NYSTROM'S POCKET-BOOK OF MECHANICS AND ENGINEERING.

REVISED AND CORRECTED BY

WILLIAM DENNIS MARKS, PH. B., C. E. [YALE S. S. S.],

WHITNEY PROFESSOR OF DYNAMICAL ENGINEERING, UNIVERSITY OF PENNSYLVANIA; HONORARY LIFE-MEMBER OF THE FRANKLIN INSTITUTE; MEMBER OF THE AMERICAN PHILOSOPHICAL SOCIETY, PHILADELPHIA; MEMBER OF THE ELECTRIC CLUB, N. Y.; AND CONSULTING MECHANICAL AND CIVIL ENGINEER.

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WITH

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PREFACE TO THE NINETEENTH EDITION.

THE very large number of this POCKET-BOOK sold in past years seems to prove its utility in its present shape. The Reviser, therefore, has principally confined himself to corrections of obvious errors in the English and the formulæ.

He cannot hope to have found and corrected every error, and will esteem it as the greatest possible favor if any person will call his attention to oversights.

An elementary article on dynamic electricity has been added by him, as also one on the expansion of steam.

In the form of notes, the Reviser has at times stated his different opinion, and also referred the reader to the literature of topics which required more space than can be given to them in a pocket-book.

A single branch of mechanical engineering may require half a lifetime of study and experience before the engineer is competent to undertake designing in it.

The writer cannot too strongly call the attention of engineers to the deplorable results of over-confidence and superficial reasoning. It would be far better that our laws should hold him a criminal who, through ignorance or dishonest pretension to knowledge, makes blunders resulting in the loss of life and property, than that the present loose ideas as to the dignity and responsibility of an engineer's position and work should prevail.

A pocket-book serves as a useful memorandum for the experienced engineer, and should guide the inexperienced engineer to higher and wider fields of thought and research.

Modestly remember, in engineering, that the possibility of error or oversight in our premises is so great that the result of unverified mathematical investigation remains only a "presumption" until verified by experiment. It makes no difference what the *intention* or *wish* of the designer may be, matter and force blindly and inevitably follow the laws of nature with certainty and precision. The designer must learn these laws, and *all* of them that affect the particular case he has, before he can effect a safe and general solution.

WILLIAM DENNIS MARKS.

University of Pennsylvania,
PHILADELPHIA, 1887.

PREFACE.

EVERY Engineer should make his own Pocket-Book, as he proceeds in study and practice, to suit his particular business. The present work has been accumulated in that way during the author's professional career. It was originally not intended for publication, but grew too large for the pocket in form of manuscript, which circumstance, combined with repeated requests to publish it, first placed it before the public in the year 1854.

The author claims to have given a goodly share of original matter, and has spent much labor and money in experiments on subjects requiring elucidation.

The authors consulted are distinguished experimenters, such as Dalton, on air and heat; Regnault, on steam; Kopp, on the expansion of water; Morin, on friction and strength of materials; Joule, on the mechanical equivalent of heat; the Franklin Institute, on the strength of iron and copper at different temperatures; the Royal Technological Institute, Stockholm, on dynamics; and various others of equal authority; but these savans are not responsible for the formulas and tables which are herein deduced from their experiments.

The solution of mathematical formulas leads to powerful presumptions in the revelation of physical laws, which could never be attained or realized from mere observation of facts in experiments and practice. All observation and contemplation which involve mind, involves theory, which is the foundation of our practice and progress.

A knowledge of algebra is not necessary for the use of the formulas, and it is satisfactory to know that most engineers who are not versed in mathematics have acquired the very important habit of inserting numerical values for the corresponding letters, which they prefer to cumbrous written rules, which are impracticable in extensive problems. If all the formulas herein were explained in words, the book would exceed in volume Webster's unabbreviated Dictionary, and the matter would be only so much the more complicated. The algebraical formulas herein are solved into all their functions, ready to receive what is given and refund what is required. They not only tell what is to be done, but at a glance impress the mind with the complete operation.

JOHN W. NYSTROM.

1010 Spruce street, PHILADELPHIA.

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INTRODUCTION.

Quantity is that which can be increased or diminished by addition or subtraction of homogeneous parts. Quantities are of two essential kinds, *Geometrical* and *Physical.*

1st, *Geometrical* quantities are those which occupy space; as *lines, surfaces, solids, liquids, gases,* &c.

2nd, *Physical* quantities are those which exist, but occupy no space, they are known by their character and action upon geometrical quantities; as *attraction, light, heat, electricity* and *magnetism, colors, force, power,* &c., &c.

To obtain the magnitude of a quantity we compare it with a part of the same, this part is imprinted in our mind as a *unit,* by which the whole is measured and conceived. No quantity can be measured by a quantity of another kind, but any quantity can be compared with any other quantity, and by such comparison arises what we call *calculation* or *Mathematics.*

MATHEMATICS.

Mathematics is a science by which the comparative value of quantities is investigated; it is divided into:

1st, **Arithmetic,**—that branch of Mathematics, which treats of the nature and property of numbers; it is subdivided into *Addition, Subtraction, Multiplication, Division, Involution, Evolution and Logarithms.*

2nd, **Algebra,**—that branch of Mathematics which employs letters to represent quantities, and by that means performs solutions without knowing or noticing the *value* of the quantities. The subdivisions of Algebra are the same as in Arithmetic.

3rd, **Geometry,**—that branch of Mathematics which investigates the relative property of quantities that occupies space; its subdivisions are *Longimetry, Planimetry, Stereometry, Trigonometry, and Conic Sections.*

4th, **Differential-calculus,**—that branch of Mathematics which ascertains the ultimate ratio of two or more variables connected by an equation.

5th, **Integral-calculus,**—the contrary of Differential, or that branch of Mathematics which effects the summation of infinitesimal quantities.

ARITHMETIC.

The art of computation with known quantities.

Figures—1, 2, 3, 4, 5, 6, 7, 8, 9. Arabic digits, about nine hundred years old.

Ciphers—0, 0, 0. Sometimes called noughts.

Number is the expression of one or more figures and ciphers.

Integer is a whole number or unit.

Fraction is a part of a number or unit.

When figures are joined together in a number, the relative dignity expressed by each figure, depends upon its position to the others. Thus,

Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Hundreds. Tens. Units.
674,385 ;	496,345 ;	695,216 ;	505,310 :	685,	367 ;

ROMAN NOTATION.

The Romans expressed their numbers by various repetitions and combinations of seven letters in the alphabet; as,

1 = I.
2 = II.
3 = III.
4 = IV.
5 = V.
6 = VI.
7 = VII.
8 = VIII.
9 = IX.
10 = X.
20 = XX.
30 = XXX.
40 = XL.
50 = L.
60 = LX.
70 = LXX.
80 = LXXX.
90 = XC.
100 = C.

EXAMPLES.—1872.—MDCCCLXXII.

500 = D, or LƆ.
1,000 = M, or CƆ.
2,000 = MM, or IIƆƆƆ.
5,000 = V, or LƆƆ.
6,000 = VI, or MMM
10,000 = $\overline{\text{X}}$, or CƆƆ.
50,000 = $\overline{\text{L}}$, or LƆƆƆ.
60,000 = $\overline{\text{LX}}$, or MMMƆ.
100,000 = $\overline{\text{C}}$, or CƆƆƆ.
1,000,000 = $\overline{\text{M}}$, or CƆƆƆƆ.
2,000,000 = $\overline{\text{MM}}$, or MMƆƆƆ.

A bar, thus, — over any number, increases it 1000 times.

524,365, $\overline{\text{DXXIV}}$CCCLXV.

An imperfection in the Roman Notation consists in the fact that there is no signification for the cipher, as in the Arabic Notation.

Signification of Characters.

= Equality, as 6 = 6, reads 6 is equal to 6.
+ *Plus*, Addition, . 3 + 6 = 9
— *Minus*, Subtraction, 6 — 2 = 4
× Multiplication, . 3 × 4 = 12
÷ or : Division, . . 15 : 5 = 3
$\sqrt{}$ Square root, . . . $\sqrt{9} = 3$
$\sqrt[3]{}$ Cube root, . . . $\sqrt[3]{8} = 2$
> Greater, ... 8 > 4
< Less, ... 6 < 9
∞ Infinite, ... $\frac{1}{0}$

∫ Integral, . . . $\int dy = y$.
dy Differential, . . $dy = dx +$.
3/4 Fraction, ... = ¾.
⋈ Ship sign, dead flat,
≡ Furnace fire-grate.
○ Boiler heating-surface.
♯ Sharp. High.
♭ Flat. Low.
π Periphery.

Astronomical Characters.

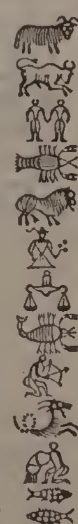
Planets.

☉ The Sun.
☾ The Moon.
☿ Mercury,
♀ Venus,
⊕ The Earth.
♂ Mars.
⚳ Ceres.
⚴ Pallas.
⚵ Juno.
⚶ Vesta.
♃ Jupiter.
♄ Saturn.
♅ Uranus.
♆ Neptune.

☌ Conjunction in the same degree or sign, or having the same longitude or Right Ascension.
⚹ Sextile, when two signs distant, or differing 60° in longitude or Right Ascension.
□ Quartile, when three signs distant, or differing 90° in Longitude or Right Ascension.
☍ Opposition, when six signs distant, or differing 180° in Longitude or Right Ascension.
☊ Ascending Node.
☋ Descending Node.
R. A. Right Ascension.

Signs of the Zodiac.

♈ Aries, . .
♉ Taurus, . .
♊ Gemini, . .
♋ Cancer, . .
♌ Leo, . . .
♍ Virgo, . . .
♎ Libra, . .
♏ Scorpio, . .
♐ Sagittarius,
♑ Capricornus,
♒ Aquarius, .
♓ Pisces, . . .



ALGEBRA.

In **Algebra** we employ certain *characters* or *letters* to represent quantities. These characters are separated by *signs*, which describe the operations; and by that means, simplify the solution.

1. Whatever the value of *any* quantity may be, it can be represented by a character, as a. Another quantity of the same kind, but of different value, being represented by b. The sum of these two quantities is of the same kind but of different value.

For **Addition** we have the algebraical sign +, (plus) which, when placed between quantities, denotes they shall be added; as $a+b$, reads in the algebraical language, "a plus b," or a is to be added to b.

Another algebraical sign =, (Equal) denotes that quantities which are placed on each side of this sign, are *equal*. Let the sum of a and b be denoted by the letter c; then we have,

$$a+b=c.$$

This composition is called an *algebraical equation*. The quantity on each side of the equal sign is called a *member*, as $a+b$, is one member, and c, the other. When one of the members contains only one quantity, that member is generally placed on the first side of the equal sign, and its value commonly unknown; but the value of the quantities in the other member being given, as $a=4$, and $b=5$, then the practical mode, to insert numerical values in algebraical equations, will appear; as,

Equation, $c=a+b$,
$4+5=9$, the value of c.

2. The sum of three quantities a, b, and c, is equal to d, then

Equation, $d=a+b+c$,
$4+5+9=18$, the value of d.

3. For **Subtraction** we have the algebraical sign, —, (minus) which, when placed before a quantity, denotes it is to be subtracted as, $a-b$, reads in the algebraical language "a minus b," or from a, subtract b. Let the difference be denoted by the letter c; and $a=8$. $b=3$

Equation, $c=a-b$,
$8-3=5$, the value of c.

4. From the sum of a and b, subtract c, and the result will be d; then,

Equation, $d=a+b-c$,
$8+3-5=6$, the value of d.

5. When two equal quantities are to be added, as $a+a$. it is the same as to take one of them twice, and is marked thus $2a$. The number 2 is called the coefficient of the quantity a. If there are more than *two* equal quantities to be added, the coefficient denotes how many there are of them; as,

Equation, - - - - $a+a=2a$,
" $a+a+a=3a$,
" $a+a+a+a=4a$,
&c., &c.

When the quantities are separated by the signs, *plus*, or *minus*, they are called *terms*.

6. **Multiplication.**—When a quantity a, is to be multiplied by another quantity b, then a and b are called *factors*; and separated by no sign as ab; which denotes that a is to be multiplied by b; but when the values of a and b are expressed by numbers, they are separated by the sign × (Multiplication); the result from Multiplication is called the product. Let $a=8$, and $b=6$, and the product of a and b, to be c, then,

Equation, $c=ab$,
$8\times6=48$, the value of c.

7. The product of a and b, is to be multiplied by c, and the latter product will be equal to d; then,

Equation, $d=abc$,
$8\times6\times48=2304$, the value of d.

8. The sum of a and b, is to be multiplied by c, and the product will be d; then.

$$\textit{Equation,}\quad d=c\,(a+b),$$
$$48\,(8+6)=672 \text{ the value of } d.$$

When the sum of two or more quantities is to be multiplied by another quantity, the sum is to be enclosed in parentheses, and is called one factor. The other factor is to be placed on the outside of the parentheses, as seen in the preceding example.

9. To the product of a and c, add b, and the result will be d; then,

$$\textit{Equation,}\quad d= ac+b,$$
$$8\times48+6=390 \text{ the value of } d.$$

Be particular to distinguish the two Examples 8, and 9.

10. The sum of a and b, to be multiplied by the sum of a and c; the product will be d; then,

$$\textit{Equation,}\quad d=(a+b)\,(a+c),$$
$$(8+6)\,(8+48)=784.$$

11. The sum of c and b, to be multiplied by the difference of c and a; the result will be d; then,

$$\textit{Equation,}\quad d=(c+b)\,(c-a),$$
$$(48+6)\,(48-8)=2160.$$

12. **Division**.—When a quantity a, is to be separated into b equal parts, the numbers of parts or b, is called the *divisor*, and the value of each part, is called the *quotient*. The sum of the parts or the whole quantity a, is called the *dividend*; a and b, is separated by the sign : (Division); as $a:b$, reads in the algebraical language, "a divided by b." Let the *quotient* be denoted by the letter c; and $a=18$, $b=6$, then,

$$\textit{Equation,}\quad c=a:b,$$
$$18:6=3 \text{ the quotient } c.$$

In Algebra it is found more convenient to set up Division as a fraction, then it will appear as,

13. Divide a, by c, and the quotient will be b. Then,

$$\textit{Equation,}\quad b=\frac{a}{c},$$
$$\frac{18}{3}=6 \text{ the quotient } b.$$

14. The product of a and b, to be divided by c; and the product will be d. Then,

$$\textit{Equation,}\quad d=\frac{ab}{c},$$
$$\frac{18\times6}{3}=36.$$

15. The sum of d and b, to be multiplied by c, and the product divided by a; then the result will be e.

$$\textit{Equation,}\quad e=\frac{c\,(d+b)}{a},$$
$$\frac{3\,(36+6)}{18}=7.$$

16. From the product of a and c, subtract $3b$; divide the remainder by the difference of a, and c; the result will be h.

$$\textit{Equation, } h = \frac{ac - 3b}{a - c}, \qquad \frac{18 \times 3 - 3 \times 6}{18 - 3} = 2.4.$$

An old man said to a boy, "How old are you?" to which he replied,—"To seven times my father's age add yours, divide the sum by double the difference of yours and his, and the result will be my age."

Letters will denote,
a = the old man's age,
b = the father's age,
c = the boy's age. Then,

$$\textit{Equation, } c = \frac{7b + a}{2(a - b)} \text{ the boy's age.}$$

$a = 73$ years, the age of the old man,
$b = 57$ years, the father's age.
Required the boy's age.

$$c = \frac{7 \times 57 + 73}{2(73 - 57)} = 14\tfrac{3}{4} \text{ years.}$$

Powers.—When a number or a quantity is to be multiplied by itself, the operation is called power and denoted by a small number at the right-hand corner of the quantity, like a^2, which denotes that a or the numerical value of a must be multiplied by itself. Suppose $a = 4$; then $a^2 = a \times a = 4 \times 4 = 16$. When a represents the length of the side of a square, then a^2 represents the area of that square. Suppose the side of the square is $a = 12$ inches; then the area of that square will be $a^2 = 12^2 = 12 \times 12 = 144$ sq. in. The small number is called the *exponent* of the power. When the exponent is 2, the power is called the square; when 3, it is called the cube; and when 4, the bisquare. When the side of a cube is $a = 12$ inches, then the volume of that cube is $a^3 = a \times a \times a = 12^3 = 12 \times 12 \times 12 = 1728$ cubic inches. The squares and cubes of numbers will be found in tables farther on.

When quantities are separated by signs, + or —, enclosed within parentheses and with an exponent outside of the parentheses—like this: $(a + b)^2$—then a and b must first be added and the sum squared. If $a = 4$ and $b = 9$, then $(4 + 9)^2 = 13^2 = 169$. $(ab - cd)^3$, $a = 4$, $b = 9$, $d = 5$, and $c = 3$.

Then, $$(4 \times 9 - 3 \times 5)^3 = (36 - 15)^3 = 21^3 = 9261.$$

The operation within the parentheses must be accomplished before the power is used.

Roots.—A root is a number from which a given power is raised, and is denoted by the sign $\sqrt{\ }$, which means the square root. That is, $\sqrt{4} = 2$, because $2^2 = 4$. The number 4 is the power and 2 the root. $\sqrt{16} = 4$, because $4^2 = 16$. Roots of higher order than the square root are indicated by a small number in the root-mark; as, $\sqrt[3]{\ }$, which means the cube root. Thus, the $\sqrt[3]{8} = 2$, because $2^3 = 2 \times 2 \times 2 = 8$.

The fourth root is marked $\sqrt[4]{\ }$, or $\sqrt[4]{16} = 2$, because $2^4 = 2 \times 2 \times 2 \times 2 = 16$. The small number in the root-mark is called the *index* of the root.

A common formula for a right-angled triangle is $a = \sqrt{b^2 + c^2}$, in which a is the *hypothenuse*, b and c the two legs forming the right angles. Suppose $b = 4$ and $c = 3$; then $a = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$. That is to say, the hypothenuse is 5. The operation under the root-mark must be accomplished before the root is extracted.

The extraction of roots by arithmetic is very complicated and not often resorted to in practice, but tables and logarithms are generally used for that purpose.

PROPORTION.

THE *relative value* of two quantities is obtained by dividing one into the other, and the quotient is called the *ratio* of their relationship. If the ratio of two quantities is equal to the ratio of two other quantities, they are said to be in the same proportion; as,

$$a : b = c : d,$$

reads in the algebraical language "a is to b as c is to d." a, b, c, and d are called *terms*, of which a is the first, b the second, c the third, and d the fourth *term*. The first and fourth are called "the *outer terms*," and the second and third "the *inner terms*." The whole is called an "*analogy*."

A property in the nature of analogies is that the product of the outer terms, $a\,d$, is equal to the product of the inner, $b\,c$. Suppose $a = 4$, $b = 9$, $c = 12$, $d = 27$.

$$4 : 9 = 12 : 27,$$
$$a\,d = b\,c, \quad 4 \times 27 = 9 \times 12.$$

If any one of the four quantities is unknown, its value can be calculated by the other three; as,

$$a = \frac{b\,c}{d} = \frac{9 \times 12}{27} = 4,$$

$$b = \frac{a\,d}{c} = \frac{4 \times 27}{12} = 9,$$

$$c = \frac{a\,d}{b} = \frac{4 \times 27}{9} = 12,$$

$$d = \frac{b\,c}{a} = \frac{9 \times 12}{4} = 27.$$

Proportion is generally used in commercial calculations, and in arithmetics it is called the *rule of three*, because in simple proportion there are three given quantities by which the fourth one is calculated. The fourth or unknown quantity is generally denoted by x.

Example. If 3 yards of cloth cost 11 dollars, how much will 7 yards of the same cloth cost?

From these three given quantities we can find the fourth or unknown price of 7 yards.

Proportion, $\quad 3 : 7 = 11 : x,$

the product of the inner terms being equal to that of the outer ones, or $3\,x = 7 \times 11$. Move the 3 of the first member under the second member; thus,

$$x = \frac{7 \times 11}{3} = 25.66, \text{ or \$25 and 66 cents, the price of 7 yards.}$$

Mean Proportion.

The mean proportion between two quantities, a and b, is set up as follows:

$$a : x = x : b, \quad x^2 = a\,b, \quad \text{or } x = \sqrt{a\,b}.$$

The square root of the product of the two quantities is the mean proportion x.

Suppose $a = 2$ and $b = 8$; then the mean proportion, x, between 2 and 8, is

$$x = \sqrt{2 \times 8} = \sqrt{16} = 4.$$

Mean Difference.

The mean difference or average between two quantities is the sum of the quantities divided by 2. Let the quantities be $a = 2$ and $b = 8$; then the mean difference, x, is

$$x = \frac{a + b}{2} = \frac{2 + 8}{2} = \frac{10}{2} = 5.$$

The mean proportion between 2 and 8 is 4, but the mean difference is 5.

It is of great importance to clearly distinguish mean proportion from mean difference, for otherwise calculation may lead to erroneous results.

SIMPLE INTEREST.

Interest is money paid for use of money which is lent for a certain time.

Notation.

$c =$ the amount lent,
$r =$ interest on the amount, c,
$p =$ per cent. in the certain time.

Analogy, $c : r = 100 : p$.

If p is the per cent. on 100, in one year, then $t =$ time in years for the standing capital c, and the interest r.

Analogy, $c : r = 100 : pt$.

From this analogy we obtain the equations,

Interest, $r = \frac{cpt}{100}$, - - - - - 1,

Per cent., $p = \frac{100\,r}{tc}$, - - - - - 2,

Capital, $c = \frac{100\,r}{pt}$, - - - - - 3,

Time in years, $t = \frac{100\,r}{cp}$, - - - - - 4.

Now for any question in Simple Interest, there is one equation which gives the answer. If the time is given in *months, weeks,* or *days,* multiply the 100 correspondingly by 12, 52, 365.

Example 1. What is the interest on \$3789.35, for 3 years and five months, at 6 per cent. per annum?

$t = 3\times12+5 = 41$ months, from the Equation 1, we have,

Interest, $r = \frac{3789.35\times6\times41}{12\times100} = 776.81$ Dollars.

Example 2. A capital $c = \$469.78$, returned interest $r = 150.72$ dollars, in time $t = 4$ years and 7 months. Required the per centage per annum?

$t = 4 \times 12 + 7 = 55$ months, from Equation 2, we have,

Per cent., $p = \frac{12 \times 100 \times 150.72}{469.78 \times 55} = 7$ per cent.

Example 3. What amount is required to return interest $r = 345$ dollars in 6 years, at 5 per cent. per annum? From the Equation 3, we have,

Capital, $c = \frac{100 \times 345}{5 \times 6} = \$1150$.

Example 4. An amount $c = \$2365$ is to stand until the interest $r = 550$ dollars, at $p = 6$ per cent. per annum. How long must the amount stand?

From the Equation 4, we have,

Time, $t = \frac{100\times550}{2365\times6} = 3.876$ years.

$12\times0.876 = 10.512$ months, $4\times0.512 = 2.048$ weeks, the time $t = 3$ years, 10 months, and 2 weeks.

Five per cent. per Annum.

Time.	$100.	$200.	$300.	$400.	$500.	$600.	$700.	$800.	$900.	$1000.
1 day.	0.01	0.03	0.04	0.06	0.07	0.08	0.10	0.11	0.13	0.14
2 days.	0.03	0.06	0.08	0.11	0.14	0.17	0.19	0.22	0.25	0.28
3 “	0.04	0.08	0.13	0.17	0.21	0.25	0.29	0.33	0.37	0.42
4 “	0.06	0.11	0.17	0.22	0.28	0.33	0.39	0.45	0.50	0.56
5 “	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.70
6 “	0.08	0.17	0.25	0.33	0.42	0.50	0.58	0.67	0.75	0.83
7 “	0.10	0.19	0.29	0.39	0.49	0.58	0.68	0.78	0.88	0.97
8 “	0.11	0.22	0.33	0.44	0.56	0.67	0.78	0.89	1.00	1.11
9 “	0.13	0.25	0.38	0.50	0.63	0.75	0.88	1.00	1.12	1.25
10 “	0.14	0.28	0.42	0.55	0.69	0.83	0.97	1.11	1.25	1.39
11 “	0.15	0.31	0.46	0.61	0.76	0.92	1.07	1.22	1.38	1.53
12 “	0.17	0.33	0.50	0.67	0.83	1.00	1.17	1.33	1.50	1.67
13 “	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62	1.80
14 “	0.20	0.39	0.59	0.79	0.98	1.18	1.38	1.57	1.77	1.96
15 “	0.21	0.43	0.62	0.83	1.04	1.25	1.45	1.67	1.87	2.08
16 “	0.22	0.44	0.67	0.89	1.11	1.33	1.56	1.78	2.00	2.22
17 “	0.24	0.47	0.71	0.94	1.18	1.41	1.65	1.89	2.12	2.36
18 “	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
19 “	0.26	0.53	0.79	1.06	1.32	1.59	1.85	2.11	2.38	2.64
20 “	0.28	0.56	0.83	1.11	1.39	1.67	1.95	2.22	2.50	2.78
21 “	0.29	0.58	0.88	1.17	1.46	1.75	2.04	2.33	2.63	2.92
22 “	0.31	0.61	0.92	1.22	1.53	1.83	2.14	2.45	2.75	3.06
23 “	0.32	0.64	0.96	1.28	1.60	1.92	2.24	2.56	2.87	3.19
24 “	0.33	0.67	1.00	1.33	1.67	2.00	2.33	2.67	3.00	3.33
25 “	0.35	0.69	1.04	1.39	1.74	2.09	2.43	2.78	3.13	3.47
26 “	0.36	0.72	1.08	1.44	1.80	2.17	2.53	2.89	3.25	3.61
27 “	0.38	0.75	1.13	1.50	1.88	2.25	2.63	3.00	3.37	3.75
28 “	0.40	0.78	1.17	1.56	1.94	2.33	2.72	3.10	3.50	3.89
29 “	0.40	0.81	1.21	1.61	2.02	2.42	2.82	3.23	3.63	4.03
30 “	0.42	0.83	1.25	1.67	2.08	2.50	2.92	3.33	3.75	4.17
1 month.	0.42	0.83	1.25	1.67	2.08	2.50	2.92	3.33	3.75	4.17
2 months	0.83	1.67	2.50	3.33	4.17	5.00	5.83	6.67	7.50	8.33
3 “	1.25	2.50	3.75	5.00	6.25	7.50	8.75	10.00	11.25	12.50
4 “	1.67	3.33	5.00	6.67	8.33	10.00	11.67	13.33	15.00	16.67
5 “	2.08	4.17	6.25	8.33	10.41	12.50	14.58	16.67	18.75	20.83
6 “	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00	22.50	25.00
7 “	2.92	5.83	8.75	11.67	14.58	17.50	20.42	23.33	26.25	29.17
8 “	3.33	6.67	10.00	13.33	16.67	20.00	23.33	26.67	30.00	33.33
9 “	3.75	7.50	11.25	15.00	18.75	22.50	26.25	30.00	33.75	37.50
10 “	4.17	8.33	12.50	16.67	20.83	25.00	29.17	33.33	37.50	41.67
11 “	4.58	9.17	13.75	18.33	22.92	27.50	32.08	36.67	41.25	45.83
1 year.	5.00	10.00	15.00	20.00	25.00	30.00	35.00	40.00	45.00	50.00

Example.—Required the interest on $8978 in 27 days at five per cent. per annum?

Interest on $8000 = 30.00
“ 900 = 3.37
“ 70 = 0.26
“ 8 = 0.03
“ $8978 = 33.66 the answer.

Six per cent. per Annum.

Time.	$100.	$200.	$300.	$400.	$500.	$600.	$700.	$800.	$900.	$1000.
1 day.	0.02	0.03	0.05	0.07	0.08	0.10	0.12	0.13	0.15	0.17
2 days.	0.03	0.07	0.10	0.13	0.17	0.20	0.23	0.27	0.30	0.33
3 "	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
4 "	0.07	0.13	0.20	0.27	0.33	0.40	0.47	0.53	0.60	0.67
5 "	0.08	0.17	0.25	0.33	0.42	0.50	0.58	0.67	0.75	0.83
6 "	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
7 "	0.12	0.23	0.35	0.47	0.58	0.70	0.81	0.93	1.05	1.17
8 "	0.13	0.27	0.40	0.53	0.67	0.80	0.93	1.07	1.20	1.33
9 "	0.15	0.30	0.45	0.60	0.75	0.90	1.05	1.20	1.35	1.50
10 "	0.17	0.33	0.50	0.67	0.83	1.00	1.17	1.33	1.50	1.67
11 "	0.18	0.37	0.55	0.73	0.91	1.10	1.28	1.47	1.65	1.83
12 "	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
13 "	0.22	0.43	0.65	0.87	1.08	1.30	1.52	1.73	1.95	2.17
14 "	0.23	0.47	0.70	0.93	1.17	1.40	1.63	1.87	2.10	2.33
15 "	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
16 "	0.27	0.53	0.80	1.07	1.33	1.60	1.86	2.13	2.40	2.67
17 "	0.28	0.57	0.85	1.13	1.41	1.70	1.98	2.27	2.55	2.83
18 "	0.30	0.60	0.90	1.20	1.50	1.80	2.10	2.40	2.70	3.00
19 "	0.32	0.63	0.95	1.27	1.58	1.90	2.21	2.53	2.85	3.17
20 "	0.33	0.67	1.00	1.33	1.67	2.00	2.33	2.67	3.00	3.33
21 "	0.35	0.70	1.05	1.40	1.75	2.10	2.45	2.80	3.15	3.50
22 "	0.37	0.73	1.10	1.47	1.83	2.20	2.53	2.93	3.30	3.67
23 "	0.38	0.77	1.15	1.53	1.92	2.30	2.68	3.07	3.45	3.83
24 "	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20	3.60	4.00
25 "	0.42	0.83	1.25	1.67	2.08	2.50	2.91	3.33	3.75	4.17
26 "	0.43	0.87	1.30	1.73	2.17	2.60	3.03	3.47	3.90	4.33
27 "	0.45	0.90	1.35	1.80	2.25	2.70	3.13	3.60	4.05	4.50
28 "	0.47	0.93	1.40	1.87	2.33	2.80	3.26	3.73	4.20	4.67
29 "	0.48	0.97	1.45	1.93	2.42	2.90	3.38	3.87	4.35	4.83
30 "	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
1 month.	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
2 months	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
3 "	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.40	15.00
4 "	2.00	4.00	6.00	8.00	10.00	12.00	14.00	16.00	18.00	20.00
5 "	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00	22.50	25.00
6 "	3.00	6.00	9.00	12.00	15.00	18.00	21.00	24.00	27.00	30.00
7 "	3.50	7.00	10.50	14.00	17.50	21.00	24.50	27.00	31.50	35.00
8 "	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00
9 "	4.50	9.00	13.50	18.00	22.50	27.00	31.50	36.00	40.50	45.00
10 "	5.00	10.00	15.00	20.00	25.00	30.00	35.00	40.00	45.00	50.00
11 "	5.50	11.00	16.50	22.00	27.50	33.00	38.50	44.00	49.50	55.00
1 year.	6.00	12.00	18.00	24.00	30.00	36.00	42.00	48.00	54.00	60.00

Example.—The interest of $700, at 6 per cent. per annum, for five months, is $17.50. The interest on $70 in the same time is $1.75, and for $7, 17½ cents. For $7000 the interest is $175 in five months. Thus, the six per cent. interest for any sum and time can be found by this table.

INTEREST LAWS OF ALL THE STATES.

States and Territories.	Penalty of Usury.	Legal.	Special.
Alabama	Loss of interest	8	
Arizona	No penalty	10	No limit.
Arkansas	" "	6	" "
California	" "	10	" "
Colorado	" "	10	" "
Connecticut	Forfeiture of all interest	7	" "
Dakota	" of contract	7	18 per ct.
Delaware	" " "	6	6 per ct.
Dist. of Columbia	" of all interest	6	10 per ct.
Florida	No penalty	8	No limit.
Georgia	Forfeiture of excess	7	12 per ct.
Idaho	$300 fine or imprisonment 6 ms. or both*	10	24 per ct.
Illinois	Forfeiture of all interest	6	10 per ct.
Indiana	" of interest and costs	6	10 per ct.
Iowa	" of excess	6	10 per ct.
Kansas	" " over 12 per ct.	7	12 per ct.
Kentucky	" of all interest	6	10 per ct.
Louisiana	" of interest	5	8 per ct.
Maine	No penalty	6	No limit.
Maryland	Forfeiture of excess	6	6 per ct.
Massachusetts	No penalty—6 per ct. on judgments	6	No limit.
Michigan	Forfeiture of excess	7	10 per ct.
Minnesota	" " " over 7 per ct	7	12 per ct.
Mississippi	Forfeiture of all interest	6	10 per ct.
Missouri	" " "	6	10 per ct.
Montana	No penalty	10	
Nebraska	Forfeiture of all interest and costs	10	12 per ct.
Nevada	No penalty	10	No limit.
New Hampshire	Forfeit of three times interest received	6	6 per ct.
New Jersey	Forfeit of all interest	7	7 per ct.
New Mexico	No penalty	6	12 per ct.
New York	Forfeiture of contract †	6	6 per ct.
North Carolina	Forfeiture of interest	6	8 per ct.
Ohio	" of excess	6	8 per ct.
Oregon	" of principal, int., and costs	10	12 per ct.
Pennsylvania	" of excess; Act of 1858	6	6 per ct.
Rhode Island	" unless by contract ‡	6	No limit.
South Carolina	No penalty	10	" "
Tennessee	Forfeit of over 6 per ct. and $100 fine	6	10 per ct.
Texas	No penalty	8	No limit.
Utah	" "	10	" "
Vermont	Forfeit of excess on R. R. bonds	6	7 per ct.
Virginia	" of contract	6	12 per ct.
Washington	No penalty	10	No limit.
West Virginia	Forfeit of excess	6	6 per ct.
Wisconsin	" of all interest	7	10 per ct.
Wyoming	No penalty	10	No limit.

* Liable to arrest for misdemeanor. † Also punishable as a misdemeanor. Banks forfeit interest only, or double the interest if charged in advance. ‡ Also 6 per ct. on judgments.

Par Value being $100, Bearing Interest at

Price paid.	4 per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	10 per ct.
$50	8.00	10.00	12.00	14.00	16.00	18.00	20.00
55	7.28	9.09	10.90	12.72	14.55	16.36	18.18
60	6.66	8.33	10.00	11.66	13.33	14.99	16.66
65	6.15	7.69	9.23	10.76	12.30	13.85	15.38
70	5.71	7.14	8.57	10.00	11.42	12.85	14.28
75	5.33	6.66	8.00	9.33	10.66	12.00	13.35
80	5.00	6.25	7.50	8.75	10.00	11.25	12.50
82½	4.85	6.06	7.27	8.48	9.69	10.91	12.12
85	4.71	5.88	7.05	8.23	9.41	10.58	11.76
87½	4.57	5.71	6.85	8.00	9.14	10.28	11.42
90	4.44	5.55	6.66	7.77	8.88	10.00	11.11
92½	4.32	5.40	6.48	7.56	8.64	9.72	10.80
95	4.21	5.26	6.31	7.36	8.42	9.47	10.52
96	4.16	5.20	6.25	7.29	8.33	9.37	10.41
97	4.12	5.15	6.16	7.21	8.24	9.27	10.30
97½	4.10	5.12	6.15	7.17	8.20	9.22	10.25
98	4.08	5.10	6.12	7.14	8.16	9.18	10.20
99	4.04	5.05	6.06	7.07	8.08	9.09	10.10
100	4.00	5.00	6.00	7.00	8.00	9.00	10.00
101	3.96	4.95	5.94	6.93	7.92	8.91	9.90
102	3.92	4.90	5.88	6.86	7.84	8.82	9.80
103	3.88	4.85	5.82	6.79	7.76	8.73	9.70
104	3.84	4.80	5.76	6.73	7.69	8.65	9.61
105	3.80	4.76	5.71	6.66	7.61	8.57	9.52
110	3.63	4.54	5.45	6.36	7.27	8.18	9.09
115	3.47	4.34	5.21	6.08	6.95	7.82	8.69
120	3.33	4.16	5.00	5.83	6.66	7.50	8.33
125	3.20	4.00	4.80	5.60	6.40	7.20	8.00
130	3.07	3.84	4.61	5.38	6.15	6.92	7.69
135	2.96	3.70	4.44	5.18	5.92	6.66	7.40
140	2.86	3.57	4.28	5.00	5.71	6.43	7.14
145	2.76	3.44	4.13	4.82	5.51	6.20	6.89
150	2.66	3.33	4.00	4.66	5.33	6.00	6.66
155	2.58	3.23	3.87	4.52	5.17	5.80	6.45
160	2.25	3.13	3.75	4.38	5.00	5.62	6.25
165	2.42	3.03	3.63	4.24	4.85	5.45	6.06
170	2.35	2.95	3.53	4.12	4.71	5.30	5.88
175	2.28	2.86	3.42	4.00	4.57	5.14	5.71
180	2.22	2.78	3.33	3.89	4.45	5.00	5.55
185	2.16	2.70	3.24	3.79	4.33	4.86	5.40
190	2.11	2.64	3.16	3.69	4.22	4.73	5.26
195	2.05	2.57	3.08	3.60	4.11	4.62	5.13
200	2.00	2.50	3.00	3.50	4.00	4.50	5.00

DISCOUNT.

Discount is interest on money which is paid before due. a = amount of money to be paid at the time t. By agreement the amount is paid with a capital c, at the beginning of the time t, but discounted a Rebate r, at p per cent., so that the interest on the capital c, at p per cent., should be equal to the Rebate r, in the time t.

$$a = c + r.$$

Discount. $r = \dfrac{a\,p\,t}{100 + pt}$. . . . 5.

Capital, $c = \dfrac{100a}{100 + p\,t}$. . . . 6.

Per cent., $p = \dfrac{100(a - c)}{c\,t}$. . . 7.

Time, $t = \dfrac{100(a - c)}{c\,p}$. . . . 8.

Amount, $a = \dfrac{c}{100}(100 + p\,t)$. . . 9.

Amount, $a = \dfrac{r}{p\,t}(100 + p\,t)$. . . 10.

Now, for any question in Rebate or Discount, there is one equation that will give the answer.

Example 5. A sum of money, $a = 78460$ dollars, is to be paid after 3 years and 6 months, but by agreement payment is to be made at the present time. What will be the Rebate, at 7 per cent.?

Amount of discount, $r = \dfrac{78460 \times 7 \times 3.5}{100 + 7 \times 3.5} = \$15439.91.$

PARTNERSHIP.

Partnership or **Fellowship** is a rule by which companies ascertain each partner's profit or loss by their stock. Each partner's part in the stock is called his *share*. The sum of shares is called the *stock*.

Partnerships are of two kinds, *Simple* and *Double*.

Simple Partnership, when there is no regard to the time the shares or stock is employed.

Letters denote,

A = *share* of either one.
a = profit or loss on the share A.
S = stock or the sum of the shares.
s = gain or loss on the stock S.

Then, $A : a = S : s.$

Stock, $S = \dfrac{A\,s}{a}$. . . . 11.

Gain or loss, $s = \dfrac{a\,S}{A}$. . . . 12.

Share, $A = \dfrac{a\,S}{s}$. . . . 13.

Profit or loss, $a = \dfrac{A\,s}{S}$. . . . 14.

Example 1. A person had invested $A = \$11645$ in a stock $S = \$64800$, which gave a gain of $s = \$13864$. What will be the profit of the person's share?

Profit, $a = \dfrac{11645 \times 13864}{64800} = \$2491.45.$

When the different shares are employed at a different length of time, each share is multiplied by its time employed, and the product is the *effect* of the share.

Letters denote,

t = time for the employed share A.
T = mean time for the employed stock S.
e = effect of the share A.
a = profit of the effect e.
E = effect of the stock.
s = gain of the effect E.

Then, $e : a = E : s.$

Effect of A,	$e=\frac{a\,E}{s}$.	15.	*Time,*	$t=\frac{a\,E}{A\,s}$.	19.
Profit of e,	$a=\frac{e\,s}{E}$.	16.	*Share,*	$A=\frac{a\,E}{t\,s}$.	20.
Effect of S,	$E=\frac{e\,s}{a}$.	17.	*Meantime,*	$T=\frac{e\,s}{a\,S}$.	21.
Gain of E,	$s=\frac{a\,E}{e}$.	18.	*Stock,*	$S=\frac{e\,s}{a\,T}$.	22,

Example 2. A canal is to be dug, and requires an effect $E=76850$ (men and days) to be accomplished; after that it will give a gain $s=12390$ dollars. An employer has $A=168$ laborers. How many days must those laborers be employed at the canal, that the employer will obtain a profit $a=5000$ dollars?

Time, $$t=\frac{5000\times76850}{168\times12390}=184.6 \text{ days.}$$

PERMUTATION.

Permutation is to arrange a number of things in every possible position. It is commonly used in *games*.

Example 1. How many different values can be written by the three figures 1, 2, 3.

$1\times2\times3=6$ different values, namely,

123, 132, 213, 231, 312, 321.

With any three different figures can be written six different values. Any three things can be placed in 6 different positions.

Example 2. How many names can be written by the three syllables, *mo, ta, la?* The answer is, Motala, Molata, Tamola, Talamo, Lamota, Latamo.

Example 3. How many words can be written by the five syllables, *mul, tip, li, ca, tion?*

$1\times2\times3\times4\times5=120$ words, the answer.

COMBINATION.

Combination is to arrange a less number of things out of a greater in every possible position. It is commonly used in *games*.

Example 1. How many different numbers can be set up by the nine figures, 1, 2, 3, 4, 5, 6, 7, 8, 9, and three figures in each number?

$$\frac{9\times8\times7}{1\times2\times3}=84 \text{ different numbers.}$$

Example 2. How many different variations can a player obtain his cards, when the set contains 52 cards, of which he receives 8 at a time?

$$\frac{52\times51\times50\times49\times48\times47\times46\times45}{1\times2\times3\times4\times5\times6\times7\times8}=752538150 \text{ variations.}$$

If there are four players, and $p\,r.\;4=24$, they can play $24\times752538150=$ 18,060,915,600 different plays.

If it takes half an hour for each play, and they play 8 hours per day, it will take

$$\frac{18060915600}{2\times8}=1128807225 \text{ days}=3092622 \text{ years.}$$

ARITHMETICAL PROGRESSION.

Arithmetical Progression is a series of numbers, as 2, 4, 6, 8, 10, 12, &c., or 18, 15, 12, 9, 6, 3, in which every successive term is increased or diminished by a constant number.

Letters denote,

a = the first term of the series.
b = any other term whose number from a is n.
n = number of terms within a and b.
δ = the difference between the terms.
S = the sum of all the terms.

In the series, 2, 5, 8, 11, $a = 2$, $b = 11$, $n = 4$, $\delta = 3$, and $S = 26$.

☞ When the series is decreasing, take the first term $= b$ and the last term $= a$.

The accompanying Table contains all the formulas or questions in Arithmetical Progressions, and the nature of the question will tell which formula is to be used.

Formulas for Arithmetical Progressions.

$$a = b - \delta\,(n-1), \qquad 1,$$

$$a = \frac{2S}{n} - b, \qquad 2,$$

$$a = \frac{S}{n} - \frac{\delta}{2}(n-1), \qquad 3,$$

$$b = a + \delta\,(n-1), \qquad 4,$$

$$b = \frac{2S}{n} - a, \qquad 5,$$

$$b = \frac{S}{n} + \frac{\delta}{2}(n-1), \qquad 6,$$

$$n = \frac{b-a}{\delta} + 1, \qquad 7,$$

$$n = \frac{2S}{a+b}, \qquad 8,$$

$$\delta = \frac{b-a}{n-1}, \qquad 9,$$

$$\delta = \frac{(b+a)(b-a)}{2S-a-b}, \qquad 10,$$

$$\delta = \frac{2\,(S-an)}{n\,(n-1)}, \qquad 11,$$

$$\delta = \frac{2\,(bn-S)}{n\,(n-1)}, \qquad 12,$$

$$S = \frac{n\,(a+b)}{2}, \qquad 13,$$

$$S = \frac{(a+b)(b+\delta-a)}{2\delta}, \qquad 14,$$

$$S = n\left[a + \frac{\delta}{2}\,(n-1)\right] \qquad 15,$$

$$S = n\left[b - \frac{\delta}{2}(n-1)\right] \qquad 16,$$

$$a = \frac{\delta}{2} \pm \sqrt{\left(b + \frac{\delta}{2}\right)^2 - 2\delta S}, \qquad 17,$$

$$b = -\frac{\delta}{2} \pm \sqrt{\left(a - \frac{\delta}{2}\right)^2 + 2\delta S}, \qquad 18,$$

$$n = \frac{1}{2} - \frac{a}{\delta} \pm \sqrt{\left(\frac{1}{2} - \frac{a}{\delta}\right)^2 + \frac{2S}{\delta}}, \qquad 19,$$

$$n = \frac{1}{2} + \frac{b}{\delta} \pm \sqrt{\left(\frac{1}{2} + \frac{b}{\delta}\right)^2 - \frac{2S}{\delta}}, \qquad 20.$$

Examples in Arithmetical Progression.

The preceding table furnishes a means for practicing the insertion of numerical values in algebraical formulas which illustrate the different arithmetical operations.

Example 1. An arithmetical progression of $n=20$ terms has a difference of $\delta=2$ between each term, and the last term is $b=166$. Required the value of the first a?

Find in the table the formula containing the given quantities, and of which the first member is a. Formula 1 corresponds to this.

The first term $a=b-\delta(n-1)=166-2(20-1)=166-2\times 19=166-38=128$, which is the required first term.

The calculation is given in detail merely for illustration, but in practice we write only

$$a=166-2(20-1)=128.$$

Example 2. The sum of $n=9$ terms in an arithmetical progression is $S=1787$, and the last term is $b=360$. Required the value of the first term a?

$$a=\frac{2S}{n}-b=\frac{2\times 1787}{9}-360=37.1.$$

Example 9. Required the difference δ in the progression of example 2?

$$\delta=\frac{b-a}{n-1}=\frac{360-37.1}{9-1}=\frac{322.9}{8}=40.3625.$$

Example 7. In an arithmetical progression the first term is $a=85$, the last term $b=163$, and the difference between each term is $\delta=17$. Required the number of terms n?

Find the proper formula in the table; insert the given quantities and perform the calculation, which will give $n=6$.

Example 8. The first term of a progression is $a=44$, and the last term $b=256$. How many terms must be inserted between a and b, inclusive, in order that the sum of all the terms shall be $S=1800$?

Find the proper formula in the table; insert the given quantities, and the calculation will give $n=12$ terms.

Example 15. A man was engaged to dig a well at one dollar ($1) for the first foot of the depth of the well, $1.84 for the second, and 84 cents more per every successive foot in depth until he reached the water, which was found at a depth of 25 feet. How much money is due to the man?

This will be answered by the formula 15, in which $a=1$, $d=0.84$, and $n=25$; then the sum,

$$S=25\left[1+\frac{0.84}{2}(25-1)\right]=\$277 \text{ the answer.}$$

Example 20. A ship cost $116,500, of which the company agreed to pay on account $14,075 on her first trip; and for every successive trip they paid $650 less than the former. How many trips must the vessel make till fully paid for?

This will be answered by the formula 20, in which $b=\$14,075$, $\delta=650$, and $S=116,500$.

$$n=\frac{1}{2}+\frac{14075}{650}-\sqrt{\left(\frac{14075}{650}+\frac{1}{2}\right)^2-\frac{2\times 116500}{650}}=10.6 \text{ or } 11 \text{ trips.}$$

Arithmetical Progressions of a Higher Order.

Arithmetical Progressions are of the first order, when the difference δ is a constant number, but when the difference δ progresses itself with a constant number, the Progression is of the *second order*.

When the difference δ progresses in a second order, the Progression is of the *third order*, &c., &c., and is thus explained:

1, 2, 3, 4, 5, 6, ... n, . . Arith. Prog., first order.

1, 3, 6, 10, 15, 21, . . . $\dfrac{n(n+1)}{2}$ . . . . 2d. order.

1, 4, 10, 20, 35, 56, . . $\dfrac{n(n+1)(n+2)}{2\times3}$, ... 3d. order.

1, 5, 15, 35, 70, 126, . $\dfrac{n(n+1)(n+2)(n+3)}{2\times3\times4}$, . . . 4th. order.

Here you will discover that the sum of n terms in one order, is equal to the same nth term in the next higher order. Arithmetical Progressions of the *first*, *second*, and *third orders*, are applied to

PILES OF BALLS AND SHELLS.

Triangular Piling.

Example 1. A complete triangular pile of balls has $n = 12$ balls in each side. Require how many balls in the base, and how many in the whole pile?

In the base, . $= \dfrac{12(12+1)}{2} = 78$ balls, . . . 2d. order.

Whole pile, . . $= \dfrac{12(12+1)(12+2)}{2\times3} = 364$ balls, . . 3d. order.

Square Piling.

1, 4, 9, 16, 25, 36, ... n^2 . . . . . . 2d. order.

1, 5, 14, 30, 55, 91, . . $\dfrac{n(n+1)(2n+1)}{2\times3}$, . . . 3d. order.

Example 2. How many balls are contained in a complete square pile, $n = 10$ rows?

$$\frac{10(10+1)(2\times10+1)}{2\times3} = \frac{10\times11\times21}{6} = 385 \text{ balls.}$$

Rectangular Piling.

Let m be the number of balls on the top of the complete pile, and n = number of rows in the same, then the number of balls in the whole pile will be,

$$\frac{n(n+1)(2n+3m-2)}{2\times3}, \quad . . . . \text{ 3d. order.}$$

The number of balls in the longest bottom side will be $= m+n-1$.

Example 3. The rectangular pile having 15 rows and 23 balls on the the top, how many in the whole pile?

$$\frac{15(15+1)(2\times15+3\times23-2)}{2\times3} = \frac{15\times16\times67}{6} = 2680 \text{ balls.}$$

ALLIGATION.

Alligation is to mix together a number of different things of different price or value, and ascertain the mean value of the mixture; or from a given mean value of a mixture ascertain the proportion and value of each ingredient.

Let the different things be a, b, c and d, etc., their respective price or value per unit, z, y, x and w, etc.

$A = a + b + c + d$, etc., the sum of the things.
$P =$ mean value or price per unit of A.

Then, $$A\,P = a\,z + b\,y + c\,x + d\,w +, \text{etc.} \qquad 1.$$

and $$P = \frac{a\,z + b\,y + c\,x + d\,w +}{A}, \text{etc.} \qquad 2.$$

Example 1. If 3 gallons of wine, at \$1.37 per gallon, 2, at \$2.18, and 5, at \$1.75, be mixed together, what is a gallon of the mixture worth?

$$A = 3 + 2 + 5 = 10 \text{ gallons.}$$
$$P = \frac{3 \times 1.37 + 2 \times 2.18 + 5 \times 1.75}{10} = \$1.72 \text{ per gallon.}$$

Alligation of two ingredients, a and b, with their respective prices or value per unit, z and y. $z > P > y$. $A = a + b$.

$$a : b = (P - y) : (z - P). \qquad 3.$$

$$a = \frac{b(P - y)}{(z = P)}, \quad \text{and} \quad a = \frac{A(P - y)}{(z - y)} \qquad 4\ \&\ 5.$$

Example 2. A silversmith mixes two sorts of silver, one at 54 and one at 64 cents per ounce. How much must be taken of each sort to make the mixture worth 60 cents per ounce? (Formula 3.) $P = 60$. $x = 54$. $y = 64$.

$$a : b = (60 - 54) : (64 - 60) = 6 : 4, \text{ or}$$

4 ounces, at 54 cents, and 6 ounces, at 64 cents.

Alligation of three ingredients, a, b and c, with their prices or value per unit, z, y and x.

$$a' : c' = (P - x) : (z - P). \qquad 6$$
$$a'' : b = (P - y) : (z - P) \text{ when } z > P > y > x. \qquad 7.$$
$$b : c'' = (P - x) : (y - P) \text{ when } z > y > P > x. \qquad 8.$$
$$a = a' + a'', \quad c = c' + c''.$$

Example 3. A farmer mixes wheat, at 94 cents per bushel, with barley, at 72 cents, rye, at 64 cents per bushel. How much of each sort must be taken to make the mixture worth 80 cents per bushel?

(Formula 6.) $z = 94$, $y = 72$, $x = 64$, and $P = 80$.

$$a' : c' = (80 - 64) : (94 - 80) = 16 : 14.$$
$$a' : b = (80 - 72) : (94 - 80) = 8 : 14.$$

The wheat $a = 16 + 8 = 24$ bushels, at 94 cents per bushel.
" barley $b = 14$ " " 72 " "
" rye $c = 14$ " " 64 " "

Alligation of four ingredients a, b, c and d, respective prices or value per unit, z, y, x and w.

$$\left.\begin{aligned} a : d &= (P - w) : (z - P) \\ b : c &= (P - y) : (x - P) \end{aligned}\right\} \text{when } z > y > P > x > w > \qquad \left\{\begin{aligned} &9. \\ &10. \end{aligned}\right.$$

$$\left.\begin{aligned} a' : d &= (P - w) : (z - P) \\ a'' : b &= (P - y) : (z - P) \\ a''' : c &= (P - x) : (z - P) \end{aligned}\right\} \text{when } z > P > y > x > w \qquad \left\{\begin{aligned} &11. \\ &12. \\ &13. \end{aligned}\right.$$
$$a = a' + a'' + a'''.$$

$$\left.\begin{aligned} a : d' &= (P - w) : (z - P) \\ b : d'' &= (P - w) : (y - P) \\ c : d''' &= (P - w) : (x - P) \end{aligned}\right\} \text{when } z > y > x > P > w \qquad \left\{\begin{aligned} &14. \\ &15. \\ &16. \end{aligned}\right.$$
$$d = d' + d'' + d'''.$$

In the same manner, formulæ can be set up for any number of ingredients.

GEOMETRICAL PROGRESSION.

Geometrical Progression is a series of numbers, as 2 : 4 : 8 : 16 : 32 : &c., or 729 : 243 : 81 : 27 : 9 : &c., in which every successive term is multiplied or divided by a constant factor.

Notation.

a = the first term of the series.
b = any other term whose number from a is n.
n = number of terms within a and b.
r = ratio, or the factor by which the terms are multiplied or divided.
S = Sum of the terms.

In the series 1 : 3 : 9 : 27 : $a = 1$, $b = 27$, $n = 4$, $r = 3$, $S = 40$.

The accompanying Table contains all the formulas or questions in Geometrical Progressions. The nature of the question will tell which formula is to be used.

Formulas for Geometrical Progressions.

$$a = \frac{b}{r^{n-1}}, \quad \ldots \quad 1,$$

$$a = S - r(S - b), \quad \ldots \quad 2,$$

$$a = S\frac{r-1}{r^n-1}, \quad \ldots \quad 3,$$

$$r = \sqrt[n-1]{\frac{b}{a}}, \quad \ldots \quad 7,$$

$$r = \frac{S-a}{S-b}, \quad \ldots \quad 8,$$

$$ar^n + S - rS - a = 0, \quad \ldots \quad 9,$$

$$b = ar^{n-1}, \quad \ldots \quad 4,$$

$$b = S - \frac{S-a}{r}, \quad \ldots \quad 5,$$

$$b = S\left(\frac{r-1}{r^n-1}\right)r^{n-1}, \quad 6,$$

$$S = \frac{br-a}{r-1}, \quad \ldots \quad 10,$$

$$S = \frac{a(r^n-1)}{r-1}, \quad \ldots \quad 11,$$

$$S = \frac{b(r^n-1)}{(r-1)r^{n-1}}, \quad \ldots \quad 12,$$

$$n = 1 + \frac{\log.b - \log.a}{\log.r}, \quad \ldots \quad 13,$$

$$n = 1 + \frac{\log.b - \log.a}{\log.(S-a) - \log.(S-b)}, \quad \ldots \quad 14,$$

$$n = \frac{\log.[a + S(r-1)] - \log.a}{\log.r.}, \quad \ldots \quad 15,$$

$$n = 1 + \frac{\log.b - \log.[br - S(r-1)]}{\log.r.}, \quad \ldots \quad 16,$$

$$S = \frac{b\sqrt[n-1]{b} - a\sqrt[n-1]{a}}{\sqrt[n-1]{b} - \sqrt[n-1]{a}}, \quad \ldots \quad 17,$$

Example 1. Required the 10th term in the Geometrical Progression 4 : 12 : 36.... ?
Given $a = 4$, $n = 10$, and $r = 3$. We have,
Formula 4. $b = ar^{n-1} = 4 \times 3^9 = 78732$, the tenth term.

Example 2. Required the sum of the 10 terms in the preceding example?

Formula 11, $S = \frac{a(r^n - 1)}{r - 1} = \frac{4(3^{10} - 1)}{2} = 118096$, the sum.

Example 3. Insert 6 proportional terms between 3 and 384?
Given $a = 3$, $b = 384$, and $n = 6+2 = 8$.

Formula 7, $r = \sqrt[n-1]{\frac{b}{a}} = \sqrt[7]{\frac{384}{3}} = 2,$

then 3 : 6 : 12 : 24 : 48 : 96 : 192 : 384, the answer.

Example 4. A man had 16 twenty dollar gold pieces, which he agreed to exchange for copper in such a way, that he gets one cent on the first $20, two on the second, four on the third, and eight on the fourth, &c., &c.; until the sixteen $20 pieces were covered. How many cents will come on the sixteenth gold piece, and what will be the whole amount of copper on the gold?

In the progression 1 : 2 : 4 : 8 : &c., we have,
Given $n = 16$, $r = 2$, and $a = 1$, then,

Formula 4. $b = 1 \times 2^{16-1} = \frac{2^{16}}{2} = \frac{4^8}{2} = \frac{16^4}{2} = \frac{256^2}{2} = 32768$ cents, on the sixteenth piece.

The total sum of cents will be found by the

Formula 10. $S = \frac{32768 \times 2 - 1}{2 - 1} = 65535$ cents $= \$655{\cdot}35$.

Table of Geometrical Progression.

The ratio $r = 2$.

1	1	16	32768	31	1073741824	46	35184372088832
2	2	17	65536	32	2147483648	47	70368744177664
3	4	18	131072	33	4294967296	48	140737488355328
4	8	19	262144	34	8589934592	49	281474976710656
5	16	20	524288	35	17179869184	50	562949953421312
6	32	21	1048576	36	34359738368	51	1125899906842624
7	64	22	2097152	37	68719476736	52	2251799813685248
8	128	23	4194304	38	137438953472	53	4503599627370496
9	256	24	8388608	39	274877906944	54	9007199254740992
10	512	25	16777216	40	549755813888	55	18014398509481984
11	1024	26	33554432	41	1099511627776	56	36028797018963968
12	2048	27	67108864	42	2199023255552	57	72057594037927936
13	4096	28	134217728	43	4398046511104	58	144115188075855872
14	8192	29	268435456	44	8796093022208	59	290790376151711744
15	16384	30	536870912	45	17592186044416	60	581580752303423488

Any power of 2 can be found by this table, up to the 60th power.

Example. The 10th power of 2, or 2^{10}, = 1024. $2^8 = 256$.
The 9th root of 512 = 2. The 20th root of 1048576 = 2.

COMPOUND INTEREST.

Compound Interest is when the interest is added to the capital for each year, and the sum is the capital for the following year.

Amount, $a = c(1+p)^n$. . . 1.	*Percentage*, $p = \sqrt[n]{\frac{a}{c}} - 1$. . 3.
Capital, $c = \frac{a}{(1+p)^n}$. . . 2.	*Number of years*, $n = \frac{log.a - log.c}{log.(1+p)}$. 4.

☞ In these formulas p must be expressed in hundredths.

Example 1. A capital $c = 8650$ standing with compound interest, at $p = 5$ per cent. What will it amount to in $n = 9$ years?

Amount $a = 8650\ (1.05)^9 = 13419$ dollars.

Example 2. A man commenced business with $c = 300$ dollars; after $n = 5$ years he had $a = 6875$ dollars. At what rate did his money increase, and how soon will he have a fortune of 50000 dollars?

The first question, or the percentage, will be answered by the formula 3.

$$p = \sqrt[5]{\frac{6875}{300}} - 1 = \sqrt[5]{22.9166} - 1 = 0.87, \text{ or } 87 \text{ per cent.}$$

The time from the commencement of business until the fortune is completed will be answered from the formula 4.

$$n = \frac{log.50000 - log.300}{log.1.87} = \frac{4.69897 - 2.47712}{0.2720048} = 8.169 \text{ years,}$$

or 8 years and 2 months.

Compound Interest Table, CALCULATED FROM FORMULA 1.

n	COMPOUND INTEREST.		
Years.	5 per ct.	6 per ct.	7 per ct.
1	1.0500	1.0600	1.0700
2	1.1025	1.1236	1.1449
3	1.1576	1.1910	1.2250
4	1.2155	1.2625	1.3108
5	1.2770	1.3382	1.4025
6	1.3400	1.4185	1.5007
7	1.4071	1.5036	1.6058
8	1.4774	1.5938	1.7182
9	1.5513	1.6895	1.8385
10	1.6289	1.7908	1.9671
11	1.7103	1.8983	2.1048
12	1.7958	2.0122	2.2522
13	1.8856	2.1329	2.4098
14	1.9799	2.2609	2.5785
15	2.0789	2.3965	2.7599
16	2.1829	2.5403	2.9522
17	2.2920	2.6928	3.1588
18	2.4066	2.8543	3.3799
19	2.5269	3.0256	3.6165
20	2.6533	3.2071	3.8697
21	2.7859	3.3995	4.1406
22	2.9252	3.6035	4.4304
23	3.0715	3.8197	4.7405
24	3.2251	4.0487	5.0724
25	3.3864	4.2919	5.4274
30	4.3219	5.7435	7.6123
35	5.5166	7.6861	10.6766
40	7.0400	10.2858	14.9745
45	8.9850	13.7646	21.0025
50	11.6792	18.4190	29 4570
60	18.6792	32.9878	57.9466

This table shows the value of one unit of money at the rates of 5, 6 and 7 per cent. per annum, compound interest, up to 60 years.

Example 1. What is the amount of 864 pounds sterling for 12 years, at 6 per cent. compound interest?

Table, $2.01219 \times 864 = 1738.53216$, or £1738 10 s. 7.7 d.

Example 2. What is the amount of 3450 dollars for 18 years, at 5 per cent. compound interest?

Table, $2.40661 \times 3450 = 8302.80$ dollars.

When the interest is compounded in more or less than one year, at the rate of interest per year, and m = the number of months in which the interest is compounded;

Then, instead of p in the formulas, put $\frac{m\ p}{12}$, and instead of n, put $\frac{12n}{m}$.

Example 3. A capital of 500 dollars bears compound interest semi-annually, at 5 per cent. per annum; what will it amount to in 10 years?

$m = 6$ months, $p = \frac{m\ p}{12} = \frac{6 \times 0.05}{12} = 0.025$,

and $n = \frac{12 \times 10}{6} = 20$,

then, $a = c(1+p)^n = 500(1 + 0.025)^{20} = 8193.11$ dollars, the answer.

	log.(1 + 0.025) =	0.0107239
		20
		0.2144780
	log. 500 =	2.6989700
Amount,	8193.11 =	2.9134480

ANNUITIES.

Annuity is a certain sum of money to be paid at regular intervals.

A yearly payment or annuity b is standing for n years; to find the whole amount a, at p per cent. interest.

Amount, $$a = b\,n\left[1 + \frac{p}{2}(n+1)\right] \text{ Simple Int.} \quad . \quad . \quad . \quad . \quad 1.$$

Amount, $$a = \frac{b}{p}\left[(1+p)^n - 1\right] \text{ Comp. Int.} \quad . \quad . \quad . \quad . \quad 2.$$

A yearly payment or annuity b is to be paid for n years; to find the present worth, or the amount a, which would pay it in full at the beginning of the time n, deducting p per cent. interest.

Amount, $$a = bn\left[1 - \frac{p}{2}\left(\frac{n+1}{1+pn}\right)\right] \text{ Simple Int.} \quad . \quad . \quad . \quad 3.$$

Amount, $$a = \frac{b}{p}\left[1 - \frac{1}{(1+p)^n}\right] \text{ Comp. Int.} \quad . \quad . \quad . \quad . \quad 4.$$

A debt D, standing for interest, is diminished yearly by a sum b; to find the debt d after n years, and the time n when it is fully paid?

The debt d after n years will be—

$$d = \frac{(Dp - b)(1+p)^n + b}{p} \text{ Comp. Int.} \quad . \quad . \quad . \quad 5.$$

The time n until fully paid will be—

$$n = \frac{log.b - log.(b - Dp)}{log.(1+p)}. \quad . \quad . \quad . \quad . \quad . \quad 6.$$

If $b = Dp$, then $n = \infty$, or the debt D will never be paid. If $b < Dp$, the debt D will be increased.

To find the yearly annuity b which will pay a debt D in n years, at p per cent. compound interest?

$$b = \frac{Dp(1+p)^n}{(1+p)^n - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad 7.$$

Annuity Table,

Showing the present worth of an annuity or rent of one unit of money. at 5, 6 and 7 per cent. compound interest for years up to 60, calculated from formula 4.

Years.	5 per ct.	6 per ct.	7 per ct.	Years.	5 per ct.	6 per ct.	7 per ct.
1	0.9524	0.9434	0.9345	17	11.2741	10.4772	9.7632
2	1.8594	1.8333	1.8080	18	11.6896	10.8276	10.0591
3	2.7232	2.6730	2.6243	19	12.0853	11.1581	10.3356
4	3.5459	3.4651	3.3872	20	12.4622	11.4699	10.5940
5	4.3295	4.2123	4.1001	21	12.8211	11.7641	10.8355
6	5.0757	4.9173	4.7665	22	13.1630	12.0416	11.0612
7	5.7864	5.5824	5.3892	23	13.4881	12.3034	11.2722
8	6.4632	6.2098	5.9712	24	13.7986	12.5503	11.4693
9	7.1078	6.8017	6.5152	25	14.0939	12.7833	11.6536
10	7.7217	7.3601	7.0235	30	15.3724	13.7648	12,4090
11	8.3064	7.8868	7.4986	35	16.3742	14.4982	12.9476
12	8.8632	8.3838	7.9426	40	17.1591	15.0463	13,3317
13	9.3936	8.8527	8.3576	45	17.7741	15.4558	13.6055
14	9.8986	9.2950	8.7454	50	18.2559	15.7618	13.8007
15	10.3796	9.7122	9.1079	55	18.6334	15.9905	13.9399
16	10.8378	10.1059	9.4466	60	18,9292	16.1614	14.0389

UNITED STATES' STANDARD MEASURES AND WEIGHTS.

MEASURE OF LENGTH.

THE *Standard Measure of Length* is a brass rod = 1 yard at the temperature of 32° Fahrenheit. The length of a pendulum vibrating seconds in vacuo, at Philadelphia is 1·08614 yards, at + 32° Fahrenheit.

The **Surveying Chain** is = 22 yards = 66 feet. It consists of 100 links, and each link = 7·92 inches.

ROPES AND CABLES.

1 Cable length = 120 fathoms = 720 feet.
1 fathom = 6 feet.

GEOGRAPHICAL AND NAUTICAL MEASURES.

1 Degree of the great circle of the Earth round the Equator = 69·032 statute miles = 60 Nautical miles.
1 Statute mile = 5280 feet = 0·86875 Nautical miles.
1 Nautical mile = 6037·424 = 1·150 Statute miles.

LOG LINE.

The **Log Line** should be about 150 fathoms long, and 10 fathoms from the Log to the first knot on the line. If half a minute glass is used, it will be 51 feet between each succeeding knot. For 28 seconds glass it will be 47·6 feet = 7·93 fathoms per knot. This is the length of knot by calculation, but practically it is shortened to 7·5 fathoms per knot for 28 seconds glass.

MEASURE OF CAPACITY.

Gallon. The standard Gallon measures 231 cubic inches, and contains 8·3388822 pounds Avoirdupois = 58372·1757 grains Troy, of distilled water, at its maximum density 39·83° Fahrenheit, and 30 inches barometer height.

Bushel. The standard Bushel measures 2150·42 cubic inches = 77·627413 pounds Avoirdupois of distilled water at 39·83° Fahrenheit, barometer 30 inches. Its dimensions are 18½ inches inside diameter, 19½ inches outside, and 8 inches deep; and when heaped, the cone must not be less than 6 inches high, equal 2747·70 cubic inches for a true cone.

Pound. The standard Pound Avoirdupois is the weight of 27·7015 cubic inches of distilled water, at 39·83° Fahrenheit, barometer 30 inches, and weighed in the air.

MEASURE OF LENGTH.

Miles.	Furlongs.	Chains.	Rods.	Yards.	Feet.	Inches.
1	8	80	320	1760	5280	63360
0·125	1	10	40	220	660	7920
0·0125	0·1	1	4	22	66	792
0·003125	0·025	0·25	1	5.5	16·5	198
0·00056818	0·0045454	0·045454	0·181818	1	3	36
0·00018939	0·00151515	0 01515151	0·0606060	0·33333	1	12
0 000015783	0·000126262	0·00126262	0·00505050	0·0277777	0·083333	1

MEASURE OF SURFACE.

Sq. Miles.	Acres	S.Chains.	Sq. Rods.	Sq. Yards.	Sq. Feet.	Sq. Inches.
1	640	6400	102400	3097600	27878400	4014489600
0·001562	1	10	160	4840	43560	6272640
0·0001562	0·1	1	16	484	4356	627264
0·000009764	0·00625	0·0625	1	30·25	272·25	39204
0·000000323	0·0002066	0·002066	0·0330	1	9	1296
0·0000000358	0·00002296	0·0002296	0·00367	0·1111111	1	144
0·00000000025	0 000000159	0·00000159	0·00002552	20·0007716	0·006944	1

MEASURE OF CAPACITY.

Cub. Yard.	Bushel.	Cub. Feet.	Pecks.	Gallons.	Cub. inch.
1	21·6962	27	100·987	201·974	46656
0·03961	1	1·24445	4	9·30918	2150·42
0·037037	0·803564	1	3·21425	7·4805	1728
0·009259	0·25	0·31114	1	2·32729	537·605
———	0·107421	0·133681	0·429684	1	231
———	———	0·000547	0·001860	0·004329	1

MEASURE OF LIQUIDS.

Gallon.	Quarts.	Pints.	Gills.	Cub. inch.
1	4	8	32	231
0·25	1	2	8	57·75
0·125	0·5	1	4	28·875
0.03125	0·125	0·25	1	7·21875
0·004329	0·017315	0·03463	0·13858	1

MEASURES OF WEIGHTS.

AVOIRDUPOIS.

Ton.	Cwt.	Pounds.	Ounces.	Drams.
1	20	2240	35840	573440
0·05	1	112	1792	28672
0·00044642	0·0089285	1	16	256
0·00002790	0·000558	0·0625	1	16
0·00000174	0·0000348	0·0016	0·0625	1

TROY.

Pounds.	Ounces.	Dwt.	Grains.	Pound Avoir.
1	12	240	5760	0·822861
0·083333	1	20	480	0·068571
0·004166	0·05000	1	24	0·0034285
0·0001736	0·002083333	0·0416666	1	0·00014285
1·215275	14·58333	291·6666	7000	1

APOTHECARIES.

Pounds.	Ounces.	Drams.	Scruples.	Grains.
1	12	96	288	5760
0·08333	1	8	24	480
0·01041666	0·125	1	3	60
0·0034722	0·0416666	0·3333	1	20
0·00017361	0·0020833	0·016666	0·05	1

WEIGHT PER BUSHEL OF GRAIN, ETC.

THE following table shows the number of pounds per bushel required by law or custom, in the sale of articles specified, in the several States of the Union. (Official.)

STATES.	Barley.	Buckwheat.	Coal.	Corn, shelled.	Corn-meal.	Onions.	Oats.	Potatoes.	Rye.	Wheat.	Salt.	Turnips.	Beans, whole.	Clover-seed.	Timothy.
Maine	48	48	...	56	50	52	30	60	...	60	...	50	64	...	...
New Hampshire	...	...	...	56	50	...	30	60	56	60	...	...	60	...	...
Vermont	48	48	...	...	...	...	32	60	56	60	70	...	64	60	42
Massachusetts	48	48	...	56	50	52	32	60	56	60	...	...	...	...	...
Connecticut	...	45	...	56	...	...	28	60	56	56	...	...	...	...	...
New York	48	48	...	58	...	...	32	60	56	60	...	...	62	60	44
New Jersey	48	50	...	56	...	...	30	60	56	60	...	...	...	64	...
Pennsylvania	47	48	...	56	...	...	30	56	56	60	85	...	...	62	...
Delaware	...	...	...	56	...	...	...	...	...	60	...	...	...	...	...
Maryland	48	48	...	56	...	57	32	60	56	60	56	...	62	64	45
Dist. Columbia	47	48	...	56	48	57	32	56	56	60	50	55	62	60	45
Virginia	48	48	...	56	50	...	32	60	56	60	...	56	60	64	45
West Virginia	48	52	80	56	48	...	32	60	56	60	...	60	60	60	45
North Carolina	48	50	...	54	46	...	30	...	56	60	...	...	...	64	...
South Carolina	48	56	80	56	50	57	33	60	56	60	50	...	60	60	...
Georgia	40	...	80	56	48	75	35	56	...	60	56	...	...	60	45
Louisiana	32	...	...	56	...	...	32	...	32	60	...	...	...	...	...
Arkansas	48	52	80	56	50	57	32	60	56	60	50	...	60	60	45
Tennessee	48	50	...	56	50	56	32	60	56	60	...	...	60	...	45
Kentucky	48	52	...	56	50	57	33	56	56	60	50	...	60	60	45
Ohio	48	50	...	56	...	...	32	60	56	60	...	...	60	60	45
Michigan	48	48	80	56	...	54	32	60	56	60	56	58	60	60	45
Indiana	48	50	70	56	50	48	32	60	56	60	50	...	60	60	...
Illinois	48	52	...	56	48	57	32	60	56	60	50	...	60	60	...
Wisconsin	48	50	...	56	...	...	32	60	56	60	...	...	...	60	...
Minnesota	48	42	...	56	...	...	32	60	56	60	...	...	...	60	...
Iowa	48	52	...	56	...	57	33	60	56	60	50	...	60	60	45
Missouri	48	52	...	56	...	57	32	60	56	60	50	...	60	60	45
Kansas	50	50	...	56	50	57	32	60	56	60	50	55	60	...	45
Nebraska	48	52	...	56	50	57	34	60	56	60	50	55	60	60	45
California	50	40	...	52	...	...	32	...	54	60	...	...	...	...	...
Oregon	46	42	...	56	...	...	36	60	56	60	...	...	...	60	...

Year.	Month.	Week.	Day.	Year.	Month.	Week.	Day.
$100	$8.33	$1.92	$0.33	$560	$46.67	$10.56	$1.87
110	9.17	2.12	0.37	570	47.50	10.96	1.90
120	10.00	2.31	0.40	580	48.30	11.15	1.93
130	10.83	2.50	0.43	590	49.17	11.34	1.97
140	11.67	2.70	0.47	600	50.00	11.53	2.00
150	12.50	2.89	0.50	610	50.83	11.73	2.03
160	13.33	3.08	0.53	620	51.67	11.93	2.07
170	14.17	3.27	0.57	630	52.50	12.12	2.10
180	15.00	3.46	0.60	640	53.33	12.31	2.13
190	15.83	3.65	0.63	650	54.17	12.50	2.17
200	16.67	3.84	0.67	660	55.00	12.70	2.20
210	17.50	4.04	0.70	670	55.83	12.88	2.23
220	18.33	4.23	0.73	680	56.67	13.07	2.27
230	19.17	4.42	0.77	690	57.50	13.26	2.30
240	20.00	4.61	0.80	700	58.33	13.45	2.33
250	20.83	4.80	0.83	710	59.17	13.64	2.37
260	21.57	5.00	0.87	720	60.00	13.84	2.40
270	22.50	5.19	0.90	730	60.83	14.03	2.43
280	23.33	5.38	0.93	740	61.67	14.23	2.47
290	24.17	5.57	0.97	750	62.50	14.42	2.50
300	25.00	5.76	1.00	760	63.33	14.60	2.53
310	25.83	5.95	1.03	770	64.17	14.80	2.57
320	26.67	6.14	1.07	780	65.00	15.00	2.60
330	27.50	6.34	1.10	790	65.83	15.19	2.63
340	28.33	6.53	1.13	800	66.67	15.38	2.67
350	29.17	6.72	1.17	810	67.50	15.57	2.70
360	30.00	6.91	1.20	820	68.33	15.77	2.73
370	30.83	7.10	1.23	830	69.17	15.96	2.77
380	31.67	7.30	1.27	840	70.00	16.15	2.80
390	32.50	7.49	1.30	850	70.83	16.34	2.83
400	33.33	7.68	1.33	860	71.67	16.54	2.87
410	34.17	7.87	1.37	870	72.50	16.73	2.90
420	35.00	8.06	1.40	880	73.33	16.92	2.93
430	35.83	8.25	1.43	890	74.17	17.11	2.97
440	36.67	8.44	1.47	900	75.00	17.30	3.00
450	37.50	8.64	1.50	910	75.83	17.49	3.03
460	38.33	8.83	1.53	920	76.67	17.68	3.07
470	39.17	9.02	1.57	930	77.50	17.87	3.10
480	40.00	9.23	1.60	940	78.33	18.06	3.13
490	40.83	9.42	1.63	950	79.17	18.25	3.17
500	41.67	9.61	1.67	960	80.00	18.44	3.20
510	42.50	9.80	1.70	970	80.83	18.64	3.23
520	43.33	10.00	1.73	980	81.67	18.83	3.27
530	44.17	10.19	1.77	990	82.50	19.03	3.30
540	45.00	10.38	1.80	1000	83.33	19.23	3.33
550	45.83	10.57	1.83	1010	84.17	19.42	3.37

When the salary per year exceeds hundreds and is less than $10,000, then move the decimal-point one space to the right in the columns Months, Weeks, and Days; for instance, if the salary is $4200 per year, then it will be $350 per month, $80.60 per week, and $14 per day.

MONEY AND COINS OF THE UNITED STATES.

10 mills = 1 cent. | 10 dimes = 1 dollar.
10 cents = 1 dime. | 10 dollars = 1 eagle.

The standard gold and silver coins contain 900 parts of pure metal and 100 parts of base metal in 1000 parts of the alloy.

The remedy of the Mint is the allowance for deviation from the exact standard fineness and weight of coins.

The nickel cent contains 88 parts of copper and 12 of nickel.

The new bronze cent contains 95 parts of copper and 5 of tin and zinc.

Pure gold, 23.22 grains = $1, or $20.67.183 = 1 ounce.

Pure silver, 357.03 grains = $1, or $1.36.166 = 1 ounce.

Silver coins of less value than one dollar are issued at the rate of 384 grains to the dollar.

Standard alloyed gold = $18.60.465, and silver = $1.22.5 per ounce.

Gold coins.	*Grains.*	*Silver coins.*	*Grains.*	*Copper coins.*	*Grains.*
Double eagle,	516.	One dollar,	412.5	Cent (old),	168.
Eagle,	258.	Fifty cents,	192.	Cent (new),	72.
Dollar,	25.8	Twenty-five cents,	96.	Cent (bronze),	48.

For silver and gold tables see pages 000.

WEIGHT AND FINENESS OF DIFFERENT COINS, AND THEIR VALUE IN AMERICAN MONEY.

Country.	*Piece and Divisions.*	Weight. Grains.	Fineness in 1000.	Fine metal. Grains.	UNITED STATES. $ Cts.
Austria,	Crown,	171.36	900.	154.22	6.64.19
	Florins,	190.56	900.	171.5	0.48.63
Baden,	Ducat,	47.5	987.	46.9	2.00.70
Belgium,	25 Francs,	121.92	899.	109.6	4.72.03
Brazil,	2000 Reis,	393.6	918.5	361.5	1.02.53
Canada,	20 Cents, 1851,	96.0	925.	88.8	0.18.87
China,	Tael,	...	...	...	1.43.00
Chili,	10 Pesos, 1855,	236.16	900.	212.5	9.15.35
	1 Peso, 1854–6,	384.48	900.5	346.2	0.98.17
Denmark,	2 Rix dollars,	444.96	877.	390.2	1.10.65
England,	Pound sterling = 20 shillings,	123.21	916.5	112.9	4.86.34
East Indies,	Company's Rupee,	180.	892.	16.5	5.10.49
France,	Napoleon, 20 Francs,	99.5	898.	87.4	3.85.00
Greece,	20 Drachms,	88.8	900.	80.9	3.44.29
Hamburg,	Rix Dollar,	450.	860.	397.5	1.17.66
Holland,	Ducat,	53.75	982.	52.77	2.29.7
	Florin,	50.	787.	39.32	1.69.30
Italy,	20 Lire,	99.36	898.	89.22	3.84.26
Mexico,	Doubloon = 8 Escudos,	416.4	870.5	362.5	15.61.05
	Peso = 8 Reals,	415.68	901.	374.5	1.06.20
Norway,	2 Rigsdaler,	444.96	877.	390.2	1.10.65
Peru,	1 Sol = 100 Centavos,	385.82	900.	347.24	0.95.41
Portugal,	Corona (Crown), 1838,	147.84	912.	134.8	5.80.66
	1000 Reis,	45.6	912.	41.6	1.18.00
Prussia,	Thaler.	268.46	900.	243.6	0.72.89
Rome,	2.5 Scudi = 250 Bajochi,	67.2	900.	60.5	2.60.47
Russia,	Imperial = 5 Roubles,	100.8	916.	92.3	3.97.64
	Rouble silver = 100 Copecks,	320.16	875.	286.8	0.79.44
Spain,	100 Reals,	128.64	896.	115.2	4.96.39
	80 Reals = 4 Dollars,	103.2	869.5	89.73	3.86.44
Sweden,	Ducat,	53.	979.	51.9	2.23.50
	Rix Dollar = 100 Ore,	112.3	873.	97.15	0.26.10
Turkey,	Piastres, 1845,	110.88	900.	99.79	4.36.93

THE CURRENCY OF DIFFERENT COUNTRIES COMPARED WITH ENGLISH AND AMERICAN MONEY.

Engl'nd.			France. Belgi'm. Sw'land.		Prussia.			Austria. (in notes.)		Sweden.	Germany.		Russia. (in paper)		Hamburg.		U. S.
£	s.	d.	Frs.	Cts.	Th.	Sgr.	Pf.	Fl.	Kr.	Rix. Ore.	Fl.	Kr.	Rhl.	Kop.	Mrk	Sch.	$ Cts.
0	0	1	0	10½	0	0	10	0	5	0.07	0	3	0	3	0	0	0.02
0	0	2	0	21	0	1	8	0	10	0.14	0	6	0	5	0	2	0.04
0	0	3	0	32	0	2	6	0	16	0.21	0	9	0	8	0	2¾	0.06
0	0	4	0	42	0	3	4	0	21½	0.28	0	12	0	12	0	2½	0.08
0	0	5	0	53	0	4	2	0	27	0.36	0	15	0	16	0	4½	0.10
0	0	6	0	64	0	5	1	0	31	0.44	0	18	0	19	0	5¼	0.12
0	0	7	0	74	0	5	11	0	36¼	0.51	0	21	0	22	0	6¼	0.14
0	0	8	0	85	0	6	10	0	42½	0.59	0	24	0	26	0	7	0.16
0	0	9	0	96	0	7	7	0	47½	0.66	0	27	0	27	0	8	0.18
0	0	10	1	6	0	8	6	0	53	0.73	0	30	0	33	0	8¾	0.20
0	0	11	1	16	0	9	5	0	57½	0.80	0	34	0	36	0	9¾	0.22
0	1	0	1	27	0	10	3	0	62	0.89	0	36	0	39	0	11	0.24
0	2	0	2	55	0	20	6	1	25	1.78	1	13	0	79	1	6	0.48
0	3	0	3	82	1	0	9	1	87	2.67	1	49	1	18	2	1	0.72
0	4	0	5	10	1	10	11	2	50	3.56	2	24	1	58	2	12	0.96
0	5	0	6	36	1	21	3	3	12	4.45	2	59	1	97	3	7	1.21
0	6	0	7	64	2	1	6	3	74	5.34	3	38	2	37	4	2	1.45
0	7	0	8	92	2	11	9	4	36	6.23	4	12	2	77	4	12	1.69
0	8	6	10	20	2	22	0	4	95	7.12	4	47	3	18	5	7	1.93
0	9	0	11	46	3	2	0	5	58	8.09	5	22	3	58	6	2½	2.18
0	10	0	12	72	3	12	4	6	25	8.90	5	58	3	94	6	13½	2.42
0	11	0	13	99	3	22	6	6	87	9.79	6	34	4	38	7	8½	2.66
0	12	0	15	27	4	2	9	7	49	10.68	7	11	4	75	8	3½	2.90
0	13	0	16	55	4	13	0	8	12	11.57	7	46	5	15	8	14½	3.14
0	14	0	17	84	4	23	3	8	75	12.66	8	24	5	55	9	9½	3.39
0	15	0	19	8	5	3	5	9	37	13.45	8	57	5	96	10	4½	3.63
0	16	0	20	40	5	13	8	10	0	14.24	9	33	6	35	10	15½	3.87
0	17	0	21	66	5	23	11	10	65	15.13	10	9	6	74	11	10½	3.12
0	18	0	22	92	6	4	2	11	28	16.02	10	46	7	14	12	5½	4.36
0	19	0	24	18	6	14	4	11	88	17.01	11	21	7	44	13	0½	4.60
1	0	0	25	45	6	24	6	12	50	17.80	11	57	7	88	13	9	4.84
2	0	0	50	90	13	19	0	25	0	35.60	23	54	15	77	27	2	9.68
3	0	0	76	35	20	13	6	37	50	53.40	35	51	23	65	40	11	14.52
4	0	0	101	80	27	8	0	50	0	71.20	47	48	31	54	54	4	17.36
5	0	0	127	25	34	3	0	62	50	89.00	59	46	39	42	67	11	24.20
6	0	0	152	70	40	27	6	75	0	106.80	71	42	47	31	81	4	29.04
7	0	0	178	15	47	22	6	87	50	124.60	83	39	55	20	94	13	33.88
8	0	0	202	60	54	16	6	100	0	142.40	95	36	63	9	108	6	38.72
9	0	0	229	5	61	11	6	112	50	160.20	107	34	70	96	121	15	43.56
10	0	0	254	50	68	6	0	125	0	178.00	119	30	78	84	135	8	48.40

The mark of Finland is equal to the French franc.

DIAMOND.

Carat.	Grain.	Parts.	Grains (Troy).
1.	4.	64	3.2
0.25	1.	16	0.8
0.015625	0.0625	1	0 05
0.3125	12.5	20	1.

Conversion of Inches and Eighths into Decimals of a Foot.

Inches.	Fractions of an Inch. 0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
0	.0000	.01041	.02083	.03125	.04166	.05208	.0625	.07291
1	.08333	.09375	.10416	.11458	.125	.13541	.14588	.15639
2	.16666	.17707	.1875	.19792	.20832	.21873	.22914	.23965
3	.25	.26041	.270	.28125	.29166	.30208	.3125	.32291
4	.33333	.34375	.35416	.364	.375	.38541	.39588	.40639
5	.41666	.42707	.437	.44792	.45832	.46873	.47914	.48965
6	.5	.51041	.520	.53125	.54166	.55208	.5625	.57291
7	.58333	.59375	.60416	.614	.625	.63541	.64588	.65639
8	.66666	.67707	.685	.69792	.70832	.71773	.72914	.73965
9	.75	.76041	.770	.78125	.79169	.80208	.8425	.82291
10	.83333	.84375	.85416	.864	.875	.88541	.89588	.90639
11	.91666	.92707	.937	.94792	.95832	.96873	.97914	.98965
12	1 foot.	foot.	foot.	foot.	foot.	foot.	foot.	foot.

$\frac{1}{16}$ in. = 0.005208 ft.; $\frac{1}{32}$ in. = 0.00265 ft.; $\frac{1}{64}$ in. = 0.001375 ft.

Angle Measurement by the Opening of a Two-foot Rule.

Opening Rule.	Fractions of an Inch. 0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
Inch's.	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	00 00	0 36	1 12	1 47	2 23	2 59	3 35	4 11
1	4 46	5 22	5 59	6 34	7 10	7 46	8 22	8 58
2	9 34	10 10	10 46	11 22	11 58	12 34	13 10	13 46
3	14 22	14 58	15 34	16 10	16 46	17 22	17 59	18 35
4	19 12	19 48	20 24	21 0	21 37	22 13	22 50	23 27
5	24 3	24 39	25 16	25 53	26 30	27 7	27 44	28 21
6	28 58	29 35	30 12	30 49	31 26	32 3	32 40	33 17
7	33 54	34 33	35 8	35 46	36 25	37 3	37 40	38 18
8	38 56	39 34	40 12	40 50	41 29	42 7	42 46	43 24
9	44 4	44 42	45 21	45 59	46 38	47 17	47 56	48 35
10	49 15	49 54	50 34	51 13	51 53	52 33	53 13	53 53
11	54 34	55 14	55 55	56 35	57 16	57 57	58 38	59 19
12	60 0	60 41	61 23	62 5	62 47	63 28	64 10	64 52
13	65 35	66 18	67 1	67 44	68 28	69 12	69 55	70 38
14	71 20	72 6	72 51	73 35	74 21	75 6	75 51	76 36
15	77 20	78 8	78 54	79 40	80 27	81 14	82 1	82 49
16	83 38	84 26	85 14	86 3	86 52	87 41	88 31	89 21
17	90 12	91 3	91 55	92 41	93 39	94 31	95 34	96 17
18	97 11	98 5	99 0	99 55	100 51	101 47	102 44	103 42
19	104 40	105 39	105 39	107 40	108 41	109 43	110 46	111 49
20	112 53	113 58	115 4	116 11	117 20	118 30	119 41	120 53

Conversion of Vulgar Fractions into Decimals.

Fract'ns.	Decimals.	Fract'ns.	Decimals.	Fract'ns.	Decimals.	Fract'ns.	Decimals.
1:2	.5	1:16	.0625	1:32	.03125	1:64	.015625
1:3	.33333	3:16	.1875	3:32	.09375	3:64	.046875
2:3	.66666	5:16	.3125	5:32	.15625	5:64	.078125
1:4	.25	7:16	.4375	7:32	.21875	7:64	.109375
3:4	.75	9:16	.5625	9:32	.28125	9:64	.140625
1:5	.2	11:16	.6875	11:32	.34375	11:64	.171875
3:5	.6	13:16	.8125	13:32	.40625	15:64	.234375
1:6	.16666	15:16	.9375	15:32	.46875	19:64	.296875
5:6	.83333	1:24	.04166	17:32	.53125	23:64	.359375
1:8	.125	5:24	.20833	19:32	.59375	27:64	.421875
3:8	.375	7:24	.29166	21:32	.65625	31:64	.484375
5:8	.625	11:24	.45833	23:32	.71875	35:64	.546875
7:8	.875	13:24	.54166	25:32	.78125	39:64	.609375
5:12	.41666	17:24	.70833	27:32	.84375	43:64	.671875
7:12	.58333	19:24	.79166	39:32	.90625	57:64	.891625
11:12	.925	23:24	.95833	31:32	.96875	61:64	.953125

To Determine an Angle by the Aid of a Two-foot Rule.

b = opening of the rule in inches;

v = angle formed by the rule;

$$\text{Sin.}\ \tfrac{1}{2}v = \frac{b}{24}; \quad \text{and} \quad b = 24\ \text{sin.}\ \tfrac{1}{2}v.$$

Example 1. How much (b = ?) must a two-foot rule be opened to form an angle of 48° 40′?

$$b = 24 \times \text{sin.}\ 24° 20' = 24 \times 0.412 = 9.888 \text{ inches.}$$

Example 2. A two-foot rule is opened to $b = 8$ inches. Required the angle formed by the rule.

$$\text{Sin.}\ \tfrac{1}{2}v = \frac{8}{24} = 0.3333 = \text{sin.}\ 19° 30', \text{ and } v = 39°.$$

THE FRENCH METRICAL SYSTEM.

The French units of weight, measure and coin are arranged into a perfect decimal system, except those of time and the circle. The division and multiplication of the units are expressed by Latin and Greek names, as follow:

Latin, Division.	*Greek, Multiplication.*
Milli = 1000th of the unit.	Deca = 10 times the unit.
Centi = 100th of the unit.	Hecato = 100 times the unit.
Deci = 10th of the unit.	Kilio = 1000 times the unit.
Metre, Litre, Stere, Are, Franc, Gramme.	Myrio = 10000 times the unit.

French Measure of Length.

1 Millimetre	= 0.03937079 inches.	1 Metre (unit)	= 3.280899 feet.
1 Centimetre	= 0.3937079 inches.	1 Decametre	= 32.80899 feet.
1 Decimetre	= 3.937079 inches.	1 Hectometre	= 328.0899 feet
1 Metre (unit)	= 39.37079 inches.	1 Kilometre	= 3280.899 ft. = 0.62138 mile.
1 Sea mile or knot	= 1.8472 kilometre.	1 Statute mile	= 1.609315 kilometres.
1 Kilometre	= 0.541343 sea miles.	1 Kilometre	= 49.7106 chains.

French Measure of Surface.

1 Sq. metre	= 10.7643 square feet.	1 Are	= 1076.43 square feet.
1 Are	= 100 square metres.	1 Decare	= 107.643 square feet.
1 Decare	= 10 ares.	1 Hectare	= 2.47114 Eng. acres.
1 Hectare	= 100 ares.	1 Sq. mile	= 258.989 hectares.

French Measure of Volume.

1 Stere (cubic metre)	= 10 decasteres.	1 Stere	= 35.3166 Eng. cubic feet.
1 Stere	= 1000 litres.	1 Litre	= 61.0271 Eng. cubic inches.
1 Litre	= 1 cubic decimetre.	1 Gallon	= 3.7852 litres.
1 Decistere	= 3.53166 cubic feet.	1 Decistere	= 2.84 bushels.

French Measure of Weight.

1 Ton	= 1 cubic metre distilled water.	1 Gramme	= 10 decigrammes.
1 Ton	= 1000 kilogrammes.	1 Decigramme	= 10 centigrammes.
1 Kilogramme	= 1000 grammes.	1 Centigramme	= 10 milligrammes.
1 Hectogramme	= 100 grammes.	1 Kilogramme	= 2.2047 pounds avoirdupois.
1 Decagramme	= 10 grammes.	1 Eng. pound	= 0.45358 kilogrammes.
1 Gramme	= 1 cubic centimetre distilled water.	1 Gramme	= 15.43315 grains troy.
1 French ton	= 0.984274 Eng. tons.	1 English ton	= 1.01598 French tons.

French Coin.

1 Franc 100 centimes = 19.06 cents of an American dollar.

A TABLE OF UNITS FOR ENGLISH LONG MEASURE.

League.	Geographical Miles.	Statute Miles.	Furlongs.	Chains.	Rods.	Fathoms.	Yards.	Feet.	Links.	Hands.	Inches.	Barley-corns.
1	2.59	3	24	240	960	2640	5280	15840	2400	47520	190080	570240
0.38066	**1**	1.1532	9.266	92⅔	370⅔	1019⅓	2038⅔	6116	9266⅔	18348	73392	220176
0.3333	0.8634	**1**	8	80	320	880	1760	5280	8000	15840	63360	190080
0.04166	0.10792	0.125	**1**	10	40	110	220	660	1000	1980	7920	23760
0.00416	0.01079	0.0125	0.1	**1**	4	11	22	66	100	198	792	2376
0.00104	0.00269	0.00312	0.025	¼	**1**	2¾	5½	16½	25	49½	198	594
0.00073	0.00098	0.00113	0.00909	0.0909	0.3636	**1**	2	6	9.09	18	72	216
0.00019	0.00049	0.00057	0.00454	0.04545	0.1818	½	**1**	3	4.545	9	36	108
0.00006	0.00016	0.00019	0.00151	0.01515	0.0606	0.165	0.3333	**1**	1.515	3	12	36
0.00004	0.00011	0.00012	0.001	0.01	0.04	0.11	0.22	0.6666	**1**	1.98	7.92	23.76
0.00002	0.00005	0.00006	0.00050	0.00505	0.0202	0.0556	0.1111	0.3333	0.5555	**1**	4	12
0.000005	0.000013	0.000012	0.00012	0.00126	0.00505	0.0139	0.0278	0.0833	0.126	¼	**1**	3
0.000002	0.000004	0.000005	0.00004	0.00042	0.00167	0.0046	0.0092	0.0278	0.042	0.0833	⅓	**1**

A TABLE OF METRIC UNITS OF LENGTH.

Kilo-metre.	Hecto-metre.	Deca-metre.	Metre.	Deci-metre.	Centi-metre.	Milli-metre.
1	10	100	1,000	10,000	100,000	1,000,000
0.1	**1**	10	100	1,000	10,000	100,000
0.01	0.1	**1**	10	100	1,000	10,000
0.001	0.01	0.1	**1**	10	100	1,000
0.0001	0.001	0.01	0.1	**1**	10	100
0.00001	0.0001	0.001	0.01	0.1	**1**	10
0.000001	0.00001	0.0001	0.001	0.01	0.1	**1**

These two tables represent a fair comparison of the English and French systems of measurement.

In a lecture on "Wave Theory of Light," delivered at the Academy of Music, Philadelphia, Sept. 29, 1884, Sir William Thomson said, "I look upon our English system as a wickedly brain-destroying piece of bondage under which we suffer. I say this seriously. I do not think any one knows how seriously I speak of it."

The English table is a heap of rubbish unworthy the title of system, whilst the French table exhibits a regular and simple system.

The metric system is now generally used in European scientific and technical publications, for which the following tables have been calculated, to enable the reader to convert quantities from one system to the other.

Conversion of English Inches into Centimetres.

Inch's	0	1	2	3	4	5	6	7	8	9
	Ct.mt.	Ct.mt.	Ct.mt.	Ct.mt.	Ct.mt.	Ct.mt.	Ct.mt.	Ct.mt.	Ct.mt.	Ct.mt.
0	0.000	2.540	5.080	7.620	10.16	12.70	15.24	17.78	20.32	22.86
10	25.40	27.94	30.48	33.02	35.56	38.10	40.64	43.18	45.72	48.26
20	50.80	53.34	55.88	58.42	60.96	63.50	66.04	68.58	71.12	73.66
30	76.20	78.74	81.28	83.82	86.36	88.90	91.44	93.98	96.52	99.06
40	101.60	104.14	106.68	109.22	111.76	114.30	116.84	119.38	121.92	124.46
50	127.00	129.54	132.08	134.62	137.16	139.70	142.24	144.78	147.32	149.86
60	152.40	154.94	157.48	160.02	162.56	165.10	167.64	170.18	172 72	175.26
70	177.80	180.34	182.88	185.42	187.96	190.50	193.04	195.58	198.12	200.96
80	203.20	205.74	208.28	210.82	213.36	215.90	218.44	220.98	223.52	226.06
90	228.60	231.14	233.68	236.22	238.76	241.30	243.84	246.38	248.92	251.46
100	254.00	256.54	259.08	261.62	264.16	266.70	269.24	271.78	274.32	276.85

Conversion of Centimetres into English Inches.

Ct. mt.	0	1	2	3	4	5	6	7	8	9
	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
0	0.000	0.394	0.787	1.181	1.575	1.969	2.362	2.756	3.150	3.543
10	3.937	4.331	4.742	5.118	5.512	5.906	6.299	6.693	7.087	7.480
20	7.874	8.268	8.662	9.055	9.449	9.843	10.236	10.630	11.024	11.418
30	11.811	12.205	12.599	12.992	13.386	13.780	14.173	14.567	14.961	15.355
40	15.748	16.142	16.536	16.929	17.323	17.717	18.111	18.504	18.898	19.292
50	19.685	20 079	20.473	20.867	21.260	21.654	22.048	22.441	22.835	23.229
60	23.622	24.016	24.410	24.804	25.197	25.591	25.985	26.378	26.772	27.166
70	27.560	27.953	28.347	28.741	29.134	29.528	29.922	30.316	30.709	31.103
80	31.497	31.890	32.284	32.678	33.071	33.465	33.859	34.253	34.646	35.040
90	35.434	35.827	36.221	36.615	37.009	37.402	37.796	38.190	38.583	38.977
100	39.370	39.764	40.158	40.552	40.945	41.339	41.733	42.126	42.520	42.914

Conversion of English Feet into Metres.

Feet.	0	1	2	3	4	5	6	7	8	9
	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.
0	0.000	0.3048	0.6096	0.9144	1.2192	1.5239	1.8287	2.1335	2.4383	2.7431
10	3.0479	3.3527	3.6575	3.9623	4.2671	4.5719	4.8767	5.1815	5.4863	5.7911
20	6.0359	6.4006	6.7055	7.0102	7.3150	7.6198	7.9246	8.2294	8.5342	8.8390
30	9.1438	9.4486	9.7534	10.058	10.363	10.668	10.972	11.277	11.582	11.887
40	12.192	12.496	12.801	13.106	13.411	13.716	14.020	14.325	14.630	14.935
50	15.239	15.544	15.849	16.154	16.459	16.763	17.068	17.373	17.678	17.983
60	18 287	18.592	18.897	19.202	19.507	19.811	20.116	20.421	20.726	21.031
70	21.335	21.640	21.945	22.250	22.555	22.859	23.164	23.469	23.774	24.079
80	24.383	24.688	24.993	25.298	25.602	25.907	26.212	26.517	26.822	27.126
90	27.431	27.736	28.041	28.346	28.651	28.955	29.260	29.565	29.870	30.174
100	30.479	30.784	31.089	31.394	31.698	32.003	32.308	32.613	32.918	33.222

Conversion of Metres into English Feet.

Metres.	0	1	2	3	4	5	6	7	8	9
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
0	0.000	3.2809	6.5618	9.8427	13.123	16.404	19.685	22.966	26.247	29.528
10	32.809	36.090	39.371	42.651	45.932	49.213	52.494	55.775	59.056	62.337
20	65 618	68 899	72.179	75 461	78.741	82.022	85.303	88.584	91.865	95.146
30	98.427	101.71	104.99	108 27	111.55	114.83	118.11	121.39	124.67	127.96
40	131.24	134.52	137.80	141.08	144.36	147.64	150.92	154.20	157.48	160.76
50	164 04	167.33	170.61	173.89	177.17	180.45	183.73	187.01	190.29	193.57
60	196.85	200.13	203.42	206.70	209.98	213 26	216.54	219.82	223.10	226.38
70	229.66	232.94	236.22	239.51	242.79	246.07	249.35	252.63	255.91	259.19
80	262.47	265.75	269.03	272.31	275.60	278.88	282.16	285.44	288.72	292.00
90	295.28	298.56	391.84	305.12	308.40	311.69	314.97	318.25	321.53	324.81
100	328.09	331.37	334.65	337.93	341.21	344.49	347.78	351.06	354.34	357.62

Conversion of English Statute-miles into Kilometres.

Miles.	0	1	2	3	4	5	6	7	8	9
	Kilom.	Kilom.	Kilom.	Kilom.	Kilom.	Kilom.	Kilom.	Kilom.	Kilom.	Kilom.
0	0.0000	1.6093	3.2186	4.8279	6.4372	8.0465	9.6558	11.2652	12.8745	14.4848
10	16.093	17.702	19.312	20.921	22.530	24.139	25.749	27.358	28.967	30.577
20	32.186	33.795	35.405	37.014	38.623	40.232	41.842	43.451	45.060	46.670
30	48.279	49.888	51.498	53.107	54.716	56.325	57.935	59.544	61.153	62.763
40	64.372	65.981	67.591	69.200	70.809	72.418	74.028	75.637	77.246	78.856
50	80.465	82.074	83.684	85.293	86.902	88.511	90.121	91.730	93.339	94.949
60	96.558	98.167	99.777	101.39	102.99	104.60	106.21	107.82	109.43	111.04
70	112.65	114.26	115.87	117.48	119.08	120.69	122.30	123.91	125.52	127.13
80	128.74	130.35	131.96	133.57	135.17	136.78	138.39	140.00	141.61	143.22
90	144.85	146.44	148.05	149.66	151.26	152.87	154.48	156.09	157.70	159.31
100	160.93	162.53	164.14	165.75	167.35	168.96	170.57	172.18	173.79	175.40

Conversion of Kilometres into English Statute-miles.

Kilom.	0	1	2	3	4	5	6	7	8	9
	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.
0	0.0000	0.6214	1.2427	1.8641	2.4855	3.1069	3.7282	4.3497	4.9711	5.5924
10	6.2138	6.8352	7.4565	8.0780	8.6994	9.3208	9.9421	10.562	11.185	11.805
20	12.427	13.049	13.670	14.292	14.913	15.534	16.156	16.776	17.399	18.019
30	18.641	19.263	19.884	20.506	21.127	21.748	22.370	22.990	23.613	24.233
40	24.855	25.477	26.098	26.720	27.341	27.962	28.584	29.204	29.827	30.447
50	31.069	31.690	32.311	32.933	33.554	34.175	34.797	35.417	36.040	36.660
60	37.282	37.904	38.525	39.147	39.768	40.389	41.011	41.631	42.254	42.874
70	43.497	44.118	44.739	45.361	45.982	46.603	47.225	47.845	48.468	49.088
80	49.711	50.332	50.953	51.575	52.196	52.817	53.439	54.059	54.682	55.302
90	55.924	56.545	57.166	57.788	58.409	59.030	59.652	60.272	60.895	61.515
100	62.138	62.759	63.380	64.002	64.623	65.244	65.866	66.486	67.109	67.729

Conversion of Sea-miles, Knots or Minutes into Kilometres.

Knots.	0	1	2	3	4	5	6	7	8	9
	Kilom.	Kilom.	Kilom.	Kilom.	Kilom.	Kilom.	Kilom.	Kilom.	Kilom.	Kilom.
0	0.0000	1.8472	3.6944	5.5416	7.3888	9.2361	11.083	12.930	14.777	16.625
10	18.472	20.319	22.166	24.013	25.861	27.708	29.555	31.402	33.249	35.097
20	36.944	38.791	40.638	42.485	44.333	46.180	48.027	49.874	51.721	53.569
30	55.416	57.263	59.110	60.957	62.805	64.652	66.499	68.346	70.193	72.041
40	73.888	75.735	77.582	79.429	81.277	83 124	84.971	86.818	88.665	90.513
50	92.361	94 207	96.054	97.901	99.749	101.59	103.44	105.29	107.14	108.98
60	110.83	112.68	114.53	116.37	118.22	120.06	121.91	123.76	125.61	127.45
70	129.30	131.15	133.00	134.84	136.70	138.54	140.39	142.24	144.09	145.94
80	147.77	149.62	151.47	153.31	155.18	157.02	158.87	160.72	162.57	164.43
90	166.25	168.09	169.94	171.78	173.65	175.49	177.34	179.19	181.04	182.90
100	184.72	186.56	188.41	190.25	192.12	193.96	195.81	198.66	199.51	201.37

Conversion of Kilometres into Sea-miles, Knots or Minutes.

Kilom.	0	1	2	3	4	5	6	7	8	9
	Knots.	Knots.	Knots.	Knots.	Knots.	Knots.	Knots.	Knots.	Knots.	Knots.
0	0.0000	0.5413	1.0827	1.6240	2.1653	2.7066	3.2480	3.7894	4.3307	4.8721
10	5.4134	5.9547	6.4961	7.0374	7.5787	8.1200	8.6614	9.2028	9.7441	10.285
20	10.827	11.368	11.909	12.451	12.992	13.533	14.075	14.616	15.157	15.702
30	16.24	16.781	17.322	17.864	18.406	18.946	19.488	20.029	20.570	21.115
40	21.653	22.194	22.735	23.277	23.819	24.359	24 901	25.442	25.983	26.528
50	27.066	27.607	28.148	28.690	29.232	29.772	30.314	30.855	31.396	31.941
60	32.480	33.020	33.561	34.103	34.645	35.185	35.727	36.268	36.809	37.364
70	37.894	38.433	38.974	39.516	40.058	40.598	41.140	41.681	42.222	42.777
80	43.307	43.846	44.387	44.929	45.471	46.011	46.553	47.094	47.635	48.190
90	48.721	49.259	49.800	50.342	50.884	51.424	51.966	52.507	53.048	54.003
100	54.134	54.672	55.213	55.755	56.297	56.837	57.379	57.920	58.461	60.016

Conversion of Square Inches into Square Centimetres.

In².	0	1	2	3	4	5	6	7	8	9
	Cm².	Cm².	Cm².	Cm².	Cm².	Cm².	Cm².	Cm².	Cm².	Cm².
0	0.0000	6.4515	12.903	19.354	25.806	32.257	38.709	45.160	51.612	58.063
10	64.515	70.967	77.418	83.869	95.321	96.772	103.22	109.67	116.12	122.57
20	129.03	135.48	141.93	148.38	154.83	161.29	167.74	174.19	180.64	187.09
30	193.54	199.99	206.44	212.89	219.34	225.80	231.25	238.70	245.15	251.60
40	258.06	264.51	270.96	277.41	283.86	290.32	296.77	303.22	309.67	316.12
50	322.57	329.02	335.47	341.92	348.37	354.83	361.28	367.73	374.18	380.63
60	387.09	393.54	399.99	406.44	412.89	419.35	425.80	432.25	438.70	445.15
70	451.60	458.05	464.50	470.95	477.40	483.86	490.31	495.76	503.21	509.66
80	516.12	522.57	529.02	535.47	541.92	548.38	554.83	561.28	567.73	574.18
90	580.63	587.08	593.53	599.98	606.43	612.89	619.34	625.79	632.24	638.69
100	645.15	651.60	658.05	664.50	670.95	677.41	683.86	690 31	696.76	703.21

Conversion of Square Centimetres into Square Inches.

Cm².	0	1	2	3	4	5	6	7	8	9
	In².	In².	In².	In².	In².	In².	In².	In².	In².	In².
0	0.0000	0.1550	0.3100	0.4650	0.6200	0.7750	0.9300	1.0850	1.2400	1.3950
10	1.5500	1.7050	1.8600	2.0150	2.1700	2.3250	2.4800	2.6350	2.7900	2.9450
20	3.1000	3.2550	3.4100	3.5650	3.7200	3.8750	4.0300	4.1850	4.3400	4.4950
30	4.6501	4.8051	4.9601	5.1151	5.2701	5.4251	5.5801	5.7351	5.8901	6.0451
40	6.2001	6.3551	6.5101	6.6651	6.8201	6.9751	7.1301	7.2851	7.4401	7.5951
50	7.7501	7.9051	8.0601	8.2151	8.3701	8.5251	8.6801	8.8351	8.9901	9.1451
60	9.3002	9.4552	9.6102	9.7652	9.9202	10.075	10.230	10.385	10.540	10.695
70	10.850	11.040	11.160	11.315	11.470	11.625	11.780	11.935	12.090	12.245
80	12.400	12.555	12.710	12.865	13.020	13.175	13.330	13.485	13.640	13.795
90	13.950	14.105	14.260	14.415	14.570	14.725	14.880	15.035	15.190	15.345
100	15.500	15.655	15.810	15.965	16.120	16.275	16.430	16.585	16.740	16.895

Conversion of Cubic Inches into Cubic Centimetres.

Cub. In.	0	1	2	3	4	5	6	7	8	9
	Cm³.	Cm³.	Cm³.	Cm³.	Cm³.	Cm³.	Cm³.	Cm³.	Cm³.	Cm³.
0	0.0000	16.383	32.773	49.160	65.546	81.933	98.320	114.71	131.01	147.48
10	163.87	180.26	196.64	213.03	229.41	245.80	262.19	278.58	294.88	311.35
20	327.73	344.12	360.50	376.89	393.27	409.66	426.05	442.44	458.74	475.21
30	491.60	507.99	524.37	519.76	557.14	573.53	569.92	606.31	622.61	639.08
40	655.46	671.85	688.23	704.52	721.00	737.39	753.78	770.17	786.47	802.94
50	819.33	835.72	851.10	868.49	884.87	901.26	917.65	934.04	950.34	966.81
60	983.20	999.59	1016.0	1032.4	1048.7	1065.1	1081.5	1097.9	1114.2	1130.7
70	1147.1	1153.5	1179.9	1196.3	1212.6	1229.0	1245.4	1261.8	1278.1	1294.6
80	1310.9	1327.3	1343.7	1360.1	1376.4	1392.8	1409.2	1425.6	1441.9	1458.4
90	1474.8	1491.2	1507.6	1524.0	1540.3	1556.7	1573.1	1589.5	1605.8	1622.3
100	1638.7	1655.1	1671.5	1687.9	1704.2	1720.6	1737.0	1753.4	1769.7	1786.2

Conversion of Cubic Centimetres into Cubic Inches.

Cm³.	0	1	2	3	4	5	6	7	8	9
	In³.	In³.	In³.	In³.	In³.	In³.	In³.	In³.	In³.	In³.
0	0.0000	0.0610	0.1221	0.1831	0.2441	0.3051	0.3661	0.4272	0.4882	0.5492
10	0.6102	0.6712	0.7323	0.7933	0.8543	0.9153	0.9763	1.0374	1.0984	1.1594
20	1.2205	1.2215	1.3426	1.4036	1.4646	1.5256	1.5866	1.6477	1.7087	1.7697
30	1.8308	1.8918	1.9529	2.0139	2.0749	2.1359	2.1969	2.2580	2.3190	2.3800
40	2.4110	2.5020	2.5631	2.6241	2.6851	2.7461	2.8071	2.8682	2.9292	2.9902
50	3.0513	3.1123	3.1734	3.2344	3.2954	3.3564	3.4174	3.4785	3.5395	3.6005
60	3.6615	3.7225	3.7836	3.8446	3.9056	3.9666	4.0276	4.0887	4.1497	4.2107
70	4.2718	4.3328	4.3939	4.4549	4.5159	4.5769	4.6379	4.6990	4.7600	4.8210
80	4.8820	4.9430	5.0041	5.0651	5.1261	5.1871	5.2481	5.3092	5.3702	4.4312
90	5.4923	5.5533	5.6144	5.6754	5.7364	5.7974	5.8584	5.9195	5.9805	6.0415
100	6.1025	6.1085	6.2246	6.2856	6.3166	6.4076	6.4686	6.5297	6.5907	6.6517

Conversion of Cubic Yards into Cubic Metres.

Cub. yd.	0	1	2	3	4	5	6	7	8	9
	M³.	M³.	M³.	M³.	M³.	M³.	M³.	M³.	M³.	M³.
0	0.0000	0.7645	1.5291	2.2936	3.0581	3.8226	4.5872	5.3517	6.1163	6.8808
10	7.6453	8.4098	9.1744	9.9389	10.703	11.468	12.232	12.997	13.761	14.526
20	15.291	16.055	16.820	17.585	18.349	19.114	19.878	20.643	21.407	22.172
30	22.936	23.700	24.455	25.230	25.994	26.759	27.523	28.288	29.052	29.817
40	30.581	31.345	32.110	32.875	33.639	34.404	35.168	35.933	36.797	37.462
50	38.226	38.990	39.755	40.520	41.284	42.049	42.813	43.578	44.342	45.107
60	45.872	46.636	47.401	48.166	48.930	49.695	50.459	51.224	51.988	52.753
70	53.517	54.281	55.046	55.811	56.575	57.340	58.104	58.869	59.633	60.398
80	61.163	61.927	62.692	63.457	64.221	64.986	65.750	66.515	67.279	68.044
90	68.808	69.572	70.337	71.102	71.866	72.631	73.395	74.160	74.924	75.689
100	76.453	77.217	77.982	78.747	79.511	80.276	81.040	81.805	82.569	83.334

Conversion of Cubic Metres into Cubic Yards.

M³.	0	1	2	3	4	5	6	7	8	9
	Yd³.	Yd³.	Yd³.	Yd³.	Yd³.	Yd³.	Yd³.	Yd³.	Yd³.	Yd³.
0	0.0000	1.3080	2.6160	3.9240	5.2329	6.5399	7.8479	9.1559	10.464	11.772
10	13.080	14.388	15.696	17.004	18.313	19.620	20.928	22.236	23.544	24.852
20	26.160	27.468	28.776	30.084	31.393	32.700	34.008	35.316	36.624	37.932
30	39.240	40.548	41.856	43.164	44.473	45.780	47.088	48.396	49.704	51.012
40	52.319	53.627	54.935	56.243	57.552	58.859	60.167	61.475	62.783	63.091
50	65.399	66.707	68.015	69.323	70.632	71.939	73.247	74.545	75.863	77.171
60	78.479	79.787	81.095	82.403	83.712	85.019	86.327	87.535	88.943	90.251
70	91.559	92.867	94.175	95.483	96.792	98.099	99.407	100.71	102.02	103.33
80	104.63	105.94	107.25	108.56	109.87	111.17	112.48	113.79	115.10	116.41
90	117.72	119.03	120.34	121.64	122.95	124.26	125.57	126.88	128.18	129.49
100	130.80	132.11	133.42	134.72	136.03	137.34	138.65	139.96	141.26	142.57

Conversion of Gallons into Litres.

Gall.	0	1	2	3	4	5	6	7	8	9
	Lit.	Lit.	Lit.	Lit.	Lit.	Lit.	Lit.	Lit.	Lit.	Lit.
0	0.0000	3.7853	7.5706	11.356	15.141	18.946	22.712	26.497	30.282	34.068
10	37.853	41.638	45.423	49.209	52.994	56.799	60.565	64.350	68.135	71.921
20	75.706	79.491	83.276	87.062	90.847	94.652	98.418	102.20	105.99	109.77
30	113.56	117.34	121.13	124.92	128.66	132.50	136.27	140.06	143.84	147.63
40	151.42	155.22	158.99	162.78	166.56	170.36	174.13	177.92	181.70	185.49
50	189.46	193.24	197.03	200.82	204.60	208.40	212.17	215.96	219.74	223.53
60	227.12	230.90	234.69	238.48	242.26	246.06	249.83	253.62	257.40	261.19
70	264.97	268.75	272.54	276.33	280.11	283.91	286.68	291.47	295.25	299.04
80	302.82	306.60	310.39	314.18	317.96	321.76	324.53	329.32	333.10	336.89
90	440.68	444.46	448.25	452.04	455.82	459.62	463.39	467.18	470.96	474.75
100	478.53	482.31	486.10	789.89	493.67	497.47	501.24	505.03	508.81	512.60

Conversion of Litres into Gallons.

Lit.	0	1	2	3	4	5	6	7	8	9
	Gal.	Gal.	Gal.	Gal.	Gal.	Gal.	Gal.	Gal.	Gal.	Gal.
0	0.0000	0.2642	0.5284	0.7925	1.0567	1.3209	1.5851	1.8492	2.1134	2.3776
10	2.6418	2.9060	3.1702	3.4343	3.6985	3.9627	4.2269	4.4910	4.7552	5.0194
20	5.2836	5.5478	5.8120	6.0761	6.3403	6.6045	6.8687	7.1328	7.3970	7.6612
30	7.9254	8.1896	8.4538	8.7179	8.9821	9.2463	9.5105	9.8746	10.030	10.303
40	10.567	10.831	11.095	11.360	11.624	11.888	12.152	12.416	12.680	12.945
50	13.209	13.473	13.737	14.002	14.266	14.530	14.794	15.058	15.322	15.587
60	15.851	16.115	16.379	16.644	16.908	17.172	17.436	17.700	17.964	18.229
70	18.492	18.756	19.020	19.281	19.549	19.813	20.077	20.341	20.605	20.870
80	21.134	21.398	21.662	21.926	22.191	22.455	22.719	22.983	23.247	23.512
90	23.776	24.040	24.304	24.568	24.832	25.097	25.361	25.625	25.889	26.154
100	26.418	26.682	26.946	27.210	27.475	27.739	28.003	28.267	28.531	28.796

Conversion of Yards into Metres.

Yards.	0	1	2	3	4	5	6	7	8	9
	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.
0	0.0000	0.9144	1.8288	2.7432	3.6576	4.5719	5.4863	6.4007	7.3151	8.2295
10	9.1439	10.058	10.973	11.887	12.801	13.716	14.630	15.544	16.458	17.373
20	18.288	19.202	20.117	21.031	21.945	22.860	23.774	24.689	25.603	26.518
30	27.432	28.346	29.260	30.174	31.088	32.003	32.917	33.832	34.746	35.661
40	36.576	37.490	38.404	39.318	40.232	41.147	42.061	42.976	43.890	44.805
50	45.719	46.634	47.548	48.462	49.376	50.291	51.205	52.120	53.034	53.949
60	51.863	55.778	56.692	57.606	58.520	59.435	60.349	61.264	62.178	63.093
70	64.007	64.922	65.836	66.750	67.664	68.578	69.493	70.408	71.322	72.237
80	73.151	74.066	74.980	75.894	76.808	77.723	78.637	79.552	80.466	81.381
90	82.295	83.210	84.124	85.038	85.952	86.867	87.781	88.696	89.610	90.525
100	91.439	92.353	93.267	94.181	95.095	96.010	96.924	97.839	98.753	99.668

Conversion of Metres into Yards.

Metres.	0	1	2	3	4	5	6	7	8	9
	Yds.	Yds.	Yds.	Yds.	Yds.	Yds.	Yds.	Yds.	Yds.	Yds.
0	0.0000	1.0936	2.1872	3.2809	4.3745	5.1681	6.5617	7.6553	8.7490	9.8426
10	10.936	12.029	13.122	14.217	15.310	16.404	17.498	18.591	19.685	20.778
20	21.872	22.966	24.059	25.153	26.217	27.340	28.434	29.527	30.621	31.715
30	32.809	33.900	34.993	36.090	37.184	38.277	39.371	40.464	41.558	42.652
40	43.745	44.839	45.932	47.026	48.120	49.213	50.307	51.409	52.544	53.588
50	54.681	55.775	56.868	57.962	59.056	60.149	61.243	62.336	63.430	64.524
60	65.617	66.711	67.804	68.898	69.992	71.085	72.179	73.272	74.366	75.460
70	76.553	77.647	78.740	79.834	80.928	82.021	83.115	84.208	85.302	86.396
80	87.490	88 584	89.677	90.771	91.865	92.958	94.052	95.145	96.239	97.333
90	98.426	99.520	100.61	101.71	102.80	103.89	104.99	106.08	107.17	108.27
100	109.36	110.45	111.55	112.64	113.73	114.83	115.92	117.02	118.11	119.20

Conversion of Square Yards into Square Metres.

Sq. Yds.	0	1	2	3	4	5	6	7	8	9
	M^2.	M^2.	M^2.	M^2.	M^2.	M^2.	M^2.	M^2.	M^2.	M^2.
0	0.0000	0.8361	1.6722	2.5803	3.3414	4.1805	5.0167	5.8528	6.6889	7.5250
10	8.3611	9.1972	9.0333	10.941	11.706	12.542	13.378	14.214	15.050	15.886
20	16.722	17.558	19.230	19.302	20.066	20.903	21.739	22.575	23.411	24.247
30	25.083	25.919	26.755	27 663	28.431	29.264	30.100	30.936	31.772	32.608
40	33.444	34.280	35.116	36.024	36.788	37.625	38.461	39.297	40.133	40.969
50	41.805	42.641	43.477	44.385	45.149	45.986	46.822	47.658	48.494	49.330
60	50.167	51.003	51.839	52.747	53.511	51.348	55.184	56.020	56.856	57.692
70	58.528	59.364	60.190	61.108	61.872	62.709	63.545	64.381	65.217	66.053
80	66.889	67.725	68.561	69.469	70.233	71.070	71.906	72.742	73.578	74.414
90	75.250	76.086	76.922	77.830	78.594	79.431	80.267	81.103	81.939	82.775
100	83.611	84.447	85.283	86.191	86.955	87.792	88.628	89.464	90.300	91.136

Conversion of Square Metres into Square Yards.

Sq. M.	0	1	2	3	4	5	6	7	8	9
	Yd^2.	Yd^2.	Yd^2.	Yd^2.	Yd^2.	Yd^2.	Yd^2.	Yd^2.	Yd^2.	Yd^2.
0	0.0000	1.1960	2.3920	3.5880	4.7840	5.9800	7.1760	8.3720	9.5681	10.764
10	11.960	13.156	14.352	15.548	16.744	17.940	19.136	20.332	21.528	22.724
20	23.920	25.116	26.312	27.508	28.704	29.900	31.096	32.292	33.488	34.684
30	35.880	37.076	38.272	39.468	40.664	41.860	43.056	41.252	45.448	46.644
40	47.840	49.036	50.232	51.428	52.624	53.820	55.016	56.212	57.408	58.604
50	59.800	60.996	62,192	63,388	63.584	65.780	66.976	68.172	69.368	70.564
60	71,760	72.956	74.152	75.348	76,544	77.740	78.936	80.132	81.328	82.524
70	83.721	84.917	86,113	87.309	87.505	89.701	90.897	92.093	93.289	94.485
80	95,681	96.877	98,073	99,269	100.46	101.66	102.86	104.06	105.25	106.44
90	107.64	108.84	110,03	111,24	112.42	113,62	114.81	116.01	117.21	118.40
100	119.60	120.80	121.99	123.19	124.38	125.58	126.77	127.97	129.17	130.36

Conversion of Acres into Hectares.

Acres.	0	1	2	3	4	5	6	7	8	9
	Hect.	Hect.	Hect.	Hect.	Hect.	Hect.	Hect.	Hect.	Hect.	Hect.
0	0.0000	2.4711	4.9422	7.4133	9.8844	12.355	14.836	17.298	19.769	22.240
10	24.711	27.182	29.653	32.124	34.695	37.046	39.547	42.009	44.480	46.951
20	49.422	51.893	54.364	56.835	59.306	61.757	64.258	66.720	68.191	71.662
30	74.133	76.604	79.075	81.546	84.017	86.468	88.969	91.431	93.902	96.373
40	98.844	101.31	103.79	106.26	108.73	111.18	113.68	116.14	118.61	121.08
50	123.55	126.02	128.49	130.96	133.43	135.88	138.38	140.85	143.32	145.79
60	148.36	150.83	153.30	155.77	158.24	160.69	163.19	155.66	168.13	170.60
70	172.95	175.45	177.92	180.39	182.86	185.31	187.81	190.28	191.75	195.22
80	197.69	200.16	202.63	205.10	207.57	210.02	212.52	214.99	217.46	219.93
90	222.40	224.87	227.34	229.81	232.28	234.73	237.23	239.70	242.17	244.64
100	247.11	249.58	252.05	254.52	256.99	259.44	261.94	264.41	266.88	269.35

Conversion of Hectares into Acres.

Hect.	0	1	2	3	4	5	6	7	8	9
	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.
0	0.0000	0.4047	0.8093	1.2140	1.6187	2.0234	2.4280	2.8327	3.2374	3.6420
10	4.0468	4.4515	4.8561	5.2608	5.6655	6.0702	6.4748	6.8795	7.2782	7.6888
20	8.0936	8.4983	8.9029	9.3076	9.7123	10.117	10.521	10.926	11.331	11.735
30	12.140	12.545	12.949	13.354	13.759	14.163	14.568	14.973	15.377	15.782
40	16.187	16.592	16.996	17.401	17.806	18.210	18.615	19.020	19.414	19.829
50	20.234	20.639	21.043	21.448	21.853	22.257	22.662	23.067	23.471	23.876
60	24.280	24.685	25.089	25.494	25.899	26.303	26.708	27.113	27.517	27.922
70	28.327	28.732	29.136	29.541	29.946	30.350	30.755	31.160	31.564	31.969
80	32.374	32.779	33.183	33.588	33.993	34.397	34.802	35.207	35.611	36.016
90	36.420	36.825	37.229	37.634	38.039	38.443	38.848	39.253	39.657	40.062
100	40.468	40.873	41.277	41.682	42.087	42.491	42.896	43.301	43.695	44.110

Conversion of Square Miles into Square Kilometres.

Sq. Mil.	0	1	2	3	4	5	6	7	8	9
	Kil².	Kil².	Kil².	Kil².	Kil².	Kil².	Kil².	Kil².	Kil².	Kil².
0	0.0000	2.5899	5.1798	7.7697	10.359	12.929	15.539	18.129	20.718	23.309
10	25.899	28.490	31.079	33.669	37.259	38.829	41.439	44.029	46.619	49.209
20	51.798	54.388	56.978	59.568	63.158	64.728	67.338	69.928	72.518	75.108
30	77.697	80.287	82.877	85.467	89.057	90.627	93.238	96.828	98.417	101.01
40	103.59	106.18	108.77	111.36	114.95	116.52	119.13	121.72	124.31	126.90
50	129.29	131.88	134.47	137.06	140.65	142.22	144.83	147.42	150.01	152.50
60	155.39	157.98	160.57	163.16	166.75	168.32	170.93	173.52	176.11	178.70
70	181.29	183.88	186.47	188.06	192.65	194.22	196.83	199.42	202.01	204.60
80	207.19	209.77	212.36	214.95	218.55	220.11	222.73	225.31	227.91	230.50
90	233.09	235.68	238.27	240.86	244.45	246.02	248.63	251.22	253.81	256.40
100	258.99	261.58	264.17	266.76	270.35	271.92	274.53	277.12	279.71	282.20

Conversion of Square Kilometres into Square Miles.

Kil².	0	1	2	3	4	5	6	7	8	9
	Sq. M.	Sq. M.	Sq. M.	Sq. M.	Sq. M.	Sq. M.	Sq. M.	Sq. M.	Sq. M.	Sq. M.
0	0.0000	0.3861	0.7722	1.1583	1.5445	1.9304	2.3166	2.7028	3.0890	3.4749
10	3.8612	4.1873	4.6334	5.0195	5.4057	5.7916	6.1778	6.5640	6.9502	7.3362
20	7.7224	8.0485	8.4946	8.8807	9.2669	9.6528	10.039	10.425	10.811	11.197
30	11.583	11.909	12.355	12.741	13.127	13.513	13.899	14.286	14.672	15.058
40	15.445	15.771	16.217	16.603	16.989	17.375	17.761	18.146	18.534	18.920
50	19.304	19.630	20.076	20.462	20.848	21.234	21.620	22.007	22.393	22.779
60	23.166	23.492	23.938	24.324	24.710	25.096	25.482	25.869	26.245	26.641
70	27.028	27.354	27.800	28.186	28.572	28.958	29.344	29.731	30.117	30.503
80	30.890	31.216	31.662	32.048	32.434	32.820	33.206	33.593	33.979	34.365
90	34.749	35.075	35.521	35.907	36.293	36.679	37.065	37.452	37.838	38.224
100	38.612	38.938	39.384	39.770	40.156	40.542	40.928	41.315	41.701	42.087

Conversion of Cubic Feet into Cubic Decimetres.

Cub. ft.	0	1	2	3	4	5	6	7	8	9
	dm³.	dm³.	dm³.	dm³.	dm³.	dm³.	dm³.	dm³.	dm³.	dm³.
0	0.0000	28.316	56.632	84.948	113.26	141.58	169.90	198.21	226.53	254.84
10	283.16	305.48	339.79	268.11	396.42	424.74	453.06	481.37	509.69	538.00
20	566.32	688.64	622.95	551.27	679.58	707.90	736.22	764.53	792.85	821.16
30	849.48	871.80	906.14	934.43	962.74	991.06	1019.4	1047.7	1076.0	1104.3
40	1132.6	1154.9	1189.2	1217.5	1245.9	1274.2	1302.5	1330.8	1359.1	1387.4
50	1415.8	1438.1	1472.4	1500.7	1529.1	1557.4	1585.7	1614.0	1642.3	1670.6
60	1698.9	1721.2	1755.5	1783.8	1712.2	1840.5	1868.8	1897.1	1925.4	1953.7
70	1982.1	2004.4	2038.7	2067.0	2095.4	2123.7	2152.0	2180.3	2208.6	2236.9
80	2265.3	2287.6	2321.9	2350.2	2378.6	2406.9	2435.2	2463.5	2491.8	2520.1
90	2548.4	2570.7	2605.0	2633.3	2661.6	2690.0	2718.3	2746.6	2774.9	2803.2
100	2831.6	2853.9	2888.2	2916.5	2944.9	2973.2	3001.5	3029.8	3058.1	3086.4

Conversion of Cubic Decimetres into Cubic Feet.

Dm³.	0	1	2	3	4	5	6	7	8	9
	ft³.	ft³.	ft³.	ft³.	ft³.	ft³.	ft³.	ft³.	ft³.	ft³.
0	0.0000	0.0353	0.0706	0.1059	0.1413	0.1766	0.2119	0.2472	0.2825	0.3178
10	0.3531	0.3884	0.4237	0.4590	0.4944	0.5297	0.5549	0.6003	0.6356	0.6709
20	0.7063	0.7416	1.4069	0.8122	0.8476	0.8829	0.9182	0.9535	0.9888	1.1241
30	1.0594	1.0947	1.1300	1.1653	1.2007	1.2360	1.2713	1.3066	1.3419	1.3772
40	1.4126	1.4479	1.4832	1.5185	1.5539	1.5892	1.6245	1.6608	1.6951	1.7304
50	1.7658	1.8011	1.8364	1.8717	1.9071	1.9424	1.9777	2.0130	2.0483	2.0836
60	2.1189	2.1542	2.1895	2.2248	2.2602	2.2955	2.3308	2.3661	2.4014	2.4367
70	2.4721	2.5074	2.5427	2.5780	2.6131	2.6487	2.6840	2.7193	2.7746	2.7899
80	2.8252	2.8605	2.8958	2.9311	2.9665	3.0018	3.0371	3.0724	3.1077	3.1430
90	3.1784	3.2137	3.2490	3.2843	3.3197	3.3550	3.3903	3.4256	3.4609	3.4962
100	3.5315	3.5668	3.6021	3.6374	3.6728	3.7081	3.7434	3.7787	3.8140	3.8493

Pounds per Sq. Foot into Kilogrammes per Sq. Metre.

Lbs. ft².	0	1	2	3	4	5	6	7	8	9
	kg. m²	kg. m²	kg. m²	kg. m²	kg. m²	kg. m²	kg. m²	kg. m²	kg. m²	kg. m²
0	0.0000	4.8825	9.7650	14.647	19.530	24.413	29.295	34.177	39.006	43.943
10	48.825	53.707	58.590	63.472	68.355	73.238	78.120	83.002	87.831	92.768
20	97.650	102.53	107.41	112.30	117.18	122.06	126.94	131.83	136.66	141.59
30	146.47	151.35	156.23	161.12	165.90	170.88	175.76	180.65	185.47	190.41
40	195.30	200.13	205.06	209.95	214.83	219.71	224.59	229.48	234.30	239.24
50	244.13	249.01	253.89	258.78	263.66	268.51	273.42	278.31	283.13	288.08
60	292.95	297.83	302.71	307.60	312.48	317.36	322.24	327.13	331.95	336.89
70	341.77	346.65	351.53	356.42	361.20	366.18	371.06	375.95	380.77	385.71
80	390.06	394.94	399.82	404.71	409.59	414.47	419.35	421.21	429.06	434.00
90	439.43	444.31	449.19	454.08	458.96	464.31	468.72	473.61	478.43	483.37
100	488.25	493.13	498.01	502.90	507.78	512.66	517.54	522.43	527.25	532.19

Kilogrammes per Sq. Metre into Pounds per Sq. Foot.

Kg. m².	0	1	2	3	4	5	6	7	8	9
	lbs.ft².	lbs.ft².	lbs.ft².	lbs.ft².	lbs.ft².	lbs.ft².	lbs.ft².	lbs.ft².	lbs.ft².	lbs.ft²
0	0.0000	0.2048	0.4096	0.6144	0.8192	1.0240	1.2289	1.4337	1.6385	1.8433
10	2.0481	2.2529	2.4577	2.6625	2.8673	3.0721	3.2770	3.4818	3.6866	3.8914
20	4.0962	4.3010	4.5058	4.7106	4.9154	5.1202	5.3251	5.5299	5.7347	5.9395
30	6.1444	6.3492	6.5540	6.7588	6.9636	7.1684	7.3733	7.5781	7.7829	7.9877
40	8.1925	8.3973	8.6021	8.8069	9.0117	9.2165	9.4214	9.6262	9.8310	10.036
50	10.240	10.445	10.649	10.854	11.059	11.264	11.469	11.674	11.878	12.083
60	12.289	12.494	12.698	12.903	13.108	13.313	13.518	13.723	13.927	14.132
70	14.337	14.542	14.746	14.951	15.156	15.364	15.566	15.771	15.975	16.180
80	16.385	16.590	16.794	16.999	17.204	17.409	17.614	17.819	18.023	18.228
90	18.433	18.638	18.842	19.047	19.252	19.457	19.662	19.867	20.071	20.276
100	20.481	20.686	20.890	21.095	21.300	21.505	21.710	21.915	22.119	22.324

Pressure per Sq. Inch into Atmospheric Pressure.

Lbs. p. in.	0	1	2	3	4	5	6	7	8	9
	at.	at.	at.	at.	at.	at.	at.	at.	at.	at.
0	0.0000	0.0680	0.1361	0.2041	0.2722	0.3402	0.4082	0.4763	0.5443	0.6124
10	0.6804	0.7484	0.8165	0.8845	0.9526	1.0266	1.0886	1.1567	1.2247	1.2928
20	1.3608	1.4288	1.4969	1.5649	1.6330	1.7070	1.7690	1.8371	1.9051	1.9732
30	2.0413	2.1093	2.1774	2.2454	2.3135	2.3875	2.4495	2.5176	2.5856	2.6537
40	2.7217	2.7897	2.8578	2.9258	2.9939	3.0679	3.1299	3.1980	3.2660	3.3341
50	3.4021	3.4701	3.5382	3.6062	3.6743	3.7483	3.8103	3.8784	3.9464	4.0145
60	4.0825	4.1505	4.2186	4.2866	4.3547	4.4287	4.4907	4.5588	4.6268	4.6949
70	4.7630	4.8310	4.8991	4.9671	5.0352	5.1092	5.1712	5.2393	5.3073	5.3754
80	5.4434	5.5114	5.5795	5.6475	5.7156	5.7896	5.8516	5.9197	5.9877	6.0558
90	6.1238	6.1918	6.2599	6.3279	6.3960	6.4690	6.5320	6.6001	6.6681	6.7362
100	6.8042	6.8722	6.9103	7.0083	7.0764	7.1504	7.2124	7.2805	7.3485	7.4166

Atmospheric Pressure into Pressure per Sq. Inch.

At. p.	0	1	2	3	4	5	6	7	8	9
	lbs. in.	lbs. in.	lbs. in.	lbs. in.	lbs. in.	lbs. in.	lbs. in.	lbs. in.	lbs. in.	lbs. in.
0	0.0000	14.697	29.393	44.090	58.787	73.483	88.180	102.87	117.57	132.27
10	146.97	161.67	176.36	191.06	205.76	220.45	235.15	249.84	264.54	279.24
20	293.93	308.63	323.32	338.02	352.72	367.41	382.11	396.80	411.50	426.20
30	440.90	455.60	470.29	484.99	499.69	514.38	529.08	543.77	558.47	573.17
40	587.87	602.57	617.26	631.96	646.66	661.35	676.05	690.74	705.44	720.14
50	734.83	749.53	764.22	778.92	793.62	808.31	823.01	837.70	852.40	867.10
60	881.80	896.50	911.19	925.89	940.59	955.28	969.98	984.67	999.37	1014.1
70	1028.7	1043.4	1058.1	1072.8	1087.5	1102.2	1116.9	1131.6	1146.3	1161.0
80	1175.7	1190.4	1205.1	1219.8	1234.5	1249.2	1263.9	1278.6	1293.3	1308.0
90	1322.7	1337.4	1352.1	1366.8	1381.5	1396.2	1410.9	1425.6	1439.3	1455.0
100	1469.7	1484.4	1499.1	1513.8	1528.5	1543.2	1557.9	1572.6	1586.3	1602.0

Pounds per Sq. In. into Kilogrammes per Sq. Centimetre.

Lbs. in^2.	0	1	2	3	4	5	6	7	8	9
	k.cm^2.	k.cm^2.	k.cm^2.	k.cm^2.	k.cm^2.	k.cm^2.	k.cm^2.	k.cm^2.	k.cm^2.	k. cm^2.
0	0.0000	0.0703	0.1406	0.2109	0.2812	0.3515	0.4218	0.4921	0.5625	0.6328
10	0.7031	0.7734	0.8437	0.9140	0.9843	1.0546	1.1249	1.1952	1.2655	1.3358
20	1.4062	1.4765	1.5468	1.6171	1.6874	1.7577	1.8280	1.8983	1.9686	2.0389
30	2.1092	2.1795	2.2498	2.3202	2.3905	2.4608	2.5311	2.6014	2.6717	2.7420
40	2.8123	2.8826	2.9529	3.0232	3.0935	3.1639	3.2342	3.3045	3.3748	3.4451
50	3.5151	3.5857	3.6560	3.7263	3.7966	3.8669	3.9372	4.0075	4.0779	4.1482
60	4.2185	4.2888	4.3591	4.4294	4.4997	4.5700	4.6403	4.7106	4.7809	4.8512
70	4.9216	4.9919	5.0622	5.1325	5.2028	5.2731	5.3434	5.4137	5.4840	5.5543
80	5.6246	5.6949	5.7652	5.8356	5.9059	5.9762	6.0465	6.1168	6.1871	6.2574
90	6.3277	6.3980	6.4683	6.5386	6.6089	6.6793	6.7496	6.8199	6.8902	6.9605
100	7.0308	7.1011	7.1714	7.2417	7.3120	7.3823	7.4526	7.5229	7.5933	7.6636

Kilogrammes per Sq. Centimetre into Pounds per Sq. In.

K. cm^2.	0	1	2	3	4	5	6	7	8	9
	lbs.in^2	lbs.in^2	lbs.in^2	lbs.in^2	lbs.in^2	lbs.in^2	.bs.in^2	lbs.in^2	lbs.in^2	lbs. in.^{2}
0	0.0000	14.223	28.446	42.670	56.893	71.116	85.339	99.562	113.78	128.01
10	142.23	156.45	170.68	184.90	199.12	213.35	227.57	241.79	256.02	270.24
20	281.46	298.69	312.91	327.13	341.36	355.58	369.80	384.03	398.25	412.47
30	426.70	440.92	455.14	469.36	483.59	497.81	512.03	526.26	540.48	554.70
40	568.93	583.15	597.37	611.60	625.82	640.01	654.27	668.49	682.71	696.94
50	711.16	725.38	739.61	753.83	768.05	782.28	796.50	810.72	824.94	839.17
60	853.39	867.61	881.84	896.06	910.28	924.51	938.73	952.95	967.18	981.40
70	995.62	1009.8	1021.1	1038.3	1052.5	1066.7	1081.0	1095.2	1109.4	1123.6
80	1137.8	1152.1	1166.3	1180.5	1194.7	1209.0	1223.2	1237.4	1251.6	1265.9
90	1280.1	1294.3	1308.5	1322.7	1337.0	1351.2	1365.4	1379.6	1393.9	1408.1
100	1422.3	1436.5	1450.8	1465.0	1479.2	1493.4	1507.7	1521.9	1536.1	1550.3

Conversion of English Pounds into Kilogrammes.

Eng. Lbs.	0	1	2	3	4	5	6	7	8	9
	Kilos.	Kilos.	Kilos.	Kilos.	Kilos.	Kilos.	Kilos.	Kilos.	Kilos.	Kilos.
0	0.000	0.453	0.907	1.361	1.814	2.268	2.722	3.175	3.629	4.082
10	4.536	4.989	5.443	5.897	6.350	6.804	7.258	7.711	8.165	8.618
20	9.072	9.525	9.979	10.43	10.89	11.34	11.79	12.25	12.70	13.15
30	13.61	14.06	14.52	14.97	15.42	15.88	16.33	16.78	17.24	17.69
40	18.14	18.59	19.05	19.50	19.95	20.41	20.86	21.31	21.77	22.22
50	22.68	23.13	23.59	24.04	24.49	24.95	25.40	25.85	26.31	26.76
60	27.22	27.67	28.13	28.58	29.03	29.49	29.94	30.39	30.85	31.30
70	31.75	32.20	32.66	33.11	33.56	34.02	34.47	34.92	35.38	35.83
80	36.29	36.74	37.20	37.65	38.10	38.56	39.01	39.46	39.92	40.37
90	40.82	41.27	41.73	42.18	42.63	43.09	43.54	43.99	44.45	44.90
100	45.36	45.81	46.27	46.72	47.17	47.63	48.08	48.53	48.99	49.44

Conversion of Kilogrammes into English Pounds.

Fr. Kil.	0	1	2	3	4	5	6	7	8	9
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
0	0.000	2.205	4.410	6.615	8.820	11 02	13.23	15.43	17.64	19.84
10	22.05	24.25	26.46	28.67	30.87	33.07	35.28	37.48	39.69	41.89
20	44.10	46.30	48.51	50.72	52.92	55.12	57.33	59.53	61.74	63.94
30	66.15	68.35	70.56	72.77	74.97	77.17	79.38	81.58	83.79	85.99
40	88.20	90.40	92.61	94.82	97.02	99.22	101.4	93.63	105.8	90.04
50	110.2	112.5	114.6	116.8	119.0	121.2	123.4	125.6	127.8	130.0
60	132.3	134.5	136.7	138.9	141.1	143.3	145.5	147.7	149.9	152.1
70	154.3	156.5	158.7	160.9	163.1	165.3	167.5	169.7	171.9	174.1
80	176.4	178.6	180.8	183.0	185.2	187.4	189.6	191.8	194.0	196.2
90	198.4	200.6	202.8	205.0	207.2	209.4	211.6	213.8	216.0	218.2
100	220.5	222.7	224.9	227.1	229.3	231.5	233.7	235.9	238.1	240.3

Conversion of English Tons into Metric Tons.

Eng. Tons.	0	1	2	3	4	5	6	7	8	9
	M.tons.	M.tons.	M.tons.	M.tons.	M.tons.	M.tons.	M.tons.	M.tons.	M.tons.	M.tons.
0	0.000	1.016	2.032	3.048	4.064	5.080	6.096	7.112	8.128	9.144
10	10.16	11.18	12.19	13.21	14.12	15.24	16.26	17.27	18.29	19.30
20	20.32	21.34	22.35	23.37	24.38	25.40	26.42	27.43	28.45	29.46
30	30.48	31.50	32.51	33.53	34.54	35.56	36.58	37.59	38.61	39.62
40	40.64	41.66	42.67	43.69	44.70	45.74	46.74	47.75	48.77	49.78
50	50.80	51.82	52.83	53.85	54.86	55.88	56.90	57.90	58.93	59.94
60	60.96	61.97	62.99	64.01	65.02	66.04	67.06	68.07	69.09	70.10
70	71.12	72.14	73.15	74.17	75.18	76.20	77.22	78.23	79.25	80.26
80	81.28	82.29	83.31	84.33	85.34	86.36	87.38	88.39	89.41	90.42
90	91.44	92.46	93.47	94.49	95.50	96.52	97.54	98.55	99.57	100.6
100	101.6	102.6	103.6	104.6	105.7	106.7	107.7	108.7	109.7	110.7

Conversion of Metric Tons into English Tons.

Fr.M.Tons.	0	1	2	3	4	5	6	7	8	9
	E. tons.	E. tons.	E. tons.	E. tons.	E. tons.	E. tons.	E. tons	E. tons.	E. tons.	E. tons.
0	0.000	0.984	1.969	2.953	3.937	4.921	5.906	6.890	7.874	8.858
10	9.843	10.83	11.81	12.79	13.78	14.76	15.75	16.73	17.72	18.70
20	19.69	20.67	21.66	22.64	23.63	24.61	25.60	26.58	27.56	28.55
30	29.53	30.51	31.50	32.48	33.47	34.45	35.44	36.42	37.40	38.39
40	39.37	40.35	41.34	42.32	43.31	44.29	45.28	46.26	47.24	48.23
50	49.21	50.19	51.18	52.16	53.15	54.13	55.12	56.10	57.08	58.07
60	59.06	60.04	61.03	62.01	63.00	63.98	64.97	65.95	66.93	67.92
70	68.90	69.88	70.87	71.85	72.84	73.82	74.81	75.79	76.77	77.76
80	78.74	79.72	80.71	81.69	82.68	83.66	84.65	85.63	86.61	87.60
90	88.58	89.56	90.55	91.53	92.52	93.50	94.49	95.47	96.45	97.44
100	98.43	99.41	100.4	101.4	102.4	103.3	104.3	105.3	106.3	107.3

Conversion of Eng. Ounces Avoirdupois into Fr. Grammes.

English Ounces.	0 Grams	1 Grams	2 Grams	3 Grams	4 Grams	5 Grams	6 Grams	7 Grams	8 Grams	9 Grams
0	0.0000	28.348	56.697	85.046	113.39	141.74	170.09	198.44	226.79	255.14
10	283.48	311.83	340.18	368.52	396.87	425.22	453.57	481.92	510.27	538.62
20	566.97	595.32	623.67	652.01	680.36	708.71	737.06	765.41	793.76	822.11
30	850.46	878.81	907.16	93,550	963.85	992.20	1020.5	1048.9	1077.2	1105.6
40	1133.9	1162.2	1190.6	1218.9	1247.3	1275.6	1304.0	1332.3	1360.7	1389.0
50	1417.4	1445.7	1474.1	1502.4	1530.8	1559.1	1587.5	1615.8	1644.2	1672.5
60	1700.9	1729.2	1756.6	1785.9	1814.3	1842.9	1871.0	1899.3	1927.7	1956.0
70	1984.4	2012.7	2041.1	2079.4	2097.8	2126.1	2154.5	2182.8	2211.2	2239.5
80	2267.9	2296.2	2324.6	2352.9	2381.3	2409.6	2438.0	2466.3	2494.7	2523.0
90	2551.4	2579.7	2608.1	2636.4	2664.8	2693.1	2721.5	2739.8	2778.2	2806.5
100	2834.8	2863.1	2891.5	2919.8	2948.2	2976.5	3004.9	3033.2	3061.6	3089.9

Conversion of Fr. Grammes into Eng. Ounces Avoirdupois.

French Grammes.	0 Oz.	1 Oz.	2 Oz.	3 Oz.	4 Oz.	5 Oz.	6 Oz.	7 Oz.	8 Oz.	9 Oz.
0	0.0000	0.0353	0.0705	0.1058	0.1411	0.1768	0.2116	0.2469	0.2822	0.3175
10	0.3527	0.3880	0.4232	0.4585	0.4938	0.5295	0.5643	0.5996	0.6349	0.6702
20	0.7055	0.7408	0.7760	0.8113	0.8466	0.8823	0.9171	0.9524	0.9877	1.0230
30	1.0582	1.0935	1.1287	1.1640	1.1993	1.2350	1.2698	1.3051	1.3404	1.3757
40	1.4110	1.4463	1.4815	1.5168	1.5521	1.5878	1.6226	1.6579	1.6932	1.7285
50	1.7687	1.8040	1.8392	1.8745	1.9098	1.9455	1.9803	2.0156	2.0509	2.0862
60	2.1165	2.1518	2.1870	2.2223	2.2576	2.2933	2.3281	2.3634	2.3987	2.4340
70	2.4692	2.5045	2.5397	2.5750	2.6103	2.6460	2.6808	2.7161	2.7514	2.7867
80	2.8220	2.8573	2.8925	2.9278	2.9631	2.9988	3.0336	3.0689	3.1042	3.1395
90	3.1747	3.2100	3.2452	3.2805	3.3158	3.3515	3.3863	3.4216	3.4569	3.4922
100	3.5275	3.5628	3.5980	3.6333	3.6686	3.7043	3.7391	3.7744	3.8097	3.8450

Conversion of Eng. Grains Troy into Fr. Grammes.

English Grains.	0 Grams	1 Grams	2 Grams	3 Grams	4 Grams	5 Grams	6 Grams	7 Grams	8 Grams	9 Grams
0	0.0000	0.0648	0.1296	0.1944	0.2592	0.3240	0.3888	0.4535	0.5183	0.5831
10	0.6479	0.7127	0.7775	0.8423	0.9071	0.9719	1.0367	1.1014	1.1662	1.2310
20	1.2959	1.3607	1.4255	1.4903	1.5551	1.6199	1.6847	1.7494	1.8142	1.8890
30	1.9438	2.0086	2.0734	2.1382	2.2030	2.2678	2.3326	2.3973	2.4621	2.5269
40	2.5918	2.6566	2.7214	2.7862	2.8510	2.9158	2.9806	3.0453	3.1101	3.1749
50	3.2398	3.3046	3.3694	3.4342	3.4990	3.5638	3.6286	3.6933	3.7581	3.8229
60	3.8877	3.9525	4.0173	4.0821	4.1469	4.2117	4.2765	4.3412	4.4060	4.4708
70	4.5357	4.6005	4.6653	4.7301	4.7949	4.8597	4.9245	4.9892	5.0540	5.1188
80	5.1830	5.2484	5.3132	5.3780	5.4428	5.5076	5.5724	5.6371	5.7019	5.7667
90	5.8316	5.8964	5.9612	6.0260	6.0908	6.1556	6.2204	6.2851	6.3499	6.4147
100	6.4795	6.5443	6.6091	6.6739	6.7387	6.8035	6.8683	6.9330	6.9978	7.0626

Conversion of Fr. Grammes into Eng. Grains Troy.

French Grammes.	0 Grs.	1 Grs.	2 Grs.	3 Grs.	4 Grs.	5 Grs.	6 Grs.	7 Grs.	8 Grs.	9 Grs.
0	0.0000	15.433	30.866	46.299	61.732	77.165	92.599	105.03	123.46	138.90
10	154.33	169.76	185.19	200.63	216.06	231.49	246.93	259.36	277.79	293.23
20	308.66	324.09	339.52	354.96	370.39	385.82	401.26	413.69	432.12	447.56
30	462.99	478.42	493.86	509.29	524.72	540.15	555.59	568.02	586.45	601.89
40	617.65	633.08	648.51	663.95	679.38	694.81	710.25	722.68	741.11	756.55
50	771.65	787.08	802.52	817.95	833.38	848.82	864.25	876.68	895.11	910.55
60	925.99	941.42	956.85	972.29	987.72	1003.1	1018.6	1031.0	1049.4	1064.9
70	1050.3	1065.7	1081.1	1096.6	1112.0	1127.5	1142.9	1155.3	1173.7	1189.2
80	1234.6	1258.7	1274.2	1289.6	1305.0	1320.4	1335.9	1348.3	1366.7	1382.2
90	1389.0	1404.4	1419.8	1435.3	1450.7	1466.1	1481.6	1494.0	1512.4	1527.9
100	1543.3	1558.7	1574.1	1589.6	1605.0	1620.4	1635.9	1643.3	1666.7	1681.2

Horse-power into Puissance de Cheval.

HP	0	1	2	3	4	5	6	7	8	9
	P. C.	P. C.	P. C.	P. C.	P. C.	P. C.	P. C.	P. C.	P. C.	P. C.
0	0.0000	1.0136	2.0272	3.0408	4.0544	5.0680	6.0816	7.0952	8.1088	9.1224
10	10.136	11.150	12.163	13.176	14.190	15.204	16.218	17.231	18.245	19.258
20	20.272	21.308	22.299	23.313	24.326	25.240	26.354	27.367	28.381	29.394
30	30.408	31.422	32.435	33.449	34.462	35.476	36.490	37.503	38.517	39.530
40	40.544	41.557	42.571	43.585	44.598	45.612	46.626	47.639	48.653	49.666
50	50.680	50.781	52.707	53.721	54.734	55.748	56.762	57.775	58.789	59.802
60	60.816	61.829	62.843	63.857	64.870	65.884	66.898	67.911	68.925	69.938
70	70.952	71.965	72.979	73.993	75.006	76.020	77.034	78.047	79.061	80.074
80	81.088	82.102	83.115	84.129	85.142	85.156	87.170	88.183	89.197	90.210
90	91.224	92.338	93.251	94.265	95.278	96.292	97.306	98.319	99.333	100.34
100	101.36	102.37	103.30	104.40	105.41	106.43	107.44	108.45	109.47	110.48

Puissance de Cheval into Horse-power.

P. C.	0	1	2	3	4	5	6	7	8	9
	HP	HP	HP	HP	HP	HP	HP	HP	HP	HP
0	0.0000	0.9863	1.9726	2.9589	3.9452	4.9315	5.9178	6.9041	7.8904	8.8767
10	9.8630	10.849	11.835	12.822	13.808	14.794	15.781	16.767	17.753	18.739
20	19.726	20.712	21.698	22.685	23.671	24.657	25.644	26.630	27.616	28.602
30	29.589	30.575	31.561	32.548	33.534	34.520	35.507	36.493	37.479	38.465
40	39.452	40.438	41.424	42.411	43.397	44.383	45.370	46.356	47.342	48.328
50	49.315	50.301	51.287	52.274	53.260	54.246	55.233	56.219	57.205	58.191
60	59.178	60.164	61.150	62.137	63.123	64.109	65.096	66.082	67.068	68.654
70	69.041	70.027	71.013	71.990	72.986	73.972	74.959	75.945	76.941	77.917
80	78.904	79.890	80.876	81.863	82.849	83.835	84.822	85.808	86.794	87.780
90	88.767	89.753	90.739	91.726	92.712	93.698	94.785	95.671	96.657	97.643
100	98.630	99.616	100.60	101.59	102.57	103.56	104.55	105.53	106.52	107.50

Power or Work. Foot-pounds into Kilogrammetres.

Ft. lbs.	0	1	2	3	4	5	6	7	8	9
	k. m.	k. m.	k. m.	k. m.	k. m.	k. m.	k. m.	k. m.	k. m.	k. m.
0	0.0000	0.1382	0.2764	0.4146	0.5528	0.6910	0.8292	0.9674	1.1056	1.2438
10	1.3820	1.5202	1.6584	1.7966	1.9348	2.0731	2.2112	2.3494	2.4876	2.6259
20	2.7640	2.9022	3.0404	3.1786	3.3168	3.4552	3.5933	3.7315	3.8696	4.0078
30	4.1460	4.2842	4.4224	4.5606	4.6988	4.8370	4.9751	5.1131	5.2517	5.3897
40	5.5280	5.6666	5.8044	5.9426	6.0808	6.2191	6.3572	6.4954	6.6336	6.7718
50	6.9100	7.0482	7.1864	7.3246	7.4628	7.6010	7.7393	7.8775	8.0155	8.1538
60	8.2920	8.4303	8.5684	8.7066	8.8448	8.9830	9.1212	9.2594	9.3976	9.5359
70	9.6740	9.8122	9.9504	10.088	10.227	10.365	10.503	10,641	10.779	10.918
80	11.056	11.194	11.322	11.570	11.609	11.747	11.885	12.023	12.161	12.300
90	12.438	12.576	12.714	12.855	12.991	13.129	13.267	13.405	13.544	13.682
100	13.820	13.958	14.096	14.235	14.373	14.511	14.649	14.787	14.925	14.064

Power or Work. Kilogrammetres into Foot-pounds.

K. m.	0	1	2	3	4	5	6	7	8	9
	ft. lbs.	ft. lbs.	ft. lbs.	ft. lbs.	ft. lbs.	ft. lbs.	ft. lbs.	ft. lbs.	ft. lbs.	ft. lbs.
0	0.0000	7.2334	14.467	21.700	28.934	36.166	43.400	50.734	57.868	65.100
10	72.334	79.567	87.101	91.034	101.27	108.50	115.74	123.07	130.20	137.43
20	144.67	151.90	158.43	166.37	173.60	180.81	188.08	195.40	202.54	209.77
30	217.00	224.23	231.77	238.70	245.93	253.17	260.41	267.73	274.87	282.10
40	289.34	296.57	304.11	311.04	318.27	325.50	332.75	310.07	347.21	354.44
50	361.66	368.89	376.43	383.36	390.59	397.82	405.07	412.39	419.53	426.76
60	434.00	441.23	448.77	455.70	462.93	470.17	477.41	484.73	491.87	499.10
70	507.34	514 57	522.11	529.04	536.27	543.50	550.75	558.07	565.21	572.44
80	578.68	585.91	593.45	599.38	607.61	614.85	622.09	629.41	636.55	643.78
90	651.00	658.23	665.77	672.70	679.93	687.17	694.41	701.73	708.87	716.10
100	723.34	730.57	738.11	745.04	752.27	759.51	766.75	774.07	781.21	788.44

Conversion of Foot-Tons into Tonnes-Metres.

Ft. tn.	0	1	2	3	4	5	6	7	8	9
	t. m.	t. m.	t. m.	t. m.	t. m.	t. m.	t. m.	t. m.	t. m.	t. m.
0	0.0000	0.3097	0.6194	0.9291	1.2382	1.5484	1.8581	2.1678	2.4775	2.7872
10	3.0969	3.3166	3.7163	4.0260	4.3356	4.6453	4.9550	5.2667	5.5744	5.8841
20	6.1938	6.4135	6.8132	7.1229	7.4325	7.7422	8.0519	8.3636	8.6713	8.9810
30	9.2906	9.6003	9.9100	10.219	10.529	10.839	11.149	11.460	11.768	12.078
40	12.387	12.697	13.006	13.316	13.626	13.935	14.245	14.557	14.864	15.174
50	15.484	15.794	16.103	16.413	16.723	17.032	17.342	17.654	17.961	18.271
60	18.581	18.891	19.200	19.510	19.820	20.129	20.439	20.751	21.058	21.368
70	21.678	21.988	22.297	22.607	22.917	23.226	23.536	23.848	24.155	24.465
80	24.775	24.085	25.394	25.704	26.014	26.323	26.633	26.945	27.252	27.562
90	27.872	28.182	28.491	28.801	29.111	29.420	29.730	30.042	30.349	30.659
100	30.969	31.279	31.588	31.898	32.208	32.517	32.827	33.139	33.446	33.756

Conversion of Tonnes-Metres into Foot-Tons.

T. M.	0	1	2	3	4	5	6	7	8	9
	ft. t.	ft. t.	ft. t.	ft. t.	ft. t.	ft. t.	ft. t.	ft. t.	ft. t.	ft. t.
0	0.0000	3.2290	6.4581	9.6871	12.916	16.145	19.374	22.603	25.832	29.061
10	32.290	35.519	38.758	41.977	45.206	48.435	51.664	54.893	58.122	61.351
20	64.581	67.810	71.049	74.268	77.497	80.726	83.955	87.184	90.413	93.642
30	96.871	100.10	103.34	106.56	109.79	113.01	116.24	119.47	122.70	125.93
40	129.16	133.39	135.63	138.85	142.07	145.30	148.53	151.76	154.99	158.22
50	161.45	164.68	167.92	171.14	174.36	177.59	180.82	184.05	187.28	190.51
60	193.74	196.97	200.21	203.43	206.65	209.88	213.11	216.34	219.57	222.80
70	226.03	229.26	232.50	235.72	238.94	242.17	245.40	248.63	251.86	255.09
80	258.32	261.55	264.79	268.01	271.23	274.46	277.69	280.92	284.15	287.38
90	290.61	293.84	297.08	300.30	303.52	306.75	309.98	313.21	316.44	319.67
100	322.90	326.13	329.37	332.59	335.81	339.04	342.27	345.50	348.73	351.96

English Units of Heat into French Calories.

Heat.	0	1	2	3	4	5	6	7	8	9
	cal.	cal.	cal.	cal.	cal.	cal.	cal.	cal.	cal.	cal.
0	0.0000	0.2520	0.5040	0.7560	1.0080	1.2600	1.5120	1.7640	2.0160	2.2680
10	2.5200	2.7720	3.0240	3.2760	3.5280	3.7800	4.0320	4.2840	4.5360	4.7880
20	5.0399	5.2919	5.5439	5.7959	6.0478	6.2699	6.5419	6.8039	7.0559	7.3079
30	7.5600	7.8120	8.0640	8.3160	8.5680	8.8200	9.0720	9.3340	9.5760	9.8280
40	10.080	10.332	10.584	10.836	11.088	11.340	11.512	11.844	12.096	12.348
50	12.600	12.852	13.104	13.356	13.608	13.860	14.112	14.364	14.616	14.868
60	15.120	15.372	15.624	15.876	16.128	16.380	16.632	16.884	17.136	17.388
70	17.640	17.892	18.144	18.396	18.648	18.900	19.152	19.404	19.656	19.908
80	20.160	20.412	20.664	20.916	21.168	21.420	21.672	21.924	22.176	22.428
90	22.680	22.932	23.184	23.436	23.688	23.940	24.192	24.444	24.696	24.948
100	25.200	25.452	25.704	25.956	26.208	26.460	26.712	26.964	27.216	27.468

French Calories into English Units of Heat.

Calories.	0	1	2	3	4	5	6	7	8	9
	ht.	ht.	ht.	ht.	ht.	ht.	ht.	ht.	ht.	ht.
0	0.0000	3.9683	7.9366	11.905	15.873	19.842	22.810	27.778	31.746	35.715
10	39.683	43.651	47.620	51.598	55.520	59.525	62.493	67.461	71.429	75.398
20	79.366	83.334	87.303	91.271	95.203	99.208	102.24	107.14	111.11	115.08
30	119.05	123.02	126.98	130.95	134.89	138.89	141.86	146.83	150.80	154.77
40	158.73	162.70	166.66	170.62	174.57	178.57	181.54	186.51	190.48	194.45
50	198.42	202.39	206.35	210.39	214.26	218.26	221.23	226.20	230.16	234.14
60	228.10	232.07	236.03	240.00	243.94	248.94	250.91	255.88	259.85	263.82
70	277.78	281.75	285.72	280.68	293.62	297.62	300.59	305.56	309.53	313.50
80	317.46	321.43	325.40	329.36	323.29	337.30	340.27	345.24	319.20	353.18
90	357.15	361.12	365.09	369.05	372.98	376.99	379.96	384.93	388.90	392.87
100	396.83	400.80	404.77	408.73	412.67	416.67	419.64	424.61	428.58	432.55

ABBREVIATIONS OF METRIC NOMENCLATURE.

THE following abbreviations have been adopted by the International Metrical Congress at Paris, and are recommended for general use:

Length.

km means kilometre.
m " metre.
dm " decimetre.
cm " centimetre.
mm " millimetre.

Surface.

km^2 means square kilometres.
m^2 " square metre.
dm^2 " square decimetre.
cm^2 " square centimetre.
mm^2 " square millimetre.

Volume.

km^3 means cubic kilometre.
m^3 " cubic metre.
dm^3 " cubic decimetre.
cm^3 " cubic centimetre.
mm^3 " cubic millimetre.

Land Measures.

ha means hectare.
a " are.

Hollow Measures.

hl means hectolitre.
l " litre.
dl " decilitre.
cl " centilitre.

Weights.

t means tons.
q " hundredweight.
kg " kilogramme.
dkg " decagramme.
g " gramme.
dg " decigramme.
cg " centigramme.
mg " milligramme.

The abbreviations should invariably appear in italic letters, and no stop to be used at the right of them except when at the end of a sentence. The abbreviations succeed the figures to which they refer, on the same line, and after the last decimal when such are used.

To the above abbreviations the writer proposes to add the following:

ef. for power or effects in kilogrammetre per second.
pc. for power puissance de cheval.
kgm. for work in kilogrammetres.

Puissance de Cheval is the correct expression for what the French call *force de cheval*. They do not mean *force*, but *power*. Force de cheval is the force with which the horse pulls, and not the effect, 75 kilogrammetre per second, which is *power* or *puissance*.

It would be advisable to adopt a similar system of abbreviations for English measures—namely, as follows:

Length.

in. for inches.
ft. " feet.
yd. " yards.
ch. " chains.
ml. " miles.

Surface.

in^2. for square inches.
ft^2. " square feet.
yd^2. " square yards.
ch^2. " square chains.
ml^2. " square miles.

Volume.

in^3. for cubic inches.
ft^3. " cubic feet.
yd^3. " cubic yards.

Weights.

oz. for ounces.
lbs. " pounds.
cwt. " hundredweights.
tn. " ton.

FORMULAS.		DIFFERENTIALS.	
$y = x$		$dy = dx,$	1
$y = ax^2$		$dy = 2\,a\,x\,dx,$	2
$y = x^n$		$dy = nx^{n-1}dx,$	3
$3\,a\,b\,x^3$	$=$	$9\,a\,b\,x^2\,dx,$	4
$4\,a\,b^2\,x^n$	$=$	$4\,n\,a\,b^2 x^{n-1}\,dx,$	5
$a + x^3$	$=$	$3\,x^2\,dx,$	6
$(a+b)x^2$	$=$	$2\,x(a+b)dx,$	7
$6a\,b^4\,x^3 - c$	$=$	$18\,a\,b^4\,x^2\,dx,$	8
$x + 3\,z^3 - v$	$=$	$dx + 6\,z\,dz - dv,$	9
$6x^3 + 4a^{x2} - 3x$	$=$	$(18x^2 + 8a\,x - 3)dx,$	10
$x\,v^2$	$=$	$v\,dx + 2\,x\,v\,dv,$	11
$x\,v\,z$	$=$	$x\,v\,z\left(\frac{dx}{x}+\frac{dv}{v}+\frac{dz}{z}\right)$	12
$x(x^2 - x\,b^2)$	$=$	$(3\,x^2 - b^2 x)\,dx,$	13
$\frac{x^2}{v}$	$=$	$\frac{2\,x\,v\,dx - x^2\,dv}{v^2},$	14
$\frac{a}{x}$	$=$	$\frac{a\,dx}{x^3},$	15
$\frac{a}{x^n}$	$=$	$\frac{n\,a\,x^{n-1}\,dx}{x^{2n}},$	16
$(a+\sqrt{x})^3$	$=$	$\frac{3(a+\sqrt{x})^2 dx}{2\sqrt{x}}$	17
$(a+\sqrt[n]{x})^m$	$-$	$m\,(a+\sqrt[n]{x})^{n-1}\frac{1}{n}x^{\frac{1}{n}-1}dx,$	18
$\frac{1}{4(a-x)}$	$=$	$\frac{dx}{(a-x)^{n+1}},$	19
$\frac{2\sqrt{2\,a\,x - x^2}}{x}$	$=$	$\frac{2\,a\,dx}{x\sqrt{2\,ax - x^2}},$	20

FORMULAS.		DIFFERENTIALS.	
a^x	$=$	$a^x\,l\cdot a\,dx,$	21
$d\cdot l\cdot x$	$=$	$\frac{dx}{x},$	22
$x\,l\cdot x$	$=$	$(1 + l\cdot x)dx,$	23
$\frac{l\cdot x}{x^n}$	$=$	$\frac{(1 - l\cdot x)dx}{x^{n+1}},$	24
$\frac{x}{l\cdot x}$	$=$	$\frac{(l\cdot x - 1)dx}{(l\cdot x)^2},$	25
$\frac{a\,y}{\sqrt{x^2+y^2}}$	$=$	$\frac{a\,y\,x\,dx - a\,x^2 dy}{\sqrt{(x^2+y^2)^3}},$	26
$\frac{a - 2\,b\,x}{(a+b\,x)^2}$	$=$	$\frac{2\,b^2\,x\,d\,x}{(a+bx)^3},$	27
$\sqrt{x} = x^{\frac{1}{2}}$	$=$	$\frac{dx}{2\sqrt{x}},$	28
$(a\,x + x^2)^n$	$=$	$n(ax+x^2)^{n-1}(a+2x)dx$	29
$\sqrt{a^2+bx^2}$	$=$	$\frac{b\,x\,d\,x}{\sqrt{a^2+b\,x^2}},$	30
$d\cdot^2(a\,x^3)$	$=$	$6\,a\,x\,dx^2,$	31
$d^3(a\,x^3)$	$=$	$6\,a\,dx^3,$	32
$d^4(a\,x^3)$	$=$	$o\,b\,a\,x^{o-1}dx^4 = o,$	33
$\sin.v$	$=$	$+\ \cos.v\,d\,v,$	34
$\cos.v$	$=$	$-\ \sin.v\,dv$	35
$\tan.v$	$=$	$+\ \frac{dv}{\cos.^2 v},$	36
$\cot.v$	$=$	$-\ \frac{dv}{\sin.^2 v},$	37
$\sec.v$	$=$	$+\ \frac{\cos.v\,dv}{\cos.^2 v},$	38
$\mathrm{cosec}.v$	$=$	$-\ \frac{\cos.v\,dv}{\sin.^2 v},$	39

Tan. for any curve $t = y\sqrt{1+\frac{dx^2}{dy^2}}$, 40

See page 66.

Comparison between Foot-measures of Different Nations.

Linear Feet.

English.	Metre.	Prussia.	Saxony.	Baden.	Austria.	Hanover	Sweden.
1	0.3048	0.9711	1.0763	1.0160	0.9642	1.0435	1.0265
3.2809	1	3.1862	3.5312	3.3333	3.1634	3.4235	3.3678
1.0297	0.3138	1	1.1083	1.0462	0.9929	1.0745	1.0572
0.9291	0.2832	0.9023	1	0.9440	0.8959	0.9695	0.9538
0.9843	0.3000	0.9559	1.0594	1	0.9490	1.0271	1.0164
1.0371	0.3161	1.0072	1.1163	1.0537	1	1.0822	1.0963
0.9583	0.2921	0.9307	1.0314	0.9736	0.9240	1	0.9838
0.9741	0.2969	0.9459	1.0484	0.9838	0.9122	1.0165	1

Square Feet.

English.	Metre.	Prussia.	Saxony.	Baden.	Austria.	Hanover	Sweden.
1	0.0929	0.9431	1.1584	1.0322	0.9297	1.0888	1.0537
10.764	1	10.152	12.469	11.111	10.007	11.721	11.342
1.0603	0.0985	1	1.2283	1.0945	0.9858	1.1545	1.1130
0.8603	0.0802	0.8141	1	0.8911	0.8026	0.9400	0.9097
0.9688	0.0900	0.9137	1.1222	1	0.9007	1.0549	1.0330
1.0756	0.0999	1.0144	1.2460	1.1103	1	1.1712	1.2019
0.9184	0.0853	0.8661	1.0639	0.9480	0.8538	1	0.9679
0.9489	0.0881	0.8947	1.0941	0.9679	0.8321	1.0331	1

Cubic Feet.

English.	Metre.	Prussia.	Saxony.	Baden.	Austria.	Hanover	Sweden.
1	0.0283	0.9159	1.2468	1.0487	0.8964	1.1362	1.1018
35.316	1	32.346	44.032	37.037	31.658	40.126	38.198
1.0918	0.0309	1	1.3613	1.1450	0.9787	1.2405	1.1816
0.8021	0.0227	0.7346	1	0.8411	0.7190	0.9113	0.8677
0.9535	0.0270	0.8733	1.1889	1	0.8548	1.0834	1.0501
1.0756	0.0999	1.0144	1.2460	1.1103	1	1.1712	1.3176
0.8801	0.0249	0.8061	1.0973	0.9230	0.7890	1	0.9522
0.9243	0.0262	0.8483	1.1444	0.9522	0.7590	1.0501	1

Conversion of Pounds of Different Nations.

Eng. av.	Kilogram.	Prussia.	Austria.	Spain.	Hanover	Russia.	Sweden.
1	0.4536	0.9072	0.8110	0.9839	0.9320	1.1076	1.0664
2.2046	1	2.0000	1.7857	2.1692	1.9842	2.4419	2.3511
1.1023	0.5000	1	0.8929	1.0857	1.0271	1.2209	1.1755
1.2346	0.5600	1.1209	1	1.2132	1.1490	1.3675	1.3166
1.0164	0.4610	0.9211	0.8243	1	0.9470	1.1257	1.0839
1.0730	0.4696	0.9752	0.8596	1.0557	1	1.1884	1.1442
0.9028	0.4095	0.8190	0.7313	0.8883	0.8414	1	0.9628
0.9377	0.4253	0.8508	0.7595	0.9226	0.8738	1.0386	1

Ancient Measures of Length.

	Feet.	Inches.
Scripture.		
Digit, ...	. . .	0.912
Palm = 4 Digits, .	. . .	3.648
Span = 3 Palms, . .	. . .	10.94
Cubit = 2 Spans, .	1	9.888
Fathom = 3.46 Cubits, .	7	3.552
Egyptian. Finger,	. . .	.7374
Nahud Cubit, . .	1	5.71
Royal Cubit, . .	1	8.66
Grecian.		
Digit, ...	. . .	0.754
Pous = 16 Digits, .	1	.0875
Cubit, ...	1	1.598
Stadium, ...	604	4.5
Mile = 8 Stadiums, .	4835	. . .

	Feet.	Inches.
Hebrew.		
Cubit, ...	1	9.868
Sabbath day's journey,	3648	. . .
Mile = 4000 Cubits, .	7296	. . .
Day's journey = 33.164 mi.	. . .	. . .
Sacred Cubit, . . .	2	0.24
Roman.		
Digit, ...	. . .	.7257
Uncia (Inch), . . .	. . .	.967
Pes (foot) = 12 Uncias, .	. . .	11.60
Cubit = 24 Digits, .	1	5.406
Passus = 3.33 Cubits, .	4	10.02
Millarium (mile), . .	4842	. . .
Arabian. Foot, .	1	1.14
Babylonian. Foot,	1	1.68

Foreign Measures of Length Compared with American.

Places.	Measures.	Inches.	Places.	Measures.	Inches.
Amsterdam,	Foot. . . .	11·14	Malta, . .	Foot, . .	11·17
Antwerp, .	" . . .	11·24	Moscow, .	" . . .	13·17
Bavaria, .	" . . .	11·42	Naples, . .	Palmo, . .	10·38
Berlin, . .	" . . .	12·19	Prussia, .	Foot, . . .	12·36
Bremen, .	" . . .	11·38	Persia, . .	Arish, . .	38·27
Brussels, .	" . . .	11·45	Rhineland,	Foot, . . .	12·35
China, . .	" mathematic,	13·12	Riga, . .	" . .	10·79
" . .	" builder's, .	12·71	Rome,	" . . .	11·60
" .	" tradesman's,	13·32	Russia, . .	" . .	13·75
" . .	" surveyor's .	12·58	Sardinia,	Palmo, . .	9·78
Copenhagen,	" . . .	12·35	Sicily, . .	" . .	9·53
Dresden, .	" . . .	11·14	Spain, .	Foot, . . .	11·03
England, .	" . . .	12·00	" . .	Toesas, . .	66·72
Florence, .	Braccio. . .	21·69	" .	Palmo, . .	8·64
France, .	Pied de Roi, .	12·79	Strasburg,	Foot, . .	11·39
" . .	Metre, . . .	39·381	Sweden, .	" . . .	11·69
Geneva, .	Foot, . .	19·20	Turin, .	" . .	12·72
Genoa, . .	Palmo, . .	9·72	Venice, . .	" . . .	13·40
Hamburg,	Foot, . .	11·29	Vienna, .	" . .	12·45
Hanover, .	" . . .	11·45	Zurich, . .	" . . .	11·81
Leipsic, .	" . . .	11·11	Utrecht, .	" . .	10·74
Lisbon, . .	" . . .	12·96	Warsaw, .	" . . .	14·03
" . .	Palmo, . .	8·64			

Foreign Road Measures Compared with American.

Places.	Measures.	Yards.	Places.	Measures.	Yards.
Arabia, . .	Mile, . . .	2148	Hungary, .	Mile, . . .	9113
Bohemia, .	" . .	10137	Ireland, .	" . . .	3038
China, . .	Li, . . .	629	Netherlands,	" . . .	1093
Denmark,	Mile, . .	8244	Persia, . .	Parasang, .	6086
England, .	" statute, .	1760	Poland, .	Mile, long, . .	8101
" .	" geographical,	2025	Portugal, .	League, . .	6760
Flanders, .	" . . .	6869	Prussia, .	Mile, . . .	8468
France, .	League, marine,	6075	Rome, . .	" . .	2025
" . .	" common, .	4861	Russia, .	Verst, . . .	1167
" . .	" post, .	4264	Scotland, .	Mile, . .	1984
Germany, .	Mile, long, . .	10126	Spain, . .	League, common,	7416
Hamburg,	" . .	8244	Sweden, .	Mile, . . .	11700
Hanover, .	" . . .	11559	Switzerland,	" . .	9153
Holland, .	" . .	6395	Turkey, . .	Berri, . . .	1826

Foreign Measures of Surface Compared with American.

Places.	Measures.	Sq. Yds.	Places.	Measures.	Sq. Yds.
Amsterdam,	Morgen, . .	9722	Portugal, .	Geira, . .	6970
Berlin, . .	" great,	6786	Prussia, .	Morgen, .	3053
" .	" small, .	3054	Rome, . .	Pezza, . .	3158
Canary Isles,	Fanegada, .	2422	Russia, .	Dessetina, .	13066·6
England, .	Acre, . . .	4840	Scotland, .	Acre. . . .	6150
Geneva, .	Arpent, .	6179	Spain, . .	Fanegada, .	5500
Hamburg, .	Morgen, . .	11545	Sweden, .	Tunneland, .	5900
Hanover, .	" . .	3100	Switzerland,	Faux, . .	7855
Ireland, . .	Acre, . .	7840	Vienna, . .	Joch, . .	6889
Naples, .	Moggia, . .	3998	Zurich, .	Common acre,	3875·0

Foreign Liquid Measures Compared with American.

Places.	Measures.	Cub. In.	Places.	Measures.	Cub. In.
Amsterdam,	Anker,	2331	Naples,	Wine Barille,	2544
"	Stoop,	146	"	Oil Stajo,	1133
Antwerp,	"	194	Oporto,	Almude,	1555
Bordeaux,	Barrique,	14033	Rome,	Wine Barille,	2560
Bremen,	Stubgens,	194·5	"	Oil "	2240
Canaries,	Arrobas,	949	"	Boccali,	80
Constantinople,	Almud,	319	Russia,	Weddras,	752
Copenhagen,	Anker,	2355	"	Kunkas,	94
Florence,	Oil Barille,	1946	Scotland,	Pint,	103·5
"	Wine "	2427	Sicily,	Oil Caffiri,	662
France,	Litre,	61·07	Spain,	Azumbras,	22·5
Geneva,	Setier,	2760	"	Quartillos,	30·5
Genoa,	Wine Barille,	4530	Sweden,	Eimer,	4794
"	Pinte,	90·5	"	Kanna,	159·57
Hamburg,	Stubgen,	221	Trieste,	Orne,	4007
Hanover,	"	231	Tripoli,	Mattari,	1376
Hungary,	Eimer,	4474	Tunis,	Oil "	1157
Leghorn,	Oil Barille,	1942	Venice,	Secchio,	628
Lisbon,	Almude,	1040	Vienna,	Eimer,	3452
Malta,	Caffiri,	1270	"	Maas,	86·33

Foreign Dry Measures Compared with American.

Places.	Measures.	Cub. In.	Places.	Measures.	Cub. In.
Alexandria,	Rebele,	9587	Malta,	Salme,	16930
"	Kislos,	10418	Marseilles,	Charge,	9411
Algiers,	Tarrie,	1219	Milan,	Moggi,	8444
Amsterdam,	Mudde,	6596	Naples,	Temoli,	3122
"	Sack,	4947	Oporto,	Alquiere,	1051
Antwerp,	Viertel,	4705	Persia,	Artaba,	4013
Azores,	Alquiere,	731	Poland,	Zorzec,	3120
Berlin,	Scheffel,	3180	Riga,	Loop,	3978
Bremen,	"	4339	Rome,	Rubbio,	16904
Candia,	Charge,	9288	"	Quarti,	4226
Constantinople,	Kislos,	2023	Rotterdam,	Sach,	6361
Copenhagen,	Toende,	8489	Russia,	Chetwert,	12448
Corsica,	Stajo,	6014	Sardinia,	Starelli,	2988
Florence,	Stari,	1449	Scotland,	Firlot,	2197
Geneva,	Coupes,	4739	Sicily,	Salme gros,	21014
Genoa,	Mina,	7382	"	" generale,	16886
Greece,	Medimni,	2390	Smyrna,	Kislos,	2141
Hamburg,	Scheffel,	6426	Spain,	Catrize,	41269
Hanover,	Malter,	6868	Sweden,	Tunna,	8940
Leghorn,	Stajo,	1501	Trieste,	Stari,	4521
"	Sacco,	4503	Tripoli,	Caffiri,	19780
Lisbon,	Alquiere,	817	Tunis,	"	21855
"	Fanega,	3268	Venice,	Stajo,	4945
Madeira,	Alquiere,	684	Vienna,	Metzen,	3753
Malaga,	Fanaga,	3783			

English Measures of Capacity.

The *Imperial gallon* measures 277·274 cubic inches, containing 10 lbs. Avoirdupois of distilled water, weighed in air, at the temperature of 62°, the barometer at 30 inches.

For Grain. 8 bushels = 1 quarter.
1 quarter = 10·2694 cubic feet.

Coal, or heaped measure. 3 bushels = 1 sack.
12 sacks = 1 chaldron.

Imperial bushel = 2218·192 cubic inches.

Heaped bushel*, $19\frac{1}{2}$ ins. diam., cone 6 ins. high = **2812·4872 cubic ins.

1 chaldron = 58·658 cubic feet, and weighs 3136 pounds.

1 chaldron (Newcastle) = 5936 pounds.

Foreign Weights Compared with American.

Places.	Weights.	Lbs. per 100 avoir.	Places.	Weights.	Lbs. per 100 avoir.
Aleppo, . . .	Rottoli, . .	20.46	Hanover, . .	Pound, . .	93.20
" . .	Oke, . . .	35.80	Japan, . .	Catty, . .	76.92
Alexandria, .	Rottoli, . .	107.	Leghorn, . .	Pound, . .	133.56
Algiers, . .	" . .	84.	Leipsic, . .	" (common)	97.14
Amsterdam, .	Pound, . . .	91.8	Lyons, . . .	" (silk), .	98.81
Antwerp, .	" . .	96.75	Madeira, .	" . .	143.20
Barcelona, . .	" . . .	112.6	Mocha, . .	Maund, . .	33.33
Batavia, . .	Catty, . .	76.78	Morea, . .	Pound, . .	90.79
Bengal, . . .	Seer, . . .	53.57	Naples, . .	Rottoli, . .	50.91
Berlin, . .	Pound, . .	96.8	Rome, . .	Pound, . .	133.69
Bologna, . .	" . . .	125.3	Rotterdam, .	" . . .	91.80
Bremen, . .	" . .	90.93	Russia, . .	" . . .	110.86
Brunswick, .	" . . .	97.14	Sicily, . . .	" . . .	142.85
Cairo, . . .	Rottoli, . .	105.	Smyrna, .	Oke, . . .	36.51
Candia, . . .	" . .	85.9	Sumatra, . .	Catty, . .	35.56
China, . .	Catty, . . .	75.45	Sweden, . .	Pound, . .	106.67
Constantinople	Oke, . . .	35.55	" . .	" (miner's),	120.68
Copenhagen,	Pound, . .	90.80	Tangiers, .	" . . .	94.27
Corsica, . . .	" . . .	131.72	Tripoli, . .	Rottoli, . .	89.28
Cyprus, . .	Rottoli, . .	19.07	Tunis, . .	" . . .	90.09
Damascus, . .	" . .	25.28	Venice, . .	Pound (heavy)	94.74
Florence, . .	Pound, . . .	133.56	" . .	" (light)	150.
Geneva, . .	" (heavy),	82.35	Vienna, . .	" . .	81.
Genoa, . .	" "	92.86	Warsaw, .	" . . .	112.25
Hamburgh, .	" "	93.63			

A Uniform System of Metrology much Needed.

The preceding variety of tables of weights, measures and coins shows the great need of a uniform system of metrology throughout the world.

The French are the first in adopting a uniform decimal system of metrology, and an International Decimal Association has been formed for the special purpose of advocating the introduction of the French system into other countries, which Association has now labored on that subject for some twenty years with but slow success.

The metric system is now adopted all over the continent of Europe, and in South and Central America. Among the English-speaking nations the metric system is legalized, but not enforced.

The principal difficulties in the way appear to be prejudices and jealousy. It must be admitted that the introduction of a new system of metrology causes some temporary inconveniences, but the objection is only *temporary*. Some few countries have decimated their old units in preference to adopting the French system.

One difficulty of the decimal system is, that the base 10 does not admit of more than one binary division without fraction. See A New System of Arithmetic, page 54.

DIFFERENTIALS.	INTEGRALS.

$$\int dx = x + c \qquad \int x\, dx = \frac{x^3}{2} + C, \qquad 1$$

$$\int 4\, a x^3\, dx = 4\, a \int x^3\, dx = a x^4 + C, \qquad 2$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, \qquad 3$$

$$\int \sqrt{x}\, dx \int x^{\frac{1}{2}}\, dx = \frac{2 x^{1\frac{1}{2}}}{3} + C, \qquad 4$$

$$\int \frac{dx}{\sqrt{x}} = \int x^{-\frac{1}{2}}\, dx = 2\sqrt{x} + C, \qquad 5$$

$$\int \frac{dx}{x^2} = \int x^{-1}\, dx = l\, x + C, \qquad 6$$

$$\int \frac{dx}{x^2} = \int x^{-2}\, dx = -\frac{1}{x} + C, \qquad 7$$

$$\int \frac{dx}{x^3} = \int x^{-3}\, dx = -\frac{1}{2x^2} + C, \qquad 8$$

$$\int \left(a x^3 + \frac{b}{2\sqrt{x}}\right) dx = \frac{a x^4}{4} + b\sqrt{x} + C, \qquad 9$$

$$\int \frac{a\, dx}{x} = a\, l\, x + C, \qquad 10$$

$$\int \frac{b\, dx}{a + x} = b\, l\, (a + x) + C, \qquad 11$$

$$\int \frac{3 a x^2\, dx}{b + a x^3} = l\, (b + a x^3) + C, \qquad 12$$

$$\int a x dx + 3x^2 dx - b^2 dx = \frac{a x^2}{2} + x^3 - b^2 x + C, \qquad 13$$

$$\int (a^2 + b^2) = x(a^2 + b^2) + C, \qquad 14$$

$$\int (a x - 2x^2)^2 dx = x^3\left(\frac{a^2}{3} - a x + \frac{4x^2}{5}\right) + C, \qquad 15$$

$$\int 3(a x - x^2)^2 (a - 2x) dx = (a x - x^2)^3 + C, \qquad 16$$

$$\int \frac{n(x^{n-1} dx)}{\sqrt{a + x^n}} = \sqrt{a^2 + x^n} + C, \qquad 17$$

$$\int \frac{2 a\, dx}{a^2 - x^2} = l.\, \frac{a + x}{a - x} + C, \qquad 18$$

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\, l\, (+ \sqrt{a^2 + x^2}), \qquad 19$$

$$\int \sqrt{a + b x}\, dx = \frac{2}{3} b (\sqrt{a + b x})^3 + C, \qquad 20$$

DIFFERENTIALS.	INTEGRALS.

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = l\, (x + \sqrt{a^2 + x^2}), \qquad 21$$

$$\int_a^b 3 m x^2\, dx = m b^3 - m a^2, \qquad 22$$

$$\int_a^b m x\, dx = \frac{m}{2}(b^2 - a^2), \qquad 23$$

$$\int_0^\infty \frac{dx}{a^2 + x^2} = \frac{\pi}{2 a}, \qquad 24$$

$$\int_0^a \frac{dx}{\sqrt{a^2 - x^2}} = \frac{\pi}{2}, \qquad 25$$

$$\int_a^b x = -\int_b^a \qquad \int_a^c = \int_a^b + \int_b^c \qquad 26$$

$$\int \sin. x\, dx = -\cos. x + C, \qquad 27$$

$$\int \cos. x\, dx = \sin. x + C, \qquad 28$$

$$\int \tan. x\, dx = -\, l\, \cos. x + C, \qquad 29$$

$$\int \cot. x\, dx = -\, l\, \sin. x + C, \qquad 30$$

$$\int \frac{dx}{\sin. x} = l\, \tan. \frac{x}{2} + C, \qquad 31$$

$$\int \frac{dx}{\cos. x} = l\, \tan. \left(\frac{\pi}{4} + \frac{x}{2}\right) + C, \qquad 32$$

$$\int \sin. x \cos. x\, dx = \frac{1}{2} \sin^2 x + C, \qquad 33$$

$$\int^\infty \frac{\sin. b x}{x}\, dx = \frac{\pi}{2}, \qquad 34$$

$$\int_0^\infty \frac{\cos. b x}{x}\, dx = \infty, \qquad 35$$

$$\int \frac{dt}{1 + t^2} = \text{circle arc of which } t = \tan.$$

$$\int \frac{dx}{\sqrt{2x - x^2}} = \text{circle arc of which } x = \text{sin. versus.} \qquad 37$$

$$\iiint 6 a\, dx^3 = \iint 6 a x\, dx^2 = \int 3 a x^2\, dx = a x^3 + C,$$

$$\iint 2 (a + b)\, dx^2 = (a + b) x^2 + C, \qquad 39$$

$$\iint 2 v^2\, dx^2 + 8 v x\, dx\, dv + 2 x^2 dv^2 = x v^2, \qquad 40$$

See page 101.

GEOMETRY.

DEFINITIONS.

Demonstration is a course of reasoning by which a truth is established. It consists of,

Thesis, the truth to be established, and,

Hypothesis, something assumed and to be demonstrated.

Axiom is that which is self-evident and requires no demonstration.

Theorem is something to be proved by demonstration.

Postulate is something to be *done*, but is self-evident and requires no demonstration.

Problem is a question proposed and requires solution.

Proposition is either a Theorem or a Problem.

Corolary is an obvious conseqence deduced from something that has gone before.

Scolium is a remark on preceding propositions, commonly demonstrated by algebraical formulæ.

Lemma is something premised for a following demonstration.

Geometrical Quantities.

Point has position, but no magnitude.

A Line has length, without breadth or thickness.

A Straight Line is the shortest distance between two points.

Curved line is a length which in every point changes its direction.

Superficies, Surface, Area, is that which has length and breadth, but no thickness.

Plane surface is a surface which coincides with a straight line in every direction.

Curved surface is a plane which coincides with a curved line.

A Solid has length, breadth and thickness.

Circle.

Circle, Cirumference, Periphery, is a curved line drawn on a plane surface, and bounded at a common distance from one point in the plane, (centre.)

Radius is a line* drawn from the centre in a circle to the periphery.

Diameter is a line drawn through the centre to the periphery, or the longest line in a circle.

Chord is any line extending its both ends to the periphery of a circle. and does not go through the centre.

Arc is a part of a periphery.

Circle plane, is a plane surface bounded within a circumference.

Sector is a part of a circle-plane bounded within an arc and two radii.

Segment is a part of a circle plane bounded within a chord and an arc.

Zone is a part of a circle included between two parallel chords.

Lune is the space between the intersecting arcs of two eccentric circles.

Oval is a round figure having one long and one short diameter at right angles to one another.

Semicircle is a half circle.

Quadrant is a quarter of a circle.

Angles.

Angle is the opening or inclination of two lines which meet in one point.

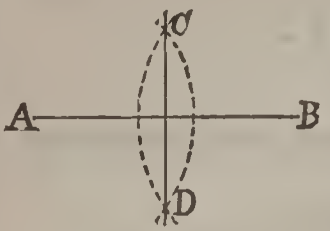
If two radii being drawn from the extremities of a circle arc, to the centre; the arc, is a measure of the angle at the centre.

Right angle is when the opening is a quarter of a circle.

Acute angle is less than a right angle.

Obtuse angle is greater than a right angle.

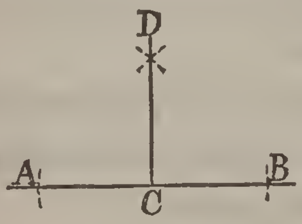
* *Line* by itself means a straight line.



1.

To divide a given line $A B$ into two equal parts; and to erect a perpendicular through the middle.

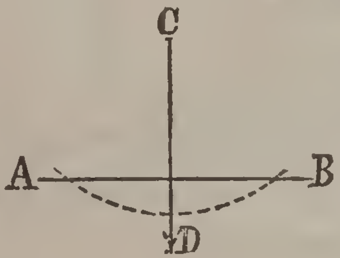
With the end A and B as centres, draw the dotted circle arcs with a radius greater than half the line. Through the crossings of the arcs draw the perpendicular $C D$, which divides the line into two equal parts.



2.

From a given point C on the line $A B$, to erect a perpendicular $C D$.

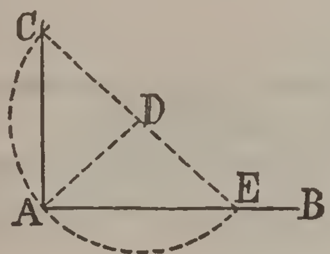
With C as a centre, draw the dotted circle arcs at A and B equal distances from C. With A and B as centres, draw the dotted circle arcs at D. From the crossing D draw the required perpendicular, $D C$.



3.

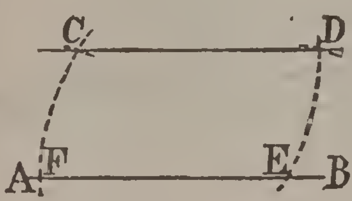
From a given point C at a distance from the line $A B$, to draw a perpendicular to the line.

With C as a centre, draw the dotted circle arc so that it cuts the line at A and B. With A and B as centres, draw the dotted cross arcs at D with equal radii. Draw the required perpendicular through C and D.



4.

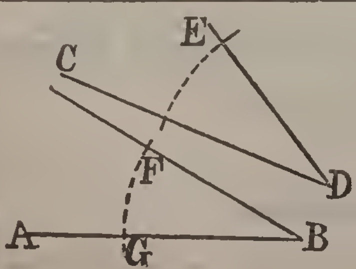
At the end A of a given line $A B$, to erect a perpendicular $A C$. With the point D as a centre at a distance from the line, and with $A D$ as radius, draw the dotted circle arc so that it cuts the line at E; through E and D, draw the diameter $E C$; then join C and A, which will be the required perpendicular.



5.

Through a given point C at a distance from the line $A B$, to draw a line $C D$ parallel to $A B$.

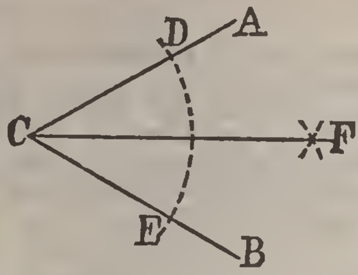
With C as a centre, draw the dotted arc $E D$; with E as a centre, draw through C the dotted arc $F C$. With the radius $F C$ and E as a centre, draw the cross arc at D. Join C with the cross at D, which will be the required parallel line.



6.

On a given line $A B$ and at the point B, to construct an angle equal to the angle $C D E$.

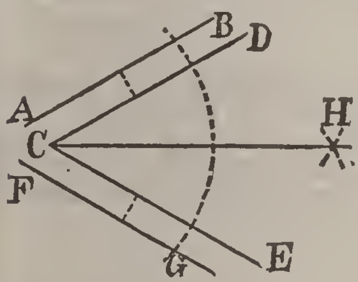
With D as a centre, draw the dotted arc $C E$; and with the same radius and B as a centre, draw the arc $G F$; then make $G F$ equal to $C E$; then join $B F$, which will form the required angle, $F B G = C D E$.



7.

To divide the angle $A\ B\ C$ into two equal parts. With C as a centre, draw the dotted arc $D\,E$; with D and E as centres, draw the cross arcs at F with equal radii. Join $C\,F$, which divides the angle into the required parts.

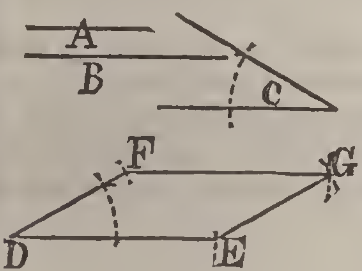
Angles $A\ C\ F = F\ C\ B = \frac{1}{2}(A\ C\ B)$.



8.

To divide an angle into two equal parts, when the lines do not extend to a meeting point.

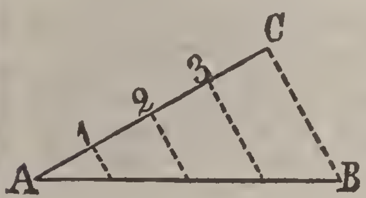
Draw the lines $C\,D$ and $C\,E$ parallel, and at equal distances from the lines $A\ B$ and $F\,G$. With C as a centre, draw the dotted arc $B\ G$; and with B and G as centres, draw the cross arcs H. Join $C\,H$, which divides the angle into the required equal parts.



9.

To construct a parallelogram, with the given sides A and B and angle C.

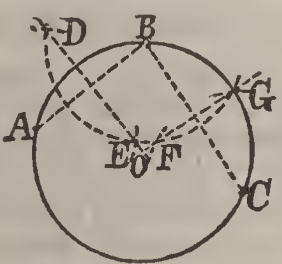
Draw the base line $D\,E$, and make the angle $F\,D\,E = C$; lines $D\,E = B$ and $D\,F = A$; complete the parallelogram by cross arcs at G, and the problem is thus solved.



10.

To divide the line $A\ B$ in the same proportion of parts as $A\ C$.

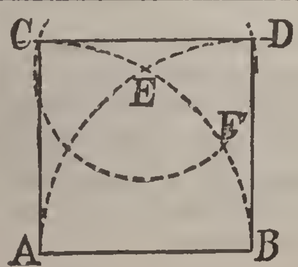
Join C and B, and through the given divisions 1, 2, and 3 draw lines parallel with $C\,B$, which solves the problem.



11.

To find the centre of a circle which will pass through three given points A, B, and C.

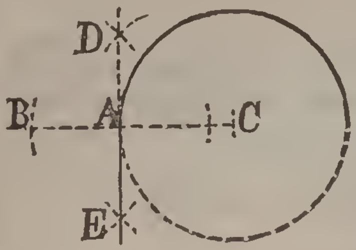
With B as a centre, draw the arc $D\ E\ F\ G$; and with the same radius and A as a centre, draw the cross arcs D and F; also with C as a centre, draw the cross arcs E and G. Join D and E. and also F and G, and the crossing 0 is the required centre of the circle.



12.

To construct a square upon a given line $A\ B$.

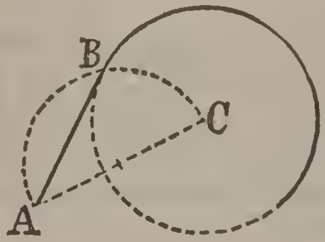
With $A\ B$ as radius and A and B as centres, draw the circle arcs $A\ E\ D$ and $B\ E\ C$. Divide the arc $B\ E$ in two equal parts at F, and with $E\ F$ as radius, and E as centre, draw the circle $C\ F\ D$. Join A and C, B and D, C and D, which completes the required square.



13.

Through a given point *A* in a circumference. to draw a tangent to the circle.

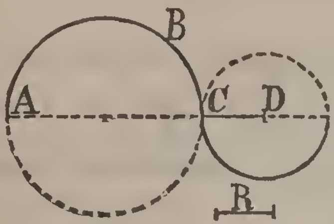
Through the given point *A* and centre *C*, draw the line *B C*. With *A* as a centre, draw the circle arcs *B* and *F*; with *B* and *F* as centres, draw the cross arcs *D* and *E*; then join *D* and *E*, which is the required tangent.



14.

From a given point *A* outside of a circumference, to draw a tangent to the circle.

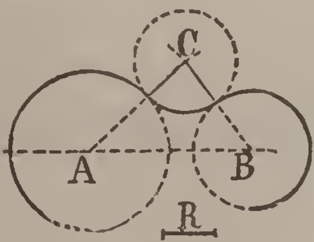
Join *A* and *C*, and upon *A C* as a diameter draw the half circle *A B C*, which cuts the given circle at *B*. Join *A* and *B*, which is the required tangent.



15.

To draw a circle with a given radius *R*, that will be tangent to the circle *A B C* at *C*.

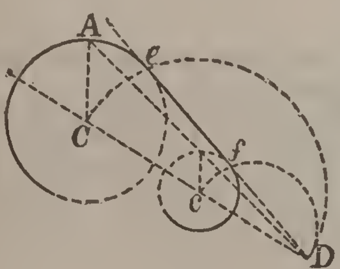
Through the given point *C*, draw the diameter *A C* extended beyond *D*; from *C* set off the given radius *R* to *D*; then *D* is the centre of the required circle, which is tangent to the given circle at *C*.



16.

To draw a circle with a given radius *R*, that will be tangent to two given circles.

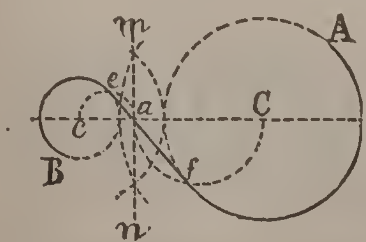
Join the centres *A* and *B* of the given circles. Add the given radius *R* to each of the radii of the given circle, and draw the cross arcs *C*, which is the centre of the circle required to be tangent to the other two.



17.

To draw a tangent to two circles of different diameters.

Join the centres *C* and *c* of the given circles, and extend the line to *D*; draw the radii *A C* and *a c* parallel with one another. Join *A a*, and extend the line to *D*. On *C D* as a diameter, draw the half circle *C e D*; on *c D* as a diameter, draw the half circle *c f D*; then the crossings *e* and *f* are the tangential points of the circles.

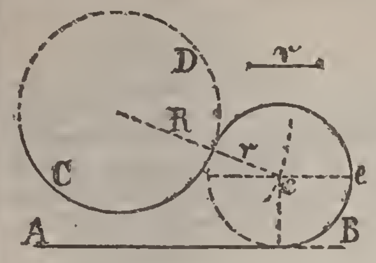


18.

To draw a tangent between two circles.

Join the centres *C* and *c* of the given circles; draw the dotted circle arcs, and join the crossing *m*, *n*, which line cuts the centre line at *a*. With *a C* as a diameter, draw the half circle *a f C*; and with *a c* as a diameter, draw the half circle *c e a*; then the crossings *e* and *f* are the tangential points of the circles.

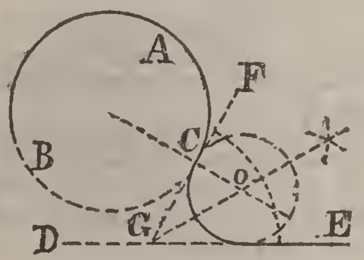
19.



With a given radius r, to draw a circle that will be tangent to the given line $A\,B$ and the given circle $C\,D$.

Add the given radius r to the radius R of the circle, and draw the arc $c\,d$. Draw the line $c\,e$ parallel with and at a distance r from the line $A\,B$. Then the crossing c is the centre of the required circle that will be tangent to the given line and circle.

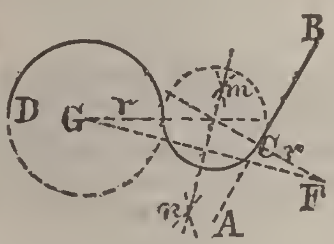
20.



To find the centre and radius of a circle that will be tangent to the given circle $A\,B$ at C, and the line $D\,E$.

Through the given point C, draw the tangent $G\,F$; bisect the angle $F\,G\,E$; then o is the centre of the required circle that will be tangent to $A\,B$ at C, and the line $D\,E$.

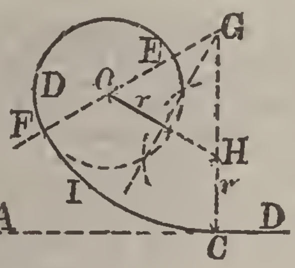
21.



To find the centre and radius of a circle that will be tangent to the given line $A\,B$ at C, and the circle $D\,E$.

Through the point C, draw the line $E\,F$ at right angles to $A\,B$; set off from C the radius $r = C\,F$ of the given circle. Join G and F. With G and F as centres, draw the arc crosses m and n. Join $m\,n$, and where it crosses the line $E\,F$ is the centre for the required circles.

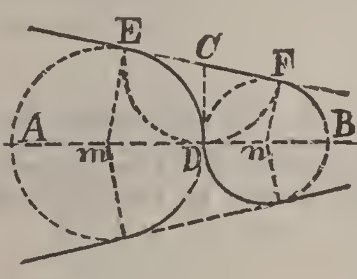
22.



To find the centre and radius of a circle that will be tangent to the given line $A\,B$ at C, and the circle $D\,E$.

From C, erect the perpendicular $C\,G$; set off the given radius r from C to H. With H as a centre and r as radius, draw the cross arcs on the circle. Through the cross arcs draw the line $I\,G$; then G is the centre of the circle arc $F\,I\,C$, which tangents the line at C and the circle at F.

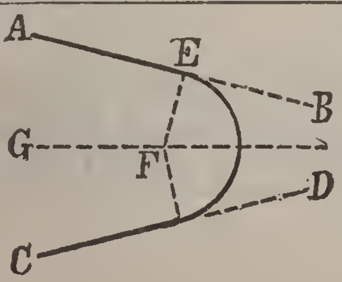
23.



Between two given lines, to draw two circles that will be tangent to each other and the lines.

Draw the centre line $A\,B$ between the given lines; assume D to be the tangenting point of the circles; draw $D\,C$ at right angles to $A\,B$. With C as centre and $C\,D$ as radius, draw the circle $E\,D\,F$. From E, draw $E\,m$ at right angles to $E\,F$; and from F, draw $F\,n$ at right angles to $F\,E$; then m and n are the centres for the required circles.

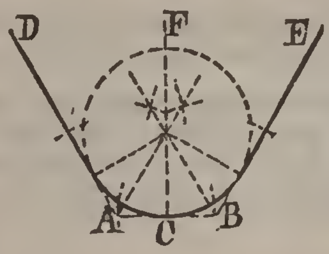
24.



To draw a circular arc that will be tangent to two given lines $A\,B$ and $C\,D$ inclined to one another, one tangential point E being given.

Draw the centre line $G\,F$. From E draw $E\,F$ at right angles to $A\,B$; then F is the centre of the circle required.

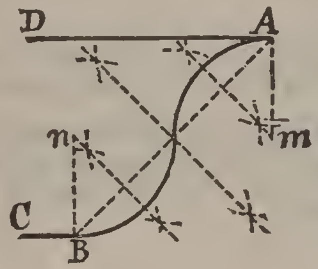
25.



To draw a circle that will be tangent to two lines and pass through a given point *C* on the line *F C*, which bisects the angle of the lines.

Through *C* draw *A B* at right angles to *C F*; bisect the angles *D A B* and *E B A*, and the crossing on *C F* is the centre of the required circle.

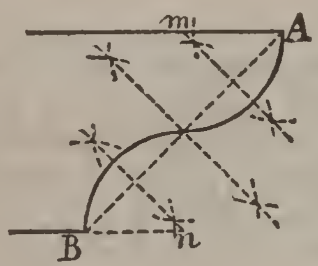
26.



To draw a *cyma*, or two circular arcs that will be tangent to each other, and two parallel lines at given points *A* and *B*.

Join *A* and *B*; divide *A B* into four equal parts and erect perpendiculars. Draw *A m* at right angles from *A*, and *B n* at right angles from *B*; then *m* and *n* are the centres of the circle arcs of the required *cyma*.

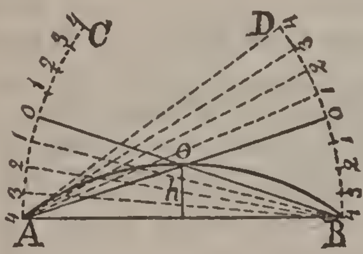
27.



To draw a *talon*, or two circular arcs, that will be tangent to each other, and meet two parallel lines at right angles in the given points *A* and *B*.

Join *A* and *B*; divide *A B* into four equal parts and erect perpendiculars; then *m* and *n* are the centres of the circle arcs of the required *talon*.

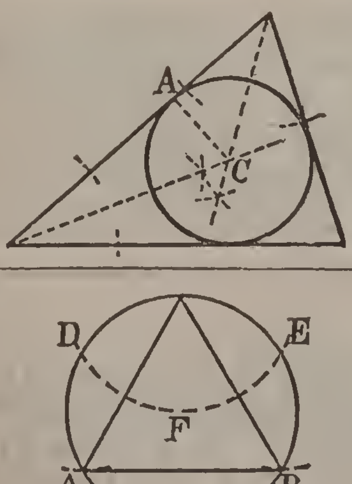
28.



To plot a circular arc without recourse to its centre, its chord *A B* and height *h* being given.

With the chord as radius, and *A* and *B* as centres, draw the dotted circular arcs *A C* and *B D*. Through the point *O* draw the lines *A O o* and *B O o*. Make the arcs *C o* = *A o* and *D o* = *B o*. Divide these arcs into any desired number of equal parts, and number them as shown on the illustration. Join *A* and *B* with the divisions, and the crossings of equal numbers are points in the circle arc.

29.



To find the centre and radius of a circle that will be tangent to the three sides in a triangle.

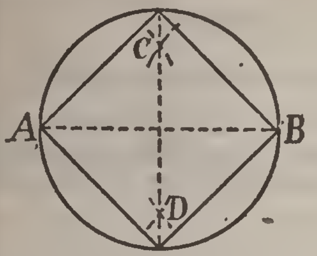
Bisect two of the angles in the triangle, and the crossing *C* is the centre of the required circle.

30.

To inscribe an equilateral triangle in a circle.

With the radius of the circle and centre *C* draw the arc *D F E*; with the same radius, and *D* and *E* as centres, set off the points *A* and *B*. Join *A* and *B*, *B* and *C*, *C* and *A*, which will be the required triangle.

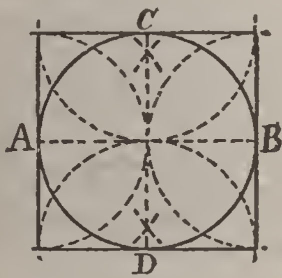
31.



To inscribe a square in a given circle.

Draw the diameter *A B*, and through the centre erect the perpendicular *C D*, and complete the square as shown in the illustration.

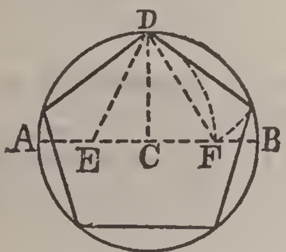
32.



To describe a square about a given circle.

Draw the diameters *A B* and *C D* at right angles to one another; with the radius of the circle, and *A*, *B*, *C*, and *D* as centres, draw the four dotted half circles which cross one another in the corners of the square, and thus solve the problem.

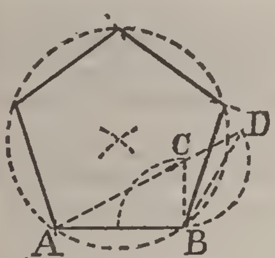
33.



To inscribe a *pentagon* in a given circle.

Draw the diameter *A B*, and from the centre *C* erect the perpendicular *C D*. Bisect the radius *A C* at *E*: with *E* as centre, and *D E* as radius, draw the arc *D E*, and the straight line *D F* is the length of the side of the pentagon.

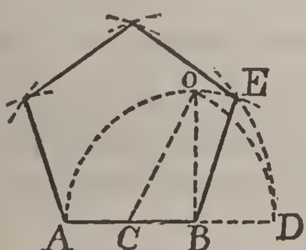
34.



To construct a *pentagon* on a given line *A B*.

From *B* erect *B C* perpendicular to and half the length of *A B*; join *A* and *C* prolonged to *D*; with *C* as a centre and *C B* as radius, draw the arc *B D*; then the chord *B D* is the radius of the circle circumscribing the pentagon. With *A* and *B* as centres, and *B D* as radius, draw the cross *O* in the centre.

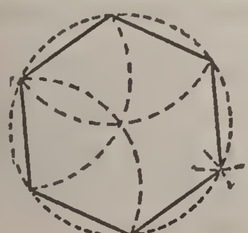
35.



To construct a pentagon on a given line *A B* without resort to its centre.

From *B* erect *B o* perpendicular and equal to *A B*; with *C* as centre and *C o* as radius, draw the arc *D o*; then *A D* is the diagonal of the pentagon. With *A D* as radius and *A* as centre, draw the arc *D E*; and with *B* as centre and *A B* as radius, finish the cross *E*, and thus complete the pentagon.

36.

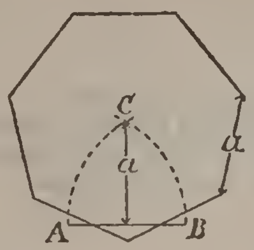


To construct a *hexagon* in a given circle.

The radius of the circle is equal to the side of the hexagon.

37.

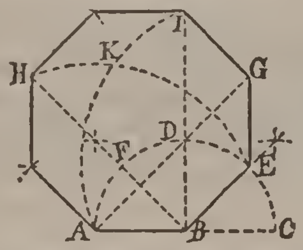
To construct a Heptagon.



The appotem *a* in a hexagon is the length of the side of a heptagon.

Set off *A B* equal to the radius of the circle; draw *a* from the centre *C* at right angles to *A B*; then *a* is the required side of the heptagon.

38.



To construct an *octagon* on the given line *A B*.

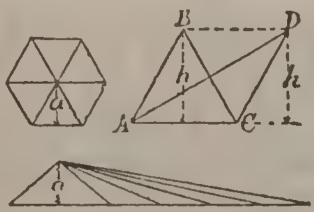
Prolong *A B* to *C*. With *B* as centre and *A B* as radius, draw the circle *A F D E C*; from *B*, draw *B I* at right angles to *A B*; divide the angles *A B D* and *D B C* each into two equal parts; then *B E* is one side of the octagon. With *A* and *E* as centres, draw the arcs *H K E* and *A K I*, which determine the points *H* and *I*, and thus complete the octagon as shown in the illustration.

39.

To cut off the corners of a square, so as to make it a regular *octagon*.

With the corners as centres, draw circle arcs through the centre of the square to the side, which determines the cut-off.

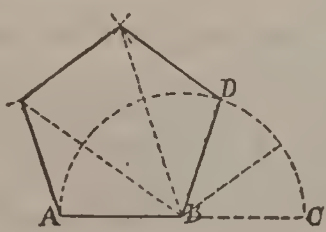
40.



The area of a regular polygon is equal to the area of a triangle whose base is equal to the sum of all the sides, and the height *a* equal to the appotem of the polygon.

The reason of this is that the area of two or more triangles *A B C* and *A D C* having a common or equal base *b* and equal height *h* are alike.

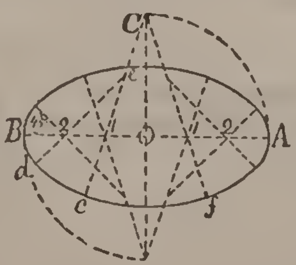
41.



To construct any regular *polygon* on a given line *A B* without resort to its centre.

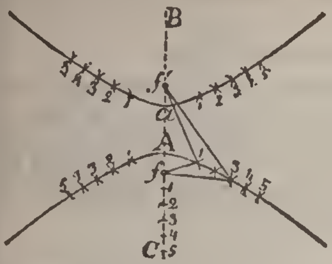
Extend *A B* to *C*, and, with *B* as centre, draw the half circle *A D B*. Divide the half circle into as many parts as the number of sides in the polygon, and complete the construction as shown in the illustration.

42.



To construct an isometric ellipse by compasses and six circle arcs.

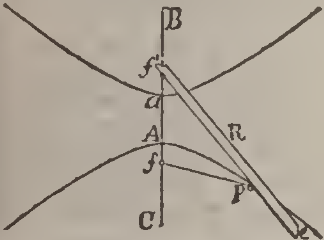
Divide *O A* and *O B* each into three equal parts; draw the quadrant *A C*. From *C*, draw the line *C c* through the point 1. Through the points 2 draw *d e* at an angle of 45° with the major axis. Then 2 is the centre for the ends of the ellipse; *e* is the centre for the arc *d c*; and *C* is the centre for the arc *c f*.



43.

To construct an Hyperbola by plotting,

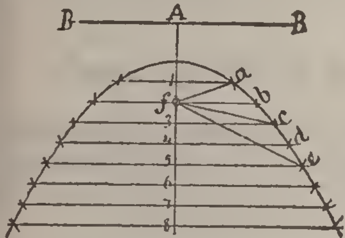
Having given the transverse axis $B\,C$, vertexes $A\,a$, and foci $f\,f'$. Set off any desired number of parts on the axis below the focus, and number them 1, 2, 3, 4, 5, etc. Take the distance $a\,1$ as radius, and, with f' as centre, strike the cross 1 with $f'\,1 = a\,1$. With the distance $A\,1$, and the focus f as centre, strike the cross 1 with the radius $F1 = A\,1$, and the cross 1 is a point in the hyperbola.



44.

To draw an Hyperbola by a pencil and a string,

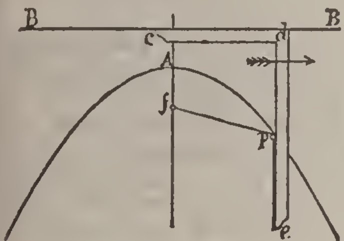
Having given the transverse axis $B\,C$, foci f' and f, and the vertexes A and a. Take a rule B, and fix to it a string at e; fix the other end of the string at the focus f. The length of the string should be such that when the rule R is in the position $f'\,C$, the loop of the string should reach to A; then move the rule on the focus f', and a pencil at P, stretching string, will trace the hyperbola.



45.

To construct a Parabola by plotting,

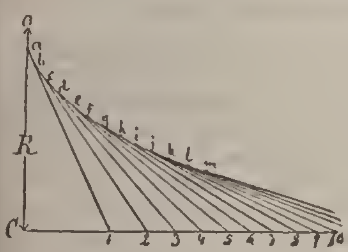
Having given the axes, vertex, and focus of the parabola. Divide the transverse axis into any desired number of parts 1, 2, 3, etc., and draw ordinates through the divisions; take the distance $A\,1$, and set it off on the 1st ordinate from the focus f to a, so that $A\,1 = f\,a$. Repeat the same operation with the other ordinates—that is, set off the distance $A\,5$ from f to e, so that $A\,5 = f\,e$; and so the parabola is constructed.



46.

To draw a Parabola by a pencil and a string,

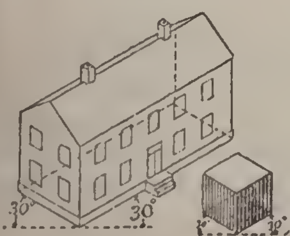
Having given the two axes, vertex, and focus of the parabola. Take a square $c\,d\,e$, and fix to it a string at e; fix the other end of the string at the focus f. The length of the string should be such that when the square is in the position of the axis $A\,f$, the string should reach to the vertex A. Move the square along $B\,B$, and the pencil P will describe the parabola.



47.

The Tractrix or Shield's anti-friction curve.

R represents the radius of the shaft, and C 1, 2, 3, etc., is the centre line of the shaft. From o, set off the small distance $o\,a$; and set off $a\,1 = R$. Set off the same small distance from a to b, and make $b\,2 = R$. Continue in the same way with the other points, and the anti-friction curve is thus constructed.

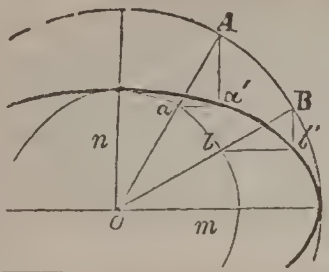


48.

Isometric Perspective.

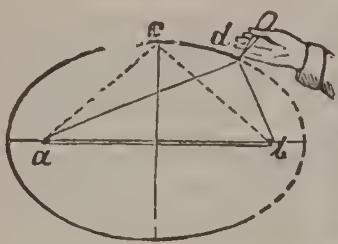
This kind of perspective admits of scale measurements the same as any ordinary drawing, and gives a clear representation of the object. It is easily learned. All horizontal rectangular lines are drawn at an angle of 30°.

All circles are ellipses drawn as shown on the preceding page (No. 42).



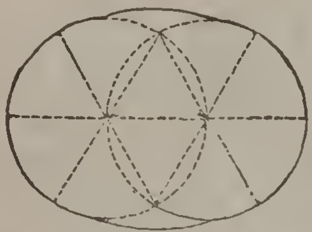
49. *To construct an ellipse.*

With *o* as a centre, draw two concentric circles with diameters equal to the long and short axes of the desired ellipse. Draw from *o* any number of radii, *A*, *B*, &c. Draw the line *B b′* parallel to *n* and *b b′* parallel to *m*, then *b′* is a point in the desired ellipse.



50. *To draw an ellipse with a string.*

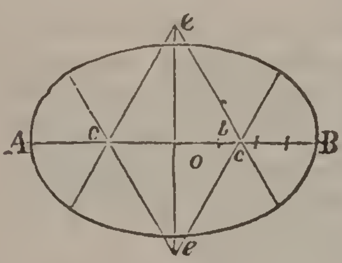
Having given the two axes, set off from *c* half the great axis at *a* and *b*, which are the two foci in the ellipse. Take an endless string as long as the three sides in the triangle *a*, *b*, *c*, fix two pins or nails in the foci, one in *a* and one in *b*, lay the string round *a* and *b*, stretch it with a pencil *d*, which then will describe the desired ellipse.



51

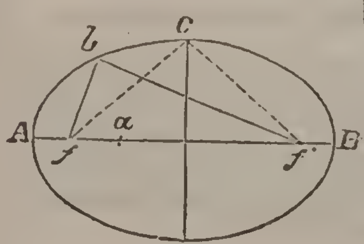
To draw an oval by circular arcs.

Divide the long axis into three equal parts, draw the two circles and where they intersect one another are the centres for the tangent arcs of the oval as shown by the figures.



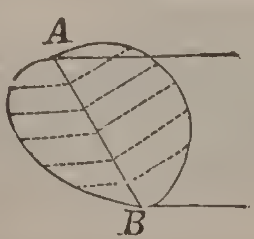
52 *To draw an oval by circular arcs.*

Given the two axes, set off the short axis from *A* to *b*, divide *b B* into three equal parts, set off two of these parts from *o* towards *c* and *c* which are the centres for the ends of the oval. Make equilateral triangles on *c c*, when *e e* will be the centres for the sides of the oval. If the long axis is more than twice the short one, this construction will not make a good oval.



53 *To construct an ellipse.*

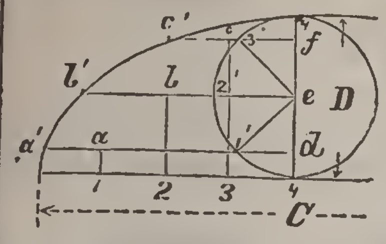
Given the two axes, set off half the long axis from *c* to *f f*, which will be the two focuses in the ellipse. Divide the long axis into any number of parts, say *a* to be a division point. Take *A a* as radius and *f* as centre and describe a circle arc about *b*, take *a B* as radius and *f* as centre describe another circle arc about *b*, then the intersection *b* is a point in the ellipse, and so the whole ellipse can be constructed.



54

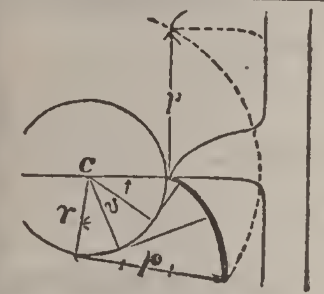
To draw an ellipse that will be tangent to two parallel lines in A and B.

Draw a semicircle on *A B*, draw ordinates in the circle at right angle to *A B*, the corresponding and equal ordinates for the ellipse to be drawn parallel to the lines, and thus the elliptic curve is obtained as shown by the figure.



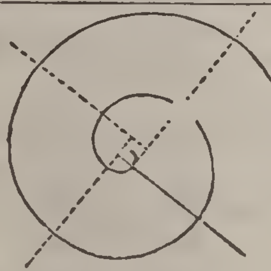
55 *To construct a cycloid.*

The circumference $C = 3{\cdot}14\ D$. Divide the rolling circle and base line C into a number of equal parts, draw through the division point the ordinates and abscissas, make $a\,a' = 1'\,d$, $b\,b' = 2'\,e$, $c\,c' = 3\ f$, then a' b' and c' are points in the cycloid. In the *Epicycloid* and *Hypocycloid* the abscissas are circles and the ordinates are radii to one common centre.



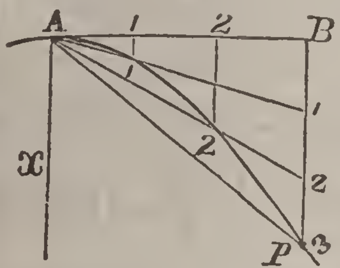
56 *Evolute of a circle.*

Given the pitch p, the angle v, and radius r. Divide the angle v into a number of equal parts, draw the radii and tangents for each part, divide the pitch p into an equal number of equal parts, then the first tangent will be one part, second two parts, third three parts, &c., and so the *Evolute* is traced.



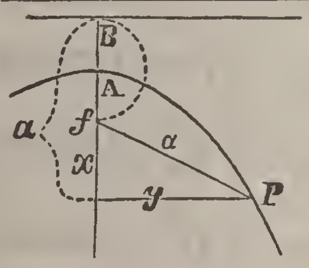
57 *To construct a spiral with compasses and four centres.*

Given the pitch of the spiral, construct a square about the centre, with the four sides together equal to the pitch. Prolong the sides in one direction as shown by the figure, the corners are the centres for each arc of the external angles.



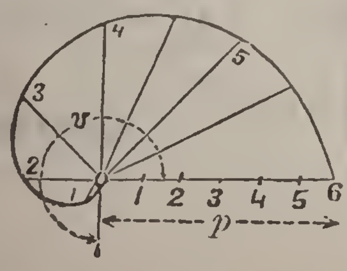
58 *To construct a Parabola.*

Given the vertex A, axis x, and a point P. Draw $A\ B$ at right angle to x, and $B\ P$ parallel to x, divide $A\ B$ and $B\ P$ into an equal number of equal parts. From the vertex A draw lines to the divisions on $B\ P$, from the divisions on $A\ B$ draw the ordinates parallel to x, the corresponding intersections are points in the *parabola*.



59 *To construct a Parabola.*

Given the axis of ordinates B, and vertex A. Take A as a centre and describe a semicircle from B which gives the focus of the *parabola* at f. Draw any ordinate y at right angle to the abscissa $A\ x$, take a as radius and the focus f as a centre, then intersect the ordinate y, by a circle-arc in P which will be a point in the *parabola*. In the same manner the whole Parabola is constructed.



60

To draw an arithmetic spiral.

Given the pitch p and angle v, divide them into an equal number of equal parts say 6. make $0\,1 = 0\,1$, $0\,2 = 02$, $0\,3 = 0\,3$, $0\,4 = 04$, $05 = 0\,5$, and $0\,6 =$ the pitch p; then join the points 1, 2, 3, 4, 5, and 6, which will form the spiral required.

THE CIRCLE.

Notation.

d = diameter of the circle.
r = radius of the circle.
p = periphery or circumference.
a = area of a circle or part thereof.
b = length of a circle-arc.
c = chord of a segment, length of.
h = height of a segment.
s = side of a regular polygon.
v = centre angle.
w = polygon angle.

All measures must be expressed by the same unit.

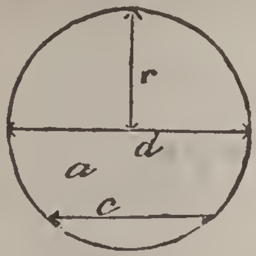
Formulas for the Circle.

Periphery or Circumference.	*Diameter and Radius.*	*Area of the Circle.*
$p = \pi d = 3.14d.$	$d = \frac{p}{\pi} = \frac{p}{3.14}.$	$a = \frac{\pi d^2}{4} = 0.785d^2.$
$p = 2\pi r = 6.28r.$	$r = \frac{p}{2\pi} = \frac{p}{6.28}.$	$a = \pi r^2 = 3.14r^2.$
$p = 2\sqrt{\pi a} = 3.54\sqrt{a}.$	$d = 2\sqrt{\frac{a}{\pi}} = 1.128\sqrt{a}.$	$a = \frac{p^2}{4\pi} = \frac{p^2}{12.56}.$
$p = \frac{2a}{r} = \frac{4a}{d}.$	$r = \sqrt{\frac{a}{\pi}} = 0.564\sqrt{a}.$	$a = \frac{pr}{2} = \frac{pd}{4}.$

$$\pi = 3.14159265358979323846264338327950288419716939 9$$

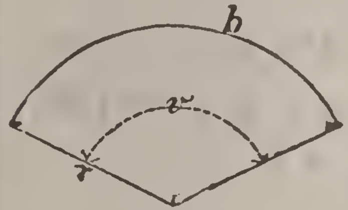
$2\pi = 6.283185$	$\frac{1}{4}\pi = 0.785398$	$\frac{1}{\pi} = 0.318310$	$\frac{360}{\pi} = 114.5915$
$3\pi = 9.424778$	$\frac{1}{3}\pi = 1.047197$	$\frac{2}{\pi} = 0.636619$	$\pi^2 = 9.869650$
$4\pi = 12.566370$	$\frac{1}{2}\pi = 1.570796$	$\frac{3}{\pi} = 0.954929$	$\sqrt{\pi} = 1.772453$
$5\pi = 15.707963$	$\frac{1}{8}\pi = 0.392699$	$\frac{4}{\pi} = 1.273239$	$\sqrt{\frac{1}{\pi}} = 0.564189$
$6\pi = 18.849556$	$\frac{1}{6}\pi = 0.523599$	$\frac{6}{\pi} = 1.909859$	$\sqrt{\frac{\pi}{2}} = 1.253314$
$7\pi = 21.991148$	$\frac{1}{12}\pi = 0.261799$	$\frac{8}{\pi} = 2.546478$	$\sqrt{\frac{2}{\pi}} = 0.797884$
$8\pi = 25.132741$	$\frac{2}{3}\pi = 2.094394$	$\frac{12}{\pi} = 3.819718$	Log. π = 0.49714987
$9\pi = 28.274334$	$\frac{1}{360}\pi = 0.008726$		

61



The periphery of a Circle is commonly expressed by the *Greek* letter $\pi = 3{\cdot}14$ when the diameter $d = 1$ or the unit. For any other value of the diameter d, we will denote the periphery by the letter p, r = radius, and a = area of the circle. The periphery of a circle is equal to $3\frac{14}{100}$ times its diameter, c = chord.

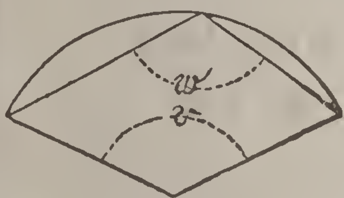
62



$$b = \frac{\pi r v}{180} = 0{\cdot}0175 rv,$$

$$v = \frac{180b}{\pi r} = 57.296\frac{b}{r}.$$

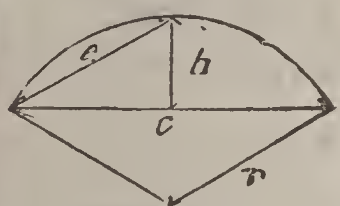
63



$$w = 180 - \frac{v}{2},$$

$$v = 2(180^\circ - w).$$

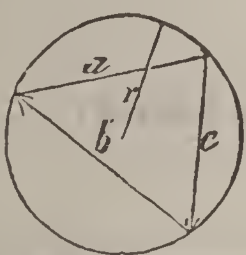
64



$$r = \frac{c^2+4h^2}{8h} = \frac{e^2}{2h},$$

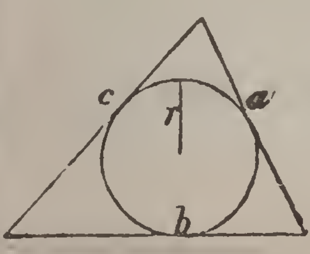
$$c = 2\sqrt{2hr - h^2}.$$

65

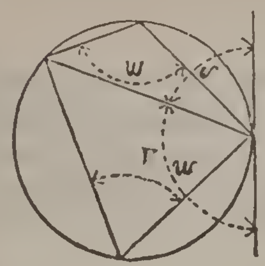


$$r = \frac{ac}{2\sqrt{a^2 - \left(\frac{a^2+b^2-c^2}{2b}\right)^2}}.$$

66

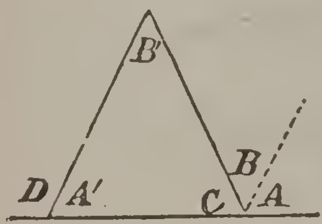


$$r = \frac{b\sqrt{a^2 - \left(\frac{a^2+b^2-c^2}{2b}\right)^2}}{a+b+c}.$$



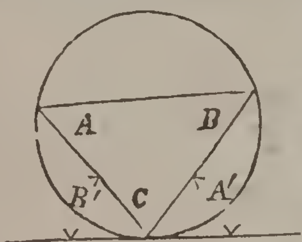
67

$$v = v, \quad w = w.$$
$$w + v = 180^\circ, \; w > v.$$



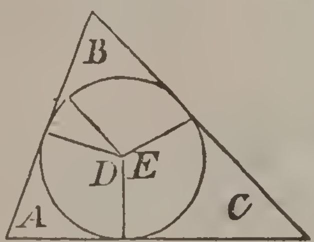
68

$$D = B + C, \quad A' + B' + C = 180^\circ,$$
$$B = D - C, \quad A + B + C = 180^\circ,$$
$$A' = A, \quad B' = B'.$$



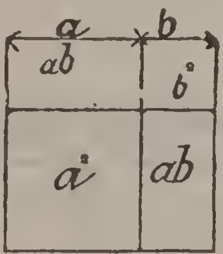
69

$$A + B + C = 180^\circ,$$
$$A' = A, \quad B' = B.$$



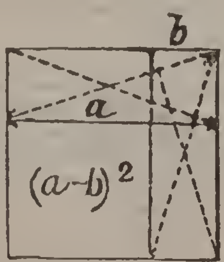
70

$$E + C = A + D = 180^\circ,$$
$$D = B + C,$$
$$E = A + B.$$



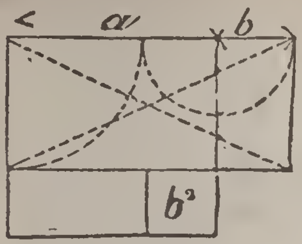
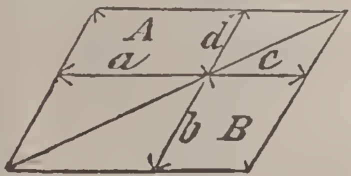
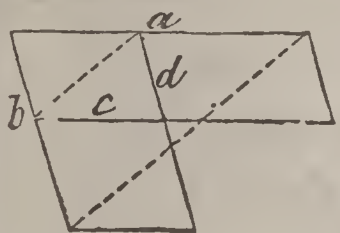
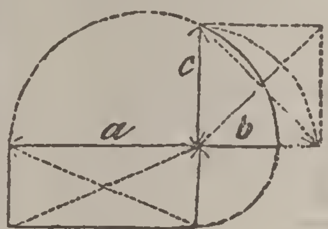
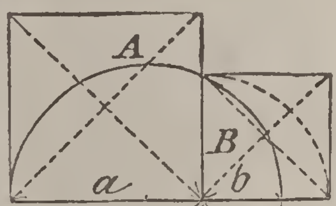
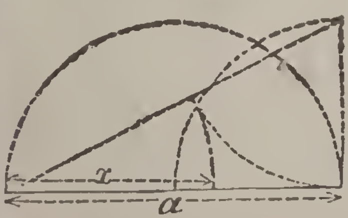
71

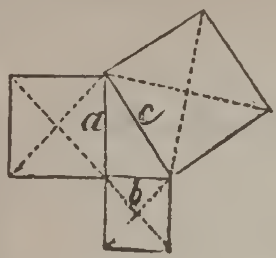
$$(a + b)^2 = a^2 + 2ab + b^2.$$



72

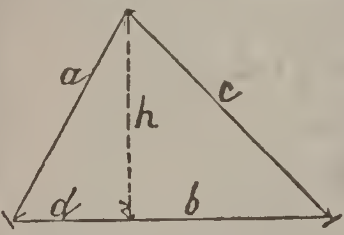
$$(a - b)^2 = a^2 - 2ab + b^2.$$

	73 $(a + b)(a - b) = a^2 - b^2.$
	74 $a : b = c : d,$ $ad = bc,$ $A = B.$
	75 $a : b = c : d.$ $ad = bc.$
	76 $a : c = c : b,$ $ab = c^2,$ $c = \sqrt{ab}.$
	77 $A : B = a : b.$
	78 $a : x = x : a - x,$ $x = \pm\sqrt{a^2 + \left(\frac{a}{2}\right)^2} + \frac{a}{2}.$



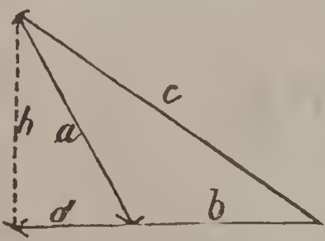
79

$$c^2 = a^2 + b^2,$$
$$a^2 = c^2 - b^2,$$
$$b^2 = c^2 - a^2.$$



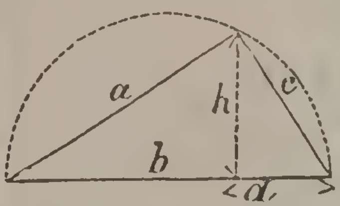
80

$$c^2 = a^2 + b^2 - 2bd,$$
$$h = \sqrt{a^2 - d^2}.$$
$$d = \frac{a^2 + b^2 - c^2}{2b}.$$



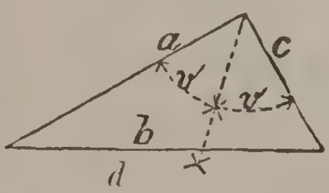
81

$$c^2 = a^2 + b^2 + 2bd,$$
$$h = \sqrt{a^2 - d^2},$$
$$d = \frac{c^2 - a^2 - b^2}{2b}.$$



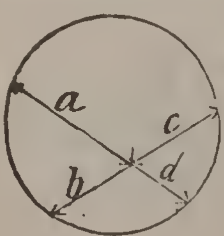
82

$$a : b = h : c,$$
$$h = \frac{ac}{b} = \frac{ad}{c},$$
$$d = \frac{c^2}{b} = \frac{ch}{a}.$$



83

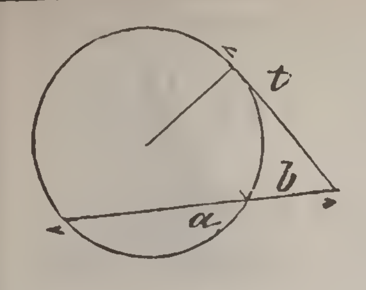
$$a : c = d : (b - d),$$
$$d = \frac{ab}{c + a},$$
$$v = v.$$



84

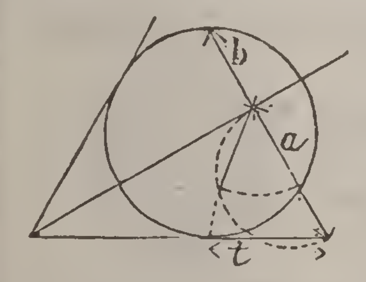
$$a : c = b : d,$$
$$ad = bc.$$

85



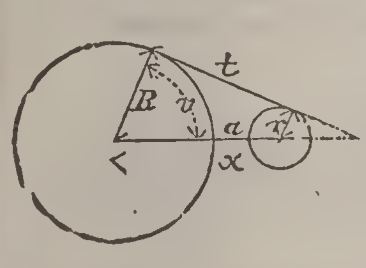
$$a : t = t : b,$$
$$t^2 = ab.$$

86



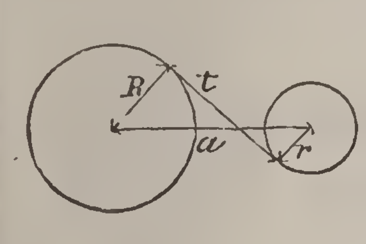
$$t^2 = (a + b)(a - b),$$
$$t = \sqrt{a^2 - b^2}.$$

87



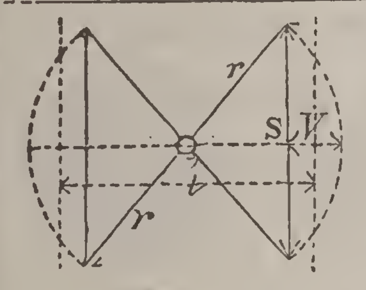
$$x = \frac{aR}{R - r}, \quad a = \sqrt{t^2 + (R - r)^2},$$
$$t = \sqrt{a^2 - (R - r)^2}, \quad \sin.v = \frac{t}{a}.$$

88



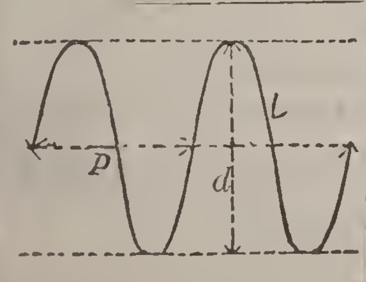
$$t = \sqrt{a^2 - (R + r)^2},$$
$$a = \sqrt{t^2 + (R + r)^2}.$$

89



$$V = r - \sqrt{r^2 - \frac{S^2}{4}} \quad l = 2r - V,$$
$$S = 2\sqrt{r^2 - (r - V)^2}. \quad r = \tfrac{1}{2}(l + V).$$

90



$$P = \sqrt{\frac{l^2}{n^2} - \pi^2 d^2},$$ P = Pitch. n = no. turns.
$$l = n\sqrt{\pi^2 d^2 + P^2},$$
$$n = \frac{l}{\sqrt{\pi^3 d^2 + P^2}}.$$

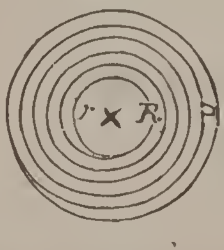
91 *To find the length of a Solid Spiral.*

$$l = \pi r n = \frac{\pi r^2}{P}, \qquad n = \frac{l}{\pi r} = \frac{r}{P},$$

$$P = \frac{\pi r^2}{l} = \frac{r}{n}.$$

$P =$ *Pitch.*
$n =$ *no. turns.*

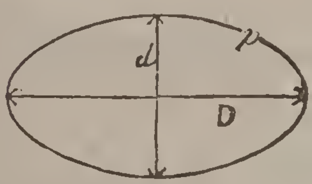
92 *To find the length of an Open Spiral.*



$$l = \pi\, n\, (R + r),$$

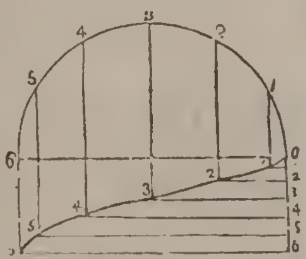
$$l = \frac{\pi}{P}\,(R^2 - r^2).$$

93 ***Periphery of an Ellipse.***

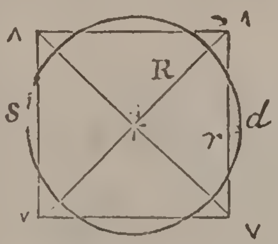


$$p = 2\sqrt{D^2 + 1{\cdot}4674 d^2}.$$

94 To construct a screw or *Helix*.



95 *To rectify a Circumference.*

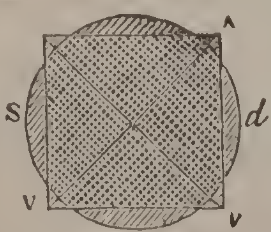


$R = 0{\cdot}555355\, d = 1{\cdot}1107\, r = 0{\cdot}7071\, S.$

$S = 0{\cdot}785398\, d = 1{\cdot}57079\, r = 1{\cdot}4142\, R.$

$d = 1{\cdot}27322\, S = 1{\cdot}79740\, R = 2\, r.$

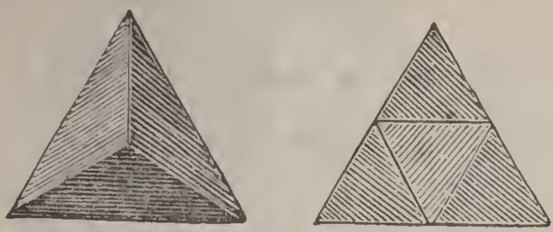
96 *To square a Circle.*



$R = 0{\cdot}626657\, d = 1{\cdot}253314\, r = 0{\cdot}7071\, S.$

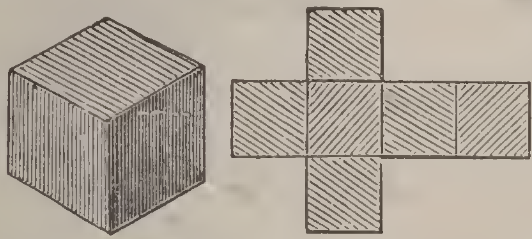
$S = 0{\cdot}886226\, d = 1{\cdot}77245\, r = 1{\cdot}4142\, R.$

$d = 1{\cdot}12838\, S = 1{\cdot}5367\, R = 2\, r.$



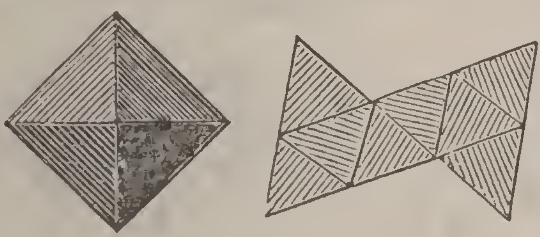
97 *Tetrahedron.*

$r = 0{\cdot}20413\ \mathbf{s}.$
$R = 0{\cdot}61237\ \mathbf{s}.$
$\mathbf{a} = 1{\cdot}73205\ \mathbf{s}^2.$
$\mathbf{c} = 0{\cdot}11785\ \mathbf{s}^3.$



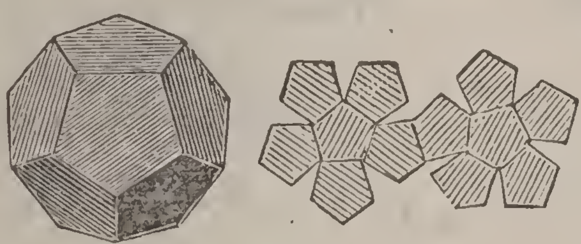
98 *Hexahedron.*

$r = 0{\cdot}50000\ \mathbf{s}.$
$R = 0{\cdot}86602\ \mathbf{s}.$
$\mathbf{a} = 6{\cdot}00000\ \mathbf{s}^2.$
$\mathbf{c} = 1{\cdot}00000\ \mathbf{s}^3.$



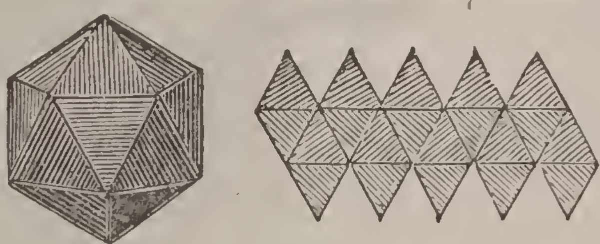
99 *Octahedron.*

$r = 0{\cdot}40721\ \mathbf{s}.$
$R = 0{\cdot}70710\ \mathbf{s}.$
$\mathbf{a} = 3{\cdot}46410\ \mathbf{s}^2.$
$\mathbf{c} = 0{\cdot}47140\ \mathbf{s}^3$



100 *Dodecahedron.*

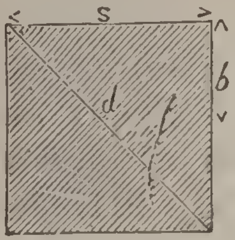
$r = 1{\cdot}11350\ \mathbf{s}.$
$R = 1{\cdot}40122\ \mathbf{s}.$
$\mathbf{a} = 20{\cdot}6457\ \mathbf{s}^2.$
$\mathbf{c} = 7{\cdot}66312\ \mathbf{s}^3.$



101 *Icosahedron.*

$r = 0{\cdot}7558\ \mathbf{s}.$
$R = 0{\cdot}9510\ \mathbf{s}.$
$\mathbf{a} = 8{\cdot}66025\ \mathbf{s}^2.$
$\mathbf{c} = 2{\cdot}18169\ \mathbf{s}^3.$

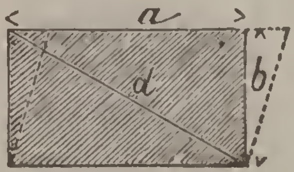
r = Radius of an inscribed Sphere.
R = Radius of circumscribed Sphere.
$\mathbf{a}$ = Area of the Polyhedrons.
$\mathbf{c}$ = Cubic contents of the Polyhedrons.
$\mathbf{s}$ = Side or edge of the Polyhedrons.



102 *Square.*

$$\mathbf{a} = \mathbf{s}^2 = 4b^2.$$

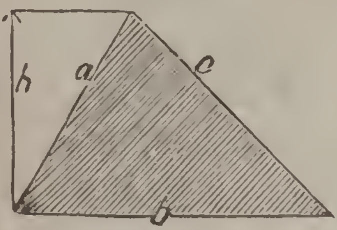
$$\mathbf{a} = 0{\cdot}5d^2$$



103 *Rectangle.*

$$\mathbf{a} = a\,b,$$

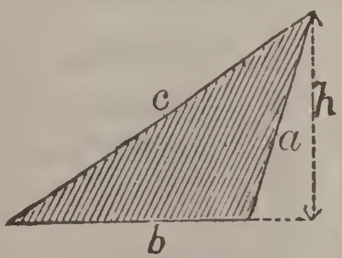
$$\mathbf{a} = b\,\sqrt{d^2 - b^2}.$$



104 *Triangle.*

$$\mathbf{a} = \frac{b\,h}{2} = \tfrac{1}{2}b\,h,$$

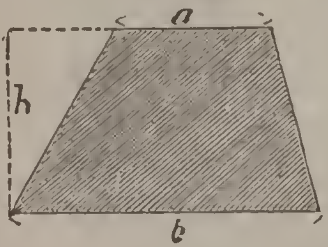
$$\mathbf{a} = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2\,b}\right)^2}.$$



105 *Triangle.*

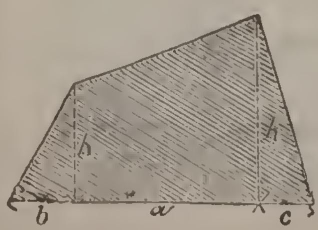
$$\mathbf{a} = \tfrac{1}{2}b\,h,$$

$$\mathbf{a} = \frac{b}{2}\sqrt{a^2 - \left(\frac{c^2 - a^2 - b^2}{2\,b}\right)^2}.$$



106 *Quadrangle.*

$$\mathbf{a} = \tfrac{1}{2}h(a + b).$$



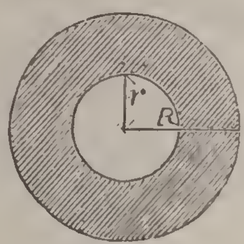
107 *Quadrangle.*

$$\mathbf{a} = \tfrac{1}{2}(a\,[h + h'] + b\,h' + c\,h).$$

108 *Circle Plane.*

$$\mathbf{a} = \pi r^2 = 0{\cdot}785\, d^2,$$

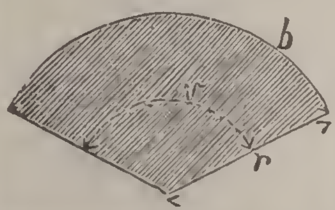
$$\mathbf{a} = \frac{p\,r}{2} = 0.0796\, P^2.$$



109 *Circle Ring.*

$$\mathbf{a} = \pi(R^2 - r^2) = \pi(R + r)(R - r),$$

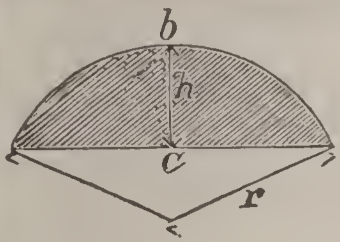
$$\mathbf{a} = 0{\cdot}785(D^2 - d^2).$$



110 *Sector.*

$$\mathbf{a} = \tfrac{1}{2} b\, r,$$

$$\mathbf{a} = \frac{\pi r^2 v}{360} = \frac{r^2 v}{114{\cdot}5}.$$



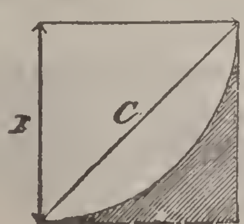
111 *Segment.*

$$\mathbf{a} = \tfrac{1}{2}[b\, r - c\,(r - h)],$$

$$\mathbf{a} = \frac{\pi r^2 v}{360} \mp \frac{c}{2}(r - h).$$

112 *Quadrant.*

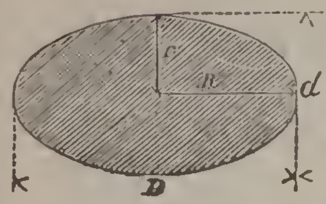
$$\mathbf{a} = 0{\cdot}785\, r^2 = 0{\cdot}3927\, \mathbf{c}^2.$$



113

$$\mathbf{a} = 0{\cdot}215\, r^2 = 0{\cdot}1075\, \mathbf{c}^2.$$

114 *Ellipse.*



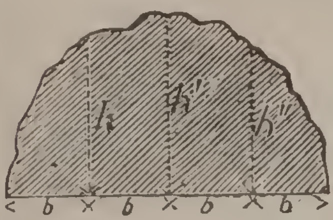
$$\mathbf{a} = \pi R r = 0{\cdot}785\, D\, d.$$

115 *Parabola.*

$$\mathbf{a} = \tfrac{2}{3}\, b\, h,$$

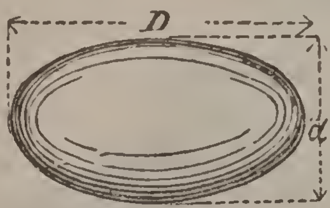
$$\mathbf{a} = \tfrac{4}{3} h \sqrt{p\, h}.$$

116 *Irregular Figure.*



$$\mathbf{a} = b\,(h + h' + h'').$$

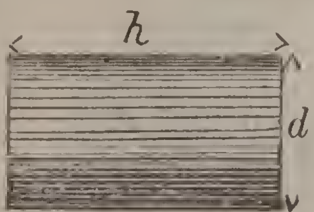
117 *Ellipsoid.*



$$\mathbf{a} = 8{\cdot}88\, r \sqrt{R^2 + r^2},$$

$$\mathbf{a} = 2{\cdot}22\, d \sqrt{D^2 + d^2}.$$

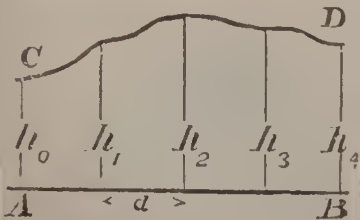
118 *Cylinder.*



$$\mathbf{a} = 2\,\pi\, r\, h = \pi\, d\, h,$$

$$h = \frac{\mathbf{a}}{2\,\pi\, r} = \frac{\mathbf{a}}{\pi\, d}.$$

119

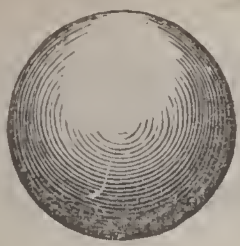


Simpson's Rule for Irregular Curves.

Divide $A\,B$ into any even number of parts of a length d. Erect the ordinates h_0, h_1, h_2, h_3, and h_4 at the divisions. Then

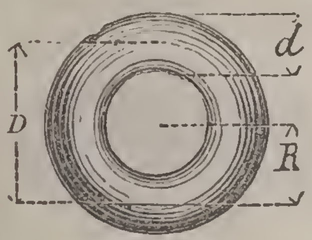
$$a = \frac{d}{3}(h_0 + 4h_1 + 2h_2 + 4h_3 + h_4).$$

This is more accurate than 116.



120 *Sphere.*

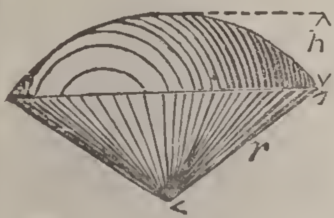
$$\mathbf{a} = 4\pi r^2 = 12{\cdot}56\, r^2 = \pi d^2.$$



121 *Forus.*

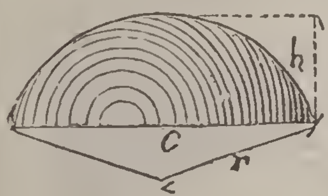
$$\mathbf{a} = 4\pi^2 R r = 39{\cdot}44\, R r,$$

$$\mathbf{a} = 9{\cdot}86\, D d.$$



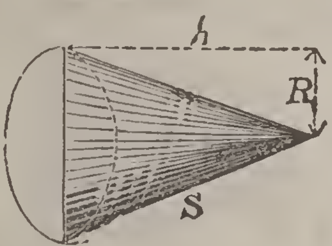
122 *Sphere Sector.*

$$\mathbf{a} = \frac{\pi r}{2}(4h + c).$$



123 *Circle Zone.*

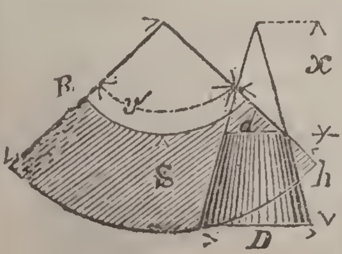
$$\mathbf{a} = 2\pi r h = \frac{\pi}{4}(c^2 + 4h^2)$$



124 *Cone.*

$$\mathbf{a} = \pi R s,$$

$$\mathbf{a} = \pi R \sqrt{R^2 + h^2}.$$



125 *Cone.*

$$x = \frac{d h}{D - d}, \qquad R = s + \frac{d s}{D - d},$$

$$\mathbf{a} = \frac{\pi s}{2}(D + d),$$

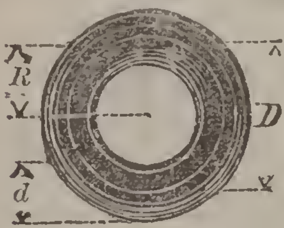
$$v = \frac{180\, D}{R} = \frac{180(D - d)}{s}.$$

126 *Sphere.*

$$\mathbf{c} = \frac{4\,\pi\,r^3}{3} = 4{\cdot}189\;\mathbf{r^3},$$

$$\mathbf{c} = \frac{\pi\,d^3}{6} = 0{\cdot}523\;\mathbf{d^3}.$$

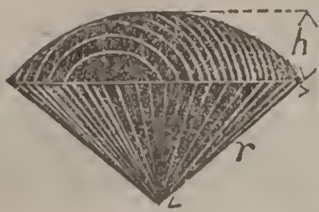
127 *Forus.*



$$\mathbf{c} = 2\,\pi^2\,R\,r^2 = 19{\cdot}74\;R\,\mathbf{r^2},$$

$$\mathbf{c} = 2{\cdot}463\;D\,d^2.$$

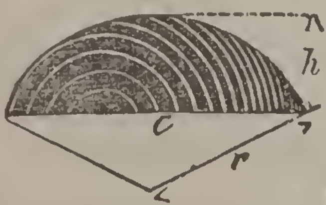
128 *Sphere Sector.*



$$\mathbf{c} = \tfrac{2}{3}\,\pi\,r^2\,h = 2{\cdot}0944\;r^2\,h,$$

$$\mathbf{c} = \tfrac{2}{3}\,\pi\,r^2(r \mp \sqrt{r^2 - \tfrac{1}{4}\,c^2}).$$

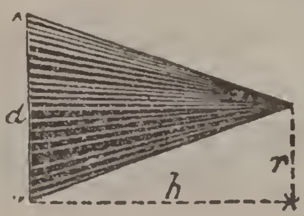
129 *Section of Sphere.*



$$\mathbf{c} = \pi\,h^2(r - \tfrac{1}{3}\,h),$$

$$\mathbf{c} = \pi\,h^2\left(\frac{c^2 + 4h^2}{8\,h} - \tfrac{1}{3}\,h\right).$$

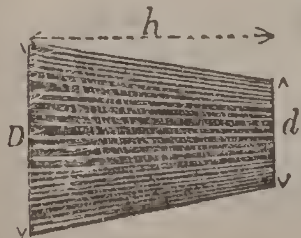
130 **Cone.**



$$\mathbf{c} = \frac{\pi\,r^2\,h}{3} = 1{\cdot}047\;\mathbf{r^2\,h},$$

$$\mathbf{c} = 0{\cdot}2618\;d^2\,h.$$

131 *Conic Frustum.*



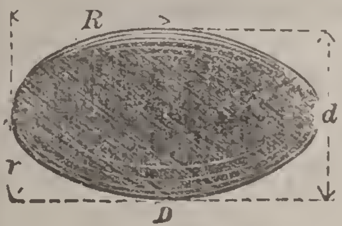
$$\mathbf{c} = \tfrac{1}{3}\,\pi\,h(R^2 + R\,r + r^2),$$

$$\mathbf{c} = \tfrac{1}{12}\,\pi\,h(D^2 + D\,d + d^2)$$

132 *Cylinder.*

$$\mathbf{c} = \pi r^2 h = 0.785\, d^2 h,$$

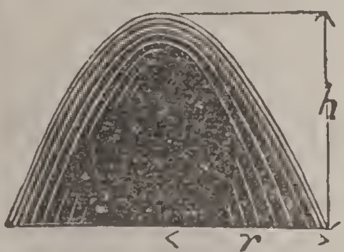
$$\mathbf{c} = \frac{p^2 h}{4\pi} = 0.0796\, p^2 h.$$



133 *Ellipsoid.*

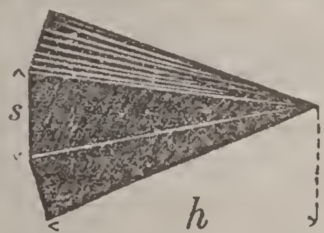
$$\mathbf{c} = 0{\cdot}424\, \pi^2 R r^2 = 4{\cdot}1847\, R r^2,$$

$$\mathbf{c} = 0{\cdot}053\, \pi^2 D d^2 = 0{\cdot}5231\, D d^2$$



134 *Paraboloid.*

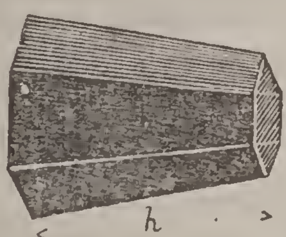
$$\mathbf{c} = \tfrac{1}{2} \pi r^2 h = 1{\cdot}5707\, r^2 h.$$



135 *Pyramid.*

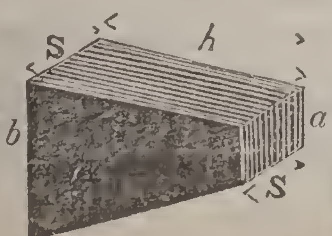
$$\mathbf{c} = \tfrac{1}{3}\, \mathbf{a}\, h,$$

$$\mathbf{c} = \frac{n s h}{6} \sqrt{r^2 - \frac{s^2}{4}}.$$



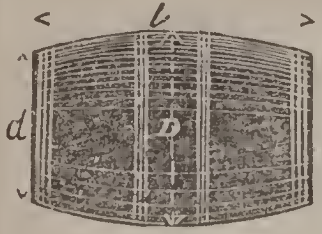
136 *Pyramidic Frustum.*

$$\mathbf{c} = \frac{h}{3}(A + \mathbf{a} + \sqrt{A\,\mathbf{a}}).$$



137 *Wedge Frustum.*

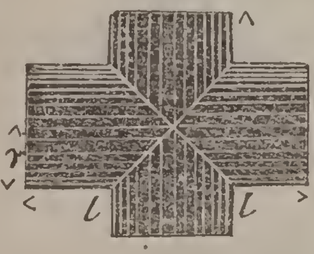
$$\mathbf{c} = \frac{h s}{2}(a + b).$$



138 *Cask.*

$$c = 1{\cdot}0453\, l\left(0{\cdot}4\, D^2 + 0{\cdot}2\, D\, d + 0{\cdot}15 d^2\right),$$

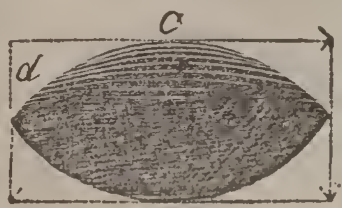
$$Gallon = \frac{l}{2200}\left(4\, D^2 + 2\, Dd + 1{\cdot}5\, d^2\right).$$



139 *Cylinder Sections.*

$$\mathbf{c} = \pi\, r^2(l + l' - \tfrac{2}{3} r),$$

$$\mathbf{c} = \pi\, r^2(l + l') - 2{\cdot}1\, r^3.$$



140 *Circular Spindle.*

$$\mathbf{c} = \pi\left(\tfrac{1}{6}\, c^3 - 0{\cdot}2\, d\left[c + \tfrac{2}{3}\sqrt{c^2 + d^2}\right]\sqrt{d^2 + c^2}\right)$$

Example 1. Fig. 104. The base of a Triangle is $b = 8$ feet, 3 inches, and the height, $h = 5$ feet, 6 inches. What is the area $\mathbf{a} = ?$

$$\mathbf{a} = \frac{b\, h}{2} = \frac{8{\cdot}25 \times 5{\cdot}5}{2} = 22{\cdot}6875 \text{ square feet.}$$

Example 2. Fig. 110. A Circle Sector having an angle $v = 39^\circ$ and the radius $\mathbf{r} = 67\frac{3}{4}$ inches. What is the area of the sector $\mathbf{a} = ?$

$$\mathbf{a} = \frac{\pi\, r^2\, v}{360} = \frac{3{\cdot}14 \times 67{\cdot}75^2 \times 39^\circ}{360} = 1562{\cdot}1 \text{ square feet.}$$

Example 3. Fig. 123. A Spherical Zone having its diameter $c = 18\frac{1}{2}$ inches and height $h = 7\frac{3}{4}$ inches. What is the convex surface of the Zone?

$$\mathbf{a} = \frac{\pi}{4}\left(c^2 + 4h^2\right) = \frac{3{\cdot}14}{4}\left(18{\cdot}5^2 + 4 \times 7{\cdot}75^2\right) = 614{\cdot}87 \text{ square inches.}$$

Example 4. Fig. 100. Require the radius R of a Sphere that will circumscribe a Dodecahedron with the side $s = 9$ inches.

$R = 1{\cdot}36428 \times 9 = 12{\cdot}27852$ inches, the answer.

Example 5. Fig. 131. A Frustrum of a Cone having its bottom diameter $D = 13$ inches, the top diameter $d = 5\frac{1}{4}$ inches, and the height $h = 25$ inches. What is the cubic contents $\mathbf{c} = ?$

$\mathbf{c} = \frac{1}{12}\pi\, h\left(D^2 + D\, d + d^2\right) = 0{\cdot}2618 \times 25\left(13^2 + 13 \times 5{\cdot}25 + 5{\cdot}25^2\right) = 20995$ cubic inches.

Example 6. Fig. 138. A Cask having its bung diameter $D = 36$ inches, head diameter $d = 28$ inches, and length $l = 56$ inches, (inside measurement) how many gallons of liquid can be contained in the cask? (The gallon = 231 cub. in.)

$$\text{Gallons} = \frac{56}{2200}\left(4 \times 36^2 + 2 \times 36 \times 28 + 1{\cdot}5 \times 28^2\right) = 214.$$

Example 7. Fig. 62. Required the length of the circular arc b, when the angle $v = 42°$, and the radius $r = 4$ feet, 3 inches?

$$b = \frac{\pi r v}{180} = \frac{3{\cdot}14 \times 4{\cdot}25 \times 42}{180} = 3{\cdot}113 \text{ feet.}$$

Example 8. Fig. 64. Required the radius of a circular arc, whose chord is 9 feet, 4 inches, and height, $h = 1$ foot, 8 inches?

$$r = \frac{c^2 + 4h^2}{8h} = \frac{9{\cdot}33^2 + 4 \times 1{\cdot}66^2}{8 \times 1{\cdot}66} = \frac{98{\cdot}0711}{13{\cdot}28} = 7{\cdot}384 \text{ feet.}$$

Example 9. Fig. 80. The three sides in a triangle being, $a = 6{\cdot}42$, $b = 7{\cdot}75$ and $c = 8{\cdot}66$ feet. How high is the triangle over the base b?

$$d = \frac{a^2 + b^2 - c^2}{2b} = \frac{6{\cdot}42^2 + 7{\cdot}75^2 - 8{\cdot}66^2}{2 \times 8{\cdot}66} = 1{\cdot}5175 \text{ feet,}$$

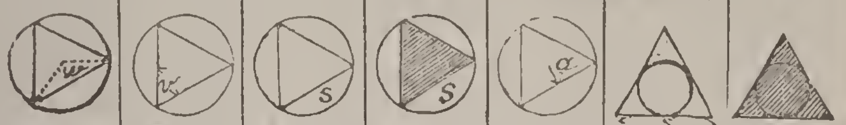
the height $h = \sqrt{a^2 - d^2} = \sqrt{6.42^2 - 1{\cdot}5175^2} = 6{\cdot}24$ feet, the answer

Example 10. Fig. 89. The radius of a working beam is, $r = 8{\cdot}36$ feet, the stroke $S = 5{\cdot}5$ feet. How much is the vibration $V = ?$

Vibration, $$V = r - \sqrt{r^2 - \frac{S^2}{4}} = 8{\cdot}36 - \sqrt{8{\cdot}36^2 - \frac{5{\cdot}5^2}{4}}$$

$$= 0{\cdot}471 \text{ feet} = 5{\cdot}65 \text{ inches} = 5{\cdot}\tfrac{21}{32}, \text{ the answer.}$$

TABLE OF POLYGONS.



Number of sides in the Polygon.		Centre Angle w.	Polygon Angle v.	Side $= k\,R$.	Area $= k\,S^2$.	Apotem $= k\,R$.	Side $= k\,r$.	Area $= k\,r^2$.
Trigon.	3	120°	60°	1·732	0·4330	0·5000	3·4641	5·1961
Tetragon.	4	90°	90°	1·4142	1·0000	0·7071	2·0000	4·0000
Pentagon.	5	72°	108°	1·1755	1·7205	0·8090	1·4536	3·6327
Hexagon.	6	60°	120°	1·0000	2·5980	0·8660	1·1547	3·4640
Heptagon.	7	51°43′	128°17′	0·8677	3·6339	0·9009	0·9631	3·3710
Octagon.	8	45°	135°	0·7653	4·8284	0·9238	0·8284	3·3130
Nonagon.	9	40°	140°	0·6840	6·1820	0·9396	0·7279	3·2750
Decagon.	10	36°	144°	0·6180	7·6942	0·9510	0·6498	3·2490
Undecagon.	11	32°13′	147°47′	0·5634	9·3656	0·9595	0·5872	3·2290
Dodecagon.	12	30°	150°	0·5176	11·196	0·9659	0·5359	3·2152
	14	25°43′	154°17′	0·4450	15·334	0·9762	0·4562	3·1935
	15	24°	156°	0·4158	17·642	0·9781	0·4250	3·1882
	16	22°30′	157°30′	0·3900	20·128	0·9807	0·4068	3·1824
	18	20°	160°	0·3472	25·534	0·9848	0·3526	3·1737
	20	18°	162°	0·3130	40·634	0·9877	0·3166	3·1676
	24	15°	165°	0·2610	45·593	0·9914	0·2632	3·1596

Explanation of the Table for Polygons.

The number of sides in the polygon is noted in the first column.

k = *tabular coefficient*, to be multiplied as noted on the top of the columns.

Example 1. How long is the side of an inscribed Pentagon, when the radius of the circle is 3 feet, and 4 inches? (4 inches = 0·333 feet.)

$$3{\cdot}333 \times 1{\cdot}1755 = 3{\cdot}9179 \text{ feet, the answer.}$$

***Example* 2.** What is the area of a Heptagon when one of its sides is 13·75 inches

$$13{\cdot}75^2 \times 3{\cdot}6339 = 687{\cdot}02 \text{ square inches.}$$

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
1	3·1416	0·7854	51	160·22	2042·8	101	317·30	8011·9
2	6·2832	3·1416	52	163·36	2123.7	102	320·44	8171·3
3	9·4248	7·0686	53	166·50	2206.2	103	323·58	8332·3
4	12·566	12·5664	54	169·65	2290·2	104	326·73	8494·9
5	15·708	19·6350	55	172·79	2375·8	105	329·87	8659·0
6	18·850	28·2743	56	175·93	2463·0	106	333·01	8824·7
7	21·991	38·4845	57	179·07	2551·8	107	336·15	8992·0
8	25·133	50·2655	58	182·21	2642·1	108	339·29	9160·9
9	28·274	63·6173	59	185·35	2734·0	109	342·43	9331·3
10	31·416	78·54	60	188·50	2827·4	110	345·58	9503·3
11	34·558	95·03	61	191·64	2922·5	111	348·72	9676·9
12	37·699	113·10	62	194·78	3019·1	112	351·86	9852.0
13	40·841	132·73	63	197·92	3117·2	113	355.00	10028·8
14	43·982	153·94	64	201·06	3217·0	114	358·14	10207·0
15	47·124	176·71	65	204·20	3318·3	115	361·28	10386·9
16	50·265	201·06	66	207·35	3421·2	116	364·42	10568.3
17	53·407	226·98	67	210·49	3525·7	117	367·57	10751·3
18	56·549	254·47	68	213·63	3631·7	118	370·71	10935·9
19	59·690	283·53	69	216·77	3739·3	119	373·85	11122·0
20	62·832	314·16	70	219·91	3848·5	120	376·99	11310
21	65·973	346·36	71	223·05	3959·2	121	380.13	11499
22	69·115	380·13	72	226·19	4071.5	122	383·27	11690
23	72·257	415·48	73	229·34	4185·4	123	386·42	11882
24	75·398	452·39	74	232·48	4300·8	124	389·56	12076
25	78·540	490·87	75	235.62	4417·9	125	392·70	12272
26	81·681	530·93	76	238·76	4536·5	126	395·84	12469
27	84·823	572·56	77	241·90	4656·6	127	398·98	12668
28	87·965	615·75	78	245·04	4778·4	128	402·12	12868
29	91·106	660·52	79	248·19	4901·7	129	405·27	13070
30	94·248	706·86	80	251·33	5026·6	130	408·41	13273
31	97·389	754·77	81	254·47	5153·0	131	411·55	13478
32	100·53	804·25	82	257·61	5281·0	132	414·69	13685
33	103·67	855·30	83	260·75	5410·6	133	417·83	13893
34	106·81	907·92	84	263·89	5541·8	134	420·97	14103
35	109·96	962·11	85	267.04	5674·5	135	424·12	14314
36	113·10	1017·88	86	270·18	5808·8	136	427.26	14527
37	116·24	1075·21	87	273·32	5944·7	137	430·40	14741
38	119·38	1134·11	88	276·46	6082·1	138	433·54	14957
39	122·52	1194·59	89	279·60	6221·1	139	436·68	15175
40	125·66	1256·63	90	282·74	6361·7	140	439·82	15394
41	128·81	1320·25	91	285·88	6503·9	141	442·96	15615
42	131·95	1385·44	92	289·03	6647·6	142	446·11	15837
43	135·09	1452·20	93	292·17	6792·9	143	449·25	16061
44	138·23	1520·52	94	295·31	6939·8	144	452·39	16286
45	141·37	1590·43	95	298·45	7088·2	145	455·53	16513
46	144·51	1661·90	96	301·59	7238·2	146	458·67	16742
47	147·65	1734·94	97	304·73	7389·8	147	461·81	16972
48	150·80	1809·55	98	307·88	7543·0	148	464·96	17203
49	153.94	1885·74	99	311·02	7697·7	149	468·10	17437
50	157·08	1963·5	100	314·16	7854·0	150	471·24	17671

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
151	474·38	17908	201	631·46	31731	251	788·54	49481
152	477·52	18146	202	634·60	32047	252	791·68	49876
153	480·66	18385	203	637·74	32365	253	794·82	50273
154	483·81	18627	204	640·89	32685	254	797·96	50671
155	486·95	18869	205	644·03	33006	255	801·11	51071
156	490·09	19113	206	647·17	33329	256	804·25	51472
157	493·23	19359	207	650·31	33654	257	807·39	51875
158	496·37	19607	208	653·45	33979	258	810·53	52279
159	499·51	19856	209	656·59	34307	259	813·67	52685
160	502·65	20106	210	659·73	34636	260	816·81	53093
161	505·80	20358	211	662·88	34967	261	819·96	53502
162	508·94	20612	212	666·02	35299	262	823·10	53913
163	512·08	20867	213	669·16	35633	263	826·24	54325
164	515·22	21124	214	672·30	35968	264	829·38	54739
165	518·36	21382	215	675·44	36305	265	832·52	55155
166	521·50	21642	216	678·58	36644	266	835·66	55572
167	524·65	21904	217	681·73	36984	267	838·81	55990
168	527·79	22167	218	684·87	37325	268	841·95	56410
169	530·93	22432	219	688·01	37668	269	845·09	56832
170	534·07	22698	220	691·15	38013	270	848·23	57256
171	537·21	22966	221	694·29	38360	271	851·37	57680
172	540·35	23235	222	697·43	38708	272	854·51	58107
173	543·50	23506	223	700·58	39057	273	857·66	58535
174	546·64	23779	224	703·72	39408	274	860·80	58965
175	549·78	24053	225	706·86	39761	275	863·94	59396
176	552·92	24328	226	710·00	40115	276	867·08	59828
177	556·06	24606	227	713·14	40471	277	870·22	60263
178	559·20	24885	228	716·28	40828	278	873·36	60699
179	562·35	25165	229	719·42	41187	279	876·50	61136
180	565·49	25447	230	722·57	41548	280	879·65	61575
181	568·63	25730	231	725·71	41910	281	882·79	62016
182	571·77	26016	232	728·85	42273	282	885·93	62458
183	574·91	26302	233	731·99	42638	283	889·07	62902
184	578·05	26590	234	735·13	43005	284	892·21	63347
185	581·19	26880	235	738·27	43374	285	895·35	63794
186	584·34	27172	236	741·42	43744	286	898·50	64242
187	587·48	27465	237	744·56	44115	287	901·64	64692
188	590·62	27759	238	747·70	44488	288	904·78	65144
189	593·76	28055	239	750·84	44863	289	907·92	65597
190	596·90	28353	240	753·98	45239	290	911·06	66052
191	600·04	28652	241	757·12	45617	291	914·20	66508
192	603·19	28953	242	760·27	45996	292	917·35	66966
193	606·33	29255	243	763·41	46377	293	920·49	67426
194	609·47	29559	244	766·55	46759	294	923·63	67887
195	612·61	29865	245	769·69	47144	295	926·77	68349
196	615·75	30172	246	772·83	47529	296	929·91	68813
197	618·89	30481	247	775·97	47916	297	933·05	69279
198	622·04	30791	248	779·12	48305	298	936·19	69747
199	625·18	31103	249	782·26	48695	299	939·34	70215
200	628·32	31416	250	785·40	49087	300	942·48	70686

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
301	945·62	71158	351	1102·70	96762	401	1259·78	126293
302	948·76	71631	352	1105·84	97314	402	1262·92	126923
303	951·90	72107	353	1108·98	97868	403	1266·06	127556
304	955·04	72583	354	1112·12	98423	404	1269·20	128190
305	958·19	73062	355	1115·27	98980	405	1272·35	128825
306	961·33	73542	356	1118·41	99538	406	1275·49	129462
307	964·47	74023	357	1121·55	100098	407	1278·63	130100
308	967·61	74506	358	1124·69	100660	408	1281·77	130741
309	970·75	74991	359	1127·83	101223	409	1284·91	131382
310	973·89	75477	360	1130·97	101788	410	1288·05	132025
311	977·04	75964	361	1134·11	102354	411	1291·19	132670
312	980·18	76454	362	1137·26	102922	412	1294·34	133317
313	983·32	76945	363	1140·40	103491	413	1297·48	133965
314	986·46	77437	364	1143·54	104062	414	1300·62	134614
315	989·60	77931	365	1146·68	104635	415	1303·76	135265
316	992·74	78427	366	1149·82	105209	416	1306·90	135918
317	995·88	78924	367	1152·96	105785	417	1310·04	136572
318	999·03	79423	368	1156·11	106362	418	1313·19	137228
319	1002·17	79923	369	1159·25	106941	419	1316·33	137885
320	1005·31	80425	370	1162·39	107521	420	1319·47	138544
321	1008·45	80928	371	1165·53	108103	421	1322·61	139205
322	1011·59	81433	372	1168·67	108687	422	1325·75	139867
323	1014·73	81940	373	1171·81	109272	423	1328·89	140531
324	1017·88	82448	374	1174·96	109858	424	1332·04	141196
325	1021·02	82958	375	1178·10	110447	425	1335·18	141863
326	1024·16	83469	376	1181·24	111036	426	1338·32	142531
327	1027·30	83982	377	1184·38	111628	427	1341·46	143201
328	1030·44	84496	378	1187·52	112221	428	1344·60	143872
329	1033·58	85012	379	1190·66	112815	429	1347·74	144545
330	1036·73	85530	380	1193·81	113411	430	1350·88	145220
331	1039·87	86049	381	1196·95	114009	431	1354·03	145896
332	1043·01	86570	382	1200·09	114608	432	1357·17	146574
333	1046·15	87092	383	1203·23	115209	433	1360·31	147254
334	1049·29	87616	384	1206·37	115812	434	1363·45	147934
335	1052·43	88141	385	1209·51	116416	435	1366·59	148617
336	1055·58	88668	386	1212·65	117021	436	1369·73	149301
337	1058·72	89197	387	1215·80	117628	437	1372·88	149987
338	1061·86	89727	388	1218·94	118237	438	1376·02	150674
339	1065·00	90259	389	1222·08	118847	439	1379·16	151363
340	1068·14	90792	390	1225·22	119459	440	1382·30	152053
341	1071·28	91327	391	1228·36	120072	441	1385·44	152745
342	1074·42	91863	392	1231·50	120687	442	1388·58	153439
343	1077·57	92401	393	1234·65	121304	443	1391·73	154134
344	1080·71	92941	394	1237·79	121922	444	1394·87	154830
345	1083·85	93482	395	1240·93	122542	445	1398·01	155528
346	1086·99	94025	396	1244·07	123163	446	1401·15	156228
347	1090·13	94569	397	1247·21	123786	447	1404·29	156930
348	1093·27	95115	398	1250·35	124410	448	1407·43	157633
349	1096·42	95662	399	1253·50	125036	449	1410·58	158337
350	1099·56	96211	400	1256·64	125664	450	1413·72	159043

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
451	1416·86	159751	501	1573·94	197136	551	1731·02	238448
452	1420·00	160460	502	1577·08	197923	552	1734·16	239314
453	1423·14	161171	503	1580·22	198713	553	1737·30	240182
454	1426·28	161883	504	1583·36	199504	554	1740·44	241051
455	1429·42	162597	505	1586·50	200296	555	1743·58	241922
456	1432·57	163313	506	1589·65	201090	556	1746·73	242795
457	1435·71	164030	507	1592·79	201886	557	1749·87	243669
458	1438·85	164748	508	1595·93	202683	558	1753·01	244545
459	1441·99	165468	509	1599·07	203482	559	1756·15	245422
460	1445·13	166190	510	1602·21	204282	560	1759·29	246301
461	1448·27	166914	511	1605·35	205084	561	1762·43	247181
462	1451·42	167639	512	1608·50	205887	562	1765·58	248063
463	1454·56	168365	513	1611·64	206692	563	1768·72	248947
464	1457·70	169093	514	1614·78	207499	564	1771·86	249832
465	1460·84	169823	515	1617·92	208307	565	1775·00	250719
466	1463·98	170554	516	1621·06	209117	566	1778·14	251607
467	1467·12	171287	517	1624·20	209928	567	1781·28	252497
468	1470·27	172021	518	1627·35	210741	568	1784·42	253388
469	1473·41	172757	519	1630·49	211556	569	1787·57	254281
470	1476·55	173494	520	1633·63	212372	570	1790·71	255176
471	1479·69	174234	521	1636·77	213189	571	1793·85	256072
472	1482·83	174974	522	1639·91	214008	572	1796·99	256970
473	1485·97	175716	523	1643·05	214829	573	1800·13	257869
474	1489·11	176460	524	1646·20	215651	574	1803·27	258770
475	1492·26	177205	525	1649·34	216475	575	1806·42	259672
476	1495·40	177952	526	1652·48	217301	576	1809·56	260576
477	1498·54	178701	527	1655·62	218128	577	1812·70	261482
478	1501·68	179451	528	1658·76	218956	578	1815·84	262389
479	1504·82	180203	529	1661·90	219787	579	1818·98	263298
480	1507·96	180956	530	1665·04	220618	580	1822·12	264208
481	1511·11	181711	531	1668·19	221452	581	1825·27	265120
482	1514·25	182467	532	1671·33	222287	582	1828·41	266033
483	1517·39	183225	533	1674·47	223123	583	1831·55	266948
484	1520·53	183984	534	1677·61	223961	584	1834·69	267865
485	1523·67	184745	535	1680·75	224801	585	1837·83	268783
486	1526·81	185508	536	1683·89	225642	586	1840·97	269702
487	1529·96	186272	537	1687·04	226484	587	1844·11	270624
488	1533·10	187038	538	1690·18	227329	588	1847·26	271547
489	1536·24	187805	539	1693·32	228175	589	1850·40	272471
490	1539·38	188574	540	1696·46	229022	590	1853·54	273397
491	1542·52	189345	541	1699·60	229871	591	1856·68	274325
492	1545·66	190117	542	1702·74	230722	592	1859·82	275254
493	1548·81	190890	543	1705·88	231574	593	1862·96	276184
494	1551·95	191665	544	1709·03	232428	594	1866·11	277117
495	1555·09	192442	545	1712·17	233283	595	1869·25	278051
496	1558·23	193221	546	1715·31	234140	596	1872·39	278986
497	1561·37	194000	547	1718·45	234998	597	1875·53	279923
498	1564·51	194782	548	1721·59	235858	598	1878·67	280862
499	1567·65	195565	549	1724·73	236720	599	1881·81	281802
500	1570·80	196350	550	1727·88	237583	600	1884·96	282743

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
601	1888·10	283687	651	2045·18	332853	701	2202·26	385945
602	1891·24	284631	652	2048·32	333876	702	2205·40	387047
603	1894·38	285578	653	2051·46	334901	703	2208·54	388151
604	1897·52	286526	654	2054·60	335927	704	2211·68	389256
605	1900·66	287475	655	2057·74	336955	705	2214·82	390363
606	1903·81	288426	656	2060·88	337985	706	2217·96	391471
607	1906·95	289379	657	2064·03	339016	707	2221·11	392580
608	1910·09	290333	658	2067·17	340049	708	2224·25	393692
609	1913·23	291289	659	2070·31	341083	709	2227·39	394805
610	1916·37	292247	660	2073·45	342119	710	2230·53	395919
611	1919·51	293206	661	2076·59	343157	711	2233·67	397035
612	1922·65	294166	662	2079·73	344196	712	2236·81	398153
613	1925·80	295128	663	2082·88	345237	713	2239·96	399272
614	1928·94	296092	664	2086·02	346279	714	2243·10	400393
615	1932·08	297057	665	2089·16	347323	715	2246·24	401515
616	1935·22	298024	666	2092·30	348368	716	2249·38	402639
617	1938·36	298992	667	2095·44	349415	717	2252·52	403765
618	1941·50	299962	668	2098·58	350464	718	2255·66	404892
619	1944·65	300934	669	2101·73	351514	719	2258·81	406020
620	1947·79	301907	670	2104·87	352565	720	2261·95	407150
621	1950·93	302882	671	2108·01	353618	721	2265·09	408282
622	1954·07	303858	672	2111·15	354673	722	2268·23	409416
623	1957·21	304836	673	2114·29	355730	723	2271·37	410550
624	1960·35	305815	674	2117·43	356788	724	2274·51	411687
625	1963·50	306796	675	2120·58	357847	725	2277·65	412825
626	1966·64	307779	676	2123·72	358908	726	2280·80	413965
627	1969·78	308763	677	2126·86	359971	727	2283·94	415106
628	1972·92	309748	678	2130·00	361035	728	2287·08	416248
629	1976·06	310736	679	2133·14	362101	729	2290·22	417393
630	1979·20	311725	680	2136·28	363168	730	2293·36	418539
631	1982·35	312715	681	2139·42	364237	731	2296·50	419686
632	1985·49	313707	682	2142·57	365308	732	2299·65	420835
633	1988·63	314700	683	2145·71	366380	733	2302·79	421986
634	1991·77	315696	684	2148·85	367453	734	2305·93	423139
635	1994·91	316692	685	2151·99	368528	735	2309·07	424292
636	1998·05	317690	686	2155·13	369605	736	2312·21	425447
637	2001·19	318690	687	2158·27	37068[illegible]	737	2315·35	426604
638	2004·34	319692	688	2161·42	371764	738	2318·50	427762
639	2007·48	320695	689	2164·56	372845	739	2321·64	428922
640	2010·62	321699	690	2167·70	373928	740	2324·78	430084
641	2013·67	322705	691	2170·84	375013	741	2327·92	431247
642	2016·90	323713	692	2173·98	376099	742	2331·06	432412
643	2020·04	324722	693	2177·12	377187	743	2334·30	433578
644	2023·19	325733	694	2180·27	378276	744	2337·34	434746
645	2026·33	326745	695	2183·41	379367	745	2340·49	435916
646	2029·47	327759	696	2186·55	380459	746	2343·63	437087
647	2032·61	328775	697	2189·69	381554	747	2346·77	438259
648	2035·75	329792	698	2192·83	382649	748	2349·91	439433
649	2038·89	330810	699	2195·97	383746	749	2353·05	440609
650	2042·04	331831	700	2199·11	384845	750	2356·19	441786

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
751	2359·34	442965	801	2516·42	503912	851	2673·50	568786
752	2362·48	444146	802	2519·56	505171	852	2676·64	570124
753	2365·62	445328	803	2522·70	506432	853	2679·78	571463
754	2368·76	446511	804	2525·84	507694	854	2682·92	572803
755	2371·90	447697	805	2528·98	508958	855	2686·06	574146
756	2375·04	448883	806	2532·12	510223	856	2689·20	575490
757	2378·19	450072	807	2535·27	511490	857	2692·34	576835
758	2381·33	451262	808	2538·41	512758	858	2695·49	578182
759	2384·47	452453	809	2541·55	514028	859	2698·63	579530
760	2387·61	453646	810	2544·69	515300	860	2701·77	580880
761	2390·75	454841	811	2547·83	516573	861	2704·91	582232
762	2393·89	456037	812	2550·97	517848	862	2708·05	583585
763	2397·04	457234	813	2554·11	519124	863	2711·19	584940
764	2400·18	458434	814	2557·26	520402	864	2714·34	586297
765	2403·32	459635	815	2560·40	521681	865	2717·48	587655
766	2406·46	460837	816	2563·54	522962	866	2720·62	589014
767	2409·60	462041	817	2566·68	524245	867	2723·76	590375
768	2412·74	463247	818	2569·82	525529	868	2726·90	591738
769	2415·88	464454	819	2572·96	526814	869	2730·04	593102
770	2419·03	465663	820	2576·11	528102	870	2733·19	594468
771	2422·17	466873	821	2579·25	529391	871	2736·33	595835
772	2425·31	468085	822	2582·39	530681	872	2739·47	597204
773	2428·45	469298	823	2585·53	531973	873	2742·61	598575
774	2431·59	470513	824	2588·67	533267	874	2745·75	599947
775	2434·73	471730	825	2591·81	534562	875	2748·89	601320
776	2437·88	472948	826	2594·96	535858	876	2752·04	602696
777	2441·02	474168	827	2598·10	537157	877	2755·18	604073
778	2444·16	475389	828	2601·24	538456	878	2758·32	605451
779	2447·30	476612	829	2604·38	539758	879	2761·46	606831
780	2450·44	477836	830	2607·52	541061	880	2764·60	608212
781	2453·58	479062	831	2610·66	542365	881	2767·74	609595
782	2456·73	480290	832	2613·81	543671	882	2770·88	610980
783	2459·87	481519	833	2616·95	544979	883	2774·03	612366
784	2463·01	482750	834	2620·09	546288	884	2777·17	613754
785	2466·15	483982	835	2623·23	547599	885	2780·31	615143
786	2469·29	485216	836	2626·37	548912	886	2783·45	616534
787	2472·43	486451	837	2629·51	550226	887	2786·59	617927
788	2475·58	487688	838	2632·65	551541	888	2789·73	619321
789	2478·72	488927	839	2635·80	552858	889	2792·88	620717
790	2481·86	490167	840	2638·94	554177	890	2796·02	622114
791	2485·00	491409	841	2642·08	555497	891	2799·16	623513
792	2488·14	492652	842	2645·22	556819	892	2802·30	624913
793	2491·28	493897	843	2648·36	558142	893	2805·44	626315
794	2494·42	495143	844	2651·50	559467	894	2808·58	627718
795	2497·57	496391	845	2654·65	560794	895	2811·73	629124
796	2500·71	497641	846	2657·79	562122	896	2814·87	630530
797	2503·85	498892	847	2660·93	563452	897	2818·01	631938
798	2506·99	500145	848	2664·07	564783	898	2821·15	633348
799	2510·13	501399	849	2667·21	566116	899	2824·29	634760
800	2513·27	502655	850	2670·35	567450	900	2827·43	636173

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
901	2830·58	637587	934	2934·25	685147	967	3037·92	734417
902	2833·72	639003	935	2937·39	686615	968	3041·06	735937
903	2836·86	640421	936	2940·53	688084	969	3044·20	737458
904	2840·00	641840	937	2943·67	689555	970	3047·34	738981
905	2843·14	643261	938	2946·81	691028	971	3050·49	740506
906	2846·28	644683	939	2949·96	692502	972	3053·63	742032
907	2849·42	646107	940	2953·10	693978	973	3056·77	743559
908	2852·57	647533	941	2956·24	695455	974	3059·91	745088
909	2855·71	648960	942	2959·38	696934	975	3063·05	746619
910	2858·85	650388	943	2962·52	698415	976	3066·19	748151
911	2861·99	651818	944	2965·66	699897	977	3069·34	749685
912	2865·13	653250	945	2968·81	701380	978	3072·48	751221
913	2868·27	654684	946	2971·95	702865	979	3075·62	752758
914	2871·42	656118	947	2975·09	704352	980	3078·76	754296
915	2874·56	657555	948	2978·23	705840	981	3081·90	755837
916	2877·70	658993	949	2981·37	707330	982	3085·04	757378
917	2880·84	660433	950	2984·51	708822	983	3088·19	758922
918	2883·98	661874	951	2987·65	710315	984	3091·33	760466
919	2887·12	663317	952	2990·80	711809	985	3094·47	762013
920	2890·27	664761	953	2993·94	713307	986	3097·61	763561
921	2893·41	666207	954	2997·08	714803	987	3100·75	765111
922	2896·55	667654	955	3000·22	716303	988	3103·89	766662
923	2899·69	669103	956	3003·36	717804	989	3107·04	768215
924	2902·83	670554	957	3006·50	719306	990	3110·18	769769
925	2905·97	672006	958	3009·65	720810	991	3113·32	771325
926	2909·11	673460	959	3012·79	722316	992	3116·46	772882
927	2912·26	674915	960	3015·93	723823	993	3119·60	774441
928	2915·40	676372	961	3019·07	725332	994	3122·74	776002
929	2918·54	677831	962	3022·21	726842	995	3125·88	777564
930	2921·68	679291	963	3025·35	728354	996	3129·03	779128
931	2924·82	680752	964	3028·50	729867	997	3132·17	780693
932	2927·96	682216	965	3031·64	731382	998	3135·31	782260
933	2931·11	683680	966	3034·78	732899	999	3138·45	783828

Explanation of the Preceding Table.

