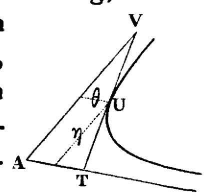
ferred to, in which matter generally considered inanimate assumes additional properties and becomes alive. Many arguments were then brought forward to show that organic life is the result of exalting inorganic life by combination of elementary properties.

The following extract of a letter from Sir William Rowan Hamilton to the Rev. Charles Graves was read to the Academy:

"If I had been more at leisure when last writing, I should have remarked that besides the construction of the ellipsoid by the two *sliding spheres*, which, in fact, led me last summer to an equation nearly the same as that lately submitted to the Academy, a simple interpretation may be given to the equation,



$$\mathrm{TV}\,\frac{\eta\rho-\rho\theta}{\mathrm{U}\,(\eta-\theta)}=\theta^2-\eta^2; \qquad (1)$$

which may also be thus written,

$$\mathrm{TV}\frac{\rho\eta-\theta\rho}{\eta-\theta}=\frac{\theta^2-\eta^2}{\mathrm{T}\,(\eta-\theta)}. \qquad (2)$$

"At an umbilic U, draw a tangent TUV to the focal hyperbola, meeting the asymptotes in T and V; then I can shew *geometrically*, as also in other ways,—what, indeed, is likely enough to be known,—that the sides of the triangle TAV are, as respects their *lengths*,

$$\overline{\mathrm{AV}}=a+c;\ \overline{\mathrm{AT}}=a-c;\ \overline{\mathrm{TV}}=2b. \qquad (3)$$

Now my η and θ are precisely the halves of the sides AV and AT of this triangle; or they are the two co-ordinates of the umbilic U, referred to the two asymptotes, when *directions* as well as lengths are attended to. This explains several of my formulæ, and accounts for the remarkable circumstance that we can pass to a *confocal surface*, by changing η and θ to $t^{-1}\eta$ and $t\theta$ respectively, where t is a scalar.

"Again we have, identically,

$$\mathrm{V}\frac{\rho\eta-\theta\rho}{\eta-\theta}=\rho_1+\rho_2; \tag{4}$$

if for conciseness we write

$$\rho_1=(\eta-\theta)^{-1}\,\mathrm{S}\,.\,(\eta-\theta)\rho; \tag{5}$$

$$\rho_2=\mathrm{V}\,.\,(\eta-\theta)^{-1}\,\mathrm{V}\,.\,(\eta+\theta)\rho. \tag{6}$$

But ρ_1 is the perpendicular from the centre A of the ellipsoid on the plane of a circular section, through the extremity of any vector or semidiameter ρ; and ρ_2 may be shewn (by a process similar to that which I used to express Mac Cullagh's mode of generation) to be a radius of that circular section, multiplied by the scalar coefficient $\mathrm{S}\,.\,(\eta-\theta)^{-1}\,(\eta+\theta)$, which is equal to

$$\frac{\theta^2-\eta^2}{-(\eta-\theta)^2}=\frac{\mathrm{T}\eta^2-\mathrm{T}\theta^2}{\mathrm{T}\,(\eta-\theta)^2}=\frac{ac}{b^2}. \tag{7}$$

If, then, from the foot of the perpendicular let fall, as above, on the plane of a circular section, we draw a right line in that plane, which bears to the radius of that section the constant ratio of the rectangle (ac) under the two extreme semiaxes to the square (b^2) of the mean semiaxis of the ellipsoid, the equation (2) expresses that the line so drawn will terminate on a spheric surface, which has its centre at the centre of the ellipsoid, and has its radius $=\frac{ac}{b}$; this last being the value of the second member of that equation (2). And, in fact, it is not difficult to prove *geometrically* that this construction conducts to this spheric locus, namely, to the sphere concentric with the ellipsoid, which touches at once the four umbilicar tangent planes."

The Rev. Charles Graves communicated the following theorems relating to the principal circular sections of a central surface of the second order, and the sphero-conics traced upon it.

In a central surface of the second order, the arc of a principal section P, *included between the two principal circular sections* C, C′, *is bisected at the point* t, *where it touches a sphero-conic of the surface.*

The proof of this theorem is extremely simple. The semidiameter drawn to the point t is obviously a semiaxis of the section P; and the semidiameters drawn to the points c and c', in which P meets C and C', are equal to one another, being each equal to the mean semiaxis of the surface; consequently the arc cc' is bisected at t. Precisely in the same manner it might be shown that

The arc of a diametral section, included within one of the sphero-conics of the surface, is bisected at the point where it touches a second sphero-conic of the same system.

From the theorem first stated we deduce the following: *The sector of the surface included between the two principal planes of circular section, and any diametral plane which touches a fixed sphero-conic, is of constant volume.*

For, if we draw a second diametral plane P', infinitely near to P, and touching the same sphero-conic, the two elementary sectors respectively included between P, P', and each of the two principal planes of circular section, will evidently be equal: and for this same reason

The sector included between a cone generated by a semidiameter moving along one of the sphero-conics of the surface, and any diametral plane which touches a fixed sphero-conic, is of constant volume.