrams. Observing how closely the curve of centres of buoyancy for an ordinary ship approximates (as we have seen previously) to a straight line—especially where it corresponds to draughts of water which are not very remote from the load draught—Mr. Jenkins observes that we can determine the straight line to which the locus of the centres of buoyancy will then closely approximate, by getting an expression for the *tangent* to that curve in terms of the elements usually ascertained in the calculations for displacement, &c. Such an expression may readily be obtained, indeed it may be written down at once by any one familiar with the subject, and with the doctrine of limits, in the form

$$\frac{\mathrm{B_1\,C}}{\mathrm{B\,C}} = \cot.\theta = \frac{\mathrm{A}\,h}{\mathrm{V}} \text{ (Fig. 101)},$$

in which $\mathrm{B\,B_1}$ are centres of buoyancy at two draughts indefinitely

Fig.101.

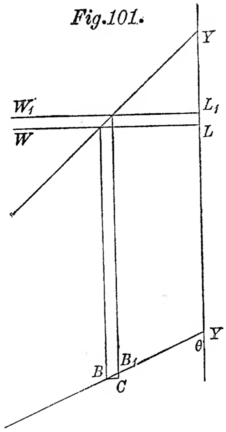
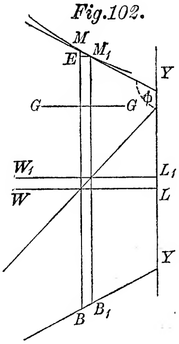


Fig.102.



* In the Session of 1884.

near, C is the point in which a horizontal line through B meets a vertical line through B_1, θ is the angle which the curve of buoyancy makes with the vertical, A is the area of water-plane, V is the displacement, and h is the height of the water-plane above B. It follows that for each foot of change in draught, within the limits in which the tangent may be taken to represent the locus of B, the rise or fall of the centre of buoyancy is $\frac{A\,h}{V}$. We have here, evidently, a ready and useful means of approximating to a ship's curve of buoyancy.

A corresponding equation can be obtained for the tangent to the curve of metacentres, but, as Mr. Jenkins points out, this cannot be reduced to an available form, "owing to the fact that it involves the ratio of the increment of the moment of inertia of the water-plane to the increment of the volume of displacement, or its equivalent the radius of curvature of the curve of flotation." The equation referred to, which Mr. Jenkins gives—using A, V, and h, as before, calling the angle with the vertical θ, M and M_1 being immediately adjacent metacentres, E the point of intersection of a horizontal through M_1 and a vertical through M, and I the moment of inertia of the water-plane (see Fig. 102)—is

$$\frac{M\,E}{M_1\,E} = -\cot.\phi = \frac{A\,I - V\,A\,h - A\,V\,\frac{d\,I}{d\,V}}{V^2};$$

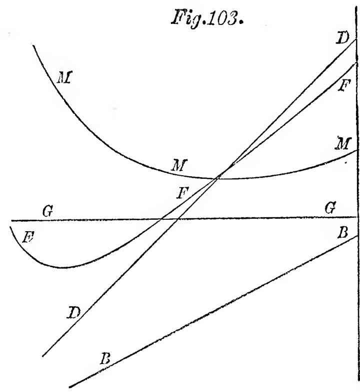
or, putting for $\frac{d\,I}{d\,V}$, r, the radius of curvature of the curve of flotation, and m for the height of the metacentre M, above the centre of buoyancy,

$$\cot.\phi = -\frac{A}{V}(m - h - r) = \frac{A}{V}\,k,$$

where k is the distance of the metacentre, M, below the centre of curvature of the curve of flotation.

The resemblance of this expression for the cotangent of the angle which the locus of metacentre makes at a given point with the upright, to that for the cotangent of the angle which we just saw that the locus of the centres of buoyancy makes therewith is notable. The relation between the two expressions is thus geometrically illustrated by Mr. Jenkins. If, in Fig. 103, the curve M M M represent the locus of the metacentre, F F F the locus of the centre of curvature of the curve of flotation, and B B the locus of the centre of buoyancy, the cotangent of the inclination of the curve to the line Y Y is expressed in the one case by the product of $\frac{A}{V}$ into the corresponding ordinate between the curves M M M and F F F, and in the other by the product of $\frac{A}{V}$ into the ordinate between the diagonal D D and the curve B B; observing that if the centre of curvature of the curve of flotation be above M, cot. ϕ is positive, and *vice versâ.*

Fig.103.



The relation between θ and ϕ may be obtained from the foregoing equations, and is

$$\cot. \phi = \cot. \theta \frac{k}{h},$$

and as cot. θ is very nearly constant, cot. ϕ will practically vary as the ratio of k to h varies. Where the curves M M M and F F F intersect, $k = o$, ϕ becomes 90°, and the tangent to the curve M M M at that point is parallel to the water-plane. This agrees with the result of Mr. White, which has been previously set forth in this chapter. It is also evident that where the curve F F F intersects the diagonal D D, $\frac{d\,I}{d\,V}$ becomes zero, the centre of curvature of the curve of flotation lies in the water-plane, and the moment of inertia of that plane is a maximum or a minimum. Above that point of intersection, Fig. 103, the centre of curvature of the curve of flotation lies below the water-plane, and above it at lighter draughts.

From what has gone before, and more especially from the expression first obtained for cot. ϕ, it will be seen that the radius of curvature of the curve of flotation may be obtained directly from the tangent to the locus of metacentres, and, therefore, without constructing a curve of moments of inertia of the water-planes.

In illustrating the relations which exist between the position of the centre of gravity of a body and the positions of those of its water-planes which give to the initial righting moment maximum and minimum values, Mr. Jenkins points out that these relations are all conformable to a fixed law, which he defines as follows:—"For any given position of the centre of gravity, the initial righting moment is either a maximum or a minimum, when the water-plane is so placed that the centre of curvature of the curve of flotation is at the same height in the vessel as the centre of gravity." He gives a demonstration of this law, and a set of curves illustrating it. He also refers to the cross curves of stability given in Professor Elgar's Royal Society Papers, which will be fully explained in a later chapter, and shows, as we shall see there, the application to such curves of the above principle.

CHAPTER VIII.

Elements to be Determined in Calculating Stability—Atwood's Method of Calculating Volumes of Wedges—Mr. Samuel Read's Method—Mr. Barnes' Method—Moments of the Wedges—Modifications of Mr. Barnes' Method by Messrs. White and John—Mr. Amsler-Laffon's Mechanical Integrator—Example of Mr. Barnes' Method—Explanation of Preliminary and Combination Tables—Tables—Mr. Benjamin's Method of Calculating Statical Stability—Loci of Centres of Buoyancy—Curves of Draughts—Stability Model—Tables—Modification of Mr. Benjamin's Method—Direct Explanation of Diagram.

WE have previously seen that the calculation of a ship's stability at any considerable angle of inclination resolves itself, in accordance with what has gone before, into the determination of the following elements, viz.:—

1. The whole volume of displacement below the given water-line plane when the ship is upright.

2. The position of a new water-line plane inclined to the given one at the required angle, and having below it precisely the same volume as in 1.*

3. The volume of displacement comprised between the first and second water-line planes, on either side of the fore and aft line in which these planes intersect; or, in other words, the volume of the immersed wedge of displacement, or of the emersed wedge, which is equal to that immersed. (The wedge immersed is usually called the "*In*," and that emersed the "*Out*.")

4. The horizontal distance apart, measured parallel to the new water-line, of the centres of gravity of the volumes of the wedges of immersion and emersion.

If the process of calculation adopted is such (as it sometimes is)

* We shall hereafter describe systems of calculation in which it is not necessary to provide for cutting off fixed volumes of displacement.

as to give at once the product of the volume mentioned in (3), and of the horizontal distance mentioned in (4), or, in other words, the *moment* of the wedge about the longitudinal axis, there is obviously no necessity for obtaining these quantities separately.

5. The height of the centre of gravity of the ship above the centre of buoyancy in the upright position.

The sine of the angle of inclination needs no separate calculation, being readily taken from a table of natural sines.

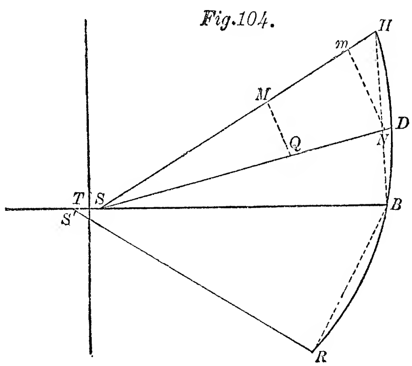
With these items ascertained, the righting arm or lever of stability can be at once determined by substituting these ascertained values for V, A, b, and d in the formula

$$G Z = \frac{b A}{V} - d \sin. \theta.$$

The methods of calculating the displacement of the whole immersed ship when upright, and of its centre of gravity (the centre of buoyancy) need not be discussed here; nor need we stay at this point to explain either how the centre of gravity of the ship is approximately calculated, or how it is experimentally ascertained from the actual ship. But this is a suitable place in which to speak of the methods of calculating the volumes of the wedges, and the positions of their centres of gravity.

The manner in which Atwood's method may be applied has been summarised as follows:*—Let H D B (Fig. 104) be part of the half of a vertical transverse section of a ship, T B being the projection of the water section corresponding to the upright position, and H S the projection of the inclined plane of flotation, cutting T B in the point, S, so that the volume immersed, of which H D B S is the base, may be equal to the volume emersed (brought from the opposite side of the ship), of which B S′ R is the base. Join H B, B R. Then conceive the area, H D B S, to be made up of the triangle, S H B, and a portion, H D B N, of a common parabola. By drawing S D so as

Fig.104.



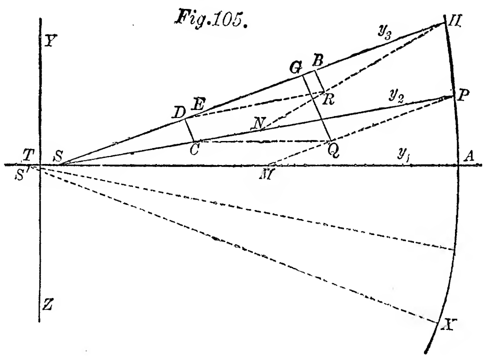
* See Report of Messrs. Read, Chatfield, and Creuze.

to bisect H B in N, and taking $S Q = \frac{2}{3} S N$, we obtain, Q, the centre of gravity of the triangle. The centre of gravity of the parabola, H D B N, is ascertained by taking a point at $\frac{2}{5}$ of N D from N. Then, drawing lines from this point, and from Q at right angles to S H, and meeting it in m and M, we have the horizontal distances of the centres of gravity of the triangle, H S B, and the parabola, H D B H, respectively shown by S M and S m. Hence, the horizontal moment of the "sectorial" area, S H B, about the point, S, is equal to triangle, S H B × S M, plus the parabola H D B × S m. Similarly all the sectorial areas for all the transverse sections, and their horizontal moments about the axis through S may be obtained, and then summed up by any suitable approximate method. In like manner the whole moment of the wedge of emersion, B S′ R, may be obtained, and the two moments be then added together, thus giving the quantity, b A, of the formula.

In order to simplify the calculations, the late Mr. Samuel Read, of the School of Naval Architecture, devised and often employed a method of calculating the wedges of immersion and emersion by means of radial ordinates, the following account of which is by himself:—*

The theorem upon which this novel method is founded may be thus demonstrated:—Let H A X (Fig. 105) be a transverse vertical section of a ship, above or

Fig. 105.



below the water's surface, as the case may be, the point, S, being the intersection of the two planes of flotation corresponding to the vertical and inclined positions of the ship respectively, passing through S A and S′ X on opposite sides of the middle line, Y Z, and A S T, coincident with the water's surface; and let the angle, A S H,

* See *Transactions of Institution of Naval Architects*, vol. ii., for 1861.

of inclination be bisected by the line, S P, passing through the point, P, in the ship's side, A P H. Let the middle vertical line be Y T Z, and A S′ X be the corresponding inclined position of the volume emerged; then the hydrostatical conditions of the ship involve the consequence of the equality of this volume with that of the volume immersed in the volume of the total displacement, which is a constant quantity. Now bisect the side, A S, of the triangle, A S P, in the point, M, and the side, S P, in the point, N; join P M and H N; and take $MQ = \frac{1}{3}MP$, and $NR = \frac{1}{3}NH$; then Q will be the centre of gravity of triangle, P S A, and R that of H S P. Draw Q G and R B perpendicular to H S, and Q C parallel to A S, C D perpendicular to H S; also R E parallel to P S. Then* putting $AS = y_1$; $PS = y_2$; and $HS = y_3$; and the angle $ASH = \theta$; we shall have the moment of the triangle, A S P, from the point, S, in the direction S H, equal to

$$\left(\frac{y_2}{3}\cos.\frac{\theta}{2} + \frac{y_1}{3}\cos.\theta\right)\frac{y_1 y_2}{2}\sin.\frac{\theta}{2};$$

or,

$$\left(y_2\cos.\frac{\theta}{2} + y_1\cos.\theta\right)\frac{y_1 y_2}{6}\sin.\frac{\theta}{2};$$

and the moment of the triangle, P S H, in the same direction, will be

$$\left(\frac{y_3}{3} + \frac{y_2}{3}\cos.\frac{\theta}{2}\right)\frac{y_3 y_2}{2}\sin.\frac{\theta}{2};$$

or,

$$\left(y_3 + y_2\cos.\frac{\theta}{2}\right)\frac{y_3 y_2}{6}\sin.\frac{\theta}{2}.$$

Consequently, the aggregate moments of the two triangles, or the moment of the triangle, A S H, from the point, S, in the horizontal direction, S H, will be found equal to

$$\frac{1}{6}\left\{\left(y_3 + y_2\cos.\frac{\theta}{2}\right)y_3 + \left(y_2\cos.\frac{\theta}{2} + y_1\cos.\theta\right)y_1\right\}y_2\sin.\frac{\theta}{2}.$$

This is not a valuable formula at the present time, when it has become the general and laudable practice to calculate the stability at large angles of inclination; but it has been given as an example of the treatment of the sectional wedge areas by means of radial ordinates. When devised and employed by the late Mr. Read, it was simplified by limiting the supposed inclination to 8 degrees, below which it was assumed that no serious error was introduced by supposing the cosines of $\frac{\theta}{2}$ and θ to be equal to the radius, or 1. The expression then became

$$\frac{1}{6}\left\{(y_3 + y_2)\,y_3 + (y_2 + y_1)\,y_1\right\}y_2\sin.\frac{\theta}{2};$$

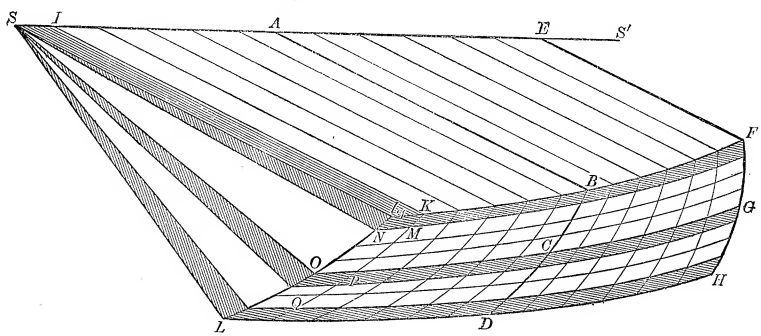
and the operations pointed out by it in this form were not laborious.

* $SC = \frac{1}{3}SP$, and $SE = \frac{1}{3}SH$; $DG = CQ \times \cos.\theta$, and $EB = ER \times \cos.\frac{\theta}{2}$.

Mr. F. K. Barnes, of the Admiralty, produced at the Institution of Naval Architects in 1861 a very practical and satisfactory method of calculating the volumes and moments of the wedges of immersion and emersion for any angle of inclination, giving the following account of it (which we condense), observing that it "admits of any degree of accuracy, and can be readily applied by persons who are but indifferent mathematicians." It has been generally employed in this country ever since its introduction, as it well deserved to be.

Let S H, Fig. 106, represent a portion only of one of the wedges

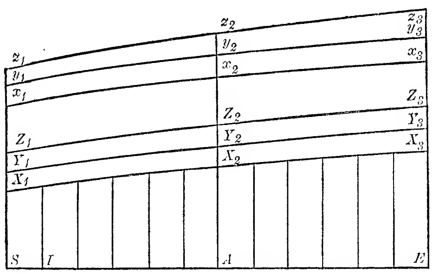
Fig. 106.



of immersion or emersion, cut off by planes, S L_1 L, E F H, perpendicular to S S′, the intersection of two planes of flotation. Imagine the wedge to be divided into a large number of thin slices by equidistant planes, all perpendicular to S S′, or parallel to the bounding planes, S L L_1 and E F H, and imagine, further, each of these thin slices to be divided into the same large number of thin wedges by planes passing through S S′, each such plane being inclined to its adjacent planes at the small angle, a. The volume of the whole wedge, S L_1 H, is equal to the sum of all the very small wedges into which it is divided by the two sets of planes. The volume of one of these very small wedges, as S L_1 M shaded in the figure, is equal to the small triangular figure, S L_1 N, multiplied by the breadth of the wedge, which is the same as the distance, S I, between the transverse planes of division. Now, since the angle, L_1 S N $(= a)$, is very small, the side, L_1 N, of the triangle, L_1 S N, may be regarded as a small portion of an arc of a circle with centre, S, and is therefore equal to S $L_1 \times a$; and the area of the small triangle (or sector), S L_1 N (which is equal to $\frac{1}{2}$ S $L_1 \times L_1$ N), is equal to $\frac{a}{2} \times \mathrm{S\,L}_1^2$; and the volume of the one small wedge is $\frac{a}{2} \times \mathrm{S\,I} \times \mathrm{S\,L}_1^2$.

In proceeding to find the aggregate volume of all the similar small wedges in the upper series (bounded above by the inclined water section and below by the adjacent plane), Mr. Barnes resorted to the device of setting off the wedge volumes as ordinates of a curve, and then finding the area of the curvilinear area so obtained. To do this he took a line, S E, Fig. 107, equal to S E in Fig. 106, and divided it into two equal parts at A, as S E is divided by the transverse plane, A B D, in Fig. 106, and through the points of division in Fig. 107 he drew lines perpendicular to S E, setting off upon them quantities equal at each point to the square of the corresponding ordinate in Fig. 106. The ordinate,

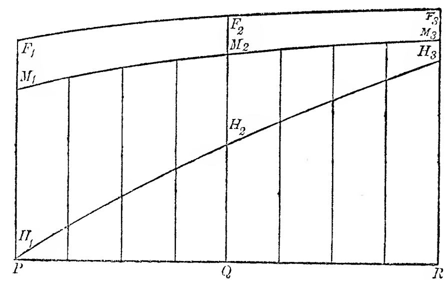
Fig.107.



SX_1, at S, Fig. 107, was, of course, equal to SL_1^2, that at A was AB^2, and that at E was EF^2, the parabolic curve, $X_1 X_2 X_3$, being drawn through the points. The whole curvilinear area, $S X_3$, when multiplied by $\frac{a}{2}$, was equal to the whole volume of the very thin wedge, S N F. Represent this area, $S X_1 X_3 E$ by B. In the same manner curvilinear areas may be found to represent each of the series of thin wedges which together make up the whole wedge, $S L_1 H$, and the sum of all these curvilinear areas, multiplied by $\frac{a}{2}$, will give the volume of the whole wedge, $S L_1 H$.

But in order to reduce the work thus set before us, Mr. Barnes introduces a further artifice. He takes a line, P Q R, Fig. 108, to represent the angle, I, of the ship's inclination, and divides it into as many equal portions as the whole wedge, $S L_1 H$, Fig. 106, is divided into very thin wedges, S N F, &c., drawing, at the points of division, ordinates perpendicular to P R. On these ordinates he sets off distances numerically equal to the curvilinear areas (of Fig. 107) which, when multiplied by $\frac{a}{2}$, represent the several volumes of the wedges comprised in $S L_1 H$. Joining the extremities of the ordinates so determined by a parabolic curve, he obtains a curvilinear area, the divisions of which (viz., the small areas between the consecutive ordinates) represent the volumes of the successive wedge-slices, S N F, &c., and the whole curvilinear area of which will represent the volume of the whole wedge, SL_1H. In this case, however, as in the previous one, instead of dealing with every ordinate, he deals only with the two extreme ones, and an intermediate one, which here is S O G, Fig. 106, dividing the angle, LSL_1, into two equal portions. Q is the corresponding point in Fig. 108. The curvilinear areas, SY_1Y_3E and SZ_1Z_3E, in Fig. 107, are found representing the volumes of the thin wedges adjacent to S O G, and to the upright water-plane respectively, and these are represented by C and D respectively, so that $C \times \frac{a}{2}$ and $D \times \frac{a}{2}$ will be numerically equal to the volumes of these thin wedges. Then in Fig. 108 set off perpendicular to P R, $PM_1 = B$, $QM_2 = C$, and $RM_3 = D$, and pass a parabolic curve through M_1, M_2, and M_3. The area of the curvilinear area bounded by this line will be equal to the entire volume of the wedge, SL_1H.

Fig. 108.



The above processes must, of course, be applied throughout the whole length of the ship, and the complete *volume* of the wedge of immersion or of emersion may be thus obtained. Like processes are pursued for obtaining the *moments* of the wedges of immersion and emersion. Mr. Barnes pointed out the great difficulty of getting these wedges exactly equal, and adopted the usual method of equalising them by drawing a new water-plane parallel to the inclined water-plane, and at a distance, *d*, from it given by the formula

$$d = \frac{\text{Difference of wedge volumes.}}{\text{Area of inclined water section.}}$$

The determination of the exact water-line by this means is sometimes not practised, the inclined water-plane section being supposed to pass through the middle point of the upright water-section (as projected on the plane of the ship's body-plan), corrections for the error thus involved being subsequently introduced.

In proceeding to find the *moments* of the *ins* and *outs*, *i.e.*, the moments of all the small wedges into which the large wedge, SL_1H, has been divided, Mr. Barnes points out that it is manifest that the distance of the centre of gravity of one of the small wedges, S K N from $S S_1$, Fig. 106, is the same as that of the small triangular area, $S L_1 N_1$ or $\frac{2}{3} S L_1$; and, therefore, the moment of that wedge about $S S_1$ is

$$= S I \times \frac{a}{2} \times S L_1^2 \times \frac{2}{3} S L_1,$$

$$= S I \times \frac{a}{3} \times S L_1^3.$$

Also, the moment of any other small wedge in the upper series about $S S'$ is similarly equal to $S I \times \frac{a}{3} \times$ cube of ordinate measured on the water-line section parallel to $S L_1$.

The *moment* of the whole thin wedge may, therefore, be found in the same manner as its volume was found.

Using the same figure, 107, set off the distance $S x_1$, numerically equal to $S L_1^3$; also, make $A x_2$ numerically equal to $A B_1^3$ and $E x_3$ to $E F^3$; through the points x_1, x_2, and x_3 draw a parabolic curve, and the area thus enclosed will represent the moment of the upper thin wedge about $S S'$, and if multiplied by $\frac{a}{3}$ will be numerically equal to it.

Similarly the moment of the wedge, S P O, may be dealt with, observing that, as the distance of its centre of gravity from S has to be measured in the direction of $S L_1$, or parallel thereto, and *not* in the direction of S O, we shall have to multiply the distance $\frac{2}{3}$ S O by the cosine of the angle $\left(\frac{I}{2}\right)$ which its centre line makes with S L; and, therefore, the expression for the moment of this wedge, S P O, becomes

$$S I \times \frac{a}{3} \times \cos. \frac{I}{2} \times S O^3.$$

The co-efficient $S I \times \frac{a}{3} \times \cos. \frac{I}{2}$ will be the same for all the small wedges lying next (below) the plane, S O G, and their moments will vary only as the cubes of the ordinates. Proceeding as before, therefore, we may construct the curve, y_1, y_2, y_3, Fig. 107, and the area lying within it will represent the moment of the entire thin wedge adjacent to the plane, $S O G_1$, and when multiplied by $\frac{a}{3} \cos. \frac{I}{2}$ will be numerically equal to it.

In like manner the area $S z_1 z_3 E$ may be made to represent the moment of the lowest series of their wedges, and when multiplied by $\frac{a}{3} \cos. I_1$ will be numerically equal to it.

Now, let P R, Fig. 108, be taken to represent the ship's angle of inclination; bisect it in Q, and draw ordinates through P Q R perpendicular to P R—all as before. Make $P F_1$ numerically equal to the curvilinear area, $S x_3$; $Q F_2$ equal to the area, $S y_3$, multiplied by $\cos. \frac{I}{2}$; and $R F_3$ to the area, $S z_3$, multiplied by cos. I. Draw the parabolic curve, F_1, F_2, F_3, and the area lying within it will represent the moment of the entire wedge, $S L_1 H$ about $S S'$, and $\frac{1}{3}$rd the area obtained by the common rule will be numerically equal to the moment.

This calculation, like that for the volume of the wedge, must, of course, be extended for the whole wedge extending from end to end of the ship, and for both the wedges (*ins* and *outs*). Their sum will be the quantity

$$v \times h h'$$

of Atwood's fundamental formula.

In the Paper of Messrs. White & John, read at the Institution of Naval Architects in 1871, an account was given of the extensions and modifications which the above system of calculation, introduced by Mr. Barnes many years before, had subsequently undergone in meeting the wholly modern conditions under which stability has now frequently to be calculated, more especially in the Admiralty Office. With the sanction of the Admiralty Council of Construction, an official specimen calculation was produced, and to this we shall presently revert when we come to explain the formal methods of conducting calculations of stability.

All the time such calculations had to be effected without direct mechanical aid, it was a matter of much importance to facilitate them by the introduction of whatever devices tended to simplify, shorten, and relieve from the risk of errors and oversights the actual processes of calculation. But the invention and employment for this purpose of Mr. J. Amsler-Laffon's mechanical integrator has brought about a great change, enabling the draughtsman, as it does, to ascertain the *area*, the *statical moment*, and the *moment of inertia* of any closed curvilinear area, by simply passing a pointer along its boundaries, and reading off the results. In using this instrument, the areas and moments of the wedges of immersion and emersion are found directly from the body-plan of the ship. The readings of these areas and moments are set off as ordinates of new curvilinear figures of which the abscissæ are given by the positions of the stations at which the sectional areas have been taken. The areas of these new curvilinear figures are then obtained by means of the instrument, and are equal to the volumes and moments of the wedges. When this instrument was first brought to the notice of naval architects, it was thought that it would be chiefly valuable as furnishing a ready check upon calculations made by the method of Mr. Barnes. But those calculators who have now had great experience with it, claim for it, not only greater simplicity, but greater accuracy likewise, and both these combined with much economy of time, and with the further economy of enabling the calculations to be effected by young and inexpensive persons. The perfect success with which young apprentice boys (and in some instances girls) are found to make calculations of stability with the Amsler Integrator has frequently been mentioned to the author, one ingenious friend observing—"I find boys more accurate in these calculations than men, who have a lot to think about." But further remarks on this instrument must be deferred until the ordinary process of calculation has been described.

We will now give an illustration, taken from the author's office records, of Mr. Barnes' method of calculating stability. We select the case of the *Kaiser*, an ironclad central-battery ship, 285 feet long, 62 feet beam, and about 7,600 tons displacement, designed by the author for the Imperial German Government. The example shown consists of the "Preliminary" and "Combination"* tables, for the ship inclined to an angle of 34 degrees from the upright, and is accompanied by a description of the tables.

It may be again noted that the calculation relating to the alteration in the position of the longitudinal centre of gravity, caused by the heeling of the vessel, is found in general practice to be unnecessary, as the alteration is so slight as not to practically affect the righting moment, even at large angles of heel. The following is a description of the method of calculating the stabilities set forth in the accompanying tables:—

* See Messrs. White & John's Paper on "Stability," *Transactions, I. N. A.*, 1871.

The vessel is assumed to be floating in still water. The body-plan is prepared from transverse sections 26·64 feet apart, with additional sections 13·32 feet apart, at the extremities of the ship. The load water-line of the vessel is drawn, and through the intersection of this and the middle line, radial lines are drawn at such angular intervals as to allow one of them to pass through the edge of the deck when that is just immersed. This interval is in the example $8\frac{1}{2}$ degrees.

The lengths of the ordinates of the transverse sections measured along the radial lines are arranged in the "Preliminary" table (page 155), as also the squares and cubes of the ordinates, under the heading of the respective wedges, both "Immersed" and "Emerged." These quantities are then treated by Simpson's rule, from which the functions of the ordinates of the inclined water-plane are obtained, in order to find its area; the functions of the squares of the ordinates, in order to obtain the position of its centre of gravity, and also the excess of the volume of one of the wedges over the other; and the sum of the functions of the cubes of the ordinates, in order to determine the moment of inertia.

It will be observed that the functions above referred to are divided by 3, so that the common factor for use in the "Combination" table for areas, moments, &c., is the longitudinal interval.

In the "Combination" table (pages 156–7), the various angles of inclination, from the body-plan, are entered up to the one to which it is desired to calculate the stability, and the results or functions from the "Preliminary" table above described are arranged in the proper columns opposite their respective angles, observing that it is necessary to prepare a "Preliminary" table for each angle of inclination used in the calculation. These are treated by the correct multipliers, and care must be taken to make the angle at which the deck becomes immersed a stopping-point in the integration in order to avoid discontinuity. The area of the inclined water-section is now ascertained, as also the position of its centre of gravity; the calculation showing the excess of the volume of either of the wedges over the other is next proceeded with, and is obtained from the functions of the squares of the ordinates for volume of wedges, for the various angles of inclination, taken from the respective preliminary tables.

Should this excess prove to be on the "Emerged" side, as in our example, it shows that the total displacement of the ship at this inclined line is less than when in the upright position.

In order to make the necessary correction for this deficiency, and find the true water-line at this inclination, the volume of the layer, after correcting for appendages, is found and divided by the area of the inclined water-section, which gives its thickness. This thickness is added to the assumed height of the water-section, and gives the true position of the inclined water-line. If, on the contrary, the "Immersed" wedge is in excess, the thickness of the layer must be deducted.

The sums of the functions of the cubes of the ordinates as taken from the respective "Preliminary" tables are now dealt with by the same multipliers, and their products multiplied by the cosines of the angles of inclination. The sum of the functions thus obtained being divided by 3, and multiplied by the longitudinal interval, and also by one-third the angular interval, gives the moment of the wedges of "Immersion" and "Emersion," uncorrected, taken about the intersection of the middle line and the inclined water-section. The corrections for the appendages and layer are now taken into account, observing that for the latter the centre of gravity of the layer lies toward the immersed side, and the emerged wedge being the greater, the correction must be added. Had this centre of gravity been toward the emerged side the correction would have had to be deducted. Dividing the result, after the necessary corrections have been made, by the total displacement in cubic feet, the horizontal distance (BR in the table) between the centre of buoyancy in the upright position and the centre of buoyancy in the inclined position is obtained. Deducting from this distance the vertical height between the centre of buoyancy and the centre of gravity of the ship when in the upright position (B G in the table), multiplied into the sine of the angle of inclination, the length of the arm of the righting lever (G Z in the table) is obtained, and this, multiplied by the displacement, gives the righting moment in foot-tons.

The moment of the wedges, uncorrected, for the dynamical stability is obtained by multiplying the same products of the sums of the functions of cubes of the ordinates as were used for the statical stability, by the sines of the angles of inclination, dividing the sum of the functions so obtained by 3, and multiplying by the longitudinal interval and by $\frac{1}{3}$ of the angular interval. Corrections for the appendages and layer are now made, observing that in finding the dynamical stability the correction for the latter is always deducted, irrespective of the position of the centre of gravity of the layer.

PRELIMINARY TABLE.

WATER-SECTION INCLINED AT 34 DEGREES.

IMMERSED WEDGE.

No. of Section.	Ordinates.	Multipliers.	Functions of Ordinates.	Squares of Ordinates.	Multipliers.	Functions of Squares.	Cubes of Ordinates.	Multipliers.	Functions of Cubes.
1	0	½	0	0	½	0	0	½	0
1½	12·8	2	25·6	163·8	2	327·6	2,097·2	2	4,194·4
2	21·3	1½	31·9	453·7	1½	680·5	9,663·6	1½	14,495·4
3	26·0	4	104·0	676·0	4	2,704·0	17,576·0	4	70,304·0
4	27·5	2	55·0	756·2	2	1,512·4	20,796·9	2	41,593·8
5	29·1	4	116·4	846·8	4	3,387·2	24,642·2	4	98,568·8
6	30·1	2	60·2	906·0	2	5,436·0	27,270·9	2	163,625·4
7	30·1	4	120·4	906·0	4		27,270·9	4	
8	30·0	2	60·0	900·0	2	1,800·0	27,000·0	2	54,000·0
9	29·9	4	119·6	894·0	4	3,576·0	26,730·9	4	106,923·6
10	28·9	1½	43·3	835·2	1½	1,252·8	24,137·6	1½	36,206·4
10½	26·3	2	52·6	691·7	2	1,383·4	18,191·4	2	36,382·8
11	20·5	½	10·2	420·2	½	210·1	8,615·1	½	4,307·5
			3)799·2			3)22,270·0			3)630,602·1
			266·4			7,423·3	Immersed, .		210,200·7
							Emerged, . .		204,298·3
							Sum, . . .		414,499·0

EMERGED WEDGE.

No. of Section.	Ordinates.	Multipliers.	Functions of Ordinates.	Squares of Ordinates.	Multipliers.	Functions of Squares.	Cubes of Ordinates.	Multipliers.	Functions of Cubes.
1	·3	½	·1	·1	½	0	0	½	0
1½	9·1	2	18·2	82·8	2	165·6	753·6	2	1,507·2
2	15·6	1½	23·4	243·4	1½	365·1	3,796·4	1½	5,694·6
3	25·0	4	100·0	625·0	4	2,500·0	15,625·0	4	62,500·0
4	29·6	2	59·2	876·2	2	1,752·4	25,934·3	2	51,868·6
5	31·4	4	125·6	985·9	4	3,943·6	30,959·1	4	123,836·4
6	32·3	2	64·6	1,043·3	2	6,259·8	33,698·3	2	202,189·8
7	32·3	4	129·2	1,043·3	4		33,698·3	4	
8	31·2	2	62·4	973·4	2	1,946·8	30,371·3	2	60,742·6
9	27·7	4	110·8	767·3	4	3,069·2	21,253·9	4	85,015·6
10	20·6	1½	30·9	424·4	1½	636·6	8,741·8	1½	13,112·7
10½	14·7	2	29·4	216·1	2	432·2	3,176·5	2	6,353·0
11	5·3	½	2·6	28·1	½	14·0	148·9	½	74·4
			3)756·4			3)21,085·3			3)612,894·9
			252·1			7,028·4			204,298·3

COMBINATION TABLE.

Calculation of Stability at an Inclination of 34 Degrees.

	IMMERSED WEDGE.				BOTH WEDGES.						
								Statical Stability.		Dynamical Stability.	
Inclination of Radial Planes.	Functions of Ordinates of Area of Inclined Water-Section.	Functions of Squares of Ordinates.	Multipliers.	Functions of Squares of Ordinates for Vol. of Wedges.	Sums of Functions of Cubes of Ordinates.	Multipliers.	Products of Functions of Cubes.	Cosines of Inclination.	Functions of Cubes for Moments of Wedges.	Sines of Inclination.	Functions of Cubes for Moments of Wedges.
0	...	6,983·5	1	6,983·5	394,243·0	1	394,243·0	·82904	326,843·2	·55919	220,456·7
$8\frac{1}{2}°$	...	6,690·2	4	26,760·8	392,841·8	4	1,571,367·2	·90258	1,418,284·6	·43051	676,489·3
17°	...	6,546·4	2	13,092·8	393,174·9	2	786,349·8	·95630	751,986·2	·29237	229,905·1
$25\frac{1}{2}°$	...	6,740·9	4	26,963·6	401,047·2	4	1,604,188·8	·98902	1,586,574·8	·14781	237,115·1
34°	266·4	7,423·3	1	7,423·3	414,499·0	1	414,499·0	1·00000	414,499·0	·00000	0·0
	For Immersed Wedge, . . .			81,224·0				3)	4,498,187·8	3)	1,363,966·2
	For Emerged Wedge, . . .			87,036·1					1,499,395·9		454,655·4
				2)5,812·1	Longitudinal Interval, ...				26·64		26·64
Difference in favour of Emerged Wedge,				2,906·0					39,943,906·8		12,112,019·8
	Longitudinal Interval, . . .			26·64	$\frac{1}{3}\times$ Angular Interval, ...				·04945		·04945
				77,415·84	Moment of Wedges, uncorrected, . . .				1,975,226·2		597,939·3
	$\frac{1}{3}\times$ Angular Interval, ...			·04945	Correction for appendages, . . .				88,338·7		7,361·5
	Excess in volume of Emerged Wedge without appendage, . . .			3,828·21	Correction for layer, . . .				383·1		8·6
								264,556·25)	2,063,948·0	264,556·25)	605,292·2
					B R =				7·802	B_1 R =	2·287
					B G sin. θ = 9·98 × ·5592 =				5·585	B G vers. θ (9·98 × ·171)	1·706
					G Z =				2·217		·581
					Displacement, =				7,558·75		7,558·75
					Righting Moment in Foot-tons, =				16,757·7	Work in Foot-tons,	4,391·6

COMBINATION TABLE—*continued.*

CALCULATION OF STABILITY AT AN INCLINATION OF 34 DEGREES.

EMERGED WEDGE.

0	...	6,983·5	1	6,983·5
$8\frac{1}{2}$	...	7,229·4	4	28,917·6
17	...	7,356·9	2	14,713·8
$25\frac{1}{2}$	...	7,348·2	4	29,392·8
34	252·1	7,028·4	1	7,028·4
				87,036·1

OUTLYING APPENDAGE.

Volume, Abaft No. 11, Ordinate, .	549·3 c. ft.
Volume, In wake of Battery, .	2,774·4
Both on immersed side, . .	3,324·1
Moments of Volumes, Abaft No. 11, Ordinate,	6,027·1 sq. ft.
Moments of Volumes, In wake of Battery, .	82,311·6
	88,338·7
Area of Water-Section, Abaft No. 11, Ord., .	268·1 sq. ft.
Area of Water-Section, In wake of Battery, .	127·2
	395·9
Moments of Areas, Abaft No. 11, Ordinate, .	1,704·3
Moments of Areas, In wake of Battery, .	3,781·1
	5,485·4

INCLINED WATER-SECTION.

Functions of Ordinates, Immersed, . .	266·4
Functions of Ordinates, Emerged, . .	252·1
	518·5
Longitudinal Interval, . . .	26·64
Area without Appendage, . .	13,812·8
,, of ,, . .	395·9
Total Area, ...	14,208·7
Functions of Squares of Ordinates, Immersed,	7,423·3
Functions of Squares of Ordinates, Emerged,	7,028·4
	2) 394·9
	197·4
Longitudinal Interval, . .	26·64
Moment of Area Immersed, .	5,258·7
,, Appendage ,, . .	5,485·4
	14,208·7) 10,744·1
C G of Area towards the Immersed Side, . . .	·76

CORRECTING LAYER.

Excess in Volume of Immersed Wedge, . .	3,828·2
Volume of Appendages Immersed Side, . . .	3,324·1
Total Volume of Layer, .	504·1

Thickness of Layer $= \dfrac{504·1}{14,208·7}$ = ·035 foot.

Correction for Statical Stability, = 504·1 × ·76 = 383·1.

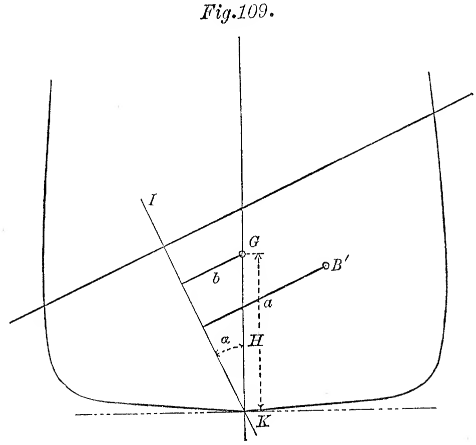
Correction for Dynamical Stability, = 504·1 × ·017 = 8 6.

This result divided by the displacement in cubic feet gives the distance ($B_1 R$ in the table), on a vertical line drawn through the centre of buoyancy in the inclined position, between that centre and a horizontal line drawn through the position of the centre of buoyancy in the upright position. The vertical height between the centre of gravity of the ship and its centre of buoyancy in the upright position, multiplied by the versed sine of the angle of inclination is now deducted, and the remainder multiplied by the total displacement in tons represents the work expended in foot-tons in heeling the vessel to the angle of 34 degrees.

Mr. Ludwig Benjamin has recently devised a method of arranging and carrying out the calculations of statical stability in a novel manner, which is employed by some firms upon the Clyde, and is marked by considerable skill and ingenuity. Mr. Benjamin's object has been to indicate in a single diagram "the stability for every angle of heel, every draught, and every position of the centre of gravity," and to accomplish this object "in less time than the usual method required for one draught only." Mr. Benjamin further regards his results as at least as correct as those of the usual method. An account of his system will be of interest, at least as a means of regarding the doctrines of stability from somewhat novel points of view.

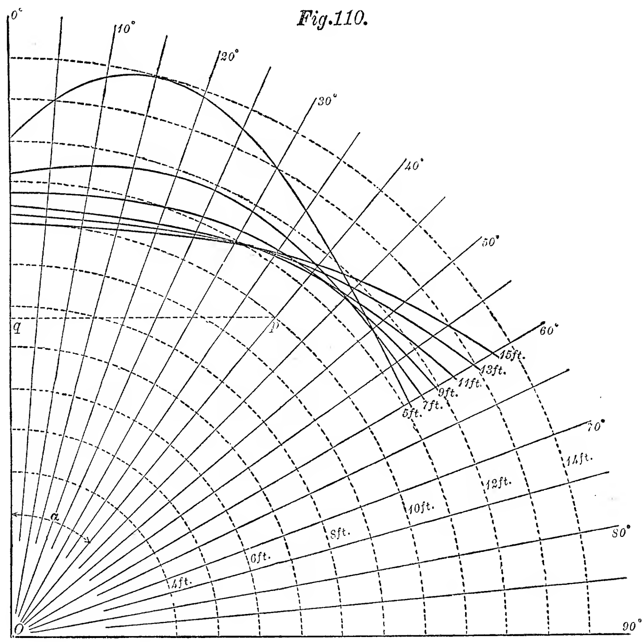
In Fig. 109 is illustrated the first step taken in following his method, a line, K I, being drawn through the upright axis of the

Fig.109.



ship at the height of the top of the keel, at any given inclination of the ship, say *a*, from the upright. From the centre of gravity, G, and also from the new centre of buoyancy, B′, draw perpendiculars upon K I. Calling (as he does) the perpendicular from B′, a, and that from G, b, and the height, K G, H, we have the lever of stability $= a - b$; and $b =$ H sin. a. Values of the latter (H sin. a) may be conveniently obtained for varying values of H and of sin. a by constructing a diagram, Fig. 110, in which the lines radiating from a fixed point, O, are drawn for numerous values of a, from 0 up to 90°, and circular arcs are described about the point O, with radii corresponding to various heights of the centre of gravity above the top of keel. It is obvious that if we measure the horizontal distance from the upright line of any point of intersection of a radial line with a circular arc, we shall have the value of H sin. a

Fig. 110.



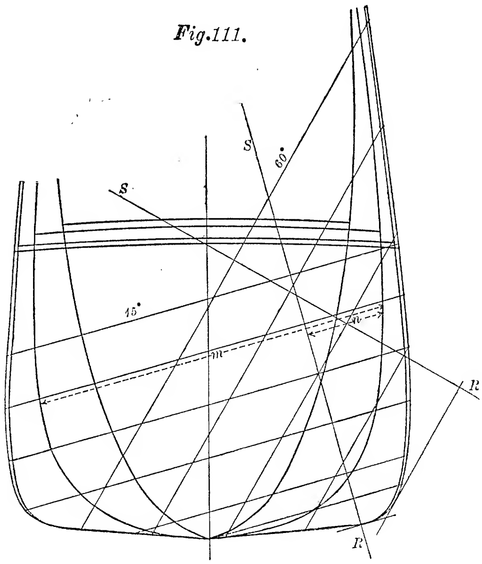
for the corresponding height of G and the corresponding draught of water. For example, $p\,q$ (Fig. 110) $=$ H sin. a, when H $= 10$ ft., and $a = 40°$.

Mr. Benjamin next calculates the Displacement and the Centre of Buoyancy of the ship for various draughts of water, as in Fig. 111, and further calculates the position of the centre of buoyancy for a series of successive inclinations, and describes the loci of these centres for each of the successive given inclinations, and at the respective draughts, as illustrated by the curves, $l\,c\,b$, in Fig. 112.*

* See also Fig. 38, chap. iv. We think it right to state that, although Mr. Benjamin's method finds a place here, and Mons. Daymard's comes later in this work,

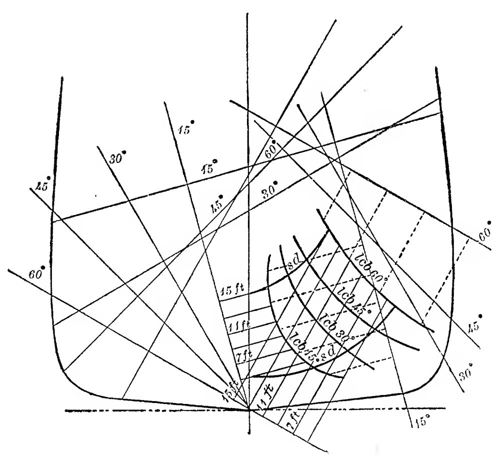
Upon these loci, or curves, are marked the points which correspond to the different upright draughts. To facilitate this operation, Mr.

Fig. 111.



Benjamin makes use of curves such as those shown in Fig. 113,

Fig. 112.

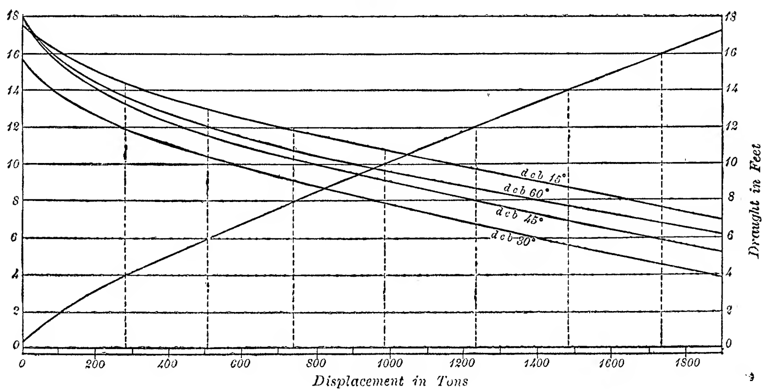


marked *d c b*, which represent the distances, at various inclinations, of the centre of buoyancy from any given water-line (the first in

the MS. of Mons. Daymard was in our hands several months before Mr. Benjamin's system came to our knowledge.

this case). An ordinary displacement scale being made from the results of the calculations, the distances from the given water-line of the centres of buoyancy corresponding to the calculated displacements are also set up to scale from the base-line, and curves passed through them. From these curves the distance from the given

Fig. 113.



water-line of the centre of buoyancy for any displacement and inclination can be readily obtained.

The curves marked *s d* in Fig. 112 are obtained by joining those centres of buoyancy at the different inclinations which correspond to the same displacement, and are used only as a means of checking the calculations.

By aid of Fig. 112 measurements of the value of *a* (the perpendicular from B′ upon K I, Fig. 109) can now be made for different draughts and different inclinations, and can be set off horizontally in Fig. 110 from the upright line through O, and curves being then passed through the points thus obtained corresponding to each draught, a new series of curves is described which Mr. Benjamin designates "Curves of Draughts." The diagram, Fig. 110, being thus finished, constitutes a complete representation of the statical stability for every inclination up to 90 degrees, and for every draught of water between the limits employed in the construction of the figure. To find in this diagram the lever of stability for a given draught and inclination, it is only necessary to mark the two points upon which the line for the given angle of inclination is intersected by the "curve of draughts," and by the circle of centres of gravity respectively, and to measure the distances of these points from the vertical axis, $Oo°$, of the figure: the difference of these distances is the stability lever. When the circle lies within the curve of draughts the lever is a *righting* lever, because b will be less than a in the previous expression for the lever, $a - b$; when the circle falls outside of the curve of draughts, the lever will be an *upsetting* one. In dealing with the case of the deck-edge becoming immersed, Mr. Benjamin, in order to get his curves exact, in the first place supposes the side continued up (as is often done in other systems), afterwards making deductions for the deficiency caused by the deck entering the water. Quarter-decks, bridge-houses, &c., are accounted for in like manner.

Mr. Benjamin has conceived and carried out the idea of employing the "curves of draughts" in the construction of a model whose horizontal sections correspond to the respective draughts of water, the planes of which consequently may be supposed to intersect the model, and may be easily drawn upon it. Radial planes corresponding to the respective inclinations are also supposed to intersect the model, and their intersections may likewise be readily drawn upon it. If the model be built up of annular sections, corresponding to the circular arcs in Fig. 110 (which represent heights of centres of gravity), the intersections of such arcs with the surface of the model will also be shown. From such a model measurements giving the levers of stability may obviously be readily measured. Mr. Benjamin considers that this model will facilitate comparisons between ships, and help to convey to captains of ships and others, in a more impressive form than a drawing, the necessary information with respect to the stability of the vessels in which they are concerned.

The calculations necessary to obtain the displacements and positions of centres of buoyancy required for the construction of such a diagram as Fig. 110 can be readily made by the aid of Amsler's Integrator; but as this instrument is not in general use, we give below a specimen of each of two tables used by Mr. Benjamin in making his calculations directly from the body-plan of a ship. His mode of applying well-known rules will be found somewhat different from that usually pursued in such cases, but will be easily understood by means of the following explanation.

The two Tables given, which contain the figures for an actual ship, are used to find the required data for one inclination only (15°), and similar tables would have to be used for each inclination for which the data may be required. Referring to Fig. 111, the water-lines (arbitrarily chosen) used for this inclination are there shown upon the body-plan of the ship, as well as the axis, R S (arbitrarily fixed), relative to which the abscissæ of the centres of buoyancy are to be calculated, this axis being so chosen that all moments taken about it shall be to the same side of it. In the column to the extreme left of Table I. the numbers of the transverse sections used are placed, and in the next "Simpson's multipliers" corresponding to them. It will be seen that each water-line has four columns devoted to it. In the first of these are entered the breadths of the ordinates at the several sections (as m, Fig. 111), and in the third the distances of their centres from the axis, R S $\left(\frac{m}{2} - n \text{ or } \frac{m}{2} + n, \text{as the case may be}\right)$. The ordinates are multiplied by Simpson's multipliers, and the products placed in the second column; their sum being divided by the sum of the multipliers, the length of the mean ordinate is obtained. The fourth column contains the functions of moments of the ordinates about the axis, R S, these being arrived at by multiplying the quantities in the second column by those in the third: their sum is divided by the sum of the multipliers, the result being the mean moment.

In Table II. the first column on the left contains the numbers of the water-lines used, and the second their distances (in multiples of the interval) from the 1st water-line; in the third are placed the mean ordinates for the several water-lines, and in the fourth the mean moments, the two latter being taken from Table I. The fifth, sixth, seventh, and eighth columns form a group headed "Up to 1st W. L.," and the remaining columns comprise similar groups. The fifth column contains Simpson's multipliers corresponding to the numbers of the water-lines, by which the mean ordinates in the third column are multiplied; and the results (functions of mean ordinates) are placed in the sixth column; their sum is divided by the sum of the multipliers, and the mean of all the ordinates is thus obtained, which, being multiplied by the distance between the extreme water-lines, and by the length of the ship, gives the volume of displacement. The seventh column contains the functions of vertical moments, arrived at by multiplying the quantities in the sixth by the numbers in the second column, and the sum of these functions being divided by the sum of the functions of mean ordinates (sixth column), the result is the vertical distance (in multiples of the interval) of the centre of buoyancy below the 1st water-line; this multiplied by the interval gives the same distance in feet. In the eighth column are placed the functions of mean level moments, obtained by multiplying the mean level moments (fourth column) by the multipliers (fifth column); and their sum being divided by the sum of the functions of mean ordinates (sixth column), the result is the distance, parallel to the water-lines, of the centre of buoyancy from the axis, R S. The results obtained in the groups headed "between 1st and 4th W. L." and "between 2nd and 4th W. L." are combined with the results of other groups, in order to give the positions of the centre of buoyancy for displacements up to 2nd water-line and up to 4th water-line.

The displacements and vertical and level distances of the centres of buoyancy are recorded at the foot of the table, and these results are used for constructing the loci of centres of buoyancy in Fig. 112 —as already explained.

SPECIMEN CALCULATION.

Inclination, 15°.)

TABLE I.

Length, 195′ 0″. *W. Lines*, 3′ 6″ *apart*.

No. of Sections.	Simpson's Multipliers.	1st W. L.				2nd W. L.				3rd W. L.				4th W. L.				5th W. L.				6th W. L.			
		Ord.		Dist.		Ord.		Dist.		Ord.		Dist.		Ord.		Dist.		Ord.		Dist.		Ord.		Dist.	
1	½	0·	...	...	...	0·	...	...	...	0·	...	...	...	0·	...	...	...	0·	...	...	...	0·	...	...	...
2	2	22·95	45·9	5·45	250	20·7	41·4	5·45	226	16·6	33·2	6·15	204	10·75	21·5	7·6	163	4·8	9·6	9·05	87	0·	...	...	...
3	1½	25·9	38·85	6·0	233	25·6	38·4	6·55	251	24·05	36·07	6·77	244	20·6	30·9	6·65	205	12·9	19·35	6·65	129	1·	1·5	6·7	10
4	4	27·2	108·8	6·05	658	27·55	110·2	6·92	763	27·5	110·0	7·7	847	26·2	104·8	7·9	828	18·75	75·0	5·22	391	12·1	48·4	3·	145
5	2	27·4	54·8	6·1	334	27·7	55·4	6·9	382	27·8	55·6	7·7	428	26·8	53·6	8·1	43	18·75	37·5	5·17	194	12·6	25·2	2·8	71
6	4	27·1	108·4	6·1	661	27·4	109·6	6·9	750	27·3	109·2	7·65	835	26·2	104·8	8·0	838	18·5	74·0	5·25	388	12·1	48·4	3·	145
7	1½	24·45	36·67	5·82	214	24·2	36·3	6·6	240	23·15	34·72	7·2	250	20·7	31·05	7·4	230	12·6	18·9	5·9	111	6·7	10·5	5·75	58
8	2	18·6	37·2	5·45	203	18·	36·0	6·3	227	14·9	29·8	7·1	212	11·95	23·9	7·87	188	6·7	13·4	9·25	124	0·	...	...	...
9	½	0·	...	...	...	0·	...	...	...	0·	...	...	...	0·	...	...	...	0·	...	...	...	0·	...	...	...
...	18	...	430·62	...	2553	...	427·3	...	2845	...	408.59	...	3020	...	370·55	...	288	...	247·75	...	1424	...	133·55	...	429
Means		...	23·9	...	142	...	23·7	...	158	...	22·7	...	168	...	20·6	...	16	...	13·76	...	79	...	7·42	...	24

Table II.

No. of Water-Lines.	Multiplier for Vert. Moment.	From Table I.		Up to 1st W. L.				Up to 3rd W. L.				Up to 5th W. L.				Between 1st and 4th W. L.				Between 2nd and 4th W. L.			
		Mean Ordinate.	Mean Level Moment.		Function of Ordinate.	Function of Vert. Moment.	Function of Level Moment.		Function of Ordinate.	Function of Vert. Moment.	Function of Level Moment.		Function of Ordinate.	Function of Vert. Moment.	Function of Level Moment.		Function of Ordinate.	Function of Vert. Moment.	Function of Level Moment.		Function of Ordinate.	Function of Vert. Moment.	Function of Level Moment.
1	0	23·9	142	1	23·9	...	142	...	...	...	...	...	...	...	...	1	23·9	...	142	...	...	...	...
2	1	23·7	158	4	94·8	94·8	632	...	...	...	...	...	...	...	...	3	71·1	71·1	474	1	23·7	3·7	158
3	2	22·7	168	2	45·4	90·8	336	1	22·7	45·4	168	...	...	...	...	3	68·1	136·2	504	4	90·8	181·6	672
4	3	20·6	160	4	82·4	247·2	640	4	82·4	247·2	640	...	...	...	...	1	20·6	61·8	160	1	20·6	61·8	168
5	4	13·76	79	$1\frac{1}{2}$	20·64	82·56	118	$1\frac{1}{2}$	20·64	82·56	118	$\frac{1}{2}$	6·88	27·52	40	...	...	...	...	...	...	...	...
6	$4\frac{1}{2}$	7·42	24	2	14·84	66·78	48	2	14·84	66·78	48	2	14·84	66·78	48	...	...	...	...	...	...	...	...
7	5	0	...	$\frac{1}{2}$	...	...	...	$\frac{1}{3}$	...	...	...	$\frac{1}{2}$	...	...	...	...	...	...	...	...	...	...	...
				15	281·98	582·14	1916	9	140·58	441·94	974	3	21·72	94·30	88	8	183·7	269·1	1280	6	135·1	267·1	998
					...	2·06	...	...	...	3·13	...	...	...	4·34	...	...	...	1·46	...	...	...	1·98	...
Distance of C. B. . .					...	7·21	6·79	...	...	10·96	6·90	...	...	15·19	4·06	...	...	5·11	6·95	...	...	6·93	7·33
Mean Ordinate . .					18·80	...	...	...	15·62	...	...	...	7·24	...	...	...	22·96	...	...	...	22·5	...	...
Volume in Cubic Feet					64,155	...	...	...	31,982	...	...	...	4,941	...	...	4	47,011	...	...	...	32,906	...	...

Results.	1st W. L.	2nd W. L.	3rd W. L.	4th W. L.	5th W. L.
Displacement . .	1,833 tons.	1,430 tons.	914 tons.	490 tons.	141 tons.
Vertical Dist. of C.B.	7·21 feet.	8·98 feet.	10·96 feet.	12·97 feet.	15·19 feet.
Level Distance of C.B.	6·79 ,,	7·04 ,,	6·90 ,,	6·49 ,,	4·06 ,,

Subsequent to the writing of the above account of Mr. Benjamin's method of calculating the statical stability of vessels, that gentleman has communicated to the author a modification of his system, by means of which he dispenses with the calculation of the position of the centre of buoyancy in relation to the water-line, requiring only its position in relation to the vertical axis which he uses, in order to determine the line of action of the resultant pressure of the water upon the immersed hull of the vessel. Mr. Benjamin's own exposition of his modified method is as follows :—

"If a vessel is immersed in water in an inclined position there are two forces acting—viz., the weight of the vessel, and the resultant pressure of the water, which is equal and opposite to the resultant of the weight of that body of water which is displaced by the vessel. Both forces are acting in lines vertical to the surface of the water, and equal to the displacement, D. If the distance between their lines of action is a, the moment of the couple thus formed is

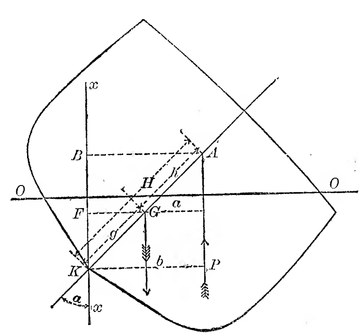
$$D\,a,$$

which is the moment of stability. In Fig. 114 the line of resultant pressure is shown by P. Its position is known, if the point, A, is given, in which it intersects the middle line of the section representing the vessel. I shall call this the point of action. Let its height above the top of keel be = H, and that of the centre of gravity = g, then

$$H - g = h = \frac{a}{\sin. \alpha}$$

when α is the angle of inclination; and thus, for a given angle of heel, the distance between G and A bears a certain proportion to the lever of stability. When G lies below A, the stability is positive; when G lies above A, it is negative; while, when G and A fall together, there is equilibrium. The value of h depends on two items—viz., the height of the centre of gravity, G, and the height of the point of action, A. The first is due to the distribution of weights, the latter to the shape of the vessel. Only H, therefore, needs to be calculated. But

Fig.114.



$$H = \frac{b}{\sin. a}.$$

Where b is the distance of the line of the pressure, P, from an axis, $x\,x$, through the top of the keel, and vertical to the surface of the water; and we need, therefore, only to calculate the different values of b.

"Now, if the inclination is constant and the displacement variable, as indicated in Fig. 115 from 0_1 to 0_5, the line of pressure passes through the different positions, P_1 to P_5, and the point of action from A_1 to A_5. The height of the latter above the top of keel varies from H_1 to H_5, while the distances of the line of pressure from the axis, $x\,x$, vary from b_1 to b_5.

"Suppose the values of b for different displacements to be calculated, and divided by sin. a; then, as

$$H = \frac{b}{\sin. a}$$

we have as the result of the calculations the displacements, D_1, D_2, D_3, &c., and the heights, H_1, H_2, H_3, &c. We can construct a curve (Fig. 116) using the displacements as abscissæ and the heights as

Fig. 115.

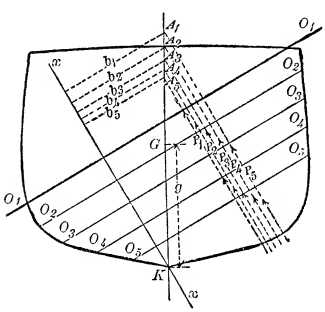
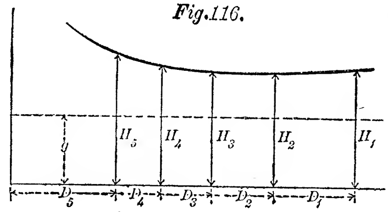


Fig. 116.



ordinates. If now any position of the centre of gravity is given, the height, h, which is wanted, is

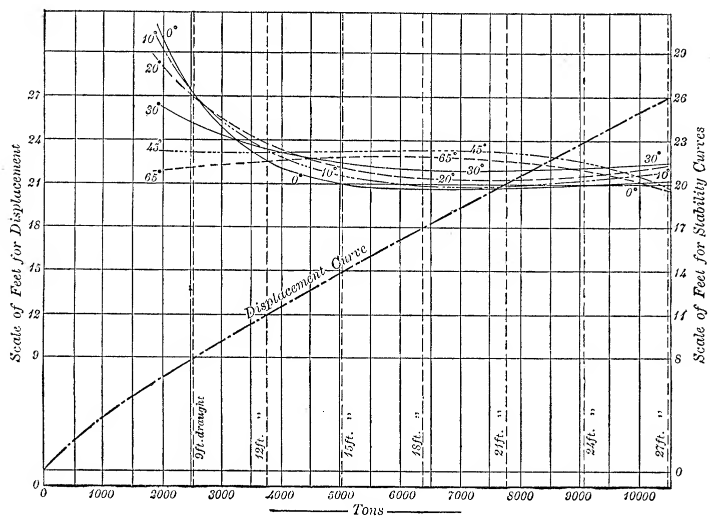
$$H - g;$$

and if we draw a straight line parallel to the axis of Fig. 116 at the distance, g, the distances of the points of the curve from this line represent the heights, h. If this is carried through for different angles of keel, the diagram (Fig. 117) is produced.

"By means of a displacement curve, as shown in Fig. 117, we find those abscissæ which correspond to given upright draughts, and the corresponding ordinates give the heights, H and h, respectively.

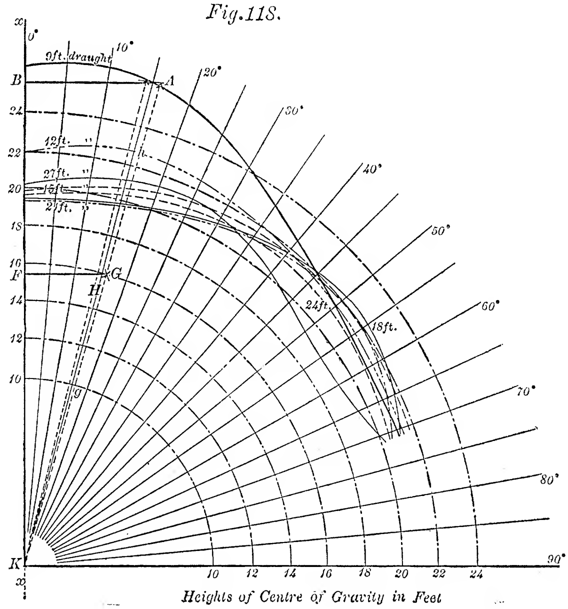
"As this diagram is not very handy for use, and only gives the heights of the points of action above the centre of gravity, and not

Fig. 117.



the levers of stability themselves, and as the lines are generally confusing, I transform it into another diagram (Fig. 118). In this diagram the heights, H, are used as ordinates, and the angles of

Fig. 118.



heel are used as abscissæ, which form, according to their nature, a system of polar co-ordinates starting from the point, K, on the axis, $x\,x$.

"The ordinates for that abscissa in Fig. 117, which corresponds to a given displacement, are placed in Fig. 118, on the different lines for angles of heel, measuring from K, and form thus a curve for that displacement, and any number of such curves can be produced without further calculation.

"Any given height of the centre of gravity evidently produces a circle, as it remains constant for all angles of keel.

"Thus, for a given condition of the vessel, the heights, h, are found as the distances between one of the circles and one of the curves of the diagram, while the righting lever

$$A = h \sin. \alpha,$$

is found as the difference of the distances of the points in which the line for the angle intersects the curve and the circle.

"A model is formed by placing each of the curves of Fig. 118 in parallel planes; this model is intersected by a system of *polar* planes, which pass through the angled lines, and are vertical to the system of parallel planes, and the intersections of these polar planes with the surface of the model evidently are the curves of Fig. 117. In the model the circles for the centres of gravity produce cylindrical surfaces, and using these the model can be constructed of annular cylindrical pieces, which can be readily separated.

"On each of these, and on the curved surface, the lines are to be shown in which they are intersected by the two sets of planes.

"For a given centre of gravity, only that part of the model which lies between the corresponding cylindrical and the curved surface has to be considered; it gives by the distances between the sets of lines the heights, h, for any displacement and any angle of keel.

"The only calculations necessary for this method are to find the positions of the lines of action for different draughts at given angles of heel. Taking the moments round the axis, $x\,x$, Fig. 114, we have

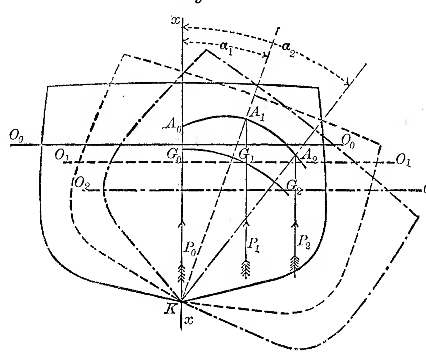
$$b\,D = \int y\,d\,D,$$

where y is the distance from $x\,x$ of a particle, $d\,D$, of the displaced volume of water; therefore, only a common moment calculation is needed to find b. This can be done for all angles of heel by means of the tables already given, leaving out those columns used for the calculation of the vertical position of the centre of buoyancy. For an indefinitely small angle of heel, *i.e.*, for the upright position, it is necessary to consider that the point of action, A, falls into the metacentre, and that the curve in Fig. 117, for the heights, H, in the upright position is, therefore, a curve of metacentric heights, which can be calculated by *Atwood's* formula.