When the diameter is expressed in more or less units than in the table, add or subtract so many figures more in the circumference; add or subtract twice as many in the area.

Examples.

Diameter.	Circumference.	Area.
9370	29436.7	68955500
93.7	294.367	6895.55
9.37	29.4367	68.9555
0.937	2.94367	0.689555

Two variable quantities x and y depended on one another, to find the value of one, when the other is a *maxima* or *minima*.

$$\left.\begin{matrix} x \\ y \end{matrix}\right\} \text{is a maxima or minima when its first differential coefficient} \left\{\begin{matrix} \frac{dx}{dy} = o. \\ \frac{dy}{dx} = o. \end{matrix}\right.$$

When the second $d\cdot$ coef. $\frac{d^2y}{dx^2}$ is positive, y is a minimum, and when negative y is a maximum. The variables may have both maximums and minimums, as formulas will indicate.

Example 1. Find the value of x when y is a maximum or minimum, in the formula $y = x^3 - 12x + 22$? $dy = (3x^2 - 12)\,dx$, $\frac{dy}{dx} = 3x^2 - 12 = o$.

Of which $x = \sqrt{\frac{12}{3}} = 2$ the answer. $\frac{d^2y}{dx^2} = 6x$, which is positive, consequently $y = 2^3 - 12 \times 2 + 22 = 6$, a minimum, when $x = 2$.

Example 2. It is required to cut out the strongest possible beam of height h and breadth b, from a log of diameter D, fig. 221, page 174? The strength of a beam is in proportion to bh^2 which is to be a maximum.

$D^2 = b^2 + h^2$, $h^2 = D^2 - b^2$, $b\,h^2 = b\,(D^2 - h^2)$, $d(bh^2) = (D^2 - 3\,b^2)d\,b$.

$\frac{d\,(bh^2)}{d\,b} = D^2 - 3\,b^2 = o$, of which the breadth $b = D\sqrt{\frac{1}{3}} = 0.577\,D$, and height $h = \sqrt{D^2 - \frac{1}{3}} = D\sqrt{0\cdot6666} = 0\cdot8164\,D$, the answer. The second $d\cdot$ coef. $\frac{d^2(b\,h^2)}{d\,b^2} = -6\,b$, which is negative, and therefore $b\,h^2$ is a maximum when $b = 0\cdot577\,D$.

Example 3. It is required to know the proportion of height h and diameter D of a cylinder, having the greatest cubic content v, with the smallest surface z including top and bottom? $z = \frac{\pi\,D^2}{2} + \pi\,D\,h = \frac{2\,v}{h} + \pi\,D\,h$, which is to be a minimum. Let $v = 1$ and $D = 1$, then $z = \frac{2}{h} + \pi\,h$, and

$d\,z = \left(\pi - \frac{4}{h^2}\right)dh$, $\frac{dz}{dh} = \pi - \frac{4}{h^2} = o$, when $h = \sqrt{\frac{4}{\pi}} = 1\cdot1284\,D$, the answer.

The second $d\cdot$ coef. $\frac{d^2z}{dh^2} = +\frac{4 \times 2}{h^4}$, which is positive, and z a minimum when $h = 1\cdot1284\,D$.

Maclaurin's Theorem.

Maclaurin's Theorem, explains how to develop into a series a function with one variable, as

$$u = (u) + \frac{x}{1}\left(\frac{du}{dx}\right) + \frac{x^2}{2}\left(\frac{d^2u}{dx^2}\right) + \frac{x^3}{2 \times 3}\left(\frac{d^3u}{dx^3}\right) \dots + \frac{x^n}{2 \times 3 \times \dots n}\left(\frac{d^nu}{dx^n}\right) \text{etc.}$$

where the factor in the parenthesis is that which it assumes when $x = o$.

The function $u = \frac{1}{a + x}$ developed into a series will be

$$\frac{1}{a + x} = \frac{1}{a} - \frac{x}{a^2} + \frac{x^2}{a^3} - \frac{x^3}{a^4} \dots \frac{x^n}{a^{n+1}}, \text{ etc.}$$

Taylor's Theorem.

Taylor's Theorem, explains how to develop into a series a function of the sum or difference of two variable as $u = x \pm y$.

$$\mathrm{F}(x \pm y) = u \pm \frac{du}{dx}y + \frac{d^2u}{dx^2} \cdot \frac{y^2}{2} \pm \frac{d^3u}{dx^3} \cdot \frac{y^3}{2 \times 3} + \dots \frac{d^nu}{dx^n} \cdot \frac{y^n}{2 \times 3 \dots \times n}.$$

where u represents the value of the function when $y = o$.

Diameter.	Circ.	Area.	Diameter.	Circ.	Area	Diameter.	Circ.	Area.
1/32	·0981	·00076	5	15·70	19·635	.11	34.55	95·033
1/16	·1963	.00306		16·10	20·629		34·95	97·205
1/8	·3926	·01227	1/4	16·49	21·647	1/4	35·34	99·402
3/16	·5890	·02761		16·88	22·690		35·73	101·62
1/4	·7854	·04908	1/2	17·27	23·758	1/2	36·12	103·86
5/16	·9817	·07669		17·67	24·850		36·52	106·13
3/8	1·178	·1104	3/4	18·06	25·967	3/4	36·91	108·43
7/16	1·374	·1503		18·45	27·108		37·30	110·75
1/2	1·570	·1963	6	18·84	28·274	12	37·69	113·09
9/16	1·767	·2485		19·24	29·464		38·09	115·46
5/8	1·963	·3067	1/4	19·63	30·679	1/4	38·48	117·85
11/16	2·159	·3712		20·02	31·919		38·87	120·27
3/4	2·356	·4417	1/2	20·42	33·183	1/2	39·27	122·71
13/16	2·552	·5184		20·81	34·471		39·66	125·18
7/8	2.748	·6013	3/4	21·20	35·784	3/4	40·05	127·67
15/16	2·945	·6902		21·57	37·122		40·44	130·19
1	3·141	·7854	7	21·99	38·484	13	40·84	132·73
	3·534	·9940		22·38	39·871		41·23	135·29
1/4	3·927	1·227	1/4	22·77	41·282	1/4	41·62	137·88
	4·319	1·484		23·16	42·718		42·01	140·50
1/2	4·712	1·767	1/2	23·56	44·178	1/2	42·41	143·13
	5·105	2·073		23·95	45·663		42·80	145·80
3/4	5·497	2·405	3/4	24·34	47·173	3/4	43·19	148·48
	5·890	2·761		24·74	48·707		43·58	151·20
2	6·283	3·141	8	25·13	50·265	14	43·98	153·93
	6·675	3·546		25·52	51·848		44·37	156·69
1/4	7·068	3·976	1/4	25·91	53·456	1/4	44·76	159·48
	7·461	4·430		26·31	55·088		45·16	162·29
1/2	7·854	4·908	1/2	26·70	56·745	1/2	45·55	165·13
	8·246	5·411		27·09	58·426		45·94	167·98
3/4	8·639	5·939	3/4	27·48	60·132	3/4	46·33	170·87
	9·032	6·491		27·88	61·862		46·73	173·78
3	9·424	7·068	9	28·27	63·617	15	47·12	176·71
	9·817	7·669		28·66	65·396		47·51	179·67
1/4	10·21	8·295	1/4	29·05	67·200	1/4	47·90	182·65
	10·60	8·946		29·45	69·029		48·30	185·66
1/2	10·99	9·621	1/2	29·84	70·882	1/2	48·69	188·69
	11·38	10·320		30·23	72·759		49·08	191·74
3/4	11·78	11·044	3/4	30·63	74·662	3/4	49·48	194·82
	12·17	11·793		31·02	76·588		49·87	197·93
4	12·56	12·566	10	31·41	78·539	16	50·26	201·06
	12·95	13·364		31·80	80·515		50·65	204·21
1/4	13·35	14·186	1/4	32·20	82·516	1/4	51·05	207·39
	13·74	15·033		32·59	84·540		51·44	210·59
1/2	14·13	15·904	1/2	32·98	86·590	1/2	51·83	213·82
	14·52	16·800	·	33·37	88·664		52·22	217·07
3/4	14·92	17·720	3/4	33·77	90·762	3/4	52·62	220·35
	15.31	18·665		34·16	92·885		53·01	223·65

Diameter.	Circ.	Area.	Diameter.	Circ.	Area	Diameter.	Circ.	Area.
17	53·40	226·98	23	72·25	415·47	29	91·10	660·52
	53·79	230·33		72·64	420·00		91·49	666·22
¼	54·19	233·70	¼	73·04	424·55	¼	91·89	671·95
	54·58	237·10		73·43	429·13		92·28	677·71
½	54·97	240·52	½	73·82	433·73	½	92·67	683·49
	55·37	243·97		74·21	438·30		93·06	689·29
¾	55·76	247·45	¾	74·61	443·01	¾	93·46	695·12
	56·16	250·94		75·	447·69		93·85	700·98
18	56·54	254·46	24	75·39	452·39	30	94·24	706·86
	56·94	258·01		75·79	457·11		94·64	712·76
¼	57·33	261·58	¼	76·18	461·86	¼	95·03	718·69
	57·72	265·18		76·57	466·63		95·42	724·64
½	58·11	268·80	½	76·96	471·43	½	95·81	730·61
	58·51	272·44		77·36	476·25		96·21	736·61
¾	58·90	276·11	¾	77·75	481·10	¾	96·60	742·64
	59·29	279·81		78·14	485·97		96·99	748·69
19	59·69	283·52	25	78·54	490·87	31	97·38	754·76
	60·08	287·27		78·93	495·79		97·78	760·86
¼	60·47	291·03	¼	79·32	500·74	¼	98·17	766·99
	60·86	294·83		79·71	505·71		98·56	773·14
½	61·26	298·64	½	80·10	510·70	½	98·96	779·31
	61·65	302·48		80·50	515·72		99·35	785·51
¾	62·04	306·35	¾	80·89	520·70	¾	99·74	791·73
	62·43	310·24		81·28	525·83		100·1	797·97
20	62·83	314·16	26	81·68	530·93	32	100·5	804·24
	63·22	318 09		82·07	536·04		100·9	810·54
¼	63·61	322·06	¼	82·46	541·18	¼	101·3	816·86
	64·01	326·05		82·85	546·35		101·7	823·21
½	64.40	330·06	½	83·25	551·54	½	102·1	829·57
	64·79	334·10		83·64	556·76		102·4	835·97
¾	65·18	338·16	¾	84·03	562·00	¾	102·8	842·39
	65·58	342·25		84·43	567·26		103·2	848·83
21	65·97	346·36	27	84·82	572·55	33	103·6	855·30
	66·36	350·49		85·21	577·87		104·	861·79
¼	66·75	354·65	¼	85·60	583·20	¼	104·4	868·30
	67·15	358·84		86·	588·57		104·8	874·84
½	67·54	363·05	½	86·39	593·95	½	105·2	881·41
	67·93	367·28		86·78	599·37		105·6	888·00
¾	68·32	371·54	¾	87·17	604·80	¾	106·	894·61
	68·72	375·82		87·57	610·26		106·4	901·25
22	69·11	380·13	28	87·96	615·75	34	106·8	907·92
	69·50	384·46		88·35	621·26		107·2	914·61
¼	69·90	388·82	¼	88·75	626·79	¼	107·5	921·32
	70·29	393·20		89·14	632·35		107·9	928·06
½	70·68	397·60	½	89·53	637·94	½	108·3	934·82
	71·07	402·03		89·92	643·54		108·7	941·60
¾	71·47	406·49	¾	90·32	649·18	¾	109·1	948·41
	71·86	410·97		90·71	654·83		109·5	955·25

Diameter.	Circ.	Area.	Diameter.	Circ.	Area.	Diameter.	Circ.	Area.
35	109·9	962·11	41	128.8	1320·2	47	147·6	1734·9
	110·3	968·99		129·1	1328·3		148·	1744·1
¼	110·7	975·90	¼	129·5	1336·4	¼	148·4	1753·4
	111·1	982·84		129·9	1344·5		148·8	1762·7
½	111·5	989·80	½	130·3	1352·6	½	149·2	1772·0
	111·9	996·78		130·7	1360·8		149·6	1781·3
¾	112·3	1003·7	¾	131·1	1369·0	¾	150·	1790·7
	112·7	1010·8		131·5	1377·2		150·4	1800·1
36	113·	1017·8	42	131·9	1385·4	48	150·7	1809·5
	113·4	1024·9		132·3	1393·7		150·1	1818·9
¼	113·8	1032·0	¼	132·7	1401·9	¼	151·5	1828·4
	114·2	1039·1		133·1	1410·2		151·9	1837·9
½	114·6	1046·3	½	133·5	1418·6	½	152·3	1847·4
	115·	1053·5		133·9	1426·9		152·7	1856·9
¾	115·4	1060·7	¾	134·3	1435·3	¾	153·1	1866·5
	115·8	1067·9		134·6	1443·7		153·5	1876·1
37	116·2	1075·2	43	135·	1452·2	49	153·9	1885·7
	116·6	1082·4		135·4	1460·6		154·3	1895·3
¼	117·	1089·7	¼	135·8	1469·1	¼	154·7	1905·0
	117·4	1097·1		136·2	1477·6		155·1	1914·7
½	117·8	1104·4	½	136·6	1486·1	½	155·5	1924·4
	118·2	1111·8		137·	1494·7		155·9	1934·1
¾	118·6	1119·2	¾	137·4	1503·3	¾	156·2	1943·9
	118·9	1126·6		137·8	1511·9		156·6	1953·6
38	119·3	1134·1	44	138·2	1520·5	50	157·	1963·5
	119·7	1141·5		138·6	1529·1		157·4	1973·3
¼	120·1	1149·0	¼	139·	1537·8	¼	157·8	1983·1
	120·5	1156·6		139·4	1546·5		158·2	1993·0
½	120·9	1164·1	½	139·8	1555·2	½	158·6	2002·9
	121·3	1171·7		140·1	1564·0		159·	2012·8
¾	121·7	1179·3	¾	140·5	1572·8	¾	159·4	2022·8
	122·1	1186·9		140·9	1581·6		159·8	2032·8
39	122·5	1194·5	45	141·3	1590·4	51	160·2	2042·8
	122·9	1202·2		141·7	1599·2		160·6	2052·8
¼	123·3	1209·9	¼	142·1	1608·1	¼	161·	2062·9
	123·7	1217·6		142·5	1617·0		161·3	2072·9
½	124·	1225·4	½	142·9	1625·9	½	161·7	2083·0
	124·4	1233·1		143·3	1634·9		162·1	2093·2
¾	124·8	1240·9	¾	143·7	1643·8	¾	162·5	2103·3
	125·2	1248·7		144·1	1652·8		162·9	2113·5
40	125·6	1256·6	46	144·5	1661·9	52	163·3	2123·7
	126·	1264·5		144·9	1670·9		163·7	2133·9
¼	126·4	1272·3	¼	145·2	1680·0	¼	164·1	2144·1
	126·8	1280·3		145·6	1689·1		164·5	2154·4
½	127·2	1288·2	½	146·	1698·2	½	164·9	2164·7
	127·6	1296·2		146·4	1707·3		165·3	2175·0
¾	128·	1304·2	¾	146·8	1716·5	¾	165·7	2185·4
	128·4	1312·2		147·2	1725·7		166·1	2195·7

Diameter.	Circ.	Area.	Diameter.	Circ.	Area.	Diameter.	Circ.	Area.
53	166·5	2206·1	59	185·3	2733·9	65	204·2	3318·3
	166·8	2216·6		185·7	2745·5		204·5	3331·0
¼	167·2	2227·0	¼	186·1	2757·1	¼	204·9	3343·8
	167·6	2237·5		186·5	2768·8		205·3	3356·7
½	168·	2248·0	½	186·9	2780·5	½	205·7	3369·5
	168·4	2258·5		187·3	2792·2		206·1	3382·4
¾	168·8	2269·0	¾	187·7	2803·9	¾	206·5	3395·3
	169·2	2279·6		188·1	2815·6		206·9	3408·2
54	169·6	2290·2	60	188·4	2827·4	66	207·3	3421·2
	170·	2300·8		188·8	2839·2		207·7	3434·1
¼	170·4	2311·4	¼	189·2	2851·0	¼	208·1	3447·1
	170·8	2322·1		189·6	2862·8		208·5	3460·1
½	171·2	2332·8	½	190·	2874·7	½	208·9	3473·2
	171·6	2343·5		190·4	2886·6		209·3	3486·3
¾	172·	2354·2	¾	190·8	2898·5	¾	209·7	3499·3
	172·3	2365·0		191·2	2910·5		210·	3512·5
55	172·7	2375·8	61	191·6	2922·4	67	210·4	3525·6
	173·1	2386·6		192·	2934·4		210·8	3538·8
¼	173·5	2397·4	¼	192·4	2946·4	¼	211·2	3552·0
	173·9	2408·3		192·8	2958·5		211·6	3565·2
½	174·3	2419·2	½	193·2	2970·5	½	212·	3578·4
	174·7	2430·1		193·6	2982·6		212·4	3591·7
¾	175·1	2441·0	¾	193·9	2994·7	¾	212·8	3605·0
	175·5	2452·0		194·3	3006·9		213·2	3618·3
56	175·9	2463·0	62	194·7	3019·0	68	213·6	3631·6
	176·3	2474·0		195·1	3031·2		214·	3645·0
¼	176·7	2485·0	¼	195·5	3043·4	¼	214·4	3658·4
	177·1	2496·1		195·9	3055·7		214·8	3671·8
½	177·5	2507·1	½	196·3	3067·9	½	215·1	3685·2
	177·8	2518·2		196·7	3080·2		215·5	3698·7
¾	178·2	2529·4	¾	197·1	3092·5	¾	215·9	3712·2
	178·6	2540·5		197·5	3104·8		216·3	3725·7
57	179·	2551·7	63	197·9	3117·2	69	216·7	3739·2
	179·4	2562·9		198·3	3129·6		217·1	3752·8
¼	179·8	2574·1	¼	198·7	3142·0	¼	217·5	3766·4
	180·2	2585·4		199·	3154·4		217·9	3780·0
½	180·6	2596·7	½	199·4	3166·9	½	218·3	3793·6
	181·	2608·0		199·8	3179·4		218·7	3807·3
¾	181·4	2619·3	¾	200·2	3191·9	¾	219·1	3821·0
	181·8	2630·7		200·6	3204·4		219·5	3834·7
58	182·2	2642·0	64	201·	3216·9	70	219·9	3848·4
	182·6	2653·4		201·4	3229·5		220·3	3862·2
¼	182·9	2664·9	¼	201·8	3242·1	¼	220·6	3875·9
	183·3	2676·3		202·2	3254·8		221·	3889·8
½	183·7	2687·8	½	202·6	3267·4	½	221·4	3903·6
	184·1	2699·3		203·	3280·1		221·8	3917·4
¾	184·5	2710·8	¾	203·4	3292·8	¾	222·2	3931·3
	184·9	2722·4		203·8	3305·5		222·6	3945·2

Diameter.	Circ.	Area.	Diameter.	Circ.	Area.	Diameter.	Circ.	Area.
71	223·	3959·2	77	241·9	4656·6	83	260·7	5410·6
	223·4	3973·1		242·2	4671·7		261·1	5426·9
¼	223·8	3987·1	¼	242·6	4686·9	¼	261·5	5443·2
	224·2	4001·1		243·	4702·1		261·9	5459·6
½	224·6	4015·1	½	243·4	4717·3	½	262·3	5476·0
	225·	4029·2		243·8	4732·5		262·7	5492·4
¾	225·4	4043·2	¾	244·2	4747·7	¾	263·1	5508·8
	225·8	4067·3		244·6	4763·0		263·5	5525·3
72	226·1	4071·5	78	245·	4778·3	84	263·8	5541·7
	226·5	4085·6		245·4	4793·7		264·2	5558·2
¼	226·9	4099·8	¼	245·8	4809·0	¼	264·6	5574·8
	227·3	4114·0		246·2	4824·4		265·	5591·3
½	227·7	4128·2	½	246·6	4839·8	½	265·4	5607·9
	228·1	4142·5		247·	4855·2		265·8	5624·5
¾	228·5	4156·7	¾	247·4	4870·7	¾	266·2	5641·1
	228·9	4171·0		247·7	4886·1		266·6	5657·8
73	229·3	4185·3	79	248·1	4901·6	85	267·	5674·5
	229·7	4199·7		248·5	4917·2		267·4	5691·2
¼	230·1	4214·1	¼	248·9	4932·7	¼	267·8	5707·9
	230·5	4228·5		249·3	4948·3		268·2	5724·6
½	230·9	4242·9	½	249·7	4963·9	½	269·6	5741·4
	231·3	4257·3		250·1	4979·5		268·9	5758·2
¾	231·6	4271·8	¾	250·5	4995·1	¾	269·3	5775·0
	232·	4286·3		250·9	5010·8		269·7	5791·9
74	232·4	4300·8	80	251·3	5026·5	86	270·1	5808·8
	232·8	4315·3		251·7	5042·2		270·5	5825·7
¼	233·2	4329·9	¼	252·1	5058·0	¼	270·9	5842·6
	233·6	4344·5		252·5	5073·7		271·3	5859·5
½	234·	4359·1	½	252·8	5089·5	½	271·7	5876·5
	234·4	4373·8		253·2	5105·4		272·1	5893·5
¾	234·8	4388·4	¾	253·6	5121·2	¾	272·5	5910·5
	235·2	4403·1		254·	5137·1		272·9	5927·6
75	235·6	4417·8	81	254·4	5153·0	87	273·3	5944·6
	236·	4432·6		254·8	5168·9		273·7	5961·7
¼	236·4	4447·3	¼	255·2	5184·8	¼	274·1	5978·9
	236·7	4462·1		255·6	5200·8		274·4	5996·0
½	237·1	4476·9	½	256·	5216·8	½	274·8	6013·2
	237·5	4491·8		256·4	5232·8		275·2	6030·4
¾	237·9	4506·6	¾	256·8	5248·8	¾	275·6	6047·6
	238·3	4521·5		257·2	5264·9		276·	6064·8
76	238·7	4536·4	82	257·6	5281·0	88	276·4	6082·1
	239·1	4551·4		258·	5297·1		276·8	6099·4
¼	239·5	4566·3	¼	258·3	5313·2	¼	277·2	6116·7
	239·9	4581·3		258·7	5329·4		277·6	6134·0
½	240·3	4596·3	½	259·1	5345·6	½	278·	6151·4
	240·7	4611·3		259·5	5361·8		278·4	6168·8
¾	241·1	4626·4	¾	259·9	5378·0	¾	278·8	6186·2
	241·5	4641·5		260·3	5394·3		279·2	6203·6

Diameter.	Circ.	Area.	Diameter.	Circ.	Area.	Diameter.	Circ.	Area.
89	279·6	6221·1	93	292·1	6792·9	97	304·7	7389·8
	279·9	6238·6		292·5	6811·1		305·1	7408·8
¼	280·3	6256·1	¼	292·9	6829·4	¼	305·5	7427·9
	280·7	6273·6		293·3	6847·8		305·9	7447·0
½	281·1	6291·2	½	293·7	6866·1	½	306·3	7466·2
	281·5	6308·8		294·1	6884·5		306·6	7485·3
¾	281·9	6326·4	¾	294·5	6902·9	¾	307·	7504·5
	282·3	6344·0		294·9	6921·3		307·4	7523·7
90	282·7	6361·7	94	295·3	6939·7	98	307·8	7542·9
	283·1	6379·4		295·7	6958·2		308·2	7562·2
¼	283·5	6397·1	¼	296·	6976·7	¼	308·6	7581·5
	283·9	6414·8		296·4	6995·2		309·0	7600·8
½	284·3	6432·6	½	296·8	7013·8	½	309·4	7620·1
	284·7	6450·4		297·2	7032·3		309·8	7639·4
¾	285·1	6468·2	¾	297·6	7050·9	¾	310·2	7658·8
	285·4	6486·0		298·	7069·5		310·6	7678·2
91	285·8	6503·8	95	298·4	7088·2	99	311·0	7697·7
	286·2	6521·7		298·8	7106·9		311·4	7717·1
¼	286·6	6539·6	¼	299·2	7125·5	¼	311·8	7736·6
	287·	6557·6		299·6	7144·3		312·1	7756·1
½	287·4	6575·5	½	300·	7163·0	½	312·5	7775·6
	287·8	6593·5		300·4	7181·8		312·9	7795·2
¾	288·2	6611·5	¾	300·8	7200·5	¾	313·3	7814·7
	288·6	6629·5		301·2	7219·4		313·7	7834·3
92	289·	6647·6	96	301·5	7238·2	100	314·1	7853·9
	289·4	6665·7		301·9	7257·1		314·5	7853·6
¼	289·8	6683·8	¼	302·3	7275·9	¼	314·9	7893·3
	290 2	6701·9		302·7	7294·9		315·3	7913·1
½	290·5	6720·0	½	303·1	7313·8	½	315·7	7932·7
	290·9	6738·2		303·5	7332·8		316·0	7942·4
¾	291·3	6756·4	¾	303·9	7351·7	¾	316·4	7972·2
	291·7	6776·4		304·3	7370·7		316·8	7991·9

EXPLANATION OF THE TABLE FOR SEGMENTS, &c.

The *chord divided by the height* is the gauge in the Table, the quotient in the first column.

k = *tabular coefficient*, always to be multiplied by the chord.

To find the angle of an arc of a circle.

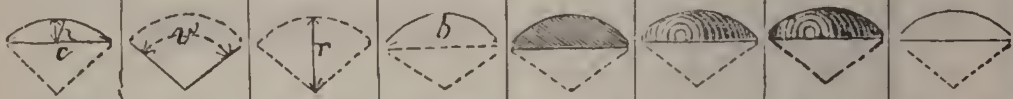
RULE. Divide the base (chord) of the arc by its height, (*sine verse*) and find the quotient in the first column. The corresponding number in the second column is the angle of the arc in degrees of the circle.

To find the radius of an arc of a circle.

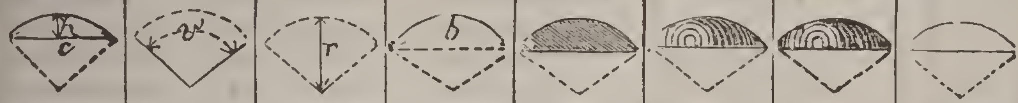
RULE. Divide the chord of the arc by its height, and find the quotient in the first column. The corresponding number in the third column, multiplied by the chord, is the radius of the arc.

Chord div. by height.	Centre Angle v.	Radius r = k c.	Cir. Arc. b = k c.	Area Seg. a = k c².	Surface a = k c².	Solidity C = k c³.	Chord c = k r.
							
458·08	1	57·296	1·0000	·01091	·78539	·00085	·01744
229·18	2	28·649	1·0000	·00218	·78549	·00172	·03490
152·77	3	19·101	1·0000	·00327	·78462	·00255	·05234
114·57	4	14·327	1·0000	·00436	·78574	·00310	·06978
84·747	5	11·462	1·0001	.00647	·78586	·00401	·08722
76·375	6	9·5530	1·0003	·00741	·78599	·00514	·10466
65·943	7	8·1902	1·0004	·00910	·78621	·00592	·12208
57·273	8	7·1678	1·0006	·01089	·78630	·00686	·13950
50·902	9	6·3728	1·0008	·01254	·78665	·00772	·15690
45·807	10	5·7368	1·0011	·01407	·78695	·00857	·17430
41·203	11	5·2167	1·0013	·01552	·78730	·00964	·19168
38·133	12	4·7834	1·0016	·01695	·78725	·01031	·20904
35·221	13	4·4168	1·0019	·01841	·78794	·01114	·22640
32·742	14	4·1027	1·0023	·02000	·78832	·01199	·24372
30·514	15	3·8307	1·0027	·02157	·78889	·01288	·26104
28·601	16	3·5927	1·0029	·02269	·78909	·01375	·27834
26·915	17	3·3827	1·0034	·02434	·78969	·01462	·29560
25·412	18	3·1962	1·0039	·02592	·79028	·01542	·31286
24·068	19	3·0293	1·0044	·02744	·79084	·01635	·33008
22·860	20	2·8793	1·0048	·02878	·79140	·01722	·34728
21·760	21	2·7440	1·0054	·03040	·79234	·01802	·36446
20·777	22	2·6222	1·0059	·03178	·79300	·01897	·38160
19·862	23	2·5080	1·0066	·03343	·79340	·01984	·39872
19·028	24	2·4050	1·0072	·03493	·79416	·02072	·41582
18·261	25	2·3101	1·0078	·03639	·79486	·02159	·43286
17·553	26	2·2233	1·0084	·03784	·79530	·02248	·44990
16·970	27	2·1418	1·0091	·03970	·79639	·02315	·46688
16·288	28	2·0673	1·0101	·04115	·79748	·02424	·48384
15·721	29	1·9969	1·0105	·04230	·79811	·02511	·50076
15·191	30	1·9319	1·0113	·04385	·79907	·02600	·51762
14·970	31	1·8710	1·0121	·04476	·78530	·02692	·53446
14·230	32	1·8140	1·0129	·04710	·80098	·02778	·55126
13·796	33	1·7605	1·0138	·04842	·80181	·02866	·56802
13·382	34	1·7102	1·0146	·04989	·80300	·02956	.58479
12·994	35	1·6628	1·0155	·05137	·80405	·03046	·60140
12·733	36	1·6184	1·0167	·05311	·80531	·03137	·61802
12·473	37	1·5758	1·0174	·05401	·80622	·03226	·63460
11·931	38	1·5358	1·0184	·05628	·80713	·03328	·65112
11·621	39	1·4979	1·0194	·05755	·80850	·03418	·66760
11·342	40	1·4619	1·0204	·05899	·80987	·03506	·68404
11·060	41	1·4266	1·0207	·06001	·81016	·03589	·70040
10·791	42	1·3952	1·0226	·06196	·81240	·03680	·71672
10·534	43	1·3643	1·0237	·06359	·81377	·03773	·73300
10·289	44	1·3347	1·0248	·06574	·81505	·03864	·74920
10·043	45	1·3066	1·0260	·06628	·81756	·03890	·76536
9·8303	46	1·2797	1·0272	·06826	.81795	·04050	·78146
9·6153	47	1·2539	1·0290	·06998	·81939	·04143	·79748
9·4092	48	1·2289	1·0297	·09138	·82064	·04247	·81346

Chord div by height.	Centre Angle v.	Radius $r = k\,c$.	Cir. Arc. $b = k\,c$.	Area Seg. $a = k\,c^2$.	Surface $a = k\,c^2$.	Solidity $\mathfrak{C} = k\,c^3$.	Chord $c = k\,r$.
9·2113	49	1·2057	1·0309	·07290	·82244	·04330	·82938
9·0214	50	1·1831	1·0323	·07453	·82384	·04424	·84522
8·8387	51	1·1614	1·0336	·07611	·82562	·04519	·86102
8·6629	52	1·1406	1·0349	·07758	·82729	·04614	·87674
8·4462	53	1·1206	1·0364	·07959	·83363	·04685	·89238
8·3306	54	1·1014	1·0378	·08083	·83072	·04805	·90798
8 1733	55	1·0828	1·0393	·08246	·83249	·04901	·92348
8·0215	56	1·0650	1·0407	·08400	·83422	·05002	·93894
7·8750	57	1·0478	1·0422	·08579	·83602	·05098	·95430
7·7334	58	1·0313	1·0431	·08680	·83796	·05191	·96960
7·5895	59	1·0154	1·0454	·08891	·84064	·05299	·98484
7·4565	60	1·0000	1·0470	·09106	·84266	·05400	1·0000
7·3358	61	·98515	1·0486	·09209	·84380	·05476	1·0150
7·2118	62	·97080	1·0503	·09375	·84581	·05583	1·0300
7·0914	63	·95694	1·0520	·09540	·84791	·05684	1·0450
6·9748	64	·94352	1·0537	·09697	·84996	·05784	1·0598
6·8616	65	·93058	1·0555	·09865	·85215	·05885	1·0746
6·7512	66	·91804	1·0573	·10036	·85441	·05987	1·0892
6·6453	67	·90590	1·0591	·10201	·85640	·06088	1·1038
6·5469	68	·89415	1·0610	·10367	·85815	·06181	1·1184
6·4902	69	·88276	1·0629	·10520	·85464	·06201	1·1328
6·3431	70	·87172	1·0648	·10710	·86350	·06396	1·1471
6·2400	71	·86102	1·0668	·10887	·86699	·06515	1·1614
6·1553	72	·85065	1·0687	·11046.	·86834	·06604	1·1755
6·0652	73	·84058	1·0708	·11225	·87081	·06709	1·1896
5·9773	74	·83082	1·0728	·11385	·87935	·06815	1·2036
5·8918	75	·82134	1·0749	·11563	·87590	·06921	1·2175
5·8084	76	·81213	1·0770	·11736	·87853	·07037	1·2313
5·7271	77	·80319	1·0792	·11910	·88120	·07136	1·2450
5·6478	78	·79449	1·0814	·12072	·88389	·07244	1·2586
5·5704	79	·78606	1·0836	·12281	·88677	·07352	1·2721
5·4949	80	·77786	1·0859	·12441	·88949	·07462	1·2855
5·4254	81	·76988	1·0882	·12660	·89161	·07512	1·2989
5·3492	82	·76212	1·0905	·12793	·89520	·07683	1·3121
5·2705	83	·75458	1·0920	·12958	·89958	·07819	1·3252
5·2101	84	·74724	1·0953	·13157	·90095	·07907	1·3383
5·1429	85	·74009	1·0977	·13330	·90420	·07960	1·3512
5·0772	86	·73314	1·1012	·13546	·90734	·08102	1·3639
5·0134	87	·72637	1·1027	·13704	·91036	·08340	1·3767
4·9501	88	·71978	1·1054	·13893	·91363	·08436	1·3893
4·8886	89	·71336	1·1079	·14078	·91696	·08530	1·4818
4·8216	90	·70710	1·1105	·14279	·92210	·08621	1·4142
4·7694	91	·70101	1·1132	·14449	·92352	·08716	1·4265
4·7117	92	·69508	1·1159	·14643	·92476	·08798	1·4387
4·6615	93	·68930	1·1186	·14817	·92914	·08932	1·4507
4·5999	94	·68366	1·1211	·15009	·93385	·09076	1·4627
4·5453	95	·67817	1·1242	·15211	·93746	·09197	1·4745
4·4845	96	·67282	1·1271	·15375	·94272	·09348	1·4863



Chord div by height.	Centre Angle *v*.	Radius *r* = *k c*.	Cir. Arc. *b* = *k c*.	Area Seg. *a* = *k c*².	Surface *a* = *k c*².	Solidity C = *k c*³.	Chord *c* = *k r*.
4·4398	97	·66760	1·1300	·15600	·94470	·09442	1·4979
4·3859	98	·66250	1·1329	·15801	·94852	·09567	1·5094
4·3383	99	·65754	1·1359	·15995	·95236	·09693	1·5208
4·2862	100	·65270	1·1382	·16180	·95682	·09831	1·5321
4·2406	101	·64798	1·1420	.16393	·96011	·09856	1·5432
4·1930	102	·64338	1·1451	·16610	·96412	·10076	1·5543
4·1570	103	·63889	1·1483	·16925	·96568	·10557	1·5652
4·1006	104	·63450	1·1515	·17001	·97246	·10273	1·5760
4·0555	105	·63023	1·1547	·17204	·97643	·10471	1·5867
4·0113	106	·62607	1·1580	·17414	·98067	·10601	1·5973
3·9679	107	·62200	1·1614	·17619	·98495	·10735	1·6077
3·9252	108	·61803	1·1648	·17832	·98931	·10870	1·6180
3·8832	109	·61416	1·1682	·18041	·99376	·11007	1·6282
3·8419	110	·61039	1·1716	·18257	·98827	·11149	1·6383
3·8013	111	·60670	1·1752	·18472	1·0028	·11284	1·6482
3·7612	112	·60325	1·1790	·18696	1·0077	·11426	1·6581
3·7221	113	·59960	1·1823	·18900	1·0122	·11566	1·6677
3·6837	114	·59618	1·1859	·19117	1·0169	·11709	1·6773
3·6454	115	·59284	1·1897	·19339	1·0218	·11853	1·6867
3·6086	116	·58959	1·1934	·19559	1·0266	·11995	1·6961
3·5712	117	·58641	1·1972	·19787	1·0317	·12145	1·7053
3·5349	118	·58331	1·2011	·20009	1·0368	·12294	1·7143
3·4992	119	·58030	1·2050	·20227	1·0417	·12444	1·7232
3·4641	120	·57735	1·2089	·20453	1·0472	·12596	1·7320
3·4296	121	·57450	1·2130	·20678	1·0525	·12748	1·7407
3·3953	122	·57168	1·2177	·20945	1·0578	·12903	1·7492
3·3616	123	·56895	1·2213	·21175	1·0634	·13060	1·7576
3·3285	124	·56628	1·2253	·21399	1·0690	·13218	1·7659
3·2940	125	·56370	1·2295	·21538	1·0753	·13391	1·7740
3·2637	126	·56116	1·2338	·21859	1·0803	·13558	1·7820
3·2319	127	·55870	1·2381	·22121	1·0862	·13701	1·7898
3·2006	128	·55630	1·2425	·22370	1·0921	·13866	1·7976
3·1716	129	·55396	1·2470	·22617	1·0974	·14028	1·8051
3·1393	130	·55169	1·2515	·22865	1·1040	·14202	1·8126
3·1093	131	·54947	1·2561	·23113	1·1104	·14371	1·8199
3·0805	132	·54732	1·2607	·23372	1·1164	·14537	1·8271
3·0555	133	·54522	1·2654	·23603	1·1212	·14676	1·8341
3·0216	134	·54318	1·2701	·23892	1·1295	·14894	1·8410
2·9777	135	·54120	1·2749	·24198	1·1420	·15209	1·8477
2·9651	136	·53927	1·2798	·24364	1·1428	·15252	1·8543
2·9374	137	·53740	1·2847	·24676	1·1495	·15422	1·8608
2·9115	138	·53557	1·2897	·24938	1·1558	·15605	1·8671
2·8829	139	·53380	1·2948	·25222	1·1634	·15807	1·8733
2·8562	140	·53209	1·2999	·25485	1·1705	·15996	1·8794
2·8299	141	·53042	1·3051	·25759	1·1777	·16201	1·8853
2·8038	142	·52881	1·3065	·25936	1·1851	·16381	1·8910
2·7781	143	·52724	1·3157	·26320	1·1925	·16577	1·8966
2·7527	144	·52573	1·3211	·26604	1·2000	·16776	1·9021



Chord div. by height.	Centre Angle v.	Radius $r = k\,c$.	Cir. Arc. $b = k\,c$.	Area Seg. $a = k\,c^2$.	Surface $a = k\,c^2$.	Solidity $C = k\,c^3$.	Chord $c = k\,r$.
2·7276	145	·52426	1·3265	·26889	1·2077	·16965	1·9074
2·7002	146	·52284	1·3320	·27196	1·2166	·17209	1·9126
2·6816	147	·52147	1·3377	·27449	1·2219	·17205	1·9176
2·6533	148	·52015	1·3433	·27772	1·2318	·17605	1·9225
2·6301	149	·51887	1·3491	·28168	1·2396	·17809	1·9272
2·6064	150	·51764	1·3549	·28369	1·2476	·18023	1·9318
2·5830	151	·51645	1·3608	·28674	1·2563	·18666	1·9363
2·5598	152	·51530	1·3668	·28983	1·2648	·18751	1·9406
2·5239	153	·51420	1·3729	·29397	1·2801	·18845	1·9447
2·5143	154	·51315	1·3790	·29607	1·2824	·18913	1·9487
2·4919	155	·51214	1·3852	·29928	1·2914	·19147	1·9526
2·4699	156	·51117	1·3919	·30259	1·3004	·19374	1·9563
2·4478	157	·51014	1·3973	·30560	1·3094	·19607	1·9598
2·4262	158	·50936	1·4043	·30905	1·3191	·20029	1·9632
2·4047	159	·50851	1·4109	·31239	1·3287	·20095	1·9663
2·3835	160	·50771	1·4175	·31575	1·3368	·20342	1·9696
2·3613	161	·50695	1·4243	·31931	1·3490	·20609	1·9725
2·3417	162	·50623	1·4311	·32263	1·3583	·20847	1·9753
2·3211	163	·50555	1·4380	·32618	1·3682	·21105	1·9780
2·3004	164	·50491	1·4450	·32969	1·3791	·21371	1·9805
2·2805	165	·50431	1·4520	·33327	1·3895	·21634	1·9829
2·2605	166	·50374	1·4592	·33684	1·4021	·21904	1·9851
2·2408	167	·50323	1·4665	·34048	1·4111	·22177	1·9871
2·2212	168	·50275	1·4739	·34422	1·4222	·21946	1·9890
2·2013	169	·50231	1·4813	·34802	1·4344	·22766	1·9908
2·1826	170	·50191	1·4889	·35230	1·4476	·23028	1·9924
2·1636	171	·50154	1·4966	·35563	1·4565	·23266	1·9938
2·1447	172	·50122	1·5044	·35953	1·4684	·23650	1·9951
2·1271	173	·50093	1·5123	·36337	1·4797	·23900	1·9962
2·1075	174	·50068	1·5202	·36747	1·4927	·24225	1·9972
2·0892	175	·50047	1·5283	·37152	1·5052	·24537	1·9981
2·0710	176	·50030	1·5365	·37562	1·5179	·24856	1·9988
2·0530	177	·50017	1·5448	·37974	1·5308	·25179	1·9993
2·0352	178	·50007	1·5533	·38401	1·5439	·25531	1·9996
2·0175	179	·50002	1·5618	·38828	1·5573	·25840	1·9999
2·0000	180	·50000	1·5707	·39269	1·5708	·26179	2·0000

To find the length of an arc of a circle.

RULE. Divide the chord of the arc by its height, and find the quotient in the first column. The corresponding number in the fourth column multiplied by the chord is the length of the arc.

To find the area of a segment of a circle,

RULE. Divide the chord of the segment by its height, and find the quotient in the first column. The corresponding number in the fifth column multiplied by the square of the chord, is the area of the segment.

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
1	1	1	1·0000000	1·0000000	1.00000000
2	4	8	1·4142136	1·2599210	·500000000
3	9	27	1·7320508	1·4422496	·333333333
4	16	64	2·0000000	1·5874011	·250000000
5	25	125	2·2360680	1·7099759	·200000000
6	36	216	2·4494897	1·8171206	·166666667
7	49	343	2·6457513	1·9129312	·142857143
8	64	512	2·8284271	2·0000000	·125000000
9	81	729	3·0000000	2·0800837	·111111111
10	100	1000	3·1622777	2·1544347	·100000000
11	121	1331	3·3166248	2·2239801	·090909091
12	144	1728	3·4641016	2·2894286	·083333333
13	169	2197	3·6055513	2·3513347	·076923077
14	196	2744	3·7416574	2·4101422	·071428571
15	225	3375	3·8729833	2·4662121	·066666667
16	256	4096	4·0000000	2·5198421	·062500000
17	289	4913	4·1231056	2·5712816	·058823529
18	324	5832	4·2426407	2·6207414	·055555556
19	361	6859	4·3588989	2·6684016	·052631579
20	400	8000	4·4721360	2·7144177	·050000000
21	441	9261	4·5825757	2·7589243	·047619048
22	484	10648	4·6904158	2·8020393	·045454545
23	529	12167	4·7958315	2·8438670	·043478261
24	576	13824	4·8989795	2·8844991	·041666667
25	625	15625	5·0000000	2·9240177	·040000000
26	676	17576	5·0990195	2·9624960	·038461538
27	729	19683	5·1961524	3·0000000	·037037037
28	784	21952	5·2915026	3·0365889	·035714286
29	841	24389	5·3851648	3·0723168	·034482759
30	900	27000	5·4772256	3·1072325	·033333333
31	961	29791	5·5677644	3·1413806	·032258065
32	1024	32768	5·6568542	3·1748021	·031250000
33	1089	35937	5·7445626	3·2075343	·030303030
34	1156	39304	5·8309519	3·2396118	·029411765
35	1225	42875	5·9160798	3·2710663	·028571429
36	1296	46656	6·0000000	3·3019272	·027777778
37	1369	50653	6·0827625	3·3322218	·027027027
38	1444	54872	6·1644140	3·3619754	·026315789
39	1521	59319	6·2449980	3·3912114	·025641026
40	1600	64000	6·3245553	3·4199519	·025000000
41	1681	68921	6·4031242	3·4482172	·024390244
42	1764	74088	6·4807407	3·4760266	·023809524
43	1849	79507	6·5574385	3·5033981	·023255814
44	1936	85184	6·6332496	3·5303483	·022727273
45	2025	91125	6·7082039	3·5568933	·022222222
46	2116	97336	6·7823300	3·5830479	·021739130
47	2209	103823	6·8556546	3·6088261	·021276600
48	2304	110592	6·9282032	3·6342411	·020833333
49	2401	117649	7·0000000	3·6593057	·020408163
50	2500	125000	7·0710678	3·6840314	·020000000
51	2601	132651	7·1414284	3·7084298	·019607843
52	2704	140608	7·2111026	3·7325111	·019230769

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
53	2809	148877	7·2801099	3·7562858	018867925
54	2916	157464	7·3484692	3·7797631	·018518519
55	3025	166375	7·4161985	3·8029525	·018181818
56	3136	175616	7·4833148	3·8258624	·017857143
57	3249	185193	7·5498344	3·8485011	·017543860
58	3364	195112	7·6157731	3·8708766	·017241379
59	3481	205379	7·6811457	3·8929965	·016949153
60	3600	216000	7·7459667	3·9148676	·016666667
61	3721	226981	7·8102497	3·9304972	·016393443
62	3844	238328	7·8740079	3·9578915	·016129032
63	3969	250047	7·9372539	3·9790571	·015873016
64	4096	262144	8·0000000	4·0000000	·015625000
65	4225	274625	8·0622577	4·0207256	·015384615
66	4356	287496	8·1240384	4·0412401	·015151515
67	4489	300763	8·1853528	4·0615480	·014925373
68	4624	314432	8·2462113	4·0816551	·014705882
69	4761	328509	8·3066239	4·1015661	·014492754
70	4900	343000	8·3666003	4·1212853	·014285714
71	5041	357911	8·4261498	4·1408178	·014084517
72	5184	373248	8·4852814	4·1601676	·013888889
73	5329	389017	8·5440037	4·1793390	·013698630
74	5476	405224	8·6023253	4·1983364	·013513514
75	5625	421875	8·6602540	4·2171633	·013333333
76	5776	438976	8·7177979	4·2358236	·013157895
77	5929	456533	8·7749644	4·2543210	·012987013
78	6084	474552	8·8317609	4·2726586	·012820513
79	6241	493039	8·8881944	4·2908404	·012658228
80	6400	512000	8·9442719	4·3088695	·012500000
81	6561	531441	9·0000000	4·3267487	·012345679
82	6724	551368	9·0553851	4·3444815	·012195122
83	6889	571787	9·1104336	4·3620707	·012048193
84	7056	592704	9·1651514	4·3795191	·011904762
85	7225	614125	9·2195445	4·3968296	·011764706
86	7396	636056	9·2736185	4·4140049	·011627907
87	7569	658503	9·3273791	4·4310476	·011494253
88	7744	681472	9·3808315	4·4479692	·011363636
89	7921	704969	9·4339811	4·4647451	·011235955
90	8100	729000	9·4868330	4·4814047	·011111111
91	8281	753571	9·5393920	4·4979414	·010989011
92	8464	778688	9·5916630	4·5143574	·010869565
93	8649	804357	9·6436508	4·5306549	·010752688
94	8836	830584	9·6953597	4·5468359	·010638298
95	9025	857375	9·7467943	4·5629026	·010526316
96	9216	884736	9·7979590	4·5788570	·010416667
97	9409	912673	9·8488578	4·5947009	·010309278
98	9604	941192	9·8994949	4·6104363	·010204082
99	9801	970299	9·9498744	4·6260650	·010101010
100	10000	1000000	10·0000000	4·6415888	·010000000
101	10201	1030301	10·0498756	4·6570095	·009900990
102	10404	1061208	10·0995049	4·6723287	009803922
103	10609	1092727	10·1488916	4·6875482	·009708738
104	10816	1124864	10·1980390	4·7026694	·009615385

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
105	11025	1157625	10·2469508	4·7176940	·009523810
106	11236	1191016	10·2956301	4·7326235	·009433962
107	11449	1225043	10·3440804	4·7474594	·009345794
108	11664	1259712	10·3923048	4·7622032	·009259259
109	11881	1295029	10·4403065	4·7768562	·009174312
110	12100	1331000	10·4880885	4·7914199	·009090909
111	12321	1367631	10·5356538	4·8058995	·009009009
112	12544	1404928	10·5830052	4·8202845	·008928571
113	12769	1442897	10·6301458	4·8345881	·008849558
114	12996	1481544	10·6770783	4·8488076	·008771930
115	13225	1520875	10·7238053	4·8629442	·008695652
116	13456	1560896	10·7703296	4·8769990	·008620690
117	13689	1601613	10·8166538	4·8909732	·008547009
118	13924	1643032	10·8627805	4·9048681	·008474576
119	14161	1685159	10·9087121	4·9186847	·008403361
120	14400	1728000	10·9544512	4·9324242	·008333333
121	14641	1771561	11·0000000	4·9460874	·008264463
122	14884	1815848	11·0453610	4·9596757	·008196721
123	15129	1860867	11·0905365	4·9731898	·008130081
124	15376	1906624	11·1355287	4·9866310	·008064516
125	15625	1953125	11·1803399	5·0000000	·008000000
126	15876	2000376	11·2249722	5·0132979	·007936508
127	16129	2048383	11·2694277	5·0265257	·007874016
128	16384	2097152	11·3137085	5·0396842	·007812500
129	16641	2146689	11·3578167	5·0527743	·007751938
130	16900	2197000	11·4017543	5·0657970	·007692308
131	17161	2248091	11·4455231	5·0787531	·007633588
132	17424	2299968	11·4891253	5·0916434	·007575758
133	17689	2352637	11·5325626	5·1044687	·007518797
134	17956	2406104	11·5758369	5·1172299	·007462687
135	18225	2460375	11·6189500	5·1299278	·007407407
136	18496	2515456	11·6619038	5·1425632	·007352941
137	18769	2571353	11·7046999	5·1551367	·007299270
138	19044	2628072	11·7473401	5·1676493	·007246377
139	19321	2685619	11·7898261	5·1801015	·007194245
140	19600	2744000	11·8321596	5·1924941	·007142857
141	19881	2803221	11·8743421	5·2048279	·007092199
142	20164	2863288	11·9163753	5·2171034	·007042254
143	20449	2924207	11·9582607	5·2293215	·006993007
144	20736	2985984	12·0000000	5·2414828	·006944444
145	21025	3048625	12·0415946	5·2535879	·006896552
146	21316	3112136	12·0830460	5·2656374	·006849315
147	21609	3176523	12·1243557	5·2776321	·006802721
148	21904	3241792	12·1655251	5·2895725	·006756757
149	22201	3307949	12·2065556	5·3014592	·006711409
150	22500	3375000	12·2474487	5·3132928	·006666667
151	22801	3442951	12·2882057	5·3250740	·006622517
152	23104	3511008	12·3288280	5·3368033	·006578947
153	23409	3581577	12·3693169	5·3484812	·006535948
154	23716	3652264	12·4096736	5·3601084	·006493506
155	24025	3723875	12·4498996	5·3716854	·006451613
156	24336	3796416	12·4899960	5·3832126	·006410256

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
157	24649	3869893	12·5299641	5·3946907	·006369427
158	24964	3944312	12·5698051	5·4061202	·006329114
159	25281	4019679	12·6095202	5·4175015	·006289308
160	25600	4096000	12·6491106	5·4288352	·006250000
161	25921	4173281	12·6885775	5·4401218	·006211180
162	26244	4251528	12·7279221	5·4513618	·006172840
163	26569	4330747	12·7671453	5·4625556	·006134969
164	26896	4410944	12·8062485	5·4737037	·006097561
165	27225	4492125	12·8452326	5·4848066	·006060606
166	27556	4574296	12·8840987	5·4958647	·006024096
167	27889	4657463	12·9228480	5·5068784	·005988024
168	28224	4741632	12·9614814	5·5178484	·005952381
169	28561	4826809	13·0000000	5·5287748	·005917160
170	28900	4913000	13·0384048	5·5396583	·005882353
171	29241	5000211	13·0766968	5·5504991	·005847953
172	29584	5088448	13·1148770	5·5612978	·005813953
173	29929	5177717	13·1529464	5·5720546	·005780347
174	30276	5268024	13·1909060	5·5827702	·005747126
175	30625	5359375	13·2287566	5·5934447	·005714286
176	30976	5451776	13·2664992	5·6040787	·005681818
177	31329	5545233	13·3041347	5·6146724	·005649718
178	31684	5639752	13·3416641	5·6252263	·005617978
179	32041	5735339	13·3790882	5·6357408	·005586592
180	32400	5832000	13·4164079	5·6462162	·005555556
181	32761	5929741	13·4536240	5·6566528	·005524862
182	33124	6028568	13·4907376	5·6670511	·005494505
183	33489	6128487	13·5277493	5·6774114	·005464481
184	33856	6229504	13·5646600	5·6877340	·005434783
185	34225	6331625	13·6014705	5·6980192	·005405405
186	34596	6434856	13·6381817	5·7082675	·005376344
187	34969	6539203	13·6747943	5·7184791	·005347594
188	35344	6644672	13·7113092	5·7286543	·005319149
189	35721	6751269	13·7477271	5·7387936	·005291005
190	36100	6859000	13·7840488	5·7488971	·005263158
191	36481	6967871	13·8202750	5·7589652	·005235602
192	36864	7077888	13·8564065	5·7689982	·005208333
193	37249	7189517	13·8924400	5·7789966	·005181347
194	37636	7301384	13·9283883	5·7889604	·005154639
195	38025	7414875	13·9642400	5·7988900	·005128205
196	38416	7529536	14·0000000	5·8087857	·005102041
197	38809	7645373	14·0356688	5·8186479	·005076142
198	39204	7762392	14·0712473	5·8284867	·005050505
199	39601	7880599	14·1067360	5·8382725	·005025126
200	40000	8000000	14·1421356	5·8480355	·005000000
201	40401	8120601	14·1774469	5·8577660	·004975124
202	40804	8242408	14·2126704	5·8674673	·004950495
203	41209	8365427	14·2478068	5·8771307	·004926108
204	41616	8489664	14·2828569	5·8867653	·004901961
205	42025	8615125	14·3178211	5·8963685	·004878049
206	42436	8741816	14·3527001	5·9059406	·004854369
207	42849	8869743	14·3874946	5·9154817	·004830918
208	43264	8998912	14·4222051	5·9249921	·004807692

Number.	Squares	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
209	43681	9129329	14·4568323	5·9344721	·004784689
210	44100	9261000	14·4913767	5·9439220	·004761905
211	44521	9393931	14·5258390	5·9533418	·004739336
212	44944	9528128	14·5602198	5·9627320	·004716981
213	45369	9663597	14·5945195	5·9720926	·004694836
214	45796	9800344	14·6287388	5·9814240	·004672897
215	46225	9938375	14·6628783	5·9907264	·004651163
216	46656	10077696	14·6969385	6·0000000	·004629630
217	47089	10218313	14·7309199	6·0092450	·004608295
218	47524	10360232	14·7648231	6·0184617	·004587156
219	47961	10503459	14·7986486	6·0276502	·004566210
220	48400	10648000	14·8323970	6·0368107	·004545455
221	48841	10793861	14·8660687	6·0459435	·004524887
222	49284	10941048	14·8996644	6·0550489	·004504505
223	49729	11089567	14·9331845	6·0641270	·004484305
224	50176	11239424	14·9666295	6·0731779	·004464286
225	50625	11390625	15·0000000	6·0824020	·004444444
226	51076	11543176	15·0332964	6·0991994	·004424779
227	51529	11697083	15·0665192	6·1001702	·004405286
228	51984	11852352	15·0996689	6·1091147	·004385965
229	52441	12008989	15·1327460	6·1180332	·004366812
230	52900	12167000	15·1657509	6·1269257	·004347826
231	53361	12326391	15·1986842	6·1357924	·004329004
232	53824	12487168	15·2315462	6·1446337	·004310345
233	54289	12649337	15·2643375	6·1534495	·004291845
234	54756	12812904	15·2970585	6·1622401	·004273504
235	55225	12977875	15·3297097	6·1710058	·004255319
236	55696	13144256	15·3622915	6·1797466	·004237288
237	56169	13312053	15·3948043	6·1884628	·004219409
238	56644	13481272	15·4272486	6·1971544	·004201681
239	57121	13651919	15·4596248	6·2058218	·004184100
240	57600	13824000	15·4919334	6·2144650	·004166667
241	58081	13997521	15·5241747	6·2230843	·004149378
242	58564	14172488	15·5563492	6·2316797	·004132231
243	59049	14348907	15·5884573	6·2402515	·004115226
244	59536	14526784	15·6204994	6·2487998	·004098361
245	60025	14706125	15·6524758	6·2573248	·004081633
246	60516	14886936	15·6843871	6·2658266	·004065041
247	61009	15069223	15·7162336	6·2743054	·004048583
248	61504	15252992	15·7480157	6·2827613	·004032258
249	62001	15438249	15·7797338	6·2911946	·004016064
250	62500	15625000	15·8113883	6·2996053	·004000000
251	63001	15813251	15·8429795	6·3079935	·003984064
252	63504	16003008	15·8745079	6·3163596	·003968254
253	64009	16194277	15·9059737	6·3247035	·003952569
254	64516	16387064	15·9373775	6·3330256	·003937008
255	65025	16581375	15·9687194	6·3413257	·003921569
256	65536	16777216	16·0000000	6·3496042	·003906250
257	66049	16974593	16·0312195	6·3578611	·003891051
258	66564	17173512	16·0623784	6·3660968	·003875969
259	67081	17373979	16·0934769	6·3743111	·003861004
260	67600	17576000	16·1245155	6·3825043	**·003846154**

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
261	68121	17779581	16·1554944	6·3906765	·003831418
262	68644	17984728	16·1864141	6·3988279	·003816794
263	69169	18191447	16·2172747	6,4069585	·003802281
264	69696	18399744	16·2480768	6·4150687	·003787879
265	70225	18609625	16·2788206	6·4231583	·003773585
266	70756	18821096	16·3095064	6·4312276	·003759398
267	71289	19034163	16·3401346	6·4392767	·003745318
268	71824	19248832	16·3707055	6·4473057	·003731343
269	72361	19465109	16·4012195	6·4553148	·003717472
270	72900	19683000	16.4316767	6·4633041	·003703704
271	73441	19902511	16·4620776	6·4712736	·003690037
272	73984	20123648	16·4924225	6·4792236	·003676471
273	74529	20346417	16·5227116	6·4871541	·003663004
274	75076	20570824	16·5529454	6·4950653	·003649635
275	75625	20796875	16·5831240	6·5029572	·003636364
276	76176	21024576	16·6132477	6·5108300	·003623188
277	76729	21253933	16·6433170	6·5186839	·003610108
278	77284	21484952	16·6783320	6·5265189	·003597122
279	77841	21717639	16·7032931	6·5343351	·003584229
280	78400	21952000	16·7332005	6·5421326	·003571429
281	78961	22188041	16·7630546	6·5499116	·003558719
282	79524	22425768	16·7928556	6·5576722	·003546099
283	80089	22665187	16·8226038	6·5654144	·003533569
284	80656	22906304	16·8522995	6·5731385	·003521127
285	81225	23149125	16·8819430	6·5808443	·003508772
286	81796	23393656	16·9115345	6·5885323	·003496503
287	82369	23639903	16·9410743	6·5962023	·003484321
288	82944	23887872	16·9705627	6·6038545	·003472222
289	83521	24137569	17·0000000	6·6114890	·003460208
290	84100	24389000	17·0293864	6·6191060	·003448276
291	84681	24642171	17·0587221	6·6267054	·003436426
292	85264	24897088	17·0880075	6·6342874	·003424658
293	85849	25153757	17·1172428	6·6418522	·003412969
294	86436	25412184	17·1464282	6·6493998	·003401361
295	87025	25672375	17·1755640	6·6569302	·003389831
296	87616	25934836	17·2046505	6·6644437	·003378378
297	88209	26198073	17·2336879	6·6719403	·003367003
298	88804	26463592	17·2626765	6.6794200	·003355705
299	89401	26730899	17·2916165	6.6868831	·003344482
300	90000	27000000	17·3205081	6.6943295	·003333333
301	90601	27270901	17·3493516	6.7017593	·003322259
302	91204	27543608	17·3781472	6·7091729	·003311258
303	91809	27818127	17·4068952	6·7165700	·003301330
304	92416	28094464	17·4355958	6·7239508	·003289474
305	93025	28372625	17·4642492	6·7313155	·003278689
306	93636	28652616	17·4928557	6·7386641	·003267974
307	94249	28934443	17·5214155	6·7459967	·003257329
308	94864	29218112	17·5499288	6·7533134	·003246753
309	95481	29503609	17·5783958	6·7606143	·003236246
310	96100	29791000	17·6068169	6·7678995	·003225806
311	96721	30080231	17·6351921	6·7751690	·003215434
312	97344	30371328	17·6635217	6·7824229	·003205128

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
313	97969	30664297	17·6918060	6·7896613	·003194888
314	98596	30959144	17·7200451	6·7968844	·003184713
315	99225	31255875	17·7482393	6·8040921	·003174603
316	99856	31554496	17·7763888	6·8112847	·003164557
317	100489	31855013	17·8044938	6·8184620	.003154574
318	101124	32157432	17·8325545	6·8256242	·003144654
319	101761	32461759	17·8605711	6·8327714	·003134796
320	102400	32768000	17·8885438	6·8399037	·003125000
321	103041	33076161	17·9164729	6·8470213	·003115265
322	103684	33386248	17·9443584	6·8541240	·003105590
323	104329	33698267	17·9722008	6·8612120	·003095975
324	104976	34012224	18·0000000	6·8682855	·003086420
325	105625	34328125	18·0277564	6·8753433	·003076923
326	106276	34645976	18·0554701	6·8823888	·003067485
327	106929	34965783	18·0831413	6·8894188	·003048104
328	107584	35287552	18·1107703	6·8964345	·003048780
329	108241	35611289	18·1383571	6·9034359	·003039514
330	108900	35937000	18·1659021	6·9104232	·003030303
331	109561	36264691	18·1934054	6·9173964	·003021148
332	110224	36594368	18·2208672	6·9243556	·003012048
333	110889	36926037	18·2482876	6·9313088	·003003003
334	111556	37259704	18·2756669	6·9382321	·002994012
335	112225	37595375	18·3030052	6·9451496	·002985075
336	112896	37933056	18·3303028	6·9520533	·002976190
337	113569	38272753	18·3575598	6·9589434	·002967359
338	114244	38614472	18·3847763	6·9658198	·002958580
339	114921	38958219	18·4119526	6·9726826	·002949853
340	115600	39304000	18·4390889	6·9795321	·002941176
341	116281	39651821	18·4661853	6·9863681	·002932551
342	116964	40001688	18·4932420	6·9931906	·002923977
343	117649	40353607	18·5202592	7·0000000	·002915452
344	118336	40707584	18·5472370	7·0067962	·002906977
345	119025	41063625	18·5741756	7·0135791	·002898551
346	119716	41421736	18·6010752	7·0203490	·002890173
347	120409	41781923	18·6279360	7·0271058	·002881844
348	121104	42144192	18·6547581	7·0338497	·002873563
349	121801	42508549	18·6815417	7·0405860	·002865330
350	122500	42875000	18·7082869	7·0472987	·002857143
351	123201	43243551	18·7349940	7·0540041	·002849003
352	123904	43614208	18·7616630	7·0606967	·002840909
353	124609	43986977	18·7882942	7·0673767	·002832861
354	125316	44361864	18·8148877	7·0740440	·002824859
355	126025	44738875	18·8414437	7·0806988	·002816901
356	126736	45118016	18·8679623	7·0873411	·002808989
357	127449	45499293	18·8944436	7·0939709	·002801120
358	128164	45882712	18·9208879	7·1005885	·002793296
359	128881	46268279	18·9472953	7·1071937	·002785515
360	129600	46656000	18·9736660	7·1137866	·002777778
361	130321	47045831	19·0000000	7·1203674	·002770083
362	131044	47437928	19·0262976	7·1269360	·002762431
363	131769	47832147	19·0525589	7·1334925	·002754821
364	132496	48228544	19·0787840	7·1400370	·002747253

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
365	133225	48627125	19·1049732	7·1465695	·002739726
366	133956	49027896	19·1311265	7·1530901	·002732240
367	134689	49430863	19·1572441	7·1595988	·002724796
368	135424	49836032	19·1833261	7·1660957	·002717391
369	136161	50243409	19·2093727	7·1725809	·002710027
370	136900	50653000	19·2353841	7·1790544	·002702703
371	137641	51064811	19·2613603	7·1855162	·002695418
372	138384	51478848	19·2873015	7·1919663	·002688172
373	139129	51895117	19·3132079	7·1984050	·002680965
374	139876	52313624	19·3390796	7·2048322	·002673797
375	140625	52734375	19·3649167	7·2112479	·002666667
376	141376	53157376	19·3907194	7·2176522	·002659574
377	142129	53582633	19·4164878	7·2240450	·002652520
378	142884	54010152	19·4422221	7·2304268	·002645503
379	143641	54439939	19·4679223	7·2367972	·002638521
380	144400	54872000	19·4935887	7·2431565	·002631579
381	145161	55306341	19·5192213	7·2495045	·002624672
382	145924	55742968	19·5448203	7·2558415	·002617801
383	146689	56181887	19·5703858	7·2621675	·002610966
384	147456	56623104	19·5959179	7·2684824	·002604167
385	148225	57066625	19·6214169	7·2747864	·002597403
386	148996	57512456	19·6468827	7·2810794	·002590674
387	149769	57960603	19·6723156	7·2873617	·002583979
388	150544	58411072	19·6977156	7·2936330	·002577320
389	151321	58863869	19·7230829	7·2998936	·002570694
390	152100	59319000	19·7484177	7·3061436	·002564103
391	152881	59776471	19·7737199	7·3123828	·002557545
392	153664	60236288	19·7989899	7·3186114	·002551020
393	154449	60698457	19·8242276	7·3248295	·002544529
394	155236	61162984	19·8494332	7·3310369	·002538071
395	156025	61629875	19·8746069	7·3372339	·002531646
396	156816	62099136	19·8997487	7·3434205	·002525253
397	157609	62570773	19·9248588	7·3495966	·002518892
398	158404	63044792	19·9499373	7·3557624	·002512563
399	159201	63521199	19·9749844	7·3619178	·002506266
400	160000	64000000	20·0000000	7·3680630	·002500000
401	160801	64481201	20·0249844	7·3741979	·002493766
402	161604	64964808	20·0499377	7·3803227	·002487562
403	162409	65450827	20·0748599	7·3864373	·002481390
404	163216	65939264	20·0997512	7·3925418	·002475248
405	164025	66430125	20·1246118	7·3986363	·002469136
406	164836	66923416	20·1494417	7·4047206	·002463054
407	165649	67419143	20·1742410	7·4107950	·002457002
408	166464	67917312	20·1990099	7·4168595	·002450980
409	167281	68417929	20·2237484	7·4229142	·002444988
410	168100	68921000	20·2484567	7·4289589	·002439024
411	168921	69426531	20·2731349	7·4349938	·002433090
412	169744	69934528	20·2977831	7·4410189	·002427184
413	170569	70444997	20·3224014	7·4470343	·002421308
414	171396	70957944	20·3469899	7·4530399	·002415459
415	172225	71473375	20·3715488	7·4590359	·002409639
416	173056	71991296	20·3960781	7·4650223	·002403846

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
417	173889	72511713	20·4205779	7·4709991	·002398082
418	174724	73034632	20·4450483	7·4769664	·002392344
419	175561	73560059	20·4694895	7·4829242	·002386635
420	176400	74088000	20·4939015	7·4888724	·002380952
421	177241	74618461	20·5182845	7·4948113	·002375297
422	178084	75151448	20·5426386	7·5007406	·002369668
423	178929	75686967	20·5669638	7·5066607	·002364066
424	179776	76225024	20·5912603	7·5125715	·002358491
425	180625	76765625	20·6155281	7·5184730	·002352941
426	181476	77308776	20·6397674	7·5243652	·002347418
427	182329	77854483	20·6639783	7·5302482	·002341920
428	183184	78402752	20·6881609	7·5361221	·002336449
429	184041	78953589	20·7123152	7·5419867	·002331002
430	184900	79507000	20·7364414	7·5478423	·002325581
431	185761	80062991	20·7605395	7·5536888	·002320186
432	186624	80621568	20·7846097	7·5595263	·002314815
433	187489	81182737	20·8086520	7·5653548	·002309469
434	188356	81746504	20·8326667	7·5711743	·002304147
435	189225	82312875	20·8566536	7·5769849	·002298851
436	190096	82881856	20·8806130	7·5827865	·002293578
437	190969	83453453	20·9045450	7·5885793	·002288330
438	191844	84027672	20·9284495	7·5943633	·002283105
439	192721	84604519	20·9523268	7·6001385	·002277904
440	193600	85184000	20·9761770	7·6059049	·002272727
441	194481	85766121	21·0000000	7·6116626	·002267574
442	195364	86350888	21·0237960	7·6174116	·002262443
443	196249	86938307	21·0475652	7·6231519	·002257336
444	197136	87528384	21·0713075	7·6288837	·002252252
445	198025	88121125	21·0950231	7·6346067	·002247191.
446	198916	88716536	21·1187121	7·6403213	·002242152
447	199809	89314623	21·1423745	7·6460272	·002237136
448	200704	89915392	21·1660105	7·6517247	·002232143
449	201601	90518849	21·1896201	7·6574138	·002227171
450	202500	91125000	21·2132034	7·6630943	·002222222
451	203401	91733851	21·2367606	7·6687665	·002217295
452	204304	92345408	21·2602916	7·6744303	·002212389
453	205209	92959677	21·2837967	7·6800857	·002207506
454	206116	93576664	21·3072758	7·6857328	·002202643
455	207025	94196375	21·3307290	7·6913717	·0021 7802
456	207936	94818816	21·3541565	7·6970023	·002192982
457	208849	95443993	21·3775583	7·7026246	·002188184
458	209764	96071912	21·4009346	7·7082388	·002183406
459	210681	96702579	21·4242853	7·7188448	·002178649
460	211600	97336000	21·4476106	7·7194426	·002173913
461	212521	97972181	21·4709106	7·7250325	·002169197
462	213444	98611128	21·4941853	7·7306141	·002164502
463	214369	99252847	21·5174348	7·7361877	·002159827
464	215296	99897344	21·5406592	7·7417532.	·002155172
465	216225	100544625	21·5638587	7·7473109	·002150538
466	217156	101194696	21·5870331	7·7528606	·002145923
467	218089	101847563	21·6101828	7·7584023	·002141328
468	219024	102503232	21·6333077	7·7639361	·002136752

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
469	219961	103161709	21·6564078	7·7694620	·002132196
470	220900	103823000	21·6794834	7·7749801	·002127660
471	221841	104487111	21·7025344	7·7804904	·002123142
472	222784	105154048	21·7255610	7·7859928	·002118644
473	223729	105828817	21·7485632	7·7914875	·002114165
474	224676	106496424	21·7715411	7·7969745	·002109705
475	225625	107171875	21·7944947	7·8024538	·002105263
476	226576	107850176	21·8174242	7·8079254	·002100840
477	227529	108531333	21·8403297	7·8133892	·002096436
478	228484	109215352	21·8632111	7·8188456	·002092050
479	229441	109902239	21·8860686	7·8242942	·002087683
480	230400	110592000	21·9089023	7·8297353	·002083333
481	231361	111284641	21·9317122	7·8351688	·002079002
482	232324	111980168	21·9544984	7·8405949	·002074689
483	233289	112678587	21·9772610	7·8460134	·002070393
484	234256	113379904	22·0000000	7·8514244	·002066116
485	235225	114084125	22·0227155	7·8568281	·002061856
486	236196	114791256	22·0454077	7·8622242	·002057613
487	237169	115501303	22·0680765	7·8676130	·002053388
488	238144	116214272	22·0907220	7·8729944	·002049180
489	239121	116930169	22·1133444	7·8783684	·002044990
490	240100	117649000	22·1359436	7·8837352	·002040816
491	241081	118370771	22·1585198	7·8890946	·002036660
492	242064	119095488	22·1810730	7·8944468	·002032520
493	243049	119823157	22·2036033	7·8997917	·002028398
494	244036	120553784	22·2261108	7·9051294	·002024291
495	245025	121287375	22·2485955	7·9104599	·002020202
496	246016	122023936	22·2710575	7·9157832	·002016129
497	247009	122763473	22·2934968	7·9210994	·002012072
498	248004	123505992	22·3159136	7·9264085	·002008032
499	249001	124251499	22·3383079	7·9317104	·002004008
500	250000	125000000	22·3606798	7·9370053	·002000000
501	251001	125751501	22·3830293	7·9422931	·001996008
502	252004	126506008	22·4053565	7·9475739	·001992032
503	253009	127263527	22·4276615	7·9528477	·001988072
504	254016	128024064	22·4499443	7·9581144	·001984127
505	255025	128787625	22·4722051	7·9633743	·001980198
506	256036	129554216	22·4944438	7·9686271	·001976285
507	257049	130323843	22·5166605	7·9738731	·001972387
508	258064	131096512	22·5388553	7·9791122	·001968504
509	259081	131872229	22·5610283	7·9843444	·001964637
510	260100	132651000	22·5831796	7·9895697	·001960784
511	261121	133432831	22·6053091	7·9947883	·001956947
512	262144	134217728	22·6274170	8·0000000	·001953125
513	263169	135005697	22·6495033	8·0052049	·001949318
514	264196	135796744	22·6715681	8·0104032	·001945525
515	265225	136590875	22·6936114	8·0155946	·001941748
516	266256	137388096	22·7156334	8·0207794	·001937984
517	267289	138188413	22·7376341	8·0259574	·001934236
518	268324	138991832	22·7596134	8·0311287	·001930502
519	269361	139798359	22·7815715	8·0362935	·001926782
520	270400	140608000	22·8035085	8·0414515	·001923077

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
521	271441	141420761	22·8254244	8·0466030	·001919386
522	272484	142236648	22·8473193	8·0517479	·001915709
523	273529	143055667	22·8691933	8·0568862	·001912046
524	274576	143877824	22·8910463	8·0620180	·001908397
525	275625	144703125	22·9128785	8·0671432	·001904762
526	276676	145531576	22·9346899	8·0722620	·001901141
527	277729	146363183	22·9564806	8·0773743	·001897533
528	278784	147197952	22·9782506	8·0824800	·001893939
529	279841	148035889	23·0000000	8·0875794	·001890359
530	280900	148877001	23·0217289	8·0926723	·001886792
531	281961	149721291	23·0434372	8·0977589	·001883239
532	283024	150568768	23·0651252	8·1028390	·001879699
533	284089	151419437	23·0867928	8·1079128	·001876173
534	285156	152273304	23·1084400	8·1129803	·001872659
535	286225	153130375	23·1300670	8·1180414	·001869159
536	287296	153990656	23·1516738	8·1230962	·001865672
537	288369	154854153	23·1732605	8·1281447	·001862197
538	289444	155720872	23·1948270	8·1331870	·001858736
539	290521	156590819	23·2163735	8·1382230	·001855288
540	291600	157464000	23·2379001	8·1432529	·001851852
541	292681	158340421	23·2594067	8·1482765	·001848429
542	293764	159220088	23·2808935	8·1532939	·001845018
543	294849	160103007	23·3023604	8·1583051	·001841621
544	295936	160989184	23·3238076	8·1633102	·001838235
545	297025	161878625	23·3452351	8·1683092	·001834862
546	298116	162771336	23·3666429	8·1733020	·001831502
547	299209	163667323	23·3880311	8·1782888	·001828154
548	300304	164566592	23·4093998	8·1832695	·001824818
549	301401	165469149	23·4307490	8·1882441	·001821494
550	302500	166375000	23·4520788	8·1932127	·001818182
551	303601	167284151	23·4733892	8·1981753	·001814882
552	304704	168196608	23·4946802	8·2031319	·001811594
553	305809	169112377	23·5159520	8·2080825	·001808318
554	306916	170031464	23·5372046	8·2130271	·001805054
555	308025	170953875	23·5584380	8·2179657	·001801802
556	309136	171879616	23·5796522	8·2228985	·001798561
557	310249	172808693	23·6008474	8·2278254	·001795332
558	311364	173741112	23·6220236	8·2327463	·001792115
559	312481	174676879	23·6431808	8·2376614	·001788909
560	313600	175616000	23·6643191	8·2425706	·001785714
561	314721	176558481	23·6854386	8·2474740	·001782531
562	315844	177504328	23·7065392	8·2523715	·001779359
563	316969	178453547	23·7276210	8·2572635	·001776199
564	318096	179406144	23·7486842	8·2621492	·001773050
565	319225	180362125	23·7697286	8·2670294	·001769912
566	320356	181321496	23·7907545	8·2719039	·001766784
567	321489	182284263	23·8117618	8·2767726	·001763668
568	322624	183250432	23·8327506	8·2816255	·001760563
569	323761	184220009	23·8537209	8·2864928	·001757469
570	324900	185193000	23·8746728	8·2913444	·001754386
571	326041	186169411	23·8956063	8·2961903	·001751313
572	327184	187149248	23·9165215	8·3010304	·001748252

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
573	328329	188132517	23·9374184	8·3058651	·001745201
574	329476	189119224	23·9582971	8·3106941	·001742160
575	330625	190109375	23·9791576	8·3155175	·001739130
576	331776	191102976	24·0000000	8·3203353	·001736111
577	332927	192100033	24·0208243	8·3251475	·001733102
578	334084	193100552	24·0416306	8·3299542	·001730104
579	335241	194104539	24·0624188	8·3347553	·001727116
580	336400	195112000	24·0831891	8·3395509	·001724138
581	337561	196122941	24·1039416	8·3443410	·001721170
582	338724	197137368	24·1246762	8·3491256	·001718213
583	339889	198155287	24·1453929	8·3539047	·001715266
584	341056	199176704	24·1660919	8·3586784	·001712329
585	342225	200201625	24·1867732	8·3634466	·001709402
586	343396	201230056	24·2074369	8·3682095	·001706485
587	344569	202262003	24·2280829	8·3729668	·001703578
588	345744	203297472	24·2487113	8·3777188	·001700680
589	346921	204336469	24·2693222	8·3824653	·001697793
590	348100	205379000	24·2899156	8·3872065	·001694915
591	349281	206425071	24·3104996	8·3919428	·001692047
592	350464	207474688	24·3310501	8·3966729	·001689189
593	351649	208527857	24·3515913	8·4013981	·001686341
594	352836	209584584	24·3721152	8·4061180	·001683502
595	354025	210644875	24·3926218	8·4108326	·001680672
596	355216	211708736	24·4131112	8·4155419	·001677852
597	356409	212776173	24·4335834	8·4202460	·001675042
598	357604	213847192	24·4540385	8·4249448	·001672241
599	358801	214921799	24·4744765	8·4296383	·001669449
600	360000	216000000	24·4948974	8·4343267	·001666667
601	361201	217081801	24·5153013	8·4390098	·001663894
602	362404	218167208	24·5356883	8·4436877	·001661130
603	363609	219256227	24·5560583	8·4483605	·001658375
604	364816	220348864	24·5764115	8·4530281	·001655629
605	366025	221445125	24·5967478	8·4576906	·001652893
606	367236	222545016	24·6170673	8·4623479	·001650165
607	368449	223648543	24·6373700	8·4670001	·001647446
608	369664	224755712	24·6576560	8·4716471	·001644737
609	370881	225866529	24·6779254	8·4762892	·001642036
610	372100	226981000	24·6981781	8·4809261	·001639344
611	373321	228099131	24·7184142	8·4855579	·001636661
612	374544	229220928	24·7386338	8·4901848	·001633987
613	375769	230346397	24·7588368	8·4948065	·001631321
614	376996	231475544	24·7790234	8·4994233	·001628664
615	378225	232608375	24·7991935	8·5040350	·001626016
616	379456	233744896	24·8193473	8·5086417	·001623377
617	380689	234885113	24·8394847	8·5132435	·001620746
618	381924	236029032	24·8596058	8·5178403	·001618123
619	383161	237176659	24·8797106	8·5224331	·001615509
620	384400	238328000	24·8997992	8·5270189	·001612903
621	385641	239483061	24·9198716	8·5316009	·001610306
622	386884	240641848	24·9399278	8·5361780	·001607717
623	388129	241804367	24·9599679	8·5407501	·001605136
624	389376	242970624	24·9799920	8·5453173	001602564

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
625	390625	244140625	25·0000000	8·5498797	·001600000
626	391876	245134376	25·0199920	8·5544372	·001597444
627	393129	246491883	25·0399681	8·5589899	·001594896
628	394384	247673152	25·0599282	8·5635377	·001592357
629	395641	248858189	25·0798724	8·5680807	·001589825
630	396900	250047000	25·0998008	8·5726189	·001587302
631	398161	251239591	25·1197134	8·5771523	·001584786
632	399424	252435968	25·1396102	8·5816809	·001582278
633	400689	253636137	25·1594913	8·5862247	·001579779
634	401956	254840104	25·1793566	8·5907238	·001577287
635	403225	256047875	25·1992063	8·5952380	·001574803
636	404496	257259456	25·2190404	8·5997476	·001572327
637	405769	258474853	25·2388589	8·6042525	·001569859
638	407044	259694072	25·2586619	8·6087526	·001567398
639	408321	260917119	25·2784493	8·6132480	·001564945
640	409600	262144000	25·2982213	8·6177388	·001562500
641	410881	263374721	25·3179778	8·6222248	·001560062
642	412164	264609288	25·3377189	8·6267063	·001557632
643	413449	265847707	25·3574447	8·6311830	·001555210
644	414736	267089984	25·3771551	8·6356551	·001552795
645	416025	268336125	25·3968502	8·6401226	·001550388
646	417316	269585136	25·4165302	8·6445855	·001547988
647	418609	270840023	25·4361947	8·6490437	·001545595
648	419904	272097792	25·4558441	8·6534974	·001543210
649	421201	273359449	25·4754784	8·6579465	·001540832
650	422500	274625000	25·4950976	8·6623911	·001538462
651	423801	275894451	25·5147013	8·6668310	·001536098
652	425104	277167808	25·5342907	8·6712665	·001533742
653	426409	278445077	25·5538647	8·6756974	·001531394
654	427716	279726264	25·5734237	8·6801237	·001529052
655	429025	281011375	25·5929678	8·6845456	·001526718
656	430336	282300416	25·6124969	8·6889630	·001524390
657	431649	283593393	25·6320112	8·6933759	·001522070
658	432964	284890312	25·6515107	8·6977843	·001519757
659	434281	286191179	25·6709953	8·7021882	·001517451
660	435600	287496000	25·6904652	8·7065877	·001515152
661	436921	288804781	25.7099203	8·7109827	·001512859
662	438244	290117528	25·7293607	8·7153734	·001510574
663	439569	291434247	25·7487864	8·7197596	·001508296
664	440896	292754944	25·7681975	8·7241414	·001506024
665	442225	294079625	25·7875939	8·7285187	·001503759
666	443556	295408296	25·8069758	8·7328918	·001501502
667	444889	296740963	25·8263431	8·7372604	·001499250
668	446224	298077632	25·8456960	8·7416246	·001497006
669	447561	299418309	25·8650343	8·7459846	·001494768
670	448900	300763000	25·8843582	8·7503401	·001492537
671	450241	302111711	25·9036677	8·7546913	·001490313
672	451584	303464448	25·9229628	8·7590383	·001488095
673	452929	304821217	25·9422435	8·7633809	·001485884
674	454276	306182024	25·9615100	8·7677192	·001483680
675	455625	307546875	25·9807621	8·7720532	·001481481
676	456976	308915776	26·0000000	8·7763830	·001479290

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
677	458329	310288733	26·0192237	8·7807084	·001477105
678	459684	311665752	26·0384331	8·7850296	·001474926
679	461041	313046839	26·0576284	8·7893466	·001472754
680	462400	314432000	26·0768096	8·7936593	·001470588
681	463761	315821241	26·0959767	8·7979679	·001468429
682	465124	317214568	26·1151297	8·8022721	·001466276
683	466489	318611987	26·1342687	8·8065722	·001464129
684	467856	320013504	26·1533937	8·8108681	·001461988
685	469225	321419125	26·1725047	8·8151598	·001459854
686	470596	322828856	26·1916017	8·8194474	·001457726
687	471969	324242703	26·2106848	8·8237307	·001455604
688	473344	325660672	26·2297541	8·8280099	·001453488
689	474721	327082769	26·2488095	8·8322850	·001451379
690	476100	328509000	26·2678511	8·8365559	·001449275
691	477481	329939371	26·2868789	8·8408227	·001447178
692	478864	331373888	26·3058929	8·8450854	·001445087
693	480249	332812557	26·3248932	8·8493440	·001443001
694	481636	334255384	26·3438797	8·8535985	·001440922
695	483025	335702375	26·3628527	8·8578489	·001438849
696	484416	337153536	26·3818119	8·8620952	·001436782
697	485809	338608873	26·4007576	8·8663375	·001434720
698	487204	340068392	26·4196896	8·8705757	·001432665
699	488601	341532099	26·4386081	8·8748099	·001430615
700	490000	343000000	26·4575131	8·8790400	·001428571
701	491401	344472101	26·4764046	8·8832661	·001426534
702	492804	345948408	26·4952826	8·8874882	·001424501
703	494209	347428927	26·5141472	8·8917063	·001422475
704	495616	348913664	26·5329983	8·8959204	·001420455
705	497025	350402625	26·5518361	8·9001304	·001418440
706	498436	351895816	26·5706605	8·9043366	·001416431
707	499849	353393243	26·5894716	8·9085387	·001414427
708	501264	354894912	26·6082694	8·9127369	·001412429
709	502681	356400829	26·6270539	8·9169311	·001410437
710	504100	357911000	26·6458252	8·9211214	·001408451
711	505521	359425431	26·6645833	8·9253078	·001406470
712	506944	360944128	26·6833281	8·9294902	·001404494
713	508369	362467097	26·7020598	8·9336687	·001402525
714	509796	363994344	26·7207784	8·9378433	·001400560
715	511225	365525875	26·7394839	8·9420140	·001398601
716	512656	367061696	26·7581763	8·9461809	·001396648
717	514089	368601813	26·7768557	8·9503438	·001394700
718	515524	370146232	26·7955220	8·9545029	·001392758
719	516961	371694959	26·8141754	8·9586581	·001390821
720	518400	373248000	26·8328157	8·9628095	·001388889
721	519841	374805361	26·8514432	8·9669570	·001386963
722	521284	376367048	26·8700577	8·9711007	·001385042
723	522729	377933067	26·8886593	8·9752406	·001383126
724	524176	379503424	26·9072481	8·9793766	·001381215
725	525625	381078125	26·9258240	8·9835089	·001379310
726	527076	382657176	26·9443872	8·9876373	·001377410
727	528529	384240583	26·9629375	8·9917620	·001375516
728	529984	385828352	26·9814751	8·9958899	·001373626

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
729	531441	387420489	27·0000000	9·0000000	001371742
730	532900	389017000	27·0185122	9·0041134	·001369863
731	534361	390617891	27·0370117	9·0082229	·001367989
732	535824	392223168	27·0554985	9·0123288	·001366120
733	537289	393832837	27·0739727	9·0164309	·001364256
734	538756	395446904	27·0924344	9·0205293	·001362398
735	540225	397065375	27·1108834	9·0246239	·001360544
736	541696	398688256	27·1293199	9·0287149	·001358696
737	543169	400315553	27·1477149	9·0328021	·001356852
738	544644	401947272	27·1661554	9·0368857	·001355014
739	546121	403583419	27·1845544	9·0409655	·001353180
740	547600	405224000	27·2029140	9·0450419	·001351351
741	549081	406869021	27·2213152	9·0491142	·001349528
742	550564	408518488	27·2396769	9·0531831	·001347709
743	552049	410172407	27·2580263	9·0572482	·001345895
744	553536	411830784	27·2763634	9 0613098	·001344086
745	555025	413493625	27·2946881	9·0653677	·001342282
746	556516	415160936	27·3130006	9·0694220	·001340483
747	558009	416832723	27·3313007	9·0734726	·001338688
748	559504	418508992	27·3495887	9·0775197	·001336898
749	561001	420189749	27·3678644	9·0815631	·001335113
750	562500	421875000	27·3861279	9·0856030	·001333333
751	564001	423564751	27·4043792	9·0896352	·001331558
752	565504	425259008	27·4226184	9·0936719	·001329787
753	567009	426957777	27·4408455	9·0977010	·001328021
754	568516	428661064	27·4590604	9·1017265	·001326260
755	570025	430368875	27·4772633	9·1057485	·001324503
756	571536	432081216	27·4954542	9·1097669	·001322751
757	573049	433798093	27·5136330	9·1137818	·001321004
758	574564	435519512	27·5317998	9·1177931	·001319261
759	576081	437245479	27·5499546	9·1218010	·001317523
760	577600	438976000	27·5680975	9·1258053	·001315789
761	579121	440711081	27·5862284	9·1298061	·001314060
762	580644	442450728	27·6043475	9·1338034	·001312336
763	582169	444194947	27·6224546	9·1377971	·001310616
764	583696	445943744	27·6405499	9·1417874	·001308901
765	585225	447697125	27·6586334	9·1457742	·001307190
766	586756	449455096	27·6767050	9·1497576	·001305483
767	588289	451217663	27·6947648	9·1537375	·001303781
768	589824	452984832	27·7128129	9·1577139	·001302083
769	591361	454756609	27·7308492	9·1616869	·001300390
770	592900	456533000	27·7488739	9·1656565	·001298701
771	594441	458314011	27·7668868	9·1696225	·001297017
772	595984	460099648	27·7848880	9·1735852	·001295337
773	597529	461889917	27·8028775	9·1775445	·001293661
774	599076	463684824	27·8208555	9·1815003	·001291990
775	600625	465484375	27·8388218	9·1854527	·001290323
776	602176	467288576	27·8567766	9·1894018	·001288660
777	603729	469097433	27·8747197	9·1933474	·001287001
778	605284	470910952	27·8926514	9·1972897	·001285347
779	606841	472729139	27·9105715	9·2012286	·001283697
780	608400	474552000	27·9284801	9·2051641	·001282051

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
781	609961	476379541	27·9463772	9·2090962	·001280410
782	611524	478211768	27·9642629	9·2130250	·001278772
783	613089	480048687	27·9821372	9·2169505	·001277139
784	614656	481890304	28·0000000	9·2208726	·001275510
785	616225	483736625	28·0178515	9·2247914	·001273885
786	617796	485587656	28·0356915	9·2287068	·001272265
787	619369	487443403	28·0535203	9·2326189	·001270648
788	620944	489303872	28·0713377	9·2365277	·001269036
789	622521	491169069	28·0891438	9·2404333	·001267427
790	624100	493039000	28·1069386	9·2443355	·001265823
791	625681	494913671	28·1247222	9·2482344	·001264223
792	627264	496793088	28·1424946	9·2521300	·001262626
793	628849	498677257	28·1602557	9·2560224	·001261034
794	630436	500566184	28·1780056	9·2599114	·001259446
795	632025	502459875	28·1957444	9·2637973	·001257862
796	633616	504358336	28·2134720	9·2676798	·001256281
797	635209	506261573	28·2311884	9·2715592	·001254705
798	636804	508169592	28·2488938	9·2754352	·001253133
799	638401	510082399	28·2665881	9·2793081	·001251564
800	640000	512000000	28·2842712	9·2831777	·001250000
801	641601	513922401	28·3019434	9·2870444	·001248439
802	643204	515849608	28·3196045	9·2909072	·001246883
803	644809	517781627	28·3372546	9·2947671	·001245330
804	646416	519718464	28·3548938	9·2986239	·001243781
805	648025	521660125	28·3725219	9·3024775	·001242236
806	649636	523606616	28·3901391	9·3063278	·001240695
807	651249	525557943	28·4077454	9·3101750	·001239157
808	652864	527514112	28·4253408	9·3140190	·001237624
809	654481	529475129	28·4429253	9·3178599	·001236094
810	656100	531441000	28·4604989	9·3216975	·001234568
811	657721	533411731	28·4780617	9·3255320	·001233046
812	659344	535387328	28·4956137	9·3293634	·001231527
813	660969	537367797	28·5131549	9·3331916	·001230012
814	662596	539353144	28·5306852	9·3370167	·001228501
815	664225	541343375	28·5482048	9·3408386	·001226994
816	665856	543338496	28·5657137	9·3446575	·001225499
817	667489	545338513	28·5832119	9·3484731	·001223990
818	669124	547343432	28·6006993	9·3522857	·001222494
819	670761	549353259	28·6181760	9·3560952	·001221001
820	672400	551368000	28·6356421	9·3599016	·001219512
821	674041	553387661	28·6530976	9·3637049	·001218027
822	675684	555412248	28·6705424	9·3675051	·001216545
823	677329	557441767	28·6879716	9·3713022	·001215067
824	678976	559476224	28·7054002	9·3750963	·001213592
825	680625	561515625	28·7228132	9·3788873	·001212121
826	682276	563559976	28·7402157	9·3826752	·001210654
827	683929	565609283	28·7576077	9·3864600	·001209190
828	685584	567663552	28·7749891	9·3902419	·001207729
829	687241	569722789	28·7923601	9·3940206	·001206273
830	688900	571787000	28·8097206	9·3977964	·001204819
831	690561	573856191	28·8270706	9·4015691	·001203369
832	692224	575930368	28·8444102	9·4053387	·001201923

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals
833	693889	578009537	28·8617394	9·4091054	·001200480
834	695556	580093704	28·8790582	9·4128690	·001199041
835	697225	582182875	28·8963666	9·4166297	·001197605
836	698896	584277056	28·9136646	9·4203873	·001196172
837	700569	586376253	28·9309523	9·4241420	·001194743
838	702244	588480472	28·9482297	9·4278936	·001193317
839	703921	590589719	28·9654967	9·4316423	·001191895
840	705600	592704000	28·9827535	9·4353800	·001190476
841	707281	594823321	29·0000000	9·4391307	·001189061
842	708964	596947688	29·0172363	9·4428704	·001187648
843	710649	599077107	29·0344623	9·4466072	·001186240
844	712336	601211584	29·0516781	9·4503410	·001184834
845	714025	603351125	29·0688837	9·4540719	·001183432
846	715716	605495736	29·0860791	9·4577999	·001182033
847	717409	607645423	29·1032644	9·4615249	·001180638
848	719104	609800192	29·1204396	9·4652470	·001179245
849	720801	611960049	29·1376046	9·4689661	·001177856
850	722500	614125000	29·1547595	9·4726824	·001176471
851	724201	616295051	29·1719043	9·4763957	·001175088
852	725904	618470208	29·1890390	9·4801061	·001173709
853	727609	620650477	29·2061637	9·4838136	·001172333
854	729316	622835864	29·2232784	9·4875182	·001170960
855	731025	625026375	29·2403830	9·4912200	·001169591
856	732736	627222016	29·2574777	9·4949188	·001168224
857	734449	629422793	29·2745623	9·4986147	·001166861
858	736164	631628712	29·2916370	9·5023078	·001165501
859	737881	633839779	29·3087018	9·5059980	·001164144
860	739600	636056000	29·3257566	9·5096854	·001162791
861	741321	638277381	29·3428015	9·5133699	·001161440
862	743044	640503928	29·3598365	9·5170515	·001160093
863	744769	642735647	29·3768616	9·5207303	·001158749
864	746496	644972544	29·3938769	9·5244063	·001157407
865	748225	647214625	29·4108823	9·5280794	·001156069
866	749956	649461896	29·4278779	9·5317497	·001154734
867	751689	651714363	29·4448637	9·5354172	·001153403
868	753424	653972032	29·4618397	9·5390818	·001152074
869	755161	656234909	29·4788059	9·5427437	·001150748
870	756900	658503000	29·4957624	9·5464027	·001149425
871	758641	660776311	29·5127091	9·5500589	·001148106
872	760384	663054848	29·5296461	9·5537123	·001146789
873	762129	665338617	29·5465734	9·5573630	·001145475
874	763876	667627624	29·5634910	9·5610108	·001144165
875	765625	669921875	29·5803989	9·5646559	·001142857
876	767376	672221376	29·5972972	9·5682782	·001141553
877	769129	674526133	29·6141858	9·5719377	·001140251
878	770884	676836152	29·6310648	9·5755745	·001138952
879	772641	679151439	29·6479342	9·5792085	·001137656
880	774400	681472000	29·6647939	9·5828397	·001136364
881	776161	683797841	29·6816442	9·5864682	·001135074
882	777924	686128968	29·6984848	9·5900937	·001133787
883	779689	688465387	29·7153159	9·5937169	·001132503
884	781456	690807104	29·7321375	9·5973373	·001131222

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
885	783225	693154125	29·7489496	9·6009548	·001129944
886	784996	695506456	29·7657521	9·6045696	·001128668
887	786769	697864103	29·7825452	9·6081817	·001127396
888	788544	700227072	29·7993289	9·6117911	·001126126
889	790321	702595369	29·8161030	9·6153977	·001124859
890	792100	704969000	29·8328678	9·6190017	·001123596
891	793881	707347971	29·8496231	9·6226030	·001122334
892	795664	707932288	29·8663690	9·6262016	·001121076
893	797449	712121957	29·8831056	9·6297975	·001119821
894	799236	714516984	29·8998328	9·6333907	·001118568
895	801025	716917375	29·9165506	9·6369812	·001117818
896	802816	719323136	29·9332591	9·6405690	·001116071
897	804609	721734273	29·9499583	9·6441542	·001114827
898	806404	724150792	29·9666481	9·6477367	·001113586
899	808201	726572699	29·9833287	9·6513166	·001112347
900	810000	729000000	30·0000000	9·6548938	·001111111
901	811801	731432701	30·0166621	9·6584684	·001109878
902	813604	733870808	30·0333148	9·6620403	·001108647
903	815409	736314327	30·0499584	9·6656096	·001107420
904	817216	738763264	30·0665928	9·6691762	·001106195
905	819025	741217625	30·0832179	9·6727403	·001104972
906	820836	743677416	30·0998339	9·6763017	·001103753
907	822649	746142643	30·1164407	9·6798604	·001102536
908	824464	748613312	20·1330383	9·6834166	·001101322
909	826281	751089429	30·1496269	9·6869701	·001100110
910	828100	753571000	30·1662063	9·6905211	·001098901
911	829921	756058031	30·1827765	9·6940694	·001097695
912	831744	758550828	30·1993377	9·6976151	·001096491
913	833569	761048497	30·2158899	9·7011583	·001095290
914	835396	763551944	30·2324329	9·7046989	·001094092
915	837225	766060875	30·2489669	9·7082369	·001092896
916	839056	768575296	30·2654919	9·7117723	·001091703
917	840889	771095213	30·2820079	9·7153051	·001090513
918	842724	773620632	30·2985148	9·7188354	·001089325
919	844561	776151559	30·3150128	9·7223631	·001088139
920	846400	778688000	30·3315018	9·7258883	·001086957
921	848241	781229961	30·3479818	9.7294109	·001085776
922	850084	783777448	30·3644529	9·7329309	·001084599
923	851929	786330467	30·3809151	9·7364484	·001083423
924	853776	788889024	30·3973683	9·7399634	·001082251
925	855625	791453125	30·4138127	9·7434758	·001081081
926	857476	794022776	30·4302481	9·7469857	·001079914
927	859329	796597983	30·4466747	9·7504930	·001078749
928	861184	799178752	30·4630924	9·7539979	·001077586
929	863041	801765089	30·4795013	9·7575002	·001076426
930	864900	804357000	30·4959014	9·7610001	·001075269
931	866761	806954491	30·5122926	9·7644974	·001074114
932	868624	809557568	30·5286750	9·7679922	·001072961
933	870489	812166237	30·5450487	9·7714845	·001071811
934	872356	814780504	30·5614136	9·7749743	·001070664
935	874225	817400375	30·5777697	9·7784616	·001069519
936	876096	820025856	30·5941171	9·7819466	·001068376

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
937	877969	822656953	30·6104557	9·7854288	·001067236
938	879844	825293672	30·6267857	9·7889087	·001066098
939	881721	827936019	30·6431069	9·7923861	·001064963
940	883600	830584000	30·6594194	9·7958611	·001063830
941	885481	833237621	30·6757233	9·7993336	·001062699
942	887364	835896888	30·6920185	9·8028036	·001061571
943	889249	838561807	30·7083051	9·8062711	·001060445
944	891136	841232384	30·7245830	9·8097362	·001059322
945	893025	843908625	30·7408523	9·8131989	·001058201
946	894916	846590536	30·7571130	9·8166591	·001057082
947	896809	849278123	30·7733651	9·8201169	·001055966
948	898704	851971392	30·7896086	9·8235723	·001054852
949	900601	854670349	30·8058436	9·8270252	·001053741
950	902500	857375000	30·8220700	9·8304757	·001052632
951	904401	860085351	30·8382879	9·8339238	·001051525
952	906304	862801408	30·8544972	9·8373695	·001050420
953	908209	865523177	30·8706981	9·8408127	·001049318
954	910116	868250664	30·8868904	9·8442536	·001048218
955	912025	870983875	30·9030743	9·8476920	·001047120
956	913936	873722816	30·9192477	9·8511280	·001046025
957	915849	876467493	30·9354166	9·8545617	·001044932
958	917764	879217912	30·9515751	9·8579929	·001043841
959	919681	881974079	30·9677251	9·8614218	·001042753
960	921600	884736000	30·9838668	9·8648483	·001041667
961	923521	887503681	31·0000000	9·8682724	·001040583
962	925444	890277128	31·0161248	9·8716941	·001039501
963	927369	893056347	31·0322413	9·8751135	·001038422
964	929296	895841344	31·0483494	9·8785305	·001037344
965	931225	898632125	31·0644491	9·8819451	·001036269
966	933156	901428696	31·0805405	9·8853574	·001035197
967	935089	904231063	31·0966236	9·8887673	·001034126
968	937024	907039232	31·1126984	9·8921749	·001033058
969	938961	909853209	31·1287648	9·8955801	·001031992
970	940900	912673000	31·1448230	9·8989830	·001030928
971	942841	915498611	31·1608729	9·9023835	·001029866
972	944784	918330048	31·1769145	9·9057817	·001028807
973	946729	921167317	31·1929479	9·9091776	·001027749
974	948676	924010424	31·2089731	9·9125712	·001026694
975	950625	926859375	31·2249900	9·9159624	·001025641
976	952576	929714176	31·2409987	9·9193513	·001024590
977	954529	932574833	31·2569992	9·9227379	·001023541
978	956484	935441352	31·2729915	9·9261222	·001022495
979	958441	938313739	31·2889757	9·9295042	·001021450
980	960400	941192000	31·3049517	9·9328839	·001020408
981	962361	944076141	31·3209195	9·9362613	·001019168
982	964324	946966168	31·3368792	9·9396363	·001018330
983	966289	949862087	31·3528308	9·9430092	·001017294
984	968256	952763904	31·3687743	9·9463797	·001016260
985	970225	955671625	31·3847097	9·9497479	·001015228
986	972196	958585256	31·4006369	9·9531138	·001014199
987	974169	961504803	31·4165561	9·9564775	·001013171
988	976144	964430272	31·4324673	9·9598389	·001012146

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
989	978121	967361669	31·4483704	9·9631981	·001011122
990	980100	970299000	31·4642654	9·9665549	·001010101
991	982081	973242271	31·4801525	9·9699055	·001009082
992	984064	976191488	31·4960315	9·9732619	·001008065
993	986049	979146657	31·5119025	9·9766120	·001007049
994	988036	982107784	31·5277655	9·9799599	·001006036
995	990025	985074875	31·5436206	9·9833055	·001005025
996	992016	988047936	31·5594677	9·9866488	·001004016
997	994009	991026973	31·5753068	9·9899900	·001003009
998	996004	994011992	31·5911380	9·9933289	·001002004
999	998001	997002999	31·6069613	9·9966656	·001001001
1000	1000000	1000000000	31·6227766	10·0000000	·001000000
1001	1002001	1003003001	31·6385840	10·0033222	·0009990010
1002	1004004	1006012008	31·6543866	10·0066622	·0009980040
1003	1006009	1009027027	31·6701752	10·0099899	·0009970090
1004	1008016	1012048064	31·6859590	10 0133155	·0009960159
1005	1010025	1015075125	31·7017349	10·0166389	·0009950249
1006	1012036	1018108216	31·7175030	10·0199601	·0009940358
1007	1014049	1021147343	31·7332633	10·0232791	·0009930487
1008	1016064	1024192512	31·7490157	10·0265958	·0009920635
1009	1018081	1027243729	31·7647603	10·0299104	·0009910803
1010	1020100	1030301000	31·7804972	10·0332228	·0009900990
1011	1022121	1033364331	31·7962262	10·0365330	·0009891197
1012	1024144	1036433728	31·8119474	10·0398410	·0009881423
1013	1026169	1039509197	31·8276609	10·0431469	·0009871668
1014	1028196	1042590744	31·8433666	10·0464506	·0009861933
1015	1030225	1045678375	31·8590646	10·0497521	·0009852217
1016	1032256	1048772096	31·8747549	10·0530514	·0009842520
1017	1034289	1051871913	31·8904374	10·0563485	·0009832842
1018	1036324	1054977832	31·9061123	10·0596435	·0009823183
1019	1038361	1058089859	31·9217794	10·0629364	·0009813543
1020	1040400	1061208000	31·9374388	10·0662271	·0009803922
1021	1042441	1064332261	31·9530906	10·0695156	·0009794319
1022	1044484	1067462648	31·9687347	10·0728020	·0009784736
1023	1046529	1070599167	31·9843712	10·0760863	·0009775171
1024	1048576	1073741824	32·0000000	10·0793684	·0009765625
1025	1050625	1076890625	32·0156212	10·0826484	·0009756098
1026	1052676	1080045576	32·0312348	10·0859262	·0009746589
1027	1054729	1083206683	32·0468407	10·0892019	·0009737098
1028	1056784	1086373952	32·0624391	10·0924755	·0009727626
1029	1058841	1089547389	32·0780298	10·0957469	·0009718173
1030	1060900	1092727000	32·0936131	10·0990163	·0009708738
1031	1062961	1095912791	32·1091887	10·1022835	·0009699321
1032	1065024	1099104768	32·1247568	10·1055487	·0009689922
1033	1067089	1102302937	32·1403173	10·1088117	·0009680542
1034	1069156	1105507304	32·1558704	10·1120726	·0009671180
1035	1071225	1108717875	32·1714159	10·1153314	·0009661836
1036	1073296	1111934656	32·1869539	10·1185882	·0009652510
1037	1075369	1115157653	32·2024844	10·1218428	·0009643202
1038	1077444	1118386872	32·2180074	10·1250953	·0009633911
1039	1079521	1121622319	32·2335229	10·1283457	·0009624639
1040	1081600	1124864000	32·2490310	10·1315941	·0009615385

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1041	1083681	1128111921	32·2645316	10·1348403	·0009606148
1042	1085764	1131366088	32·2800248	10·1380845	·0009596929
1043	1087849	1134626507	32·2955105	10·1413266	·0009587728
1044	1089936	1137893184	32·3109888	10·1445667	·0009578544
1045	1092025	1141166125	32·3264598	10·1478047	·0009569378
1046	1094116	1144445336	32·3419233	10·1510406	·0009560229
1047	1096209	1147730823	32·3573794	10·1542744	·0009551098
1048	1098304	1151022592	32·3728281	10·1575062	·0009541985
1049	1100401	1154320649	32·3882695	10·1607359	·0009532888
1050	1102500	1157625000	32·4037035	10·1639636	·0009523810
1051	1104601	1160935651	32·4191301	10·1671893	·0009514748
1052	1106704	1164252608	32·4345495	10·1704129	·0009505703
1053	1108809	1167575877	32·4499615	10·1736344	·0009496676
1054	1110916	1170905464	32·4653662	10·1768539	·0009487666
1055	1113025	1174241375	32·4807635	10·1800714	·0009478673
1056	1115136	1177583616	32·4961536	10·1832868	·0009469697
1057	1117249	1180932193	32·5115364	10·1865002	·0009460738
1058	1119364	1184287112	32·5269119	10·1897116	·0009451796
1059	1121481	1187648379	32·5422802	10·1929209	·0009442871
1060	1123600	1191016000	32·5576412	10·1961283	·0009433962
1061	1125721	1194389981	32·5729949	10·1993336	·0009425071
1062	1127844	1197770328	32·5883415	10·2025369	·0009416196
1063	1129969	1201157047	32·6035807	10·2057382	·0009407338
1064	1132096	1204550144	32·6190129	10·2089375	·0009398496
1065	1134225	1207949625	32·6343377	10·2121347	·0009389671
1066	1136356	1211355496	32·6496554	10·2153300	·0009380863
1067	1138489	1214767763	32·6649659	10·2185233	·0009372071
1068	1140624	1218186432	32·6802693	10·2217146	·0009363296
1069	1142761	1221611509	32·6955654	10·2249039	·0009354537
1070	1144900	1225043000	32·7108544	10·2280912	·0009345794
1071	1147041	1228480911	32·7261363	10·2312766	·0009337068
1072	1149184	1231925248	32·7414111	10·2344599	·0009328358
1073	1151329	1235376017	32·7566787	10·2376413	·0009319664
1074	1153476	1238833224	32·7719392	10·2408207	·0009310987
1075	1155625	1242296875	32·7871926	10·2439981	·0009302326
1076	1157776	1245766976	32·8024398	10·2471735	·0009293680
1077	1159929	1249243533	32·8176782	10·2503470	·0009285051
1078	1162084	1252726552	32·8329103	10·2535186	·0009276438
1079	1164241	1256216039	32·8481354	10·2566881	·0009267841
1080	1166400	1259712000	32·8633535	10·2598557	·0009259259
1081	1168561	1263214441	32·8785644	10·2630213	·0009250694
1082	1170724	1266723368	32·8937684	10·2661850	·0009242144
1083	1172889	1270238787	32·9089653	10·2693467	·0009233610
1084	1175056	1273760704	32·9241553	10·2725065	·0009225092
1085	1177225	1277289125	32·9393382	10·2756644	·0009216590
1086	1179396	1280824056	32·9545141	10·2788203	·0009208103
1087	1181569	1284365503	32·9696830	10·2819743	·0009199632
1088	1183744	1287913472	32·9848450	10·2851264	·0009191176
1089	1185921	1291467969	33·0000000	10·2882765	·0009182736
1090	1188100	1295029000	33·0151480	10·2914247	·0009174312
1091	1190281	1298596571	33·0302891	10·2945709	·0009165903
1092	1192464	1302170688	33·0454233	10·2977153	·0009157509

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
1093	1194649	1305751357	33·0605505	10·3008577	·0009149131
1094	1196836	1309338584	33·0756708	10·3039982	·0009140768
1095	1199025	1312932375	33·0907842	10·3071368	·0009132420
1096	1201216	1316532736	33·1058907	10·3102735	·0009124008
1097	1203409	1320139673	33·1209903	10·3134083	·0009115770
1098	1205604	1323753192	33·1360830	10·3165411	·0009107468
1099	1207801	1327373299	33·1511689	10·3196721	·0009099181
1100	1210000	1331000000	33·1662479	10·3228012	·0009090909
1101	1212201	1334633301	33·1813200	10·3259284	·0009082652
1102	1214404	1338273208	33·1963853	10·3290537	·0009074410
1103	1216609	1341919727	33·2114438	10·3321770	·0009066183
1104	1218816	1345572864	33·2266955	10·3352985	·0009057971
1105	1221025	1349232625	33·2415403	10·3384181	·0009049774
1106	1223236	1352899016	33·2565783	10·3415358	·0009041591
1107	1225449	1356572043	33·2716095	10·3446517	·0009033424
1108	1227664	1360251712	33·2866339	10·3477657	·0009025271
1109	1229881	1363938029	33·3016516	10·3508778	·0009017133
1110	1232100	1367631000	33·3166625	10·3539880	·0009009009
1111	1234321	1371330631	33·3316666	10·3570964	·0009000900
1112	1236544	1375036928	33·3466640	10·3602029	·0008992806
1113	1238769	1378749897	33·3616546	10·3633076	·0008984726
1114	1240996	1382469544	33·3766385	10·3664103	·0008976661
1115	1243225	1386195875	33·3916157	10·3695113	·0008968610
1116	1245456	1389928896	33·4065862	10·3726103	·0008960753
1117	1247689	1393668613	33·4215499	10·3757076	·0008952551
1118	1249924	1397415032	33·4365070	10·3788030	·0008944544
1119	1252161	1401168159	33·4514573	10·3818965	·0008936550
1120	1254400	1404928000	33·4664011	10·3849882	·0008928571
1121	1256641	1408694561	33·4813381	10·3880781	·0008960607
1122	1258884	1412467848	33·4962684	10·3911661	·0008922656
1123	1261129	1416247867	33·5111921	10·3942527	·0008904720
1124	1263376	1420034624	33·5261092	10·3973366	·0008896797
1125	1265625	1423828125	33·5410196	10·4004192	·0008888889
1126	1267876	1427628376	33·5559234	10·4034999	·0008880995
1127	1270129	1431435383	33·5708206	10·4065787	·0008873114
1128	1272384	1435249152	33·5857112	10·4096557	·0008865248
1129	1274641	1439069689	33·6005952	10·4127310	·0008857396
1130	1276900	1442897000	33·6154726	10·4158044	·0008849558
1131	1279161	1446731091	33·6303434	10·4188760	·0008841733
1132	1281424	1450571968	33·6452077	10·4219458	·0008833922
1133	1283689	1454419637	33·6600653	10·4250138	·0008826125
1134	1285956	1458274104	33·6749165	10·4280800	·0008818342
1135	1288225	1462135375	33·6897610	10·4311443	·0008810573
1136	1290496	1466003456	83·7045991	10·4342069	·0008802817
1137	1292769	1469878353	33·7174306	10·4372677	·0008795075
1138	1295044	1473760072	33·7340556	10·4403677	·0008787346
1139	1297321	1477648619	33·7490741	10·4433839	·0008779631
1140	1299600	1481544000	33·7638860	10·4464393	·0008771930
1141	1301881	1485446221	33·7786915	10·4494929	·0008764242
1142	1304164	1489355288	33·7934905	10·4525448	·0008756567
1143	1306449	1493271207	33·8082830	10·4555948	·0008748906
1144	1308736	1497193984	33·8230691	10·4586431	·0008741259

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1145	1311025	1501123625	33·8378486	10·4616896	·0008733624
1146	1313316	1505060136	33·8526218	10·4647343	·0008726003
1147	1315609	1509003523	33·8673884	10·4677773	·0008718396
1148	1317904	1512953792	33·8821487	10·4708158	·0008710801
1149	1320201	1516910949	33·8969025	10·4738579	·0008703220
1150	1322500	1520875000	33·9116499	10·4768955	·0008695652
1151	1324801	1524845951	33·9263909	10·4799314	·0008688097
1152	1327104	1528823808	33·9411255	10·4829656	·0008680556
1153	1329409	1532808577	33·9558537	10·4859980	·0008673027
1154	1331716	1536800264	33·9705755	10·4890286	·0008665511
1155	1334025	1540798875	33·9852910	10·4920575	·0008658009
1156	1336336	1544804416	34·0000000	10·4950847	·0008650519
1157	1338649	1548816893	34·0147027	10·4981101	·0008643042
1158	1340964	1552836312	34·0293990	10·5011337	·0008635579
1159	1343281	1556862679	34·0440890	10·5041556	·0008628128
1160	1345600	1560896000	34·0587727	10·5071757	·0008620690
1161	1347921	1564936281	34·0734501	10·5101942	·0008613264
1162	1350244	1568983528	34·0881211	10·5132109	·0008605852
1163	1352569	1573037747	34·0127858	10·5162259	·0008598452
1164	1354896	1577098944	34·1174442	10·5192391	·0008591065
1165	1357225	1581167125	34·1320963	10·5222506	·0008583691
1166	1359556	1585242296	34·1467422	10·5252604	·0008576329
1167	1361889	1589324463	34·1613817	10·5282685	·0008568980
1168	1364224	1593413632	34·1760150	10·5312749	·0008561644
1169	1366561	1597509809	34·1906420	10·5342795	·0008554320
1170	1368900	1601613000	34·2052627	10·5372825	·0008547009
1171	1371241	1605723211	34·2198773	10·5402837	·0008539710
1172	1373584	1609840448	34·2344855	10·5432832	·0008532423
1173	1375929	1613964717	34·2490875	10·5462810	·0008525149
1174	1378276	1618096024	34·2636834	10·5492771	·0008517888
1175	1380625	1622234375	34·2782730	10·5522715	·0008510638
1176	1382976	1626379776	34·2928564	10·5552642	·0008503401
1177	1385329	1630532233	34·3074336	10·5582552	·0008496177
1178	1387684	1634691752	34·3220046	10·5612445	·0008488964
1179	1390041	1638858339	34·3365694	10·5642322	·0008481764
1180	1392400	1643032000	34·3511281	10·5672181	·0008471576
1181	1394761	1647212741	34·3656805	10·5702024	·0008467401
1182	1397124	1651400568	34·3802268	10·5731849	·0008460237
1183	1399489	1655595487	34·3947670	10·5761658	·0008453085
1184	1401856	1659797504	34·4093011	10·5791449	·0008445946
1185	1404225	1664006625	34·4238289	10·5821225	·0008438819
1186	1406596	1668222856	34·4383507	10·5850983	·0008431703
1187	1408969	1672446203	34·4528663	10·5880725	·0008424600
1188	1411344	1676676672	34·4673759	10·5910450	·0008417508
1189	1413721	1680914629	34·4818793	10·5940158	·0008410429
1190	1416100	1685159000	34·4963766	10·5969850	·0008403361
1191	1418481	1689410871	34·5108678	10·5999525	·0008396306
1192	1420864	1693669888	34·5253530	10·6029184	·0008389262
1193	1423249	1697936057	34·5398321	10·6058826	·0008382320
1194	1425636	1702209384	34·5543051	10·6088451	·0008375209
1195	1428025	1706489875	34·5687720	10·6118060	·0008368201
1196	1430416	1710777536	34·5832329	10·6147652	·0008361204

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1197	1432809	1715072373	34·5976879	10·6177228	·0008354219
1198	1435204	1719374392	34·6121366	10·6206788	·0008347245
1199	1437601	1723683599	34·6265794	10·6236331	·0008340284
1200	1440000	1728000000	34·6410162	10·6265857	·0008333333
1201	1442401	1732323601	34·6554469	10·6295367	·0008326395
1202	1444804	1736654408	34·6698716	10·6324860	·0008319468
1203	1447209	1740992427	34·6842904	10·6354338	·0008312552
1204	1449616	1745337664	34·6987031	10·6383799	·0008305648
1205	1452025	1749690125	34·7131099	10·6413244	·0008298755
1206	1454436	1754049816	34·7275107	10·6442672	·0008291874
1207	1456849	1758416743	34·7419055	10·6472085	·0008285004
1208	1459264	1762790912	34·7562944	10·6501480	·0008278146
1209	1461681	1767172329	34·7706773	10·6530860	·0008271299
1210	1464100	1771561000	34·7850543	10·6560223	·0008264463
1211	1466521	1775956931	34·7994253	10·6589570	·0008257638
1212	1468944	1780360128	34·8137904	10·6618902	·0008250825
1213	1471369	1784770597	34·8281495	10·6648217	·0008244023
1214	1473796	1789188344	34·8425028	10·6677516	·0008237232
1215	1476225	1793613375	34·8568501	10·6706799	·0008230453
1216	1478656	1798045696	34·8711915	10·6736066	·0008223684
1217	1481089	1802485313	34·8855271	10·6765317	·0008216927
1218	1483524	1806932232	34·8998567	10·6794552	·0008210181
1219	1485961	1811386459	34·9141805	10·6823771	·0008203445
1220	1488400	1815848000	34·9284984	10·6852973	·0008196721
1221	1490841	1820316861	34·9428104	10·6882160	·0008190008
1222	1493284	1824793048	34·9571166	10·6911331	·0008183306
1223	1495729	1829276567	34·9714169	10·6940486	·0008176615
1224	1498176	1833764247	34·9857114	10·6969625	·0008169935
1225	1500625	1838265625	35·0000000	10·6998748	·0008163265
1226	1503276	1842771176	35·0142828	10·7027855	·0008156607
1227	1505529	1847284083	35·0285598	10·7056947	·0008149959
1228	1507984	1851804352	35·0428309	10·7086023	·0008143322
1229	1510441	1856331989	35·0570963	10·7115083	·0008136696
1230	1512900	1860867000	35·0713558	10·7144127	·0008130081
1231	1515361	1865409391	35·0856096	10·7173155	·0008123477
1232	1517824	1869959168	35·0998575	10·7202168	·0008116883
1233	1520289	1874516337	35·1140997	10·7231165	·0008110300
1234	1522756	1879080904	35·1283361	10·7260146	·0008103728
1235	1525225	1883652875	35·1425668	10·7289112	·0008097166
1236	1527696	1888232256	35·1567917	10·7318062	·0008090615
1237	1530169	1892819953	35·1710108	10·7346997	·0008084074
1238	1532644	1897413272	35·1852242	10·7375916	·0008077544
1239	1535121	1902014919	35·1994318	10·7404819	·0008071025
1240	1537600	1906624000	35·2136337	10·7433707	·0008064516
1241	1540081	1911240521	35·2278299	10·7462579	·0008058018
1242	1542564	1915864488	35·2420204	10·7491436	·0008051530
1243	1545049	1920495907	35·2562051	10·7520277	·0008045052
1244	1547536	1925134784	35·2703842	10·7549103	·0008038585
1245	1550025	1929781125	35·2845575	10·7577913	·0008032129
1246	1552516	1934434936	35·2987252	10·7606708	·0008025682
1247	1555009	1939096223	35·3128872	10·7635488	·0008019246
1248	1557504	1943764992	35·3270435	10·7664252	·0008012821

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1249	1560001	1948441249	35·3411941	10·7693001	·0008006405
1250	1562500	1953125000	35·3553391	10·7721735	·0008000000
1251	1565001	1957816251	35·3694784	10·7750453	·0007993605
1252	1567504	1962515008	35·3836120	10·7779156	·0007987220
1253	1570009	1967221277	35·3977400	10·7807843	·0007980846
1254	1572516	1971935064	35·4118624	10·7836516	·0007974482
1255	1575025	1976656375	35·4259792	10·7865173	·0007968127
1256	1577536	1981385216	35·4400903	10·7893815	·0007961783
1257	1580049	1986121593	35·4541958	10·7922441	·0007955449
1258	1582564	1990865512	35·4682957	10·7951053	·0007949126
1259	1585081	1995616979	35·4823900	10·7979649	0007942812
1260	1587600	2000376000	35·4964787	10·8008230	·0007936508
1261	1590121	2005142581	35·5105618	10·8036797	·0007930214
1262	1592644	2009916728	35·5246393	10·8065348	·0007923930
1263	1595169	2014698447	35·5387113	10·8093884	·0007917656
1264	1597696	2019487744	35·5527777	10·8122404	·0007911392
1265	1600225	2024284625	35·5668385	10·8150909	·0007905138
1266	1602756	2029089096	35·5808937	10·8179400	·0007898894
1267	1605289	2033901163	35·5949434	10·8207876	·0007892660
1268	1607824	2038720832	35·6089876	10·8236336	·0007886435
1269	1610361	2043548109	35·6230262	10·8264782	·0007880221
1270	1612900	2048383000	35·6370593	10·8293213	·0007874016
1271	1615441	2053225511	35·6510869	10·8321629	·0007867821
1272	1617984	2058075648	35·6651090	10·8350030	·0007861635
1273	1620529	2062933417	35·6791255	10·8378416	·0007855460
1274	1623076	2067798824	35·6931366	10·8406788	·0007849294
1275	1625625	2072671875	35·7071421	10·8435144	·0007843137
1276	1628176	2077552576	35·7211422	10·8463485	·0007836991
1277	1630729	2082440933	35·7351367	10·8491812	·0007830854
1278	1633284	2087336952	35·7491258	10·8520125	·0007824726
1279	1635841	2092240639	35·7631095	10·8548422	·0007818608
1280	1638400	2097152000	35·7770876	10·8576704	·0007812500
1281	1640961	2102071841	35·7910603	10·8604972	·0007806401
1282	1643524	2106997768	35·8050276	10·8633225	·0007800312
1283	1646089	2111932187	35·8189894	10·8661454	·0007794232
1284	1648656	2116874304	35·8329457	10·8689687	·0007788162
1285	1651225	2121824125	35·8468966	10·8717897	·0007782101
1286	1653796	2126781656	35·8608421	10·8746091	·0007776050
1287	1656369	2131746903	35·8747822	10·8774271	·0007770008
1288	1658944	2136719872	35·8887169	10·8802436	·0007763975
1289	1661521	2141700569	35·9026461	10·8830587	·0007757952
1290	1664100	2146689000	35·9165699	10·8858723	·0007751938
1291	1666681	2151685171	35·9304884	10·8886845	·0007745933
1292	1669264	2156689088	35·9444015	10·8914952	·0007739938
1293	1671849	2161700757	35·9583092	10·8943044	·0007733952
1294	1674436	2166720184	35·9722115	10·8971123	·0007727975
1295	1677025	2171747375	35·9861084	10·8999186	·0007722008
1296	1679616	2176782336	36·0000000	10·9027235	·0007716049
1297	1682209	2181825073	36·0138862	10·9055269	·0007710100
1298	1684804	2186875592	36·0277671	10·9083290	·0007704160
1299	1687401	2191933899	36·0416426	10·9111296	·0007698229
1300	1690000	2197000000	36·0555128	10·9139287	·0007692308

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
1301	1692601	2202073901	36·0693776	10·9167265	·0007686395
1302	1695204	2207155608	36·0832371	10·9195228	·0007680492
1303	1697809	2212245127	36·0970913	10·9223177	·0007674579
1304	1700416	2217342464	36·1109402	10·9251111	·0007668712
1305	1703025	2222447625	36·1247837	10·9279031	·0007662835
1306	1705636	2227560616	36·1386220	10·9306937	·0007656968
1307	1708249	2232681443	36·1524550	10·9334829	·0007651109
1308	1710864	2237810112	36·1662826	10·9362706	·0007645260
1309	1713481	2242946629	36·1801050	10·9390569	·0007639419
1310	1716100	2248091000	36·1939221	10·9418418	·0007633588
1311	1718721	2253243231	36·2077340	10·9446253	·0007627765
1312	1721344	2258403328	36·2215406	10·9475074	·0007621951
1313	1723969	2263571297	36·2353419	10·9501880	·0007616446
1314	1726596	2268747144	36·2491379	10·9529673	·0007610350
1315	1729225	2273930875	36·2626287	10·9557451	·0007604563
1316	1731856	2279122496	36·2767143	10·9585215	·0007598784
1317	1734489	2284322013	36·2904246	10·9612965	·0007593014
1318	1737124	2289529432	36·3042697	10·9640701	·0007587253
1319	1739761	2294744759	36·3180396	10·9668423	·0007581501
1320	1742400	2299968000	36·3318042	10·9696131	·0007575758
1321	1745041	2305199161	36·3455637	10·9723825	·0007570023
1322	1747684	2310438248	36·3593179	10·9751505	·0007564297
1323	1750329	2315685267	36·3730670	10·9779171	·0007558579
1324	1752976	2320940224	36·3868108	10·9806823	·0007552870
1325	1755625	2326203125	36·4005494	10·9834462	·0007547170
1326	1758276	2331473976	36·4142829	10·9862086	·0007541478
1327	1760929	2336752783	36·4280112	10·9889696	·0007535795
1328	1763584	2342039552	36·4417343	10·9917293	·0007530120
1329	1766241	2347334289	36·4554523	10·9944876	·0007524454
1330	1768900	2352637000	36·4691650	10·9972445	·0007518797
1331	1771561	2357947691	36·4828727	11·0000000	·0007513148
1332	1774224	2363266368	36·4965752	11·0027541	·0007507508
1333	1776889	2368593037	36·5102725	11·0055069	·0007501875
1334	1779556	2373927704	36·5239647	11·0082583	·0007496252
1335	1782225	2379270375	36·5376518	11·0110082	·0007490637
1336	1784896	2384621056	36·5513388	11·0137569	·0007485030
1337	1787569	2389979753	36·5650106	11·0165041	·0007479432
1338	1790244	2395346472	36·5786823	11·0192500	·0007473842
1339	1792921	2400721219	36·5923489	11·0219945	·0007468260
1340	1795600	2406104000	36·6060104	11·0247377	·0007462687
1341	1798281	2411494821	36·6196668	11·0274795	·0007457122
1342	1800964	2416893688	36·6333181	11·0302199	·0007451565
1343	1803649	2422300607	36·6469144	11·0329590	·0007446016
1344	1806336	2427715584	36·6606056	11·0356967	·0007440476
1345	1809025	2433138625	36·6742416	11·0384330	·0007434944
1346	1811716	2438569736	36·6878726	11·0411680	·0007429421
1347	1814409	2444008923	36·7014986	11·0439017	·0007423905
1348	1817104	2449456192	36·7151195	11·0466339	·0007418398
1349	1819801	2454911549	36·7287353	11·0493649	·0007412898
1350	1822500	2460375000	36·7423461	11·0520945	·0007407407
1351	1825201	2465846551	36·7559519	11·0548227	·0007401924
1352	1827904	2471326208	36·7695526	11·0575497	·0007396450

Number	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1353	1830609	2476813977	36·7831483	11·0602752	·0007390983
1354	1833316	2482309864	36·7967390	11·0629994	·0007385524
1355	1836025	2487813875	36·8103246	11·0657222	·0007380074
1356	1838736	2493326016	36·8239053	11·0684437	·0007374631
1357	1841449	2498846293	36·8374809	11·0711639	·0007369197
1358	1844164	2504374712	36·8510515	11·0738828	·0007363770
1359	1846881	2509911279	36·8646172	11·0766003	·0007358352
1360	1849600	2515456000	36·8781778	11·0793165	·0007352941
1361	1852321	2521008881	36·8917335	11·0820314	·0007347539
1362	1855044	2526569928	36·9052842	11·0847449	·0007342144
1363	1857769	2532139147	36·9188299	11·0874571	·0007336757
1364	1860496	2537716544	36·9323706	11·0901679	·0007331378
1365	1863225	2543302125	36·9459064	11·0928775	·0007326007
1366	1865956	2548895896	36·9594372	11·0955857	·0007320644
1367	1868689	2554497863	36·9729631	11·0982926	·0007315289
1368	1871424	2560108032	36·9864840	11·1009982	·0007309942
1369	1874161	2565726409	37·0000000	11·1037025	·0007304602
1370	1876900	2571353000	37·0135110	11·1064054	·0007299270
1371	1879641	2576987811	37·0270172	11·1091070	·0007293946
1372	1882384	2582630848	37·0405184	11·1118073	·0007288630
1373	1885129	2588282117	37·0540146	11·1145064	·0007283321
1374	1887876	2593941624	37·0675060	11·1172041	·0007278020
1375	1890625	2599609375	37·0809924	11·1199004	·0007272727
1376	1893376	2605285376	37·0944740	11·1225955	·0007267442
1377	1896129	2610969633	37·1079506	11·1252893	·0007262164
1378	1898884	2616662152	37·1214224	11·1279817	·0007256894
1379	1901641	2622362939	37·1348893	11·1306729	·0007251632
1380	1904400	2628072000	37·1483512	11·1333628	·0007246377
1381	1907161	2633789341	37·1618084	11·1360514	·0007241130
1382	1909924	2639514968	37·1752606	11·1387386	·0007235890
1383	1912689	2645248887	37·1887079	11·1414246	·0007230658
1384	1915456	2650991104	37·2021505	11·1441093	·0007225434
1385	1918225	2656741625	37·2155881	11·1467926	·0007220217
1386	1920996	2662500456	37·2290209	11·1494747	·0007215007
1387	1923769	2668267603	37·2424489	11·1521555	·0007209805
1388	1926544	2674043072	37·2558720	11·1548350	·0007204611
1389	1929321	2679826869	37·2692903	11·1575133	·0007199424
1390	1932100	2685619000	37·2827037	11·1601903	·0007194245
1391	1934881	2691419471	37·2961124	11·1628659	·0007189073
1392	1937664	2697228288	37·3095162	11·1655403	·0007183908
1393	1940449	2703045457	37·3229152	11·1682134	·0007178751
1394	1943236	2708870984	37·3363094	11·1708852	·0007173601
1395	1946025	2714704875	37·3496988	11·1735558	·0007168459
1396	1948816	2720547136	37·3630834	11·1762250	·0007163324
1397	1951609	2726397773	37·3764632	11·1788930	·0007158196
1398	1954404	2732256792	37·3898382	11·1815598	·0007153076
1399	1957201	2738124199	37·4032084	11·1842252	·0007147963
1400	1960000	2744000000	37·4165738	11·1868894	·0007142857
1401	1962801	2749884201	37·4299345	11·1895523	·0007137759
1402	1965604	2755776808	37·4432904	11·1922139	·0007132668
1403	1968409	2761677827	37·4566416	11·1948743	·0007127584
1404	1971216	2767587264	37·4699880	11·1975334	·0007122507

Number	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1405	1974025	2773505123	37·4833296	11·2001913	·0007117438
1406	1976836	2779431416	37·4966665	11·2028479	·0007112376
1407	1979649	2785366143	37·5099987	11·2055032	·0007107321
1408	1982464	2791309312	37·5233261	11·2081573	·0007102273
1409	1985281	2797260929	37·5366487	11·2108101	·0007097232
1410	1988100	2803221000	37·5499667	11·2134617	·0007092199
1411	1990921	2809189531	37·5632799	11·2161120	·0007087172
1412	1993744	2815166528	37·5765885	11·2187611	·0007082153
1413	1996569	2821151997	37·5898922	11·2214089	·0007077141
1414	1999396	2827145944	37·6031913	11·2240054	·0007072136
1415	2002225	2833148375	37·6164857	11·2267007	·0007067138
1416	2005056	2839159296	37·6297754	11·2293448	·0007062147
1417	2007889	2845178713	37·6430604	11·2319876	·0007057163
1418	2010724	2851206632	37·6563407	11·2346292	·0007052186
1419	2013561	2857243059	37·6696164	11·2372696	·0007047216
1420	2016400	2863288000	37·6828874	11·2399087	·0007042254
1421	2019241	2869341461	37·6961536	11·2425465	·0007037298
1422	2022084	2875403448	37·7094153	11·2451831	·0007032349
1423	2024929	2881473967	37·7226722	11·2478185	·0007027407
1424	2027776	2887553024	37·7359245	11·2504527	·0007022472
1425	2030625	2893640625	37·7491722	11·2530856	·0007017544
1426	2033476	2899736776	37·7624152	11·2557173	·0007012623
1427	2036329	2905841483	37·7756535	11·2583478	·0007007708
1428	2039184	2911954752	37·7888873	11·2609770	·0007002801
1429	2042041	2918076589	37·8021163	11·2636050	·0006997901
1430	2044900	2924207000	37·8153408	11·2662318	·0006993007
1431	2047761	2930345991	37·8285606	11·2688573	·0006988120
1432	2050624	2936493568	37·8417759	11·2714816	·0006983240
1433	2053489	2942649737	37·8549864	11·2741047	·0006978367
1434	2056356	2948814504	37·8681924	11·2767266	·0006973501
1435	2059225	2954987875	37·8813938	11·2793472	·0006968641
1436	2062096	2961169856	37·8945906	11·2819666	·0006963788
1437	2064969	2967360453	37·9077828	11·2845849	·0006958942
1438	2067844	2973559672	37·9209704	11·2872019	·0006954103
1439	2070721	2979767519	37·9341535	11·2898177	·0006949270
1440	2073600	2985984000	37·9473319	11·2924323	·0006944444
1441	2076481	2992209121	37·9605058	11·2950457	·0006939625
1442	2079364	2998442888	37·9736751	11·2976579	·0006934813
1443	2082249	3004685307	37·9868398	11·3002688	·0006930007
1444	2085136	3010936384	38·0000000	11·3028786	·0006925208
1445	2088025	3017196125	38·0131556	11·3054871	·0006920415
1446	2090916	3023464536	38·0263067	11·3080945	·0006915629
1447	2093809	3029741623	38·0394532	11·3107006	·0006910850
1448	2096704	3036027392	38·0525952	11·3133056	·0006906078
1449	2099601	3042321849	38·0657326	11·3159094	·0006901312
1450	2102500	3048625000	38·0788655	11·3185119	·0006896552
1451	2105401	3054936851	38·0919939	11·3211132	·0006891799
1452	2108304	3061257408	38·1051178	11·3237134	·0006887052
1453	2111209	3067586777	38·1182371	11·3263124	·0006882312
1454	2114116	3073924664	38·1313519	11·3289102	·0006877579
1455	2117025	3080271375	38·1444622	11·3315067	·0006872852
1456	2119936	3086626816	38·1575681	11·3341022	·0006368132

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1457	2122849	3092990993	38·1706693	11·3366964	·0006863412
1458	2125764	3099363912	38·1837662	11·3392894	·0006858711
1459	2128681	3105745579	38·1968585	11·3418813	·0006854010
1460	2131600	3112136000	38·2099463	11·3444719	·0006849315
1461	2134521	3118535181	38·2230297	11·3470614	·0006844627
1462	2137444	3124943128	38·2361085	11·3496497	·0006839945
1463	2140369	3131359847	38·2491829	11·3522368	·0006835270
1464	2143296	3137785344	38·2622529	11·3548227	·0006830601
1465	2146225	3144219625	38·2753184	11·3574075	·0006825939
1466	2149156	3150662696	38·2883794	11·3599911	·0006821282
1467	2152089	3157114563	38·3014360	11·3625735	·0006816633
1468	2155024	3163575232	38·3144881	11·3651547	·0006811989
1469	2157961	3170044709	38·3275358	11·3677347	·0006807352
1470	2160900	3176523000	38·3405790	11·3703136	·0006802721
1471	2163841	3183010111	38·3536178	11·3728914	·0006798097
1472	2166784	3189506048	38·3666522	11·3754679	·0006793478
1473	2169729	3196010817	38·3796821	11·3780433	·0006788866
1474	2172676	3202524424	38·3927076	11·3806175	·0006784261
1475	2175625	3209046875	38·4057287	11·3831906	·0006779661
1476	2178576	3215578176	38·4187454	11·3857625	·0006775068
1477	2181529	3222118333	38·4317577	11·3883332	·0006770481
1478	2184484	3228667352	38·4447656	11·3909028	·0006765900
1479	2187441	3235225239	38·4577691	11·3934712	·0006761325
1480	2190400	3241792000	38·4707681	11·3960384	·0006756757
1481	2193361	3248367641	38·4837627	11·3986045	·0006752194
1482	2196324	3254952168	38·4967530	11·4011695	·0006747638
1483	2199289	3261545587	38·5097390	11·4037332	·0006743088
1484	2202256	3268147904	38·5227206	11·4062959	·0006738544
1485	2205225	3274759125	38·5356977	11·4088574	·0006734007
1486	2208196	3281379256	38·5486705	11·4114177	·0006729474
1487	2211169	3288008303	38·5616389	11·4139769	·0006724950
1488	2214144	3294646272	38·5746030	11·4165349	·0006720430
1489	2217121	3301293169	38·5875627	11·4190918	·0006715917
1490	2220100	3307949000	38·6005181	11·4206476	·0006711409
1491	2223081	3314613771	38·6134691	11·4242022	·0006796908
1492	2226064	3321287488	38·6264158	11·4267556	·0006702413
1493	2229049	3327970157	38·6393582	11·4293079	·0006697924
1494	2232036	3334661784	38·6522962	11·4318591	·0006693440
1495	2235025	3341362375	38·6652299	11·4344092	·0006688963
1496	2238016	3348071936	38·6781593	11·4369581	·0006684492
1497	2241009	3354790473	38·6910843	11·4395059	·0006680027
1498	2244004	3361517992	38·7040050	11·4420525	·0006675567
1499	2247001	3368254499	38·7169214	11·4445980	·0006671114
1500	2250000	3375000000	38·7298335	11·4471424	·0006666667
1501	2253001	3381754501	38·7427412	11·4496857	·0006662225
1502	2256004	3388518008	38·7556447	11·4522278	·0006657790
1503	2259009	3395290527	38·7685439	11·4547688	·0006553360
1504	2262016	3402072064	38·7814389	11·4573087	·0006648936
1505	2265025	3408862625	38·7943294	11·4598476	·0006644518
1506	2268036	3415662216	38·8072158	11·4623850	·0006640106
1507	2271049	3422470843	38·8200978	11·4649215	·0006635700
1508	2274064	3429288512	38·8329757	11·4674568	·0006631300

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
1509	2277081	3436115229	38·8458491	11·4699911	·0006626905
1510	2280100	3442951000	38·8587184	11·4725242	·0006622517
1511	2283121	3449795831	38·8715834	11·4750562	·0006618134
1512	2286144	3456649728	38·8844442	11·4775871	·0006613757
1513	2289169	3463512697	38·8973006	11·4801169	·0006609385
1514	2292196	3470384744	38·9101529	11·4826455	·0006605020
1515	2295225	3477265875	38·9230009	11·4851731	·0006600660
1516	2298256	3484156096	38·9358447	11·4876995	·0006596306
1517	2301289	3491055413	33·9486841	11·4902249	·0006591958
1518	2304324	3597963832	38·9615194	11·4927491	·0006587615
1519	2307361	3504881359	38·9743505	11·4952722	·0006583278
1520	2310400	3511808000	38·9871774	11·4977942	·0006578947
1521	2313441	3518743761	39·0000000	11·5003151	·0006574622
1522	2316484	3525688648	39·0128184	11·5028318	·0006570302
1523	2319529	3532642667	39·0256326	11·5053535	·0006565988
1524	2322576	3539605824	39·0384426	11·5078711	·0006561680
1525	2325625	3546578125	39·0512483	11·5103876	·0006557377
1526	2328676	3553559576	39·0640499	11·5129030	·0006553080
1527	2331729	3560558183	39·0768473	11·5154173	·0006548788
1528	2334784	3567549552	39·0896406	11·5179305	·0006544503
1529	2337841	3574558889	39·1024296	11·5204425	·0006540222
1530	2340900	3581577000	39·1152144	11·5229535	·0006535948
1531	2343961	3588604291	39·1279951	11·5254634	·0006531679
1532	2347024	3595640768	39·1407716	11·5279722	·0006527415
1533	2350089	3602686437	39·1535439	11·5304799	·0006523157
1534	2353156	3609741304	39·1663120	11·5329865	·0006518905
1535	2356225	3616805375	39·1790760	11·5354920	·0006514658
1536	2359296	3623878656	39·1918359	11·5379965	·0006510417
1537	2362369	3630961153	39·2045915	11·5404998	·0006506181
1538	2365444	3638052872	39·2173431	11·5430021	·0006501951
1539	2368521	3645153819	39·2300905	11·5455033	·0006497726
1540	2371600	3652264000	39·2428337	11·5480034	·0006493506
1541	2374681	3657983421	39·2555728	11·5505025	·0006489293
1542	2377764	3666512088	39·2683078	11·5530004	·0006485084
1543	2380849	3673650007	39·2810387	11·5554972	·0006480881
1544	2383936	3680797184	39·2937654	11·5579931	·0006476684
1545	2387025	3687953625	39·3064880	11·5604878	·0006472492
1546	2390116	3695119336	39·3192065	11·5629815	·0006468305
1547	2393209	3702294323	39·3319208	11·5654740	·0006464124
1548	2396304	3709478592	39·3446311	11·5679655	·0006459948
1549	2399401	3716672149	39·3573373	11·5704559	·0006455778
1550	2402500	3723875000	39·3700394	11·5729453	·0006451613
1551	2405601	3731087151	39·3827373	11·5754336	·0006447453
1552	2408704	3738308608	39·3954312	11·5779208	·0006443299
1553	2411809	3745539377	39·4081210	11·5804069	·0006439150
1554	2414916	3752779464	39·4208067	11·5828919	·0006435006
1555	2418025	3760028875	39·4334883	11·5853759	·0006430868
1556	2421136	3767287616	39·4461658	11·5878588	·0006426735
1557	2424249	3774555693	39·4588393	11·5903407	·0006422608
1558	2427364	3781833112	39·4715087	11·5928215	·0006418485
1559	2430481	3789119879	39·4841740	11·5953013	·0006414368
1560	2433600	3796416000	39·4968353	11·5977799	·0006410256

Number	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1561	2436721	3803721481	39·5094925	11 6002576	·0006406150
1562	2439844	3811036328	39·5221457	11·6027342	·0006402049
1563	2442969	3818360547	39·5347948	11·6052097	·0006397953
1564	2446096	3825641444	39·5474399	11·6076841	·0006393862
1565	2449225	3833037125	39·5600809	11·6101575	·0006389776
1566	2452356	3840389496	39·5727179	11·6126299	·0006385696
1567	2455489	3847751263	39·5853508	11·6151012	·0006381621
1568	2458624	3855123432	39·5979797	11·6175715	·0006377551
1569	2461761	3862503009	39·6106046	11·6200407	·0006373486
1570	2464900	3869883000	39·6232255	11·6225088	·0006369427
1571	2468041	3877292411	39·6358424	11·6249759	·0006365372
1572	2471184	3884701248	39·6484552	11·6274420	·0006361323
1573	2474329	3892119157	39·6610640	11·6299070	·0006357279
1574	2477476	3899547224	39·6736688	11·6323710	·0006353240
1575	2480625	3906984375	39·6862696	11·6348339	·0006349206
1576	2483776	3914430976	39·6988665	11·6372957	·0006345178
1577	2486929	3921887033	39·7114593	11·6397566	·0006341154
1578	2490084	3929352552	39·7240481	11·6422164	·0006337136
1579	2493241	3936827539	39·7366329	11·6446751	·0006333122
1580	2496400	3944312000	39·7492138	11·6471329	·0006329114
1581	2499561	3951805941	39·7617907	11·6495895	·0006325111
1582	2502724	3959309368	39·7743636	11·6520452	·0006321113
1583	2505889	3966822287	39·7869325	11·6544998	·0006317119
1584	2509056	3974344704	39·7994976	11·6569534	·0006313131
1585	2512225	3981876625	39·8120585	11·6594059	·0006309148
1586	2515396	3989418056	39·8246155	11·6618574	·0006305170
1587	2518569	3996969003	39·8371686	11·6643079	·0006301197
1588	2521744	4004529472	39·8497177	11·6667574	·0006297229
1589	2524921	4012099469	39·8622628	11·6692058	·0006293266
1590	2528100	4014679000	39·8748040	11·6716532	·0006289308
1591	2531281	4027268071	39·8873413	11·6740996	·0006285355
1592	2534464	4034866688	39·8998747	11·6765449	·0006281407
1593	2537649	4042474857	39·9124041	11·6789892	·0006277464
1594	2540836	4050092584	39·9249295	11·6814325	·0006273526
1595	2544025	4057719875	39·9374511	11·6838748	·0006269592
1596	2547216	4065356736	39·9499687	11·6863161	·0006265664
1597	2550409	4073003173	39·9624824	11·6887563	·0006261741
1598	2553604	4080659192	39·9749922	11·6911955	·0006257822
1599	2556801	4088324799	39·9874980	11·6936337	·0006253909
1600	2560000	4096000000	40·0000000	11·6960709	·0006250000

N.	N^2.	N^3.	$\sqrt{N}$.	$\sqrt[3]{N}$.	$\frac{1}{N}$.
$\sqrt{N}$.	N.	$\sqrt{N^3}$.	$\sqrt[4]{N}$.	$\sqrt[6]{N}$.	$\frac{1}{\sqrt{N}}$.
$\sqrt[3]{N}$.	$\sqrt[3]{N^2}$.	N.	$\sqrt[6]{N}$.	$\sqrt[9]{N}$.	$\frac{1}{\sqrt[3]{N}}$.
N^2.	N^4.	N^6.	N.	$\sqrt[3]{N^2}$.	$\frac{1}{N^2}$.
N^3.	N^6.	N^9.	$\sqrt{N^3}$.	N.	$\frac{1}{N^3}$.
$\frac{1}{N}$.	$\frac{1}{N^2}$.	$\frac{1}{N^3}$.	$\sqrt{\frac{1}{N}}$.	$\sqrt[3]{\frac{1}{N}}$.	N.

When the number contains Integer and Decimals.

Example 5. Required the Square Root of 7845·45? In the column of *Squares* you will find,

+7849·96 = $88·6^2$,	+7849·96 = $88·6^2$,
—7845·45 = $88·5^{2\cdots}$,	—7832·25 = $88·5^2$,
451 divided by	1771 = 00·0256.
	$\sqrt{7845·45}$ = 88·5256 nearly.

☞When the number of ciphers in the integer is *even*, the number of figures taken in the Square column must also be *even*; but when the number of figures in the integer is *odd*, the number taken in the Square column must also be *odd*.

To find the Cube Root of Numbers exceeding 1600.

Example 6. Required the Cube Root of 5694958? In the *Cube* column you will find,

+5735339 = 179^3	+5735339 = 179^3.
—5694958 = $178^{3\cdots}$	—5639752 = 178^3.
40381 divided by	95587 = 000·4225,
	$\sqrt[3]{5694958}$ = 178·4225 nearly.

When the number contains Integer and Decimals.

Example 7. Required the Cube Root of 4186·586? In the column of *Cubes* you will find,

+4251·528 = $16·2^3$	4251·528 = $16·2^3$
—4186·585 = $16·1^{3\cdots}$	4173·281 = $16·1^3$
64942	78247 = 00·083
	$\sqrt[3]{4186·586}$ = 16·183 nearly.

☞The following notice must be particularly attended to, when extracting *Cube Root* of numbers with decimals.

2	ciphers in the integer	must	be	5, 8, or 11	ciphers in the	Cube column.
3	"	"	"	3, 6, or 9	"	"
4	"	"	"	4, or 7	"	"
5	"	"	"	5, or 8	"	"
6	"	"	"	6, or 9	"	"
7	"	"	"	7, or 10	"	"

Example 8. Required the Cube Root of 61358·75? In the *Cube* column and 8 figures you will find,

+61629·875 = 395^3	+61629875 = $39·5^3$
—61358·750 = $394^{3\cdots}$	—61162984 = $39·4^3$
271·125 divided by	466891 = 00·05807
	$\sqrt[3]{61358·75}$ = 39·45807.

To find the Fourth Root.

Rule. Extract the Square Root of the number as before described, and of that root extract the Square Root again, then the last is the Fourth root of the number.

Example 9. Required the fourth root of 2469781?

$$\sqrt[4]{2469781} = \sqrt{\sqrt{2469781}} = \sqrt{1571·4463} = 39·6467, \text{ the answer.}$$

To find the Sixth Root.

Rule. Find the Cube Root of the number as before described, and of that root extract the Square Root, and then the last is the Sixth root of the number.

To Find Powers and Roots not Noted in the Table.

At the foot of the last page of square and cube roots is an algebraic table which indicates how higher power and root can be found in the above tables.

Example 1. Suppose the 6th root is required of the number 3914430976. On the 4th line in the algebraic table will be found $\sqrt[6]{N}$; on the same line find the letter N, which is in the column of cubes. Find the given number 3914430976 in the column of cubes, which answers to 39.6988 in the column of square roots; therefore $\sqrt[6]{3914430976} = 39.6988$.

Example 2. Find the $\sqrt[3]{5735339^2}$. On the third line in the algebraic table find $\sqrt{N^2}$, which is in the columns of squares; and N is in the column of cubes. Find the number 5735339 in the column of cubes; and in the column of squares is found the required number, namely $\sqrt[3]{5735339^2} = 32041$.

Thus, a variety of powers and roots can be found by reference to the algebraic table.

Table of the First Nine Powers of Numbers.

1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

Table of Permutation. *See page 29.*

1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800
11	39916800
12	479001600
13	6227020800
14	87178291200
15	1307674368000
16	20922789888000
17	355687428096000
18	6402373705728000
19	121645100408832000
20	2432902008176640000
21	51090942171709440000
22	1124000727777607680000
23	25852016738884976640000
24	620448401733239439360000

How many different numbers can be written by the nine Arabic digits? From the table we have the permutation 9 = 362880 different numbers.

How many different words can be written by the seven letters *algebra?*

The permutation of 7 is 5040, but there are two *a*'s; and the permutation of 2 is 2. Therefore, 5040 : 2 = 2520 different words.

No.	4th Power.	5th Power.	No.	4th Power.	5th Power.	No.	4th Power.	5th Power.
1	1	1	51	6765201	345025251	101	104060401	10510100501
2	16	32	52	7311616	380204032	102	108243216	11040808032
3	81	243	53	7890481	418195493	103	112550881	11592740743
4	256	1024	54	8503056	459165024	104	116985856	12166529024
5	625	3125	55	9150625	503284375	105	121550625	12762815625
6	1296	7776	56	9834496	550731776	106	126247696	13382255776
7	2401	16807	57	10556001	601692057	107	131079601	14025517307
8	4096	32768	58	11316496	656356768	108	136048896	14693280768
9	6561	59049	59	12117361	714924299	109	141158161	15386239549
10	10000	100000	60	12960000	777600000	110	146410000	16105100000
11	14641	161051	61	13845841	844596301	111	151807041	16850581551
12	20736	248832	62	14776336	916132832	112	157351936	17623416832
13	28561	371293	63	15752961	992436543	113	163047361	18424351793
14	38416	537824	64	16777216	1073741824	114	168896016	19254145824
15	50625	759375	65	17850625	1160290625	115	174900625	20113571875
16	65536	1048576	66	18974736	1252332576	116	181063936	21003416576
17	83521	1419857	67	20151121	1350125107	117	187388721	21924480357
18	104976	1889568	68	21381376	1453933568	118	193877776	22877577568
19	130321	2476099	69	22667121	1564031349	119	200533921	23863536599
20	160000	3200000	70	24010000	1680700000	120	207360000	24883200000
21	194481	4084101	71	25411681	1804229351	121	214358881	25937424601
22	234256	5153632	72	26873856	1934917632	122	221533456	27027081632
23	279841	6436343	73	28398241	2073071593	123	228886641	28153056843
24	331776	7962624	74	29986576	2219006624	124	236421376	29316250624
25	390625	9765625	75	31640625	2373046875	125	244140625	30517578125
26	456976	11881376	76	33362176	2535525376	126	252047376	31757969376
27	531441	14348907	77	35153041	2706784157	127	260144641	33038369407
28	614656	17210368	78	37015056	2887174368	128	268435456	34359738368
29	707281	20511149	79	38950081	3077056399	129	276922881	35723051649
30	810000	24300000	80	40960000	3276800000	130	285610000	37129300000
31	923521	28629151	81	43046721	3486784401	131	294499921	38579489651
32	1048576	33554432	82	45212176	3707398432	132	303595776	40074642432
33	1185921	39135393	83	47458321	3939040643	133	312900721	41615795893
34	1336336	45435424	84	49787136	4182119424	134	322417936	43204003424
35	1500625	52521875	85	52200625	4437053125	135	332150625	44840334375
36	1679616	60466176	86	54708016	4704270176	136	342102016	46525874176
37	1874161	69343957	87	57289761	4984209207	137	352275361	48261724457
38	2085136	79235168	88	59969536	5277319168	138	362673936	50049003168
39	2313441	90224199	89	62742241	5584059449	139	373301041	51888844699
40	2560000	102400000	90	65610000	5904900000	140	384160000	53782400000
41	2825761	115856201	91	68574961	6240321451	141	395254161	55730836701
42	3111696	130691232	92	71639296	6590815232	142	406586896	57735339232
43	3418801	147008443	93	74805201	6956883693	143	418161601	59797108943
44	3748096	164916224	94	78074896	7339040224	144	429981696	61917364224
45	4100625	184528125	95	81450625	7737809375	145	442050625	64097340625
46	4477456	205962976	96	84934656	8153726976	146	454371856	66338290976
47	4879681	229345007	97	88529281	8587340257	147	466948881	68641485507
48	5308416	254803968	98	92236816	9039207968	148	479785216	71008211968
49	5764801	282475249	99	96059601	9509900499	149	492884401	73439775749
50	6250000	312500000	100	100000000	10000000000	150	506250000	75937500000

To find the Area and Solidity of Irregular Figures.

Chapman's rule in the construction of ships, Stockholm, 1775.

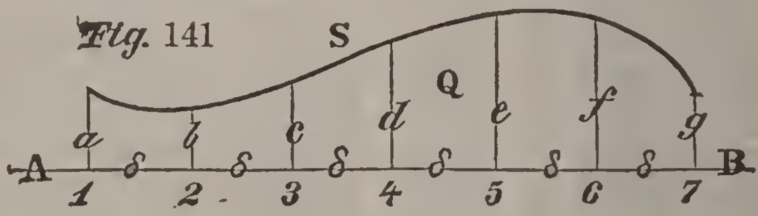


Fig. 141

Divide the base $A\ B$ into any even number of equal parts. $\delta =$ distance between the ordinates; $Q =$ area of the projecting figure.

$$Q = \frac{\delta}{3}(a + 4b + 2c + 4d + 2e + 4f + g). \quad . \quad . \quad . \quad 1.$$

Suppose this area to revolve around the axis $A\ B$ and form a solid figure like a handle, an urn or a gun; then the solidity C of the figure will be—

$$C = \frac{\pi\,\delta}{3}(a^2 + 4b^2 + 2c^2 + 4d^2 + 2e^2 + 4f^2 + g^2). \quad . \quad . \quad . \quad 2.$$

The practical calculation of these formulas is set up as in table for Formula 1. Suppose $a = 1.25$. $b = 1.15$, $c = 1.52$, $d = 1.86$. $e = 2$, $f = 1.77$, and $g = 1.20$.

The distance between the ordinates being $\delta = 2$, then the area will be, $Q = \frac{2}{3} \times 28.51 = 19$ square of whatever measure used.

The convex surface S of the figure will be, $S = 2\pi Q = 2 \times 3.14 \times 19 = 119.3$ square.

Ordinates.		Mult.	Product.
a	1.25	1	1.25
b	1.15	4	4.50
c	1.52	2	3.04
d	1.86	4	7.44
e	2.	2	4.00
f	1.77	4	7.08
g	1.20	1	1.20
Q	9.506	⅓	28.51

This rule can also be employed in calculating the cubic contents of earth-work in excavations and embankments, in which the ordinates are expressed in areas of the sections.

Suppose $a = 36$ square feet, yards, metres, or whatever unit of measure, $b = 30$, $c = 42$, $d = 56$, $e = 84$, $f = 72$, and $g = 50$, the distance between the sections being, say, 50 feet. The calculation is set up as in the preceding table, namely:

Volume $C = 50 \times 323.3 = 1616.5$ cubics of whatever unit of measure used.

Ordinates.		Mult.	Product.
a	36	1	36
b	30	4	120
c	42	2	84
d	56	4	224
e	84	2	168
f	72	4	288
g	50	1	50
C	323.3	⅓	970

This rule is universally employed for calculating the areas of water-lines, cross-sections and cubic contents of displacement in ships (*known as Simpson's rule*).

When the cubic content is required between each section, calculate it as explained in Excavation and Embankment.

Surface of Revolution.

The surface generated by a line revolving around an axis, is equal to the length of the line multiplied by the circumference of its centre of gravity.

N. B. The line, whether straight or curved, must be in the same plane as the axis.

Solidity of Revolution.

The solidity generated by a plane revolving around an axis, is equal to the area of the plane multiplied by the circumference of its centre of gravity.

N. B. The revolving plane must be in the same plane as the axis.

Angle. W	Ordinates. 1. 7.	2. 6.	3. 5.	4. h.	Angle. W	Ordinates. 1. 7.	2. 6.	3. 5.	4. h.
1°	·00084	·00164	·00193	·00218	53°	·05313	·08932	·11063	·11773
2	·00191	·00327	·00409	·00436	54	·05422	·09130	·11318	·12003
3	·00299	·00522	·00561	·00659	55	·05531	·09308	·11510	·12235
4	·00382	·00654	·00818	·00872	56	·05646	·09487	·11731	·12466
5	·00437	·00818	·01023	·01091	57	·05760	·09673	·11950	·12698
6	·00573	·00928	·01228	·01309	58	·05875	·09853	·12170	·12932
7	·00675	·01173	·01432	·01527	59	·05989	·10037	·12393	·13162
8	·00764	·01309	·01639	·01746	60	·06094	·10220	·12612	·13397
9	·00845	·01474	·01842	·01964	61	·06261	·10427	·12840	·13631
10	·00955	·01637	·02047	·02183	62	·06331	·10593	·13054	·13866
11	·01053	·01801	·02250	·02402	63	·06451	·10781	·13281	·14101
12	·01146	·01965	·02456	·02620	64	·06570	·10964	·13505	·14337
13	·01245	·02129	·02662	·02839	65	·06681	·11101	·13765	·14573
14	·01284	·02271	·02861	·03058	66	·06805	·11342	·13956	·14810
15	·01438	·02461	·03081	·03282	67	·06914	·11532	·14181	·15048
16	·01535	·02625	·03277	·03496	68	·07040	·11721	·14409	·15286
17	·01630	·02789	·03484	·03715	69	·07168	·11912	·14637	·15526
18	·01730	·02956	·03693	·03935	70	·07284	·12103	·14864	·15765
19	·01858	·03125	·03996	·04154	71	·07407	·12294	·15087	·16005
20	·01922	·03286	·04103	·04374	72	·07535	·12485	·15323	·16245
21	·02022	·03453	·04309	·04594	73	·07656	·12685	·15555	·16487
22	·02119	·03619	·04522	·04814	74	·07784	·12877	·15785	·16729
23	·02215	·03787	·04720	·05034	75	·07912	·13078	·16016	·16972
24	·02311	·03934	·04930	·05255	76	·08040	·13292	·16247	·17216
25	·02413	·04117	·05138	·05475	77	·08168	·13472	·16482	·17460
26	·02508	·04283	·05346	·05696	78	·08297	·13670	·16716	·17706
27	·02610	·04457	·05552	·05917	79	·08426	·13868	·16951	·17951
28	·02708	·04621	·05761	·06139	80	·08560	·14070	·17187	·18198
29	·02813	·04793	·05970	·06361	81	·08695	·14274	·17423	·18445
30	·02911	·04970	·06188	·06582	82	·08829	·14477	·17660	·18694
31	·03005	·05125	·06386	·06804	83	·08944	·14681	·17901	·18943
32	·03107	·05298	·06596	·07027	84	·09105	·14888	·18140	·19193
33	·03191	·05464	·06806	·07250	85	·09235	·15120	·18379	·19444
34	·03310	·05637	·07016	·07477	86	·09377	·15304	·18622	·19695
35	·03412	·05804	·07424	·07695	87	·09518	·15509	·18865	·19946
36	·03515	·05992	·07452	·07919	88	·09660	·15756	·19108	·20201
37	·03616	·06147	·07646	·08143	89	·09780	·15931	·19350	·20555
38	·03718	·06327	·07858	·08367	90	·09944	·16144	·19597	·20710
39	·03821	·06492	·08069	·08591	91	·10098	·16359	·19842	·20966
40	·03905	·06631	·08243	·08816	92	·10240	·16575	·20092	·21223
41	·04030	·06836	·08494	·09041	93	·10384	·16787	·20338	·21481
42	·04133	·07012	·08707	·09266	94	·10537	·17005	·20589	·21740
43	·04241	·07182	·08920	·09492	95	·10692	·17224	·20837	·22000
44	·04363	·07353	·09130	·09719	96	·10851	·17444	·21091	·22262
45	·01522	·07531	·09346	·09945	97	·10997	·17666	·21342	·22523
46	·04556	·07706	·09562	·10172	98	·11150	·17888	·21596	·22786
47	·04682	·07894	·09790	·10400	99	·11310	·18111	·22800	·23050
48	·04833	·08059	·09991	·10627	100	·11468	·18354	·22107	·23315
49	·04879	·08236	·00207	·10856	101	·11626	·18500	·22364	·23596
50	·04982	·08413	·00422	·11085	102	·11791	·18793	·22623	·23848
51	·05096	·08593	·10639	·11314	103	·11959	·19021	·22876	·24107
52	·05204	·08768	·10855	·11543	104	·12116	·19256	·23147	·24386

RAIL ROAD CURVES.

WHEN Railroads are to be connected by curves, we commonly have given the *distance* (Fig. 142,) between the two ends $o\ o$ of the tracks, and the tangential angle v. By these the curve is to be constructed.

Example 1. Fig. 142. The chord $C = 168$ feet, and the tangential angle $v = 19° 30'$. Required the centre angle $w =$, and the radius $R = ?$

$$w = 2(19° 30') = 39°. \quad R = {}^{39}k\,c = 1{\cdot}4979 \times 168 = 251{\cdot}647 \text{ feet.}$$

$k =$ See Table for Segments, &c., of a circle.

By Tangential Angles.

The curve to be laid out by the three tangential angles $r\,o\,r$, $r\,o\,n$, and $n\,o\,o$, each angle $= \frac{1}{3}v = 6° 30'$. Required the chord $r = ?$

The centre angle for the chord r is

$$2 \times (6° 30') = 13°, \text{ and } r = {}^{13}k\,R = 0{\cdot}2264 \times 251{\cdot}647 = 56{\cdot}974 \text{ feet.}$$

By Angles of Deflexion.

Divide the centre angle w into an even number of parts $= z$. Set off at o the angle $z = r\,o\,n$, and bisect it into $r\,o\,r$ and $r\,o\,n$.—find the chord r, and sub-chord a, and continue as shown by Figure.

Example 2. Fig. 142. The tangential angle $v = 78°$, and the chord $C = 638$ feet. Required the centre-angle $w = ?$ Radius $R = ?$ Chord $r = ?$ and the sub-chord $a = ?$

$$w = 2 \times 78° = 156°. \quad R = {}^{156}k\,c = 0{\cdot}51117 \times 638 = 326{\cdot}126 \text{ feet.}$$

Let the curve be laid out by 6 angles of deflexion, and $z = \frac{1}{6} \times 156° = 26°$, and

$$r = {}^{26}k\,R = 0{\cdot}4499 \times 326{\cdot}126 = 146{\cdot}73 \text{ feet.}$$
$$a = {}^{26}k\,r = 0{\cdot}4495 \times 146{\cdot}73 = 66{\cdot}012 \text{ feet.}$$

By Ordinates.

Example 3. Fig. 143. The chord $C = 368$ feet, and $v = 36°$. Required the height $h = ?$

$$h = \tfrac{1}{2}C(\text{cosec.}v - \text{cot.}v).$$

From	- - - - - - - -	$\text{cosec.}36° = 1{\cdot}70130$
Subtract	- - - - - - - -	$\text{cot.}36° = 1{\cdot}37638$
The height		$h = 0{\cdot}32492 \times 184 = 59{\cdot}785$ feet.

At $x = 92$ feet from h. Required the ordinate y?

$$\sin.z = \frac{2 \times 92 \sin.36°}{368} = 0{\cdot}2938926 = \sin.17° 6'.$$

$$y = \tfrac{1}{2} \times 368\left(\frac{\cos.17° 6'}{\sin.36°} - \text{cot.}36°\right) = 45{\cdot}9448 \text{ feet.}$$

By Sub-Chords.

Example 4. Fig. 144. The ends o and o of the tracks form different angles w and W to the chord C, and therefore must be connected by two curves of different radii, R and r. The chord $C = 869$ feet, $w = 38°$, and $W = 86°$. Required the distance from o to the height h, $n = ?$ sub-chord $b = ?$ sub-chord $a = ?$ radii R and $r = ?$

$$v = \tfrac{1}{2} \times 38° = 19°, \quad \text{and } V = \tfrac{1}{2} \times 86° = 43°.$$

$$n = \frac{869 \tan.19°}{\tan.19° + \tan.43°} = 234{\cdot}35 \text{ feet.}$$

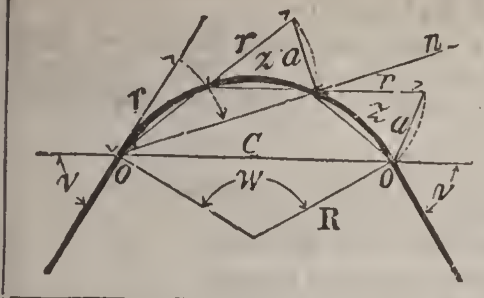
$b = 234{\cdot}35 \sec.43° = 320{\cdot}42$ feet.
$a = \sec.19°(869 - 234{\cdot}35) = 671{\cdot}21$ ft.

$R = {}^{38}k\,a = 1{\cdot}5358 \times 671{\cdot}21 = 1030{\cdot}2$ ft.
$r = {}^{86}k\,b = 0{\cdot}73314 \times 320{\cdot}42 = 234{\cdot}91$ ft.

By Eight Ordinates.

Example 5. Fig. 146. Required 8 ordinates for a curve of chord $C = 710$ feet and the centre angle $w = 69°$? (See Table on the preceding page.)

1st and 7th Ordinates	$0{\cdot}07168 \times 710 =$	$50{\cdot}8928$	feet.
2nd " 6th "	$0{\cdot}11912 \times 710 =$	$84{\cdot}5752$	"
3rd " 5th "	$0{\cdot}14637 \times 710 =$	$103{\cdot}9227$	"
4th or height h	$0{\cdot}15526 \times 710 =$	$110{\cdot}2346$	"

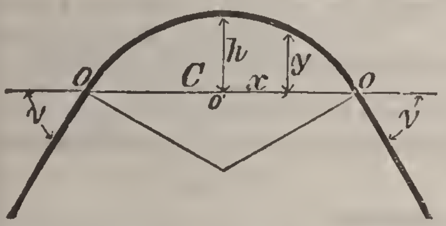


142

By angles of deflexion.

$$w = 2v, \quad R = {}^{w}k\,C = \tfrac{1}{2}C\,\text{cosec.}v.$$

$$r = {}^{z}k\,R, \quad a = {}^{z}k\,r = 2r\,\sin.\tfrac{1}{2}z.$$



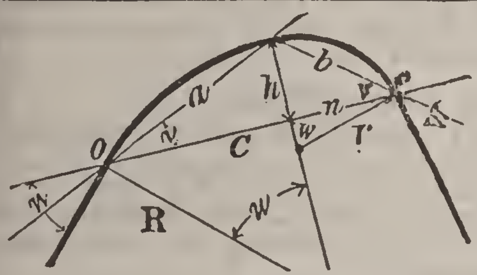
143

By Ordinates.

$$h = \tfrac{1}{2}C(\text{cosec.}v - \cot.v).$$

$$y = \tfrac{1}{2}C\left(\frac{\cos.z}{\sin.v} - \cot.v\right),$$

$$\sin.z = \frac{2x\,\sin.v}{C}.$$



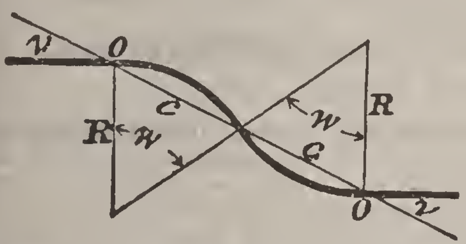
144 *By Sub-chords.*

$$n = \frac{C\,\tan.v}{\tan.v + \tan.V}, \quad h = n\,\tan.V,$$

$$b = n\,\sec.V, \quad w = 2v$$

$$W = 2V$$

$$a = \sec.v(C - n),$$

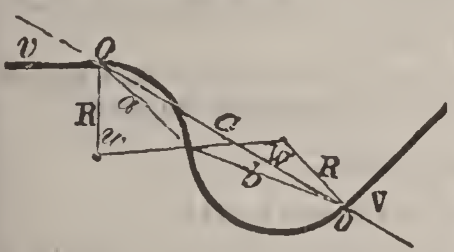


145

Parallel tracks by a reverse curve.

Formulas same as above.

The length $o\,o = 2c$, length of a circle arc $l = 0{\cdot}035v\,R$.



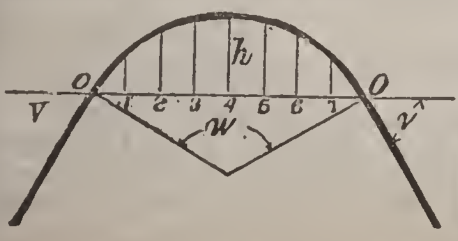
146

The greatest radius in a reverse curve.

$$w = \tfrac{1}{2}(V+3v), \quad W = w+V - v,$$

$$a = {}^{w}k\,R, \quad b = {}^{w}k\,R,$$

$$R = C\,\sec.w(\sin.V - \sqrt{\sin.^2V - \cos.^2w}).$$



147

Curve by 8 Ordinates.

The ordinates are calculated in the accompanying Table, the chord $C = 1$ or the unit.

If the angle w is large, or there be some obstacle on the chord C, find the height h and lay out the curve by two or more sets of 8 ordinates.

By Ordinates and Subchords.

Example 6. Fig. 148. The tangents t being prolonged to where they meet at a, divide that angle into two equal parts, say $W=75°$. Required the tangents $t=?$ external secant $S=?$ chords $C=?$ and the angle $w=?$ Radius of the curve $R=1500$ feet.

$t=R$ cot.$75° = 1500 \times 0{\cdot}26794 = 401{\cdot}91$ feet.
Centre angle $w=90-75°=15°$ for half the curve.
$S=R$ (sec.$15°-1$) $=1500\ (1{\cdot}0352-1) = 52{\cdot}8$ feet.
The chords $C=k\ R=0{\cdot}26104 \times 1500 = 391{\cdot}56$ feet.

Measure off from a the tangents and the external secant.

Draw the chords C C, and divide them each into eight equal parts. In the table of ordinates under $w=15°$ will be found the

1st. 7th. $0{\cdot}01438 \times 391{\cdot}56 = 5{\cdot}631$,
2nd. 6th. $0{\cdot}02461 \times 391{\cdot}56 = 9{\cdot}636$,
3rd. 5th. $0{\cdot}03081 \times 391{\cdot}56 = 12{\cdot}063$,
4th. $0{\cdot}03282 \times 39{\cdot}56 = 12{\cdot}851$,

Thus by only four multiplications, 16 ordinates in the curve is obtained.

Should there be any obstacles for the chords C. C. as is often the case in excavations and on embankments, a line can be drawn further in on the track parallel to the chord and the ordinates obtained by subtraction, readily understood by the Engineer.

Ellipse by Ordinates.

By this arrangement ellipses can be constructed of any proportions. One of the two axes is divided into 16 equal parts. The ordinates drawn and calculated as shown by the figure 102.

Parallel Tracks by a Semi-Ellipse,

Example 7. Fig. 150. The instrument placed at b and b', divide the angles W and w each into two equal parts, prolong the chords which will meet at a, a point in the curve. Divide the chords each into eight equal parts, and draw the ordinates parallel to the tracks as shown in the figure. The grand chord C is the unit for calculating the ordinates, which latter are alike on both the chords c', c''.

1st	2nd.	3rd.	4th.	5th.	6th.	7th.
$0{\cdot}1795C$	$0{\cdot}2058C$	$0{\cdot}2029C$	$0{\cdot}1830C$	$0{\cdot}1477C$	$0{\cdot}1091C$	$0{\cdot}0586C$.

Suppose the grand chord to be $C=2050$ feet.

Required the length of the 6th ordinate? $0{\cdot}1091 \times 2050 = 223{\cdot}655$ feet.

Tracks not Parallel by Elliptic, arc,

Example 8. Fig. 151. Divide the angles W and w each into two equal parts, prolong the subchords until they intersect one another at a, which is a point in the curve. Divide the chord C into eight equal parts, join a with the 4th division and draw the other ordinates parallel thereto.

Suppose the angles are $W=18°$ and $w=12°$, the centre angle will be 30° for which the ordinates are to be calculated from the table. The chord $C=125$ feet. Required the 3rd and 5th ordinates? $0{\cdot}06188 \times 125 = 7{\cdot}335$ feet.

Springing of Rails.

Example 9. Fig. 152. A rail of $L=21$ feet is to be curved to a radius of $R=1250$ feet. Required the spring $S=?$ in sixteenths of an inch.

$$S=\frac{24 \times 21^2}{1250}=8{\cdot}47 \text{ sixteenths.}$$

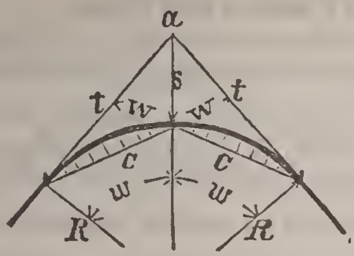
Super Elevation of the External Rail.

Example 10. Fig. 153. A train running $M=30$ miles per hour on a curve of $R=1550$ feet radii, the gauge of the track is $G=5$ feet. Required the angle of inclination $v=?$ and the super elevation of the external rail $h=?$

$$tan.v=\frac{30^2}{15 \times 1550}=0{\cdot}0387=\text{tan. } 2°\ 13'.$$

$h=G$ sin.$1°\ 21'=5 \times 0{\cdot}02356=0{\cdot}1178$ feet, or nearly $1\frac{1}{2}$ inches.

It is practically impossible to lay the super elevation to suit the different speeds of trains. If a mean speed is taken, the faster passenger trains will wear the outer rail, and the slow or freight train will wear the inner rail.



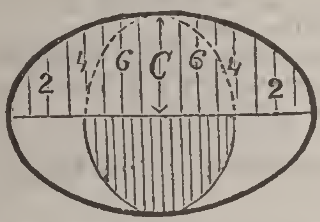
148

By ordinates and subchords.

$$t = R\cot. W = R\tan. w, \quad W = 90 - w,$$

$$S = R(\sec. w - 1) = R(\text{cosec}.\ W - 1)$$

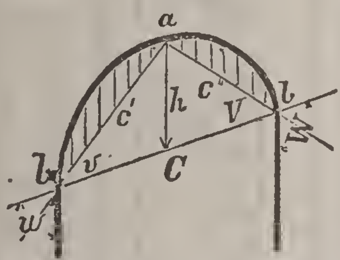
$C = k\ R$. For k, see table of segments.



149

Ellipse by ordinates.

$1 = 0{\cdot}4840\,C$	$5 = 0{\cdot}9204\,C$
$2 = 0{\cdot}6616\,C$	$6 = 0{\cdot}9682\,C$
$3 = 0{\cdot}7808\,C$	$7 = 0{\cdot}9922\,C$
$4 = 0{\cdot}8660\,C$	$8 = C$ the unit.



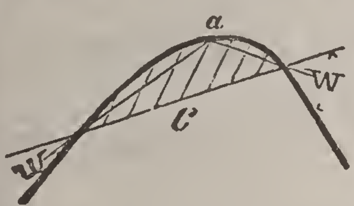
150

Parallel tracks by elliptic curve.

$$h = \tfrac{1}{2}\,C. \quad w = 2\,v. \quad W = 2\,V,$$

$$c' = \frac{C\sin. W}{2\sin. v}, \qquad c'' = \frac{C\ \sin. w}{2\ \sin. V},$$

See example for ordinates.

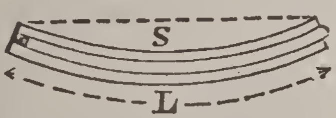


151

Tracks not parallel by elliptic arc.

Angle of the arc $= W + w$.

Ordinates to be calculated from the table, page 147.

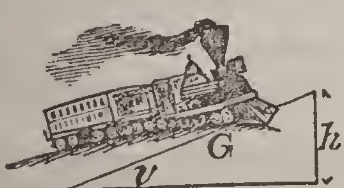


152

Spring of Rails.

$$S = \frac{1{\cdot}5\ L^2}{R} = \text{spring in inches.}$$

$$S = \frac{2\ 4\ L^2}{R} = \text{16ths of an inch.}$$



153

Inclination of tracks in curves.

$$\tan. v = \frac{M^2}{15\ R}. \quad h = G\sin. v.$$

Meaning of letters, see example.

Explanation of the Figures on the Following Page.

The most correct and accurate ways of laying out railway curves are by external secant or by ver sines, either to be employed, as the ground permits. The operation is well understood from figures 154 and 155.

The natural secant and versin. are found in the trigonometrical tables. Subtract 1 from the natural secant, and the remainder will be the external secant. Multiply the external secant by the assumed radius, and the product is the external secant s in the same unit of measure as the radius.

The centre angle is divided by 2 and 2 as many times as may be required for setting out the curve.

Fig. 154 is used when there are obstacles inside the curve, and Fig. 155 when the outside is inaccessible. The sinus-versus in the tables, multiplied by the assumed radius, will be the height of the curve above the chord.

When the inside of the curve is obstructed, and the point T of intersection is also inaccessible, then the curve can be laid out as illustrated by Fig. 156.

Fig. 157 illustrates how to lay out a curve by chords of 100 feet.

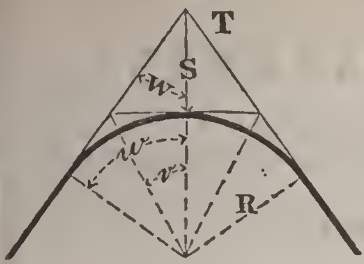
Tangential angles for a chord of c = 100 *feet, and different radii R from* 500 *feet to* 3 *miles* (fig. 157).

R. Feet.	tan. angle. °	′	″	R. Feet.	tan. angle. °	′	″	R. Miles.	tan. angle. °	′	″
500	5	43	46	3000	0	57	18	0.125	4	20	26
600	4	46	29	3500	0	49	6	0.25	2	10	13
700	4	5	33	4000	0	42	58	0.5	1	5	6
800	3	34	52	4500	0	38	12	0.75	0	43	25
900	3	10	59	5000	0	34	23	1 mile.	0	32	33
1000	2	51	53	5500	0	31	15	1.25	0	26	2
1100	2	36	16	6000	0	28	39	1.5	0	21	42
1200	2	23	15	7000	0	24	34	1.75	0	18	42
1500	1	54	35	8000	0	21	30	2	0	16	17
2000	1	25	56	9000	0	19	6	2½	0	13	1
2500	1	8	46	10000	0	17	12	3	0	10	51

Fig. 158 illustrates a section of a cut or embankment through sloping ground. The meaning of letters is the same as that on the following pages on excavation and embankment.

Fig. 159. *Sidings for parallel tracks.*—D = distance over tangent points; W = width between centres of tracks, and R = radius of curvature; v = angle of frog-plates.

The different operations of laying out the curves are so well understood by railroad engineers that it is considered unnecessary to enter into detailed description. The formulas and figures are intended only as a memorandum.

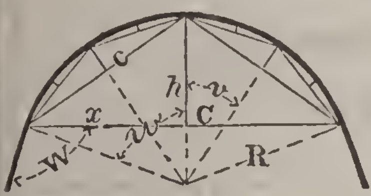


154 *By external secants.*

External secant $s = R(\sec. w - 1)$.

$W = 90 - w;\ w = 90 - W,\ w = 2v.$

tangent $t = R \cot. W = R \tan.\ w.$



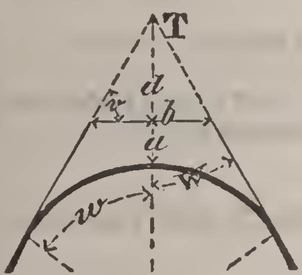
155 *By sinus-versus.*

$w = 180 - W. \qquad c = 2R \sin. w.$

$R = \dfrac{C}{2 \sin. w}. \qquad c = 2R \sin. v.$

$\text{ver} \sin. h = R\ \text{ver}.\sin. w.$

$x = 90 - \dfrac{w}{2}.$

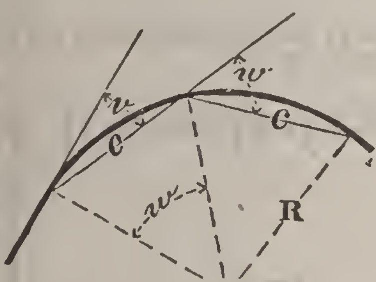


156 *When the point T is inaccessible.*

$W = 90 - v. \qquad b = 2d \cot. v.$

$a + d = R\ (\sec. w - 1). \quad d = \tfrac{1}{2} b \tan. v.$

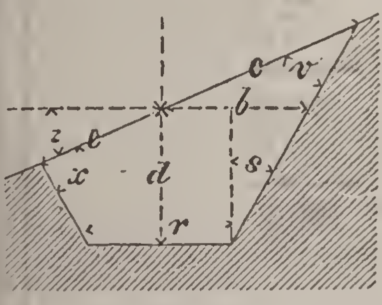
$a = R\,(\sec. w - 1) - \tfrac{1}{2} b \tan. v.$



157 *Tangential angle for a chord of c = 100 feet, and different radii R from 500 feet to 3 miles.*

$w = 2v. \qquad \sin. \tfrac{1}{2} w = \dfrac{c}{2R}.$

$R = \dfrac{c}{2 \sin. \tfrac{1}{2} w}. \qquad c = 2R \sin. \tfrac{1}{2} w.$



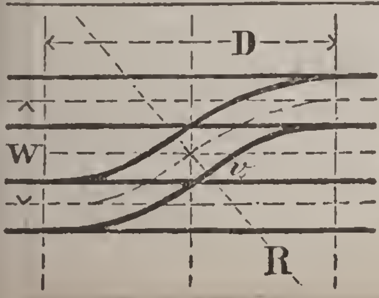
158 *Railway cut or embankment through side slopes.*

$b = \dfrac{r}{2} + d \tan. s. \quad c = \dfrac{b \sin.(90 + 2z - s)}{\sin.(90 - z - s)}.$

$x = 90 + z - s. \ v = 90 - z - s.$

$e = \dfrac{b \cos. s}{\sin.(90 + z - s)}.$

$\mathfrak{a} = d(d \tan. s + r).$



159 *Sidings for parallel tracks.*

$D = 2\sqrt{W(R - \tfrac{1}{4} W)}.$

$R = \dfrac{D^2}{4W} + \tfrac{1}{4} W. \qquad \sin. v = \dfrac{D}{2R}.$

EXCAVATION AND EMBANKMENT.

Example 1. The Road-way of an excavated channel is $r = 15$ feet, the depth $D = 9$ feet, and the breadth at the top $b = 46\frac{1}{2}$ feet. Require the slope $S = ?$

Formula 6. $$S = \frac{46{\cdot}5 - 15}{2 \times 9} = 1{\cdot}75 \text{ or } 1\tfrac{3}{4} \text{ to } 1{\cdot}$$

Example 2. The Road way is to be $r = 15$, $D = 18$, and the slope $S = 1\frac{1}{4}$, Require the breadth $b = ?$ and the cross-section $\mathbf{A} = ?$

Formula 4. $$b = 2 \times 18 \times 1{\cdot}25 + 15 = 60 \text{ feet.}$$

Formula 7. $$\mathbf{A} = \frac{18}{2}\left(60 + 15\right) = 675 \text{ square feet.}$$

Example 3. The Road-way is to be $r = 16$ feet, the slope $S = 1\frac{1}{2}$, and the depth $D = 11$ feet. Required the area of Cross-section $\mathbf{A} = ?$

Formula 9. $$\mathbf{A} = 11\,(11 \times 1\tfrac{3}{4} + r) = 357{\cdot}5 \text{ square feet.}$$

Excmple 4. The Road-way $r = 18$ feet, slope $S = 1\frac{1}{4}$, $d = 14$ feet 6 inches, and the length from o is $l = 55$ feet. Required the cubic contents $\mathbf{c} = ?$

Formula 11. $\mathbf{c} = 55 \times 14{\cdot}5\left(\frac{14{\cdot}5 \times 1{\cdot}25}{3} + \frac{18}{2}\right) = 11995{\cdot}676$ cubic feet, divided by $27 = 444.28$ cubic yards.

Example 5. The Road-way is $r = 16$ feet, slope $S = 1\frac{1}{2}$ feet, $D = 17{\cdot}5$, $d = 7{\cdot}4$ and the length $L = 100$ feet. Required the cubic content $\mathbf{C} = ?$

Formula 12. $$\mathbf{C} = 100\left[1\tfrac{1}{2}\left(\frac{17.5^2 + 7{\cdot}4^2 + 17{\cdot}5 \times 7{\cdot}4}{3}\right) + \frac{16}{2}\left(17{\cdot}5 + 7{\cdot}4\right)\right]$$
$= 44445$ cubic feet, or $1645{\cdot}4$ cubic yards.

The computation is executed thus.

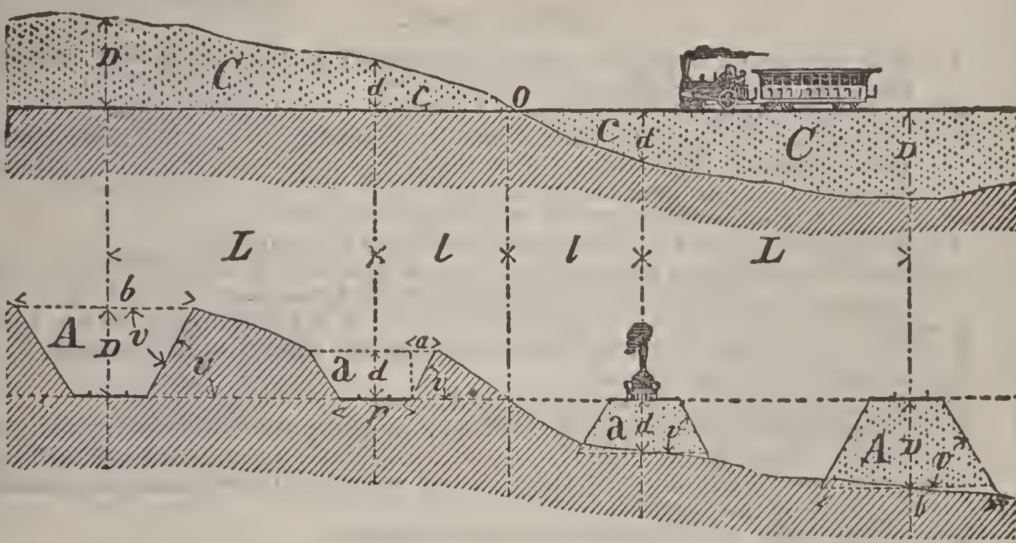
$$\begin{array}{r} 17{\cdot}5 \\ 7{\cdot}4 \\ \hline 700 \\ 1225 \\ \hline 129{\cdot}50 \end{array} \qquad\qquad \begin{array}{r} 17{\cdot}5 \\ 7.4 \\ \hline 24{\cdot}9 \\ 8 \\ \hline 199{\cdot}2 \end{array}$$

$$\begin{array}{r} 17{\cdot}5^2 = 306{\cdot}25 \\ 7{\cdot}4^2 = 54{\cdot}76 \end{array} \Big\} \text{ From table of Squares.}$$

$$3)\ 490{\cdot}51\ (163{\cdot}5$$
$$\begin{array}{r} 81{\cdot}75 \end{array} \Big\} \text{ Slope, add } \tfrac{1}{2}$$
$$\begin{array}{r} 199{\cdot}2 \\ \hline \end{array}$$

$$\times\, 100 = 44445. \text{ cubic feet.}$$

160.



Letters in the Formulas correspond with the Figure.

$$S = \cot. v, \qquad 1.$$

$$a = D\,S, \qquad 2.$$

$$a = D \cot. v, \qquad 3.$$

$$b = 2\,D\,S + r, \qquad 4.$$

$$S = \frac{a}{D}, \qquad 5.$$

$$S = \frac{b - r}{2\,D}, \qquad 6.$$

$$\mathbf{A} = \frac{D}{2}(b + r), \qquad 7.$$

$$\mathbf{a} = \frac{d}{2}(b + r), \qquad 8.$$

$$\mathbf{A} = D(D\,S + r), \qquad 9.$$

$$\mathbf{a} = d(d\,S + r), \qquad 10.$$

$$\mathbf{c} = l\,d\left(\frac{d\,S}{3} + \frac{r}{2}\right), \qquad 11.$$

$$\mathbf{C} = L\left[S\left(\frac{D^2 + d^2 + Dd}{3}\right) + \frac{r}{2}(D + d)\right]. \qquad 12.$$

Letters Denote,

A and **a** = Cross-Sections in square feet, of the excavated channel or embankment.

D and *d* = depth in feet, of the Sections.

r = width in feet of the Road-Way.

b = Base in feet of the embankment, or top breadth of the channel.

L = length in feet, between the two Sections **A** and **a**.

l = length in feet, from the Section **a** to the point **o** where the ground is level with the road.

C = cubic contents in feet, between **A** and **a**.

c = cubic contents in feet, between **a** and **o**.

S = slope of the sides. The slope is commonly given in proportions, thus: "Slope = 1½ to 1," which means, that the side slopes 1½ feet horizontally for 1 foot vertical.

v = angle of the slope.

TRACTION ON ROADS.

Letters denote.

F = tractive force in pound avoir., necessary to overcome the rolling friction, and ascending inclined planes.
M = miles per hour of the train or force F.
T = weight of the load in tons, including the weight of the carriages. On rail-roads T includes the weight of the locomotive and tender.
t = weight of the locomotive resting on the driving wheels in tons.
h = vertical rise in feet per 100 of inclined roads.
b = base in feet per 100 of the inclined road or plane.
k = tractive coefficient in pound per ton of the load T, as noted in the accompanying Table, under the different conditions of the road. (P. 158.)
A = area of *one* of the two cylinder pistons in a locomotive, in sq. in.
P = mean pressure of steam in lbs. per sq. in. on cylinder pistons.
S = stroke of pistons in feet.
D = diameter of driving wheel in feet.
IP = actual horse power of a locomotive or the power necessary for the load. About 25 per cent. is allowed for friction and working pumps.
f = adherence coefficient of the driving wheels to the rails, in pounds per ton of the weight t.
n = revolutions per minute of driving wheels.
d = continued working hours of a horse.
v = velocity in feet per second. t' = weight of a horse in pounds.

Example 11. Fig. 161. The area of one of the two cylinder pistons in a locomotive is $A=314$ square inches, stroke of piston $P=2$ feet, mean-pressure $P=80$ lbs. per square inch. Driving wheels $D=4$ feet diameter. Required the tractive force $F=?$ of a locomotive.

$$F=\frac{314\times2\times80}{4}=12560 \text{ lbs. the answer.}$$

The adhesive force of the driving wheels to the rails, $f\,t$, must always be greater than the tractive force of the locomotive, otherwise the wheels will slip on the track.

Example 12. Fig. 162. A locomotive of $t=15$ tons on an inclined plane rising $h=10$ feet, and the base $b=99{\cdot}5$ feet per 100. $f=560$, other dimensions being the same as in the preceding example. Required the tractive, retroactive and adhesive forces?

Tractive, $$F=\frac{314\times2\times80}{4}-22{\cdot}4\times15\times10=9200 \text{ lbs.}$$

Retroactive, $$F=22{\cdot}4\times15\times10=3360 \text{ lbs.}$$

Adhesive, $$F=\frac{560\times15\times99{\cdot}5}{100}=8358 \text{ lbs.}$$

Consequently the locomotive can ascend the inclined plane with a pulling force of $8358-3360=4998$ lbs., without slip in the driving wheels.

Example 13. Fig. 163. A train of $T-200$ tons is to be drawn $M=20$ miles per hour on a horizontal track in good condition, $k-4$. Required retroactive force $F=?$

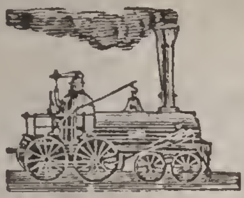
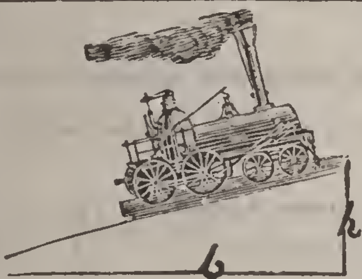
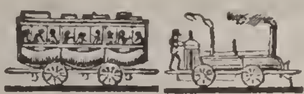
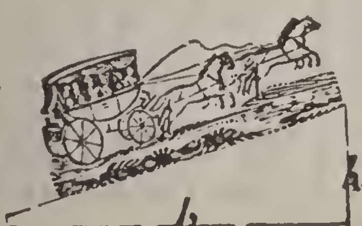
$$F=200\,(4+\sqrt{20})=1694{\cdot}4 \text{ lbs. the answer.}$$

Example 14. Fig. 164. A train of $T=150$ tons is to be drawn up an inclined plane of $h=9$ feet in 100, with a speed of $M=16$ miles per hour, $k=4$. Required the necessary horse power of the locomotive $IP=?$

$$IP=\frac{16\times150}{375}\,(22{\cdot}4\times9+4+\sqrt{16})=1342{\cdot}144 \text{ horses.}$$

Example 15. Fig. 165. Required the tractive ability $F=?$ of a horse, running $M=7$ miles per hour, in $d=4$ continued hours.

$$F=\frac{375}{7\sqrt{4}}=26{\cdot}8 \text{ lbs. the answer.}$$

	161 $F=\frac{A\ S\ P}{D}$. $n=\frac{28\ M}{D}$, $HP=\frac{A\ S\ P\ n}{11000}=\frac{A\ S\ P\ M}{376\ D}$. Adhesive force $=f\,t$.
	162 $F=\frac{A\ S\ P}{D}-22{\cdot}4th$. $M=\frac{D\ n}{28}$, Adhesive, $\frac{f\,t\,b}{100}>22{\cdot}4th$. retractive.
	163 $F=T\,(k+\sqrt{M})$. $<f\,t=$ Adhesive. $HP=\frac{M\ T}{375}(k+\sqrt{M})$,
	164 $F=T(22{\cdot}4\,h+k+\sqrt{M})$. $<\frac{f\,t\,b}{100}=$ Ad. $HP=\frac{M\ T}{375}(22{\cdot}4\ h+k+\sqrt{M})$.
	165 $F=T\,(k+\sqrt{M})$. $v=1{\cdot}466\ M$. $F=\frac{550}{v\sqrt{d}}=\frac{375}{M\sqrt{d}}$. ability of a horse.
	166 $F=T\,(22.4\ h+k+\sqrt{M})$. $F=\frac{550}{v\ \sqrt{d}}-\frac{t'\,h}{100}$ $M=0{\cdot}6821\ v$.

Example 16. Fig. 166. Required, the tractive force $F = ?$ of a load $T = 5.25$ tons to be drawn $M = 2$ miles per hour up a turnpike of $h = 9$ feet in 100, the road being newly laid with coarse gravel $k = 50$?

$$F = 5.25\,(22.4 \times 9 + 50 + \sqrt{2}) = 1328.30 \text{ lbs.}$$

Suppose a horse to weigh $t' = 1000$ lbs., working continually in $d = 1$ hour up the turnpike. Required, the tractive ability $F = ?$ per horse?

$$F = \frac{375}{2\sqrt{1}} - \frac{1000 \times 9}{100} = 97.5 \text{ lbs.}$$

Number of horses $= \dfrac{1328.25}{97.5} = 13.6$, say 14 horses, which will be necessary for the load under the mentioned circumstances. In these examples it is necessary to take $M > 1$. and $d > 1$.

Traction Coefficient at very Slow Speed.

	k
On railroads in good condition, carriage axles well lubricated,	4
On railroads under ordinary, not very good condition,	8
On very smooth stone pavement,	12
On ordinary street pavements in good condition,	20
On street pavements and turnpikes,	30
On turnpikes new laid with coarse gravel and broken stones,	50
On common roads in bad condition,	150
On natural loose ground or sand.	560

Adhesion Coefficient.

	f
On rails of maximum dryness,	672
" very dry,	560
" under ordinary circumstances,	450
" in wet weather,	315
" with snow or frost,	224

In railway curves the retractive force is augmented so many per cent. as the whole train occupies degrees in the curve.

Railway Gauges.

	Gauge feet.	in.
The most general gauge in coal mines,	2	6
Denver and Rio Grande railway,	3	
Rio Grande and Texas,	3	6
The most general gauge in the United States, England, France, Prussia, Sweden, Mexico, Chili and Peru,	4	8½
The compromised gauge,	4	9
Camden and Amboy,	4	9″
In the Southern States and in Russia,	5	
Irish railways,	5	3
Louisiana and Texas, also in Canada and India,	5	6
Great Western in England,	7	

Rain-fall in Inches at Different Seasons of the Year.

Locations.	Year.	Spring.	Summer.	Fall.	Winter.
Nishny, Taguilsk, Russia,	18.26	3.35	9.28	3.70	1.93
Tobolsk, Siberia,	17.76	2.29	9.05	4.02	2.40
Nertchinsk, Asia,	18.13	2.32	10.5	4.96	0.35
Yakoutsk, East Siberia,	10.25	1.46	3.35	3.59	1.85
Peking, China,	23.88	2.17	17.7	3.50	0.51
Macao, Quang-tong,	67.81	18.8	28.0	17.7	3.31
Saigon, India,	62.80	5.86	28.9	28.0	0.04
Yokohama, Japan,	35.02	7.52	12.0	15.2	0.295
Manilla, Philip. Islands,	71.31	4.77	34.1	25.6	4.84

For rain-fall, see page 491.

TRAVERSE SAILING AND SURVEYING.

To navigate a vessel upon the supposition that the earth is a level plane, on which the meridians are drawn north and south, parallel with each other; and the parallels east and west, at right-angles to the former.

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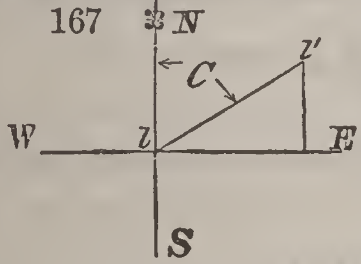


The line NS represents a meridian north and south; the line WE represents a parallel east and west.

A ship in l sailing in the direction of $l\,l'$, and having reached l', it is required to know her position to the point l, which is measured by the line $l\,l'$, and the angle $N l\,l'$; and located by the lines $l\,a$ and $a\,l'$

While the vessel is running from l to l', the distance is measured by the log and time; and the course $N l\,l'$ is measured by the compass commonly expressed in points.

These four quantities bear the following names.

$d = l\,l'$, *distance* from l to l' in miles.
$C = N l\,l'$, *course*, or points from the meridian.
$\mathfrak{d} = l\,a$, *departure* or difference in longitudes, in miles.
$u = a\,l'$, *difference in latitudes*, in miles.
$l =$ *latitude* in degrees.
$L =$ difference in longitude, in degrees or time.

Traverse Formulas.

$$\mathfrak{d} = d \sin.C, \quad 1,$$
$$\mathfrak{d} = u \tan.C, \quad 2,$$
$$\mathfrak{d} = 60 \cos.l\, L, \quad 3,$$
$$\mathfrak{d} = \sqrt{d^2 - u^2}, \quad 4,$$
$$u = d \cos.C, \quad 5,$$
$$u = \mathfrak{d} \cot.C, \quad 6,$$
$$u = \frac{60L \cos.l}{\tan.C}, \quad 7,$$
$$u = \sqrt{d^2 - \mathfrak{d}^2}, \quad 8,$$
$$d = \frac{\mathfrak{d}}{\sin.C}, \quad 9,$$
$$d = \frac{u}{\cos.C}, \quad 10,$$
$$d = \frac{60L \cos.l}{\sin.C}, \quad 11,$$
$$d = \sqrt{\mathfrak{d}^2 + u^2}, \quad 12,$$
$$\cos.l = \frac{\mathfrak{d}}{60L}, \quad 13,$$
$$\cos.l = \frac{d \sin.C}{60L}, \quad 14,$$
$$\cos.l = \frac{u \tan.C}{60L}, \quad 15,$$
$$L = \frac{\mathfrak{d}}{60\cos.l}, \quad 16,$$
$$L = \frac{d \sin.C}{60\cos.l}, \quad 17,$$
$$L = \frac{u \tan.C}{60\cos.l}, \quad 18,$$
$$\cos.C = \frac{u}{d}, \quad 19,$$
$$\sin.C = \frac{\mathfrak{d}}{d}, \quad 20,$$
$$\tan.C = \frac{\mathfrak{d}}{u}, \quad 21,$$
$$\sin.C = \frac{60L \cos.l}{d}, \quad 22,$$
$$\tan.C = \frac{60L \cos.l}{u}, \quad 23,$$

Example 1. A vessel sails *east-north-east* (6 points) 236 miles. Required her departure $\mathfrak{d}$, and difference in latitude u. (See page 162.)

Formula 1. $\mathfrak{d} = d \sin. C = 236 \times \sin. 6$ points $= 218$ miles departure, and $u = d \cos. c. = 236 \times \cos. 6$ points $= 90.3$ miles difference in latitude.

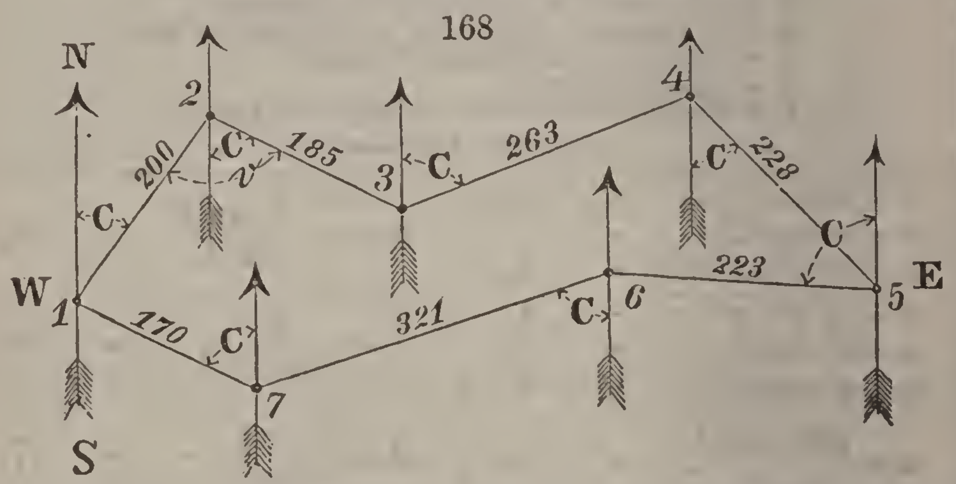
Example 2. A ship sails in north latitude in a course $C = ESE\tfrac{3}{4}E = 6\tfrac{3}{4}$ points; at a distance of 132 miles she made a difference in longitude of $L = 3^\circ\ 34'$. What latitude is she in?

Formula 14. $$\cos. l = \frac{d \sin. C}{60L} = \frac{132 \times \sin. 6\tfrac{3}{4}}{60 \times (3\tfrac{34}{60}^\circ)} = 0.59832;$$
or $l = 53^\circ\ 15'$ the latitude.

In high latitudes and very long distances, the preceding formulas will not give correct results. (See Spherical Trignometry.)

LAND SURVEYING.

Application of formulas on the preceding page.



The operation is readily understood by the illustration. When only an azimuth compass is used, the course C at each station is measured from the magnetic needle or meridian to the direction of the survey. When a theodolite is employed, the course C is read as carefully as possible from the compass at the first station, but at the second station the angle v between the distances is measured, from which subtract the first course, and the remainder will be the second course. At the third station subtract the second course from the angle between the distances, and the remainder will be the third course, and so on. The calculated course is compared with that shown by the compass at each station; if a difference is observed, there may be some errors in the subtraction or angle measurement, or some local attraction of the magnetic needle, which is sometimes the case near great deposits of iron ores. The angles and courses are measured by the theodolite because they cannot be read so delicately on the compass.

At the 5th station, where the 4th and 6th stations are on the same side of the meridian and both north of 5, add the 4th course to the angle 4, 5, 6, and the sum is the new course. On return to the 1st station, where the 7th and 2d stations are both on the same side of the meridian, and one north and the other south, add the angle 2, 1, 7, to the 7th course, subtract the sum from 180°, and the remainder should be the 1st course, which shows the accuracy of the survey.

At station 1 when a transit is used the vernier is clamped at zero, and then the telescope turned until the compass needle is at zero, and the instrument clamped below. The vernier is then loosened and the angle read both from needle and vernier. At the 2d station the telescope is transited for a back sight, fixed below, the vernier loosened again, and the course read from vernier and needle as before. This is called traversing.

Traverse Table for the Survey.

Station.	Course C.	Sin. or cos. C.	Dist. d.	Latitude. N.	Latitude. S.	Departure. E.	Departure. W.
1	N. 35°42′ E.,	cos. 8121 / sin. 5835	200	162.42	. . .	116.70	. . .
2	S. 63 48 E.,	cos. 4415 / sin. 8972	185	. . .	81.68	165.98	. . .
3	N. 68 38 E.,	cos. 3643 / sin. 9312	263	95.81	. . .	244.90	. . .
4	S. 42 25 E.,	cos. 7382 / sin. 6747	228	. . .	168.31	153.78	. . .
5	N. 85 51 W.,	cos. 0723 / sin. 9974	223	16.12	. . .	. . .	222.42
6	S. 72 18 W.,	cos. 3040 / sin. 9526	321	. . .	97.58	. . .	305.78
7	N. 64 27 W.,	cos. 4313 / sin. 9022	170	73.32	. . .	. . .	153.37
Sum of N. S. E. and W., . .				347.67	347.57	681.36	681.57
Subtract the smallest, . .				347.57			681.36
Errors in the measurement, . .				0.10			0.21

Find the natural *sines* and *cosines* in the trigonometrical tables.

The distance, *d*, multiplied by the *cosine* for the course *C*, will be the difference in latitude formula 5.

The distance, *d*, multiplied by the *sine* for the course *C*, will be the departure formula 1.

The formulas and traverse table will answer for any unit of measure, but if the above traverse had been made in miles, whether on land or sea, each departure should be divided by *cosine* for the mean latitude between each two stations, formula 16, in order to obtain the true difference in longitude. To divide by *cosine* is the same as to multiply by the *secant* for the same angle.

Length of a Degree in Parallel of Latitude.

Multiply the length of a degree at the equator (60 sea-miles = 69.03 statute miles = 110.83 kilometres) by *cosine* for the latitude, and the product will be the length of a degree in parallel of latitude.

The length of a minute or second at the equator, multiplied by the *cosine* for the latitude, will be the corresponding length in the parallel of that latitude.

Measurement over Sloping Ground.

d = *Sloping distance.*	b = *Base, or horizontal distance.*	h = *Difference in height.*	v = *Angle of the slopes.*
$d = h$ cosec. v.	$b = d$ cos. v.	$h = d$ sin. v.	sin. $v = \frac{h}{d}$.
$d = b$ sec. v.	$b = h$ cot. v.	$h = b$ tan. v.	tan. $v = \frac{h}{b}$.

The horizontal distance b is equal to the inclined distance d, multiplied by *cosine* for the sloping angle v.

The vertical height h is equal to the inclined distance d, multiplied by *sine* of the sloping angle v.

The natural sine and cosine for any slope will be found in the tables.

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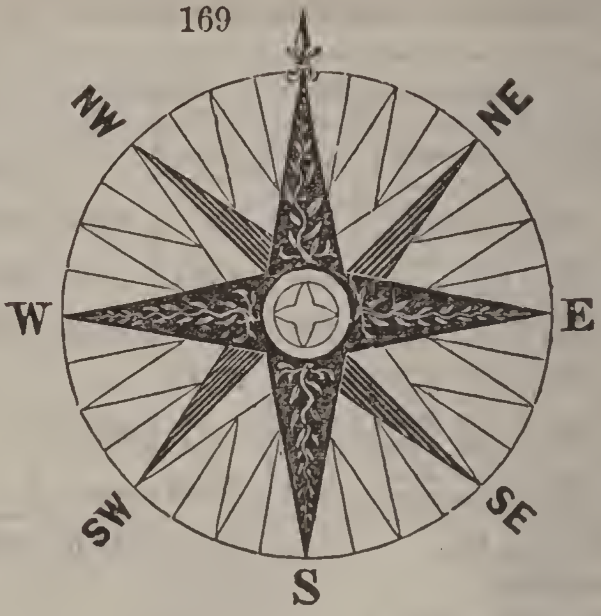

North.	South.	Points.	Angle.	*sineC.*	*Cos.C.*	*tan.C.*
N.	S.	¼	2° 49′	·0491	·9988	·0492
		½	5 37	·0979	·9952	·0983
		¾	8 26	·1544	·9880	·1982
N. by E. and N. by W.	S. by E. and S. by W.	1	11 15	·1936	·9811	·1989
		1¼	14 4	·2430	·9700	·2505
		1½	16 52	·2901	·9570	·3032
		1¾	19 41	·3368	·9416	·3577
N. N. E. and N. N. W.	S. S. E. and S. S. W.	2	22 30	·3827	·9239	·4142
		2¼	25 19	·4276	·9039	·4730
		2½	28 7	·4713	·8820	·5343
		2¾	30 56	·5140	·8577	·5993
N. E. by N. and N. W. by N.	S. E. by S. and S. W. by S.	3	33 45	·5555	·8314	·6883
		3¼	36 44	·5981	·8014	·7463
		3½	39 22	·6343	·7731	·8204
		3¾	42 11	·6715	·7410	·9062
N. E. and N. W.	S. E. and S. W	4	45 0	·7071	·7071	1·000
		4¼	47 49	·7410	·6715	1·103
		4½	50 37	·7731	·6345	1·218
		4¾	53 26	·8014	·5981	1·348
N. E. by E. and N. W. by W.	S. E. by E. and S. W. by W.	5	56 15	·8314	·5555	1·496
		5¼	59 4	·8577	·5140	1·668
		5½	61 52	·8820	·4713	1·870
		5¾	64 41	·9039	4276	2·114
E. N. E and W. N. W.	E. S. E. and W. S. W.	6	67 30	·9239	·3827	2·414
		6¼	70 19	·9416	·3368	2·795
		6½	73 7	·9570	·2901	3·295
		6¾	75 56	·9700	·2430	3·991
E. by N. and W. by N.	E. by S. and W. by S.	7	78 45	·9811	·1936	5·027
		7¼	81 34	·9880	·1544	6·744
		7½	84 22	·9952	·0979	11·14
		7¾	87 11	·9988	·0491	20·32
East or West . .		8	90°	1.000	0.000	∞

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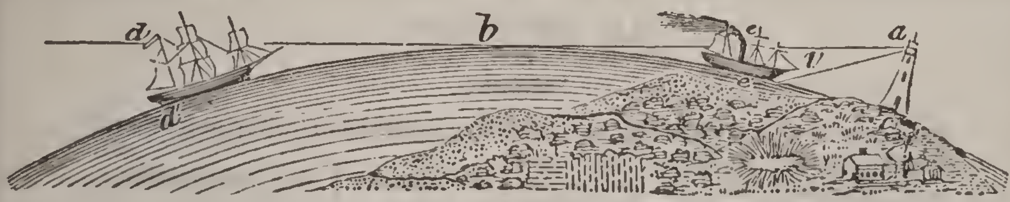


Distance and Dip of Horizon,

from different heights above the surface of the ocean.

Height. Feet.	Distance. Miles.	Dip. ′ ″	Height. Feet.	Distance. Miles.	Dip. ′ ″	Height. Feet.	Distance. Miles.	Dip. ° ′ ″
0.582	1 mile.	0 59	16	5.29	3 56	150	16.22	0 14 07
1*	1.31	0 59	17	5.45	4 03	200	18.72	0 16 18
2	1.87	1 24	18	5.61	4 11	300	22.91	0 19 56
3	2.29	1 42	19	5.77	4 17	400	26.46	0 23 03
4	2.63	1 58	20	5.92	4 24	500	29.58	0 25 46
5	2.96	2 12	25	6.61	4 55	1000	32.41	0 28 18
6	3.24	2 25	30	7.25	5 23	2000	59 20	0 51 42
7	3.49	2 36	35	7.83	5 49	3000	72.50	1 3 24
8	3.73	2 47	40	8.37	6 14	4000	83.70	1 14 15
9	3.96	2 57	45	8.67	6 36	5000	93.50	1 21 54
10	4.18	3 07	50	9.35	6 58	1 mile.	96.10	1 24 01
11	4.39	3 16	60	10.25	7 37	$1\frac{1}{2}$ "	108.96	1 35 40
12	4.58	3 25	70	11.07	8 14	2 "	123.23	1 48 20
13	4.77	3 33	80	11.83	8 48	$2\frac{1}{2}$ "	140.64	2 3 50
14	4.95	3 41	90	12.55	9 20	3 "	154.10	2 15 50
15	5.12	3 49	100	13.23	9 51	5 "	199.15	2 57 15

* For smaller heights, see *Curvature of the Earth.*

The refraction is included in the dip of horizon.

The distance being the tangent *a b* in statute miles, at the elevation *a c*, in feet.

Example 1. The lighthouse at *a* is 100 feet above the level of the sea. Required the distance *a b*.

Height 100 feet = 13.23 miles.

Example 2. The flag of a ship is seen from *a* in *d*. Required the distance *a d*, when the flag is known to be 50 feet above the level *d′* of the sea?

Height of the light 100 = 13.23 miles *a b*,
Height of the flag 50 = 9.35 " *b d*,
Distance to the ship = 22.58 miles *a d*.

Example 3. A steamer is seen at *e*; the horizon *b* seen in the masts is assumed to be 16 feet above the level *e′*. Required the distance to the ship?

Height of the light 100 = 13.23 miles *a b*,
The assumed height 16 = 5.29 " *e b*,
Distance to the ship = 7.94 miles *a e*.

Particular attention is called to page 159, to find the *distance d*, *course C*, *departure* 𝔡, *difference in latitude u*, and *difference in longitude L.* When the *course C* is given by the compass, use the compass table on page 162 for *sin.C*, *cos.C*, and *tan.C*, which is handier than the full trignometrical tables.

CORRECTION FOR CURVATURE OF THE EARTH

IN LEVELING.

Notation.

D = distance in miles from the level to the stave or other object, and
d = the same distance in feet.
C = correction for curvature in feet at the stave; always negative.
c = the same correction in inches.

$$C = \frac{2D^2}{3}. \qquad D = 1.2247\sqrt{C}.$$

$$c = \frac{d^2}{3486543}. \qquad d = 1867.3\sqrt{c}.$$

The accompanying table gives the curvature for distances from 100 feet to 20 miles. For greater distances see table of *Distances and Dip of Horizon.*

Difference of Apparent and True Level or Curvature of the Earth, with and without Refraction.

Distance. Feet.	Curvature. Inches.	Curv. and ref. Feet.	Distance. Miles.	Curvature. Feet.	Curv. and ref. Feet.
100	.0028	.0002	1	0.666	0.575
200	.0115	.0008	2	2.666	2.283
300	.0258	.0018	3	6.000	5.141
400	.0489	.0033	4	10.675	9.150
500	.0717	.0051	5	16.675	14.291
600	.1032	.0073	6	24.083	20.583
700	.1405	.0100	7	32.683	28.167
800	.1835	.0130	8	42.691	36.591
900	.2223	.0158	9	54.025	46.031
1000	.2868	.0204	10	66.700	57.175
1500	.6453	.0459	11	80.708	69.175
2000	1.147	.0817	12	96.050	82.325
2500	1.792	.1276	13	112.716	96.616
3000	2.581	.1836	14	130.732	112.058
3500	3.513	.2500	15	150.075	126.633
4000	4.589	.372	16	170.750	147.191
4500	5.557	.396	17	192.766	165.225
5000	7.170	.5110	18	216.108	185.233
5500	8.676	.6185	19	240.783	206.391
6000	10.324	.7360	20	266.800	228.683

TO FIND THE DIVERGENCY OF THE PARALLEL

FROM THE PRIME VERTICAL.

Notation.

l = latitude of the parallel in degrees.

v = distance on the prime vertical, expressed in angle of the great circle from the base-meridian.

c = divergency in feet of the parallel at the angle v.

$$c = 729000 \sin.^2 \tfrac{1}{2}v \times l.$$

The divergency is calculated in the accompanying table for distances from one second to one degree, also expressed in feet and miles on the prime vertical. The coefficient $c = 729000 \sin.^2 \frac{1}{2}v$, which, multiplied by the latitude of the parallel in degrees, gives the divergency in feet.

Example 1. Suppose the distance on the prime vertical to be $v = 6' = 6$ miles and 4770 feet, the latitude of the parallel being 48°. Required the divergency.

From the table, $0.5551 \times 48° = 26.6448$ feet, the divergency required.

Divergency of the Parallel from the Prime Vertical.

Distance on prime vertical.		Coefficient.	Distance on prime vertical.			Coefficient.
Seconds v.	Feet.	C.	Minutes v.	Miles.	Feet.	C.
1	101.25	0.00000434	1	1	795	0.0154213
2	202.5	0.00001735	1½	1	3832.5	0.0346979
3	303.75	0.00003855	2	2	1590	0.061685
4	405	0.00006916	2½	2	4627.5	0.0964
5	506.25	0.0001071	3	3	2585	0.1387917
6	607.5	0.0001542	4	4	3180	0.24674
7	708.75	0.0002099	5	5	3975	0.3855
8	810	0.00027665	6	6	4770	0.55516
9	911.25	0.0003470	7	8	285	0.75564
10	1012.5	0.0004284	8	9	1080	0.98696
11	1113.75	0.00051833	9	10	1875	1.2491253
12	1215	0.0006168	10	11	2670	1.5420
13	1316.25	0.00072394	11	12	3465	1.865820
14	1417.50	0.0008396	12	13	4260	2.220604
15	1518.75	0.0009638	13	14	5055	2.6062
16	1620	0.0010966	14	16	570	3.02256
17	1721.25	0.0012380	15	17	1365	3.4696
18	1822.5	0.0013879	16	18	2160	3.94783
19	1923.75	0.0015464	18	20	3750	4.9965012
20	2025	0.0017135	20	23	60	6.1680
25	2531.25	0.002677	25	28	4035	9.637500
30	3037.5	0.0038553	30	34	2730	13.8785
35	3543.75	0.0052475	35	40	1425	18.8895
40	4050	0.0068539	40	45	120	24.6720
45	4556.25	0.0086742	45	51	4095	31.22815
50	5062.5	0.010709	50	57	2790	38.5500
55	5568.75	0.012958	55	63	1485	46.6455
60	6075	0.154213	60	69	180	55.5151

The length of minutes and seconds on the parallel is equal to that in the table, multiplied by cosine for the latitude.

These calculations are necessary in running a parallel of latitude by fore and back sighting, and also for laying out the parallels and meridians on a map.

Empirical Formulas.

In making use of empirical formulas, whether derived by the method of interpolation following or otherwise, it must always be remembered that they are valueless beyond the limits of the observation on which they are based.

Interpolation is to insert numerical values between given data, for constructing tables or empirical formulas expressing the probable relative variation of quantities. Let x and y be two variable quantities depending on one another and measured in simultaneous stages of their progress, as

$$x_1 \; x_2 \; x_3 \; x_4 \text{ and } x_5$$

$$y_1 \; y_2 \; y_3 \; y_4 \text{ and } y_5$$

We have $y = Ay_1 + By_2 + Cy_3 + Dy_4 + Ey_5 + \&c.$ - - - - 1

2, 3, 4, 5 given data.

$$A = \frac{(x - x_2)\,(x - x_3)\,(x - x_4)\,(x - x_5)}{(x_1 - x_2)\,(x_1 - x_3)\,(x_1 - x_4)\,(x_1 - x_5)}$$

$$B = \frac{(x - x_1)\,(x - x_3)\,(x - x_4)\,(x - x_5)}{(x_2 - x_1)\,(x_2 - x_3)\,(x_2 - x_4)\,(x_2 - x_5)}$$

2 > - - - - -

$$C = \frac{(x - x_1)\,(x - x_2)\,(x - x_4)\,(x - x_5)}{(x_3 - x_1)\,(x_3 - x_2)\,(x_3 - x_4)\,(x_3 - x_5)}$$

3 > - - - - - - - - - -

$$D = \frac{(x - x_1)\,(x - x_2)\,(x - x_3)\,(x - x_5)}{(x_4 - x_1)\,(x_4 - x_2)\,(x_4 - x_3)\,(x_4 - x_5)}$$

4 > - - - - - - - - - - - - - -

$$E = \frac{(x - x_1)\,(x - x_2)\,(x - x_3)\,(x - x_4)}{(x_5 - x_1)\,(x_5 - x_2)\,(x_5 - x_3)\,(x_5 - x_4)}$$

5 > - - - - - - - - - - - - - - - - - - -

Number of given data.

The values of the coefficients A, B, C, D, and E, with their given data, inserted in formula 1 gives an empirical formula for the variation of x and y. The number of observations or given data of x and y should be one more than the order of progression. In arithmetical progression two observations are sufficient for a correct formula. For all curves in the conic sections, or others which are of the second order, there should be at least three observations. Pressure of steam progresses with the temperature in the 6th order, for which requires seven observations to make a correct formula. When the order of progression is not known, the more observations gives the most correct result.

Example. Let y represent the boiling-point of salt water and x the percentage of salt in solution. It is found in three experiments,

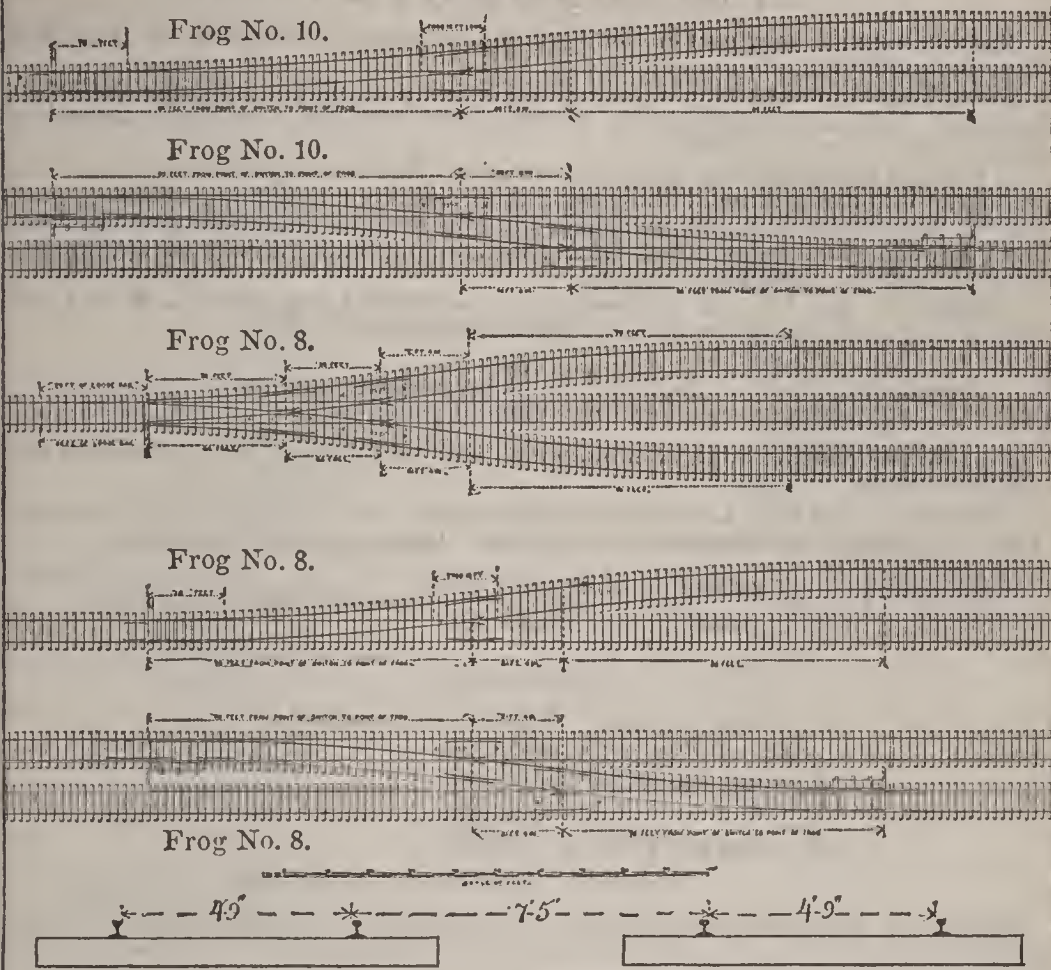
that $x_1 = \mathbf{3}$, $x_2 = 18$, $x_3 = 36$ per cent. salt.
when $y_1 = 213{\cdot}2$ $y_2 = 219^\circ$, $y_3 = 226^\circ$ boiling-point.

Find a formula that will give any intermediate value of x and y?

$$A = \frac{(x-18)(x-36)}{(3-18)(3-36)}, \quad B = \frac{(x-3)(x-36)}{(18-3)(18-36)}, \quad C = \frac{(x-3)(x-18)}{(36-3)(36-18)},$$

$$y = 213{\cdot}2\,A + 219\,B + 226\,C. \qquad y = 0{\cdot}4x + 212$$

Standard Turnouts and Crossings, P. R. R.



MEASUREMENTS ON SECTION OF TRACK.

No. of Frog.	Angle.	Curve. Degree.	Curve. Radius.	Pt. of Frog to Heel of Switch. g(in ft.)$\times 2n$.	Pt. of Frog to do. measured on Tangent. Approx. $(d-g)\times n$.	Pt. of Frog to do. measured on Tangent. Actual $\frac{(d-g)n}{\cos.^2 a}+\frac{g}{2n}$.	Between Frogs on Cen. Line. Actual $\frac{(d-g)\,n}{\cos.^2 a}-\frac{d}{2n}$.
	° ′ ″	° ′ ″					
4	14 15 00	37 47 46	154.4	38	$10\frac{2}{3}$	11.42	9.90
6	9 31 38	16 41 48	344.4	57	16	16.51	15.49
8	7 9 10	9 23 52	610.4	76	$21\frac{1}{3}$	21.71	20.96
10	5 43 29	6 1 8	952.4	95	$26\frac{2}{3}$	26.97	26.36
12	4 46 18	4 10 54	1370.4	113	32	32.24	31.75
15	3 49 6	2 40 40	2139.9	142.5	40	40.22	39.79

d = distance between tracks.
g = gauge.
n = number of frog.
$a = \frac{1}{2}$ frog angle.

The frogs are all 15 feet long.

Approved as standard April 1, 1883.

W. H. BROWN,
Chief Engineer.

TRIGONOMETRY.

TRIGONOMETRY is that part of Geometry which treats of triangles. It is divided into two parts—viz., *plane* and *spherical.*

Plane Trigonometry treats of triangles which are drawn (or imagined to be) on a plane. Spherical Trigonometry treats of the triangles which are drawn (or imagined to be) on a sphere.

A triangle contains seven quantities—namely, three sides, three angles and the surface. When any three of these quantities are given, the four remaining ones can by them be ascertained (one side or the area must be one of the given quantities), and the operation is called *solving the triangle*, which is only an application of arithmetic to geometrical objects.

Example 1. Fig. 173. An inclined plane $a = 150$ feet long, and $c = 27$ feet, the height over its base. What is the angle of inclination $C = ?$

Formula 14. $$\sin.C = \frac{c}{a} = \frac{27}{150} = 0.18000.$$

Find 9.18000* in the table of *sines*, which will be found at $10° 30'$, which is the angle C nearly.

Example 2. Fig. 174. An oblique-angled triangle has the sides $c = 27.6$ feet, the angle $C = 34° 10'$, and the angle $A = 47° 40'$. How long is the side $a = ?$

Formula 1. $$a = \frac{c \sin.A}{\sin.C} = \frac{27.6 \times \sin. 47° 40'}{\sin. 34° 10'} = 36.33 \text{ feet, the answer.}$$

By Logarithms.

$$\log.a = \log.c + \log.\sin.A - \log.\sin.C.$$

$$\begin{array}{rr} c + \qquad \log. 27.6 = & 1.44090 \\ A + \log.\sin. 47° 40' = & \underline{9.86878} \\ & 1.30968^* \\ C - \log.\sin. 34° 10' = & \underline{9.74942} \\ \log. 36.4 = & 1.56026, \text{ or } a = 36.4 \text{ feet.} \end{array}$$

Example 3. Two ships A and B are 800 yards apart; the angles between each other and a castle C are $A = 63° 45'$, $B = 75° 50'$. What are the two distances from the castle?

$$C = 180 - 63° 45' - 75° 50' = 40° 25'.$$

Form. 4.

To A the distance will be,

$$b = \frac{c \sin.B}{\sin.C} = \frac{800 \times \sin. 75° 50'}{\sin. 40° 25'} = 1195.75 \text{ yards.}$$

Form 1.

To B the distance will be,

$$a = \frac{c \sin.A}{\sin.C} = \frac{800 \times \sin. 63° 45'}{\sin. 40° 25'} = 1106.6 \text{ yards.}$$

* The index of a logarithm for a fraction is negative; but in the logarithms for the trigonometrical functions, 10 is added to the index, for which it appears so much less than 10 as the real negative index. Therefore, when trigonometrical logarithms are added, 10's must be rejected from the sum of the index, which will be understood by the examples.

$$\cot.B = \frac{c}{b \cos.A} - \cot.A. \qquad \tan.A = \frac{a \sin.C}{b + a \cos.C}.$$

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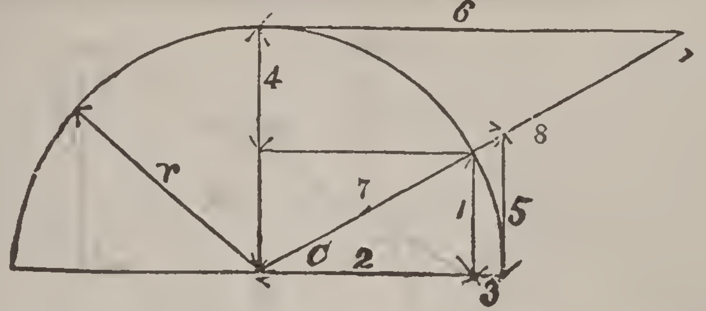


1 *Sine*	abbreviated	*sin.C.*
2 *Cosine*	"	*cos.C.*
3 *Versed sine*	"	*ver.sin. C.*
4 *Coversed sine*	"	*co.ver.sin.C.*
5 *Tangent*	"	*tan.C.*
6 *Cotangent*	"	*cot.C.*
7 *Secant*	"	*sec.C.*
8 *Cosecant*	"	*cosec.C.*

r = *Radius* of the circle, which is the unit by which the functions are measured.

$$r^2 = \sin.^2 C + \cos.^2 C.$$

$$\tan. C = \frac{\sin. C}{\cos. C},$$

$$\tan. C = \frac{1}{\cot. C},$$

$$\cot. C = \frac{\cos. C.}{\sin. C},$$

$$\cot. C = \frac{1}{\tan. C},$$

$$\sec. C = \frac{1}{\cos. C},$$

$$\text{cosec}. C = \frac{1}{\sin. C},$$

$$\text{sinv}. C = 1 - \cos. C,$$

$$\text{cosv}. C = 1 - \sin. C,$$

$$\sin. 2C = 2 \sin. C \cos. C,$$

$$\sin. \tfrac{1}{2}C = \tfrac{1}{2}\sqrt{(\sin.^2 C + \text{sinv}.^2 C)},$$

$$\sin.(C \pm B) = \sin. C \cos. B \pm \sin. B \cos. C.$$

Positive and Negative Signs.

Angles.	sin.	cos.	ver.sin.	co.ver.sin.	tan.	cot.	sec.	cosec.
+0°	+0	+1	+0	+1	+0	+∞	+1	+∞
+90°	+1	±0	+1	+0	±∞	±0	±∞	+1
+180°	±0	−1	+2	+1	∓0	∓∞	−1	±∞
+270°	−1	∓0	+1	+2	±∞	±	∓∞	−1
+360°	∓0	+1	+0	+1	∓0	−∞	+1	−∞

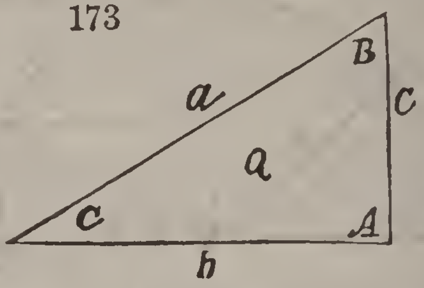
When a quantity has reached 0 or ∞, it has ceased to be comprehensible, because it cannot be increased or diminished.

Example. What is the length of the secant for an angle of 74° 18′?

$$\text{Secant C} = \frac{1}{\cos. 74^\circ 18'} = 3{\cdot}695.$$

FORMULAS FOR RIGHT-ANGLED TRIANGLES.

173



$$a = \sqrt{b^2 + c^2}, \qquad 1,$$

$$a = \frac{c}{\sin. C}, \qquad 2,$$

$$a = \frac{b}{\cos. C}, \qquad 3,$$

$$a = 2\sqrt{\frac{Q}{\sin. 2C}}, \qquad 4,$$

$$b = a \cos. C, \qquad 5,$$

$$b = c \cot. C, \qquad 6,$$

$$b = a \sin. B, \qquad 7,$$

$$b = c \tan. B, \qquad 8,$$

$$b = \sqrt{\frac{2Q}{\tan. C}}, \qquad 9,$$

$$Q = \frac{a^2 \sin. 2C}{4}, \qquad 10,$$

$$Q = \tfrac{1}{2}\, b^2 \tan. C, \qquad 11,$$

$$Q = \tfrac{1}{2}\, c^2 \cot. C, \qquad 12,$$

$$Q = \tfrac{1}{2}\, c\sqrt{(a+c)(a-c)} \qquad 13,$$

$$\sin. C = \frac{c}{a}, \qquad 14,$$

$$\cos. C = \frac{b}{a}, \qquad 15,$$

$$\tan. C = \frac{c}{b}, \qquad 16,$$

$$\sin. 2C = \frac{4Q}{a^2}, \qquad 17,$$

$$\tan. C = \frac{2Q}{b^2}, \qquad 18,$$

Say the angle to be $C = 60°$. In the first column of the table of *sines*, 60° corresponds with 0·86602 in the next column, which is the length of *sin.* 60°, when the radius of the circle is one, or the unit, and the expression *sin.* $60° \times 36$ means $0{\cdot}86602 \times 36 = 31{\cdot}17672$, and likewise with all the other Trigonometrical expressions.

In a triangle the functions of an angle have a certain relation to the opposite side; it is this relationship which enables us to solve the triangle by the application of Simple Arithmetic.

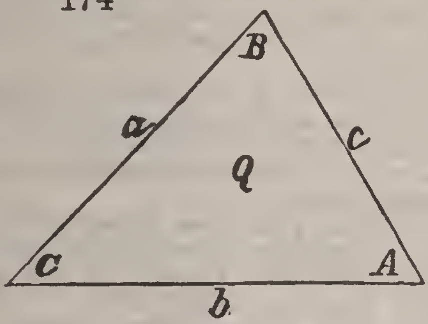
In triangles the sides are denoted by the letters *a*, *b*, and *c*; their respective opposite angles are denoted by *A*, *B*, and *C*, and the area by *Q*.

Example 1. Fig. 173 The side *c* in a right angled Triangle being 365 feet, and the angle $C = 39° 20'$. How long is the side $a = ?$

Formula 2. $a = \dfrac{c}{sin. C} = \dfrac{365}{sin.\ 39°.20'} = \dfrac{365}{0{\cdot}63383} = 575{\cdot}86$ feet, the answer.

FORMULAS FOR OBLIQUE-ANGLED TRIANGLES.

174



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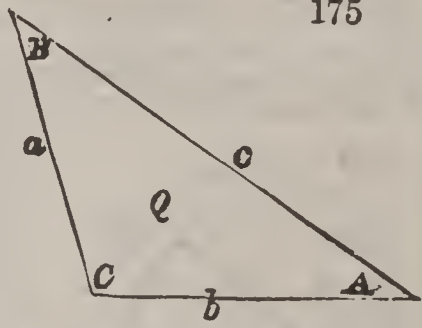

$a : b = \sin.A : \sin.B$, and $b : c = \sin.B : \sin.C$.
$a : c = \sin.A : \sin.C$, and $Q : ab = \sin.C : 2$.

$$a = \frac{c \sin.A}{\sin.C}, \qquad 1,$$

$$a = \frac{c \sin.A}{\sin.(A+B)}, \qquad 2,$$

$$a = \frac{2Q}{b \sin.C}, \qquad 3,$$

$$b = \frac{c \sin.B}{\sin.C}, \qquad 4,$$

$$b = \frac{2Q}{c \sin.A}, \qquad 5,$$

$$\sin.C = \frac{c \sin.B}{b}, \qquad 6,$$

$$\sin.C = \frac{c \sin.A}{a}, \qquad 7,$$

$$\sin.A = \frac{2Q}{bc}, \qquad 8,$$

$$\sin.A = \frac{a \sin.C}{c}, \qquad 9,$$

$$a = \sqrt{b^2+c^2-2b\,c \cos.A}, \qquad 10,$$

$$a = \sqrt{\frac{2Q \sin.A.}{\sin.B \sin.(A+B)}} \qquad 11,$$

$$S = \tfrac{1}{2}(a+b+c) \qquad 12,$$

$$\sin.\tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}, \qquad 13,$$

$$\sin.\tfrac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{ac}}, \qquad 14,$$

$$\cos.\tfrac{1}{2}A = \sqrt{\frac{s\,(s-a)}{bc}}, \qquad 15,$$

$$\cos.\tfrac{1}{2}B = \sqrt{\frac{s\,(s-b)}{ac}}, \qquad 16,$$

$$Q = \frac{bc \sin.A}{2}, \qquad 17,$$

$$Q = \frac{ab \sin.C}{2}, \qquad 18,$$

$$Q = \frac{c^2 \sin.A \sin.B}{2 \sin.(A+B)}, \qquad 19,$$

$$Q = \sqrt{S-a)(S-b)(S-c)S} \qquad 20,$$

$$b = \sqrt{\frac{2Q \sin.(A+C)}{\sin.A \sin.C}}, \qquad 21,$$

$$c = \sqrt{\frac{2Q \sin.C}{\sin.A \sin.(A+C)}}, \qquad 22$$

SPHERICAL TRIGONOMETRY.

Spherical Trigonometry treats of triangles which are drawn (or imagined to be) on the surface of a sphere. Their sides are arcs of the great circle of the sphere, and measure by the angle of the arc. Therefore the trigonometrical functions bear quite a different relation to the sides.

Every section of a sphere cut by a plane is a *circle*. A line drawn through the centre and at right angles to the sectional circle is called an *axis*, and the two points where the axis meets the surface of the sphere are called the *poles* of the sectional circle.

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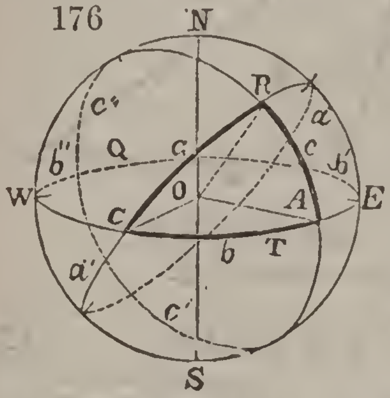


When the cutting plane goes through the centre of the sphere, it will pass through the great circle, and is then called the *Equator* for the poles. Axis = *N.S.* Equator — *G.E.T.W.*

Three great circle-planes, $a\,a'a''a'''$, $b\,b'b''$, and $c\,c'c''$, cutting a sphere, *NESW*, will form a solid angle at the centre *O*, and a triangle *ABC* on the surface of the sphere, in which the arcs a, b, c, are the sides. The angles formed by each two planes are congruent to each of the appertinent angles *A*, *B* and *C*.

Spherical Distances.

For the spherical distances, letters will denote,

l = lower latitude, in degrees from the equator.
l' = highest latitude, " " "
C = course, from the latitude l to l'.
C' = course, from " l' to l.
d = shortest distance between l and l' in degrees of the great circle.
L = difference in longitude between l and l', in degrees, or time.
$\tan. m = \cot. l' \cos. L.$
$n = 90 \mp l - m.$
$-\,l$, when l and l' are on one side of the equator.
$+\,l$, when l is on one side, and l' on the other. Then

$$\cos. d = \frac{\sin. l' \cos. n}{\cos. m}, \quad \ldots \quad 1.$$

$$\sin. C = \frac{\sin. L \cos. l'}{\sin. d}, \quad \ldots \quad 2.$$

$$\sin. C' = \frac{\sin. L \cos. l}{\sin. d}, \quad \ldots \quad 3.$$

Example. Required the shortest distance and course from New York to Liverpool.

$l = 40^\circ 42'$ N. latitude, $74^\circ 42'$ W. longitude, } New York.

$l' = 53^\circ 22'$ N. latitude, $2^\circ 52'$ W. longitude, } Liverpool.

$L = 71^\circ 8'$ difference in longitude.

$$\tan. m = \cot. 53^\circ 22' \times \cos. 71^\circ 8' = 13^\circ 31'.$$

$$n = 90^\circ - 13^\circ 31' - 40^\circ 42' = 35^\circ 47'.$$

Formula 1. $\cos. d = \frac{\sin. 53^\circ 22' \times \cos. 35^\circ 47'}{\cos. 13^\circ 31''} = 47^\circ 58'.$

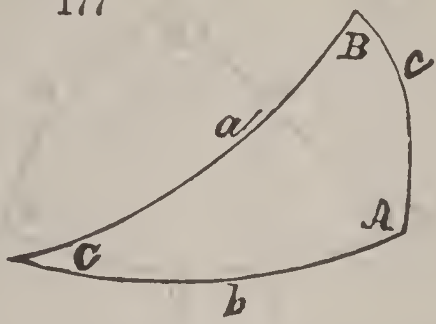
Shortest distance $= 47^\circ \times 60 + 58 = 2878$ geographical miles.

$$\sin. C = \frac{\sin. 71^\circ 8' \times \cos. 53^\circ 22'}{\sin. 47^\circ 58'} = 49^\circ 23' = 4\tfrac{3}{8} \text{ points},$$

course from New York $NE\tfrac{3}{8}E$.

RIGHT-ANGLED SPHERICAL TRIANGLE.

177



$$\sin.b = \sin.a \sin.B, \qquad 1,$$

$$\tan.c = \tan.a \cos.B, \qquad 2,$$

$$\cot.C = \cos.a \tan.B, \qquad 3,$$

$$\tan.c = \sin.b \tan.C, \qquad 4,$$

$$\cos.a = \cos.b \cos.c, \qquad 5,$$

$$\cos.B = \cos.b \sin.C, \qquad 6,$$

$$\tan.a = \frac{\tan.b}{\cos.C}, \qquad 7,$$

$$\sin.c = \frac{\tan.b}{\tan.B}, \qquad 8,$$

$$\sin.a = \frac{\sin.b}{\sin.B}, \qquad 9,$$

$$\sin.C = \frac{\cos.B}{\cos.b}, \qquad 10,$$

$$\cos.c = \frac{\cos.a}{\cos.b}, \qquad 11,$$

$$\sin.B = \frac{\sin.b}{\sin.a}, \qquad 12,$$

$$\cos.C = \frac{\tan.b}{\tan.a} \qquad 13,$$

$$\tan.C = \frac{\tan.c}{\sin.b}, \qquad 14,$$

$$\tan.B = \frac{\tan.b}{\sin.c}, \qquad 15,$$

$$\cos.c = \frac{\cos.C}{\sin.B}, \qquad 16,$$

$$\cos.b = \frac{\cos.B}{\sin.C}, \qquad 17,$$

$$\cos.a = \frac{\cot.C}{\tan.B}, \qquad 18.$$

The sum of the three angles in a spherical triangle is greater than two right angles, and less than six right angles.

By Spherical Trigonometry we ascertain distances and courses on the surface of the earth; positions and motions of the heavenly bodies, &c., &c. Examples will be furnished in Geography and Astronomy.

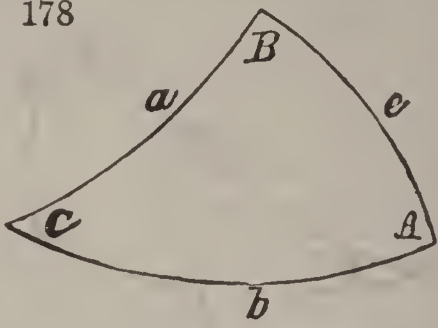
Example 1. Fig. 177 In a right-angled spherical triangle the side or hypothenuse $a = 36° 20'$, the angle $B = 68° 50'$. How long is the side b =?

Formula 1. $\sin.b = \sin.a.\sin.B = \sin.36°20' \times \sin.68°60'$.

a	log.sin. 36° 20′ = 9:77267	
B	log.sin. 68° 50′ = 9:96966	
The answer,	log.sin. 33° 32′ = 9:74233	**or $b = 33° 32'$.**

OBLIQUE-ANGLED SPHERICAL TRIANGLE.

178



$$\sin.a : \sin.b = \sin.A : \sin.B, \qquad \sin.a = \frac{\sin.b \sin.A}{\sin.B}, \qquad 19$$

$$\sin.b : \sin.c = \sin.B : \sin.C, \qquad \sin.b = \frac{\sin.c \sin.B}{\sin.C}, \qquad 20$$

$$\tan.\tfrac{1}{2}(a+b) = \tan.\tfrac{1}{2}c \frac{\cos.\frac{1}{2}(A-B)}{\cos.\frac{1}{2}(A+B)}, \quad - \quad - \quad - \quad 21,$$

$$\tan.\tfrac{1}{2}(a-b) = \tan.\tfrac{1}{2}c \frac{\sin.\frac{1}{2}(A-B)}{\sin.\frac{1}{2}(A+B)}, \quad - \quad - \quad 22,$$

$$\tan.\tfrac{1}{2}(B+C) = \cot.\tfrac{1}{2}A \frac{\cos.\frac{1}{2}(b-c)}{\cos.\frac{1}{2}(b+c)}, \quad - \quad - \quad 23,$$

$$\tan.\tfrac{1}{2}(B-C) = \cot.\tfrac{1}{2}A \frac{\sin.\frac{1}{2}(b-c)}{\sin.\frac{1}{2}(b+c)}, \quad - \quad - \quad 24,$$

$$\cot.\tfrac{1}{2}A = \tan.\tfrac{1}{2}(B-C) \frac{\sin.\frac{1}{2}(b+c)}{\sin.\frac{1}{2}(b-c)} \quad - \quad - \quad 25,$$

$$\tan.\tfrac{1}{2}c = \tan.\tfrac{1}{2}(a-b) \frac{\sin.\frac{1}{2}(A+B)}{\sin.\frac{1}{2}(A-B)}, \quad - \quad 26,$$

Example 2. Fig. 178 Oblique angled spherical triangle. $c = 72° 30'$. $B = 17° 30'$. $C = 79° 50'$.
How long is the side $b = ?$

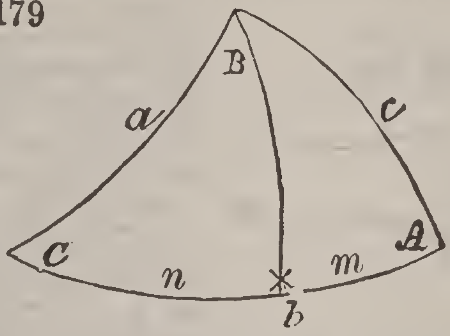
Formula 20. $\sin.b = \dfrac{\sin.c \sin.B}{\sin.C} = \dfrac{\sin.72° 30' \times \sin.17° 30'}{\sin.79° 50'}$

c	+ log.sin. 72° 30′ =	9:97942
B	+ log.sin. 17° 30′ =	9:47812
	+ =	1:45754
C	+ log. sin. 79° 50′ =	9:99312
The answer	log.sin. 16° 56′ =	9:46442

or $b = 16° 56'$

OBLIQUE-ANGLED SPHERICAL TRIANGLE.

179



$$\tan.\tfrac{1}{2}(m+n)\tan\tfrac{1}{2}(m-n = \tan.\tfrac{1}{2}(a+c)\tan\tfrac{1}{2}(a-c)$$

$$\tan.m = \tan.c\cos.A, \quad \text{- - - - } \quad 27,$$

$$\tan.C = \frac{\sin.m\tan.A}{\sin.(b-m)}, \quad \text{- - - - - } \quad 28,$$

$$\cos.a = \frac{\cos.c\cos.(b-m)}{\cos.m}, \quad \text{- - - - } \quad 29,$$

$$\cos.n = \frac{\cos.a\cos.m}{\cos.c}, \quad \text{- - - - - } \quad 30,$$

$$b = m \pm n.$$

$$\cot.m = \frac{\cos.c\tan.A}{\tan.a}, \quad \text{- - - - - - } \quad 31,$$

$$s = \frac{a+b+c}{2} \qquad S = \frac{A+B+C}{2},$$

$$\sin.\tfrac{1}{2}A = \sqrt{\frac{\sin.(s-c)\sin.(s-b)}{\sin.b\sin.c}}, \quad \text{- - } \quad 32,$$

$$\sin.\tfrac{1}{2}a = \sqrt{\frac{\cos.S\cos.(S-A)}{\sin.B\sin.C}}, \quad \text{- - } \quad 33,$$

To Find the Area of a Spherical Triangle.

Let Q be the area of the triangle in square degrees; if R = radius of the sphere, the length of one degree will be,

$$= \frac{2\pi R}{360}, \text{ or one square degree} = \frac{R^2}{3285{\cdot}58}.$$

$$\cot.\tfrac{1}{2}Q = \frac{\cot.\tfrac{1}{2}c\cot.\tfrac{1}{2}a + \cos.B}{\sin.B}, \quad \text{- - - - } \quad 1,$$

$$\sin.\tfrac{1}{2}Q = \frac{\sin.\tfrac{1}{2}c\sin.\tfrac{1}{2}a\sin.B}{\cos.\tfrac{1}{2}b}, \quad \text{- - - - } \quad 2,$$

ANALYTICAL GEOMETRY AND CONIC SECTIONS.

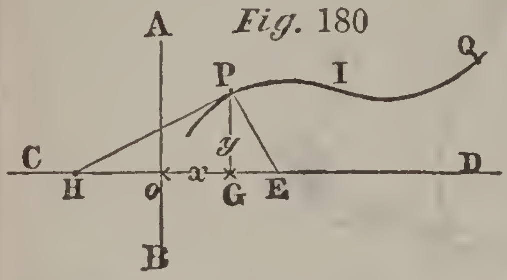


Fig. 180

An equation of a line is generally referred to rectangular lines, A B = axis of ordinates and C D = axis of abscissas. The position of any point P in the curved line P I Q is defined by the rectangular distances, y the ordinate and x the abscissa; x and y are variables, depending on one another. Any change in either of them will produce a change in the other, in accordance with the formulæ for the line. The position of a number of points can be determined, located and joined into the required line of the equation.

The ordinate y generally constitutes the first member of the equation, and its value is determined by assumed values of the abscissa x.

The junction of the two axes is called *origin*, and denoted by *o*. The line will not pass through the *origin* when the equation has a constant term.

Properties of Lines Referred to Rectangular Co-ordinates.

The tangent of any curve, ... $HP = y\sqrt{1 + \frac{dx^2}{dy^2}}$. . 1.

The subtangent of any curve, . . . $HG = y\frac{dx}{dy}$. . . 2.

The normal of any curve, ... $PE = y\sqrt{1 + \frac{dy^2}{dx^2}}$. . 3.

The subnormal of any curve, . . . $GE = y\frac{dy}{dx}$. . . 4.

The point of inflection, I, where convex and concave curves tangent, or where a curve reverses, is when . . . . $\frac{d^2y}{dx^2} = o$, or ∞. . . 5.

Let z denote the length of any curve, then $dz = \sqrt{dx^2 + dy^2}$. . 6.

The radius of curvature of any curve is . $R = \frac{dz^3}{dx\, d^2y}$. . . 7.

The ordinate y is a maximum or minimum when $\frac{dy}{dx} = 0$. . . . 8.

(See Maxima and Minima.)

A curve is *convex* to the axis of abscissas when the ordinate and second differential coefficient have the same sign, but *concave* when either of them is positive and the other negative I Q is convex, and P I concave, to the abscissa C D.

A Conic Section is the section obtained when a plane cuts a cone.

The conic sections are of five different kinds, namely:

1st. *Triangle.* When the plane cuts the cone through its axis.
2d. *Circle.* When the plane cuts the cone at right angles to its axis.
3d. *Ellipse.* When the plane cuts the cone obliquely, passing through the *two sides.*
4th. *Parabola.* When the plane cuts the cone parallel to one side.
5th. *Hyperbola.* When the plane cuts the cone at an angle to the axis less than the angle of the axis and the side of the cone.

181.

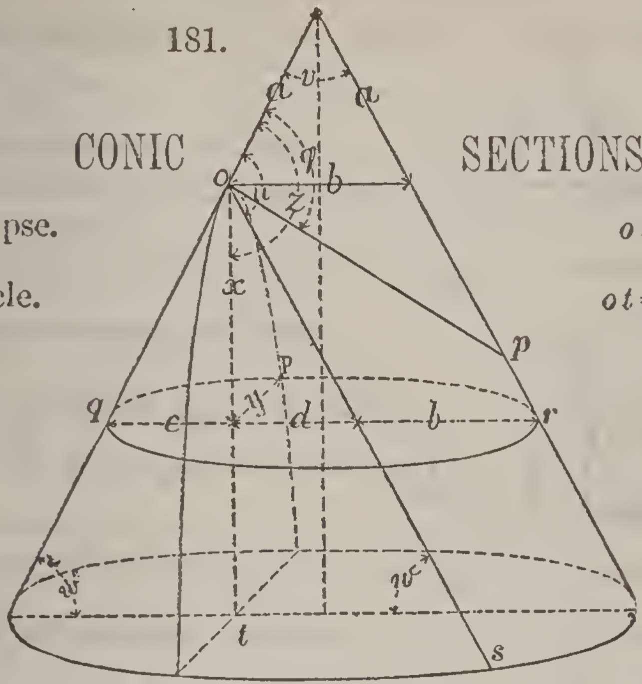
CONIC SECTIONS.

op = ellipse.

qr = circle.

os = parabola.

ot = hyperbola.



$a : b = \sin.w : \sin.v.$

$x : c = \sin.w : \sin.z.$

$x : d = \sin.w : \sin.(z+v).$

$y^2 = c\,(d+b), \quad . \quad . \quad . \quad . \quad . \quad . \quad 4.$

$b = \dfrac{a \sin.v}{\sin.w}, \quad . \quad . \quad . \quad . \quad . \quad . \quad 1.$

$c = \dfrac{x \sin.z}{\sin.w}, \quad . \quad . \quad . \quad . \quad . \quad . \quad 2.$

$d = \dfrac{x \sin.(z+v)}{\sin.w}, \quad . \quad . \quad . \quad . \quad . \quad 3.$

$$y^2 = \frac{x \sin z}{\sin.^2 w}\left[x \ \sin.(z+v) + a \ \sin.v\right], \quad . \quad . \quad . \quad 5.$$

This is the general formula for all conic sections.

Any point P in any curve of the conic sections can be calculated by the formula 5. The formula will, however, be greatly simplified for the different sections.

For a parabola $z+v=180°$, and $\sin.z = \sin.v$, for which

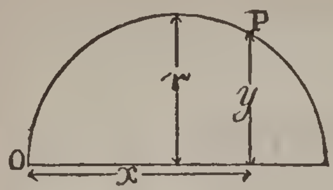
$$y^2 = \frac{a x \sin.^2 v}{\sin.^2 w}.$$

The *parameter* of the ellipse, parabola, and hyperbola is the ordinate y passing through the focus of the curve, and is generally denoted by the letter p.

The *origin* is the point where the rectangular co-ordinates meet, and from which the ordinates and abscissas are measured. The origin is generally denoted by the letter o.

The radius of curvature at the vertex of any conic section is equal to half the parameter.

182.



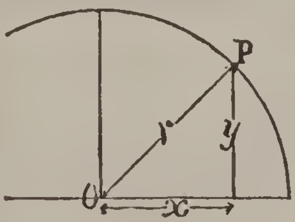
Circle. Origin in Circumference and Diameter.

$y = \sqrt{2rx - x^2}$, $\qquad r = \dfrac{x^2 + y^2}{2x}$,

$x = r + \sqrt{r^2 - y^2}$, $\quad r =$ radius of the circle,

$x =$ abscissa and $y =$ ordinate for the circle.

183.

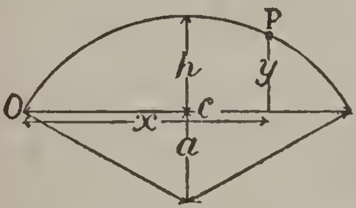


Circle. Origin in the Centre.

$y = \sqrt{r^2 - x^2}$, $\quad x = \sqrt{r^2 - y^2}$, $\quad r = \sqrt{x^2 + y^2}$.

The circle can be plotted by these formulas.

184.

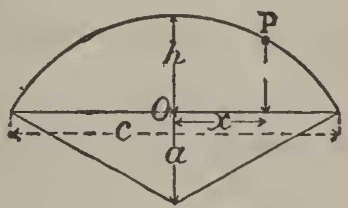


Circle Arc. Origin in the Arc.

$y = \sqrt{a^2 + cx - x^2} - a$,

$a = \dfrac{c^2 - 4h^2}{8h}$, the distance of the chord from the centre.

185.

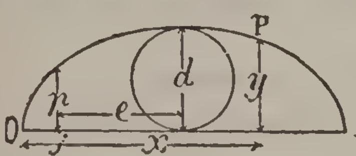


Circle Arc. Origin in Centre of Chord.

$$y = \sqrt{\left(\frac{c^2 + 4h^2}{8h}\right)^2 - x^2} - a,$$

$$a = \frac{c^2 - 4h^2}{8h}.$$

186.



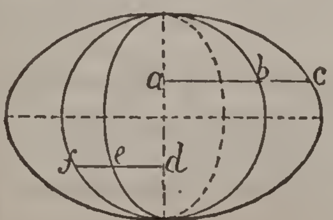
Cycloid.

$y = 0.637\sqrt{x(\pi d - x)}$, $\qquad e = 1.211\, d$,

$p = 0.632\, d$. $\qquad p =$ parameter.

$f =$ focus of the cycloid.

187. *Circle and Ellipse.*



If a circle be described on the minor axis of an ellipse, any ordinate drawn from the minor axis, such as ab and ac, will be

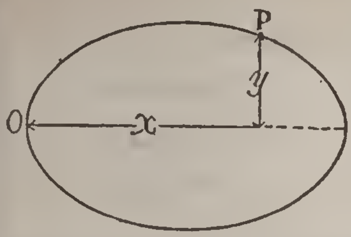
$$ab : ac = n : m.$$

When a circle is described on the major axis, we have

$$de : df = n : m.$$

An ellipse can be considered a circle seen in perspective. An ellipse seen in perspective along the major axis can appear like a circle.

188.

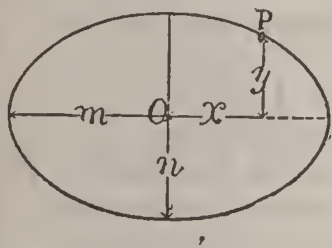


Ellipse.

$$y = \frac{n}{m}\sqrt{2\,m\,x - x^2}.$$

m = major semi-axis.
n = minor semi-axis.

189.

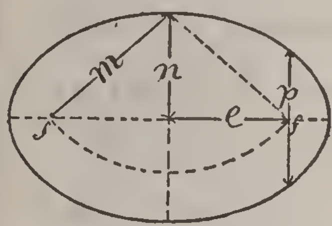


Ellipse.

$$y = \sqrt{n^2 - \left(\frac{n\,x}{m}\right)^2}.$$

m = major semi-axis.
n = minor semi-axis.

190.

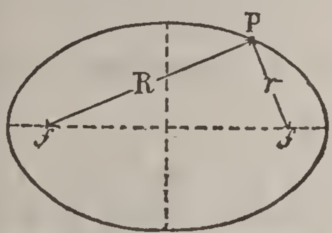


Focus and Parameter of an Ellipse.

$$e = \sqrt{m^2 - n^2}, \qquad p = \frac{2\,n^2}{m}.$$

e = distance from centre for focus f.
p = parameter of the ellipse.

191.

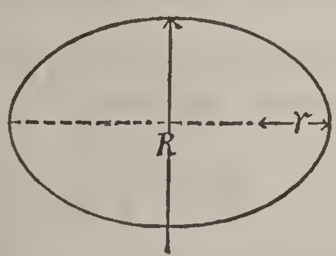


Radius Vector of an Ellipse.

$$R = 2\,m - r, \qquad r = 2\,m - R, \qquad 2\,m = R + r.$$

The major axis is the sum of $R + r$.

192.

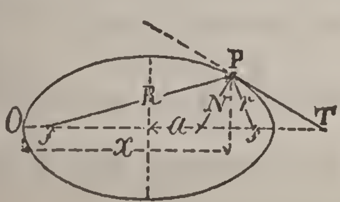


Radius of Curvature of an Ellipse.

$$r = \frac{n^2}{m}, \qquad R = \frac{m^2}{n}, \qquad r = \tfrac{1}{2}p.$$

m = major semi-axis.
n = minor semi-axis.

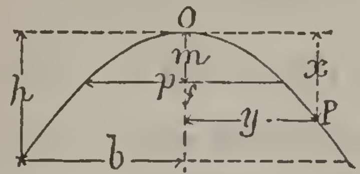
193.



The normal PN at any point of an ellipse bisects the angle formed by the radii vectors R and r, and a line PT drawn through P at right angles to the normal N will be tangent to the ellipse in P.

$$a = \frac{x\,(m^2 - n^2)}{m^2}.$$

194.



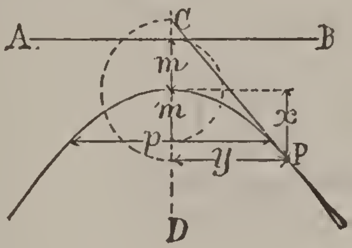
Parabola.

$y = \sqrt{p\,x}$, $p = 4\,m$,

$p = \dfrac{b^2}{h}$, $m = \dfrac{b^2}{4\,h}$.

m = distance from focus f to vertex o.

195.



Parabola, Tangent, and Radius.

Tangent $C\,P = \sqrt{4\,x^2 + y^2}$,

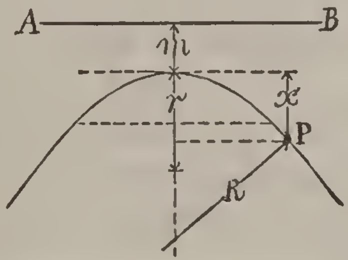
or $C\,P = 2\sqrt{x(x+m)}$.

$r = x + m$, the radius $f\,P$.

$A\,B$ = axis of ordinates.

$C\,D$ = axis of abscissas.

196.

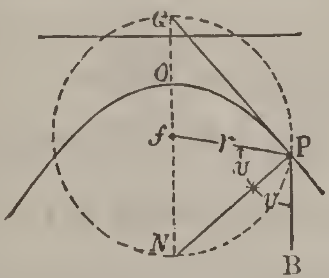


Radius of Curvature of a Parabola.

$$R = \frac{(16\,m^2 + 16\,m\,x)^{\frac{3}{2}}}{32\,m^2} = \frac{2(m^2 + m\,x)^{\frac{3}{2}}}{m^2}.$$

For the vertex $r = 2\,m$, or $r = \frac{1}{2}\,p$.

197.

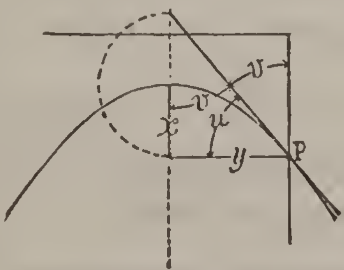


The normal $P\,N$ of a parabola bisects the angle $f\,P\,B$.

$f\,N = r$.

With r as radius, and the focus f as centre, draw the dotted circle $N\,P\,Q$, which determines the tangent and normal.

198.

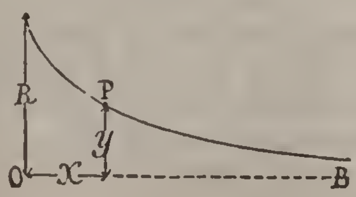


To find the angle of the parabola, with the axis of abscissas at any point P.

$\sin.v = \dfrac{y}{\sqrt{4\,x^2 + y^2}}$. $\sin.u = \dfrac{2\,x}{\sqrt{4\,x^2 + y^2}}$.

u = angle with ordinate y.

199.

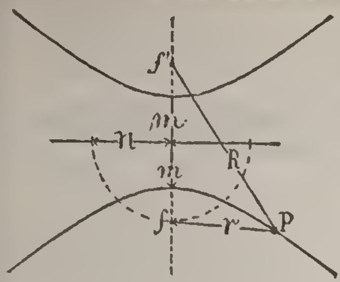


Shield's Anti-Friction Curve.

$$y = R\left(\frac{R}{x+R}\right)^2.$$

The line $O\,B$ is the centre line of the shaft.

R = radius of the shaft.

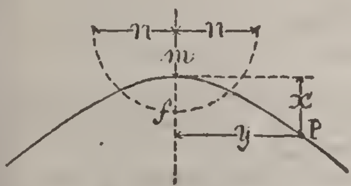


200.

In an hyperbola the transverse axis $m+m$ is equal to the difference between the two radii $(R-r)$.

$$2\,m = R - r. \qquad r = R - 2\,m.$$

The hyperbola can thus be plotted.

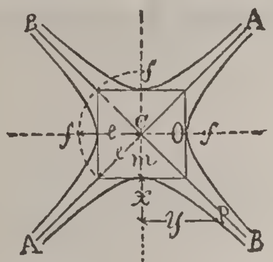


201. *Formula for the Hyperbola.*

$$y = \frac{n}{m}\sqrt{2\,m\,x + x^2}. \qquad n = \frac{m\,y}{\sqrt{2\,m\,x + x^2}}.$$

m = major semi-axis.
n = minor semi-axis.

$$\text{Parameter } p = \frac{2\,n^2}{m}.$$

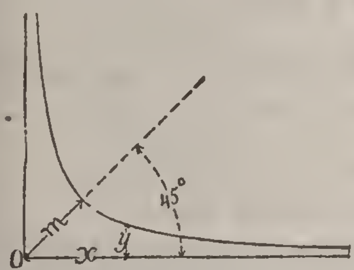


202. *Equilateral Hyperbola.*

$$y = \sqrt{2\,m x + x^2}. \quad n = m. \quad e = m\sqrt{2}.$$

$A\,A$ and $B\,B$ are at right angles and called *asymptotes.*

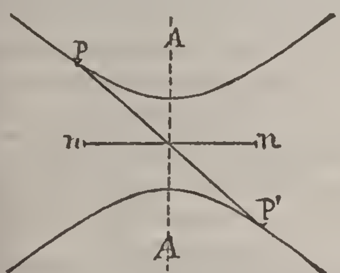
Steam-indicator diagrams are approximately equilateral hyperbolas on the expansion and compression lines.



203. *Equilateral Hyperbola referred to its Asymptotes.*

$$y = \frac{m^2}{2\,x}. \qquad m = \sqrt{2\,x\,y}.$$

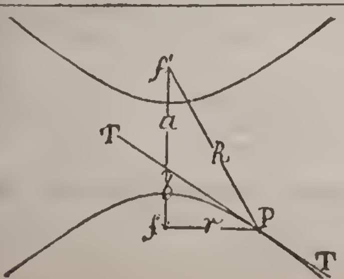
In this formula— $x\,y$ = constant
x = part of the stroke of piston.
y = steam-pressure.



204. *Diameter of an Hyperbola.*

Every diameter $P\,P'$ of an hyperbola is bisected in the centre by the minor axis $n\,n$.

$A\,A$ is called the transverse axis.



205.

A Tangent $T\,T'$ to an Hyperbola bisects the Angle formed by the Radii R and r.

$$a = \frac{R(a+b)}{R+r}. \qquad f\,f' = a + b.$$

LOGARITHMS.

A Common or Briggs Logarithm is the exponent of the power to which 10 must be raised to give a certain number. Thus,

Number,.... Logarithm, Base,......... Exponent,...

$$\text{log. } 100 = 2 \quad \text{because } 10^2 = 100.$$
$$\text{log. } 10000 = 4 \quad \text{"} \quad 10^4 = 10000.$$
$$\text{log. } 5012 = 3.7 \quad \text{"} \quad 10^{3.7} = 5012.$$

The *unit* of the logarithm is called the *characteristic* or *index*, and the *decimal part* is called the *mantissa*, the sum of the *characteristic* and *mantissa* is the *Logarithm*. 10 is the *base* of the system of Logarithms commonly used.

It is not necessary that the *base* should be 10, it can be any number, but all the tables of Logarithms now in common use are calculated with 10 as the base.

The nature of logarithms in connection with their numbers is such, that the *index* of the logarithm is always one less than the number of figures in the number, (when the base of the logarithm is 10,) as,

$$\text{index } 5012 = 3$$
$$\text{mantissa } 5012 = 0{\cdot}7$$
$$\text{logarithm } 5012 = 3{\cdot}7$$

Let 10 be raised to any power x, and

$$10^x = a \text{ or log. } a = x,$$
$$10^z = b \text{ or log. } b = z.$$

Let the product of $ab = c$ and the quotient $\frac{a}{b} = d$.

$$10^x \times 10^z = 10^{x \cdot z} = ab = c \quad \text{or} \quad \text{log. } c = x + z.$$
$$\frac{10^x}{10^z} = 10^{x-z} = \frac{a}{b} = d \quad \text{or} \quad \text{log. } d = x - z.$$
$$a = m^z \quad \text{or} \quad \text{log. } m = z \times \text{log. } a.$$
$$\sqrt[3]{a} = n \quad \text{or} \quad \text{log. } n = \text{log. } a : 3.$$

Any number represented by the letters a, b, c, or d, can be a power of 10, whose exponent is the logarithm of the number. Logarithms are given for numbers with three figures in the accompanying Table, by which any operation in Multiplication, Division, Involution and Evolution can be performed by simple Addition or Subtraction of Logarithms. Tables of Logarithms are commonly more extensive, and calculated for any number of four or five figures, which would occupy too much room in this book; but by the proportional parts, the logarithm can be found by this Table, to four or five figures. The *index* of the logarithms does not appear in the Table, only the *mantissa*. It is easily remembered that *the index is one less than the number of figures in the number;* then when the number is only one figure, the index is 0; and when the number is a fraction, the index is negative.

When the logarithm is to be found for a fraction, we commonly have the fraction expressed in a decimal; and then the negative index is equal to one more than the number of ciphers before the first figure, and is sometimes placed after the mantissa; thus,

log. 365 = 2·56229	log. 0·365 = ·56229—1.
log. 46·7 = 1·66931	log. 0·0467 = ·66931—2.
log. 7·59 = 0·88024	log 0·00759 = ·88024—3.

In the accompanying Table of Logarithms, for the trigonometrical lines the negative index is shown thus, 10 being added to the characteristic:

$$\text{log. sin. } 35^\circ\, 40' = \text{log. } 0{\cdot}58306 = 9{:}76572.$$

To find the Logarithm of Numbers.

Example 1. Find the logarithm of 45.

To 45 in the first column of the Table, answers 65321 in the next column, which is the mantissa; index = 1 because 45 is two figures.

Then, log. 45 = 1·65321, the answer.

Example 2. Find the logarithm of 768?

Opposite 76 in the first column, answers 88536 in the column marked 8 on the top or bottom. Index = 2 because 768 is three figures.

Then, log. 768 = 2·88536.

Example 3. Find the Logarithm of 6846?

	log. 6840 =	3·83505
Proportional part,	64×0·6 =	384
	log. 6846 =	3·835434 the answer.

To find the number for a given Logarithm.

Example 1. What number answers to the logarithm 3·87157?

In the Table you will find in the column of logarithms, that log. 7440 = 3·87157.

Example 2. What number answers to the logarithm 3·801884?

Given logarithm	3·801884,	
Subt. nearest table log.	3·801400 = log. 6330,	
Divided by proportional part,	69|484| - - - - - -	7,
		6337 the req. numb.

Multiplication by Logarithms.

Rule. Add together the logarithms of the factors, and the sum is the logarithm of the product.

Example 1. Multiply 425 by 48.

To	log. 425 =	2·62839,
Add	log. 48 =	1·68124,
The product,	log. 20400 =	4·30963.

Example 2. Multiply 79600 by 0·435.

To	log. 79600 =	4·90091,
Add,	log. 0·435 =	·63848—1,
The product	log. 34690 =	4·53939.

Division by Logarithms.

Rule. From the logarithm of the dividend subtract the logarithm of the divisor, and the difference is the logarithm of the quotient.

Example 1. Divide 43800 by 368.

From	log. 43800 =	4·64147,
Subtract	log. 368 =	2·56584,
The quotient	log. 119 =	2·07563.

Example 2. Divide 36 by 0.625.

From	log. 36 =	1·55636,
Subtract,	log. 0·625 =	·79588-1.
The quotient,	log. 57·6 =	1·76048.

A *negative index* has the reverse operation of its mantissa, as if the mantissa is subtracted, add the negative index, and *vice versâ*.

Involution by Logarithms.

Rule. Multiply the logarithm of the number by its exponent, and the product is the logarithm of the power of the number.

Evolution by Logarithms.

Rule. Divide the logarithm of the number by the index of the root, and the quotient is the logarithm of the root of the number.

No. 100 to 1600. **Logarithms.** 00000 to 20412.

No.	0	1	2	3	4	5	6	7	8	9
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389
101	0432	0475	0518	0561	0604	0647	0689	0732	0775	0817
102	0860	0903	0945	0988	1030	1072	1115	1157	1199	1242
103	1284	1326	1368	1410	1452	1494	1536	1578	1620	1662
104	1703	1745	1787	1828	1870	1912	1953	1995	2036	2078
105	02119	02160	02202	02243	02284	02325	02366	02407	02449	02490
106	2531	2572	2612	2653	2694	2735	2776	2816	2857	2898
107	2938	2979	3019	3060	3100	3141	3181	3222	3262	3302
108	3342	3383	3423	3463	3503	3543	3583	3623	3663	3703
109	3743	3782	3822	3862	3902	3941	3981	4021	4060	4100
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493
111	4532	4571	4610	4650	4689	4727	4766	4805	4844	4883
112	4922	4961	4999	5038	5077	5115	5154	5192	5231	5269
113	5308	5346	5385	5423	5461	5500	5538	5576	5614	5652
114	5690	5729	5767	5805	5843	5881	5918	5956	5994	6032
115	06070	06108	06145	06183	06221	06258	06296	06333	06371	06408
116	6446	6483	6521	6558	6595	6633	6670	6707	6744	6781
117	6819	6856	6893	6930	6967	7004	7041	7078	7115	7151
118	7188	7225	7262	7298	7335	7372	7408	7445	7482	7518
119	7555	7591	7628	7664	7700	7737	7773	7809	7846	7882
120	07918	07954	07990	08027	08063	08099	08135	08171	08207	08243
121	8279	8314	8350	8386	8422	8458	8493	8529	8565	8600
122	8636	8672	8707	8743	8778	8814	8849	8884	8920	8955
123	8991	9026	9061	9096	9132	9167	9202	9237	9272	9307
124	9342	9377	9412	9447	9482	9517	9552	9587	9621	9656
125	09691	09726	09760	09795	09830	09864	09899	09934	09968	10003
126	10037	10072	10106	10140	10175	10209	10243	10278	10312	0346
127	0380	0415	0449	0483	0517	0551	0585	0619	0653	0687
128	0721	0755	0789	0823	0857	0890	0924	0958	0992	1025
129	1059	1093	1126	1160	1193	1227	1261	1294	1327	1361
130	11394	11428	11461	11494	11528	11561	11594	11628	11661	11694
131	1727	1760	1793	1826	1860	1893	1926	1959	1992	2024
132	2057	2090	2123	2156	2189	2222	2254	2287	2320	2352
133	2385	2418	2450	2483	2516	2548	2581	2613	2646	2678
134	2710	2743	2775	2808	2840	2872	2905	2937	2969	3001
135	13033	13066	13098	13130	13162	13194	13226	13258	13290	13322
136	3354	3386	3418	3450	3481	3513	3545	3577	3609	3640
137	3672	3704	3735	3767	3799	3830	3862	3893	3925	3956
138	3988	4019	4051	4082	4114	4145	4176	4208	4239	4270
139	4301	4333	4364	4395	4426	4457	4489	4520	4551	4582
140	14613	14644	14675	14706	14737	14768	14799	14829	14860	14891
141	4922	4953	4983	5014	5045	5076	5106	5137	5168	5198
142	5229	5259	5290	5320	5351	5381	5412	5442	5473	5503
143	5534	5564	5594	5625	5655	5685	5715	5746	5776	5806
144	5836	5866	5897	5927	5957	5987	6017	6047	6077	6107
145	16137	16167	16197	16227	16256	16286	16316	16346	16376	16406
146	6435	6465	6495	6524	6554	6584	6613	6643	6673	6702
147	6732	6761	6791	6820	6850	6879	6909	6938	6967	6997
148	7026	7056	7085	7114	7143	7173	7202	7231	7260	7289
149	7319	7348	7377	7406	7435	7464	7493	7522	7551	7580
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869
151	7898	7926	7955	7984	8013	8041	8070	8099	8127	8156
152	8184	8213	8241	8270	8298	8327	8355	8384	8412	8441
153	8469	8498	8526	8554	8583	8611	8639	8667	8696	8724
154	8752	8780	8808	8837	8865	8893	8921	8949	8977	9005
155	19033	19061	19089	19117	19145	19173	19201	19229	19257	19285
156	9312	9340	9368	9396	9424	9451	9479	9507	9535	9562
157	9590	9618	9645	9673	9700	9728	9756	9783	9811	9838
158	9866	9893	9921	9948	9976	20003	20030	20058	20085	20112
159	20140	20167	20194	20222	20249	0276	0303	0330	0358	0385
No.	0	1	2	3	4	5	6	7	8	9

	43	41	39	37	35	33
1	4	4	4	4	4	3
2	9	8	8	7	7	7
3	13	12	12	11	11	10
4	17	16	16	15	14	13
5	22	21	20	19	18	17
6	26	25	23	22	21	20
7	30	29	27	26	25	23
8	34	33	31	30	28	26
9	39	37	35	33	32	30

No. 1600 to 2200. **Logarithms.** 20412 to 34242.

No.	0	1	2	3	4	5	6	7	8	9
160	20412	20439	20466	20493	20520	20548	20575	20602	20629	20656
161	0683	0710	0737	0763	0790	0817	0844	0871	0898	0925
162	0952	0978	1005	1032	1059	1085	1112	1139	1165	1192
163	1219	1245	1272	1299	1325	1352	1378	1405	1431	1458
164	1484	1511	1537	1564	1590	1617	1643	1669	1696	1722
165	21748	21775	21801	21827	21854	21880	21906	21932	21958	21985
166	2011	2037	2063	2089	2115	2141	2167	2194	2220	2246
167	2272	2298	2324	2350	2376	2401	2427	2453	2479	2505
168	2531	2557	2583	2608	2634	2660	2686	2712	2737	2763
169	2789	2814	2840	2866	2891	2917	2943	2968	2994	3019
170	23045	23070	23096	23121	23147	23172	23198	23223	23249	23274
171	3300	3325	3350	3376	3401	3426	3452	3477	3502	3528
172	3553	3578	3603	3629	3654	3679	3704	3729	3754	3779
173	3805	3830	3855	3880	3905	3930	3955	3980	4005	4030
174	4055	4080	4105	4130	4155	4180	4204	4229	4254	4279
175	24304	24329	24353	24378	24403	24428	24452	24477	24502	24527
176	4551	4576	4601	4625	4650	4674	4699	4724	4748	4773
177	4797	4822	4846	4871	4895	4920	4944	4969	4993	5018
178	5042	5066	5091	5115	5139	5164	5188	5212	5237	5261
179	5285	5310	5334	5358	5382	5406	5431	5455	5479	5503
180	25527	25551	25575	25600	25624	25648	25672	25696	25720	25744
181	5768	5792	5816	5840	5864	5888	5912	5935	5959	5983
182	6007	6031	6055	6079	6102	6126	6150	6174	6198	6221
183	6245	6269	6293	6316	6340	6364	6387	6411	6435	6458
184	6482	6505	6529	6553	6576	6600	6623	6647	6670	6694
185	26717	26741	26764	26788	26811	26834	26858	26881	26905	26928
186	6951	6975	6998	7021	7045	7068	7091	7114	7138	7161
187	7184	7207	7231	7254	7277	7300	7323	7346	7370	7393
188	7416	7439	7462	7485	7508	7531	7554	7577	7600	7623
189	7646	7669	7692	7715	7738	7761	7784	7807	7830	7852
190	27875	27898	27921	27944	27967	27989	28012	28035	28058	28081
191	8103	8126	8149	8171	8194	8217	8240	8262	8285	8307
192	8330	8353	8375	8398	8421	8443	8466	8488	8511	8533
193	8556	8578	8601	8623	8646	8668	8691	8713	8735	8758
194	8780	8803	8825	8847	8870	8892	8914	8937	8959	8981
195	29003	29026	29048	29070	29092	29115	29137	29159	29181	29203
196	9226	9248	9270	9292	9314	9336	9358	9380	9403	9425
197	9447	9469	9491	9513	9535	9557	9579	9601	9623	9645
198	9667	9688	9710	9732	9754	9776	9798	9820	9842	9863
199	9885	9907	9929	9951	9973	9994	30016	30038	30060	30081
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298
201	0320	0341	0363	0384	0406	0428	0449	0471	0492	0514
202	0535	0557	0578	0600	0621	0643	0664	0685	0707	0728
203	0750	0771	0792	0814	0835	0856	0878	0899	0920	0942
204	0963	0984	1006	1027	1048	1069	1091	1112	1133	1154
205	31175	31197	31218	31239	31260	31281	31302	31323	31345	31366
206	1387	1408	1429	1450	1471	1492	1513	1534	1555	1576
207	1597	1618	1639	1660	1681	1702	1723	1744	1765	1785
208	1806	1827	1848	1869	1890	1911	1931	1952	1973	1994
209	2015	2035	2056	2077	2098	2118	2139	2160	2181	2201
210	32222	32243	32263	32284	32305	32325	32346	32366	32387	32408
211	2428	2449	2469	2490	2510	2531	2552	2572	2593	2613
212	2634	2654	2675	2695	2715	2736	2756	2777	2797	2818
213	2838	2858	2879	2899	2919	2940	2960	2980	3001	3021
214	3041	3062	3082	3102	3122	3143	3163	3183	3203	3224
215	33244	33264	33284	33304	33325	33345	33365	33385	33405	33425
216	3445	3465	3486	3506	3526	3546	3566	3586	3606	3626
217	3646	3666	3686	3706	3726	3746	3766	3786	3806	3826
218	3846	3866	3885	3905	3925	3945	3965	3985	4005	4025
219	4044	4064	4084	4104	4124	4143	4163	4183	4203	4223
No.	0	1	2	3	4	5	6	7	8	9

	31	29	27	25	23	21
1	3	3	3	3	2	2
2	6	6	5	5	5	4
3	9	9	8	8	7	6
4	12	12	11	10	9	8
5	16	15	14	13	12	11
6	19	17	16	15	14	13
7	22	20	19	18	16	15
8	25	23	22	20	18	17
9	28	26	24	23	21	19

No. 2200 to 2800. **Logarithms.** 34242 to 44716.

No.	0	1	2	3	4	5	6	7	8	9
220	34242	34262	34282	34301	34321	34341	34361	34380	34400	34420
221	4439	4459	4479	4498	4518	4537	4557	4577	4596	4616
222	4635	4655	4674	4694	4713	4733	4753	4772	4792	4811
223	4830	4850	4869	4889	4908	4928	4947	4967	4986	5005
224	5025	5044	5064	5083	5102	5122	5141	5160	5180	5199
225	35218	35238	35257	35276	35295	35315	35334	35353	35372	35392
226	5411	5430	5449	5468	5488	5507	5526	5545	5564	5583
227	5603	5622	5641	5660	5679	5698	5717	5736	5755	5774
228	5793	5813	5832	5851	5870	5889	5908	5927	5946	5965
229	5984	6003	6021	6040	6059	6078	6097	6116	6135	6154
230	36173	36192	36211	36229	36248	36267	36286	36305	36324	36342
231	6361	6380	6399	6418	6436	6455	6474	6493	6511	6530
232	6549	6568	6586	6605	6624	6642	6661	6680	6698	6717
233	6736	6754	6773	6791	6810	6829	6847	6866	6884	6903
234	6922	6940	6959	6977	6996	7014	7033	7051	7070	7088
235	37107	37125	37144	37162	37181	37199	37218	37236	37254	37273
236	7291	7310	7328	7346	7365	7383	7401	7420	7438	7457
237	7475	7493	7511	7530	7548	7566	7585	7603	7621	7639
238	7658	7676	7694	7712	7731	7749	7767	7785	7803	7822
239	7840	7858	7876	7894	7912	7931	7949	7967	7985	8003
240	38021	38039	38057	38075	38093	38112	38130	38148	38166	38184
241	8202	8220	8238	8256	8274	8292	8310	8328	8346	8364
242	8382	8399	8417	8435	8453	8471	8489	8507	8525	8543
243	8561	8578	8596	8614	8632	8650	8668	8686	8703	8721
244	8739	8757	8775	8792	8810	8828	8846	8863	8881	8899
245	38917	38934	38952	38970	38987	39005	39023	39041	39058	39076
246	9094	9111	9129	9146	9164	9182	9199	9217	9235	9252
247	9270	9287	9305	9322	9340	9358	9375	9393	9410	9428
248	9445	9463	9480	9498	9515	9533	9550	9568	9585	9602
249	9620	9637	9655	9672	9690	9707	9724	9742	9759	9777
250	39794	39811	39829	39846	39863	39881	39898	39915	39933	39950
251	9967	9985	40002	40019	40037	40054	40071	40088	40106	40123
252	40140	40157	0175	0192	0209	0226	0243	0261	0278	0295
253	0312	0329	0346	0364	0381	0398	0415	0432	0449	0466
254	0483	0500	0518	0535	0552	0569	0586	0603	0620	0637
255	40654	40671	40688	40705	40722	40739	40756	40773	40790	40807
256	0824	0841	0858	0875	0892	0909	0926	0943	0960	0976
257	0993	1010	1027	1044	1061	1078	1095	1111	1128	1145
258	1162	1179	1196	1212	1229	1246	1263	1280	1296	1313
259	1330	1347	1363	1380	1397	1414	1430	1447	1464	1481
260	41497	41514	41531	41547	41564	41581	41597	41614	41631	41647
261	1664	1681	1697	1714	1731	1747	1764	1780	1797	1814
262	1830	1847	1863	1880	1896	1913	1929	1946	1963	1979
263	1996	2012	2029	2045	2062	2078	2095	2111	2127	2144
264	2160	2177	2193	2210	2226	2243	2259	2275	2292	2308
265	42325	42341	42357	42374	42390	42406	42423	42439	42455	42472
266	2488	2504	2521	2537	2553	2570	2586	2602	2619	2635
267	2651	2667	2684	2700	2716	2732	2749	2765	2781	2797
268	2813	2830	2846	2862	2878	2894	2911	2927	2943	2959
269	2975	2991	3008	3024	3040	3056	3072	3088	3104	3120
270	43136	43152	43169	43185	43201	43217	43233	43249	43265	43281
271	3297	3313	3329	3345	3361	3377	3393	3409	3425	3441
272	3457	3473	3489	3505	3521	3537	3553	3569	3584	3600
273	3616	3632	3648	3664	3680	3696	3712	3727	3743	3759
274	3775	3791	3807	3823	3838	3854	3870	3886	3902	3917
275	43933	43949	43965	43981	43996	44012	44028	44044	44059	44075
276	4091	4107	4122	4138	4154	4170	4185	4201	4217	4232
277	4248	4264	4279	4295	4311	4326	4342	4358	4373	4389
278	4404	4420	4436	4451	4467	4483	4498	4514	4529	4545
279	4560	4576	4592	4607	4623	4638	4654	4669	4685	4700
No.	0	1	2	3	4	5	6	7	8	9

	20	19	18	17	16	15
1	2	2	2	2	2	2
2	4	4	4	3	3	3
3	6	6	5	5	5	5
4	8	8	7	7	6	6
5	10	10	9	9	8	8
6	12	11	11	10	10	9
7	14	13	13	12	11	11
8	16	15	14	14	13	12
9	18	17	16	15	14	14

No. 2800 to 3400. **Logarithms.** 44716 to 53148.

No.	0	1	2	3	4	5	6	7	8	9		**16**
280	44716	44731	44747	44762	44778	44793	44809	44824	44840	44855	1	2
281	4871	4886	4902	4917	4932	4948	4963	4979	4994	5010	2	3
282	5025	5040	5056	5071	5086	5102	5117	5133	5148	5163	3	5
283	5179	5194	5209	5225	5240	5255	5271	5286	5301	5317	4	6
284	5332	5347	5362	5378	5393	5408	5423	5439	5454	5469	5	8
285	45484	45500	45515	45530	45545	45561	45576	45591	45606	45621	6	10
286	5637	5652	5667	5682	5697	5712	5728	5743	5758	5773	7	11
287	5788	5803	5818	5834	5849	5864	5879	5894	5909	5924	8	13
288	5939	5954	5969	5984	6000	6015	6030	6045	6060	6075	9	14
289	6090	6105	6120	6135	6150	6165	6180	6195	6210	6225		
290	46240	46255	46270	46285	46300	46315	46330	46345	46359	46374		
291	6389	6404	6419	6434	6449	6464	6479	6494	6509	6523		
292	6538	6553	6568	6583	6598	6613	6627	6642	6657	6672		**15**
293	6687	6702	6716	6731	6746	6761	6776	6790	6805	6820	1	2
294	6835	6850	6864	6879	6894	6909	6923	6938	6953	6967	2	3
295	46982	46997	47012	47026	47041	47056	47070	47085	47100	47114	3	5
296	7129	7144	7159	7173	7188	7202	7217	7232	7246	7261	4	6
297	7276	7290	7305	7319	7334	7349	7363	7378	7392	7407	5	8
298	7422	7436	7451	7465	7480	7494	7509	7524	7538	7553	6	9
299	7567	7582	7596	7611	7625	7640	7654	7669	7683	7698	7	11
300	47712	47727	47741	47756	47770	47784	47799	47813	47828	47842	8	12
301	7857	7871	7885	7900	7914	7929	7943	7958	7972	7986	9	14
302	8001	8015	8029	8044	8058	8073	8087	8101	8116	8130		
303	8144	8159	8173	8187	8202	8216	8230	8244	8259	8273		
304	8287	8302	8316	8330	8344	8359	8373	8387	8401	8416		
305	48430	48444	48458	48473	48487	48501	48515	48530	48544	48558		**14**
306	8572	8586	8601	8615	8629	8643	8657	8671	8686	8700	1	1
307	8714	8728	8742	8756	8770	8785	8799	8813	8827	8841	2	3
308	8855	8869	8883	8897	8911	8926	8940	8954	8968	8982	3	4
309	8996	9010	9024	9038	9052	9066	9080	9094	9108	9122	4	6
310	49136	49150	49164	49178	49192	49206	49220	49234	49248	49262	5	7
311	9276	9290	9304	9318	9332	9346	9360	9374	9388	9402	6	8
312	9415	9429	9443	9457	9471	9485	9499	9513	9527	9541	7	10
313	9554	9568	9582	9596	9610	9624	9638	9651	9665	9679	8	11
314	9693	9707	9721	9734	9748	9762	9776	9790	9803	9817	9	13
315	49831	49845	49859	49872	49886	49900	49914	49927	49941	49955		
316	9969	9982	9996	50010	50024	50037	50051	50065	50079	50092		
317	50106	50120	50133	0147	0161	0174	0188	0202	0215	0229		
318	0243	0256	0270	0284	0297	0311	0325	0338	0352	0365		**13**
319	0379	0393	0406	0420	0433	0447	0461	0474	0488	0501	1	1
320	50515	50529	50542	50556	50569	50583	50596	50610	50623	50637	2	3
321	0651	0664	0678	0691	0705	0718	0732	0745	0759	0772	3	4
322	0786	0799	0813	0826	0840	0853	0866	0880	0893	0907	4	5
323	0920	0934	0947	0961	0974	0987	1001	1014	1028	1041	5	7
324	1055	1068	1081	1095	1108	1121	1135	1148	1162	1175	6	8
325	51188	51202	51215	51228	51242	51255	51268	51282	51295	51308	7	9
326	1322	1335	1348	1362	1375	1388	1402	1415	1428	1441	8	10
327	1455	1468	1481	1495	1508	1521	1534	1548	1561	1574	9	12
328	1587	1601	1614	1627	1640	1654	1667	1680	1693	1706		
329	1720	1733	1746	1759	1772	1786	1799	1812	1825	1838		
330	51851	51865	51878	51891	51904	51917	51930	51943	51957	51970		**12**
331	1983	1996	2009	2022	2035	2048	2061	2075	2088	2101	1	1
332	2114	2127	2140	2153	2166	2179	2192	2205	2218	2231	2	2
333	2244	2257	2270	2284	2297	2310	2323	2336	2349	2362	3	4
334	2375	2388	2401	2414	2427	2440	2453	2466	2479	2492	4	5
335	52504	52517	52530	52543	52556	52569	52582	52595	52608	52621	5	6
336	2634	2647	2660	2673	2686	2699	2711	2724	2737	2750	6	7
337	2763	2776	2789	2802	2815	2827	2840	2853	2866	2879	7	8
338	2892	2905	2917	2930	2943	2956	2969	2982	2994	3007	8	10
339	3020	3033	3046	3058	3071	3084	3097	3110	3122	3135	9	11
No.	0	1	2	3	4	5	6	7	8	9		

No. 3400 to 4000. **Logarithms.** Log. 53148 to 60206.

No.	0	1	2	3	4	5	6	7	8	9		13
340	53148	53161	53173	53186	53199	53212	53224	53237	53250	53263	1	1
341	3275	3288	3301	3314	3326	3339	3352	3364	3377	3390	2	3
342	3403	3415	3428	3441	3453	3466	3479	3491	3504	3517	3	4
343	3529	3542	3555	3567	3580	3593	3605	3618	3631	3643	4	5
344	3656	3668	3681	3694	3706	3719	3732	3744	3757	3769	5	7
345	53782	53794	53807	53820	53832	53845	53857	53870	53882	53895	6	8
346	3908	3920	3933	3945	3958	3970	3983	3995	4008	4020	7	9
347	4033	4045	4058	4070	4083	4095	4108	4120	4133	4145	8	10
348	4158	4170	4183	4195	4208	4220	4233	4245	4258	4270	9	12
349	4283	4295	4307	4320	4332	4345	4357	4370	4382	4394		
350	54407	54419	54432	54444	54456	54469	54481	54494	54506	54518		
351	4531	4543	4555	4568	4580	4593	4605	4617	4630	4642		
352	4654	4667	4679	4691	4704	4716	4728	4741	4753	4765		
353	4777	4790	4802	4814	4827	4839	4851	4864	4876	4888		
354	4900	4913	4925	4937	4949	4962	4974	4986	4998	5011		
355	55023	55035	55047	55060	55072	55084	55096	55108	55121	55133		
356	5145	5157	5169	5182	5194	5206	5218	5230	5242	5255		**12**
357	5267	5279	5291	5303	5315	5328	5340	5352	5364	5376	1	1
358	5388	5400	5413	5425	5437	5449	5461	5473	5485	5497	2	2
359	5509	5522	5534	5546	5558	5570	5582	5594	5606	5618	3	4
360	55630	55642	55654	55666	55678	55691	55703	55715	55727	55739	4	5
361	5751	5763	5775	5787	5799	5811	5823	5835	5847	5859	5	6
362	5871	5883	5895	5907	5919	5931	5943	5955	5967	5979	6	7
363	5991	6003	6015	6027	6038	6050	6062	6074	6086	6098	7	8
364	6110	6122	6134	6146	6158	6170	6182	6194	6205	6217	8	10
365	56229	56241	56253	56265	56277	56289	56301	56312	56324	56336	9	11
366	6348	6360	6372	6384	6396	6407	6419	6431	6443	6455		
367	6467	6478	6490	6502	6514	6526	6538	6549	6561	6573		
368	6585	6597	6608	6620	6632	6644	6656	6667	6679	6691		
369	6703	6714	6726	6738	6750	6761	6773	6785	6797	6808		
370	56820	56832	56844	56855	56867	56879	56891	56902	56914	56926		
371	6937	6949	6961	6972	6984	6996	7008	7019	7031	7043		
372	7054	7066	7078	7089	7101	7113	7124	7136	7148	7159		
373	7171	7183	7194	7206	7217	7229	7241	7252	7264	7276		**11**
374	7287	7299	7310	7322	7334	7345	7357	7368	7380	7392	1	1
375	57403	57415	57426	57438	57449	57461	57473	57484	57496	57507	2	2
376	7519	7530	7542	7553	7565	7576	7588	7600	7611	7623	3	3
377	7634	7646	7657	7669	7680	7692	7703	7715	7726	7738	4	4
378	7749	7761	7772	7784	7795	7807	7818	7830	7841	7852	5	6
379	7864	7875	7887	7898	7910	7921	7933	7944	7955	7967	6	7
380	57978	57990	58001	58013	58024	58035	58047	58058	58070	58081	7	8
381	8092	8104	8115	8127	8138	8149	8161	8172	8184	8195	8	9
382	8206	8218	8229	8240	8252	8263	8274	8286	8297	8309	9	10
383	8320	8331	8343	8354	8365	8377	8388	8399	8410	8422		
384	8433	8444	8456	8467	8478	8490	8501	8512	8524	8535		
385	58546	58557	58569	58580	58591	58602	58614	58625	58636	58647		
386	8659	8670	8681	8692	8704	8715	8726	8737	8749	8760		
387	8771	8782	8794	8805	8816	8827	8838	8850	8861	8872		
388	8883	8894	8906	8917	8928	8939	8950	8961	8973	8984		
389	8995	9006	9017	9028	9040	9051	9062	9073	9084	9095		
390	59106	59118	59129	59140	59151	59162	59173	59184	59195	59207		**10**
391	9218	9229	9240	9251	9262	9273	9284	9295	9306	9318	1	1
392	9329	9340	9351	9362	9373	9384	9395	9406	9417	9428	2	2
393	9439	9450	9461	9472	9483	9494	9506	9517	9528	9539	3	3
394	9550	9561	9572	9583	9594	9605	9616	9627	9638	9649	4	4
395	59660	59671	59682	59693	59704	59715	59726	59737	59748	59759	5	5
396	9770	9780	9791	9802	9813	9824	9835	9846	9857	9868	6	6
397	9879	9890	9901	9912	9923	9934	9945	9956	9966	9977	7	7
398	9988	9999	60010	60021	60032	60043	60054	60065	60076	60086	8	8
399	60097	60108	0119	0130	0141	0152	0163	0173	0184	0195	9	9
No.	0	1	2	3	4	5	6	7	8	9		

No. 4000 to 4600. **Logarithms.** Log. 60206 to 66276.

No.	0	1	2	3	4	5	6	7	8	9	11
400	60206	60217	60228	60239	60249	60260	60271	60282	60293	60304	1 1
401	0314	0325	0336	0347	0358	0369	0379	0390	0401	0412	2 2
402	0423	0433	0444	0455	0466	0477	0487	0498	0509	0520	3 3
403	0531	0541	0552	0563	0574	0584	0595	0606	0617	0627	4 4
404	0638	0649	0660	0670	0681	0692	0703	0713	0724	0735	5 6
405	60746	60756	60767	60778	60788	60799	60810	60821	60831	60842	6 7
406	0853	0863	0874	0885	0895	0906	0917	0927	0938	0949	7 8
407	0959	0970	0981	0991	1002	1013	1023	1034	1045	1055	8 9
408	1066	1077	1087	1098	1109	1119	1130	1140	1151	1162	9 10
409	1172	1183	1194	1204	1215	1225	1236	1247	1257	1268	
410	61278	61289	61300	61310	61321	61331	61342	61352	61363	61374	
411	1384	1395	1405	1416	1426	1437	1448	1458	1469	1479	
412	1490	1500	1511	1521	1532	1542	1553	1563	1574	1584	
413	1595	1606	1616	1627	1637	1648	1658	1669	1679	1690	
414	1700	1711	1721	1731	1742	1752	1763	1773	1784	1794	
415	61805	61815	61826	61836	61847	61857	61868	61878	61888	61899	
416	1909	1920	1930	1941	1951	1962	1972	1982	1993	2003	
417	2014	2024	2034	2045	2055	2066	2076	2086	2097	2107	
418	2118	2128	2138	2149	2159	2170	2180	2190	2201	2211	
419	2221	2232	2242	2252	2263	2273	2284	2294	2304	2315	
420	62325	62335	62346	62356	62366	62377	62387	62397	62408	62418	
421	2428	2439	2449	2459	2469	2480	2490	2500	2511	2521	
422	2531	2542	2552	2562	2572	2583	2593	2603	2613	2624	
423	2634	2644	2655	2665	2675	2685	2696	2706	2716	2726	
424	2737	2747	2757	2767	2778	2788	2798	2808	2818	2829	10
425	62839	62849	62859	62870	62880	62890	62900	62910	62921	62931	1 1
426	2941	2951	2961	297	2982	2992	3002	3012	3022	3033	2 2
427	3043	3053	3063	3073	3083	3094	3104	3114	3124	3134	3 3
428	3144	3155	3165	3175	3185	3195	3205	3215	3225	3236	4 4
429	3246	3256	3266	3276	3286	3296	3306	3317	3327	3337	5 5
430	63347	63357	63367	63377	63387	63397	63407	63417	63428	63438	6 6
431	3448	3458	3468	3478	3488	3498	3508	3518	3528	3538	7 7
432	3548	3558	3568	3579	3589	3599	3609	3619	3629	3639	8 8
433	3649	3659	3669	3679	3689	3699	3709	3719	3729	3739	9 9
434	3749	3759	3769	3779	3789	3799	3809	3819	3829	3839	
435	63849	63859	63869	63879	63889	63899	63909	63919	63929	63939	
436	3949	3959	3969	3979	3988	3998	4008	4018	4028	4038	
437	4048	4058	4068	4078	4088	4098	4108	4118	4128	4137	
438	4147	4157	4167	4177	4187	4197	4207	4217	4227	4237	
439	4246	4256	4266	4276	4286	4296	4306	4316	4326	4335	
440	64345	64355	64365	64375	64385	64395	64404	64414	64424	64434	
441	4444	4454	4464	4473	4483	4493	4503	4513	4523	4532	
442	4542	4552	4562	4572	4582	4591	4601	4611	4621	4631	
443	4640	4650	4660	4670	4680	4689	4699	4709	4719	4729	
444	4738	4748	4758	4768	4777	4787	4797	4807	4816	4826	
445	64836	64846	64856	64865	64875	64885	64895	64904	64914	64924	
446	4933	4943	4953	4963	4972	4982	4992	5002	5011	5021	
447	5031	5040	5050	5060	5070	5079	5089	5099	5108	5118	
448	5128	5137	5147	5157	5167	5176	5186	5196	5205	5215	
449	5225	5234	5244	5254	5263	5273	5283	5292	5302	5312	
450	65321	65331	65341	65350	65360	65369	65379	65389	65398	65408	9
451	5418	5427	5437	5447	5456	5466	5475	5485	5495	5504	1 1
452	5514	5523	5533	5543	5552	5562	5571	5581	5591	5600	2 2
453	5610	5619	5629	5639	5648	5658	5667	5677	5686	5696	3 3
454	5706	5715	5725	5734	5744	5753	5763	5772	5782	5792	4 4
455	65801	65811	65820	65830	65839	65849	65858	65868	65877	65887	5 5
456	5896	5906	5916	5925	5935	5944	5954	5963	5973	5982	6 5
457	5992	6001	6011	6020	6030	6039	6049	6058	6068	6077	7 6
458	6087	6096	6106	6115	6124	6134	6143	6153	6162	6172	8 7
459	6181	6191	6200	6210	6219	6229	6238	6247	6257	6266	9 8
No.	0	1	2	3	4	5	6	7	8	9	

No. 4600 to 5200. **Logarithms.** Log. 66276 to 71600.

No.	0	1	2	3	4	5	6	7	8	9		10
460	66276	66285	66295	66304	66314	66323	66332	66342	66351	66361	1	1
461	6370	6380	6389	6398	6408	6417	6427	6436	6445	6455	2	2
462	6464	6474	6483	6492	6502	6511	6521	6530	6539	6549	3	3
463	6558	6567	6577	6586	6596	6605	6614	6624	6633	6642	4	4
464	6652	6661	6671	6680	6689	6699	6708	6717	6727	6736	5	5
465	66745	66755	66764	66773	66783	66792	66801	66811	66820	66829	6	6
466	6839	6848	6857	6867	6876	6885	6894	6904	6913	6922	7	7
467	6932	6941	6950	6960	6969	6978	6987	6997	7006	7015	8	8
468	7025	7034	7043	7052	7062	7071	7080	7089	7099	7108	9	9
469	7117	7127	7136	7145	7154	7164	7173	7182	7191	7201		
470	67210	67219	67228	67237	67247	67256	67265	67274	67284	67293		
471	7302	7311	7321	7330	7339	7348	7357	7367	7376	7385		
472	7394	7403	7413	7422	7431	7440	7449	7459	7468	7477		
473	7486	7495	7504	7514	7523	7532	7541	7550	7560	7569		
474	7578	7587	7596	7605	7614	7624	7633	7642	7651	7660		
475	67669	67679	67688	67697	67706	67715	67724	67733	67742	67752		
476	7761	7770	7779	7788	7797	7806	7815	7825	7834	7843		
477	7852	7861	7870	7879	7888	7897	7906	7916	7925	7934		
478	7943	7952	7961	7970	7979	7988	7997	8006	8015	8024		
479	8034	8043	8052	8061	8070	8079	8088	8097	8106	8115		
480	68124	68133	68142	68151	68160	68169	68178	68187	68196	68205		
481	8215	8224	8233	8242	8251	8260	8269	8278	8287	8296		
482	8305	8314	8323	8332	8341	8350	8359	8368	8377	8386		
483	8395	8404	8413	8422	8431	8440	8449	8458	8467	8476		
484	8485	8494	8502	8511	8520	8529	8538	8547	8556	8565		
485	68574	68583	68592	68601	68610	68619	68628	68637	68646	68655		9
486	8664	8673	8681	8690	8699	8708	8717	8726	8735	8744	1	1
487	8753	8762	8771	8780	8789	8797	8806	8815	8824	8833	2	2
488	8842	8851	8860	8869	8878	8886	8895	8904	8913	8922	3	3
489	8931	8940	8949	8958	8966	8975	8984	8993	9002	9011	4	4
490	69020	69028	69037	69046	69055	69064	69073	69082	69090	69099	5	5
491	9108	9117	9126	9135	9144	9152	9161	9170	9179	9188	6	5
492	9197	9205	9214	9223	9232	9241	9249	9258	9267	9276	7	6
493	9285	9294	9302	9311	9320	9329	9338	9346	9355	9364	8	7
494	9373	9381	9390	9399	9408	9417	9425	9434	9443	9452	9	8
495	69461	69469	69478	69487	69496	69504	69513	69522	69531	69539		
496	9548	9557	9566	9574	9583	9592	9601	9609	9618	9627		
497	9636	9644	9653	9662	9671	9679	9688	9697	9705	9714		
498	9723	9732	9740	9749	9758	9767	9775	9784	9793	9801		
499	9810	9819	9827	9836	9845	9854	9862	9871	9880	9888		
500	69897	69906	69914	69923	69932	69940	69949	69958	69966	69975		
501	9984	9992	70001	70010	70018	70027	70036	70044	70053	70062		
502	70070	70079	0088	0096	0105	0114	0122	0131	0140	0148		
503	0157	0165	0174	0183	0191	0200	0209	0217	0226	0234		
504	0243	0252	0260	0269	0278	0286	0295	0303	0312	0321		
505	70329	70338	70346	70355	70364	70372	70381	70389	70398	70406		
506	0415	0424	0432	0441	0449	0458	0467	0475	0484	0492		
507	0501	0509	0518	0526	0535	0544	0552	0561	0569	0578		
508	0586	0595	0603	0612	0621	0629	0638	0646	0655	0663		
509	0672	0680	0689	0697	0706	0714	0723	0731	0740	0749		
510	70757	70766	70774	70783	70791	70800	70808	70817	70825	70834		8
511	0842	0851	0859	0868	0876	0885	0893	0902	0910	0919	1	1
512	0927	0935	0944	0952	0961	0969	0978	0986	0995	1003	2	2
513	1012	1020	1029	1037	1046	1054	1063	1071	1079	1088	3	2
514	1096	1105	1113	1122	1130	1139	1147	1155	1164	1172	4	3
515	71181	71189	71198	71206	71214	71223	71231	71240	71248	71257	5	4
516	1265	1273	1282	1290	1299	1307	1315	1324	1332	1341	6	5
517	1349	1357	1366	1374	1383	1391	1399	1408	1416	1425	7	6
518	1433	1441	1450	1458	1466	1475	1483	1492	1500	1508	8	6
519	1517	1525	1533	1542	1550	1559	1567	1575	1584	1592	9	7
No.	0	1	2	3	4	5	6	7	8	9		

No. 5200 to 5800. **Logarithms.** Log. 71600 to 76343.

No.	0	1	2	3	4	5	6	7	8	9		
520	71600	71609	71617	71625	71634	71642	71650	71659	71667	71675		9
521	1684	1692	1700	1709	1717	1725	1734	1742	1750	1759	1	1
522	1767	1775	1784	1792	1800	1809	1817	1825	1834	1842	2	2
523	1850	1858	1867	1875	1883	1892	1900	1908	1917	1925	3	3
524	1933	1941	1950	1958	1966	1975	1983	1991	1999	2008	4	4
525	72016	72024	72032	72041	72049	72057	72066	72074	72082	72090	5	5
526	2099	2107	2115	2123	2132	2140	2148	2156	2165	2173	6	5
527	2181	2189	2198	2206	2214	2222	2230	2239	2247	2255	7	6
528	2263	2272	2280	2288	2296	2304	2313	2321	2329	2337	8	7
529	2346	2354	2362	2370	2378	2387	2395	2403	2411	2419	9	8
530	72428	72436	72444	72452	72460	72469	72477	72485	72493	72501		
531	2509	2518	2526	2534	2542	2550	2558	2567	2575	2583		
532	2591	2599	2607	2616	2624	2632	2640	2648	2656	2665		
533	2673	2681	2689	2697	2705	2713	2722	2730	2738	2746		
534	2754	2762	2770	2779	2787	2795	2803	2811	2819	2827		
535	72835	72843	72852	72860	72868	72876	72884	72892	72900	72908		
536	2916	2925	2933	2941	2949	2957	2965	2973	2981	2989		
537	2997	3006	3014	3022	3030	3038	3046	3054	3062	3070		
538	3078	3086	3094	3102	3111	3119	3127	3135	3143	3151		
539	3159	3167	3175	3183	3191	3199	3207	3215	3223	3231		
540	73239	73247	73255	73263	73272	73280	73288	73296	73304	73312		
541	3320	3328	3336	3344	3352	3360	3368	3376	3384	3392		
542	3400	3408	3416	3424	3432	3440	3448	3456	3464	3472		
543	3480	3488	3496	3504	3512	3520	3528	3536	3544	3552		
544	3560	3568	3576	3584	3592	3600	3608	3616	3624	3632		
545	73640	73648	73656	73664	73672	73679	73687	73695	73703	73711		8
546	3719	3727	3735	3743	3751	3759	3767	3775	3783	3791	1	1
547	3799	3807	3815	3823	3830	3838	3846	3854	3862	3870	2	2
548	3878	3886	3894	3902	3910	3918	3926	3933	3941	3949	3	2
549	3957	3965	3973	3981	3989	3997	4005	4013	4020	4028	4	3
550	74036	74044	74052	74060	74068	74076	74084	74092	74099	74107	5	4
551	4115	4123	4131	4139	4147	4155	4162	4170	4178	4186	6	5
552	4194	4202	4210	4218	4225	4233	4241	4249	4257	4265	7	6
553	4273	4280	4288	4296	4304	4312	4320	4327	4335	4343	8	6
554	4351	4359	4367	4374	4382	4390	4398	4406	4414	4421	9	7
555	74429	74437	74445	74453	74461	74468	74476	74484	74492	74500		
556	4507	4515	4523	4531	4539	4547	4554	4562	4570	4578		
557	4586	4593	4601	4609	4617	4624	4632	4640	4648	4656		
558	4663	4671	4679	4687	4695	4702	4710	4718	4726	4733		
559	4741	4749	4757	4764	4772	4780	4788	4796	4803	4811		
560	74819	74827	74834	74842	74850	74858	74865	74873	74881	74889		
561	4896	4904	4912	4920	4927	4935	4943	4950	4958	4966		
562	4974	4981	4989	4997	5005	5012	5020	5028	5035	5043		
563	5051	5059	5066	5074	5082	5089	5097	5105	5113	5120		
564	5128	5136	5143	5151	5159	5166	5174	5182	5189	5197		
565	75205	75213	75220	75228	75236	75243	75251	75259	75266	75274		
566	5282	5289	5297	5305	5312	5320	5328	5335	5343	5351		
567	5358	5366	5374	5381	5389	5397	5404	5412	5420	5427		
568	5435	5442	5450	5458	5465	5473	5481	5488	5496	5504		
569	5511	5519	5526	5534	5542	5549	5557	5565	5572	5580		
570	75587	75595	75603	75610	75618	75626	75633	75641	75648	75656		7
571	5664	5671	5679	5686	5694	5702	5709	5717	5724	5732	1	1
572	5740	5747	5755	5762	5770	5778	5785	5793	5800	5808	2	1
573	5815	5823	5831	5838	5846	5853	5861	5868	5876	5884	3	2
574	5891	5899	5906	5914	5921	5929	5937	5944	5952	5959	4	3
575	75967	75974	75982	75989	75997	76005	76012	76020	76027	76035	5	4
576	6042	6050	6057	6065	6072	6080	6087	6095	6103	6110	6	4
577	6118	6125	6133	6140	6148	6155	6163	6170	6178	6185	7	5
578	6193	6200	6208	6215	6223	6230	6238	6245	6253	6260	8	6
579	6268	6275	6283	6290	6298	6305	6313	6320	6328	6335	9	6
No.	0	1	2	3	4	5	6	7	8	9		

No. 5800 to 6400. **Logarithms.** Log. 76343 to 80618.

No.	0	1	2	3	4	5	6	7	8	9		8
580	76343	76350	76358	76365	76373	76380	76388	76395	76403	76410	1	·1
581	6418	6425	6433	6440	6448	6455	6462	6470	6477	6485	2	2
582	6492	6500	6507	6515	6522	6530	6537	6545	6552	6559	3	2
583	6567	6574	6582	6589	6597	6604	6612	6619	6626	6634	4	3
584	6641	6649	6656	6664	6671	6678	6686	6693	6701	6708	5	4
585	76716	76723	76730	76738	76745	76753	76760	76768	76775	76782	6	5
586	6790	6797	6805	6812	6819	6827	6834	6842	6849	6856	7	6
587	6864	6871	6879	6886	6893	6901	6908	6916	6923	6930	8	6
588	6938	6945	6953	6960	6967	6975	6982	6989	6997	7004	9	7
589	7012	7019	7026	7034	7041	7048	7056	7063	7070	7078		
590	77085	77093	77100	77107	77115	77122	77129	77137	77144	77151		
591	7159	7166	7173	7181	7188	7195	7203	7210	7217	7225		
592	7232	7240	7247	7254	7262	7269	7276	7283	7291	7298		
593	7305	7313	7320	7327	7335	7342	7349	7357	7364	7371		
594	7379	7386	7393	7401	7408	7415	7422	7430	7437	7444		
595	77452	77459	77466	77474	77481	77488	77495	77503	77510	77517		
596	7525	7532	7539	7546	7554	7561	7568	7576	7583	7590		
597	7597	7605	7612	7619	7627	7634	7641	7648	7656	7663		
598	7670	7677	7685	7692	7699	7706	7714	7721	7728	7735		
599	7743	7750	7757	7764	7772	7779	7786	7793	7801	7808		
600	77815	77822	77830	77837	77844	77851	77859	77866	77873	77880		
601	7887	7895	7902	7909	7916	7924	7931	7938	7945	7952		
602	7960	7967	7974	7981	7988	7996	8003	8010	8017	8025		
603	8032	8039	8046	8053	8061	8068	8075	8082	8089	8097		
604	8104	8111	8118	8125	8132	8140	8147	8154	8161	8168		
605	78176	78183	78190	78197	78204	78211	78219	78226	78233	78240		7
606	8247	8254	8262	8269	8276	8283	8290	8297	8305	8312	1	1
607	8319	8326	8333	8340	8347	8355	8362	8369	8376	8383	2	1
608	8390	8398	8405	8412	8419	8426	8433	8440	8447	8455	3	2
609	8462	8469	8476	8483	8490	8497	8504	8512	8519	8526	4	3
610	78533	78540	78547	78554	78561	78569	78576	78583	78590	78597	5	4
611	8604	8611	8618	8625	8633	8640	8647	8654	8661	8668	6	4
612	8675	8682	8689	8696	8704	8711	8718	8725	8732	8739	7	5
613	8746	8753	8760	8767	8774	8781	8789	8796	8803	8810	8	6
614	8817	8824	8831	8838	8845	8852	8859	8866	8873	8880	9	6
615	78888	78895	78902	78909	78916	78923	78930	78937	78944	78951		
616	8958	8965	8972	8979	8986	8993	9000	9007	9014	9021		
617	9029	9036	9043	9050	9057	9064	9071	9078	9085	9092		
618	9099	9106	9113	9120	9127	9134	9141	9148	9155	9162		
619	9169	9176	9183	9190	9197	9204	9211	9218	9225	9232		
620	79239	79246	79253	79260	79267	79274	79281	79288	79295	79302		
621	9309	9316	9323	9330	9337	9344	9351	9358	9365	9372		
622	9379	9386	9393	9400	9407	9414	9421	9428	9435	9442		
623	9449	9456	9463	9470	9477	9484	9491	9498	9505	9511		
624	9518	9525	9532	9539	9546	9553	9560	9567	9574	9581		
625	79588	79595	79602	79609	79616	79623	79630	79637	79644	79650		
626	9657	9664	9671	9678	9685	9692	9699	9706	9713	9720		
627	9727	9734	9741	9748	9754	9761	9768	9775	9782	9789		
628	9796	9803	9810	9817	9824	9831	9837	9844	9851	9858		
629	9865	9872	9879	9886	9893	. 9900	9906	9913	9920	9927		
630	79934	79941	79948	79955	79962	79969	79975	79982	79989	79996		6
631	80003	80010	80017	80024	80030	80037	80044	80051	80058	80065	1	1
632	0072	0079	0085	0092	0099	0106	0113	0120	0127	0134	2	1
633	0140	0147	0154	0161	0168	0175	0182	0188	0195	0202	3	2
634	0209	0216	0223	0229	0236	0243	0250	0257	0264	0271	4	2
635	80277	80284	80291	80298	80305	80312	80318	80325	80332	80339	5	3
636	0346	0353	0359	0366	0373	0380	0387	0393	0400	0407	6	4
637	0414	0421	0428	0434	0441	0448	0455	0462	0468	0475	7	4
638	0482	0489	0496	0502	0509	0516	0523	0530	0536	0543	8	5
639	0550	0557	0564	0570	0577	0584	0591	0598	0604	0611	9	5
No.	0	1	2	3	4	5	6	7	8	9		

No. 6400 to 7000. **Logarithms.** Log. 80618 to 84510.

No.	0	1	2	3	4	5	6	7	8	9	
640	80618	80625	80632	80638	80645	80652	80659	80665	80672	80679	7
641	0686	0693	0699	0706	0713	0720	0726	0733	0740	0747	1 1
642	0754	0760	0767	0774	0781	0787	0794	0801	0808	0814	2 1
643	0821	0828	0835	0841	0848	0855	0862	0868	0875	0882	3 2
644	0889	0895	0902	0909	0916	0922	0929	0936	0943	0949	4 3
645	80956	80963	80969	80976	80983	80990	80996	81003	81010	81017	5 4
646	1023	1030	1037	1043	1050	1057	1064	1070	1077	1084	6 4
647	1090	1097	1104	1111	1117	1124	1131	1137	1144	1151	7 5
648	1158	1164	1171	1178	1184	1191	1198	1204	1211	1218	8 6
649	1224	1231	1238	1245	1251	1258	1265	1271	1278	1285	9 6
650	81291	81298	81305	81311	81318	81325	81331	81338	81345	81351	
651	1358	1365	1371	1378	1385	1391	1398	1405	1411	1418	
652	1425	1431	1438	1445	1451	1458	1465	1471	1478	1485	
653	1491	1498	1505	1511	1518	1525	1531	1538	1544	1551	
654	1558	1564	1571	1578	1584	1591	1598	1604	1611	1617	
655	81624	81631	81637	81644	81651	81657	81664	81671	81677	81684	
656	1690	1697	1704	1710	1717	1723	1730	1737	1743	1750	
657	1757	1763	1770	1776	1783	1790	1796	1803	1809	1816	
658	1823	1829	1836	1842	1849	1856	1862	1869	1875	1882	
659	1889	1895	1902	1908	1915	1921	1928	1935	1941	1948	
660	81954	81961	81968	81974	81981	81987	81994	82000	82007	82014	
661	2020	2027	2033	2040	2046	2053	2060	2066	2073	2079	
662	2086	2092	2099	2105	2112	2119	2125	2132	2138	2145	
663	2151	2158	2164	2171	2178	2184	2 91	2197	2204	2210	
664	2217	2223	2230	2236	2243	2249	2256	2263	2269	2276	
665	82282	82289	82295	82302	82308	82315	82321	82328	82334	82341	
666	2347	2354	2360	2367	2373	2380	2387	2393	2400	2406	
667	2413	2419	2426	2432	2439	2445	2452	2458	2465	2471	
668	2478	2484	2491	2497	2504	2510	2517	2523	2530	2536	
669	2543	2549	2556	2562	2569	2575	2582	2588	2595	2601	
670	82607	82614	82620	82627	82633	82640	82646	82653	82659	82666	
671	2672	2679	2685	2692	2698	2705	2711	2718	2724	2730	
672	2737	2743	2750	2756	2763	2769	2776	2782	2789	2795	
673	2802	2808	2814	2821	2827	2834	2840	2847	2853	2860	
674	2866	2872	2879	2885	2892	2898	2905	2911	2918	2924	
675	82930	82937	82943	82950	82956	82963	82969	82975	82982	82988	
676	2995	3001	3008	3014	3020	3027	3033	3040	3046	3052	
677	3059	3065	3072	3078	3085	3091	3097	3104	3110	3117	
678	3123	3129	3136	3142	3149	3155	3161	3168	3174	3181	
679	3187	3193	3200	3206	3213	3219	3225	3232	3238	3245	
680	83251	83257	83264	83270	83276	83283	83289	83296	83302	83308	
681	3315	3321	3327	3334	3340	3347	3353	3359	3366	3372	
682	3378	3385	3391	3398	3404	3410	3417	3423	3429	3436	
683	3442	3448	3455	3461	3467	3474	3480	3487	3493	3499	
684	3506	3512	3518	3525	3531	3537	3544	3550	3556	3563	
685	83569	83575	83582	83588	83594	83601	83607	83613	83620	83626	6
686	3632	3639	3645	3651	3658	3664	3670	3677	3683	3689	1 1
687	3696	3702	3708	3715	3721	3727	3734	3740	3746	3753	2 1
688	3759	3765	3771	3778	3784	3790	3797	3803	3809	3816	3 2
689	3822	3828	3835	3841	3847	3853	3860	3866	3872	3879	4 2
690	83885	83891	83897	83904	83910	83916	83923	83929	83935	83942	5 3
691	3948	3954	3960	3967	3973	3979	3985	3992	3998	4004	6 4
692	4011	4017	4023	4029	4036	4042	4048	4055	4061	4067	7 4
693	4073	4080	4086	4092	4098	4105	4111	4117	4123	4130	8 5
694	4136	4142	4148	4155	4161	4167	4173	4180	4186	4192	9 5
695	84198	84205	84211	84217	84223	84230	84236	84242	84248	84255	
696	4261	4267	4273	4280	4286	4292	4298	4305	4311	4317	
697	4323	4330	4336	4342	4348	4354	4361	4367	4373	4379	
698	4386	4392	4398	4404	4410	4417	4423	4429	4435	4442	
699	4448	4454	4460	4466	4473	4479	4485	4491	4497	4504	
No.	0	1	2	3	4	5	6	7	8	9	

No. 7000 to 7600. **Logarithms.** Log. 84510 to 88081.

No.	0	1	2	3	4	5	6	7	8	9		
700	84510	84516	84522	84528	84535	84541	84547	84553	84559	84566		**7**
701	4572	4578	4584	4590	4597	4603	4609	4615	4621	4628	1	1
702	4634	4640	4646	4652	4658	4665	4671	4677	4683	4689	2	1
703	4696	4702	4708	4714	4720	4726	4733	4739	4745	4751	3	2
704	4757	4763	4770	4776	4782	4788	4794	4800	4807	4813	4	3
705	84819	84825	84831	84837	84844	84850	84856	84862	84868	84874	5	4
706	4880	4887	4893	4899	4905	4911	4917	4924	4930	4936	6	4
707	4942	4948	4954	4960	4967	4973	4979	4985	4991	4997	7	5
708	5003	5009	5016	5022	5028	5034	5040	5046	5052	5058	8	6
709	5065	5071	5077	5083	5089	5095	5101	5107	5114	5120	9	6
710	85126	85132	85138	85144	85150	85156	85163	85169	85175	85181		
711	5187	5193	5199	5205	5211	5217	5224	5230	5236	5242		
712	5248	5254	5260	5266	5272	5278	5285	5291	5297	5303		
713	5309	5315	5321	5327	5333	5339	5345	5352	5358	5364		
714	5370	5376	5382	5388	5394	5400	5406	5412	5418	5425		
715	85431	85437	85443	85449	85455	85461	85467	85473	85479	85485		
716	5491	5497	5503	5509	5516	5522	5528	5534	5540	5546		
717	5552	5558	5564	5570	5576	5582	5588	5594	5600	5606		
718	5612	5618	5625	5631	5637	5643	5649	5655	5661	5667		
719	5673	5679	5685	5691	5697	5703	5709	5715	5721	5727		
720	85733	85739	85745	85751	85757	85763	85769	85775	85781	85788		
721	5794	5800	5806	5812	5818	5824	5830	5836	5842	5848		
722	5854	5860	5866	5872	5878	5884	5890	5896	5902	5908		
723	5914	5920	5926	5932	5938	5944	5950	5956	5962	5968		
724	5974	5980	5986	5992	5998	6004	6010	6016	6022	6028		**6**
725	86034	86040	86046	86052	86058	86064	86070	86076	86082	86088	1	1
726	6094	6100	6106	6112	6118	6124	6130	6136	6141	6147	2	1
727	6153	6159	6165	6171	6177	6183	6189	6195	6201	6207	3	2
728	6213	6219	6225	6231	6237	6243	6249	6255	6261	6267	4	2
729	6273	6279	6285	6291	6297	6303	6308	6314	6320	6326	5	3
730	86332	86338	86344	86350	86356	86362	86368	86374	86380	86386	6	4
731	6392	6398	6404	6410	6415	6421	6427	6433	6439	6445	7	4
732	6451	6457	6463	6469	6475	6481	6487	6493	6499	6504	8	5
733	6510	6516	6522	6528	6534	6540	6546	6552	6558	6564	9	5
734	6570	6576	6581	6587	6593	6599	6605	6611	6617	6623		
735	86629	86635	86641	86646	86652	86658	86664	86670	86676	86682		
736	6688	6694	6700	6705	6711	6717	6723	6729	6735	6741		
737	6747	6753	6759	6764	6770	6776	6782	6788	6794	6800		
738	6806	6812	6817	6823	6829	6835	6841	6847	6853	6859		
739	6864	6870	6876	6882	6888	6894	6900	6906	6911	6917		
740	86923	86929	86935	86941	86947	86953	86958	86964	86970	86976		
741	6982	6988	6994	6999	7005	7011	7017	7023	7029	7035		
742	7040	7046	7052	7058	7064	7070	7075	7081	7087	7093		
743	7099	7105	7111	7116	7122	7128	7134	7140	7146	7151		
744	7157	7163	7169	7175	7181	7186	7192	7198	7204	7210		
745	87216	87221	87227	87233	87239	87245	87251	87256	87262	87268		
746	7274	7280	7286	7291	7297	7303	7309	7315	7320	7326		
747	7332	7338	7344	7349	7355	7361	7367	7373	7379	7384		
748	7390	7396	7402	7408	7413	7419	7425	7431	7437	7442		
749	7448	7454	7460	7466	7471	7477	7483	7489	7495	7500		
750	87506	87512	87518	87523	87529	87535	87541	87547	87552	87558		**6**
751	7564	7570	7576	7581	7587	7593	7599	7604	7610	7616	1	1
752	7622	7628	7633	7639	7645	7651	7656	7662	7668	7674	2	1
753	7679	7685	7691	7697	7703	7708	7714	7720	7726	7731	3	2
754	7737	7743	7749	7754	7760	7766	7772	7777	7783	7789	4	2
755	87795	87800	87806	87812	87818	87823	87829	87835	87841	87846	5	3
756	7852	7858	7864	7869	7875	7881	7887	7892	7898	7904	6	3
757	7910	7915	7921	7927	7933	7938	7944	7950	7955	7961	7	4
758	7967	7973	7978	7984	7990	7996	8001	8007	8013	8018	8	4
759	8024	8030	8036	8041	8047	8053	8058	8064	8070	8076	9	5
No.	0	1	2	3	4	5	6	7	8	9		

No. 7600 to 8200. **Logarithms.** Log. 88081 to 91381.

No.	0	1	2	3	4	5	6	7	8	9
760	88081	88087	88093	88098	88104	88110	88116	88121	88127	88133
761	8138	8144	8150	8156	8161	8167	8173	8178	8184	8190
762	8195	8201	8207	8213	8218	8224	8230	8235	8241	8247
763	8252	8258	8264	8270	8275	8281	8287	8292	8298	8304
764	8309	8315	8321	8326	8332	8338	8343	8349	8355	8360
765	88366	88372	88377	88383	88389	88395	88400	88406	88412	88417
766	8423	8429	8434	8440	8446	8451	8457	8463	8468	8474
767	8480	8485	8491	8497	8502	8508	8513	8519	8525	8530
768	8536	8542	8547	8553	8559	8564	8570	8576	8581	8587
769	8593	8598	8604	8610	8615	8621	8627	8632	8638	8643
770	88649	88655	88660	88666	88672	88677	88683	88689	88694	88700
771	8705	8711	8717	8722	8728	8734	8739	8745	8750	8756
772	8762	8767	8773	8779	8784	8790	8795	8801	8807	8812
773	8818	8824	8829	8835	8840	8846	8852	8857	8863	8868
774	8874	8880	8885	8891	8897	8902	8908	8913	8919	8925
775	88930	88936	88941	88947	88953	88958	88964	88969	88975	88981
776	8986	8992	8997	9003	9009	9014	9020	9025	9031	9037
777	9042	9048	9053	9059	9064	9070	9076	9081	9087	9092
778	9098	9104	9109	9115	9120	9126	9131	9137	9143	9148
779	9154	9159	9165	9170	9176	9182	9187	9193	9198	9204
780	89209	89215	89221	89226	89232	89237	89243	89248	89254	89260
781	9265	9271	9276	9282	9287	9293	9298	9304	9310	9315
782	9321	9326	9332	9337	9343	9348	9354	9360	9365	9371
783	9376	9382	9387	9393	9398	9404	9409	9415	9421	9426
784	9432	9437	9443	9448	9454	9459	9465	9470	9476	9481
785	89487	89492	89498	89504	89509	89515	89520	89526	89531	89537
786	9542	9548	9553	9559	9564	9570	9575	9581	9586	9592
787	9597	9603	9609	9614	9620	9625	9631	9636	9642	9647
788	9653	9658	9664	9669	9675	9680	9686	9691	9697	9702
789	9708	9713	9719	9724	9730	9735	9741	9746	9752	9757
790	89763	89768	89774	89779	89785	89790	89796	89801	89807	89812
791	9818	9823	9829	9834	9840	9845	9851	9856	9862	9867
792	9873	9878	9883	9889	9894	9900	9905	9911	9916	9922
793	9927	9933	9938	9944	9949	9955	9960	9966	9971	9977
794	9982	9988	9993	9998	90004	90009	90015	90020	90026	90031
795	90037	90042	90048	90053	90059	90064	90069	90075	90080	90086
796	0091	0097	0102	0108	0113	0119	0124	0129	0135	0140
797	0146	0151	0157	0162	0168	0173	0179	0184	0189	0195
798	0200	0206	0211	0217	0222	0227	0233	0238	0244	0249
799	0255	0260	0266	0271	0276	0282	0287	0293	0298	0304
800	90309	90314	90320	90325	90331	90336	90342	90347	90352	90358
801	0363	0369	0374	0380	0385	0390	0396	0401	0407	0412
802	0417	0423	0428	0434	0439	0445	0450	0455	0461	0466
803	0472	0477	0482	0488	0493	0499	0504	0509	0515	0520
804	0526	0531	0536	0542	0547	0553	0558	0563	0569	0574
805	90580	90585	90590	90596	90601	90607	90612	90617	90623	90628
806	0634	0639	0644	0650	0655	0660	0666	0671	0677	0682
807	0687	0693	0698	0703	0709	0714	0720	0725	0730	0736
808	0741	0747	0752	0757	0763	0768	0773	0779	0784	0789
809	0795	0800	0806	0811	0816	0822	0827	0832	0838	0843
810	90849	90854	90859	90865	90870	90875	90881	90886	90891	90897
811	0902	0907	0913	0918	0924	0929	0934	0940	0945	0950
812	0956	0961	0966	0972	0977	0982	0988	0993	0998	1004
813	1009	1014	1020	1025	1030	1036	1041	1046	1052	1057
814	1062	1068	1073	1078	1084	1089	1094	1100	1105	1110
815	91116	91121	91126	91132	91137	91142	91148	91153	91158	91164
816	1169	1174	1180	1185	1190	1196	1201	1206	1212	1217
817	1222	1228	1233	1238	1243	1249	1254	1259	1265	1270
818	1275	1281	1286	1291	1297	1302	1307	1312	1318	1323
819	1328	1334	1339	1344	1350	1355	1360	1365	1371	1376
No.	0	1	2	3	4	5	6	7	8	9

	6
1	1
2	1
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4	2
5	3
6	4
7	4
8	5
9	5

	5
1	1
2	1
3	2
4	2
5	3
6	3
7	4
8	4
9	5

No. 8200 to 8800. **Logarithms.** Log. 91381 to 94448.

No.	0	1	2	3	4	5	6	7	8	9		6
820	91381	91387	91392	91397	91403	91408	91413	91418	91424	91429	1	1
821	1434	1440	1445	1450	1455	1461	1466	1471	1477	1482	2	1
822	1487	1492	1498	1503	1508	1514	1519	1524	1529	1535	3	2
823	1540	1545	1551	1556	1561	1566	1572	1577	1582	1587	4	2
824	1593	1598	1603	1609	1614	1619	1624	1630	1635	1640	5	3
825	91645	91651	91656	91661	91666	91672	91677	91682	91687	91693	6	4
826	1698	1703	1709	1714	1719	1724	1730	1735	1740	1745	7	4
827	1751	1756	1761	1766	1772	1777	1782	1787	1793	1798	8	5
828	1803	1808	1814	1819	1824	1829	1834	1840	1845	1850	9	5
829	1855	1861	1866	1871	1876	1882	1887	1892	1897	1903		
830	91908	91913	91918	91924	91929	91934	91939	91944	91950	91955		
831	1960	1965	1971	1976	1981	1986	1991	1997	2002	2007		
832	2012	2018	2023	2028	2033	2038	2044	2049	2054	2059		
833	2065	2070	2075	2080	2085	2091	2096	2101	2106	2111		
834	2117	2122	2127	2132	2137	2143	2148	2153	2158	2163		
835	92169	92174	92179	92184	92189	92195	92200	92205	92210	92215		
836	2221	2226	2231	2236	2241	2247	2252	2257	2262	2267		
837	2273	2278	2283	2288	2293	2298	2304	2309	2314	2319		
838	2324	2330	2335	2340	2345	2350	2355	2361	2366	2371		
839	2376	2381	2387	2392	2397	2402	2407	2412	2418	2423		
840	92428	92433	92438	92443	92449	92454	92459	92464	92469	92474		
841	2480	2485	2490	2495	2500	2505	2511	2516	2521	2526		
842	2531	2536	2542	2547	2552	2557	2562	2567	2572	2578		
843	2583	2588	2593	2598	2603	2609	2614	2619	2624	2629		
844	2634	2639	2645	2650	2655	2660	2665	2670	2675	2681		5
845	92686	92691	92696	92701	92706	92711	92716	92722	92727	92732	1	1
846	2737	2742	2747	2752	2758	2763	2768	2773	2778	2783	2	1
847	2788	2793	2799	2804	2809	2814	2819	2824	2829	2834	3	2
848	2840	2845	2850	2855	2860	2865	2870	2875	2881	2886	4	2
849	2891	2896	2901	2906	2911	2916	2921	2927	2932	2937	5	3
850	92942	92947	92952	92957	92962	92967	92973	92978	92983	92988	6	3
851	2993	2998	3003	3008	3013	3018	3024	3029	3034	3039	7	4
852	3044	3049	3054	3059	3064	3069	3075	3080	3085	3090	8	4
853	3095	3100	3105	3110	3115	3120	3125	3131	3136	3141	9	5
854	3146	3151	3156	3161	3166	3171	3176	3181	3186	3192		
855	93197	93202	93207	93212	93217	93222	93227	93232	93237	93242		
856	3247	3252	3258	3263	3268	3273	3278	3283	3288	3293		
857	3298	3303	3308	3313	3318	3323	3328	3334	3339	3344		
858	3349	3354	3359	3364	3369	3374	3379	3384	3389	3394		
859	3399	3404	3409	3414	3420	3425	3430	3435	3440	3445		
860	93450	93455	93460	93465	93470	93475	93480	93485	93490	93495		
861	3500	3505	3510	3515	3520	3526	3531	3536	3541	3546		
862	3551	3556	3561	3566	3571	3576	3581	3586	3591	3596		
863	3601	3606	3611	3616	3621	3626	3631	3636	3641	3646		
864	3651	3656	3661	3666	3671	3676	3682	3687	3692	3697		
865	93702	93707	93712	93717	93722	93727	93732	93737	93742	93747		
866	3752	3757	3762	3767	3772	3777	3782	3787	3792	3797		
867	3802	3807	3812	3817	3822	3827	3832	3837	3842	3847		
868	3852	3857	3862	3867	3872	3877	3882	3887	3892	3897		
869	3902	3907	3912	3917	3922	3927	3932	3937	3942	3947		
870	93952	93957	93962	93967	93972	93977	93982	93987	93992	93997		4
871	4002	4007	4012	4017	4022	4027	4032	4037	4042	4047	1	0
872	4052	4057	4062	4067	4072	4077	4082	4086	4091	4096	2	1
873	4101	4106	4111	4116	4121	4126	4131	4136	4141	4146	3	1
874	4151	4156	4161	4166	4171	4176	4181	4186	4191	4196	4	2
875	94201	94206	94211	94216	94221	94226	94231	94236	94240	94245	5	2
876	4250	4255	4260	4265	4270	4275	4280	4285	4290	4295	6	2
877	4300	4305	4310	4315	4320	4325	4330	4335	4340	4345	7	3
878	4349	4354	4359	4364	4369	4374	4379	4384	4389	4394	8	3
879	4399	4404	4409	4414	4419	4424	4429	4433	4438	4443	9	4
No.	0	1	2	3	4	5	6	7	8	9		

No. 8800 to 9400. **Logarithms.** Log. 94448 to 97313.

No.	0	1	2	3	4	5	6	7	8	9
880	94448	94453	94458	94463	94468	94473	94478	94483	94488	94493
881	4498	4503	4507	4512	4517	4522	4527	4532	4537	4542
882	4547	4552	4557	4562	4567	4571	4576	4581	4586	4591
883	4596	4601	4606	4611	4616	4621	4626	4630	4635	4640
884	4645	4650	4655	4660	4665	4670	4675	4680	4685	4689
885	94694	94699	94704	94709	94714	94719	94724	94729	94734	94738
886	4743	4748	4753	4758	4763	4768	4773	4778	4783	4787
887	4792	4797	4802	4807	4812	4817	4822	4827	4832	4836
888	4841	4846	4851	4856	4861	4866	4871	4876	4880	4885
889	4890	4895	4900	4905	4910	4915	4919	4924	4929	4934
890	94939	94944	94949	94954	94959	94963	94968	94973	94978	94983
891	4988	4993	4998	5002	5007	5012	5017	5022	5027	5032
892	5036	5041	5046	5051	5056	5061	5066	5071	5075	5080
893	5085	5090	5095	5100	5105	5109	5114	5119	5124	5129
894	5134	5139	5143	5148	5153	5158	5163	5168	5173	5177
895	95182	95187	95192	95197	95202	95207	95211	95216	95221	95226
896	5231	5236	5240	5245	5250	5255	5260	5265	5270	5274
897	5279	5284	5289	5294	5299	5303	5308	5313	5318	5323
898	5328	5332	5337	5342	5347	5352	5357	5361	5366	5371
899	5376	5381	5386	5390	5395	5400	5405	5410	5415	5419
900	95424	95429	95434	95439	95444	95448	95453	95458	95463	95468
901	5472	5477	5482	5487	5492	5497	5501	5506	5511	5516
902	5521	5525	5530	5535	5540	5545	5550	5554	5559	5564
903	5569	5574	5578	5583	5588	5593	5598	5602	5607	5612
904	5617	5622	5626	5631	5636	5641	5646	5650	5655	5660
905	95665	95670	95674	95679	95684	95689	95694	95698	95703	95708
906	5713	5718	5722	5727	5732	5737	5742	5746	5751	5756
907	5761	5766	5770	5775	5780	5785	5789	5794	5799	5804
908	5809	5813	5818	5823	5828	5832	5837	5842	5847	5852
909	5856	5861	5866	5871	5875	5880	5885	5890	5895	5899
910	95904	95909	95914	95918	95923	95928	95933	95938	95942	95947
911	5952	5957	5961	5966	5971	5976	5980	5985	5990	5995
912	5999	6004	6009	6014	6019	6023	6028	6033	6038	6042
913	6047	6052	6057	6061	6066	6071	6076	6080	6085	6090
914	6095	6099	6104	6109	6114	6118	6123	6128	6133	6137
915	96142	96147	96152	96156	96161	96166	96171	96175	96180	96185
916	6190	6194	6199	6204	6209	6213	6218	6223	6227	6232
917	6237	6242	6246	6251	6256	6261	6265	6270	6275	6280
918	6284	6289	6294	6298	6303	6308	6313	6317	6322	6327
919	6332	6336	6341	6346	6350	6355	6360	6365	6369	6374
920	96379	96384	96388	96393	96398	96402	96407	96412	96417	96421
921	6426	6431	6435	6440	6445	6450	6454	6459	6464	6468
922	6473	6478	6483	6487	6492	6497	6501	6506	6511	6515
923	6520	6525	6530	6534	6539	6544	6548	6553	6558	6562
924	6567	6572	6577	6581	6586	6591	6595	6600	6605	6609
925	96614	96619	96624	96628	96633	96638	96642	96647	96652	96656
926	6661	6666	6670	6675	6680	6685	6689	6694	6699	6703
927	6708	6713	6717	6722	6727	6731	6736	6741	6745	6750
928	6755	6759	6764	6769	6774	6778	6783	6788	6792	6797
929	6802	6806	6811	6816	6820	6825	6830	6834	6839	6844
930	96848	96853	96858	96862	96867	96872	96876	96881	96886	96890
931	6895	6900	6904	6909	6914	6918	6923	6928	6932	6937
932	6942	6946	6951	6956	6960	6965	6970	6974	6979	6984
933	6988	6993	6997	7002	7007	7011	7016	7021	7025	7030
934	7035	7039	7044	7049	7053	7058	7063	7067	7072	7077
935	97081	97086	97090	97095	97100	97104	97109	97114	97118	97123
936	7128	7132	7137	7142	7146	7151	7155	7160	7165	7169
937	7174	7179	7183	7188	7192	7197	7202	7206	7211	7216
938	7220	7225	7230	7234	7239	7243	7248	7253	7257	7262
939	7267	7271	7276	7280	7285	7290	7294	7299	7304	7308
No.	0	1	2	3	4	5	6	7	8	9

	5
1	1
2	1
3	2
4	2
5	3
6	3
7	4
8	4
9	5

	4
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2	1
3	1
4	2
5	2
6	2
7	3
8	3
9	4

No. 9400 to 10000. **Logarithms.** Log. 97313 to 99996.

No.	0	1	2	3	4	5	6	7	8	9		5
940	97313	97317	97322	97327	97331	97336	97340	97345	97350	97354	1	1
941	7359	7364	7368	7373	7377	7382	7387	7391	7396	7400	2	1
942	7405	7410	7414	7419	7424	7428	7433	7437	7442	7447	3	2
943	7451	7456	7460	7465	7470	7474	7479	7483	7488	7493	4	2
944	7497	7502	7506	7511	7516	7520	7525	7529	7534	7539	5	3
945	97543	97548	97552	97557	97562	97566	97571	97575	97580	97585	6	3
946	7589	7594	7598	7603	7607	7612	7617	7621	7626	7630	7	4
947	7635	7640	7644	7649	7653	7658	7663	7667	7672	7676	8	4
948	7681	7685	7690	7695	7699	7704	7708	7713	7717	7722	9	5
949	7727	7731	7736	7740	7745	7749	7754	7759	7763	7768		
950	97772	97777	97782	97786	97791	97795	97800	97804	97809	97813		
951	7818	7823	7827	7832	7836	7841	7845	7850	7855	7859		
952	7864	7868	7873	7877	7882	7886	7891	7896	7900	7905		
953	7909	7914	7918	7923	7928	7932	7937	7941	7946	7950		
954	7955	7959	7964	7968	7973	7978	7982	7987	7991	7996		
955	98000	98005	98009	98014	98019	98023	98028	98032	98037	98041		
956	8046	8050	8055	8059	8064	8068	8073	8078	8082	8087		
957	8091	8096	8100	8105	8109	8114	8118	8123	8127	8132		
958	8137	8141	8146	8150	8155	8159	8164	8168	8173	8177		
959	8182	8186	8191	8195	8200	8204	8209	8214	8218	8223		
960	98227	98232	98236	98241	98245	98250	98254	98259	98263	98268		
961	8272	8277	8281	8286	8290	8295	8299	8304	8308	8313		
962	8318	8322	8327	8331	8336	8340	8345	8349	8354	8358		
963	8363	8367	8372	8376	8381	8385	8390	8394	8399	8403		
964	8408	8412	8417	8421	8426	8430	8435	8439	8444	8448		
965	98453	98457	98462	98466	98471	98475	98480	98484	98489	98493		
966	8498	8502	8507	8511	8516	8520	8525	8529	8534	8538		
967	8543	8547	8552	8556	8561	8565	8570	8574	8579	8583		
968	8588	8592	8597	8601	8605	8610	8614	8619	8623	8628		
969	8632	8637	8641	8646	8650	8655	8659	8664	8668	8673		
970	98677	98682	98686	98691	98695	98700	98704	98709	98713	98717		
971	8722	8726	8731	8735	8740	8744	8749	8753	8758	8762		
972	8767	8771	8776	8780	8784	8789	8793	8798	8802	8807		
973	8811	8816	8820	8825	8829	8834	8838	8843	8847	8851		
974	8856	8860	8865	8869	8874	8878	8883	8887	8892	8896		
975	98900	98905	98909	98914	98918	98923	98927	98932	98936	98941		
976	8945	8949	8954	8958	8963	8967	8972	8976	8981	8985		
977	8989	8994	8998	9003	9007	9012	9016	9021	9025	9029		
978	9034	9038	9043	9047	9052	9056	9061	9065	9069	9074		
979	9078	9083	9087	9092	9096	9100	9105	9109	9114	9118		
980	99123	99127	99131	99136	99140	99145	99149	99154	99158	99162		
981	9167	9171	9176	9180	9185	9189	9193	9198	9202	9207		
982	9211	9216	9220	9224	9229	9233	9238	9242	9247	9251		
983	9255	9260	9264	9269	9273	9277	9282	9286	9291	9295		
984	9300	9304	9308	9313	9317	9322	9326	9330	9335	9339		
985	99344	99348	99352	99357	99361	99366	99370	99374	99379	99383		4
986	9388	9392	9396	9401	9405	9410	9414	9419	9423	9427	1	0
987	9432	9436	9441	9445	9449	9454	9458	9463	9467	9471	2	1
988	9476	9480	9484	9489	9493	9498	9502	9506	9511	9515	3	1
989	9520	9524	9528	9533	9537	9542	9546	9550	9555	9559	4	2
990	99564	99568	99572	99577	99581	99585	99590	99594	99599	99603	5	2
991	9607	9612	9616	9621	9625	9629	9634	9638	9642	9647	6	2
992	9651	9656	9660	9664	9669	9673	9677	9682	9686	9691	7	3
993	9695	9699	9704	9708	9712	9717	9721	9726	9730	9734	8	3
994	9739	9743	9747	9752	9756	9760	9765	9769	9774	9778	9	4
995	99782	99787	99791	99795	99800	99804	99808	99813	99817	99822		
996	9826	9830	9835	9839	9843	9848	9852	9856	9861	9865		
997	9870	9874	9878	9883	9887	9891	9896	9900	9904	9909		
998	9913	9917	9922	9926	9930	9935	9939	9944	9948	9952		
999	9957	9961	9965	9970	9974	9978	9983	9987	9991	9996		
No.	0	1	2	3	4	5	6	7	8	9		

0h	0°			Logarithms.				179°	11h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
00	0	Inf.Neg.	Infinite.	Inf. Neg.	Infinite.	10.00000	10.00000	60	**60**
4	1	6.46373	13.53627	6.46373	13.53627	00000	00000	59	56
8	2	76476	23524	76476	23524	00000	00000	58	52
12	3	94085	05915	94085	05915	00000	00000	57	48
16	4	7.06579	12.93421	7.06579	12.93421	00000	00000	56	44
20	5	7.16270	12.83730	7.16270	12.83730	10.00000	10.00000	55	40
24	6	24188	75812	24188	75812	00000	00000	54	36
28	7	30882	69118	30882	69118	00000	00000	53	32
32	8	36682	63318	36682	63318	00000	00000	52	28
36	9	41797	58203	41797	58203	00000	00000	51	24
40	10	7.46373	12.53627	7.46373	12.53627	10.00000	10.00000	50	20
44	11	50512	49488	50512	49488	00000	00000	49	16
48	12	54291	45709	54291	45709	00000	00000	48	12
52	13	57767	42233	57767	42233	00000	00000	47	8
56	14	60985	39015	60986	39014	00000	00000	46	4
1	15	7.63982	12.36018	7.63982	12.36018	10.00000	10.00000	45	**59**
4	16	66784	33216	66785	33215	00000	00000	44	56
8	17	69417	30583	69418	30582	00001	9.99999	43	52
12	18	71900	28100	71900	28100	00001	99999	42	48
16	19	74248	25752	74248	25752	00001	99999	41	44
20	20	7.76475	12.23525	7.76476	12.23524	10.00001	9.99999	40	40
24	21	78594	21406	78595	21405	00001	99999	39	36
28	22	80615	19385	80615	19385	00001	99999	38	32
32	23	82545	17455	82546	17454	00001	99999	37	28
36	24	84393	15607	84394	15606	00001	99999	36	24
40	25	7.86166	12.13834	7.86167	12.13833	10.00001	9.99999	35	20
44	26	87870	12130	87871	12129	00001	99999	34	16
48	27	89509	10491	89510	10490	00001	99999	33	12
52	28	91088	08912	91089	08911	00001	99999	32	8
56	29	92612	07388	92613	07387	00002	99998	31	4
2	30	7.94084	12.05916	7.94086	12.05914	10.00002	9.99998	30	**58**
4	31	95508	04492	95510	04490	00002	99998	29	56
8	32	96887	03113	96889	03111	00002	99998	28	52
12	33	98223	01777	98225	01775	00002	99998	27	48
16	34	99520	00480	99522	00478	00002	99998	26	44
20	35	8.00779	11.99221	8.00781	11.99219	10.00002	9.99998	25	40
24	36	02002	97998	02004	97996	00002	99998	24	36
28	37	03192	96808	03194	96806	00003	99997	23	32
32	38	04350	95650	04353	95647	00003	99997	22	28
36	39	05478	94522	05481	94519	00003	99997	21	24
40	40	8.06578	11.93422	8.06581	11.93419	10.00003	9.99997	20	20
44	41	07650	92350	07653	92347	00003	99997	19	16
48	42	08696	91304	08700	91300	00003	99997	18	12
52	43	09718	90282	09722	90278	00003	99997	17	8
56	44	10717	89283	10720	89280	00004	99996	16	4
3	45	8.11693	11.88307	8.11696	11.88304	10.00004	9.99996	15	**57**
4	46	12647	87353	12651	87349	00004	99996	14	56
8	47	13581	86419	13585	86415	00004	99996	13	52
12	48	14495	85505	14500	85500	00004	99996	12	48
16	49	15391	84609	15395	84605	00004	99996	11	44
20	50	8.16268	11.83732	8.16273	11.83727	10.00005	9.99995	10	40
24	51	17128	82872	17133	82867	00005	99995	9	36
28	52	17971	82029	17976	82024	00005	99995	8	32
32	53	18798	81202	18804	81196	00005	99995	7	28
36	54	19610	80390	19616	80384	00005	99995	6	24
40	55	8.20407	11.79593	8.20413	11.79587	10.00006	9.99994	5	20
44	56	21189	78811	21195	78805	00006	99994	4	16
48	57	21958	78042	21964	78036	00006	99994	3	12
52	58	22713	77287	22720	77280	00006	99994	2	8
56	59	23456	76544	23462	76538	00006	99994	1	4
4	60	24186	75814	24192	75808	00007	99993	0	**56**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6h	90°							89°	5h

0h	1°			Logarithms.				178°	11h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
4	0	8.24186	11.75814	8.24192	11.75808	10.00007	9.99993	60	**56**
4	1	24903	75097	24910	75090	00007	99993	59	56
8	2	25609	74391	25616	74384	00007	99993	58	52
12	3	26304	73696	26312	73688	00007	99993	57	48
16	4	26988	73012	26996	73004	00008	99992	56	44
20	5	8.27661	11.72339	8.27669	11.72331	10.00008	9.99992	55	40
24	6	28324	71676	28332	71668	00008	99992	54	36
28	7	28977	71023	28986	71014	00008	99992	53	32
32	8	29621	70379	29629	70371	00008	99992	52	28
36	9	30255	69745	30263	69737	00009	99991	51	24
40	10	8.30879	11.69121	8.30888	11.69112	10.00009	9.99991	50	20
44	11	31495	68505	31505	68495	00009	99991	49	16
48	12	32103	67897	32112	67888	00010	99990	48	12
52	13	32702	67298	32711	67289	00010	99990	47	8
56	14	33292	66708	33302	66698	00010	99990	46	4
5	15	8.33875	11.66125	8.33886	11.66114	10.00010	9.99990	45	**55**
4	16	34450	65550	34461	65539	00011	99989	44	56
8	17	35018	64982	35029	64971	00011	99989	43	52
12	18	35578	64422	35590	64410	00011	99989	42	48
16	19	36131	63869	36143	63857	00011	99989	41	44
20	20	8.36678	11.63322	8.36689	11.63311	10.00012	9.99988	40	40
24	21	37217	62783	37229	62771	00012	99988	39	36
28	22	37750	62250	37762	62238	00012	99988	38	32
32	23	38276	61724	38289	61711	00013	99987	37	28
36	24	38796	61204	38809	61191	00013	99987	36	24
40	25	8.39310	11.60690	8.39323	11.60677	10.00013	9.99987	35	20
44	26	39818	60182	39832	60168	00014	99986	34	16
48	27	40320	59680	40334	59666	00014	99986	33	12
52	28	40816	59184	40830	59170	00014	99986	32	8
56	29	41307	58693	41321	58679	00015	99985	31	4
6	30	8.41792	11.58208	8.41807	11.58193	10.00015	9.99985	30	**54**
4	31	42272	57728	42287	57713	00015	99985	29	56
8	32	42746	57254	42762	57238	00016	99984	28	52
12	33	43216	56784	43232	56768	00016	99984	27	48
16	34	43680	56320	43696	56304	00016	99984	26	44
20	35	8.44139	11.55861	8.44156	11.55844	10.00017	9.99983	25	40
24	36	44594	55406	44611	55389	00017	99983	24	36
28	37	45044	54956	45061	54939	00017	99983	23	32
32	38	45489	54511	45507	54493	00018	99982	22	28
36	39	45930	54070	45948	54052	00018	99982	21	24
40	40	8.46366	11.53634	8.46385	11.53615	10.00018	9.99982	20	20
44	41	46799	53201	46817	53183	00019	99981	19	16
48	42	47226	52774	47245	52755	00019	99981	18	12
52	43	47650	52350	47669	52331	00019	99981	17	8
56	44	48069	51931	48089	51911	00020	99980	16	4
7	45	8.48485	11.51515	8.48505	11.51495	10.00020	9.99980	15	**53**
4	46	48896	51104	48917	51083	00021	99979	14	56
8	47	49304	50696	49325	50675	00021	99979	13	52
12	48	49708	50292	49729	50271	00021	99979	12	48
16	49	50108	49892	50130	49870	00022	99978	11	44
20	50	8.50504	11.49496	8.50527	11.49473	10.00022	9.99978	10	40
24	51	50897	49103	50920	49080	00023	99977	9	36
28	52	51287	48713	51310	48690	00023	99977	8	32
32	53	51673	48327	51696	48304	00023	99977	7	28
36	54	52055	47945	52079	47921	00024	99976	6	24
40	55	8.52434	11.47566	8.52459	11.47541	10.00024	9.99976	5	20
44	56	52810	47190	52835	47165	00025	99975	4	16
48	57	53183	46817	53208	46792	00025	99975	3	12
52	58	53552	46448	53578	46422	00026	99974	2	8
56	59	53919	46081	53945	46055	00026	99974	1	4
8	60	54282	45718	54308	45692	00026	99974	0	**52**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6h	91°							88°	5h

0h	2°			Logarithms.				177°	11h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
8	0	8.54282	11.45718	8.54308	11.45692	10.00026	9.99974	60	**52**
4	1	54642	45358	54669	45331	00027	99973	59	56
8	2	54999	45001	55027	44973	00027	99973	58	52
12	3	55354	44646	55382	44618	00028	99972	57	48
16	4	55705	44295	55734	44266	00028	99972	56	44
20	5	8.56054	11.43946	8.56083	11.43917	10.00029	9.99971	55	40
24	6	56400	43600	56429	43571	00029	99971	54	36
28	7	56743	43257	56773	43227	00030	99970	53	32
32	8	57084	42916	57114	42886	00030	99970	52	28
36	9	57421	42579	57452	42548	00031	99969	51	24
40	10	8.57757	11.42243	8.57788	11.42212	10.00031	9.99969	50	20
44	11	58089	41911	58121	41879	00032	99968	49	16
48	12	58419	41581	58451	41549	00032	99968	48	12
52	13	58747	41253	58779	41221	00033	99967	47	8
56	14	59072	40928	59105	40895	00033	99967	46	4
9	15	8.59395	11.40605	8.59428	11.40572	10.00033	9.99967	45	**51**
4	16	59715	40285	59749	40251	00034	99966	44	56
8	17	60033	39967	60068	39932	00034	99966	43	52
12	18	60349	39651	60384	39616	00035	99965	42	48
16	19	60662	39338	60698	39302	00036	99964	41	44
20	20	8.60973	11.39027	8.61009	11.38991	10.00036	9.99964	40	40
24	21	61282	38718	61319	38681	00037	99963	39	36
28	22	61589	38411	61626	38374	00037	99963	38	32
32	23	61894	38106	61931	38069	00038	99962	37	28
36	24	62196	37804	62234	37766	00038	99962	36	24
40	25	8.62497	11.37503	8.62535	11.37465	10.00039	9.99961	35	20
44	26	62795	37205	62834	37166	00039	99961	34	16
48	27	63091	36909	63131	36869	00040	99960	33	12
52	28	63385	36615	63426	36574	00040	99960	32	8
56	29	63678	36322	63718	36282	00041	99959	31	4
10	30	8.63968	11.36032	8.64009	11.35991	10.00041	9.99959	30	**50**
4	31	64256	35744	64298	35702	00042	99958	29	56
8	32	64543	35457	64585	35415	00042	99958	28	52
12	33	64827	35173	64870	35130	00043	99957	27	48
16	34	65110	34890	65154	34846	00044	99956	26	44
20	35	8.65391	11.34609	8.65435	11.34565	10.00044	9.99956	25	40
24	36	65670	34330	65715	34285	00045	99955	24	36
28	37	65947	34053	65993	34007	00045	99955	23	32
32	38	66223	33777	66269	33731	00046	99954	22	28
36	39	66497	33503	66543	33457	00046	99954	21	24
40	40	8.66769	11.33231	8.66816	11.33184	10.00047	9.99953	20	20
44	41	67039	32961	67087	32913	00048	99952	19	16
48	42	67308	32692	67356	32644	00048	99952	18	12
52	43	67575	32425	67624	32376	00049	99951	17	8
56	44	67841	32159	67890	32110	00049	99951	16	4
11	45	8.68104	11.31896	8.68154	11.31846	10.00050	9.99950	15	**49**
4	46	68367	31633	68417	31583	00051	99949	14	56
8	47	68627	31373	68678	31322	00051	99949	13	52
12	48	68886	31114	68938	31062	00052	99948	12	48
16	49	69144	30856	69196	30804	00052	99948	11	44
20	50	8.69400	11.30600	8.69453	11.30547	10.00053	9.99947	10	40
24	51	69654	30346	69708	30292	00054	99946	9	36
28	52	69907	30093	69962	30038	00054	99946	8	32
32	53	70159	29841	70214	29786	00055	99945	7	28
36	54	70409	29591	70465	29535	00056	99944	6	24
40	55	8.70658	11.29342	8.70714	11.29286	10.00056	9.99944	5	20
44	56	70905	29095	70962	29038	00057	99943	4	16
48	57	71151	28849	71208	28792	00058	99942	3	12
52	58	71395	28605	71453	28547	00058	99942	2	8
56	59	71638	28362	71697	28303	00059	99941	1	4
12	60	71880	28120	71940	28060	00060	99940	0	**48**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6h	92°							87°	5h

0h	3°			Logarithms.			176°		11h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
12	0	8.71880	11.28120	8.71940	11.28060	10.00060	9.99940	60	**48**
4	1	72120	27880	72181	27819	00060	99940	59	56
8	2	72359	27641	72420	27580	00061	99939	58	52
12	3	72597	27403	72659	27341	00062	99938	57	48
16	4	72834	27166	72896	27104	00062	99938	56	44
20	5	8.73069	11.26931	8.73132	11.26868	10.00063	9.99937	55	40
24	6	73303	26697	73366	26634	00064	99936	54	36
28	7	73535	26465	73600	26400	00064	99936	53	32
32	8	73767	26233	73832	26168	00065	99935	52	28
36	9	73997	26003	74063	25937	00066	99934	51	24
40	10	8.74226	11.25774	8.74292	11.25708	10.00066	9.99934	50	20
44	11	74454	25546	74521	25479	00067	99933	49	16
48	12	74680	25320	74748	25252	00068	99932	48	12
52	13	74906	25094	74974	25026	00068	99932	47	8
56	14	75130	24870	75199	24801	00069	99931	46	4
13	15	8.75353	11.24647	8.75423	11.24577	10.00070	9.99930	45	**47**
4	16	75575	24425	75645	24355	00071	99929	44	56
8	17	75795	24205	75867	24133	00071	99929	43	52
12	18	76015	23985	76087	23913	00072	99928	42	48
16	19	76234	23766	76306	23694	00073	99927	41	44
20	20	8.76451	11.23549	8.76525	11.23475	10.00074	9.99926	40	40
24	21	76667	23333	76742	23258	00074	99926	39	36
28	22	76883	23117	76958	23042	00075	99925	38	32
32	23	77097	22903	77173	22827	00076	99924	37	28
36	24	77310	22690	77387	22613	00077	99923	36	24
40	25	8.77522	11.22478	8.77600	11.22400	10.00077	9.99923	35	20
44	26	77733	22267	77811	22189	00078	99922	34	16
48	27	77943	22057	78022	21978	00079	99921	33	12
52	28	78152	21848	78232	21768	00080	99920	32	8
56	29	78360	21640	78441	21559	00080	99920	31	4
14	30	8.78568	11.21432	8.78649	11.21351	10.00081	9.99919	30	**46**
4	31	78774	21226	78855	21145	00082	99918	29	56
8	32	78979	21021	79061	20939	00083	99917	28	52
12	33	79183	20817	79266	20734	00083	99917	27	48
16	34	79386	20614	79470	20530	00084	99916	26	44
20	35	8.79588	11.20412	8.79673	11.20327	10.00085	9.99915	25	40
24	36	79789	20211	79875	20125	00086	99914	24	36
28	37	79990	20010	80076	19924	00087	99913	23	32
32	38	80189	19811	80277	19723	00087	99913	22	28
36	39	80388	19612	80476	19524	00088	99912	21	24
40	40	8.80585	11.19415	8.80674	11.19326	10.00089	9.99911	20	20
44	41	80782	19218	80872	19128	00090	99910	19	16
48	42	80978	19022	81068	18932	00091	99909	18	12
52	43	81173	18827	81264	18736	00091	99909	17	8
56	44	81367	18633	81459	18541	00092	99908	16	4
15	45	8.81560	11.18440	8.81653	11.18347	10.00093	9.99907	15	**45**
4	46	81752	18248	81846	18154	00094	99906	14	56
8	47	81944	18056	82038	17962	00095	99905	13	52
12	48	82134	17866	82230	17770	00096	99904	12	48
16	49	82324	17676	82420	17580	00096	99904	11	44
20	50	8.82513	11.17487	8.82610	11.17390	10.00097	9.99903	10	40
24	51	82701	17299	82799	17201	00098	99902	9	36
28	52	82888	17112	82987	17013	00099	99901	8	32
32	53	83075	16925	83175	16825	00100	99900	7	28
36	54	83261	16739	83361	16639	00101	99899	6	24
40	55	8.83446	11.16554	8.83547	11.16453	10.00102	9.99898	5	20
44	56	83630	16370	83732	16268	00102	99898	4	16
48	57	83813	16187	83916	16084	00103	99897	3	12
52	58	83996	16004	84100	15900	00104	99896	2	8
56	59	84177	15823	84282	15718	00105	99895	1	4
16	60	84358	15642	84464	15536	00106	99894	0	**44**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6h	93°							86°	5h

0h	4°			Logarithms.				175°	11h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S
16	0	8.84358	11.15642	8.84464	11.15536	10.00106	9.99894	60	**44**
4	1	84539	15461	84646	15354	00107	99893	59	56
8	2	84718	15282	84826	15174	00108	99892	58	52
12	3	84897	15103	85006	14994	00109	99891	57	48
16	4	85075	14925	85185	14815	00109	99891	56	44
20	5	8.85252	11.14748	8.85363	11.14637	10.00110	9.99890	55	40
24	6	85429	14571	85540	14460	00111	99889	54	36
28	7	85605	14395	85717	14283	00112	99888	53	32
32	8	85780	14220	85893	14107	00113	99887	52	28
36	9	85955	14045	86069	13931	00114	99886	51	24
40	10	8.86128	11.13872	8.86243	11.13757	10.00115	9.99885	50	20
44	11	86301	13699	86417	13583	00116	99884	49	16
48	12	86474	13526	86591	13409	00117	99883	48	12
52	13	86645	13355	86763	13237	00118	99882	47	8
56	14	86816	13184	86935	13065	00119	99881	46	4
17	15	8.86987	11.13013	8.87106	11.12894	10.00120	9.99880	45	**43**
4	16	87156	12844	87277	12723	00121	99879	44	56
8	17	87325	12675	87447	12553	00121	99879	43	52
12	18	87494	12506	87616	12384	00122	99878	42	48
16	19	87661	12339	87785	12215	00123	99877	41	44
20	20	8.87829	11.12171	8.87953	11.12047	10.00124	9.99876	40	40
24	21	87995	12005	88120	11880	00125	99875	39	36
28	22	88161	11839	88287	11713	00126	99874	38	32
32	23	88326	11674	88453	11547	00127	99573	37	28
36	24	88490	11510	88618	11382	00128	99872	36	24
40	25	8.88654	11.11346	8.88783	11.11217	10.00129	9.99871	35	20
44	26	88817	11183	88948	11052	00130	99870	34	16
48	27	88980	11020	89111	10889	00131	99869	33	12
52	28	89142	10858	89274	10726	00132	99868	32	8
56	29	89304	10696	89437	10563	00133	99867	31	4
18	30	8.89464	11.10536	8.89598	11.10402	10.00134	9.99866	30	**42**
4	31	89625	10375	89760	10240	00135	99865	29	56
8	32	89784	10216	89920	10080	00136	99864	28	52
12	33	89943	10057	90080	09920	00137	99863	27	48
16	34	90102	09898	90240	09760	00138	99862	26	44
20	35	8.90260	11.09740	8.90399	11.09601	10.00139	9.99861	25	40
24	36	90417	09583	90557	09443	00140	99860	24	36
28	37	90574	09426	90715	09285	00141	99859	23	32
32	38	90730	09270	90872	09128	00142	99858	22	28
36	39	90885	09115	91029	08971	00143	99857	21	24
40	40	8.91040	11.08960	8.91185	11.08815	10.00144	9.99856	20	20
44	41	91195	08805	91340	08660	00145	99855	19	16
48	42	91349	08651	91495	08505	00146	99854	18	12
52	43	91502	08498	91650	08350	00147	99853	17	8
56	44	91655	08345	91803	08197	00148	99852	16	4
19	45	8.91807	11.08193	8.91957	11.08043	10.00149	9.99851	15	**41**
4	46	91959	08041	92110	07890	00150	99850	14	56
8	47	92110	07890	92262	07738	00152	99848	13	52
12	48	92261	07739	92414	07586	00153	99847	12	48
16	49	92411	07589	92565	07435	00154	99846	11	44
20	50	8.92561	11.07439	8.92716	11.07284	10.00155	9.99845	10	40
24	51	92710	07290	92866	07134	00156	99844	9	36
28	52	92859	07141	93016	06984	00157	99843	8	32
32	53	93007	06993	93165	06835	00158	99842	7	28
36	54	93154	06846	93313	06687	00159	99841	6	24
40	55	8.93301	11.06699	8.93462	11.06538	10.00160	9.99840	5	20
44	56	93448	06552	93609	06391	00161	99839	4	16
48	57	93594	06406	93756	06244	00162	99838	3	12
52	58	93740	06260	93903	06097	00163	99837	2	8
56	59	93885	06115	94049	05951	00164	99836	1	4
20	60	94030	05970	94195	05805	00166	99834	0	**40**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6h	94°							85°	5h

0h	5°			Logarithms.				174°	11h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
20	0	8.94030	11.05970	8.94195	11.05805	10.00166	9.99834	60	**40**
4	1	94174	05826	94340	05660	00167	99833	59	56
8	2	94317	05683	94485	05515	00168	99832	58	52
12	3	94461	05539	94630	05370	00169	99831	57	48
16	4	94603	05397	94773	05227	00170	99830	56	44
20	5	8.94746	11.05254	8.94917	11.05083	10.00171	9.99829	55	40
24	6	94887	05113	95060	04940	00172	99828	54	36
28	7	95029	04971	95202	04798	00173	99827	53	32
32	8	95170	04830	95344	04656	00175	99825	52	28
36	9	95310	04690	95486	04514	00176	99824	51	24
40	10	8.95450	11.04550	8.95627	11.04373	10.00177	9.99823	50	20
44	11	95589	04411	95767	04233	00178	99822	49	16
48	12	95728	04272	95908	04092	00179	99821	48	12
52	13	95867	04133	96047	03953	00180	99820	47	8
56	14	96005	03995	96187	03813	00181	99819	46	4
21	15	8.96143	11.03857	8.96325	11.03675	10.00183	9.99817	45	**39**
4	16	96280	03720	96464	03536	00184	99816	44	56
8	17	96417	03583	96602	03398	00185	99815	43	52
12	18	96553	03447	96739	03261	00186	99814	42	48
16	19	96689	03311	96877	03123	00187	99813	41	44
20	20	8.96825	11.03175	8.97013	11.02987	10.00188	9.99812	40	40
24	21	96960	03040	97150	02850	00190	99810	39	36
28	22	97095	02905	97285	02715	00191	99809	38	32
32	23	97229	02771	97421	02579	00192	99808	37	28
36	24	97363	02637	97556	02444	00193	99807	36	24
40	25	8.97496	11.02504	8.97691	11.02309	10.00194	9.99806	35	20
44	26	97629	02371	97825	02175	00196	99804	34	16
48	27	97762	02238	97959	02041	00197	99803	33	12
52	28	97894	02106	98092	01908	00198	99802	32	8
56	29	98026	01974	98225	01775	00199	99801	31	4
22	30	8.98157	11.01843	8.98358	11.01642	10.00200	9.99800	30	**38**
4	31	98288	01712	98490	01510	00202	99798	29	56
8	32	98419	01581	98622	01378	00203	99797	28	52
12	33	98549	01451	98753	01247	00204	99796	27	48
16	34	98679	01321	98884	01116	00205	99795	26	44
20	35	8.98808	11.01192	8.99015	11.00985	10.00207	9.99793	25	40
24	36	98937	01063	99145	00855	00208	99792	24	36
28	37	99066	00934	99275	00725	00209	99791	23	32
32	38	99194	00806	99405	00595	00210	99790	22	28
36	39	99322	00678	99534	00466	00212	99788	21	24
40	40	8.99450	11.00550	8.99662	11.00338	10.00213	9.99787	20	20
44	41	99577	00423	99791	00209	00214	99786	19	16
48	42	99704	00296	99919	00081	00215	99785	18	12
52	43	99830	00170	9.00046	10.99954	00217	99783	17	8
56	44	99956	00044	00174	99826	00218	99782	16	4
23	45	9.00082	10.99918	9.00301	10.99699	10.00219	9.99781	15	**37**
4	46	00207	99793	00427	99573	00220	99780	14	56
8	47	00332	99668	00553	99447	00222	99778	13	52
12	48	00456	99544	00679	99321	00223	99777	12	48
16	49	00581	99419	00805	99195	00224	99776	11	44
20	50	9.00704	10.99296	9.00930	10.99070	10.00225	9.99775	10	40
24	51	00828	99172	01055	98945	00227	99773	9	36
28	52	00951	99049	01179	98821	00228	99772	8	32
32	53	01074	98926	01303	98697	00229	99771	7	28
36	54	01196	98804	01427	98573	00231	99769	6	24
40	55	9.01318	10.98682	9.01550	10.98450	10.00232	9.99768	5	20
44	56	01440	98560	01673	98327	00233	99767	4	16
48	57	01561	98439	01796	98204	00235	99765	3	12
52	58	01682	98318	01918	98082	00236	99764	2	8
56	59	01803	98197	02040	97960	00237	99763	1	4
24	60	01923	98077	02162	97838	00239	99761	0	**36**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6h	95°							84°	5h

0h	6°			Logarithms.			173°		11h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
24	0	9.01923	10.98077	9.02162	10.97838	10.00239	9.99761	60	36
4	1	02043	97957	02283	97717	00240	99760	59	56
8	2	02163	97837	02404	97596	00241	99759	58	52
12	3	02283	97717	02525	97475	00243	99757	57	48
16	4	02402	97598	02645	97355	00244	99756	56	44
20	5	9.02520	10.97480	9.02766	10.97234	10.00245	9.99755	55	40
24	6	02639	97361	02885	97115	00247	99753	54	36
28	7	02757	97243	03005	96995	00248	99752	53	32
32	8	02874	97126	03124	96876	00249	99751	52	28
36	9	02992	97008	03242	96758	00251	99749	51	24
40	10	9.03109	10.96891	9.03361	10.96639	10.00252	9.99748	50	20
44	11	03226	96774	03479	96521	00253	99747	49	16
48	12	03342	96658	03597	96403	00255	99745	48	12
52	13	03458	96542	03714	96286	00256	99744	47	8
56	14	03574	96426	03832	96168	00258	99742	46	4
25	15	9.03690	10.96310	9.03948	10.96052	10.00259	9.99741	45	35
4	16	03805	96195	04065	95935	00260	99740	44	56
8	17	03920	96080	04181	95819	00262	99738	43	52
12	18	04034	95966	04297	95703	00263	99737	42	48
16	19	04149	95851	04413	95587	00264	99736	41	44
20	20	9.04262	10.95738	9.04528	10.95472	10.00266	9.99734	40	40
24	21	04376	95624	04643	95357	00267	99733	39	36
28	22	04490	95510	04758	95242	00269	99731	38	32
32	23	04603	95397	04873	95127	00270	99730	37	28
36	24	04715	95285	04987	95013	00272	99728	36	24
40	25	9.04828	10.95172	9.05101	10.94899	10.00273	9.99727	35	20
44	26	04940	95060	05214	94786	00274	99726	34	16
48	27	05052	94948	05328	94672	00276	99724	33	12
52	28	05164	94836	05441	94559	00277	99723	32	8
56	29	05275	94725	05553	94447	00279	99721	31	4
26	30	9.05386	10.94614	9.05666	10.94334	10.00280	9.99720	30	34
4	31	05497	94503	05778	94222	00282	99718	29	56
8	32	05607	94393	05890	94110	00283	99717	28	52
12	33	05717	94283	06002	93998	00284	99716	27	48
16	34	05827	94173	06113	93887	00286	99714	26	44
20	35	9.05937	10.94063	9.06224	10.93776	10.00287	9.99713	25	40
24	36	06046	93954	06335	93665	00289	99711	24	36
28	37	06155	93845	06445	93555	00290	99710	23	32
32	38	06264	93736	06556	93444	00292	99708	22	28
36	39	06372	93628	06666	93334	00293	99707	21	24
40	40	9.06481	10.93519	9.06775	10.93225	10.00295	9.99705	20	20
44	41	06589	93411	06885	93115	00296	99704	19	16
48	42	06696	93304	06994	93006	00298	99702	18	12
52	43	06804	93196	07103	92897	00299	99701	17	8
56	44	06911	93089	07211	92789	00301	97699	16	4
27	45	9.07018	10.92982	9.07320	10.92680	10.00302	9.99698	15	33
4	46	07124	92876	07428	92572	00304	99696	14	56
8	47	07231	92769	07536	92464	00305	99695	13	52
12	48	07337	92663	07643	92357	00307	99693	12	48
16	49	07442	92558	07751	92249	00308	99692	11	44
20	50	9.07548	10.92452	9.07858	10.92142	10.00310	9.99690	10	40
24	51	07653	92347	07964	92036	00311	99689	9	36
28	52	07758	92242	08071	91929	00313	99687	8	32
32	53	07863	92137	08177	91823	00314	99686	7	28
36	54	07968	92032	08283	91717	00316	99684	6	24
40	55	9.08072	10.91928	9.08389	10.91611	10.00317	9.99683	5	20
44	56	08176	91824	08495	91505	00319	99681	4	16
48	57	08280	91720	08600	91400	00320	99680	3	12
52	58	08383	91617	08705	91295	00322	99678	2	8
56	59	08486	91514	08810	91190	00323	99677	1	4
28	60	08589	91411	08914	91086	00325	99675	0	32
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6h	96°							83°	5h

0h	7°			Logarithms.				172°	11h
M. S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M. S.
28	0	9.08589	10.91411	9.08914	10.91086	10.00325	9.99675	60	**32**
4	1	08692	91308	09019	90981	00326	99674	59	56
8	2	08795	91205	09123	90877	00328	99672	58	52
12	3	08897	91103	09227	90773	00330	99670	57	48
16	4	08999	91001	09330	90670	00331	99669	56	44
20	5	9.09101	10.90899	9.09434	10.90566	10.00333	9.99667	55	40
24	6	09202	90798	09537	90463	00334	99666	54	36
28	7	09304	90696	09640	90360	00336	99664	53	32
32	8	09405	90595	09742	90258	00337	99663	52	28
36	9	09506	90494	09845	90155	00339	99661	51	24
40	10	9.09606	10.90394	9.09947	10.90053	10.00341	9.99659	50	20
44	11	09707	90293	10049	89951	00342	99658	49	16
48	12	09807	90193	10150	89850	00344	99656	48	12
52	13	09907	90093	10252	89748	00345	99655	47	8
56	14	10006	89994	10353	89647	00347	99653	46	4
29	15	9.10106	10.89894	9.10454	10.89546	10.00349	9.99651	45	**31**
4	16	10205	89795	10555	89445	00350	99650	44	56
8	17	10304	89696	10656	89344	00352	99648	43	52
12	18	10402	89598	10756	89244	00353	99647	42	48
16	19	10501	89499	10856	89144	00355	99645	41	44
20	20	9.10599	10.89401	9.10956	10.89044	10.00357	9.99643	40	40
24	21	10697	89303	11056	88944	00358	99642	39	36
28	22	10795	89205	11155	88845	00360	99640	38	32
32	23	10893	89107	11254	88746	00362	99638	37	28
36	24	10990	89010	11353	88647	00363	99637	36	24
40	25	9.11087	10.88913	9.11452	10.88548	10.00365	9.99635	35	20
44	26	11184	88816	11551	88449	00367	99633	34	16
48	27	11281	88719	11649	88351	00368	99632	33	12
52	28	11377	88623	11747	88253	00370	99630	32	8
56	29	11474	88526	11845	88155	00371	99629	31	4
30	30	9.11570	10.88430	9.11943	10.88057	10.00373	9.99627	30	**30**
4	31	11666	88334	12040	87960	00375	99625	29	56
8	32	11761	88239	12138	87862	00376	99624	28	52
12	33	11857	88143	12235	87765	00378	99622	27	48
16	34	11952	88048	12332	87668	00380	99620	26	44
20	35	9.12047	10.87953	9.12428	10.87572	10.00382	9.99618	25	40
24	36	12142	87858	12525	87475	00383	99617	24	36
28	37	12236	87764	12621	87379	00385	99615	23	32
32	38	12331	87669	12717	87283	00387	99613	22	28
36	39	12425	87575	12813	87187	00388	99612	21	24
40	40	9.12519	10.87481	9.12909	10.87091	10.00390	9.99610	20	20
44	41	12612	87388	13004	86996	00392	99608	19	16
48	42	12706	87294	13099	86901	00393	99607	18	12
52	43	12799	87201	13194	86806	00395	99605	17	8
56	44	12892	87108	13289	86711	00397	99603	16	4
31	45	9.12985	10.87015	9.13384	10.86616	10.00399	9.99601	15	**29**
4	46	13078	86922	13478	86522	00400	99600	14	56
8	47	13171	86829	13573	86427	00402	99598	13	52
12	48	13263	86737	13667	86333	00404	99596	12	48
16	49	13355	86645	13761	86239	00405	99595	11	44
20	50	9.13447	10.86553	9.13854	10.86146	10.00407	9.99593	10	40
24	51	13539	86461	13948	86052	00409	99591	9	36
28	52	13630	86370	14041	85959	00411	99589	8	32
32	53	13722	86278	14134	85866	00412	99588	7	28
36	54	13813	86187	14227	85773	00414	99586	6	24
40	55	9.13904	10.86096	9.14320	10.85680	10.00416	9.99584	5	20
44	56	13994	86006	14412	85588	00418	99582	4	16
48	57	14085	85915	14504	85496	00419	99581	3	12
52	58	14175	85825	14597	85403	00421	99579	2	8
56	59	14266	85734	14688	85312	00423	99577	1	4
32	60	14356	85644	14780	85220	00425	99575	0	**28**
M. S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M. S.
6h	97°							82°	5h

0h	8°			Logarithms.				171°	11h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
32	0	9.14356	10.85644	9.14780	10.85220	10.00425	9.99575	60	**28**
4	1	14445	85555	14872	85128	00426	99574	59	56
8	2	14535	85465	14963	85037	00428	99572	58	52
12	3	14624	85376	15054	84946	00430	99570	57	48
16	4	14714	85286	15145	84855	00432	99568	56	44
20	5	9.14803	10.85197	9.15236	10.84764	10.00434	9.99566	55	40
24	6	14891	85109	15327	84673	00435	99565	54	36
28	7	14980	85020	15417	84583	00437	99563	53	32
32	8	15069	84931	15508	84492	00439	99561	52	28
36	9	15157	84843	15598	84402	00441	99559	51	24
40	10	9.15245	10.84755	9.15688	10.84312	10.00443	9.99557	50	20
44	11	15333	84667	15777	84223	00444	99556	49	16
48	12	15421	84579	15867	84133	00446	99554	48	12
52	13	15508	84492	15956	84044	00448	99552	47	8
56	14	15596	84404	16046	83954	00450	99550	46	4
33	15	9.15683	10.84317	9.16135	10.83865	10.00452	9.99548	45	**27**
4	16	15770	84230	16224	83776	00454	99546	44	56
8	17	15857	84143	16312	83688	00455	99545	43	52
12	18	15944	84056	16401	83599	00457	99543	42	48
16	19	16030	83970	16489	83511	00459	99541	41	44
20	20	9.16116	10.83884	9.16577	10.83423	10.00461	9.99539	40	40
24	21	16203	83797	16665	83335	00463	99537	39	36
28	22	16289	83711	16753	83247	00465	99535	38	32
32	23	16374	83626	16841	83159	00467	99533	37	28
36	24	16460	83540	16928	83072	00468	99532	36	24
40	25	9.16545	10.83455	9.17016	10.82984	10.00470	9.99530	35	20
44	26	16631	83369	17103	82897	00472	99528	34	16
48	27	16716	83284	17190	82810	00474	99526	33	12
52	28	16801	83199	17277	82723	00476	99524	32	8
56	29	16886	83114	17363	82637	00478	99522	31	4
34	30	9.16970	10.83030	9.17450	10.82550	10.00480	9.99520	30	**26**
4	31	17055	82945	17536	82464	00482	99518	29	56
8	32	17139	82861	17622	82378	00483	99517	28	52
12	33	17223	82777	17708	82292	00485	99515	27	48
16	34	17307	82693	17794	82206	00487	99513	26	44
20	35	9.17391	10.82609	9.17880	10.82120	10.00489	9.99511	25	40
24	36	17474	82526	17965	82035	00491	99509	24	36
28	37	17558	82442	18051	81949	00493	99507	23	32
32	38	17641	82359	18136	81864	00495	99505	22	28
36	39	17724	82276	18221	81779	00497	99503	21	24
40	40	9.17807	10.82193	9.18306	10.81694	10.00499	9.99501	20	20
44	41	17890	82110	18391	81609	00501	99499	19	16
48	42	17973	82027	18475	81525	00503	99497	18	12
52	43	18055	81945	18560	81440	00505	99495	17	8
56	44	18137	81863	18644	81356	00506	99494	16	4
35	45	9.18220	10.81780	9.18728	10.81272	10.00508	9.99492	15	**25**
4	46	18302	81698	18812	81188	00510	99490	14	56
8	47	18383	81617	18896	81104	00512	99488	13	52
12	48	18465	81535	18979	81021	00514	99486	12	48
16	49	18547	81453	19063	80937	00516	99484	11	44
20	50	9.18628	10.81372	9.19146	10.80854	10.00518	9.99482	10	40
24	51	18709	81291	19229	80771	00520	99480	9	36
28	52	18790	81210	19312	80688	00522	99478	8	32
32	53	18871	81129	19395	80605	00524	99476	7	28
36	54	18952	81048	19478	80522	00526	99474	6	24
40	55	9.19033	10.80967	9.19561	10.80439	10.00528	9.99472	5	20
44	56	19113	80887	19643	80357	00530	99470	4	16
48	57	19193	80807	19725	80275	00532	99468	3	12
52	58	19273	80727	19807	80193	00534	99466	2	8
56	59	19353	80647	19889	80111	00536	99464	1	4
36	60	19433	80567	19971	80029	00538	99462	0	**24**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6h	98°							81°	5h

0h	9°			Logarithms.			170°		11h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
36	0	9.19433	10.80567	9.19971	10.80029	10.00538	9.99462	60	**24**
4	1	19513	80487	20053	79947	00540	99460	59	56
8	2	19592	80408	20134	79866	00542	99458	58	52
12	3	19672	80328	20216	79784	00544	99456	57	48
16	4	19751	80249	20297	79703	00546	99454	56	44
20	5	9.19830	10.80170	9.20378	10.79622	10.00548	9.99452	55	40
24	6	19909	80091	20459	79541	00550	99450	54	36
28	7	19988	80012	20540	79460	00552	99448	53	32
32	8	20067	79933	20621	79379	00554	99446	52	28
36	9	20145	79855	20701	79299	00556	99444	51	24
40	10	9.20223	10.79777	9.20782	10.79218	10.00558	9.99442	50	20
44	11	20302	79698	20862	79138	00560	99440	49	16
48	12	20380	79620	20942	79058	00562	99438	48	12
52	13	20458	79542	21022	78978	00564	99436	47	8
56	14	20535	79465	21102	78898	00566	99434	46	4
37	15	9.20613	10.79387	9.21182	10.78818	10.00568	9.99432	45	**23**
4	16	20691	79309	21261	78739	00571	99429	44	56
8	17	20768	79232	21341	78659	00573	99427	43	52
12	18	20845	79155	21420	78580	00575	99425	42	48
16	19	20922	79078	21499	78501	00577	99423	41	44
20	20	9.20999	10.79001	9.21578	10.78422	10.00579	9.99421	40	40
24	21	21076	78924	21657	78343	00581	99419	39	36
28	22	21153	78847	21736	78264	00583	99417	38	32
32	23	21229	78771	21814	78186	00585	99415	37	28
36	24	21306	78694	21893	78107	00587	99413	36	24
40	25	9.21382	10.78618	9.21971	10.78029	10.00589	9.99411	35	20
44	26	21458	78542	22049	77951	00591	99409	34	16
48	27	21534	78466	22127	77873	00593	99407	33	12
52	28	21610	78390	22205	77795	00596	99404	32	8
56	29	21685	78315	22283	77717	00598	99402	31	4
38	30	9.21761	10.78239	9.22361	10.77639	10.00600	9.99400	30	**22**
4	31	21836	78164	22438	77562	00602	99398	29	56
8	32	21912	78088	22516	77484	00604	99396	28	52
12	33	21987	78013	22593	77407	00606	99394	27	48
16	34	22062	77938	22670	77330	00608	99392	26	44
20	35	9.22137	10.77863	9.22747	10.77253	10.00610	9.99390	25	40
24	36	22211	77789	22824	77176	00612	99388	24	36
28	37	22286	77714	22901	77099	00615	99385	23	32
32	38	22361	77639	22977	77023	00617	99383	22	28
36	39	22435	77565	23054	76946	00619	99381	21	24
40	40	9.22509	10.77491	9.23130	10.76870	10.00621	9.99379	20	20
44	41	22583	77417	23206	76794	00623	99377	19	16
48	42	22657	77343	23283	76717	00625	99375	18	12
52	43	22731	77269	23359	76641	00628	99372	17	8
56	44	22805	77195	23435	76565	00630	99370	16	4
39	45	9.22878	10.77122	9.23510	10.76490	10.00632	9.99368	15	**21**
4	46	22952	77048	23586	76414	00634	99366	14	56
8	47	23025	76975	23661	76339	00636	99364	13	52
12	48	23098	76902	23737	76263	00638	99362	12	48
16	49	23171	76829	23812	76188	00641	99359	11	44
20	50	9.23244	10.76756	9.23887	10.76113	10.00643	9.99357	10	40
24	51	23317	76683	23962	76038	00645	99355	9	36
28	52	23390	76610	24037	75963	00647	99353	8	32
32	53	23462	76538	24112	75888	00649	99351	7	28
36	54	23535	76465	24186	75814	00652	99348	6	24
40	55	9.23607	10.76393	9.24261	10.75739	10.00654	9.99346	5	20
44	56	23679	76321	24335	75665	00656	99344	4	16
48	57	23752	76248	24410	75590	00658	99342	3	12
52	58	23823	76177	24484	75516	00660	99340	2	8
56	59	23895	76105	24558	75442	00663	99337	1	4
40	60	23967	76033	24632	75368	00665	99335	0	**20**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6h	99°							80°	5h

0^h	10°			Logarithms.			169°		11^h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
40	0	9.23967	10.76033	9.24632	10.75368	10.00665	9.99335	60	**20**
4	1	24039	75961	24706	75294	00667	99333	59	56
8	2	24110	75890	24779	75221	00669	99331	58	52
12	3	24181	75819	24853	75147	00672	99328	57	48
16	4	24253	75747	24926	75074	00674	99326	56	44
20	5	9.24324	10.75676	9.25000	10.75000	10.00676	9.99324	55	40
24	6	24395	75605	25073	74927	00678	99322	54	36
28	7	24466	75534	25146	74854	00681	99319	53	32
32	8	24536	75464	25219	74781	00683	99317	52	28
36	9	24607	75393	25292	74708	00685	99315	51	24
40	10	9.24677	10.75323	9.25365	10.74635	10.00687	9.99313	50	20
44	11	24748	75252	25437	74563	00690	99310	49	16
48	12	24818	75182	25510	74490	00692	99308	48	12
52	13	24888	75112	25582	74418	00694	99306	47	8
56	14	24958	75042	25655	74345	00696	99304	46	4
41	15	9.25028	10.74972	9.25727	10.74273	10.00699	9.99301	45	**19**
4	16	25098	74902	25799	74201	00701	99299	44	56
8	17	25168	74832	25871	74129	00703	99297	43	52
12	18	25237	74763	25943	74057	00706	99294	42	48
16	19	25307	74693	26015	73985	00708	99292	41	44
20	20	9.25376	10.74624	9.26086	10.73914	10.00710	9.99290	40	40
24	21	25445	74555	26158	73842	00712	99288	39	36
28	22	25514	74486	26229	73771	00715	99285	38	32
32	23	25583	74417	26301	73699	00717	99283	37	28
36	24	25652	74348	26372	73628	00719	99281	36	24
40	25	9.25721	10.74279	9.26443	10.73557	10.00722	9.99278	35	20
44	26	25790	74210	26514	73486	00724	99276	34	16
48	27	25858	74142	26585	73415	00726	99274	33	12
52	28	25927	74073	26655	73345	00729	99271	32	8
56	29	25995	74005	26726	73274	00731	99269	31	4
42	30	9.26063	10.73937	9.26797	10.73203	10.00733	9.99267	30	**18**
4	31	26131	73869	26867	73133	00736	99264	29	56
8	32	26199	73801	26937	73063	00738	99262	28	52
12	33	26267	73733	27008	72992	00740	99260	27	48
16	34	26335	73665	27078	72922	00743	99257	26	44
20	35	9.26403	10.73597	9.27148	10.72852	10.00745	9.99255	25	40
24	36	26470	73530	27218	72782	00748	99252	24	36
28	37	26538	73462	27288	72712	00750	99250	23	32
32	38	26605	73395	27357	72643	00752	99248	22	28
36	39	26672	73328	27427	72573	00755	99245	21	24
40	40	9.26739	10.73261	9.27496	10.72504	10.00757	9.99243	20	20
44	41	26806	73194	27566	72434	00759	99241	19	16
48	42	26873	73127	27635	72365	00762	99238	18	12
52	43	26940	73060	27704	72296	00764	99236	17	8
56	44	27007	72993	27773	72227	00767	99233	16	4
43	45	9.27073	10.72927	9.27842	10.72158	10.00769	9.99231	15	**17**
4	46	27140	72860	27911	72089	00771	99229	14	56
8	47	27206	72794	27980	72020	00774	99226	13	52
12	48	27273	72727	28049	71951	00776	99224	12	48
16	49	27339	72661	28117	71883	00779	99221	11	44
20	50	9.27405	10.72595	9.28186	10.71814	10.00781	9.99219	10	40
24	51	27471	72529	28254	71746	00783	99217	9	36
28	52	27537	72463	28323	71677	00786	99214	8	32
32	53	27602	72398	28391	71609	00788	99212	7	28
36	54	27668	72332	28459	71541	00791	99209	6	24
40	55	9.27734	10.72266	9.28527	10.71473	10.00793	9.99207	5	20
44	56	27799	72201	28595	71405	00796	99204	4	16
48	57	27864	72136	28662	71338	00798	99202	3	12
52	58	27930	72070	28730	71270	00800	99200	2	8
56	59	27995	72005	28798	71202	00803	99197	1	4
44	60	28060	71940	28865	71135	00805	99195	0	**16**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6^h	100°						79°		5^h

0ʰ	11°			Logarithms.				168°	11ʰ
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
44	0	9.28060	10.71940	9.28865	10.71135	10.00805	9.99195	60	**16**
4	1	28125	71875	28933	71067	00808	99192	59	56
8	2	28190	71810	29000	71000	00810	99190	58	52
12	3	28254	71746	29067	70933	00813	99187	57	48
16	4	28319	71681	29134	70866	00815	99185	56	44
20	5	9.28384	10.71616	9.29201	10.70799	10.00818	9.99182	55	40
24	6	28448	71552	29268	70732	00820	99180	54	36
28	7	28512	71488	29335	70665	00823	99177	53	32
32	8	28577	71423	29402	70598	00825	99175	52	28
36	9	28641	71359	29468	70532	00828	99172	51	24
40	10	9.28705	10.71295	9.29535	10.70465	10.00830	9.99170	50	20
44	11	28769	71231	29601	70399	00833	99167	49	16
48	12	28833	71167	29668	70332	00835	99165	48	12
52	13	28896	71104	29734	70266	00838	99162	47	8
56	14	28960	71040	29800	70200	00840	99160	46	4
45	15	9.29024	10.70976	9.29866	10.70134	10.00843	9.99157	45	**15**
4	16	29087	70913	29932	70068	00845	99155	44	56
8	17	29150	70850	29998	70002	00848	99152	43	52
12	18	29214	70786	30064	69936	00850	99150	42	48
16	19	29277	70723	30130	69870	00853	99147	41	44
20	20	9.29340	10.70660	9.30195	10.69805	10.00855	9.99145	40	40
24	21	29403	70597	30261	69739	00858	99142	39	36
28	22	29466	70534	30326	69674	00860	99140	38	32
32	23	29529	70471	30391	69609	00863	99137	37	28
36	24	29591	70409	30457	69543	00865	99135	36	24
40	25	9.29654	10.70346	9.30522	10.69478	10.00868	9.99132	35	20
44	26	29716	70284	30587	69413	00870	99130	34	16
48	27	29779	70221	30652	69348	00873	99127	33	12
52	28	29841	70159	30717	69283	00876	99124	32	8
56	29	29903	70097	30782	69218	00878	99122	31	4
46	30	9.29966	10.70034	9.30846	10.69154	10.00881	9.99119	30	**14**
4	31	30028	69972	30911	69089	00883	99117	29	56
8	32	30090	69910	30975	69025	00886	99114	28	52
12	33	30151	69849	31040	68960	00888	99112	27	48
16	34	30213	69787	31104	68896	00891	99109	26	44
20	35	9.30275	10.69725	9.31168	10.68832	10.00894	9.99106	25	40
24	36	30336	69664	31233	68767	00896	99104	24	36
28	37	30398	69602	31297	68703	00899	99101	23	32
32	38	30459	69541	31361	68639	00901	99099	22	28
36	39	30521	69479	31425	68575	00904	99096	21	24
40	40	9.30582	10.69418	9.31489	10.68511	10.00907	9.99093	20	20
44	41	30643	69357	31552	68448	00909	99091	19	16
48	42	30704	69296	31616	68384	00912	99088	18	12
52	43	30765	69235	31679	68321	00914	99086	17	8
56	44	30826	69174	31743	68257	00917	99083	16	4
47	45	9.30887	10.69113	9.31806	10.68194	10.00920	9.99080	15	**13**
4	46	30947	69053	31870	68130	00922	99078	14	56
8	47	31008	68992	31933	68067	00925	99075	13	52
12	48	31068	68932	31996	68004	00928	99072	12	48
16	49	31129	68871	32059	67941	00930	99070	11	44
20	50	9.31189	10.68811	9.32122	10.67878	10.00933	9.99067	10	40
24	51	31250	68750	32185	67815	00936	99064	9	36
28	52	31310	68690	32248	67752	00938	99062	8	32
32	53	31370	68630	32311	67689	00941	99059	7	28
36	54	31430	68570	32373	67627	00944	99056	6	24
40	55	9.31490	10.68510	9.32436	10.67564	10.00946	9.99054	5	20
44	56	31549	68451	32498	67502	00949	99051	4	16
48	57	31609	68391	32561	67439	00952	99048	3	12
52	58	31669	68331	32623	67377	00954	99046	2	8
56	59	31728	68272	32685	67315	00957	99043	1	4
48	60	31788	68212	32747	67253	00960	99040	0	**12**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6ʰ	101°							78°	5ʰ

0h	12°			Logarithms.				167°	11h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
48	0	9.31788	10.68212	9.32747	10.67253	10.00960	9.99040	60	**12**
4	1	31847	68153	32810	67190	00962	99038	59	56
8	2	31907	68093	32872	67128	00965	99035	58	52
12	3	31966	68034	32933	67067	00968	99032	57	48
16	4	32025	67975	32995	67005	00970	99030	56	44
20	5	9.32084	10.67916	9.33057	10.66943	10.00973	9.99027	55	40
24	6	32143	67857	33119	66881	00976	99024	54	36
28	7	32202	67798	33180	66820	00978	99022	53	32
32	8	32261	67739	33242	66758	00981	99019	52	28
36	9	32319	67681	33303	66697	00984	99016	51	24
40	10	9.32378	10.67622	9.33365	10.66635	10.00987	9.99013	50	20
44	11	32437	67563	33426	66574	00989	99011	49	16
48	12	32495	67505	33487	66513	00992	99008	48	12
52	13	32553	67447	33548	66452	00995	99005	47	8
56	14	32612	67388	33609	66391	00998	99002	46	4
49	15	9.32670	10.67330	9.33670	10.66330	10.01000	9.99000	45	**11**
4	16	32728	67272	33731	66269	01003	98997	44	56
8	17	32786	67214	33792	66208	01006	98994	43	52
12	18	32844	67156	33853	66147	01009	98991	42	48
16	19	32902	67098	33913	66087	01011	98989	41	44
20	20	9.32960	10.67040	9.33974	10.66026	10.01014	9.98986	40	40
24	21	33018	66982	34034	65966	01017	98983	39	36
28	22	33075	66925	34095	65905	01020	98980	38	32
32	23	33133	66867	34155	65845	01022	98978	37	28
36	24	33190	66810	34215	65785	01025	98975	36	24
40	25	9.33248	10.66752	9.34276	10.65724	10.01028	9.98972	35	20
44	26	33305	66695	34336	65664	01031	98969	34	16
48	27	33362	66638	34396	65604	01033	98967	33	12
52	28	33420	66580	34456	65544	01036	98964	32	8
56	29	33477	66523	34516	65484	01039	98961	31	4
50	30	9.33534	10.66466	9.34576	10.65424	10.01042	9.98958	30	**10**
4	31	33591	66409	34635	65365	01045	98955	29	56
8	32	33647	66353	34695	65305	01047	98953	28	52
12	33	33704	66296	34755	65245	01050	98950	27	48
16	34	33761	66239	34814	65186	01053	98947	26	44
20	35	9.33818	10.66182	9.34874	10.65126	10.01056	9.98944	25	40
24	36	33874	66126	34933	65067	01059	98941	24	36
28	37	33931	66069	34992	65008	01062	98938	23	32
32	38	33987	66013	35051	64949	01064	98936	22	28
36	39	34043	65957	35111	64889	01067	98933	21	24
40	40	9.34100	10.65900	9.35170	10.64830	10.01070	9.98930	20	20
44	41	34156	65844	35229	64771	01073	98927	19	16
48	42	34212	65788	35288	64712	01076	98924	18	12
52	43	34268	65732	35347	64653	01079	98921	17	8
56	44	34324	65676	35405	64595	01081	98919	16	4
51	45	9.34380	10.65620	9.35464	10.64536	10.01084	9.98916	15	**9**
4	46	34436	65564	35523	64477	01087	98913	14	56
8	47	34491	65509	35581	64419	01090	98910	13	52
12	48	34547	65453	35640	64360	01093	98907	12	48
16	49	34602	65398	35698	64302	01096	98904	11	44
20	50	9.34658	10.65342	9.35757	10.64243	10.01099	9.98901	10	40
24	51	34713	65287	35815	64185	01102	98898	9	36
28	52	34769	65231	35873	64127	01104	98896	8	32
32	53	34824	65176	35931	64069	01107	98893	7	28
36	54	34879	65121	35989	64011	01110	98890	6	24
40	55	9.34934	10.65066	9.36047	10.63953	10.01113	9.98887	5	20
44	56	34989	65011	36105	63895	01116	98884	4	16
48	57	35044	64956	36163	63837	01119	98881	3	12
52	58	35099	64901	36221	63779	01122	98878	2	8
56	59	35154	64846	36279	63721	01125	98875	1	4
52	60	35209	64791	36336	63664	01128	98872	0	**8**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6h	102°							77°	5h

0h	13°			Logarithms.			166°		11h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
52	0	9.35209	10.64791	9.36336	10.63604	10.01128	9.98872	60	**8**
4	1	35263	64737	36394	63606	01131	98869	59	56
8	2	35318	64682	36452	63548	01133	98867	58	52
12	3	35373	64627	36509	63491	01136	98864	57	48
16	4	35427	64573	36566	63434	01139	98861	56	44
20	5	9.35481	10.64519	9.36624	10.63376	10.01142	9.98858	55	40
24	6	35536	64464	36681	63319	01145	98855	54	36
28	7	35590	64410	36738	63262	01148	98852	53	32
32	8	35644	64356	36795	63205	01151	98849	52	28
36	9	35698	64302	36852	63148	01154	98846	51	24
40	10	9.35752	10.64248	9.36909	10.63091	10.01157	9.98843	50	20
44	11	35806	64194	36966	63034	01160	98840	49	16
48	12	35860	64140	37023	62977	01163	98837	48	12
52	13	35914	64086	37080	62920	01166	98834	47	8
56	14	35968	64032	37137	62863	01169	98831	46	4
53	15	9.36022	10.63978	9.37193	10.62807	10.01172	9.98828	45	**7**
4	16	36075	63925	37250	62750	01175	98825	44	56
8	17	36129	63871	37306	62694	01178	98822	43	52
12	18	36182	63818	37363	62637	01181	98819	42	48
16	19	36236	63764	37419	62581	01184	98816	41	44
20	20	9.36289	10.63711	9.37476	10.62524	10.01187	9.98813	40	40
24	21	36342	63658	37532	62468	01190	98810	39	36
28	22	36395	63605	37588	62412	01193	98807	38	32
32	23	36449	63551	37644	62356	01196	98804	37	28
36	24	36502	63498	37700	62300	01199	98801	36	24
40	25	9.36555	10.63445	9.37756	10.62244	10.01202	9.98798	35	20
44	26	36608	63392	37812	62188	01205	98795	34	16
48	27	36660	63340	37868	62132	01208	98792	33	12
52	28	36713	63287	37924	62076	01211	98789	32	8
56	29	36766	63234	37980	62020	01214	98786	31	4
54	30	9.36819	10.63181	9.38035	10.61965	10.01217	9.98783	30	**6**
4	31	36871	63129	38091	61909	01220	98780	29	56
8	32	36924	63076	38147	61853	01223	98777	28	52
12	33	36976	63024	38202	61798	01226	98774	27	48
16	34	37028	62972	38257	61743	01229	98771	26	44
20	35	9.37081	10.62919	9.38313	10.61687	10.01232	9.98768	25	40
24	36	37133	62867	38368	61632	01235	98765	24	36
28	37	37185	62815	38423	61577	01238	98762	23	32
32	38	37237	62763	38479	61521	01241	98759	22	28
36	39	37289	62711	38534	61466	01244	98756	21	24
40	40	9.37341	10.62659	9.38589	10.61411	10.01247	9.98753	20	20
44	41	37393	62607	38644	61356	01250	98750	19	16
48	42	37445	62555	38699	61301	01254	98746	18	12
52	43	37497	62503	38754	61246	01257	98743	17	8
56	44	37549	62451	38808	61192	01260	98740	16	4
55	45	9.37600	10.62400	9.38863	10.61137	10.01263	9.98737	15	**5**
4	46	37652	62348	38918	61082	01266	98734	14	56
8	47	37703	62297	38972	61028	01269	98731	13	52
12	48	37755	62245	39027	60973	01272	98728	12	48
16	49	37806	62194	39082	60918	01275	98725	11	44
20	50	9.37858	10.62142	9.39136	10.60864	10.01278	9.98722	10	40
24	51	37909	62091	39190	60810	01281	98719	9	36
28	52	37960	62040	39245	60755	01285	98715	8	32
32	53	38011	61989	39299	60701	01288	98712	7	28
36	54	38062	61938	39353	60647	01291	98709	6	24
40	55	9.38113	10.61887	9.39407	10.60593	10.01294	9.98706	5	20
44	56	38164	61836	39461	60539	01297	98703	4	16
48	57	38215	61785	39515	60485	01300	98700	3	12
52	58	38266	61734	39569	60431	01303	98697	2	8
56	59	38317	61683	39623	60377	01306	98694	1	4
56	60	38368	61632	39677	60323	01310	98690	0	**4**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6h	103°							76°	5h

0h	14°			Logarithms.				165°	11h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
56	0	9.38368	10.61632	9.39677	10.60323	10.01310	9.98690	60	**4**
4	1	38418	61582	39731	60269	01313	98687	59	56
8	2	38469	61531	39785	60215	01316	98684	58	52
12	3	38519	61481	39838	60162	01319	98681	57	48
16	4	38570	61430	39892	60108	01322	98678	56	44
20	5	9.38620	10.61380	9.39945	10.60055	10.01325	9.98675	55	40
24	6	38670	61330	39999	60001	01329	98671	54	36
28	7	38721	61279	40052	59948	01332	98668	53	32
32	8	38771	61229	40106	59894	01335	98665	52	28
36	9	38821	61179	40159	59841	01338	98662	51	24
40	10	9.38871	10.61129	9.40212	10.59788	10.01341	9.98659	50	20
44	11	38921	61079	40266	59734	01344	98656	49	16
48	12	38971	61029	40319	59681	01348	98652	48	12
52	13	39021	60979	40372	59628	01351	98649	47	8
56	14	39071	60929	40425	59575	01354	98646	46	4
57	15	9.39121	10.60879	9.40478	10.59522	10.01357	9.98643	45	**3**
4	16	39170	60830	40531	59469	01360	98640	44	56
8	17	39220	60780	40584	59416	01364	98636	43	52
12	18	39270	60730	40636	59364	01367	98633	42	48
16	19	39319	60681	40689	59311	01370	98630	41	44
20	20	9.39369	10.60631	9.40742	10.59258	10.01373	9.98627	40	40
24	21	39418	60582	40795	59205	01377	98623	39	36
28	22	39467	60533	40847	59153	01380	98620	38	32
32	23	39517	60483	40900	59100	01383	98617	37	28
36	24	39566	60434	40952	59048	01386	98614	36	24
40	25	9.39615	10.60385	9.41005	10.58995	10.01390	9.98610	35	20
44	26	39664	60336	41057	58943	01393	98607	34	16
48	27	39713	60287	41109	58891	01396	98604	33	12
52	28	39762	60238	41161	58839	01399	98601	32	8
56	29	39811	60189	41214	58786	01403	98597	31	4
58	30	9.39860	10.60140	9.41266	10.58734	10.01406	9.98594	30	**2**
4	31	39909	60091	41318	58682	01409	98591	29	56
8	32	39958	60042	41370	58630	01412	98588	28	52
12	33	40006	59994	41422	58578	01416	98584	27	48
16	34	40055	59945	41474	58526	01419	98581	26	44
20	35	9.40103	10.59897	9.41526	10.58474	10.01422	9.98578	25	40
24	36	40152	59848	41578	58422	01426	98574	24	36
28	37	40200	59800	41629	58371	01429	98571	23	32
32	38	40249	59751	41681	58319	01432	98568	22	28
36	39	40297	59703	41733	58267	01435	98565	21	24
40	40	9.40346	10.59654	9.41784	10.58216	10.01439	9.98561	20	20
44	41	40394	59606	41836	58164	01442	98558	19	16
48	42	40442	59558	41887	58113	01445	98555	18	12
52	43	40490	59510	41939	58061	01449	98551	17	8
56	44	40538	59462	41990	58010	01452	98548	16	4
59	45	9.40586	10.59414	9.42041	10.57959	10.01455	9.98545	15	**1**
4	46	40634	59366	42093	57907	01459	98541	14	56
8	47	40682	59318	42144	57856	01462	98538	13	52
12	48	40730	59270	42195	57805	01465	98535	12	48
16	49	40778	59222	42246	57754	01469	98531	11	44
20	50	9.40825	10.59175	9.42297	10.57703	10.01472	9.98528	10	40
24	51	40873	59127	42348	57652	01475	98525	9	36
28	52	40921	59079	42399	57601	01479	98521	8	32
32	53	40968	59032	42450	57550	01482	98518	7	28
36	54	41016	58984	42501	57499	01485	98515	6	24
40	55	9.41063	10.58937	9.42552	10.57448	10.01489	9.98511	5	20
44	56	41111	58889	42603	57397	01492	98508	4	16
48	57	41158	58842	42653	57347	01495	98505	3	12
52	58	41205	58795	42704	57296	01499	98501	2	8
56	59	41252	58748	42755	57245	01502	98498	1	4
60	60	41300	58700	42805	57195	01506	98494	0	**0**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
6h	104°							75°	5h

1h	15°			Logarithms.			164°		10h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
0	0	9.41300	10.58700	9.42805	10.57195	10.01506	9.98494	60	60
4	1	41347	58653	42856	57144	01509	98491	59	56
8	2	41394	58606	42906	57094	01512	98488	58	52
12	3	41441	58559	42957	57043	01516	98484	57	48
16	4	41488	58512	43007	56993	01519	98481	56	44
20	5	9.41535	10.58465	9.43057	10.56943	10.01523	9.98477	55	40
24	6	41582	58418	43108	56892	01526	98474	54	36
28	7	41628	58372	43158	56842	01529	98471	53	32
32	8	41675	58325	43208	56792	01533	98467	52	28
36	9	41722	58278	43258	56742	01536	98464	51	24
40	10	9.41768	10.58232	9.43308	10.56692	10.01540	9.98460	50	20
44	11	41815	58185	43358	56642	01543	98457	49	16
48	12	41861	58139	43408	56592	01547	98453	48	12
52	13	41908	58092	43458	56542	01550	98450	47	8
56	14	41954	58046	43508	56492	01553	98447	46	4
1	15	9.42001	10.57999	9.43558	10.56442	10.01557	9.98443	45	**59**
4	16	42047	57953	43607	56393	01560	98440	44	56
8	17	42093	57907	43657	56343	01564	98436	43	52
12	18	42140	57860	43707	56293	01567	98433	42	48
16	19	42186	57814	43756	56244	01571	98429	41	44
20	20	9.422 2	10.57768	9.43806	10.56194	10.01574	9.98426	40	40
24	21	42278	57722	43855	56145	01578	98422	39	36
28	22	42324	57676	43905	56095	01581	98419	38	32
32	23	42370	57630	43954	56046	01585	98415	37	28
36	24	42416	57584	44004	55996	01588	98412	36	24
40	25	9.42461	10.57539	9.44053	10.55947	10.01591	9.98409	35	20
44	26	42507	57493	44102	55898	01595	98405	34	16
48	27	42553	57447	44151	55849	01598	98402	33	12
52	28	42599	57401	44201	55799	01602	98398	32	8
56	29	42644	57356	44250	55750	01605	98395	31	4
2	30	9.42690	10.57310	9.44299	10.55701	10.01609	9.98391	30	**58**
4	31	42735	57265	44348	55652	01612	98388	29	56
8	32	42781	57219	44397	55603	01616	98384	28	52
12	33	42826	57174	44446	55554	01619	98381	27	48
16	34	42872	57128	44495	55505	01623	98377	26	44
20	35	9.42917	10.57083	9.44544	10.55456	10.01627	9.98373	25	40
24	36	42962	57038	44592	55408	01630	98370	24	36
28	37	43008	56992	44641	55359	01634	98366	23	32
32	38	43053	56947	44690	55310	01637	98363	22	28
36	39	43098	56902	44738	55262	01641	98359	21	24
40	40	9.43143	10.56857	9.44787	10.55213	10.01644	9.98356	20	20
44	41	43188	56812	44836	55164	01648	98352	19	16
48	42	43233	56767	44884	55116	01651	98349	18	12
52	43	43278	56722	44933	55067	01655	98345	17	8
56	44	43323	56677	44981	55019	01658	98342	16	4
3	45	9.43367	10.56633	9.45029	10.54971	10.01662	9.98338	15	**57**
4	46	43412	56588	45078	54922	01666	98334	14	56
8	47	43457	56543	45126	54874	01669	98331	13	52
12	48	43502	56498	45174	54826	01673	98327	12	48
16	49	43546	56454	45222	54778	01676	98324	11	44
20	50	9.43591	10.56409	9.45271	10.54729	10.01680	9.98320	10	40
24	51	43635	56365	45319	54681	01683	98317	9	36
28	52	43680	56320	45367	54633	01687	98313	8	32
32	53	43724	56276	45415	54585	01691	98309	7	28
36	54	43769	56231	45463	54537	01694	98306	6	24
40	55	9.43813	10.56187	9.45511	10.54489	10.01698	9.98302	5	20
44	56	43857	56143	45559	54441	01701	98299	4	16
48	57	43901	56099	45606	54394	01705	98295	3	12
52	58	43946	56054	45654	54346	01709	98291	2	8
56	59	43990	56010	45702	54298	01712	98288	1	4
4	60	44034	55966	45750	54250	01716	98284	0	**56**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7h	105°							74°	4h

1h	16°		Logarithms.					163°	10h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
4	0	9.44034	10.55966	9.45750	10.54250	10.01716	9.98284	60	56
4	1	44078	55922	45797	54203	01719	98281	59	56
8	2	44122	55878	45845	54155	01723	98277	58	52
12	3	44166	55834	45892	54108	01727	98273	57	48
16	4	44210	55790	45940	54060	01730	98270	56	44
20	5	9.44253	10.55747	9.45987	10.54013	10.01734	9.98266	55	40
24	6	44297	55703	46035	53965	01738	98262	54	36
28	7	44341	55659	46082	53918	01741	98259	53	32
32	8	44385	55615	46130	53870	01745	98255	52	28
36	9	44428	55572	46177	53823	01749	98251	51	24
40	10	9.44472	10.55528	9.46224	10.53776	10.01752	9.98248	50	20
44	11	44516	55484	46271	53729	01756	98244	49	16
48	12	44559	55441	46319	53681	01760	98240	48	12
52	13	44602	55398	46366	53634	01763	98237	47	8
56	14	44646	55354	46413	53587	01767	98233	46	4
5	15	9.44689	10.55311	9.46460	10.53540	10.01771	9.98229	45	55
4	16	44733	55267	46507	53493	01774	98226	44	56
8	17	44776	55224	46554	53446	01778	98222	43	52
12	18	44819	55181	46601	53399	01782	98218	42	48
16	19	44862	55138	46648	53352	01785	98215	41	44
20	20	9.44905	10.55095	9.46694	10.53306	10.01789	9.98211	40	40
24	21	44948	55052	46741	53259	01793	98207	39	36
28	22	44992	55008	46788	53212	01796	98204	38	32
32	23	45035	54965	46835	53165	01800	98200	37	28
36	24	45077	54923	46881	53119	01804	98196	36	24
40	25	9.45120	10.54880	9.46928	10.53072	10.01808	9.98192	35	20
44	26	45163	54837	46975	53025	01811	98189	34	16
48	27	45206	54794	47021	52979	01815	98185	33	12
52	28	45249	54751	47068	52931	01819	98181	32	8
56	29	45292	54708	47114	52886	01823	98177	31	4
6	30	9.45334	10.54666	9.47160	10.52840	10.01826	9.98174	30	54
4	31	45377	54623	47207	52793	01830	98170	29	56
8	32	45419	54581	47253	52747	01834	98166	28	52
12	33	45462	54538	47299	52701	01838	98162	27	48
16	34	45504	54496	47346	52654	01841	98159	26	44
20	35	9.45547	10.54453	9.47392	10.52608	10.01845	9.98155	25	40
24	36	45589	54411	47438	52562	01849	98151	24	36
28	37	45632	54368	47484	52516	01853	98147	23	32
32	38	45674	54326	47530	52470	01856	98144	22	28
36	39	45716	54284	47576	52424	01860	98140	21	24
40	40	9.45758	10.54242	9.47622	10.52378	10.01864	9.98136	20	20
44	41	45801	54199	47668	52332	01868	98132	19	16
48	42	45843	54157	47714	52286	01871	98129	18	12
52	43	45885	54115	47760	52240	01875	98125	17	8
56	44	45927	54073	47806	52194	01879	98121	16	4
7	45	9.45969	10.54031	9.47852	10.52148	10.01883	9.98117	15	53
4	46	46011	53989	47897	52103	01887	98113	14	56
8	47	46053	53947	47943	52057	01890	98110	13	52
12	48	46095	53905	47989	52011	01894	98106	12	48
16	49	46136	53864	48035	51965	01898	98102	11	44
20	50	9.46178	10.53822	9.48080	10.51920	10.01902	9.98098	10	40
24	51	46220	53780	48126	51874	01906	98094	9	36
28	52	46262	53738	48171	51829	01910	98090	8	32
32	53	46303	53697	48217	51783	01913	98087	7	28
36	54	46345	53655	48262	51738	01917	98083	6	24
40	55	9.46386	10 53614	9.48307	10.51693	10.01921	9.98079	5	20
44	56	46428	53572	48353	51647	01925	98075	4	16
48	57	46469	53531	48398	51602	01929	98071	3	12
52	58	46511	53489	48443	51557	01933	98067	2	8
56	59	46552	53448	48489	51511	01937	98063	1	4
8	60	46594	53406	48534	51466	01940	98060	0	52
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7h	106°							73°	4h

1h	17°			Logarithms.			162°		10h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
8	0	9.46594	10.53406	9.48534	10.51466	10.01940	9.98060	60	**52**
4	1	46635	53365	48579	51421	01944	98056	59	56
8	2	46676	53324	48624	51376	01948	98052	58	52
12	3	46717	53283	48669	51331	01952	98048	57	48
16	4	46758	53242	48714	51286	01956	98044	56	44
20	5	9.46800	10.53200	9.48759	10.51241	10.01960	9.98040	55	40
24	6	46841	53159	48804	51196	01964	98036	54	36
28	7	46882	53118	48849	51151	01968	98032	53	32
32	8	46923	53077	48894	51106	01971	98029	52	28
36	9	46964	53036	48939	51061	01975	98025	51	24
40	10	9.47005	10.52995	9.48984	10.51016	10.01979	9.98021	50	20
44	11	47045	52955	49029	50971	01983	98017	49	16
48	12	47086	52914	49073	50927	01987	98013	48	12
52	13	47127	52873	49118	50882	01991	98009	47	8
56	14	47168	52832	49163	50837	01995	98005	46	4
9	15	9.47209	10.52791	9.49207	10.50793	10.01999	9.98001	45	**51**
4	16	47249	52751	49252	50748	02003	97997	44	56
8	17	47290	52710	49296	50704	02007	97993	43	52
12	18	47330	52670	49341	50659	02011	97989	42	48
16	19	47371	52629	49385	50615	02014	97986	41	44
20	20	9.47411	10.52589	9.49430	10.50570	10.02018	9.97982	40	40
24	21	47452	52548	49474	50526	02022	97978	39	36
28	22	47492	52508	49519	50481	02026	97974	38	32
32	23	47533	52467	49563	50437	02030	97970	37	28
36	24	47573	52427	49607	50393	02034	97966	36	24
40	25	9.47613	10.52387	9.49652	10.50348	10.02038	9.97962	35	20
44	26	47654	52346	49696	50304	02042	97958	34	16
48	27	47694	52306	49740	50260	02046	97954	33	12
52	28	47734	52266	49784	50216	02050	97950	32	8
56	29	47774	52226	49828	50172	02054	97946	31	4
10	30	9.47814	10.52186	9.49872	10.50128	10.02058	9.97942	30	**50**
4	31	47854	52146	49916	50084	02062	97938	29	56
8	32	47894	52106	49960	50040	02066	97934	28	52
12	33	47934	52066	50004	49996	02070	97930	27	48
16	34	47974	52026	50048	49952	02074	97926	26	44
20	35	9.48014	10.51986	9.50092	10.49908	10.02078	9.97922	25	40
24	36	48054	51946	50136	49864	02082	97918	24	36
28	37	48094	51906	50180	49820	02086	97914	23	32
32	38	48133	51867	50223	49777	02090	97910	22	28
36	39	48173	51827	50267	49733	02094	97906	21	24
40	40	9.48213	10.51787	9.50311	10.49689	10.02098	9.97902	20	20
44	41	48252	51748	50355	49645	02102	97898	19	16
48	42	48292	51708	50398	49602	02106	97894	18	12
52	43	48332	51668	50442	49558	02110	97890	17	8
56	44	48371	51629	50485	49515	02114	97886	16	4
11	45	9.48411	10.51589	9.50529	10.49471	10.02118	9.97882	15	**49**
4	46	48450	51550	50572	49428	02122	97878	14	56
8	47	48490	51510	50616	49384	02126	97874	13	52
12	48	48529	51471	50659	49341	02130	97870	12	48
16	49	48568	51432	50703	49297	02134	97866	11	44
20	50	9.48607	10.51393	9.50746	10.49254	10.02139	9.97861	10	40
24	51	48647	51353	50789	49211	02143	97857	9	36
28	52	48686	51314	50833	49167	02147	97853	8	32
32	53	48725	51275	50876	49124	02151	97849	7	28
36	54	48764	51236	50919	49081	02155	97845	6	24
40	55	9.48803	10.51197	9.50962	10.49038	10.02159	9.97841	5	20
44	56	48842	51158	51005	48995	02163	97837	4	16
48	57	48881	51119	51048	48952	02167	97833	3	12
52	58	48920	51080	51092	48908	02171	97829	2	8
56	59	48959	51041	51135	48865	02175	97825	1	4
12	60	48998	51002	51178	48822	02179	97821	0	**48**
M.S.	M.	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7h	107°							72°	4h

1h	18°			Logarithms.			161°		10h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
12	0	9.48998	10.51002	9.51178	10.48822	10.02179	9.97821	60	**48**
4	1	49037	50963	51221	48779	02183	97817	59	56
8	2	49076	50924	51264	48736	02188	97812	58	52
12	3	49115	50885	51306	48694	02192	97808	57	48
16	4	49153	50847	51349	48651	02196	97804	56	44
20	5	9.49192	10.50808	9.51392	10.48608	10.02200	9.97800	55	40
24	6	49231	50769	51435	48565	02204	97796	54	36
28	7	49269	50731	51478	48522	02208	97792	53	32
32	8	49308	50692	51520	48480	02212	97788	52	28
36	9	49347	50653	51563	48437	02216	97784	51	24
40	10	9.49385	10.50615	9.51606	10.48394	10.02221	9.97779	50	20
44	11	49424	50576	51648	48352	02225	97775	49	16
48	12	49462	50538	51691	48309	02229	97771	48	12
52	13	49500	50500	51734	48266	02233	97767	47	8
56	14	49539	50461	51776	48224	02237	97763	46	4
13	15	9.49577	10.50423	9.51819	10.48181	10.02241	9.97759	45	**47**
4	16	49615	50385	51861	48139	02246	97754	44	56
8	17	49654	50346	51903	48097	02250	97750	43	52
12	18	49692	50308	51946	48054	02254	97746	42	48
16	19	49730	50270	51988	48012	02258	97742	41	44
20	20	9.49768	10.50232	9.52031	10.47969	10.02262	9.97738	40	40
24	21	49806	50194	52073	47927	02266	97734	39	36
28	22	49844	50156	52115	47885	02271	97729	38	32
32	23	49882	50118	52157	47843	02275	97725	37	28
36	24	49920	50080	52200	47800	02279	97721	36	24
40	25	9.49958	10.50042	9.52242	10.47758	10.02283	9.97717	35	20
44	26	49996	50004	52284	47716	02287	97713	34	16
48	27	50034	49966	52326	47674	02292	97708	33	12
52	28	50072	49928	52368	47632	02296	97704	32	8
56	29	50110	49890	52410	47590	02300	97700	31	4
14	30	9.50148	10.49852	9.52452	10.47548	10.02304	9.97696	30	**46**
4	31	50185	49815	52494	47506	02309	97691	29	56
8	32	50223	49777	52536	47464	02313	97687	28	52
12	33	50261	49739	52578	47422	02317	97683	27	48
16	34	50298	49702	52620	47380	02321	97679	26	44
20	35	9.50336	10.49664	9.52661	10.47339	10.02326	9.97674	25	40
24	36	50374	49626	52703	47297	02330	97670	24	36
28	37	50411	49589	52745	47255	02334	97666	23	32
32	38	50449	49551	52787	47213	02338	97662	22	28
36	39	50486	49514	52829	47171	02343	97657	21	24
40	40	9.50523	10.49477	9.52870	10.47130	10.02347	9.97653	20	20
44	41	50561	49439	52912	47088	02351	97649	19	16
48	42	50598	49402	52953	47047	02355	97645	18	12
52	43	50635	49365	52995	47005	02360	97640	17	8
56	44	50673	49327	53037	46963	02364	97636	16	4
15	45	9.50710	10.49290	9.53078	10.46922	10.02368	9.97632	15	**45**
4	46	50747	49253	53120	46880	02372	97628	14	56
8	47	50784	49216	53161	46839	02377	97623	13	52
12	48	50821	49179	53202	46798	02381	97619	12	48
16	49	50858	49142	53244	46756	02385	97615	11	44
20	50	9.50896	10.49104	9.53285	10.46715	10.02390	9.97610	10	40
24	51	50933	49067	53327	46673	02394	97606	9	36
28	52	50970	49030	53368	46632	02398	97602	8	32
32	53	51007	48993	53409	46591	02403	97597	7	28
36	54	51043	48957	53450	46550	02407	97593	6	24
40	55	9.51080	10.48920	9.53492	10.46508	10.02411	9.97589	5	20
44	56	51117	48883	53533	46467	02416	97584	4	16
48	57	51154	48846	53574	46426	02420	97580	3	12
52	58	51191	48809	53615	46385	02424	97576	2	8
56	59	51227	48773	53656	46344	02429	97571	1	4
16	60	51264	48736	53697	46303	02433	97567	0	**44**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7h	108°							71°	4h

1h	19°			Logarithms.			160°		10h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
16	0	9.51264	10.48736	9.53697	10.46303	10.02433	9.97567	60	**44**
4	1	51301	48699	53738	46262	02437	97563	59	56
8	2	51338	48662	53779	46221	02442	97558	58	52
12	3	51374	48626	53820	46180	02446	97554	57	48
16	4	51411	48589	53861	46139	02450	97550	56	44
20	5	9.51447	10.48553	9.53902	10.46098	10.02455	9.97545	55	40
24	6	51484	48516	53943	46057	02459	97541	54	36
28	7	51520	48480	53984	46016	02464	97536	53	32
32	8	51557	48443	54025	45975	02468	97532	52	28
36	9	51593	48407	54065	45935	02472	97528	51	24
40	10	9.51629	10.48371	9.54106	10.45894	10.02477	9.97523	50	20
44	11	51666	48334	54147	45853	02481	97519	49	16
48	12	51702	48298	54187	45813	02485	97515	48	12
52	13	51738	48262	54228	45772	02490	97510	47	8
56	14	51774	48226	54269	45731	02494	97506	46	4
17	15	9.51811	10.48189	9.54309	10.45691	10.02499	9.97501	45	**43**
4	16	51847	48153	54350	45650	02503	97497	44	56
8	17	51883	48117	54390	45610	02508	97492	43	52
12	18	51919	48081	54431	45569	02512	97488	42	48
16	19	51955	48045	54471	45529	02516	97484	41	44
20	20	9.51991	10.48009	9.54512	10.45488	10.02521	9.97479	40	40
24	21	52027	47973	54552	45448	02525	97475	39	36
28	22	52063	47937	54593	45407	02530	97470	38	32
32	23	52099	47901	54633	45367	02534	97466	37	28
36	24	52135	47865	54673	45327	02539	97461	36	24
40	25	9.52171	10.47829	9.54714	10.45286	10.02543	9.97457	35	20
44	26	52207	47793	54754	45246	02547	97453	34	16
48	27	52242	47758	54794	45206	02552	97448	33	12
52	28	52278	47722	54835	45165	02556	97444	32	8
56	29	52314	47686	54875	45125	02561	97439	31	4
18	30	9.52350	10.47650	9.54915	10.45085	10.02565	9.97435	30	**42**
4	31	52385	47615	54955	45045	02570	97430	29	56
8	32	52421	47579	54995	45005	02574	97426	28	52
12	33	52456	47544	55035	44965	02579	97421	27	48
16	34	52492	47508	55075	44925	02583	97417	26	44
20	35	9.52527	10.47473	9.55115	10.44885	10.02588	9.97412	25	40
24	36	52563	47437	55155	44845	02592	97408	24	36
28	37	52598	47402	55195	44805	02597	97403	23	32
32	38	52634	47366	55235	44765	02601	97399	22	28
36	39	52669	47331	55275	44725	02606	97394	21	24
40	40	9.52705	10.47295	9.55315	10.44685	10.02610	9.97390	20	20
44	41	52740	47260	55355	44645	02615	97385	19	16
48	42	52775	47225	55395	44605	02619	97381	18	12
52	43	52811	47189	55434	44566	02624	97376	17	8
56	44	52846	47154	55474	44526	02628	97372	16	4
19	45	9.52881	10.47119	9.55514	10.44486	10.02633	9.97367	15	**41**
4	46	52916	47084	55554	44446	02637	97363	14	56
8	47	52951	47049	55593	44407	02642	97358	13	52
12	48	52986	47014	55633	44367	02647	97353	12	48
16	49	53021	46979	55673	44327	02651	97349	11	44
20	50	9.53056	10.46944	9.55712	10.44288	10.02656	9.97344	10	40
24	51	53092	46908	55752	44248	02660	97340	9	36
28	52	53126	46874	55791	44209	02665	97335	8	32
32	53	53161	46839	55831	44169	02669	97331	7	28
36	54	53196	46804	55870	44130	02674	97326	6	24
40	55	9.53231	10.46769	9.55910	10.44090	10.02678	9.97322	5	20
44	56	53266	46734	55949	44051	02683	97317	4	16
48	57	53301	46699	55989	44011	02688	97312	3	12
52	58	53336	46664	56028	43972	02692	97308	2	8
56	59	53370	46630	56067	43933	02697	97303	1	4
20	60	53405	46595	56107	43893	02701	97299	0	**40**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7h	109°							70°	4h

1h	20°			Logarithms.			159°		10h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
20	0	9.53405	10.46595	9.56107	10.43893	10.02701	9.97299	60	**40**
4	1	53440	46560	56146	43854	02706	97294	59	56
8	2	53475	46525	56185	43815	02711	97289	58	52
12	3	53509	46491	56224	43776	02715	97285	57	48
16	4	53544	46456	56264	43736	02720	97280	56	44
20	5	9.53578	10.46422	9.56303	10.43697	10.02724	9.97276	55	40
24	6	53613	46387	56342	43658	02729	97271	54	36
28	7	53647	46353	56381	43619	02734	97266	53	32
32	8	53682	46318	56420	43580	02738	97262	52	28
36	9	53716	46284	56459	43541	02743	97257	51	24
40	10	9.53751	10.46249	9.56498	10.43502	10.02748	9.97252	50	20
44	11	53785	46215	56537	43463	02752	97248	49	16
48	12	53819	46181	56576	43424	02757	97243	48	12
52	13	53854	46146	56615	43385	02762	97238	47	8
56	14	53888	46112	56654	43346	02766	97234	46	4
21	15	9.53922	10.46078	9.56693	10.43307	10.02771	9.97229	45	**39**
4	16	53957	46043	56732	43268	02776	97224	44	56
8	17	53991	46009	56771	43229	02780	97220	43	52
12	18	54025	45975	56810	43190	02785	97215	42	48
16	19	54059	45941	56849	43151	02790	97210	41	44
20	20	9.54093	10.45907	9.56887	10.43113	10.02794	9.97206	40	40
24	21	54127	45873	56926	43074	02799	97201	39	36
28	22	54161	45839	56965	43035	02804	97196	38	32
32	23	54195	45805	57004	42996	02808	97192	37	28
36	24	54229	45771	57042	42958	02813	97187	36	24
40	25	9.54263	10.45737	9.57081	10.42919	10.02818	9.97182	35	20
44	26	54297	45703	57120	42880	02822	97178	34	16
48	27	54331	45669	57158	42842	02827	97173	33	12
52	28	54365	45635	57197	42803	02832	97168	32	8
56	29	54399	45601	57235	42765	02837	97163	31	4
22	30	9.54433	10.45567	9.57274	10.42726	10.02841	9.97159	30	**38**
4	31	54466	45534	57312	42688	02846	97154	29	56
8	32	54500	45500	57351	42649	02851	97149	28	52
12	33	54534	45466	57389	42611	02855	97145	27	48
16	34	54567	45433	57428	42572	02860	97140	26	44
20	35	9.54601	10.45399	9.57466	10.42534	10.02865	9.97135	25	40
24	36	54635	45365	57504	42496	02870	97130	24	36
28	37	54668	45332	57543	42457	02874	97126	23	32
32	38	54702	45298	57581	42419	02879	97121	22	28
36	39	54735	45265	57619	42381	02884	97116	21	24
40	40	9.54769	10.45231	9.57658	10.42342	10.02889	9.97111	20	20
44	41	54802	45198	57696	42304	02893	97107	19	16
48	42	54836	45164	57734	42266	02898	97102	18	12
52	43	54869	45131	57772	42228	02903	97097	17	8
56	44	54903	45097	57810	42190	02908	97092	16	4
23	45	9.54936	10.45064	9.57849	10.42151	10.02913	9.97087	15	**37**
4	46	54969	45031	57887	42113	02917	97083	14	56
8	47	55003	44997	57925	42075	02922	97078	13	52
12	48	55036	44964	57963	42037	02927	97073	12	48
16	49	55069	44931	58001	41999	02932	97068	11	44
20	50	9.55102	10.44898	9.58039	10.41961	10.02937	9.97063	10	40
24	51	55136	44864	58077	41923	02941	97059	9	36
28	52	55169	44831	58115	41885	02946	97054	8	32
32	53	55202	44798	58153	41847	02951	97049	7	28
36	54	55235	44765	58191	41809	02956	97044	6	24
40	55	9.55268	10.44732	9.58229	10.41771	10.02961	9.97039	5	20
44	56	55301	44699	58267	41733	02965	97035	4	16
48	57	55334	44666	58304	41696	02970	97030	3	12
52	58	55367	44633	58342	41658	02975	97025	2	8
56	59	55400	44600	58380	41620	02980	97020	1	4
24	60	55433	44567	58418	41582	02985	97015	0	**36**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7h	110°							69°	4h

1h	21°			Logarithms.				158°	10h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
24	0	9.55433	10.44567	9.58418	10.41582	10.02985	9.97015	60	**36**
4	1	55466	44534	58455	41545	02990	97010	59	56
8	2	55499	44501	58493	41507	02995	97005	58	52
12	3	55532	44468	58531	41469	02999	97001	57	48
16	4	55564	44436	58569	41431	03004	96996	56	44
20	5	9.55597	10.44403	9.58606	10.41394	10.03009	9.96991	55	40
24	6	55630	44370	58644	41356	03014	96986	54	36
28	7	55663	44337	58681	41319	03019	96981	53	32
32	8	55695	44305	58719	41281	03024	96976	52	28
36	9	55728	44272	58757	41243	03029	96971	51	24
40	10	9.55761	10.44239	9.58794	10.41206	10.03034	9.96966	50	20
44	11	55793	44207	58832	41168	03038	96962	49	16
48	12	55826	44174	58869	41131	03043	96957	48	12
52	13	55858	44142	58907	41093	03048	96952	47	8
56	14	55891	44109	58944	41056	03053	96947	46	4
25	15	9.55923	10.44077	9.58981	10.41019	10.03058	9.96942	45	**35**
4	16	55956	44044	59019	40981	03063	96937	44	56
8	17	55988	44012	59056	40944	03068	96932	43	52
12	18	56021	43979	59094	40906	03073	96927	42	48
16	19	56053	43947	59131	40869	03078	96922	41	44
20	20	9.56085	10.43915	9.59168	10.40832	10.03083	9.96917	40	40
24	21	56118	43882	59205	40795	03088	96912	39	36
28	22	56150	43850	59243	40757	03093	96907	38	32
32	23	56182	43818	59280	40720	03097	96903	37	28
36	24	56215	43785	59317	40683	03102	96898	36	24
40	25	9.56247	10.43753	9.59354	10.40646	10.03107	9.96893	35	20
44	26	56279	43721	59391	40609	03112	96888	34	16
48	27	56311	43689	59429	40571	03117	96883	33	12
52	28	56343	43657	59466	40534	03122	96878	32	8
56	29	56375	43625	59503	40497	03127	96873	31	4
26	30	9.56408	10.43592	9.59540	10.40460	10.03132	9.96868	30	**34**
4	31	56440	43560	59577	40423	03137	96863	29	56
8	32	56472	43528	59614	40386	03142	96858	28	52
12	33	56504	43496	59651	40349	03147	96853	27	48
16	34	56536	43464	59688	40312	03152	96848	26	44
20	35	9.56568	10.43432	9.59725	10.40275	10.03157	9.96843	25	40
24	36	56599	43401	59762	40238	03162	96838	24	36
28	37	56631	43369	59799	40201	03167	96833	23	32
32	38	56663	43337	59835	40165	03172	96828	22	28
36	39	56695	43305	59872	40128	03177	96823	21	24
40	40	9.56727	10.43273	9.59909	10.40091	10.03182	9.96818	20	20
44	41	56759	43241	59946	40054	03187	96813	19	16
48	42	56790	43210	59983	40017	03192	96808	18	12
52	43	56822	43178	60019	39981	03197	96803	17	8
56	44	56854	43146	60056	39944	03202	96798	16	4
27	45	9.56886	10.43114	9.60093	10.39907	10.03207	9.96793	15	**33**
4	46	56917	43083	60130	39870	03212	96788	14	56
8	47	56949	43051	60166	39834	03217	96783	13	52
12	48	56980	43020	60203	39797	03222	96778	12	48
16	49	57012	42988	60240	39760	03228	96772	11	44
20	50	9.57044	10.42956	9.60276	10.39724	10.03233	9.96767	10	40
24	51	57075	42925	60313	39687	03238	96762	9	36
28	52	57107	42893	60349	39651	03243	96757	8	32
32	53	57138	42862	60386	39614	03248	96752	7	28
36	54	57169	42831	60422	39578	03253	96747	6	24
40	55	9.57201	10.42799	9.60459	10.39541	10.03258	9.96742	5	20
44	56	57232	42768	60495	39505	03263	96737	4	16
48	57	57264	42736	60532	39468	03268	96732	3	12
52	58	57295	42705	60568	39432	03273	96727	2	8
56	59	57326	42674	60605	39395	03278	96722	1	4
28	60	57358	42642	60641	39359	03283	96717	0	**32**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7h	111°							68°	4h

1ʰ	22°	Logarithms.						157°	10ʰ
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
28	0	9.57358	10.42642	9.60641	10.39359	10.03283	9.96717	60	**32**
4	1	57389	42611	60677	39323	03289	96711	59	56
8	2	57420	42580	60714	39286	03294	96706	58	52
12	3	57451	42549	60750	39250	03299	96701	57	48
16	4	57482	42518	60786	39214	03304	96696	56	44
20	5	9.57514	10.42486	9.60823	10.39177	10.03309	9.96691	55	40
24	6	57545	42455	60859	39141	03314	96686	54	36
28	7	57576	42424	60895	39105	03319	96681	53	32
32	8	57607	42393	60931	39069	03324	96676	52	28
36	9	57638	42362	60967	39033	03330	96670	51	24
40	10	9.57669	10.42331	9.61004	10.38996	10.03335	9.96665	50	20
44	11	57700	42300	61040	38960	03340	96660	49	16
48	12	57731	42269	61076	38924	03345	96655	48	12
52	13	57762	42238	61112	38888	03350	96650	47	8
56	14	57793	42207	61148	38852	03355	96645	46	4
29	15	9.57824	10.42176	9.61184	10.38816	10.03360	9.96640	45	**31**
4	16	57855	42145	61220	38780	03366	96634	44	56
8	17	57885	42115	61256	38744	03371	96629	43	52
12	18	57916	42084	61292	38708	03376	96624	42	48
16	19	57947	42053	61328	38672	08381	96619	41	44
20	20	9.57978	10.42022	9.61364	10.38636	10.03386	9.96614	40	40
24	21	58008	41992	61400	38600	03392	96608	39	36
28	22	58039	41961	61436	38564	03397	96603	38	32
32	23	58070	41930	61472	38528	03402	96598	37	28
36	24	58101	41899	61508	38492	09407	96593	36	24
40	25	9.58131	10.41869	9.61544	10.38456	10.03412	9.96588	35	20
44	26	58162	41838	61579	38421	03418	96582	34	16
48	27	58192	41808	61615	38385	03423	96577	33	12
52	28	58223	41777	61651	38349	03428	96572	32	8
56	29	58253	41747	61687	38313	03433	96567	31	4
30	30	9.58284	10.41716	9.61722	10.38278	10.03438	9.96562	30	**30**
4	31	58314	41686	61758	38242	03444	96556	29	56
8	32	58345	41655	61794	38206	03449	96551	28	52
12	33	58375	41625	61830	38170	03454	96546	27	48
16	34	58406	41594	61865	38135	03459	96541	26	44
20	35	9.58436	10.41564	9.61901	10.38099	10.03465	9.96535	25	40
24	36	58467	41533	61936	38064	03470	96530	24	36
28	37	58497	41503	61972	38028	03475	96525	23	32
32	38	58527	41473	62008	37992	03480	96520	22	28
36	39	58557	41443	62043	37957	03486	96514	21	24
40	40	9.58588	10.41412	9.62079	10.37921	10.03491	9.96509	20	20
44	41	58618	41382	62114	37886	03496	96504	19	16
48	42	58648	41352	62150	37850	03502	96498	18	12
52	43	58678	41322	62185	37815	03507	96493	17	8
56	44	58709	41291	62221	37779	03512	96488	16	4
31	45	9.58739	10.41261	9.62256	10.37744	10.03517	9.96483	15	**29**
4	46	58769	41231	62292	37708	03523	96477	14	56
8	47	58799	41201	62327	37673	03528	96472	13	52
12	48	58829	41171	62362	37638	03533	96467	12	48
16	49	58859	41141	62398	37602	03539	96461	11	44
20	50	9.58889	10.41111	9.62433	10.37567	10.03544	9.96456	10	40
24	51	58919	41081	62468	37532	03549	96451	9	36
28	52	58949	41051	62504	37496	03555	96445	8	32
32	53	58979	41021	62539	37461	03560	96440	7	28
36	54	59009	40991	62574	37426	03565	96435	6	24
40	55	9.59039	10.40961	9.62609	10.37391	10.03571	9.96429	5	20
44	56	59069	40931	62645	37355	03576	96424	4	16
48	57	59098	40902	62680	37320	03581	96419	3	12
52	58	59128	40872	62715	37285	03587	96413	2	8
56	59	59158	40842	62750	37250	03592	96408	1	4
32	60	59188	40812	62785	37215	03597	96403	0	**28**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7ʰ	112°							67°	4ʰ

1ʰ	23°			Logarithms.				156°	10ʰ
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
32	0	9.59188	10.40812	9.62785	10.37215	10.03597	9.96403	60	**28**
4	1	59218	40782	62820	37180	03603	96397	59	56
8	2	59247	40753	62855	37145	03608	96392	58	52
12	3	59277	40723	62890	37110	03613	96387	57	48
16	4	59307	40693	62926	37074	03619	96381	56	44
20	5	9.59336	10.40664	9.62961	10.37039	10.03624	9.96376	55	40
24	6	59366	40634	62996	37004	03630	96370	54	36
28	7	59396	40604	63031	36969	03635	96365	53	32
32	8	59425	40575	63066	36934	03640	96360	52	28
36	9	59455	40545	63101	36899	03646	96354	51	24
40	10	9.59484	10.40516	9.63135	10.36865	10.03651	9.96349	50	20
44	11	59514	40486	63170	36830	03657	96343	49	16
48	12	59543	40457	63205	36795	03662	96338	48	12
52	13	59573	40427	63240	36760	03667	96333	47	8
56	14	59602	40398	63275	36725	03673	96327	46	4
33	15	9.59632	10.40368	9.63310	10.36690	10.03678	9.96322	45	**27**
4	16	59661	40339	63345	36655	03684	96316	44	56
8	17	59690	40310	63379	36621	03689	96311	43	52
12	18	59720	40280	63414	36586	03695	96305	42	48
16	19	59749	40251	63449	36551	03700	96300	41	44
20	20	9.59778	10.40222	9.63484	10.36516	10.03706	9.96294	40	40
24	21	59808	40192	63519	36481	03711	96289	39	36
28	22	59837	40163	63553	36447	03716	96284	38	32
32	23	59866	40134	63588	36412	03722	96278	37	28
36	24	59895	40105	63623	36377	09727	96273	36	24
40	25	9.59924	10.40076	9.63657	10.36343	10.03733	9.96267	35	20
44	26	59954	40046	63692	36308	03738	96262	34	16
48	27	59983	40017	63726	36274	03744	96256	33	12
52	28	60012	39988	63761	36239	03749	96251	32	8
56	29	60041	39959	63796	36204	03755	96245	31	4
34	30	9.60070	10.39930	9.63830	10.36170	10.03760	9.96240	30	**26**
4	31	60099	39901	63865	36135	03766	96234	29	56
8	32	60128	39872	63899	36101	03771	96229	28	52
12	33	60157	39843	63934	36066	03777	96223	27	48
16	34	60186	39814	63968	36032	03782	96218	26	44
20	35	9.60215	10.39785	9.64003	10.35997	10.03788	9.96212	25	40
24	36	60244	39756	64037	35963	03793	96207	24	36
28	37	60273	39727	64072	35928	03799	96201	23	32
32	38	60302	39698	64100	35894	03804	96196	22	28
36	39	60331	39669	64140	35860	03810	96190	21	24
40	40	9.60359	10.39641	9.64175	10.35825	10.03815	9.96185	20	20
44	41	60388	39612	64209	35791	03821	96179	19	16
48	42	60417	39583	64243	35757	03826	96174	18	12
52	43	60446	39554	64278	35722	03832	96168	17	8
56	44	60474	39526	64312	35688	03838	96162	16	4
35	45	9.60503	10.39497	9.64346	10.35654	10.03843	9.96157	15	**25**
4	46	60532	39468	64381	35619	03849	96151	14	56
8	47	60561	39439	64415	35585	03854	96146	13	52
12	48	60589	39411	64449	35551	03860	96140	12	48
16	49	60618	39382	64483	35517	03865	96135	11	44
20	50	9.60646	10.39354	9.64517	10.35483	10.03871	9.96129	10	40
24	51	60675	39325	64552	35448	03877	96123	9	36
28	52	60704	39296	64586	35414	03882	96118	8	32
32	53	60732	39268	64620	35380	03888	96112	7	28
36	54	60761	39239	64654	35346	03893	96107	6	24
40	55	9.60789	10.39211	9.64688	10.35312	10.03899	9.96101	5	20
44	56	60818	39182	64722	35278	03905	96095	4	16
48	57	60846	39154	64756	35244	03910	96090	3	12
52	58	60875	39125	64790	35210	03916	96084	2	8
56	59	60903	39097	64824	35176	03921	96079	1	4
36	60	60931	39069	64858	35142	03927	96073	0	**24**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7ʰ	113°							66°	4ʰ

1ʰ	24°			Logarithms.				155°	10ʰ
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
36	0	9.60931	10.39069	9.64858	10.35142	10.03927	9.96073	60	**24**
4	1	60960	39040	64892	35108	03933	96067	59	56
8	2	60988	39012	64926	35074	03938	96062	58	52
12	3	61016	38984	64960	35040	03944	96056	57	48
16	4	61045	38955	64994	35006	03950	96050	56	44
20	5	9.61073	10.38927	9.65028	10.34972	10.03955	9.96045	55	40
24	6	61101	38899	65062	34938	03961	96039	54	36
28	7	61129	38871	65096	34904	03966	96034	53	32
32	8	61158	38842	65130	34870	03972	96028	52	28
36	9	61186	38814	65164	34836	03978	96022	51	24
40	10	9.61214	10.38786	9.65197	10.34803	10.03983	9.96017	50	20
44	11	61242	38758	65231	34769	03989	96011	49	16
48	12	61270	38730	65265	34735	03995	96005	48	12
52	13	61298	38702	65299	34701	04000	96000	47	8
56	14	61326	38674	65333	34667	04006	95994	46	4
37	15	9.61354	10.38646	9.65366	10.34634	10.04012	9.95988	45	**23**
4	16	61382	38618	65400	34600	04018	95982	44	56
8	17	61411	38589	65434	34566	04023	95977	43	52
12	18	61438	38562	65467	34533	04029	95971	42	48
16	19	61466	38534	65501	34499	04035	95965	41	44
20	20	9.61494	10.38506	9.65535	10.34465	10.04040	9.95960	40	40
24	21	61522	38478	65568	34432	04046	95954	39	36
28	22	61550	38450	65602	34398	04052	95948	38	32
32	23	61578	38422	65636	34364	04058	95942	37	28
36	24	61606	38394	65669	34331	04063	95937	36	24
40	25	9.61634	10.38366	9.65703	10.34297	10.04069	9.95931	35	20
44	26	61662	38338	65736	34264	04075	95925	34	16
48	27	61689	38311	65770	34230	04080	95920	33	12
52	28	61717	38283	65803	34197	04086	95914	32	8
56	29	61745	38255	65837	34163	04092	95908	31	4
38	30	9.61773	10.38227	9.65870	10.34130	10.04098	9.95902	30	**22**
4	31	61800	38200	65904	34096	04103	95897	29	56
8	32	61828	38172	65937	34063	04109	95891	28	52
12	33	61856	38144	65971	34029	04115	95885	27	48
16	34	61883	38117	66004	33996	04121	95879	26	44
20	35	9.61911	10.38089	9.66038	10.33962	10.04127	9.95873	25	40
24	36	61939	38061	66071	33929	04132	95868	24	36
28	37	61966	38034	66104	33896	04138	95862	23	32
32	38	61994	38006	66138	33862	04144	95856	22	28
36	39	62021	37979	66171	33829	04150	95850	21	24
40	40	9.62049	10.37951	9.66204	10.33796	10.04156	9.95844	20	20
44	41	62076	37924	66238	33762	04161	95839	19	16
48	42	62104	37896	66271	33729	04167	95833	18	12
52	43	62131	37869	66304	33696	04173	95827	17	8
56	44	62159	37841	66337	33663	04179	95821	16	4
39	45	9.62186	10.37814	9.66371	10.33629	10.04185	9.95815	15	**21**
4	46	62214	37786	66404	33596	04190	95810	14	56
8	47	62241	37759	66437	33563	04196	95804	13	52
12	48	62268	37732	66470	33530	04202	95798	12	48
16	49	62296	37704	66503	33497	04208	95792	11	44
20	50	9.62323	10.37677	9.66537	10.33463	10.04214	9.95786	10	40
24	51	62350	37650	66570	33430	04220	95780	9	36
28	52	62377	37623	66603	33397	04225	95775	8	32
32	53	62405	37595	66636	33364	04231	95769	7	28
36	54	62432	37568	66669	33331	04237	95763	6	24
40	55	9.62459	10.37541	9.66702	10.33298	10.04243	9.95757	5	20
44	56	62486	37514	66735	33265	04249	95751	4	16
48	57	62513	37487	66768	33232	04255	95745	3	12
52	58	62541	37459	66801	33199	04261	95739	2	8
56	59	62568	37432	66834	33166	04267	95733	1	4
40	60	62595	37405	66867	33133	04272	95728	0	**20**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S
7ʰ	114°							65°	4ʰ

1h	25°	Logarithms.						154°	10h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
40	0	9.62595	10.37405	9.66867	10.33133	10.04272	9.95728	60	**20**
4	1	62622	37378	66900	33100	04278	95722	59	56
8	2	62649	37351	66933	33067	04284	95716	58	52
12	3	62676	37324	66966	33034	04290	95710	57	48
16	4	62703	37297	66999	33001	04296	95704	56	44
20	5	9.62730	10.37270	9.67032	10.32968	10.04302	9.95698	55	40
24	6	62757	37243	67065	32935	04308	95692	54	36
28	7	62784	37216	67098	32902	04314	95686	53	32
32	8	62811	37189	67131	32869	04320	95680	52	28
36	9	62838	37162	67163	32837	04326	95674	51	24
40	10	9.62865	10.37135	9.67196	10.32804	10.04332	9.95668	50	20
44	11	62892	37108	67229	32771	04337	95663	49	16
48	12	62918	37082	67262	32738	04343	95657	48	12
52	13	62945	37055	67295	32705	04349	95651	47	8
56	14	62972	37028	67327	32673	04355	95645	46	4
41	15	9.62999	10.37001	9.67360	10.32640	10.04361	9.95639	45	**19**
4	16	63026	36974	67393	32607	04367	95633	44	56
8	17	63052	36948	67426	32574	04373	95627	43	52
12	18	63079	36921	67458	32542	04379	95621	42	48
16	19	63106	36894	67491	32509	04385	95615	41	44
20	20	9.63133	10.36867	9.67524	10.32476	10.04391	9.95609	40	40
24	21	63159	36841	67556	32444	04397	95603	39	36
28	22	63186	36814	67589	32411	04403	95597	38	32
32	23	63213	36787	67622	32378	04409	95591	37	28
36	24	63239	36761	67654	32346	04415	95585	36	24
40	25	9.63266	10.36734	9.67687	10.32313	10.04421	9.95579	35	20
44	26	63292	36708	67719	32281	04427	95573	34	16
48	27	63319	36681	67752	32248	04433	95567	33	12
52	28	63345	36655	67785	32215	04439	95561	32	8
56	29	63372	36628	67817	32183	04445	95555	31	4
42	30	9.63398	10.36602	9.67850	10.32150	10.04451	9.95549	30	**18**
4	31	63425	36575	67882	32118	04457	95543	29	56
8	32	63451	36549	67915	32085	04463	95537	28	52
12	33	63478	36522	67947	32053	04469	95531	27	48
16	34	63504	36496	67980	32020	04475	95525	26	44
20	35	9.63531	10.36469	9.68012	10.31988	10.04481	9.95519	25	40
24	36	63557	36443	68044	31956	04487	95513	24	36
28	37	63583	36417	68077	31923	04493	95507	23	32
32	38	63610	36390	68109	31891	04500	95500	22	28
36	39	63636	36364	68142	31858	04506	95494	21	24
40	40	9.63662	10.36338	9.68174	10 31826	10.04512	9.95488	20	20
44	41	63689	36311	68206	31794	04518	95482	19	16
48	42	63715	36285	68239	31761	04524	95476	18	12
52	43	63741	36259	68271	31729	04530	95470	17	8
56	44	63767	36233	68303	31697	04536	95464	16	4
43	45	9.63794	10.36206	9.68336	10.31664	10.04542	9.95458	15	**17**
4	46	63820	36180	68368	31632	04548	95452	14	56
8	47	63846	36154	68400	31600	04554	95446	13	52
12	48	63872	36128	68432	31568	04560	95440	12	48
16	49	63898	36102	68465	31535	04566	95434	11	44
20	50	9.63924	10.36076	9.68497	10.31503	10.04573	9.95427	10	40
24	51	63950	36050	68529	31471	04579	95421	9	36
28	52	63976	36024	68561	31439	04585	95415	8	32
32	53	64002	35998	68593	31407	04591	95409	7	28
36	54	64028	35972	68626	31374	04597	95403	6	24
40	55	9.64054	10.35946	9.68658	10.31342	10.04603	9.95397	5	20
44	56	64080	35920	68690	31310	04609	95391	4	16
48	57	64106	35894	68722	31278	04616	95384	3	12
52	58	64132	35868	68754	31246	04622	95378	2	8
56	59	64158	35842	68786	31214	04628	95372	1	4
44	60	64184	35816	68818	31182	04634	95366	0	**16**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7h	115°							64°	4h

1^h	26°			**Logarithms.**				153°	10^h
M.S	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
44	0	9.64184	10.35816	9.68818	10.31182	10.04634	9.95366	60	**16**
4	1	64210	35790	68850	31150	04640	95360	59	56
8	2	64236	35764	68882	31118	04646	95354	58	52
12	3	64262	35738	68914	31086	04652	95348	57	48
16	4	64288	35712	68946	31054	04659	95341	56	44
20	5	9.64313	10.35687	9.68978	10.31022	10.04665	9.95335	55	40
24	6	64339	35661	69010	30990	04671	95329	54	36
28	7	64365	35635	69042	30958	04677	95323	53	32
32	8	64391	35609	69074	30926	04683	95317	52	28
36	9	64417	35583	69106	30894	04690	95310	51	24
40	10	9.64442	10.35558	9.69138	10.30862	10.04696	9.95304	50	20
44	11	64468	35532	69170	30830	04702	95298	49	16
48	12	64494	35506	69202	30798	04708	95292	48	12
52	13	64519	35481	69234	30766	04714	95286	47	8
56	14	64545	35455	69266	30734	04721	95279	46	4
45	15	9.64571	10.35429	9.69298	10.30702	10.04727	9.95273	45	**15**
4	16	64596	35404	69329	30671	04733	95267	44	56
8	17	64622	35378	69361	30639	04739	95261	43	52
12	18	64647	35353	69393	30607	04746	95254	42	48
16	19	64673	35327	69425	30575	04752	95248	41	44
20	20	9.64698	10.35302	9.69457	10.30543	10.04758	9.95242	40	40
24	21	64724	35276	69488	30512	04764	95236	39	36
28	22	64749	35251	69520	30480	04771	95229	38	32
32	23	64775	35225	69552	30448	04777	95223	37	28
36	24	64800	35200	69584	30416	04783	95217	36	24
40	25	9.64826	10.35174	9.69615	10.30385	10.04789	9.95211	35	20
44	26	64851	35149	69647	30353	04796	95204	34	16
48	27	64877	35123	69679	30321	04802	95198	33	12
52	28	64902	35098	69710	30290	04808	95192	32	8
56	29	64927	35073	69742	30258	04815	95185	31	4
46	30	9.64953	10.35047	9.69774	10.30226	10.04821	9.95179	30	**14**
4	31	64978	35022	69805	30195	04827	95173	29	56
8	32	65003	34997	69837	30163	04833	95167	28	52
12	33	65029	34971	69868	30132	04840	95160	27	48
16	34	65054	34946	69900	30100	04846	95154	26	44
20	35	9.65079	10.34921	9.69932	10.30068	10.04852	9.95148	25	40
24	36	65104	34896	69963	30037	04859	95141	24	36
28	37	65130	34870	69995	30005	04865	95135	23	32
32	38	65155	34845	70026	29974	04871	95129	22	28
36	39	65180	34820	70058	29942	04878	95122	21	24
40	40	9.65205	10.34795	9.70089	10.29911	10.04884	9.95116	20	20
44	41	65230	34770	70121	29879	04890	95110	19	16
48	42	65255	34745	70152	29848	04897	95103	18	12
52	43	65281	34719	70184	29816	04903	95097	17	8
56	44	65306	34694	70215	29785	04910	95090	16	4
47	45	9.65331	10.34669	9.70247	10.29753	10.04916	9.95084	15	**13**
4	46	65356	34644	70278	29722	04922	95078	14	56
8	47	65381	34619	70309	29691	04929	95071	13	52
12	48	65406	34594	70341	29659	04935	95065	12	48
16	49	65431	34569	70372	29628	04941	95059	11	44
20	50	9.65456	10.34544	9.70404	10.29596	10.04948	9.95052	10	40
24	51	65481	34519	70435	29565	04954	95046	9	36
28	52	65506	34494	70466	29534	04961	95039	8	32
32	53	65531	34469	70498	29502	04967	95033	7	28
36	54	65556	34444	70529	29471	04973	95027	6	24
40	55	9.65580	10.34420	9.70560	10.29440	10.04980	9.95020	5	20
44	56	65605	34395	70592	29408	04986	95014	4	16
48	57	65630	34370	70623	29377	04993	95007	3	12
52	58	65655	34345	70654	29346	04999	95001	2	8
56	59	65680	34320	70685	29315	05005	94995	1	4
48	60	65705	34295	70717	29283	05012	94988	0	**12**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7^h	116°							63°	4^h

1ʰ	27°	Logarithms.						152°	10ʰ
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
48	0	9.65705	10.34295	9.70717	10.29283	10.05012	9.94988	60	**12**
4	1	65729	34271	70748	29252	05018	94982	59	56
8	2	65754	34246	70779	29221	05025	94975	58	52
12	3	65779	34221	70810	29190	05031	94969	57	48
16	4	65804	34196	70841	29159	05038	94962	56	44
20	5	9.65828	10.34172	9.70873	10.29127	10.05044	9.94956	55	40
24	6	65853	34147	70904	29096	05051	94949	54	36
28	7	65878	34122	70935	29065	05057	94943	53	32
32	8	65902	34098	70966	29034	05064	94936	52	28
36	9	65927	34073	70997	29003	05070	94930	51	24
40	10	9.65952	10.34048	9.71028	10.28972	10.05077	9.94923	50	20
44	11	65976	34024	71059	28941	05083	94917	49	16
48	12	66001	33999	71090	28910	05089	94911	48	12
52	13	66025	33975	71121	28879	05096	94904	47	8
56	14	66050	33950	71153	28847	05102	94898	46	4
49	15	9.66075	10.33925	9.71184	10.28816	10.05109	9.94891	45	**11**
4	16	66099	33901	71215	28785	05115	94885	44	56
8	17	66124	33876	71246	28754	05122	94878	43	52
12	18	66148	33852	71277	28723	05129	94871	42	48
16	19	66173	33827	71308	28692	05135	94865	41	44
20	20	9.66197	10.33803	9.71339	10.28661	10.05142	9.94858	40	40
24	21	66221	33779	71370	28630	05148	94852	39	36
28	22	66246	33754	71401	28599	05155	94845	38	32
32	23	66270	33730	71431	28569	05161	94839	37	28
36	24	66295	33705	71462	28538	05168	94832	36	24
40	25	9.66319	10.33681	9.71493	10.28507	10.05174	9.94826	35	20
44	26	66343	33657	71524	28476	05181	94819	34	16
48	27	66368	33632	71555	28445	05187	94813	33	12
52	28	66392	33608	71586	28414	05194	94806	32	8
56	29	66416	33584	71617	28383	05201	94799	31	4
50	30	9.66441	10.33559	9.71648	10.28352	10.05207	9.94793	30	**10**
4	31	66465	33535	71679	28321	05214	94786	29	56
8	32	66489	33511	71709	28291	05220	94780	28	52
12	33	66513	33487	71740	28260	05227	94773	27	48
16	34	66537	33463	71771	28229	05233	94767	26	44
20	35	9.66562	10.33438	9.71802	10.28198	10.05240	9.94760	25	40
24	36	66586	33414	71833	28167	05247	94753	24	36
28	37	66610	33390	71863	28137	05253	94747	23	32
32	38	66634	33366	71894	28106	05260	94740	22	28
36	39	66658	33342	71925	28075	05266	94734	21	24
40	40	9.66682	10.33318	9.71955	10.28045	10.05273	9.94727	20	20
44	41	66706	33294	71986	28014	05280	94720	19	16
48	42	66731	33269	72017	27983	05286	94714	18	12
52	43	66755	33245	72048	27952	05293	94707	17	8
56	44	66779	33221	72078	27922	05300	94700	16	4
51	45	9.66803	10.33197	9.72109	10.27891	10.05306	9.94694	15	**9**
4	46	66827	33173	72140	27860	05313	94687	14	56
8	47	66851	33149	72170	27830	05320	94680	13	52
12	48	66875	33125	72201	27799	05326	94674	12	48
16	49	66899	33101	72231	27769	05333	94667	11	44
20	50	9.66922	10.33078	9.72262	10.27738	10.05340	9.94660	10	40
24	51	66946	33054	72293	27707	05346	94654	9	36
28	52	66970	33030	72323	27677	05353	94647	8	32
32	53	66994	33006	72354	27646	05360	94640	7	28
36	54	67018	32982	72384	27616	05366	94634	6	24
40	55	9.67042	10.32958	9.72415	10.27585	10.05373	9.94627	5	20
44	56	67066	32934	72445	27555	05380	94620	4	16
48	57	67090	32910	72476	27524	05386	94614	3	12
52	58	67113	32887	72506	27494	05393	94607	2	8
56	59	67137	32863	72537	27463	05400	94600	1	4
52	60	67161	32839	72567	27433	05407	94593	0	**8**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7ʰ	117°							62°	4ʰ

1h	28°			Logarithms.				151°	10h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
52	0	9.67161	10.32839	9.72567	10.27433	10.05407	9.94593	60	**8**
4	1	67185	32815	72598	27402	05413	94587	59	56
8	2	67208	32792	72628	27372	05420	94580	58	52
12	3	67232	32768	72659	27341	05427	94573	57	48
16	4	67256	32744	72689	27311	05433	94567	56	44
20	5	9.67280	10.32720	9.72720	10.27280	10.05440	9.94560	55	40
24	6	67303	32697	72750	27250	05447	94553	54	36
28	7	67327	32673	72780	27220	05454	94546	53	32
32	8	67350	32650	72811	27189	05460	94540	52	28
36	9	67374	32626	72841	27159	05467	94533	51	24
40	10	9.67398	10.32602	9.72872	10.27128	10.05474	9.94526	50	20
44	11	67421	32579	72902	27098	05481	94519	49	16
48	12	67445	32555	72932	27068	05487	94513	48	12
52	13	67468	32532	72963	27037	05494	94506	47	8
56	14	67492	32508	72993	27007	05501	94499	46	4
53	15	9.67515	10.32485	9.73023	10.26977	10.05508	9.94492	45	**7**
4	16	67539	32461	73054	26946	05515	94485	44	56
8	17	67562	32438	73084	26916	05521	94479	43	52
12	18	67586	32414	73114	26886	05528	94472	42	48
16	19	67609	32391	73144	26856	05535	94465	41	44
20	20	9.67633	10.32367	9.73175	10.26825	10.05542	9.94458	40	40
24	21	67656	32344	73205	26795	05549	94451	39	36
28	22	67680	32320	73235	26765	05555	94445	38	32
32	23	67703	32297	73265	26735	05562	94438	37	28
36	24	67726	32274	73295	26705	05569	94431	36	24
40	25	9.67750	10.32250	9.73326	10.26674	10.05576	9.94424	35	20
44	26	67773	32227	73356	26644	05583	94417	34	16
48	27	67796	32204	73386	26614	05590	94410	33	12
52	28	67820	32180	73416	26584	05596	94404	32	8
56	29	67843	32157	73446	26554	05603	94397	31	4
54	30	9.67866	10.32134	9.73476	10.26524	10.05610	9.94390	30	**6**
4	31	67890	32110	73507	26493	05617	94383	29	56
8	32	67913	32087	73537	26463	05624	94376	28	52
12	33	66936	32064	73567	26433	05631	94369	27	48
16	34	67959	32041	73597	26403	05638	94362	26	44
20	35	9.67982	10.32018	9.73627	10.26373	10.05645	9.94355	25	40
24	36	68006	31994	73657	26343	05651	94349	24	36
28	37	68029	31971	73687	26313	05658	94342	23	32
32	38	68052	31948	73717	26283	05665	94335	22	28
36	39	68075	31925	73747	26253	05672	94328	21	24
40	40	9.68098	10.31902	9.73777	10.26223	10.05679	9.94321	20	20
44	41	68121	31879	73807	26193	05686	94314	19	16
48	42	68144	31856	73837	26163	05693	94307	18	12
52	43	68167	31833	73867	26133	05700	94300	17	8
56	44	68190	31810	73897	26103	05707	94293	16	4
55	45	9.68213	10.31787	9.73927	10.26073	10.05714	9.94286	15	**5**
4	46	68237	31763	73957	26043	05721	94279	14	56
8	47	68260	31740	73987	26013	05727	94273	13	52
12	48	68283	31717	74017	25983	05734	94266	12	48
16	49	68305	31695	74047	25953	05741	94259	11	44
20	50	9.68328	10.31672	9.74077	10.25923	10.05748	9.94252	10	40
24	51	68351	31649	74107	25893	05755	94245	9	36
28	52	68374	31626	74137	25863	05762	94238	8	32
32	53	68397	31603	74166	25834	05769	94231	7	28
36	54	68420	31580	74196	25804	05776	94224	6	24
40	55	9.68443	10.31557	9.74226	10.25774	10.05783	9.94217	5	20
44	56	68466	31534	74256	25744	05790	94210	4	16
48	57	68489	31511	74286	25714	05797	94203	3	12
52	58	68512	31488	74316	25684	05804	94196	2	8
56	59	68534	31466	74345	25655	05811	94189	1	4
56	60	68557	31443	74375	25625	05818	94182	0	**4**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7h	118°							61°	4h

1h	29°			Logarithms.			150°		10h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
56	0	9.68557	10.31443	9.74375	10.25625	10.05818	9.94182	60	**4**
4	1	68580	31420	74405	25595	05825	94175	59	56
8	2	68603	31397	74435	25565	05832	94168	58	52
12	3	68625	31375	74465	25535	05839	94161	57	48
16	4	68648	31352	74494	25506	05846	94154	56	44
20	5	9.68671	10.31329	9.74524	10.25476	10.05853	9.94147	55	40
24	6	68694	31306	74554	25446	05860	94140	54	36
28	7	68716	31284	74583	25417	05867	94133	53	32
32	8	68739	31261	74613	25387	05874	94126	52	28
36	9	68762	31238	74643	25357	05881	94119	51	24
40	10	9.68784	10.31216	9.74673	10.25327	10.05888	9.94112	50	20
44	11	68807	31193	74702	25298	05895	94105	49	16
48	12	68829	31171	74732	25268	05902	94098	48	12
52	13	68852	31148	74762	25238	05910	94090	47	8
56	14	68875	31125	74791	25209	05917	94083	46	4
57	15	9.68897	10.31103	9.74821	10.25179	10.05924	9.94076	45	**3**
4	16	68920	31080	74851	25149	05931	94069	44	56
8	17	68942	31058	74880	25120	05938	94062	43	52
12	18	68965	31035	74910	25090	05945	94055	42	48
16	19	68987	31013	74939	25061	05952	94048	41	44
20	20	9.69010	10.30990	9.74969	10.25031	10.05959	9.94041	40	40
24	21	69032	30968	74998	25002	05966	94034	39	36
28	22	69055	30945	75028	24972	05973	94027	38	32
32	23	69077	30923	75058	24942	05980	94020	37	28
36	24	69100	30900	75087	24913	05988	94012	36	24
40	25	9.69122	10.30878	9.75117	10.24883	10.05995	9.94005	35	20
44	26	69144	30856	75146	24854	06002	93998	34	16
48	27	69167	30833	75176	24824	06009	93991	33	12
52	28	69189	30811	75205	24795	06016	93984	32	8
56	29	69212	30788	75235	24765	06023	93977	31	4
58	30	9.69234	10.30766	9.75264	10.24736	10.06030	9.93970	30	**2**
4	31	69256	30744	75294	24706	06037	93963	29	56
8	32	69279	30721	75323	24677	06045	93955	28	52
12	33	69301	30699	75353	24647	06052	93948	27	48
16	34	69323	30677	75382	24618	06059	93941	26	44
20	35	9.69345	10.30655	9.75411	10.24589	10.06066	9.93934	25	40
24	36	69368	30632	75441	24559	06073	93927	24	36
28	37	69390	30610	75470	24530	06080	93920	23	32
32	38	69412	30588	75500	24500	06088	93912	22	28
36	39	69434	30566	75529	24471	06095	93905	21	24
40	40	9.69456	10.30544	9.75558	10.24442	10.06102	9.93898	20	20
44	41	69479	30521	75588	24412	06109	93891	19	16
48	42	69501	30499	75617	24383	06116	93884	18	12
52	43	69523	30477	75647	24353	06124	93876	17	8
56	44	69545	30455	75676	24324	06131	93869	16	4
59	45	9.69567	10.30433	9.75705	10.24295	10.06138	9.93862	15	**1**
4	46	69589	30411	75735	24265	06145	93855	14	56
8	47	69611	30389	75764	24236	06153	93847	13	52
12	48	69633	30367	75793	24207	06160	93840	12	48
16	49	69655	30345	75822	24178	06167	93833	11	44
20	50	9.69677	10.30323	9.75852	10.24148	10.06174	9.93826	10	40
24	51	69699	30301	75881	24119	06181	93819	9	36
28	52	69721	30279	75910	24090	06189	93811	8	32
32	53	69743	30257	75939	24061	06196	93804	7	28
36	54	69765	30235	75969	24031	06203	93797	6	24
40	55	9.69787	10.30213	9.75998	10.24002	10.06211	9.93789	5	20
44	56	69809	30191	76027	23973	06218	93782	4	16
48	57	69831	30169	76056	23944	06225	93775	3	12
52	58	69853	30147	76086	23914	06232	93768	2	8
56	59	69875	30125	76115	23885	06240	93760	1	4
60	60	69897	30103	76144	23856	06247	93753	0	**0**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
7h	119°							60°	4h

2h	30°			Logarithms.				149°	9h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
0	0	9.69897	10.30103	9.76144	10.23856	10.06247	9.93753	60	**60**
4	1	69919	30081	76173	23827	06254	93746	59	56
8	2	69941	30059	76202	23798	06262	93738	58	52
12	3	69963	30037	76231	23769	06269	93731	57	48
16	4	69984	30016	76261	23739	06276	93724	56	44
20	5	9.70006	10.29994	9.76290	10.23710	10.06283	9.93717	55	40
24	6	70028	29972	76319	23681	06291	93709	54	36
28	7	70050	29950	76348	23652	06298	93702	53	32
32	8	70072	29928	76377	23623	06305	93695	52	28
36	9	70093	29907	76406	23594	06313	93687	51	24
40	10	9.70115	10.29885	9.76435	10.23565	10.06320	9.93680	50	20
44	11	70137	29863	76464	23536	06327	93673	49	16
48	12	70159	29841	76493	23507	06335	93665	48	12
52	13	70180	29820	76522	23478	06342	93658	47	8
56	14	70202	29798	76551	23449	06350	93650	46	4
1	15	9.70224	10.29776	9.76580	10.23420	10.06357	9.93643	45	**59**
4	16	70245	29755	76609	23391	06364	93636	44	56
8	17	70267	29733	76639	23361	06372	93628	43	52
12	18	70288	29712	76668	23332	06379	93621	42	48
16	19	70310	29690	76697	23303	06386	93614	41	44
20	20	9.70332	10.29668	9.76725	10.23275	10.06394	9.93606	40	40
24	21	70353	29647	76754	23246	06401	93599	39	36
28	22	70375	29625	76783	23217	06409	93591	38	32
32	23	70396	29604	76812	23188	06416	93584	37	28
36	24	70418	29582	76841	23159	06423	93577	36	24
40	25	9.70439	10.29561	9.76870	10.23130	10.06431	9.93569	35	20
44	26	70461	29539	76899	23101	06438	93562	34	16
48	27	70482	29518	76928	23072	06446	93554	33	12
52	28	70504	29496	76957	23043	06453	93547	32	8
56	29	70525	29475	76986	23014	06461	93539	31	4
2	30	9.70547	10.29453	9.77015	10.22985	10.06468	9.93532	30	**58**
4	31	70568	29432	77044	22956	06475	93525	29	56
8	32	70590	29410	77073	22927	06483	93517	28	52
12	33	70611	29389	77101	22899	06490	93510	27	48
16	34	70633	29367	77130	22870	06498	93502	26	44
20	35	9.70654	10.29346	9.77159	10.22841	10.06505	9.93495	25	40
24	36	70675	29325	77188	22812	06513	93487	24	36
28	37	70697	29303	77217	22783	06520	93480	23	32
32	38	70718	29282	77246	22754	06528	93472	22	28
36	39	70739	29261	77274	22726	06535	93465	21	24
40	40	9.70761	10.29239	9.77303	10.22697	10.06543	9.93457	20	20
44	41	70782	29218	77332	22668	06550	93450	19	16
48	42	70803	29197	77361	22639	06558	93442	18	12
52	43	70824	29176	77390	22610	06565	93435	17	8
56	44	70846	29154	77418	22582	06573	93427	16	4
3	45	9.70867	10.29133	9.77447	10.22553	10.06580	9.93420	15	**57**
4	46	70888	29112	77476	22524	06588	93412	14	56
8	47	70909	29091	77505	22495	06595	93405	13	52
12	48	70931	29069	77533	22467	06603	93397	12	48
16	49	70952	29048	77562	22438	06610	93390	11	44
20	50	9.70973	10.29027	9.77591	10.22409	10.06618	9.93382	10	40
24	51	70994	29006	77619	22381	06625	93375	9	36
28	52	71015	28985	77648	22352	06633	93367	8	32
32	53	71036	28964	77677	22323	06640	93360	7	28
36	54	71058	28942	77706	22294	06648	93352	6	24
40	55	9.71079	10.28921	9.77734	10.22266	10.06656	9.93344	5	20
44	56	71100	28900	77763	22237	06663	93337	4	16
48	57	71121	28879	77791	22209	06671	93329	3	12
52	58	71142	28858	77820	22180	06678	93322	2	8
56	59	71163	28837	77849	22151	06686	93314	1	4
4	60	71184	28816	77877	22123	06693	93307	0	**56**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
8h	120°							59°	3h

2h	31°		Logarithms.				148°		9h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
4	0	9.71184	10.28816	9.77877	10.22123	10.06693	9.93307	60	**56**
4	1	71205	28795	77906	22094	06701	93299	59	56
8	2	71226	28774	77935	22065	06709	93291	58	52
12	3	71247	28753	77963	22037	06716	93284	57	48
16	4	71268	28732	77992	22008	06724	93276	56	44
20	5	9.71289	10.28711	9.78020	10.21980	10.06731	9.93269	55	40
24	6	71310	28690	78049	21951	06739	93261	54	36
28	7	71331	28669	78077	21923	06747	93253	53	32
32	8	71352	28648	78106	21894	06754	93246	52	28
36	9	71373	28627	78135	21865	06762	93238	51	24
40	10	9.71393	10.28607	9.78163	10.21837	10.06770	9.93230	50	20
44	11	71414	28586	78192	21808	06777	93223	49	16
48	12	71435	28565	78220	21780	06785	93215	48	12
52	13	71456	28544	78249	21751	06793	93207	47	8
56	14	71477	28523	78277	21723	06800	93200	46	4
5	15	9.71498	10.28502	9.78306	10.21694	10.06808	9.93192	45	**55**
4	16	71519	28481	78334	21666	06816	93184	44	56
8	17	71539	28461	78363	21637	06823	93177	43	52
12	18	71560	28440	78391	21609	06831	93169	42	48
16	19	71581	28419	78419	21581	06839	93161	41	44
20	20	9.71602	10.28398	9.78448	10.21552	10.06846	9.93154	40	40
24	21	71622	23378	78476	21524	06854	93146	39	36
28	22	71643	28357	78505	21495	06862	93138	38	32
32	23	71664	28336	78533	21467	06869	93131	37	28
36	24	71685	28315	78562	21438	06877	93123	36	24
40	25	9.71705	10.28295	9.78590	10.21410	10.06885	9.93115	35	20
44	26	71726	28274	78618	21382	06892	93108	34	16
48	27	71747	28253	78647	21353	06900	93100	33	12
52	28	71767	28233	78675	21325	06908	93092	32	8
56	29	71788	28212	78704	21296	06916	93084	31	4
6	30	9.71809	10.28191	9.78732	10.21268	10.06923	9.93077	30	**54**
4	31	71829	28171	78760	21240	06931	93069	29	56
8	32	71850	28150	78789	21211	06939	93061	28	52
12	33	71870	28130	78817	21183	06947	93053	27	48
16	34	71891	28109	78845	21155	06954	93046	26	44
20	35	9.71911	10.28089	9.78874	10.21126	10.06962	9.93038	25	40
24	36	71932	28068	78902	21098	06970	93030	24	36
28	37	71952	28048	78930	21070	06978	93022	23	32
32	38	71973	28027	78959	21041	06986	93014	22	28
36	39	71994	28006	78987	21013	06993	93007	21	24
40	40	9.72014	10.27986	9.79015	10.20985	10.07001	9.92999	20	20
44	41	72034	27966	79043	20957	07009	92991	19	16
48	42	72055	27945	79072	20928	07017	92983	18	12
52	43	72075	27925	79100	20900	07024	92976	17	8
56	44	72096	27904	79128	20872	07032	92968	16	4
7	45	9.72116	10.27884	9.79156	10.20844	10.07040	9.92960	15	**53**
4	46	72137	27863	79185	20815	07048	92952	14	56
8	47	72157	27843	79213	20787	07056	93944	13	52
12	48	72177	27823	79241	20759	07064	93036	12	48
16	49	72198	27802	79269	20731	07071	92929	11	44
20	50	9.72218	10.27782	9.79297	10.20703	10.07079	9.92921	10	40
24	51	72238	27762	79326	20674	07087	92913	9	36
28	52	72259	27741	79354	20646	07095	92905	8	32
32	53	72279	27721	79382	20618	07103	92897	7	28
36	54	72299	27701	79410	20590	07111	92889	6	24
40	55	9.72320	10.27680	9.79438	10.20562	10.07119	9.92881	5	20
44	56	72340	27660	79466	20534	07126	92874	4	16
48	57	72360	27640	79495	20505	07134	92866	3	12
52	58	72381	27619	79523	20477	07142	92858	2	8
56	59	72401	27599	79551	20449	07150	92850	1	4
8	60	72421	27579	79579	20421	07158	92842	0	**52**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
8h	121°							58°	3h

2^h	32°			Logarithms.			147°		9^h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
8	0	9.72421	10.27579	9.79579	10.20421	10.07158	9.92842	60	**52**
4	1	72441	27559	79607	20393	07166	92834	59	56
8	2	72461	27539	79635	20365	07174	92826	58	52
12	3	72482	27518	79663	20337	07182	92818	57	48
16	4	72502	27498	79691	20309	07190	92810	56	44
20	5	9.72522	10.27478	9.79719	10.20281	10.07197	9.92803	55	40
24	6	72542	27458	79747	20253	07205	92795	54	36
28	7	72562	27438	79776	20224	07213	92787	53	32
32	8	72582	27418	79804	20196	07221	92779	52	28
36	9	72602	27398	79832	20168	07229	92771	51	24
40	10	9.72622	10.27378	9.79860	10.20140	10.07237	9.92763	50	20
44	11	72643	27357	79888	20112	07245	92755	49	16
48	12	72663	27337	79916	20084	07253	92747	48	12
52	13	72683	27317	79944	20056	07261	92739	47	8
56	14	72703	27297	79972	20028	07269	92731	46	4
9	15	9.72723	10.27277	9.80000	10.20000	10.07277	9.92723	45	**51**
4	16	72743	27257	80028	19972	07285	92715	44	56
8	17	72763	27237	80056	19944	07293	92707	43	52
12	18	72783	27217	80084	19916	07301	92699	42	48
16	19	72803	27197	80112	19888	07309	92691	41	44
20	20	9.72823	10.27177	9.80140	10.19860	10.07317	9.92683	40	40
24	21	72843	27157	80168	19832	07325	92675	39	36
28	22	72863	27137	80195	19805	07333	92667	38	32
32	23	72883	27117	80223	19777	07341	92659	37	28
36	24	72902	27098	80251	19749	07349	92651	36	24
40	25	9.72922	10.27078	9.80279	10.19721	10.07357	9.92643	35	20
44	26	72942	27058	80307	19693	07365	92635	34	16
48	27	72962	27038	80335	19665	07373	92627	33	12
52	28	72982	27018	80363	19637	07381	92619	32	8
56	29	73002	26998	80391	19609	07389	92611	31	4
10	30	9.73022	10.26978	9.80419	10.19581	10.07397	9.92603	30	**50**
4	31	73041	26959	80447	19553	07405	92595	29	56
8	32	73061	26939	80474	19526	07413	92587	28	52
12	33	73081	26919	80502	19498	07421	92579	27	48
16	34	73101	26899	80530	19470	07429	92571	26	44
20	35	9.73121	10.26879	9.80558	10.19442	10.07437	9.92563	25	40
24	36	73140	26860	80586	19414	07445	92555	24	36
28	37	73160	26840	80614	19386	07454	92546	23	32
32	38	73180	26820	80642	19358	07462	92538	22	28
36	39	73200	26800	80669	19331	07470	92530	21	24
40	40	9.73219	10.26781	9.80697	10.19303	10.07478	9.92522	20	20
44	41	73239	26761	80725	19275	07486	92514	19	16
48	42	73259	26741	80753	19247	07494	92506	18	12
52	43	73278	26722	80781	19219	07502	92498	17	8
56	44	73298	26702	80808	19192	07510	92490	16	4
11	45	9.73318	10.26682	9.80836	10.19164	10.07518	9.92482	15	**49**
4	46	73337	26663	80864	19136	07527	92473	14	56
8	47	73357	26643	80892	19108	07535	92465	13	52
12	48	73377	26623	80919	19081	07543	92457	12	48
16	49	73396	26604	80947	19053	07551	92449	11	44
20	50	9.73416	10.26584	9.80975	10.19025	10.07559	9.92441	10	40
24	51	73435	26565	81003	18997	07567	92433	9	36
28	52	73455	26545	81030	18970	07575	92425	8	32
32	53	73474	26526	81058	18942	07584	92416	7	28
36	54	73494	26506	81086	18914	07592	92408	6	24
40	55	9.73513	10.26487	9.81113	10.18887	10.07600	9.92400	5	20
44	56	73533	26467	81141	18859	07608	92392	4	16
48	57	73552	26448	81169	18831	07616	92384	3	12
52	58	73572	26428	81196	18804	07624	92376	2	8
56	59	73591	26409	81224	18776	07633	92367	1	4
12	60	73611	26389	81252	18748	07641	92359	0	**48**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
8^h	122°							57°	3^h

2h	33°	Logarithms.						146°	9h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
12	0	9.73611	10.26389	9.81252	10.18748	10.07641	9.92359	60	**48**
4	1	73630	26370	81279	18721	07649	92351	59	56
8	2	73650	26350	81307	18693	07657	92343	58	52
12	3	73669	26331	81335	18665	07665	92335	57	48
16	4	73689	26311	81362	18638	07674	92326	56	44
20	5	9.73708	10.26292	9.81390	10.18610	10.07682	9.92318	55	40
24	6	73727	26273	81418	18582	07690	92310	54	36
28	7	73747	26253	81445	18555	07698	92302	53	32
32	8	73766	26234	81473	18527	07707	92293	52	28
36	9	73785	26215	81500	18500	07715	92285	51	24
40	10	9.73805	10.26195	9.81528	10.18472	10.07723	9.92277	50	20
44	11	73824	26176	81556	18444	07731	92269	49	16
48	12	73843	26157	81583	18417	07740	92260	48	12
52	13	73863	26137	81611	18389	07748	92252	47	8
56	14	73882	26118	81638	18362	07756	92244	46	4
13	15	9.73901	10.26099	9.81666	10.18334	10.07765	9.92235	45	**47**
4	16	73921	26079	81693	18307	07773	92227	44	56
8	17	73940	26060	81721	18279	07781	92219	43	52
12	18	73959	26041	81748	18252	07789	92211	42	48
16	19	73978	26022	81776	18224	07798	92202	41	44
20	20	9.73997	10.26003	9.81803	10.18197	10.07806	9.92194	40	40
24	21	74017	25983	81831	18169	07814	92186	39	36
28	22	74036	25964	81858	18142	07823	92177	38	32
32	23	74055	25945	81886	18114	07831	92169	37	28
36	24	74074	25926	81913	18087	07839	92161	36	24
40	25	9.74093	10.25907	9.81941	10.18059	10.07848	9.92152	35	20
44	26	74113	25887	81968	18032	07856	92144	34	16
48	27	74132	25868	81996	18004	07864	92136	33	12
52	28	74151	25849	82023	17977	07873	92127	32	8
56	29	74170	25830	82051	17949	07881	92119	31	4
14	30	9.74189	10.25811	9.82078	10.17922	10.07889	9.92111	30	**46**
4	31	74208	25792	82106	17894	07898	92102	29	56
8	32	74227	25773	82133	17867	07906	92094	28	52
12	33	74246	25754	82161	17839	07914	92086	27	48
16	34	74265	25735	82188	17812	07923	92077	26	44
20	35	9.74284	10.25716	9.82215	10.17785	10.07931	9.92069	25	40
24	36	74303	25697	82243	17757	07940	92060	24	36
28	37	74322	25678	82270	17730	07948	92052	23	32
32	38	74341	25659	82298	17702	07956	92044	22	28
36	39	74360	25640	82325	17675	07965	92035	21	24
40	40	9.74379	10.25621	9.82352	10.17648	10.07973	9.92027	20	20
44	41	74398	25602	82380	17620	07982	92018	19	16
48	42	74417	25583	82407	17593	07990	92010	18	12
52	43	74436	25564	82435	17565	07998	92002	17	8
56	44	74455	25545	82462	17538	08007	91993	16	4
15	45	9.74474	10.25526	9.82489	10.17511	10.08015	9.91985	15	**45**
4	46	74493	25507	82517	17483	08024	91976	14	56
8	47	74512	25488	82544	17456	08032	91968	13	52
12	48	74531	25469	82571	17429	08041	91959	12	48
16	49	74549	25451	82599	17401	08049	91951	11	44
20	50	9.74568	10.25432	9.82626	10.17374	10.08058	9.91942	10	40
24	51	74587	25413	82653	17347	08066	91934	9	36
28	52	74606	25394	82681	17319	08075	91925	8	32
32	53	74625	25375	82708	17292	08083	91917	7	28
36	54	74644	25356	82735	17265	08092	91908	6	24
40	55	9.74662	10.25338	9.82762	10.17238	10.08100	9.91900	5	20
44	56	74681	25319	82790	17210	08109	91891	4	16
48	57	74700	25300	82817	17183	08117	91883	3	12
52	58	74719	25281	82844	17156	08126	91874	2	8
56	59	74737	25263	82871	17129	08134	91866	1	4
16	60	74756	25244	82899	17101	08143	91857	0	**44**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
8h	123°							56°	3h

2ʰ	34°			Logarithms.			145°		9ʰ
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
16	0	9.74756	10.25244	9.82899	10.17101	10.08143	9.91857	60	**44**
4	1	74775	25225	82926	17074	08151	91849	59	56
8	2	74794	25206	82953	17047	08160	91840	58	52
12	3	74812	25188	82980	17020	08168	91832	57	48
16	4	74831	25169	83008	16992	08177	91823	56	44
20	5	9.74850	10.25150	9.83035	10.16965	10.08185	9.91815	55	40
24	6	74868	25132	83062	16938	08194	91806	54	36
28	7	74887	25113	83089	16911	08202	91798	53	32
32	8	74906	25094	83117	16883	08211	91789	52	28
36	9	74924	25076	83144	16856	08219	91781	51	24
40	10	9.74943	10.25057	9.83171	10.16829	10.08228	9.91772	50	20
44	11	74961	25039	83198	16802	08237	91763	49	16
48	12	74980	25020	83225	16775	08245	91755	48	12
52	13	74999	25001	83252	16748	08254	91746	47	8
56	14	75017	24983	83280	16720	08262	91738	46	4
17	15	9.75036	10.24964	9.83307	10.16693	10.08271	9.91729	45	**43**
4	16	75054	24946	83334	16666	08280	91720	44	56
8	17	75073	24927	83361	16639	08288	91712	43	52
12	18	75091	24909	83388	16612	08297	91703	42	48
16	19	75110	24896	83415	16585	08305	91695	41	44
20	20	9.75128	10.24872	9.83442	10.16558	10.08314	9.91686	40	40
24	21	75147	24853	83470	16530	08323	91677	39	36
28	22	75165	24835	83497	16503	08331	91669	38	32
32	23	75184	24816	83524	16476	08340	91660	37	28
36	24	75202	24798	83551	16449	08349	91651	36	24
40	25	9.75221	10.24779	9.83578	10.16422	10.08357	9.91643	35	20
44	26	75239	24761	83605	16395	08366	91634	34	16
48	27	75258	24742	83632	16368	08375	91625	33	12
52	28	75276	24724	83659	16341	08383	91617	32	8
56	29	75291	24706	83686	16314	08392	91608	31	4
18	30	9.75313	10.24687	9.83713	10.16287	10.08401	9.91599	30	**42**
4	31	75331	24669	83740	16260	08409	91591	29	56
8	32	75350	24650	83768	16232	08418	91582	28	52
12	33	75368	24632	83795	16205	08427	91573	27	48
16	34	75386	24614	83822	16178	08435	91565	26	44
20	35	9.75405	10.24595	9.83849	10.16151	10.08444	9.91556	25	40
24	36	75423	24577	83876	16124	08453	91547	24	36
28	37	75441	24559	83903	16097	08462	91538	23	32
32	38	75459	24541	83930	16070	08470	91530	22	28
36	39	75478	24522	83957	16043	08479	91521	21	24
40	40	9.75496	10.24504	9.83984	10.16016	10.08488	9.91512	20	20
44	41	75514	24486	84011	15989	08496	91504	19	16
48	42	75533	24467	84038	15962	08505	91495	18	12
52	43	75551	24449	84065	15935	08514	91486	17	8
56	44	75569	24431	84092	15908	08523	91477	16	4
19	45	9.75587	10.24413	9.84119	10.15881	10.08531	9.91469	15	**41**
4	46	75605	24395	84146	15854	08540	91460	14	56
8	47	75624	24376	84173	15827	08549	91451	13	52
12	48	75642	24358	84200	15800	08558	91442	12	48
16	49	75660	24340	84227	15773	08567	91433	11	44
20	50	9.75678	10.24322	9.84254	10.15746	10.08575	9.91425	10	40
24	51	75696	24304	84280	15720	08584	91416	9	36
28	52	75714	24286	84307	15693	08593	91407	8	32
32	53	75733	24267	84334	15666	08602	91398	7	28
36	54	75751	24249	84361	15639	08611	91389	6	24
40	55	9.75769	10.24231	9.84388	10.15612	10.08619	9.91381	5	20
44	56	75787	24213	84415	15585	08628	91372	4	16
48	57	75805	24195	84442	15558	08637	91363	3	12
52	58	75823	24177	84469	15531	08646	91354	2	8
56	59	75841	24159	84496	15504	08655	91345	1	4
20	60	75859	24141	84523	15477	08664	91336	0	**40**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
8ʰ	124°							55°	3ʰ

2h	35°			Logarithms.			144°		9h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
20	0	9.75859	10.24141	9.84523	10.15477	10.08664	9.91336	60	**40**
4	1	75877	24123	84550	15450	08672	91328	59	56
8	2	75895	24105	84576	15424	08681	91319	58	52
12	3	75913	24087	84603	15397	08690	91310	57	48
16	4	75931	24069	84630	15370	08699	91301	56	44
20	5	9.75949	10.24051	9.84657	10.15343	10.08708	9.91292	55	40
24	6	75967	24033	84684	15316	08717	91283	54	36
28	7	75985	24015	84711	15289	08726	91274	53	32
32	8	76003	23997	84738	15262	08734	91266	52	28
36	9	76021	23979	84764	15236	08743	91257	51	24
40	10	9.76039	10.23961	9.84791	10.15209	10.08752	9.91248	50	20
44	11	76057	23943	84818	15182	08761	91239	49	16
48	12	76075	23925	84845	15155	08770	91230	48	12
52	13	76093	23907	84872	15128	08779	91221	47	8
56	14	76111	23889	84899	15101	08788	91212	46	4
21	15	9.76129	10.23871	9.84925	10.15075	10.08797	9.91203	45	**39**
4	16	76146	23854	84952	15048	08806	91194	44	56
8	17	76164	23836	84979	15021	08815	91185	43	52
12	18	76182	23818	85006	14994	08824	91176	42	48
16	19	76200	23800	85033	14967	08833	91167	41	44
20	20	9.76218	10.23782	9.85059	10.14941	10.08842	9.91158	40	40
24	21	76236	23764	85086	14914	08851	91149	39	36
28	22	76253	23747	85113	14887	08859	91141	38	32
32	23	76271	23729	85140	14860	08868	91132	37	28
36	24	76289	23711	85166	14834	08877	91123	36	24
40	25	9.76307	10.23693	9.85193	10.14807	10.08886	9.91114	35	20
44	26	76324	23676	85220	14780	08895	91105	34	16
48	27	76342	23658	85247	14753	08904	91096	33	12
52	28	76360	23640	85273	14727	08913	91087	32	8
56	29	76378	23622	85300	14700	08922	91078	31	4
22	30	9.76395	10.23605	9.85327	10.14673	10.08931	9.91069	30	**38**
4	31	76413	23587	85354	14646	08940	91060	29	56
8	32	76431	23569	85380	14620	08949	91051	28	52
12	33	76448	23552	85407	14593	08958	91042	27	48
16	34	76466	23534	85434	14566	08967	91033	26	44
20	35	9.76484	10.23516	9.85460	10.14540	10.08977	9.91023	25	40
24	36	76501	23499	85487	14513	08986	91014	24	36
28	37	76519	23481	85514	14486	08995	91005	23	32
32	38	76537	23463	85540	14460	09004	90996	22	28
36	39	76554	23446	85567	14433	09013	90987	21	24
40	40	9.76572	10.23428	9.85594	10.14406	10.09022	9.90978	20	20
44	41	76590	23410	85620	14380	09031	90969	19	16
48	42	76607	23393	85647	14353	09040	90960	18	12
52	43	76625	23375	85674	14326	09049	90951	17	8
56	44	76642	23358	85700	14300	09058	90942	16	4
23	45	9.76660	10.23340	9.85727	10.14273	10.09067	9.90933	15	**37**
4	46	76677	23323	85754	14246	09076	90924	14	56
8	47	76695	23305	85780	14220	09085	90915	13	52
12	48	76712	23288	85807	14193	09094	90906	12	48
16	49	76730	23270	85834	14166	09104	90896	11	44
20	50	9.76747	10.23253	9.85860	10.14140	10.09113	9.90887	10	40
24	51	76765	23235	85887	14113	09122	90878	9	36
28	52	76782	23218	85913	14087	09131	90869	8	32
32	53	76800	23200	85940	14060	09140	90860	7	28
36	54	76817	23183	85967	14033	09149	90851	6	24
40	55	9.76835	10.23165	9.85993	10.14007	10.09158	9.90842	5	20
44	56	76852	23148	86020	13980	09168	90832	4	16
48	57	76870	23130	86046	13954	09177	90823	3	12
52	58	76887	23113	86073	13927	09186	90814	2	8
56	59	76904	23096	86100	13900	09195	90805	1	4
24	60	76922	23078	86126	13874	09204	90796	0	**36**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
8h	125°							54°	3h

2h	36°	Logarithms.						143°	9h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
24	0	9.76922	10.23078	9.86126	10.13874	10.09204	9.90796	60	**36**
4	1	76939	23061	86153	13847	09213	90787	59	56
8	2	76957	23043	86179	13821	09223	90777	58	52
12	3	76974	23026	86206	13794	09232	90768	57	48
16	4	76991	23009	86232	13768	09241	90759	56	44
20	5	9.77009	10.22991	9.86259	10.13741	10.09250	9.90750	55	40
24	6	77026	22974	86285	13715	09259	90741	54	36
28	7	77043	22957	86312	13688	09269	90731	53	32
32	8	77061	22939	86338	13662	09278	90722	52	28
36	9	77078	22922	86365	13635	09287	90713	51	24
40	10	9.77095	10.22905	9.86392	10.13608	10.09296	9.90704	50	20
44	11	77112	22888	86418	13582	09306	90694	49	16
48	12	77130	22870	86445	13555	09315	90685	48	12
52	13	77147	22853	86471	13529	09324	90676	47	8
56	14	77164	22836	86498	13502	09333	90667	46	4
25	15	9.77181	10.22819	9.86524	10.13476	10.09343	9.90657	45	**35**
4	16	77199	22801	86551	13449	09352	90648	44	56
8	17	77216	22784	86577	13423	09361	90639	43	52
12	18	77233	22767	86603	13397	09370	90630	42	48
16	19	77250	22750	86630	13370	09380	90620	41	44
20	20	9.77268	10.22732	9.86656	10.13344	10.09389	9.90611	40	40
24	21	77285	22715	86683	13317	09398	90602	39	36
28	22	77302	22698	86709	13291	09408	90592	38	32
32	23	77319	22681	86736	13264	09417	90583	37	28
36	24	77336	22664	86762	13238	09426	90574	36	24
40	25	9.77353	10.22647	9.86789	10.13211	10.09435	9.90565	35	20
44	26	77370	22630	86815	13185	09445	90555	34	16
48	27	77387	22613	86842	13158	09454	90546	33	12
52	28	77405	22595	86868	13132	09463	90537	32	8
56	29	77422	22578	86894	13106	09473	90527	31	4
26	30	9.77439	10.22561	9.86921	10.13079	10.09482	9.90518	30	**34**
4	31	77456	22544	86947	13053	09491	90509	29	56
8	32	77473	22527	86974	13026	09501	90499	28	52
12	33	77490	22510	87000	13000	09510	90490	27	48
16	34	77507	22493	87027	12973	09520	90480	26	44
20	35	9.77524	10.22476	9.87053	10.12947	10.09529	9.90471	25	40
24	36	77541	22459	87079	12921	09538	90462	24	36
28	37	77558	22442	87106	12894	09548	90452	23	32
32	38	77575	22425	87132	12868	09557	90443	22	28
36	39	77592	22408	87158	12842	09566	90434	21	24
40	40	9.77609	10.22391	9.87185	10.12815	10.09576	9.90424	20	20
44	41	77626	22374	87211	12789	09585	90415	19	16
48	42	77643	22357	87238	12762	09595	90405	18	12
52	43	77660	22340	87264	12736	09604	90396	17	8
56	44	77677	22323	87290	12710	09614	90386	16	4
27	45	9.77694	10.22306	9.87317	10.12683	10.09623	9.90377	15	**33**
4	46	77711	22289	87343	12657	09632	90368	14	56
8	47	77728	22272	87369	12631	09642	90358	13	52
12	48	77744	22256	87396	12604	09651	90349	12	48
16	49	77761	22239	87422	12578	09661	90339	11	44
20	50	9.77778	10.22222	9.87448	10.12552	10.09670	9.90330	10	40
24	51	77795	22205	87475	12525	09680	90320	9	36
28	52	77812	22188	87501	12499	09689	90311	8	32
32	53	77829	22171	87527	12473	09699	90301	7	28
36	54	77846	22154	87554	12446	09708	90292	6	24
40	55	9.77862	10.22138	9.87580	10.12420	10.09718	9.90282	5	20
44	56	77879	22121	87606	12394	09727	90273	4	16
48	57	77896	22104	87633	12367	09737	90263	3	12
52	58	77913	22087	87659	12341	09746	90254	2	8
56	59	77930	22070	87685	12315	09756	90244	1	4
28	60	77946	22054	87711	12289	09765	90235	0	**32**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
8h	126°							53°	3h

2h | 37° — Logarithms. — 142° | 9h

M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
28	0	9.77946	10.22054	9.87711	10.12289	10.09765	9.90235	60	**32**
4	1	77963	22037	87738	12262	09775	90225	59	56
8	2	77980	22020	87764	12236	09784	90216	58	52
12	3	77997	22003	87790	12210	09794	90206	57	48
16	4	78013	21987	87817	12183	09803	90197	56	44
20	5	9.78030	10.21970	9.87843	10.12157	10.09813	9.90187	55	40
24	6	78047	21953	87869	12131	09822	90178	54	36
28	7	78063	21937	87895	12105	09832	90168	53	32
32	8	78080	21920	87922	12078	09841	90159	52	28
36	9	78097	21903	87948	12052	09851	90149	51	24
40	10	9.78113	10.21887	9.87974	10.12026	10.09861	9.90139	50	20
44	11	78130	21870	88000	12000	09870	90130	49	16
48	12	78147	21853	88027	11973	09880	90120	48	12
52	13	78163	21837	88053	11947	09889	90111	47	8
56	14	78180	21820	88079	11921	09899	90101	46	4
29	15	9.78197	10.21803	9.88105	10.11895	10.09909	9.90091	45	**31**
4	16	78213	21787	88131	11869	09918	90082	44	56
8	17	78230	21770	88158	11842	09928	90072	43	52
12	18	78246	21754	88184	11816	09937	90063	42	48
16	19	78263	21737	88210	11790	09947	90053	41	44
20	20	9.78280	10.21720	9.88236	10.11764	10.09957	9.90043	40	40
24	21	78296	21704	88262	11738	09966	90034	39	36
28	22	78313	21687	88289	11711	09976	90024	38	32
32	23	78329	21671	88315	11685	09986	90014	37	28
36	24	78346	21654	88341	11659	09995	90005	36	24
40	25	9.78362	10.21638	9.88367	10.11633	10.10005	9.89995	35	20
44	26	78379	21621	88393	11607	10015	89985	34	16
48	27	78395	21605	88420	11580	10024	89976	33	12
52	28	78412	21588	88446	11554	10034	89966	32	8
56	29	78428	21572	88472	11528	10044	89956	31	4
30	30	9.78445	10.21555	9.88498	10.11502	10.10053	9.89947	30	**30**
4	31	78461	21539	88524	11476	10063	89937	29	56
8	32	78478	21522	88550	11450	10073	89927	28	52
12	33	78494	21506	88577	11423	10082	89918	27	48
16	34	78510	21490	88603	11397	10092	89908	26	44
20	35	9.78527	10.21473	9.88629	10.11371	10.10102	9.89898	25	40
24	36	78543	21457	88655	11345	10112	89888	24	36
28	37	78560	21440	88681	11319	10121	89879	23	32
32	38	78576	21424	88707	11293	10131	89869	22	28
36	39	78592	21408	88733	11267	10141	89859	21	24
40	40	9.78609	10.21391	9.88759	10.11241	10.10151	9.89849	20	20
44	41	78625	21375	88780	11214	10160	89840	19	16
48	42	78642	21358	88812	11188	10170	89830	18	12
52	43	78658	21342	88838	11162	10180	89820	17	8
56	44	78674	21326	88864	11136	10090	89810	16	4
31	45	9.78691	10.21309	9.88890	10.11110	10.10199	9.89801	15	**29**
4	46	78707	21293	88916	11084	10209	89791	14	56
8	47	78723	21277	88942	11058	10219	89781	13	52
12	48	78739	21261	88968	11032	10229	89771	12	48
16	49	78756	21244	88994	11006	10239	89761	11	44
20	50	9.78772	10.21228	9.89020	10.10980	10.10248	9.89752	10	40
24	51	78788	21212	89046	10954	10258	89742	9	36
28	52	78805	21195	89073	10927	10268	89732	8	32
32	53	78821	21179	89099	10901	10278	89722	7	28
36	54	78837	21163	89125	10875	10288	89712	6	24
40	55	9.78853	10.21147	9.89151	10.10849	10.10298	9.89702	5	20
44	56	78869	21131	89177	10823	10307	89693	4	16
48	57	78886	21114	89203	10797	10317	89683	3	12
52	58	78902	21098	89229	10771	10327	89673	2	8
56	59	78918	21082	89255	10745	10337	89663	1	4
32	60	78934	21066	89281	10719	10347	89653	0	**28**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.

8h | 127° — 52° | 3h

Logarithms.

2ʰ	38°							141°	9ʰ
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
32	0	9.78934	10.21066	9.89281	10.10719	10.10347	9.89653	60	**28**
4	1	78950	21050	89307	10693	10357	89643	59	56
8	2	78967	21033	89333	10667	10367	89633	58	52
12	3	78983	21017	89359	10641	10376	89624	57	48
16	4	78999	21001	89385	10615	10386	89614	56	44
20	5	9.79015	10.20985	9.89411	10.10589	10.10396	9.89604	55	40
24	6	79031	20969	89437	10563	10406	89594	54	36
28	7	79047	20953	89463	10537	10416	89584	53	32
32	8	79063	20937	89489	10511	10426	89574	52	28
36	9	79079	20921	89515	10485	10436	89564	51	24
40	10	9.79095	10.20905	9.89541	10.10459	10.10446	9.89554	50	20
44	11	79111	20889	89567	10433	10456	89544	49	16
48	12	79128	20872	89593	10407	10466	89534	48	12
52	13	79144	20856	89619	10381	10476	89524	47	8
56	14	79160	20840	89645	10355	10486	89514	46	4
33	15	9.79176	10.20824	9.89671	10.10329	10.10496	9.89504	45	**27**
4	16	79192	26808	89697	10303	10505	89495	44	56
8	17	79208	20792	89723	10277	10515	89485	43	52
12	18	79224	20776	89749	10251	10525	89475	42	48
16	19	79240	20760	89775	10225	10535	89465	41	44
20	20	9.79256	10.20744	9.89801	10.10199	10.10545	9.89455	40	40
24	21	79272	20728	89827	10173	10555	89445	39	36
28	22	79288	20712	89853	10147	10565	89435	38	32
32	23	79304	20696	89879	10121	10575	89425	37	28
36	24	79319	20681	89905	10095	10585	89415	36	24
40	25	9.79335	10.20665	9.89931	10.10069	10.10595	9.89405	35	20
44	26	79351	20649	89957	10043	10605	89395	34	16
48	27	79367	20633	89983	10017	10615	89385	33	12
52	28	79383	20617	90009	09991	10625	89375	32	8
56	29	79399	20601	90035	09965	10636	89364	31	4
34	30	9.79415	10.20585	9.90061	10.09939	10.10646	9.89354	30	**26**
4	31	79431	20569	90086	09914	10656	89344	29	56
8	32	79447	20553	90112	09888	10666	89334	28	52
12	33	79463	20537	90138	09862	10676	89324	27	48
16	34	79478	20522	90164	09836	10686	89314	26	44
20	35	9.79494	10.20506	9.90190	10.09810	10.10696	9.89304	25	40
24	36	79510	20490	90216	09784	10706	89294	24	36
28	37	79526	20474	90242	09758	10716	89284	23	32
32	38	79542	20458	90268	09732	10726	89274	22	28
36	39	79558	20442	90294	09706	10736	89264	21	24
40	40	9.79573	10.20427	9.90320	10.09680	10.10746	9.89254	20	20
44	41	79589	20411	90346	09654	10756	89244	19	16
48	42	79605	20395	90371	09629	10767	89233	18	12
52	43	79621	20379	90397	09603	10777	89223	17	8
56	44	79636	20364	90423	09577	10787	89213	16	4
35	45	9.79652	10.20348	9.90449	10.09551	10.10797	9.89203	15	**25**
4	46	79668	20332	90475	09525	10807	89193	14	56
8	47	79684	20316	90501	09499	10817	89183	13	52
12	48	79699	20301	90527	09473	10827	89173	12	48
16	49	79715	20285	90553	09447	10838	89162	11	44
20	50	9.79731	10.20269	9.90578	10.09422	10.10848	9.89152	10	40
24	51	79746	20254	90604	09396	10858	89142	9	36
28	52	79762	20238	90630	09370	10868	89132	8	32
32	53	79778	20222	90656	09344	10878	89122	7	28
36	54	79793	20207	90682	09318	10888	89112	6	24
40	55	9.79809	10.20191	9.90708	10.09292	10.10899	9.89101	5	20
44	56	79825	20175	90734	09266	10909	89091	4	16
48	57	79840	20160	90759	09241	10919	89081	3	12
52	58	79856	20144	90785	09215	10929	89071	2	8
56	59	79872	20128	90811	09189	10940	89060	1	4
36	60	79887	20113	90837	09163	10950	89050	0	**24**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
8ʰ	128°							51°	3ʰ

2h	39°			Logarithms.				140°	9h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
36	0	9.79887	10.20113	9.90837	10.09163	10.10950	9.89050	60	**24**
4	1	79903	20097	90863	09137	10960	89040	59	56
8	2	79918	20082	90889	09111	10970	89030	58	52
12	3	79934	20066	90914	09086	10980	89020	57	48
16	4	79950	20050	90940	09060	10991	89009	56	44
20	5	9.79965	10.20035	9.90966	10.09034	10.11001	9.88999	55	40
24	6	79981	20019	90992	09008	11011	88989	54	36
28	7	79996	20004	91018	08982	11022	88978	53	32
32	8	80012	19988	91043	08957	11032	88968	52	28
36	9	80027	19973	91069	08931	11042	88958	51	24
40	10	9.80043	10.19957	9.91095	10.08905	10.11052	9.88948	50	20
44	11	80058	19942	91121	08879	11063	88937	49	16
48	12	80074	19926	91147	08853	11073	88927	48	12
52	13	80089	19911	91172	08828	11083	88917	47	8
56	14	80105	19895	91198	08802	11094	88906	46	4
37	15	9.80120	10.19880	9.91224	10.08776	10.11104	9.88896	45	**23**
4	16	80136	19864	91250	08750	11114	88886	44	56
8	17	80151	19849	91276	08724	11125	88875	43	52
12	18	80166	19834	91301	08699	11135	88865	42	48
16	19	80182	19818	91327	08673	11145	88855	41	44
20	20	9.80197	10.19803	9.91353	10.08647	10.11156	9.88844	40	40
24	21	80213	19787	91379	08621	11166	88834	39	36
28	22	80228	19772	91404	08596	11176	88824	38	32
32	23	80244	19756	91430	08570	11187	88813	37	28
36	24	80259	19741	91456	08544	11197	88803	36	24
40	25	9.80274	10.19726	9.91482	10.08518	10.11207	9.88793	35	20
44	26	80290	19710	91507	08493	11218	88782	34	16
48	27	80305	19695	91533	08467	11228	88772	33	12
52	28	80320	19680	91559	08441	11239	88761	32	8
56	29	80336	19664	91585	08415	11249	88751	31	4
38	30	9.80351	10.19649	9.91610	10.08390	10.11259	9.88741	30	**22**
4	31	80366	19634	91636	08364	11270	88730	29	56
8	32	80382	19618	91662	08338	11280	88720	28	52
12	33	80397	19603	91688	08312	11291	88709	27	48
16	34	80412	19588	91713	08287	11301	88699	26	44
20	35	9.80428	10.19572	9.91739	10.08261	10.11312	9.88688	25	40
24	36	80443	19557	91765	08235	11322	88678	24	36
28	37	80458	19542	91791	08209	11332	88668	23	32
32	38	80473	19527	91816	08184	11343	88657	22	28
36	39	80489	19511	91842	08158	11353	88647	21	24
40	40	9.80504	10.19496	9.91868	10.08132	10.11364	9.88636	20	20
44	41	80519	19481	91893	08107	11374	88626	19	16
48	42	80534	19466	91919	08081	11385	88615	18	12
52	43	80550	19450	91945	08055	11395	88605	17	8
56	44	80565	19435	91971	08029	11406	88594	16	4
39	45	9.80580	10.19420	9.91996	10.08004	10.11416	9.88584	15	**21**
4	46	80595	19405	92022	07978	11427	88573	14	56
8	47	80610	19390	92048	07952	11437	88563	13	52
12	48	80625	19375	92073	07927	11448	88552	12	48
16	49	80641	19359	92099	07901	11458	88542	11	44
20	50	9.80656	10.19344	9.92125	10.07875	10.11469	9.88531	10	40
24	51	80671	19329	92150	07850	11479	88521	9	36
28	52	80686	19314	92176	07824	11490	88510	8	32
32	53	80701	19299	92202	07798	11501	88499	7	28
36	54	80716	19284	92227	07773	11511	88489	6	24
40	55	9.80731	10.19269	9.92253	10.07747	10.11522	9.88478	5	20
44	56	80746	19254	92279	07721	11532	88468	4	16
48	57	80762	19238	92304	07696	11543	88457	3	12
52	58	80777	19223	92330	07670	11553	88447	2	8
56	59	80792	19208	92356	07644	11564	88436	1	4
40	60	80807	19193	92381	07619	11575	88425	0	**20**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
8h	129°							50°	3h

2h	40°			Logarithms.			139°		9h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
40	0	9.80807	10.19193	9.92381	10.07619	10.11575	9.88425	60	**20**
4	1	80822	19178	92407	07593	11585	88415	59	56
8	2	80837	19163	92433	07567	11596	88404	58	52
12	3	80852	19148	92458	07542	11606	88394	57	48
16	4	80867	19133	92484	07516	11617	88383	56	44
20	5	9.80882	10.19118	9.92510	10.07490	10.11628	9.88372	55	40
24	6	80897	19103	92535	07465	11638	88362	54	36
28	7	80912	19088	92561	07439	11649	88351	53	32
32	8	80927	19073	92587	07413	11660	88340	52	28
36	9	80942	19058	92612	07388	11670	88330	51	24
40	10	9.80957	10.19043	9.92638	10.07362	10.11681	9.88319	50	20
44	11	80972	19028	92663	07337	11692	88308	49	16
48	12	80987	19013	92689	07311	11702	88298	48	12
52	13	81002	18998	92715	07285	11713	88287	47	8
56	14	81017	18983	92740	07260	11724	88276	46	4
41	15	9.81032	10.18968	9.92766	10.07234	10.11734	9.88266	45	**19**
4	16	81047	18953	92792	07208	11745	88255	44	56
8	17	81061	18939	92817	07183	11756	88244	43	52
12	18	81076	18924	92843	07157	11766	88234	42	48
16	19	81091	18909	92868	07132	11777	88223	41	44
20	20	9.81106	10.18894	9.92894	10.07106	10.11788	9.88212	40	40
24	21	81121	18879	92920	07080	11799	88201	39	36
28	22	81136	18864	92945	07055	11809	88191	38	32
32	23	81151	18849	92971	07029	11820	88180	37	28
36	24	81166	18834	92996	07004	11831	88169	36	24
40	25	9.81180	10.18820	9.93022	10.06978	10.11842	9.88158	35	20
44	26	81195	18805	93048	06952	11852	88148	34	16
48	27	81210	18790	93073	06927	11863	88137	33	12
52	28	81225	18775	93099	06901	11874	88126	32	8
56	29	81240	18760	93124	06876	11885	88115	31	4
42	30	9.81254	10.18746	9.93150	10.06850	10.11895	9.88105	30	**18**
4	31	81269	18731	93175	06825	11906	88094	29	56
8	32	81284	18716	93201	06799	11917	88083	28	52
12	33	81299	18701	93227	06773	11928	88072	27	48
16	34	81314	18686	93252	06748	11939	88061	26	44
20	35	9.81328	10.18672	9.93278	10.06722	10.11949	9.88051	25	40
24	36	81343	18657	93303	06697	11960	88040	24	36
28	37	81358	18642	93329	06671	11971	88029	23	32
32	38	81372	18628	93354	06646	11982	88018	22	28
36	39	81387	18613	93380	06620	11993	88007	21	24
40	40	9.81402	10.18598	9.93406	10.06594	10.12004	9.87996	20	20
44	41	81417	18583	93431	06569	12015	87985	19	16
48	42	81431	18569	93457	06543	12025	87975	18	12
52	43	81446	18554	93482	06518	12036	87964	17	8
56	44	81461	18539	93508	06492	12047	87953	16	4
43	45	9.81475	10.18525	9.93533	10.06467	10.12058	9.87942	15	**17**
4	46	81490	18510	93559	06441	12069	87931	14	56
8	47	81505	18495	93584	06416	12080	87920	13	52
12	48	81519	18481	93610	06390	12091	87909	12	48
16	49	81534	18466	93636	06364	12102	87898	11	44
20	50	9.81549	10.18451	9.93661	10.06339	10.12113	9.87887	10	40
24	51	81563	18437	93687	06313	12123	87877	9	36
28	52	81578	18422	93712	06288	12134	87866	8	32
32	53	81592	18408	93738	06262	12145	87855	7	28
36	54	81607	18393	93763	06237	12156	87844	6	24
40	55	9.81622	10.18378	9.93789	10.06211	10.12167	9.87833	5	20
44	56	81636	18364	93814	06186	12178	87822	4	16
48	57	81651	18349	93840	06160	12189	87811	3	12
52	58	81665	18335	93865	06135	12200	87800	2	8
56	59	81680	18320	93891	06109	12211	87789	1	4
44	60	81694	18306	93916	06084	12222	87778	0	**16**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
8h	130°							49°	3h

2h 41° **Logarithms.** 138° 9h

M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
44	0	9.81694	10.18306	9.93916	10.06084	10.12222	9.87778	60	**16**
4	1	81709	18291	93942	06058	12233	87767	59	56
8	2	81723	18277	93967	06033	12244	87756	58	52
12	3	81738	18262	93993	06007	12255	87745	57	48
16	4	81752	18248	94018	05982	12266	87734	56	44
20	5	9.81767	10.18233	9.94044	10.05956	10.12277	9.87723	55	40
24	6	81781	18219	94069	05931	12288	87712	54	36
28	7	81796	18204	94095	05905	12299	87701	53	32
32	8	81810	18190	94120	05880	12310	87690	52	28
36	9	81825	18175	94146	05854	12321	87679	51	24
40	10	9.81839	10.18161	9.94171	10.05829	10.12332	9.87668	50	20
44	11	81854	18146	94197	05803	12343	87657	49	16
48	12	81868	18132	94222	05778	12354	87646	48	12
52	13	81882	18118	94248	05572	12365	87635	47	8
56	14	81897	18103	94273	05727	12376	87624	46	4
45	15	9.81911	10.18089	9.94299	10.05701	10.12387	9.87613	45	**15**
4	16	81926	18074	94324	05676	12399	87601	44	56
8	17	81940	18060	94350	05650	12410	87590	43	52
12	18	81955	18045	94375	05625	12421	87579	42	48
16	19	81969	18031	94401	05599	12432	87568	41	44
20	20	9.81983	10.18017	9.94426	10.05574	10.12443	9.87557	40	40
24	21	81998	18002	94452	05548	12454	87546	39	36
28	22	82012	17988	94477	05523	12465	87535	38	32
32	23	82026	17974	94503	05497	12476	87524	37	28
36	24	82041	17959	94528	05472	12487	87513	36	24
40	25	9.82055	10.17945	9.94554	10.05446	10.12499	9.87501	35	20
44	26	82069	17931	94579	05421	12510	87490	34	16
48	27	82084	17916	94604	05396	12521	87479	33	12
52	28	82098	17902	94630	05370	12532	87468	32	8
56	29	82112	17888	94655	05345	12543	87457	31	4
46	30	9.82126	10.17874	9.94681	10.05319	10.12554	9.87446	30	**14**
4	31	82141	17859	94706	05294	12566	87434	29	56
8	32	82155	17845	94732	05268	12577	87423	28	52
12	33	82169	17831	94757	05243	12588	87412	27	48
16	34	82184	17816	94783	05217	12599	87401	26	44
20	35	9.82198	10.17802	9.94808	10.05192	10.12610	9.87390	25	40
24	36	82212	17788	94834	05166	12622	87378	24	36
28	37	82226	17774	94859	05141	12633	87367	23	32
32	38	82240	17760	94884	05116	12644	87356	22	28
36	39	82255	17745	94910	05090	12655	87345	21	24
40	40	9.82269	10.17731	9.94935	10.05065	10.12666	9.87334	20	20
44	41	82283	17717	94961	05039	12678	87322	19	16
48	42	82297	17703	94986	05014	12689	87311	18	12
52	43	82311	17689	95012	04988	12700	87300	17	8
56	44	82326	17674	95037	04963	12712	87288	16	4
47	45	9.82340	10.17660	9.95062	10.04938	10.12723	9.87277	15	**13**
4	46	82354	17646	95088	04912	12734	87266	14	56
8	47	82368	17632	95113	04887	12745	87255	13	52
12	48	82382	17618	95139	04861	12757	87243	12	48
16	49	82396	17604	95164	04836	12768	87232	11	44
20	50	9.82410	10.17590	9.95190	10.04810	10.12779	9.87221	10	40
24	51	82424	17576	95215	04785	12791	87209	9	36
28	52	82439	17561	95240	04760	12802	87198	8	32
32	53	82453	17547	95266	04734	12813	87187	7	28
36	54	82467	17533	95291	04709	12825	87175	6	24
40	55	9.82481	10.17519	9.95317	10.04683	10.12836	9.87164	5	20
44	56	82495	17505	95342	04658	12847	87153	4	16
48	57	82509	17491	95368	04632	12859	87141	3	12
52	58	82523	17477	95393	04607	12870	87130	2	8
56	59	82537	17463	95418	04582	12881	87119	1	4
48	60	82551	17449	95444	04556	12893	87107	0	**12**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.

8h 131° 48° 3h

2h	42°			Logarithms.				137°	9h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
48	0	9.82551	10.17449	9.95444	10.04556	10.12893	9.87107	60	**12**
4	1	82565	17435	95469	04531	12904	87096	59	56
8	2	82579	17421	95495	04505	12915	87085	58	52
12	3	82593	17407	95520	04480	12927	87073	57	48
16	4	82607	17393	95545	04455	12938	87062	56	44
20	5	9.82621	10.17379	9.95571	10.04429	10.12950	9.87050	55	40
24	6	82635	17365	95596	04404	12961	87039	54	36
28	7	82649	17351	95622	04378	12972	87028	53	32
32	8	82663	17337	95647	04353	12984	87016	52	28
36	9	82677	17323	95672	04328	12995	87005	51	24
40	10	9.82691	10.17309	9.95698	10.04302	10.13007	9.86993	50	20
44	11	82705	17295	95723	04277	13018	86982	49	16
48	12	82719	17281	95748	04252	13030	86970	48	12
52	13	82733	17267	95774	04226	13041	86959	47	8
56	14	82747	17253	95799	04201	13053	86947	46	4
49	15	9.82761	10.17239	9.95825	10.04175	10.13064	9.86936	45	**11**
4	16	82775	17225	95850	04150	13076	86924	44	56
8	17	82788	17212	95875	04125	13087	86913	43	52
12	18	82802	17198	95901	04099	13098	86902	42	48
16	19	82816	17184	95926	04074	13110	86890	41	44
20	20	9.82830	10.17170	9.95952	10.04048	10.13121	9.86879	40	40
24	21	82844	17156	95977	04023	13133	86867	39	36
28	22	82858	17142	96002	03998	13145	86855	38	32
32	23	82872	17128	96028	03972	13156	86844	37	28
36	24	82885	17115	96053	03947	13168	86832	36	24
40	25	9.82899	10.17101	9.96078	10.03922	10.13179	9.86821	35	20
44	26	82913	17087	96104	03896	13191	86809	34	16
48	27	82927	17073	96129	03871	13202	86798	33	12
52	28	82941	17059	96155	03845	13214	86786	32	8
56	29	82955	17045	96180	03820	13225	86775	31	4
50	30	9.82968	10.17032	9.96205	10.03795	10.13237	9.86763	30	**10**
4	31	82982	17018	96231	03769	13248	86752	29	56
8	32	82996	17004	96256	03744	13260	86740	28	52
12	33	83010	16990	96281	03719	13272	86728	27	48
16	34	83023	16977	96307	03693	13283	86717	26	44
20	35	9.83037	10.16963	9.96332	10.03668	10.13295	9.86705	25	40
24	36	83051	16949	96357	03643	13306	86694	24	36
28	37	83065	16935	96383	03617	13318	86682	23	32
32	38	83078	16922	96408	03592	13330	86670	22	28
36	39	83092	16908	96433	03567	13341	86659	21	24
40	40	9.83106	10.16894	9.96459	10.03541	10.13353	9.86647	20	20
44	41	83120	16880	96484	03516	13365	86635	19	16
48	42	83133	16867	96510	03490	13376	86624	18	12
52	43	83147	16853	96535	03465	13388	86612	17	8
56	44	83161	16839	96560	03440	13400	86600	16	4
51	45	9.83174	10.16826	9.96586	10.03414	10.13411	9.86589	15	**9**
4	46	83188	16812	96611	03389	13423	86577	14	56
8	47	83202	16798	96636	03364	13435	86565	13	52
12	48	83215	16785	96662	03338	13446	86554	12	48
16	49	83229	16771	96687	03313	13458	86542	11	44
20	50	9.83242	10.16758	9.96712	10.03288	10.13470	9.86530	10	40
24	51	83256	16744	96738	03262	13482	86518	9	36
28	52	83270	16730	96763	03237	13493	86507	8	32
32	53	83283	16717	96788	03212	13505	86495	7	28
36	54	83297	16703	96814	03186	13517	86483	6	24
40	55	9.83310	10.16690	9.96839	10.03161	10.13528	9.86472	5	20
44	56	83324	16676	96864	03136	13540	86460	4	16
48	57	83338	16662	96890	03110	13552	86448	3	12
52	58	83351	16649	96915	03085	13564	86436	2	8
56	59	83365	16635	96940	03060	13575	86425	1	4
52	60	83378	16622	96966	03034	13587	86413	0	**8**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
8h	132°							47°	3h

2h	43°		Logarithms.				136°		9h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
52	0	9.83378	10.16622	9.96966	10.03034	10.13587	9.86413	60	**8**
4	1	83392	16608	96991	03009	13599	86401	59	56
8	2	83405	16595	97016	02984	13611	86389	58	52
12	3	83419	16581	97042	02958	13623	86377	57	48
16	4	83432	16568	97067	02933	13634	86366	56	44
20	5	9.83446	10.16554	9.97092	10.02908	10.13646	9.86354	55	40
24	6	83459	16541	97118	02882	13658	86342	54	36
28	7	83473	16527	97143	02857	13670	86330	53	32
32	8	83486	16514	97168	02832	13682	86318	52	28
36	9	83500	16500	97193	02807	13694	86306	51	24
40	10	9.83513	10.16487	9.97219	10.02781	10.13705	9.86295	50	20
44	11	83527	16473	97244	02756	13717	86283	49	16
48	12	83540	16460	97269	02731	13729	86271	48	12
52	13	83554	16446	97295	02705	13741	86259	47	8
56	14	83567	16433	97320	02680	13753	86247	46	4
53	15	9.83581	10.16419	9.97345	10.02655	10.13765	9.86235	45	**7**
4	16	83594	16406	97371	02629	13777	86223	44	56
8	17	83608	16392	97396	02604	13789	86211	43	52
12	18	83621	16379	97421	02579	13800	86200	42	48
16	19	83634	16366	97447	02553	13812	86188	41	44
20	20	9.83648	10.16352	9.97472	10.02528	10.13824	9.86176	40	40
24	21	83661	16339	97497	02503	13836	86164	39	36
28	22	83674	16326	97523	02477	13848	86152	38	32
32	23	83688	16312	97548	02452	13860	86140	37	28
36	24	83701	16299	97573	02427	13872	86128	36	24
40	25	9.83715	10.16285	9.97598	10.02402	10.13884	9.86116	35	20
44	26	83728	16272	97624	02376	13896	86104	34	16
48	27	83741	16259	97649	02351	13908	86092	33	12
52	28	83755	16245	97674	02326	13920	86080	32	8
56	29	83768	16232	97700	02300	13932	86068	31	4
54	30	9.83781	10.16219	9.97725	10.02275	10.13944	9.86056	30	**6**
4	31	83795	16205	97750	02250	13956	86044	29	56
8	32	83808	16192	97776	02224	13968	86032	28	52
12	33	83821	16179	97801	02199	13980	86020	27	48
16	34	83834	16166	97826	02174	13992	86008	26	44
20	35	9.83848	10.16152	9.97851	10.02149	10.14004	9.85996	25	40
24	36	83861	16139	97877	02123	14016	85984	24	36
28	37	83874	16126	97902	02098	14028	85972	23	32
32	38	83887	16113	97927	02073	14040	85960	22	28
36	39	83901	16099	97953	02047	14052	85948	21	24
40	40	9.83914	10.16086	9.97978	10.02022	10.14064	9.85936	20	20
44	41	83927	16073	98003	01997	14076	85924	19	16
48	42	83940	16060	98029	01971	14088	85912	18	12
52	43	83954	16046	98054	01946	14100	85900	17	8
56	44	83967	16033	98079	01921	14112	85888	16	4
55	45	9.83980	10.16020	9.98104	10.01896	10.14124	9.85876	15	**5**
4	46	83993	16007	98130	01870	14136	85864	14	56
8	47	84006	15994	98155	01845	14149	85851	13	52
12	48	84020	15980	98180	01820	14161	85839	12	48
16	49	84033	15967	98206	01794	14173	85827	11	44
20	50	9.84046	10.15954	9.98231	10.01769	10.14185	9.85815	10	40
24	51	84059	15941	98256	01744	14197	85803	9	36
28	52	84072	15928	98281	01719	14209	85791	8	32
32	53	84085	15915	98307	01693	14221	85779	7	28
36	54	84098	15902	98332	01668	14234	85766	6	24
40	55	9.84112	10.15888	9.98357	10.01643	10.14246	9.85754	5	20
44	56	84125	15875	98383	01617	14258	85742	4	16
48	57	84138	15862	98408	01592	14270	85730	3	12
52	58	84151	15849	98433	01567	14282	85718	2	8
56	59	84164	15836	98458	01542	14294	85706	1	4
56	60	84177	15823	98484	01516	14307	85693	0	**4**
M.S.	M	Cosine.	Secant.	Cotangent	Tangent.	Cosecant.	Sine.	M	M.S.
8h	133°							46°	3h

2h	44°			Logarithms.				135°	9h
M.S.	M	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M	M.S.
56	0	9.84177	10.15823	9.98484	10.01516	10.14307	9.85693	60	**4**
4	1	84190	15810	98509	01491	14319	85681	59	56
8	2	84203	15797	98534	01466	14331	85669	58	52
12	3	84216	15784	98560	01440	14343	85657	57	48
16	4	84229	15771	98585	01415	14355	85645	56	44
20	5	9.84242	10.15758	9.98610	10.01390	10.14368	9.85632	55	40
24	6	84255	15745	98635	01365	14380	85620	54	36
28	7	84269	15731	98661	01339	14392	85608	53	32
32	8	84282	15718	98686	01314	14404	85596	52	28
36	9	84295	15705	98711	01289	14417	85583	51	24
40	10	9.84308	10.15692	9.98737	10.01263	10.14429	9.85571	50	20
44	11	84321	15679	98762	01238	14441	85559	49	16
48	12	84334	15666	98787	01213	14453	85547	48	12
52	13	84347	15653	98812	01188	14466	85534	47	8
56	14	84360	15640	98838	01162	14478	85522	46	4
57	15	9.84373	10.15627	9.98863	10.01137	10.14490	9.85510	45	**3**
4	16	84385	15615	98888	01112	14503	85497	44	56
8	17	84398	15602	98913	01087	14515	85485	43	52
12	18	84411	15589	98939	01061	14527	85473	42	48
16	19	84424	15576	98964	01036	14540	85460	41	44
20	20	9.84437	10.15563	9.98989	10.01011	10.14552	9.85448	40	40
24	21	84450	15550	99015	00985	14564	85436	39	36
28	22	84463	15537	99040	00960	14577	85423	38	32
32	23	84476	15524	99065	00935	14589	85411	37	28
36	24	84489	15511	99090	00910	14601	85399	36	24
40	25	9.84502	10.15498	9.99116	10.00884	10.14614	9.85386	35	20
44	26	84515	15485	99141	00859	14626	85374	34	16
48	27	84528	15472	99166	00834	14639	85361	33	12
52	28	84540	15460	99191	00809	14651	85349	32	8
56	29	84553	15447	99217	00783	14663	85337	31	4
58	30	9.84566	10.15434	9.99242	10.00758	10.14676	9.85324	30	**2**
4	31	84579	15421	99267	00733	14688	85312	29	56
8	32	84592	15408	99293	00707	14701	85299	28	52
12	33	84605	15395	99318	00682	14713	85287	27	48
16	34	84618	15382	99343	00657	14726	85274	26	44
20	35	9.84630	10.15370	9.99368	10.00632	10.14738	9.85262	25	40
24	36	84643	15357	99394	00606	14750	85250	24	36
28	37	84656	15344	99419	00581	14763	85237	23	32
32	38	84669	15331	99444	00556	14775	85225	22	28
36	39	84682	15318	99469	00531	14788	85212	21	24
40	40	9.84694	10.15306	9.99495	10.00505	10 14800	9.85200	20	20
44	41	84707	15293	99520	00480	14813	85187	19	16
48	42	84720	15280	99545	00455	14825	85175	18	12
52	43	84733	15267	99570	00430	14838	85162	17	8
56	44	84745	15255	99596	00404	14850	85150	16	4
59	45	9.84758	10.15242	9.99621	10.00379	10.14863	9 85137	15	**1**
4	46	84771	15229	99646	00354	14875	85125	14	56
8	47	84784	15216	99672	00328	14888	85112	13	52
12	48	84796	15204	99697	00303	14900	85100	12	48
16	49	84809	15191	99722	00278	14913	85087	11	44
20	50	9.84822	10.15178	9.99747	10.00253	10.14926	9.85074	10	40
24	51	84835	15165	99773	00227	14938	85062	9	36
28	52	84847	15153	99798	00202	14951	85049	8	32
32	53	84860	15140	99823	00177	14963	85037	7	28
36	54	84873	15127	99848	00152	14976	85024	6	24
40	55	9.84885	10.15115	9.99874	10.00126	10.14988	9.85012	5	20
44	56	84898	15102	99899	00101	15001	84999	4	16
48	57	84911	15089	99924	00076	15014	84986	3	12
52	58	84923	15077	99949	00051	15026	84974	2	8
56	59	84936	15064	99975	00025	15039	84961	1	4
60	60	84949	15051	10.00000	00000	15051	84949	0	**0**
M.S.	M	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M	M.S.
8h	134°							45°	3h

EXPLANATION OF THE TABLES.

The outer columns in the trigonometrical tables contain the angle in time of hours, minutes and seconds, corresponding to the same angle in degrees and minutes in the next columns. The hour is noted at the top and bottom, the minutes in black, and the seconds in ordinary figures.

To find the Logarithm and Natural Line for Seconds exceeding Minutes of a Degree.

Example 1. Find the logarithm for sin. 38° 47′ 55″.

From table, { log. sin. 38° 48′ = 9.79699 ; " " 38° 47′ = 9.79684 } diff. 15.
Correction, 15 × 55 : 60 = + 14 nearly.
The required log. sin. 38° 47′ 55″ = 9.79698

In practice, the difference is subtracted direct from the tables.

Example 2. Find the natural cos. 43° 29′ 19″.

From table, cos. 43° 29′ = 0.72557
Correction, 20 × 19 : 60 = — 6 nearly.
The required cos. 43° 29′ 19″ = 0.72551

The correction is added when the function is increasing, and subtracted when decreasing.

To find the Angle corresponding to a given Logarithm or Natural Line.

Example 3. Log. sin. = 9.56429. Required the angle.

From table, { log. sin. 21° 31′ = 9.56440 ; " " 21° 30′ = 9.56408 } diff. 32.
The angle required, " " 21° 30′ 29″ = 9.56429 } " 21.
Correction, 21 × 60 : 32 = 29 seconds nearly.

Example 4. Cosine = 0.35254. Required the angle.

From table, { cos. 69° 22′ = 0.35239 ; " 69° 21′ = 0.35266 } diff. 27.
The required angle, " 69° 21′ 27″ = 0.35254 } " 12.
Correction, 12 × 60 : 27 = 27 seconds, nearly.

Conversion of Minutes and Seconds into Decimals of a Degree or of an Hour.

M.	Decimal.	M.	Decimal.	M.	Decimal.	S.	Decimal.	S.	Decimal.	S.	Decimal.
1	.016666	21	.350000	41	.683333	1	.000277	21	.005833	41	.011388
2	.033333	22	.366666	42	.700000	2	.000555	22	.006111	42	.011666
3	.050000	23	.383333	43	.716666	3	.000833	23	.006388	43	.011944
4	.066666	24	.400000	44	.733333	4	.001111	24	.006666	44	.012222
5	.083333	25	.416666	45	.750000	5	.001388	25	.006944	45	.012500
6	.100000	26	.433333	46	.766666	6	.001666	26	.007222	46	.012777
7	.116666	27	.450000	47	.783333	7	.001944	27	.007500	47	.013055
8	.133333	28	.466666	48	.800000	8	.002222	28	.007777	48	.013333
9	.150000	29	.483333	49	.816666	9	.002500	29	.008055	49	.013611
10	.166666	30	.500000	50	833333	10	.002777	30	.008333	50	.013888
11	.183333	31	.516000	51	.850000	11	.003055	31	.008611	51	.014166
12	.200000	32	.533333	52	.866666	12	.003333	32	.008888	52	.014444
13	.216666	33	.550000	53	.883333	13	.003611	33	.009166	53	.014722
14	.233333	34	.566666	54	.900000	14	.003888	34	.009444	54	.015000
15	.250000	35	.583333	55	.916666	15	.004166	35	.009722	55	.015277
16	.266666	36	.600000	56	.933333	16	.004444	36	.010000	56	.015555
17	.283333	37	.616666	57	.950000	17	.004722	37	.010277	57	.015833
18	.300000	38	.633333	58	.966666	18	.005000	38	.010555	58	.016111
19	.316666	39	.650000	59	.983333	19	.005277	39	.010833	59	.016388
20	.333333	40	.666666	60	1.000000	20	.005555	40	.011111	60	.016666

0h	0°	Natural Trigonometrical Functions.							179°	11h
M.S.	M.	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin.	Cosine.	M.	M.S.
0	0	.00000	1.0000	Infinite	.00000	Infinite	1.0000	.00000	1.0000	60	**60**
4	1	. 0029	.99971	3437.7	. 0029	3437.7	.0000	. 0000	.0000	59	56
8	2	. 0058	. 9942	1718.9	. 0058	1718.9	.0000	. 0000	.0000	58	52
12	3	. 0087	. 9913	1145.9	. 0087	1145.9	.0000	. 0000	.0000	57	48
16	4	. 0116	. 9884	859.44	. 0116	859.44	.0000	. 0000	.0000	56	44
20	5	.00145	.99854	687.55	.00145	687.55	1.0000	.00000	1.0000	55	40
24	6	. 0174	. 9825	572.96	. 0174	572.96	.0000	. 0000	.0000	54	36
28	7	. 0204	. 9796	491.11	. 0204	491.11	.0000	. 0000	.0000	53	32
32	8	. 0233	. 9767	429.72	. 0233	429.72	.0000	. 0000	.0000	52	28
36	9	. 0262	. 9738	381.97	. 0262	381.97	.0000	. 0000	.0000	51	24
40	10	.00291	.99709	343.77	.00291	343.77	1.0000	.00000	.99999	50	20
44	11	. 0320	. 9680	312.52	. 0320	312.52	.0000	. 0000	. 9999	49	16
48	12	. 0349	. 9651	286.48	. 0349	286.48	.0000	. 0001	. 9999	48	12
52	13	. 0378	. 9622	64.44	. 0378	64.44	.0000	. 0001	. 9999	47	8
56	14	. 0407	. 9593	45.55	. 0407	45.55	.0000	. 0001	. 9999	46	4
1	15	.00436	.99564	229.18	.00436	229.18	1.0000	.00001	.99999	45	**59**
4	16	. 0465	. 9534	14.86	. 0465	14.86	.0000	. 0001	. 9999	44	56
8	17	. 0494	. 9505	02.22	. 0494	02.22	.0000	. 0001	. 9999	43	52
12	18	. 0524	. 9476	190.99	. 0524	190.98	.0000	. 0001	. 9999	42	48
16	19	. 0553	. 9447	180.93	. 0553	180.93	.0000	. 0001	. 9998	41	44
20	20	.00582	.99418	171.89	.00582	171.88	1.0000	.00002	.99998	40	40
24	21	. 0611	. 9389	63.70	. 0611	63.70	.0000	. 0002	. 9998	39	36
28	22	. 0640	. 9360	56.26	. 0640	56.26	.0000	. 0002	. 9998	38	32
32	23	. 0669	. 9331	49.47	. 0669	49.46	.0000	. 0002	. 9998	37	28
36	24	. 0698	. 9302	43.24	. 0698	43.24	.0000	. 0002	. 9997	36	24
40	25	.00727	.99273	137.51	.00727	137.51	1.0000	.00003	.99997	35	20
44	26	. 0766	. 9244	32.22	. 0756	32.22	.0000	. 0003	. 9997	34	16
48	27	. 0785	. 9215	27.32	. 0785	27.32	.0000	. 0003	. 9997	33	12
52	28	. 0814	. 9185	22.78	. 0814	22.77	.0000	. 0003	. 9997	32	8
56	29	. 0843	. 9156	18.54	. 0844	18.54	.0000	. 0003	. 9996	31	4
2	30	.00873	.99127	114.59	.00873	114.59	1.0000	.00004	.99996	30	**58**
4	31	. 0902	. 9098	10.90	. 0902	10.89	.0000	. 0004	. 9996	29	56
8	32	. 0931	. 9069	07.43	. 0931	07.43	.0000	. 0004	. 9996	28	52
12	33	. 0960	. 9040	04.17	. 0960	04.17	.0000	. 0005	. 9995	27	48
16	34	. 0989	. 9011	01.11	. 0989	01.11	.0000	. 0005	. 9995	26	44
20	35	.01018	.98982	98.223	.01018	98.218	1.0000	.00005	.99995	25	40
24	36	. 1047	. 8953	5.495	. 1047	5.489	.0000	. 0005	. 9994	24	36
28	37	. 1076	. 8924	2.914	. 1076	2.908	.0000	. 0006	. 9994	23	32
32	38	. 1105	. 8895	0.469	. 1105	0.463	.0001	. 0006	. 9994	22	28
36	39	. 1134	. 8865	88.149	. 1134	88.143	.0001	. 0006	. 9993	21	24
40	40	.01163	.98836	85.946	.01164	85.940	1.0001	.00007	.79993	20	20
44	41	. 1193	. 8807	3.849	. 1193	3.843	.0001	. 0007	. 9993	19	16
48	42	. 1222	. 8778	1.853	. 1222	1.847	.0001	. 0007	. 9992	18	12
52	43	. 1251	. 8749	79.950	. 1251	79.943	.0001	. 0008	. 9992	17	8
56	44	. 1280	. 8720	78.133	. 1280	78.126	.0001	. 0008	. 9992	16	4
3	45	.01309	.98691	76.396	.01309	76.390	1.0001	.00008	.99991	15	**57**
4	46	. 1338	. 8662	4.736	. 1338	4.729	.0001	. 0009	. 9991	14	56
8	47	. 1367	. 8633	3.146	. 1367	3.139	.0001	. 0009	. 9991	13	52
12	48	. 1396	. 8604	1.622	. 1396	1.615	.0001	. 0010	. 9990	12	48
16	49	. 1425	. 8575	0.160	. 1425	0.153	.0001	. 0010	. 9990	11	44
20	50	.01454	.98546	68.757	.01454	68.750	1.0001	.00010	.99989	10	40
24	51	. 1483	. 8516	7.409	. 1484	7.402	.0001	. 0011	. 9989	9	36
28	52	. 1512	. 8487	6.113	. 1513	6.105	.0001	. 0011	. 9988	8	32
32	53	. 1542	. 8458	4.866	. 1542	4.858	.0001	. 0012	. 9988	7	28
36	54	. 1571	. 8429	3.664	. 1571	3.657	.0001	. 0012	. 9988	6	24
40	55	.01600	.98400	62.507	.01600	62.499	1.0001	.00013	.99987	5	20
44	56	. 1629	. 8371	1.391	. 1629	1.383	.0001	. 0013	. 9987	4	16
48	57	. 1658	. 8342	0.314	. 1658	0.306	.0001	. 0014	. 9987	3	12
52	58	. 1687	. 8313	59.274	. 1687	59.266	.0001	. 0014	. 9986	2	8
56	59	. 1716	. 8284	8.270	. 1716	8.261	.0001	. 0015	. 9985	1	4
4	60	. 1745	. 8255	7.299	. 1745	7.290	.0001	. 0015	. 9985	0	**56**
M.S.	M.	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos.	Sine.	M.	M.S.
6h	90°	**Natural.**								89°	5h

0ʰ	1°	Natural Trigonometrical Functions.							178°		11ʰ
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
4	0	.01745	.98255	57.299	.01745	57.290	1.0001	.00015	.99985	60	**56**
4	1	.1774	.8226	56.359	.1775	56.350	.0001	.0016	.9984	59	56
8	2	.1803	.8196	55.450	.1804	55.441	.0002	.0016	.9984	58	52
12	3	.1832	.8167	54.570	.1833	54.561	.0002	.0017	.9983	57	48
16	4	.1861	.8138	53.718	.1862	53.708	.0002	.0017	.9983	56	44
20	5	.01891	.98109	52.891	.01891	52.882	1.0002	.00018	.99982	55	40
24	6	.1920	.8080	2.090	.1920	2.081	.0002	.0018	.9981	54	36
28	7	.1949	.8051	1.313	.1949	1.303	.0002	.0019	.9981	53	32
32	8	.1978	.8022	0.558	.1978	0.548	.0002	.0019	.9980	52	28
36	9	.2007	.7993	49.826	.2007	49.816	.0002	.0020	.9980	51	24
40	10	.02036	.97964	49.114	.02036	49.104	1.0002	.00021	.99979	50	20
44	11	.2065	.7935	8.422	.2066	8.412	.0002	.0021	.9979	49	16
48	12	.2094	.7906	7.750	.2095	7.739	.0002	.0022	.9978	48	12
52	13	.2123	.7877	7.096	.2124	7.085	.0002	.0022	.9977	47	8
56	14	.2152	.7847	6.460	.2153	6.449	.0002	.0023	.9977	46	4
5	15	.02181	.97818	45.840	.02182	45.829	1.0002	.00024	.99976	45	**55**
4	16	.2210	.7789	5.237	.2211	5.226	.0002	.0024	.9975	44	56
8	17	.2240	.7760	4.650	.2240	4.638	.0002	.0025	.9975	43	52
12	18	.2269	.7731	4.077	.2269	4.066	.0002	.0026	.9974	42	48
16	19	.2298	.7702	3.520	.2298	3.508	.0003	.0026	.9974	41	44
20	20	.02327	.97673	42.976	.02327	42.964	1.0003	.00027	.99973	40	40
24	21	.2356	.7644	2.445	.2357	2.433	.0003	.0028	.9972	39	36
28	22	.2385	.7615	1.928	.2386	1.916	.0003	.0028	.9971	38	32
32	23	.2414	.7586	1.423	.2415	1.410	.0003	.0029	.9971	37	28
36	24	.2443	.7557	0.930	.2444	0.917	.0003	.0030	.9970	36	24
40	25	.02472	.97528	40.448	.02473	40.436	1.0003	.00030	.99969	35	20
44	26	.2501	.7499	39.978	.2502	39.965	.0003	.0031	.9969	34	16
48	27	.2530	.7469	9.518	.2531	9.506	.0003	.0032	.9968	33	12
52	28	.2559	.7440	9.069	.2560	9.057	.0003	.0033	.9967	32	8
56	29	.2589	.7411	8.631	.2589	8.618	.0003	.0033	.9966	31	4
6	30	.02618	.97382	38.201	.02618	38.188	1.0003	.00034	.99966	30	**54**
4	31	.2647	.7353	7.782	.2648	7.769	.0003	.0035	.9965	29	56
8	32	.2676	.7324	7.371	.2677	7.358	.0003	.0036	.9964	28	52
12	33	.2705	.7295	6.969	.2706	6.956	.0004	.0036	.9963	27	48
16	34	.2734	.7266	6.576	.2735	6.563	.0004	.0037	.9963	26	44
20	35	.02763	.97237	36.191	.02764	36.177	1.0004	.00038	.99962	25	40
24	36	.2792	.7208	5.814	.2793	5.800	.0004	.0039	.9961	24	36
28	37	.2821	.7179	5.445	.2822	5.431	.0004	.0040	.9960	23	32
32	38	.2850	.7150	5.084	.2851	5.069	.0004	.0041	.9959	22	28
36	39	.2879	.7121	4.729	.2880	4.715	.0004	.0041	.9958	21	24
40	40	.02908	.97091	34.382	.02910	34.368	1.0004	.00042	.99958	20	20
44	41	.2937	.7062	4.042	.2939	4.027	.0004	.0043	.9957	19	16
48	42	.2967	.7033	3.708	.2968	3.693	.0004	.0044	.9956	18	12
52	43	.2996	.7004	3.381	.2997	3.366	.0004	.0045	.9955	17	8
56	44	.3025	.6975	3.060	.3026	3.045	.0004	.0046	.9954	16	4
7	45	.03054	.96946	32.745	.03055	32.730	1.0005	.00046	.99953	15	**53**
4	46	.3083	.9692	2.437	.3084	2.421	.0005	.0047	.9952	14	56
8	47	.3112	.6888	2.134	.3113	2.118	.0005	.0048	.9951	13	52
12	48	.3141	.6859	1.836	.3143	1.820	.0005	.0049	.9951	12	48
16	49	.3170	.6830	1.544	.3172	1.528	.0005	.0050	.9950	11	44
20	50	.03199	.96801	31.257	.03201	31.241	1.0005	.00051	.99949	10	40
24	51	.3228	.6772	0.976	.3230	0.960	.0005	.0052	.9948	9	36
28	52	.3257	.6743	0.699	.3259	0.683	.0005	.0053	.9947	8	32
32	53	.3286	.6713	0.428	.3288	0.411	.0005	.0054	.9946	7	28
36	54	.3315	.6684	0.161	.3317	0.145	.0005	.0055	.9945	6	24
40	55	.03344	.96655	29.899	.03346	29.882	1.0005	.00056	.99944	5	20
44	56	.3374	.6626	9.641	.3375	9.624	.0006	.0057	.9943	4	16
48	57	.3403	.6597	9.388	.3405	9.371	.0006	.0058	.9942	3	12
52	58	.3432	.6568	9.139	.3434	9.122	.0006	.0059	.9941	2	8
56	59	.3461	.6539	8.894	.3463	8.877	.0006	.0060	.9940	1	4
8	60	.3490	.6510	8.654	.3492	8.636	.0006	.0061	.9939	0	**52**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
6ʰ	91°	Natural.							88°		5ʰ

0h	2°	Natural Trigonometrical Functions.							177°		11h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
8	0	.03490	.96510	28.654	.03492	28.636	1.0006	.00061	.99939	60	**52**
4	1	. 3519	. 6481	8.417	. 3521	8.399	.0006	. 0062	. 9938	59	56
8	2	. 3548	. 6452	8.184	. 3550	8.166	.0006	. 0063	. 9937	58	52
12	3	. 3577	. 6423	7.955	. 3579	7.937	.0006	. 0064	. 9936	57	48
16	4	. 3606	. 6394	7.730	. 3608	7.712	.0006	. 0065	. 9935	56	44
20	5	.03635	.96365	27.508	.03638	27.490	1.0007	.00066	.99934	55	40
24	6	. 3664	. 6336	7.290	. 3667	7.271	.0007	. 0067	. 9933	54	36
28	7	. 3693	. 6306	7.075	. 3696	7.056	.0007	. 0068	. 9932	53	32
32	8	. 3722	. 6277	6.864	. 3725	6.845	.0007	. 0069	. 9931	52	28
36	9	. 3751	. 6248	6.655	. 3754	6.637	.0007	. 0070	. 9930	51	24
40	10	.03781	.96219	26.450	.03783	26.432	1.0007	.00071	.99928	50	20
44	11	. 3810	. 6190	6.249	. 3812	6.230	.0007	. 0073	. 9927	49	16
48	12	. 3839	. 6161	6.050	. 3842	6.031	.0007	. 0074	. 9926	48	12
52	13	. 3868	. 6132	5.854	. 3871	5.835	.0007	. 0075	. 9925	47	8
56	14	. 3897	. 6103	5.661	. 3900	5.642	.0008	. 0076	. 9924	46	4
9	15	.03926	.96074	25.471	.03929	25.452	1.0008	.00077	.99923	45	**51**
4	16	. 3955	. 6045	5.284	. 3958	5.264	.0008	. 0078	. 9922	44	56
8	17	. 3984	. 6016	5.100	. 3987	5.080	.0008	. 0079	. 9921	43	52
12	18	. 4013	. 5987	4.918	. 4016	4.898	.0008	. 0080	. 9919	42	48
16	19	. 4042	. 5958	4.739	. 4045	4.718	.0008	. 0082	. 9918	41	44
20	20	.04071	.95929	24.562	.04075	24.542	1.0008	.00083	.99917	40	40
24	21	. 4100	. 5900	4.388	. 4104	4.367	.0008	. 0084	. 9916	39	36
28	22	. 4129	. 5870	4.216	. 4133	4.196	.0008	. 0085	. 9915	38	32
32	23	. 4158	. 5841	4.047	. 4162	4.026	.0009	. 0086	. 9913	37	28
36	24	. 4187	. 5812	3.880	. 4191	3.859	.0009	. 0088	. 9912	36	24
40	25	.04217	.95783	23.716	.04220	23.694	1.0009	.00089	.99911	35	20
44	26	. 4246	. 5754	3.553	. 4249	3.532	.0009	. 0090	. 9910	34	16
48	27	. 4275	. 5725	3.393	. 4279	3.372	.0009	. 0091	. 9908	33	12
52	28	. 4304	. 5696	3.235	. 4308	3.214	.0009	. 0093	. 9907	32	8
56	29	. 4333	. 5667	3.079	. 4337	3.058	.0009	. 0094	. 9906	31	4
10	30	.04362	.95638	22.925	.04366	22.904	1.0009	.00095	.99905	30	**50**
4	31	. 4391	. 5609	2.774	. 4395	2.752	.0010	. 0096	. 9903	29	56
8	32	. 4420	. 5580	2.624	. 4424	2.602	.0010	. 0098	. 9902	28	52
12	33	. 4449	. 5551	2.476	. 4453	2.454	.0010	. 0099	. 9901	27	48
16	34	. 4478	. 5522	2.330	. 4483	2.308	.0010	. 0100	. 9900	26	44
20	35	.04507	.95493	22.186	.04512	22.164	1.0010	.00102	.99898	25	40
24	36	. 4536	. 5464	2.044	. 4541	2.022	.0010	. 0103	. 9897	24	36
28	37	. 4565	. 5435	1.904	. 4570	1.881	.0010	. 0104	. 9896	23	32
32	38	. 4594	. 5405	1.765	. 4599	1.742	.0010	. 0106	. 9894	22	28
36	39	. 4623	. 5376	1.629	. 4628	1.606	.0011	. 0107	. 9893	21	24
40	40	.04652	.95347	21.494	.04657	21.470	1.0011	.00108	.99892	20	20
44	41	. 4681	. 5318	1.360	. 4687	1.337	.0011	. 0110	. 9890	19	16
48	42	. 4711	. 5289	1.228	. 4716	1.205	.0011	. 0111	. 9889	18	12
52	43	. 4740	. 5260	1.098	. 4745	1.075	.0011	. 0112	. 9888	17	8
56	44	. 4769	. 5231	0.970	. 4774	0.946	.0011	. 0114	. 9886	16	4
11	45	.04798	.95202	20.843	.04803	20.819	1.0011	.00115	.99885	15	**49**
4	46	. 4827	. 5173	0.717	. 4832	0.693	.0012	. 0116	. 9883	14	56
8	47	. 4856	. 5144	0.593	. 4862	0.569	.0012	. 0118	. 9882	13	52
12	48	. 4885	. 5115	0.471	. 4891	0.446	.0012	. 0119	. 9881	12	48
16	49	. 4914	. 5086	0.350	. 4920	0.325	.0012	. 0121	. 9879	11	44
20	50	.04943	.95057	20.230	.04949	20.205	1.0012	.00122	.99878	10	40
24	51	. 4972	. 5028	0.112	. 4978	0.087	.0012	. 0124	. 9876	9	36
28	52	. 5001	. 4999	19.995	. 5007	19.970	.0012	. 0125	. 9875	8	32
32	53	. 5030	. 4970	9.880	. 5037	9.854	.0013	. 0127	. 9873	7	28
36	54	. 5059	. 4941	9.766	. 5066	9.740	.0013	. 0128	. 9872	6	24
40	55	.05088	.94912	19.653	.05095	19.627	1.0013	.00129	.99870	5	20
44	56	. 5117	. 4883	9.541	. 5124	9.515	.0013	. 0131	. 9869	4	16
48	57	. 5146	. 4853	9.431	. 5153	9.405	.0013	. 0132	. 9867	3	12
52	58	. 5175	. 4824	9.322	. 5182	9.296	.0013	. 0134	. 9866	2	8
56	59	. 5204	. 4795	9.214	. 5212	9.188	.0013	. 0135	. 9864	1	4
12	60	. 5234	. 4766	9.107	. 5241	9.081	.0014	. 0137	. 9863	0	**48**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
6h	92°	Natural.								87°	5h

0h	3°	Natural Trigonometrical Functions.							176°		11h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
12	0	.05234	.94766	19.107	.05241	19.081	1.0014	.00137	.99863	60	**48**
4	1	. 5263	. 4737	9.002	. 5270	8.975	.0014	. 0138	. 9861	59	56
8	2	. 5292	. 4708	8.897	. 5299	8.871	.0014	. 0140	. 9860	58	52
12	3	. 5321	. 4679	8.794	. 5328	8.768	.0014	. 0142	. 9858	57	48
16	4	. 5350	. 4650	8.692	. 5357	8.665	.0014	. 0143	. 9857	56	44
20	5	.05379	.94621	18.591	.05387	18.564	1.0014	.00145	.99855	55	40
24	6	. 5408	. 4592	8.491	. 5416	8.464	.0015	. 0146	. 9854	54	36
28	7	. 5437	. 4563	8.393	. 5445	8.365	.0015	. 0148	. 9852	53	32
32	8	. 5466	. 4534	8.295	. 5474	8.268	.0015	. 0149	. 9850	52	28
36	9	. 5495	. 4505	8.198	. 5503	8.171	.0015	. 0151	. 9849	51	24
40	10	.05524	.94476	18.103	.05532	18.075	1.0015	.00153	.99847	50	20
44	11	. 5553	. 4447	8.008	. 5562	7.980	.0015	. 0154	. 9846	49	16
48	12	. 5582	. 4418	7.914	. 5591	7.886	.0016	. 0156	. 9844	48	12
52	13	. 5611	. 4389	7.821	. 5620	7.793	.0016	. 0157	. 9842	47	8
56	14	. 5640	. 4360	7.730	. 5649	7.701	.0016	. 0159	. 9841	46	4
13	15	.05669	.94331	17.639	.05678	17.610	1.0016	.00161	.99839	45	**47**
4	16	. 5698	. 4302	7.549	. 5707	7.520	.0016	. 0162	. 9837	44	56
8	17	. 5727	. 4273	7.460	. 5737	7.431	.0016	. 0164	. 9836	43	52
12	18	. 5756	. 4244	7.372	. 5766	7.343	.0017	. 0166	. 9834	42	48
16	19	. 5785	. 4214	7.285	. 5795	7.256	.0017	. 0167	. 9832	41	44
20	20	.05814	.94185	17.198	.05824	17.169	1.0017	.00169	.99831	40	40
24	21	. 5843	. 4156	7.113	. 5853	7.084	.0017	. 0171	. 9829	39	36
28	22	. 5872	. 4127	7.028	. 5883	6.999	.0017	. 0172	. 9827	38	32
32	23	. 5902	. 4098	6.944	. 5912	6.915	.0017	. 0174	. 9826	37	28
36	24	. 5931	. 4069	6.861	. 5941	6.832	.0018	. 0176	. 9824	36	24
40	25	.05960	.94040	16.779	.05970	16.750	1.0018	.00178	.99822	35	20
44	26	. 5989	. 4011	6.698	. 5999	6.668	.0018	. 0179	. 9820	34	16
48	27	. 6018	. 3982	6.617	. 6029	6.587	.0018	. 0181	. 9819	33	12
52	28	. 6047	. 3953	6.538	. 6058	6.507	.0018	. 0183	. 9817	32	8
56	29	. 6076	. 3924	6.459	. 6087	6.428	.0018	. 0185	. 9815	31	4
14	30	.06105	.93895	16.380	.06116	16.350	1.0019	.00186	.99813	30	**46**
4	31	. 6134	. 3866	6.303	. 6145	6.272	.0019	. 0188	. 9812	29	56
8	32	. 6163	. 3837	6.226	. 6175	6.195	.0019	. 0190	. 9810	28	52
12	33	. 6192	. 3808	6.150	. 6204	6.119	.0019	. 0192	. 9808	27	48
16	34	. 6221	. 3777	6.075	. 6233	6.043	.0019	. 0194	. 9806	26	44
20	35	.06250	.93750	16.000	.06262	15.969	1.0019	.00195	.99804	25	40
24	36	. 6279	. 3721	5.926	. 6291	5.894	.0020	. 0197	. 9803	24	36
28	37	. 6308	. 3692	5.853	. 6321	5.821	.0020	. 0199	. 9801	23	32
32	38	. 6337	. 3663	5.780	. 6350	5.748	.0020	. 0201	. 9799	22	28
36	39	. 6366	. 3634	5.708	. 6379	5.676	.0020	. 0203	. 9797	21	24
40	40	.06395	.93605	15.637	.06408	15.605	1.0020	.00205	.99795	20	20
44	41	. 6424	. 3576	5.566	. 6437	5.534	.0021	. 0206	. 9793	19	16
48	42	. 6453	. 3547	5.496	. 6467	5.464	.0021	. 0208	. 9791	18	12
52	43	. 6482	. 3518	5.427	. 6496	5.394	.0021	. 0210	. 9790	17	8
56	44	. 6511	. 3489	5.358	. 6525	5.325	.0021	. 0212	. 9788	16	4
15	45	.06540	.93460	15.290	.06554	15.257	1.0021	.00214	.99786	15	**45**
4	46	. 6569	. 3431	4.222	. 6583	5.189	.0022	. 0216	. 9784	14	56
8	47	. 6598	. 3402	5.155	. 6613	5.122	.0022	. 0218	. 9782	13	52
12	48	. 6627	. 3373	5.089	. 6642	5.056	.0022	. 0220	. 9780	12	48
16	49	. 6656	. 3343	5.023	. 6671	4.990	.0022	. 0222	. 9778	11	44
20	50	.06685	.93314	14.958	.06700	14.924	1.0022	.00224	.99776	10	40
24	51	. 6714	. 3285	4.893	. 6730	4.860	.0023	. 0226	. 9774	9	36
28	52	. 6743	. 3256	4.829	. 6759	4.795	.0023	. 0228	. 9772	8	32
32	53	. 6772	. 3227	4.765	. 6788	4.732	.0023	. 0230	. 9770	7	28
36	54	. 6801	. 3198	4.702	. 6817	4.668	.0023	. 0231	. 9768	6	24
40	55	.06830	.93169	14.640	.06846	14.606	1.0023	.00233	.99766	5	20
44	56	. 6859	. 3140	4.578	. 6876	4.544	.0024	. 0235	. 9764	4	16
48	57	. 6888	. 3111	4.517	. 6905	4.482	.0024	. 0237	. 9762	3	12
52	58	. 6918	. 3082	4.456	. 6934	4.421	.0024	. 0239	. 9760	2	8
56	59	. 6947	. 3053	4.395	. 6963	4.361	.0024	. 0241	. 9758	1	4
16	60	. 6976	. 3024	4.335	. 6993	4.301	.0024	. 0243	. 9756	0	**44**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
6h	93°	Natural.							86°		5h

0^h	4°	Natural Trigonometrical Functions.							175°		11^h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
16	0	.06976	.93024	14.335	.06993	14.301	1.0024	.00243	.99756	60	**44**
4	1	. 7005	. 2995	4.276	. 7022	4.241	.0025	. 0246	. 9754	59	56
8	2	. 7034	. 2966	4.217	. 7051	4.182	.0025	. 0248	. 9752	58	52
12	3	. 7063	. 2937	4.159	. 7080	4.123	.0025	. 0250	. 9750	57	48
16	4	. 7092	. 2908	4.101	. 7110	4.065	.0025	. 0252	. 9748	56	44
20	5	.07121	.92879	14.043	.07139	14.008	1.0025	.00254	.99746	55	40
24	6	. 7150	. 2850	3.986	. 7168	3.951	.0026	. 0256	. 9744	54	36
28	7	. 7179	. 2821	3.930	. 7197	3.894	.0026	. 0258	. 9742	53	32
32	8	. 7208	. 2792	3.874	. 7226	3.838	.0026	. 0260	. 9740	52	28
36	9	. 7237	. 2763	3.818	. 7256	3.782	.0026	. 0262	. 9738	51	24
40	10	.07266	.92734	13.763	.07285	13.727	1.0026	.00264	.99736	50	20
44	11	. 7295	. 2705	3.708	. 7314	3.672	.0027	. 0266	. 9733	49	16
48	12	. 7324	. 2676	3.654	. 7343	3.617	.0027	. 0268	. 9731	48	12
52	13	. 7353	. 2647	3.600	. 7373	3.563	.0027	. 0271	. 9729	47	8
56	14	. 7382	. 2618	3.547	. 7402	3.510	.0027	. 0273	. 9727	46	4
17	15	.07411	.92589	13.494	.07431	13.457	1.0027	.00275	.99725	45	**43**
4	16	. 7440	. 2560	3.441	. 7460	3.404	.0028	. 0277	. 9723	44	56
8	17	. 7469	. 2531	3.389	. 7490	3.351	.0028	. 0279	. 9721	43	52
12	18	. 7498	. 2502	3.337	. 7519	3.299	.0028	. 0281	. 9718	42	48
16	19	. 7527	. 2473	3.286	. 7548	3.248	.0028	. 0284	. 9716	41	44
20	20	.07556	.92444	13.235	.07577	13.197	1.0029	.00286	.99714	40	40
24	21	. 7585	. 2415	3.184	. 7607	3.146	.0029	. 0288	. 9712	39	36
28	22	. 7614	. 2386	3.134	. 7636	3.096	.0029	. 0290	. 9710	38	32
32	23	. 7643	. 2357	3.084	. 7665	3.046	.0029	. 0292	. 9707	37	28
36	24	. 7672	. 2328	3.034	. 7694	2.996	.0029	. 0295	. 9705	36	24
40	25	.07701	.92299	12.985	.07724	12.947	1.0030	.00297	.99703	35	20
44	26	. 7730	. 2270	2.937	. 7753	2.898	.0030	. 0299	. 9701	34	16
48	27	. 7759	. 2241	2.888	. 7782	2.849	.0030	. 0301	. 9698	33	12
52	28	. 7788	. 2212	2.840	. 7812	2.801	.0030	. 0304	. 9696	32	8
56	29	. 7817	. 2183	2.793	. 7841	2.754	.0031	. 0306	. 9694	31	4
18	30	.07846	.92154	12.745	.07870	12.706	1.0031	.00308	.99692	30	**42**
4	31	. 7875	. 2125	2.698	. 7899	2.659	.0031	. 0310	. 9689	29	56
8	32	. 7904	. 2096	2.652	. 7929	2.612	.0031	. 0313	. 9687	28	52
12	33	. 7933	. 2067	2.606	. 7958	2.566	.0032	. 0315	. 9685	27	48
16	34	. 7962	. 2038	2.560	. 7987	2.520	.0032	. 0317	. 9682	26	44
20	35	.07991	.92009	12.514	.08016	12.474	1.0032	.00320	.99680	25	40
24	36	. 8020	. 1980	2.469	. 8046	2.429	.0032	. 0322	. 9678	24	36
28	37	. 8049	. 1951	2.424	. 8075	2.384	.0032	. 0324	. 9675	23	32
32	38	. 8078	. 1922	2.379	. 8104	2.339	.0033	. 0327	. 9673	22	28
36	39	. 8107	. 1893	2.335	. 8134	2.295	.0033	. 0329	. 9671	21	24
40	40	.08136	.91864	12.291	.08163	12.250	1.0033	.00331	.99668	20	20
44	41	. 8165	. 1835	2.248	. 8192	2.207	.0033	. 0334	. 9666	19	16
48	42	. 8194	. 1806	2.204	. 8221	2.163	.0034	. 0336	. 9664	18	12
52	43	. 8223	. 1777	2.161	. 8251	2.120	.0034	. 0339	. 9661	17	8
56	44	. 8252	. 1748	2.118	. 8280	2.077	.0034	. 0341	. 9659	16	4
19	45	.08281	.91719	12.076	.08309	12.035	1.0034	.00343	.99656	15	**41**
4	46	. 8310	. 1690	2.034	. 8339	1.992	.0035	. 0346	. 9654	14	56
8	47	. 8339	. 1661	1.992	. 8368	1.950	.0035	. 0348	. 9652	13	52
12	48	. 8368	. 1632	1.950	. 8397	1.909	.0035	. 0351	. 9649	12	48
16	49	. 8397	. 1603	1.909	. 8426	1.867	.0035	. 0353	. 9647	11	44
20	50	.08426	.91574	11.868	.08456	11.826	1.0036	.00356	.99644	10	40
24	51	. 8455	. 1545	1.828	. 8485	1.785	.0036	. 0358	. 9642	9	36
28	52	. 8484	. 1516	1.787	. 8514	1.745	.0036	. 0360	. 9639	8	32
32	53	. 8513	. 1487	1.747	. 8544	1.704	.0036	. 0363	. 9637	7	28
36	54	. 8542	. 1458	1.707	. 8573	1.664	.0037	. 0365	. 9634	6	24
40	55	.08571	.91429	11.668	.08602	11.625	1.0037	.00368	.99632	5	20
44	56	. 8600	. 1400	1.628	. 8632	1.585	.0037	. 0370	. 9629	4	16
48	57	. 8629	. 1371	1.589	. 8661	1.546	.0037	0373	. 9627	3	12
52	58	. 8658	. 1342	1.550	. 8690	1.507	.0038	. 0375	. 9624	2	8
56	59	. 8687	. 1313	1.512	. 8719	1.468	.0038	. 0378	. 9622	1	4
20	60	. 8715	. 1284	1.474	. 8749	1.430	.0038	. 0380	. 9619	0	**40**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
6^h	94°	Natural.							85°		5^h

0h	5°	Natural Trigonometrical Functions.								174°	11h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
20	0	.08715	.91284	11.474	.08749	11.430	1.0038	.00380	.99619	60	**40**
4	1	. 8744	. 1255	1.436	. 8778	1.392	.0038	. 0383	. 9617	59	56
8	2	. 8773	. 1226	1.398	. 8807	1.354	.0039	. 0386	. 9614	58	52
12	3	. 8802	. 1197	1.360	. 8837	1.316	.0039	. 0388	. 9612	57	48
16	4	. 8831	. 1168	1.323	. 8866	1.279	.0039	. 0391	. 9609	56	44
20	5	.08860	.91139	11.286	.08895	11.242	1.0039	.00393	.99607	55	40
24	6	. 8889	. 1110	1.249	. 8925	1.205	.0040	. 0396	. 9604	54	36
28	7	. 8918	. 1082	1.213	. 8954	1.168	.0040	. 0398	. 9601	53	32
32	8	. 8947	. 1053	1.176	. 8983	1.132	.0040	. 0401	. 9599	52	28
36	9	. 8976	. 1024	1.140	. 9013	1.095	.0040	. 0404	. 9596	51	24
40	10	.09005	.90995	11.104	.09042	11.059	1.0041	.00406	.99594	50	20
44	11	. 9034	. 0966	1.069	. 9071	1.024	.0041	. 0409	. 9591	49	16
48	12	. 9063	. 0937	1.033	. 9101	0.988	.0041	. 0411	. 9588	48	12
52	13	. 9092	. 0908	0.998	. 9130	0.953	.0041	. 0414	. 9586	47	8
56	14	. 9121	. 0879	0.963	. 9159	0.918	.0042	. 0417	. 9583	46	4
21	15	.09150	.90850	10.929	.09189	10.883	1.0042	.00419	.99580	45	**39**
4	16	. 9179	. 0821	0.894	. 9218	0.848	.0042	. 0422	. 9578	44	56
8	17	. 9208	. 0792	0.860	. 9247	0.814	.0043	. 0425	. 9575	43	52
12	18	. 9237	. 0763	0.826	. 9277	0.780	.0043	. 0427	. 9572	42	48
16	19	. 9266	. 0734	0.792	. 9306	0.746	.0043	. 0430	. 9570	41	44
20	20	.09295	.90705	10.758	.09335	10.712	1.0043	.00433	.99567	40	40
24	21	. 9324	. 0676	0.725	. 9365	0.678	.0044	. 0436	. 9564	39	36
28	22	. 9353	. 0647	0.692	. 9394	0.645	.0044	. 0438	. 9562	38	32
32	23	. 9382	. 0618	0.659	. 9423	0.612	.0044	. 0441	. 9559	37	28
36	24	. 9411	. 0589	0 626	. 9453	0.579	.0044	. 0444	. 9556	36	24
40	25	.09440	.90560	10.593	.09482	10.546	1.0045	.00446	.99553	35	20
44	26	. 9469	. 0531	0.561	. 9511	0.514	.0045	. 0449	. 9551	34	16
48	27	. 9498	. 0502	0 529	. 9541	0.481	.0045	. 0452	. 9548	33	12
52	28	. 9527	. 0473	0.497	. 9570	0.449	.0046	. 0455	. 9545	32	8
56	29	. 9556	. 0444	0.465	. 9599	0.417	.0046	. 0458	. 9542	31	4
22	30	.09584	.90415	10.433	.09629	10.385	1.0046	.00460	.99540	30	**38**
4	31	. 9613	. 0386	0.402	. 9658	0.354	.0046	. 0463	. 9537	29	56
8	32	. 9642	. 0357	0.371	. 9688	0.322	.0047	. 0466	. 9534	28	52
12	33	. 9671	. 0328	0.340	. 9717	0.291	.0047	. 0469	. 9531	27	48
16	34	. 9700	. 0300	0.309	. 9746	0.260	.0047	. 0472	. 9528	26	44
20	35	.09729	.90271	10.278	.09776	10.229	1.0048	.00474	.99525	25	40
24	36	. 9758	. 0242	0.248	. 9805	0.199	.0048	. 0477	. 9523	24	36
28	37	. 9787	. 0213	0.217	. 9834	0.168	.0048	. 0480	. 9520	23	32
32	38	. 9816	. 0184	0.187	. 9864	0.138	.0048	. 0483	. 9517	22	28
36	39	. 9845	. 0155	0.157	. 9893	0.108	.0049	. 0486	. 9514	21	24
40	40	.09874	.90126	10.127	.09922	10.078	1.0049	.00489	.99511	20	20
44	41	. 9903	. 0097	0.098	. 9952	0.048	.0049	. 0491	. 9508	19	16
48	42	. 9932	. 0068	0.068	. 9981	0.019	.0050	. 0494	. 9505	18	12
52	43	. 9961	. 0039	0.039	.10011	9.9893	.0050	. 0497	. 9503	17	8
56	44	. 9990	. 0010	0.010	.10040	.9601	.0050	. 0500	. 9500	16	4
23	45	.10019	.89981	9.9812	.10069	.9310	1.0050	.00503	.99497	15	**37**
4	46	. 0048	. 9952	.9525	. 0099	.9021	.0051	. 0506	. 9494	14	56
8	47	. 0077	. 9923	.9239	. 0128	.8734	.0051	. 0509	. 9491	13	52
12	48	. 0106	. 9894	.8955	. 0158	.8448	.0051	. 0512	. 9488	12	48
16	49	. 0134	. 9865	.8672	. 0187	.8164	.0052	. 0515	. 9485	11	44
20	50	.10163	.89836	9.8391	.10216	9.7882	1.0052	.00518	.99482	10	40
24	51	. 0192	. 9807	.8112	. 0246	.7601	.0052	. 0521	. 9479	9	36
28	52	. 0221	. 9779	.7834	. 0275	.7322	.0053	. 0524	. 9476	8	32
32	53	. 0250	. 9750	.7558	. 0305	.7044	.0053	. 0527	. 9473	7	28
36	54	. 0279	. 9721	.7283	. 0334	.6768	.0053	. 0530	. 9470	6	24
40	55	.10308	.89692	9.7010	.10363	9.6493	1.0053	.00533	.99467	5	20
44	56	. 0337	. 9663	.6739	. 0393	.6220	.0054	. 0536	. 9464	4	16
48	57	. 0366	. 9634	.6469	. 0422	.5949	.0054	. 0539	. 9461	3	12
52	58	. 0395	. 9605	.6200	. 0452	.5679	.0054	. 0542	. 9458	2	8
56	59	. 0424	. 9576	.5933	. 0481	.5411	.0055	. 0545	. 9455	1	4
24	60	. 0453	. 9547	.5668	. 0510	.5144	.0055	. 0548	. 9452	0	**36**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
6h	95°	Natural.								84°	5h

0^h | 6° **Natural Trigonometrical Functions.** 173° | 11^h

M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
24	0	.10453	.89547	9.5668	.10510	9.5144	1.0055	.00548	.99452	60	**36**
4	1	. 0482	. 9518	.5404	. 0540	.4878	.0055	. 0551	. 9449	59	56
8	2	. 0511	. 9489	.5141	. 0569	.4614	.0056	. 0554	. 9446	58	52
12	3	. 0540	. 9460	.4880	. 0599	.4351	.0056	. 0557	. 9443	57	48
16	4	. 0568	. 9431	.4620	. 0628	.4090	.0056	. 0560	. 9440	56	44
20	5	.10597	.89402	9.4362	.10657	9.3831	1.0057	.00563	.99437	55	40
24	6	. 0626	. 9373	.4105	. 0687	.3572	.0057	. 0566	. 9434	54	36
28	7	. 0655	. 9345	.3850	. 0716	.3315	.0057	. 0569	. 9431	53	32
32	8	. 0684	. 9316	.3596	. 0746	.3060	.0057	. 0572	. 9428	52	28
36	9	. 0713	. 9287	.3343	. 0775	.2806	.0058	. 0575	. 9424	51	24
40	10	.10742	.89258	9.3092	.10805	9.2553	1.0058	.00579	.99421	50	20
44	11	. 0771	. 9229	.2842	. 0834	.2302	.0058	. 0582	. 9418	49	16
48	12	. 0800	. 9200	.2593	. 0863	.2051	.0059	. 0585	. 9415	48	12
52	13	. 0829	. 9171	.2346	. 0893	.1803	.0059	. 0588	. 9412	47	8
56	14	. 0858	. 9142	.2100	. 0922	.1555	.0059	. 0591	. 9409	46	4
25	15	.10887	.89113	9.1855	.10952	9.1309	1.0060	.00594	.99406	45	**35**
4	16	. 0916	. 9084	.1612	. 0981	.1064	.0060	. 0597	. 9402	44	56
8	17	. 0944	. 9055	.1370	. 1011	.0821	.0060	. 0601	. 9399	43	52
12	18	. 0973	. 9026	.1129	. 1040	.0579	.0061	. 0604	. 9396	42	48
16	19	. 1002	. 8998	.0890	. 1069	.0338	.0061	. 0607	. 9393	41	44
20	20	.11031	.88969	9.0651	.11099	9.0098	1.0061	.00110	.99390	40	40
24	21	. 1060	. 8940	.0414	. 1128	8.9860	.0062	. 0613	. 9386	39	36
28	22	. 1089	. 8911	.0179	. 1158	.9623	.0062	. 0617	. 9383	38	32
32	23	. 1118	. 8882	.9944	. 1187	.9387	.0062	. 0620	. 9380	37	28
36	24	. 1147	. 8853	8.9711	. 1217	.9152	.0063	. 0623	. 9377	36	24
40	25	.11176	.88824	8.9479	.11246	8.8918	1.0063	.00626	.99373	35	20
44	26	. 1205	. 8795	.9248	. 1276	.8686	.0063	. 0630	. 9370	34	16
48	27	. 1234	. 8766	.9018	. 1305	.8455	.0064	. 0633	. 9367	33	12
52	28	. 1262	. 8737	.8790	. 1335	.8225	.0064	. 0636	. 9364	32	8
56	29	. 1291	. 8708	.8563	. 1364	.7996	.0064	. 0639	. 9360	31	4
26	30	.11320	.88680	8.8337	.11393	8.7769	1.0065	.00643	.99357	30	**34**
4	31	. 1349	. 8651	.8112	. 1423	.7542	.0065	. 0646	. 9354	29	56
8	32	. 1378	. 8622	.7888	. 1452	.7317	.0065	. 0649	. 9350	28	52
12	33	. 1407	. 8593	.7665	. 1482	.7093	.0066	. 0653	. 9347	27	48
16	34	. 1436	. 8564	.7444	. 1511	.6870	.0066	. 0656	. 9344	26	44
20	35	.11465	.88535	8.7223	.11541	8.6648	1.0066	.00659	.99341	25	40
24	36	. 1494	. 8506	.7004	. 1570	.6427	.0067	. 0663	. 9337	24	36
28	37	. 1523	. 8477	.6786	. 1600	.6208	.0067	. 0666	. 9334	23	32
32	38	. 1551	. 8448	.6569	. 1629	.5989	.0067	. 0669	. 9330	22	28
36	39	. 1580	. 8420	.6353	. 1659	.5772	.0068	. 0673	. 9327	21	24
40	40	.11609	.88391	8.6138	.11688	8.5555	1.0068	.00676	.99324	20	20
44	41	. 1638	. 8362	.5924	. 1718	.5340	.0068	. 0679	. 9320	19	16
48	42	. 1667	. 8333	.5711	. 1747	.5126	.0069	. 6683	. 9317	18	12
52	43	. 1696	. 8304	.5499	. 1777	.4913	.0069	. 0686	. 9314	17	8
56	44	. 1725	. 8272	.5289	. 1806	.4701	.0069	. 0690	. 9310	16	4
27	45	.11754	.88246	8.5079	.11836	8.4489	1.0070	.00693	99307	15	**33**
4	46	. 1783	. 8217	.4871	. 1865	.4279	.0070	. 0696	. 9303	14	56
8	47	. 1811	. 8188	.4663	. 1895	.4070	.0070	. 0700	. 9300	13	52
12	48	. 1840	. 8160	.4457	. 1924	.3862	.0071	. 0703	. 9296	12	48
16	49	. 1869	. 8131	.4251	. 1954	.3655	.0071	. 0707	. 9293	11	44
20	50	.11898	.88102	8.4046	.11983	8.3449	1.0071	.00710	.99290	10	40
24	51	. 1927	. 8073	.3843	. 2013	.3244	.0072	. 0714	. 9286	9	36
28	52	. 1956	. 8044	.3640	. 2042	.3040	.0072	. 0717	. 9283	8	32
32	53	. 1985	. 8015	.3439	. 2072	.2837	.0073	. 0721	. 9279	7	28
36	54	. 2014	. 7986	.3238	. 2101	.2635	.0073	. 0724	. 9276	6	24
40	55	.12042	.87957	8.3039	.12131	8.2434	1.0073	.00728	.99272	5	20
44	56	. 2071	. 7928	.2840	. 2160	.2234	.0074	. 0731	. 9269	4	16
48	57	. 2100	. 7900	.2642	. 2190	.2035	.0074	. 0735	. 9265	3	12
52	58	. 2129	. 7871	.2446	. 2219	.1837	.0074	. 0738	. 9262	2	8
56	59	. 2158	. 7842	.2250	. 2249	.1640	.0075	. 0742	. 9258	1	4
28	60	. 2187	. 7813	.2055	. 2278	.1443	.0075	. 0745	. 9255	0	**32**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M.S.