Direct Explanation of the Diagram.

"In Fig. 119 the vessel is shown under different angles of heel, in such a manner that the top of keel remains a fixed point, and that the angles are measured from an axis, $x x$, which coincides with the middle line in the upright position of the vessel; thus, this axis always remains vertical to the surface of the water, and the latter takes the positions O_0, O_1, O_2, O_3, successively. The line of pressure passes from P_0 to P_1, P_2, &c., and the centre of gravity from G_0 to G_1, G_2, &c. The point of action passes on a curve A_0, A_1, A_2, the distances of which are measured on the angled lines, given by the successive middle lines of the vessel, which all pass through K, forming with $x x$ the angles of heel, a_1, a_2, &c., and A moves therefore on one of the curves, as shown by the diagram, Fig. 118, while the centre of gravity moves evidently on a circle, so that, in heeling the vessel in this manner, the diagram is produced.

Fig. 119.



"It will be distinctly understood that the centre of buoyancy has no bearing on the question of stability. However, if for comparison with other methods, its position is wanted, it must be remembered that it always lies on the line of action, and that while for a constant angle the latter passes through P_1, P_2, &c., Fig. 115, it moves on the locus of centre of buoyancy, as shown in Fig. 112. Here a double moment calculation would be necessary, and this can be made, if Simpson's Rules are adhered to, by the tables already given. To get on these curves the points which correspond to given draughts, the diagram Fig. 118 needs to be constructed, and the points on the different loci of centre of buoyancy corresponding to the same displacement give the usual curves of centres of buoyancy."

CHAPTER IX.

Mr. Macfarlane Gray's System of Diagrams and Calculations—Stability Curve—Alteration of Position of Centre of Gravity—Stability Diagrams—Polar Diagram—Construction of Same—Template Arc—Example of Practical Application of Mr. Gray's System—Importance of Position of Centre of Gravity—Stability of Example compared with that of the *Captain*—Stability Diagrams for the *Athulchni*—Forms for Amsler's Integrator—Co-ordinates of Centres of Buoyancy—Explanation of Use of Forms—Determination of Righting Levers from Functions of Displacements—Other Forms—Explanation of Use of Same.

A SPECIAL system of diagrams and calculations now used in the Board of Trade for stability (and longitudinal strength) has been contrived by Mr. J. Macfarlane Gray, an able officer of that Department. A paper on "Polar Diagrams of Stability" by him was published in the *Transactions of the Institution of Naval Architects for* 1875, giving "practicable methods of constructing stability diagrams that might be adopted by naval architects, and drawn upon the midship-section in the plans furnished to the owner, to be a guide in stowage, and a help in settling freeboard." Only the last of the methods there given has been used practically; the others are now regarded by Mr. Gray as merely interesting and suggestive features in the geometry of stability,* by which he was led to the simple practical diagram now used. Since the date of that paper Mr. Macfarlane Gray's system has been improved and extended, and the following account of the methods pursued is now prepared from notes kindly supplied by him, with the permission of the authorities of the Board of Trade. We give also the preliminary features, although these are not discernible in the practical diagram.

Beginning with Fig. 120, the well-known diagram, G is the centre of gravity of the hull; B is the centre of buoyancy; M B is the vertical line through B; and G Z, the perpendicular from G upon M B, is the righting arm. We might have indicated G Z (as in

* Other gentlemen are nevertheless now engaged, as we have just seen, and shall see again hereafter, in carrying out systems of calculation, based upon the same fundamental considerations, and turning them to practical account for determining exhaustively the stability of ships.

Fig. 121) upon the hull upright on the page, but with the water-line inclined. If the inclination of the water-line alone is to be noted, the water-line itself may be erased, and G Z will sufficiently denote the inclination.

Fig.120.

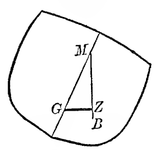
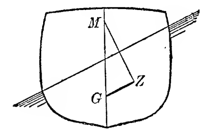
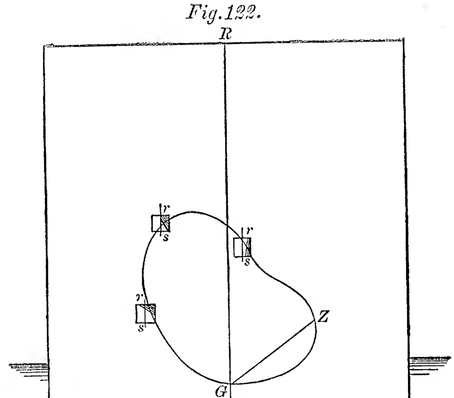


Fig.121.



Let these G Z arms be drawn for a number of inclinations, with the centre of gravity in assumed coincidence with the upright centre of buoyancy for a given displacement, the line drawn through a series of these Z points is the stability curve given in Fig. 122, which represents a prism of square section immersed $\frac{3}{25}$ths of its depth. The G Z lines may now be erased, and for any required

Fig.122.

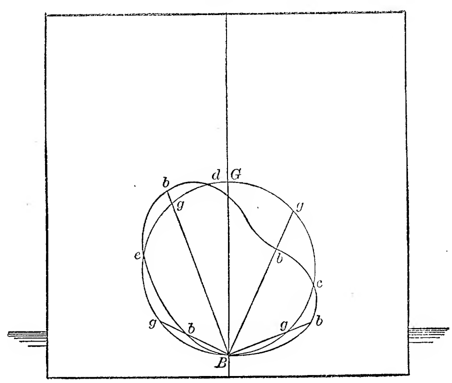


inclination a G Z line can be drawn (limited in length by the stability curve) giving the righting arm for that inclination with the centre of gravity at G, the centre of the upright buoyancy. The curve as drawn is for a capsize through 180°.

The reading of the curve is indicated by the skeleton hulls sketched in; the shading marks the part of the hull that is immersed, and the radial line through G gives the inclination of the water-line. The water is always at the same side of that line as it turns round. The skeleton hulls are all vertical on the page.

A stability diagram for any point, G, as origin is at once converted into a stability diagram for any other position of the centre of gravity by merely describing a circle on the line joining these two points, and taking the G Z, shortened by the chord of the circle at each inclination, as the stability lever. The reason of this will be seen by reverting to the general expression for G Z, which is the difference of two lengths, the one being the length, B R, which is equivalent to G Z, presuming G to coincide with B, and the other is B G sin. θ. This latter length, B G sin. θ, is clearly equal to the chord of a semi-circle described upon the height B G, the said chord being drawn from B at any angle, θ, to the horizontal. As the stability curve is drawn through the centre of buoyancy for the upright position, that point is now, Fig. 123, lettered B, and the

Fig. 123.

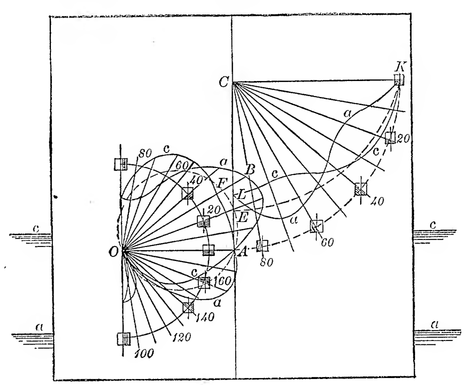


actual position of the centre of gravity is lettered G. For any inclination, at the same displacement, the righting arm is the segment of the inclined line between the circle and the stability curve. If the intersections of the radial line and the circle are lettered g, and those with the curve are lettered b, then, reading from the centre, $g\,b$ is a righting arm, and $b\,g$ is an upsetting arm. Reading round the diagram through 180° from the right, the arms are $g\,b$ at first, or *righting*. At c the arm is nil; this is a position of unstable equilibrium, and the hull will turn over from that to d, impelled by the capsizing force acting on the upsetting arm, $b\,g$. At d the condition is stable equilibrium, that is to say, the vessel will settle with the masts at the angle, d B G, below the water level, but perhaps at the opposite side to that they entered at, according as d is near to B, or distant from B, having regard to the dynamic feature of the diagram. From d to e the arms are righting levers (with reference to the last position of stable equilibrium corresponding to d) as indicated by $g\,b$: at e, another position of unstable equilibrium occurs, and beyond that point the arms are upsetting levers until the position B is reached, when the vessel is bottom up and in stable equilibrium.

The usefulness of this diagram, as was pointed out by Mr. Gray in 1875, is impaired by the acuteness of the angles at the intersections near the upright position. At an inclination of 90°, however, the intersections are nearly at right angles, and if we could transfer this quality to the neighbourhood of the upright position, we should then have the sharp definition where it is most required in a stability diagram.

In Fig. 124, on the left side, stability diagrams are drawn, as described above, from the prism whose transverse section is given.

Fig. 124.



The stability curves, O a A and O c A, are for the centre of gravity at the point O, and for the two displacements indicated by the two water levels shown.

"It may seem a ridiculous position," says Mr. Gray, "for the centre of gravity, so far to one side; but that is only a device to get a curve that will give definite intersections at moderate inclinations. For the centre of gravity at the point, E, describe a circle on O E, as diameter, viz., O F A, that circle and the stability curve for the immersion constitute a stability diagram. At the immersion, a, the righting arm at 30°, inclination is F B. In this figure, the skeleton hulls are shown on an arbitrary circle of degrees."

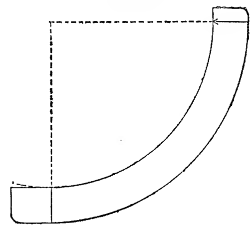
The sense in which the arm acts in the first quadrant is that when the circle intersection is within the curve, it is a righting arm, when beyond the curve, it is a capsizing arm. At 160°, G Z is a capsizing arm, because the action in the second quadrant, being represented at the opposite end of the radius vector, has the opposite sense.

The outcome of all these methods of representing stability for different positions of the centre of gravity is the polar diagram, given on the right-hand side of Fig. 124, which Mr. Macfarlane Gray regards as practically accurate, but justly describes as "not mathematically correct." About the centre, C, a point in the vertical centre line, an arc of 90° is described through the point A, the mean position of the centre of gravity. (The radius may be conveniently 8 inches, if the beam is more than 32 feet, and the scale half-inch to a foot.) The righting arms are set off on the radial lines from the circular arc towards the centre, C, on their respective lines of inclination, and a curve drawn through the points, thus obtained, is the stability curve for the corresponding immersion, with the centre of gravity at A. The stability diagram for any position of the centre of gravity, E, and the same immersion, is completed by describing an arc *with the same radius* through K and E. Practically the original quadrant arc is erased, and a card-board template applied to the fixed point, K, and to the position of the centre of gravity, giving at once, without drawing, the complete diagram of stability. The assumption which underlies this device obviously is, that the originally assumed centre of gravity and the new one, and consequently the centres of the circular arc in the two cases, will in practice be sufficiently near to each other to justify the assumption that the radial distance between the two arcs may be taken as always equal to the distance apart of the two centres of gravity multiplied by the sine of the angle of inclination — an assumption which may fairly enough be made for the average run of office-work, but which it would be unsafe to make in many supposable cases of ship stability.

The two curves shown, Fig. 124, are for water levels, *a* and *c*. It is obvious that the first part of the curve from K, is a portion of a circular arc that would pass through the upright metacentre for that immersion, and that its radius is C K. It is also obvious that the height of the point, L, is determined by the intersection of the horizontal normal of the curve of buoyancy with the vertical centre line.

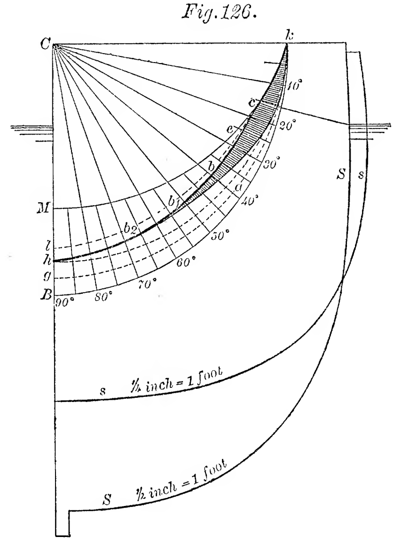
Since the date of the paper referred to, the template arc has been modified, as shown in Fig. 125. The quadrant is completed, but beyond the quadrant the template line falls outside the circular arc. This makes the template applicable for positions of the centre of gravity below as well as above the original point, A. For use in the Board of Trade, the range of stability does not require to be considered through even 90°, and a further refinement is then available. The template is made to lie slightly outside the circular arc from 78°, touching the arc again at 90°. This is so very little that its effect is generally not appreciable, and for any practical use this may be neglected. The set off beyond 90° is of greater amount, and it does not in any other way impair the usefulness of the template.

Fig. 125.



The following is an example of the practical application of Mr. Macfarlane Gray's system to a case which came under the consideration of the Board of Trade, taken in substance from a Parliamentary Paper.* S S, Fig. 126, represents a midship section of the vessel. From any point, C, in the vertical axis, with any radius, C k, an arc of a circle was described cutting the axis in g. (The point, g, should be near to the lowest position of the centre of gravity.) Radial lines were drawn at equal angles, and upon these were set off from the arc, kg, the righting arms for the centre of gravity at the point, g; that is, ab was made equal to the length of the righting arm for 40° inclination, &c., the points, h, b_2, b_1, b, k, were joined by a fair curve, which was the curve of stability for radius, C k. As the point, g, was taken arbitrarily, and in ignorance of the actual position of the centre of gravity, the arc, $k a g$, was then erased.

Fig. 126.



* We do not reproduce the full particulars of the case, or the name of the vessel, as there was a misunderstanding involved as to the draught and freeboard contemplated; but the example is none the less instructive and interesting as an illustration of the system.

The peculiar property of this stability diagram is that, if a hollow curve (circular arc) be cut out of paper to radius, C *k*, and applied to the point, *k*, and also to any point on the vertical centre line, as B for example, it will cut the radial lines at a distance from the stability curve equal to the length of the righting arms at the respective angles, if the centre of gravity be at the point, B. When the radius, C *k*, is great in proportion to the length of righting arm, as in this diagram, the departure from mathematical exactness is very small. In Fig. 126 the points, M and B, are respectively the metacentre and the centre of buoyancy of the vessel, and the distance, M B, is 3 feet 8 inches. The usual position of the centre of gravity was assumed for this case about 4 feet below the metacentre, and the circular arc, *k* B, was therefore drawn with radius = C *k*; *k* B *h b k* was then the complete diagram of stability for centre of gravity at B, with centre of gravity 3 feet 8 inches below the metacentre. The stability curve begins at *k*, as a portion of the diagram arc through M, "diagram arc" signifying an arc of a circle with C *k* radius; the same arc is used for all positions of the centre of gravity.

In his description of this case, Mr. M'Farlane Gray said:—"The stability curve at first inclines to the inside of the arc, that is, to give greater stability than that due to a constant metacentre. At the point, *c*, however, the gunwale being just immersed, the stability curve begins to turn outward, giving less stability. With one foot more freeboard the stability curve would have kept within the arc for another 5°, and it would have terminated at the point *l*, and the stability curve* would have been *k e l*. The radial lines between the curve *k e l* and the arc *k* B, increasing up to 60° of heeling, represent the righting arms at the respective angles in an ordinary sea-going vessel of this size, according to the authorities cited above.† By reason of the unusually low freeboard, the stability curve is moved from *k e l* to *k b h*; that side of the diagram is the result of form of hull, the arc side, as *k* B, is the result of stowage. . . . The importance of the position of the centre of gravity is seen by considering the condition of stability that would obtain with the centre of gravity placed at *h*, that is, 6 feet below the load waterline. The maximum righting arm would be at 34°—it would be only one foot, or about the length of righting arm usual for 15° of

* This curve has not been constructed except for points *k*, *e*, *l*.

† Rankine, Barnes, Napier, and Watts, in *Shipbuilding: Theoretical and Practical.*

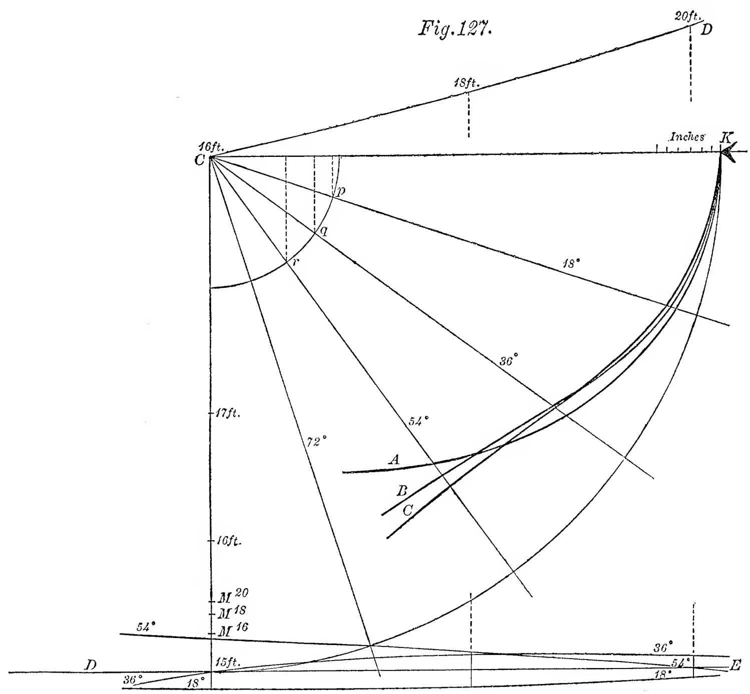
heel in ordinary vessels; and another way of stating this is: the heeling force which would incline an ordinary vessel of this size 15°, would capsize the present vessel, even if the centre of gravity were as much as 6 feet below the load water-line."

Mr. Gray further says:—"In comparing the stability of a special vessel with that of other vessels of known character, it is advisable to draw all the diagrams to one radius; say, as in this example, to 10 feet radius, or 5 inches, using a scale $\frac{1}{2}$-inch = 1 foot. An arc of horn, 5 inches radius convex, rather more than a quadrant, is the standard arc, and it can have engraved upon it the curves for some reference vessels, as that of the *Captain*. The point C is not necessarily always on the deck-line. To make the annexed diagram complete in itself, I have applied," he adds, "the diagram for the *Captain* to the curve for the other vessel, viz., the portion of the diagram that is shaded. It is shown by the arc, $k\,h$, falling within the field of the diagram of the *Captain's* stability, that the righting arm in the other vessel, with centre of gravity at h, would be for these angles less than that of the *Captain*. The metacentric depth of the centre of gravity was, in the *Captain*, 2 feet 8 inches; with the centre of gravity at h in this case the metacentric depth of the centre of gravity is $\mathrm{M}\,h = 2$ feet 4 inches. It may be thought that that would be quite right, this being a smaller vessel. The reverse of that is the proper relation; the smaller the vessel the longer should be the righting arm." $s\,s$ in the figure represents the midship section of the *Captain*.

These Board of Trade methods of calculating stability are further illustrated by the polar stability diagrams, Fig. 127 (next page), for the s.s. *Athulchni*, a vessel (which disappeared at sea when grain laden) having an equivalent of about 200 tons of empty cellular bottom space. A, B, and C are the curves of stability calculated for draughts of water of 16, 18, and 20 feet respectively from top of keel, M^{16}, M^{18}, M^{20} being the corresponding metacentres; the curves marked 18°, 36°, and 54° being the curves of righting levers at those inclinations. D E is the curve of displacement.

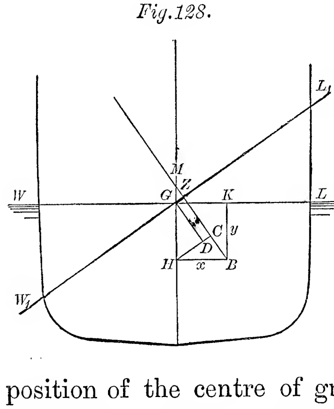
The righting arms are calculated by means of Amsler's Integrator, the readings of this instrument being entered on Forms similar to that which follows this description (page 187). It will be observed that the labour of calculation is greatly reduced by using logarithms, and the arrangement of the tables in these Forms is such that they can be worked out by practically unskilled persons. It is immaterial that the inclined water-lines by this method should cut off any particular displacement, as we obtain for each inclination a function of the displacement cut off, and also the length of the righting arm

Fig. 127.



corresponding to that inclination and displacement, and this is adapted to intermediate displacements or draughts of water by a method which will be described hereafter. Thus we do away with the necessity of calculating moments of correcting layers, and the wedges of immersion and emersion are ignored, as we deal only with the total displacements up to the several inclined water-lines, and with the centres of buoyancy corresponding thereto.

Fig. 128.



It is very clear that by fully determining the position of the centre of buoyancy, and having an assumed position of the centre of gravity at the intersection of the upright water-line and the middle line of the body-plan, we can readily determine the length of the righting lever in terms of the co-ordinates of the centre of buoyancy. In Fig. 128 let G be the position of the centre of gravity, and B the position of the centre of buoyancy corresponding to the inclined water-line, $W_1 L_1$, with reference to which it is desired to obtain the length of the righting arm for this position of G. Through G draw G Z perpendicular to the upward line of action of buoyancy through B, and draw B H, B K, parallel to W L and H G respectively, and through H draw H C parallel to G Z. Then, if θ=inclination, and x and y are the co-ordinates of B referred to the middle-line of the body, and to the upright water-line as axes, we have $GZ = x \cos. \theta - y \sin. \theta$. From this value are obtained the lengths of the righting levers for any possible position of the centre of gravity by means of the template mould previously explained.

In order to obtain the values of the co-ordinates, x, y, the body-plan is first prepared by taking transverse sections at equal distances apart, and so that the number of ordinates shall always be odd; they are numbered 0, 1, 2, 3, 4, 5, 6, 7, 8. . . . . 12, commencing from the foremost ordinate, and arranged in pairs; thus, if there are 13 ordinates, they are grouped 1 and 11, 3 and 9, 5 and 7, 7 and 5; and 0 and 12, 2 and 10, 4 and 8, 6; and each group is styled *odds* or *evens*, according as the numbers representing the ordinates are odd or even. It will be observed that the midship ordinate, or number 6, is not combined with any other ordinate, there being none left for it to be paired with; this will cause a slight modification of the system of traversing the sections, which we shall refer to hereafter. This simple form of numbering the ordinates has been found to be very convenient in practice, as the sum of the numbers of any pair of ordinates is equal to the number of the last ordinate, and thus affords a very simple aid to memory when traversing the sections with the pointer of the Integrator.

A circle is now drawn sufficiently large to enclose the whole of the body-plan,* and diameters of this circle are drawn, having the same inclination to one another as the angles for which it is desired to calculate the righting arms. In our example (given in the following Table) these angles are 18 degrees; but, of course, if greater accuracy is desired, the respective angles of inclination must be smaller. The tracing of the body-plan is now fixed over the

* This is usually done on a separate sheet of paper, and a tracing of the body-plan is placed over it.

circle, with the intersection of the upright water-line and the middle line of the body coincident with its centre, and with the upright water-line in the body-plan corresponding with the horizontal or zero diameter, or the diameter from which the inclinations are set off, observing that this water-line is taken at about the mean position of the centre of gravity.

We will now explain how the Form (shown on page 187) is practically employed. The guide-batten of the Integrator* is set parallel to the middle-line of the body-plan by means of the gauge which accompanies the instrument, and fixed in that position; the Integrator is then placed on the guide-batten, and its pointer brought to the centre of the circle. Just previous to starting, the readings of the instrument for area and moments are taken and recorded in the 1st and 3rd columns respectively at the top of the table headed "axis vertical." The letters O and E on the left-hand side of the table indicate *odds* and *evens* respectively, the inclined line against them thus $\overset{O}{\underset{E}{\backslash}}$ $\overset{O}{\underset{E}{/}}$ indicating the direction of the radial line under consideration, with reference to the upright water-line. The signs $\urcorner$ $\ulcorner$ are guides to indicate to the operator that the upper column has to be subtracted from the lower, and the lower subtracted from the upper respectively, the horizontal arm of these signs being in a line with the column on the right-hand side, where the results of the subtraction are to be placed. The sign $\rangle$ signifies "repeat in next column below," and the sign + on the right-hand side of the column means that the figures in the column are to be added together, and the result tabulated below. The thick line on each of these sets of columns denotes the position of the decimal point of the figures tabulated.

The Forms arranged by Mr. Macfarlane Gray, as described, are convenient for applying a slightly modified form of Simpson's Rule to the finding of the volume and centre of gravity of displacement. The first and last ordinates being usually very small, and their effect upon the final results being insignificant, they are ignored by Mr. Gray when preparing the columns, so that the only multipliers necessary are 2 and 1, and their application is easily recognised in the Form itself.

* The reader who is unacquainted with the Integrator will find its construction and use fully described later on in this work.

As the tables are exactly the same for all inclined water-lines at any draught of water, and also applicable for determining the displacement and the centre of buoyancy at any draught of water, when the vessel is upright, it will be quite sufficient to describe the manner of dealing with one of them; observing that when the ship is upright, the value of sin. θ is struck out.

Proceeding, we traverse the pointer of the integrator along the radial line corresponding to the inclination under consideration, until we reach the 1st odd numbered section, and then continue round this section to the middle line of the body-plan; thence along the section paired with the one already traced, to the previous radial line produced through the centre of the circle, and along this radial line to the starting point. We now traverse the same radial line as at first, until we come to the second odd numbered section, and proceed similarly until we have traversed the whole of the *odds*, and the pointer reaches the starting point. The readings are now taken for areas and moments, and placed directly under the corresponding readings taken previously. We traverse the *evens* in exactly the same manner as for the *odds*, but previous to starting we take the readings of the instrument and record them in their proper columns, as in the event of the operator being interrupted the instrument may possibly be shifted. We include in the *evens* the midship ordinate previously adverted to; in this case, however, we traverse round the section to the middle line of its body-plan, and then up this to the starting point, when the readings are taken, and entered in proper columns directly under the previous corresponding readings. It should be stated that the pointer is always traced round the sections in the direction of the movement of the hands of a clock. Having traced the whole of the *odds* and *evens* for this inclination of the radial line, we proceed to perform the same tracing of the sections for the opposite inclination of the same radial line, and record the readings as before.

It is evident that by subtracting the readings for area and moment obtained at starting, from those readings when the complete traverse has been made, we determine the exact readings due to the traverse, which would have been the readings had the indices been set at zero before commencing to traverse in each case; we thus avoid the necessity of setting each index at zero before starting. It must be observed that the change from subtracting the upper reading for moments from the lower, to subtracting the lower reading from the upper in the columns for axis vertical is due to the fact that, although the instrument indicates statical moments, it gives no indication of the position of the centre of gravity relatively to the axis, and on reference to a body-plan with inclined water-line in the position indicated by the tables at this place, it will be seen that the centre of gravity of the volume will evidently lie on the off side of the axis, and therefore should have a negative sign, and to make the columns for moments additive, recourse is had to the system adopted in the tables. We repeat the operation of tracing, &c., for all the inclined water-lines for this depth of immersion, and also for any other draught of water, both when the vessel is upright and when she is inclined, for which we should wish to calculate the curve of stability, before the guide-batten is removed from its position parallel to the middle line of the body-plan. It should, however, be observed that since the axes remain the same for all draughts of water, and the position of the centre of buoyancy obtained with reference to these axes in each case, the lengths of the righting arms obtained from the tables are all for the same assumed position of the centre of gravity.

The guide-batten is now placed in a position parallel to the upright water-line, using the same means as before, and the operation of reading and recording is again proceeded with, the readings being put in the columns headed "axis horizontal" in this case. We thus have determined submultiples of areas, and statical moments of the immersed sections up to the radial line under consideration, and when these submultiples are affected as described and indicated by the tables, the results marked A, in columns headed "axis vertical" and "axis horizontal" should practically agree, because each is the same function of the volume of displacement; of course the functions of moments in each set of tables, marked B, will not agree, because they are taken about different axes—viz., the upright water-line and the middle line of body. It is evident that if B be divided by A, in each case, and the result multiplied by a number which depends on the integrator and the scale of the drawing,* we thus find the distance of the centre of buoyancy from the two axes, or, in other words, the co-ordinates of the centre of buoyancy. Its position for a displacement, of which A is

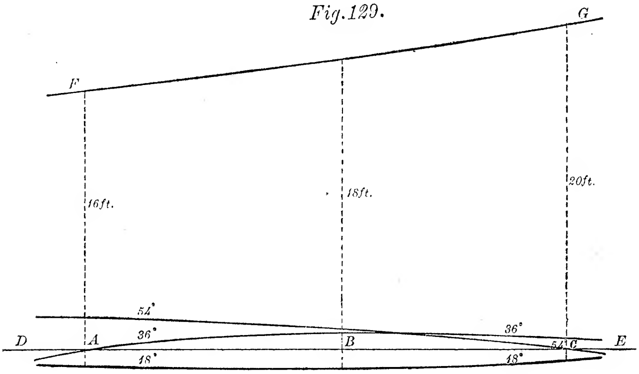
* In this case the integrator readings for areas should be multiplied by 15 to give actual results, and the readings for moments by 40, if the scale of the drawing is 1 inch = 1 foot. The scale however being $\frac{1}{4}$ inch = 1 foot, we have $\frac{40\,B \times 4}{15\,A} = \frac{32\,B}{3\,A}$; therefore, $10\frac{2}{3}$ is the multiplier in the case before us.

the function, the vessel being inclined to the angle under consideration, is therefore fully determined.

We can now determine in the manner described previously, the righting arm corresponding to the assumed position of the centre of gravity from the data before us. In the table herewith it will be seen that logarithms are used to determine the lengths of the righting levers and the amount of the displacement; the several steps of the operation being of the usual character, and so clearly shown, need not be detailed—they are self-evident.

Although the displacement may be readily found from the tables, it is not necessary for determining the curve of stability, the function of the displacement being quite sufficient. It must be clearly borne in mind that the length of the righting lever obtained as described is the righting lever for a displacement of which A is the function, and for the inclination we are considering, with the centre of gravity in the assumed position, and *not* corresponding to the displacement cut off by the upright water-plane, which, however, as we have indicated, must be calculated by similar tables. This point should not be overlooked, as the whole merit of this method of calculating stability is intimately connected with it. The expressions marked A, in the tables calculated for each angle of inclination, and each draught of water, are evidently proportional to the displacements; if therefore the numbers representing them are set off from any point on a base-line to a certain scale, they will have the same relative position one to the other as the corresponding displacements would. In Fig. 129, let A B and C be the positions

Fig. 129.



of numbers representing functions of displacements marked A in the tables, corresponding to draughts of water of 16, 18, and 20 feet respectively from top of keel, and set off on the base-line, D E, to a certain scale from a fixed point, the vessel being upright in each case. If through these points we draw ordinates representing in length the corresponding draughts of water, and pass a curve through their extremities, we shall thus determine the curve of displacements, F G, for the vessel upright, such that any ordinate represents the draught of water, and the corresponding abscissa the the function of displacement. On the base-line we now set off, from the fixed point, the functions of displacements taken from the tables for the inclined water-lines at each inclination, and through these points so obtained draw ordinates equal in length to the corresponding righting levers obtained from the tables, and pass curves through the extremities of the ordinates for the same angle of inclination. We thus obtain curves of righting levers for each angle of inclination as indicated in the figure, and for any draught of water comprised between the limits of the curve of displacement. To determine the length of the righting levers for any draught of water comprised between these limits, we have to determine the point on the base-line, D E, whose ordinate to the curve of displacement represents the draught of water under consideration, the length of this ordinate between the base-line and any curve of righting levers is evidently the length of the righting lever for that inclination and immersion, and for the assumed position of the centre of gravity.

The mode of setting off the lengths thus obtained to form the polar curve of stability, and the use of the template mould for applying this curve for any position of the centre of gravity have been previously described.

The polar stability diagrams for the steamship *Athulchni*, previously adverted to in Fig. 127, are slightly modified from the previous description, the mode of calculating the lengths of the righting levers, however, being exactly the same. It was found after calculating the lengths of the righting levers with the centre of gravity at a height of 16 feet, that it would make the diagram more serviceable to make the centre of gravity normally at a height of 15 feet. To attain this end a quadrant was drawn from C, Fig. 127, with a radius equal to the difference in height of the centre of gravity, viz., 1 foot, and the 8-inch radius quadrant was described for the datum-line. The length of the righting arms were set off from the arc or datum-line on the corresponding radii, with the additions due to the 1 foot difference obtained by measuring from the points, p, q, r, respectively, normally to the line C K.

STABILITY BY INTEGRATOR FOR S.S. *ATHULCHNI.*

INCLINATION 18° = θ. AXIS VERTICAL. *Scale $\frac{1}{4}$ inch = 1 foot.*

O	5	4	9	0
\	8	1	0	3
E	7	4	1	9
\	9	7	4	0
O	4	9	9	7
/	7	4	1	9
E	9	7	4	0
/	1	9	5	6

1	2	6	1	3	
1	2	6	1	3	
1	2	3	2	1	
					+
1	2	4	2	2	
1	2	4	2	2	
1	2	4	2	2	
7	4	6	0	7	= A

8	8	7	8
0	9	0	0
9	1	7	0
0	1	4	9
0	8	6	1
9	1	7	0
0	1	4	9
7	5	5	2

	2	0	2	2	
	2	0	2	2	
	0	9	7	9	
					+
	1	6	9	1	
	1	6	9	1	
	2	5	9	7	
1	1	0	0	2	= B

Log. A =	1	8	7	2	7	7	9	6
Log. d =								
Log. $\frac{32}{7}$ =		6	6	0	0	5	2	0
Dispt. in Tons.								

x cos. θ = | 1 | 4 | 9 | 5 |

Distance between Sections d = 12 feet.

Odds.	Evens.
1·23	2·22
3·21	4·20
5·19	6·18
7·17	8·16
9·15	10·14
11·13	12

Logs. $10\frac{2}{3}$ cos θ.
18° = 1·0062350
36° = ·9359863
54° = ·7972474
72° = ·5180111

1	0	0	6	2	3	5	0	= Log. $10\frac{2}{3}$ cos θ.
1	0	4	1	4	7	1	6	+ = Log. B.
$\bar{2}$	1	2	7	2	2	0	4	= Compt. Log. A.
	1	7	4	9	2	7	0	= Log. x cos θ.

W. L. INTERSECTIONS AT HEIGHT 20 FEET. AXIS HORIZONTAL, AT HEIGHT 16 FEET.

O	5	8	6	0
\	8	3	7	1
E	8	3	7	1
\	0	6	7	4
O	1	0	6	0
/	3	3	5	2
E	3	3	5	2
/	5	8	3	2

1	2	5	1	1	
1	2	5	1	1	
1	2	3	0	3	
					+
1	2	2	9	2	
1	2	2	9	2	
1	2	4	8	0	
7	4	3	8	9	= A

0	3	3	7
6	0	3	9
6	0	3	9
1	8	5	7
1	9	5	5
7	7	4	1
7	7	4	1
3	4	7	2

	5	7	0	2	
	5	7	0	2	
	5	8	1	8	
					+
	5	7	8	6	
	5	7	8	6	
	5	7	3	1	
3	4	5	2	5	= B

Log. A =	1	8	7	1	5	0	8	7
Log. d =								
Log. $\frac{32}{7}$ =		6	6	0	0	5	2	0
Dispt. in Tons.								

y sin. θ =		1	5	2	9
x cos. θ =		1	4	9	5
Arm. =	–		0	3	4

Logs. $10\frac{2}{3}$ sin. θ.
18° = ·5180111
36° = ·7972474
54° = ·9359863
72° = 1·0062350
90° = 1·0280287 = Log. $10\frac{2}{3}$

	5	1	8	0	1	1	1	= Log. $10\frac{2}{3}$ sin. θ.
1	5	3	8	1	3	3	7	+ = Log. B.
$\bar{2}$	1	2	8	4	9	1	3	= Compt. Log. A.
	1	8	4	6	3	6	1	= Log. y sin. θ.

When θ=0° strike out sin. θ throughout for G at intersection of axes.

The method described in the preceding pages may be applied to ascertain the lengths of the righting levers when the vessel has a list to one side, by applying the marked point on the template to the line, C K, as far from the point K, Fig. 127, towards the centre C, as the centre of gravity of the vessel has shifted, keeping the other end of the template to the original position of the centre of gravity; the template then cuts the stability curve at the angle of list, and the lengths of the righting levers may be taken as previously described.

The lengths of the arms of the righting levers for the several angles of inclination may also be obtained by means of a set of tables similar to those previously described, by dealing with a zone or belt of displacement, in order to determine the position of its centre of buoyancy. The first set of tables are much preferable when we are dealing with several draughts of water; but when only one draught of water is necessary, the method we are about to describe is employed.

By reference to the set of tables (shown on page 191) it will be observed that they are arranged specially for inclined water-sections, but the same form is adapted for determining the displacement and the centre of buoyancy when the vessel is upright. Although for our purpose we do not require the actual displacement in cubic feet or tons, still it can be determined easily, and the several steps are indicated very clearly in the tables. The sheet contains two forms; the upper one is for the sections, and the lower for the zone or belt. When the vessel is upright the upper form only is used, and the righting arm is the distance between the centre of gravity and centre of buoyancy. In order to fix the guide-batten in position, throughout the work, the body-plan is arranged in a manner which we will proceed to describe.

The body-plan being prepared, and the sections numbered and paired, as previously described, a position of the centre of gravity is assumed such that if possible the lengths of the righting levers may all be positive. A circle, with diameters inclined at the angles of inclination under consideration, is fixed on the drawing board, and a tracing of the body-plan placed over it, with the assumed position of the centre of gravity coincident with the centre. The tracing is so arranged on a pivot or turn-table at the centre of the circle that it can move about that point. The guide-batten for the integrator is fixed so that the axis of the integrator shall pass through the centre of gravity, and also allow the tracing of the body-plan to move under it with a covering of tracing paper. The covering paper has the axis of the instrument marked on it, and is secured so that it can only move parallel to that axis. It has also two lines square to the axis in a convenient position, these lines in every case representing the inclined water-line and the corresponding water-line of the zone or belt of displacement on moving the lines into position corresponding thereto. By proceeding as described, the guide-batten may be fixed throughout the whole operation of tracing.

We first deal with the upright water-section. The body-plan being placed in position with its water-line parallel to the axis of the instrument, with the bilge towards the guide-batten, we proceed to read the indexes, record the readings, traverse the sections, and so on, as detailed previously. In this case we traverse the sections twice, when the difference of readings in the two cases will be the same, if the readings are taken correctly. Proceeding as indicated in the Form, we determine the function of displacement and the length of the righting lever corresponding to that draught of water.

The inclined water-lines are all taken to pass through the intersection of the upright water-section and the middle line of the body-plan for the draught of water under consideration. We treat each inclined water-section in the manner previously described, and we obtain in the upper part of the Forms the length of the righting lever and the function of displacement. The layers for each set of sections are affected similarly to the sections, and we obtain similar results for them in the lower part of the forms. We now obtain the difference between the functions of displacement when the vessel is upright, and when she is inclined at the angle under consideration. This difference is affected by the submultiple of the distance of the centre of buoyancy of the zone or belt from the axis, in order to obtain the function of moment due to the excess or defect of the displacement up to the inclined water-line, the assumption in this step being that the centre of buoyancy of the zone or belt of displacement agrees with the centre of buoyancy of the layer required to correct the displacement up to the inclined water-line.

The function of moment due to the layer is added to or subtracted from the function of moment for the inclined water-plane, and we thus obtain the function of moment due to the inclined position, corresponding to the displacement when the vessel is upright at the draught of water we are considering. It will always be indicated by the tables if the function of moment for the layer is positive or negative. From this result we can readily deduce the distance of the centre of buoyancy in the inclined position from the axis of the instrument, and this distance is evidently the length of the righting lever for the inclination we are considering, with the centre of gravity in the assumed position. These lengths of the righting levers are adapted for any special position of the centre of gravity by an application of the template mould previously described.

The several steps in the calculation which we have described may be clearly seen by aid of the table herewith, where, as before, Logarithms are used to reduce the labour of calculation.

STABILITY BY INTEGRATOR FOR S.S.

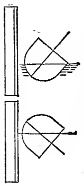
INCLINATION $\theta = 18°$.

SECTIONS APART, $d = 23$ FEET.

HORIZONTAL LINE 21 FEET ABOVE KEEL.

Scale $\frac{1}{4}$-*inch* = 1 *Foot.*

SECTIONS.



2	9	2	1
8	4	1	6
8	4	1	6
3	8	5	4
4	7	0	3
0	1	4	4
0	1	4	4
5	1	7	2

	5	4	9	5
	5	4	9	5
	5	4	3	8
	5	4	4	1
	5	4	4	1
	5	0	2	8
A =	3	2	3	3	8

Log. A =	1	5	0	9	7	1	3	2	+
Log. d =									+
Log. $\frac{128}{21}$ =	0	7	8	4	9	9	0	7	
Tons Dispct.									

7	6	8	4
6	3	1	1
6	3	1	1
3	5	6	9
3	9	9	1
3	1	8	4
3	1	8	4
3	8	9	1

		1	3	7	3
		1	3	7	3
		2	7	4	2
		0	8	0	7
		0	8	0	7
	$\bar{1}$	9	2	9	3
M ±		6	3	9	5

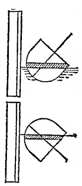
$\mathcal{L}$ =	0	8	0	5	5	3	4	8	+
Compt. Log. A =	$\bar{2}$	4	9	0	2	8	6	8	+
Log. 8 =	0	9	0	3	0	9	0	0	
$\mathcal{L}$		1	9	8	9	1	1	6	

Righting Arm = | 1 | 5 | 8 | 0 | 0 |

For vessel upright write H for A and G B for Righting Arm.

E	
2	12
4	10
6	8
0	
1	13
3	11
5	9
7	

LAYERS.



3	8	5	0
4	1	7	5
4	1	7	5
4	4	8	9
4	4	8	9
5	5	0	8
5	5	0	8
5	8	0	9

	0	3	2	5
	0	3	2	5
	0	3	1	4
	1	0	1	9
	1	0	1	9
	0	3	0	1
a =	3	6	0	3

Log. a = | | 7 | 8 | 0 | 3 | 1 | 7 | 3 |

3	5	7	1
3	4	9	2
3	4	9	2
3	5	1	7
3	5	1	7
3	8	3	1
3	8	3	1
3	7	1	4

		0	0	7	9
		0	0	7	9
	$\bar{1}$	9	9	7	5
	$\bar{1}$	9	6	8	6
	$\bar{1}$	9	6	8	6
		0	1	1	7
m = ±	—	0	3	7	8

H =			3	3	0	9	6		
A =			3	2	3	3	8		
+				0	7	5	8		
$\mathcal{L}$	$\bar{1}$	8	7	9	6	6	9	2	+
$\mathcal{L}og.$	$\bar{1}$	5	7	7	4	9	1	8	+
Compt. Log. a =		2	1	9	6	8	2	7	
	$\bar{1}$	6	7	6	8	4	3	7	
	0	9	0	3	0	9	0	0	= Log. 8
		8	3	6	9	5	6	7	+
	$\bar{2}$	4	8	0	2	2	4	5	= Compt. Log. A.
	0	2	2	0	2	7	1	2	

Keep G_0 low enough to give *all righting* if the size will permit.

M = ±	+		4	7	5	
±	+	6	3	9	5	+
±	+	6	8	7	0	

Righting Arm $\pm$ = | + | 1 | 6 | 6 | 0 |

CHAPTER X.

Professor Elgar's Vertical or Cross-Curves of Stability—For Homogeneous Prismatic Bodies—For an Actual Ship—Variation in Position of Centre of Gravity—Analogy between Light-draught and Deep-draught Stability—Remarks of Mr. Jenkins on these Curves—Cross-Curves of Stability described by Mr. Denny—Mr. Fellows' Method—Mr. Couwenberg's Method.

SINCE the manuscript of this volume drew near to completion, Professor Elgar has contributed to the Royal Society a Paper on "The Variation of Stability with Draught of Water in Ships," from which it appears that it has been his practice for some time past to construct what we may call vertical curves of stability, or curves crossing the ordinary curves of stability, and formed from them. After calculating ordinary curves of stability at load draught, at light draught, and at an intermediate draught or two, he has constructed from them cross-curves exhibiting the righting forces at different draughts, each such curve corresponding, be it observed, to some one given angle of inclination. In accomplishing this object, Professor Elgar has availed himself of the fact that the ordinates of the ordinary curve of stability represent (as we have fully seen in previous chapters) the differences between the B R's, and B G sin. θ's, so to speak, or between the levers of stability of form, and the lever due to the height of the centre of gravity above the centre of buoyancy. It is obvious that when a curve of stability, with B R's as ordinates, has been formed for each of several different draughts of water, it is easy to take a vertical line, to set off horizontal lines intersecting it at corresponding draughts of water, and to set off upon these horizontal lines, from the vertical line, lengths corresponding to the ordinates of the curves of stability (B R's) at a given angle of inclination. Lines drawn through the points so obtained will form a vertical curve of stability at that angle of inclination. The stability, at that angle, and at any draught of water (within the range of the curve) may then be measured by merely taking the length of the horizontal ordinate of this curve at a point corresponding to the given draught. Similar cross-curves, or vertical curves of stability may be constructed for other angles of inclination by a like graphic process. By employing these curves in their turn, as furnishing B R's for different inclinations and different draughts of water, the number of ordinary curves of stability may be increased to any desired extent. In this way a very full record of a ship's stability may be quickly obtained.

As Professor Elgar's "Royal Society" Paper furnishes the first exposition of this essentially English method of investigating the stability of ships at different draughts, and as he appears to have been, and doubtless was, the first person to devise it and put it in practice, we shall now give his own description of his diagrams:—

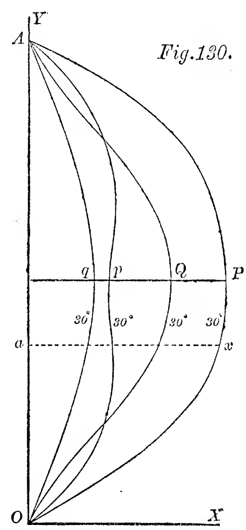
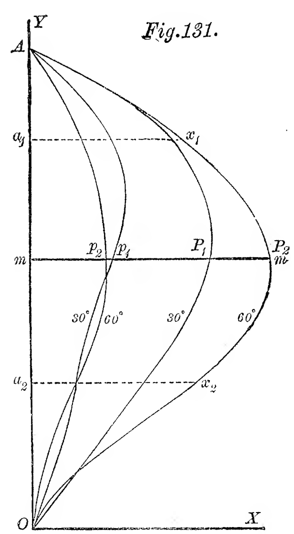


Fig. 130.

"Fig. 130 shows cross-curves of stability* at an angle of inclination of 30° for two prismatic forms of homogeneous floating bodies; one being rectangular and the other elliptical in cross sections. The shorter axis is vertical when the bodies are upright, and is two-thirds of the longer axis, or extreme breadth. The measurements in the direction of O Y give the depths of immersion, and those in the direction of O X represent moments. The curve A P O gives the values of the horizontal shift of the centre of buoyancy multiplied by the immersed volume or B R × V. Thus if O a be any draught of water, the ordinate $a\ x$ gives the value of B R × V at that draught. The curve A Q O is the corresponding curve of

* "It should be remarked here that in dealing with cross-curves of stability, and thus considering the variation of stability with draught of water, the curves of righting moments require to be constructed, and not merely curves of lengths of righting arm, as G Z. The ordinary curve of stability usually has for its ordinates the lengths of G Z at the various angles of inclination. This is right enough for the conditions under which such curves are constructed, because the displacement is then constant, and the curves represent either lengths of righting arm or righting moments, according to the scale upon which the ordinates are measured. In the cross-curves of stability, however, draught is one of the variable conditions, and the displacement varies accordingly. A cross-curve whose ordinates represent the lengths of righting arm at various draughts, is therefore quite different in character from a cross-curve of righting moments, whose ordinates are length of righting arm, or G Z, × displacement. It is necessary, in order to judge accurately of the variation of stability with draught of water, to use curves of righting moments, and not merely curves of G Z, such as are considered sufficient when the draught of water is fixed."

moments for a prismatic body of elliptical section, and equal length to the above. The axes of the ellipse are of the same length as the sides of the rectangular section taken in the former case; the minor axis being two-thirds of the major, and the minor axes being vertical when the body is upright. A p O and A q O are the corresponding curves of G Z × V at the various draughts of water, and are obtained by deducting V × B G sin 30° from the ordinates of the curves A P O and A Q O. The bodies being homogeneous G is taken at the middle of their depth. These curves therefore represent the actual righting moments of the two bodies under consideration at all draughts of water. The ordinates measured to the right of A O give righting moments, and those to the left, if there were any, would be upsetting moments. It will be seen that the whole of the curves in Fig. 130 are similar with respect to a line drawn parallel to O X at one-half the depth of total immersion. The elliptical figure tends to return to the upright, when at the inclination of 30°, at all draughts of water, and exerts the maximum efforts to do so when immersed to the middle of its depth. The rectangular figure, when inclined to the same angle, also tends to return to the upright at all depths of immersion, but the maximum righting moment is not when floating at the middle of its depth, but at draughts which are at equal distances above and below it.

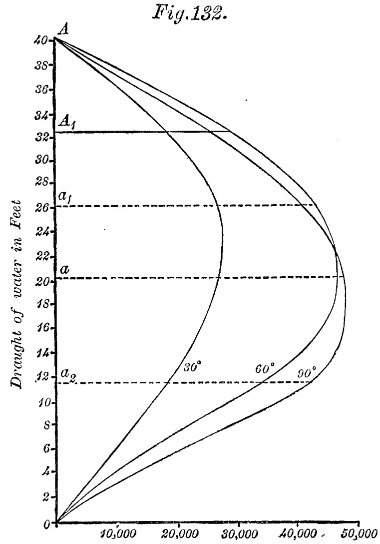


"Fig 131 represents similar curves for a prismatic body, the upper half of whose section is rectangular, and the lower half elliptical; the extreme dimensions of the section being the same as in the previous cases. This form of section is an example of the kind of departure from symmetry of form which exists in ships. It has been seen that if homogeneous bodies of symmetrical form be altered in density so as to float alternately at water-lines which are at equal distances above and below the centres of such bodies, the righting moments at equal angles of inclination will in each case be the same at these draughts. In the body for which the curves in Fig. 131 have been constructed, the departure from similarity between the immersed and out of water volume, causes a variation in the righting moments at the draughts described. A P_1 O and A P_2 O represent the curves of B R × V at angles of inclination of 30° and 60° respectively, and A p_1 O, A p_2 O are the corresponding curves of G Z × V, or curves of righting moments when G is taken in the position it would have if the body were homogeneous. The lines, $a_1\,x_1$ and $a_2\,x_2$, indicate draughts at which equal volumes are cut off above and below water, and $m\,m$ shows the depth at which the immersed volume is one-half of the total displacement of the body.

"It will be seen that the righting moments are greater at 30° and 60° of inclination when the body is deeply immersed, than when it is floating at light draught with equal volumes below water to what there are above in the other case. The relation between the righting moments at the two extremes of draught in a ship is, however, largely determined by the position of the centre of gravity, which in this case has been taken for a homogeneous body. This will be seen by the next example.

"Fig. 132 gives curves of B R × V for an actual ship, at 30°, 60°, and 90° of inclination respectively. The vessel for which these have been constructed is 400 feet in length, 44 feet in breadth, and 32 feet 6 inches in moulded depth. The extreme depth from the top of keel to the highest point of the sheer of the upper deck, is 40 feet. The point O is the top of keel, A is the highest point of the sheer of the deck, and A_1 the lowest point of the upper deck at side, from which the freeboard is measured. The horizontal ordinates of these curves represent the moments B R × V at the draughts to which they correspond. The displacement of the vessel when wholly immersed is 11,800 tons; and when displacing half this amount—or 5,900 tons—she draws 20 feet 6 inches of water, and the depth of flotation, with the corresponding value of B R × V are shown by the ordinate drawn at the point, a. a_2 represents the draught of water at which the vessel was launched, and a_1 the draught at which there is an equal volume out of water

Fig. 132.



to what there is below water at the draught a_2. The draught of water at the point a_2 is 11 feet, and the freeboard above the point a_1 is 7 feet.

"If the centre of gravity be taken at 19 feet above the top of keel at all draughts of water—it always varies, and in some cases considerably, with the draught, as has been stated, but 19 feet is found to be a fair mean height for the ship in question—and the moment, $V \times BG \sin. \theta$, be deducted from the ordinates of the curves in Fig. 132, we obtain new ordinates which represent the curves of righting moments, $V \times GZ$; and these are shown in Fig. 133. It will be seen that the curves of righting moments, which correspond with the ordinary curves of stability show much larger moments at deep draughts than at light draughts. For instance, the ordinary curve of stability for the launching draught of 11 feet at a_2, gives very much smaller moments than the corresponding curve for the deep draught at a_1, where there is an equal volume above water to what there is below in the other case, and the freeboard is only 7 feet.

"The centre of gravity taken in Fig. 133 is 1·1 feet below

Fig. 133.

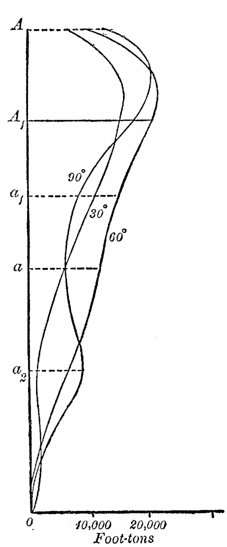


Fig. 134.

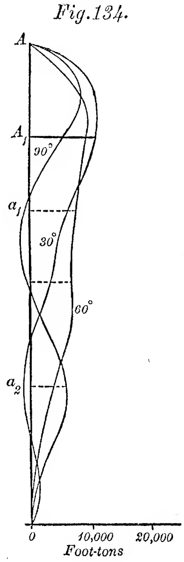
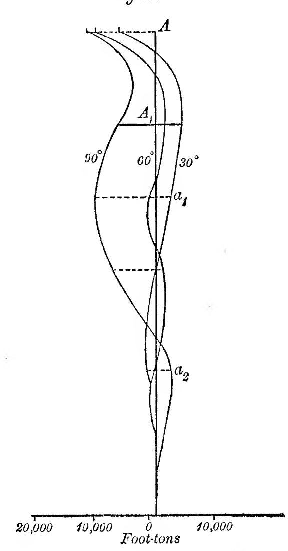


Fig. 135.



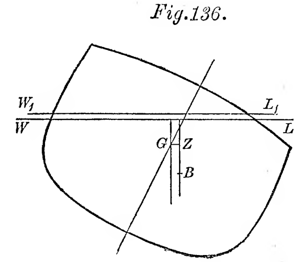
where it would be if the external surface of the ship enclosed a homogeneous volume. The relation which it bears to the position of the centre of gravity of a homogeneous body of the same form, largely determines the stability at all except small angles of inclination, and also the relation between the stability of light and deep draughts. Figs. 134 and 135 show what Fig. 4 becomes changed into if the centre of gravity is, first, raised 1·1 feet so as to be in the same position as if the ship were a homogeneous body, and 2nd, if it is raised a further 1·1 feet so as to be as much above this point in Fig. 135 as it is below in Fig. 133. A comparison of these figures will show that, except in the case of a very high centre of gravity, the stability at light draughts, with various positions of centre of gravity, is less than at deep draughts.

It appears therefore that in the case of the ship in question, and she is a type of many mercantile passenger steamers, the proposition respecting the equality of the stability at light and deep draughts in homogeneous symmetrical bodies, requires modification in a direction which is disadvantageous to light draughts. When there are equal volumes above and below water in this vessel, the righting moments at the light draughts are generally much less than at the deep draughts, except when the centre of gravity is raised excessively, and for this class of ship unusually, high.

"The analogy that exists between light-draught and deep-draught stability in forms that are approximately symmetrical, and particularly the point of resemblance afforded by the fact that what is a wedge of immersion in one case is that of emersion in the other, and *vice versâ*, cannot fail to have struck some who have had to calculate the stability of bodies floating at light draughts, but attention has never been prominently called to it. It is time, however, that the connection between the two cases were fully realised, and the dangers peculiar to very light draughts of water appreciated as thoroughly as are those which attach to low freeboard."

It is of these curves of Professor Elgar that Mr. Jenkins remarked (in his Paper on "Metacentric Diagrams," previously quoted)—"These curves exhibit the peculiarities to which I have referred in connection with curves of initial righting moment, and the conditions which ensure maximum and minimum values of the moment are even more simple than those which hold in the upright condition." In illustrating and commenting upon this, he proceeds as follows:—

Fig. 136.



"In Fig. 136, if W L be the water-plane at which a vessel inclined through an angle, θ, is floating, the righting moment will be proportional to V.G Z. If, now, the draught be increased by Δx to $W_1 L_1$, ΔV be the increase in volume, and y the distance of the centre of gravity of the layer from the vertical through B, we obtain by taking moment about the vertical through B

$$\Delta GZ = \frac{y \cdot \Delta V}{V + \Delta V},$$

and

$$H + \Delta H = (V + \Delta V)(GZ + \Delta GZ) = y \Delta V + V \cdot GZ + \Delta V \cdot GZ.$$

If V . G Z be subtracted from this expression, we obtain—

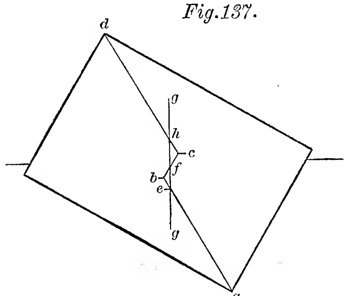
$$\Delta H = y \Delta V + GZ \cdot \Delta V.$$

$$\therefore \frac{\Delta H}{\Delta V} = y + GZ, \text{ and } \frac{dH}{dV} = y + GZ.$$

When the righting moment is a maximum or minimum $y = -GZ$, and the centre of gravity of the water-plane, and therefore also the centre of curvature of the curve of flotation and G lie in the same vertical line.

"Maximum values of the righting moment occur where the locus of the centre of gravity of the water-plane cuts the vertical through the centre of gravity of the vessel in passing from right to left in the diagram as the draught increases, and minimum values occur where the locus of the centre of gravity of the water-plane cuts the vertical through the centre of gravity of the vessel in passing from the left to the right as the draught increases. A very good illustration of the above property can be afforded by reference to one of the bodies for which curves were given by Professor Elgar. The rectangle, Fig. 137, is a section of a homogeneous prism, which is immersed at an inclination of 30°. The depth is two-thirds the breadth, and the line $a\,b\,c\,d$ is the locus of the centre of gravity of the water-plane, as the draught changes. The line $g\,g$ is the vertical through the centre of gravity of the prism. At the point e, where the two lines first intersect, the locus of the centre of gravity of the plane of flotation is passing from the right to the left as the draught increases, and the righting moment is a maximum; at the next point of intersection, f, the moment is a minimum, and at h it is again a maximum.

Fig. 137.



"The direction of the tangent to a cross-curve of stability at any draught is capable of being readily expressed. Thus, we have in the above equation $\frac{dH}{dV} = y + GZ$, and this reduces to the form $\frac{dH}{dx} = A(y + GZ)$, which gives the inclination of the tangent at any point of the curve. As the area of the water-plane and the position of its centre of gravity are both estimated in the determination of the value of G Z at a given draught of water, the drawing of the tangent for each value of G Z calculated will involve but little additional labour, and will enable the curve to be drawn with a fewer number of spots.

"If the practice of constructing cross-curves of stability, to which an impulse has already been given, becomes more general—and they appear to be indispensable to a complete knowledge of the stability of a vessel under all the conditions of draught and lading in which she is liable to be placed—the above property will be of assistance in their construction, and in determining the exact depths of immersion at which the righting moment attains its maximum and minimum values."

Mr. W. Denny has also devoted himself, and the scientific staff of the firm of Messrs. Denny Brothers of Dumbarton, of which he is a member, to the production of cross-curves of stability. "Shortly after the *Daphne* inquiry," he writes, "it occurred to me that as stability curves were required for at least four draughts for each steamer, it would be well if some method of obtaining these curves could be arrived at which would facilitate their construction." In pursuing this idea, Mr. Denny observed that the four conditions of draught at which such curves are usually required are the launching condition, the finished condition without weights on board, the fully loaded condition with coals in bunkers, and the same condition but with the coals consumed; and it occurred to him that, if in addition to the stability curves corresponding to these conditions, a curve were calculated for a draught intermediate between the finished condition without weights on board and the fully laden condition with coals consumed, he would then possess curves of stability at five fairly-distributed draughts, from which cross-curves could readily be constructed. As a matter of fact, he has since found it better to adopt six different draughts of water instead of five. He has also developed the fact that, with the systems of calculation pursued at Dumbarton, it is preferable to calculate these "cross-curves" first, and from them to measure off the righting levers requisite for constructing the usual curves of stability at given draughts. The account of his system, with which Mr. Denny has been good enough to furnish the author, involves such close reference to the use of the Amsler Integrator that we deem it better not to attempt here a complete record of his improvements, but to content ourselves with describing the characteristics of the Dumbarton system.

Mr. Denny and his scientific assistant, Mr. F. Purvis, have employed two methods of proceeding—one suggested by Mr. Fellows, and the other by Mr. Couwenberg, both draughtsmen and calculators engaged under them. Mr. Denny thus describes the method of Mr. Fellows:—"A tracing or drawing, having upon it the ordinary displacement sections of the steamer, was pinned to the disc,* a centre of gravity being assumed in its middle line, and made coincident with the centre of the disc. The disc was then turned round, so that the centre line of the drawing formed an angle of 10° or 15° with the axis of the integrator, and a number of parallel water-lines

* This disc or turntable enables the tracing or drawing to be turned round, thus obviating the necessity of shifting the axis of the integrator.

were drawn across the sections at right angles to the axis-line of the integrator, dividing the range of draught of water into a suitable number of intervals. These extended from a draught somewhat under the launching condition to a draught somewhat over the loaded condition, so as to afford a sufficient number of spots for the cross-curves. The disc being fixed at the given angle, work was begun, each section up to the lowest water-line being circumscribed by the pointer of the integrator, and the results for the area, and the moment of the area, noted upon forms prepared for the purpose. The areas and moments of areas up to each water-line were then plotted off upon a base-line representing the length of the steamer between the extreme displacement ordinates, and upon this were set up at the proper distances of the sections, ordinates of length corresponding to the areas and moments of areas at these different sections. Curves being drawn through these points, the integrator was again employed for their integration. The quotient obtained by dividing the areas of the curve of moments by the area of the curve of areas is the length of the righting or upsetting arm, as the case may be, with the assumed centre of gravity. The product of the area of the curve of areas and a suitable multiplier is the displacement at a given draught." The same operations being performed for each draught at the given angle of the inclination, the righting arms thus obtained are set up as ordinates from a horizontal base-line, each ordinate having for its abscissa the calculated displacement to which it corresponds. A curve passed through the extremities of these ordinates is the cross-curve of stability for the given angle. Any variation in the position of the centre of gravity from that assumed in the calculations can, of course, be readily taken into account, as in other methods. It is the custom of Mr. Denny's staff to work out cross-curves for angles at intervals of 15° up to an inclination of 90°, though in special cases it may be desirable to reduce the interval to 10°. For this method complete sections for both the fore and after bodies of the ship are necessary.

Mr. Couwenberg arrives at the same results as Mr. Fellows, but by a different method. "The general principle upon which he proceeded was," says Mr. Denny, "that for any inclination the integral of the statical moments of the number of parallel water-lines about a given axis will give the statical moment of the immersed part of the ship about that axis. The integral of the areas of these water-lines will give the displacement of the same part of the ship. From these two integrals we therefore get the means of finding the righting arm at any inclined draught by dividing the integral of the statical moments by the integral of the areas of the water-lines." For this method half-sections only for the fore and after bodies, such as are ordinarily used, are required. The number and positions of the water-lines for any given angle having been chosen, an axis normal to them is fixed upon, about which to take the moments. The inclined water-lines are drawn out in a manner similar to that for the water-lines of an ordinary half-breadth plan, a horizontal line representing the fixed axis being taken to correspond to the ordinary centre line, and the widths of the various sections measured on either side of it. When the water-lines for the given inclination have been thus set out, the integrator is used to integrate them, by which means are obtained their areas and their moments about the fixed axis. These areas and moments are then set up as ordinates from a horizontal base-line, their abscissæ being the distances between the water-lines, and curves are passed through the extremities of the ordinates. Integrating these curves by means of the integrator up to any draught, and dividing the area of the curve of moments by that of the curve of areas, the righting lever for the given inclination is obtained for that draught. In this method, as in that of Mr. Fellows, the centre of gravity is assumed to be in a certain position on the middle line of the vessel, and the normal axes to the various water-lines are made to pass through that point. The displacement and righting arms for the different draughts at the required inclinations having been ascertained, it is easy to construct cross-curves of stability, as already explained.

CHAPTER XI.

Brief Historical Survey of Geometrical Aspect of Stability—Bouguer's Investigations —Position of Centre of Gravity—Metacentre and Metacentric—Exposition of First Principles of Stability—Determination of Height of Metacentre—Expression for Same—Locus of Centre of Buoyancy—Change in Form of Displacement —Change in Dimensions of Displacement—Method of Finding Centre of Gravity —Great Value of Bouguer's Work—Bernoulli's Investigations—*Differential* Expression for Righting Moment—Rolling and Pitching—Don Juan d'Ulloa's Investigations—Formula for Inclining Experiment—Euler's Investigations—Three Different States of Equilibrium—"Measure" of Stability—Moments of the Wedges—Moment of Inertia—Position of Centre of Gravity—Euler's Expression for Stability—Chapman's Investigations—Romme's Investigations—Romme's Expression for Stability—Determination of Centre of Gravity.

In the progress we have thus far made, we have been brought more than once to considerations which might be regarded more especially as unfolding the geometrical aspect of our subject, and much advantage will result if from this point we take a rapid survey of the developments which the doctrines of stability originally underwent in this respect. This survey is essential to a proper apprehension of the recent developments of the science with which we shall have presently to deal.

Although much was written on the subject at the end of the 17th, and in the first half of the 18th century, and even great and famous controversies then took place—notably that originated in the great work of the famous Jesuit, Père l' Hoste, in 1693—it is not necessary for our purpose to go back farther than the middle of the last century, when the illustrious Frenchman, Bouguer, of the Royal Academy of Sciences, formerly Hydrographer Royal at Croisie and Havre-de-Grace, published his famous *Traité du Navire,* in which, as we have seen, the "Metacentre" was first described, and the Metacentric Theory of Stability was given to the world. This great work treated of many subjects connected with the construction and movements of ships, besides that of stability, but upon these we have not here even to touch. It is in his second book that he enters upon the consideration of "the ship afloat," and under this head most of his remarks on stability are ranged.

Bouguer first shows that a ship afloat in still water must displace a volume of water equal in weight to itself, and that the resultant fluid pressure exerted upon the immersed bottom of the ship must act upwards through the centre of gravity of the volume of displacement. He explains that the weight of the ship in tons may be directly ascertained by dividing this volume of displacement expressed in cubic feet, by the number of cubic feet of water which weigh a ton; and he describes various methods of approximating to the volume of displacement from the drawings of a ship. One of these methods differs from the method at present employed only in the use of the common "trapezoidal rule" instead of the ordinary "Simpson's Rule" for calculating irregular areas, &c.*

Bouguer next proceeds to treat of the distribution of the weight of a ship, and of the position which should be given to the centre of gravity. He first describes the method of determining a ship's centre of buoyancy. This, he says, can be done practically by suspending a solid block model of the immersed portion of the ship in different positions, and ascertaining the intersection of the lines of suspension; or it can be done by calculations based upon the ship's drawings, the method of doing this which he describes, resembling that now practiced by Naval Architects in all respects excepting the use of Simpson's Rules.