6^h | 96° **Natural.** 83° | 5^h

0h	7°	Natural Trigonometrical Functions.							172°		11h
M.S.	M	Sine.	Vrs. Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
28	0	.12187	.87813	8.2055	.12278	8.1443	1.0075	.00745	.99255	60	**32**
4	1	. 2216	. 7787	.1861	. 2308	.1248	.0075	. 0749	. 9251	59	56
8	2	. 2245	. 7755	.1668	. 2337	.1053	.0076	. 0752	. 9247	58	52
12	3	. 2273	. 7726	.1476	. 2367	.0860	.0076	. 0756	. 9244	57	48
16	4	. 2302	. 7697	.1285	. 2396	.0667	.0076	. 0760	. 9240	56	44
20	5	.12331	.87669	8.1094	.12426	8.0476	1.0077	.00763	.99237	55	40
24	6	. 2360	. 7649	.0905	. 2456	.0285	.0077	. 0767	. 9233	54	36
28	7	. 2389	. 7611	.0717	. 2485	.0095	.0078	. 0770	. 9229	53	32
32	8	. 2418	. 7582	.0529	. 2515	7.9906	.0078	. 0774	. 9226	52	28
36	9	. 2447	. 7553	.0342	. 2544	7.9717	.0078	. 0778	. 9222	51	24
40	10	.12476	.87524	8.0156	.12574	7.9530	1.0079	.00781	.99219	50	20
44	11	. 2504	. 7495	7.9971	. 2603	.9344	.0079	. 0785	. 9215	49	16
48	12	. 2533	. 7467	.9787	. 2633	.9158	.0079	. 0788	. 9211	48	12
52	13	. 2562	. 7438	.9604	. 2662	.8973	.0080	. 0792	. 9208	47	8
56	14	. 2591	. 7409	.9421	. 2692	.8789	.0080	. 0796	. 9204	46	4
29	15	.12620	.87380	7.9240	.12722	7.8606	1.0080	.00799	.99200	45	**31**
4	16	. 2649	. 7351	.9059	. 2751	.8424	.0081	. 0803	. 9197	44	56
8	17	. 2678	. 7322	.8879	. 2781	.8243	.0081	. 0807	. 9193	43	52
12	18	. 2706	. 7293	.8700	. 2810	.8062	.0082	. 0810	. 9189	42	48
16	19	. 2735	. 7265	.8522	. 2840	.7882	.0082	. 0814	. 9186	41	44
20	20	.12764	.87236	7.8344	.12869	7.7703	1.0082	.00818	.99182	40	40
24	21	. 2793	. 7207	.8168	. 2899	.7525	.0083	. 0822	. 9178	39	36
28	22	. 2822	. 7178	.7992	. 2928	.7348	.0083	. 0825	. 9174	38	32
32	23	. 2851	. 7149	.7817	. 2958	.7171	.0084	. 0829	. 9171	37	28
36	24	. 2879	. 7120	.7642	. 2988	.6996	.0084	. 0833	. 9167	36	24
40	25	.12908	.87091	7.7469	.13017	7.6821	1.0084	.00837	.99163	35	20
44	26	. 2937	. 7063	.7296	. 3047	.6646	.0085	. 0840	. 9160	34	16
48	27	. 2966	. 7034	.7124	. 3076	.6473	.0085	. 0844	. 9156	33	12
52	28	. 2995	. 7005	.6953	. 3106	.6300	.0085	. 0848	. 9152	32	8
56	29	. 3024	. 6976	.6783	. 3136	.6129	.0086	. 0852	. 9148	31	4
30	30	.13053	.86947	7.6613	.13165	7.5957	1.0086	.00855	.99144	30	**30**
4	31	. 3081	. 6918	.6444	. 3195	.5787	.0087	. 0859	. 9141	29	56
8	32	. 3110	. 6890	.6276	. 3224	.5617	.0087	. 0863	. 9137	28	52
12	33	. 3139	. 6861	.6108	. 3254	.5449	.0087	. 0867	. 9133	27	48
16	34	3168	. 6832	.5942	. 3284	.5280	.0088	. 0871	. 9129	26	44
20	35	.13197	.86803	7.5776	.13313	7.5113	1.0088	.00875	.99125	25	40
24	36	. 3226	. 6774	.5611	. 3343	.4946	.0089	. 0878	. 9121	24	36
28	37	. 3254	. 6745	.5446	. 3372	.4780	.0089	. 0882	. 9118	23	32
32	38	. 3283	. 6717	.5282	. 3402	.4615	.0089	. 0886	. 9114	22	28
36	39	. 3312	. 6688	.5119	. 3432	.4451	.0090	. 0890	. 9110	21	24
40	40	.13341	.86659	7.4957	.13461	7.4287	1.0090	.00894	.99106	20	20
44	41	. 3370	. 6630	.4795	. 3491	.4124	.0090	. 0898	. 9102	19	16
48	42	. 3399	. 6601	.4634	. 3520	.3961	.0091	. 0902	. 9098	18	12
52	43	. 3427	. 6572	.4474	. 3550	.3800	.0091	. 0905	. 9094	17	8
56	44	. 3456	. 6544	.4315	. 3580	.3639	.0092	. 0909	. 9090	16	4
31	45	.13485	.86515	7.4156	.13609	7.3479	1.0092	.00913	.99086	15	**29**
4	46	. 3514	. 6486	.3998	. 3639	.3319	.0092	. 0917	. 9083	14	56
8	47	. 3543	. 6457	.3840	. 3669	.3160	.0093	. 0921	. 9079	13	52
12	48	. 3571	. 6428	.3683	. 3698	.3002	.0093	. 0925	. 9075	12	48
16	49	. 3600	. 6400	.3527	. 3728	.2844	.0094	. 0929	. 9070	11	44
20	50	.13629	.86371	7.3372	.13757	7.2687	1.0094	.00933	.99067	10	40
24	51	. 3658	. 6342	.3217	. 3787	.2531	.0094	. 0937	. 9063	9	36
28	52	. 3687	. 6313	.3063	. 3817	.2375	.0095	. 0941	. 9059	8	32
32	53	. 3716	. 6284	.2909	. 3846	.2220	.0095	. 0945	. 9055	7	28
36	54	. 3744	. 6255	.2757	. 3876	.2066	.0096	. 0949	. 9051	6	24
40	55	.13773	.86227	7.2604	.13906	7.1912	1.0096	.00953	.99047	5	20
44	56	. 3802	. 6198	.2453	. 3935	.1759	.0097	. 0957	. 9043	4	16
48	57	. 3831	. 6169	.2302	. 3965	.1607	.0097	. 0961	. 9039	3	12
52	58	. 3860	. 6140	.2152	. 3995	.1455	.0097	. 0965	. 9035	2	8
56	59	. 3888	. 6111	.2002	. 4024	.1304	.0098	. 0969	. 9031	1	4
32	60	. 3917	. 6083	.1853	. 4054	.1154	.0098	. 0973	. 9027	0	**28**
M.S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M.S.
6h	97°	Natural.							82°		5h

0h	8°			Natural Trigonometrical Functions.						171°	11h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
32	0	.13917	.86083	7.1853	.14054	7.1154	1.0098	.00973	.99027	60	**28**
4	1	. 3946	. 6054	.1704	. 4084	.1004	.0099	. 0977	. 9023	59	56
8	2	. 3975	. 6025	.1557	. 4113	.0854	.0099	. 0981	. 9019	58	52
12	3	. 4004	. 5996	.1409	. 4143	.0706	.0099	. 0985	. 9015	57	48
16	4	. 4032	. 5967	.1263	. 4173	.0558	.0100	. 0989	. 9010	56	44
20	5	.14061	.85939	7.1117	.14202	7.0410	1.0100	.00993	.99006	55	40
24	6	. 4090	. 5910	.0972	. 4232	.0264	.0101	. 0998	. 9002	54	36
28	7	. 4119	. 5881	.0827	. 4262	.0117	.0101	. 1002	. 8998	53	32
32	8	. 4148	. 5852	.0683	. 4291	6.9972	.0102	. 1006	. 8994	52	28
36	9	. 4176	. 5823	.0539	. 4321	6.9827	.0102	. 1010	. 8990	51	24
40	10	.14205	.85795	7.0396	.14351	6.9682	1.0102	.01014	.98986	50	20
44	11	. 4234	. 5766	.0254	. 4380	.9538	.0103	. 1018	. 8982	49	16
48	12	. 4263	. 5737	.0112	. 4410	.9395	.0103	. 1022	. 8978	48	12
52	13	. 4292	. 5708	6.9971	. 4440	.9252	.0104	. 1026	. 8973	47	8
56	14	. 4320	. 5679	6.9830	. 4470	.9110	.0104	. 1031	. 8969	46	4
33	15	.14349	.85651	6.9690	.14499	6.8969	1.0104	.01035	.98965	45	**27**
4	16	. 4378	. 5622	.9550	. 4529	.8828	.0105	. 1039	. 8961	44	56
8	17	. 4407	. 5593	.9411	. 4559	.8687	.0105	. 1043	. 8957	43	52
12	18	. 4436	. 5564	.9273	. 4588	.8547	.0106	. 1047	. 8952	42	48
16	19	. 4464	. 5536	.9135	. 4618	.8408	.0106	. 1052	. 8948	41	44
20	20	.14493	.85507	6.8998	.14648	6.8269	1.0107	.01056	.98944	40	40
24	21	. 4522	. 5478	.8861	. 4677	.8131	.0107	. 1060	. 8940	39	36
28	22	. 4551	. 5449	.8725	. 4707	.7993	.0107	. 1064	. 8936	38	32
32	23	. 4579	. 5420	.8589	. 4737	.7856	.0108	. 1068	. 8931	37	28
36	24	. 4608	. 5392	.8454	. 4767	.7720	.0108	. 1073	. 8927	36	24
40	25	.14637	.85363	6.8320	.14796	6.7584	1.0109	.01077	.98923	35	20
44	26	. 4666	. 5334	.8185	. 4826	.7448	.0109	. 1081	. 8919	34	16
48	27	. 4695	. 5305	.8052	. 4856	.7313	.0110	. 1085	. 8914	33	12
52	28	. 4723	. 5277	.7919	. 4886	.7179	.0110	. 1090	. 8910	32	8
56	29	. 4752	. 5248	.7787	. 4915	.7045	.0111	. 1094	. 8906	31	4
34	30	.14781	.85219	6.7655	.14945	6.6911	1.0111	.01098	.98901	30	**26**
4	31	. 4810	. 5190	.7523	. 4975	.6779	.0111	. 1103	. 8897	29	56
8	32	. 4838	. 5161	.7392	. 5004	.6646	.0112	. 1107	. 8893	28	52
12	33	. 4867	. 5133	.7262	. 5034	.6514	.0112	. 1111	. 8889	27	48
16	34	. 4896	. 5104	.7132	. 5064	.6383	.0113	. 1116	. 8884	26	44
20	35	.14925	.85075	6.7003	.15094	6.6252	1.0113	.01120	.98880	25	40
24	36	. 4953	. 5046	.6874	. 5123	.6122	.0114	. 1124	. 8876	24	36
28	37	. 4982	. 5018	.6745	. 5153	.5992	.0114	. 1129	. 8871	23	32
32	38	. 5011	. 4989	.6617	. 5183	.5863	.0115	. 1133	. 8867	22	28
36	39	. 5040	. 4960	.6490	. 5213	.5734	.0115	. 1137	. 8862	21	24
40	40	.15068	.84931	6.6363	.15243	6.5605	1.0115	.01142	.98858	20	20
44	41	. 5097	. 4903	.6237	. 5272	.5478	.0116	. 1146	. 8854	19	16
48	42	. 5126	. 4874	.6111	. 5302	.5350	.0116	. 1151	. 8849	18	12
52	43	. 5155	. 4845	.5985	. 5332	.5223	.0117	. 1155	. 8845	17	8
56	44	. 5183	. 4816	.5860	. 5362	.5097	.0117	. 1159	. 8840	16	4
35	45	.15212	.84788	6.5736	.15391	6.4971	1.0118	.01164	.98836	15	**25**
4	46	. 5241	. 4759	.5612	. 5421	.4845	.0118	. 1168	. 8832	14	56
8	47	. 5270	. 4730	.5488	. 5451	.4720	.0119	. 1173	. 8827	13	52
12	48	. 5298	. 4701	.5365	. 5481	.4596	.0119	. 1177	. 8823	12	48
16	49	. 5328	. 4672	.5243	. 5511	.4472	.0119	. 1182	. 8818	11	44
20	50	.15356	.84644	6.5121	.15540	6.4348	1.0120	.01186	.98814	10	40
24	51	. 5385	. 4615	.4999	. 5570	.4225	.0120	. 1190	. 8809	9	36
28	52	. 5413	. 4586	.4878	. 5600	.4103	.0121	. 1195	. 8805	8	32
32	53	. 5442	. 4558	.4757	. 5630	.3980	.0121	. 1199	. 8800	7	28
36	54	. 5471	. 4529	.4637	. 5659	.3859	.0122	. 1204	. 8796	6	24
40	55	.15500	.84500	6.4517	.15689	6.3737	1.0122	.01208	.98791	5	20
44	56	. 5528	. 4471	.4398	. 5719	.3616	.0123	. 1213	. 8787	4	16
48	57	. 5557	. 4443	.4279	. 5749	.3496	.0123	. 1217	. 8782	3	12
52	58	. 5586	. 4414	.4160	. 5779	.3376	.0124	. 1222	. 8778	2	8
56	59	. 5615	. 4385	.4042	. 5809	.3257	.0124	. 1227	. 8773	1	4
36	60	. 5643	. 4356	.3924	. 5838	.3137	.0125	. 1231	. 8769	0	**24**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Sine.	Vrs.Cos	M	M.S.
6h	98°			Natural.						81°	5h

0h	9°	Natural Trigonometrical Functions.							170°		11h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
36	0	.15643	.84356	6.3924	.15838	6.3137	1.0125	.01231	.98769	60	**24**
4	1	. 5672	. 4328	.3807	. 5868	.3019	.0125	. 1236	. 8764	59	56
8	2	. 5701	. 4299	.3690	. 5898	.2901	.0125	. 1240	. 8760	58	52
12	3	. 5730	. 4270	.3574	. 5928	.2783	.0126	. 1245	. 8755	57	48
16	4	. 5758	. 4242	.3458	. 5958	.2665	.0126	. 1249	. 8750	56	44
20	5	.15787	.84213	6.3343	.15987	6.2548	1.0127	.01254	.98746	55	40
24	6	. 5816	. 4184	.3228	. 6017	.2432	.0127	. 1259	. 8741	54	36
28	7	. 5844	. 4155	.3113	. 6047	.2316	.0128	. 1263	. 8737	53	32
32	8	. 5873	. 4127	.2999	. 6077	.2200	.0128	. 1268	. 8732	52	28
36	9	. 5902	. 4098	.2885	. 6107	.2085	.0129	. 1272	. 8727	51	24
40	10	.15931	.84069	6.2772	.16137	6.1970	1.0129	.01277	.98723	50	20
44	11	. 5959	. 4041	.2659	. 6167	.1856	.0130	. 1282	. 8718	49	16
48	12	. 5988	. 4012	.2546	. 6196	.1742	.0130	. 1286	. 8714	48	12
52	13	. 6017	. 3983	.2434	. 6226	.1628	.0131	. 1291	. 8709	47	8
56	14	. 6045	. 3954	.2322	. 6256	.1515	.0131	. 1296	. 8704	46	4
37	15	.16074	.83926	6.2211	.16286	6.1402	1.0132	.01300	.98700	45	**23**
4	16	. 6103	. 3897	.2100	. 6316	.1290	.0132	. 1305	. 8695	44	56
8	17	. 6132	. 3868	.1990	. 6346	.1178	.0133	. 1310	. 8690	43	52
12	18	. 6160	. 3840	.1880	. 6376	.1066	.0133	. 1314	. 8685	42	48
16	19	. 6189	. 3811	.1770	. 6405	.0955	.0134	. 1319	. 8681	41	44
20	20	.16218	.83782	6.1661	.16435	6.0844	1.0134	.01324	.98676	40	40
24	21	. 6246	. 3753	.1552	. 6465	.0734	.0135	. 1328	. 8671	39	36
28	22	. 6275	. 3725	.1443	. 6495	.0624	.0135	. 1333	. 8667	38	32
32	23	. 6304	. 3696	.1335	. 6525	.0514	.0136	. 1338	. 8662	37	28
36	24	. 6333	. 3667	.1227	. 6555	.0405	.0136	. 1343	. 8657	36	24
40	25	.16361	.83639	6.1120	.16585	6.0296	1.0136	.01347	.98652	35	20
44	26	. 6390	. 3610	.1013	. 6615	.0188	.0137	. 1352	. 8648	34	16
48	27	. 6419	. 3581	.0906	. 6644	.0080	.0137	. 1357	. 8643	33	12
52	28	. 6447	. 3553	.0800	. 6674	5.9972	.0138	. 1362	. 8638	32	8
56	29	. 6476	. 3524	.0694	. 6704	5.9865	.0138	. 1367	. 8633	31	4
38	30	.16505	.83495	6.0588	.16734	5.9758	1.0139	.01371	.98628	30	**22**
4	31	. 6533	. 3466	.0483	. 6764	.9651	.0139	. 1376	. 8624	29	56
8	32	. 6562	. 3438	.0379	. 6794	.9545	.0140	. 1381	. 8619	28	52
12	33	. 6591	. 3409	.0274	. 6824	.9439	.0140	. 1386	. 8614	27	48
16	34	. 6619	. 3380	.0170	. 6854	.9333	.0141	. 1391	. 8609	26	44
20	35	.16648	.83352	6.0066	.16884	5.9228	1.0141	.01395	.98604	25	40
24	36	. 6677	. 3323	5.9963	. 6914	.9123	.0142	. 1400	. 8600	24	36
28	37	. 6705	. 3294	.9860	. 6944	.9019	.0142	. 1405	. 8595	23	32
32	38	. 6734	. 3266	.9758	. 6973	.8915	.0143	. 1410	. 8590	22	28
36	39	. 6763	. 3237	.9655	. 7003	.8811	.0143	. 1411	. 8585	21	24
40	40	.16791	.83208	5.9554	.17033	5.8708	1.0144	.01420	.98580	20	20
44	41	. 6820	. 3180	.9452	. 7063	.8605	.0144	. 1425	. 8575	19	16
48	42	. 6849	. 3151	.9351	. 7093	.8502	.0145	. 1430	. 8570	18	12
52	43	. 6878	. 3122	.9250	. 7123	.8400	.0145	. 1434	. 8565	17	8
56	44	. 6906	. 3094	.9150	. 7153	.8298	.0146	. 1439	. 8560	16	4
39	45	.16935	.83065	5.9049	.17183	5.8196	1.0146	.01444	.98556	15	**21**
4	46	. 6964	. 3036	.8950	. 7213	.8095	.0147	. 1449	. 8551	14	56
8	47	. 6992	. 3008	.8850	. 7243	.7994	.0147	. 1454	. 8546	13	52
12	48	. 7021	. 2979	.8751	. 7273	.7894	.0148	. 1459	. 8541	12	48
16	49	. 7050	. 2950	.8652	. 7303	.7793	.0148	. 1464	. 8536	11	44
20	50	.17078	.82922	5.8554	.17333	5.7694	1.0149	.01469	.98531	10	40
24	51	. 7107	. 2893	.8456	. 7363	.7594	.0150	. 1474	. 8526	9	36
28	52	. 7136	. 2864	.8358	. 7393	.7495	.0150	. 1479	. 8521	8	32
32	53	. 7164	. 2836	.8261	. 7423	.7396	.0151	. 1484	. 8516	7	28
36	54	. 7193	. 2807	.8163	. 7453	.7297	.0151	. 1489	. 8511	6	24
40	55	.17221	.82778	5.8067	.17483	5.7199	1.0152	.01494	.98506	5	20
44	56	. 7250	. 2750	.7970	. 7513	.7101	.0152	. 1499	. 8501	4	16
48	57	. 7279	. 2721	.7874	. 7543	.7004	.0153	. 1504	. 8496	3	12
52	58	. 7307	. 2692	.7778	. 7573	.6906	.0153	. 1509	. 8491	2	8
56	59	. 7336	. 2664	.7683	. 7603	.6809	.0154	. 1514	. 8486	1	4
40	60	. 7365	. 2635	.7588	. 7633	.6713	.0154	. 1519	. 8481	0	**20**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec nt	Vrs.Cos	Sine.	M	M.S.
6h	99°	Natural.							80°		5h

0h	10°	Natural Trigonometrical Functions.							169°		11h
M.S.	M	Sine.	Vrs. Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
40	0	.17365	.82635	5.7588	.17633	5.6713	1.0154	.01519	.98481	60	**20**
4	1	. 7393	. 2606	.7493	. 7663	.6616	.0155	. 1524	. 8476	59	56
8	2	. 7422	. 2578	.7398	. 7693	.6520	.0155	. 1529	. 8471	58	52
12	3	. 7451	. 2549	.7304	. 7723	.6425	.0156	. 1534	. 8465	57	48
16	4	. 7479	. 2521	.7210	. 7753	.6329	.0156	. 1539	. 8460	56	44
20	5	.17508	.82492	5.7117	.17783	5.6234	1.0157	.01544	.98455	55	40
24	6	. 7537	. 2463	.7023	. 7813	.6140	.0157	. 1550	. 8450	54	36
28	7	. 7565	. 2435	.6930	. 7843	.6045	.0158	. 1555	. 8445	53	32
32	8	. 7594	. 2406	.6838	. 7873	.5951	.0158	. 1560	. 8440	52	28
36	9	. 7622	. 2377	.6745	. 7903	.5857	.0159	. 1565	. 8435	51	24
40	10	.17651	.82349	5.6653	.17933	5.5764	1.0159	.01570	.98430	50	20
44	11	. 7680	. 2320	.6561	. 7963	.5670	.0160	. 1575	. 8425	49	16
48	12	. 7708	. 2291	.6470	. 7993	.5578	.0160	. 1580	. 8419	48	12
52	13	. 7737	. 2263	.6379	. 8023	.5485	.0161	. 1585	. 8414	47	8
56	14	. 7766	. 2234	.6288	. 8053	.5393	.0162	. 1591	. 8409	46	4
41	15	.17794	.82206	5.6197	.18083	5.5301	1.0162	.01596	.98404	45	**19**
4	16	. 7823	. 2177	.6107	. 8113	.5209	.0163	. 1601	. 8399	44	56
8	17	. 7852	. 2148	.6017	. 8143	.5117	.0163	. 1606	. 8394	43	52
12	18	. 7880	. 2120	.5928	. 8173	.5026	.0164	. 1611	. 8388	42	48
16	19	. 7909	. 2091	.5838	. 8203	.4936	.0164	. 1617	. 8383	41	44
20	20	.17937	.82062	5.5749	.18233	5.4845	1.0165	.01622	.98378	40	40
24	21	. 7966	. 2034	.5660	. 8263	.4755	.0165	. 1627	. 8373	39	36
28	22	. 7995	. 2005	.5572	. 8293	.4665	.0166	. 1632	. 8368	38	32
32	23	. 8023	. 1977	.5484	. 8323	.4575	.0166	. 1638	. 8362	37	28
36	24	. 8052	. 1948	.5396	. 8353	.4486	.0167	. 1643	. 8357	36	24
40	25	.18080	.81919	5.5308	.18383	5.4396	1.0167	.01648	.98352	35	20
44	26	. 8109	. 1891	.5221	. 8413	.4308	.0168	. 1653	. 8347	34	16
48	27	. 8138	. 1862	.5134	. 8444	.4219	.0169	. 1659	. 8341	33	12
52	28	. 8166	. 1834	.5047	. 8474	.4131	.0169	. 1664	. 8336	32	8
56	29	. 8195	. 1805	.4960	. 8504	.4043	.0170	. 1669	. 8331	31	4
42	30	.18223	.81776	5.4874	.18534	5.3955	1.0170	.01674	.98325	30	**18**
4	31	. 8252	. 1748	.4788	. 8564	.3868	.0171	. 1680	. 8320	29	56
8	32	. 8281	. 1719	.4702	. 8594	.3780	.0171	. 1685	. 8315	28	52
12	33	. 8309	. 1691	.4617	. 8624	.3694	.0172	. 1690	. 8309	27	48
16	34	. 8338	. 1662	.4532	. 8654	.3607	.0172	. 1696	. 8304	26	44
20	35	.18366	.81633	5.4447	.18684	5.3521	1.0173	.01701	.98299	25	40
24	36	. 8395	. 1605	.4362	. 8714	.3434	.0174	. 1706	. 8293	24	36
28	37	. 8424	. 1576	.4278	. 8745	.3349	.0174	. 1712	. 8288	23	32
32	38	. 8452	. 1548	.4194	. 8775	.3263	.0175	. 1717	. 8283	22	28
36	39	. 8481	. 1519	.4110	. 8805	.3178	.0175	. 1722	. 8277	21	24
40	40	.18509	.81490	5.4026	.18835	5.3093	1.0176	.01728	.98272	20	20
44	41	. 8538	. 1462	.3943	. 8865	.3008	.0176	. 1733	. 8267	19	16
48	42	. 8567	. 1433	.3860	. 8895	.2923	.0177	. 1739	. 8261	18	12
52	43	. 8595	. 1405	.3777	. 8925	.2839	.0177	. 1744	. 8256	17	8
56	44	. 8624	. 1376	.3695	. 8955	.2755	.0178	. 1749	. 8250	16	4
43	45	.18652	.81348	5.3612	.18985	5.2671	1.0179	.01755	.98245	15	**17**
4	46	. 8681	. 1319	.3530	. 9016	.2588	.0179	. 1760	. 8240	14	56
8	47	. 8709	. 1290	.3449	. 9046	.2505	.0180	. 1766	. 8234	13	52
12	48	. 8738	. 1262	.3367	. 9076	.2422	.0180	. 1771	. 8229	12	48
16	49	. 8767	. 1233	.3286	. 9106	.2339	.0181	. 1777	. 8223	11	44
20	50	.18795	.81205	5.3205	.19136	5.2257	1.0181	.01782	.98218	10	40
24	51	. 8824	. 1176	.3124	. 9166	.2174	.0182	. 1788	. 8212	9	36
28	52	. 8852	. 1147	.3044	. 9197	.2092	.0182	. 1793	. 8207	8	32
32	53	. 8881	. 1119	.2963	. 9227	.2011	.0183	. 1799	. 8201	7	28
36	54	. 8909	. 1090	.2883	. 9257	.1929	.0184	. 1804	. 8196	6	24
40	55	.18938	.81062	5.2803	.19287	5.1848	1.0184	.01810	.98190	5	20
44	56	. 8967	. 1033	.2724	. 9317	.1767	.0185	. 1815	. 8185	4	16
48	57	. 8995	. 1005	.2645	. 9347	.1686	.0185	. 1821	. 8179	3	12
52	58	. 9024	. 0976	.2566	. 9378	.1606	.0186	. 1826	. 8174	2	8
56	59	. 9052	. 0948	.2487	. 9408	.1525	.0186	. 1832	. 8168	1	4
44	60	. 9081	. 0919	.2408	. 9438	.1445	.0187	. 1837	. 8163	0	**16**
M.S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M.S.
6h	100°	Natural.							79°		5h

0^h 11° Natural Trigonometrical Functions. 168° 11^h

M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
44	0	.19081	.80919	5.2408	.19438	5.1445	1.0187	.01837	.98163	60	**16**
4	1	. 9109	. 0890	.2330	. 9468	.1366	.0188	. 1843	. 8157	59	56
8	2	. 9138	. 0862	.2252	. 9498	.1286	.0188	. 1848	. 8152	58	52
12	3	. 9166	. 0833	.2174	. 9529	.1207	.0189	. 1854	. 8146	57	48
16	4	. 9195	. 0805	.2097	. 9559	.1128	.0189	. 1859	. 8140	56	44
20	5	.19224	.80776	5.2019	.19589	5.1049	1.0190	.01865	.98135	55	40
24	6	. 9252	. 0748	.1942	. 9619	.0970	.0191	. 1871	. 8129	54	36
28	7	. 9281	. 0719	.1865	. 9649	.0892	.0191	. 1876	. 8124	53	32
32	8	. 9309	. 0691	.1788	. 9680	.0814	.0192	. 1882	. 8118	52	28
36	9	. 9338	. 0662	.1712	. 9710	.0736	.0192	. 1887	. 8112	51	24
40	10	.19366	.80634	5.1636	.19740	5.0658	1.0193	.01893	.98107	50	20
44	11	. 9395	. 0605	.1560	. 9770	.0581	.0193	. 1899	. 8101	49	16
48	12	. 9423	. 0576	.1484	. 9800	.0504	.0194	. 1904	. 8095	48	12
52	13	. 9452	. 0548	.1409	. 9831	.0427	.0195	. 1910	. 8090	47	8
56	14	. 9480	. 0519	.1333	. 9861	.0350	.0195	. 1916	. 8084	46	4
45	15	.19509	.80491	5.1258	.19891	5.0273	1.0196	.01921	.98078	45	**15**
4	16	. 9537	. 0462	.1183	. 9921	.0197	.0196	. 1927	. 8073	44	56
8	17	. 9566	. 0434	.1109	. 9952	.0121	.0197	. 1933	. 8067	43	52
12	18	. 9595	. 0405	.1034	. 9982	.0045	.0198	. 1938	. 8061	42	48
16	19	. 9623	. 0377	.0960	.20012	4.9969	.0198	. 1944	. 8056	41	44
20	20	.19652	.80348	5.0886	.20042	4.9894	1.0199	.01950	.98050	40	40
24	21	. 9680	. 0320	.0812	. 0073	.9819	.0199	. 1956	. 8044	39	36
28	22	. 9709	. 0291	.0739	. 0103	.9744	.0200	. 1961	. 8039	38	32
32	23	. 9737	. 0263	.0666	. 0133	.9669	.0201	. 1967	. 8033	37	28
36	24	. 9766	. 0234	.0593	. 0163	.9594	.0201	. 1973	. 8027	36	24
40	25	.19794	.80206	5.0520	.20194	4.9520	1.0202	.01979	.98021	35	20
44	26	. 9823	. 0177	.0447	. 0224	.9446	.0202	. 1984	. 8016	34	16
48	27	. 9851	. 0149	.0375	. 0254	.9372	.0203	. 1990	. 8010	33	12
52	28	. 9880	. 0120	.0302	. 0285	.9298	.0204	. 1996	. 8004	32	8
56	29	. 9908	. 0092	.0230	. 0315	.9225	.0204	. 2002	. 7998	31	4
46	30	.19937	.80063	5.0158	.20345	4.9151	1.0205	.02007	.97992	30	**14**
4	31	. 9965	. 0035	.0087	. 0375	.9078	.0205	. 2013	. 7987	29	56
8	32	. 9994	. 0006	.0015	. 0406	.9006	.0206	. 2019	. 7981	28	52
12	33	.20022	.79978	4.9944	. 0436	.8933	.0207	. 2025	. 7975	27	48
16	34	.20051	.79949	4.9873	. 0466	.8860	.0207	. 2031	. 7969	26	44
20	35	.20079	.79921	4.9802	.20497	4.8788	1.0208	.02037	.97963	25	40
24	36	. 0108	. 9892	.9732	. 0527	.8716	.0208	. 2042	. 7957	24	36
28	37	. 0136	. 9863	.9661	. 0557	.8644	.0209	. 2048	. 7952	23	32
32	38	. 0165	. 9835	.9591	. 0588	.8573	.0210	. 2054	. 7946	22	28
36	39	. 0193	. 9807	.9521	. 0618	.8501	.0210	. 2060	. 7940	21	24
40	40	.20222	.79778	4.9452	.20648	4.8430	1.0211	.02066	.97934	20	20
44	41	. 0250	. 9750	.9382	. 0679	.8359	.0211	. 2072	. 7928	19	16
48	42	. 0279	. 9721	.9313	. 0709	.8288	.0212	. 2078	. 7922	18	12
52	43	. 0307	. 9693	.9243	. 0739	.8217	.0213	. 2084	. 7916	17	8
56	44	. 0336	. 9664	.9175	. 0770	.8147	.0213	. 2089	. 7910	16	4
47	45	.20364	.79636	4.9106	.20800	4.8077	1.0214	.02095	.97904	15	**13**
4	46	. 0393	. 9607	.9037	. 0830	.8007	.0215	. 2101	. 7899	14	56
8	47	. 0421	. 9579	.8969	. 0861	.7937	.0215	. 2107	. 7893	13	52
12	48	. 0450	. 9550	.8901	. 0891	.7867	.0216	. 2113	. 7887	12	48
16	49	. 0478	. 9522	.8833	. 0921	.7798	.0216	. 2119	. 7881	11	44
20	50	.20506	.79493	4.8765	.20952	4.7728	1.0217	.02125	.97875	10	40
24	51	. 0535	. 9465	.8697	. 0982	.7659	.0218	. 2131	. 7869	9	36
28	52	. 0563	. 9436	.8630	. 1012	.7591	.0218	. 2137	. 7863	8	32
32	53	. 0592	. 9408	.8563	. 1043	.7522	.0219	. 2143	. 7857	7	28
36	54	. 0620	. 9379	.8496	. 1073	.7453	.0220	. 2149	. 7851	6	24
40	55	.20649	.79351	4.8429	.21104	4.7385	1.0220	.02155	.97845	5	20
44	56	. 0677	. 9323	.8362	. 1134	.7317	.0221	. 2161	. 7839	4	16
48	57	. 0706	. 9294	.8296	. 1164	.7249	.0221	. 2167	. 7833	3	12
52	58	. 0734	. 9266	.8229	. 1195	.7181	.0222	. 2173	. 7827	2	8
56	59	. 0763	. 9237	.8163	. 1225	.7114	.0223	. 2179	. 7821	1	4
48	60	. 0791	. 9209	.8097	. 1256	.7046	.0223	. 2185	. 7815	0	**12**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.

6^h 101° **Natural.** 78° 5^h

0^h	12°	Natural Trigonometrical Functions.								167°	11^h
M.S	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
48	0	.20791	.79209	4.8097	.21256	4.7046	1.0223	.02185	.97815	60	**12**
4	1	. 0820	. 9180	.8032	. 1286	.6979	.0224	. 2191	. 7809	59	56
8	2	. 0848	. 9152	.7966	. 1316	.6912	.0225	. 2197	. 7803	58	52
12	3	. 0876	. 9123	.7901	. 1347	.6845	.0225	. 2203	. 7806	57	48
16	4	. 0905	. 9105	.7835	. 1377	.6778	.0226	. 2209	. 7790	56	44
20	5	.20933	.79066	4.7770	.21408	4.6712	1.0226	.02215	.97784	55	40
24	6	. 0962	. 9038	.7706	. 1438	.6646	.0227	. 2222	. 7778	54	36
28	7	. 0990	. 9010	.7641	. 1468	.6580	.0228	. 2228	. 7772	53	32
32	8	. 1019	. 8981	.7576	. 1499	.6514	.0228	. 2234	. 7766	52	28
36	9	. 1047	. 8953	.7512	. 1529	.6448	.0229	. 2240	. 7760	51	24
40	10	.21076	.78924	4.7448	.21560	4.6382	1.0230	.02246	.97754	50	20
44	11	. 1104	. 8896	.7384	. 1590	.6317	.0230	. 2252	. 7748	49	16
48	12	. 1132	. 8867	.7320	. 1621	.6252	.0231	. 2258	. 7741	48	12
52	13	. 1161	. 8839	.7257	. 1651	.6187	.0232	. 2264	. 7735	47	8
56	14	. 1189	. 8811	.7193	. 1682	.6122	.0232	. 2271	. 7729	46	4
49	15	.21218	.78782	4.7130	.21712	4.6057	1.0233	.02277	.97723	45	**11**
4	16	. 1246	. 8754	.7067	. 1742	.5993	.0234	. 2283	. 7717	44	56
8	17	. 1275	. 8725	.7004	. 1773	.5928	.0234	. 2289	. 7711	43	52
12	18	. 1303	. 8697	.6942	. 1803	.5864	.0235	. 2295	. 7704	42	48
16	19	. 1331	. 8668	.6879	. 1834	.5800	.0235	. 2302	. 7698	41	44
20	20	.21360	.78640	4.6817	.21864	4.5736	1.0236	.02308	.97692	40	40
24	21	. 1388	. 8612	.6754	. 1895	.5673	.0237	. 2314	. 7686	39	36
28	22	. 1417	. 8583	.6692	. 1925	.5609	.0237	. 2320	. 7680	38	32
32	23	. 1445	. 8555	.6631	. 1956	.5546	.0238	. 2326	. 7673	37	28
36	24	. 1473	. 8526	.6569	. 1986	.5483	.0239	. 2333	. 7667	36	24
40	25	.21502	.78508	4.6507	.22017	4.5420	1.0239	.02339	.97661	35	20
44	26	. 1530	. 8470	.6446	. 2047	.5357	.0240	. 2345	. 7655	34	16
48	27	. 1559	. 8441	.6385	. 2078	.5294	.0241	. 2351	. 7648	33	12
52	28	. 1587	. 8413	.6324	. 2108	.5232	.0241	. 2358	. 7642	32	8
56	29	. 1615	. 8384	.6263	. 2139	.5169	.0242	. 2364	. 7636	31	4
50	30	.21644	.78356	4.6202	.22169	4.5107	1.0243	.02370	.97630	30	**10**
4	31	. 1672	. 8328	.6142	. 2200	.5045	.0243	. 2377	. 7623	29	56
8	32	. 1701	. 8299	.6081	. 2230	.4983	.0244	. 2383	. 7617	28	52
12	33	. 1729	. 8271	.6021	. 2261	.4921	.0245	. 2389	. 7611	27	48
16	34	. 1757	. 8242	.5961	. 2291	.4860	.0245	. 2396	. 7604	26	44
20	35	.21786	.78214	4.5901	.22322	4.4799	1.0246	.02402	.97598	25	40
24	36	. 1814	. 8186	.5841	. 2353	.4737	.0247	. 2408	. 7592	24	36
28	37	. 1843	. 8154	.5782	. 2383	.4676	.0247	. 2415	. 7585	23	32
32	38	. 1871	. 8129	.5722	. 2414	.4615	.0248	. 2421	. 7579	22	28
36	39	. 1899	. 8100	.5663	. 2444	.4555	.0249	. 2427	. 7573	21	24
40	40	.21928	.78072	4.5604	.22475	4.4494	1.0249	.02434	.97566	20	20
44	41	. 1956	. 8043	.5545	. 2505	.4434	.0250	. 2440	. 7560	19	16
48	42	. 1985	. 8015	.5486	. 2536	.4373	.0251	. 2446	. 7553	18	12
52	43	. 2013	. 7987	.5428	. 2566	.4313	.0251	. 2453	. 7547	17	8
56	44	. 2041	. 7959	.5369	. 2597	.4253	.0252	. 2459	. 7541	16	4
51	45	.22070	.77930	4.5311	.22628	4.4194	1.0253	.02466	.97534	15	**9**
4	46	. 2098	. 7902	.5253	. 2658	.4134	.0253	. 2472	. 7528	14	56
8	47	. 2126	. 7873	.5195	. 2689	.4074	.0254	. 2479	. 7521	13	52
12	48	. 2155	. 7845	.5137	. 2719	.4015	.0255	. 2485	. 7515	12	48
16	49	. 2183	. 7817	.5079	. 2750	.3956	.0255	. 2491	. 7508	11	44
20	50	.22211	.77788	4.5021	.22781	4.3897	1.0256	.02498	.97502	10	40
24	51	. 2240	. 7760	.4964	. 2811	.3838	.0257	. 2504	. 7495	9	36
28	52	. 2268	. 7732	.4907	. 2842	.3779	.0257	. 2511	. 7489	8	32
32	53	. 2297	. 7703	.4850	. 2872	.3721	.0258	. 2517	. 7483	7	28
36	54	. 2325	. 7675	.4793	. 2903	.3662	.0259	. 2524	. 7476	6	24
40	55	.22353	.77647	4.4736	.22934	4.3604	1.0260	.02530	.97470	5	20
44	56	. 2382	. 7618	.4679	. 2964	.3546	.0260	. 2537	. 7463	4	16
48	57	. 2410	. 7590	.4623	. 2995	.3488	.0261	. 2543	. 7457	3	12
52	58	. 2438	. 7561	.4566	. 3025	.3430	.0262	. 2550	. 7450	2	8
56	59	. 2467	. 7533	.4510	. 3056	.3372	.0262	. 2556	. 7443	1	4
52	60	. 2495	. 7505	.4454	. 3087	.3315	.0263	. 2563	. 7437	0	**8**
M. S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S
6^h	102°	Natural.								77°	5^h

0h | 13° **Natural Trigonometrical Functions.** 166° | 11h

M.S.	M	Sine.	Vrs. Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
52	0	.22495	.77505	4.4454	.23087	4.3315	1.0263	.02563	.97437	60	**8**
4	1	. 2523	. 7476	.4398	. 3117	.3257	.0264	. 2569	. 7430	59	56
8	2	. 2552	. 7448	.4342	. 3148	.3200	.0264	. 2576	. 7424	58	52
12	3	. 2580	. 7420	.4287	. 3179	.3143	.0265	. 2583	. 7417	57	48
16	4	. 2608	. 7391	.4231	. 3209	.3086	.0266	. 2589	. 7411	56	44
20	5	.22637	.77363	4.4176	.23240	4.3029	1.0266	.02596	.97404	55	40
24	6	. 2665	. 7335	.4121	. 3270	.2972	.0267	. 2602	. 7398	54	36
28	7	. 2693	. 7306	.4065	. 3301	.2916	.0268	. 2609	. 7391	53	32
32	8	. 2722	. 7278	.4011	. 3332	.2859	.0268	. 2616	. 7384	52	28
36	9	. 2750	. 7250	.3956	. 3363	.2803	.0269	. 2622	. 7378	51	24
40	10	.22778	.77221	4.3901	.23393	4.2747	1.0270	.02629	.97371	50	20
44	11	. 2807	. 7193	.3847	. 3424	.2691	.0271	. 2635	. 7364	49	16
48	12	. 2835	. 7165	.3792	. 3455	.2635	.0271	. 2642	. 7358	48	12
52	13	. 2863	. 7136	.3738	. 3485	.2579	.0272	. 2649	. 7351	47	8
56	14	. 2892	. 7108	.3684	. 3516	.2524	.0273	. 2655	. 7344	46	4
53	15	.22920	.77080	4.3630	.23547	4.2468	1.0273	.02662	.97338	45	**7**
4	16	. 2948	. 7052	.3576	. 3577	.2413	.0274	. 2669	. 7331	44	56
8	17	. 2977	. 7023	.3522	. 3608	.2358	.0275	. 2675	. 7324	43	52
12	18	. 3005	. 6995	.3469	. 3639	.2303	.0276	. 2682	. 7318	42	48
16	19	. 3033	. 6967	.3415	. 3670	.2248	.0276	. 2689	. 7311	41	44
20	20	.23061	.76938	4.3362	.23700	4.2193	1.0277	.02695	.97301	40	40
24	21	. 3090	. 6910	.3309	. 3731	.2139	.0278	. 2702	. 7298	39	36
28	22	. 3118	. 6882	.3256	. 3762	.2084	.0278	. 2709	. 7291	38	32
32	23	. 3146	. 6853	.3203	. 3793	.2030	.0279	. 2716	. 7284	37	28
36	24	. 3175	. 6825	.3150	. 3823	.1976	.0280	. 2722	. 7277	36	24
40	25	.23203	.76797	4.3098	.23854	4.1921	1.0280	.02729	.97271	35	20
44	26	. 3231	. 6769	.3045	. 3885	.1867	.0281	. 2736	. 7264	34	16
48	27	. 3260	. 6740	.2993	. 3916	.1814	.0282	. 2743	. 7257	33	12
52	28	. 3288	. 6712	.2941	. 3946	.1760	.0283	. 2749	. 7250	32	8
56	29	. 3316	. 6684	.2888	. 3977	.1706	.0283	. 2756	. 7244	31	4
54	30	.23344	.76655	4.2836	.24008	4.1653	1.0284	.02763	.97237	30	**6**
4	31	. 3373	. 6627	.2785	. 4039	.1600	.0285	. 2770	. 7230	29	56
8	32	. 3401	. 6599	.2733	. 4069	.1546	.0285	. 2777	. 7223	28	52
12	33	. 3429	. 6571	.2681	. 4100	.1493	.0286	. 2783	. 7216	27	48
16	34	. 3458	. 6542	.2630	. 4131	.1440	.0287	. 2790	. 7210	26	44
20	35	.23486	.76514	4.2579	.24162	4.1388	1.0288	.02797	.97203	25	40
24	36	. 3514	. 6486	.2527	. 4192	.1335	.0288	. 2804	. 7196	24	36
28	37	. 3542	. 6457	.2476	. 4223	.1282	.0289	. 2811	. 7189	23	32
32	38	. 3571	. 6429	.2425	. 4254	.1230	.0290	. 2818	. 7182	22	28
36	39	. 3599	. 6401	.2375	. 4285	.1178	.0291	. 2824	. 7175	21	24
40	40	.23627	.76373	4.2324	.24316	4.1126	1.0291	.02831	.97169	20	20
44	41	. 3655	. 6344	.2273	. 4346	.1073	.0292	. 2838	. 7162	19	16
48	42	. 3684	. 6316	.2223	. 4377	.1022	.0293	. 2845	. 7155	18	12
52	43	. 3712	. 6288	.2173	. 4108	.0970	.0293	. 2852	. 7148	17	8
56	44	. 3740	. 6260	.2122	. 4439	.0918	.0294	. 2859	. 7141	16	4
55	45	.23768	.76231	4.2072	.24470	4.0867	1.0295	.02866	.97134	15	**5**
4	46	. 3797	. 6203	.2022	. 4501	.0815	.0296	. 2873	. 7127	14	56
8	47	. 3825	. 6175	.1972	. 4531	.0764	.0296	. 2880	. 7120	13	52
12	48	. 3853	. 6147	.1923	. 4562	.0713	.0297	. 2886	. 7113	12	48
16	49	. 3881	. 6118	.1873	. 4593	.0662	.0298	. 2893	. 7106	11	44
20	50	.23910	.76090	4.1824	.24624	4.0611	1.0299	.02900	.97099	10	40
24	51	. 3938	. 6062	.1774	. 4655	.0560	.0299	. 2907	. 7092	9	36
28	52	. 3966	. 6034	.1725	. 4686	.0509	.0300	. 2914	. 7086	8	32
32	53	. 3994	. 6005	.1676	. 4747	.0458	.0301	. 2921	. 7079	7	28
36	54	. 4023	. 5977	.1627	. 4747	.0408	.0302	. 2928	. 7072	6	24
40	55	.24051	.75949	4.1578	.24778	4.0358	1.0302	.02935	.97065	5	20
44	56	. 4079	. 5921	.1529	. 4809	.0307	.0303	. 2942	. 7058	4	16
48	57	. 4107	. 5892	.1481	. 4840	.0257	.0304	. 2949	. 7051	3	12
52	58	. 4136	. 5864	.1432	. 4871	.0207	.0305	. 2956	. 7044	2	8
56	59	. 4164	. 5836	.1384	. 4902	.0157	.0305	. 2963	. 7037	1	4
56	60	. 4192	. 5808	.1336	. 4933	.0108	.0306	. 2970	. 7029	0	**4**
M.S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M.S.

6h | 103° **Natural.** 76° | 5h

0h	14°	Natural Trigonometrical Functions.							165°	11h
M. S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
56	0	.24192	.75808	4.1336	.24933	4.0108	1.0306	.02970	.97029	60	**4**
4	1	. 4220	. 5779	.1287	. 4964	.0058	.0307	. 2977	. 7022	59	56
8	2	. 4249	. 5751	.1239	. 4995	.0009	.0308	. 2984	. 7015	58	52
12	3	. 4277	. 5723	.1191	. 5025	.9959	.0308	. 2991	. 7008	57	48
16	4	. 4305	. 5695	.1144	. 5056	3.9910	.0309	. 2999	. 7001	56	44
20	5	.24333	.75667	4.1096	.25087	3.9861	1.0310	.03006	.96994	55	40
24	6	. 4361	. 5638	.1048	. 5118	.9812	.0311	. 3013	. 6987	54	36
28	7	. 4390	. 5610	.1001	. 5149	.9763	.0311	. 3020	. 6980	53	32
32	8	. 4418	. 5582	.0953	. 5180	.9714	.0312	. 3027	. 6973	52	28
36	9	. 4446	. 5554	.0906	. 5211	.9665	.0313	. 3034	. 6966	51	24
40	10	.24474	.75526	4.0859	.25242	3.9616	1.0314	.03041	.96959	50	20
44	11	. 4502	. 5497	.0812	. 5273	.9568	.0314	. 3048	. 6952	49	16
48	12	. 4531	. 5469	.0765	. 5304	.9520	.0315	. 3055	. 6944	48	12
52	13	. 4559	. 5441	.0718	. 5335	.9471	.0316	. 3063	. 6937	47	8
56	14	. 4587	. 5413	.0672	. 5366	.9423	.0317	. 3070	. 6930	46	4
57	15	.24615	.75385	4.0625	.25397	3.9375	1.0317	.03077	.96923	45	**3**
4	16	. 4643	. 5356	.0579	. 5428	.9327	.0318	. 3084	. 6916	44	56
8	17	. 4672	. 5328	.0532	. 5459	.9279	.0319	. 3091	. 6909	43	52
12	18	. 4700	. 5300	.0486	. 5490	.9231	.0320	. 3098	. 6901	42	48
16	19	. 4728	. 5272	.0440	. 5521	.9184	.0320	. 3106	. 6894	41	44
20	20	.24756	.75244	4.0394	.25552	3.9136	1.0321	.03113	.96887	40	40
24	21	. 4784	. 5215	.0348	. 5583	.9089	.0322	. 3120	. 6880	39	36
28	22	. 4813	. 5187	.0302	. 5614	.9042	.0323	. 3127	. 6873	38	32
32	23	. 4841	. 5159	.0256	. 5645	.8994	.0323	. 3134	. 6865	37	28
36	24	. 4869	. 5131	.0211	. 5676	.8947	.0324	. 3142	. 6858	36	24
40	25	.24897	.75103	4.0165	.25707	3.8900	1.0325	.03149	.96851	35	20
44	26	. 4925	. 5075	.0120	. 5738	.8853	.0326	. 3156	. 6844	34	16
48	27	. 4953	. 5046	.0074	. 5769	.8807	.0327	. 3163	. 6836	33	12
52	28	. 4982	. 5018	.0029	. 5800	.8760	.0327	. 3171	. 6829	32	8
56	29	. 5010	. 4990	3.9984	. 5831	.8713	.0328	. 3178	. 6822	31	4
58	30	.25038	.74962	3.9939	.25862	3.8667	1.0329	.03185	.96815	30	**2**
4	31	. 5066	. 4934	.9894	. 5893	.8621	.0330	. 3192	. 6807	29	56
8	32	. 5094	. 4906	.9850	. 5924	.8574	.0330	. 3200	. 6800	28	52
12	33	. 5122	. 4877	.9805	. 5955	.8528	.0331	. 3207	. 6793	27	48
16	34	. 5151	. 4849	.9760	. 5986	.8482	.0332	. 3214	. 6785	26	44
20	35	.25179	.74821	3.9716	.26017	3.8436	1.0333	.03222	.96778	25	40
24	36	. 5207	. 4793	.9672	. 6048	.8390	.0334	. 3229	. 6771	24	36
28	37	. 5235	. 4765	.9627	. 6079	.8345	.0334	. 3236	. 6763	23	32
32	38	. 5263	. 4737	.9583	. 6110	.8299	.0335	. 3244	. 6756	22	28
36	39	. 5291	. 4709	.9539	. 6141	.8254	.0336	. 3251	. 6749	21	24
40	40	.25319	.74680	3.9495	.26172	3.8208	1.0337	.03258	.96741	20	20
44	41	. 5348	. 4652	.9451	. 6203	.8163	.0338	. 3266	. 6734	19	16
48	42	. 5376	. 4624	.9408	. 6234	.8118	.0338	. 3273	. 6727	18	12
52	43	. 5404	. 4596	.9364	. 6266	.8073	.0339	. 3281	. 6719	17	8
56	44	. 5432	. 4568	.9320	. 6297	.8027	.0340	. 3288	. 6712	16	4
59	45	.25460	.74540	3.9277	.26328	3.7983	1.0341	.03295	.96704	15	**1**
4	46	. 5488	. 4512	.9234	. 6359	.7938	.0341	. 3303	. 6697	14	56
8	47	. 5516	. 4483	.9190	. 6390	.7893	.0342	. 3310	. 6690	13	52
12	48	. 5544	. 4455	.9147	. 6421	.7848	.0343	. 3318	. 6682	12	48
16	49	. 5573	. 4427	.9104	. 6452	.7804	.0344	. 3325	. 6675	11	44
20	50	.25601	.74399	3.9061	.26483	3.7759	1.0345	.03332	.96667	10	40
24	51	. 5629	. 4371	.9018	. 6514	.7715	.0345	. 3340	. 6660	9	36
28	52	. 5657	. 4344	.8976	. 6546	.7671	.0346	. 3347	. 6652	8	32
32	53	. 5685	. 4315	.8933	. 6577	.7627	.0347	. 3355	. 6645	7	28
36	54	. 5713	. 4287	.8890	. 6608	.7583	.0348	. 3362	. 6638	6	24
40	55	.25741	.74259	3.8848	.26639	3.7539	1.0349	.03370	.96630	5	20
44	56	. 5769	. 4230	.8805	. 6670	.7495	.0349	. 3377	. 6623	4	16
48	57	. 5798	. 4202	.8763	. 6701	.7451	.0350	. 3385	. 6615	3	12
52	58	. 5826	. 4174	.8721	. 6732	.7407	.0351	. 3392	. 6608	2	8
56	59	. 5854	. 4146	.8679	. 6764	.7364	.0352	. 3400	. 6600	1	4
60	60	. 5882	. 4118	.8637	. 6795	.7320	.0353	. 3407	. 6592	0	**0**
M. S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
6h	104°	Natural.							75°	5h

1^h 15° Natural Trigonometrical Functions. 164° 10^h

M. S.	M	Sine.	Vrs. Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M. S.
0	0	.25882	.74118	3.8637	.26795	3.7320	1.0353	.03407	.96592	60	**60**
4	1	. 5910	. 4090	.8595	. 6826	.7277	.0353	. 3415	. 6585	59	56
8	2	. 5938	. 4062	.8553	. 6857	.7234	.0354	. 3422	. 6577	58	52
12	3	. 5966	. 4034	.8512	. 6888	.7191	.0355	. 3430	. 6570	57	48
16	4	. 5994	. 4006	.8470	. 6920	.7147	.0356	. 3438	. 6562	56	44
20	5	.26022	.73978	3.8428	.26951	3.7104	1.0357	.03445	.96555	55	40
24	6	. 6050	. 3949	.8387	. 6982	.7062	.0358	. 3453	. 6547	54	36
28	7	. 6078	. 3921	.8346	. 7013	.7019	.0358	. 3460	. 6540	53	32
32	8	. 6107	. 3893	.8304	. 7044	.6976	.0359	. 3468	. 6532	52	28
36	9	. 6135	. 3865	.8263	. 7076	.6933	.0360	. 3475	. 6524	51	24
40	10	.26163	.73837	3.8222	.27107	3.6891	1.0361	.03483	.96517	50	20
44	11	. 6191	. 3809	.8181	. 7138	.6848	.0362	. 3491	. 6509	49	16
48	12	. 6219	. 3781	.8140	. 7169	.6806	.0362	. 3498	. 6502	48	12
52	13	. 6247	. 3753	.8100	. 7201	.6764	.0363	. 3506	. 6494	47	8
56	14	. 6275	. 3725	.8059	. 7232	.6722	.0364	. 3514	. 6486	46	4
1	15	.26303	.73697	3.8018	.27263	3.6679	1.0365	.03521	.96479	45	**59**
4	16	. 6331	. 3669	.7978	. 7294	.6637	.0366	. 3529	. 6471	44	56
8	17	. 6359	. 3641	.7937	. 7326	.6596	.0367	. 3536	. 6463	43	52
12	18	. 6387	. 3613	.7897	. 7357	.6554	.0367	. 3544	. 6456	42	48
16	19	. 6415	. 3585	.7857	. 7388	.6512	.0368	. 3552	. 6448	41	44
20	20	.26443	.73556	3.7816	.27419	3.6470	1.0369	.03560	.96440	40	40
24	21	.. 6471	. 3528	.7776	. 7451	.6429	.0370	. 3567	. 6433	39	36
28	22	. 6499	. 3500	.7736	. 7482	.6387	.0371	. 3575	. 6425	38	32
32	23	. 6527	. 3472	.7697	. 7513	.6346	.0371	. 3583	. 6417	37	28
36	24	. 6556	. 3444	.7657	. 7544	.6305	.0372	. 3590	. 6409	36	24
40	25	.26584	.73416	3.7617	.27576	3.6263	1.0373	.03598	.96402	35	20
44	26	. 6612	. 3388	.7577	. 7607	.6222	.0374	. 3606	. 6394	34	16
48	27	. 6640	. 3360	.7538	. 7638	.6181	.0375	. 3614	. 6386	33	12
52	28	. 6668	. 3332	.7498	. 7670	.6140	.0376	. 3621	. 6378	32	8
56	29	. 6696	. 3304	.7459	. 7701	.6100	.0376	. 3629	. 6371	31	4
2	30	.26724	.73276	3.7420	.27732	3.6059	1.0377	.03637	.96363	30	**58**
4	31	. 6752	. 3248	.7380	. 7764	.6018	.0378	. 3645	. 6355	29	56
8	32	. 6780	. 3220	.7341	. 7795	.5977	.0379	. 3652	. 6347	28	52
12	33	. 6808	. 3192	.7302	. 7826	.5937	.0380	. 3660	. 6340	27	48
16	34	. 6836	. 3164	.7263	. 7858	.5896	.0381	. 3668	. 6332	26	44
20	35	.26864	.73136	3.7224	.27889	3.5856	1.0382	.03676	.96324	25	40
24	36	. 6892	. 3108	.7186	. 7920	.5816	.0382	. 3684	. 6316	24	36
28	37	. 6920	. 3080	.7147	. 7952	.5776	.0383	. 3691	. 6308	23	32
32	38	. 6948	. 3052	.7108	. 7983	.5736	.0384	. 3699	. 6301	22	28
36	39	. 6976	. 3024	.7070	. 8014	.5696	.0385	. 3707	. 6293	21	24
40	40	.27004	.72996	3.7031	.28046	3.5656	1.0386	.03715	.96285	20	20
44	41	. 7032	. 2968	.6993	. 8077	.5616	.0387	. 3723	. 6277	19	16
48	42	. 7060	. 2940	.6955	. 8109	.5576	.0387	. 3731	. 6269	18	12
52	43	. 7088	. 2912	.6917	. 8140	.5536	.0388	. 3739	. 6261	17	8
56	44	. 7116	. 2884	.6878	. 8171	.5497	.0389	. 3746	. 6253	16	4
3	45	.27144	.72856	3.6840	.28203	3.5457	1.0390	.03754	.96245	15	**57**
4	46	. 7172	. 2828	.6802	. 8234	.5418	.0391	. 3762	. 6238	14	56
8	47	. 7200	. 2800	.6765	. 8266	.5378	.0392	. 3770	. 6230	13	52
12	48	. 7228	. 2772	.6727	. 8297	.5339	.0393	. 3778	. 6222	12	48
16	49	. 7256	. 2744	.6689	. 8328	.5300	.0393	. 3786	. 6214	11	44
20	50	.27284	.72716	3.6651	.28360	3.5261	1.0394	.03794	.96206	10	40
24	51	. 7312	. 2688	.6614	. 8391	.5222	.0395	. 3802	. 6198	9	36
28	52	. 7340	. 2660	.6576	. 8423	.5183	.0396	. 3810	. 6190	8	32
32	53	. 7368	. 2632	.6539	. 8454	.5144	.0397	. 3818	. 6182	7	28
36	54	. 7396	. 2604	.6502	. 8486	.5105	.0398	. 3826	. 6174	6	24
40	55	.27424	.72576	3.6464	.28517	3.5066	1.0399	.03834	.96166	5	20
44	56	. 7452	. 2548	.6427	. 8549	.5028	.0399	. 3842	. 6158	4	16
48	57	. 7480	. 2520	.6390	. 8580	.4989	.0400	. 3850	. 6150	3	12
52	58	. 7508	. 2492	.6353	. 8611	.4951	.0401	. 3858	. 6142	2	8
56	59	. 7536	. 2464	.6316	. 8643	.4912	.0402	. 3866	. 6134	1	4
4	60	. 7564	. 2436	.6279	. 8674	.4874	.0403	. 3874	. 6126	0	**56**
M. S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M. S.

7^h 105° **Natural.** 74° 4^h

1h 16° Natural Trigonometrical Functions. 163° 10h

M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
4	0	.27564	.72436	3.6279	.28674	3.4874	1.0403	.03874	.96126	60	**56**
4	1	. 7592	. 2408	.6243	. 8706	.4836	.0404	. 3882	. 6118	59	56
8	2	. 7620	. 2380	.6206	. 8737	.4798	.0405	. 3890	. 6110	58	52
12	3	. 7648	. 2352	.6169	. 8769	.4760	.0406	. 3898	. 6102	57	48
16	4	. 7675	. 2324	.6133	. 8800	.4722	.0406	. 3906	. 6094	56	44
20	5	.27703	.72296	3.6096	.28832	3.4684	1.0407	.03914	.96086	55	40
24	6	. 7731	. 2268	.6060	. 8863	.4646	.0408	. 3922	. 6078	54	36
28	7	. 7759	. 2240	.6024	. 8895	.4608	.0409	. 3930	. 6070	53	32
32	8	. 7787	. 2213	.5987	. 8926	.4570	.0410	. 3938	. 6062	52	28
36	9	. 7815	. 2185	.5951	. 8958	.4533	.0411	. 3946	. 6054	51	24
40	10	.27843	.72157	3.5915	.28990	3.4495	1.0412	.03954	.96045	50	20
44	11	. 7871	. 2129	.5879	. 9021	.4458	.0413	. 3962	. 6037	49	16
48	12	. 7899	. 2101	.5843	. 9053	.4420	.0413	. 3971	. 6029	48	12
52	13	. 7927	. 2073	.5807	. 9084	.4383	.0414	. 3979	. 6021	47	8
56	14	. 7955	. 2045	.5772	. 9116	.4346	.0415	. 3987	. 6013	46	4
5	15	.27983	.72017	3.5736	.29147	3.4308	1.0416	.03995	.96005	45	**55**
4	16	. 8011	. 1989	.5700	. 9179	.4271	.0417	. 4003	. 5997	44	56
8	17	. 8039	. 1961	.5665	. 9210	.4234	.0418	. 4011	. 5989	43	52
12	18	. 8067	. 1933	.5629	. 9242	.4197	.0419	. 4019	. 5980	42	48
16	19	. 8094	. 1905	.5594	. 9274	.4160	.0420	. 4028	. 5972	41	44
20	20	.28122	.71877	3.5559	.29305	3.4124	1.0420	.04036	.95964	40	40
24	21	. 8150	. 1849	.5523	. 9337	.4087	.0421	. 4044	. 5956	39	36
28	22	. 8178	. 1822	.5488	. 9368	.4050	.0422	. 4052	. 5948	38	32
32	23	. 8206	. 1794	.5453	. 9400	.4014	.0423	. 4060	. 5940	37	28
36	24	. 8234	. 1766	.5418	. 9432	.3977	.0424	. 4069	. 5931	36	24
40	25	.28262	.71738	3.5383	.29463	3.3941	1.0425	.04077	.95923	35	20
44	26	. 8290	. 1710	.5348	. 9495	.3904	.0426	. 4085	. 5915	34	16
48	27	. 8318	. 1682	.5313	. 9526	.3868	.0427	. 4093	. 5907	33	12
52	28	. 8346	. 1654	.5279	. 9558	.3832	.0428	. 4101	. 5898	32	8
56	29	. 8374	. 1626	.5244	. 9590	.3795	.0428	. 4110	. 5890	31	4
6	30	.28401	.71608	3.5209	.29621	3.3759	1.0429	.04118	.95882	30	**54**
4	31	. 8429	. 1570	.5175	. 9653	.3723	.0430	. 4126	. 5874	29	56
8	32	. 8457	. 1543	.5140	. 9685	.3687	.0431	. 4134	. 5865	28	52
12	33	. 8485	. 1515	.5106	. 9716	.3651	.0432	. 4143	. 5857	27	48
16	34	. 8513	. 1487	.5072	. 9748	.3616	.0433	. 4151	. 5849	26	44
20	35	.28541	.71459	3.5037	.29780	3.3580	1.0434	.04159	.95840	25	40
24	36	. 8569	. 1431	.5003	. 9811	.3544	.0435	. 4168	. 5832	24	36
28	37	. 8597	. 1403	.4969	. 9843	.3509	.0436	. 4176	. 5824	23	32
32	38	. 8624	. 1375	.4935	. 9875	.3473	.0437	. 4184	. 5816	22	28
36	39	. 8652	. 1347	.4901	. 9906	.3438	.0438	. 4193	. 5807	21	24
40	40	.28680	.71320	3.4867	.29938	3.3402	1.0438	.04201	.95799	20	20
44	41	. 8708	. 1292	.4833	. 9970	.3367	.0439	. 4209	. 5791	19	16
48	42	. 8736	. 1264	.4799	.30001	.3332	.0440	. 4218	. 5782	18	12
52	43	. 8764	. 1236	.4766	. 0033	.3296	.0441	. 4226	. 5774	17	8
56	44	. 8792	. 1208	.4732	. 0065	.3261	.0442	. 4234	. 5765	16	4
7	45	.28820	.71180	3.4698	.30096	3.3226	1.0443	.04243	.95757	15	**53**
4	46	. 8847	. 1152	.4665	. 0128	.3191	.0444	. 4251	. 5749	14	56
8	47	. 8875	. 1125	.4632	. 0160	.3156	.0445	. 4260	. 5740	13	52
12	48	. 8903	. 1097	.4598	. 0192	.3121	.0446	. 4268	. 5732	12	48
16	49	. 8931	. 1069	.4565	. 0223	.3087	.0447	. 4276	. 5723	11	44
20	50	.28959	.71041	3.4532	.30255	3.3052	1.0448	.04285	.95715	10	40
24	51	. 8987	. 1013	.4498	. 0287	.3017	.0448	. 4293	. 5707	9	36
28	52	. 9014	. 0985	.4465	. 0319	.2983	.0449	. 4302	. 5698	8	32
32	53	. 9042	. 0958	.4432	. 0350	.2948	.0450	. 4310	. 5690	7	28
36	54	. 9070	. 0930	.4399	. 0382	.2914	.0451	. 4319	. 5681	6	24
40	55	.29098	.70902	3.4366	.30414	3.2879	1.0452	.04327	.95673	5	20
44	56	. 9126	. 0874	.4334	. 0446	.2845	.0453	. 4335	. 5664	4	16
48	57	. 9154	. 0846	.4301	. 0478	.2811	.0454	. 4344	. 5656	3	12
52	58	. 9181	. 0818	.4268	. 0509	.2777	.0455	. 4352	. 5647	2	8
56	59	. 9209	. 0791	.4236	. 0541	.2742	.0456	. 4361	. 5639	1	4
8	60	. 9237	. 0763	.4203	. 0573	.2708	.0457	. 4369	. 5630	0	**52**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.

7h 106° **Natural.** 73° 4h

1h	17°	Natural Trigonometrical Functions.							162°	10h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
8	0	.29237	.70763	3.4203	.30573	3.2708	1.0457	.04369	.95630	60	**52**
4	1	. 9265	. 0735	.4170	. 0605	.2674	.0458	. 4378	. 5622	59	56
8	2	. 9293	. 0707	.4138	. 0637	.2640	.0459	. 4386	. 5613	58	52
12	3	. 9321	. 0679	.4106	. 0668	.2607	.0460	. 4395	. 5605	57	48
16	4	. 9348	. 0651	.4073	. 0700	.2573	.0461	. 4404	. 5596	56	44
20	5	.29376	.70624	3.4041	.30732	3.2539	1.0461	.04412	.95588	55	40
24	6	. 9404	. 0596	.4009	. 0764	.2505	.0462	. 4421	. 5579	54	36
28	7	. 9432	. 0568	.3977	. 0796	.2472	.0463	. 4426	. 5571	53	32
32	8	. 9460	. 0540	.3945	. 0828	.2438	.0464	. 4438	. 5562	52	28
36	9	. 9487	. 0512	.3913	. 0859	.2405	.0465	. 4446	. 5554	51	24
40	10	.29515	.70485	3.3881	.30891	3.2371	1.0466	.04455	.95545	50	20
44	11	. 9543	. 0457	.3849	. 0923	.2338	.0467	. 4463	. 5536	49	16
48	12	. 9571	. 0429	.3817	. 0955	.2305	.0468	. 4472	. 5528	48	12
52	13	. 9598	. 0401	.3785	. 0987	.2271	.0469	. 4481	. 5519	47	8
56	14	. 9626	. 0374	.3754	. 1019	.2238	.0470	. 4489	. 5511	46	4
9	15	.29654	.70346	3.3722	.31051	3.2205	1.0471	.04498	.95502	45	**51**
4	16	. 9682	. 0318	.3690	. 1083	.2172	.0472	. 4507	. 5493	44	56
8	17	. 9710	. 0290	.3659	. 1115	.2139	.0473	. 4515	. 5485	43	52
12	18	. 9737	. 0262	.3627	. 1146	.2106	.0474	. 4524	. 5476	42	48
16	19	. 9765	. 0235	.3596	. 1178	.2073	.0475	. 4532	. 5467	41	44
20	20	.29793	.70207	3.3565	.31210	3.2041	1.0476	.04541	.95459	40	40
24	21	. 9821	. 0179	.3534	. 1242	.2008	.0477	. 4550	. 5450	39	36
28	22	. 9848	. 0151	.3502	. 1274	.1975	.0478	. 4558	. 5441	38	32
32	23	. 9876	. 0124	.3471	. 1306	.1942	.0478	. 4567	. 5433	37	28
36	24	. 9904	. 0096	.3440	. 1338	.1910	.0479	. 4576	. 5424	36	24
40	25	.29932	.70068	3.3409	.31370	3.1877	1.0480	.04585	.95415	35	20
44	26	. 9959	. 0040	.3378	. 1402	.1845	.0481	. 4593	. 5407	34	16
48	27	. 9987	. 0013	.3347	. 1434	.1813	.0482	. 4602	. 5398	33	12
52	28	.30015	.69982	.3316	. 1466	.1780	.0483	. 4611	. 5389	32	8
56	29	.30043	. 9957	.3286	. 1498	.1748	.0484	. 4619	. 5380	31	4
10	30	.30070	.69929	3.3255	.31530	3.1716	1.0485	.04628	.95372	30	**50**
4	31	. 0098	. 9902	.3224	. 1562	.1684	.0486	. 4637	. 5363	29	56
8	32	. 0126	. 9874	.3194	. 1594	.1652	.0487	. 4646	. 5354	28	52
12	33	. 0154	. 9846	.3163	. 1626	.1620	.0488	. 4654	. 5345	27	48
16	34	. 0181	. 9818	.3133	. 1658	.1588	.0489	. 4663	. 5337	26	44
20	35	.30209	.69791	3.3102	.31690	3.1556	1.0490	.04672	.95328	25	40
24	36	. 0237	. 9763	.3072	. 1722	.1524	.0491	. 4681	. 5319	24	36
28	37	. 0265	. 9735	.3042	. 1754	.1492	.0492	. 4690	. 5310	23	32
32	38	. 0292	. 9707	.3011	. 1786	.1460	.0493	. 4698	. 5301	22	28
36	39	. 0320	. 9680	.2981	. 1818	.1429	.0494	. 4707	. 5293	21	24
40	40	.30348	.69652	3.2951	.31850	3.1397	1.0495	.04716	.95284	20	20
44	41	. 0375	. 9624	.2921	. 1882	.1366	.0496	. 4725	. 5275	19	16
48	42	. 0403	. 9597	.2891	. 1914	.1334	.0497	. 4734	. 5266	18	12
52	43	. 0431	. 9569	.2861	. 1946	.1303	.0498	. 4743	. 5257	17	8
56	44	. 0459	. 9541	.2831	. 1978	.1271	.0499	. 4751	. 5248	16	4
11	45	.30486	.69513	3.2801	.32010	3.1240	1.0500	.04760	.95239	15	**49**
4	46	. 0514	. 9486	.2772	. 2042	.1209	.0501	. 4769	. 5231	14	56
8	47	. 0542	. 9458	.2742	. 2074	.1177	.0502	. 4778	. 5222	13	52
12	48	. 0569	. 9430	.2712	. 2106	.1146	.0503	. 4787	. 5213	12	48
16	49	. 0597	. 9403	.2683	2138	.1115	.0504	. 4796	. 5204	11	44
20	50	.30625	.69375	3.2653	.32171	3.1084	1.0505	.04805	.95195	10	40
24	51	. 0653	. 9347	.2624	. 2203	.1053	.0506	. 4814	. 5186	9	36
28	52	. 0680	. 9320	.2594	. 2235	.1022	.0507	. 4823	. 5177	8	32
32	53	. 0708	. 9292	.2565	. 2267	.0991	.0508	. 4832	. 5168	7	28
36	54	. 0736	. 9264	.2535	. 2299	.0960	.0509	. 4840	. 5159	6	24
40	55	.30763	.69237	3.2506	.32331	3.0930	1.0510	.04849	.95150	5	20
44	56	. 0791	. 9209	.2477	. 2363	.0899	.0511	. 4858	. 5141	4	16
48	57	. 0819	. 9181	.2448	. 2395	.0868	.0512	. 4867	. 5132	3	12
52	58	. 0846	. 9154	.2419	. 2428	.0838	.0513	. 4876	. 5124	2	8
56	59	. 0874	. 9126	.2390	. 2460	.0807	.0514	. 4885	. 5115	1	4
12	60	. 0902	. 9098	.2361	. 2492	.0777	.0515	. 4894	. 5106	0	**48**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
7h	107°				Natural.					72°	4h

1h	18°	Natural Trigonometrical Functions.								161°	10h
M.S.	M	Sine.	Vrs. Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
12	0	.30902	.69098	3.2361	.32492	3.0777	1.0515	.04894	.95106	60	**48**
4	1	. 0929	. 9071	.2332	. 2524	.0746	.0516	. 4903	. 5097	59	56
8	2	. 0957	. 9043	.2303	. 2556	.0716	.0517	. 4912	. 5088	58	52
12	3	. 0985	. 9015	.2274	. 2588	.0686	.0518	. 4921	. 5079	57	48
16	4	. 1012	. 8988	.2245	. 2621	.0655	.0519	. 4930	. 5070	56	44
20	5	.31040	.68960	3.2216	.32653	3.0625	1.0520	.04939	.95061	55	40
24	6	. 1068	. 8932	.2188	. 2685	.0595	.0521	. 4948	. 5051	54	36
28	7	. 1095	. 8905	.2159	. 2717	.0565	.0522	. 4957	. 5042	53	32
32	8	. 1123	. 8877	.2131	. 2749	.0535	.0523	. 4966	. 5033	52	28
36	9	. 1150	. 8849	.2102	. 2782	.0505	.0524	. 4975	. 5024	51	24
40	10	.31178	.68822	3.2074	.32814	3.0475	1.0525	.04985	.95015	50	20
44	11	. 1206	. 8794	.2045	. 2846	.0445	.0526	. 4994	. 5006	49	16
48	12	. 1233	. 8766	.2017	. 2878	.0415	.0527	. 5003	. 4997	48	12
52	13	. 1261	. 8739	.1989	. 2910	.0385	.0528	. 5012	. 4988	47	8
56	14	. 1289	. 8711	.1960	. 2943	.0356	.0529	. 5021	. 4979	46	4
13	15	.31316	.68684	3.1932	.32975	3.0326	1.0530	.05030	.94970	45	**47**
4	16	. 1344	. 8656	.1904	. 3007	.0296	.0531	. 5039	. 4961	44	56
8	17	. 1372	. 8628	.1876	. 3039	.0267	.0532	. 5048	. 4952	43	52
12	18	. 1399	. 8601	.1848	. 3072	.0237	.0533	. 5057	. 4942	42	48
16	19	. 1427	. 8573	.1820	. 3104	.0208	.0534	. 5066	. 4933	41	44
20	20	.31454	.68545	3.1792	.33136	3.0178	1.0535	.05076	.94924	40	40
24	21	. 1482	. 8518	.1764	. 3169	.0149	.0536	. 5085	. 4915	39	36
28	22	. 1510	. 8490	.1736	. 3201	.0120	.0537	. 5094	. 4906	38	32
32	23	. 1537	. 8463	.1708	. 3233	.0090	.0538	. 5103	. 4897	37	28
36	24	. 1565	. 8435	.1681	. 3265	.0061	.0539	. 5112	. 4888	36	24
40	25	.31592	.68407	3.1653	.33298	3.0032	1.0540	.05121	.94878	35	20
44	26	. 1620	. 8380	.1625	. 3330	3.0003	.0541	. 5131	. 4869	34	16
48	27	. 1648	. 8352	.1598	. 3362	2.9974	.0542	. 5140	. 4860	33	12
52	28	. 1675	. 8325	.1570	. 3395	.9945	.0543	. 5149	. 4851	32	8
56	29	. 1703	. 8297	.1543	. 3427	.9916	.0544	. 5158	. 4841	31	4
14	30	.31730	.68269	3.1515	.33459	2.9887	1.0545	.05168	.94832	30	**46**
4	31	. 1758	. 8242	.1488	. 3492	.9858	.0546	. 5177	. 4823	29	56
8	32	. 1786	. 8214	.1461	. 3524	.9829	.0547	. 5186	. 4814	28	52
12	33	. 1813	. 8187	.1433	. 3557	.9800	.0548	. 5195	. 4805	27	48
16	34	. 1841	. 8159	.1406	. 3589	.9772	.0549	. 5205	. 4795	26	44
20	35	.31868	.68132	3.1379	.33621	2.9743	1.0550	.05214	.94786	25	40
24	36	. 1896	. 8104	.1352	. 3654	.9714	.0551	. 5223	. 4777	24	36
28	37	. 1923	. 8076	.1325	. 3686	.9686	.0552	. 5232	. 4767	23	32
32	38	. 1951	. 8049	.1298	. 3718	.9657	.0553	. 5242	. 4758	22	28
36	39	. 1978	. 8021	.1271	. 3751	.9629	.0554	. 5251	. 4749	21	24
40	40	.32006	.67994	3.1244	.33783	2.9600	1.0555	.05260	.94740	20	20
44	41	. 2034	. 7966	.1217	. 3816	.9572	.0556	. 5270	. 4730	19	16
48	42	. 2061	. 7939	.1190	. 3848	.9544	.0557	. 5279	. 4721	18	12
52	43	. 2089	. 7911	.1163	. 3880	.9515	.0558	. 5288	. 4712	17	8
56	44	. 2116	. 7884	.1137	. 3913	.9487	.0559	. 5297	. 4702	16	4
15	45	.32144	.67856	3.1110	.33945	2.9459	1.0560	.05307	.94693	15	**45**
4	46	. 2171	. 7828	.1083	. 3978	.9431	.0561	. 5316	. 4684	14	56
8	47	. 2199	. 7801	.1057	. 4010	.9403	.0562	. 5326	. 4674	13	52
12	48	. 2226	. 7773	.1030	. 4043	.9375	.0563	. 5335	. 4665	12	48
16	49	. 2254	. 7746	.1004	4075	.9347	.0565	. 5344	. 4655	11	44
20	50	.32282	.67718	3.0977	.34108	2.9319	1.0566	.05354	.94646	10	40
24	51	. 2309	. 7691	.0951	. 4140	.9291	.0567	. 5363	. 4637	9	36
28	52	. 2337	. 7663	.0925	. 4173	.9263	.0568	. 5373	. 4627	8	32
32	53	. 2364	. 7636	.0898	. 4205	.9235	.0569	. 5382	. 4618	7	28
36	54	. 2392	. 7608	.0872	. 4238	.9208	.0570	. 5391	. 4608	6	24
40	55	.32419	.67581	3.0846	.34270	2.9180	1.0571	.05401	.94599	5	20
44	56	. 2447	. 7553	.0820	. 4303	.9152	.0572	. 5410	. 4590	4	16
48	57	. 2474	. 7526	.0793	. 4335	.9125	.0573	. 5420	. 4580	3	12
52	58	. 2502	. 7498	.0767	. 4368	.9097	.0574	. 5429	. 4571	2	8
56	59	. 2529	. 7471	.0741	. 4400	.9069	.0575	. 5439	. 4561	1	4
16	60	. 2557	. 7443	.0715	. 4433	.9042	.0576	. 5448	. 4552	0	**44**
M.S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M.S.
7h	108°	Natural.								71°	4h

1h 19° Natural Trigonometrical Functions. 160° 10h

M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
16	0	.32557	.67443	3.0715	.34433	2.9042	1.0576	.05448	.94552	60	**44**
4	1	. 2584	. 7416	.0690	. 4465	.9015	.0577	. 5458	. 4542	59	56
8	2	. 2612	. 7388	.0664	. 4498	.8987	.0578	. 5467	. 4533	58	52
12	3	. 2639	. 7361	.0638	. 4530	.8960	.0579	. 5476	. 4523	57	48
16	4	. 2667	. 7333	.0612	. 4563	.8933	.0580	. 5486	. 4514	56	44
20	5	.32694	.67306	3.0586	.34595	2.8905	1.0581	.05495	.94504	55	40
24	6	. 2722	. 7278	.0561	. 4628	.8878	.0582	. 5505	. 4495	54	36
28	7	. 2749	. 7251	.0535	. 4661	.8851	.0584	. 5515	. 4485	53	32
32	8	. 2777	. 7223	.0509	. 4693	.8824	.0585	. 5524	. 4476	52	28
36	9	. 2804	. 7196	.0484	. 4726	.8797	.0586	. 5534	. 4466	51	24
40	10	.32832	.67168	3.0458	.34758	2.8770	1.0587	.05543	.94457	50	20
44	11	. 2859	. 7141	.0433	. 4791	.8743	.0588	. 5553	. 4447	49	16
48	12	. 2887	. 7113	.0407	. 4824	.8716	.0589	. 5562	. 4438	48	12
52	13	. 2914	. 7086	.0382	. 4856	.8689	.0590	. 5572	. 4428	47	8
56	14	. 2942	. 7058	.0357	. 4889	.8662	.0591	. 5581	. 4418	46	4
17	15	.32969	.67031	3.0331	.34921	2.8636	1.0592	.05591	.94409	45	**43**
4	16	. 2996	. 7003	.0306	. 4954	.8609	.0593	. 5601	. 4399	44	56
8	17	. 3024	. 6976	.0281	. 4987	.8582	.0594	. 5610	. 4390	43	52
12	18	. 3051	. 6948	.0256	. 5019	.8555	.0595	. 5620	. 4380	42	48
16	19	. 3079	. 6921	.0231	. 5052	.8529	.0596	. 5629	. 4370	41	44
20	20	.33106	.66894	3.0206	.35085	2.8502	1.0598	.05639	.94361	40	40
24	21	. 3134	. 6866	.0181	. 5117	.8476	.0599	. 5649	. 4351	39	36
28	22	. 3161	. 6839	.0156	. 5150	.8449	.0600	. 5658	. 4341	38	32
32	23	. 3189	. 6811	.0131	. 5183	.8423	.0601	. 5668	. 4332	37	28
36	24	. 3216	. 6784	.0106	. 5215	.8396	.0602	. 5678	. 4322	36	24
40	25	.33243	.66756	3.0081	.35248	2.8370	1.0603	.05687	.94313	35	20
44	26	. 3271	. 6729	.0056	. 5281	.8344	.0604	. 5697	. 4303	34	16
48	27	. 3298	. 6701	.0031	. 5314	.8318	.0605	. 5707	. 4293	33	12
52	28	. 3326	. 6674	.0007	. 5346	.8291	.0606	. 5716	. 4283	32	8
56	29	. 3353	. 6647	2.9982	. 5379	.8265	.0607	. 5726	. 4274	31	4
18	30	.33381	.66619	2.9957	.35412	2.8239	1.0608	.05736	.94264	30	**42**
4	31	. 3408	. 6592	.9933	. 5445	.8213	.0609	. 5745	. 4254	29	56
8	32	. 3435	. 6564	.9908	. 5477	.8187	.0611	. 5755	. 4245	28	52
12	33	. 3463	. 6537	.9884	. 5510	.8161	.0612	. 5765	. 4235	27	48
16	34	. 3490	. 6510	.9859	. 5543	.8135	.0613	. 5775	. 4225	26	44
20	35	.33518	.66482	2.9835	.35576	2.8109	1.0614	.05784	.94215	25	40
24	36	. 3545	. 6455	.9810	. 5608	.8083	.0615	. 5794	. 4206	24	36
28	37	. 3572	. 6427	.9786	. 5641	.8057	.0616	. 5804	. 4196	23	32
32	38	. 3600	. 6400	.9762	. 5674	.8032	.0617	. 5814	. 4186	22	28
36	39	. 3627	. 6373	.9738	. 5707	.8006	.0618	. 5823	. 4176	21	24
40	40	.33655	.66345	2.9713	.35739	2.7980	1.0619	.05833	.94167	20	20
44	41	. 3682	. 6318	.9689	. 5772	.7954	.0620	. 5843	. 4157	19	16
48	42	. 3709	. 6290	.9665	. 5805	.7929	.0622	. 5853	. 4147	18	12
52	43	. 3737	. 6263	.9641	. 5838	.7903	.0623	. 5863	. 4137	17	8
56	44	. 3764	. 6236	.9617	. 5871	.7878	.0624	. 5872	. 4127	16	4
19	45	.33792	.66208	2.9593	.35904	2.7852	1.0625	.05882	.94118	15	**41**
4	46	. 3819	. 6181	.9569	. 5936	.7827	.0626	. 5892	. 4108	14	56
8	47	. 3846	. 6153	.9545	. 5969	.7801	.0627	. 5902	. 4098	13	52
12	48	. 3874	. 6126	.9521	. 6002	.7776	.0628	. 5912	. 4088	12	48
16	49	. 3901	. 6099	.9497	. 6035	.7751	.0629	. 5922	. 4078	11	44
20	50	.33928	.66071	2.9474	.36068	2.7725	1.0630	.05932	.94068	10	40
24	51	. 3956	. 6044	.9450	. 6101	.7700	.0632	. 5941	. 4058	9	36
28	52	. 3983	. 6017	.9426	. 6134	.7675	.0633	. 5951	. 4049	8	32
32	53	. 4011	. 5989	.9402	. 6167	.7650	.0634	. 5961	. 4039	7	28
36	54	. 4038	. 5962	.9379	. 6199	.7625	.0635	. 5971	. 4029	6	24
40	55	.34065	.65935	2.9355	.36232	2.7600	1.0636	.05981	.94019	5	20
44	56	. 4093	. 5907	.9332	. 6265	.7574	.0637	. 5991	. 4009	4	16
48	57	. 4120	. 5880	.9308	. 6298	.7549	.0638	. 6001	. 3999	3	12
52	58	. 4147	. 5853	.9285	. 6331	.7524	.0639	. 6011	. 3989	2	8
56	59	. 4175	. 5825	.9261	. 6364	.7500	.0641	. 6021	. 3979	1	4
20	60	. 4202	. 5798	.9238	. 6397	.7475	.0642	. 6031	. 3969	0	**40**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.

7h 109° **Natural.** 70° 4h

1^h	20°	Natural Trigonometrical Functions.						159°		10^h
M.S	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
20	0	.34202	.65798	2.9238	.36397	2.7475	1.0642	.06031	.93969	60	**40**
4	1	. 4229	. 5771	.9215	. 6430	.7450	.0643	. 6041	. 3959	59	56
8	2	. 4257	. 5743	.9191	. 6463	.7425	.0644	. 6051	. 3949	58	52
12	3	. 4284	. 5716	.9168	. 6496	.7400	.0645	. 6061	. 3939	57	48
16	4	. 4311	. 5689	.9145	. 6529	.7376	.0646	. 6071	. 3929	56	44
20	5	.34339	.65661	2.9122	.36562	2.7351	1.0647	.06080	.93919	55	40
24	6	. 4366	. 5634	.9098	. 6595	.7326	.0648	. 6090	. 3909	54	36
28	7	. 4393	. 5607	.9075	. 6628	.7302	.0650	. 6100	. 3899	53	32
32	8	. 4421	. 5579	.9052	. 6661	.7277	.0651	. 6110	. 3889	52	28
36	9	. 4448	. 5552	.9029	. 6694	.7252	.0652	. 6121	. 3879	51	24
40	10	.34475	.65525	2.9006	.36727	2.7228	1.0653	.06131	.93869	50	20
44	11	. 4502	. 5497	.8983	. 6760	.7204	.0654	. 6141	. 3859	49	16
48	12	. 4530	. 5470	.8960	. 6793	.7179	.0655	. 6151	. 3849	48	12
52	13	. 4557	. 5443	.8937	. 6826	.7155	.0656	. 6161	. 3839	47	8
56	14	. 4584	. 5415	.8915	. 6859	.7130	.0658	. 6171	. 3829	46	4
21	15	.34612	.65388	2.8892	.36892	2.7106	1.0659	.06181	.93819	45	**39**
4	16	. 4639	. 5361	.8869	. 6925	.7082	.0660	. 6191	. 3809	44	56
8	17	. 4666	. 5334	.8846	. 6958	.7058	.0661	. 6201	. 3799	43	52
12	18	. 4693	. 5306	.8824	. 6991	.7033	.0662	. 6211	. 3789	42	48
16	19	. 4721	. 5279	.8801	. 7024	.7009	.0663	. 6221	. 3779	41	44
20	20	.34748	.65252	2.8778	.37057	2.6985	1.0664	.06231	.93769	40	40
24	21	. 4775	. 5225	.8756	. 7090	.6961	.0666	. 6241	. 3758	39	36
28	22	. 4803	. 5197	.8733	. 7123	.6937	.0667	. 6251	. 3748	38	32
32	23	. 4830	. 5170	.8711	. 7156	.6913	.0668	. 6262	. 3738	37	28
36	24	. 4857	. 5143	.8688	. 7190	.6889	.0669	. 6272	. 3728	36	24
40	25	.34884	.65115	2.8666	.37223	2.6865	1.0670	.06282	.93718	35	20
44	26	. 4912	. 5088	.8644	. 7256	.6841	.0671	. 6292	. 3708	34	16
48	27	. 4939	. 5061	.8621	. 7289	.6817	.0673	. 6302	. 3698	33	12
52	28	. 4966	. 5034	.8599	. 7322	.6794	.0674	. 6312	. 3687	32	8
56	29	. 4993	. 5006	.8577	. 7355	.6770	.0675	. 6323	. 3677	31	4
22	30	.35021	.64979	2.8554	.37388	2.6746	1.0676	.06333	.93667	30	**38**
4	31	. 5048	. 4952	.8532	. 7422	.6722	.0677	. 6343	. 3657	29	56
8	32	. 5075	. 4925	.8510	. 7455	.6699	.0678	. 6353	. 3647	28	52
12	33	. 5102	. 4897	.8488	. 7488	.6675	.0679	. 6363	. 3637	27	48
16	34	. 5130	. 4870	.8466	. 7521	.6652	.0681	. 6373	. 3626	26	44
20	35	.35157	.64843	2.8444	.37554	2.6628	1.0682	.06384	.93616	25	40
24	36	. 5184	. 4816	.8422	. 7587	.6604	.0683	. 6394	. 3606	24	36
28	37	. 5211	. 4789	.8400	. 7621	.6581	.0684	. 6404	. 3596	23	32
32	38	. 5239	. 4761	.8378	. 7654	.6558	.0685	. 6414	. 3585	22	28
36	39	. 5266	. 4734	.8356	. 7687	.6534	.0686	. 6425	. 3575	21	24
40	40	.35293	.64707	2.8334	.37720	2.6511	1.0688	.06435	.93565	20	20
44	41	. 5320	. 4680	.8312	. 7754	.6487	.0689	. 6445	. 3555	19	16
48	42	. 5347	. 4652	.8290	. 7787	.6464	.0690	. 6456	. 3544	18	12
52	43	. 5375	. 4625	.8269	. 7820	.6441	.0691	. 6466	. 3534	17	8
56	44	. 5402	. 4598	.8247	. 7853	.6418	.0692	. 6476	. 3524	16	4
23	45	.35429	.64571	2.8225	.37887	2.6394	1.0694	.06486	.93513	15	**37**
4	46	. 5456	. 4544	.8204	. 7920	.6371	.0695	. 6497	. 3503	14	56
8	47	. 5483	. 4516	.8182	. 7953	.6348	.0696	. 6507	. 3493	13	52
12	48	. 5511	. 4489	.8160	. 7986	.6325	.0697	. 6517	. 3482	12	48
16	49	. 5538	. 4462	.8139	8020	.6302	.0698	. 6528	. 3472	11	44
20	50	.35565	.64435	2.8117	.38053	2.6279	1.0699	.06538	.93462	10	40
24	51	. 5592	. 4408	.8096	. 8086	.6256	.0701	. 6548	. 3451	9	36
28	52	. 5619	. 4380	.8074	. 8120	.6233	.0702	. 6559	. 3441	8	32
32	53	. 5647	. 4353	.8053	. 8153	.6210	.0703	. 6569	. 3431	7	28
36	54	. 5674	. 4326	.8032	. 8186	.6187	.0704	. 6579	. 3420	6	24
40	55	.35701	.64299	2.8010	.38220	2.6164	1.0705	.06590	.93410	5	20
44	56	. 5728	. 4272	.7989	. 8253	.6142	.0707	. 6600	. 3400	4	16
48	57	. 5755	. 4245	.7968	. 8286	.6119	.0708	. 6611	. 3389	3	12
52	58	. 5782	. 4217	.7947	. 8320	.6096	.0709	. 6621	. 3379	2	8
56	59	. 5810	. 4190	.7925	. 8353	.6073	.0710	. 6631	. 3368	1	4
24	60	. 5837	. 4163	.7904	. 8386	.6051	.0711	. 6642	. 3358	0	**36**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
7^h	110°	Natural.						69°			4^h

1^{h}	21°	Natural Trigonometrical Functions.								158°	10^{h}
M.S	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
24	0	.35837	.64163	2.7904	.38386	2.6051	1.0711	.06642	.93358	60	**36**
4	1	. 5864	. 4136	.7883	. 8420	.6028	.0713	. 6652	. 3348	59	56
8	2	. 5891	. 4109	.7862	. 8453	.6006	.0714	. 6663	. 3337	58	52
12	3	. 5918	. 4082	.7841	. 8486	.5983	.0715	. 6673	. 3327	57	48
16	4	. 5945	. 4055	.7820	. 8520	.5960	.0716	. 6684	. 3316	56	44
20	5	.35972	.64027	2.7799	.38553	2.5938	1.0717	.06694	.93306	55	40
24	6	. 6000	. 4000	.7778	. 8587	.5916	.0719	. 6705	. 3295	54	36
28	7	. 6027	. 3973	.7757	. 8620	.5893	.0720	. 6715	. 3285	53	32
32	8	. 6054	. 3946	.7736	. 8654	.5871	.0721	. 6726	. 3274	52	28
36	9	. 6081	. 3919	.7715	. 8687	.5848	.0722	. 6736	. 3264	51	24
40	10	.36108	63892	2.7694	.38720	2.5826	1.0723	.06747	.93253	50	20
44	11	. 6135	. 3865	.7674	. 8754	.5804	.0725	. 6757	. 3243	49	16
48	12	. 6162	. 3837	.7653	. 8787	.5781	.0726	. 6768	. 3232	48	12
52	13	. 6189	. 3810	.7632	. 8821	.5759	.0727	. 6778	. 3222	47	8
56	14	. 6217	. 3783	.7611	. 8854	.5737	.0728	. 6789	. 3211	46	4
25	15	.36244	.63756	2.7591	.38888	2.5715	1.0729	.06799	.93201	45	**35**
4	16	. 6271	. 3729	.7570	. 8921	.5693	.0731	. 6810	. 3190	44	56
8	17	. 6298	. 3702	.7550	. 8955	.5671	.0732	. 6820	. 3180	43	52
12	18	. 6325	. 3675	.7529	. 8988	.5649	.0733	. 6831	. 3169	42	48
16	19	. 6352	. 3648	.7509	. 9022	.5627	.0734	. 6841	. 3158	41	44
20	20	.36379	.63621	2.7488	.39055	2.5605	1.0736	.06852	.93148	40	40
24	21	. 6406	. 3593	.7468	. 9089	.5583	.0737	. 6863	. 3137	39	36
28	22	. 6433	. 3566	.7447	. 9122	.5561	.0738	. 6873	. 3127	38	32
32	23	. 6460	. 3539	.7427	. 9156	.5539	.0739	. 6884	. 3116	37	28
36	24	. 6488	. 3512	.7406	. 9189	.5517	.0740	. 6894	. 3105	36	24
40	25	.36515	.63485	2.7386	.39223	2.5495	1.0742	.06905	.93095	35	20
44	26	. 6542	. 3458	.7366	. 9257	.5473	.0743	. 6916	. 3084	34	16
48	27	. 6569	. 3431	.7346	. 9290	.5451	.0744	. 6926	3074	33	12
52	28	. 6596	. 3404	.7325	. 9324	.5430	.0745	. 6937	. 3063	32	8
56	29	. 6623	. 3377	.7305	. 9357	.5408	.0747	. 6947	. 3052	31	4
26	30	.36650	.63350	2.7285	.39391	2.5386	1.0748	.06958	.93042	30	**34**
4	31	. 6677	. 3323	.7265	. 9425	.5365	.0749	. 6969	. 3031	29	56
8	32	. 6704	. 3296	.7245	. 9458	.5343	.0750	. 6979	. 3020	28	52
12	33	. 6731	. 3269	.7225	. 9492	.5322	.0751	. 6990	. 3010	27	48
16	34	. 6758	. 3242	.7205	. 9525	.5300	.0753	. 7001	. 2999	26	44
20	35	.36785	.63214	2.7185	.39559	2.5278	1.0754	.07012	.92988	25	40
24	36	. 6812	. 3187	.7165	. 9593	.5257	.0755	. 7022	. 2978	24	36
28	37	. 6839	. 3160	.7145	. 9626	.5236	.0756	. 7033	. 2967	23	32
32	38	. 6866	. 3133	.7125	. 9660	.5214	.0758	. 7044	. 2956	22	28
36	39	. 6893	. 3106	.7105	. 9694	.5193	.0759	. 7054	. 2945	21	24
40	40	.36921	.63079	2.7085	.39727	2.5171	1.0760	.07065	.92935	20	20
44	41	. 6948	. 3052	.7065	. 9761	.5150	.0761	. 7076	. 2924	19	16
48	42	. 6975	. 3025	.7045	. 9795	.5129	.0763	. 7087	. 2913	18	12
52	43	. 7002	. 2998	.7026	. 9828	.5108	.0764	. 7097	. 2902	17	8
56	44	. 7029	. 2971	.7006	. 9862	.5086	.0765	. 7108	. 2892	16	4
27	45	.37056	.62944	2.6986	.39896	2.5065	1.0766	.07119	.92881	15	**33**
4	46	. 7083	. 2917	.6967	. 9930	.5044	.0768	. 7130	. 2870	14	56
8	47	. 7110	. 2890	.6947	. 9963	.5023	.0769	. 7141	. 2859	13	52
12	48	. 7137	. 2863	.6927	. 9997	.5002	.0770	. 7151	. 2848	12	48
16	49	. 7164	. 2836	.6908	.40031	.4981	.0771	. 7162	. 2838	11	44
20	50	.37191	.62809	2.6888	.40065	2.4960	1.0773	.07173	.92827	10	40
24	51	. 7218	. 2782	.6869	. 0098	.4939	.0774	. 7184	. 2816	9	36
28	52	. 7245	. 2755	.6849	. 0132	.4918	.0775	. 7195	. 2805	8	32
32	53	. 7272	. 2728	.6830	. 0166	.4897	.0776	. 7205	. 2794	7	28
36	54	. 7299	. 2701	.6810	. 0200	.4876	.0778	. 7216	. 2784	6	24
40	55	.37326	.62674	2.6791	.40233	2.4855	1.0779	.07227	.92773	5	20
44	56	. 7353	. 2647	.6772	. 0267	.4834	.0780	. 7238	. 2762	4	16
48	57	. 7380	. 2620	.6752	. 0301	.4813	.0781	. 7249	. 2751	3	12
52	58	. 7407	. 2593	.6733	. 0335	.4792	.0783	. 7260	. 2740	2	8
56	59	. 7434	. 2566	.6714	. 0369	.4772	.0784	. 7271	. 2729	1	4
28	60	. 7461	. 2539	.6695	. 0403	.4751	.0785	. 7282	. 2718	0	**32**
M.S.	M	Cosine.	Vrs.Sin	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
7^{h}	111°	Natural.								68°	4^{h}

1h	22°	Natural Trigonometrical Functions.							157°		10h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
28	0	.37461	.62539	2.6695	.40403	2.4751	1.0785	.07282	.92718	60	**32**
4	1	. 7488	. 2512	.6675	. 0436	.4730	.0787	. 7292	. 2707	59	56
8	2	. 7514	. 2485	.6656	. 0470	.4709	.0788	. 7303	. 2696	58	52
12	3	. 7541	. 2458	.6637	. 0504	.4689	.0789	. 7314	. 2686	57	48
16	4	. 7568	. 2431	.6618	. 0538	.4668	.0790	. 7325	. 2675	56	44
20	5	.37595	.62404	2.6599	.40572	2.4647	1.0792	.07336	.92664	55	40
24	6	. 7622	. 2377	.6580	. 0606	.4627	.0793	. 7347	. 2653	54	36
28	7	. 7649	. 2351	.6561	. 0640	.4606	.0794	. 7358	. 2642	53	32
32	8	. 7676	. 2324	.6542	. 0673	.4586	.0795	. 7369	. 2631	52	28
36	9	. 7703	. 2297	.6523	. 0707	.4565	.0797	. 7380	. 2620	51	24
40	10	.37730	.62270	2.6504	.40741	2.4545	1.0798	.07391	.92609	50	20
44	11	. 7757	. 2243	.6485	. 0775	.4525	.0799	. 7402	. 2598	49	16
48	12	. 7784	. 2216	.6466	. 0809	.4504	.0801	. 7413	. 2587	48	12
52	13	. 7811	. 2189	.6447	. 0843	.4484	.0802	. 7424	. 2576	47	8
56	14	. 7838	. 2162	.6428	. 0877	.4463	.0803	. 7435	. 2565	46	4
29	15	.37865	.62135	2.6410	.40911	2.4443	1.0804	.07446	.92554	45	**31**
4	16	. 7892	. 2108	.6391	. 0945	.4423	.0806	. 7457	. 2543	44	56
8	17	. 7919	. 2081	.6372	. 0979	.4403	.0807	. 7468	. 2532	43	52
12	18	. 7946	. 2054	.6353	. 1013	.4382	.0808	. 7479	. 2521	42	48
16	19	. 7972	. 2027	.6335	. 1047	.4362	.0810	. 7490	. 2510	41	44
20	20	.37999	.62000	2.6316	.41081	2.4342	1.0811	.07501	.92499	40	40
24	21	. 8026	. 1974	.6297	. 1115	.4322	.0812	. 7512	. 2488	39	36
28	22	. 8053	. 1947	.6279	. 1149	.4302	.0813	. 7523	. 2477	38	32
32	23	. 8080	. 1920	.6260	. 1183	.4282	.0815	. 7534	. 2466	37	28
36	24	. 8107	. 1893	.6242	. 1217	.4262	.0816	. 7545	. 2455	36	24
40	25	.38134	.61866	2.6223	.41251	2.4242	1.0817	.07556	.92443	35	20
44	26	. 8161	. 1839	.6205	. 1285	.4222	.0819	. 7567	. 2432	34	16
48	27	. 8188	. 1812	.6186	. 1319	.4202	.0820	. 7579	. 2421	33	12
52	28	. 8214	. 1785	.6168	. 1353	.4182	.0821	. 7590	. 2410	32	8
56	29	. 8241	. 1758	.6150	. 1387	.4162	.0823	. 7601	. 2399	31	4
30	30	.38268	.61732	2.6131	.41421	2.4142	1.0824	.07612	.92388	30	**30**
4	31	. 8295	. 1705	.6113	. 1455	.4122	.0825	. 7623	. 2377	29	56
8	32	. 8322	. 1678	.6095	. 1489	.4102	.0826	. 7634	. 2366	28	52
12	33	. 8349	. 1651	.6076	. 1524	.4083	.0828	. 7645	. 2354	27	48
16	34	. 8376	. 1624	.6058	. 1558	.4063	.0829	. 7657	. 2343	26	44
20	35	.38403	.61597	2.6040	.41592	2.4043	1.0830	.07668	.92332	25	40
24	36	. 8429	. 1570	.6022	. 1626	.4023	.0832	. 7679	. 2321	24	36
28	37	. 8456	. 1544	.6003	. 1660	.4004	.0833	. 7690	. 2310	23	32
32	38	. 8483	. 1517	.5985	. 1694	.3984	.0834	. 7701	. 2299	22	28
36	39	. 8510	. 1490	.5967	. 1728	.3964	.0836	. 7712	. 2287	21	24
40	40	.38537	.61463	2.5949	.41762	2.3945	1.0837	.07724	.92276	20	20
44	41	. 8564	. 1436	.5931	. 1797	.3925	.0838	. 7735	. 2265	19	16
48	42	. 8591	. 1409	.5913	. 1831	.3906	.0840	. 7746	. 2254	18	12
52	43	. 8617	. 1382	.5895	. 1865	.3886	.0841	. 7757	. 2242	17	8
56	44	. 8644	. 1356	.5877	. 1899	.3867	.0842	. 7769	. 2231	16	4
31	45	.38671	.61329	2.5859	.41933	2.3847	1.0844	.07780	.92220	15	**29**
4	46	. 8698	. 1302	.5841	. 1968	.3828	.0845	. 7791	. 2209	14	56
8	47	. 8725	. 1275	.5823	. 2002	.3808	.0846	. 7802	. 2197	13	52
12	48	. 8751	. 1248	.5805	. 2036	3789	.0847	. 7814	. 2186	12	48
16	49	. 8778	. 1222	.5787	2070	.3770	.0849	. 7825	. 2175	11	44
20	50	.38805	.61195	2.5770	.42105	2.3750	1.0850	.07836	.92164	10	40
24	51	. 8832	. 1168	.5752	. 2139	.3731	.0851	. 7847	. 2152	9	36
28	52	. 8859	. 1141	.5734	. 2173	.3712	.0853	. 7859	. 2141	8	32
32	53	. 8886	. 1114	.5716	. 2207	.3692	.0854	. 7870	. 2130	7	28
36	54	. 8912	. 1088	.5699	. 2242	.3673	.0855	. 7881	. 2118	6	24
40	55	.38939	.61061	2.5681	.42276	2.3654	1.0857	.07893	.92107	5	20
44	56	. 8966	. 1034	.5663	. 2310	.3635	.0858	. 7904	. 2096	4	16
48	57	. 8993	. 1007	.5646	. 2344	.3616	.0859	. 7915	. 2084	3	12
52	58	. 9019	. 0980	.5628	. 2379	.3597	.0861	. 7927	. 2073	2	8
56	59	. 9046	. 0954	.5610	. 2413	.3577	.0862	. 7938	. 2062	1	4
32	60	. 9073	. 0927	.5593	. 2447	.3558	.0864	. 7949	. 2050	0	**28**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
7h	112°	Natural.								67°	4h

1h 23° Natural Trigonometrical Functions. 156° 10h

M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
32	0	.39073	.60927	2.5593	.42447	2.3558	1.0864	.07949	.92050	60	**28**
4	1	. 9100	. 0900	.5575	. 2482	.3539	.0865	. 7961	. 2039	59	56
8	2	. 9126	. 0873	.5558	. 2516	.3520	.0866	. 7972	. 2028	58	52
12	3	. 9153	. 0846	.5540	. 2550	.3501	.0868	. 7984	. 2016	57	48
16	4	. 9180	. 0820	.5523	. 2585	.3482	.0869	. 7995	. 2005	56	44
20	5	.39207	.60793	2.5506	.42619	2.3463	1.0870	.08006	.91993	55	40
24	6	. 9234	. 0766	.5488	. . 2654	.3445	.0872	. 8018	. 1982	54	36
28	7	. 9260	. 0739	.5471	. 2688	.3426	.0873	. 8029	. 1971	53	32
32	8	. 9287	. 0713	.5453	. 2722	.3407	.0874	. 8041	. 1959	52	28
36	9	. 9314	. 0686	.5436	. 5757	.3388	.0876	. 8052	. 1948	51	24
40	10	.39341	60659	2.5419	.42791	2.3369	1.0877	.08063	.91936	50	20
44	11	. 9367	. 0632	.5402	. 2826	.3350	.0878	. 8075	. 1925	49	16
48	12	. 9394	. 0606	.5384	. 2860	.3332	.0880	. 8086	. 1913	48	12
52	13	. 9421	. 0579	.5367	. 2894	.3313	.0881	. 8098	. 1902	47	8
56	14	. 9448	. 0552	.5350	. 2929	.3294	.0882	. 8109	. 1891	46	4
33	15	.39474	.60526	2.5333	.42963	2.3276	1.0884	.08121	.91879	45	**27**
4	16	. 9501	. 0499	.5316	. 2998	.3257	.0885	. 8132	. 1868	44	56
8	17	. 9528	. 0452	.5299	. 3032	.3238	.0886	. 8144	. 1856	43	52
12	18	. 9554	. 0445	.5281	. 3067	.3220	.0888	. 8155	. 1845	42	48
16	19	. 9581	. 0419	.5264	. 3101	.3201	.0889	. 8167	. 1833	41	44
20	20	.39608	.60392	2.5247	.43136	2.3183	1.0891	.08178	.91822	40	40
24	21	. 9635	. 0365	.5230	. 3170	.3164	.0892	. 8190	. 1810	39	36
28	22	. 9661	. 0339	.5213	. 3205	.3145	.0893	. 8201	. 1798	38	32
32	23	. 9688	. 0312	.5196	. 3239	.3127	.0895	. 8213	. 1787	37	28
36	24	. 9715	. 0285	.5179	. 3274	.3109	.0896	. 8224	. 1775	36	24
40	25	.39741	.60258	2.5163	.43308	2.3090	1.0897	.08236	.91764	35	20
44	26	. 9768	. 0232	.5146	. 3343	.3072	.0899	. 8248	. 1752	34	16
48	27	. 9795	. 0205	.5129	. 3377	.3053	.0900	. 8259	. 1741	33	12
52	28	. 9821	. 0178	.5112	. 3412	.3035	.0902	. 8271	. 1729	32	8
56	29	. 9848	. 0152	.5095	. 3447	.3017	.0903	. 8282	. 1718	31	4
34	30	.39875	.60125	2.5078	.43481	2.2998	1.0904	.08294	.91706	30	**26**
4	31	. 9901	. 0098	.5062	. 3516	.2980	.0906	. 8306	. 1694	29	56
8	32	. 9928	. 0072	.5045	. 3550	.2962	.0907	. 8317	. 1683	28	52
12	33	. 9955	. 0045	.5028	. 3585	.2944	.0908	. 8329	. 1671	27	48
16	34	. 9981	. 0018	.5011	. 3620	.2925	.0910	. 8340	. 1659	26	44
20	35	.40008	.59992	2.4995	.43654	2.2907	1.0911	.08352	.91648	25	40
24	36	. 0035	. 9965	.4978	. 3689	.2889	.0913	. 8364	. 1636	24	36
28	37	. 0061	. 9938	.4961	. 3723	.2871	.0914	. 8375	. 1625	23	32
32	38	. 0088	. 9912	.4945	. 3758	.2853	.0915	. 8387	. 1643	22	28
36	39	. 0115	. 9885	.4928	. 3793	.2835	.0917	. 8399	. 1601	21	24
40	40	.40141	.59858	2.4912	.43827	2.2817	1.0918	.08410	.91590	20	20
44	41	. 0168	. 9832	.4895	. 3862	.2799	.0920	. 8422	. 1578	19	16
48	42	. 0195	. 9805	.4879	. 3897	.2781	.0921	. 8434	. 1566	18	12
52	43	. 0221	. 9778	.4862	. 3932	.2763	.0922	. 8445	. 1554	17	8
56	44	. 0248	. 9752	.4846	. 3966	.2745	.0924	. 8457	. 1543	16	4
35	45	.40275	.59725	2.4829	.44001	2.2727	1.0925	.08469	.91531	15	**25**
4	46	. 0301	. 9699	.4813	. 4036	.2709	.0927	. 8480	. 1519	14	56
8	47	. 0328	. 9672	.4797	. 4070	.2691	.0928	. 8492	. 1508	13	52
12	48	. 0354	. 9645	.4780	. 4105	.2673	.0929	. 8504	. 1496	12	48
16	49	. 0381	. 9619	.4764	. 4140	.2655	.0931	. 8516	. 1484	11	44
20	50	.40408	.59592	2.4748	.44175	2.2637	1.0932	.08527	.91472	10	40
24	51	. 0434	. 9566	.4731	. 4209	.2619	.0934	. 8539	. 1461	9	36
28	52	. 0461	. 9539	.4715	. 4244	.2602	.0935	. 8551	. 1449	8	32
32	53	. 0487	. 9512	.4699	. 4279	.2584	.0936	. 8563	. 1437	7	28
36	54	. 0514	. 9486	.4683	. 4314	.2566	.0938	. 8575	. 1425	6	24
40	55	.40541	.59469	2.4666	.44349	2.2548	1.0939	.08586	.91414	5	20
44	56	. 0567	. 9433	.4650	. 4383	.2531	.0941	. 8598	. 1402	4	16
48	57	. 0594	. 9406	.4634	. 4418	.2513	.0942	. 8610	. 1390	3	12
52	58	. 0620	. 9379	.4618	. 4453	.2495	.0943	. 8622	. 1378	2	8
56	59	. 0647	. 9353	.4602	. 4488	.2478	.0945	. 8634	. 1366	1	4
36	60	. 0674	. 9326	.4586	. 4523	.2460	.0946	. 8645	. 1354	0	**24**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.

7h 113° **Natural.** 66° 4h

1h	24°			Natural Trigonometrical Functions.					155°		10h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
36	0	.40674	.59326	2.4586	.44523	2.2460	1.0946	.08645	.91354	60	**24**
4	1	.0700	.9300	.4570	.4558	.2443	.0948	.8657	.1343	59	56
8	2	.0727	.9273	.4554	.4593	.2425	.0949	.8669	.1331	58	52
12	3	.0753	.9247	.4538	.4627	.2408	.0951	.8681	.1319	57	48
16	4	.0780	.9220	.4522	.4662	.2390	.0952	.8693	.1307	56	44
20	5	.40806	.59193	2.4506	.44697	2.2373	1.0953	.08705	.91295	55	40
24	6	.0833	.9167	.4490	.4732	.2355	.0955	.8716	.1283	54	36
28	7	.0860	.9140	.4474	.4767	.2338	.0956	.8728	.1271	53	32
32	8	.0886	.9114	.4458	.4802	.2320	.0958	.8740	.1260	52	28
36	9	.0913	.9087	.4442	.4837	.2303	.0959	.8752	.1248	51	24
40	10	.40939	59061	2.4426	.44872	2.2286	1.0961	.08764	.91236	50	20
44	11	.0966	.9034	.4411	.4907	.2268	.0962	.8776	.1224	49	16
48	12	.0992	.9008	.4395	.4942	.2251	.0963	.8788	.1212	48	12
52	13	.1019	.8981	.4379	.4977	.2234	.0965	.8800	.1200	47	8
56	14	.1045	.8955	.4363	.5012	.2216	.0966	.8812	.1188	46	4
37	15	.41072	.58928	2.4347	.45047	2.2199	1.0968	.08824	.91176	45	**23**
4	16	.1098	.8901	.4332	.5082	.2182	.0969	.8836	.1164	44	56
8	17	.1125	.8875	.4316	.5117	.2165	.0971	.8848	.1152	43	52
12	18	.1151	.8848	.4300	.5152	.2147	.0972	.8860	.1140	42	48
16	19	.1178	.8822	.4285	.5187	.2130	.0973	.8872	.1128	41	44
20	20	.41204	.58795	2.4269	.45222	2.2113	1.0975	.08884	.91116	40	40
24	21	.1231	.8769	.4254	.5257	.2096	.0976	.8896	.1104	39	36
28	22	.1257	.8742	.4238	.5292	.2079	.0978	.8908	.1092	38	32
32	23	.1284	.8716	.4222	.5327	.2062	.0979	.8920	.1080	37	28
36	24	.1310	.8689	.4207	.5362	.2045	.0981	.8932	.1068	36	24
40	25	.41337	.58663	2.4191	.45397	2.2028	1.0982	.08944	.91056	35	20
44	26	.1363	.8636	.4176	.5432	.2011	.0984	.8956	.1044	34	16
48	27	.1390	.8610	.4160	.5467	.1994	.0985	.8968	.1032	33	12
52	28	.1416	.8584	.4145	.5502	.1977	.0986	.8980	.1020	32	8
56	29	.1443	.8557	.4130	.5537	.1960	.0988	.8992	.1008	31	4
38	30	.41469	.58531	2.4114	.45573	2.1943	1.0989	.09004	.90996	30	**22**
4	31	.1496	.8504	.4099	.5608	.1926	.0991	.9016	.0984	29	56
8	32	.1522	.8478	.4083	.5643	.1909	.0992	.9028	.0972	28	52
12	33	.1549	.8451	.4068	.5678	.1892	.0994	.9040	.0960	27	48
16	34	.1575	.8425	.4053	.5713	.1875	.0995	.9052	.0948	26	44
20	35	.41602	.58398	2.4037	.45748	2.1859	1.0997	.09064	.90936	25	40
24	36	.1628	.8372	.4022	.5783	.1842	.0998	.9076	.0924	24	36
28	37	.1654	.8345	.4007	.5819	.1825	.1000	.9088	.0911	23	32
32	38	.1681	.8319	.3992	.5854	.1808	.1001	.9101	.0899	22	28
36	39	.1707	.8292	.3976	.5889	.1792	.1003	.9113	.0887	21	24
40	40	.41734	.58266	2.3961	.45924	2.1775	1.1004	.09125	.90875	20	20
44	41	.1760	.8240	.3946	.5960	.1758	.1005	.9137	.0863	19	16
48	42	.1787	.8213	.3931	.5995	.1741	.1007	.9149	.0851	18	12
52	43	.1813	.8187	.3916	.6030	.1725	.1008	.9161	.0839	17	8
56	44	.1839	.8160	.3901	.6065	.1708	.1010	.9173	.0826	16	4
39	45	.41866	.58134	2.3886	.46101	2.1692	1.1011	.09186	.90814	15	**21**
4	46	.1892	.8108	.3871	.6136	.1675	.1013	.9198	.0802	14	56
8	47	.1919	.8081	.3856	.6171	.1658	.1014	.9210	.0790	13	52
12	48	.1945	.8055	.3841	.6206	.1642	.1016	.9222	.0778	12	48
16	49	.1972	.8028	.3826	.6242	.1625	.1017	.9234	.0765	11	44
20	50	.41998	.58002	2.3811	.46277	2.1609	1.1019	.09247	.90753	10	40
24	51	.2024	.7975	.3796	.6312	.1592	.1020	.9259	.0741	9	36
28	52	.2051	.7949	.3781	.6348	.1576	.1022	.9271	.0729	8	32
32	53	.2077	.7923	.3766	.6383	.1559	.1023	.9283	.0717	7	28
36	54	.2103	.7896	.3751	.6418	.1543	.1025	.9296	.0704	6	24
40	55	.42130	.57870	2.3736	.46454	2.1527	1.1026	.09308	.90692	5	20
44	56	.2156	.7844	.3721	.6489	.1510	.1028	.9320	.0680	4	16
48	57	.2183	.7817	.3706	.6524	.1494	.1029	.9332	.0668	3	12
52	58	.2209	.7791	.3691	.6560	.1478	.1031	.9345	.0655	2	8
56	59	.2235	.7764	.3677	.6595	.1461	.1032	.9357	.0643	1	4
40	60	.2262	.7738	.3662	.6631	.1445	.1034	.9369	.0631	0	**20**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
7h	114°				Natural.				65°		4h

1h	25°	Natural Trigonometrical Functions.							154°		10h
M.S.	M	Sine.	Vrs. Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
40	0	.42262	.57738	2.3662	.46631	2.1445	1.1034	.09369	.90631	60	**20**
4	1	. 2288	. 7712	.3647	. 6666	.1429	.1035	. 9381	. 0618	59	56
8	2	. 2314	. 7685	.3632	. 6702	.1412	.1037	. 9394	. 0606	58	52
12	3	. 2341	. 7659	.3618	. 6737	.1396	.1038	. 9406	. 0594	57	48
16	4	. 2367	. 7633	.3603	. 6772	.1380	.1040	. 9418	. 0581	56	44
20	5	.42394	.57606	2.3588	.46808	2.1364	1.1041	.09431	.90569	55	40
24	6	. 2420	. 7580	.3574	. 6843	.1348	.1043	. 9443	. 0557	54	36
28	7	. 2446	. 7554	.3559	. 6879	.1331	.1044	. 9455	. 0544	53	32
32	8	. 2473	. 7527	.3544	. 6914	.1315	.1046	. 9468	. 0532	52	28
36	9	. 2499	. 7501	.3530	. 6950	.1299	.1047	. 9480	. 0520	51	24
40	10	.42525	.57475	2.3515	.46985	2.1283	1.1049	.09492	.90507	50	20
44	11	. 2552	. 7448	.3501	. 7021	.1267	.1050	. 9505	. 0495	49	16
48	12	. 2578	. 7422	.3486	. 7056	.1251	.1052	. 9517	. 0483	48	12
52	13	. 2604	. 7396	.3472	. 7092	.1235	.1053	. 9530	. 0470	47	8
56	14	. 2630	. 7369	.3457	. 7127	.1219	.1055	. 9542	. 0458	46	4
41	15	.42657	.57343	2.3443	.47163	2.1203	1.1056	.09554	.90445	45	**19**
4	16	. 2683	. 7317	.3428	. 7199	.1187	.1058	. 9567	. 0433	44	56
8	17	. 2709	. 7290	.3414	. 7234	.1171	.1059	. 9579	. 0421	43	52
12	18	. 2736	. 7264	.3399	. 7270	.1155	.1061	. 9592	. 0408	42	48
16	19	. 2762	7238	.3385	. 7305	.1139	.1062	. 9604	. 0396	41	44
20	20	.42788	.57212	2.3371	.47341	2.1123	1.1064	.09617	.90383	40	40
24	21	. 2815	. 7185	.3356	. 7376	.1107	.1065	. 9629	. 0371	39	36
28	22	. 2841	. 7159	.3342	. 7412	.1092	.1067	. 9641	. 0358	38	32
32	23	. 2867	. 7133	.3328	. 7448	.1076	.1068	. 9654	. 0346	37	28
36	24	. 2893	. 7106	.3313	. 7483	.1060	.1070	. 9666	. 0333	36	24
40	25	.42920	.57080	2.3299	.47519	2.1044	1.1072	.09679	.90321	35	20
44	26	. 2946	. 7054	.3285	. 7555	.1028	.1073	. 9691	. 0308	34	16
48	27	. 2972	. 7028	.3271	. 7590	.1013	.1075	. 9704	. 0296	33	12
52	28	. 2998	. 7001	.3256	. 7626	.0997	.1076	. 9716	. 0283	32	8
56	29	. 3025	. 6975	.3242	. 7662	.0981	.1078	. 9729	. 0271	31	4
42	30	.43051	.56949	2.3228	.47697	2.0965	1.1079	.09741	.90258	30	**18**
4	31	. 3077	. 6923	.3214	. 7733	.0950	.1081	. 9754	. 0246	29	56
8	32	. 3104	. 6896	.3200	. 7769	.0934	.1082	. 9766	. 0233	28	52
12	33	. 3130	. 6870	.3186	. 7805	.0918	.1084	. 9779	. 0221	27	48
16	34	. 3156	. 6844	.3172	. 7840	.0903	.1085	. 9792	. 0208	26	44
20	35	.43182	.56818	2.3158	.47876	2.0887	1.1087	.09804	.90196	25	40
24	36	. 3208	. 6791	.3143	: 7912	.0872	.1088	. 9817	. 0183	24	36
28	37	. 3235	. 6765	.3129	. 7948	.0856	.1090	. 9829	. 0171	23	32
32	38	. 3261	. 6739	.3115	. 7983	.0840	.1092	. 9842	. 0158	22	28
36	39	. 3287	. 6713	.3101	. 8019	.0825	.1093	. 9854	. 0145	21	24
40	40	.43313	.56686	2.3087	.48055	2.0809	1.1095	.09867	.90133	20	20
44	41	. 3340	. 6660	.3073	. 8091	.0794	.1096	. 9880	. 0120	19	16
48	42	. 3366	. 6634	.3059	. 8127	.0778	.1098	. 9892	. 0108	18	12
52	43	. 3392	. 6608	.3046	. 8162	.0763	.1099	. 9905	. 0095	17	8
56	44	. 3418	. 6582	.3032	. 8198	.0747	.1101	. 9917	. 0082	16	4
43	45	.43444	.56555	2.3018	.48234	2.0732	1.1102	.09930	.90070	15	**17**
4	46	. 3471	. 6529	.3004	. 8270	.0717	.1104	. 9943	. 0057	14	56
8	47	. 3497	. 6503	.2990	. 8306	.0701	.1106	. 9955	. 0044	13	52
12	48	. 3523	. 6477	.2976	. 8342	.0686	.1107	. 9968	. 0032	12	48
16	49	. 3549	. 6451	.2962	8378	.0671	.1109	. 9981	. 0019	11	44
20	50	.43575	.56424	2.2949	.48414	2.0655	1.1110	.09993	.90006	10	40
24	51	. 3602	. 6398	.2935	. 8449	.0640	.1112	.10006	.89994	9	36
28	52	. 3628	. 6372	.2921	. 8485	.0625	.1113	. 0019	. 9981	8	32
32	53	. 3654	. 6346	.2907	. 8521	.0609	.1115	. 0031	. 9968	7	28
36	54	. 3680	. 6320	.2894	. 8557	.0594	.1116	. 0044	. 9956	6	24
40	55	.43706	.56294	2.2880	.48593	2.0579	1.1118	.10057	.89943	5	20
44	56	. 3732	. 6267	.2866	. 8629	.0564	.1120	. 0070	. 9930	4	16
48	57	. 3759	. 6241	.2853	. 8665	.0548	.1121	. 0082	. 9918	3	12
52	58	. 3785	. 6215	.2839	. 8701	.0533	.1123	. 0095	. 9905	2	8
56	59	. 3811	. 6189	.2825	. 8737	.0518	.1124	. 0108	. 9892	1	4
44	60	. 3837	. 6163	.2812	. 8773	.0503	.1126	. 0121	. 9879	0	**16**
M.S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M.S.
7h	115°	Natural.							64°		4h

1ʰ	26°	Natural Trigonometrical Functions.								153°	10ʰ
M. S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M. S.
44	0	.43837	.56163	2.2812	.48773	2.0503	1.1126	.10121	.89879	60	**16**
4	1	. 3863	. 6137	.2798	. 8809	.0488	.1127	. 0133	. 9867	59	56
8	2	. 3889	. 6111	.2784	. 8845	.0473	.1129	. 0146	. 9854	58	52
12	3	. 3915	. 6084	.2771	. 8881	.0458	.1131	. 0159	. 9841	57	48
16	4	. 3942	. 6058	.2757	. 8917	.0443	.1132	. 0172	. 9828	56	44
20	5	.43968	.56032	2.2744	.48953	2.0427	1.1134	.10184	.89815	55	40
24	6	. 3994	. 6006	.2730	. 8989	.0412	.1135	. 0197	. 9803	54	36
28	7	. 4020	. 5980	.2717	. 9025	.0397	.1137	. 0210	. 9790	53	32
32	8	. 4046	. 5954	.2703	. 9062	.0382	.1139	. 0223	. 9777	52	28
36	9	. 4072	. 5928	.2690	. 9098	.0367	.1140	. 0236	. 9764	51	24
40	10	.44098	.55902	2.2676	.49134	2.0352	1.1142	.10248	.89751	50	20
44	11	. 4124	. 5875	.2663	. 9170	.0338	.1143	. 0261	. 9739	49	16
48	12	. 4150	. 5849	.2650	. 9206	.0323	.1145	. 0274	. 9726	48	12
52	13	. 4177	. 5823	.2636	. 9242	.0308	.1147	. 0287	. 9713	47	8
56	14	. 4203	. 5797	.2623	. 9278	.0293	.1148	. 0300	. 9700	46	4
45	15	.44229	.55771	2.2610	.49314	2.0278	1.1150	.10313	.89687	45	**15**
4	16	. 4255	. 5745	.2596	. 9351	.0263	.1151	. 0326	. 9674	44	56
8	17	. 4281	. 5719	.2583	. 9387	.0248	.1153	. 0338	. 9661	43	52
12	18	. 4307	. 5693	.2570	. 9423	.0233	.1155	. 0351	. 9649	42	48
16	19	. 4333	. 5667	.2556	. 9459	.0219	.1156	. 0364	. 9636	41	44
20	20	.44359	.55641	2.2543	.49495	2.0204	1.1158	.10377	.89623	40	40
24	21	. 4385	. 5615	.2530	. 9532	.0189	.1159	. 0390	. 9610	39	36
28	22	. 4411	. 5589	.2517	. 9568	.0174	.1161	. 0403	. 9597	38	32
32	23	. 4437	. 5562	.2503	. 9604	.0159	.1163	. 0416	. 9584	37	28
36	24	. 4463	. 5536	.2490	. 9640	.0145	.1164	. 0429	. 9571	36	24
40	25	.44489	.55510	2.2477	.49677	2.0130	1.1166	.10442	.89558	35	20
44	26	. 4516	. 5484	.2464	. 9713	.0115	.1167	. 0455	. 9545	34	16
48	27	. 4542	. 5458	.2451	. 9749	.0101	.1169	. 0468	. 9532	33	12
52	28	. 4568	. 5432	.2438	. 9785	.0086	.1171	. 0481	. 9519	32	8
56	29	. 4594	. 5406	.2425	. 9822	.0071	.1172	. 0493	. 9506	31	4
46	30	.44620	.55380	2.2411	.49858	2.0057	1.1174	.10506	.89493	30	**14**
4	31	. 4646	. 5354	.2398	. 9894	.0042	.1176	. 0519	. 9480	29	56
8	32	. 4672	. 5328	.2385	. 9931	.0028	.1177	. 0532	. 9467	28	52
12	33	. 4698	. 5302	.2372	. 9967	.0013	.1179	. 0545	. 9454	27	48
16	34	. 4724	. 5276	.2359	.50003	1.9998	.1180	. 0558	. 9441	26	44
20	35	.44750	.55250	2.2346	.50040	1.9984	1.1182	.10571	.89428	25	40
24	36	. 4776	. 5224	.2333	. 0076	.9969	.1184	. 0584	. 9415	24	36
28	37	. 4802	. 5198	.2320	. 0113	.9955	.1185	. 0598	. 9402	23	32
32	38	. 4828	. 5172	.2307	. 0149	.9940	.1187	. 0611	. 9389	22	28
36	39	. 4854	. 5146	.2294	. 0185	.9926	.1189	. 0624	. 9376	21	24
40	40	.44880	.55120	2.2282	.50222	1.9912	1.1190	.10637	.89363	20	20
44	41	. 4906	. 5094	.2269	. 0258	.9897	.1192	. 0650	. 9350	19	16
48	42	. 4932	. 5068	.2256	. 0295	.9883	.1193	. 0663	. 9337	18	12
52	43	. 4958	. 5042	.2243	. 0331	.9868	.1195	. 0676	. 9324	17	8
56	44	. 4984	. 5016	.2230	. 0368	.9854	.1197	. 0689	. 9311	16	4
47	45	.45010	54990	2.2217	.50404	1.9840	1.1198	.10702	.89298	15	**13**
4	46	. 5036	. 4964	.2204	. 0441	.9825	.1200	. 0715	. 9285	14	56
8	47	. 5062	. 4938	.2192	. 0477	.9811	.1202	. 0728	. 9272	13	52
12	48	. 5088	. 4912	.2179	. 0514	.9797	.1203	. 0741	. 9258	12	48
16	49	. 5114	. 4886	.2166	. 0550	.9782	.1205	. 0754	. 9245	11	44
20	50	.45140	.54860	2.2153	.50587	1.9768	1.1207	.10768	.89232	10	40
24	51	. 5166	. 4834	.2141	. 0623	.9754	.1208	. 0781	. 9219	9	36
28	52	. 5191	. 4808	.2128	. 0660	.9739	.1210	. 0794	. 9206	8	32
32	53	. 5217	. 4782	.2115	. 0696	.9725	.1212	. 0807	. 9193	7	28
36	54	. 5243	. 4756	.2103	. 0733	.9711	.1213	. 0820	. 9180	6	24
40	55	.45269	.54730	2.2090	.50769	1.9697	1.1215	.10833	.89166	5	20
44	56	. 5295	. 4705	.2077	. 0806	.9683	.1217	. 0846	. 9153	4	16
48	57	. 5321	. 4679	.2065	. 0843	.9668	.1218	. 0860	. 9140	3	12
52	58	. 5347	. 4653	.2052	. 0879	.9654	.1220	. 0873	. 9127	2	8
56	59	. 5373	. 4627	.2039	. 0916	.9640	.1222	. 0886	. 9114	1	4
48	60	. 5399	. 4601	.2027	. 0952	.9626	.1223	. 0899	. 9101	0	**12**
M. S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M. S.
7ʰ	116°	Natural.								63°	4ʰ

1^h	27°	**Natural Trigonometrical Functions.**							152°		10^h
M. S.	M	Sine.	Vrs. Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M. S.
48	0	.45399	.54601	2.2027	.50952	1.9626	1.1223	.10899	.89101	60	**12**
4	1	.5425	.4575	.2014	.0989	.9612	.1225	.0912	.9087	59	56
8	2	.5451	.4549	.2002	.1026	.9598	.1226	.0926	.9074	58	52
12	3	.5477	.4523	.1989	.1062	.9584	.1228	.0939	.9061	57	48
16	4	.5503	.4497	.1977	.1099	.9570	.1230	.0952	.9048	56	44
20	5	.45528	.54471	2.1964	.51136	1.9556	1.1231	.10965	.89034	55	40
24	6	.5554	.4445	.1952	.1172	.9542	.1233	.0979	.9021	54	36
28	7	.5580	.4420	.1939	.1209	.9528	.1235	.0992	.9008	53	32
32	8	.5606	.4394	.1927	.1246	.9514	.1237	.1005	.8995	52	28
36	9	.5632	.4368	.1914	.1283	.9500	.1238	.1018	.8981	51	24
40	10	.45658	.54342	2.1902	.51319	1.9486	1.1240	.11032	.88968	50	20
44	11	.5684	.4316	.1889	.1356	.9472	.1242	.1045	.8955	49	16
48	12	.5710	.4290	.1877	.1393	.9458	.1243	.1058	.8942	48	12
52	13	.5736	.4264	.1865	.1430	.9444	.1245	.1072	.8928	47	8
56	14	.5761	.4238	.1852	.1466	.9430	.1247	.1085	.8915	46	4
49	15	.45787	.54213	2.1840	.51503	1.9416	1.1248	.11098	.88902	45	**11**
4	16	.5813	.4187	.1828	.1540	.9402	.1250	.1112	.8888	44	56
8	17	.5839	.4161	.1815	.1577	.9388	.1252	.1125	.8875	43	52
12	18	.5865	.4135	.1803	.1614	.9375	.1253	1138	.8862	42	48
16	19	.5891	.4109	.1791	.1651	.9361	.1255	.1152	.8848	41	44
20	20	.45917	.54083	2.1778	.51687	1.9347	1.1257	.11165	.88835	40	40
24	21	.5942	.4057	.1766	.1724	.9333	.1258	.1178	.8822	39	36
28	22	.5968	.4032	.1754	.1761	.9319	.1260	.1192	.8808	38	32
32	23	.5994	.4006	.1742	.1798	.9306	.1262	.1205	.8795	37	28
36	24	.6020	.3980	.1730	.1835	.9292	.1264	.1218	.8781	36	24
40	25	.46046	.53954	2.1717	.51872	1.9278	1.1265	.11232	.88768	35	20
44	26	.6072	.3928	.1705	.1909	.9264	.1267	.1245	.8755	34	16
48	27	.6097	.3902	.1693	.1946	.9251	.1269	.1259	.8741	33	12
52	28	.6123	.3877	.1681	.1983	.9237	.1270	.1272	.8728	32	8
56	29	.6149	.3851	.1669	.2020	.9223	.1272	.1285	.8714	31	4
50	30	.46175	.53825	2.1657	.52057	1.9210	1.1274	.11299	.88701	30	**10**
4	31	.6201	.3799	.1645	.2094	.9196	.1275	.1312	.8688	29	56
8	32	.6226	.3773	.1633	.2131	.9182	.1277	.1326	.8674	28	52
12	33	.6252	.3748	.1620	.2168	.9169	.1279	.1339	.8661	27	48
16	34	.6278	.3722	.1608	.2205	.9155	.1281	.1353	.8647	26	44
20	35	.46304	.53696	2.1596	.52242	1.9142	1.1282	.11366	.88634	25	40
24	36	.6330	.3670	.1584	.2279	.9128	.1284	.1380	.8620	24	36
28	37	.6355	.3645	.1572	.2316	.9115	.1286	1393	.8607	23	32
32	38	.6381	.3619	.1560	.2353	.9101	.1287	.1407	.8593	22	28
36	39	.6407	.3593	.1548	.2390	.9088	.1289	.1420	.8580	21	24
40	40	.46433	.53567	2.1536	.52427	1.9074	1.1291	.11434	.88566	20	20
44	41	.6458	.3541	.1525	.2464	.9061	.1293	.1447	.8553	19	16
48	42	.6484	.3516	.1513	.2501	.9047	.1294	.1461	.8539	18	12
52	43	.6510	.3490	.1501	.2538	.9034	.1296	.1474	.8526	17	8
56	44	.6536	.3464	.1489	.2575	.9020	.1298	.1488	.8512	16	4
51	45	.46561	.53438	2.1477	.52612	1.9007	1.1299	.11501	.88499	15	**9**
4	46	.6587	.3413	.1465	.2650	.8993	.1301	.1515	.8485	14	56
8	47	.6613	.3387	.1453	.2687	.8980	.1303	.1528	.8472	13	52
12	48	.6639	.3361	.1441	.2724	.8967	.1305	.1542	.8458	12	48
16	49	.6664	.3336	.1430	.2761	.8953	.1306	.1555	.8444	11	44
20	50	.46690	.53310	2.1418	.52798	1.8940	1.1308	.11569	.88431	10	40
24	51	.6716	.3284	.1406	.2836	.8927	.1310	.1583	.8417	9	36
28	52	.6741	.3258	.1394	.2873	.8913	.1312	.1596	.8404	8	32
32	53	.6767	.3233	.1382	.2910	.8900	.1313	.1610	.8390	7	28
36	54	.6793	.3207	.1371	.2947	.8887	.1315	.1623	.8376	6	24
40	55	.46819	.53181	2.1359	.52984	1.8873	1.1317	.11637	.88363	5	20
44	56	.6844	.3156	.1347	.3022	.8860	.1319	.1651	.8349	4	16
48	57	.6870	.3130	.1335	.3059	.8847	.1320	.1664	.8336	3	12
52	58	.6896	.3104	.1324	.3096	.8834	.1322	.1678	.8322	2	8
56	59	.6921	.3078	.1312	.3134	.8820	.1324	.1691	.8308	1	4
52	60	.6947	.3053	.1300	.3171	.8807	.1326	.1705	.8295	0	**8**
M. S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt.	Vrs. Cos	Sine.	M	M. S.
7^h	117°	**Natural.**							62°		4^h

1^h	28°	Natural Trigonometrical Functions.							151°	10^h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
52	0	.46947	.53053	2.1300	.53171	1.8807	1.1326	.11705	.88295	60	8
4	1	.6973	.3027	.1289	.3208	.8794	.1327	.1719	.8281	59	56
8	2	.6998	.3001	.1277	.3245	.8781	.1329	.1732	.8267	58	52
12	3	.7024	.2976	.1266	.3283	.8768	.1331	.1746	.8254	57	48
16	4	.7050	.2950	.1254	.3320	.8754	.1333	.1760	.8240	56	44
20	5	.47075	.52924	2.1242	.53358	1.8741	1.1334	.11774	.88226	55	40
24	6	.7101	.2899	.1231	.3395	.8728	.1336	.1787	.8213	54	36
28	7	.7127	.2873	.1219	.3432	.8715	.1338	.1801	.8199	53	32
32	8	.7152	.2847	.1208	.3470	.8702	.1340	.1815	.8185	52	28
36	9	.7178	.2822	.1196	.3507	.8689	.1341	.1828	.8171	51	24
40	10	.47204	.52796	2.1185	.53545	1.8676	1.1343	.11842	.88158	50	20
44	11	.7229	.2770	.1173	.3582	.8663	.1345	.1856	.8144	49	16
48	12	.7255	.2745	.1162	.3619	.8650	.1347	.1870	.8130	48	12
52	13	.7281	.2719	.1150	.3657	.8637	.1349	.1883	.8117	47	8
56	14	.7306	.2694	.1139	.3694	.8624	.1350	.1897	.8103	46	4
53	15	.47332	.52668	2.1127	.53732	1.8611	1.1352	.11911	.88089	45	7
4	16	.7357	.2642	.1116	.3769	.8598	.1354	.1925	.8075	44	56
8	17	.7383	.2617	.1104	.3807	.8585	.1356	.1938	.8061	43	52
12	18	.7409	.2591	.1093	.3844	.8572	.1357	1952	.8048	42	48
16	19	.7434	.2565	.1082	.3882	.8559	.1359	.1966	.8034	41	44
20	20	.47460	.52540	2.1070	.53919	1.8546	1.1361	.11980	.88020	40	40
24	21	.7486	.2514	.1059	.3957	.8533	.1363	.1994	.8006	39	36
28	22	.7511	.2489	.1048	.3995	.8520	.1365	.2007	.7992	38	32
32	23	.7537	.2463	.1036	.4032	.8507	.1366	.2021	.7979	37	28
36	24	.7562	.2437	.1025	.4070	.8495	.1368	.2035	.7965	36	24
40	25	.47588	.52412	2.1014	.54107	1.8482	1.1370	.12049	.87951	35	20
44	26	.7613	.2386	.1002	.4145	.8469	.1372	.2063	.7937	34	16
48	27	.7639	.2361	.0991	.4183	.8456	.1373	.2077	.7923	33	12
52	28	.7665	.2335	.0980	.4220	.8443	.1375	.2090	.7909	32	8
56	29	.7690	.2310	.0969	.4258	.8430	.1377	.2104	.7895	31	4
54	30	.47716	.52284	2.0957	.54295	1.8418	1.1379	.12118	.87882	30	6
4	31	.7741	.2258	.0946	.4333	.8405	.1381	.2132	.7868	29	56
8	32	.7767	.2233	.0935	.4371	.8392	.1382	.2146	.7854	28	52
12	33	.7792	.2207	.0924	.4409	.8379	.1384	.2160	.7840	27	48
16	34	.7818	.2182	.0912	.4446	.8367	.1386	.2174	.7826	26	44
20	35	.47844	.52156	2.0901	.54484	1.8354	1.1388	.12188	.87812	25	40
24	36	.7869	.2131	.0890	.4522	.8341	.1390	.2202	.7798	24	36
28	37	.7895	.2105	.0879	.4559	.8329	.1391	2216	.7784	23	32
32	38	.7920	.2080	.0868	.4597	.8316	.1393	.2229	.7770	22	28
36	39	.7946	.2054	.0857	.4635	.8303	.1395	.2243	.7756	21	24
40	40	.47971	.52029	2.0846	.54673	1.8291	1.1397	.12257	.87742	20	20
44	41	.7997	.2003	.0835	.4711	.8278	.1399	.2271	.7728	19	16
48	42	.8022	.1978	.0824	.4748	.8265	.1401	.2285	.7715	18	12
52	43	.8048	.1952	.0812	.4786	.8253	.1402	.2299	.7701	17	8
56	44	.8073	.1927	.0801	.4824	.8240	.1404	.2313	.7687	16	4
55	45	.48099	.51901	2.0790	.54862	1.8227	1.1406	.12327	.87673	15	5
4	46	.8124	.1876	.0779	.4900	.8215	.1408	.2341	.7659	14	56
8	47	.8150	.1850	.0768	.4937	.8202	.1410	.2355	.7645	13	52
12	48	.8175	.1825	.0757	.4975	.8190	.1411	.2369	.7631	12	48
16	49	.8201	.1799	.0746	.5013	.8177	.1413	.2383	.7617	11	44
20	50	.48226	.51774	2.0735	.55051	1.8165	1.1415	.12397	.87603	10	40
24	51	.8252	.1748	.0725	.5089	.8152	.1417	.2411	.7588	9	36
28	52	.8277	.1723	.0714	.5127	.8140	.1419	.2425	.7574	8	32
32	53	.8303	.1697	.0703	.5165	.8127	.1421	.2439	.7560	7	28
36	54	.8328	.1672	.0692	.5203	.8115	.1422	.2453	.7546	6	24
40	55	.48354	.51646	2.0681	.55241	1.8102	1.1424	.12468	.87532	5	20
44	56	.8379	.1621	.0670	.5279	.8090	.1426	.2482	.7518	4	16
48	57	.8405	.1595	.0659	.5317	.8078	.1428	.2496	.7504	3	12
52	58	.8430	.1570	.0648	.5355	.8065	.1430	.2510	.7490	2	8
56	59	.8455	.1544	.0637	.5393	.8053	.1432	.2524	.7476	1	4
56	60	.8481	.1519	.0627	.5431	.8040	.1433	.2538	.7462	0	4
M.S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt.	Vrs. Cos	Sine.	M	M.S.
7^h	118°	Natural.							61°	4^h

1h	29°	Natural Trigonometrical Functions.							150°	10h
M. S.	M	Sine.	Vrs. Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M. S.
56	0	.48481	.51519	2.0627	.55431	1.8040	1.1433	.12538	.87462	60	**4**
4	1	. 8506	. 1493	.0616	. 5469	.8028	.1435	. 2552	. 7448	59	56
8	2	. 8532	. 1468	.0605	. 5507	.8016	.1437	. 2566	. 7434	58	52
12	3	. 8557	. 1443	.0594	. 5545	.8003	.1439	. 2580	. 7420	57	48
16	4	. 8583	. 1417	.0583	. 5583	.7991	.1441	. 2594	. 7405	56	44
20	5	.48608	.51392	2.0573	.55621	1.7979	1.1443	.12609	.87391	55	40
24	6	. 8633	. 1366	.0562	. 5659	.7966	.1445	. 2623	. 7377	54	36
28	7	. 8659	. 1341	.0551	. 5697	.7954	.1446	. 2637	. 7363	53	32
32	8	. 8684	. 1316	.0540	. 5735	.7942	.1448	. 2651	. 7349	52	28
36	9	. 8710	. 1290	.0530	. 5774	.7930	.1450	. 2665	. 7335	51	24
40	10	.48735	.51265	2.0519	.55812	1.7917	1.1452	.12679	.87320	50	20
44	11	. 8760	. 1239	.0508	. 5850	.7905	.1454	. 2694	. 7306	49	16
48	12	. 8786	. 1214	.0498	. 5888	.7893	.1456	. 2708	. 7292	48	12
52	13	. 8811	. 1189	.0487	. 5926	.7881	.1458	. 2722	. 7278	47	8
56	14	. 8837	. 1163	.0476	. 5964	.7868	.1459	. 2736	. 7264	46	4
57	15	.48862	.51138	2.0466	.56003	1.7856	1.1461	.12750	.87250	45	**3**
4	16	. 8887	. 1112	.0455	. 6041	.7844	.1463	. 2765	. 7235	44	56
8	17	. 8913	. 1087	.0444	: 6079	.7832	.1465	. 2779	. 7221	43	52
12	18	. 8938	. 1062	.0434	. 6117	.7820	.1467	. 2793	. 7207	42	48
16	19	. 8964	. 1036	.0423	. 6156	.7808	.1469	. 2807	. 7193	41	44
20	20	.48989	.51011	2.0413	.56194	1.7795	1.1471	.12821	.87178	40	40
24	21	. 9014	. 0986	.0402	. 6232	.7783	.1473	. 2836	. 7164	39	36
28	22	. 9040	. 0960	.0392	. 6270	.7771	.1474	. 2850	. 7150	38	32
32	23	. 9065	. 0935	.0381	. 6309	.7759	.1476	. 2864	. 7136	37	28
36	24	. 9090	. 0910	.0370	. 6347	.7747	.1478	. 2879	. 7121	36	24
40	25	.49116	.50884	2.0360	.56385	1.7735	1.1480	.12893	.87107	35	20
44	26	. 9141	. 0859	.0349	. 6424	.7723	.1482	. 2907	. 7093	34	16
48	27	. 9166	. 0834	.0339	. 6462	.7711	.1484	. 2921	. 7078	33	12
52	28	. 9192	. 0808	.0329	. 6500	.7699	.1486	. 2936	. 7064	32	8
56	29	. 9217	. 0783	.0318	. 6539	.7687	.1488	. 2950	. 7050	31	4
58	30	.49242	.50758	2.0308	.56577	1.7675	1.1489	.12964	.87035	30	**2**
4	31	. 9268	. 0732	.0297	. 6616	.7663	.1491	. 2979	. 7021	29	56
8	32	. 9293	. 0707	.0287	. 6654	.7651	.1493	. 2993	. 7007	28	52
12	33	. 9318	. 0682	.0276	. 6692	.7639	.1495	. 3007	. 6992	27	48
16	34	. 9343	. 0656	.0266	. 6731	.7627	.1497	. 3022	. 6978	26	44
20	35	.49369	.50631	2.0256	.56769	1.7615	1.1499	.13036	.86964	25	40
24	36	. 9394	. 0606	.0245	. 6808	.7603	.1501	. 3050	. 6949	24	36
28	37	. 9419	. 0580	.0235	. 6846	.7591	.1503	3065	. 6935	23	32
32	38	. 9445	. 0555	.0224	. 6885	.7579	.1505	. 3079	. 6921	22	28
36	39	. 9470	. 0530	.0214	. 6923	.7567	.1507	. 3094	. 6906	21	24
40	40	.49495	.50505	2.0204	.56962	1.7555	1.1508	.13108	.86892	20	20
44	41	. 9521	. 0479	.0194	. 7000	.7544	.1510	. 3122	. 6877	19	16
48	42	. 9546	. 0454	.0183	. 7039	.7532	.1512	. 3137	. 6863	18	12
52	43	. 9571	. 0429	.0173	. 7077	.7520	.1514	. 3151	. 6849	17	8
56	44	. 9596	. 0404	.0163	. 7116	.7508	.1516	. 3166	. 6834	16	4
59	45	.49622	.50378	2.0152	.57155	1.7496	1.1518	.13180	.86820	15	**1**
4	46	. 9647	. 0353	.0142	. 7193	.7484	.1520	. 3194	. 6805	14	56
8	47	. 9672	. 0328	.0132	. 7232	.7473	.1522	. 3209	. 6791	13	52
12	48	. 9697	. 0303	.0122	. 7270	.7461	.1524	. 3223	. 6776	12	48
16	49	. 9723	. 0277	.0111	. 7309	.7449	.1526	. 3238	. 6762	11	44
20	50	.49748	.50252	2.0101	.57348	1.7437	1.1528	.13252	.86748	10	40
24	51	. 9773	. 0227	.0091	. 7386	.7426	.1530	. 3267	. 6733	9	36
28	52	. 9798	. 0202	.0081	. 7425	.7414	.1531	. 3281	. 6719	8	32
32	53	. 9823	. 0176	.0071	. 7464	.7402	.1533	. 3296	. 6704	7	28
36	54	. 9849	. 0151	.0061	. 7502	.7390	.1535	. 3310	. 6690	6	24
40	55	.49874	.50126	2.0050	.57541	1.7379	1.1537	.13325	.86675	5	20
44	56	. 9899	. 0101	.0040	. 7580	.7367	.1539	. 3339	. 6661	4	16
48	57	. 9924	. 0076	.0030	. 7619	.7355	.1541	. 3354	. 6646	3	12
52	58	. 9950	. 0050	.0020	. 7657	.7344	.1543	. 3368	. 6632	2	8
56	59	. 9975	. 0025	.0010	. 7696	.7332	.1545	. 3383	. 6617	1	4
60	60	.50000	. 0000	.0000	. 7735	.7320	.1547	. 3397	. 6602	0	**0**
M. S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M. S.
7h	119°	Natural.								60°	4h

2^h 30° Natural Trigonometrical Functions. 149° 9^h

M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
0	0	.50000	.50000	2.0000	.57735	1.7320	1.1547	.13397	.86602	60	**60**
4	1	. 0025	.49975	1.9990	. 7774	.7309	.1549	. 3412	. 6588	59	56
8	2	. 0050	. 9950	.9980	. 7813	.7297	.1551	. 3426	. 6573	58	52
12	3	. 0075	. 9924	.9970	. 7851	.7286	.1553	. 3441	. 6559	57	48
16	4	. 0101	. 9899	.9960	. 7890	.7274	.1555	. 3456	. 6544	56	44
20	5	.50126	.49874	1.9950	.57929	1.7262	1.1557	.13470	.86530	55	40
24	6	. 0151	. 9849	.9940	. 7968	.7251	.1559	. 3485	. 6515	54	36
28	7	. 0176	. 9824	.9930	. 8007	.7239	.1561	. 3499	. 6500	53	32
32	8	. 0201	. 9799	.9920	. 8046	.7228	.1562	. 3514	. 6486	52	28
36	9	. 0226	. 9773	.9910	. 8085	.7216	.1564	. 3529	. 6471	51	24
40	10	.50252	.49748	1.9900	.58123	1.7205	1.1566	.13543	.86457	50	20
44	11	. 0277	. 9723	.9890	. 8162	.7193	.1568	. 3558	. 6442	49	16
48	12	. 0302	. 9698	.9880	. 8201	.7182	.1570	. 3572	. 6427	48	12
52	13	. 0327	. 9673	.9870	. 8240	.7170	.1572	. 3587	. 6413	47	8
56	14	. 0352	. 9648	.9860	. 8279	.7159	.1574	. 3602	. 6398	46	4
1	15	.50377	.49623	1.9850	.58318	1.7147	1.1576	.13616	.86383	45	**59**
4	16	. 0402	. 9597	.9840	. 8357	.7136	.1578	. 3631	. 6369	44	56
8	17	. 0428	. 9572	.9830	. 8396	.7124	.1580	. 3646	. 6354	43	52
12	18	. 0453	. 9547	.9820	. 8435	.7113	.1582	. 3660	. 6339	42	48
16	19	. 0478	. 9522	.9811	. 8474	.7101	.1584	. 3675	. 6325	41	44
20	20	.50503	.49497	1.9801	.58513	1.7090	1.1586	.13690	.86310	40	40
24	21	. 0528	. 9472	.9791	. 8552	.7079	.1588	. 3704	. 6295	39	36
28	22	. 0553	. 9447	.9781	. 8591	.7067	.1590	. 3719	. 6281	38	32
32	23	. 0578	. 9422	.9771	. 8630	.7056	.1592	. 3734	. 6266	37	28
36	24	. 0603	. 9397	.9761	. 8670	.7044	.1594	. 3749	. 6251	36	24
40	25	.50628	.49371	1.9752	.58709	1.7033	1.1596	.13763	.86237	35	20
44	26	. 0653	. 9346	.9742	. 8748	.7022	.1598	. 3778	. 6222	34	16
48	27	. 0679	. 9321	.9732	. 8787	.7010	.1600	. 3793	. 6207	33	12
52	28	. 0704	. 9296	.9722	. 8826	.6999	.1602	. 3807	. 6192	32	8
56	29	. 0729	. 9271	.9713	. 8865	.6988	.1604	. 3822	. 6178	31	4
2	30	.50754	.49246	1.9703	.58904	1.6977	1.1606	.13837	.86163	30	**58**
4	31	. 0779	. 9221	.9693	. 8944	.6965	.1608	. 3852	. 6148	29	56
8	32	. 0804	. 9196	.9683	. 8983	.6954	.1610	. 3867	. 6133	28	52
12	33	. 0829	. 9171	.9674	. 9022	.6943	.1612	. 3881	. 6118	27	48
16	34	. 0854	. 9146	.9664	. 9061	.6931	.1614	. 3896	. 6104	26	44
20	35	.50879	.49121	1.9654	.59100	1.6920	1.1616	.13911	.86089	25	40
24	36	. 0904	. 9096	.9645	. 9140	.6909	.1618	. 3926	. 6074	24	36
28	37	. 0929	. 9071	.9635	. 9179	.6898	.1620	. 3941	. 6059	23	32
32	38	. 0954	. 9046	.9625	. 9218	.6887	.1622	. 3955	. 6044	22	28
36	39	. 0979	. 9021	.9616	. 9258	.6875	.1624	. 3970	. 6030	21	24
40	40	.51004	.48996	1.9606	.59297	1.6864	1.1626	.13985	.86015	20	20
44	41	. 1029	. 8971	.9596	. 9336	.6853	.1628	. 4000	. 6000	19	16
48	42	. 1054	. 8946	.9587	. 9376	.6842	.1630	. 4015	. 5985	18	12
52	43	. 1079	. 8921	.9577	. 9415	.6831	.1632	. 4030	. 5970	17	8
56	44	. 1104	. 8896	.9568	. 9454	.6820	.1634	. 4044	. 5955	16	4
3	45	.51129	.48871	1.9558	.59494	1.6808	1.1636	.14059	.85941	15	**57**
4	46	. 1154	. 8846	.9549	. 9533	.6797	.1638	. 4074	. 5926	14	56
8	47	. 1179	. 8821	.9539	. 9572	.6786	.1640	. 4089	. 5911	13	52
12	48	. 1204	. 8796	.9530	. 9612	.6775	.1642	. 4104	. 5896	12	48
16	49	. 1229	. 8771	.9520	. 9651	.6764	.1644	. 4119	. 5881	11	44
20	50	.51254	.48746	1.9510	.59691	1.6753	1.1646	.14134	.85866	10	40
24	51	. 1279	. 8721	.9501	. 9730	.6742	.1648	. 4149	. 5851	9	36
28	52	. 1304	. 8696	.9491	. 9770	.6731	.1650	. 4164	. 5836	8	32
32	53	. 1329	. 8671	.9482	. 9809	.6720	.1652	. 4178	. 5821	7	28
36	54	. 1354	. 8646	.9473	. 9849	.6709	.1654	. 4193	. 5806	6	24
40	55	.51379	.48621	1.9463	.59888	1.6698	1.1656	.14208	.85791	5	20
44	56	. 1404	. 8596	.9454	. 9928	.6687	.1658	. 4223	. 5777	4	16
48	57	. 1429	. 8571	.9444	. 9967	.6676	.1660	. 4238	. 5762	3	12
52	58	. 1454	. 8546	.9435	.60007	.6665	.1662	. 4253	. 5747	2	8
56	59	. 1479	. 8521	.9425	. 0046	.6654	.1664	. 4268	. 5732	1	4
4	60	. 1504	. 8496	.9416	. 0086	.6643	.1666	. 4283	. 5717	0	**56**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.

8^h 120° **Natural.** 59° 3^h

2h	31°	Natural Trigonometrical Functions.								148°	9h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
4	0	.51504	.48496	1.9416	.60086	1.6643	1.1666	.14283	.85717	60	56
4	1	. 1529	. 8471	.9407	. 0126	.6632	.1668	. 4298	. 5702	59	56
8	2	. 1554	. 8446	.9397	. 0165	.6621	.1670	. 4313	. 5687	58	52
12	3	. 1578	. 8421	.9388	. 0205	.6610	.1672	. 4328	. 5672	57	48
16	4	. 1603	. 8396	.9378	. 0244	.6599	.1674	. 4343	. 5657	56	44
20	5	.51628	.48371	1.9369	.60284	1.6588	1.1676	.14358	.85642	55	40
24	6	. 1653	. 8347	.9360	. 0324	.6577	.1678	. 4373	. 5627	54	36
28	7	. 1678	. 8322	.9350	. 0363	.6566	.1681	. 4388	. 5612	53	32
32	8	. 1703	. 8297	.9341	. 0403	.6555	.1683	. 4403	. 5597	52	28
36	9	. 1728	. 8272	.9332	. 0443	.6544	.1685	. 4418	. 5582	51	24
40	10	.51753	.48247	1.9322	.60483	1.6534	1.1687	.14433	.85566	50	20
44	11	. 1778	. 8222	.9313	. 0522	.6523	.1689	. 4448	. 5551	49	16
48	12	. 1803	. 8197	.9304	. 0562	.6512	.1691	. 4463	. 5536	48	12
52	13	. 1827	. 8172	.9295	. 0602	.6501	.1693	. 4479	. 5521	47	8
56	14	. 1852	. 8147	.9285	. 0642	.6490	.1695	. 4494	. 5506	46	4
5	15	.51877	.48123	1.9276	.60681	1.6479	1.1697	.14509	.85491	45	55
4	16	. 1902	. 8098	.9267	. 0721	.6469	.1699	. 4524	. 5476	44	56
8	17	. 1927	. 8073	.9258	. 0761	.6458	.1701	. 4539	. 5461	43	52
12	18	. 1952	. 8048	.9248	. 0801	.6447	.1703	. 4554	. 5446	42	48
16	19	. 1977	8023	.9239	. 0841	.6436	.1705	. 4569	. 5431	41	44
20	20	.52002	.47998	1.9230	.60881	1.6425	1.1707	.14584	.85416	40	40
24	21	. 2026	. 7973	.9221	. 0920	.6415	.1709	. 4599	. 5400	39	36
28	22	. 2051	. 7949	.9212	. 0960	.6404	.1712	. 4615	. 5385	38	32
32	23	. 2076	. 7924	.9203	. 1000	.6393	.1714	. 4630	. 5370	37	28
36	24	. 2101	. 7899	.9193	. 1040	.6383	.1716	. 4645	. 5355	36	24
40	25	.52126	.47874	1.9184	.61080	1.6372	1.1718	.14660	.85340	35	20
44	26	. 2151	. 7849	.9175	. 1120	.6361	.1720	. 4675	. 5325	34	16
48	27	. 2175	. 7824	.9166	. 1160	.6350	.1722	. 4690	. 5309	33	12
52	28	. 2200	. 7800	.9157	. 1200	.6340	.1724	. 4706	. 5294	32	8
56	29	. 2225	. 7775	.9148	. 1240	.6329	.1726	. 4721	. 5279	31	4
6	30	.52250	.47750	1.9139	.61280	1.6318	1.1728	.14736	.85264	30	54
4	31	. 2275	. 7725	.9130	. 1320	.6308	.1730	. 4751	. 5249	29	56
8	32	. 2299	. 7700	.9121	. 1360	.6297	.1732	. 4766	. 5234	28	52
12	33	. 2324	. 7676	.9112	. 1400	.6286	.1734	. 4782	. 5218	27	48
16	34	. 2349	. 7651	.9102	. 1440	.6276	.1737	. 4797	. 5203	26	44
20	35	.52374	.47626	1.9093	.61480	1.6265	1.1739	.14812	.85188	25	40
24	36	. 2398	. 7601	.9084	. 1520	.6255	.1741	. 4827	. 5173	24	36
28	37	. 2423	. 7577	.9075	. 1560	.6244	.1743	. 4842	. 5157	23	32
32	38	. 2448	. 7552	.9066	. 1601	.6233	.1745	. 4858	. 5142	22	28
36	39	. 2473	. 7527	.9057	. 1641	.6223	.1747	. 4873	. 5127	21	24
40	40	.52498	.47502	1.9048	.61681	1.6212	1.1749	.14888	.85112	20	20
44	41	. 2522	. 7477	.9039	. 1721	.6202	.1751	. 4904	. 5096	19	16
48	42	. 2547	. 7453	.9030	. 1761	.6191	.1753	. 4919	. 5081	18	12
52	43	. 2572	. 7428	.9021	. 1801	.6181	.1756	. 4934	. 5066	17	8
56	44	. 2597	. 7403	.9013	. 1842	.6170	.1758	. 4949	. 5050	16	4
7	45	.52621	.47379	1.9004	.61882	1.6160	1.1760	.14965	.85035	15	53
4	46	. 2646	. 7354	.8995	. 1922	.6149	.1762	. 4980	. 5020	14	56
8	47	. 2671	. 7329	.8986	. 1962	.6139	.1764	. 4995	. 5004	13	52
12	48	. 2695	. 7304	.8977	. 2003	.6128	.1766	. 5011	. 4989	12	48
16	49	2720	. 7280	.8968	2043	.6118	.1768	. 5026	. 4974	11	44
20	50	.52745	.47255	1.8959	.62083	1.6107	1.1770	.15041	.84959	10	40
24	51	. 2770	. 7230	.8950	. 2123	.6097	.1772	. 5057	. 4943	9	36
28	52	. 2794	. 7205	.8941	. 2164	.6086	.1775	. 5072	. 4928	8	32
32	53	. 2819	. 7181	.8932	. 2204	.6076	.1777	. 5087	. 4912	7	28
36	54	. 2844	. 7156	.8924	. 2244	.6066	.1779	. 5103	. 4897	6	24
40	55	.52868	.47131	1.8915	.62285	1.6055	1.1781	.15118	.84882	5	20
44	56	. 2893	. 7107	.8906	. 2325	.6045	.1783	. 5133	. 4866	4	16
48	57	. 2918	. 7082	.8897	. 2366	.6034	.1785	. 5149	. 4851	3	12
52	58	. 2942	. 7057	.8888	. 2406	.6024	.1787	. 5164	. 4836	2	8
56	59	. 2967	. 7033	.8879	. 2446	.6014	.1790	. 5180	. 4820	1	4
8	60	. 2992	. 7008	.8871	. 2487	.6003	.1792	. 5195	. 4805	0	52
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
8h	121°	Natural.								58°	3h

2h 32° Natural Trigonometrical Functions. 147° 9h

M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
8	0	.52992	.47008	1.8871	.62487	1.6003	1.1792	.15195	.84805	60	**52**
4	1	. 3016	. 6983	.8862	. 2527	.5993	.1794	. 5211	. 4789	59	56
8	2	. 3041	. 6959	.8853	. 2568	.5983	.1796	. 5226	. 4774	58	52
12	3	. 3066	. 6934	.8844	. 2608	.5972	.1798	. 5241	. 4758	57	48
16	4	. 3090	. 6909	.8836	. 2649	.5962	.1800	. 5257	. 4743	56	44
20	5	.53115	.46885	1.8827	.62689	1.5952	1.1802	.15272	.84728	55	40
24	6	. 3140	. 6860	.8818	. 2730	.5941	.1805	. 5288	. 4712	54	36
28	7	. 3164	. 6835	.8809	. 2770	.5931	.1807	. 5303	. 4697	53	32
32	8	. 3189	. 6811	.8801	. 2811	.5921	.1809	. 5319	. 4681	52	28
36	9	. 3214	. 6786	.8792	. 2851	.5910	.1811	. 5334	. 4666	51	24
40	10	.53238	.46762	1.8783	.62892	1.5900	1.1813	.15350	.84650	50	20
44	11	. 3263	. 6737	.8775	. 2933	.5890	.1815	. 5365	. 4635	49	16
48	12	. 3288	. 6712	.8766	. 2973	.5880	.1818	. 5381	. 4619	48	12
52	13	. 3312	. 6688	.8757	. 3014	.5869	.1820	. 5396	. 4604	47	8
56	14	. 3337	. 6663	.8749	. 3055	.5859	.1822	. 5412	. 4588	46	4
9	15	.53361	.46638	1.8740	.63095	1.5849	1.1824	.15427	.84573	45	**51**
4	16	. 3386	. 6614	.8731	. 3136	.5839	.1826	. 5143	. 4557	44	56
8	17	. 3411	. 6589	.8723	. 3177	.5829	.1828	. 5458	. 4542	43	52
12	18	. 3435	. 6565	.8714	. 3217	.5818	.1831	. 5474	. 4526	42	48
16	19	. 3460	. 6540	.8706	. 3258	.5808	.1833	. 5489	. 4511	41	44
20	20	.53484	.46516	1.8697	.63299	1.5798	1.1835	.15505	.84495	40	40
24	21	. 3509	. 6491	.8688	. 3339	.5788	.1837	. 5520	. 4479	39	36
28	22	. 3533	. 6466	.8680	. 3380	.5778	.1839	. 5536	. 4464	38	32
32	23	. 3558	. 6442	.8671	. 3421	.5768	.1841	. 5552	. 4448	37	28
36	24	. 3583	. 6417	.8663	. 3462	.5757	.1844	. 5567	. 4433	36	24
40	25	.53607	.46393	1.8654	.63503	1.5747	1.1846	.15583	.84417	35	20
44	26	. 3632	. 6368	.8646	. 3543	.5737	.1848	. 5598	. 4402	34	16
48	27	. 3656	. 6344	.8637	. 3584	.5727	.1850	. 5614	. 4386	33	12
52	28	. 3681	. 6319	.8629	. 3625	.5717	.1852	. 5630	. 4370	32	8
56	29	. 3705	. 6294	.8620	. 3666	.5707	.1855	. 5645	. 4355	31	4
10	30	.53730	.46270	1.8611	.63707	1.5697	1.1857	.15661	.84339	30	**50**
4	31	. 3754	. 6245	.8603	. 3748	.5687	.1859	. 5676	. 4323	29	56
8	32	. 3779	. 6221	.8595	. 3789	.5677	.1861	. 5692	. 4308	28	52
12	33	. 3803	. 6196	.8586	. 3830	.5667	.1863	. 5708	. 4292	27	48
16	34	. 3828	. 6172	.8578	. 3871	.5657	.1866	. 5723	. 4276	26	44
20	35	.53852	.46147	1.8569	.63912	1.5646	1.1868	.15739	.84261	25	40
24	36	. 3877	. 6123	.8561	. 3953	.5636	.1870	. 5755	. 4245	24	36
28	37	. 3901	. 6098	.8552	. 3994	.5626	.1872	. 5770	. 4229	23	32
32	38	. 3926	. 6074	.8544	. 4035	.5616	.1874	. 5786	. 4214	22	28
36	39	. 3950	. 6049	.8535	. 4076	.5606	.1877	. 5802	. 4198	21	24
40	40	.53975	.46025	1.8527	.64117	1.5596	1.1879	.15817	.84182	20	20
44	41	. 3999	. 6000	.8519	. 4158	.5586	.1881	. 5833	. 4167	19	16
48	42	. 4024	. 5976	.8510	. 4199	.5577	.1883	. 5849	. 4151	18	12
52	43	. 4048	. 5951	.8502	. 4240	.5567	.1886	. 5865	. 4135	17	8
56	44	. 4073	. 5927	.8493	. 4281	.5557	.1888	. 5880	. 4120	16	4
11	45	.54097	.45902	1.8485	.64322	1.5547	1.1890	.15896	.84104	15	**49**
4	46	. 4122	. 5878	.8477	. 4363	.5537	.1892	. 5912	. 4088	14	56
8	47	. 4146	. 5854	.8468	. 4404	.5527	.1894	. 5927	. 4072	13	52
12	48	. 4171	. 5829	.8460	. 4446	.5517	.1897	. 5943	. 4057	12	48
16	49	. 4195	. 5805	.8452	. 4487	.5507	.1899	. 5959	. 4041	11	44
20	50	.54220	.45780	1.8443	.64528	1.5497	1.1901	.15975	.84025	10	40
24	51	. 4244	. 5756	.8435	. 4569	.5487	.1903	. 5991	. 4009	9	36
28	52	. 4268	. 5731	.8427	. 4610	.5477	.1906	. 6006	. 3993	8	32
32	53	. 4293	. 5707	.8418	. 4652	.5467	.1908	. 6022	. 3978	7	28
36	54	. 4317	. 5682	.8410	. 4693	.5458	.1910	. 6038	. 3962	6	24
40	55	.54342	.45658	1.8402	.64734	1.5448	1.1912	.16054	.83946	5	20
44	56	. 4366	. 5634	.8394	. 4775	.5438	.1915	. 6070	. 3930	4	16
48	57	. 4391	. 5609	.8385	. 4817	.5428	.1917	. 6085	. 3914	3	12
52	58	. 4415	. 5585	.8377	. 4858	.5418	.1919	. 6101	. 3899	2	8
56	59	. 4439	. 5560	.8369	. 4899	.5408	.1921	. 6117	. 3883	1	4
12	60	. 4464	. 5536	.8361	. 4941	.5399	.1922	. 6133	. 3867	0	**48**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.

8h 122° **Natural.** 57° 3h

2^h	33°	Natural Trigonometrical Functions.							146°	9^h
M.S.	M	Sine.	Vrs. Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
12	0	.54464	.45536	1.8361	.64941	1.5399	1.1924	.16133	.83867	60	**48**
4	1	. 4488	. 5512	.8352	. 4982	.5389	.1926	. 6149	. 3851	59	56
8	2	. 4513	. 5487	.8344	. 5023	.5379	.1928	. 6165	. 3835	58	52
12	3	. 4537	. 5463	.8336	. 5065	.5369	.1930	. 6180	. 3819	57	48
16	4	. 4561	. 5438	.8328	. 5106	.5359	.1933	. 6196	. 3804	56	44
20	5	.54586	.45414	1.8320	.65148	1.5350	1.1935	.16212	.83788	55	40
24	6	. 4610	. 5390	.8311	. 5189	.5340	.1937	. 6228	. 3772	54	36
28	7	. 4634	. 5365	.8303	. 5231	.5330	.1939	. 6244	. 3756	53	32
32	8	. 4659	. 5341	.8295	. 5272	.5320	.1942	. 6260	. 3740	52	28
36	9	. 4683	. 5317	.8287	. 5314	.5311	.1944	. 6276	. 3724	51	24
40	10	.54708	.45292	1.8279	.65355	1.5301	1.1946	.16292	.83708	50	20
44	11	. 4732	. 5268	.8271	. 5397	.5291	.1948	. 6308	. 3692	49	16
48	12	. 4756	. 5244	.8263	. 5438	.5282	.1951	. 6323	. 3676	18	12
52	13	. 4781	. 5219	.8255	. 5480	.5272	.1953	. 6339	. 3660	47	8
56	14	. 4805	. 5195	.8246	. 5521	.5262	.1955	. 6355	. 3644	46	4
13	15	.54829	.45171	1.8238	.65563	1.5252	1.1958	.16371	.83629	45	**47**
4	16	. 4854	. 5146	.8230	. 5604	.5243	.1960	. 6387	. 3613	44	56
8	17	. 4878	. 5122	.8222	. 5646	.5233	.1962	. 6403	. 3597	43	52
12	18	. 4902	. 5098	.8214	. 5688	.5223	.1964	. 6419	. 3581	42	48
16	19	. 4926	5073	.8206	. 5729	.5214	.1967	. 6435	. 3565	41	44
20	20	.54951	.45049	1.8198	.65771	1.5204	1.1969	.16451	.83549	40	40
24	21	. 4975	. 5025	.8190	. 5813	.5195	.1971	. 6467	. 3533	39	36
28	22	. 4999	. 5000	.8182	. 5854	.5185	.1974	. 6483	. 3517	38	32
32	23	. 5024	. 4976	.8174	. 5896	.5175	.1976	. 6499	. 3501	37	28
36	24	. 5048	. 4952	.8166	. 5938	.5166	.1978	. 6515	. 3485	36	24
40	25	.55072	.44928	1.8158	.65980	1.5156	1.1980	.16531	.83469	35	20
44	26	. 5097	. 4903	.8150	. 6021	.5147	.1983	. 6547	. 3453	34	16
48	27	. 5121	. 4879	.8142	. 6063	.5137	.1985	. 6563	. 3437	33	12
52	28	. 5145	. 4855	.8134	. 6105	.5127	.1987	. 6579	. 3421	32	8
56	29	. 5169	. 4830	.8126	. 6147	.5118	.1990	. 6595	. 3405	31	4
14	30	.55194	.44806	1.8118	.66188	1.5108	1.1992	.16611	.83388	30	**46**
4	31	. 5218	. 4782	.8110	. 6230	.5099	.1994	. 6627	. 3372	29	56
8	32	. 5242	. 4758	.8102	. 6272	.5089	.1997	. 6643	. 3356	28	52
12	33	. 5266	. 4733	.8094	. 6314	.5080	.1999	. 6660	. 3340	27	48
16	34	. 5291	. 4709	.8086	. 6356	.5070	.2001	. 6676	. 3324	26	44
20	35	.55315	.44685	1.8078	.66398	1.5061	1.2004	.16692	.83308	25	40
24	36	. 5339	. 4661	.8070	. 6440	.5051	.2006	. 6708	. 3292	24	36
28	37	. 5363	. 4637	.8062	. 6482	.5042	.2008	. 6724	. 3276	23	32
32	38	. 5388	. 4612	.8054	. 6524	.5032	.2010	. 6740	. 3260	22	28
36	39	. 5412	. 4588	.8047	. 6566	.5023	.2013	. 6756	. 3244	21	24
40	40	.55436	.44564	1.8039	.66608	1.5013	1.2015	.16772	.83228	20	20
44	41	. 5460	. 4540	.8031	. 6650	.5004	.2017	. 6788	. 3211	19	16
48	42	. 5484	. 4515	.8023	. 6692	.4994	.2020	. 6804	. 3195	18	12
52	43	. 5509	. 4491	.8015	. 6734	.4985	.2022	. 6821	. 3179	17	8
56	44	. 5533	. 4467	.8007	. 6776	.4975	.2024	. 6837	. 3163	16	4
15	45	.55557	.44443	1.7999	.66818	1.4966	1.2027	.16853	.83147	15	**45**
4	46	. 5581	. 4419	.7992	. 6860	.4957	.2029	. 6869	. 3131	14	56
8	47	. 5605	. 4395	.7984	. 6902	.4947	.2031	. 6885	. 3115	13	52
12	48	. 5629	. 4370	.7976	. 6944	.4938	.2034	. 6901	. 3098	12	48
16	49	. 5654	. 4346	.7968	6986	.4928	.2036	. 6918	. 3082	11	44
20	50	.55678	.44322	1.7960	.67028	1.4919	1.2039	.16934	.83066	10	40
24	51	. 5702	. 4298	.7953	. 7071	.4910	.2041	. 6950	. 3050	9	36
28	52	. 5726	. 4274	.7945	. 7113	.4900	.2043	. 6966	. 3034	8	32
32	53	. 5750	. 4250	.7937	. 7155	.4891	.2046	. 6982	. 3017	7	28
36	54	. 5774	. 4225	.7929	. 7197	.4881	.2048	. 6999	. 3001	6	24
40	55	.55799	.44201	1.7921	.67239	1.4872	1.2050	.17015	.82985	5	20
44	56	. 5823	. 4177	.7914	. 7282	.4863	.2053	. 7031	. 2969	4	16
48	57	. 5847	. 4153	.7906	. 7324	.4853	.2055	. 7047	. 2952	3	12
52	58	. 5871	. 4129	.7898	. 7366	.4844	.2057	. 7064	. 2936	2	8
56	59	. 5895	. 4105	.7891	. 7408	.4835	.2060	. 7080	. 2920	1	4
16	60	. 5919	. 4081	.7883	. 7451	.4826	.2062	. 7096	. 2904	0	**44**
M.S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M.S.
8^h	123°	**Natural.**								56°	3^h

2ʰ	34°	Natural Trigonometrical Functions.								145°	9ʰ
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M.	M.S.
16	0	.55919	.44081	1.7883	.67451	1.4826	1.2062	.17096	.82904	60	**44**
4	1	. 5943	. 4057	.7875	. 7493	.4816	.2064	. 7112	. 2887	59	56
8	2	. 5967	. 4032	.7867	. 7535	.4807	.2067	. 7129	. 2871	58	52
12	3	. 5992	. 4008	.7860	. 7578	.4798	.2069	. 7145	. 2855	57	48
16	4	. 6016	. 3984	.7852	. 7620	.4788	.2072	. 7161	. 2839	56	44
20	5	.56040	.43960	1.7844	.67663	1.4779	1.2074	.17178	.82822	55	40
24	6	. 6064	. 3936	.7837	. 7705	.4770	.2076	. 7194	. 2806	54	36
28	7	. 6088	. 3912	.7829	. 7747	.4761	.2079	. 7210	. 2790	53	32
32	8	. 6112	. 3888	.7821	. 7790	.4751	.2081	. 7227	. 2773	52	28
36	9	. 6136	. 3864	.7814	. 7832	.4742	.2083	. 7243	. 2757	51	24
40	10	.56160	.43840	1.7806	.67875	1.4733	1.2086	.17259	.82741	50	20
44	11	. 6184	. 3816	.7798	. 7917	.4724	.2088	. 7276	. 2724	49	16
48	12	. 6208	. 3792	.7791	. 7960	.4714	.2091	. 7292	. 2708	48	12
52	13	. 6232	. 3768	.7783	. 8002	.4705	.2093	. 7308	. 2692	47	8
56	14	. 6256	. 3743	.7776	. 8045	.4696	.2095	. 7325	. 2675	46	4
17	15	.56280	.43719	1.7768	.68087	1.4687	1.2098	.17341	.82659	45	**43**
4	16	. 6304	. 3695	.7760	. 8130	.4678	.2100	. 7357	. 2643	44	56
8	17	. 6328	. 3671	.7753	. 8173	.4669	.2103	. 7374	. 2626	43	52
12	18	. 6353	. 3647	.7745	. 8215	.4659	.2105	. 7390	. 2610	42	48
16	19	. 6377	. 3623	.7738	. 8258	.4650	.2107	. 7406	. 2593	41	44
20	20	.56401	.43599	1.7730	.68301	1.4641	1.2110	.17423	.82577	40	40
24	21	. 6425	. 3575	.7723	. 8343	.4632	.2112	. 7439	. 2561	39	36
28	22	. 6449	. 3551	.7715	. 8386	.4623	.2115	. 7456	. 2544	38	32
32	23	. 6473	. 3527	.7708	. 8429	.4614	.2117	. 7472	. 2528	37	28
36	24	. 6497	. 3503	.7700	. 8471	.4605	.2119	. 7489	. 2511	36	24
40	25	.56521	.43479	1.7693	.68514	1.4595	1.2122	.17505	.82495	35	20
44	26	. 6545	. 3455	.7685	. 8557	.4586	.2124	. 7521	. 2478	34	16
48	27	. 6569	. 3431	.7678	. 8600	.4577	.2127	. 7538	. 2462	33	12
52	28	. 6593	. 3407	.7670	. 8642	.4568	.2129	. 7554	. 2445	32	8
56	29	. 6617	. 3383	.7663	. 8685	.4559	.2132	. 7571	. 2429	31	4
18	30	.56641	.43359	1.7655	.68728	1.4550	1.2134	.17587	.82413	30	**42**
4	31	. 6664	. 3335	.7648	. 8771	.4541	.2136	. 7604	. 2396	29	56
8	32	. 6688	. 3311	.7640	. 8814	.4532	.2139	. 7620	. 2380	28	52
12	33	. 6712	. 3287	.7633	. 8857	.4523	.2141	. 7637	. 2363	27	48
16	34	. 6736	. 3263	.7625	. 8899	.4514	.2144	. 7653	. 2347	26	44
20	35	.56760	.43239	1.7618	.68942	1.4505	1.2146	.17670	.82330	25	40
24	36	. 6784	. 3216	.7610	. 8985	.4496	.2149	. 7686	. 2314	24	36
28	37	. 6808	. 3192	.7603	. 9028	.4487	.2151	. 7703	. 2297	23	32
32	38	. 6832	. 3168	.7596	. 9071	.4478	.2153	. 7719	. 2280	22	28
36	39	. 6856	. 3144	.7588	. 9114	.4469	.2156	. 7736	. 2264	21	24
40	40	.56880	.43120	1.7581	.69157	1.4460	1.2158	.17752	.82247	20	20
44	41	. 6904	. 3096	.7573	. 9200	.4451	.2161	. 7769	. 2231	19	16
48	42	. 6928	. 3072	.7566	. 9243	.4442	.2163	. 7786	. 2214	18	12
52	43	. 6952	. 3048	.7559	. 9286	.4433	.2166	. 7802	. 2198	17	8
56	44	. 6976	. 3024	.7551	. 9329	.4424	.2168	. 7819	. 2181	16	4
19	45	.57000	.43000	1.7544	.69372	1.4415	1.2171	.17835	.82165	15	**41**
4	46	. 7023	. 2976	.7537	. 9415	.4406	.2173	. 7852	. 2148	14	56
8	47	. 7047	. 2952	.7529	. 9459	.4397	.2175	. 7868	. 2131	13	52
12	48	. 7071	. 2929	.7522	. 9502	.4388	.2178	. 7885	. 2115	12	48
16	49	. 7095	. 2905	.7514	. 9545	.4379	.2180	. 7902	. 2098	11	44
20	50	.57119	.42881	1.7507	.69588	1.4370	1.2183	.17918	.82082	10	40
24	51	. 7143	. 2857	.7500	. 9631	.4361	.2185	. 7935	. 2065	9	36
28	52	. 7167	. 2833	.7493	. 9674	.4352	.2188	. 7951	. 2048	8	32
32	53	. 7191	. 2809	.7485	. 9718	.4343	.2190	. 7968	. 2032	7	28
36	54	. 7214	. 2785	.7478	. 9761	.4335	.2193	. 7985	. 2015	6	24
40	55	.57238	.42761	1.7471	.69804	1.4326	1.2195	.18001	.81998	5	20
44	56	. 7262	. 2738	.7463	. 9847	.4317	.2198	. 8018	. 1982	4	16
48	57	. 7286	. 2714	.7456	. 9891	.4308	.2200	. 8035	. 1965	3	12
52	58	. 7310	. 2690	.7449	. 9934	.4299	.2203	. 8051	. 1948	2	8
56	59	. 7334	. 2666	.7442	. 9977	.4290	.2205	. 8068	. 1932	1	4
20	60	. 7358	. 2642	.7434	.70021	.4281	.2208	. 8085	. 1915	0	**40**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
8ʰ	124°	Natural.								55°	3ʰ

2h	35°	Natural Trigonometrical Functions.							144°		9h
M.S.	M	Sine.	Vrs. Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
20	0	.57358	.42642	1.7434	.70021	1.4281	1.2208	.18085	.81915	60	**40**
4	1	. 7381	. 2618	.7427	. 0064	.4273	.2210	. 8101	. 1898	59	56
8	2	. 7405	. 2595	.7420	. 0107	.4264	.2213	. 8118	. 1882	58	52
12	3	. 7429	. 2571	.7413	. 0151	.4255	.2215	. 8135	. 1865	57	48
16	4	. 7453	. 2547	.7405	. 0194	.4246	.2218	. 8151	. 1848	56	44
20	5	.57477	.42523	1.7398	.70238	1.4237	1.2220	.18168	.81832	55	40
24	6	. 7500	. 2499	.7391	. 0281	.4228	.2223	. 8185	. 1815	54	36
28	7	. 7524	. 2476	.7384	. 0325	.4220	.2225	. 8202	. 1798	53	32
32	8	. 7548	. 2452	.7377	. 0368	.4211	.2228	. 8218	. 1781	52	28
36	9	. 7572	. 2428	.7369	. 0412	.4202	.2230	. 8235	. 1765	51	24
40	10	.57596	.42404	1.7362	.70455	1.4193	1.2233	.18252	.81748	50	20
44	11	. 7619	. 2380	.7355	. 0499	.4185	.2235	. 8269	. 1731	49	16
48	12	. 7643	. 2357	.7348	. 0542	.4176	.2238	. 8285	. 1714	48	12
52	13	. 7667	. 2333	.7341	. 0586	.4167	.2240	. 8302	. 1698	47	8
56	14	. 7691	. 2309	.7334	. 0629	.4158	.2243	. 8319	. 1681	46	4
21	15	.57714	.42285	1.7327	.70673	1.4150	1.2245	.18336	.81664	45	**39**
4	16	. 7738	. 2262	.7319	. 0717	.4141	.2248	. 8353	. 1647	44	56
8	17	. 7762	. 2238	.7312	. 0760	.4132	.2250	. 8369	. 1630	43	52
12	18	. 7786	. 2214	.7305	. 0804	.4123	.2253	. 8386	. 1614	42	48
16	19	. 7809	2190	.7298	. 0848	.4115	.2255	. 8403	. 1597	41	44
20	20	.57833	.42167	1.7291	.70891	1.4106	1.2258	.18420	.81580	40	40
24	21	. 7857	. 2143	.7284	. 0935	.4097	.2260	. 8437	. 1563	39	36
28	22	. 7881	. 2119	.7277	. 0979	.4089	.2263	. 8453	. 1546	38	32
32	23	. 7904	. 2096	.7270	. 1022	.4080	.2265	. 8470	. 1530	37	28
36	24	. 7928	. 2072	.7263	. 1066	.4071	.2268	. 8487	. 1513	36	24
40	25	.57952	.42048	1.7256	.71110	1.4063	1.2270	.18504	.81496	35	20
44	26	. 7975	. 2024	.7249	. 1154	.4054	.2273	. 8521	. 1479	34	16
48	27	. 7999	. 2001	.7242	. 1198	.4045	.2276	. 8538	. 1462	33	12
52	28	. 8023	. 1977	.7234	. 1241	.4037	.2278	. 8555	. 1445	32	8
56	29	. 8047	. 1953	.7227	. 1285	.4028	.2281	. 8571	. 1428	31	4
22	30	.58070	.41930	1.7220	.71329	1.4019	1.2283	.18588	.81411	30	**38**
4	31	. 8094	. 1906	.7213	. 1373	.4011	.2286	. 8605	. 1395	29	56
8	32	. 8118	. 1882	.7206	. 1417	.4002	.2288	. 8622	. 1378	28	52
12	33	. 8141	. 1859	.7199	. 1461	.3994	.2291	. 8639	. 1361	27	48
16	34	. 8165	. 1835	.7192	. 1505	.3985	.2293	. 8656	. 1344	26	44
20	35	.58189	.41811	1.7185	.71549	1.3976	1.2296	.18673	.81327	25	40
24	36	. 8212	. 1788	.7178	. 1593	.3968	.2298	. 8690	. 1310	24	36
28	37	. 8236	. 1764	.7171	. 1637	.3959	.2301	. 8707	. 1293	23	32
32	38	. 8259	. 1740	.7164	. 1681	.3951	.2304	. 8724	. 1276	22	28
36	39	. 8283	. 1717	.7157	. 1725	.3942	.2306	. 8741	. 1259	21	24
40	40	.58307	.41693	1.7151	.71769	1.3933	1.2309	.18758	.81242	20	20
44	41	. 8330	. 1669	.7144	. 1813	.3925	.2311	. 8775	. 1225	19	16
48	42	. 8354	. 1646	.7137	. 1857	.3916	.2314	. 8792	. 1208	18	12
52	43	. 8378	. 1622	.7130	. 1901	.3908	.2316	. 8809	. 1191	17	8
56	44	. 8401	. 1599	.7123	. 1945	.3899	.2319	. 8826	. 1174	16	4
23	45	.58425	.41575	1.7116	.71990	1.3891	1.2322	.18843	.81157	15	**37**
4	46	. 8448	. 1551	.7109	. 2034	.3882	.2324	. 8860	. 1140	14	56
8	47	. 8472	. 1528	.7102	. 2078	.3874	.2327	. 8877	. 1123	13	52
12	48	. 8496	. 1504	.7095	. 2122	.3865	.2329	. 8894	. 1106	12	48
16	49	. 8519	. 1481	.7088	2166	.3857	.2332	. 8911	. 1089	11	44
20	50	.58543	.41457	1.7081	.72211	1.3848	1.2335	.18928	.81072	10	40
24	51	. 8566	. 1433	.7075	. 2255	.3840	.2337	. 8945	. 1055	9	36
28	52	. 8590	. 1410	.7068	. 2299	.3831	.2340	. 8962	. 1038	8	32
32	53	. 8614	. 1386	.7061	. 2344	.3823	.2342	. 8979	. 1021	7	28
36	54	. 8637	. 1363	.7054	. 2388	.3814	.2345	. 8996	. 1004	6	24
40	55	.58661	.41339	1.7047	.72432	1.3806	1.2348	.19013	.80987	5	20
44	56	. 8684	. 1316	.7040	. 2477	.3797	.2350	. 9030	. 0970	4	16
48	57	. 8708	. 1292	.7033	. 2521	.3789	.2353	. 9047	. 0953	3	12
52	58	. 8731	. 1268	.7027	. 2565	.3781	.2355	. 9064	. 0936	2	8
56	59	. 8755	. 1245	.7020	. 2610	.3772	.2358	. 9081	. 0919	1	4
24	60	. 8778	. 1221	.7013	. 2654	.3764	.2361	. 9098	. 0902	0	**36**
M.S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M.S.
8h	125°				Natural.					54°	3h

2h M. S.	36° M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	143° M	9h M. S.
24	0	.58778	.41221	1.7013	.72654	1.3764	1.2361	.19098	.80902	60	**36**
4	1	. 8802	. 1198	.7006	. 2699	.3755	.2363	. 9115	. 0885	59	56
8	2	. 8825	. 1174	.6999	. 2743	.3747	.2366	. 9132	. 0867	58	52
12	3	. 8849	. 1151	.6993	. 2788	.3738	.2368	. 9150	. 0850	57	48
16	4	. 8873	. 1127	.6986	. 2832	.3730	.2371	. 9167	. 0833	56	44
20	5	.58896	.41104	1.6979	.72877	1.3722	1.2374	.19184	.80816	55	40
24	6	. 8920	. 1080	.6972	. 2921	.3713	.2376	. 9201	. 0799	54	36
28	7	. 8943	. 1057	.6965	. 2966	.3705	.2379	. 9218	. 0782	53	32
32	8	. 8967	. 1033	.6959	. 3010	.3697	.2382	. 9235	. 0765	52	28
36	9	. 8990	. 1010	.6952	. 3055	.3688	.2384	. 9252	. 0747	51	24
40	10	.59014	.40986	1.6945	.73100	1.3680	1.2387	.19270	.80730	50	20
44	11	. 9037	. 0963	.6938	. 3144	.3672	.2389	. 9287	. 0713	49	16
48	12	. 9060	. 0939	.6932	. 3189	.3663	.2392	. 9304	. 0696	48	12
52	13	. 9084	. 0916	.6925	. 3234	.3655	.2395	. 9321	. 0679	47	8
56	14	. 9107	. 0892	.6918	. 3278	.3647	.2397	. 9338	. 0662	46	4
25	15	.59131	.40869	1.6912	.73323	1.3638	1.2400	.19355	.80644	45	**35**
4	16	. 9154	. 0845	.6905	. 3368	.3630	.2403	. 9373	. 0627	44	56
8	17	. 9178	. 0822	.6898	. 3412	.3622	.2405	. 9390	. 0610	43	52
12	18	. 9201	. 0799	.6891	. 3457	.3613	.2408	. 9407	. 0593	42	48
16	19	. 9225	. 0775	.6885	. 3502	.3605	.2411	. 9424	. 0576	41	44
20	20	.59248	.40752	1.6878	.73547	1.3597	1.2413	.19442	.80558	40	40
24	21	. 9272	. 0728	.6871	. 3592	.3588	.2416	. 9459	. 0541	39	36
28	22	. 9295	. 0705	.6865	. 3637	.3580	.2419	. 9476	. 0524	38	32
32	23	. 9318	. 0681	.6858	. 3681	.3572	.2421	. 9493	. 0507	37	28
36	24	. 9342	. 0658	.6851	. 3726	.3564	.2424	. 9511	. 0489	36	24
40	25	.59365	.40635	1.6845	.73771	1.3555	1.2427	.19528	.80472	35	20
44	26	. 9389	. 0611	.6838	. 3816	.3547	.2429	. 9545	. 0455	34	16
48	27	. 9412	. 0588	.6831	. 3861	.3539	.2432	. 9562	. 0437	33	12
52	28	. 9435	. 0564	.6825	. 3906	.3531	.2435	. 9580	. 0420	32	8
56	29	. 9459	. 0541	.6818	. 3951	.3522	.2437	. 9597	. 0403	31	4
26	30	.59482	.40518	1.6812	.73996	1.3514	1.2440	.19614	.80386	30	**34**
4	31	. 9506	. 0494	.6805	. 4041	.3506	.2443	. 9632	. 0368	29	56
8	32	. 9529	. 0471	.6798	. 4086	.3498	.2445	. 9649	. 0351	28	52
12	33	. 9552	. 0447	.6792	. 4131	.3489	.2448	. 9666	. 0334	27	48
16	34	. 9576	. 0424	.6785	. 4176	.3481	.2451	. 9683	. 0316	26	44
20	35	.59599	.40401	1.6779	.74221	1.3473	1.2453	.19701	.80299	25	40
24	36	. 9622	. 0377	.6772	. 4266	.3465	.2456	. 9718	. 0282	24	36
28	37	. 9646	. 0354	.6766	. 4312	.3457	.2459	9736	. 0264	23	32
32	38	. 9669	. 0331	.6759	. 4357	.3449	.2461	. 9753	. 0247	22	28
36	39	. 9692	. 0307	.6752	. 4402	.3440	.2464	. 9770	. 0230	21	24
40	40	.59716	.40284	1.6746	.74447	1.3432	1.2467	.19788	.80212	20	20
44	41	. 9739	. 0261	.6739	. 4492	.3424	.2470	. 9805	. 0195	19	16
48	42	. 9762	. 0237	.6733	. 4538	.3416	.2472	. 9822	. 0177	18	12
52	43	. 9786	. 0214	.6726	. 4583	.3408	.2475	. 9840	. 0160	17	8
56	44	. 9809	. 0191	.6720	. 4628	.3400	.2478	. 9857	. 0143	16	4
27	45	.59832	.40167	1.6713	.74673	1.3392	1.2480	.19875	.80125	15	**33**
4	46	. 9856	. 0144	.6707	. 4719	.3383	.2483	. 9892	. 0108	14	56
8	47	. 9879	. 0121	.6700	. 4764	.3375	.2486	. 9909	. 0090	13	52
12	48	. 9902	. 0098	.6694	. 4809	.3367	.2488	. 9927	. 0073	12	48
16	49	. 9926	. 0074	.6687	. 4855	.3359	.2495	. 9944	. 0056	11	44
20	50	.59949	.40051	1.6681	.74900	1.3351	1.2494	.19962	.80038	10	40
24	51	. 9972	. 0028	.6674	. 4946	.3343	.2497	. 9979	. 0021	9	36
28	52	. 9995	. 0004	.6668	. 4991	.3335	.2499	. 9997	. 0003	8	32
32	53	.60019	.39981	.6661	. 5037	.3327	.2502	.20014	.79986	7	28
36	54	. 0042	. 9958	.6655	. 5082	.3319	.2505	. 0031	. 9968	6	24
40	55	.60065	.39935	1.6648	.75128	1.3311	1.2508	.20049	.79951	5	20
44	56	. 0088	. 9911	.6642	. 5173	.3303	.2510	. 0066	. 9933	4	16
48	57	. 0112	. 9888	.6636	. 5219	.3294	.2513	. 0084	. 9916	3	12
52	58	. 0135	. 9865	.6629	. 5264	.3286	.2516	. 0101	. 9898	2	8
56	59	. 0158	. 9842	.6623	. 5310	.3278	.2519	. 0119	. 9881	1	4
28	60	. 0181	. 9818	.6616	. 5355	.3270	.2521	. 0136	. 9863	0	**32**
M. S.	M	Cosine.	Vrs. Sin	Secante.	Cotang.	Tangent.	Cosec'nt.	Vrs. Cos	Sine.	M	M S.
8h	126°				Natural.					53°	3h

2^h	37°			Natural Trigonometrical Functions.					142°		9^h
M. S.	M	Sine.	Vrs. Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M. S.
28	0	.60181	.39818	1.6616	.75355	1.3270	1.2521	.20136	.79863	60	**32**
4	1	. 0205	. 9795	.6610	. 5401	.3262	.2524	. 0154	. 9846	59	56
8	2	. 0228	. 9772	.6603	. 5447	.3254	.2527	. 0171	. 9828	58	52
12	3	. 0251	. 9749	.6597	. 5492	.3246	.2530	. 0189	. 9811	57	48
16	4	. 0274	. 9726	.6591	. 5538	.3238	.2532	. 0206	. 9793	56	44
20	5	.60298	.39702	1.6584	.75584	1.3230	1.2535	.20224	.79776	55	40
24	6	. 0320	. 9679	.6578	. 5629	.3222	.2538	. 0242	. 9758	54	36
28	7	. 0344	. 9656	.6572	. 5675	.3214	.2541	. 0259	. 9741	53	32
32	8	. 0367	. 9633	.6565	. 5721	.3206	.2543	. 0277	. 9723	52	28
36	9	. 0390	. 9610	.6559	. 5767	.3198	.2546	. 0294	. 9706	51	24
40	10	.60413	.39586	1.6552	.75812	1.3190	1.2549	.20312	.79688	50	20
44	11	. 0437	. 9563	.6546	. 5858	.3182	.2552	. 0329	. 9670	49	16
48	12	. 0460	. 9540	.6540	. 5904	.3174	.2554	. 0347	. 9653	48	12
52	13	. 0483	. 9517	.6533	. 5950	.3166	.2557	. 0365	. 9635	47	8
56	14	. 0506	. 9494	.6527	. 5996	.3159	.2560	. 0382	. 9618	46	4
29	15	.60529	.39471	1.6521	.76042	1.3151	1.2563	.20400	.79600	45	**31**
4	16	. 0552	. 9447	.6514	. 6088	.3143	.2565	. 0417	. 9582	44	56
8	17	. 0576	. 9424	.6508	. 6134	.3135	.2568	. 0435	. 9565	43	52
12	18	. 0599	. 9401	.6502	. 6179	.3127	.2571	. 0453	. 9547	42	48
16	19	. 0622	. 9378	.6496	. 6225	.3119	.2574	. 0470	. 9530	41	44
20	20	.60645	.39355	1.6489	.76271	1.3111	1.2577	.20488	.79512	40	40
24	21	. 0668	. 9332	.6483	. 6317	.3103	.2579	. 0505	. 9494	39	36
28	22	. 0691	. 9309	.6477	. 6364	.3095	.2582	. 0523	. 9477	38	32
32	23	. 0714	. 9285	.6470	. 6410	.3087	.2585	. 0541	. 9459	37	28
36	24	. 0737	. 9262	.6464	. 6456	.3079	.2588	. 0558	. 9441	36	24
40	25	.60761	.39239	1.6458	.76502	1.3071	1.2591	.20576	.79424	35	20
44	26	. 0784	. 9216	.6452	. 6548	.3064	.2593	. 0594	. 9406	34	16
48	27	. 0807	. 9193	.6445	. 6594	.3056	.2596	. 0611	. 9388	33	12
52	28	. 0830	. 9170	.6439	. 6640	.3048	.2599	. 0629	. 9371	32	8
56	29	. 0853	. 9147	.6433	. 6686	.3040	.2602	. 0647	. 9353	31	4
30	30	.60876	.39124	1.6427	.76733	1.3032	1.2605	.20665	.79335	30	**30**
4	31	. 0899	. 9101	.6420	. 6779	.3024	.2607	. 0682	. 9318	29	56
8	32	. 0922	. 9078	.6414	. 6825	.3016	.2610	. 0700	. 9300	28	52
12	33	. 0945	. 9055	.6408	. 6871	.3009	.2613	. 0718	. 9282	27	48
16	34	. 0963	. 9031	.6402	. 6918	.3001	.2616	. 0735	. 9264	26	44
20	35	.60991	.39008	1.6396	.76964	1.2993	1.2619	.20753	.79247	25	40
24	36	. 1014	. 8985	.6389	. 7010	.2985	.2622	. 0771	. 9229	24	36
28	37	. 1037	. 8962	.6383	. 7057	.2977	.2624	0789	. 9211	23	32
32	38	. 1061	. 8939	.6377	. 7103	.2970	.2627	. 0806	. 9193	22	28
36	39	. 1084	. 8916	.6371	. 7149	.2962	.2630	. 0824	. 9176	21	24
40	40	.61107	.38893	1.6365	.77196	1.2954	1.2633	.20842	.79158	20	20
44	41	. 1130	. 8870	.6359	. 7242	.2946	.2636	. 0860	. 9140	19	16
48	42	. 1153	. 8847	.6352	. 7289	.2938	.2639	. 0878	. 9122	18	12
52	43	. 1176	. 8824	.6346	. 7335	.2931	.2641	. 0895	. 9104	17	8
56	44	. 1199	. 8801	.6340	. 7382	.2923	.2644	. 0913	. 9087	16	4
31	45	.61222	.38778	1.6334	.77428	1.2915	1.2647	.20931	.79069	15	**29**
4	46	. 1245	. 8755	.6328	. 7475	.2907	.2650	. 0949	. 9051	14	56
8	47	. 1268	. 8732	.6322	. 7521	.2900	.2653	. 0967	. 9033	13	52
12	48	. 1290	. 8709	.6316	. 7568	.2892	.2656	. 0984	. 9015	12	48
16	49	. 1314	. 8686	.6309	. 7614	.2884	.2659	. 1002	. 8998	11	44
20	50	.61337	.38663	1.6303	.77661	1.2876	1.2661	.21020	.78980	10	40
24	51	. 1360	. 8640	.6297	. 7708	.2869	.2664	. 1038	. 8962	9	36
28	52	. 1383	. 8617	.6291	. 7754	.2861	.2667	. 1056	. 8944	8	32
32	53	. 1405	. 8594	.6285	. 7801	.2853	.2670	. 1074	. 8926	7	28
36	54	. 1428	. 8571	.6279	. 7848	.2845	.2673	. 1091	. 8908	6	24
40	55	.61451	.38548	1 6273	.77895	1.2838	1.2676	.21109	.78890	5	20
44	56	. 1474	. 8525	.6267	. 7941	.2830	.2679	. 1127	. 8873	4	16
48	57	. 1497	. 8503	.6261	. 7988	.2822	.2681	. 1145	. 8855	3	12
52	58	. 1520	. 8480	.6255	. 8035	.2815	.2684	. 1163	. 8837	2	8
56	59	. 1543	. 8457	.6249	. 8082	.2807	.2687	. 1181	. 8819	1	4
32	60	. 1566	. 8434	.6243	. 8128	.2799	.2690	. 1199	. 8801	0	**28**
M. S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt.	Vrs. Cos	Sine.	M	M. S.
8^h	127°			Natural.					52°		3^h

2h 38° Natural Trigonometrical Functions. 141° 9h

M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
32	0	.61566	.38434	1.6243	.78128	1.2799·	1.2690	.21199	.78801	60	**28**
4	1	. 1589	. 8411	.6237	. 8175	.2792	.2693	. 1217	. 8783	59	56
8	2	. 1612	. 8388	.6231	. 8222	.2784	.2696	. 1235	. 8765	58	52
12	3	. 1635	. 8365	.6224	. 8269	.2776	.2699	. 1253	. 8747	57	48
16	4	. 1658	. 8342	.6218	. 8316	.2769	.2702	. 1271	. 8729	56	44
20	5	.61681	.38319	1.6212	.78363	1.2761	1.2705	.21288	.78711	55	40
24	6	. 1703	. 8296	.6206	. 8410	.2753	.2707	. 1306	. 8693	54	36
28	7	. 1726	. 8273	.6200	. 8457	.2746	.2710	. 1324	. 8675	53	32
32	8	. 1749	. 8251	.6194	. 8504	.2738	.2713	. 1342	. 8657	52	28
36	9	. 1772	. 8228	.6188	. 8551	.2730	.2716	. 1360	. 8640	51	24
40	10	.61795	.38205	1.6182	.78598	1.2723	1.2719	.21378	.78622	50	20
44	11	. 1818	. 8182	.6176	. 8645	.2715	.2722	. 1396	. 8604	49	16
48	12	. 1841	. 8159	.6170	. 8692	.2708	.2725	. 1414	. 8586	48	12
52	13	. 1864	. 8136	.6164	. 8739	.2700	.2728	. 1432	. 8568	47	8
56	14	. 1886	. 8113	.6159	. 8786	.2692	.2731	. 1450	. 8550	46	4
33	15	.61909	.38091	1.6153	.78834	1.2685	1.2734	.21468	.78532	45	**27**
4	16	. 1932	. 8068	.6147	. 8881	.2677	.2737	. 1486	. 8514	44	56
8	17	. 1955	. 8045	.6141	. 8928	.2670	.2739	. 1504	. 8496	43	52
12	18	. 1978	. 8022	.6135	. 8975	.2662	.2742	. 1522	. 8478	42	48
16	19	. 2001	. 7999	.6129	. 9022	.2655	.2745	. 1540	. 8460	41	44
20	20	.62023	.37976	1.6123	.79070	1:2647	1.2748	.21558	.78441	40	40
24	21	. 2046	. 7954	.6117	. 9117	.2639	.2751	. 1576	. 8423	39	36
28	22	. 2069	. 7931	.6111	. 9164	.2632	.2754	. 1594	. 8405	38	32
32	23	. 2092	. 7908	.6105	. 9212	.2624	.2757	. 1612	. 8387	37	28
36	24	. 2115	. 7885	.6099	. 9259	.2617	.2760	. 1631	. 8369	36	24
40	25	.62137	.37862	1.6093	.79306	1.2609	1.2763	.21649	.78351	35	20
44	26	. 2160	. 7840	.6087	. 9354	.2602	.2766	. 1667	. 8333	34	16
48	27	. 2183	. 7817	.6081	. 9401	.2594	.2769	. 1685	. 8315	33	12
52	28	. 2206	. 7794	.6077	. 9449	.2587	.2772	. 1703	. 8297	32	8
56	29	. 2229	. 7771	.6070	. 9496	.2579	.2775	. 1721	. 8279	31	4
34	30	.62251	.37748	1.6064	.79543	1.2572	1.2778	.21739	.78261	30	**26**
4	31	. 2274	. 7726	.6058	. 9591	.2564	.2781	. 1757	. 8243	29	56
8	32	. 2297	. 7703	.6052	. 9639	.2557	.2784	. 1775	. 8224	28	52
12	33	. 2320	. 7680	.6046	. 9686	.2549	.2787	. 1793	. 8206	27	48
16	34	. 2342	. 7657	.6040	. 9734	.2542	.2790	. 1812	. 8188	26	44
20	35	.62365	.37635	1.6034	.79781	1.2534	1.2793	.21830	.78170	25	40
24	36	. 2388	. 7612	.6029	. 9829	.2527	.2795	. 1848	. 8152	24	36
28	37	. 2411	. 7589	.6023	. 9876	.2519	.2798	. 1866	. 8134	23	32
32	38	. 2433	. 7566	.6017	. 9924	.2512	.2801	. 1884	. 8116	22	28
36	39	. 2456	. 7544	.6011	. 9972	.2504	.2804	. 1902	. 8097	21	24
40	40	.62479	.37521	1.6005	.80020	1.2497	1.2807	.21921	.78079	20	20
44	41	. 2501	. 7498	.6000	. 0067	.2489	.2810	. 1939	. 8061	19	16
48	42	. 2524	. 7476	.5994	. 0115	.2482	.2813	. 1957	. 8043	18	12
52	43	. 2547	. 7453	.5988	. 0163	.2475	.2816	. 1975	. 8025	17	8
56	44	. 2570	. 7430	.5982	. 0211	.2467	.2819	. 1993	. 8007	16	4
35	45	.62592	.37408	1.5976	.80258	1.2460	1.2822	.22011	.77988	15	**25**
4	46	. 2615	. 7385	.5971	. 0306	.2452	.2825	. 2030	. 7970	14	56
8	47	. 2638	. 7362	.5965	. 0354	.2445	.2828	. 2048	. 7952	13	52
12	48	. 2660	. 7340	.5959	. 0402	.2437	.2831	. 2066	. 7934	12	48
16	49	. 2683	. 7317	.5953	. 0450	.2430	.2834	. 2084	. 7915	11	44
20	50	.62706	.37294	1.5947	.80498	1.2423	1.2837	.22103	.77897	10	40
24	51	. 2728	. 7272	.5942	. 0546	.2415	.2840	. 2121	. 7879	9	36
28	52	. 2751	. 7249	.5936	. 0594	.2408	.2843	. 2139	. 7861	8	32
32	53	. 2774	. 7226	.5930	. 0642	.2400	.2846	. 2157	. 7842	7	28
36	54	. 2796	. 7204	.5924	. 0690	.2393	.2849	. 2176	. 7824	6	24
40	55	.62819	.37181	1.5919	.80738	1.2386	1.2852	.22194	.77806	5	20
44	56	. 2841	. 7158	.5913	. 0786	.2378	.2855	. 2212	. 7788	4	16
48	57	. 2864	. 7136	.5907	. 0834	.2371	.2858	. 2230	. 7769	3	12
52	58	. 2887	. 7113	.5901	. 0882	.2364	.2861	. 2249	. 7751	2	8
56	59	. 2909	. 7090	.5896	. 0930	.2356	.2864	. 2267	. 7733	1	4
36	60	. 2932	. 7068	.5890	. 0978	.2349	.2867	. 2285	. 7715	0	**24**
M. S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.

8h 128° **Natural.** 51° 3h

2h	39°			Natural Trigonometrical Functions.					140°		9h
M.S.	M	Sine.	Vrs. Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
36	0	.62932	.37068	1.5890	.80978	1.2349	1.2867	.22285	.77715	60	**24**
4	1	. 2955	. 7045	.5884	. 1026	.2342	.2871	. 2304	. 7696	59	56
8	2	. 2977	. 7023	.5879	. 1075	.2334	.2874	. 2322	. 7678	58	52
12	3	. 3000	. 7000	.5873	. 1123	.2327	.2877	. 2340	. 7660	57	48
16	4	. 3022	. 6977	.5867	. 1171	.2320	.2880	. 2359	. 7641	56	44
20	5	.63045	.36955	1.5862	.81219	1.2312	1.2883	.22377	.77623	55	40
24	6	. 3067	. 6932	.5856	. 1268	.2305	.2886	. 2395	. 7605	54	36
28	7	. 3090	. 6910	.5850	. 1316	.2297	.2889	. 2414	. 7586	53	32
32	8	. 3113	. 6887	.5845	. 1364	.2290	.2892	. 2432	. 7568	52	28
36	9	. 3135	. 6865	.5839	. 1413	.2283	.2895	. 2450	. 7549	51	24
40	10	.63158	.36842	1.5833	.81461	1.2276	1.2898	.22469	.77531	50	20
44	11	. 3180	. 6820	.5828	. 1509	.2268	.2901	. 2487	. 7513	49	16
48	12	. 3203	. 6797	.5822	. 1558	.2261	.2904	. 2505	. 7494	48	12
52	13	. 3225	. 6774	.5816	. 1606	.2254	.2907	. 2524	. 7476	47	8
56	14	. 3248	. 6752	.5811	. 1655	.2247	.2910	. 2542	. 7458	46	4
37	15	.63270	.36729	1.5805	.81703	1.2239	1.2913	.22561	.77439	45	**23**
4	16	. 3293	. 6707	.5799	. 1752	.2232	.2916	. 2579	. 7421	44	56
8	17	. 3315	. 6684	.5794	. 1800	.2225	.2919	. 2597	. 7402	43	52
12	18	. 3338	. 6662	.5788	. 1849	.2218	.2922	. 2616	. 7384	42	48
16	19	. 3360	6639	.5783	. 1898	.2210	.2926	. 2634	. 7365	41	44
20	20	.63383	.36617	1.5777	.81946	1.2203	1.2929	.22653	.77347	40	40
24	21	. 3405	. 6594	.5771	. 1995	.2196	.2932	. 2671	. 7329	39	36
28	22	. 3428	. 6572	.5766	. 2043	.2189	.2935	. 2690	. 7310	38	32
32	23	. 3450	. 6549	.5760	. 2092	.2181	.2938	. 2708	. 7292	37	28
36	24	. 3473	. 6527	.5755	. 2141	.2174	.2941	. 2727	. 7273	36	24
40	25	.63495	.36504	1.5749	.82190	1.2167	1.2944	.22745	.77255	35	20
44	26	. 3518	. 6482	.5743	. 2238	.2160	.2947	. 2763	. 7236	34	16
48	27	. 3540	. 6459	.5738	. 2287	.2152	.2950	. 2782	. 7218	33	12
52	28	. 3563	. 6437	.5732	. 2336	.2145	.2953	. 2800	. 7199	32	8
56	29	. 3585	. 6415	.5727	. 2385	.2138	.2956	. 2819	. 7181	31	4
38	30	.63608	.36392	1.5721	.82434	1.2131	1.2960	.22837	.77162	30	**22**
4	31	. 3630	. 6370	.5716	. 2482	.2124	.2963	. 2856	. 7144	29	56
8	32	. 3653	. 6347	.5710	. 2531	.2117	.2966	. 2874	. 7125	28	52
12	33	. 3675	. 6325	.5705	. 2580	.2109	.2969	. 2893	. 7107	27	48
16	34	. 3697	. 6302	.5699	. 2629	.2102	.2972	. 2912	. 7088	26	44
20	35	.63720	.36280	1.5694	.82678	1.2095	1.2975	.22930	.77070	25	40
24	36	. 3742	. 6258	.5688	. 2727	.2088	.2978	. 2949	. 7051	24	36
28	37	. 3765	. 6235	.5683	. 2776	.2081	.2981	. 2967	. 7033	23	32
32	38	. 3787	. 6213	.5677	. 2825	.2074	.2985	. 2986	. 7014	22	28
36	39	. 3810	. 6190	.5672	. 2874	.2066	.2988	. 3004	. 6996	21	24
40	40	.63832	.26168	1.5666	.82923	1.2059	1.2991	.23023	.76977	20	20
44	41	. 3854	. 6146	.5661	. 2972	.2052	.2994	. 3041	. 6958	19	16
48	42	. 3877	. 6123	.5655	. 3022	.2045	.2997	. 3060	. 6940	18	12
52	43	. 3899	. 6101	.5650	. 3071	.2038	.3000	. 3079	. 6921	17	8
56	44	. 3921	. 6078	.5644	. 3120	.2031	.3003	. 3097	. 6903	16	4
39	45	.63944	.36056	1.5639	.83169	1.2024	1.3006	.23116	.76884	15	**21**
4	46	. 3966	. 6034	.5633	. 3218	.2016	.3010	. 3134	. 6865	14	56
8	47	. 3989	. 6011	.5628	. 3267	.2009	.3013	. 3153	. 6847	13	52
12	48	. 4011	. 5989	.5622	. 3317	.2002	.3016	. 3172	. 6828	12	48
16	49	. 4033	. 5967	.5617	. 3366	.1995	.3019	. 3190	6810	11	44
20	50	.64056	.35944	1.5611	.83415	1.1988	1.3022	.23209	.76791	10	40
24	51	. 4078	. 5922	.5606	. 3465	.1981	.3025	. 3227	. 6772	9	36
28	52	. 4100	. 5900	.5600	. 3514	.1974	.3029	. 3246	. 6754	8	32
32	53	. 4123	. 5877	.5595	. 3563	.1967	.3032	. 3265	. 6735	7	28
36	54	. 4145	. 5855	.5590	. 3613	.1960	.3035	. 3283	. 6716	6	24
40	55	.64167	.35833	1.5584	.83662	1.1953	1.3038	.23302	.76698	5	20
44	56	. 4189	. 5810	.5579	. 3712	.1946	.3041	. 3321	. 6679	4	16
48	57	. 4212	. 5788	.5573	. 3761	.1939	.3044	. 3339	. 6660	3	12
52	58	. 4234	. 5766	.5568	. 3811	.1932	.3048	. 3358	. 6642	2	8
56	59	. 4256	. 5743	.5563	. 3860	.1924	.3051	. 3377	. 6623	1	4
40	60	. 4279	. 5721	.5557	. 3910	.1917	.3054	. 3395	6604	0	**20**
M.S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M.S.
8h	129°			Natural.						50°	3h

2h	40°	Natural Trigonometrical Functions.							139°		9h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
40	0	.64279	.35721	1.5557	.83910	1.1917	1.3054	.23395	.76604	60	**20**
4	1	. 4301	. 5699	.5552	. 3959	.1910	.3057	. 3414	. 6586	59	56
8	2	. 4323	. 5677	.5546	. 4009	.1903	.3060	. 3433	. 6567	58	52
12	3	. 4345	. 5654	.5541	. 4059	.1896	.3064	. 3452	. 6548	57	48
16	4	. 4368	. 5632	.5536	. 4108	.1889	.3067	. 3470	. 6530	56	44
20	5	.64390	.35610	1.5530	.84158	1.1882	1.3070	.23489	.76511	55	40
24	6	. 4412	. 5588	.5525	. 4208	.1875	.3073	. 3508	. 6492	54	36
28	7	. 4435	. 5565	.5520	. 4257	.1868	.3076	. 3527	. 6473	53	32
32	8	. 4457	. 5543	.5514	. 4307	.1861	.3080	. 3545	. 6455	52	28
36	9	. 4479	. 5521	.5509	. 4357	.1854	.3083	. 3564	. 6436	51	24
40	10	.64501	.35499	1.5503	.84407	1.1847	1.3086	.23583	.76417	50	20
44	11	. 4523	. 5476	.5498	. 4457	.1840	.3089	. 3602	. 6398	49	16
48	12	. 4546	. 5454	.5493	. 4506	.1833	.3092	. 3620	. 6380	48	12
52	13	. 4568	. 5432	.5487	. 4556	.1826	.3096	. 3639	. 6361	47	8
56	14	. 4590	. 5410	.5482	. 4606	.1819	.3099	. 3658	. 6342	46	4
41	15	.64612	.35388	1.5477	.84656	1.1812	1.3102	.23677	.76323	45	**19**
4	16	. 4635	. 5365	.5471	. 4706	.1805	.3105	. 3695	. 6304	44	56
8	17	. 4657	. 5343	.5466	. 4756	.1798	.3109	. 3714	. 6286	43	52
12	18	. 4679	. 5321	.5461	. 4806	.1791	.3112	. 3733	. 6267	42	48
16	19	. 4701	. 5299	.5456	. 4856	.1785	.3115	. 3752	. 6248	41	44
20	20	.64723	.35277	1.5450	.84906	1.1778	1.3118	.23771	.76229	40	40
24	21	. 4745	. 5254	.5445	. 4956	.1771	.3121	. 3790	. 6210	39	36
28	22	. 4768	. 5232	.5440	. 5006	.1764	.3125	. 3808	. 6191	38	32
32	23	. 4790	. 5210	.5434	. 5056	.1757	.3128	. 3827	. 6173	37	28
36	24	. 4812	. 6188	.5429	. 5107	.1750	.3131	. 3846	. 6154	36	24
40	25	.64834	.35166	1.5424	.85157	1.1743	1.3134	.23865	.76135	35	20
44	26	. 4856	. 5144	.5419	. 5207	.1736	.3138	. 3884	. 6116	34	16
48	27	. 4878	. 5121	.5413	. 5257	.1729	.3141	. 3903	. 6097	33	12
52	28	. 4900	. 5099	.5408	. 5307	.1722	.3144	. 3922	. 6078	32	8
56	29	. 4923	. 5077	.5403	. 5358	.1715	.3148	. 3940	. 6059	31	4
42	30	.64945	.35055	1.5398	.85408	1.1708	1.3151	.23959	.76041	30	**18**
4	31	. 4967	. 5033	.5392	. 5458	.1702	.3154	. 3978	. 6022	29	56
8	32	. 4989	. 5011	.5387	. 5509	.1695	.3157	. 3997	. 6003	28	52
12	33	. 5011	. 4989	.5382	. 5559	.1688	.3161	. 4016	. 5984	27	48
16	34	. 5033	. 4967	.5377	. 5609	.1681	.3164	. 4035	. 5965	26	44
20	35	.65055	.34945	1.5371	.85660	1.1674	1.3167	.24054	.75946	25	40
24	36	. 5077	. 4922	.5366	. 5710	.1667	.3170	. 4073	. 5927	24	36
28	37	. 5099	. 4900	.5361	. 5761	.1660	.3174	. 4092	. 5908	23	32
32	38	. 5121	. 4878	.5356	. 5811	.1653	.3177	. 4111	. 5889	22	28
36	39	. 5144	. 4856	.5351	. 5862	.1647	.3180	. 4130	. 5870	21	24
40	40	.65166	.34834	1.5345	.85912	1.1640	1.3184	.24149	.75851	20	20
44	41	. 5188	. 4812	.5340	. 5963	.1633	.3187	. 4168	. 5832	19	16
48	42	. 5210	. 4790	.5335	. 6013	.1626	.3190	. 4186	. 5813	18	12
52	43	. 5232	. 4768	.5330	. 6064	.1619	.3193	. 4205	. 5794	17	8
56	44	. 5254	. 4746	.5325	. 6115	.1612	.3197	. 4224	. 5775	16	4
43	45	.65276	.34724	1.5319	.86165	1.1605	1.3200	.24243	.75756	15	**17**
4	46	. 5298	. 4702	.5314	. 6216	.1599	.3203	. 4262	. 5737	14	56
8	47	. 5320	. 4680	.5309	. 6267	.1592	.3207	. 4281	. 5718	13	52
12	48	. 5342	. 4658	.5304	. 6318	.1585	.3210	. 4300	. 5699	12	48
16	49	. 5364	. 4636	.5299	. 6368	.1578	.3213	. 4319	. 5680	11	44
20	50	.65386	.34614	1.5294	.86419	1.1571	1.3217	.24338	.75661	10	40
24	51	. 5408	. 4592	.5289	. 6470	.1565	.3220	. 4357	. 5642	9	36
28	52	. 5430	. 4570	.5283	. 6521	.1558	.3223	. 4376	. 5623	8	32
32	53	. 5452	. 4548	.5278	. 6572	.1551	.3227	. 4396	. 5604	7	28
36	54	. 5474	. 4526	.5273	. 6623	.1544	.3230	. 4415	. 5585	6	24
40	55	.65496	.34504	1.5268	.86674	1.1537	1.3233	.24434	.75566	5	20
44	56	. 5518	. 4482	.5263	. 6725	.1531	.3237	. 4453	. 5547	4	16
48	57	. 5540	. 4460	.5258	. 6775	.1524	.3240	. 4472	. 5528	3	12
52	58	. 5562	. 4438	.5253	. 6826	.1517	.3243	. 4491	. 5509	2	8
56	59	. 5584	. 4416	.5248	. 6878	.1510	.3247	. 4510	. 5490	1	4
44	60	. 5606	. 4394	.5242	. 6929	.1504	.3250	. 4529	. 5471	0	**16**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
8h	130°	Natural.							49°		3h

2h	41°	Natural Trigonometrical Functions.								138°	9h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M.S.
44	0	.65606	.34394	1.5242	.86929	1.1504	1.3250	.24529	.75471	60	**16**
4	1	. 5628	. 4372	.5237	. 6980	.1497	.3253	. 4548	. 5452	59	56
8	2	. 5650	. 4350	.5232	. 7031	.1490	.3257	. 4567	. 5433	58	52
12	3	. 5672	. 4328	.5227	. 7082	.1483	.3260	. 4586	. 5414	57	48
16	4	. 5694	. 4306	.5222	. 7133	.1477	.3263	. 4605	. 5394	56	44
20	5	.65716	.34284	1.5217	.87184	1.1470	1.3267	.24624	.75375	55	40
24	6	. 5737	. 4262	.5212	. 7235	.1463	.3270	. 4644	. 5356	54	36
28	7	. 5759	. 4240	.5207	. 7287	.1456	.3274	. 4663	. 5337	53	32
32	8	. 5781	. 4219	.5202	. 7338	.1450	.3277	. 4682	. 5318	52	28
36	9	. 5803	. 4197	.5197	. 7389	.1443	.3280	. 4701	. 5299	51	24
40	10	.65825	.34175	1.5192	.87441	1.1436	1.3284	.24720	.75280	50	20
44	11	. 5847	. 4153	.5187	. 7492	.1430	.3287	. 4739	. 5261	49	16
48	12	. 5869	. 4131	.5182	. 7543	.1423	.3290	. 4758	. 5241	48	12
52	13	. 5891	. 4109	.5177	. 7595	.1416	.3294	. 4778	. 5222	47	8
56	14	. 5913	. 4087	.5171	. 7646	.1409	.3297	. 4797	. 5203	46	4
45	15	.65934	.34065	1.5166	.87698	1.1403	1.3301	.24816	.75184	45	**15**
4	16	. 5956	. 4043	.5161	. 7749	.1396	.3304	. 4835	. 5165	44	56
8	17	. 5978	. 4022	.5156	. 7801	.1389	.3307	. 4854	. 5146	43	52
12	18	. 6000	. 4000	.5151	. 7852	.1383	.3311	. 4873	. 5126	42	48
16	19	. 6022	. 3978	.5146	. 7904	.1376	.3314	. 4893	. 5107	41	44
20	20	.66044	.33956	1.5141	.87955	1.1369	1.3318	.24912	.75088	40	40
24	21	. 6066	. 3934	.5136	. 8007	.1363	.3321	. 4931	. 5069	39	36
28	22	. 6087	. 3912	.5131	. 8058	.1356	.3324	. 4950	. 5049	38	32
32	23	. 6109	. 3891	.5126	. 8110	.1349	.3328	. 4970	. 5030	37	28
36	24	. 6131	. 3869	.5121	. 8162	.1343	.3331	. 4989	. 5011	36	24
40	25	.66153	.33847	1.5116	.88213	1.1336	1.3335	.25008	.74992	35	20
44	26	. 6175	. 3825	.5111	. 8265	.1329	.3338	. 5027	. 4973	34	16
48	27	. 6197	. 3803	.5106	. 8317	.1323	.3342	. 5047	. 4953	33	12
52	28	. 6218	. 3781	.5101	. 8369	.1316	.3345	. 5066	. 4934	32	8
56	29	. 6240	. 3760	.5096	. 8421	.1309	.3348	. 5085	. 4915	31	4
46	30	.66262	.33738	1.5092	.88472	1.1303	1.3352	.25104	.74895	30	**14**
4	31	. 6284	. 3716	.5087	. 8524	.1296	.3355	. 5124	. 4876	29	56
8	32	. 6305	. 3694	.5082	. 8576	.1290	.3359	. 5143	. 4857	28	52
12	33	. 6327	. 3673	.5077	. 8628	.1283	.3362	. 5162	. 4838	27	48
16	34	. 6349	. 3651	.5072	. 8680	.1276	.3366	. 5181	. 4818	26	44
20	35	.66371	.33629	1.5067	.88732	1.1270	1.3369	.25201	.74799	25	40
24	36	. 6393	. 3607	.5062	. 8784	.1263	.3372	. 5220	. 4780	24	36
28	37	. 6414	. 3586	.5057	. 8836	.1257	.3376	. 5239	. 4760	23	32
32	38	. 6436	. 3564	.5052	. 8888	.1250	.3379	. 5259	. 4741	22	28
36	39	. 6458	. 3542	.5047	. 8940	.1243	.3383	. 5278	. 4722	21	24
40	40	.66479	.33520	1.5042	.88992	1.1237	1.3386	.25297	.74702	20	20
44	41	. 6501	. 3499	.5037	. 9044	.1230	.3390	. 5317	. 4683	19	16
48	42	. 6523	. 3477	.5032	. 9097	.1224	.3393	. 5336	. 4664	18	12
52	43	. 6545	. 3455	.5027	. 9149	.1217	3397	. 5355	. 4644	17	8
56	44	. 6566	. 3433	.5022	. 9201	.1211	3400	. 5375	. 4625	16	4
47	45	.66588	.33412	1.5018	.89253	1.1204	1.3404	.25394	.74606	15	**13**
4	46	. 6610	. 3390	.5013	. 9306	.1197	.3407	. 5414	. 4586	14	56
8	47	. 6631	. 3368	.5008	. 9358	.1191	.3411	. 5433	. 4567	13	52
12	48	. 6653	. 3347	.5003	. 9410	.1184	.3414	. 5452	. 4548	12	48
16	49	. 6675	. 3325	.4998	. 9463	.1178	.3418	. 5472	4528	11	44
20	50	.66697	.33303	1.4993	.89515	1.1171	1.3421	.25491	.74509	10	40
24	51	. 6718	. 3282	.4988	. 9567	.1165	.3425	. 5510	. 4489	9	36
28	52	. 6740	. 3260	.4983	. 9620	.1158	.3428	. 5530	. 4470	8	32
32	53	. 6762	. 3238	.4979	. 9672	.1152	.3432	. 5549	. 4450	7	28
36	54	. 6783	. 3217	.4974	. 9725	.1145	.3435	. 5569	. 4431	6	24
40	55	.66805	.33195	1.4969	.89777	1.1139	1.3439	.25588	.74412	5	20
44	56	. 6826	. 3173	.4964	. 9830	.1132	.3442	. 5608	. 4392	4	16
48	57	. 6848	. 3152	.4959	. 9882	.1126	.3446	. 5627	. 4373	3	12
52	58	. 6870	. 3130	.4954	. 9935	.1119	.3449	. 5647	. 4353	2	8
56	59	. 6891	. 3108	.4949	. 9988	.1113	.3453	. 5666	. 4334	1	4
48	60	. 6913	. 3087	.4945	.90040	.1106	.3456	. 5685	. 4314	0	**12**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs. Cos	Sine.	M	M.S.
8h	131°	Natural.								48°	3h

2h	42°	Natural Trigonometrical Functions.							137°		9h
M. S.	M	Sine.	Vrs. Cos.	Cosce'nte	Tang.	Cotang.	Secante.	Vrs. Sin	Cosine.	M	M. S.
48	0	.66913	.33087	1.4945	.90040	1.1106	1.3456	.25685	.74314	60	**12**
4	1	. 6935	. 3065	.4940	. 0093	.1100	.3460	. 5705	. 4295	59	56
8	2	. 6956	. 3044	.4935	. 0146	.1093	.3463	. 5724	. 4275	58	52
12	3	. 6978	. 3022	.4930	. 0198	.1086	.3467	. 5744	. 4256	57	48
16	4	. 6999	. 3060	.4925	. 0251	.1080	.3470	. 5763	. 4236	56	44
20	5	.67021	.32979	1.4921	.90304	1.1074	1.3474	.25783	.74217	55	40
24	6	. 7043	. 2957	.4916	. 0357	.1067	.3477	. 5802	. 4197	54	36
28	7	. 7064	. 2936	.4911	. 0410	.1061	.3481	. 5822	. 4178	53	32
32	8	. 7086	. 2914	.4906	. 0463	.1054	.3485	. 5841	. 4158	52	28
36	9	. 7107	. 2893	.4901	. 0515	.1048	.3488	. 5861	. 4139	51	24
40	10	.67129	.32871	1.4897	.90568	1.1041	1.3492	.25880	.74119	50	20
44	11	. 7150	. 2849	.4892	. 0621	.1035	.3495	. 5900	. 4100	49	16
48	12	. 7172	. 2828	.4887	. 0674	.1028	.3499	. 5919	. 4080	48	12
52	13	. 7194	. 2806	.4882	. 0727	.1022	.3502	. 5939	. 4061	47	8
56	14	. 7215	. 2785	.4877	. 0780	.1015	.3506	. 5959	. 4041	46	4
49	15	.67237	.32763	1.4873	.90834	1.1009	1.3509	.25978	.74022	45	**11**
4	16	. 7258	. 2742	.4868	. 0887	.1003	.3513	. 5998	. 4002	44	56
8	17	. 7280	. 2720	.4863	. 0940	.0996	.3517	. 6017	. 3983	43	52
12	18	. 7301	. 2699	.4858	. 0993	.0990	.3520	. 6037	. 3963	42	48
16	19	. 7323	. 2677	.4854	. 1046	.0983	.3524	. 6056	. 3943	41	44
20	20	.67344	.32656	1.4849	.91099	1.0977	1.3527	.26076	.73924	40	40
24	21	. 7366	. 2634	.4844	. 1153	.0971	.3531	. 6096	. 3904	39	36
28	22	. 7387	. 2613	.4839	. 1206	.0964	.3534	. 6115	. 3885	38	32
32	23	. 7409	. 2591	.4835	. 1259	.0958	.3538	. 6135	. 3865	37	28
36	24	. 7430	. 2570	.4830	. 1312	.0951	.3542	. 6154	. 3845	36	24
40	25	.67452	.32548	1.4825	.91366	1.0945	1.3545	.26174	.73826	35	20
44	26	. 7473	. 2527	.4821	. 1419	.0939	.3549	. 6194	. 3806	34	16
48	27	. 7495	. 2505	.4816	. 1473	.0932	.3552	. 6213	. 3787	33	12
52	28	. 7516	. 2484	.4811	. 1526	.0926	.3556	. 6233	. 3767	32	8
56	29	. 7537	. 2462	.4806	. 1580	.0919	.3560	. 6253	. 3747	31	4
50	30	.67559	.32441	1.4802	.91633	1.0913	1.3563	.26272	.73728	30	**10**
4	31	. 7580	. 2419	.4797	. 1687	.0907	.3567	. 6292	. 3708	29	56
8	32	. 7602	. 2398	.4792	. 1740	.0900	.3571	. 6311	. 3688	28	52
12	33	. 7623	. 2377	.4788	. 1794	.0894	.3574	. 6331	. 3669	27	48
16	34	. 7645	. 2355	.4783	. 1847	.0888	.3578	. 6351	. 3649	26	44
20	35	.67666	.32334	1.4778	.91901	1.0881	1.3581	.26371	.73629	25	40
24	36	. 7688	. 2312	.4774	. 1955	.0875	.3585	. 6390	. 3610	24	36
28	37	. 7709	. 2291	.4769	. 2008	.0868	.3589	6410	. 3590	23	32
32	38	. 7730	. 2269	.4764	. 2062	.0862	.3592	. 6430	. 3570	22	28
36	39	. 7752	. 2248	.4760	. 2116	.0856	.3596	. 6449	. 3551	21	24
40	40	.67773	.32227	1.4755	.92170	1.0849	1.3600	.26469	.73531	20	20
44	41	. 7794	. 2205	.4750	. 2223	.0843	.3603	. 6489	. 3511	19	16
48	42	. 7816	. 2184	.4746	. 2277	.0837	.3607	. 6508	. 3491	18	12
52	43	. 7837	. 2163	.4741	. 2331	.0830	.3611	. 6528	. 3472	17	8
56	44	. 7859	. 2141	.4736	. 2385	.0824	.3614	. 6548	. 3452	16	4
51	45	.67880	.32120	1.4732	.92439	1.0818	1.3618	.26568	.73432	15	**9**
4	46	. 7901	. 2098	.4727	. 2493	.0812	.3622	. 6587	. 3412	14	56
8	47	. 7923	. 2077	.4723	. 2547	.0805	.3625	. 6607	. 3393	13	52
12	48	. 7944	. 2056	.4718	. 2601	.0799	.3629	. 6627	. 3373	12	48
16	49	. 7965	. 2034	.4713	. 2655	.0793	.3633	. 6647	. 3353	11	44
20	50	.67987	.32013	1.4709	.92709	1.0786	1.3636	.26666	.73333	10	40
24	51	. 8008	. 1992	.4704	. 2763	.0780	.3640	. 6686	. 3314	9	36
28	52	. 8029	. 1970	.4699	. 2817	.0774	.3644	. 6706	. 3294	8	32
32	53	. 8051	. 1949	.4695	. 2871	.0767	.3647	. 6726	. 3274	7	28
36	54	. 8072	. 1928	.4690	. 2926	.0761	.3651	. 6746	. 3254	6	24
40	55	.68093	.31907	1.4686	.92980	1.0755	1.3655	.26765	.73234	5	20
44	56	. 8115	. 1885	.4681	. 3034	.0749	.3658	. 6785	. 3215	4	16
48	57	. 8136	. 1864	.4676	. 3088	.0742	.3662	. 6805	. 3195	3	12
52	58	. 8157	. 1843	.4672	. 3143	.0736	.3666	. 6825	. 3175	2	8
56	59	. 8178	. 1821	.4667	. 3197	.0730	.3669	. 6845	. 3155	1	4
52	60	. 8200	. 1800	.4663	. 3251	.0724	.3673	. 6865	. 3135	0	**8**
M. S.	M	Cosine.	Vrs. Sin.	Secante.	Cotang	Tangent.	Cosec'nt.	Vrs. Cos	Sine.	M	M S
8h	132°	Natural.								47°	3h

2h	43°	Natural Trigonometrical Functions.								136°	9h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
52	0	.68200	.31800	1.4663	.93251	1.0724	1.3673	.26865	.73135	60	**8**
4	1	. 8221	. 1779	.4658	. 3306	.0717	.3677	. 6884	. 3115	59	56
8	2	. 8242	. 1758	.4654	. 3360	.0711	.3681	. 6904	. 3096	58	52
12	3	. 8264	. 1736	.4649	. 3415	.0705	.3684	. 6924	. 3076	57	48
16	4	. 8285	. 1715	.4644	. 3469	.0699	.3688	. 6944	. 3056	56	44
20	5	.68306	.31694	1.4640	.93524	1.0692	1.3692	.26964	.73036	55	40
24	6	. 8327	. 1673	.4635	. 3578	.0686	.3695	. 6984	. 3016	54	36
28	7	. 8349	. 1651	.4631	. 3633	.0680	.3699	. 7004	. 2996	53	32
32	8	. 8370	. 1630	.4626	. 3687	.0674	.3703	. 7023	. 2976	52	28
36	9	. 8391	. 1609	.4622	. 3742	.0667	.3707	. 7043	. 2956	51	24
40	10	.68412	.31588	1.4617	.93797	1.0661	1.3710	.27063	.72937	50	20
44	11	. 8433	. 1566	.4613	. 3851	.0655	.3714	. 7083	. 2917	49	16
48	12	. 8455	. 1545	.4608	. 3906	.0649	.3718	. 7103	. 2897	48	12
52	13	. 8476	. 1524	.4604	. 3961	.0643	.3722	. 7123	. 2877	47	8
56	14	. 8497	. 1503	.4599	. 4016	.0636	.3725	. 7143	. 2857	46	4
53	15	.68518	.31482	1.4595	.94071	1.0630	1.3729	.27163	.72837	45	**7**
4	16	. 8539	. 1460	.4590	. 4125	.0624	.3733	. 7183	. 2817	44	56
8	17	. 8561	. 1439	.4586	. 4180	.0618	.3737	. 7203	. 2797	43	52
12	18	. 8582	. 1418	.4581	. 4235	.0612	.3740	. 7223	. 2777	42	48
16	19	. 8603	. 1397	.4577	. 4290	.0605	.3744	. 7243	. 2757	41	44
20	20	.68624	.31376	1.4572	.94345	1.0599	1.3748	.27263	.72737	40	40
24	21	. 8645	. 1355	.4568	. 4400	.0593	.3752	. 7283	. 2717	39	36
28	22	. 8666	. 1333	.4563	. 4455	.0587	.3756	. 7302	. 2697	38	32
32	23	. 8688	. 1312	.4559	. 4510	.0581	.3759	. 7322	. 2677	37	28
36	24	. 8709	. 1291	.4554	. 4565	.0575	.3763	. 7342	. 2657	36	24
40	25	.68730	.31270	1.4550	.94620	1.0568	1.3767	.27362	.72637	35	20
44	26	. 8751	. 1249	.4545	. 4675	.0562	.3771	. 7382	. 2617	34	16
48	27	. 8772	. 1228	.4541	. 4731	.0556	.3774	. 7402	. 2597	33	12
52	28	. 8793	. 1207	.4536	. 4786	.0550	.3778	. 7422	. 2577	32	8
56	29	. 8814	. 1186	.4532	. 4841	.0544	.3782	. 7442	. 2557	31	4
54	30	.68835	.31164	1.4527	.94896	1.0538	1.3786	.27462	.72537	30	**6**
4	31	. 8856	. 1143	.4523	. 4952	.0532	.3790	. 7482	. 2517	29	56
8	32	. 8878	. 1122	.4518	. 5007	.0525	.3794	. 7503	. 2497	28	52
12	33	. 8899	. 1101	.4514	. 5062	.0519	.3797	. 7523	. 2477	27	48
16	34	. 8920	. 1080	.4510	. 5118	.0513	.3801	. 7543	. 2457	26	44
20	35	.68941	.31059	1.4505	.95173	1.0507	1.3805	.27563	.72437	25	40
24	36	. 8962	. 1038	.4501	. 5229	.0501	.3809	. 7583	. 2417	24	36
28	37	. 8983	. 1017	.4496	. 5284	.0495	.3813	. 7603	. 2397	23	32
32	38	. 9004	. 0996	.4492	. 5340	.0489	.3816	. 7623	. 2377	22	28
36	39	. 9025	. 0975	.4487	. 5395	.0483	.3820	. 7643	. 2357	21	24
40	40	.69046	.30954	1.4483	.95451	1.0476	1.3824	.27663	.72337	20	20
44	41	. 9067	. 0933	.4479	. 5506	.0470	.3828	. 7683	. 2317	19	16
48	42	. 9088	. 0912	.4474	. 5562	.0464	.3832	. 7703	. 2297	18	12
52	43	. 9109	. 0891	.4470	. 5618	.0458	.3836	. 7723	. 2277	17	8
56	44	. 9130	. 0870	.4465	. 5673	.0452	.3839	. 7743	. 2256	16	4
55	45	.69151	.30849	1.4461	.95729	1.0446	1.3843	.27764	.72236	15	**5**
4	46	. 9172	. 0828	.4457	. 5785	.0440	.3847	. 7784	. 2216	14	56
8	47	. 9193	. 0807	.4452	. 5841	.0434	.3851	. 7804	. 2196	13	52
12	48	. 9214	. 0786	.4448	. 5896	.0428	.3855	. 7824	. 2176	12	48
16	49	. 9235	. 0765	.4443	. 5952	.0422	.3859	. 7844	. 2156	11	44
20	50	.69256	.30744	1.4439	.96008	1.0416	1.3863	.27864	.72136	10	40
24	51	. 9277	. 0723	.4435	. 6064	.0410	.3867	. 7884	. 2115	9	36
28	52	. 9298	. 0702	.4430	. 6120	.0404	.3870	. 7901	. 2095	8	32
32	53	. 9319	. 0681	.4426	. 6176	.0397	.3874	. 7925	. 2075	7	28
36	54	. 9340	. 0660	.4422	. 6232	.0391	.3878	. 7945	. 2055	6	24
40	55	.69361	.30639	1.4417	.96288	1.0385	1.3882	.27965	.72035	5	20
44	56	. 9382	. 0618	.4413	. 6344	.0379	.3886	. 7985	. 2015	4	16
48	57	. 9403	. 0597	.4408	. 6400	.0373	.3890	. 8005	. 1994	3	12
52	58	. 9424	. 0576	.4404	. 6456	.0367	.3894	. 8026	. 1974	2	8
56	59	. 9445	. 0555	.4400	. 6513	.0361	.3898	. 8046	. 1954	1	4
56	60	. 9466	. 0534	.4395	. 6569	.0355	.3902	. 8066	. 1934	0	**4**
M. S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt	Vrs.Cos	Sine.	M	M.S.
8h	133°	Natural.								46°	3h

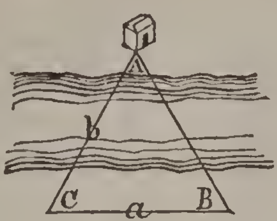
2h	44°	Natural Trigonometrical Functions.							135°		9h
M.S.	M	Sine.	Vrs.Cos.	Cosec'nte	Tang.	Cotang.	Secante.	Vrs.Sin	Cosine.	M	M.S.
56	0	.69466	.30534	1.4395	.96569	1.0355	1.3902	.28066	.71934	60	**4**
4	1	. 9487	. 0513	.4391	. 6625	.0349	.3905	. 8086	1914	59	56
8	2	. 9508	. 0492	.4387	. 6681	.0343	.3909	. 8106	. 1893	58	52
12	3	. 9528	. 0471	.4382	. 6738	.0337	.3913	. 8127	. 1873	57	48
16	4	. 9549	. 0450	.4378	. 6794	.0331	.3917	. 8147	. 1853	56	44
20	5	.69570	.30430	1.4374	.96850	1.0325	1.3921	.28167	.71833	55	40
24	6	. 9591	. 0409	.4370	. 6907	.0319	.3925	. 8187	. 1813	54	36
28	7	. 9612	. 0388	.4365	. 6963	.0313	.3929	. 8208	. 1792	53	32
32	8	. 9633	. 0367	.4361	. 7020	.0307	.3933	. 8228	. 1772	52	28
36	9	. 9654	. 0346	.4357	. 7076	.0301	.3937	. 8248	. 1752	51	24
40	10	.69675	.30325	1.4352	.97133	1.0295	1.3941	.28268	.71732	50	20
44	11	. 9696	. 0304	.4348	. 7189	.0289	.3945	. 8289	. 1711	49	16
48	12	. 9716	. 0283	.4344	. 7246	.0283	.3949	. 8309	. 1691	48	12
52	13	. 9737	. 0263	.4339	. 7302	.0277	.3953	. 8329	. 1671	47	8
56	14	. 9758	. 0242	.4335	. 7359	.0271	.3957	. 8349	. 1650	46	4
57	15	.69779	.30221	1.4331	.97416	1.0265	1.3960	.28370	.71630	45	**3**
4	16	. 9800	. 0200	.4327	. 7472	.0259	.3964	. 8390	. 1610	44	56
8	17	. 9821	. 0179	.4322	. 7529	.0253	.3968	. 8410	. 1589	43	52
12	18	. 9841	. 0158	.4318	. 7586	.0247	.3972	. 8431	. 1569	42	48
16	19	. 9862	. 0138	.4314	. 7643	.0241	.3976	. 8451	. 1549	41	44
20	20	.69883	.30117	1.4310	.97699	1.0235	1.3980	.28471	.71529	40	40
24	21	. 9904	. 0096	.4305	. 7756	.0229	.3984	. 8492	. 1508	39	36
28	22	. 9925	. 0075	.4301	. 7813	.0223	.3988	. 8512	. 1488	38	32
32	23	. 9945	. 0054	.4297	. 7870	.0218	.3992	. 8532	. 1468	37	28
36	24	. 9966	. 0034	.4292	. 7927	.0212	.3996	. 8553	. 1447	36	24
40	25	.69987	.30013	1.4288	.97984	1.0206	1.4000	.28573	.71427	35	20
44	26	.70008	.29992	.4284	. 8041	.0200	.4004	. 8593	. 1406	34	16
48	27	. 0029	. 9971	.4280	. 8098	.0194	.4008	. 8614	. 1386	33	12
52	28	. 0049	. 9950	.4276	. 8155	.0188	.4012	. 8634	. 1366	32	8
56	29	. 0070	. 9930	.4271	. 8212	.0182	.4016	. 8654	. 1345	31	4
58	30	.70091	.29909	1.4267	.98270	1.0176	1.4020	.28675	.71325	30	**2**
4	31	. 0112	. 9888	.4263	. 8327	.0170	.4024	. 8695	. 1305	29	56
8	32	. 0132	. 9867	.4259	. 8384	.0164	.4028	. 8716	. 1284	28	52
12	33	. 0153	. 9847	.4254	. 8441	.0158	.4032	. 8736	. 1264	27	48
16	34	. 0174	. 9826	.4250	. 8499	.0152	.4036	. 8756	. 1243	26	44
20	35	.70194	.29805	1.4246	.98556	1.0146	1.4040	.28777	.71223	25	40
24	36	. 0215	. 9785	.4242	. 8613	.0141	.4044	. 8797	. 1203	24	36
28	37	. 0236	. 9764	.4238	. 8671	.0135	.4048	8818	. 1182	23	32
32	38	. 0257	. 9743	.4233	. 8728	.0129	.4052	. 8838	. 1162	22	28
36	39	. 0277	. 9722	.4229	. 8786	.0123	1.4056	. 8859	. 1141	21	24
40	40	.70298	.29702	1.4225	.98843	1.0117	.4060	.28879	.71121	20	20
44	41	. 0319	. 9681	.4221	. 8901	.0111	.4065	. 8899	. 1100	19	16
48	42	. 0339	. 9660	.4217	. 8958	.0105	.4069	. 8920	. 1080	18	12
52	43	. 0360	. 9640	.4212	. 9016	.0099	.4073	. 8940	. 1059	17	8
56	44	. 0381	. 9619	.4208	. 9073	.0093	.4077	. 8961	. 1039	16	4
59	45	.70401	.29598	1.4204	.99131	1.0088	1.4081	.28981	.71018	15	**1**
4	46	. 0422	. 9578	.4200	. 9189	.0082	.4085	. 9002	. 0998	14	56
8	47	. 0443	. 9557	.4196	. 9246	.0076	.4089	. 9022	. 0977	13	52
12	48	. 0463	. 9536	.4192	. 9304	.0070	.4093	. 9043	. 0957	12	48
16	49	. 0484	. 9516	.4188	. 9362	.0064	.4097	. 9063	. 0936	11	44
20	50	.70505	.29495	1.4183	.99420	1.0058	1.4101	.29084	.70916	10	40
24	51	. 0525	. 9475	.4179	. 9478	.0052	.4105	. 9104	. 0895	9	36
28	52	. 0546	. 9454	.4175	. 9536	.0047	.4109	. 9125	. 0875	8	32
32	53	. 0566	. 9433	.4171	. 9593	.0041	.4113	. 9145	. 0854	7	28
36	54	. 0587	. 9413	.4167	. 9651	.0035	.4117	. 9166	. 0834	6	24
40	55	.70608	.29392	1.4163	.99709	1.0029	1.4122	.29186	.70813	5	20
44	56	. 0628	. 9372	.4159	. 9767	.0023	.4126	. 9207	. 0793	4	16
48	57	. 0649	. 9351	.4154	. 9826	.0017	.4130	. 9228	. 0772	3	12
52	58	. 0669	. 9330	.4150	. 9884	.0012	.4134	. 9248	. 0752	2	8
56	59	. 0690	. 9310	.4146	. 9942	.0006	.4138	. 9269	. 0731	1	4
60	60	. 0711	. 9289	.4142	1.0000	1.0000	.4142	. 9289	. 0711	0	**0**
M.S.	M	Cosine.	Vrs.Sin.	Secante.	Cotang.	Tangent.	Cosec'nt.	Vrs.Cos	Sine.	M	M.S.
8h	134°	Natural.							45°		3h

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
1.01	.0099503	1.65	.5007752	2.29	.8285518	2.93	1.0750024
1.02	.0198026	1.66	.5068175	2.30	.8329091	2.94	1.0784095
1.03	.0295588	1.67	.5128236	2.31	.8372475	2.95	1.0818051
1.04	.0392207	1.68	.5187937	2.32	.8415671	2.96	1.0851892
1.05	.0487902	1.69	.5247285	2.33	.8458682	2.97	1.0885619
1.06	.0582689	1.70	.5306282	2.34	.8501509	2.98	1.0919233
1.07	.0676586	1.71	.5364933	2.35	.8544153	2.99	1.0952733
1.08	.0769610	1.72	.5423242	2.36	.8586616	3.00	1.0986123
1.09	.0861777	1.73	.5481214	2.37	.8628899	3.01	1.1019400
1.10	.0953102	1.74	.5538851	2.38	.8671004	3.02	1.1052568
1.11	.1043600	1.75	.5596157	2.39	.8712933	3.03	1.1085626
1.12	.1133287	1.76	.5653138	2.40	.8754687	3.04	1.1118575
1.13	.1222176	1.77	.5709795	2.41	.8796267	3.05	1.1151415
1.14	.1310283	1.78	.5766133	2.42	.8837675	3.06	1.1184149
1.15	.1397619	1.79	.5822156	2.43	.8878912	3.07	1.1216775
1.16	.1484200	1.80	.5877866	2.44	.8919980	3.08	1.1249295
1.17	.1570037	1.81	.5933268	2.45	.8960880	3.09	1.1281710
1.18	.1655144	1.82	.5988365	2.46	.9001613	3.10	1.1314021
1.19	.1739533	1.83	.6043159	2.47	.9042181	3.11	1.1346227
1.20	.1823215	1.84	.6097655	2.48	.9082585	3.12	1.1378330
1.21	.1906203	1.85	.6151856	2.49	.9122826	3.13	1.1410330
1.22	.1988508	1.86	.6205764	2.50	.9162907	3.14	1.1442227
1.23	.2070141	1.87	.6259384	2.51	.9202827	3.15	1.1474024
1.24	.2151113	1.88	.6312717	2.52	.9242589	3.16	1.1505720
1.25	.2231435	1.89	.6365768	2.53	.9282193	3.17	1.1537315
1.26	.2311117	1.90	.6418538	2.54	.9321640	3.18	1.1568811
1.27	.2390169	1.91	.6471032	2.55	.9360933	3.19	1.1600209
1.28	.2468600	1.92	.6523251	2.56	.9400072	3.20	1.1631508
1.29	.2546422	1.93	.6575200	2.57	.9439058	3.21	1.1662709
1.30	.2623642	1.94	.6626879	2.58	.9477893	3.22	1.1693813
1.31	.2700271	1.95	.6678293	2.59	.9516578	3.23	1.1724821
1.32	.2776317	1.96	.6729444	2.60	.9555114	3.24	1.1755733
1.33	.2851789	1.97	.6780335	2.61	.9593502	3.25	1.1786549
1.34	.2926696	1.98	.6830968	2.62	.9631743	3.26	1.1817271
1.35	.3001045	1.99	.6881346	2.63	.9669838	3.27	1.1847899
1.36	.3074846	2.00	.6931472	2.64	.9707789	3.28	1.1878434
1.37	.3148107	2.01	.6981347	2.65	.9745596	3.29	1.1908875
1.38	.3220834	2.02	.7030974	2.66	.9783261	3.30	1.1939224
1.39	.3293037	2.03	.7080357	2.67	.9820784	3.31	1.1969481
1.40	.3364722	2.04	.7129497	2.68	.9858167	3.32	1.1999647
1.41	.3435897	2.05	.7178397	2.69	.9895411	3.33	1.2029722
1.42	.3506568	2.06	.7227059	2.70	.9932517	3.34	1.2059707
1.43	.3576744	2.07	.7275485	2.71	.9969486	3.35	1.2089603
1.44	.3646431	2.08	.7323678	2.72	1.0006318	3.36	1.2119409
1.45	.3715635	2.09	.7371640	2.73	1.0043015	3.37	1.2149127
1.46	.3784364	2.10	.7419373	2.74	1.0079579	3.38	1.2178757
1.47	.3852624	2.11	.7466879	2.75	1.0116008	3.39	1.2208299
1.48	.3920420	2.12	.7514160	2.76	1.0152306	3.40	1.2237754
1.49	.3987761	2.13	.7561219	2.77	1.0188473	3.41	1.2267122
1.50	.4054651	2.14	.7608058	2.78	1.0224509	3.42	1.2296405
1.51	.4121096	2.15	.7654678	2.79	1.0260415	3.43	1.2325605
1.52	.4187103	2.16	.7701082	2.80	1.0296194	3.44	1.2354714
1.53	.4252677	2.17	.7747271	2.81	1.0331844	3.45	1.2383742
1.54	.4317824	2.18	.7793248	2.82	1.0367368	3.46	1.2412685
1.55	.4382549	2.19	.7839015	2.83	1.0402766	3.47	1.2441545
1.56	.4446858	2.20	.7884573	2.84	1.0438040	3.48	1.2470322
1.57	.4510756	2.21	.7929925	2.85	1.0473189	3.49	1.2499017
1.58	.4574248	2.22	.7975071	2.86	1.0508216	3.50	1.2527629
1.59	.4637340	2.23	.8020015	2.87	1.0543120	3.51	1.2556160
1.60	.4700036	2.24	.8064758	2.88	1.0577902	3.52	1.2584609
1.61	.4762341	2.25	.8109302	2.89	1.0612564	3.53	1.2612978
1.62	.4824261	2.26	.8153648	2.90	1.0647107	3.54	1.2641266
1.63	.4885800	2.27	.8197798	2.91	1.0681530	3.55	1.2669475
1.64	.4946962	2.28	.8241754	2.92	1.0715836	3.56	1.2697605

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
3.57	1.2725655	4.21	1.4374626	4.85	1.5789787	5.49	1.7029282
3.58	1.2753627	4.22	1.4398351	4.86	1.5810384	5.50	1.7047481
3.59	1.2781521	4.23	1.4422020	4.87	1.5830939	5.51	1.7065646
3.60	1.2809338	4.24	1.4445632	4.88	1.5851452	5.52	1.7083778
3.61	1.2837077	4.25	1.4469189	4.89	1.5871923	5.53	1.7101878
3.62	1.2864740	4.26	1.4492691	4.90	1.5892352	5.54	1.7119944
3.63	1.2892326	4.27	1.4516138	4.91	1.5912739	5.55	1.7137979
3.64	1.2919836	4.28	1.4539530	4.92	1.5933085	5.56	1.7155981
3.65	1.2947271	4.29	1.4562867	4.93	1.5953389	5.57	1.7173950
3.66	1.2974631	4.30	1.4586149	4.94	1.5973653	5.58	1.7191887
3.67	1.3001916	4.31	1.4609379	4.95	1.5993875	5.59	1.7209792
3.68	1.3029127	4.32	1.4632553	4.96	1.6014057	5.60	1.7227666
3.69	1.3056264	4.33	1.4655675	4.97	1.6034198	5.61	1.7245507
3.70	1.3083328	4.34	1.4678743	4.98	1.6054298	5.62	1.7263316
3.71	1.3110318	4.35	1.4701758	4.99	1.6074358	5.63	1.7281094
3.72	1.3137236	4.36	1.4724720	5.00	1.6094379	5.64	1.7298840
3.73	1.3164082	4.37	1.4747630	5.01	1.6114359	5.65	1.7316555
3.74	1.3190856	4.38	1.4770487	5.02	1.6134300	5.66	1.7334238
3.75	1.3217558	4.39	1.4793292	5.03	1.6154200	5.67	1.7351891
3.76	1.3244189	4.40	1.4816045	5.04	1.6174060	5.68	1.7369512
3.77	1.3270749	4.41	1.4838746	5.05	1.6193882	5.69	1.7387102
3.78	1.3297240	4.42	1.4861396	5.06	1.6213664	5.70	1.7404661
3.79	1.3323660	4.43	1.4883995	5.07	1.6233408	5.71	1.7422189
3.80	1.3350010	4.44	1.4906543	5.08	1.6253112	5.72	1.7439687
3.81	1.3376291	4.45	1.4929040	5.09	1.6272778	5.73	1.7457155
3.82	1.3402504	4.46	1.4951487	5.10	1.6292405	5.74	1.7474591
3.83	1.3428648	4.47	1.4973883	5.11	1.6311994	5.75	1.7491998
3.84	1.3454723	4.48	1.4996230	5.12	1.6331544	5.76	1.7509374
3.85	1.3480731	4.49	1.5018527	5.13	1.6351056	5.77	1.7526720
3.86	1.3506671	4.50	1.5040774	5.14	1.6370530	5.78	1.7544036
3.87	1.3532544	4.51	1.5062971	5.15	1.6389967	5.79	1.7561323
3.88	1.3558351	4.52	1.5085119	5.16	1.6409365	5.80	1.7578579
3.89	1.3584091	4.53	1.5107219	5.17	1.6428726	5.81	1.7595805
3.90	1.3609765	4.54	1.5129269	5.18	1.6448050	5.82	1.7613002
3.91	1.3635373	4.55	1.5151272	5.19	1.6467336	5.83	1.7630170
3.92	1.3660916	4.56	1.5173226	5.20	1.6486586	5.84	1.7647308
3.93	1.3686394	4.57	1.5195132	5.21	1.6505798	5.85	1.7664416
3.94	1.3711807	4.58	1.5216990	5.22	1.6524974	5.86	1.7681496
3.95	1.3737156	4.59	1.5238800	5.23	1.6544112	5.87	1.7698546
3.96	1.3762440	4.60	1.5260563	5.24	1.6563214	5.88	1.7715567
3.97	1.3787661	4.61	1.5282278	5.25	1.6582280	5.89	1.7732559
3.98	1.3812818	4.62	1.5303947	5.26	1.6601310	5.90	1.7749523
3.99	1.3837912	4.63	1.5325568	5.27	1.6620303	5.91	1.7766458
4.00	1.3862943	4.64	1.5347143	5.28	1.6639260	5.92	1.7783364
4.01	1.3887912	4.65	1.5368672	5.29	1.6658182	5.93	1.7800242
4.02	1.3912818	4.66	1.5390154	5.30	1.6677068	5.94	1.7817091
4.03	1.3937663	4.67	1.5411590	5.31	1.6695918	5.95	1.7833912
4.04	1.3962446	4.68	1.5432981	5.32	1.6714733	5.96	1.7850704
4.05	1.3987168	4.69	1.5454325	5.33	1.6733512	5.97	1.7867469
4.06	1.4011829	4.70	1.5475625	5.34	1.6752256	5.98	1.7884205
4.07	1.4036429	4.71	1.5496879	5.35	1.6770965	5.99	1.7900914
4.08	1.4060969	4.72	1.5518087	5.36	1.6789639	6.00	1.7917594
4.09	1.4085449	4.73	1.5539252	5.37	1.6808278	6.01	1.7934247
4.10	1.4109869	4.74	1.5560371	5.38	1.6826882	6.02	1.7950872
4.11	1.4134230	4.75	1.5581446	5.39	1.6845453	6.03	1.7967470
4.12	1.4158531	4.76	1.5602476	5.40	1.6863989	6.04	1.7984040
4.13	1.4182774	4.77	1.5623462	5.41	1.6882491	6.05	1.8000582
4.14	1.4206957	4.78	1.5644405	5.42	1.6900958	6.06	1 8017098
4.15	1.4231083	4.79	1.5665304	5.43	1.6919391	6.07	1.8033586
4.16	1.4255150	4.80	1.5686159	5.44	1.6937790	6.08	1.8050047
4.17	1.4279160	4.81	1.5706971	5.45	1.6956155	6.09	1.8066481
4.18	1.4303112	4.82	1.5727739	5.46	1.6974487	6.10	1.8082887
4.19	1.4327007	4.83	1.5748464	5.47	1.6992786	6.11	1.8099267
4.20	1.4350845	4.84	1.5769147	5.48	1.7011051	6.12	1.8115621

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
6.13	1.8131947	6.77	1.9125011	7.41	2.0028305	8.05	2.0856720
6.14	1.8148247	6.78	1.9139771	7.42	2.0041790	8.06	2.0869135
6.15	1.8164520	6.79	1.9154509	7.43	2.0055258	8.07	2.0881534
6.16	1.8180767	6.80	1.9169226	7.44	2.0068708	8.08	2.0893918
6.17	1.8196988	6.81	1.9183921	7.45	2.0082140	8.09	2.0906287
6.18	1.8213182	6.82	1.9198594	7.46	2.0095553	8.10	2.0918640
6.19	1.8229351	6.83	1.9213247	7.47	2.0108949	8.11	2.0930984
6.20	1.8245493	6.84	1.9227877	7.48	2.0122327	8.12	2.0943306
6.21	1.8261608	6.85	1.9242486	7.49	2.0135687	8.13	2.0955613
6.22	1.8277699	6.86	1.9257074	7.50	2.0149030	8.14	2.0967905
6.23	1.8293763	6.87	1.9271641	7.51	2.0162354	8.15	2.0980182
6.24	1.8309801	6.88	1.9286186	7.52	2.0175661	8.16	2.0992444
6.25	1.8325814	6.89	1.9300710	7.53	2.0188950	8.17	2.1004691
6.26	1.8341801	6.90	1.9315214	7.54	2.0202221	8.18	2.1016923
6.27	1.8357763	6.91	1.9329696	7.55	2.9215475	8.19	2.1029140
6.28	1.8373699	6.92	1.9344157	7.56	2.0228711	8.20	2.1041341
6.29	1.8389610	6.93	1.9358598	7.57	2.0241929	8.21	2.1053529
6.30	1.8405496	6.94	1.9373017	7.58	2.0255131	8.22	2.1065702
6.31	1.8421356	6.95	1.9387416	7.59	2.0268315	8.23	2.1077861
6.32	1.8437191	6.96	1.9401794	7.60	2.0281482	8.24	2.1089998
6.33	1.8453002	6.97	1.9416152	7.61	2.0294631	8.25	2.1102128
6.34	1.8468787	6.98	1.9430489	7.62	2.0307763	8.26	2.1114243
6.35	1.8484547	6.99	1.9444805	7.63	2.0320878	8.27	2.1126343
6.36	1.8500283	7.00	1.9459101	7.64	2.0333976	8.28	2.1138428
6.37	1.8515994	7.01	1.9473376	7.65	2.0347056	8.29	2.1150499
6.38	1.8531680	7.02	1.9487632	7.66	2.0360119	8.30	2.1162555
6.39	1.8547342	7.03	1.9501866	7.67	2.0373166	8.31	2.1174596
6.40	1.8562979	7.04	1.9516080	7.68	2.0386195	8.32	2.1186622
6.41	1.8578592	7.05	1.9530275	7.69	2.0399207	8.33	2.1198634
6.42	1.8594181	7.06	1.9544449	7.70	2.0412203	8.34	2.1210632
6.43	1.8609745	7.07	1.9558604	7.71	2.0425181	8.35	2.1222615
6.44	1.8625285	7.08	1.9572739	7.72	2.0438143	8.36	2.1234584
6.45	1.8640801	7.09	1.9586853	7.73	2.0451088	8.37	2.1246539
6.46	1.8656293	7.10	1.9600947	7.74	2.0464016	8.38	2.1258479
6.47	1.8671761	7.11	1.9615022	7.75	2.0476928	8.39	2.1270405
6.48	1.8687205	7.12	1.9629077	7.76	2.0489823	8.40	2.1282317
6.49	1.8702625	7.13	1.9643112	7.77	2.0502701	8.41	2.1294214
6.50	1.8718021	7.14	1.9657127	7.78	2.0515563	8.42	2.1306098
6.51	1.8733394	7.15	1.9671123	7.79	2.0528408	8.43	2.1317967
6.52	1.8748743	7.16	1.9685099	7.80	2.0541237	8.44	2.1329822
6.53	1.8764069	7.17	1.9699056	7.81	2.0554049	8.45	2.1341664
6.54	1.8779371	7.18	1.9712993	7.82	2.0566845	8.46	2.1353491
6.55	1.8794650	7.19	1.9726911	7.83	2.0579624	8.47	2.1365304
6.56	1.8809906	7.20	1.9740810	7.84	2.0592388	8.48	2.1377104
6.57	1.8825138	7.21	1.9754689	7.85	2.0605135	8.49	2.1388889
6.58	1.8840347	7.22	1.9568549	7.86	2.0817866	8.50	2.1400661
6.59	1.8855533	7.23	1.9782390	7.87	2.0630580	8.51	2.1412419
6.60	1.8870696	7.24	1.9796212	7.88	2.0643278	8.52	2.1424163
6.61	1.8885837	7.25	1.9810014	7.89	2.0655961	8.53	2.1435893
6.62	1.8900954	7.26	1.9823798	7.90	2.0668627	8.54	2.1447609
6.63	1.8916048	7.27	1.9837562	7.91	2.0681277	8.55	2.1459312
6.64	1.8931119	7.28	1.9851308	7.92	2.0693911	8.56	2.1471001
6.65	1.8946168	7.29	1.9865035	7.93	2.0706530	8.57	2.1482676
6.66	1.8961194	7.30	1.9878743	7.94	2.0719132	8.58	2.1494339
6.67	1.8976198	7.31	1.9892432	7.95	2.0731719	8.59	2.1505987
6.68	1.8991179	7.32	1.9906103	7.96	2.0744290	8.60	2.1517622
6.69	1.9006138	7.33	1.9919754	7.97	2.0756845	8.61	2.1529243
6.70	1.9021075	7.34	1.9933387	7.98	2.0769384	8.62	2.1540851
6.71	1.9035989	7.35	1.9947002	7.99	2.0781907	8.63	2.1552445
6.72	1.9050881	7.36	1.9960599	8.00	2.0794415	8.64	2.1564026
6.73	1.9065751	7.37	1.9974177	8.01	2.0806907	8.65	2.1575593
6.74	1.9080600	7.38	1.9987736	8.02	2.0819384	8.66	2.1587147
6.75	1.9095425	7.39	2.0001278	8.03	2.0831845	8.67	2.1598687
6.76	1.9110228	7.40	2.0014800	8.04	2.0844290	8.68	2.1610215

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
8.69	2.1621729	9.33	2.2332350	9.97	2.2995806	71	4.2626799
8.70	2.1633230	9.34	2.2343062	9.98	2.3005831	72	4.2766661
8.71	2.1644718	9.35	2.2353763	9.99	2.3015846	73	4.2904594
8.72	2.1656192	9.36	2.2364452	10	2.3025851	74	4.3040651
8.73	2.1667653	9.37	2.2375130	11	2.3978953	75	4.3174881
8.74	2.1679101	9.38	2.2385797	12	2.4849067	76	4.3307333
8.75	2.1690536	9.39	2.2396452	13	2.5649494	77	4.3438054
8.76	2.1701959	9.40	2.2407096	14	2.6390573	78	4.3567088
8.77	2.1713367	9.41	2.2417729	15	2.7080502	79	4.3694479
8.78	2.1724763	9.42	2.2428350	16	2.7725887	80	4.3820266
8.79	2.1736146	9.43	2.2438960	17	2.8332133	81	4.3944492
8.80	2.1747517	9.44	2.2449559	18	2.8903718	82	4.4067193
8.81	2.1758874	9.45	2.2460147	19	2.9444390	83	4.4188406
8.82	2.1770218	9.46	2.2470723	20	2.9957323	84	4.4308168
8.83	2.1781550	9.47	2.2481288	21	3.0445224	85	4.4426513
8.84	2.1792868	9.48	2.2491843	22	3.0910425	86	4.4543473
8.85	2.1804174	9.49	2.2502386	23	3.1354942	87	4.4659081
8.86	2.1815467	9.50	2.2512917	24	3.1780538	88	4.4773368
8.87	2.1826747	9.51	2.2523438	25	3.2188758	89	4.4886364
8.88	2.1838015	9.52	2.2533948	26	3.2580965	90	4.4998097
8.89	2.1849270	9.53	2.2544446	27	3.2958369	91	4.5108595
8.90	2.1860512	9.54	2.2554934	28	3.3322045	92	4.5217886
8.91	2.1871742	9.55	2.2565411	29	3.3672958	93	4.5325995
8.92	2.1882959	9.56	2.2575877	30	3.4011974	94	4.5432948
8.93	2.1894163	9.57	2.2586332	31	3.4339872	95	4.5538769
8.94	2.1905355	9.58	2.2596776	32	3.4657359	96	4.5643482
8.95	2.1916535	9.59	2.2607209	33	3.4965076	97	4.5747110
8.96	2.1927702	9.60	2.2617631	34	3.5263605	98	4.5849675
8.97	2.1938856	9.61	2.2628042	35	3.5553481	99	4.5951199
8.98	2.1949998	9.62	2.2638442	36	3.5835189	100	4.6051702
8.99	2.1961128	9.63	2.2648832	37	3.6109179	101	4.6151205
9.00	2.1972245	9.64	2.2659211	38	3.6375862	102	4.6249728
9.01	2.1983350	9.65	2.2669579	39	3.6635617	103	4.6347290
9.02	2.1994443	9.66	2.2679936	40	3.6888795	104	4.6443909
9.03	2.2005523	9.67	2.2690282	41	3.7135721	105	4.6539604
9.04	2.2016591	9.68	2.2700618	42	3.7376696	106	4.6634391
9.05	2.2027647	9.69	2.2710944	43	3.7612001	107	4.6728288
9.06	2.2038691	9.70	2.2721258	44	3.7841896	108	4.6821312
9.07	2.2049722	9.71	2.2731562	45	3.8066525	109	4.6913479
9.08	2.2060741	9.72	2.2741856	46	3.8286414	110	4.7004804
9.09	2.2071748	9.73	2.2752138	47	3.8501476	111	4.7095302
9.10	2.2082744	9.74	2.2762411	48	3.8712010	112	4.7184989
9.11	2.2093727	9.75	2.2772673	49	3.8918203	113	4.7273878
9.12	2.2104697	9.76	2.2782924	50	3.9120230	114	4.7361985
9.13	2.2115656	9.77	2.2793165	51	3.9318256	115	4.7449321
9.14	2.2126603	9.78	2.2803395	52	3.9512437	116	4.7535902
9.15	2.2137538	9.79	2.2813614	53	3.9702919	117	4.7621739
9.16	2.2148461	9.80	2.2823823	54	3.9889841	118	4.7706846
9.17	2.2159372	9.81	2.2834022	55	4.0073332	119	4.7791235
9.18	2.2170272	9.82	2.2844211	56	4.0253517	120	4.7874917
9.19	2.2181160	9.83	2.2854389	57	4.0430513	121	4.7957906
9.20	2.2192034	9.84	2.2864556	58	4.0604430	122	4.8040210
9.21	2.2202898	9.85	2.2874714	59	4.0775374	123	4.8121844
9.22	2.2213750	9.86	2.2884861	60	4.0943446	124	4.8202816
9.23	2.2224590	9.87	2.2894998	61	4.1108739	125	4.8283137
9.24	2.2235418	9.88	2.2905124	62	4.1271344	126	4.8362819
9.25	2.2246235	9.89	2.2915241	63	4.1431347	127	4.8441871
9.26	2.2257040	9.90	2.2925347	64	4.1588839	128	4.8520303
9.27	2.2267833	9.91	2.2935443	65	4.1743873	129	4.8598124
9.28	2.2278615	9.92	2.2945529	66	4.1896547	130	4.8675345
9.29	2.2289385	9.93	2.2955604	67	4.2046926	131	4.8751973
9.30	2.2300144	9.94	2.2965670	68	4.2195077	132	4.8828019
9.31	2.2310890	9.95	2.2975725	69	4.2341065	133	4.8903491
9.32	2.2321626	9.96	2.2985770	70	4.2484952	134	4.8978398



206.

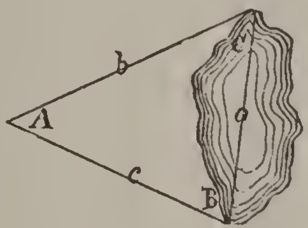
To find the distance b from the point C to an inaccessible house.

Measure a base line a and the angles B and C. Then $a : b = \sin.A : \sin.B$.

$b = \frac{a \sin.B}{\sin.A}$. Suppose $a = 160$ feet, $C = 87° 45'$, $B = 65° 35'$. Required the distance b?

$A = 180 - (B + C) = 180 - (87° 45' + 65° 35') = 26° 40'$.

$$b = \frac{160 \times \sin.65° 35'}{\sin.26° 40'} = \frac{160 \times 0.91056}{0.4488} = 325.56 \text{ feet.}$$

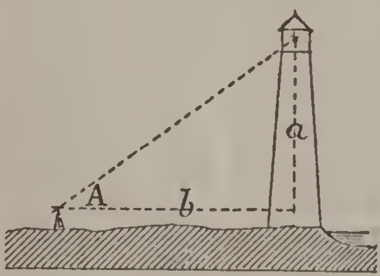


207.

To find the distance a over a lake.

Measure the angle A and the two sides b and c. Say $A = 34° 25'$, $b = 1684$ feet, and $c = 1310$ feet. Required the distance a?

$$\cot.B = \frac{c}{b \sin.A} - \cot.A = \frac{1310}{1684 \times \sin.34° 25'}$$
$$- \cot.34° 25' = 0.083 = \cot.94° 45'.$$
$$a : b = \sin.34° 25' : \sin.94° 15'.$$
$$a = \frac{1684 \times \sin.34° 25'}{\sin.94° 15} = 955.1 \text{ feet.}$$



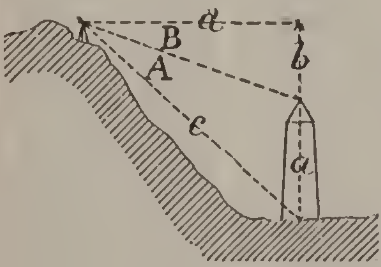
208.

To measure the height of a lighthouse.

Measure the distance b and angle A; then the height a will be $a = b \tan.A$. Suppose $b = 134$ feet and the angle $A = 56° 34'$. Required the height a?

$a = 134 \times \tan.56° 34' = 134 \times 1.5147 = 202.97$ feet.

The height of the line b above the ground can be measured separately and added.



209.

To find the height of an object from a height.

The distance c and the angles A and B are given.

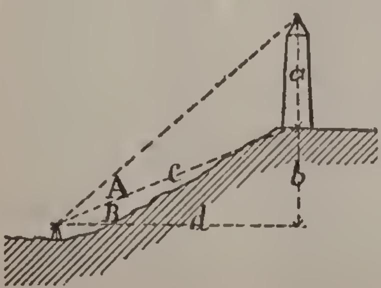
$a + b = c \sin.(A + B)$. Suppose $A = 36° 42'$, $B = 21° 22'$, and the side $c = 115$ feet. Required the height a?

$a + b = 115 \sin.(36° 42' + 21° 22') = 115 \sin.58° 4'$
$= 115 \times 0.84866 = 97.216$ feet.

The horizontal distance $d = c \cos.58° 4' = 115 \times 0.52893 = 60.827$ feet.

$b = 60.827 \times \tan.21° 22' = 60.827 \times 0.36433 = 22.162$ feet.

The height $a = 97.216 - 22.162 = 75.054$ feet.

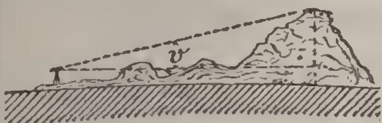


210.

To find the height of an object from a point below the base.

Measure the angles A and B and the distance c. $b = c \sin.B$, and $d = c \cos.B$. Suppose $c = 86$ feet, $A = 42° 20'$, $B = 10° 15'$. Required the horizontal distance d and the height b?

$$b = 86 \sin.10° 15' = 15.3028 \text{ feet.}$$
$$d = 86 \cos.10° 15' = 84.627.$$
$$(a + b) = d \tan.(A + B) = 84.627 \times \tan.52° 35'$$
$$= 110.616 \text{ feet.}$$
$$a = 110.616 - 15.3021 = 95.313 \text{ feet.}$$

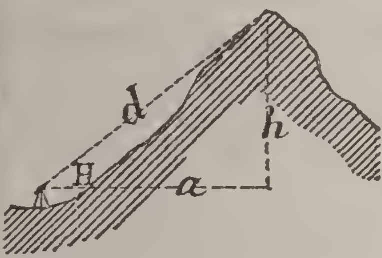


211.

To find the height of a distant mountain.

The distance from the instrument to the mountain is approximately known by a map to be say 12 miles, and the angle v measured from the top of the mountain to the horizontal line is $1^\circ 44'$. Required the height of the mountain? $12 \times 5280 = 63360$ feet.

	Feet.
Height of mountain $h = 63360 \tan.1^\circ 44' =$	1917.27
For curvature and refraction add	82.32
Height of instrument	4.
The required height	2003.59



212.

To find the height h of a mountain, and horizontal distance a from the instrument to under the highest point.

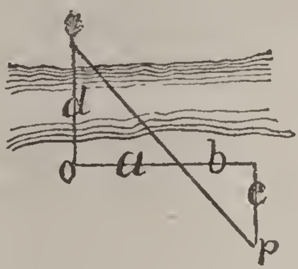
Measure the angle H and distance d.

$$h = d \sin.H, \quad \text{and} \quad a = d \cos.H.$$

Suppose $d = 1560$ feet and the angle $H = 42^\circ 54'$. Required the height h and distance a?

$$h = 1560 \times \sin.42^\circ 54' = 1061.9 \text{ feet.}$$

$$a = 1560 \times \cos.42^\circ 54' = 1142.7 \text{ feet.}$$

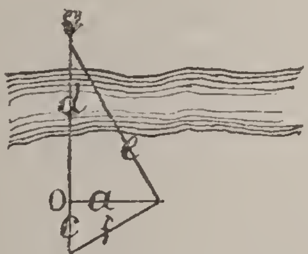


213.

To measure the distance d from o to a tree on the other side of the stream.

Draw from o the line $a + b$ at right angles to d. Draw c at right angles to $a + b$. From a point p sight the tree, which cuts the line $a + b$,

so that $a : b = d : c$. Distance, $d = \dfrac{a\,c}{b}$.

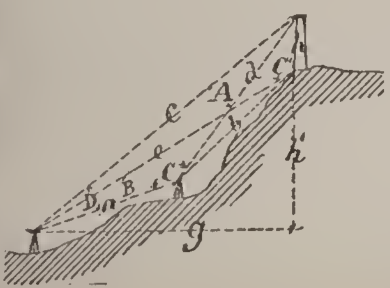


214.

To find the distance d from o to a tree on the other side of a stream.

Make a at right angles to d, and f at right angles to e; then $a : d = c : a$.

$$d = \frac{a^2}{c}.$$



215.

To find the height h of an inaccessible object, also its horizontal distance from the instrument.

Measure the base line a and the angles B, C, D, and H.

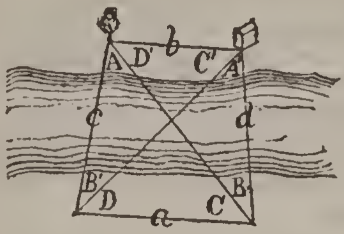
$a : c = \sin.A : \sin.C$. $c = \dfrac{a \sin.C}{\sin.A}$. $A = 180 - (D + C)$.

$a : e = \sin.A' : \sin.E$. $\quad e = \dfrac{a \sin.E}{\sin.A'}$.

$h : c = \sin.H : \sin.C$. $\quad h = \dfrac{c \sin.H}{\sin.C}$.

$g = c \cos.(c\,g)$. $\quad h' = e \sin.(e\,g)$.

216.



To find the distance b between a tree and a house on the other side of the river.

$$a : d = \sin.A' : \sin.D. \qquad d = \frac{a \sin.D}{\sin.A'}.$$

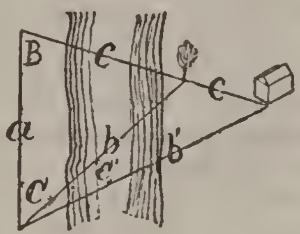
$$A' = 180 - (B + C + D).$$

$$a : e = \sin.A : \sin.(B' + D). \qquad e = \frac{a \sin.(B' + D)}{\sin.A}.$$

$$\cot.D' = \frac{e}{d \sin.B} - \cot.B.$$

$$b : d = \sin.B : \sin.D'. \qquad b = \frac{d \sin.B}{\sin.D'}.$$

217.



To find the distance c' between a house and a tree.

Put the instrument at B, in line with the tree and house; measure the angle B, and lay out the base line a; measure the angles C and c'.

$$a : c = \sin.A' : \sin.C. \qquad c = \frac{a \sin.C}{\sin.A'}.$$

$$A' = 180 - (B + C).$$

$$a : c + c' = \sin.A : \sin.(C + c'). \qquad c + c' = \frac{a \sin.(C + c')}{\sin.A}.$$

$$c' = \frac{a \sin.(C + c')}{\sin.A} - \frac{a \sin.C}{\sin.A} = \frac{a}{\sin.A}\Big(\sin.(C + c') - \sin.C\Big).$$

ELEMENTS OF MECHANICS.

Mechanics is that branch of natural philosophy which treats of forces in equilibrium and in motion; it is divided into two distinct parts, namely—

STATICS AND DYNAMICS.

Statics is the science of forces in equilibrium or at rest; it is subdivided into various branches bearing upon solids, liquids, gases, and imponderable fluids.

Statics, strictly speaking, refers to forces in equilibrium in regard to solids.

Hydrostatics treats of forces in equilibrium of liquids.

Aërostatics treats of pressure and equilibrium of air or other gases.

Electrostatics refers to static electricity held or stored in a body.

Thermostatics refers to stationary temperatures in bodies.

The science of statics is very well developed in text-books, and particularly so in works on roofs, girders, bridges, and on strength of materials; but such cannot be said about dynamics, which is yet in a confused condition.

Dynamics is the science of forces in motion, producing power and work, and is also subdivided into various branches like statics, bearing upon solids, liquids, gases, and imponderable fluids.

Dynamics, strictly speaking, refers to power and work of solids.

Hydrodynamics, or *Hydraulics*, treats of motion, power, and work of liquids.

Aërodynamics treats of power and work of air and other gases.

Electrodynamics treats of power and work of electricity and magnetism.

Thermodynamics treats of power and work produced directly by heat like that of steam.

Imponderable Matter.

The term *imponderable* implies that the body has no weight, but is, nevertheless, a material substance which is in such condition as to constitute the connection of attraction or repulsion between ponderable bodies. We know that imponderable substances are matter by their manifestation of possessing *inertia*, and, whatever aggregate form a body may be converted into, its inertia can never be destroyed or interfered with in the least.

All matter can be resolved into four aggregate forms—namely, *solid*, *liquid*, *gaseous*, and *imponderable;* but there is no sharp line of distinction between these four forms—that is, a body may be in a semifluid state, like *glaciers*, which flow in a solid state; and many soft substances are neither solid nor liquid, but may be called *plastic*.

The compositions of imponderable fluids are as varied as those of ponderable matter, and the probability is that they are all binary compounds of the same matter which we know when in a ponderable state. Each kind of imponderable substance manifests a nature peculiar to itself, and some kinds cannot be displaced by ponderable bodies passing through them, as is the case with that imponderable fluid which forms the connection of universal attraction, and also that constituting magnetic attraction and repulsion, which fluid passes through the ponderable like light passes through a perfectly transparent substance. The spectrum is a fair representation of known ponderable matter in imponderable states. It is also manifest by the spectrum that some imponderables are not refracted, but pass straight through as if the prism were not there, whilst other imponderables do not pass through the prism, but are reflected therefrom.

This is purely speculative on the part of Mr. Nystrom, and is not to be accepted as established fact.—W. D. M.

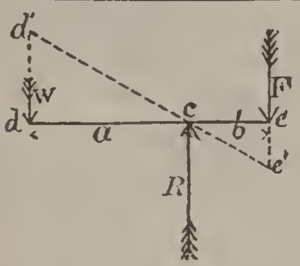
STATICS. FORCES IN EQUILIBRIUM.

In mechanics, the term *force* means any action which can be expressed simply by weight, and which can be realized only by an equal amount of reaction. Forces are derived from a great variety of sources; but whenever it is simply *force*, it can invariably be expressed by weight, without regard to motion, time, power, or work.

The magnitude and direction of a force can be represented by a straight line, but no force can be realized without an equal amount of reaction in the opposite direction, which can likewise be represented by a straight line.

1. *Action and Reaction.*

Let the line F represent the magnitude and direction of a force acting on the point c at rest or in motion, and let the line R represent an equal amount of resistance acting in the opposite direction; then the force F and resistance R are said to be in equilibrium. R is called resultant.



2. *Resultant of two Parallel Forces.*

W is a weight whose magnitude and direction are represented by the arrow, and F is a force acting parallel to W, both acting on the inflexible bar $a\,b$. Make $d\,d'$ equal to F and $e\,e'$ equal to W; join $d'\,e$, and the crossing c is the fulcrum for the resultant R. Make R equal to $W+F$ and draw it from c parallel to W and F.

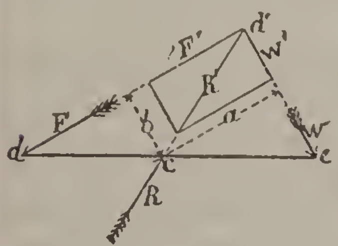
Static Moment and Lever.

$F : W = a : b.$ Static moments $F b = W a.$

a and b are called levers; the force or weight multiplied by the lever acted upon is called static moment.

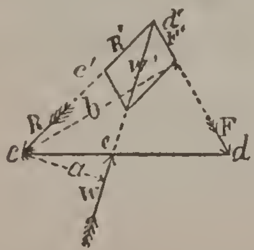
A lever is the rectangular distance from the direction of a force to the fulcrum.

Fulcrum is the point on which a lever turns.



3. *Forces Acting Obliquely.*

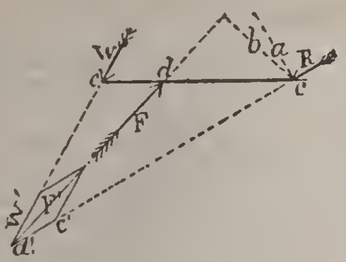
F and W represent the magnitude and direction of two forces acting at the points d and e. Continue the direction of the two forces until they meet at d'. From d' set off the F' and W' equal to F and W respectively; complete the parallelogram, and the diagonal R' represents the magnitude and direction of the resultant R acting at the fulcrum c. The levers for these forces are the rectangular distances a and b. Static moments $F b = W a.$



4. *Forces Acting Against One Another.*

Prolong the forces F and W until they meet at d', from which set off the forces W' and F' equal to W and F respectively. Complete the parallelogram, and $c'\ d'$ is the magnitude and direction of the resultant R, which, acting in the fulcrum c, will balance the two forces. The levers a and b are drawn from the fulcrum c at right angles to the direction of the respective forces.

Static moments $F\,b = W\,a.$

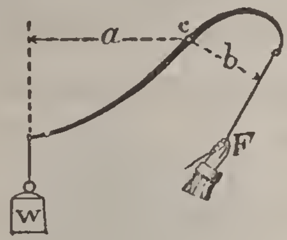


5.

Two Forces Acting Against One Another.

Prolong F and W until they meet at d'; set off W' and F' equal to W and F respectively and complete the parallelogram; then $c' d'$ represents the direction and magnitude of the resultant R acting at the fulcrum c. The levers a and b are drawn from the fulcrum c at right angles to the respective forces.

Static moments $Fb = Wa$.

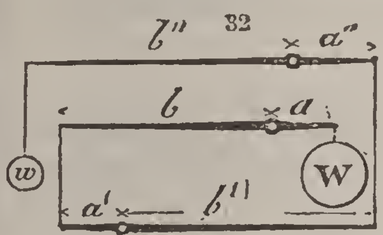


6.

Form of Bar Independent of Levers.

The bar upon which the forces act can be of any desired form, such as $e\,c\,d$, but the actual levers will still be as before defined—namely, the rectangular distances from the fulcrum to the direction of the respective forces.

Static moments $Fb = Wa$.



7.

Combination of Levers.

The static moments in this combination will be

$$W\,a\,a'\,a'' = w\,b\,b'\,b''.$$

$$W = \frac{w\,b\,b'\,b''}{a\,a'a''}. \qquad w = \frac{W\,a\,a'\,a''}{b\,b'\,b''}.$$

Moment of Stability.

8.

A body W is acted upon by a force F to turn the body over the fulcrum A. Required its moment of stability.

Moment of stability $Wa = Fb$ static moment. The weight of the body is considered concentrated in its centre of gravity W, and the lever a is the rectangular distance from the fulcrum c to the direction of gravity.

$$F = \frac{W\,a}{b}, \quad W = \frac{F\,b}{a}, \quad a = \frac{F\,b}{W}, \quad \text{and } b = \frac{W\,a}{F}.$$

9.

Formulas are the same as above.

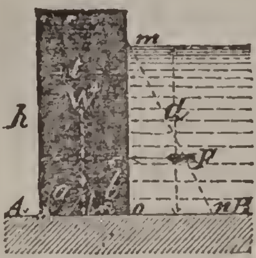
Example.—The weight of the body is $W = 36{,}390$ pounds, the lever $a = 6$ feet, and the lever $b = 18$ feet. Required the force F to turn the body over the fulcrum B.

$$F = \frac{W\,a}{b} = \frac{36390 \times 6}{18} = 12{,}130 \text{ pounds},$$

the answer.

Retaining Walls for Water.

10.



The moment of stability of a retaining wall should in practice be at least four times the static moment of the water-pressure. Let $o\,n$ represent the water-pressure per unit of surface at the bottom; then the triangle $o\,n\,m$ represents the total water-pressure on the retaining wall. When the linear dimensions are expressed in feet, we can express the moments per foot of length of wall. One cubic foot of fresh water at 60° Fahr. weighs 62.33 pounds. The centre of pressure of the water is at the height of the centre of gravity of the triangle $o\,n\,m$, or one-third of the depth d. The total water-pressure or force F on the wall will then be $F = \frac{1}{2} \times 62.33 d^2 = 31.16 d^2$. The lever of the force F is $b = \frac{1}{3} d$. The static moment of the force F will then be $F b = \frac{1}{3} F d$; but $F = 31.16 d^2$, when the static moment will be $\frac{1}{3} \times 31.16\, d^3 = 10.3866\, d^3$.

The weight w per cubic foot of hydraulic masonry is about 90 pounds for brick and 140 pounds for granite.

h = height and t = thickness of the retaining wall.

Weight of the wall $W = h\,t\,w$.

a = lever for the weight W of the wall.

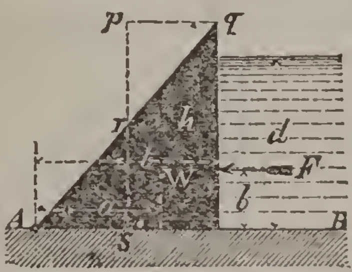
The moment of stability of the wall without water-pressure will then be $W a = h\,t\,w\,a$.

The real moment of stability of the wall with water is

$$W a - F b = h\,t\,w\,a - 10.3866\, d^3.$$

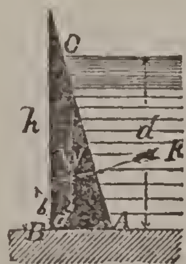
In practice, make $W a = 4 F b$.

11.



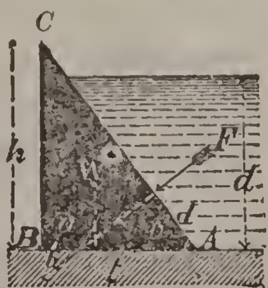
When the cross-section of the retaining wall is in the form of a triangle of the same area as the rectangular one, it will be $2\frac{2}{3}$ stronger in moment of stability, on account of the lever a being that much increased. The static moment of the water-pressure will be the same as in the foregoing case.

12.



When the section of the retaining wall is in form of a triangle, but the inclined side is on the water side, the moment of stability and static moment are both diminished, but that of stability suffers most.

13.



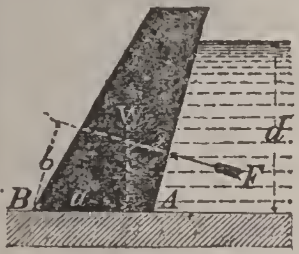
Mt. Stb. $W a = \frac{1}{2} h\,t\,w \times \frac{1}{3} t = \frac{1}{6} h\,t^2 w$.

R = hypothenuse $A\,C$.

$$\text{Force } F = \frac{1}{2} d^2 \frac{R}{h} 62.33.$$

$$F b = \frac{d^2 \left(\mp t^2 \pm \dfrac{d\,R^2}{3\,h} \right)}{2\,h} 62.33.$$

When walls are used to retain earth-work, etc., it is impossible to say what the action of the force may be. For practical purposes its action is assumed similar to water, with certain modifications given below.

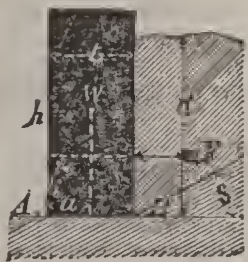


14.

Whatever may be the shape and position of the retaining wall, its moment of stability is always Wa, and the static pressure of the water acts at right angles to the surface of the wall. The force of the water-pressure in pounds is equal to the area of the pressed surface in square feet multiplied by half the depth in feet, and the product by 62.33. The centre of pressure is at one-third of the depth from the bottom.

Retaining Walls for Earthwork.

15.



The action of earth or other granular substances, like sand, gravel, grain, etc., on retaining walls is similar to that described for water, and the moments are calculated by the same formulas, with the only exception that the natural slope of the granular materials diminishes the force F as the cosine for that slope.

The natural slope of a granular substance is the greatest angle with the horizon at which it will repose in a heap. Let s denote the angle of natural slope and $W =$ weight per cubic foot of the material retained by the wall; which values for some substances are contained in the following table:

Natural Slope and Weight of Granular Substances.

Granular Substances Loosely Heaped.	Slope. s.	$\cos.s$.	Weight, w.
Lime (powder)	45	0.70711	
Saw-dust, wheat-flour	44	0.71934	
Broken stone or coal	43	0.73135	
Malt-flour	40	0.76604	
Sand (moist)	39	0.77715	95
Sand (dry)	38	0.78801	94
Malt-corn	37	0.79863	47
Wheat, rye, and corn	36	0.80902	45
Peas	35	0.81915	48
Gravel and earth	35 to 40	0.8 to 0.75	80 to 100

The force F per foot of length of wall will be $F = \frac{1}{2} w d \cos.s$. For safe calculation, we may assume $w = 100$ pounds per cubic foot of earth or gravel pressing against the wall, and $s = 32° 51'$, safety angle of natural slope, which cosine is 0.84; then the force F will be $F = \frac{1}{2} \times 100 \times 0.84\, d = 42\, d$.

Static moment $Fb = \frac{1}{3} Fd$, but $F = 42\, d$. $Fb = 14\, d^2$.

Moment of stability $Wa = h t w a = \frac{1}{2} h w t^2$.

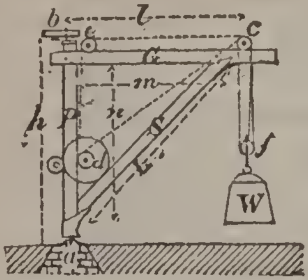
$W =$ weight per cubic foot of materials in the wall, which is 90 for brick, 120 for rubble concrete, and 140 for granite.

The real stability of the wall will be

$$Wa - Fb = \tfrac{1}{2} h w t^2 - 14 d^2.$$

In practice, make $Wa = 4\, Fb$ or $h w t^2 = 28\, d^2$.

16.



HOISTING-CRANES.

THE ordinary foundry crane consists of the post P, resting in a shoe a, and cap b, jib G, and stay S.

The weight W acts on the lever l, which is reacted in the supports a and b on the lever h. Consider the support b as the fulcrum, and the force F acting on the lever h in the shoe a. Static moments $Fh = Wl$.

$$F = \frac{Wl}{h}. \quad W = \frac{Fh}{l}. \quad h = \frac{Wl}{F}. \quad l = \frac{Fh}{W}.$$

S = strain of compression on the stay S, and L = length of the stay.

$$S = \frac{WL}{h} = \frac{WlL}{mn}. \qquad \text{Force of tension of the jib, } G = \frac{Wl}{n}.$$

The horizontal strain in the cap b is equal to that in the shoe a. The vertical pressure in the shoe a is equal to the weight W and weight of the crane added together. In foundry cranes the block c is moved in or out to suit the location of the weight to be lifted; and the jib and stay are both made double, so that the chain can pass between the parts.

The chain in cranes should be kept constantly moist with grease for two good reasons—namely:

1st. Lubrication decreases the friction and wear of the chain.

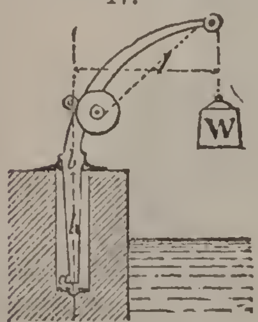
2d. It prevents, to a certain extent, the crystallization of the chain and the hook. For a foundry crane in constant use, the chain should be greased with lubricating oil about once a month. It has been found advantageous to anneal the chain about once a year for preventing crystallization, but the annealing itself injures the fibres of the iron after having been repeated a few times, so that greasing answers a better purpose.

Supposing the crane is strong enough, the weight W which can be lifted with it depends upon the proportion of gearings in the windlass and pulleys.

A man working a crank can exert a force of about 40 pounds moving with a velocity of 2 feet per second, which is 80 effects. If the gearing in the crane is say 100 to 1, the one man can lift 800 pounds. The gearing in large foundry cranes varies between 200 and 300 to 1.

The loss of power by friction is about $10 + \sqrt[3]{n}$ per cent. when n is the proportion of gearing.

17.



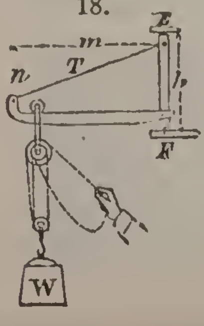
Wharf Cranes.

This kind of crane is often used for loading and unloading vessels. Its static moments are the same as for the foundry crane—namely, $Fh = Wl$.

$$F = \frac{Wl}{h}. \quad W = \frac{Fh}{l}. \quad h = \frac{Wl}{F}. \quad l = \frac{Fh}{W}.$$

The lateral strength of the curved part of the post must compensate the stay in the foundry crane.

18.



Shop Cranes.

L = length of the tension-rod.
T = force of tension.
F = pressure in each of the post journals.
f = force of compression of the jib.
l = distance between the centres of the block and the post.

$$F = \frac{Wl}{h}. \quad W = \frac{Fh}{l}. \quad h = \frac{Wl}{F}. \quad l = \frac{Fh}{W}.$$

$$T = \frac{WLl}{mn}. \quad f = \frac{Wl}{n}. \quad F = \frac{fn}{h}. \quad W = \frac{fn}{l}.$$

All forces, weight, and measures are in pounds and feet.

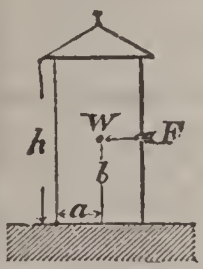
Stability of Towers to the Force of Wind.

The moment of stability of a tower or other structure exposed to the wind should be at least four times the static moment of the greatest storm to which the object is exposed. The moment of stability is like that in retaining walls, but the static moment depends upon the velocity of the wind.

Let A denote the area in square feet of the structure exposed to the wind, the greatest force of which will be $50\,A$; b = height of the centre of gravity of the exposed surface above the ground. The greatest static moment of the wind will then be $50\,A\,b$, and the minimum stability of the structure should be limited to

$$4 \times 50\,A\,b = 200\,A\,b.$$

19.



Stability, $Wa = Fb$ static moment.

Practically, the stability Wa ought to be four times the static moment Fb.

$$W\,a = 4\,Fb, \quad \text{or} \quad W\,a = 200\,A\,b,$$

$$\text{and } W = \frac{200\,A\,b}{a},$$

in which W = the whole weight of the tower in pounds, A = area of one side of the tower facing the wind.

Example.—Suppose the tower to be 4 feet square and 20 feet high, from which $a = 2$ and $b = 10$ feet. The area of the side will be $A = 4 \times 20 = 80$ square feet. What weight of the tower is required to maintain it stable to a tornado?

$$W = \frac{200\,A\,b}{a} = \frac{200 \times 80 \times 10}{2} = 80{,}000 \text{ pounds.}$$

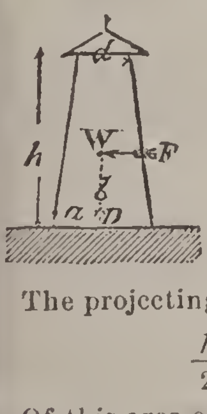
It is supposed that the wind acts at right angles on one side of the tower; but if acting in the direction of the diagonal of the square section, a greater surface will be exposed, but at such angle to the wind that the acting force will be the same as when blowing directly on only one side.

Stability of a Round Tower.

On a round tower of diameter equal to the side of the square the force of the wind is only one-half of that on the square tower.

The illustration represents a tower or chimney of the form of a conic frustum of diameters d at the top and D at the base; h = height of the tower; all in feet.

20.



The height of the centre of gravity b is calculated from the following formula:

$$b = \frac{h}{2} - \frac{h}{6}\left(\frac{D-d}{D+d}\right).$$

Example.—The height of the tower $h = 260$ feet, diameter at the top $d = 10$ feet, and $D = 25$ feet at the base. Required the height of the centre of gravity of the surface of the tower?

$$b = \frac{260}{2} - \frac{260}{6}\left(\frac{25-10}{25+10}\right) = 111.43 \text{ feet.}$$

The projecting area to the wind is

$$\frac{h}{2}(D+d) = \frac{260}{2}(25+10) = 4550 \text{ square feet.}$$

Of this area only one-half is effective to the force of the wind, or $A = 2275$ square feet. The static moment of a tornado will then be

$$Fb = 50\,A\,b = 50 \times 2275 \times 111.43 = 12675162.5 \text{ foot-pounds.}$$

If the material in the tower and that in the base it stands on were hard enough to stand the crushing force at the fulcrum in upsetting the tower, the lever of the moment of stability would be half the diameter D of the base; but as such is not the case, in practice a deduction from the leverage must be made to allow for the softness of the materials acted upon at the fulcrum.

When the base of the tower is square, and the force of the wind acts at right angles to one of its sides, the fulcrum will be a line; whilst on a circular base the fulcrum will be a point in which the whole weight of the tower acts to crush the materials.

In ordinary good brickwork the lever of the moment of stability may be taken at 0.7 of the radius of the circular base.

The weight of the tower in the preceding example should then be

$$W = \frac{4\,Fb}{a} = \frac{4 \times 12675162.5}{0.7 \times 12.5} = 5794368 \text{ pounds.}$$

A small deduction ought also to be made from the lever in a square base, where it may be taken at 0.9 of half the side of the square.

The cohesive force of the materials increases the stability of the structure when the masonry is perfectly solid at the base.

Levers.

Levers are of three distinct kinds, with reference to the relative positions of the *Force* F, *Weight* W, and *Fulcrum* C.

1st. *Fulcrum* C is between the *force* F and the *weight* W.
2d. *Weight* W is between the *fulcrum* C and the *force* F.
3d. *Force* F is between the *fulcrum* C and the *weight* W.

Example 1. Figure 21. The weight $W = 68$ pounds, the lever $l = 3.86$ feet, and $L = 10$ feet 6 inches.

Required the force $F = ?$

$$\textit{Formula 1.}\quad F = \frac{W\,l}{L} = \frac{68 \times 3.86}{10.5} = 25 \text{ pounds nearly.}$$

a = distance between the force F and the weight W.

The formulas 3, 4, 7, 8, 11, 12 are for finding the fulcrum C when the *force* F, *weight* W, and the distance a are given.

Example 2. Fig. 22. The force $F = 360$ pounds, $W = 1870$, and $a = 8$ feet 4 inches.

Required the position of the fulcrum c?

$$\textit{Formula 7.}\quad l = \frac{F\,a}{W - F} = \frac{360 \times 8.333}{1870 - 360} = \frac{2999.988}{1510} = 19.86 \text{ feet.}$$

$$L = 8.333 + 19.86 = 28.193 \text{ feet, the answer.}$$

Example 3. Fig. 26. The weight of the lever is $Q = 18$ pounds. The centre of gravity is $x = 2.25$ feet from the fulcrum. $W = 299$ pounds, $l = 5.5$ feet, and $L = 11.95$.

Required the force $F = ?$ in pounds.

$$F = \frac{W\,l - Q\,x}{L} = \frac{299 \times 5.5 - 18 \times 2.25}{11.95} = 134.25 \text{ pounds.}$$

Inclined Plane.

Example 4. Fig. 45. A load $W = 3466$ pounds is to be drawn up an inclined plane $l = 638$ feet long and $h = 86$ feet high.

What force is required to keep the load on the inclined plane?

$$F = \frac{h\,W}{l} = \frac{86 \times 3466}{638} = 467.2 \text{ pounds.}$$

Example 4. Fig. 46. A Cylinder of cast iron, weighing $W = 5245$ pounds, is to be rolled up an inclined plane; the angles $v = 18° \, 20'$ and $v' = 8° \, 10'$

What force is required to keep the cylinder on the plane?

$$F = W. \sin.(v+v') = 5245 \times \sin.(18° \, 20' + 8° \, 10') = 2340 \text{ pounds.}$$

Example 5. Fig. 47. An iron ball which weighs 398 pounds, is tied to an inclined plane with a rope; the angle of the rope and the inclined plane is $v' = 16° \, 40'$, and $v = 14° \, 30'$. What force is acting on the rope?

$$F = \frac{W \sin.v}{\cos.v'} = \frac{398 \times \sin.14° \, 30'}{\cos.16° \, 40'} = 104 \text{ pounds.}$$

Example 6. Fig. 35. What force F is required to raise a weight $W = 8469$ pounds, by a double moveable pulley?

$$F = \tfrac{1}{4} W = \tfrac{1}{4} \times 8469 = 2117 \cdot 25 \text{ pounds.}$$

Example 7. Fig. 38. How much weight can a force $F = 269$ pounds lift by three compound moveable pulleys?

$$W = 2^n F = 2^3 \times 269 = 2152 \text{ pounds, the answer.}$$

Screw.

Example 8. Fig. 54. What force is required to lift a weight $W = 16785$ pounds, by a screw, with a pitch $P = 0 \cdot 125$ feet, the lever being $r = 5$ feet. 4 inches?

$$F = \frac{WP}{2 \pi r} = \frac{16785 \times 0 \cdot 125}{2 \times 3 \cdot 14 \times 5 \cdot 333} = 62 \cdot 62 \text{ pounds, the answer.}$$

Including friction the force F will be

$$F = \frac{W(P + f d \pi)}{2 \pi r}.$$

Find the friction f on page 267. d diameter of the screw in feet.

Wedge.

Example 9. Fig. 51. The head of the wedge $a = 3$ inches, and length $l = 16\tfrac{1}{2}$ inches; the resistance to be separated is $R = 4846$ pounds. Required the force $F = ?$ (Friction omitted.)

$$F = \frac{4846 \times 3}{16 \cdot 5} = 881 \text{ pounds.}$$

Including friction the force F will be, 146

$$F = R \left[\frac{a}{l} + f \left(2 + \frac{a^2}{l \, s} \right) \right]$$

in which the friction f is to be found on page 267.

Catenaria.

Example 10. An iron chain 256 feet long, weighing 1560 pounds, is to be suspended between two points in the same horizontal line, but 196 feet apart.

How *deep* will the chain hang under the line of suspension, and with what *force* will the chain act at the points of suspension?

Figure and *Formula* 43. we have given,

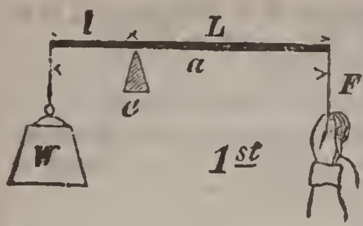
$W = \tfrac{1}{2} \times 1560 = 780$ pounds, $l = \tfrac{1}{2} \times 256 = 128$ feet, and $a = \tfrac{1}{2} \times 196 = 98$ feet.

$h = 0.\tfrac{3}{4} \sqrt{128^2 - 98^2} = 61.7$ feet, the required depth under the horizontal line.

$$\cot.v = \frac{2 \times 61.7}{98} = 1.259, \quad \text{or } v = 38° \, 27', \text{ and } 2v = 76° \, 54'.$$

The required force will be,

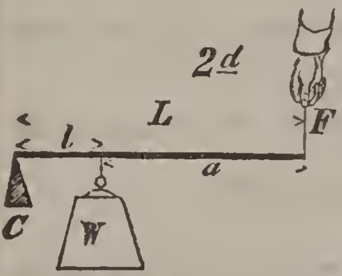
$$F = \frac{780 \times \sin.38° \, 27'}{\sin.76° \, 54'} = 507 \text{ pounds.}$$



21 $F : W = l : L, \quad F\,L = W\,l,$

$$F = \frac{W\,l}{L}, \qquad W = \frac{F\,L}{l}, \qquad 1, 2,$$

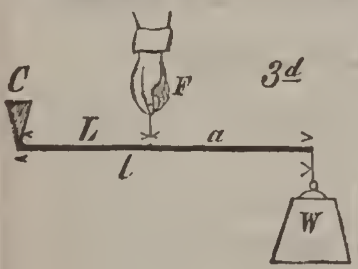
$$l = \frac{F\,a}{W+F}, \qquad L = \frac{W\,a}{W+F}, \qquad 3, 4,$$



22 $F : W = l : L, \quad F\,L = W\,l,$

$$F = \frac{W\,l}{L}, \qquad W = \frac{F\,L}{l}, \qquad 5, 6,$$

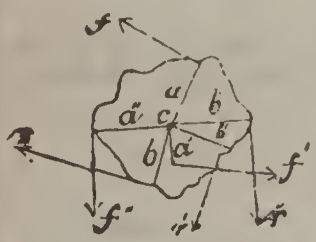
$$L = \frac{W\,a}{W-F}, \qquad l = \frac{F\,a}{W-F} \qquad 7, 8,$$



23 $F : W = l : L, \quad FL = W\,l,$

$$F = \frac{W\,l}{L}, \qquad W = \frac{F\,L}{l}, \qquad 9, 10,$$

$$L = \frac{W\,a}{F-W}, \qquad l = \frac{F\,a}{F-W} \qquad 11, 12.$$

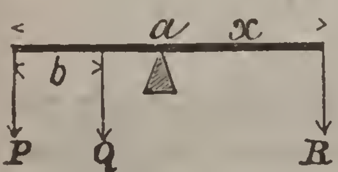


24.

Static moments.

$$a\,f + a'\,f' + a''\,f'' = b\,r + b'\,r' + b''\,r''.$$

If the sum of the moments that act to move the body in one direction are equal to the sum of the moments that act opposite, the acting forces will be in equilibrium; c being the centre or fulcrum.

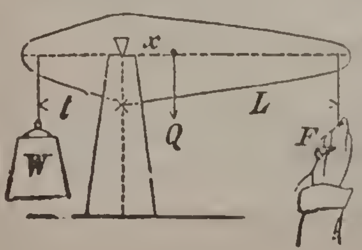


25

To find the fulcrum c when three forces act on one lever

$$R\,x = Q(a - b - x) + P(a - x),$$

$$x = \frac{Q\,a + P\,a - Q\,b}{R + Q + P}.$$

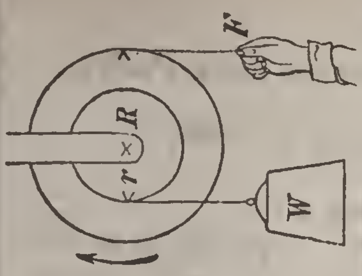


26

Q = weight of the lever. x = distance from the centre of gravity of the lever to the fulcrum. Balance the lever over a sharp edge, and the centre of gravity is found.

$$F = \frac{W\,l - Q\,x}{L}, \quad W = \frac{F\,L + Q\,x}{l}.$$

27

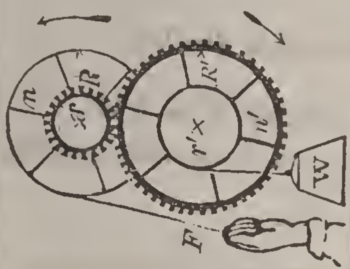


$$F : W = r : R. \quad F R = W r,$$

$$F = \frac{W r}{R}, \quad R = \frac{W r}{F},$$

$$W = \frac{R F}{r}, \quad r = \frac{R F}{W}.$$

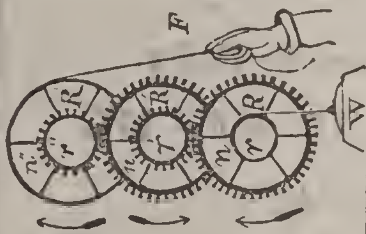
28



$$F = \frac{W r r'}{R R'}, \quad W = \frac{F R R'}{r r'},$$

n = number of revolutions of the wheels,
$n : n' = R' : r, \quad v : v' = r r' : R R',$
v = velocity of W, v' = velocity of F.

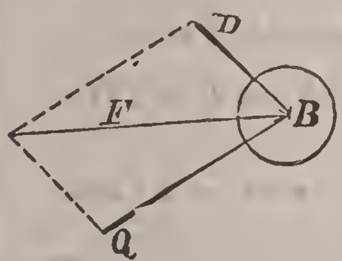
29



$$F = \frac{W r r' r''}{R R' R''}, \quad W = \frac{F R R' R''}{r r' r''},$$

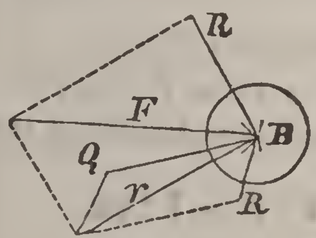
$n : n'' = r' r'' : R R', \; v : v' = r r' r'' : R R' R''.$
$r\, r' r''$ &c. = radii of the pinions.
$R\, R' R''$ &c. = radii of the wheels.

30



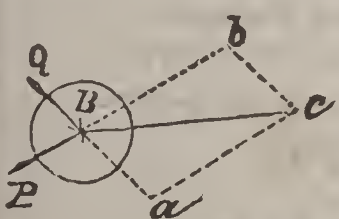
Let P and Q represent the magnitudes and directions of two forces which act to move the body B. By completing the parallelogram, there will be obtained a diagonal force F, whose magnitude and direction is equal to the resultant of P and Q. F is called the resultant of P and Q.

31

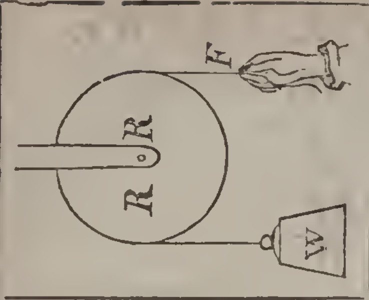


If three or more forces act in different directions to move a body B, find the resultant of any two of them, and consider it as a single force. Between this and the next force find a second resultant, thus: P, Q, and R are magnitudes and directions of the forces. $P+Q=r$, $r+R=F=P+Q+R$, or F is the magnitude and direction of the three forces, P, Q, and R.

32



A force Q acting (alone) on the body B, can move it to a in a unit of time, another force P is able to move it to b in the same time; now if the two forces act at the same time, they will move the body to c. c is the resultant of a and b.



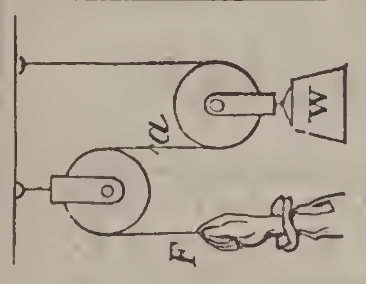
33

PULLEYS.—*A single fixed Pulley.*

$$F : W = R : R, \text{ or } F = W,$$

$$v = v'.$$

v' = velocity of F, v = vel. of W.



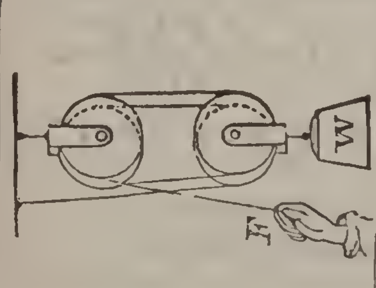
34

A single moveable Pulley.

$$F : W = R : 2R, \text{ or } F = \tfrac{1}{2}W,$$

If the force F being applied at a and act upwards, the result will be the same.

$$v' = 2v.$$

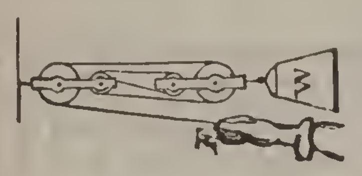


35

A double moveable Pulley.

$$F : W = R : 4R, \text{ or } F = \tfrac{1}{4}W,$$

$$v' = 4v.$$



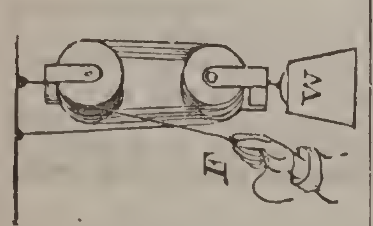
36

A double moveable Pulley.

$$F : W = R : 4R, \text{ or } F = \tfrac{1}{4}W,$$

$$F = \frac{W}{2u}, \qquad v : v' = 1 : 2u.$$

u = number of moveable pulleys.



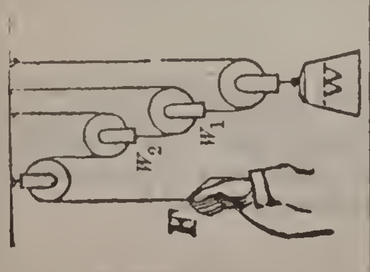
37

Quadruple moveable Pulley.

$$F = \tfrac{1}{8}W. \qquad F : W = R : 8R.$$

Let u = any number of moveable pulleys, then

$$F = \frac{W}{2u}, \qquad v : v' = 1 : 2u.$$



38

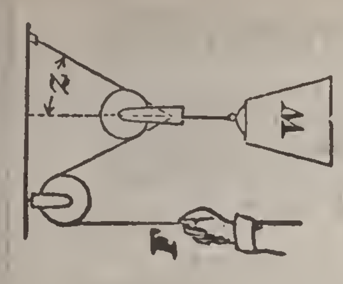
Compound Pulleys.

$$F = \tfrac{1}{2}W_2 = \tfrac{1}{4}W_1 = \tfrac{1}{8}W.$$

u = number of moveable pulleys.

$$F = \frac{W}{2^u}, \qquad W = 2^u F,$$

$$v : v' = 1 : 2^u.$$

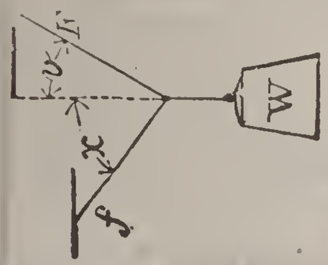


39

An oblique fixed Pulley.

$F : W = \sec.z : 2,$

$$W = \frac{2F}{\sec.z}, \qquad F = \frac{W \sec.z}{2}$$

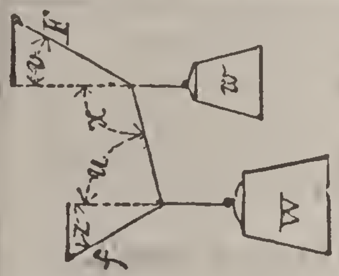


40

$$W : F = \sin.(x+v) : \sin.x,$$

$$F = \frac{W \sin.x}{\sin.(x+v)}, \qquad f = \frac{W \sin.v}{\sin.(x+v)},$$

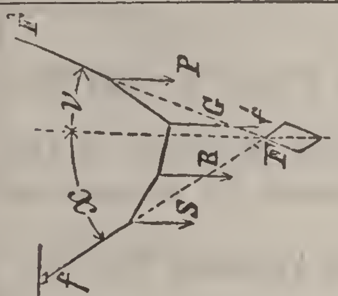
$$W : f = \sin.(x+v) : \sin.v.$$



41

$$F = \frac{w \sin.x}{\sin.(x+v)},$$

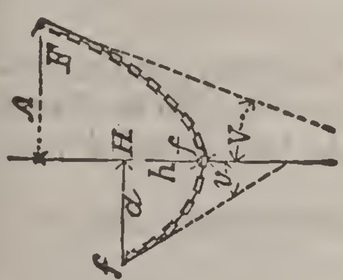
$$f = \frac{W \sin.u}{\sin.(u+z)}.$$



42

$$W = P+Q+R+S, \quad F = F', f = f',$$

$$F = \frac{W \sin.x}{\sin.(x+v)}, \qquad f = \frac{W \sin.v}{\sin.(x+v)}.$$

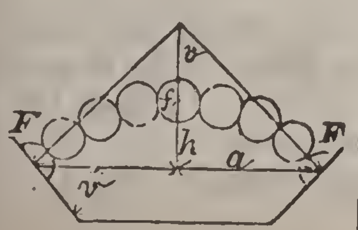


43

$h = \frac{3}{4}\sqrt{l^2 - a^2}.$

l = length, and W = weight of half the chain, ff'

$$\cot.v = \frac{2h}{a}, \; F = 2\frac{W \sin.V}{\sin.2V}, \; F = W \sec.V.$$

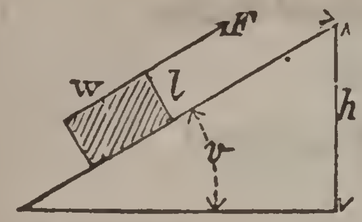


44

$$F = W \sec.V, \qquad f = W \tan.v,$$

$$\cot.v = \frac{2h}{a}.$$

W = weight of half the number of balls.



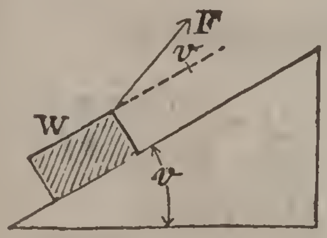
45

$$F = \frac{W\,h}{l} = W \sin.v,$$

$$W = \frac{F\,l}{h} = \frac{F}{\sin.v},$$

$$w = \frac{W\,b}{l} = W \cos.v.$$

w = normal component.

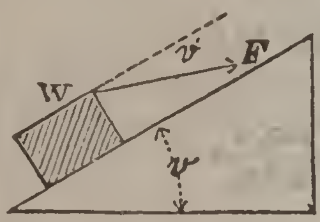


46

$$F = W\frac{\sin.v}{\cos.v'},$$

$$W = \frac{F\cos.v'}{\sin.v},$$

$$w = W(\cos.v - \sin.v \tan.v').$$

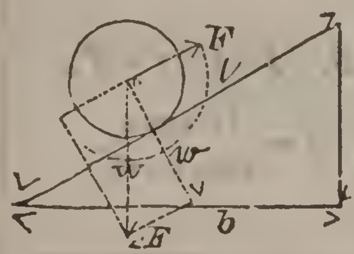


47

$$F = \frac{W\sin.v}{\cos.v'},$$

$$W = \frac{F\cos.v'}{\sin.v},$$

$$w = W(\cos.v + \sin.v.\ \tan v').$$

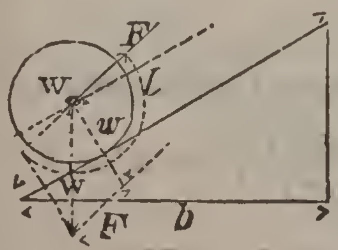


48

To solve an Inclined Plane by diagrams.

F = magnitude and direction of the force, which is obtained by completing the parallelogram.

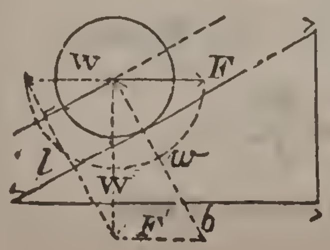
By calculation see Formula, Fig. 45.



49

W = weight of the body, and direction of the force of gravity; to be drawn at right-angles to the base b, and F parallel to F.

By calculation see Formula, Fig. 46.



50

w = the force with which the body presses against the plane, to be drawn at right-angles to the plane l; then the parallelogram is completed.

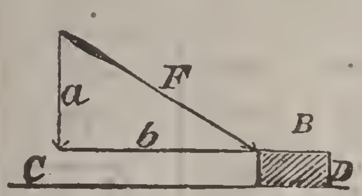
By calculation see Formula, Fig. 47.

51

Wedge.

$$F = \frac{R\,a}{l}, \qquad R \doteq \frac{F\,l}{a}.$$

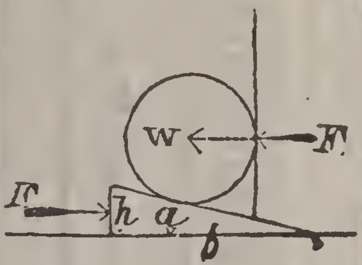
F = force required to drive the wedge.



52

Let the line F represent the magnitude and direction of a force acting to move the body B on the line CD; then the line a represents a part of F which presses the body B against CD, and the line b represents the magnitude of the force which actually moves the body B.

$$b = \sqrt{F^2 - a^2}, \qquad b = F \cos.v.$$

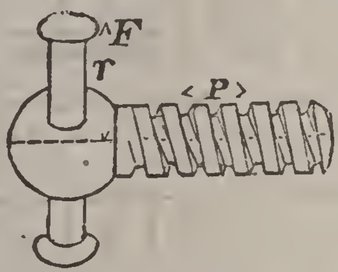


53

$$F : W = h : b = \sin.v : \cos.v = \tan.v.$$

$$F = \frac{Wh}{b} = W \tan.v. \qquad F' = F.$$

$$W = \frac{Fb}{h} = \frac{F}{\tan.v} = F \cot.v.$$



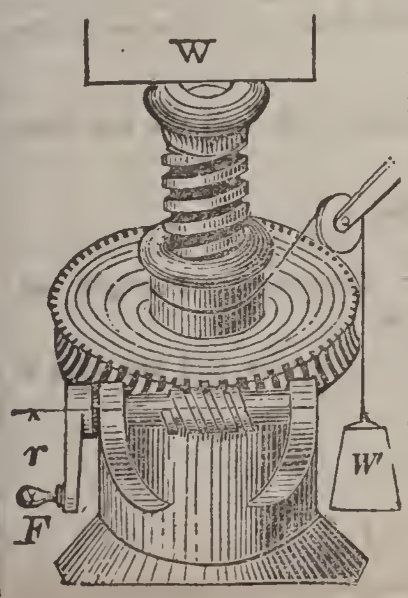
54 *Force by a Screw.*

P = Pitch of the screw,

r = radius on which the force F acts.

$$F : W = P : 2\pi\,r.$$

$$F = \frac{WP}{2\pi r}, \qquad W = \frac{F 2\pi r}{P}.$$



55 *Force by Compound Screws.*

P = Pitch of the large screw,

p = Pitch of the endless screw.

R = radius of spur-wheel for the endless screw.

$$W : F = 4\pi^2\,R\,r : P\,p.$$

$$F = \frac{W\,P\,p}{4\pi^2\,R\,r}, \qquad W = \frac{F 4\,\pi^2\,R r}{P\,p}.$$

On the spur-wheel is a cylinder by which the weight W' is wound up, the formula will be, (r' = radius of the cylinder,) and

$$F : W' = p\ \ r' : 2\pi\ R\,r.$$

$$F = \frac{W'\ p\ r'}{2\pi\ R\ r}, \qquad W = \frac{F 2\pi\ R\,r}{p\,r'}.$$

STRENGTH OF MATERIALS.

TABLE I., shows the weight a column can bear with *safety;* when the weight presses through the length of the column. The tabular number is the weight in pounds or tons per square inch on the transverse section of a column of a length less than 12 times its smallest thickness.

Table I.

RESISTANCE FOR COMPRESSION. 56

Kind of Materials.	*Pounds.*	*Tons.*
Oak, of good quality,	432	0·1885
Oak, common,	280	0·125
Spruce, red (Sapin rouge),	540	0·241
" white, (Sapin blanc),	140	0·0256
Iron, wrought,	14400	6·43
Iron, cast,	28750	12·85
Basalt,	2875	1·285
Granite, hard,	1000	0·446
" common,	575	0·256
Marble, hard,	1435	0·640
" common,	431	0·192
Sandstone, hard,	1295	0·577
" loose,	5·6	0·0025
Brick, good quality,	175	0·078
" common,	58	0·0259
Lime-stone, of hardest kind,	720	0·321
" common,	432	0·193
Plaster-Paris,	86	0·0384
Mortar, good quality, and 18 months old,	58	0·0259
Do. common,	36	0·016

When the length or height of the column is more than 12 times its smallest thickness, divide the tabular weight by the corresponding number in this Table.

Length×thickness	12	18	24	30	36	42	48	54	60
Divide by	1·2	1·6	2	2·8	4	5	6	8	12

Example. A building which is to weigh 2000 tons is to be supported by piles of Sapin rouge Spruce 18 feet in length, and 12 inches diameter. How many piles are required to support the building?

$$\frac{12^2 \times 0{\cdot}785 \times 0{\cdot}241}{1{\cdot}6} = 17 \text{ tons, the weight which each pile can bear,}$$

and

$$\frac{2000}{17} = 118 \text{ piles.}$$

Professor Hodgkinson's Formulæ for Crushing Strength of Cast Iron Pillars.

The ends of the pillars should be perfectly flat and square, and the load to bear even on the whole surface.

T=crushing weight in tons.

D=outside and d inside diameters in inches.

l=length or height of pillar in feet.

$$T = 46{\cdot}65 \left(\frac{D^{3.55} - d^{3.55}}{l^{1.7}}\right)$$

Table showing the Weight in tons which Cast Iron Pillars or Tubes can bear with Safety.

Diameters in Inches. For Tubes subtract the weight due to the bore.

L	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	20	24
	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons	tons,
1	22	218	1156	3256	7272	14...	244..	395..	6.....	88...	124..	169 .	226..	296..	379..	478..	59....	73...	1......	2.....
2	6·8	82·5	355	1000	2238	4315	7500	1215	85.1	2714	38...	523..	697..	910..	116..	147..	18....	22....	32....	63....
3	3·5	41·5	178	500	1123	2166	3744	6101	9322	136..	162..	262..	350..	457..	586..	739..	920..	113..	165..	318..
4	2·1	25·5	110	308	689	1328	2313	3741	5716	8354	1177	161..	2148	280..	359..	453..	564..	693..	101..	195..
5	1·4	17·4	75	211	471	908	1583	2560	3912	5716	8056	110..	1470	1919	246..	310..	386..	474..	693..	133..
6	1·0	12·8	55	155	346	666	1161	1878	2869	4192	5909	8082	1078	1407	1804	2277	283..	347..	508..	980..
7	0·81	9·8	42	119	266	513	894	1444	2207	3226	4547	6219	8296	1083	1388	1752	217..	267..	391..	754..
8	0·65	7·8	36	95	212	408	712	1151	1760	2571	3624	4956	6611	8633	1106	1396	173..	213..	311..	601..
9	0·53	6·4	28	78	174	335	583	942	1440	2104	2966	4057	5412	7066	9059	1142	142..	174..	255..	491..
10	0·44	5·3	23	65	145	280	487	788	1200	1760	2479	3391	4524	5907	7574	9554	118..	146..	213..	411..
11	0·38	4·6	20	55	123	238	414	670	1024	1496	2108	2884	3847	5024	6441	8125	101..	124..	181..	349..
12	0·32	3·9	17	48	106	205	357	578	883	1292	1818	2487	3318	4333	5555	7008	8717	107..	156..	301..
13	0·28	3·4	15	42	93	179	312	507	771	1126	1587	2171	2896	3782	4848	6116	7608	9346	136..	263..
14	0·25	3·0	13	37	82	158	275	445	679	993	1400	1914	2553	3333	4274	5392	6707	8240	120..	232..
15	0·22	2·7	11·5	33	73	140	250	396	604	883	1244	1702	2271	2965	3801	4795	5965	7328	102..	206..
16	0·20	2·4	10	28	65	126	218	354	541	792	1115	1526	2035	2657	3406	4297	5345	6551	9590	184..
17	0·18	2·3	9·4	26	58	113	193	320	489	714	1006	1376	1836	2397	3073	3876	4712	5923	8656	166..
18	0·16	2·0	8·5	24	53	103	174	290	443	648	913	1248	1665	2175	2788	3517	4375	5375	7854	151..
19	0·14	1·8	7·8	22	49	93	163	265	404	591	832	1139	1519	1984	2543	3208	3991	4903	7165	138..
20	0·13	1·6	7·1	20	45	86	150	243	371	541	763	·1044	1392	1818	2331	2940	3658	4497	6066	126..

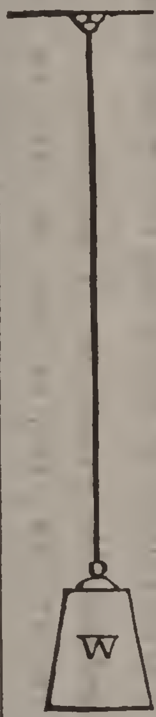
These values are about half of that by Prof. Hodgkinson's formula. The points after the numbers mean ciphers.

Table II.

COHESIVE STRENGTH PER SQ. INCH OF CROSS-SECTION.

Kind of Materials.	*Just tear asunder.*		*With safety.*	
	Pounds.	*Tons.*	*Pounds.*	*Tons.*
Cast Steel, - - -	134256	59·93	33600	14·98
Blistered Steel, - -	133152	59·43	33300	14·86
Steel, Shear, - - -	128632	56·97	32160	14·24
Iron, Swedish bar, - -	65000	29·2	16260	7·3
" Russian, - -	59470	26·7	14900	6·7
" English, - -	56000	25·0	14000	6·25
" common, over 2 in. sq.,	36000	16·00	9000	4·0
" sheet, parallel rolling,	40000	17·85	10000	4·46
" at right angles to roll,	34400	15·35	8600	3·84
Cast iron, good quality, -	45000	20·05	11250	5·00
" inferior, - -	18000	8·03	4500	2·0
Copper, cast, - - -	32500	14·37	8130	3·6
" rolled, - -	61200	27·2	15300	6·8
Tin, cast, - - - -	5000	2·23	12500	0·56
Lead, cast, - - -	880	0·356	220	0·09
" rolled, - - -	3320	1·48	830	0·37
Platinum, wire, - -	53000	23·6	13250	5·9
Brass, common, - -	45000	20·05	11250	5·0
Wood.				
Ash, - - - -	16000	7·14	4000	1·87
Beach, - - - -	11500	5·13	2875	1·28
Box, - - - - -	20000	8·93	5000	2·23
Cedar, - - - -	11400	5·09	2850	1·27
Mahogany, - - -	21000	9·38	5250	2·34
" Spanish, -	12000	5·36	3000	1·34
Oak, American white, -	11500	5·13	2875	1·28
" English " -	10000	4·46	2500	1·11
" seasoned, - - -	13600	6·07	3400	1·52
Pine, pitch, . - -	12000	5·35	3000	1·34
" Norway, - - -	13000	5·8	3250	1·45
Walnut, - - - -	7800	3·48	1950	0·87
Whalebone, - - -	7600	3·40	1900	0·85
Hemp ropes, good, - -	6400	2·86	2130	0·95
Manilla ropes, - -	3200	1·43	1100	0·49
Wire ropes, - - .	38000	17	12600	5·36
Iron chain, - - -	65000	29	21600	9·38
" with cross pieces, -	90000	40	30000	13·4

57



To Find the Cohesive Strength.

RULE.—Multiply the cross-section of the materials in square inches by the tabular number in Table II., and the product is the cohesive strength.

Example. An iron-bar has a cross-section of 2·27 sq. in. How many tons are required to tear it asunder, and how many pounds can it bear with safety?

English iron $2{\cdot}27 \times 25 = 56{\cdot}75$ tons, which will tear it asunder, and it will bear with safety

$$2{\cdot}27 \times 14000 = 31780 \text{ pounds.}$$

Safety proof.	Inches and 16ths.			Wht. per fathom.			Price per fathom.			Ultimate
	Chain.	Hemp.	Wire.	Chain.	Hemp.	Wire.	Chain.	Hemp.	Wire.	Strain.
Cwt.	Diam.	Circ'm.	Circ'm.	Pounds	Pounds	Pounds	$ cts.	$ cts.	$ cts.	Cwt.
1.3	1	0·10	0·4	0·23	0·08	0·06	0·15	0·06	0·08	2·6
4·5	2	1.6	0·8	0·93	0·47	0·24	0·25	0·12	0·15	9
10	3	2·1	0·12	2·11	1·06	0·54	0·36	0·17	0·22	20
18	4	2·12	1·1	3·75	1·89	1·10	0·48	0·25	0·32	35
28	5	3·7	1·6	5·86	2·94	1·83	0·60	0·33	0·43	55
40	6	4·2	1·10	8·45	4·52	2·56	0·96	0·42	0·54	80
55	7	4·15	1·14	11·5	6.09	3·42	1·25	0·48	0·62	109
69	8	5·8	2·2	15·0	7·55	4·39	1·44	0·60	0·78	138
80	9	6·3	2·6	18·8	9·56	5·48	1·60	0·76	0·90	160
94	10	6·14	2·11	23·0	11·8	7·00	1·86	0·95	1·20	218
109	11	7·9	2·15	27·7	14·3	8·38	2·16	1·14	1·50	187
127	12	8·4	3·3	33·0	17·1	9.90	2·43	1·37	1·80	254
147	13	8·15	3·8	38·5	19·9	11·9	2·70	1·60	2·10	293
168	14	9·10	3·12	44·7	23·1	13·6	3·06	1·85	2·28	335
199	15	10·5	4·1	51·1	26·3	16·0	3·70	2·10	2·45	397
220	1 in.	11·	4·6	58·0	30·2	18·6	4·33	2·42	2·73	440
246	1·1	11·11	4·11	65·6	34·1	21·3	4·68	2·73	3·10	492
278	1·2	12·6	5 in.	73·7	38·2	24·2	5·58	3·06	3·50	545
302	1·3	13·1	5·5	82·1	42·6	27·4	5·86	3·40	3·91	604
332	1·4	13·12	5·10	91·0	47·1	30·7	6·42	3·77	4·35	663
365	1·5	14·7	6 in	100	52·0	35·	7.08	4·16	4·89	730
399	1·6.	15·2	6·5	110	57·1	38·7	7·75	4·57	5·35	798
435	1·7	15·15	6·10	120	63·4	42·6	8·42	5·07	5·86	869
472	1·8	16·8	6·15	131	67·9	46·7	9·15	5·44	6·35	944
553	1·10	17·14	7·10	154	79·8	56·4	10·07	6·38	7·63	1105
638	1·12	19·4	8·4	178	92·6	66·0	12·38	7·40	8·83	1275
729	1·14	20·10	8·14	205	106	76·5	14·15	8·48	10·00	1457
825	2 in	22·	9.8	232	121	88·0	16·00	9·70	11·50	1650
1072	2·4	24·12	10·12	293	153	112	20·75	10·25	14·60	2141
1288	2·8	27·8	12 in.	363	189	140	25·	15·10	18·00	2575
1559	2·12	30·4	13·4	438	229	172	30·25	18·30	21·80	3117
1854	3 in.	33·	14·8	522	272	205	36·00	21·80	25·90	3708

The prices of the chains are taken from that in England and added 50 per cent. Price of hemp ropes from Weaver, Fitler & Co., Rope manufacturers, Philadelphia. The prices of Wire ropes are deduced from the price list of John A. Roebling, Patent Wire Rope Manufacturer, Trenton, N. J.

The safe strength is here taken one half of the ultimate strength which may be trusted for new ropes, but when much in use only one quarter or less should be used.

LATERAL STRENGTH OF MATERIALS.

The formulas for lateral strength are here reduced to the simplest possible form, and are in consequence subject to conditions which must be particularly attended to. In calculating the strength of beams of irregular sections, as shown by the figures 68 to 75 on page 318, it is necessary to maintain the proportions marked on the figures and the calculation will be correct. For the sections 206 to 209 any proportion will answer in the formulas. The weight of the beam itself has not here been taken into consideration, for which allowance must be made if considerable. See pages 317 and 318.

Notation.

l = length of beam in feet. See figures 59 to 64.
h = height, b = breadth or thickness in inches of the beam, where the strain is acting.
k = coefficient for the different materials and sections of beams, to be found in the tables.
x = modulus of elasticity of materials. See Table.
f = elastic deflection in inches.
W = weight in pounds which the beam can bear with safety, being about one quarter of the ultimate strain at which the beam would break.

Example 1. Fig. 58. A rectangular beam of oak fastened in a wall projects out $l=6$ feet 4 inches, $h=8$ inches, and $b=5$ inches. Required what weight it can bear on the end $W=?$

$$W=\frac{30\times5\times8^2}{6\cdot333}=1509 \text{ pounds, with perfect safety.}$$

Example 2. Fig. 59. A beam of section fig. 69, with thickness $b=1\cdot25$ inches, height $h=22\cdot5$ inches, supported at the two ends in a length $l=25$ feet. Required what weight $W=?$ it can bear in the middle. For cast iron coefficient $k=260$.

$$W=\frac{4\times260\times1\cdot25\times22\cdot5^2}{25}=26325 \text{ lbs.}=11\cdot8 \text{ tons nearly.}$$

Example 3. Required the elastic deflection for the same beam and condition as in the foregoing example? See Table, modulus of elasticity $x=2285$ for cast iron. See page 320.

$$f=\frac{26325\times25^3}{16\times2285\times1\cdot25\times22\cdot5^3}=0\cdot80 \text{ inches, nearly.}$$

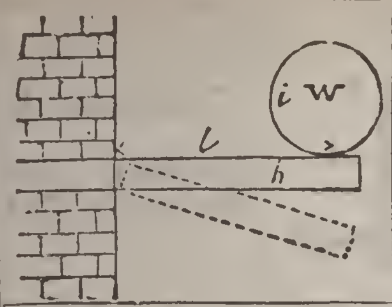
Example 4. Fig. 62. A wrought iron girder of section fig. 75. consisting of four angle irons of $a=3\cdot5\times0\cdot5\times2\times4=14$ square inches, the plate being $0\cdot5:1\cdot35=0\cdot37$ inches thick, and $h=18$ inches deep by $l=22$ feet. Required how much weight evenly distributed the girder can bear with safety?

$$W=\frac{8\times800\times14\times18}{22}=73309 \text{ lbs.}=32\cdot75 \text{ tons.}$$

If plates being riveted to the angle iron at top and bottom, add that area to a.

Example 5. Fig. 80. The crank $R=3\cdot5$ feet, force $F=3860$ lbs., length of the shaft $l=64$ feet, diameter $D=5\cdot25$ inches. Required the twisting in degrees. The shaft being of wrought iron for which $x=4110$. Page 320.

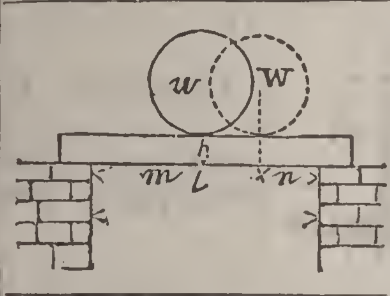
$$\text{Degrees}=\frac{425\times3860\times3\cdot5\times64}{4110\times5\cdot25^4}=11\cdot76°.$$



58

$$W = \frac{k\,b\,h^2}{l},$$

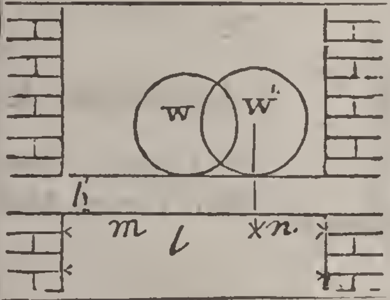
$$f = \frac{W\,l^3}{x\,b\,h^3}.$$ elastic set in inches.



59

$$W = \frac{4\,k\,b\,h^2}{l}, \qquad W' = \frac{W\,l^2}{4\,m\,n}.$$

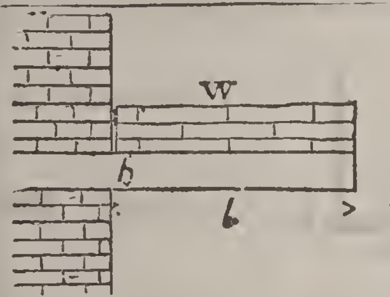
$$f = \frac{W\,l^3}{16\,x\,b\,h^3}.$$ elastic set in inches.



60

$$W = \frac{8\,k\,b\,h^2}{l}, \qquad W' = \frac{W\,l^2}{4\,m\,n}.$$

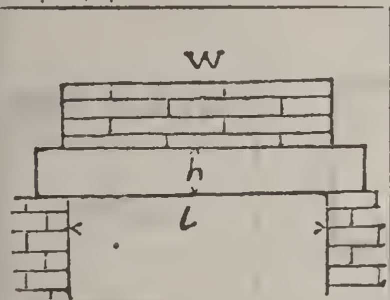
$$f = \frac{W\,l^3}{32\,x\,b\,h^3}.$$ elastic set in inches.



61

$$W = \frac{2\,k\,b\,h^2}{l},$$

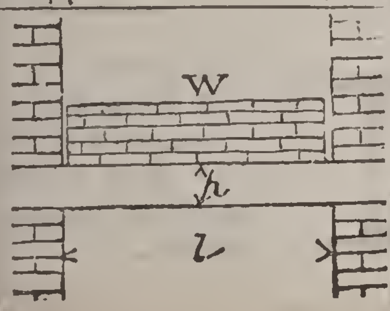
$$f = \frac{2\,W\,l^3}{5\,x\,b\,h^3}.$$ elastic set in inches.



62

$$W = \frac{8\,k\,b\,h^2}{l},$$

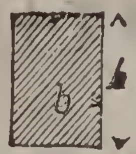
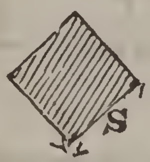
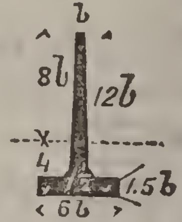
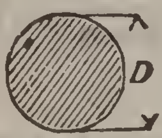
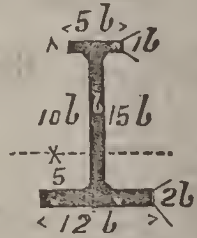
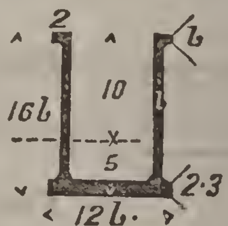
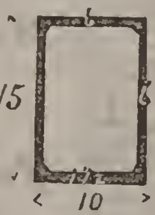
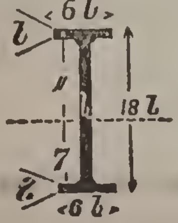
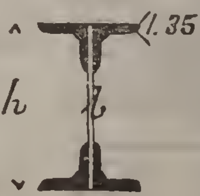
$$f = \frac{W\,l^3}{40\,x\,b\,h^3}.$$ elastic set in inches.

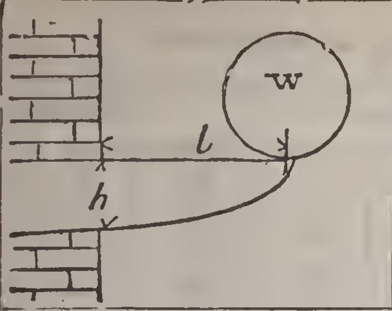


63

$$W = \frac{16\,k\,b\,h^2}{l},$$

$$f = \frac{W\,l^3}{80\,x\,b\,h^3}.$$ elastic set in inches.

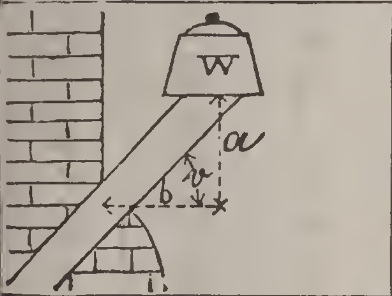
	64 Coefficient k. Cast iron, 150 Wro't iron, 120 Wood, 30		70 Coefficient k. Cast iron, 236 Wro't iron, 189
	65 Coefficient k. Cast iron, 150 Wro't iron, 120 Wood, 30 $b\ h^2 = S^3$.		71 Coefficient k. Cast iron, 250 Wro't iron, 200
	66 Coefficient k. Cast iron, 88 Wro't iron, 70 Wood, 18 $b\ h^2 = D^2$.		72 Coefficient k. Cast iron, 700 Wro't iron, 560
	67 Coefficient k. Cast iron, 88 Wro't iron, 70 Wood, 18 $b\ h^2 = D^3 - d^3$.		73 Coefficient k. Cast iron, 900
	68 Coefficient k. Wro't iron, 700 $b\ h^2 = a\ h$.		74 Cast iron tube, $k = 800$.
	69 Coefficient k. Cast iron, 260 Wro't iron, 208		75 $k = 800$, $b\ h^2 = a\ h$, a = area of all the four angle irons in square inches.



76

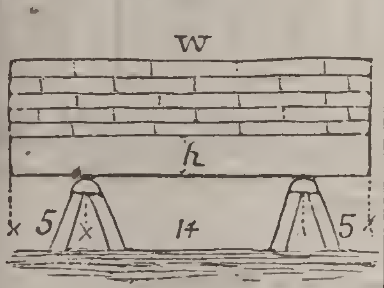
A beam fixed in one end and loaded at the other, should have the form of a Parabola, in which l = abscissa and h = ordinate. y = depth, x = length from W.

$$y = h\sqrt{\frac{x}{l}}.$$



77

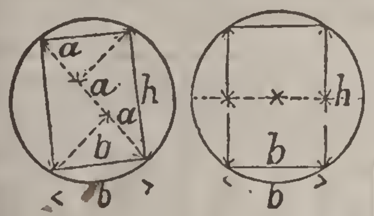
$$W = \frac{k\,b\,h^2}{l\cos.v} = \frac{k\,b\,h^2}{b'}.$$



78

$$W = \frac{36\,k\,b\,h^2}{l}.$$

Divide the length into 24 equal parts, place 14 in the middle and 5 at each end.



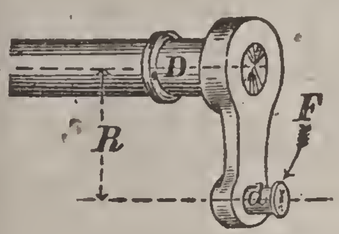
79 *To cut out the stoutest rectangular beam from a log.*

1st case, divide the diameter in 3 equal parts, and draw lines at right-angles as represented.

2d, divide the diameter in 4 equal parts.

1, $b = 1{\cdot}414\,b$, non-elastic.

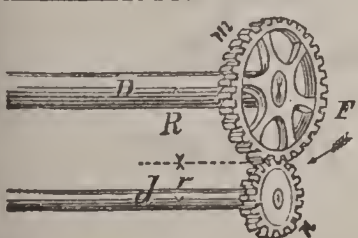
2, $h = 1{\cdot}73\,b$, elastic beams.



80

$$D = 4\sqrt[3]{\frac{F\,R}{x}} = 80\sqrt[3]{\frac{H}{x\,n}},$$

$$\text{Twisting in degrees} = \frac{425\,F\,R\,l}{x\,D^4}.$$



81

$$D : d = \sqrt[3]{R} : \sqrt[3]{r},$$

$$D = 80\sqrt[3]{\frac{H}{x\,n}},$$

$$\text{Twisting in degrees} = \frac{2233000\,H\,l}{x\,n\,D^4}.$$

Absolute and Ultimate Strength of Materials.

Kind of Materials.	Coefficient k.				Elasticity.
	Safety.	Inter.	Pr. cir.	Ultimate.	x
Wrought iron, ...	120	162	240	488	4110
Cast iron, ...	150	200	300	600	2285
Cast steel, soft, ...	385	519	170	1540	4300
Cast steel, hardened, . .	1050	1400	2100	4200	6000
Blistered steel, soft, . . .	175	233	350	700	4200
Brass, ...	58	75	113	226	1280
Copper, ...	53	71	106	212	2160
Zinc, ...	15	20	30	61	2360
Tin, ...	17	23	34	69	. . .
Lead, ...	4	6	9	18	100
Ash, ...	45	56	85	170	221
Hickory, ...	67	90	135	270	. . .
Chestnut, sweet, . . .	42	56	85	170	. . .
Oak, white, ...	50	66	100	200	300
Oak, English, ...	25	33	50	100	248
Canadian oak, . . .	37	49	73	147	283
Pine, white, ...	34	45	67	135	. . .
Yellow pine, . . .	38	50	75	150	268
Teak, ...	51	68	102	205	316

The safe stress is here taken one-quarter of the ultimate breaking weight, but when the weight is acting at short intervals one-third might be relied upon, or in pressing circumstances one-half, when the materials in the beams are known to be of good quality; but the latter should never be exceeded.

Properties of some South American Woods,

Taken from the borders of the rivers Perene and Madre de Dios, and experimented upon by the author of this Pocket-Book.

Peruvian Names of the Woods.	Color.	Specific gravity.	Wt. per cub. foot. lbs.	Hardness. H	Ultimate strength. k	Elasticity. x
Chonta (Palm), . . .	Black, . . .	1.564	96.75	28	450	640
Balsamo, ...	Brown, . .	1.207	75.25	22	422	492
Shacaranda, ...	Brown stripes,	0.991	61.75	18	343	322
Jebe (Ind.-rubber tree)*	Light yellow,	0.797	49.65	15	351	305
Amarillo, ...	Yellow, . .	0.734	45.75	13	334	300
Caoba, ...	Light brown,	0.613	38.20	11	128	. . .
Huachapeli, ...	Oak, . . .	0.566	35.25	10	134	180
Nogal, ...	Dark brown,	0.551	34.35	10	131	158
Jebe (best Ind.-rubber)*	White, . . .	0.527	32.85	9	162	262
M. Barigon, ...	White, . .	0.282	17.58	6	62	92

* There are different kinds of trees which give India-rubber, but of different quantity and quality.

The woods were perfectly dry. Four experiments on each were made.

The hardness, H, is compared with that of substances on page 333.

The coefficient, k, is the ultimate lateral strength of the woods.

x = modulus of elasticity determined near the ultimate strength.

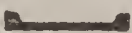
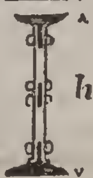
$$k = \frac{Wl}{4bh^2} \quad \text{and} \quad x = \frac{Wl^3}{16fbh^3}. \quad \text{Fig. 59, p. 317.}$$

Meaning of letters is the same as that on page 272.

Angle Iron.

Dimensions.	Per Foot. Weight.	Price.
Inches.	lbs.	cents.
$\frac{3}{16}(1\frac{1}{2}+1\frac{1}{2})$	1·77	5·13
$\frac{1}{4}(1\frac{1}{2}+1\frac{1}{2})$	2·32	6·74
$\frac{3}{16}(1\frac{3}{4}+1\frac{3}{4})$	2·09	6·07
$\frac{5}{16}(1\frac{3}{4}+1\frac{3}{4})$	3·49	10·1
$\frac{1}{4}(2+2)$	3·17	8·60
$\frac{3}{8}(2+2)$	4·59	13·3
$\frac{5}{16}(2\frac{1}{2}+2\frac{1}{2})$	4.97	14·4
$\frac{7}{16}(2\frac{1}{2}+2\frac{1}{2})$	6.84	19·9
$\frac{3}{8}(3+3)$	7·13	20·7
$\frac{1}{2}(3+3)$	9·32	27·1
$\frac{3}{8}(3\frac{1}{2}+3\frac{1}{2})$	8·40	24·4
$\frac{9}{16}(3\frac{1}{2}+3\frac{1}{2})$	12·2	34·9
$\frac{7}{16}(4+4)$	11·2	32·5
$\frac{5}{8}(4+4)$	15·5	45·0
ShipFrames.		
$\frac{1}{4}\cdot1\frac{1}{2}+\frac{3}{16}\cdot2\frac{1}{4}$	2.5	7·95
$\frac{5}{16}\cdot2+\frac{1}{4}\cdot3$	4·36	13·8
$\frac{3}{8}\cdot2\frac{1}{2}+\frac{5}{16}\cdot3\frac{3}{4}$	6.68	21·2
$\frac{1}{2}\cdot3\frac{1}{2}+\frac{5}{16}\cdot4\frac{1}{2}$	8.85	28·1
$\frac{7}{16}\cdot3+\frac{3}{8}\cdot5\frac{1}{2}$	11·0	35·0
$\frac{5}{8}\cdot4+\frac{7}{16}\cdot6$	16·4	51·0

Variety of Forms.

Section. Figure.	Area. Sq. In.	Wt.p.ft. lbs.	Wt. p. Mile. Tons	Price Per Ft. $ cts.
1 	7·4	25	117·86	0·56
2 	5·9	20	92·7	0·45
3 	7·1 *	24	113.14	0·54
4 	1·95	6·6	31·45	0·16
5 	5·41	18·3	86·43	0·45
6 	4·44	15	70·71	0·37
7 	4·22	14·3	67·75	0·35
8 	7·00	23·6	111·57	0·58
9 	5·32	18	Chair.	0·72
10 	9·65	32.6	Channel.	1·16
11 	5·41	18·3	Channel.	0·65
12 	2·66	9	Purlin.	0·35
13 	2·66	9	T iron.	0·32
14 	0·65	2·2	Window-	12
15 	0·50	1·7	Sashes.	12
16 	0·89	3	Sash bar.	12
17 	2·07	7	Shoe.	0·25
18 	6·66	22·5	Girder.	0·80
20 	This is the beam for which the formulas and table are set up. Top and bottom are alike.			
21 	This compound Girder is made to order of any size, for about 6 cents per pound.			

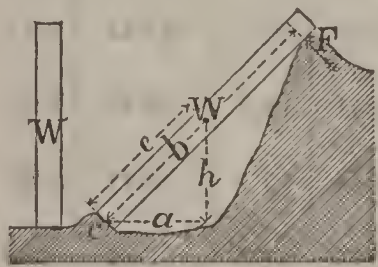
* 20 intermediate sizes,

FORCES IN STRUCTURES.

[*Important structures should not be undertaken without consulting the works of Stoney, Du Bois or Greene, or others of equal authority.*]

A vertical pole, Fig. 1, presses on its support with a force equal to its weight; but when the pole has an incline, Fig. 2, resting with its upper end on a second support at F, the action is divided into two equal static moments—namely,

Fig. 1. Fig. 2.



$$F : W = a : b, \qquad Wa = Fb.$$

$$F = \frac{Wa}{b}, \qquad W = \frac{Fb}{a}, \qquad w = \frac{We}{h}.$$

W = weight of the pole, supposed to act in its centre of gravity.

a = lever of the weight W, drawn horizontally from the fulcrum c to the vertical direction of the centre of gravity of the pole.

F = force holding the pole at F.

b = lever of the force F.

h = vertical height of the centre of gravity of the pole above the fulcrum c.

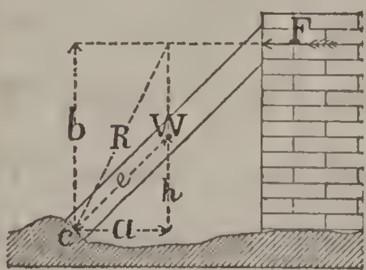
e = distance from the centre of gravity to the fulcrum.

w = pressure at c in the direction of the pole.

These notations of letters will be the same in the following figures.

A Beam Resting Against a Vertical Wall.

Fig. 3.



$$F : W = a : b, \qquad Fb = Wa.$$

$$F = \frac{Wa}{b}, \qquad W = \frac{Fb}{a}, \qquad a = \frac{Fb}{W},$$

$$b = \frac{Wa}{F}, \qquad w = \frac{We}{h}.$$

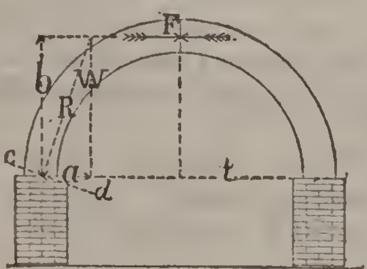
The horizontal pressure at the fulcrum c is equal to the force F.

The vertical pressure at the fulcrum c is equal to W.

The diagonal R shows the direction of the resultant of the two forces F and W.

Force of a Half-Circle Arc Resting on Two Walls.

Fig. 4.

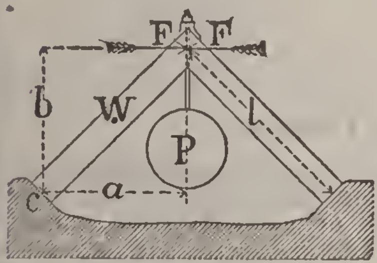


It appears in this case that the arc presses vertically down upon the two walls, but such is not the case. The formulas are the same as those for Fig. 3. If the walls are not stable enough to stand with safety the force F, then a tie-rod t must be inserted. If the walls are sufficiently stable without the tie t, then they should not be level on the top, but inclined like $c\,d$, so as to be at right angles to the direction of the resultant R.

It is supposed that there is no lateral strength in the arc, but that the centre of gravity of half the arc acts like that in Fig. 3.

A Weight Suspended on a Pair of Shears.

Fig. 5.



The weight P is hung from the angle of two spars.

$$F = \frac{Pa}{2b}, \text{ omitting the weight of the spars.}$$

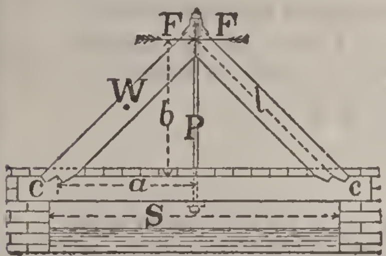
$$F = \frac{a}{2b}(P + 2W), \text{ when } W = \text{weight of one spar.}$$

The vertical pressure at c will be $\frac{P}{2} + W$.

Thrust on spar $= \frac{l}{b}(W + P)$.

Truss-Bridge with Two Rafters and a King-Rod.

Fig. 6.



The bridge consists of two rafters Wl, a tie-beam cc, king-rod P. The span S is divided into two parts by the king-rod, which bears one-half of the load uniformly distributed on the bridge. The tie-beam must be strong enough to bear with safety half the load in half the span.

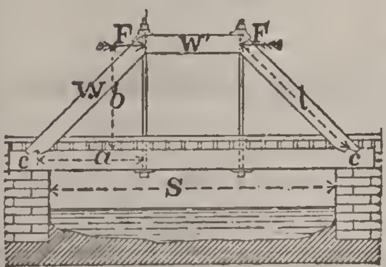
P = weight uniformly distributed on the bridge, including the weights of the tire-beam and flooring.

$$F = \frac{a}{b}\left(\frac{P}{2} + W\right).$$

Thrust on each rafter $w = \frac{l}{b}\left(W + \frac{P}{2}\right)$.

Truss-Bridge with two Queen-Rods.

Fig. 7.



The truss-bridge consists of a tie-beam cc, two rafters W and l, two queen-rods, and a horizontal straining-beam W'.

The span S is divided into three equal parts by the queen-rods. Bridges have usually two trusses, and therefore the stresses must be halved.

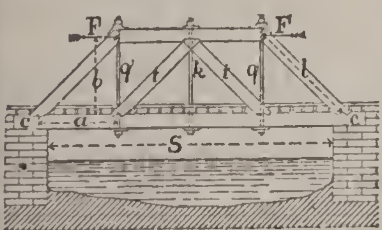
$$F = \frac{3a}{4b}(P + 2W + W').$$

Thrust on rafter $w = \frac{l}{2b}(P + 2W + W')$.

The force F and thrust w are divided on each side of the bridge.
The tension on the tie-beam is equal to F.

Truss-Bridge with one King- and two Queen-Rods.

Fig. 8.



The span S is divided into four equal parts by the king- and queen-rods, making the load on each of the rods $\frac{1}{4}P$. The weight on the king-rod is supported by the diagonal trusses tt, by which it is transmitted to the two queen-rods—that is, $\frac{1}{8}P$ on each—making $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}P$ on each queen-rod.

$$F = \frac{a}{b}\left(P + W\right).$$

W = weight of all the trusses and horizontal straining-beam.

$P =$ the whole load on the bridge, uniformly distributed, and including that of tie-beam and flooring.

Thrust on t and t, $w' = \frac{t P}{8 b}$.

Thrust on rafters $w = \frac{3 l P}{8 b}$.

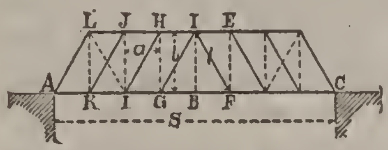
The tension on the tie-beam is equal to F.

Truss-Bridge of Many Panels.

The span S is divided into so many parts as to make the lateral strength of the tie-beam or lower chord in each division sufficiently strong to bear the load with safety. The rectangular space of each division, such as $B\ I\ E\ F$, is called a *panel.*

The heavy-drawn lines represent compression members, except the lower chord $A\ C$, and the dotted lines are tension-rods.

Fig. 9.



$P =$ whole load uniformly distributed on the bridge, including the weight of the bridge unloaded.

$p =$ load and weight on each panel.

$n =$ number of panels in the span S.

$t =$ the strain on each tension-rod, which will be $p = \frac{P}{n} + x$.

$a = \frac{S}{n}$, the width of each division or panel.

$b =$ height of the bridge or panel. $l =$ length of each strut.

$x =$ the additional weight thrown upon each tie-rod by its neighbor toward the middle $B\ I$ of the bridge, that is, half the tension on $B\ D$ is thrown on $G\ H$; the whole tension on $G\ H$ is thrown on $I\ J$; the whole tension on $I\ J$ is thrown on $K\ L$; and lastly, the bearing of the bridge on the pier at A is the sum of all the tensions $+ \frac{1}{2} p$, which will be half the weight and load of the bridge.

The tension on the lower chord in each panel is equal to the compression on the upper chord in the same panel.

The compression of the diagonal struts $= \frac{l t}{b}$.

Assume the span to be $S = 96$ feet, which, divided into eight parts, makes $a = 12$ feet; and if the angle of the struts are 60°, then

$$b = a \tan.60° = 12 \times 1.732 = 20.784 \text{ feet.}$$

The length of the struts $l = 12 \times \sec.60° = 12 \times 2 = 24$ feet.

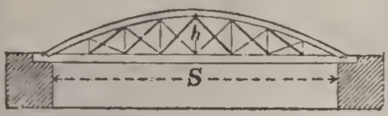
TABLE OF CALCULATION OF COMPRESSION AND TENSION.

Tension t on Rods.	*Compression and Tension.*	*Struts, compression.*
$t = p + x.$	$A\ K = 3\frac{1}{2} p \times \frac{12}{20.784} = 2.019\ p.$	$\frac{l t}{b}.$
$B\ I = p + o = p.$	$K\ I = L\ J = (3\frac{1}{2} + 2\frac{1}{2})\ p \times \frac{12}{20.784} = 3.462\ p.$	$I\ G = \frac{24 \times p}{20.784} \frac{1}{2} = .577\ p.$
$G\ H = p + \frac{1}{2} p = 1\frac{1}{2} p.$	$I\ G = J\ H = (3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2})\ p \times \frac{12}{20.784} = 4.327\ p.$	$H\ I = \frac{24 \times 1\frac{1}{2} p}{20.784} = 1.732\ p.$
$I\ J = p + 1\frac{1}{2} p = 2\frac{1}{2} p.$	$G\ B = H\ I = (3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2} + \frac{1}{2})\ p \times \frac{12}{20.784} = 4.616\ p.$	$J\ K = \frac{24 \times 2\frac{1}{2} p}{20.784} = 2.887\ p.$
$K\ L = p + 2\frac{1}{2} p = 3\frac{1}{2} p.$		$A\ L = \frac{24 \times 3\frac{1}{2} p}{20.784} = 4.042\ p.$
$A\ t\ A = \frac{1}{2} p + 3\frac{1}{2} p = 4\ p.$		

The tension and compression of the chords at mid-span can be checked by multiplying the quarter span into the half load on bridge and dividing by height of bridge.

Bowstring Bridge.

Fig. 10.



This is a strong and cheap bridge suitable for small spans; it is, in fact, an arched girder, generally built of timber, but is also made of iron.

The arch should be made in the form of a parabola; the tie-beam must be made strong enough to bear the maximum load on each panel.

The bracing between the arch and tie-beam may be all ties, in which case the compression and tension will be uniform and alike in the arch and tie-beam namely, $\frac{P S}{8 h}$; that is, the maximum load P on the bridge, including the weight of the bridge, multiplied by the span S, and the product divided by eight times the height h of the girder, is the tension on the tie-beam or compression of the arch.

When the bracing consists of ties and struts, then the tension compression is calculated like that for fig. 9.

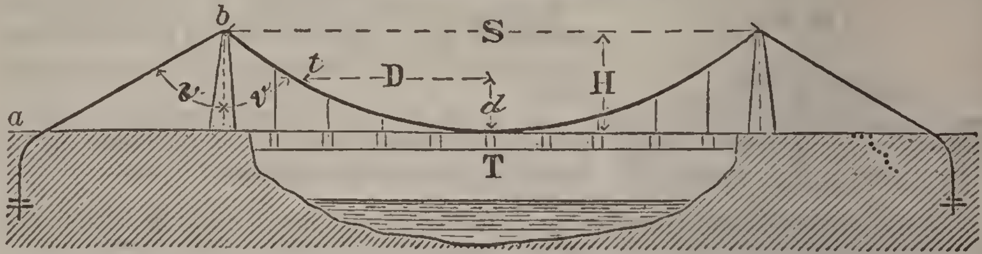
STRENGTH OF TIMBER.

(*From Mr. Laslett's experiments.*)

NAMES OF DIFFERENT KINDS OF WOODS. (The strengths mean per square inch of cross-section.)	Transverse breaking weight.	Tensile strength.	Compressive strength.
	Lbs.	Tons.	Tons.
English Oak (Mean)	687	2.546	3.337
Ironwood	1273	4.311	5.208
Chow	975	3.214	5.621
Iron Bark	1407	3.740	4.601
Blue Gum	712	2.700	3.078
Canadian Ash	638	2.453	2.453
Average of the above hardwoods	949	3.161	3.615
Baltimore Oak, African Teak, Green Heart, Sabicu, American and Eucalyptus Mahogany, and English Ash. Average hardwoods	967	2.120	3.493
Dantzic Fir, Riga Fir, Spruce Fir, Larch, Cedar, Red Pine, Yellow Pine, Pitch Pine, Kauri Pine. Average soft woods	683	1.597	2.486
Average twenty-nine hard and soft woods	830	2.416	3.168

The strength of wood or timber is so varied that it is impossible to give correct data, for even in the same set of experiments on the same kind of wood, taken from the same tree, the variation reaches 50 per cent. or more. Green timber is about half as strong as sun-dried timber. The compressive strength per unit of cross-section of the same kind of wood is 10 to 15 per cent. greater than the tensile strength.

SUSPENSION BRIDGES.



When a chain or flexible cord is suspended between two supports, the form of the curve taken by the chain is called a *catenary*, which resembles very much that of the *parabola* of the conic sections.

For the same height H and span S the catenary has a slightly larger radius of curvature at the lowest point than has the parabola; but when a bridge is suspended on the catenary, the weight per unit of length of the chain is greater at T in the middle of the span S than it is at and near the points of suspension. This greater weight depresses the catenary in the middle, so as to practically make it the form of a parabola.

The formulas for the catenary are very difficult to manage, whilst those for a parabola are very simple, and will here be used.

Notation.

W = total load on the bridge.
T = tension on the chain in the centre.
t = tension at any point of distance D from the centre.
t' = tension at abutments.
S = span.
L = length of chain between the piers.
H = versed sine, or height of abutments above centre of chain.
D and d = co-ordinates for any point of the chain.
v = angle of suspension at piers. (The angle of the counter-chain ought to be equal to that of suspension.)

The formulas will answer for any system of weights and measures.

$$D = \frac{S}{2}\sqrt{\frac{d}{H}}. \quad \ldots \quad 1.$$

$$d = \frac{4\,D^2 H}{S^2}. \quad \ldots \quad 2.$$

$$\tan.v = \frac{S}{4\,H}. \quad \ldots \quad 3.$$

$$T = \tfrac{1}{2} W \tan.v. \quad \ldots \quad 4.$$

$$T = \frac{WS}{8\,H}. \quad \ldots \quad 5.$$

$$H = \frac{WS}{8\,T}. \quad \ldots \quad 6.$$

$$H = \frac{S}{13}, \text{ generally.} \quad \ldots \quad 7.$$

$$L = 2\sqrt{\frac{S^2}{4} + \frac{16}{9}H^2}, \text{ approx.} \quad \ldots \quad 8.$$

$$t' = \frac{W}{2}\sec.v \quad \ldots \quad 9.$$

$$W = 2\,t' \cos.v. \quad \ldots \quad 10.$$

$$\cos.v = \frac{W}{2\,t'}. \quad \ldots \quad 11.$$

$$\sin.v = \frac{T}{t'}. \quad \ldots \quad 12.$$

A bridge is generally suspended on two or more chains, and the tensions T, t', and t mean that on all the chains.

On account of the tension on the backstays, a, b is equal to that on the chains or cables at the top of the piers; the vertical pressure on each pier will be equal to the weight W of the whole bridge, to which must be added the weight of the pier itself to get the total pressure on the base.

The anchorage must be firm enough to hold with safety the tension t'. It is best to curve the backstay into the anchorage, as shown at a, and the anchors should be set in stone masonry laid in cement.

Example. The chains or cables for a suspension bridge of $S=400$ feet span are to hang $H=32$ feet below the supports on the piers, and to support a weight $W=600{,}000$ pounds, uniformly distributed. Required the length of the cables between the piers, the angle v, and the strain t' on the cables at the top of the piers?

The length of the cable will be found by Formula 8.

$$L=2\sqrt{\frac{160000}{4}+\frac{16}{9}\times 1024}=409 \text{ feet.}$$

The angle v will be found by Formula 3.

$$\tan.v=\frac{S'}{4\,H}=\frac{400}{4\times 32}=3.1111.$$

This tangent answers to the angle $v=72^\circ\ 11'$. The tension t' at the piers will then be, Formula 9:

$$t=\frac{W}{2}\sec.v=\frac{600000}{2}\times 3.2683=980{,}490 \text{ pounds.}$$

If the bridge is to be suspended on say four cables that is, two on each side of the roadway, then each cable must be strong enough to bear with safety $980490:4=245142.5$ pounds.

The tension T in the middle of the span is found by Formula 4, namely:

$$T=\tfrac{1}{2}\,W\tan.v=\tfrac{1}{2}\times 600000\times 3.1111=933{,}333 \text{ pounds.}$$

The ultimate strength of the bridge, so far as the strength of the cables is concerned, will be found by Formula 10, in which case t' means the ultimate strength of the cables, say $980490\times 6=5{,}882{,}940$.

$$W=2\times 5882940\times 0.30597=4{,}532{,}300 \text{ pounds.}$$

The ordinates D for the curve can be calculated for assumed values of the abscissa d by Formula 1.

The parameter of the parabola is $p=\dfrac{S^2}{4\,H}$.

The focus of the parabola is $d=\dfrac{S^2}{16\,H}$.

$d=$ height of focus above the lowest part of the cable.

Dimensions and Cost of Large Bridges.

Name of Bridge.	Maximum. App. Height.	Maximum. Span.	Length.	Reputed Cost. Total Amount.	Reputed Cost. Per ft. run.	Nature of Bridge.
	Ft.	Ft.	Ft.	$	$	
Britannia.........	125	460	1511	3,009,325	1990	Two lines R.R. Tubular.
Charing Cross..	50	154	1365	900,000	655	Four lines R. R. Double Warren.
Boyne...............	90	264	550	700,000	1270 ?	Four lines R. R. Lattice.
Crumlin	200	150	1800	195,000	105	Two lines R. R. Lattice on open-work piers.
Craige Machie.	20	200	413	61,000	145.5	One line R. R. Lattice and plate girders.
Grand River (Mauritius)	130	12	620	150,000	250	One line R. R. Plate girders.
Deepdale	150 ?	60	740	101,330	135	Two lines R. R. Lattice on open-work piers.
Westminster....	20	120	1160	1,175,000	1010	Road. Cast and wrought iron arch, 83 feet wide.
Freeburg.........	167	......	808	120,000	145	Wire-rope suspension bridge. Road only.
Niagara....... ...	245	821	800	400,000	500	Wire-rope suspension bridge. Road and R. R.
Landore	75	110	1760	143,600	16.3	Wooden trusses.
East River.......	154	1596	10043	9,000,000	900	Wire-rope suspension.

THE CATENARY.

THE curve formed by a flexible rope or chain suspended from two points is called a *catenary* or *chain-line.*

Let v denote the angle of the curve with the vertical in any point P whose abscissa is x and ordinate y; l = length of the curve OP; W = weight of the whole chain; F = force of tension at the angle v. The formulas for the catenary will then be

$$y = \frac{l \sin.^2 v}{\sin.2v} \, hyp.log.cot.\tfrac{1}{2}v.$$

$$x = \frac{l \sin.^2 v}{\sin.2v} (cosec.v - 1).$$

$$y = \frac{x \; hyp.log.cot.\tfrac{1}{2}v}{(cosec.v - 1)}.$$

$$x = \frac{y(cosec.v - 1)}{hyp.log.cot.\tfrac{1}{2}v}.$$

$$l = \frac{x \sin.2v}{\sin.v(1 - \sin.v)}.$$

$$F = \frac{W \sin.v}{\sin.2v}.$$

The formulas for the catenary are very difficult to manage, because the angle v must be given; but by the aid of the following table the solution becomes very simple:

Table for the Catenary Curve.

Angle v.	Abscissa x.	Ordinate y.	Curve l.	$\frac{y}{x}$.	$\frac{l}{y}$.
30	1.00000	1.31690	1.73210	1.3169	1.3153
40	0.55573	1.01068	1.19175	1.8186	1.1792
45	0.41421	0.88137	1.00000	2.1278	1.1346
50	0.30540	0.76291	0.83910	2.4981	1.1000
54	0.22078	0.65284	0.70021	2.9570	1.0725
60	0.15470	0.54930	0.57735	3.5507	1.0511
62	0.13257	0.50940	0.53171	3.8425	1.0438
64	0.11260	0.47021	0.48773	4.1759	1.0372
66	0.09484	0.43169	0.44523	4.5518	1.0314
68	0.07853	0.39376	0.40403	5.0141	1.0261
70	0.06418	0.35637	0.36397	5.5527	1.0213
71	0.05762	0.33786	0.34433	5.8636	1.0192
72	0.05146	0.31946	0.32492	6.2079	1.0171
73	0.04569	0.30116	0.30573	6.5914	1.0152
74	0.04030	0.28296	0.28675	7.0213	1.0134
75	0.03528	0.26484	0.26795	7.5068	1.0117
76	0.03061	0.24681	0.24933	8.0631	1.0102
77	0.02630	0.22887	0.23087	8.7023	1.0088
78	0.02234	0.21099	0.21256	9.4445	1.0073
79	0.01872	0.19318	0.19438	10.820	1.0062
80	0.01543	0.17542	0.17633	11.372	1.0052
81	0.01247	0.15773	0.15838	12.654	1.0041
82	0.00983	0.14008	0.14054	14.254	1.0033
83	0.00751	0.12248	0.12278	16.309	1.0025
84	0.00551	0.10491	0.10510	19.046	1.0018
85	0.00382	0.08738	0.08749	22.874	1.0013
86	0.00244	0.06987	0.06993	28.613	1.0008
87	0.00137	0.05238	0.05241	38.171	1.0005
88	0.00061	0.03491	0.03492	57.279	1.0002
89	0.00015	0.01745	0.01745	114.586	1.0000

Application of the Catenary Table.

The chain for a suspension bridge of 300 feet span is to hang 60 feet below its supports on the piers. The chain is to support a weight of 52,000 pounds, uniformly distributed in its length. Required the length of the chain and the angle v and strain at the supports? Half the span or $y = 150$ feet, for which $x = 60$ feet.

$$\frac{y}{x} = \frac{150}{60} = 2.5,$$

which corresponds nearly with an angle of $v = 50°$ in the table, and the length of half the chain will be $l = 150 \times 1.1 = 165$ feet.

The strain at the supports will be

$$F = \frac{W \sin.v}{\sin.2v} = \frac{52000 \times \sin.50°}{\sin.100°} = 40449 \text{ pounds.}$$

The ordinates x and y and the length l for any angle v in the table are found as follows:

When $v = 50°$ at the support, find x and y where $v = 70°$?

$$0.30540 : 0.06418 = 60 : x. \qquad x = \frac{0.06418 \times 60}{0.30540} = 12.609 \text{ feet.}$$

$$0.76291 : 0.35637 = 150 : y. \qquad y = \frac{0.35637 \times 150}{0.76291} = 70.068 \text{ feet.}$$

$$\text{Length } l = 70.068 \times 1.0213 = 71.56 \text{ feet.}$$

The ordinates can thus be calculated for a sufficient number of points in the catenary to define the course of the curve.

The strain at the lowest point or centre of the catenary will be

$$w \tan.v = 26000 \times \tan.50° = 30984 \text{ pounds};$$

when v = angle at the piers, and w = half the weight on the whole chain.

The catenary is not a line of the conic sections; its figure has the appearance of a parabola, but is a little fuller at the vertex.

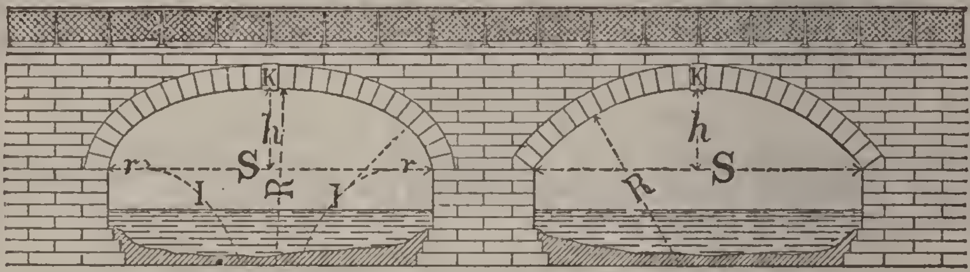
All the curves of the conic sections are of the second order, or of the exponent $n = 2$; whilst the exponent of the catenary is nearly $n = 2.3$.

The formula for any parabola is $y = \sqrt[n]{p\,x}$,

when that of the catenary will approach $y = \sqrt[2.3]{p\,x}$.

Length of the curve OP or $l = \frac{1}{3}(2y + \sqrt{y^2 + 9x^2})$.

STONE BRIDGES.



THE above illustration represents an ordinary stone bridge with one elliptic and one circular arch. The construction of the elliptic arch can be made by the methods explained in geometry or in conic sections, or by ordinates, as shown on pages 150 and 151. When the rise h is $\frac{1}{8}$th or more of the span S, circle arcs may be resorted to in constructing the ellipse; but when $h < \frac{1}{8}$th S, the ellipse should be very accurately constructed by Formula 188 or 189, page 179. The radius of curvature of the intrados of the ellipse at the key-stone is $R = \frac{S^2}{4h}$, and the smallest radius of curvature at the sides is $r = \frac{2h^2}{S}$. The depth of arc at the crown or key-stone should be $0.35\sqrt{R}$ when the bridge has only one span, and for several spans the depth of key-stone should be at least $0.4\sqrt{R}$, according to average practice.

The involute I of the ellipse should be drawn as shown by the dotted lines, in order to know the proper direction of the seams in the arch-stones, which should tangent the involute.

Having given the span S and rise h of a circular span, the radius of curvature will be $R = \frac{S^2}{8h} + \frac{h}{2}$.

Bridge Glossary.

Abutment is the stone-work against which the arches of a bridge abut. (See Piers.)

Arch-stones, the stone blocks of which the arch is built.

Back, the upper or outside surface of the arch.

Crown, the vertex of the arch.

Extrados, the same as back or the outside surface of the arch.

Faces, the two projecting areas of the arch.

Haunches, the extrados from the crown to abutment.

Intrados, the inside surface of the arch.

Key-stone, the centre stone at the crown of the arch.

Piers, the stone-work upon which a bridge rests. When the bridge is in the form of a girder, like Figs. 00, 00, which does not press sideways on the piers, the piers are not called abutments.

Rise, the vertical height h of the intrados above the abutments.

Skew-backs, the seats for the arch.

Soffit, the same as intrados.

Span, the horizontal distance S between the piers.

Spandrels, the filled-in space above the piers.

Spandrel-fillings, the materials filled in the spandrels.

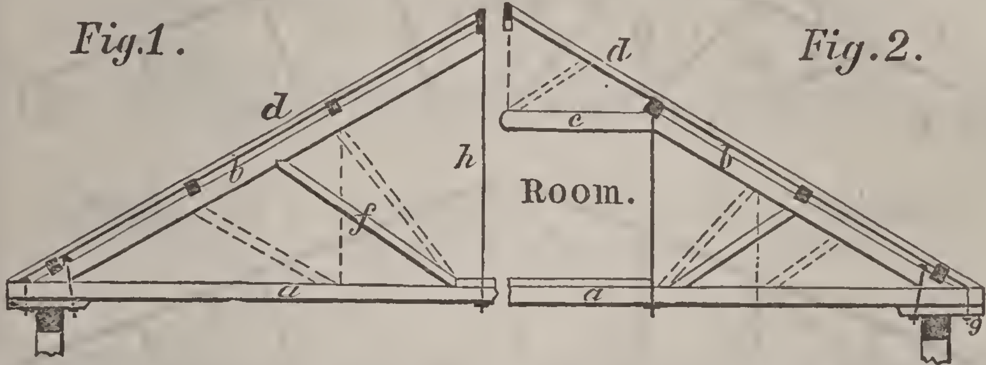
Springing line, the inner junction between the arch and the pier.

Springs, the same as springing lines.

Springers, the foot-soles of the arch.

Voussoirs, the same as arch-stones.

ROOFS OF WOOD AND IRON.



The Figs. 1 and 2 illustrate the common form of wooden roofs, as constructed over spans of from 30 to 80 feet. When the span exceeds 60 feet, a proportionate number of struts and tie-rods must be inserted, as shown by the dotted lines, or as illustrated for iron roofs.

Table of Timber Dimensions, in Inches, for Roofs over Spans from 30 to 80 feet.

Name of timbers.		Span in feet.								
		30	35	40	45	50	55	60	70	80
Tie-beams,	*a*	5×6	6×7	6×8	7×8	8×9	8×12	9×11	10×11	10×12
Truss rafter,	*b*	5×5	5×6	6×7	7×7	8×8	8× 9	9× 9	9×10	10×11
Collar-beams	*c*	5×5	5×6	6×7	7×7	8×8	8× 9	9× 9	9×10	10×11
Com. rafter,	*d*	2×5	2×5	2×6	2×6	2×6	2× 7	2× 7	$2\frac{1}{2}$× 8	3× 9
Purlins, .	*e*	5×6	5×6	5×7	6×7	6×8	6× 8	6× 9	6× 9	6× 9
Struts, . .	*f*	3×4	3×5	3×6	4×7	4×8	5× 8	5× 9	6× 9	6× 9
King's rod,	*h*	1	1	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Bolts, . .		$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$

Load on Roofs in Pounds per Square Foot, exclusive of Framing.

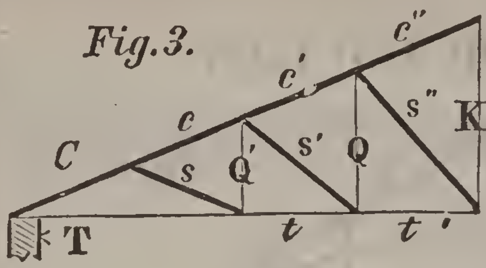
	Pounds.		Pounds.
Lead covering, ...	8	Tiles, ...	9 to 16
Zinc covering, ...	2	Boarding, $\frac{3}{4}$ thick, .	3
Corrugated Iron, . . .	3.5	Boarding, $1\frac{1}{4}$ thick, .	6
Slates, ...	10	Pressure of wind, .	40

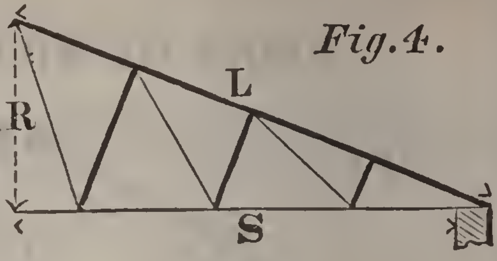
In high latitudes the roofs may be covered with snow, which makes a pressure of 10 pounds per square foot per foot of depth of the snow.

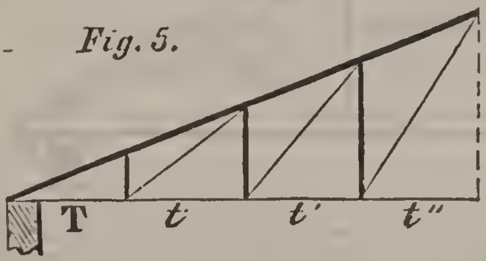
Empirical statements of sizes of timber are valuable only as a check. Each truss should be graphically solved as set forth by Du Bois, *On Roof and Bridge Construction*, or some other writer of equal authority.

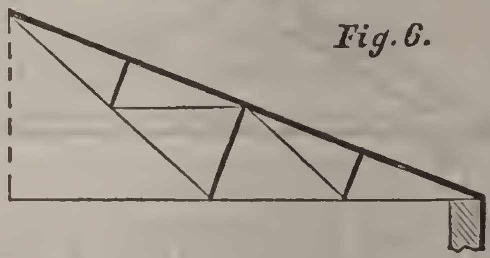
Engineers cannot be too careful in the calculations for structures upon whose stability life depends.

In addition to the graphical calculation of the framed structure, careful calculation of the strength of each member should be made. The rolling or partial loads should also be carefully calculated and tabulated, to avoid possibility of weak points. The computation and erection of bridges require broad and thorough knowledge of statical theories, and the engineer undertaking such work without adequate knowledge should be held as a murderer should life be lost by his negligence or ignorance.







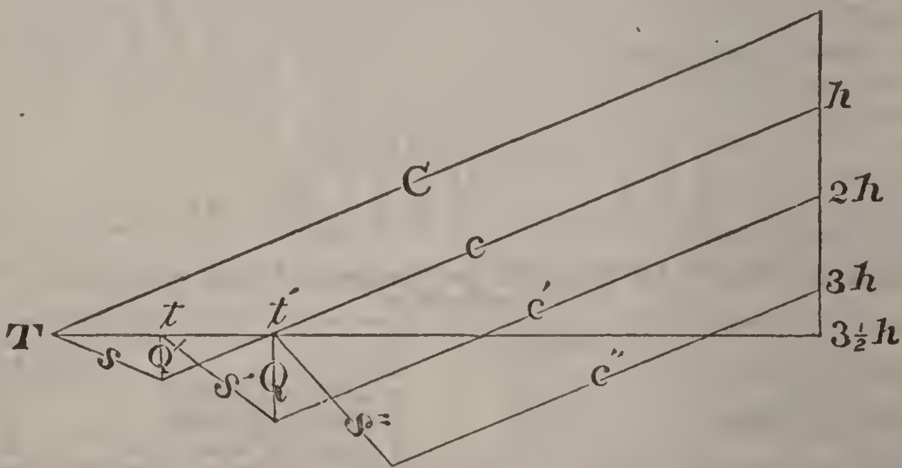


STRAINS ON ROOFS.

The above figures illustrate four different kinds of pointed iron roofs, of which figure 3 is most in use.

Maxwell's Graphical Method (Du Bois).

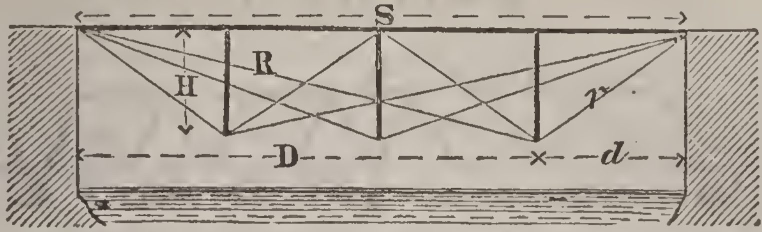
Solution of Fig. 3.



Determine the whole load on each truss. Take the load as concentrated at the apexes, The reaction at each pier is $\frac{1}{2}$ of $7h = 3\frac{1}{2}h$. To any convenient scale lay down the line $3\frac{1}{2}h$. Draw lines parallel to C and T from the extremities of $3\frac{1}{2}h$. They will intersect and give the stresses to the same scale for T and C. Take the first upper vertex; its polygon of forces is $C p c s$ in the diagram. Take the first lower vertex; its polygon of forces is $T s Q' t$.

In this manner treat each apex, measuring the stresses on each member.

For a thorough exposition of this method and others used in framed structures, suspension bridges, stone arches, etc., etc., see "The Strains in Framed Structures" (Du Bois).



BOLLMAN'S AMERICAN TRUSS BRIDGES.

Notation of Letters.

W = total load uniformly distributed on the bridge.
w = load on each point of suspension.
S = span.
D and d = distances from abutments A and B to point of suspension.
A and a = cross areas of the tension and counter-tension rods in square inches.
R and r = lengths of tension and counter-tension rods.
H = depth of truss, which is usually one-seventh of the span.
N = number of points of suspension.
T and t = tensions on the rods R and r respectively.
C = compression on the top at centre.

These formulas will answer for any system of weights and measures.

$$w = \frac{W}{(N+1)}. \qquad T = \frac{w\,D\,R}{S\,H}. \qquad t = \frac{w\,d\,r}{S\,H}. \qquad C = \frac{S\,W}{8H}.$$

When T and t are tons, A and a = square inches, D, d, S, H, R and r = feet, then

$$A = \frac{w\,D\,R}{5\,S\,H}. \qquad a = \frac{w\,d\,r}{5\,S\,H}.$$

STEAM HAMMER.

A heavy steam hammer with short fall produces a better forging than a light hammer with a high fall, although the dynamical work may be the same in both cases. This is accounted for by the inertia of the ingot forged.

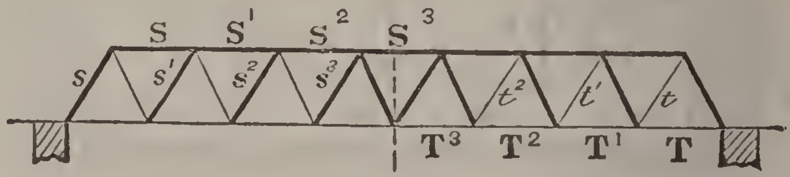
The effect of blows of a heavy hammer and short fall will penetrate through the metal, and nearly with the same effect on the anvil side, while a light hammer and high fall will affect the metal on or near the surface of the blow, because most of the work is in the latter case discharged in the inertia of the ingot forged. In forging a large shaft, it is generally piled up with iron bars sometimes rolled into a segment form to suit the pile. When placed under the hammer in a welding heat, very light and gentle blows are first given. The work of a light hammer will be discharged in the bars nearest to the blow, while a heavy hammer will squeeze the whole mass together throughout, and a sound welding will be produced.

The additional expense of a heavy hammer is fully compensated by the waste of labor and materials under a small one. I have often seen, in broken shafts, the bars in the centre as clear and unwelded as when first piled, which is a sure indication that the shaft has been forged by a too light hammer. In crank-shafts for propeller engines forged under a light hammer, when brought to the machine-shop the best part of the metal is worked away by planing and turning, and the poorest left for the engine; but if forged under a heavy hammer, the difference in quality of metal will not be so great.

Weight of Steam Hammers.

The weight of a steam hammer in pounds should be at least eighty times the square of the diameter of the shaft in inches.

BRIDGES.



The Warren Girder. (See Stoney's *Theory of Strains.*)

FINE LINES IN TENSION AND THICK LINES IN COMPRESSION.

The Warren girder consists of equilateral triangles formed by the trusses and ties, which make divisions or bays in the span.

The depth of the girder is 0.10825 of the span.

For a uniformly distributed load Stoney's method may be used. For a rolling load, such as a train of cars, a table will have to be made. See Du Bois, *Graphical Statics* The theory is too elaborate to be discussed in a general Pocket-Book,

Weight of one pair of Warren's Girders in tons,

for a single track of railway on the top or on the bottom (approximate).

On the	Span of the girder in feet.											
	50	60	70	80	90	100	110	120	130	140	150	160
Top,	11	15	18	23	27	32	38	44	51	58	66	75
Bottom,	15	19	24	29	35	41	48	56	64	72	80	89

Table of Dimensions in inches of Rolled Iron,

for roofs on spans from 30 to 80 feet (figures 3 to 6, page 332).

Name of iron.		Span in feet.					
		30	40	50	60	70	80
Rafter T-iron,	*L*	$2\frac{3}{4}\times2\frac{1}{2}\times\frac{1}{2}$	$3\frac{1}{2}\times3\times\frac{1}{2}$	$4\times3\frac{1}{2}\times\frac{5}{8}$	$5\times4\frac{1}{2}\times\frac{3}{4}$	$5\frac{1}{2}\times5\times\frac{3}{4}$	$6\times6\times\frac{3}{4}$
Struts T-iron,	*s*	$2\frac{1}{2}\times2\frac{1}{2}\times\frac{3}{8}$	$3\times2\frac{1}{2}\times\frac{1}{2}$	$3\times3\times\frac{5}{8}$	$4\times4\times\frac{5}{8}$	$4\frac{1}{2}\times4\frac{1}{2}\times\frac{5}{8}$	$5\times4\frac{1}{2}\times\frac{3}{4}$
King bolt,	*K*	1	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$
Queen bolt,	*Q*	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$
	Q'	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$
Tie-rod,	*T*	$1\frac{1}{8}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$
	t'	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$
Weight, lbs.,		1500	3000	4800	7000	9550	12400

The last line shows the approximate weight in pounds of each principal when the rise of the roof is 0.2 of the span.

ELEMENTS OF PHŒNIX BEAMS.

RIGIDITY is a different quality from strength. A beam may be quite strong enough to carry a given load, but under this load it may deflect more than is desirable. About one-thirtieth of an inch per foot of clear span is the usual maximum of deflexion that is permissible. Under ordinary loads this is attained when the clear span is about 26 times the depth of the beam, and the dividing lines in the table show where this limit is passed for each beam.

Like the load-factor, the deflexion-factor is dependent upon the depth and flange area of the beam to which it is to be applied; the general formula for the deflexion of any beam under an equally distributed load being

$$\delta' = \frac{.004\ W.\ L^3}{\left(a + \frac{a'}{6}\right)d^2}.$$

By inserting the values proper to each beam, the results given in the following tables have been obtained. A close approximation to the actual deflexion may be obtained by dividing the square of the length of the span in feet by 62 times the depth of the beam in inches.

Definition of Terms used in Formulæ.

W = Equally distributed load on any beam in net tons.
L = Length of clear span, expressed in feet.
a = Area of top, or bottom, flange, in square inches.
a' = Area of stem of beam, in square inches.
D = Effective depth of beam, expressed in feet.
d = Effective depth of beam, expressed in inches.
S = Safe strain per square inch of effective section $\left(a + \frac{a'}{6}\right)$ in tons of 2000 pounds.
δ = Deflexion in inches at middle for a central load.
δ' = Deflexion in inches at middle for a uniformly distributed load.

General formula for any I beam under an equally distributed load:

$$W = \frac{8\,D\left(a + \frac{a'}{6}\right)S}{L}.$$

Now, in this formula, it is only necessary to insert the proper values for "effective depth" and "effective section," given in the table for each particular beam, in order to determine its strength.

The load-factor for each beam is thus dependent upon its depth and the quantity of metal in its flanges. This load-factor, when divided by the clear span, gives a quotient that indicates the number of tons that the beam will carry with perfect safety. The table gives the safe loads per foot of clear span for various-sized beams.

TABLE I.—ELEMENTS OF PHŒNIX BEAMS.

Beam.	Dimensions, Inches.			Area, Square Inches.		
	Width of Flange.	Average Thickness of Flange.	Thickness of Stem.	a of Flange.	a' of Stem.	Sum of $a+\frac{a'}{6}$.
15″ 200	5$\frac{5}{16}$	1.156	.65	6.142	7.715	7.428
15″ 150	4¾	.911	.50	4.330	6.340	5.386
12″ 170	5½	1.050	.59	5.777	5.446	6.684
12″ 125	4¾	.802	.49	3.810	4.880	4.623
10½″ 135	5	.875	.50	4.375	4.750	5.166
10½″ 105	4½	.745	.44	3.353	3.793	3.986
9″ 150	5⅜	1.039	.60	5.586	3.828	6.224
9″ 84	4	.700	.40	2.800	2.800	3.261
9″ 70	3½	.680	.31	2.381	2.238	2.754
8″ 81	4½	.625	.38	2.812	2.476	3.225
8″ 65	4	.527	.35	2.109	2.282	2.489
7″ 69	4	.625	.37	2.500	1.900	2.816
7″ 55	3½	.507	.35	1.775	1.949	2.100
6″ 50	3½	.531	.31	1.858	1.284	2.072
6″ 40	2¾	.517	.25	1.421	1.158	1.614
5″ 36	3	.400	.30	1.200	1.200	1.400
5″ 30	2¾	.375	.25	1.000	1.000	1.166
4″ 30	2¾	.410	.25	1.135	.730	1.257
4″ 18	2	.281	.21	.562	.682	.676

Beam.	Effective Depth,		Load Factor. $8D\left(a+\frac{a'}{6}\right)S$ When $S=6$ Tons.	Deflection Factor. $\left(a+\frac{a'}{6}\right)d^2$.
	D Feet.	d Inches.		
15″ 200	1.150	13.80	410	1415
15″ 150	1.170	14.04	302	1062
12″ 170	.910	10.92	292	797
12″ 125	.930	11.16	208	576
10½″ 135	.800	9.62	178	478
10½″ 105	.812	9.74	155	378
9″ 150	.658	7.90	197	388
9″ 84	.691	8.30	108	225
9″ 70	.698	8.38	92	193
8″ 81	.610	7.37	94	175
8″ 65	.618	7.42	74	137
7″ 69	.530	6.37	72	114
7″ 55	.537	6.44	54	87
6″ 50	.456	5.47	45	62
6″ 40	.458	5.50	35	49
5″ 36	.383	4.60	25	30
5″ 30	.385	4.62	21	25
4″ 30	.298	3.58	18	16
4″ 18	.304	3.65	10	9

PHŒNIX BEAMS.

Their Adaptation and Duty as Flooring Joists.

Clear Span.	3′ apart	3½′ apart	4′ apart	4½′ apart	5′ apart	5½′ apart	6′ apart
10 feet.	30 □′	35 □′	40 □′	45 □′	50 □′	55 □′	60 □′
Load lbs.	**4,200**	**4,900**	**5,600**	**6,300**	**7,000**	**7,700**	**8,400**
I		6″				7 or 8″	
12 feet.	36 □′	42	48	54	60	66	72
Load lbs.	**5,040**	**5,880**	**6,720**	**7,560**	**8,400**	**9,240**	**10,080**
I	6 or 7″			7″		8″	
14 feet.	42 □′	49	56	63	70	77	84
Load lbs.	**5,880**	**6,860**	**7,840**	**8,820**	**9,800**	**10,780**	**11,760**
I	7 or 8″			8 or 9″^{70}		9″^{70}	
16 feet.	48 □′	56	64	72	80	88	96
Load lbs.	**6,720**	**7,840**	**8,960**	**10,080**	**11,200**	**12,320**	**13,440**
I	8″		9″^{70}	9″^{84}		10½″^{105}	
18 feet.	54 □′	63	72	81	90	99	108
Load lbs.	**7,560**	**8,820**	**10,080**	**11,340**	**12,600**	**13,860**	**15,120**
I	8 or 9″^{70}	9″^{84}			10½″^{105}		
20 feet.	60 □′	70	80	90	100	110	120
Load lbs.	**8,400**	**9,800**	**11,200**	**12,600**	**14,000**	**15,400**	**16,800**
I	9^{84} or 10½		10½″^{105}			12″^{125}	
22 feet.	66 □′	77	88	99	110	121	132
Load lbs.	**9,240**	**10,780**	**12,320**	**13,860**	**15,400**	**16,940**	**18,480**
I		10½″^{105}			12″^{125}		12″^{170}
24 feet.	72 □′	84	96	108	120	132	144
Load lbs.	**10,080**	**11,760**	**13,440**	**15,120**	**16,800**	**18,480**	**20,160**
I	10½ or 12″^{125}		12″^{125}		12″^{170} or 15″^{150}		
26 feet.	78 □′	91	104	117	130	143	156
Load lbs.	**10,928**	**12,740**	**14,560**	**16,380**	**18,240**	**20,020**	**21,840**
I	10½ or 12	12″^{125}		12″^{170} or 15″^{150}			15″^{150}
28 feet.	84 □′	98	112	126	140	154	168
Load lbs.	**11,760**	**13,720**	**15,680**	**17,640**	**19,600**	**21,560**	**23,520**
I	12″^{125} or 15″^{150}		12″^{170} or 15″^{150}		15″^{150}	15″^{200}	
30 feet.	90 □′	105	120	135	150	165	180
Load lbs.	**12,600**	**14,700**	**16,800**	**18,900**	**21,000**	**23,100**	**25,200**
I	12 or 15^{150}	12″^{170} or 15″^{150}		15″^{150}		15″^{200}	

In the above table the load is taken at 140 lbs. per □ foot of floor.

	15″ 200 Lbs. $W=\frac{410}{L}$.			15″ 150 Lbs. $W=\frac{302}{L}$.			12″ 170 Lbs. $W=\frac{292}{L}$.			12″ 125 Lbs. $W=\frac{208}{L}$.		
Clear Span, in Feet.	Safe Load, Net Tons.	Correspon'g Deflection.	Wt. of Beam, in Lbs.	Safe Load, Net Tons.	Correspon'g Deflection.	Wt. of Beam, in Lbs.	Safe Load, Net Tons.	Correspon'g Deflection.	Wt. of Beam, in Lbs.	Safe Load, Net Tons.	Correspon'g Deflection.	Wt. of Beam, in Lbs.
		″			″			″			″	
10	41.0	.116	667	30.2	.114	500	29.2	.147	567	20.8	.144	417
11	37.2	.140	733	27.4	.138	550	26.6	.177	623	18.8	.174	458
12	34.2	.167	800	25.2	.154	600	24.3	.210	680	17.3	.207	500
13	31.6	.196	867	23.2	.182	650	22.4	.246	737	16.0	.243	542
14	29.3	.227	933	21.6	.212	700	20.9	.286	793	14.9	.282	583
15	27.4	.261	1000	20.0	.254	750	19.4	.328	850	13.8	.325	625
16	25.6	.296	1067	18.9	.289	800	18.3	.374	907	13.0	.360	667
17	24.1	.334	1133	17.8	.327	850	17.2	.423	963	12.2	.408	708
18	22.8	.376	1200	16.8	.367	900	16.2	.475	1020	11.5	.459	750
19	21.6	.419	1267	15.9	.410	950	15.4	.530	1077	10.9	.513	792
20	20.5	.463	1333	15.1	.455	1000	14.6	.587	1133	10.4	.578	833
21	19.5	.510	1400	14.4	.502	1050	13.9	.648	1190	9.9	.636	875
22	18.6	.560	1467	13.7	.551	1100	13.3	.711	1247	9.4	.698	917
23	17.8	.612	1533	13.1	.602	1150	12.7	.777	1303	9.0	.763	958
24	17.1	.667	1600	12.6	.656	1200	12.2	.846	1360	8.7	.832	1000
25	16.4	.725	1667	12.1	.712	1250	11.7	.918	1417	8.3	.903	1042
26	15.8	.785	1733	11.6	.769	1300	11.2	.992	1473	8.0	.977	1083
27	15.2	.846	1800	11.2	.828	1350	10.8	1.068	1530	7.7	1.053	1125
28	14.6	.906	1867	10.8	.889	1400	10.4	1.147	1587	7.4	1.131	1167
29	14.1	.972	1933	10.4	.942	1450	10.0	1.230	1643	7.1	1.211	1208
30	13.7	1.04	2000	10.0	1.017	1500	9.7	1.314	1700	6.9	1.294	1250

	10½″ 135 Lbs. $W=\frac{178}{L}$.			10½″ 105 Lbs. $W=\frac{155}{L}$.			9″ 150 Lbs. $W=\frac{197}{L}$.			9″ 84 Lbs. $W=\frac{108}{L}$.		
10	17.8	.149	450	15.5	.164	350	19.7	.203	500	10.8	.192	280
11	16.2	.180	495	14.0	.197	385	17.8	.243	550	9.8	.231	308
12	14.8	.214	540	12.9	.236	420	16.4	.296	600	9.0	.276	336
13	13.7	.251	585	11.8	.278	455	15.2	.347	650	8.3	.324	364
14	12.7	.291	630	11.1	.322	490	14.1	.402	700	7.7	.376	392
15	11.8	.333	675	10.2	.364	525	13.2	.459	750	7.2	.432	420
16	11.1	.380	720	9.7	.414	560	12.3	.530	800	6.7	.488	448
17	10.5	.431	765	9.1	.470	595	11.6	.585	850	6.3	.550	476
18	9.9	.481	810	8.6	.528	630	10.9	.654	900	6.0	.622	504
19	9.3	.533	855	8.1	.589	665	10.3	.737	950	5.7	.695	532
20	8.9	.595	900	7.7	.652	700	9.8	.807	1000	5.4	.768	560
21	8.5	.658	945	7.3	.719	735	9.3	.891	1050	5.1	.839	588
22	8.1	.721	990	7.0	.788	770	8.9	.980	1100	4.9	.927	616
23	7.7	.784	1035	6.7	.862	805	8.5	1.07	1150	4.7	1.01	644
24	7.4	.856	1080	6.5	.941	840	8.2	1.17	1200	4.5	1.10	672
25	7.1	.928	1125	6.2	1.025	875	7.9	1.27	1250	4.3	1.19	700
26	6.8	1.00	1170	5.9	1.105	910	7.6	1.38	1300	4.1	1.27	728
27	6.6	1.08	1215	5.7	1.187	945	7.3	1.48	1350	3.9	1.36	756
28	6.3	1.16	1260	5.5	1.271	980	7.0	1.59	1400	3.8	1.48	784
29	6.1	1.24	1305	5.3	1.360	1015	6.8	1.70	1450	3.7	1.60	812
30	5.9	1.33	1350	5.1	1.455	1050	6.6	1.83	1500	3.6	1.73	840

Clear Span, in Feet.	9″ 70 Lbs. $W = \frac{92}{L}$.			8″ 81 Lbs. $W = \frac{94}{L}$.			8″ 65 Lbs. $W = \frac{74}{L}$.			7″ 69 Lbs. $W = \frac{72}{L}$.		
	Safe Load, Net Tons.	Correspon'g Deflection.	Wt. of Beam, in Lbs.	Safe Load, Net Tons.	Correspon'g Deflection.	Wt. of Beam, in Lbs.	Safe Load, Net Tons.	Correspon'g Deflection.	Wt. of Beam, in Lbs.	Safe Load, Net Tons.	Correspon'g Deflection.	Wt. of Beam, in Lbs.
		″			″			″			″	
10	9.2	.190	233	9.4	.215	270	7.4	.215	216	7.2	.252	230
11	8.4	.231	256	8.5	.258	297	6.8	.264	238	6.5	.303	253
12	7.7	.275	280	7.8	.308	324	6.2	.312	260	6.0	.363	276
13	7.0	.318	303	7.2	.361	351	5.7	.365	282	5.5	.424	299
14	6.7	.380	326	6.7	.420	378	5.3	.424	303	5.1	.491	322
15	6.2	.432	350	6.2	.478	405	4.9	.475	325	4.8	.568	345
16	5.7	.448	373	5.9	.546	432	4.6	.549	347	4.5	.645	368
17	5.4	.548	396	5.5	.617	459	4.3	.616	368	4.2	.724	391
18	5.1	.615	420	5.2	.693	486	4.1	.697	390	4.0	.818	414
19	4.8	.690	443	5.0	.783	513	3.9	.780	412	3.8	.914	437
20	4.6	.761	466	4.7	.859	540	3.7	.863	433	3.6	1.01	460
21	4.4	.842	490	4.5	.952	567	3.5	.946	455	3.4	1.10	483
22	4.2	.925	513	4.2	1.02	594	3.4	1.05	477	3.2	1.19	506
23	4.0	1.01	536	4.1	1.14	621	3.2	1.13	498	3.1	1.32	529
24	3.8	1.08	560	3.9	1.23	648	3.1	1.25	520	3.0	1.45	552
25	3.6	1.16	583	3.7	1.32	675	2.9	1.32	542	2.9	1.59	575
26	3.5	1.27	606	3.6	1.44	702	2.8	1.43	563	2.8	1.72	598
27	3.4	1.38	630	3.5	1.57	729	2.7	1.55	585	2.7	1.86	621
28	3.3	1.49	653	3.3	1.65	756	2.6	1.66	607	2.6	2.00	644
29	3.2	1.60	676	3.2	1.78	783	2.5	1.77	628	2.5	2.14	667
30	3.1	1.73	700	3.1	1.91	810	2.4	1.88	650	2.4	2.27	690

	7″ 55 Lbs. $W = \frac{54}{L}$.			6″ 40 Lbs. $W = \frac{35}{L}$.			5″ 30 Lbs. $W = \frac{21}{L}$.			4″ 30 Lbs. $W = \frac{18}{L}$.		
10	5.4	.248	183	3.5	.286	133	2.1	.336	100	1.80	.448	100
11	4.8	.293	201	3.2	.348	146	1.9	.405	110	1.63	.545	110
12	4.5	.357	220	2.9	.410	160	1.7	.471	120	1.50	.643	120
13	4.2	.423	238	2.7	.486	173	1.6	.563	130	1.38	.752	130
14	3.9	.491	256	2.5	.562	186	1.5	.660	140	1.28	.872	140
15	3.6	.558	275	2.3	.636	200	1.4	.757	150	1.20	1.00	150
16	3.4	.651	293	2.2	.738	213	1.3	.854	160	1.12	1.13	160
17	3.2	.722	311	2.0	.805	226	1.2	.945	170	1.06	1.29	170
18	3.0	.803	330	1.9	.907	240	1.2	1.12	180	1.00	1.44	180
19	2.8	.882	348	1.8	1.01	253	1.1	1.21	190	.95	1.62	190
20	2.7	.992	366	1.7	1.11	266	1.0	1.28	200	.90	1.79	200
21	2.5	1.06	385	1.6	1.21	280	1.0	1.45	210	.85	1.95	210
22	2.4	1.17	403	1.6	1.39	293	.95	1.62	220	.81	2.14	220
23	2.3	1.28	421	1.5	1.49	306	.90	1.75	230	.78	2.35	230
24	2.2	1.39	440	1.5	1.58	320	.85	1.88	240	.75	2.57	240
25	2.1	1.50	458	1.4	1.79	333	.82	2.05	250	.72	2.79	250
26	2.1	1.69	476	1.3	1.87	346	.80	2.25	260	.69	3.01	260
27	2.0	1.80	495	1.3	2.09	360	.77	2.43	270	.66	3.26	270
28	1.9	1.90	513	1.2	2.15	373	.75	2.64	280	.64	3.51	280
29	1.8	2.01	531	1.2	2.39	386	.72	2.81	290	.62	3.77	290
30	1.8	2.23	550	1.1	2.43	400	.70	3.03	300	.60	4.02	300

FLOORS WITH IRON JOISTS.

(Phœnix Iron Company, Philadelphia.)

HAVING given the weight a floor is to bear per unit of surface, select the proper size I girders and joists required for sustaining the floor with safety.

To find the distance from centre to centre of beams for laying in floors when the span and load per square foot are given: The total load for each beam may be represented by $W = P\,L\,B$; in which P = load per square foot; L = clear span in feet; B = distance from centre to centre of beams. In the equation $W = \dfrac{8\,D\left(a + \frac{a'}{6}\right)S}{L}$ substitute the value of W, as above.

Then $$P\,L\,B = \frac{8\,D\left(a + \frac{a'}{6}\right)S}{L}, \text{ whence } B = \frac{8\,D\left(a + \frac{a'}{6}\right)S}{P\,L^2},$$

and from this formula B may be determined for any given values of P, L, and S.

Assuming P at 140 pounds per square foot and S at 6 tons safe strain per effective square inch, the proper spaces for each beam may be determined from the following table:

TABLE III.

Beam.	Value of $8\,D\left(a+\frac{a'}{6}\right)6.$	Clear Span, in Feet.								
		10	12	14	16	18	20	22	24	26
15″ -200	410	....	....	....	....	18.10	14.64	12.10	10.17	8.66
15″ -150	302	....	....	....	16.86	13.32	10.80	9.00	7.49	6.40
12″ -170	292	....	....	....	16.30	12.87	10.43	8.62	7.24	6.17
12″ -125	208	....	....	15.09	11.55	9.15	7.39	6.11	5.13	4.38
10½″-135	178	....	....	13.05	9.93	7.88	6.36	5.22	4.41	3.76
10½″-105	155	....	....	11.28	8.65	6.83	5.54	4.58	3.84	3.28
9″ -150	197	....	....	14.00	10.70	8.68	7.04	5.81	4.88	4.16
9″ - 84	108	....	10.71	7.87	6.02	4.76	3.86	3.18	2.68	
9″ - 70	92	13.00	9.02	6.70	5.14	4.05	3.28	2.72		
8″ - 81	94	13.44	9.32	6.85	5.24	4.14	3.36	2.77		
8″ - 65	74	10.60	7.34	5.39	4.13	3.28	2.64			
7″ - 69	72	10.29	7.14	5.25	4.02	3.13				
7″ - 55	54	7.70	5.36	3.94	3.01					
6″ - 50	45	6.43	4.45	3.28						
6″ - 40	35	5.00	3.47	2.55						
5″ - 36	25	3.57	2.48							
5″ - 30	21	3.00	2.08							

For any given span, and when it is equally convenient to set the beams apart at the distances B, B, B, corresponding to their several sizes in the above table, *the cheapest beam to use is the deepest of the series, even with the extra charge for size and length* added.

Example.—For a 26 feet span, what is comparative weight of iron and cost per square foot of floor sustained of a 10½″–105, a 12″–125, and a 15″–150 beam placed apart at their respective distances, B, in table?

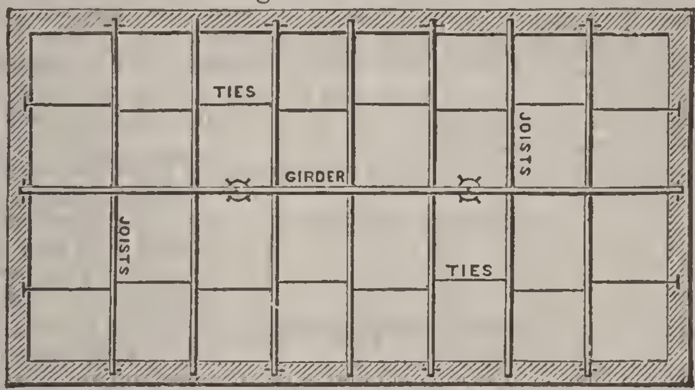
	Iron per □′.	Per lb.	Cents.	Ratio.
10½″	10.57 lbs. @	= 6 c.	= .63[42]	= $1.00
12″	9.51 lbs. @ 6¼ + ¼	= 6½c.	= .61[81]	= .97½
15″	7.81 lbs. @ 6½ + ¼ + ¼	= 7 c.	= .54[67]	= .86¼

The *rigidity* of a floor is greater when a suitable number of deep beams are used in preference to a greater number of shallower ones of equal strength. For any given span, to find the rigidity of a floor, ascertain from the table the bending moments of the several beams under comparison; and from table III. their respective distances B for given span, and divide the former by the latter. The quotients will represent their respective ratios of rigidity in the floor.

When P is any other weight than 140 pounds per square foot of flooring, then calculate the distance B from centre to centre of the beams by the given formula.

Under no circumstances should the floor-beams be strained beyond the limits of their elasticity, or, in other words, so strained that on the removal of the load they will not return to their original condition without set or permanent deflection.

Fig. 82.



If a beam is required to sustain its load at the centre, the figures in the table must be divided by 2; if at any other point, the weight at the centre is to the weight at any other point as the product of the segments of the

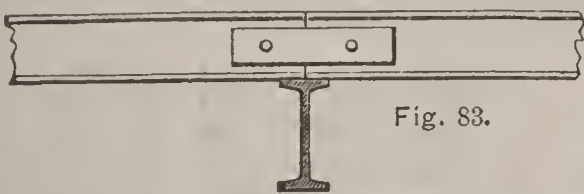


Fig. 83.

span at the given point is to the square of half the span. The coefficient of safety or load factor is placed above each beam in the table, and this divided by the clear span in feet gives the strength of the beam for a distributed load in net tons of 2000 pounds.

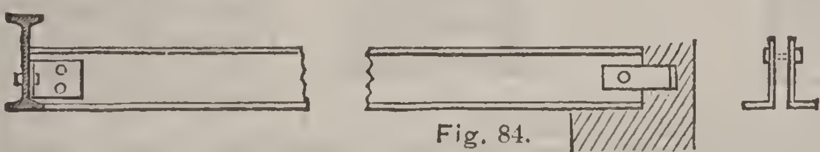


Fig. 84.

The deflection for each beam corresponding to this load will be found in the next line, and the weight of the beam itself, for the given length, should be deducted from the safe load. For any less load uniformly distributed, the

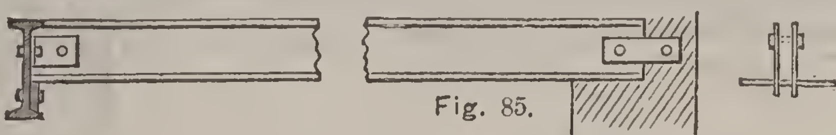


Fig. 85.

deflection will be directly proportionate to that given in the table. The deflection should not exceed one-thirtieth of an inch per foot of clear span, and the dividing lines in table II. show where this limit is passed for each beam.

Beams are generally laid in floors as shown in Fig. 82, the joists either resting on top of the girders, as in Fig. 83, or bolted to the sides of the girders.

Fig. 84 shows the detail of connection when the under sides are made flush, Fig. 85 the joint when the upper sides are flush, and Fig. 86 shows the form usually adopted when the beams are of the same size or when the centre lines are brought together. Arrangements of this kind are also used to connect the trimmer-beams of hatchways, jambs, and stairways. Fig. 87 shows the end of a double girder resting on a cast shoe plate, the beams being joined with a cast separator and bolts.

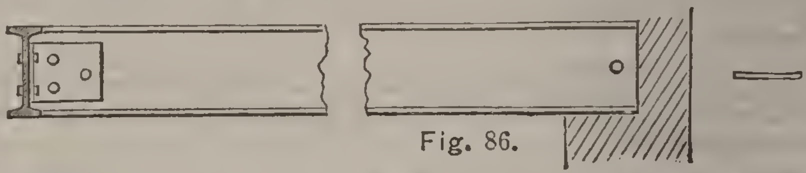
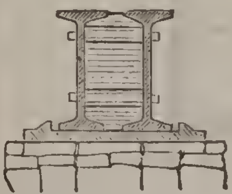


Fig. 86.

The wall end of the joists should also be provided with a *shoe or bearing* plate of iron or stone, as the brickwork is apt to crush under the end of the beam unless the load is distributed by this means over a sufficient surface. *Anchor straps* should be bolted *to the end of each girder*, and *to the wall end* of every alternate joist, thus binding the walls firmly from falling outwards in the event of fire or other accident.

Several simple modes of anchorage are shown in Figs. 84, 85, and 86.



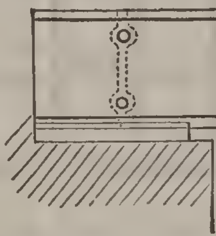


Fig. 87.

When one beam does not give sufficient strength for a girder, it is customary to bolt together two or more with cast separators between them. (Figs. 87 and 88.) For carrying a wall nine inches thick, two beams laid close together, of depths proportioned to the span, and for thirteen-inch walls the same beams, with a space between them, make a very good arrangement, care being taken to bind the two beams firmly together with separators and bolts every six or eight feet.

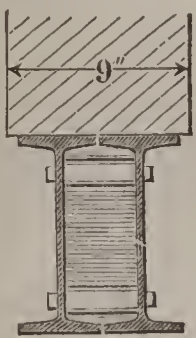
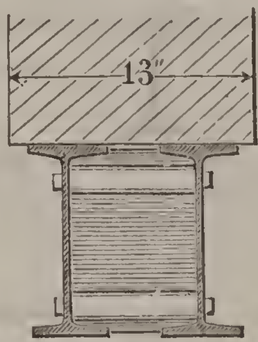


Fig. 88.



When the length of the span becomes too great for the girder, and posts are introduced for intermediate supports, joint boxes of simple pattern are provided at each floor, forming caps and bases for the wrought-iron columns, and at the same time serving to unite the girders continuously through the length of the building.

The cap or base may be of any ornamental pattern desired to give a finish to the column.

Between the joists the spaces are filled up with brick arches, resting on the lower flanges against cast-iron or brick skew-backs.

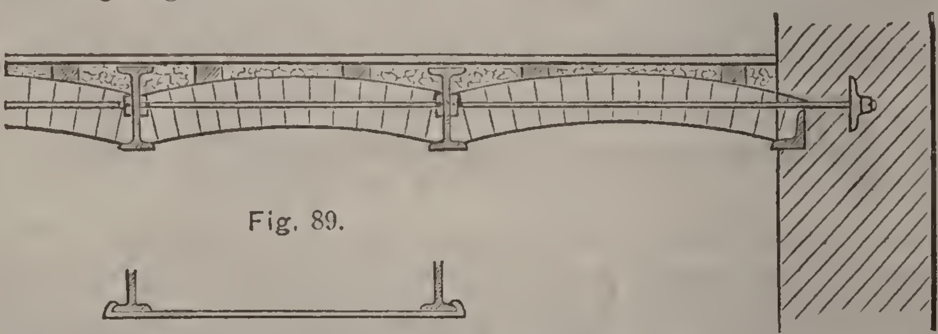


Fig. 89.

The bricks should be moulded with a slight taper to suit the arch, and be laid in place with as little mortar as possible. Above the arch the space is filled with grouting, in which wooden strips 2″ × 1″ are bedded for nailing the flooring to. The thrust of the arches is taken up by a series of tie-rods, placed in lines from 6 to 8 feet apart, and usually from $\frac{3}{4}$ to 1 inch in diameter, [as shown in plan,] that run from beam to beam from one end of the building to the other, being anchored into each end wall with stout washers, an angle bar or channel serving as a wall-plate for distributing the strain produced by the thrust of the first arch.

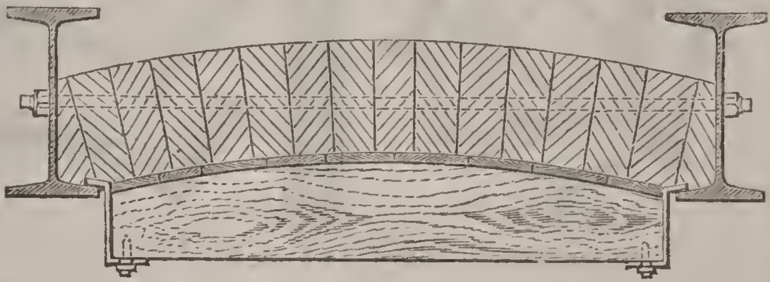


Fig. 90.

Instead of the brick arches corrugated iron is sometimes used to fill in the spaces. It is placed on the lower flanges of the beams and filled in above with cement in place of brick-work.

The centres for turning the arches can be suspended by iron straps hooked on the lower flange, and detachable on one side so that the frames can be shifted from point to point as the work progresses. If a flush surface is preferred for the ceiling, it may be obtained by wedging strips of pine between the beams, and tacking the laths diagonally to the under side of these, finishing with a smooth and fair surface of plastering, and thus entirely concealing the iron-work above.

Plate- and Box-Girders.

For large halls in which columns cannot be allowed, the floor-beams must be made deep and strong enough to bear the floor above. For this purpose, *plate-girders*, as represented by Fig. 91, also *box-girders*, as represented by Fig. 92, are made.

Fig. 91.

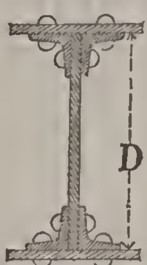


Fig. 92.

The strength of these girders is calculated by the standard formula for I beams.

a = area of all the four angle-irons and top and bottom plates.

a' = area of the stem or stems, in square inches.

In this kind of girders the iron should not be strained to more than $\frac{1}{6}$th, or better, $\frac{1}{8}$th, of the breaking strength of the iron.

W = equally distributed load on the girder, including its own weight.

The weight of the girder, including rivets and lap-plates, will be

$$w = 3.6\,L\,(a + a').$$

The length of a girder which can bear with safety only its own weight will be

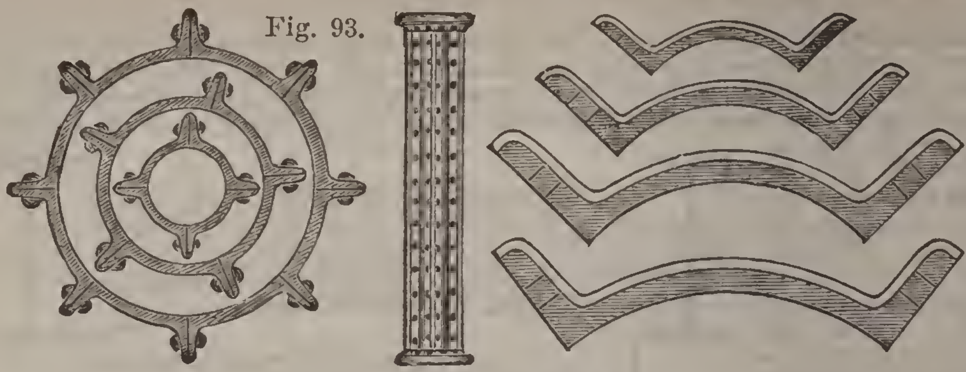
$$L = \sqrt{\frac{2.22\,D\,S\left(a + \frac{a'}{b}\right)}{a + a'}}.$$

The same rule will hold good for the different sections of beams shown on page 318.

For lattice-girders the area a' is omitted in the formula for strength.

$$\text{Lattice-girder } W = \frac{8\,D\,S\,a}{L}.$$

Fig. 93.



PHŒNIX COLUMNS.

Mark.	One Segment. Thickness, in Inches.	One Segment. Weight in Pounds, per Yard.	One Column. Area in Sq. Inches.	One Column. Weight in Pounds, per Foot.
A	3/16	9½	3.8	12.6
4 Segment.	1/4	12	4.8	16.0
3⅝″ diam.	5/16	14½	5.8	19.3
	3/8	17	6.8	22.6
B1	1/4	16	6.4	21.3
4 Segment.	5/16	19½	7.8	26.0
4 13/16″ diam.	3/8	23	9.2	30.6
	7/16	26½	10.6	35.3
	1/2	30	12.0	40.0
	9/16	33½	13.4	44.6
	5/8	37	14.8	49.3
B2	1/4	18½	7.4	24.6
4 Segment.	5/16	22½	9.0	30.0
5 15/16″ diam.	3/8	26½	10.6	35.3
	7/16	30½	12.2	40.6
	1/2	34½	13.8	46.0
	9/16	38½	15.4	51.3
	5/8	42½	17.0	56.6
C	1/4	25	10.0	33.3
4 Segment.	5/16	30	12.0	40.0
7 3/16″ diam.	3/8	35	14.0	46.0
	7/16	40	16.0	53.0
	1/2	45	18.0	60.0
	9/16	48	19.2	64.0
	5/8	53	21.2	70.6
	11/16	58	23.2	77.3
	3/4	63	25.2	84.0
	13/16	68	27.2	90.6
	7/8	73	29.2	97.3
	1	83	33.2	110.6
	1⅛	93	37.2	124.0
	1¼	103	41.2	137.3
D	1/4	28	14.	46.6
5 Segment.	5/16	32	16.	53.3
9⅛″ diam.	3/8	36	18.	66.0
	7/16	40	20.	66.6
	1/2	44	22.	73.3
	9/16	48	24.	80.0
E	1/4	28	16.8	56.
6 Segment.	5/16	32	19.2	64.
11″ diam.	3/8	36	21.6	72.
	7/16	40	24.0	80.
	1/2	44	26.4	88.
	9/16	48	28.8	96.
	5/8	53	31.8	106.
	11/16	58	38.8	116.
	3/4	63	37.8	126.
	13/16	68	40.8	136.
	7/8	73	43.8	146.
	1	83	49.8	166.
G	5/16	30	24.	80.0
8 Segment.	3/8	35	28.	93.3
14⅜″ diam.	7/16	40	32.	106.6
	1/2	45	36.	120.0
	9/16	50	40.	133.3
	5/8	55	41.	146.6
	11/16	60	48.	160.0
	3/4	65	52.	173.3
	13/16	70	56.	186.6
	7/8	75	60.	200.0
	1	85	68.	226.6
	1⅛	95	76.	253.3
	1¼	105	84.	280.0
	1⅜	115	92.	306.6

NOTES.—1. Diameters given are for the interior of columns, and are constant for all thicknesses of the same column. 2. The weight of rivet-heads adds from 2 to 5 per cent. to the weight of finished columns.

For wrought-iron columns . . . . $W = \dfrac{FA}{1 + \frac{1}{3000}\left(\frac{l}{h}\right)^2}$.

For cast-iron . . . . $W = \dfrac{FA}{1 + \frac{1}{400}\left(\frac{l}{h}\right)^2}$.

W = Breaking load in lbs; $\dfrac{w}{4}$ = Safe load for wrought

and $\dfrac{w}{6}$ = Safe load for cast iron.

$F = \begin{cases} 50{,}000 \text{ lbs for wrought-iron.} \\ 80{,}000 \text{ lbs. for cast-iron.} \end{cases}$

A = Sectional area of metal, in square inches. l = length.

h = diameter. $\dfrac{l}{h}$ = length in terms of the diameter.

In order to find the load which a cast- or wrought-iron column will sustain with safety, ascertain first the number of times its diameter will divide into its length; seek for the quotient in column I. of the table, and on the same line, in column IV. or V. (according as the material shall be cast or wrought iron), the safe load on each square inch of its cross section may be taken; multiply by the number of square inches contained in the cross section for the total safe load.

Ratio of Length to Diameter.	Maximum Load. Per Square Inch.		Safe Load. Per Square Inch.	
$\frac{l}{h}$	Cast. W = Lbs.	Wrought. W = Lbs.	Cast. Factor $\frac{1}{6} = \frac{w}{6}$.	Wrought. Factor $\frac{1}{4} = \frac{w}{6}$.
I.	II.	III.	IV.	V.
8	68 965	48 971	11 494	12 243
9	66 528	48 685	11 088	12 171
10	64 000	48 387	10 666	12 100
11	61 420	48 076	10 236	12 019
12	58 823	47 709	9 804	11 927
13	56 289	47 333	9 381	11 837
14	53 691	46 948	8 948	11 737
15	51 200	46 511	8 533	11 628
16	48 780	46 082	8 130	11 520
17	46 444	45 620	7 741	11 405
18	44 198	45 248	7 349	11 212
19	42 050	44 642	7 008	11 160
20	40 000	44 130	6 666	11 032
21	38 049	43 591	6 341	10 898
22	36 199	43 066	6 033	10 766
23	34 445	42 517	5 741	10 629
24	32 789	41 946	5 465	10 486
25	31 219	41 390	5 203	10 347
26	29 739	40 816	4 957	10 204
27	28 343	40 225	4 724	10 056
28	27 027	39 651	4 505	9 913
29	25 785	39 062	4 298	9 765
30	24 617	38 461	4 103	9 615
31	23 512	37 878	3 919	9 469
32	22 479	37 285	3 747	9 321
33	21 491	36 683	3 582	9 171
34	20 565	36 101	3 428	9 025
35	19 692	35 511	3 282	8 878
36	18 869	34 916	3 145	8 729
37	18 090	34 340	3 015	8 585
38	17 353	33 760	2 892	8 440
39	16 658	33 178	2 777	8 294
40	16 000	32 616	2 667	8 154

Table Showing the Proper Size of Rolled *I* Beams to be Used for Different Loads and Spans.

Load in Centre.	Distance between Supports, in Feet.													Load uniformly dist.
	8	9	10	11	12	13	14	15	16	17	18	19	20	
1,000	5 *l*	5 *l*	5 *l*	5 *l*	6 *l*	6 *l*	6 *l*	7	7	7	7	7	7	2,000
1,500	5 *l*	5 *l*	6 *l*	6 *l*	6 *l*	7	7	7	7	7	8 *l*	8 *l*	8 *l*	3,000
2,000	6 *l*	6 *l*	6 *l*	7	7	7	7	7	8 *l*	8 *l*	8 *l*	8 *h*	8 *h*	4,000
2,500	6 *l*	6 *l*	7	7	7	7	8 *l*	8 *l*	8 *l*	8 *h*	8 *h*	8 *h*	10½ *l*	5,000
3,000	6 *l*	7	7	7	7	8 *l*	8 *l*	8 *h*	8 *h*	8 *h*	10½ *l*	10½ *l*	10½ *l*	6,000
3,500	7	7	7	7	8 *l*	8 *l*	8 *h*	8 *h*	8 *h*	10½ *l*	10½ *l*	10½ *l*	10½ *l*	7,000
4,000	7	7	7	8 *l*	8 *l*	8 *h*	8 *h*	10½ *l*	10½ *l*	10½ *l*	10½ *l*	10½ *l*	12¼ *l*	8,000
4,500	7	7	8 *l*	8 *l*	8 *h*	8 *h*	10½ *l*	10½ *l*	10½ *l*	10½ *l*	10½ *l*	12¼ *l*	12¼ *l*	9,000
5,000	7	8 *l*	8 *l*	8 *h*	8 *h*	10½ *l*	10½ *l*	10½ *l*	10½ *l*	10½ *l*	12¼ *l*	12¼ *l*	12¼ *l*	10,000
6,000	8 *l*	8 *l*	8 *h*	8 *h*	10½ *l*	10½ *l*	10½ *l*	10½ *l*	12¼ *l*	12¼ *l*	12¼ *l*	12¼ *l*	15 *l*	12,000
7,000	8 *l*	8 *h*	8 *h*	10½ *l*	10½ *l*	10½ *l*	10½ *l*	12¼ *l*	12¼ *l*	12¼ *l*	15 *l*	15 *l*	15 *l*	14,000
8,000	8 *h*	8 *h*	10½ *l*	10½ *l*	10½ *l*	10½ *l*	12¼ *l*	12¼ *l*	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *l*	16,000
9,000	8 *h*	10½ *l*	10½ *l*	10½ *l*	10½ *l*	12¼ *l*	12¼ *l*	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *l*	15 *h*	18,000
10,000	10½ *l*	10½ *l*	10½ *l*	10½ *l*	12¼ *l*	12¼ *l*	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *l*	15 *h*	15 *h*	20,000
11,000	10½ *l*	10½ *l*	10½ *l*	12¼ *l*	12¼ *l*	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *l*	15 *h*	15 *h*	15 *h*	22,000
12,000	10½ *l*	10½ *l*	10½ *l*	12¼ *l*	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *l*	15 *h*	15 *h*	15 *h*	15 *h*	24,000
13,000	10½ *l*	10½ *l*	12¼ *l*	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *l*	15 *h*	15 *h*	15 *h*	15 *h*		26,000
14,000	10½ *l*	12¼ *l*	12¼ *l*	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *h*	15 *h*	15 *h*	15 *h*			28,000
15,000	10½ *l*	12¼ *l*	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *l*	15 *h*	15 *h*	15 *h*				30,000
16,000	12¼ *l*	12¼ *l*	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *h*	15 *h*	15 *h*	15 *h*				32,000
17,000	12¼ *l*	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *l*	15 *h*	15 *h*	15 *h*					34,000
18,000	12¼ *l*	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *h*	15 *h*	15 *h*						36,000
19,000	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *l*	15 *h*	15 *h*	15 *h*						38,000
20,000	12¼ *l*	15 *l*	15 *l*	15 *l*	15 *h*	15 *h*	15 *h*							40,000

h means heavy, *l* means light.

NAILS AND SPIKES.

Size, Length, and Number to the Pound.

(*Cumberland Nail and Iron Co.*)

Ordinary.

Size.	Length.	No. to Lb.
	"	
2d	7/8	716
3 fine.	1 1/16	588
3	1 1/16	448
4	1 3/8	336
5	1 3/4	216
6	2	166
7	2 1/4	118
8	2 1/2	94
10	2 3/4	72
12	3 1/8	50
20	3 3/4	32
30	4 1/4	20
40	4 3/4	17
50	5	14
60	5 1/2	10

Light.

Size.	Length.	No. to Lb.
	"	
4d	1 3/8	373
5	1 3/4	272
6	2	196

Brads.

Size.	Length.	No. to Lb.
	"	
6d	2	163
8	2 1/2	96
10	2 3/4	74
12	3 1/8	50

Clinch.

Length.	No. to Lb.
"	
2	152
2 1/4	133
2 1/2	92
2 3/4	72
3	60
3 1/4	43

Fence.

Length.	No. to Lb.
"	
2	96
2 1/4	66
2 1/2	56
2 3/4	50
3	40

Spikes.

Length.	No. to Lb.
"	
3 1/2	19
4	15
4 1/2	13
5	10
5 1/2	9
6	7

Boat.

Length.	No. to Lb.
"	
1 1/2	206

Finishing.

Size.	Length.	No. to Lb.
	"	
4d	1 3/4	384
5	1 1/8	256
6	2	204
8	2 1/2	102
10	3	80
12	3 5/8	65
20	3 7/8	46

Core.

Size.	Length.	No. to Lb.
	"	
6d	2	143
8	2 1/2	68
10	2 1/3	60
12	3 1/8	42
20	3 3/4	25
30	4 1/4	18
40	4 3/4	14
W H	2 1/2	69
W H L	2 1/4	72

Slate.

Size.	Length.	No. to Lb.
	"	
3d	1 5/16	288
4	1 7/16	244
5	1 3/4	187
6	2	146

Square and Hexagon Nuts.

Number of each Size in 100 Lbs.

(*Hoopes & Townsend, Philadelphia.*)

Size of Bolt.	Width.	Thickness.	Square. No. in 100 Lbs.	Square. Weight each in Lbs.	Hexagon. No. in 100 Lbs.	Hexagon. Weight each in Lbs.
1/4	1/2	1/4	8140	.012	9300	.011
5/16	19/32	5/16	3000	.033	6200	.016
3/8	11/16	3/8	2320	.043	3120	.032
7/16	25/32	7/16	1940	.052	2200	.045
1/2	7/8	1/2	1180	.085	1350	.074
9/16	31/32	9/16	920	.109	1000	.100
5/8	1 1/16	5/8	738	.135	830	.120
3/4	1 1/4	3/4	420	.238	488	.205
7/8	1 7/16	7/8	280	.357	309	.32
1	1 5/8	1	180	.556	216	.46
1 1/8	1 13/16	1 1/8	130	.769	148	.68
1 1/4	2	1 1/4	96	1.04	111	.90
1 3/8	2 3/16	1 3/8	70	1.43	85	1.18
1 1/2	2 3/8	1 1/2	60	1.67	70	1.43

These nuts are chamfered and trimmed.

Tacks.

Size.	Length.	No. to Lb.	Size.	Length.	No. to Lb.	Size.	Length.	No. to Lb.
1 oz.	$\frac{1}{8}$	16000	4 oz.	$\frac{7}{16}$	4000	14 oz.	$\frac{13}{16}$	1143
$1\frac{1}{2}$	$\frac{3}{16}$	10066	6	$\frac{9}{16}$	2666	16	$\frac{7}{8}$	1000
2	$\frac{1}{4}$	8000	8	$\frac{5}{8}$	2000	18	$\frac{15}{16}$	888
$2\frac{1}{2}$	$\frac{5}{16}$	6400	10	$\frac{11}{16}$	1600	20	1	800
3	$\frac{3}{8}$	5333	12	$\frac{3}{4}$	1333	22	$1\frac{1}{16}$	727

Railroad Spikes.

Length and Thickness in a Keg of 150 Pounds.

Length.	Thickness.	Number.	Length.	Thickness.	Number.
$4\frac{1}{2}$	$\frac{7}{16}$	527	$5\frac{1}{2}$	$\frac{1}{2}$	356
$4\frac{1}{2}$	$\frac{1}{2}$	400	$5\frac{1}{2}$	$\frac{9}{16}$	290
5	$\frac{3}{8}$	710	$5\frac{1}{2}$	$\frac{5}{8}$	219
5	$\frac{7}{16}$	489	6	$\frac{1}{2}$	311
5	$\frac{1}{2}$	390	6	$\frac{9}{16}$	263
5	$\frac{9}{16}$	296	6	$\frac{5}{8}$	197
5	$\frac{5}{8}$	258			

Splices and Bolts for One Mile of Track.

Rails	30 feet	long take	704	splices,	1408	bolts.
"	28	"	"	754	"	1508	"
"	27	"	"	782	"	1564	"
"	25	"	"	844	"	1688	"
"	24	"	"	880	"	1760	"

Railroad Iron.

To find the number of tons of rails for one mile of track, divide the weight per yard by 7 and multiply by 11. Thus: for 56 lb. rail, $56 \div 7 = 8$, and $8 \times 11 = 88$ tons per mile.

Hoopes & Townsend's Regular Sizes.

Square.				Hexagon.			
Width.	Thickness.	No. in 100 lbs.	Wt. each in lbs.	Width.	Thickness.	No. in 100 lbs.	Wt. each in lbs.
$\frac{1}{2}$	$\frac{1}{4}$	6680	.015	$\frac{1}{2}$	$\frac{1}{4}$	8600	.012
$\frac{5}{8}$	$\frac{5}{16}$	3540	.028	$\frac{5}{8}$	$\frac{5}{16}$	4260	.023
$\frac{3}{4}$	$\frac{3}{8}$	2050	.049	$\frac{3}{4}$	$\frac{3}{8}$	2500	.040
$\frac{7}{8}$	$\frac{3}{8}$	1380	.072	$\frac{3}{4}$	$\frac{7}{16}$	2180	.046
1	$\frac{1}{2}$	840	.119	1	$\frac{9}{16}$	900	.111
$1\frac{1}{8}$	$\frac{1}{2}$	650	.154	1	$\frac{9}{16}$	880	.114
$1\frac{1}{4}$	$\frac{5}{8}$	410	.244	$1\frac{1}{8}$	$\frac{3}{4}$	535	.187
$1\frac{3}{8}$	$\frac{3}{4}$	270	.370	$1\frac{3}{8}$	$\frac{7}{8}$	295	.339
$1\frac{1}{2}$	$\frac{7}{8}$	215	.465	$1\frac{1}{2}$	1	224	.446
$1\frac{3}{4}$	1	140	.714	$1\frac{3}{4}$	$1\frac{1}{8}$	150	.667
2	$1\frac{1}{8}$	95	1.05	2	$1\frac{3}{8}$	100	1.00
$2\frac{1}{4}$	$1\frac{1}{4}$	72	1.39	2	$1\frac{3}{8}$	96	1.04
$2\frac{1}{2}$	$1\frac{1}{8}$	45	2.22	$2\frac{1}{4}$	$1\frac{1}{4}$	72	1.39
3	$1\frac{1}{8}$	32	3.12	$2\frac{3}{4}$	$1\frac{5}{8}$	43	2.33

Bolts with Square Heads and Nuts.

Weight of 100 of the Enumerated Sizes.

(*Hoopes & Townsend, Philadelphia.*)

Lengths.	¼ in.	⅜ in.	½ in.	⅝ in.	¾ in.	⅞ in.	1 in.	1⅛ in.
Inch.								
1½	4.16	10.62	23.87	39.31				
1¾	4.22	11.72	25.06	41.38				
2	4.75	12.38	26.44	45.69	73.62			
2¼	5.34	12.90	28.62	49.50	76.			
2½	5.97	14.69	29.50	51.25	79.75			
2¾	6.50	16.47	31.16	53.	83.			
3	......	17.87	32.44	56.	85.38	127.25		
3½	......	18.94	39.75	63.12	93.44	140.56		
4	......	20.59	42.50	74.87	108.12	148.37	228.	296.
4½	......	21.69	44.87	79.62	113.12	158.76	239.	310.
5	......	23.62	48.81	83.	122.	167.25	250.	324.
5½	......	25.81	51.38	87.88	128.62	174.88	261.	338.
6	......	26.87	53.31	92.38	131.75	204.25	272.	352.
6½	......	......	56.87	96.88	139.56	214.69	283.	366.
7	......	......	59.12	99.87	145.50	228.44	294.	370.
7½	......	......	61.87	105.75	150.88	235.31	305.	384.
8	......	......	64.44	109.50	157.12	239.88	316.	398.
9	......	......	70.50	118.12	169.62	258.12	338.	426.
10	......	......	77.	128.13	184.	276.18	360.	454.
11	......	......	82.88	136.19	195.13	295.69	382.	482.
12	......	......	86.37	144.87	209.75	311.94	404.	510.
13	......	......	92.	155.50	219.37	335.81	426.	538.
14	......	......	97.75	163.58	237.50	351.88	448.	566.
15	......	......	103.25	170.75	249.06	391.75	470.	594.

FLAGGING.

Weight per Cubic Foot, 168 Pounds.

Weight per Square Foot.

Thickness	1	2	3	4	5	6	7	8 inch.
Weight	14	28	42	56	70	84	98	112 lbs.

ROOFING SLATE.

General Rule for the Computation of Slate.

FROM the length of the slate take 3 inches, or as many as the third covers the first; divide the remainder by 2, and multiply the quotient by the width of the slate, and the product will be the number of square inches in a single slate. Divide the number of square inches thus procured by 144, the number of square inches in a square foot, and the quotient will be the number of feet and inches required. A square of slate is what will cover 100 square feet when laid upon the roof.

Weight per Cubic Foot, 174 Pounds.

Weight per Square Foot.

Thickness	⅛	3/16	¼	⅜	½	⅝	¾	1 inch.
Weight	1.81	2.71	3.62	5.43	7.25	9.06	10.87	14.5 lbs.

Table of Sizes and Number of Slate in One Square.

Size in Inches.	No. of Slate in Square.	Size in Inches.	No. of Slate in Square.	Size in Inches.	No. of Slate in Square.
6 × 12	53	8 × 16	277	12 × 20	141
7 12	457	9 16	246	14 20	121
8 12	400	10 16	221	11 22	137
9 12	355	12 16	184	12 22	126
10 12	320	9 18	213	14 22	108
12 12	266	10 18	192	12 24	114
7 14	374	11 18	174	14 24	98
8 14	327	12 18	160	16 24	86
9 14	291	14 18	137	14 26	89
10 14	261	10 20	169	16 26	78
12 14	218	11 20	154		

Iron Rivets.

Weight per 100.

Length under Head.	Diameters.						
	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
1	1.895	4.848	.966	16.79	26.49	39.3	55.2
$\frac{1}{8}$	2.067	5.235	10.34	17.86	27.99	41.4	57.9
$\frac{1}{4}$	2.238	5.616	11.04	18.96	29.61	43.5	60.7
$\frac{3}{8}$	2.410	6.003	11.73	20.03	31.13	45.6	63.4
$\frac{1}{2}$	2.582	6.402	12.43	21.04	32.74	47.8	66.2
$\frac{5}{8}$	2.754	6.789	13.12	22.11	34.25	49.9	68.9
$\frac{3}{4}$	2.926	7.179	13.81	23.21	35.86	52.0	71.7
$\frac{7}{8}$	3.098	5.566	14.50	24.28	37.37	54.1	74.4
2	3.269	7.956	15.19	25.48	38.99	56.3	77.2
$\frac{1}{8}$	3.441	8.343	15.88	26.56	40.40	58.4	79.9
$\frac{1}{4}$	3.613	8.733	16.57	27.65	42.11	60.5	82.7
$\frac{3}{8}$	3.785	9.120	17.26	28.73	43.67	62.6	85.4
$\frac{1}{2}$	3.957	9.511	17.95	29.82	45.24	64.8	88.2
$\frac{5}{8}$	4.129	9.898	18.64	30.90	46.80	66.9	90.9
$\frac{3}{4}$	4.301	10.29	19.33	31.99	48.36	69.0	93.7
$\frac{7}{8}$	4.473	10.67	20.02	33.08	49.92	71.1	96.4
3	4.644	11.06	20.71	34.18	51.49	73.3	99.2
$\frac{1}{8}$	4.816	11.44	21.40	35.27	53.05	75.4	101.9
$\frac{1}{4}$	4.988	11.84	22.09	36.35	54.61	77.5	104.7
$\frac{3}{8}$	5.160	12.23	22.78	37.44	56.17	79.6	107.4
$\frac{1}{2}$	5.332	12.62	23.48	38.52	57.74	81.8	110.2
$\frac{5}{8}$	5.504	13.01	24.17	39.60	59.30	83.9	112.9
$\frac{3}{4}$	5.676	13.39	24.86	40.69	60.86	86.0	116.7
$\frac{7}{8}$	5.848	13.78	25.55	41.78	62.42	88.1	119.4
4	6.019	14.17	26.24	42.87	63.99	90.3	121.2
$\frac{1}{8}$	6.191	14.56	26.93	43.94	65.55	92.4	123.9
$\frac{1}{4}$	6.363	14.95	27.62	45.01	67.11	94.5	126.6
100 Heads.	.519	1.74	4.14	8.10	13.99	22.27	33.15

Length of rivet required to make one head = $1\frac{1}{2}$ diameters of round bar.

Price per Pound or Ton.

Per lb.	Price per Ton.	Per lb.	Price per Ton.	Per lb.	Price per Ton.	Per lb.	Price per Ton.
$\frac{1}{10}$ cts.	$2.24	$3\frac{1}{10}$ cts.	$69.44	$6\frac{1}{10}$ cts.	$136.64	$9\frac{1}{10}$ cts.	$203.84
$\frac{2}{10}$	4.48	$\frac{2}{10}$	71.68	$\frac{2}{10}$	138.88	$\frac{2}{10}$	206.08
$\frac{3}{10}$	6.72	$\frac{3}{10}$	73.92	$\frac{3}{10}$	141.12	$\frac{3}{10}$	208.32
$\frac{4}{10}$	8.96	$\frac{4}{10}$	76.16	$\frac{4}{10}$	143.36	$\frac{4}{10}$	210.56
$\frac{5}{10}$	11.20	$\frac{5}{10}$	78.40	$\frac{5}{10}$	145.60	$\frac{5}{10}$	212.80
$\frac{6}{10}$	13.44	$\frac{6}{10}$	80.64	$\frac{6}{10}$	147.84	$\frac{6}{10}$	215.04
$\frac{7}{10}$	15.68	$\frac{7}{10}$	82.88	$\frac{7}{10}$	150.08	$\frac{7}{10}$	217.28
$\frac{8}{10}$	17.92	$\frac{8}{10}$	85.12	$\frac{8}{10}$	152.32	$\frac{8}{10}$	219.52
$\frac{9}{10}$	20.16	$\frac{9}{10}$	87.36	$\frac{9}{10}$	154.56	$\frac{9}{10}$	221.76
1	22.40	4	89.60	7	156.80	10	224.00
$\frac{1}{10}$	24.64	$\frac{1}{10}$	91.84	$\frac{1}{10}$	158.04	$\frac{1}{10}$	226.24
$\frac{2}{10}$	26.88	$\frac{2}{10}$	94.08	$\frac{2}{10}$	161.28	$\frac{2}{10}$	228.48
$\frac{3}{10}$	29.12	$\frac{3}{10}$	96.32	$\frac{3}{10}$	163.52	$\frac{3}{10}$	230.72
$\frac{4}{10}$	31.36	$\frac{4}{10}$	98.56	$\frac{4}{10}$	165.76	$\frac{4}{10}$	232.96
$\frac{5}{10}$	33.60	$\frac{5}{10}$	100.80	$\frac{5}{10}$	168.00	$\frac{5}{10}$	235.20
$\frac{6}{10}$	35.84	$\frac{6}{10}$	103.04	$\frac{6}{10}$	170.24	$\frac{6}{10}$	237.44
$\frac{7}{10}$	38.08	$\frac{7}{10}$	105.28	$\frac{7}{10}$	172.48	$\frac{7}{10}$	239.68
$\frac{8}{10}$	40.32	$\frac{8}{10}$	107.52	$\frac{8}{10}$	174.72	$\frac{8}{10}$	241.92
$\frac{9}{10}$	42.56	$\frac{9}{10}$	109.76	$\frac{9}{10}$	176.96	$\frac{9}{10}$	244.16
2	44.80	5	112.00	8	179.20	11	246.40
$\frac{1}{10}$	47.04	$\frac{1}{10}$	114.24	$\frac{1}{10}$	181.44	$\frac{1}{10}$	248.64
$\frac{2}{10}$	49.28	$\frac{2}{10}$	116.48	$\frac{2}{10}$	183.68	$\frac{2}{10}$	250.88
$\frac{3}{10}$	51.52	$\frac{3}{10}$	118.62	$\frac{3}{10}$	185.92	$\frac{3}{10}$	253.12
$\frac{4}{10}$	53.76	$\frac{4}{10}$	120.96	$\frac{4}{10}$	188.16	$\frac{4}{10}$	255.36
$\frac{5}{10}$	56.00	$\frac{5}{10}$	123.20	$\frac{5}{10}$	190.40	$\frac{5}{10}$	257.60
$\frac{6}{10}$	58.24	$\frac{6}{10}$	125.44	$\frac{6}{10}$	192.64	$\frac{6}{10}$	259.84
$\frac{7}{10}$	60.48	$\frac{7}{10}$	127.68	$\frac{7}{10}$	194.88	$\frac{7}{10}$	262.08
$\frac{8}{10}$	62.72	$\frac{8}{10}$	129.92	$\frac{8}{10}$	197.12	$\frac{8}{10}$	264.32
$\frac{9}{10}$	64.96	$\frac{9}{10}$	132.16	$\frac{9}{10}$	199.36	$\frac{9}{10}$	266.56
3	67.20	6	134.40	9	201.60	12	268.80

SKYLIGHT AND FLOOR GLASS.

Weight per Cubic Foot, 156 Pounds.

(*Lennox Plate Glass Co., Ward & Co., Agents, Philadelphia.*)

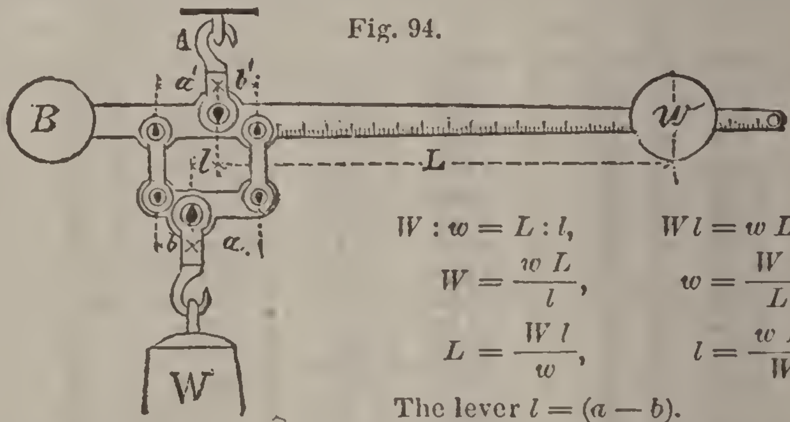
Weight per Square Foot.

Thickness . . .	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	1 inch.
Weight	1.62	2.43	3.25	4.88	6.50	8.13	9.75	13 lbs.

DIFFERENTIAL BALANCE.

FIG. 94 represents a convenient balance-scale for weighing heavy weights. It is much used in iron-foundries, where the balance with the weight is hoisted in a crane for weighing.

Fig. 94.



$$W : w = L : l, \qquad W l = w L.$$

$$W = \frac{w L}{l}, \qquad w = \frac{W l}{L}.$$

$$L = \frac{W l}{w}, \qquad l = \frac{w L}{W}.$$

The lever $l = (a - b)$.

The object of the links and the short lever is to bring the weight close to the fulcrum, or, more correctly, to obtain a short lever-arm l.

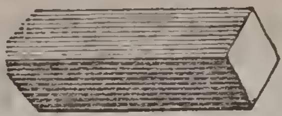
There is not room enough on the main-balance to bring the direction of the action of the weight W sufficiently close to the fulcrum for weighing heavy weights. The levers a and b ought to be equal to a' and b' respectively.

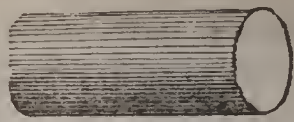
$$a + b = a' + b', \qquad \text{and } a - b = a' - b'.$$

The difference between a and b is generally not made so great as shown in the illustration. For a lever $L = 8$ feet, the lever l is only a fraction of an inch, and can be made as small as desired by making $(a - b)$ small. The scale should be well balanced by the ball B, without the weights W and w.

Spheres, Balls—Surfaces, Capacity and Weight of.

Diameter. Inches.	Surface. Sq. inches.	Capacity. Cub. Inches.	Cast iron. Pounds.	Lead. Pounds.	Water. Pounds.
1 in.	3.1416	0.5236	0.1365	0.2147	0.0188
1.125	3.9760	0.7455	0.1943	0.3062	0.0264
1.25	4.9087	1.0226	0.2673	0.4200	0.0368
1.375	5.9395	1.3611	0.3550	0.5579	0.0490
1.5	7.0686	1.7671	0.4607	0.7248	0.0636
1.625	8.2957	2.2467	0.5861	0.9227	0.0809
1.75	9.6211	2.8061	0.7325	1.1528	0.1050
1.875	11.044	3.4514	0.8000	1.4156	0.1242
2 in.	12.566	4.1888	1.0920	1.7180	0.1508
2.125	14.186	5.0243	1.3124	2.0631	0.1809
2.25	15.904	5.9640	1.5592	2.4482	0.2147
2.375	17.720	7.0143	1.8334	2.8811	0.2525
2.5	19.635	8.1812	2.1328	3.3554	0.2945
2.625	21.647	9.4708	2.4725	3.8892	0.3410
2.75	23.758	10.889	2.8400	4.4623	0.3920
2.875	25.967	12.442	3.2512	5.1056	0.4479
3 in.	28.274	14.137	3.6855	5.7982	0.5089
3.125	30.680	15.979	4.1721	6.5568	0.5752
3.25	33.183	17.974	4.6835	7.3623	0.6471
3.375	35.785	20.129	5.2612	8.2521	0.7246
3.5	38.484	22.449	5.8525	9.2073	0.8081
3.625	41.282	24.941	6.5089	10.231	0.8979
3.75	44.179	27.612	7.2135	11.323	0.9941
3.875	47.173	30.466	7.9556	12.500	1.0968
4 in.	50.265	33.510	8.7361	13.744	1.2064
4.25	56.745	40.194	10.510	16.482	1.4470
4.5	63.617	47.713	12.439	19.569	1.7177
4.75	70.882	56.115	14.666	23.035	2.0202
5 in.	78.540	65.450	17.063	26.843	2.3562
5.25	86.590	75.766	19.810	31.089	2.7276
5.5	95.033	87.114	22.720	35.729	3.1361
5.75	103.87	99.541	26.000	40.856	3.5835
6 in.	113.10	113.10	29.484	46.385	4.0716
6.5	132.73	143.79	37.453	58.976	5.1765
7.	153.94	179.59	46.820	73.659	6.4653
7.5	176.71	220.89	57.587	90.598	7.9520
8 in.	201.06	268.08	69.889	109.95	9.6509
8.5	226.98	321.55	83.839	131.38	11.576
9 in.	254.47	381.70	99.510	156.55	13.741
9.5	283.53	448.92	117.03	184.12	16.161
10	314.16	523.60	136.50	214.75	18.850
11	380.13	696.91	181.76	285.83	26.289
12	452.39	904.78	235.87	371.09	32.572
13	530.92	1150.3	299.62	471.80	41.411
14	615.72	1436.7	374.56	589.27	51.721
15	706.84	1767.1	460.69	724.78	63.616
16	804.24	2144.6	559.11	879.61	77.206
17	853.96	2572.4	670.71	1055.0	92.607
18	1017.8	3053.6	796.08	1252.4	109.93
19	1134.1	3591.3	936.27	1472.9	129.29
20	1256.6	4188.8	1092.0	1718.0	150.80





Side in inches	Weight in pounds.	Side in inches.	Weight in pounds	Diameter in inches.	Weight in pounds.	Diameter in inches.	Weight in pounds.
1/16	0·013	3 5/8	44·418	1/16	0·010	3 5/8	34·886
1/8	0·53	3 3/4	47·534	1/8	0·041	3 3/4	37·332
3/16	0·118	3 7/8	50·756	3/16	0·119	3 7/8	39·864
1/4	0·211	4	54·084	1/4	0·165	4	42·464
3/8	0·475	4 1/8	57·517	3/8	0·373	4 1/8	45·174
1/2	0·845	4 1/4	61·055	1/2	0·663	4 1/4	47·952
5/8	1·320	4 3/8	64·700	5/8	1·043	4 3/8	50·815
3/4	1·901	4 1/2	68·448	3/4	1·493	4 1/2	53·760
7/8	2·588	4 5/8	72·305	7/8	2·032	4 5/8	56·788
1	3·380	4 3/4	76·264	1	2·654	4 3/4	59·900
1 1/8	4·278	4 7/8	80·333	1 1/8	3·360	4 7/8	63·094
1 1/4	5·280	5	84·480	1 1/4	4·172	5	66·752
1 3/8	6·390	5 1/8	88·784	1 3/8	5·019	5 1/8	69·731
1 1/2	7·604	5 1/4	93·168	1 1/2	5·972	5 1/4	73·172
1 5/8	8·926	5 3/8	97·657	1 5/8	7·010	5 3/8	76·700
1 3/4	10·325	5 1/2	102·24	1 3/4	8·128	5 1/2	80·304
1 7/8	11·883	5 5/8	106·95	1 7/8	9·333	5 5/8	84·001
2	13·520	5 3/4	111·75	2	10·616	5 3/4	87·776
2 1/8	15·263	5 7/8	116·67	2 1/8	11·988	5 7/8	91·634
2 1/4	17·112	6	121·66	2 1/4	13·440	6	95·552
2 3/8	19·066	6 1/4	132·04	2 3/8	14·975	6 1/4	103·70
2 1/2	21·120	6 1/2	142·82	2 1/2	16·688	6 1/2	112·16
2 5/8	23·292	6 3/4	154·01	2 5/8	18·293	6 3/4	120·96
2 3/4	25·56	7	165·63	2 3/4	20·076	7	130·05
2 7/8	27·939	7 1/2	190·14	2 7/8	21·944	7 1/2	149·33
3	30·416	8	216·34	3	23·888	8	169·85
3 1/8	33·010	8 1/2	244·22	3 1/8	25·926	8 1/2	191·81
3 1/4	35·704	9	273·79	3 1/4	28·040	9	215·04
3 3/8	38·503	10	337·92	3 3/8	30·240	10	266·29
3 1/2	41·408	12	486·66	3 1/2	32·512	12	382·21

Rule for Finding the Weight of Pipes.

The diameter of the pipe in inches, measured from inside to outside, multiplied by the coefficient for the metal, will be the weight in pounds per linear foot.

Coefficients.

Lead,	0.1005	Brass, rolled,	0.0985
Copper,	0.0989	Iron, rolled,	0.0876
Brass, cast,	0.0882	Cast iron,	0.0811
Cast steel,	0.0891	Tin, rolled,	0.0821
Clay, burnt,	0.0214	Zinc, rolled,	0.0808

Diam.	0	⅛	¼	⅜	½	⅝	¾	⅞	Diam.
0	00000	·03804	15418	34675	61669	96352	1·3876	1·8975	0
1	2·5132	3·1227	3·9047	4·6620	5·5512	6·5476	7·5414	8·7012	1
2	9·8989	11·145	12·491	13·947	15·419	16·999	18·658	20·392	2
3	22·205	24·093	26·059	28·104	30·225	32·420	34·695	37·038	3
4	39·544	41·984	44·566	47·227	49·963	52·778	55·629	58·637	4
5	61·584	64·807	68·005	71·282	74·537	78.068	81·577	84·848	5
6	88·825	92·564	96·380	100·27	104·24	108·29	112·42	116·62	6
7	120·90	125·26	129·69	134·20	138·79	143·45	148·19	153·02	7
8	157·91	162·88	168·15	173·06	178·29	183·55	188·91	194·34	8
9	199·86	205·44	211·11	216·86	222·68	228·57	234·56	240·50	9
10	246·73	252·94	259·23	265·59	272·03	278·54	285·13	291·81	10
11	298·55	305·38	312·28	319·24	326·28	333·40	340·64	347·92	11
12	355·29	362·72	370·23	377·83	390·50	393·26	401·08	408·69	12
13	416·98	425·02	433·15	441·39	449·64	458·04	466·46	475·00	13
14	483·73	492·24	501·02	509·84	518·77	527·72	536·80	545·94	14
15	528·15	564·44	573·81	583·76	592·78	602·36	612·04	621·71	15
16	631·64	641·54	651·53	661·58	671·73	681·94	692·24	702·61	16
17	712·79	723·59	734·19	744·86	755·80	766·44	777·38	788·35	17
18	799·30	810·56	821·79	838·17	844·45	855·86	867·42	879·04	18
19	890·70	902·48	914·29	926·23	938·20	950·27	962·42	974·64	19
20	986·95	999·30	1011·6	1024·3	1036·9	1049·5	1062·3	1075·0	20
21	1088·1	1104·2	1114·6	1127·3	1140·5	1153·8	1167·2	1180·7	21
22	1194·2	1207·8	1221·5	1235·2	1249·1	1263·0	1277·0	1291·1	22
23	1305·2	1319·4	1333·7	1348·1	1362·6	1376·9	1391·7	1406·4	23
24	1421·5	1436·0	1451·0	1466·1	1481·0	1496·1	1511·4	1526·7	24
25	1492·1	1557·5	1572·1	1588·7	1604·4	1620·2	1635·8	1651·9	25
26	1667·9	1683·9	1700·1	1716·4	1732·7	1749·1	1765·5	1782·1	26
27	1798·7	1815·5	1832·2	1849·0	1865·9	1882·9	1900·0	1917·2	27
28	1934.4	1951·7	1969·1	1986·5	2004·1	2021·7	2039·4	2057·2	28
29	2075·1	2093·0	2111·0	2129·1	2147·2	2165·4	2183·8	2202·2	29
30	2220·6	2239·2	2257·8	2276·5	2295·2	2314·1	2333·1	2352·0	30
31	2371·1	2390·3	2409·6	2428·9	2448·3	2467·9	2461·3	2506·9	31
32	2526·6	2545·7	2566·2	2586·2	2606·1	2626·3	2646·4	2666·7	32
33	2687·0	2707·4	2727·8	2748·4	2769·0	2789·7	2810·4	2831·3	33
34	2852·3	2873·3	2894·4	2927·3	2936·8	2958·1	2979·5	3001·0	34
35	3022·5	3044·2	3065·9	3087·7	3109·5	3131·5	3143·7	3175·5	35
36	3197·5	3219·4	3242·0	3264·3	3286·9	3309·5	3332·2	3354·3	36
37	3377·8	3400·4	3423·3	3446·6	3469·5	3492·7	3516·0	3539·2	37
38	3562·9	3586·1	3609·6	3633·5	3657·0	3680·9	3704·8	3728·6	38
39	3752·2	3776·8	3801·0	3835·1	3849·7	3873·8	3898·3	3922·8	39
40	3947·7	3972·5	3987·0	4022·1	4046·9	4071·6	4094·1	4122·3	40
41	4147·5	4173·0	4198·4	4223·8	4249·2	4275·0	4300·7	4326·5	41
42	4352·3	4378·4	4404·1	4430·2	4456·6	4482·6	4509·0	4540·5	42
43	4562·2	4588·6	4615·2	4641·9	4668·6	4695·6	4722·7	4749·7	43
44	4776·7	4803·7	4831·1	4858·4	4885·7	4913·3	4941·1	4968·7	44
45	4996·3	5024·0	5051·9	5079·9	5107·8	5136·1	5164·1	5192·4	45
46	5220·9	5249·2	5277·8	5306·3	5335·0	5363·6	5393·5	5421·4	46
47	5450·2	5473·1	5508·4	5537·6	5566·8	5596·1	5625·6	5655·1	47
48	5684·6	5714·1	5744·1	5773·9	5803·7	5833·5	5863·7	5893·8	48

A solid cast-iron cylinder 42⅝ in. diameter weighs 4482·6 pounds per foot.
Subtract inside cylinder 40⅛ in. diameter weight 3972·5 " "
Weight of pipe 1¼ in. thick will be 510·1 " "

Weight of Flat Rolled Iron per Foot.

Look first for the thickness, and follow its line to the column headed by the given breadth. The value there found is the number of pounds per foot.

																				2½
																			2⅜	20·06
																		2¼	18·06	19·01
																	2⅛	16·16	17·05	17·95
																2	12·36	15·21	16·05	16·89
															1⅞	12·67	13·46	14·26	15·05	15·84
														1¾	11·09	11·83	12·57	13·31	14·04	14·78
													1⅝	9·610	10·30	10·98	11·67	12·36	13·04	13·73
												1½	8.237	8·871	9·505	10·14	10·77	11·40	12·04	12·67
											1⅜	6·970	7·551	8·132	8·713	9·294	9·874	10·45	11·03	11·62
										1¼	5·808	6·337	6·864	7·393	7·921	8·448	8·977	9·505	10·03	10·56
									1⅛	4·752	5·227	5·703	6·178	6·653	7·129	7·604	8·079	8·554	9·028	9·504
								1	3·802	4·224	4·646	5·069	5·492	5·914	6·336	6·758	7·181	7·604	8·025	8·448
							⅞	2·956	3·326	3·696	4·065	4·435	4·805	5·178	5·544	5·914	6·283	6·653	7·022	7·392
						¾	2·217	2·534	2·850	3·168	3·484	3·802	4·119	4 435	4·752	5·069	5·386	5·703	6·019	6·336
					⅝	1·584	1·846	4·112	2·375	2·640	2·904	3·168	3·432	3·696	3·960	4·224	4·488	4·752	5·916	5·280
				½	1·056	1·265	1·477	1·690	1·901	2·112	2·325	2·535	2·746	2·957	3·168	3·379	3·591	3·802	4·013	4·224
			⅜	0·634	0·792	0·950	1·108	1·267	1·425	1·584	1·742	1·900	2·059	2·218	2·376	2·534	2·693	2·851	3·009	3·168
		¼	0·317	0·422	0·528	0·633	0·738	0·845	0·950	1·056	1·161	1·266	1·372	1·479	1·584	1·689	1·795	1·900	2·006	2·112
	$\frac{3}{16}$	0·162	0·237	0·316	0·396	0·474	0·553	0·633	0·712	0·792	0·870	0·949	1·029	1·109	1·188	1·267	1·346	1·425	1·504	1·584
⅛	0·081	0·108	0·158	0·211	0·264	0·316	0·369	0·422	0·475	0·528	0·580	0·633	0·686	0·739	0·792	0·845	0·950	0 898	1·00	1·056

Weight of Flat Rolled Iron per Foot,

The thickness is in the first column, and the breadth in the **top line.**

	2½	2⅝	2¾	2⅞	3	3⅛	3¼	3⅜	3½	3⅝	3¾	3⅞	4	4¼	4½	4¾	5	5¼	5½	5¾	6
2⅜	20·06	21·07	22·07	23·07	24·07	25·07	26·08	27·18	28·09	28·84	29·59	30·84	32·10	34·11	36·12	38·12	40·13	42·13	44·14	45·95	47·76
2¼	19·01	19·96	20·91	21·86	22·81	23·76	24·71	25·66	26·61	27·56	28·51	29·46	30·41	32·32	34·22	36·12	38·02	39·92	41·82	43·32	44·82
2⅛	17·95	18·85	19·75	20·65	21·54	22·44	23·34	24·23	25·13	25·78	26·43	27·57	28·72	30·52	32·31	34·11	35·90	37·70	39·49	41·09	42·69
2	16·89	17·74	18·58	19·43	20·28	21·12	21·97	22·81	23·65	24·50	25·35	26·19	27·04	28·72	30·41	32·10	33·79	35·48	37·17	38·86	39·55
1⅞	15·84	16·63	17·42	18·22	19·01	19·80	20·59	21·38	22·18	22·97	23·76	24·55	25·35	26·92	28·51	30·09	31·68	33·26	34·85	36·43	38·01
1¾	14·78	15·52	16·26	17·00	17·74	18·48	19·22	19·96	20·70	21·44	22·18	22·92	23·66	25·13	26·61	28·09	29·57	31·05	32·53	34·01	36·49
1⅝	13·73	14·42	15·10	15·79	16·47	17·16	17·85	18·53	19·22	19·90	20·59	21·28	21·97	23·33	24·71	26·08	27·46	28·83	30·20	31·53	32·86
1½	12·67	13·31	13·94	14·57	15·21	15·84	16·47	17·10	17·74	18·37	19·01	19·64	20·28	21·54	22·81	24·08	25·35	26·61	27·88	29·15	30·42
1⅜	11·62	12·20	12·78	13·36	13·94	14·52	15·10	15·68	16·26	16·84	17·42	18·00	18·59	19·74	20·91	22·07	23·23	24·39	25·55	26·72	27·90
1¼	10·56	11·09	11·62	12·14	12·67	13·20	13·73	14·25	14·78	15·31	15·84	16·37	16·90	17·95	19·01	20·07	21·12	22·18	23·23	24·29	25·35
1⅛	9·504	9·931	10·45	10·93	11·41	11·88	12·36	12·83	13·31	13·79	14·27	14·74	15·21	16·15	17·11	18·06	19·01	19·96	20·91	21·86	22·81
1	8·448	8·872	9·294	9·716	10·14	10·55	10·98	11·40	11·83	12·20	12·67	13·09	13·52	14·36	15·21	16·05	16·90	17·74	18·59	19·43	20·27
⅞	7·392	7·763	8·132	8·502	8·871	9·740	9·610	9·980	10·35	10·72	11·09	11·46	11·83	12·56	13·31	14·04	14·78	15·52	16·26	17·00	17·74
¾	6·336	6·654	6·970	7·287	7·604	7·920	8·237	8·554	8·871	9·188	9·505	9.822	10·14	10·77	11·41	12·04	12·67	13·31	13·94	14·57	15·20
⅝	5·280	5·545	5·808	6·072	6·337	6·601	6·865	7·129	7·393	7·657	7·921	8·183	8·445	8·975	9·507	10·03	10·56	11·09	11·61	12·14	12·67
½	4·224	4·436	4·647	4·858	5·069	5·280	5·492	5·703	5·914	6·125	6·336	6·547	6·759	7·181	7·604	8·026	8·449	8·871	9·294	9·716	10·23
⅜	3·168	3·372	3·485	3·644	3·802	3·960	4·119	4·277	4·436	4·594	4·752	4·910	5·069	5·386	5·703	6·019	6·386	6·653	6·970	7·287	7·594
¼	2·112	2·218	2·320	2·429	2·535	2·640	2·746	2·851	2·957	3·062	3·168	3·274	3·380	3·591	3·802	4·013	4·224	4·436	4·647	4·858	5·069
3/16	1·584	1·663	1·741	1·822	1·901	1·980	2·059	2·138	2·218	2·297	2·376	2·455	2·535	2·693	2·851	3·009	3·168	3·327	3·485	3·643	3·801
⅛	1·056	1·109	1·162	1·215	1·267	1·320	1·373	1·426	1·479	1·531	1·584	1·637	1·690	1·795	1·901	2·006	2·112	2·218	2·323	2·429	2·535

Weight of Flat Rolled Iron per Foot.

Look first for the thickness, and follow its line to the column headed by the given breadth. The value there found is the number of pounds per foot.

																				6
																			5¾	116·6
																		5½	106·9	111·5
																	5¼	97·58	102·0	106·4
																5	88·71	92·93	97·16	101·4
															4¾	80·26	84·27	88·29	92·30	96·41
														4½	72·23	76·04	79·84	83·64	87·44	91·24
													4¼	64·63	68·22	71·81	75·40	78·99	82·58	86·17
												4	57·45	60·83	64·21	67·59	70·97	74·35	77·73	81·11
											3⅞	52·38	55·65	58·93	62·20	65·97	68·75	72·02	75·30	78·58
										3¾	49·09	50·69	53·86	57·03	60·20	63.36	66·53	69·70	72·87	76·04
									3⅝	45·93	47·46	49·00	52·06	55·13	58·19	61·25	64·31	67·37	70·44	73·51
								3½	42·87	44·35	45·83	47·31	50·27	53·23	56·18	59·14	62·10	65·05	68·01	70·97
							3⅜	39·92	41·34	42·77	44·19	45·62	48·47	51·24	54·17	57·03	59·88	62·73	65·58	68·43
						3¼	37·07	38·44	39·81	41·19	42·51	43·93	46·68	49·25	52·17	54·92	57·66	60·41	63·15	65·89
					3⅛	34·33	35·64	36·96	38·28	39·60	40·92	42·24	44·88	47·43	50·16	52·80	55·44	58·08	60·72	63·36
				3	31·94	32·95	34·21	35·48	36·75	38·02	39·28	40·55	43·09	45·62	48·16	50·69	53·23	55·76	58·30	60·84
			2⅞	29·42	30·49	31·57	32·78	34·00	35·21	36·43	37·64	38·86	41·29	43·72	46·15	48·58	51·01	53·43	55·87	58·31
		2¾	26·72	27·88	29·04	30·20	31·36	32·53	33·69	34·85	36·01	37·17	39·50	41·82	44·14	46·47	48·79	51·11	53·44	55·77
	2⅝	24·39	25·50	26·61	27·72	28·83	29·93	31·04	32·15	33·26	34·37	35·48	37·70	39·92	42·13	44·35	46·57	48·79	51·01	53·23
2½	22·18	23·23	24·29	25·34	26·40	27·46	28·51	29·57	30·62	31·68	32·73	33·79	35·91	38·02	40·13	42·24	44·35	46·47	48·58	50·69

Weight Per Square Foot in Pounds.

Thickness in inches.	Cast Iron.	Wrought or Sheet Iron.	Sheet Copper.	Sheet Lead.	Sheet Zinc
$\frac{1}{16}$	2·346	2·517	2·890	3·694	2·320
$\frac{1}{8}$	4·693	5·035	5·781	7·382	4·642
$\frac{3}{16}$	7·039	7·552	8·672	11·074	6·961
$\frac{1}{4}$	9·386	10·070	11·562	14·765	9·275
$\frac{5}{16}$	11·733	12·588	14·453	18·456	11.61
$\frac{3}{8}$	14·079	15·106	17·344	22·148	13·93
$\frac{7}{16}$	16·426	17·623	20·234	25·839	16·23
$\frac{1}{2}$	18·773	20 141	23·125	29·530	18·55
$\frac{9}{16}$	21·119	22·659	26·016	33·222	20·87
$\frac{5}{8}$	23·466	25·176	28·906	36·913	23·19
$\frac{11}{16}$	25·812	27·694	31·797	40·604	25·53
$\frac{3}{4}$	28·159	30·211	34·688	44·296	27·85
$\frac{13}{16}$	30·505	32·729	37·578	47·987	30·17
$\frac{7}{8}$	32·852	35·247	40·469	51·678	32·47
$\frac{15}{16}$	35·199	37·764	43·359	55·370	34·81
1	37·545	40·282	46·250	59·061	37·13
$1\frac{1}{8}$	42·238	45·317	52·031	66·444	41·78
$1\frac{1}{4}$	46·931	50·352	57·813	73·826	46·42
$1\frac{3}{8}$	51·625	55·387	63·594	63·594	51·04
$1\frac{1}{2}$	56·317	60·422	69·375	88·592	55·48
$1\frac{5}{8}$	61·011	65·458	75·156	95·975	60·35
$1\frac{3}{4}$	65·704	70·493	80·938	103·358	65.00
$1\frac{7}{8}$	70·397	75·528	86·719	110·740	69·61
2	75·090	80·563	92:500	118·128	74·25

Weight of Copper Rods or Bolts per Foot,

Diameter. Inches.	Weight. Pounds.	Diameter. Inches	Weight. Pounds	Diameter. Inches.	Weight. Pounds.	Diameter Inches.	Weight. Pounds.
$\frac{1}{4}$	0·1892	1	3·0270	$1\frac{7}{8}$	10·642	$3\frac{3}{8}$	34·487
$\frac{5}{16}$	0·2956	$1\frac{1}{16}$	3·4170	2	12·108	$3\frac{1}{2}$	37·081
$\frac{3}{8}$	0·4256	$1\frac{1}{8}$	3·8912	$2\frac{1}{8}$	13·668	$3\frac{5}{8}$	39·737
$\frac{7}{16}$	0·5794	$1\frac{3}{16}$	4·2688	$2\frac{1}{4}$	15·325	$3\frac{3}{4}$	42.568
$\frac{1}{2}$	0·7567	$1\frac{1}{4}$	4·7298	$2\frac{3}{8}$	17·075	$3\frac{7}{8}$	45·455
$\frac{9}{16}$	0·9578	$1\frac{5}{16}$	5·2140	$2\frac{1}{2}$	18·916	4	48·433
$\frac{5}{8}$	1·1824	$1\frac{3}{8}$	5·7228	$2\frac{5}{8}$	20·856	$4\frac{1}{4}$	53·550
$\frac{11}{16}$	1·4307	$1\frac{7}{16}$	6·2547	$2\frac{3}{4}$	22·891	$4\frac{1}{2}$	61·321
$\frac{3}{4}$	1·7027	$1\frac{1}{2}$	6·8109	$2\frac{7}{8}$	25·019	$4\frac{3}{4}$	68·312
$\frac{13}{16}$	1·9982	$1\frac{9}{16}$	7·3898	3	27·243	5	76·130
$\frac{7}{8}$	2·3176	$1\frac{5}{8}$	7·9931	$3\frac{1}{8}$	29·559	$5\frac{1}{2}$	91·550
$\frac{15}{16}$	2·6605	$1\frac{3}{4}$	9·2702	$3\frac{1}{4}$	31·972	6	109·

Gauge num.	Size inches.	Rolled Plates. Weight per square foot.				Drawn Wire. Weight per 1000 feet.			
No.	In.	Iron. Lbs.	Steel. Lbs.	Copper Lbs.	Brass. Lbs.	Iron. Lbs.	Steel. Lbs.	Copper Lbs.	Brass. Lbs.
0000	.4600	18.75	18.97	21.36	20.84	566.3	571.7	646.8	634.1
000	.4096	16.70	16.90	19.01	18.56	449.1	453.3	512.9	502.9
00	.3648	14.87	15.05	16.93	16.52	356.1	359.5	406.8	398.8
0	.3249	13.24	13.40	15.08	14.72	282.4	285.1	322.5	316.3
1	.2893	11.79	11.93	13.43	13.11	224.5	226.1	255.8	250.8
2	.2576	10.50	10.63	11.96	11.67	177.6	179.3	202.9	198.9
3	.2294	9.354	9.464	10.65	10.39	140.8	142.2	160.8	157.8
4	.2043	8.330	8.428	9.486	9.255	111.7	112.7	127.5	125.1
5	.1819	7.418	7.505	8.448	8.242	88.59	89.43	101.2	99.20
6	.1620	6.606	6.683	7.523	7.340	70.26	70.92	80.25	78.67
7	.1443	5.882	5.952	6.699	6.536	55.71	56.24	63.64	62.38
8	.1285	5.238	5.300	5.966	5.821	44.18	44.60	50.46	49.48
9	.1144	4.665	4.720	5.313	5.184	35.04	35.37	40.02	39.24
10	.1019	4.154	4.203	4.731	4.616	28.26	28.05	31.73	31.11
11	.0907	3.700	3.743	4.213	4.110	22.03	22.24	25.16	24.68
12	.0808	3.294	3.333	3.752	3.661	17.47	17.64	19.95	19.57
13	.0720	2.934	2.968	3.341	3.260	13.85	13.99	15.82	15.52
14	.0641	2.613	2.643	2.978	2.903	10.99	11.09	12.55	12.31
15	.0571	2.327	2.354	2.650	2.585	8.717	8.899	9.953	9.761
16	.0508	2.072	2.096	2.359	2.302	6.913	6.978	7.896	7.741
17	.0452	1.845	1.867	2.101	2.050	5.481	5.532	6.261	6.137
18	.0403	1.643	1.662	1.872	1.826	4.347	4.387	4.965	4.867
19	.0359	1.463	1.480	1.666	1.626	3.447	3.479	3.937	3.861
20	.0320	1.303	1.318	1.484	1.448	2.735	2.761	3.125	3.064
21	.0285	1.160	1.174	1.321	1.289	2.168	2.188	2.476	2.428
22	.0253	1.033	1.045	1.176	1.148	1.720	1.736	1.964	1.926
23	.0226	.9203	.9310	1.048	1.023	1.363	1.376	1.557	1.527
24	.0201	.8195	.8291	.9334	.9105	1.081	1.091	1.235	1.211
25	.0179	.7298	.7383	.8311	.8109	.8575	.8656	.9795	.9603
26	.0159	.6499	.6575	.7401	.7221	.6801	.6864	.7768	.7616
27	.0142	.5787	.5855	.6591	.6430	.5393	.5444	.6160	.6039
28	.0126	.5154	.5214	.5869	.5726	.4277	.4317	.4885	.4789
29	.0113	.4580	.4643	.5227	.5099	.3391	.3422	.3873	.3797
30	.0100	.4087	.4135	.4654	.4541	.3690	.2714	.3072	.3012
31	.0089	.3640	.3683	.4145	.4044	.2134	.2153	.2437	.2389
32	.0080	.3241	.3279	.3691	.3601	.1691	.1707	.1932	.1894
33	.0071	.2887	.2920	.3287	.3207	.1341	.1354	.1532	.1502
34	.0063	.2570	.2600	.2927	.2856	.1063	.1073	.1216	.1192
35	.0056	.2289	.2316	.2606	.2543	.0845	.0853	.0965	.0947
36	.0050	.2039	.2062	.2322	.2265	.0669	.0675	.0764	.0750
37	.0045	.1816	.1837	.2067	.2017	.0531	.0536	.0606	.0594
38	.0040	.1617	.1636	.1841	.1796	.0118	.0424	.0480	.0471
39	.0035	.1440	.1456	.1640	.1600	.0334	.0337	.0381	.0374
40	.0031	.1282	.1297	.1460	.1424	.0268	.0267	.0302	.0297
Spec. grav.		7.828	7.92	8.917	8.70	7.85	7.93	8.96	8.78

The American Wire Gauge is introduced and manufactured by J. R. Brown & Sharpe, of Providence, R. I., and is to be had in the principal hardware stores in the country. It is adopted by most manufacturers of plates and wire, and is now considered the American Standard Gauge.

Birmingham Gauge for Wire, Sheet Iron and Steel.

Thickness by the gauge.	Thickness in inches.	Weight per Square Foot in Pounds. Sheet and Boiler Iron.	Sheet Cast Steel.	Sheet Copper.	Sheet Lead.	Thickness in inches.
No. 0000	0.454	18.267	18.259	20.566	26.75	7 : 16
000	0.425	17.053	17.280	19.252	25.06	27 : 64
00	0.380	15.247	15.451	17.214	22.42	3 : 8
0	0.340	13.7	14.0	15.6	20.06	11 : 32
1	0.300	12.1	12.4	13.8	17.72	5 : 16
2	0.284	11.4	11.7	13.0	16.75	9 : 32
3	0.259	10.4	10.6	11.9	15.26	1 : 4
4	0.238	9.60	9.80	11.0	14.02	7 : 32
5	0.220	8.85	9.02	10.1	12.98	7 : 32
6	0.203	8.17	8.33	9.32	11.98	7 : 32
7	0.180	7.24	7.38	8.25	10.63	3 : 16
8	0.165	6.65	6.78	7.59	9.73	3 : 16
9	0.148	5.96	6.08	6.80	8.72	5 : 32
10	0.134	5.40	5.51	6.16	7.90	5 : 32
11	0.120	4.83	4.93	5.51	7.08	1 : 8
12	0.109	4.40	4.50	5.02	6.42	1 : 8
13	0.095	3.83	3.91	4.37	5.60	3 : 32
14	0.083	3.34	3.41	3.81	4.90	3 : 32
15	0.072	2.90	2.96	3.31	4.25	1 : 16
16	0.065	2.62	2.67	3.00	3.83	1 : 16
17	0.058	2.34	2.39	2.67	3.42	1 : 16
18	0.049	1.97	2.01	2.25	2.90	1 : 16
19	0.042	1.69	1.72	1.93	2.48	3 : 64
20	0.035	1.41	1.42	1.61	2.04	3 : 64
21	0.032	1.29	1.31	1.47	1.89	3 : 64
22	0.028	1.13	1.15	1.29	1.65	1 : 32
23	0.025	1.00	1.02	1.14	1.47	1 : 32
24	0.022	0.885	0.903	1.01	1.30	1 : 32
25	0.020	0.805	0.820	0.918	1.18	1 : 32
26	0.018	0.724	0.738	0.826	1.06	1 : 64
27	0.016	0.644	0.657	0.735	0.945	1 : 64
28	0.014	0.563	0.574	0.642	0.826	
29	0.013	0.523	0.533	0.597	0.767	
30	0.012	0.483	0.493	0.551	0.708	
31	0.010	0.402	0.410	0.480	0.600	
32	0.009	0.362	0.370	0.420	0.532	
33	0.008	0.322	0.328	0.370	0.472	
34	0.007	0.282	0.288	0.323	0.413	
35	0.005	0.230	0.235	0.262	0.309	
36	0.004	0.170	0.173	0.194	0.236	

New English Gauge. See Electro-Dynamics.

Birmingham Gauge for Silver and Gold.

No.	Thick. Inch.	No.	Thick. Inch.	No.	Thick. Inch.	No.	Thick. Inch.	No.	Thick. Inch.	No.	Thick. Inch.
1	.004	7	.015	13	.036	19	.064	25	.095	31	.133
2	.005	8	.016	14	.041	20	.067	26	.103	32	.143
3	.008	9	.019	15	.047	21	.072	27	.113	33	.145
4	.010	10	.024	16	.051	22	.074	28	.120	34	.148
5	.013	11	.029	17	.057	23	.077	29	.120	35	.158
6	.013	12	.034	18	.061	24	.082	30	.126	36	.167

Proportions of Bolts and Nuts. Number of Threads per In.

Diameter of Bolt.						Diameter.	Number of Threads per Inch.	
3 inch.	$4\frac{1}{2}$	$5\frac{1}{4}$	5	$7\frac{1}{16}$	$2\frac{1}{2}$	$\frac{1}{4}$	20	10
$2\frac{3}{4}$	$4\frac{1}{8}$	$4\frac{3}{4}$	$4\frac{1}{2}$	$6\frac{3}{8}$	$2\frac{1}{4}$	$\frac{5}{16}$	18	9
$2\frac{1}{2}$	$3\frac{3}{4}$	$4\frac{3}{8}$	$4\frac{1}{8}$	$5\frac{13}{16}$	2	$\frac{3}{8}$	16	9
$2\frac{1}{4}$	$3\frac{3}{8}$	$3\frac{7}{8}$	$3\frac{3}{4}$	$5\frac{5}{16}$	$1\frac{3}{4}$	$\frac{7}{16}$	14	8
2	3	$3\frac{1}{2}$	$3\frac{3}{8}$	$4\frac{3}{4}$	$1\frac{5}{8}$	$\frac{1}{2}$	12	7
$1\frac{7}{8}$	$2\frac{3}{4}$	$3\frac{1}{4}$	3	$4\frac{3}{16}$	$1\frac{1}{2}$	$\frac{5}{8}$	11	7
$1\frac{3}{4}$	$2\frac{5}{8}$	3	$2\frac{3}{4}$	$3\frac{7}{8}$	$1\frac{3}{8}$	$\frac{3}{4}$	10	6
$1\frac{5}{8}$	$2\frac{1}{2}$	$2\frac{7}{8}$	$2\frac{5}{8}$	$3\frac{11}{16}$	$1\frac{1}{4}$	$\frac{7}{8}$	9	6
$1\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{5}{8}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{1}{8}$	1	8	5
$1\frac{3}{8}$	2	$2\frac{3}{8}$	$2\frac{1}{4}$	$3\frac{3}{16}$	$1\frac{1}{16}$	$1\frac{1}{8}$	7	4
$1\frac{1}{4}$	$1\frac{7}{8}$	$2\frac{1}{8}$	2	$2\frac{7}{8}$	1	$1\frac{1}{4}$	7	$3\frac{1}{2}$
$1\frac{1}{8}$	$1\frac{5}{8}$	$1\frac{7}{8}$	$1\frac{7}{8}$	$2\frac{5}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	6	3
1	$1\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{5}{8}$	$2\frac{5}{16}$	$\frac{7}{8}$	$1\frac{3}{4}$	5	$2\frac{1}{2}$
$\frac{7}{8}$	$1\frac{5}{16}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{8}$	$\frac{3}{4}$	2	$4\frac{1}{2}$	$2\frac{1}{4}$
$\frac{3}{4}$	$1\frac{3}{16}$	$1\frac{3}{8}$	$1\frac{5}{16}$	$1\frac{7}{8}$	$\frac{5}{8}$	$2\frac{1}{2}$	4	2
$\frac{5}{8}$	1	$1\frac{1}{8}$	$1\frac{3}{16}$	$1\frac{11}{16}$	$\frac{1}{2}$	3	$3\frac{1}{2}$	
$\frac{9}{16}$	$\frac{7}{8}$	1	1	$1\frac{7}{16}$	$\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{4}$	
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8}$	$1\frac{1}{4}$	$\frac{7}{16}$	4	3	
$\frac{7}{16}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{1}{16}$	$\frac{3}{8}$	$4\frac{1}{2}$	$2\frac{7}{8}$	
$\frac{3}{8}$	$\frac{7}{16}$	$\frac{5}{8}$	$\frac{8}{4}$	$1\frac{1}{16}$	$\frac{5}{16}$	5	$2\frac{7}{8}$	
$\frac{5}{16}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{5}{16}$	$5\frac{1}{2}$	$2\frac{5}{8}$	
$\frac{1}{4}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{1}{4}$	6	$2\frac{1}{2}$	

The above proportions of bolts and nuts were established by Sir Joseph Whitworth.

Weight in Pounds of Nut and Bolt-Head.

	Diameter of Bolt in Inches.												
Head and Nut.	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3
Hexagon, .	.017	.057	.128	.267	.43	.73	1.1	2.14	3.77	5.62	8.75	17.2	28.8
Square, . .	.021	.070	.164	.321	.553	.882	1.31	2.56	4.42	7.00	10.5	21.	36.4

Proportions of Screw-Threads, Nuts and Boltheads.

Diam. of Screw.	Threads per inch.	Diamet. of core.	Width of flat.	Outside diamet.	Inside diamet.	Diagonal.	height of h'd.
·1/4	20	·185	·0062	·9/16	·1/2	·11/16	·1/4
·5/16	18	·240	·0070	·11/16	·19/32	·13/16	·19/64
·3/8	16	·294	·0078	·25/32	·11/16	·31/32	·11/32
·7/16	14	·344	·0089	·43/48	·25/32	1·1/16	·25/64
·1/2	13	·400	·0096	1·	·7/8	1·1/4	·7/16
·9/16	12	·454	·0104	1·7/64	·31/32	1·5/16	·31/64
·5/8	11	·507	·0113	1·7/32	1·1/16	1·1/2	·17/32
·3/4	10	·620	·0125	1·7/16	1·1/4	1·3/4	·5/8
·7/8	9	·731	·0140	1·21/32	1·7/16	2·1/32	·23/32
1·	8	·837	·0156	1·7/8	1·5/8	2·5/16	·13/16
1·1/8	7	·940	·0180	2·3/32	1·13/16	2·1/2	·29/64
1·1/4	7	1·065	·0180	2·5/16	2·	2·27/32	1·
1·3/8	6	1·160	·0210	2·1/2	2·3/16	3·1/10	1·3/32
1·1/2	6	1·284	·0210	2·3/4	2·3/8	3·3/8	1·3/16
1·5/8	5·1/2	1·389	·0227	2·15/16	2·9/16	3·5/8	1·9/32
1·3/4	5·	1·490	·0250	3·3/16	2·3/4	3·29/32	1·3/8
1·7/8	5·	1·615	·0250	3·13/32	2·15/16	4·3/16	1·15/32
2·	4·1/2	1·712	·0280	3·5/8	3·1/8	4·7/16	1·9/16
2·1/4	4·1/2	1·962	·0280	4·1/16	3·1/2	4·31/32	1·3/4
2·1/2	4·	2·175	·0310	4·1/2	3·7/8	5·1/2	1·15/16
2·3/4	4·	2·425	·0310	4·29/32	4·1/4	6·	2·1/8
3·	3·1/2	2·628	·0357	5·3/8	4·5/8	6·9/16	2·5/16
3·1/4	3·1/2	2·878	·0357	5·3/4	5·	7·1/8	2·1/2
3·1/2	3·1/4	3·100	·0384	6·7/64	5·3/8	7·5/8	2·11/16
3·3/4	3·	3·317	·0410	6·5/8	5·3/4	8·3/16	2·7/8
4·	3·	3·566	·0410	7·3/64	6·1/8	8·11/16	3·1/16
4·1/4	2·7/8	3·798	·0435	7·1/2	6·1/2	9·1/4	3·1/4
4·1/2	2·3/4	4·027	·0460	7·31/32	6·7/8	9·3/4	3·7/16
4·3/4	2·5/8	4·255	·0480	8·3/8	7·1/4	10·9/32	3·5/8
5·	2·1/2	4·480	·0500	8·13/16	7·5/8	10·13/16	3·13/16
5·1/4	2·1/2	4·730	·0500	9·1/4	8·	11·3/8	4·
5·1/2	2·3/8	4·953	·0526	9·11/16	8·3/8	11·29/32	4·3/16
5·3/4	2·3/8	5·203	·0526	10·1/8	8·3/4	12·7/16	4·3/8
6·	2·1/4	5·423	·0555	10·9/16	9·1/8	12·9/10	4·9/16

Englishmen make the angle of the thread 55°, with round top and bottom; whilst in the U. S. we make the angle of the thread 60°, with flat top and bottom, and of the following proportions, which were recommended by a special committee appointed by the Franklin Institute of Philadelphia. For full information see Journal of the Institute, May, 1864, and Jan., 1865.

Notation. All dimensions in inches.

D = outside diameter of screw.
d = diameter of root of thread, or of hole in the nut.
p = pitch of screw.
t = number of threads per inch.
f = flat top and bottom.
o = outside diameter of hexagon nut or bolthead.
i = inside diameter of hexagon, or side of square nut or bolthead.
s = diagonal of square nut or bolthead.
h = height of rough or unfinished bolthead.

The height of finished nut or bolthead is made equal to the diameter D of the screw.*

$$p = \frac{\sqrt{16\,D + 10} - 2{\cdot}909}{16{\cdot}64}. \qquad t = \frac{1}{p}. \qquad s = 1{\cdot}414\,i.$$

$$d = D - \frac{1{\cdot}299}{t}. \qquad i = \frac{3\,D}{2} + \frac{1}{8} \qquad o = 1{\cdot}155\,i. \qquad f = \frac{p}{8}.$$

* Whitworth makes the height of the nut about half the hexagon diameter o.

Weight, Size, Price and Surface of Copper and Brass Tubes,

10 feet long.

Outside Diameter.	Bir. W. Gauge	Weight of tube. Brass.	Weight of tube. Cop.	Price per tube.	Whole Surface.	Outside Diameter.	Bir. W. Gauge	Weight of tube. Brass.	Weight of tube. Cop.	Price per tube.	Whole Surface.
Inches.	No.	Lbs.	Lbs.	$ cts.	Sq. Ft.	Inches.	No.	Lbs.	Lbs.	$ cts.	Sq. Ft.
0.625	18	3.478	3.681	2 30	1.636	2.	14	18.84	19.95	8 00	5.236
0.75	17	4.950	5.241	2 97	1.963	2.125	14	19.07	20.18	8 20	5.563
.8125	17	5.372	5.679	3 00	2.127	2.25	14	21.18	22.42	8 90	5.890
0.875	17	5.775	6.114	3 40	2.290	3.375	14	22.32	23.65	9 45	6.217
.9375	16	6.954	7.362	3 60	2.454	2.5	14	23.53	24.89	9 95	6.544
1.	16	7.418	5.854	3 92	2.618	2.625	14	24.67	26.12	10 45	6.872
1.125	16	8.354	8.835	4 40	2.945	2.75	14	25.83	27.35	11 00	7.200
1.25	15	10.21	10.83	4 70	3.272	3.	13	37.00	39.17	13 70	7.854
1.375	15	11.23	11.91	4 95	3.600	3.25	13	40.00	42.34	14 85	8.508
1.5	15	12.28	13.00	5 20	3.927	3.5	13	43.10	45.61	16 00	9.163
1.625	15	13.30	14.08	5 65	4.254	4.	12	49.39	52.30	17 00	10.47
1.75	14	16.5	17.45	7 00	4.581	4.5	12	55.55	58.8	19 10	11.78
1.812	14	17.08	18.08	7 20	4.745	5.	12	61.44	65.00	21 00	13.08
1.875	14	17.72	18.75	7 50	4.908	6.	11	81.58	86.35	26 00	15.71
1.937	14	18.26	19.32	7 75	5.072	8.	11	108.8	115.0	34 50	20.95

Seamless-Drawn Brass Tubes for Plumbing,

In lengths of 10 feet. Screw-coupling on one end of each length. Price per tube.

Diameters, inches.	5/8	3/4	7/8	1	1 1/4	1 1/2
	$ cts.	$ cts.	$ cts.	$ cts.	$ cts.	$ cts.
Plain tubes, . . .	2 50	3 00	4 50	6 00	7 00	8 00
Tinned tubes, . .	3 00	3 50	5 00	7 00	8 00	9 00

Price of Taps, Dies and Stocks.

Diameters, inches.	5/8	3/4	7/8	1	1 1/4	1 1/2
	$ cts.	$ cts.	$ cts.	$ cts.	$ cts.	$ cts.
Taps, ...	2 50	2 75	3 00	3 50	4 50	6 00
Solid dies, . .	3 50	3 50	3 50	3 50	3 50	4 00

Stocks, $8, net.

Price for Each Extra Coupling.

Diameters, inches.	5/8	3/4	7/8	1	1 1/4	1 1/2
	$ cts.	$ cts.	$ cts.	$ cts.	$ cts.	$ cts.
Straight couplings, .	20	25	35	40	45	50
Elbows, ...	26	35	48	53	65	80
Tees, ...	30	40	55	60	85	1 00
Cross couplings, .	45	60	85	90	1 50	2 20

The prices are only approximate. The price of copper tubes is 12 to 13 per cent. more than of brass.

Brass and copper tubes are manufactured at the American Tube Works, Boston, Mass.; Merchant & Co., 507 Market street, Philadelphia, agents.

Proportionate Tensile Strength of Rolled Iron and Copper,

In pounds per square inch, at different temperatures, Fahr. and Centigrade.

Fahr.	Cent.	Iron.	Copper.	Fahr.	Cent.	Iron.	Copper.
32	0.	55,000	32,800	800	427	51,800	17,200
100	37.7	58,200	32,300	900	483	45,000	14,000
200	93.3	62,800	31,000	1000	540	37,000	11,000
300	149.	65,750	29,500	1200	650	25,000	7,000
400	205.	67,000	27,400	1500	820	16,500	3,000
500	260.	66,000	25,300	2000	1090	7,000	0.0000
600	316.	62,700	23,000	2500	1370	2,500	Fused to
700	370.	57,800	20,100	3000	1650	Fused.	liquid.

Composition Nails, Copper and Iron Rivets.

Gauge.	Composition Nails.			Braziers' Copper Rivets.			Gauge.	Iron Rivets.		
	Thick.	Length.	In 1 lb.	Diameter.	Length.	In 10 lbs.		Diameter.	Length.	In 10 lbs.
No.	Inches.	Inches.	Num	Inches.	Inches.	Num.	No.	Inches.	Inches.	Num.
1	0.04	3/4	290	3/16	1/2	2384	0	3/16	1/2	3280
2	0.05	7/8	260	1/4	1/2	1018	1	1/4	1/2	1276
3	0.06	1 inch.	212	1/4	9/16	983	2	1/4	9/16	1130
4	0.07	1.1/8	201	5/16	9/16	573	3	5/16	9/16	654
5	0.08	1.1/4	199	5/16	5/8	516	4	5/16	5/8	589
6	0.09	1 inch.	190	3/8	7/8	357	5	3/8	7/8	407
7	0.10	1.1/8	184	3/8	15/16	334	6	3/8	15/16	380
8	0.10	1.1/4	168	7/16	1 inch.	210	7	7/16	1 inch.	239
9	0.11	1.1/2	110	1/2	1.3/16	141	8	1/2	1.3/16	160
10	0.11	1.5/8	101	9/16	1.5/16	99.5	9	9/16	1.5/16	112
11	0.12	1.3/4	74	5/8	1.7/16	71.9	10	5/8	1.7/16	81.7
12	0.12	2 inches	64	11/16	1.9/16	53.8	11	11/16	1.9/16	61.3
13	0.13	2.1/4	59	3/4	1.3/4	41.6	12	3/4	1.3/4	47.3
14	0.14	2.1/2	51	13/16	1.13/16	32.8	13	13/16	1.13/16	37.3
15	0.15	2.3/4	43	7/8	2.1/16	26.3	14	7/8	2.1/16	30.
16	0.16	3 inches	35	1 inch.	2.3/8	16.7	15	1 inch.	2.3/8	19.

Length in Inches of Penny Nails.

1 in.	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	4	4.25	5	5.5	6
2 d.	*3 d.*	*4 d.*	*5 d.*	*6 d.*	*7 d.*	*8 d.*	*9 d.*	10	12	16	20	30	40	50	60

Sheet Zinc and Iron.

SHEET ZINC.
Size 84 in. by 24, 28, 32, 36 and 40 inches.

Zinc gauge.	Width of Sheet. 24	32	40	Bir. W. gauge.
No.	Pounds.	Pounds.	Pounds.	No.
8	6.23	9.68	12.1	28
9	7.20	11.2	14.0	27
10	8.00	12.4	15.6	26
11	8.90	13.8	17.3	25
12	10.1	15.7	19.7	24½
13	11.1	17.3	21.6	23
14	12.4	19.3	24.1	22
15	16.2	25.2	31.6	21
16	17.4	27.1	33.9	20
18	21.9	34.0	42.6	18

RUSSIA SHEET IRON.
Size 28 × 56 in. = 10.88 sq. feet.

Russian gauge.	Weight per Sheet.	Weight per Sq. Ft.	Bir. W. gauge.
No.	Pounds.	Pounds.	No.
7	6.25	0.574	29
8	7.25	0.666	28
9	8.	0.735	27
10	9.	0.827	26
11	10.	0.918	25
12	10.75	0.987	24½
13	11.75	1.08	24
14	12.5	1.15	23¼
15	13.5	1.24	22⅜
16	14.5	1.33	21½

To find the Weight of Castings, by the Weight of Pine Patterns.

RULE.—

Multiply the weight of the Pattern by { 12 for Cast Iron, 13 for Brass, 19 for Lead, 12.2 for Tin, 11.4 for Zinc, } and the product is the weight of the Castings.

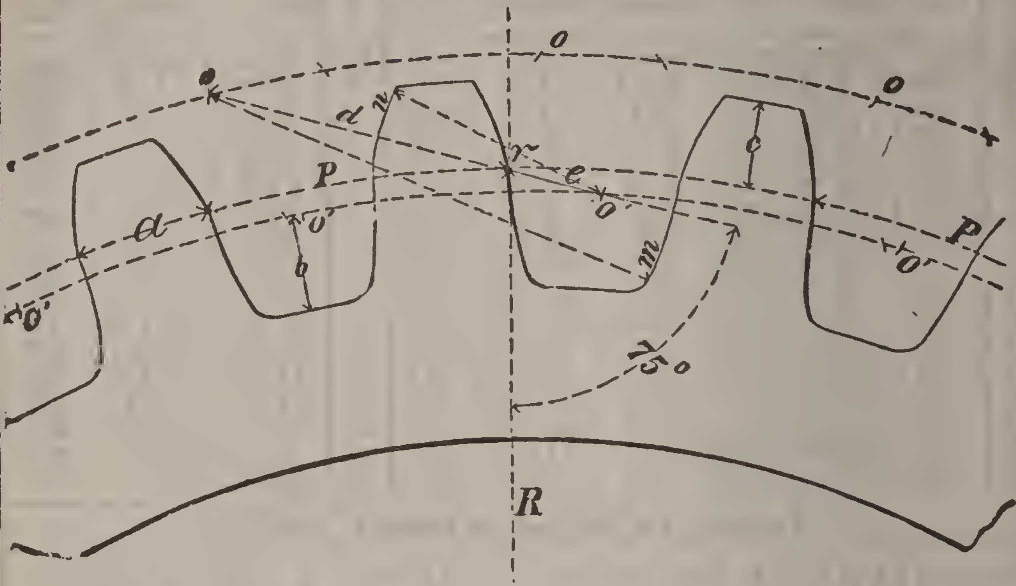
Reductions for Round Cores and Core-prints.

Rule. Multiply the square of the diameter by the length of the Core in inches, and the product by 0.017, is the weight of the pine core, to be deducted from the weight of the pattern.

Shrinking of Castings.

Pattern-Makers' Rule should be for { Cast Iron, . $\frac{1}{8}$ Brass, . . $\frac{3}{16}$ Lead, . . . $\frac{1}{8}$ Tin, . . . $\frac{1}{12}$ Zinc, . . . $\frac{3}{16}$ } of an inch longer per linear foot.

GEARING.



Notation.

P = pitch,—the distances between the centres of two teeth in the pitch circle.

D = diameter
C = circumference
M = number of teeth
N = number of revolutions
} of the wheel.

d = diameter
c = circumference
m = number of teeth
n = number of revolutions
} of the pinion.

Pitch: $P = \frac{C}{M}$ . . 1 ; $P = \frac{\pi D}{M}$ . . 2

No. of teeth: $M = \frac{C}{P}$ . . 3 ; $M = \frac{\pi D}{P}$ . . 4

Circum.: $C = P M$ . . 5 ; $C = \pi D$ . . 6

Diameter: $D = \frac{P M}{\pi}$ . . 7 ; $D = \frac{C}{\pi}$ . . 8

$$D : d = C : c = M : m = n : N$$

Example 1. A wheel of $D = 40$ inches in diameter, is to have $M =$ 75 teeth. Required the pitch $P = ?$

Formula 2. Pitch $P = \frac{3{\cdot}14 \times 40}{75} = 1{\cdot}66$ inches nearly.

Example 2. The pitch of teeth in a wheel, is to be $P = 1{\cdot}71$ inches, and having $M = 48$ teeth. Required the diameter $D = ?$ of the wheel.

Formula 7. Diam. D of pitch circle $= \frac{1{\cdot}71 \times 48}{3{\cdot}14} = 26{\cdot}14$ in.

Construction of Teeth for Wheels.

Draw the radius $R\ r$, and pitch circle $P\ P$. Through r draw the line $o\ o'$ at an angle of 75° to the radius $R\ r$.

Half the angle between two teeth in the
- wheel, $v = \frac{180}{M}$. 1
- pinion, $V = \frac{180}{m}$. 2

$$D : d = sin.\ V : sin.\ v.$$

Diameter of the
- wheel, $D = \frac{d\ sin.\ V}{sin.\ v}$. 3
- pinion, $d = \frac{D\ sin.\ v}{sin.\ V}$. 4

Pitch (chord) of teeth in the pitch circle
- wheel, $P = D\ sin.\ v$. . . . 5
- pinion, $P = d\ sin.\ V$. . . . 6

Approximate pitch in the wheel $P = 0{\cdot}028\ D$. . . . 7

Number of teeth
- wheel, $M = \frac{3{\cdot}14\ D}{P}$. 8
- pinion, $m = \frac{d\ M}{D}$. . . . 9

Thickness of tooth, $a = 0{\cdot}46\ P$* 10

Bottom clearance, $b = 0{\cdot}4\ P$. 11

Depth to pitch line, $c = 0{\cdot}3\ P$. 12

Distance $r\ o$, $d = \frac{P\ (m + 6)}{2\ (m - 11)}$. 13

Distance $r\ o'$, $e = 0{\cdot}11\ P \sqrt[3]{m}$ 14

* If a wheel of more than 80 teeth is to gear a pinion of less than 20 teeth, and the wheel and pinion are of the same kind of materials; take the thickness

of the tooth in the
- wheel, $a = P\left(0{\cdot}42 + \frac{m}{700}\right)$. . 15
- pinion, $a = 0{\cdot}5\ P\left(1 - \frac{m}{350}\right)$. . 16

A rack is to be considered as a wheel of 200 teeth.

Example with Plate I.

Example. A wheel of $D = 48$ inches diameter is to gear a pinion about 8 revolutions to 1. Required a complete construction of the gearing?

Approximate pitch $P = 0{\cdot}028 \times 48 = 1{\cdot}34$ in. - - - 7

Number of teeth in the
- wheel, $M = \dfrac{3{\cdot}14 \times 48}{1{\cdot}34} = 112.$ - 8
- pinion, $m = \dfrac{112}{8} = 14$ - - 9

Half the angle between two teeth in the
- wheel, $v = \dfrac{180}{112} = 1°36'.\ sin = 0{\cdot}028.$ 1
- pinion $V = \dfrac{180}{14} = 12°51'.\ sin = 0{\cdot}2224.$ 2

Diameter of pinion $d = \dfrac{48 \times 0{\cdot}028}{0{\cdot}2224} = 6{\cdot}043$ in. - 4

Draw the pitch circle for the wheel and pinion so that they are tangent to one another at r on a straight line between the centres of the circles.

Pitch in the gearing $P = 48 \times 0{\cdot}028 = 1{\cdot}344$ in. - 5

Take this chordial pitch in a pair of compasses, and set it off in the pitch circles.

Thickness of tooth
- wheel $a = 1{\cdot}344\left(0{\cdot}42 + \dfrac{14}{700}\right) = 0{\cdot}592$ in. 15
- pinion $a = 0{\cdot}5 \;\; 1{\cdot}344\left(1 - \dfrac{14}{350}\right) = 0{\cdot}645$ in. 16

Set off the thickness of tooth in the corresponding pitch circles.

Bottom clearance $b = 0{\cdot}4 \times 1{\cdot}344 = 0{\cdot}5376$ in. - 11

Depth to pitch line $c = 0{\cdot}3 \;\; 1{\cdot}344 = 0{\cdot}4032$ in. - - 12

Distances $r\,o$ and $r\,o'$ in the wheel
- $d = \dfrac{1{\cdot}344(112 + 6)}{2(112 - 11)} = 0{\cdot}7851$ in. - 13
- $e = 0{\cdot}11 \times 1{\cdot}344 \sqrt{112} = 0{\cdot}7126$ in. 14

Set off these distances on the line $o\,o'$ from r,—d beyond and e within the pitch circle for the wheel; then o is the centre and $o\,m$ radius for the flank m. o' the centre and $o'\,n$ radius for the face n. Draw circles through o and o' concentric with the pitch circle of the wheel.

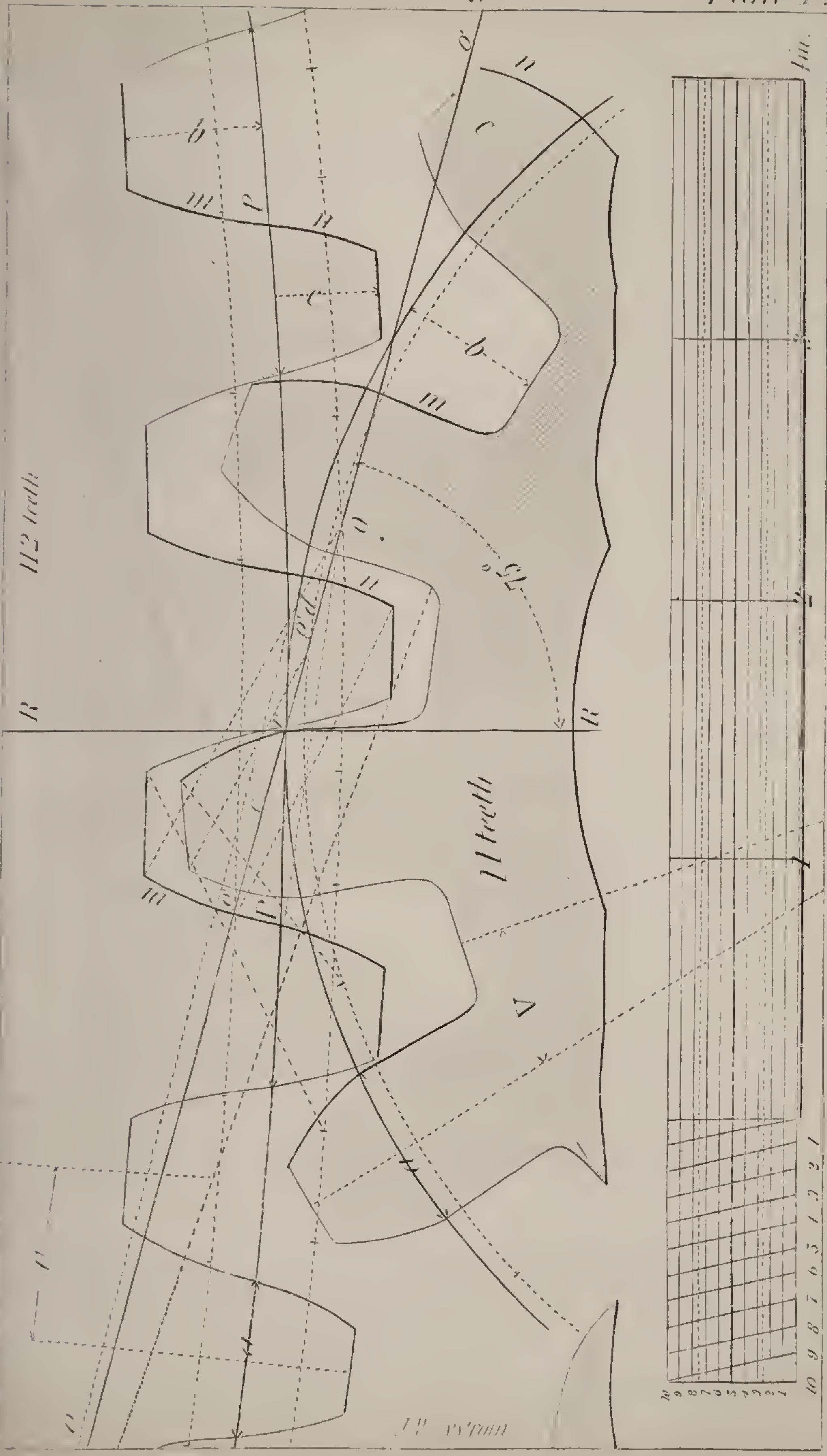
Distances $r\,o$ and $r\,o'$ in the pinion
- $d = \dfrac{1{\cdot}344(14 + 6)}{2(14 - 11)} = 4{\cdot}48$ in. - 13
- $e = 0{\cdot}11 \times 1{\cdot}344 \sqrt[3]{14} = 0{\cdot}356$ in. 14

Proceed with the pinion in a similar manner as with the wheel.

On the plate is a scale of inches and decimals, which will be convenient for the above measurements.

Gearing.

Plate I.



On the Strength of Teeth in Cast-iron Wheels.

The strength and durability of cast-iron teeth require that they shall transmit a force of 80 lbs. per inch of pitch and per inch breadth of face.

Let HP = indicated horse-power transmitted.
" D = diameter of pitch circle in inches.
" N = number of turns of wheel per minute.
" F = stress on one tooth in lbs.

We have—

$$F = 126050 \frac{HP}{D\,N}.$$

Having thus found F, we can assume the pitch and find the breadth of face, or we can assume the breadth of face and find the pitch.

Williams & Brown, Tenth and Chestnut streets, Phila., Pa., have for sale, at 50 cents each, Epicycloidal and Involute Odontographs devised so as to furnish without formulæ all the data required to correctly lay out both forms of teeth of wheels. The writer's methods do not agree with Mr. Nystrom's.—W. D. M.

To find the Diameter of Axles and Shafts.

Notation.

d = diameter in inches, in the bearing; and the length of the bearing $1.5d$.
W = weight in pounds, acting in the bearing.

Empirical Formulas.

Water-wheels.
$$d = \frac{\sqrt{W}}{18} \text{ of cast iron.} \qquad d = \frac{\sqrt{W}}{21} \text{ of wrought iron.}$$

Common Machinery in good order.
$$d = \frac{\sqrt{W}}{24} \text{ of cast iron.} \qquad d = \frac{\sqrt{W}}{28} \text{ of wrought iron.}$$

Example 1. A water-wheel weighs 58,680 pounds, and is supported in two bearings. Required the diameter of the wheel axles? The weight acting in each bearing will be 58680 : 2 = 29340 pounds, and

$$\text{diameter } d = \frac{\sqrt{29340}}{21} = 8.15 \text{ inches of wrought iron.}$$

Example 2. Required the diameter of an axle in a wheel, when the weight = 4864 pounds? If the wheel is supported in two bearings $W = 4864 : 2 = 2432$ pounds.

$$\text{diameter } d = \frac{\sqrt{2432}}{28} = 1.76 \text{ inches of wrought iron.}$$

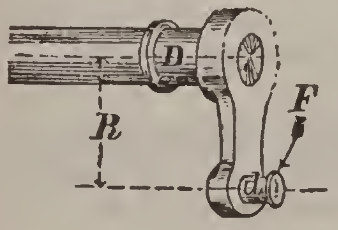
Example 3. The pressure on the steam piston in a working beam engine is 25,000 pounds. Required the diameter of the beam journals?

Rational Method.

(See *The Relative Proportions of the Steam Engine*, Marks, J. B. Lippincott Co.)

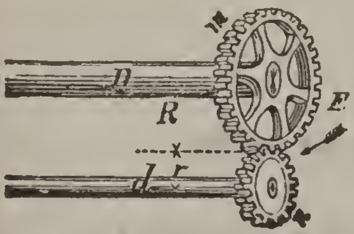
$$\text{Projected area} = \frac{25000}{500} = 50 \text{ sq. inches for end bearings.}$$

$$\text{Projected area} = \frac{50000}{500} = 100 \text{ sq. inches for centre bearings.}$$



$$D = \frac{\sqrt[3]{FR}}{4} = 5\sqrt[3]{\frac{H}{n}}$$

n = number of turns per minute.
D = inches wrought iron.
R = radius of crank in feet.
F = force from the steam piston, lbs.



$$D : d = \sqrt[3]{R} : \sqrt[3]{r}$$

$$D = 4{\cdot}35\sqrt[3]{\frac{H}{n}}$$

H = horse-power transmitted.
n = number revolutions per minute.

When an axle or shaft not only serves as a fulcrum, but effect is transmitted by the act of twisting it, the diameter is to be calculated as follows.

Example 4. The pressure on the piston in a steam engine is $F = 45{,}600$ pounds, applied direct on a crank of $R = 3$ feet radius. Required the diameter of the shaft and crank pin?

$$\text{Diameter of the shaft } D = \frac{\sqrt[3]{45600 \times 3}}{4} = 12{\cdot}9 \text{ inches.}$$

For diameter of crank-pin, see *The Relative Proportions of the Steam-Engine.*

Example 5. A steam engine of 368 horse-power is to make 32 revolutions per minute. Required the diameter of the main shaft?

$$\text{Diameter } D = 5\sqrt[3]{\frac{368}{32}} = 11\tfrac{1}{4} \text{ inches.}$$

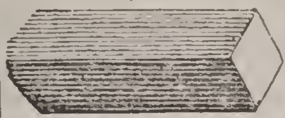
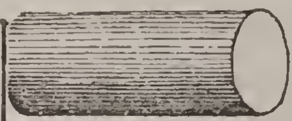
Example 6. A cog wheel of $R = 6{\cdot}5$ feet radius is to gear with a pinion of $r = 1{\cdot}25$ feet radius, and to transmit an effect of 231 horse-power with 42 revolutions per minute. Required the diameter of the wheel and pinion shafts? The force F is acting uniformly at the periphery.

$$\text{Diameter of wheel shaft } D = 4{\cdot}35\sqrt[3]{\frac{231}{42}} = 7{\cdot}66 \text{ inches}$$

$$D : d = \sqrt[3]{R} : \sqrt[3]{r}$$

$$\text{Diameter of pinion shaft } d = 7{\cdot}66\sqrt[3]{\frac{1{\cdot}25}{6{\cdot}5}} = 4{\cdot}41 \text{ inches.}$$

Coefficients for Capacity and Weight,


Names of Substances.	*FFF.*	*Fii.*	*iii.*	*FF²*.	*Fi²*.	*ii²*.	*F³*.	*i³*.
Cubic inches, - -	1728	12	1	1356	9·42	0·78	903·7	0·523
Cubic feet, - - -	1	·..694	·.....58	0·785	·..549	·.....44	0·523	·.....3
Gallons, - - - -	7·476	0·052	·...433	5·868	·.408	·.....34	3·91	·..226
Water, fresh, - -	62·5	0·433	0·036	49	0·34	·.283	32·7	0·019
Water, salt, - - -	64·3	0·445	0·037	50·4	0·35	0·029	33·6	0·02
Oil, - - - - - -	57·5	0·4	0·033	45·1	0·313	0·026	30	0·017
Cast-iron, - - -	450	3·12	0·26	353	2·45	0·204	235	0·136
Wrought-iron, - -	487	3·37	0·281	382	2·65	0·221	255	0·147
Steel, - - - - -	490	3·4	0·283	385	2·67	0·222	257	0·149
Brass, - - - - -	532	3·68	0·307	417	2·9	0·241	278	0·161
Tin, - - - - -	456	3·16	0·263	358	2·48	0·207	239	0·138
Lead, - - - - -	710	4·92	0·41	557	3·87	0·322	371	0·215
Zinc, - - - - -	440	3·05	0·254	345	2·4	0·2	230	0·133
Copper, - - - -	556	3·85	0·321	436	3·03	0·252	291	0·168
Mercury, . - - -	850	5·9	0·491	666	4·63	0·385	445	0·257
Stone, common, -	156	1·08	0·09	122	0·85	0·071	82	0·047
Clay, - - - - -	135	0·936	0·078	106	0·735	0·061	70	0·04
Earth, compact, -	127	0·88	0·0733	99	0·692	0·058	66	0·038
Earth, loose, - -	95	0·66	0·055	74	0·517	0·043	50	0·029
Oak, dry, - - -	58	0·4	0·033	44	0·316	0·026	30	0·017
Pine, - - - - -	30	0·208	0·017	24	0·163	0·014	16	0·009
Mahogany, - - -	66	0·457	0·038	52	0·36	0·03	34	0·02
Coal, stone, - - -	54	0·375	0·031	42	0·294	0·024	28·2	0·016
Charcoal, - - - -	27·5	0·19	0·016	21	0·15	0·012	14·4	0·008

To Find the Weight and Capacity by this Table

RULE. The product of the dimensions in feet or in inches, as noted in the columns, multiplied by the tabular coefficient, is the capacity of the solid, or weight in pounds avoirdupois.

Example 1. A cistern is 6 feet long, 27 inches wide, and 20 inches deep. How many gallons of liquid can be contained in it?

$$6 \times 27 \times 20 \times 0{\cdot}052 = 168{\cdot}48 \text{ gallons.}$$

Example 2. A cast-iron cylinder is 4·5 feet long, and 7·5 inches diameter. Required the weight of it?

$$4{\cdot}5 \times 7{\cdot}5^2 \times 2{\cdot}45 = 620 \text{ pounds.}$$

Preliminary Note on the Pitch of Gearing.

The Reviser of this Pocket-Book does not agree with Mr. Nystrom as to the necessity or advantage in the use of "chordal pitch."

Diametral pitch affords a very easy method of computing the teeth of wheels, and should be used as much as possible.

The tables are left to the judgment of the reader.

STANDARD PITCH OF GEAR-WHEELS.

The difficulty in finding cog-wheel patterns made at different establishments to gear correctly into one another is well known, and much time and money is lost for the want of a standard scale of pitch in gearings. The pitch of teeth in a cog-wheel should always be understood to mean the *chord-pitch*, and not the arc-pitch, because equal arc-pitch in wheels of widely different diameters will not gear well.

The pitch of gear-wheels should be even measures of the inch and binary fractions thereof, and the number of teeth and diameter of pitch-circle should be regulated accordingly.

The following pitch-table is offered or proposed as standard, in which the first column varies with sixteenths of an inch from 0 to 1 inch, with eighths from 1 to 3 inches, and with quarters from 3 to 7 inches. The pitch of wheels from $\frac{1}{16}$ to 7 inches should not be made of any other measure than of those in the table, and the fractions with the most decimals should be avoided as much as possible:

Standard Pitch for Gear-Wheels.

Pitch from $\frac{1}{16}$ to 1 inch.	Pitch from 1 to 3 inches.	Pitch from 3 to 7 inches.
Binary Decimals.	1 in. Decimals.	3 in. Decimals.
$\frac{1}{16}$ = 0·0625	$1\frac{1}{8}$ = 1·125	$3\frac{1}{4}$ = 3·25
$\frac{1}{8}$ = 0·125	$1\frac{1}{4}$ = 1·25	$3\frac{1}{2}$ = 3·5
$\frac{3}{16}$ = 0·1875	$1\frac{3}{8}$ = 1·375	$3\frac{3}{4}$ = 3·75
$\frac{1}{4}$ = 0·25	$1\frac{1}{2}$ = 1·5	4 in.
$\frac{5}{16}$ = 0·3125	$1\frac{5}{8}$ = 1·625	$4\frac{1}{4}$ = 4·25
$\frac{3}{8}$ = 0·375	$1\frac{3}{4}$ = 1·75	$4\frac{1}{2}$ = 4·5
$\frac{7}{16}$ = 0·4375	$1\frac{7}{8}$ = 1·875	$4\frac{3}{4}$ = 4·75
$\frac{1}{2}$ = 0·5	2 in.	5 in.
$\frac{9}{16}$ = 0·5625	$2\frac{1}{8}$ = 2.125	$5\frac{1}{4}$ = 5·25
$\frac{5}{8}$ = 0·625	$2\frac{1}{4}$ = 2·25	$5\frac{1}{2}$ = 5·5
$\frac{11}{16}$ = 0·6875	$2\frac{3}{8}$ = 2·375	$5\frac{3}{4}$ = 5·75
$\frac{3}{4}$ = 0·75	$2\frac{1}{2}$ = 2·5	6 in.
$\frac{13}{16}$ = 0·8125	$2\frac{5}{8}$ = 2·625	$6\frac{1}{4}$ = 6·25
$\frac{7}{8}$ = 0·875	$2\frac{3}{4}$ = 2·75	$6\frac{1}{2}$ = 6·5
$\frac{15}{16}$ = 0·9375	$2\frac{7}{8}$ = 2.875	$6\frac{3}{4}$ = 6·75
1 in.	3 in.	7 in.

The width of the face should be two and a half the pitch.

The following table contains the proportions of number of teeth, diameter, and chord-pitch of wheels from 6 to 250 teeth. The first column is the number of teeth, the second is the diameter when the chord-pitch is unity, and the third column is the chord-pitch when the diameter is the unity.

Example 1.—What diameter is required for a wheel of 45 teeth and chord-pitch $1\frac{3}{4}$ inches? Opposite 45 teeth we find the diameter.

$$14.3356 \times 1.75 = 25.0874 \text{ inches in pitch-line.}$$

Example 2.—A wheel of 62.35 inches diameter in the pitch-line has 198 teeth. What is the chord-pitch?

$$\text{Pitch} = 62.35 \times 0.015866 = 0.989 \text{ of an inch.}$$

That wheel will not work with the standard gear.

The number of teeth multiplied by the pitch gives the length of the pitch-polygon and *not* the pitch-circle. The difference between the length of the two pitch-lines is greater the less the number of teeth in the wheels. For 250 teeth and one-inch pitch the difference is only $\frac{8}{10000}$ part of an inch in the whole pitch-line.

For properly-constructed *cut*-gearing there should be no clearance between the teeth, as shown in Plate I., but the thickness a of the teeth should be half the pitch.

No. Teeth.	Diameter when $P=1$.	Pitch when $D=1$.	No. Teeth.	Diameter when $P=1$.	Pitch when $D=1$.	No. Teeth.	Diameter when $P=1$.	Pitch when $D=1$.	No. Teeth.	Diameter when $P=1$.	Pitch when $D=1$.
6	2.0000	.50000	66	21.016	.04758	126	40.111	.02493	186	59.208	.01689
7	2.3068	.43358	67	21.334	.04687	127	40.429	.02473	187	59.527	.01680
8	2.6131	.38268	68	21.652	.04618	128	40.748	.02454	188	59.845	.01671
9	2.9238	.34202	69	21.970	.04552	129	41.066	.02435	189	60.163	.01662
10	3.2361	.30902	70	22.289	.04486	130	41.384	.02416	190	60.482	.01653
11	3.5490	.28177	71	22.607	.04423	131	41.702	.02398	191	60.800	.01645
12	3.8637	.25882	72	22.925	.04361	132	42.021	.02380	192	61.118	.01636
13	4.1785	.23932	73	23.243	.04307	133	42.330	.02362	193	61.436	.01628
14	4.4940	.22252	74	23.562	.04242	134	42.657	.02344	194	61.755	.01619
15	4.8097	.20791	75	23.880	.04187	135	42.976	.02327	195	62.073	.01611
16	5.1259	.19509	76	24.198	.04131	136	43.294	.02310	196	62.391	.01603
17	5.4423	.18374	77	24.516	.04091	137	43.612	.02293	197	62.710	.01595
18	5.7588	.17365	78	24.835	.04026	138	43.931	.02276	198	63.028	.01587
19	6.0756	.16460	79	25.153	.03976	139	44.250	.02260	199	63.346	.01579
20	6.3925	.15643	80	25.471	.03926	140	44.567	.02244	200	63.665	.01571
21	6.7095	.14904	81	25.789	.03878	141	44.885	.02228	201	63.983	.01563
22	7.0266	.14231	82	26.108	.03830	142	45.204	.02212	202	64.301	.01555
23	7.3338	.13636	83	26.426	.03784	143	45.522	.02197	203	64.620	.01547
24	7.6613	.13053	84	26.744	.03739	144	45.840	.02182	204	64.938	.01540
25	7.9787	.12533	85	27.062	.03695	145	46.158	.02167	205	65.256	.01532
26	8.2962	.12054	86	27.381	.03652	146	46.477	.02152	206	65.575	.01525
27	8.6138	.11609	87	27.699	.03611	147	46.795	.02137	207	65.893	.01517
28	8.9315	.11196	88	28.017	.03569	148	47.113	.02122	208	66.211	.01510
29	9.2493	.10811	89	28.335	.03529	149	47.432	.02108	209	66.529	.01503
30	9.5668	.10453	90	28.654	.03490	150	47.750	.02094	210	66.848	.01496
31	9.8845	.10117	91	28.972	.03452	151	48.068	.02080	211	67.166	.01488
32	10.202	.09800	92	29.290	.03414	152	48.386	.02067	212	67.484	.01482
33	10.520	.09506	93	29.608	.03377	153	48.705	.02553	213	67.802	.01475
34	10.838	.09226	94	29.927	.03341	154	49.023	.02039	214	68.121	.01470
35	11.156	.08964	95	30.245	.03306	155	49.341	.02029	215	68.439	.01461
36	11.474	.08716	96	30.563	.03272	156	49.660	.02014	216	68.757	.01454
37	11.792	.08480	97	30.881	.03238	157	49.978	.02001	217	69.076	.01448
38	12.110	.08257	98	31.200	.03205	158	50.296	.01988	218	69.394	.01441
39	12.427	.08049	99	31.518	.03173	159	50.614	.01976	219	69.712	.01434
40	12.745	.07846	100	31.836	.03141	160	50.933	.01963	220	70.031	.01428
41	13.064	.07653	101	32.154	.03100	161	51.251	.01951	221	70.349	.01421
42	13.382	.07476	102	32.473	.03079	162	51.569	.01939	222	70.667	.01415
43	13.700	.07299	103	32.791	.03049	163	51.888	.01927	223	70.985	.01409
44	14.018	.07134	104	33.109	.03021	164	52.206	.01915	224	71.304	.01402
45	14.336	.06976	105	33.427	.02992	165	52.524	.01904	225	71.622	.01396
46	14.654	.06826	106	33.745	.02963	166	52.842	.01892	226	71.940	.01390
47	14.972	.06679	107	34.064	.02936	167	53.161	.01881	227	72.259	.01384
48	15.290	.06540	108	34.382	.02908	168	53.479	.01870	228	72.577	.01378
49	15.608	.06407	109	34.700	.02882	169	53.797	.01859	229	72.895	.01372
50	15.926	.06279	110	35.018	.02856	170	54.116	.01848	230	73.213	.01366
51	16.244	.06156	111	35.337	.02830	171	54.434	.01837	231	73.532	.01360
52	16.562	.06038	112	35.655	.02805	172	54.752	.01826	232	73.850	.01354
53	16.880	.05925	113	35.973	.02780	173	55.070	.01816	233	74.168	.01348
54	17.198	.05815	114	36.292	.02755	174	55.389	.01805	234	74.487	.01342
55	17.517	.05709	115	36.610	.02731	175	55.707	.01795	235	74.805	.01337
56	17.835	.05607	116	36.928	.02708	176	56.025	.01785	236	75.123	.01331
57	18.153	.05509	117	37.246	.02685	177	56.344	.01775	237	75.442	.01325
58	18.471	.05414	118	37.565	.02662	178	56.662	.01765	238	75.760	.01320
59	18.789	.05322	119	37.883	.02640	179	56.980	.01755	239	76.078	.01314
60	19.107	.05234	120	38.201	.02618	180	57.299	.01745	240	76.396	.01309
61	19.425	.05148	121	38.520	.02596	181	57.617	.01736	241	76.715	.01303
62	19.744	.05065	122	38.838	.02575	182	57.935	.01726	242	77.033	.01298
63	20.062	.04982	123	39.156	.02554	183	58.253	.01717	243	77.351	.01293
64	20.380	.04907	124	39.475	.02533	184	58.572	.01707	244	77.670	.01287
65	20.698	.04831	125	39.793	.02513	185	58.890	.01698	245	77.988	.01282

Number Teeth.	Chordal Pitch of Wheel or Pinion.							
	⅛	¼	⅜	½	⅝	¾	⅞	1 inch.
6	0.2500	0.5000	0.7500	1.0000	1.2500	1.5000	1.7500	2.0000
7	0.2783	0.5767	0.8550	1.1534	1.4317	1.7301	2.0085	2.3068
8	0.3266	0.6532	0.9799	1.3065	1.6332	1.9597	2.2864	2.6131
9	0.3654	0.7309	1.0964	1.4619	1.8264	2.1928	2.5583	2.9238
10	0.4015	0.8090	1.2136	1.6180	2.0225	2.4270	2.8315	3.2361
11	0.4436	0.8872	1.3309	1.7745	2.2182	2.6617	3.1053	3.5490
12	0.4829	0.9659	1.4489	1.9318	2.4148	2.8977	3.3807	3.8637
13	0.5223	1.0446	1.5670	2.0892	2.6116	3.1338	3.6562	4.1785
14	0.5617	1.1235	1.6852	2.2470	2.8087	3.3705	3.9323	4.4940
15	0.6012	1.2024	1.8037	2.4048	3.0061	3.6072	4.2085	4.8097
16	0.6407	1.2814	1.9222	2.5629	3.2037	3.8443	4.4851	5.1259
17	0.6803	1.3605	2.0408	2.7211	3.4014	4.0816	4.7620	5.4423
18	0.7198	1.4397	2.1596	2.8794	3.5992	4.3191	5.0390	5.7588
19	0.7594	1.5189	2.2783	3.0378	3.7973	4.5567	5.3162	6.0756
20	0.7991	1.5981	2.3972	3.1962	3.9953	4.7943	5.5934	6.3925
21	0.8387	1.6774	2.5160	3.3547	4.1934	5.0320	5.8708	6.7095
22	0.8783	1.7566	2.6350	3.5133	4.3916	5.2699	6.1483	7.0266
23	0.9167	1.8334	2.7502	3.6669	4.5836	5.5003	6.4171	7.3338
24	0.9576	1.9153	2.8730	3.8306	4.7883	5.7459	6.7036	7.6613
25	0.9973	1.9946	2.9920	3.9893	4.9866	5.9839	6.9813	7.9787
26	1.0370	2.0740	3.1111	4.1481	5.1851	6.2221	7.2592	8.2962
27	1.0767	2.1534	3.2302	4.3069	5.3836	6.4603	7.5371	8.6138
28	1.1164	2.2328	3.3493	4.4657	5.5822	6.6985	7.8150	8.9315
29	1.1561	2.3123	3.4685	4.6246	5.7808	6.9369	8.0931	9.2493
30	1.1958	2.3917	3.5875	4.7834	5.9792	7.1751	8.3710	9.5668
31	1.2355	2.4711	3.7067	4.9122	6.1778	7.4133	8.6488	9.8845
32	1.2753	2.5506	3.8258	5.1012	6.3764	7.6516	8.9270	10.2023
33	1.3150	2.6300	3.9450	5.2600	6.5750	7.8900	9.2051	10.5201
34	1.3547	2.7095	4.0642	5.4190	6.7737	8.1285	9.4833	10.8380
35	1.3944	2.7889	4.1834	5.5778	6.9723	8.3667	9.7612	11.1557
36	1.4342	2.8684	4.3027	5.7368	7.1711	8.6052	10.0395	11.4737
37	1.4739	2.9479	4.4219	5.8958	7.3698	8.8437	10.3177	11.7917
38	1.5137	3.0274	4.5411	6.0548	7.5685	9.0812	10.5949	12.1096
39	1.5534	3.1068	4.6603	6.2137	7.7672	9.3205	10.8740	12.4275
40	1.5932	3.1864	4.7795	6.3728	7.9659	9.5590	11.1522	12.7455
41	1.6329	3.2659	4.8988	6.5318	8.1647	9.8077	11.4407	13.0636
42	1.6727	3.3454	5.0181	6.6908	8.3635	10.0362	11.7089	13.3816
43	1.7124	3.4249	5.1373	6.8498	8.5622	10.2747	11.9872	13.6996
44	1.7522	3.5044	5.2567	7.0088	8.7611	10.5132	12.2655	14.0177
45	1.7919	3.5839	5.3759	7.1678	8.9598	10.7517	12.5437	14.3357
46	1.8317	3.6634	5.4952	7 3269	9.1586	10.9903	12.8221	14.6538
47	1.8714	3.7429	5.6144	7.4859	9.3573	11.2288	13.1003	14.9718
48	1.9112	3.8224	5.7337	7.6449	9.5561	11.4673	13.3786	15.2898
49	1.9509	3.9019	5.8529	7.8039	9.7549	11.7058	13.6568	15.6079
50	1.9907	3.9815	5.9722	7.9630	9.9537	11.9445	13.9352	15.9260
51	2.0305	4.0610	6.0916	8.1220	10.1525	12.1830	14.2141	16.2441
52	2.0702	4.1405	6.2108	8.2811	10.3513	12.4216	14.4919	16.5622
53	2.1105	4.2210	6.3315	8.4420	10.5525	12.6630	14.7735	16.8840
54	2.1498	4.2996	6.4495	8.5992	10.7491	12.8988	15.0487	17.1985
55	2.1895	4.3791	6.5687	8.7583	10.9479	13.1374	15.3270	17.5167
56	2.2293	4.4587	6.6881	8.9174	11.1468	13.3761	15.6055	17.8349
57	2.2691	4.5382	6.8075	9.0765	11.3456	13.6147	15.8839	18.1530
58	2.3088	4.6177	6.9266	9.2355	11.5444	13.8532	16.1621	18.4711
59	2.3486	4.6973	7.0459	9.3946	11.7432	14.0919	16.4406	18.7892
60	2.3884	4.7768	7.1653	9.5536	11.9421	14.3304	16.7189	19.1073
61	2.4282	4.8563	7.2845	9.7127	12.1409	14.5690	16.9972	19.4255
62	2.4679	4.9359	7.4038	9.8718	12.3397	14.8077	17.2757	19.7436
63	2.5077	5.0154	7.5232	10.0309	12.5387	15.0463	17.5541	20.0618
64	2.5474	5.0949	7.6424	10.1899	12.7374	15.2848	17.8323	20.3799
65	2.5872	5.1745	7.7617	10.3490	12.9362	15.5235	18.1108	20.6980

Number Teeth.	Chordal Pitch of Wheel or Pinion. $\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1 inch.
66	2.6270	5.2540	7.8810	10.5081	13.1351	15.7621	18.3891	21.0161
67	2.6668	5.3336	8.0004	10.6672	13.3340	16.0008	18.6675	21.3343
68	2.7065	5.4131	8.1196	10.8262	13.5327	16.2393	18.9459	21.6524
69	2.7463	5.4926	8.2490	10.9853	13.7316	16.4779	19.2242	21.9705
70	2.7861	5.5722	8.3583	11.1444	13.9305	16.7166	19.5026	22.2887
71	2.8258	5.6518	8.4776	11.3035	14.1293	16.9553	19.7812	22.6070
72	2.8656	5.7313	8.5970	11.4627	14.3283	17.1940	20.0597	22.9253
73	2.9054	5.8109	8.7163	11.6218	14.5272	17.4327	20.3381	23.2435
74	2.9452	5.8904	8.8356	11.7809	14.7261	17.6713	20.6165	23.5617
75	2.9850	5.9700	8.9550	11.9400	14.9250	17.9100	20.8950	23.8800
76	3.0248	6.0495	9.0743	12.0991	15.1238	18.1486	21.1734	24.1982
77	3.0646	6.1291	9.1937	12.2583	15.3230	18.3874	21.4519	24.5165
78	3.1043	6.2087	9.3131	12.4174	15.5217	18.6261	21.7305	24.8348
79	3.1441	6.2882	9.4324	12.5765	15.7206	18.8647	22.0089	25.1530
80	3.1839	6.3678	9.5518	12.7357	15.9196	19.1035	22.2874	25.4713
81	3.2237	6.4474	9.6711	12.8948	16.1185	19.3422	22.5658	25.7895
82	3.2635	6.5269	9.7904	13.0539	16.3173	19.5808	22.8443	26.1078
83	3 3032	6.6065	9.9098	13.2130	16.5162	19.8195	23.1228	26.4260
84	3.3430	6.6861	10.0292	13.3722	16.7152	20.0583	23.4013	26.7443
85	3.3828	6.7656	10.1485	13.5313	16.9141	20.2969	23.6797	27.0625
86	3.4226	6.8452	10.2678	13.6904	17.1130	20.5356	23.9581	27.3807
87	3.4624	6.9247	10.3871	13.8495	17.3118	20.7642	24.2266	27.6990
88	3.5021	7.0043	10.5065	14.0086	17.5107	21.0129	24.5151	28.0172
89	3.5419	7.0838	10.6258	14.1677	17.7096	21.2515	24.7934	28.3354
90	3.5817	7.1634	10.7452	14.3269	17.9086	21.4903	25.0720	28.6537
91	3.6215	7.2430	10.8645	14.4860	18.1075	21.7290	25.3505	28.9719
92	3.6612	7.3225	10.9838	14.6451	18.3063	21.9676	25.6289	29.2902
93	3.7011	7.4021	11.1032	14.8042	18.5052	22.2063	25.9074	29.6084
94	3.7408	7.4817	11.2223	14.9633	18.7038	22.4440	26.1847	29.9267
95	3.7806	7.5612	11.3419	15.1225	18.9031	22.6837	26.4643	30.2449
96	3.8204	7.6408	11.4612	15.2816	19.1020	22.9224	26.7428	30.5632
97	3.8602	7.7203	11.5805	15.4407	19.3009	23.1610	27.0213	30.8814
98	3.8999	7.7999	11.6999	15.5999	19.4998	23.3998	27.2998	31.1997
99	3.9397	7.8794	11.8192	15.7589	19.6986	23.6383	27.5780	31.5179
100	3.9795	7.9590	11.9386	15.9181	19.8976	23.8771	27.8566	31.8362
101	4.0193	8.0386	12.0579	16.0772	20.0965	24.1168	28.1351	32.1544
102	4.0591	8.1181	12.1772	16.2363	20.2954	24.3544	28.4135	32.4726
103	4.0964	8.1927	12.2891	16.3954	20.4917	24.5881	28.6845	32.7909
104	4.1361	8.2723	12.4085	16.5546	20.6907	24.8269	28.9630	33.1091
105	4.1784	8.3568	12.5353	16.7137	20.8821	25.0695	29.2479	33.4273
106	4.2182	8.4364	12.6546	16.8728	21.0910	25.3092	29.5273	33.7455
107	4.2579	8.5159	12.7739	17.0318	21.2897	25.5477	29.8057	34.0637
108	4.2977	8.5954	12.8932	17.1909	21.4886	25.7863	30.0841	34.3819
109	4.3375	8.6750	13.0126	17.3501	21.6876	26.0251	30.3626	34.7001
110	4.3773	8.7546	13.1319	17.5092	21.8865	26.2638	30.6411	35.0183
111	4.4171	8.8341	13.2512	17.6683	22.0854	26.5024	30.9195	35.3366
112	4.4569	8.9137	13.3706	17.8275	22.2844	26.7412	31.1981	35.6550
113	4.4967	8.9933	13.4900	17.9867	22.4834	26.9800	31.4766	35.9733
114	4.5364	9.0729	13.6094	18.1458	22.6823	27.2187	31.7552	36.2916
115	4.5762	9.1525	13.7288	18.3050	22.8812	27.4575	32.0338	36.6100
116	4.6160	9.2321	13.8482	18.4642	23.0802	27.6963	32.3123	36.9283
117	4.6558	9.3116	13.9675	18.6233	23.2791	27.9349	32.6908	37.2466
118	4.6956	9.3912	14.0869	18.7825	23.4781	28.1737	32.8694	37.5650
119	4.7354	9.4708	14.2063	18.9417	23.6771	28.4125	33.1479	37.8833
120	4.7752	9.5504	14.3256	19.1008	23.8760	28.6512	33.4264	38.2016
121	4.8149	9.6299	14.4449	19.2599	24.0748	28.8898	33.7048	38.5198
122	4.8548	9.7095	14.5643	19.4191	24.2739	29.1286	33.9833	38.8381
123	4.8945	9.7891	14.6837	19.5782	24.4728	29.3673	34.2619	39.1564
124	4.9393	9.8687	14.8081	19.7374	24.6767	29.6061	34.5454	39.4747
125	4.9741	9.9482	14.9224	19.8965	24.8706	29.8447	34.8189	39.7930

Number Teeth.	Chordal Pitch of Wheel or Pinion.							
	⅛	¼	⅜	½	⅝	¾	⅞	1 inch.
126	5.0139	10.0278	15.0417	20.0556	25.0695	30.0834	35.0973	40.1112
127	5.0537	10.1074	15.1611	20.2148	25.2685	30.3222	35.3759	40.4295
128	5.0934	10.1869	15.2803	20.3739	25.4673	30.5608	35.6542	40.7478
129	5.1332	10.2665	15.3997	20.5330	25.6662	30.7995	35.9327	41.0660
130	5.1730	10.3461	15.5191	20.6922	25.8652	31.0383	36.2113	41.3843
131	5.2128	10.4256	15.6384	20.8513	26.0641	31.2769	36.4897	41.7026
132	5.2526	10.5052	15.7578	21.0104	26.2630	31.5156	36.7682	42.0208
133	5.2924	10.5848	15.8792	21.1696	26.4620	31.7544	37.0468	42.3391
134	5.3321	10.6643	15.9964	21.3287	26.6608	31.9930	37.3251	42.6574
135	5.3719	10.7439	16.1158	21.4879	26.8598	32.2318	37.6037	42.9757
136	5.4117	10.8234	16.2351	21.6469	27.0586	32.4903	37.8820	43.2939
137	5.4515	10.9030	16.3545	21.8061	27.2576	32.7091	38.1606	43.6122
138	5.4913	10.9826	16.4739	21.9653	27.4566	32.9479	38.4392	43.9305
139	5.5312	11.0624	16.5936	22.1249	27.6561	33.1873	38.7185	44.2498
140	5.5709	11.1418	16.7127	22.2836	27.8545	33.4254	38.9963	44.5671
141	5.6106	11.2213	16.8319	22.4427	28.0533	33.6040	39.2746	44 8854
142	5.6504	11.3009	16.9513	22.6019	28.2523	33.9528	39.5532	45.2037
143	5.6900	11.3800	17.0700	22.7601	28.4591	34.1401	39.8301	45.5202
144	5.7300	11.4600	17.1909	22.9201	28.6501	34.3801	40.1101	45.8402
145	5.7698	11.5396	17.3094	23.0793	28.8491	34.6189	40.3887	46.1585
146	5.8096	11.6192	17.4288	23.2384	29.0480	34.8576	40.6672	46.4768
147	5.8494	11.6988	17.5482	23.3976	29.2470	35.0964	40.9458	46.7951
148	5.8891	11.7783	17.6674	23.5567	29.4458	35.3350	41.2241	47.1134
149	5.9289	11.8579	17.7868	23.7159	29.6448	35.5738	41.5027	47.4317
150	5.9687	11.9375	17.8962	23.8750	29.8437	35.8125	41.7812	47.7500
151	6.0080	12.0171	18.0251	24.0342	30.1222	36.0513	42.1393	48.0683
152	6.0483	12.0966	18.1449	24.1933	30.2416	36.2899	42.3382	48.3866
153	6.0881	12.1762	18.2645	24.3525	30.4406	36.5287	42.6168	48.7049
154	6.1279	12.2558	18.3837	24.5116	30.6395	36.7674	42.8953	49.0232
155	6.1676	12.3353	18.5029	24.6707	30.8383	37.0060	43.1736	49.3414
156	6.2074	12.4149	18.6223	24.8299	31.0373	37.2448	43.4522	49.6597
157	6.2472	12.4945	18.7417	24.9890	31.2362	37.4835	43.7307	49.9780
158	6.2870	12.5741	18.8611	25.1482	31.4352	37.7223	44.0093	50.2963
159	6.3268	12.6536	18.9804	25.3073	31.6341	37.9609	44.2877	50.6146
160	6.3666	12.7332	19.0998	25.4665	31.8331	38.1997	44.5663	50.9329
161	6.4064	12.8128	19.2192	25.6256	32.0320	38.4384	44.8448	51.2512
162	6.4462	12.8924	19.3386	25.7848	32.2310	38.6772	45.1234	51.5695
163	6.4859	12.9719	19.4578	25.9439	32.4298	38.9158	45.4017	51.8878
164	6.5257	13.0515	19.5772	26.1030	32.6287	39.1545	45.6802	52.2069
165	6.5655	13.1311	19.6966	26.2622	32.8277	39.3933	45.9588	52.5243
166	6.6053	13.2106	19.8159	26.4213	33.0266	39.6319	46.2372	52.8426
167	6.6450	13.2901	19.9351	26.5804	33.2254	39.8705	46.5155	53.1608
168	6.6849	13.3698	20.0547	26.7396	33.4245	40.1094	46.7943	53.4791
169	6.7244	13.4488	20.1732	26.8977	33.6221	40.3465	47.1109	53.7974
170	6.7644	13.5289	20.2933	27.0578	33.8222	40.5867	47.3511	54.1157
171	6.8042	13.6085	20.4127	27.2170	34.0212	40.8255	47.6297	54.4340
172	6.8440	13.6881	20.5321	27.3762	34.2202	41.0643	47.9083	54.7523
173	6.8838	13.7676	20.6514	27.5353	34.4191	41.3029	48.1867	55.0706
174	6.9236	13.8472	20.7708	27.6945	34.6181	41.5417	48.4653	55.3889
175	6.9634	13.9268	20.8902	27.8536	34.8170	41.7804	48.7438	55.7072
176	7.0032	14.0064	21.0096	28.0128	35.0160	42.0172	49.0224	56.0255
177	7.0429	14.0859	21.1288	28.1719	35.2148	42.2578	49.3007	56.3438
178	7.0827	14.1655	21.2482	28.3311	35.4138	42.4966	49.5793	56.6621
179	7.1225	14.2451	21.3676	28.4902	35.6127	42.7353	49.8578	56.9804
180	7.1623	14.3247	21.3870	28.6494	35.8117	42.9741	50.1359	57.2987
181	7.2021	14.4042	21.6063	28.8085	36.0106	43.2127	50.4148	57.6170
182	7.2419	14.4838	21.7257	28.9677	36.2096	43.4515	50.6934	57.9353
183	7.2817	14.5634	21.8451	29.1268	36.4085	43.6902	50.9719	58.2536
184	7.3215	14.6430	21.9645	29.2860	36.6075	43.9290	51.2505	58 5720
185	7.3613	14.7226	22.0839	29.4452	36.8065	44.1678	51.5291	58.8903

Number Teeth.	Chordal Pitch of Wheel or Pinion. $\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1 inch.
186	7.4010	14.8021	22.2031	29.6043	37.0053	44.4064	51.8074	59.2086
187	7.4408	14.8817	22.3225	29.7635	37.2043	44.6452	52.0860	59.5269
188	7.4806	14.9613	22.4419	29.9226	37.4032	44.8839	52.3655	59.8452
189	7.5204	15.0109	22.5613	30.0818	37.6022	45.1227	52.6431	60.1635
190	7.5702	15.1404	22.7106	30.2409	37.8111	45.3813	52.9515	60.4818
191	7.6000	15.2000	22.8000	30.4001	38.0001	45.6001	53.2001	60.8001
192	7.6398	15.2796	22.9194	30.5592	38.1990	45.8388	53.4786	61.1184
193	7.6795	15.3591	23.0386	30.7183	38.3978	46.0774	53.7569	61.4366
194	7.7193	15.4387	23.1580	30.8775	38.5968	46.3162	54.0355	61.7549
195	7.7591	15.5183	23.2774	31.0366	38.7967	46.5549	54.3150	62.0732
196	7.7989	15.5979	23.3968	31.1958	38.9947	46.7937	54.5926	62.3916
197	7.8387	15.6775	23.5162	31.3550	39.1937	47.0325	54.8712	62.7099
198	7.8785	15.7571	23.6356	31.5142	39.3927	47.2713	55.1498	63.0283
199	7.9183	15.8366	23.7549	31.6733	39.5916	47.5099	55.4282	63.3466
200	7.9581	15.9162	23.8743	31.8325	39.7906	47.7487	55.7068	63.6649
201	7.9979	15.9958	23.9937	31.9916	39.9895	47.9874	55.9853	63.9832
202	8.0376	16.0753	24.1129	32.1507	40.1883	48.2260	56.2636	64.3014
203	8.0773	16.1547	24.2320	32.3094	40.3867	48.4641	56.5414	64.6197
204	8.1172	16.2345	24.3480	32.4690	40.5862	48.7035	56.8207	64.9380
205	8.1570	16.3141	24.4711	32.6282	40.7852	48.9423	57.0993	65.2563
206	8.1968	16.3936	24.5904	32.7873	40.9841	49.1809	57.3777	65.5746
207	8.2366	16.4732	24.6348	32.9465	41.1831	49.4197	57.6563	65.8929
208	8.2764	16.5528	24.8292	33.1056	41.3820	49.6584	57.9348	66.2112
209	8.3162	16.6324	24.9486	33.2648	41.5810	49.8972	58.2134	66.5295
210	8.3559	16.7119	25.0678	33.4239	41.7798	50.1358	58.4917	66.8478
211	8.3957	16.7915	25.1872	33.5831	41.9788	50.3746	58.7703	67.1661
212	8.4355	16.8711	25.3066	33.7422	42.1777	50.6133	59.0488	67.4844
213	8.4753	16.9506	25.4259	33.9013	42.3766	50.8519	59.3272	67.8026
214	8.5151	17.0302	25.5453	34.0605	42.5756	51.0907	59.6058	68.1209
215	8.5549	17.1098	25.6647	34.2196	42.7735	51.3294	59.8833	68.4391
216	8.5947	17.1894	25.7841	34.3789	42.9736	51.5683	60.1630	68.7574
217	8.6344	17.2689	25.9033	34.5378	43.1722	51.8067	60.3411	69.0757
218	8.6742	17.3485	26.0227	34.6970	43.3712	52.0455	60.7197	69.3940
219	8.7140	17.4281	26.1421	34.8562	43.5702	52.2843	60.9983	69.7123
220	8.7538	17.5076	26.2614	35.0153	43.7691	52.5229	61.2767	70.0306
221	8.7936	17.5872	26.3808	35.1745	43.9681	52.7617	61.5553	70.3489
222	8.8334	17.6668	26.5002	35.3336	44.1670	52.0004	61.8338	70.6672
223	8.8732	17.7464	26.6296	35.4928	44.3660	53.2392	62.1124	70.9855
224	8.9129	17.8259	26.7388	35.6519	44.5648	53.4778	62.4907	71.3038
225	8.9527	17.9055	26.8582	35.8111	44.7638	53.7166	62.6693	71.6221
226	8.9925	17.9851	26.9776	35.9702	44.9627	53.9553	62.9478	71.9404
227	9.0323	18.0647	27.0970	36.1294	45.1617	54.1941	63.2264	72.2587
228	9.0721	18.1442	27.2163	36.2885	45.3606	54.4327	63.5048	72.5770
229	9.1119	18.2238	27.3357	36.4477	45.5596	54.6715	63.7834	72.8953
230	9.1517	18.3034	27.4551	36.6068	45.7585	54.9102	64.0619	73.2136
231	9.1914	18.3829	27.5743	36.7659	45.9573	55.1488	64.3402	73.5319
232	9.2312	18.4625	27.6937	36.9251	46.1563	55.3876	64.6188	73.8502
233	9.2710	18.5421	27.8131	37.0843	46.3553	55.6264	64.8974	74.1685
234	9.3108	18.6217	27.9325	37.2434	46.5542	55.8651	65.1759	74.4867
235	9.3506	18.7012	28.0518	37.4025	46.7531	56.1037	65.4543	74.8050
236	9.3904	18.7808	28.1712	37.5617	46.9521	56.3425	65.7329	75.1233
237	9.4302	18.8604	28.2906	37.7208	47.1510	56.5812	66.0114	75.4416
238	9.4700	18.9400	28.4100	37.8800	47.3500	56.8200	66.2900	75.7599
239	9.5097	19.0195	28.5282	38.0391	47.5488	57.0586	66.5683	76.0782
240	9.5495	19.0991	28.6486	38.1983	47.7478	57.2974	66.8469	76.3965
241	9.5894	19.1788	28.7682	38.3574	47.9468	57.5362	67.1256	76.7148
242	9.6291	19.2583	28.8874	38.5166	48.1457	57.7649	67.4040	77.0331
243	9.6689	19.3378	29.0067	38.6757	48.3446	58.0135	67.6814	77.3514
244	9.7087	19.4174	29.1261	38.8349	48.5436	58.2523	67.9610	77.6697
245	9.7485	19.4970	29.2455	38.9940	48.7425	58.4910	68.2395	77.9880

Number Teeth.	Chordal Pitch of Wheel or Pinion.							
	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2 inches.	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	3 inches.
6	2.5000	3.0000	2.5000	4.0000	4.5000	5.0000	5.5000	6.0000
7	2.8634	3.4602	4.0170	4.6136	5.1903	5.7268	6.3035	6.9204
8	3.2664	3.9194	4.4728	5.2262	5.8794	6.5328	7.1860	7.8393
9	3.6528	4.3856	5.1166	5.8476	6.5785	7.3056	8.0365	8.7714
10	4.0450	4.8540	5.6630	6.1722	7.2812	8.0900	8.8990	9.7083
11	4.4364	5.3234	6.2106	7.0980	7.9852	8.8728	9.7600	10.6470
12	4.8296	5.7954	6.7614	7.7274	8.6933	9.6592	10.6251	11.5911
13	5.2232	6.2676	7.3124	8.3570	9.4016	10.4464	11.4910	12.5355
14	5.6171	6.7410	7.8646	8.9880	10.1115	11.2348	12.3583	13.4820
15	6.0122	7.2144	8.4170	9.6194	10.8218	12.0244	13.2268	14.4291
16	6.4074	7.6886	8.9702	10.2518	11.5332	12.8148	14.0962	15.3777
17	6.8028	8.1632	9.5240	10.8846	12.2451	13.6056	14.9661	16.3260
18	7.1984	8.6382	10.0780	11.5176	12.9573	14.3968	15.8365	17.2764
19	7.5946	9.1134	10.6324	12.1512	13.6701	15.1892	16.7081	18.2268
20	7.9916	9.5886	11.1868	12.7850	14.3831	15.9832	17.5813	19.1775
21	8.3868	10.0640	11.7416	13.4190	15.0964	16.7736	18.4510	20.1285
22	8.7832	10.5398	12.2966	14.0532	15.8098	17.5664	19.3230	21.0798
23	9.1672	11.0006	12.8342	14.6676	16.5010	18.3344	20.1678	22.0014
24	9.5766	11.4918	13.4072	15.3226	17.2379	19.1532	21.0685	22.9839
25	9.9732	11.9678	13.9626	15.9574	17.9520	19.9464	21.9410	23.9361
26	10.3702	12.4442	14.5184	16.5924	18.6664	20.7404	22.8144	24.8886
27	10.7672	12.9206	15.0742	17.2276	19.3810	21.5344	23.6878	25.8414
28	11.1644	13.3970	15.6300	17.8630	20.0958	22.3288	24.5616	26.7945
29	11.5616	13.8738	16.1862	18.4986	20.8109	23.1232	25.4355	27.7479
30	11.9584	14.3502	15.7420	19.1336	21.5253	23.9168	26.3085	28.7004
31	12.3556	14.8266	17.2976	19.7690	22.2401	24.7112	27.1823	29.6535
32	12.7528	15.3032	17.8540	20.4046	22.9552	25.5056	28.0562	30.6069
33	13.1500	15.7800	18.4102	21.0402	23.6702	26.3000	28.9300	31.5603
34	13.5474	16.2570	18.9666	21.6760	24.3855	27.0948	29.8043	32.5140
35	13.9446	16.7334	19.5224	22.3114	25.1003	27.8892	30.6781	33.4671
36	14.3422	17.2104	20.0790	22.9474	25.8158	28.6844	31.5528	34.4211
37	14.7396	17.6874	20.6354	23.5834	26.5313	29.4792	32.4271	35.3751
38	15.1370	18.1624	21.1898	24.2192	27.2466	30.2740	33.3014	36.3288
39	15.5344	18.6410	21.7480	24.8550	27.9618	31.0688	34.1756	37.2825
40	15.9318	19.1180	22.3044	25.4910	28.6774	31.8636	35.0500	38.2365
41	16.3294	19.6154	22.8814	26.1272	29.3931	32.6588	35.9247	39.1908
42	16.7270	20.0724	23.4178	26.7632	30.1086	33.4540	36.7994	40.1448
43	17.1244	20.5494	23.9744	27.3992	30.8241	34.2488	37.6737	41.0988
44	17.5222	21.0264	24.5310	28.0354	31.5398	35.0444	38.5488	42.0531
45	17.9196	21.5034	25.0874	28.6714	32.2553	35.8392	39.4231	43.0071
46	18.3172	21.9806	25.6442	29.3076	32.9710	36.6344	40.2978	43.9614
47	18.7146	22.4576	26.2006	29.9436	33.6865	37.4292	41.1721	44.9154
48	19.1122	22.9346	26.7572	30.5796	34.4020	38.2244	42.0468	45.8694
49	19.5098	23.4116	27.3136	31.2158	35.1177	39.0196	42.9215	46.8167
50	19.9074	23.8890	27.8704	31.8520	35.8335	39.8148	43.7963	47.7780
51	20.3050	24.3660	28.4282	32.4882	36.5492	40.6100	44.6710	48.7323
52	20.7026	24.8432	28.9838	33.1244	37.2649	41.4052	45.5457	49.6866
53	21.1050	25.3260	29.5470	33.7680	37.9890	42.2100	46.3505	50.6520
54	21.4982	25.7976	30.0974	34.3970	38.6966	42.9964	47.2960	51.5955
55	21.8958	26.2748	30.6540	35.0334	39.4125	43.7916	48.1707	52.5501
56	22.2936	26.7522	31.2110	35.6698	40.1285	44.5872	49.0459	53.5047
57	22.6912	27.2294	31.7678	36.3060	40.8442	45.3824	49.9206	54.4590
58	23.0888	27.7064	32.3242	36.9422	41.5599	46.1776	50.7953	55.4133
59	23.4864	28.1838	32.8812	37.5784	42.2757	46.9728	51.6701	56.3676
60	23.9842	28.6608	33.4378	38.2146	42.9914	47.9684	52.7452	57.3219
61	24.2818	29.1380	33.9944	38.8510	43.7073	48.5636	53.4199	58.2765
62	24.6794	29.6154	34.5514	39.4872	44.4231	49.3588	54.2947	59.2308
63	25.0774	30.0926	35.1082	40.1236	45.1390	50.1548	55.1702	60.1854
64	25.4748	30.5696	35.6646	40.7598	45.8547	50.9496	56.0445	61.1397
65	25.8724	31.0470	36.2216	41.3960	46.5705	51.7448	56.9193	62.0940

Number Teeth.	Chordal Pitch of Wheel or Pinion. $1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2 inches.	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	3 inches.
66	26.2701	31.5242	36.7782	42.0322	47.2862	52.5443	57.7943	63.0483
67	26.6679	32.0015	37.3351	42.6686	48.0022	53.3358	58.6694	64.0029
68	27.0655	32.4786	37.8917	43.3048	48.7179	54.1310	59.5441	64.9572
69	27.4631	32.9558	38.4484	43.9410	49.4330	54.9263	60.4189	65.9115
70	27.8609	33.4331	39.0053	44.5774	50.1496	55.7218	61.2940	66.8661
71	28.2588	33.9105	39.5623	45.2140	50.8658	56.5175	62.1693	67.8210
72	28.6566	34.3880	40.1193	45.8506	51.5819	57.3133	63.0446	68.7759
73	29.0544	34.8653	40.6762	46.4870	52.2979	58.1088	63.9193	69.7305
74	29.4521	35.3426	41.2330	47.1234	53.0138	58.9043	64.7947	70.6851
75	29.8500	35.8200	41.7900	47.7600	53.7300	59.7000	65.6700	71.6400
76	30.2477	36.2973	42.3468	48.3964	54.4459	60.4955	66.5450	72.5946
77	30.6456	36.7748	42.9039	49.0330	55.1621	61.2913	67.4204	73.5495
78	31.0435	37.2522	43.4609	49.6696	55.8783	62.0870	68.2957	74.5044
79	31.4412	37.7295	44.0177	50.3060	56.5942	62.8825	69.1707	75.4590
80	31.8391	38.2070	44.6048	50.9426	57.3104	63.6783	70.0461	76.4139
81	32.2369	38.6843	45.1317	51.5790	58.0564	64.4738	70.9212	77.3685
82	32.6347	39.1617	45.6886	52.2156	58.7425	65.2695	71.7964	78.3234
83	33.0325	39.6390	46.2455	52.8520	59.4585	66.0650	72.6715	79.2780
84	33.4304	40.1165	46.8026	53.4886	60.1747	66.8608	73.5469	80.2329
85	33.8281	40.5938	47.3594	54.1250	60.8906	67.6563	74.4219	81.1875
86	34.2259	41.0711	47.9163	54.7614	61.6066	68.4518	75.2970	82.1421
87	34.6237	41.5485	48.4632	55.3980	62.3227	69.2475	76.1622	83.0970
88	35.0215	42.0258	49.0301	56.0344	63.0387	70.0430	77.0473	84.0516
89	35.4192	42.5031	49.5869	56.6708	63.7546	70.8385	77.9223	85.0062
90	35.8171	42.9806	50.1440	57.3074	64.4708	71.6343	78.7977	85.9611
91	36.2149	43.4579	50.7009	57.9438	65.1868	72.4298	79.6728	86.9157
92	36.6127	43.9353	51.2578	58.5804	65.9029	73.2255	80.5480	87.8706
93	37.0105	44.4126	51.8147	59.2168	66.6189	74.0210	81.4231	88.8252
94	37.4084	44.8900	52.3707	59.8534	67.3351	74.8167	82.2974	89.7801
95	37.8061	45.3674	52.9286	60.4898	68.0510	75.6123	83.1735	90.7347
96	38.2040	45.8448	53.4856	61.1264	68.7672	76.4080	84.0488	91.6896
97	38.6017	46.3221	54.0424	61.7628	69.4831	77.2035	84.9238	92.6442
98	38.9996	46.7996	54.5995	62.3994	70.1993	77.9993	85.7992	93.5991
99	39.3973	47.2768	55.1562	63.0358	70.9152	78.7947	86.6741	94.5537
100	39.7952	47.7543	55.7133	63.6724	71.6314	79.5905	87.5495	95.5086
101	40.1930	48.2316	56.2702	64.3088	72.3474	80.3860	88.4646	96.4632
102	40.5907	48.7089	56.8270	64.9432	73.0613	81.1795	89.2976	97.4178
103	40.9836	49.1863	57.3790	65.5918	73.7845	81.9872	90.1799	98.3727
104	41.3814	49.6637	57.9360	66.2182	74.4905	82.7728	91.0451	99.3273
105	41.7841	50.1410	58.4968	66.8546	75.2114	83.5683	91.9241	100.2819
106	42.1819	50.6183	59.0547	67.4910	75.9274	84.3638	92.8002	101.2365
107	42.5796	51.0955	59.6114	68.1274	76.6433	85.1592	93.6751	102.1911
108	42.9773	51.5728	60.1682	68.7638	77.3592	85.9547	94.5501	103.1457
109	43.3751	52.0502	60.7252	69.4002	78.0752	86.7503	95.4253	104.1003
110	43.7729	52.5275	61.2821	70.0366	78.7912	87.5458	96.3004	105.0549
111	44.1707	53.0049	61.8390	70.6732	79.5073	88.3415	97.1766	106.0098
112	44.5687	53.4825	62.3962	71.3100	80.2237	89.1375	98.0212	106.9650
113	44.9666	53.9600	62.9533	71.9466	80.9399	89.9333	98.9266	107.9199
114	45.3645	54.4374	63.5103	72.5832	81.6561	90.7290	99.8019	108.8748
115	45.7625	54.9150	64.0675	73.2200	82.3725	91.5250	100.6775	109.8300
116	46.1604	55.3925	64.6246	73.8566	83.0887	92.5208	101.5559	110.7849
117	46.5582	55.8699	65.1815	74.4932	83.8048	93.1165	102.4281	111.7398
118	46.9562	56.3475	65.7387	75.1300	84.5212	93.9125	103.3037	112.6950
119	47.3541	56.8250	66.2958	75.7666	85.2374	94.7083	104.1791	113.6499
120	47.7520	57.3024	66.8528	76.4032	85.9536	95.5040	105.0544	114.6048
121	48.1497	57.7797	67.4096	77.0396	86.6695	96.2995	105.9294	115.5594
122	48.5476	58.2572	67.9667	77.6762	87.3857	97.0953	106.8048	116.5143
123	48.9455	58.7346	68.5237	78.3128	88.1016	97.8910	107.6801	117.4692
124	49.3434	59.2121	69.0808	78.9494	88.8181	98.6868	108.5555	118.4241
125	49.7412	59.6895	69.6377	79.5860	89.5342	99.4825	109.4307	119.3790

Number Teeth.	Chordal Pitch of Wheel or Pinion.							
	1¼	1½	1¾	2 inches.	2¼	2½	2¾	3 inches.
126	50.1390	60.1668	70.1946	80.2224	90.2502	100.2780	110.3058	120.3336
127	50.5369	60.6443	70.7517	80.8590	90.9664	101.1808	111.1812	121.2885
128	50.9347	61.1217	71.3086	81.4956	91.5825	101.8695	112.0564	122.2434
129	51.3325	61.5990	71.8655	82.1320	92.3985	102.6650	112.9315	123.1980
130	51.7304	62.0765	72.4226	82.7678	93.1147	103.4608	113.8069	124.1529
131	52.1282	62.5539	72.9795	83.4052	93.8308	104.2565	114.6821	125.1078
132	52.5260	63.0312	73.5364	84.0416	94.5468	105.0520	115.5572	126.0624
133	52.9239	63.5087	74.0945	84.6782	95.2630	105.8478	116.4326	127.0173
134	53.3217	63.9861	74.6504	85.3148	95.9791	106.6435	117.3078	127.9722
135	53.7196	64.4636	75.2075	85.9514	96.6953	107.1393	118.1832	128.9271
136	54.1173	64.9408	75.7842	86.5878	97.4112	108.1347	119.0781	129.8817
137	54.5152	65.4183	76.3213	87.2244	98.1274	108.9305	119.9335	130.8366
138	54.9131	65.8958	76.8784	87.8610	98.8436	109.8263	120.8089	131.7915
139	55.3122	66.3747	77.4371	88.4996	99.5620	110.7245	121.6869	132.7494
140	55.7089	66.8507	77.9925	89.1342	100.2760	111.4178	122.5596	133.7013
141	56.1067	67.3281	78.5494	89.7708	100.9921	112.2135	123.4348	134.6562
142	56.5046	67.8056	79.1665	90.4074	101.7083	113.0093	124.3702	135.6111
143	56.9002	68.2803	79.6603	91.0404	102.4204	113.8005	125.1805	136.5606
144	57.3002	68.7603	80.2203	91.6804	103.1404	114.6005	126.0605	137.5206
145	57.6981	69.2378	80.7774	92.3170	103.8566	115.3963	126.9359	138.4755
146	58.0960	69.7152	81.3344	92.9536	104.5728	116.1920	127.8112	139.4304
147	58.4939	70.1827	81.8915	93.5902	105.2890	116.9878	128.6866	140.3853
148	58.8917	70.6701	82.4484	94.2268	106.0051	117.7835	129.5618	141.3402
149	59.2896	71.1476	83.0055	94.8634	106.7213	118.5793	130.4372	142.2951
150	59.6875	71.6250	83.5625	95.5000	107.4375	119.3750	131.3125	143.2500
151	60.0854	72.1025	84.1196	96.1366	108.1537	120.1708	132.1879	144.2049
152	60.4832	72.5799	84.6765	96.7732	108.8698	120.9665	133.0631	145.1598
153	60.8811	73.0574	85.2336	97.4098	109.5860	121.7623	133.9385	146.1147
154	61.2790	73.5348	85.7906	98.0464	110.3022	122 5580	134.8138	147.0696
155	61.6767	74.0121	86.3474	98.6828	111.0181	123.3535	135.6888	148.0242
156	62.0746	74.4896	86.9045	99.3194	111.7343	124.1493	136.5642	148.9791
157	62.4725	74.9670	87.4615	99.9560	112.4505	124.9450	137.4395	149.9340
158	62.8704	75.4445	88.0186	100.5926	113.1667	125.7408	138.3149	150.8889
159	63.2682	75.9219	88.5755	101.2292	113.8828	126.5365	139.1901	151.8438
160	63.6661	76.3994	89 1326	101.8658	114.6990	127.3323	140.0655	152.7987
161	64.0640	76.8768	89.6896	102.5024	115.3152	128.1280	140.9408	153.7536
162	64.4619	77.3543	90.2467	103.1390	116.0314	128.9238	141.8162	154.7085
163	64.8597	77.8317	90.8036	103.7756	116.7475	129.7195	142.6914	155.6634
164	65.2575	78.3090	91.3605	104.4120	117.4635	130.5150	143.5665	156.6180
165	65.6554	78.7865	91.9176	105.0486	118.1597	131.3108	144.4419	157.5729
166	66.0532	79.2639	92.4745	105.6852	118.8958	132.1065	145.3171	158.5278
167	66.4509	79.7412	93.0313	106.3216	119.6117	132.9020	146.1921	159.4824
168	66.8489	80.2187	93.5885	106.9582	120.3280	133.6978	147.0676	160.4373
169	67.2462	80.6951	94.1439	107.5948	121.0436	134.4925	147.9413	161.3922
170	67.6446	81.1735	94.7024	108.2314	121.7603	135.2892	148.8181	162.3471
171	68.0425	81.6510	95.2595	108.8680	122.4765	136.0850	149.6935	163.3020
172	68.4404	82.1285	95.8166	109.5046	123.1927	136.8808	150.5689	164.2569
173	68.8382	82.6059	96.3735	110.1412	123.9088	137.6765	151.4441	165.2118
174	69.2361	83.0834	96.9306	110.7778	124.6250	138.4723	152.3195	166.1667
175	69.6340	83.5608	97.4876	111.4144	125.3412	139.2680	153.1948	167.1216
176	70.0319	84.0383	98.0427	112.0510	126.0574	140.0638	154.0182	168.0765
177	70.4297	84.5157	98.6016	112.6876	126.7735	140.8595	154.9454	169.0314
178	70.8276	84.9932	99.1587	113.3242	127.4897	141.6553	155.8208	169.9863
179	71.2255	85.4766	99.7157	113.9608	128.2059	142.4510	156.6961	170.9412
180	71.6234	85.9481	100.2728	114.5974	128.9221	143.2468	157.5715	171.8961
181	72.0212	86.4255	100.8297	115.2340	129.6382	144.0425	158.4467	172.8510
182	72.4191	86.9030	101.3868	115.8706	130.3544	144.8383	159.3221	173.8059
183	72.8170	87.3804	101.9438	116.5072	131.0706	145.6340	160.1974	174.7608
184	73.2150	87.8580	102.5010	117.1540	131.7970	146.4400	161.0830	175.7160
185	73.6129	88.3355	103.0581	117.7806	132.5032	147.2258	161.9484	176.6709

Number Teeth.	Chordal Pitch of Wheel or Pinion.							
	1¼	1½	1¾	2 inches.	2¼	2½	2¾	3 inches.
186	74.0107	88.8129	103.6150	118.4172	133.2193	148.0215	162.8236	177.6258
187	74.4086	89.2903	104.1721	119.0538	133.9355	148.8173	163.6990	178.5807
188	74.8065	89.7678	104.7291	119.6904	134.6517	149.6130	164.5743	179.5356
189	75.2044	90.2453	105.2862	120.3270	135.3679	150.4088	165.4497	180.4905
190	75.6222	90.7227	105.8631	120.9636	136.1040	151.2045	166.3449	181.4448
191	76.0001	91.2002	106.4002	121.6002	136.8002	152.0003	167.2003	182.4003
192	76.3980	91.6776	106.9572	122.2368	137.5164	152.7960	168.0756	183.3552
193	76.7957	92.1549	107.5140	122.8732	138.2323	153.5955	168.9506	184.3098
194	77.1936	92.6324	108.0711	123.5098	138.9485	154.3873	169.8260	185.2647
195	77.5915	93.1098	108.6281	124.1464	139.6647	155.1830	170.7013	186.2196
196	77.9895	93.5874	109.1853	124.7832	140.3811	155.9790	171.5769	187.1748
197	78.3874	94.0649	109.7424	125.4198	141.0973	156.7748	172.4523	188.1297
198	78.7854	94.5425	110.2996	126.0566	141.8137	157.5708	173.3279	189.0849
199	79.1832	95.0199	110.8565	126.6932	142.5298	158.3665	174.2031	190.0398
200	79.5811	95.5974	111.4136	127.3298	143.2460	159.1623	175.0785	190.9947
201	79.9790	95.9748	111.9706	127.9664	143.9622	159.9580	175.9538	191.9496
202	80.3772	96.4521	112.5274	128.6028	144.6881	160.7535	176.8288	192.9042
203	80.7744	96.9291	113.0838	129.2394	145.3941	161.5488	177.7035	193.8591
204	81.1725	97.4070	113.6415	129.8760	146.1105	162.3450	178.5795	194.8140
205	81.5704	97.8845	114.1986	130.5126	146.8267	163.1408	179.4549	195.7689
206	81.9682	98.3619	114.7555	131.1492	147.5428	163.9365	180.3301	196.7238
207	82.3661	98.8394	115.3126	131.7858	148.2590	164.7323	181.2055	197.6787
208	82.7640	99.3168	115.8696	132.4224	148.9752	165.5280	182.0808	198.6336
209	83.1619	99.7943	116.4267	133.0590	149.6914	166.3238	182.9662	199.5885
210	83.5597	100.2717	116.9836	133 6956	150.4075	167.1195	183.8314	200.5434
211	83.9576	100.7492	117.5407	134.3322	151.1237	167.9153	184.7068	201.4983
212	84.3555	101.2266	118.0977	134.9688	151.8399	168.7110	185.5821	202.4532
213	84.7532	101.7039	118.6545	135.6052	152.5558	169.5065	186.4571	203.4078
214	85.1511	102.1814	119.2116	136.2418	153.2720	170.3023	187.3325	204.3627
215	85.5489	102.6587	119.7685	136.8782	153.9880	171.0978	188.2076	205.3173
216	85.9468	103.1363	120.3257	137.5148	154.7042	171.8937	189.0831	206.2722
217	86.3446	103.6135	120.8824	138.1514	155.4203	172.6892	189.9581	207.2271
218	86.7425	104.0910	121.4395	138.7880	156.1365	173.4850	190.8335	208.1820
219	87.1404	104.5685	121.9966	139.4246	156.8527	174.2808	191.7089	209.1369
220	87.5382	105.0459	122.5535	140.0612	157.5688	175.0765	192.5841	210.0918
221	87.9361	105.5234	123.1106	140.6978	158.2850	175.8723	193.4595	211.0467
222	88.3340	106.0008	123.6676	141.3344	159.0012	176.6680	194.3348	212.0016
223	88.7319	106.4783	124.2247	141.9710	159.7174	177.4638	195.2102	212.9565
224	89.1297	106.9557	124.7816	142.6076	160.4335	178.2595	196.0854	213.9114
225	89.5276	107.4332	125.3387	143.2442	161.1497	179.0553	196.9608	214.8663
226	89.9255	107.9106	125.8957	143.8808	161.8659	179.8510	197.8361	215.8224
227	90.3234	108.3881	126.4528	144.5174	162.5821	180.6468	198.7115	216.7761
228	90.7212	108.8655	127.0097	145.1540	163.2982	181.4425	199.5867	217.7310
229	91.1191	109.3430	127.5668	145.7906	164.0144	182.2383	200.4621	218.6859
230	91.5170	109.8204	128.1238	146.4272	164.7306	183.0340	201.3374	219.6408
231	91.9148	110.2978	128.6807	147.0638	165.4467	183.8297	202.2126	220.5957
232	92.3127	110.7753	129.2378	147.7004	166.1629	184.6255	203.0880	221.5506
233	92.7106	111.2528	129.7949	148.3370	166.8791	185.4213	203.9634	222.5055
234	93.1084	111.7301	130.3518	148.9734	167.5951	186.2168	204.8385	223.4601
235	93 5062	112.2075	130.9087	149.6100	168.3112	187.0125	205.7137	224.4150
236	93.9041	112.6850	131.4658	150.2466	169.0274	187.8083	206.5891	225.3699
237	94.3020	113.1624	132.0228	150.8832	169.7436	188.6040	207 4644	226.3248
238	94.7999	113.6399	132.5799	151.5198	170.4998	189.3998	208.3398	227.2797
239	95.0977	114.1173	133.1368	152.1564	171.1759	190.1955	209.2150	228.2346
240	95.4956	114.5948	133.6939	152.7930	171.8921	190.9913	210.0904	229.1895
241	95.8936	115.0722	134.2510	153.4296	172.6084	191.7870	210.9658	230.1444
242	96.2914	115.5997	134.7980	154.0662	173.3245	192.5828	211.8311	231.0993
243	96.6892	116.0271	135.3649	154.7028	174 0406	193.3785	212.7163	232.0542
244	97.0871	116.5046	135.9220	155.3394	174.7568	194.1643	213.5917	233.0091
245	97.4850	116.9820	136.4790	155.9760	175.4730	194.9700	214.4670	233.9640

Number Teeth.	Chordal Pitch of Wheel or Pinion.							
	3½	4 inches.	4½	5 inches.	5½	6 inches.	6½	7 inches.
6	7.0000	8.0000	9.0000	10.0000	11.0000	12.0000	13.0000	14.0000
7	8.0738	9.2272	10.3806	11.5340	12.6874	13.8408	14.9942	16.1476
8	9.1458	10.4524	11.7589	13.0655	14.3720	15.6786	16.9851	18.2917
9	10.2333	11.6952	13.1571	14.6190	16.0809	17.5528	19.0147	20.4666
10	11.3263	12.9444	14.5624	16.1805	17.7985	19.4166	21.0346	22.6527
11	12.4215	14.1960	15.9705	17.7450	19.5195	21.2940	23.0685	24.8430
12	13.5229	15.4548	17.3866	19.3185	21.2503	23.1822	25.1140	27.0459
13	14.6247	16.7140	18.8032	20.8925	22.9817	25.0710	27.1602	29.2495
14	15.7290	17.9760	20.2230	22.4700	24.7170	26.9640	29.2110	31.4580
15	16.8339	19.2388	21.6436	24.0485	26.4533	28.8582	31.2630	33.5679
16	17.9406	20.5036	23.0665	25.6295	28.1924	30.7554	33.3183	35.8813
17	19.0480	21.7692	24.4903	27.2115	29.9326	32.6538	35.3749	38.0961
18	20.1558	23.0352	25.9146	28.7940	31.6934	34.5528	37.4322	40.3116
19	21.2646	24.3024	27.3402	30.3780	33.4158	36.4536	39.4914	42.5292
20	22.3737	25.5700	28.7662	31.9625	35.1587	38.3550	41.5512	44.7475
21	23.4832	26 8380	30.4327	33.5475	36.9022	40.2570	43.6117	46.9665
22	24.5931	28.1064	31.6197	35.1330	38.6463	42.1596	45.6729	49.1862
23	25.6683	29.3352	33.0021	36.6690	40.3359	44.0028	47.6697	51.3366
24	26.8145	30.6452	34.4758	38.3065	42.1371	45.9678	49.7984	53.6291
25	27.9254	31.9148	35.9041	39.8935	43.8828	47.8722	51.8615	55.8509
26	29.0367	33.1848	37.3329	41.4810	45.6291	49.7772	53.9253	58.0754
27	30.1483	34.4552	38.7621	43.0690	47.3759	51.6828	55.9897	60.2966
28	31.2602	35.7260	40.1917	44.6575	49.1232	53.5890	58.0547	62.5205
29	32.3725	36.9972	41.6218	46.2465	50.8711	55.4958	60.1204	64.7451
30	33.4838	38.2672	43.0506	47.8340	52.6174	57.4008	62.1842	66.9676
31	34.5957	39.5380	44.4802	49.4225	54.3647	59.3070	64.2492	69.1915
32	35.7081	40.8092	45.9014	51.0115	56.1127	61.2138	66.3150	71.4161
33	36.8203	42.0804	47.3404	52.6005	57.8605	63.1206	68.3806	73.6407
34	37.9330	43.3520	48.7710	54.1900	59.6090	65.0280	70.4470	75.8660
35	39.0449	44.6228	50.2006	55.7785	61.3563	66.9342	72.5120	78.0899
36	40.1579	45.8948	51.6316	57.3685	63.1053	68.8422	74.5790	80.3159
37	41.2709	47.1668	53.0626	58.9585	64.8543	70.7502	76.6460	82.5419
38	42.3836	48.4384	54.4932	60.5480	66.5928	72.6576	78.7124	84.7672
39	43.4962	49.7100	55.9237	62.1375	68.3512	74.5650	80.7787	86.9925
40	44.6093	50.9820	57.3548	63.7275	70.1003	76.4730	82.8458	89.2185
41	45.7226	52.2544	58.7862	65.3180	71.8498	78.3816	84.9134	91.4452
42	46.8356	53.5264	60.2172	66.9080	73.5988	80.2896	86.9804	93.6712
43	47.9486	54.7984	61.6482	68.4980	75.3478	82.1976	89.0474	95.8972
44	49.0619	56.0708	63.0796	70.0885	77.0973	84.1062	91.1150	98.1229
45	50.1749	57.3428	64.5106	71.6785	78.8463	86.0242	93.1920	100.3499
46	51.2883	58.6152	65.9421	73.2690	80.5959	87.9228	95.2497	102.5766
47	52.4013	59.8872	67.3731	74.8590	82.3449	89.8308	97.3167	104.8026
48	53.5143	61.1592	68.8041	76.4490	84.0939	91.7388	99.3837	107.0286
49	54.6206	62.4316	70.2355	78.0158	85.8197	93.6474	101.4513	109.2553
50	55.7410	63.7040	71.6670	79.6300	87.5930	95.5560	103.5190	111.4820
51	56.8543	64.9764	73.0984	81.2205	89.3425	97.4646	105.5866	113.7087
52	57.9677	66.2488	74.5299	82.8110	91.0921	99.3732	107.6543	115.9354
53	59.0940	67.5360	75.9780	84.4200	92.8620	101.3040	109.7460	118.1880
54	60.1947	68.7940	77.3932	85.9925	94.5917	103.1910	111.7802	120.3895
55	61.3084	70.0668	78.8251	87.5835	96.3418	105.1002	113.8595	122.6169
56	62.4221	71.3396	80.2570	89.1745	98.0919	107.0294	115.9468	124.8443
57	63.5355	72.6120	81.6885	90.7650	99.8415	108.9180	117.9945	127.0710
58	61.6488	73.8844	83.1199	92.3555	101.5910	110.8266	119.0621	129.2977
59	65.7622	75.1568	84.5514	93.9460	103.3406	112.7352	122.1298	131.5244
60	66.8755	76.4292	85.9828	95.5365	105.0901	114.6438	124.1974	133.7511
61	67.9892	77.7020	87.4147	97.1275	106 8402	116.5530	126.2657	135.9785
62	69.1026	78.9744	88.8462	98.7180	108.5898	118.4616	128.3334	138.2052
63	70.2163	80.2472	90.2781	100.3090	110.3399	120.3708	130.4017	140.4326
64	71.3296	81.5196	91.7095	101.8995	112.0894	122.2794	132.4693	142.6593
65	72.4430	82.7920	93.1410	103.4900	113.8390	124.1880	134.5370	144.8860

Number Teeth.	Chordal Pitch of Wheel or Pinion.							
	3½	4 inches.	4½	5 inches.	5½	6 inches.	6½	7 inches.
66	73.5564	84.0644	94.5725	105.0805	115.5886	126.0966	136.6047	147.1127
67	74.6701	85.3372	96.0044	106.6715	117.3387	128.0058	138.6730	149.3401
68	75.7834	86.6096	97.4358	108.2620	119.0882	129.9144	140.7406	151.5668
69	76.8968	87.8820	98.8673	109.8525	120.8378	131.8230	142.8083	153.7935
70	78.0105	89.1548	100.2992	111.4435	122.5879	133.7322	144.8766	156.0209
71	79.1245	90.4280	101.7315	113.0350	124.3385	135.6420	146.9455	158.2490
72	80.2386	91.7012	102.1639	114.6265	120.0892	137.5518	149.0135	160.4771
73	81.3523	92.9740	103.5958	116.2175	127.8393	139.4610	151.0828	162.7045
74	82.4660	94.2468	105.0277	117.8085	129.5894	141.3702	153.1511	164.9319
75	83.5800	95.5200	107.4600	119.4400	131.3800	143.2800	155.2200	167.1600
76	84.6937	96.7928	108.8919	120.9910	133.0901	145.1892	157.2883	169.3874
77	85.8078	98.0660	110.3243	122.5825	134.8408	147.0990	159.3573	171.6155
78	86.9218	99.3392	111.7566	124.1740	136.5914	149.0088	161.4262	173.8436
79	88.0355	100.6120	113.1885	125.7650	138.3415	150.9180	163.4945	176.0710
80	89.1496	101.8852	114.6209	127.3565	140.0922	152.8278	165.5635	178.2991
81	90.2633	103.1580	116.0528	128.9475	141.8423	154.7370	167.6318	180.5265
82	91.3773	104.4312	117.4851	130.5390	143.5929	156.6468	169.7007	182.7546
83	92.4910	105.7040	118.9170	132.1560	145.3690	158.5560	171.7690	184.9820
84	93.6051	106.9772	120.3494	133.7215	147.0937	160.4658	173.8380	187.2101
85	94.7188	108.2500	121.7813	135.3125	148.8438	162.3750	175.9063	189.4375
86	95.8325	109.5228	122.9132	136.9035	150.5939	164.2842	177.9746	191.6649
87	96.9465	110.7960	124.6455	138.4950	152.3445	166.1940	180.0435	193.8930
88	98.0602	112.0688	126.0774	140.0860	154.0946	168.1032	182.1118	196.1204
89	99.1739	113.3416	127.5093	141.6770	155.8447	170.0124	184.1801	198.3478
90	100.2880	114.6148	128.9417	143.2685	157.5954	171.9222	186.2491	200.5759
91	101.4017	115.8876	130.3736	144.8595	159.3455	173.8314	188.3174	202.8233
92	102.5157	117.1608	131.8059	146.4510	161.0961	175.7412	190.3863	205.0314
93	103.6294	118.4336	133.2378	148.0420	162.8462	177.6504	192.4546	207.2588
94	104.7434	119.7068	134.6701	149.6335	164.5968	179.5602	194.5235	209.4869
95	105.8572	120.9796	136.1021	151.2245	166.3470	181.4694	196.5919	211.7143
96	106.9712	122.2528	137.5344	152.8160	168.0976	183.3792	198.6608	213.9424
97	108.0849	123.5256	138.9663	154.4070	169.8877	185.2884	200.7291	216.1698
98	109.1990	124.7988	140.3987	155.9985	171.5984	187.1982	202.7981	218.3979
99	110.3126	126.0716	141.8305	157.5895	173.3484	189.1074	204.8663	220.6253
100	111.4267	127.3448	143.2629	159.1810	175.0991	191.0172	206.9353	222.8534
101	112.5404	128.6352	144.7124	160.7720	176.8492	192.9264	209.0036	225.0808
102	113.6541	129.8904	146.1267	162.3630	178.5993	194.8356	211.0719	227.3082
103	114.7681	131.1636	147.5590	163.9545	180.3499	196.7454	213.1408	229.5363
104	115.8819	132.4364	148.9910	165.5455	181.9001	198.6546	215.2092	231.7637
105	116.9956	133.7092	149.4229	167.1365	183.8502	200.5638	217.2775	233.9911
106	118.1093	134.9820	151.8548	168.7275	185.6003	202.4730	219.3458	236.2185
107	119.2229	136.2548	153.2866	170.3185	187.3503	204.3822	221.4140	238.4459
108	120.3366	137.5276	154.7185	171.9095	189.1004	206.2914	223.4823	240.6733
109	121.4504	138.8004	156.1505	173.5005	190.8506	208.2006	225.5507	242.9007
110	122.5641	140.0732	157.5824	175.0915	192.6007	210.1098	227.6190	245.1281
111	123.6781	141.3464	159.0147	176.6830	194.3513	212.0196	229.6879	247.3562
112	124.7925	142.6200	160.4475	178.2750	196.1025	213.9300	231.7575	249.5850
113	125.9066	143.8932	161.8799	179.7265	197.7132	215.8398	233.8265	251.8131
114	127.0206	145.1664	163.3122	181.4580	199.6038	217.7496	235.8954	254.0412
115	128.1350	146.4600	164.7650	183.0500	201.3550	219.6600	237.9650	256.2700
116	129.2491	147.7132	166.1774	184.6415	203.1057	221.5698	240.0340	258.4981
117	130.3631	148.9864	167.6097	186.2330	204.8563	223.4796	242.1029	260.7262
118	131.4775	150.2600	169.0425	187.8250	206.6075	225.3900	244.1725	262.9550
119	132.5916	151.3332	170.2749	189.4165	208.3572	227.2998	246.2415	265.1831
120	133.7056	152.8064	171.9072	191.0080	210.1088	229.2096	248.3104	267.4112
121	134.8193	154.0792	173.3391	192.5990	211.8989	231.1188	250.3787	269.6386
122	135.9334	155.3524	174.7715	194.1905	213.6096	233.0286	252.4477	271.8667
123	137.0474	156.6956	176.2038	195.7820	215.3602	234.9384	254.5166	274.0948
124	138.1615	157.8988	177.6362	197.3735	217.1109	236.8482	256.5856	276.3229
125	139.2755	159.1720	179.0685	198.9650	218.8615	238.7580	258.6545	278.5510

Number Teeth.	Chordal Pitch of Wheel or Pinion.							
	3½	4 inches.	4½	5 inches.	5½	6 inches.	6½	7 inches.
126	140.3892	160.4148	180.5004	200.5560	220.6116	240.6672	260.7228	280.7784
127	141.5033	161.7180	181.9328	202.1475	222.3623	242.5770	262.7918	283.0065
128	142.6173	162.9912	183.3651	203.7390	224.1129	244.4868	264.8607	285.2346
129	143.7310	164.2640	184.7970	205.3300	225.8630	246.3960	266.9290	287.4620
130	144.8451	165.5372	186.2294	206.9215	227.6137	248.3058	268.9980	289.6901
131	145.9591	166.8104	187.6617	208.5130	229.3643	250.2156	271.0669	291.9182
132	147.0728	168.0832	189.0936	210.1040	231.1144	252.1248	273.1352	294.1456
133	148.1869	169.3564	190.5260	211.6955	232.8651	254.0346	275.2042	296.3737
134	149.3009	170.6296	191.9583	213.2870	234.6157	255.9444	277.2731	298.6018
135	150.4150	171.9028	193.3907	214.8785	236.3664	257.8542	279.3421	300.8299
136	151.5286	173.1756	194.8225	216.4695	238.1164	259.7634	281.4103	303.0573
137	152.6427	174.4488	196.2549	218.0610	239.8671	261.6732	283.4793	305.2854
138	153.7568	175.7220	197.6873	219.6525	241.6178	263.5830	285.5483	307.5135
139	154.8743	176.9992	199.1241	221.2490	243.3739	265.4988	287.6237	309.7486
140	155.9849	178.2684	200.5520	222.8355	245.1191	267.4026	289.6862	311.9697
141	157.0989	179.5416	201.9843	224.4270	246.8697	269.3124	291.7551	314.1878
142	158.2130	180.8148	203.4167	226.0185	248.6204	271.2222	293.8241	316.4259
143	159.3207	182.0808	204.8409	227.6010	250.3611	273.1212	295.8813	318.6414
144	160.4407	183.3608	206.2809	229.2010	252.1211	275.0412	297.9613	320.8814
145	161.5548	184.6340	207.7133	230.7925	253.8718	276.9510	300.0303	323.1095
146	162.6688	185.9072	209.1456	232.3840	255.6224	278.8608	302.0992	325.3376
147	163.7829	187.0804	210.4780	233.9755	257.3731	280.7706	304.1682	327.5657
148	164.8969	188.4536	212.0103	235.5670	259 1237	282.6804	306.2371	329.7938
149	166.0110	189.7268	213.4427	237.1585	260.8744	284.5902	308.3061	332.0219
150	167.1250	191.0000	214.8759	238.7500	262.6250	286.5000	310.3750	334.2500
151	168.2391	192.2732	216.3074	240.3415	264.3757	288.4098	312.4440	336.4781
152	169.3531	193.5464	217.7397	241.9330	266.1263	290.3196	314.5129	338.7062
153	170.4672	194.8196	219.1721	243.5245	267.8770	292.2294	316.5819	340.9243
154	171.5812	196.0928	220.6044	245.1160	269.6276	294.1392	318.6508	343.1624
155	172.6949	197.3656	222.0363	246.9070	271.5777	296.0484	320.7191	345.3898
156	173.8090	198.6388	223.4687	248.2985	273.1284	297.9582	322.7881	347.6179
157	174.9230	199.9120	224.9010	249.8900	274.8790	299.8680	324.8570	349.8460
158	176.0371	201.1852	226.3334	251.4815	276.6297	301.7778	326.9260	352.0741
159	177.1511	202.4584	227.7657	253.0730	278.3803	303.6876	328.9949	354.3022
160	178.2652	203.7316	229.1981	254.6645	280.1310	305 5974	331.0639	356.5303
161	179.3892	205.0048	230.6304	256.2560	281.8816	307.5072	333.1328	358.7584
162	180.4933	206.2780	232.0628	257.8475	283.6323	309.4170	335.2018	360.9865
163	181.6073	207.5512	233.4951	259.4390	285.3829	311.3268	337.2707	363.2146
164	182.7210	208.8240	234.9270	261.0300	287.1330	313.2360	339.3390	365.4420
165	183.8351	210.0972	236.3594	262.6215	288.8837	315.1458	341.4080	367.6701
166	184.9491	211.3704	237.7917	264.2130	290.6343	317.0556	343.4769	369.8982
167	186.0628	212.6432	239.2236	265.8040	292.3844	319.9648	345.5452	372.1256
168	187.1769	213.9164	240.6560	267.3955	294.1351	320.8746	347.6142	374.3537
169	188.2899	215.1896	242.0873	268.9870	295.8847	322.7844	349.6821	376.5818
170	189.4049	216.4628	243.5206	270.5785	297.6363	324.6942	351.7520	378.8099
171	190.5190	217.7360	244.9530	272.1700	299.3870	326.6040	353.8210	380.0380
172	191.6331	219.0092	246.3854	273.7615	301.1377	328.5138	355.8900	383.2661
173	192.7471	220.2824	247.8177	275.3530	302.8883	330.4248	357.9601	385.4942
174	193.8612	221.5556	249.2501	276.9445	304.6390	332.3334	360.0279	387.7223
175	194.9752	222.8288	250.6824	278.5360	305.3896	334.2432	362.0968	389.9504
176	196.0893	224.1020	252.1148	280.1275	308.1403	336.1530	364.1658	392.1785
177	197.2033	225.3652	253.5371	281.7190	309.8909	338.0628	366.2347	394.4066
178	198.3174	226.6484	254.9795	283.3105	311.6416	339.9726	368.3037	396.6347
179	199.4314	227.9216	256.4118	284.9020	313.3922	341.8824	370.3726	398.8628
180	200.5455	229.1948	257.8442	286.4935	315.1429	343.7822	372.4316	401.0909
181	201.6595	230.4680	259.2765	288.0850	316.8935	345.7020	374.5105	403.3190
182	202.7736	231.7412	260.7089	289.6765	318.6442	347.6128	376.5705	405.5471
183	203.8876	233.0144	262.1412	291.2680	320.3948	349.5216	378.6484	407.7752
184	205.0020	234.2880	263.5740	292.8600	322.1460	351.4320	380.7580	410.0040
185	206.1161	235.5612	265.0064	294.4515	323.8967	353.3418	382.7870	412.2321

Number Teeth.	Chordal Pitch of Wheel or Pinion.							
	3½	4 inches.	4½	5 inches.	5½	6 inches.	6½	7 inches.
186	207.2301	236.8344	266.4387	296.0430	325.6473	355.2516	384.8549	414.4602
187	208.3442	238.1076	267.8711	297.6345	327.3980	357.1614	366.9249	416.6883
188	209.4582	239.4208	269.3434	299.2260	329.1486	359.0712	388.9938	418.9164
189	210.5723	240.6540	270.7538	300.8175	330.8993	360.9810	391.0628	421.1445
190	211.6857	241.9272	272.1681	302.4090	332.6499	362.8908	393.1317	423.3726
191	212.8004	243.2004	273.6005	304.0005	334.4006	364.8006	395.2007	425.6007
192	213.9144	244.4736	275.0328	305.5920	336.1512	366.7104	397.2696	427.8288
193	215.0281	245.7464	276.4647	307.1830	337.9013	368.6196	399.3379	430.0262
194	216.1422	247.0196	277.8971	308.7745	339.6520	370.5294	401.4069	432.2843
195	217.2562	248.2928	279.3294	310.3660	341.4026	372.4396	403.4762	434.5124
196	218.3706	249.5664	280.7622	311.9580	343.1538	374.3496	405 5454	436.7412
197	219.4847	250.8396	282.1946	313.5495	344.9045	376.2595	407.6145	438.9693
198	220.5991	252.1132	283.6274	315.1415	346.6557	378.1698	409.6840	441.1981
199	221.7131	253.3864	285.0597	316.7330	348.4063	380.0796	411.7529	443.4262
200	222.8272	254.6596	286.4921	318.3245	350.1570	381.9894	413.8219	445.6543
201	223.9412	255.9328	287.9244	319.9160	351.9076	383.8992	415.8908	447.8824
202	225.0549	257.2056	289.3563	321.5070	353.6577	385.8084	417.9591	450.1098
203	226.1685	258.4788	290.7882	323.0985	355.4079	387.7182	420.0276	452.3379
204	227.2830	259.7520	292.2210	324.6900	357.1590	389.6280	422.0970	454.5660
205	228.3971	261.0252	293.6534	326.2815	358.9097	391.5378	424.1660	456.7941
206	229.5111	262.2984	295.0857	327.8730	360.6603	393.4476	426.2349	459.0122
207	230.6252	263.5716	296.5181	329.4645	362.4110	395.3574	428.3039	461.2503
208	231.7392	264.8448	297.9504	331.0560	364.1616	397.2672	430.3728	463.4784
209	232.8533	266.1180	299.3828	332.6480	365.9128	397.1770	432.4418	465.7065
210	233.9673	267.3912	300.8151	334.2390	367.6629	401.0868	434.5107	467.9346
211	235.0814	268.6644	302.2475	335.8305	369.4136	402.9966	436.5797	470.1627
212	236.1954	269.9376	303.6798	337.4220	371.1642	404.9064	438.6486	472.3908
213	237.3091	271.2104	305.1117	339.0130	372.9143	406.8156	440.7169	474.6182
214	238.4232	272.4836	306.5441	340.6045	374.6650	408.7254	442.7859	476.8463
215	239.5369	273.7564	307.9760	342.1955	376.4151	410.6346	444.8542	479.0737
216	240.6511	275.0296	309.4085	343.7870	378.1659	412.5444	446.9233	481.3018
217	241.7649	276.3028	310.8406	345.3785	379.9163	414.4542	448.9920	483.5299
218	242.8790	277.5760	312.2730	346.9700	381.6670	416.3640	451.0610	485.7580
219	243.9931	278.8492	313.7054	348.5615	383.4177	418.2738	453.1300	487.9861
220	245.1071	280.1224	315.1377	350.1530	385.1683	420.1836	455.1989	490.2142
221	246.2212	281.3956	316.5701	351.7445	386.9190	422.0934	457.2679	492.3423
222	247.3352	282.6688	318.0024	353.3360	388.6696	424.0032	459.3368	494.6704
223	248.4493	283.9420	319.4348	354.9275	390.4203	425.9130	461.4058	496.8985
224	249.5633	285.2152	320.8671	356.5190	392.1709	427.8228	463.4747	499.1266
225	250.6774	286.4884	322.2995	358.1105	393.9216	429.7326	465.5436	501.3547
226	251.7926	287.7616	323.7318	359.7020	395.6722	431.6424	467.6126	503.5828
227	252.9055	289.0348	325.1642	361.2935	397.4229	433.5522	469.6816	505.8109
228	254.0195	290.3080	326.5965	362.8850	399.1735	435.4620	471.7505	508.0390
229	255.1336	291.5812	328.0289	364.4765	400.9242	437.3718	473.8195	510.2671
230	256.2476	292.8544	329.4612	366.0680	402.6748	439.2816	475.8884	512.4952
231	257.3616	294.1276	330.8935	367.6585	404.4244	441.1914	477.9573	514.8233
232	258.4757	295.4008	332.3259	369.2510	406.1761	443.1012	480.0263	516.9514
233	259.5898	296.6740	333.7583	370.8415	407.9258	445.0110	482.0953	519.1795
234	260.7035	297.9468	335.1902	372.4335	409.6769	446 9202	484.1636	521.4069
235	261.8175	299.2200	336.6225	374.0250	411.4275	448.8300	486.2325	523.6350
236	262.9316	300.4932	338.0549	375.6165	413.1782	450.7398	488.3015	525.8631
237	264.0456	301.7664	339.4872	377.2080	414.9288	452.6496	490.3704	528.0912
238	265.1597	303.0396	340.9196	378.7995	416.6795	454.5594	492.4394	530.2193
239	266.2737	304.1128	342.1519	380.2910	418.3301	456.4692	494.5083	532.5474
240	267.3878	305.5860	343.7843	381.9825	420.1808	458.3790	496.5773	534.7755
241	268.5018	306.8592	345.2166	383.5740	421.9314	460.2888	498.6462	537.0036
242	269.6159	308.1324	346.6490	385.1655	423.6821	462.1986	500.7152	539.2317
243	270.7299	309.4056	348.0813	386.7570	425.4327	464.1084	502.7841	531.4598
244	271.8440	310.6788	349.5137	388.3485	427.1834	466.0182	504 8531	533.6879
245	272.9580	311.9520	350.9460	389.9400	428.9340	467.9280	506.9220	535.9160

DYNAMICS.