We next arrive at an investigation of the maximum height at which the centre of gravity of a ship may be placed; in which phrase is shadowed forth that conception of the "Metacentre" which he proceeds to develop, and which obviously involves the idea of a *limit*. We are therefore not left in doubt, as some have supposed, of the true meaning of the word metacentre, as it was originally, and long afterwards employed. At the same time it must be borne in mind that Bouguer himself, as we saw previously, extended the meaning of the word so as to embrace within it the points of intersection of adjacent verticals through the centre of buoyancy at large angles of inclination. His 5th chapter (book ii., section 2), is headed "On more extended investigations on the metacentres, and on the curved line which these points form when the ship is inclined," which curve he designates the metacentric *(métacentrique)*. It is described as being what we have in previous chapters seen it to be, viz., the locus of the centres of curvature of the curve

* These and cognate matters occupy the 1st, 2nd, and 3rd chapters of the 1st section of book ii. In chapters 4, 5, and 6, Bouguer digresses into a discussion of the Tonnage of Ships, exposing the faults of the Tonnage Rules then employed.

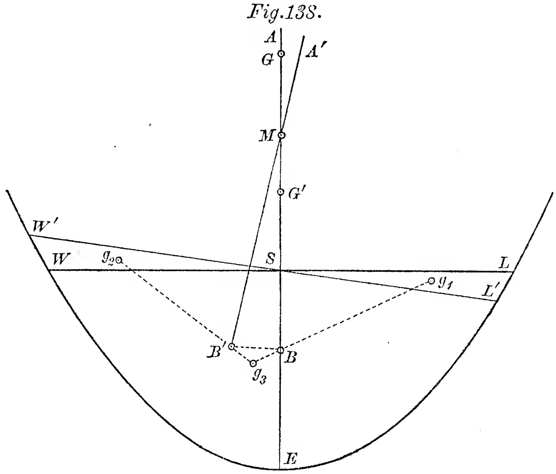
of buoyancy, and each point of it, even for considerable angles from the upright, is undoubtedly regarded by Bouguer as itself a metacentre. In fact, with him, the *metacentric* was neither more nor less than the locus of his metacentres, just as it is now with us, in this work, the locus of pro-metacentres.

In giving the following summaries of Bouguer's investigations, we shall take the liberty of modifying the diagrams, and especially their lettering as far as may seem convenient.*

Having determined the centre of gravity of the volume of displacement, he says:—

"The point is now known in which the pressure of the water is concentrated, and through which the vertical line of action of that pressure passes. The centre of gravity of the ship is always situated in the same vertical line, otherwise the pressure of the water would not be directly opposed to the weight and could not sustain it; these two forces could not counterbalance or neutralise (suspendroient) each other's action. This does not suffice however to keep the ship in a permanent position; because the particles of water, like those of all other liquids, are in continual motion, and so it happens constantly that some of these particles strike the ship's bottom harder on one side than on the other; † and this suffices to produce an inclination which is perhaps insensible at first, but would not fail to increase if the centre of

Fig. 138.



gravity of the ship were too high." He illustrates this by referring to the instability of a rod made to float end-on in water, and then proceeds:—"Suppose W E L (Fig.

* In translating and abridging the substance of what is said in this chapter on Stability by Bouguer, Euler, and one or two other early writers, I avail myself in some degree of certain abridgments prepared for me some years ago by Mr. W. H. White, whose competence to perform such work with skill and accuracy is well known.

† "Bouguer considered this force" (the disturbing force of the moving water) "as exclusively produced by the blows exerted by the crests of the waves on the topsides, and made it pass above the centre of gravity."—*Bertin.*

138) to represent a transverse section of a ship, G being the centre of gravity, and B the centre of buoyancy in the upright position, for the displacement W E L, B A being then vertical. If the ship is inclined to an infinitely small extent (which may result from the irregular shock of the smallest particle of air or water) and W′ L′ be the new position of the water surface, the upward pressure of the water will no longer be concentrated in B, the centre of gravity of W E L, but in B′, the centre of gravity of W′ E L′, the part actually immersed; and as the new vertical, B′ A′, instead of passing through the centre of gravity, G, of the ship passes on the side opposite to that inclination, it is clear that the upward pressure, instead of tending to restore the ship to the upright tends to incline her more. The ship cannot, therefore, remain upright, there being no force to retain her in that position, while the smallest force will make her begin to move from it, after which commencement she inclines further by herself. . . .

"But if the centre of gravity of the ship were below the intersection M of B A and B′ A′, as at G′, the pressure of the water would always tend to restore her to the upright if she were inclined, because its direction would always be placed on the inclined* side with respect to G′. Consequently there would always be a power to keep the ship upright, or at least to tend to restore her, however little she might be inclined, and this would increase according to the necessity. Hence is seen the great importance of the knowledge of the point of intersection, M, which not only shows the maximum height which may be given to the centre of gravity of the ship, but also distinguishes the case of a ship which will maintain an upright position from that of one which will upset even in harbour, and which cannot remain upright a single instant.

"The point, M, which may justly be styled the *metacentre*, is the limit which the height of the centre of gravity of the ship must not pass or even reach; † for if the centre of gravity coincides with M, the ship will no more seek the upright than an inclined position—the two positions will be equally indifferent to her, and she will consequently be incapable of righting herself if heeled over by any outside influence."

Having thus clearly stated the conditions of the problem, Bouguer proceeds to illustrate his remarks by considering the case of a hemisphere, and then describes in detail "the method of determining the metacentre." To find the metacentre, M, at a very small inclination it is necessary to determine the corresponding centre of buoyancy, B′; and the shift of this centre from B to B′ he shows to be due to the immersion of W S W′ and the emersion of L S L′. His investigation of the change of position of the centre of buoyancy is both simple and correct. Assuming that g_1 and g_2, Fig. 138, are the centres of gravity of the wedges of emersion and immersion, and that g_3 is the centre of gravity of the common part, W S L′ E, he shows that, since the wedges are of equal volume, the ratio of g_3 B to B g, must be the same as that of g_3 B′ to B′ g_2; so that the line B B′ must be parallel to the line $g_1 g_2$, joining the centres of gravity of

* Here and in the preceding remarks Bouguer uses the phrase *côtés de l'inclination* in the sense of *immersed side.*

† The precise language here employed is important, as it is the original definition of the word "metacentre." Bouguer's words are—"*Le point qu'on peut à juste titre nommer métacentre est la terme que la hauteur du centre de gravité* G, *ne doit pas passer, et ne doit pas même attendre.*"

the wedges. He says:—"The whole displacement, that of the small part L S L′, and the distance $g_1 g_2$ of the centres of gravity, of L S L′ and W S W′, form three terms of a proportion of which the length B B′ is the fourth term." This is a perfectly sound conclusion, as we have seen, and upon it he bases his further investigations.*

In these he assumes that the form of the load-water section is known; that for any longitudinal abscissa, x, measured along the middle-line of that section the corresponding half-breadth ordinate is y, and that the infinitesimal calculus may be used. He then proceeds to calculate the volumes and moments of the small wedges. For the volume of an element of the wedge he obtains the expression $\frac{e}{2b} y^2 dx$; and for the moment he has $\frac{e}{2b} y^2 dx \times \frac{2}{3} y$, or $\frac{e}{3b} y^3 dx$; $\frac{e}{b}$ being the tangent of the infinitely small angle of inclination. Integrating these quantities, and making use of the proposition previously stated for finding the length B B′, he obtains the expression,

$$\text{B B}' = \frac{2}{3}\frac{e}{b}\frac{\int y^3 dx}{\text{P}},$$

where P = the volume of displacement; and thence gets B M = B B′ × cotangent of angle of inclination,

$$= \frac{2}{3}\frac{\int y^3 dx}{\text{P}},$$

the well-known expression for the metacentric height still in use.

In applying this formula to the case of an actual ship, Bouguer proposes to integrate the cubes of equidistant ordinates measured on the load-water section by means of the trapezoidal rule; here, again, differing from present practice only in the *rule* he uses. He further illustrates his method by applying it to particular forms.

In the next chapter (5) (to which we have already referred, and the heading of which we have quoted) Bouguer, after stating that the limitation of the inclination in his previous solution rendered it insufficient, and pointing out the fact that, as the inclination of the ship increased, the line of action of the fluid pressure, which passed *above* the centre of gravity at small inclinations, might pass *below* it and so tend to upset the ship, goes on to show that the investigation previously given for finding the shift of the centre of buoyancy is

* All this agrees with what we saw from Atwood in a former chapter, but it must be remembered that Bouguer wrote a half-a-century earlier than Atwood.

general, holding even for irregular bodies. Hence, he concludes that the locus of the centre of buoyancy is a curve of which any infinitely small piece is parallel to the corresponding water surface, and of which the curvature is continuous. Also, that "the pressure of the water always acts along a normal to the curve, which is the locus of the centre of buoyancy;" and that "all these normals must by their intersection form another curve, which may be termed the *metacentric*, of which we as yet only know the point M (Fig. 138), and which is the involute of the curve of the centres of buoyancy." In afterwards dealing with this curve he falls into a serious error, as we shall presently see.

Fig. 139.

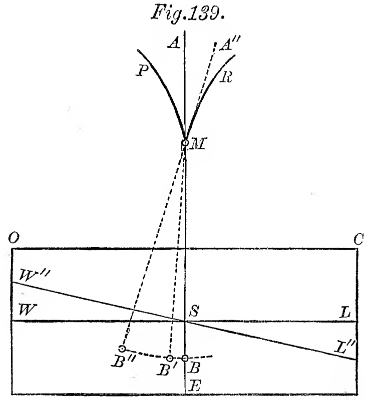
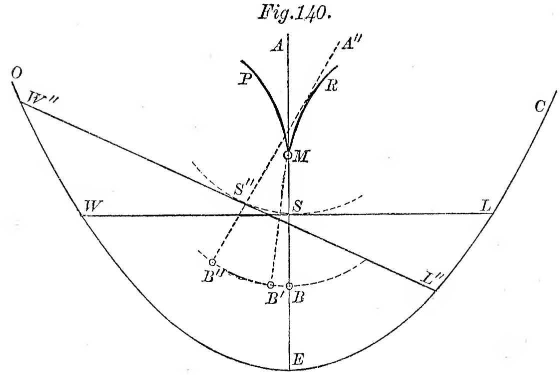


Fig. 140.



Applying the conclusions just reached, he says—"If it be desired to know whether any vessel, O E C (Figs. 139 and 140), of which the centre of gravity is placed below the metacentre M, in the upright position, will be safe, it is necessary first to determine all portions of equal displacement with W E L (when the ship is upright). . . . Then to determine the locus, B B′ B″, of the centre of buoyancy, which can be done by determining the distances of various points on the curve from the line E A and another line at right angles to it." Throughout all these remarks Bouguer grasps the subject effectively, and illustrates his meaning by the simple cases of vessels with rectangular cross-sections, as well as of others with straight sides near the water-line, describing the character of the loci of the centres of buoyancy, as well as that of the loci of the intersection of consecutive planes of flotation. Passing thence to the general case, he says—"The method previously explained for finding the height of the metacentre is generally applicable. It is only necessary to consider the plane of flotation when the ship is inclined: the axis or greatest diameter of this plane will not coincide with the middle-line because of its irregularity; but that axis must be made to pass through the centre of gravity of the plane, in order that the two small solids of immersion and emersion may be always equal, and the volume of displacement remain constant."

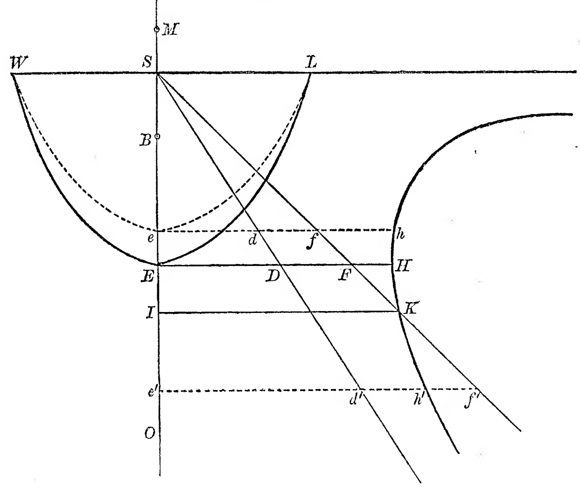
He then shows that all the change consequent on this want of symmetry is that the cubes of the half-breadth measurements on either side of the axis of an inclined water-section have to be integrated separately, and summed, instead of one side only being dealt with, and the result doubled, as in the upright position. His practical deduction from these considerations is "that the form of a ship ought to gain in breadth, or at least maintain its full breadth, up to the height to which the side is immersed when the ship is most inclined;" because the curve of centres of buoyancy would then be either a hyperbola or a parabola, and the branches, M P, M R, of the metacentric curve (Fig. 139) would rise above M, the metacentre in the upright position. In such a case, he adds, up to the inclination considered, "the centre of gravity of the vessel would depart further from the direction B″ A″ of the upward fluid pressure . . . and would contribute to increase the length of the arm of the lever to which the upward pressure of the water is continually applied." It is here that Bouguer falls into the serious error to which we have adverted, and which was clearly pointed out by Atwood. Bouguer assumes quite erroneously that the mere *rise* of the metacentre (using the term here to express any point on the metacentric curve) will be attended by a decrease of stability. This is not necessarily the case, as we have already seen more than once.

Having laid down his general principles, Bouguer proceeds to urge most strongly the desirability of calculating carefully the positions of the centre of gravity and metacentre in a newly-designed ship, and argues that the amount of work involved is probably over-estimated, in proof of which he gives an account of the calculations made by him for a frigate. He supplements this by some statements respecting the position of the centres of gravity in the ships of that period, and illustrates the variations produced in that position by changes in the weights carried.

Less practical departments of the subject next come under consideration, viz., the changes in position of the metacentre resulting from changes in the form of the displacement. Here Bouguer displays considerable ingenuity, and uses his fundamental formula for the height of the metacentre in the solution of several of the questions he propounds. He supposes changes to be made in the length, the breadth, and the draught of a ship, and (with certain assumptions) he investigates the effects produced in the positions of the centre of buoyancy and metacentre. All these are, however, little more than geometrical exercises, and have no great value now beyond the evidence they afford that Bouguer's investigations extended over most branches of the subject.

The most noteworthy of his examples is that where the length

Fig.141.



and breadth of a ship are supposed to be unchanged, while the draught is diminished. Assuming that the displacement is made to vary directly as the draught, he describes a graphical method of representing the various positions occupied by the metacentre as follows:—

> "If a perpendicular, E H (Fig. 141) be drawn at the foot of the vertical, S E, and E D be made equal to the height of the centre of buoyancy, B, while E H equals the height of the metacentre, M, above E, then D H = B M; and if the hyperbola, *h* H *h′*, be traced, passing through H, and having S L and S D for asymptotes, all the other ordinates, such as *e h*, drawn parallel to E H will mark the height of the metacentre above the bottom, *e*, of the ship for all the different draughts."

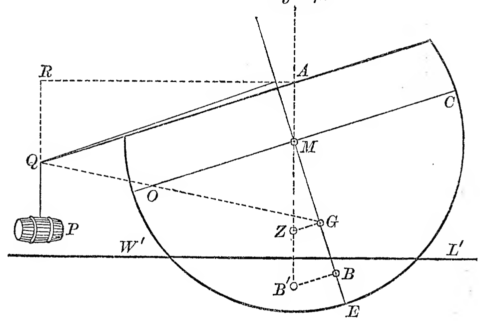
This assumes that the centre of buoyancy always divides the total draught of the ship in the same ratio, and then, as Bouguer points out, the straight line, S D, will represent the locus of the centres of buoyancy at different draughts, while the line, S *f*, drawn at an angle of 45 degrees with S L will represent the corresponding water surface at any draught. Bouguer in this particular case indicates, therefore, a method of graphical representation very similar to that now used for the metacentric and centre of buoyancy curves of ships floating at different draughts.

Going a step further, he considers "the changes in the force of the ship to keep upright resulting from changes in the dimensions of the displacement." He here defines the "stability" of a ship as the "force" with which she attempts to return to the upright position when moved out of it; and he fully explains the now familiar fact that the righting force on a ship inclined equals the product of the displacement into the distance of the centre of gravity from the vertical line up which the fluid pressure acts. His various cases of changes in length, breadth, and draught, closely resemble those previously taken in connection with their effects upon the height of the metacentre, and rest upon similar assumptions, so that they have little practical value.

Some of his remarks are, however, interesting. For example, he says—"As very large breadths are now given to ships, it would not be proper to increase those breadths much, except towards the bow and stern, and when thus increased, the ship's stability would be made greater." Again, for ships of the same length and draught, but of different breadths, he says, "the stabilities are as the cubes of the breadths," provided that the centre of gravity coincides with the centre of buoyancy—"a condition," said Atwood 50 years afterwards, with needless asperity, perhaps, "which may be deemed amongst the most extreme cases that can be devised, and such as is rarely known to exist." Attention is also given by Bouguer to the effect upon the stability of various arrangements of ballast.

Next follows a clearly stated account of a method of finding the centre of gravity of a ship by inclining her in smooth water. Fig. 142 illustrates this account. P is the weight used to incline the

Fig. 142.



ship, B′ A is the vertical line along which the fluid pressure acts when the ship is heeled, and G is the centre of gravity of the ship. "As the inclination increases," says Bouguer, "the distance of G from the vertical B′ A increases, and the distance, G Z, from this line, is continually proportional to the sine of the inclination, so long as the angle is small. Knowing this distance, as well as the total weight of the ship, the moment of the force tending to restore her to the upright can be found. And since the situation and weight of P producing the inclination are equally ascertainable, the moment of this weight about B′ A must equal that of the ship's stability, and so it will be easily discovered whether the centre of gravity is in the position desired." He proceeds to urge the necessity for great care in measuring the angle of inclination of the ship, and in keeping "all the circumstances absolutely unchanged throughout the experiment, in order that the inclination may be due solely to the weight P." People on board should occupy fixed positions during the time all the observations and measurements are being made; and the angle of inclination might be taken by means of a long plumb-line hung from the mast-head. He thinks that in some cases the weight of the crew of a ship might be sufficient to produce the necessary inclination. Having thus determined the position of the centre of gravity, G, "it will be easy," he says, "to discover how much it is below the metacentre."

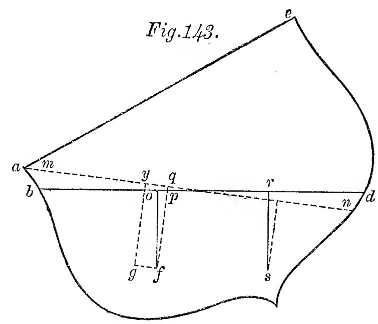
Bouguer assigns the credit of first suggesting such an experiment to Père l' Hoste, but he is himself evidently entitled to the honour of putting it into the form described. His recommendation is to use this experimental method as a test of the accuracy of the *calculated* position of the centre of gravity, in cases where such a calculation is made when the design is prepared. In cases where no such calculation is made, and where the metacentre is unknown, he suggests that the inclining experiment might be made as a test of the proper storage of the weights carried. He concludes this section of his subject by remarking that it is necessary to know more than the weight and position of the centre of gravity of a ship, in order to deal with the question of rolling; and in the next section devotes attention to the "distribution of a ship's weight in relation to her rolling motion."

Those who are familiar with the condition of naval science before the appearance of the *Traité du Navire* will be in the best position for fully estimating the value of Bouguer's work; but the foregoing account of his investigations, however imperfect, can hardly fail to convey to the reader the just impression that they were as brilliant as they were original, and such as to furnish a sound and lasting foundation for the science of the stability of ships.

We do not think full justice has been done in this country to Bouguer's labours in connection with this particular branch of our science. Too much stress has been laid upon his errors, which are rare; and too frequently he has been spoken of as having disregarded the differences that may exist, and do exist, between the amounts of stability existing at small and at large angles of inclination. We have seen that he by no means deserves this reproach, at least in so far as a perfect appreciation of those differences, their causes, and their magnitudes are concerned.

Soon after the production of Bouguer's Treatise, appeared in 1757, Daniel Bernoulli's famous work upon *Hydrostatical and Mechanical Principles; or, Memoir on the Means of Diminishing Rolling and Pitching,* in which, notwithstanding some serious misconceptions, was laid the foundation of the accepted theory of "Rolling," and which treats preliminarily of the question of "Statical Stability." The memoir first lays down the fundamental conditions of equilibrium for a floating body, then follows a discussion of the *change* in the moment of stability produced by a very small inclination. Bernoulli follows a method very much resembling Bouguer's investigation for the metacentre, only he does not consider the position from which the inclination is measured to be one of equilibrium, but obtains what may be termed the *differential* expression for the righting moment. Using Fig. 143, he supposes $b\,d$ to be the water-line for an inclination, σ, from the vertical, and $m\,n$ to be the water-line, making an angle, $d\ \sigma$, with $d\,b$. Then, taking s as the centre of gravity, and f and g as the positions of the centres of buoyancy corresponding respectively to the water-lines, $b\,d$ and $m\,n$, he obtains an expression for the *difference* between the two values of what we should call the corresponding arms of the levers (G Z) for statical stability. He expresses this as

Fig. 143.



$$d\,r = \left(\frac{q^3}{12m} - s\right) d\ \sigma,$$

where $q =$ breadth $b\,d$ of water-line; $r =$ G Z in our notation; $m =$ area of immersed section; and $s =$ vertical height of the centre of gravity above the centre of buoyancy. It will be obvious that for a very small inclination from the upright this formula would become identical with Bouguer's for metacentric stability.

Having established the formula for a particular cross-section, Bernoulli applies it to several prismatic bodies of known geometrical forms of cross-section, putting these investigations forward simply as examples illustrating the fact that the two most important elements in stability are the form of the water-section and the height of the centre of gravity above the centre of buoyancy. He does not give any practical rule applicable to ships.

In the second chapter the connection between stability and rolling or pitching motions is discussed, the opening passage being as follows :—

"An uniform and constant force acting upon a ship can only incline her to a certain angle depending solely upon her stability; and the greater her stability the less will she be inclined. For ordinary ships the sines of the angles of inclination will be nearly proportional to the forces producing them. In this respect increased stability is advantageous, because it is desirable for the vessel to remain as nearly as possible upright. A ship sailing in a wind may sometimes be heeled over 15 or even 20 degrees, if the spread of sail is not reduced; and when thus heeled she will still continue subject to oscillations similar to those occurring when she is upright. The masting will obviously be much more strained, however, when the mean position is greatly inclined than when it coincides with the upright. . . . Besides, the ship would always be in danger of capsizing, and the least accident might be fatal, more especially because the righting moments only grow with the sine of the angle of inclination, and consequently increase less rapidly as greater inclinations are reached. . . . Hence it appears that only an advantage can result from an increase in the stability to the greatest extent consistent with other good qualities in the ship."

Into the question of the oscillations of ships it is not our present purpose to follow Bernoulli.

In 1771 was published in Spanish, and in 1783 was translated into French by M. Levêque, the great treatise, entitled *Examen Maritime,* of Don George Juan d' Ulloa. This work was chiefly notable for an elaborate theory of the resistance of fluids; but it treats, although in a limited way, of stability, and deserves a brief notice. The treatise consists of two volumes, the first being wholly devoted to demonstrations of the principles of mechanics, hydrostatics, &c., and the second containing the applications of the investigations made in the first volume to the qualities and performances of ships, of the properties of the metacentre, and an investigation of a formula for its height above the centre of buoyancy. In these respects he scarcely goes beyond Bouguer, except in suggesting that the *longitudinal,* as well as the *transverse,* metacentre should be found, and in determining an expression for its height. His proposed method for making the numerical calculations incidental to determining the metacentres is complicated, and not so good as Bouguer's rule for the transverse metacentre. Like Bouguer, Don Juan urges the importance of finding the centre of gravity of a ship, and gives an example of such a calculation. He also suggests that an inclining experiment should be made on a ship similar to the new design, in order to reduce the labour of finding the centre of gravity of the latter by calculation. His formula for the inclining experiment is—

$$\begin{array}{c}\text{Metacentric height}\\ \text{(above C G)}\end{array} = \frac{\text{Weights moved} \times \text{their distances,}}{\text{Displacement} \times \text{sine of inclination,}}$$

and differs from that still in use only in having the *sine* instead of the *tangent* of the angle of inclination—an unimportant matter, so long as only small angles are reached. He proposes to determine the inclination by "measuring carefully the extent to which the ship's side amidships is emerged from the water by the inclination." Lengthy calculations follow, illustrating the use of the formula, and various estimates are made of the value of $\overline{\text{G M}}$ (height of metacentre above centre of gravity) for different classes of the ships of that period; but the results appear for the most part to be hypothetical, and not based upon experiment. Don Juan considered Bouguer's estimate of 1 or 2 feet as the value of GM in three-deckers to be far too small, and thought 8 or 9 feet nearer the truth.

We have now to notice the very remarkable work of Leonard Euler, entitled *Complete Theory of Construction and Properties of Vessels,* which was published in Petersburgh in 1773, and translated into English in 1790, by Colonel Henry Watson.* Euler's method of investigation is more analytical than that of Bouguer, but the results arrived at by him are substantially the same. After stating the fundamental conditions of equilibrium in a floating body, Euler defines the three different states of equilibrium, and illustrates them by the case of a ship which is made to alter her *trim* slightly. He says, however, that the same kind of reasoning applies to transverse heeling, and remarks that ships should be stable in the upright position. After showing that for any inclined position the stability depends upon the relative positions of the vertical lines passing respectively through the centre of gravity and the centre of buoyancy, he says:—

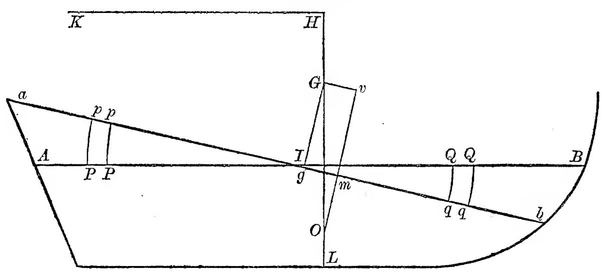
"In order to form proper conclusions respecting the state of equilibrium in a vessel, it is necessary to make researches respecting both the axes (longitudinal and transverse), for the case might easily happen that a vessel had sufficient stability with respect to one of these axes, whilst its equilibrium with respect to the other might be indifferent or even unstable. It is likewise as certain that when a vessel shall have a sufficient stability with respect to the two principal axes, it will also have sufficient with respect to all the other intermediate axes round which the vessel may receive any inclination."

Passing on, Euler remarks that the "measure" of stability must be the "moment of the forces with respect to the axis round which the inclination is made," which axis he considers to be some horizontal line passing through the centre of gravity of the vessel, although he does not assume the centre of gravity to be a *fixed* point. Using Fig. 144 to represent a longitudinal section of a ship,

* Of this work, Dr. Woolley in his celebrated Paper, read at the opening of the Institution of Naval Architects in 1860, said:—"Making allowances for the imperfection of his theory of resistances, there was no work published before the commencement of the nineteenth century which would better repay perusal than this. His mode of treating his subject is simple, philosophical, and highly instructive." Dr. Woolley notes, however, that Euler's propositions have appeared in a more modern form in later works.

of which A B was the water-line previously, and $a\,b$ is the water-line corresponding to an inclined position produced by the action of a force, K, along the line K H, he says, that K $\times$ $\overline{GH}$ measures the moment of this force, if G be the centre of gravity, which moment must be balanced by "the efforts which the vessel exerts in order to re-establish itself in its state of equilibrium," or, as we should say, by "the moment of stability" at that inclination. For small incli-

Fig. 144.



nations, he says, the "moment of force requisite to maintain the vessel in its inclined state will always have the form S $\times$ t $\times$ sin. i; where S denotes a certain absolute force, t a certain line, and sin. i the sine of the angle i, the radius being supposed unity." The product, S t, is what he "means by the term *stability*," and he justifies its separation from the sine of the angle of inclination by the remark, that in speaking of stability, "we form to ourselves an idea that no ways depends on the quantity of the inclination, seeing that the same idea must belong as well to the state of equilibrium itself (the upright) as to all possible inclinations."

Upon this assumption he shows how to apply his measure in calculating the comparative stability of two ships when inclined to equal, but very small, angles; and for estimating the stability of the same ship at different angles, supposing S t to remain constant. This reasoning, he admits, is applicable only to very small angles of inclination, and he urges the importance of providing a large reserve of stability, "many times greater than the greatest efforts which the vessel can ever be exposed to."

Again, making use of Fig. 144, and taking O as the centre of buoyancy of the original volume of displacement, A L B, Euler considers separately the effect of the fluid pressures corresponding to that volume, and the volumes of the wedges of immersion and emersion. For A L B he shows that the moment about G equals M $\times$ $\overline{OG}$ sin. I, where M is the weight of the ship, and I is the angle of inclination. This would tend to augment the inclination, and it is pointed out that if the wedges "should not furnish a moment of force opposite to and greater than the effects of the first, the vessel would not have any stability, and the least inclination would overturn it entirely."

Supposing the angle, I, to be very small, Euler investigates an expression for the moments of these wedges, using differential coefficients, and ultimately arrives at an expression for the righting moment, including both the wedges and the volume, A L B (so as to measure the effect of the new volume of displacement, *a* L *b*). This is done in a manner very similar to that described by Bouguer, and the result, as stated by Euler (but put into modern notation) gives, as his fundamental expression for stability.

Righting moment =

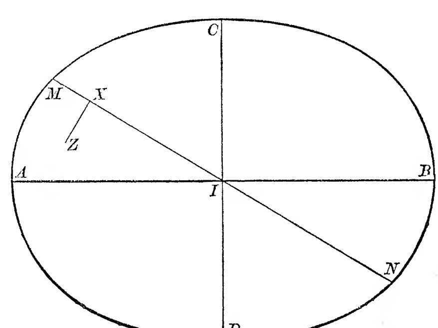
$$\frac{\mathrm{M}}{\mathrm{V}} \sin. \mathrm{I} \left\{ \int y_1^2 \, dy_1 \times \int y_2^2 \, dy_2 \right\} - \mathrm{M} \times \mathrm{O\,G} \sin. \mathrm{I},$$

where M = weight of ship, V = volume of displacement, y_1 = I P, and y_2 = I Q, P p and Q q being elementary pieces of the two wedges.

Euler draws attention to the fact that the terms under the integral signs depend principally upon the form of the water-section, and that they express the *moment of inertia* of that section, or the sums of all the particles contained in the section of the water (A B, Fig. 144), each multiplied by the square of its distance from the intersection, I, which intersection, on account of the equality of the wedges, must be a line passing through the centre of gravity of the section A B.

Fig. 145 is taken as a plan of the water-section represented by A B in Fig. 144, and on this plan, AB represents the fore and aft middle line, I corresponds to the same letter in Fig. 144, and C D is perpendicular to A B. Euler then demonstrates at length the well-known principle in dynamics, that when the moments of inertia of the plane about the two principal axes, AB and CD, are known, it is easy to calculate the moment

Fig. 145.



of inertia about any other axis, such as MN, of which the inclination to AB is known. As bounding cases, between which the actual form of a ship's water-section would lie, he takes a *rectangle* and a rhombus formed by setting two equal isosceles triangles base to base [CD in Fig. 145, corresponding to this base]. For these two figures he finds the moments of inertia with respect to the axis, AB, the results being

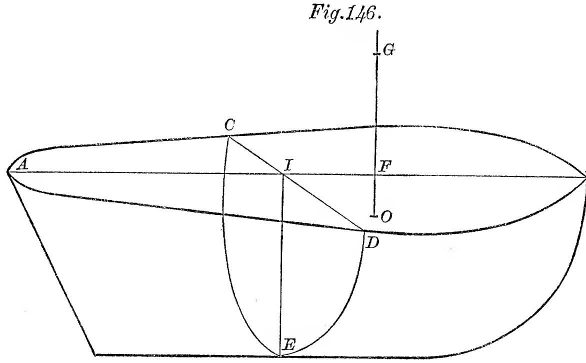
$$\text{For rectangle} = \tfrac{1}{12}\,AB \times \overline{CD}^3,$$
$$\text{,, rhombus} = \tfrac{1}{48}\,AB \times \overline{CD}^3,$$

and he remarks that similar expressions would represent the *moments of inertia* about CD. Calling attention to the fact that the numerical coefficients in these expressions bear to one another the same proportion as the *squares of the areas of the water-sections,* he proposes to avoid the labour of integrating for the moments of inertia in an actual ship by the following device:—Find the proportion which the squares of the areas of the water-section and of the circumscribing rectangle bear to one another, and take this as the numerical coefficient of the product, $AB \times \overline{CD}^3$,

or, $CD \times \overline{AB}^3$.

He tests this on an ellipse, and finds it approximately true. Assuming that this method removes all difficulty in finding the moment of inertia of the water-section, Euler proceeds to consider the remaining terms in his formula for the stability. He shows

Fig. 146.



that, other things being equal, the stability "is always proportional to the weight of the vessel." Then he takes Fig. 146 to represent the immersed part of a ship, and approximates to what would now be termed limiting values of "coefficients of fineness" for the area of the water-section and for the volume of displacement. For the water-section he concludes that in the area $= a \times \mathrm{AB} \times \mathrm{CD}$, Fig. 146, a is less than 1 and greater than $\frac{1}{2}$; for the volume of displacement, say $= a\beta \times \mathrm{AB} \times \mathrm{CD} \times \mathrm{IE}$, $a\beta$ is less than 1 and greater than $\frac{1}{4}$; and for the depth, OF, of the centre of buoyancy, (O), below the water he thinks a good approximation might be made from the equation:

$$\mathrm{OF} = \frac{\beta}{\beta + 1}\,\mathrm{IE},$$

where β is less than 1 and greater than $\frac{1}{2}$.

This latter equation is based upon three cases only, viz.—ships with rectangular or with triangular cross-sections, and a ship which is a pyramid with its apex in E. Respecting the position of the centre of gravity, G, he says—"FG depends upon the weight of the whole vessel, and according to different distributions it may happen that G will be more or less elevated above the section of the water, or that it may even fall below it."

Calling the quotient of the moment of inertia divided by the volume of displacement, l, Euler writes the expression for stability in the form,

$$\mathrm{M}\,(l - \overline{\mathrm{OG}}),$$

and says that for stable equilibrium, l must be greater than OG. The least value of l being that for transverse inclinations (about the axis, AB), he concludes that this is the critical case, and that if it gives stable equilibrium, the ship will be stable with respect to all other axes.

Acting on this conclusion, Euler considers this critical case with a view to providing the requisite stability. His methods of approximation are continuations of those previously described, the same coefficients of fineness being used, and his practical rule for moments of inertia being applied. With these assumptions he reduces his fundamental expression for stability to the form—

$$\mathrm{M}\left\{\frac{a}{12\,\beta}\,\frac{\overline{\mathrm{CD}}^2}{\overline{\mathrm{IE}}} - \frac{\beta}{1 \times \beta}\,\overline{\mathrm{IE}} - \overline{\mathrm{FG}}\right\}$$

and for stable equilibrium gets the condition—

$$\overline{\mathrm{CD}}^2 > m\,.\,\overline{\mathrm{IE}}^2 + n\,.\,\overline{\mathrm{IE}} \times \overline{\mathrm{FG}},$$

$$\text{where } m = \frac{12\,\beta^2}{a\,(1 \times \beta)}\,;\; n = \frac{12\,\beta}{a}.$$

Illustrations follow of the application of this condition, with *assumed* values for α, β and F G. No interest attaches to them now, but the evident intention on Euler's part was to fix some proportion between the breadth, C D, and draught, I E.

Chapman's famous *Treatise on Shipbuilding* was published in Sweden in 1775, and possessed the merit of being much less speculative than most of the works that had been previously published. It added but little, however, to the doctrines of stability. His method of determining the height of the metacentre was essentially the same as Bouguer's; but in the numerical example which he gave, he improved upon Bouguer's process by making use of Simpson's ordinary rule, in lieu of the trapezoidal, for integrating the cubes of the half-breadths of the load water-line section. In a later work Chapman adopted and detailed a method of finding the vertical position of the centre of gravity by arranging the weights into two divisions, having their centres of gravity respectively above the centre of buoyancy, and coincident with or below it. This is followed by an investigation (resembling Bouguer's) of the effects produced upon the height of the metacentre by changes in the under-water form and displacement of a ship. Into this discussion of rolling motions, and of the rise and fall of the centre of gravity during rolling, we need not enter.*

In his notable work, *L'Art de la Marine*, published by M. Romme in 1787, a chapter is devoted to the stability and rolling of ships. Romme lays down correctly the conditions of equilibrium for a floating body, and his investigation of an expression for the stability is very like Bouguer's in principle, and in being limited to very small angles of inclination, but no use is made of the metacentre. Romme's expression for the stability takes the form—

* Chapman's work was translated into French by M. Vial de Clairbois, who was *Chef des Constructions Navales et Directeur d' études de l'Ecole d' application du Génie Maritime*, and who somewhat later published, himself, an *Elementary Treatise on the Construction of Ships*, which seems to have been designed as a text-book for general use. It does not add materially to the other existing knowledge of the subject. Chapman was also translated into English in 1820 by Dr. Inman, then at the head of the School of Naval Architecture, who appended many useful notes and comments, with which we have not this place to deal.

$$\frac{1}{12}\int l^3 \,.\, \sin.\,\theta \,.\, d\,\mathrm{E} - 2\int (e\,c\,m)\ \mathrm{G\,P}\sin.\,\theta\,d\,\mathrm{E},^*$$

or, as it might be better arranged in modern notation,

$$\text{Stability} = \left\{\begin{array}{l}\text{Moment of inertia of load-water section}\\ -\text{Volume of displacement} \times \overline{\mathrm{B\,G}}\end{array}\right\}\sin.\,\theta.$$

It will be seen, therefore, that Romme's final result agrees with Bouguer's, only Romme prefers to keep separate the two terms which depend respectively upon the moment of inertia of the load-water section and the distance (B G, as we should say) between the centres of buoyancy and gravity.

Assuming that, as a rule, the centre of gravity of a ship lies above the centre of buoyancy, Romme arrives at the following practical deductions:—"To ensure stability to a vessel, the transverse vertical sections must be made upright in the neighbourhood of the water-line on the midship section; this should be done from two or three feet above water to an equal distance below: for sections nearer the extremities the upright part need not be so long as amidships, but may be reduced in the ratio of the respective breadths of the sections at the water-line." He again and again, in subsequent remarks, insists upon the advantages to be gained from following this course.

Turning to the first term in his expression, he shows that a rectangle would be the form of water-section giving a maximum value to the moment of inertia; but adds that such a "form is inadmissible, seeing that it would be detrimental to other good qualities of the ship." As regards the second term, he advocates a reduction, so far as possible, in the distance between the centres of gravity and buoyancy, in order to increase the stability. He expresses the opinion that, in ships of war especially, little can be done towards *lowering* the centre of gravity, and, therefore recommends that suitable modifications should be made in the form of the ship, filling her as much as possible at the water-line, and fining her below in order to raise the centre of buoyancy, and at the same time to increase the moment of inertia of the water-section.

* In this formula l = breadth of cross-section at L . W . L.
$(e\ c\ m)$ = half-area of immersed cross-section.
G P = distance between centres of gravity and buoyancy for that cross-section.
d E = element of length.
θ = angle of inclination.

Throughout his treatise Romme illustrates the points he raises by reference to the cases of actual ships; and here he cites the case of a 74-gun vessel named the *Scipion*, which proved very crank when completed in 1779. At first, attempts were made to remedy the error by altering the stowage, and putting in additional ballast instead of a part of the water supply originally carried; but these failing, thick doubling planks were worked in the neighbourhood of the water-line, and with more satisfactory results. Romme goes very fully into this case, and contrasts the vessel with other more successful ships. The only noteworthy passage occurring in this connection is, however, that in which he suggests that the vertical position of the centre of gravity may be found by experiment. Supposing the ship to be heeled under sail to a certain angle, and then to be further heeled by shifting known weights—such as guns—through known distances, from the windward to the leeward side, he shows how the *difference* in the stabilities at the two inclinations may be made to determine the position of the centre of gravity in relation to the centre of buoyancy, and thence in relation to the water-line.

Up to this point Romme considers only *transverse* inclinations, but he remarks in passing, that his method of investigation is just as applicable to inclinations which take place about any other axis besides a longitudinal one. He also declines to enter into the discussion of the effect produced upon the stability of a ship by her progression through the water, expressing the opinion that it would be "very little different from, or superior to, that of the same ship floating at rest in still water," and sums up as follows:—

"The general principles by which the stability of a sea-going ship may be assured, leave undetermined the form of the greater part of the displacement, preserving only a large area for the plane of flotation, and for the water-lines lying near to it, and a smaller area for the lower water-lines. The contours of the displacement consequently remain unfixed, and may be determined in accordance with the particular conditions which secure the other qualities that are desirable in sea-going ships."

CHAPTER XII.

Dupin's Investigations; their Generality—Positions of Equilibrium—Surface of Buoyancy—Surface of Flotation—Geometrical Nature of these Surfaces—Surface of Buoyancy a Closed Surface—Tangent Planes to Surface of Buoyancy—Normals to Surface of Buoyancy Correspond to Positions of Equilibrium—The Three Kinds of Equilibrium—Direction of Greatest and Least Curvature—*Indicatrix* Curve of Surface of Buoyancy—Planes of Symmetry—Relation of Surface of Buoyancy to Surface of Flotation—Greatest and Least Radius of Curvature—General Theorems—Value of Radius of Curvature—Possible Positions of Equilibrium of Floating Body—Positions of Equilibrium Round a Fixed Axis—Resulting Theorems—Mixed Equilibrium—Absolute Number of Positions of Equilibrium when Axis is not Fixed—Absolute and Relative Stability—Intermediate Positions of Stability—M. Leclert's Formulæ.

It has been already intimated that the eminent French investigator, Dupin, took a very broad and general view of the question, and his researches brought him lasting fame, both as a geometer and as a participator in the higher developments of the theory of naval architecture.* Dupin was both a very able and a very eloquent writer, and his productions were received with unwonted approval and admiration by some of the most distinguished men of science of his day. It is not possible to do justice either to our subject or to him without devoting considerable attention to the results of his labours, although this book will probably have many readers by whom a mastery of all that follows in this chapter need not be regarded as essential.

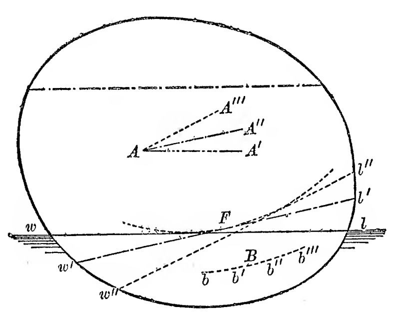
It will be seen that Dupin sought to give the utmost generality to his work when we say that he started his inquiries from the centre of the earth as a fixed point, and proceeded to show the value of a vertical axis, and of a horizontal plane perpendicular thereto, to geometers and mechanics. We may here, however, pass over his remarks upon centres of gravity and the equilibrium of fluids, and take note at once of the fact that, for the purpose of

* Charles Dupin, it appears, was an "ingénieur de la marine" when in 1814 he submitted his "Memoir on the Stability of Floating Bodies" to the Institution of France, and on the title-page of the volume in which it appeared, with other "Memoirs," in 1822, he is described as an "*Officier Supérieur au Corps du Génie Maritime.*"

developing his process of investigation, he assumed, for the time being, as he clearly was entitled to do, that the centre of gravity of a floating body may be shifted about without thereby changing the *weight* of the body; and in connection with this assumption he shows, what we need only state, viz., that no merely *vertical* movement of the centre of gravity, either up or down, can either produce equilibrium, if it does not exist, or take it away, if it does exist. It is equally obvious that if a body be floating in equilibrium, no mere movement of the body up or down can bring it into a new position of equilibrium, because it cannot thus be brought into any other position in which the weight and the buoyancy will be equal. From this reasoning it follows that in the case of a partially-immersed floating body, say Fig. 147, by suitably placing its centre of gravity, it is possible to find a position of equilibrium in which the plane of flotation, $w\,l$, $w'\,l'$, or $w''\,l''$, for example, shall be parallel to any plane, $A\,A'$, $A\,A''$, $A\,A'''$, given in position in the solid. This proposition is equally true if the body be turned upside down. It further follows that if we suppose the centre of gravity of the solid, instead of remaining in the same vertical, to be moved into another straight line, which we then regard as a vertical, a position of equilibrium can be found for the body to suit these new conditions when floating, both in the new upright position and upside down. The new planes of flotation will be perpendicular, not to the original vertical, but to the new vertical.

Let us conceive, says Dupin (whom we render freely and briefly)* that the divers planes of flotation, $w\,l$, $w'\,l'$, $w''\,l''$, &c. (Fig. 147), have been determined respectively parallel to all the planes, $A\,A'$, $A\,A''$, $A\,A'''$, &c., that can be drawn from the point, A, of the solid. To each new plane of flotation thus determined will correspond a centre of buoyancy, b or b' or b'', &c. All these centres of buoyancy will form in the aggregate a surface, called the *surface of centres of buoyancy*, B, or more simply, the *surface of buoyancy*.† All the planes of flotation envelope or form

Fig.147.



* We also modify the lettering of the diagrams as seems convenient.

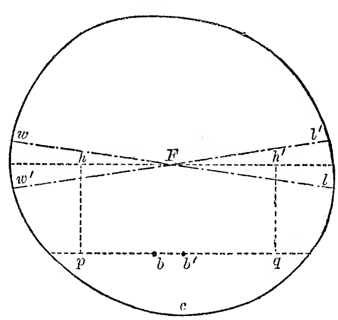
† This is a usual abbreviation, not Dupin's.

by their lines of intersection another surface, F, similarly determined, called the *surface envelope of flotations*, or more simply, *surface of flotation.* These two surfaces (of buoyancy and flotation) possess remarkable properties in relation to the equilibrium of floating bodies.

The planes of flotation are determined, as we have seen, by the single condition that they each cut off from the solid a segment of given and constant volume; each centre of buoyancy is merely the *centre of the mean distances* of the geometric volume of such a segment. Dupin says, therefore, that the "surface of buoyancy" and the "surface of flotation" have a definition purely geometrical, and "altogether independent of the gravity of the floating body."* The foregoing property, which is based on geometrical considerations, serves as the basis of all Dupin's subsequent work.

Adverting for a moment to the question of centres of gravity, and more especially to the fact that the centre of gravity of two masses is always situated between them, he points out that, from an extension of this consideration, it follows that the centre of gravity of any continuous volume whatever must always lie within the limit of the volume itself. This is not, however, strictly correct; many examples to the contrary will readily occur to the reader; for example, a rod much bent, or a curved frame timber of a ship, or a twin-vessel may have its centre of gravity altogether outside of it; but the limits of the author's meaning will be seen without difficulty, and are substantially supplied by himself in a foot-note, and in a more lengthy note appended to his treatise. He goes on to say that, for the reason stated, the surface of buoyancy, B (Fig. 147), necessarily has all its points within the interior of the floating body. Therefore, in bodies of finite extent (such as ships) the surface of buoyancy must always itself be of finite extent, and usually *closed* in all its parts.

Fig.148.



Let w F l, w' F l', Fig. 148, be two planes of flotation indefinitely

* This last phrase must be taken with due limitation, because the gravity of the body obviously determines the magnitude of the volume to be cut off by the plane of flotation. What Dupin intended was correct, as he clearly had in his mind not the *amount, but the disposition of gravity.*

near each other. In order that the buoyancy may be the same with each, it is necessary that the two segments, w F w', l F l', by means of which alone they could differ, should be equal, the part below w' F l being common to both cases. Let h, h' be the centres of gravity of the equal volumes, w F w', l F l' respectively, and draw through the centre of buoyancy, b, of the volume $w\,c\,l$, the straight line, $p\,b\,q$, parallel to $h\,h'$. The distance from the centre of buoyancy of $w'\,c\,l'$, to the line $p\,b\,q$, will evidently be proportional to the difference of the moments of w F w' and l F l' about that line. Now, this difference is nothing, since w F w' is equal to l F l', and $h\,p$ is equal to $h'\,q$. Therefore, the centre of buoyancy, b' of $w'\,c\,l'$, is situated in the line $p\,b\,q$, which is parallel to $h\,h'$. The angle of inclination being infinitely small, and the line $h\,h'$ being nevertheless comprised within that angle, it will follow that its parallel, $p\,b\,b'\,q$, can depart but infinitely little from parallelism to the plane of flotation, w F l. Therefore, the tangents through b to the surface of buoyancy are parallel to the corresponding plane of flotation. Hence follows the general property that: If through one of its points b, considered as an individual centre of buoyancy, we draw a plane parallel to the plane of flotation, which limits the volume of buoyancy, this will be the tangent plane to the surface of buoyancy, B, at the point b.

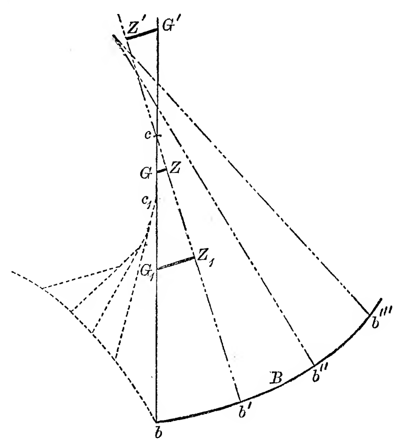
Hence, also, for each position of equilibrium, the tangent plane to the surface of buoyancy through the corresponding centre of buoyancy is *horizontal*, because the plane of flotation, or fluid surface to which it is parallel, is necessarily horizontal.

The surface of buoyancy also possesses the property of having for *normals* the straight lines drawn through each of the centres of buoyancy of which it is formed, and through that position of the centre of gravity of the floating body which, when the body is in equilibrium, corresponds to the centre of buoyancy. The condition of equilibrium is, that the centres of gravity and buoyancy shall be in the same vertical; and when the tangent plane of a surface is horizontal, the corresponding normals must be vertical.

Since the straight lines which join corresponding centres of buoyancy and gravity, in positions of equilibrium, are normals to the surface of buoyancy, B, all the properties which belong to normals of surfaces belong equally to these straight lines. Therefore, these normals taken together present two distinct systems of developable surfaces; the developable surfaces of one system are cut at right angles by all the developables of the other system; each of these cut the surface B through one of its lines of curvature, &c. It is, says Dupin, from these developable surfaces, and the curves and centres of curvature, which correspond to them, that we learn the general conditions of the stability of floating bodies.

Next, suppose that a floating body, placed first in one of its positions of equilibrium, is very slightly disturbed, without any change in the position of its centre of gravity. Suppose, further, for greater facility, that the weight of the body, and the immersed volume also remain unchanged during this very slight disturbance. We have to see whether the equilibrium still subsists, or tends to re-establish itself, or tends, so to speak, to further disturbance. Let b, Fig. 149, be the centre of buoyancy corresponding to the given position of equilibrium of which the vertical is $b\,G$, G being the centre of gravity. In consequence of the slight disturbance, the centre of buoyancy finds itself now at b', immediately consecutive to b, on the surface of buoyancy, and the new vertical, $b'\,c$, like the former, is normal to the surface of buoyancy, B. Further, suppose for facility's sake, that the plane of projection of Fig. 149 is parallel to the two consecutive normals, $b\,c$ and $b'\,c$. Then the shortest distance between these two consecutive normals will have for its vertical projection the single point, c, the intersection of the two projections. It is evident that $c\,b = c\,b'$, since c is on the normal, $b\,c$, the centre of curvature of the element, $b\,b'$. This point, therefore, was neither raised nor lowered by the inclination, and the movement of the floating body was the same as if the shortest distance between $c\,b$ and $c\,b'$ had been a fixed axis round which the body had been infinitesimally turned. The whole system may be supposed to receive any vertical motion of translation, either downward or upward.

Fig. 149.



It will now be seen: (1.) that if the centre of gravity is at G, below the centre of curvature c, the weight will operate to bring back the normal $c\,b$ to its primitive vertical position; (2.) that if the centre of gravity is at G′, above the centre of curvature of the element, $b\,b'$, it will operate to carry the normal, $c\,b$, still farther away from its primitive position; and (3.) if the centre of gravity be at c, the centre of curvature of $b\,b'$, it will have no perturbing effect at all, and the body will not be solicited to turn either way by any force. We have here respectively the three cases of stable, unstable, and indifferent equilibrium. It is evident that in the first case the centre of gravity, G, is nearer the corresponding centre of buoyancy, b, than it is to any other centre of buoyancy, b' or b'', &c., on the line $b\,b'\,b''\,b'''$, &c. In the second case, the centre of gravity, G′, is farther from b than from any other of those centres of buoyancy (supposed always exceedingly near to b). In the last case (3) the distances, $c\,b$, $c\,b'$ are equal. Hence, Dupin deduces the following theorem:—In comparing a position of equilibrium of a floating body with positions very near thereto, the distance of its centre of gravity from its centre of buoyancy is a minimum or maximum, according as the equilibrium is stable or unstable; in indifferent equilibrium this distance is constant, or more correctly, the difference of the distances is of an order infinitely less than in the case of the maximum or minimum.

The action of the weight of the body concentrated in G′, or in G_1 is evidently proportional to the distance, G′ Z′ or $G_1\,Z_1$, of G′ or G_1 from the axis fixed at c. Therefore the more the centre of gravity is raised the more is the stability diminished; on the contrary, the more the centre of curvature, c, is raised, the more is the stability increased.

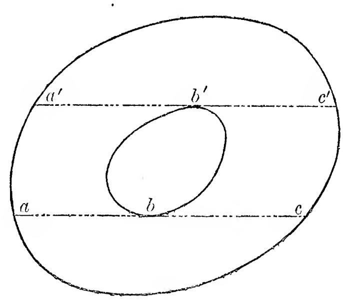
If the different centres which, on the same normal, $b\,c$, belong to the normal sections of the surface of buoyancy, B, are considered, it will be seen that they must all lie between two centres,* c and c_1, the one of the least and the other of greatest curvature of the *surface* B. The line which we have regarded as a horizontal axis of moments (and which is defined by the condition of its being perpendicular to $c\,b$ and $c\,b'$ passing through each) appertains to the position of maximum stability when it passes through the centre, c, of least curvature of B. On the contrary, when it passes through c_1, the centre of greatest curvature of B, it will appertain to the position of minimum stability. Therefore, when the stability is greatest, the

* The centre, c, of least curvature is very important for the determination of the stability of ships, says Dupin in a foot-note. It is the point that Bouguer has named the *metacentre*.

axis of rotation is parallel to the direction of greatest curvature of the surface of buoyancy; and when the stability is least, that axis is parallel to the direction of least curvature of that surface.

We have previously seen Dupin regard the surface of buoyancy as *closed,* and incapable of having more than two tangent planes, $a\,b\,c$, $a'\,b'\,c'$, Fig. 150, parallel to each other; or rather, of having more than one other tangent plane parallel to any given tangent plane, for that is what is meant. These properties require that the two curvatures of the surface (meeting at the point of contact with a tangent plane) shall always be directed towards the same side, *i.e.*, both diverge from their common tangent plane. The proposition is correct, but Dupin's attempted proof of it is unsatisfactory.

Fig.150.



The two curvatures of the surface of buoyancy are nevertheless on the same side, and in surfaces possessing this property, the centres of mean curvature are always situated on the normal between the two centres of greatest and least curvature. It follows that if the centre of gravity, G (Fig. 149) of the floating body finds itself below the centre of greatest curvature, c_1, the stability, for the position of equilibrium which is under consideration, will take place in all possible directions (*i.e.*, in whatever direction the body may be inclined); and this stability Dupin, therefore, designates *absolute.* If the centre of gravity is at G, above the centre, c_1, of greatest curvature, but remains below the centre, c, of least curvature, the equilibrium will be stable in the direction of least curvature (*i.e.*, stable on taking for its axis of rotation the horizontal axis perpendicular to that direction), but it will no longer be stable in the direction of greatest curvature. If, then, it is desired to determine which, among all the intermediate sections, are those that separate the directions in which there is stability from those in which there is not, it is necessary to regard the centre of gravity, G, as the centre of curvature of a normal section through b of the surface of buoyancy, and to find the position of this section. Dupin shows how this may be done by means of the *indicatrix* curve of the surface of buoyancy, as set forth in his *Développements de Géométrie, Mémoire Premier.*

It is not necessary to pursue the matter further here.* Finally, if the centre of gravity of the body finds itself at G′, Fig. 149, above the centre, *c*, of least curvature, the equilibrium will not be stable in any direction. In all cases for each (very small) inclination of the floating body supposed to be turning round an axis parallel to the directions of greatest or least stability, there will correspond a second inclination symmetrically situated in relation to those directions, as well as to the centres of gravity and buoyancy. The degree of stability will be the same for both. Whence is inferred the following theorem:—"In the equilibrium of a body floating on a fluid, and bounded by any surface whatever (regular or irregular) the stability, considered in the divers directions in which it may be disturbed, is always symmetrical with respect to two vertical planes normal to each other."

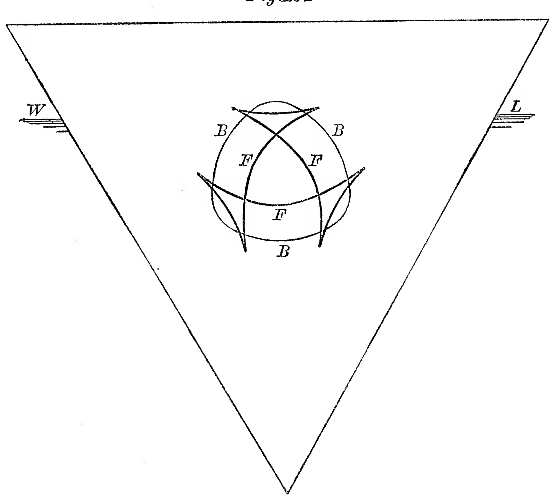
These planes of symmetry (passing, of course, through the centres of both gravity and buoyancy) are identical in direction with the two principal curvatures of the surface of buoyancy. In the case of ships, formed symmetrically with respect to a vertical longitudinal plane, this plane is one of the planes of symmetry, and the other is normal to it. In bodies which are not thus symmetrical, and in which the planes of symmetry are not therefore indicated *a priori*, it is necessary to determine the principal curvatures through the centre of buoyancy, corresponding to the given position of equilibrium. For this purpose it is necessary to take the planes of flotation, &c., into consideration.

The "Surface of Flotation," as we have seen, is the envelope of all possible planes of flotation which can be obtained by placing a body, without change of total weight, or of exterior form, into all conceivable positions of equilibrium. This surface of flotation, like the surface of buoyancy, is, says Dupin, closed in all its parts, and

* "The investigation of the amount of stability when the vessel is displaced round an axis intermediate between these (axes of greatest and least stability) is very elegant, and is made to depend on the properties of the indicatrix, which is the section of the locus of centres of buoyancy made by a plane parallel to the plane touching this surface in the inclined position, *i.e.*, parallel to the inclined water-line, and indefinitely near it. This curve is always an ellipse, whose greater and lesser axes are parallel to the directions of greatest and least stability. The stabilities in different directions are proportional to the squares of the corresponding diameters of the indicatrix, and these are proportional to the corresponding radii of curvature of the surface."—From Dr. Joseph Woolley's Paper "On the Mathematical Theory of Naval Architecture," *Transactions I. N. A.* 1860.

the demonstration before employed in connection with the latter surface, shows, he considers, that this also has two principal curvatures always directed towards the same side. This, however, is inaccurate. He next attempted to demonstrate that the surface of buoyancy and the surface of flotation of a floating body can never penetrate each other, but must always wholly surround each other, either after the manner shown in section only in our Figs. 2 and 3, chap. i., where the cylindrical portion of the surface of flotation, F F′, wholly embraces the surface of buoyancy, B B′, or after the manner similarly shown in Fig. 4, where the reverse is the case. Dupin, in a note, more conveniently employs a sphere for illustrating this principle, but obviously our Figs. 2, 3, and 4, chap. i., will serve perfectly well for that purpose, representing the sectional state of the sphere in every conceivable position. The proposition itself, however, is not correct, and his attempted demonstration is, in our

Fig. 151.



opinion, altogether invalid, as can be made to appear at once by the simple process of reversing the figure which he employs in his demonstration. In the interesting Paper on Stability,* previously adverted to, Messrs. White and John have adduced repeated cases in which the surfaces of buoyancy and of flotation penetrate each other.

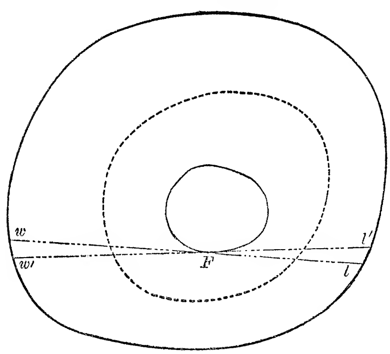
* "On the Calculation of the Stability of Ships, and some Matters of Interest connected therewith."—*Transactions of Institution of Naval Architects*, vol. xii., for 1873.

One of them is shown in Fig. 151, which is that of a prism of triangular and equilateral section, immersed as indicated. The curve of flotation, F F, has six cusps, and intersects the curve of buoyancy, B B, at 12 points, each curve being made up of six branches of hyperbolas. The authors of the Paper here referred to have also pointed out that Dupin does not appear to have contemplated the possibility of cusps in the curve of flotation resulting from the immersion of the edges of the decks of ships, and add, "For ships, the occurrence of cusps in the curve of flotation may, however, be regarded as the rule rather than as the exception; and for other bodies of various forms the same thing is possible."

It follows from what has just been said and shown that the proposition of Dupin, which affirms that the two principal curvatures of the surface of flotation always lie on the same side, cannot be sustained. The existence of cusps in sections of the surface of flotation is incompatible with this condition.

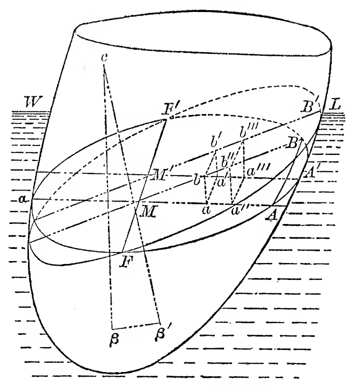
Dupin next considers the question of the point of contact of the plane of flotation with the surface of flotation; and, supposing two planes of flotation, *w* F *l*, *w′* F *l′*, Fig. 152, indefinitely near each other, shows that, in accordance with the geometrical theory of the envelopes of surfaces, the line of contact of the plane, *w* F *l*, with the surface envelope is necessarily identical with the intersection of the two consecutive tangent planes, this line of contact being represented in projection by the point, F, the tangent planes being represented (in the plane of the figure) by their traces, *w* F *l*, *w′* F *l′*. He goes on to show that the volumes of the infinitesimally small wedges, *w* F *w′*, *l* F *l′*, must equal each other, and thence (as did Bouguer long before) deduces the property that "the intersection of two planes of flotation, infinitely near, always passes through the centre of gravity of the area of the plane of flotation of the body;" and hence infers the following definition, viz., "The Surface of Flotation is the Locus of the Centres of Gravity of the Areas of all the Planes of Flotation," these areas being, of course, bounded on all sides by the exterior surface of the body.

Fig. 152.



Having first shown that, taking Fig. 153 to represent a solid floating body, and supposing F A A′ F′, F B B′ F′, two water-line planes, infinitely near each other, the volume of the very small wedge, F A F′ B, divided by the tangent of A M B, is equal to the sum of the moments of the area, F A F′, about the axis, F F′, Dupin proceeds as follows:—If now we multiply the area, $a\,a'\,a''\,a''' = a\,a' \times a\,a''$ by $\frac{1}{4}$ (Ma + Ma'')^2, we shall have the moment of that element about the axis, F F′. The sum of all such moments, extended over the whole area, F A F′, will be the moment of inertia of that area. But the volume of the little solid, $a\,b'''$, being

Fig. 153.



$$\tfrac{1}{2}\left\{a\,a' \times a\,a''\left(\mathrm{M}a + \mathrm{M}a''\right) \tan.\ \mathrm{A\,M\,B}\right\},$$

its moment about the same axis will be

$$\tfrac{1}{4}\,a\,a' \times a\,a''\left(\mathrm{M}a + \mathrm{M}a''\right)^2 \tan.\ \mathrm{A\,M\,B},$$

whence results the important theorem:—

The moment of the wedge, F A F′ B (Fig. 153), divided by the tangent of the angle, A M B, is equal to the moment of inertia of the area, F A F′, F F′ being the axis of the moments.

Dupin shows later on that the moments of the two infinitesimal wedges on opposite sides of F F′ divided by tan. A M B, and by the displacement, are together equal to the radius of curvature of the arc, $\beta\,\beta'$, of the curve of buoyancy (Fig. 153), when the plane of flotation ceases to be F A A′ F′, and becomes F B B′ F′. Therefore, the radius of curvature of $\beta\,\beta'$ is equal to the moment of inertia of the total area, F A F′ a (about F F′), divided by the constant volume of displacement. (This is a more general form of the expression for the height of the metacentre given by Bouguer, who anticipated substantially the steps of Dupin's demonstration.) As the radius of curvature and the sum of the moments of inertia bear a constant relation, they will have at the same time maximum and minimum values.

These results are summed up in the following *théorèmes remarquables*:—

I. The greatest radius of curvature of the surface of buoyancy at a given point is equal to the greatest moment of inertia of the area of the corresponding plane of flotation, divided by the volume of displacement.

II. The least radius is, on the contrary, equal to the least of these moments of inertia, divided by the displacement.*

III. The direction of greatest curvature of the surface of buoyancy is that of the axis of the greatest moment of inertia of the area of the plane of flotation.

IV. The direction of least curvature of that surface is that of the axis of the least moment of inertia of the area of the plane of flotation.

Since the lines of greatest and least curvature of any surface whatever always cross each other at right angles, the principal axes of the greatest and least moments of inertia of the area of the plane of flotation are always at right angles to each other, whatever may be the form of that area. We thus come, says Dupin, by another route, to the admirable properties which Euler discovered relative to the principal axes of bodies.

After demonstrating what he just before virtually assumed (as we saw) that the small shift of the centre of buoyancy from β to β', in Fig. 153, is equal to the sum of the moments of the small wedges divided by the volume of displacement, Dupin points out that the vertical, βc, does not pass, except in very special cases, through the intersection of consecutive planes of flotation, and says that this does not affect the generality of his proof. We can always pass, he says, and shows, from the general case in which the plane of the moments does not contain the vertical, βc, to the particular case in which it does contain it; therefore, under all hypotheses, the expression given for the radius of curvature, βc, is correct. He adds that we can at will displace horizontally the area of the plane of flotation, so long as the centres of gravity and of buoyancy remain at the same height, without varying either of the radii of curvature of the surface of buoyancy for the given position of equilibrium.