ALGEBRAICAL AND GEOMETRICAL EXPRESSIONS OF THE FUNDAMENTAL PRINCIPLES OF DYNAMICS.

Elements.	Functions.
Force $=F$.	Power $P=F\,V$.
Space $=S$.	Velocity $V=\frac{S}{T}$.
Time $=T$.	Work $K=F\,V\,T$.
Mass $=M$.	Work $K=\frac{1}{2}\,M\,V^2$.
$F:M=V:T$.	$F:M=\frac{1}{2}\,V^2:S$.
Momentum.	**Work.**
$F\,T=M\,V$.	$F\,S=\frac{M\,V^2}{2}$.

These are the fundamental principles in Mechanics.

Dynamics is that branch of mechanics which treats of *forces in* motion, producing *power* and *work*. It comprehends the action of all kinds of machinery, manual and animal labor in the transformation of physical work.

Quantity is that which can be increased or diminished by addition or subtraction of homogeneous parts, and which can be expressed by a number.

Element is that which cannot be resolved into two or more different things.

A **Function** is composed of two or more different elements.

A function is resolved by dividing it into one or more of its elements.

Force, Space, and **Time** are simple physical elements.

Power, Velocity, and **Work** are functions of those elements.

Force (F).

Force is any action which can be expressed simply by weight, without regard to motion, time, power, or work; it is an ultimate thing which cannot be resolved into two or more things, and is therefore a simple physical element, corresponding with length in geometry.

Force is expressed by a great variety of terms, such as *attraction*, *repulsion*, *gravity*, *pressure*, *tension*, *compression*, *cohesion*, *adhesion*, *resistance*, *inertia*, *strain*, *stress*, *strength*, *thrust*, *burden*, *load*, *squeeze*, *pull*, *push*, *pinch*, *punch*, etc., the magnitude of which can be expressed by any established unit of weight.

Motive force is that which produces motion, but otherwise it is the same as static force, and is denoted by the letter F. Force is the first element in mechanics.

Motion.

Motion is a continuous change of position in regard to assumed fixed objects. Motion or rest is only relative; that is to say, when two bodies change their relative position, either one of them can be considered at rest and the other in motion. There is no absolute rest known in the universe.

Motion is expressed by the following terms: *move*, *going*, *walking*, *passing*, *transit*, *involution*, *evolution*, *run*, *locomotion*, *flux*, *rolling*, *flow*, *sweep*, *wander*, *shift*, *flight*, *current*, etc.

Velocity (V).

Velocity is speed or rate of motion; it is the space passed over in the unit of time in uniform motion, or in variable motion is the space which would be passed over in a unit of time if the velocity were rendered constant at any instant.

$$\text{Velocity} = \frac{S}{T}.$$

Velocity or rate of motion is expressed by many terms.

Quick Motion.

Speed, *swiftness*, *rapidity*, *fleetness*, *speediness*, *quickness*. *haste*, *hurry*, *race*, *forced march*, *gallop*, *trot*, *run*, *rush*, *scud*, *dash*, *spring*, etc.

Slow Motion.

Slowness, *tardiness*, *dilatoriness*, *slackness*, *drawl*, *retardation*, *hobbling*, *creeping*, *lounging*, *linger*, *sluggish*, *crawl*, *loiter*, *glide*, *languid*, *drowsy*, etc.

Angular velocity is the curvilinear velocity of a point at a unit's distance from the axis around which a body turns this point, turning with the body.

Time (T).

Time implies a continuous perception, recognized as duration.

Chronology is the science of time.

Instant and moment are points of time.

Epoch is the beginning of any time marked with some remarkable events and recorded by historians or chronologists. Era is nearly the same as epoch, except that it is generally fixed by nations or denominations, as the Christian era.

Time is expressed by a great variety of units—namely, *millennium*, a thousand years; *century*, one hundred years; *score*, twenty years; *year*, *season*, *month*, *fortnight*, *week*, *day*, *hour*, *minute*, and *second*.

Time is an ultimate thing which cannot be resolved into two or more different things, and is therefore a simple physical element.

Power ($P = F\,V$).

Power is the product of force and velocity, and is therefore a function.

A force multiplied by the velocity with which it is acting is the power in operation.

The English unit for measuring power is a force of one pound acting with a velocity of one foot per second, called foot-pound.

Man-power is a unit of power established by Morin to be equivalent to 50 foot-pounds of power, or 50 effects; that is to say, a man turning a crank with a force of 50 pounds and with a velocity of one foot per second is a standard man-power, or a force of 25 pounds by two feet per second is a man-power.

An ordinary workingman can exert this power eight hours per day without overstraining himself.

Horse-power is a unit of power established by James Watt to be equivalent to a force of 33,000 pounds acting with a velocity of one foot per minute, which is the same as a force of 550 pounds acting with a velocity of one foot per second.

That is to say, one horse-power is 550 foot-pounds per second, or 11 man-power.

The product of any force in pounds and its velocity in feet per second divided by 550 gives the horse-power.

Power is the differential of work or any action which produces work, whether mental or physical.

Power multiplied by the time of action is work; work divided by time is power. Writers on dynamics have heretofore assumed "*power is the work done in a unit of time.*"

The number which expresses the work done in a unit of time is equal to the number which expresses the power in operation.

When we say "in a certain time," which is equivalent to the expression "per unit of time," we divide by the time.

Work is the product of the two elements **Force** and **Space.** When we divide work by the time of its operation, the result is *power*, which is the product of *force* and *velocity*.

Power may be expressed by the following terms:

Traction, *propulsion*, *impulsion*, *capability*, *puissance*, *labor*, *haul*, *drag*, *draw*, *heave*, *occupation*, *activity*, *vigor*, *energy*, etc., or any action which implies force and motion with regard to time.

Space ($S=VT$).

Space in dynamics means linear space; it is an element.
Space is herein denoted by

$$S=VT,$$

which means that the space S, expressed in linear feet, is the product obtained by multiplying together the velocity V and time T.

Mr. Nystrom says velocity is an element and space is a function of velocity and time.

There does not appear to be any difficulty in conceiving space to be an element apart from time or velocity.—W. D. M.

Geometrical spaces are magnitudes of three different kinds—namely, *linear*, *superficial*, and *voluminous*.

Linear space is that generated by the motion of a point.

Superficial space is that generated by the lateral motion of a line.

Voluminous space is that generated by the lateral motion of a plane.

Space in dynamics means the *generation* of that space by velocity and time. A line of any kind cannot be drawn without velocity and time.

A locomotive running with a uniform velocity of 30 miles per hour will make 2640 feet per minute or 44 feet per second; and if we diminish the spaces and time to infinitely small values, or, say, absolutely nothing, the velocity is still constant when passing that time and space reduced to a point.

Work ($K=FS$).

Work is the product obtained by multiplying together the elements *force* F and *space* S.

Work may also be expressed by $K=PT$, or the product of power and time.

The work of a steam-engine operating with a constant power will be directly as the time of operation, and so with all labor, whether it be mechanical or manual.

Moment of a Force (Fl). The moment of a force is its lever arm at right angles to its direction of action multiplied by its intensity in pounds or tons.

Momentum (MV).

The momentum of a moving body is the intensity of that *constant force* which, resisting its movement, will bring it to rest in *one second*.

$$M=\frac{weight}{32.2}.$$

$V=$ velocity in feet per second.

Moment of Inertia (MVr).

The moment of inertia of a rotating body is the moment of its momentum, and is equal to its momentum MV multiplied by its radius of oscillation r.

It is the universal custom to consider the angular velocity as unity, and we thus obtain a mathematical expression which is the *comparative measure* of the moment of inertia only.

Nevertheless, the true moment of inertia is a real thing, and can be expressed as the statical moment of a *constant* force acting for *one second*.

Many able writers on mechanics do not seem to have a clear physical conception of its true meaning.—W. D. M.

Virtual Velocities.

By the theorem of virtual velocities is meant the instantaneous equality of elementary quantities of power transmitted.

The reason for its name does not appear clearly. It is used in older works on mechanics and higher forms of analytical mechanics.

Radius of Oscillation.

The radius of oscillation is the mean lever-arm of the momentum of a revolving body. It is equal to the moment of inertia divided by the momentum of the revolving body.

Radius of Gyration.

The square of the radius of gyration of an oscillating body is equal to the product of the radius of oscillation and of the distance of the centre of gravity of the suspended body from its point of suspension.

The intensity of the force of momentum is proportional to the distance of the centre of gravity from the axis of suspension, and the mean leverage of the momentum is the radius of oscillation. The square of the "radius of gyration," then, is a convenient product of these two quantities, as including both, and therefore giving them in a convenient mathematical form. If a straight rod be balanced at its middle, we are obliged to consider each half separately and add them together.

While we can locate both centres of gravity and of oscillation, we cannot locate a centre of gyration, nor has it an actual physical existence, being a product of two quantities only.

This quantity has proved a constant stumbling-block to students of mechanism, but a little reflection and the solution of a few examples will make its nature clear.—W. D. M.

Units of Work.—Foot-Pound.

The English unit of work is assumed to be that accomplished by a force of one pound raising an equal weight one foot high, which unit is called a **foot-pound.** Then a force of 6 pounds working through a space of 4 feet is equivalent to 24 foot-pounds of work.

This unit is very convenient for small amounts of work, but it is too small for many purposes in practice.

Foot-Ton.

English ordnance officers have adopted a larger unit for work—namely, **foot-ton,** which is used for expressing work of heavy ordnance. It means the work of lifting one ton one foot high.

Workmanday.

A laborer working eight hours per day can exert a power of 50 foot-pounds per second. A day's work will then be $50 \times 8 \times 60 \times 60 = 1{,}440{,}000$ foot-pounds of work, which may be termed a *workmanday.*

All kinds of heavy work can be estimated in workmandays, such as the building of a house, a bridge, a steamboat, canal and railroad excavations and embankments, loading or unloading a ship, powder and steam-boiler explosions, the capability of heavy ordnance, etc.

The magnitude of the unit *workmanday* is easily conceived, because it is that amount of work which a laborer can accomplish in one day. Work expressed in foot-pounds, divided by 1,440,000, gives the work in *workmandays.*

A work of 20 workmandays can be accomplished by 20 men in 1 day, by one man in 20 days, by 4 men in 5 days, or by 10 men in 2 days.

Work done is expressed by the following terms:

Hauled, dragged, raised, heaved, cultivated, tilled, broken, crushed, thrown, wrought, fermented, labored, embroidered, etc., or any expression which implies the three simple elements *force, velocity,* and *time.*

Power is the differential of work.

Work is the integral of power.

DYNAMICAL FORMULAS.

Force or Pressure in Pounds.

$F=\frac{P}{V}$. . . . . . . 1.

$F=\frac{550\,HP}{V}$. . . . . . 2.

$F=\frac{K}{S}$. . . . . . . 3.

$F=\frac{K}{VT}$. . . . . . . 4.

Velocity in Feet per Second. For Uniform Motion.

$V=\frac{S}{T}$. . . . . . . 5.

$V=\frac{P}{F}$. . . . . . . 6.

$V=\frac{550\,HP}{F}$. . . . . . 7.

$V=\frac{K}{FT}$. . . . . . . 8.

Time of Action in Seconds. For Uniform Motion.

$T=\frac{S}{V}$. . . . . . . 9.

$T=\frac{FS}{P}$. . . . . . . 10.

$T=\frac{FS}{550\,HP}$. . . . . . 11.

$T=\frac{K}{FV}$. . . . . . . 12.

Power in Effects. Work per Second.

$P=FV$. . . . . . . 13.

$P=\frac{FS}{T}$. . . . . . . 14.

$P=550\,HP$. . . . . . 15.

$P=\frac{K}{T}$. . . . . . . 16.

Space Passed Through in the Time T.

$S=VT$. . . . . . . 17.

$S=\frac{PT}{F}$. . . . . . . 18.

$S=\frac{550\,T\,HP}{F}$. . . . . . 19.

$S=\frac{K}{F}$. . . . . . . 20.

Horse-Power.

$HP=\frac{P}{550}$. . . . . . . 21.

$HP=\frac{FV}{550}$. . . . . . . 22.

$HP=\frac{FS}{550\,T}$. . . . . . 23.

$HP=\frac{K}{550\,T}$. . . . . . 24.

Work in Foot-Pounds.

$K=FVT$ in time T. . . 25.

$K=PT$ in time T. . . 26.

$K=FS$. . . . . . . 27.

$K=550\,HP\,T$ in time T. . 28.

It will be observed in the preceding formulas that an element is never divided by an element, but a function is divided by an element only when that function contains that element.

Power divided by velocity gives force, because power contains the elements force and velocity; but power cannot be divided by time, because time is not a constituent element of power.

Work can be divided by either one or two of its three constituent factors. When work is divided by either two of its elements, the product will be the third element.

Different elements or functions cannot be added to or subtracted from one another. Power or space cannot be added to or subtracted from work. Force, velocity, or time cannot be added to or subtracted from space.

When a formula contains several terms, all the terms must be of the same kind; for instance:

$$\text{Work } K = T\left(FV + P - \frac{K}{T}\right).$$

The terms within the parentheses are all power, which multiplied by time gives work.

Mistakes in dynamical formulas are easily detected by the above rules.

No element can be converted into an element of a different kind.

Different Kinds of Foot-Pounds.

There are two different kinds of foot-pounds in mechanics—namely,

1st. A foot-pound of static moment, which is force in pounds multiplied by by its lever of action in feet.

2d. A foot-pound of work is force in pounds multiplied by space in feet.

It will be observed that foot-pounds of static moment and foot-pounds of work are both the product of force and linear space, from which it would appear that these two functions are substantially alike; but they are of entirely different nature.

Static moment is force multiplied by the geometrical element length, without regard to velocity and time; in which case the force has nothing to do with the generation of that length.

Work is force multipied by space.

EXAMPLES CORRESPONDING WITH THE FORMULAS.

Force or Pressure in Pounds.

Example 1. A power $P = 6400$ effects is operating with a velocity of $V = 12$ feet per second. Required the force F?

$$F = \frac{P}{V} = \frac{6400}{12} = 533 \text{ pounds.}$$

Example 2. The piston of a steam-engine of $IP = 24$ horses is moving at the rate of $V = 8$ feet per second. Required the force F?

$$F = \frac{550\ IP}{V} = \frac{550 \times 24}{8} = 1650 \text{ pounds.}$$

Example 3. A work of $K = 3266$ foot-pounds is accomplished in a space $S = 16$ feet. Required the force F?

$$F = \frac{K}{S} = \frac{3266}{16} = 204 \text{ pounds.}$$

Example 4. A work of $K = 183600$ foot-pounds was accomplished with a velocity $V = 18$ feet per second in a time of 3 minutes, or $T = 3 \times 60 = 180$ seconds. Required the force F?

$$F = \frac{K}{V\,T} = \frac{183600}{18 \times 180} = 56.6 \text{ pounds.}$$

Velocity in Feet per Second.

Example 5. A body moves through a space of $S = 160$ feet in a time o $T = 40$ seconds. Required the velocity V?

$$V = \frac{S}{T} = \frac{160}{40} = 4 \text{ feet per second.}$$

Example 6. A power of $P = 4266$ effects is operating with a force $F = 76$ pounds. Required the velocity V?

$$V = \frac{P}{F} = \frac{4266}{760} = 5.6 \text{ feet per second.}$$

Example 7. The cylinder of a steam-engine of $IP = 160$ horse-power is 2 inches in diameter, and the effective steam-pressure is 30 pounds to the square inch. Required the velocity of the steam-piston?

The area of the piston is 452.39 square inches, which multiplied by 30 pounds to the square inch will be a force of

$$F = 13570.8 \text{ pounds.}$$

$$V = \frac{550\ IP}{F} = \frac{550 \times 160}{13570.8} = 6.5 \text{ feet per second.}$$

Example 8. A work of $K = 864360$ foot-pounds is accomplished with a force of $F = 68$ pounds in a time of 5 minutes. Required the velocity V?

The time $T = 5 \times 60 = 300$ seconds.

$$V = \frac{K}{F\,T} = \frac{864360}{68 \times 300} = 42.4 \text{ feet per second.}$$

Time of Action in Seconds.

Example 9. A space of $S = 2896$ feet is generated with a velocity of $V = 25$ feet per second. Required the time T?

$$T = \frac{S}{V} = \frac{2896}{25} = 115.84 \text{ seconds.}$$

Example 10. A force of $F = 4596$ pounds is working through a space $S = 960$ feet. What time is required for the force to generate a power of $P = 840680$ effects?

$$T = \frac{F\,S}{P} = \frac{4596 \times 960}{840680} = 5.25 \text{ seconds.}$$

Example 11 *a*. The stroke of a steam-piston is four feet, and the effective pressure of steam is $F = 46360$ pounds. The power of the engine is $IP = 500$ horse-power. What time is required of the engine to make 64 double strokes?

The space $S = 4 \times 2 \times 64 = 512$ feet.

$$T = \frac{F\,S}{550\ IP} = \frac{46360 \times 512}{550 \times 500} = 86 \text{ seconds.}$$

Example 11 *b*. What time is required to raise a weight of 200 tons to a height of $S = 50$ feet with an engine of $IP = 8$ horse-power?

$$F = 200 \times 2240 = 448000 \text{ pounds.}$$

$$T = \frac{F\,S}{550\ IP} = \frac{448000 \times 50}{550 \times 8} = 509 \text{ seconds,}$$

or 8 minutes and 29 seconds.

Example 12. What time is required to accomplish a work of $K = 96236000$ foot-pounds, with a force $F = 88$ pounds, moving with a velocity of $V = 1.5$ feet per second?

$$T = \frac{96236000}{88 \times 1.5} = 729066 \text{ seconds,}$$

or 202 hours 31 minutes and 6 seconds.

Assuming a workmanday to be 1,440,000 foot-pounds, it would require about 67 such units to accomplish the work; that is to say, one man could do the work in 67 days, or 67 men could accomplish it in one day.

Power in Effects or Foot-Pounds.

Example 13. A weight of five tons is raised vertically at the rate of $1\frac{1}{2}$ inches per second. Required the power P?

The force $F = 5 \times 2240 = 11200$ pounds.

Velocity $V = 0.125$ feet per second.

$$P = 11200 \times 0.125 = 1400 \text{ ft.-lbs. per second.}$$

One man-power is 50 effects, and it would require $1400 : 50 = 28$ men to raise five tons with a velocity of $1\frac{1}{2}$ inches per second at continued work.

One horse-power is 550 effects, and it would require $1400 : 550 = 2.55$ horse-power for the same work.

Example 14. What power is required to lift a weight of three tons a space of $S = 5$ feet in a time of 10 minutes?

$$P = \frac{FS}{T} = \frac{3 \times 2240 \times 5}{10 \times 60} = 56 \text{ ft.-lbs. per second.}$$

Example 15. How many effects are there in $IP = 30$ horse-power?

$$P = 550 \times 30 = 16500 \text{ effects, or ft.-lbs. per second.}$$

Example 16. What power is required to do a work of $K = 186000$ foot-pounds in one minute? $T = 60$.

$$P = \frac{186000}{60} = 31000 \text{ effects.}$$

Space Passed Through in the Time T.

Example 17. A body moving with a velocity of $V = 960$ feet per second for a time of $T = 5$ seconds. Required the space passed through?

$$S = V\,T = 4800 \text{ feet.}$$

Example 18. A power of $P = 6500$ effects is operating for a time of $T = 12$ seconds with a force $F = 240$ pounds. Required the space passed through?

$$S = \frac{P\,T}{F} = \frac{6500 \times 12}{240} = 325 \text{ feet.}$$

Example 19. To what height can a steam-engine of $IP = 6$ horse-power lift a weight of 25 tons in a time of 5 minutes?

$$F = 25 \times 2240 = 56000 \text{ pounds.}$$

$$T = 5 \times 60 = 300 \text{ seconds.}$$

$$\text{The height } S = \frac{550 \times IP\;T}{F} = \frac{550 \times 8 \times 300}{56000} = 23.6 \text{ feet.}$$

Example 20. A work of $K = 7280$ foot-pounds is to be accomplished by a force of $F = 24$ pounds. In what space can the force do the work?

$$S = \frac{K}{F} = \frac{7280}{24} = 304 \text{ feet.}$$

Horse-Power.

Example 21. How many horse-power are there in $P = 56680$ effects?

$$IP = \frac{P}{550} = \frac{56680}{550} = 103 \text{ horse-power.}$$

Example 22. A weight of three tons is to be raised with a velocity of $V = 6$ feet per second. Required the horse-power?

$$IP = \frac{F\,V}{550} = \frac{3 \times 2240 \times 6}{550} = 73.3 \text{ horse-power.}$$

Example 23. A steam-crane is to be constructed to lift 30 tons 12 feet high in 5 minutes. Required the horse-power?

Force $F = 30 \times 2240 = 67200$ pounds.

Time $T = 5 \times 60 = 300$ seconds.

$$IP = \frac{FS}{550\,T} = \frac{67200 \times 12}{550 \times 300} = 5 \text{ horse-power, nearly.}$$

Example 24. What horse-power is required to accomplish a work of $K=346000$ foot-pounds in $T=5$ seconds?

$$IP = \frac{K}{550\ T} = \frac{346000}{550 \times 5} = 12.6 \text{ horse-power.}$$

WORK IN FOOT-POUNDS.

Example 25. How much work is accomplished with a force of $F=280$ pounds, moving with a velocity of $V=9$ feet per second for a time of $T=1200$ seconds, or 20 minutes?

$$K = F\ V\ T = 280 \times 9 \times 1200 = 3024000 \text{ foot-pounds.}$$

Example 26. How much work can be accomplished by a power of $P=36$ effects during $T=4$ seconds?

$$K = P\ T = 36 \times 4 = 144 \text{ foot-pounds.}$$

Example 27. A weight of 25 tons is lifted $S=18$ feet. Required the work?

$$K = F\ S = 25 \times 2240 \times 18 = 1008000 \text{ foot-pounds.}$$

Example 28. How much work is accomplished per minute by an engine of $IP=48$ horse-power?

$$K = 550\ IP\ T = 550 \times 48 \times 60 = 1584000 \text{ foot-pounds.}$$

Dynamics of Circular or Rotary Motion.

In circular motion it is supposed that the motive force is applied in the direction of the tangent to the circle of radius R in feet, like that of a belt or rope over a pulley or in all kinds of gearing.

$n=$ revolutions of the circle per minute.

$N=$ total number of revolutions in the time T, or for generating a definite circular space S, and also for accomplishing a definite work K.

Example 33. The radius of a wheel or crank-pin is $R=2.5$ feet, and makes $n=56$ revolutions per minute. Required the velocity in the circumference?

$$V = \frac{2\ \pi\ R\ n}{60} = 0.10472\ R\ n = 0.10472 \times 2.5 \times 56 = 20.6 \text{ feet per second.}$$

Example 35. A pulley of 54 inches diameter, or $R=2.25$ feet, is to run a belt $V=60$ feet per second. Required the revolutions per minute?

$$\text{Revolutions } n = \frac{9.55\ V}{R} = \frac{9.55 \times 60}{2.25} = 254\tfrac{2}{3} \text{ per minute.}$$

Example 41. A pulley is to make $n=150$ revolutions per minute with a velocity of the belt $V=50$ feet per second. Required the radius of the pulley?

$$\text{Radius } R = \frac{9.55\ V}{n} = \frac{9.55 \times 50}{150} = 3.183 \text{ feet.}$$

Example 34. Find the velocity of the circumference of a pulley 27 inches diameter, making $n=250$ revolutions per minute? 27 inches $=2.25$ feet. $R=1.125$.

$$\text{Velocity } V = 0.1047 \times 1.125 \times 250 = 29.45 \text{ feet per second.}$$

Example 36. A pulley of $R=1.5$ feet radius is to transmit $IP=4.8$ horse-power with a motive force $F=64$ pounds. Required the number of revolutions per minute?

$$n = \frac{5250\ IP}{F\ R} = \frac{5250 \times 4.8}{64 \times 1.5} = 262.5 \text{ per minute.}$$

Force F Acting in the Direction of the Tangent.

$$F = \frac{60\ P}{2\ \pi\ R\ n} \quad . \quad . \quad . \quad 29.$$

$$F = \frac{9.55\ P}{R\ n} \quad . \quad . \quad . \quad 30.$$

$$F = \frac{9.55\ K}{R\ n\ T} \quad . \quad . \quad . \quad 31.$$

$$F = \frac{5252\ IP}{R\ n} \quad . \quad . \quad . \quad 32.$$

Circumferential Velocity and Revolutions per Minute.

$$V = \frac{2\ \pi\ R\ n}{60} \quad . \quad . \quad . \quad 33.$$

$$V = 0.10472\ R\ n. \quad . \quad . \quad . \quad 34.$$

$$n = \frac{9.55\ V}{R} \quad . \quad . \quad . \quad 35.$$

$$n = \frac{5252\ IP}{F\ R} \quad . \quad . \quad . \quad 36.$$

Time of Operation in Seconds.

$$T = \frac{9.55\ S}{R\ n} \quad . \quad . \quad . \quad 37.$$

$$T = \frac{9.55\ K}{F\ R\ n} \quad . \quad . \quad . \quad 38.$$

$$T = \frac{F\ R\ n}{9.55\ P} \quad . \quad . \quad . \quad 39.$$

$$T = \frac{F\ R\ N}{87.5\ IP} \quad . \quad . \quad . \quad 40.$$

Radius of Revolution.

$$R = \frac{9.55\ V}{n} \quad . \quad . \quad . \quad 41.$$

$$R = \frac{9.55\ P}{F\ n} \quad . \quad . \quad . \quad 42.$$

$$R = \frac{5252\ IP}{F\ n} \quad . \quad . \quad . \quad 43.$$

$$R = \frac{9.55\ K}{F\ n\ T} \quad . \quad . \quad . \quad 44.$$

Power Generated in Effects, or Foot-pounds per Second.

$$P = \frac{2\ \pi\ R\ n\ F}{60} \quad . \quad . \quad . \quad 45.$$

$$P = \frac{F\ R\ n}{9.55} \quad . \quad . \quad . \quad 46.$$

$$P = \frac{F\ R\ N}{9.55\ T} \quad . \quad . \quad . \quad 47.$$

$$N = \frac{9.55\ P\ T}{F\ R} \quad . \quad . \quad . \quad 48.$$

Space Generated in Feet.

$$S = \frac{2\ \pi\ R\ n\ T}{60} \quad . \quad . \quad . \quad 49.$$

$$S = \frac{R\ n\ T}{9.55} \quad . \quad . \quad . \quad 50.$$

$$S = \frac{F\ n\ N}{755.625\ IP} \quad . \quad . \quad . \quad 51.$$

$$S = N\ 2\ \pi\ R. \quad . \quad . \quad . \quad 52.$$

Horse-Power Generated.

$$IP = \frac{F\ R\ n}{5252} \quad . \quad . \quad . \quad 53.$$

$$IP = \frac{F\ R\ N}{87.5\ T} \quad . \quad . \quad . \quad 54.$$

$$N = \frac{87.5\ IP\ T}{F\ R} \quad . \quad . \quad . \quad 55.$$

$$N = \frac{S}{2\ \pi\ R} \quad . \quad . \quad . \quad 56.$$

Work Accomplished in Foot-pounds in Time T.

$$K = \frac{F R n\ T}{9.55} \quad . \quad . \quad . \quad 57.$$

$$K = F\ 2\ \pi\ R\ N. \quad . \quad . \quad . \quad 58.$$

$$N = \frac{K}{F\ 2\ \pi\ R} \quad . \quad . \quad . \quad 59.$$

$$R = \frac{K}{F\ 2\ \pi\ N} \quad . \quad . \quad . \quad 60.$$

OBSERVED RESULTS OF POWER.

Description of Works.	Work-hrs. per day.	Force. F	Veloc'y V	Effects, or ft.-lbs. per sec. P.	Horses. H
A man can raise a weight by a single fixed pulley,	6	50	0.8	40	0.072
A man working a crank,	8	20	2.5	50	0.090
A man on a tread-wheel (horizontal),	8	144	0.5	72	0.130
A man in a tread-wheel (axis 24° from vertical),	8	30	2.3	69	0.125
A man draws or pushes in a horizontal direction,	8	30	2	60	0.109
A man pulls up or down,	8	12	3.7	44.4	0.080
A man can bear on his back,	7	95	2.5	237.5	. . .
A horse in a horse-mill, walking moderately,	8	106	3	318	0.577
A horse in a horse-mill, running fast,	5	72	9	648	1.178
An ox in a horse-mill, walking moderately,	8	154	2	308	0.558
A mule " " "	8	71	3	213	0.308
An ass " " "	8	33	2.65	87.4	0.160
On bad Foot-roads, like those in Peru.					
A man can bear,	10	50	3.5	175	
Llama of Peru can bear,	10	100	3.5	350	
Donkey can bear,	10	200	3.5	700	
Mule can bear,	10	400	5	2000	
Flour Mills,					
For every 100 pounds of fine flour ground per hour, require,				550	1.000
One pair of mill-stones of 4 feet diameter, making 120 revolutions per minute, can grind 5 bushels of wheat to fine flour per hour,				2400	4.36
One pair of mill-stones of 4 feet diameter, making 120 revolutions per minute, can grind 5 bushels of rye to coarse flour per hour,				1600	2.91
Saw Mills, reciprocating.					
For every 20 square feet sawed per hour, in dry oak, there requires,				550	1.000
Dry pine, 30 square feet per hour,				550	1.000
Circular Saw.					
A saw 2.5 feet in diameter, and making 270 revolutions per minute, will saw 40 square feet in oak per hour, with				550	1.000
In dry spruce, 70 square feet per hour,				550	1.000
Threshing Machine.					
Velocity of the feed-rollers at the circumference, 0.55 feet per second. Diameter of threshing-cylinder 3.5 feet and 4½ feet long, making 300 revolutions per minute, can thresh from 30 to 40 bushels of oats, and from 25 to 35 bushels of wheat, per hour,				2200	4.000
One man with a flail can thresh half a bushel per hour (wheat),				70	0.127
Rolling Mills.					
Bar iron-mills. Two pair of rough rollers, two pair of finishing rollers, six puddle furnaces, two welding furnaces, making 10 tons of bar iron per 24 hours, rollers making 70 revolutions per minute, require,				29000	52.7
Plate-mill requires about five *IP* per square foot of plates rolled. Largest size plate rollers should not make over 30 revolutions per minute.					

DREDGING-MACHINES.

Ladder-Dredge.—The ladder-dredge consists of an endless chain upon which a number of buckets are fixed and work continually like a Noria. This appears to be the best form of dredge for deepening harbors, but is not so well suited for docks, where the dipper and grapple dredges are the best.

Notation.

T = tons of materials excavated and raised per hour.
h = height in feet to which the materials are raised above the bottom of the excavated channel.
k = 0.1 for hard clay with gravel.
k = 0.07 for hard pure clay.
k = 0.05 for common clay or sand.
k = 0.04 for soft clay or loose sand.
k = 0.03 for very loose materials.
IP = horse-power required for excavating and raising the materials.
F = force in pounds required to feed the dredge ahead.
v = velocity of the buckets in feet per second.

Formulas.

$$IP = T\left(\frac{h}{700} + k\right) \qquad 1.$$

$$T = \frac{700\,IP}{h + 700\,k}. \qquad 2.$$

$$F = \frac{550\,IP}{v}. \qquad 3.$$

$$F = \frac{550\,Tk}{v}. \qquad 4.$$

$$k = \frac{IP}{T} + \frac{h}{700}. \qquad 5.$$

Example 1. What power is required to excavate $T = 160$ tons of hard pure clay per hour, and raise it up $h = 25$ feet above the bottom of the channel? For hard clay $k = 0.07$.

$$IP = 160\left(\frac{25}{700} + 0.07\right) = 16.9, \text{ or 17 horses.}$$

Example 2. What force F = ? is required to feed the dredge ahead for the above example when the buckets move $v = 1$ foot per second?

$$F = \frac{550 \times 16.9}{1} = 9295 \text{ pounds.}$$

Dipper-Dredge, consisting of one scoop, worked with a triple chain wound on a 15-inch drum, and driven by a pair of engines 10 inches in diameter by 15 inches stroke of cylinders. Under ordinary work the scoop makes 30 to 40 dips per hour, and takes up about two cubic yards, or three tons, of materials each dip.

The dipper-dredge is used in harbors and docks, and also in railroad excavations.

Grapple-Dredge, consisting of a double scoop opening in the bottom like a mouth, takes up about five tons of materials each grapple. It is worked by a single chain wound on a drum three feet in diameter, with a pair of engines 14 inches diameter by 20 inches stroke of cylinders. Under ordinary work it makes 50 to 60 grapples per hour.

BELTING.

Preliminary Note by Reviser.

The English rule for belting is, A single-thickness leather belt, one inch wide, running at a speed of 1000 feet per minute, will transmit about one horse-power. Double-thickness belts will do twice as much. However tightly drawn, belts slack themselves in running, and it is not safe to overload them or there will be trouble shortly from slipping. At high speeds centrifugal force greatly diminishes the adhesion of belts to pulleys. Belts should never be run on cast-iron pulleys at a higher speed than 8000 feet per minute. Formulæ 13 and 26, page 399, give rational formulæ for belts, taken from Rankine.

BELTING.

Fig. 1, Plate II., represents a pulley hung from the points a and b by the belt T, t, forming an angle of contact $2z$ on the pulley. The weight W is h... on the pulley for stretching the belt from a and b, in which case the tens... T and t will be alike. The letters on the illustrations correspond with ... letters in the formulas, and the number of each example corresponds w... the number of the formula used.

W = weight in pounds hung on the belt.
T and t = respective tensions of the belt in pounds.
Z = half the angle of contact of the belt on the pulley.
C = force of contact of the belt on the pulley in pounds.
F = motive force in pounds transmitted by the belt.
f = friction coefficient of the surfaces in contact.

$F=(T-t)$. $T=(F+t)$. $t=(T-F)$. $T+t=F+2t=2T-F$.

Fig. 2, Plate II., represents two pulleys of different sizes and connec... by a belt T, t, in which case the smallest pulley will be in the same condit... as that in Fig. 1, but the weight W is the pressure in the journal boxes, a... the system is arranged for transmitting power. The greatest motive fo... F that can be transmitted by the belt cannot exceed the product of the fo... of contact C and the friction f without slipping of the belt.

When $F > fC$, the belt will slide.
When $F < fC$, there is no sliding.

In good practice the motive force F should not exceed 75 per cent. of fC

Example 1. Fig. 1. The weight $W = 84$ pounds, and half the angle of co...tact $Z = 45°$. Required the sum of tensions.

$$\text{Tension } T+t=\frac{84}{\sin. 45°}=\frac{84}{0.70711}=118.7928 \text{ pounds.}$$

That is, $118.7928 : 2 = 59.3964$ pounds, the tension at each point of suspensi... a and b.

Example 2. It is found by experiment that the tension at a is 41.36 poun... that is $T+t=41.36\times 2=82.72$ pounds, half the angle of contact being Z ... 48° 36′. Required the weight W.

Weight $W = 82.72\times \sin. 48° 36' = 82.72\times 0.75011 = 61.05$ pounds.

Example 4. The suspended weight $W = 86$ pounds and the angle of conta... 120°, making $Z = 60°$. Required the force of contact of the belt on the pull...

$$C=\frac{86\times 3.1416\times 60}{180\times \sin.60°}=104 \text{ pounds.}$$

Example 9. Fig. 2. What motive force F can be transmitted by a leath... belt of tension $T+t=450$ pounds when at rest, half the angle of contact Z ... 78° on a smooth cast-iron pulley of friction $f=0.35$?

$$\text{Motive force } F=\frac{0.35\times 3.1416\times 78\times 450}{180}=214.4 \text{ pounds.}$$

This is the maximum motive force that can be applied under the given co...ditions, but only 75 per cent. of it should be applied in practice, or 214.4 ... $0.75 = 160.8$ pounds.

When at rest, $(T+t)=450$ pounds, or $T=t=225$ pounds, but when in m...tion half the motive force is added to $T=250+107.2=357.2$ pounds, th... pulling tension, and the other half of the motive force is subtracted from t ... $225-107.2=117.8$ pounds, the slack tension. This rule is, however, infl...enced by the grade of elasticity of the belt.

Example 10. How much tension must be given to a belt when at rest, i... order to transmit when in motion a motive force $F=500$ pounds, when th... angle of contact is 165°, or $Z = 82° 30'$, and the friction coefficient $f=0.4$?

$$T+t=\frac{180\times 500}{0.4\times 3.1416\times 82.5}=868.1 \text{ pounds.}$$

The tension of the belt should be $868 : 2 = 434$ pounds, to which must b... added $\frac{1}{3}$d for practical working, making the required tension 579 pounds.

The friction coefficient is found by formulas 13 and 26.

The formulas will answer equally well for any system of weights an... measures.

Formulas for Oblique Belting, Figs. 1 and 2.

$$(T+t) : W = 1 : \sin. Z.$$

Sum of tensions $(T+t) = \dfrac{W}{\sin. Z}$ . . 1.

Weight $W = (T+t) \sin. Z.$ . . . . . 2.

Half-angl. cont. $\sin. Z = \dfrac{W}{(T+t)}$. . . 3.

*Force of contact $C = \dfrac{W \pi Z}{180^\circ \sin. Z}$ . . 4.

*Force of contact $C = \dfrac{\pi Z (T+t)}{180}$. . . 5.

Sum of tensions $(T+t) = \dfrac{180\, c}{\pi Z}$. . . 6.

Half-ang. cont. $Z = \dfrac{180^\circ C}{\pi (T+t)}$. . . . . 7.

Weight suspended $W = \dfrac{180^\circ C \sin. Z}{\pi Z}$. 8.

Motive force $F = \dfrac{f \pi Z (T+t)}{180^\circ}$. . . . . 9.

Sum of tensions $(T+t) = \dfrac{180^\circ F}{f \pi Z}$. 10.

Greatest tension $T = t\left(\dfrac{180^\circ + f\pi Z}{180^\circ - f\pi Z}\right)$. 11.

Slack tension $t = T\left(\dfrac{180^\circ - f\pi Z}{180^\circ + f\pi Z}\right)$. . 12.

(Weisbach's formula.)

Motive force $F = t\,(e^{2fz} - 1) = \dfrac{550\, HP}{V}$. 13.

$e = 2.7183$ V = speed of belt in ft. per sec.

Formulas for Parallel Belting on Pulleys of Equal Diameters.

Sum of tensions $(T+t) = W$ . . . . 14.

Pressure in journals $W = (T+t)$. . 15.

Half-angl. cont. $\sin. Z = \dfrac{W}{(T+t)} = 1$ 16.

*Force of contact $C = \dfrac{W\pi}{2}$ . . . . . . 17.

*Force of contact $C = \dfrac{\pi (T+t)}{2}$ . . . 18.

Sum of tensions $(T+t) = \dfrac{2C}{\pi}$. . . . 19.

Half-angle cont. $Z = 90^\circ$. . . . . . . 20.

Pressure in journals $W = \dfrac{2C}{\pi}$. . . . 21.

Motive force $F = \dfrac{f\pi (T+t)}{2}$. . . . . . 22.

Sum of tensions $(T+t) = \dfrac{2F}{f\pi}$ . . . 23.

Formulas for Oblique Belting (Weisbach's formulæ).

Greatest tension $T = t\, e^{2fz}$. . . . . . 24.

Slack tension $t = \dfrac{F}{e^{2fz} - 1}$. . . . . . 25.

Motive force $F_{z=90^\circ} = t\,(e^{\pi f} - 1)$

$= \dfrac{550\, HP}{V}$. . . . . . . 26.

* Factor of adhesion of belt to pulley.

Friction Coefficient for Different Surfaces in Contact.

Surface of pulley.	Condition of leather belt.				India-rubber belt.	Canvas belt.	Gutta-percha belt.
	Hair side on pulley.	Flesh side on pulley.	Wet belt.	Good adhesive.			
	f	f	f	f	f	f	f
Rubber..............	0.50	0.46	0.42	......	0.43	0.30	0.42
Leather.............	0.48	0.45	0.50	0.60	0.42	0.27	0.40
Wood	0.46	0.40	0.48	0.55	0.41	0.23	0.38
Iron..................	0.40	0.35	0.45	0.50	0.38	0.20	0.35

TABLE I.—Motive Force F, when the Pulling Tension T = 1.

Angle of contact.	Friction coefficient *f*, for the surfaces in contact on the smallest pulley.									
	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
2 Z	F	F	F	F	F	F	F	F	F	F
60°	0.147	0.189	0.231	0.272	0.310	0.346	0.367	0.415	0.447	0.462
70°	0.171	0.219	0.269	0.316	0.352	0.393	0.455	0.467	0.481	0.536
80°	0.189	0.245	0.297	0.346	0.393	0.436	0.478	0.514	0.555	0.598
90°	0.202	0.274	0.330	0.388	0.431	0.477	0.522	0.544	0.620	0.648
100°	0.231	0.300	0.358	0.415	0.468	0.517	0.564	0.608	0.648	0.687
110°	0.254	0.325	0.392	0.453	0.503	0.542	0.602	0.648	0.688	0.731
120°	0.272	0.346	0.416	0.478	0.536	0.590	0.640	0.687	0.731	0.793
130°	0.292	0.373	0.445	0.515	0.568	0.624	0.676	0.724	0.768	0.810
140°	0.310	0.393	0.468	0.536	0.594	0.656	0.709	0.758	0.797	0.846
150°	0.330	0.418	0.498	0.570	0.628	0.687	0.741	0.791	0.837	0.880
160°	0.346	0.436	0.517	0.591	0.656	0.717	0.793	0.822	0.869	0.912
170°	0.365	0.461	0.545	0.623	0.683	0.745	0.801	0.852	0.898	0.942
180°	0.380	0.478	0.564	0.640	0.709	0.772	0.828	0.880	0.927	0.970
190°	0.399	0.499	0.592	0.671	0.727	0.797	0.854	0.906	0.953	0.997
200°	0.415	0.517	0.607	0.687	0.758	0.822	0.880	0.932	0.975	1.000
210°	0.433	0.539	0.633	0.717	0.781	0.846	0.904	0.956	1.000	1.000
220°	0.447	0.555	0.668	0.731	0.803	0.868	0.926	0.979	1.000	1.000
230°	0.464	0.571	0.674	0.758	0.825	0.890	0.949	1.000	1.000	1.000
240°	0.478	0.590	0.687	0.772	0.845	0.912	0.970	1.000	1.000	1.000
250°	0.492	0.612	0.706	0.795	0.866	0.932	0.991	1.000	1.000	1.000

TABLE II.—Pulling Tension T, when the Motive Force F = 1.

Angle of contact.	Friction coefficient *f* for the surfaces in contact on the smallest pulley.									
	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
2 Z	T	T	T	T	T	T	T	T	T	T
60°	6.779	5.291	4.321	3.680	3.227	2.887	2.724	2.419	2.236	2.163
70°	5.855	4.558	3.711	3.165	2.838	2.546	2.198	2.140	2.078	1.863
80°	5.274	4.077	3.363	2.887	2.546	2.290	2.092	1.944	1.802	1.671
90°	4.703	3.646	3.028	2.580	2.319	2.092	1.915	1.838	1.613	1.543
100°	4.321	3.361	2.792	2.410	2.136	1.933	1.773	1.645	1.542	1.455
110°	3.941	3.079	2.548	2.206	1.988	1.845	1.660	1.542	1.453	1.368
120°	3.680	2.887	2.403	2.091	1.864	1.693	1.561	1.455	1.368	1.261
130°	3.421	2.683	2.246	1.942	1.759	1.601	1.479	1.381	1.302	1.234
140°	3.227	2.546	2.136	1.864	1.699	1.523	1.411	1.319	1.254	1.182
150°	3.032	2.390	2.009	1.755	1.591	1.455	1.349	1.264	1.195	1.137
160°	2.887	2.290	1.932	1.690	1.523	1.395	1.261	1.216	1.151	1.097
170°	2.739	2.169	1.835	1.581	1.463	1.342	1.249	1.174	1.113	1.062
180°	2.631	2.091	1.773	1.561	1.410	1.296	1.207	1.136	1.079	1.030
190°	2.506	2.004	1.688	1.490	1.374	1.253	1.170	1.103	1.049	1.003
200°	2.410	1.932	1.646	1.455	1.319	1.216	1.136	1.073	1.026	1.000
210°	2.307	1.853	1.580	1.395	1.280	1.182	1.106	1.046	1.000	1.000
220°	2.236	1.802	1.495	1.368	1.244	1.151	1.080	1.021	1.000	1.000
230°	2.153	1.730	1.489	1.318	1.212	1.123	1.053	1.000	1.000	1.000
240°	2.091	1.693	1.455	1.296	1.196	1.098	1.030	1.000	1.000	1.000
250°	2.023	1.633	1.416	1.257	1.155	1.073	1.009	1.000	1.000	1.000

It is assumed in the above tables that the friction gripe on the smallest pulley just balances the motive force, for which allowance must be made to prevent slipping.

The slack tension t is the difference between the pulling tension T and the motive force F, or $t = T - F$.

When the friction and angle of contact are great, the pulling tension is equal to the motive force, and no slack tension is then required.

TABLE III.—Pressure P in the Shaft Journals, when the Motive Force F = 1 and the System in Motion.

Angle of contact.	Friction coefficient *f* for the surfaces in contact on the smallest pulley.									
	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
2 Z	P	P	P	P	P	P	P	P	P	P
60°	6.779	5.291	4.325	3.680	3.227	2.887	2.724	2.410	2.236	2.163
70°	6.570	5.082	4.110	3.484	3.109	2.774	2.414	2.308	2.135	1.989
80°	6.495	4.956	4.038	3.352	2.988	2.658	2.339	2.214	2.030	1.863
90°	6.237	4.742	3.868	3.235	2.865	2.544	2.294	2.093	1.867	1.768
100°	6.088	4.624	3.746	3.100	2.740	2.430	2.184	1.988	1.829	1.697
110°	5.818	4.406	3.536	2.976	2.619	2.384	2.081	1.888	1.701	1.602
120°	5.642	4.268	3.430	2.890	2.497	2.200	1.972	1.788	1.637	1.451
130°	5.388	4.051	3.259	2.708	2.376	2.089	1.868	1.691	1.547	1.424
140°	5.185	3.906	3.135	2.624	2.257	1.983	1.772	1.600	1.477	1.342
150°	4.925	3.685	2.949	2.456	2.142	1.879	1.674	1.510	1.377	1.264
160°	4.717	3.541	2.836	2.359	2.030	1.778	1.513	1.431	1.297	1.191
170°	4.465	3.329	2.664	2.157	1.922	1.681	1.496	1.351	1.224	1.122
180°	4.262	3.182	2.546	2.122	1.824	1.592	1.414	1.272	1.158	1.060
190°	4.000	3.000	2.371	1.976	1.745	1.504	1.339	1.169	1.097	1.005
200°	3.777	2.836	2.261	1.896	1.628	1.425	1.268	1.134	1.050	1.000
210°	3.525	2.648	2.120	1.763	1.541	1.352	1.205	1.101	1.000	1.000
220°	3.323	2.507	1.930	1.692	1.458	1.284	1.150	1.020	1.000	1.000
230°	3.090	2.323	1.886	1.576	1.384	1.223	1.066	1.000	1.000	1.000
240°	2.890	2.200	1.788	1.513	1.340	1.170	1.032	1.000	1.000	1.000
250°	2.676	2.037	1.681	1.421	1.254	1.120	1.015	1.000	1.000	1.000

TABLE IV.—Pressure P in the Shaft Journals, when the Motive Force F = 1 and the System at Rest.

Angle of contact.	Friction coefficient *f* for the surfaces in contact on the smallest pulley.									
	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
2 Z	p	p	p	p	p	p	p	p	p	p
60°	5.779	4.291	3.325	2.680	2.227	1.887	1.724	1.410	1.236	1.163
70°	5.570	4.082	3.110	2.484	2.109	1.774	1.414	1.308	1.135	0.989
80°	5.495	3.956	3.038	2.352	1.988	1.658	1.339	1.214	1.030	0.862
90°	5.237	3.742	2.863	2.235	1.865	1.544	1.294	1.093	0.867	0.768
100°	5.088	3.624	2.746	2.100	1.740	1.430	1.184	0.988	0.829	0.697
110°	4.818	3.406	2.536	1.976	1.619	1.384	1.081	0.888	0.701	0.602
120°	4.642	3.268	2.430	1.890	1.497	1.200	0.972	0.788	0.637	0.451
130°	4.388	3.051	2.259	1.708	1.376	1.089	0.868	0.691	0.547	0.424
140°	4.185	2.906	2.135	1.624	1.257	0.983	0.772	0.600	0.477	0 342
150°	3.925	2.685	1.949	1.456	1.142	0.879	0.674	0.510	0.377	0 264
160°	3.717	2.541	1.836	1.359	1.030	0.778	0.513	0.431	0.297	0.191
170°	3.465	2.329	1.664	1.157	0.922	0.681	0.496	0.351	0.224	0.122
180°	3.262	2.182	1.546	1.122	0.824	0.592	0.414	0.272	0.158	0.060
190°	3.000	2.000	1.371	0.976	0.745	0.504	0.339	0.169	0.097	0.005
200°	2.777	1.836	1.261	0.896	0.628	0.425	0.268	0.134	0.050	0.000
210°	2.525	1.648	0.120	0.763	0.541	0.352	0.205	0.101	0.000	0.000
220°	2.323	1.507	0.930	0.692	0.458	0.284	0.150	0.020	0.000	0.000
230°	2.090	1.323	0.886	0.576	0.384	0.223	0.066	0.000	0.000	0.000
240°	1.890	1.200	0.788	0.513	0.340	0.170	0.032	0.000	0.000	0.000
250°	1.676	1.037	0.681	0.421	0.254	0.120	0.015	0.000	0.000	0.000

The belt should be tightened to the pressure p in the journals when the system is at rest, to enable it to transmit the motive force F when the system is in motion. The friction gripe on the smallest pulley will then just balance the motive force, or $C = F = P - p$. The pressure p should therefore be made ⅙ greater for safe working.

TABLE V.—Motive Force F when the Slack Tension $t = 1$.

Angle Contact.	Friction Coefficient f for the Surfaces in Contact.									
	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
$2Z$	F'	F'	F'	F'	F'	F'	F'	F'	F'	F'
60°	.1730	.2330	.3011	.3731	.4464	.5319	.5800	.7092	0.8091	0.860
70°	.2060	.2810	.3689	.4619	.5441	.6468	.8347	.8792	0.928	1.158
80°	.2340	.3250	.4232	.5299	.6468	.7752	.9157	1.059	1.247	1.490
90°	.2700	.3779	.4931	.6329	.7581	.9157	1.093	1.293	1.631	1.842
100°	.3011	.4230	.5580	.7092	.8803	1.072	1.294	1.550	1.845	2.198
110°	.3400	.4810	.6460	.8292	1.012	1.183	1.515	1.845	2.336	2.717
120°	.3731	.5299	.7127	.9166	1.157	1.443	1.782	2.198	2.717	3.831
130°	.4130	.5942	.8026	1.061	1.317	1.664	2.087	2.625	3.311	4.273
140°	.4490	.6468	.8803	1.157	1.495	1.912	2.433	3.135	3.937	5.494
150°	.4921	.7194	.9911	1.324	1.692	2.198	2.865	3.788	5.128	7.299
160°	.5299	.7740	1.073	1.449	1.912	2.531	4.831	4.629	6.622	10.31
170°	.5750	.8598	1.198	1.721	2.160	2.934	4.016	5.747	8.849	16.13
180°	.6131	.9166	1.294	1.782	2.415	3.378	4.831	7.353	12.66	33.33
190°	.6640	.9960	1.453	2.041	2.673	3.952	5.882	9.709	20.41	333.3
200°	.7092	1.073	1.562	2.198	3.135	4.629	7.353	13.70	38.46	0.000
210°	.7651	1.172	1.724	2.532	3.571	5.494	9.434	21.74	0.000	0.000
220°	.8091	1.247	2.020	2.717	4.098	6.622	12.5	47.62	0.000	0.000
230°	.8673	1.370	2.045	3.144	4.717	8.130	18.87	0.000	0.000	0.000
240°	.9166	1.443	2.198	3.378	5.102	10.20	33.33	0.000	0.000	0.000
250°	.9775	1.580	2.404	3.891	6.451	13.70	111.1	0.000	0.000	0.000

TABLE VII.—Slack Tension t when the Motive Force $F = 1$.

Angle Contact.	Friction Coefficient f for the Surfaces in Contact.									
	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
$2Z$	t'	t'	t'	t'	t'	t'	t'	t'	t'	t'
60°	5.779	4.291	3.321	2.680	2.227	1.887	1.724	1.410	1.236	1.163
70°	4.855	3.558	2.711	2.165	1.838	1.546	1.198	1.140	1.078	0.863
80°	4.274	3.077	2.363	1.887	1.546	1.290	1.092	0.944	0.802	0.671
90°	3.703	2.646	2.028	1.580	1.319	1.092	0.915	0.773	0.613	0.543
100°	3.321	2.364	1.792	1.410	1.136	0.933	0.773	0.645	0.542	0.455
110°	2.941	2.079	1.548	1.206	0.988	0.845	0.660	0.542	0.428	0.368
120°	2.680	1.887	1.403	1.091	0.864	0.693	0.561	0.455	0.368	0.261
130°	2.421	1.683	1.246	0.942	0.759	0.601	0.479	0.381	0.302	0.234
140°	2.227	1.546	1.136	0.864	0.669	0.523	0.411	0.319	0.254	0,182
150°	2.032	1.390	1.009	0.755	0.591	0.455	0.349	0.264	0.195	0.137
160°	1.887	1.290	0.932	0.690	0.523	0.395	0.261	0.216	0.151	0.097
170°	1.739	1.169	0.835	0.581	0.463	0.342	0.249	0.174	0.113	0.062
180°	1.631	1.091	0.773	0.561	0.414	0.296	0.207	0.136	0,079	0.030
190°	1.506	1.094.	0.688	0.490	0.374	0.253	0.170	0.103	0.049	0.003
200°	1.410	0.932	0.640	0.455	0.319	0.216	0.136	0.073	0.026	0.000
210°	1.307	0.853	0.580	0.395	0.280	0.182	0.106	0.046	0.000	0.000
220°	1.236	0.802	0.495	0.368	0.244	0.151	0.080	0.021	0.000	0.000
230°	1.153	0.730	0.489	0.318	0.212	0.123	0.053	0.000	0.000	0.000
240°	1.091	0.693	0.455	0.296	0.196	0.098	0.030	0.000	0.000	0.000
250°	1.023	0.633	0.416	0.257	0.155	0.073	0.009	0.000	0.000	0.000

TABLE VII.—Slack Tension t when the Pulling Tension $T=1$.

Angle Contact.	Friction Coefficient f for the Surfaces in Contact.									
	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
2 Z	t'	t'	t'	t'	t'	t'	t'	t'	t'	t'
60°	.8525	.8110	.7686	.7283	.6901	.6536	.6329	.5851	.5528	.5376
70°	.8292	.7806	.7305	.6840	.6477	.6072	.5449	.5328	.5187	.4634
80°	.8104	.7547	.7027	.6536	.6072	.5634	.5219	.4826	.4150	.4017
90°	.7874	.7257	.6698	.6124	.5688	.5219	.4778	.4361	.3799	.3518
100°	.7686	.7027	.6418	.5851	.5319	.4826	.4361	.3923	.3514	.3127
110°	.7463	.6752	.6075	.5467	.4970	.4581	.3976	.3515	.2997	.2691
120°	.7283	.6536	.5838	.5219	.4636	.4095	.3594	.3127	.2691	.2070
130°	.7077	.6273	.5549	.4850	.4316	.3756	.3241	.2762	.2320	.1900
140°	.6901	.6072	.5319	.4636	.4010	.3435	.2912	.2417	.2025	.1540
150°	.6702	.5817	.5023	.4393	.3716	.3127	.2587	.2088	.1633	.1202
160°	.6536	.5634	.4826	.4085	.3434	.2832	.2070	.1778	.1312	.0883
170°	.6349	.5391	.4550	.3765	.3164	.2552	.1993	.1483	.1014	.0581
180°	.6200	.5219	.4361	.3594	.2909	.2283	.1718	.1202	.0730	.0296
190°	.6010	.5010	.4078	.3288	.2725	.2024	.1454	.09354	.0468	.0026
200°	.5851	.4826	.3925	.3127	.2417	.1777	.1202	.06807	.0251	.0000
210°	.5666	.4604	.3672	.2833	.2187	.1542	.09597	.04374	.0000	.0000
220°	.5528	.4452	.3314	.2691	.1964	.1314	.07305	.02052	.0000	.0000
230°	.5356	.4219	.3286	.2417	.1749	.1094	.05089	.0000	.0000	.0000
240°	.5219	.4095	.3127	.2283	.1543	.08826	.02969	.0000	.0000	.0000
250°	.5058	.3879	.2938	.2045	.1341	.06793	.00927	.0000	.0000	.0000

TABLE VIII.—Pulling Tension T when the Slack Tension $t=1$.

Angle Contact.	Friction Coefficient f for the Surfaces in Contact.									
	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
2 Z	T'	T'	T'	T'	T'	T'	T'	T'	T'	T'
60°	1.173	1.233	1.301	1.373	1.449	1.530	1.580	1.709	1.809	1.860
70°	1.206	1.281	1.369	1.462	1.544	1.647	1.835	1.877	1 928	2.157
80°	1.234	1.325	1.423	1.530	1.647	1.775	1.916	2.072	2.247	2.489
90°	1.270	1.378	1.493	1.633	1.758	1.916	2.093	2.293	2.632	2.842
100°	1.301	1.423	1.558	1.709	1.880	2.072	2.293	2.549	2.846	3.198
110°	1.340	1.181	1.646	1.829	2.012	2.183	2.515	2.845	3.237	3.716
120°	1.373	1.530	1.713	1.916	2.157	2.442	2.782	3.198	3.716	4.831
130°	1.413	1.594	1.802	2.062	2.317	2.662	3.085	3.621	4.311	5.263
140°	1.449	1.647	1.880	2.157	2.494	2.911	3.434	4.138	4.939	6.492
150°	1.492	1.719	1.991	2.324	2.691	3.198	3.866	4.788	6.124	8.319
160°	1.530	1.775	2.072	2.448	2.912	3.531	4.832	5.624	7.619	11.33
170°	1.575	1.855	2.198	2.656	3.160	3.919	5.018	6.742	9.866	17.20
180°	1.613	1.916	2.293	2.782	3.437	4.381	5.822	8.317	13.70	33.78
190°	1.664	1.996	2.452	3.041	3.670	4.941	6.877	10.69	21.70	386.1
200°	1.709	2.072	2.548	3.198	4.138	5.625	8.317	14.69	39.77	
210°	1.764	2.172	2.723	3.530	4.573	6.484	10.42	22.86		
220°	1.809	2.246	2.846	3.716	5.091	7.612	13.69	48 72		
230°	1.867	2.370	3.043	4.137	5.716	9.141	19.65	0.000		
240°	1.916	2.442	3.198	4.380	6.481	11.33	33.68	0.000		
250°	1.977	2.578	3.404	4.890	7.457	14.72	107.8	0.000		

CONE PULLEYS.

The illustration Fig. 2, Plate II., represents the largest and smallest diameters of a pair of cone pulleys, of which the letters denote as follows:

R = radius of the largest step or pulley.
r = radius of the smallest step or pulley.
a = distance between the centres of rotation of the pulleys.
b = distance or length of the belt between the tangential points on the two pulleys.
L = whole length of the belt.
x = distance from the centre of the small pulley to the vertex c'.
Z' = half the angle of contact of the belt on the large pulley.
Z = half the angle of contact of the belt on the small pulley.

For open belts the sum of the whole angles of contact is always 360°, and $Z' + Z = 180°$, omitting the slack of belt.

$$b = \sqrt{a^2 - (R - r)^2} \quad . \; . \; . \; . \; 1.$$

$$\text{Sin. } Z = \frac{b}{a} \quad . \; . \; . \; . \; . \; . \; . \; 2.$$

$$x = a\left(\frac{R}{R - r} - 1\right) \quad . \; . \; . \; . \; 3.$$

$$L = \frac{\pi}{90}(RZ' + rZ) + 2b \quad . \; . \; 4.$$

Three-step Pulley, Fig. 3, Plate II.

The object is to make two cone pulleys cast from one pattern, and to have three steps on each pulley. Having given the diameters of the largest and smallest steps, the problem is to find the diameter of the middle step, so proportioned that the belt will have the same tension on all the three steps.

D = diameter of the middle step.

$$D = R + r + \frac{(R - r)^2}{\pi a} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; 5.$$

Four-step Pulley, Fig. 4, Plate II.

Having given the radii of the largest and smallest pulleys 1 and 4, the problem is to find the diameters of the inner pulleys 2 and 3.

d = diameter of the pulley 3, next to the smallest.
d' = diameter of the pulley 2, next to the largest pulley.

$$d = \tfrac{2}{3}(R + 2r) + \frac{(R - r)^2}{4a} \quad . \; . \; . \; . \; . \; . \; . \; . \; 6.$$

$$d' = \tfrac{2}{3}(2R + r) + \frac{(R - r)^2}{4a} \quad . \; . \; . \; . \; . \; . \; . \; . \; 7.$$

Five-step Pulley, Fig. 5, Plate II.

Cone pulleys of five steps are constructed as follows: The largest and smallest pulleys, 1 and 5, are assumed to be given from the first start. The diameter of the middle step 3 is calculated by formula 5, and the problem remains to find the diameters of the steps 2 and 4.

R' = radius of the middle step 3; d and d' = diameters of the respective steps 4 and 2.

$$d = R' + r + \frac{(R' - r)^2}{\pi a} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; 8.$$

$$d' = R + R' + \frac{(R - R')^2}{\pi a} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; 9.$$

It is supposed in the above that each pair of cone pulleys is cast from one pattern.

To find the Proper Distance between Centres.

Having given two equal cone pulleys, to find at what distance they ought to be placed to make the belt of equal tension on all the steps.

$$a = \frac{(R - r)^2}{\pi(D - R - r)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; 10.$$

This formula can be used only for pulleys of an odd number of steps, of which D = diameter of the middle step, and R and r are the respective radii of the largest and smallest pulleys.

Cone Pulleys of any Number of Steps and Proportions, Figs. 6 and 7, Plate II.

The conoids $a, b, c\, d\, B$ and $A\, d\, c\, b\, a$ are drawn alike and inverted to one another, as represented by Fig. 6. The centre lines $A\, B$ can be divided into any number of steps, and the conoids determine the corresponding diameters on each pulley. The pulleys can thus be made of widely different sizes, as required in many cases, particularly in foot-lathes.

The construction of the conoid is represented by Fig. 7. Draw the centre-line $A\, B$, in the middle of which erect the mean diameter D, which will be alike on each pulley. The largest radius R is determined by the formula

$$R = \sqrt{\frac{\pi^2 a}{4} + \pi a D} - \frac{\pi a}{2}. \quad . \quad . \quad . \quad . \quad . \quad 11.$$

Divide the centre line $A\, B$ into four equal parts and draw the diameters d and d', the lengths of which are determined by the following formulas:

$$d = \tfrac{1}{2}D + \frac{\tfrac{1}{4}D^2}{\pi a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 12.$$

$$d' = R + \tfrac{1}{2}D + \frac{(R - \tfrac{1}{2}D)^2}{\pi a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 13.$$

Draw a regular curve through the points a, b, c, d and B which form the sides of the conoid. Draw also the inverted conoid as shown by the dotted lines. Any line drawn through the conoids at right angles to $A\, B$ determines the diameters of the corresponding steps on each cone pulley. The diameter $b\, o$ on one pulley corresponds to the diameter $m\, n$ on the other pulley. It is not intended that the conoid should determine the width of the steps, as may be inferred from Fig. 6, for it only determines the diameters of the steps.

In practice the conoids should be laid out on a full-size scale to make the measurements correct. The length of the centre-line $A\, B$ should not be less than the diameter D, but better to make it $2\, D$, for a less inclination of the conoid makes sharper measurements for the diameters of the steps.

The whole length of the belt will be,

$$L = \pi D + 2a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 14.$$

For different lengths of the belt and diameter D, with the same distance a between the shafts, the conoid a, b, c, d, B will be of more or less curvature. When $a = \infty$, the conoid becomes a straight line, and when $a = D$, the radius $R = 0.8D$, which forms the greatest curvature of the conoid.

For a given length of belt the diameter of the middle step should be

$$D = \frac{L - 2a}{\pi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 15.$$

The diameter of the middle step can also be obtained by assuming a definite value for the greatest radius R—namely:

$$D = R + \frac{R^2}{\pi a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 16.$$

The length of the belt will then be—

$$L = \pi R + \frac{R^2}{a} + 2a. \quad . \quad . \quad . \quad . \quad . \quad 17.$$

Having given the greatest radius R and middle diameter D, the distance between the shafts should be—

$$a = \frac{R^2}{\pi(D - R)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 18.$$

Dimensions, Strength, and Power of Belts.

The strain on a belt is its pulling tension T, and not only the motive force F, as is often considered.

The motive force, under some conditions, is only a small fraction of the pulling tension, as seen in the Tables I. and II., page 400.

The strength of the belt must, therefore, be in proportion to the pulling tension T.

The following formula 1 is more correct for calculating the breadth of belts than the formulas on page 407. The difference is, however, very small.

S = maximum strain in pounds per inch of width of belt, which should not exceed the safety strength given in the accompanying tables, but may be made less.

B = breadth of belt in inches. T = pulling tension in pounds.

$$\text{Breadth of belt } B = \frac{T}{S} \quad \ldots \quad 1.$$

$$\text{Pulling tension } T = BS. \quad \ldots \quad 2.$$

$$\text{Strain per inch } S = \frac{T}{B} \quad \ldots \quad 3.$$

India-rubber belts are best in wet or damp places where leather belts cannot be used.

Dimensions and Strength of India-rubber Belts.

Number of plies.	Wt. per. sq. ft. Pounds.	Thickness. Inches.	Ult. strength. Lbs. per in.	Safety strength.
2 ply.	1.25	$\frac{3}{16} = 0.1875$	625	104
3 ply.	1.66	$\frac{5}{24} = 0.2083$	830	138
4 ply.	2	$\frac{5}{16} = 0.3125$	1000	166
5 ply.	2.4	$\frac{5}{12} = 0.4166$	1200	200
6 ply.	2.8125	$\frac{7}{16} = 0.4375$	1400	233

Thickness in Inches, and Strength in Pounds, of Belts.

Kind of material in belts.	Thickness.	Strength. Break.	Strength. Safety.
Oak-tanned leather	0.25	1000	166
" "	0.1875	780	130
" "	0.125	560	95
Ordinary tanned leather	0.25	740	125
" " "	0.1875	560	95
" " "	0.125	290	50
Raw hide, best quality	...	1250	225
" " ordinary	...	1100	185
Horse-skin	...	800	135
Calf's-skin	...	360	60
Sheep-skin	...	322	54
Cowhide	...	790	130
Cotton duck	...	200	66
Flax, woven belt	...	1250	200

The above data are for new belts, and cannot be trusted for old and worn-out belts.

Care should be taken to prevent animal oil or fat from coming in contact with the working surfaces of the belt and pulleys, for it reduces the friction, and if it once permeates the leather it is difficult to get rid of.

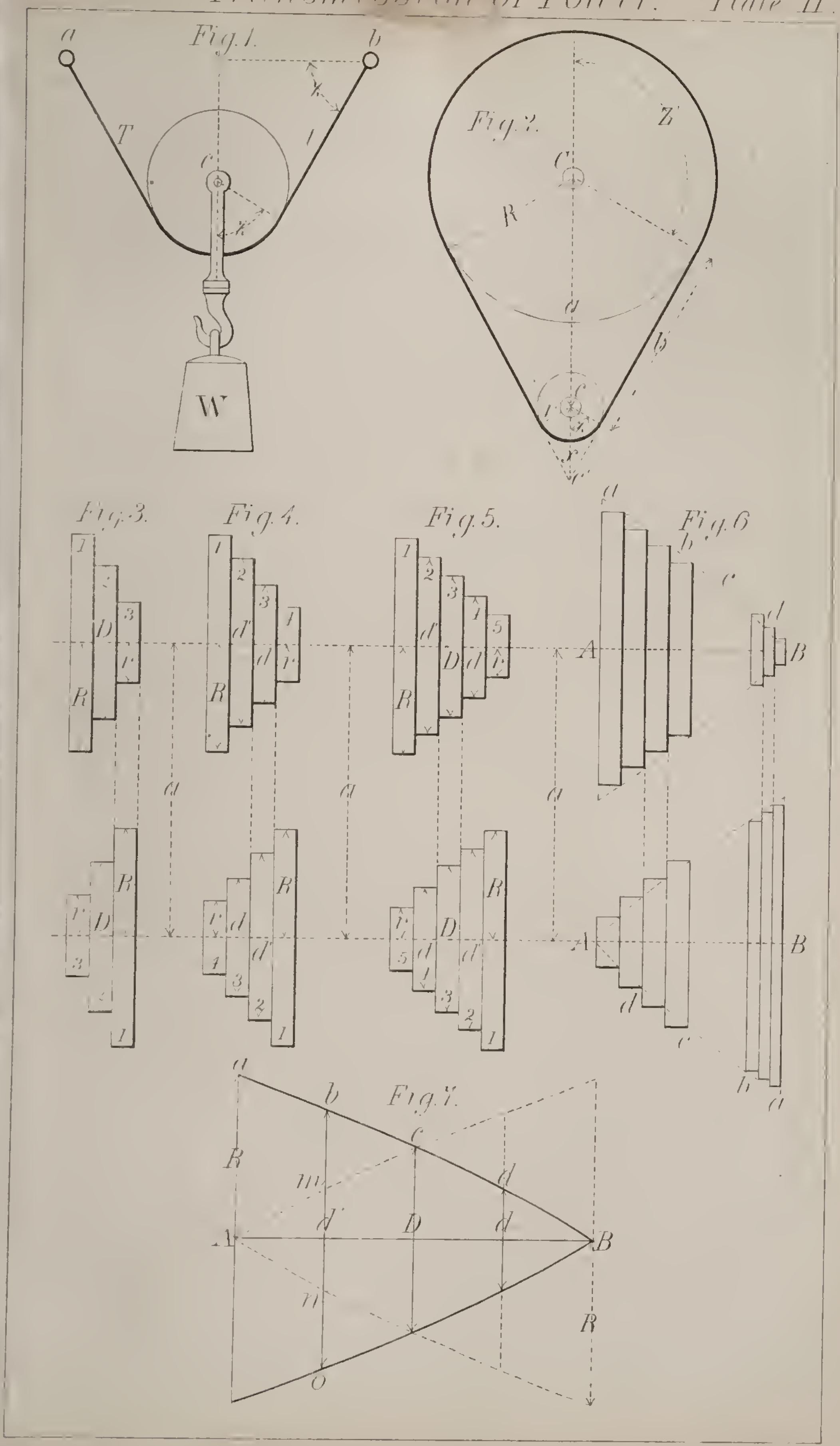


Fig. 1.
W
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6
Fig. 7.

HORSE-POWER AND BREADTH OF LEATHER BELTS.

B = breadth of belt in inches.
$H\!P$ = horse-power transmitted by the belt.
V = velocity in feet per second of the belt.
d = diameter in inches } of the smallest pulley.
n = revolutions per minute } of the smallest pulley.
F = motive force in pounds transmitted by the belt.
Z = half angle of contact of belt on the small pulley.
S = safe working strength in pounds per inch of width of belt, which for oak-tanned leather $\frac{1}{4}$ inch thick, cemented and riveted joints, can be taken at 100 pounds, and less in proportion for weaker belts.

$$H\!P = \frac{d\,n\,F}{126050} = \frac{60\,B\,V}{1000} \text{ for single thickness.} \quad 1.$$

$$H\!P = \frac{B\,d\,n\,Z\,S}{15000000}. \quad 2.$$

$$H\!P = \frac{B\,V\,Z\,S}{130000}. \quad 3.$$

$$B = \frac{15000000\,H\!P}{d\,n\,Z\,S}. \quad 4.$$

$$B = \frac{T+t}{2\,S}. \quad 5.$$

$$F = \frac{126050\,H\!P}{d\,n} = \frac{550\,H\!P}{V}. \quad 6.$$

Example. A leather belt is to transmit $H\!P = 75$ horse-power over a pulley $d = 36$ inches in diameter, making $n = 80$ revolutions per minute; angle of contact $Z = 85°$, and the safe working strength $S = 100$ pounds per inch of width. Required the width of the belt?

Formula 1. Width $b = \frac{15000000 \times 75}{36 \times 80 \times 85 \times 100} = 46$ inches, nearly.

Horse-power of Iron and Steel Ropes.

Iron. Diam.	Velocity of Rope in Feet per Second. 10	20	30	40	50	60	70	80	90	100	Steel. Diam.
Inches.	$H\!P$	$H\!P$	$H\!P$	$H\!P$	$H\!P$	$H\!P$	$H\!P$	$H\!P$	$H\!P$	$H\!P$	Inches.
$\frac{1}{4}$	4	8	12	16	20	24	28	32	36	40	$\frac{3}{16}$
$\frac{5}{16}$	6.250	12.50	18.75	25.00	31.25	37.50	43.75	50.00	56.25	62.50	$\frac{1}{4}$
$\frac{3}{8}$	9.000	18	27	36	45	54	63	72	81	90.00	$\frac{5}{16}$
$\frac{7}{16}$	12.25	24.5	36.75	49.00	61.25	73.50	85.75	98.00	101.2	122.5	$\frac{3}{8}$
$\frac{1}{2}$	16.00	32	48	64	80	96	112	128	144	160.0	$\frac{3}{8}$
$\frac{9}{16}$	20.25	40.5	60.75	81.00	101.2	121.5	141.7	162.0	182.2	202.5	$\frac{7}{16}$
$\frac{5}{8}$	25.00	50	75	100	125	150	175	200	225	250.0	$\frac{1}{2}$
$\frac{11}{16}$	30.25	60.50	90.75	121	151.2	181.5	211.7	242	272.2	302.5	$\frac{9}{16}$
$\frac{3}{4}$	36.00	72.00	108.0	144.0	180.0	216.0	252.0	288.0	324.0	360.0	$\frac{9}{16}$
$\frac{13}{16}$	42.25	84.5	126.7	169	211.2	253.5	295.7	338	380.2	422.5	$\frac{5}{8}$
$\frac{7}{8}$	49.00	98	147	196	245	294	343	392	441	490.0	$\frac{11}{16}$
$\frac{15}{16}$	56.25	112.5	168.7	225.0	281.2	337.5	393.7	450	506.2	562.5	$\frac{3}{4}$
1 in.	64	128	192	256	320	384	448	512	576	640.0	$\frac{13}{16}$
$1\frac{1}{8}$	81	162	243	324	405	486	567	648	729	810.0	$\frac{7}{8}$
$1\frac{1}{4}$	100	200	300	400	500	600	700	800	900	1000	1 in.
$1\frac{3}{8}$	121	242	363	484	605	726	847	968	1089	1210	$1\frac{1}{8}$
$1\frac{1}{2}$	144	288	432	576	720	864	1008	1152	1296	1440	$1\frac{3}{16}$
$1\frac{5}{8}$	169	338	507	676	845	1014	1183	1352	1521	1690	$1\frac{1}{4}$
$1\frac{3}{4}$	196	392	588	784	980	1176	1372	1568	1764	1960	$1\frac{3}{8}$
$1\frac{7}{8}$	225	450	675	900	1125	1350	1575	1800	2025	2250	$1\frac{1}{2}$
2 in.	256	512	768	1024	1280	1536	1792	2048	2304	2560	$1\frac{9}{16}$
$2\frac{1}{4}$	324	648	974	1296	1620	1944	2268	2602	2916	3240	$1\frac{3}{4}$
$2\frac{1}{2}$	400	800	1200	1600	2000	2400	2800	3200	3600	4000	2 in.
$2\frac{3}{4}$	484	968	1452	1936	2420	2904	3388	3882	4356	4840	$2\frac{3}{16}$
3 in.	576	1152	1728	2304	2880	3456	4032	4608	5184	5760	$2\frac{1}{2}$

Breadth of Belts in Inches for Different Motive Forces and Angles of Contact.

Motive Force.	Whole Angle of Contact 2 Z.									
	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°
F lbs.	B. in.	B. in.	B. in.	B. in.	B. in.	B. in.	B. in.	B. in.	B. in.	B. in.
10	0.424	0.372	0.331	0.300	0.275	0.254	0.238	0.223	0.211	0.200
20	0.847	0.743	0.662	0.599	0.549	0.509	0.475	0.426	0.421	0.400
30	1.271	1.115	0.993	0.898	0.824	0.763	0.713	0.670	0.632	0.600
40	1.695	1.487	1.324	1.198	1.099	1.018	0.950	0.893	0.842	0.800
50	2.119	1.859	1.655	1.497	1.374	1.272	1.188	1.116	1.053	1.000
60	2.542	2.230	1.987	1.796	1.648	1.526	1.425	1.339	1.263	1.200
70	2.966	2.602	2.318	2.095	1.923	1.780	1.663	1.562	1.474	1.400
80	3.390	2.974	2.648	2.396	2.198	2.036	1.900	1.786	1.684	1.600
90	3.813	3.345	2.980	2.695	2.472	2.290	2.138	2.009	1.895	1.800
100	4.237	3.717	3.311	2.994	2.747	2.544	2.375	2.232	2.105	2.000
120	5.084	4.460	3.974	3.592	3.296	3.052	2.850	2.678	2.526	2.400
140	5.932	5.204	4.636	4.190	3.846	3.560	3.326	3.124	2.948	2.800
160	6.780	5.948	5.296	4.792	4.396	4.072	3.800	3.572	3.368	3.200
180	7.626	6.690	5.960	5.390	4.944	4.580	4.276	4.018	3.790	3.600
200	8.474	7.434	6.622	5.988	5.494	5.088	4.750	4.464	4.210	4.000
220	9.321	8.177	7.284	6.586	6.043	5.596	5.225	4.910	4.631	4.400
240	10.17	8.920	7.948	7.184	6.592	6.104	5.700	5.356	5.052	4.800
260	11.02	9.663	8.610	7.783	7.141	6.613	6.175	5.800	5.473	5.200
280	11.86	10.41	9.272	8.380	7.692	7.120	6.652	6.248	5.896	5.600
300	12.71	11.15	9.933	8.982	8.241	7.632	7.125	6.696	6.315	6.000
320	13.56	11.90	10.59	9.584	8.692	8.144	7.600	7.144	6.736	6.400
340	14.41	12.64	11.21	10.18	9.241	8.652	8.075	7.590	7.157	6.800
360	15.25	13.38	11.92	10.78	9.988	9.160	8.552	8.036	7.580	7.200
380	16.10	14.12	12.58	11.38	10.53	9.669	9.027	8.482	8.001	7.600
400	16.95	14.87	13.24	11.98	10.99	10.18	9.500	8.928	8.420	8.000
420	17.80	15.61	13.90	12.58	11.54	10.69	9.975	9.374	8.841	8.400
440	18.64	16.35	14.57	13.17	12.09	11.19	10.45	9.820	9.262	8.800
460	19.49	17.09	15.23	13.77	12.64	11.70	10.93	10.27	9.683	9.200
480	20.31	16.84	15.90	14.37	13.18	12.21	11.40	10.71	10.10	9.600
500	21.19	18.59	16.55	14.97	13.74	12.72	11.88	11.16	10.53	10.00
600	25.42	22.30	19.87	17.96	16.48	15.26	14.25	13.39	12.63	12.00
700	29.66	26.02	23.18	20.95	19.23	17.80	16.63	15.62	14.74	14.00
800	33.90	29.74	26.48	23.96	21.98	20.36	19.00	17.86	16.84	16.00
900	38.13	33.45	29.80	26.95	24.72	22.90	21.38	20.09	18.95	18.00
1000	42.37	37.17	33.11	29.94	27.47	25.44	23.75	22.32	21.05	20.00
1100	46.61	40.89	36.42	32.93	29.22	27.98	26.12	24.55	23.15	22.00
1200	50.84	44.60	39.74	35.92	32.96	30.52	28.50	26.78	25.26	24.00
1300	55.08	48.32	40.05	38.91	35.71	33.06	30.87	29.01	27.36	26.00
1400	59.32	52.04	46.36	41.90	38.56	35.60	33.26	31.24	29.48	28.00
1500	63.56	55.76	49.67	44.89	41.31	38.14	36.63	33.47	31.58	30.00
1600	67.80	59.48	52.96	47.92	43.96	40.72	38.00	35.72	33.68	32.00
1700	72.04	63.20	56.27	50.91	46.71	43.26	40.38	37.95	35.78	34.00
1800	76.26	66.90	59.60	53.90	49.44	45.80	42.76	40.18	37.90	36.00
1900	80.50	70.62	62.91	56.89	52.19	48.34	45.14	42.41	40.01	38.00
2000	84.74	74.34	66.22	59.88	54.94	50.88	47.50	44.64	42.10	40.00
2100	88.98	78.06	69.53	62.87	57.69	53.42	49.88	46.87	44.21	42.00
2200	93.21	81.77	72.84	65.86	60.43	55.96	52.25	49.10	46.31	44.00
2300	97.45	85.49	76.15	68.85	63.18	58.50	54.63	51.33	48.42	46.00
2400	101.7	89.20	79.48	71.84	65.92	61.04	57.00	53,56	50.52	48.00
2500	105.5	92.95	82.75	74.85	68.70	61.50	59.40	55.80	52.65	50.00

Breadth of Belts in Inches for Different Motive Forces and Angles of Contact.

Motive Force.	Whole Angle of Contact 2 *Z*.									
	160°	170°	180°	190°	200°	210°	220°	230°	240°	250°
F lbs.	B. in.	B. in.	B. in.	B. in.	B. in.	B. in.	B. in.	B. in.	B. in.	B. in.
10	0.190	0.182	0.175	0.163	0.162	0.157	0.152	0.148	0.144	0.140
20	0.381	0.365	0.350	0.337	0.325	0.314	0.304	0.295	0.287	0.280
30	0.571	0.547	0.525	0.505	0.487	0.472	0.457	0.442	0.431	0.420
40	0.762	0.730	0.700	0.673	0.649	0.629	0.609	0.591	0.574	0.560
50	0.952	0.912	0.875	0.841	0.811	0.786	0.761	0.738	0.718	0.700
60	1.143	1.094	1.051	1.010	0.974	0.943	0.913	0.884	0.862	0.840
70	1.333	1.276	1.226	1.178	1.136	1.100	1.065	1.032	1.005	0.980
80	1.524	1.459	1.401	1.346	1.298	1.258	1.218	1.182	1.149	1.120
90	1.714	1.842	1.576	1.515	1.461	1.415	1.370	1.326	1.292	1.260
100	1.905	1.824	1.751	1.683	1.623	1.572	1.522	1.477	1.436	1.400
120	2.286	2.188	2.102	2.020	1.948	1.886	1.826	1.768	1.723	1.680
140	2.667	2.553	2.452	2.357	2.273	2.200	2.130	2.063	2.010	1.960
160	3.048	2.918	2.802	2.692	2.596	2.516	2.436	2.264	2.298	2.240
180	3.429	3.283	3.152	3.029	2.921	2.830	2.740	2.659	2.585	2.520
200	3.810	3.648	3.502	3.366	3.246	3.144	3.044	2.954	2.872	2.800
220	4.191	4.013	3.852	3.703	3.571	3.458	3.348	3.249	3.159	3.080
240	4.572	4.376	4.204	4.040	3.896	3.772	3.652	3.536	3.446	3.260
260	4.953	4.741	4.554	4.377	4.221	4.086	3.956	3.831	3.733	3.540
280	5.334	5.105	4.904	4.714	4.546	4.400	4.260	4.126	4.020	3.820
300	5.715	5.472	5.253	5.049	4.869	4.716	4.566	4.421	4.308	4.200
320	6.096	5.836	5.604	5.384	5.192	5.032	4.872	4.528	4.596	4.480
340	6.477	6.201	5.954	5.720	5.516	5.346	5.176	4.823	4.883	4.760
360	6.858	6.564	6.306	6.060	5.944	5.658	5.478	5.304	5.169	5.040
380	7.239	6.929	6.656	6.397	6.269	5.972	5.782	5.599	5.456	5.320
400	7.620	7.296	7.004	6.732	6.732	6.288	6.088	5.908	5.744	5.600
420	8.001	7.661	7.354	7.068	6.816	6.602	6.492	6.203	6.031	5.880
440	8.382	8.026	7.704	7.406	7.142	6.916	6.696	6.498	6.318	6.160
460	8.763	8.391	8.054	7.743	7.466	7.230	7.000	6.793	6.605	6.440
480	9.144	8.752	8.408	8.080	7.792	7.544	7.304	7.072	6.892	6.520
500	9.525	9.120	8.755	8.415	8.115	7.860	7.610	7.385	7.180	7.000
600	11.43	10.94	10.51	10.10	9.738	9.432	9.132	8.842	8.616	8.400
700	13.33	12.76	12.26	11.78	11.36	11.00	10.65	10.32	10.04	9.800
800	15.24	14.59	14.01	13.46	12.98	12.58	12.18	11.82	11.49	11.20
900	17.14	16.42	15.76	15.15	14.61	14.15	13.70	13.26	12.92	12.60
1000	19.05	18.24	17.51	16.83	16.23	15.72	15.22	14.77	14.36	14.00
1100	20.95	20.06	19.26	18.51	17.85	17.29	16.74	16.25	15.80	15.40
1200	22.86	21.88	21.02	20.20	19.48	18.86	18.26	17.68	17.23	16.80
1300	24.76	23.70	22.77	21.88	21.10	20.43	19.78	19.16	18.67	18.20
1400	26.66	25.52	24.52	23.56	22.72	22.00	21.30	20.64	20.08	19.60
1500	28.56	27.34	26.27	25.24	24.34	23.57	22.82	22.12	21.52	21.00
1600	30.48	29.18	28.02	26.92	25.96	25.16	24.36	22.64	22.98	22.40
1700	32.38	30.90	29.77	28.60	27.58	26.73	25.88	24.12	24.42	23.80
1800	34.29	32.83	31.52	30.29	29.21	28.30	27.40	26.59	25.85	25.20
1900	36.19	34.65	33.27	31.97	30.83	29.87	28.92	28.06	27.28	26.60
2000	38.10	36.48	35.02	33.66	32.46	31.44	30.44	29.54	28.72	28.00
2100	40.00	38.30	36.77	35.34	34.08	33.01	31.96	31.02	30.15	29.40
2200	41.90	40.12	38.52	37.02	35.60	34.58	33.48	32.50	31.60	30.80
2300	43.80	41.94	40.27	38.70	37.22	36.15	35.00	33.98	33.04	32.20
2400	45.72	43.76	42.04	40.40	38.96	37.72	36.52	35.36	34.46	32.60
2500	47.62	44.78	43.79	41.08	40.58	39.29	38.04	36.84	35.90	34.00

Velocity in Feet per Second of Belts, Wire Ropes, or of Circumference of Revolving Wheels or Pulleys.

Diam. Pulley.	Revolutions per Minute of Wheel or Pulley.									
	10	20	30	40	50	60	70	80	90	100
Inches.	V	V	V	V	V	V	V	V	V	V
1	.04363	.08727	.13090	.17453	.21817	.26180	.30543	.34906	.39270	.43633
2	.08727	.17453	.26180	.34906	.43633	.52360	.61086	.69813	.78540	.87266
3	.13090	.26180	.39270	.52360	.65450	.78540	.91630	1.0472	1.1781	1.3090
4	.17453	.31906	.52360	.69813	.87266	1.0472	1.2217	1.3963	1.5708	1.7453
5	.21817	.43633	.65450	.87266	1.0908	1.3090	1.5272	1.7453	1.9635	2.1817
6	.26180	.52360	.78540	1.0472	1.3090	1.5708	1.8326	2.0944	2.3562	2.6180
7	.30543	.61086	.91630	1.2217	1.5274	1.8326	2.1380	2.4434	2.7489	3.0543
8	.34906	.69813	1.0472	1.3963	1.7453	2.0944	2.4434	2.7926	3.1416	3.4906
9	.39270	.78540	1.1781	1.5708	1.9635	2.3562	2.7489	3.1416	3.5343	3.9270
10	.43633	.87266	1.3090	1.7453	2.1817	2.6180	3.0543	3.4906	3.9270	4.3633
11	.47996	.95992	1.4398	1.9198	2.3998	2.8798	3.3597	3.8396	4.3194	4.7996
12	.52360	1.0472	1.5708	2.0944	2.6180	3.1416	3.6652	4.1888	4.7124	5.2360
13	.56723	1.1345	1.7017	2.2690	2.8361	3.4034	3.9706	4.5380	5.1051	5.6723
14	.61086	1.2217	1.8326	2.4434	3.0543	3.6652	4.2760	4.8868	5.4978	6.1086
15	.65450	1.3090	1.9635	2.6180	3.2725	3.9270	4.5815	5.2360	5.8905	6.5450
16	.69813	1.3963	2.0944	2.7926	3.4906	4.1888	4.8869	5.5852	6.2832	6.9813
18	.78540	1.5708	2.3562	3.1416	3.9270	4.7124	5.4978	6.2832	7.0686	7.8540
20	.87266	1.7453	2.6180	3.4906	4.3633	5.2360	6.1086	6.9813	7.8540	8.7266
21	.91630	1.8326	2.7489	3.6652	4.5815	5.4978	6.4141	7.3304	8.2467	9.1630
24	1.0472	2.0944	3.1416	4.1888	5.2360	6.2832	7.3304	8.3776	9.4248	10.472
27	1.1781	2.3562	3.5343	4.7124	5.8905	7.0686	8.2467	9.4248	10.603	11.781
30	1.3090	2.6180	3.9270	5.2360	6.5450	7.8540	9.1630	10.742	11.781	13.090
33	1.4398	2.8798	4.3194	5.7596	7.1994	8.6388	10.079	11.519	12.958	14.398
36	1.5708	3.1416	4.7124	6.2832	7.8540	9.4248	10.996	12.566	14.137	15.708
39	1.7017	3.4034	5.1051	6.8068	8.5084	10.210	11.912	13.614	15.315	17.017
42	1.8326	3.6652	5.4978	7.3304	9.1630	10.996	12.828	14.661	16.493	18.326
45	1.9635	3.9270	5.8905	7.8540	9.8175	11.781	13.744	15.708	17.671	19.635
48	2.0944	4.1888	6.2832	8.3776	10.472	12.566	14.661	16.755	18.850	20.944
51	2.2253	4.4506	6.6759	8.9012	11.126	13.352	15.577	17.802	20.028	22.253
54	2.3562	4.7124	7.0686	9.4248	11.781	14.137	16.493	18.850	21.206	23.562
60	2.6180	5.2360	7.8540	10.472	13.090	15.708	18.326	20.944	23.562	26.180
66	2.8798	5.7596	8.6394	11.519	14.398	17.279	20.159	23.038	25.918	28.798
72	3.1416	6.2832	9.4248	12.566	15.708	18.850	21.991	25.132	28.274	31.416
78	3.4034	6.8068	10.210	13.614	17.017	20.420	23.824	27.228	30.630	34.034
84	3.6652	7.3304	10.996	14.661	18.326	21.992	25.656	29.322	32.988	36.652
90	3.9270	7.8540	11.781	15.708	19.635	23.562	27.489	31.416	35.343	39.270
96	4.1888	8.3776	12.566	16.755	20.944	25.132	29.322	33.510	37.698	41.888
102	4.4506	8.9012	13.352	17.802	22.253	26.704	31.154	35.604	40.056	44.506
108	4.7124	9.4248	14.137	18.850	23.562	28.274	32.987	37.700	42.411	47.124
114	4.9742	9.9484	14.923	19.897	24.871	29.846	34.819	39.794	44.769	49.742
120	5.2360	10.472	15.708	20.944	26.180	31.416	36.652	41.888	47.124	52.360
126	5.4978	10.995	16.493	21.990	27.489	32.986	38.485	43.980	49.479	54.978
132	5.7596	11.519	17.279	23.038	28.798	34.558	40.317	46.076	51.837	57.596
138	6.0214	12.043	18.064	24.086	30.107	36.128	42.150	48.172	54.192	60.214
144	6.2832	12.566	18.850	25.132	31.416	37.700	43.982	50.264	56.650	62.832
150	6.5450	13.090	19.635	26.180	32.725	39.270	45.815	52.360	58.905	65.450
160	6.9813	13.963	20.944	27.926	34.906	41.888	48.869	55.852	62.832	69.813
180	7.8540	15.708	23.562	31.416	39.270	47.124	54.978	62.832	70.686	78.540
200	8.7266	17.453	26.180	34.906	43.633	52.360	61.086	69.813	78.540	87.266
240	10.472	20.944	31.416	41.888	52.360	62.832	73.304	83.776	94.248	104.72

Velocity in Feet per Second of Belts, Wire Ropes, or of Circumference of Revolving Wheels or Pulleys.

Diam. Pulley.	Revolutions per Minute of Wheel or Pulley.									
	110	120	130	140	150	160	170	180	190	200
Inches.	V	V	V	V	V	V	V	V	V	V
1	.47996	.52360	.56723	.61086	.65450	.69813	.74176	.78540	.82903	.87266
2	.95993	1.0472	1.1345	1.2217	1.3090	1.3963	1.4835	1.5708	1.6581	1.7453
3	1.4399	1.5708	1.7017	1.8326	1.9635	2.0944	2.2253	2.3562	2.4870	2.6180
4	1.9199	2.0944	2.2689	2.4435	2.6180	2.7925	2.9671	3.1416	3.3160	3.4906
5	2.3998	2.6180	2.8362	3.0543	3.2725	3.4907	3.7088	3.9270	4.1451	4.3633
6	2.8800	3.1416	3.4034	3.6652	3.9270	4.1888	4.4506	4.7124	4.9742	5.2360
7	3.3597	3.6652	3.9706	4.2760	4.5815	4.8869	5.1924	5.4978	5.8032	6.1086
8	3.8397	4.1888	4.5378	4.8869	5.2360	5.5851	5.9341	6.2832	6.6322	6.9813
9	4.3196	4.7124	5.1051	5.4978	5.8905	6.2832	6.6759	7.0686	7.4612	7.8540
10	4.7996	5.2360	5.6723	6.1086	6.5450	6.9813	7.4177	7.8540	8.2903	8.7266
11	5.2796	5.7596	6.2395	6.7195	7.1990	7.6794	8.1994	8.6394	9.1193	9.5992
12	6.7596	6.2832	6.8068	7.3304	7.8540	8.3776	8.9012	9.4248	9.9483	10.472
13	6.2395	6.8060	7.3740	7.9412	8.5085	9.0757	9.6429	10.210	10.777	11.345
14	6.7195	7.3304	7.9412	8.5521	9.1630	9.7738	10.385	10.996	11.606	12.217
15	7.1995	7.8540	8.5085	9.1630	9.8175	10.472	11.126	11.781	12.435	13.090
16	7.6794	8.3776	9.0757	9.7738	10.472	11.170	11.868	12.566	13.264	13.963
18	8.6394	9.4248	10.210	10.996	11.781	12.566	13.352	14.137	14.422	15.708
20	9.5993	10.472	11.345	12.217	13.090	13.963	14.835	15.708	16.580	17.453
21	10.079	10.966	11.912	12.828	13.744	14.661	15.577	16.493	17.409	18.326
24	11.519	12.566	13.613	14.661	15.709	16.755	17.802	18.850	19.897	20.944
27	12.959	14.137	15.315	16.493	17.671	18.850	20.027	21.206	22.384	23.562
30	14.399	15.708	17.017	18.326	19.635	20.944	22.253	23.562	24.871	26.180
33	15.839	17.278	18.718	20.159	21.597	23.038	24.478	25.918	27.358	28.798
36	17.280	18.850	20.420	21.991	23.562	25.133	26.704	28.274	29.845	31.416
39	18.719	20.420	22.122	23.824	25.525	27.227	28.929	30.631	32.332	34.034
42	20.159	21.992	23.824	25.656	27.489	29.322	31.154	32.987	34.819	36.652
45	21.599	23.562	25.525	27.489	29.452	31.416	33.379	35.343	37.306	39.270
48	23.039	25.132	27.227	29.322	31.416	33.510	35.605	37.699	39.792	41.888
51	24.579	26.704	28.929	31.154	33.379	35.605	37.830	40.055	42.279	44.506
54	26.019	28.274	30.621	32.987	35.343	37.699	40.055	42.411	44.767	47.124
60	28.800	31.416	34.034	36.652	39.270	41.888	44.506	47.124	49.742	52.360
66	31.678	34.558	37.437	40.317	43.197	46.077	48.956	51.836	54.716	57.596
72	34.557	37.700	40.841	43.982	47.124	50.265	53.407	56.549	59.690	62.832
78	37.437	40.840	44.244	47.647	51.050	54.454	57.858	61.261	64.664	68.068
84	40.317	43.984	47.647	51.313	54.980	58.643	62.308	65.973	69.639	73.304
90	43.197	47.125	51.051	54.978	58.905	62.832	66.759	70.686	74.613	78.540
96	46.076	50.264	54.454	58.643	62.830	67.021	71.209	75.398	79.587	83.776
102	48.956	53.404	57.858	62.308	66.760	71.209	75.660	80.111	84.561	89.012
108	51.836	56.548	61.261	65.973	70.685	75.398	80.111	84.823	89.535	94.248
114	54.716	59.692	64.664	69.638	74.615	79.587	84.561	89.535	94.509	99.484
120	57.595	62.832	68.068	73.304	78.540	83.776	89.012	94.248	99.484	104.72
126	60.476	65.972	71.471	76.969	82.465	87.965	93.462	98.960	104.46	109.95
132	63.355	69.116	74.875	80.634	86.395	92.153	97.913	103.67	109.43	115.19
138	66.235	72.256	78.278	84.299	90.320	96.342	102.36	108.38	114.41	120.43
144	69.115	75.400	81.681	87.965	94.250	100.53	106.81	113.10	119.38	125.66
150	71.995	78.540	85.085	91.630	98.175	104.72	111.26	117.81	124.35	130.90
160	76.794	83.776	90.757	97.738	104.72	111.70	118.68	125.66	132.64	139.63
180	86.394	94.248	102.10	109.96	117.81	125.66	133.52	141.37	149.22	150.08
200	87.266	104.72	113.45	122.17	130.90	139.63	148.35	157.08	165.80	174.53
240	115.19	126.66	136.13	146.61	157.08	167.55	178.02	188.50	198.97	209.44

Velocity in Feet per Second of Belts, Wire Ropes, or of Circumference of Revolving Wheels or Pulleys.

Diam. Pulley.	Revolutions per Minute of Wheel or Pulley.									
	210	220	230	240	250	260	270	280	290	300
Inches.	V	V	V	V	V	V	V	V	V	V
1	.91630	.95993	1.0036	1.0472	1.0908	1.1345	1.1781	1.2218	1.2654	1.3090
2	1.8326	1.9199	2.0071	2.0944	2.1816	2.2689	2.3562	2.4435	2.5307	2.6180
3	2.7489	2.8798	3.0107	3.1416	3.2724	3.4034	3.5343	3.6652	3.7961	3.9270
4	3.6652	3.8397	4.0142	4.1888	4.3633	4.5378	4.7124	4.8869	5.0614	5.2360
5	4.5814	4.7997	5.0178	5.2360	5.4580	5.6723	5.8905	6.1086	6.3268	6.5450
6	5.4977	5.7596	6 0214	6.2832	6.5450	6.8068	7.0686	7.3304	7.5921	7.8540
7	6.4140	6.7195	7.0249	7.3304	7.6335	7.9412	8.2467	8.5521	8.8575	9.1630
8	7.3303	7.6794	8.0285	8.3776	8.7265	9.0757	9.4248	9.7738	10.123	10.472
9	8.2466	8.6394	9.0320	9.4248	9.8175	10.210	10.603	10.995	11.388	11.781
10	9.1610	9.5993	10.036	10.472	10.908	11.345	11.781	12.218	12.654	13.090
11	10.079	10.559	11.039	11.519	11.999	12.479	12.959	13.439	13.919	14.398
12	10.996	11.519	12.043	12.566	13.090	13.613	14.137	14.660	15.184	15.708
13	11.912	12.479	13.016	13.615	14.180	14.748	15.315	15.882	16.450	17.017
14	12.828	13.439	14.050	14.662	15.271	15.882	16.493	17.104	17.715	18.326
15	13.744	14.399	15.052	15.709	16.362	17.017	17.671	18.326	18.980	19.635
16	14.661	15.359	16.054	16.776	17.453	18.151	18.850	19.548	20.246	20.944
18	16.493	17.279	18.061	18.851	19.635	20.420	21.206	21.991	22.776	23.562
20	18.326	19.199	20.071	20.944	21.816	22.689	23.562	24.435	25.307	26.180
21	19.242	20.159	21.748	21.991	22.907	23.824	24.790	25.656	26.573	27.489
24	21.991	23.038	24.085	25.133	26.180	27.227	28.274	29,321	30.369	31.416
27	24.740	25.918	27.096	28.274	29.452	30.630	31.809	32.987	34.165	35.343
30	27.489	28.798	30.107	31.416	32.725	34.034	35.343	36.652	37.961	39.270
33	30.238	31.678	33.117	34.558	35.997	37.437	38.877	40.317	41.757	43.194
36	32.987	34.557	36.128	37.699	39.270	40.841	42.412	43 982	45.553	47.124
39	35.735	37.437	39.138	40.841	42.542	44.244	45.946	47.647	49.349	51.051
42	38.485	40.317	42.149	43.982	45.815	47.647	49.480	51.313	53.145	54.978
45	41.233	43.197	45.160	47.124	49.085	51.051	53.014	54.978	56.941	58.905
48	43.982	46.077	48.171	50.266	52.360	54.454	56.549	58.643	60.737	62.832
51	46.731	48.956	51.182	53.407	55.630	57.857	60.083	62.308	64.533	66.759
54	49.480	51.836	54.192	56.549	58.905	61.261	63.617	65.973	68.329	70.686
60	54.978	57.596	60.214	62.832	65.450	68.068	70.686	73.304	75.922	78.540
66	60.476	63.356	66.235	69.115	71.990	74.874	77.755	80.634	83.514	86.394
72	65.973	69.116	72.256	75.399	58.540	81.681	84.823	87.964	91.106	94.248
78	71.471	74.876	78.278	81.682	58.085	88.488	91.892	95.295	98.698	102.10
84	76.969	80.635	84.299	87.965	91.630	95.294	98.960	102.63	106.29	109.96
90	82.466	86.395	90.320	94.248	98.175	102.10	106.03	109.55	113.88	117.81
96	87.964	92.154	96.342	100.53	104.72	108.91	113.10	117.29	121.47	125.66
102	93 462	97.914	102.36	106.81	111.26	115.71	120.17	124.61	129.07	133.52
108	98.960	103.67	108.38	113.10	117.61	122.52	127.23	131.95	136.66	141.37
114	104.46	109.43	114.40	119.38	124.35	129.33	134.30	139.28	144.25	149.23
120	109.96	115.19	120.43	125.66	130.90	136.13	141.37	146.66	151.84	157.08
126	115.45	120.95	126.45	131.95	137.44	142.94	148.44	153.94	159.44	164.93
132	120.95	126.67	132.47	138.23	143.99	149.80	155.51	161.27	167.03	172.79
138	126.45	132.47	138.49	144.51	150.53	156.60	162.58	168.60	174.62	180.64
144	131.94	138.23	144.51	150.80	157.09	163.41	169.65	175.93	182.21	188.50
150	137.44	143.99	150.52	157.09	163.62	170.17	176.71	183.26	189.80	196.35
160	146.61	153.59	160.54	167.76	174.53	181.51	188.56	195.48	202.46	209.44
180	164.93	172.79	180.61	188.51	191.35	204.20	212.06	219.91	227.76	235.62
200	183.26	191.99	200.71	209.44	218.16	226.89	247.90	244.35	253.07	261.80
240	219.91	230.38	240.85	251.33	261.80	272.27	282.74	293.21	303.69	314.16

Velocity in Feet per Second of Belts, Wire Ropes, or of Circumference of Revolving Wheels or Pulleys.

Diam. Pulley.	Revolutions per Minute of Wheel or Pulley.									
	310	320	330	340	350	360	370	380	390	400
Inches.	V	V	V	V	V	V	V	V	V	V
1	1.3526	1.3963	1.4399	1.4835	1.5271	1.5708	1.6144	1.6581	1.7017	1.7454
2	2.7052	2.7925	2.8798	2.9761	3.0543	3.1416	3.2289	3.3162	3.4034	3.4906
3	4.0579	4.888	4.3197	4.4506	4.5815	4.7124	4.8433	4.9743	5.1051	5.2360
4	5.4105	5.5850	5.7596	5.9341	6.1080	6.2832	6.4577	6.6324	6.8068	6.9813
5	6.7632	6.9813	7.1995	7.4176	7.6360	7.8540	8.0722	8.2905	8.5085	8.7266
6	8.1158	8.3776	8.6394	8.9012	9.1630	9.4248	9.6866	9.948	10.210	10.472
7	9.4684	9.7738	10.079	10.385	10.690	10.996	11.301	11.606	11.912	12.217
8	10.821	11.170	11.519	11.868	12.217	12.566	12.915	13.265	13.613	13.963
9	12.174	12.566	12.959	13.352	13.744	14.137	14.530	14.922	15.315	15.708
10	13.526	13.963	14.399	14.835	15.271	15.708	16.144	16.581	17.017	17.453
11	14.879	15.359	15.839	16.319	16.798	17.279	17.759	18.239	18.718	19.198
12	16.232	16.755	17.279	17.802	18.326	18.850	19.373	19.897	20.420	20.944
13	17.584	18.151	18.719	19.286	19.853	20.420	20.988	21.555	22.122	22.690
14	18.937	19.548	20.159	20.769	21.384	21.991	22.602	23.213	23.824	24.434
15	20.289	20.944	21.599	22.253	22.907	23.562	24.216	24.872	25.525	26.180
16	21.652	22.340	23.038	23.736	24.434	25.133	25.831	26.530	27.227	27.296
18	24.347	25.133	25.918	26.704	27.489	28.274	29.060	29.846	30.630	31.416
20	27.052	27.925	28.798	29.671	30.543	31.416	32.289	33.162	34.034	34.906
21	28.405	29.321	30.238	31.154	32.071	32.987	33.903	34.820	35.735	36.652
24	32.463	33.510	34.558	35.605	36.652	37.699	38.746	39.795	40.841	41.888
27	36.521	37.699	38.867	40.055	41.234	42.412	43.590	44.769	45.946	47.124
30	40.579	41.888	43.187	44.506	45.815	47.124	48.433	49.743	51.051	52.360
33	44.637	46.077	47.507	48.956	50.395	51.836	53.276	54.717	56.156	57.596
36	48.695	50.265	51.826	53.407	54.980	56.549	58.119	59.691	61.261	62.832
39	52.753	54.454	56.146	57.858	59.560	61.261	62.963	64.666	66.366	68.068
42	56.810	58.643	60.466	62.308	64.140	65.974	67.806	69.640	71.471	73.304
45	60.868	62.832	64.786	66.759	68.720	70 686	72.649	74.614	76.576	78.540
48	64.926	67.020	69.105	71.209	73.305	75.398	77.493	79.589	81.681	83.776
51	68.984	71.209	73.425	75.660	77.885	80.111	82.336	84.563	86.787	89.012
54	73.042	75.398	77.745	80.111	82.465	84.823	87.179	89.537	91.892	94.248
60	81.158	83.776	86.394	89.012	91.630	94.248	96.866	99.486	102.10	104.72
66	89.274	92.153	95.033	97.913	100.79	103.67	106.55	109.43	112.31	115.19
72	97.389	100.53	103.67	106.81	109.95	113.10	116.24	119.38	122.52	125.66
78	105.51	108.91	112.31	115.72	119.12	122.52	125.93	129.33	132.73	136.14
84	113.62	117.29	120.95	124.62	120.28	131.95	135.61	139.28	142.94	146.61
90	121.74	125.66	129.59	133.52	137.44	141.37	145.30	149.22	153.15	157.08
96	129.85	134.04	138.23	142.42	146.61	150.80	154.99	159.17	163.36	167.55
102	137.97	142.42	146.87	151.32	155.72	160.22	164.67	169.12	173.57	178.02
108	146.08	150.80	155.51	160.22	164.93	163.65	174.36	179.07	183.78	188.50
114	154.20	159.17	164.15	169.12	174.20	179.07	184.05	189.02	193.99	198.97
120	162.32	167.55	172.79	178.02	183.26	188.50	193.73	198.97	204.20	209.44
126	170.31	175.93	181.43	186.92	192.42	197.92	203.42	208.92	214.41	219.90
132	178.55	184.31	190.07	195.83	201.58	207.35	213.10	218.87	224.62	230.38
138	186.66	192.68	198.71	204.73	210.75	216.77	222.79	228.82	234.83	240.86
144	194.78	201.06	207.35	213.63	219.91	226.19	232.48	238.76	245.04	251.32
150	202.89	209.44	215.99	222.53	229.07	235.62	242.16	248.72	255.25	261.80
160	216.52	223.40	230.38	237.36	244.34	251.33	258.31	265.30	272.27	279.26
180	243.47	251.33	259.18	267.04	274.89	282.74	290.60	298.46	306.30	314.16
200	270.52	279.25	287.98	296.71	305.43	314.16	322.89	331.62	340.34	349.06
240	284.05	335.10	345.58	356.05	366.52	376.99	387.46	397.95	408.41	418.88

Motive Force in Pounds per Horse-Power transmitted in the Periphery of Revolving Wheels or Pulleys.

Diam. Pulley.	Revolutions n per Minute of Wheel or Pulley.									
	10	20	30	40	50	60	70	80	90	100
Inches.	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*
1	1260.5	6302.5	4201.6	3151.2	2521.0	2100.8	1800.7	1578.6	1400.6	1260.5
2	6302.5	3151.2	2100.8	1578.6	1260.5	1050.4	900.35	789.30	700.30	630.25
3	4201.6	2100.8	1400.6	1050.4	840.33	700.30	600.23	525.20	466.86	430.16
4	3151.2	1575.6	1050.4	789.30	630.25	525.20	150.17	394.65	350.15	315.12
5	2521.0	1260.5	840.33	630.25	504.20	420.16	360.14	318.12	280.12	252.10
6	2100.8	1050.4	700.30	525.20	420.16	350.13	300.12	263.10	233.10	210.10
7	1800.7	900.35	600.23	450.17	360.14	300.12	257.24	225.10	200.08	180.07
8	1578.6	789.30	525.20	394.65	315.72	263.10	225.51	197.32	175.40	157.86
9	1400.6	700.30	466.86	350.15	280.12	233.43	200.10	175.07	155.62	140.06
10	1260.5	630.25	420.16	315.12	252.10	210.08	180.07	157.56	140.05	126.05
11	1146.0	573.00	382.00	286.50	229.20	191.00	163.43	133.25	127.11	114.60
12	1050.4	525.20	350.13	262.60	210.08	175.07	150.06	131.30	116.81	105.04
13	969.61	484.80	323.20	242.40	193.92	161.60	138.51	121.20	107.73	96.961
14	900.35	450.17	300.11	225.09	180.07	150.06	128.62	112.32	100.04	90.035
15	840.33	420.16	280.11	210.08	168.06	140.05	120.05	105.04	93.37	84.033
16	789.30	394.65	263.10	197.57	157.86	131.55	112.75	98.66	87.700	78.930
18	700.30	350.15	233.43	175.08	140.06	116.71	100.04	87.537	78.812	70.030
20	630.25	315.12	210.08	157.56	126.05	105.04	90.039	78.781	70.028	63.025
21	600.23	300.11	200.08	150.06	120.05	100.04	85.747	75.030	66.692	60.023
24	525.20	262.60	175.06	131.55	105.04	87.533	75.020	65.650	59.400	52.520
27	466.86	233.43	155.62	116.71	93.372	77.810	66.694	58.357	51.855	46.686
30	420.16	210.08	140.05	105.04	84.032	70.027	60.026	52.520	46.686	42.016
33	382.00	191.00	127.33	95.500	76.400	63.666	54.559	47.750	42.444	38.200
36	350.15	175.07	116.71	87.54	70.038	58.360	50.021	43.769	37.794	35.015
39	323.20	161.60	107.73	80.800	64.640	53.866	46.171	40.400	35.911	32.320
42	300.12	150.06	100.04	75.030	60.240	50.020	42.874	37.515	33.343	30.012
45	280.12	140.06	93.373	70.030	56.024	46.687	40.017	35.015	31.123	28.012
48	263.10	131.55	87.700	65.775	52.620	43.850	37.586	32.885	28.145	26.310
51	247.16	123.58	82.386	61.790	49.432	41.193	35.310	30.895	27.462	24.716
54	233.43	116.71	77.810	58.357	46.686	38.905	33.347	29.180	25.825	23.343
60	210.08	105.04	70.026	52.520	42.016	35.013	30.011	26.010	23.342	21.008
66	191.00	95.500	63.666	47.750	38.200	31.833	27.286	23.875	21.222	19.100
72	175.40	87.700	58.466	43.850	35.080	29.233	25.057	21.925	19.490	17.540
78	161.60	80.800	53.866	40.400	32.320	26.933	23.086	20.200	18.955	16.160
84	150.06	75.030	50.020	37.515	30.012	25.010	21.437	18.757	16.682	15.006
90	140.06	70.030	46.686	35.015	28.012	23.343	20.008	17.507	15.562	14.006
96	131.55	65.775	43.850	32.887	26.310	21.925	18.793	16.444	14.616	13.155
102	123.58	61.790	41.193	30.895	24.716	20.597	17.654	15.447	13.731	12.358
108	116.71	58.355	38.901	29.175	23.342	19.452	16.387	14.586	12.968	11.671
114	110.57	55.285	36.856	27.642	22.114	18.428	15.996	13.821	12.285	11.057
120	105.04	51.020	35.013	26.260	21.008	17.507	15.006	13.130	11.671	10.504
126	100.04	50.020	33.346	25.010	20.008	16.673	14.291	12.505	11.115	10.004
132	95.500	47.750	31.833	23.875	19.100	15.916	13.643	11.938	10.611	9.5500
138	90.978	45.489	30.326	22.744	18.196	15.163	12.997	11.372	10.109	9.0978
144	87.533	43.766	29.177	21.883	17.506	14.589	12.505	10.942	9.7260	8.7533
150	84.033	42.016	28.011	21.008	16.807	14.005	12.005	10.504	93.370	8.4033
160	78.930	39.465	26.310	19.742	15.786	13.155	11.261	9.8662	8.8662	7.8930
180	70.030	35.015	23.343	17.507	14.006	11.672	10.004	8.7537	7.7811	7.0030
200	63.025	31.512	21.008	15.756	12.605	10.504	9.004	7.8781	7.0030	6.3025
240	52.520	26.260	17.506	13.130	10.504	8.7533	7.5028	6.5650	5.8355	5.2520

Horse-Power for Different Motive Forces *F* and Velocities *V*.

Motive Force.	Velocity *V* in Feet per Second.									
	10	20	30	40	50	60	70	80	90	100
F lbs.	*HP*	*HP*	*HP*	*HP*	*HP*	*HP*	*HP*	*HP*	*HP*	*HP*
510	9.2727	18.545	27.818	37.091	46.363	55.636	64.909	74.182	84.454	92.727
520	9.4545	18.909	28.363	37.818	47.273	56.727	66.182	75.636	86.091	94.545
530	9.6363	19.273	28.909	38.545	48.182	57.818	67.454	77.091	87.727	96.363
540	9.8181	19.636	29.454	39.273	49.091	58.909	68.727	78.545	88.363	98.181
550	10.000	20.000	30.000	40.000	50.000	60.000	70.000	80.000	90.000	100.00
560	10.181	20.363	30.545	40.727	50.909	61.091	71.273	81.454	91.636	101.81
570	10.363	20.727	31.091	41.454	51.818	62.182	72.545	82.909	93.273	103.63
580	10.545	21.091	31.636	42.182	52.727	63.273	73.818	84.363	94.909	105.45
590	10.626	21.444	32.182	42.909	53.636	64.363	75.091	85.818	96.545	106.26
600	10.909	21.818	32.727	43.636	54.545	65.454	76.363	87.273	98.182	109.09
610	11.091	22.182	33.273	44.363	55.454	66.545	77.636	88.727	99.82	110.91
620	11.273	22.545	33.818	45.091	56.363	67.636	78.909	90.182	101.45	112.73
630	11.454	22.909	34.363	45.818	57.273	68.727	80.182	91.636	103.09	114.54
640	11.636	23.273	34.909	46.545	58.182	69.818	81.454	93.091	104.73	116.36
650	11.818	23.636	35.454	47.273	59.091	70.909	82.727	94.546	106.26	118.18
660	12.000	24.000	36.000	48.000	60.000	72.000	84.000	96.000	108.00	120.00
670	12.182	24.363	36.545	48.727	60.909	73.091	85.273	97.454	109.63	121.82
680	12.364	24.727	37.091	49.454	61.818	74.182	86.545	98.909	111.27	123.64
690	12.545	25.091	37.636	50.182	62.727	75.273	87.818	100.36	112.91	125.45
700	12.727	25.444	38.182	50.909	63.636	76.363	89.091	101.82	114 54	127.27
710	12.909	25.818	38.727	51.636	64.545	77.454	90.363	103.27	116.18	129.09
720	13.090	25.182	39.273	52.363	65.454	78.545	91.636	104.73	117.82	130.90
730	13.273	25.545	39.818	53.091	66.363	79.636	92.909	106.18	119.45	132.73
740	13.454	25.909	40.363	53.818	67.273	80.727	94.182	107.63	121.09	134.54
750	13.636	26.273	40.909	54.545	68.182	81.818	95.454	109.09	122.73	136.36
760	13.818	26.636	41.454	55.273	69.091	82.909	96.727	110.54	124.36	138.18
770	14.000	28.000	42.000	56.000	70.000	84.000	98.000	112.00	126 00	140.00
780	14.182	28.363	42.545	56.727	70.909	85.091	99.273	113.45	127.63	141.82
790	14.363	28.727	43.091	57.454	71.818	86.182	100.54	114.91	129.27	143.63
800	14.545	29.091	43.636	58.182	72.727	87.273	101.82	116.36	130.91	145.45
810	14.727	29.444	44.182	58.909	73.636	88.363	103.09	117.82	132.54	147.27
820	14.909	29.818	44.727	59.636	74.545	89 454	104.36	119.27	134.18	149.09
830	15.091	30.182	45.273	60.363	75.454	90.545	105.63	120.72	135.82	150.91
840	15.273	30.545	45.818	61.091	76.363	91.636	106.91	122.18	137.45	152.73
850	15.454	30.909	46.363	61.818	77.273	92.727	108.18	123.63	139.09	154.54
860	15.636	31.273	46.909	62.545	78.182	93.818	109.45	125.09	140.73	156.36
870	15.818	31.363	47.454	63.273	79.090	94.909	110.73	126.54	142.36	158.18
880	16.000	32.000	48.000	64.000	80.000	96.000	112.00	128.00	144.00	160.00
890	16.182	32.363	48.545	64.727	80.909	97.091	113.27	129.45	145.63	161.82
900	16.364	32.727	49.091	65.454	81.818	98.182	114.54	130.91	147.27	163.64
910	16.545	33.091	49.636	66.182	82.727	99.273	115.82	132.36	148.91	165.45
920	16.727	33.444	50.182	66.909	83.363	100.36	117.09	133.82	150.54	167.27
930	16.909	33.818	50.727	67.636	84.545	101.45	118.36	135.27	152.18	169.09
940	17.091	34.182	51.273	68.363	85.454	102.54	119.63	136.73	153.82	170.91
950	17.273	34.545	51.818	69.091	86.363	103.63	120.91	138.18	155.45	172.73
960	17.454	34.909	52.363	69.818	87.273	104.73	122.18	139.63	157.09	174.54
970	17.636	35.273	52.909	70.545	88.182	105.82	123.45	141.09	158.73	176.36
980	17.818	35.636	53.454	71.273	89.091	106.91	124.72	142.54	160.36	178.18
990	18.000	36.000	54.000	72.000	90.000	108.00	126.00	144.00	162.00	180.00
1000	18.182	36.363	54.545	72.727	90.909	109.1	128.27	145.45	163.63	181.82

Diameters in Inches of Wrought-Iron Shafts.

Horse power	Number of revolutions per minute of wrought-iron shafts.												
	10	15	20	25	30	35	40	45	50	55	60	70	80
IP.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
1	2.32	2.07	1.84	1.71	1.61	1.53	1.46	1.41	1.36	1.31	1.28	1.22	1.16
2	2.92	2.55	2.32	2.16	2.03	1.92	1.84	1.72	1.71	1.66	1.61	1.53	1.46
3	3.40	2.72	2.66	2.47	2.33	2.21	2.11	2.03	1.91	1.90	1.71	1.75	1.67
4	3.68	3.22	2.92	2.72	2.55	2.43	2.32	2.23	2.16	2.09	1.80	1.93	1.84
5	3.97	3.48	3.15	2.92	2.75	2.61	2.50	2.41	2.33	2.25	1.88	2.08	1.99
6	4.22	3.68	3.35	3.11	2.92	2.78	2.66	2.56	2.47	2.39	2.06	2.21	2.11
7	4.44	3.88	3.52	3.27	3.06	2.92	2.80	2.69	2.60	2.52	2.12	2.33	2.22
8	4.64	4.05	3.69	3.42	3.22	3.06	2.92	2.82	2.72	2.63	2.27	2.43	2.33
9	4.82	4.22	3.83	3.56	3.35	3.18	3.04	2.93	2.82	2.74	2.40	2.52	2.42
10	5.00	4.36	3.98	3.69	3.47	3.29	3.15	3.03	2.92	2.83	2.75	1.61	2.50
12	5.34	4.64	4.22	3.92	3.68	3.50	3.35	3.22	3.11	3.01	2.92	2.78	2.61
15	5.72	5.00	4.54	4.22	3.97	3.76	3.60	3.47	3.35	3.24	3.15	3.00	2.86
20	6.30	5.50	5.00	4.64	4.37	4.14	3.97	3.82	3.96	3.57	3.47	3.29	3.15
25	6.79	5.93	5.39	5.00	4.71	4.46	4.28	4.11	3.97	3.84	3.74	3.55	3.40
30	7.21	6.30	5.72	5.32	5.00	5.74	4.54	4.37	4.22	4.08	3.97	3.77	3.61
35	7.59	6.63	6.03	5.60	5.26	5.00	4.78	4.60	4.44	4.30	4.18	3.97	3.80
40	7.94	6.93	6.30	5.85	5.50	5.22	5.00	4.81	4.65	4.50	4.37	4.15	3.98
45	8.25	7.20	6.55	6.09	5.73	5.43	5.20	5.00	4.83	4.67	4.54	4.31	4.13
50	8.55	7.47	6.79	6.30	5.93	5.62	5.38	5.18	5.00	4.84	4.71	4.47	4.28
60	9.08	7.93	7.21	6.70	6.30	5.98	5.72	5.50	5.32	5.15	5.08	4.75	4.55
70	9.57	8.36	7.59	7.05	6.64	6.30	6.03	5.80	5.60	5.42	5.27	5.00	4.79
80	10.0	8.75	7.94	7.37	6.94	6.58	6.30	6.07	5.85	5.67	5.51	5.23	5.00
90	10.4	9.11	8.25	7.67	7.22	6.84	6.55	6.30	6.10	5.90	5.73	5.44	5.20
100	10.8	9.41	8.55	7.94	7.47	7.09	6.78	6.53	6.31	6.10	5.93	5.63	5.39
120	11.4	10.0	9.09	8.44	7.95	7.53	7.21	6.94	6.70	6.49	6.31	6.00	5.73
140	12.0	11.5	9.57	8.89	8.37	7.93	7.59	7.31	7.06	6.83	6.64	6.30	6.03
160	12.6	11.0	10.0	9.29	8.74	8.29	7.94	7.64	7.38	7.14	6.94	6.59	6.31
180	13.1	11.4	10.4	9.67	9.09	8.62	8.26	7.95	7.67	7.42	7.21	6.85	6.55
200	13.6	11.9	10.8	10.0	9.42	8.94	8.55	8.23	7.94	7.69	7.48	7.10	6.79
250	14.6	12.8	11.6	10.8	10.1	9.63	9.22	8.87	8.56	8.29	8.06	7.65	7.32
300	15.5	13.6	12.3	11.5	10.8	10.2	9.80	9.43	9.10	8.80	8.57	8.12	7.77
350	16.3	14.3	13.0	12.0	11.3	10.8	10.3	9.92	9.58	9.27	9.00	8.55	8.18
400	17.1	15.0	13.6	12.6	11.9	11.2	10.8	10.4	10.0	9.69	9.42	8.94	8.55
450	17.8	15.5	14.1	13.1	12.3	11.7	11.2	10.8	10.4	10.1	9.80	9.30	8.89
500	18.4	16.1	14.6	13.6	12.8	12.1	11.6	11.2	10.8	10.4	10.1	9.64	9.21
550	19.0	16.6	15.1	14.0	13.2	12.5	12.0	11.5	11.1	10.8	10.5	9.94	9.50
600	19.6	17.1	15.5	14.4	13.5	12.9	12.3	11.9	11.5	11.1	10.8	10.2	9.79
700	20.7	18.0	16.4	15.2	14.3	13.6	13.0	12.5	12.1	11.7	11.4	10.8	10.3
800	21.5	18.9	17.1	15.9	15.0	14.2	13.6	13.1	12.6	12.2	11.9	11.9	10.8
1000	23.3	20.4	18.5	17.1	16.1	15.3	14.6	14.1	13.6	13.2	12.8	12.2	11.6
1200	24.7	21.6	19.6	19.2	17.1	16.3	15.6	14.9	14.5	14.0	13.6	12.9	12.4
1500	26.6	23.3	21.1	19.6	18.5	17.5	16.7	16.1	15.5	15.1	14.6	13.9	13.3
2000	29.3	25.5	23.5	21.5	20.3	19.3	18.4	17.7	17.1	16.6	16.1	15.3	14.6
2500	31.5	27.5	25.0	23.3	21.9	20.8	19.8	19.1	18.4	17.9	17.3	16.5	15.8
3000	33.5	29.3	26.6	24.8	23.3	22.1	21.1	20.3	19.6	19.0	18.4	17.5	16.7
3500	35.2	30.8	28.0	26.0	24.4	23.3	22.2	21.4	20.7	20.0	19.4	18.4	17.6
4000	36.8	32.2	29.3	27.2	25.6	24.3	23.3	22.3	21.6	20.9	20.3	19.3	18.5
4500	38.4	33.5	30.4	28.3	26.6	25.2	24.1	23.3	22.4	21.7	21.1	20.0	19.2
5000	39.6	34.7	31.5	29.3	27.2	26.1	25.0	24.1	23.3	22.5	21.9	20.8	19.9

Motive Force in Pounds per Horse-Power transmitted in the Periphery of Revolving Wheels or Pulleys.

Diam. Pulley.	Revolutions *n* per Minute of Wheel or Pulley.									
	110	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
Inches.	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*
1	1145.9	1050.4	969.62	900.35	840.33	789.30	741.71	700.30	663.43	630.25
2	572.96	525.20	484.81	450.17	420.16	394.65	370.85	350.15	331.71	315.12
3	381.97	350.15	323.21	300.11	280.11	262.60	247.23	233.43	221.14	210.08
4	286.48	263.10	242.41	225.08	210.08	197.32	185.43	175.75	165.85	157.56
5	229.18	210.08	193.92	180.07	168.06	159.06	148.34	140.06	132.68	126.05
6	190.98	175.40	161.61	150.06	140.05	131.55	123.61	116.55	110.57	105.04
7	163.70	150.06	138.52	128.62	120.05	112.55	105.67	100.04	94.779	90.036
8	143.24	131.55	121.21	112.75	105.04	98.660	92.715	87.700	82.925	78.780
9	127.33	116.71	107.74	100.05	93.370	87.535	82.410	77.810	73.713	70.026
10	114.59	105.04	96.962	90.035	84.033	78.780	74.171	70.025	66.343	63.025
11	101.17	95.500	88.147	81.715	76.392	66.625	67.428	63.550	60.311	57.300
12	95.500	87.533	80.805	75.030	70.025	65.650	61.805	58.405	55.285	52.520
13	88.147	80.800	74.586	69.255	64.641	60.600	57.056	53.865	51.033	48.481
14	81.851	75.028	69.260	64.210	60.025	56.160	52.835	50.020	47.389	45.018
15	76.394	70.030	64.640	60.025	56.020	52.520	49.446	46.685	44.226	42.016
16	71.620	65.650	60.605	56.375	52.520	49.330	46.357	43.850	41.462	39.390
18	63.664	58.360	53.870	50.020	46.685	43.768	41.205	38.906	36.856	35.013
20	57.296	52.520	48.481	45.020	42.016	39.391	37.085	35.014	33.171	31.512
21	54.567	50.021	46.173	42.873	40.016	37.515	35.223	33.346	31.593	30.012
24	47.750	43.850	40.402	37.510	35.012	32.825	30.403	29.500	27.642	26.260
27	42.443	37.794	35.913	33.347	31.125	29.178	27.470	25.927	24.571	23.342
30	38.197	35.015	32.321	30.013	28.010	26.260	24.723	23.343	22.114	21.008
33	34.725	31.833	29.382	27.279	25.462	23.875	22.476	21.222	20.104	19.100
36	31.832	29.177	26.935	25.011	23.342	21.884	20.603	18.897	18.428	17.506
39	29.382	26.933	24.862	23.085	21.547	20.200	19.019	17.955	17.011	16.160
42	27.283	25.010	23.086	21.437	20.008	18.757	17.611	16.676	15.796	15.006
45	25.466	23.346	21.548	20.008	18.673	17.507	16.482	15.561	14.745	14.005
48	23.875	21.925	20.201	18.793	17.506	16.442	15.202	14.072	13.821	13.130
51	22.469	20.596	19.012	17.655	16.477	15.447	14.543	13.731	13.008	12.358
54	21.222	18.897	17.956	16.673	15.562	14.590	13.735	12.912	12.285	11.671
60	19.098	17.540	16.169	15.005	14.005	13.005	12.361	11.671	11.057	10.504
66	17.363	15.916	14.691	13.643	12.731	11.937	11.238	10.611	10.052	9.5500
72	15.916	14.585	13.467	12.528	11.671	10.962	10.302	9.7450	9.2140	8.7530
78	14.691	13.466	12.431	11.543	10.773	10.100	9.5095	9.4775	8.5055	8.0800
84	13.642	12.505	11.543	10.718	10.004	9.3785	8.8055	8.3410	7.8980	7.5030
90	12.733	11.671	10.774	10.004	9.3370	8.7535	8.2410	7.7810	7.3713	7.0026
96	11.938	10.962	10.100	9.3965	8.7530	8.2220	7.6010	7.3080	6.911	6.5650
102	11.235	10.898	9.5060	8.8270	8.2385	7.7235	7.2715	6.8655	6.5040	6.1790
108	10.611	9.4485	8.9780	8.1935	7.7810	7.2930	6.8675	6.4840	6.1425	5.8355
114	10.052	9.2141	8.5055	7.9980	7.3712	6.9105	6.5062	6.1425	5.8195	5.5285
120	9.5500	8.7533	8.0805	7.5030	7.0025	6.5650	6.1805	5.8355	5.5285	5.2520
126	9.0946	8.3666	7.6800	7.1455	6.6692	6.2525	5.8865	5.5575	5.2653	5.0020
132	8.6815	7.9580	7.3455	6.8215	6.3666	5.9640	5.6190	5.3055	5.0260	4.7750
138	8.3038	7.5815	7.0263	6.4985	6.0652	5.6860	5.3747	5.0545	4.8060	4.5489
144	7.9580	7.2940	6.7325	6.2525	5.8354	5.4710	5.1510	4.8630	4.6070	4.3766
150	7.6394	7.0030	6.4640	6.0025	5.6020	5.2520	4.9446	4.6685	4.4226	4.2016
160	7.1620	6.5650	6.0605	5.6305	5.2525	4.9331	4.6357	4.4331	4.1462	3.9390
180	6.3664	5.8360	5.3870	5.0020	4.6685	4.3768	4.1205	3.8905	3.6856	3.5013
200	5.7296	5.2520	4.8481	4.5020	4.2016	3.9891	3.7085	3.5015	3.3171	3.1512
240	4.7750	4.3850	4.0402	3.7514	3.5012	3.2825	3.0103	2.9177	2.7642	2.6260

Motive Force in Pounds per Horse-Power transmitted in the Periphery of Revolving Wheels or Pulleys.

Diam. Pulley.	Revolutions *n* per Minute of Wheel or Pulley.									
	210	**220**	**230**	**240**	**250**	**260**	**270**	**280**	**290**	**300**
Inches.	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*
1	600.23	572.96	548.05	525.20	504.20	484.81	466.86	450.17	434.66	420.16
2	300.11	286.48	274.02	262.60	252.40	242.41	233.13	225.09	217.33	210.08
3	200.08	190.99	182.68	175.07	168.06	164.64	155.62	150.06	144.89	140.05
4	150.06	143.24	137.01	131.55	126.05	121.21	116.71	112.54	108.16	105.04
5	120.05	114.59	109.64	105.04	100.84	96.960	93.040	90.035	86.932	84.032
6	100.04	95.490	91.340	87.700	84.030	80.805	77.700	75.030	72.445	70.025
7	85.746	81.650	78.293	75.030	72.029	69.260	66.693	64.340	62.094	60.023
8	75.170	71.620	68.505	65.775	63.025	60.605	58.466	56.378	54.080	52.520
9	66.700	63.665	60.893	58.355	56.020	53.870	51.873	50.025	48.296	46.683
10	60.023	57.795	54.805	52.520	50.420	48.481	46.683	45.018	43.466	42.016
11	54.477	52.085	49.822	47.750	45.836	44.073	42.370	40.358	39.514	38.200
12	50.020	47.750	45.670	43.766	42.015	40.402	38.933	37.515	36.222	35.012
13	46.170	41.079	42.157	40.400	38.784	37.293	35.910	34.628	33.435	32.320
14	42.873	40.926	39.146	37.514	36.014	34.630	33.346	32.155	31.047	30.011
15	40.016	38.197	36.536	35.015	33.613	32.320	31.123	30.012	28.977	28.071
16	37.583	35.810	34.252	32.825	31.512	30.302	29.233	28.188	27.040	26.260
18	33.343	31.842	30.446	29.180	28.010	26.935	25.937	25.040	24.148	23.341
20	30.013	28.618	27.402	26.260	25.210	24.241	23.343	22.510	21.733	21.008
21	28.582	27.284	26.097	25.011	24.010	23.086	22.231	21.439	20.698	20.008
24	25.006	23.875	22.835	21.925	21.007	20.201	19.800	18.755	18.111	17.506
27	22.231	21.222	20.297	18.897	18.673	17.956	17.285	16.673	16.099	15.561
30	20.009	19.099	18.268	17.507	16.806	16.161	15.562	15.069	14.489	14.005
33	18.186	17.363	16.607	15.916	15.280	14.691	14.148	13.640	13.171	12.733
36	16.673	15.916	15.223	14.588	14.005	13.467	12.598	12.505	12.074	11.677
39	15.390	14.691	14.052	13.466	12.928	12.431	11.970	11.542	11.145	10.740
42	14.291	13.642	13.018	12.505	12.005	11.543	11.114	10.718	10.349	10.004
45	13.339	12.733	12.179	11.673	11.204	10.774	10.374	10.004	9.6590	9.337
48	12.529	11.938	11.417	10 962	10.503	10.100	9.3817	9.3965	9.0555	8.7530
51	11.770	11.234	10.746	10.298	9.8862	9.5060	9.1510	8.8275	8.5227	8.2188
54	11.116	10.611	10.148	9.4485	9.3365	8.9780	8.6083	8.3388	8.0495	7.7805
60	10.004	9.5190	9.293	8.7200	8.4030	8.0845	7.7806	8.5028	7.2145	7.0025
66	9.1953	8.6815	8.3035	7.9580	7.6400	7.3455	7.0740	6.8215	6.5855	6.3666
72	8.352	7.9580	7.6115	7.2940	7.0025	6.7335	6.4956	6.2642	6.0370	5.8383
78	7.6953	7.3455	7.0260	6.7330	6.4640	6.2155	6.3183	5.7715	5.5725	5.3700
84	7.1456	6.821	6.5240	6.2525	6.0025	5.7722	5.5606	5.3592	5.1745	5.0020
90	6.6693	6.3665	6.089	5.8355	5.6020	5.3870	5.1873	5.0029	4.8296	4.6683
96	6.2643	5.9687	5.7085	5.4810	5.2515	5.050	4.8720	4.6982	4.5278	4.3765
102	5.8846	5.6175	5.3730	5.1490	4.9431	4.7530	4.5770	4.4735	4.2613	4.1094
108	5.4623	5.3055	5.0740	4.7242	4.6682	4.4890	4.3226	4.0968	4.0248	3.8902
114	5.3320	5.0260	4.8074	4.6071	4.4228	4.2527	4.0950	3.6660	3.8128	3.6856
120	5.0020	4.7750	4.5670	4.3766	4.2015	4.0402	3.8903	3.7565	3.6222	3.5012
126	4.7633	4.5473	4.3496	4.1833	4.0016	3.8400	3.7050	3.5725	3.4197	3.3346
132	4.5476	4.3408	4.1517	3.9790	3.8200	3.6727	3.5370	3.4108	3.2928	3.1833
138	4.3323	4.1519	3.9713	3.7907	3.6392	3.5131	3.3696	3.2492	3.1555	3.0326
144	4.1683	3.9790	3.8057	3.6472	3.5012	3.3662	3.2420	3.1262	3.0185	2.9177
150	4.0016	3.8197	3.6536	3.5015	3.3613	3.2320	3.1123	3.0612	2.8977	2.8011
160	3.7533	3.5840	3.4252	3.2825	3.1512	3.0302	2.9887	2.8152	2.7040	2.6260
180	3.3316	3.1832	3.0146	2.9180	2.8010	2.6935	2.5937	2.5010	2.4148	2.3341
200	3.0013	2.8648	2.7402	2.6260	2.5210	2.4241	2.3343	2.255	2.1733	2.1008
240	2.5009	2.3875	2.2835	2.1925	2.1007	2.0201	1.9451	1.8757	1.8111	1.7506

Motive Force in Pounds per Horse-Power transmitted in the Periphery of Revolving Wheels or Pulleys.

Diam. Pulley.	Revolutions *n* per Minute of Wheel or Pulley.									
	310	320	330	340	350	360	370	380	390	400
Inches.	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*
1	406.62	394.65	381.97	370.85	360.14	350.15	340.68	331.71	323.21	315.12
2	203.31	197.33	190.99	185.43	180.07	175.07	170.34	165.85	161.60	157.56
3	135.54	131.30	127.32	123.62	120.05	116.71	113.23	110.57	107.73	105.04
4	101.66	98.660	95.495	92.715	90.035	87.537	85.170	82.925	80.803	78.780
5	81.324	79.530	76.394	74.170	72.028	70.030	68.136	66.340	64.840	63.024
6	67.770	65.775	63.660	61.805	60.025	58.275	56.515	55.285	53.533	52.520
7	58.090	56.275	54.566	52.835	51.449	50.020	48.669	47.389	46.173	45.017
8	50.830	49.330	47.747	46.357	45.017	43.850	42.585	41.462	40.403	39.390
9	45.180	43.767	42.443	41.205	40.016	38.905	37.743	36.856	35.913	35.013
10	40.662	39.390	38.197	37.085	36.014	35.012	34.068	33.172	32.321	31.512
11	36.965	33.312	34.723	33.714	32.740	31.775	30.971	30.155	29.382	28.650
12	33.885	32.825	31.833	30.902	30.012	29.202	28.308	27.642	26.935	26.260
13	31.278	30.300	29.382	28.528	27.703	26.932	26.206	25.516	24.862	24.240
14	29.045	28.080	27.284	26.417	25.724	25.010	24.335	23.699	23.086	22.508
15	27.108	26.260	25.465	24.723	24.093	23.342	22.712	22.113	21.546	21.008
16	25.415	24.665	23.873	23.178	22.508	21.925	21.293	20.731	20.202	19.695
18	22.590	21.884	21.221	20.602	20.008	19.453	18.871	18.428	17.956	17.506
20	20.331	19.695	19.099	18.543	18.007	17.507	17.034	16.585	16.160	15.756
21	19.363	18.757	18.189	17.612	17.149	16.673	16.223	15.796	15.391	15.006
24	16.942	16.412	15.916	15.202	15.066	14.850	14.154	13.821	13.467	13.130
27	15.060	14.589	14.146	13.735	13.338	12.963	12.581	12.285	11.971	11.671
30	13.554	13.130	12.733	12.362	12.046	11.671	11.323	11.057	10.774	10.504
33	12.322	11.937	11.575	11.238	10.914	10.611	10.323	10.052	9.7940	9.5500
36	11.295	10.942	10.611	10.302	10.004	9.4485	9.4355	9.2140	8.9783	8.7533
39	10.426	10.100	9.7940	9.5095	9.2343	8.9775	8.7353	8.5055	8.2873	8.0800
42	9.6815	9.3785	9.0943	8.8055	8.5745	8.3380	8.1115	7.8980	7.6953	7.5030
45	9.0360	8.7535	8.4887	8.2410	8.0310	7.7805	7.5707	7.3725	7.1826	7.0026
48	8.4710	8.2210	7.9583	7.6010	7.5030	7.0360	7.0770	6.9105	6.7336	6.5650
51	7.9728	7.7235	7.4897	7.2715	7.0620	6.8655	6.7995	6.5040	6.3373	6.1790
54	7.530	7.2950	7.0740	6.8675	6.6690	6.4560	6.2905	6.1425	5.9853	5.8355
60	6.7770	6.5025	6.3660	6.1805	6.0025	5.8355	5.6615	5.5285	5.3583	5.2520
66	6.1610	5.9685	5.7843	5.6190	5.4570	5.3055	5.1615	5.0260	4.8970	4.7750
72	5.6475	5.4810	5.3053	5.1510	5.0020	4.8725	4.7178	4.6070	4.4890	4.3766
78	5.2130	5.0500	4.8970	4.7047	4.6171	4.7387	4.3677	4.2527	4.1436	4.0400
84	4.8408	4.6892	4.5473	4.4027	4.2870	4.1705	4.0558	3.9490	3.8476	3.7515
90	4.5180	4.3767	4.2443	4.1205	4.0016	3.8905	3.7743	3.6856	3.5913	3.5013
96	4.2355	4.1110	3.9793	3.8005	3.7515	3.6540	3.5385	3.4555	3.3666	3.2825
102	3.9864	3.8617	3.7450	3.6357	3.5310	3.4325	3.3998	3.2520	3.1686	3.0895
108	3.7650	3.6465	3.5370	3.4337	3.3345	3.2420	3.1453	3.0712	2.9926	2.9179
114	3.5668	3.4552	3.3507	3.2531	3.1992	3.0712	2.9814	2.9097	2.8352	2.7642
120	3.3885	3.2825	3.1833	3.0903	3.0012	2.9177	2.8308	2.7642	2.6935	2.6260
126	3.2272	3.1262	3.0315	2.9433	2.8582	2.7787	2.7038	2.6326	2.5600	2.5010
132	3.0805	2.9820	2.8938	2.8095	2.7286	2.6527	2.5808	2.5130	2.4485	2.3875
138	2.9465	2.8430	2.7679	2.6873	2.5994	2.5272	2.4686	2.4030	2.3421	2.2744
144	2.8237	2.7355	2.6526	2.5755	2.5010	2.4315	2.3589	2.3035	2.2442	2.1883
150	2.7108	2.6260	2.5465	2.4726	2.4093	2.3342	2.2712	2.2113	2.1546	2.1008
160	2.5415	2.4665	2.3873	2.3178	2.2508	2.2165	2.1293	2.0731	2.0202	1.9695
180	2.2090	2.1884	2.1221	2.0603	2.0008	1.9452	1.8871	1.8428	1.7956	1.7506
200	2.0331	1.9945	1.9099	1.8543	1.8007	1.7507	1.7034	1.6585	1.6160	1.5756
240	1.6942	1.6412	1.5917	1.5202	1.5006	1.4588	1.4154	1.3821	1.3467	1.3130

Horse-Power for Different Motive Forces *F* and Velocities *V*.

Motive Force.	Velocity *V* in Feet per Second.									
	10	20	30	40	50	60	70	80	90	100
F lbs.	*IP*	*IP*	*IP*	*IP*	*IP*	*IP*	*IP*	*IP*	*IP*	*IP*
10	.18182	.36364	.54545	.72727	.90909	1.0909	1.2727	1.4545	1.6364	1.8182
20	.36364	.72727	1.0909	1.4545	1.8182	2.1818	2.5454	2.9091	3.3636	3.6364
30	.54545	1.0909	1.6363	2.1818	2.7273	3.2727	3.8182	4.3636	4.9091	5.4545
40	.72727	1.4444	2.1818	2.9091	3.6363	4.3636	5.0909	5.8182	6.545	7.2727
50	.90909	1.8182	2.7273	3.6363	4.5454	5.4545	6.3636	7.2727	8.1818	9.0909
60	1.0909	2.1818	3.2727	4.3636	5.4545	6.5454	7.6363	8.7272	9.8182	10.909
70	1.2727	2.5454	3.8182	5.0909	6.3636	7.6363	8.9091	10.181	11.454	12.727
80	1.4545	2.9091	4.3636	5.8182	7.2727	8.7273	10.182	11.636	13.091	14.545
90	1.6364	3.2727	4.9091	6.5454	8.1818	9.8182	11.454	13.091	14.727	16.364
100	1.8182	3.6364	5.4545	7.2727	9.0909	10.909	12.727	14.545	16.364	18.182
110	2.0000	4.0000	6.0000	8.0000	10.000	12.000	14.000	16.000	18.000	20.000
120	2.1818	4.3636	6.5454	8.7273	10.909	13.091	15.273	17.454	19.636	21.818
130	2.3636	4.7273	7.0909	9.4545	11.818	14.182	16.545	18.909	21.273	23.636
140	2.5454	5.0909	7.6363	10.182	12.727	15.273	17.818	20.363	22.909	25.454
150	2.7273	5.4444	8.1818	10.909	13.636	16.363	19.091	21.818	24.545	27.273
160	2.9091	5.8182	8.7273	11.636	14.545	17.454	20.363	23.272	26.182	29.091
170	3.0909	6.1818	9.2727	12.363	15.454	18.545	21.636	24.727	27.818	30.909
180	3.3636	6.5454	9.8182	13.091	16.363	19.636	22.909	26.182	29.454	33.636
190	3.5454	6.9091	10.364	13.818	17.273	20.727	24.182	27.636	31.091	35.454
200	3.6364	7.2727	10.909	14.545	18.182	21.818	25.454	29.091	33.636	36.364
210	3.8182	7.6363	11.454	15.273	19.091	22.909	26.727	30.545	35.363	38.182
220	4.0000	8.0000	12.000	16.000	20.000	24.000	28.000	32.000	36.000	40.000
230	4.1818	8.3636	12.545	16.727	20.909	25.091	29.273	33.454	37.636	41.818
240	4.3634	8.7272	13.091	17.454	21.818	26.182	30.545	34.909	39.273	43.634
250	4.5454	9.0909	13.636	18.182	22.727	27.273	31.818	36.363	40.909	45.454
260	4.7273	9.4444	14.181	18.909	23.636	28.363	33.091	37.818	42.545	47.273
270	4.9091	9.8182	14.727	19.636	24.545	29.454	34.363	39.273	44.182	49.091
280	5.0909	10.182	15.273	20.363	25.454	30.545	35.636	40.727	45.818	50.909
290	5.2727	10.545	15.818	21.091	26.363	31.636	36.909	42.182	47.454	52.727
300	5.4545	10.909	16.363	21.818	27.273	32.727	38.182	43.636	49.091	54.545
310	5.6364	11.273	16.909	22.545	28.182	33.818	39.454	45.091	50.727	56.364
320	5.8182	11.636	17.454	23.273	29.091	34.909	40.727	46.545	52.363	58.182
330	6.0000	12.000	18.000	24.000	30.000	36.000	42.000	48.000	54.000	60.000
340	6.1818	12.363	18.545	24.727	30.909	37.091	43.273	49.454	55.636	61.818
350	6.3636	12.727	19.091	25.454	31.818	38.182	44.545	50.909	57.273	63.636
360	6.5454	13.091	19.636	26.182	32.727	39.273	45.818	52.363	58.909	65.454
370	6.7273	13.444	20.182	26.901	33.636	40.363	47.091	53.818	60.545	67.273
380	6.9091	13.818	20.727	27.636	34.545	41.454	48.363	55.273	62.182	69.091
390	7.0909	14.182	21.273	28.363	35.454	42.545	49.636	56.727	63.818	70.909
400	7.2727	14.545	21.818	29.091	36.363	43.636	50.909	58.182	65.454	72.727
410	7.4545	14.909	22.363	29.818	37.273	44.727	52.182	59.636	67.091	74.545
420	7.6364	15.273	22.909	30.545	38.182	45.818	53.454	61.091	68.727	76.364
430	7.8182	15.636	23.454	31.273	39.091	46.909	54.727	62.545	70.363	78.182
440	8.0000	16.000	24.000	32.000	40.000	48.000	56.000	64.000	72.000	80.000
450	8.1818	16.363	24.545	32.727	40.909	49.091	57.272	65.454	73.636	81.818
460	8.3636	16.727	25.091	33.454	41.818	50.182	58.545	66.909	75.273	83.636
470	8.5454	17.091	25.636	34.182	42.727	51.273	59.818	68.363	77.909	85.454
480	8.7273	17.444	26.182	34.909	43.636	52.363	61.091	69.818	79.545	87.273
490	8.9091	17.818	26.727	35.636	44.545	53.454	62.363	71.273	81.182	89.091
500	9.0909	18.182	27.273	36.363	45.454	54.545	63.636	72.727	82.818	90.909

Diameters in Inches of Wrought-Iron Shafts.

Horse power.	Number of revolutions per minute of wrought-iron shafts. 100	125	150	175	200	250	300	350	400	500	600	800	1000
IP.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
1	1.08	1.00	0.94	0.90	0.85	0.80	0.75	0.71	0.68	0.63	0.59	0.54	0.5
2	1.35	1.26	1.19	1.13	1.08	1.00	0.94	0.90	0.86	0.80	0.75	0.68	0.63
3	1.55	1.44	1.36	1.29	1.24	1.15	1.08	1.03	0.98	0.91	0.86	0.78	0.72
4	1.71	1.59	1.50	1.42	1.36	1.26	1.19	1.13	1.08	1.00	0.94	0.86	0.80
5	1.84	1.71	1.61	1.53	1.46	1.36	1.28	1.22	1.16	1.08	1.02	0.92	0.86
6	1.96	1.82	1.71	1.62	1.56	1.44	1.36	1.29	1.24	1.15	1.08	0.98	0.91
7	2.06	1.92	1.80	1.71	1.64	1.52	1.43	1.36	1.30	1.21	1.14	1.03	0.96
8	2.15	2.00	1.88	1.79	1.71	1.59	1.49	1.42	1.36	1.26	1.19	1.08	1.00
9	2.24	2.08	1.96	1.86	1.78	1.65	1.55	1.48	1.41	1.31	1.24	1.12	1.04
10	2.22	2.16	2.03	1.93	1.85	1.71	1.61	1.53	1.47	1.36	1.28	1.16	1.08
12	2,47	2.29	2.15	2.05	1.96	1.82	1.71	1.63	1.56	1.44	1.36	1.24	1.15
15	2.66	2.47	2.32	2.20	2.11	1.96	1.85	1.75	1.67	1.55	1.46	1.33	1.24
20	2.92	2.71	2.56	2.43	2.33	2.15	2.03	1.93	1.84	1.71	1.61	1.46	1.36
25	3.15	2.92	2.75	2.61	2.5	2.33	2.19	2.06	1.98	1.84	1.74	1.57	1.46
30	3.34	3.11	2.92	2.78	2.66	2.47	2.33	2.21	2.11	1.96	1.84	1.68	1.55
35	3.52	3,27	3.08	2.93	2.80	2.60	2.44	2.33	2.12	2.06	1.94	1.76	1.64
40	3.68	3.42	3.22	3.06	2.92	2.71	2.56	2.43	2.33	2.15	2.03	1.85	1.71
45	3.83	3.56	3.34	3.18	3.04	2.82	2.66	2.53	2.42	2.24	2.11	1.92	1.78
50	3.97	3.69	3.47	3.29	3.15	2.92	2.75	2.62	2.50	2.33	2.19	1.99	1.84
60	4.21	3.91	3.68	3.50	3.35	3.11	2.93	2.78	2.66	2.47	2.33	2.11	1.96
70	4.44	4.12	3.88	3.69	3.53	3.27	3.06	2.93	2.80	2.60	2.44	2.22	2.06
80	4.64	4.31	4.05	3.86	3.63	3.42	3.22	3.06	2.93	2.71	2.55	2.33	2.15
90	4.82	4.48	4.22	4.01	3.83	3.56	3.35	3.18	3.04	2.82	2.61	2.42	2.24
100	5.00	4.64	4.37	4.15	3.97	3.68	3.47	3.29	3.15	2.92	2.75	2.50	2.31
120	5.31	4.94	4.64	4.41	4.22	3.92	3.68	3.50	3.35	3.11	2.92	2.66	2.46
140	5.59	5.20	4.89	4.64	4.44	4.12	3.88	3.68	3.52	3.27	3.08	2.80	2.59
160	5.85	5.43	5.10	4.85	4.64	4.31	4.05	3.85	3.68	3.42	3.22	2.92	2.71
180	6.08	5.65	5.31	5.05	4.83	4.48	4.22	4.01	3.83	3.56	3.35	3.04	2.82
200	6.30	5.85	5.50	5.23	5.00	4.64	4.37	4.15	3.97	3.68	3.47	3.15	2.92
250	6.78	6.30	5.93	5.51	5.39	5.00	4.70	4.47	4.27	3.97	3.73	3.39	3.15
300	7.21	6.69	6.30	6.00	5.73	5.31	5.00	4.75	4.54	4.11	3.97	3.61	3.35
350	7.59	7.05	6.63	6.30	6.03	5.59	5.41	5.00	4.78	4.44	4.28	3.80	3.52
400	7.94	7.37	6.93	6.59	6.30	5.85	5.50	5.23	5.00	4.64	4.37	3.92	3.68
450	8.26	7.66	7.21	6.85	6.55	6.08	5.72	5.44	5.20	4.83	4.54	4.13	3.88
500	8.55	7.94	7.46	7.10	6.79	6.30	5.93	5.63	5.39	5.00	4.71	4.27	3.97
550	8.82	8.19	7.71	7.32	7.01	6.50	6.12	5.81	5.56	5.16	4.86	4.41	4.10
600	9.08	8.43	7.92	7.54	7.21	6.69	6.30	6.00	5.72	5.32	5.00	4.54	4.22
700	9.56	8.88	8.35	7.94	7.59	7.05	6.63	6.30	6.03	5.59	5.26	4.78	4.44
800	10.0	9.28	8.74	8.30	7.94	7.37	6.93	6.59	6.30	5.85	5.50	5.00	4.64
1000	10.8	10.0	9.41	8.94	8.55	7.94	7.47	7.09	6.79	6.30	5.93	5.39	5.00
1200	11.5	10.6	10.0	9.50	9.09	8.43	7.94	7.54	7.21	6.69	6.30	5.72	5.31
1500	12.3	11.5	10.7	10.3	9.79	9.09	8.55	8.12	7.77	7.21	6.79	6.17	5.72
2000	13.5	12.6	11.8	11.2	10.8	10.0	9.41	8.94	8.55	7.94	7.47	6.79	6.30
2500	14.6	13.5	12.8	12.2	11.6	10.8	10.2	9.63	9.21	8.55	8.05	7.31	6.73
3000	15.5	14.4	13.5	12.9	12.3	11.4	10.8	10.2	9.79	9.09	8.55	7.77	7.41
3500	16.4	15.2	14.3	13.5	13.0	12.1	11.3	10.8	10.3	9.56	9.00	8.18	7.59
4000	17.1	15.9	15.0	14.2	13.6	12.6	11.8	11.3	10.8	10.0	9.41	8.55	7.94
4500	17.8	16.5	15.5	14.7	14.1	13.1	12.3	11.7	11.2	10.4	9.79	8.89	8.25
5000	18.4	17.1	16.1	15.3	14.6	13.5	12.8	12.1	11.6	11.8	10.1	9.21	8.55

PILE DRIVING.

Notation.

$M =$ weight of the ram in pounds.
$S =$ fall of the ram in feet.
$V =$ velocity of pile and ram together.
$s =$ space in inches which the pile sinks by the blow.
$r =$ resistance of the ground in pounds to entrance of the pile.
$a =$ section area in sq. in. of the pile, sharpened to a point **not more** than 45°.
$k =$ coefficient for the hardness of the ground.
$h =$ depth to which the pile is driven.
$W =$ weight in pounds which a driven pile can bear with **safety after** the last blow when the pile sunk s inches.
$V =$ velocity in feet per second by which the ram strikes the pile.
Ram and pilehead considered non-elastic and perfectly hard.

$$V = 8\sqrt{S} \quad \ldots \quad 1.$$

$$s = \frac{12\,M\,S}{r}. \quad \ldots \quad 2.$$

$$W = \frac{2\,M\,S}{s}. \quad \ldots \quad 3.$$

$$v = \frac{8\,M\sqrt{S}}{M+m}. \quad \ldots \quad 4.$$

$$r = \frac{12\,M\,S}{s}. \quad \ldots \quad 5.$$

$$r = a\,k\sqrt{h}. \quad \ldots \quad 6.$$

Example 1. A wooden pile 18 feet long by 12 inches square, driven $h=12$ feet into common natural ground imbedded with tenacious clay for which may be assumed the coefficient $k=50$. Required how much the pile will set $s=?$ into the ground at a blow with a ram of $M=3500$ lbs. falling $S=42$ inches.

The weight of the wooden pile will be about $m=18\times40=720$ lbs.
Area of the pile $a=144$ square inches.

Resistance $r=144\times50\sqrt{12}=23840$ lbs.

The resistance sought from this formula 6, cannot be depended upon for calculating the weight the pile can bear with safety.

$$\text{The set } s = \frac{12\times3\tfrac{1}{2}\times3500}{23840} = 6''.1.$$

Suppose the set to be $s=6$ inches at the last blow, required what weight the pile can bear with safety?

$$W = \frac{2\times3\tfrac{1}{2}\times3500}{6} = 4083 \text{ lbs.}$$

This can be depended on with safety, if calculated from the actual set of the pile at the last blow.

For ordinary pile driving a heavy ram and short fall is the most effective, but in some cases when the ground itself is elastic, or when driving piles in pure sand it is found more advantageous to use a high fall of the ram.

Approximate Coefficients.

	k
In coral formations,	120
In hard clay with gravel,	100
In hard pure clay,	70
In common clay or sand,	50
In soft clay or loose sand,	40
In very loose materials,	30

ROPES FOR TRANSMITTING POWER.

THE following tables of properties of ropes for transmitting power are deduced from experiments made by the author expressly for this Pocket-book. John A. Roebling's Sons & Co., of Trenton, N. J., kindly furnished the wire ropes experimented upon, and Edwin H. Fitler & Co., of Philadelphia, furnished a variety of hemp, manilla, and cotton ropes.

The heavy iron and steel cables were kindly furnished by the Philadelphia and Reading Railroad Company.

The subject of transmission of power with ropes is an important one, but too extensive to be fully treated in this Pocket-book. The most important points are given in the accompanying tables.

Column D contains the minimum diameter of pulley, wheel, or drum that should be used for the maximum diameter of rope in the next column, d. The pulley may be made larger for the same size or smaller rope, and the larger the better for the rope. The heading of each column explains the contents.

The wear of a rope is proportionate to its stiffness—that is, as the cube d^3 of its diameter, and inversely as the square D^2 of the diameter of the pulley.

$$\text{Wear of rope} = \frac{d^3}{D^2},$$

d = diameter in inches of the rope, and D = diameter in feet of the pulley.

Column Φ contains the stiffness of the rope in pounds for both winding and unwinding on each pulley; when the rope runs over a number of pulleys, as when power is transmitted for long distances, the stiffness must be added for each pulley.

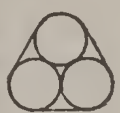
The dynamics of transmission of power by belting, ropes, or chains is calculated by the formulas for circular motion.

Circumference of Ropes.

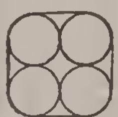
The circumference of a rope, as practically measured by a tape-line, is not 3.14 times the diameter of the rope, but is considerably less, depending upon the number of strands in the rope.

d = diameter and c = circumference of the rope.

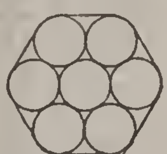
Two strands. $\pi = 2.57$. $c = 2.57\,d$. $d = 0.389\,c$.



Three strands. $\pi = 2.86$. $c = 2.86\,d$. $d = 0.35\,c$.



Four strands. $\pi = 2.96$. $c = 2.96\,d$. $d = 0.338\,c$.



Seven strands. $\pi = 3$. $c = 3\,d$. $d = \frac{1}{3}\,c$.

The diameter of the rope is that of the circle tangenting the strands, whilst the circumference is the sum of the lines drawn between the strands.

Hemp Ropes, White. Three Strands.

Diam. Pulley. Feet.	Size of Rope. Diam. Inches.	Circum. Inches.	Strength. Break. Pounds.	Safety. Pounds.	Weight per Ft. Pounds.	Length per Lb. Feet.	Stiffness. Wind-ing.	Wind and Unwind.
D	*d*	*c*	*S*	*T*	*w*	*l*	Φ	Φ
21.	6 in.	17.1	324000	81000	9.4	.1064	7.6	12.7
19.	5½	15.7	272000	68000	7.9	.1266	6.6	11.
16.5	5 in.	14.25	225000	56250	6.52	.1533	6.	10.
14.	4½	12.1	182000	45500	5.28	.1894	5.4	9.
12.	4 in.	11.4	144000	36000	4.18	.2392	4.6	7.7
11.	3¾	10.7	126000	31500	3.67	.2725	4.2	7.
10.	3½	10.	110000	27500	3.2	.3125	3.9	6.5
9.	3¼	9.27	95000	23750	2.76	.3613	3.6	6.
8.	3 in.	8.57	81000	20250	2.35	.4255	3.36	5.6
7.	2¾	7.85	68000	17000	1.97	.5076	3.05	5.1
6.	2½	7.14	56200	14050	1.63	.6135	2.82	4.7
5.25	2¼	6.43	45500	11375	1.32	.7575	2.4	4.
4.25	2 in.	5.70	36000	9000	1.04	.9615	2.25	3.75
3.4	1¾	5.00	27500	6875	0.80	1.25	2.1	3.5
2.75	1½	4.28	20200	5050	0.588	1.700	1.74	2.9
2.1	1¼	3.97	14000	3500	0.407	2.457	1.44	2.4
1.5	1 in.	2.86	9000	2250	0.261	3.831	1.16	1.93
1.22	⅞	2.5	6900	1725	0.200	5.000	1.02	1.7
0.97	¾	2.14	5050	1262	0.147	6.803	0.87	1.46
0.74	⅝	1.78	3500	875	0.102	9.803	0.72	1.21
0.53	½	1.43	2240	560	0.065	15.38	0.58	0.97
0.34	⅜	1.07	1260	315	0.036	27.77	0.45	0.75
0.18	¼	0.71	560	140	0.016	62.5	0.31	0.52

Manilla Ropes. Three Strands.

Diam. Pulley. Feet.	Size of Rope. Diam. Inches.	Circum. Inches.	Strength. Break. Pounds.	Safety. Pounds.	Weight per Ft. Pounds.	Length per Lb. Feet.	Stiffness. Wind-ing.	Wind and Unwind.
D	*d*	*c*	*S*	*T*	*w*	*l*	Φ	Φ
26.4	6 in.	17.1	216000	54000	8.64	.1157	5.37	8.87
23.2	5½	15.7	181500	45375	7.26	.1377	5.00	8.26
20.	5 in.	14.25	150000	37500	6.00	.1666	4.5	7.45
17.2	4½	12.1	121000	30250	4.86	.2057	4.00	6.62
14.4	4 in.	11.4	96000	24000	3.84	.2604	3.57	5.9
13.	3¾	10.7	84400	21100	3.38	.2958	3.37	5.56
11.8	3½	10.	73600	18400	2.94	.3401	3.10	5.15
10.5	3¼	9.27	63500	15875	2.53	.3952	2.93	4.85
9.35	3 in.	8.57	54000	13500	2.16	.4629	2.68	4.43
8.2	2¾	7.85	45400	11350	1.81	.5524	2.45	4.06
7.1	2½	7.14	37500	9375	1.5	.6666	2.24	3.70
6.	2¼	6.43	30400	7600	1.21	.8264	2.06	3.4
5.	2 in.	5.70	24000	6000	0.96	1.041	1.85	3.07
4.	1¾	5.00	18400	4600	0.725	1.379	1.700	2.8
3.3	1½	4.28	13500	3350	0.54	1.852	1.40	2.32
2.5	1¼	3.57	9380	2345	0.375	2.666	1.11	1.84
1.8	1 in.	2.86	6000	1500	0.24	4.166	0.8870	1.47
1.46	⅞	2.5	4600	1150	0.184	5.435	0.894	1.31
1.17	¾	2.14	3380	845	0.135	7.407	0.6666	1.10
0.89	⅝	1.78	2350	587	0.093	10.75	0.558	0.92
0.63	½	1.43	1500	375	0.060	16.66	0.454	0.75
0.41	⅜	1.07	845	211	0.033	30.30	0.36	0.56
0.22	¼	0.71	375	93	0.015	66.66	0.23	0.38

Tarred Hemp Ropes. Four Strands.

Diam. Pulley. Feet.	Size of Rope. Diam. Inches.	Circum. Inches.	Strength. Break. Pounds.	Safety. Pounds.	Weight per Ft. Pounds.	Length per Lb. Feet.	Stiffness. Wind-ing.	Wind and Unwind.
D	d	c	S	T	w	l	ϕ	ϕ
36.	6 in.	18 in.	230000	57500	15.1	.0662	13.3	18.9
32.	5½	16½	194000	48500	12.7	.0784	12.	17.
28.	5 in.	15 in.	160000	40000	10.5	.0952	10.6	15.1
24.	4½	13½	130000	32500	8.52	.1174	9.5	13.5
20.	4 in.	12 in.	102500	25625	6.72	.1488	8.5	12.1
18.	3¾	11¼	90000	22500	5.92	.1689	8.1	11.5
16.	3½	10½	78500	19625	5.16	.1938	7.8	11.1
14.6	3¼	9¾	67700	16925	4.44	.2252	7.	10.
12.9	3 in.	9 in.	57700	14425	3.78	.2645	6.46	9.2
11.4	2¾	8¼	48400	12100	3.18	.3144	5.83	8.3
9.9	2½	7½	40000	10000	2.63	.3802	5.27	7.5
8.4	2¼	6¾	32400	8100	2.13	.4695	4.83	6.87
7.	2 in.	6 in.	25600	6400	1.68	.5952	4.34	6.18
5.8	1¾	5¼	19600	4900	1.29	.7752	3.70	5.26
4.6	1½	4½	14400	3600	0.945	1.058	3.18	4.53
3.5	1¼	3¾	10000	2500	0.656	1.524	2.64	3.76
2.5	1 in.	3 in.	6400	1600	0.420	2.381	2.13	3.03
2.	⅞	2⅝	4900	1225	0.322	3.105	1.95	2.78
1.6	¾	2¼	3600	900	0.236	4.237	1.64	2.34
1.2	⅝	1⅞	2500	625	0.164	6.097	1.40	2.
0.9	½	1½	1600	400	0.105	9.523	1.02	1.46
0.58	⅜	1⅛	900	225	0.059	16.95	0.77	1.10
0.31	¼	¾	400	100	0.026	38.46	0.53	0.76

Cotton Ropes. Three Strands of Fine Yarns.

Diam. Pulley. Feet.	Size of Rope. Diam. Inches.	Circum. Inches.	Strength. Break. Pounds.	Safety. Pounds.	Weight per Ft. Pounds.	Length per Lb. Feet.	Stiffness. Wind-ing.	Wind and Unwind.
D	d	c	S	T	w	l	ϕ	ϕ
14.7	6 in.	18 in.	18000	4500	7.2	0.1389	4.	6.
12.9	5½	16½	15125	3781	6.05	0.1653	3.68	5.5
11.2	5 in.	15 in.	12500	3125	5.00	0.2000	3.3	5.
9.5	4½	13½	10125	2531	4.05	0.2469	3.	4.5
8.0	4 in.	12 in.	8000	2000	3.20	0.3125	2.66	4.
7.2	3¾	11¼	7030	1782	2.81	0.3559	2.55	3.83
6.5	3½	10½	6125	1531	2.45	0.4082	2.37	3.56
5.8	3¼	9¾	5281	1320	2.11	0.4739	2.22	3.33
5.2	3 in.	9 in.	4500	1125	1.80	0.5555	2.	3.
4.5	2¾	8¼	3781	945	1.52	0.6579	1.89	2.84
4.	2½	7½	3125	781	1.25	0.8000	1.63	2.45
3.4	2¼	6¾	2531	633	1.01	0.9901	1.48	2.23
2.8	2 in.	6 in.	2000	500	0.80	1.250	1.36	2.05
2.3	1¾	5¼	1531	383	0.61	1.639	1.18	1.78
1.8	1½	4½	1125	281	0.45	2.222	1.04	1.58
1.4	1¼	3¾	781	195	0.31	3.226	0.83	1.25
1 ft.	1 in.	3 in.	500	125	0.20	5.000	0.66	1.
0.82	⅞	2⅝	383	96	0.15	6.666	0.59	0.89
0.65	¾	2¼	281	70	0.11	9.009	0.5	0.75
0.5	⅝	1⅞	195	49	0.078	12.82	0.4	0.61
0.35	½	1½	125	31	0.05	20.00	0.34	0.51
0.23	⅜	1⅛	70	17	0.028	35.71	0.25	0.37
0.125	¼	¾	31	8	0.012	83.33	0.16	0.25

Iron Ropes. 19 × 7 = 133 Wires and Wire Centre.

Diam. Pulley. Feet.	Size of Rope.		Strength.		Weight per Ft. Pounds.	Length per Lb. Feet.	Stiffness.	
	Diam. Inches.	Circum. Inches.	Break. Pounds.	Safety. Pounds.			Winding.	Wind and Unwind.
D	d	c	S	T	w	l	Φ	Φ
41.6	3 in.	9	300000	75000	16.83	.0594	5.4	7.4
36.5	2$\frac{3}{4}$	8$\frac{1}{4}$	252500	63125	12.45	.0803	4.95	6.8
31.5	2$\frac{1}{2}$	7$\frac{1}{2}$	209000	51250	10.3	.0971	4.54	6.24
27.	2$\frac{1}{4}$	6$\frac{3}{4}$	169000	42250	8.34	.1199	4.06	5.58
22.6	2 in.	6	133000	33250	6.62	.1510	3.60	4.97
20.5	1$\frac{7}{8}$	5$\frac{5}{8}$	117500	29375	5.78	.1730	3.37	4.64
18.5	1$\frac{3}{4}$	5$\frac{1}{4}$	102000	25500	5.04	.1984	3.15	4.34
16.5	1$\frac{5}{8}$	4$\frac{7}{8}$	88400	44100	4.35	.2299	2.95	4.05
14.75	1$\frac{1}{2}$	4$\frac{1}{2}$	75200	18800	3.70	.2703	2.65	3.64
13.	1$\frac{3}{8}$	4$\frac{1}{8}$	63200	15800	3.12	.3205	2.42	3.33
11.2	1$\frac{1}{4}$	3$\frac{3}{4}$	52200	18050	2.57	.3891	2.22	3.06
9.55	1$\frac{1}{8}$	3$\frac{3}{8}$	42300	10575	2.08	.4807	1.91	2.77
8.	1 in.	3	33300	8325	1.65	.6061	1.80	2.47
6.56	$\frac{7}{8}$	2$\frac{5}{8}$	25600	6400	1.26	.7936	1.56	2.15
5.2	$\frac{3}{4}$	2$\frac{1}{4}$	18800	4700	0.927	1.078	1.34	1.85
3.96	$\frac{5}{8}$	1$\frac{7}{8}$	13000	3250	0.644	1.553	1.11	1.54
2.83	$\frac{1}{2}$	1$\frac{1}{2}$	8360	2090	0.412	2.427	0.90	1.23
2.31	$\frac{7}{16}$	1$\frac{5}{16}$	6400	1600	0.315	3.174	0.79	1.09
1.83	$\frac{3}{8}$	1$\frac{1}{8}$	4710	1177	0.231	4.329	0.68	0.93
1.4	$\frac{5}{16}$	$\frac{15}{16}$	3270	812	0.160	6.250	0.55	0.76
1.	$\frac{1}{4}$	$\frac{3}{4}$	2090	522	0.102	9.804	0.44	0.61
0.65	$\frac{3}{16}$	$\frac{9}{16}$	1180	295	0.057	17.54	0.33	0.46
0.35	$\frac{1}{8}$	$\frac{3}{8}$	522	130	0.025	40.00	0.23	0.32

Cast-Steel Ropes. 19 × 7 = 133 Wires and Wire Centre.

Diam. Pulley. Feet.	Size of Rope.		Strength.		Weight per Ft. Pounds.	Length per Lb. Feet.	Stiffness.	
	Diam. Inches.	Circum. Inches.	Break. Pounds.	Safety. Pounds.			Winding.	Wind and Unwind.
D	d	c	S	T	w	l	Φ	Φ
50.	3 in.	9	486000	121500	16.83	.0594	9.37	11.7
45.	2$\frac{3}{4}$	8$\frac{1}{4}$	416000	104000	12.45	.0803	8.19	10.2
38.7	2$\frac{1}{2}$	7$\frac{1}{2}$	341000	86000	10.3	.0971	7.59	9.45
33.	2$\frac{1}{4}$	6$\frac{3}{4}$	279000	69750	8.34	.1199	6.84	8.52
27.7	2 in.	6	220000	55000	6.62	.1510	6.36	7.92
25.	1$\frac{7}{8}$	5$\frac{5}{8}$	193500	48375	5.78	.1730	5.73	7.15
22.7	1$\frac{3}{4}$	5$\frac{1}{4}$	168500	42125	5.04	.1984	5.28	6.58
20.3	1$\frac{5}{8}$	4$\frac{7}{8}$	145500	36375	4.35	.2299	4.87	6.08
18.	1$\frac{1}{2}$	4$\frac{1}{2}$	123500	30875	3.70	.2703	4.52	5.63
15.8	1$\frac{3}{8}$	4$\frac{1}{8}$	104000	26000	3.12	.3205	4.12	5.13
13.7	1$\frac{1}{4}$	3$\frac{3}{4}$	86000	21500	2.57	.3891	3.77	4.70
11.7	1$\frac{1}{8}$	3$\frac{3}{8}$	69600	17400	2.08	.4807	3.39	4.23
9.8	1 in.	3	55000	13750	1.65	.6061	3.00	3.75
8.	$\frac{7}{8}$	2$\frac{5}{8}$	42200	10550	1.26	.7936	2.65	3.30
6.4	$\frac{3}{4}$	2$\frac{1}{4}$	31000	7750	0.927	1.078	2.24	2.79
4.87	$\frac{5}{8}$	1$\frac{7}{8}$	21500	5375	0.644	1.553	1.87	2.33
3.46	$\frac{1}{2}$	1$\frac{1}{2}$	13750	3687	0.412	2.427	1.51	1.89
2.84	$\frac{7}{16}$	1$\frac{5}{16}$	10500	2625	0.315	3.174	1.32	1.65
2.25	$\frac{3}{8}$	1$\frac{1}{8}$	7740	1935	0.231	4.329	1.13	1.41
1.71	$\frac{5}{16}$	$\frac{15}{16}$	5380	1345	0.160	6.250	0.94	1.18
1.22	$\frac{1}{4}$	$\frac{3}{4}$	3440	860	0.102	9.804	0.75	0.94
0.8	$\frac{3}{16}$	$\frac{9}{16}$	1935	484	0.057	17.54	0.56	0.7
0.433	$\frac{1}{8}$	$\frac{3}{8}$	860	215	0.025	40.00	0.38	0.48

Iron Ropes. 19 × 6 = 114 Wires and Hemp Centre.

Diam. Pulley. Feet.	Size of Rope.		Strength.		Weight per Ft. Pounds.	Length per Lb. Feet.	Stiffness.	
	Diam. Inches.	Circum. Inches.	Break. Pounds.	Safety. Pounds.			Winding.	Wind and Unwind.
D	d	c	S	T	w	l	Φ	Φ
31.	3 in.	9	287500	71875	13.5	.0741	7.2	10.1
27.	$2\frac{3}{4}$	$8\frac{1}{4}$	241500	60375	11.3	.0885	6.77	9.46
23.7	$2\frac{1}{2}$	$7\frac{1}{2}$	200000	50000	9.36	.1068	6.09	8.51
20.	$2\frac{1}{4}$	$6\frac{3}{4}$	161500	40750	7.60	.1316	5.51	7.71
17.	2 in.	6	128000	32000	6.02	.1661	4.75	6.65
15.4	$1\frac{7}{8}$	$5\frac{5}{8}$	112000	28000	5.27	.1807	4.47	6.25
14.	$1\frac{3}{4}$	$5\frac{1}{4}$	98000	24500	4.58	.2183	4.11	5.75
12.4	$1\frac{5}{8}$	$4\frac{7}{8}$	84400	21100	3.96	.2525	3.90	5.45
11.	$1\frac{1}{2}$	$4\frac{1}{2}$	72000	18000	3.37	.2967	3.66	5.12
9.7	$1\frac{3}{8}$	$4\frac{1}{8}$	60400	15100	2.83	.3533	3.25	4.55
7.8	$1\frac{1}{4}$	$3\frac{3}{4}$	50000	12500	2.34	.4273	2.96	4.13
7.1	$1\frac{1}{8}$	$3\frac{3}{8}$	40400	10100	1.89	.5291	2.73	3.82
6.	1 in.	3	32000	8000	1.50	.6666	2.39	3.34
5.	$\frac{7}{8}$	$2\frac{5}{8}$	24250	6062	1.14	.8772	2.00	2.81
4.	$\frac{3}{4}$	$2\frac{1}{4}$	18000	4500	0 844	1.184	1.70	2.38
3.	$\frac{5}{8}$	$1\frac{7}{8}$	12500	3125	0.586	1.706	1.45	2.03
2.1	$\frac{1}{2}$	$1\frac{1}{2}$	8000	2000	0.375	2.666	1.22	1.70
1.7	$\frac{7}{16}$	$1\frac{5}{16}$	6120	1530	0.287	3.484	1.09	1.52
1.3	$\frac{3}{8}$	$1\frac{1}{8}$	4500	1125	0.211	4.739	1.01	1.41
1.	$\frac{5}{16}$	$\frac{15}{16}$	3129	780	0.146	6.849	0.81	1.14
0.75	$\frac{1}{4}$	$\frac{3}{4}$	2000	500	0.093	10.75	0.59	0.83
0.5	$\frac{3}{16}$	$\frac{9}{16}$	1120	280	0.052	19.23	0.42	0.60
0.27	$\frac{1}{8}$	$\frac{3}{8}$	500	125	0.023	43.48	0.29	0.41

Cast-Steel Ropes. 19 × 6 = 114 Wires and Hemp Centre.

Diam. Pulley. Feet.	Size of Rope.		Strength.		Weight per Ft. Pounds.	Length per Lb. Feet.	Stiffness.	
	Diam. Inches.	Circum. Inches.	Break. Pounds.	Safety. Pounds.			Winding.	Wind and Unwind.
D	d	c	S	T	w	l	Φ	Φ
41.6	3 in.	9	432000	108000	13.5	.0741	6.	8.25
36.5	$2\frac{3}{4}$	$8\frac{1}{4}$	363000	90750	11.3	.0885	5.45	7.50
31.5	$2\frac{1}{2}$	$7\frac{1}{2}$	300000	75000	9.36	.1068	5.04	6.93
27.	$2\frac{1}{4}$	$6\frac{3}{4}$	243000	67500	7.60	.1316	4.51	6.20
22.6	2 in.	6	192000	48000	6.02	.1661	4.00	5.49
20.5	$1\frac{7}{8}$	$5\frac{5}{8}$	168500	42125	5.27	.1807	3.77	5.18
18.5	$1\frac{3}{4}$	$5\frac{1}{4}$	146500	36625	5.08	.2183	3.50	4.82
16 5	$1\frac{5}{8}$	$4\frac{7}{8}$	126500	31625	3.96	.2525	3.27	4.50
14.75	$1\frac{1}{2}$	$4\frac{1}{2}$	108000	27000	3.37	.2967	2.98	4.10
13.	$1\frac{3}{8}$	$4\frac{1}{8}$	90700	12675	2.83	.3533	2.70	3.72
11.2	$1\frac{1}{4}$	$3\frac{3}{4}$	75000	18750	2.34	.4273	2.50	3.43
9.55	$1\frac{1}{8}$	$3\frac{3}{8}$	60700	15175	1.89	.5291	2.24	3.08
8.	1 in.	3	48000	12000	1.50	.6666	2.00	2.76
6.56	$\frac{7}{8}$	$2\frac{5}{8}$	36800	9200	1.14	.8772	1.74	2.40
5.2	$\frac{3}{4}$	$2\frac{1}{4}$	27000	6750	0.844	1.184	1.50	2.06
3.96	$\frac{5}{8}$	$1\frac{7}{8}$	18750	4687	0.586	1.706	1.24	1.71
2.83	$\frac{1}{2}$	$1\frac{1}{2}$	12000	3000	0.375	2.666	1.00	1.38
2.31	$\frac{7}{16}$	$1\frac{5}{16}$	9200	2300	0.287	3.484	0.88	1.21
1.83	$\frac{3}{8}$	$1\frac{1}{8}$	6750	1687	0.211	4.739	0.75	1.04
1.4	$\frac{5}{16}$	$\frac{15}{16}$	4680	1170	0.146	6.849	0.61	0.85
1.	$\frac{1}{4}$	$\frac{3}{4}$	3000	750	0.093	10.75	0.49	0.68
0.65	$\frac{3}{16}$	$\frac{9}{16}$	1685	421	0.052	19.23	0.37	0.51
0.35	$\frac{1}{8}$	$\frac{3}{8}$	750	187	0.023	43.48	0.24	0.33

Iron Ropes. $7 \times 7 = 49$ Wires and Wire Centre.

Diam. Pulley. Feet.	Size of Rope. Diam. Inches.	Size of Rope. Circum. Inches.	Strength. Break. Pounds.	Strength. Safety. Pounds.	Weight per Ft. Pounds.	Length per Lb. Feet.	Stiffness. Winding.	Stiffness. Wind and Unwind.
D	*d*	*c*	*S*	*T*	*w*	*l*	Φ	Φ
62.5	3 in.	9	300000	75000	16.83	.0594	8.8	12.1
54.5	2¾	8.25	252500	63125	12.45	.0803	7.9	10.9
47.	2½	7.5	209000	51250	10.3	.0971	7.45	10.2
40.	2¼	6.75	169000	42250	8.34	.1199	6.8	9.30
34.	2 in.	6	133000	33250	6.62	.1510	5.86	8.04
30.	1⅞	5⅝	117500	29375	5.78	.1730	5.60	7.68
27.	1¾	5¼	102000	25500	5.04	.1984	5.30	7.26
25.	1⅝	4⅞	88400	44100	4.35	.2299	4.72	6.48
22.	1½	4½	75200	18800	3.70	.2703	4.41	6.05
19.	1⅜	4⅛	63200	15800	3.12	.3205	4.18	5.74
16.5	1¼	3¾	52200	18050	2.57	.3891	3.78	5.19
14.	1⅛	3⅜	42300	10575	2.08	.4807	3.45	4.74
12.	1 in.	3	33300	8325	1.65	.6061	2.93	4.03
10.	⅞	2⅝	25600	6400	1.26	.7936	2.48	3.40
8.	¾	2¼	18800	4700	0.927	1.078	2.05	2.82
6.	⅝	1⅞	13000	3250	0.644	1.553	1.79	2.46
4.25	½	1½	8360	2090	0.412	2.427	1.53	2.10
3.5	7/16	1 5/16	6400	1600	0.315	3.174	1.26	1.73
2.75	⅜	1⅛	4710	1177	0.231	4.329	1.10	1.52
2.1	5/16	15/16	3270	812	0.160	6.250	0.91	1.25
1.5	¼	¾	2090	522	0.102	9.804	0.73	1.00
1.	3/16	9/16	1180	295	0.057	17.54	0.52	0.72
0.5	⅛	⅜	522	130	0.025	40.00	0.4	0.56

Iron Ropes. $7 \times 6 = 42$ Wires and Hemp Centre.

Diam. Pulley. Feet.	Size of Rope. Diam. Inches.	Size of Rope. Circum. Inches.	Strength. Break. Pounds.	Strength. Safety. Pounds.	Weight per Ft. Pounds.	Length per Lb. Feet.	Stiffness. Winding.	Stiffness. Wind and Unwind.
D	*d*	*c*	*S*	*T*	*w*	*l*	Φ	Φ
52.	3 in.	9	287500	71875	13.5	.0741	5.00	7.02
45.	2¾	8¼	241500	60375	11.3	.0885	4.72	6.62
39.	2½	7½	200000	50000	9.36	.1068	4.26	6.00
34.	2¼	6¾	161500	40750	7.60	.1316	3.72	5.21
28.	2 in.	6	128000	32000	6.02	.1661	3.43	4.81
25.	1⅞	5⅝	112000	28000	5.27	.1807	3.17	4.44
23.	1¾	5¼	98000	24500	4.58	.2183	2.93	4.12
21.	1⅝	4⅞	84400	21100	3.96	.2525	2.64	3.70
18.	1½	4½	72000	18000	3.37	.2967	2.5	3.49
16.	1⅜	4⅛	60400	15100	2.83	.3533	2.33	3.27
14.	1¼	3¾	50000	12500	2.34	.4273	2.10	2.95
12.	1⅛	3⅜	40400	10100	1.89	.5291	1.85	2.60
10.	1 in.	3	32000	8000	1.50	.6666	1.60	2.34
8.2	⅞	2⅝	24250	6062	1.14	.8772	1.45	2.04
6.5	¾	2¼	18000	4500	0.844	1.184	1.25	1.75
5.	⅝	1⅞	12500	3125	0.586	1.706	1.02	1.43
3.5	½	1½	8000	2000	0.375	2.666	0.85	1.20
3.	7/16	1 5/16	6120	1530	0.287	3.484	0.67	0.95
2.3	⅜	1⅛	4500	1125	0.211	4.739	0.62	0.87
1.7	5/16	15/16	3120	780	0.146	6.849	0.55	0.77
1.25	¼	¾	2000	500	0.093	10.75	0.41	0.58
0.8	3/16	9/16	1120	280	0.052	19.23	0.32	0.45
0.4	⅛	⅜	500	125	0.023	43.48	0.25	0.35

Iron Rope. 7 × 6 × 6 = 252 Wires. Cotton Centre in each Rope Strand and Hemp in the Centre.

Diam. Pulley. Feet.	Size of Rope.		Strength.		Weight per Ft. Pounds.	Length per Lb. Feet.	Stiffness.	
	Diam. Inches.	Circum. Inches.	Break. Pounds.	Safety. Pounds.			Winding.	Wind and Unwind.
D	d	c	S	T	w	l	Φ	Φ
21.	3 in.	9	270000	67500	12.2	.0819	5.40	7.80
18.	$2\frac{3}{4}$	$8\frac{1}{4}$	226500	56625	10.2	.0980	4.88	7.04
16.	$2\frac{1}{2}$	$7\frac{1}{2}$	187000	46750	8.57	.1167	4.40	6.37
13.5	$2\frac{1}{4}$	$6\frac{3}{4}$	152000	38000	6.84	.1462	3.98	5.74
11.	2 in.	6	120000	30000	5.42	.1845	3.74	5.39
10.	$1\frac{7}{8}$	$5\frac{5}{8}$	105400	26350	4.76	.2101	3.50	5.03
9.	$1\frac{3}{4}$	$5\frac{1}{4}$	91700	22925	4.13	.2421	3.26	4.70
8.	$1\frac{5}{8}$	$4\frac{7}{8}$	79000	19750	3.56	.2809	3.08	4.44
7.	$1\frac{1}{2}$	$4\frac{1}{2}$	67500	16875	3.04	.3289	2.87	4.14
6.25	$1\frac{3}{8}$	$4\frac{1}{8}$	56600	14250	2.55	.3921	2.58	3.72
5.5	$1\frac{1}{4}$	$3\frac{3}{4}$	46800	11700	2.11	.4739	2.30	3.30
4.75	$1\frac{1}{8}$	$3\frac{3}{8}$	38000	9500	1.71	.5848	2.00	2.90
4.	1 in.	3	30000	7500	1.35	.7407	1.77	2.55
3.25	$\frac{7}{8}$	$2\frac{5}{8}$	23000	5750	1.03	.9709	1.57	2.27
2.6	$\frac{3}{4}$	$2\frac{1}{4}$	16850	4212	0.760	1.316	1.32	1.91
2.	$\frac{5}{8}$	$1\frac{7}{8}$	11700	2925	0.528	1.894	1.09	1.57
1.4	$\frac{1}{2}$	$1\frac{1}{2}$	7500	1875	0.338	2.958	0.90	1.30
1.15	$\frac{7}{16}$	$1\frac{5}{16}$	5740	1435	0.258	3.876	0.79	1.14
0.9	$\frac{3}{8}$	$1\frac{1}{8}$	4220	1055	0.190	5.263	0.69	1.00
0.7	$\frac{5}{16}$	$\frac{15}{16}$	2930	732	0.132	7.576	0.55	0.80
0.5	$\frac{1}{4}$	$\frac{3}{4}$	1870	467	0.084	11.90	0.44	0.64
0.3	$\frac{3}{16}$	$\frac{9}{16}$	1050	262	0.047	21.27	0.38	0.56
0.18	$\frac{1}{8}$	$\frac{3}{8}$	468	117	0.021	47.62	0.20	0.30

Copper Ropes. 7 × 6 = 42 Wires. Cotton Centre.

Diam. Pulley. Feet.	Size of Rope.		Strength.		Weight per Ft. Pounds.	Length per Lb. Feet.	Stiffness.	
	Diam. Inches.	Circum. Inches.	Break. Pounds.	Safety. Pounds.			Winding.	Wind and Unwind.
D	d	c	S	T	w	l	Φ	Φ
26.	3 in.	9	306000	76500	15.25	.0656	9.60	13.4
22.5	$2\frac{3}{4}$	$8\frac{1}{4}$	257000	64250	12.9	.0775	9.00	12.6
20.	$2\frac{1}{2}$	$7\frac{1}{2}$	212500	53125	10.6	.0943	8.50	11.9
17.	$2\frac{1}{4}$	$6\frac{3}{4}$	172000	43000	8.44	.1185	7.14	10.0
14.	2 in.	6	136000	34000	6.82	.1466	6.44	9.00
12.5	$1\frac{7}{8}$	$5\frac{5}{8}$	120000	30000	5.97	.1675	6.16	8.60
11.5	$1\frac{3}{4}$	$5\frac{1}{4}$	104000	26000	5.20	.1923	5.43	7.8
10.5	$1\frac{5}{8}$	$4\frac{7}{8}$	90000	22500	4.48	.2232	5.00	7.0
9.	$1\frac{1}{2}$	$4\frac{1}{2}$	76500	19125	3.82	.2618	4.70	6.1
8.	$1\frac{3}{8}$	$4\frac{1}{8}$	64200	16050	3.21	.3115	4.44	6.2
7.	$1\frac{1}{4}$	$3\frac{3}{4}$	53100	13275	2.88	.3472	3.96	5.54
6.	$1\frac{1}{8}$	$3\frac{3}{8}$	42800	10700	2.15	.4651	3.50	4.90
5.	1 in.	3	34000	8500	1.70	.5882	3.22	4.50
4.	$\frac{7}{8}$	$2\frac{5}{8}$	26000	6500	1.30	.7692	2.94	4.10
3.25	$\frac{3}{4}$	$2\frac{1}{4}$	19100	4775	0.956	1.046	2.44	3.42
2.5	$\frac{5}{8}$	$1\frac{7}{8}$	13250	3312	0.673	1.486	1.83	2.70
1.75	$\frac{1}{2}$	$1\frac{1}{2}$	8500	2125	0.424	2.358	1.65	2.30
1.5	$\frac{7}{16}$	$1\frac{5}{16}$	6510	1627	0.325	3.077	1.29	1.80
1.15	$\frac{3}{8}$	$1\frac{1}{8}$	4780	1195	0.239	4.184	1.20	1.67
0.85	$\frac{5}{16}$	$\frac{15}{16}$	3320	830	0.166	6.024	1.05	1.47
0.62	$\frac{1}{4}$	$\frac{3}{4}$	2120	530	0.106	9.433	0.81	1.13
0.4	$\frac{3}{16}$	$\frac{9}{16}$	1200	300	0.059	16.95	0.62	0.87
0.2	$\frac{1}{8}$	$\frac{3}{8}$	530	132	0.26	38.46	0.48	0.67

THE ART OF TYING KNOTS.

FOR illustrations of the explanations here given, see cut on succeeding page.

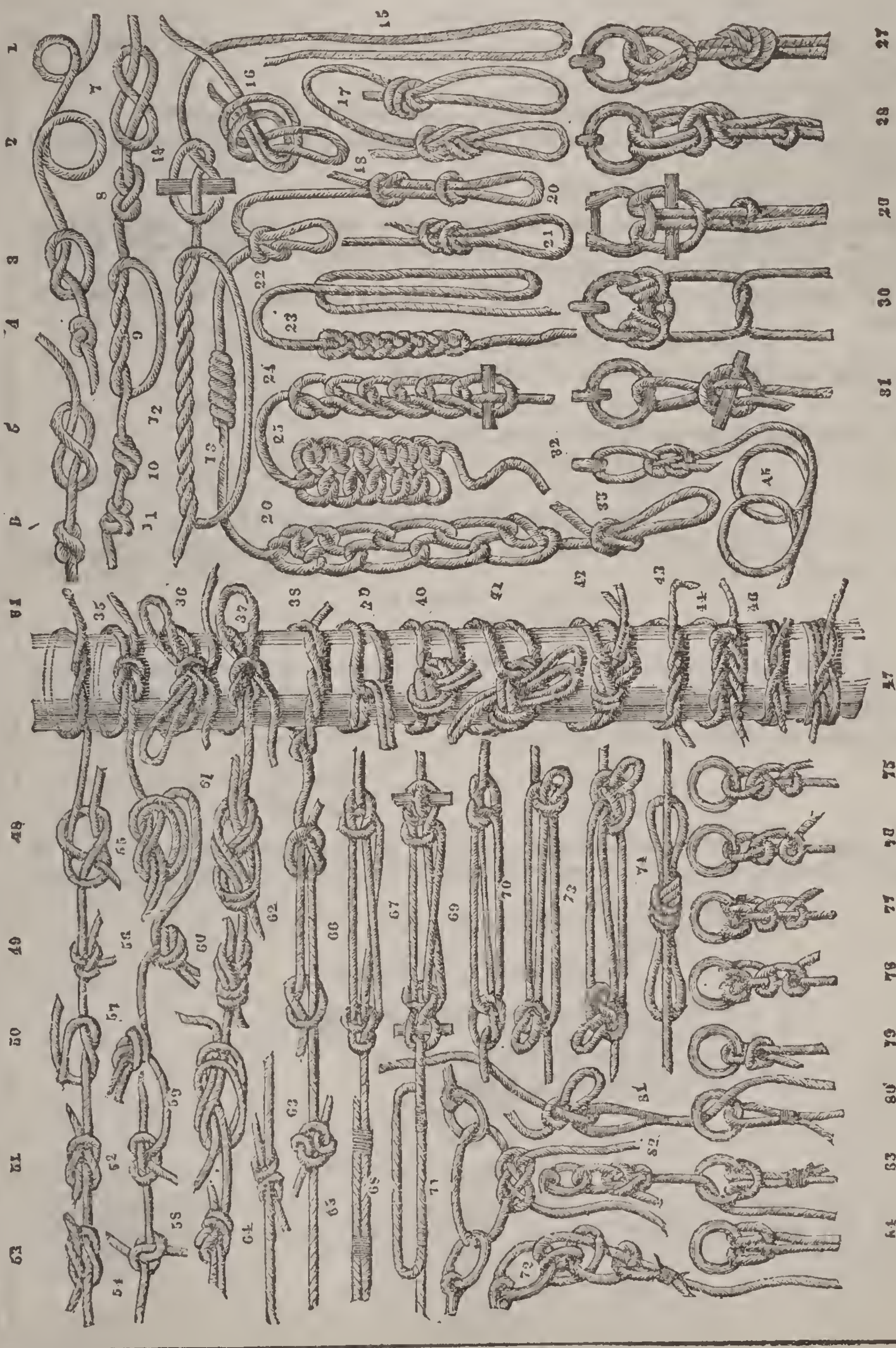
1 and 2 are simple loops, showing the elements of the simplest knot.

3. Simple knot commenced.
4. The same completed.
5. Flemish knot commenced.
6. The same completed.
7. Rope knot commenced.
8. The same completed.
9. Double knot commenced.
10. The same completed.
11. Double knot, back view.
12. Six-fold knot commenced.
13. The same completed. This is closed or "nipped," drawing the two ends with equal force.
14. A "boat" knot, made with the aid of a stick. This is a good knot for handling weights which may want instant detachment. Lift the weight very slightly, push out the stick, and the knot is untied.
15. Simple hitch (or double) used in making loop holes.
16. Loop knot commenced.
17. Loop knot finished.
18. Flemish loop or "Dutch" double knot.
19. Running knot.
20. Running knot to hold; the end knot nearest the bend of the rope is the check knot.
21. Running knot "checked."
22. Double loop for twist knot.
23. The twist knot completed. It is made by taking a half turn on both the right-hand and left-hand cords and passing the end through the "bight" so made.
24. Chain knot, a series of loops. The end of the cord is fastened, a simple loop made and passed over the left hand, the right hand retaining hold of the free end. The left hand then seizes the cord above the right and draws a loop through the loop already formed; the left then finishes the knot by drawing it tight. This is repeated until you have all the knot wanted, when it is secured by passing the free end entirely through the last loop. This is a kind of knot much in vogue for the knotting of leather whip-lashes, etc. It is very convenient.
25. Double chain.
26. Double chain secured and pulled out as when in use. Notice the mode in which the end is thrust through the last loop.
27. Lark's head; useful to sailors as a mooring knot.
28. The same, double looped.
29. The same on a ring of a boat. The advantage of instant release by the use of the stick has been noted in No. 14.
30. A treble lark's head. First tie a single lark's head, and then divide the two ends and use each singly as shown in the cut.
31. Simple boat knot with one turn.
32. Crossed running knot, strong and handy. Looks difficult, but by taking a cord about one-eighth of an inch in diameter and tying the same two or three times with the picture, you will find no difficulty in mastering it. It is a common knot in some parts of the country.
33. A knotted loop for end of rope. Use various, to prevent the end of the rope from slipping, etc. Very readily untied.
34. Simple (lashing) knot commenced.
35. The same finished. (See 51.) In making 34 it is necessary to hold the simple knot, as shown in 33, by some pressure on the knot until it is ready to draw tight for the finish.
36. Is the same knot with two turns, sometimes called a rosette. This is very easily untied, as will be seen by tracing the loose ends back in the illustration.
37. Knot with single turn; unties as easily as 36, but the "strands"—that is to say, all parts of the knot—must be laid as in the true or reef knot (see 50 and 51) or a "granny" knot will be produced which will not hold. One who ties this knot well will be a master of this art.
38. Timber hitch or slip knot, with double hitch. The greater the strain the tighter this knot will hold. It looks as if it might give way, but it will not.
39. Running knot with two ends.
40. The same with check knot, which cannot be opened except with a marlinspike.
41. Running knot with two ends, with a check knot (to the running loops), which can be untied by drawing both ends of the cord.
42. Running knot with two ends, fixed by a double Flemish. When an object is to be encircled by this knot, pass the end on which the check knot is to be through the cords.
43. Ordinary twist knot. 44. Double.
45. Form of loop for builder's knot.
46. Builder's knot finished, used by workmen in securing building materials.
47. Double builder's knot.
48. Weaver's knot. On the small scale, lay the ends of the two cords to be united between the thumb and first finger of the left hand, the right end undermost; pass the right-hand cord back over the thumb to form a loop, and bring it back under the thumb and hold it fast. Now put the end of the upper or left-hand cord over the right-hand cord and through the loop. Catch it with thumb and finger of the left hand, and tighten by drawing the right hand.
49. Weaver's knot completed.
50. True or reef knot commenced.
51. The same completed. Useful for small ropes, but if ropes are unequal in size it is apt to draw out into the shape shown by 52. To obviate this the two ends issuing from each side of the knot are whipped or lashed together.
53. A "granny" knot, the ends not lying alongside of each other.
54. Granny knot with a strain in it, showing its uselessness.
55, 56, 57. Commencement, finished front view, and finished back view. This is a common knot. The two ends to be united are seized together and tied in a common simple knot.
58. And the ordinary knot, the ends used separately.
59. The same knot open. This knot is made by making No. 3 on one rope, holding it open so that we can pass the end of the other cord through the first loop of the last, making it with a second loop. Then draw it tight.
60 and 61. Knot used for the same purpose as the simple Flemish. 60 is the tightened or finished knot.
62. English knot commenced
63. English knot tightened (front view).
64. English knot tightened (back view).
65. Splice, with two ties.
66. Shortening by loops and turns where the end of the rope is free.
67. Shortening knot, can be used when either end is free.
68. The same, with double bend and ties.
69. The same, passing through the knots.
70. Another method of shortening, called making a "sheep shank" or dog shank. Unsafe unless the shank (the loose loop) is attached to the contiguous rope by a stout "seizing," that is, a cord tied around it.
71. Shows a dog shank that will hold without seizing.

From 73 to 84 explain themselves without especial allusion to them.

THE ART OF KNOT-TYING.

(*From the "Scientific American."*)



FRICTION.

The resistance occasioned by Friction is independent of the velocity of motion; but the re-effect of friction is proportional to the velocity. Friction is independent of the extent of surface in contact when the pressure remains the same, but is proportional to the pressure. This law was established from experiments by Arthur Morin in the years 1831–32 and 1833, from which a summary is contained in the accompanying Table.

Notation.

a = Fibres of the woods are parallel to themselves, and to the direction of motion.
b = Fibres at right-angles to fibres.
c = Fibres vertical on the fibres which are parallel to the motion.
d = Fibres parallel to themselves, but at right-angles to the motion, length by length.
e = Fibres vertical, end to end.

Example. A vessel of 800 tons is to be hauled up an inclined plane, which inclines 9° 40′ from the horizon; the plane is of oak, and greased with tallow. What power is required to haul her up?

The coefficient for oak on oak with continued motion is $f = 0{\cdot}097$, say 0·1, then,

$$800 \times \sin.9^\circ\, 40' = 800 \times 0{\cdot}16791 = 124{\cdot}328 \text{ tons},$$

the force required if there were no friction, and

$$800 \times \cos.9^\circ\, 40' \times 0{\cdot}1 = 800 \times 0{\cdot}9858 \times 0{\cdot}1 = 78{\cdot}864 \text{ tons},$$

the force required for the friction only, and

$$\begin{array}{r} 134{\cdot}328 \\ 78{\cdot}864 \\ \hline 213{\cdot}192 \end{array}$$ tons, the force required to haul her up.

The work lost per sec. by friction in axle and bearings is expressed simply by the formula

$$P = \frac{\pi\, d\, W n f}{12{\cdot}60} = \frac{W d n f}{230},$$

in which W = the weight of pressure in the bearing, d = diameter on which the friction acts in inches, n = number of revolutions per minute, and f = coefficient of friction from the Table. In common machinery kept in good order the coefficient of friction can be assumed to $f = 0{\cdot}065$, then

$$P = \frac{W d n}{3530}, \qquad H = \frac{W d n}{1941500}$$

Example. The pressure on a steam-piston is 20000 pounds, and makes $n = 40$ double strokes per minute. Required the friction in the shaft of $d = 8$ inches?

$$H = \frac{20000 \times 8 \times 40}{1941500} = 3{\cdot}3 \text{ horse-power, the by friction.}$$

Friction in Guides.

W = pressure on the steam piston in pounds.
S = stroke of piston in feet.
l = length of connecting rod in feet
H = horse power of the friction.

$$H = \frac{f\, W S^2 n}{16500\sqrt{4\, l^2 - S^2}}.$$

Example. The pressure on a steam piston being W = 30,000 pounds, stroke S = 4 feet, length of connecting rod l = 7 feet, and making 50 revolutions per minute. Required the horse power of the friction H?

$$H = \frac{.065 \times 30000 \times 16 \times 50}{16500\sqrt{4 \times 49 - 16}} = 7 \text{ HP}.$$

TABLE OF FRICTION FOR PLANE SURFACES IN CONTACT.

Kind of Materials in contact.		Lubricated with.	Coefficient in Motion.	Coefficient in Starting.
Oak on Oak,	a	o	0·478	0·625
" "	"	tallow	0·097	0·160
" "	"	lard	0·067	....
" "	b	o	0·324	0·540
" "	"	unctuous	0·143	0·314
" "	"	tallow	0·083	0·254
" "	"	water,	0·25	....
" "	d	o	0·336	....
" "	c	o	0·192	0·271
" "	e	o	....	0·43
Cast-iron on Oak,	a	o	0·400	0·570
" "	"	soap	0·214	....
" "	"	tallow	0·078	0·108
Wrought-iron on Oak,	"	o	0·252	....
" "	"	tallow	0·078	....
Wrought iron, together,	a	o	0·138	0·137
" "	a	unctuous	0·177	....
" "	"	tallow	0·082	....
" "	"	olive oil	0·070	0·115
Wrought on cast-iron,	a	o	0·194	0·194
" "	"	unctuous	0·18	0·118
" "	"	tallow	0·103	0·10
" "	"	olive oil	0·066	0·100
Cast-iron on cast-iron,	a	water	0·314	0·314
" "	"	soap	0·197	....
" "	"	tallow	0·100	0·100
" "	"	olive oil	0·064	....
Wrought-iron on brass,	a	o	0·172	....
" "	"	unctuous	0·160	....
" "	"	tallow	0·103	....
" "	"	lard	0·075	....
" "	"	olive oil,	0·078	....
Cast-iron on brass,	a	o	0·147	...
" "	"	unctuous	0·132	....
" "	"	tallow	0·103	....
" "	"	lard	0·075	....
" "	"	olive oil	0·078	....
Brass on brass,	a	o	0·201	....
" "	"	unctuous	0·134	....
" "	"	olive oil	0·053	....
Steel on cast-iron,	"	o	0·202	....
" "	"	tallow	0·105	....
" "	"	lard	0 081	....
" "	a	olive oil	0·079	. .

FRICTION OF AXLES IN MOTION.

Designation of surface in contact.	Dry or slightly greasy, or wet.	Oil, Tallow, or Hog's Lard. Supplied in the ordinary manner.	Oil, Tallow, or Hog's Lard. The grease continually running.
Brass on Brass,	.....	0·079	.....
" on cast-iron,	.....	0·072	0·049
Iron on Brass,	0·251	0·075	0·054
" on cast-iron,	.....	0·075	0·054
Cast-iron on cast-iron,	0·137	0·075	0·054
" on Brass,	0·194	0·075	0·054
Iron on lignum-vitæ,	0·188	0·125	.....
Cast-iron on "	0.185	0·100	0·092
Lignum-vitæ on cast-iron,	.....	0·116	0·170

PAPER.

1 ream = 20 quires = 480 sheets.
1 quire = 24 sheets.

Drawing Paper.

Cap, ...	13 × 16 inches.	Columbier, . . .	34 × 23 inches.
Demy, ...	20 " 15 "	Atlas, ...	33 " 26 "
Medium, . . .	22 " 17 "	Theorem, . . .	34 " 28 "
Royal, ...	24 " 19 "	Double Elephant, .	40 " 26 "
Super Royal, .	27 " 19 "	Antiquarian, . .	52 " 31 "
Imperial, . . .	30 " 21 "	Emperor, . . .	40 " 60 "
Elephant, . . .	28 " 22 "	Uncle Sam, . . .	48 " 120 "

Continuous Colossal Drawing Paper, No. A and No. B, 56 inches wide, and of any required length. No A of this paper is excellent for mechanical drawings. Price, from 40 to 50 cents per yard.

Tracing Paper.

Double Crown, ...	30 by 20 inches.	Glazed or Crystal. Yellow or Blue Wove.
Double Double Crown, . . .	40 " 30 "	
Double Double Double Crown, . .	60 " 40 "	

Finest French Vegetable Tracing Paper.

Grand Raisin (or Royal), 24 in. by 18. Grand Aigle, 40 in. by 27.

Mounted Tracing Paper.

This paper is mounted on cloth, and is still transparent; it will take ink and water-colors. It is 38 inches wide, and of any required length.

Vellum Writing Cloth.

Adapted for every description of tracing; it is transparent, durable and strong. It is 18 to 38 inches wide, and of any required length.

Weight and Marks of English Tin-plates.

Brand.	Plates per Box. No.	Length and Breadth. In.	Weight per Box. Lbs.	Brand.	Plates per Box. No.	Length and Breadth. In.	Weight per Box. Lbs.
1 C. .	225	13¾×10	112	1 XX. . .	225	13¾×10	161
2 C. . .	225	13¼ " 9¾	105	1 XXX. .	225	13¾ " 10	182
3 C. .	225	12¾ " 9½	98	1 XXXX. .	225	13¾ " 10	203
H C. . .	225	13¾ " 10	119	1 XXXXX.	225	13¾ " 10	224
H X. .	225	13¾ " 10	157	1 XXXXXX.	225	13¾ " 10	245
1 X. . .	225	13¾ " 10	140	DC. . .	100	16¾ " 12½	98
2 X. .	225	13¼ " 9¾	133	DX. . .	100	16¾ " 12½	126
3 X. . .	225	12¾ " 9½	126	DXX. . .	100	16¾ " 12½	147
Leaded IC.	112	20 " 14	112	DXXX. .	100	16¾ " 12½	168
" IX.	112	20 " 14	140	DXXXX. .	100	16¾ " 12½	189
ICW. .	225	13¾ " 10	112	SDC. .	200	15 " 11	168
IXW. .	225	13¾ " 10	140	SDX. . .	200	15 " 11	188
CSDW.	200	15 " 11	168	SDXX. .	200	15 " 11	209
CHW. .	100	16¾ " 12½	105	SDXXX. .	200	15 " 11	230
XHW. .	100	16¾ " 12½	126	SDXXXX.	200	15 " 11	251
TT. . .	450	13¾ " 10	112	SDXXXXX.	200	15 " 11	272
XTT. .	450	13¾ " 10	126	SDXXXXXX.	200	15 " 11	293

When the plates are 14 by 20 inches, there are 112 in a box.

Thickness and Weight of Window Glass.

Number of the glass or weight in ounces per square foot.

12	13	15	16	17	19	21	24	26	32	36	42
.059	.063	.071	.077	.083	.091	.100	.111	.125	.154	.167	.200

Thickness in decimals of an inch.

GRAVITATION.

Gravity or **Gravitation** is a mutual property which all bodies in nature possess of attracting one another; or *Gravity* is he tforce by which all bodies tend to approach each other. A large body attracting a comparatively very small one, and their distance apart being inconsiderable, the force of gravity in the small body will be very sensible compared with that in the large one; such is the case with the body, our earth, attracting small bodies on or near her surface.

Gravitation is not periodical, it acts continually *ever* and *ever*. A body placed unsupported at a distance from the earth, the *force of gravity* is instantly operating to draw it down, and then we say, "the body fell down" If it were possible to withdraw the attraction between the body and the earth, it would not fall down, but remain unsupported in the space where it was placed;—giving the body a motion upwards, it would continue moving, and never come back to the earth again.

Law of Gravity.

The force of Gravity is directly proportional to the product of the masses of the attracting bodies, and inversely to the square of their distance apart.

This law was discovered by Sir Isaac Newton. It is this law that supports the condition of the whole universe, and enables us to calculate the distances, motions and masses, &c., of the heavenly bodies.

The unit or measure of force of gravity is assumed to be the velocity a falling body has attained at the end of the first second of descent; this unit is commonly denoted by the letter g; its value at the level of the sea in New York is $g = 32{\cdot}17$ feet per second, in vacuum. g is called *the acceleratrix of gravity*. The space fallen through in the first second is $\frac{1}{2}g = 16{\cdot}085$ feet.

This value increases with the latitude, and decreases with the elevation above the level of the sea.

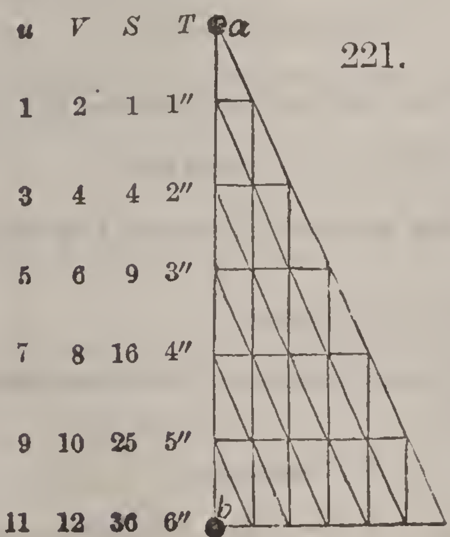
$l =$ *latitude*, $h =$ *height* in feet above the level of the sea, and $r =$ *radius* of the earth in feet, at the given latitude l.

$$r = 20887510\,(1 + 0{\cdot}00164 \cos.2l),$$

$$g = 32{\cdot}16954\,(1 - 0{\cdot}00284 \cos.2l)\left(1 - \frac{2h}{r}\right)$$

Notation.

$S =$ the *space* in feet, which the falling body passes through in the time T.
$u =$ the *space* in feet, which the body falls in the Tth second.
$V =$ *velocity* in feet per second, of the falling body at the end of the time T.
$T =$ *time* in seconds the body is falling.



221.

The accompanying Diagram is a good illustration of the acceleration of a falling body. The body is supposed to fall from a to b, every small triangle represents the space 16·08 feet which the body falls in the first second; when the body has reached the line 3″ seconds, it will be found that it has passed 9 triangles, and $9 \times 16{\cdot}08 = 144{\cdot}72$ feet the space which a body will fall in 3″ seconds. The number of triangles between each line is the space u which the body has fallen in that second. Between 3″ and 4″ are 7 triangles and $7 \times 16{\cdot}08 = 112{\cdot}56$ feet, the space fallen through in the fourth second. Under the line 3″ will be found 6 triangles, which represents the velocity V the body has obtained at the end of the third second or $6 \times 16{\cdot}08 = 96{\cdot}48$ feet per second. For every successive second the body will gain two triangles or $2 \times 16{\cdot}08 = 32{\cdot}16$ feet per second.

FORMULAS FOR ACCELERATED MOTION.

Velocity *V* in Feet per Second.

$V = g\,T.$ . . . . . . 1.

$V = \frac{2\,S}{T}.$ . . . . . . 2.

$V = \sqrt{2\,g\,S}.$ . . . . . 3.

$V = 8.02\,\sqrt{S}.$ . . . . . 4.

Space *S* Fallen through in Feet.

$S = \frac{g\,T^2}{2}.$ . . . . . . 5.

$S = \frac{V\,T}{2}.$ . . . . . . 6.

$S = \frac{V^2}{2\,g}.$ . . . . . . 7.

$S = \frac{V^2}{64.33}.$ . . . . . . 8.

Time of Fall in Seconds.

$T = \frac{V}{g}.$ . . . . . . 9.

$T = \frac{2\,S}{V}$ . . . . . . 10.

$T = \sqrt{\frac{2\,S}{g}}.$ . . . . . 11.

$T = \frac{\sqrt{S}}{4.01}.$ . . . . . 12.

Space Fallen through in the *T*th Second.

$u = g\left(T - \frac{1}{2}\right).$ . . . . 13.

$T = \frac{u}{g} + \frac{1}{2}.$ . . . . . 14.

Example 1. What velocity has a body attained after having fallen freely for a time of $T = 2\frac{1}{2}$ seconds?

Velocity $V = 32.17 \times 2.5 = 80.2$ feet per second.

Example 4. A body is dropped from a height of $S = 98$ feet. What velocity will it have on reaching the ground, and what time is required for its fall?

Formula 4. Velocity $V = 8.02\,\sqrt{98} = 79.3939$ feet per second.

Formula 12. Time $T = \frac{\sqrt{S}}{4.01} = \frac{\sqrt{98}}{4.01} = 2.46$ seconds.

Example 5. A body was dropped at the opening of a hole in a rock, and reached the bottom in $T = 3.5$ seconds. Required the depth of the hole?

Formula 5. Depth $S = g\,\frac{T^2}{2} = \frac{32.17 \times 3.5^2}{2} = 196.98$ feet.

Example 8. What space must a body fall through in order to acquire a velocity $V = 369$ feet per second?

Space $S = \frac{V^2}{64.33} = \frac{369^2}{64.33} = 2116.6$ feet.

Example 10. What time is required for a body to fall $S = 2116.6$ feet when the final velocity $V = 369$ feet per second?

Time $T = \frac{2\,S}{V} = \frac{2 \times 2116.6}{369} = 11.472$ seconds.

Example 13. A body falls freely for a time of $T = 4\frac{1}{2}$ seconds. How much will it fall in the last second?

Formula 13. $u = g\,(T - \frac{1}{2}) = 32.17\,(4.5 - 0.5) = 128.68$ feet.

RETARDED MOTION.

A body thrown up vertically will obtain inversely the same motion as when it falls down, because it is the same force that acts upon it, and causes *retarded motion* when it ascends, and *accelerated motion* when it descends.

V = the *velocity* at which the body starts to ascend.
v = *velocity* at the end of the time t.
T = *time* in seconds in which the body will ascend.
t = any *time* less than T.
S = *height* in feet to which the body will ascend.
s = the space it ascends in the time t.

Velocity in Feet per Second at the End of the Time t.

$$v = V - g\,t. \quad \ldots \quad 15.$$

$$v = \frac{s}{t} - \frac{g\,t}{2}. \quad \ldots \quad 16.$$

Height of Ascension in the Time t.

$$s = t\left(V - g\,\frac{t}{2}\right). \quad \ldots \quad 17.$$

$$s = t\left(v + g\,\frac{t}{2}\right). \quad \ldots \quad 18.$$

Starting Velocity in Feet per Second.

$$V = v + g\,t. \quad \ldots \quad 19.$$

$$V = \frac{s}{t} + g\,\frac{t}{2}. \quad \ldots \quad 20.$$

Time of Ascension in Seconds.

$$t = \frac{V - v}{g}. \quad \ldots \quad 21.$$

$$t = \frac{V}{g} - \sqrt{\frac{V^2}{g^2} - \frac{2\,s}{g}}. \quad \ldots \quad 22.$$

Starting and Ending Velocities.

$$v = \sqrt{V^2 - 2\,g\,s}. \quad \ldots \quad 23.$$

$$V = \sqrt{v^2 + 2\,g\,s}. \quad \ldots \quad 24.$$

Formulas for T and S are the same as for accelerated motion.

Example 22. A ball starts to ascend with a velocity of 135 feet per second. At what velocity will it strike an object 60 feet above? Find the time t by the Formula 22.

$$t = \frac{135}{32.16} - \sqrt{\frac{135^2}{32.16} - \frac{2 \times 60}{32.16}} = 0.41 \text{ seconds,}$$

until it strikes; and from Formula 15 we have

$$v = 135 - 32.16 \times 0.41 = 121.83 \text{ feet per second.}$$

Example 24. With what velocity must a body start to ascend in order to strike an object s = 15 feet above with a velocity v = 10 feet per second?

$$\text{Velocity } V = \sqrt{10^2 + 2 \times 32.17 \times 15} = 32.63 \text{ feet per second.}$$

Example 5. A ball thrown up vertically from a cannon, occupied 20 seconds, until it arrived at the same place it started from. How high up was the ball, and at what velocity did it start?

One-half of 20 = 10 seconds. Formula 2.

$$S=\frac{32.16\times 10^2}{2}=1608 \text{ feet high.}$$

$$V=32.16\times 10=321.6 \text{ feet per second.}$$

If a cannon-ball be shot from A, in the direction AB, at an angle BAC to the horizon, there are two forces acting on the ball at the same time, namely — the force of gunpowder, which would propel the ball uniformly in the direction AB, and the force of gravity, which only acts to draw the ball down at an accelerated motion; these two different (uniform and accelerated) motions will cause the ball to move in a curved line (Parabola) AaC. Fig. 225.

V = velocity of the ball at A. W = weight of the ball in pounds.
S = the greatest height of ball over the horizontal line AC.
t = time from A to C, viâ a. p = pounds of powder in the charge.
b = the distance from A to C, called *horizontal range*.

$$V=2800\sqrt{\frac{p}{W}},\qquad p=\frac{W V^2}{7840000},\qquad b=87.06 \, sin.x \, cos.x \, \frac{p}{W}.$$

Example The cannon being loaded sufficiently to give the ball a velocity of 900 feet per second, the angle $x = 45°$. Required, the distance $b = ?$ and the time $t = ?$

$$b=\frac{900^2\times \sin.45°\times \cos.45°}{32.16}=1259 \text{ feet, the distance from } A \text{ to } C.$$

It will be observed that the distance b will be longest when the angle x is 45°, because the product of *sine* and *cosine* is greatest for that angle. $\sin.45° \times \cos.45° = 0.5$.

Example What time will it take for a ball to roll 38 feet on an inclined plane, angle $x = 12° 20'$, and what velocity has it at 38 feet from the starting-point? Fig. 222.

$$T=\sqrt{\frac{2S}{g \sin.x}}=\sqrt{\frac{2\times 38}{32.16\times \sin.12°\ 20'}}=3.33 \text{ seconds.}$$

$$V=g\ T \sin.x=32.16\times 3.33\times \sin.12°\ 20'=22.8 \text{ feet per second.}$$

Resistance of Air to the Flight of Projectiles.

A = area of resistance of the projectile in square inches.
ϕ = angle of resistance of the projectile, which for flat surfaces $\sin.^2\phi = 1$, for sphere $\sin.^2\phi = 0.5$.

For a pointed projectile of parabolic form, and when the ordinate is double the abscissa $\sin.^2\phi = 0.25$.

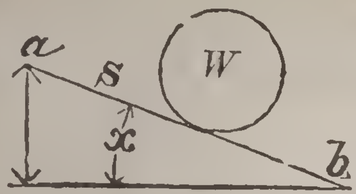
V = velocity of the projectile in feet per second.
R = resistance to the projectile in pounds.

$$R=\frac{A V^2 \sin.^2\phi}{57000}.$$

Let T denote the time of flight in seconds, and W = weight in pounds of the projectile.

D = distance in feet which the projectile is retarded by resistance of air in the time T.

$$D=\frac{32.166 R T^2}{2W}=\frac{16 R T^2}{W}.$$

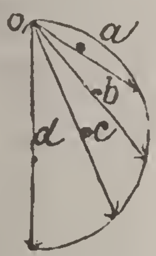


222.

$$V = g\,T \sin.x = \sqrt{2g\,S \sin.x},$$

$$S = \frac{g\,T^2}{2 \sin.x} = \frac{V^2}{2\,g \sin.x},$$

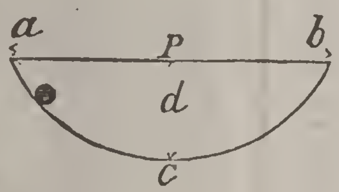
$$T = \frac{V}{g \sin.x} = \sqrt{\frac{2\,S \sin.x}{g}}.$$



223.

A body will fall from *o* the distances *a*, *b*, *c* and *d*, in equal times.

$$T = \sqrt{\frac{2d}{g}}.$$

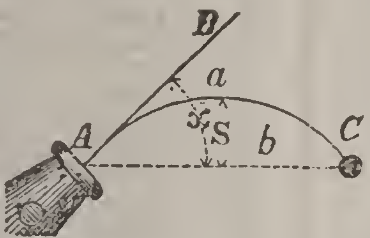


224.

A body will fall from *a* to *b* via *c* in the shortest time, if the curve is a Cycloid.

$S = 4d$, the length of the Cycloid.

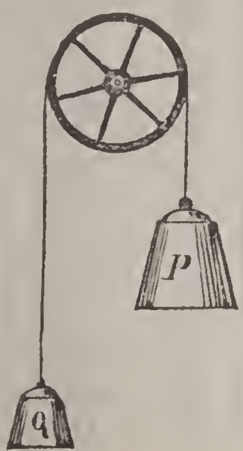
$$T = \pi\sqrt{\frac{d}{2g}} = \pi\sqrt{\frac{p}{2\pi\,g}}.$$



225.

$$b = \frac{2V^2 \sin.x \cos.x}{g},$$

$$T = \frac{V \sin.x}{g}, \qquad S = \frac{V^2 \sin.^2 x}{2g}.$$



226.

$$S = g\,\frac{T^2\,F}{2\,W} = \frac{V^2\,W}{2g\,F},$$

$$V = g\,T\,\frac{F}{W} = \sqrt{\frac{2g\,S\,F}{W}},$$

$$T = \frac{V\,W}{g\,F} = \sqrt{\frac{2S\,W}{g\,F}},$$

$$F = \frac{V\,W}{g\,T} = \frac{2S\,W}{g\,T^2},$$

$$W = P + Q, \text{ and } F = P - Q.$$

Falling Bodies.

V = velocity in feet per second at the end of fall.

T = time in seconds of the fall.

S = space fallen through in feet.

V	T	S	V	T	S	V	T	S
0.1	0.0031	.00015	5.1	0.1585	0.4042	11	0.3419	1.8804
0.2	0.0062	.00031	5.2	0.1616	0.4202	12	0.3730	2.2380
0.3	0.0093	0.0014	5.3	0.1647	0.4364	13	0.4041	2.6266
0.4	0.0124	0.0025	5.4	0.1678	0.4530	14	0.4352	3.0464
0.5	0.0155	0.0039	5.5	0.1709	0.4700	15	0.4663	3.4975
0.6	0.0186	0.0055	5.6	0.1740	0.4872	16	0.4973	3.9784
0.7	0.0217	0.0076	5.7	0.1771	0.5047	17	0.5284	4.4914
0.8	0.0248	0.0099	5.8	0.1802	0.5226	18	0.5595	5.0355
0.9	0.0279	0.0125	5.9	0.1833	0.5407	19	0.5906	5.6107
1.	0.0311	0.0155	6.	0.1865	0.5595	20	0.6217	6.2170
1.1	0.0342	0.0188	6.1	0.1896	0.5782	21	0.6527	6.8502
1.2	0.0373	0.0224	6.2	0.1927	0.5973	22	0.6838	7.5218
1.3	0.0404	0.0262	6.3	0.1958	0.6168	23	0.7149	8.2213
1.4	0.0435	0.0304	6.4	0.1989	0.6365	24	0.7460	8.9520
1.5	0.0446	0.0335	6.5	0.2020	0.6565	25	0.7771	9.7125
1.6	0.0477	0.0381	6.6	0.2051	0.6768	26	0.8082	10.566
1.7	0.0508	0.0432	6.7	0.2082	0.6975	27	0.8393	11.330
1.8	0.0539	0.0485	6.8	0.2113	0.7184	28	0.8704	12.185
1.9	0.0580	0.0551	6.9	0.2144	0.7397	29	0.9015	13.072
2.	0.0622	0.0622	7.	0.2176	0.7616	30	0.9325	13.987
2.1	0.0653	0.0685	7.1	0.2207	0.7835	31	0.9636	14.936
2.2	0.0684	0.0756	7.2	0.2238	0.8057	32	0.9947	15.915
2.3	0.0715	0.0822	7.3	0.2269	0.8282	33	1.0258	16.926
2.4	0.0746	0.0895	7.4	0.2300	0.8510	34	1.0569	17.967
2.5	0.0777	0.0971	7.5	0.2331	0.8741	35	1.0879	19.038
2.6	0.0808	0.1050	7.6	0.2362	0.8975	36	1.1190	20.142
2.7	0.0839	0.1135	7.7	0.2393	0.9213	37	1.1501	21.277
2.8	0.0870	0.1218	7.8	0.2424	0.9453	38	1.1812	22.443
2.9	0.0901	0.1305	7.9	0.2455	0.9697	39	1.2123	23.640
3.	0.0932	0.1398	8.	0.2487	0.9948	40	1.2434	24.868
3.1	0.0963	0.1492	8.1	0.2518	1.0168	41	1.2745	26.127
3.2	0.0994	0.1590	8.2	0.2549	1.0451	42	1.3056	27.417
3.3	0.1025	0.1691	8.3	0.2580	1.0707	43	1.3367	28.739
3.4	0.1054	0.1795	8.4	0.2611	1.0966	44	1.3678	29.407
3.5	0.1087	0.1886	8.5	0.2642	1.1228	45	1.3989	31.475
3.6	0.1118	0.2012	8.6	0.2673	1.1494	46	1.4300	32.890
3.7	0.1149	0.2125	8.7	0.2704	1.1762	47	1.4611	34.336
3.8	0.1170	0.2223	8.8	0.2735	1.2034	48	1.4922	35.813
3.9	0.1201	0.2355	8.9	0.2766	1.2259	49	1.5233	37.321
4.	0.1243	0.2486	9.	0.2797	1.2586	50	1.5544	38.830
4.1	0.1274	0.2611	9.1	0.2828	1.2867	51	1.5854	40.413
4.2	0.1305	0.2740	9.2	0.2859	1.3151	52	1.6165	42.029
4.3	0.1336	0.2872	9.3	0.2890	1.3438	53	1.6475	43.659
4.4	0.1367	0.2939	9.4	0.2921	1.3729	54	1.6786	45.322
4.5	0.1398	0.3145	9.5	0.2952	1.4022	55	1.7097	47.017
4.6	0.1429	0.3286	9.6	0.2983	1.4318	56	1.7407	48.740
4.7	1.1460	0.3431	9.7	0.3014	1.4618	57	1.7718	50.396
4.8	0.1491	0.3578	9.8	0.3045	1.4920	58	1.8029	52.284
4.9	0.1522	0.3729	9.9	0.3076	1.5226	59	1.8340	54.103
5.	0.1554	0.3885	10.	0.3108	1.5540	60	1.8651	55.953

Falling Bodies.

$$V=\frac{2\,S}{T}. \qquad T=\sqrt{\frac{2\,S}{g}}. \qquad S=\frac{g\,T^2}{2}.$$

V	T	S	V	T	S	V	T	S
65	2.0206	65.669	530	16.478	4366.6	1030	32.027	16494
70	2.1769	76.260	540	16.788	4452.8	1040	32.338	16815
75	2.3314	87.427	550	17.099	4701.7	1050	32.649	17141
80	2.4868	97.472	560	17.409	4874.5	1060	32.950	17463
85	2.6422	112.29	570	17.720	5050.2	1070	33.261	17794
90	2.7976	125.89	580	18.030	5228.7	1080	33.572	18129
95	2.9530	140.27	590	18.341	5410.6	1090	33.883	18446
100	3.1085	155.42	600	18.651	5595.3	1100	34.194	18806
110	3.4194	188.07	610	18.961	5783.1	1110	34.504	19149
120	3.7302	223.81	620	19.271	5974.0	1120	34.815	19496
130	4.0411	262.67	630	19.582	6168.3	1130	35.126	19846
140	4.3519	304.63	640	19.893	6365.7	1140	35.436	20198
150	4.6627	349.70	650	20.204	6566.3	1150	35.747	20504
160	4.9736	397.88	660	20.515	6770.0	1160	36.058	20913
170	5.2844	449.18	670	20.826	6976.7	1170	36.369	21275
180	5.5953	503.36	680	21.137	7186.6	1180	36.680	21641
190	5.9061	561.08	690	21.448	7399.5	1190	36.991	22009
200	6.2170	621.70	700	21.759	7615.6	1200	37.302	22381
210	6.5279	689.43	710	22.070	7834.8	1210	37.613	22755
220	6.8387	752.26	720	22.380	8056.8	1220	37.924	23133
230	7.1496	822.20	730	22.691	8282.2	1230	38.235	23514
240	7.4604	895.25	740	23.002	8510.7	1240	38.546	23898
250	7.7713	971.41	750	23.313	8742.4	1250	38.857	24285
260	8.0821	1050.6	760	23.623	8976.7	1260	39.168	24676
270	8.3930	1133.1	770	23.934	9214.6	1270	39.479	25069
280	8.7038	1218.5	780	24.245	9455.5	1280	39.780	25459
290	9.0147	1308.2	790	24.556	9699.6	1290	40.090	25855
300	9.3255	1398.8	800	24.868	9947.2	1300	40.411	26267
310	9.6363	1493.7	810	25.179	10197	1310	40.722	26673
320	9.9472	1591.6	820	25.490	10451	1320	41.033	27081
330	10.258	1690.6	830	25.801	10707	1330	41.343	27493
340	10.569	1791.7	840	26.112	10967	1340	41.654	27908
350	10.879	1903.8	850	26.423	11230	1350	41.965	28326
360	11.190	2014.2	860	26.733	11495	1360	42.276	28747
370	11.501	2127.7	870	27.044	11764	1370	42.587	29172
380	11.812	2244.3	880	27.354	12035	1380	42.897	29599
390	12.123	2364.0	890	27.665	12311	1390	43.208	30029
400	12.434	2486.8	900	27.976	12589	1400	43.519	30463
410	12.745	2612.7	910	28.287	12871	1410	43.820	30893
420	13.055	2741.5	920	28.598	13155	1420	44.131	31333
430	13.366	2873.7	930	28.908	13442	1430	44.442	31776
440	13.677	3008.9	940	29.219	13733	1440	44.753	32222
450	13.989	3144.8	950	29.530	14027	1450	45.064	32671
460	14.300	3289.0	960	29.841	14323	1460	45.375	33123
470	14.611	3433.6	970	30.152	14623	1470	45.686	33579
480	14.922	3581.3	980	30.463	14927	1480	45.997	34037
490	15.233	3732.1	990	30.774	15233	1490	46.308	34499
500	15.545	3886.2	1000	31.085	15542	1500	46.631	34973
510	15.856	4043.3	1010	31.396	15855	1510	46.732	35082
520	16.167	4203.4	1020	31.707	16179	1520	47.043	35752

Dynamics of Matter.

Matter is that of which bodies are composed, and occupies space.

Matter is recognized as substance in contradistinction from geometrical quantities and physical phenomena, such as color, shadow, light, heat, electricity, and magnetism.

Chemistry has, thus far, dissolved matter into some sixty-five distinct elements, but in the philosophy of mechanics we treat matter only as one simple element in relation to the three physical elements *force*, *motion*, and *time*.

These four elements, force F, motion V, time T, and mass M, are what constitute nature, and their different combinations cause the phenomena which we study and observe.

Mass and Weight.

Mass is the real quantity of matter in a body, and is proportional to weight when compared in one or the same locality. The mass of a body is a constant quantity, whilst the weight of the same body varies with the force of gravity which produces it.

Inertia is the incapability of a dead body to change its own state of motion or rest.

Force of inertia is the resistance a body free to move presents to any external force acting to change its state of motion or rest.

Let a constant force F be applied to a body W free to move; then the body will start and continue with an accelerated velocity until the force ceases to act, when it will continue in the same direction with a uniform velocity equal to that of the final action of the force, and will never stop unless a force act upon it in the opposite direction. If the force F were equal to the weight W, then the acceleration would be the same as when falling freely under the action of gravity—namely, 32.17 feet per second; and if the force F is greater or smaller than the weight of the mass, the acceleration will be proportionally greater or smaller.

Any force, however small, is able to set in motion any body free to move, or to bring to rest any moving body, however large.

No force is required to maintain a uniform motion of a body free to move, but force is required to bring a body from rest into a uniform motion. If force is applied to maintain a body not free to move in uniform motion, such force is expended in overcoming the friction and resistance of the medium in which the body moves. A steamboat or a railway train in motion is thus suspended between the action of two opposite forces—namely, the driving force on the one side, and the friction and resistance on the other. When the opposite forces are equal, the motion will be uniform; and any change of velocity is due to a disparity between these opposing forces.

Now we are ready to combine the four physical quantities, *force*, *velocity*, *time*, and *mass*, into their functions in dynamics, where they bear the following relations:

$$M : F = T : V.$$

Momentum $MV = FT$, the acting force into time.

When F is expressed in pounds, T in seconds, and V in feet per second, then the unit mass, or M, will be 32.17 pounds, which is equal to the acceleratrix g for a falling body at the surface of the earth.

$$\text{Mass } M = \frac{W}{g} = \frac{W}{32.17}.$$

To get the mass of a body is only to divide its weight in pounds by the acceleratrix 32.17, and the quotient is the mass.

Force, Power, and Work in Moving Bodies.

It requires force, power, and work to change the state of motion or rest of a body.

In the dynamic expression $M V = F T$ we have

Force $F = \frac{M V}{T}$. . . . . 1.	$M = \frac{F T}{V}$. . . . . . . 3.
$T = \frac{M V}{F}$. . . . . 2.	$V = \frac{F T}{M}$. . . . . . . 4.

The force F required to set a mass M in motion with velocity V depends inversely on the time T of action. The more time the less need the force be for a certain velocity, and therefore it cannot be determined what force has set a mass in motion without knowing its time of action; but when the mass and its velocity are given, then we can determine the exact amount of work bestowed on the motion.

Multiply the dynamic momentum by the velocity V, and we have

$$M V^2 = F V T.$$

Here we recognize the work $\frac{V}{2} F T$, which is that bestowed on the mass M in giving it the velocity V, or the mass multiplied by $\frac{1}{2}$ the square of its velocity is the work stored in it.

Vis-viva.—The term $M V^2$ has formerly been called *vis-viva*, but that term is now seldom used.

The real work in foot-pounds is $\frac{1}{2} M V^2 = \frac{1}{2} F V T$. The space S in which the mass was set in motion is $S = \frac{1}{2} V T$, which inserted in the formula gives the

$$\text{Work } K = \frac{1}{2} M V^2 = F S.$$

The following table of formulas will show what a variety of problems are connected with a force acting on a body free to move.

When a body is left free to the action of gravity in falling or rising, the acceleratrix $G = g$, and the force $F = W$.

Example 1. What force $F = ?$ is required to give a body $W = 1689$ pounds a velocity of $V = 36$ feet per second in a time $T = 5.6$ seconds?

Find in the formulas under *constant force* the one which contains the given quantities W, V, and T, which is the second formula.

$$F = \frac{W V}{g T} = \frac{1689 \times 36}{32.166 \times 5.6} = 337.55 \text{ pounds, the answer.}$$

Example 2. A projectile of $W = 150$ pounds is fired horizontally from a rifled gun of $S = 11$ feet in length, in which it receives a velocity of $V = 950$ feet per second. Required the mean force $F = ?$ of the powder acting on the projectile, when the friction in the rifle is 230 pounds.

$F = \frac{W V^2}{2g S} = \frac{150 \times 950^2}{2 \times 32.166 \times 11} = 191302 + 230 = 191532$ pounds, the force required.

Example $2\frac{1}{2}$. What force is required to give a mass $W = 6386$ pounds a velocity of $V = 160$ feet per second when acting in a space $S = 15$ feet?

$$F = \frac{W V^2}{2 g S} = \frac{6386 \times 160^2}{2 \times 32.17 \times 15} = 425500 \text{ pounds.}$$

Example $2\frac{3}{4}$. What force is required on a mass free to move $W = 1500$ pounds to move it $S = 60$ feet in $T = 2\frac{1}{2}$ seconds?

$$F = \frac{W S}{g T^2} = \frac{1500 \times 60}{32.17 \times 2.5^2} = 447.63 \text{ pounds.}$$

Example 3.—The moving parts in a propeller steam-engine, such as the steam-piston, piston-rods, cross-heads, connecting-rod, &c. &c., weigh $W = 8456$ pounds. Stroke of piston = 4 feet, making $n = 52$ revolutions per minute. What force F is required for each stroke, to set in motion and bring to rest the moving mass?

The velocity of the moving mass at half stroke will be (formula , page 263)

$$V = \frac{2\pi r n}{60} = \frac{2 \times 3{\cdot}1416 \times 2 \times 52}{60} = 10{\cdot}79 \text{ feet per second.}$$

The time for each half stroke will be

$$T = \frac{60}{4 \times 52} = 0{\cdot}28846 \text{ seconds.}$$

Then the required mean force will be

$$F = \frac{W V}{g T} = \frac{8456 \times 10{\cdot}79}{32{\cdot}166 \times 0{\cdot}2846} = 9966{\cdot}8 \text{ pounds.}$$

For high grade of expansion of steam, this force acts beneficially to the movement of the engine.

Example 4.—The mean force of gunpowder in a rifled gun is known to be 231400 pounds, on a projectile $W = 180$ lbs. The friction of the projectile through the gun is estimated at 264 pounds, leaving $F = 231400 - 264 = 231136$ pounds. The length of the gun is $S = 12$ feet, elevated to an angle $x = 6° 30'$. Required the velocity $V = ?$ of the projectile when it leaves the gun.

$$V = \sqrt{2 g S \left(\frac{F}{W} - \sin.x\right)} = \sqrt{2 \times 32{\cdot}166 \times 12\left(\frac{231136}{180} - \sin.6° 30'\right)}$$

= 995·64 feet per second, the answer.

Example 5.—What velocity V can a steam-engine of $H = 56$ horses impart to a body $W = 9$ tons in a time $T = 30$ seconds?

$$P = 56 \times 550 = 19800 \text{ effects, and } W = 9 \times 2240 = 20160 \text{ lbs.}$$

$$V = \sqrt{\frac{2 g P T}{W}} = \sqrt{\frac{2 \times 32{\cdot}166 \times 19800 \times 30}{20160}} = 43{\cdot}538 \text{ feet per second.}$$

Example 6.—A body $W = 3685$ lbs. is moving with a velocity $V = 56$ feet per second. What time $T = ?$ is required to bring that body to rest, with a force $F = 128$ pounds?

$$T = \frac{W V}{g F} = \frac{3685 \times 56}{32{\cdot}166 \times 128} = 50{\cdot}121 \text{ pounds, the answer.}$$

Example 7.—What power P is required to drive a centrifugal gun to throw out balls of $W = 50$ lbs. every $T = 8$ seconds, with a velocity $V = 785$ feet per second (friction omitted)?

$$P = \frac{W V^2}{2 g T} = \frac{50 \times 785^2}{2 \times 32{\cdot}166 \times 8} = 59867 \text{ effects,}$$

divided by 550 = 108·85 horses, the power required.

Example 8.—A sledge of $W = 20$ lbs. strikes a spike into a log $S = 0{\cdot}08$ foot, with a velocity of $V = 25$ feet per second. Required the force $F = ?$ with which the spike was driven into the log, omitting the weight of the spike.

$$F = \frac{W V^2}{2 g S} = \frac{20 \times 25^2}{2 \times 32{\cdot}166 \times 0{\cdot}08} = 2628{\cdot}9 \text{ pounds.}$$

Example 9.—A body starts to ascend vertically with a velocity of 860 feet per second. What will be its velocity at the end of $T = 5$ seconds?

$$V = G T = 32{\cdot}166 \times 5 = 160{\cdot}830 \text{ feet per second,}$$

and 860 — 160·83 = 699·17 feet per second, the answer.

Dynamical Formulas for Accelerated or Retarded Motion.

Constant Force in Pounds acting on a Body free to move.

$$F=\frac{G\,W}{g}=\frac{W\,V}{g\,T}=\frac{2\,W\,S}{g\,T^2}=\frac{W\,V^2}{2\,g\,S}=\frac{P\,T}{S}=\sqrt{\frac{2\,P\,W}{g\,T}}=\frac{2\,K}{G\,T^2}=\frac{K}{S}$$

Final Velocity in the Time T, or Uniform Velocity of a Moving Body.

$$V=G\,T=\frac{g\,F\,T}{W}=\frac{2\,S}{T}=\sqrt{\frac{2\,g\,S\,F}{W}}=\sqrt{2\,G\,S}=\frac{P\,T}{K}=\sqrt{\frac{2\,g\,P\,T}{W}}=\sqrt{\frac{2\,g\,K}{W}}$$

Time in Seconds in which the Force acts on the Body free to move.

$$T=\frac{V}{G}=\frac{W\,V}{g\,F}=\sqrt{\frac{2\,W\,S}{g\,F}}=\sqrt{\frac{2\,S}{G}}=\frac{2\,F\,S^2}{V\,K}=\frac{K}{P}=\frac{2\,S\,W}{g\,T\,F}=\sqrt{\frac{2\,W\,K}{g\,F^2}}$$

Constant Acceleration of the Force F in Feet per Second.

$$G=\frac{g\,F}{W}=\frac{2\,S}{T^2}=\frac{V}{T}=\frac{V^2}{2\,S}=\frac{g\,P\,T}{W\,S}=\frac{F\,V^2}{P\,T}=\frac{g\,K}{W\,S}=\frac{2\,K}{F\,T^2}.$$

Space in Feet in which the Force acts on the Body free to move.

$$S=\frac{G\,T^2}{2}=\frac{V\,T}{2}=\frac{V^2}{2\,G}=\frac{g\,F\,T^2}{2\,W}=\frac{P\,T}{F}=\frac{g\,P\,T^2}{W\,V}=\frac{g\,K}{G\,W}=\frac{K}{F}.$$

Weight in Pounds of the Moving Body.

$$W=\frac{g\,F}{G}=\frac{g\,F\,T^2}{2\,S}=\frac{2\,g\,F\,S}{V^2}=\frac{g\,F\,T}{V}=\frac{g\,P\,T^3}{2\,S^2}=\frac{g\,F^2\,T}{2\,P}=\frac{2\,g\,K}{V^2}=\frac{g\,T^2\,K}{2\,S^2}$$

Mean Power in Effects during the Time T, or in the Space S.

$$P=\frac{F\,S}{T}=\frac{g\,F^2\,T}{2\,W}=\frac{2\,W\,S^2}{g\,T^3}=\frac{W\,V^2}{2\,g\,T}=\frac{2\,K}{T}=\frac{T\,K}{2\,S}=\frac{V\,K}{S}=\frac{F\,V^2}{G\,T}$$

Work in Footpounds concentrated in a Moving Body.

$$K=F\,S=\frac{W\,V^2}{2\,g}=\frac{F\,V\,T}{2}=\frac{G\,W\,V\,T}{2\,g}=\frac{F\,G\,T^2}{2}=\frac{g\,F^2\,T^2}{2\,W}=\frac{2\,S\,P}{T}=P\,T.$$

The Body moving in an Inclined Direction of an Angle x.

Applied Constant Force in Pounds.

$$F=W\left(\frac{V}{g\,T}\pm\sin.x\right)=W\left(\frac{2\,S}{g\,T^2}\pm\sin.x\right)=W\left(\frac{V^2}{2\,g\,S}\pm\sin.x\right).$$

Final Velocity in Feet per Seconds when the Force F ceases to act.

$$V=g\,T\left(\frac{F}{W}\mp\sin.x\right)=\frac{2\,S\sin.x}{T}=\sqrt{2\,g\,S\left(\frac{F}{W}\mp\sin.x\right)}.$$

Time of Action in Seconds.

$$T=\frac{W\,V}{g\,(F\mp W\sin.x)}=\sqrt{\frac{2\,W\,S}{g\,(F\mp W\sin.x)}}.$$

Acceleration.

$$G=g\left(\frac{F}{W}\mp\sin.x\right).$$

Space in Feet.

$$S=\frac{g\,T^2}{2}\left(\frac{F}{W}\mp\sin.x\right).$$

Work done by F.

$$K=W\,S\left(\frac{V}{g\,T}\mp\sin.x\right)=\frac{F\,g\,T^2}{2}\left(\frac{F}{W}\mp\sin.x\right).$$

Use the upper sign when the direction of motion rises above the horizon, and the lower sign when the direction of motion dips under the horizon.

Force and Work in Revolving Bodies. Centre of Gyration. Fly-Wheels.

Centre of gyration is a point in revolving bodies in which, if all the revolving matter were there collected, it would obtain equal *angular velocity* from, and sustain equal resistance to, the force that gives it the rotary motion.

The *centre of gyration* in different forms of bodies will be found by the formulas on pages 316 and 317.

$F =$ constant force in pounds, acting to rotate the body as in figs. 249 and 250, or the mean force on a steam-piston.
$r =$ radius in feet upon which the force F acts. For a steam-engine the mean radius will be $r = 0{\cdot}63661 \times$ the radius of the crank, or $0.3183\ S$, when
$S =$ stroke of the steam-piston in feet.
$W =$ weight in pounds of a fly-wheel, or other rotating body.
$x =$ radius of centre gyration in feet.
$T =$ time in seconds in which the force F is applied from the first start, or the time in which the velocity is accelerated.
$N =$ number of revolutions in the time T.
$n =$ number of revolutions per minute.
$K =$ work concentrated in the revolving body.
$f =$ irregularity in a fraction of the mean revolutions n.

For a double-acting single-cylinder engine, the fly-wheel in its regular course of running has an irregular velocity through each revolution. Its smallest velocity is when the crank is at an angle of 40° from the beginning of the stroke, and its greatest velocity when at 40° from the end of the stroke. The larger the fly-wheel is for a given velocity, the more regular will the machinery run. But the fly-wheel may be made so small that its accumulated work cannot carry the machinery around, which will be the case when the irregularity $f = 1$. In ordinary practice make irregularity $f = 0{\cdot}1$ to $0{\cdot}01$.

Example 1.—What force F is required to give a body $W = 3600$ pounds a velocity $n = 76$ revolutions per minute in a time $T = 24$ seconds, the radius of gyration being $x = 12$ feet, and the force F acting on a radius $r = 3$ feet?

$$F = \frac{W x^2 n}{307{\cdot}1\ T r} = \frac{3600 \times 12^2 \times 76}{307{\cdot}1 \times 24 \times 3} = 1779{\cdot}5 \text{ pounds, the answer.}$$

Example 2.—Required the weight W of a fly-wheel for an engine of $D = 36$ inches diameter of cylinder double acting, with steam-pressure $p = 50$ lbs. per sq. in. $S = 6$ feet, the stroke of piston. Area of steam-piston 1017·8 sq. in., and the force $F = 1017{\cdot}8 \times 50 = 50890$ pounds. Radius of gyration $x = 10$ feet, and $n = 48$ revolutions per minute. Assume $f = 0{\cdot}05$.

$$W = \frac{2542\ F S}{n^2 x^2 f} = \frac{2542 \times 50890 \times 6}{48^2 \times 10^2 \times 0{\cdot}05} = 67376{\cdot}2 \text{ pounds, the weight required.}$$

Should the steam be used expansively, the fly-wheel ought to be so much heavier, as the initial pressure is greater than the mean pressure.

The radius of gyration in a fly-wheel, including the arms, can in practice be assumed to be the inner radius of the ring.

Example 3.—What time from the start of engine is required to give the fly-wheel in *Ex.* 2 a velocity of $n = 48$ turns per minute? $r = 0{\cdot}3183\ S = 1{\cdot}9098$ ft.

$$T = \frac{W x^2 n}{307{\cdot}17\ F r} = \frac{67376{\cdot}2 \times 10^2 \times 48}{307{\cdot}17 \times 50890 \times 1{\cdot}9098} = 10{\cdot}85 \text{ seconds.}$$

Example 4.—Let the steam-engine in the preceding examples be applied to a rolling-mill, geared two to one of the rollers. An iron plate is rolled through with $N = 8$ revolutions of the engines, after which the revolutions were found to be reduced to $n_1 = 36$ per minute. Required the work done in rolling the plate; and what time is required for the engine to regain the $n = 48$ revolutions?

Work done by engine, $K = 2\ F S N = 2 \times 67376{\cdot}2 \times 6 \times 8 = 6468115{\cdot}2$ footps.

Work done by fly-wheel,

$$K = \frac{W x^2 (n^2 - n_1^2)}{5866{\cdot}5} = \frac{67376{\cdot}2 \times 10^2 (48^2 - 36^2)}{5866{\cdot}5} = 1157671 \text{ footpounds,}$$

to which add $6468115{\cdot}2 = 7625786{\cdot}2$ footpounds, work consumed in rolling plate.

The time required for the engine to make up the $n = 48$ revolutions will be

$$T = \frac{W x^2 (n - n_1)}{307{\cdot}17\ F r} = \frac{67376{\cdot}2 \times 10^2 (48 - 36)}{307{\cdot}17 \times 50890 \times 1{\cdot}9098} = 2{\cdot}71 \text{ seconds.}$$

Formulas for Accelerated Circular Motion.

Force F, in pounds, acting on the Lever or Radius r, to rotate the Body.

$$F = \frac{W\,x^2\,n}{307{\cdot}49\,T\,r} = \frac{W\,x^2\,N}{2{\cdot}562\,T^2\,r} = \frac{60\,K}{\pi\,r\,n\,T} = \frac{K}{2\,\pi\,r\,N}.$$

Final Revolutions per Minute in the Time T.

$$n = \frac{120\,N}{T} = \frac{307{\cdot}49\,F\,T\,r}{W\,x^2} = \frac{60\,K}{\pi\,r\,T\,F} = \sqrt{\frac{5872{\cdot}2\,K}{W\,x^2}}.$$

Total Number of Revolutions in the Time T.

$$N = \frac{T\,n}{120} = \frac{2{\cdot}562\,F\,T^2\,r}{W\,x^2} = \frac{K}{2\,\pi\,r\,F} = \frac{T}{1{\cdot}565\,x}\sqrt{\frac{K}{W}}.$$

Time of Acceleration, in Seconds, from the Start of Change of Motion.

$$T = \frac{W\,x^2\,n}{307{\cdot}49\,F\,r} = \sqrt{\frac{W\,x^2\,N}{2{\cdot}562\,F\,r}} = \frac{60\,K}{\pi\,r\,n\,F} = \frac{x\sqrt{W\,K}}{4{\cdot}09\,F\,r}.$$

Radius of Gyration, in Feet, of the Revolving Body.

$$x = \sqrt{\frac{307{\cdot}49\,F\,r\,T}{W\,n}} = \sqrt{\frac{2{\cdot}562\,F\,r\,T^2}{W\,N}} = \frac{K\,T}{3{\cdot}9\,N\sqrt{W\,N\,F\,r}} = \frac{334{\cdot}9\,K}{n\sqrt{W\,n\,T\,F\,r}}.$$

Weight, in Pounds, of the Revolving Body.

$$W = \frac{307{\cdot}49\,T\,F\,r}{x^2\,n} = \frac{2{\cdot}562\,T^2\,F\,r}{x^2\,N} = \frac{5872{\cdot}2\,K}{n^2\,x^2} = \frac{K\,T^2}{2{\cdot}452\,x^2\,N^2}.$$

Work in Footpounds, concentrated in a Revolving Body.

$$K = \frac{W\,x^2\,n^2}{5872{\cdot}2} = \frac{2{\cdot}452\,W\,x^2\,N^2}{T^2} = \frac{\pi\,r\,n\,F\,T}{60} = 2\,\pi\,r\,N\,F$$

Fly-Wheels for Steam-Engines.

Fly-Wheel for a Single Acting Steam-Engine for Uniform Work.

$$W = \frac{5872.2\,F\,S}{n^2\,x^2\,f}.\quad n = \frac{76{\cdot}6}{x}\sqrt{\frac{F\,S}{W\,f}}.\quad x = \frac{76{\cdot}6}{n}\sqrt{\frac{F\,S}{W\,f}}.\quad f = \frac{5872{\cdot}2\,F\,S}{x^2\,n^2\,W}.$$

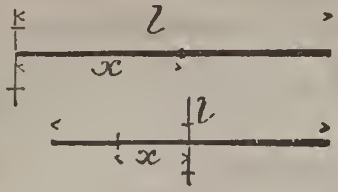
Fly-Wheel for a Double Acting Steam-Engine for Uniform Work.

$$W = \frac{2542\,F\,S}{n^2\,x^2\,f}.\quad n = \frac{50.42}{x}\sqrt{\frac{F\,S}{W\,f}}.\quad x = \frac{50{\cdot}42}{n}\sqrt{\frac{F\,S}{W\,f}}.\quad f = \frac{2542\,F\,S}{x^2\,n^2\,W}.$$

Fly-Wheel for a Double Acting Two-Cylinder Engine for Uniform Work.

$$W = \frac{1172\,F\,S}{n^2\,x^2\,f}.\quad n = \frac{34{\cdot}23}{x}\sqrt{\frac{F\,S}{W\,f}}.\quad x = \frac{34{\cdot}23}{n}\sqrt{\frac{F\,S}{W\,f}}.\quad f = \frac{1172\,F\,S}{x^2\,n^2\,W}.$$

Whenever accurate work for expanding steam is required, consult the *Relative Proportions of the Steam-Engine* (Marks). These formulæ give only rough averages.

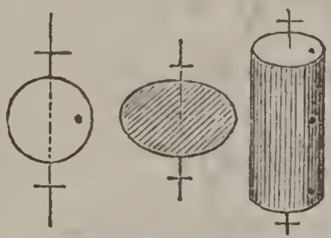


239.

A line or bar.

$$x = 0{\cdot}5773l,$$

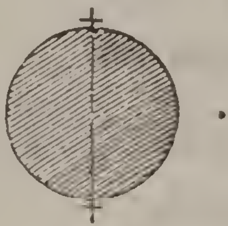
$$x = 0{\cdot}2887l.$$



240.

A circumference round its diameter,
A disk round its centre,
A cylinder round its axis.

$$x = 0{\cdot}7072r.$$



241.

A disk round its diameter.

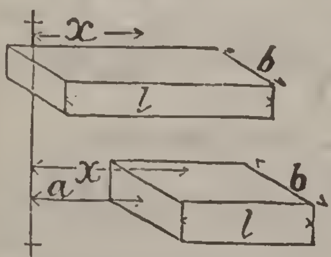
$$x = 0{\cdot}5r.$$

242.

A Sphere round its diameter.

Spherical shell, $x = 0{\cdot}8165r$,

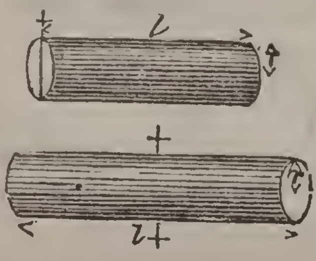
Solid, . . . $x = 0{\cdot}6324r$.



243. *Parallelopipedon.*

$$x = \sqrt{\frac{4l^2+b^2}{12}},$$

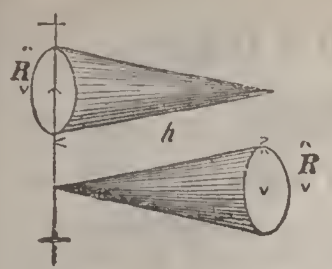
$$x = \sqrt{\frac{4l^2+b^2}{12}+a^2+a\,l}.$$



244. *Cylinder.*

$$x = \sqrt{\frac{4l^2+3r^2}{12}},$$

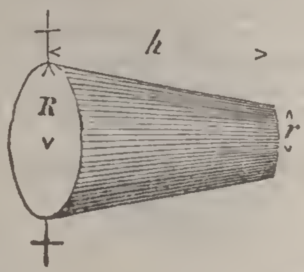
$$x = \sqrt{\frac{l^2+3r^2}{12}}.$$



245. *Cone.*

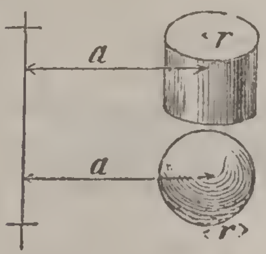
$$x = \sqrt{\frac{2h^2+3R^2}{20}},$$

$$x = \sqrt{\frac{12h^2+3R^2}{20}}.$$



246. *Conic Frustum.*

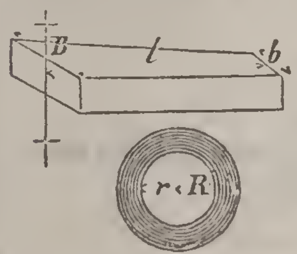
$$x = \sqrt{\frac{h}{10}\left(\frac{R^2+3Rr+Rr^2}{R^2+Rr+r^2}\right)+\frac{3}{20}\left(\frac{R^5-r^5}{R^3-r^3}\right)}$$



247. *Cylinder and Sphere.*

$$x = \sqrt{a^2+\tfrac{1}{2}r^2},$$

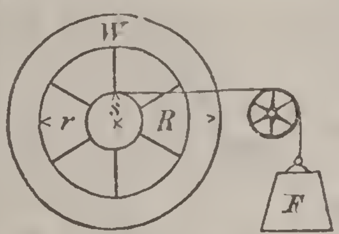
$$x = \sqrt{a^2+\tfrac{2}{5}r^2}.$$



248. *Wedge and Ring.*

$$x = 0{\cdot}204\sqrt{12l^2+B^2+b^2},$$

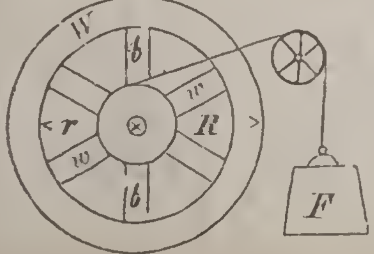
$$x = \sqrt{\frac{R^2+r^2}{2}}.$$



249. *Fly Wheel.*

$$x = \sqrt{\frac{R^2+r^2}{2}},$$

$$F\,G : W\,g = x^2 : s^2.$$



250. *Fly Wheel with Arms.*

$$x^2(W+w) = W\frac{R^2+r^2}{2}+w\frac{4r^2+b^2}{12},$$

$$x = \sqrt{\frac{6W(R^2+r^2)+w(4r^2+b^2)}{12(W+w)}}.$$

CENTRIFUGAL FORCE.

Central Forces are of two kinds, *centrifugal* and *centripetal.*

Centrifugal Force is the resistance which a revolving body offers to being moved in the arc of a circle.

Centripetal Force is that by which a revolving body is attracted or attached to its centre of motion.

The *Centrifugal* and *Centripetal forces* are opposites to each other, and when equal the body revolves in a circle; but when they differ the body will revolve in other curved lines, as the Ellipse, the Parabola, &c., according to the nature of the difference in the forces. If the *centrifugal force* is *o* while the other is acting, the body will move straight to the centre of motion; and if the *centripetal force* is *o* while the other is acting, the body will depart from the circle in a straight line, tangent to the circle in the point where the *centripetal force* ceased to act. The *central forces* are distinct from the force that has set the body in motion.

If the *centrifugal force* be made use of to produce an effect, such effect will be at the expense of the one producing the rotary motion.

Notation.

F = *Centrifugal force*, in pounds.
W = the weight of the revolving body, in pounds.
v = *Velocity* of the revolving body, in feet per second.
R = *Radius* of the circle in which the body revolves, in feet.
n = number of revolutions per minute.

Example 1. Required the *centrifugal force* of a body weighing 63 pounds, and making 163 revolutions per minute, in a circle of 4 feet, 4 inches radius?

$$F = \frac{W R n^2}{2933} = \frac{63 \times 4{\cdot}33 \times 163^2}{2933} = 2475 \text{ pounds.}$$

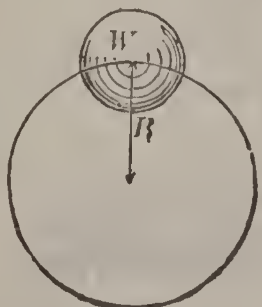
Example 2. A Railroad train runs 43 miles per hour on a curved track of 115 feet radii. What should be the obliquity of the track?

$$\tan. x = \frac{\text{Miles}^2}{69R} = \frac{43^2}{69 \times 115} = 0{\cdot}233,$$

or $x = 13° 10'$, the obliquity of the track.

Example 3. A governor having its arms l = 1 foot, 6 inches, how many revolutions must it make per minute to form an angle $x = 30°$?

$$n = \frac{54{\cdot}16}{\sqrt{1{\cdot}5 \times \cos. 30°}} = 47{\cdot}5 \text{ revolutions per minute.}$$



227.

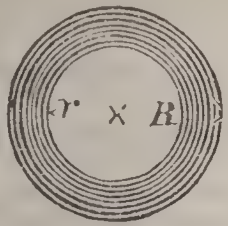
$$F = \frac{W v^2}{g\, R} = \frac{W v^2}{32{\cdot}2 R}, \quad 1,$$

$$F = \frac{4 W R \pi^2 n^2}{60^2 g} = \frac{W R n^2}{2936}, \quad 2,$$

$$W = \frac{F g R}{v^2} = \frac{2936 F}{R n^2}, \quad 3,$$

$$R = \frac{W v^2}{F g} = \frac{2936 F}{W n^2}, \quad 4,$$

$$n = \sqrt{\frac{2936\ F}{W R}}, \quad v = \sqrt{\frac{F R g}{W}}, \quad 5,$$



228.

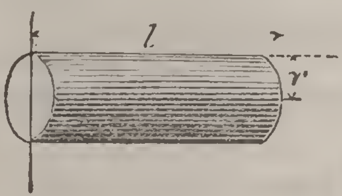
Centrifugal force of a ring.

$$F = \frac{W n^2 \sqrt{R^2 + r^2}}{4153}.$$

229.

Centrifugal force of a grinding stone, circle-plane, cylinder, rotating round its centre.

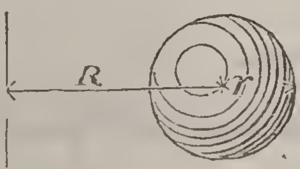
$$F = \frac{W R n^2}{4153}.$$



230.

Centrifugal force of a cylinder rotating round the diameter of its base.

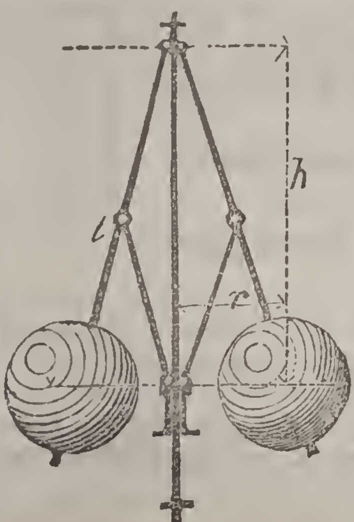
$$F = \frac{W n^2 l}{5872}.$$



231.

Centrifugal force of a ball,

$$F = \frac{W n^2 R}{2936}.$$



232.

Governor.

$$n = \frac{60}{2\pi}\sqrt{\frac{g}{h}} = \frac{54{\cdot}16}{\sqrt{h}} = \frac{54{\cdot}16}{\sqrt{l \cos x}},$$

$$h = \frac{2936}{n^2}, \quad l = \frac{2936}{n^2 \cos.x} = \frac{h}{\cos.x},$$

$$\cos.x = \frac{2936}{n^2 l} = \frac{h}{l}, \quad r = \sqrt{l^2 - h^2}.$$

PENDULUM.

Simple Pendulum is a material point under the action of gravitation, and suspended at a fixed point by a line of no weight.

Compound Pendulum is a suspended rod and body of sensible magnitude, fixed as the simple pendulum.

Centre of Oscillation is a point at which if all the matter in the compound pendulum were there collected, it would make a simple pendulum oscillate at the same times.

Angle of Oscillation is the space a pendulum describes when in motion.

The velocity of an oscillating body through the vertical position, is equal to the velocity a body would obtain by falling vertically the distance *versed sine* of half the angle of oscillation.

Notation.

l = *length* of the simple pendulum, or the distance between the centre of suspension, and centre of oscillation in inches.

t = *time* in seconds for n oscillations.

n = number of single oscillations in the time t.

Example 1. Required the length of a pendulum that will vibrate seconds? here $n = 1$, and $t = 1''$.

$$l = 39{\cdot}109 \frac{t^2}{n^2} = 39{\cdot}109 \text{ inches, the length of a pendulum for seconds.}$$

Example 2. Require the length of a pendulum that will make 180 vibrations per minute? here $t = 60''$ and $n = 180$.

$$l = \frac{39{\cdot}109 t^2}{n^2} = \frac{39{\cdot}109 \times 60^2}{180^2} = 4{\cdot}346 \text{ inches.}$$

Example 3. How many vibrations will a pendulum of 25 inches length make in 8 seconds?

$$n = \frac{6{\cdot}254 t}{\sqrt{l}} = \frac{6{\cdot}254 \times 8}{\sqrt{25}} = 10 \text{ vibrations.}$$

Example 4. A pendulum is 137·67 inches long and makes 8 vibrations in 15 seconds. Required the unit or accelleratrix $g = ?$

$$g = \frac{0{\cdot}8225 l\, n^2}{t^2} = \frac{0{\cdot}8225 \times 137{\cdot}67 \times 8^2}{15^2} = 32{\cdot}209.$$

Example 5. A compound pendulum of two iron balls P and Q, having the centre of suspension between themselves: see Fig. 238. $P = 38$ pounds, $Q = 12$ pounds, $a = 25$ inches, and $b = 18$ inches. How long is the simple pendulum, and how many vibrations will the pendulum make in 10 seconds?

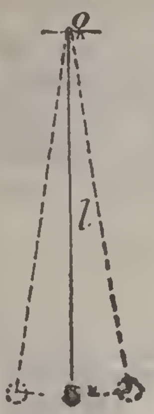
$$x = \frac{a\,P - b\,Q}{P + Q} = \frac{25 \times 38 - 18 \times 12}{38 + 12} = 14{\cdot}68 \text{ inches.}$$

$$l = \frac{a^2 P + b^2 Q}{x(P + Q)} = \frac{25^2 \times 38 + 18^2 \times 12}{14{\cdot}68(38 + 12)} = 37{\cdot}68 \text{ inches,}$$

the length of the single pendulum.

$$n = \frac{6{\cdot}254 t}{\sqrt{l}} = \frac{6{\cdot}254 \times 10}{\sqrt{37{\cdot}68}} = 10{\cdot}193 \text{ vibrations in 10 seconds.}$$

If a compound pendulum is hung up at its centre of oscillation, the former centre of suspension will be the centre of oscillation, and the pendulum will oscillate the same time.



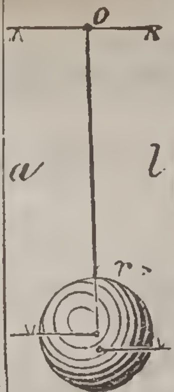
233.

Simple Pendulum.

$$l = \frac{12g\, t^2}{\pi^2 n^2} = \frac{39{\cdot}1 t^2}{n^2},$$

$$t = \frac{n\sqrt{l}}{6.25},$$

$$n = \frac{6.254 t}{\sqrt{l}},$$



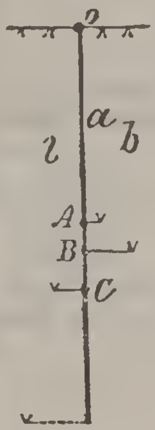
236.

$$g = \frac{l\pi^2 n^2}{12 t^2},$$

$$g = \frac{0{\cdot}8225\, l\, n^2}{t^2},$$

o = centre of suspension.

$$l = a + \frac{2r^2}{5a}.$$



234.

A = centre of gravity.

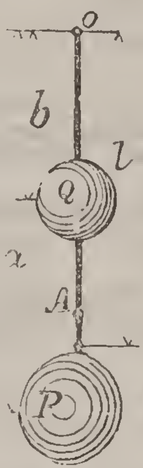
B = centre of gyration.

C = centre of oscillation.

$$a : b = b : l,$$

$$b = \sqrt{al} = 1{\cdot}414a,$$

$$l = 1\tfrac{1}{3}a.$$

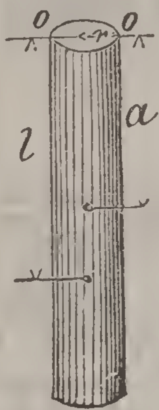


237.

$$l = \frac{a^2 P + b^2 Q}{a P + b Q}.$$

P and Q expressed in pounds, or cubic contents.

Connecting wire neglected.



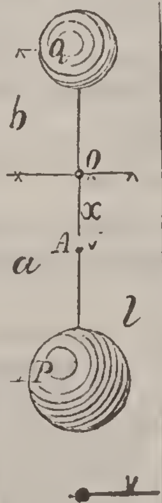
235.

Compound Pendulum.

r = radius of cylinder.

$$l = \frac{16a^2 + 3r^2}{12a},$$

$$l = \frac{4a}{3} + \frac{r^2}{4a}.$$



238.

$$x = \frac{a P - b Q}{P + Q}.$$

$$l = \frac{a^2 P + b^2 Q}{x(P + Q)}.$$

Connecting wire neglected.

Length of a Pendulum vibrating seconds at the level of the sea, in various places.

At the Equator, lat. 0° 0′ 0″	39·0152 inches.
" Washington, lat. 38° 53′ 23″	39·0958 "
" New York, lat. 40° 42′ 40″	39·1017 "
" London, lat. 51° 31′	39·1393 "
" lat. 45°	39·1270 "
" Stockholm, lat. 59° 21′ 30″	39·1845 "

l = 39·127 — 0·09982 cos.2 lat. for seconds.

COLLISION OF BODIES IN MOTION.

When bodies in motion come in collision with each other, the sum of their concentrated momentum will be the same after the collision as before, but their velocities and sometimes their directions of motion will differ.

On the accompanying page the bodies are supposed to move in the same straight line, and the formula illustrates the consequences after collision.

Notation.

M and m = weight of the bodies in pounds.
V and v = their respective velocities in feet per second.
V' and v' = respective velocities of the bodies after impact.
K and k = coefficient of elasticity, which for perfectly hard bodies $k=0$ and for perfectly elastic bodies $k=1$, therefore the elastic coefficient will always be between 0 and 1. When the bodies are perfectly hard their velocities after impact will be common.

$$\text{For } M, \quad K=\frac{MV}{M(V-V')}, \qquad \text{For } m, \quad k=\frac{mv}{m(v-V')}.$$

Example 1. Fig 191. The non-elastic body weighs $M=25$ pounds, and moves at a velocity $V=12$ feet per second; $m=16$ pounds, and $v=9$. Required the bodies' common velocities, $v'=?$ after impact.

$$v'=\frac{MV+mv}{M+m}=\frac{25\times12+16\times9}{25+16}=10{\cdot}83 \text{ feet per second.}$$

Example 2. Fig. 195. The perfect elastic body $M=84$ pounds, $V=18$, $m=48$, and $v=27$. Required the velocity $V'=?$ after impact with the body m.

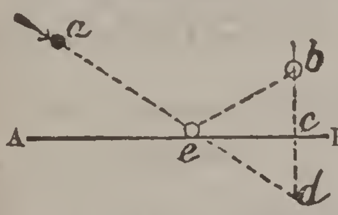
$$V=\frac{18\,(84-48)-2\times48\times27}{84+48}=-23{\cdot}64.$$

the negative sign denotes that the body will return after the collision with a velocity of 23·63 feet per second.

Example 3. Fig. 196. The partly elastic body $M=38$ pounds and $V=79$ feet per second, will strike the body in rest $m=24$ pounds; what will be the velocity $v'=?$ of the body m, its elasticity being $k'=0{\cdot}6$.

$$v'=\frac{79\times38\,(1+0{\cdot}6)}{38+24}=70{\cdot}6 \text{ feet per second.}$$

When a moving body strikes a stationary elastic plane, its course of departure from the plane will be equal to its course of incidence.



Problem. A body in a is to strike the plane AB so that it will depart to the given point b; required its course of incident from a?

Draw bd, at right angles through AB, make $cd=bc$ join a and d; then ad is the course of incident, and cb, the course of departure, and the body will strike in e.

In this article the common error of most text-books arising from ignorance of the true meaning of momentum MV is shared by the author. Example 1 should be written:

$$V'^2=\frac{MV^2+mv^2}{M+m}=\frac{25\times144+16\times81}{25+16}=119{\cdot}41. \qquad V'=10{\cdot}93.$$

This remark holds with regard to all the formulæ following, which are allowed to remain with this caution. De Volson Wood's *Elementary Mechanics* is an elaborate example of this form of error.—W. D. M.

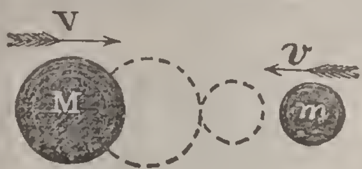
The bodies perfectly hard.

191.

The bodies move in the same direction.

$$v'(M+m) = MV+mv,$$

$$v' = \frac{MV+mv}{M+m}.$$



192.

The bodies move in opposite directions.

$$v'(M+m) = MV-mv,$$

$$v' = \frac{MV-mv}{M+m}.$$

193.

Only one body in motion.

$$v'(M+m) = MV,$$

$$v' = \frac{MV}{M+m}.$$

The bodies elastic.

194.

The bodies move in the same direction.

$$V' = \frac{V(M-Km)+vm(1+K)}{M+m},$$

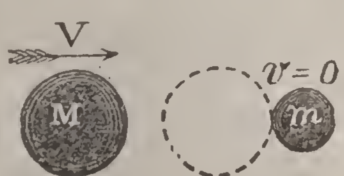
$$v' = \frac{MV(1+k)+v(m-kM)}{M+m}.$$

195.

The bodies move in opposite directions.

$$V' = \frac{V(M-Km)-vm(1+K)}{M+m},$$

$$v' = \frac{MV(1+k)-v(m-kM)}{M+m.}$$



196.

Only one body in motion.

$$V' = \frac{V(M-Km)}{M+m},$$

$$v' = \frac{VM(1+k)}{M+m}.$$

CENTRE OF PERCUSSION.

Centre of Percussion is a point in which the momentums of a moving body are concentrated. *Centre of Percussion* is the same as *centre of oscillation*, and to be calculated by the same formulas.

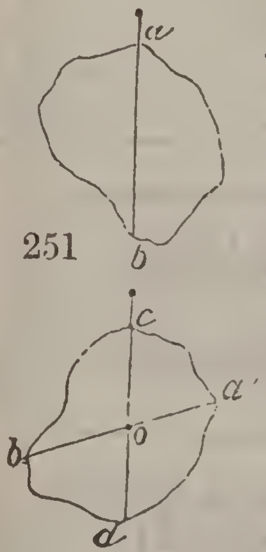
Take an iron bar in one hand, and strike it heavily upon a sharp edge; if the *centre of percussion* of the bar strikes upon the edge, the whole momentum will there be discharged, but if it strikes at a distance from the *centre of percussion* a part of the momentum will be discharged in the hand, and a shock will be felt.

It is sometimes of great importance to properly place the *centre of percussion*. If it is dislocated, the moving body not only fails to properly transmit its effect, but the lost momentum acts to wear out the machinery.

CENTRE OF GRAVITY.

Centre of Gravity is a point around which the moments of all elements (under the action of the force of gravity) in a body, or system of bodies, are equally divided.

251



A body or system of bodies suspended at its *centre of gravity*, will be in equilibrium in all positions.

A body or system of bodies, suspended in a point *out* of its *centre* of *gravity*, will hang with its *centre of gravity* vertical under the point of suspension.

A body or system of bodies suspended successively at two points out of its *centre of gravity*, the two vertical lines through the points of suspension will meet in the *centre of gravity*; thus if a plane be hung up in two different positions, the vertical lines a, b, and c, d, will meet in the *centre of gravity* o.

z = distance to the centre of gravity as noted in the figures.

Example 1. The radius of a circle being 3 feet, how far is its centre of gravity from the centre of the half circle?

$$z = 0{\cdot}6367 \times 3 = 1{\cdot}91 \text{ feet.}$$

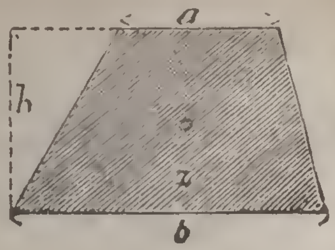
Example 2. How far from the bottom of a cylindric shell, open at one end, is its centre of gravity? The cylinder is 4 feet long, radius $r = 0{\cdot}8$ feet.

$$z = \frac{h}{r+2h} = \frac{4}{0{\cdot}8+2\times4} = 0{\cdot}625 \text{ feet.}$$

Example 3. Fig. 264. An irregular figure weighing $P = 138$ pounds, is suspended between a fulcrum and a weight, $l = 5{\cdot}6$ feet, $W = 57$ pounds. Required the distance to the centre of gravity $z = ?$

$$z = \frac{57\times5{\cdot}6}{138} = 2{\cdot}31 \text{ feet.}$$

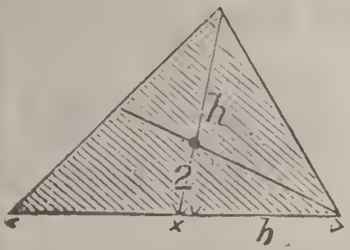
252.



Quadrangle.—a and b parallel.

$$z = \frac{h}{2} - \frac{h}{6}\left(\frac{b-a}{b+a}\right).$$

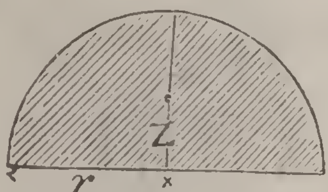
253.



Triangle.

$$z = \frac{h}{3}.$$

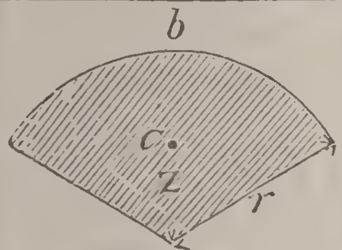
254.



Half a circle plane or Elliptic ***plane.***

$$z = 0{\cdot}4244r.$$

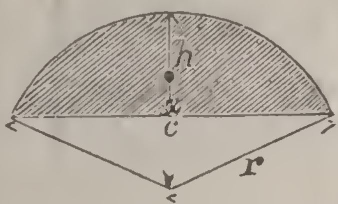
255.



Circle sector.

$$z = \frac{2c\,r}{3b}.$$

256.

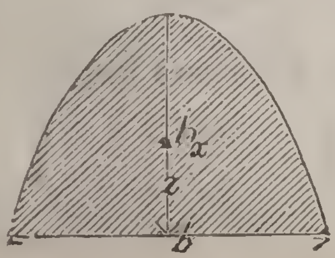


Circle Segment. **a** = *area.*

$$z = \frac{c^3}{12\mathbf{a}}.$$

$$x = h + z - r.$$

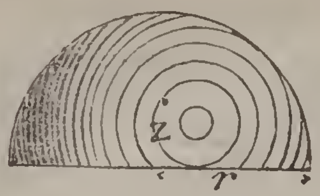
257.



Parabola.

$$z = \frac{2h}{5}.$$

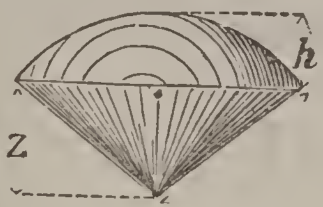
For half a Parabola $x = \frac{1}{5}b$.



258.

Half Sphere.

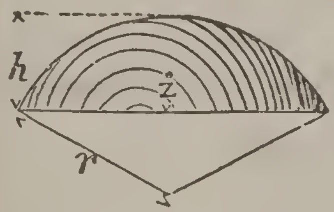
Convex surface . . . . $z = \frac{1}{2}r$.
Solid . . . . $z = \frac{3}{8}r$.



259.

Spherical Sector.

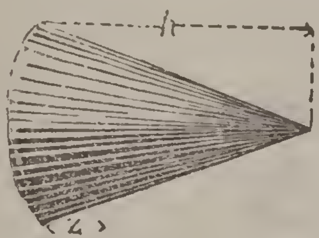
Solid, $z = \frac{3}{4}\left(r - \frac{h}{2}\right)$.



260. *Spherical Segment.*

Convex surface $z = \frac{h}{2}$,

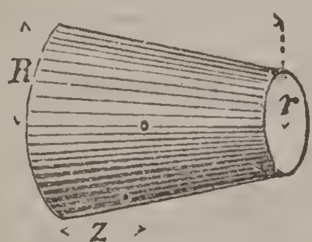
Solid $z = \frac{h}{4} \cdot \left[\frac{4r-h}{3r-h}.\right]$



261. *Cone.*

Convex surface $z = \frac{h}{3}$,

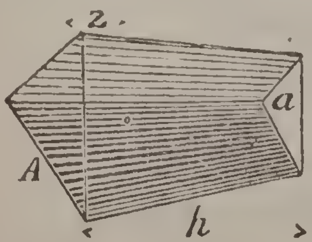
Solid $z = \frac{h}{4}$.



262. *Conic Frustum.*

Con. sur. $z = \frac{h}{2} - \frac{h}{6}\left[\frac{R-r}{R+r}.\right]$

Solid $z = \frac{h}{4}.\left[\frac{R^2+r(2R+3r)}{R^2+r(R+r)}.\right]$



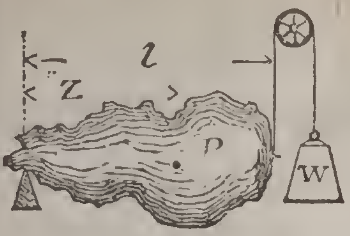
263.

Pyramidic Frustum.

A and a = area of the two bases.

Solid $z = \frac{h}{4}\left[\frac{A+3a+2\sqrt{A\,a}}{A+a+\sqrt{A\,a}}.\right]$

264.

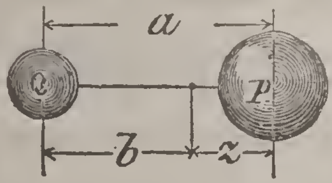


Irregular Figure.

$$P : W = l : z.$$

$$z = \frac{W\,l}{P}.$$

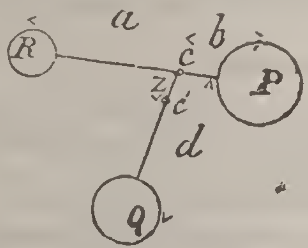
265.



To find the Centre of Gravity of two bodies, P and Q.

$$z = \frac{Q\,a}{P+Q}, \qquad b = \frac{P\,a}{P+Q}.$$

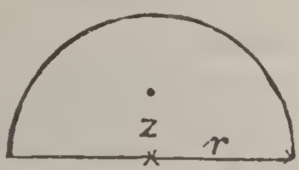
266.



To find the Centre of Gravity of a system of bodies.

$$b = \frac{R\,a}{P+R}, \qquad z = \frac{Q\,d}{P+R+Q}.$$

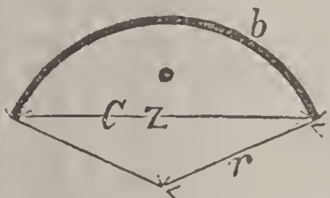
267.



Half a circumference of a Circle or Ellipse.

$$z = 0{\cdot}4244r.$$

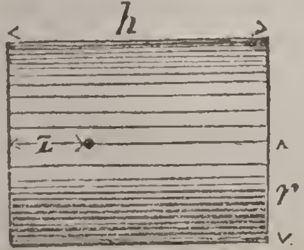
268.



Circle arc or Elliptic arc.

$$z = \frac{c\,r}{b} = \frac{c(c^2+4h^2)}{8h\,b}.$$

269.



Cylindric Surface with a bottom in one end.

$$z = \frac{h^2}{r+2h}.$$

SPECIFIC GRAVITY.

Specific Gravity is the comparative density of substances. The unit for measuring the specific gravity is assumed to be the density of rain or of distilled water.

One cubic foot of distilled water weighs 1000 ounces, or 62·5 pounds avoirdupois.

To Find the Weight of a Body.

RULE 1. Multiply the contents of the body in cubic feet by 62·5, and the product by its specific gravity, will be the weight of the body in pounds avoirdupois.

RULE 2. Multiply the contents of the body in cubic inches by 0·03616, and the product by its specific gravity, will be the weight of the body in pounds avoirdupois.

RULE 3. Divide the specific gravity by 0·016 and the quotient is the weight of a cubic foot.

Example 1. A bottle full of mercury is 3 inches, inside diameter, and 6 inches high. How much mercury is there in the bottle in pounds?

One cubic inch of mercury weighs 0·491 pounds, and by the formula for Fig. 84 we have the

$$\text{weight} = 0{\cdot}491 \times 0{\cdot}785 \times 3^2 \times 6 = 20{\cdot}85 \text{ pounds.}$$

Example 2. Required the weight of a cone of cast iron, diameter at the base $d = 1{\cdot}33$ feet, height $h = 4$ feet? One cubic foot of cast iron weighs 450·5 pounds, and by formula for Fig. 82 we have the

$$\text{weight} = 450{\cdot}5 \times 0{\cdot}2616 \times 1{\cdot}33^2 \times 4 = 834 \text{ pounds.}$$

Example 3. The section area of the lower hold in a steamboat is 245 square feet; how much space must be taken in the length of the hold for 131 tons of anthracite coal?

Anthracite coal weighs 42·3 cubic feet per ton.

$$\text{length} = \frac{42{\cdot}3 \times 131}{245} = 22{\cdot}6 \text{ feet, the space required.}$$

Weight and Bulk of Substances.

Names of Substances.	*Cubic foot in pounds.*	*Cubic feet per ton.*	*Names of Substances.*	*Cubic foot in pounds.*	*Cubic feet per ton.*
Cast iron, - -	450·5	4·97	Sand, - - -	94·5	23·7
Wrought iron, -	486·6	4·60	Granite, - - -	165	13·5
Steel, - - -	489·8	4·57	Earth, loose, - -	78·6	28·5
Copper, - - -	555·	4·03	Water, salt, (sea) -	64·3	34·8
Lead, - - -	707·7	3·16	" fresh - -	62·5	35·9
Brass, - - -	537·7	4·16	Ice, - - - -	58·08	38·56
Tin, - - - -	456	4·91	Gold, - - -	1013	2·21
Pine, white - -	29·56	75·6	Silver, - - -	551	4·07
" yellow, - -	33·81	66·2	Coal, Anthracite -	53	42·3
Mahogany, - -	66·4	33·8	" Bituminous -	50	44·8
Marble, common, -	165	13·6	" Cumberland -	53	42·3
Mill-stone, - -	130	17·2	" Charcoal -	18·2	123
Oak, live - - -	70	32·0	Coke, Midlothian -	32·70	68·5
" white, - -	45.2	49·5	" Cumberland -	31·57	70·9
Clay, - - -	101·3	22·1	" Natural Virginia	46·64	48·3
Cotton Bales, - -			Conventional rate of		
Brick, - - -	100	22·4	Stone coal, 28 bushels		
Plaster Paris, - -	105	21·3	(5 pecks) = 1 ton, -		43·56

To Find the Specific Gravity.

W = weight of a body in the air.
w = weight of the body (heavier than water) immersed in water.
S = specific gravity of the body. Then,

$$W - w : W = 1 : S. \quad S = \frac{W}{W - w}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 1,$$

Example 4. Required the specific gravity of a piece of iron-ore weighing 6·345 pounds in the air, and 4·935 pounds in water, $S = ?$

$$S = \frac{6{\cdot}345}{6{\cdot}345 - 4{\cdot}935} = 4{\cdot}5 \text{ the specific gravity.}$$

When the body is lighter than water, annex to it a heavier body that is able to sink the lighter one.

S = specific gravity of the heavier annexed body.
s = specific gravity of the lighter body.
W = weight of the two bodies in air.
w = weight of the two bodies in water.
V = weight of the heavier body in air.
v = weight of the lighter body in air.

$$s = \frac{v}{W - w - \frac{V}{S}}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 2,$$

Example 5. To a piece of wood, which weighs $v = 14$ pounds in the air, is fastened a piece of cast-iron $V = 28$ pounds; the two bodies together weigh $w = 11{\cdot}7$ pounds in water. Required the specific gravity of the wood?

$W = V + v = 28 + 14 = 42$ pounds.
$S = 7{\cdot}2$ specific gravity of cast-iron.

Formula 2. $$S = \frac{14}{42 - 11{\cdot}7 - \frac{28}{7{\cdot}2}} = 0{\cdot}529, \text{ the specific}$$

gravity of the wood, (Poplar White Spanish.)

A simple way to obtain the specific gravity of wood is to make it into a rod and place it vertically in water; then, when in equilibrium, the immersed end is to the whole rod as the specific gravity is to 1.

Example 6. A cylinder of wood is 6 feet, 3 inches long, when immersed vertically in water it will sink 3 feet, 9 inches by its own weight. Required its specific gravity.

$$3{\cdot}75 : 6{\cdot}25 = S : 1, \qquad S = \frac{3{\cdot}75}{6{\cdot}25} = 0{\cdot}600.$$

To discover the Adulteration in Metals. or to find the proportions of two Ingredients in a Compound.

$$V = \frac{W - s(W - w)}{1 - \frac{s}{S}}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 3,$$

Example 7. A metal compounded of silver and gold weighs $W = 6$ pounds in the air, and in water $w = 5{\cdot}636$ pounds. Require the proportions of silver and gold?

$S = 19{\cdot}36$ specific gravity of gold.
$s = 10{\cdot}51$ specific gravity of silver.

$$\text{weight } V = \frac{6 - 10.51(6 - 5.636)}{1 - \frac{10{\cdot}51}{19{\cdot}36}} = 4{\cdot}755 \text{ pounds of gold.}$$

and 1·245 pounds of silver.

Names of Substances.	*Specific gravity.*	*Weight per cubic inch.*
Metals.		*lbs.*
Platinum, rolled	22·669	·798
" wire,	21·042	·761
" hammered,	20·337	·736
" purified,	19·50	·706
" crude, grains	15·602	·565
Gold, hammered	19·361	·700
" pure cast	19·258	·697
" 22 carats fine	17·486	·733
" 20 " "	15·702	·568
Mercury, solid at — 40°	15·632	·566
" at +32° Fahr.	13·619	·493
" " 60° "	13·580	·491
" " 212° "	13·375	·484
Lead, pure	11·330	·410
" hammered	11·388	·412
Silver, hammered	10·511	·381
" pure	10·474	·379
Bismuth,	9·823	·355
Red Lead,	8·940	·324
Cinnabar,	8·098	·293
Manganese,	8·030	·290
Copper, wire and rolled	8·878	·321
" pure	8·788	·318
Bronze, gun metal	8·700	·315
Brass, common	7·820	·282
Steel, cast steel	7·919	·286
" common soft	7·833	·283
" hardened & temp.	7·818	·283
Iron, pure	7·768	·281
" wrought and rolled	7·780	·282
" hammered	7·789	·282
" cast-iron	7·207	·261
Tin, from Böhmen	7·312	·265
" English	7·291	·264
Zinc, rolled	7·191	·260
" cast	6·861	·248
Antimony,	6·712	·244
Aluminium	2·5	0·09
Arsenic,	5·763	·208
Stones and Earths.		
Topaz, oriental	4·011	·145
Emery,	4·000	·144
Diamond,	3·521	·127
Limestone, green	3·180	·115
" white	3·156	·114
Asbestos, starry	3·073	·111
Glass, flint	2·933	·106
" white	2·892	·104
" bottle	2·732	·0987
" green	2·642	·0954
Marble, Parian	2·838	·103
" African	2·708	·0978
" Egyptian	2·668	·0964
Mica,	2·800	·1000
Hone, white razor	2·838	·104
Chalk,	2·784	·100
Porphyry,	2·765	·0999
Spar, green	2·704	·0976
" blue	2·693	·0971

Names of Substances.	*Specific gravity.*	*Weight per cubic inch.*
		lbs.
Alabaster, white	2·730	·0987
" yellow	2·699	·0974
Coral, red	2·700	·0974
Granite, Susquehanna	2·704	·0976
" Quincy	2·652	·0958
" Patapsco	2·640	·0954
" Scotch	2·625	·0948
Marble, white Italian	2·708	·0978
" common	2·686	·0968
Talc, black	2·900	·0105
Quartz,	2·660	·0962
Slate,	2·672	·0965
Pearl, oriental	2·650	·0957
Shale,	2·600	·0940
Flint, white	2·594	·0936
" black	2·582	·0933
Stone, common	2·520	·0910
" Bristol	2·510	·0906
" Mill	2·484	·0897
" Paving	2·416	·0873
Gypsum, opaque	2·168	·0783
Grindstone,	2·143	·0775
Salt, common	2·130	·0770
Saltpetre,	2·090	·0755
Sulphur, native	2·033	·0735
Common soil,	1·984	·0717
Rotten stone,	1·981	·0416
Clay,	1·930	·0698
Brick,	1·900	·0686
Nitre,	1·900	·0636
Plaster Paris,	1·872 2·473	·0677 ·0894
Ivory,	1·822	·0659
Sand,	1·800	·0651
Phosphorus,	1·770	·0640
Borax,	1·714	·0620
Coal, Anthracite	1·640 1·436	·0593 ·0592
" Maryland	1·355	·0490
" Scotch	1·300	·0470
" New Castle	1·270	·0460
" Bituminous	1·270	·0460
Charcoal, triturated	1·380	·0500
Earth, loose	1·500	·0542
Amber,	1·078	·0387
Pimstone,	1·647	·0596
Lime, quick	0·804	·0291
Charcoal,	0·441	·0160
Woods (Dry.)		
Alder,	·800	·0289
Apple-tree,	·793	·0287
Ash, the trunk	·845	·0306
Bay-tree,	·822	·0297
Beech,	·852	·0308
Box, French	·912	·0330
" Dutch	1·328	·0480
" Brazilian red	1·031	·0373
Cedar, wild	·596	·0219
" Palestine	·613	·0222

Names of Substances.	Specific gravity.	Weight per cubic inch.
Cedar, Indian - -	1·315	·0476
" American	·561	·0203
Citron, - - -	·726	·0263
Cocoa-wood, - -	1·040	·0376
Cherry-tree, - -	·715	·0259
Cork, - - -	·240	·0087
Cypress, Spanish -	·644	·0233
Ebony, American -	1·331	·0481
" Indian -	1·209	·0437
Elder-tree, - -	·695	·0252
Elm, trunk of - -	·671	·0243
Filbert-tree, - -	·600	·0217
Fir, male - - -	·550	·0199
" female - -	·498	·0180
Hazel, - - -	·600	·0217
Jasmine, Spanish -	·770	·0279
Juniper-tree, - -	·556	·0201
Lemon-tree, - -	·703	·0254
Lignum-vitæ, - -	1·333	·0482
Linden-tree, - -	·604	·0219
Log-wood, -	·913	·0331
Mastic-tree - -	·849	·0307
Mahogany, - -	1·063	·0385
Maple, - - -	·750	·0271
Medlar, - - -	·944	·0342
Mulberry - -	·897	·0324
Oak, heart of, 60 old	1·170	·0423
Orange-tree, - -	·705	.0255
Pear-tree, - -	·661	·0239
Pomegranate-tree, -	1·354	·0490
Poplar, - - -	·383	·0138
" white Spanish	·529	·0191
Plum-tree, - -	·785	·0284
Quince-tree, - -	·705	·0255
Sassafras, - -	·482	·0174
Spruce, - - -	·500	·0181
" old - -	·460	·0166
Pine, yellow - -	·660	·0239
" white - -	·554	·0200
Vine, - - -	1·327	·0480
Walnut, - - -	·671	·0243
Yew, Dutch - -	·788	·0285
" Spanish - -	·807	·0292
Liquids.		
Acid, Acetic - -	1·062	·0384
" Nitric -	1·217	·0440
" Sulphuric -	1·841	·0666
" Muriatic -	1·200	·0434
" Fluoric - -	1·500	·0542
" Phosphoric -	1·558	·0563
Alcohol, commercial	·833	·0301
" pure -	·792	·0287
Ammoniac, liquid -	·897	·0324
Beer, lager - -	1·034	·0374
Champagne, - -	9·97	·0360
Cider, - - -	1·018	·0361
Ether, sulphuric -	·739	·0267
Egg, - - - -	1·090	·0394
Honey, - - -	1·450	·0524
Human blood -	1·054	·0381
Milk, - - -	1·032	·0373

Names of Substances.	Specific gravity.	Weight per cubic inch.
Oil, Linseed - -	·940	·0340
" Olive - - -	·915	·0331
" Turpentine -	·870	·0314
" Whale - -	·932	·0337
Proof Spirit, - -	·925	·0334
Vinegar, - - -	1·080	·0390
Water, distilled	1·000	·0361
" Sea - -	1·030	·0371
" Dead sea -	1·240	·0448
Wine, - - -	·992	·0359
" Port - -	·997	·0361
Miscellaneous.		
Asphaltum, - -	·905 1·650	·0327 0597
Beeswax, - - -	·965	·0349
Butter, - - -	·942	·0341
Camphor, - -	·988	·0357
India rubber, - -	·933	0338
Fat of Beef, - -	·923	·0334
" Hogs, - -	·936	·0338
" Mutton, -	·923	·0334
Gamboge, - -	1·222	·0442
Gunpowder, loose -	·900	·0325
" shaken	1·000	·0361
" solid -	1·550 1·800	·0561 ·0650
Gum Arabic, -	1·452	·0525
Indigo, - - -	1·009	·0365
Lard, - - -	·947	·0343
Mastic, - - -	1·074	·0388
Spermaceti, - -	·943	·0341
Sugar, - - -	1·605	·0580
Tallow, sheep - -	·924	·0334
" calf - -	·934	·0338
" ox, - -	·923	·0334
Atmospheric air, -	·0012	43
Gases. Vapours.		*Weight cub. ft. grains.*
Atmospheric air, -	1·000	527·0
Ammoniacal gas, -	·500	263·7
Carbonic acid, - -	1·527	805·3
Carbonic oxid, -	·972	512·7
Carburetted hydrogen,	·972	512·7
Chlorine, - - -	2·500	1316
Chlorocarbonous acid,	3·472	1828
Chloroprussic acid,	2·152	1134
Fluoboric acid,	2·371	1250
Hydriodic acid, -	4·346	2290
Hydrogen, - -	·069	36·33
Oxygen, - - -	1·104	581·8
Sulphuretted hydrogen,	1·777	9370
Nitrogen, - -	·972	512·0
Vapour of Alcohol,	1·613	851·0
" turpen'e spir.,	5·013	2642
" water, -	·623	328·0
Smoke of bitumin. coal,	·102	53·80
" wood, -	·90	474·0
Steam at 212° - -	·488	257·3

ALLOYS.

A = Antimony, *B* = Bismuth, *C* = Copper, *G* = Gold, *I* = Iron, *L* = Lead, *N* = Nickel, *S* = Silver, *T* = Tin, and *Z* = Zinc.

Name.	Alloy.
Brass, common yellow,	2*C*, 1*Z*.
Brass, to be rolled,	32*C*, 10*Z*, 1.5*T*.
Brass castings, com.,	20*C*, 1.25*Z*, 2.5*T*.
" " hard,	25*C*, 2*Z*, 4.5*T*.
Brass propellers,	8*C*, 0.5*Z*, 1*T*.
Gun-metal,	8*C*, 1*T*.
Copper-flanges,	9*C*, 1*Z*, 0.26*T*.
Muntz's metal,	6*C*, 4*Z*.
Statuary,	91.4*C*, 5.53*Z*, 1.7*T*, 1.37*L*.
German Silver,	2*C*, 7.9*N*, 6.3*Z*, 6.5*I*.
Britannia metal,	50*A*, 25*T*, 25*B*.
Chinese Silver,	65.1*C*, 19.3*Z*, 13*N*, 2.58*S*, 12*I*.
Chi. wht. Copper,	20.2*C*, 12.7*Z*, 1.3*T*, 15.8*N*.
Medals,	100*C*, 8*Z*.
Pinchbeck,	5*C*, 1*Z*.
Babbitt's metal,	25*T*, 2*A*, 0.5 *C*.
Bell metal, large,	3*C*, 1*T*.
" " small,	4*C*, 1*T*.
Chinese gongs,	40.5*C*, 9.2*T*.
Telescope mirrors,	33.3*C*, 16.7*T*.
White metal, ord.,	3.7*C*, 3.7*Z*, 14.2*T*, 28.4*A*.
" " hard,	35*C*, 13*Z*, 2.2*T*.
Sheeting metal,	56*C*, 45*Z*, 12 arsenic.
Metal, expand in cooling,	75*L*, 16.7*A*, 8.3*B*.

Imitation of Gold.

Melt separately,	*x* = 21*C*, 13*T*. *y* = 62*C*, 9*Z*.
Gold imitation.	71*y*, 9*x*.

Type Metals.

Name.	Alloy.
Smallest type,	3*L*, 1*A*.
Small type,	4*L*, 1*A*.
Medium type,	5*L*, 1*A*.
Large type,	6*L*, 1*A*.
Largest type,	7*L*, 1*A*.
Metal which can be forged at red heat, and strong as good iron,	38.2*Z*, 60*C*, 1.75*I*.

Alloys for Solders.

Name.	Alloy.	Melts.
Newton's fusible,	8*B*, 5*L*, 3*T*,	212°
Rose's "	2*B*, 1*L*, 1*T*,	201°
A more "	5*B*, 3*L*, 2*T*,	199°
Still more "	12*T*, 25*L*, 50*B*, 13 cadium,	155°
For tin solder, coarse,	1*T*, 3*L*,	500°
For tin solder, ordinary,	2*T*, 1*L*,	360°
For brass, soft spelter,	1*C*, 1*Z*,	550°
Hard, for iron,	2*C*, 1*Z*,	700°
For steel,	19*S* 3*C*, 1*Z*.	
For fine brass work,	1*S*, 8*C*, 8*Z*.	
Pewterer's soft solder,	2*B*, 4*L*, 3*T*.	
Pewterer's soft solder,	1*B*, 1*L*, 2*T*.	
Gold solder,	24*G*, 2*S*, 1*C*.	
Silver solder, hard,	4*S*, 1*C*.	
" " soft,	2*S*, 1 brass wire.	
For Lead,	16*T*, 33*L*.	

Tempering of Steel.

The ability of heat to color steel or iron can be applied for ascertaining the temperature in flues and chimneys of steam-boilers, and for other temperatures between 430° and 600° Fah.

Yellow, very faint, for lancets,	430°
" pale straw, for razors, scalpels,	450°
" full, for penknives and chisels for cast iron,	470°
Brown, for scissors and chisels for wrought iron,	490°
Red, for carpenters' tools in general,	510°
Purple, for fine watch-springs and table-knives,	530°
Blue, bright, for swords, lock-springs,	550°
" full, for daggers, fine saws, needles,	560°
" dark, for common saws,	600°

When tempering a tool, heat to a dull red; plunge the point to be hardened into water half an inch; withdraw immediately. Emery point, so as to watch point for color from heat of shank; at color drop into tub altogether.

Relative Hardness, H, of Substances.

MINERALS.	H.	METALS.	H.	WOODS, DRY.	H.
Diamond, Ormuz,	100	Cast steel, hardened,	65	Chonta, S. Am., .	28
Diamond, Pink,	97	Cast steel, unhard.,	40	Lignum vitæ, .	25
Diamond, Yellow,	94	Cast iron, . .	38	Ebony, . . .	24
Diamond, Cubic,	92	Iron, hammered, .	37	Pomegranate, .	23
Sapphire, . .	90	Pure iron, . .	35	Boxwood, . .	22
Topaz, . .	80	Antimony, ham., .	36	Oak, very old, .	22
Garnet, . . .	72	Antimony, cast,	32	Oak, ordinary, .	21
Agate, . .	71	Platinum, cast, .	40	Mulberry, . .	20
Amethyst, . .	71	Platinum, ham.,	45	Cedar, India, .	20
Quartz, . .	70	Brass, common .	32	Beech, . .	19
Ruby, pale Brazil,	65	White metal, hard,	38	Ash, . . .	18
Ruby, . .	64	Gold, hammered, .	30	Alder, . .	18
Iron pyrites, .	63	Gold, cast, . .	26	Apple tree, . .	17
Opal, . . .	62	Copper, ham., .	34	Plum tree, .	16
Felspar, . .	60	Copper, cast, .	29	Yew, . . .	15
Fluor spar, .	40	Silver, ham., . .	32	Maple, . .	14
Copper pyrites, .	38	Silver, cast, .	27	Pine, yellow, .	14
Calcareous spar,	30	Zinc, . . .	26	Hazel, . .	13
Anthracite coal, .	28	Aluminum, .	24	Cedar, wild, . .	13
Galena, . .	27	Tin, ham., . .	24	Birch, . .	12
Amber, . . .	23	Tin, cast, . .	20	Fir, . . .	12
Granite, . .	22	Babbitt's metal, .	20	Pine, white, .	11
Gypsum, . .	20	Silenium, . .	22	Spruce, . . .	10
Bituminous coal,	16	Bismuth, . .	20	Sassafras, . .	9
Chalk, . . .	15	Lead, ham., .	18	Hemlock, . .	8
Talc, . . .	10	Lead, cast, . .	15	Cork, . . .	5

Mr. Chapman has arranged a scale for the hardness of minerals, as follows:

1 yields easily to the nail.

2 yields with difficulty to the nail, or merely receives an impression from it. Does not scratch a copper coin.

3 scratches a copper coin, but is also scratched by it, being of about the same hardness.

4 not scratched by a copper coin. Does not scratch glass.

5 scratches glass, though rather with difficulty, leaving its powder on it. Yields easily to the knife.

6 scratches glass easily. Yields with difficulty to the knife.

7 does not yield to the knife. Yields to the edge of a file, though with difficulty.

8, **9** and **10**, harder than flint.

The numbers in Chapman's scale multiplied by 10 will correspond with the hardness in the preceding table.

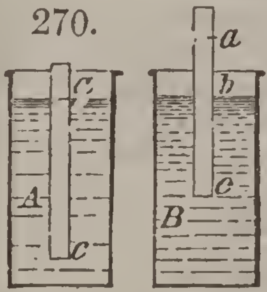
Charcoal from 1000 Weights of Dry Wood.

Oak, . . .	226	Beech, . .	200	Ash, . . .	179
Chestnut, .	232	Fir, . .	156	Norwegian Pine,	192
Mahogany, .	254	Cedar, . .	198	Sallow, . .	184
Walnut, .	206	Pine, . .	200	Birch, . .	174
Elm, . .	195	Scotch Pine, .	164	Sycamore, . .	197

HYDROMETER.

A BODY wholly immersed in a liquid will lose as much of its weight, as the weight of the liquid it displaces.

270.

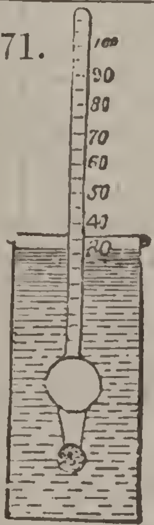


A floating body will displace its own weight of the liquid in which it floats.

A cylindrical rod of wood or some light materials, being set down in two liquids, A and B, of different specific gravities, when in equilibrium it will sink to the mark a in the liquid A, and to b in the liquid B; then the specific gravity of $A : B = b, c : a, c$, or inverse as the immersed part of the rod. This is the principle upon which a hydrometer is constructed.

Table showing the comparative Scales of Gay Lussac and Baumè, with the Specific Gravity and Proof, at the temperature of 60° Fahr.

271.



Gay Lussac's. (Per centage of pure alcohol.)	Baumè's	Specific Grav.	Proof.	
100	46	·796	100	Per centage over Proof.
95	40	·815	92	
90	36	·833	82	
85	33	·848	72	
80	31	·863	62	
75	28	·876	52	
70	26	·889	42	
65	24	·901	32	
60	23	·912	22	
55	21	·923	12	
50	19	·933	0	Proof.
45	18	·942	8	Under Proof.
40	17	·951	18	
35	16	·958	29	
30	15	·964	35	
25	14	·970	48	

HYDROSTATICS.

Notation.

A and **a** = areas of the pressed surfaces in square feet.

P and p = hydrostatic pressure in pounds.

d = depth of the centre of gravity of **A** or **a** under the surface of the liquids in feet.

S = specific gravity of the liquid.

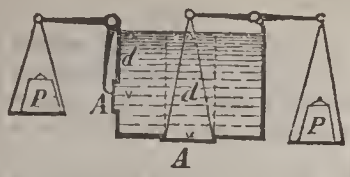
Example 1. Fig. 272. The plane **A** = 3·3 square feet, at a depth of $d = 6$ feet under the surface of fresh water. Required the pressure $P = ?$ Specific gravity of fresh water $S = 1$.

$$P = 62{\cdot}5\mathbf{A}\,d = 62{\cdot}5 \times 3{\cdot}3 \times 6 = 1237{\cdot}5 \text{ pounds.}$$

Example 2. Fig. 275. The area of the pistons **A** = 8·5 square feet, **a** = 0·02 square feet, $l = 4$ feet, $e = 9$ inches, and $F = 18$ pounds. Required the pressure $P = ?$

$$P = \frac{F\,l\,\mathbf{A}}{e\,\mathbf{a}} = \frac{18 \times 4 \times 8{\cdot}5}{0{\cdot}75 \times 0{\cdot}02} = 40800 \text{ pounds.}$$

It must be distinguished that the centre of pressure and centre of gravity of the planes, are two different points; the centre of pressure is below the centre of gravity, when the plane is inclined or vertical.

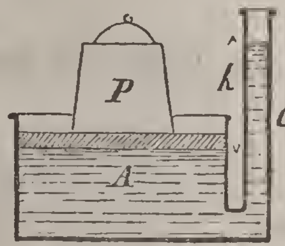


272.

$$P = 62{\cdot}5\ SAd,$$

$$A = \frac{P}{62{\cdot}5\ S\,d},$$

$$d = \frac{P}{62{\cdot}5\ S\,A}.$$

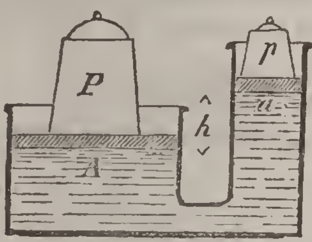


273.

The Hydrostatic paradox.

The pressure P is independent of the width of column C.

$P = 62{\cdot}5\ S\ A\ h.$ (same as above.)

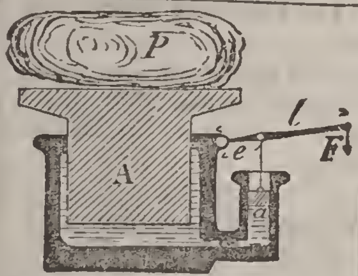


274.

$$P = A\left(62{\cdot}5 S h + \frac{p}{a}\right)$$

$$p = a\left(\frac{P}{A} - 62{\cdot}5 S h\right),$$

$$h = \frac{P\,a - p\,A}{62{\cdot}5\ SA\,a}$$

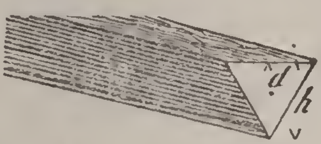


275. *Bramah's Hydraulic Press.*

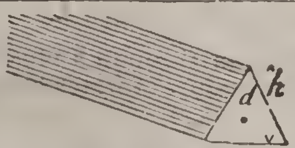
$$P = \frac{F\,l\,A}{e\,a}, \qquad A = \frac{P\,e\,a}{F\,l},$$

$$F = \frac{P\,e\,a}{A}, \qquad a = \frac{F\,A\,l}{P\,e}.$$

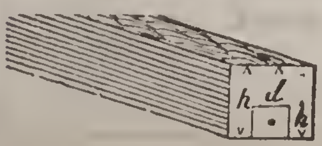
276. *Centre of Pressure of a rectangle, the upper edge at the surface of the liquid* $d = \frac{2}{3}\,h$.



277. *Centre of Pressure of a triangle, the base being at the surface of the liquid,* $d = \frac{1}{2}\,h$.



278. *Centre of Pressure of a triangle, the vertex being at the surface of the liquid.* $d = \frac{3}{4}\,h$.



279.

$$d = \frac{2}{3}\cdot\frac{3\,h^2 - 3\,h\,h_1 + h_1^2}{2h - h_1}$$

HYDRAULICS.

Let the vessel A, Fig. 284, be kept constantly full of water up to the water line w. In two horizontal faces lower than the water line w, are made orifices a and a', through which the water will pass up vertical nearly to the water line w. Omitting the resistance of air, &c., the jet should theoretically reach the water line w; practically it reaches $0{\cdot}967h$.

It is evident that the velocity of the *jet* through the orifices, must be the velocity due to a body falling the height h, according to the law of force of gravity.

Notation.

Q = actual quantity of water discharged per second or in the time t, in cubic feet.
h = head, or height of water over the orifice.
t = operating time in seconds.
a = area of the orifice in square feet.
m = the coefficient for contraction. (See Fig. 299)
G = gallon of 231 cubic inches discharged in the time t.
V = velocity through the orifice in feet per second.

Example 1. Fig. 284. How many gallons of water will be discharged in five minutes, through an orifice of 0·025 square feet, applied at 8 feet under the level of the water?

$$G = 37{\cdot}75\,a\,t\,\sqrt{h} = 37{\cdot}75 \times 0{\cdot}025 \times 5 \times 60\,\sqrt{8} = 800 \text{ gallons.}$$

Fig. 285. The weight P can represent the weight of a column of water whose

height $= \dfrac{P}{62{\cdot}5A} = \dfrac{h'}{0{\cdot}967}$, acting on the area A.

Fig. 286. n = number of down strokes per minute, s = stroke of piston; the air vessel $C = 6A\,s$ at the pressure of the atmosphere.

Example 2. Fig. 286. How many double strokes must be made per minute by the lever of a hand pump, to throw up 22 cubic feet of water 18 feet high, in the time of 8 minutes and 15 seconds; the levers $l = 30$ inches, $e = 8$ inches, $s = 0{\cdot}6$ feet, $F = 20$ pounds? $8 \times 60 + 15 = 495$ seconds.

$$n = \frac{3630\,Q\,h'\,e}{t\,s\,F\,l} = \frac{3630 \times 22 \times 18 \times 8}{495 \times 0{\cdot}6 \times 20 \times 30} = 64{\cdot}5 \text{ strokes per minute.}$$

Example 3. Fig. 294. A vessel of rectangular form is of dimensions $A = 6$ square feet, the height $h = 5$ feet. What time will it take the water level to sink 2 feet, when the orifice $a = 0{\cdot}212$ square feet.

$$t = \frac{A\,(h - h')}{2{\cdot}52a(\sqrt{h} + \sqrt{h'})} = \frac{6(5-3)}{2{\cdot}52 \times 0{\cdot}212(\sqrt{5} + \sqrt{3})} = 5{\cdot}66.$$

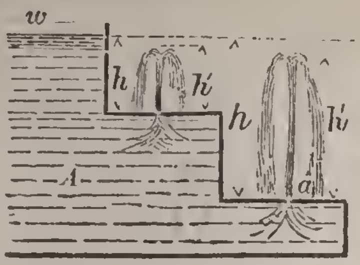
Motion of Water in Pipes,

Notation.

L = extreme length of the pipe in feet.
d = inside diameter in feet, and uniform throughout the length L.

Example 4. Fig. 287. What will be the velocity of the water through a pipe of 0·45 feet inside diameter, and $L = 68$ feet long, the head pressure of water being $h = 8$ feet?

$$V = 48\sqrt{\frac{0{\cdot}45 \times 8}{68 + 50 \times 0{\cdot}45}} = 9{\cdot}6 \text{ feet per second.}$$



284.

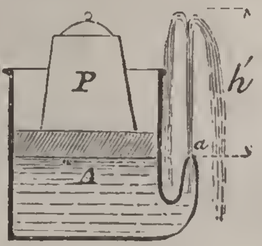
$$V = 8{\cdot}02\sqrt{h}$$

$$Q = m\,a\,t\,8{\cdot}02\sqrt{h} = 5{\cdot}05\,a\,t\sqrt{h}$$

$$G = 37{\cdot}77\,a\,t\sqrt{h}, \quad m = 0{\cdot}63,$$

$$jet = 0{\cdot}967\,h,$$

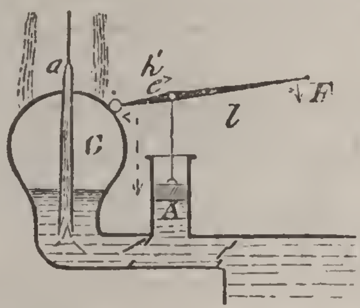
$$h = \frac{Q^2}{25{\cdot}5\,a^2\,t^2}.$$



285.

$$V = 1{\cdot}015\sqrt{\frac{P}{A}}, \quad Q = 1{\cdot}015\,a\,t\sqrt{\frac{P}{A}},$$

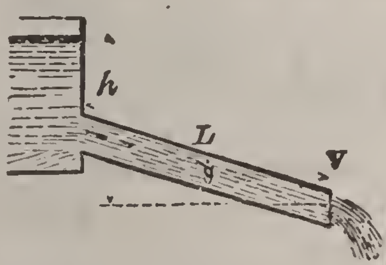
$$G = 7{\cdot}5\,a\,t\sqrt{\frac{P}{A}}, \quad h' = \frac{P}{60{\cdot}5\,A}.$$



286.

$$Q = a\,t\sqrt{\frac{F\,l}{A\,e}}, \quad n = \frac{3630\,Q\,h'\,e}{t\,s\,F\,l},$$

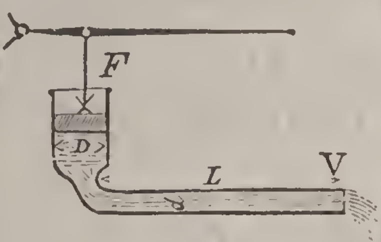
$$G = 7{\cdot}5\,a\,t\sqrt{\frac{F\,l}{A\,e}}, \quad h' = \frac{F\,l}{60{\cdot}5\,A\,e}.$$



287. *Motion of Water in Pipes.*

$$V = 48\sqrt{\frac{d\,h}{L+50d}}, \quad Q = 37{\cdot}7d^2\sqrt{\frac{d\,h}{L+50d}},$$

$$d = 0{\cdot}24\sqrt[5]{\frac{L\,Q^2}{h}}, \quad h = \frac{Q^2\,(L+5d)}{142\,d^5}.$$

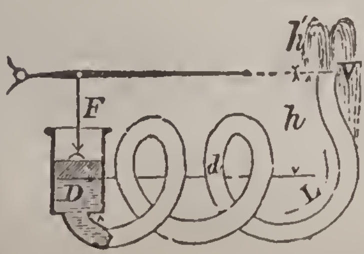


288.

Motion of Water in Pipes.

$$V = 6{\cdot}86\sqrt{\frac{d\,F}{D(L+50d)}}, \quad Q = 5{\cdot}38d^2\sqrt{\frac{d\,F}{D(L+50d)}}$$

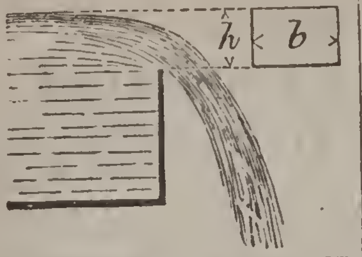
$$d = 1{\cdot}68\sqrt[5]{\frac{L\,D\,Q'}{F}}, \quad F = \frac{Q^2D(L+5d)}{2{\cdot}9d^5}.$$



289.

$$V = 6{,}86\sqrt{\frac{d(F-49D^2h)}{D(L+50d)}} \quad Q = 5.38\,Vd^2t$$

$$h' = \frac{V^2}{66{\cdot}5}, \quad h' = \frac{D\sqrt{s\,n}}{57{\cdot}65d.}$$

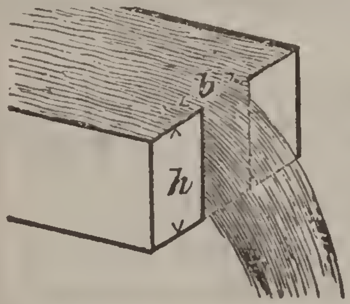


290.

Weirs.

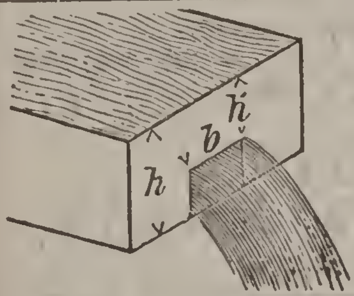
$Q = k\,b\,t.$ See Table for *Weirs.*

$$t = \frac{Q}{k\,b}, \quad b = \frac{Q}{k\,t},$$



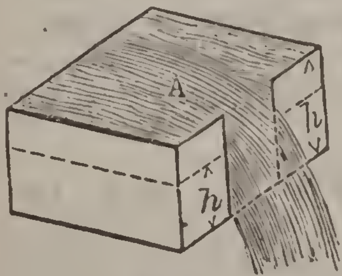
291.

$$Q = 5{\cdot}35m\,b\,h\,t\sqrt{h},$$
$$G = 40m\,b\,h\,t\sqrt{h},$$
$$t = \frac{Q}{5{\cdot}35\,m\,b\,h\sqrt{h}},$$



292.

$$Q = 5{\cdot}35m\,b\,t(h\sqrt{h} - h'\sqrt{h'}),$$
$$G = 40m\,b\,t(h\sqrt{h} - h'\sqrt{h'}),$$
$$t = \frac{Q}{5{\cdot}35m\,b(h\sqrt{h} - h'\sqrt{h'})},$$

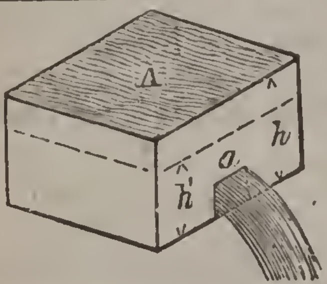


293.

$$t = \frac{0{\cdot}95m\,A(\sqrt{h} - \sqrt{h'})}{b\sqrt{h\,h'}},$$

A = area of the vessel in square feet.

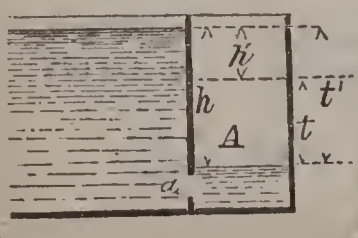
t = time in seconds, in which the water level will sink the space $h - h'$.



294.

$$t = \frac{A(h - h')}{4m\,a\,(\sqrt{h} + \sqrt{h'})},$$

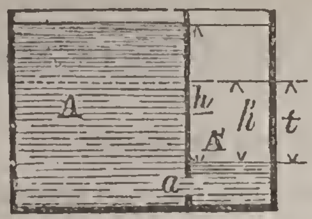
$$Q = 4m\,a\,t(\sqrt{h} + \sqrt{h'}),$$



295.

$$t = \frac{A}{3{\cdot}85a\,m}(\sqrt{h} - \sqrt{h'}),$$

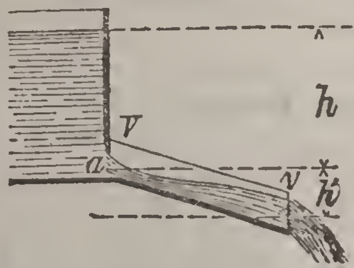
$$a = \frac{A\sqrt{h}}{3{\cdot}85t'\,m}, \quad t' = \frac{A\sqrt{h}}{3{\cdot}85a\,m},$$



296.

$$t = \frac{A\,A'\sqrt{h}}{13{\cdot}7\,m\,a\sqrt{A+A'}},$$

$$h = \frac{A\,h'}{A + A'},$$

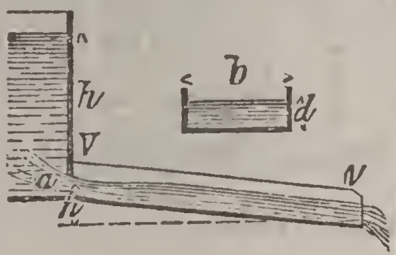


297. *Short Drain.*

$$V = \frac{8{\cdot}02\,\sqrt{h}}{\sqrt{1+(\frac{1}{m}-1)^2}},$$

$$v = \sqrt{V^2 + 64{\cdot}4\,h'},$$

$Q = a\,mVt.$ *from V to v about* $6\sqrt{a}$,

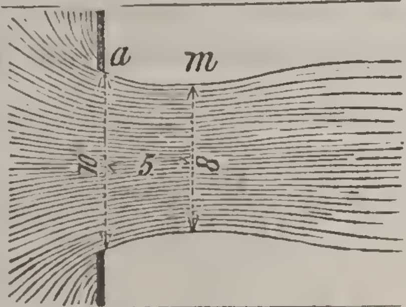


298. *Long Drain.*

$$V = \frac{8{\cdot}02\,\sqrt{h}}{\sqrt{1+(\frac{1}{m}-1)^2}},$$

$$v = 3\left(\sqrt{h} + 14.145\sqrt{\frac{h's}{l\,a}}\right)$$

$s = b + 2d$ $\qquad$ $l = V$ *to* v, *feet.*



299. *Proportions of the contracted Vein.*

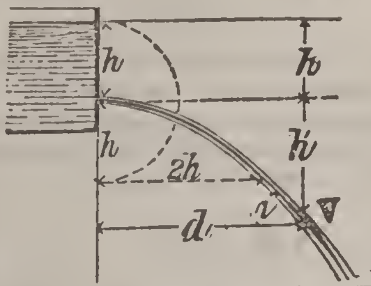
$a : m = 10^2 : 8^2.$ $\quad$ $m = 0{\cdot}64\,a.$

$m = 0{\cdot}64$ *when contracted on* 4 *sides.*

$m = 0{\cdot}72$ " " " 3 *sides.*

$m = 0{\cdot}8$ " " " 2 *sides.*

$m = 0{\cdot}9$ " " " 1 *side.*



300.

The form of the Vein is a Parabola.

$d = 2\sqrt{hh'}$, $\qquad$ $V = 8\sqrt{h + h'}$

$Q = 8\,m\,a\,t\,\sqrt{h}$,

$$\tan v = \frac{2\,h'}{d}, \qquad h = \frac{d^2}{4\,h'}.$$

All of the above formulæ give theoretical results assuming smooth surfaces and perfect orifices, and may give results far above actual results in practice. Mr. Nystrom does not give authorities for coefficients of friction or efflux.

W. D. M.

Example 5. Fig. 289. Required the velocity and quantity of water discharged in a long pipe or hose of $L = 135$ feet long, and $d = 0.17$ feet, attached to a hand-pump of $D = 0.2$ feet in diameter P = 44 pounds, and the end of the pipe elevated $h = 20$ feet above the piston D?

$$V = 6.86\sqrt{\frac{0.17(44 - 49 \times 0.2^2 \times 20)}{0.2(135 + 50 \times 0.17)}} = 1.95 \text{ feet per second.}$$

$Q = 1.95 \times 5.38 \times 0.2^2 = 0.042$ per second $\times 60 = 2.52$ cubic feet per minute.
$s = 0.8$ feet, the stroke of piston, we shall have

$$n = \frac{2.52}{0.8 \times 0.785 \times 0.2^2} = 100 \text{ strokes per minute.}$$

Table for Water flowing over Weirs.

This table is set up from careful experiments on a large scale, and is suited for *weirs* only. See Fig. 290. $Q = 4.327\, b\, \sqrt{h^3}.$

RULE. Multiply the width b, in feet, of the *weir* by the coefficient k, and the product is the quantity of water discharged per second, in cubic feet. h is the height as represented by Fig. 290. The width b should be $b > h$.

Example 6. How much water will flow over a weir of $b = 5$ feet, $h = 0.5$ feet, in one minute?
$Q = k\,b\,t = 1.1295 \times 5 \times 60 = 338.35$ cubic feet.

h. inches.	h. feet.	m.	k.
0.4	0.033	0.424	0.01365
0.8	0.066	0.417	0.05452
1.2	0.100	0.412	0.10592
1.6	0.133	0.407	0.16616
2.4	0.200	0.401	0.29171
3.2	0.266	0.397	0.44480
4.	0.333	0.395	0.63111
6.	0.500	0.393	1.1295
8.	0.666	0.390	1.7464
9.	0.750	0.385	2.0331
12.	1.000	0.376	3.1350

On the Velocity of Water in Rivers.

Notation.

F = fall of the river in feet per mile.
R = hydraulic radius in feet, or the area of the cross-section of the river in square feet, divided by the wet perimeter in feet.
V = mean velocity of the water in inches per second.
M = mean velocity in miles per hour.

$$V = 10.9\sqrt{FR}, \qquad F = \frac{V^2}{118.8\,R}.$$

$$M = 0.619\sqrt{FR}, \qquad F = \frac{M^2}{3.83\,R}.$$

The mean velocity of the water throughout the whole section of the river is to the velocity at the surface in the middle of the river as 84 : 100, or as 100 : 120.

Example 1. The cross-section of a river is measured to be 560 square feet, and the wet perimeter 196 feet; the fall of the river is 5 feet per mile. Required, the hydraulic radius and the mean velocity of the water in miles per hour?

$$\text{Hydraulic radius } R = \frac{560}{196} = 2.86 \text{ feet.}$$

$$\text{Mean velocity } M = 0.619\sqrt{5 \times 2.86} = 2.34 \text{ miles per hour.}$$

Example 2. The velocity of the surface in the middle of a river is 36 inches per second; hydraulic radius $R = 2$ feet. Required, the mean velocity and the fall of the river per mile?

$$\text{Mean Velocity } V = 36 \times 0.84 = 30.24 \text{ inches per second.}$$

$$\text{Fall } F = \frac{30.24^2}{118.8 \times 2} = 3.8487 \text{ feet per mile.}$$

Obstruction in Rivers.

R = rise of water in feet caused by obstruction.

A = sectional area in square feet of river unobstructed, and a = that when obstructed. V = velocity in feet per second of the water without obstruction.

$$R = \left(\frac{V^2}{5.86} + 0.05\right)\left(\left(\frac{A}{a}\right)^2 - 1\right).$$

Resistance to a Plane Facing a Current of Water

or Moving in Still Water.

A = area of the plane in square feet.
R = resistance in pounds.
V = velocity in feet per second.

$$R = A\,V^2, \text{ in fresh water.}$$

$$R = 1.032\,A\,V^2, \text{ in salt or sea water.}$$

When the plane is set at an angle of less than 90° to the direction of motion, the resistance will be, when ϕ = angle of the plane,

$$R = A\,(V \sin.\phi)^2, \text{ in fresh water.}$$

$$R = 1.032\,A\,(V \sin.\phi)^2, \text{ in salt water.}$$

Theoretical Velocity of Water, due to Head or Fall.

See table for falling bodies, page 308, in which the column S represents the head of fall in feet.

To find the Number of Gallons of Water G which can be raised per Hour from a Well of Depth D,

By a Suitable Double-action Force-and-lift-pump.

D may also denote the height to which water may be raised in water-works.

One man working a crank, $G = \frac{18000}{D}$.

A donkey, $G = \frac{36000}{D}$.

A horse, $G = \frac{126000}{D}$.

Per steam horse-power, $G = \frac{190000}{D}$,

or 0.8 of the natural effect.

Example 1. How many gallons of water can be raised per hour from a well 150 feet deep by a horse?

$$G = \frac{126000}{150} = 840 \text{ gallons, the answer.}$$

Example 2. How many gallons of water can be raised per hour to a height of D = 150 feet by a steam-engine of 120 actual horse-power?

$$G = \frac{190000 \times 120}{150} = 152{,}000 \text{ gallons, the answer.}$$

MOTION OF WATER IN PIPES.

Notation.

Q = cubic feet of water passed through the pipe per minute.
D = inside diameter of the pipe in feet.
L = length of the pipe in feet, increased by 50 diameters.
H = head or fall in feet.
V = velocity of the water in the pipe in feet per minute.

$$Q = 2356\sqrt{\frac{HD^5}{L}}, \qquad D = \frac{1}{22.329}\sqrt[5]{\frac{Q^2L}{H}}, \qquad V = 3000\sqrt{\frac{HD}{L}}.$$

Example 1. A water-pipe of $D = 1.75$ feet in diameter, $L = 36{,}000 + 50 \times 1.75 = 36087.5$ feet long, head pressure $H = 390$ feet. Required, how much water it can discharge per minute?

$$Q = 2356\sqrt{\frac{390 \times 1.75^5}{36087.5}} = 992.26.$$

Example 2. At a distance of 27960 feet from a water-work is required $Q = 564$ cubic feet of water per minute, head pressure being $H = 256$ feet. Required, the diameter of the pipe? $L = 27960 + 50 = 28010$ feet.

$$D = \frac{1}{22.329}\sqrt[5]{\frac{564^2 \times 28010}{256}} = 1.4436 \text{ feet.}$$

Example 3. A water-pipe of $D = 0.75$ feet in diameter, $L = 8650 + 50 = 8700$ feet, has a head pressure of $H = 128$ feet. Required the velocity v of the discharge.

$$V = 3000\sqrt{\frac{128 \times 0.75}{8700}} = 315.13 \text{ feet per minute.}$$

Consumption of Water in Cubic Feet per head of Population,

Including all Uses, as for Manufactories, Fires, etc., in 24 hours.

January,	2.58	April,	2.73	July,	4.58	October,	4.46
February,	2.40	May,	3.37	August,	4.75	November,	4.12
March,	2.64	June,	3.50	Sept.,	4.61	December,	3.61

On the Flow of Water in Bends of Pipes.

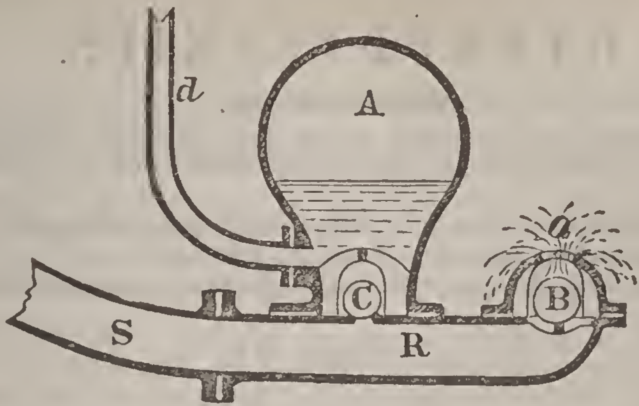
Notation.

L = the whole length of pipe in feet, straight and curved or bent, increased by 50 D.
R = radius in feet of the bend of the centre-line of the pipe.
ϕ = angle of deflection or bend of pipe in degrees. Should the pipe have several bends, add all the angles to ϕ.
Sin. ϕ to be used only up to 90°, and disappears in the formulæ for greater angles.
D = inside diameter of the pipe in feet; V = velocity of the water in feet per minute; and H = head or fall in feet; Q = cubic feet of water discharged per minute.

$$Q = 2356\sqrt{\frac{HD^5}{L}\left(\frac{90}{\phi + 90}\right)\left(1 + \frac{R \text{ Sin. } \phi}{R + 10}\right)}.$$

$$V = 3000\sqrt{\frac{HD}{L}\left(\frac{90}{\phi + 90}\right)\left(1 + \frac{R \text{ Sin. } \phi}{R + 10}\right)}.$$

The formulæ will answer for a pipe of the form of a screw-spiral.



THE HYDRAULIC RAM.

This hydraulic motor appears to be too little known in many parts or the world. The author of this book has been in the interior of many countries where water is raised in a very rude and laborious way, and where the hydraulic ram would be of great utility. The useful effect of the ram, like that of water-wheels and turbines, depends much upon its construction. In ordinary cases it returns about 50 per cent. of the natural effect. That is, the quantity of water (q) multiplied by the height (h) of the delivery above the ram will be about 50 per cent. of the quantity of water (Q) working the ram, multiplied by the head of fall (F), in the same unit of time.

$$qh = 0.5QF. \qquad q = \frac{0.5QF}{h}. \qquad Q = \frac{2qh}{F}.$$

Q and q can be expressed in any unit of volume or weight.

F and h can be expressed in any unit of length.

But let us assume Q and q to be cubic feet per minute,

F and h = fall and height in feet,

L = length in feet and D = diameter in inches of the supply-pipe S,

l = length and d = diameter of the delivery-pipe d.

$$\text{Then } D = \sqrt[5]{\frac{2Q^2(L+5D)}{F}}, \text{ and } d = \sqrt[5]{\frac{4q^2(l+5d)}{h}}.$$

Description of the Hydraulic Ram.

Reference to the figure above.

The water working the ram is supplied through the pipe S, and escapes through an opening at o, until it has gained a velocity sufficient to raise the valve or ball B, which suddenly stops the current, and causes an excessive pressure in the ram R, which opens the valve or ball C; the water is forced into the vessel and air-chamber A, and finally through the delivery-pipe d to its destination. When equilibrium of pressure is restored between S and R, the ball B falls, and the operation is repeated. The ram can make as much as 200 strokes per minute, depending upon its size.

The length of the supply-pipe S should not be less than five times the height of the fall F, because it is the dynamic action (see page 310) of the water in the pipe which works the ram. The delivery-pipe may be made 10 or more times the height of the fall.

HYDRODYNAMICS.

Water Power.

The natural power contained in a fall of water is equal to the weight of the quantity of water passing over per second, multiplied by the vertical space through which it falls.

Fig. 291. Let Q be the quantity of water which passes through the orifice a in the time $t = 1''$ second, in cubic feet of 62.5 pounds each.

$h =$ the vertical space the water falls; then the value or natural effect of the fall is at the orifice a.

$$P = 62.5\, Q\, h,$$

But, $$Q = 5.06\, a\sqrt{h};$$

Then we have $$P = 315.5\, a\, h\sqrt{h}.$$

This will be in horse power. $$H = 0.573\, a\, h\sqrt{h}, \qquad H = 0.1136\, Q\, h,$$

$$h = 1.14\sqrt[3]{\frac{H^2}{a^2}}, \qquad h = \frac{H}{0.1136\, Q}.$$

Example 1. In a creek passes 18 cubic feet of water per second. How high must that creek be dammed up to produce an effect of ten horses?

$$h = \frac{10}{0.1134 \times 18} = 4.9 \text{ feet, the answer.}$$

Comparison of Columns of Water in Feet.

Mercury in inches, and pressure in pounds, per square inch.

Pounds Pr. sq. in.	Water. Feet.	Merc'ry. Inches.	Water. Feet.	Merc'ry. Inches.	Pounds. Pr. sq. in.	Merc'ry Inches.	Water. Feet.	Pounds. Pr. sq. in.
1	2.311	2.046	1	0.8853	0.4327	1	1.1295	0.4887
2	4.622	4.092	2	1.7706	0.8654	2	2.2590	0.9775
3	6.933	6.138	3	2.6560	1.2981	3	3.3885	1.4662
4	9.244	8.184	4	3.5413	1.7308	4	4.5181	1.9550
5	11.555	10.230	5	4.4266	2.1635	5	5.6476	2.4437
6	13.866	12.2276	6	5.3120	2.5962	6	6.7771	2.9325
7	16.177	14.322	7	6.1973	3.0289	7	7.9066	3.4212
8	18.488	16.368	8	7.0826	3.4616	8	9.0361	3.9100
9	20.800	18.414	9	7.9680	3.8942	9	10.165	4.3987
10	23.111	20.462	10	8.8533	4.3273	10	11.295	4.8875
11	25.422	22.508	11	9.7386	4.7600	11	12.424	5.3762
12	27.733	24.554	12	10.624	5.1927	12	13.554	5.8650
13	30.044	26.600	13	11.509	5.6255	13	14.683	6.3537
14	32.355	28.646	14	12.394	6.0582	14	15.813	6.8425
15	34.666	30.692	15	13.280	6.4909	15	16.942	7.3312
16	36.977	32.738	16	14.165	6.9236	16	18.072	7.8200
17	39.288	34.784	17	15.050	7.3563	17	19.201	8.3087
18	41.599	36.830	18	15.936	7.7890	18	20.331	8.7975
19	43.910	38.876	19	16.821	8.2217	19	21.460	9.2862
20	46.221	40.922	20	17.706	8.6544	20	22.590	9.7750
21	48.532	42.968	21	18.591	9.0871	21	23.719	10.264
22	50.843	45.014	22	19.477	9.5198	22	24.849	10.752
23	53.154	47.060	23	20.362	9.9525	23	25.978	11.241
24	55.465	49.106	24	21.247	10.385	24	27.108	11.7300
25	57.776	51.152	25	22.133	10.818	25	28.237	12.219
26	60.087	53.198	26	23.018	11.251	26	29.367	12.707
27	62.398	55.244	27	23.903	11.683	27	30.496	13.196
28	64.709	57.290	28	24.789	12.116	28	31.626	13.685
29	67.020	59.336	29	25.674	12.549	29	32.755	14.174
30	69.331	61.386	30	26.560	12.981	30	33.885	14.662

WATER-WHEELS.

Water-wheels are of two essential kinds, namely, *Vertical* and *Horizontal*.

The *Vertical* are subdivided into

Overshot-wheels, *Undershot-wheels*, *Breast-wheels*, and *High-breast* and *Low-breast wheels*.

The *Horizontal* are *with Floats*, *Screw-wheels*, *Turbine*, *Reaction-wheels*, *&c.*

Waterwheels do not transmit in full the natural effect concentrated in a fall of water; under most favourable circumstances 80 per cent. has been utilized, but with poor arrangements only 20 per cent. may be expected.

Example 1. Fig. 302. The vertical section of the immersed floats of an undershot-wheel in a mid-stream is $\mathbf{a} = 27$ square feet, velocity of the stream $V = 8{\cdot}6$, and $v = 4$ feet per second. Required the horse-power of the wheel H?

$$H = \frac{\mathbf{a}\, v}{200}(V - v)^2 = \frac{27 \times 4}{200}(8{\cdot}6 - 4)^2 = 11{\cdot}4\ H\!P.$$

Example 2. Fig. 307. On a breast-wheel is acting $Q = 88$ cubic feet of water per second, the head $h = 8$ feet, velocity of the wheel at the centre of the buckets $v = 5$ feet per second; the water strikes the buckets at an angle $u = 8^\circ$ and velocity $V = 7$ feet per second. Required the horse-power of the wheel, H?

$$H = \frac{88}{11{\cdot}4}\left(8 + \frac{5}{25}(7 \times \cos.8^\circ - 5)\right) = 65\ H\!P.$$

Example 3. Required the effect of Poncelet's wheel, the head $h = 4$ feet, and the orifice $a = 5$ square feet, the velocity of the wheel at the centre of pressure of the floats is $v = 6{\cdot}78$ feet per second?

$$V = 6{\cdot}91\sqrt{4} = 13{\cdot}82 \text{ feet per second.}$$

$$Q = 6{\cdot}5 \times 5 \times \sqrt{4} = 65 \text{ cubic feet per second.}$$

$$H = \frac{65 \times 6{\cdot}78}{197}(13{\cdot}82 - 6{\cdot}78) = 15{\cdot}8\ H\!P.$$

Example 4. Fig. 309. A saw-mill wheel is to be built under a fall of $h = 18$ feet, and to make $n = 110$ revolutions per minute. Required the proper diameter of the wheel.

$$D = \frac{100}{110}\sqrt{18} = 3{\cdot}857 \text{ feet,}$$

at the centre of pressure of the buckets.

Velocity $V = 8\sqrt{18} = 33{\cdot}94$ feet per second.

Velocity $$v = \frac{3{\cdot}14 \times 3{\cdot}857 \times 110}{60} = 22{\cdot}2 \text{ feet per second.}$$

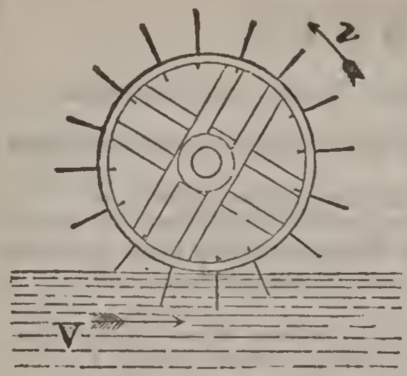
The fall discharged 30 cubic feet of water per second. Required the horse power of the wheel. H?

$$H = \frac{30 \times 22{\cdot}2}{200}(33{\cdot}94 - 22{\cdot}2) = 39\ H\!P.$$

How many square feet of dry Pine can it saw per hour?

See page 264. $30 \times 39 = 1170$ square feet.

The saw is meant to be applied direct on the wheel shaft.



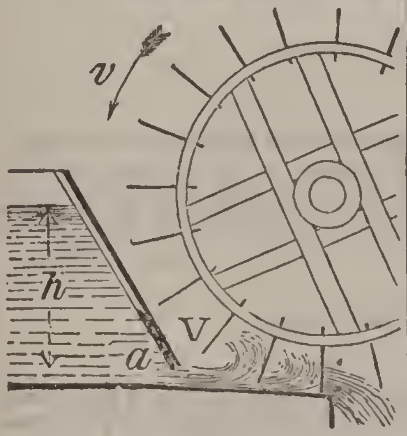
302.

Undershot wheel in a mid-stream.

$$H = \frac{\mathbf{a}\, v}{200}(V - v)^2,$$

When $V = 2v$ about, the effect will be,

$$H = \frac{\mathbf{a}\, V^3}{1600}, \quad \mathbf{a} = \text{area of float.}$$



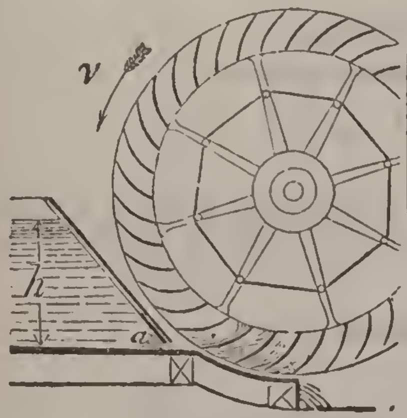
303.

Undershot-Wheel.

$$H = \frac{Q\, v}{454}(V - v),$$

$$H = \frac{m\, a\, v}{56{\cdot}6}(V - v)\sqrt{h},$$

When $V = 2v$, about, $H = \dfrac{a\, h \sqrt{h}}{3{\cdot}9}$.



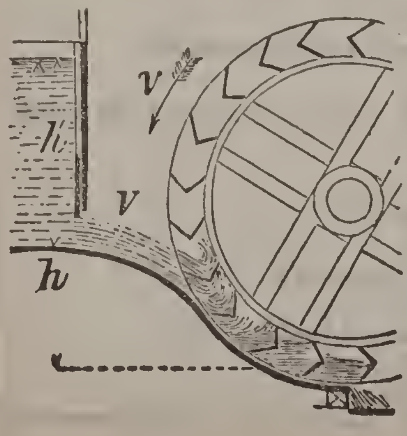
304.

Poncelet's Wheel.

$$H = \frac{Q\, v}{228}(V - v), \text{ when } h > 5 \text{ feet},$$

$$H = \frac{Q\, v}{200}(V - v) \text{ when } h < 5 \text{ feet},$$

$$Q = 8m\, a\sqrt{h}, \qquad V = 6{\cdot}91\sqrt{h}.$$

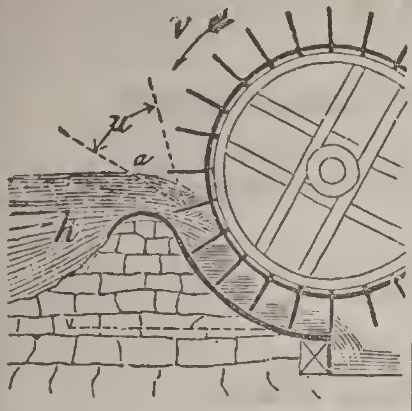


305.

Breast-Wheel with Parabolic drain.

$$H = \frac{Q}{12}\left[h + \frac{v}{28}(V - v)\right],$$

$$Q = 6{\cdot}5a\sqrt{h'}.$$

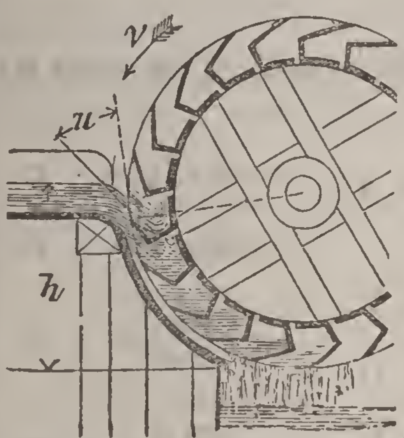


306. *Low-breast Wheel.*

$$H = \frac{Q}{11 \cdot 2}\left[h + \frac{v}{32}(V \cos u - v)\right]$$

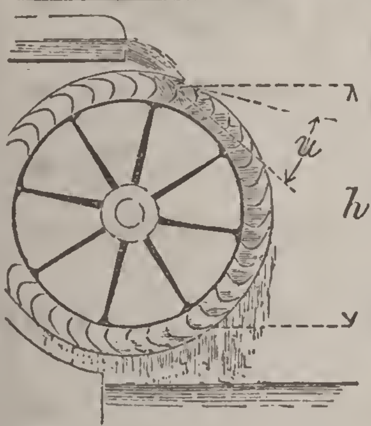
$Q = kb.\ \ V = \frac{Q}{a}.$ See table for weirs.

See page 283, Weisbach's *Hydraulic Motors.*



307. *Breast Wheel.*

$$H = \frac{Q}{11 \cdot 4}\left[h + \frac{v}{25}(V \cos u - v)\right]$$

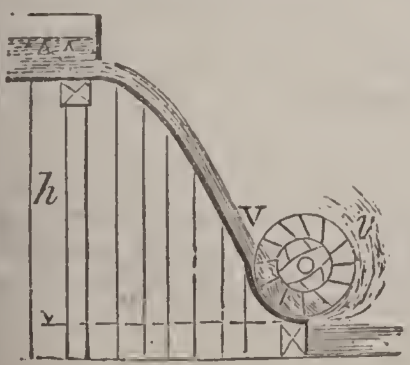


308. *Over-shot Wheel.*

$$H = \frac{Q}{13 \cdot 7}\left[h + \frac{v}{21 \cdot 5}(V \cos u - v)\right]$$

Proper velocity about $n = \frac{35\,D + 100}{D}$, *revolutions per minute.*

See page 247, Weisbach's *Hyd. Motors.*



309. *Saw-Mill Wheel.*

$$H = \frac{Q\,v}{200}(V - v)$$

Proper diameter of the Wheel,

$D = \frac{100}{n}\sqrt{h}$, *in feet,*

n = *revolutions per min.*

TURBINES.

Notation.

Q = cubic feet of water passed through the turbine per second.
h = height of fall in feet.
D = diameter in inches of circle of effort in the turbine.
a = area in sq. in. of the conduit passage into the turbine wheel.
b = depth in inches of turbine buckets.
c = depth in inches of leading buckets.
r = breadth of turbine buckets in inches.
m = number of buckets in the turbine wheel.
m' = number of leading buckets.
n = number of revolutions of turbine per minute.
S and s = height of conduit and discharge in inches.
t = thickness of steel plate buckets in 16ths of an inch.
H = actual horse power of the turbine.
l = length in feet } of conduit pipe.
d = diameter in inches } of conduit pipe.
d' = diameter in inches of the discharge pipe.
W = Hydraulic pressure on the turbine wheel bearing on the end of the shaft.

$D = \frac{k\sqrt{h}}{n}$, - - - 1

$D = \frac{a}{0{\cdot}436\, r}$, - - 2

$n = \frac{k\sqrt{h}}{D}$, - - - 3

$n = \frac{20\, k\, Q}{a\, D}$, - - 4

$r = \frac{a}{0{\cdot}436 D}$, - - 5

$r = \frac{46\, k\, Q}{D^2 n}$, - - 6

$r = \frac{D}{5}$ to $\frac{D}{8}$, - - 7

$t = \frac{m}{10}$, - - - - 8

$a = \frac{20\, Q}{\sqrt{h}}$, - - - 9

$a = \frac{20\, k\, Q}{D\, n}$, - - 10

$a = 0{\cdot}436\, Dr$, - 11

$a = m'rs$, - - - 12

$a' = m\, r\, s$, - - - 13

$a' = 0{\cdot}98\, a$, - - 14

$Q = \frac{a\sqrt{h}}{20}$, - - - 15

$Q = \frac{a\, D\, n}{20\, k}$, - - 16

$m = 5\sqrt{D}$, - - 17

$m' = 4{\cdot}5\sqrt{D}$, - - 18

$b = \frac{0{\cdot}625\, D}{\sqrt{m}}$, - 19

$c = \frac{0{\cdot}78\, D}{\sqrt{m'}}$, - - 20

$s = 0{\cdot}86\, S$, - - 21

$d = D + r + \sqrt[3]{l}$, 22

$d' = D + 2\, r$, - - 23

$W = \frac{D^2 h}{3}$, - - 24

$H = 0{\cdot}1134\, Q\, h$ natural effect of the fall, - - - 25

$H = \frac{30\, Q^3}{a^2}$, } actual horse power, 26

$H = \frac{a\, h\sqrt{h}}{267{\cdot}5}$, } 66 per cent of the natural. 27

The coefficient k can vary from 800 to 1200 without seriously affecting the percentage of the utilized power, but it is best between 900 and 1000. These formulæ cannot apply to all forms of turbines. Work out the entire theory of particular wheel, Weisbach's *Hydraulic Motors*.—W. D. M.

Jonval's Turbine has so many advantages above other hydraulic motors that it is considered sufficient to describe the construction of that one only, but the principal formulas will answer for any kind of turbines.

On the accompanying plate is a drawing of a Jonval Turbine such as the Author of this Pocket Book has built in Russia. The buckets are not supported by concentric rings, but are fastened only on one side, which is considered more simple and convenient for replacing new buckets. For falls over 30 feet it may be better to make it with concentric rings. When a turbine is to be constructed we have on the one side given the natural effect of the fall, and on the other side the actual work to be done, which latter should not exceed 66 per cent. of the former. Between these two points the turbine is to be so proportioned as to utilize the greatest possible effect with smallest expense of Machinery.

Jonval's turbine in good condition generally utilizes 60 to 80 per cent. Suppose a fall of $h=25$ feet, discharging $Q=12$ cubic feet of water per second, the natural effect will be,

$$H=0{\cdot}1134\times12\times25=34 \text{ horses},$$

of which $34\times{\cdot}66=22{\cdot}4$ horses to be counted upon as the actual effect of the turbine.

Turbine shaft to make $n=200$ revolutions per minute with the assumed coefficient $k=960$. From these dates we will obtain all the principal dimensions of the turbine, namely,

$$D=\frac{960\sqrt{25}}{200}=24 \text{ inches.} \quad \text{- - 1}$$

$$a=\frac{20\times960\times12}{24\times200}=48 \text{ sq. in.} \quad 10$$

$$m=5\sqrt{24}=24{\cdot}5 \text{ say } 25. \quad \text{- - 17}$$

$$m'=4{\cdot}5\sqrt{24}=22 \text{ buckets.} \quad \text{- 18}$$

$$r=\frac{48}{0{\cdot}436\times24}=4{\cdot}6 \text{ in.} \quad \text{- - - 5}$$

$$b=\frac{0{\cdot}625\times24}{\sqrt{25}}=3 \text{ in.} \quad \text{- - - - 19}$$

$$c=\frac{0{\cdot}78\times24}{\sqrt{22}}=4 \text{ inches.} \quad \text{- - 20}$$

$$t=\frac{25}{10}=2{\cdot}5, \text{ 16ths.} \quad \text{- - - - 8}$$

In calculating the breadth r from formula 5, it must come inside of formula 7, if not the diameter D must be altered.

Now proceed with the construction as shown at the bottom of the plate, which represents a section of the buckets through the circle of effort of the turbine.

The drawing of the turbine is ⅜ of an inch to the foot, and the construction of the buckets 3 inches to the foot.

Draw the base line AB, set off the angle of the leading buckets $=10°$. The distance between the leading buckets will in this case be $24\times3{\cdot}14:22=3{\cdot}43$ inches, set off this from S towards A, draw the straight part of the second bucket parallel to the first one, draw from S the line $d\ d$ at right angle to the buckets, and c will be the centre for the curved part. From the centre of S draw the line o to the end of the second buckets, divide this line into eight equal parts take five of them as radus and draw from the end of the second bucket a circlearc of about 50°, which will be the propelling part of the turbine wheel bucket.

Distance between the wheel buckets will be $24\times3{\cdot}14:25=3{\cdot}02$ inches, set off this from A towards S, draw the second propelling arc. Set off from A the depth of the wheel buckets $b=3$ inches, set off $2\ b$ to s, which will be the length of the first wheel bucket. Set off from s to u the distance between the buckets 3·02 inches. Make $s=0{\cdot}86\ S$. Draw from u a curved line in the form of a parabola that will leave the space s and tangent the propelling circlearc somewhere about x. Care must be taken that the discharging area a' of all the wheel buckets will be about 2 per cent. less than the conduit area a of all the leading buckets. The surface of the buckets should be made as smooth as possible, or even polished.

For very high falls the Hydraulic pressure W becomes very considerable and may necessitate another arrangement, namely, to lay the shaft horizontally and place on it two turbines, so that the leading buckets are either between or outside of the wheels; but then comes another disadvantage, namely, that the number of revolutions will be greatly increased and may be required to gear it down 10 to 20 times to the proper speed of the main shaft.

To avoid this as much as possible, take $k = 800$, and make $r = \frac{D}{8}$.

One great advantage with Jonval's turbine is that it can be placed almost anywhere between the high and low levels to suit the location, though it should not be more than 20 feet above the lower level; then, in order to utilize the whole fall, care must be taken to make the discharge-pipe perfectly air-tight. It is not necessary to make the discharge straight down from the turbine: it can be carried horizontally or inclined, as may suit the location. The author has built turbines similar to that represented on the accompanying plate, at General Maltzof's establishment, Kaluga, Russia.

Approximate or Proportionate Price of Turbines,

as fitted and delivered at the foundry, without shaftings or gearings, is—

$$\$ = \frac{400\sqrt{H}}{\sqrt[3]{F}},$$

in which H = horse power of the turbine and F the height of fall in feet.

Example. Required the price of a turbine, $H = 100$ horses, to work under a fall of $F = 25$ feet.

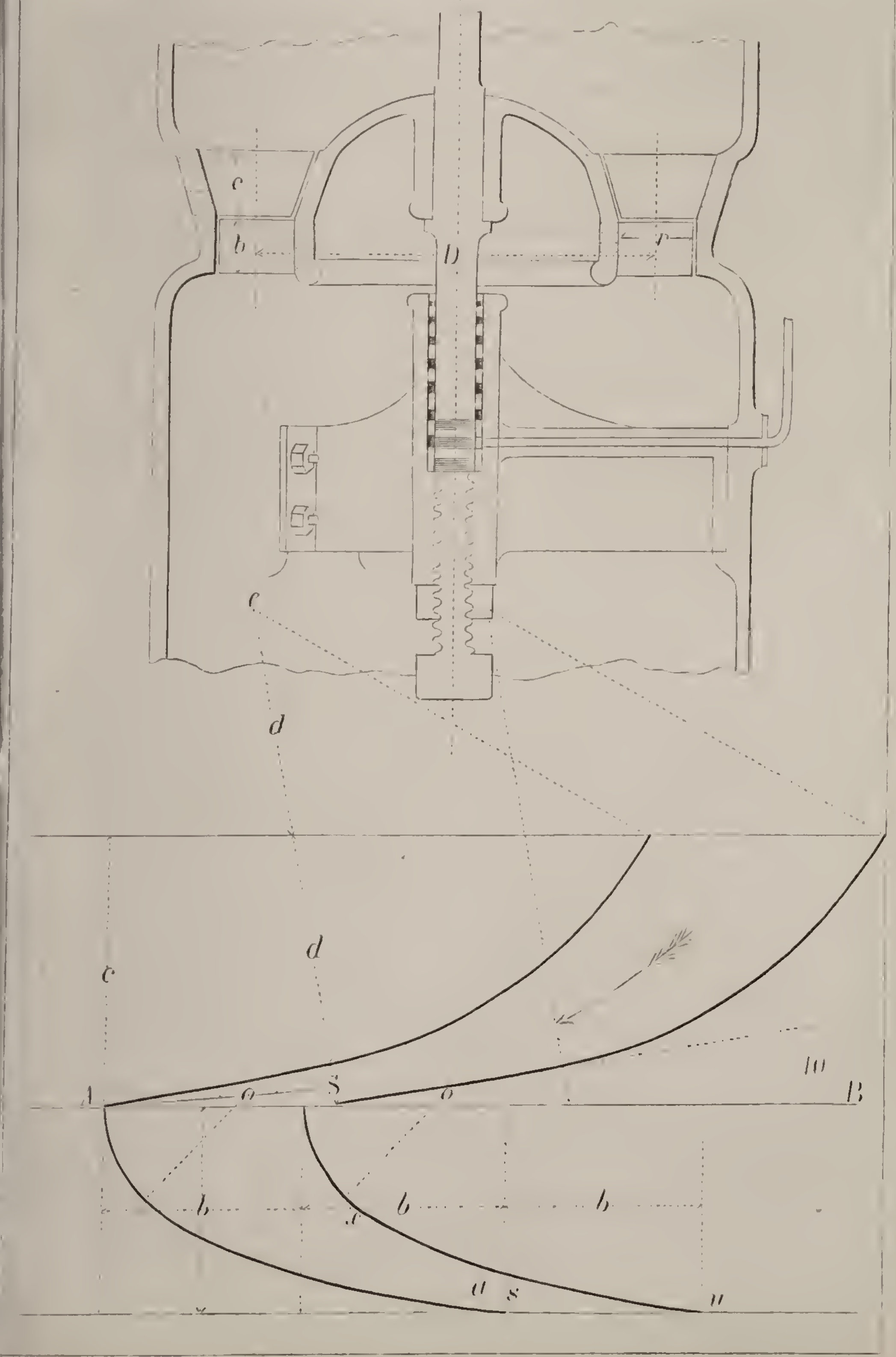
$$\$ = \frac{400\sqrt{100}}{\sqrt[3]{25}} = \frac{400 \times 10}{2.92} = 1375 \text{ dollars.}$$

Price List of Turbines in Dollars.

Horse power.	Head of fall in feet, *F*.									
	5	10	15	20	30	40	50	75	100	150
H	$	$	$	$	$	$	$	$	$	$
1	234	186	163	148	130	117	110	95	86	76
2	330	263	231	209	183	167	154	134	122	107
4	467	372	326	295	258	235	218	190	172	151
6	552	455	400	262	316	288	266	232	211	185
8	660	526	462	418	365	332	308	269	244	213
12	808	642	565	510	447	405	377	329	300	261
16	935	742	654	590	516	468	434	380	345	302
20	1045	830	730	660	577	524	485	425	385	338
30	1280	1020	894	810	705	642	595	520	472	414
40	1480	1180	1035	932	815	740	686	600	545	476
50	1650	1320	1155	1045	913	828	768	671	610	532
60	1810	1440	1264	1140	1000	908	840	735	668	584
80	2090	1645	1460	1320	1155	1050	975	848	770	674
100	2340	1860	1630	1480	1295	1175	1090	948	860	753
125	2620	2080	1820	1650	1440	1310	1220	1060	984	845
150	2860	2280	2000	1810	1580	1440	1330	1170	1060	924
175	3100	2460	2150	1950	1700	1550	1440	1260	1140	1000
200	3310	2630	2300	2090	1825	1655	1540	1350	1220	1066
225	3510	2790	2440	2215	1935	1755	1630	1430	1300	1130
250	3700	2940	2570	2335	2035	1850	1720	1500	1365	1195
275	3890	3090	2700	2450	2135	1945	1800	1580	1430	1255
300	4060	3225	2820	2560	2235	2030	1890	1650	1500	1300
350	4380	3480	3035	2760	2410	2190	2035	1780	1610	1410
400	4680	3720	3250	2950	2580	2310	2175	1900	1725	1510
450	5080	3950	3450	3130	2740	2480	2310	2015	1825	1600
500	5240	4160	3610	3300	2890	2620	2535	2125	1925	1685

JONVAL'S TURBINE,

as constructed by John W. Nystrom.



Weir Measurement of Water-Flow.

Q = cubic feet of water flowing over the weir per second.
G = gallons of 231 cubic inches each flowing over the weir per hour.
h = height of water-level over the weir in inches.
L = width of weir in feet.

$$Q = \frac{L h \sqrt{h}}{14.187}. \quad \text{Theoretic formula, } Q = \frac{L h \sqrt{h}}{7.76}. \quad G = 1898.15\, h \sqrt{h}.$$

When the water-level over the weir is fluctuating during the experimental measurement, it will not be correct to take the average of h for the calculation, but the average of $h\sqrt{h}$ must be used.

The following table gives the value of $h\sqrt{h}$ up to 24 inches and for every 10th of an inch.

Example. How many cubic feet of water will flow per second over a weir of $L = 3$ feet wide and $h = 6.3$ inches of water over the comb?

$$Q = \frac{3 \times 15.812}{14.187} = 3.3435 \text{ cubic feet.}$$

Value of $h\sqrt{h}$ for Weir Measurements.

h	Tenths of an Inch of the Height h.									
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0.	0.0316	0.0894	0.1643	0.2530	0.3536	0.4647	0.5858	0.7155	0.8531
1	1.	1.1537	1.3145	1.4830	1.6565	1.8371	2.0238	2.2164	2.4149	2.6189
2	2.8284	3.0432	3.2631	3.4882	3.7180	3.9526	4.1927	4.4365	4.6868	4.9385
3	5.1961	5.4581	5.7243	5.9947	6.2692	6.5479	6.8305	7.1171	7.4074	7.7020
4	8.0000	8.3018	8.6074	8.9166	9.2285	9.5459	9.8658	10.189	10.516	10.846
5	11.180	11.520	11.858	12.196	12.550	12.897	13.258	13.610	13.975	14.331
6	14.697	15.065	15.437	15.812	16.192	16.575	16.955	17.343	17.734	18.120
7	18.522	18.924	19.324	19.724	20.133	20.540	20.950	21.367	21.875	22.210
8	22.624	23.056	23.488	23.910	24.350	24.781	25.222	25.663	26.156	26.558
9	27.000	27.455	27.910	28.361	28.820	29.282	29.747	30.211	30.678	31.151
10	31.623	32.111	32.570	33.060	33.538	34.027	34.513	35.000	35.495	35.985
11	36.483	36.985	37.483	37.999	38.494	39.000	39.500	40.018	40.548	41.040
12	41.569	42.090	42.610	43.145	43.675	44.192	44.725	45.255	45.800	46.333
13	46.872	47.418	47.958	48.506	49.055	49.600	49.934	50.707	51.266	51.824
14	52.383	52.944	53.508	54.075	54.644	55.219	55.786	56.368	56.936	57.514
15	58.094	58.676	59.262	59.846	60.439	61.022	61.614	62.206	62.803	63.400
16	64.000	64.600	65.200	65.809	66.413	67.024	67.632	68.245	68.853	69.473
17	70.092	70.714	71.333	71.954	72.584	73.210	73.835	74.463	75.095	75.730
18	76.367	77.005	77.643	78.284	78.930	79.570	80.216	80.865	81.514	82.169
19	82.819	83.472	84.130	84.789	85.450	86.110	86.772	87.440	87.970	88.771
20	89.442	90.114	90.790	91.463	92.140	92.819	93.496	94.180	94.862	95.548
21	96.234	96.922	97.614	98.303	99.000	99.690	100.39	101.09	101.79	102.49
22	103.19	103.89	104.60	105.30	106.00	106.70	107.41	108.15	108.85	109.59
23	110.30	111.04	111.73	112.48	113.19	113.90	114.63	115.38	116.12	116.80
24	117.57	118.35	119.06	119.79	120.51	121.25	122.00	122.75	123.50	124.25

LIGHT AND COLORS.

Light is the sensation transmitted to the eye, and produces the sense of seeing. Light is a component part of heat, and a compound imponderable substance whose ingredients depend upon the composition of the burning substance; or, burning substances can be analyzed by decomposition of its light in a spectrum.

Decomposition of Light in the Spectrum.

Colors.	Maximum ray.	Combination of Colors.		
		Primary.	*Secondary.*	*Tertiary.*
Violet.	Chemical.	Blue. Yellow.	Green.	Dark Green.
Indigo.				
Blue.	Electrical.	Blue. Red.	Purple.	
Green.				
Yellow.	Light.	Yellow. Red.	Orange.	Brown.
Orange.				
Red.	Heat.			

All the colors of the spectrum mixed together make white, which is proved by the decomposition of white light, which makes the seven colors.

The velocity of light in planetary space is 192500 miles per second. The velocity of light through transparent bodies is not known, but probably varies inverse as the square root of the specific gravity of the transparent substance.

Light passes from the sun to the earth, 95000000 miles, in eight minutes, at which rate of velocity light can pass around the earth in one-eighth of a second.

The intensity of light is inversely proportional to the square of the distance from the luminous body.

The standard unit for measuring the intensity of light is assumed to be that produced by a sperm candle, "short 6," burning 120 grains per hour.

A spermatic candle 0.85 in diameter burns about 1 inch per hour.

MOTION OF GAS IN PIPES.

Notation.

Q = cubic feet of gas passed through the gas-pipe per hour.
L = length in feet, D = diameter in inches, of the pipe.
H = head of water in inches which presses the gas through the pipe.
S = specific gravity of the gas, air being 1.
n = number of candles required for giving the same light as Q cubic feet of gas per hour. (Rudely approximate for 16-candle gas).

$$Q = 780\,\pi D^2\sqrt{\frac{H\,D}{S\,L}}, \qquad Q = \sqrt{n} + 1, \quad n = (Q-1)^2 \qquad D = \frac{1}{22.65}\sqrt[5]{\frac{S\,L\,Q^2}{H}}.$$

Example. At a distance of $L = 6450$ feet from the gas-work is required $Q = 940$ cubic feet of gas per hour. Head of water being $H = 1$ inch, specific gravity $s = 0.5$. Required, the diameter of the pipe $D = ?$

$$D = \frac{1}{22.65}\sqrt[5]{\frac{0.5 \times 6450 \times 940^2}{1}} = 3\tfrac{1}{2}\text{ inches.}$$

Each light in a room consumes about 4 cubic feet of gas per hour, and ordinary street-lights 5 cubic feet.

Differing formulæ are given by authorities. See Clegg on Coal-Gas.

THE ATMOSPHERE.

The mean height of the atmosphere is about 302 feet greater at the equator than at the poles, which is caused by the difference of the earth's attraction at the two places, and also by centrifugal force.

The mean height of the atmosphere in 45° latitude is 60158.6 feet; at the poles, 60007.6 feet, and at the equator, 60309.6 feet.

The temperature of the atmosphere is greatest at the surface of the earth, and decreases with the height above the surface. The compression of the air by the upper layers of the atmosphere generates heat in the lower layers, as explained in the article on Air and Heat. The rays of light from the sun, passing through a denser air near the surface of the earth, also generate more heat by friction, as it were. The temperature of congelation of water is 32°, which is marked by the perpetual snow-line on high mountains, as shown in the accompanying table.

Heights of Snow-Line in Different Latitudes.

Latitudes of snow-line on high mountains.

5°	15°	25°	35°	40°	45°	55°	65°	75°	85°
15,210	14,760	12,560	10,290	9,000	7,670	5,030	2,230	1,016	120

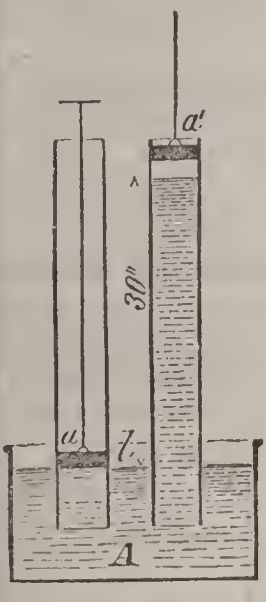
Heights of snow-line in feet above the sea.

New-fallen snow occupies eight times its volume in water.

Heat is constantly absorbed from the atmosphere by evaporation of water on the surface of the seas, which heat is carried up and warms the atmosphere above; heat is also absorbed by support of the growth of vegetation on land. It is this operation of consuming and generating heat which causes the winds and difference of weather.

As the atmosphere is a material substance, it is subject to the action of the force of gravity, which causes a pressure of 14.75 pounds to the square inch at the level of the sea; or a column of air one square inch base and of the height of the atmosphere weighs 14.75 pounds, which balances an equal weight of a column of mercury 30 inches high at the temperature of 60° Fahr., or a column of water of 34 feet high.

Columns of Air, Mercury and Water.



A is a vessel full of mercury, in which is placed vertically a glass tube about 3 feet high above the surface *l*; in the glass tube is fitted an air-tight piston *a*, just one square inch area, which can be moved by the piston-rod *c*. Now let the piston stand at *a* on the level *l*, and in contact with the mercury in the tube. Raise the piston by the piston-rod and handle *c*. The mercury in the tube will follow until the height of 30 inches is reached, and there remain even if the motion of the piston is continued. Now it may be supposed that it is some force of the piston that draws the mercury up in the tube. If so, why did it separate at 30 inches? If the column becomes too heavy, it could separate at *l*, and the 30 inches of mercury follow the piston. But this is not the case. The weight of the atmosphere pressing on the surface *l* forces the mercury up the tube until the weight of the column of mercury and the external pressure are in equilibrium, which occurs when the column has reached a height of 30 inches. The piston only served to overcome the atmospheric pressure in the tube. We have then the weight of a column of atmospheric air with one square inch base equal to the weight of a column of mercury 30 inches high and one sq. in. base. One cubic inch of mercury at 60° Fahr. weighs 0.941 pounds; this, multiplied by the height, 30 inches, gives 14.73 pounds, the weight of the column of mercury or atmosphere; this is generally termed "the atmospheric pressure per square inch."

The specific gravity of mercury at 60° Fahr. is 13.58, and

$$\frac{13.58 \times 30}{12} = 33.95 \text{ feet,}$$

the height of a column of water required to balance the atmosphere.

WIND, AERODYNAMICS.

The motions and effects of gases by the force of gravity are analagous to that of liquids. (See Hydraulics.) (See Weisbach's *Mechanics of Engineering*, 1st vol.)

The altitude or head of the atmosphere at uniform density will be the altitude of a column of water 33.95 feet, divided by the specific gravity of the air, 0.0012046, or,

$$\frac{33.95}{0.0012046} = 28133 \text{ feet.}$$

The velocity due to this head will be—

$V = 8.02\sqrt{28133} = 1346.4$ feet per second, the velocity with which the air will pass into a vacuum.

Velocity of Wind.

When air passes into an air of less density, the velocity of its passage is measured by the difference of their density.

H and h = density of the air in inches of mercury; t = temperature at the time of passage; and V = velocity of the wind in feet per second.

$$V = 1346.4\sqrt{\frac{H-h}{h}\left(1 + 0.00208t\right)}, \quad . \quad . \quad . \quad 6.$$

The force of wind increases as the square of its velocity.

a = area exposed at right angles to the wind in square feet; F = force of the wind in pounds; H = horse-power, and v = velocity of the plane **a** in direction of the wind, + when it moves opposite, and — when it moves with the wind.

$$F = 0.002288\mathbf{a}\,V^2, \qquad \text{when } v = o, \quad . \quad . \quad 7.$$

$$F = 0.002288\mathbf{a}(V \pm v)^2, \quad 8. \qquad H = \frac{\mathbf{a}v(V \pm v)^2}{240384.6}, \quad 9.$$

Example. A rail-train running *ENE* 25 miles per hour exposes a surface of 1000 square feet to a pleasant brisk gale *NE* by *E*. Required the resistance to the train in the direction it moves, and the horse-power lost.

$ENE - NE$ by $N = 3$ points $= 33° 45'$; $V = 14$ feet per second, a brisk gale; $v = 25 \times 1.467 = 36.6$ feet per second, and $F = 0.002288 \sin.^2 33° 46' \times 1000\ (14 + \cos. 33° 45' \times 36.6)^2 = 305.1$ pounds.

$$H = \frac{305.1 \times 36.6}{550} = 20 \text{ horses.}$$

Table of Velocity and Force of Wind, in Pounds per Square Inch.

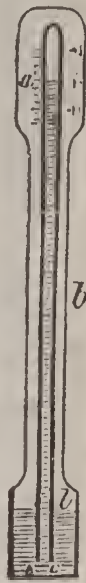
Miles per hour.	Feet per second	Force per sq. ft. pound.	Common Appellations of the force of Winds.	Miles per hour.	Feet per second	Force per sq. ft. pound.	Common Appellations of the force of Winds.
1	1.47	0.005	Hardly perceptible.	18	26.4	1.55	Very brisk.
2	2.93	0.020	Just perceptible.	20	29.34	1.968	
3	4.4	0.044		25	36.67	3.075	
4	5.87	0.079	Gentle pleasant wind.	30	44.01	4.429	High wind.
5	7.33	0.123		35	51.34	6.027	
6	8.8	0.177		40	58.68	7.873	
7	10.25	0.241		45	66.01	9.963	Very high.
8	11.75	0.315	Pleasant brisk gale.	50	73.35	12.30	
9	13.2	0.400		55	80.7	14.9	
10	14.67	0.492		60	88.02	17.71	Storm.
12	17.6	0.708		66	95.4	20.85	
14	20.5	0.964		70	102.5	24.1	Great storm.
15	22.00	1.107		75	110	27.7	Hurricane.
16	23.45	1.25		80	117.36	31.49	
				100	140.66	50.	Tornado.

THE BAROMETER.

The barometer measures the pressure of the atmosphere, as described in the former editions of this Pocket Book.

The English have graduated the barometer to indicate weather as follows:

Barometer in inches.		*Weather.*
At 28.3	=	Stormy.
At 28.7	=	Much rain.
At 29.1	=	Rain.
At 29.5	=	Change of weather.
At 29.9	=	Fair weather.
At 30.3	=	Set fair.
At 30.7	=	Very dry.



The following guides in predicting weather-changes are selected from the "Barometer Manual" of the London Board of Trade:

I. If the mercury, standing at thirty inches, rises gradually while the thermometer falls, and dampness becomes less, N.W., N. or N.E. wind; less wind or less snow and rain may be expected.

II. If a fall take place with a rising thermometer and increasing dampness, wind and rain may be expected from S.E., S. or S.W. A fall in winter with a low thermometer foretells snow.

III. An impending north wind, before which the barometer often rises, may be accompanied with rain, hail or snow, and so forms an apparent exception to the above rules, for the barometer always rises with a north wind.

IV. The barometer being at 29½ inches, a rise foretells less wind or a change of it northward, or less dampness. But if at 29 inches, a fast first rise precedes strong winds or squalls from N.W., N. or N.E., after which a gradual rise with falling thermometer, a S. or S.W. wind will follow, especially if the rise of the barometer has been sudden.

V. A rapid barometric rise indicates unsettled, and a rapid fall stormy, weather with rain or snow; while a steady barometer, with dryness, indicates continued fine weather.

VI. The greatest barometric depressions indicate gales from S.E., S. or S.W.; the greatest elevations foretell wind from N.W., N. or N.E., or calm weather.

VII. A sudden fall of the barometer, with a westerly wind, is sometimes followed with a violent storm from the N.W., N. or N.E.

VIII. If the wind veer to the south during a gale from the E. to S.E., the barometer will continue to fall until the wind is near a marked change, when a lull may occur. The gale may afterward be renewed, perhaps suddenly and violently; and if the wind then veer to the N.W., N. or N.E., the barometer will rise and the thermometer fall.

IX. The maximum height of the barometer occurs during a north-east wind, and the minimum during one from the south-west; hence these points may be considered the poles of the wind. The range between these two heights depends on the direction of the wind, which causes, on an average, a change of half an inch; on the moisture of the air, which produces, in extreme cases, a change of half an inch; and on the strength of the wind, which may influence the barometer to the extent of two inches. These causes, separately or conjointly with the temperature, produce either steady or rapid barometric variations, according to their force.

HYGROMETRY.

On the Humidity and other Properties of Air, deduced from Glaisher's Tables of the Greenwich Observatory.

Mason's hygrometer, consisting of wet and dry bulb thermometers, is considered the best for determining the dew-point and the humidity of the air.

Example. The temperature of the air being 75°, and the wet-bulb thermometer showing 63°, or 12° cold; barometer 30 inches. Required, the humidity of the air, the dew-point, weight of vapor per cubic foot, and the weight of a cubic foot of the air in grains troy?

Table I., 75° and 12° cold = 55 per cent. of humidity.
Table II., " " = 57° temperature of dew-point.
Table III., weight of dry air = 516.7 grains per cubic foot.
" " " " saturated air = 511.4 " " "
Difference = 5.3 × 0.55 = 2.915 grains.
Weight of the air 511.4 + 2.9 = 514.3 grains per cubic foot.
Table III., 9.31 + 0.55 = 5.12 grains of vapor per cubic foot.

The weight of air of equal temperature and humidity varies inversely as the height of the barometer.

TABLE I.

Humidity of the Air, or Percentage of Full Saturation,

At Different Temperatures, indicated by the Dry and Wet Bulbs of the Hygrometer (Glaisher).

Temp. of the air, Fahr.	Difference in Temperature, or Cold on the Wet-bulb Thermometer.																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
30	86	73	55																				
35	91	83	76	70	64	57	53	48															
40	93	86	79	74	68	63	58	53	50	46													
45	93	86	80	74	69	64	59	55	51	48													
50	93	87	81	76	71	66	61	57	53	49	46												
55	94	88	83	78	75	69	65	60	56	53	50	49	46	44	41	39	36						
60	94	89	84	80	75	71	67	63	59	56	53	50	47	45	42	40	38	35	33				
65	95	89	85	81	76	72	69	65	61	58	55	52	49	47	44	42	40	37	35	34	32	30	28
70	95	91	86	82	78	74	71	67	64	61	58	55	52	49	47	45	42	40	37	35	34	31	29
75	95	90	86	82	78	74	71	68	64	61	58	55	52	49	48	47	44	41	39	37	35	32	30
80	95	90	87	83	79	75	72	68	65	62	59	56	53	50	49	48	44	42	40	38	36	33	31
85	96	91	87	83	79	75	72	68	65	62	59	56	54	51	49	46	44	42	40	38	36	34	32
90	96	91	87	83	79	75	72	68	65	62	59	56	54	51	49	46	44	42	40	38	36	34	32
	Percentage of Humidity.																						

TABLE II.

Temperature of the Dew-point,

At Different States of the Hygrometer.

Temp. of the air, Fahr.	Difference in Temperature, or Cold on the Wet-bulb Thermometer.																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
30	25	21																					
35	32	30	27	25	22	20	17	15	13														
40	37	35	33	31	29	27	25	22	20	18													
45	43	41	39	37	34	32	30	28	26	24	22												
50	48	46	44	42	40	38	36	34	32	30	28	26											
55	53	52	50	48	46	45	43	41	40	38	36	34	33	31	29	28	26	24					
60	58	56	55	53	51	50	48	46	45	43	41	39	38	36	34	33	31	29	28	26			
65	63	62	60	58	57	55	54	52	50	49	47	46	44	43	41	39	38	36	34	33	31	30	28
70	68	67	65	64	62	61	59	58	56	55	53	52	50	49	47	46	44	43	41	40	38	37	35
75	73	72	70	69	67	66	64	63	61	60	58	57	55	54	52	51	49	48	46	45	43	42	40
80	78	77	75	74	72	71	69	68	66	65	63	62	60	59	57	56	54	53	51	50	48	47	45
85	84	83	82	81	80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62
90	89	88	87	86	85	84	83	82	81	80	79	78	77	76	75	74	73	72	71	70	69	68	67
	Temperature of Dew-point.																						

TABLE III.

Properties of Air, by Glaisher, Greenwich Observatory.

Barometer 30 inches, at 60° Fahrenheit.

Temp. of the air.	Force of vapor in inches of mercury	Weight of vapor per cub. ft. of sat. air.	Wt. per cub. ft. Dry air.	Wt. per cub. ft. Saturated air.	Temp. of the air.	Force of vapor in inches of mercury	Weight of vapor per cub. foot of sat. air.	Wt. per cub. ft. Dry air.	Wt. per cub. ft. Sat'd air.
Fahr.	Inches.	Grains.	Grains.	Grains.	Fahr.	Inches.	Grains.	Grains.	Grains.
10°	0.089	1.11	590.0	589.4	52°	0.400	4.56	540.5	537.9
11	0.093	1.15	588.7	588.1	53	0.414	4.71	539.4	536.7
12	0.096	1.19	587.5	586.8	54	0.428	4.86	538.3	535.5
13	0.100	1.24	586.2	585.5	55	0.442	5.02	537.3	534.4
14	0.104	1.28	584.9	584.2	56	0.458	5.18	536.2	533.2
15	0.108	1.32	583.7	582.9	57	0.473	5.34	535.1	532.1
16	0.112	1.37	582.4	581.6	58	0.489	5.51	534.1	530.9
17	0.116	1.41	581.1	580.3	59	0.506	5.69	533.0	529.8
18	0.120	1.47	579.9	579.1	60	0.523	5.87	532.0	528.6
19	0.125	1.52	578.7	577.8	61	0.541	6.06	530.9	527.5
20	0.129	1.58	577.4	576.5	62	0.559	6.25	529.9	526.3
21	0.134	1.63	576.2	575.3	63	0.578	5.45	528.8	525.2
22	0.139	1.69	575.0	574.0	64	0.597	6.65	527.8	524.0
23	0.144	1.75	573.7	572.7	65	0.617	6.87	526.9	522.9
24	0.150	1.81	572.5	571.5	66	0.638	7.08	525.8	521.7
25	0.155	1.87	571.3	570.2	67	0.659	7.30	524.7	520.6
26	0.161	1.93	570.1	569.0	68	0.681	7.53	523.7	519.4
27	0.167	2.00	568.9	567.7	69	0.704	7.76	522.7	518.3
28	0.173	2.07	567.7	566.5	70	0.727	8.00	521.7	517.2
29	0.179	2.14	566.5	565.3	71	0.751	8.25	520.7	516.0
30	0.186	2.21	565.3	564.1	72	0.776	8.50	519.7	514.9
31	0.192	2.29	564.2	562.8	73	0.801	8.76	518.7	513.7
32	0.199	2.37	563.0	561.6	74	0.827	9.04	517.7	512.6
33	0.207	2.45	561.8	566.4	75	0.854	9.31	516.7	511.4
34	0.214	2.53	560.7	559.2	76	0.882	9.60	515.7	510.3
35	0.222	2.62	559.5	558.0	77	0.910	9.89	514.7	509.2
36	0.230	2.71	558.3	556.8	78	0.940	10.19	513.8	508.0
37	0.238	2.80	557.2	555.6	79	0.970	10.50	512.8	506.9
38	0.246	2.89	556.0	554.4	80	1.001	10.81	511.8	505.7
39	0.255	2.99	554.9	553.2	81	1.034	11.14	510.9	504.6
40	0.264	3.09	553.8	552.0	82	1.067	11.47	509.9	503.4
41	0.274	3.19	552.6	550.8	83	1.101	11.82	508.9	502.3
42	0.283	3.30	551.5	549.6	84	1.136	12.17	508.0	501.1
43	0.293	3.41	550.4	548.4	85	1.171	12.53	507.0	500.0
44	0.304	3.52	549.3	547.2	86	1.209	12.91	506.1	498.9
45	0.315	3.64	548.1	546.1	87	1.247	13.29	505.1	497.7
46	0.326	3.76	547.0	544.9	88	1.286	13.68	504.2	496.6
47	0.337	3.88	546.0	543.7	89	1.326	14.08	503.2	495.4
48	0.349	4.01	544.8	542.5	90	1.368	14.50	502.3	494.3
49	0.361	4.14	543.7	541.3	91	1.411	14.91	501.3	493.2
50	0.373	4.28	542.6	540.2	92	1.456	15.33	500.4	492.0
51	0.386	4.42	541.5	539.0	93	1.502	15.76	499.4	491.9

MEAN TEMPERATURE AT DIFFERENT SEASONS OF THE YEAR.

LOCATIONS.	Mean Temperature, Fahr.					Hemisphere.	Height ab. sea. Feet.
	Year.	Spring.	Sum.	Autm.	Wint'r.		
Algiers,	63.0	63.0	74.5	70.5	54.0	N.	310
Berlin,	47.5	46.4	63.1	47.8	30.6	N.	128
Berne,	46.0	45.8	60.4	47.3	30.4	N.	1918
Boston,	49	48	66	53	28	N.	71
Buenos Ayres,	62.5	59.4	73.0	64.6	52.5	S.	. . .
Cairo,	72.3	71.6	84.6	74.3	58.5	N.	. . .
Calcutta,	78.4	82.6	83.3	80.0	67.8	N.	. . .
Canton,	69.8	69.8	82.0	72.9	54.8	N.	10
Christiania,	41.7	39.2	59.5	42.4	25.2	N.	74
Cape of Good Hope,	66.4	63.5	74.1	66.9	58.6	S.	. . .
Constantinople,	56.7	51.8	73.4	60.4	40.6	N.	150
Copenhagen,	46.8	43.7	63.0	48.7	31.3	N.	20
Edinburgh,	47.5	45.7	57.9	48.0	38.5	N.	288
Jerusalem,	62.2	60.6	72.6	66.3	49.6	N.	2500
Jamaica (Kingston),	79.0	78.3	81.3	80.0	76.3	N.	10
Lima, Peru,	66.2	63.0	73.2	69.6	59.0	S.	511
Lisbon,	61.5	59.9	71.1	62.6	52.3	N.	236
London,	50.7	49.1	62.8	51.3	39.6	N.	50
Madeira (Funchal),	65.7	63.5	70.0	67.6	61.3	N.	. . .
Madrid,	57.6	57.6	74.1	56.7	42.1	N.	2175
Mexico, City,	60.5	53.6	63.4	65.2	60.1	N.	6990
Montreal,	43.7	44.2	69.1	47.1	17.5	N.	. . .
Moscow,	38.5	43.3	62.6	34.9	13.5	N.	480
Naples,	61.5	59.4	74.8	62.2	49.6	N.	180
New Orleans,	72	73	84	72	58	N.	20
New York,	53	50	72	56	33	N.	20
New Zealand,	59.6	60.1	66.7	58.0	53.5	S.	. . .
Nice,	60.1	55.9	72.5	63.0	48.7	N.	. . .
Nicolaief (Russia),	48.7	49.3	71.2	50.0	25.9	N.	. . .
Paramatta (Australia),	64.6	66.6	73.9	64.8	54.5	S.	. . .
Palermo,	63.0	59.0	74.3	66.2	52.5	N.	180
Pekin, China,	52.6	56.6	77.8	54.9	29	N.	97
Paris,	51.4	50.5	64.6	52.2	37.9	N.	210
Philadelphia,	55	52	76	57	34	N.	30
Quito, Ecuador,	60.1	60.3	60.1	62.5	59.7	S.	9560
Rio Janeiro,	73.6	72.5	79.0	74.5	68.5	S.	10
Rome,	59.7	57.4	73.2	61.7	46.6	N.	174
San Francisco,	57.5	58	59	60	53	N.	150
St. Petersburg,	38.3	35.1	60.3	40.5	16.7	N.	10
Stockholm,	42.1	38.3	61.0	43.7	25.5	N.	134
Trieste,	55.8	53.8	71.5	56.7	39.4	N.	288
Turin,	53.1	53.1	71.6	53.8	33.4	N.	915
Vienna,	50.7	49.1	62.8	51.3	39.6	N.	480
Warsaw,	45.5	44.6	63.5	46.4	27.5	N.	397
Washington,	59	60	79	58	38	N.	. . .

Seasons.

Southern Latitude.			Seasons.	Northern Latitude.		
December,	January,	February,	Summer.	June,	July,	August.
March,	April,	May,	Fall.	September,	October,	November.
June,	July,	August,	Winter.	December,	January,	February.
September,	October,	November,	Spring.	March,	April,	May.

Rain and Melted Snow.

Fall in Inches at Different Places.

LOCATIONS.	Year.	Spring.	Summ'r.	Fall.	Winter.
Albany, North America, . .	40.67	9.79	12.3	10.3	8.30
Algiers, ...	37.04	8.34	0.60	10.3	17.8
Baltimore, North America, .	42.00	11.2	11.1	10.52	9.31
Berlin, Prussia, ...	23.56	5.66	7.21	5.45	5.24
Bergen, Norway, ...	87.61	15.7	18.6	29.8	23.5
Bombay, India.	110.	. . .	. . .	. . .	. . .
Boston, North America, . .	44.48	10.8	11.8	12.57	9.89
Buffalo, " . .	27.35	5.90	8.45	7.48	5.52
Canton, China, ...	69.3	18.8	27.9	19.3	3.3
Charleston, North America, .	48.29	8.60	18.7	11.6	9.40
Copenhagen, ...	18.35	2.84	6.86	5.13	3.52
Dover, England, ...	38.	. . .	. . .	. . .	. . .
Dublin, Ireland, ...	25.	. . .	. . .	. . .	. . .
Edinburgh, Scotland, . . .	28.	. . .	. . .	. . .	. . .
England, ...	33.	. . .	. . .	. . .	. . .
Glasgow, ...	28.9	5.43	7.13	8.95	7.39
Granada (Colombia), . .	115.	. . .	. . .	. . .	. . .
Liverpool, ...	34.1	6.19	9.78	10.8	7.32
Lima, Peru, ...	13.5	5.1	0.2	1.2	7.0
London, ...	20.69	4.09	6.00	6.15	4.45
Madeira Islands, ...	30.87	5.11	2.30	6.96	16.5
Manchester, England, . .	36.	7.	9.	11.	9.
Milano, Italy, ...	38.	9.04	9.18	11.7	8.05
Mississippi State, ...	53.00	10.9	14.2	9.50	18.4
New York, ...	42.23	11.5	11.3	10.3	9.63
New Orleans, ...	52.31	13.3	16.1	10.8	12.6
Ohio, State, ...	39.69	10.4	10.9	9.03	6.91
Pekin, China, ...	26.9	2.67	20.5	3.22	0.53
Peru (Interior), Carabaya, .	355.	88.	120.	87.	60.
St. Petersburg, ...	17.65	2.89	6.73	5.11	2.93
Paris, ...	22.64	5.53	5.92	6.51	4.68
Philadelphia, ...	48.00	13.	12.	11.	12.
Rio Janeiro, Brazil, . . .	. . .	. . .	. . .	. . .	10.76
Rome, Italy, ...	30.87	7.27	3.4	10.9	9.3
Stockholm, ...	19.67	2.17	7.81	6.94	2.75
Tiflis, Caucasus, ...	19.26	6.25	7.62	3.51	1.88
Washington, ...	41.20	10.4	10.5	10.2	11.1
San Francisco. California, . .	83.	22.	1.	15.	45.

Volume of Evaporation and Rain-Fall.

Inches × 2.323 200 = cubic feet per square mile.
Inches × 17,335,019 = gallons per square mile.
Inches × 3630 = cubic feet per acre.

Length in Miles of the Principal Rivers.

EUROPE.		NORTH AND SOUTH AMERICA.		ASIA AND AFRICA.	
Volga, Russia, . .	2000	Missouri, . . .	2900	Yang-tse-kiang, .	2800
Danube, ...	1600	Mississippi, . .	2800	Lena, ...	2600
Don and Dnieper, .	1000	Mackenzie's, . .	2500	Obe, Hoangho, .	2500
Rhine, ...	950	St. Lawrence, .	2200	Yenesei, . . .	2300
Dwina, ...	700	Rio Grande, . .	1800	Amor, ...	2200
Petchora, Elbe, Loire,	600	Colorado, Cal., .	1100	Cambodia, . . .	2000
Vistula. Tagus, . .	550	Alabama, . . .	600	Indus, Irrawaddy,	1700
Dniester, Guadiana,	500	Amazon, . . .	3600	Nile, ...	3000
Rhone, Po, Seine, .	450	Rio de la Plata, .	2250	Niger or Joliba,	2600
Mezene, Desna, .	400	Orinoco, . . .	1500	Senegal, . . .	1200
Dahl, Bug, ...	300	Araguay, . . .	1100	Orange, . . .	1000
Thames, ...	233	Magdalena, . .	900	Gambia, . . .	700

Evaporation on the Surface of Water in the Open Air.

When the surface of water is freely exposed to the atmosphere, the dry air in contact with it becomes charged with vapor, and consequently becomes lighter (see Table, page 357), rises, and gives place to drier air, which repeats the same operation. In this way moisture is constantly carried up into the air from the surface of the water. The rate of this evaporation depends upon the temperature of the water, the dryness, the temperature and the velocity of the air.

Evaporation of Water in Decimals of an Inch, per 24 Hours,

on the surface of fresh-water lakes, rivers and canals, at different temperatures of the water and currents of the air.

Water. Temp.	Velocity of wind in miles per hour on the water. Calm.	10	20	30	40	50	60
32°	0.012	0.014	0.016	0.017	0.019	0.021	0.023
35	0.020	0.023	0.026	0.029	0.032	0.035	0.038
40	0.040	0.046	0.052	0.058	0.064	0.070	0.076
45	0.068	0.078	0.088	0.098	0.109	0.119	0.129
50	0.100	0.115	0.130	0.145	0.160	0.175	0.190
55	0.133	0.153	0.173	0.193	0.213	0.233	0.253
60	0.177	0.203	0.230	0.256	0.283	0.310	0.336
65	0.225	0.259	0.292	0.326	0.360	0.394	0.427
70	0.278	0.320	0.361	0.404	0.444	0.486	0.527
75	0.335	0.385	0.435	0.485	0.535	0.585	0.635
80	0.400	0.460	0.520	0.580	0.640	0.700	0.760
85	0.468	0.538	0.608	0.679	0.749	0.819	0.889
90	0.540	0.621	0.703	0.784	0.865	0.946	1.025
95	0.620	0.713	0.808	0.900	0.995	1.088	1.180
100	0.700	0.805	0.912	1.015	1.123	1.225	1.332

The evaporation on the surface of salt water on the ocean is about 0.8 of that in the table.

The quantity of water evaporated on the surface of all the waters on the earth is equal to the quantity of rain-fall.

Area in Square Miles of the largest Inland Lakes.

LAKES.	Sq. Miles.
Eastern Hemisphere.	
Aral Sea, Tartary,	16650
Azov Sea, Russia,	8800
Baikal Sea, Siberia,	13000
Balkash, Mongolia,	5200
Black Sea, Turkey,	113000
Caspian Sea, Russia,	138000
Constance, Switzerland,	456
Dead Sea, Palestine,	370
Dembia, Abyssinia,	13000
Enare, Lapland,	870
Geneva, Switzerland,	400
Hjelmaren, Sweden,	900
Tchad, Africa,	11600
Ladoga, Russia,	6200
Loch Lomond, Scotland,	27
Lough Neagh, Ireland,	80
Onega, Russia,	3300
Ouroomia, Persia,	1000

LAKES.	Sq. Miles.
Tonting, China,	1200
Wenern, Sweden,	2400
Wettern, Sweden,	1045
Zaizan, Mongolia,	1600
Western Hemisphere.	
Athabasca, N. America,	3200
Erie Lake, N. America,	7000
Great Bear, N. America,	4000
Great Slave, N. America,	12000
Great Salt Lake,	1880
Huron, N. America,	22800
Maracaibo, S. America,	6000
Michigan, N. America,	22600
Nicaragua, Cent. America,	3905
Ontario, N. America,	4950
Otehenantekane, N. Amer.,	2500
Superior, N. America,	30000
Titicaca, Peru,	5400
Winnipeg, N. America,	7200

BAROMETRICAL OBSERVATIONS.

For Determining Difference of Levels.

Notation for the complete formulæ of La Place, in French and English measures.

$$\text{Lower station,}\ \left\{\begin{array}{l} h = \text{height of barometer} = h' \\ T = \text{temp. of barometer} = T' \\ t = \text{temp. of the air} = t' \end{array}\right\}\ \text{Upper station.}$$

H = height of barometer at the upper station reduced to the temperature of the barometer at the lower station. When the height is read on a brass scale, the reduction will be in

French measures.	English measures.
$H = h'\,[1 + 0.0001614\,(T - T')]$.	$H = h'\,[1 + 0.00008967\,(T - T')]$.

Mean radius of the earth = 6,366,200 metres = 20,886,860 feet.

Mean height of the atmosphere = 18,336 metres = 60,158.6 feet.

L = mean latitude between the two stations.

Z = difference of level between the two stations.

French measures.

$$Z = \underbrace{\log.\frac{h}{H} \times 18336}_{1} \times \left\{\begin{array}{ll} \left(1 + \frac{t + t'}{533.2}\right) \times \quad \ldots & 2. \\ (1 + 0.00251 \times \cos. 2L) \times \quad \ldots & 3. \\ \left(1 + \frac{Z + 15926}{6366200}\right). \quad \ldots & 4. \end{array}\right.$$

English measures.

$$Z = \underbrace{\log.\frac{h}{H} \times 60158.6}_{1} \times \left\{\begin{array}{ll} \left(1 + \frac{t + t' - 64}{960}\right) \times \quad \ldots & 2. \\ (1 + 0.00251 \times \cos. 2L) \times \quad \ldots & 3. \\ \left(1 + \frac{Z + 52252}{20886860}\right). \quad \ldots & 4. \end{array}\right.$$

The factor (1) gives the difference of level when the observations are made in a temperature of 32° Fahr., or 0° Cent., the freezing-point of water, and in latitude 45°, without the factors of correction (2), (3) and (4).

The factor (2) is the correction for temperature of the air above or below the freezing-point.

The factor (3) is the correction for latitude above or below 45°.

The factor (4) is the correction for the decrease of the earth's attraction. This correction is included in the following Table I., to suit any level of the stations.

There are some other barometrical corrections not included in the above formulæ, such as for humidity of the air, capularity and boiling of the glass tube, for the hour of the day and season of the year, all of which are so insignificant, uncertain and complicated that they have been omitted here.

Explanation of the Barometrical Tables.

The tables have been calculated in Peru, and practically used by the author.

Table I. is calculated from the factors (1) and (2), which gives the approximate heights above the level of the sea, in English and French measures, for every tenth of an inch from 11 to 31 inches. The mean temperature of the air and of the barometer is assumed to be 60° Fahrenheit = 15.555 Centigrade, and in latitude 45°. The barometer is assumed to be 30 inches = 760 millimetres at the level of the sea, but when it is observed to be higher or lower, make the corresponding addition or subtraction for difference of levels in the table.

Table II. contains the correction for difference of level in feet or metres at different temperatures of the air above or below 60° Fahr.

Table III. contains the correction for heights in different latitudes above or below 45°.

Tables IV. and V. are logarithmic corrections for temperature and latitude.

Table VII. gives the height of a column of air in metres, corresponding to a difference of one milimetre of mercury at different heights of the barometer.

Table VIII. gives the height of a column of air in feet. corresponding to a difference of one-tenth of an inch of mercury at different heights of the barometer.

Tables X. and XI. contain the correction for the mercurial column at different temperatures of the barometer above or below 60° Fahr. = 15.555 Cent. This correction must be made before the barometrical height is applied to Table I.

Table XII. contains the approximate mean temperature of the air at the level of the sea for every month of the year in different latitudes. This table has been deduced from observations of Mr. Dove, Humboldt, Raimondi, and other distinguished authors. The table agrees very well with the mean temperatures on the Atlantic and Pacific coasts, but will not answer for the North Sea and the Baltic, where the temperatures are much higher. A great deal of inconvenience may arise for want of a table of this kind.

When barometrical observations are made far inland, some means must be resorted to for estimating the temperature of the air at the level of the sea in the latitude of observation, in order to make proper corrections for difference of level. From all the meteorological observations of different authors it appears that the mean temperature of 24 successive hours is near 9 o'clock in the morning, and that the mean temperature of the day from 9 to 5 P. M. is at noon.

The variation of temperature throughout the day varies with the latitude, that is, the higher the latitude, the greater is the variation.

Example 1. On the 14th of March, 1869, 2h. 15m. P. M., in Oroya, Peru, latitude 11° 30′, the barometer stood 19.46 inches, the temperature of the air 62°, and that of the barometer 60°. Required, the height of Oroya above the level of the sea in feet?

Table I.	Barometer 19.4 in.,		= 12099 6 feet.
	Correction 0.6 × diff. 142.6 feet,		= 85.5 feet.
	Approximate height,		= 12014.1 feet.
	Temperature at Oroya,	62° Fahr.	
Table XII.	Latitude 11° 30′, 14th of March,	79°	
	Mean temp. of the column of air,	141 = 70 5°.	
Table II.	Correct mean temp. 70° of the air,	10000 feet =	208.2 feet.
		2000 feet =	41.6 feet.
		10 feet =	0.2 feet.
Table III.	Correct for lat. 11° 30′,	10000 feet =	23.6 feet.
		2000 feet =	4.8 feet.
		10 feet =	0.0 feet.
Sum of corrections,	. . . .		278.4 feet.
Approximate height,	. . . .		12014.1 feet.
Height of Oroya,	. . . .		12292.5 feet.

Example 2. In the city of Paucartambo, Peru, the barometer was observed to stand 21.272 inches, the temperature of the air 70°, and that of the barometer 69°, in latitude 13° 18′ south. About three miles from the city, on the mountain Huanacaury, the barometer stood 18.224 inches, the temperature of the air 62°, and that of the barometer 64°. Required, the height of the mountain above the city of Paucartambo?

Barometer at the lower station, .	21.272 inches.
Correction for 69°, Table XI., subtract	.017 inches.
Height of barometer at 60°, . .	21.255 inches.
Barometer at the upper station, .	18.224 inches.
Correction for 64°, Table XI., subtract	.006 inches.
Height of barometer at 60°, .	18.218 inches.

	Barometer.	Heights.
Table I.	18.218	13845.0, upper station.
	21.255	9565.3, lower station.

Logarithms	3.6314133 =	4279.7 feet, approximate height.
Table IV.	0.0053929 =	66° mean temperature.
Table V.	0.0009888 =	13° 18′ latitude.
	3.6377950 =	4343.1 feet, the height required.

Aneroid Barometer.

The aneroids made by Negretti & Zambra, London, are compensated, and show the height of a column of mercury at the temperature of the freezing-point of water, 32° Fahr., or zero Centigrade. The aneroid is not affected by different temperatures. When the aneroid is used with the accompanying Table I., a correction must be made to convert the column of mercury from 32° to 60° Fahr., namely:

Height of a column of mercury as indicated by the aneroid.

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
.046	.048	.050	.053	.056	.059	.061	.064	.067	.070	.073	.075	.078	.081	.084

Correction in fraction of an inch, always additive.

Example. Suppose the aneroid to indicate 25.261 inches.
Correction from the table, . .070
Height of a column of mercury, 25.331 inches at 60° Fahr.

Heights of the Principal Mountains and Volcanoes.

NORTH AMERICA.	Feet.	EUROPE.	Feet.	Volcanoes, Active.	Feet.
Mount St. Elias,	17,860	Elbruz, Caucasus,	17,776	Aconcagua, Chili,	23,100
Mt. Brown, R. M.,	16,000	Mont Blanc, Alps,	15,668	Gualatieri, Peru,	22,000
Sierra Nevada, Cal.,	15,500	Malhaven, Spain,	11,678	Cotapaxi, Equador,	19,500
Fremont's Peak,	13,470	Mt. Maladetta, Py.,	11,436	Misti, Peru*, . .	18,136
Long's Peak, R. M.,	12,500	Mt. Caballo, Alps,	10,154	Popocatapetl, .	17,735
Cibao Mt., Hayti,	8,600	Mt. Scardus, Tur.,	10,000	Pichincha, Equa.,	16,000
Cierra del Cobre,	7,200	Ural Mts., Russia,	5,397	Kliutchewaskaja,	15,763
Black Mt., S. C.,	6,476	ASIA.		Volcan de Fuego,	14,000
Mt. Washington.	6,234	Kunchinginga, Hy.	28,176	Mauna Loa, S. Isl.,	13,440
Mansfield Mt., Vt.,	4,280	Dhawalaghiri, Hy.,	28,000	St. Helen's, Oreg'n,	13,300
Peak of Otter, Vt.,	4,260	Hindo Koo, Cabul,	20,000	Indrapura, Sum.,	12,300
		Mt. Ararat, Tur.,	17,210	Teneriffe, Can. Isl.,	12,182
SOUTH AMERICA.		Mt. Lebanon, Syr.,	12,000	Erebus, Vic. Land,	12,400
Illimani, Bolivia,*	24,100	AFRICA.		Cartago, C. Amer.,	11,480
Ansangati, Peru,*	22,150	Abba Yared, Aby.,	15,200	Etna, Sicily, . .	10,874
Chimborazo, Eq.,	21,960	Piton des Neiges,	12,500	Hecla, Iceland, .	5,110
Sorato, Bolivia,	21,500	Talba Waha, Aby.,	12,000	Souffriere, Guad.,	5,108
Tolima, N. Gran.,	18,250	OCEANICA.		Jurollo, Mexico,	4,205
Cerro de Potosi,	16,150	Mt. Ophir, Sum.,	13,842	Vesuvius, Italy,	3,948
Cerro de Pasco,	13,780	Mt. Semero, Java,	13,000		
Organ Mt., Brazil,	7,500				

* Measured by the author of this Pocket-book.

TABLE I.

Barometric and Atmospheric Heights.

	French.		English.		
Dif.	Altitude. metres.	Bar. m.m.	Bar. in.	Altitude. feet.	Dif.
76.59	8488.09	279.4	**11.**	27848.5	251.3
75.87	8411.48	281.9	.1	27597.2	248.9
75.19	8335.61	284.5	.2	27348.3	246.7
74.56	8260.42	287.0	.3	27101.6	244.6
73.88	8185.86	289.5	.4	26857.0	242.4
73.24	8111.96	292.2	.5	26614.6	240.3
72.63	8038.74	294.7	.6	26374.3	238.3
71.98	7966.11	297.3	.7	26136.0	236.2
71.43	7894.13	299.8	.8	25899.8	234.3
70.77	7822.70	302.2	.9	25665.5	232.2
70.23	7751.93	304.8	**12.**	25433.3	230.4
69.61	7681.70	307.3	.1	25202.9	228.4
69.07	7612.09	309.8	.2	24974.5	226.6
68.48	7543.02	312.4	.3	24747.9	224.7
67.94	7474.54	314.9	.4	24523.2	222.9
67.42	7406.60	317.5	.5	24300.3	221.2
66.88	7339.18	320.0	.6	24079.1	219.4
66.38	7272.30	322.5	.7	23859.7	217.8
65.84	7205.92	325.1	.8	23641.9	216.0
65.32	7140.08	327.6	9	23425.9	214.3
64.80	7074.76	330.2	**13.**	23211.6	212.6
64.34	7009.96	332.7	.1	22999.0	211.1
63.88	6945.62	335.2	.2	22787.9	209.6
63.36	6881.74	337.8	.3	22578.3	207.9
62.91	6818.38	340.3	.4	22370.4	206.4
62.46	6755.47	342.9	.5	22164.0	204.9
61.97	6693.01	345.4	.6	21959.1	203.3
61.50	6631.04	347.9	.7	21755.8	201.8
61.09	6569.54	350.5	.8	21554.0	200.4
60.65	6508.45	353.0	.9	21353.6	199.0
60.23	6447.80	355.6	**14.**	21154.6	197.6
59.76	6387.57	358.1	.1	20957.0	196.1
59.34	6327.81	360.6	.2	20760.9	194.7
58.92	6268.47	363.2	.3	20566.2	193.8
58.68	6209.55	365.7	.4	20372.9	192.5
58.12	6150.87	368.3	.5	20180.4	190.9
57.79	6092.75	370.8	.6	19989.7	189.6
57.40	6034.96	373.3	.7	19800.1	188.3
56.90	5977.56	375.9	.8	19611.8	187.2
56.57	5920.66	378.4	.9	19425.1	185.6
56.20	5864.00	381.0	**15**	19239.5	184.4
55.84	5807.89	383.5	.1	19055.1	183.2
55.47	5752.05	386.0	.2	18871.9	182.0
55.11	5696.58	388.6	.3	18689.9	180.3
51.74	5641.47	391.1	.4	18509.1	179.6
54.41	5586.73	393.6	.5	18329.5	178.5
54.10	5532.32	396.2	.6	18151.0	177.5
53.70	5478.22	398.7	.7	17973.5	176.2
53.37	5424.52	401.3	.8	17797.3	175.1
53.04	5371.15	403.8	.9	17622.2	174.0

	French.		English.		
Dif.	Altitude. metres.	Bar. m.m.	Bar. in.	Altitude. feet.	Dif.
52.72	5318.11	406.4	**16.**	17448.2	173.0
52.49	5265.39	408.9	.1	17275.2	171.9
51.94	5212.90	411.4	.2	17103.3	170.7
51.78	5160.96	414.0	.3	16932.6	169.9
51.39	5109.18	416.5	.4	16762.7	168.6
51.14	5057.79	419.1	.5	16594.1	167.7
50.82	5006.65	421.6	.6	16426.3	166.7
50.50	4955.83	424.1	.7	16259.6	165.7
50.20	4905.33	426.7	.8	16093.9	164.7
49.90	4855.13	429.2	.9	15929.2	163.7
49.65	4805.23	431.8	**17.**	15765.5	162.8
49.34	4755.58	434.3	.1	15602.6	161.9
49.07	4706.24	436.8	.2	15440.7	161.0
48.77	4657.17	439.4	.3	15279.7	160.0
48.47	4608.40	441.9	.4	15119.7	159.0
48.18	4559.93	444.5	.5	14960.7	158.1
47.91	4511.75	447.0	.6	14802.6	157.2
47.64	4463.84	449.5	.7	14645.4	156.3
47.40	4416.20	452.1	.8	14489.1	155.5
47.15	4368.80	454.6	.9	14333.6	154.7
46.85	4321.65	457.1	**18.**	14178.9	153.7
46.60	4274.80	459.7	.1	14025.2	152.9
46.36	4228.20	462.2	.2	13872.3	152.1
46.11	4181.84	464.8	.3	13720.2	151.3
45.88	4135.73	467.3	.4	13568.9	150.5
45.60	4089.85	469.9	.5	13418.4	149.6
45.35	4044.25	472.4	.6	13268.8	148.8
45.10	3998.90	474.9	.7	13120.0	148.0
44.90	3953.80	477.5	.8	12972.0	147.3
44.60	3908.90	480.0	.9	12824.7	146.6
44.44	3864.21	482.6	**19.**	12678.1	145.8
44.19	3819.77	485.1	.1	12532.3	145.0
43.95	3775.58	487.6	.2	12387.3	144.2
43.74	3731.63	490.2	.3	12243.1	143.5
43.47	3687.89	492.7	.4	12099.6	142.6
43.22	3644.42	495.3	.5	11957.0	141.8
43.00	3601.20	497.8	.6	11815.2	141.2
42.82	3558.20	500.3	.7	11674.1	140.5
42.62	3515.38	502.9	.8	11533.6	139.8
42.42	3472.76	505.4	.9	11393.8	139.2
42.24	3430.34	508.0	**20.**	11254.6	138.6
42.04	3388.10	510.5	.1	11116.0	137.9
41.82	3346.06	513.0	.2	10978.1	137.2
41.60	3304.24	516.6	.3	10840.9	136.5
41.39	3262.64	518.1	.4	10704.4	135.8
41.15	3221.25	520.7	.5	10568.6	135.0
40.94	3180.10	523.2	.6	10433.6	134.3
40.75	3139.16	525.7	.7	10299.3	133.7
40.54	3098.41	528.3	.8	10165.6	133.0
40.37	3057.87	530.8	.9	10032.6	132.3

The columns *Bar.* is the height of the Barometer in inches and milimetres.

The columns Altitude is the corresponding height of level above the sea in feet and metres.

The altitude in metres can be read from the barometer in inches; or, the altitude in feet can be read from the barometer in milimetres.

TABLE I.

Barometric and Atmospheric Heights.

Dif.	FRENCH. Altitude. metres.	Bar. m.m.	ENGLISH. Bar. in.	Altitude. feet.	Dif.
40.17	3017.50	533.4	**21.**	9900.1	131.7
40.09	2977.33	535.9	.1	9768.3	131.9
39.81	2937.34	538.4	.2	9637.1	130.6
39.65	2897.53	541.0	.3	9506.5	130.1
39.44	2857.88	543.5	.4	9376.4	129.4
39.23	2818.44	546.1	.5	9247.0	128.7
39.11	2779.21	548.6	.6	9118.3	128.3
38.99	2740.10	551.1	.7	8990.0	127.6
38.74	2701.21	553.7	.8	8862.4	127.1
38.53	2662.47	556.2	.9	8735.3	126.4
38.37	2623.94	558.8	**22.**	8608.9	125.9
38.20	2585.57	561.3	.1	8483.0	125.3
38.00	2547.37	563.8	.2	8357.7	124.7
37.88	2509.37	566.4	.3	8233.0	124.3
37.67	2471.49	568.9	.4	8108.7	123.6
37.52	2433 82	571.5	.5	7985.1	123.1
37.37	2396.30	574.0	.6	7862.0	122.6
37.16	2358.93	576.5	.7	7739.4	121 9
37.06	2321.77	579.1	.8	7617.5	121.6
36.82	2284.71	581.6	.9	7495.9	120.8
36.69	2247.89	584.2	**23.**	7375.1	120.4
36.59	2211.20	586.7	.1	7254.7	120.0
36.39	2174.61	589.2	.2	7134.7	119.4
36.21	2138.22	591.8	.3	7015.3	118.8
36.09	2102.01	594.3	.4	6896.5	118.4
35.93	2065.92	596.9	.5	6778.1	117.9
35.79	2029.09	599.4	.6	6660.2	117.4
35.60	1994.20	601.9	.7	6542.8	116.8
35.46	1958.60	604.6	.8	6426.0	116.4
35.30	1923.14	607.0	.9	6309.6	115.8
35.21	1887.34	609.6	**24.**	6193.8	115.5
35.02	1852.63	612.1	.1	6078.3	114.9
34.90	1817.61	614.6	.2	5963.4	114.5
34.75	1782.71	617.2	.3	5848.9	114.0
34.59	1747.96	619 7	.4	5734.9	113.5
34.47	1713.37	622.3	.5	5621.4	113.1
34.32	1678.90	624.8	.6	5508.3	112.6
34.17	1644.58	627.3	.7	5395.7	112.1
34.05	1610.41	629.9	.8	5283.6	111.7
33 92	1576.36	632.4	.9	5171.9	111.3
33.78	1542.44	635 0	**25.**	5060.6	110.8
33.61	1508.66	637.5	.1	4949.8	110.3
33.50	1475.05	640.0	.2	4839.5	109.9
33.37	1441.55	642.6	.3	4729.6	109.5
33.25	1408.18	645.1	.4	4620.1	109.1
33.13	1374.93	647.7	.5	4511.0	108.7
33.01	1341.80	650.3	.6	4402.3	108.3
32.83	1308.79	652.8	.7	4294.0	107.7
32.74	1275 96	655.3	.8	4186.3	107.4
32.61	1243.22	657.9	.9	4078.9	107.0

Dif.	FRENCH. Altitude. metres.	Bar. m.m.	ENGLISH. Bar. in.	Altitude. feet.	Dif.
32.46	1210.61	660.4	**26.**	3971.9	106.5
32.34	1178.15	662.9	.1	3865.4	106.1
32.22	1145.81	665.4	.2	3759.3	105.7
32.09	1113.59	668.0	.3	3653.6	105 3
31.98	1081.50	670.5	.4	3548.3	104.9
31.88	1049.52	673.1	.5	3443.4	104 6
31.76	1017.64	675 6	.6	3338.8	104.2
31.64	985.888	678.1	.7	3234 6	103.8
31.52	954.251	680.7	.8	3130.8	103.4
31.39	922.734	683.2	.9	3027.4	103.0
31 27	891.341	685.8	**27.**	2924.4	102.6
31.15	860.070	688.3	.1	2821.8	102.2
31.03	828.919	690.8	.2	2719.6	101.8
30.94	797.891	693.4	.3	2617.8	101.5
30.81	766 953	695.9	.4	2516.3	101.1
30.72	736.140	698 5	.5	2415.2	100.8
30.60	705.416	701.0	.6	2314.4	100.4
30.48	674.815	703.5	.7	2214.0	100.0
30.41	644 335	706.1	.8	2114.0	99.7
30.24	613.927	708.6	.9	2014.3	99.3
30.18	583.682	711.2	**28.**	1915.0	99.0
30.05	553.503	713.7	.1	1816.0	98.6
29.93	523.454	716.2	.2	1717.4	98.2
29.84	493 523	718 8	.3	1619.2	97.9
29.75	463.683	721.3	.4	1521.3	97.6
29 63	433 935	723.9	.5	1423 7	97.2
29 52	404.309	726.4	.6	1326.5	96.9
29 45	374.785	728.9	.7	1229.6	96.6
29.32	345.332	731.5	.8	1133.0	96.2
29.23	316.010	734.0	.9	1036.8	95.9
29.11	286.781	736.6	**29.**	940.9	95.5
29.02	257.672	739.1	.1	845.4	95.2
28.92	228.657	741.6	.2	750.2	94.9
28.84	199.731	744.2	.3	655.3	94.5
28.71	170.898	746.7	.4	560.7	94.2
28.62	142.186	749.3	.5	466.5	93.9
28.53	113.566	751.8	.6	372.6	93.6
28 43	85.037	754.3	.7	279 0	93.3
28 35	56.600	756.9	.8	185.7	93.0
28.25	28.254	759.4	.9	92.7	92.7
28.19	0.0000	762.0	**30.**	0.0000	92.5
28.10	28.193	764.5	.1	92.5	92.2
28.01	56.295	767.0	.2	184.7	91.0
27.92	84.305	769.6	.3	276.6	91.6
27.83	112.225	772.1	.4	368.2	91.3
27 77	140.053	774.7	.5	459.5	91.1
27.67	167.820	777.2	.6	550.6	90.8
27.58	195.495	779.7	.7	641.4	90.5
27.52	223.079	782.3	.8	731.9	90.3
27.43	250.601	784.8	.9	822.2	90.0
	278.033	787.4	**31.**	912.2	

The difference in the Bar. m.m. column is 2.5 milimetres; therefore, multiply the difference of altitude in metres by the exceeding milimetres and by 0.4; subtract the product from the tabular altitude, and the remainder will be the altitude in metres, corresponding to the reading of the barometer in millimetres.

TABLE II.—Correction for Mean Temperature.

Temp. Fahr.	Temp. Cent	1000	2000	3000	4000	5000	6000	7000	8000	9000	10000	Temp. Cent.	Temp. Fahr.
		Height in feet or metres.											
61	16.1	2.08	4.17	6.25	8.34	10.42	12.50	14.59	16.68	18.76	20.85	15.0	59
62	16.6	4.17	8.31	12.49	16.61	20.82	24.98	29.15	33.32	37.48	41.65	14.4	58
63	17.2	6.25	12.49	18.74	24.98	31.23	37.48	43.72	49.97	56.21	62.46	13.8	57
64	17.7	8.33	16.67	24.99	33.34	41.65	49.98	58.31	66.61	74.97	83.30	13.3	56
65	18.3	10.41	20.82	31.24	41.65	52.06	62.48	72.89	83.30	93.72	104.1	12.7	55
66	18.8	12.49	24.99	37.48	49.98	62.47	74.96	87.46	99.96	112.4	124.9	12.2	54
67	19.4	14.58	29.15	43.43	58.31	72.89	86.86	102.1	116.6	130.3	145.8	11.6	53
68	20.0	16.66	33.32	49.61	66.64	83.30	99.22	116.6	133 3	148.8	166.6	11.1	52
69	20.5	18.74	37.49	56.23	74.98	93.71	112.4	131.2	149.9	168.7	187.4	10.5	51
70	21.1	20.82	41.64	62.46	83.28	104.1	124.9	145.7	166.5	187.4	208.2	10.0	50
71	21.6	22.91	45.81	68.72	91.6 3	114.5	137.4	160.3	183.2	206.1	229.1	9.4	49
72	22.2	24.99	49.98	74.77	99.96	124.9	149.5	174.9	199.9	224.9	249.9	8.8	48
73	22.7	27.07	54.14	81.21	108.3	135.3	162.4	189.5	216.6	243.6	270.7	8.3	47
74	23.3	29.15	58.31	87.46	116.6	145.7	174.9	204.1	233.2	262.4	291.5	7.7	46
75	23.8	31.24	62.47	93.71	124.9	156.2	187.4	218.6	249.9	281.1	312.4	7.2	45
76	24.4	33.32	66.64	99.96	133.3	166.6	199.9	233.2	266.5	299.9	333.2	6.6	44
77	25.0	35.40	70.80	106.2	141.6	177.0	212.4	247.8	283.2	318.6	354.0	6.1	43
78	25.5	37.48	74.98	112.4	149.9	187.4	224.9	261.4	299.8	337.3	374.8	5.5	42
79	26.1	39.57	79.13	118.7	158.2	197.8	237.4	277.0	316.5	356.1	395.7	5.0	41
80	26.6	41.65	83.30	124.9	166.6	208.2	249.9	291.5	333.2	374.8	416.5	4.4	40
81	27.2	43.73	87.46	131.2	174.9	218.6	262.4	306.1	349.8	393.6	437.3	3.8	39
82	27.7	45.81	91.63	137.4	183.2	229.0	274.8	320.7	366.5	412.3	458.1	3.3	38
83	28.3	47.90	95.79	143.3	191.6	239.5	286.6	335.3	383.2	431.1	478.9	2.7	37
84	28.8	49.98	99.96	149.9	199.9	249.9	299.8	349.8	399.8	449.8	499.8	2.2	36
85	29.4	52.06	104.1	156.2	208.2	260.3	312.4	364.4	416.5	468.6	520.6	1.6	35
86	30.0	54.14	108.2	162.4	216.5	270.7	324.8	379.0	433.1	487.3	541.5	1.1	34
87	30.5	56.23	112.4	168.6	224.9	281.1	337.2	393.6	449.8	506.0	562.2	0.5	33
88	31.1	58.31	116.6	174.9	233.2	291.5	349.8	408.2	466.5	524.8	583.1	0.0	32
89	31.6	59.37	118.7	178.1	237.5	296.8	356.2	415.5	474.9	534.3	593.7	—0.5	31
90	32.2	61.77	123.5	185.3	247.1	308.8	370.6	432.4	494.1	555.9	617.7	—1.1	30
91	32.7	64.56	129.1	193.7	258.2	322.8	387.3	451.9	516.5	581.0	645.6	—1.6	29
92	33.3	66.64	133.3	199.9	266.5	333.2	399.8	466.5	533.1	599.7	666.4	—2.2	28
93	33.8	68.72	137.4	206.1	274.9	343.6	412.3	471.0	549.7	618.5	687.2	—2.7	27
94	34.4	70.80	141.6	212.4	283.2	354.0	424.8	495.6	566.4	637.2	708.0	—3.3	26
95	35.0	72.89	145.8	218.7	291.6	364.4	437.3	510.2	583.1	656.0	728.9	—3.8	25
96	35.5	74.83	149.6	224.5	299.3	374.1	449.0	523.8	598.6	673.5	748.3	—4.4	24
97	36.1	77.05	154.1	231.1	308.2	385.2	462.3	539.3	616.4	693.4	770.5	—5.0	23
98	36.6	79.13	158.2	237.4	316.5	395.6	474.8	553.9	633.0	712.2	791.3	—5.5	22
99	37.2	81.22	162.4	243.6	324.9	406.1	487.3	560.5	649.7	731.0	812.2	—6.1	21
100	37.7	83.30	166.6	249.9	333 2	416.5	499.8	583.1	666.4	749.7	833.0	—6.6	20
Fahr.	Cent	1000	2000	3000	4000	5000	6000	7000	8000	9000	10000	Cent.	Fahr.

Add the correction for this temperature. (left-hand temperatures)

Subtract the correction for this temperature. (right-hand temperatures)

TABLE III.—Correction for Mean Latitude.

Mean latitude.	1000	2000	3000	4000	5000	6000	7000	8000	9000	10000	Mean latitude.
	Heights in feet or metres.										
44	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.76	0.81	0.90	46
42	0.27	0.54	0.81	1.08	1.35	1.62	1.89	2.16	2.43	2.7	48
40	0.44	0.88	1.32	1.76	2.20	2.64	3.08	3.52	3.96	4.4	50
38	0.62	1.24	1.86	2.48	3.10	3.72	4.34	4.96	5.58	6.2	52
36	0.79	1.58	2.37	3.16	3.95	4.74	5.53	6.32	7.11	7.9	54
34	0.96	1.92	2.88	3.84	4.80	5.76	6.72	7.68	8.64	9.6	56
30	1.28	2.56	3.84	5.12	6.40	7.68	8.96	10.2	11.5	12.8	60
28	1.43	2.86	4.29	5.72	7.15	8.58	10.0	11.4	12.9	14.3	62
24	1.71	3.42	5.13	6.84	8.55	10.26	12.0	13.7	15.4	17.1	66
20	1.95	3.90	5.85	7.80	9.75	11.7	13.6	15.6	17.5	19.5	70
18	2.06	4.12	6.18	8.24	10.3	12.3	14.4	16.5	18.5	20.6	72
14	2.25	4.50	6.75	9.00	11.2	13.5	15.7	18.0	20.2	22.5	76
10	2.39	4.78	7.17	9.56	11.9	14.3	16.7	19.1	21.5	23.9	80
6	2.49	4.98	7.47	9.96	12.4	14.9	17.4	19.8	22.4	24.9	84
2	2.54	5.08	7.62	10.2	12.7	15.2	17.8	20.3	22.9	25.4	88

Add for these latitudes. (left-hand latitudes)

Subtract for these latitudes. (right-hand latitudes)

TABLE IV.—Logarithmic Correction for Temperature of the *Atmosphere. Always positive.*

Temp. Cent.	Temp. Fahr	Logarithms.	Temp. Cent.	Temp. Fahr	Logarithms.	Temp. Cent.	Temp. Fahr	Logarithms.	Temp. Cent.	Temp. Fahr	Logarithms.
—2.2	28	9.97005	8.33	47	9.98808	18.8	66	0.00539	29.4	85	0.02204
—1.6	29	9.97102	8.88	48	9.98901	19.4	67	0.00628	30.0	86	0.02290
—1.1	30	9.97198	9.44	49	9.98993	20.0	68	0.00717	30.5	87	0.02376
—0.5	31	9.97294	10.0	50	9.99086	20.5	69	0.00806	31.1	88	0.02461
0.0	32	9.97391	10.5	51	9.99178	21.1	70	0.00895	31.6	89	0.02547
+ 55	33	9.97487	11.1	52	9.99270	21.6	71	0.00984	32.2	90	0.02632
1.11	34	9.97682	11.6	53	9.99362	22.2	72	0.01072	32.7	91	0.02717
1.66	35	9.97678	12.2	54	9.99454	22.7	73	0.01160	33.3	92	0.02802
2.22	36	9.97773	12.7	55	9.99545	23.3	74	0.01248	33.8	93	0.02886
2.77	37	9.97868	13.3	56	9.99637	23.8	75	0.01336	34.4	94	0.02971
3.33	38	9.97963	13.8	57	9.99728	24.4	76	0.01423	35.0	95	0.03055
3.88	39	9.98058	14.4	58	9.99819	25.0	77	0.01511	35.5	96	0.03139
4.44	40	9.98152	15.0	59	9.99909	25.5	78	0.01598	36.1	97	0.03224
5.00	41	9.98247	15.5	60	0.00000	26.1	79	0.01685	36.6	98	0.03307
5.55	42	9.98341	16.1	61	0.00090	26.6	80	0.01772	37.2	99	0.03391
6.11	43	9.98434	16.6	62	0.00180	27.2	81	0.01859	37.7	100	0.03475
6.66	44	9.98528	17.2	63	0.00270	27.7	82	0.01945	38.3	101	0.03558
7.22	45	9.98622	17.7	64	0.00360	28.3	83	0.02032	38.8	102	0.03641
7.77	46	9.98715	18.3	65	0.00450	28.8	84	0.02118	39.4	103	0.03724

TABLE V.—Logarithmic Correction for Mean Latitude of *Observation. Always positive.*

Lat.	Log.	Lat.	Log.	Lat.	Log.	Lat.	Log.	Lat.	Log.	Lat.	Log.
0	0.00111	15	0.00096	30	0.00055	45	0.00000	60	9.99944	75	9.99904
1	0.00110	16	0.00094	31	0.00052	46	9.99996	61	9.99941	76	9.99902
2	0.00110	17	0.00092	32	0.00048	47	9.99992	62	9.99938	77	9.99900
3	0.00110	18	0.00089	33	0.00045	48	9.99988	63	9.99935	78	9.99898
4	0.00109	19	0.00087	34	0.00041	49	9.99984	64	9.99932	79	9.99897
5	0.00109	20	0.00085	35	0.00038	50	9.99981	65	9.99929	80	9.99896
6	0.00108	21	0.00082	36	0.00034	51	9.99977	66	9.99926	81	9.99894
7	0.00107	22	0.00079	37	0.00030	52	9.99973	67	9.99923	82	9.99893
8	0.00106	23	0.00077	38	0.00027	53	9.99969	68	9.99920	83	9.99892
9	0.00105	24	0.00074	39	0 00023	54	9.99966	69	9.99917	84	9.99891
10	0.00104	25	0.00071	40	0.00019	55	9.99962	70	9.99915	85	9.99891
11	0.00103	26	0.00068	41	0.00015	56	9.99958	71	9.99913	86	9.99890
12	0.00101	27	0.00065	42	0.00011	57	9.99955	72	9.99910	87	9.99889
13	0.00099	28	0.00062	43	0.00008	58	9.99951	73	9.99908	88	9.99889
14	0.00095	29	0.00059	44	0.00004	59	9.99948	74	9.99906	89	9.99889

TABLE VI.—Temperature of Boiling Water,
Corresponding to the Height of the Barometer at 60° Fahrenheit.

French Measures.			English Measures.			French Measures.			English Measures.		
Diff.	Height Barom. M. M.	Temp. Water. Cent.	Temp. Water Fahr.	Height Barom. Inches.	Diff.	Diff.	Height Barom. M. M.	Temp. Water. Cent.	Temp. Water Fahr.	Height Barom. Inches.	Diff.
9.61	434.07	85.00	185	17.090	.378	12.49	597.45	93.33	200	23.522	.492
9.77	443.68	85.55	186	17.468	.385	12.70	609.94	93.88	201	24.014	.500
9.96	453.45	86.11	187	17.853	.392	12.91	622.64	94.44	202	24.514	.508
10.14	463.41	86.66	188	18.245	.399	13.18	635.55	95.00	203	25.022	.519
10.31	473.55	87.22	189	18.644	.406	13.36	648.73	95.55	204	25.551	.526
10.47	483.86	87.77	190	19.050	.412	13.57	662.09	96.11	205	26.067	.535
10.66	494.33	88.33	191	19.462	.420	13.83	675.66	96.66	206	26.602	.544
10.85	504.99	88.99	192	19.882	.427	14.05	689.49	97.22	207	27.146	.553
11.08	515.84	89.44	193	20.309	.436	14.28	703.54	97.77	208	27.699	.562
11.24	526.92	90.00	194	20.745	.443	14.53	717.82	98.33	209	28.261	.572
11.46	538.16	90.55	195	21.188	.451	14.78	732.35	98.88	210	28.833	.582
11.84	549.62	91.11	196	21.639	.458	15.04	747.13	99.44	211	29.415	.592
11.85	561.26	91.66	197	22.097	.467	15.26	762.17	100.0	212	30.007	.601
12.07	573.11	92.22	198	22.564	.475	15.52	777.43	100.5	213	30.608	.611
12.27	585.18	92.77	199	23.039	.483		792.95	101.6	214	31.219	

TABLE VII.—Height of a Column of Air in Metres,

Corresponding to one milimetre in the barometer, at different temperatures.

Bar. M.M.	Centigrade temperature of the air and the barometer.														
	—6	—3	0	+3	6	9	12	15	18	21	24	27	30	33	36
400	20.7	20.8	20.9	21.0	21.1	21.2	21.3	21.4	21.5	21.6	21.7	21.8	21.9	22.0	22.1
420	19.7	19.8	19.9	20.0	20.1	20.2	20.3	20.4	20.5	20.6	20.7	20.8	20.9	21.0	21.1
440	18.8	18.9	19.0	19.1	19.2	19.3	19.4	19.5	19 6	19.7	19.8	19.9	20.0	20.1	20.2
460	18.0	18.1	18.2	18.3	18.4	18.4	18.5	18.6	18.7	18.8	18.8	18.9	19.0	19.1	19.2
480	17.2	17.3	17.4	17.5	17.6	17.6	17.7	17.8	17.9	18.0	18.1	18.1	18.2	18.3	18.4
500	16.5	16.6	16.7	16.8	16.9	16.9	17.0	17.1	17.2	17.3	17.3	17.4	17.5	17.6	17.7
520	15.8	15.9	16.0	16.1	16.2	16.2	16.3	16.4	16.5	16.6	16.6	16.7	16.8	16.9	17.0
540	15.3	15.3	15.4	15.5	15.6	15.6	15.7	15.8	15.9	16.0	16.0	16.1	16.2	16.3	16.3
560	14.8	14.8	14.9	15.0	15.1	15.1	15.2	15.3	15.4	15.5	15.5	15.6	15.7	15.8	15.8
580	14.3	14.3	14.4	14.5	14.6	14.6	14.7	14.8	14.9	15.0	15.0	15.1	15.2	15.3	15.3
600	13.8	13.8	13.9	14.0	14.1	14.1	14.2	14.3	14.4	14.5	14.5	14.6	14.7	14.8	14.8
620	13.3	13.3	13.4	13.5	13.6	13.6	13.7	13.8	13.9	14.0	14.0	14.1	14.2	14.3	14.3
640	12.9	12.9	13.0	13.1	13.2	13.2	13.3	13.4	13.5	13.6	13.6	13.7	13.8	13.9	13.9
660	12.5	12.6	12.6	12.7	12.8	12.8	12.9	13.0	13.1	13.2	13.2	13.3	13.3	13.4	13 4
680	12.2	12.2	12.3	12 3	12.4	12.5	12.5	12.6	12.7	12 7	12.8	12.9	12 9	13.0	13.1
700	11.9	11.9	12.0	12.0	12.1	12.2	12.2	12.3	12.4	12.4	12.5	12.6	12.6	12.7	12.7
720	11.5	11.5	11.6	11.6	11.7	11.8	11.8	11.9	11.9	12.0	12.1	12.1	12.2	12.3	12.3
740	11.2	11 2	11.3	11.3	11.4	11.5	11.5	11.6	11.7	11.7	11.8	11.9	11.9	12.0	12.0
760	10.9	10.9	11.0	11.1	11.1	11.2	11.2	11.3	11.4	11.4	11.5	11.6	11.6	11.7	11.7
780	10.6	10.6	10.7	10.8	10.8	10.9	10.9	11.0	11.1	11.1	11.2	11.3	11.3	11.4	11.4

TABLE VIII.—Height of a Column of Air in Feet,

Corresponding to one-tenth of an inch in the barometer at different temperatures.

Bar. In.	Fahrenheit temperature of the air and the barometer.														
	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	95°	100°
16	163	165	167	168	170	172	174	176	178	179	181	183	185	187	188
17	153	155	156	158	159	161	163	165	166	168	170	171	173	175	177
18	145	157	158	160	161	162	154	156	157	159	160	162	163	165	166
19	135	137	138	140	142	144	146	148	149	151	152	153	155	157	158
20	130	132	133	135	136	137	139	140	142	143	145	146	148	149	151
21	124	126	127	128	130	131	132	133	135	136	137	139	140	142	143
22	118	120	121	123	124	125	126	127	129	130	131	132	134	135	136
23	112	114	115	116	117	118	120	121	122	124	125	126	127	129	130
24	108	110	111	112	113	114	115	116	117	119	120	121	122	123	125
25	104	106	107	108	109	110	111	112	113	115	116	117	118	119	120
25.5	102	104	105	106	107	108	109	110	112	113	114	115	116	117	118
26	100	102	103	104	105	106	107	108	109	111	112	113	114	115	116
26.5	98	100	101	102	103	104	105	106	108	109	110	111	112	113	114
27	96	97	98	100	101	102	103	104	105	107	108	109	110	111	112
27.5	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109
28	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107
28.5	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
29	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104
29.5	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102
30	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101

TABLE IX.—Mean Height of the Barometer

in different countries, reduced to the level of the sea, and to 60° Fahr. temperature.

	Inches.		Inches.		Inches.
Africa, Northern, .	30.26	China, . .	30.11	Peru, ...	30.09
Atlantic coast, N. A.,		Denmark, .	29.90	Prussia, ...	30.00
Northern States,	30.10	England, .	30.03	Scotland, . . .	29.93
Southern States,	30.17	France, . .	30.00	Sicily, ...	30.11
Australia, ...	30.00	Greenland,	29.75	Spitzbergen, . .	29.87
Brazil, ...	30.15	Italy, . .	30.00	Sweden, ...	29.96
Canary Islands, . .	30.16	Iceland, .	29.70	Venezuela, . . .	30.00
Cape Good Hope, .	30.11	Norway, . .	29.89	West In. Islands,	30.02

TABLE X.—Correction for the Mercurial Column

in Millimetres at Different Temperatures of Barometer.

Tem. Cen.	415	440	465	490	515	540	565	590	615	640	665	690	715	740	765	Temp. Cent.
16	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	15
17	0.10	0.11	0.11	0.12	0.12	0.13	0.14	0.14	0.15	0.15	0.16	0.17	0.17	0.18	0.18	14
18	0.16	0.18	0.19	0.20	0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.30	13
19	0.24	0.25	0.26	0.28	0.29	0.30	0.32	0.33	0.35	0.36	0.38	0.39	0.40	0.42	0.43	12
20	0.35	0.37	0.39	0.42	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.59	0.61	0.63	0.65	11
21	0.42	0.45	0.42	0.51	0.53	0.56	0.59	0.61	0.64	0.66	0.69	0.71	0.74	0.76	0.78	10
22	0.49	0.52	0.55	0.58	0.60	0.63	0.66	0.69	0.72	0.75	0.78	0.81	0.84	0.87	0.90	9
23	0.55	0.58	0.62	0.65	0.69	0.72	0.75	0.79	0.82	0.85	0.89	0.92	0.95	0.99	1.02	8
24	0.62	0.66	0.70	0.73	0.77	0.81	0.85	0.88	0.92	0.96	1.00	1.03	1.07	1.11	1.15	7
25	0.69	0.73	0.77	0.81	0.85	0.89	0.94	0.98	1.02	1.06	1.10	1.14	1.19	1.23	1.27	6
26	0.75	0.80	0.84	0.89	0.93	0.98	1.02	1.07	1.11	1.16	1.20	1.25	1.30	1.34	1.39	5
27	0.82	0.87	0.92	0.97	1.02	1.08	1.12	1.17	1.22	1.27	1.32	1.37	1.42	1.47	1.52	4
28	0.89	0.94	1.00	1.05	1.10	1.16	1.21	1.26	1.32	1.37	1.42	1.48	1.53	1.59	1.64	3
29	0.98	1.01	1.07	1.13	1.19	1.24	1.30	1.36	1.41	1.47	1.53	1.59	1.64	1.70	1.76	2
30	1.02	1.09	1.15	1.21	1.27	1.33	1.40	1.46	1.53	1.58	1.64	1.70	1.76	1.83	1.89	1
31	1.09	1.16	1.22	1.29	1.35	1.42	1.48	1.55	1.61	1.68	1.74	1.81	1.88	1.94	2.01	0
32	1.16	1.23	1.30	1.36	1.43	1.50	1.56	1.64	1.71	1.78	1.85	1.92	1.99	2.06	2.13	—1
33	1.23	1.30	1.38	1.45	1.52	1.60	1.67	1.74	1.82	1.89	1.97	2.04	2.11	2.19	2.26	—2
34	1.29	1.37	1.44	1.52	1.60	1.68	1.76	1.83	1.91	1.99	2.07	2.14	2.22	2.30	2.38	—3
35	1.36	1.44	1.52	1.60	1.69	1.76	1.85	1.93	2.01	2.10	2.17	2.25	2.33	2.42	2.50	—4

Height of barometer in millimetres.

Subtract the correction for these temperatures. — Add the correction for these temperatures.

TABLE XI.—Correction for the Mercurial Column in Thousands of an Inch, at Different Temperatures of the Barometer above or below 60°.

Temp. Fahr.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	Temp Fahr.
62	003	003	003	003	003	003	004	004	004	004	005	005	005	005	005	58
64	006	006	006	007	007	007	008	008	008	009	009	009	010	010	010	56
66	009	009	009	010	011	011	012	012	012	013	014	014	015	015	016	54
68	011	012	013	013	014	015	016	016	017	018	018	019	020	020	021	52
70	014	015	016	017	018	019	020	020	021	022	023	024	025	026	026	50
72	017	018	019	020	021	022	023	024	024	027	028	029	030	031	032	48
74	020	021	.022	024	025	026	027	028	029	031	032	034	035	036	038	46
76	023	024	026	027	028	030	031	032	034	036	037	039	040	041	043	44
78	026	027	029	030	032	033	035	036	038	040	042	044	045	046	048	42
80	029	030	032	034	036	037	039	041	043	045	046	048	050	052	054	40
82	031	033	035	037	039	041	043	045	048	049	051	053	055	057	059	38
84	034	036	039	041	043	045	047	049	052	054	056	058	060	062	064	36
86	037	040	042	044	046	049	051	053	056	058	060	063	065	067	070	34
88	040	043	045	047	050	052	055	057	061	063	065	068	070	072	075	32
90	043	046	048	051	054	056	059	061	065	067	070	072	075	077	080	30
92	046	049	051	054	057	060	063	065	069	071	074	077	080	083	086	28
94	049	052	054	057	060	064	067	069	074	076	079	082	085	088	091	26
96	051	055	058	060	063	067	071	073	078	080	084	087	090	093	097	24
98	054	058	061	064	067	071	075	077	082	085	088	092	095	098	102	22
100	057	061	064	068	071	075	079	082	086	089	093	097	100	104	107	20

Height of the barometer in inches.

Subtract the correction from the mercurial column. — Add the correction to the mercurial column.

Heights in Feet of the Principal Waterfalls.

Gavarny, Pyrenees,	1260	Gray Mare's Tail,	350	Rupin, Himalayas,	120
Lauterbrun, Switz.,	912	Hepste. . .	300	Kakabika, S. Am.,	115
Staubbach, Switz.,	900	Nakchikin, Kamch.	300	Lidford, England,	100
Ruican, Norway,	800	Terni, Italy, .	270	Genesee, N. York,	100
Secnlego, Pyrenees,	795	Montmorency, Can.,	242	Oyapock, S. Amer.,	80
Luleå, Sweden, .	600	Foyers, Scotland,	207	Rhine Lauffen, Swi.	65
Teqnendama, Colum.	540	Wilberforce, N. A.,	160	Trollhetta, Sweden,	60
Tosa, Piedmont, .	470	Cetina, Dalmatia,	150	Parana, Paraguay,	52
Missouri, N. Amer.,	400	Niagara Falls, .	145	Tivoli, Italy, .	50
Powerscaurt, Irel.,	380	Tendon, France,	125	Cataracts of Nile,	40

TABLE XII.—Mean Temperature of the Air

at the Level of the Sea.

Months in the year.	North Latitude. 60	50	40	30	20	10	South Latitude. 0	10	20	30	40	Ther.
January, .	25	46	62	72	78	80	80	80	77	75	72	Fahrenheit.
February,	28	48	63	73	78	81	80	80	77	74	71	
March, .	32	50	64	74	79	82	81	79	76	72	69	
April, .	38	55	67	76	81	83	82	79	75	70	64	
May, . .	48	61	72	78	83	84	83	78	74	68	61	
June, .	58	67	75	80	84	85	84	78	72	66	65	
July, . .	61	69	76	81	85	86	84	77	73	64	52	
August, .	59	68	75	80	84	85	83	78	73	64	51	
September, .	52	64	72	78	83	84	82	78	72	62	54	
October, .	44	57	68	76	81	83	81	79	71	63	59	
November, .	35	52	65	74	80	82	80	79	73	66	66	
December,	28	48	63	73	79	81	80	79	75	71	71	
January, .	—3.8	7.7	16.6	22.2	25.5	26.6	26.6	26.4	25.3	23.8	22.2	Centigrade.
February,	—2.2	8.8	17.2	22.7	25.8	27.2	26.8	26 6	25.	23.3	21.6	
March, .	0.0	10.	17.7	23 3	26.1	27.7	27.2	26.1	24.4	22.2	20.5	
April, .	+3.3	12.7	19.4	24.4	27.2	28 3	27.7	26.1	23.8	21.1	17.7	
May, . .	8.8	16.1	22.2	25.5	28.3	28.8	28.3	25.5	23.3	20.	16.1	
June, .	14.4	19.4	23.8	26.6	28.8	29.4	28.8	25.5	22.2	18.8	18.3	
July, . .	16.1	20.5	24.4	27 2	29.4	30.	28.8	25.	22.7	17.7	11.1	
August, .	15.	20.	23.8	26.6	28.8	29.4	28.3	25.5	22.7	17.7	10.5	
September, .	11.1	17.7	22.2	25.5	28.3	28.8	27.7	25.5	22.2	16.6	12.2	
October, .	6.6	13.8	20.	24.4	27.2	28.3	27 2	25.8	21.6	17.2	15.	
November, .	1.6	11.1	18.3	23 3	26.6	27.7	26.8	26.	22.7	18.8	18.8	
December,	—2.2	8.8	17.2	22.7	26.1	27.2	26.6	26.1	23.8	21.6	21.6	

Heights of Natural and Artificial Works.

HEIGHTS ABOVE LEVEL OF THE SEA.	Feet.	HEIGHTS ABOVE THE GROUND.	Feet.
Green in a balloon, 1837, . .	27 000	Tower of Babel, said to have been	680
Gay-Lussac, Paris, 1804, .	22 900	Pyramid Cheops, Egypt, . .	520
Highest flight of condor, .	21.000	Tower of Baalbec, Syria, .	500
Humboldt in the Andes, .	19,500	St. Peter's Cathedral, Rome, .	500
Growth of vegetation, . .	17.000	Spire of Strasbourg, . .	486
The author in the Andes,* .	15 120	Cathedral, Antwerp, . .	476
Lake Manasarooa, Thibet, .	14.500	St. Stephen's spire, Vienna,	465
Pine and birch grow, . .	14.000	Highest chimney, Glasgow, .	455
Highest habitation of people,*	14.000	Spire of Salisbury, . .	450
Potosi silver mine, Bolivia, .	13 350	Cathedral, Milan, . . .	438
Lake Titicaca, Peru,* . .	13.000	St. Mary, Lübeck, . . .	404
La Paz, Bolivia,* . . .	12.400	Cathedral, Florence, . . .	384
Poplar grows at . .	12.000	St. Paul, London, . . .	366
City of Cuzco, Peru,* . .	11.500	Hotel des Invalides, Paris, .	344
Oak grows at	11.000	Cathedral, New York, . .	325
City Riobamba, Andes, . .	10.800	Dome of Capitol, Washington,	287
Quito, Equador, . . .	9.560	Trinity Church, New York,	286
City St. Bernard, Switzerland,	8,600	Notre Dame, Paris, . . .	220
City Santa Fe de Bogota, .	8.350	Column City of London, .	202
Wild monkeys found at* . .	8.000	Porcelain, China, . . .	200
City of Mexico, ...	6.9?0	Leaning Tower of Pisa, .	188
St. Gothard, Alps, . . .	6.900	Alexander Column, St. Petersb'g,	175
Lake Lucon, France, . .	6.220	July Column, Paris, . .	157
Palm and bananas grow at .	2.500	Column Napoleon, Paris, . .	138

* Measured by the author of this Pocket-book.

HEAT.

Heat resembles *light*, *electricity*, and *magnetism*. It is convertible into dynamic work, and can consequently be resolved into the two physical elements, *force* and *motion*. Temperature is convertible into force, which is only one element of heat, and is no measure of quantity of heat. (See Dynamics and Units of Heat.)

The temperature or intensity of heat is measured in various ways, but most generally by the expansion of mercury and alcohol, or the thermometer.

Thermometers.

There are three differently-graduated thermometers in use—namely, *Fahrenheit*, *Centigrade* and *Reaumur*. The last named is gradually being abolished, and now used only in Peru.

Graduation.

Zero Fahr. = —17.77° Cent.—14.22° Reau.

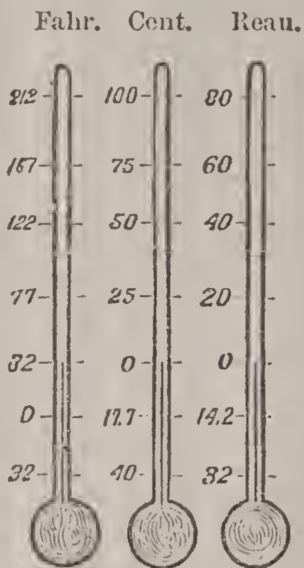
Freezing-Point of Water.

Zero Cent. = 32 Fahr. = zero Reau.

Boiling-Point of Water.

212° Fahr. = 100° Cent. = 80° Reau.

9° Fahr. = 5° Cent. = 4° Reau.



Formulas.

Cent. $= \frac{5}{9}$ (Fahr. $\mp$ 32) $= \frac{5}{4}$ Reau.

Fahr. $= \frac{9}{5}$ Cent. $\pm$ 32 $= \frac{9}{4}$ Reau. $\pm$ 32.

Reau. $= \frac{4}{5}$ Cent. $= \frac{4}{9}$ (Fahr. $\mp$ 32).

The accompanying tables give the equivalents of Centigrade's and Fahrenheit's thermometers. The first numbers in the table of comparison,—276* and 461*, are the absolute zero of temperature.

Example 1. How many degrees on Fahr. scale is 964.5° Cent.?

Table comparison,	Cent. 960°	=	1760°	Fahr.
Table Centigrade,	Cent. 4.5	=	8.1	"
The required,	Cent. 964.5		1768.1	"

Example 2. How many degrees is 2136.7° Fahr. on Centigrade thermometer?

Table comparison,	Fahr. 2120°	=	1160°	Cent.
Table Fahrenheit,	" 16°	=	8.90	"
" "	" 0.7	=	0.389	"
The required degrees,	Fahr. 2136.7	=	1169.289	"

* Deduced by Nystrom.

Comparison of Fahrenheit and Centigrade Thermometers.

Fahr.	Centig.	Fahr.	Centig.	Fahr.	Centig.	Fahr.	Centig.	Fahr.	Centig.
—5	—20.55	57	13.88	119	48.33	181	82.77	243	117.22
—4	—20.00	58	14.44	120	48.88	182	83.33	244	117.77
—3	—19.44	59	15.00	121	49.44	183	83.88	245	118.33
—2	—18.88	60	15.55	122	50.00	184	84 44	246	118.88
—1	—18.33	61	16.11	123	50.55	185	85.00	247	119.44
Zero.	—17.77	62	16.66	124	51.11	186	85.55	248	120.00
+1	—17.22	63	17.22	125	51.66	187	86.11	249	120.55
2	—16.66	64	17.77	126	52.22	188	86.66	250	121.11
3	—16.11	65	18.33	127	52.77	189	87.22	251	121.66
4	—15.55	66	18.88	128	53.33	190	87.77	252	122.22
5	—15.00	67	19.44	129	53.88	191	88.33	253	122.77
6	—14.44	68	20.00	130	54.44	192	88.88	254	123.33
7	—13.88	69	20.55	131	55.00	193	89.44	255	123.88
8	—13.33	70	21.11	132	55.55	194	90.00	256	124.44
9	—12.77	71	21.66	133	56.11	195	90.55	257	125.00
10	—12.22	72	22.22	134	56.66	196	91.11	258	125.55
11	—11.66	73	22.77	135	57.22	197	91.66	259	126.11
12	—11.11	74	23.33	136	57.77	198	92.22	260	126.66
13	—10.55	75	23.88	137	58.33	199	92.77	261	127.22
14	—10.00	76	24.44	138	58.88	200	93.33	262	127.77
15	— 9.44	77	25.00	139	59.44	201	93.88	263	128.33
16	— 8.88	78	25.55	140	60.00	202	94.44	264	128.88
17	— 8 33	79	26.11	141	60.55	203	95.00	265	129.44
18	— 7.77	80	26.66	142	61.11	204	95.55	266	130.00
19	— 7.22	81	27.22	143	61.66	205	96.11	267	130.55
20	— 6.66	82	27.77	144	62.22	206	96.66	268	131.11
21	— 6.11	83	28.33	145	62.77	207	97.22	269	131.66
22	— 5.55	84	28.88	146	63.33	208	97.77	270	132.22
23	— 5.00	85	29.44	147	63.88	209	98.33	271	132.77
24	— 4.44	86	30.00	148	64.44	210	98.88	272	133.33
25	— 3.88	87	30.55	149	65.00	211	99.44	273	133.88
26	— 3.33	88	31.11	150	65.55	212	100 00	274	134.44
27	— 2.77	89	31.66	151	66.11	213	100.55	275	135.00
28	— 2.22	90	32.22	152	66.66	214	101.11	276	135.55
29	— 1.66	91	32.77	153	67.22	215	101.66	277	136.11
30	— 1.11	92	33.33	154	67.77	216	102.22	278	136.66
31	— .55	93	33.88	155	68.33	217	102.77	279	137.22
32	Zero.	94	34.44	156	68.88	218	103.33	280	137.77
33	+0.55	95	35.00	157	69.44	219	103.88	281	138.33
34	1.11	96	35.55	158	70.00	220	104.44	282	138.88
35	1.66	97	36.11	159	70.55	221	105.00	283	139.44
36	2.22	98	36.66	160	71.11	222	105.55	284	140.00
37	2.77	99	37 22	161	71.66	223	106.11	285	140.55
38	3.33	100	37.77	162	72.22	224	106 66	286	141.11
39	3.88	101	38.33	163	72.77	225	107.22	287	141.66
40	4.44	102	38.88	164	73.33	226	107.77	288	142.22
41	5.00	103	39.44	165	73.88	227	108.33	289	142.77
42	5.55	104	40.00	166	74.44	228	108.88	290	143.33
43	6.11	105	40.55	167	75.00	229	109.44	291	143.88
44	6 66	106	41.11	168	75.55	230	110.00	292	144.44
45	7.22	107	41.66	169	76.11	231	110.55	293	145.00
46	7.77	108	42.22	170	76.66	232	111.11	294	145.55
47	8.33	109	42.77	171	77.22	233	111.66	295	146.11
48	8.88	110	43.33	172	77.77	234	112.22	296	146.66
49	9.44	111	43.88	173	78.33	235	112.77	297	147.22
50	10.00	112	44.44	174	78.88	236	113.33	298	147.77
51	10.55	113	45.00	175	79.44	237	113.88	299	148.33
52	11.11	114	45.55	176	80.00	238	114.44	300	148.88
53	11.66	115	46.11	177	80.55	239	115.00	400	204.44
54	12.22	116	46.66	178	81.11	240	115.55	600	315.55
55	12.77	117	47.22	179	81.66	241	116.11	800	433.33
56	13.33	118	47.77	180	82.22	242	116.66	1000	537.77

Comparison of Centigrade and Fahrenheit Thermometers.

Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.
276*	461*	16	60.8	330	626	950	1742	1570	2858
—260	—436	17	62.6	340	644	960	1760	1580	2876
—250	—418	18	64.4	350	662	970	1778	1590	2894
—240	—400	19	66.2	360	680	980	1796	1600	2912
—230	—382	20	68.0	370	698	990	1814	1610	2930
—220	—364	21	69.8	380	716	1000	1832	1620	2948
—210	—346	22	71.6	390	734	1010	1850	1630	2966
—200	—328	23	73.4	400	752	1020	1868	1640	2984
—190	—310	24	75.2	410	770	1030	1886	1650	3002
—180	—298	25	77.0	420	788	1040	1904	1660	3020
—170	—274	26	78.8	430	806	1050	1922	1670	3038
—160	—256	27	80.6	440	824	1060	1940	1680	3056
—150	—238	28	82.4	450	842	1070	1958	1690	3074
—140	—220	29	84.2	460	860	1080	1976	1700	3092
—130	—202	30	86.0	470	878	1090	1994	1710	3110
—120	—184	31	87.8	480	896	1100	2012	1720	3128
—110	—166	32	89.6	490	914	1110	2030	1730	3146
—100	—148	33	91.4	500	932	1120	2048	1740	3164
— 90	—130	34	93.2	510	950	1130	2066	1750	3182
— 80	—112	35	95.0	520	968	1140	2084	1760	3200
— 70	— 94	36	96.8	530	986	1150	2102	1770	3218
— 60	— 76	37	98.6	540	1004	1160	2120	1780	3236
— 50	— 58	38	100.4	550	1022	1170	2138	1790	3254
— 40	— 40	39	102.2	560	1040	1180	2156	1800	3272
— 30	— 22	40	104.0	570	1058	1190	2174	1810	3290
— 20	— 4	41	105.8	580	1076	1200	2192	1820	3308
— 19	— 2.2	42	107.6	590	1094	1210	2210	1830	3326
— 18	— 0.4	43	109.4	600	1112	1220	2228	1840	3344
17.77	Zero.	44	111.2	610	1130	1230	2246	1850	3362
— 17	+ 1.4	45	113.0	620	1148	1240	2264	1860	3380
— 16	+ 3.2	46	114.8	630	1166	1250	2282	1870	3398
— 15	+ 5.0	47	116.6	640	1184	1260	2300	1880	3416
— 14	+ 6.8	48	118.4	650	1202	1270	2318	1890	3434
— 13	+ 8.6	49	120.2	660	1220	1280	2336	1900	3452
— 12	+10.4	50	122.0	670	1238	1290	2354	1910	3470
— 11	+12.2	60	140	680	1256	1300	2372	1920	3488
— 10	+14.0	70	158	690	1274	1310	2390	1930	3506
— 9	+15.8	80	176	700	1292	1320	2408	1940	3524
— 8	+17.6	90	194	710	1310	1330	2426	1950	3542
— 7	+19.4	100	212	720	1328	1340	2444	1960	3560
— 6	+21.2	110	230	730	1346	1350	2462	1970	3578
— 5	+23.0	120	248	740	1364	1360	2480	1980	3596
— 4	+24.8	130	266	750	1382	1370	2498	1990	3614
— 3	+26.6	140	284	760	1400	1380	2516	2000	3632
— 2	+28.4	150	302	770	1418	1390	2534	2010	3650
— 1	+30.2	160	320	780	1436	1400	2552	2020	3668
Zero.	+32.	170	338	790	1454	1410	2570	2030	3686
+1	+33.8	180	356	800	1472	1420	2588	2040	3704
2	35.6	190	374	810	1490	1430	2606	2050	3722
3	37.4	200	392	820	1508	1440	2624	2060	3740
4	39.2	210	410	830	1526	1450	2642	2070	3758
5	41.0	220	428	840	1544	1460	2660	2080	3776
6	42.8	230	446	850	1562	1470	2678	2090	3794
7	44.6	240	464	860	1580	1480	2696	2100	3812
8	46.4	250	482	870	1698	1490	2714	2110	3830
9	48.2	260	500	880	1616	1500	2732	2120	3848
10	50.0	270	518	890	1634	1510	2750	2130	4166
11	51.8	280	536	900	1652	1520	2768	2140	4184
12	53.6	290	554	910	1670	1530	2786	2150	4162
13	52.4	300	572	920	1688	1540	2804	2160	4180
14	57.2	310	590	930	1706	1550	2822	2180	4216
15	59.0	320	608	940	1724	1560	2840	2200	4252

Number of Degrees Cent. = Number of Degrees Fahr.

Degrees Cent.	Tenths of a Degree—Centigrade Scale. .0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.
0	0.00	0.18	0.36	0.54	0.72	0.90	1.08	1.26	1.44	1.62
1	1.80	1.98	2.16	2.34	2.55	2.70	2.88	3.06	3.24	3.42
2	3.60	3.78	3.96	4.14	4.32	4.50	4.68	4.86	5.04	5.22
3	5.40	5.58	5.76	5.94	6.12	6.30	6.48	6.66	6.84	7.02
4	7.20	7.38	7.56	7.74	7.92	8.10	8.28	8.46	8.64	8.82
5	9.00	9.18	9.36	9.54	9.72	9.90	10.08	10.26	10.44	10.62
6	10.80	10.98	11.16	11.34	11.52	11.70	11.88	12.06	12.24	12.42
7	12.60	12.78	12.96	13.14	13.32	13.50	13.68	13.86	14.04	14.22
8	14.40	14.58	14.76	14.94	15.12	15.30	15.48	15.66	15.84	16.02
9	16.20	16.38	16.56	16.74	16.92	17.10	17.28	17.46	17.64	17.82

Number of Degrees Fahr. = Number of Degrees Cent.

Degrees Fahr.	Tenths of a Degree—Fahrenheit Scale. .0	.1	.2	.3	.4	.5	.6	7.	.8	.9
	Cent.	Cent.	Cent.	Cent.	Cent.	Cent.	Cent.	Cent.	Cent.	Cent
0	0.00	0.06	0.11	0.17	0.22	0.28	0.33	0.39	0.44	0.50
1	0.56	0.61	0.67	0.72	0.78	0.83	0.89	0.94	1.00	1.06
2	1.11	1.17	1.22	1.28	1.33	1.39	1.44	1.50	1.56	1.61
3	1.67	1.72	1.78	1.83	1.89	1.94	2.00	2.06	2.11	2.17
4	2.22	2.28	2.33	2.39	2.44	2.50	2.56	2.61	2.67	2.72
5	2.78	2.83	2.89	2.94	3.00	3.06	3.11	3.17	3.22	3.28
6	3.33	3.39	3.44	3.50	3.56	3.61	3.67	3.72	3.78	3.83
7	3.89	3.94	4.00	4.06	4.11	4.17	4.22	4.28	4.33	4.39
8	4.44	4.50	4.56	4.61	4.67	4.72	4.78	4.83	4.89	4.94
9	5.00	5.06	5.11	5.17	5.22	5.28	5.33	5.39	5.44	5.50

LATENT HEAT.

Latent heat is the number of units of heat required to change a body from one state to another whilst the temperature remains constant—that is, the heat required to melt a body from solid to liquid. and to evaporate a liquid. In the one case it is called the *latent heat of fusion*, and in the other, the *latent heat of evaporation*.

Latent Units of Heat per Pound of Substance.

Solids smelted to liquid.	Latent heat.	Liquids converted to vapor.	Latent heat.
Ice to water, ...	141	Water to steam, ...	966
Tin, ...	25.6	Ammonia.	895
Zinc.	50.6	Alcohol, pure, ...	372
Sulphur, ...	17.0	Carbonic acid, ...	298
Lead, ...	9.72	Bisulphide of carbon, . .	212
Mercury, ...	5.00	Ether, sulphuric, . . .	174
Beeswax, ...	175	Essence of turpentine, . .	137
Bismuth, ...	550	Oil of turpeutine, . . .	184
Cast iron, ...	233	Mercury, ...	157
Spermaceti, ...	46.4	Chymogene, ...	175

Fusion.

L = latent heat (units) per pound of liquid at smelting-point.
C = specific heat of the liquid.
c = specific heat of solid.
t = temperature of fusion, Fahr.

$$L = (C - c)(t + 256).$$

Evaporation.

l = latent units of heat per pound of vapor at boiling-point.
T = temperature of boiling-point, Fahr.

(*Regnault.*)

$$l = 1091.7 - 0.695(T - 32) - 0.000000103(T - 39.1)^3.$$

Temperature of Boiling or Evaporation under Atmospheric Pressure.

LIQUIDS.	Fahr.	Cent.	LIQUIDS.	Fahr.	Cent.
Wrought iron, . .	5000°	2760°	Alcohol, ...	173	78
Cast iron, . . .	3300	1815	Ether, ...	96	35
Mercury, . . .	675	352	Carbon, bisulphuretted,	116	47
Whale oil, . . .	630	332	Water, distilled, . .	212	100
Oil of linseed, . .	600	316	Salt sea water, . .	213	101
Oil of turpentine, .	357	180	Water 20 per cent. salt,	218	103
Sulphuric acid, . .	593	312	" 30 " " .	222	105
Sulphur, . . .	570	300	" 40 " saturated,	227	108
Phosphorus, . . .	557	292	Ammonia, *liquid*, . .	140	60
Sweet oil, . . .	412	211	Water in vacuo, . .	98	36
Naphtha, . . .	320	160	Chymogene, . . .	+ 38	3.3
Nitric acid, . . .	220	104	Carbonic acid, . .	—112	—80
Milk of cows, . . .	213	101	Ammonia, . . .	— 30	—34
Rectified petroleum, .	316	158			

Distillation Temperatures of Coal-oils.—(Tissandier.)

LIGHT OILS.	Fahr.	Cent.	HEAVY OILS.	Fahr.	Cent.	HEAVY OILS.	Fahr.	Cent.
Amylène,	102°	38.9°	Cumène,	304°	151°	Carbolic acid	370°	188°
Benzine,	187°	86.1°	Lutidine,	311°	155°	Nephthline,	422°	217°
Toluène,	226°	108°	Eupione,	338°	170°	Quilonèine,	462°	239°
Xylène,	271°	133°	Cymène,	347°	175°	Anthracene,	500°	260°
Pyridine,	302°	150°	Aniline.	359°	182°	Chrysène,	572°	300°

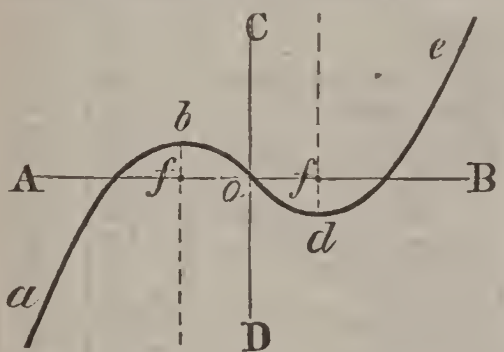
The temperature of distillation of vapors is equal to that of the boiling-point of the liquid of which the vapor is formed.

Temperature of Fusion, Freezing or Melting-Point.

SOLIDS.	Fahr.	Cent.	SOLIDS.	Fahr.	Cent.
Platinum, . . .	3080°	1693°	Puddle slag, . . .	2606°	1430°
Wrought iron, . .	2912	1600	Sulphur, . . .	228	109
Cast iron, gray. . .	2012	1100	Beeswax, white, . .	155	68
Cast iron, white, . .	1922	1050	" yellow, .	142	61
Steel, ...	2500	1371	Spermaceti, . . .	142	61
Gold, pure, . . .	2300	1260	Potassium, . . .	136	58
Gold, money, . . .	2192	1200	Sodium, ...	104	90
Copper, . . .	2160	1232	Olive oil, . . .	92	33
Brass, common, . .	1900	1038	Tallow, ...	36	2.2
Silver, ...	1850	1021	Ice of water, . .	32	0.000
Litharge, . . .	1739	954	" milk, . . .	30	— 1.1
Antimony, . . .	800	427	" sea water, . .	28	— 2.2
Zinc, ...	740	393	" vinegar, . .	28	— 2.2
Lead, ... L	600	316	" streng wine, .	20	— 6.6
Bismuth, ... B	470	254	" " brandy, .	7	— 13.9
Tin, ... T	420	215	" oil of turpentine,	14	— 10
2 Tin, 1 Lead, . . .	360	181	1 snow, 1 salt, . .	0.00	— 17.8
1 Tin, 3 Lead, . .	500	260	1 alcohol, 1 water, .	— 7	— 21.6
1 Tin, 1 Bismuth, . .	283	140	Cyangen, ...	— 30	— 34.4
3T + 2L + 5B, . .	212	100	Mercury, ...	— 40	— 40
1T + 1L + 4B, . .	200	93	Sulphuric ether, . .	— 47	— 43.9
2T + 3L + 2B, . .	199	92	Sulphurous acid, . .	—105	— 76
Slag of copper, . .	2462	1350	Nitrous oxide, . .	—150	— 101
Slag of tin, . . .	2402	1318	Nitric acid, . . .	— 55	
Nickel, ...	2800	1538			

EXPANSION OF BODIES BY HEAT.

Most bodies in nature expand when heated, and contract when cooled. Solids vary but little by the difference in temperature; liquids vary more; but gases are extremely susceptible to such differences.



There is a very singular fact connected with the expansion and contraction of substances at and near the temperature of fusion, which may be illustrated in the accompanying figure.

Let A B represent the absciss-axis of temperature, C D the ordinate axis of expansion or contraction, and the origin O the temperature of fusion, O A the temperature of the solid, and O B that of the liquid.

Let a solid of volume and temperature at a be heated, it will expand until it reaches a maximum volume at b, after which it contracts toward the temperature of fusion O. The temperature still increasing, the liquid will continue to contract until it reaches a minimum volume or maximum density at d, after which it will expand toward e. The lines $a\ b$ O and O $d\ e$ are parabolas, of which the absciss-axis A B passes through the foci f. The formula for the parabola is $y = x^n$, in which the exponent n depends upon the nature of the substance operated upon, and also whether it is linear or volume expansion, x representing the temperature and y the volume.

Ice melts at 32° Fahr., and the water reaches its maximum density at $d = 39°$ (as now accepted, but d is nearer 40°). Ice reaches its maximum volume at $b = 24°$. Ice and water are of equal density at the temperatures 16°, 32° and 48°. Ice generally floats in water, because the difference in temperature is less than 32°: but if ice of less than 16° is put into water of more than 48°, it will sink. The same phenomenon takes place with other substances; for instance, solid cast iron put into molten cast iron will float, but if the fluid cast iron is at a white heat, like that in a pneumatic furnace (Bessemer), the solid iron will sink.

The following formulas are deduced from experiments which have not extended through the temperatures of fusion, except that for water, page 392.

Notation.

L = linear expansion of solids and liquids, per degree Fahr., between any temperatures.

l = linear expansion per degree between 32° and 212°, as contained in the accompanying table.

D and d = absolute temperature in degrees Fahr.

n = exponent of expansion, which varies inversely with the rate of expansion of bodies.

Exponent n for	1.04 Water.	2.5 Glass.	2.6 Iron.	2.77 Copper.	14.1 Platinum.	15.6 Mercury.

Linear expansion per degree from 32° to T° will be

$$L = \frac{l}{10580000} \sqrt[n]{D}.$$

Linear expansion per degree between any temperature is

$$L = \frac{l}{10580000} \left(\sqrt[n]{D} - \sqrt[n]{d} \right)$$

The linear expansion per degree multiplied by 2 will be the surface expansion.

The linear expansion per degree multiplied by 3 will be the volume expansion.

Dilatation or Expansion of Substances,

Per Degree of Fahrenheit Scale.

Temperatures.	Solids.	Linear, l.	Surface, a.	Volume, v.
32° to 212°	Glass	0.00000478	0.00000956	0.00001434
212 " 392	Glass	0.00000546	0.00001093	0.00001639
392 " 572	Glass	0.00000660	0.00001320	0.00001980
32 " 212	Wrought iron	0.00000656	0.00001312	0.00001968
32 " 572	Wrought iron	0.00000895	0.00001790	0.00002686
32 " 212	Soft, good iron	0.00000680	0.00001360	0.00002040
32 " 212	Cast iron	0.00000618	0.00001236	0.00001854
32 " 212	Cast steel	0.00000600	0.00001200	0.00001800
32 " 212	Hardened steel	0.00000689	0.00001378	0.00002067
32 " 212	Copper	0.00000955	0.00001910	0.00002865
32 " 572	Copper	0.00001092	0.00002184	0.00003276
32 " 212	Lead	0.00001580	0.00003160	0.00004740
32 " 212	Gold, pure	0.00000815	0.00001630	0.00002445
32 " 212	Gold, hammered	0.00000830	0.00001650	0.00002490
33 " 212	Silver, pure	0.00001060	0.00002120	0.00003180
32 " 212	Silver, hammered	0.00001116	0.00002232	0.00003348
32 " 212	Brass, common cast	0.00001043	0.00002086	0.00003129
32 " 212	Brass, wire or sheet	0.00001075	0.00002150	0.00003225
32 " 212	Platinum, pure	0.00000491	0.00000982	0.00001473
32 " 572	Platinum, pure	0.00000520	0.00001040	0.00001560
32 " 212	Palladium	0.00000555	0.00001110	0.00001665
32 " 212	Roman cement	0.00000797	0.00001594	0.00002391
32 " 212	Platinum, hammered	0.00000530	0.00001060	0.00001590
32 " 212	Zinc, pure or cast	0.00001633	0.00003266	0.00004899
32 " 212	Zinc, hammered	0.00001722	0.00003444	0.00005166
32 " 212	Tin, cast	0.00001207	0.00002414	0.00003621
32 " 212	Tin, hammered	0.00001500	0.00003000	0.00004500
32 " 212	Fire brick	0.00000275	0.00000470	0.00000705
32 " 212	Good red brick	0.00000305	0.00000610	0.00000915
32 " 212	Marble	0.00000613	0.00001226	0.00001839
32 " 212	Granite	0.00000438	0.00000876	0.00001314
32 " 212	Bismuth	0.00000773	0.00001546	0.00002319
32 " 212	Antimony	0.00000602	0.00001204	0.00001806
32 " 212	Palladium	0.00000555	0.00001110	0.00001665
32 " 212	Mercury	0.00003333	0.00006666	0.00010000
212 " 392	Mercury	0.00003416	0.00006833	0.00010250
392 " 572	Mercury	0.00003500	0.00007000	0.00010500
32 " 212	Water	0.00008806	0.00017612	0.00026420
212 " 392	Water	0.00017066	0.00034133	0.00051020
392 " 572	Water	0.00018904	0.00037808	0.00056713
32 " 212	Salt, dissolved	0.00009250	0.00018500	0.00027780
32 " 212	Sulphuric acid	0.00011111	0.00022222	0.00033333
32 " 212	Turpentine and ether	0.00012966	0.00025933	0.00038900
32 " 212	Oil, common	0.00014814	0.00029629	0.00044444
32 " 212	Alcohol and Nitric Acid	0.00015151	0.00030302	0.00055555
32 " 212	All permanent gases	0.00069416	0.00138832	0.00208250

Force of Temperature.

It is temperature which expands bodies, and not the quantity of heat. See pages 379 and 392. Temperature is convertible into pressure. Let P denote the pressure in pounds per square inch, and T temperature Fahr.

$$\text{Then, } P = \left(\frac{T + 105.1}{202.8}\right)^6, \quad \text{and } T = 202.8\sqrt[6]{P} - 105.1.$$

This force, multiplied by the space of expansion, is the work done by the heat.

Linear Expansion or Contraction in Inches of Cast Iron, Lengths in Feet.

Length.	Difference in Temperature.—Fahrenheit.								
	100°	150°	200°	250°	300°	400°	500°	600°	800°
Feet.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
1	0.0072	0.0110	0.0150	0.0192	0.0237	0.0336	0.0444	0.0561	0.0787
2	0.0144	0.0220	0.0300	0.0384	0.0474	0.0632	0.0885	0.1123	0.1574
3	0.0216	0.0330	0.0450	0.0576	0.0711	0.1008	0.1332	0.1684	0.2361
4	0.0288	0.0440	0.0600	0.0768	0.0948	0.1344	0.1776	0.2246	0.3148
5	0.0360	0.0550	0.0750	0.0960	0.1185	0.1680	0.2220	0 2805	0.3935
6	0.0432	0.0660	0.0900	0.1152	0.1422	0.2016	0.2664	0.3368	0.4722
7	0.0504	0.0770	0.1050	0.1344	0.1659	0.2552	0 3108	0.3929	0.3509
8	0.0576	0.0880	0.1200	0.1536	0.1896	0.2688	0.3552	0.4496	0.6396
9	0.0648	0.0990	0.1350	0.1728	0.2133	0.3024	0.3996	0.5052	0.7083
10	0.0720	0.1102	0.1502	0.1926	0.2376	0.3360	0.4440	0.5616	0.7872
11	0.0792	0.1214	0.1652	0.2125	0.2615	0.3696	0.4884	0.6177	0.8659
12	0.0864	0.1316	0.1802	0.2318	0.2853	0.4032	0.5328	0.6739	0.9446
13	0.0936	0.1417	0.1952	0.2510	0.3090	0.4368	0.5772	0.7300	1.0233
14	0.1008	0.1519	0.2102	0.2703	0.3328	0.4704	0.6216	0.7862	1.1020
15	0.1080	0.1620	0.2253	0.2895	0.3565	0.5040	0.6660	0.8423	1.1808
16	0.1152	0.1722	0.2403	0.3088	0 3803	0.5376	0.7104	0.8985	1.2595
17	0.1224	0.1823	0.2553	0.3280	0.4040	0.5712	0.7548	0.9546	1.3382
18	0.1296	0.1925	0.2703	0.3472	0.4278	0.6048	0.7992	1.0108	1.4169
19	0.1368	0.2026	0.2853	0 3665	0.4515	0.6384	0.8436	1.0669	1 4956
20	0.1440	0.2203	0.3005	0.3852	0.4752	0.6720	0.8880	1.1232	1.5744
21	0.1512	0.2305	0.3155	0.4045	0.4995	0.7056	0.9324	1.1793	1.6531
22	0.1584	0.2407	0.3275	0.4238	0.5228	0.7392	0.9768	1.2394	1.7318
23	0.1656	0.2508	0.3425	0.3430	0.5465	0.7728	1.0212	1.2915	1.8105
24	0.1728	0.2610	0 3575	0.3623	0 5703	0.8064	1.0656	1.3477	1.8892
25	0.1800	0.2711	0.3725	0.3815	0.5940	0.8400	1.1100	1.4038	1.9679
26	0.1872	0.2813	0.3876	0.4008	0.6179	0.8736	1.1544	1.4600	2.0467
27	0.1944	0.2914	0.4026	0.4200	0.6415	0.9072	1.1988	1 5161	2 1254
28	0.2016	0.3016	0.4176	0.4393	0.6553	0.9408	1.2432	1.5723	2.2041
29	0.2088	0.3117	0.4326	0.4585	0.6890	0.9744	1.2876	1.6284	2 2829
30	0.2160	0.3304	0.4507	0.5778	0.7128	1.0080	1.3320	1.6848	2.3616
31	0.2232	0.3405	0.4657	0.5970	0.7365	1.0416	1.3764	1.7409	2.4403
32	0.2304	0.3507	0.4807	0.6163	0.7603	1.0752	1.4208	1.7971	2.5190
33	0.2376	0.3608	0.4957	0.6355	0.7841	1.1088	1.4652	1.8533	2 5977
34	0.2448	0.3710	0.5107	0.6548	0.8078	1.1424	1.5096	1.9094	2.6764
35	0.2520	0.3811	0.5258	0.6740	0.8316	1.1760	1.5540	1.9656	2.7552
36	0.2592	0.3913	0.5408	0.6933	0.8553	1.2096	1.5984	2.0217	2.8339
37	0.2664	0.4014	0.5558	0.7125	0.8791	1.2432	1.6428	2.0779	2.9126
38	0.2736	0.4116	0.5708	0.7298	0.9028	1.2768	1.6872	2.1340	2.9913
39	0.2808	0.4217	0.5858	0.7490	0.9266	1.3104	1.7316	2.1902	3.0701
40	0.2880	0.4406	0.6009	0.7704	0.9504	1.3440	1.7760	2.2464	3.1488
45	0 3240	0.4957	0.6760	0.8667	1.0692	1.5120	1.9980	2.5272	3.5424
50	0.3600	0.5508	0.7512	0.9630	1.1880	1.6800	2.2200	2.8080	3.9360
55	0.3960	0.6059	0.8263	1.0593	1.3068	1.8480	2.4420	3.0888	4.3296
60	0.4230	0.6610	0.9014	1.1556	1.4256	2.0160	2.6640	3.3696	4.7132
65	0.4680	0.6665	0.9765	1.2519	1.5444	2.1840	2.8860	3.6540	5.1068
70	0.5040	0.7711	1.0517	1.3482	1.6632	2.3520	3.1080	3.9312	5.5104
75	0.5400	0.8262	1.1268	1.4445	1.7820	2.5200	3.3300	4.2120	5.9040
80	0.5760	0.8813	1.2019	1.5408	1.9008	2.6880	3.5520	4.4948	6.2976
85	0.6120	0.9364	1.2770	1.6371	2.0196	2.7560	3.7740	4.7756	6.6912
90	0.6480	0.9914	1.3521	1.7334	2.1384	3.0240	3.9960	5.0544	7.0848
95	0.6840	1.0465	1.4272	1.8297	2.2572	3.1920	4.2180	5.3352	7.4784
100	0.7200	1.1016	1.5024	1.9260	2.3760	3.3600	4.4400	5.6160	7.8720
	0.00000600	612	626	642	660	700	740	780	820
	Expansion per Degree.—Fahrenheit.								

Multiply by 1.1 for wrought iron, 1.5 for copper, 1.6 for brass and 2.6 for zinc.

Conducting Power of Different Substances for Heat and Electricity.

Metals.	
Silver, fine,	100
Gold, "	98
Gold, .991,	84
Copper, ham'd,	85
Copper, cast,	81
Mercury,	68
Aluminium,	66
Zinc, hammered	64
Zinc, cast vertical,	63
Zinc, cast horizontal,	60
Lead, cast,	20
Cadmium,	57
Wrought iron,	43
Tin,	42
Steel,	40
Platinum,	40
Cast iron,	36
Antimony, cast vertical,	21
Antimony, cast horizontal,	19
German silver,	10
Bismuth,	6
Stone & Crystals.	
Marble,	12.21
Glass,	9.65
Common brick,	8.422
Fire-brick,	6.05
Fire-clay,	6.61
Porcelain,	7.55
Wood-ashes,	0.8359
Coal, anthracite	19.25
Coal, bitum.,	16.84
Coal, charred,	0.738
Coke,	19.80

Quartz sand,	35.56
Limestone,	19.8
Lime,	24.00
Quartz crystals,	80.0
Slate,	10.00
Keen's cement,	1.901
Plaster and sand,	1.870
Plaster Paris,	2.026
Roman cement,	2.080
Asphalt,	4.52
Chalk,	5.853
Woods.	
Fir, cross grain,	1.10
Fir, with the fibre,	3.10
Pine,	3.90
Oak, with the fibre,	3.30
Elm, " "	3.2
Ash, " "	3.1
Apple, " "	2.8
Ebony, " "	2.2
Lampblack,	0.112
Cross: with fibre=1:3,	
Birch,	4.10
Black oak,	3.2
Chestnut,	3.0
Spanish mahogany,	2.8
Walnut,	3.3
Fur.	
Hare's fur,	0.0946
Eider down,	0.0668
Beaver's fur,	0.0675
Raw silk,	0.0692
Wool, sheep,	0.0778
Cotton,	0.0834
Lint,	0.0846
Sewing-silk,	0.0955
Flannel,	0.395
Horse-hair,	0.304

Liquids.	
Water,	1.000
Mercury,	2.80
Proof spirit,	0.847
Alcohol, pure,	0.931
Nitric acid,	1.5
Sulphur. acid,	1.7
Sulphur. ether,	2.1
Turpentine,	3.1
Gases.	
Air,	0.9855
Radiating Power.	
Water,	100
Lampblack,	100
Paper, writing,	98
Rosin,	96
Sealing-wax,	95
Glass, common,	90
India ink,	88
Ice,	85
Red lead,	80
Graphite,	75
Lead, tempered,	45
Mercury,	20
Lead, polished,	19
Iron, polished,	15
Tin and silver,	12
Copper and gold,	12
Reflecting Powers.	
Brass,	100
Silver,	90
Tinfoil,	85
Tin,	80
Steel,	70
Lead,	60
Glass,	10
Glass, oiled or waxed,	5
Lampblack,	0

Miscellaneous Temperatures.

	Fahr.
In the Bessemer furnace,	4000°
Puddling-furnace,	3500
Cupola,	3000
Heat of common fire,	1100
Red heat in daylight,	1070
Iron red in dark,	752
Mean temp. of the earth,	50
" " " torrid zone,	75
" " " temp. "	50
" " " polar region,	20
Temp. of ignition,	636
Highest temp. of wind,	117
Temp. of the human blood,	98
A comfortable room,	70
Mean temp. of ocean,	62

		Fahr.
1 alcohol, 1 water freezes,		— 7°
Mean temp. of the poles,		— 13
Temp. outside atmosphere,		— 58
Greatest natural cold,		— 56
Vinous fermentation,		— 65
Acetous fermentation begins,		— 78
Acetification ends,		— 88
Phosphorus burns,		— 43
Greatest artificial cold produced,		—166
At 50°, Mixtures of—		Prod. cold.
Nitrate of ammonia,	1	46
Water,	1	
Sulphate of soda,	8	50
Muriatic acid,	5	
Dilute sulphuric acid,	5	23
Snow,	4	

ON AIR AND HEAT.

Dry air expands or contracts uniformly 0.00204 its volume per degree Fahr. in difference of temperature, or 0.00367 per degree Centigrade under constant pressure. Assuming the expansion per degree Fahr. as unit, the primitive volume will be—

$$\frac{1}{0.00204} = 490.196.$$

V and v = volumes of dry air of temperatures t and T Fahr., and pressure p and P above vacuum. The volumes and pressures in the following formulas may be expressed in any units of measures.

Volume and Temperature Variable under Constant Pressure.

$$V = v[1 + .00204\,(T - t)], \quad \text{and} \quad (T - t) = \frac{490.196\,(V - v)}{v}, \qquad 1.$$

Example 1. $v = 18$ cubic feet of air, of $t = 36°$, is to be heated to $T = 84°$ under constant pressure. Required, the volume V?

$$V = 18\,[1 + .00204\,(84 - 36)] = 19.798 \text{ cubic feet.}$$

Volume and Pressure Variable under Constant Temperature.

$(T - t) = 0$, *the pressures will be*—

$$\frac{P}{p} = \frac{V}{v}, \quad \text{and} \quad \frac{p}{P} = \frac{v}{V}, \qquad 2.$$

$$P = p\,\frac{v}{V}, \quad \text{and} \quad v = V\,\frac{p}{P}, \qquad 3.$$

Example 2. $V = 150$ cubic feet of air, of pressure $p = 1475$ pounds to the square inch, is to be compressed to 50 cubic feet. The heat generated in the compression to radiate through the vessel until the temperature of the compressed air is equal to that in V. Required, the pressure P?

$$\textit{Formula 3.} \quad P = p\,\frac{V}{v} = 14.75 \times \frac{150}{50} = 44.25 \text{ lbs.}$$

When the Temperature, Volume, and Pressure

are all variable, we have—

$$P = p\,\frac{V}{v}\,[1 + .00204\,(T - t)], \quad \text{and} \quad (T - t) = 490.196\left(\frac{Pv}{pV} - 1\right), \qquad 4.$$

It must be distinctly understood in all these formulas that the volume v belongs to the pressure P and temperature T, and the volume V to p and t. The primitive quantities are v, P, T. It may happen in the Formula 4 that $v > V$, $t > T$, and $p > P$.

Example 3. $V = 1000$ cubic inches of air of $p = 14.75$ and $t = 59°$, is to be reduced to $v = 320$ cubic inches, and the temperature increased to $T = 369°$. Required, the pressure P per square inch?

$$\textit{Formula 4.} \quad P = 14.75\,\frac{1000}{320}\,[1 + .00204\,(369 - 59)] = 75.23 \text{ lbs.}$$

Example 4. $v = 290$, $P = 88.5$ to be increased to $V = 838$, and $p = 18.4$. Required, the difference of temperature $(T - t)$?

$$\textit{Formula 4.} \quad (T - t) = 490.196\left(\frac{88.5 \times 290}{18.4 \times 838} - 1\right) = 325.7°.$$

HEAT IN PERMANENT GASES.

S = specific heat under constant pressure.
s = mean specific heat under any pressure and volume from 32° to $T°$.
p = 14.7 pounds to the square inch pressure of the gas at $t° = 32°$ F.
P = pressure of the same gas at the temperature $T°$.
v = volume in cubic feet of the gas at 32°.
V = volume of the same gas, but of pressure P and temperature $T°$.
W = weight in pounds of the gas experimented upon.
𝔓 = weight in a fraction of a pound per cubic foot of the gas.
h = units of heat in W pounds of gas elevated from 32° to $T°$, or from a pressure of 14.7 to P pound.

$$h = y\,S\,W\sqrt{\frac{V}{v}} \quad . \quad . \quad . \quad 1.$$

$$h = y\,S\,\mathfrak{P}\sqrt{Vv} \quad . \quad . \quad . \quad 2.$$

$$y = \frac{h}{S\,W}\sqrt{\frac{v}{V}} \quad . \quad . \quad . \quad 3.$$

$$y = \frac{h}{S\,\mathfrak{P}\sqrt{Vv}} \quad . \quad . \quad . \quad 4.$$

The value of y is calculated for different temperatures in the following tables of physical properties of permanent gases, by the aid of which the units of heat in any gas can be found.

Having given the weight W, volumes V and v, and the units of heat h, in any permanent gas, calculate the value of y by Formula 3 or 4, which gives the corresponding temperature of the gas in the table.

Example 11. How many units of heat are required to elevate the temperature of $v = 160$ cubic feet of air from 32° to $T° = 480°$, and expand the volume to $V = 240$ cubic feet?

In the table find $y = 324.29$ for 480°.

Units of heat, $h = 324.29 \times 0.25 \times 0.08042\sqrt{160 \times 240} = 1277.6.$

Example 13. What will be the temperature of $V = 36$ cubic feet of carbonic acid heated from 32° and volume $v = 24$ cubic feet when $h = 140$ units of heat has been expended on it?

$$y = \frac{140}{0.221 \times 0.1233\sqrt{36 \times 24}} = 133.8.$$

This corresponds to a temperature $T° = 185°$ in the table.

Specific Heat under Constant Pressure and Temperature 32°.

Kinds of Gases.	Pounds per cubic foot.	Cubic foot per pound.	Specific gravity. Water=1.	Specific gravity. Air=1.	Specific heat.
	𝔓	𝔈			S
Atmospheric air..........	0.08042	12.433	0.00130	1.000	0.15
Oxygen gas..................	0.08888	11.251	0.00143	1.104	0.23
Nitrogen gas................	0.07837	12.760	0.00126	0.972	0.275
Hydrogen gas...............	0.00559	178.84	0.00009	0.069	3.3
Carbonic oxide............	0.07837	12.760	0.00126	0.972	0.288
Carbonic acid..............	0.12333	8.108	0.00197	1.527	0.221
Steam..........................	0.05021	19.915	0.00634	0.488	0.475

Temp. Fahr.	$\frac{PV}{pv}$	$\frac{T-t}{\sqrt{x}}$	Temp. Fahr.	$\frac{PV}{pv}$	$\frac{T-t}{\sqrt{x}}$	Temp. Fahr.	$\frac{PV}{pv}$	$\frac{T-t}{\sqrt{x}}$	Temp. Fahr.	$\frac{PV}{pv}$	$\frac{T-t}{\sqrt{x}}$
$T°$	x	y	$T°$	x	y	$T°$	x	y	$T°$	x	y
32	1.0000	0.0000	92	1.1217	56.652	152	1.2434	107.62	212	1.3650	154.06
33	1.0020	0.9990	93	1.1237	57.555	153	1.2454	108.43	213	1.3670	154.80
34	1.0040	1.9960	94	1.1257	58.436	154	1.2475	109.23	214	1.3691	155.54
35	1.0061	2.9909	95	1.1277	59.326	155	1.2495	110.04	215	1.3711	156.28
36	1.0081	3.9839	96	1.1297	60.214	156	1.2515	110.84	216	1.3731	157.02
37	1.0101	4.9750	97	1.1318	61.098	157	1.2535	111.65	217	1.3751	157.76
38	1.0121	5.9640	98	1.1338	61.983	158	1.2556	112.45	218	1.3772	158.50
39	1.0142	6.9508	99	1.1358	62.867	159	1.2576	113.25	219	1.3792	159.24
40	1.0162	7.9360	100	1.1378	63.749	160	1.2596	114.05	220	1.3812	159.97
41	1.0182	8.9192	101	1.1399	64.627	161	1.2616	114.85	221	1.3832	160.71
42	1.0203	9.9000	102	1.1419	65.506	162	1.2637	115.64	222	1.3853	161.45
43	1.0223	10.880	103	1.1439	66.384	163	1.2657	116.44	223	1.3873	162.19
44	1.0243	11.857	104	1.1459	67.260	164	1.2677	117.24	224	1.3893	162.93
45	1.0264	12.834	105	1.1480	68.132	165	1.2697	118.04	225	1.3913	163.67
46	1.0284	13.805	106	1.1500	69.005	166	1.2717	118.83	226	1.3934	164.41
47	1.0304	14.777	107	1.1520	69.877	167	1.2738	119.62	227	1.3954	165.15
48	1.0325	15.746	108	1.1541	70.745	168	1.2758	120.41	228	1.3974	165.88
49	1.0345	16.714	109	1.1561	71.613	169	1.2778	121.20	229	1.3995	166.61
50	1.0365	17.680	110	1.1581	72.481	170	1.2798	121.98	230	1.4015	167.25
51	1.0385	18.666	111	1.1602	73.344	171	1.2818	122.77	231	1.4035	167.98
52	1.0406	19.606	112	1.1622	74.208	172	1.2839	123.56	232	1.4056	168.70
53	1.0426	20.567	113	1.1642	75.072	173	1.2859	124.35	233	1.4076	169.42
54	1.0446	21.575	114	1.1663	75.929	174	1.2879	125.13	234	1.4096	170.14
55	1.0466	22.482	115	1.1683	76.790	175	1.2899	125.91	235	1.4116	170.86
56	1.0487	23.463	116	1.1703	77.648	176	1.2920	126.69	236	1.4137	171.58
57	1.0507	24.390	117	1.1724	78.502	177	1.2940	127.47	237	1.4157	172.29
58	1.0527	25.341	118	1.1744	79.358	178	1.2960	128.25	238	1.4177	173.01
59	1.0547	26.290	119	1.1764	80.212	179	1.2980	129.02	239	1.4198	173.73
60	1.0567	27.260	120	1.1784	81.066	180	1.3001	129.80	240	1.4218	174.44
61	1.0588	28.184	121	1.1805	81.914	181	1.3021	130.57	241	1.4238	175.15
62	1.0608	29.128	122	1.1825	82.764	182	1.3041	131.34	242	1.4258	175.86
63	1.0628	30.070	123	1.1845	83.621	183	1.3062	132.11	243	1.4279	176.57
64	1.0649	31.010	124	1.1866	84.457	184	1.3082	132.88	244	1.4299	177.28
65	1.0669	31.949	125	1.1886	85.303	185	1.3102	133.65	245	1.4319	177.99
66	1.0689	32.896	126	1.1906	86.148	186	1.3122	134.42	246	1.4340	178.70
67	1.0709	33.822	127	1.1927	86.988	187	1.3143	135.19	247	1.4360	179.41
68	1.0720	34.770	128	1.1947	87.830	188	1.3163	135.96	248	1.4380	180.12
69	1.0740	35.703	129	1.1967	88.671	189	1.3184	136.73	249	1.4401	180.83
70	1.0760	36.633	130	1.1987	89.510	190	1.3204	137.50	250	1.4421	181.54
71	1.0780	37.563	131	1.2008	90.374	191	1.3224	138.27	251	1.4441	182.24
72	1.0811	38.470	132	1.2028	91.152	192	1.3244	139.04	252	1.4462	182.94
73	1.0831	39.396	133	1.2048	92.016	193	1.3265	139.81	253	1.4482	183.64
74	1.0851	40.320	134	1.2069	92.846	194	1.3285	140.58	254	1.4502	184.34
75	1.0871	41.289	135	1.2089	93.579	195	1.3305	141.35	255	1.4522	185.04
76	1.0892	42.160	136	1.2109	94.510	196	1.3326	142.12	256	1.4543	185.74
77	1.0912	43.079	137	1.2129	95.340	197	1.3346	142.89	257	1.4563	186.44
78	1.0932	43.995	138	1.2150	96.165	198	1.3366	143.66	258	1.4583	187.14
79	1.0953	44.940	139	1.2170	96.993	199	1.3386	144.42	259	1.4604	187.84
80	1.0973	45.822	140	1.2190	97.819	200	1.3407	145.19	260	1.4624	188.54
81	1.0993	46.734	141	1.2211	98.640	201	1.3427	145.95	261	1.4644	189.24
82	1.1014	47.643	142	1.2231	99.463	202	1.3447	146.70	262	1.4664	189.93
83	1.1034	48.552	143	1.2251	100.29	203	1.3468	147.44	263	1.4685	190.62
84	1.1054	49.459	144	1.2272	101.10	204	1.3488	148.18	264	1.4705	191.32
85	1.1075	50.362	145	1.2292	101.92	205	1.3508	148.92	265	1.4725	192.01
86	1.1095	51.266	146	1.2312	102.74	206	1.3529	149.66	266	1.4745	192.70
87	1.1115	52.168	147	1.2333	103.55	207	1.3549	150.39	267	1.4766	193.39
88	1.1135	53.069	148	1.2353	104.37	208	1.3569	151.12	268	1.4786	194.08
89	1.1156	53.966	149	1.2373	105.18	209	1.3589	151.85	269	1.4806	194.77
90	1.1176	54.851	150	1.2393	106.00	210	1.3610	152.58	270	1.4826	195.46
91	1.1196	55.760	151	1.2414	106.81	211	1.3630	153.32	271	1.4847	196.15

Temp. Fahr.	$\frac{PV}{pv}$	$\frac{T-t}{\sqrt{x}}$	Temp. Fahr.	$\frac{PV}{pv}$	$\frac{T-t}{\sqrt{x}}$	Temp. Fahr.	$\frac{PV}{pv}$	$\frac{T-t}{\sqrt{x}}$	Temp. Fahr.	$\frac{PV}{pv}$	$\frac{T-t}{\sqrt{x}}$
$T°$	x	y	$T°$	x	y	$T°$	x	y	$T°$	x	y
272	1.4867	196.84	332	1.6084	236.55	424	1.7950	292.59	720	2.3952	444.52
273	1.4887	197.53	333	1.6104	237.19	426	1.7990	293.75	730	2.4155	449.11
274	1.4907	198.22	334	1.6124	237.83	428	1.8031	294.91	740	2.4357	453.67
275	1.4928	198.90	335	1.6144	238.43	430	1.8071	296.07	750	2.4560	458.15
276	1.4948	199.58	336	1.6165	239.11	432	1.8112	297.23	760	2.4763	462.60
277	1.4968	200.26	337	1.6185	239.75	434	1.8152	298.39	770	2.4966	467.03
278	1.4988	200.94	338	1.6205	240.39	436	1.8193	299.54	780	2.5169	471.44
279	1.5009	201.62	339	1.6226	241.02	438	1.8234	300.68	790	2.5371	475.84
280	1.5029	202.30	340	1.6246	241.65	440	1.8274	301.82	800	2.5574	480.24
281	1.5049	202.98	341	1.6266	242.28	442	1.8315	302.96	810	2.5777	484.56
282	1.5070	203.66	342	1.6286	242.91	444	1.8355	304.10	820	2.5980	488.87
283	1.5090	204.34	343	1.6307	243.54	446	1.8396	305.24	830	2.6183	493.14
284	1.5110	205.02	344	1.6327	244.17	448	1.8436	306.37	840	2.6385	497.43
285	1.5131	205.70	345	1.6347	244.80	450	1.8477	307.51	850	2.6588	501.66
286	1.5151	206.37	346	1.6368	245.43	452	1.8518	308.65	860	2.6791	505.88
287	1.5171	207.04	347	1.6388	246.06	454	1.8558	309.79	870	2.6994	510.07
288	1.5192	207.71	348	1.6408	246.69	456	1.8599	310.91	880	2.7197	514.23
289	1.5212	208.38	349	1.6429	247.31	458	1.8639	312.03	890	2.7399	518.36
290	1.5232	209.05	350	1.6449	247.93	460	1.8680	313.15	900	2.7602	522.45
291	1.5252	209.72	351	1.6469	248.56	462	1.8720	314.27	910	2.7805	526.54
292	1.5273	210.39	352	1.6490	249.19	464	1.8761	315.39	920	2.8008	530.61
293	1.5293	211.06	353	1.6510	249.82	466	1.8801	316.51	930	2.8211	534.66
294	1.5313	211.73	354	1.6530	250.45	468	1.8842	317.63	940	2.8413	538.71
295	1.5334	212.40	355	1.6551	251.08	470	1.8882	318.75	950	2.8616	542.67
296	1.5354	213.07	356	1.6571	251.70	472	1.8923	319.37	960	2.8819	546.66
297	1.5374	213.74	357	1.6591	252.32	474	1.8963	320.99	970	2.9022	550.60
298	1.5395	214.40	358	1.6611	252.94	476	1.9004	322.09	980	2.9225	554.52
299	1.5415	215.06	359	1.6632	253.56	478	1.9044	323.19	990	2.9427	558.45
300	1.5435	215.72	360	1.6652	254.18	480	1.9085	324.29	1000	2.9630	562.36
301	1.5455	216.38	362	1.6692	255.42	482	1.9126	325.39	1010	2.9833	566.24
302	1.5476	217.04	364	1.6733	256.66	484	1.9166	326.49	1020	3.0036	570.09
303	1.5496	217.70	366	1.6773	257.90	486	1.9207	327.59	1030	3.0239	573.94
304	1.5516	218.36	368	1.6814	259.12	488	1.9248	328.69	1040	3.0441	577.73
305	1.5537	219.02	370	1.6854	260.36	490	1.9288	329.78	1050	3.0644	581.53
306	1.5557	219.68	372	1.6895	261.58	492	1.9329	330.88	1060	3.0847	585.32
307	1.5577	220.34	374	1.6935	262.80	494	1.9369	331.98	1070	3.1050	589.08
308	1.5597	221.00	376	1.6976	264.02	496	1.9410	333.06	1080	3.1253	592.82
309	1.5618	221.65	378	1.7016	265.24	498	1.9451	334.14	1090	3.1455	596.54
310	1.5638	222.30	380	1.7057	266.46	500	1.9491	335.22	1100	3.1658	600.24
311	1.5658	222.96	382	1.7097	267.71	510	1.9694	340.60	1110	3.1861	603.92
312	1.5678	223.61	384	1.7138	268.94	520	1.9898	345.95	1120	3.2064	607.62
313	1.5699	224.27	386	1.7179	270.16	530	2.0102	351.26	1130	3.2267	611.27
314	1.5719	224.93	388	1.7219	271.38	540	2.0302	356.53	1140	3.2469	614.92
315	1.5739	225.58	390	1.7260	272.50	550	2.0505	361.75	1150	3.2672	618.52
316	1.5759	226.23	392	1.7301	273.70	560	2.0708	366.93	1160	3.2875	622.13
317	1.5780	226.88	394	1.7341	274.90	570	2.0909	372.06	1170	3.3078	625.73
318	1.5800	227.53	396	1.7382	276.09	580	2.1113	377.16	1180	3.3281	629.32
319	1.5820	228.28	398	1.7422	277.29	590	2.1316	382.28	1190	3.3484	632.90
320	1.5840	228.83	400	1.7463	278.48	600	2.1519	387.20	1200	3.3687	636.38
321	1.5861	229.48	402	1.7504	279.68	610	2.1721	392.18	1300	3.5714	671.08
322	1.5881	230.13	404	1.7544	280.86	620	2.1924	397.13	1400	3.7743	704.74
323	1.5901	230.78	406	1.7585	282.04	630	2.2127	402.03	1500	3.9770	737.35
324	1.5922	231.42	408	1.7625	283.22	640	2.2329	406.89	1600	4.1798	766.95
325	1.5942	232.06	410	1.7666	284.40	650	2.2532	411.71	1700	4.3826	797.49
326	1.5962	232.71	412	1.7706	285.58	660	2.2734	416.50	1800	4.5854	826.60
327	1.5982	233.35	414	1.7747	286.76	670	2.2938	421.25	1900	4.7882	854.45
328	1.6003	233.90	416	1.7787	287.94	680	2.3141	425.98	2000	4.9910	880.91
329	1.6023	234.63	418	1.7828	289.01	690	2.3343	430.67	2100	5.1938	906.76
330	1.6043	235.27	420	1.7868	290.27	700	2.3545	435.34	2200	5.3966	931.72
331	1.6063	235.91	422	1.7909	291.43	710	2.3749	439.88	2300	5.5994	957.80

WARMING AND VENTILATION.

THE most comfortable temperature of a habitable room is about 70° F. or 21° C., and to make it wholesome 3 to 5 cubic feet of fresh air should be admitted per minute for each individual occupying the room under ordinary circumstances, but in warm weather, or when the occupants exert themselves with manual labor, double that quantity of fresh air is required. In hospitals about 50 cubic feet of fresh air is required per minute for each patient.

Respiration.—An adult in good health respires about 600 to 1000 cubic inches of air per minute under ordinary circumstances, but under great exertion double that quantity may be required. Air becomes vitiated in the act of respiration; that is, its oxygen combines with carbon, forming carbonic acid. A man makes 15 to 20 respirations per minute, using about 50 cubic inches in each respiration, or $\frac{20 \times 50 \times 60}{1728} = 35$ cubic feet per hour, nearly.

Warming.—The specific heat of air under atmospheric pressure is 0.25 of that of an equal weight of water. One pound of air at 32° F. occupies 12.433 cubic feet; or $12.433 \times 4 = 49.732$, say 50, cubic feet of air can be elevated 1° F. per unit of heat.

V = volume of air in cubic feet to be heated.
$T°$ = temperature of the heated air.
$t°$ = temperature of the cold air.
h = units of heat required to heat V volumes of air from $t°$ to $T°$.

$$h = \frac{V}{50}(T° - t°).$$

When warming is accomplished by hot water or steam conducted in pipes through the room to be warmed, the heat radiates from the pipe and thus heats the air, and at the same time heat is conducted through the walls and windows of the room.

The problem before us is to find the quantity of pipes and fuel required for heating and maintaining a desired temperature of a given volume of air.

Experiments on this subject have been made by MM. Peclet, Tredgold, Rumford, Hood, and many others, from which the following data are deduced:

L = length of pipe in feet.
d = diameter (internal) of pipe in inches.
$T°$ = temperature of the steam or water in the pipe.
t'' = temperature of the external air.
t' = the required temperature of the room.
V = volume of air in cubic feet to be warmed per minute—about 5 cubic feet for each occupant of the room—to which add $1\frac{1}{4}$ cubic feet for each square foot of glass windows.
C = consumption of coal in pounds per hour for maintaining the heat in the pipe.

$$L = \frac{d\,V(t' - t'')}{8(T° - t')}. \qquad C = \frac{d\,L\,(T° - t')}{12800}.$$

On an average, one square foot heating-surface is used for every 100 cubic feet of air to be kept warm.

The exhaust steam from steam-engines is better for heating purposes than is live steam. When exhaust steam is used, care should be taken to have the steam passages large, so as to prevent back pressure.

Heating Rooms by Stoves and Open Fires.

The quantity of heat utilized in warming apartments by open fires is from 10 to 20 per cent. of the total heat of combustion.

Compression and Expansion of a Definite Weight of Air Enclosed in a Vessel.

In the compression and expansion of air, as given in the following table, it is supposed that no heat is transmitted to or from the air operated upon. In compression, the temperature of the air rises; and if the heat is allowed to be conducted through the sides of the vessel enclosing the air, the pressure will not correspond with the table. In expanding the air the temperature is lowered, as seen in the table. The primitive volume is assumed to be at 32° Fahr.

Compression and Expansion of Air.

Compression of Air.				Expansion of Air.			
Volume. $v = 1$.	Temp. Fahr.	Pressure. Atmos.	Pressure. Lbs. p. sq.in.	Volume. $v = 1$.	Temp. Fahr.	Pressure. Atmos.	Pressure. Lbs. p. sq. in.
V	*T*°	*A*	*P*	*V*	*T*	*A*	*P*
1.	32°	1.0000	14.7	1.0	+ 32°	1.0	14.7
0.95	42.43	1.0297	15.137	1.1	+ 13.20	0.8751	12.864
0.90	53.66	1.159	17.036	1.2	— 3.3	0.7747	11.393
0.85	65.81	1.255	18.456	1.3	— 18.06	0.6926	10.181
0.80	79.01	1.366	20.090	1.4	— 31.26	0.6243	9.1778
0.75	93.43	1.496	21.991	1.5	— 39.65	0.58354	8.5780
0.70	109.26	1.647	24.215	1.6	— 54.06	0.5179	7.613
0.65	126.77	1.828	26.561	1.7	— 64.00	0.4757	6.9934
0.60	146.30	2.044	30.054	1.8	— 73.16	0.4391	6.4556
0.55	168.25	2.309	33.948	1.9	— 82.34	0.4083	6.002
0.50	193.20	2.639	38.792	2.0	— 89.47	0.3789	5.570
0.45	221.96	3.058	44.547	2.25	— 106.9	0.3213	4.7235
0.40	245.70	3.607	53.020	2.5	— 121.85	0.2779	4.0851
0.35	295.73	4.348	63.917	2.75	— 134.77	0.2426	3.5666
0.33	314.10	4.721	69.406	3.00	— 146.15	0.2148	3.1576
0.30	344.87	5.396	79.313	3.25	— 156.27	0.1920	2.8228
0.25	407.13	6.964	102.38	3.50	— 167.29	0 1731	2.5446
0.20	489.91	9.518	139.92	3.75	— 173.57	0.1572	2.3103
0.15	606.4	14.24	209.31	4.00	— 181.00	0.1436	2.1111
0.125	691.0	18.38	270.17	4.5	— 194.18	0.1218	1.7900
0.10	800.9	25.12	369.24	5	— 205.4	0.1051	1.5444
0.05	1213.5	66.289	974.45	6	— 223.74	0.0813	1.1965
0.04	1373.2	90.60	1331.8	7	— 238.26	0.0656	0.9642
0.03	1601.7	135.53	1992.3	8	— 250.03	0.0544	0.7998
0.02	1973.0	239.09	3514.6	9	— 259.92	0.0461	0.6782
0.01	4469.0	794.33	1167.600	10	— 268.39	0.0355	0.5216

On the Compression and Expansion
of a definite weight of air enclosed in a vessel.

In this treatment no heat must be lost or gained by radiation from the sides of the vessel in which the air is enclosed. Let D and d represent the degrees of absolute temperatures of volumes v and V of the air to be experimented upon.

The absolute zero is 461° below Fahr. zero, and 273° Cent. below the freezing-point of water. $D = 461 + T$, $d = 461 + t$, and $D - d = T - t$, Fahr. scale.

Volume and Temperature.

$$\frac{V}{v} = \left(\frac{D}{d}\right)^{2.45}, \quad \text{and} \quad \frac{v}{V} = \left(\frac{d}{D}\right)^{2.45} \qquad 5.$$

$$\text{Expansion} \quad V = v\left(\frac{D}{d}\right)^{2.45}, \quad \text{Compression} \quad v = \left(\frac{d}{D}\right)^{2.45} \qquad 6.$$

$$\text{Compression} \quad D = d \sqrt[2.45]{\frac{V}{v}}, \quad \text{Expansion} \quad d = D \sqrt[2.45]{\frac{v}{V}} \qquad 7.$$

Example 5. To what fraction must air of $t = 65°$ be compressed, in order to fire tinder at a temperature of $T = 550°$, $d = 461 + 65 = 526°$, $D = 550 + 461 = 1011°$?

Formula 5. $\frac{v}{V} = \left(\frac{526}{1011}\right)^{2.45} = 0.20$, the answer.

Example 6. How much must air of $T = 80°$ be expanded to reduce the temperature to $t = 32°$, or freezing-point of water?

Formula 5. $\frac{V}{v} = \left(\frac{541}{493}\right)^{2.45} = 1.3308$ times, the answer.

Example 7. $v = 360$ cubic inches of air of temperature $T = 380°$, or $D = 841°$, is to be expanded until the temperature becomes $t = 80°$ or $d = 541°$. Required, the volume V, corresponding to that temperature?

Formula 6. $V = 360\left(\frac{821}{541}\right)^{2.45} = 1025.9$ cubic feet.

Example 8. $V = 20$ cubic feet of air of $t = 32°$, or $d = 493$, is to be compressed to $v = 12$ cubic feet. Required, the temperature T of compression?

Formula 7. $D = 493 \sqrt[2.45]{\frac{20}{12}} = 60729°$, or $T = 146.29°$.

Pressure and Temperature.

$$\frac{P}{p} = \left(\frac{D}{d}\right)^{3.42}, \quad \text{and} \quad \frac{p}{P} = \left(\frac{D}{d}\right)^{3.42} \qquad 8.$$

$$\text{Compression} \quad P = p\left(\frac{D}{d}\right)^{3.42}, \quad \text{Expansion} \quad p = P\left(\frac{d}{D}\right)^{3.42} \qquad 9.$$

$$\text{Compression} \quad D = d \sqrt[3.42]{\frac{P}{p}}, \quad \text{Expansion} \quad d = D \sqrt[3.42]{\frac{p}{P}} \qquad 10.$$

Example 9. A volume of air of pressure $p = 15$ pounds to the square inch, and of temperature $t = 62°$, is to be compressed until the temperature becomes $T = 120°$. Required, the pressure P per square inch at $T = 120°$?

$$d = 461 + 62 = 523, \quad \text{and } D = 461 + 120 = 581.$$

Formula 9. $P = 15\left(\frac{581}{523}\right)^{3.42} = 21.49$ lbs. pr. sq. inch.

Example 10. A volume of air of pressure $P=45$ pounds to the square inch, and of temperature $T=250^\circ$, or $D=711^\circ$, is to be expanded to a pressure of $p=25$ pounds. Required, the temperature t of the expanded air?

Formula 10. $$d=711\sqrt[3.42]{\frac{25}{45}}=598.72^\circ, \text{ and}$$

$$t=598.72-461=137.72^\circ, \text{ the temperature required.}$$

Pressure and Volume.

$$\sqrt[.41]{\frac{V}{v}}=\sqrt[.29]{\frac{p}{P}} \quad \text{and} \quad \sqrt[.41]{\frac{v}{V}}=\sqrt[.29]{\frac{p}{P}} \quad . \quad 11.$$

$$\text{Expansion } V=v\sqrt[1.4]{\frac{P}{p}}, \quad \text{Compression } v=V\sqrt[1.4]{\frac{p}{P}} \quad . \quad . \quad 12.$$

$$\text{Compression } P=p\left(\frac{V}{v}\right)^{1.4}, \quad \text{Expansion } p=P\left(\frac{v}{V}\right)^{1.4} \quad . \quad 13.$$

Example 11. A volume $v=50$ cubic inches, and of pressure $P=80$ pounds per square inch, is to be expanded until the pressure becomes $p=15$ pounds. Required, the expanded volume V?

Formula 12. $$V=50\sqrt[1.4]{\frac{80}{15}}=165 \text{ cubic inches.}$$

Example 12. What will be the pressure of a volume of air expanded 1.3308 times?

Formula 13. $$p=\left(\frac{1}{1.3308}\right)^{1.4}=0.5324 \text{ of the primitive pressure.}$$

Volume and Weight of Dry Air

At different Temperatures, under a constant Atmospheric Pressure of 29.92 *inches in the Barometer, the Volume at* 32° *Fahr. being the unit.*

Temp. Fahr.	Volume.	Wt. per Cub. ft. Pounds.	Temp. Fahr.	Volume.	Wt. per Cub. ft. Pounds.	Temp. Fahr.	Volume.	Wt. per. Cub. ft. Pounds.
0°	.935	.0864	162°	1.265	.0368	550°	2.056	.0384
12	.960	.0842	172	1.425	.0628	600	2.150	.0376
22	.980	.0824	182	1.306	.0618	650	2.260	.0357
32	1.000	.0807	192	1.326	.0609	700	2.362	.0338
42	1.020	.0791	202	1.347	.0600	800	2.566	.0315
52	1.041	.0776	212	1.367	.0591	900	2.770	.0292
62	1.061	.0761	230	1.404	.0575	1000	2.974	.0268
72	1.082	.0747	250	1.444	.0559	1100	3.177	.0254
82	1.102	.0733	275	1.495	.0540	1200	3.381	.0239
92	1.122	.0720	300	1.546	.0522	1500	3.993	.0202
102	1.143	.0707	325	1.597	.0506	1800	4.605	.0175
112	1.163	.0694	350	1.648	.0490	2000	5.012	.0161
122	1.184	.0682	375	1.689	.0477	2200	5.420	.0149
132	1.204	.0671	400	1.750	.0461	2500	6.032	.0133
142	1.224	.0659	450	1.852	.0436	2800	6.644	.0121
152	1.245	.0649	500	1.954	.0413	3000	7.051	.0114

For Weight and Volume of air at Low Temperature, see Hygrometry, page 357.

SPECIFIC HEAT.

Different bodies require different quantities of heat to raise them to the same temperature. The amount of heat required to raise the unit weight of a substance one degree in temperature is called the specific heat of that substance. The specific heat of bodies varies nearly inversely as the specific gravity. The specific heat in all bodies increases slightly with the temperature. One pound of water raised from 32° to 212° requires 180.9 units of heat instead of 180. The specific heat increases nearly in the same ratio for all solid and liquid bodies. The specific heat of water from 32° to $T°$ will be—

$$S = 1 + \frac{(T - 32)^{1.67}}{1167713}. \qquad 1.$$

The specific heat of water between any temperatures T and t will be—

$$S = 1 + \frac{(T - 32)^{1.67} - (t - 32)^{1.67}}{1167713}. \qquad 2.$$

The following table gives the specific heat of different substances between the temperature 32° and 212°, compared with water as unit. When the specific heat of a body is required between high temperatures, it is necessary to calculate first the specific heat of water between such temperatures, which multiplied by the number in the table will give the required specific heat of the body.

Specific Heat of Substances.

Water, ...	1.000	Lead, ...	0.030	Sweet oil, . . .	0.310
Ice of water, .	0.513	Steel, ...	0.118	Oil of turpentine,	0.472
Cast iron, . . .	0.140	Diamond, . . .	0.147	*Gases of constant*	
Wrought iron, .	0.110	Arsenic, ...	0.081	*volume and under*	
Cobalt, ...	0.150	Iodine, ...	0.054	*atmospheric pres-*	
Nickel, ...	0.103	Sulphur, ...	0.200	*sure.*	
Copper, ...	0.094	Lime, burned, .	0.217	Atmospheric air, .	0.250
Zinc, ...	0.093	Glass-crystals, . .	0.193	Oxygen, . . .	0.230
Tin, ...	0.047	Glass, common,	0.177	Hydrogen, . . .	3.30
Antimony, . .	0.051	Woods, average, .	0.500	Nitrogen, . . .	0.275
Bismuth, ...	0.030	Brick, common,	0.200	Carbonic acid, . .	0.221
Tellurium, . .	0.091	Firebrick, . . .	0.220	Carbonic oxide, .	0.288
Gold, ...	0.029	Coal, ...	0.261	Olefiant gas, . .	0.421
Silver, ...	0.057	Beeswax, ...	0.450	Nitro-oxide, . .	0.237
Platinum, . . .	0.034	Alcohol, s. g. 0.81,	0.700	Gas of oils, . .	0.421
Brass, ...	0.094	Sulphuric acid, .	0.335	Sulp. hydrogen, .	0.242
Mercury, ...	0.030	Nitric acid, . .	0.661	Steam of atm. pr.,	0.475

Let two different substances of known weight or volume and temperature be mixed together; the temperature of the mixture will dissolve the relative quantity of heat in the ingredients.

Mixture of the same Substances.

W = weight or volume of a substance of temperature T.
w = weight or volume of a similar substance of temperature t.
t' = temperature of the mixture $W + w$. We shall have—

$$t'(W + w) = WT + wt, \qquad 3.$$

$$t' = \frac{WT + wt}{W + w}, \qquad 5.$$

$$W = \frac{w(t' - t)}{T - t'}, \qquad 4.$$

$$T = \frac{w(t' - t)}{W} + t'. \qquad 6.$$

Example 1. Let $W = 4.62$ cubic feet of water at $T = 150°$ be mixed with $w = 5.43$ cubic feet at $t = 46$. Required, the temperature of the mixture $t' = ?$

$$t' = \frac{4.62 \times 150° + 5.43 \times 46°}{4.62 + 5.43} = 97.6°, \text{ the answer.}$$

Example 2. How much water of $T = 107°$ must be mixed with $w = 27.3$ gallon of $t = 58°$, the mixture of the water to be $75°$?

$$W = \frac{27.3(75 - 58)}{107 - 75} = 14.5 \text{ gallons.}$$

Mixture of Different Substances.

W and w expressed by *weights* only. S and s = *specific heat* as given in the accompanying table. We shall have—

$$WS(T - t') = ws(t' - t), \quad 7.$$

$$T = \frac{t'(WS + ws) - wst}{WS}. \quad 8.$$

$$t' = \frac{WST + wst}{WS + ws}. \quad 9.$$

$$W = \frac{ws(t' - t)}{S(T - t)}, \quad 10.$$

Example 3. To what temperature must $W = 20$ pounds of cast iron be heated to raise $w = 131$ pounds of water of $t = 54°$ to a temperature $t' = 64°$? $T = ?$

From the table we have $s = 1$, and $S = 0.14$.

$$T = \frac{64(20 \times 0.14 + 131) - 131 \times 1 \times 54}{20 \times 0.14} = 532°,$$

the required temperature, supposing no vapor escapes from the water.

If any chemical action takes place in the mixture, these formulas will not answer, because part of the *sensible caloric* may become *latent*, or *latent caloric* may be set free.

Example 4. The temperature of 5 pounds of copper is to be elevated from 60° to 80°. How many heat units will be required?

See table for copper $0.094\,(80 - 60) = 1.88$ heat units, the answer.

Specific Heat of Gases.

When heat is applied to a constant volume of gas enclosed in a vessel, the specific heat of that gas increases as the square root of the pressure generated by the heat.

$$S = \frac{0.9585}{\sqrt{P}}. \quad 11.$$

When the volume, pressure and temperature are all variable, the specific heat of air will be—

$$S = \frac{0.9585}{\sqrt{\frac{VP}{v}\left(\frac{T - t}{493} + 1\right)}}. \quad 12.$$

For any permanent gas of s = specific heat under atmospheric pressure, the specific heat under any other pressure and volume will be—

$$S = \frac{3.834\, s}{\sqrt{\frac{VP}{v}\left(\frac{T - t}{493} + 1\right)}}. \quad 13.$$

The weight of 320 cubic feet of air at 59° is (see table) $0.076 \times 320 = 24.32$ pounds. The heat units required in the operation will be—

Formula 13. $h = 1.08 \times 24.32\,(369 - 59) = 1050.625$ units.

The specific heat of any other gas, under different volumes, pressures and temperatures, is equal to that of air multiplied by the specific heat in the table and divided by 0.25.

The number of heat units h required to elevate the temperature of W pounds of gas from $t°$ to $T°$, will be—

$$h = S\,W(T - t). \quad . \quad . \quad . \quad . \quad 13.$$

When the pressure is constant, and the volume is increased by heat, and $S =$ specific heat of the primitive volume of air, then the heat units will be—

$$h = S\,W(T - t) + \frac{P}{5.36}\left(\frac{V}{v} - v\right), \quad . \quad . \quad 14.$$

in which the last term expresses the heat units expended in expanding the volume under the pressure P.

Example 6. One cubic foot of air of $t = 32°$ is enclosed in a cylinder of one square foot area of piston, and under atmospheric pressure $P = 14.75$ pounds to the square inch. Let heat be applied to the air until $T = 511°$, when the volume will be about double, the piston being well balanced to move with the constant pressure. Required, the number of heat units imparted to the air, and the heat expended in moving the piston with the pressure 14.75 pounds.

Formula 14. $h = 0.25 \times 0.0807 \times (511 - 32) + \frac{14.75}{5.36}\left(\frac{2}{1} - 1\right) = 9.65 + 2.75 =$ 12.4 heat units, of which 2.75 were expended in moving the piston one foot.

UNITS OF HEAT AND WORK.

The ordinary English unit of heat is that required to elevate the temperature of one pound of distilled water one degree Fahr. from 39° to 40° (Fahr. ℔.), and called one *calorie* or heat unit.

The German unit of heat is that required to elevate the temperature of one pound (German pfund) of water one degree Centigrade, from 4° to 5° (Cent. ℔.) or (Cent. pfd.)

The French unit of heat (called *calorie*) is that required to elevate the temperature of one kilogramme (2.2047 ℔s.) of water one degree Centigrade from 4° to 5° (Cent. kilo.).

A combination of French and English units of heat is sometimes expressed by Fahr. scale and French weight (Fahr. kilo.).

Heat is dynamic work, or the product of the two elements, *force* and *motion*, in which the temperature of the heat represents *force*, and the cubic contents of the units of heat represent the product of *time* and *velocity*, which is *space*.

The English unit of *dynamic work* is one pound raised one foot, called *footpound* (Ft. ℔.). One calorie = 772 ft. ℔s.

The French unit of *dynamic work* is one kilogramme raised one metre, called kilomet.

One horse-power will consume or generate 2564 heat units per hour.

Comparison of Different Units of Heat and Work.

English Calorie.		French Calorie.		Prussian.	Dynamic Work.	
Fahr. ℔.	Cent. ℔.	Fahr. kilo.	Cent. kilo.	Cent. pfd.	Ft. ℔.	Kilomet.
1	0.5555	0.4536	0.2520	0.5769	772	106.51
1.8	1	0 8165	0.4536	1.0385	1389.6	191.71
2.2047	1.2248	1	0.555	1.2719	1702	348.066
3.968	2.2047	1.8	1	2.2894	3063.6	626.52
1.733	0.9630	0.7862	0.4368	1	1368.2	273.66
.0012953	.0007196	.0005876	.0003264	.0007473	1	0.13825
.0093896	.0005205	.0004250	.0002361	.0005405	7.233	1

Performance, Weight and Dimensions of Heavy Ordnance.

Description.	Diam. of Bore.	Length of Gun.	Weight in Pounds of Gun.	Proj'tile	Powd.	Velocity	Bore.
	Inches.	Ft. in.	Pounds.	Pounds.	Pounds	Ft. per sec	
American, rifle, . .	4¼	7′ 4″	3,089	. . .	. .	. . .	Rifle.
" " . .	6	9′ 6″	7,970	. . .	. .	. . .	Rifle.
Eng., wrought iron,	8	9′ 10″	14.560	180	30	1324	Smooth.
American, cast iron,	9	10′	9,08½	. . .	. .	. . .	"
English, "	10	11′	40.320	400	60	1298	"
American, "	11	11′ 6″	16.511	. . .	. .	. . .	"
Russian, "	11	11′ 6″	55.800	496	82.5	1362	"
English, "	12	12′	55,800	600	67	1180	"
American, "	13	13′ 3″	16,511	. . .	. .	. . .	"
" "	15	15′ 6″	42,100	. . .	. .	. . .	"
" "	20	20′ 3″	115,000	936	120	1131	"
" mortar, .	13	2′ 10″	17,198	. . .	. .	. . .	"
Rus. brass, Moscow,	30	25′	80,000	3000	. .	. . .	"

Effect of Gunpowder.

The dynamic work of different kinds of gunpowder, utilized in heavy ordnance, varies between 150,000 and 200,000 foot-pounds per pound of powder. Let K denote the dynamic work in a charge of powder, $W=$ weight in pounds of the ball or projectile, $V=$ velocity of the projectile in feet per second, then

$$V=8\sqrt{\frac{K}{W}}, \qquad W=\frac{64K}{V^2}, \qquad \text{and} \qquad K=\frac{WV^2}{64}.$$

The length of the gun for these formulas should be at least 12 times the bore.

Force of Gunpowder.

The force of gunpowder depends much upon its quickness of burning and resistance to its expansion. Gunpowder enclosed in a strong vessel, and burned in its primitive volume, may reach a pressure of 100 tons to the square inch; but when the gas of powder is subjected to an excessive pressure, it seems to condense and loses the property of expansion due to a permanent gas less strained. This is a very important fact in the use of heavy ordnance, where the gun may be double strained with a loss of effect in the projectile.

Quick powder may strain a gun over 30 tons to the square inch, whilst slower powder will strain it only 15 tons, and give a greater velocity to the projectile.

It appears that the charge ought to be so arranged in a gun that a slow powder is first ignited, and then a quicker and quicker until the quickest at last, by which the gun need not be strained to more than 15 tons to the square inch, with full benefit of the expansion property of the gas, greater velocity of the projectile and less risk of bursting the gun. The work done by the gas of powder in a gun should be treated under the same laws as that of steam in a steam cylinder.

This special subject is too extensive for proper treatment in this Pocket-book.

Composition of Gunpowder.

The composition of gunpowder varies in all proportions between the limits of 70 and 78 parts of saltpetre (nitrate of potash, KNO_3), 13 and 15 parts of charcoal, 9 and 20 parts of sulphur in 100 parts of the powder.

Chinese powder, 62 saltpetre, 23 charcoal, 15 sulphur.

The different proportions depend much upon the purpose for which the powder is used, and also upon the ideas and experience of the manufacturers and users of the powder. The quickest powder requires the highest proportions of saltpetre.

Size of Gunpowder-grains.

Use of Powder.	Size of Sieve.	Size in Inches.	Composition
Sporting, . . .	No. 1 to 2.	0.03 to 0.06	77.13.10
Mortar, . .	No. 2 to 3.	0.06 to 0.1	76.13.11
Cannon, . . .	No. 4 to 5.	0.25 to 0.35	75.13.12
Mammoth, . .	No. 6 to 7.	0.6 to 0.9	74.14.12

Fine gunpowder is also moulded into lumps to fit the chamber of the gun. The Russians mould fine powder into hexagon blocks for heavy ordnance.

PROPERTIES OF WATER AND STEAM,

In Relation to Heat.

The following six pages of tables for water and steam have been calculated by the author whilst stationed in the Bureau of Steam Engineering of the United States Navy Department, under the direction of Chief Engineer Isherwood. The tables have been improved for this Pocket-book.

Properties of Water.

Column h' contains the heat units required to raise each cubic foot of distilled water from 32° to temperature T, under the pressure P.

Column h contains the heat units required to raise each pound of water from 32° to $T°$. This column is calculated from the formula deduced from Regnault's experiments, namely:

$$h = T° - 32° + \frac{(T° - 32)^n}{p'}, \quad \text{or } h' = T - t' + \frac{(T-32)^n - (t'-32)^n}{p'}. \quad 1.$$

in which the last term is a parabola of exponent $n = 2.67$, and parameter p' 1167713. log. $p' = 6.0673350$. h'= heat units required per pound of water of temperature t', and raised to $T°$.

Column c contains the fractional cubic feet per pound of water of temperature T.

Column w contains the weight in pounds per cubic foot of water of temperature T. Water of the maximum density at 39° weighs 62.388055 pounds per cubic foot.

Column v contains the volume of water of temperature T, that at 39° being unit. This column is calculated from the Formula 2, deduced from Kopp's experiments.

$$v = 1 + \frac{(T - 39)^2}{2000000\,[0.23 + 0.0007\,(T - 39)]}. \quad . \quad 2.$$

Column t contains the temperature of the steam and water, Centigrade scale.

Columns i and p give the steam-pressure indicated on the safety-valve or mercury-gauge.

+ means pressure above the atmosphere.
— means vacuum under the atmosphere.

Properties of Steam.

Column P contains the total steam-pressure in pounds per square inch, including the pressure of the atmosphere.

Column I is the same pressure in inches of mercury, The specific gravity of mercury at 32° Fahr. is 13.5959, compared with water of maximum density at 39°. One cubic inch of mercury weighs 0.49086 pounds, of which a column of 29.9218 inches is a mean balance of the atmosphere, or 14.68757 lbs. per sq. in.

Column T contains the temperature of the steam on Fahr.'s scale, deduced from Regnault's experiments.

Column V contains the volume of steam of the corresponding temperature T, compared with that of water of maximum density at 39° Fahr. This column is calculated from the formula of Fairbairn and Tate, namely:

$$V = 25.62 + \frac{49513}{I + 0.72}. \quad . \quad . \quad . \quad 3.$$

Column W contains the weight per cubic foot in fractions of a pound; and

Column C the cubic feet per pound of saturated steam under the pressure P and temperature T.

Column H contains the heat units per pound of steam from 32° to temperature T and pressure P, calculated from the formula—

$$H = 1081.91 + 0.305\,T. \quad . \quad . \quad . \quad . \quad 4.$$

Column H' contains the heat units per cubic foot of steam from 32° to temperature T.

The columns H and H' give the heat units required to heat the water from 32° to the boiling-point and evaporate the same to steam under the pressure P and of temperature T.

Column L contains the latent units of heat per pound in steam of temperature T and pressure P. The latent heat expresses the work done in the evaporation, or the difference between the number of heat units per pound in the steam and in the water of the same temperature.

Column L' contains the latent heat per cubic foot of steam.

Latent heat $L = H - h$, the heat units required to evaporate each pound of water from the boiling-point into steam.

The maximum work K, which can be realized per heat unit in steam without expansion, is—

$$K = \frac{144\,P\,(V-1)}{H'\,V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5.$$

Example 1. Required, the maximum work K that can be realized per heat unit in steam of $P = 50$ lbs. per sq. in.? $V = 508.29$ and $H' = 143.3$.

$$K = \frac{144 \times 50\,(508.29 - 1)}{143.3 \times 508.29} = 50.14 \text{ footpounds.}$$

or, $50.14 : 772 = 0.0649$ of the total power.

The maximum work which can be realized per heat unit in steam with expansion will be—

$$K = \frac{144\,P\,(V-1)(2.3 \log. \frac{S}{l} + 1)}{H'\,V}, \quad . \quad . \quad . \quad . \quad 6.$$

in which $S =$ stroke of piston, and $l =$ part of the stroke with full steam.

The natural effect of a steam-engine in horse-power is $= \dfrac{N\,K}{\tau\,550}$, . . 7.

of which from 50 to 75 per cent. is realized in ordinary practice. $N =$ number of heat units passed through the engine in the time τ in seconds.

Example 2. Let the steam in Example 1 be expanded $S : l = 3$ times. We have log. $3 = 0.47712$, and $2.09737 \times 50.14 = 105.16$ footpounds per heat unit. Suppose each stroke of the piston to use 4 cubic feet of steam expanded 3 times, and making 90 strokes per minute.

Then $$\frac{90 \times 4 \times 143.3}{60} = 439.8 \text{ heat units per second,}$$

and the power will be $$\frac{439.8 \times 105.16}{1 \times 550} = 84 \text{ HP.}$$

This is the effect of steam when raised from water of 32°, but when the feed-water is of higher temperature, calculate the heat units from the Formula 1, h', and add the latent heat per pound of the steam; the sum will be the heat units required in generating the steam.

The author's formulas for temperature and pressure of steam are as follows:

English Measures.	*French Measures.*
$T = 200\sqrt[6]{P} - 101.$	$t = 84.5\sqrt[6]{P} - 73.9.$
$P = \left(\frac{T+101}{200}\right)^6.$	$P = \left(\frac{t+73.9}{84.5}\right)^6.$

These formulæ, as also the others given, do not accord accurately with experiment, but are approximate only.

Temp. Fahr. T°	Volume 1 at 39° v	Units of heat. pr. lb. h	Units of heat. pr. cub. ft. h'	Pounds pr. cub. ft. w	Cubic ft. pr. lb. c	Temp. Celsius. t
32	1·000109	0·000000000	0·00000	62.387	0·01603046	0·000
33	1·000077	1·000000867	62·383	62·383	0·01602994	0·555
34	1·000055	2·000000545	124·77	62·384	0·01602956	1·111
35	1·000035	3·00001609	187·16	62·385871	0·01602927	1·666
36	1·000020	4·00003468	249·55	62·386791	0·01602904	2·222
37	1·000009	5·00006294	311·99	62·387493	0·01602886	2·777
38	1·000002	6·00010241	374·33	62·387930	0·01602874	3·333
39	1·000000	7·00015455	436·72	62·388055	0·01602871	3·888
40	1·000002	8·00022076	499·12	62·387930	0·01602874	4·444
41	1·000009	9·00030234	561·51	62·387493	0·01602886	5·000
42	1·000019	10·00040056	623·89	62·386869	0·01602902	5·555
43	1·000034	11·00051663	686·28	62·385933	0·01602926	6·111
44	1·000053	12·00065175	748·66	62·384748	0·01602956	6·666
45	1·000077	13·00080704	811·03	62·383251	0·01602994	7·222
46	1·000104	14·00098362	873·40	62·381567	0·01603038	7·777
47	1·000136	15·001326	935·70	62·379571	0·01603088	8·333
48	1·000171	16·0014050	997·77	62·377388	0·01603146	8·888
49	1·000211	17·0016518	1060·0	62·374893	0·01603210	9·444
50	1·000254	18·0019242	1122·8	62·372212	0·01603278	10·000
51	1·000302	19·0022230	1185·1	62·369219	0·01603355	10·555
52	1·000353	20·0025493	1248·0	62·366039	0·01603437	11·111
53	1·000408	21·0029241	1310·1	62·362611	0·01603525	11·666
54	1·000468	22·0032880	1372·3	62·358871	0·01603621	12·222
55	1·000531	23·0037024	1434·3	62·354944	0·01603723	12·777
56	1·000597	24·0041479	1496·4	62·350831	0·01603828	13·333
57	1·000668	25·0046256	1558·6	62·346407	0·01603942	13·888
58	1·000740	26·0051362	1620·9	62·341921	0·01604057	14·444
59	1·000819	27·0056808	1683·2	62·337000	0·01604184	15·000
60	1·000901	28·0062600	1745·5	62·331893	0·01604316	15·555
61	1·000986	29·0068749	1807·8	62·326620	0·01604451	16·111
62	1·001075	30·0075263	1870·1	62·321059	0·01604594	16·666
63	1·001167	31·0082149	1932·4	62·315333	0·01604741	17·222
64	1·001262	32·0089416	1994·4	62·309420	0·01604894	17·777
65	1·001362	33·0097073	2056·6	62·303198	0·01605054	18·333
66	1·001464	34·010513	2118·7	62·296852	0·01605218	18·888
67	1·001570	35·011359	2180·8	62·290259	0·01605388	19·444
68	1·001680	36·012246	2242·9	62·283418	0·01605564	20·000
69	1·001793	37·013175	2305·0	62·276293	0·01605748	20·555
70	1·001909	38·014148	2367·1	62·269183	0·01605921	21·111
71	1·002028	39·015164	2429·2	62·261788	0·01606122	21·666
72	1·002151	40·016224	2491·2	62·254146	0·01606318	22·222
73	1·002277	41·017330	2553·2	62·246320	0·01606521	22·777
74	1·002406	42·018482	2615·2	62·238309	0·01606728	23·333
75	1·002539	43·019680	2677·1	62·230052	0·01606941	23·888
76	1·002675	44·020926	2739·2	62·221612	0·01607158	24·444
77	1·002814	45·022220	2801·0	62·212987	0·01607382	25·000
78	1·002956	46·023563	2862·8	62·204179	0·01607610	25·555
79	1·003101	47·024956	2924·6	62·195187	0·01607841	26·111
80	1·003249	48·026398	2985·4	62·186012	0·01608078	26·666
81	1·003400	49·027893	3048·2	62·176654	0·01608321	27·222
82	1·003554	50·029438	3111·0	62·167113	0·01608567	27·777
83	1·003711	51·031039	3172·8	62·157388	0·01608820	28·333
84	1·003872	52·032688	3234·4	62·147420	0·01609077	28·888
85	1·004035	53·034394	3296.2	62·137330	0·01609338	29·444
86	1·004199	54·036154	3358·2	62·127182	0·01609601	30·000
87	1·004370	55·037969	3418·7	62·116605	0·01609875	30·555
88	1·004542	56·039841	3480·4	62·105969	0·01610151	31·111
89	1·004717	57·041769	3542·1	62·095152	0·01610432	31·666
90	1·004894	58·043754	3603·8	62·084214	0·01610715	32·222

Temp. Fahr. T°	Volume 1 at 39° v	Units of heat. pr. lb. h	Units of heat. pr. cub. ft. h'	Pounds pr. cub. ft. w	Cubic ft. pr. lb. c	Temp. Celsius. t
91	1·005094	59·045797	3665·0	62·071860	0·01611036	32·777
92	1·005258	60·047899	3726·6	62·061734	0·01611298	33·333
93	1·005444	61·050061	3788·2	62·050252	0·01611597	33·888
94	1·005633	62·052282	3849·8	62·038591	0·01611900	34·444
95	1·005825	63·054564	3911·2	62·026749	0·01612208	35·000
96	1·006019	64·056907	3972·6	62·014787	0·01612519	35·555
97	1·006216	65·059312	4033·9	62·002646	0·01612834	36·111
98	1·006415	66·061780	4095·2	61·990386	0·01613153	36·666
99	1·006618	67·064311	4156·5	61·977885	0·01613478	37·222
100	1·006822	68·066906	4217·7	61·965322	0·01613806	37·777
101	1·007030	69·069565	4278·9	61·952528	0·01614140	38·333
102	1·007240	70·072290	4340·1	61·939612	0·01614475	38·888
103	1·007553	71·075080	4401·3	61·920370	0·01614977	39·444
104	1·007668	72·077937	4462·5	61·913303	0·01615161	40·000
105	1·007905	73·080861	4523·0	61·898745	0·01615541	40·555
106	1·008106	74·083852	4585·0	61·886403	0·01615863	41·111
107	1·008328	75·086912	4645·9	61·872778	0·01616220	41·666
108	1·008554	76·090044	4706·8	61·858913	0·01616581	42·222
109	1·008781	77·093239	4767·7	61·844994	0·01616946	42·777
110	1·009032	78·096509	4828·6	61·829609	0·01617348	43·333
111	1·009244	79·099846	4889·5	61·816622	0·01617677	43·888
112	1·009479	80·103255	4950·4	61·802231	0·01618064	44·444
113	1·009718	81·10674	5011·3	61·787602	0·01618447	45·000
114	1·009956	82·11029	5072·2	61·773042	0·01618829	45·555
115	1·010197	83·11392	5133·0	61·758305	0·01619216	46·111
116	1·010442	84·11762	5193·7	61·743331	0·01619608	46·666
117	1·010688	85·12140	5254·3	61·728302	0·01620003	47·222
118	1·010938	86·12525	5314·9	61·713037	0·01620403	47·777
119	1·011189	87·12918	5375·5	61·697719	0·01620806	48·333
120	1·011442	88·13318	5436·1	61·682286	0·01621211	48·888
121	1·011698	89·13726	5496·6	61·666678	0·01621621	49·444
122	1·011956	90·14141	5557·1	61·650956	0·01622034	50·000
123	1·012216	91·14565	5617·6	61·635123	0·01622451	50·555
124	1·012478	92·14996	5678·1	61·619170	0·01622871	51·111
125	1·012743	93·15435	5738·6	61·603047	0·01623296	51·666
126	1·013010	94·15882	5798·9	61·586810	0·01623724	52·222
127	1·013278	95·16338	5859·2	61·580516	0·01624153	52·777
128	1·013550	96·16801	5919·5	61·553998	0·01624590	53·333
129	1·013823	97·17272	5979·7	61·537423	0·01625027	53·888
130	1·014098	98·17752	6040·0	61·520735	0·01625468	54·444
131	1·014358	99·18239	6100·2	61·504966	0·01625884	55·000
135	1·015505	103·20274	6340·3	61·435497	0·01627724	57·222
140	1·016962	108·23009	6639·6	61·347282	0·01630064	60·000
145	1·018468	113·25965	6937·9	61·256765	0·01632473	62·777
150	1·020021	118·29147	7215·1	61·163500	0·01634961	65·555
155	1·021619	123·32562	7531·2	61·067829	0·01637523	68·333
160	1·023262	128·36217	7826·2	60·969776	0·01640156	71·111
165	1·024947	133·40119	8098·1	60·869542	0·01642857	73·888
170	1·026672	138·44273	8412·8	60·767270	0·01645623	76·666
175	1·028438	143·48687	8704·2	60·662047	0·01648477	79·444
180	1·030242	148·53666	8994·9	60·556699	0·01651345	82·222
185	1·032083	153·58316	9281·9	60·448679	0·01654296	85·000
190	1·033960	158·63545	9571·6	60·338944	0·01657305	87·777
195	1·035873	163·69057	9858·5	60·227513	0·01660370	90·555
200	1·037819	168·74858	10318	60·114581	0·01663489	93·333
205	1·039798	173·80956	10428	60·000168	0·01666662	96·111
210	1·041809	178·87355	10712	59·884350	0·01679885	98·888
212	1·042622	180·90000	18824	59·837654	0·01681160	100·000

	Water.						Indic. press.	
Temp. Fahr. Scale.	Units of heat. per cub. ft.	per pound.	Bulk cub. ft. per lb.	Weight lbs. pr. cub. ft.	Volume wat.=1 at 39°	Temp. Celsius Scale.	Atmos. excluded inches mercury	lbs. pr. sq. in.
T	h'	h	c	w	v	t	i	p
101.36	4301	69.430	.01617	61.848	1.0071	30.83	−28.52	−14
126.21	5631	94.369	.01624	61.583	1.0130	41.87	−26.48	−13
141.67	6583	109.91	.01630	61.317	1.0174	48.74	−24.44	−12
153.27	7331	121.58	.01637	61.101	1.0210	53.90	−22.41	−11
162.51	7974	130.89	.01638	60.920	1.0241	58.00	−20.37	−10
170.25	8421	138.69	.01644	60.762	1.0267	61.44	−18.33	− 9
176.97	8812	145.46	.01647	60.657	1.0288	64.43	−16.29	− 8
182.96	9203	151.52	.01652	60.514	1.0309	67.09	−14.26	− 7
188.36	9531	156.97	.01656	60.372	1.0333	69.49	−12.22	− 6
193.20	9755	161.87	.01659	60.282	1.0359	71.64	−10.18	− 5
197.60	9975	166.32	.01663	60.169	1.0369	73.60	−8.149	− 4
201.90	10183	170.67	.01666	60.072	1.0385	75.51	−6.111	− 3
205.77	10398	174.59	.01669	59.973	1.0401	77.23	−4.074	− 2
209.55	10613	178.42	.01672	59.896	1.0416	78.91	−2.037	− 1
212.00	10824	180.9	.01674	59.838	1.0426	100.00	0.0000	0
213.04	10883	181.95	.01675	59.814	1.0430	100.58	0.6365	0.3125
216.33	11047	185.29	.01677	59.735	1.0444	102.45	+2.037	+ 1
219.45	11225	188.45	.01679	59.659	1.0457	104.36	+4.074	+ 2
222.40	11389	191.44	.01680	59.592	1.0469	105.78	+6.111	+ 3
225.25	11550	194.33	.01681	59.523	1.0481	107.35	+8.149	+ 4
227.95	11718	197.08	.01684	59.459	1.0492	108.86	+10.18	+ 5
230.60	11868	199.77	.01686	59.389	1.0503	110.33	+12.22	+ 6
233.10	12012	202.40	.01688	59.329	1.0514	111.50	+14.26	+ 7
235.49	12150	204.73	.01690	59.270	1.0524	113.05	+16.29	+ 8
237.81	12282	207.10	.01692	59.212	1.0534	114.00	+18.33	+ 9
240.07	12408	209.39	.01693	59.154	1.0545	115.59	+20.37	+10
242.24	12528	211.57	.01695	59.097	1.0555	116.80	+22.41	+11
244.32	12642	213.72	.01696	59.057	1.0564	117.95	+24.44	+12
246.35	12750	215.78	.01697	59.006	1.0573	119.08	+26.48	+13
248.33	12852	217.80	.01698	58.953	1.0589	120.18	+28.52	+14
250.26	12946	219.76	.01699	58.901	1.0590	121.25	+30.55	+15
252.13	13053	221.67	.01700	58.851	1.0599	122.29	+32.59	+16
253.98	13157	223.55	.01701	58.803	1.0607	123.32	+34.63	+17
255.77	13258	225.38	.01702	58.757	1.0615	124.32	+36.67	+18
257.52	13336	227.16	.01703	58.713	1.0623	125.29	+38.71	+19
259.22	13430	228.89	.01704	58.671	1.0631	126.23	+40.74	+20
260.88	13520	230.59	.01705	58.631	1.0639	127.15	+42.78	+21
262.50	13608	232.24	.01707	58.592	1.0646	128.05	+44.82	+22
264.09	13694	233.86	.01708	58.560	1.0654	128.94	+46.85	+23
265.65	13778	235.45	.01709	58.517	1.0661	129.80	+48.89	+24
267.17	13860	237.00	.01710	58.481	1.0668	130.65	+50.93	+25
268.66	13940	238.52	.01711	58.435	1.0675	131.48	+52.97	+26
270.12	14018	240.02	.01712	58.400	1.0684	132.29	+55.00	+27
271.55	14094	241.48	.01713	58.366	1.0688	133.05	+57.04	+28
272.96	14168	242.92	.01714	58.332	1.0695	133.86	+59.08	+29
274.33	14241	244.32	.01715	58.298	1.0701	134.63	+61.11	+30
275.68	14314	245.70	.01716	58.264	1.0708	135.38	+63.15	+31
277.01	14385	247.06	.01717	58.230	1.0714	136.12	+65.19	+32
278.32	14454	248.40	.01718	58.197	1.0720	136.84	+67.23	+33
279.62	14522	249.73	.01719	58.164	1.0726	137.56	+69.20	+34
280.89	14592	251.03	.01720	58.131	1.0732	138.27	+71.30	+35
282.14	14659	252.30	.01721	58.098	1.0738	138.96	+73.34	+36
283.39	14725	253.58	.01722	58.066	1.0744	139.66	+75.38	+37
284.58	14789	254.80	.01723	58.035	1.0750	140.33	+77.41	+38
285.76	14852	256.01	.01724	58.004	1.0756	140.98	+79.45	+39
286.96	14913	257.24	.01725	57.972	1.0761	141.64	+81.49	+40
288.06	14973	258.38	.01726	57.941	1.0767	142.27	+83.52	+41
289.24	15032	259.67	.01727	57.910	1.0773	142.91	+85.56	+42
290.37	15091	260.71	.01728	57.879	1.0778	143.54	+87.61	+43
291.48	15149	261.87	.01729	57.848	1.0783	144.15	+89.64	+44
292.58	15208	262.99	.01730	57.817	1.0789	144.76	+91.67	+45

Temp. Fahr. Scale.	Water.						Indic. press.	
	Units of heat. per cub. ft.	Units of heat. per pound.	Bulk cub. ft. per lb.	Weight lbs. pr. cub. ft.	Volume wat.=1 at 39°	Temp. Celsius Scale.	Atmos. excluded inches mercury	Atmos. excluded lbs. pr. sq. in.
T	*h'*	*h*	*c*	*w*	*v*	*t*	*i*	*p*
293.66	15265	264.10	.01731	57.786	1.0794	145.37	+ 93.71	+ 46
294.73	15321	265.20	.01732	57.769	1.0799	145.96	+ 95.75	+ 47
295.78	15377	266.27	.01733	57.742	1.0804	146.54	+ 97.78	+ 48
296.82	15432	267.34	.01734	57.714	1.0809	147.12	+ 99.82	+ 49
297.84	15485	268.39	.01735	57.687	1.0814	147.69	+ 101.8	+ 50
298.85	15536	269.42	.01735	57.660	1.0820	148.25	+ 103.9	+ 51
299.85	15588	270.45	.01736	57.633	1.0825	148.80	+ 105.9	+ 52
300.84	15639	271.46	.01737	57.606	1.0830	149.34	+ 108.0	+ 53
301.81	15690	272.46	.01737	57.580	1.0835	149.89	+ 110.0	+ 54
302.77	15739	273.44	.01738	57.554	1.0840	150.43	+ 112.0	+ 55
303.72	15789	274.42	.01739	57.529	1.0844	150.95	+ 114.1	+ 56
304.69	15839	275.40	.01739	57.504	1.0849	151.48	+ 116.1	+ 57
305.60	15888	276.35	.01740	57.480	1.0854	152.00	+ 118.1	+ 58
306.52	15936	277.30	.01741	57.456	1.0859	152.51	+ 120.2	+ 59
307.42	15983	278.22	.01741	57.432	1.0863	153.01	+ 122.2	+ 60
308.38	16029	279.14	.01742	57.410	1.0867	153.51	+ 124.3	+ 61
309.22	16075	280.07	.01743	57.388	1.0871	154.01	+ 126.3	+ 62
310.11	16120	280.98	.01743	57.364	1.0875	154.50	+ 128.3	+ 63
310.99	16165	281.87	.01744	57.344	1.0880	154.99	+ 130.4	+ 64
311.86	16209	282.78	.01745	57.322	1.0884	155.48	+ 132.4	+ 65
312.72	16254	283.66	.01745	57.300	1.0888	155.95	+ 134.4	+ 66
313.57	16298	284.54	.01746	57.278	1.0892	156.42	+ 136.5	+ 67
314.42	16342	285.41	.01746	57.254	1.0897	156.90	+ 138.5	+ 68
315.25	16384	286.27	.01747	57.232	1.0901	157.36	+ 140.5	+ 69
316.08	16426	287.12	.01748	57.210	1.0905	157.82	+ 142.6	+ 70
316.90	16467	287.96	.01748	57.188	1.0909	158.28	+ 144.6	+ 71
317.71	16507	288.80	.01749	57.166	1.0913	158.73	+ 146.7	+ 72
318.51	16547	289.62	.01750	57.144	1.0918	159.17	+ 148.7	+ 73
319.31	16587	290.44	.01751	57.122	1.0921	159.62	+ 150.7	+ 74
320.10	16637	291.26	.01752	57.101	1.0926	160.05	+ 152.8	+ 75
320.88	16677	292.06	.01752	57.080	1.0929	160.49	+ 154.8	+ 76
321.66	16717	292.85	.01753	57.059	1.0935	160.92	+ 156.8	+ 77
322.42	16756	293.65	.01753	57.038	1.0937	161.34	+ 158.9	+ 78
323.18	16795	294.43	.01754	57.017	1.0941	161.76	+ 160.9	+ 79
323.94	16834	295.21	.01755	56.996	1.0945	162.17	+ 163.0	+ 80
324.67	16872	295.96	.01756	56.975	1.0949	162.59	+ 165.0	+ 81
325.43	16910	296.75	.01756	56.954	1.0953	163.02	+ 167.0	+ 82
326.17	16947	297.51	.01757	56.933	1.0956	163.43	+ 169.1	+ 83
326.90	16984	298.26	.01757	56.912	1.0960	163.83	+ 171.1	+ 84
327.63	17020	299.01	.01758	56.891	1.0964	164.24	+ 173.1	+ 85
328.35	17056	299.75	.01759	56.871	1.0968	164.64	+ 175.2	+ 86
329.07	17092	300.50	.01759	56 862	1.0972	165.04	+ 177.2	+ 87
329.78	17127	301.23	.01760	56.844	1.0975	165.43	+ 179.2	+ 88
330.48	17162	301.95	.01761	56.826	1.0979	165.82	+ 181.3	+ 89
331.18	17197	302.67	.01761	56.808	1.0982	166.21	+ 183.3	+ 90
331.87	17231	303.38	.01762	56.790	1.0986	166.59	+ 185.4	+ 91
332.56	17265	304.10	.01763	56.772	1.0989	166.98	+ 187.4	+ 92
333.24	17299	304.80	.01763	56.754	1.0993	167.35	+ 189.4	+ 93
333.92	17333	305.50	.01764	56.735	1.0996	167.77	+ 191.5	+ 94
334 59	17366	306.19	.01765	56.716	1.0999	168.10	+ 193.5	+ 95
335.26	17399	306.88	.01765	56.699	1.1003	168.47	+ 195.5	+ 96
336.58	17465	308.34	.01767	56.664	1.1010	169.21	+ 199.6	+ 98
337.23	17497	308.91	.01768	56.647	1.1013	169 57	+ 201.6	+ 99
337.89	17529	309.60	.01769	56.629	1.1017	169.94	+ 203.7	+ 100
341.0	17688	312.87	.01772	56.549	1.1035	171.70	+ 213.9	+ 105
344.1	17840	316.04	.01775	56.469	1.1050	173.40	+ 224.1	+ 110
347.1	17993	319.12	.01778	56.389	1.1065	175.06	+ 234.2	+ 115
350.0	18136	322.13	.01781	56.309	1.1090	176.68	+ 244.4	+ 120
352.8	18278	325.06	.01784	56.220	1.1095	178.25	+ 254.6	+ 125
355.6	18413	327.91	.01786	56.146	1.1100	179.78	+ 264.8	+ 130
358.4	18549	330.75	.01788	56.073	1.1124	181.35	+ 275.0	+ 135

Steam.

Total pressure lbs. pr. sq. in.	Total pressure inches mer.	Temp. Fahr. Scale.	Volume wat.=1 at 39°	Weight lbs. pr. cub. ft.	Bulk cub. ft. pr. lb.	Units of heat, from 32° to T: Total pr. pound.	Total pr. cub. ft.	Latent pr. pound.	Latent pr. cub. ft.	Press. ob. at. lbs. pr. sq. in.
P	*I*	*T*	*V*	*W*	*C*	*H*	*H*	*L*	*L'*	*p*
1	2.037	101.36	17983	.00347	288.24	1112.8	3.8614	1043.4	3.6337	— 14
2	4.074	126.21	10353	.00602	165.94	1120.4	6.7449	1026.0	6.1165	— 13
3	6.111	141.67	7283.8	.00856	116.75	1125.1	9.6308	1015.2	8.6901	— 12
4	8.149	153.27	5608.4	.01112	89.895	1128.7	12.551	1007.1	11.199	— 11
5	10.18	162.51	4565.6	.01366	73.180	1131.5	15.456	1000.6	13.714	— 10
6	12.22	170.25	3851.0	.01619	61.742	1133.8	18.156	995.17	16.113	— 9
7	14.26	176.97	3330.8	.01872	53.388	1135.9	20.846	990.44	18.194	— 8
8	16.29	182.93	2935.1	.02125	47.046	1137.7	24.176	986.22	20.957	— 7
9	18.33	188.36	2624.0	.02377	42.059	1139.4	27.083	982.41	23.352	— 6
10	20.37	193.20	2373.0	.02628	38.037	1140.8	29.980	978.99	25.728	— 5
11	22.41	197.60	2166.3	.02880	34.723	1142.2	32.895	975.88	28.099	— 4
12	24.44	201.90	1993.0	.03130	31.945	1143.5	35.791	972.84	30.450	— 3
13	26.48	205.77	1845.7	.03380	29.584	1144.7	38.691	970.11	32.789	— 2
14	28.52	209.55	1718.9	.03629	27.551	1145.8	41.581	967.43	35.435	— 1
14.7	29.92	212.00	1641.5	.03800	26.311	1146.6	43.571	965.70	36.706	0
15	30.55	213.04	1608.6	.03878	25.784	1146.9	44.476	964.95	37.421	0.3125
16	32.59	216.33	1511.7	.04123	24.230	1147.9	47.328	962.63	39.600	+ 1
17	34.63	219.45	1426.2	.04374	22.859	1148.8	50.248	960.49	42.012	+ 2
18	36.67	222.40	1349.8	.04622	21.636	1149.7	53.138	958.32	44.393	+ 3
19	38.71	225.25	1281.1	.04868	20.539	1150.6	56.011	958.30	46.698	+ 4
20	40.74	227.95	1219.7	.05119	19.550	1151.4	58.894	954.38	48.655	+ 5
21	42.78	230.60	1163.8	.05360	18.654	1152.2	61.758	952.50	51.924	+ 6
22	44.82	233.10	1112.9	.05605	17.838	1153.0	64.637	950.62	53.282	+ 7
23	46.85	235.49	1066.3	.05851	17.092	1153.7	67.503	949.03	55.529	+ 8
24	48.89	237.81	1023.6	.06095	16.407	1154.5	70.367	947.37	57.743	+ 9
25	50.93	240.07	984.23	.06338	15.776	1155.1	73.410	945.76	59.942	+ 10
26	52.97	242.24	947.86	.06582	15.193	1155.8	76.074	944.25	62.161	+ 11
27	55.00	244.32	914.14	.06824	14.652	1156.4	78.913	942.74	64.423	+ 12
28	57.04	246.35	882.80	.07067	14.150	1157.1	81.772	941.29	66.521	+ 13
29	59.08	248.33	853.60	.07308	13.682	1157.7	84.604	939.88	68.686	+ 14
30	61.11	250.26	826.32	.07550	13.245	1158.2	87.444	938.50	70.857	+ 15
31	63.15	252.13	800.79	.07791	12.835	1158.8	90.166	937.17	73.015	+ 16
32	65.19	253.98	766.83	.08031	12.451	1159.4	93.121	935.45	75.126	+ 17
33	67.23	255.77	754.31	.08271	12.090	1159.9	95.861	934.57	77.298	+ 18
34	69.26	257.52	733.09	.08510	11.750	1160.5	98.782	933.32	79.425	+ 19
35	71.30	259.22	713.08	.08749	11.429	1161.0	101.18	932.10	81.549	+ 20
36	73.34	260.88	694.17	.08987	11.127	1161.5	104.38	930.92	83.662	+ 21
37	75.38	262.50	676.27	.09225	10.840	1162.0	107.19	929.76	85.770	+ 22
38	77.41	264.09	659.31	.09462	10.568	1162.5	109.98	928.62	87.866	+ 23
39	79.45	265.65	643.21	.09700	10.310	1162.9	112.79	927.51	89.968	+ 24
40	81.49	267.17	627.91	.09936	10.064	1163.4	115.59	926.42	92.059	+ 25
41	83.52	268.66	613.34	.10172	9.8310	1163.9	118.39	925.35	94.126	+ 26
42	85.56	270.12	599.46	.10407	9.6086	1164.3	121.17	924.30	96.192	+ 27
43	87.60	271.55	586.23	.10642	9.3963	1164.7	123.95	923.28	98.255	+ 28
44	89.64	272.96	573.58	.10877	9.1938	1165.2	126.74	922.27	100.32	+ 29
45	91.67	274.33	561.50	.11111	9.0002	1165.6	129.51	921.29	102.36	+ 30
46	93.71	275.68	549.94	.11344	8.8140	1166.0	132.29	920.32	104.40	+ 31
47	95.75	277.01	538.87	.11577	8.6374	1166.4	135.07	919.36	106.43	+ 32
48	97.78	278.32	528.25	.11810	8.4673	1166.8	137.83	918.43	108.46	+ 33
49	99.82	279.62	518.07	.12042	8.3040	1167.2	140.69	917.49	110.48	+ 34
50	101.86	280.89	508.29	.12273	8.1472	1167.6	143.30	916.58	112.49	+ 35
51	103.90	282.14	498.89	.12505	7.9966	1167.9	146.08	915.68	114.50	+ 36
52	105.93	283.39	489.85	.12736	7.8517	1168.4	148.85	914.79	116.51	+ 37
53	107.97	284.58	481.15	.12966	7.7122	1168.7	151.63	913.93	118.50	+ 38
54	110.01	285.76	472.77	.13196	7.5779	1169.0	154.48	913.08	120.49	+ 39
55	112.04	286.96	464.60	.13128	7.4468	1169.4	157.02	912.22	122.47	+ 40
56	114.08	288.09	456.90	.13652	7.3236	1169.8	159.74	911.42	124.43	+ 41
57	116.12	289.21	449.38	.13883	7.2030	1170.1	162.45	910.48	126.40	+ 42
58	118.16	290.37	442.12	.14111	7.0866	1170.5	165.15	909.78	128.38	+ 43
59	120.19	291.48	435.10	.14338	6.9741	1170.8	167.84	908.97	130.33	+ 44
60	122.23	292.58	428.32	.14566	6.8654	1171.2	170.58	908.18	132.28	+ 45

Steam.