* The above propositions may be conveniently expressed by the equations—

$$r = \frac{i}{V} = \frac{2}{3}\,\frac{\int y^3\,dx}{V}\,;\ \text{and, } R = \frac{I}{V}\,;$$

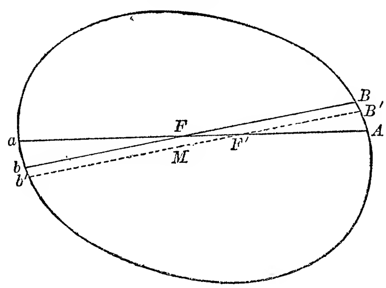
where r and R represent the principal radii of curvature, and i and I the principal moments of inertia of the plane of flotation.

Dupin next enters upon an argument which has for its object to show that no changes which it would be possible to make in the figure of a floating body would at all alter its stability, all the while they caused no variation in the distance of the centre of buoyancy from the centre of gravity, nor in the curvature of the surface of buoyancy about the given centre of buoyancy, the weight of the body remaining constant. In pursuing this course of reasoning he requires to make use of, and therefore he demonstrates, the general property of projections of centres of gravity, afterwards applying it, with much skill and ingenuity, to the purpose stated. As the inquiry is almost wholly a geometrical exercise, and as the proposition to be demonstrated admits of readier proof, it is unnecessary to reproduce the investigation here.

The next section of the *Mémoire* has to do with lines and radii of curvature of the surface of flotation. He first lays down the preliminary theorem, that, of all the tangent planes of a surface which make the same infinitely small angle with the tangent plane at a given point, the plane which lies most distant from that point appertains to the line of least curvature, and the plane which lies nearest belongs to the line of greatest curvature, observing in a foot-note that, if we suppose these two lines of curvature to revolve about the normal to the surface, at the given point, of the two lines of least and greatest curvature, the former will envelope all the surface about the point, and the latter will be constantly enveloped by it. Dupin goes on to illustrate the principle by reference to sections of curvature of bodies of cylindrical form, and proceeds to show generally for infinitesimal angles and for irregular bodies (that which he had previously shown for finite angles of inclination, viz.), that (unless of cylindrical or other like section), a floating body, when inclined from a position of equilibrium, has usually to seek a new plane of flotation, separated from the former by a small interval, in order to equalise the wedges of immersion and emersion. The distance between this plane of flotation and a parallel plane through the centre of gravity of the original plane of flotation is determined (as we have already seen) by the obvious fact that this distance multiplied by the area of the plane of flotation will give the small volume of the layer lying between the two parallel planes; and (as the whole immersed volume must remain unaltered) the volume of the layer must be equal to the difference between the volumes of the wedges of immersion and emersion, supposing the second plane of flotation to pass through the centre of gravity of the first.

As has been already said, of all the planes of flotation making an infinitesimal angle with the surface of flotation about F, Fig. 154, that which makes the distance, F F′, a *maximum*, is a tangent to the line of least curvature, and that which makes it a *minimum* is a tangent to the line of greatest curvature. Therefore, in order to find at any point, F, the directions of the lines of least and greatest curvature of the surface of flotation, it is sufficient to determine for which plane of flotation the thickness, F M, consequently the volume of the layer, b B B′ b', and consequently also the difference of the wedges, a F b, A F B is a *minimum* or a *maximum*.

Fig.154.



If the body be a cylinder floating with its axis vertical, the sides being everywhere parallel, the wedges of immersion and emersion will always be equal without change of the point, F. Consequently, if we regard such irregular figures as are shown at Figs. 155 and 156, in order that the difference of their two segments

Fig.155.

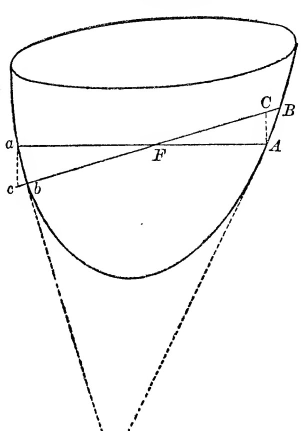
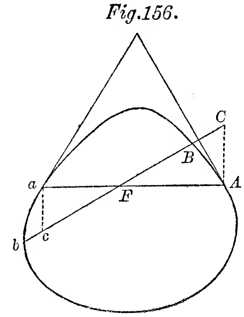


Fig.156.



a F b, A F B, may be a *maximum* or *minimum*, it is sufficient to make a *maximum* or a *minimum* of the total volumes of the triangular parts, a b c, A B C comprised between the two planes of flotation infinitesimally separated from each other, and the vertical cylinder having for its base the area of the plane of flotation, and the sides, $a\,b$, $A\,B$ of the floating body. To determine the volume of these parts, extending throughout the length of the water-line plane—for example, that of $a\,b\,c$—as the angle $a\,c\,b$ differs but extremely little from a right angle, let us call T the ratio $\frac{b\,c}{a\,c}$ the tangent of angle $b\,a\,c$. Area of $a\,b\,c = \frac{b\,c \times a\,c}{2}$, and therefore $= T \times \frac{a\,c^2}{2}$. Designating by $d\,w$ the tangent of the infinitely small angle made by the planes, $a\,F\,A$ and $b\,F\,B$, we have, $a\,c = F\,a \times d\,w$. Therefore,

$$\text{area of triangle } a\,b\,c = \frac{(d\,w)^2\,(F\,a)^2\,T}{2}.$$

If we multiply this product by $d\,E$, the thickness of an infinitesimal volume compressed between two vertical planes parallel to a given plane, we shall have,

$$\frac{(d\,w)^2\,(F\,a)^2}{2}\,T \times d\,E,$$

for the volume of this element of the solid. The sum of such elements will be the volume of the whole solid.

In order to give this expression a form more convenient for our subsequent use, he supposes the triangular element to be divided into parts of constant thickness infinitesimally small, by planes normal to the contour of the water-plane, and puts θ for T, and $d\,s$ for an element of the contour of the plane of flotation, then the sum of the elements $\frac{(d\,w)^2\,(F\,a)^2\,\theta}{2} \times d\,s$, will be the total volume required. Whence results, says our author, the remarkable theorem :—

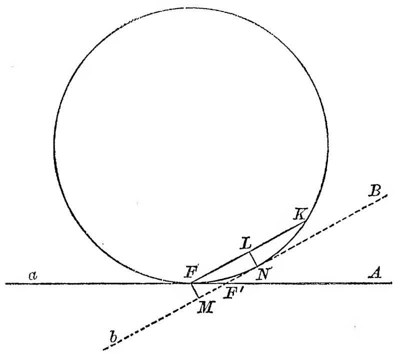
"If we apply to each point of the contour of the plane of flotation, a weight proportional to the tangent, θ, of the angle which the vertical forms, at this point, with the surface of the floating body, a heavy continuous line will be produced; and the principal axes of the greatest and least moments of inertia of this line will be respectively parallel to the lines of least and of greatest curvature of the surface of flotation."

It is evident, he says, that we shall have respectively for those two axes, represented successively in projection by the point F,

$$\int \frac{(F\,a)^2}{2}\,d\,s \times \text{the constant } (d\,w)^2 = \text{a } \textit{maximum} \text{ or a } \textit{minimum}.$$

Dupin next finds the lengths of the radii of curvature of the surface of flotation at the point F. He takes a small arc, F N K, Fig. 157, of the osculatory circle coinciding with the radius of curvature under consideration, of which arc L N is the versed sine; the tangent, $F' N = F' F$, will differ from one-fourth of the chord, F K, only by an infinitesimal quantity of the second order. Therefore, the radius of the circle, $\frac{F L^2}{2 L N}$ is equal to $\frac{2 F F'^2}{F M}$. This is the value of the radius sought. To render it explicit, it should be observed that $d w$, being the tangent of the angle formed by a A and b B (Figs. 154 to 157), we have $d w = \frac{F M}{F' M} = \frac{F M}{F F'}$ by neglecting the indefinitely small quantities of the second order. Therefore, $\frac{F F'}{F M} = \frac{1}{d w}$, and $\frac{2 F F'^2}{F M} = \frac{2 F M}{(d w)^2}$. But the layer, F b' b B B$'$, of which the volume is expressed by the primitive area of flotation, say A, multiplied by the thickness, F M, is equal in volume to the triangular element, of which the volume is the sum of the elements, $\frac{(d w)^2}{2} \int \theta \left(F a\right)^2 . d s$. Equating this with A $\times$ F M, we may thence obtain—

Fig. 157.



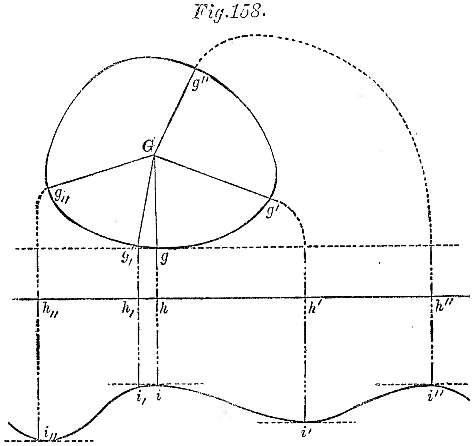
$$\frac{2 F M}{(d w)^2} = \int \frac{\theta (F a)^2 . d s}{A},$$

which last is therefore the value of the radius of curvature of the surface of flotation. But $\int \theta \left(F a\right)^2 . d s$ is the moment of inertia of the contour of the plane of flotation, supposing each element, $d s$, thereof charged with a weight, as we just now saw. It follows that if we divide the greatest or the least moment of inertia of the contour of the plane of flotation, weighted proportionally to the tangent, θ, by the area of the plane of flotation, the quotient obtained will be the radius of the least or greatest curvature of the surface of flotation (multiplied by the unit of weight).

The next subject which engages Dupin's attention is that of the divers positions of equilibrium which a given body can take up in floating upon a fluid. Hitherto he has considered the centre of gravity to be movable at will; he now regards it as occupying a fixed position. Recalling what has been previously demonstrated with respect to the surface of buoyancy, and introducing the condition of a constant position for the centre of gravity, he lays down as a first theorem this: The total number of positions of equilibrium which a floating body of invariable form can take up is equal to the number of normals to the surface of buoyancy, which can be drawn from the centre of gravity. In determining how many of these there can be, he supposes the body to be turned round any *fixed* axis, horizontal or not, but passing through the centre of gravity. For each position of the body there will be a corresponding centre of buoyancy, and the *ensemble* of these will form a closed curve of buoyancy. When a position of equilibrium is arrived at, the tangent to the curve will always be horizontal. (But the normal to this curve, drawn through the centre of gravity, may not now be an absolute vertical, because of the axis being fixed.)

Recalling the proposition before demonstrated—that the equilibrium is stable when the distance of the centre of gravity from the centre of buoyancy is a *minimum*, and unstable when that distance is a *maximum*—he takes the Fig. 158 and infers that the total

Fig.158.



number of positions of equilibrium will be equal to the number of normals which can be drawn from the centre of gravity, G, to the curve of buoyancy, $g\,g'\,g''$, &c., and the positions of equilibrium (stable or unstable) are respectively those in which the length of the normals is a *maximum* or a *minimum*. He proceeds to prove that the number of such normals which can be drawn from any point, G, whatever is generally an *even* number.

And first he shows that the *maxima* and *minima* normals, Gg, Gg', Gg'', &c., must be successively less and greater than the corresponding radii of curvature. He supposes that through G straight lines are drawn to $g_{,}$, g, g', g'', . . . &c., and that these are set down in Fig. 158 from the base-line, $h_{,}$, h, h', &c. (which is an expansion of $g_{,}$, g, g', &c.) at right angles to it. Then the points, $g_{,}$, g, g', &c., become $i_{,}$, i, i', &c., the lengths G$g_{,}$, Gg, Gg', &c., becoming $h_{,}i_{,}$, $h\,i$, $h'\,i'$, &c., respectively. If two points, $g_{,}$ and g, indefinitely near each other, are placed on the curve, g, g', g'', &c., at an equal distance from the point G, the two ordinates, $h_{,}i_{,}$ and $h\,i$, in the expansion will be equal, indefinitely near, and parallel. Therefore the element, $i_{,}i$ of $i\,i'\,i''$, &c., is, at i, perpendicular to the ordinate, $h\,i$, whence follows this principle: To each point, g, where Gg is normal to the curve, g, g', g'', &c., there corresponds a point, h, at which the straight line, $h\,i$, is normal to the curve, $i\,i'\,i''$, &c., reciprocally. But when a curve, $i\,i'\,i''$, &c., is put in relation to rectangular co-ordinates, if we consider only the ordinates, $h\,i$, $h'\,i'$, $h''\,i''$, &c., which are normal to the curve, they are alternately greater and less than those immediately adjacent to them, at least when there is no point of inflection. Therefore, if we determine the length of all the normals, Gg, G$g_{,}$, &c., which can be drawn from G on $g\,g'\,g''$, &c.; and if we pass successively from one to the other, travelling round the curve of buoyancy, these normals will be alternately a *maximum* and a *minimum* in relation to the distances from G to the intermediate points of the curve. But when one set of lines which start from the same point are alternately a *maximum* and a *minimum*, the number of *maxima* must evidently be equal to the number of *minima*. Therefore, first, the total numbers of these lines are *even;* and in any closed curve, $g\,g'\,g''$, &c., the number of normals which can be drawn from any point whatever is even.

Connecting all this with what was already proved, Dupin lays down the theorems:—

(1.) The total number of positions of equilibrium, around a fixed axis, is an *even* number;

(2.) In the equilibrium of a floating body of any figure whatever moving round a fixed axis, the number of positions of stable equilibrium is equal to the number of the unstable positions, and these positions recur alternately.

Dupin points out that the foregoing reasoning is open to the objection that it does not hold unless the assumption that the ordinates, $h\,i$, $h'\,i'$, &c., which are intersected at right angles by the curve, $i\ i'\ i''$, &c., are *maxima* or *minima*, which they are not *necessarily;* and he shows that a demonstration of the same principle given by the famous Poisson, is open to the same objection. The occurrence of a single case of mixed equilibrium, from which the body may return to a position of stable equilibrium (or unstable, as the case may be) without reaching the alternate position, so to speak, would bring about an uneven number of positions of equilibrium. To avoid this difficulty, Dupin suggests the treatment of a case of mixed equilibrium as one in which there is a reunion of stable and unstable equilibrium, which have moved up to one another. Whether this view be adopted, or whether we "suppress by thought," as he puts it, the case of mixed equilibrium, and leave it out of consideration altogether, the generality of his demonstration would remain. This he takes the pains to demonstrate in a simple manner.

The next consideration is the absolute number of positions of equilibrium which are possible when the restriction of the axis being fixed is removed. We have already seen (1) that the total number of positions is equal to the total number of possible normals to the surface of buoyancy from the centre of gravity; (2) that, either of these normals being vertical, the equilibrium is stable or unstable according as its length (from the centre of gravity to the centre of buoyancy) is a *minimum* or a *maximum*. To determine these, suppose the body to revolve about any axis whatever through the centre of gravity, and the surface of buoyancy thus made to describe a surface embracing or enveloping all its successive positions. This "envelope," and the surface of buoyancy will touch at a series of points which will together form a closed curve, and at each of these points the normal common to the surface of buoyancy and its enveloping surface will pass through the axis. Among all the normals common to the two surfaces we must find those which pass through the centre of gravity of the body—the axis, be it remembered, passing always through this centre. We must now suppose that the "*méridien*" of the envelope has been determined, and that from the centre of gravity we have to draw to this "*méridien*" all possible normals. We have just before dealt with a problem of this kind; we have seen (1) that the number of such normals to a closed curve is even; and (2) that they are alternately a *maximum* and a *minimum*, in relation to the neighbouring normals through the centre of gravity. (Here the positions of *mixed* equilibrium, which, as we just now saw, present a difficulty, are regarded as double, or "suppressed by thought," as before). We have now considered all the normals absolutely that can be drawn from the centre of gravity to the surface of buoyancy. Since the surface of revolution, which envelopes the surface of buoyancy, touches it in all the points at which the normals intersect the axis of revolution, there must be included among these points all those for which the normals pass through a particular point of that axis —the centre of gravity. But for each such point there is a position of equilibrium. Hence we have now, in the most general form possible, the principles established—(1) that the total number of positions of equilibrium of any freely-floating body whatever is always *even;* and (2) that positions of stable and unstable equilibrium (neglecting cases of mixed equilibrium) alternate.

"*Absolute stability* exists," says Dupin, "only when all around a given position of equilibrium, the floating body returns of itself to that position, whatever may be the direction of the inclination given to it. But the stability is *relative* only, if, in a single direction, the body tends to move still further from its primitive position when it undergoes the least disturbance. It follows from this that the number of positions of absolute stability cannot, at most, exceed the total number of positions of relative stability and absolute instability." He then goes on to show that the number of positions of equilibrium yielding absolute stability is always equal to the number yielding absolute instability; and also that a floating body, whatever be its figure, can take at least one position of absolute stability and one of absolute instability.

In demonstrating these propositions he employs a series of imaginary spheres, having for their centre the centre of gravity of the body, but of variable radii. When the sphere has the shortest normal to the surface of buoyancy for its radius, it is in contact tangentially with that surface internally, and indicates a position of absolute stability. Since that surface is closed and continuous, an exterior sphere, corresponding to this interior one, can be drawn, touching the surface in at least one point, and marking a position of absolute instability. If the radius of this variable sphere is supposed to increase from its lesser to its greater value, it will become, at any intermediate position of equilibrium, a tangential sphere to the surface of buoyancy, and, according to the degree of its contact, will determine a position of absolute or relative stability or instability.

If it lies completely outside the surface at the vicinity of the point of contact it makes absolute *instability,* and then, says Dupin, there may always be found another sphere lying between it and the largest sphere of all, and having internal contact, and there will always be a corresponding position of absolute *stability.* He then deals with the remaining possible cases, and rebuts some objections to its generality, which may be brought against this method of demonstration.

Hitherto have been considered only those inclinations which, for a given position of equilibrium, have corresponded to the directions of greatest and least stability. Dupin concludes by some remarks upon intermediate positions of stability. In doing this he determines, for a position of equilibrium with its corresponding centre of buoyancy, the *indicatrix (l'ellipse indicatrice)* of the surface of buoyancy. He designates as "conjugate stabilities" those which the floating body will possess when inclined successively in the direction of the two conjugate diameters of this indicatrix. These stabilities possess this property, that their sum, taken two and two, is constant, and is equal to the sum of the greatest and least stabilities of the floating body. They are, therefore, connected with the "principal stabilities" by an equation of the first degree. With the demonstration of the property aforesaid, Dupin concludes his extremely able and suggestive *Mémoire.*

In devoting to that *Mémoire* so large an amount of space, we are not unmindful that much of it is of an abstract character, and not essential to the investigation of a ship's stability, because it is sufficient for all practical purposes to calculate the stability of a ship (or other such floating body) about one axis, or at most about two axes. Still, as has been well said by Dr. Woolley, a thorough appreciation of it "cannot fail to be of the greatest service to the naval architect in giving depth and breadth to his views," and therefore we have considered it desirable to set forth its substance at some length in a comprehensive work of this description.

M. Emile Leclert, of Paris, in 1870,* placed before our Institution of Naval Architects certain theorems respecting the geometry of ships, which extended in an elegant manner Dupin's investigations concerning the Surface of Flotation. Writing Dupin's expression for the radius of curvature of the transverse section of this surface in the form

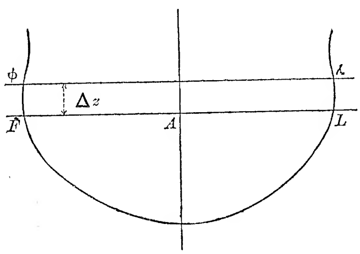
* M. Leclert was then a professor of the *Ecole Impériale du Génie Maritime,* in Paris.

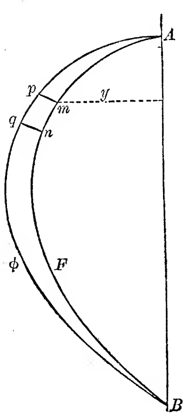
$$r_1 = \int \frac{y^2 \tan. a\, d\, s}{\Omega} \quad (1)$$

(where $d\,s$ indicates an infinitesimal element of the perimeter of the water section, a the inclination of the ship's side to the vertical, and Ω the area of the water section), he proceeded first to establish certain new formulæ of his own as consequences of (1), and afterwards demonstrated them independently.

In deducing them from (1) he took the section of a body, as in Fig. 159, with two parallel water-planes, F L and $\phi\,\lambda$, separated by the distance, $\Delta\,z$. Ω, i being respectively the area and moment of inertia of the water-plane, F L, which cuts off a volume of buoyancy, V, we have $d\,V = \Omega\, d\,z$. The volume cut off by $\phi\,\lambda$, may be called $V + \Delta\,V$, and the moment of inertia of the water-plane, $\phi\,\lambda$, about its longitudinal axes, $i + \Delta\,i$. Now, if we project the two planes of flotation through F L and $\phi\,\lambda$, upon a horizontal water section, as in Fig. 159, $\Delta\,i$ will be the moment of inertia of the area lying between the projections. Let $m\,n = d\,s$ an elementary portion of the perimeter of F L, and draw $m\,p$ and $n\,q$ normals to the perimeter. Then z, being the ordinate and a the angle made by the ship's side with the vertical through m, we have

Fig.159.





$$m\,p = \Delta\,z \tan. a.$$

$$\text{Area } m\,n\,p\,q^{*} = \Delta\,z \tan. a\, d\,s$$

$$\Delta\,i = \Sigma\, y^2\, \Delta\,z \tan. a\, d\,s$$

$$= \Delta\,z\, \Sigma\, y^2 \tan. a\, d\,s.$$

Whence $$\frac{d\,i}{d\,z} = \int y^2 \tan. a\, d\,s.$$

* Neglecting small quantities of the 2nd order

Consequently, by virtue of equation (1)

$$r_1 = \frac{1}{\Omega}\frac{d\,i}{d\,z}$$
$$= \frac{d\,i}{d\,\mathrm{V}}*$$

Transforming this equation, and employing it in connection with Dupin's expression for the radius of curvature of the curve of buoyancy, viz., $r = \frac{i}{\mathrm{V}}$, we may write

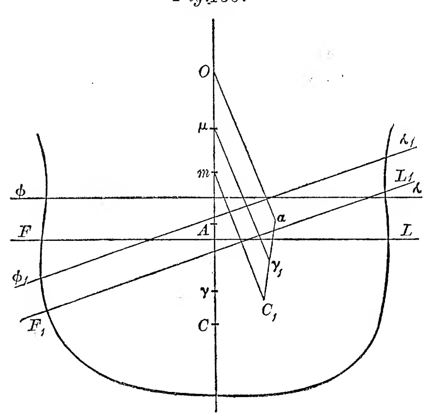
$$r_1 - r = \frac{d\,i}{d\,\mathrm{V}} - \frac{i}{\mathrm{V}}$$
$$= \frac{\mathrm{V}\,d\,i - i\,d\,\mathrm{V}}{\mathrm{V}\,d\,\mathrm{V}}.$$

Whence $$r_1 = r + \frac{d\,r}{d\,\mathrm{V}} \quad (2).$$

To this last formula M. Leclert gives preference, as it expresses r_1 in terms of the quantities r and V, which are usually shown in the French calculations for various values of z.

In establishing this formula independently, M. Leclert employed Fig. 160, and 1st, for a volume of displacement, V, took F L as the upright water section, and F_1 L_1 as a water section inclined to it at a very small angle, θ. C and C_1 were the corresponding centres of buoyancy, m the metacentre, and r the height m C. 2nd, for a volume of displacement, $\mathrm{V} + \Delta\,\mathrm{V}_1$, the upright water section was $\phi\ \lambda$, the slightly inclined one, $\phi_1\ \lambda_1$, and $r + \Delta\,r$, the height, $\gamma\ \mu$. Call the centres of buoyancy of the slices comprised between the upright and the inclined water sections respectively A and a.

Fig.160.



* Messrs. White and John make the just observation that this expression for the radius of curvature of the surface of buoyancy is quite in accordance with their own statement that cusps will occur in the curve of flotation, "seeing that whenever the edge of the deck of a ship becomes immersed there is a break in the continuity." They

The point of intersection, O, of the lines A O, perpendicular to F L and a O, perpendicular to $F_1 L_1$, tends to coincidence with the centre of curvature of the surface of flotation corresponding to V as ΔV approaches zero; and at the same time A O tends to equality with r_1; γ_1 lies on the straight line, $C_1 a$, and since $O a$, $\mu \gamma_1 m C_1$ are parallel, the point μ divides the distance, $m O$, in the same ratio as γ divides C A, and we thus obtain this set of equations, viz.:—

$$\frac{O m}{m \mu} = \frac{C A}{C \gamma} = \frac{O m - C A}{m \mu - C \gamma} = \frac{V + \Delta V}{\Delta V}.$$

We have also

$$O m - C A = (O m + A m) - (C A + A m) = O A - C m = O A - r;$$

and $m \mu - C \gamma = (\gamma m + m \mu) - (C \gamma + \gamma m) = \gamma \mu - C m = \Delta r.$

The two last terms of the set of equations among the ratios enable us to write down

$$\frac{O A - r}{\Delta r} = \frac{V + \Delta V}{\Delta V};$$

or

$$O A - r = (V + \Delta V) \frac{\Delta r}{\Delta V}.$$

In the limit this becomes

$$r_1 - r = V \frac{d r}{d V},$$

which agrees with formula (2).

"This method of calculation," says Professor Leclert, "is that which is usual in such cases—to take for $\frac{d r}{d V}$ the ratio $\frac{\Delta r}{\Delta V}$ relatively to two consecutive groups of simultaneous values of r and V. It is very easy, moreover, to draw a curve of which V is the abscissa and r the ordinate. Theoretically, formula (2) gives, with the help of this curve, a very simple construction for r, whatever be the scales on which V and r are set off. In any case such a curve will be a graphic auxiliary to the computation above-mentioned."

In what has gone before the vessel has only been considered as upright. If it were inclined, the analogous expressions corresponding to those for r, r_1 would be those for the radii of curvature of the

go on to observe that, "at the water section passing through the edge of the deck, any increase in the immersion, and consequently in the displacement, is accompanied by a *decrease* in the moment of inertia; whereas, at smaller inclinations, an increase in immersion and displacement is accompanied by an increase in the moment of inertia. Hence, it is clear that a change in sign of radius of curvature occurs at the angle where the edge of deck is immersed, and this points to a cusp, or some singular point."

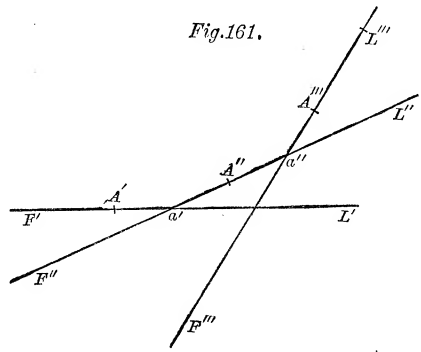
curves of buoyancy and flotation corresponding to the given inclination. This the author shows by a demonstration appended to his paper.

Professor Leclert points out also that we can pass in succession from the upright to any series of inclined positions by calculating for each of them, such as F′ L′ (Fig. 161), the position of its centre of gravity, A′, and the radius of curvature of the curve of buoyancy; because, if F″ L″, making a small angle, θ, with F′ L′ cuts off the same displacement, it will cut F′ L′ at a point, a', such that

$$A'\,a' = r_1'.\ \tan.\ \tfrac{1}{2}\,\theta,$$

$r' = \dfrac{i'}{V}$ will be the radius of curvature of the curve of buoyancy; and $r_1' = r' + V\,\dfrac{d\,r'}{d\,V}$

Fig.161.



will be the radius of curvature of the envelope of flotation. The ratio, $\dfrac{\Delta\,r'}{\Delta\,V}$, employed for getting the coefficient, $\dfrac{d\,r'}{d\,V}$, will be found by means of one or two auxiliary water-plane sections parallel to F′ L′, and very near to it.

The desirability of checking the last of the inclined sections does not escape the notice of M. Leclert, who recommends its verification by a direct calculation of the volume of displacement by some ordinary method. If a correction is found necessary, it can be attended by any other necessary corrections of the intermediate sections by means of interpolations obtained with the help of the auxiliary water sections. "They may even be settled," he observes, "by the simple consideration of the continuous character of the variation of the lengths, $A'\,a'$, $a'\,A''$, $A''\,a''$, . . . in consequence of the equality of the angles, θ, which the successive water sections, F′ L′, F″ L″, F‴ L‴, . . . make with one another." The author (doubtless having in mind the beautiful process of M. Reech*) goes on to add that there are not wanting methods of tracing the evolute of the curve of buoyancy by means of the calculated lengths of the radii of curvature, r, r', &c., of that curve. If it should

* To be fully given in the next chapter.

become necessary to plot the flotation envelope and the evolute of the curve of buoyancy for various values of V, it would obviously be easy to give the extension necessary for this purpose to the artifice of introducing the auxiliary water-plane sections just adverted to.

The great merit of Professor Leclert's formulæ no doubt is, that they connect the surfaces of buoyancy and flotation in a novel and elegant, although not at all in a surprising, manner (for the intimate relations of the two surfaces must often have been vaguely conjectured before), and thus, in addition to any value they may possess in the calculations of the naval architect, lead, as he says, "as a question of pure analysis, to some interesting considerations relating to the geometrical interdependence of these two classes of surfaces."

It is only necessary to add that by using capitals for the longitudinal radii of curvature, in lieu of the small letters employed for the transverse radii, we may complete M. Leclert's formulæ by writing down the following equations:—

$$\mathrm{R} = \frac{d\,\mathrm{I}}{d\,\mathrm{V}}\,; \text{ and } \mathrm{R}_1 = \mathrm{R} + \mathrm{V}\,\frac{d\,\mathrm{R}}{d\,\mathrm{V}}.$$

CHAPTER XIII.

French Systems of Calculating Stability—Dargnies' Method—Determination of Radii of Curvature—Reech's Method of Co-ordinates—Risbec's System of Calculation—Translation of Reech's *Mémoire*—Risbec's Note on Reech's *Mémoire*—Explanation of Risbec's Calculation Form—Ferranty's Investigations.

We have seen that the exact calculation of a ship's stability at various finite angles of inclination by the English method originated by Atwood, involves the detailed calculation of the volumes and centres of gravity of the wedges of immersion and emersion, commonly known as the *Ins* and the *Outs*, at the respective angles of inclination. The employment of the "Amsler Integrator" has so greatly reduced the expenditure of labour and time requisite for the performance of all such calculations, as previously stated, that approximate methods of estimating stability will now be less sought for or considered than they were previously. But other systems of calculation which avoid the labour of calculating the *ins* and *outs* are nevertheless of great interest and importance, even when they are but approximate, and deserve the careful study of all those who would become masters of this subject. Among the most instructive and valuable of such approximate systems are those which grow out of the consideration that the metacentric is the evolute of the curve of buoyancy, traced out by the extremities of successive radii of curvature of that curve, and that the righting levers of stability are the perpendiculars successively let fall from the centre of gravity upon these radii.

In France the science of stability has undergone some important and highly interesting developments in the direction thus suggested, the earliest of these, so far as we can trace, having been attempted by the late M. G. Dargnies, naval architect of the *Messageries Impériales* at *La Ciotat*, and formerly a pupil of the *Ecole d'application du Génie Maritime*, as long ago as 1863, at which time—some of our English naval architects will be surprised to hear—stability calculations were made in France for angles of 10, 20, 30, and 40 degrees, and for four or five different draughts of water, from the light up to the load draught. The method of M. Dargnies essentially consists in calculating the several radii of curvature of the curve of buoyancy (from the water-line area and the displacement), and in then laying down by means of them, and, by a kind of trial and error process ("*tâtonnement*") the evolute of the curve of buoyancy; or, in other words, the metacentric. He therefore dispenses with the calculation of the positions of the centres of buoyancy, and thus greatly reduces the work.

The method which he adopts is based upon the principle that the length of an arc of the evolute or "metacentric" is equal to the difference of the radii of curvature of the involute corresponding to the extremities of the arc. After having found the length of the radii of curvature (instead of the curve of buoyancy) for the inclinations 0°, 10°, 20°, &c., he constructs the metacentric evolute by means

Fig.162.

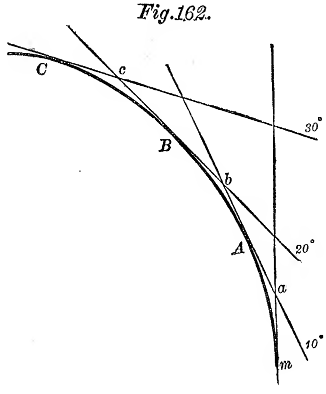
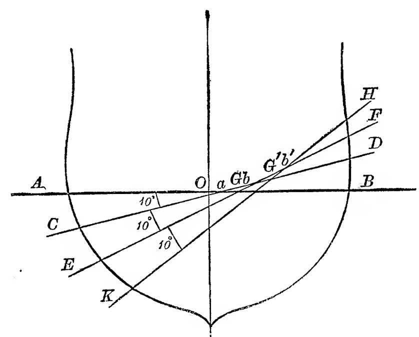


Fig.163.



of the successive differences of these radii only. Let m, Fig. 162, be the metacentre of the upright position of the ship. Supposing the lengths known are m A, A B, B C, &c., he determines the points of intersection a, b, c, &c., of the tangents to the evolute in such manner that the lengths $m\ a$, a A, A b, b B, B c, c C, form a continuous series. This process he regards as sufficiently exact, especially for the part of the curve near the point of departure, m. In order to determine the various radii of curvature of the curve of buoyancy he proceeds as follows:—A B, Fig. 163, is the horizontal plane of flotation corresponding to the given displacement; he finds (in a manner not very unlike that pursued in this country) the plane of flotation, C D, inclined at 10° and corresponding to the same displacement; also the centre of gravity, G, of this plane of flotation, and the length of the corresponding radius of curvature. The curve enveloped by the planes of flotation (or "curve of flotation") is tangential to A B at O, and to C D at G. The positions of the three points, O, a, G, enable us to determine very approximately the point, b, where the plane of flotation, C D, is intersected by the plane of flotation, E F, and thence the area of that plane of flotation and the radius of curvature at the corresponding point of the curve of buoyancy. Before tracing by the same process the plane of flotation, K H, inclined at 30°, we should modify the position of the point, b, if necessary, in accordance with the position of the point, G. The calculations may be similarly pursued as far as desired.

In order to study the evolute of a ship in a complete manner, it suffices, on this plan of M. Dargnies, to find the metacentric radii for the inclinations, 0°, 10°, 20°, 30, and 40°, for the displacements corresponding to the various water-lines. After having obtained all these values, it is well to verify them (and here we come to a very important part of the system of M. Dargnies) by drawing two series of curves, each series comprising as many curves in the one case as there are different draughts of water, and in the other as there are different angular positions, viz.:—

I. Curves corresponding to a constant displacement, having for ordinates the metacentric radii, and for abscissæ the angles of inclination.

II. Curves corresponding to a constant inclination, having for ordinates the metacentric radii, and for abscissæ the draughts of water.

The *ensembles* of these curves indicate the general form, and the transformations of the evolutes, for all the region comprised between the launching and load-line planes of flotation. The points of change of curvature (cusps) are determined by the *maxima* and *minima* of the curves of the series I., and by the envelope of the curves of series II. The curves of series II. enable us to draw immediately the evolute for any draught of water whatever comprised between the light draught of water and the load draught.

The method pursued by M. Dargnies was submitted by him to the consideration of M. Reech, then at the head of the *Ecole d' Application du Génie Maritime,* whose distinguished labours in that position are held in the highest esteem by all those who had the advantage of coming within the sphere of his professional influence. We have not been able to obtain any published record of M. Reech's writings on this subject, but we are able to give on good authority the following account of his labours in connection with this investigation, and our readers will soon discover that these labours have resulted in a very beautiful and valuable extension of the theory of stability.*

M. Reech, while recognising and allowing the merit of M. Dargnies' proposals, and attracted by the value of his *graphic* mode of representation, at the same time adopting the *polar equation* principle, was led by his further study of the subject to dismiss all consideration of the evolute, and to substitute for it a rigidly correct determination of the vertical and horizontal co-ordinates of the centre of buoyancy for the inclined positions of the ship. He formally described two methods, but as they were, in fact, really one and the same in substance, his process may here be considered as one only. It is founded on the principle of graphical representations, by which geometrical interpolations are readily made. He observed that if the value of ρ (the radius of curvature) for any angle and draught be known, the corresponding co-ordinates (vertical and horizontal, with reference to the upright position of the ship) of the corresponding inclined centre of buoyancy can be found from an application of the well-known theorem, that, if $d\,s$ be an elementary portion of the length of the curve of buoyancy, $d\,s = \rho\, d\,\theta$, and it readily follows (since this radius is a normal to the curve),

$$\text{that } d\,y = d\,s \cos.\ \theta = \rho \cos.\ \theta\, d\,\theta,$$

$$d\,z = d\,s \sin.\ \theta = \rho \sin.\ \theta\, d\,\theta,$$

y being measured horizontally from, and z vertically on, the axis of the vessel. Hence, $y_1 = \int_0^{\theta_1} \rho \cos.\ \theta\, d\,\theta$; $z_1 = \int_0^{\theta_1} \rho \sin.\ \theta\, d\,\theta$, y_1 and

* M. Reech's paper was in fact printed in the *Mémorial du Génie Maritime* (3me Livraison, 1864, page 168), but of this we have not been able to obtain a copy (1883).†

† Since the above was written, we have, through the courtesy and good offices of M. Daymard, obtained a copy of M. Reech's Paper, and the assent of its venerable author to make it public in this work. We produce it further on in this chapter (1884).

z_1 being these co-ordinates for an angle of inclination, θ_1. Hence, a sufficient number of values of ρ being found, and $\rho \cos. \theta$, $\rho \sin. \theta$ being obtained—graphically, as M. Reech proposes—the corresponding values of y_1 and z_1 are at once obtained by the usual rules for approximation.

M. Reech got over the difficulty of calculating the radii of curvature at each angle of inclination for a constant displacement very ingeniously. His process requires that a scale of displacement, extending of course beyond the load-draught, be laid down, and a sufficient number of centres of buoyancy be calculated at the same time. If, then, z, being the draught of water, and y_1, z_1 the co-ordinates of the centre of buoyancy be known, he proposes to lay down curves $F(z, y_1) = 0$, and $F'(z, z_1) = 0$. Hence, if a sufficient number of values of y_1 and z_1, corresponding to any value of z be found, curves may be constructed having their draughts for abscissæ, and for ordinates the corresponding value of y_1 and z_1, and therefore enabling us to find y_1 and z_1 for any other values of z.

Hence it follows that we are not limited to finding the radii of curvature and corresponding values of y_1 and z_1 for a constant draught (or displacement) for different angles of inclination; but, if we can find them for different draughts corresponding to different angles of inclination, the above curve can be constructed with these data, and the values of y_1 and z_1 corresponding to the same draught can be taken off from these curves; and so the curve of buoyancy, in the ordinary meaning of the term, *i.e.*, for a constant volume, can be constructed.

He therefore draws water-lines, always through the axis of symmetry of the upright line of flotation, inclined at the different angles of 10°, 20°, 30°, 40°, and more, if requisite; calculates the corresponding volumes, and therefore draughts; and the corresponding radii of curvature; and all the elements of these auxiliary curves are then found.

Thus, if for an inclination, θ, the ordinates along the *in* and *out* scale be measured, and called y' and y'' respectively, and

A be put for $\int y' \, dx + \int y'' \, dx$,

B „ „ „ $\frac{1}{2} \int y'^2 \, dx - \frac{1}{2} \int y''^2 \, dx$,

and C „ „ „ $\frac{1}{3} \int y'^3 \, dx + \frac{1}{3} \int y''^3 \, dx$;

and if v be the volume of displacement in the upright position, and V that for angle θ_1

$$V = v + \int_0^{\theta_1} B\,d\,\theta,$$

$$\text{and } \rho = \frac{C - \frac{B^2}{A}}{V} = \frac{C - \frac{B^2}{A}}{v + \int_0^{\theta_1} B\,d\,\theta}.$$

Hence, when for a sufficient number of values of B, $\int_0^{\theta_1} B\,d\,\theta$, has been obtained by summation in the ordinary way, $v + \int_0^{\theta_1} B\,d\,\theta$, or the volume of displacement is found; the corresponding draught, $z + \delta z$ (z being that for v) is taken from the scale of displacement; ρ calculated as above, can be laid down as an ordinate. The curve connecting z and ρ can be laid down, if necessary; but by using the above value of ρ in the equations,

$$y_1 = \int_0^{\theta_1} \rho \cos.\ \theta\,d\,\theta, \text{ and } z_1 = \int_0^{\theta_1} \rho \sin.\ \theta\,d\,\theta,$$

and erecting y_1 and z_1 as ordinates in the several curves corresponding to the draught, $z + \delta z$, these two subsidiary curves $F(z, z_1) = 0$, $F'(y, y_1) = 0$, can be laid down, and the values of z_1 and y_1, corresponding to the draught, z, can be taken off from the curve.

Hence all the elements for calculating stability are obtained.

This process is rigorous, and seems to offer great advantages for calculation.

When y_1 and z_1, corresponding to an angle, θ_1, and to a given draught, z, for the upright position (*i.e.*, corresponding to a volume equal to the volume in the upright position with draught z) are known; if C be the corresponding centre of buoyancy for the upright position, the length of the perpendicular from C on the normal $= y_1 \cos.\ \theta_1 + z_1 \sin.\ \theta_1$; and if ($a$) be the distance C G (G being the centre of gravity of the ship), then the arm of the righting couple $= y_1 \cos.\ \theta_1 + z_1 \sin.\ \theta_1 - a \sin.\ \theta_1$.

Those of our readers who have read with care the foregoing exposition of the system of M. Reech will probably be disposed to agree with a very eminent English authority on the subject, Dr Joseph Woolley, who, in a note to us, says—"Reech's method is so simple, and founded on so well-known a property, that it seems wonderful that it never occurred to any one before." He adds, "I am convinced that if ever the present system of calculating stability in this country be superseded, it will be by Reech's, or some other founded on it and equivalent to it."

The advantages of the system were so striking that it was not long in becoming practically adopted in France, and in 1870, M. Risbec, whose labours in naval science are well known and much esteemed in this country, prepared a paper upon the method, together with a Calculation Form for use in applying it. M. Risbec, having pointed out the length of the calculations by which alone particulars of a ship's stability could previously be ascertained, went on to say that the method of M. Reech had removed this difficulty and made it possible to carry out the full calculation of a ship's stability (with the aid of tables of squares and cubes) as easily as the ordinary calculations of displacements, &c.

M. Risbec's method involves the use of the Trapezoidal Rule in obtaining the values of $\int_0^{\theta_1} \mathrm{B}\, d\,\theta$, y_1 and z_1, which is equivalent to joining the points of a curve in each section by straight lines, and assuming the area to be equal to the sum of the triangles so found, thus ignoring the segmental areas included between these chords and the curve. This degree of approximation has been deliberately adopted by the highly-accomplished naval architects of France as being amply sufficient, and it certainly possesses the merit of making calculations simple and easy. If, however, a closer approximation be deemed desirable, by taking an additional number of water-lines, bisecting the angles contained by the others (inclined successively at 10°) we can readily apply Simpson's rule. The additional labour of taking off from the body-plan additional ordinates, and enlarging the rule, is not very great, and in the estimation of most English naval architects would be considered fully compensated by the increased accuracy of the result. We reproduce, however (later on in this chapter), on a reduced scale, the form employed in France for carrying out M. Reech's method.

Looking to the remarkable character and great value of M. Reech's *Mémoire*, we will now reproduce it, as promised a few pages back in a foot-note, with but very slight abbreviations, and still slighter departures from an exact translation of the original :—

CONSTRUCTION OF METACENTRIC EVOLUTES FOR A VESSEL UNDER DIFFERENT CONDITIONS OF LADING. By M. Reech, *late Directeur de l'Ecole d'application du Génie Maritime.*

PARIS, 16*th February*, 1864.

I.

M. Dargnies, *Ingénieur de la Cie des Messageries Impériales at la Ciotat*, late free student at the *Ecole d'applications du Génie Maritime*, has submitted for my approval some pages of text and figures relating to the construction of the metacentric evolutes . . . under different conditions of lading. The calculations have been worked out for inclinations of 10, 20, 30, and 40 degrees.

It is well known what a long and tedious operation it would be to obtain the important results arrived at by M. Dargnies, if, in conformity with the definitions and rules in common use, each of the volumes of displacement, V, were divided into parallel layers, in order to determine the co-ordinates of the centres of buoyancy. It is well known also, that in that case, the curves of centres of buoyancy could be drawn, and the evolutes of these curves determined graphically, without its being necessary to calculate the radii of curvature, ρ, of the curves of centres of buoyancy.

M. Dargnies has been able to avoid the long operations just spoken of by proceeding in an inverse manner, that is to say, by calculating the radii of curvature ρ, and constructing the metacentric evolutes immediately, without troubling himself about the positions of the centres of buoyancy.

The shortness of M. Dargnies' procedure is due to this, that the author determines simply enough, and with a degree of approximation doubtless sufficient, each of the planes of flotation, which, at inclinations of 10, 20, 30, and 40 degrees, cut off the same volume, V, of the lower part of the ship. It is necessary to calculate the position of the centre of gravity of the area of each of the planes of flotation. The author also advises the direct verification (with all the tediousness that would entail) of the position of the last plane of flotation inclined at 40°, and the making of slight rectifications, *if necessary*, before calculating definitely the values of ρ by means of the known formula—

$$s = \frac{\frac{1}{3}\int y'^3\,d\,x + \frac{1}{3}\,y''^3\,d\,x}{\mathrm{V}},$$

y', y'' being the ordinates of a plane of flotation, about an axis passed longitudinally through the centre of the area of flotation.

The values found for ρ are set off as abscissæ upon horizontal straight lines, of which the heights, z, are the draughts of water corresponding to the volume, V, of displacement in the upright position of the vessel. M. Dargnies thus obtains auxiliary curves, by means of which he can obtain graphically the values of ρ for any given value of V between the light and load conditions at inclinations respectively equal to 0, 10, 20, 30, and 40 degrees. Other auxiliary curves enable him to obtain the intermediate values of ρ for any given inclination, θ, of the ship from 0 to 40 degrees. By means of the known values of ρ for the same value of V at different inclinations, and by conforming to certain graphic rules of interpolation, the author constructs a metacentric evolute, the form of which seems to be such that it should inspire a degree of confidence generally sufficient. The work of M. Dargnies is worthy of praise, and deserves, in my opinion, to be inserted in the *Mêmorial du Génie Maritime*, and the more so, as in my last revision of the course on the stability of floating bodies, I have developed with some minuteness the following considerations:—

Let us suppose that a certain relation, such as $\rho = f(\theta)$, has been found to exist between the radius of curvature, ρ, of a curve of centres of buoyancy and the corresponding inclination, θ, of a ship. Let us take y, z, to be the rectangular co-ordinates of the extremity of an arc, s, of the curve of centres of buoyancy. Let us conceive the axis of z as being directed vertically upwards in the initial position of equilibrium, through the corresponding centre of buoyancy as origin (Fig. 164), we shall have:

$$d\,s = \rho\, d\,\theta,$$
$$d\,y = d\,s \cos.\,\theta = \rho \cos.\,\theta\, d\,\theta,$$
$$d\,z = d\,s \sin.\,\theta = \rho \sin.\,\theta\, d\,\theta,$$

and integrating from $\theta = 0$ to $\theta = \theta$,

$$y' = \int_0^{\theta} \rho \cos.\,\theta\, d\,\theta,$$
$$z' = \int_0^{\theta} \rho \sin.\,\theta\, d\,\theta.$$

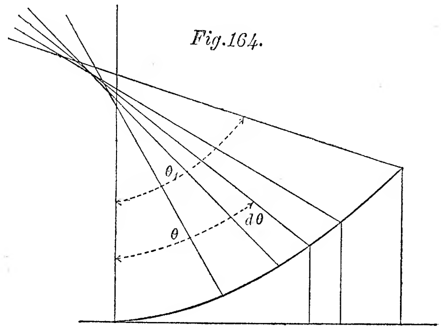


Fig.164.

It will be easy to construct a curve of which the radii vectores, starting from a point as origin, shall be the values of ρ, making angles, θ, with a fixed straight line. It will also be easy to pass through different points of such a curve straight lines perpendicular and parallel to the fixed straight line, in such a manner as to find the values of the products, $\rho \cos.\,\theta$, $\rho \sin.\,\theta$.

Again, it will be easy to trace two other curves of which the rectangular co-ordinates, x, y, shall be such as to give respectively—

$$x = \theta \qquad y = \rho \cos.\,\theta,$$
$$\text{and} \quad x = \theta \qquad y = \rho \sin.\,\theta.$$

Lastly, we shall be able to determine the areas of these curves, that is to say, the integrals,

$$\int_0^{\theta'} y\,dx,$$

which will be the value of the co-ordinates, y_1, z_1, of the curve, s, of the centres of buoyancy. When a series of points, y_1, z_1, are known, as well as the inclinations, θ, of the radii of curvature, ρ, passing through these points, we can construct at the same time a curve of centres of buoyancy, and the evolute of that curve.

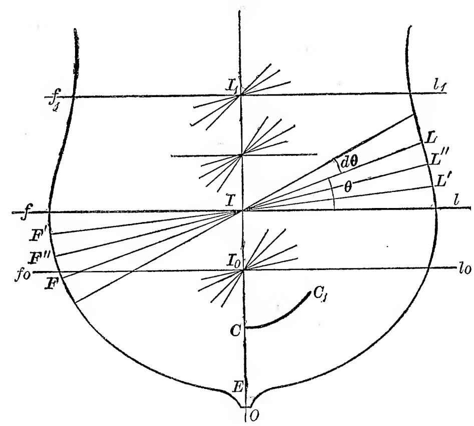
These considerations on the subject of an expression of the form, $\rho = f(\theta)$, are evidently allied to the procedure of M. Dargnies. They serve to complete this procedure in a manner rigorously satisfactory; that is why I think that the whole deserves to be inserted in the *Mémorial du Génie Maritime.*

II.

Reflection on what is new and convenient in M. Dargnies' procedure shows that the principal auxiliary curves of the author, those of which the co-ordinates are ρ and z, could be determined without its being necessary that the different inclined lines of flotation at 0°, 10°, 20°, 30°, and 40° respectively, should each cut off a volume equal to V. There is reason, therefore, to suppose that the procedure of M. Dargnies is susceptible of advantageous modification, and that the system of operation at the same time most convenient and most exact by which to determine the metacentric evolutes of a vessel is decidedly the following :—

Let us call V the volume of displacement, $f\,E\,l$, Fig. 165, which, in the upright position of the ship, will correspond to a draught of water, $O\,I = Z$. Let us suppose that the operations necessary to obtain a scale of volumes (displacements) have been performed, so that such corresponding values of V and of Z as we may require can be obtained. Let us consider particularly the line of flotation, $f\,l$, of one of the volumes V; and represent the axis of symmetry of the line, $f\,l$, in the diametral plane by I, Fig. 165. Let us draw through this axis a series of radial planes, $F'\,L'$,

Fig. 165.



$F''\,L''$ . . . making angles respectively equal to $d\,\theta$, $2\,d\,\theta$, $3\,d\,\theta$, . . . with the plane, $f\,l$.

Let F L be any one of these oblique lines of flotation. Let us take from the plan of the ship the ordinates y' on the starboard side, and the ordinates y'' on the port side of the plane, F L. Let us consider the common line of intersection of the planes, $f\,l$, $F'\,L'$, $F''\,L''$ . . . as the axis of x, and let us calculate by means of the method of quadratures, for each of the lines, F L, three quantities, A, B, C, such that we may have respectively—

$$A = \int y'\,dx + \int y''\,dx,$$

$$B = \tfrac{1}{2}\int y'^2\,dx - \tfrac{1}{2}\int y''^2\,dx,$$

$$C = \tfrac{1}{3}\int y'^3\,dx + \tfrac{1}{3}\int y''^3\,dx.$$

In this manner A will be the area of the plane of flotation, F L.

Calling η the distance of the centre of figure of the area, A, from the axis of x, we shall have

$$\eta = \frac{B}{A}.$$

Calling $d\,V$ the difference of the volumes of the wedges comprised between the plane, F L, and a subsequent plane, we shall have

$$d\,V = A\,\eta\,d\,\theta = B\,d\,\theta.$$

The volume of displacement cut off by a plane of flotation, F L, will be therefore

$$V = v + \int_0^\theta B\, d\,\theta.$$

Evidently to find the values of V, it will be necessary to take the trouble to determine the areas of an auxiliary curve of which the rectangular co-ordinates, x, y, will be $x = \theta$, $y = B$. Such an auxiliary curve being supposed to be constructed, we can have recourse to the method of quadratures in order to determine the values of the integral—

$$\int y\, dx = \int B\, d\,\theta,$$

and consequently the values of V.

The moment of inertia of an area, A, about a straight line drawn through the centre of figure of the area, A, parallel to the axis of x will be

$$C - A\,\eta^2,$$

and consequently we shall find the value of ρ by putting

$$\rho = \frac{C - A\,\eta^2}{V} = \frac{C - B\,\eta}{V} = \frac{C - \frac{B^2}{A}}{v + \int_0^\theta B\, d\,\theta}.$$

As the scale of volumes gives the draught of water, z, which, in the upright position of the ship, will correspond to any value, V, of the volume of displacement, it will be easy to construct a point of which the co-ordinates will be ρ and z for any given value of θ.

Supposing the operations just described to have been performed, first for a certain value of v and for different values of θ, then for other values of v, and for the same values of θ, we shall arrive at the construction of the same auxiliary lines, and consequently the same final lines, as those of M. Dargnies. We shall arrive at it by the aid of a regular procedure perfectly exact, without any experiment.

It should be remarked that the numerator of the expression for ρ may be calculated differently by means of the following equivalent expression:—

$$\tfrac{1}{3}\,\rho\left(y' - \eta\right)^3 dx + \tfrac{1}{3}\,\rho\left(y'' + \eta\right)^3 dx.$$

III.

If we call h the depth of the centre of buoyancy, C (Fig. 165), of one of the volumes, v, below the plane, $f\,l$, and y_1, z_1, the co-ordinates of the centre of buoyancy, C_1, of the volume, V,* with reference to a point in the axis, I, of the line $f\,l$, as origin, we can easily obtain the following relations:—

* The volume V is the sum of v, and an infinity of prisms constructed with dx for height upon a triangular base of an extent equal to the product,

$$\tfrac{1}{2}\,y^2\, d\,\theta.$$

The horizontal and vertical distances of the centre of gravity of the prism from the axis of x are—

$$\tfrac{2}{3}\,y \cos.\theta \text{ and } \tfrac{2}{3}\,y \sin.\theta.$$

The two moments of a prism are consequently—

$$\tfrac{1}{3}\,y^3 \cos.\theta\, d\,\theta\, dx \text{ and } \tfrac{1}{3}\,y^3 \sin.\theta\, d\,\theta\, dx.$$

Integrating first with regard to x, then with regard to θ, and taking account of the moment of the volume v, we obtain the expressions given above for Y_1, Z_1.

$$V\,Y_1 = \int_0^\theta C \cos.\ \theta\ d\ \theta,$$

$$V\,Z_1 = vh + \int_0^\theta \sin.\ \theta\ d\ \theta.$$

As already explained, we have

$$V = v + \int_0^\theta B\ d\ \theta.$$

Consequently it will not be difficult to find the values of Y_1, Z_1, provided that we have determined beforehand the volumes, v, as well as the positions of the centres of gravity of the volumes, v.

These new equations will be sufficiently simple to be developed numerically if, instead of calculating the values of ρ, we choose to construct auxiliary curves, of which the co-ordinates will be respectively

$$(Y_1,\ Z) \text{ and } (Z_1,\ Z),$$

for the same values of V and for a common value of θ.

Supposing such auxiliary curves to be constructed, it will be easy to find the values of Y and Z, which will correspond to the same value of V for different values of θ. It will consequently be easy to construct different curves of centres of buoyancy, and at the same time the evolutes of these lines.

We thus obtain a second mode of operations which will be still more simple than the preceding, if we are not anxious to calculate the values of ρ. On the contrary, if we do wish to calculate the values of ρ, it will be preferable to return to the expressions for Y_1, Z_1, and develop numerically those of y_1, z_1, which are—

$$y_1 = \int_0^\theta \rho \cos.\ \theta\ d\ \theta$$

$$z_1 = \int_0^\theta \rho \sin.\ \theta\ d\ \theta.$$

IV.

That which distinguishes the two processes which I have just made known, from those generally in use, is the employment of polar co-ordinates from the light line of flotation, $f_0\,l_0$, to the load line of flotation, $f_1\,l_1$; also the inutility of making oblique sections of the volume of a ship above the axis of the line, $f_0\,l_0$.

That which distinguishes the same two processes from that of M. Dargnies is simplicity and exactitude, and consequently the complete absence of tentative processes.

I do not see that it is possible to do better than use one or other of the processes which have been explained; but it may be useful to add to what has been said some means of checking the determination of the values of V, and of those of $V\,Y_1$, $V\,Z_1$.

Let us consider two volumes of displacement, V, V′, cut off by parallel planes, F L, F′ L′, Fig. 166. The volume, V′, may be considered as being the sum of V and an infinity of parallel slices, for which the differential expression is—

$$A\ dz \cos.\ \theta.$$

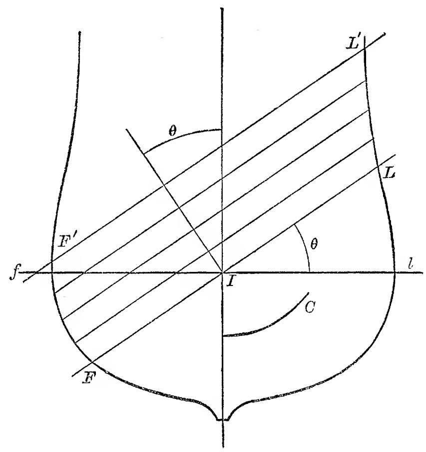
We shall have, consequently,

$$V' - V = \cos.\ \theta \int_z^{z'} A\ dz.$$

The co-ordinates of the centre of gravity of a differential slice, with reference to the origin, O, of the draughts of water, z, in the longitudinal middle-line plane are—

$$\eta \cos.\ \theta,\ z + \eta \sin.\ \theta.$$

Fig. 166.



If we call (Y, Z), (Y′, Z′), the corresponding co-ordinates of the centres of gravity of the volumes, V, V′, we shall have—

$$V' Y' - V Y = \int_z^{z'} A\,dx \cos.\ \theta,\ \eta \cos.\ \theta = \cos.^2 \theta \int_z^{z'} B\,d\eta$$

$$V' Z' - V Z = \int_z^{z'} A\,dz \cos.\ \theta\ (z + \eta \sin.\ \theta) = \cos.\ \theta \int_z^{z'} Az\ dz + \cos.\theta \sin.\theta \int_z^{z'} B\ dz.$$

These expressions will serve to find the quantities, V′, V′ Y′, V′ Z′, when the corresponding values of V, V Y, V Z, are known; but the operations to be performed will be far less simple than those of the polar integrations previously discussed.

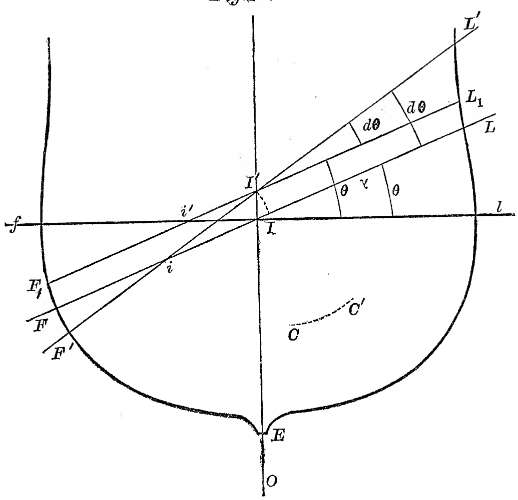
The results of the two systems of operations not being at variance, there must be certain relations between the different integrals, which will extend from z to z', and from 0 to θ; but nothing would be gained by developing these relations algebraically. Their existence is due to the fact that in taking at the same time from the plan of the vessel the horizontal ordinates, y, of the lines of flotation, $f l, f' l'$ . . ., Fig. 155, and the oblique ordinates, y', y'', of the inclined lines of flotation, F L, F′ L′, we obtain lengths which are implicitly dependent upon one another.

Consequently certain relations must exist between the values of the integrals which are dependent upon y and upon z only, and between those which are dependent upon y', y'', z and θ.

The only developments upon which, I think, I should dwell in connection with this subject, have for their object to make it clear that when we restrict the value of V to a constant, the expressions for Y_1, Z_1, agree with those for y_1, z_1.

Let us represent by f E l, F E L, F′ E L′, Fig. 167, displacements, the volumes of which are respectively v, V, and V + d V. I and I′ are the projections of two parallel straight lines through which the diametral plane is cut by the planes, F L, F′ L′. Let us draw through the straight line projected in I′, a plane, $F_1 L_1$, parallel to F L: we have—

Fig. 167.



$$Z = O\,I.$$

$$\theta = \text{angle } L\,I\,l = \text{angle } L\,i'\,l;$$

$$d\,\theta = \text{angle } L'\,I'\,L_1 = \text{angle } L'\,i\,L;$$

$$d\,z = I\,I';$$

$$\text{and } d\,V = A\,d\,z\cos.\,\theta + A\,\eta\,d\,\theta = A\,(d\,z\cos.\,\theta + \eta\,d\,\theta.)$$

It is geometrically evident from Fig. 157, that we shall have $dz \cos.\ \theta = i\,I\,.\,d\,\theta$. Thus we see that the expression, $d\,V$, is reduced to

$$d\,V = A\,(i\,I + \eta)\,d\,\theta.$$

Let γ (Fig. 167) be the centre of figure of the area A, and let us call η_1 the distance, $i\,\gamma$,

$$\text{we shall have } \eta_1 = i\,I + \eta,$$
$$\text{and consequently } d\,V = A\,\eta_1\,d\,\theta.$$

Consequently, if we wish V to be constant, we must have $\eta_1 = 0$. That means that the line of intersection, i, of the planes, F L, F′ L′, must pass through the centre of figure of the area, A, of the line of flotation, F L.

The value of $d\,z$, which will be dependent upon the condition, $\eta_1 = 0$, will be such that we shall have—

$$d\,z\cos.\,\theta = -\,\eta\,d\,\theta = -\,\frac{B}{A}\,d\,\theta.$$

This granted, $V\,Y_1$, $V\,Z_1$, being the moments of the volume, F E L, and $V'\,Y'_1$, $V'\,Z'_1$, those of the volume, F′ E L′, if we consider as the origin the point, I, we find easily,

$$V\,Y'_1 - V\,Y_1 = A\,d\,z\cos.\,\theta \times \eta\cos.\,\theta + C\cos.\,\theta\,d\,\theta;$$
$$V\,Z'_1 - V\,Z_1 = A\,d\,z\cos.\,\theta \times \eta\sin.\,\theta + C\sin.\quad d\,\theta.$$

That becomes again,

$$d\,(V\,Y_1) = B\cos.^2\,\theta\,d\,z + C\cos.\,\theta\,d\,\theta;$$
$$d\,(V\,Z_1) = B\cos.\,\theta\sin.\,\theta\,d\,z + C\sin.\,\theta\,d\,\theta.$$

If we differentiate the expressions for y_1, z_1, we find—

$$d y_1 = \rho \cos. \theta \, d \theta ;$$
$$d z_1 = \rho \sin. \theta \, d \theta .$$

But in that case it is understood that we have

$$\mathrm{V} = \text{constant}.$$

In order that V may be constant, we must have

$$d z \cos. \theta = - \eta \, d \theta = - \frac{\mathrm{B}}{\mathrm{A}} \, d \theta ;$$

and then the differential expressions for Y_1, Z_1, become again—

$$\mathrm{V} \, d \mathrm{Y}_1 = \left(\mathrm{C} - \frac{\mathrm{B}^2}{\mathrm{A}} \right) \cos. \theta \, d \theta ;$$

$$\mathrm{V} \, d \mathrm{Z}_1 = \left(\mathrm{C} - \frac{\mathrm{B}^2}{\mathrm{A}} \right) \sin. \theta \, d \theta .$$

Thus we see that it is necessary and sufficient to have the known relation,

$$\rho = \frac{\mathrm{C} - \frac{\mathrm{B}^2}{\mathrm{A}}}{\mathrm{V}} ,$$

in order to get

$$d \mathrm{Y}_1 = d y_1 \quad ; \quad d \mathrm{Z}_1 = d z_1 .$$

(Reech.)

Having thus given the valuable investigations of M. Reech, it will be well to add the following translation of part of M. Risbec's note upon it:—

"M. Reech's method rests upon this observation, that, if we consider only the planes of flotation perpendicular to the transverse vertical plane of the ship, any plane whatever will be completely determined by the angle, θ, which it makes with the horizontal, and by the volume, V, which it cuts off. It follows that all the quantities that are dependent upon the volume cut off by this plane are solely functions of the two independent variables, θ and V. So that for one of these quantities, $\mathrm{Z} = f(\theta_1 \mathrm{V})$, it is only necessary to perform a certain number of numerical operations (and the most convenient will be chosen) in order to arrive at the graphic knowledge of the surface, $\mathrm{Z} = f(\theta_1 \mathrm{V})$, and hence the possibility of finding immediately the value of Z for such values as we please of θ and of V. In reality, we do not procure first of all for independent variable the quantity, V (which must be calculated), but the draught of water (t) relative to the line in which the plane drawn at an angle, θ, cuts the vertical longitudinal plane. But we shall see that, the values of V being known, we are obliged in the graphic construction to use t in the condition of a function of V and of θ, which are the true independent variables.

"The most convenient choice of the series of volumes, upon which to perform the numerical operations, seems to be that indicated in the first case by M. Reech; it consists, starting from one of the horizontal water-lines, in drawing through its axis of symmetry a series of planes making equal angles with one another, and operating by successive summations upon the whole of the volumes thus cut off. M. Reech gives the formula by which can be calculated for each of these volumes—1st, its volume; 2nd, the co-ordinates of its centres of buoyancy; 3rd, the corresponding metacentric height.

"The Table (which will be found at the end of this volume) is only the realisation of the type of calculation which results from these formulæ after their transformation by the practical method of quadrature.

"The calculation will have to be repeated upon a certain number of identical tables for so many horizontal water-lines, starting from which we wish to perform the operations.

"The number of water-lines chosen, and consequently the number of tables to be filled, will depend upon the extent of the limits between which we wish to know with exactitude the different functions enumerated above.

"In order to represent the surfaces, of which the functions thus determined are the ordinates, the most natural method consists in making in these surfaces a series of sections parallel to one of the co-ordinate planes, and projecting upon this plane the curves obtained, affecting each of them by an index, which is nothing else than the number of the plane of the section. Thus, in the surface, $Z = f(\theta_1 V)$, we should project upon the plane (Z V) the sections made parallel to this plane, indicating upon the projected curves the values of θ corresponding to each section.

"The calculations being finished, we shall have then first to carry to a line of abscissæ all the values of V obtained.

"Above, in a parallel position, we shall draw, at a convenient distance, straight lines, representing by their intervals (to scale) the horizontal water-lines, starting from which we have operated in the calculations. It will be convenient to set off above the line of abscissæ the values of Y (abscissæ of the centres of buoyancy, or distances from the longitudinal vertical plane), and those of ρ (metacentric heights), and to set off the values of Z (ordinates of the centres of buoyancy, or distances from the horizontal water-line) downwards from the parallel lines which were previously drawn.

"Each point will be marked by the indication of the value of θ, with which it agrees.

"There will only remain then to join for each kind of curve the points which correspond to the same value of θ, and the surfaces, Y Z and ρ, will be completely represented.

"We may observe now that the parallel lines drawn in the upper part represent, by their distances above a certain origin, the different values of draught of water, t; their intersections with the ordinates erected at the abscissæ, V, which correspond to them respectively, represent, therefore, points of the surface, $t = f(\theta_1 V)$, provided that we affect each point by the index of the value of θ, with which it agrees.

"Thus, given an arbitrary value of θ and one for V, we can measure on these curves the corresponding values of the functions,

$$Y, Z, \rho \text{ and } t$$

(because, for values of θ intermediate between those indicated, it will suffice to make an auxiliary section perpendicular to the axis of the V's).

"We see, consequently, that these curves make it possible to construct the curve of centres of 'Isocarènes' (equal buoyancies, so to speak) for a given displacement of the ship (within the limits of the calculation), and for all inclinations. The metacentric heights that we obtain at the same time serve only as a graphic verification; because we know *à priori* at each point the direction of the normal to the curve of the centres of 'Isocarènes.'

"As to the curves, $t = f(\theta_1 V)$, they allow us to draw immediately the successive inclined flotations, since they give the position of the points where these flotations cut the vertical longitudinal plane. To sum up; all the quantities we require are to be obtained from the same ordinate, corresponding to the abscissæ, V, under consideration."

In order to make the Table at the end of the volume perfectly clear, we append the following remarks in further explanation of it, observing that we shall use M. Risbec's notation in all but one or two instances. We will also state at once that throughout his calculations M. Risbec, in accordance with the general practice in France, uses what is known as the trapezoidal rule.

In the column at the extreme left of the table the numbers of the transverse sections used are arranged, the midship section being called M, and the others numbered from it forward and aft, F P being the fore perpendicular, and A P the after perpendicular. The upright water-section being exceptional on account of its symmetry about the longitudinal vertical plane, we shall take the "water-line for 10°" to illustrate the method followed for each inclined water-line. In the first and second columns, marked I and E, of this division of the table, the ordinates measured along the inclined water-line on the immersed and emersed sides are respectively entered:* the sum total (s) of the sums of the two columns being multiplied by the longitudinal interval (X), the product (s X) gives the total area of the inclined water-plane (A_1). In the third and fourth columns, marked I and E, are entered the squares of the immersed and emerged ordinates† respectively, which also represent the functions of squares of ordinates, and the difference (δ) of their sums being divided by 2 and multiplied by the longitudinal interval (X), the result ($\frac{1}{2}\,\delta$ X) gives the moment (B_1) of the water-plane relative to the longitudinal axis. This moment is multiplied by half the angular interval ($\frac{1}{2}\,\theta$), and the product (b_1) is the excess of the volume of the wedge of immersion over that of the wedge of emersion, and consequently the excess of the "inclined displacement" over the "upright displacement." In the fifth and sixth columns, marked I and E, are entered respectively the cubes of the immersed and emerged ordinates‡ which also represent functions of cubes of ordinates, and the sum total (σ) of their sums, being divided by 3 and multiplied by the longitudinal interval (X), the result ($\frac{1}{3}\,\sigma$ X)

* The trapezoidal rule being used, the quantities in these columns also represent the functions of ordinates for areas, provided that only half the actual ordinate is inserted opposite F P and A P.

† Half the square of the whole ordinate must be inserted opposite F P and A P.

‡ Half the cube of the whole ordinate must be inserted opposite F P and A P.

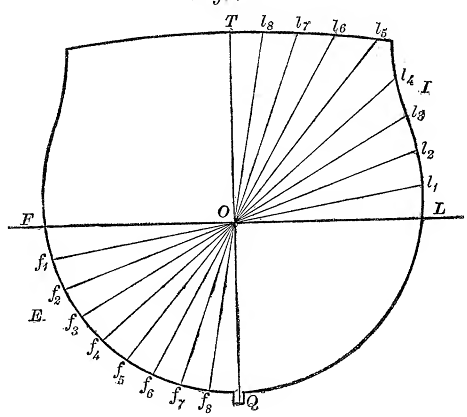
gives the moment of inertia of the water-plane (C_1). This moment is multiplied by half the angular interval, and by the sine of the angle of inclination, the product ($\frac{1}{2} C_1 \theta$ [*sin.* 10°]) being the moment (e_1) of the wedges relatively to the longitudinal plane; multiplying by the cosine instead of the sine, the product ($\frac{1}{2} C_1 \theta$ [*cos.* 10°]), is the moment (f_1) of the wedges relative to the horizontal water-plane.

The same operations having been performed for each inclined water-line, the final results are worked out at the right of the table. The displacements (v_1, v_2, &c.) cut off by the different inclined water-planes are first obtained, by adding to the displacement in the upright (v_0) the successive increments (b_1, b_2, &c.) due to the inclinations. Next, the abscissæ (Y_1, Y_2, &c.) of the centres of buoyancy corresponding to each inclination are obtained, by adding the successive moments (f_1, f_2, &c.) and dividing by the displacement corresponding to the inclination. The ordinates (Z_1, Z_2, &c.) of the centres of buoyancy corresponding to each angle of inclination are obtained by subtracting the successive moments (e_1, e_2, &c.) from the moment of the original centre of buoyancy about the horizontal water-line, and dividing the remainder by the corresponding displacement. The lengths of the radii of curvature (ρ_1, ρ_2, &c.) for each inclination are obtained by applying the formula $\rho = \dfrac{C - \dfrac{B^2}{A}}{V}$ with the value of each term proper to the inclination. The remainder of the space at the bottom of the table is occupied by general data which it is found convenient to have at hand.

We have another French investigator of this branch of naval science to notice before concluding this chapter. The late M. de Ferranty (father-in-law of M. Daymard, of Marseilles), laboured with marked ability and success at the simplification of stability calculations for large angles of inclination, down to the very eve of his death, in 1882. His latest and best method of procedure was deduced from the study of our English methods, and led both to their extension and to their simplification. In view of the other systems of calculation described in this work, and more especially of the very recent one of M. Daymard, who is familiar with M. de Ferranty's labours, we deem it unnecessary to do more here than describe in outline the course of investigation pursued by M. de Ferranty, and bear our testimony to the skill and ingenuity displayed by him in devising forms for the practical application of his results.

Suppose that through the axis of the water-line when upright, ten radial planes are drawn (F L, in Fig. 168 being the water-line,

Fig. 168.



and T Q the longitudinal vertical plane) at equal angular distances; upon them we measure off from the sections the ordinates, I, on the immersed side and, E, on the emerged side.* Let V be the volume of displacement, supposed in the first place for the upright position, and Z the distance of the centre of buoyancy from the normal water-line, F L; a very simple analysis shows first that calling Δx, the interval between the transverse sections, we have :—

$$V = \Delta x \Delta \theta \left(\tfrac{1}{2} \Sigma E_0^2 + \Sigma E_1^2 + \Sigma E_2^2 \ldots . + \Sigma E_7^2 + \Sigma E_8^2 + \tfrac{1}{2} \Sigma E_9^2\right)$$
$$= \Delta x \Delta \theta A,$$
$$V Z = \tfrac{2}{3} \Delta x \Delta \theta \left(\Sigma E_1^3 \sin. 10° + \Sigma E_2^3 \sin. 20° \ldots . + \Sigma E_8^3 \sin. 80° + \tfrac{1}{2} \Sigma E_9^3\right) = \Delta x \Delta \theta B,$$

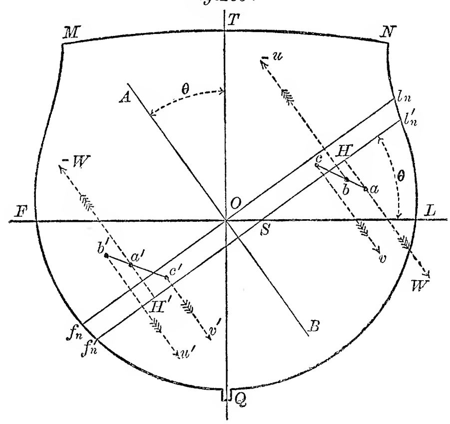
calling A, and B, respectively the quantities between brackets.

Passing on to the stability, let us consider one of the radial planes, $f_n\, l_n$, for instance, Fig. 169, at an inclination to the upright water-line of θ $(= 10_n^0)$; let $f_n'\, l_n'$ be the oblique water-line parallel to $f_n\, l_n$, cutting off the isocarène (immersed volume of equal buoyancy) of the ship, and consequently cutting off with F L, the

* "The calculations could be made for an indefinite number of radial planes, but it seems more simple to deal with ten, drawn at intervals of 10°, $\Delta\,\theta = \frac{\pi}{18}$, as practice shows that, for ships of ordinary form, this number suffices to obtain useful results with a sufficient degree of approximation."—*M. de Ferranty's MS.*

two equal wedges $L S l_n'$, and $F S f_n'$, the volume of which we will call W. What we want to find is the value of $P p$ for this isocarène; P being the displacement, and p the arm of the lever of the righting

Fig.169.



couple. Now the fundamental formula of the theory of stability for finite inclinations gives:—

$$P p = W \omega \times H H' - P a \sin. \theta, \quad . \quad . \quad . \quad . \quad (1.)$$

calling, as usual, a the distance of the centre of buoyancy from the centre of gravity of the ship, ω the density of sea-water, H and H′ the projections upon $f_n' l_n'$ of the centres of gravity a and a' of the two wedges, W.

We proceed to express (and this is the essence of the method) the quantities, W and H H′, as functions of the other quantities dependent upon the ordinates obtained from our radial planes. Let u and u' be the two unequal volumes, $L O l_n$, $F O f_n$ comprised between the radial plane, $f_n l_n$, and the water-line, F L; v and v' the two volumes, $O S l_n l_n'$, and $O S f_n f_n'$.

The figure shows that

$$u = W + v, \; u' = W - v', \text{ whence } u - u' = v + v':$$

this last equality can be written immediately, since it expresses the fact that the total volume of the larger, $f_n f_n' l_n l_n'$ is equal to the difference of the volumes, $L O l_n$ and $F O f_n$.

The water-line, $f_n' l_n'$, will be above the point, O, if the second wedge be greater than the first, and below it in the opposite case, which is that of ships of usual forms.

Let a, b, c, be the centres of gravity of the volumes W, u and v on the immersed side, and a', b', c' those of the corresponding volumes W, u' and v' on the emerged side, and A B, a straight line perpendicular to $f_n' \, l_n'$ passing through the point O. Let us apply to the three first points, respectively, the forces W, $-u$, and v, and to the three last the forces $-$W, u', and v', all in a direction parallel to A B. The three forces of each group being in equilibrium, the sum of their moments about A B is *nil*, and we have, calling respectively g, h, m and g', h', m' their distances from this straight line :

$$\mathrm{W}\, g - u\, h + v\, m = 0 \text{ for the side of immersion,}$$
$$-\mathrm{W} g' + u'\, h' + v'\, m' = 0 \text{ for the side of emersion,}$$

whence we deduce :

$$\mathrm{W}(g+g') = u\, h + u'\, h' - (v\, m - v'\, m') \quad . \ . \ . \ . \quad (2.)$$

The quantity, $v\, m - v'\, m'$, representing the moment of the whole layer, $f_n f_n'$, $l_n\, l_n'$, of which the volume is $u - u'$, as we have seen above, if we call d the distance of its centre of gravity from the line A B, we shall have

$$v\, m - v'\, m' = (u - u')\, d.$$

Now, considering the moderate thickness of the layer, $f_n f_n'$, $l_n\, l_n'$, "we may admit," says M. de Ferranty, "that d is equal to the distance of the centre of gravity of the section of the radial plane, $f_n\, l_n$, from the longitudinal axis projected in O"—an assumption which, it must be acknowledged, might, in many conceivable cases, be not very justifiable.

Adopting the usual notation of the French tables, $d = \dfrac{\delta}{2\, s}$, s being the sum of the ordinates $(\Sigma\, \mathrm{E} + \Sigma\, \mathrm{I})$, and δ the difference of the squares $(\Sigma\, \mathrm{I}^2 - \Sigma\, \mathrm{E}^2)$ . . . ., we have then :

$$\mathrm{W}(g+g') \text{or } \mathrm{W} \times \mathrm{H\, H'} = u\, h + u'\, h' - (u - u') \frac{\delta}{2\, s} \,;$$

substituting in formula (1.) we have finally :

$$\mathrm{P}\, p = \omega \left\{ u\, h + u'\, h' - (u - u') \frac{\delta}{2\, s} \right\} - \mathrm{P}\, a \sin.\, \theta \quad . \ . \ . \ . \quad (3.)$$

Under this form it becomes possible, and even easy, to calculate all the terms of the second member for the different values of θ equal to 10°, 20°, ... 80°, and 90°.

In proceeding towards this object, M. de Ferranty first puts the second member into the following form :

$$P p = M - \mu - \Phi$$

by putting

$$M = (u h + u' h')\,\omega,$$

$$\mu = (u - u')\,\omega\,\frac{\delta}{2 s},$$

$$\Phi = P\,a \sin.\,\theta.$$

and finds the values of M, μ, and Φ, for the various inclinations under consideration. He obtains by a brief analysis, which we need not here reproduce, the following general expressions for these values:—

$$M_n = (u_n h_n + u_n' h_n')\,\omega,$$

$$= \omega \times \tfrac{1}{6} \Delta x \Delta \theta [\sigma_n \cos.\,0^\circ + 2\,\sigma_{n-1} \cos.\,10^\circ + .. + 2\,\sigma_1 \cos.\,(n-1) \times 10^\circ + \sigma_0 \cos.\,n \times 10^\circ];$$

$$\text{and } \mu_n = (u_n - u_n')\,\omega\,\frac{\delta_n}{2 s_n}$$

$$= \tfrac{1}{4} \Delta x \Delta \theta\,\omega \times \frac{\delta_n}{2 s_n}\,[2\,\delta_1 + 2\delta_2 + 2\,\delta_3 + \,.\,.\,.\,.\,.\, + 2\,\delta_{n-1} + \delta_n].$$

Further, as $\theta = (10 \times n)^\circ$,

$\Phi = P\,a \sin.\,(10 \times n)^\circ$; the calculation of this last term presenting no difficulty.

CHAPTER XIV.

Merrifield's Suggested Mode of Approximating to Evolute of Curve of Buoyancy—Rankine's Improvement thereof—Dr. Woolley's Extension of Similar Methods to Stability at Large Angles of Inclination—Curve of Radii of Curvature—Dr. Woolley's Formula and its Proof—Positions of Cusps in Metacentric Evolute—Amount of Error involved in the Process.

In 1867, the late Mr. Charles W. Merrifield, then Honorary Secretary of the Institution of Naval Architects, read at that institution a Paper on Stability Calculations, with an important note thereupon by the late Professor Macquorn Rankine, improving upon and extending the method of approximately calculating stability suggested by Mr. Merrifield. The fundamental assumption which Mr. Merrifield made was that the locus of centres of buoyancy, or the curve of buoyancy, can be regarded with sufficient accuracy as a *Conic;* the stability being measured as usual by the perpendicular from the centre of gravity upon the normal due to the inclination. Of the assumed Conic we already know, or can by usual methods find, the vertex, and the tangent and curvature at the vertex, these being given by the upright centre of buoyancy and the metacentre. For the complete determination of the Conic is required the length of another radius of curvature corresponding to a known inclination. To simplify the work of obtaining this, Mr. Merrifield formed and employed a "Mean Section" for the given ship, and thus reduced the process of so fixing the water-line as to cut off the proper volume of buoyancy to an easy problem of plane geometry—an approximate process, of course, but regarded by him as sufficiently near for ordinary purposes.

Assuming the second radius of curvature corresponding to an angle of inclination, θ, thus obtained, and calling it ρ_θ, then, from the properties of the Conic we have

$$\rho_\theta = \frac{a\,(1 - e^2)}{(1 - e^2 \sin.^2 \theta)^{\frac{3}{2}}};\ \rho_0 = a\,(1 - e^2).$$

From these expressions Mr. Merrifield drew the equations necessary for completely calculating the Conic.

Professor Rankine suggested that the calculation might be simplified by assuming for the approximate form of the metacentric involute, or curve of buoyancy, not a conic, but *the involute of the involute of a circle*, the locus of its centres of curvature, or the "Metacentric evolute" being assumed itself to be the involute of a circle. He then went on to say :—

"The involute of the involute of a circle is distinguished by the following property. Let r be the radius of the circle, ρ_0, that radius of curvature of the involute of the involute which touches the involute at its cusp, and ρ another radius of curvature of the same curve making the angle, θ, with the radius, ρ_0; then,

$$\rho = \rho_0 + \frac{r\,\theta^2}{2}, \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

Having found, then, the radii of curvature of the metacentric involute in an upright position and at a given angle of inclination, θ_1, let ρ_0, and ρ_1, be those radii respectively; then make

$$r = 2\,\frac{\rho_1 - \rho_0}{\theta_1^2}, \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

This will be the radius of the required circle; and its positive or negative sign will show whether it is to be laid off downwards or upwards from the metacentre. For any given angle of inclination the radius of curvature of the metacentric involute will be given by equation (1.), which may also be put in the following form :—

$$\rho = \rho_0 + (\rho_1 - \rho_0)\frac{\theta^2}{\theta_1^2}, \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

Let λ, be the depth of the ship's centre of gravity below her metacentre, and p the perpendicular let fall from that centre of gravity upon the radius of curvature of the metacentric involute at any given angle of inclination, θ; then,

$$p = (\lambda - r)\sin.\,\theta + r\,\theta, \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

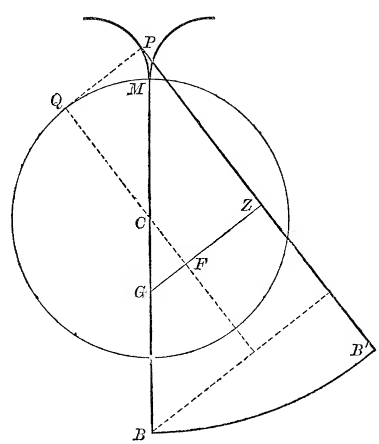
and the moment of stability is

$$p \times \text{displacement}, \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

Mr. Merrifield gave the geometry of Professor Rankine's formula as follows :—

Let M P, Fig. 170, be the metacentric evolute; M Q, the circle of which it is assumed to be the involute; B B′, the curve of buoyancy, or metacentric involute; B P B′, the angle of the ship's inclination, supposed $= \theta$. Since Q P winds off the circle, it is obvious that it is perpendicular to Q C, and equal to the arc, Q M. It is also perpendicular to the arc, M P, and its tangent, P B′. Hence, C Q is parallel to P B′, and Q C P $= \theta$. Hence, Q P $=$ arc Q M $= r\,\theta$. Hence, we obtain the values of r and ρ, quoted above in the extract from Professor Rankine's note.

Fig. 170.



Further, since Q P is the radius of curvature of the arc P M at P, the arc P M is obtained from the well-known formula, $d\,s = \rho\,d\,\theta$; and

$$\int_0^\theta \mathrm{Q\,P}\,d\,\theta = \tfrac{1}{2}\,r\,\theta_1^2.$$

Let us call P B′, ρ_θ, and M P, ρ_0; then,

$$\rho_\theta = \rho_0 + \tfrac{1}{2}\,r\,\theta_1^2.$$

Now, let G be the centre of gravity, and M G $= \lambda$; draw G F Z perpendicular to P B′. Then,

$$\begin{aligned} \mathrm{G\,C} &= \lambda - r, \\ \mathrm{G\,F} &= \mathrm{G\,C}\sin.\theta = (\lambda - r)\sin.\theta, \\ \mathrm{F\,Z} &= \mathrm{Q\,P} = r\,\theta, \\ p &= \mathrm{G\,Z} = \mathrm{G\,F} + \mathrm{F\,Z}, \\ &= (\lambda - r)\sin.\theta + r\,\theta, \\ &= \lambda\sin.\theta + r\,(\theta - \sin.\theta). \end{aligned}$$

$\lambda \sin.\theta$ gives the metacentric portion of the stability, so to speak, and the rest of the expression gives the correction for the second centre of curvature. We may write the last equation—

$$p = \lambda\sin.\theta + 2\,\frac{\rho_1 - \rho_0}{\theta_1^2}\,(\theta - \sin.\theta);$$

or,

$$p = \lambda \sin. \theta + 2 (\rho_1 - \rho_0) \frac{\theta}{\theta_1^2} - 2 (\rho_1 - \rho_0) \frac{\sin. \theta}{\theta_1^2}.$$

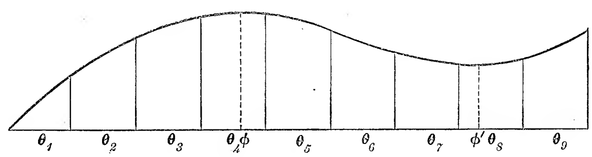
Of these three terms, each must be calculated separately.

While perfectly well aware of its essentially approximate character, and holding the process involved as inferior to that of M. Reech, previously described, which has the merit of being, not approximate, but exact, Dr. Joseph Woolley (whose complete mastery of this subject is well known, both at home and abroad) has favoured me with a very able and highly interesting extension of the above system, preserving the fundamental assumption of Rankine, that the curve of buoyancy is the involute of the involute of a circle, but rendering the process of calculation available for large angles of inclination.

Dr. Woolley's method consists in finding the radii of curvature to the curve of buoyancy at convenient angles of inclination of the vessel, not necessarily equal; then assuming that the portion of the involute of a circle lying between any two successive radii of curvature practically coincides with the corresponding portion of the evolute itself; and in then obtaining a succession of perpendiculars on the inclined normals, on subtracting from which the corresponding product, B G sin. θ, the arms of the righting levers at the several inclinations can be obtained, and thus the curve of stability readily laid down. This method differs from M. Reech's process in the circumstance that the latter obtains, from the *same elements*, the co-ordinates (parallel and perpendicular to the axis of the ship) of the successive centres of buoyancy.

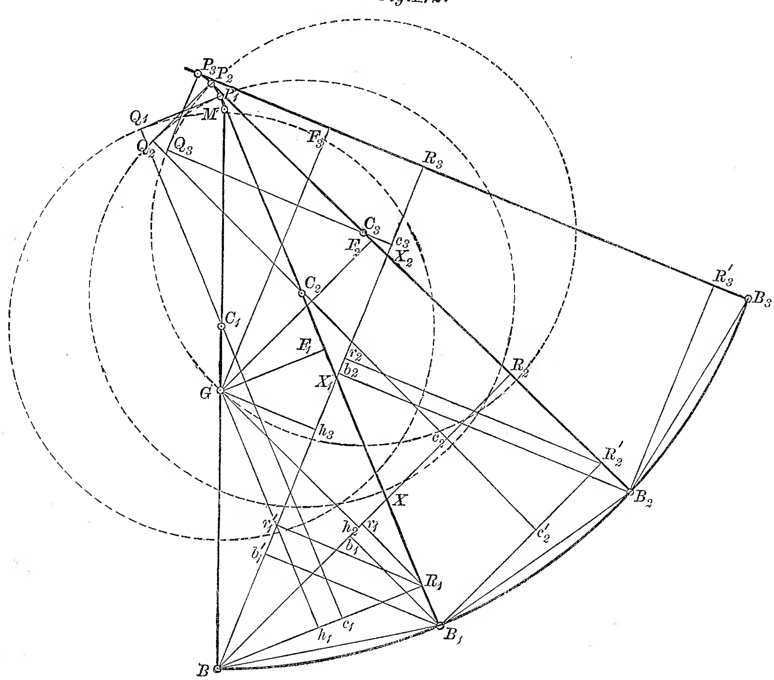
As regards the possibility of including a cusp—a point which has been suggested—and the means of knowing beforehand where such a point is reached, it may be observed that a cusp corresponds to a maximum or minimum value of the radius of curvature. The radius of curvature $= \frac{1}{3} \int y'^3 \ dx + \frac{1}{3} \int y''^3 \, dx$, depends on the greater or less dimensions of the inclined water-line; and this in turn will depend on the form of the ship. Generally speaking, so long as the sides of the ship flare out, it may be expected for general forms of ship that the successive radii of curvature will go on increasing; and as soon as they tumble home, they will begin to diminish. When the radius of curvature first begins to diminish, a cusp has been reached. If a curve be formed, of which the abscissæ are the angles of inclination, and the ordinates are the radii of curvature, the position of a cusp is readily found by the position of the maximum or minimum radius of curvature. Such a curve, it may be observed, is of the essence of M. Reech's process. Thus, in Fig. 171, represent-

Fig. 171.



ing such a curve, the dotted line at ϕ would represent a maximum, and the dotted line at ϕ' a minimum, and the corresponding angles, ϕ and ϕ', may be read off from the scale. It will be seen further on

Fig. 172.



that as the successive angles of inclination, θ_1, θ_2, θ_3, . . ., are not necessarily equal, there is no difficulty in finding the involute up to the cusp, and from the cusp, when the radii of curvature diminish; the generating circle changes in position, and the radii on which the involutes depend become negative in the formulæ, which otherwise retain the same form.

The following is Dr. Woolley's proof of the formula which he employs:—

In Fig. 172, M is the metacentre, $M P_1$, $P_1 P_2$, $P_2 P_3$, are successive arcs of involutes of circles described as hereafter explained, the points, P_1, P_2, P_3, being assumed to correspond fairly to the corresponding points of the true evolute; $M P_1$ is the arc between the inclination of 0° and θ_1; $P_1 P_2$, between θ_1 and θ_2; $P_2 P_3$, &c., between θ_2 and θ_3, &c.; and so on. $P_{n-1} P_n$ is the arc between θ_{n-1} and θ_n. B, B_1, B_2, B_3, &c., ... B_n, are the corresponding centres of buoyancy. Let ρ_0 be the metacentric height, B M, *i.e.*, radius of curvature to the curve of buoyancy at B; ρ_1, ρ_2, ρ_3, . . . &c., . . . ρ_n be the radii of curvature to the same curve at B_1, B_2, B_3, . . &c., . . B_n. Then, the angle between B M and $B_1 P_1 = \theta_1$; the angle between $B_1 P_1$ and $B_2 P_2 = \theta_2$; and so on. (In the figure, angles up to θ_3 only are shown).

$B R_1$, $B R_2$, $B R_3$, ... &c., . . . $B R_n$, are perpendiculars from B, on $P_1 B_1$, $P_2 B_2$, . . . &c., . . . $P_n B_n$.

$B_1 R'_2$, $B_2 R'_3$, &c., are perpendiculars from B_1 . B_2 . . &c., successively on $P_2 B_2$, $P_3 B_3$, &c.

Then, since the angle between the perpendiculars on two straight lines is equal to the angle between the lines themselves, the angle between $B R_1$ and $B R_2$ ($R_1 B R_2$) is equal to the angle between $P_1 B_1$ and $P_2 B_2 = \theta_2$; and so the angle between $B R_2$, or $B_1 R'_2$ and $P_3 B_3 = \theta_3$, and so on.

$M Q_1$ is a circle described through M with a centre, C_1, in B M; C_1 being so taken that $Q_1 C_1 c_1$, being drawn inclined to B M, at an angle θ_1, the arc, $M Q_1$, being unwrapped from this circle, traces out the involute, $M P_1$, corresponding to radii of curvature, ρ_0 and ρ_1. Similarly, $P_1 Q_2$, is an arc of a circle described about a point, C_2, in $P_1 B_1$, $Q_2 C_2 c_2$, making an angle, θ_2 with $P_1 B_1$; and $P_1 P_2$ is the arc of the involute corresponding to radii of curvature, ρ_1 and ρ_2, and so on.

Join $B B_1$, $B_1 B_2$, $B_2 B_3$, . . &c.

From the mode of generation, $Q_1 P_1$ is perpendicular to both $Q_1 C_1$ and $P_1 B_1$, and therefore parallel to $B R_1$. $Q_2 P_2$ is perpendicular to both $Q_2 C_2$ and $P_2 B_2$, and therefore parallel to $B R_2$ and $B_1 R'_2$; $Q_3 P_3$ similarly is parallel to $B R_3$, or $B_2 R'_2$.

Let r_1, r_2, r_3, &c., be the radii of the circles, $M C_1$, $P_1 C_2$, $P_2 C_3$, &c.

$$\text{Then, } Q_1 P_1 = \text{arc } Q_1 M = r_1 \theta_1,$$
$$Q_2 P_2 = \text{arc } Q_2 P_1 = r_2 \theta_2,$$
$$Q_3 P_3 = \text{arc } Q_3 P_2 = r_3 \theta_3.$$

$$\left.\begin{aligned}
(1.)\ B R_1 &= B c_1 + c_1 R_1 = B C_1 \sin.\theta_1 + Q_1 P_1 \\
&= (BM - M C_1) \sin.\theta + Q_1 P_1 \\
&= (\rho_0 - r_1) \sin.\theta + r_1 \theta_1. \\
R_1 B_1 &= P_1 B_1 - P_1 R'_1 = P_1 B_1 - Q_1 c_1 \\
&= P_1 B_1 - Q_1 C_1 - C_1 c_1 \\
&= \rho_1 - r_1 - (\rho_0 - r_1) \cos.\theta.
\end{aligned}\right\} A.$$

$$\left.\begin{aligned}
(2.)\ B_1 R'_2 &= B_1 c_2 + c_2 R'_2 = B_1 C_2 \sin.\theta_2 + Q_2 P_2 \\
&= (\rho_1 - r_2) \sin.\theta_2 + r_2 \theta_2. \\
R'_2 B_2 &= P_2 B_2 - P_2 R'_2 = P_2 B_2 - Q_2 c_2 \\
&= \rho_2 - r_2 - (\rho_1 - r_2) \cos.\theta_2.
\end{aligned}\right\} B.$$

$$\left.\begin{aligned}
\text{Similarly, } B_2 R'_3 &= (\rho_2 - r_3) \sin.\theta_3 + r_3 \theta_3. \\
R'_3 B_3 &= \rho_3 - r_3 - (\rho_2 - r_3) \cos.\theta_3. \\
\&c. &= \&c.
\end{aligned}\right\} C.$$

$$\left.\begin{aligned}
B_{n-1} R'_n &= (\rho_{n-1} - r_n) \sin.\theta_n + r_n \theta_n. \\
R'_n B_n &= \rho_n - r_n - (\rho_{n-1} - r_n) \cos.\theta_n.
\end{aligned}\right\} N.$$

Now, $B R_2$ is evidently the projection of $B B_1$ on $B R_2 + B_1 R'_2$ ($= b_1 R_2$); and the projection of $B B_1$ is the sum of the projections of $B R_1$ and $R_1 B_1$ on the same line, and, as shown before, the angle which $B R_1$ makes with $B R_2 = \theta_2$, and the angle which $R_1 B_1$ makes with $B R_2 = R_1 X R_2 = \frac{\pi}{2} + \theta_2$.

Therefore the projection of

$$\begin{aligned}
B B_1 &= B R_1 \cos.\theta_2 + R_1 B_1 \cos.\left(\frac{\pi}{2} + \theta_2\right) = B R_1 \cos.\theta_2 - R_1 B_1 \sin.\theta_2 \\
&= (\rho_0 - r_1) \sin.\theta_1 \cos.\theta_2 + r_1 \theta_1 \cos.\theta_2 - (\rho_1 - r_1) \sin.\theta_2 \\
&\quad + (\rho_0 - r_1) \cos.\theta_1 \sin.\theta_2 \\
&= (\rho_0 - r_1) \sin.(\theta_1 + \theta_2) - (\rho_1 - r_1) \sin.\theta_2 + r_1 \theta_1 \cos.\theta_2.
\end{aligned}$$

Hence,

$$\begin{aligned}
B R_2 &= (\rho_0 - r_1) \sin.(\theta_1 + \theta_2) - (\rho_1 - r_1) \sin.\theta_2 + r_1 \theta_1 \cos.\theta_2 \\
&\quad + (\rho_1 - r_2) \sin.\theta_2 + r_2 \theta_2 \\
&= (\rho_0 - r_1) \sin.(\theta_1 + \theta_2) - (r_2 - r_1) \sin.\theta_2 + r_1 \theta_1 \cos.\theta_2 + r_2 \theta_2 \;\ldots\; (\beta.)
\end{aligned}$$

Also,

$B R_3 =$ Projection of $B B_1$ on $B R_3 +$ projection of $B_1 B_2$ on $B R_3 + B_2 R'_3$.

Projection of $B B_1$ = Sum of projections of $B R_1$ and $R_1 B_1$, which make with $B R_3$ respectively, the angles, $\theta_2 + \theta_3$ and $\frac{\pi}{2} + (\theta_2 + \theta_3)$

$$= (\rho_0 - r_1) \sin. \theta_1 \cos. (\theta_2 + \theta_3) + r_1 \theta_1 \cos. (\theta_2 + \theta_3) - (\rho_1 - r_1) \sin. (\theta_2 + \theta_3) + (\rho_0 - r_1) \cos. \theta_1 \sin. (\theta_2 + \theta_3)$$

$$= (\rho_0 - r_1) \sin. (\theta_1 + \theta_2 + \theta_3) - (\rho_1 - r_1) \sin. (\theta_2 + \theta_3) + r_1 \theta_1 \cos. (\theta_2 + \theta_3).$$

Similarly the projection of $B_1 B_2$, or sum of projections of $B_1 R'_2$ and $R'_2 B_2$, making angles, θ_3 and $\frac{\pi}{2} + \theta_3$ respectively, with $B R_3$,

$$= (\rho_1 - r_2) \sin. (\theta_2 + \theta_3) - (\rho_2 - r_2) \sin. \theta_3 + r_2 \theta_2 \cos. \theta_3.$$

Hence,

$$\begin{aligned} BR_3 = & (\rho_0 - r_1) \sin. (\theta_1 + \theta_2 + \theta_3) - (\rho_1 - r_1) \sin. (\theta_2 + \theta_3) + r_1 \theta_1 \cos. (\theta_2 + \theta_3) \\ & + (\rho_1 - r_2) \sin. (\theta_2 + \theta_3) - (\rho_2 - r_2) \sin. \theta_3 + r_2 \theta_2 \cos. \theta_3 \\ & + (\rho_2 - r_3) \sin. \theta_2 + r_3 \theta_3 \end{aligned}$$

$$\begin{aligned} = & (\rho_0 - r_1) \sin. (\theta_1 + \theta_2 + \theta_3) - (r_2 - r_1) \sin. (\theta_2 + \theta_3) - (r_3 - r_2) \sin. \theta_3 \\ & + r_1 \theta_1 \cos. \overline{\theta_2 + \theta_3} + r_2 \theta_2 \cos. \theta_3 + r_3 \theta_3. \quad . \quad . \quad . \quad (\gamma.) \end{aligned}$$

Similarly, $B R_4$

$$\begin{aligned} = & (\rho_0 - r_1) \sin. (\theta_1 + \theta_2 + \theta_3) - (r_2 - r_1) \sin. (\theta_2 + \theta_3 + \theta_4) \\ & - (r_3 - r_2) \sin. (\theta_3 + \theta_4) - (r_4 - r_3) \sin. \theta_4 \\ & + r_1 \theta_1 \cos. (\theta_2 + \theta_3 + \theta_4) + r_2 \theta_2 \cos. (\theta_3 + \theta_4) \\ & + r_3 \theta_3 \cos. \theta_4 + r_4 \theta_4, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\delta.) \end{aligned}$$

and so on.

Generally, if there be (n) angles, $\theta_1, \theta_2, \ldots . \theta_{n-1}, \theta_n$, $B R_n$ = sum of projections of $B B_1$, $B_1 B_2$, $B_2 B_3$ . . . . $B_{n-2} B_{n-1}$ and $B_{n-1} B_n$, corresponding to which are the angles,

$$(\theta_2 + \theta_3 + \ldots . + \theta_{n-1} + \theta_n) \text{ and } \frac{\pi}{2} + (\theta_2 + \theta_3 + \ldots . + \theta_{n-1} + \theta_n),$$

$$(\theta_3 + \theta_4 + \ldots . + \theta_n) \text{ and } \frac{\pi}{2} + (\theta_3 + \theta_4 + \ldots . + \theta_n),$$

$$\ldots \ldots \ldots \text{ and } \ldots \ldots \ldots$$

$$\theta_n \text{ and } \frac{\pi}{2} + \theta_n,$$

and 0.

Hence, we get by proceeding in the same way,

$$\begin{aligned} B R_n = (\rho_0 - r_1) \sin. (\theta_1 + \theta_2 + \ldots + \theta_n) \\ - (r_2 - r_1) \sin. (\theta_2 + \theta_3 + \ldots + \theta_n) \\ - (r_3 - r_2) \sin. (\theta_3 + \theta_4 + \ldots + \theta_n) \\ - (r_{n-1} - r_n) \sin. \theta_n + r_1 \theta_1 \cos. (\theta_2 + \theta_3 + \ldots + \theta_n) \\ + r_2 \theta_2 \cos. (\theta_3 + \ldots + \theta_n) + r_3 \theta_3 \cos. (\theta_4 + \ldots + \theta_n) \\ + \ldots + r_{n-1} \theta_{n-1} \cos. \theta_n + r_n \theta_n. \end{aligned}$$

To determine r_1, r_2, &c.,

Let $P_{n-1} P_n$, Fig. 173, be any one of the involutes, so that arc $P_{n-1} P_n$ = difference of radii of curvature at P_{n-1}, and P_n.

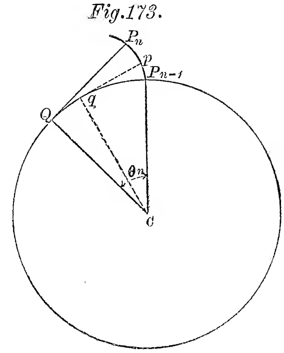


Fig. 173.

$P_{n-1} C Q = \theta_n$,

$P_{n-1} C q = \theta$, and $q p$ = arc $q P_{n-1}$.

Let $s = P_{n-1} p$.

Then, $d s = p q d \theta = r_n \theta d \theta$,

$s = \frac{1}{2} r_n \theta^2$,

from 0 to $\theta = \theta_n$,

and $P_{n-1} P_n = \frac{1}{2} r_n \theta_n^2 = \rho_n - \rho_{n-1}$ by property of evolute, whence,

$$r_n = \frac{2(\rho_n - \rho_{n-1})}{\theta_n^2}.$$

Putting $n = 1, 2, 3, \ldots \ldots n$, respectively,

$$r_1 = \frac{2(\rho_1 - \rho_0)}{\theta_1^2}; \; r_2 = \frac{2(\rho_2 - \rho_1)}{\theta_2^2}; \; r_3 = \frac{2(\rho_3 - \rho_2)}{\theta_3^2};$$

$$r_{n-1} = \frac{2(\rho_{n-1} - \rho_{n-2})}{\theta_{n-1}^2}; \; r_n = \frac{2(\rho_n - \rho_{n-1})}{\theta_n^2};$$

$\rho_0, \rho_1, \rho_2, \rho_3, \ldots \rho_n$, are to be determined as in Reech's, or any other method, being the moments of inertia about the axis of the centre of figure of the areas of the several inclined water-lines.

In this way the several arms, $B R_1$, $B R_2$, $B R_3$, ... $B R_n$, being found, the corresponding arms of righting levers may at once be found.

Thus, $G F_1 = B R_1 - B h_1 = B R_1 - B G \sin. B G h_1$

$= B R_1 - b \sin. \theta$, putting $b = B G$.

Similarly, $G F_2 = B R_2 - B h_2 = B R_2 - b \sin. (\theta_1 + \theta_2)$,

$G F_3 = B R_3 - b \sin. (\theta_1 + \theta_2 + \theta_3)$,

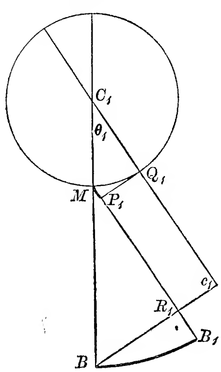
$G F_n = B R_n - b \sin. (\theta_1 + \theta_2 + \theta_3 + \ldots \ldots + \theta_n)$, and from these several values the curve of stability may be at once laid down as usual.

In this demonstration the radii of curvature are supposed to increase from ρ_0 to ρ_n.

If, on the contrary, they diminish, we shall have, $r_1 = \frac{2(\rho_1 - \rho_0)}{\theta_1^2}$ $= \frac{-2(\rho_0 - \rho_1)}{\theta_1^2}$, and is negative; $r_2 = \frac{2(\rho_2 - \rho_1)}{\theta_2^2} = \frac{-2(\rho_1 - \rho_2)}{\theta_2^2}$, a negative quantity.

In the Fig. 174, draw the circle $M Q_1$, on $B M$ produced: let $M P_1$ be the arc of the involute described by unwrapping the string, $M Q_1$, so that as before, $P_1 Q_1 = \text{arc } M Q_1 = r_1 \theta_1$, $C_1 Q_1 c_1$ being drawn at an angle, θ_1, with $B M$ produced.

Fig. 174.



Then, $M P_1 = M B - P_1 B_1 = \rho_0 - \rho_1$,

$R_1 c_1 = P_1 Q_1 = r_1 \theta_1$,

and, $B R_1 = B c_1 - R_1 c_1 = B C_1 \sin. \theta_1 - R_1 c_1$
$= (\rho_0 + r_1) \sin. \theta - r_1 \theta_1$,

and $R_1 B_1 = P_1 B_1 - P_1 R_1 = P_1 B_1 - Q_1 c_1$
$= P_1 B_1 - (C_1 c_1 - C_1 Q_1)$,
$= P_1 B_1 + C_1 Q_1 - C_1 B \cos. \theta_1$,
$= \rho_1 + r_1 - (\rho_0 + r_1) \cos. \theta_1$.

We shall obtain expressions of a similar form for $B_1 R'_2$, $R'_2 B_2$, &c., &c., and the general expression will become in this case—

$$\begin{aligned} B R_n = {} & (\rho_0 - r_1) \sin. (\theta_1 + \theta_2 + \ldots + \theta_n) + (r_1 - r_2) \sin. (\theta_2 + \theta_3 + \ldots \theta_n) \\ & + (r_2 - r_3) \sin. (\theta_3 + \theta_4 + \ldots \theta_n) + \ldots + (r_{n-1} - r_n) \sin. \theta_n \\ & - r_1 \theta_1 \cos. (\theta_2 + \theta_3 + \ldots + \theta_n) - r_2 \theta_2 \cos. (\theta_3 + \ldots + \theta_n) \\ & - r_{n-1} \theta_{n-1} \cos. \theta_n - r_n \theta_n, \end{aligned}$$

which may be obtained at once from the former by putting

$$- r_1, - r_2 \ldots\ldots - r_n, \text{ for } r_1 . r_2 \ldots\ldots r_n.$$

If the radii at first increase and then diminish, *i.e.*, if the metacentric evolute pass through a cusp, the formula must be slightly modified. Let the cusp correspond to the angle, θ_m, m being less than n.

Then, $\rho_0, \rho_1 \ldots\ldots \rho_m$, are all positive, and $\rho_{m+1}, \rho_{m+2}, \ldots\ldots\ldots \rho_{n-1}, \rho_n$, are negative, and the formula takes the form—

$$\begin{aligned}
B\,R_n = & (\rho_0 - r_1) \sin. (\theta_1 + \theta_2 + \ldots . \theta_n) \\
& - (r_2 - r_1) \sin. (\theta_2 + \theta_3 + \ldots . \theta_n) - \ldots . \\
& - (r_m - r_{m+1}) \sin. (\theta_m + \theta_{m+1} + \ldots . \theta_n) \\
& + (r_{m+1} + r_m) \sin. (\theta_{m+1} + \theta_{m+2} + \ldots . \theta_n) \\
& + (r_{m+1} - r_{m+2}) \sin. (\theta_{m+2} \ldots \theta_n) + \ldots + (r_{n-1} - r_n) \sin. \theta_n \\
& + r_1\, \theta_1 \cos. (\theta_2 + \theta_3 + \ldots . \theta_n) + r_2\, \theta_2 \cos. (\theta_3 + \theta_4 + \ldots . \theta_n) \\
& + r_m\, \theta_m \cos. (\theta_{m+1} + \ldots . \theta_n) - r_{m+1}\, \theta_{m+1} \cos. (\theta_{m+2} + \ldots . + \theta_n) \\
& - \ldots .. - r_{n-1}\, \theta_{n-1} \cos. \theta_n - r_n\, \theta_n .
\end{aligned}$$

How to find the exact position of a cusp has been pointed out; but it is essential to the success of this method that series of successive involutes shall commence and terminate at cusps.

It is hence seen that this method is not founded on the principle of finding the radius of curvature in any extreme position; but on a succession at given convenient finite angles. In this, as in other methods, such as Reech's, the successive values of $B\,R_1$, $B\,R_2$, $B\,R_3$ .... $B\,R_n$ involve all that precede it.

If, as would generally be the case, the angles, θ_1, θ_2, θ_3 .... θ_n, be taken equal, the formulæ become modified.

$$\begin{aligned}
\text{Thus, } B\,R_n = & (\rho_0 - r_1) \sin. n\, \theta_1 - (r_2 - r_1) \sin. (n - 1)\, \theta_1 \\
& - (r_3 - r_2) \sin. (n - 2)\, \theta_1 - \ldots - (r_n - r_{n-1}) \sin. \theta_1 \\
& + r_1\, \theta_1 \cos. (n - 1)\, \theta + r_2\, \theta_1 \cos. (n - 2)\, \theta + \ldots . \\
& + r_{n-1}\, \theta_1 \cos. \theta_1 + r_n\, \theta_1 .
\end{aligned}$$

But if, for convenience, supposing the series to terminate in a cusp, the first $(n - 1)$ angles be taken equal and the last, θ_n, be different, we shall have—

$$\begin{aligned}
B\,R_n = & (\rho_0 - r_1) \sin. (\overline{n - 1}\, \theta_1 + \theta_n) - (r_2 - r_1) \sin. (\overline{n - 2}\, \theta_1 + \theta_n) \\
& - \ldots\ldots - (r_n - r_{n-1}) \sin. \theta_n \\
& + r_1\, \theta_1 \cos. (\overline{n - 2}\, \theta_1 + \theta_n) + r_2\, \theta_1 \cos. (\overline{n - 3}\, \theta_1 + \theta_n) \\
& + r_{n-1}\, \theta_1 \cos. \theta_n + r_n\, \theta_n .
\end{aligned}$$

In a test case to which Dr. Woolley applied this method, viz., to a prism with a rectangular section, having a breadth of 70 feet and a depth, 29·36 feet, which just brings one of the lower angles to the surface of the water at 40°, he employed this as his maximum angle, because the error would be very sensible. Assuming the centre of gravity of the vessel to be 7·5 feet above the centre of buoyancy in the upright position, and supposing the displacement to be 10,000 tons, he found the righting moment, at an inclination of 40°, to be on the approximate method, 68,697 foot-tons; and on the direct or true method, 71,624 foot-tons, giving a defect of 2,927 foot-tons, which is about $\frac{1}{25}$th part of the true value in defect.

If the calculation is made at intervals of 5° instead of 10°, this error is reduced by about $\frac{1}{2}$, or to $\frac{1}{50}$th part of the true value. These errors are probably greater than would be found in the case of a vessel of an ordinary form. Moreover if, in order to make the approximation closer, angles of 5° are taken instead of 10°, the labour of the calculation is considerably increased, and little if any advantage is gained from this point of view. Of course, as said before, Reech's method, which obtains at once the co-ordinates of the centre of buoyancy, is to be preferred.

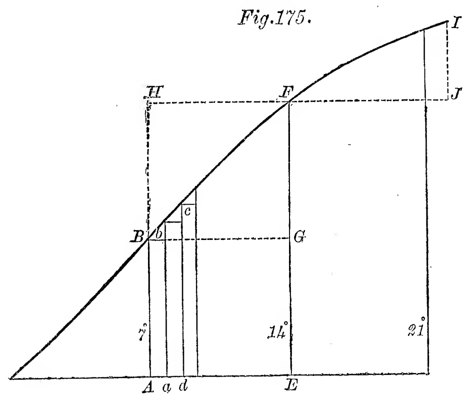
CHAPTER XV.

Dynamical Stability—Different Views of Same—Canon Moseley's Paper on Dynamical Stability—M. Moreau's *Mémoire* on the Subject—Fundamental Doctrines set forth by Canon Moseley—Vis Viva and Work—Formulæ for Dynamical Stability—Amount of Error in Same—Application of Canon Moseley's Formulæ to Ships—Criticism on Moseley's General Equation—M. Bertin's view of Dynamical Stability—View of MM. Risbec and Duhil de Benazé—M. Guyou's Paper on a New Theory of Stability—Interesting Investigation of Surface of Buoyancy and its *Podaire*—An Imaginative Method of Explaining the Doctrines of Stability—Maxima, Minima, and Mixed Normals—Application of M. Guyou's Method to Dynamical Stability.

WE come now to consider the *dynamical* aspects of the subject. Let us first endeavour to make understood, in plain terms, and with the fewest possible technical phrases, what is ordinarily meant by "Dynamical Stability." One view of it may be obtained in this way:—We have already observed that the forces which resist the inclination of the ship are vertical forces, the weight of the ship acting downwards, and the buoyancy acting upwards with an equal pressure. If we could follow the action of the inclining forces closely throughout the inclination of the ship to a given angle, we should find them continually overcoming these resisting forces (of weight and buoyancy) through successive infinitesimally small spaces, their action taking effect either in actually raising the centre of gravity, or in lowering the centre of buoyancy, or in both. If we could multiply the force (weight of ship) at every point into the very small vertical spaces through which these centres move, we should obtain the effect (known technically as the "work" done) from point to point, and adding all these small effects together (or subtracting them, as the case may be), we should get the total "work" done during the given amount of inclination. This we cannot possibly do in detail, as we cannot handle infinitesimal quantities; but what we can do is this: we can find the total vertical distance through which the centres of gravity and buoyancy have *separated* during the inclination, and multiply this by the weight of the ship, and in this way obtain the result we desire. For it is obvious that if we multiplied the weight into each infinitesimal space in succession, and then added all the products together, we should obtain exactly the same result as we obtain by adding all the small spaces together first (that is, by taking the total separation of the centres), and multiplying the sum by the weight. In either case we should obtain the equivalent of the total "*work*" done upon the ship by the inclining force, and that total work is the dynamical stability.*

Another view of the matter may be obtained as follows:—The dynamical stability developed during the inclination of the ship from one angle to another only infinitesimally greater, will obviously be equal to the product of the statical stability at the given angle into the infinitesimally small space through which it acts. And, in like manner, the dynamical stability developed during the inclination of the ship through a finite angle will be equal to the sum of a series of such products. But we have seen that the statical stability is, at every point, *proportional* to the length of the lever, G Z, and consequently the dynamical stability from point to point will be *proportional* to the product of that lever into an infinitesimally small space. For example, let Fig. 175 represent the curve of statical

Fig. 175.



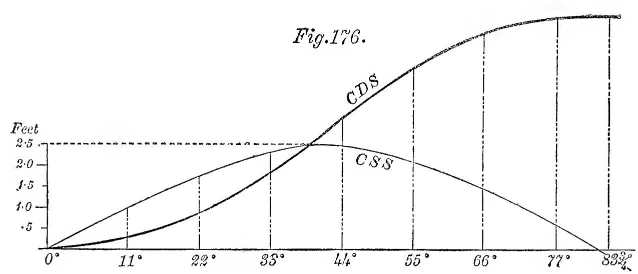
stability of a ship at moderate angles of inclination; then the dynamical stability developed by inclining the ship, say from 7° to an angle slightly exceeding 7°, will be equal to A B multiplied into

* See *Naval Science*, vol. i.

a very short portion of the space between 7 and 14°; and that product is obviously approximately equal to the very small rectangular area, A B *b a*. Similarly the dynamical stability developed during a further very small angle of inclination will be equal to the rectangular area, *a b c d*; and so we may proceed from point to point until we have covered the whole space between A B and E F with very small rectangles, the sum of which will approximately represent (by being proportional to) the whole dynamical stability or work done during the inclination of the ship from an angle of 7° to an angle of 14°. In other words, the dynamical stability exerted during the inclination of the ship from one angle to another is represented by the corresponding portion of the curvilinear *area* in such diagrams of statical stability as we have been considering.

It is obvious that the curvilinear area lying between 0° of inclination and any given angle can be calculated, and the result made the ordinate of a new curve. By repeating the operation, a series of points may be obtained, and a curve passed through all such points will represent at every point the dynamical stability that must be overcome in inclining the ship from the upright position to the angle indicated. By subtracting one ordinate from another, the dynamical stability exerted in inclining the ship from the angle indicated by the one ordinate to that indicated by the other can be found. In Fig. 176 is indicated, by C D S, the curve of dynamical stability of

Fig. 176.



an unarmoured corvette, whose section, curve of flotation, F F, curve of buoyancy, B B, and loci of movicentres, M M, are shown in Fig. 177. The curve, C D S, was obtained by graphic integration from the curve of statical stability, C S S. Fig. 178 exhibits curves of statical and dynamical stability for the armour-plated, flat-bottomed ram, whose section, curves of flotation, and buoyancy, &c., are shown in Fig. 179. The curves, No. 1, correspond to the usual condition of the ship with its sides intact; the curves, No. 2, correspond to its

Fig. 177.

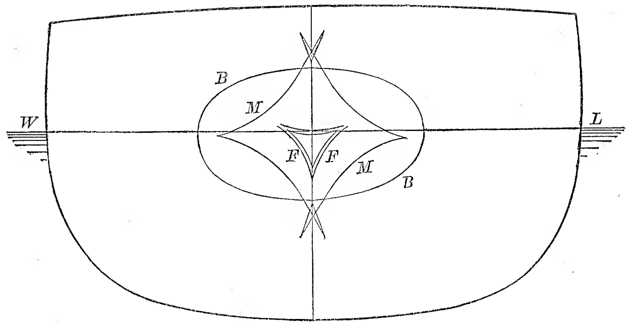


Fig. 178.

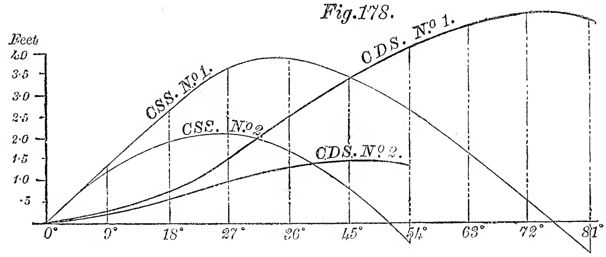
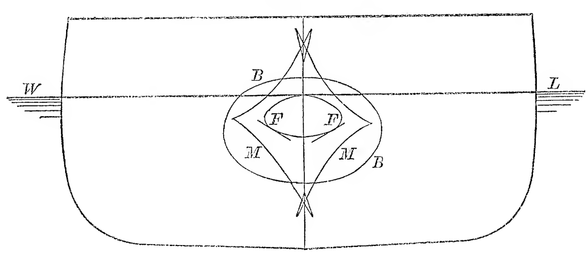


Fig. 179.



condition with certain unarmoured ends riddled. The following is a comparison between the statical and dynamical stabilities in the two cases:—*

	Maximum Righting Lever.	Angle of Maximum Stability.	Range of Stability.
Condition No. 1, ...	3·85 feet.	34 degrees.	$76\frac{1}{4}$ degrees.
,, No. 2, ...	1·7 ,,	23 ,,	$50\frac{1}{2}$,,

The question of dynamical stability may be viewed in a somewhat different manner. Reverting to Fig. 175, the ordinate (A B, say) represents the arm of the couple, G Z, at the extremities of

* See *Transactions, Institution of Naval Architects* for 1877, vol. xviii.—Mr. White's Account of the Work done by the Students of the Royal Naval College.

which the weight of the ship and the buoyancy of the ship (equal to the weight) are acting vertically; and, in the case of further inclination of the ship taking place, we may obviously assume one of these extremities of the lever, G Z, remains at rest and the other moves—no matter which. For a very small, an infinitesimal, amount of motion, the weight of the ship may be presumed to prevail, and to move through a very small vertical space, which may be represented by $d\,\theta$. Then $\mathrm{G\,Z}\,d\,\theta$ will represent the space moved through, and $\mathrm{W} \times \mathrm{G\,Z}\ d\,\theta$ will equal the "work" done by the inclining force in effecting the very small change of inclination. The whole work done in inclining the ship from the upright through an angle, θ, will therefore be

$$\int_0^\theta \mathrm{W} \times \mathrm{G\,Z}\,d\,\theta;$$

and the work done in inclining her from an angle, θ_1, to an angle, θ_2, will be

$$\int_{\theta_1}^{\theta_2} \mathrm{W} \times \mathrm{G\,Z}\,d\,\theta.$$

That this agrees perfectly with what has been said respecting Fig. 175 will be obvious, for A B there corresponds to G Z; B b to $d\,\theta$; and the areas, as already explained, are equivalent to the integrations suggested by the above expressions. But we need not further anticipate the late Canon Moseley's investigations.

At the commencement of his famous Paper on "Dynamical Stability," read at the Royal Society in 1850, Moseley, after speaking of the oscillations of floating bodies under the action of disturbing forces, and of the different amounts of "work" done upon them, the most stable requiring the greatest amount, said—"It is this condition of stability, dependent upon dynamical considerations, to which, in the following Paper, the name of Dynamical Stability is given;" adding—"I cannot find that the question has before been considered in this point of view, but only in that which determines whether any given position be one of stable, unstable, or mixed equilibrium, or which determines what pressure is necessary to retain the body at any given inclination from such a position." It has been suggested that, although Moseley was well acquainted with the works of Poisson, Poncelet, and other Continental philosophers, this passage seems to indicate that he was not in a position to do justice to the actual condition of the science of naval architecture in France at the time of writing his paper, or he must have known that the dynamical conditions of floating bodies, with reference to their oscillations in still water, had, before 1850, received both distinct recognition and skilful treatment. We are at present in possession (by the courtesy of M. E. L. Bertin) of a work published at Brest in 1830 from the pen of M. Moreau (then Professor of the *Ecole Royale du Génie Maritime*, and an *Ingénieur de première classe au Corps Royal du Génie Maritime*), entitled "*Principes Fondamentaux de l'Equilibre et du Mouvement des Corps Flottans Dans deux Milieux Résistans*," in which the dynamical aspect of the subject is kept in view throughout; and this work was not a mere geometrical exercise thrown out for the study of *savans*, but was expressly printed for the use of the pupils of the Royal School. M. Moreau commenced his treatise by stating that the mechanical principle laid down by Maupertuis, under the name of the "Law of Repose," was perfectly applicable to floating bodies, although it had never previously been applied to them, and he proceeded to set forth and to demonstrate, as one of the fundamental principles of naval science, that "in the state of equilibrium of a body immersed in ponderable fluids, the centre of gravity of the system (the floating body and the fluids) is the lowest or the highest possible." He added that this theorem, when combined with the "*formule de la dynamique*," will enable us to resolve with great facility the problem of the small oscillations of floating bodies. In investigating the equilibrium of the floating body in still water, he treated it as partly immersed in two fluids, the air as well as the water, and thus added much, and somewhat unnecessarily, to the extent and complication of his mathematical equations; he also adopted the device (which we shall presently see again resorted to more than once by French investigators) of considering the water which floats the body as contained in a basin or closed vessel—a device which he doubtless introduces for the express purpose of facilitating the *dynamical* treatment of the subject.

We have carefully considered the *Mémoire* of M. Moreau, and are willing to accord to it great merit as a sound exposition of the dynamical conditions of a floating body subject to very small oscillations; but it dealt *only* with very small motions, and dealt with them for the limited purpose of obtaining maximum and minimum values of the height of the centre of gravity. Moseley took a wholly different, and very much wider, view of the question, dealing with large angles of inclination, and applying the principles of *vis viva* and "work" for a very different object. We cannot see that it would have made any difference in his treatment of the subject had he been ever so familiar with M. Moreau's investigations, able and interesting as they undoubtedly were.

Much of Moseley's paper was occupied with an account of experiments made in Portsmouth Dockyard by the late Mr. John Fincham and Mr. Rawson, which were interesting and valuable at the time, but which would not now assist the appreciation of the doctrine of dynamical stability. It will be well, however (especially as the original paper is difficult of access to most persons), to state here the fundamental doctrines, and some of the inferences which Moseley set forth.

His investigations were based, as has been intimated, upon the principles of *vis viva* and *work*, the primary of which principles he thus rendered:—

"When, being acted upon by given forces, a body or system of bodies has been moved from a state of rest, the difference between the aggregate work of those forces whose tendencies are *in* the directions in which their points of application have been moved, and that of the forces whose tendencies are in the opposite direction, is equal to one-half the *vis viva* of the system."

If Σu_1 represent the aggregate *work* of the forces displacing the body from rest, and Σu_2 the aggregate *work* of the other forces applied to it; and if the terms composing Σu_1 and Σu_2 be taken positively or negatively, according as the forces at work tend to move their points of application in the directions in which they do move, or in opposite directions, then, putting for the aggregate *vis viva* of the body $\frac{1}{g} \Sigma w v^2$,

$$\Sigma u_1 + \Sigma u_2 = \frac{1}{2g} \Sigma w v^2 - . \quad . \quad . \quad . \quad . \quad (1).$$

Σu_2 may be presumed known, because that expression represents the aggregate work of the operating forces which move the body from rest to a new position, and Σu_1 may therefore be determined in terms of the *vis viva;* or conversely. When a body oscillates, it comes to rest for an instant in its extreme position, and there its *vis viva* consequently disappears, and we have

$$\Sigma u_1 + \Sigma u_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad (2).$$

This equation, therefore, determines either the extreme position; or, that position being given, determines the forces producing the motion.

But, by a well-known property, the *vis viva* of a system is a maximum when it passes through a *stable* position of equilibrium, and a minimum when it passes through an unstable position of equilibrium. The position of *final rest* is therefore that in which $\Sigma u_1 + \Sigma u_2$ has a maximum value. The body will therefore rest in a very different position from that which it occupies when in the extreme position of its oscillation.

Of different bodies, requiring different amounts of work to incline them to a given extent, that is the most *stable* which requires the greatest amount of work to be done upon it in inclining it to that extent, Σu_1 being in that case the greatest. If all such bodies are respectively brought into positions of unstable equilibrium, the corresponding values of Σu_1 then represent the work requisite to overthrow it. In the former case, Σu_1 represents the *relative,* and in the latter case the *absolute*, stability of the body. The absolute stability of a given body may be greater than that of another, notwithstanding that its *relative* stability, with reference to a given inclination, may be less.

Canon Moseley represents the absolute dynamical stability of a body by U, and its relative dynamical stability, corresponding to an inclination, θ, by U (θ). U is, of course, the maximum of U (θ).

The work opposed by the weight of a body to any change in its position is measured by the product of its weight by the vertical elevation of its centre of gravity. If W be its weight, Δ H the vertical displacement of the centre of gravity occasioned by the inclination of the body through the angle, θ (this displacement being in a direction opposite to that in which the force applied to it acts), we have $\Sigma u_2 = \mathrm{W} \cdot \Delta \mathrm{H}$, and, consequently (from equation (2)),

$$\mathrm{U}\,(\theta) - \mathrm{W}\,\Delta\,\mathrm{H} = \mathrm{O} \qquad . \quad . \quad . \quad . \quad (3).$$

The absolute dynamical stability of a body resting on a rigid surface is therefore measured (if no force other than its weight oppose its overthrow) by the product of its weight by the height through which its centre of gravity must be raised to bring it from a position of stable to one of unstable equilibrium.

The foregoing considerations are of general application to all heavy bodies. We will now give their application to floating bodies in Canon Moseley's own language :—

Let a body be conceived to float, acted upon by no other forces than its weight, W, and the upward pressure of the water (equal to its weight); which forces may be conceived to be applied respectively to the centre of gravity of the body, and to the centre of gravity of the displaced fluid; and let it be supposed to be subjected to the action of a third force whose direction is parallel to the surface of the fluid. Let ΔH_1 represent the vertical displacement of the centre of gravity of the body thereby produced, and ΔH_2 that of the centre of gravity of its immersed part. Let, moreover, the volume of the immersed part be conceived to remain unaltered whilst the body is in the act of displacement. If each centre of gravity be assumed to ascend, the work of the weight of the body will be represented by $-W.\Delta H_1$, and that of the upward pressure of the fluid by $+W.\Delta H_2$, the negative sign being taken in the former case, because the force acts in a direction opposite to that in which the point of application is moved, and the positive sign in the latter, because it acts in the same direction, so that the aggregate work, Σu_2 (see equation 1), of the forces which constituted the equilibrium of the body in the state from which it has been disturbed is represented by

$$-W.\Delta H_1 + W.\Delta H_2.$$

If the centre of gravity of the body or of the displaced fluid *descends* (a property which will be found to characterise a large class of vessels), ΔH_1 in the one case, and ΔH_2 in the other, must be taken with the negative sign, since the weight of the body will be applied in the same direction, and the pressure of the fluid in an opposite direction to that in which their respective points of application are moved. Moreover, the system put in motion includes, with the floating body, the particles of the fluid displaced by it as it changes its position, so that if the weight of any element of the floating body be represented by w_1, and of the fluid by w_2, and if their velocities be v_1 and v_2, the whole *vis viva* is represented by

$$\frac{1}{g}\Sigma w_1 v_1^2 + \frac{1}{g}\Sigma w_2 v_2^2,$$

and we have (by equation 1),

$$U(\theta) - W(\Delta H_1 - \Delta H_2) = \frac{1}{2g}\Sigma w_1 v_1^2 + \frac{1}{2g}\Sigma w_2 v_2^2 \quad . \; . \; . \; . \quad (4).$$

In the extreme position into which the body is made to roll and in which $\Sigma w_1 v_1^2 = 0$,

$$U(\theta) = W.(\Delta H_1 - \Delta H_2) + \frac{1}{2g} \quad w_2 v_2^2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (5);$$

or, if the inertia of the displaced fluid be neglected,

$$U(\theta) = W.(\Delta H_1 - \Delta H_2) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6).$$

Whence it follows that the work necessary to incline a floating body through any given angle is equal to that necessary to raise it bodily through a height equal to the difference of the vertical displacements of its centres of gravity and of that of its immersed part, so that, other things being the same, that ship is the most stable, the product of whose weight by this difference is the greatest. In the case in which the centre of gravity of the displaced fluid descends, the *sum* of the displacements is to be taken instead of the difference.

Canon Moseley points out that this conclusion is in error in the following respects :—

1st. It supposes that throughout the motion the weight of the displaced fluid remains equal to that of the floating body, which equality cannot accurately have been preserved by reason of the inertia of the body and of the displaced fluid. From this cause there cannot but result small vertical oscillations of the body about those positions which, whilst it is in the act of inclining, correspond to this equality, which oscillations are independent of its principal oscillation.

2ndly. It involves the hypothesis of absolute rigidity in the floating body, so that the motion of every part and its *vis viva* may cease *at once* when the principal oscillation terminates. The frame of a ship and its masts are, however, elastic, and by reason of this elasticity there cannot but result oscillations, which are independent of, and may not synchronise with, the principal oscillation of the ship as she rolls, so that the *vis viva* of every part cannot be assumed to cease and determine at the same instant, as it has been supposed to do.

3rdly. No account has been taken of the work expended in communicating *motion* to the displaced fluid, measured by half its *vis viva*, and represented by the term $\frac{1}{2g} \Sigma w_2 v_2^2$ in equation (5).

From a careful consideration of these causes of error, Moseley was led to conclude that they would not affect that practical application of the formula which he had principally in view in investigating it, especially as in certain respects they tended to neutralise one another. The experiments made at Portsmouth, previously adverted to, confirmed this view.

In applying his formulæ to ships, Moseley deals first with the case of vessels whose athwartship sections (where subject to immersion and emersion) are circular, having their centres in a common longitudinal axis; these vessels being of two types, viz. (1) such as are shown in Fig. 180, in which the lower part of the ship's section extends below the completed circle; and (2) such as are shown in Fig. 181, in which the lower part lies within the completed circle. E T F is the circular part in both figures, C being its centre. G_1 is the projection of the centre of gravity; G_2 that of the part S D R T (supposing it filled with water); $h_1 = C G_1$; $h_2 = C G_2$; W_1, weight of vessel; W_2, weight S D R T (supposed to be water); and θ the inclination of the ship. The whole volume of the fluid displaced remains constant during the inclination, and so also do that of the immersed circular part and that of the part S D R T. The water-line, P Q, is the same distance from C as was A B, so that C neither rises nor falls.

Fig. 180.

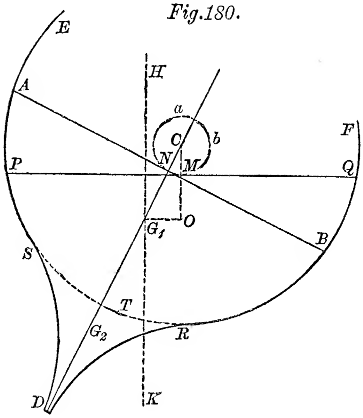
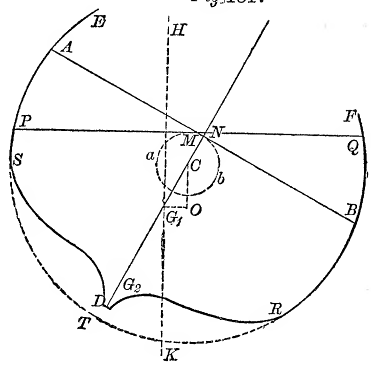


Fig. 181.



The forces operating in the upright position of equilibrium were the weight of the vessel and the equal weight of the fluid displaced. Since C has not been vertically displaced, the work of the former force (weight of vessel) is that done during the rise of G through a space $= h_1$ versin. θ. The work of the latter is equal to that of the upward pressure of the water which would occupy the circular space, having P T Q for sectional area, *increased* by that of the water, represented by S D R T in the case of Fig. 180, and *diminished* by that in the case of Fig. 181. But the centre of gravity of the circular part, P T Q, remains always at C, and neither rises nor falls; and therefore the work done by the upward force acting upon it is *zero*. The whole work done by the upward pressure of the water is therefore that done upon S D R T, which must receive the *positive* sign in Fig. 180 and the *negative* sign in Fig. 181, so that its general expression is $\pm W_2 h_2$ versin. θ. On the whole, therefore, the work Σu_2 of those forces, which, in the vertical position of the body, constituted its equilibrium, is thus expressed—

$$\Sigma u_2 = - W_1 h_1 \text{ versin. } \theta \pm W_2 h_2 \text{ versin. } \theta,$$

the minus sign being prefixed to $W_1 h_1$ versin. θ, because the point of application (G_1) of the ship's weight has been moved upward, and therefore in the opposite direction to that in which the weight tends to act.

Representing therefore the Dynamical Stability, Σu_1, by $U\theta$, we have, by equation (2),

$$U\theta = (W_1 h_1 \mp W_2 h_2) \text{ versin. } \theta, \quad . \quad . \quad . \quad (7);$$

in which expression the sign $\mp$ is to be taken according as the circular area, A T B, lies wholly within A D B, as in Fig. 180, or partly without it, as in Fig. 181. Other things being equal, therefore, Fig. 180 is a more stable form than Fig. 181.

"The work of the upward pressure of the water upon the vessel represented in Fig. 181, being," says Moseley, "a negative quantity, $-W_2 h_2$, versin. θ, it follows that the point of application of the pressure must be moved in a direction opposite to that in which the pressure acts; but the pressure acts upwards, therefore its point of application, *i.e.*, the centre of gravity of the displaced fluid *descends*. This property may be considered to distinguish *mechanically* the class of vessels whose type is Fig. 180, from that whose type is Fig. 181; as the property of including wholly or only partly, within the area of any of their athwart sections, the corresponding circular area, E T F, distinguishes them geometrically."

We now come to the general case of the Dynamical Stability of a vessel of any given form.

Conceive the vessel, at the end of an oscillation, to be for an instant at rest, and let R S, Fig. 182, represent its plane of flotation *then*, and P Q its plane of flotation when it was upright, C A D being a vertical section of the vessel; G, centre of gravity of vessel when she was vertical; H, centre of gravity of fluid then displaced; g, that of fluid displaced by the portion Q O S; h, that of fluid which would be displaced by part P O R, if immersed; G M, H N, $g\,m$, $h\,n$, K L, perpendiculars upon the plane R S. Also, let W be weight of vessel, or of fluid displaced; w, weight of water displaced by either of the equal wedge portions, P O R, Q O S; H_1, depth of centre of gravity of vessel in vertical position; H_2, depth of centre of gravity of displacement in that position; ΔH_1, elevation of centre of gravity of vessel; ΔH_2, elevation of centre of gravity of displaced water; θ, the inclination; η, the inclination of the line O, in which planes P Q and R S intersect to the line about which plane P Q is symmetrical; $z = h\,n + m\,g$; and $\lambda = $ K L.

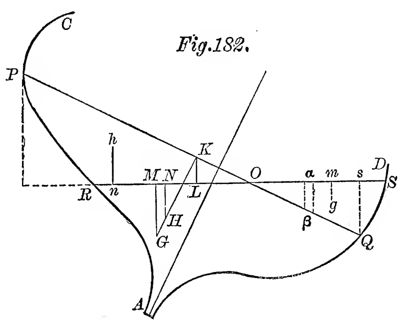


Fig. 182.

Now adopt the device of supposing the water *displaced* by the vessel to be, on the contrary, contained by it, and the water which thus would occupy the space Q O S to pass into the space P O R, the whole then becoming solid. Further, let ΔH_3 represent the corresponding elevation of the centre of gravity of the whole contained fluid, so that $\Delta H_2 + \Delta H_3$ will represent the total elevation of the centre of gravity of this fluid as it passes from the position it occupied when the vessel was vertical into the position P A Q. This is obviously the same as though the fluid had assumed the solid state when the body was vertical, and had revolved with it. It is therefore represented by K H − N H. Therefore,

$$\Delta H_2 + \Delta H_3 = KH - NH;$$

and

$$\Delta H_3 = KH - NH - \Delta H_2.$$

Further, by raising the water, Q O S (whose weight is w), to O P R, and its centre of gravity through the height $(g\,m + h\,n)$, the centre of gravity of the mass of fluid of which it forms a part (and the weight of which is W), is raised through the space, ΔH_3; it follows (from the property of the centre of gravity of a system) that,

$$W \Delta H_3 = w\,(g\,m + h\,n),$$

and therefore,

$$W\,(KH - NH - \Delta H_2) = w\,(g\,m + h\,n).$$

But,

$$NH = KH \cos.\theta - KL,$$
$$= H_2 \cos.\theta - \lambda;$$

Therefore,

$$KH - NH = H_2 \text{ versin.}\,\theta + \lambda,$$

and,

$$mg + nh = z;$$

Therefore,

$$W\,(H_2 \text{ versin.}\,\theta + \lambda - \Delta H_2) = w\,z;$$

And consequently,

$$W \Delta H_2 = W\,(H_2 \text{ versin.}\,\theta + \lambda) - w\,z;$$

Also,

$$\Delta H_1 = K - MG = H_1 - (H_1 \cos.\theta - \lambda);$$
$$= H_1 \text{ versin.}\,\theta + \lambda;$$

Therefore,

$$W\,(\Delta H_1 \mp \Delta H_2) = W\,(H_1 \mp H_2) \text{ versin.}\,\theta + w\,z;$$

And, from equation (6),

$$U\,(\theta, \eta) = W\,(H \mp H_2) \text{ versin.}\,\theta + w\,z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

"The sign $\mp$ being taken," says Moseley, "according as the vessel is of the class represented in Fig. 181, in which the centre of gravity of the displaced fluid ascends, or of that represented in Fig. 180, in which it descends."

We have reproduced all the above equations of Moseley just as he gives them, but not with the satisfaction which we could desire, as regards the signs employed in the last equation (8). It must strike the reader as strange that, in a general equation for Dynamical Stability, signs should appear requiring the explanation which we have just quoted from Moseley. No exception need be taken to the use made of the types indicated in Figs. 180 and 181, in framing and interpreting equation (7), because there the relation of the $+$ and $-$ signs are an essential part of the arguments employed. But even in relation to Figs. 180 and 181, and to equation (7) one cannot but feel that, except for prismatic bodies, the employment of the sections shown as typical of actual ships is not very satisfactory, on account of the extreme incongruity that exists between the midship section, and many other transverse sections of actual ships. In a general equation, it is quite out of the question to refer the investigator to Figs. 180 and 181, as indicating in what way the formula is to be applied to different classes of ships. To ascertain whether a particular ship should be classed with Fig. 180, or with Fig. 181, would in itself sometimes be no light and no desirable task. The fact seems to be that the term, "$H_1 \mp H_2$" was imported into equation (8) by an oversight—probably by hasty inference from equation (7), where the two signs were correctly employed. The upward vertical displacement of the centre of buoyancy, H_1, will always (with Moseley's Notation) be expressed by H_2 versin. $\theta + \lambda$, and the vertical position of the centre of buoyancy of the ship, after the given inclination, must be determined by taking account of the immersed and emersed wedges, which themselves occasion a vertical displacement of the centre of buoyancy expressed by $\frac{w}{W}(g\,m + n\,h)$ $= \frac{w}{W} z$. The general equation must therefore always be without the double $\mp$ signs which Moseley places between H_1 and H_2, and be written thus :—

$$U(\theta, \eta) = W(H_1 - H_2) \text{ versin. } \theta + w\,z.$$

This equation truly expresses the work done, and whether the centre of buoyancy rises or falls depends upon whether wz is greater or less than the quantity, H_2 versin. $\theta + \lambda$.

Canon Moseley, in the paper with which we have been dealing, employs Dupin's Surface of Flotation in determining the true axis of oscillation of a ship, and proceeds to infer thence the time of rolling, &c., but it would carry us beyond the limits prescribed for this volume to follow him in those investigations.

It is obvious that the relation between statical and dynamical stability, though always essentially the same, may be very variously considered. Writing the moment of statical stability, as we have seen, may be done in the French form,

$$M = -P(\rho - a)\sin.\phi.$$

M. E. L. Bertin, in his admirable "Notes on Waves and Rolling," regards the dynamical problem as follows:—Each element of the "work" performed by the couple of stability $P(\rho - a)\sin.\phi\, d\,\phi$ during an inclination of the ship is obviously equal to the differential of the moment $P(\rho - a)\cos.\phi$ of the weight of the water displaced, with respect to the centre of gravity, G, and consequently with respect to the water's surface. Now, every change in this last moment corresponds to an element of "work"—equal to and in the same direction as that of the weight—on the water which surrounds the ship. The work of the hydrostatic pressure is then fundamentally only the work of the weight of the portion of the liquid which ascends or descends, according as the centre of displacement ascends or descends. Upon this principle might be established the equation of motion

$$-\Sigma m r^2 \frac{d^2\theta}{dt^2} = P(\rho - a)\phi.$$

$\Sigma m r^2$ being the moment of inertia of the floating body round the axes passing through the point G. "We may remark" says M. Bertin, "that the hull exerts on the water a reaction the work of which is equal and opposite to the work of the pressure; we see that the whole work of the forces which act, whether on the ship or on the water, is limited to a work of the weight; that is evident *à priori*, since the weight is the only external force in play."

In an able and well-known memoir on "The Complete Motion of a Ship Oscillating in Still Water," presented by MM. Risbec and Duhil de Benazé to the French Academy of Sciences in June, 1873, those gentlemen presented the following view of the matter.* The

* See *Naval Science*, vol. iii., 1874.

only force in play being gravity (resistance being neglected), the effective work proceeds solely from the vertical displacements of the centre of gravity and centre of buoyancy. The displacement of the centre of gravity is evident in itself; that of the centre of buoyancy, or at least the work which it causes, is not less real. To prove the latter, let us consider at first a limited portion of the liquid of weight E, upon which a body of displacement, P, floats without motion; the moment of this liquid about the surface can be expressed by the difference of the moment of the whole volume—including the displacement of the body—and the moment of that displacement; it will be

$$(E + P)\,H - P\,h,$$

where H equals the distance of the centre of gravity of the whole volume from the surface, and h equals the distance of the centre of buoyancy of the body from the surface. If now we make the body occupy another inclined position, without altering its displacement, its centre of buoyancy will come to a distance h_1 (other than h) from the surface of the liquid; if the latter remain invariable, it is clear, since the displacement is the same, that the moment of the liquid calculated as before will become

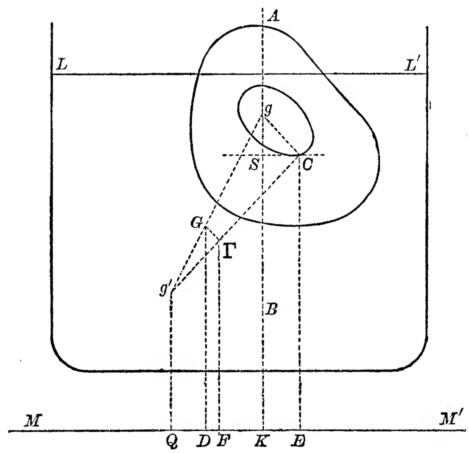
$$(E + P)\,H - P\,h_1.$$

Thus this moment will have varied by exactly the quantity $-\,P\,(h - h_1)$, by which also the moment of the displacement has varied. Consequently, the centre of gravity of the liquid will have been displaced vertically by a quantity $-\,\frac{P}{E}\,(h - h_1)$ opposite in direction to the displacement of the centre of buoyancy. The lowering of the centre of buoyancy corresponds then effectively to a certain elevation of the centre of gravity (and *vice versâ*), if the free surface does not change; and, in consequence of the equality of the moments (that is, the equality $-\,P\,(h - h_1) = \frac{P}{E}\,(h - h_1 \times E)$ the work of gravity in the liquid is precisely expressed by that of the pressure (weight taken in the contrary sense) on the centre of buoyancy.

Mons. M. E. Guyou, of the French Navy, contributed to the *Revue Maritime et Coloniale* (I believe in 1879) a highly interesting paper, entitled "Théorie Nouvelle de la Stabilité de l'Equilibre des Corps Flottans," which started from the consideration of a floating body supported by a given volume of water, like that of MM.

Risbec and Duhil de Benazé, and terminated with a somewhat novel presentation of the doctrine of dynamical stability, but one which perfectly agrees with the accepted theory of the subject. There is so much that is novel to English naval architects in M. Guyou's paper, that we readily give the following rather full account of it. He lays great stress in the opening part of his paper on the imperfections which have characterised the ordinary statical theories of stability from Bouguer downwards, and insists that the investigation of the equilibrium of a system is a purely dynamical problem which cannot be completely treated by the methods adopted by Bouguer and Dupin. When we speak of a material system being in equilibrium, we mean, he says, that if it is made to undergo a finite but small disturbance it will, after a series of oscillations, necessarily return to its first position. Coming closer to the subject, M. Guyou shows, first, that if any body whatever, floating on a given quantity of water contained in a vessel is caused to move without change of trim, the common centre of gravity of the system (liquid and floating body) cannot descend below a certain level; and its height above a given horizontal plane shall be a minimum when, the fluid surface being free, the body displaces a quantity of water equal in weight to itself. He demonstrates this, and shows that when the weight of the liquid displaced is less than that of the floating body, the common centre of gravity of the system, as well as that of the floating body will descend; but when the weight of the displaced liquid is greater than that of the floating body, the centre of gravity of the system will rise. Fig. 183 (next page) represents a body floating within a vessel as suggested; g is its centre of gravity, g' is the centre of gravity of the fluid when the body floats as shown, G is the common centre of gravity of the body of the liquid, and Γ is the centre of gravity of the interior volume of the vessel below the water-line, L L′. C is a point in the surface of buoyancy of the floating body. The internal volume of the vessel below L L′ is equal to $\frac{\mathrm{P}}{\omega}$, *i.e.*, the weight of the liquid which would fill it is equal to the weight of the system (liquid and body), which call P. M M′ is any horizontal line whatever, and C E, g K, Γ F, G D, and g' Q are verticals let fall upon it. The water-line, L L′, will not be altered by changes of position of the floating body all the time the body and the water are left free. Putting p for the weight of the floating body, and p' that of the liquid in the vessel (so that $\mathrm{P} = p + p'$), we shall have, by taking moments

Fig.183.



$$P \times GD = p' \times g' Q + p \times g K;$$

but

$$p' \times g' Q = P \times \Gamma F - p\, CE,$$

therefore

$$P \times GD = P \times \Gamma F \times p(g K - CE);$$

and if we draw through C a plane parallel to L L′, and put ρ for the distance, g S, of the point g above this plane, and h for the height, G D, we shall have

$$P h = P \times \Gamma F \times p \rho.$$

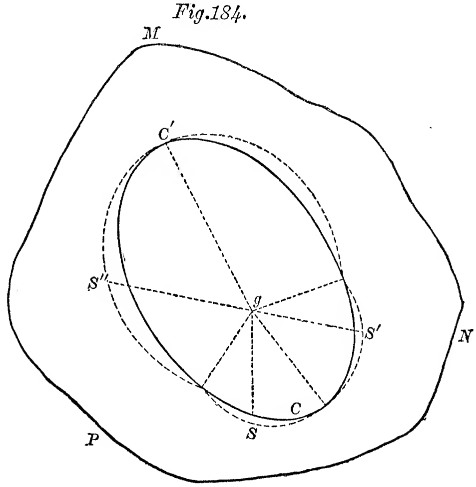
The point Γ is fixed in the system, and we can cause the horizontal plane from which we take our heights to pass through it; we shall then have $\Gamma F = 0$, and then the previous equation will become

$$P h = p \rho.$$

But as the tangent plane to the surface of buoyancy is parallel to L L′, therefore C S is a tangent to that surface at C, and ρ is the distance of the centre of gravity of the body from that tangent plane, perpendicular to the axis, A K. Consequently, if we take a quantity equal to $h \frac{P}{p}$, that is equal to ρ, and set it down from g, upon the vertical, G K, we shall obtain, as the body is rotated in all directions, a locus of all such points as S in the figure, and these will constitute a surface. It will be by its construction, the locus of the feet of the perpendiculars let fall from the centre of gravity upon the tangent planes to the surface of buoyancy, and this locus M. Guyou designates, after the manner of French men of science, the *podaire* of the surface of buoyancy, with respect to the centre of gravity, g. He calls it the surface S, and the surface of buoyancy the surface C. It follows from what has gone before, he says "that, if we refer the heights of the centre of gravity of the system to a horizontal plane passing through the fixed point Γ, the minimum value of the product $P\,h$, for any given trim of the body, will be equal to the product $p\,\rho$, or the weight of the body multiplied by the radius vector of the surface, S, corresponding to the axis of trim (orientation)."

M. Guyou next considers, in an extremely interesting manner, the different forms which the surface S can take in relation to the surface C. If the centre of gravity is situated within the surface of buoyancy, as shown in section in Fig. 184, the curve S everywhere

Fig. 184.

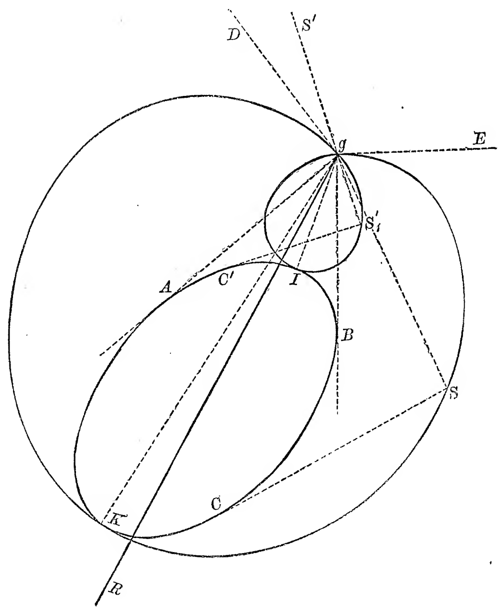


envelopes the curve C, and therefore the point g also. S S′ and S″ are points in the former curve, and C C′ points in the latter. Wherever the radius vector drawn from g is normal to both curves, the two curves touch.

If the centre of gravity lie outside of the surface of buoyancy, as shown in section in Fig. 185, the curve S has an internal loop. Let g A and g B be tangents drawn from g to the curve C, and g D and g E be perpendiculars to these tangents. These lines, g D and g E, are tangents to the internal loop of the curve S. This loop corresponds with the arc, A C′ B, of the curve of buoyancy C, and with negative values of ρ, g being below the centre of buoyancy;

the larger or external loop corresponds with the arc, A C B, of the curve C, and with positive values of ρ. By the device of adding to

Fig. 185.



the values of ρ a constant quantity greater than the greatest of the negative values, the author can transform the surface S into another surface, Σ, which will have *radii vectores* always positive, so that Σ will always envelope g. The surfaces C, S, and Σ, will have all their normals on common axes.

We now come to a very imaginative device of M. Guyou, adopted for the purpose of indicating the limits and circumstances of the stability of the floating body, observing that he resorts to it for the purpose of imparting clear conceptions of his meaning when he speaks of *normales maxima*, *normales minima*, and *normales mixtes*—which latter Dupin, it will not be forgotten, endeavoured, by a like process, to elucidate. Let us imagine, says M. Guyou, that from about point g as centre we describe a sphere of less radius than the least of the *radii vectores* of the surface, S (Fig. 184), and let us suppose that the radius of this sphere gradually increases. There arrives a moment when, as it grows, it will touch internally the surface, S, and to the point of contact will correspond a minimum normal. As the sphere still grows on, it will intersect the surface, S, in a line, surrounding on all sides the foot of the normal. But instead of speaking of "intersection," let us imagine our sphere to be a sphere of liquid, so that as it first touches the dry interior of the surface, S, it begins to wet and submerge it. If it enlarges sufficiently it will ultimately submerge the whole interior of that surface, but before this expansion is reached, and after it has begun to submerge parts of the surface, S, the latter will be divided into isles and lakes. If S_0 be the foot of the minimum normal which the sphere has first touched, there will gradually, as the sphere expands, be formed round S_0 a little lake; and as the sphere grows, the lake will extend, the tongues of land always receding, and there will arrive a moment when the original lake will be on the point of joining another. To the point of junction there will correspond a *mixed normal*, and it will obviously be relatively near to a minimum normal. If ρ_0 be the length of the minimum normal, and ρ_0' the length of the neighbouring mixed normal, we see that, so long as the radius of the sphere is comprised between ρ_0 and ρ_0', the lake around S_0 can contain only one minimum normal; and, on the contrary, when the sphere's radius exceeds ρ_0', the lake will contain at least two normals of this character.

Further, let us suppose that at the precise instant when the radius of the sphere is equal to ρ_0', we join the point, g, by straight lines with every point in the circumference of the lake surrounding S_0; we shall thus obtain a perfectly defined cone, which may be properly called the *cone of stability* corresponding to the axis, $g\,S_0$.

Returning now to the body, of weight p, floating in a given vessel containing a certain quantity of liquid, and putting H for the height of the centre of gravity of the *system* (of weight P) in any position whatever in which the free surface of the fluid is not horizontal, and letting the trim and the elevation of the floating body be any whatever, and calling R a quantity such that we have

$$P \times H = p \times R;$$

if we further call h the minimum height of G for the given trim of the body, and ρ the corresponding radius vector of the surface, S, we have, as before,

$$P h = p \rho,$$

and as H is greater than h, R must also be greater than ρ. Consequently, if from g as centre, with radius equal to R $\left(= \frac{P \times H}{p}\right)$, we trace a liquid sphere, the point, S, where the vertical axis through g pierces the surface, S, must be submerged; and if the radius, R, is less than the greatest radius vector of the surface, S, the liquid sphere will detach upon this surface a certain number of lakes, into one of which will descend the actual vertical axis through g. And, still further, if we now suppose the system to receive a movement such that the product, $P \times H$, could not surpass the value, $p \times R$, it is evident that the body could not at any time take an inclination such that its vertical axis through g would pierce the surface, S, outside of the lake which we have defined. Consequently, the oscillations of the body about the vertical would be limited by the cone before referred to.

Still keeping in view the system of a given body floating in a fixed quantity of fluid in a vessel, we know that the pressures upon the sides of the vessel balance each other, and the pressure upon the free surface of the fluid is also balanced. Presuming the weight above to be acting, let us assume the body to be floating in equilibrium, and therefore with a minimum normal, which we will call ρ_0, then, in the case of any small disturbance from the position of equilibrium, either of the liquid or of the floating body, which converts the height, h_0, of the centre of gravity of the system into H_0, and impresses upon the system a certain *vis viva*, which may be written $\Sigma m v_0^2$, and if the system be now left to itself, "the laws of the movement which the liquid and the floating body will undergo will depend," says our author, "on a multitude of circumstances that we cannot analyse with precision, but this movement will necessarily fulfil the equation—

$$\frac{1}{2}\left(\Sigma m v^2 - \Sigma m v_0^2\right) = P \times H_0 - P \times H;\text{* ''}$$

or,

$$\frac{1}{2}\,\Sigma m v^2 + P \times H = \frac{1}{2}\,\Sigma m v_0^2 + P \times H_0. \quad . \quad . \quad . \quad . \quad (a)$$

Putting

$$\frac{1}{2}\,\Sigma m v_0^2 + P \times H_0 = p\,R_0,$$

* "The difference between the aggregate work done upon the machine during any time, by those forces which tend to accelerate the motion, and the aggregate work, during the same time, of those which tend to retard the motion, is equal to the aggregate number of units of work accumulated in the moving parts of the machine during that time, if the former aggregate exceed the latter, and loss from them during that time if the former aggregate fall short of the latter."—*Moseley.* The work accumulated by a freely-falling body, and which it is capable of reproducing, is equal to one-half its *vis viva*, as we have previously seen.

this becomes,

$$\frac{1}{2}\,\Sigma\, mv^2 + \mathrm{P} \times \mathrm{H} = p \times \mathrm{R}_0.$$

As $\frac{1}{2}\,\Sigma\, mv^2$ is essentially positive, we shall always have

$$\mathrm{P} \times \mathrm{H} < p \times \mathrm{R}_0.$$

Consequently, if from g as a centre, with a radius equal to R_0, we describe a sphere, this sphere will detach around the primitive normal, ρ_0, a lake, and the cone formed by joining its contour at every point with g will define the limit beyond which the vertical axis of the floating body cannot pass during its oscillations under the conditions prescribed. In returning to its initial position (in which H_0 again becomes h_0), the work performed must be equal to $\mathrm{P} \times \mathrm{H}_0 - \mathrm{P} \times h_0$, and the *vis viva* imparted must be

$$\frac{1}{2}\,\Sigma\, mv_0^2;\ \text{and as } \mathrm{P} \times h_0 = p \times \rho_0,$$

we have

$$\frac{1}{2}\,\Sigma\, mv_0^2 + \mathrm{P} \times \mathrm{H}_0 - \mathrm{P} \times h_0 = p \times \mathrm{R}_0 - p\,\rho_0 = p\,(\mathrm{R}_0 - \rho_0). \quad . \quad (b)$$

It will be seen from this that if the work imparted to the system be small, the radius, R, will be but little greater than the primitive normal, ρ_0, and the cone of oscillation will be very small—a property belonging only to positions of equilibrium which correspond to minima normals. The author demonstrates the reverse of this for maxima normals, and shows that for mixed normals the sphere described with the radius, R_0, exceeding as little as we please the normal of equilibrium, will always cut off a lake of finite extent, composed of two parts united by a narrow strait.

In showing that a body slightly disturbed from a position of equilibrium corresponding to a minimum normal, will return necessarily to that position, M. Guyou observes that if, instead of imagining a material system to be perfect, we consider a natural system, it becomes necessary to introduce into equation (a) a term representing the work of the passive forces, and we have

$$\frac{1}{2}\,\Sigma\, mv^2 + \mathrm{P} \times \mathrm{H} + \phi = \frac{1}{2}\,\Sigma\, mv_0^2 + \mathrm{P} \times \mathrm{H}_0.$$

If we cut off $\mathrm{P}\,h_0$, having regard to equation (b), we shall have

$$\frac{1}{2}\,\Sigma\, mv^2 + \mathrm{P}\,(\mathrm{H} - h_0) = p\,(\mathrm{R}_0 - \rho_0) - \phi = p\,\left(\mathrm{R}_0 - \frac{\phi}{p} - \rho_0\right). \quad . \quad (c)$$

ϕ represents in this formula the sum of the passive forces exerted during the interval, $t - t_0$; therefore it is essentially positive. Let us suppose that at the instant, t, the system again becomes perfect, and let us call $\Sigma mv'^2$ its *vis viva*, and H′ the height of the centre of gravity at the instant, t', posterior to t, we should have

$$\frac{1}{2}\Sigma mv'^2 - \frac{1}{2}\Sigma mv^2 = \mathrm{P} \times \mathrm{H} - \mathrm{P} \times \mathrm{H}';$$

or,

$$\frac{1}{2}\Sigma mv'^2 + \mathrm{P} \times \mathrm{H}' - \mathrm{P} \times h_0 = \frac{1}{2}\Sigma mv^2 + \mathrm{P}(\mathrm{H} - h_0).$$

And finally, having regard to equation (c),

$$\frac{1}{2}\Sigma mv'^2 + \mathrm{P} \times \mathrm{H}' - \mathrm{P}\, h_0 = p\left(\mathrm{R}_0 - \frac{\phi}{p} - \rho_0\right). \quad . \quad . \quad . \quad . \quad . \quad (d)$$

This equation has the same form as (b), and we see, as before, that the movement which it represents is limited by the cone of oscillation obtained by describing the sphere with a radius, R, equal to $\mathrm{R}_0 - \frac{\phi}{p}$; that is to say, by a cone less open than that in which the system would have oscillated if the passive resistances had not diminished its *vis viva*. In other words, the passive resistances have to some extent dried the lake.

Now, if ρ_0 be a minimum normal, ρ_0' the nearest mixed normal, and the energy imparted to the system such that R_0 shall be less than ρ_0', it follows from what has gone before that the lake corresponding to this case will surround ρ_0, will gradually dry up, so to speak, and the cone of oscillation will close up round ρ_0 until it becomes identical with that normal, when we shall have (from equation, d),

$$\mathrm{R}_0 - \rho_0 - \frac{\phi}{p} = 0,$$

and necessarily,

$$\Sigma m' v'^2 = 0, \text{ and } \mathrm{H}' = h_0.$$

In fact, the second member of equation (d) being nothing, the body must float with ρ_0 upright; and as in this position H′ is always at least equal to h_0, the first member can only become *nil* when $\mathrm{H}' = h_0$, and the *vis viva* is also *nil*. The equilibrium is, of course, *stable*. If the energy imparted to the system be such that the initial cone of oscillation passes beyond the cone of stability, the corresponding lake would comprise at least no minima normals, and whether the body will come to rest (as the lakes are dried up by the diminution of the oscillations), with the axis of the cone of oscillation remaining in one or other branch of the lake, will depend upon whether it happens to be situated in the one or the other at the moment when the strait between the two lakes is left dry. The body will come to rest with the minimum normal situated in the lake in which the axis of the cone of oscillation remains in an upright position. The body *may* therefore capsize.

In order that the floating body may not capsize it is necessary that $\frac{1}{2}\Sigma mv_0^2 + P(H_0 - h_0)$ shall be such that R_0 shall be less than ρ_0'; but we have

$$\frac{1}{2}\Sigma mv_0^2 + P(H_0 - h_0) = p(R_0 - \rho_0);$$

the maximum "energy of stability" which the body can receive without capsizing is therefore equal to $p(\rho_0' - \rho_0)$, *i.e.*, to the work which would be done by the weight of the body falling through the height, $\rho_0' - \rho_0$.

Therefore, if $\rho_0, \rho_1, \rho_2, \ldots$ are different minima normals that can be drawn from the centre of gravity of the floating body to the surface, S (or to its surface of buoyancy, C), and $\rho_0', \rho_1', \rho_2', \ldots$ are their neighbouring "mixed normals," the energy of stability of the equilibrium (which is the minimum energy which the body can receive without capsizing) corresponding to any normal ρ of the first series will be equal to the work which would be done by the body falling through a height, $\rho' - \rho$.

By a similar process of reasoning M. Guyou demonstrates that under all circumstances it must be with a minimum normal alone upright that the body can come to rest, and that equilibrium about either mixed normals or maximum normals is impossible in nature.

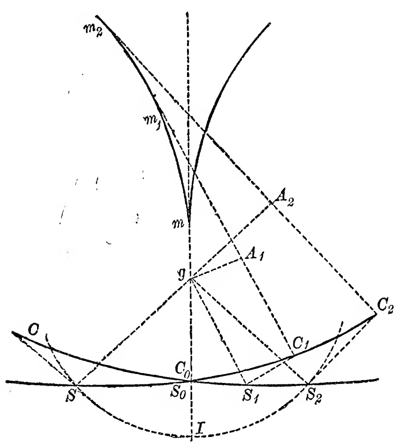
All the foregoing principles are applicable generally to the equilibrium of floating bodies, in open water as well as elsewhere, because the previous investigations have nowhere been dependent upon the dimensions of the containing vessel, or upon the quantity of water employed.

It will be seen that the whole treatment of stability by M. Guyou is thoroughly dynamical, and therefore contrasts strongly with the ordinary English methods of treatment, which were almost wholly of a statical character up to the time of the appearance of

Canon Moseley's Royal Society Paper in 1850. M. Guyou's method obviously admits, as will be seen, of special application to the measurement of what is known among us, since Moseley's time, as the Dynamical Stability of Ships, or the work performed in inclining them through finite and large angles. In giving it this application it becomes desirable to abandon the generality of the previous investigations, and to limit our inquiry to the case of inclination about a given longitudinal axis.

Fig. 186 represents a transverse plane through the centre of gravity of the floating body; let C_0, C_1, C_2, indicate the curve of buoyancy of the ship; S_0, S_1, S_2, the *podaire* of that curve, or M. Guyou's curve, S; and let C_2 and S_2 be two corresponding points on those curves. The righting couple tending to return the body from the position in which $g\,S_2$ is vertical, to the upright position shown, in which $g\,S_0$ is vertical, will have for its moment, $p \times g\,A_2$, which is equal to $p \times C_2\,S_2$. Further, the work done in inclining the ship from $g\,S_0$ upright, to $g\,S_2$ upright, we have seen to be equal to $p\,(g\,S_2 - g\,S_0)$; if, therefore, in accordance with the usage previously adopted, we call the arm of the lever of dynamical stability the quotient obtained by dividing the work requisite for inclining the ship by p, we shall have the following rule for determining in practice the two arms or leverages of a ship's stability, viz., C being the centre of buoyancy corresponding to the given inclination, if we draw at this point the tangent to the curve, C, and let fall from the point, g, a perpendicular, $g\,S$, upon this tangent, the line C S will represent the arm of the lever of statical stability, and the excess of the other side $g\,S$ (which with C S forms the right angle) over the distance $g\,S_0$, that of dynamical stability.

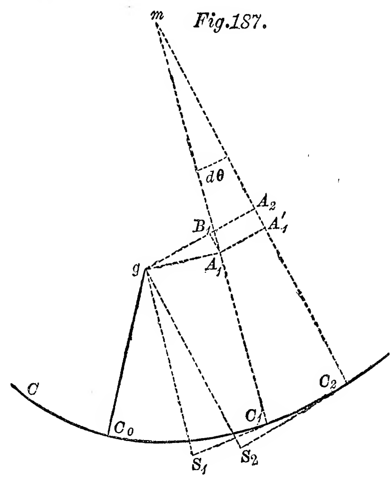
Fig.186.



M. Guyou remarks that the quantities, $g\,S$, $g\,S_0$, and C S, are given directly by the tables employed in the calculations of stability.

He then proceeds to compare the rule just enunciated with the ordinary rule—a comparison which is hardly necessary, as it is obvious that the difference between $g\,S$ and $g\,S_0$ is the measure of the vertical distance through which the centres of gravity and of buoyancy have become separated by the inclination, and that is with us, as well as with the French *savants*, the measure of dynamical stability. He proves that the arm of the lever furnished by the integration of the curve of stability is equal to the increase of the radius vector of the curve, S. In doing this he employs Fig. 187, in which C_1 and C_2 are the points of the curve of buoyancy, C, infinitely near each other; $m\,C_1$ and $m\,C_2$ are the two corresponding normals to that curve, and m is its centre of curvature. Let fall $g\,A_1$ and $g\,A_2$, perpendiculars from the point, g, upon the said normals, and draw $A_1\,B_1$ perpendicular to $g\,A_2$. Neglecting a very small quantity of the second order, we shall have

Fig.187.



$$A_1\,B_1 = g\,A_1\,d\,\theta.$$

Describe from the point, m, as centre, the infinitely small arc, $A_1\,A_1'$, and consider, as we may, that $C_1\,C_2$ is also an arc of a circle about m, and consequently, $m\,C_2 = m\,C_1$; therefore, $A_1'\,C_2 = A_1\,C_1$, and $A_1'\,A_2 = C_2\,A_2 - C_1\,A_1$; and, finally, if we observe that $g\,S_2 = C_2\,A_2$, and $g\,S_1 = C_1\,A_1$, it will follow that as

$$A_1'\,A_2 = A_1\,B_1 = g\,A_1\,d\,\theta,$$

we shall have

$$g\,A_1\,d\,\theta = g\,S - g\,S_1 = d\,g\,S\,;$$

and therefore, integrating from zero to θ,

$$\int_0^\theta g\,A_1\,d\,\theta = g\,S_1 - g\,C_0,$$

which is the equality that was to be established. "We see, therefore," says M. Guyou in conclusion, "that it is useless to make for each inclination the graphic integration of the curve of stability in order to obtain the arm of the lever of dynamic stability for ships: this arm can henceforth be taken either on the drawing by a mere opening of the compasses, or from the tables employed in calculating the elements of statical stability."

CHAPTER XVI.

M. Daymard's Process of Stability Calculation—Pantocarènes Isoclines—Fundamental Formulæ—Construction of Pantocarènes—Isocarènes Pantoclines—Deduction of Righting Levers from these Curves—Relation between Righting Levers and Co-ordinates of Centres of Buoyancy—M. Daymard's Method of Operating—Graphic Process for Facilitating the Work—Auxiliary Curves—Tracing out of the Pantocarène.

It has already been stated (in chapter iv.) that M. Daymard, of Marseilles, has devised an exhaustive process of stability calculation, in which curves passed through the extremities of what we have called the B R's, for different draughts of water, and for successive angular intervals (see Fig. 38, page 55), form a fundamental element. In explaining his process,* and the considerations which led up to it, M. Daymard states that it was while he was pursuing the investigations of his father-in-law, the late M. de Ferranty, that it first occurred to him to join by continuous lines the extremities of the arms of the righting levers (presuming the centre of gravity to be identical with the centre of buoyancy) *corresponding to the same angle of inclination.* He perceived that if he had, for all angles from 10° upwards, the curves thus drawn, which he proposes to call curves of stability, "*Pantocarènes isoclines,*" he could obtain from them at once, and with complete exactitude, and *for all possible cases,* the usual curves of statical stability.

Having obtained the means of determining the two extremities of each "*Pantocarène,*" and of calculating directly, and in a

* In a communication to the author, and subsequently in a paper contributed to the Institution of Naval Architects, session 1884.

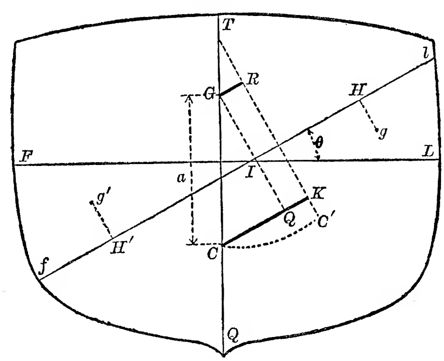
mathematical manner, as many intermediate points as he wished, he prepared a sample table of calculations by means of which he could, in a comparatively short time (about 40 hours) arrive at the complete representation of these curves for a ship, at intervals of 10°, from 0° to 180°, and for all draughts of water, and consequently at the entire solution of the problem of the stability of the ship.

The fundamental formula of statical stability for a ship of displacement P, is

$$P p = \omega \times H H' - P a \sin. \theta \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which p is the righting lever, G R, ω is the displacement of either of the two equal wedges, F I f, L I l (Fig. 188): H H′ the distance

Fig. 188.



along $f\,l$, between the perpendiculars from the centres of gravity g and g', of these wedges; and a the height of the centre of gravity above the centre of buoyancy.

Therefore,

$$p = \frac{\omega}{P} \times H H' - a \sin. \theta\,; \text{ or}$$

$$p = \frac{W}{V} \times H H' - a \sin. \theta.$$

W and V being the volumes corresponding to ω and P.

Also,

$$G R, \text{ or } p = C K - C Q\,; \text{ and } C Q = a \sin. \theta\,;$$

Consequently,

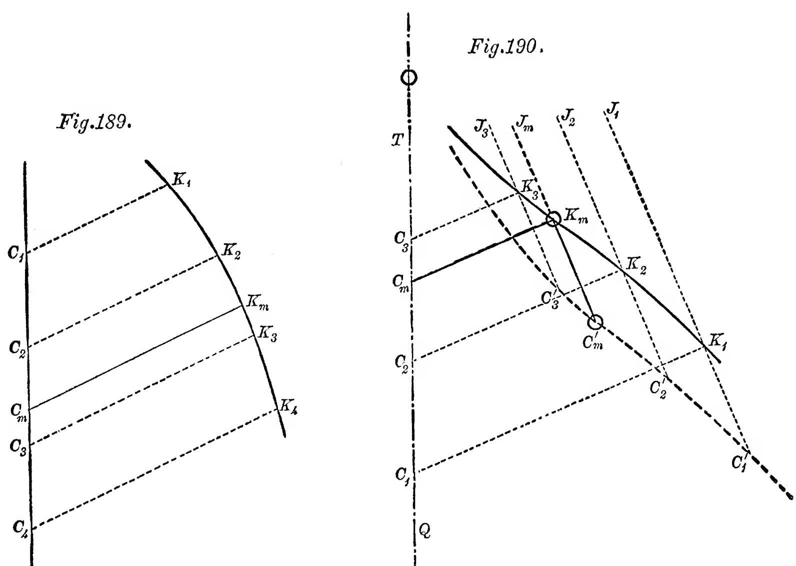
$$G R = \frac{W}{V} \times H H' \quad . \quad . \quad . \quad . \quad (2).$$

The length C K, would be the righting lever itself, if the centre of gravity coincided with the centre of buoyancy (when $a = 0$); calling it b, we have

$$b = p + a \sin. \theta \quad . \quad . \quad . \quad . \quad (3).$$

From this formula (3) it is easy to construct (for a given displacement) a curve such as that described in chapter iv., Figs. 36 and 37, which represents the value b (or B R) at successive angles of inclination; then, for each value of a, to trace (as is also done in the figure aforesaid, viz., Fig. 36, chapter iv.) the curve representing $a \sin. \theta$; the algebraic difference of the ordinates at the same angles gives the ordinates of the usual curve of stability, or of values of p.

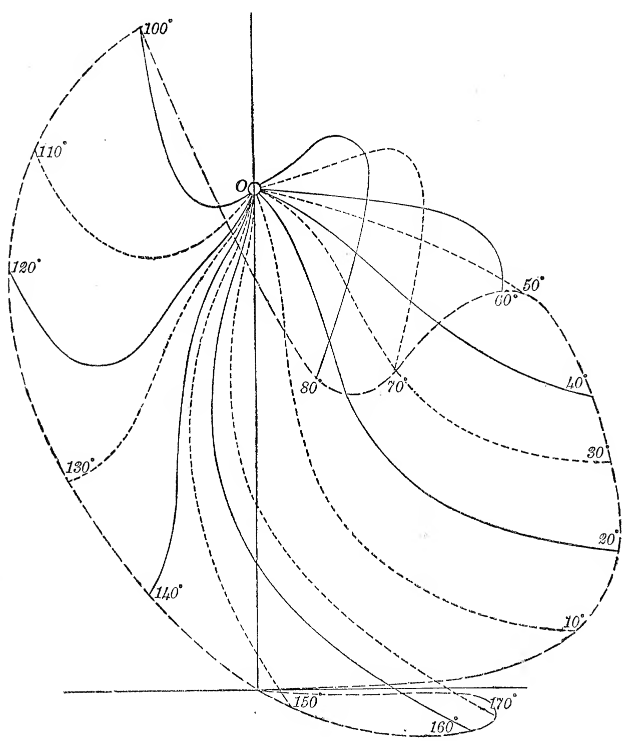
M. Daymard dispenses with the tracing of the auxiliary curves



(A B and C D, Fig. 36) in the following manner:—Suppose that, by any method, we have obtained the values of b at 10° inclination, for a series of centres of buoyancy, C_1, C_2, C_3 . . . sufficiently close together (Fig. 189); we can draw, in the vertical section, and to a convenient scale, the righting levers, $C_1 K_1$, $C_2 K_2$, $C_3 K_3$ . . . . corresponding to successive values of b, parallel to each other, and respectively equal say to $b_1^{10}\ b_2^{10}\ b_3^{10}$, &c.; joining the points K_1, K_2, K_3 . . . we shall have a curve from which we can at once obtain intermediate values of b, by drawing from any point C_m within the limits C_1, C_2, C_3, a straight line parallel to $C_1 K_1$ . . . till it meets K_1, K_2, K_3 in K_m. The length, $C_m K_m$, will represent for 10° inclination the value of the arm of the lever, b_m, for the displacement whose centre of buoyancy is, C_m, in the upright position. On account of this property M. Daymard gives to the locus, $K_1 K_2 K_3$ . . . the name "Curve of geometrical stability Pantocarène," at 10°; or, by abbreviation, "Pantocarène" at 10°.

This Pantocarène at 10° is also the locus of the feet of perpendi-

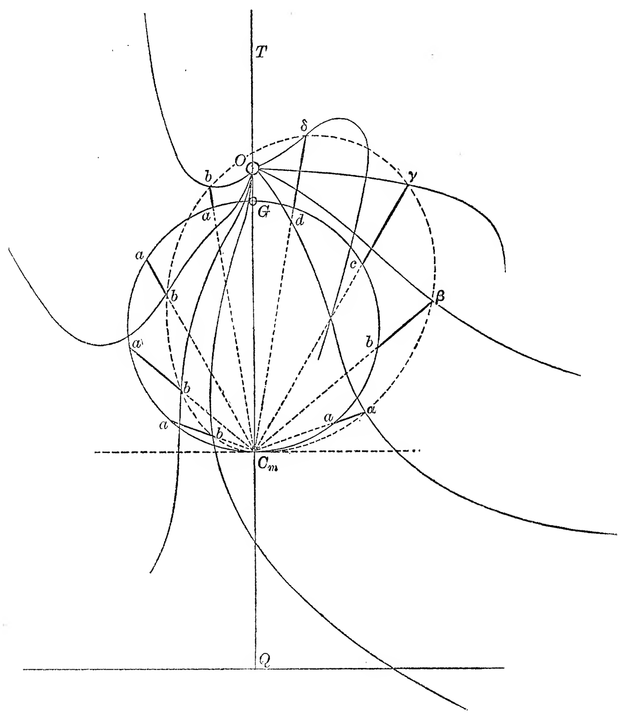
Fig.191.



culars let fall from the upright centres of buoyancy, C_1, C_2, C_m (Fig. 190) upon the normals $C'_1 J_1, C'_2 J_2$, . . . $C'_m J_m$ to the water-line at 10° inclination, drawn from the corresponding centres of buoyancy, $C'_1 C'_2$ . . . C'_m in the oblique position.

Supposing a series of Pantocarènes for 10°, 20° . . . 170°, 180° inclination to be traced, the general appearance of these curves for vessels of ordinary form is indicated in (Fig. 191), and they can be employed as follows:—If from any point, C_m (Fig. 192) of the

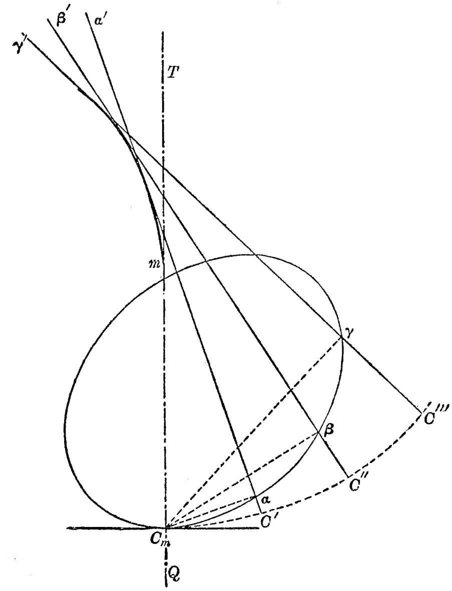
Fig.192.



vertical axis, taken as the centre of buoyancy of an upright displacement (known from ordinary calculations), we draw radii vectores, inclined at 10°, 20° . . . 170° and 180° to the horizontal, till they meet at α, β, γ . . . the corresponding Pantocarènes, we obtain a series of values of b from 10° to 180° at intervals of 10°; and if G is the centre of gravity of the ship, by describing on C_m G, $= a$, a circle which has for its polar equation (C_m being the pole) $\rho = a$ sin. θ, the values of p, or $b - a$ sin. θ, are then the portions, $a\,\alpha$, $b\,\beta$ . . . &c. (of the radii vectores) intercepted between the corresponding Pantocarène and the circumference. These lengths can be taken directly from the figure, if we desire to draw, with rectangular co-ordinates, the curve of values of p, *i.e.*, the usual curve of stability.

M. Daymard now takes another important step—Passing a continuous line through the points, α, β, γ . . . (Fig. 193), he obtains

Fig.193.



the curve of values of b in polar form. That curve is closed (with or without loop) at its pole, C_m, the centre of the upright displacement, and it is also the locus of the feet of the perpendiculars let fall from C_m on the tangents, $\alpha\alpha'$, $\beta\beta'$, $\gamma\gamma'$ . . ., to the metacentric evolute, which tangents are normal to the successive inclined water-sections, and are drawn through the corresponding oblique centres of buoyancy. He gives the name of "Isocarène Pantocline" to the locus, α, β, γ . . ., &c. It results that the values of p are the portions of the radii vectores intercepted between the circumference described on C_m G (Fig. 192) as a diameter, and the "Isocarène Pantocline" curve. It will be obvious that this curve, when drawn, enables us to determine with what height of centre of gravity the stability will vanish at a given angle, M C_m O (Fig. 194), for, by describing through the points, C_m and O, a circle having its centre on the axis, C_m G, the extremity, G, of the vertical diameter is the limit of height which the centre of gravity must not exceed, in order that the stability shall not vanish before the inclination, M C_m O, is reached. Moreover, with the Pantocarène at angle, θ, viz., the curve, P C_θ (Fig.

Fig. 194.

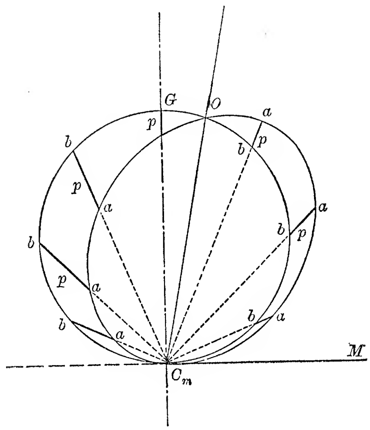
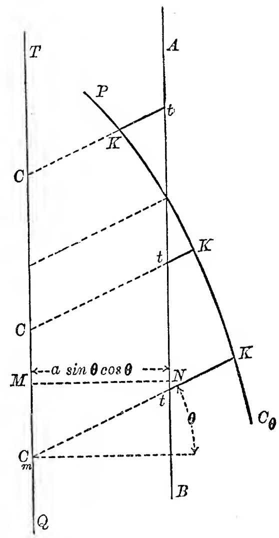
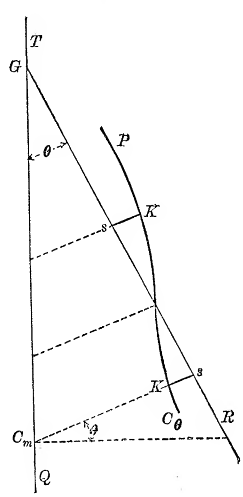


Fig. 195.



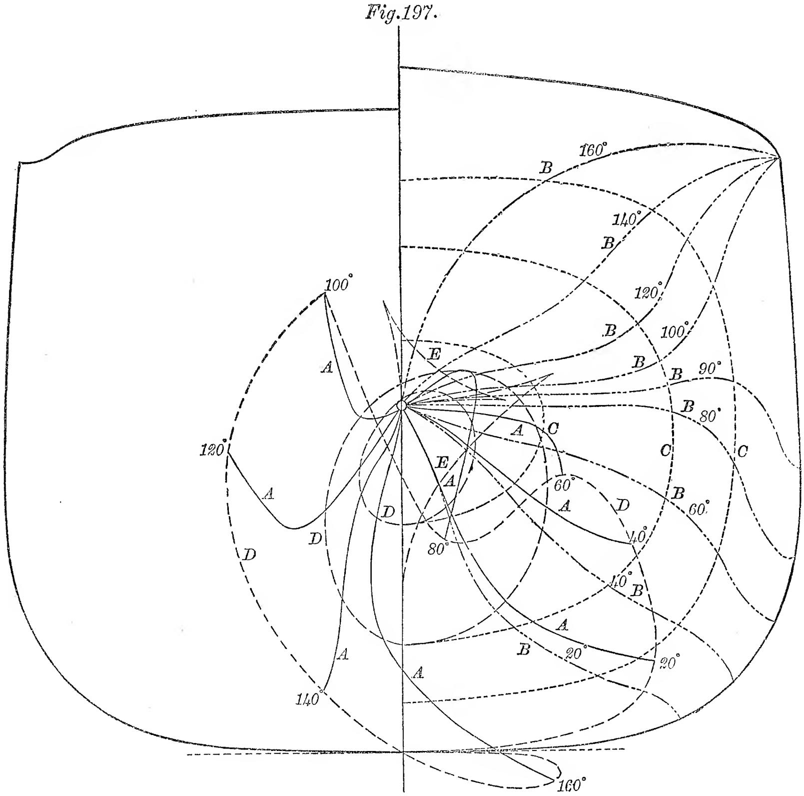
195), we can see at once the variation of p at θ inclination when the draught of water varies. We have only to draw A B parallel to the axis, T Q, at the distance, M N $= a$ sin. θ cos. θ; for any centre of buoyancy, C_m, p is the portion, t K, of C_m K, drawn at the inclination, θ, intercepted between A B and the pantocarène. Also the centre of gravity, G, being supposed fixed in the ship (Fig. 196), we obtain p for the inclination, θ, and the displacement whose upright centre of buoyancy is C_m, by drawing G R inclined to T Q at the angle, θ; the value, p, is the portion, K s, of C_m K s drawn at θ inclination, intercepted between the pantocarène and the straight line, G R.

Fig. 196.



The pantocarène curves by furnishing the points, α, β, γ . . . (Fig. 193), for a given displacement, enable us to obtain the metacentric evolute which is the envelope of the straight lines, $\alpha\alpha'$, $\beta\beta'$, $\gamma\gamma'$ . . ., of which we know the direction; and, the ordinary calculations supplying the position of the metacentre, m, for the upright flotation, we are able, using the property of evolutes, to trace the involute, C′, C″, C‴, or the locus of centres of buoyancy at various inclinations and with a constant displacement. "In short, our curves give," says M. Daymard, "for any draught of water, and whatever be the position of the centre of gravity, not only the usual curve of stability, but also all the elements of stability which can be desired for all inclinations. They are, so to speak, the synthesis of the series of metacentric evolutes, and form, with the loci of centres of buoyancy at a given angle, on the one hand, and with the isocarènes pantoclines, the metacentric evolutes, and the successive centres of buoyancy of a given displacement, on the other hand, a double system of curves (Fig. 197) capable of being deduced

Fig.197.



one from the other, but of which they, the pantocarènes, are the most useful for solving problems of naval architecture."

In Fig. 197 the curves marked A are curves of stability, "Pantocarène-isocline," *i.e.*, for varying displacements and constant inclinations. Those marked B are loci of centres of buoyancy for varying displacements and constant inclinations. Those marked C are loci of centres of buoyancy for constant displacements and varying inclinations. Those marked D are curves of stability, "Isocarène-pantocline," *i.e.*, for constant displacements and varying inclinations. Those marked E are ordinary metacentric evolutes.

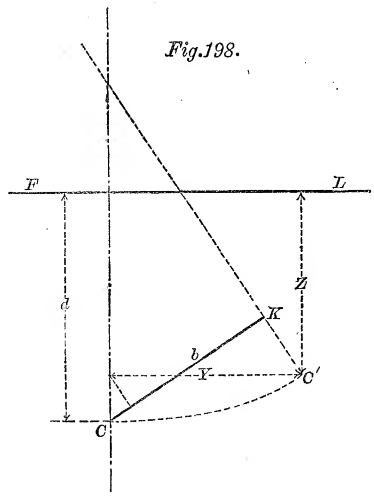
Considering floating bodies of geometrical form, their pantocarènes are of a very simple nature; for prisms of circular or polygonal section they are straight lines, branches of hyperbola, or algebraic curves of a degree not exceeding the fourth.

From Fig. 198 may be seen the relation between the values, b, of the arms of the lever, and the co-ordinates, Y and Z, of the corresponding centres of buoyancy;

$$b = Y \cos. \theta + (d - Z) \sin. \theta,$$

d being the distance from the water-line, F L, of the upright centre of buoyancy, C. Pantocarènes can therefore be constructed from the calculations giving Y and Z; but altogether, it is more convenient to get directly the values, b, in the manner to be presently indicated.

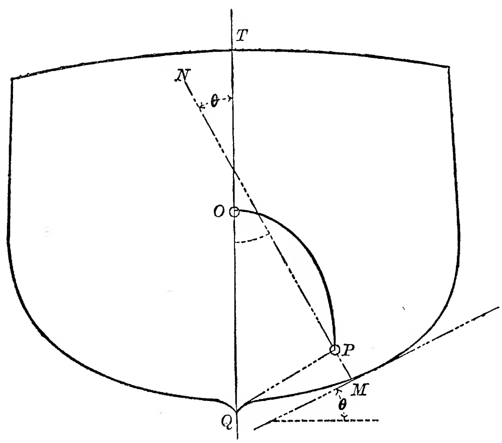
Fig.198.



The two extremities of each pantocarène are very easily determined, and that circumstance greatly facilitates the tracing of these curves. In the first place, all pantocarènes converge to the centre, O, of the total volume of the ship (centre of bulk); for, when the floating body is totally submerged, the centre of buoyancy, whatever the inclination, must coincide with the centre of form. On the other hand, with the displacement continually decreasing so as to arrive, as a limit, at displacement $= 0$, the ship in that extreme case (only possible as a geometrical conception) will be inclined by rolling on her midship or lowest section, the lowest point of which is the limit of the locus of the centres of buoyancy when the displacement tends to zero. It follows that by letting fall from the top of the keel, Q (Fig. 199), a perpendicular, Q P, upon the normal, M N, to the midship section, inclined at an angle θ to the axis T Q, the intersection, P, is the second limiting point of the pantocarène at θ.

For the calculation of intermediate points or values of b for a series of draughts of water and various inclinations, M. Daymard, following M. Reech, dispenses with water-lines corresponding exactly to a given displacement, and makes his calculations for inclined planes of flotation, radiating from a given point of the axis, and

Fig.199.



respectively inclined to the horizontal at 10°, 20°, . . . 170°, 180°. By a process of reasoning somewhat resembling that pursued by M. de Ferranty in chapter xiii., and using a similar notation, he demonstrates the equation,

$$\mathrm{W} \times \mathrm{H}_1 \mathrm{H}_1' = uh + u'h' - (u - u')\, d \sin. \theta.$$

Consequently,

$$b = \frac{uh + u'h' - \phi\, d \sin. \theta}{\mathrm{V} + \phi} \quad \ldots . (4);$$

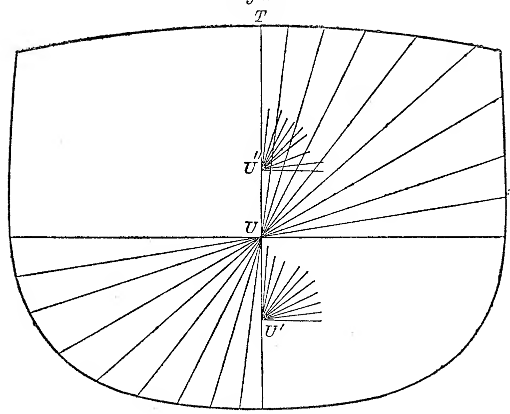
(in which ϕ is the difference between the wedges of immersion and emersion)—a formula of which we can compute all the quantities in the second member by means of ordinates, starting from V (a point in the axis) and by mathematical methods enabling any degree of approximation to be obtained.

The mode of operating is described by M. Daymard as follows :—

After having made for the upright vessel, and for the total volume, the ordinary calculations by means of horizontal water-lines giving the displacement scale, curves of centres of buoyancy, and metacentric heights, let us choose on the vertical axis different points, U, U′, U″, . . . (Fig. 200), which can be named points of stations; and for each of these points, such as U, let us perform the following operations :—

Draw from 0° to 90° a series of straight lines representing the traces of radial planes equidistant angularly, θ (in practice, $\theta = 10°$); take on those traces, starting from U, the ordinates of the equidistant sections (generally 20 in number) on the side

Fig. 200.



of immersion (I), as well on the side of emersion (E), and draw up a table like that annexed,* containing the ordinates (I) and (E), their squares, their cubes, the sums, Σ I, Σ E, $\Sigma(I^2)$, $\Sigma(E^2)$, $\Sigma(I^3)$, $\Sigma(E^3)$, and the quantities, $[\Sigma(I)^2 - \Sigma(E)^2] = \delta$ $\Sigma(I^3) + \Sigma(E^3) = \sigma$; we shall then have the necessary elements for calculating a series of values of b.

For, in the successive radial planes we have, by the usual formulæ of polar integration, Δx being the interval of the sections, $\Delta \theta$ the arc of 10° in a circle with radius, 1:

$$\text{At} \quad 10° : \phi_{10} = \frac{1}{4} \Delta x \Delta \theta \times \delta_1.$$

$$20° : \phi_{20} = \frac{1}{4} \Delta x \Delta \theta \, (2\,\delta_1 + \delta_2).$$

$$30° : \phi_{30} = \frac{1}{4} \Delta x \Delta \theta \, (2\,\delta_1 + 2\,\delta_2 + \delta_3).$$

$$. \quad . \quad . \quad . \quad = \quad . \quad . \quad . \quad .$$

$$n \times 10° : \phi_{(n \times 10)} = \phi_{(n-1)10} + \frac{1}{4} \Delta x \Delta \theta \, (\delta_{(n-1)} + \delta_n).$$

$$. \quad . \quad . \quad . \quad = \quad . \quad . \quad . \quad .$$

$$\text{Finally, at} \quad 90° : \phi_9 = \phi_8 + \frac{1}{4} \Delta x \Delta \theta \, (\delta_8 + \delta_9),$$

giving for index to δ the figure which represents the multiple of 10°, at which the corresponding radial plane is drawn.

Knowing ϕ, we shall have $(V + \phi)$ for each radial plane, V being given by the displacement scale.

On the other hand, the curves of heights of the upright centres of buoyancy supply the distances, d_1 and d_2, from $F_0 L_0$ of the centres corresponding to V and to $(V + \phi)$, and by the rule of moments we have,

* See end of Volume.

$$\phi d = V d_2 - (V + \phi) d_2;$$

it remains only in the formula : $b = \dfrac{uh + u'h' - \phi d \sin. \theta}{V + \phi}$ to calculate $uh + u'h'$.

Then a very simple analysis made by taking the moments of the elementary wedges relative to the radial plane under consideration, shows that calling M_1, M_2 . . . the sums, $uh + u'h'$, and adopting for indices of M and σ the same rule as for δ, we have,

$$\text{for } 10° : M_1 = \frac{1}{6} \Delta x \Delta \theta (\sigma_1 + \sigma_0 \cos. 10°).$$

$$20° : M_2 = \frac{1}{6} \Delta x \Delta \theta (\sigma_2 + 2 \sigma_1 \cos. 10° + \sigma_0 \cos. 20°).$$

$$30° : M_3 = \frac{1}{6} \Delta x \Delta \theta (\sigma_3 + 2 \sigma_2 \cos. 10° + 2 \sigma_1 \cos. 20° + \sigma_0 \cos. 30°). \quad . \quad . \quad .$$

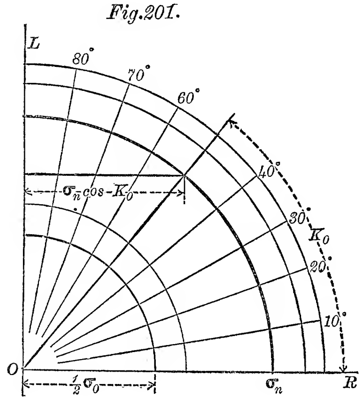
$$n \times 10° : M_n = \frac{1}{6} \Delta x \Delta \theta [\sigma_n + 2 \sigma_{(n-1)} \cos. 10° + \ . \ . \ . \ 2 \sigma_1 \cos. (n - 1) 10° + \sigma_0 \cos. (n \times 10)°]. \quad . \quad . \quad . \quad .$$

Finally,

$$90° : M_9 = \frac{1}{6} \Delta x \Delta \theta (\sigma_9 + 2 \sigma_8 \cos. 10° + \ . \ . \ . \ . \ 2 \sigma_1 \cos. 80°).$$

We can obtain the values, M_1, M_2, . . . rapidly by a graphic process; for that purpose we draw through the vertex, O, of a right angle, L O R (Fig. 201), a series of radii inclined 10°, 20°, . . . 80°, to the horizontal, O R; we describe from O as a centre, arcs with lengths, $\frac{1}{2} \sigma_0, \sigma_1, \sigma_2, \ldots \sigma_9$, to a convenient scale; and the distances from O L of the intersections of the radii with the arcs, give us the series of values, $\sigma_n \cos. k \theta$, of which are formed the quantities, M_1, M_2, . . . M_9.

Fig. 201.



We have then for each radial plane, fl (of angle, θ, and volume, $V + \phi$), the "geometrical arm of lever," $b = \dfrac{M - \phi d \sin. \theta}{V + \phi}$.

Drawing through the centre, C_1 (Fig. 202), of the upright hull a parallel to fl, and with the length, $C_1 K = b$, the point, K, is one point of the pantocarène, θ.

We determine, besides, a second point belonging to the pantocarène $(180 - \theta)$. For each oblique water-line, fl, cuts off two distinct hulls; one inclined θ of volume $(V + \phi)$, the other inclined $(180 - \theta)$, having for volume $[V_1 - (V + \phi)]$, V being the total volume of the floating body.

Let C_2 be the upright centre of the volume $[V_1 - (V + \phi)]$, the arm of lever, b_s, corresponding to C_2, and inclination $(180 - \theta)$, is given by the equation—

$$b_s = \left[D_2 + \frac{D_1 (V + \phi)}{V_1 - (V - \phi)}\right] \sin. \theta - b \frac{(V + \phi)}{V_1 - (V + \phi)},$$

D_1 and D_2 being the distances of C_1 and C_2 to the centre of bulk, O.

This equation is easily deduced from considerations of elementary geometry. In fact, the series of radial planes from 0° to 90° supplies a series of values of b

Fig.202.

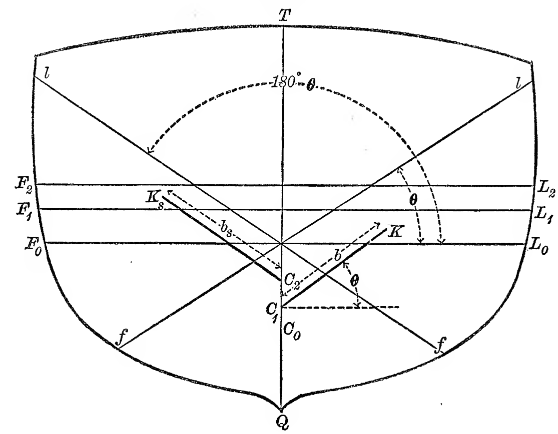
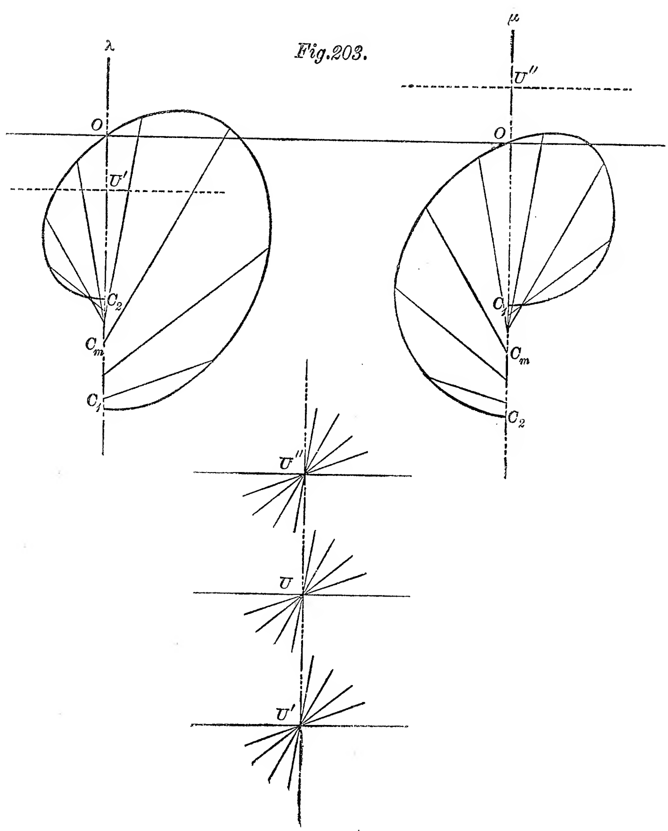


Fig.203.



and corresponding points of pantocarènes for all angles, at intervals of 10°, from 0° to 180°. As from 180° to 360° the elements of the stability are symmetrical, having the same values as from 180° to 0°, we may conclude that the radial planes from 0° to 90° give the results for all possible inclinations.

Having thus determined as many series of points, K and K_s, as we have considered stations, U, we should be able to trace out the pantocarènes.

It will be useful, however, first to join the points, K and K_s, given by a single station; by virtue of the continuity of the lines of the ship, the curves so formed ought to show a certain regularity. Moreover, they should pass through the centre of bulk, O, because whatever be the station, we return by the radial plane at 90° to the longitudinal plane of symmetry, and to the half vertical volume of the ship. Those conditions should check the results of the calculations. These auxiliary curves will have the form (λ) or (μ) (Fig. 203), according as the station is below or above the centre of bulk.

What we have said about their continuity can be more or less invalidated, when the ships are of unusual forms, with armoured breast-works or other peculiarities, but in that case we can, by proceedings analogous to those which we apply to the calculations of surfaces bounded by discontinuous curves, make use of radial planes placed more closely together, and meeting the intersections of the deck with the sides.

In any case, after having completed the operations for the number of stations deemed necessary (practice proves that the knowledge of the terminal points for three stations suffices in ordinary cases), and traced enough curves of the kind (λ) or (μ), it only remains to join the points, K, corresponding to the same inclination, and to prolong, when necessary, the lines so obtained, on one side to the centre of bulk, on the other side to the ending point, which we have given the means of determining.

We have then completed the pantocarènes of the ship.

The sample table of calculations found herewith will supply, we hope, sufficient explanations to elucidate what our exposition may have left incomplete or obscure.

This table contains every calculation for one station. With little assistance, one person may fill it in eight hours, all included. That corresponds to the complete tracing of the pantocarènes, by 10° intervals from 0° to 180°, in about forty hours.

We may remark that any method giving a sufficient number of arms of lever, CK, would enable us to trace the pantocarènes, and also in the case where these arms of levers have been found by inclining a model; a process open to objections, but nevertheless sometimes used.

Our table, of course, can be used with ships designed to any condition of trim. But we reserve the explanation of the means of obtaining from the pantocarènes corresponding to the horizontal keel the measurement of the stability, whatever be the trim.

CHAPTER XVII.

Amsler-Laffon's Mechanical Integrator—Description of the Instrument and Explanation of its Principle—Readings for Area—Readings for Statical Moment—Readings for Moment of Inertia—Improvements by Dr. Amsler—Dr. Amsler's Method of Using the Integrator in Stability Calculations—Specimen Calculations.

We propose in this chapter to give a description of Mr. J. Amsler-Laffon's Mechanical Integrator, an instrument which, as has been previously stated, is of especial value in Naval Architecture for curtailing calculations which, when otherwise made, are both lengthy and tedious. It is now largely used in the profession, as it possesses advantages which cannot be neglected. Greater accuracy is obtained, as the instrument records quite as correctly for a discontinuous curve as for a continuous one; and, operating mechanically, is but little liable to error. Much less work is necessary, as by the movement of a pointer round the boundary of any figure we determine readings for the area, statical moment, and moment of inertia, and therefore, there is a great saving of time and labour. The work by its aid may be performed by comparatively unskilled labour subject to skilled direction, with an occasional check on the accuracy of the instrument, as a broken pivot-point, or some small mishap, may give rise to very erroneous and curious readings.

It is therefore necessary that the instrument should be carefully handled in placing it on, and taking it off the drawing board, its bearings should be kept clean and free from dust; and should any inaccuracy in the readings reveal itself, the adjustments should not be tampered with by any but an expert instrument maker. As to unskilled labour being employed on the several calculations for which the instrument is used, we may observe that drawing-office apprentices are usually so employed, and generally the younger lads. Although in most private shipbuilding establishments these lads are now required to pass an examination, similar to that which

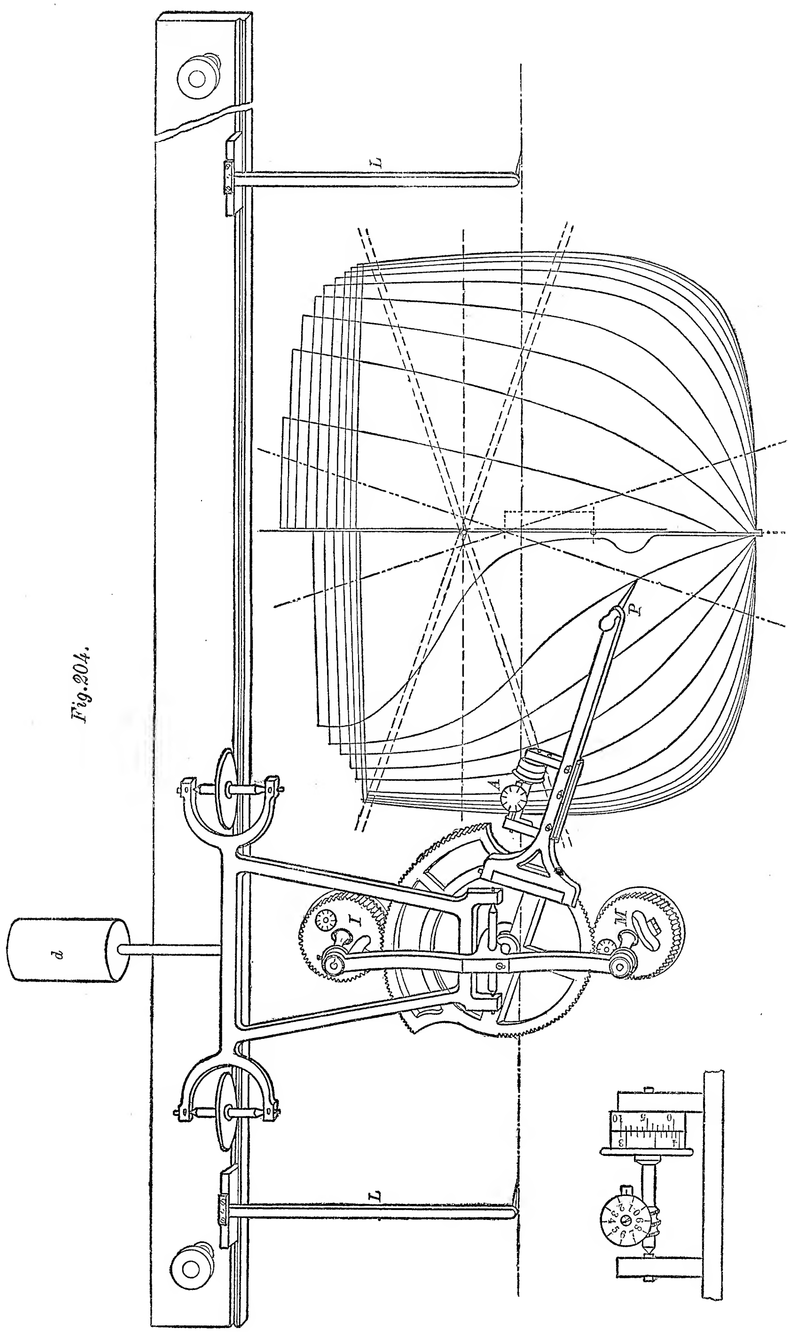


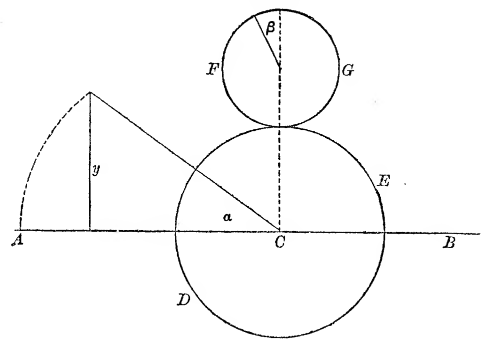
Fig. 204.

candidates for Shipwright Apprentices in Her Majesty's Dockyards undergo, they need only possess sufficient intelligence to become acquainted with the instrument, take the readings correctly, record them in prepared tables, and to perform the simple arithmetical operations necessary. This is more especially the case, when we employ the tables prepared by Mr. Macfarlane Gray, where each step is indicated in a clear and simple manner.

On reference to Fig. 204, it will be seen that the instrument consists of a pointer rigidly connected (so far as horizontal motion is concerned) to a disc formed by two arcs of circles described about the same centre with different radii. To each of these arcs, pinions are geared, and so placed that the three centres are in a line perpendicular to the axis. On the radii of these pinions, roller wheels are fitted, and each wheel is connected with a counter, so that they record the spaces traversed by their respective pinions. These vital parts, as we may call them, are secured on a carriage, and balanced by means of a weight, *d*. The whole instrument is made to run on wheels, in order to reduce the friction which would take place if sliding motion occurred, and is confined to rectilinear motion, by means of these wheels running in a V-shaped groove along a steel batten fixed parallel to the axis, about which we wish to determine the statical moment and moment of inertia.

It is evident that for any angular motion, a, of the circular arc, each pinion has an angular motion, β, such that a and β are inversely proportional to the respective radii. The principle of the machine will be more clearly seen by taking any one of the pinions in combination with the corresponding circular arc to which it is attached. Let A B, Fig. 205 (next page), represent the axis of the instrument; D E, the arc to which the pointer is attached, and F G, the pinion geared to the arc, D E. It will be seen from the preceding remarks that the centre, C, is free to move along the axis. Taking $n : 1$ as the ratio of the radius of D E, to that of F G, and c as a *constant*, dependent on the initial position of F G; for any angular motion, a, of the arc, the pinion has an angular motion, β, such that $\beta = n\,a + c$. It is evident that when the pointer traverses a closed curve, the readings from the roller due to angular motion are *nil*, because the angular movement is exactly the same in opposite directions. For a traverse of the pointer through an angle, a, however, if $d\,x$ represent the linear motion of the centres, the roller evidently records a constant multiple of $-\,d\,x \cos.\ \beta$, taking the direction of motion of the pointer as positive. The complete record due to the traverse of the pointer round the whole boundary of the curve is $-\int d\,x \cos. \beta$; or $-\int d\,x \cos. (n\,a + c)$. Referring to rectangular

Fig.205.



co-ordinates, we see that the ordinate, y, is always proportionate to sin. a, and therefore we only require to know the value of c, the constant, in order to express the value $\cos. (n\,a + c)$ in terms of y, or $\sin. a$. Now, $n : 1$ being the ratio of the radii of D E and F G, taking

$n=1$ and $c=-\frac{\pi}{2}$, we have $\cos. (n\,a+c)=\cos. \left(a - \frac{\pi}{2}\right) = \sin. a = \frac{y}{k}$

k being constant (the distance of the pointer from the centre). It is therefore seen that the difference between two readings of the rolling wheel gives a quantity always proportionate to $\int y\,d\,x$, and since $\int d\,x$, disappears for a closed curve, we have a result which gives the area.

The rolling wheel for areas, however, is attached to the pointer, as by this means the machine is made more compact than it would be if a circular arc of equal effect were used.

When $n=2$ taking $c=0$ we have

$$\cos. (n a + c) = \cos. 2\,a = 1 - 2 \sin.^2 a = 1 - 2\frac{y^2}{k^2}.$$

The difference between two readings of the rolling wheel gives a quantity in this case proportional to $\int y^2\,d\,x$, and since the term

$\int d\,x$ disappears for a closed curve, this expression evidently gives the statical moment.

Taking $n = 3$ and $c = -\frac{\pi}{2}$ we have

$$\cos.\,(n\,a + c) = \cos.\left(3\,a - \frac{\pi}{2}\right) = \sin.3\,a = 3\sin.a - 4\sin.^3 a = 3\,\frac{y}{n} - 4\,\frac{y^3}{k^3}.$$

We therefore obtain the difference between the moment of inertia and a fixed multiple of the area.

The area and statical moments are obtained direct from the readings of the instrument, but to obtain the moment of inertia the necessary subtraction has to be performed by the operator.

The adjustments of the axles for the three rollers is determined as follows:—We have shown $\beta = n\,a + c$. Now, when $a = 0$, or the pointer is in the initial position, $\beta = c$, and as we have shown that $c = -\frac{\pi}{2}$ for the determination of the area and moment of inertia, it is evident that the rollers for these items must, in their initial position, have their axles parallel to the axis, and the motion of the wheels perpendicular to the axis. On the other hand, since we have shown c must $= 0$ for statical moments, the position of the axle for the rolling wheel must be perpendicular to the axis. Now, Amsler's integrator has the circular arcs of radii compared to the smaller pinions, as 2 : 3, and they, therefore, record readings which, on being affected by multipliers dependent on the size of the instrument and on simple calculation, give the actual statical moment and moment of inertia about the axis respectively. The reading from the wheel attached to the pointer, on being similarly affected by the multiplier dependent on the size of the instrument, gives the actual area.

As before stated, the integrator is restricted to rectilinear motion by means of the wheels of the carriage running in a groove in the straight-edged batten, or guide-batten, this batten being fixed parallel to the axis by means of the gauge accompanying the instrument, and marked L in Fig. 204. The rollers are marked A, M, I, and deal with areas, statical moments about the axis, and moments of inertia about the axis, respectively. The pointer, P, is traversed round the outline of the figure to be dealt with, which causes the rollers to rotate and slide. The rotatory motion in each case can be read off from the index attached to each roller, in the manner described in the following instructions* :—

* These instructions are chiefly derived from Mr. Amsler's pamphlet.

The result of reading from any roller is a number of four figures, the counting disc gives the first figure on the left, which represents a whole number; the three following figures are directly taken from the roller and vernier, and form the decimals of the number. The reading of the roller, as shown in Fig. 204, for example, is 1·417. The mark of the roller (in the example the 41) which has just passed the fixed mark, 0, of the vernier, gives the first and second decimal; the mark of the vernier, which is opposite to a mark of the roller (the 7 in the example), gives the last decimal of the number. The counting disc counts the complete revolutions of the roller. When the mark, 0, on the roller is at the mark, 0, of the vernier, a mark of the counting disc should be opposite the fixed index mark.

Readings for Area.

The pointer of the integrator being moved round the outline of the figure in the direction of the movement of the hands of a watch, at the end position the roller, A, will have moved forward from the starting position, that is to say, in the direction of the increasing numbers. The last reading, a_1, must therefore always be larger than the reading, a_0, at the starting position, and the difference $a_1 - a_0$ will be a positive value.

It often happens that the last reading is apparently smaller than the first reading—say, for example, the first reading is 8·325 and the last 1·256. This means that the mark, 0, of the counting disc has passed the fixed index mark, so that the revolution performed by the roller will equal $11{\cdot}256 - 8{\cdot}325$, if the mark of the counting disc has passed the fixed index mark once only during the traverse; or, in other words, 10·000 must be added to the last reading for each time the 0 mark of the counting disc passes the index mark.

Readings for Statical Moment.

The manner of taking the readings for this roller differs from the preceding, owing to the peculiarity of the statical moments being positive or negative, according to the position of the centre of gravity with reference to the axis. According to usual practice the moments are positive when the centre of gravity lies between the axis and the guide-batten, and negative when it lies on the other side of the axis. If the negative moments are greater than the positive, the result is negative, and its amount is what the roller, M, has gone back. As in the readings for area, the mark, 0, of the counting disc may pass once or several times the fixed index mark. In this case, however, the number 10·000 multiplied by the number of these passages is to be added to the second reading if the roller moves forward, and added to the first reading if it moves backward. For example, let the first reading be 2·432, and last reading 5·657, and suppose the marks 2, 1, 0, 9, 8, 7, 6, of the counting disc to have passed the index in succession, then the true reading due to the traverse will be $5{\cdot}657 - 12{\cdot}432 = -6{\cdot}775$.

In most cases the position of the centre of gravity relatively to the axis may be judged without difficulty, and it is therefore not necessary to observe the movement of the roller, M, and its counting disc; when the position of the centre of gravity is near the axis, the difference of the readings at starting and finish of traverse will be small, so that a mistake about the direction of rotation of the roller is not likely to occur, even without having observed the movement of the roller.

Readings for Moment of Inertia.

These are taken in the same manner as for areas if the roller moves generally forward. If, however, the arm of the moment of inertia for the figure under consideration exceeds 4 inches, the roller moves backwards, the difference of readings then becomes negative, and they are dealt with as explained for statical moments. Let a_0, m_0, i_0, be the zero readings, or readings at starting from the indexes, A, M, and I respectively, and a_1, m_1, i_1, the readings after a complete traverse of the pointer round the figure under consideration, the inch being taken as the unit of length; the area of the figure will be $(a_1 - a_0)\ 15$; the statical moment about the axis $= 40\ (m_1 - m_0)$; the moment of inertia relatively to the axis $= 240\ (a_1 - a_0) - 100\ (i_1 - i_0)$; the distance of the neutral axis from the axis of the instrument $= 40 \left(\dfrac{m_1 - m_0}{15\ (a_1 - a_0)}\right) = 8 \left(\dfrac{m_1 - m_0}{3 (a_1 - a_0)}\right)$; the "constants" being entirely dependent on the size of the instrument and the proportion of its parts. If, however, we are dealing with a figure drawn to a scale of $\dfrac{1}{n}$ inch $= 1$ foot, then the modification necessary is as follows. The area in this case will be $15\ n^2\ (a_1 - a_0)$. The statical moment about the axis $= 40\ n^3\ (m_1 - m_0)$, and the moment of inertia about the axis $240\ n^4\ (a_1 - a_0) - 100\ n^4\ (i_1 - i_0)$, the foot being the unit of length.

From these remarks it will be seen that actual results may be obtained, even when the drawing which we are considering is made to two different scales, one in the direction of the length of the figure under consideration, and the other perpendicular to it. The modification then necessary is obvious from the preceding details.

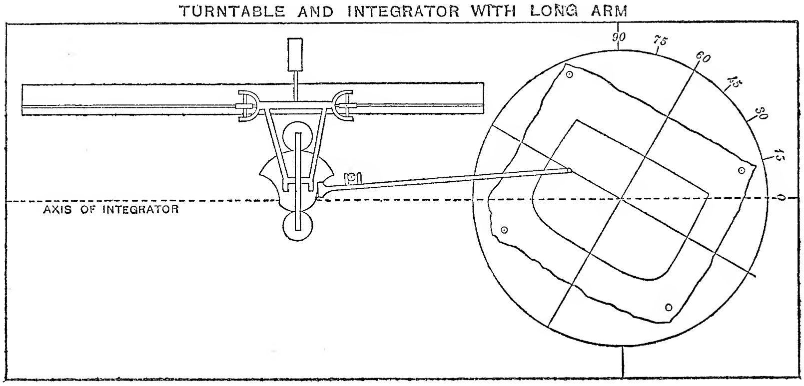
The usefulness of the integrator in naval architecture is thus apparent, it being used for determining displacement, centre of buoyancy, height of transverse and longitudinal metacentres, the neutral axis and moment of inertia of the section of the material of a ship, and also for curves of stability, both statical and dynamical.

It would occupy too much space to detail separately the manner of dealing with each item; suffice it to say that the methods for calculating curves of stability by means of this instrument are varied, some shipbuilders preferring to deal with the wedges of immersion and emersion and their moments, whilst others take the whole inclined body and its moment, determining in each of these cases, by means of layers, the exact inclined water-line corresponding to the displacement under consideration. In the Board of Trade plan, where the displacement in the inclined position is not necessarily the same as when the vessel is upright, the lengths of the righting levers are obtained for the displacement corresponding to the inclined water-line, as described in a previous chapter.

Dr. Amsler, son of the inventor of the instrument, has spent some months in the scientific branch of the works of Messrs. Denny, of Dumbarton, in studying the calculations of naval architecture, and in extending the application of the integrator to them. An arm of variable length has been supplied to Mr. Denny which can be fitted to the arm of the ordinary integrator, in order to increase the range and to allow the rollers to move on a surface independent of the paper which bears the diagram to be measured. This involves the introduction of fresh constants for its readings. On reference to Fig. 206, which shows the integrator with the long arm, it will be seen that the body-plan to be used is placed on a turntable, similar to that used by Mr. Macfarlane Gray, and the axis of the integrator therefore remains fixed throughout the work. Mr. Amsler, sen., has designed a parallelogrammatic mechanism, as shown in Fig. 207, which can be fitted to any integrator without involving any change of the constants. Though not increasing the range, this allows the rollers to move on a specially chosen paper instead of running on the drawing. The accuracy is not influenced. This latter addition to the integrator proves very useful. Further, an integrator of a

Fig.206.

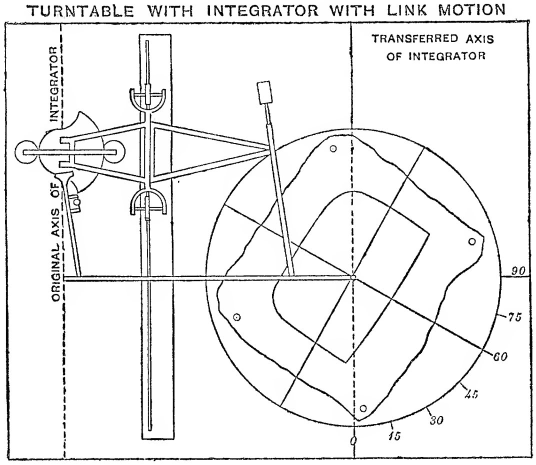
TURNTABLE AND INTEGRATOR WITH LONG ARM



new kind has been made for Mr. Denny. As in the ordinary integrator, a carriage runs to and fro on a railway, when the pointer

Fig.207.

TURNTABLE WITH INTEGRATOR WITH LINK MOTION



travels round a figure. The recording rollers, instead of moving directly on the drawing surface, move on a special disc which turns on a vertical axis at the same rate as the carriage shifts along the rail. In consequence of the rollers running on the disc independently of the drawing surface, and this disc giving the rollers by its rotation a much greater motion than they would get by rolling directly on the drawing, the accuracy acquired is considerably greater.

Dr. Amsler has favoured the author with an account of his method of using the integrator in stability calculations. He has endeavoured to reduce the taking of readings as much as possible, to avoid the shifting of the straight edge of the integrator (which may involve considerable errors and inconvenience) and to arrange the measurements so as to check each other without repetitions. We take the following from his communication* :—

"For avoiding the shifting of the straight edge, I do not measure directly the arm of the righting moment, but the coordinates of the centre of buoyancy of the inclined body relatively to the two axes. Fore-body and after-body are measured at the same time. I proceed in the following way:—The integrator is set to the centre line of the body-plan as the axis of moments; then the wedges between the water-lines from 0° up to 45° are measured, moving the pointer round all the sections to be multiplied by the same Simpson's multiplier; I thus reduce the number of readings considerably, viz., the reading at the starting position of the pointer; the reading after having gone round the first and last sections; then after the odd sections, and then after the even sections; the last reading is to be used as the starting reading for the next wedge, as the straight edge has not to be shifted. The method of checking will be obvious on the specimen I enclose. When the wedges are measured up to 45°, the volumes of the immersed and emerged wedges are compared, and layers drawn where necessary. The volumes and moments of the layers are then measured with the integrator on the body-plan, and the water-line at 45° corrected.

"This done, the straight edge of the integrator is adjusted to a horizontal axis, and the wedges from the correct water-line at 45° up to 90°, as well as the layers, are measured in the same way. This is the whole integrator work to be done. The results obtained are the *horizontal* shifts of the centre of buoyancy from the upright to 45° on the one hand, and on the other hand the *vertical* shifts of centre of buoyancy from its position at 45° up to 90°.

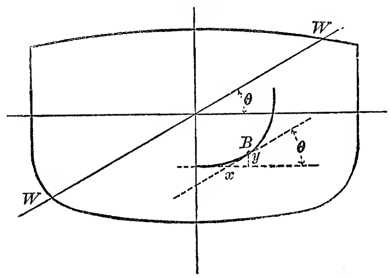
"The *vertical* shifts from 0° to 45°, and the *horizontal* shifts from 45° to 90°, are then found by the following method, which has

* Dated 18th March, 1884.

been suggested to me by Mr. Purvis, and which I put into a practical form.

"B (Fig. 208), may be the centre of buoyancy corresponding to the inclination θ; x and y the co-ordinates of B. The tangent at B

Fig. 208.



to the curve of centres of buoyancy is parallel to the corresponding water-line, W W, and contains therefore with the x axis the angle θ.

Hence $$\frac{dy}{dx} = \tan. \theta.$$

$$y = \int_0^x \tan. \theta \, dx.$$

"This integral represents the area of a curve, the ordinates of which are the tangents of the inclination, and the abscissæ the corresponding values of x found by measurement. As tan. θ grows very rapidly for values of θ beyond $60°$, I use for $\theta > 45°$ another curve. Instead of considering θ, I consider now $90° - \theta$, and

$$\frac{dx}{dy} = \tan. (90° - \theta),$$

$$x = \int_{y\,45°}^{y} \tan. (90° - \theta) \, dy.$$

The values of x taken from the curve, the area of which represents this integral, are then the horizontal shifts of centre of buoyancy from $45°$ up to the heel, θ.

"The curves for finding the two integrals are very easily drawn, and are generally fair curves. The results obtained from them are quite as exact as if found by direct measurement on the body-plan.

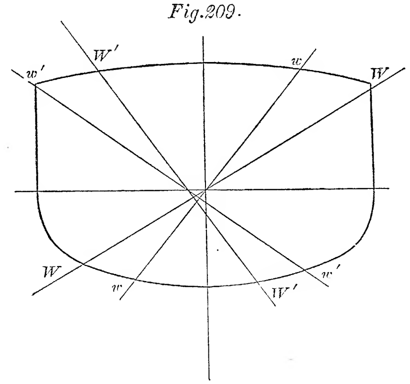
"I can, by means of the method explained, easily get out the co-ordinates of the centre of buoyancy for seven different inclinations (10°, 20°, 30°, 45°, 60°, 75°, 90°) in about seven hours—the time depending, of course, on the number of sections of the body-plan."

The specimen calculation referred to by Dr. Amsler is given on a separate page opposite.

A paper, descriptive of the integrator, giving further examples of calculations made by means of it, was read at the Institution of Naval Architects, in April, 1884, by Dr. Amsler, and of which examples we propose to give (at the end of the volume) those relating to the determination of the displacement and vertical position of the centre of buoyancy, displacement per inch of immersion, and transverse and longitudinal metacentres, in order to more fully describe the uses of this important and valuable instrument.

			Wedges between the Water-Lines, W.			Wedges between the Water-Lines, W′.			Sum of Functions.		
Sections:			1 L 17.	3, 5, 7, 9, 11, 13, 15.	2, 4, 6, 8, 10, 12, 14, 16.	1 L 17.	3, 5, 7, 11, 13, 15.	2, 4, 6, 8, 10, 12, 14, 16.			
Simpson's Multipliers.			½	1	2	½	1	2			
Inclinations.	Wedges.	Starting position	Difference ⟶ Reading.	Difference ⟶ Reading.	Difference ⟶ Reading.	Difference ⟶ Reading.	Difference ⟶ Reading.	Difference ⟶ Reading.	Positive Excesses.	Negative Excesses.	Total.
0–10°	Immersed.	7340	⟶	·714 8054	·842 8896	⟶	1·029 9925	·896 0821			
	Emerged.	0821	⟶	·995 1816	·841 2657	⟶	·689 3346	·846 4192			
	Im.—Em. (I.)			− ·281	+ ·001		+ ·340	+ ·050			
	Functions.		**0**	**− ·281**	**+ ·002**		**+ ·340**	**+ ·100**	**·442**	**− ·281**	**·161**
10°–30°	Immersed.	4192		·322 4514							
	Emerged.	9325		·479 9804							
	Im.—Em. (II.)			− ·157							
0–30°	(I.) + (II.)			− ·438							
	Functions.		**0**	**− ·438**							
30°–45°	Immersed.	4660		·772 5432							
	Emerged.	8446		·416 8862							
	Im.—Em. (III.)			+ ·356							
0–45°	(I.)+(II.)+(III.)			− ·082*							
	Functions.		**0**	**− ·082**							
0–20°	Immersed.	0637		·639 1276							
	Emerged.	6970		·844 7814							
	Im.—Em. (1.)			− ·205							
	Functions.		**0**	**− ·205**							
20°–45°	Immersed.	4192		·1168 5360							
	Emerged.	2698		·1058 3756							
	Im.—Em. (2.)			+ ·110							
Check 0–45°	(1.) + (2.)			− ·095*							

Fig. 209.



The figures in block type are to be entered after all the measurements are made.

The arrow ⟶ shows the way in which the differences of readings have to be taken.

Similar sheets are used for the moments, as well as for the measurements, about the horizontal axis.

* The two quantities marked thus * should be the same if they were rigidly exact.

CHAPTER XVIII.

Rolling of Ships at Sea—Relation of Curve of Statical Stability to Curve of Dynamical Stability—Comparison of Stability of *Monarch* and *Captain* with same Wind-Pressure—Effect of Canvas upon Ships Sailing among Waves—Limit to which Ships may be Safely Inclined in Waves—Report of Admiralty Committee on Designs—Curve of Wind-Pressures—Comparative Stability of *Monarch* and *Captain*—Objections to Foregoing Report—Quantitative Values of Wind-Pressures in Relation to Stability—Examples of Stability of Ships in Waves.

It does not lie within the scope of this work to discuss the rolling and other motions of ships at sea. To do that effectually it would be necessary to consider also, and at much length, the constitution and movements of waves, together with other matters which concern not so much the *stability* of ships, as the external relations, so to speak, of that stability. It seems desirable, nevertheless, to bring together a few considerations of a dynamical kind, which admit of ready and sufficient exposition.

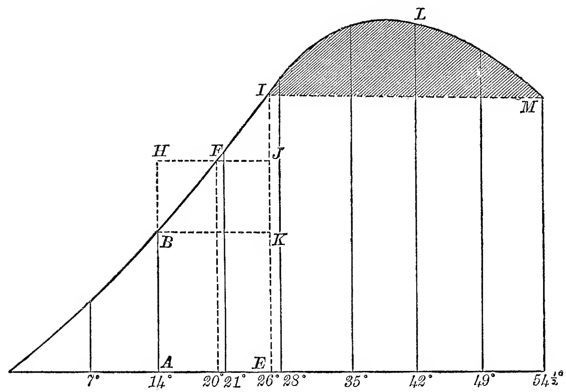
Great confusion has existed in past times, and some confusion still exists, as to the relation that holds between a ship's stability and her steadiness in a sea-way. It was formerly considered by many that the excessive rolling of ships was usually due to a too-elevated centre of gravity, and when armour-plated ships were introduced, and found frequently to roll greatly, top-heaviness was the cause most commonly assigned, both in England and in France. No improvement being found to result, however, from the lowering of the centre of gravity, but substantial improvement usually following the raising thereof, the matter soon came to be regarded with more care and judgment. In the mercantile marine the violent rolling that resulted from a too-low centre of gravity, such as attended cargoes of iron rails, or other heavy cargoes, when stowed low, also came to be well understood, and remedied by the obvious means of raising the centre of gravity. The scientific investigations of the late Mr. Froude and others have placed the true principles of the matter beyond all question. Nor is any great amount of scientific investigation needful to a true appreciation of the subject. If a ship possessing a very low centre of gravity, and consequently great stability, be forcibly moved from the upright position in still water, and then left free to go back, she will (neglecting the question of her moment of inertia) return with violence to the upright position; pass, by virtue of the moment acquired, beyond the upright, and there come to rest; return again through the upright position; and so on, oscillating about that position through decreasing angles, until the momentum imparted to her becomes expended, and she is brought eventually to rest. The whole of her movements are the result of her efforts to seek her original upright position, or of the efforts of the water to place her there, in which position her axis of symmetry is at right angles to the water's surface. If now, we imagine that, instead of the ship being moved from the upright, the water is lifted out of the horizontal, say, into the position of a wave-slope, thus immersing (with a constant displacement) more of the ship on one side than on the other, it is reasonable and just to expect that the ship will consequently be turned from the upright to an inclined position, in response to this increase of pressure on one side and diminution on the other; and also that the urgency with which she will be so moved, will be proportioned to the forces which in still water urged her to the upright position, viz., to her statical stability. This idea, while needing manifold developments and qualifications, is the fundamental idea which regulates, if we may so speak, a ship's behaviour in waves; and it points immediately and directly to the doctrine that, within certain limits, a very stable ship may be regarded as tending to violent rolling in waves, while a ship of small stability may be regarded as having much less inducement to violent rolling. The limit on the side of great stability is to be found in the fact that were the stability of a ship infinite, an exact conformity to changes of mean wave-slope would be the condition of maximum motion; the limit on the side of small stability is of course to be found in the well-understood danger of capsizing from excessive crankness.

The ordinary curve of stability furnishes a ready means of viewing the stability of a ship, under given conditions, in relation to forces tending to incline her. Fig. 175, chap. xv., represents (as an interesting illustration of the principles involved), a portion of the statical curve of stability of the late ship, *Captain*, and, as we said, the dynamical stability exerted during the inclination of the ship from one angle to another is represented by the corresponding curvilinear *area*. It follows* that when a steady breeze is keeping the ship at an inclination of 7°, if the wind slowly increases until she is inclined to an angle of 14°, the amount of dynamical stability expended during the further inclination by the increase of wind will be represented by the area B F G, for the whole dynamical stability developed during the further inclination is equal to the area A B F E, of which the portion A B G E is due to the original force of the wind, leaving the remainder as the result of the wind's increase. Next, if we suppose that the wind, instead of increasing slowly, suddenly springs up to the full force which we have been considering, the increase of work which it will perform upon the ship will be represented by the rectangle B H F G, and the ship will have to perform an equal amount of work in resisting this sudden increase of wind. This she can only do by inclining through a still further angle, and afterwards finding her way back again to the angle of 14°, at which the increased wind will ultimately keep her steady. It is easy to see how much further she will go; for what she has to do is clearly to exert an excess of work equal to the excess of work which the wind has exerted, and which is represented by B H F. She will therefore incline to such an angle that F I J shall be equal in area to B H F. We thus see why a reserve of dynamical stability is necessary, and how it is called into play by a sudden increase of wind—a mere gust or squall often lasting quite long enough to produce upon an unstable ship all the effects of a permanent increase of wind.

Let us now, as an instructive example of the use of such curves, compare the cases of the *Captain* and *Monarch*, both sailing under a steady breeze at a permanent inclination of 14°. This is a rather large inclination, but it is selected for two reasons: First, it is an angle at which the righting leverage of the *Captain* was slightly greater than that of the *Monarch*, and secondly, 14° is the angle at which the *Captain* was actually sailing a few hours before her loss. With both ships steadily inclined at 14° then, let us now suppose the wind to suddenly increase in a gust or squall, sufficiently to heel the *Monarch* over considerably further, and to keep her, say at 20°, during the continuance of the squall, or until the sail is shortened. Let us now see what happens to the *Monarch*. Fig. 210 shows us. The

* Many of the remarks which follow are reproduced from *Naval Science*, vol. i., edited by the author.

Fig. 210.

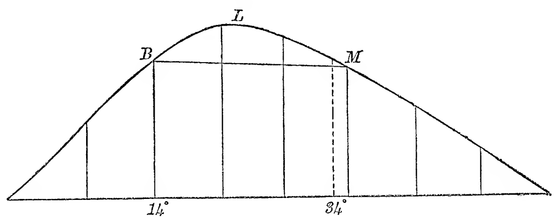


force of the wind springs suddenly from A B to A H, and drives the ship over to an angle of 26°, by which time the total extra dynamical "wind-work" (so to speak), represented by B H J K has found its equivalent in the resisting work represented by B I K, the portion B F J K being common to both, and B H F equal to F J I. But the extra wind-work having thus been absorbed by the stability, the ship now finds herself under the action of a force of stability, represented by E I, which is considerably greater than the force of the wind (A H or E J), and she will consequently be forced back to the angle of 20°, where the force of the wind and of the stability will exactly balance each other. As a matter of fact, the stability will force her back beyond this position until the excess of stability is absorbed, and then the wind will drive her back again beyond 20°, but to a less extent; and so on, the ship oscillating for a time about the balanced position—20°—and at length steadying herself there. It will be observed, however, that, even when over at nearly 26°, she still has a reserve of stability of considerable amount, represented by the area of the figure I L M.

Next, let us see what happens to a ship like the *Captain* under precisely similar circumstances—assuming, for convenience, that the weights of the two ships (which only differed by about 4 per cent.) are identical, and that the relative ordinates of the diagrams (or levers of stability) therefore represent the stability in foot-tons in each case. As the same force of wind holds the two ships at the same angle, where the righting forces are about alike, we may also assume that the increase of wind will affect both ships equally.

Hence we may consider that B I K, Fig. 210, which represents the extra work done upon the *Monarch*, represents also the extra work which will be imposed by the wind upon the *Captain*, and will be deducted from the reserve of work, or dynamical stability, which she possesses before the squall springs up, and which reserve is represented by the area, B L M,* Fig. 211. But this is not equal in

Fig. 211.



amount to B I K; the whole of B L M will therefore be absorbed, the ship will incline to an angle of 34°, and still leave a small part of the wind's demand unsatisfied. With this unsatisfied demand pressing upon her, she will, of course, go on inclining still farther, her stability getting smaller and smaller as she goes, until at $54\frac{1}{2}°$ she ceases to have any at all, and will then turn bottom upwards. Here we see that, although under slowly-increasing wind, free from gusts and squalls, the *Captain* would have been safe, though inclined to an angle of more than 20°, she was unsafe even at an angle of 14° if exposed, as all ships are at sea in squally weather, to the sudden rising of the wind. This is why we see greater cause for alarm in considering the dynamical stability of such a ship than we had already seen in considering the statical stability.

Thus far we have considered the conditions of vessels upon the assumption that the level of the sea is uniform; or, in other words, we have considered "smooth-water" circumstances only. It now becomes necessary to observe that the existence of waves must make a very considerable difference. Probably the simplest (though not the most exact) way of looking at the subject is to consider that a ship inclined under canvas is affected by waves in the same manner and degree as she would be affected by them if inclined to the same extent by any other means, and that under this aspect the effect of setting canvas upon a ship, and thus inclining her to a given angle, will simply be to diminish the angles through which

* B M should not be quite parallel to the base-line, on account of the diminution of the wind's moment as the ship inclines.

she can roll with safety by that amount. If, for example, we should, for the moment, consider the angle at which a ship attains her maximum stability to be the limit of the angle through which she can roll with safety, from the upright position to either side, whether with or without sails, then that angle, diminished by her greatest inclination under a given spread of canvas and given force of wind (allowing for gust), will be the limit through which she can roll with safety under that spread of canvas and that force of wind. The angle of maximum stability for the *Captain* was 21°; that for the *Monarch*, 40°; that for the *Vanguard*, $44\frac{3}{4}$°. When sailing under a steady breeze, capable of inclining them at 7° in smooth water, and not allowing for gusts, these ships would therefore (on the suppositions laid down) have respectively the following limits of safe rolling:—

Captain, ...	$21° - 7° = 14°$
Monarch, ...	$40° - 7° = 33°$
Vanguard, ...	$44\frac{3}{4}° - 7° = 37\frac{3}{4}°$

When sailing under a wind that would incline them 14° in smooth water (again neglecting gusts), they would have these limits respectively reduced by 7°.

We here have presented, in a mournful light (although not nearly the worst light), the danger of low freeboard in sailing ships. The *Captain's* 21° of maximum stability compare badly enough with the *Monarch's* 40° under any circumstances, seeing that she was intended for sailing; but when we come to reduce these respective amounts to an equal extent, as we undoubtedly must do when the ships are equally pressed by canvas, and consider the remainders left for safety, we see that the high freeboard ship has nearly three times the amount of stability possessed by the *Captain*. This was the normal condition of these two rigged ships at sea, and the stability remaining in her when inclined under sail (and not the stability with no sail spread) was the capital, so to speak, with which the *Captain* was to purchase immunity from squall and storm and from the action of the waves of the sea.

The view just presented of the effect of canvas upon a ship sailing among waves is not strictly correct, for several reasons. First, there is no sufficient reason for limiting the extreme aggregate inclination to the angle of maximum stability; secondly, the angle through which waves will cause a ship to roll is usually less when she has canvas spread than when she has none; thirdly, the mean angle of inclination—about which position the ship may be supposed to roll to either side—will be greater in waves than in smooth water under an equal force of wind; and fourthly, the modifications of motion resulting from causes of this nature will change the ship's relation to the waves, and to the wave-periods, and thus modify the forces impressed upon her.

To the first point we shall recur hereafter. As to the second—that a ship with canvas spread will roll less in waves than she would roll under similar circumstances without canvas, is obvious, and a matter of common experience. Even if no wind blows, the sails will act as huge atmospheric keels, so to speak, diminishing the rolling motions in the same manner as bilge keels below water diminish them. This action is obviously favourable to a ship the stability of which vanishes at moderate angles. But the third of the modifying causes just enumerated—the increase of the mean angle of inclination—is increased by the action of the wind upon the sails, as may be easily shown. First, let us suppose the ship, with her sail spread, to be set uniformly oscillating by the waves, with no wind blowing. The resistance of the air upon the canvas will obviously be the same as she moves to either side, and the vessel will oscillate about the upright position as the mean position. Next, suppose the air to move from left to right, and to have a velocity equal to the mean velocity of the sails, the consequence will be (not quite exactly, but very nearly) that the sails will experience no resistance in moving from left to right, and a greatly increased resistance in moving from right to left. The ship will therefore move further to the right of the upright position than to the left of it, and the oscillations will consequently take place, not about the upright position, but about a mean position, which will be inclined at an angle to it, and on the right side of it. The same is evidently true in various degrees whatever be the force of the wind. That the angle of inclination of this mean position in waves will be greater than that produced by the inclining force of an equal wind on the ship in smooth water, arises from the fact that the force of the wind varies with the square of the velocity, and not as the velocity itself. If we suppose, for example, that the velocity of the wind is represented by 12, and that of the sails by 4, then the relative velocities of the wind and sails in the opposite directions respectively will be 8 and 16; and the forces of the wind upon the sails moving from the wind, the sails at rest, and the sails moving against the wind, respectively, will be proportional to 64, 144, and 256, respectively. The decrease of force on the sails due to their moving away from the wind will therefore be proportional to 80 (144–64), and the increase due to their moving back against it will be proportional to 112 (256–144). The resultant effect will obviously be to increase the mean angle of inclination. While, therefore, the effect of the wind is on the one hand to diminish the angle of rolling of the ship under canvas, on the other hand its effect is to lay her over to a greater angle than she would incline to in smooth water, and compel her to make her diminished oscillations about that position. What is the precise relation between these two effects, or how far the fourth cause above referred to—the change of the wave-action which results—tends, in combination with them, to modify the simple aspect of the subject which we have given, we have not attempted to investigate.

We now revert to the question of the limit to which a ship may be safely inclined by the joint action of the wind and waves, to the consideration of which we promised to recur. In our comparison of the *Captain* and *Monarch* we for the moment assumed the angle of maximum stability to be that limit. It will be obvious, however, that a ship will not necessarily be capsized because she is driven over to that angle; on the contrary, she may with safety be laid over much further, under favourable conditions. This subject is dealt with in one of the published reports of the late Admiralty Committee on Designs, the following view of it being taken—it being necessary for the reader to observe that, as a ship is inclined by a given force of wind, the effect of the wind upon the sails gradually diminishes, and therefore the moments of the wind forces, set off to scale on a curve analogous to a curve of stability, will be represented by a curve gradually approaching the base, as we just now stated in a foot-note, and not by a straight line parallel to the base. Professor Rankine, who drew up the Committee's Report, writes as follows:—

"1. The curve of stability of a ship being given with ordinates proportional to the righting moments at different angles of heel, conceive to be drawn a curve with ordinates proportional to the moments of pressure of the wind, of an altitude such that the segment cut off by it from the top of the curve of stability shall cover an angle equal to the angle of vanishing stability required in order that the ship may be safe against the heave of the waves alone. This curve of moments of wind will divide the curve of stability into three arcs; and the angle covered by the arc cut off at the commencement of the curve of stability will be the limit which ought not to be exceeded by the greatest angle of heel produced by the wind alone, allowing for the dynamical effect of a sudden squall. The angle of steady heel ought not to exceed about one-half of the before-mentioned angle.

"2. From the limiting angle of steady heel may be deduced, by the help of the curve of stability, the greatest safe steady heeling moment of the wind pressure, and thence the greatest safe pressure of wind with a given spread of canvas.

"3. As an elementary means of performing the construction mentioned in paragraph 1, conceive the diagram showing the curve of stability to be wrapped on a cylindrical surface, so that the points marked 0° and 90° on the base-line of the diagram shall be at the two ends of a diameter of a cylinder; draw the projection of the diagrams on a plane traversing that diameter and the axis of the cylinder; then through the point marked 90°, draw a straight line by trial, so as to cut off from the top of the new diagram a segment, covering an angle equal to the angle of vanishing stability required for safety against the heave of the waves alone. The interval from the point marked 0° to the commencement of that segment will correspond to the greatest angle of safe heel by a squall; and half the ordinate at the commencement of the segment will represent, very nearly, the greatest safe leverage of the moment due to the steady pressure of the wind. That leverage multiplied by the displacement will be the required moment.

"4. The following are examples :—

"I. The *Captain*, with $6\frac{1}{2}$ feet freeboard—

"Angle of vanishing stability required for safety against the heave of the waves alone, ... say 39°

"Top segment cut off from curve C in diagram begins at . 9°

(Being the greatest safe heel by a squall when rolling in the trough of a long swell.)

"Half the lever of stability at that angle, . . . 0·23 foot.

"Displacement—say 7,700 tons; therefore greatest safe moment of pressure of wind . . . . $7{,}700 \times {\cdot}23 = 1{,}771$ foot-tons $= 3{,}967{,}000$ foot-lbs.

"Moment of all plain sail, with a pressure of 1 lb. per square foot $= 2{,}500{,}000$ foot-lbs.

"Greatest safe pressure of wind, when all plain sail is set $= 1{\cdot}59$ lb. on the square foot.

"II. The *Monarch* at 24 feet 3 inches draught—

"Angle of vanishing stability required for safety against the heave of the waves alone, ... say 39°

"Top segment cut off from diagram of stability begins at . $27\frac{1}{2}°$

(Being the greatest safe heel by a squall when rolling in a trough of a long swell.)

"Displacement—8,306 tons; therefore greatest safe moment of pressure of wind . . . $= 8{,}306 \times \cdot 74 = 6{,}146$ foot-tons $= 13{,}768{,}000$ foot-lbs.

"Moment of all plain sail, with a pressure of 1 lb. on the square foot . . . . =2,540,000 foot-lbs.

"Greatest safe steady pressure of wind when all plain sail is set =5·42 lbs. on the square foot.

"Summary of the comparative stability of the *Captain* and *Monarch*:—

	Captain.	*Monarch.*
"Least safe angle of vanishing stability without canvas, say, ...	39°	39°
Greatest safe angle of heel by a squall, when rolling in the trough of a long swell, ...	9°	$27\frac{1}{2}°$
Greatest safe moment of steady pressure of wind, (ft.-tons), ...	1,771	6,146
Greatest safe pressure of wind on all plain sail (lbs. on the sq. ft.), ...	1·59	5·42"

The Committee, having considered the preceding paper, approved of it generally, subject to the following observations:—

"I. As a means of comparing together two given designs of ships with respect to margin of stability under canvas, the method above described is valid.

"II. Hence, it appears that a heel of $4\frac{1}{2}°$ under canvas gave only the same margin of stability to the *Captain*, that a heel of $13\frac{3}{4}°$ gives to the *Monarch*.

"III. The absolute value of 39° is assumed for the least safe angle of vanishing stability without canvas, because of its having been the smallest value of that angle found to be sufficient by the Scientific Sub-Committee in their reports on unmasted ships. It does not take account of the steadying effect of friction, or of bilge-keels, or of canvas.

"IV. The estimate that the greatest safe moment of steady pressure of wind is only one-half of that corresponding to the greatest safe angle of heel by a sudden squall probably errs on the safe side; because it leaves out of account the diminution of the pressure of the wind due to the velocity with which the ship yields to it in rolling.

"V. The Committee are not prepared to say that the *Monarch* may not be safely sailed at an angle of steady heel greater than $13\frac{3}{4}°$. They consider that she will be a perfectly safe ship when managed according to the ordinary rules and usages of seamanship."

Diagrams from which Figs. 212 and 213 have been reduced are

Fig. 212.

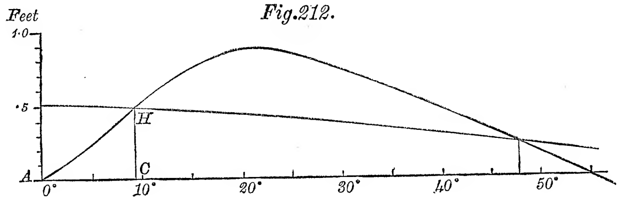
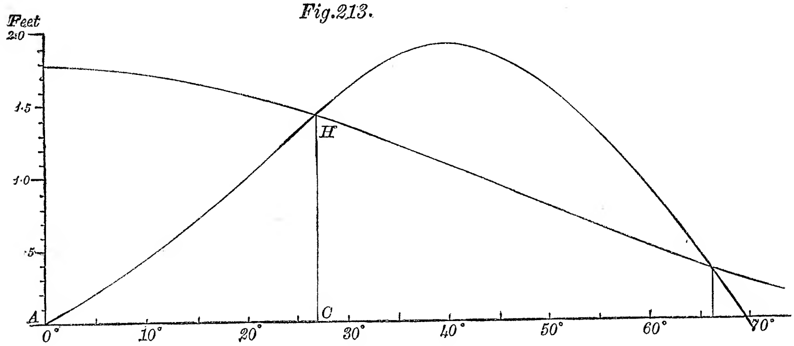


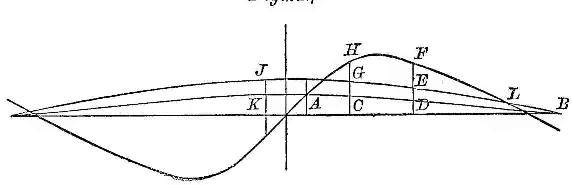
Fig. 213.



appended to the above report of the Admiralty Committee. The lettering upon these diagrams is our own, for further use presently, C H indicating the angle marked by the Committee as the "greatest safe heel by a squall," when the ship is rolling in the trough of a long swell.

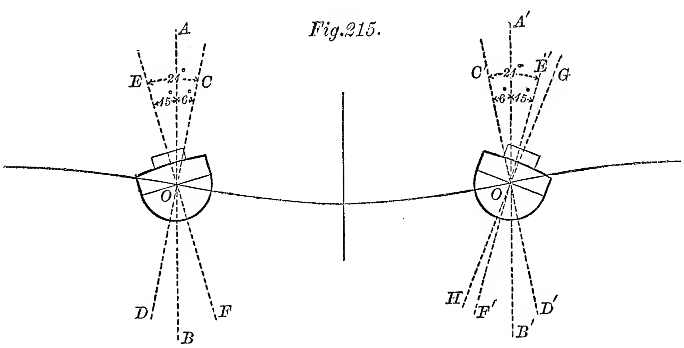
Exceptions were taken to the foregoing report. The first objection was that it fixes a definite angle as the limit of a ship's range of stability when not under sail, although no satisfactory means of determining such a limit exists. It is quite true, in our opinion, that no such means have yet been laid down with exactness, or with a sufficiently close approximation to the truth to justify a designer in fixing a much lower limit to safe rolling without sail than other considerations point to. But this objection can scarcely be opposed with fairness to the method of the Committee (although it may to some of its applications), for the method may be sound, although the means of fixing the limiting angle of wave-rolling may remain undetermined. The Committee's mode of estimating the combined effect of wind and waves has been objected to, even accepting the angle of wave-rolling as fixed. The objection is founded on the assumption that if the curve from A to H C (Fig. 212 or 213), is sufficient to provide for the effect of sail, under both breeze and squall, then all the remainder of the curve, except such as the steady breeze demands, is available to resist the roll caused by the waves or heave of the sea; and the objection is that, in the Committee's method, resistance to the squall is provided twice over—viz., in both ends of the curve. This objection is no doubt valid, for the cause of the extreme heel to H C will already have ceased before even the inclination to H is reached. Further, as a "radical objection to the whole method" of the Committee, it has been said that, "the two actions, of the sea and the wind, must generally go on together, and extend over the same portion of the curve of stability, so that it appears improper to devote one portion to the gusts of wind and another to the heave of the sea. It would be more nearly consistent with the conditions of a ship under sail and rolling in waves, to suppose her to be rolling in the worst waves she is likely to meet, and to suppose her sails struck by a squall at that part of her roll at which the gust would be most dangerous." In illustrating this view, the case of a ship "assumed to be rolling to leeward under the influence of waves alone," has been taken, a state of things which presumes that no wind is blowing, although the "worst waves" are supposed to be running, the object being to bring into view the worst possible effects of wind upon the canvas; and also the case of a ship rolling among waves with a steady pressure of wind, the power of which suddenly becomes doubled at the worst part of the roll. This latter case is represented in Fig. 214,

Fig. 214.



where K B is the wind-curve for the steady pressure, J K and H C are the extremes of the roll before the power of the wind is increased, and J G E is the wind-curve for the suddenly increased intensity which is assumed to take place when the ship is in the position J K. "The gust will cause the roll to be lengthened, say to the position D F, the area H G E F being equal to the area J K C G; and in this case the ship will be safe so long as the area H G L F is greater than the area J K C G." This is, no doubt, a very instructive way of treating the question, although not by any means free from objection, because the curve of stability has reference in all such methods to the inclination of the ship to the wave-surface, while the wind-curve has reference to her inclination to the horizon. This difficulty, however, can, we think, be pretty satisfactorily overcome by a process which we will endeavour briefly to describe. We may suppose that between the squalls the vessel rolls as she would do were she without canvas; by Professor Rankine's method we can determine approximately the angle to which she rolls—to the horizon, as well as to the wave-surface. Let us for illustration suppose she would roll, as the *Devastation* was estimated to roll, among waves with a maximum slope of 36°—viz., 15° to the horizon and 21° to the wave-slope. Fig. 215 shows the positions she would occupy at the two extremes

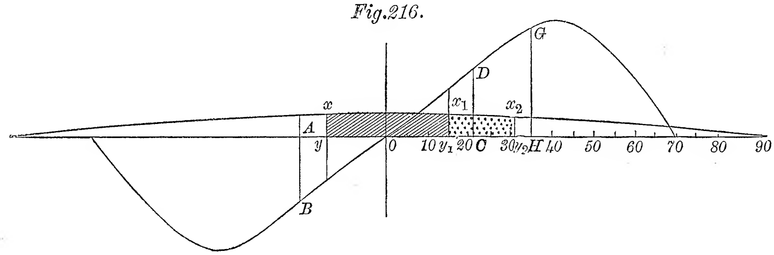
Fig. 215.



of a roll without the action of the wind, these positions occurring on the steepest parts of the wave, and the ship keeping time with the waves. Suppose now the wind blowing from left to right, and to strike the vessel when she is at the extreme of her roll to windward. It is evident that, instead of rolling to the position E′ F′, she will go over to some position beyond it, as G H, in order to absorb the work stored up by the action of the wind on the sails.

We may also fairly assume that the period of the roll is not materially altered by the action of the wind, although the range is increased, and that the extreme position still occurs about the mid-height of the wave. The position, H G, can be found as follows:—In Fig. 216, A B and C D mark the positions of the

Fig. 216.



extreme rolls without canvas, 21° to the wave-slope, $x\,y$, $x_1\,y_1$, mark the same positions of the ship, but with respect to the horizon—viz., 15°. The vessel would, by the action of the waves alone, have rolled to C D, expending an amount of work, O C D, equal to O A B, which would be stored up as *vis viva* as she passed the upright position; consequently, when she reaches C D, she has still to expend work equal to the area $x\,y$, $x_1\,y_1$, due to the wind. She therefore continues her roll to the position H G, which is determined by the condition that the area C D G H shall be equal to $x\,y$, $x_1\,y_1$, plus the area $x_1\,y_1$, $x_2\,y_2$, due to the wind through a range $y_1\,y_2$, equal to the additional range C H (Fig. 216), or the angle E′ O G, Fig. 215.

In the case of a ship liable to sudden squalls, but with a steady pressure of wind between them, the principles involved are identical with the preceding, except that the rolling between the squalls cannot be treated as that of a mastless ship. The extreme of the roll to windward would still be the worst position in which the squall could strike her, but it is impossible to determine with any degree of accuracy, in the present state of our knowledge, what the extent of that inclination might be expected to be. Could we determine that, it would only be necessary to draw the wind-curve representing the force of the squall above that of the steady wind-pressure, and to measure the area between the two wind-curves for the stored-up energy, instead of taking it to the base-line, as in the former case.

Let us now consider the matter, having regard to the quantitative values of wind-pressures in relation to the stability of ships. And, first we will take the case of a sailing vessel of 1,600 tons, whose stability has already been discussed in a previous chapter.* Her displacement at a mean load draught of 20 feet 1 inch is 3,400 tons. The area of her plain sail, *i.e.*, the courses, topsails, and topgallant sails on each mast, together with the jib and foresail, is 23,517 square feet, the total area of all her sails being 38,472 square feet. In order to determine the resistance to heeling offered by her stability with any given pressure of wind, it is necessary to assume a certain condition of stowage; we shall assume her to be laden with a cargo of such a density, and so stowed, that the common centre of gravity of the hull and cargo is $3\frac{1}{2}$ feet below the metacentre at the load line given above, viz., 20 feet 1 inch. Her curve of stability for this assumed condition is represented in Fig. 217, in which the curve, B,

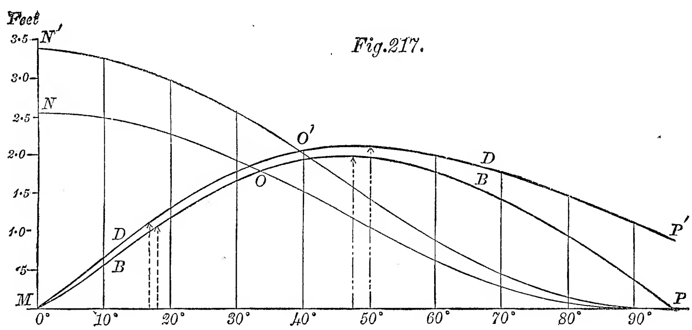


Fig.217.

shows the lengths of the arms of righting levers from 0° to 100°. The angle at which the deck-edge becomes immersed, 18°, is indicated by an ordinary dotted line, and that at which her maximum stability is reached, $47\frac{1}{2}$°, by a different kind of dotted line. From this curve of stability we can readily find, in the manner previously described, the exact angle of heel at which the ship would be held over by any given steady pressure of wind upon the sails. Assuming all plain sail to be set, a steady pressure of wind upon the beam equal to a force of one pound upon each square foot of sail would keep the ship inclined at an angle of 5°, because the moment obtained by multiplying the area of the sail set by the pressure of wind per square foot, and by the distance of the centre of effort of the sail above the centre of lateral resistance, would there be exactly equal to the moment of stability tending to restore the ship to the upright position. It must be understood that in this and in similar cases we assume the sails to be braced round in a position exactly per-

* Chapter vii., page 125, ship B.

pendicular to the direction of the wind, *i.e.*, exactly "fore and aft." With the same sail set a pressure of two pounds to the square foot would keep the ship inclined to an angle of 9°, somewhat less than twice as great an angle as that to which she was inclined by a pressure of wind of exactly one-half the force; this is, of course, partly owing to the fact that the sails are less square to the direction of the wind in the more inclined position. To keep her inclined at an angle of $47\frac{1}{2}$ degrees, *i.e.*, her angle of maximum stability, with the same sail set, a pressure of 17·6 pounds per square foot would be required, nearly equal to the force of a hurricane. Although the assumption of such a force of wind would be inconsistent with those of carrying all plain sail, and of smoothness of water, still it is instructive to consider what an enormous pressure is necessary to heel the ship over to her angle of maximum stability. The very greatly increased effect upon a ship of the sudden application of a given pressure of wind, instead of a gradual application of the same pressure, has already been shown. The pressure which, applied suddenly to this ship when in the upright position, would throw her over upon her beam-ends is represented in Fig. 217 by the curve, N O P, which is a curve of wind-pressures drawn in a manner previously explained, so that the area, M O N, is equal to the area of that portion of the stability curve comprised above the portion, O P, of the line, N O P. If this same pressure of 10 pounds on the square foot were to be applied gradually and steadily, instead of suddenly, it would incline the ship and keep her over to an angle corresponding to the point, O, where the line, N O P, intersects the curve of stability, B, *i.e.*, an angle of about 33°.

In Fig. 217 we also illustrate the case of another sailing vessel of 800 tons, whose stability has also been discussed in a previous chapter.* Her displacement at a mean load draught of 17 feet 3 inches is 1,775 tons. The area of her plain sail is 10,983 square feet, the total area of all her sails being 17,331 square feet. Her curve of stability, marked D, was calculated on the assumption that her stowage was similar to that in the previous case, *i.e.*, such as to give her a metacentric height of $3\frac{1}{2}$ feet. Her deck-edge becomes immersed at an angle of 17 degrees, as indicated, and her angle of maximum stability is reached at 50 degrees.

The wind-curve, N′ O′ P′, drawn so that the area, M O′ N′, is equal to the area, P O′ P′ (continued), determines the pressure of

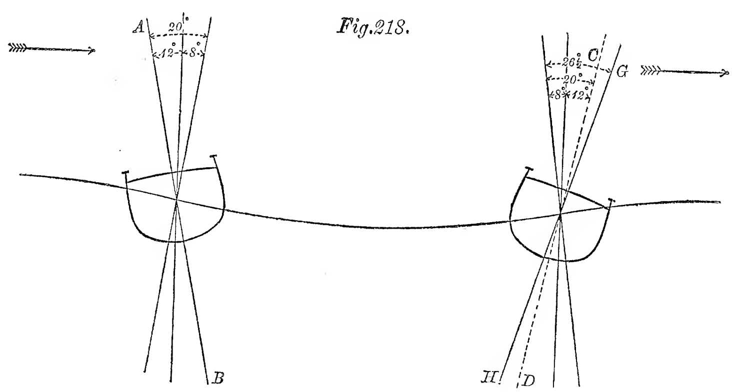
* Chapter vii., page 125, ship D.

wind which, applied suddenly to the ship when in the upright position with her plain sail set, would be sufficient to capsize her. If this same pressure, of 21·4 lbs. on the square foot, were to be applied gradually and steadily, instead of suddenly, it would incline the ship and keep her over to an angle corresponding to the point O′, where the line, N′ O′ P′, intersects the curve of stability, D, *i.e.*, an angle of about 39 degrees.

In what has been said of the last two examples, we have assumed the ships to have been acted upon by the wind when floating in still water. When we consider their action amongst waves, we find that the effects of wind-pressure previously shown may be greatly modified, for the reason that the righting power of a ship amongst waves depends upon her inclination to a normal to the effective wave-slope, instead of upon her inclination to the vertical, while the pressure of the wind upon the sails has to be considered with reference to her inclination to the vertical, as is the case when the ship is in still water. Another question to be taken into account is the relative periods of the ship and of the wave, because, if that of the ship be less than that of the wave, the ship will roll away from the slope of the wave, and if greater, she will roll towards it.

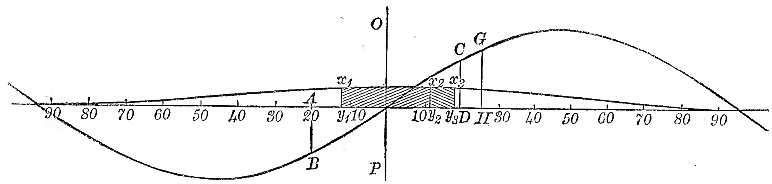
In Fig. 218 we show a vessel rolling towards the wave-slope.

Fig. 218.



She is supposed to be rolling through an angle of 20 degrees on each side of the normal to the wave-slope, the latter being inclined at 8 degrees to the vertical; in this case her greatest angle to the vertical is 12 degrees. In Fig. 223 the ship is shown rolling away from the wave-slope, her period being less than that of the wave. In this case her greatest angle to the vertical is seen to be 28 degrees. Fig. 219 represents a curve of stability for this vessel produced as a negative curve on the left of O P, which corresponds to the upright position of the ship. The ordinates marked A B and

Fig. 219.



C D, mark the extreme angles of roll of the ship, corresponding to the positions A B and C D, in Fig. 218. The curve, x_1 x_2 x_3, represents a pressure of wind of 2 lbs. per square foot, having the direction indicated by the arrows in Fig. 218. Assuming the wind to strike the ship suddenly when inclined to the position A B, the work done by the wind on the sails during the roll from this position to that of C D, would be about the same as that done during a roll from 12 degrees on one side of the vertical to 12 degrees on the other side, in still water. If we ignore the reduction of the wind-pressure due to the rolling of the ship, this work would be measured by the area, x_1 y_1 x_2 y_2. This, however, as can be easily seen, admits of considerable error.*

"In the case of a ship rolling from 12 degrees on the windward side of the vertical to 12 degrees on the leeward side in 5 seconds, with the centre of effort of the sails 70 feet above the axis of rotation, the mean velocity of the sails at this point would be about 7·3 feet per second." The speed of a wind of 2 lbs. per square foot is known to be 30 feet per second. "The relative velocity of the wind and the sails is therefore 22·7 feet per second, or about $\frac{3}{4}$ths only of the actual velocity of the wind; and as the wind-pressures vary nearly as the square of the relative velocity, the moment of the sails may be taken to be reduced to about one-half on account of the velocity of the sails away from the wind.

"This reduction is obviously still greater when the ship rolls away from the wave-crest, because her movements are quicker and the relative velocity is therefore less. The method adopted above is only a rough approximation, as will be seen from the following considerations:—

* The paragraphs which follow are quoted from the Lloyd's Report on "Masting," before referred to.

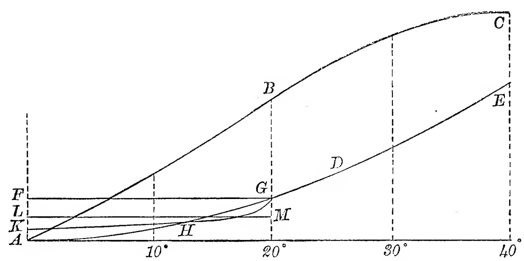
"1. The velocity of the ship in her angular motion is not uniform.

"2. Owing to the wind-pressures varying as the square of the relative velocity, it is not exact to take the motion of a centre of effort fixed in its height from the deck, as representing in effect the motion of the whole of the sails.

"The variation in the angular velocity for a still-water roll can be ascertained, neglecting resistance from the curve of stability, in the following way*:—

"In Fig. 220, A D E is the curve of statical stability, A B C is

Fig. 220.



the corresponding curve of dynamical stability, and G H K is a curve whose ordinates to the base, F G, give the angular velocity at any point. The straight line, L M, drawn so as to make the rectangle, F G L M, equal to the area, F K H G, will give the mean velocity. The variation of the wind-pressures due to the velocity of the sails varying in proportion to their height above the axis, and the consequent change in the height of the resultant of those pressures, are of not less importance than the variation of the angular velocity of the ship. As the ship rolls away from the wind, the pressures per square foot on the upper sails must be less than on the lower ones, and conversely, when the ship rolls up towards the wind, the pressures on the upper sails the greater. To find the height of the real centre of effort at any instant during the roll, if w be the angular velocity of the ship, V the velocity of the wind, and y the height of any unit of area above the axis, and if $\bar{y}$ be the height of this centre of effort, we have—

$$\bar{y} = \frac{\int (\mathrm{V} \cos. \theta - w\, y)^2\, y\, d\, y}{\int (\mathrm{V} \cos. \theta - w\, y)^2\, d\, y.}$$

* For particulars see *Transactions of Institution of Naval Architects*, vol. xii., page 95.

"The value of $\bar{y}$ will obviously be a maximum when the vessel is passing the upright towards the wind, and a minimum when in the same position, but moving in the opposite direction. In this position $\theta = 0$, and the equation becomes—

$$\bar{y} = \frac{\int (V - w\,y)^2\, y\, d\,y}{\int (V - w\,y)^2\, d\,y;}$$

and from it the range through which the centre of effort rises and falls can be determined, when the velocity of the wind and the angular velocity of the ship are known.

"The extreme angle of roll on the leaward side to the position, G H (Fig. 218) can be found by making the area of the stability curve comprised between G H (Fig. 219) and the axis, O P, equal to the whole work done by the wind in the roll of the ship from 12 degrees on the windward side to the extreme angle of inclination on the leeward side corresponding to the inclination, G H, to the wave-normal. Thus the area, C D G H, will be equal to about half the area, $x_1\,y_1\,x_3\,y_3$, where the distance between G H and $x_3\,y_3$ corresponds to the wave-slope. The corresponding case in still water would be represented by Figs. 221 and 222, and in rolling from

Fig. 221.

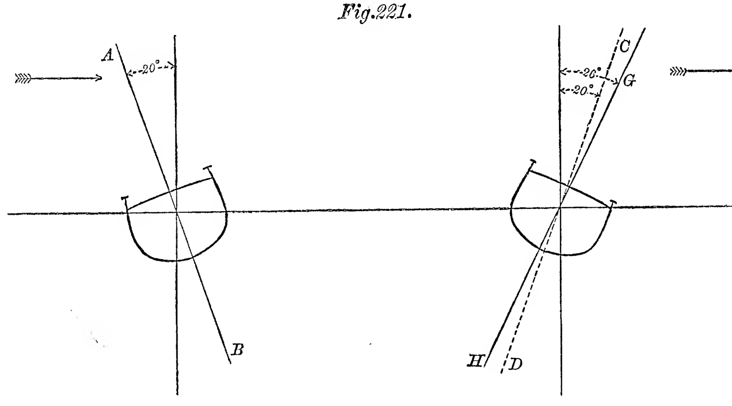
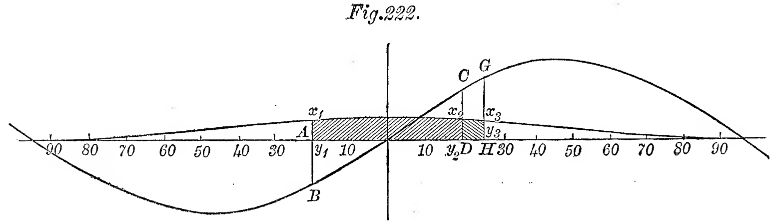


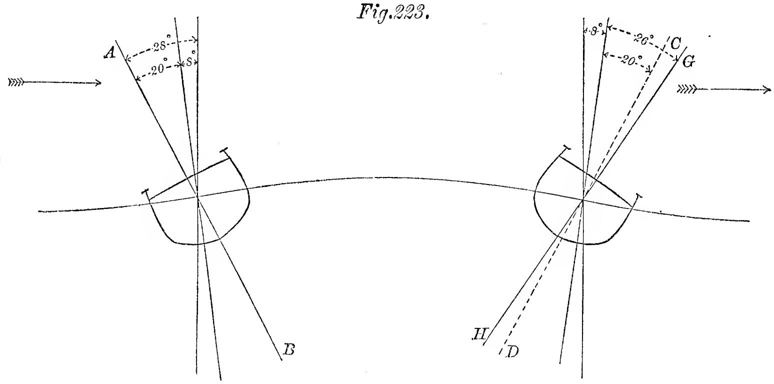
Fig. 222.



A B to C D, the work accumulated by the wind on the sails would be the area, $x_1\,y_1\,x_2\,y_2$, less a reduction for the angular velocity of the ship, and the position, G H, would be found by making C D G H equal to about one-third of the area, $x_1\,y_1\,x_3\,y_3$.

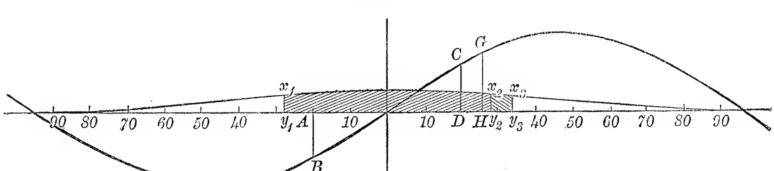
"When the vessel rolls away from the crest, it will be clear that the lee-roll will be taking place when the crest of the wave is passing the vessel, instead of when the hollow is passing the vessel, as in the former case. This is represented by Fig. 223,

Fig. 223.



where the angles of slope and the vessel's inclination to the normal remain the same as before. The work done by the wind in the roll from A B to C D will in this case be measured by a proportion found to be one-fifth of the area, $x_1\,y_1\,x_2\,y_2$ (Fig. 224) which shows

Fig. 224.



the action of the wind in a roll from 28 degrees windward to 28 degrees leeward of the vertical. And if G H in the same figure represent the position to which the vessel ultimately rolls, then the area, C D G H, will be equal to about one-fifth of the area, $x_1\,y_1\,x_3\,y_3$, where $x_3\,y_3$ is 8 degrees, or the wave-slope, beyond G H, as before."

It will be seen, from the above, that although there is very little difference in the extreme angle of leeward roll to the wave-normal, which the vessel reaches whether rolling towards the crest of the wave or away from it, there is a difference of $15\frac{1}{2}$ degrees in the extreme angle to the vertical, between the two conditions.

CHAPTER XIX.

Importance of Determining Longitudinal and Vertical Positions of Centre of Gravity—Method of doing same—Method of Determining Position of Centre of Gravity when the Vessel is Afloat—Precautions to be Taken—Account of Experiment Performed by the Author—Calculation of Position of Centre of Gravity from Drawings of Ship—Taylor's Stability Indicator.

It will have been repeatedly made clear throughout the course of this work that, although the form and the immersion determine very important elements which enter into the stability of a ship, there can be no determination of that stability, and no estimate formed of its amount, unless and until the position of the centre of gravity is either known or assumed. It remains, therefore, for us to consider by what means the position of a ship's centre of gravity can be determined. In order to fix its position accurately, we must obviously ascertain its height, its fore and aft position, and whether it lies or does not lie on the longitudinal vertical plane about which most ships are symmetrical, or supposed to be. The last of these conditions may usually be assumed, and seldom need be made a matter of calculation; but it is of great importance in designing a ship to determine its longitudinal, or fore and aft position very approximately, and to provide for it being in the same vertical as the centre of buoyancy, in order that the *trim* of the ship may be that which the designer intends. But more important even than this, is the correct determination of the *height* of the centre of gravity, for upon this depends, as the reader well knows, that transverse stability of the ship which transcends all other elements in vital interest.

The centre of gravity of a ship, or of any other body, being the common centre of gravity of all its parts, it is manifest that its position can be ascertained, by first ascertaining the weight and the centre of gravity of each part, and thence deducing the common centre of gravity. If the weight of each part be multiplied by the perpendicular distance of its centre of gravity from a given plane, the products thus obtained be all added together, and their sum divided by the aggregate weight of the parts, the quotient will be the distance of the centre of gravity of the whole ship from that plane. As the plane may be anywhere, and anyhow placed, it follows that if we take in succession a horizontal and a vertical transverse plane, and proceed with reference to each as just stated, we shall obtain respectively the vertical height and the fore and aft position of the ship's centre of gravity. This principle was fully understood by the old writers on naval architecture—Père l' Hoste, Bouguer, Don Juan d' Ulloa, Chapman, and others.

But in the days of wooden ships, the specific gravities of the timber entering into them differed so considerably, and methods of detailed calculation were so crude, or so imperfectly applied, that the position of the centre of gravity calculated as above described could not be relied upon, and consequently an experimental method of ascertaining its position after the ship was built and floated, was sought for, and devised, the nature of which we shall presently observe. In these days of iron ships, when but comparatively little timber enters into their construction, the centre of gravity can be ascertained by calculation with much more exactness, although there still remains the difficulty of dealing with the undevelopable surface of the ship's skin plating, the great variety of her fittings and equipments, the weights and centres of gravity of which usually can be but imperfectly ascertained, &c. This difficulty is not, however, insurmountable, especially by those who have the advantage of large experience, and of accumulated information, and the calculation of the centre of gravity of a new ship, from the particulars of her design and specifications, is a common practice of the best class of naval architects. In fact, a naval design cannot be prepared in a trustworthy manner without a large resort to this class of calculations in the case of ships of war, and of other ships in which the disposable weights to be carried by the completed ships are small; for in such cases both the trim and the stability of the ship are determined in the drawing office. In the case of cargo ships, where so much disposable weight remains to be dealt with after the ship is built, engined, and equipped, such calculations are not usually pressed so far; but even with them, as more or less homogeneous cargoes have often to be carried, the designer should determine beforehand by calculation, approximately, the position of the centre of gravity. When designing a vessel of a known type this need not be done; the data may be taken from a vessel of like type which has had the exact position of the centre of gravity determined in the vertical direction by experiment, the necessary modifications being made.

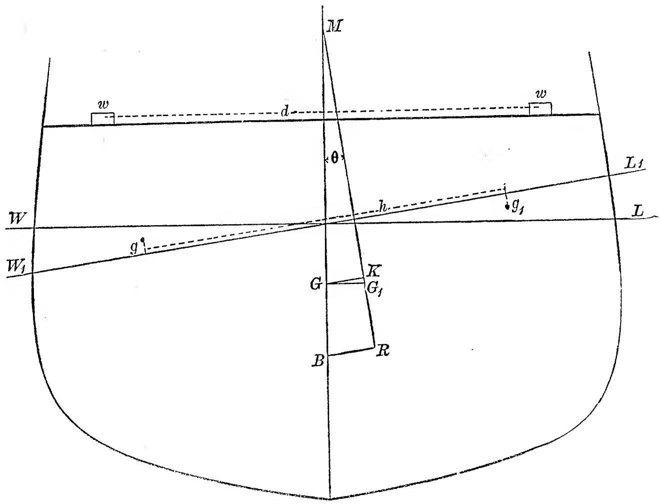
When a vessel is afloat, the vertical position of the centre of gravity may be determined with exactitude by a well-known method, which depends in principle upon an elementary doctrine of mechanics.

All the practice of the question resolves itself into merely moving a weight and measuring an angle: and even the latter is not practically necessary at the ship, the measurement of two sides of a right-angled triangle, which, of course, determine the angle, being all that is necessary. It is not usually quite so easy as it may at first sight seem, however, to perform an actual experiment upon a large ship; because, in the first place, it requires the transfer of many tons of weight to incline her to an appreciable angle; then, again, there are many things on board a ship when in progress that form no permanent part of her, or of her fittings, and these have to be accounted for; again, there are many things that do belong to her, which are often not on board at the time convenient for the experiment; there are also the disturbing operations of work in progress; and, finally, as it is necessary to cast off during the experiment all the lashings of the ship (excepting a bow and stern hawser, or both), a very little wind suffices to disturb the ship and embarrass the operation. By carefully performing the experiment, however, and by duly accounting for the several items to be placed into the vessel to complete her, and for those requiring to be taken out, an exact measurement of the vertical position of her centre of gravity may be obtained.

The principle upon which the experiment rests is a simple one. We know that if any one of a system of heavy bodies be moved through any distance in a given direction, the centre of gravity of the whole system will move in the same direction, and through a distance which bears the same ratio to the distance traversed by the moved body, as the weight of that body does to the total weight of the system.

In Fig. 225, let G represent the centre of gravity of a ship and of all weights on board; W L, the water-line, and B the centre of buoyancy. Let w represent a weight on the deck which is to be moved through a distance, d, to produce the necessary inclination. If w be moved from one side to the other, it will cause the vessel to

Fig. 225.



incline through an angle, θ, which may be accurately measured. Let $W_1 L_1$ be the new water-line, and G_1 the new position of the centre of gravity. Then, if D = the total weight of the vessel and all on board, we have—

$$D \times G G_1 = w \times d;$$

$G G_1$ being drawn parallel to the line joining the two positions of the centre of gravity of the movable weight, w, before and after it has been moved. But we have $G G_1 = G M \tan. \theta$; and therefore—

$$D \times G M \tan. \theta = w d;$$

and

$$G M = \frac{w d}{D \tan. \theta}.$$

In practice the weight, w, is so small, as compared with the whole displacement, that the point, M, or the intersection of the verticals through the consecutive centres of gravity, is the metacentre, or sufficiently near it for all practical purposes. But the height of the metacentre, M, above the centre of buoyancy, and therefore its true height, can readily be calculated from the drawings of the vessel; and thence, setting down the value of G M, we determine the position of the centre of gravity.

Should the weight, w, be sufficient to incline the vessel to any large angle, the intersections of the verticals through the original centre of gravity and the new centre of gravity may be found as follows*: Draw B R parallel to the inclined water-plane. Let v be the weight of water equivalent in volume to either of the wedges of immersion or emersion. From g and g_1, the centres of gravity of these wedges, let fall perpendiculars on the new water-line, $W_1 L_1$, and let h represent the distance along $W_1 L_1$ between these perpendiculars. Since, so to speak, the wedge of displacement has been shifted through a distance, h, due to the transfer of the weight, w, from one side of the ship to the other side, the centre of buoyancy of the vessel must have moved in the same direction through a distance, $\frac{h v}{D}$, as we saw in a former chapter; but

$$B R = B M \sin. \theta = (G M + B G) \sin. \theta ;$$

and

$$G M \sin. \theta = G K = G G_1 \cos. \theta = \frac{w d}{D} \cos. \theta.$$

Therefore,

$$B R \text{ or } \frac{h v}{D} = \frac{w d \cos. \theta}{D} + B G \sin. \theta ;$$

and

$$B G = \frac{h v - w d \cos. \theta}{D \sin. \theta}.$$

From this equation the distance, B G, can readily be found, and being set off from the centre of buoyancy, will determine the centre of gravity.

In performing the experiment at the ship great care has to be observed that the ship is cleared of all free-water; and all articles likely to shift during the experiment should be secured, as it is most important that only the weights that are to be used and accounted for should be moved during the operation of inclining. A fine day should be chosen, with the water calm. The ship is attached to the

* See Mr. Barnes' Paper on "Experiments on Board H.M. Ships in 1855-57," in vol. i., *Transactions of Institution of Naval Architects.*

shore by bow and stern hawsers, which will not affect her inclination, in a position head or stern on to the wind, should there be any, and plumbed, or made vertical, as nearly as possible. Battens are placed at the principal hatchways, usually two or three at the middle-line, and plumb-bobs hung therefrom. At a distance of 10 and sometimes 20 feet from the point of suspension, horizontal battens are fixed, so that the horizontal shift of the plumb-bob, due to the inclination, may be accurately measured. We will give the actual experiment as performed by the Author on H.M.S. *Prince Consort*, careened in Keyham Basin, with 70 tons of ballast. At the time of the experiment the ship was fitted with her armour and machinery complete; she was also masted, had her lower yards crossed on the gunwale, and her lower and topmast standing rigging in place. She had other weights on board, the amounts and positions of all of which were carefully ascertained, but into the details of which it is unnecessary to enter here. The draught of water at the time was: forward, 19 feet $5\frac{1}{2}$ inches; aft, 23 feet $0\frac{1}{2}$ inch, which gave a mean draught of 21 feet 3 inches. The displacement to this line was 5,126·3 tons; the corresponding centre of buoyancy, 8·141 feet below this line; and the height of the metacentre above the centre of buoyancy was 17·238 feet. The first experiment consisted in moving 20·718 tons of ballast from port to starboard (on the main deck), through a distance of 33·4 feet, and 4·6134 tons of fire-bars (which happened to be stacked upon the deck) through a distance, in the same direction, of 45·583 feet. This produced a horizontal traverse of the plumb-line bobs of $16\frac{15}{32}$ inches in a length of 20 feet —the measurements being taken in two different hatchways, 200 feet apart, in this case, to ensure accuracy.

The next experiment consisted in moving the above-mentioned ballast and fire-bars to their original position, and in addition 46·132 tons of ballast from starboard to port through a distance of 36·4 feet. This produced an inclination of precisely 12 inches in 20 feet. Substituting these quantities in the equation, $GM = \frac{w\,d}{D \tan. \theta}$, we obtain the following results:—

		FEET.
Centre of gravity below metacentre,	- - -	6·53
,, above centre of buoyancy,	-	10·7
,, above the then water-line,	-	2.7

These distances were calculated independently from the two separate experiments before described, and were found to differ only by $\frac{1}{100}$th part of an inch.

At the time of the experiment the equipment of the ship was incomplete, the guns, coals, provisions, ammunition, &c., having to go on board; and the articles before referred to as forming no part of the equipment, to be taken out of her. A detailed calculation of the whole of these weights was made, and their effect in altering the height of the centre of gravity determined. This effect was found to be that the centre of gravity was lowered 7·75 inches when the ship was completed for sea, and when her mean draught of water was 25 feet $5\frac{1}{2}$ inches, her displacement 6,696 tons, her centre of buoyancy below the new water-line 9·98 feet, and the height of the metacentre above the centre of buoyancy 13·84 feet. Consequently, we have when the ship is thus ready for sea, and immersed to her load-line, the following results:—

		FEET.
Centre of gravity	below metacentre, - - -	6·01
„	above centre of buoyancy, -	7·83
„	below the load water-line, -	2·15

and we therefore thus obtain the exact vertical position of the centre of gravity.*

With respect to the longitudinal position of this point, it is evidently in the transverse vertical section which contains the centre of buoyancy, and therefore is obtained directly from the drawings of the ship, and we thus obtain the full determination of the centre of gravity.

It is not within the province of this work to enter into the details of calculating the centre of gravity of a vessel complete, by means of estimating the weight and centre of gravity of each item forming part of the vessel's hull, equipment, &c., as previously described in this chapter. It may be stated, however, that these calculations are made direct from the drawings of the vessel before they leave the designer's hands, and it may be repeated that great experience and care are necessary throughout the work in order to deduce trustworthy results. Each item of the vessel's hull, equipment, &c., is calculated separately, and its weight and the distance of its centre of gravity from two planes of reference, are obtained. From these are calculated its moment relatively to the planes of reference.

* For a Paper by the Author on this subject, from which the above figures, &c., are taken, see the *Transactions of the Institution of Naval Architects*, vol. v., 1864.

These planes are usually the load water-plane, and that of the midship ordinate. The several items are summarised in tables, and the total weight and moments obtained, and thence the distance of the centre of gravity from each plane of reference.* There are, however, elements of uncertainty existing in calculations of this description, even when the greatest pains are taken. In using paint, cement, white and red lead, &c., for example, some builders are more heavy-handed than others with these items, and the estimate for each item has to be based on known weights of similar materials placed in certain ships. Similar approximations have to be made for cabin bulk-heads, mess-tables, &c., and these will of course again depend on the accommodation requisite. It has been found in practice that there are almost always some items not taken into account at all, or incorrectly estimated, and it is therefore usual to add a small percentage to these estimated weights (from 2 to 5 per cent. on the total) to cover such deficiencies. To limit error, however, these calculations are made independently by two persons.

So many examples of the heights of centres of gravity in actual ships have been given in former chapters of this work that no additions need here be made to them.

Mr. Alexander Taylor, M.I.N.A., has invented and applied to several vessels an apparatus which he designates a Stability Indicator, but which "does" he says, "exactly the same work as is done in an ordinary inclining experiment." It is intended to be fixed permanently in the ship, and to be employed under suitable conditions for determining the height of the centre of gravity for various displacements and dispositions of cargo. Its essential principle consists in the employment of two tanks of given volume, placed one on each side of the ship, and connected by a pipe, in lieu of iron ballast or other shifting weights.

* In Mr. Mackrow's very handy and useful *Naval Architect's and Shipbuilder's Pocket-Book* (pp. 159-61) are given Rules and Tables for determining the position of the centre of gravity of the bottom plating of a ship's hull when of uniform thickness throughout.

REECH'S METHOD : RISBEC'S FORM.

TABLE I.

TABLE FOR THE CALCULATION OF CO-ORDINATES OF CENTRES OF BUOYANCY FOR WATER-PLANES INCLINED AT INTERVALS OF 10° STARTING FROM THE HORIZONTAL WATER-PLANE.

Numbers of Sections.	Upright Water Section.			Water-plane for 10°.						Water-plane for 20°.						Water-plane for 30°.						Water-plane for 40°.					
	Ordinates.	Squares of Ordinates.	Cubes of Ordinates.	Ordinates.		Squares of Ordinates.		Cubes of Ordinates.		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.	
				I	E	I	E	I	E	I	E	I	E	I	E	I	E	I	E	I	E	I	E	I	E	I	E
F.P.																											
9																											
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7																											
6																											
5																											
4																											
3																											
2																											
1																											
M																											
1																											
2																											
3																											
4																											
5																											
6																											
7																											
8																											
9																											
A.P.																											
Sums		,,																									

	Ordinates.	Squares of Ordinates.	Cubes of Ordinates.	10° Ordinates.	10° Squares of Ordinates.	10° Cubes of Ordinates.	20° Ordinates.	20° Squares of Ordinates.	20° Cubes of Ordinates.	30° Ordinates.	30° Squares of Ordinates.	30° Cubes of Ordinates.	40° Ordinates.	40° Squares of Ordinates.	40° Cubes of Ordinates.
	$\frac{1}{2}s$		$\frac{1}{2}\sigma$												
,,	$s=$	Difference $\binom{\mathrm{I-E}}{\delta=0}$	$\sigma=$	Sum Total $s=$	Difference (I−E) $\delta=$	Sum Total $\sigma=$	Sum Total $s=$	Difference (I−E) $\delta=$	Sum Total $\sigma=$	Sum Total $s=$	Difference (I−E) $\delta=$	Sum Total $\sigma=$	Sum Total $s=$	Difference (I−E) $\delta=$	Sum Total $\sigma=$
,,	$s\mathrm{X}=$ A_0	$\mathrm{B}_0=0$	$\frac{1}{3}\sigma\mathrm{X}=$ C_0	$s\mathrm{X}=$ A_1	$\frac{1}{2}\delta\mathrm{X}=$ B_1	$\frac{1}{3}\sigma\mathrm{X}=$ C_1	$s\mathrm{X}=$ A_2	$\frac{1}{2}\delta\mathrm{X}=$ B_2	$\frac{1}{3}\sigma\mathrm{X}=$ C_2	$s\mathrm{X}=$ A_3	$\frac{1}{2}\delta\mathrm{X}=$ B_3	$\frac{1}{3}\sigma\mathrm{X}=$ C_3	$s\mathrm{X}=$ A_4	$\frac{1}{2}\delta\mathrm{X}=$ B_4	$\frac{1}{3}\sigma\mathrm{X}=$ C_4
,,	,,	,,	$\frac{1}{2}\mathrm{C}_0\theta=$ f_0	,,	$\frac{1}{2}\mathrm{B}_1\theta=$ b_1	$\frac{1}{2}\mathrm{C}_1\theta(\sin 10°)=$ e_1	,,	$\frac{1}{2}\mathrm{B}_2\theta=$ b_2	$\frac{1}{2}\mathrm{C}_2\theta(\sin 20°)=$ e_2	,,	$\frac{1}{2}\mathrm{B}_3\theta=$ b_3	$\frac{1}{2}\mathrm{C}_3\theta(\sin 30°)=$ e_3	,,	$\frac{1}{2}\mathrm{B}_4\theta=$ b_4	$\frac{1}{2}\mathrm{C}_4\theta(\sin 40°)=$ e_4
				,,	sin 10° = ·1736 cos 10° = ·9848	$\frac{1}{2}\mathrm{C}_1\theta(\cos 10°)=$ f_1	,,	sin 20° = ·3420 cos 20° = ·9397	$\frac{1}{2}\mathrm{C}_2\theta(\cos 20°)=$ f_2	,,	sin 30° = ·5000 cos 30° = ·8660	$\frac{1}{2}\mathrm{C}_3\theta(\cos 30°)=$ f_3	,,	sin 40° = ·6428 cos 40° = ·7660	$\frac{1}{2}\mathrm{C}_4\theta(\cos 40°)=$ f_4

DATA TAKEN FROM THE DISPLACEMENT CALCULATIONS.

Displacement up to the horizontal Water-line } $v_0=$

Distance below this Water-line of the corresponding centre of buoyancy } $h=$

GENERAL DATA { Interval between 2 sections, X=
Length of the arc of 10°, $\theta=$ ·1745

GENERAL FORMULÆ

(y' = immersed ordinates
y'' = emerged ordinates)

$$\begin{cases} \mathrm{A}=\int y'\,dx+\int y''\,dx \\ \mathrm{B}=\frac{1}{2}\int y'^2\,dx-\frac{1}{2}\int y''^2\,dx \\ \mathrm{C}=\frac{1}{3}\int y'^3\,dx+\frac{1}{3}\int y''^3\,dx \end{cases}$$

$$\mathrm{V}=v_0+\int_0^\theta \mathrm{B}\,d\theta$$
$$\mathrm{V\,Y}=\int_0^\theta \mathrm{C}\cos\theta\,d\theta$$
$$\mathrm{V\,Z}=v\,h-\int_0^\theta \mathrm{C}\sin\theta\,d\theta$$

$$\rho=\frac{\mathrm{C}-\frac{\mathrm{B}^2}{\mathrm{A}}}{\mathrm{V}}$$

BODY PLAN.

RESULTS.

DISPLACEMENTS V.

at 0° $v_0=$
,, 10° $v_1=v_0+b_1=$
,, 20° $v_2=v_1+b_2=$
,, 30° $v_3=v_2+b_3=$
,, 40° $v_4=v_3+b_4=$

NOTE.—The values b have the sign of the differences δ.

ABSCISSÆ OF CENTRES OF BUOYANCY.

at 0° $\mathrm{Y}_0=o$
,, 10° $\mathrm{Y}_1=\frac{f_0+f_1}{v_1}=$
,, 20° $\mathrm{Y}_2=\frac{f_0+2f_1+f_2}{v_2}=$
,, 30° $\mathrm{Y}_3=\frac{f_0+2f_1+2f_2+f_3}{v_3}=$
,, 40° $\mathrm{Y}_4=\frac{f_0+2f_1+2f_2+2f_3+f_4}{v_4}$

ORDINATES OF CENTRES OF BUOYANCY.

(Distances below the horizontal Water-line.)

at 0° $\mathrm{Z}_0=h$
,, 10° $\mathrm{Z}_1=\frac{v_0h-e_1}{v_1}=$
,, 20° $\mathrm{Z}_2=\frac{v_0h-(2e_1+e_2)}{v_2}=$
,, 30° $\mathrm{Z}_3=\frac{v_0h-(2e_1+2e_2+e_3)}{v_3}=$
,, 40° $\mathrm{Z}_4=\frac{v_0h-(2e_1+2e_2+2e_3+e_4}{v_4}=$

LENGTH OF RADII OF CURVATURE OF THE CURVE OF CENTRES OF BUOYANCY.

Apply the formula $\rho=\frac{\mathrm{C}-\frac{\mathrm{B}^2}{\mathrm{A}}}{\mathrm{V}}$ to the quantities A, B, C, V, corresponding to each inclination.

$\rho_0=$ $\rho_3=$
$\rho_1=$ $\rho_4=$
$\rho_2=$

DAYMARD.]

TABLE FOR THE CALCULATION OF GENERAL CURVES OF STABILITY (OR PANTOCARÈNE-ISOCLINES) OF A SHIP FROM 0° TO 180° OF INCLINATION.

General Observations.—Before commencing the table below, it is necessary to calculate, for the upright position, the scale of displacement and the curve of heights of centres of buoyancy for the total volume of the ship, that is to say, to make the ordinary calculations with horizontal water-lines taken up to the height of the upper deck.

1. The first operation consists in drawing radial planes angularly equidistant through a longitudinal horizontal axis O, and taking from each the ordinates, for 20 equidistant sections, on both the immersed and emerged sides; these ordinates are then inserted in the tables below, as well as their squares and cubes taken from tables. The number of the ordinates may be less than 20. Care should be taken in this case to insert only half of the numbers of the last ordinate.

2. For ships of ordinary form, it is sufficient to make the angle contained between the radial planes and the horizontal water-line vary from 10 to 10 degrees, and to operate for 3 stations such as O.

3. The ordinates need not be taken to more than two places of decimals. One only is sufficient for the squares, and whole numbers for the cubes, increasing the last figure by 1 when the decimal portion is equal to or greater than ½.

4. The result is that each column contains only numbers of 5 figures, which considerably reduces the calculations.

5. Only half the numbers which belong to the first and last ordinates are inserted in these tables, in order that the net total of each column may be directly obtained. Care must be taken to put in the tables the half of the squares and cubes of the whole ordinates.

6. The position of each ordinate is determined by its distance from the after perpendicular A.P.

Distance of Ordinates from after Perpendiculars, Δx being taken as Unity.	Multipliers.	Radial Plane at 0°.						Radial Plane at 10°.						Radial Plane at 20°.						Radial Plane at 30°.						Radial Plane at 40°.					
		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.	
		I_0	E_0	I_0^2	E_0^2	I_0^3	E_0^3	I_1	E_1	I_1^2	E_1^2	I_1^3	E_1^3	I_2	E_2	I_2^2	E_2^2	I_2^3	E_2^3	I_3	E_3	I_3^2	E_3^2	I_3^3	E_3^3	I_4	E_4	I_4^2	E_4^2	I_4^3	E_4^3
0 A.P.	½																														
1	1																														
2	1																														
3	1																														
4	1																														
5	1																														
6	1																														
7	1																														
8	1																														
9	1																														
10	1																														
11	1																														
12	1																														
13	1																														
14	1																														
15	1																														
16	1																														
17	1																														
18	1																														
19	1																														
20	½																														
Totals, .		,,	,,	ΣI_0^2	ΣE_0	ΣI_0^3	ΣE_0^3	,,	,,	ΣI_1^2	ΣE_1^2	ΣI_1^3	ΣE_1^3	,,	,,	ΣI_2^2	ΣE_2^2	ΣI_2^3	ΣE_2^3	,,	,,	ΣI_3^2	ΣE_3^2	ΣI_3^3	ΣE_3^3	,,	,,	ΣI_4^2	ΣE_4^2	ΣI_4^3	ΣE_4^3
$\Delta x =$		,,		$\delta_0 =$ sign ,, value 0		$\sigma_0 =$		,,		$\delta_1 =$ sign		$\sigma_1 =$		,,		$\delta_2 =$ sign		$\sigma_2 =$		,,		$\delta_3 =$ sign		$\sigma_3 =$		,,		$\delta_4 =$ sign		$\sigma_4 =$	

Distances of Ordinates from after Perpendiculars, Δx being taken as Unity.	Multipliers.	Radial Plane at 5						Radial Plane at 60°.						Radial Plane at 70°.						Radial Plane at 80°.						Radial Plane at 90°.					
		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.		Ordinates.		Squares of Ordinates.		Cubes of Ordinates.	
		I_5	E_5	I_5^2	E_5^2	I_5^3	E_5^3	I_6	E_6	I_6^2	E_6^2	I_6^3	E_6^3	I_7	E_7	I_7^2	E_7^2	I_7^3	E_7^3	I_8	E_8	I_8^2	E_8^2	I_8^3	E_8^3	I_9	E_9	I_9^2	E_9^2	I_9^3	E_9^3
0 A.P.	½																														
1	1																														
2	1																														
3	1																														
4	1																														
5	1																														
6	1																														
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14	1																														
15	1																														
16	1																														
17	1																														
18	1																														
19	1																														
20	½																														
Totals, .		,,	,,	ΣI_5^2	ΣE_5^2	ΣI_5^3	ΣE_5^3	,,	,,	ΣI_6^2	ΣE_6^2	ΣI_6^3	ΣE_6^3	,,	,,	ΣI_7^2	ΣE_7^2	ΣI_7^3	ΣE_7^3	,,	,,	ΣI_8^2	ΣE_8^2	ΣI_8^3	ΣE_8^3	,,	,,	ΣI_9^2	ΣE_9^2	ΣI_9^3	ΣE_9^3
		,,		$\delta_5 =$ sign		$\sigma =$		,,		$\delta_6 =$ sign		$\sigma_6 =$		,		$\delta_7 =$ sign		$\sigma_7 =$		,,		$\delta_8 =$ sign		$\sigma_8 =$		,,		$\delta_9 =$ sign		$\sigma_9 =$	

Calculation of $V = \Delta x \Delta \theta A$, and of $Z = \frac{B}{1 \cdot 5 A}$.

Degrees of Inclination.	Volume V of Displacement.	Distance Z of the Centre of Buoyancy from the Water-line.		
θ	$\Sigma \Sigma E^2$	$\Sigma \Sigma E^3$	$\sin \theta$	$\Sigma \Sigma E \sin \theta$
0°	$\frac{1}{2} \Sigma E_0^2 =$	,,	0	0
10°	$\Sigma E_1^2 =$		·174	
20°	$\Sigma E_2^2 =$		·342	
30°	$\Sigma E_3^2 =$		·500	
40°	$\Sigma E_4^2 =$		·643	
50°	$\Sigma E_5^2 =$		·766	
60°	$\Sigma E_6^2 =$		·866	
70°	$\Sigma E_7^2 =$		·940	
80°	$\Sigma E_8^2 =$		·989	
90°	$\frac{1}{2} \Sigma E_9^2 =$		1	$(\frac{1}{2} \Sigma E_9^3) =$
	(A)	(B)		
	$A =$	$A =$	$B =$	
	$\Delta x \Delta \theta =$			
	$\Delta x \Delta \theta =$ V	$1 \cdot 5 A =$	$\frac{B}{1 \cdot 5 A} = Z$	
	$V =$	$Z =$		

Name of the Ship:

We have, for the station under consideration:

Draught of water, mean....................

Depth of hull....................................

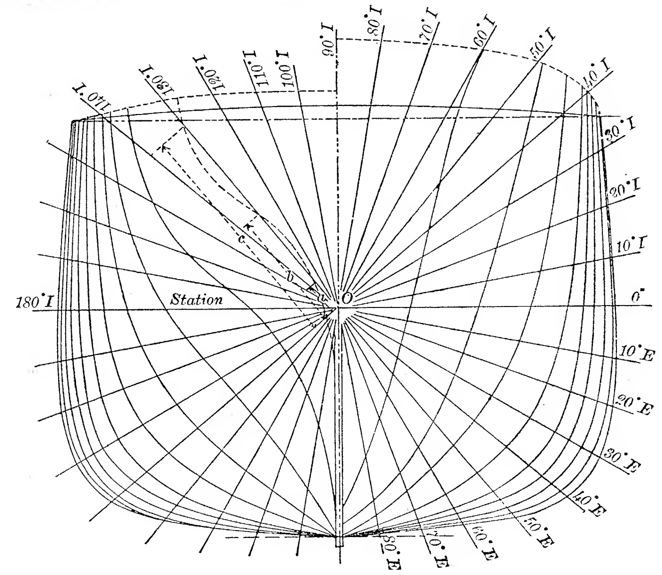
Corresponding displacement (without keel) V =

Height of transverse metacentre above keel.

Height of centre of buoyancy above keel.

N.B.—The values of V and Z in the adjoining table should not differ much from those indicated by the curves of displacement and centres of buoyancy obtained by the horizontal water-lines, at the draught of water of the station. The results of the table really serve only for verification.

It is well, for the execution of the calculations of these tables, to use paper in book form, and not in loose sheets. The operations can then be verified in case of error.



LEGEND.

Δx, ... Constant interval between two adjacent ordinates; usually equal to a twentieth part of the length of the ship from perpendicular to perpendicular.

I . . . . . Ordinates on the immersed side; I^2 squares, and I^3 cubes of these ordinates.

E . . . . . Ordinates on the emerged side; E^2 squares, and E^3 cubes of these ordinates.

ΣI, ΣI^2, ΣI^3, Sum of ordinates on the immersed side, sum of their squares, and sum of their cubes respectively.

ΣE, ΣE^2, ΣE^3, Similar sums on the emerged side.

$\delta = \Sigma I^2 - \Sigma E^2$, For each raidal plane (δ is negative when ΣE^2 is $> \Sigma I^2$). The sign should always be indicated.

$\sigma = \Sigma I^3 + \Sigma E^3$, For each radial plane.

The preceding quantities, taken in a certain direction, are designated by an index representing the ordinal number of the radial plane to which they correspond.

The method is complete in two Tables.

Remarks.—There is no difficulty in taking off the ordinates when the radial planes meet the sides of the ship; but when they cut the deck of the ship, it may, perhaps, be useful to indicate a practical and rapid way of obtaining them.

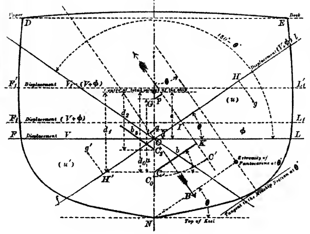
Project upon each of these planes the extremity of each of the sections, taking account of the round of the deck; which is done by joining with a square, the extremity (at the upper deck) of the midship section with the point of intersection of the radial plane with the round of the deck, and projecting with the square, in the direction thus given, the extremities of the sections upon the radial plane under consideration.

The radial plane at 90° passing through the longitudinal axis of the ship, the ordinates of this plane are limited by the sheer at middle on one side and the top of keel on the other.

It sometimes happens that certain oblique ordinates meet a section at three points (see the figure above), the value of the ordinate should then be taken as $c - b + a$; the square as $c^2 - b^2 + a^2$; the cube as $c^3 - b^3 + a^3$. The ultimate value only is inserted in the columns.

DAYMARD.]

TABLE FOR THE CALCULATION OF GENERAL CURVES OF STABILITY (OR PANTOCARÈNE-ISOCLINES) OF A SHIP FROM 0° TO 180° OF INCLINATION.



LEGEND.

D E N	Transverse section of the ship.
u, u'	Volumes of the opposite wedges L O l and F O f.
g, g'	Centres of gravity of the above wedges.
H, H′	Projection on fl of the centres g and g'.
h, h'	Distances I H, I H′.
V	Volume of displacement (ship upright).
V_1	Total volume of the ship.
C_0	Centre of buoyancy for the water-line F L.
C_1	Centre of buoyancy for the water-line $F_1 L_1$.
C_2	Centre of buoyancy for the water-line $F'_1 L'_1$.
Z	Distance of the centre of buoyancy from the water-line F L.
$\Delta\theta$	Arc between 2 consecutive radial planes; when they form an angle of 10°, it is equal to $\frac{\pi}{18} = \cdot174533$.
$d_0\, d_1\, d_2$	Distances from the centre of total volume of the centres of buoyancy $C_0\, C_1\, C_2$.
b	Arm of lever to the centre of buoyancy C_1.
b_s	Arm of lever to the centre of buoyancy C_2 for 180° − θ°.
p	Arm of lever to the centre of gravity G.

CALCULATION OF VALUES OF $M = (u\,h + u'\,h')$.

	10°	20°	30°	40°	50°	60°	70°	80°	90°
	$\frac{1}{2}\sigma_0 \cos 10° =$ $\frac{1}{2}\sigma_1 \cos 0° =$	$\frac{1}{2}\sigma_0 \cos 20° =$ $\sigma_1 \cos 10° =$ $\frac{1}{2}\sigma_2 \cos 0° =$	$\frac{1}{2}\sigma_0 \cos 30° =$ $\sigma_1 \cos 20° =$ $\sigma_2 \cos 10° =$ $\frac{1}{2}\sigma_3 \cos 0° =$	$\frac{1}{2}\sigma_0 \cos 40° =$ $\sigma_1 \cos 30° =$ $\sigma_2 \cos 20° =$ $\sigma_3 \cos 10° =$ $\frac{1}{2}\sigma_4 \cos 0° =$	$\frac{1}{2}\sigma_0 \cos 50° =$ $\sigma_1 \cos 40° =$ $\sigma_2 \cos 30° =$ $\sigma_3 \cos 20° =$ $\sigma_4 \cos 10° =$ $\frac{1}{2}\sigma_5 \cos 0° =$	$\frac{1}{2}\sigma_0 \cos 60° =$ $\sigma_1 \cos 50° =$ $\sigma_2 \cos 40° =$ $\sigma_3 \cos 30° =$ $\sigma_4 \cos 20° =$ $\sigma_5 \cos 10° =$ $\frac{1}{2}\sigma_6 \cos 0° =$	$\frac{1}{2}\sigma_0 \cos 70° =$ $\sigma_1 \cos 60° =$ $\sigma_2 \cos 50° =$ $\sigma_3 \cos 40° =$ $\sigma_4 \cos 30° =$ $\sigma_5 \cos 20° =$ $\sigma_6 \cos 10° =$ $\frac{1}{2}\sigma_7 \cos 0° =$	$\frac{1}{2}\sigma_0 \cos 80° =$ $\sigma_1 \cos 70° =$ $\sigma_2 \cos 60° =$ $\sigma_3 \cos 50° =$ $\sigma_4 \cos 40° =$ $\sigma_5 \cos 30° =$ $\sigma_6 \cos 20° =$ $\sigma_7 \cos 10° =$ $\frac{1}{2}\sigma_8 \cos 0° =$	$\frac{1}{2}\sigma_0 \cos 90° = 0$ $\sigma_1 \cos 80° =$ $\sigma_2 \cos 70° =$ $\sigma_3 \cos 60° =$ $\sigma_4 \cos 50° =$ $\sigma_5 \cos 40° =$ $\sigma_6 \cos 30° =$ $\sigma_7 \cos 20° =$ $\sigma_8 \cos 10° =$ $\frac{1}{2}\sigma_9 \cos 0° =$
Totals or values of $\frac{M}{\frac{1}{3}\Delta x \Delta\theta} =$	Σ =	Σ =	Σ =	Σ =	Σ =	Σ =	Σ =	Σ =	Σ =
Values of M............ =									
Values of μ .... =									
Values of M − μ............ =									

Inclinations.	Natural Sines.	Natural Cosines.
0° and 180°	·000	±1·000
10° ,, 170°	·174	± ·985
20° ,, 160°	·342	± ·940
30° ,, 150°	·500	± ·866
40° ,, 140°	·643	± ·766
50° ,, 130°	·766	± ·643
60° ,, 120°	·866	± ·500
70° ,, 110°	·940	± ·342
80° ,, 100°	·985	± ·174
90°	1·000	± ·000

Calculations of values of (b) for variable volumes $(V+\phi)$ and $V_1-(V+\phi)$, and for angles increasing by 10° from 0° to 180°.

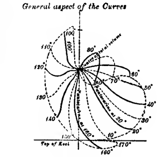
Volumes cut off by each radial plane { On one side $(V+\phi)=$; On the other $V_1-(V+\phi)=$

Height above keel of the centre of total volume =

Inclinations.	$\frac{\Delta x \Delta\theta}{4}\delta =$ $=q$	$\phi_\theta = \{\phi_{\theta-1}, q_{\theta-1}, q_\theta\}$ $(u-u')=\phi$	$V+\phi$ and $V_1-(V+\phi)$	Draughts of Water to Volumes of preceding Column.	Centres of Buoyancy above Keel for Volumes of Col. 4.	Distances of adjoining C of Buoyancy from Centre of Total Volume $=d_1$ and d_2	$\phi\,\epsilon-(V+\phi)d_1$	$\phi\,d=$ $\phi\,\epsilon-(V+\phi)d_1+Vd_0$	$\phi\,d \sin\theta$ $=\mu$	$\frac{M-\mu}{V+\phi}=$ $=b$
1	2	3	4	5	6	7	8	9	10	11
0°	,,	0	V =			$d_0 =$				0
180°		,,	$V_1 - V =$			,,	,,	,,	,,	0
10°						$d_1 =$				$b =$
170°		,,				$d_2 =$	,,	,,	,,	,,
20°						$d_1 =$				$b =$
160°		,,				$d_2 =$	,,	,,	,,	,,
30°						$d_1 =$				$b -$
150°		,,				$d_2 =$	,,	,,	,,	,,
40°						$d_1 =$				$b =$
140°		,,				$d_2 =$	,,	,,	,,	,,
50°						$d_1 =$				$b =$
130°		,,				$d_2 =$	,,	,,	,,	,,
60°						$d_1 =$				$b =$
120°		,,				$d_2 =$	,,	,,	,,	,,
70°						$d_1 =$				$b =$
110°		,,				$d_2 =$	,,	,,	,,	,,
80°						$d_1 =$				$b =$
100°		,,				$d_2 =$	,,	,,	,,	,,
90°						$d_1 = d_2 =$				$b =$

Calculation for Supplementary Angles.

Inclinations.	$d_1 \times \frac{V+\phi}{V_1(-V+\phi)}$ $=f$	$(d_2+f)\sin\theta$ $=g$	$b \times \frac{V+\phi}{V_1-(V+\phi)}$ $=r$	$g-r$ $=b_s$
12	13	14	15	16
100°				$b_s =$
110°				$b_s =$
120°				$b_s =$
130°				$b_s =$
140°				$b_s =$
150°				$b_s =$
160°				$b_s =$
170°				$b_s =$
180°	,,	,,	,,	$b_s =$



General aspect of the Curves

Name of the Ship:

We have, for the station under consideration:

Draught of water, mean..............................

Depth of hull...

Volume of displacement (ship upright) without keel...V =

Height of transverse metacentre above keel.....

Height of centre of buoyancy above keel.........

Heigbt of centre of gravity above keel...........

Total volume of the ship (without keel).......V_1 =

OBSERVATIONS.

Column 2 represents the differences of volumes of the wedges. (Be careful with the signs).

Column 3 represents the accummulations of the differences of the wedges, as its title indicates; it is, perhaps, well to note that the value of (q) and of (ϕ) is the same at 10°, as there is yet no accumulation. (Be very careful with the signs).

In the 4th column, the $(V+\phi)$ at 90°, represents the half total volume of the ship; this value should differ little for the 3 stations.

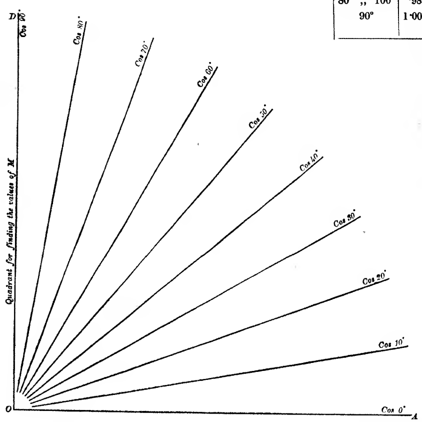
Columns 5 and 6 are deduced from the curves of displacement and centres of buoyancy obtained from the horizontal water-lines, and as stated in the general observations of Tahle No. 1.

Column 7 is easily obtained; the centre of total volume is the centre of buoyancy of the ship completely immersed.

The 8th column is obtained by the difference of the moments: of $(V+\phi)$ multiplied by the corresponding distance of the centre of buoyancy in col. 7, and (ϕ) multiplied by the height (ϵ) of the centre of total volume above the initial water-line.

To ohtain the 9th column, the constant moment ($V\,d_0$) is added to tbe preceding column. (The values of this column are always positive).

The titles of the subsequent columns sufficiently indicate how they are formed.



To use the qnadrant above, we set off along the horizontal O A (which is also the line of values of σ cos 0°), tbe values of (σ) indicated in Table No. 1, setting off only half of the values of σ_0, which are always to be divided hy 2. Through the points thus obtained we describe, from the point O, a series of arcs of circles.

The products which enter into the value of $\frac{M}{\frac{1}{3}\Delta x \Delta\theta}$ are then obtained by taking the distanee from the axis O D of the intersections of these arcs with the lines of the desired inclinations.—Even when operating with so small a scale, we obtain, with a little care, all the approximation desirable.

NOTE.—Columns 11 and 16 give, at the angle of inclination under consideration and for the displacement $(V+\phi)$, the arms of levers (b) of the righting couples (the centre of gravity being supposed to coincide with the centre of buoyancy). See the figure on the left of this tahle where (b) is shown, in length and direction, by tbe straight line C_1 K.

The Pantocarènes are the locus of the points K for each inclination; their ordinary aspect is represented by the small figure above. They are drawn as follows:

First of all a vertical is drawn, ahove, and starting from the lower edge of rabbet (top of keel for iron ships), and on this are set off tbe heights of the centres of buoyancy from column 6; through these points straight lines are drawn representing, in length and direction, the values of (b) and of (b_s) taken from columns 11 and 16; this is done for all the stations that have been calculated. The Pantocarène for 10°, 20°, &c., or in general for θ°, is then obtained by joining the extremities of the arms of levers corresponding to this inclination θ.—The drawing of the Pantocarène is facilitated hy the knowledge of its two extremities: one being the centre of total volume of the ship, and the other the foot of the perpendicular let fall from the lower edge of rabbet (or top of keel) upon the normal to the midship section, making the angle θ with the vertical axis. (It may be useful to draw, as auxiliary curves, continuous lines joining the extremities of the arms of levers given by the same station. They serve as a verification of the calculations; these curves, coincident for θ = 90°, should all cut one anotber at the centre of total volume of the ship).

The Pantocarènes once drawn, we obtain very rapidly the statical stability for all inclinations, with a given displacement, D, and any position whatever, G, of tbe centre of gravity. We have only to draw through the point C, the centre of the upright displacement, D, a series of straight lines or radii-vectores inclined 10°, 20°,......170°, 180° to the horizontal, until they meet the corresponding Pantocarènes, and to describe a circumference upon C G = a as diameter: tbe portions of the radii-vectores comprised between the circumference and the Pantocarènes, give, at the corresponding angles, tbe values of the arms of righting levers (p). We can then draw, if we wish, the usual curve of statical stability, where the angles are taken as abscissæ and the values of (p) as ordinates.

DISPLACEMENT AND VERTICAL POSITION OF CENTRE OF BUOYANCY.—[AMSLER.]

Horizontal interval between Sections, 24 ft. Height of axis of moments above keel $h = 20$ ft. Scale of body plan, $\frac{1}{4}'' = 1$ ft.

INTEGRATOR ADJUSTED TO THE HORIZONTAL AXIS.

Sections.	Readings of Roller for Areas. →	17 & 1 (first & last)		$16\frac{1}{2}, 1\frac{1}{2}$; 14, 4; 12, 6; 10, 8.		16 & 2.		15, 3; 13, 5; 11, 7; 9 & centre line.		Results. Sum of Functions. A	Displ. in tons =Factor×A (Factor=146·1).		Obtained by Ordinary Calculation.
	Reading at Starting Position.	Difference →	Reading	Difference →	Reading	Difference →	Reading	Difference →	Reading		Factor $=2\times\frac{P\,d\,n^2}{3\times35}$		
Simpson's Multipliers		$\frac{1}{2}$		2		$1\frac{1}{2}$		4					
Bottom to I.	9·385	·003	9·388	1·013	0·401	·041	0·442	1·045	1·487				
(Functions)		**·002**		**2·026**		**·061**		**4·180**		**6·269**	**916**		919
+ I. to II.	1·487	·004	1·491	1·510	3·001	·115	3·116	1·565	4·681				
= Bottom to II.		·007		2·523		·156		2·610					
(Functions)		**·004**		**5·046**		**·234**		**10·440**		**15·724**	**2297**		2301
+ II. to III.	4·394	·006	4·400	1·651	6·051	·179	6·230	1·710	7·940				
= Bottom to III.		·013		4·174		·335		4·320					
(Functions)		**·007**		**8·348**		**·502**		**17·280**		**26·137**	**3819**		3815
+ III. to IV.	7·940	·004	7·944	1·705	9·649	·236	9·885	1·776	1·661				
= Bottom to IV.		·017		5·879		·571		6·096					
(Functions)		**·009**		**11·758**		**·856**		**24·384**		**37·007**	**5407**		5399
+ IV. to V.	1·308	·005	1·313	1·772	3·085	·282	3·367	1·783	5·150				
= Bottom to V.		·022		7·651		·853		7·879					
Bottom to V. direct	5·150	·024	5·174	7·649	2·823	·855	3·678	7·876	1·554				
(Functions)		**·012**		**15·298**		**1·283**		**31·504**		**48·097**	**7027**		7029

Sections.	Readings of Roller for Moments.	17 & 1		$16\frac{1}{2}, 1\frac{1}{2}$; 14, 4; 12, 6; 10, 8.		16 & 2.		15, 3; 13, 5; 11, 7; 9 & centre line.		Results. Sum of Functions. M	Factor $\times\frac{M}{A}$ =Centre of Buoy. below Axis. (Factor =7·938.)	C. B. above Keel. (h=20)	Obtained by Ordinary Calculation.
	Reading at Starting Position.	Difference →	Reading	Difference →	Reading	Difference →	Reading	Difference →	Reading		Factor$=\frac{Q\,n}{P}$	$h-\frac{Q\,n}{P}\times\frac{M}{A}$	
Simpson's Multipliers		$\frac{1}{2}$		2		$1\frac{1}{2}$		4					
Bottom to I.	2·549	·009	2·540	2·236	0·304	·089	0·215	2·307	7·908				
(Functions)		**·005**		**4·472**		**·133**		**9·228**		**13·838**	**17·63**	**2·37**	2·38
+ I. to II.	7·908	·007	7·901	2·644	5·257	·195	5·062	2·737	2·325				
= Bottom to II.		·016		4·880		·284		5·044					
(Functions)		**·008**		**9·760**		**·426**		**20·176**		**30·370**	**15·43**	**4·57**	4·58
+ II. to III.	2·632	·005	2·627	2·060	0·567	·222	0·345	2·135	8·210				
= Bottom to III.		·021		6·940		·506		7·[illegible]79					
(Functions)		**·011**		**13·880**		**·759**		**28·716**		**43·365**	**13·25**	**6·75**	6·74
+ III. to IV.	8·210	·003	8·207	1·291	6·916	·174	6·742	1·333	5·409				
= Bottom to IV.		·024		8·231		·680		8·512					
(Functions)		**·012**		**16·462**		**1·020**		**34·048**		**51·542**	**11·12**	**8·88**	8·87
+ IV. to V.	5·565	·002	5·563	·433	5·130	·070	5·060	·450	4·610				
= Bottom to V.		·026		8·664		·750		8·962					
Bottom to V. direct	4·610	·027	4·583	8·655	5·928	·748	5·180	8·953	6·227				
(Functions)		**·014**		**17·310**		**1·122**		**35·812**		**54·258**	**9·01**	**10·99**	10·99

NOTE.—The complete Integrator and arithmetical wor took two hours.

TABLE V.

DISPLACEMENT PER INCH OF IMMERSION AND TRANSVERSE METACENTRE.—[AMSLER.]

Scale of half-breadth plan, $\frac{1}{8}'' = 1\,ft.$

$$f_1 = P \times \frac{2n^2}{35 \times 12} \qquad f_2 = R \times \frac{2n^4}{35} \qquad f_3 = S \times \frac{2n^4}{35}$$

			WATER LINES.—→	I.		II.		III.		IV.		V.	
		Factors.	Reading at After-end.	Difference. →	Reading at Fore-end.	Difference. ←	Reading at After-end.	Difference. →	Reading at Fore-end.	Difference. ←	Reading at After-end.	Difference. →	Reading at Fore-end.
Integrator adjusted to centre line of half-breadth plan. Readings of roller for Areas.	Difference of readings $= a$	...	5·175	4·351	9·526	5·009	4·517	5·322	9·839	5·520	4·319	5·668	9·987
	$f_1 \times a =$ {Displacement per in. of immersion}	$f_1 = 6{\cdot}086$	...	26·48	...	30·48	...	32·39	...	33·59	...	34·50	...
	Obtained by ordinary calculation	...	...	26·40	...	30·47	...	32·35	...	33·56	...	34·46	...
Readings of roller for Moment of Inertia.	Difference of readings $= i$	...	1·610	12·390	4·000	13·985	0·015	14·767	4·782	15·270	9·512	15·644	5·156
	$f_2 \times a$	$f_2 = 74570$	...	324454	...	373521	...	396862	...	411626	...	422663	...
	$f_3 \times i$	$f_3 = 23336$	...	289133	...	326354	...	344603	...	356341	...	365068	...
	$I_x =$ {$f_2 a - f_3 i = \frac{1}{35} \times I_x$ Moment of Inertia about centre line}	...	...	35321	...	47167	...	52259	...	55285	...	57595	...
	{Displacement in tons = D (from Displacement curve)}	...	...	916	...	2297	...	3819	...	5407	...	7027	...
	$\frac{I_x}{35\,D} =$ {Transverse Metacentre above C. B.}	...	...	38·56	...	20·53	...	13·68	...	10·22	...	8·20	...
	{C. B. above keel (from C. B. curve)}	...	...	2·37	...	4·57	...	6·75	...	8·88	...	10·99	...
	{Transverse Metacentre above keel}	...	...	40·93	...	25·10	...	20·43	...	19·10	...	19·19	...
	Obtained by ordinary calculation	...	...	41·03	...	25·13	...	20·48	...	19·15	...	19·21	...

NOTE.—This calculation took one hour.

If a check is necessary, the straight-edge of the Integrator should be set to the opposite side of the centre line and the measurements repeated. A mean between the results corrects for any inaccuracy in the adjustments of the instrument.

As stated before, it is often necessary to supply one more digit to the left of the reading. Whether this digit shall be 1, 2, 3 . . . must be decided by noticing whether the zero of the counting disc passes the index-mark 1, 2, 3 . . . times. This may be previously ascertained by going roughly round the diagram.

LONGITUDINAL METACENTRE.—[AMSLER.]

TABLE VI.

Transverse axis y amidships.

Angle between inclined axes l_1, l_2, and centre line $a = 10° 34'$.

Scale of half-breadth plan $\frac{1}{8}'' = 1$ *ft.*

$f_4 = \frac{Q}{P} \times \frac{n}{\sin a}$ $f_5 = \cos^2 a$ $f_6 = \frac{1}{2 \sin^2 a}$	Water Lines.		Centre Line.		I.		II.		III.		IV.		V.	
	Factors.	Reading at Fore-end.	Difference. ⟶	Reading at After-end.	Difference. ⟶	Reading at Fore-end.	Difference. ⟵	Reading at After-end.	Difference. ⟶	Reading at Fore-end.	Difference. ⟵	Reading at After-end.	Difference. ⟶	Reading at Fore-end.
Integrator adjusted to axis l_1. Readings of roller for areas. Difference of readings	...	5·312	1·278	6·590	3·074	9·664	3·727	5·937	4·039	9·976	4·240	5·736	4·388	0·124
Plus advance of roller for centre line $= a$	...	...	...	...	4·352	...	5·005	...	5·317	...	5·518	...	5·666	...
Readings of roller for moments. Difference of readings	...	1·233	14·923	6·156	−12·442	3·714	−11·923	5·637	−11·746	3·891	−11·662	5·553	−11·623	3·930
Plus advance of roller for centre line $= (m)_1$	...	...	...	...	2·481	...	3·000	...	3·177	...	3·261	...	3·300	...
Readings of roller for moments of inertia. Difference of readings	...	7·250	−·158	7·092	9·349	6·441	10·045	6·396	10·281	6·677	10·435	6·242	10·538	6·780
Plus advance of roller for centre line $= (i)_1$	...	...	...	...	9·191	...	9·887	...	10·123	...	10·277	...	10·380	...
Integrator adjusted to axis l_2. Readings of roller for areas. Difference of readings	...	9·273	−1·274	7·999	5·623	3·622	6·279	7·343	6·593	3·936	6·792	7·144	6·938	4·082
Plus advance of roller for centre line $= a$	...	...	...	...	4·349	...	5·005	...	5·319	...	5·518	...	5·664	...
Readings of roller for moments. Difference of readings	...	3·252	15·582	8·834	−13·303	5·531	−12·661	8·192	−12·307	5·885	−12·050	7·935	−11·845	6·090
Plus advance of roller for centre line $= (m)_2$	...	...	...	...	2·279	...	2·921	...	3·275	...	3·532	...	3·737	...
Readings of roller for moments of inertia. Difference of readings	...	4·211	·123	4·334	9·227	3·501	9·779	3·782	9·837	3·619	9·790	3·829	9·681	3·510
Plus advance of roller for centre line $= (i)_2$	...	...	...	...	9·350	...	9·902	...	9·960	...	9·913	...	9·804	...
$(m)_1 - (m)_2$	...	...	...	...	·202	...	·079	...	−·098	...	−·271	...	−·437	...
$f_4 \left((m)_1 - (m)_2 \right)$	$f_4 = 43·31$	...	...	...	8·75	...	3·42	...	−4·24	...	−11·90	...	−18·93	...
$\frac{f_4 (m_1 - m_2)}{a} = C =$ {Distance of centre of water-plane from y axis, to the right for +, to the left for −}	...	...	...	...	2·01	...	·68	...	−·80	...	−2·16	...	−3·34	...
Obtained by ordinary calculation	...	...	...	...	1·80	...	·40	...	−·92	...	−2·25	...	−3·40	...
C^2	...	...	...	...	4·04	...	·46	...	·64	...	4·67	...	11·16	...
$12 f_1 a$. (See Table for transverse metacentre)	...	...	...	...	321	...	366	...	388	...	403	...	414	...
$12 f_1 a C^2$	...	...	...	...	1297	...	168	...	248	...	1880	...	4620	...
$2 f_2 a$. (See Table for transverse metacentre)	...	...	...	...	648908	...	747042	...	793724	...	823252	...	845326	...
$(i)_1 + (i)_2$	...	...	...	...	18·541	...	19·789	...	20·083	...	20·190	...	20·184	...
$f_3 (i_1 + i_2)$	$f_3 = 23336$	...	...	...	432673	...	461796	...	468657	...	471154	...	471014	...
$2 f_2 a - f_3 (i_1 + i_2) = \frac{1}{35} (I_1 + I_2)$	...	...	...	...	216235	...	285246	...	325067	...	352098	...	374312	...
$f_5 \times \frac{I x}{35}$ See Table for transverse metacentre	$f_5 = 1·933$	...	...	...	68275	...	91174	...	101017	...	107866	...	111331	...
$\frac{1}{35} (I_1 + I_2) - f_5 \frac{I x}{35}$	...	...	...	...	147960	...	194072	...	224050	...	244232	...	262981	...
$f_6 \left\{ \frac{1}{35} (I_1 + I_2) - f_5 \frac{I x}{35} \right\} = \frac{I y}{35}$	$f_6 = \frac{1}{·067}$	...	...	...	2208358	...	2896600	...	3344030	...	3645254	...	3925090	...
$\frac{1}{35} I y - 12 f_1 a C^2 = \frac{1}{35} I n$ {$I n$ = mom. of inertia about neutral axis}	...	...	...	...	2207061	...	2896432	...	3343782	...	3643374	...	3920470	...
$\frac{I n}{35 D} =$ {Height of longitudinal metacentre above C. B.}	...	...	...	...	2409	...	1261	...	876	...	674	...	558	...
C. B. above keel (from curve)	...	...	...	...	2·37	...	4·57	...	6·75	...	8·88	...	10·99	...
Height of longitudinal metacentre above keel	...	...	...	...	2411	...	1266	...	883	...	683	...	569	...
Obtained by ordinary calculation	...	...	...	...	2409	...	1273	...	889	...	678	...	572	...

Note.—This calculation took two hours. A considerable amount of time could be saved by using the table of logarithms for the arithmetical work.

INDEX.

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