Total pressure lbs. pr. sq. in.	Total pressure inches mer.	Temp. Fahr. Scalo.	Volume wat.=1 at 39°	Weight lbs. pr. cub. ft.	Bulk cub. ft. pr. lb.	Units of heat, from 32° to T: Total pr. pound.	Total pr. cub. ft.	Latent pr. pound.	Latent pr. cub. ft.	Press ob. at. lbs. pr. sq. in.
P	*I*	*T*	*V*	*W*	*C*	*H*	*H'*	*L*	*L'*	*p*
61	124.27	293.66	421.75	.14792	6.7601	1171.5	173.27	907.40	134.22	+ 46
62	126.30	294.73	415.40	.15018	6.6583	1171.8	175.96	906.63	136.16	+ 47
63	128.34	295.78	409.25	.15244	6.5597	1172.1	178.65	905.87	138.09	+ 48
64	130.38	296.82	403.29	.15469	6.4642	1172.5	181.34	905.13	140.01	+ 49
65	132.42	297.84	397.51	.15694	6.3715	1172.8	184.03	904.39	141.93	+ 50
66	134.45	298.85	391.90	.15919	6.2817	1173.1	186.72	903.66	143.85	+ 51
67	136.49	299.85	386.47	.16130	6.1994	1173.4	189.40	902.94	145.64	+ 52
68	138.53	300.84	381.18	.16366	6.1099	1173.7	192.07	902.23	147.66	+ 53
69	140.36	301.81	376.06	.16590	6.0277	1174.0	194.74	901.53	149.56	+ 54
70	142.60	302.77	371.07	.16812	5.9478	1174.3	197.42	900.84	151.45	+ 55
71	144.64	303.72	366.24	.17035	5.8702	1174.6	200.08	900.15	153.34	+ 56
72	146.68	304.69	361.53	.17256	5.7948	1174.9	202.74	899.46	155.21	+ 57
73	148.72	305.60	356.95	.17478	5.7214	1175.1	205.40	898.79	157.09	+ 58
74	150.75	306.52	352.49	.17690	5.6500	1175.4	208.04	898.13	158.88	+ 59
75	152.79	307.42	348.15	.17919	5.5805	1175.8	210.67	897.57	160.83	+ 60
76	154.83	308.32	343.93	.18139	5.5129	1176.0	213.30	896.83	162.67	+ 61
77	156.86	309.22	339.81	.18359	5.4468	1176.2	215.93	896.18	164.56	+ 62
78	158.90	310.11	335.80	.18578	5.3825	1176.5	218.56	895.54	166.37	+ 63
79	160.94	310.99	331.89	.18797	5.3190	1176.8	221.19	894.92	168.22	+ 64
80	162.98	311.86	328.08	.19015	5.2588	1177.0	223.82	894.27	170.04	+ 65
81	165.01	312.72	324.37	.19233	5.1992	1177.3	226.44	893.65	171.87	+ 66
82	167.05	313.57	320.74	.19451	5.1410	1177.6	229.06	893.03	173.70	+ 67
83	169.09	314.42	317.20	.19668	5.0843	1177.9	231.68	892.51	175.52	+ 68
84	171.12	315.25	313.74	.19885	5.0289	1178.1	234.28	891.82	177.33	+ 69
85	173.16	316.08	310.36	.20101	4.9748	1178.3	236.89	891.22	179.14	+ 70
86	175.20	316.90	307.07	.20317	4.9219	1178.6	239.50	890.63	180.95	+ 71
87	177.24	317.71	303.85	.20532	4.8703	1178.8	242.10	890.04	182.75	+ 72
88	179.27	318.51	300.70	.20747	4.8198	1179.1	244.69	889.46	184.53	+ 73
89	181.31	319.31	297.62	.20962	4.7704	1179.3	247.29	888.88	186.33	+ 74
90	183.35	320.10	294.61	.21185	4.7222	1179.6	249.88	888.31	188.12	+ 75
91	185.38	320.88	291.66	.21390	4.6750	1179.8	252.45	887.74	189.88	+ 76
92	187.42	321.66	288.78	.21603	4.6288	1180.0	255.02	887.19	191.66	+ 77
93	189.46	322.42	285.96	.21816	4.5836	1180.3	257.58	886.63	193.43	+ 78
94	191.50	323.18	283.21	.22029	4.5394	1180.5	260.14	886.08	195.19	+ 79
95	193.53	323.94	280.50	.22241	4.4961	1180.7	262.69	885.53	196.94	+ 80
96	195.57	324.67	277.86	.22453	4.4537	1180.9	265.23	885.00	198.71	+ 81
97	197.61	325.43	275.27	.22672	4.4106	1181.2	267.77	884.45	200.49	+ 82
98	199.65	326.17	272.73	.22875	4.3715	1181.4	270.30	883.91	202.18	+ 83
99	201.68	326.90	270.24	.23085	4.3316	1181.6	273.10	883.38	203.92	+ 84
100	203.72	327.63	267.80	.23296	4.2926	1181.9	275.52	882.85	205.67	+ 85
101	205.76	328.35	265.41	.23505	4.2543	1182.1	277.85	882.33	207.39	+ 86
102	207.79	329.07	263.07	.23715	4.2167	1182.3	280.38	881.81	209.12	+ 87
103	209.83	329.78	260.77	.23924	4.1799	1182.5	282.90	881.29	210.84	+ 88
104	211.87	330.48	258.52	.24132	4.1438	1182.7	285.42	880.78	212.55	+ 89
105	213.91	331.18	256.31	.24340	4.1083	1182.9	287.93	880.27	214.26	+ 90
106	215.94	331.87	251.14	.24548	4.0736	1183.2	290.45	879.77	215.96	+ 91
107	217.98	332.56	252.01	.24750	4.0394	1183.4	292.94	879.27	217.66	+ 92
108	220.02	333.24	249.92	.24963	4.0058	1183.6	295.41	879.79	219.36	+ 93
109	222.05	333.92	247.87	.25169	3.9731	1183.8	297.91	878.28	221.05	+ 94
110	224.10	334.59	245.86	.25375	3.9408	1183.9	300.44	877.80	222.74	+ 95
111	226.13	335.26	243.88	.25581	3.9091	1184.2	302.93	877.31	224.42	+ 96
113	230.20	336.58	240.03	.25991	3.8474	1184.6	307.90	876.25	227.74	+ 98
114	232.24	337.23	238.15	.26204	3.8100	1184.8	310.36	875.88	229.51	+ 99
115	234.28	337.89	236.31	.26400	3.7878	1185.0	312.86	875.40	231.10	+ 100
120	244.4	341.0	227.56	.27421	3.6475	1185.9	325.20	873.09	239.41	+ 105
125	254.6	344.1	219.50	.28422	3.5184	1186.9	337.39	870.85	247.51	+ 110
130	264.8	347.1	212.07	.29419	3.3991	1187.8	349.44	868.68	253.55	+ 115
135	275.0	350.0	205.18	.30406	3.2880	1188.7	361.42	866.56	263.48	+ 120
140	285.2	352.8	198.78	.31385	3.1862	1189.5	373.34	864.49	271.32	+ 125
145	295.4	355.6	192.83	.32354	3.0908	1190.4	385.20	862.48	278.97	+ 130
150	305.6	358.4	187.26	.33315	3.0001	1191.2	396.86	860.45	286.66	+ 135

Water, by the Author's Formula.

Temperature of the Water. Cent.	Fahr.	Volume. Water = 1 at 40°.	Weight. lbs. per cubic ft.	Bulk. Cubic feet per pound.	Units of Heat in Water from 32° to T°. Total per pound.	cubic ft.	Latent per pound.	cu. ft.
T°	T°	𝔙	𝔚	∈	H	H'	L	L'
179.2	354.8	1.11070	56.166	0.01780	326.73	18349	3.927	220.8
180.7	357.4	1.11208	56.098	0.01782	329.41	18481	4.010	225.0
182.2	360.0	1.11344	56.031	0.01784	332.09	18607	4.099	229.0
183.7	362.5	1.11478	55.965	0.01787	334.67	18730	4.168	233.3
185.0	365.0	1.11613	55.900	0.01789	337.24	18850	4.244	237.2
186.5	367.4	1.11742	55.834	0.01791	339.72	18966	4.318	241.0
188.0	369.8	1.11869	55.770	0.01793	342.19	19080	4.390	244.6
188.5	372.0	1.11993	55.708	0.01795	344.46	19190	4.460	248.5
190.0	374.2	1.12109	55.648	0.01797	346.73	19296	4.530	252.1
191.2	376.4	1.12227	55.591	0.01799	349.00	19399	4.598	255.7
192.5	378.5	1.12343	55.534	0.01800	351.16	19501	4.666	259.1
193.7	380.6	1.12456	55.477	0.01802	353.33	19602	4.731	262.5
194.4	382.6	1.12561	55.426	0.01804	355.39	19698	4.794	265.7
197.0	386.6	1.12783	55.317	0.01807	359.54	19885	4.940	272.8
199.1	390.4	1.13000	55.211	0.01811	363.48	20068	5.082	279.8
201.1	394.0	1.13210	55.108	0.01814	367.20	20236	5.200	286.6
203.5	397.6	1.13301	55.017	0.01817	370.92	20402	5.318	292.9
205.0	401.0	1.13577	54.926	0.01821	374.44	20561	5.437	299.1
206.8	404.3	1.13760	54.838	0.01824	357.86	20720	5.558	305.2
208.7	407.5	1.13944	54.752	0.01826	381.18	20870	5.679	311.2
210.2	410.6	1.14119	54.670	0.01829	384.40	21015	5.800	317.1
211.9	413.5	1.14285	54.590	0.01832	387.40	21147	5.903	324.6
213.6	416.5	1.14441	54.514	0.01834	390.50	21273	6.006	332.0
215.1	419.2	1.14589	54.440	0.01837	393.31	21364	6.109	339.5
216.7	422.1	1.14743	54.367	0.01839	396.31	21510	6.212	346.7
218.2	424.8	1.14897	54.299	0.01841	399.11	21622	6.315	353.8
219.6	427.4	1.15050	54.230	0.01844	401.82	21751	6.418	356.9
221.1	430.0	1.15202	54.161	0.01846	404.52	21876	6.521	359.9
222.4	432.4	1.15339	54.093	0.01849	407.02	21997	6.624	362.8
223.6	434.9	1.15481	54.024	0.01851	409.63	22114	6.727	365.6
225.1	437.3	1.15621	53.959	0.01853	412.13	22238	6.830	368.5
226.4	439.6	1.15764	53.895	0.01856	414.53	22347	6.926	373.2
227.7	441.9	1.15880	53.834	0.01858	416.92	22452	7.020	377.9
228.9	444.1	1.16003	53.777	0.01859	419.21	22553	7.111	382.5
230.2	446.4	1.16127	53.721	0.01861	421.60	22650	7.200	386.9
231.4	448.5	1.16250	53.667	0.01863	423.79	22744	7.288	391.1
232.5	450.6	1.16372	53.614	0.01865	425.97	22843	7.374	395.3
233.6	452.6	1.16494	53.563	0.01867	428.06	22938	7.459	399.4
234.7	454.6	1.16571	53.513	0 01869	430.14	23029	7.542	403.6
235.9	456.7	1.16695	53.455	0.01871	432.32	23116	7.623	407.3
237.0	458.7	1.16818	53.406	0.01872	434.40	23200	7.700	411.2
238.0	460.6	1.16942	53.352	0.01874	435.38	23282	7.787	415.5
239.0	462.5	1.17066	53.293	0.01876	438.39	23363	7.893	423.3
241.1	466.1	1.17274	53.158	0.01881	442.21	23555	8.113	433.2
244.1	471.5	1.17598	53.027	0.01886	447.83	23741	8.329	442.9
246.5	475.7	1.17917	52.900	0.01890	452.24	23923	8.541	452.4
248.8	479.8	1.18231	52.768	0.01895	456.55	24091	8.747	461.6
253.1	487.6	1.18531	52.588	0.01901	464.66	24436	9.060	476.5
257.2	494.9	1.18961	52.430	0.01907	472.28	24762	9.381	491.8
261.0	501.8	1.19343	52.264	0.01913	479.51	25061	9.710	507.5
263.5	508.4	1.19472	52.102	0.01019	486.40	25577	10.00	521.0
268.1	514.6	1.20131	51.943	0.01925	492.97	25606	10.37	538.7
271.9	521.4	1.20562	51.787	0.01931	500.14	25901	10.74	556.2
273.3	526.0	1.20812	51.642	1.01936	505.00	26079	11.00	568.1
277.5	531.6	1.21147	51.498	0.01942	510.84	26307	11.242	578.8

Steam, by the Author's Formula.

Total Pressure. lbs. per sq. inch.	Total Pressure. Inches mercury.	Temperature Fahr.	Volume. Water = 1 at 40.	Weight. lbs. per cubic foot.	Bulk. Cubic feet per pound.	Units of Heat from 32° to $T°$. Total per pound.	Total per cubic ft.	Latent per pound.	Latent per cubic ft.	Pressure above atmosphere.
P	I	$T°$	V	$\mathfrak{W}$	$\mathfrak{C}$	H	H'	L	L'	p
140	285.2	354.8	194.3	0.3212	3.1139	1190.1	381.88	863.5	277.0	125
145	295.4	357.4	187.8	0.3322	3.0105	1190.9	395.16	861.5	275.8	130
150	305.6	360.0	181.8	0.3432	2.9136	1191.7	408.38	859.6	294.5	135
155	310.8	362.5	176.5	0.3534	2.8289	1192.5	421.54	857.8	303.2	140
160	325.9	365.0	171.5	0.3646	2.7432	1193.3	435.08	856.1	312.1	145
165	336.0	367.4	166.6	0.3756	2.6517	1194.0	448.64	854.3	321.0	150
170	346.3	369.8	161.1	0.3871	2.5831	1194.7	462.22	852.5	329.9	155
175	356.5	372.0	157.0	0.3973	2.5171	1195.4	475.80	851.0	338.7	160
180	366.7	374.2	152.8	0.4075	2.4541	1196.1	488.96	849.4	347.1	165
185	376.9	376.4	148.8	0.4182	2.3916	1196.8	502.10	847.8	355.5	170
190	378.1	378.5	145.0	0.4292	2.3299	1197.4	515.20	846.2	363.9	175
195	387.3	380.6	141.5	0.4409	2.2684	1198.1	528.27	844.8	372.4	180
200	407.4	382.6	138.1	0.4517	2.2137	1198.7	542.07	843.3	381.0	185
210	427.8	386.6	132.0	0.4719	2.1192	1199.8	568.40	840.3	398.0	195
220	448.2	390.4	126.3	0.4935	2.0265	1201.0	574.70	837.5	414.8	205
230	468.5	394.0	120.8	0.5165	1.9360	1202.2	620.96	835.0	431.3	215
240	488.9	397.6	116.1	0.5364	1.8646	1203.2	647.41	832.3	447.9	225
250	509.3	401.0	111.7	0.5595	1.7874	1204.2	673.85	829.8	464.4	235
260	529.7	404.3	107.5	0.4803	1.7230	1205.2	700.28	827.4	480.8	245
270	550.0	407.5	103.7	0.6016	1.6621	1206.2	726.66	825.0	497.1	255
280	570.4	410.6	100.2	0.6238	1.6031	1207.2	753.04	822.8	513.3	265
290	590.8	413.5	97.01	0.6459	1.5481	1208.1	779.40	820.7	529.4	275
300	611.1	416.5	94.22	0.6681	1.4967	1209.0	805.74	818.6	545.4	285
310	631.5	419.2	91.13	0.6896	1.4499	1209.8	832.96	816.5	561.4	295
320	651.9	422.1	88.21	0.7107	1.4071	1210.6	858.36	814.4	577.3	305
330	672.3	424.8	85.44	0.7302	1.3695	1211.5	884.63	812.4	593.2	315
340	692.6	427.4	83.19	0.7547	1.3250	1212.3	910.89	810.5	608.9	325
350	713.0	430.0	80.99	0.7745	1.2915	1213.1	937.13	808.6	624.5	335
360	733.4	432.4	78.84	0.7943	1.2590	1213.9	963.34	806.9	640.2	345
370	753.8	434.9	76.74	0.8146	1.2275	1214.7	989.51	805.1	655.8	355
380	774.1	437.3	74.66	0.8353	1.1968	1215.5	1015.7	803.4	671.3	365
390	794.5	439.6	72.90	0.8626	1.1597	1216.2	1041.8	801.7	686.7	375
400	814.9	441.9	71.19	0.8745	1.1434	1216.8	1067.9	800.0	702.0	385
410	835.2	444.1	69.52	0.8952	1.1170	1217.4	1094.0	799.4	717.2	395
420	855.6	446.4	67.90	0.9142	1.0938	1218.0	1120.2	797.7	732.4	405
430	876.0	448.5	66.34	0.9400	1.0634	1218.7	1146.3	795.0	747.6	415
440	896.4	450.6	64.91	0.9599	1.0417	1219.4	1172.3	793.5	762.8	425
450	916.7	452.6	63.55	0.9804	1.0201	1220.1	1198.3	792.0	777.9	435
460	937.1	454.6	62.22	1.0007	0.9993	1220.7	1224.3	790.5	792.9	445
470	957.5	456.7	60.94	1.0211	0.9793	1221.3	1250.4	789.0	807.8	455
480	977.8	458.7	59.72	1.0446	0.9573	1221.9	1276.5	787.5	822.7	465
490	998.2	460.6	58.54	1.0652	0.9388	1222.5	1302.3	786.1	837.4	475
500	1018.6	462.5	57.45	1.0859	0.9209	1223.0	1328.1	784.7	852.1	485
525	1069.5	466.1	54.81	1.1381	0.8786	1224.5	1392.6	782.3	881.8	510
550	1120.4	471.5	52.47	1.1890	0.8410	1225.8	1456.9	778.0	921.3	535
575	1171.4	475.7	50.32	1.2397	0.8066	1227.2	1521.0	775.0	960.4	560
600	1222.3	479.8	48.35	1.2901	0.7751	1228.3	1584.8	771.8	1000	585
650	1324.2	487.6	44.75	1.3943	0.7172	1230.6	1709.5	766.0	1082	635
700	1426.0	494.9	41.70	1.4961	0.6684	1232.7	1933.8	760.4	1157	685
750	1527.9	501.8	39.05	1.5977	0.6259	1234.9	2057.7	755.4	1234	735
800	1629.8	508.4	36.73	1.6986	0.5887	1237.0	2101.2	750.6	1307	785
850	1731.6	514.6	34.68	1.7989	0.5554	1238.9	2228.3	745.9	1374	835
900	1833.5	521.4	32.87	1.8979	0.5269	1241.0	2355.4	740.0	1435	885
950	1935.5	526.0	31.21	1.9992	0.5002	1242.4	2482.5	737.4	1490	935
1000	2037.2	531.6	29.73	2.0986	0.4765	1243.5	2609.6	732.3	1538	985

Mean Pressure above Vacuum of Expanding Steam.

Absolute Steam Pressure.	Grade of Expansion of Steam, denoted by X.							
	1.333	1.5	1.6	2	2.666	3	4	8
	Steam Cut-off at l from Beginning of Stroke.							
P	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{8}$
25	24.130	23.481	22.938	21.164	18.567	17.488	19.913	9.6232
30	28.956	28.100	27.524	25.396	22.280	20.986	17.897	11.548
35	33.782	32.874	32.110	29 630	25.992	24.484	20.880	13.472
40	38.608	37.468	36.700	33.862	28.964	27.982	23.862	15.396
45	43.474	42.151	41.287	38.095	32.677	31.479	26.845	17.320
50	48.262	46.835	45.875	42.328	37.133	34.977	29.828	19.246
55	53.088	51.518	50.462	46.561	40.846	38.474	32.811	21.170
60	57.914	56.202	55.050	50.794	44.559	41.972	35.794	23.095
65	62.740	60.885	59.637	55.027	48.273	45.470	38.777	25.020
70	67.566	65.569	64.225	59.260	51.986	48.967	41.760	26.944
75	72.393	70.252	68.812	63.493	55.700	52.465	44.743	28.869
80	77.216	74.936	73.400	67.726	59.413	55.963	47.726	30.794
85	82.042	79.619	77.987	71.959	63.126	59.461	50.709	32.718
90	86.866	85.303	82.574	76.192	66.840	62.958	53.692	34.643
95	91.699	89.986	87.163	80.425	70.553	66.456	56.675	36.568
100	96.524	93.670	91.750	84.657	74.267	69.954	59.657	38.493
105	101.35	98.353	96.337	88.890	77.981	73.451	62.640	40.417
110	106.17	103.04	100.92	93.123	81.694	76.949	65.622	42.342
115	111.00	107.72	105.51	97.356	85.407	80.447	68.606	44.267
120	115.83	112.40	110.10	101.59	89.121	83.944	71.589	46.191
125	120.65	117.08	114.68	105.82	92.834	87.442	74.572	48.116
130	125.48	121.77	119.27	110.05	96.548	90.940	77.555	50.041
135	130.30	126.45	123.86	114.28	100.26	94.437	80.538	51.966
140	135.13	131.13	128.45	118.52	103.97	97.935	83.520	53.890
145	139.96	135.82	133.03	122.75	107.68	101.43	86.502	55.815
150	144.78	140.50	137.62	126.98	111.40	104.93	89.485	57.739
155	149.60	145.18	142.20	131.22	115.11	108.42	92.468	59.663
160	154.43	149.87	146.79	135.45	118.82	111.92	95.451	61.588
165	159.26	154.55	151.38	139.68	122.54	115.42	98.434	63.513
170	164.08	159.23	155.97	143.92	126.25	118.92	101.41	65.437
175	168.91	163.92	160.55	148.15	129.96	122.42	104.40	67.362
180	173.73	168.60	165.14	152.38	133.68	125.91	107.38	69.287
185	178.56	173.28	169.73	156.61	137.39	129.41	110.36	71.212
190	183.39	177.97	174.32	160.85	141.10	132.91	113.35	73.136
195	188.21	182.65	178.90	165.08	144.82	136.41	116.33	75.061
200	193.04	187.34	183.50	169.31	148.53	139.91	119.31	76.986
210	202.69	196.71	192.68	177.78	155.96	146.90	125.27	80.835
220	212.34	205.08	201.85	186.25	163.39	153.90	131.24	84.684
230	221.99	215.45	211.03	194.71	170.82	160.89	137.20	88.534
240	231.65	224.81	220.20	203.18	178.23	167.89	143.17	92.383
250	241.30	234.18	229.38	211.64	185.67	174.88	149.13	96.232
260	250.96	243.55	238.55	220.11	193.18	181.88	155.11	100.08
270	260.61	252.91	247.73	228.57	200.52	188.87	161.07	103.93
280	270.26	262.28	256.90	237.04	207.95	195.87	167.04	107.78
300	289.56	281.00	275.24	253.96	222.80	209.86	178.97	115.48

Mean Pressure for High-Pressure Engines Above Atmosphere.

Pressure above Atmosphere.	Grade of Expansion of Steam, denoted by *X*.							
	$1\frac{1}{3}$	$1\frac{1}{2}$	1.6	2	$2\frac{2}{3}$	3	4	8
	Steam Cut-off at *l* from Beginning of Stroke.							
p	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{8}$
25	23.908	22.768	22.000	19.162	14.264	13.282	9.162	0.696
30	28.774	27.451	26.587	23.395	17.977	16.779	12.145	2.620
35	33.562	32.135	31.175	27.628	22.433	20.277	15.128	4.546
40	38.388	36.818	35.762	31.861	26.146	23.774	18.111	6.470
45	43.214	41.502	40.350	36.094	29.859	27.272	21.094	8.395
50	48.040	46.185	44.937	40.327	33.573	30.770	24.077	10.320
55	52.866	50.869	49.625	44.560	37.286	34.267	27.060	12.244
60	57.693	55.552	54.112	48.793	41.000	37.765	30.043	14.169
65	62.516	60.236	58.700	53.026	44.713	41.263	33.026	16.094
70	67.342	64.919	63.287	57.259	48.426	44.761	36.009	18.018
75	72.166	70.603	67.874	61.492	52.140	48.258	38.992	19.943
80	76.999	75.286	72.463	65.725	55.853	51.756	41.975	21.868
85	81.824	78.970	77.050	69.957	59.567	55.254	44.957	23.793
90	86.650	83.653	81.637	74.190	63.281	58.751	47.940	25.717
95	91.470	88.340	86.220	78.423	66.994	62.249	50.922	27.642
100	96.300	93.020	90.810	82.656	70.707	65.747	53.906	29.567
105	101.13	97.700	95.400	86.890	74.421	69.244	56.889	31.491
110	105.95	102.38	99.980	91.120	78.134	72.742	59.872	33.416
115	110.78	107.07	104.57	95.350	81.848	76.240	62.855	35.341
120	115.60	111.75	109.16	99.580	85.560	79.737	65.838	37.266
125	120.43	116.43	113.75	103.82	89.270	83.235	68.820	39.190
130	125.26	121.12	118.33	108.05	92.980	86.730	71.802	41.115
135	130.08	125.80	122.92	112.28	96.700	90.230	74.785	43.039
140	134.90	130.48	127.50	116.52	100.41	93.720	77.768	44.963
145	139.73	135.17	132.09	120.75	104.12	97.220	80.751	46.888
150	144.56	139.85	136.68	124.98	107.84	100.72	83.734	48.813
155	149.38	144.83	141.27	129.22	111.85	104.22	86.710	50.737
160	154.21	149.22	145.85	133.45	115.26	107.72	89.700	52.662
165	159.03	153.90	150.44	137.68	118.98	111.21	92.680	54.587
170	163.86	158.58	155.03	141.91	122.69	114.71	95.660	56.812
175	168.69	163.27	159.62	146.15	126.40	118.21	98.650	58.436
180	173.51	167.95	164.20	150.38	130.12	121.71	101.63	60.361
185	178.34	172.64	168.80	154.81	133.83	125.21	104.61	62.286
190	183.16	177.32	173.39	158.81	137.54	128.71	107.59	64.210
195	187.99	182.01	177.98	163.08	141.26	132.20	110.57	66.135
200	192.81	186.69	182.58	167.31	144.97	135.70	113.55	68.060
210	202.46	195.06	191.74	175.78	152.40	142.70	119.52	71.908
220	212.11	205.43	200.93	184.24	159.83	149.69	125.48	75.758
230	221.77	214.79	210.10	192.71	167.24	156.69	131.39	79.603
240	231.42	224.16	219.27	201.17	174.68	163.68	137.41	83.456
250	241.08	233.57	228.45	209.64	182.19	170.68	143.39	87.300
260	250.73	242.89	237.62	218.10	189.53	177.67	149.35	91.150
270	260.38	252.26	246.79	226.57	196.96	184.67	155.32	95.000
280	270.04	261.62	255.94	235.03	204.39	191.66	161.29	98.860
300	289.34	280.35	264.30	251.95	219.24	205.56	173.22	106.550

Mean Pressure *f* during the Expansion.

Pres. P.	1/4	1/3	3/8	1/2	5/8	2/3	3/4	7/8
30	28·549	24	23·50	20·79	17·60	16·31	13·86	8·9097
35	33·308	28	27·41	24·25	20·54	19·02	16·17	10·394
40	38·066	32	31·83	27·72	23·47	21·73	18·48	11·879
45	42·824	36	35·25	31·18	26·40	24·46	20·79	13·364
50	47·582	40	39·16	34·65	29·33	27·16	23·10	14·849
55	52·340	44	43·08	38·11	32·24	30·17	25·41	16·334
60	57·098	48	47·00	41·58	35·20	32·62	27·72	17·819
65	61·853	52	50·91	45·04	38·14	35·33	30·03	19·303
70	66·616	56	54·83	48·51	41·07	38·04	32·34	20·788
75	71·371	60	58·75	51·90	44·00	40·75	34·65	22·263
80	76·128	64	62·66	55·44	46·94	43·47	36·96	23·758
85	80·885	68	66·18	58·90	49·87	46·19	39·27	25·243
90	86·448	72	70·50	62·37	52·80	48·93	41·58	26·729
95	90·391	76	74·41	65·73	55·73	51·62	43·89	28·213
100	95·160	80	78·33	69·30	58·66	54·33	46·20	29·699
105	99·910	84	82·24	72·76	61·57	57·33	48·51	31·183
110	104·68	88	86·16	76·23	64·48	60·35	50·82	32·669
115	109·40	92	90·08	79·69	67·44	62·79	53·13	34·153
125	118·95	100	97·91	97·02	73·34	67·95	57·75	37·122
140	133·23	112	109·6	97·02	82·14	76·08	64·68	41·576
150	142·74	120	117·5	103·9	88·00	81·50	69·30	44·548
200	190·32	160	156·6	138.6	117·3	108·6	92·40	59·398
250	237·07	200	195·7	173.2	146·6	135.8	115·5	74·247
300	288·16	240	235·0	207.9	176·0	163.1	138·6	89·097

Table III. Economy of Expansion and high Steam.

Fuel saved or effect gained per cent.

Pres. P.	0	1/4	1/3	3/8	1/2	5/8	2/3	3/4	7/8
30	0	12	29·5	32	41	49·3	52	58	67·5
35	1·6	13·6	31	33·6	42·6	51	53·6	59·6	69·1
40	2·5	14·5	32	34·5	43·5	51·8	54·5	60·5	70
45	3·4	15·4	33	35·4	44·4	52·7	55·4	61·4	71
50	4.3	16·3	33·8	36·3	45·3	53·6	56·3	62·3	71·8
55	5·2	17·2	34·7	37·2	46·2	54·5	57·2	63·2	72·7
60	6	18	35·7	38	47·	55·3	58	64	73·5
65	6·7	18·7	36.2	38·7	47·7	56	58·7	64·7	74·2
70	7·3	19.3	36.8	39·3	48·3	56·6	59·3	65·3	74·8
75	7·8	19·8	37·3	39·8	48·8	57·1	59·8	65·8	75·3
80	8·5	20·5	38	40·5	49·5	57·8	60·5	66·5	76
85	9	21	38·5	41	50	58·3	61	67	76·5
90	9·5	21·5	39	41·5	50·5	58·8	61·5	67·5	77
95	10	22	39·5	42	51	59·3	62	68	77·5
100	10·4	22·4	40	42.4	51.4	59·7	62·4	68·4	78
105	10·7	22·7	40·2	42·7	51·7	60·	62·7	68·7	78·2
115	11	23	40·5	43	52	60·3	63	69	78·5
125	11·7	23·7	41·2	43·7	52·7	61	63·7	69·7	79·2
150	14	26	43·5	46	55	63·3	66	72	81·5
200	16	28	45·5	48	57	65·3	68	74	83·5
250	17·7	29·7	46·2	49·7	58·7	67	69·7	75·7	85·2
300	19	31	48·5	51	60	68·3	71	77	86.5

Table IV.

Consumption of Coal in pounds per horse power per hour,

Grade of Expansion.

Pres. P.	0	1/4	1/3	3/8	1/2	5/8	2/3	3/4	7/8
lbs,	lbs,	lbs,	lbs,	lbs,	lbs,	lbs,	lbs,	lbs,	lbs,
30	5·6	4·93	3·95	3·81	3·30	2·84	2·69	2·35	1·82
35	5·51	4·84	3·86	3 72	3·21	2·74	2·60	2·26	1·73
40	5·46	4·79	3·81	3·67	3·16	2·70	2·55	2·21	1·68
45	5·41	4·73	3·75	3·62	3·11	2·65	2·50	2·16	1·62
50	5·36	4·68	3·71	3·57	3·06	2·60	2·45	2·11	1·58
55	5·31	4·63	3·66	3·51	3·01	2·55	2·40	2·06	1·53
60	5·26	4·59	3·60	3·47	2·97	2·50	2·35	2·02	1·49
65	5·20	4·55	3·57	3·43	2·93	2·46	2·31	1·98	1·45
70	5·19	4·52	3·54	3·40	2·90	2·43	2·28	1·94	1·41
75	5·16	4.49	3·51	3·37	2·87	2·40	2·25	1·91	1·39
80	5·12	4·45	3·47	3·33	2·83	2·36	2·21	1·88	1·35
85	5·09	4·42	3·44	3·30	2·80	2·33	2·18	1·85	1·32
90	5·07	4·39	3·41	3·28	2·77	2·31	2·16	1·82	1·29
95	5·04	4·37	3·39	3·25	2·74	2·28	2·13	1·79	1·26
100	5·01	4·34	3·36	3·23	2·72	2·26	2·10	1·77	1·23
105	5·00	4·32	3·35	3·21	2·70	2·24	2·09	1·75	1·22
115	4·98	4·31	3·33	3·19	2·69	2·22	2·07	1·73	1·20
125	4·94	4·27	3·29	3·15	2·65	2·19	2·03	1·70	1·17
150	4·81	4·14	3·16	3·02	2·52	2·05	1·90	1·57	1·04
200	4·70	4·03	3·05	2·91	2 41	1·94	1·79	1·46	0·92
250	4·60	3·93	3·01	2·81	2·31	1·85	1·70	1·36	0·83
300	4·54	3·87	2·89	2·75	2·24	1·78	1·62	1·29	0·75

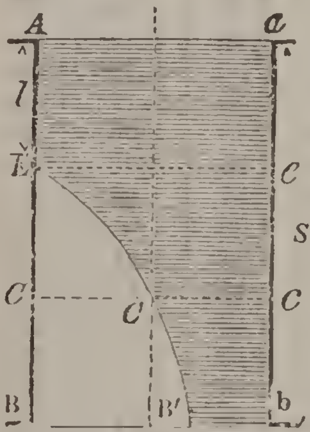
Table V.

Consumption of Coal in tons per 100 horses in 24 hours.

Pres. P.	0	1/4	1/3	3/8	1/2	5/8	2/3	3/4	7/8
lbs.	tons	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,
30	6·00	5·29	4·23	4·09	3·54	3·04	2·88	2·52	1·95
35	5·90	5·19	4·13	3·99	3·44	2·94	2·79	2·42	1·86
40	5·85	5·13	4·08	3·93	3·39	2·90	2·73	2 37	1·80
45	5·80	5·07	4·02	3·88	3·34	2·84	2·68	2·31	1·73
50	5·75	5 01	3·97	3·83	3·28	2·79	2·63	2·26	1·69
55	5·70	4·96	3·92	3·77	3·22	2·73	2·57	2·21	1·64
60	5·64	4·92	3·87	3·72	3·18	2·68	2·52	2·17	1·60
65	5·58	4·88	3·82	3·68	3·14	2·63	2·48	2·12	1·55
70	5·56	4·84	3·79	3·64	3·11	2·60	2·44	2·08	1·51
75	5·53	4·81	3·76	3·61	3·07	2·57	2·41	2·05	1·49
80	5·49	4·77	3·72	3·57	3·03	2·53	2·37	2·01	1·44
85	5·46	4·74	3·69	3·54	3·00	2·50	2·33	1·98	1·41
90	5·43	4·70	3·66	3·51	3·97	2·47	2·31	1·95	1·38
95	5·40	4·68	3·63	3·48	2·94	2·44	2·28	1·92	1·35
100	5·37	4·65	3·60	3·46	2·91	2·42	2·26	1·90	1·32
105	5·36	4·63	3·59	3·44	2·89	2·40	2·24	1·88	1·31
115	5·34	4·61	3·57	3·42	2·88	2·38	2·22	1·85	1·29
125	5·30	4·58	3·53	3·38	2·84	2·34	2·18	1·82	1·25
150	5·16	4·44	3.39	3·34	2·81	2·30	2·04	1·68	1·11
200	5·04	4·32	3·27	3·12	2·59	2·19	1·92	1·56	0·99
250	4·93	4·21	3·22	3·01	2·47	2·09	1·82	1·46	0·89
300	4·87	4·15	3·10	2 95	2·40	2·01	1·74	1·38	0·83

EXPANSION OF STEAM.

In order to save steam, or more correctly to employ its effect to a higher degree, the admittance of steam to the cylinder is shut off when the piston has moved a part of the stroke; from the cut-off point the steam acts expansively with a decreasing pressure on the piston, as represented by the accompanying figure.



Let the steam be cut off at ⅓ of the stroke, and *Aa* represent the total pressure, say 20 pounds per square inch which will continue to the point *E* where the admittance of steam is shut off at one-third the stroke *S*. The steam *Aa eE*, is now acting expansively on the piston, and the pressure decreases as the volume increases, when the piston has attained *Cc* or two-thirds of *S*, the pressure *C′c*=10 pounds, only half the pressure *Aa*=20 because the volume *Aa eE* is only half of *Aa cC*, and so on until the piston has attained *B b* the pressure *B′b*=⅓ ×20=6·66 pounds.

The mean pressure, or the effectual pressure, throughout the stroke, will be about 13·33 pounds per square inch, or 66 per cent., but the quantity of steam used is only 33 per cent., hence 33 per cent. is gained by using the steam expansively.

l = part of the stroke S in feet, at which the steam is cut off.
P = pressure per square inch under full admittance of steam.
F = mean pressure per square inch throughout the stroke S.
f = mean pressure per square inch during the expansion, which in double expansion cylinder engines will be the average pressure per square inch on the large piston A.
p = end pressure per square inch after expansion.
S = stroke of the cylinder Piston in feet.

$$F' = \frac{Pl}{S}\left[2.3(\log. S - \log. l) + 1\right] \qquad f = \frac{FS - Pl}{S - l}, \qquad p = \frac{Pl}{S}.$$

The following Tables are calculated from these formulas.

On page 594 *et seq.* will be found a rational and practical method of computing the most economic number of expansions for steam-engines of any type, which is abstracted from *The Relative Proportions of the Steam-Engine*, by Professor Marks.

Example 3. Required the mean pressure f=? for an initial pressure P=43 lbs. under ¾ expansion?

For P = 40 lbs.	f = 18·48	Table II.
P = 30 or 3 lbs.	f = 1·38	
P = 43 lbs.	f = 19·86 the answer.	

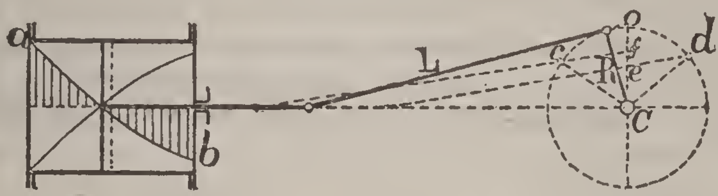
The effect gained or fuel saved by expansion and high steam is calculated from the following formulæ, in which it is supposed as a unit the work of an engine with P=30 pounds per square inch, or an indicated pressure of 15 lbs. without expansion.

c = per cent on 100, of effect gained or fuel saved.

$$\text{For expansion } c = 100\left(1 - \frac{lP}{SF}\right). \qquad \text{For high steam } c = 100\left(1 - \frac{26490}{VP}\right).$$

The preceding Table III. is calculated from these formulæ, in which the first line from 30 contains the economy per cent. from expansion alone, and the column *o* contains the economy per cent. from high steam above P = 30 lbs. The balance of the table contains the conjoined economy of expansion and high steam. Required the conjoined economy of P = 90 lbs. under ½ expansion? 50·5 per cent. the answer.

Inertia of Reciprocating Masses.



When bodies are moved rapidly forward and backward like the reciprocating parts of a steam-engine, the force of inertia plays an important part in the operation. The reciprocating parts must be started and stopped at each end of the cylinder. The accompanying illustration shows the inertia diagram *a b* drawn in the cylinder; the ordinates drawn from the centre line represent the force of inertia, which is greatest at the ends and vanishes to nothing near the middle of the stroke. For a definite length of connecting-rod the inertia is greater at the back than at the front end of the cylinder; but if the connecting-rod was infinitely long (like in slot motion), the inertia would be alike at both ends of the stroke.

F = force in pounds consumed in starting the reciprocating parts at the beginning of the stroke, which force is restored by bringing the moving masses to rest at the end of the stroke.
W = weight in pounds of all the reciprocating parts.
R = radius of the crank in feet.
L = length of connecting-rod in feet.
n = revolutions of crank per minute.

Force of inertia: $F = \dfrac{W R n^2}{2933.54}$ . . . . 1.

Examples 1 and 2. The reciprocating parts of an engine weigh $W = 1000$ pounds. Radius of crank $R = 1$ foot, $n = 200$ revolutions per minute. Required the force of inertia at the back and front ends of the stroke?

$$F = \frac{1000 \times 1 \times 200^2}{2933.54} = 13635 \text{ pounds.}$$

d = distance in fraction of a foot from the centre of the stroke to where the inertia diagram crosses the centre line.

$$d = \sqrt{L^2 + R^2} - L. \quad . \quad . \quad . \quad . \quad . \quad 3.$$

The piston and the other reciprocating parts move fastest when the crank and connecting-rod are at right angles.

The velocity of the piston when the crank passes at d is to the velocity of the piston when the crank passes at c as Ce is to Cf. Where the direction of the connecting-rod crosses the line CO is the measure of the velocity of the piston.

For a connecting-rod of infinite length the formula for inertia is

$$F = \frac{W R n^2}{2933.54},$$

which is the same as the formula for centrifugal force.

Force or Feed Pumps.

Approximate Formulæ.

d = diameter, s = stroke } of the force-pump, single acting.
D = diameter, S = stroke } of the steam-cylinder piston, in inches, double acting,
V = volume of steam given in the table at the given pressure.

The stroke of the steam-piston is only that under which steam is fully admitted to the cylinder.

$$d = 2D\sqrt{\frac{S}{V s}}, \qquad s = 4\frac{D^2 S}{V d^2}, \quad . \quad . \quad . \quad 4, 5.$$

Slip-water included in the formulas.

Example. Required, the diameter of a force-pump having the same stroke as the cylinder piston $s = 38$ inches, diameter of cylinder $D = 30$ inches. The steam is cut off at $\frac{1}{2}$ the stroke, and the steam pressure + 50 pounds per square inch. Here $V = 437$, and $S = 19$ inches, because steam is cut off at $\frac{1}{2}$ the stroke.

$$d = 2 \times 30\sqrt{\frac{19}{437 \times 38}} = 2.03 \text{ inches.}$$

To find the Quantity of Condensing Water.

q = condensing water of temp. t in cubic feet.
Q = steam of temperature T in cubic feet.
t' = temperature in the condenser.

$$q = \frac{1.4Q(990 + T - t')}{V(t' - t)}, \quad . \quad 6.$$

Dimensions of the Air-Pump.

d = diameter, s = stroke } of the air-pump, single acting.
D = diameter, S = stroke } of the steam cylinder, double acting.

$$d = 2.3D\sqrt{\frac{S(990 + T - t')}{V s(t' - t)}}, \quad . \quad . \quad 7.$$

Assume $t' = 100°$, and $t = 50°$, we shall have—

Single acting air-pumps.

$$d = 0.326D\sqrt{\frac{S(940 + T)}{V s}}, \quad 8.$$

$$s = 0.106D^2\frac{S(940 + T)}{V d^2}, \quad 9.$$

Double acting air-pumps.

$$d = 0.23D\sqrt{\frac{S(940 + T)}{V s}}, \quad 10.$$

$$s = 0.053D^2\frac{S(940 + T)}{V d^2}, \quad 11.$$

Example. A single acting air-pump is to be constructed for an engine $D = 38$ inches, $S = 45$ inches stroke of the cylinder: the stroke of the air-pump can be 32 inches, and the exhaust steam is 261°. Required, the diameter of the air-pump? $V = 767$.

$$d = 0.326 \times 38\sqrt{\frac{45(940 + 261)}{767 \times 32}} = 18.38 \text{ inches.}$$

☞ Slip-water included. T and V must be taken for the exhaust steam, as the steam may have worked expansively; the area of the foot valve must be calculated from the following formulas.

Foot Valve in the Air-Pump.

To enable an air-pump to work well and with the greatest advantage, it is necessary to pay particular attention to the following formulas. The force by which the water is driven from the condenser through the foot valve into the air-pump is limited by the pressure in the condenser; this absolute pressure is the vacuum subtracted from 14.7 pounds; it is noted in the third column, where the temperature in the condenser is opposite, in the first column. Every pound of this pressure per square inch balances a column of water 27 inches high, which is the head that presses the water from the condenser.

Foot-Valves in Air-Pumps.

$\mathfrak{A}$ = area of the air-pump piston.
$\mathfrak{a}$ = area of the foot-valve, or bucket-valve.
$\mathfrak{D}$ = diameter of the air-pump-piston.
$\mathfrak{d}$ = diameter of the foot-valve, when round.
$\mathfrak{S}$ = stroke of air-pump piston, in feet.
$\mathfrak{P}$ = pressure in the condenser at the temperature T.
n = number of strokes of the air-pump piston per minute.

$$\mathfrak{a} = \frac{D^2 S\, n\, (890 + T)}{23000\, m V \sqrt{\mathfrak{p}}}.$$

$m = 0{\cdot}6$ to $0{\cdot}8$

$$\mathfrak{a} = \frac{\mathfrak{A}\,\mathfrak{S}\,n}{100\sqrt{\mathfrak{P}}}, \quad 12,$$

$$\mathfrak{S} = \frac{100\mathfrak{a}\sqrt{\mathfrak{P}}}{n\,\mathfrak{A}}, \quad 13,$$

$$n = \frac{100\mathfrak{a}\sqrt{\mathfrak{P}}}{\mathfrak{A}\,\mathfrak{S}}, \quad 14,$$

$$\mathfrak{d} = \frac{\mathfrak{D}\sqrt{\mathfrak{S}\,n}}{10\sqrt[4]{\mathfrak{P}}}, \quad 15,$$

$$\mathfrak{S} = \frac{100\mathfrak{d}^2\sqrt{\mathfrak{P}}}{n\,\mathfrak{D}^2}, \quad 16,$$

$$n = \frac{100\mathfrak{d}^2\sqrt{\mathfrak{P}}}{\mathfrak{D}^2\,\mathfrak{S}}, \quad 17.$$

Example. The area $\mathfrak{A}$ of an air-pump-piston is 2·35 square feet, stroke of piston $\mathfrak{S} = 3{\cdot}6$ feet, to make $n = 40$ strokes per minute, and the pressure to be $\mathfrak{P} = 3{\cdot}2$ pounds. Required the area of the foot-valve.

$$\mathfrak{a} = \frac{2{\cdot}35 \times 3{\cdot}6 \times 40}{100\sqrt{3{\cdot}2}} = 1{\cdot}85 \text{ square feet.}$$

To Find the Velocity and Quantity of the Injection Water through the Adjutage into the Condenser.

Letters denote.

v = velocity in feet per second.
h = head of the press water; + when above, and — below the adjustage.
F = vacuum, noted — or negative in the last column, but is positive in the formulas.
q = quantity of water discharged in cubic feet, per second.
a = area of all the holes in the adjustage in square feet.
d = diameter, L = length } of the injection pipe, in feet.
n = double strokes of cylinder-piston, or revolutions per minute.
A, D, and S, dimensions of the steam cylinder, in feet.
T = temperature, and v = volume coefficient of the exhaust steam.

$$a = \frac{q}{5\sqrt{2F \pm h}}, \quad 18,$$

$$v = 8\sqrt{2F \pm h} \quad 19,$$

$$q = \frac{n\,S\,D^2(940+T)}{55V}, \quad 20,$$

$$q = 5a\sqrt{2F \pm h}, \quad 21$$

$$d = 0{\cdot}35\sqrt[5]{\frac{L\,q^2}{2F \pm h}}, \quad 22,$$

$$a = \frac{n\,S\,D^2(940+T)}{275V\sqrt{2F+h}}, \quad 23,$$

The above are approximate formulæ only.—W. D. M.

Example. Required the diameter of an injection-pipe $L = 10$ feet long, which shall supply $q = 1.3$ cubic feet of water per second into a vacuum of 12 pounds per square inch, the head of press water $h = 2$ feet?

$$d = 0.35\sqrt[5]{\frac{10 \times 1.3^2}{2 \times 12}} = 0.3211 \text{ feet} = 3.84 \text{ inches.}$$

Area of Steam Passages.

n = number of revolutions per minute.
$\mathbf{a}$ = area of the steam pipe, sq. in,
A = area of the cylinder piston, sq. in.
d = diameter of the pipe, in inches.
D = diameter, S = stroke of cylinder, in inches.

$$\mathbf{a} = \frac{A\,S\,n}{40000}, \qquad d = \frac{D\sqrt{S\,n}}{200}. \quad . \quad . \quad . \; 24, 25.$$

Example. Required the diameter of a steam pipe for a cylinder $D = 40$ inches. Stroke of piston $S = 48$ inches, and $n = 38$ revolutions per minute?

$$d = \frac{40\sqrt{48 \times 38}}{200} = 8.54 \text{ inches.}$$

Steam Ports to the Cylinder.

$$\text{Steam port,} \quad \mathbf{a} = \frac{A\,S\,n}{35000}, \qquad \text{Exhaust port,} \quad \mathbf{a} = \frac{A\,S\,n}{27000}. \quad . \quad . \; 26.$$

Safety Valve.

Three-fourths of the fire grate in square feet is a good proportion for the safety valve in square inches. Make area of safety valve $\frac{1}{3000}$ of boiler heating surface. Weisbach, Vol. II.

Notation of Letters corresponds with Figure 3, Plate V.

$\mathbf{a}$ = area of safety valve in square inches.
P = pressure per square inch in the boiler, W = weight on the safety valve lever, Q = weight of the safety valve and lever } in pounds.
l = lever for W, e = " $\mathbf{a}\,P$, x = " Q } in inches.

Balance the lever over a sharp edge, and the centre of gravity Q is found; measure the distance x from the fulcrum C.

$$\mathbf{a}\,Pe = Wl + Qx. \quad . \quad . \; 27.$$

$$P = \frac{Wl + Qx}{\mathbf{a}\,e}. \quad . \quad . \; 28.$$

$$W = \frac{\mathbf{a}\,Pe - Qx}{l}. \quad . \quad . \; 29.$$

$$l = \frac{\mathbf{a}\,Pe - Qx}{W}. \quad . \quad . \; 30.$$

Example. Area of the safety valve $\mathbf{a} = 9$ square inches, $e = 4\frac{1}{2}$ inches, $W = 50$ pounds, weight of the lever and safety valve $Q = 15$ pounds, and $x = 17$ inches. Required at what distances l, l', and l'' will the weight W indicate pressures of $P = 30$, $P' = 40$, and $P'' = 50$ pounds?

$$l = \frac{9 \times 30 \times 4.5 - 15 \times 17}{50} = 19.2 \text{ inches,}$$

from the fulcrum C the weight W will indicate $P = 30$ pounds.

$l' = 27.3$ inches, when $P' = 40$ pounds.
$l'' = 35.4$ " " $P'' = 50$ "

and thus the lever can be graduated.

SURFACE CONDENSER.

THE fresh-water or surface-condenser is now considered indispensable on sea-going steamers, as it not only saves 15 to 20 per cent. in the consumption of fuel, but saves the boiler from dangerous incrustation. It is also advisable to use surface-condensers on rivers with muddy water, and on lakes with hard water, which is very injurious and treacherous to steam-boilers.

The condensing surface in a fresh-water condenser should be about five-eighths ($\frac{5}{8}$) of the heating surface in the steam-boiler, or about two square feet per indicated horse-power.

The jet-condenser cannot be used for steam of 35 pounds to the square inch above atmospheric pressure, because when sea-water is raised to the temperature of 280° Fahr. its insoluble salts, principally sulphate of lime, wholly precipitate and form scale upon the hot surfaces in the boiler. This scale sticks fast to the surface of the iron and cannot be dissolved.

Scale formed of carbonate of lime and salts of soda and magnesia has a soft or plastic consistence, which can to a great extent be blown out from the boiler when in port.

For the surface-condenser any steam-pressure can be used, but it is generally expanded down to a low pressure before it reaches the condenser.

The fresh-water condenser is more expensive, occupying more room, and requires an extra circulating-pump.

The air-pump need not be so large for a surface-condenser as for the jet-condenser. The capacity of the circulating-pump should be about 0.6 of that of the air-pump. The velocity of the cold water through the valves of the circulating-pump should not be over 450 feet per minute.

HORSE-POWER IN STEAM-ENGINES.

Horse-power in machinery is assumed to be about the effect a horse is able to produce, and has been estimated and established by Mr. Watt to be 33,000 lbs. raised one foot per minute for one horse, which will be the same as 550 lbs. raised one foot per second. Mr. Watt adopted a standard steam-pressure of 7 lbs. per square inch, established a simple rule for the nominal horse-power of engines, which is, "*The square of the diameter of the cylinder in inches multiplied by the cube root of the stroke in feet, and divided by the constant number* 47, *is the nominal horse-power.* This rule agreed very near to the actual performance of engines in those days, but as improvements were made the steam-pressure and piston-speed were increased. The rule is not now applicable.

Nominal Horse-Power.

Assume a standard steam-pressure of 30 lbs. per square inch expanded two-thirds, the velocity of the steam-piston to be $200\sqrt[3]{S}$ feet and revolutions per minute $n = \dfrac{100}{\sqrt[3]{S^2}}$, we will arrive at a formula of nominal horse-power.

$$H = \frac{D^2\sqrt[3]{S}}{10}$$ for condensing engines, which will agree very near with the actual performance of our present condensing engines. The following tables are calculated from this formula.

For high-pressure engines I will assume the steam-pressure to be 80 lbs. per square inch, expanded one-half, which will give the nominal horse-power—

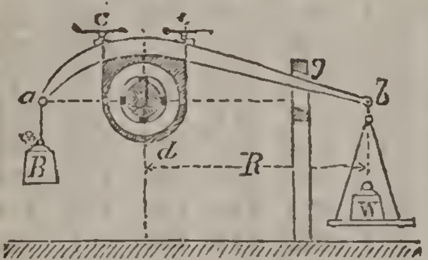
$$H = \frac{D^2\sqrt[3]{S}}{4},$$ high-pressure engines.

The horse-power in the accompanying table, divided by 0.4, gives the nominal power of high-pressure engines. The diameters D are contained in the first column in inches, and the stroke S in feet and inches on the top line.

Diam. D in.	Stroke of Cylinder Piston S in feet. 1′	1′ 3″	1′ 6″	1′ 9″	2′	2′ 3″	2′ 6″	2′ 9″	3′	3′ 6″	4′	4′ 6″	5′
	H	H	H	H	H	H	H	H	H	H	H	H	H
6	3·6	3·88	4·12	4·33	4·53	4·72	4·88	5·04	5·19	5·47	5·71	5·94	6·16
7	4·9	5·27	5·61	5·90	6·17	6·43	6·65	6·86	7·07	7·44	7·78	8·10	8·38
8	6·4	6·90	7·32	7·71	8·06	8·39	8·68	8·96	9·23	9·72	10·1	10·6	11·0
9	8·1	8·72	9·27	9·75	10·2	10·6	11·0	11·3	11·7	12·3	12·9	13·4	13·9
10	10	10·8	11·4	12·0	12·6	13·1	13·6	14·0	14·4	15·2	15·9	16·5	17·1
11	12·1	13·0	13·9	14·6	15·2	15·8	16·4	16·9	17·4	18·3	19·2	20·0	20·7
12	14·4	15·5	16·5	17·4	18·1	18·9	19·5	20·2	20·8	21·9	22·9	23·8	24·6
13	16·9	18·2	19·3	20·3	21·3	22·1	22·9	23·7	24·4	25·6	26·8	27·9	28·9
14	19·6	21·1	22·4	23·6	24·7	25·7	26 6	27·4	28·3	29·7	31·1	32·4	33·5
15	2·25	24·2	25·8	27·1	28·3	29·5	30·5	31·5	32·4	34·1	35·7	37·1	38·5
16	25·6	27·4	29·3	30·8	32·2	33·5	34·7	35·8	37·0	38·9	40·6	42·2	43·8
17	28·9	31·1	33·1	34·8	36·4	37·9	39·2	40·5	41·7	43·9	45·9	47·7	49·4
18	32·4	34·9	37·1	39·0	40·8	42·5	44·0	45·4	46·8	49·2	51·4	53·5	55·4
19	36·1	38·9	41·3	43·5	45·5	47·3	49·0	50·6	52·1	54·8	57·3	59·6	61·7
20	40·0	43·1	45·8	48·2	50·4	52·4	54·3	56·0	57·7	60·7	63·5	66·0	68·4
21	44·1	47·5	50·5	53·1	55·6	57·8	59·8	61·7	63·6	67·0	70·0	72·8	75.4
22	48·4	52·1	55·4	58·3	61·0	63·4	65·6	64·8	69·8	73·5	76·8	80 0	82·8
23	52·9	57·0	60·5	63·7	66·7	69·3	71·8	74·1	76·3	80·3	84·0	87·4	90·5
24	57·6	62·0	65·9	69·4	72·6	75·5	78·1	80·7	83·1	87·4	91·5	95·2	98·6
25	62·5	67·3	71·5	75·3	78·7	81·9	84·8	87·5	90·2	94·8	99·2	103	107
26	67·6	72·8	77·3	81·5	85·2	88·6	91·7	94·7	97·5	102	107	111	116
27	72·9	78·5	83·5	87·8	91·9	95·6	99·0	102	105	111	116	120	125
28	78·4	84·4	89·8	94·5	98·8	102	106	110	113	119	124	129	134
29	84.1	90·5	96·2	101	106	110	114	118	121	128	133	139	144
30	90·0	96·9	103	108	113	118	122	126	130	137	143	149	154
31	96·1	103	110	116	121	126	130	134	139	146	153	159	164
32	102	110	117	123	129	134	138	143	148	155	163	170	175
33	109	117	124	131	137	142	147	152	157	165	173	180	186
34	115	124	132	139	145	151	157	162	167	175	183	190	198
35	122	132	140	148	154	160	166	172	177	186	194	202	210
36	129	140	148	156	163	170	176	182	187	197	205	214	222
37	137	147	156	165	172	180	186	192	198	208	217	226	234
38	144	155	165	174	182	190	196	202	209	218	229	238	247
39	152	164	174	183	192	200	206	213	220	231	241	251	260
40	160	172	183	193	202	210	217	224	231	243	254	264	274
42	176	190	202	212	222	231	240	347	254	268	280	291	302
44	193	208	221	233	244	254	263	271	280	294	307	320	331
46	211	228	242	255	266	277	287	297	306	321	336	350	362
48	230	248	264	277	290	302	313	323	332	350	366	380	394
50	250	269	286	301	315	328	339	350	360	380	397	413	427
52	270	291	309	326	340	354	367	378	390	410	429	446	463
54	291	314	333	351	367	382	396	408	420	443	463	481	500
60	360	388	412	433	453	472	488	504	519	547	571	594	616
66	435	468	498	525	548	571	591	610	628	661	690	718	744
72	518	558	593	626	653	679	704	726	748	787	822	856	886
78	608	655	696	734	766	784	825	852	877	924	964	1003	1039
84	705	759	807	851	888	924	957	989	1015	1071	1116	1166	1206
90	810	872	927	975	1020	1062	1100	1134	1168	1229	1284	1336	1385
96	921	991	1053	1110	1160	1206	1249	1291	1327	1400	1460	1505	1575

D	Stroke of Cylinder Piston S in feet.												
	6′	7′	8′	9′	10′	11′	12′	13′	14′	15′	16′	18′	20′
in.	H	H	H	H	H	H	H	H	H	H	H	H	H
30	163	172	180	187	194	200	206	211	217	222	226	236	244
32	186	196	204	213	220	227	252	241	246	253	258	268	278
34	210	221	231	240	249	257	264	272	276	285	291	303	313
36	235	248	259	269	273	288	296	306	312	319	326	339	351
38	262	276	289	299	311	321	330	341	348	356	363	378	391
40	290	306	320	333	344	355	366	377	385	395	403	419	434
42	320	336	352	365	380	392	404	416	425	435	444	462	478
44	352	371	387	402	417	430	453	461	466	477	494	507	525
46	384	405	423	440	460	470	484	497	510	522	533	554	587
48	418	441	461	479	496	512	527	541	555	569	580	603	625
50	554	478	500	520	538	555	572	588	602	617	630	655	677
52	491	518	541	562	582	601	619	635	651	667	681	708	733
54	529	558	583	606	628	648	667	685	700	719	734	764	790
56	570	600	637	652	675	697	718	737	755	775	790	811	850
58	611	644	673	700	724	648	770	791	810	830	847	880	912
60	654	689	720	749	775	800	824	846	867	888	907	943	977
62	698	736	769	800	828	855	879	903	925	948	968	1007	1043
64	744	784	819	852	882	911	938	963	987	1010	1024	1073	1101
66	791	834	871	906	938	968	997	1024	1049	1074	1099	1141	1182
68	840	885	925	960	996	1023	1059	1087	1114	1140	1165	1211	1254
70	890	938	980	1019	1055	1089	1122	1152	1171	1208	1234	1283	1329
72	942	994	1037	1078	1116	1153	1187	1218	1249	1278	1306	1358	1406
74	995	1048	1095	1139	1179	1218	1254	1287	1319	1350	1380	1434	1485
76	1050	1105	1155	1201	1244	1284	1322	1358	1392	1424	1455	1512	1567
78	1105	1165	1219	1265	1310	1353	1393	1430	1466	1500	1533	1594	1649
80	1162	1225	1280	1331	1378	1423	1465	1504	1542	1578	1612	1676	1737
84	1282	1350	1411	1467	1520	1569	1615	1658	1700	1741	1778	1848	1914
88	1407	1423	1549	1610	1668	1722	1773	1820	1866	1909	1951	2029	2100
92	1538	1619	1693	1761	1823	1882	1938	1990	2039	2086	2133	2258	2297
96	1674	1763	1843	1917	1985	2049	2010	2166	2221	2272	2322	2414	2474
100	1817	1913	2000	2080	2154	2224	2290	2351	2400	2466	2520	2620	2714
104	1964	1969	2163	2250	2349	2405	2477	2542	2608	2666	2725	2833	2935
108	2119	2231	2333	2426	2512	2594	2671	2742	2806	2873	2939	3056	3165
112	2279	2399	2509	2609	2702	2790	2871	2949	3023	3092	3161	3286	3404
116	2445	2574	2691	2799	2898	2992	3081	3163	3243	3315	3391	3525	3651
120	2616	2754	2880	2995	3312	3202	3297	3385	3471	3550	3628	3772	3908
124	2793	2941	3075	3198	3101	3419	3521	3614	3706	3790	3874	4028	4172
128	2977	3133	3277	3408	3529	3643	3752	3852	3949	4038	4128	4292	4446
132	3166	3333	3485	3624	3753	3875	3990	4096	4190	4295	4390	4565	4728
136	3360	3538	3699	3847	3984	4113	4235	4348	4457	4557	4654	4846	5019
140	3561	3749	3920	4077	4222	4359	4488	4608	4716	4832	4939	5135	5319
144	3767	3966	4147	4313	4466	4611	4748	4875	4997	5111	5225	5432	5681
148	3980	4190	4381	4556	4718	4871	5016	5179	5279	5399	5519	5739	5944
152	4198	4402	4621	4805	4976	5138	5291	5431	5568	5696	5821	6053	6270
156	4421	4655	4867	5062	5242	5412	5573	5721	5865	6000	6132	6376	6604
162	4768	5020	5249	5458	5653	5836	5010	6170	6324	6469	6613	6876	7122
168	5128	5399	5645	5870	6079	6277	6463	6635	6802	6958	7112	7094	7659
174	5494	5791	6055	6297	6521	6733	6933	7117	7296	7464	7629	7932	8216
180	5887	6198	6480	6539	6979	7205	7419	7617	7809	7989	8164	8488	8793

Prony's Friction Dynamometer.



This dynamometer consists of a friction brake, as shown by the illustration. It is keyed on the shaft *A*, which transmits the power and work to be measured.

The lever of the brake should be balanced at *B* before the weight *W* is put on the scale; and if it is not balanced, the weight of the lever and scale should be weighed at the scale and added to the weight *W*.

The weight *W* on the scale is the force acting on the lever or radius *R*.

It is supposed that all the power and work transmitted by the shaft is consumed by the friction in the brake. When the shaft is running with its average speed of *n* revolutions per minute, the strap is tightened up with the screws, so that the lever will barely lift the weight *W*, which is also adjusted to suit the motion. When the weight and friction are well balanced, count the revolutions per minute of the shaft.

The power transmitted through the shaft is equal to the weight *W* multiplied by the velocity of the circumference of a wheel of radius *R*, making the same revolutions as the shaft.

The velocity in feet per second is

$$V = \frac{2\,\pi\,R\,n}{60}.$$

$$\text{Power} \quad P = W\,V = \frac{2\,\pi\,R\,n\,W}{60} \text{ in effects,}$$

which divided by 550 give the

$$\text{Horse-power} \quad HP = \frac{2\,\pi\,R\,n\,W}{60 \times 550} = \frac{W\,R\,n}{5252.2}.$$

The work *K* in foot-pounds consumed by the friction of the brake in the time *T* in seconds will be

$$\text{Work} \quad K = \frac{2\,\pi\,R\,n\,W\,T}{60} = \frac{W\,R\,n\,T}{9.55}.$$

All this work consumed by the friction is converted into heat, which makes the brake so hot that a constant stream of water must run on it to absorb the heat whilst the experiment is made, otherwise the wood in the brake would take fire.

When convenient, it is best to make the lever $R = 10.5$ feet, or 10 feet 6 inches, which will make the circumference 66 feet; in which case, the horse-power will be

$$HP = \frac{66\,n\,W}{550 \times 60} = \frac{2\,n\,W}{1000}.$$

That is to say, the product of the revolutions per minute and weight *W*, multiplied by 2 and point off three places, will be the horse-power of the experiment.

A lever of $R = 5$ feet 3 inches will make the circumference 33 feet, and the horse-power

$$HP = \frac{n\,W}{1000}.$$

ACTUAL HORSE POWER.

One actual horse power is 33000 lbs. raised one foot in one minute. This applied to steam engines will be the mean steam pressure on cylinder piston in pounds, multiplied by the velocity of piston in feet per minute, divided by 33,000, is the horse power imparted by the steam. From this we shall deduct 25 per cent. in condensing engines, and 13·1 per cent. in high pressure engines, for working friction and pumps, the balance to be termed the actual horse power.

Example 1. Fig. and formulæ 318. Area of steam cylinder $A=1809$ square inches, stroke of piston $S=4$ feet, indicated pressure of steam 30 lbs. to which add the atmospheric pressure 15 lbs. or $P=45$ lbs. expanded $\frac{2}{3}$, the mean pressure will be $F=31·459$ lbs. (see Expansion Table I.), vacuum $v=12$ lbs. the engine making $n=45$ revolutions or double stroke per minute. Required the actual horse power, $H=?$ $W=31·459+12-14·7=$ 28·759 lbs.

$$H=\frac{1809\times4\times28·759\times45}{22000}=425·6 \text{ horses.}$$

In this example the actual horse power is 11·6 per cent. more than the nominal power from the table.

Example 2. Fig. 318. A high pressure engine of cylinder piston $A=314$ square inches, stroke $S=3$ feet, steam pressure 80 lbs, per square inch, to which add 15 lbs. $P=95$ lbs. expanded $\frac{1}{2}$, the engine making $n=56$ revolutions per minute. Required the actual horse power? From the expansion table we have the mean pressure $F=80·412$ lbs., from which subtract the atmospheric pressure 14·7 lbs. $W=65·712$ lbs.

$$H=\frac{314\times3\times65·12\times56}{19000}=180·8 \text{ horses.}$$

Annular Expansion Double Cylinder, Fig. 319.

These kind of engines are now sometimes made in Europe with a view to economise fuel, and to extend the expansion of steam. The outer cylinder A, A, is annular, similar to that made by Mouslay, but in this case it is employed only for expansion, the inner cylinder a is used for high pressure only. It is so arranged by steam valves that the high steam is acting the whole stroke on the small piston a, after which it is conducted to the annular cylinder where it acts expansively on the large piston A, A. The two pistons being connected by rods to one common crosshead as shown by Fig. 319. This arrangement has been successfully carried out by Mr. Jägerfelt in Nyköping, Sweden. The inner cylinder can be considered an ordinary high pressure engine where the utilized steam is set free into the air at each stroke; but in this case the exhaust steam accomplishes a second engagement in the annular cylinder, which according to the grade of expansion may greatly exceed the original effect imparted in the small cylinder during the first engagement.

Example 3. Fig. 319. Area of the high pressure cylinder piston $a=254·4$ square inches, the annular expansive piston $A=763·2$ square inches, stroke of pistons $S=3$ feet, the high steam pressure $P=60$ lbs. vacuum $v=12$ lbs., making $n=65$ revolutions per minute. Required the actual horse power of the engine $H=?$ The grade of expansion will be

$1-\frac{763·2}{254·4}=\frac{2}{3}$, for which the mean pressure on the annular piston will be $f=32·62$ lbs. See Expansion Table II. The effective pressure on the two pistons will be $V=763·2\ (32·62+12-14·7)+254·4\ (60-32·62)=30337$ lbs.

$$H=\frac{29800\times3\times65}{22000}=269 \text{ horses.}$$

Example 4. Now we will reject the annular expansion cylinder, and take the effect of the steam without expansion, when the effectual pressure will be $60-14·7=45·3$ lbs. and the actual power,

$$H=\frac{254·4\times3\times45·3\times65}{19000}=118 \text{ horses.}$$

If we consider the last result as unit we shall have 269−118=151 horses or nearly 128 per cent. gained by the expansion, omiting the loss of steam in the steam passages.

In the first case about 11 per cent. was gained by vacuum, but that advantage is rather in favour of the utility of expansion, because the high steam cannot so well be introduced into the condenser.

The economy will be in the same proportion when the same grade of expansion is used in one cylinder.

I do not mean to maintain that this high per centage of economy is always fully realized in practice, as I am well aware of cases where expansion is of little use, caused by misconception and carelessness in its employment. (Very little confidence can be placed in computations of economy of expansion when the theory is incomplete, as in the above examples. On page 594 *et. seq.* will be found a natural and practical theory of expansion.—W. D. M.)

Half Trunk Expansion Engines. Fig. 320.

This kind of engines has been introduced by Mr. Carlsund, and are extensively used in Sweden, they are well suited for Gunboats where the machinery is required to be below the water line. The high steam is employed throughout the stroke in the annular space around the trunk, after which it is conducted to act expansively on the large piston A Fig. 320.

Example 5. Fig. 320. Area of the annular piston a=562 square inches, and A=2248 square inches, stroke of piston S=4 feet, steam pressure P=90 lbs., making n=68 revolutions per minute. Required the actual horse power?

$$\text{Grade of expansion} = 1-\frac{562}{2248} = \tfrac{3}{4},$$

From the Expansion Table II. we have f=41·58 lbs. mean pressure on A.

The effectual pressure will be V=2248 (41·58—14·7) + 562 (90—41·58) = 87639 lbs., high pressure

$$H = \frac{87639 \times 4 \times 68}{38000} = 627{\cdot}3 \text{ horses.}$$

Double Cylinder Expansion Engines, Fig. 321.

This kind of engines are now made in England and are said to be very economical. The small cylinder is used for high pressure, from which the steam is conveyed to expand in the large cylinder. In the figure it is arranged so that the pistons follow one another in one direction, when the steam must be conveyed from the top of the small cylinder to the bottom of the large one, and vice-versa; but it is sometimes arranged so that the pistons move in opposite direction, when the steam is conveyed direct at the same ends from the small cylinder to the large one, which has the advantage of making the steam passages shorter, but is more complicated in concentrating the motion.

Example 6.

High pressure cylinder, $\begin{cases} a = 962. \text{ square inches.} \\ s = 5 \text{ feet.} \end{cases}$

Expansion cylinder, $\begin{cases} A = 3848 \text{ square inches.} \\ S = 10 \text{ feet.} \end{cases}$

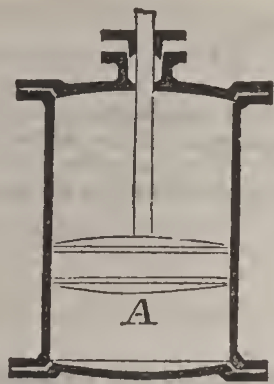
Steam pressure in the small cylinder P=40 lbs., vacuum v=12 lbs., making n=21 revolutions per minute. Required the actual horse power, H=?

$$\text{Grade of expansion} = 1-\frac{962 \times 5}{3848 \times 10} = \tfrac{7}{8}.$$

From the Expansion Table II. we have f=11.879 lbs., mean pressure on A.

w = 3848×10 (11.879+12−14:7) + 962×5 (40−11.879) = 488438 lbs.

$$H = \frac{366767 \times 21}{22000} = 466 \text{ horses.}$$

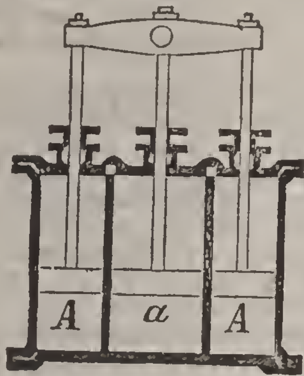


318. *One double acting Cylinder.*

Actual horse power. $\begin{cases} H = \dfrac{A\ S\ W\ n}{22000}, & \text{cond. engs.} \\ H = \dfrac{A\ S\ W\ n}{19000}, & \text{high pr. engines.} \end{cases}$

$W = F + v - 14{\cdot}7$ for cond. engines.

$W = F - 14{\cdot}7$ for high pressure engines.

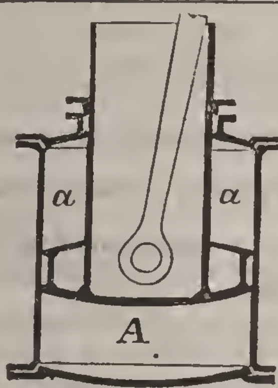


319. *Annular expansion double Cylinder.*

$$F = \frac{P\ a}{A}[2{\cdot}3\,(log.A - log.a) + 1].$$

$$f = \frac{F\ A - P\ a}{A - a}, \qquad V = A(f + v - 14{\cdot}7) + a(P - f).$$

Actual horse power. $\begin{cases} H = \dfrac{V\ S\ n}{22000}, & \text{cond. engines.} \\ H = \dfrac{V\ S\ n}{19000}, & \text{high pr. engs.} \end{cases}$

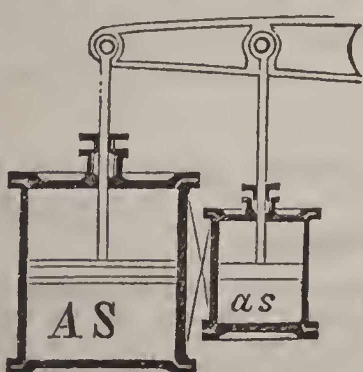


320. *Halftrunk expansion Cylinder.*

$$F = \frac{P\ a}{A}\ [2{\cdot}3(log.A - log.a) + 1].$$

$$f = \frac{F\ a - P\ a}{A - a}. \qquad V = A(f + v - 14{\cdot}7) + a(P - f).$$

Actual horse power. $\begin{cases} H = \dfrac{V\ S\ n}{44000}, & \text{cond. engines.} \\ H = \dfrac{V\ S\ n}{38000}, & \text{high pr. engs.} \end{cases}$



321. *Double Cylinder expansion.*

$$F = \frac{P\ a\ s}{A\ S}[2{\cdot}3(log.AS - log.as) + 1]$$

$$f = \frac{F\ A - P\ a}{A - a}, \qquad w = AS(f + v - 14{\cdot}7) + a\ s(P - f).$$

Actual horse power. $\begin{cases} H = \dfrac{w\ n}{22000}, & \text{cond. engines.} \\ H = \dfrac{w\ n}{19000}, & \text{high pr. engines.} \end{cases}$

INDICATOR DIAGRAMS.

Page 553.

FIG. 1 represents an ordinary diagram taken from a condensing-engine. The line a, b i, e, B, B is drawn by the indicator; the space U is added for clearance between the piston and cylinder-head and volume of steam-ports. In ordinary engines this clearance is between 6 and 12 per cent., to be added to the stroke S for proper analysis of the diagram.

c = volume of clearance in cubic inches.

A = area of steam-piston in square inches.

Clearance, $$U = \frac{c}{A}.$$

The line b, i, f, which indicates the steam-pressure during the expansion, is an equilateral hyperbola which Rankin has given the name of isothermal *curve*, or a curve of constant temperature. The lines Va and Vd are the *asymptotes* and m the axis of the hyperbola. (See Conic Sections, page 181, Figs. 202 and 203.) The axis m forms an angle of 45° with the asymptotes, and the hyperbola passes m at right angles.

The diagram shows that the steam is cut off when the piston is at b, and the hyperbola should then be the line b, q, h, t, e; but for the sake of a better explanation I have purposely selected a case in which the steam leaked into the cylinder during the expansion; so that the actual line of pressure is b i, f, e, which shows that the steam was wire-drawn. When the steam was cut off at b, the end-pressure should have been d e; but the steam which leaked in during the expansion raised the end-pressure to d f. The work done by the leakage is represented by the area bounded by b, i, f e t h q b. Had there been no leakage, but the same amount of steam admitted from beginning of the stroke and cut-off at c, the end-pressure would have still been d, f, with the additional work represented by the area bounded by b, c, g, f, i b, or that much work was lost by the leakage.

A e is the atmospheric line which should always be drawn by the indicator.

Vd is the perfect vacuum line, and the vertical height from Vd to A e represents the atmospheric pressure, about 14.7 pounds to the square inch.

u represents the pressure in the condenser, or the deficiency of vacuum.

v = vacuum in the condenser.

p = steam-pressure in pounds per square inch above atmospheric pressure at that part of the stroke.

P represents the absolute initial steam-pressure in pounds per square inch in the cylinder.

The work done for each single stroke of the piston is represented by the area bounded by the diagram a, b, i, f, e, B, B, a, of which the vertical heights are expressed in pounds per square inch and the stroke S in feet.

The variable steam-pressure must be reduced to a mean pressure before it can be applied for calculating the power of the engine. When the point of cut-off is correctly known, the expansion tables give the mean pressure, from which deduct the deficiency of vacuum u.

F = mean effective steam-pressure in pounds per square inch on the cylinder piston.

A = area of piston in square inches, double acting.

V = velocity of piston in feet per minute.

Indicated horse-power, $$IP = \frac{A\ F\ V}{33000}.$$

If the velocity is given in feet per second, then divide by 550 instead of 33000.

To Construct the Indicator Diagram.

Having given the point of cut-off b, stroke S, pressure P, and clearance U, draw the rectangle a, v, d. From b draw b, k parallel with a v. Divide the rectangle into 10 equal parts and number them as shown. From v draw the dotted lines to the top of the ordinates, as v o p'; from o draw the line o q; then q is a point in the diagram, and the other points are obtained in the same way, and so the hyperbola is obtained.

To Find the Point of Cut-off when the Final Pressure is Given.

Measure the end-pressure by any convenient scale (a diagonal decimal scale of inches is the best, such as that on plate I., page 368), say $de = 0.35$ of an inch; then divide 0.35 by the number of each ordinate, and the quotient is the ordinate pressure; for instance, $0.35 : 0.8 = 0.4375$ of an inch is the height of the ordinate $8t$; $0.35 : 0.4 = 0.875$ of an inch, the height of the ordinate $4q$; and thus the curve can be plotted out until it reaches the cut-off point ab. The curve c, g, f is obtained in the same way. The indicator curve b, i is continued to f to get the end-pressure df.

To Find the Final Pressure when the Point of Cut-off is Given.

l = length or part of stroke with full steam.
f = end-pressure at end of stroke.
S = stroke and P = initial steam-pressure.

$$P : (U + S) = f : l.$$

End-pressure, $f = \dfrac{Pl}{U + S}$. Cut-off at $l = \dfrac{f(U + S)}{P}$.

To Find the Effective Mean Pressure by Measurement.

Fig. 2 represents a fairly good card taken from a condensing-engine. The stroke is divided into 10 equal parts, and each part measured in the middle as indicated by the dotted lines and arrows. Use a decimal diagonal scale of inches like that on plate I., page 368. Add all the dotted ordinates together, and point off one decimal from the sum, which gives the mean length of all the ordinates. Multiply this mean length by the scale of the indicator-spring, and the product will be the effective mean pressure.

Scale of the Indicator-Spring.

The scale of the spring means the number of pounds of steam-pressure per square inch corresponding with one inch on the diagram, which scale is, or should be, marked on the spring.

When the vacuum and atmospheric lines are known on the diagram, then the scale of the spring can be determined, as there is 14.7 pounds pressure between these two lines.

The vacuum line cannot be drawn by the indicator, because there is never a perfect vacuum in the condenser, but the atmospheric line is always given by the indicator.

AMSLER'S PLANIMETER.

Area and Mean Pressure of Diagrams.

The best method of finding the area and mean pressure of indicator cards is by using the Amsler Planimeter, which instrument not only gives the area correctly, but also the mean ordinate in inches, from which the mean pressure is calculated by the scale of the indicator-spring.

Divide the area of the diagram in square inches by its length in inches; the result is the mean ordinate.

Remarks on Card, Fig. 2.

An indicator card as represented by Fig. 2 is considered to be a good card. The inclination of the part a, b shows that the steam has been slightly throttled or wire-drawn, or that the steam-port was not large enough for the speed of the engine. The steam appears to have been cut off at b, from which the nearly regular hyperbola extends to c, where the exhaust opens and rounds off the diagram at the end of the stroke. On the return stroke the diagram runs nearly parallel with the vacuum line $v\ d$, and rounds off the corner at C, which is called *compression.* The compression is caused by the exhaust being closed before the piston reaches the end of the stroke, and also by lead of the main valve.

Diagram Fig. 3.

The diagram Fig. 3 is taken from a high-pressure engine and from both ends of the cylinder, and is considered a bad diagram. The rise *c* of the exhaust is caused by the exhaust-opening being too small, or when two cylinders are connected at right angles the exhaust of one cylinder opens when the piston of the other cylinder is at half stroke and has its greatest velocity, which may interfere with the free exhaust, and thus raise the back-pressure. The vacuum line is not shown in the high-pressure diagram, and the horse-power cannot be calculated without knowing the pressure or scale of the indicator-spring.

Diagram Fig. 4.

Fig. 4 represents a diagram taken from a compound engine, of which the upper part *a, b, c, f, g, a* is from the high-pressure cylinder, and *e, h, V, i, e* from the low-pressure cylinder. After the steam has worked through the high-pressure cylinder, it is expanded into the low-pressure cylinder.

When the high- and low-pressure cylinders are connected at right angles, there must be a steam space between the cylinders for the high-pressure exhaust, and it is generally so arranged that the steam entering the low-pressure cylinder is cut off at the moment the exhaust of the high steam is opened, for otherwise there will be a loss of power. The steam is generally expanded some in the high-pressure cylinder before it enters the other.

When the high- and low-pressure cylinders are connected to opposite cranks, then the steam can expand directly from one to the other without more space between the cylinders than that of the conducting ports.

Compound engines are generally so proportioned that the power of the high-pressure cylinder is nearly equal to that of the low-pressure cylinder, and on account of the high grade of expansion of steam used in them, and the reduction of initial condensation, they sometimes prove very economical.

Nominal Horse-Power of Compound Engines.

The English have adopted a rule for designating the size of compound engines in nominal horse-power, as follows:

D = diameter of low-pressure cylinder in inches.
d = diameter of high-pressure cylinder.
S = stroke of piston in inches.

$$\text{Nominal } HP = \frac{D^2 + d^2}{100} \sqrt[3]{S}.$$

The diameter of the low-pressure cylinder is made about double that of the high-pressure cylinder.

Diagram Fig. 5.

The diagram Fig. 5 is taken from a locomotive engine, and is a fair average of the steam-engineering existing in our day's locomotive practice. About fifty per cent. of the power and fuel is uselessly wasted in the locomotive.

The defect of the valve-gear is clearly illustrated by the diagram Fig. 5; namely, that there is no sharp cut-off, the steam is wire-drawn, the exhaust is choked and compressed by cutting off the steam with the main valve and link-motion. There is plenty of ingenuity among us to devise a locomotive valve-gear that would distribute the steam as correctly as required, but how to make such a gear simple and substantial is a problem not easily solved.

To compound the two cylinders of a non-condensing locomotive, when properly done, would result in a great economy of steam. There is no practical difficulty in attaining far better results than have as yet been reached. —W. D. M.

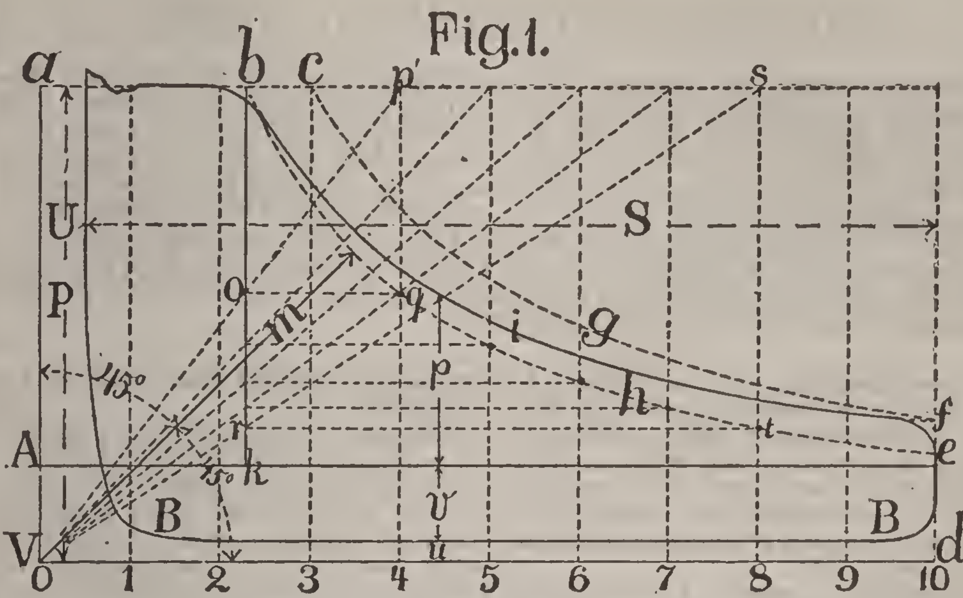


Fig.1.
a
b
c
p'
s
U
S
P
o
q
m
i
g
p
h
45°
f
r
t
A
e
45°
k
v
B
B
V
u
d
0
1
2
3
4
5
6
7
8
9
10

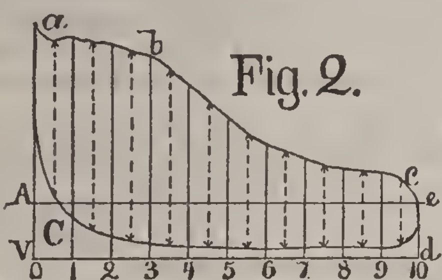


Fig. 2.
a
b
c
A
e
C
V
d
0
1
2
3
4
5
6
7
8
9
10

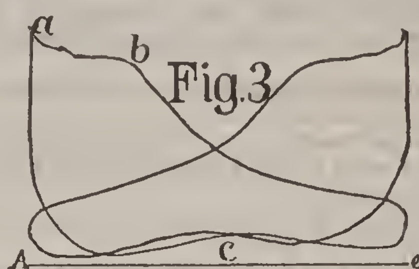


Fig.3
a
b
A
C
e

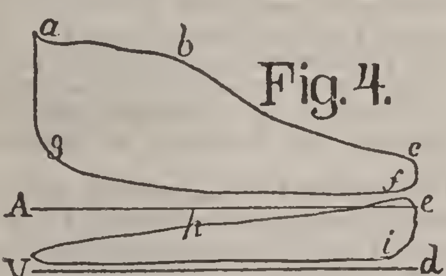


Fig.4.
a
b
g
c
f
A
e
h
i
V
d

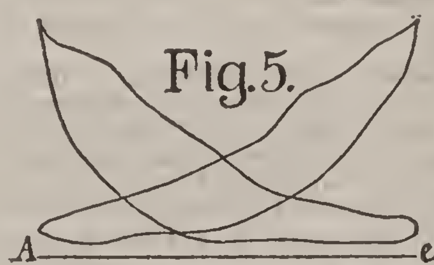


Fig.5.
A
e

SCIENTIFIC AND TECHNICAL TERMS.

CURIOUS names are often given to principles supposed to be newly discovered, but which in reality are old and have proper technical names. One vagary of scientists has been the giving individual names to physical conceptions which have technical names when those principles are properly understood. This annoyance exists more in the electrical profession than in any other branch of science, the result of which is sometimes ridiculous.

The Electrical Congress, which met in Paris in 1881, occupied much time in discussing which individual names should be adopted for electrical conceptions. Many superficial talkers and writers have much bothered earnest students by this form of solemn nonsense.

Acceleration, the increment of velocity per second of a moving body. The acceleration of gravity is generally denoted by the letter g and amounts to about 32.17 feet, or 9.81 metres, per second, the velocity attained at the end of one second for a body free to fall.

Adiabatic curve, a curve representing volume and pressure of a gas or vapor without transmission of heat.

Ampere, the unit of measurement of electrical intensity.

Amplitude, the deviation from east or west toward north or south in the horizon.

Azimuth, the deviation from the meridian east or west.

Battery of steam-boilers is applied to a number of boilers working together.

Binnacle, a case in which a mariner's compass is set on board a vessel.

Binary means doubling or halving. A binary system of numbers is that whose base can be divided by two repeatedly without leaving fractions. The metric or decimal system is not binary, because the base 10 can be divided only once by 2 without fractions.

Colomb, unit of quantity of electricity which is equivalent to work, and can be expressed in foot-pounds or kilogrammeters.

Dynamic quantity, some quantity containing both force and motion.

Dynamic effect, used for expressing either power or work.

Dyne, a unit of electromotive force established by the British Association for the Advancement of Science.

Electro-dynamics, the science of electricity producing power and work.

Electrolysis, the science of analyzing substances by electricity.

Electrolyte, any substance that can be decomposed and analyzed by electricity.

Energy means power, but is most frequently and erroneously applied to work, and even to force. Scientific writers have a great many kinds of energies, distinguished as *potential*, *actual*, *equality of*, *intrinsic*, *mechanical*, *kinetic*, etc., which appellations have no definite meaning.

Erg, a unit of electric work equivalent to one dyne lifted one centimetre; established by the British Association.

Farad, the unit of electric capacity, which capacity is equal to one colomb divided by one volt.

Field, magnetic field, is the space between the poles of the magnets in a dynamo or electric motor.

Field magnets, the stationary magnets in a dynamo or electric motor.

Galvanic current, a current of electricity direct from a battery.

Galvanometer, a magnetic needle acted upon by an electric current for measuring the strength and determining the direction of electric flow.

Isothermal line, a curve representing volume and pressure of a gas or vapor whilst the temperature remains constant.

Kinetic. In mechanics kinetic means motion or the science of cause of motion. It is superfluous in mechanics.

Mechanical effect means power or work.

Micro-farad, a unit of electric capacity, or a one-millionth part of a *farad*.

Moment of activity means simply power.

Moment of inertia means the moment of the momentum.

Momentum, Mv, is the intensity of that constant force which will in one second give to a body at rest, of mass M, the final velocity v.

Ohm, the unit of resistance of a conductor to the flow of electricity. Resistance is the loss of electro-motive force per ampère. The latest unit ohm established by the Electrical Congress is the resistance of a column of mercury 1.06 metres long by 1 square millimetre section.

Origin is the point where the rectangular co-ordinates of a curve meet, and from which the ordinates and abscissas are measured.

Parameter is the ordinate which passes through the focus of a curve.

Potential. In electricity, *potential* means the available electro-motive force, including the combined action of both positive and negative electricity, and called *electrical potential.*

Quantity of action means simply power.

Quantity of moving force means motive force.

Quantity of motion is often used for momentum.

Rate of work means simply power.

Rheostat, an instrument containing a number of resistance coils for comparing electrical resistances of conductors. Each coil has a known resistance, marked in ohms. The rheostat is analogous to the friction dynamometer in mechanics.

Vis-viva means living force said to be stored in a moving body. An old term meaning twice the work stored in a moving body.

Volt, unit of electro-motive force corresponding nearly with a force or weight of one milligramme.

Watt, unit of electric power established by the British Association; it is the rate of working one *erg* per second. There are 746 *watts* per horse-power.

Work produced or consumed by combustion of one ounce avoirdupois or one gramme of coal, gunpowder, zinc, copper, and hydrogen:

One ounce	coal	=	695,000	foot-pounds.
" "	gunpowder	=	100,000	" "
" "	zinc	=	113,000	" "
" "	copper	=	69,000	" "
" "	hydrogen	=	2,925,000	" "
One gramme	coal	=	3,390	kilogrammeter.
" "	gunpowder	=	487	" "
" "	zinc	=	550	" "
" "	copper	=	336	" "
" "	hydrogen	=	14,225	" "

Approximate Horse-Power

of small high-pressure engines. $H = 0.1 D^2 \sqrt[3]{S}$. *Steam pressure not less than* **80** *pounds to the square inch.*

Diam. D Inches.	Stroke S of piston in inches. 3	4	5	6	7	8	9	10	12	14	15	16	18
2	.577	.634	.684	.727	.765	.800	.832	.862	.915	.964	.985	1.00	1.05
2½	.900	.990	1.07	1.13	1.20	1.25	1.30	1.34	1.42	1.50	1.54	1.57	1.63
3	1.30	1.43	1.54	1.64	1.72	1.80	1.87	1.94	2.06	2.17	2.22	2.27	2.36
3½	1.77	1.94	2.10	2.22	2.34	2.45	2.55	2.64	2.80	2.95	3.00	3.09	3.21
4	2.31	2.54	2.74	2.90	3.06	3.20	3.33	3.44	3.66	3.85	3.94	4.05	4.19
4½	2.92	3.21	3.47	3.68	3.87	4.05	4.42	4.36	4.64	4.88	5.00	5.10	5.30
5	3.60	3.96	4.27	4.54	4.78	5.00	5.20	5.38	5.72	6.02	6.16	6.30	6.55
6	5.19	5.70	6.15	6.53	6.89	7.20	7.55	7.82	8.31	8.75	8.95	9.15	9.50
7	7.08	7.78	8.40	8.92	9.40	9.80	10.2	10.6	11.2	11.8	12.1	12.3	12.9
8	9.25	10.1	11.0	11.6	12.2	12.8	13.3	13.8	14.6	15.4	15.7	16.1	16.8
9	11.7	12.9	13.9	14.7	15.5	16.2	16.8	17.4	18.5	19.5	20.0	20.4	21.2
10	14.4	15.9	17.1	18.2	19.1	20.0	20.8	21.5	22.9	24.1	24.6	25.2	26.2
11	17.5	19.2	20.8	22.0	23.2	24.2	25.2	26.1	27.7	29.2	29.9	30.5	31.6
12	20.8	22.9	24.7	26.2	27.6	28.8	30.0	31.0	33.0	34.8	35.5	36.3	37.8

The horse-power of small engines, as counted by the English, is only 0.4 of that in this table for the same size cylinders.

To Approximate the Size of Steam-Engines.

Example 1. It is required to build a river steamer of displacement $T = 1000$ tons to run $M = 16$ nautical miles per hour. Required, the size of the cylinder for an ordinary overbeam engine? From the table of steamship performance will be found the required actual power $H = 1798$ *IP*.

From the table of Nominal horse-power select the approximate size of cylinder, which may be $D = 88$ inches, diameter of cylinder by $S = 14$ feet stroke, which answers to $H = 1866$ horses nominal. In this case the nominal horse-power can be considered the same as the actual.

Example 2. A propeller steamer is to run $M = 10$ nautical miles per hour, with a displacement $T = 3400$ tons. Required, the size of the cylinders?

From table of steamship performance $H = 992$ horses, to be divided into two cylinders of 406 each. Select from table of Nominal horse-power $D = 60$ inches diameter of cylinders and $S = 2' \, 19''$ stroke of piston, which answers to $H = 504$, or $504 \times 2 = 1008$ horses of the two cylinders. After these approximations are made, make a careful calculation from the original formulas.

Example 3. Suppose the propeller for the steamer in the preceding Example 2 makes $n = 60$ revolutions per minute. Required, the diameter of the propeller-shaft? See Table, page 418, for wrought-iron shafts, for 1000 horses and 60 revolutions, the shaft should be 12.8 inches.

Example 4. A steamer of $T = 2500$ tons is to run $M = 9$ nautical miles per hour with an indicated steam-pressure of 20 lbs., or $P = 35$ lbs. per square inch, expanded ½. Required, the consumption of fuel in tons per 24 hours?

Table of steamship performance $H = 585$ *IP*.

Table V., page 405, consumption of fuel, 3.44 tons.

The required consumption will be $5.85 \times 3.44 = 20.124$ tons per 24 hours' steaming.

[The resistance of vessels or the work to be done is to be calculated and the proper size of engine, with margin for frictional loss, computed from indicated horse-power. W. D. M.]

PROPORTIONATE PRICES OF MACHINERY.

MACHINES or engines made of different sizes, but of uniform proportion, generally vary in prices as the square root of the cube of any linear dimension. Suppose two engines, one of exactly double the linear dimensions of the other; then the proportionate prices will be as $1 : \sqrt{2^3} = 1 : 2.828$.

For steam-engines the volume of the cylinder—that is, the displacement of the steam-piston—is a good representation of the cube of any linear dimension of the engine.

C = volume of the steam-cylinder in cubic inches.
X = a coefficient to be determined by the manufacturer.
\$ = price of the engine in dollars.

$$\$ = X\sqrt{C}.$$

For a double engine it will not answer to add the volumes of the two cylinders, but the price to be calculated for one engine, and then doubled.

For ordinary stationary engines the coefficient X is between 20 and 30; for highly-finished high-speed engines it runs up as high as $X = 50$; and for donkey-pumps as low as $X = 15$.

Example 1. What will be the price of an ordinary stationary engine of $D = 30$ inches diameter of the cylinder, by $S = 48$ inches stroke?

Assume the coefficient to be $X = 25$.

Area of cylinder piston, 706.86 sq. in. Volume, $C = 33929.28$ cub. in.

$$\text{Price, } \$ = 25\sqrt{33929.28} = 4605 \text{ dollars.}$$

Example 2. What will be the price of a similar engine to that in Example 1, but $D = 15$ inches diameter of the cylinder and $S = 24$ inches stroke?

Area of cylinder piston, 176.71 sq. in. Volume, $C = 4241.04$ cub. in.

For a similar engine use the same coefficient, $X = 25$.

$$\text{Price, } \$ = 25\sqrt{4241.04} = 1628.125 \text{ dollars.}$$

Thus a regular scale of prices can be made for different-sized engines.

Example 3. What will be the price of a donkey-pump of $D = 6$ inches diameter of cylinder and $S = 9$ inches stroke of piston?

Assume the coefficient $X = 16$.

$$\text{Volume of cylinder, } 28.27 \times 9 = 254.43 \text{ cub. inches.}$$

$$\text{Price, } \$ = 16\sqrt{254.43} = 255.20 \text{ dollars.}$$

Example 4. What will be the price of a two-cylinder donkey-pump connected so that they work one another valve motion, when $D = 9$ inches diameter of cylinders and $S = 15$ inches stroke of pistons?

$$\text{Volume of one cylinder, } 63.617 \times 15 = 954.255 \text{ cub. inches.}$$

$$\text{Price for one, } \$ = 16\sqrt{954.255} = 494.24 \text{ dollars.}$$

$$\text{Price for the double engine, } 494.24 \times 2 = 998.48 \text{ dollars.}$$

For compound engines calculate the price for each cylinder separately; then add the two prices, and the sum is the price of the compound engine.

When the price of one engine is determined, the coefficient X will be

$$X = \frac{\$}{\sqrt{C}}.$$

Then fix the prices of other sizes, but similar engines, by the same coefficient.

Example 5. An engine of $D = 12$ inches diameter of cylinder by $S = 24$ inches stroke costs \$ = 1500 dollars. Required the coefficient X?

The volume of the cylinder is $C = 2714.16$ cub. inches.

$$\text{Coefficient } X = \frac{1500}{\sqrt{2714.16}} = 28.8; \text{ say } 29.$$

The weight of engines of different sizes should be as the cube of any linear dimension, but the smaller engines are generally made heavier in proportion to the larger ones; so that in ordinary practice the weight varies nearly as the price.

W = weight of the engine in pounds.

$$W = 2.5\,X\sqrt{C}.$$

SLIDE VALVES.

The slide valve motion is one of the most important features in the steam engine. Plate IV. is a drawing of the Gonzenbach valve motion as used in Europe. (See Zeuner's *Treatise on Valve Gears.*)

Main Valve.

It will be best to assume a certain size cylinder, and at the same time give the proportions for any size.

$D = 34$ inches, diameter of the cylinder.
$S = 18$ inches, stroke of piston.*
$n = 56$ double strokes per minute.

We have the area of the steamports m, from Formula 26, page 542.

$$\mathbf{a} = \frac{34^2 \times 0{\cdot}785 \times 18 \times 56}{35000} = 26 \text{ square inches, nearly.}$$

$$m = \frac{D + S}{26} = \frac{34 + 18}{26} = 2 \text{ inches,}$$

the width of the steamport; if the quotient gives a fraction take the nearest quarter or eighth.

$$\frac{\mathbf{a}}{m} = \frac{26}{2} = 13 \text{ inches, breadth of steamport.}$$

$r = \frac{1}{2} m$ about $= 1$ inch, the exhaust port $o = 2m - \frac{1}{2}r = 3\frac{1}{2}$ inches, and $f = o + 2r = 5\frac{1}{2}$ inches, $h = f - \frac{1}{4}r = 5\frac{1}{4}$ inches, $k = 1\frac{1}{2}m = 3$ inches, and $i = h + 2k = 11\frac{1}{4}$ inches, $e = m = 2$ inches.

* The stroke and diameter is here rather out of proportion, but we will maintain them in the calculations as they suit the drawing, which is purposely made to show the *slide valves* on a large scale. The rules will however suit any proportions of diameter and stroke.

To Find the Stroke of the Eccentric.

$s =$ stroke of the eccentric in inches.
$s = i - f - \frac{1}{4}r = 5\frac{1}{2}$ inches.

The *lap* $L = \frac{1}{2}(i - f - 2m) = \frac{7}{8}$ inches.

The *lead* of the valve, or opening of the steamport when the crank pin stands on the centre should be about

$$l = \frac{m\sqrt{n}}{80} = \frac{2\sqrt{56}}{80} = \frac{1}{4} \text{ inches, nearly.}$$

Having finished the main valve and ascertained the stroke of the eccentric, it is now required to find the position of the centre b, (Plate V.,) of the eccentric, to the crank-pin. Suppose the crank pin of the engine stands also on the centre nearest to the cylinder, and the eccentric rods are attached direct to the valve rods; draw the line dd, at right-angle to the centre-line aa'' of the engine, then

$$\text{the angle, sin.} W = \frac{2(L + l)}{s} = \frac{2(\frac{7}{8} + \frac{1}{4})}{5\frac{1}{2}} = 0{\cdot}409, \text{ or } W = 24^\circ\ 10'.$$

See Plates IV. and V.

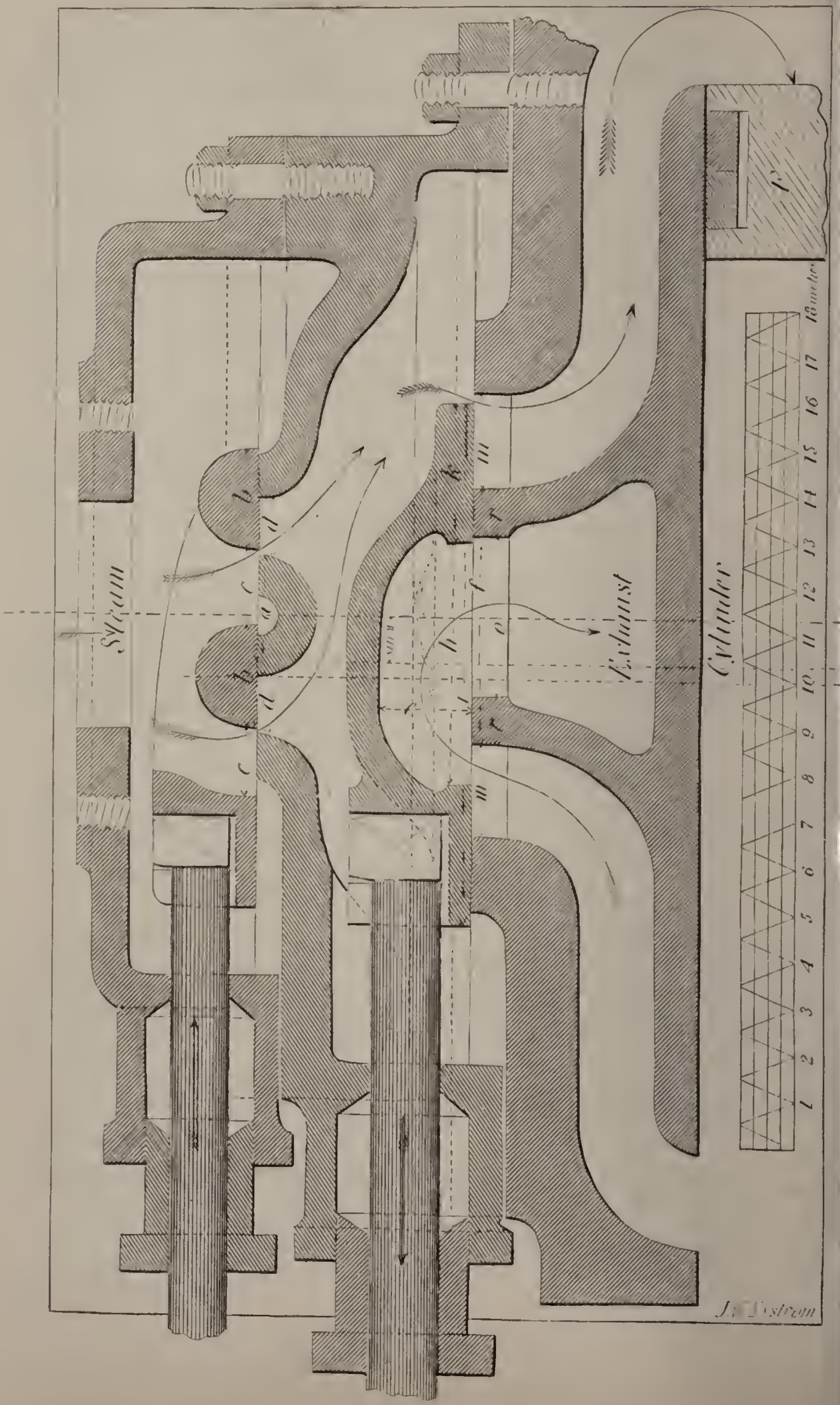
To Find the position of the Crank-Pin at the moment the Main Valve opens.

$$y = \frac{S\,l}{s \cos. W} = \frac{18 \times 0{\cdot}25}{5{\cdot}5 \times 0{\cdot}9123} = 0{\cdot}9 \text{ inches, nearly,}$$

from the centre line.

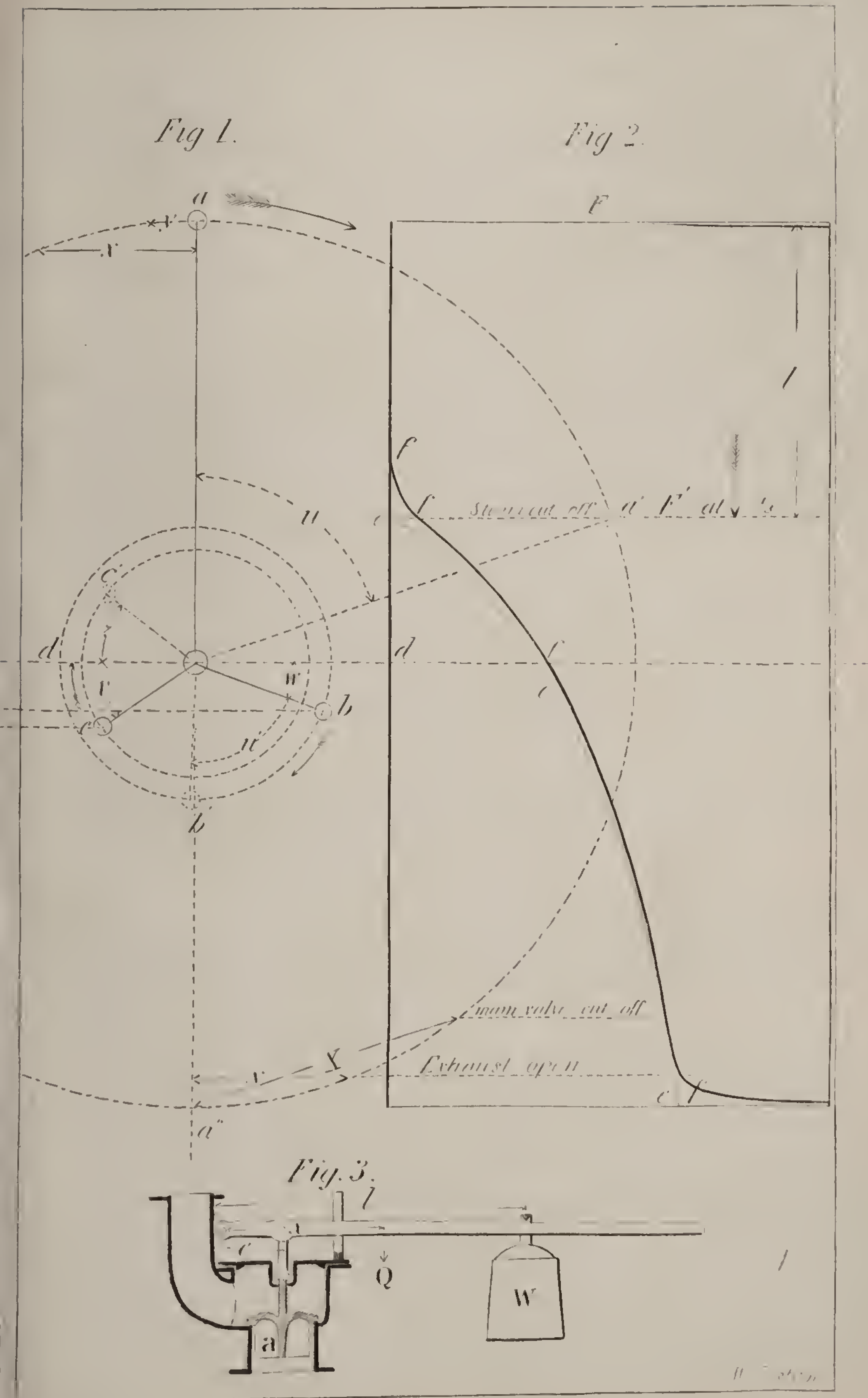
Slide Valves.

Plate II



Eccentrics.

Plate V



To Find the position of the Crank at the moment the Exhaust opens.

$$x = \frac{S}{2}\left(\sin. W - \frac{1}{s}(f-h)\right) = \frac{18}{2}\left(0{\cdot}409 - \frac{1}{5{\cdot}5}(5{\cdot}5 - 5{\cdot}25)\right) = 3{\cdot}27 \text{ inches}$$

from the centre line.

To Find the position of the Crank Pin when the Main Valve cuts off the Steam.

$$X = \frac{2S\,L}{s} = \frac{2 \times 18 \times \frac{7}{8}}{5{\cdot}5} = 5{\cdot}727 \text{ inches.}$$

To Find at what part of the Stroke the Main Valve Cuts off the Steam,

$$\text{Will cut off at} = 1 - \frac{4L^2}{s^2} = 1 - \left(\frac{2\times\frac{7}{8}}{5{\cdot}5}\right)^2 = 0{\cdot}899 \text{ of the stroke.}$$

The greater the lap is, the sooner will the main-valve cut off, but if the lap is increased the stroke of the eccentric must also be equally increased. It does not work well to cut off much by the main-valve, especially when the engine works fast; for very slow motion it may answer to cut off at $\frac{3}{4}$ the stroke.

It will be noticed that the centre of the eccentric is always ahead of the crank pin with an angle $90°+w$. Hence when the engine is to be reversed, the centre b must have the same position on the opposite side of the centre-line, or the eccentric must be moved forwards an angle of $90° - 2w$.

Cut-off Valve.

The width of the cut off ports should be about $d = \frac{5}{8}m = 1\frac{1}{4}$ inch, and their

$$\text{breadth } \frac{a}{2d} = \frac{30}{2\times 1\frac{1}{4}} = 12 \text{ inches, when two ports are used.}$$

Proportions of the Valve.

$a - b = c - d$, $a+d = b+c$, and $a = 2d$, and the stroke of the cut-off valve eccentric $s = 2b$, we shall have $a = 2\frac{1}{8}$, $b = 2\frac{1}{4}$, $c = 1\frac{1}{2}$, $c = 1\frac{1}{8}$, and $s = 4\frac{1}{2}$ inches.

Let us assume the steam to be cut off at $\frac{1}{3} = l$ of the stroke S, the position of the crank-pin a' will then be $\sin. u = 2l = 0{\cdot}666$, or $u = 70°\ 30'$; at the same time the position of the centre c' of the cut off eccentric will be

$$\sin. z = \frac{d+c}{s} = \frac{1\frac{1}{4}+1\frac{1}{8}}{4\frac{1}{2}} = 0{\cdot}612, \text{ or } z = 37°\ 50',$$

and $V = u - z = 70°\ 30' - 37°\ 50' = 32°\ 40'$, the position of the centre c when the crank-pin a is on the centre. This Table will show the positions of the centre a and c, at different cut offs. Letters correspond with Figure 1, Plate VI.

Cut off at l.	v	$\sin. v$	*stroke of eccen. s.*	z	u	*F.*	*p.*
$\frac{1}{4}$	22° 10′	0·377	$2b$	37° 50′	60°	0·5880	0·250
$\frac{1}{3}$	32° 40′	0·539	$2b$	37° 50′	70° 30′	0·6914	0·333
$\frac{3}{8}$	31° 55′	0·527	$c+a$	43° 35′	75° 30′	0·7332	0·375
$\frac{1}{2}$	42° 35′	0·675	$b+c$	47° 25′	90°	0 8350	0·500
$\frac{5}{8}$	46° 30′	0·7193	$a+b-c$	58°	104° 30′	0·910	0·625
$\frac{2}{3}$	50° 30′	0·7933	$a+b-c$	58° 30′	109° 30′	0·985	0·666

It will now be observed that the effectual pressure F in this Table is less than in the Table on page 403, owing to the valve not cutting off the steam instantly, but gradually, so that the density of the steam in the cylinder is already diminished at the *cut off* point. The valve will cut off quicker the less the angle z is.

See Figure 2, Plate VIII. The actual pressure will not form a sharp corner at e, or follow the line e,e,e, as would be due when cut off at $\frac{1}{3}$ the stroke, but the line $ff'ff$ will be the true diagram. Including the steam in the ports and steamchest, the density at the end of the stroke will correspond nearly with the Table.

STEAM-BOILERS.

The accompanying proportions are averages of a great number of good marine boilers.

Letters Denote.

D = diameter of the steam-cylinder in inches.
S = stroke of piston under which steam is fully admitted in inches.
n = number of double strokes or revolutions per minute.
w = pounds of water evaporated per pound of coal per hour.
V = volume coefficient from the steam table.
$\boxminus$ = fire grate in square feet for each cylinder and with natural draft.

To Find the Area of Fire Grate.

$$\boxminus = \frac{D^2 S n}{4.66\, w V}. \qquad n = \frac{4.66\, w V \boxminus}{D^2 S}. \qquad . \quad . \quad . \; 1, 2.$$

Example 1. A steam-engine of $D = 54$ inches diameter of the cylinder, and stroke of piston 96 inches, cut off at $\frac{1}{2}$ $S = 48$ inches, is to make 22 revolutions per minute. Anthracite coal to be used, that evaporates $w = 7$ pounds of water per pound of coal, and to carry 27 pounds of steam per square inch, $V = 649$. Required the area of fire grate $\boxminus$ = ? in square feet.

$$\boxminus = \frac{54^2 \times 48 \times 22}{4.66 \times 7 \times 649} = 145.34 \text{ square feet.}$$

Example 2. A steam-boiler of $\boxminus = 128$ square feet is to be used for an engine of $D = 36$ inches diameter and 64 inches stroke, cut off the steam at $\frac{2}{3}$, then $S = 42.66$ inches. Steam-pressure to be kept at 25 pounds per square inch $V = 679$, $w = 6.5$. Required for how many revolutions per minute can the steam be kept at 25 pounds?

$$n = \frac{4.66 \times 6.5 \times 679 \times 128}{36^2 \times 42.66} = 47.6 \text{ revolutions.}$$

Horse-Power of the Fire Grate.

IP = horse-power of the fire grate.
P = pressure in the boiler in pounds per square inch excluding the atmosphere.
p = vacuum in the condenser in pounds per square inch.

$$\boxminus = \frac{IP\, x}{V w (P + 0.8\, p)}, \qquad IP = \frac{\boxminus V w (P + 0.8\, p)}{x}. \; . \quad . \; 3, 4.$$

Cut off the steam at				
$\frac{1}{4}$ the stroke,	$x = 27700$,	saves	55	per cent. of fuel.
$\frac{1}{3}$ "	$x = 31400$,	"	49	
$\frac{1}{2}$ "	$x = 38400$,	"	38	
$\frac{2}{3}$ "	$x = 45500$,	"	26	
$\frac{3}{4}$ "	$x = 49100$,	"	20	

Steam admitted throughout the stroke $x = 61700$, saves 0 per cent.

Example 3. Steam-boilers are to be constructed for an engine of 650 horses, the steam to be cut off at $\frac{1}{2}$ the stroke; and $P = 36$ pounds per square inch, $V = 544$, $w = 7.5$ pounds of water evaporated per pound of coal. Required the fire grate in the boilers $\boxminus$ = ? in square feet.

$$\boxminus = \frac{650 \times 38400}{554 \times 7.5(36 + 0.8 \times 11)} = 134 \text{ square feet.}$$

Example 4. Required the horse-power of a fire grate $\boxminus = 112$ square feet to carry 18 pounds steam, and cut off at $\frac{2}{3}$ the stroke? $V = 810$, $w = 7$ pounds.

$$IP = \frac{112 \times 18 \times 810 \times 7}{45500} = 251.2 \text{ horses.}$$

Consumption of Coal.

$C =$ coal consumed in pounds per hour.

$$C = \frac{3\ D^2\ S\, n}{w\ V}, \qquad C = \frac{14\ IP\ x}{V\, w(P + 0.8\, p)}. \quad . \quad . \quad . \ 5, 6.$$

Example 5. A steam-engine of $D = 42$ inches diameter and 48 inches stroke, cut off the steam at $\frac{1}{3}$, $S = 16$ inches is to make $n = 65$ revolutions per minute with a pressure of 34 pounds per square inch, $V = 564$ and $w = 6$ pounds. Required the consumption of coal in pounds per hour $C = ?$

$$C = \frac{3 \times 42^2 \times 16 \times 65}{6 \times 564} = 1625 \text{ pounds per hour.}$$

Example 6. A pair of steam-engines of $IP = 260$ horses are to be worked with $P = 28$ pounds per square inch, cut off at $\frac{1}{4}$ the stroke, $V = 635$, the coal to evaporate $w = 6.5$ pounds of water per pound of coal. Required the consumption of coal in pounds per hour $C = ?$

$$C = \frac{14 \times 260 \times 31400}{630 \times 6.5\ (28 + 0.8 \times 10)} = 775 \text{ pounds per hour.}$$

It will be observed in the Formulas 4 and 6, that the higher steam used the less fuel and fire-grate is required for the same power; the proportion of fuel will be nearly as the square root of the steam-pressure, and still more fuel is saved by cutting off the steam at an early part of the stroke.

Heating Surface $\bigcirc$ Compared with Grate.

In common stationary boilers, ... $\bigcirc = 20\ \boxminus$.
Returning flue boilers, ... $\bigcirc = 25\ \boxminus$.
Tubular boilers (marine), ... $\bigcirc = 30\ \boxminus$.
With vertical tubes (Martin), ... $\bigcirc = 35\ \boxminus$.

Cross-Area of Flues (Calorimeter).

In the common single returning flue boilers the cross-section area of the first row should be, ... 0.18 $\boxminus$.
Returning row, flues or tubes, ... 0.13 $\boxminus$.
Cross-section area of chimney at the top, $A =$ . . . 0.16 $\boxminus$.

Height of Chimney.

$$h = \frac{C^2}{4\ \boxminus^2} - 2, \qquad h = \frac{IP^2}{2.1\ A}, \qquad C = 2\ \boxminus \sqrt{h + 2},$$

$$IP = 1.45\ A\sqrt{h}, \qquad A = \frac{IP}{1.45\ \sqrt{h}}, \qquad \boxminus = \frac{C}{2\sqrt{h + 2}}.$$

Example. Area of fire-grate $\boxminus = 140$ square feet to consume $C = 2100$ pounds of coal per hour. Required the height h of the chimney?

$$h = \frac{2100^2}{4 \times 140^2} = 56.3 \text{ feet, the answer.}$$

HORSE-POWER OF STEAM-BOILERS BY EVAPORATION.

HP = horse-power of evaporation.

P = steam-pressure in pounds per square inch above vacuum.

W = cubic feet of feed-water evaporated per hour from 32° F.

V = steam volume compared with that of water at 32°.

$$HP = W \frac{144\,P\,(V-1)}{1980000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1.$$

This formula gives the natural effect of the evaporation without expanding the steam.

$$\text{With expansion} \quad HP = \frac{W\,P\,(V-1)(1 + hyp.log.\ X)}{13748.4}.$$

X = grade of expansion.

$$W = \frac{13748.4\ HP}{P\,(V-1)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 2.$$

Natural Effect of Evaporation without Expanding the Steam.

Steam-pressure above vacuum.	Water evaporated per hour per horse-power. Cubic feet.	Cubic in.	Pounds.	Horse-power per cubic foot.	Equivalent work per unit of heat.
P	W	w	*lbs.*	HP	J
5	0.6024	1041.0	29.852	1.6600	46.584
10	0.5796	1002.0	28.723	1.7253	48.032
14.7	0.5701	985.2	28.252	1.7540	48.583
20	0.5641	974.7	27.954	1.7727	48.902
25	0.5593	966.5	27.717	1.7879	49.040
30	0.5553	959.6	27.518	1.8008	49.403
35	0.5516	953.2	27.337	1.8130	49.665
40	0.5483	947.4	27.170	1.8238	49.832
45	0.5451	941.9	27.012	1.8345	50.150
50	0.5420	936.6	26.861	1.8540	50.244
55	0.5391	931.5	26.715	1.8549	50.440
60	0.5362	926.6	26.573	1.8649	50.651
65	0.5334	921.6	26.429	1.8747	50.861
70	0.5305	917.1	26.300	1.8850	51.060
75	0.5280	912.5	26.168	1.8936	51.265
80	0.5254	907.9	26.038	1.9033	51.470
85	0.5228	903.5	25.910	1.9127	51.670
90	0.5203	899.1	25.783	1.9219	51.865
95	0.5178	894.7	25.660	1.9312	52.077
100	0.5153	890.5	25.537	1.9406	52.264
105	0.5129	886.2	25.415	1.9497	52.513
110	0.5104	882.0	25.295	1.9592	52.722
115	0.5081	877.9	25.177	1.9681	53.053
120	0.5057	873.8	25.060	1.9774	53.137
125	0.5034	869.8	24.945	1.9865	53.351
130	0.5008	865.3	24.815	1.9968	53.572
135	0.4988	861.9	24.718	2.0048	53.788
140	0.4965	858.0	24.606	2.0140	54.000
145	0.4943	854.1	24.494	2.0230	54.206
150	0.4921	850.4	24.387	2.0321	54.427

Legal Horse-Power of Steam-Boilers.

The legal horse-power of a steam-boiler fired with a given kind or quality of fuel should be the power passing from the boiler into the steamer-pipe with pressure above that of the atmosphere, because the boilermaker is not responsible for how the steam-user employs that steam. The only difference in the formulas for horse-power and evaporation will then be in taking the steam-pressure above that of the atmosphere.

$$HP = \frac{W P (V - 1)}{13748.4}. \qquad W = \frac{13748.4\ HP}{P (V - 1)}.$$

The last column (J) in the tables gives the equivalent work in foot-pounds per unit of heat as realized in steam without expansion.

For further information on this subject, see Nystrom's *Steam Engineering.*

Reduction for Temperature of Feed-Water.

Temp. t.	Reduction R.	Logarithm.	Temp. t.	Reduction R.	Logarithm.
40	0.9932	9.9970367	130	0.9105	9.9592620
50	0.9851	9.9934803	140	0.9000	9.9546693
60	0.9761	9.9895039	150	0.8912	9.9499637
70	0.9671	9.9854546	160	0.8815	9.9451979
80	0.9577	9.9812455	170	0.8719	9.9404765
90	0.9486	9.9770612	180	0.8625	9.9357359
100	0.9392	9.9727643	190	0.8529	9.9308916
110	0.9296	9.9683116	200	0.8432	9.9259440
120	0.9199	9.9637468	212	0.8317	9.9199515

Legal Horse-Power of Steam-Boilers per Rate of Evaporation of Water to Steam without Expansion.

Steam-pressure above atmosphere.	Water evaporated per hour per horse-power. Cubic feet.	Cubic in.	Pounds.	Horse-power per cubic foot.	Work. ft.-lbs. per unit of heat.
p	W	w	*lbs.*	HP	J
5	2.2562	3898.8	140.76	0.4433	12.225
10	1.3983	2416.2	87.235	0.7150	19.616
15	1.1106	1919.0	69.284	0.9005	24.701
20	0.9654	1668.1	60.226	1.0358	28.380
25	0.9770	1515.4	54.711	1.1403	31.145
30	0.8176	1411.9	51.010	1.2231	33.433
35	0.7743	1338.1	48.308	1.2914	35.171
40	0.7412	1280.9	46.244	1.3490	36.683
45	0.7150	1235.5	44.605	1.3986	37.988
50	0.6935	1198.3	43.264	1.4420	39.124
55	0.6755	1167.2	42.140	1.4804	40.118
60	0.6600	1140.6	41.180	1.5150	41.012
65	0.6467	1117.5	40.345	1.5463	41.819
70	0.6349	1097.1	39.607	1.5750	42.551
75	0.6243	1078.9	38.951	1.6016	43.221
80	0.6149	1062.5	38.360	1.6263	43.854
85	0.6062	1046.6	37.822	1.6495	44.425
90	0.5983	1033.9	37.328	1.6713	45.011
95	0.5910	1021.3	36.873	1.6919	45.533
100	0.5847	1009.6	36.451	1.7115	46.027
105	0.5779	998.67	36.056	1.7303	46.495
110	0.5720	988.44	35.686	1.7482	46.949
115	0.5664	978.75	35.337	1.7655	47.390
120	0.5611	969.62	35.007	1.7822	47.812
125	0.5561	960.96	34.964	1.7982	48.213
130	0.5513	952.65	34.394	1.7083	48.604
135	0.5468	945.00	34.111	1.8288	49.737

The actual quantity of feed-water of temperature t°, multiplied by the reduction in the table, gives the quantity of water that would have been evaporated when heated from temperature 32° F.

Example 11. A steam-boiler evaporates $W = 125$ cubic feet of water per hour under a pressure of $P = 75$ pounds to the square inch above vacuum, or 60 pounds above the atmosphere, the temperature of the feed-water being $t^\circ = 110^\circ$. Required the natural effect or horse-power of the evaporation?

Formula 11. $$IP = \frac{125 \times 75(348.15 - 1)}{13748.4} = 236.73 \text{ horses.}$$

That is, 0.528 cubic feet of water evaporated per hour per horse-power, or 1.893 horse-power per cubic foot of water evaporated per hour.

Making correction for the temperature of the feed-water 110° (see table), the horse-power will be $236.73 \times .9392 = 222.4$ horse-power, the natural effect of the evaporation.

Example 12. What quantity of water of temperature $t^\circ = 90^\circ$ must be evaporated under a pressure of $P = 90$ pounds to the square inch in order to generate a natural effect of $IP = 150$ horse-power?

Formula 12. $$W = \frac{13748.4 \times 150}{90(294.61 - 1)} = 78.043 \text{ cubic feet.}$$

This volume corrected for temperature gives $78.043 : 0.9486 = 82.275$ cubic feet, the quantity of water required.

Standard Horse-Power of Steam-Boilers.

The power of a steam-boiler ought to be graded by the dimensions of the areas of the fire grate and heating surface, like that of a steam-engine is graded by the diameter and stroke of the steam-piston, without taking into consideration the evaporative power of the fuel, expansion of the steam, etc., which are independent of the size of the boiler, as well as that of the engine.

Let $\equiv$ denote the area of the fire grate.

$\bigcirc$ = the area of the heating surface in square feet.

P = pressure of steam in pounds per square inch above vacuum.

Then the standard nominal horse-power H of a steam-boiler can be expressed by

$$IP = \sqrt{\frac{\equiv \bigcirc \sqrt{P}}{10}} \quad \ldots \quad 1. \qquad \bigcirc = \frac{10\, IP^2}{\equiv \sqrt{P}} \quad \ldots \quad 3.$$

$$\equiv = \frac{10\, IP^2}{\bigcirc \sqrt{P}} \quad \ldots \quad 2. \qquad P = \left(\frac{10\, IP^2}{\equiv \bigcirc}\right) \quad \ldots \quad 4.$$

Example. Suppose $\equiv = 100$, $\bigcirc = 3000$, and $P = 75$.

Then, $$IP = \sqrt{\frac{100 \times 3000 \sqrt{75}}{10}} = 510,$$ the standard nominal horse-power.

Ordinary Performance of Steam-Boilers.

Natural draft consumes about 12 to 15 pounds of coal per square foot of grate per hour, and generates about 4 to 5 horse-power per square foot of grate.

The heating surface should be about 4 to 5 square feet per horse-power, and evaporate 4 to 5 pounds, or 92.5 to 115.5 cubic inches, of sea water per hour, at the above-mentioned rate of combustion.

Good coal evaporates about 6 to 8 pounds of water per pound of coal.

Each horse-power requires the consumption of about 3 to 4 pounds of coal per hour.

Locomotive boilers with forced draft *dispose* of 80 to 120 pounds of coal per hour per square foot of grate surface, and evaporate less water per pound of coal.

Ultimate Strength of Tubes and Flues

for External Pressure against Collapsing.

Notation.

D = diameter of tube or flue in inches.
L = length of the tube or flue in feet.
t = thickness of iron in decimals of an inch.
P = external collapsing pressure in pounds per square inch.

$$P = \frac{200.000\,t^2}{D\sqrt{L}}, \qquad \text{and} \qquad t = \frac{\sqrt{P D \sqrt{L}}}{447.2}.$$

Example 1. A flue of $D = 15$ inches diameter, and $L = 12$ feet long, thickness of iron $t = 0.25$. Required, the collapsing pressure?

$$P = \frac{200,000 \times 0.25^2}{15 \times \sqrt{12}} = 241 \text{ pounds to the square inch.}$$

Example 2. $D = 9$, $L = 10$ and $t = 0.2$.

$$P = \frac{200,000 \times 0.04}{9\sqrt{10}} = 282 \text{ pounds.}$$

Example 3. $D = 6$, $L = 6$, and $t = 0.2$. Required the pressure P?

$$P = \frac{200,000 \times 0.04}{6 \times \sqrt{6}} = 552 \text{ pounds.}$$

Staying Steam-Boilers.

d = diameter of good iron stay-bolts in inches. $\qquad d = \dfrac{D\sqrt{P}}{74}.$

D = distance apart in inches in salt water on flat surfaces. . . . . . . $\qquad D = \dfrac{74\,d}{\sqrt{P}}.$

P = pressure of steam in pounds per square inch. $\qquad P = \dfrac{5476\,d^2}{D^2}.$

The following table is given by Mr. Fairbairn, as exhibiting the strongest form and best proportions of rivet joints, as deduced from experiments and actual practice:

Thickness of plate.		Diameter of rivet.		Length of rivet from head.		Distance from centre to cent.		Quantity of lap in single riveted.		Quantity of lap in double riveted.	
in.	16ths.	in.	Ratio.	in.	Ratio.	in.	Ratio.	in.	Ratio.	in.	Ratio.
0.19 =	3	0.38	2	0.88	4.5	1.25	6	1.25	6	2.10	10
0.25 =	4	0.50	2	1.13	4.5	1.50	6	1.50	6	2.50	10
0.31 =	5	0.63	2	1.38	4.5	1.63	5	1.88	6	3.15	10
0.38 =	6	0.75	2	1.63	4.5	1.75	5	2.00	5.5	3.33	9.2
0.50 =	8	0.81	1.5	2.25	4.5	2.00	4	2.25	4.5	3.75	7.5
0.63 =	10	0.94	1.5	2.75	4.5	2.50	4	2.75	4.5	4.58	7.5
0.75 =	12	1.13	1.5	3.25	4.5	3.00	4	3.25	4.5	5.42	7.5

STRENGTH OF BOILER-SHELLS.

THE steam-pressure per square inch in the boiler, multiplied by the inside diameter of the shell in inches, is the strain on the plates per inch of length of the shell; and as this strain is borne by two sides of the shell, only one-half of it is borne by each side.

S = ultimate strength in pounds per square inch of section of the plate.
t = thickness of the plate in fractions of an inch.
D = inside diameter of the boiler in inches.
p = steam-pressure in pounds per square inch above that of the atmosphere.

Coefficients X for Safety Strength of Lap-joints.

Construction of Shell.	X	Per cent. of strength.
Solid plate without joints	0.5	100
Double-riveted drilled holes	0.4	80
Double-riveted punched holes	0.35	70
Single-riveted drilled holes	0.3	60
Single-riveted punched holes	0.25	50

Steam-pressure, $p = \frac{X t S}{D}$ . . . . 1.

Diameter of boiler, $D = \frac{X t S}{p}$ . . . . 2.

Thickness of plate, $t = \frac{D p}{X S}$ . . . . 3.

Breaking-strain, $S = \frac{D p}{X t}$ . . . . 4.

The safety strength is taken one-quarter ($\frac{1}{4}$) of the bursting strength.

The static condition of riveted joints is that the sheering strain on the rivet is equal and opposite to the tearing strain on the plate, and the strength to resist these two strains must therefore be alike for the greatest strength of the joint.

It has been found by experiments that the sheering and tearing strengths of wrought iron are nearly alike per section strained, and the slight difference varies either way according to the particular iron experimented upon, but on an average the sheering strength appears to have some advantage over that of tearing.

Assuming these two strengths to be alike, the section of the rivet should be equal to the section of the plate between the rivets.

d = diameter of the rivets.
δ = distance between centres of rivets.
t = thickness of plate.

Areas of sections, $0.7854\, d^2 = t\,(\delta - d)$. $\delta = \frac{d}{t}\,(0.7854\, d + t)$.

The English Board of Trade has adopted a very good and proper rule for determining the strength of riveted joints; namely:

Strength of plate % $= \frac{100\,(\delta - d)}{d}$ per cent. of solid sheet.

Strength of rivet % $= \frac{100\, a\, n}{\delta\, t}$ per cent. of solid sheet.

a = area of rivet (cross-section) in square inches.
n = number of rows of rivets.
p = safety pressure allowed in the boiler or on safety-valve.
S = breaking strength per square inch of the iron.

$$p = \frac{S \times \text{smallest} \% + 2t}{D F}$$

The factor of safety F varies between 5 and 7 under different conditions of the boiler.

For drilled holes make the distance between the centres of the rivets one-eighth ($\frac{1}{8}$) of an inch less than for punched holes.

Proportion of Single-riveted Lap-joints with Punched Holes.

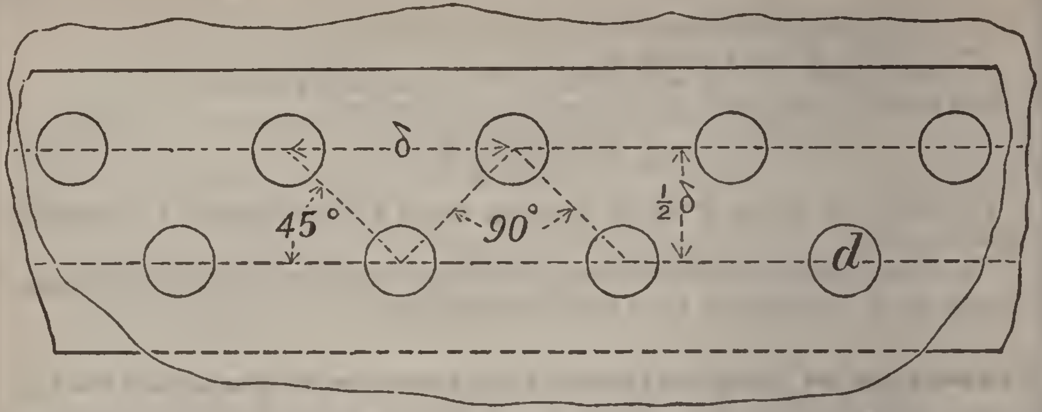
Thickness of plate.	Rivets. Diameter.	Rivets. Length.	Distance between cent.	Lap of joint.	Area of rivet.	Area of plate.	Per cent. of solid
t	d	l	δ	inches.	sq. inch.	sq. inch.	plate.
1/8	5/16	1/2	7/8	1.1/4	0.0767	0.07031	64
3/16	7/16	3/4	1.5/16	1.1/2	0.1503	0.16406	66
1/4	1/2	1.1/8	1.1/2	1.3/4	0.1963	0.25000	66
5/16	5/8	1.3/8	1.7/8	2 in.	0.3067	0.39062	66
3/8	3/4	1.11/16	2.1/4	2.1/4	0.4417	0.56250	66
7/16	13/16	1.15/16	2.3/8	2.3/8	0.5184	0.68359	65
1/2	7/8	2.1/4	2.1/2	2.1/2	0.6013	0.75250	64
9/16	1 in.	2.1/2	2.5/8	2.5/8	0.7854	0.91406	63
5/8	1.1/16	2.13/16	2.3/4	2.7/8	0.8904	1.05468	62
11/16	1.1/8	3.1/8	2.7/8	3.1/8	0.9940	1.03125	61
3/4	1.3/16	3.5/8	3 in.	3.3/8	1.3603	1.35937	60
13/16	1.5/16	3.11/16	3.1/4	3.5/8	1.3605	1.57422	60
7/8	1.3/8	3.15/16	3.1/2	4 in.	1.4840	1.85937	60
15/16	1.1/2	4.1/4	3.3/4	4.1/4	1.767	2.10937	60
1 in.	1.5/8	4.1/2	4 in.	4.5/8	2.073	2.375	60

Double-Riveted Lap-Joints.

Double-riveted joints, if properly proportioned, increase the strength of the boiler about 40 per cent. on account of the rivets being spaced farther apart, leaving more section of plate between them to resist the strain. The rivets are arranged in two rows, zig-zag, over one another, as shown in the accompanying illustration. For the greatest strength the distance between the rivets in the direction of the joint should be double the distance between the centre lines of the two rows, and the rivets will then form a right angle, or 90°, with one another.

The distance between the rivets in the direction of the joint can be made 42 to 50 per cent. greater than between rivets in single-riveted joints.

The diagonal distance between centres of rivet should be made equal to the distance in the direction of the joints in single riveting. (See page 568.)



Double-riveted joints with punched holes, proportioned according to this rule, should be 40 per cent. stronger than single-riveted joints, and with drilled holes about 60 per cent. stronger.

A. Proportions of Double-Riveted Lap-Joints with Drilled Holes.

Thickness of Plate.	Rivets.		Dist. between Rivets.		Distance between Cent. Lines.	Lap of Joint.
	Diameter.	Length.	Central.	Diagonal.		
t	d	l	δ			
1/8	5/16	1/2	1.1/4	7/8	5/8	1.5/8
3/16	7/16	3/4	1.7/8	1.5/16	15/16	2.3/16
1/4	1/2	1.1/8	2.1/8	1.1/2	1.1/16	2.9/16
5/16	5/8	1.3/8	2.5/8	1.7/8	1.5/16	3.1/4
3/8	3/4	1.11/16	3.3/16	2.1/4	1.3/8	3.7/16
7/16	13/16	1.15/16	3.3/8	2.3/8	1.11/16	4 inches.
1/2	7/8	2.1/4	3.9/16	2.1/2	1.13/16	4.1/4
9/16	1 inch.	2.1/2	3.3/4	2.5/8	1.7/8	4.1/2
5/8	1.1/16	2.13/16	3.7/8	2.3/4	1.15/16	4.7/16
11/16	1.1/8	3.1/8	4.1/16	2.7/8	2.1/16	5.1/8
3/4	1.3/16	3.5/8	4.1/4	3 inches.	2.1/8	5.7/16
13/16	1.5/16	3.11/16	4.9/16	3.1/4	2.5/16	5.7/8
7/8	1.3/8	3.15/16	4.15/16	3.1/2	2.1/2	6.7/16
15/16	1.1/2	4.1/4	5.5/16	3.3/4	2.11/16	6.15/16
1 inch.	1.5/8	4.1/2	5.5/8	4 inches.	2.7/8	7.1/2

B. Proportions of Double-Riveted Lap-Joints with Punched Holes.

Thickness of Plate.	Rivets.		Dist. between Rivets.		Distance between Cent. Lines.	Lap of Joint.
	Diameter.	Length.	Central.	Diagonal.		
t	d	l	δ			
1/8	5/16	1/2	1.3/8	1 inch.	11/16	1.7/8
3/16	7/16	3/4	2 inches.	1.7/16	1 inch.	2.1/8
1/4	1/2	1.1/8	2.1/4	1.9/16	1.1/8	2.3/8
5/16	5/8	1.3/8	2.13/16	2 inches.	1.7/16	2.3/4
3/8	3/4	1.11/16	3.3/8	2.3/8	1.11/16	3.3/8
7/16	13/16	1.15/16	3.9/16	2.1/2	1.13/16	3.1/4
1/2	7/8	2.1/4	3.13/16	2.11/16	1.15/16	3.3/4
9/16	1 inch.	2.1/2	4 inches.	2.13/16	2 inches.	4.1/4
5/8	1.1/16	2.13/16	4.1/8	2.15/16	2.1/16	4.3/4
11/16	1.1/8	3.1/8	4.5/16	3.1/16	2.3/16	5.1/8
3/4	1.3/16	3.5/8	4.1/2	3.3/16	2.1/4	5.3/8
13/16	1.5/16	3.11/16	4.7/8	3.7/16	2.7/16	5.5/8
7/8	1.3/8	3.15/16	5.1/4	3.11/16	2.5/8	6.1/8
15/16	1.1/2	4.1/4	5.5/8	3.15/16	2.9/16	6.5/8
1 inch.	1.5/8	4.1/2	6 inches.	4.3/16	3 inches.	7 inches.

Government Inspector's Table for Steam-Pressure in Marine Boilers.

Diameter of Boiler.	Thickness of Boiler Plate.	Tensile Strength of Iron per sq. in., Stamped on Plate. 50,000 lbs. Riveted. Single	50,000 lbs. Riveted. Double.	60,000 lbs. Riveted. Single.	60,000 lbs. Riveted. Double.	70,000 lbs. Riveted. Single.	70,000 lbs. Riveted. Double.
Inches.	Inches.	Pounds.	Pounds.	Pounds.	Pounds.	Pounds.	Pounds.
36	0.25	115.74	138.88	138.88	166.65	162.03	194.43
	0.29	134.25	161.11	161.11	193.33	187.90	225.48
	0.3125	144.67	173.60	173.6	208.32	202.5	243.04
	0.375	173.61	208.33	208.33	249.99	243.05	291.66
38	0.25	109.64	131.56	131.57	157.88	153.5	184.2
	0.29	127.19	152.62	152.63	183.15	178.06	213.67
	0.3125	137.00	164.46	164.47	197.36	191.88	230.25
	0.375	164.73	197.67	197.36	236.83	230.26	276.31
40	0.25	104.16	124.99	125.	150.	145.83	174.99
	0.29	120.83	144.99	145.	174.	169.16	202.99
	0.3125	130.2	156.24	156.25	187.45	182.29	218.74
	0.375	156.24	187.48	187.5	225.	218.74	262.48
42	0.25	99.2	119.04	119.04	142.84	138.88	166.65
	0.29	115.07	138.08	138.09	165.7	161.11	193.33
	0.3125	124.00	148.8	148.74	178.56	173.61	208.23
	0.375	148.8	178.56	178.57	214.28	208.33	249.99
44	0.25	94.69	113.62	113.63	136.35	132.56	159.07
	0.29	109.84	131.80	131.81	158.17	153.78	184.53
	0.3125	118.36	142.03	142.04	170.44	165.71	198.85
	0.375	142.04	170.44	170.45	204.54	198.86	238.63
46	0.25	90.57	108.68	108.69	130.42	126.8	152.16
	0.29	105.07	126.	126.09	151.3	147.1	176.52
	0.3125	113.21	135.86	135.86	163.03	158.51	190.21
	0.375	135.86	163.03	163.04	195.64	190.21	228.25
48	0.25	86.8	104.16	104.16	124.99	121.52	145.82
	0.29	100.69	120.82	120.83	144.99	140.97	169.16
	0.3125	108.5	130.2	130.21	156.25	151.9	182.28
	0.375	130.2	156.24	156.25	187.50	182.29	218.74
54	0.25	77.16	92.59	92.57	111.10	108.02	129.62
	0.29	89.5	107.4	107.41	128.88	125.3	150.36
	0.3125	96.44	115.72	115.55	138.66	135.03	162.03
	0.375	115.74	138.88	138.88	166.65	162.03	194.43
60	0.25	69.44	83.32	83.33	99.99	97.22	116 66
	0.29	80.55	96.66	96.66	115.99	112.77	135.32
	0.3125	86.8	104.16	104.18	124.99	121.52	145.82
	0.375	104.16	124.99	125.	150.	145.83	174.99
66	0.25	63.13	75.75	75.75	90.90	88.37	106.04
	0.29	73.23	87.87	87.87	105.44	102.52	123.02
	0.3125	78.91	94.69	94.69	113.62	110.47	132.56
	0.375	94.69	113.62	113.62	136.34	132.57	159.08
72	0.25	57.87	69.87	69.44	83.32	81.01	97.21
	0.29	67.12	80.54	80.55	96.66	93.98	112.77
	0.3125	72.33	86.8	86.8	104.16	101.27	121.52
	0.375	86.8	104.16	104.16	124.99	121.52	145.82
78	0.25	53.41	64.09	64.4	76.92	74.78	89.73
	0.29	61.96	74.35	74.35	89.22	86.75	104.1
	0.3125	66.77	80.12	80.12	96.14	93.48	112.17
	0.375	80.12	96.14	96.15	115.38	112.17	134.6
84	0.25	49.6	59.52	59.52	71.42	69.44	83.32
	0.29	57.53	69.03	69.04	82.84	80.55	96.66
	0.3125	62.	74.4	74.4	89.28	86.8	104.16
	0.375	74.4	89.28	89.28	107.13	104.16	124.99
90	0.25	46.29	55.54	55.55	66.66	64.81	77.77
	0.29	53.7	64.44	64.44	77.32	75.18	90.21
	0.3125	57.86	69.43	69.44	83.32	81.01	97.21
	0.375	69.44	83.32	83.33	99.99	97.22	116.66
96	0.25	43.4	52.08	52.08	62.49	60.76	72.91
	0.29	50.34	60.4	60.41	72.49	70.48	84.57
	0.3125	54.25	65.1	65.1	78.12	75.95	91.14
	0.375	65.1	78.12	78.12	93.74	91.14	109.6

LLOYD'S RULES FOR BOILERS (BRITISH), ETC.

Cylindrical Shells.

THE strength of circular shells to be calculated from the strength of the longitudinal joints by the following formula:

$$\frac{C \times T \times B}{D} = \text{working pressure,}$$

where C = coefficient as per following table; T = thickness of plate in inches; D = mean diameter of shell in inches; B = percentage of strength of joint found as follows (the least percentage to be taken):

For plate at joint $B = \frac{p - d}{p} \times 100.$

For rivets at joint $B = \frac{n \times a}{p \times T} \times 100$ with iron rivets in iron plates with punched holes.

$B = \frac{n \times a}{p \times T} \times 90$ with iron rivets in iron plates with drilled holes.

$B = \frac{n \times a}{p \times T} \times 85$ with steel rivets in steel plates.

$B = \frac{n \times a}{p \times T} \times 70$ with iron rivets in steel plates.

(In case of rivets being in double shear, 1.75 a is to be used instead of a.)

where p = pitch of rivets; d = diameter of rivets; a = sectional area of rivets; n = number of rows of rivets.

MEM.—In any case where the strength of the longitudinal joint is satisfactorily shown by experiment to be greater than that given by this formula, the actual strength may be taken in the calculation.

Table of Coefficients.

IRON BOILERS.

Description of Longitudinal Joint.	For Plates ½ inch thick and under.	For Plates ¾ thick and above ½ inch.	For Plates above ¾ inch thick.	
Lap joint, punched holes	155	165	170	
Lap joint, drilled holes	170	180	190	
Double butt strap joint, punched holes	170	180	190	
Double butt strap joint, drilled holes	180	190	200	

STEEL BOILERS.

Description of Longitudinal Joint.	For Plates ⅜ thick and under.	For Plates $\frac{9}{16}$ thick and above ⅜.	For Plates ¾ thick and above $\frac{9}{16}$	For Plates above ¾ thick.
Lap joints	200	215	230	240
Double butt strap joints	215	230	250	260

NOTE.—The inside butt strap to be at least ¾ the thickness of the plate.

NOTE.—For the shell plates of superheaters or steam chests enclosed in the uptakes or exposed to the direct action of the flame, the coefficients should be ⅝ of those given in the above tables.

Proper deductions are to be made for openings in shell.

All manholes in circular shells to be stiffened with compensating rings.

The shell plates under domes in boilers so fitted, to be stayed from the top of the dome or otherwise stiffened.

Stays.

The strength of stays supporting flat surfaces is to be calculated from the weakest part of the stay or fastening, and the strain upon them is not to exceed the following limits; namely,

Iron Stays.—For screw stays, and for other stays not exceeding $1\frac{1}{2}$ inches effective diameter, and for all stays which are welded, 6000 lbs. per square inch; for unwelded stays above $1\frac{1}{2}$ inches effective diameter, 7500 lbs. per square inch.

Steel Stays.—For screw stays, and for other stays not exceeding $1\frac{1}{2}$ inches effective diameter, 8000 lbs. per square inch; for stays above $1\frac{1}{2}$ inches effective diameter, 9000 lbs. per square inch. No steel stays are to be welded.

Flat Plates.

The strength of flat plates supported by stays to be taken from the following formula:

$$\frac{C \times T^2}{P^2} = \text{working pressure in lb. per square inch,}$$

where T = thickness of plate in sixteenths of an inch.
P = greatest pitch in inches.
C = 90 for plates $\frac{7}{16}$ thick and below fitted with screw stays with riveted heads.
C = 100 for plates above $\frac{7}{16}$ fitted with screw stays with riveted heads.
C = 110 for plates $\frac{7}{16}$ thick and under fitted with screw stays and nuts.
C = 120 for plates above $\frac{7}{16}$ fitted with screw stays and nuts.
C = 140 for plates fitted with stays with double nuts.
C = 160 for plates fitted with stays with double nuts, and washers at least $\frac{1}{2}$ thickness of plates and a diameter of $\frac{2}{5}$ of the pitch, riveted to the plates.

NOTE.—In the case of front plates of boilers in the steam space, these numbers should be reduced 20 per cent., unless the plates are guarded from the direct action of the heat.

Girders.

The strength of girders supporting the tops of combustion chambers and other flat surfaces to be taken from the following formula:

$$\frac{C \times d^2 \times T}{(L - P) \times D \times L} = \text{working pressure in lb. per square inch,}$$

where L = length of girder; P = pitch of stays; D = distance apart of girders; d = depth of girder at centre; T = thickness of girder at centre. All these dimensions to be taken in inches. C = 6000, if there is one stay to each girder; 9000, if there are two or three stays to each girder; 10,200, if there are four stays to each girder.

Circular Furnaces.

The strength of plain furnaces to resist collapsing to be calculated from the following formula:

$$\frac{89600 \times T^2}{L \times D} = \text{working pressure in lb. per square inch,}$$

where 89600 = constant.
T = thickness of plates in inches.
D = outside diameter of furnace in inches.
L = length of furnaces in feet. If rings are fitted, the length between rings to be taken.

The pressure in no case to exceed $\frac{8000 \times T}{D}$.

The strength of the corrugated furnaces (corrugations $1\frac{1}{2}$ inches deep) to be calculated from the following formula:

$$\frac{1000 \times (T - 2)}{D} = \text{working pressure in lb. per square inch.}$$

where T = thickness of plate in sixteenths of an inch.
D = greatest diameter of furnace in inches.

CALORIMETERS.

G. A. HIRN of Mulhouse devised a simple method in 1868 for measuring the humidity of steam by mixing it with cold water. The apparatus used is called *calorimeter*, consisting of a common wooden barrel of about 8 cubic feet capacity, which is filled with water, and a steam-hose leads from the boiler through the water to the bottom, where the steam is condensed and its humidity determined by the weight and temperature of the water.

w = pounds of cold water put into the barrel.
h = units of heat per pound of w when cold and above 32°.
W = pounds of heated water in the barrel after the completion of the experiment; that is, including the weight of the condensed steam.
h' = units of heat per pound of W above 32°.
f = pounds of priming or water carried over with the steam into the barrel.
S = pounds of saturated steam blown into the barrel.
H = units of heat per pound of the steam S.
H' = units of heat per pound of the priming f.
p = pounds of steam and priming carried over from the boiler into the barrel.
P = units of heat passed over with the steam and priming into the barrel.

The weight p must then be equal to the sum of the weights of the steam S and priming f, which is evidently the same as the difference between the weights W and w.

That is, $$p = S + f = W - w. \qquad 1.$$

The total units of heat P passed over with the steam S and priming f must then be:

$$P = HS + H'f = Wh' - wh. \qquad 2.$$

By solving this formula for the steam S, we have:

$$S = \frac{P - H'f}{H} = W - w - f. \qquad 3.$$

$$S = p - f.$$

$$H(p - f) = P - H'f. \qquad 4.$$

$$Hp - Hf = P - H'f. \qquad 5.$$

$$f(H - H') = Hp - P. \qquad 6.$$

From this formula we have the weight of priming carried over with the steam from the boiler into the barrel; namely,

$$f = \frac{Hp - P}{H - H'}. \qquad 7.$$

But $P = Wh' - wh$, which, inserted in Formula 7, gives:

Pounds of priming, $$f = \frac{Hp + wh - Wh'}{H - H'}. \qquad 8.$$

The percentage of humidity of the steam will then be:

$$\% = \frac{f}{p} = \frac{f}{W - w}. \qquad 9.$$

Formula 8 is ready for use of the data obtained by the calorimeter when $p = W - w$.

Another form of calorimeter depends upon the specific volume of the steam from tables.

A vessel of accurately measured capacity is inserted into the steam-pipe on the way to the engine. It is surrounded by the steam or a steam jacket carefully clothed, through which the steam is forced to pass.

This vessel is opened to the steam from the boiler for a sufficient period to become thoroughly warmed to the temperature of the steam inside and out, so that condensation is not caused by it.

It is then closed to the steam from the boiler, and opened to a closed coil surface condenser. A steam-gauge is on the vessel and a vacuum-gauge on the condenser.

Let V = volume of vessel in cubic feet.
" S = specific volume for steam pressure.
" S_1 = specific volume for vacuum pressure.
" W = weight calculated for each evacuation of vessel.
" W_1 = observed weight of each evacuation of vessel.

Then $W_1 - W$ = weight of priming

$$W = 62\tfrac{1}{2} V \left\{ \frac{1}{S} - \frac{1}{S_1} \right\}$$

$$\frac{W_1 - W}{W_1} = \text{priming in hundredths.}$$

To Approximate the Horse-Power of Horizontal Tubular Steam-Boilers.

D = diameter, and L = length in feet of the boiler.
For 3″ tubes $x = 5$. $3\tfrac{1}{2}$″ tubes $x = 6$. 4″ tubes $x = 7$.

$$HP = \frac{D^2 L}{x}.$$

To Approximate the Weight of Horizontal Tubular Steam-Boilers.

$$W = \frac{144\ D^2 L}{x} \text{ in pounds.}$$

To Approximate the Heating Surface of Horizontal Tubular Steam-Boilers.

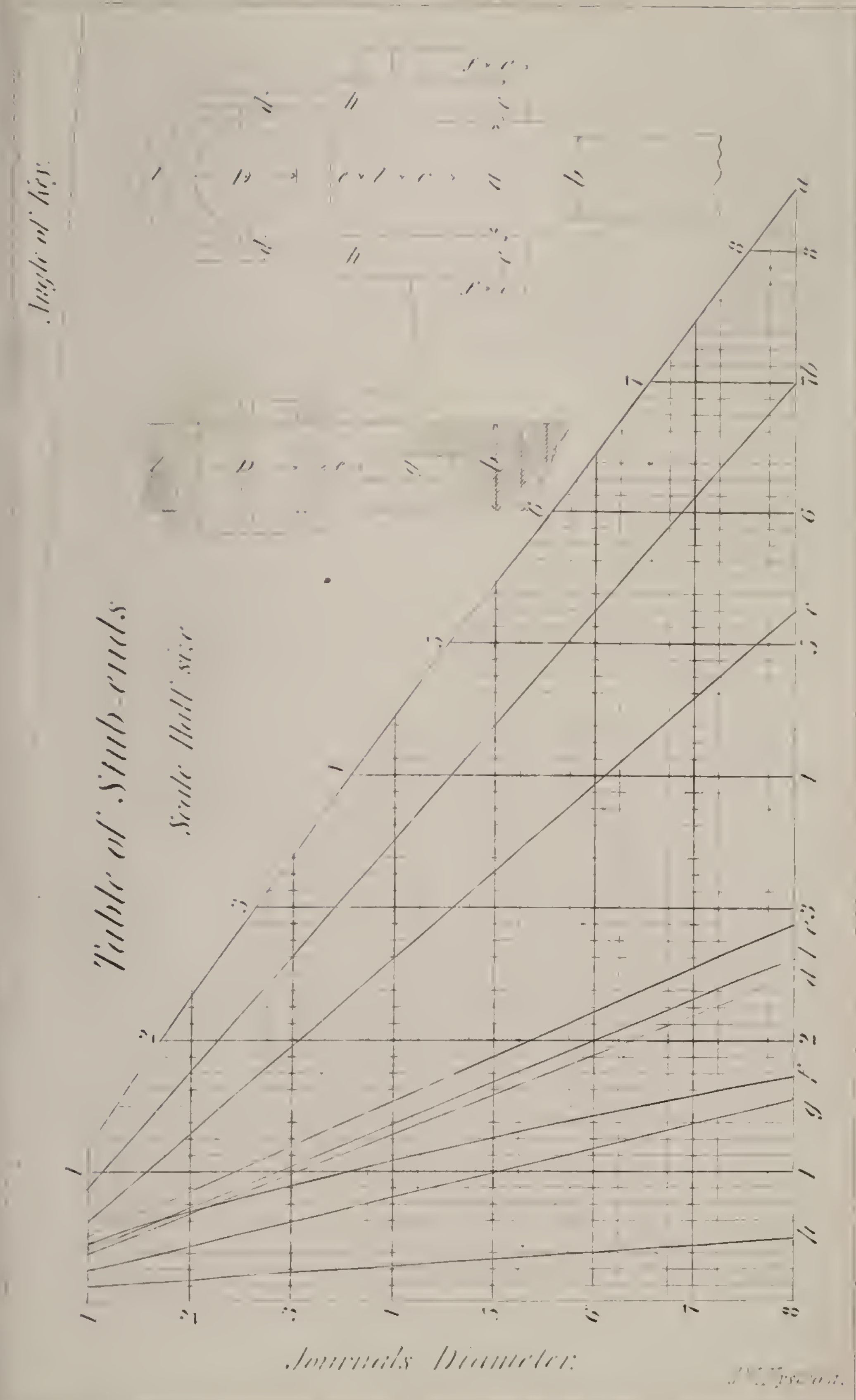
$$\square = \frac{15\ D^2 L}{x} \text{ in square feet.}$$

Horse-Power of Chimneys.

For safety this table gives the horse-power about 25 per cent. less than may be attained in practice.

Height chimney.	Area of chimney in square feet at the top.									
	0.5	1	2	4	6	10	15	20	30	40
Feet.	HP	HP	HP	HP	HP	HP	HP	HP	HP	HP
20	3.35	6.7	13.4	26.8	40.2	67	100.5	134	201	268
25	3.7	7.4	14.8	29.6	44.4	74	111.0	148	222	296
30	4.0	8.0	16.0	32.0	48.0	80	120.0	160	240	320
35	4.25	8.5	17.0	34.0	51.0	85	127.5	170	255	340
40	4.5	9.0	18.0	36.0	54.0	90	135.0	180	270	360
45	4.75	9.5	19.0	38.0	57.0	95	142.5	190	285	380
50	5.0	10.0	20.0	40.0	60.0	100	150.0	200	300	400
55	5.2	10.4	20.8	41.6	62.4	104	156.0	208	312	416
60	5.4	10.8	21.6	43.2	64.8	108	162.0	216	324	432
65	5.6	11.2	22.4	44.8	67.2	112	168.0	224	336	448
70	5.8	11.6	23.2	46.4	69.6	116	174.0	232	348	464
75	6.0	12.0	24.0	48.0	72.0	120	180.0	240	360	480
80	6.15	12.3	24.6	49.2	73.8	123	184.5	246	369	492
85	6.35	12.7	25.4	50.8	76.2	127	190.5	254	381	508
90	6.5	13.0	26.0	52.0	78.0	130	195.0	260	390	520
95	6.65	13.3	26.6	53.2	79.8	133	199.5	266	399	532
100	6.8	13.6	27.2	54.4	82.8	136	204.0	272	414	544
110	7.1	14.2	28.4	56.8	85.2	142	213.0	284	426	568
120	7.4	14.8	29.6	59.2	88.8	148	222.0	296	444	592
130	7.65	15.3	30.6	61.2	91.8	153	229.5	306	459	612
140	7.9	15.8	31.6	63.2	94.8	158	237.0	316	474	632
150	8.15	16.3	32.8	65.2	97.8	163	244.5	326	489	652
160	8.4	16.8	33.6	67.2	100.8	168	252.0	336	504	672
170	8.65	17.3	34.6	69.2	103.8	173	259.5	346	519	692
180	8.9	17.8	35.6	71.2	106.8	178	267.0	356	534	712
190	9.2	18.2	36.4	72.8	109.2	182	273.0	364	546	728
200	9.3	18.6	37.2	74.4	111.6	186	279.0	372	558	744
210	9.5	19.0	38.0	76.0	114.0	190	285.0	380	570	760
220	9.7	19.4	38.8	77.6	116.4	194	291.0	388	582	776
230	9.9	19.8	39.6	79.2	118.8	198	297.0	396	594	792
240	10.1	20.2	40.4	80.8	121.2	202	303.0	404	606	808
250	10.3	20.6	41.2	82.4	123.6	206	309.0	412	618	824
260	10.5	21.0	42.0	84.0	126.0	210	315.0	420	630	840
270	10.65	21.3	42.6	85.2	127.8	213	319.5	426	639	852
280	10.8	21.6	43.2	86.4	129.6	216	324.0	432	648	864
290	11.0	22.0	44.0	88.0	132.0	220	330.0	440	660	880
300	11.15	22.3	44.6	89.2	133.8	223	334.5	446	669	892
310	11.35	22.7	45.4	90.8	136.2	227	340.5	454	681	908
320	11.5	23.0	46.0	92.0	138.0	230	345.0	460	690	920
330	11.65	23.3	46.6	93.2	139.8	233	349.5	466	699	932
340	11.8	23.6	47.2	94.4	141.6	236	354.0	472	708	944
350	12.0	24.0	48.0	96.0	144.0	240	360.0	480	720	960
360	12.15	24.3	48.6	97.2	145.8	243	364.5	486	729	972
370	12.3	24.6	49.2	98.4	147.6	246	369.0	492	738	984
380	12.45	24.9	49.8	99.6	149.4	249	373.5	498	747	996
390	12.6	25.2	50.4	100.8	151.2	252	387.0	504	756	1008
400	12.75	25.5	51.0	102.0	153.0	255	382.5	510	765	1020

Plate VI


Angle of Key
Table of Stub-ends
Scale Half size
Journals Diameter

To Reduce Actual Evaporation to the Standard at and from 212° Fahr.

The quantity of heat required for evaporating one pound of water under atmospheric pressure at and from 212° Fahr. is 965.66 units.

W = actual evaporation in pounds of water per unit of time.
w = standard evaporation at and from 212° Fahr.
H = units of heat per pound of the steam actually evaporated, and to be found in the steam table.
h = units of heat per pound of the feed-water, to be found in the water tables.

$$w = \frac{W(H - h)}{965.66}.$$

Example.—A steam-boiler evaporated $W = 36000$ pounds of water per hour under a pressure of 50 pounds to the square inch indicated by gauge; the temperature of feed-water, 180°. Required the equivalent standard evaporation at and from 212°?

From steam table, $H = 1172.8$
From water table, $h = 148.5$
1024.3

$$w = \frac{36000\,(1172.8 - 148.5)}{965.66} = 38186 \text{ pounds.}$$

The actual evaporation multiplied by the tabular number is the standard evaporation.

Temp. feed-water.	Steam-pressure in boiler above atmosphere.												
	30	40	50	60	70	80	90	100	110	120	130	140	150
32°	1.207	1.211	1.214	1.217	1.220	1.223	1.225	1.227	1.229	1.231	1.233	1.234	1.236
40°	1.199	1.203	1.206	1.209	1.212	1.214	1.217	1.219	1.221	1.223	1.224	1.226	1.228
50°	1.188	1.192	1.196	1.199	1.201	1.204	1.206	1.208	1.210	1.212	1.214	1.216	1.217
60°	1.178	1.182	1.185	1.188	1.191	1.194	1.196	1.198	1.200	1.202	1.204	1.205	1.207
70°	1.167	1.171	1.175	1.178	1.181	1.183	1.185	1.188	1.190	1.191	1.193	1.195	1.196
80°	1.157	1.161	1.165	1.168	1.170	1.173	1.175	1.177	1.179	1.181	1.183	1.185	1.186
90°	1.147	1.151	1.154	1.157	1.160	1.162	1.165	1.167	1.169	1.171	1.172	1.174	1.176
100°	1.136	1.140	1.144	1.147	1.150	1.152	1.154	1.156	1.158	1.160	1.162	1.164	1.165
110°	1.126	1.130	1.133	1.136	1.139	1.142	1.144	1.146	1.148	1.150	1.152	1.153	1.155
120°	1.116	1.120	1.123	1.126	1.129	1.131	1.134	1.136	1.138	1.140	1.141	1.143	1.145
130°	1.105	1.109	1.113	1.116	1.118	1.121	1.123	1.125	1.127	1.129	1.131	1.133	1.134
140°	1.095	1.099	1.102	1.105	1.108	1.110	1.113	1.115	1.117	1.119	1.120	1.122	1.124
150°	1.085	1.088	1.092	1.095	1.098	1.100	1.102	1.104	1.106	1.108	1.110	1.112	1.113
160°	1.074	1.078	1.081	1.084	1.087	1.090	1.092	1.094	1.096	1.098	1.100	1.101	1.103
170°	1.064	1.067	1.071	1.074	1.077	1.079	1.081	1.084	1.086	1.087	1.089	1.091	1.092
180°	1.053	1.057	1.060	1.064	1.066	1.069	1.071	1.073	1.075	1.077	1.079	1.080	1.082
190°	1.043	1.047	1.050	1.053	1.056	1.058	1.061	1.063	1.065	1.067	1.068	1.070	1.072
200°	1.032	1.036	1.040	1.043	1.045	1.048	1.050	1.052	1.054	1.056	1.058	1.059	1.061
210°	1.022	1.026	1.030	1.032	1.035	1.037	1.040	1.042	1.044	1.046	1.047	1.049	1.051

WOOD FOR COMBUSTION.

A cord of wood is 8 feet wide by 4 feet high and 4 feet deep, or the wood is 4 feet long. The cord contains $8 \times 4 \times 4 = 128$ cubic feet, of which only 74 cubic feet is solid wood and 54 cubic feet of space.

Two cords of wood evaporate about the same quantity of water as one ton of anthracite coal.

The best pine wood evaporates 5 pounds of water per pound of wood consumed in a steam-boiler furnace. One cord of wood can be consumed per hour on 60 square feet of grate.

Weight in Pounds per Cord of Different Woods.

Woods, Seasoned.	lbs.	*Woods, Seasoned.*	lbs.	*Woods, Seasoned.*	lbs.
Shell-bark Hickory.	4469	Hard Maple . . .	2878	Cedar	1910
White Oak	3821	Beech	2875	Yellow Pine . . .	1904
Red-heart Hickory .	3705	Hazel	2870	White Pine . . .	1868
Southern Pine . .	3375	Virginia Pine . .	2689	Spruce	1685
Red Oak	3254	New Jersey Pine .	2137	Hemlock	1240

Wood requires 32 per cent. more fire grate than mineral coal, for equal generation of steam. The furnace should be 60 per cent. of cubical space more for wood than for coal, or about 4.5 cubic feet per square foot of grate.

Properties of Fuel.

Kind of Fuel.	Units of heat per pound of fuel.	Pounds of water evaporated per lb. of coal.	Per cent. of carbon.	Cubic feet of air requ. for one lb. of coal.	Weight per cubic foot.	Cubic feet to stow a ton.
Bituminous coal . .	11600	7 to 9	80	265	50	44
Anthracite coal . . .	13340	8 to 10	92	282	54	40
Coke	12420	8 to 10	86	245	31	72
Coke, nat. Virginia . .	11600	8 to 9	80	260	48	48
Coke, Cumberland . .	11600	8 to 10	80	250	32	70
Charcoal	13920	5 to 6	96	265	24	104
Dry wood	6380	4 to 5	44	147	20	100
Wood with 20 per ct. water	4930	4	34	115	25	100
Turf, dry	7395	6	51	165	28	80
Turf, 20 per ct. water .	5800	5	40	132	30	75
Oil, Wax, Tallow . .	11165	14	77	200	59	37
Alcohol (from market) .	8410	9.56	58	154	52	42

Chemically, one pound of carbon burnt to carbonic acid requires the oxygen of 153 cubic feet of atmospheric air.

Timber, Green and Seasoned.

Timber.	Green.	Seasoned.	
American Pine	44.75	30.69	Comparative weight per cubic foot in pounds of green and seasoned timber.
Ash	58.19	50.00	
Beech	60.00	53.37	
Cedar	32.00	28.25	
English Oak . . .	71.62	43.50	
Riga Fir	48.75	35.50	

Board Measure.

Multiply together the three dimensions, width and thickness in inches and the length of the lumber in feet; divide the product by 12, and the quotient will be the board measure.

Combustion and Effect of Fuel.

Combustion is the rapid chemical combination of substances with oxygen. Carbon C and hydrogen H, are the substances most generally employed for generating heat. Carbon is fully consumed when combined with oxygen O, to form carbonic acid gas CO_2, and partly consumed when in the form of carbonic oxide gas CO or smoke. $h =$ units of heat generated by one pound of fuel. The heat necessary to raise one pound of water one degree Fah. is one unit of heat. $w =$ pounds of water at 212° evaporated per pound of fuel. $A =$ volume in cubic feet and $a =$ weight in pounds of atmospheric air required for the perfect combustion of one pound of fuel.

From Mr. Johnson's *Experiments for the U. S. Government* it would appear that 24 pounds of air is required for each pound of fuel burned in order to get complete combustion with chimney draught, and 18 pounds with forced draught. That is, with chimney allow 300 cubic feet of air per pound of fuel, and with forced draught allow 225 cubic feet.

The following theoretical table is taken from Rankin's *The Steam Engine.*

One unit of heat = 772 foot pounds, if generated per second will be $H = 1{\cdot}4$ horses, of which we in present practice utilize about one-twentieth. The following table will show how important it is to fully consume the combustibles to acid. One pound of carbon consumed to oxide will generate only 1·72 horses, instead of 5·66 when consumed to acid.

Properties of Combustion, per Hour.

C	CO	CO_2	O	a	A	h	w	H
lbs.	lbs.	lbs.	lbs.	lbs.	cub. ft.	heat.	lbs.	horses.
1		3·666	2·666	12	149	14500	15	5·660
1	2·666		1·333	6	74·50	4400	4·55	1·720
0·433	**1**		0·566	2·550	31·65	1650	5·633	1·200
0·272		**1**	0·727	3·275	40·56	3960	4·100	1·545
	1·730	1.375	**1**	3·500	43·33	5440	5·633	2·125
	0·445	0·392	0·222	**1**	12·38	1210	1·250	0·472
	·0358	·0246	·0261	·0808	**1**	97·3	0·100	0·038
	0·584	0·244	0·170	0·800	9·920	966	**1**	0·378
	1·550	0·651	0·470	2·120	26·30	2558	2·645	**1**

RADIATION OF HEAT FROM STEAM-PIPES,

Boilers or Steam Cylinders.

Notation.

D = outside diameter of steam-pipe, without casing and limited to not more than 12 inches.

T = temperature of the steam, Fahr. degrees.

t = temperature of the external air.

h = heat units radiated per square foot per hour, on uncovered pipe.

A = outside area in square feet of steam-pipe.

H = horse-power lost by radiation of heat.

n = exponent of the wind, which varies with the current of air or draft about the steam-pipe, as in the following table:

Wind.	*Exp. n.*
Calm.	1.20
Gentle.	1.22
Brisk.	1.24
Storm.	1.26

The loss of heat will then be per hour (empirical formula)—

$$h = 0.001122\,[450 + (12 - D)^2]\,(T - t)^n. \quad \text{1.}$$

$$\text{Horse-power lost } H = \frac{A\,h}{2564}. \quad \text{2.}$$

One horse-power consumes or generates 2564 heat units per hour.
By logarithms the Formula 1 is reduced to—

$$\log. h = \log. k + n \log. (T - t). \quad \text{3.}$$

The log. k is contained in the second column of the accompanying table for different diameters of pipes.

For any uncovered plane or cylindrical surface above 12 inches in diameter the radiation in units of heat per square foot per hour will be—

$$h = 0.505\,(T - t)^n. \quad \text{4.}$$

The effect of thickness of metal is inappreciable for practical purposes.

Example 1. The California S. N. Co.'s steamer Julia has a steam-pipe 40 feet long by $D = 9$ inches in diameter, and two branch-pipes 12 feet long by $D = 4$ inches each, all uncovered. Pressure of steam, 100 lbs. $T = 337°$. Temperature of the external air $t = 70°$. Required, the loss of heat and power by radiation?

In calm wind $n = 1.2$. See table for n.

$$h = 0.001122\,[450 + (12 - 9)^2]\,(337° - 70°)^{1.2} = 420.34,$$

the units of heat lost per square foot per hour.

Area of pipe, $A = 0.75 \times 3.14 \times 40 = 94.24$ square feet.

Power lost, $H = \dfrac{94.24 \times 420.34}{2564} = 15.5$ horses.

The branch-pipes lose 4.6 horses.

The total loss of power 20.1 horses.

The same pipes covered with 2-inch-thick felt would gain $20.1 \times 0.93 = 18.7$ horse-power.

Example 2. In the factory of Bellavista, Peru, are 150 feet of uncovered steam-pipes $D = 3$ inches in diameter. Steam-pressure, 45 lbs. $T = 292°$ Fahr. Temperature of external air $t = 68$, and wind gentle, $n = 1.22$. Required, the horse-power and fuel lost?

Formula 3. $\log. h = 0.77408 - 1 + 1.22 \log. (292 - 68) = 2.50708$, or 178 units of heat lost per hour.

Area of steam-pipes, $A = 0.785 \times 150 = 117.8$ square feet.

Formula 2. $H = \dfrac{117.8 \times 173}{2564} = 8$ horse-power nearly, which is lost by radiation.

The same pipes covered with one-inch felt will gain 8 × 0.89 (see table) = 7.12 horse-power. The steam-engine in Bellavista works without expansion, and consumes about 10 lbs. of coal per horse-power per hour = 7.12 × 10 = 71.2 pounds, and for 8 hours' working = 569.6 lbs. of coal lost per day.

The radiation of heat from steam-pipes causes a condensation of steam to water, and the weight in pounds of water so condensed is equal to the units of heat radiated, divided by the latent heat of the steam in the pipe. The Formula 1 will also answer for calculating the quantity of heat radiated from steam or water-pipes for heating rooms.

[Many experiments have been made proving no practical gain with a thickness of covering exceeding one inch.—W. D. M.]

Percentage of Heat or Power Gained

by covering steam-pipes with felt and canvas outside.

Diam D	Logarithm k.	Thickness of felt covering in inches. $\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	2	3	4	6
1	0.80663—1	65	76	81	86	92	94	96	98	99	100
2	0.79561—1	63	74	80	85	90	93	95	97	98	99
3	0.77408—1	61	72	79	84	89	92	95	96	98	99
4	0.76096—1	59	71	77	83	88	92	94	96	97	99
5	0.74809—1	57	69	76	82	87	91	94	96	97	99
6	0.73670—1	54	67	74	81	86	91	94	95	97	99
7	0.72668—1	52	66	73	81	85	90	93	95	97	99
8	0.71838—1	50	64	71	80	85	90	93	95	97	99
9	0.71179—1	47	62	70	79	85	89	93	95	97	99
10	0.70705—1	45	61	69	78	84	89	92	95	96	99
11	0.70417—1	42	59	67	78	83	88	92	94	96	98
12	0.70321—1	40	58	66	77	83	88	92	94	96	98

Lap-welded American Charcoal Iron Boiler Tubes.

PASCAL IRON WORKS
Philadelphia.

TASKER IRON WORKS,
New Castle, Del.

Diameter of the tube. Outside	Inside.	Heating surface per foot of length. Outside	Inside.	Thickness of metal. Wg.	inches.	Length of tube per square foot. Outside.	Inside.	Area of cross-section. Outside	Inside.	Weight per foot of length.
Inches.	Inches.	Sq. ft.	Sq. ft.	Wg.	inches.	Feet.	Feet.	Sq in.	Sq. in.	Pounds.
1	0.856	0.2618	0.2241	**15**	0.072	3.819	4.460	0.785	0.575	0.708
1.25	1.106	0.3272	0.2895	**15**	0.072	3 056	3.455	1.227	0.960	0 9
1.50	1.334	0.3926	0.3492	**14**	0.083	2 547	2.863	1.767	1.396	1.250
1.75	1.560	0.4580	0.4084	**13**	0.095	2.183	2.448	2.405	1.911	1.665
2	1.804	0.5236	0.4723	**13**	0 098	1.909	2.118	3.142	2.556	1.981
2.25	2.054	0.5890	0.5377	**13**	0.098	1.698	1.850	3.976	3.314	2 238
2 50	2.283	0.6545	0.5977	**12**	0.109	1.528	1.673	4.909	4.094	2.755
2.75	2.533	0.7200	0.6631	**12**	0.109	1.390	1.508	5.940	5.039	3.045
3	2.783	0.7853	0 7285	**12**	0.109	1.273	1.373	7.069	6 083	3.333
3.25	3.012	0.8508	0.7885	**11**	0.119	1.175	1.268	8.296	7 125	3.958
3.50	3.262	0.9163	0 8430	**11**	0.119	1.091	1.171	9.621	8.357	4.272
3.75	3.512	0.9817	0.9194	**11**	0.119	1.018	1.088	11.045	9.687	4.590
4	3.741	1.0472	0.9794	**10**	0.130	0.955	1.023	12 566	10.992	5.320
4.50	4.241	1.1781	1.1105	**10**	0.130	0.849	0 901	15.904	14.126	6.010
5	4.720	1 3680	1.2357	**9.5**	0.140	0.764	0.809	19.635	17.497	7.226
6	5.699	1.5708	1.4920	**9**	0.151	0.637	0.670	28.274	25.509	9.346
7	6 657	1.8326	1.7428	**7.5**	0.172	0.545	0.574	38.484	34.875	12.435
8	7.636	2.0944	1.9991	**7**	0.182	0.478	0.500	50.265	45.795	15.109
9	8.615	2.3562	2.2553	**6.5**	0.193	0.424	0.444	63.617	58.291	18.002
10	9.573	2.5347	2.5022	**5.5**	0.214	0.382	0 399	78.540	71.975	22.19

The length of tube and thickness of metal can be varied to suit orders.
The heating surface of a boiler tube is that exposed to the fire.
Safe ends of thicker metal welded on the ends of tubes as may be required.

BLOWING OFF. SALT WATER. INCRUSTATION.

Sea water contains about 0.03 its weight of salt. When salt water boils, fresh water evaporates and the salt remains in the boiler; consequently the proportion of salt increases as the water evaporates, until it has reached 0.36 weight to the water; the salt will then commence to crystallize in the boiler, and the water in solution will hold 0.36 weight of salt to 1 of water.

To prevent deposit in the boiler, it is necessary to keep the salt below this proportion, which is overcome by withdrawing (blowing off) part of the supersalted water, while less salted (feed) water is replaced. It is found in practice that when the proportions are kept 0.12 of salt to 1 weight of water, the deposit will be very slight. To obtain this it will be necessary to blow off—

$$\frac{0.03}{0.12} = 0.25 \text{ parts of the feed water, or,}$$

if a brine-pump is used, it should be at least 0.25 of the feed-pump.

W = cubic feet of supersalted water to be blown off per minute. D, S, n and V, as before, we shall have—

$$W = \frac{D^2 S n}{3000 V}.$$

Example. $D = 30$ inches, stroke of piston 36 inches, cut off at half stroke $S = 18$, making 14 revolutions per minute, with a pressure of 30 pounds per square inch, $V = 610$. How much water must be blown off per minute?

$$W = \frac{30^2 \times 18 \times 14}{3000 \times 610} = 0.124 \text{ cubic feet.}$$

Heat Wasted by Blowing Off.

Letters denote,

w = water evaporated } in cubic feet per unit of time.
and W = water blown off }
t = temperature of the feed water.
T = " " blowing off.
H = heat wasted, per cent.

$$H = \frac{W(T - t)}{w(990 + T - t)}.$$

Example. Let the quantity of water blown off be ⅓ of the feed water, we have $W = 1$, and $w = 2$; the boiling-point of the water will then be $T = 215.5°$; let the feed water taken from the hot-well be $t = 100°$. Required, the quantity of heat lost?

$$H = \frac{1(215.5° - 100)}{2(990 + 215.5 - 100)} = 0.066 \text{ or } 6.6 \text{ per cent.}$$

This is a very trifling quantity of heat lost.

Heat Wasted by Incrustation.

The conducting power of iron for heat is about 30 times that of scale; hence a considerable portion of heat is lost when the scale becomes thick in a boiler.

t = thickness of the scale in 16ths of an inch.
H = per cent. of heat wasted.

$$H = \frac{t^2}{32 + t^2}.$$

Example. The scale in a boiler is 5-sixteenths of an inch thick. How much heat is lost by it?

$$H = \frac{5^2}{32 + 5^2} = 0.438, \text{ or 44 per cent., nearly,}$$

which goes out through the chimney.

This is merely to show that the heat lost by blowing off is but trifling compared with the heat lost by scale formation, which additionally injures the boiler by softening and fracturing the iron.

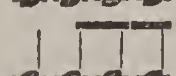
When boilers are taken good care of by cleaning and blowing off at short intervals, the scales need not exceed 1-sixteenth of an inch.

Proportions of Salt in Water:
its boiling-point and weight per cubic foot.

Salt in 100 Weights.	Boiling temp. Fahr.	Weight per cub. ft.	Specific grav.	Salt in 100 weights.	Boiling temp. Fahr.	Weight per cub. ft.	Specific grav.
		pounds.				pounds.	
0	212°	59.837	1.00	21	218.304	72.224	1.21
1	212.205	60.431	1.01	22	218.690	72.728	1.22
2	212.422	61.024	1.02	23	219.082	73.395	1.23
3	212.649	61.617	1.03	24	219.483	73.980	1.24
4	212.887	62.209	1.04	25	219.887	74.565	1.25
5	213.136	62.801	1.05	26	220.296	75.148	1.26
6	213.394	63.393	1.06	27	220.713	75.732	1.27
7	213.664	63.984	1.07	28	221.131	76.316	1.28
8	213.942	64.575	1.08	29	221.558	76.899	1.29
9	214.229	65.166	1.09	30	221.984	77.482	1.30
10	214.526	65.756	1.10	31	222.419	78.064	1.31
11	214.801	66.346	1.11	32	222.857	78.646	1.32
12	215.145	66.935	1.12	33	223.302	79.228	1.33
13	215.446	67.524	1.13	34	223.733	79.810	1.34
14	215.797	68.113	1.14	35	224.208	80.390	1.35
15	216.132	68.701	1.15	36	224.668	80.970	1.36
16	216.477	69.289	1.16	37	225.139	81.550	1.37
17	216.826	69.877	1.17	38	225.611	82.130	1.38
18	217.186	70.464	1.18	39	226.087	82.709	1.39
19	217.550	71.051	1.19	40	226.572	83.288	1.40
20	217.924	71.377	1.20	Saturates with 40 parts of salt.			

Water does not increase in volume by addition of the above proportions of salt.

Code of Signals for Engine-Room.

Go ahead, . .		. . . one stroke.
Back, . . .		. . . two strokes.
Stop, . . .		. . . one stroke.
Slowly, . . .		. . . two short.
Full speed, .		. . three short.
Go ahead slowly,		. . . one long, two short.
Back slowly, .		. . two long, two short.
Go ahead, full speed,		. . one long, three short.
Back fast, . .		. two long, three short.
Hurry, . . .		. three short repeated.

It is also customary to have two bells in the engine-room—a large bell for the long strokes, and a smaller for the short strokes.

On Driving a Nail into a Piece of Wood.

Fig. 1.

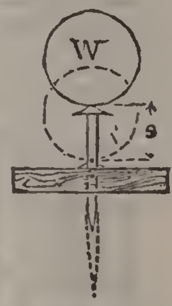
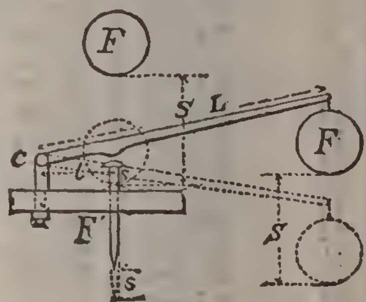


Fig. 2.



The illustration represents a nail driven through a piece of wood by a weight W resting on the head of the nail. It is supposed that the resistance to the nail in the wood is equal to the weight W, so that the slightest additional force would cause the weight to drive the nail down to its head, as shown by the dotted lines.

In driving a nail into a piece of wood the resistance is not uniform, for the deeper the nail is driven in the greater is the resistance; but the mean force of resistance will always be as the following Formula 2.

Let s denote the space which the nail is driven into the wood by the weight.

Let the same nail be driven into the same piece of wood by the aid of a lever, as represented by Fig. 2. The force F, acting on the long lever L, presses on the nail equally to the weight W. The force of resistance F' to the nail in the wood, which is equal to the weight W, Fig. 1, acts on the short lever l. The fulcrum of the lever is at c.

The force F with the lever L is adjusted so that it balances the resistance F' acting on the lever l.

From the well-known law of levers we have

$$F : F' = l : L.$$

That is to say, the weight F is as much smaller than the weight W as the lever l is smaller than L.

Let S represent the space which the weight falls in pressing down the nail in the wood, and $s =$ the space the nail was driven in, which is the same as the space s.

It is well known in geometry that

$$s : S = l : L,$$

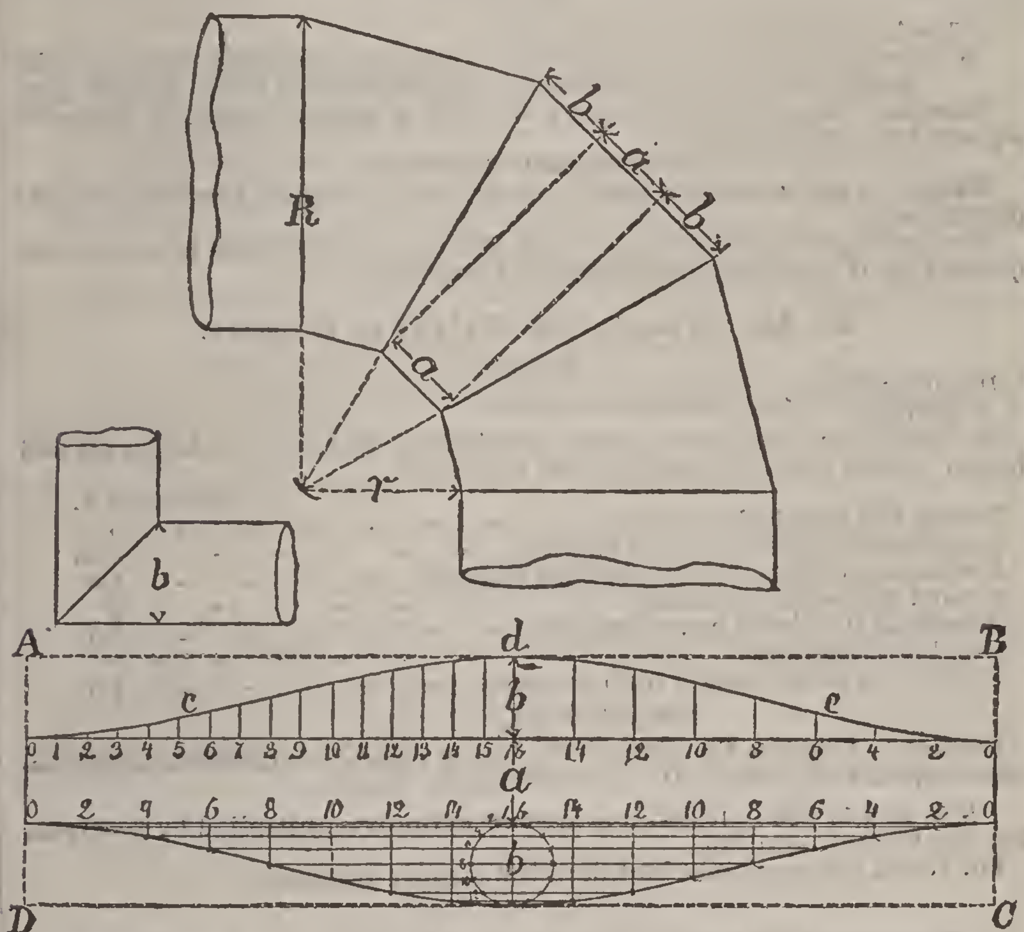
and, as $F : F' = l : L,$

we have $F : F' = s : S$, and $FS = F's$.

$$F = \frac{F's}{S}. \quad \dots \quad 1. \qquad S = \frac{F's}{F}. \quad \dots \quad 3.$$

$$F' = \frac{FS}{s}. \quad \dots \quad 2. \qquad s = \frac{FS}{F'}. \quad \dots \quad 4.$$

To Construct a Circular Elbow.



Having given the radii *R* and *r* of the circular elbow, divide the arc into as many sections as desired, say three, as shown on the illustration. Each section is cut out of a rectangular plate *A B C D*, of which the length *A B* is equal to the circumference of the pipe, and the breadth *B C* is equal to the outer side ($a + 2b$) of each section. The curve *c d e* is the same as the wave-line and constructed in the same way, as will be understood by the illustration.

The wave-line can be laid out by ordinates, for which divide half the length *A B* into 16 equal parts; and number them as shown. The adjoining table gives the length of each ordinate when the height *b* is the unit. Multiply each tabular ordinate with the actual height *b*, and the product is the actual ordinate to be laid down for the curve.

It is not necessary to use all the 16 ordinates, for half of them, or those with even numbers, may be sufficiently correct.

For a rectangular elbow pipe the height *b* is equal to the diameter of the pipe, and the length *A B* is equal to the circumference of the pipe.

Ab.	Ordinates.	Ab.	Ordinates.
1	.009607	9	.59755
2	.03806	10	.69134
3	.08426	11	.77778
4	.14644	12	.85355
5	.22221	13	.91573
6	.30866	14	.96194
7	.40245	15	.99039
8	.50000	16	1.0000

To Approximate the Weight of Steam Boilers.

The area of fire grate gives a nearer approximation to the weight of Marine boilers, than the heating surface.

Notation.

$\boxminus$ = total fire grate in square feet.
W = weight of the boiler in pounds, including fire bars, doors, smoke pipe, fire tools and appendages, but without water. $W = 800 \boxminus$.

Example. Required the weight W=? of a steam boiler of $\boxminus = 250$ square feet, grate surface.

$$W = 800 \times 250 = 200{,}000 \text{ lbs.}$$

Weight of the water is about three-fourths of W or of the total weight of boilers.

Weight of rivets, braces or stays, doors and fire bars, is about one quarter of W or of the total weight of boilers.

To Approximate the Weight of Engines.

Notation.

D = diameter } of cylinder in inches.
S = stroke }

W = weight of engine in pounds, including engine room tools, oil and tallow tanks, wheels, propeller and shafts.

	coefficient k.
Trunk and oscillating engines, - - - - - - - -	4
Direct action paddle wheel engines, - - - - - - -	4·25
Horizontal direct action propeller engine, - - - - -	4·5
Geared propeller engines, - - - - - - - - -	5·
American overhead beam engines, - - - - - - -	5·5
Side lever engines, - - - - - - - - - - -	6·
Horizontal direct action high pressure, - - - - - -	3·5

$$W = k\, D^2 \sqrt{S}.$$

Example. Require the weight W=? of a pair of Horizontal direct action propeller engines of $D=72$, $S=36$ inches, $k=4·5$.

$W = 4·5 \times 72^2 \sqrt{36} = 139968$ lbs. for one cylinder, multiplied by 2=279936 lbs. the weight required.

For trunk engines must be taken the largest diameter.

Practical Thickness in Decimals of an Inch of Good Plate Iron in Steam-boilers, Single Riveted.

P = steam pressure in pounds per square inch above atmosphere.

Press.	Diameter of Boiler in Inches.															
P.	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**120**	**150**	**200**
10	.10	.10	.11	.11	.12	.12	.13	.13	.14	.14	.15	.15	.15	.16	.17	.20
15	.10	.10	.11	.12	.13	.13	.13	.14	.15	.15	.16	.18	.19	.19	.22	.25
20	.11	.11	.12	.12	.13	.14	.14	.15	.16	.17	.18	.20	.20	.22	.26	.30
25	.11	.12	.12	.13	.14	.15	.15	.16	.18	.19	.20	.22	.23	.25	.30	.35
30	.12	.13	.13	.14	.14	.15	.16	.18	.19	.20	.22	.24	.25	.28	.33	.40
40	.12	.13	.14	.15	.16	.16	.18	.20	.22	.24	.26	.28	.30	.34	.40	.50
50	.13	.14	.15	.16	.18	.18	.20	.22	.25	.28	.30	.33	.35	.40	.47	.60
60	.14	.14	.16	.17	.19	.20	.22	.25	.28	.32	.34	.37	.40	.46	.55	.70
70	.14	.15	.17	.18	.20	.22	.24	.28	.31	.35	.38	.42	.45	.52	.60	.80
80	.15	.16	.18	.20	.22	.23	.26	.30	.34	.38	.42	.46	.50	.58	.70	.90
90	.15	.17	.19	.21	.24	.25	.28	.32	.37	.42	.46	.50	.55	.60	.77	1.0
100	.15	.18	.20	.22	.25	.27	.30	.35	.40	.45	.50	.55	.60	.70	.85	1.1
120	.16	.19	.22	.25	.28	.31	.34	.40	.46	.52	.58	.60	.70	.80	1.0	1.3
150	.17	.22	.26	.30	.33	.36	.40	.47	.55	.60	.70	.77	.85	1.0	1.2	1.6
200	.20	.25	.30	.35	.40	.45	.50	.60	.70	.80	.90	1.0	1.1	1.3	1.6	2.1

Punching Iron Plates.

To punch iron plates of from ¼ to 1 inch thick requires 24 tons per square inch of metal cut; that is, the circumference of the hole multiplied by the thickness of the plate is the area cut through.

Notation.

d = diameter of the punch or hole.
D = diameter of the hole in the die.
t = thickness of the iron plate.
All dimensions in 16ths of an inch.
W = weight or force in pounds required to punch the hole.
$W = 660\,t\,d$. $D = d + 0{\cdot}2\,t$.

Example 1. An iron plate of $t=6$ sixteenths of an inch thick, and the hole to be $d=12$ sixteenths in diameter. Required the force $W=$?

$$W = 660 \times 6 \times 12 = 47520 \text{ lbs., the answer.}$$

Example 2. Under the same conditions require the diameter $D=$? of the die.

$$D = 12 + 0{\cdot}2 \times 6 = 13{\cdot}2 \text{ sixteenths.}$$

Example 3. Required the diameter of piston for a direction action steam punch, for the plate and hole as in example 1, pressure of steam to be 50 lbs. per square inch.

Force $47520 = A \times 50$ of which area of piston will be $A = \frac{47520}{50} = 950{\cdot}4$ square inches, which answers to a diameter of 34·8, say 36 inches.

Shearing Iron Plates.

It requires the same force per section cut, for shearing as for punching, namely, 20 to 24 tons per square inch. If the shears are good, sharp, and well adjusted, 16 tons may be sufficient.

When the cutters in the shears are inclined to one another, the area cut, will be the square of the thickness of the plate multiplied by half the cotagent for the angle of the shears. Let v=angle of the shears, W and t same as for punching.

$$W = 88\,t^2 \cot.v.$$

Example 4. What force is required to cut a half inch plate $t=8$ sixteenths with a pair of shears forming an angle of $v=12°$? Cot.12°=4·7.

$$W = 88 \times 8^2 \times 4{\cdot}7 = 26470 \text{ lbs.}$$

Atmospheric Columns.

Water=33·95 feet.	2·3 feet for 1 lbs. per square inch.	
Seawater=33·05 ft.	2·23 "	"
Mercury at 60°=30 inches.	2·05 inches,	"
Atm. air=28183 feet.	1912 feet, "	"

Atmospheric air Required for each.

Blacksmith's forge,	100 to 200	Cubic feet per minute.
Charcoal forge,	400 to 500	
Finery forge,	800 to 1000	
Charcoal furnace,	1000 to 3000	
Anthracite furnace	2000 to 5000	

Cupola.

In a cupola of 3 feet 4 inches diameter, and 10 feet high, can be melted down 1000 lbs. of cast iron, 200 lbs. of bitumninous coal per hour, with a blowing machine of 4·5 horses making 1700 cubic feet of air per minute into a pressure of 2·25 inches of mercury at which the temperature of the blast will be about 70° Fah.

Steam-boiler Explosions.

THE steam-boiler is a reservoir of work. Each unit of heat in the steam and water is equivalent to a work of 772 footpounds.

The steam-table gives the units of heat per cubic foot, or per pound, in the steam and water at different temperatures and pressures. Work is the product of the elements force F and space, or $K = F\,V\,T$, when the force of the work will be $F = \frac{K}{V\,T}$. When the pressure in any part of a steam-boiler is suddenly removed, the entire work in the steam and water is at the same time started with a velocity proportionate to the removed pressure. The steam and water, in the form of a foam, strike the sides of the boiler, by which the work is suddenly arrested. If the time of arresting the work is infinitely small, we see from the above formula that the force of the blow will be infinitely great, and thus the boiler explodes.

Steam-boiler explosions are caused in various ways, namely:

1st. By long use boilers become corroded and, from neglect, give way in some unexpected place.

2d. The general construction with staying and bracing of steam-boilers is often very carelessly executed and results in explosion. This kind of explosion is often indicated, long before the accident occurs, by leakage of the boiler; when the engineer, not suspecting the approaching danger, limits the remedy generally to efforts to stop the leak. The leakage from bad calking, or packing, is easily distinguished from that of bad or insufficient bracing. In the latter case the fire ought to be hauled out, the steam blown off, and the boiler secured with proper bracing.

3d. Explosion is sometimes caused from low water in the boiler, but more rarely than generally supposed. When the fire-crown and tubes, subjected to a strong heat and not covered with water, the steam does not absorb the heat fast enough to prevent the iron from becoming so hot that it cannot withstand the pressure, but collapses from weakness.

4th. Steam-boilers often burst by strain in uneven expansion or shrinkage, occasioned by the fire being too quickly lighted or extinguished.

5th. It is a very bad practice to make boiler-ends of cast-iron, composed of a flat disc of from two to three inches thick, with a flange of from one to two inches thick, with cast rivet-holes. The first shrinkage in the cooling of such a plate causes a great strain, which is increased by riveting the boiler to it. Any sudden change of temperature, therefore, either in starting or putting out the fire, might crack the plate and thus occasion an explosion.

6th. In cold weather, when the boilers have been at rest for some time, they may be frozen full of ice: then, when fire is made in them, some parts are suddenly heated and expand, whilst other parts still remain cold, causing an undue strain, which may also burst the boilers. Such accident can be avoided by a slow and cautious firing.

7th. Sometimes a great many boilers are joined together by solid connections of cast-iron steam-pipes, which expand when heated, whilst the masonry enclosing the boilers contracts. Should such a steam-pipe burst from expansion or shrinkage, explosion will likely follow in all the connected boilers, of which numerous examples have occurred. Such accident may be avoided by making the connection elastic, or free to expand or contract without moving the boilers.

Steam-boiler explosion is thus not always caused by the pressure of steam alone, but often by the expansion and contraction of the materials of the boiler. A steam-boiler which is perfectly safe with a working pressure of 200 lbs. may explode with a pressure of 20 lbs. to the square inch.

The bursting of a boiler is a preliminary process to explosion. A boiler may burst without exploding. A boiler full of steam may burst, but never explode. It is the work in the heated water which makes the explosion.

It is evident from the results of explosions that a much higher pressure had been acting than the normal working pressure.

Destructive Work of Steam-boiler Explosion.

When a steam-boiler explosion takes place, the enclosed water is resolved into boiling hot water and steam.

The greatest precautions against explosion are sometimes unavailing. Careful attendants, regular cleaning, inspection by the authorized inspectors of the government or boiler insurance companies, will do much to prevent accidents. Close attention should be given to the safety-valve, which must be free and of sufficient area to blow out all the steam if the demands upon the boiler suddenly cease.

Opinions differ very widely as to the theoretical action of the steam and water in an explosion, but are of little consequence, as THE LEGAL DUTY OF EVERY USER IS TO TAKE STEPS WHICH SHALL PREVENT AN EXPLOSION.

Precautions against Fire on Steamboats.

Each steamer should have three buckets for every 100 tons measurement, plus 10 buckets. That is, a steamer of 800 tons should have $8 \times 3 + 10 = 34$ buckets. Also one axe for every 5 buckets.

U. S. Steam-boilers Inspector's Rule for Strength of Boilers.

Multiply one-sixth ($\frac{1}{6}$) of the lowest tensile strength found stamped on any plate in the cylindrical shell, by the thickness expressed in parts of an inch of the thinnest plate in the same cylindrical shell; and divide the product by the radius or half the diameter of the shell expressed in inches, and the quotient will be the steam pressure in pounds per square inch allowable in single riveted boilers, to which add twenty per centum for double riveting.

S = breaking-strain in pounds per square inch stamped on the plate.
t = thickness of the plate in fraction of an inch.
D = diameter of the boiler in inches.
P = steam pressure in pounds per square inch.

$$P = \frac{St}{3D} \qquad D = \frac{St}{3P} \qquad t = \frac{3DP}{S} \qquad S = \frac{3DP}{t}$$

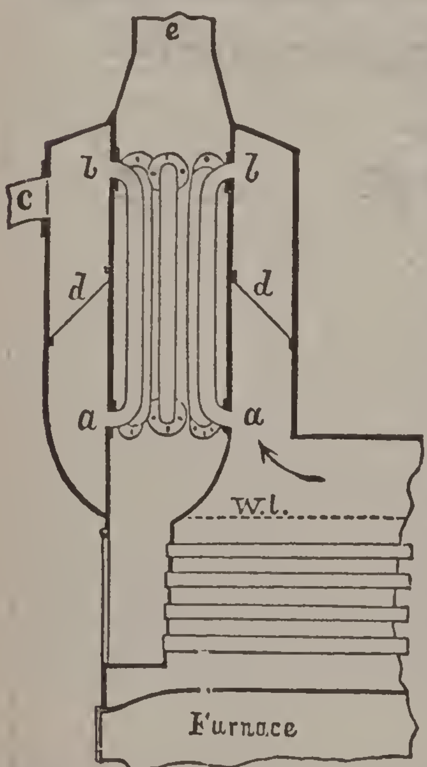
SUPERHEATED STEAM.

The Author's experience in superheated steam has been sufficient to convince him of its great importance. It appears that in order to utilize the maximum effect of steam or at least to attain the maximum result of expansion, it is not necessary to overheat it after a pure steam is formed, that is, when all the small particles and bubbles of water are evaporated. Water which accompanies the steam in such a form has the same temperature as that due to the surrounding steam pressure, preventing its vaporization; but when it passes through the superheating apparatus more heat is added, while the pressure remains the same because it being in connection with the steamroom in the boiler allows the water to vaporize and a pure steam may be formed.

If steam with particles of water is admitted into the cylinder part of the stroke and then allowed to expand, it is generally found that the end pressure does not come up to that of theory, from which it has been said that the expansive quality of steam does not follow that of a perfect gas, and that steam has condensed during the stroke; but if we knew the cubic contents of all the particles of water and subtracted that from the cubic contents of the steam it might be found that its expansive quality is not so far from that of a perfect gas. It appears also that the expansive quality is diminished by overheating pure steam.

The small particles of water contain a great deal more heat per volume than the surrounding steam, consequently when admitted into the condenser a good vacuum cannot be formed so well as with pure steam. It is therefore of great importance to pay particular attention to the superheating of steam, otherwise economy by expansion will not be realized to the extent herein given by formulas and tables. It is also of great importance that the piston and steam valves be perfectly tight.

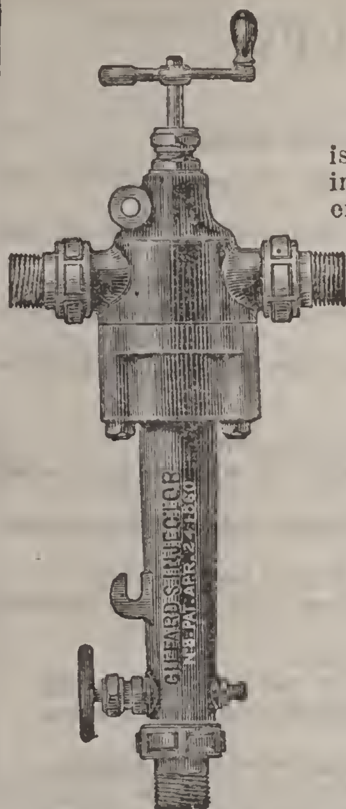
SUPERHEATING APPARATUS.



The accompanying figure represents a superheating apparatus such as the Author has built it in Russia, and is found to answer exceedingly well. The figure is a section of the forend of an ordinary tubular boiler with steamdrum and uptake. The chimney is made a great deal wider in the steamdrum and contracted to the usual size at *e*, of 0·16 times the area of the firegrate; if a strong fan blast is applied it may be better to contract it to 0·11 □. In the inside of the chimney are placed a number of copper tubes *a*, *a*, *b*, *b*, with flanges screwed to the side; the area of these tubes should be about four times that of the steampipe *c*. In the steamdrum is riveted steamtight a conical plate *d*, *d*, so that the steam cannot pass to the top without passing the superheating pipes. This superheating apparatus is in successful operation in three first class passenger steamers on the River Volga in Russia, each of 500 horse-power, and one in a steamer of 100 horse-power on the Black Sea.

The steamdrum can be placed around the chimney separately from the boiler and the steam led either above or below the plate *d*, *d*, by pipes from the steamroom, as may suit the circumstances.

This superheating apparatus may also be well suited for locomotives.

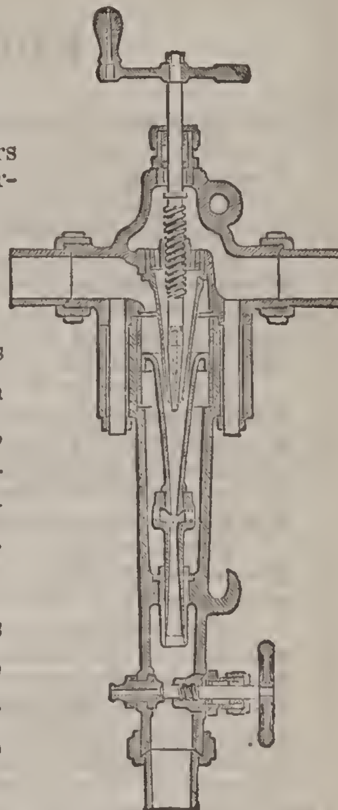


The Giffard Injector

is made by many manufacturers in different forms and at different prices.

The following table has been furnished by William Sellers & Co., Philadelphia, manufacturers of this injector. It gives the quantity of water injected per hour in cubic feet.

The first column No. is the size or diameter of the throat in French millimeters. The last column is the size in 16ths of an inch.



Capacity and Size of Giffard's Injector.

Size No.	Pressure of steam in pounds per square inch above atmosphere. 10	20	30	40	50	60	70	80	90	100	110	120	130	150	Size 16ths
2	8.3	9	9.7	10.4	11.1	11.8	12.5	13.2	13.9	14.6	15.3	16.0	16.7	18.1	1,26
3	19.3	21.0	22.8	24.6	26.3	28.1	29.9	31.6	33.4	35.2	37.0	38.7	40.5	44.1	1.89
4	36.6	39.6	42.7	45.9	49.0	52.1	55.3	58.4	61.6	64.7	67.8	71.0	74.1	80.4	2.5
5	57.6	62.5	67.4	72.3	77.2	82.2	87.1	92.0	96.9	102	107	112	116	126	3.6
6	83.5	90.6	97.7	105	112	119	126	133	140	147	155	162	169	183	3.78
7	114	124	133	143	153	162	172	182	192	201	211	221	231	250	4.41
8	149	162	174	187	200	213	226	239	251	264	277	290	303	328	5.04
9	189	205	221	237	254	270	286	302	318	334	351	367	383	415	5.67
10	234	254	274	294	313	333	353	373	393	413	433	453	473	513	6.30
12	337	366	395	423	452	481	510	539	567	596	625	654	682	740	7.56
14	451	491	531	571	611	651	691	731	771	811	851	891	931	1011	8.80
16	600	651	703	784	805	857	908	959	1010	1062	1113	1164	1215	1318	10.1
18	760	825	890	955	1020	1085	1149	1214	1279	1344	1409	1474	1539	1669	11.3
20	939	1019	1099	1179	1259	1339	1420	1500	1580	1660	1740	1820	1900	2061	12.6

Method of Working the Injector.

FIRST.—See that the steam-plug is closed down, and waste-valve stem is raised.

SECOND.—Admit steam from boiler to Injector, which should cause the water to flow from the waste pipe.

THIRD.—Turn up the steam-plug until the waste valve can be closed without causing the Injector to cease working.

FOURTH.—Turn up the steam-plug to increase the delivery, and down to decrease it. When this Injector has to lift its supply water, the steam valve between the Injector and boiler must be opened very slowly, until the water flows out of the waste pipe.

N. B.—A failure to work will always be indicated by an escape of steam and water from the waste check attached to check valve in water-supply pipe.

BLOWING ENGINES.

Notation.

D = diameter in inches, } of blowing cylinder double acting.
S = stroke in feet, }

l = part of the stroke S under which the air compresses from the atmospheric density to that in the reservoir.

F = mean resistance in pounds per square inch of the air on the cylinder piston (average for one stroke).

P = pressure in pounds per square inch of the blast in the reservoir.

C = cubic feet of air of atmospheric density, delivered from the blowing cylinder to the reservoir per minute.

H = actual horse power required to work the blowing engine, including 13 per cent. for friction.

d = diameter of blast pipe in inches.

n = number of revolutions or double stroke per minute.

A = area of supply valve to the blowing cylinder in square inches, at each end of cylinder.

p = vacuum in pounds per square inch, on the supply side of the cylinder piston, which should not exceed 0·1 lbs.

V = velocity of the blast through the tuyeres in feet per second.

v = velocity of the air through the supply valve A, in feet per second, which should not exceed 100 feet.

a = area of the orifice or tuyeres in square inches.

h = height of mercury in inches, in the gauge on the blast reservoir.

L = length of the blast pipe in feet from the receiver to the tuyeres.

k = volume coefficient, see Table.

t = temperature Fah. of the blast caused by compression or heating.

Example 1. *Formulæ* 8. For an Anthracite blast furnace is required 4000 cubic feet of air per minute, under a pressure of 6 inches mercury. Required the horse power necessary for the blowing machine? The effectual resistance F=2·365 lbs. see Table. Assume the vacuum to be v=0·09 lbs.

$$\text{We have } H = \frac{4000(2{\cdot}365 + 0{\cdot}09)}{198} = 49{\cdot}6 \text{ actual horse-powers.}$$

Example 2. *Formula* 10. Suppose the blast cylinder to be $D = 144$ inches diameter with $S = 15$ feet stroke. Required the number of double strokes per minute $n = ?$

$$n = \frac{96 \times 4000}{144^2 \times 15} = 1{\cdot}23.$$

Example 3. *Formulæ* 9. Under the above conditions, require the area of the supply valves A=? when the velocity v=105 feet, per second.

$$A = \frac{144^2 \times 15 \times 12{\cdot}3}{40 \times 105} = 911 \text{ square inches.}$$

Capacity of Blast Reservoir should not be less than the following proportions, but more is better.

For one single acting cylinder, 20 }
For one double acting cylinder, 10 } times the capacity of one cylind'r.
Two double act. cyl. cranks at 90° 5 }

One double acting cylinder, same as two single acting. The more cylinders the less capacity required for blast reservoir.

$$V = 246\sqrt{h\,(1+0{\cdot}00208\,t)}, \qquad t = 32 + 493\,(k-1),$$

$$P = 14{\cdot}7\,(k-1), \qquad t = 33{\cdot}55\,P + 32,$$

$$P = \frac{t-32}{33{\cdot}55}, \qquad k = \frac{P}{14{\cdot}7},$$

Approximate Formulas for Blowing Machines.

$$l=\frac{S\,h}{30+h}, \quad 1$$

$$P=0{\cdot}49h, \quad 2$$

$$C=\frac{D^2\,S\,n}{96}, \quad 3$$

$$C=\frac{198\,H}{F+p}, \quad 4$$

$$C=\frac{a\,k\,V}{26}, \quad 5$$

$$C=1{\cdot}83ah(30+h), \quad 6$$

$$H=\frac{D^2\,S\,n\,(F+p)}{19000}, \quad 7$$

$$H=\frac{C\,(F+p)}{198}, \quad 8$$

$$A=\frac{D^2\,S\,n}{40v}, \quad 9$$

$$n=\frac{96\,C}{D^2\,S}, \quad 10$$

$$d=\frac{\sqrt{C+10\,L}}{3} \quad 11$$

$$v=350\sqrt{p} \quad 12$$

$$v=\frac{D^2\,S\,n}{40\,A}, \quad 13$$

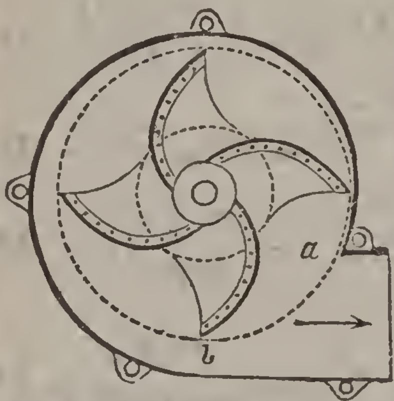
$$p=\frac{D^4\,S^4\,n^2}{180000000\,A^2}, \quad 14$$

$$k=\frac{30+h}{30}, \quad 15$$

Table for Blast and Blowing Machines.

Volume and temperat.		Guage in inches		Pressure lbs. sq inch.		Stroke.	Velocity.
k	*t*	*water.*	*h*	*P*	*F*	*l*	*V*
1·002	33°	1	0·073	0·036	0·032	0·0024	72
1·005	34·5	2	0·147	0·079	0·063	0·0049	102
1·007	35·5	3	0·220	0·108	0·095	0·0073	125
1·010	37	4	0·294	0·144	0·128	0·0097	144
1·012	38	5	0·368	0·180	0·159	0·0121	161
1·015	39·5	6	0·441	0·216	0·191	0·0145	176
1·020	42	8	0·588	0·288	0·253	0·0192	204
1·025	44·5	10	0·736	0·360	0·309	0·0239	228
1·030	47	12	0·884	0·432	0·379	0·0287	249
1·035	49·5	14	1·030	0·503	0·437	0·0334	269
1·043	53·5	17	1·250	0·612	0·531	0·0400	297
1·051	57·5	20	1·470	0·719	0·623	0·0467	322
1·062	63	24	1·766	0·863	0·745	0·0556	352
1·074	69	28	2·060	1·008	0·865	0·0643	381
1·082	73	31	2·281	1·116	0·955	0·0706	401
1·091	77.3	34	2·501	1·223	1·043	0·0769	420
1·100	82	37	2·720	1·332	1·130	0·0833	438
1·109	86·5	46	3·000	1·470	1·205	0·0908	460
1·116	90	47·5	3·500	1·715	1·431	0·1045	496
1·132	98	54·3	4·000	1·961	1·636	0·1178	530
1·165	114·5	67·7	5·000	2·450	2·010	0·1431	593
1·200	132	81·4	6·000	2·941	2·365	0·1667	650
1·265	164·5	108·5	8·000	3·925	3·088	0·2105	751
1·400	232	163	12·00	5·900	4·389	0·2859	918
1·500	282	203·7	15·00	7·375	6·875	0·3333	1077
1·625	344·5	254·6	18·75	9·217	8·831	0·3846	1393
1·750	407	305·5	22·49	11·06	10·67	0·4285	1590
1·875	469·5	356·4	26·24	13·90	11·64	0·4666	1760
2·000	532	407·4	30·00	14·75	12·50	0·5000	1955

FAN OR VENTILATOR.



Fans constructed as the accompanying figure have been found by the Author who has made several of them, to be the most effective.

The vanes are each one quarter of an arithmetical spiral with a pitch twice the diameter of the fan, that is, each vane should be constructed in an angle of 90° from centre to tip. Length of fan to be from $\frac{1}{3}$ to $\frac{1}{4}$ the diameter. Inlet to be half the diameter of the fan. Number of vanes to be not more than six, and not less than four. Six vanes work softer and better, but they give no better effect than four.

The housing should be an arithmetical spiral with sufficient clearing for the fan at a, and leaving a space at b about $\frac{1}{8}$ of the diameter. Fans of this construction make no noise.

Notation.

d = diameter, l = length } of fan in inches.
A = area in sq. in., L = length in feet, } of blast pipe, to be as straight as possible.
a = area in sq. in. tuyeres or outlet.
C = cubic feet of air delivered per minute.
h = inches of mercury.
v = velocity in feet per second through a.
k = volume coefficient, see Table, page 441.
n = revolutions of fan per minute.
H = actual horse power required to drive the fan.

Formulas for Fans.

$$h = \frac{d\,n^2}{50000000}\sqrt{\frac{d\,l}{25\,a + d\,l}}, \qquad 1$$

$$H = \frac{d\,l\,h\,n}{24000}, \qquad 2$$

$$h = \frac{24000\,H}{d\,l\,n}, \qquad 3$$

$$v = 244\sqrt{h} \qquad 4$$

$$A = a\sqrt{L} \qquad 5$$

$$v = \frac{n\sqrt{d}}{28{\cdot}86}\sqrt[4]{\frac{d\,l}{25\,a + d\,l}}, \qquad 6$$

$$n = \frac{24000\,H}{d\,l\,h}, \qquad 7$$

$$C = \frac{v\,a\,k}{2{\cdot}6}, \qquad 8$$

$$C = 94\,a\,k\sqrt{h} \qquad 9$$

$$A = \frac{C}{94\,k}\sqrt{\frac{L}{h}}. \qquad 10$$

Example 1. A fan of d=36 inches diameter, l=12 inches, making n=725 revolutions per minute, area of tuyere being a=25 square inches. Required the density of the blast in inches of mercury h=?

$$\textit{Formulæ 1.}\quad h = \frac{36 \times 725^2}{50000000}\sqrt{\frac{36 \times 12}{25 \times 25 + 36 \times 12}} = 0{\cdot}242 \text{ inches.}$$

Example 2. Under the same conditions require the cubic contents or air delivered per minute, C=? k=1·01 the nearest in the Table.

Formulæ 9. $C = 94 \times 25 \times 1{\cdot}01\sqrt{0{\cdot}242} = 1167{\cdot}7$ cubic feet. Required the horse power H=?

$$\textit{Formulæ 2.}\quad H = \frac{36 \times 12 \times 0{\cdot}242 \times 725}{24000} = 3{\cdot}16 \text{ horses.}$$

BLAST OR IRON FURNACES.

It is almost impossible to foresee the many variable circumstances connected with the performances of Blast Furnaces. The data herein given are deduced as an average from the performances of a great many furnaces both in America and Europe.

The accompanying Tables are so arranged that the numbers in Table I., multiplied by the numbers in Table II., gives the corresponding charge of Iron ore, lime stone, coal, and the produce of pig iron in pounds per 24 hours, with the consumption of air in cubic feet per minute.

Table II. contains the effective capacity of blast furnaces in cubic yards.

Example. It is required to construct a blowing machine for an Anthracite blast furnace of 12 feet diameter of boshes, height of stack 45 feet, to be worked with hot blast. Required the produce of pig iron per 24 hours, cubic feet of air per minute and actual horse power of the blowing engine?

Produce of pig iron 155 Table I.×123 Table II.=19065 lbs. or 8·5 tons per 24 hours.

Consumption of air 20×123=2460 cubic feet per minute. Suppose the blast to be blown into the furnace at a pressure of $P=2{\cdot}94$ lbs., vacuum in the supply side in cylinder to be $p=0{\cdot}07$ lbs. we shall have the required actual power.

$$H = \frac{2460\ (2{\cdot}36 + 0{\cdot}07)}{198} = 30{\cdot}2 \text{ horses.}$$

Formula 8, p. 591.

Consult Bauerman, *Metallurgy of Iron.*

Table I. Iron or Blast Furnaces.

The unit being the capacity of the Furnace in cubic yards.		*Charge and produce per 24 hours.* Iron Ore.	Pig Iron.	Lime Stone.	Coal.	*Air per minute.*
		lbs.	lbs.	lbs.	lbs.	cub. feet.
Soft charcoal	Cold blast,	535	215	196	400	24
	Warm blast,	700	292	256	350	19
Hard charcoal	Cold blast,	670	270	245	400	24
	Warm blast,	875	365	320	350	19
Goke	Cold blast,	268	108	98	515	26
	Warm blast,	350	146	128	397	20
Bituminous	Cold blast,	252	101	92	515	24
	Warm blast,	327	136	120	397	19
Anthracite	Cold blast,	287	115	105	515	26
	Warm blast,	373	155	137	597	20

Table II. Capacity and Dimensions of Iron Furnaces.

Diameter of Boshes in ft.	*Height of stack in feet.* 25	30	35	40	45	50	55	60
8	40	44	47	51	54	58	62	65
9	50	55	60	64	69	73	78	83
10	62	68	74	79	75	91	96	102
11	75	82	89	96	103	110	117	123
12	90	98	106	114	123	130	139	147
13	105	115	125	134	144	153	163	172
14	121	133	145	155	167	178	189	200
15	140	153	166	178	191	204	217	230
16	160	174	189	203	217	232	247	261
17	280	197	213	229	245	262	279	295
18	202	220	239	257	275	293	312	330

THE EXPANSION OF STEAM.

(*Continued from page 538.*)

To find the most economical point of cut-off—that is, its inverse that number of expansions which will result in the greatest economy of steam from the boiler per horse-power per hour.

Notation.

e = the true point of cut-off = the reciprocal of the true number of expansions.

B = the absolute back-pressure during exhaust in pounds per square inch.

P_b = the absolute pressure at cut-off.

s = the stroke of cylinder in feet.

d = the diameter of cylinder in feet.

$A = \frac{62{\cdot}5}{S}$.

S = the specific volume of steam at cut-off.

$D = 2\,\frac{T_b - T_e}{N}\,C$.

T_b = the temperature of the steam at cut-off (Fahr.).

T_e = the temperature of the steam during exhaust.

N = the number of strokes per minute = twice the revolutions of crank.

C = the constant of condensation = ·018 pounds of steam for about 82 pounds gauge-pressure.

$$e = \frac{B}{P_b} + \left(\frac{1}{s} + \frac{{\cdot}194}{d}\right)\frac{Dd}{Ad + D}\ \text{nat. log.}\ \frac{1}{e}.$$

Example:

Let P_b = 100 pounds absolute.
" B = 15 " "
" s = 4 feet.
" d = 1·5 "
" N = 150 per minute.

We have—

$A = {\cdot}233$.
$D = {\cdot}0274$.

$$e = 0{\cdot}15 + \left(\frac{1}{4} + \frac{{\cdot}194}{1{\cdot}5}\right)\frac{{\cdot}0274 \times 1{\cdot}5 \times 2{\cdot}3026}{{\cdot}233 \times 1{\cdot}5 + {\cdot}0274}\ \text{com. log.}\ \frac{1}{e}.$$

$$e = 0{\cdot}15 + {\cdot}3793\,\frac{{\cdot}0944}{{\cdot}3764}\ \text{log.}\ \frac{1}{e}.$$

$$e = {\cdot}15 + {\cdot}0952\ \text{log.}\ \frac{1}{e}.$$

We must solve this transcendental equation tentatively, trying values until the two members balance.

Assume $e = \frac{1}{5}$ of stroke plus clearance. We have

$${\cdot}20 = {\cdot}15 + {\cdot}066 = {\cdot}216.$$

This error of ·016 is closer work than can be realized in practice, and we can take 5 expansions as the best number.

Between $\frac{1}{5}$ and $\frac{1}{4}$ would have been near enough for all practical purposes.

To find the proper ratio of stroke to diameter under the given conditions, assuming 5 expansions and diameter = 1½ feet.

Inverting the above equation, we have

$$s = \frac{d}{\left(\frac{A}{D}d + 1\right)\left(\frac{e - \frac{B}{P_b}}{\text{nat. log. } \frac{1}{e}}\right) - 0{\cdot}194.}$$

$\frac{A}{D} = 8{\cdot}56.$

$$s = \frac{1{\cdot}5}{\left(8{\cdot}56 \times 1{\cdot}5 + 1\right)\left(\frac{{\cdot}20 - {\cdot}15}{1{\cdot}61}\right) - {\cdot}194} = 6.4 \text{ ft., nearly.}$$

With slow-moving engines it will be found that long stroke is most economical, while on the other hand high-speed engines require short stroke for greatest economy. If we double the speed of this engine, making $N = 300$, the stroke $s = 2{\cdot}4$ feet, for greatest economy.

Every detail of the steam-engine will be found discussed in a rational and practical manner in *The Relative Proportions of the Steam-Engine*, by Wm. D. Marks (3d Ed., J. B. Lippincott Co.).

The subject requires more space than can be given in a pocket book.

ELECTRO-DYNAMICS.

(*Begun on page* 644.)

These results show that this high efficiency is not extraordinary, but is and should be attained by all dynamo-makers building similar types.

In the case of the Weston (7 M.) dynamo, already specified, the power applied was distributed as follows in the first full load test:

Friction and wind resistance of armature	·0106	total.
Electrical energy lost as heat in armature	·0559	"
" " " in creating field	·0170	"
" " " in external circuit	·8992	"
Total of power accounted for	·9827	
Electrical energy lost in eddy currents, heat, and otherwise unaccounted for	·0173	
Total power as per dynamometer	1·0000	

This differs from the average already quoted, because slight variations of the conditions would cause any of the machines to vary somewhat in their percentages.

The greatest cause of uncertainty in experiments heretofore made upon the transformation of mechanical power has been our lack of certainty of accuracy in the measurements of the mechanical power driving the machine. The dynamometer must sum up the whole power yielded to the dynamo with as great accuracy as is possible for all other measurements in part.

This dynamometer must be capable of being standardized by absolute measurement, and, after being standardized, the machines to be tested must be able to be attached to it or removed from it without altering the centres or adjustments of the dynamometer. It must be of great sensitiveness to small variations of load, while measuring large amounts of power with great steadiness.

All of these conditions were fulfilled by the dynamometer invented by Mr. Wm. P. Tatham, President of the Franklin Institute. Its extreme capacity is 100 horse-power, and yet, while making 1040 revolutions per minute, carrying a load of 29 horse-power, it was possible to measure with certainty the difference of power required by an Edison voltmeter requiring two-tenths of a horse-power. It announced at once the making and breaking of the circuit of this voltmeter, measuring the work lost in it with accuracy. Still other tests showed its capability to promptly register small changes of power while carrying great loads, and proved that the slight and rapid jar of the parts, due to a high speed, increased its sensitiveness of measurement.

Finally, this dynamometer was calibrated by the agitation of water, heating something over five tons of water through 15·5° C., giving, as the mechanical equivalent of heat, 772·81 foot-pounds per British unit of heat.

While less can be claimed in the way of originality of apparatus or methods used in the electrical measurements of these tests, an examination of the precautions will convince one of the extreme care taken to obtain correct results. (See "Competitive Tests of Dynamo Electric Machines," *Journal Franklin Institute*, Nov., 1885.)

The dynamo electric machine has grown out of the fact that, if we move a dead wire in the field of another fixed wire, through which a current is passing, the dead wire will have a current generated in it whose electro-motive force is proportional to (1) the intensity of the current in the fixed wire, (2) to the velocity of motion of the moving wire, and (3) to the acting length of the moving wire.

If we take a single wire, Fig. 2, and pass a current through it, its field will resemble a whirlpool, of which the wire is the centre.

If we take two wires and place them a short distance apart, Fig. 3, and pass a current in the same direction through both, their fields will combine to form an oval field, and any number of adjacent wires, with currents in the same direction, will do the same thing, forming a field of an intensity proportional to the number of wires and the intensity of the current in each. The field would, however, be of loose texture, so to say, and the lines of force far apart if the lines of force formed themselves around the wires in the air.

Iron, because of its great permeability to the lines of magnetic force, enables us to concentrate this field, and to place it, so to speak, where we desire to use it.

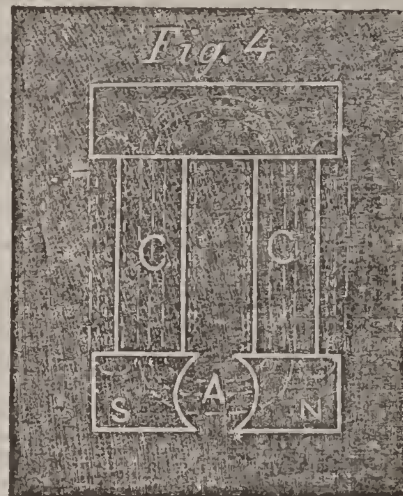
Pure, soft wrought iron may be said to be 20,000 times more permeable than air.

We will see from Fig. 4 what is meant. The wrought-iron cores *C* afford the easiest path for the lines of force, and they therefore follow them until they reach the armature-space *A*, between *N* and *S*, where they take their airy path across, *because the lines of force must always close.*

We see that we have thus managed to concentrate the lines of the field of a large number of coils in a small space *A*. In this space the wires of the armature are revolved so as to generate a current which is either alternating or approximately continuous.

The details of armatures and winding of them, as well as of the commutators, will be found described at considerable length in the works on "Dynamo Electric Machinery," written by Dr. Schellen or Prof. S. P. Thompson.

There is nothing written better calculated to give the novice a clear idea of the principles involved in a continuous-current machine than Pacinotti's own description of his machine, which can be found in the translation of Schellen by Dr. Keith, on page 209.



Taking matters as they are, the most economical engine used for the purpose of driving dynamos at the Electrical Exhibition of the Franklin Institute required about 30 pounds of steam at 90 to 100 pounds pressure, and the most economical boiler evaporated about 8 pounds of water per pound of anthracite coal at the same pressures. That is, an indicated horse-power required 3¾ pounds of average anthracite.

It can be assumed, with close approximation to average correctness, that 15 per cent. of the indicated horse-power is lost in the most direct method of transmission of power from engine to dynamo.

So we can say that one utilizable electrical horse-power per hour may, in good practice, be obtained from

$$\frac{3{\cdot}75}{85} \times \frac{100}{\cdot 90} = 4{\cdot}\tfrac{90}{100} \text{ pounds of coal}$$

(such as is sold in the open market as chestnut anthracite), and neglect the loss of electrical energy in the conductors.

The carbon equivalent of the coal used was 91 per cent. by weight.

Assuming 14,500 British units as the heat per pound of carbon, we have

$$4{\cdot}90 \times {\cdot}91 \times 14{,}500 = 64{,}655 \text{ British units of heat.}$$

Assuming the mechanical equivalent of one British unit as 774·1 foot-pounds, we have very nearly 2558 British units for one horse-power per hour. Dividing the last by the first, we find that nearly 4 per cent. of the power latent in the coal appears as electrical power in the circuit; 96 per cent. of our potential energy is lost, principally in the steam-engine.

These facts, taken from the labors of many impartial and skilful workers in scientific research, do not correspond with the alluring statements frequently set before us, but are reliable and practical.

The broad lesson to be drawn from them is that we do not obtain $\frac{1}{25}$ of the power in coal in the form of electricity, and that $\frac{24}{25}$ remain to be obtained by the discoverer of an economical method of direct conversion of heat into electricity.

When the direct method of conversion of heat into electrical energy yields a larger percentage of the power in coal than the indirect method which has just been described, at the same cost, then will the dynamo supplant the steam-engine. Until then it must remain what it is—a distributor of power for the steam-engine or other mechanical motor.

The Conversion of Electrical Power into Light.

There are at present in use two methods of converting electrical power into light. The first and apparently the most economical is by means of the voltaic arc between carbon points; the second, by means of the incandescence of a carbon filament in a vacuum.

THE ARC LIGHT.

The first method is open to severe criticism, save on the point of economy, and for lighting large spaces.

The briefest look at the intense spot of light formed by the arc between the points of carbon causes a painful and persistent image on the eye. The light has a vicious way of hissing, which becomes unendurable to sensitive nerves, and it varies the monotony of this noise by sudden jumps and flickers. Its ghastly effects are due to its bluish color and the deep, sharply-defined shadows.

In some cases the arc has a way of rotating around the axis of the carbons, which also causes variations of the intensity of the light in different directions.

Opal glass globes, which cut off something more than one-half the light, are required to make the light tolerable; and as for the lamp itself, the greatest skill and taste of designers has not yet rendered it ornamental when not lighted.

As a rule, the arc light is most intense when viewed at an angle of 45° from the vertical, and for this reason it is usually used for lighting open spaces from a considerable height. Its power is assumed as an average of the illumination at 30°, 45°, and 60° from the vertical.

From the report on Electric Lamps of the Franklin Institute, June, 1885, the following data are taken:

MACHINE.	Angle with Vertical.	Candles.	Candles per El. H. P.	Average Candles per El. H. P.
Arago Disc	30°	645	783	
	45	583	708	685
	60	465	565	
Ball	30	182	421	
	45	485	1123	916
	60	520	1204	
Brush (1200 e.-p.)	30	355	762	
	45	613	1316	1076
	60	537	1152	
Brush (2000 e.-p.)	30	1200	1529	
	45	1373	1750	1553
	60	1082	1379	
Diehl	30	887	1176	
	45	830	1101	1079
	60	725	961	
Richter	30	603	743	
	45	894	1101	1009
	60	960	1183	
Van Depoele, 20 lights	30	670	780	
	45	1377	1604	1206
	60	1060	1235	
Van Depoele, 60 lights	30	500	612	
	45	1162	1423	1045
	60	900	1101	
Western Electric	30	75	121	
	45	266	431	376
	60	355	575	
Average				994

The average candles per electrical horse-power obtained from measurements upon the Arago disc, Ball, Brush, Diehl, Richter, Van Depoele, and Western electric machines was 994.

The efficiency of these arc-light machines was not obtained, but we are justified in assuming that 70 per cent. of the absorbed power should reappear as electrical power in the circuit, neglecting its losses.

That is, $\frac{3{\cdot}75}{{\cdot}85 \times {\cdot}70} = 6{\cdot}3$ pounds of ordinary anthracite coal per electrical horse-power per hour.

$$\frac{994}{6{\cdot}3} = 158 \text{ candles.}$$

If we divide the candles per electrical horse-power by the weight of coal required to produce them, we find in the arc system that we obtain 158 candles per pound of coal for the naked light, and something less than 75 candles if ground glass or opal globes are used and the light seen from the most favorable position.

THE INCANDESCENT ELECTRIC LIGHT.

Very different from the arc light is the incandescent. Its light is so soft that we do not realize its brilliancy until we submit it to measurement. It gives out no products of combustion to poison our air; it shows colors truly. A delicate hair of carbon, sealed within a vacuum by walls of glass, glitters and glows until at almost limpid incandescence it gives us a light steady, clear, colorless as daylight.

If we take a book and hold it from one to two yards away from a sixteen-candle light, we find the light sufficiently diffused to read with comfort.

Now, all know that the intensity of illumination varies inversely as the square of the distance. Therefore, roughly estimating a shaded arc light at 500 candles, the same book would have to be held somewhere between five and one-half and eleven yards away from it to be read with equal comfort, assuming the light to be steady. We can then say that a sixteen-candle incandescent light will illuminate a circle of 12½ square yards area, and that a shaded arc light giving 500 candles out of 994 will illuminate a circle of 400 square yards area, or 32 times as great. That is to say, about 32 sixteen-candle lamps would supply an equal illumination with a vastly better distribution of light for the use of the eyes.

We can therefore say that 500 candle-power from incandescent lamps will far more than replace 1000 candle-power from the arc light, under the conditions of actual usage.

We can safely say that, for all purposes save that of obtaining light to dispel darkness, the incandescent light is twice as valuable, light for light, as the arc light, and therefore should be multiplied by 2 when compared with it.

The objections most vehemently urged against incandescent lamps have been their short life and lack of economy; this is not true of them in all cases.

The first public test of the life of incandescent lamps was made by the Franklin Institute in the early months of 1885 (*Journal of the Franklin Institute*, Sept., 1885). The record of these tests is given in a pamphlet of some 130 pages, and with a detail which renders it impossible, in our limited space, to do more than gather from its averages such general lessons as we may learn.

From the efficiency test, which was preliminary to the prolonged-duration test, we find that 194·1 spherical candles were realized per electrical horse-power.

	Spher. Candles.
Edison's 97-volt lamps, per El. H.-P.	169·2
Stanley's 96 " " " "	189·1
" 44 " " " "	216·1
Woodhouse & Rawson's 55-volt lamps, per El. H.-P.	209·0
" " " 55 " " " "	210·8
White's 50-volt lamps, per El. H.-P.	182·6
Weston's 110½-volt lamps, per El. H.-P.	209·8
" 70 " " " "	166·3
Average, per El. H.-P.	194·1

The committee was forced by the different forms of carbon filament used to take the illuminating power of the lamps from all points, and to call the mean the spherical intensity of illumination. This procedure perhaps gives a better idea of the practical value of the incandescent lamp, because it is customary to place these lamps in any position that convenience may dictate.

The incandescent lamp, by reason of its smaller quantity of light and better distribution, is worth at least twice as much as the arc light. One electrical horse-power costs, with Weston's incandescent dynamo electric machine, about 4·9 pounds of ordinary anthracite. Therefore one pound of coal will give about 40 candles by the incandescent lamp, and this is equivalent to 80, and probably many more, candles by the arc light, whenever we have to use our eyes for any purpose save guarding our footsteps.

You will recall that, under assumptions most favorable to the arc light, we probably do not get more than 75 candles per pound of anthracite from the shaded arc light. Had the Committee on Arc Lights obtained the spherical intensity of illumination of these lamps, their showing could have been made much less favorable than the one given. The present method of arc lighting must ultimately give way before the incandescent light, save for large spaces not requiring a close use of the eyes.

The low potential and larger current of the incandescent dynamos render necessary a lower resistance in the conductors, and so the cost of wiring for incandescent-lamps is much greater, because of the increased weight of copper wire demanded to convey the current without too great a loss in the form of heat. This is the pecuniary obstacle, and about the only one that prevents the entire disappearance of the arc light before the incandescent light. Could an incandescent lamp be made of sufficiently high resistance to enable the use of high potentials, the last objection to the system would vanish.

The Edison 97-volt lamp in this test outlived all the others, demanded the least weight of conductors, and was 13 per cent. less economical of power. It was the only lamp in the test that justified a claim to 1000 hours of life.

Out of 20 lamps entered by this company, 19 survived a continuous test of 1006 hours.

The more successful lamps were found to undergo a process of gradual degradation which is attributable to two causes—an increase of the resistance of the carbon filament, and a deposit of carbon upon the interior of the glass of the lamp.

The discoloration of the various lamps was carefully compared after their life had ceased, and was remarkably deep in the case of the Woodhouse & Rawson and the Stanley 44-volt lamps. Indeed, it would seem as if this discoloration was in some wise proportional to the economy of the lamp, as these two were the most economical of the makes of lamps entered.

A lamp may live a long time and yet be of little value for the purpose of giving light, because of this degradation. If you will take a lamp which has been used some time, and lay it upon a white handkerchief, the gray coloring-matter on the globe will be brought out quite distinctly.

Thus we see that great length of life with little usefulness may be attained by lamps. Indeed, the Edison lamps, which outlasted all others, had lost 36 per cent. of their illuminating power at the end of 1006 hours.

The direct conversion of heat into electrical energy by Clamond's stoves only produced 26 candle-power per pound of coke, as against 40 candles per pound of anthracite in the usual way with incandescent lamps.

It will be a surprise should not the direct conversion of heat into electrical power prove to have quite as many difficulties and as narrow limits as the conversion of heat into mechanical power by means of the steam-engine.

The Conversion of Electrical Power into Mechanical Power.

The problem which just now is demanding of electricians their most earnest effort is the transmission of work by means of electricity. This effort will be repaid by the utilization of otherwise inaccessible water-powers, and the problems of locomotion will have their simplest and least objectionable solution when it is an accomplished fact.

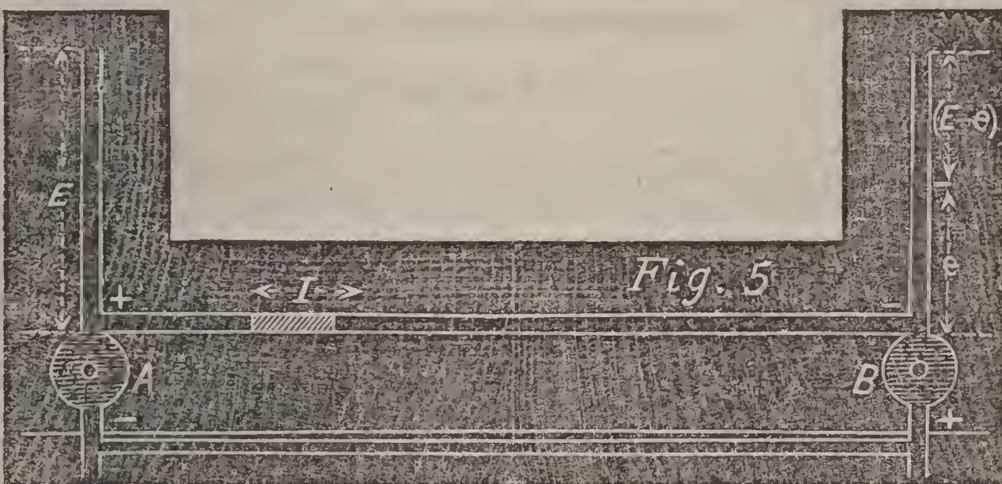
Marcel Deprez has recently transmitted 60 horse-power from Creil to Paris with a mechanical efficiency of 53 per cent.

The expense attendant upon an experiment of this magnitude has been very great, but, the scientific possibility once proved, we can rely upon the progress of manufactures to reduce this expense and to define the limits within which power can be economically delivered.

Seventy-five per cent. of the indicated power of the engine is not an overestimate of the power required to move the cable alone for our cable-cars on a road of two or three miles' length, but it would at once condemn an electric railway, which should be made to yield a practical efficiency of over 50 per cent.

Before discussing the details of the transmission of power, we will, with the aid of our previously-used hypothetical fluid and pipe, and with two pumps to represent the dynamo and motor, endeavor to make clear the laws controlling the transmission of power by electricity.

Assume two pumps, Fig. 5, *A* and *B*, connected by a closed line of pipe, so that the fluid must be pumped round a closed circuit. Let the pump *A* be driven by means of any external power. Let the pump *B* be reversed and acting as a motor. Let each of these pumps have a vertical standpipe projecting from its top, which will show the head *E* or *e*



resulting from its action. The pump *A* acts under the law that its head *E* is proportional to the speed at which it is driven. The motor *B* acts under a similar law that its counterhead *e* is proportional to the speed at which it is allowed to run. The weight of fluid per second passing through the conduit is directly proportional to the difference of these heads, and inversely to the resistance.

Let *I* equal the weight of fluid passing along the pipe each second.

The fluid passing along the pipes between *A* and *B*, and through the pumps, will lose, each second, a certain amount of head per unit of weight because of the resistances.

Let *R* equal this resistance. Then we can say

The weight of fluid per second is then directly proportional to the effective head, and inversely proportional to the resistance. This is Ohm's law, which for electricity is, *The intensity of the current is directly proportional to the difference of potential, and inversely as the resistance.*

The loss of power per second in friction in the pipes is the loss of head multiplied by the weight per second. This is Joule's law for loss in heat for electricity:

Work per second of pump A = head $\times$ wt. per second.

" " " " " " $= E\,I = \frac{E\,(E-e)}{R}$.

" " " lost in pipe $A\,B = I^2\,R = \frac{(E-e)^2}{R}$.

R = head lost per unit of weight per second.

Ohm's law:

$$I = \frac{E-e}{R} \left\{ \begin{matrix}\text{Weight} \\ \text{per second}\end{matrix} \right. = \left\{ \frac{\text{diff. head}}{\text{head lost per unit of weight per second}} \right\}.$$

Joule's law:

$$\text{Work lost per second} = I\,(E-e) = \frac{(E-e)^2}{R} = I^2\,R.$$

$$\text{Work per second of motor } B = \text{head} \times \text{wt. per second.}$$

$$\text{“ “ “ “ “ “} = e\,I = \frac{e\,(E-e)}{R}.$$

$$\text{Theoretical efficiency} = \xi = \frac{e}{E}.$$

Work of motor B per second a maximum for $e\,(E-e)$ maximum; that is, $e = \frac{E}{2}$.

$$\text{Greatest work of motor } B = \frac{E^2}{4\,R}.$$

$$\text{“ “ pump } A = \frac{E^2}{2\,R}.$$

The practical efficiency of this combination of pump and motor will be diminished because the head E will require a coefficient greater than unity, and the counterhead e a coefficient less than unity.

$$\xi = \frac{e\,(1-x)}{E\,(1+X)}.$$

The value of $\frac{1-x}{1+X}$ must be determined by experiment.

It will at once be seen that when the motor is acting at its greatest horse-power the theoretical efficiency is 50 per cent. and its practical efficiency still less, for we must introduce $\frac{(1-x)}{(1+X)}$ as a factor of $\frac{e}{E}$.

On the other hand, if we increase the counterhead e, the efficiency of the motor B increases proportionally, but the weight of fluid per second $\frac{E-e}{R}$ becomes less and less, and the work of the motor B per second decreases as $\frac{e\,(E-e)}{R}$ decreases.

But the work of the pump A per second also decreases as $\frac{E\,(E-e)}{R}$ decreases, and the lost work due to resistance to flow through pipe and pumps decreases as $\frac{(E-e)^2}{R}$ decreases.

With a theoretical efficiency of 50 per cent. an electrical motor is doing the largest amount of work in horse-power of which it is capable, but it demands of the generating dynamo twice as much power as it gives out. With a greater theoretical efficiency, it does not turn out so much work per second, but it makes a demand of less than twice its work upon the generating dynamo.

This statement requires to be modified somewhat because of the imperfections of machines, and becomes more accurate in proportion to the perfection of the machines used.

This work lost in resistance of the pipe is plainly a minimum for $E = e$, and a maximum for $e = o$ if R remains constant. Let us separate it into its component parts.

We see from the above equation that $\frac{(E-e)^2}{R}$ = constant, when R varies as the square of the difference of the heads, or when the square root of R varies as the difference of heads.

If now we assume the resistance of the pumps as trifling in comparison with that of a long pipe or pipes connecting them, we see that we must double the difference of heads $(E - e)$ in order to have the same loss of work per second with a pipe four times as long.

Increasing the resistance four times gives us only one-quarter the weight of fluid per second assumed to be passing through the pipe, but doubling the difference of heads also doubles the weight of fluid per second, so that, under the altered conditions, we obtain one-half the weight of fluid per second, and twice the effective head. Therefore, the work per second lost in the pipe is

$$2(E - e) \times \tfrac{1}{2} \frac{(E - e)}{R} = \frac{(E - e)^2}{R}.$$

as before.

The work done by the pump and motor, each working with twice its former head, remains the same as before, and their relative efficiency is the same.

This is what Marcel Deprez meant when he said,

"*The useful mechanical work and the efficiency remain the same whatever be the distance of transmission, provided the electro-motive forces, positive and negative, vary proportionately to the square root of the circuit's resistance.*"

Your attention should be called to the deadly nature of the very high electro-motive forces demanded by this law in the case of great distances.

The experiments of Marcel Deprez on the transmission of power by electricity have been conducted on a larger scale than others have yet attempted.

In 1881, at the Paris Electrical Exhibition, he exhibited in the Palace of Industry one dynamo furnishing power to 27 different pieces of apparatus. No measurements of efficiency were made, as the question of distribution was the only one then to be solved. He, however, then stated that it was possible to transport a useful work of 10 horse-power 31 miles by means of an ordinary telegraph wire, with the expenditure of only 16 horse-power on the generating dynamo, realizing 62½ per cent. mechanical efficiency.

At the Munich Electrical Exhibition of 1882, over a line of telegraph-wire 36 miles, he obtained an electrical efficiency of 39 per cent. and an actual mechanical efficiency of 30 per cent. In his experiments on the lines of the Chemin de fer du Nord, March 4, 1883, he transported 5·6 horse-power 8½ miles over ordinary telegraph-wires with 9·7 horse-power at the generating dynamo, realizing an electrical efficiency of 69½ per cent. and a mechanical efficiency of 58·3 per cent.

In his experiments announced October 16, 1885, he obtained from his first 77 per cent. electrical and 47·7 mechanical efficiency. In the second experiment he obtained 78 per cent. electrical and 53·4 mechanical efficiency by means of dynamometric measurements. The distance between these two points is 56 kilometres—about 35 miles. The speed of the generator varied from 170 to 190 times a minute, and there was no appreciable heating.

Tabulated results of experiments of Marcel Deprez—Convection of work between Creil and Paris:

	FIRST EXPERIMENT.		SECOND EXPERIMENT.	
	Generator.	*Motor.*	*Generator.*	*Motor.*
Turns per minute	190	248	170	277
Difference of potential	5469 volts.	4242	5717 volts.	4441
Current	7·21 ampères.	7·21	7·20 ampères.	7·20
Work in field	9·20 H.-P.	3·75	10·30 H.-P.	3·80
Work in armature	53·59 "	41·44	55·90 "	43·40
Measured mechanical work	62·10 "	35·80	61·	40·
Electrical efficiency	77 per cent.		78 per cent.	
Mechanical efficiency	47·7 per cent.		53·4 per cent.	

Resistance of line, 100 ohms.
" " generator, 33 ohms.
" " motor, 36 ohms.
Diameter of copper wire, 5 millimetres.

$$\frac{35{\cdot}80}{62{\cdot}10 + 9{\cdot}20 + 3{\cdot}75} = \frac{35{\cdot}80}{75{\cdot}05} = 0{\cdot}477, \text{ for first experiment.}$$

$$\frac{40}{61 + 10{\cdot}30 + 3{\cdot}80} = \frac{40}{75{\cdot}1} = 0{\cdot}534, \text{ for second experiment.}$$

The labors of Marcel Deprez have both theoretically and practically opened the way and proved the entire feasibility of transporting great amounts of power for long distances. Much remains and will yet be accomplished in the way of cheapening the first cost of apparatus required, and also of rendering it automatic.

Perhaps the first condition to be placed upon a motor used in manufactures is that its speed shall be regular under all variations of load. Now, we know that with a constant field intensity, H, and length, L, of armature wire, the speed, V, and the counter electro-motive force, e, vary together.

$$e = HLV; \qquad \frac{e}{HL} = V = \text{constant.}$$

We see, then, that if we demand a constant speed and cannot vary the length of the armature wire, the intensity of the field must vary with the counter electro-motive force.

This can be accomplished by means of double enrollment, commonly called "compound winding," patented by Marcel Deprez in 1881.

We know how the lines of force of the field are led by iron cores surrounded by coils of wire to the spot where the armature in revolving can cut them. If the whole current generated in the armature is led through the coil around the magnet and then through the external circuit, the winding is technically called series winding. If only a part of the current is taken off at the binding-posts of the machine, and led through the coils around the magnet and back to the armature, the winding is technically called shunt winding.

The resistance of the shunt-wound magnet-coils is usually much greater than the external circuit, but the number of turns also is greater, and so we attain a field of equal intensity.

Compound winding consists of the joint use of these two methods.

Mr. F. J. Sprague has recently (April 7, 1885) patented a very clever combination of shunt and series winding for the purpose of obtaining a constant speed of motion for a constant potential circuit, such as is ordinarily used for incandescent lighting.

$$\frac{e}{H} = \text{constant} = \frac{e_1}{H_1}.$$

Let R_s = resistance of shunt field coils.
" N_s = number of turns of shunt field coils.
" R_d = resistance of series field coils.
" N_d = number of turns of series field coils.
" E = potential at terminals of motor.
" I = intensity of current through series coils.
" R = resistance of armature.
$E - R_d I$ = potential at shunt terminals = E_s.

$$\left.\begin{array}{l} \dfrac{E - R_d I}{R_s} = \text{ampères in shunt coils.} \\ \dfrac{E - R_d I - e}{R} = \text{ampères in armature.} \end{array}\right\} = \text{ampères in series coils.}$$

From the first equation we have

$$\frac{e}{N_s \frac{E_s}{R_s} N_d \left[\frac{E_s - e}{R} + \frac{E_s}{R_s}\right]} = \frac{e_1}{N_s \frac{E_s}{R_s} - N_d \left[\frac{E_s - e_1}{R} - \frac{E_s}{R_s}\right]}$$

$$\text{Eliminating, } \frac{N_s}{N_d} = \frac{R + R_d}{R}.$$

The magnetizing currents in shunt and series windings are sent in opposite directions, and the number of shunt windings is to the number of series windings as the sum of the resistances of the series windings and the armature is to the resistance of the armature.

This condition produces a magnetic field whose intensity is directly proportional to the counter electro-motive force, provided the magnets have not reached saturation.

Mr. Sprague, by ingenious devices, causes the currents to act together to start the motor with a very strong effort, and, once started, reverses one current and sets the contrary currents in the field coils to balancing each other, so as to produce a constant speed.

For constant potential circuits this motor will not govern if its theoretical efficiency is less than 50 per cent. On the other hand, for constant current circuits such as are used for arc lighting, this motor will not govern if the theoretical efficiency is greater than 50 per cent. We need not discuss it.

To avoid sparking at the brushes, Mr. Sprague has added a third series coil, which causes, in the case of dynamos having consequent poles, a counter-distortion of the poles of the field magnet proportional to the increase of strength of the armature magnet.

For economical reasons, motors running on arc circuits with a constant current should have other methods of governing than the use of compound reversed coils.

Mr. Weston uses two methods for obtaining a constant speed. The first is by using belts upon reversed cone-pulleys, which, with the aid of a centrifugal governor, shift so as to retain a constant speed for the driven machine, whatever be the variations of speed in the motor. The second is to vary the intensity of the field by means of resistance controlled by a governor or other automatic device. In our equation of condition for a constant speed we observed two suggested methods of procuring this constant speed. The first was to vary the intensity of the field with the counter electro-motive force. The second was to vary the length of the wire in the armature coils.

This latter is manifestly impossible with the ordinary forms of machines, although it is not impossible that part of the field might be cut off, or the armature itself partially removed from a constant field.

Another way is to vary the counter electro-motive force of the motor by shifting the brushes around the commutator, but this is usually productive of sparking, and results in injury to both brushes and commutator.

The number of variations of this method is legion.

For the purpose of locomotion special arrangements to produce a uniform speed are not required. From all parts of the civilized world we learn the steady progress of the successful application of dynamic electricity to problems of locomotion.

In the transmission of power by electricity the ends to be reached can well be stated under these heads:

(*A*) Each receiving apparatus should receive its part of the generated power, and, whatever be its action, should not influence other apparatus on the same circuit.

(*B*) The efficiency must be independent of the number of apparatuses in action.

(*C*) When a regular speed is desired, the regulation should be automatic and instantaneous, and should not require the intervention of an attendant.

THE CONSTRUCTION OF SHIPS.

FROM Mr. Froude's experiments, it would appear that the midship section has very little effect, if any, upon the resistance of ships to propulsion.

This resistance arises from two causes:

(1) The skin resistance, due to the area and nature of the surface exposed to water.

(2) The wave resistance, due to the dimensions of the waves formed by the vessel.

At a speed in knots equal to the square root of the length of the vessel in feet, which should not be exceeded, its resistance in pounds is one-200th of the whole weight of the vessel.

The entire resistances of similar ships at corresponding speeds are as the cubes of their corresponding lengths or dimensions.

The resistance increases enormously when the speed given above is exceeded.

Froude's estimate of the distribution of the indicated horse-power of marine-engines is as follows:

Resistance of naked hull	40	per cent.
" due to suction of screw	16	" "
" " friction "	04	" "
" " " of machinery	27	" "
" " air-pumps	01	" "
Loss by slip of screw	12	" "
	100	

It is impossible in the limits of a pocket-book to give more than useful memoranda for those fully qualified by education and experience to undertake the designs of vessels. The elaborate works of Scott-Russel and Rankine, as also the smaller works of Thearle and many others, should be carefully read. An engineer who undertakes the design of a ship without exhausting all the attainable sources of knowledge of naval architecture is not honest. It would be well if all engineers professing to undertake designs for structures upon which human life depends should be held liable to trial for their lives in case of failure through their lack of knowledge or vigilance. —W. D. M.

FAST OCEAN PASSAGES.

The Cunard steamer Etruria holds the championship of the Atlantic, having made the fastest ocean passage each way, in August, 1885. On the 7th she arrived off the Fastnet Light, from Sandy Hook, in 6 d. 5 h. 35 m. Returning, she left Queenstown August 16th, and made the run to Sandy Hook in 6 d. 5 h. 46 m. From Fastnet to Queenstown the run is 2 h. 47 m. On several days the steamer ran 465 miles in the twenty-four hours, showing about the same speed as an ordinary railroad train, say 20 miles an hour. The following table shows some of the fast passages made before the Etruria changed the record:

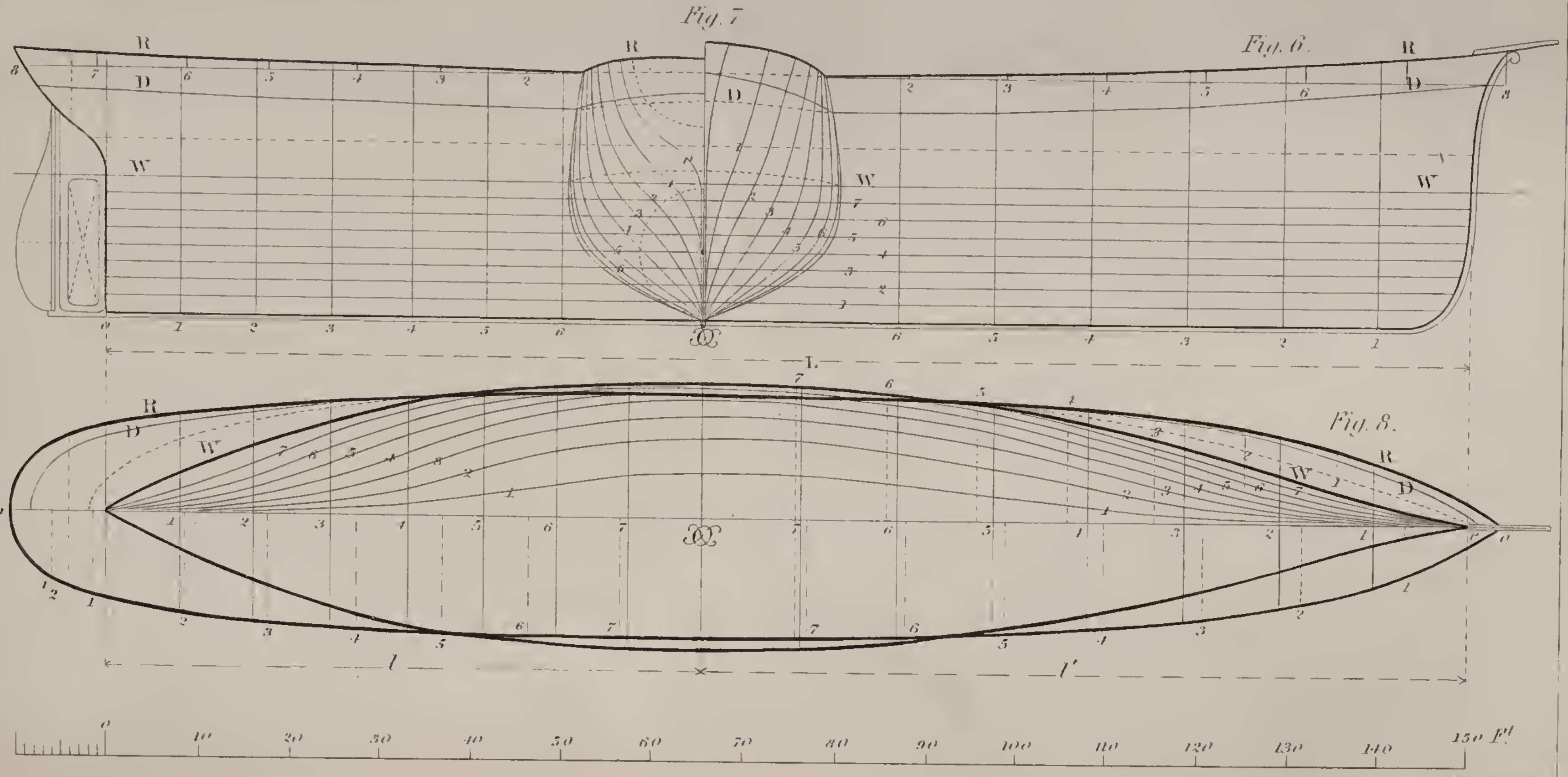
	To New York.			To Queenstown.		
	D.	H.	M.	D.	H.	M.
Oregon	6	10	10	6	15	57
America	6	15	41	6	14	18
Alaska	6	21	40	6	18	37
Servia	7	0	55	7	1	25
Urania	7	7	0	7	10	54
Britannic	7	7	11	7	12	41
Arizona	7	8	32	7	7	48
Austral	7	16	0	7	9	0
Gallia	7	16	32	7	18	38

$\mathfrak{M}$ = area of greatest immersed section.

The following tables will prove useful as giving approximate dimensions.

Plate X.

Nystrom's Parabolic Construction of Ships.



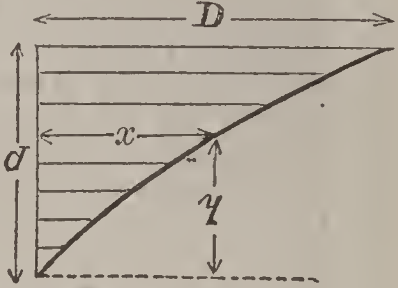
TO CONSTRUCT A DISPLACEMENT SCALE.

D = displacement of the vessel in cubic feet.
δ = displacement in cubic feet per inch of difference of draft.
$\mathbf{a}$ = area of load water line in square feet. d = draft of water in feet.
a = area of any water line at draft y and displacement x.
n = exponent of the displacement scale.

$$n = \frac{\mathbf{a}\,d}{D}.$$

$$x = \frac{y^n}{d^n} D. \qquad y = \sqrt[n]{\frac{x}{D}}.$$

$$a = \frac{D\,y^{n-1}}{d^n}. \qquad \delta = \frac{D\,y^{n-1}}{12\,d^n}.$$



Example. The area of the load water line of a vessel is $\mathbf{a} = 6400$ square feet; draft of water $d = 17$ feet, and the load draft displacement $D = 80{,}500$ cubic feet. Required the draft exponent $n = ?$ and at what draft y the displacement is $x = 45{,}000$ cubic feet?

$$n = \frac{6400 \times 17}{80500} = 1.35, \qquad y = 17\sqrt[1\cdot35]{\frac{45000}{80500}} = 11.05 \text{ feet, the draft required.}$$

Construct a scale as shown by the accompanying figure, and draw the ordinates x; the draft d being divided into eight equal parts.

Assuming the displacement as unit, the ordinates x are found in the following table, opposite the given exponent n.

After the exponent is known, the displacement can be expressed in tons, and the load draft displacement multiplied by the tabular number gives the displacement x at the corresponding draft y.

RULE. Multiply the load draft displacement, expressed either in tons or cubic feet, by the tabular number for the given exponent and water line, and the product is the corresponding displacement.

Displacement Scale.

$n = \frac{\mathbf{a}\,d}{D}$	Ordinate Waterlines.							Dead rise.
	1	2	3	4	5	6	7	
1.00	.1250	.2500	.3750	.5000	.6250	.7500	.8750	Flat bottom.
1.05	.1127	.2333	.3571	.4830	.6105	.7393	.8692	
1.10	.1015	.2176	.3300	.4665	.5963	.7287	.8634	
1.15	.0915	.2031	.3237	.4506	.5824	.7183	.8577	
1.20	.0825	.1895	.3082	.4353	.5689	.7080	.8512	
1.25	.0743	.1768	.2935	.4205	.5557	.6980	.8463	
1.30	.0669	.1649	.2794	.4061	.5428	.6880	.8407	
1.35	.0604	.1539	.2660	.3923	.5302	.6782	.8351	
1.40	.0544	.1436	.2533	.3789	.5179	.6685	.8295	The higher the exponent is, the higher is the dead rise.
1.45	.0490	.1340	.2303	.3660	.5047	.6589	.8240	
1.50	.0447	.1250	.2297	.3535	.4941	.6495	.8185	
1.55	.0398	.1166	.2187	.3415	.4826	.6402	.8130	
1.60	.0359	.1088	.2082	.3299	.4714	.6311	.8076	
1.65	.0323	.1015	.1982	.3186	.4605	.6221	.8023	
1.70	.0291	.0947	.1887	.3078	.4498	.6132	.7969	
1.75	.0257	.0884	.1797	.2985	.4393	.6044	.7916	
1.80	.0233	.0824	.1711	.2872	.4291	.5958	.7864	
1.85	.0213	.0769	.1629	.2774	.4129	.5873	.7811	
1.90	.0192	.0718	.1551	.2680	.4094	.5789	.7759	
1.95	.0173	.0670	.1477	.2588	.4008	.5706	.7708	Highest
2.00	.0156	.0625	.1406	.2500	.3906	.5625	.7656	dead rise.

Sharp Vessels. 𝔐 = 3.

Length L, Beam B, and draft d = ½ B, all in feet.

Displacem'nt in tons. T	L = 5B L	L = 6B L	L = 7B L	L = 8B L	L = 9B L	L = 10B L
1	16·6	18·3	20·3	22·2	24·0	25·8
2	21·0	22·9	25·6	28·0	30·3	32·5
3	24·0	26·4	29·3	32·0	34·6	37·2
4	26·4	29·0	32·3	35·3	38·2	41·0
5	28·4	31·4	34·8	38·0	42·0	44·2
6	30·3	33·3	37·0	40·4	43·8	47·0
7	31·7	34·9	38·8	42·4	46·0	49·3
8	33·2	36·6	40·6	44·4	48·0	51·4
9	34·6	38·0	42·2	46·2	50·0	53·7
10	35·7	39·3	43·7	47·8	51·7	55·5
11	36·9	40·6	45·1	49·3	53·4	57·3
12	38·0	41·9	46·6	50·9	55·0	59·1
13	39·0	43·0	47·8	52·2	56·5	60·6
14	40·0	44·0	49·0	53·6	58·0	62·2
15	40·9	45·0	50·0	54·7	59·0	63·5
16	41·8	46·0	51·2	56·0	60·6	65·0
17	42·7	46·9	52·2	57·0	61·8	66·3
18	43·6	47·8	53·2	58·0	63·0	67·7
19	44·4	48·6	54·2	59·0	64·2	69·0
20	45·0	49·5	55·1	60·0	65·2	70·0
25	48·5	53·4	59·2	64·8	70·2	75·4
30	51·6	57·0	63·3	69·0	74·7	80·2
35	54·3	59·6	66·4	72·7	78·6	84·3
40	56·8	62·6	69·5	76·0	82·2	88·3
45	59·2	65·0	72·3	79·0	85·5	92·0
50	61·2	67·2	74·8	81·8	88·4	95·0
55	63·1	69·4	77·3	84·4	91·4	98·0
60	64·9	71·4	79·4	86·8	93·9	101
65	66·8	73·5	81·7	89·3	96·6	104
70	68·4	75·3	83·7	91·6	98·0	107
75	70·1	77·1	85·8	93·8	101	109
80	71·6	78·9	87·6	95·7	103	111
85	73·0	80·5	89·5	97·7	106	114
90	74·5	81·9	91·1	99·6	108	116
95	75·8	83·4	92·7	111	100	118
100	77·0	84·7	94·4	103	112	120
110	79·7	87·6	97·4	107	115	124
125	83·0	91·5	100	111	120	129
150	88·3	97·0	108	113	128	137
175	93·0	102	114	124	134	144
200	97·3	107	119	130	140	151
225	101	111	124	135	146	157
250	105	115	128	140	152	163
275	108	119	132	145	156	168
300	111	122	136	149	161	173
325	114	126	140	153	165	178
350	117	129	143	157	170	182
375	120	130	147	161	173	186
400	122	135	150	164	177	191
450	128	140	156	170	184	198
500	132	145	161	176	191	205
550	136	150	166	182	197	211
600	140	154	171	188	202	217
650	144	158	176	193	208	223
700	148	162	180	197	213	229
750	151	166	185	202	218	235
800	154	170	189	206	223	240
850	158	173	193	210	227	245
900	160	176	196	215	232	250
950	163	180	200	219	236	254

Displacement. T	L = 5B L	L = 6B L	L = 7B L	L = 8B L	L = 9B L	L = 10B L
1000	166	183	203	222	240	258
1100	171	189	210	230	248	267
1200	177	194	216	236	255	274
1300	181	199	222	242	262	281
1400	186	204	226	249	269	288
1500	190	210	233	255	275	295
1600	195	214	238	260	281	302
1700	199	218	243	265	286	308
1800	202	223	248	270	292	314
1900	206	227	252	275	298	320
2000	210	230	256	280	302	325
2100	212	234	260	285	307	330
2200	216	238	265	290	312	335
2300	220	242	269	295	317	341
2400	223	245	272	300	322	346
2500	226	248	276	304	326	351
2600	229	252	280	308	330	355
2700	232	255	283	311	334	360
2800	235	259	287	315	338	365
2900	238	261	290	318	342	369
3000	240	264	294	321	347	373
3100	243	267	297	324	350	377
3200	246	270	300	327	354	381
3300	248	272	303	330	358	385
3400	250	275	306	333	362	389
3500	253	278	309	337	365	393
3600	255	281	312	340	369	396
3700	257	283	314	344	372	399
3800	260	285	317	347	375	403
3900	262	288	320	350	378	406
4000	265	291	323	353	380	409
4100	268	293	325	356	383	413
4200	270	296	328	359	386	417
4300	272	298	331	362	389	420
4400	273	300	334	365	392	423
4500	275	302	337	368	395	426
4600	277	304	339	370	398	429
4700	279	306	331	372	401	432
4800	281	308	343	374	404	435
4900	283	310	345	376	407	438
5000	286	312	348	378	411	441
5250	289	318	354	386	418	450
5500	294	323	359	392	424	457
5750	298	331	364	398	430	463
6000	303	336	370	404	437	469
6250	307	340	375	409	443	476
6500	310	345	380	414	448	482
6750	315	349	384	420	454	488
7000	319	354	389	425	460	494
7250	322	358	394	430	465	500
7500	326	361	398	435	470	506
7750	330	365	402	440	475	512
8000	333	370	407	445	480	517
8250	337	374	410	449	485	522
8500	340	377	415	453	490	527
8750	343	380	419	457	495	532
9000	347	384	423	461	500	537
9250	350	388	427	466	504	542
9500	353	392	430	470	509	547
10000	359	399	438	478	517	556

Full Vessels.

Length L, Beam B, and draft $d = \frac{1}{2}$ B, all in feet.

Displacem'nt in tons.	L = 5B	L = 6B	L = 7B	L = 8B	L = 9B	L = 10B	Displacement.	L = 5B	L = 6B	L = 7B	L = 8B	L = 9B	L = 10B
T	**L**	**L**	**L**	**L**	**L**	**L**	**T**	**L**	**L**	**L**	**L**	**L**	**L**
1	10·9	14·6	16·2	17·8	19·2	20·6	**1000**	109	146	162	178	192	206
2	13·7	18·3	20·4	22·4	24·2	26·0	**1100**	112	151	166	184	199	213
3	15·7	21·0	23·4	25·7	27·7	29·7	**1200**	116	155	173	190	205	219
4	17·3	23·2	25·8	28·3	30·5	32·8	**1300**	119	160	177	195	210	225
5	18·7	25·0	27·7	30·4	32·8	35·2	**1400**	122	164	180	199	216	230
6	19·8	26·6	29·5	32·4	34·9	37·6	**1500**	125	167	184	204	221	235
7	20·8	28·0	31·0	34·0	36·7	39·4	**1600**	127	171	188	208	226	240
8	21·8	29·2	32·4	35·6	38·4	41·2	**1700**	130	174	193	212	230	245
9	22·7	30·4	33·7	37·0	40·0	42·8	**1800**	132	177	198	217	235	250
10	23·5	31·4	34·9	38·3	41·3	44·3	**1900**	135	181	201	220	239	255
11	24·2	32·4	36·0	39·5	42·6	45·7	**2000**	137	184	205	224	243	260
12	25·0	33·3	37·1	40·7	44·0	47·2	**2100**	139	188	208	228	247	264
13	25·6	34·3	38·1	41·8	45·1	48·4	**2200**	141	190	211	231	251	268
14	26·3	35·2	39·0	42·9	46·2	49·6	**2300**	144	193	214	234	254	272
15	26·8	36·0	39·9	43·8	47·2	50·7	**2400**	146	195	217	238	258	276
16	27·5	36·8	40·9	44·8	48·4	51·8	**2500**	148	198	220	242	261	280
17	28·0	37·5	41·8	45·7	49·4	52·9	**2600**	150	200	223	245	264	284
18	28·5	38·3	42·6	46·6	50·3	54·0	**2700**	152	203	226	248	267	287
19	29·0	39·0	43·4	47·4	51·3	56·0	**2800**	154	205	228	251	270	290
20	29·5	39·6	44·2	48·2	52·0	58·0	**2900**	155	208	231	254	274	294
25	31·8	42·6	45·4	52·0	56·1	60·0	**3000**	157	211	234	257	277	297
30	33·9	45·5	50·5	55·3	59·9	64·0	**3100**	159	213	237	260	280	300
35	35·6	47·8	53·1	58·1	62·9	67·2	**3200**	160	215	240	262	284	304
40	37·3	50·0	55·6	60·8	65·7	70·4	**3300**	162	218	242	264	287	307
45	38·8	52·0	57·8	63·3	68·4	73·3	**3400**	164	220	245	267	290	310
50	40·1	53·8	59·8	65·5	70·7	75·7	**3500**	166	222	248	270	292	313
55	41·4	55·5	61·7	67·7	73·1	78·2	**3600**	167	225	250	272	295	316
60	42·6	57·1	63·5	69·5	75·2	80·4	**3700**	169	226	252	275	297	319
65	43·8	58·7	65·2	71·5	77·4	82·7	**3800**	171	228	254	278	299	322
70	45·0	60·2	66·9	73·4	79·2	85·0	**3900**	172	230	255	280	302	325
75	46·0	61·6	68·6	75·2	81·2	87·0	**4000**	173	232	258	283	305	327
80	47·0	63·0	70·0	77·0	82·9	88·8	**4100**	174	234	261	285	308	330
85	47·9	64·2	71·5	78·2	84·6	90·5	**4200**	175	236	263	287	310	333
90	48·8	65·5	72·8	79·7	86·3	92·3	**4300**	177	238	265	289	312	336
95	49·7	66·6	74·2	81·3	87·8	94·0	**4400**	178	240	267	291	314	338
100	50·5	67·8	75·4	82·7	89·2	95·6	**4500**	179	242	269	294	317	340
110	52·2	70·0	78·0	85·3	92·2	98·5	**4600**	181	244	271	296	320	343
125	55·0	73·0	81·0	89·0	96·0	103	**4700**	182	246	273	298	322	345
150	57·8	77·5	86·4	94·5	102	110	**4800**	184	248	275	300	324	347
175	60·8	81·6	90·8	99·5	108	115	**4900**	185	249	276	302	326	350
200	63·6	85·4	95·2	104	113	120	**5000**	187	250	277	304	329	352
225	66·2	88·9	99·0	108	117	125	**5250**	190	254	278	309	334	354
250	68·7	92·0	102	112	121	130	**5500**	193	258	287	315	339	356
275	70·8	95·0	106	116	125	134	**5750**	196	262	291	319	345	369
300	72·8	97·8	109	119	129	138	**6000**	198	266	295	324	350	374
325	74·0	100	112	123	132	142	**6250**	200	269	300	328	354	383
350	76·8	103	115	126	136	146	**6500**	203	273	303	332	359	384
375	78·7	106	118	129	139	149	**6750**	206	276	307	336	364	390
400	80·3	108	120	132	142	152	**7000**	209	280	311	340	368	394
450	85·5	112	125	137	148	158	**7250**	211	283	314	344	372	399
500	86·4	116	130	142	153	164	**7500**	213	286	318	348	376	404
550	89·2	120	133	146	158	169	**7750**	216	289	322	352	380	407
600	91·9	123	136	150	162	174	**8000**	218	292	325	356	384	412
650	94·4	127	140	154	167	178	**8250**	220	296	328	360	388	417
700	96·6	130	144	158	170	183	**8500**	222	299	331	364	392	421
750	98·9	133	148	162	173	187	**8750**	224	301	334	367	396	425
800	101	136	151	165	177	191	**9000**	226	304	337	370	400	429
850	103	139	154	169	182	195	**9250**	229	307	340	374	404	432
900	105	142	157	172	186	199	**9500**	231	310	344	377	408	437
950	107	144	159	175	189	203	**10000**	235	315	350	384	414	446

Horsepower in Steamship Performance.

Displacement in tons.	Nautical miles or knots per hour. 1	2	3	4	5	6	7	8	9	10
T	H	H	H	H	H	H	H	H	H	H
1	0·004	0·035	0·118	0·280	0·550	0·949	1·50	2·24	3·20	4·38
2	0·007	0·055	0·190	0·444	0·870	1·51	2·40	3·55	5·08	6 96
3	0·009	0·075	0·248	0·598	1·14	1·98	3·12	4·79	6·91	9·12
4	0·010	0·084	0·300	0·673	1·40	2·40	3·80	5·39	8 06	11·10
5	0·012	0·102	0·348	0·818	1·52	2·78	4·40	6·55	9·36	12 2
6	0·014	0·115	0·390	0·924	1·81	3·12	4·96	7·39	10 6	14·5
7	0·016	0·128	0·435	1·025	2·01	3 48	5·50	8·20	11·7	16·1
8	0·017	0·138	0·479	1·125	2 20	3·80	6·01	8·96	12·8	17·5
9	0·019	0·151	0·501	1·211	2·38	4·12	6·51	9·69	13·8	19·0
10	0·020	0·161	0·552	1·30	2·54	4·42	6·98	10·4	14·9	20·3
11	0·022	0 175	0·590	1·40	2·72	4·70	7·46	11·1	15·9	21·8
12	0·023	0·185	0 624	1·48	2·88	4·99	7·90	11·8	16·8	23·0
13	0·024	0·195	0·654	1·56	3·04	5·25	8·33	12·5	17·7	24·3
14	0·024	0 198	0·690	1·62	3·18	5·52	8·75	13·0	18 6	25·4
15	0·026	0·213	0·725	1·70	3·32	5·80	9·20	13·6	19·5	26·6
16	0·028	0·223	0·780	1·78	3·49	6·04	9·55	14·2	20·4	27·9
17	0·029	0·236	0·785	1·89	3·64	6·28	9·95	15·0	21·2	29·1
18	0·030	0·242	0·815	1·94	3·78	6·52	10·3	15 5	22·0	30·2
19	0·031	0·250	0·850	2·00	3·90	6·80	10·7	16·0	22 8	31·2
20	0·032	0·258	0·875	2·06	4·02	7·00	11·1	16·5	23·0	32·2
25	0 038	0·300	1·015	2·40	4·14	8·12	12·9	19·2	24·2	33·1
30	0·042	0·338	1 14	2·70	5·30	9·18	14 6	21·6	31 0	42·4
35	0·047	0·375	1·26	3·00	5·89	10·1	16·2	24·0	34·2	47·1
40	0·050	0·409	1·39	3·27	6·41	11·1	17·6	26·2	37·5	51·3
45	0·056	0·445	1 50	3·56	6·95	12 0	19 0	28·5	40·5	55·6
50	0·056	0·474	1·61	3·79	7·44	12 9	20·5	30·3	43 2	59·5
55	0·062	0·501	1·72	4·06	7·95	13·8	21·8	32·5	46·2	63·6
60	0·067	0·538	1·80	4·30	8·41	14·4	23·1	34·4	49·1	67·3
65	0·071	0·570	1·90	4·56	8·88	15·1	24·4	36·5	51·8	71·0
70	0·074	0·597	2·02	4·77	9·36	16·2	25·5	38·2	54·4	74·9
75	0·078	0·625	2·12	5·00	9 77	16·9	26·8	40·0	56·8	78·0
80	0·081	0·650	2·20	5·20	10·2	17 6	28 0	41·6	58·1	81·6
85	0·085	0 680	2·30	5·44	10·6	18 4	29·2	43·5	62 0	85·0
90	0·088	0·705	2·38	5 64	11 0	19·1	30 5	45·2	64·5	88·4
95	0·088	0·710	2·49	5 68	11 4	19·9	31·3	47·0	66·6	91·5
100	0·094	0·755	2·56	6·04	11·8	20·5	32·4	48·4	68 5	94·5
110	0·101	0 810	2·73	6·48	12·6	21·9	34 6	51·8	73·2	101
125	0·109	0·877	2·98	7·02	13·7	23·8	37·5	56·2	80·0	110
150	0·124	0 990	3·38	7·72	15 5	27 0	42·8	61·7	90·5	124
175	0·138	1·10	3·72	8·81	17·2	29·8	47·2	70·5	100	138
200	0·150	1·20	4·06	9·60	18·8	32·5	51 5	76·9	110	150
225	0·162	1 30	4·39	10·4	20·2	35·1	56·0	83·3	118	162
250	0·175	1·40	4·70	11·2	21·9	37 6	59·8	89 2	127	175
275	0·188	1·50	5·04	11·9	23·2	40·3	63·8	95·2	136	186
300	0·196	1·57	5·31	12 6	24·5	42·5	67·5	100	142	196
325	0·201	1·66	5·63	13·3	26 0	45 0	71·2	106	152	208
350	0 220	1·75	5 91	14 0	27 4	47·3	75·0	112	159	219
375	0·228	1·82	6·12	14·6	28·6	49·0	78·4	117	166	229
400	0·240	1·91	6·42	15·3	29 8	51·4	81·7	122	172	238
450	0·250	2·06	6·98	16·5	32·2	55·8	88 5	132	188	258
500	0·276	2·21	7·45	17·7	34·6	59·6	94·3	141	200	276
550	0·295	2·36	7 98	18·9	36·9	63·8	101	151	215	295
600	0·312	2·50	8·40	20·0	39·0	67·2	107	160	226	313
650	0·330	2·64	8 90	21·1	41·2	71·2	113	169	240	329
700	0·348	2·78	9·32	22·2	43·3	74 6	119	177	250	337
750	0·362	2·90	9·80	23·2	45·2	78·4	124	186	264	352
800	0·380	3·03	10·2	24·2	47·3	81·5	130	194	274	378
850	0·394	3·15	10 6	25·2	49·2	85·0	135	202	288	394
900	0·410	3·28	11·0	26·2	51·1	88·1	140	210	296	409
950	0·422	3 41	11·4	27·3	53·1	91·8	146	218	310	445

Horsepower in Steamship Performance.

Displacement in tons.	Nautical miles or knots per hour.									
	11	12	13	14	15	16	17	18	19	20
T	H	H	H	H	H	H	H	H	H	H
1	5·85	7·59	9·63	12·0	14·8	17·9	21·6	25·6	30·1	35·1
2	9·28	12·0	15 3	19 1	23·5	28·4	34 2	40 6	47·8	54·7
3	12·2	15·8	20 0	25·0	30·8	38·3	44·8	53·3	62·6	73·0
4	14·8	19·2	24 4	30·3	37·4	43·1	54·3	64·5	75·9	88·4
5	17·2	22·2	28 3	35·2	43·4	52·4	63 0	74·9	88·0	97·8
6	19·4	25·1	31·9	39 7	49·0	59·1	71·1	84·5	99·2	116
7	21·4	27·8	35·3	44·0	54·0	65·5	79·0	93·7	110	128
8	23·4	30·4	38·6	48·1	59·3	68·7	86·2	102	121	140
9	25·3	32·9	41·8	52·1	64·0	77·5	93·2	110	130	152
10	27 2	35·3	44 8	55·8	68·8	83·2	100	119	140	163
11	29·0	37·6	47·8	59·7	73·5	89·0	107	127	150	174
12	30·7	39·9	50 6	63·2	77·7	94·4	113	134	158	184
13	32·4	42·0	53 3	66·6	82·0	99·6	120	142	167	194
14	34 0	44·2	56·0	70·0	86·0	105	126	149	176	203
15	35·6	46·3	58·7	73·5	90·0	109	131	156	183	213
16	37·2	48·3	61·3	76·5	94·0	114	137	163	192	223
17	38·7	50·2	63·8	79·6	98·0	120	143	170	200	233
18	40·2	52·2	66·2	82·7	102	124	148	176	207	242
19	41·7	54·0	68·7	85·8	106	128	154	182	215	250
20	43·2	56·0	71 0	88·9	111	132	159	189	222	258
25	50·0	65·0	82·5	103	127	154	184	194	258	265
30	56·5	73·4	93·2	117	143	173	208	248	291	339
35	62·6	81·3	103	130	159	192	230	274	322	377
40	68·4	88·8	113	141	173	209	252	300	350	410
45	74·0	96 2	122	152	188	228	273	324	382	445
50	79·4	103	131	164	201	242	293	346	410	476
55	84·6	110	140	174	215	260	312	370	437	509
60	90·0	117	149	185	226	285	330	393	464	538
65	94·7	123	156	195	240	292	349	414	488	568
70	99 6	130	164	206	252	306	367	437	512	599
75	104	135	171	214	264	320	383	455	536	624
80	109	141	180	224	276	333	400	467	561	653
85	113	147	187	234	287	348	417	496	584	680
90	118	153	194	243	298	362	433	516	607	707
95	122	158	201	251	309	376	448	533	629	732
100	126	164	207	259	318	387	464	551	648	756
110	135	175	222	277	340	414	495	588	693	807
125	146	190	241	300	370	450	539	640	753	878
150	165	215	273	342	420	494	609	724	852	992
175	183	238	302	378	464	564	675	802	946	1100
200	200	260	330	412	506	615	737	875	1027	1201
225	217	281	358	447	548	666	800	947	1118	1300
250	232	301	384	478	588	714	855	1016	1200	1400
275	248	322	409	510	627	762	912	1087	1286	1490
300	262	340	432	540	662	806	966	1146	1347	1573
325	277	360	457	570	700	852	1010	1213	1428	1665
350	290	378	480	600	737	896	1073	1276	1500	1750
375	305	395	502	627	770	936	1122	1332	1570	1830
400	317	412	522	654	803	976	1170	1402	1632	1907
450	343	446	567	708	870	1060	1265	1500	1770	2065
500	368	478	607	759	932	1131	1358	1611	1896	2213
550	393	510	648	810	995	1210	1450	1720	2025	2362
600	415	540	684	856	1036	1280	1532	1820	2140	2500
650	440	570	724	905	1111	1350	1618	1923	2265	2636
700	460	599	759	938	1166	1417	1700	2016	2373	2770
750	483	627	797	995	1220	1485	1780	2113	2490	2900
800	503	654	830	1038	1274	1548	1857	2206	2593	3026
850	525	680	866	1080	1330	1620	1935	2300	2710	3152
900	545	708	898	1123	1380	1675	2009	2385	2803	3274
950	565	734	933	1170	1430	1740	2080	2478	2920	3400

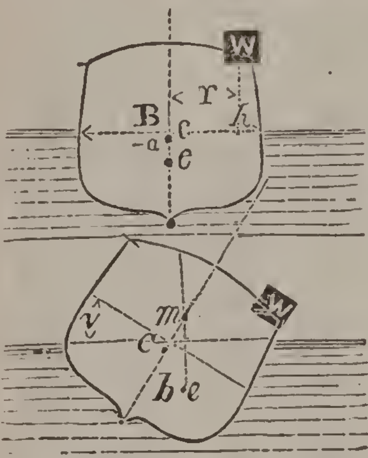
Horsepower in Steamship Performance.

Displacement in tons.	Nautical miles or knots per hour. 1	2	3	4	5	6	7	8	9	10
T	H	H	H	H	H	H	H	H	H	H
1000	0·438	3·50	11·8	28·0	54·9	94·6	150	225	318	439
1100	0·456	3·75	12·5	30·0	58·4	100	160	239	338	467
1200	0·500	4·00	13·4	32·0	62·0	107	170	254	359	495
1300	0·515	4·12	14·0	33·0	65·3	112	179	267	378	523
1400	0·548	4·38	14·9	35·0	68·7	119	189	281	398	549
1500	0·562	4·50	15·5	36·0	71·9	124	197	295	417	575
1600	0·578	4·62	16·2	37·0	75·0	130	206	307	435	600
1700	0·594	4·75	16·9	38·0	78·1	135	215	320	453	625
1800	0·625	5·00	17·5	40·0	81·2	140	224	332	470	649
1900	0·634	5·25	18·1	42·0	84·2	145	231	345	488	673
2000	0·700	5·60	18·8	44·0	87·0	150	239	356	504	696
2100	0·719	5·75	19·4	46·0	90·0	155	247	369	521	720
2200	0·735	5·88	20·0	47·0	92·7	160	255	380	537	741
2300	0·765	6·12	20·6	49·0	95·6	165	262	391	554	764
2400	0·788	6·28	21·1	50·2	98·4	170	270	402	569	786
2500	0·805	6·44	21·8	51·5	101	174	277	414	585	808
2600	0·828	6·62	22·4	53·0	104	179	285	424	600	826
2700	0·851	6·81	23·0	54·5	106	184	292	436	616	850
2800	0·872	6·98	23·5	55·8	109	188	299	446	631	871
2900	0·876	7·12	24·0	57·1	111	192	306	457	646	893
3000	0·909	7·35	24·6	58·8	114	197	313	467	660	913
3100	0·931	7·45	25·1	59·8	117	201	320	478	676	933
3200	0·952	7·62	25·6	61·0	119	205	327	488	690	952
3300	0·972	7·78	26·1	62·2	121	209	334	498	704	972
3400	0·992	7·94	26·8	63·5	124	214	340	508	718	992
3500	1·01	8·10	27·2	64·8	127	218	347	518	733	1010
3600	1·03	8·25	27·8	66·0	129	222	354	528	746	1025
3700	1·05	8·39	28·2	67·1	131	226	360	538	759	1049
3800	1·08	8·60	27·8	68·5	133	230	367	548	774	1070
3900	1·09	8·70	28·9	69·6	135	234	373	558	787	1087
4000	1·11	8·85	29·9	70·8	138	238	380	567	801	1105
4100	1·13	9·01	30·4	71·1	140	242	386	577	814	1122
4200	1·14	9·14	30·9	73·1	142	246	392	586	827	1141
4300	1·16	9·30	31·4	74·4	145	250	398	595	840	1160
4400	1·18	9·42	31·9	75·5	147	254	404	604	853	1179
4500	1·19	9·56	32·4	76·5	150	258	410	613	866	1198
4600	1·22	9·72	32·8	77·7	152	261	416	622	879	1216
4700	1·23	9·86	33·4	78·9	154	266	422	631	881	1232
4800	1·25	10·0	33·9	80·0	156	270	428	640	904	1248
4900	1·28	10·1	34·4	81·1	158	274	434	649	916	1265
5000	1·30	10·3	34·8	82·7	160	277	440	658	929	1282
5250	1·32	10·6	35·6	85·0	165	283	455	670	959	1324
5500	1·36	10·9	36·4	87·5	171	290	469	700	990	1367
5750	1·40	11·2	37·5	90·0	176	298	483	721	1024	1408
6000	1·42	11·4	38·0	92·8	181	303	497	742	1050	1448
6250	1·47	11·9	40·2	95·2	188	322	512	762	1065	1488
6500	1·52	12·2	41·2	97·8	191	330	526	782	1078	1526
6750	1·56	12·5	42·4	100	196	339	540	802	1123	1567
7000	1·60	12·9	43·2	103	202	346	554	822	1174	1616
7250	1·64	13·1	44·4	105	205	355	566	842	1198	1644
7500	1·68	13·5	45·5	108	210	364	579	861	1226	1682
7750	1·72	13·8	46·5	110	215	372	599	879	1253	1719
8000	1·75	14·0	47·4	112	220	379	603	899	1280	1757
8250	1·78	14·2	48·4	115	224	387	615	918	1306	1793
8500	1·81	14·5	49·4	116	229	395	628	929	1333	1829
8750	1·84	14·9	50·0	119	233	403	640	955	1354	1865
9000	1·88	15·2	51·1	122	238	411	653	973	1385	1902
9250	1·92	15·4	52·2	124	242	418	668	991	1411	1937
9500	1·95	15·6	53·2	126	246	426	683	1008	1437	1972
10000	2·05	16·4	55·1	131	255	441	714	1044	1488	2012

Horsepower in Steamship Performance.

Displacement in tons.	Nautical miles or knots per hour.									
	11	12	13	14	15	16	17	18	19	20
T	H	H	H	H	H	H	H	H	H	H
1000	585	759	963	1206	1480	1798	2157	2560	3008	3514
1100	622	806	1024	1284	1574	1913	2295	2723	3203	3736
1200	660	858	1090	1360	1670	2030	2435	2890	3400	3967
1300	696	903	1147	1432	1758	2136	2564	3043	3576	4178
1400	732	950	1204	1508	1850	2248	2697	3200	3762	4394
1500	766	995	1264	1580	1938	2355	2825	3252	3943	4605
1600	800	1038	1317	1648	2020	2458	2948	3500	4113	4803
1700	833	1083	1374	1718	2107	2561	3072	3646	4286	5006
1800	864	1123	1422	1784	2188	2660	3140	3785	4448	5195
1900	897	1166	1479	1850	2270	2760	3310	3928	4615	5390
2000	927	1205	1527	1913	2345	2854	3420	4060	4770	5570
2100	958	1247	1582	1979	2382	2948	3535	4195	4935	5762
2200	988	1284	1628	2037	2500	3038	3642	4325	5084	5935
2300	1017	1324	1680	2102	2578	3134	3755	4460	5241	6120
2400	1047	1360	1723	2160	2646	3220	3860	4580	5386	6290
2500	1077	1400	1777	2222	2725	3313	3970	4715	5542	6470
2600	1102	1435	1820	2280	2796	3400	4075	4835	5655	6637
2700	1131	1473	1870	2338	2868	3486	4180	4960	5832	6813
2800	1160	1508	1911	2395	2935	3568	4280	5076	5970	6970
2900	1189	1545	1960	2452	3010	3655	4385	5200	6115	7142
3000	1215	1582	2000	2508	3075	3740	4485	5318	6255	7300
3100	1242	1614	2048	2565	3145	3822	4585	5440	6394	7470
3200	1268	1648	2092	2616	3210	3905	4680	5550	6525	7622
3300	1296	1683	2134	2671	3280	3985	4775	5670	6666	7781
3400	1320	1717	2178	2725	3343	4063	4870	5784	6784	7936
3500	1347	1750	2220	2779	3408	4143	4965	5893	6936	8090
3600	1373	1783	2264	2830	3475	4222	5060	6010	7061	8250
3700	1398	1815	2303	2881	3534	4300	5155	6115	7184	8400
3800	1422	1848	2348	2941	3606	4385	5250	6238	7333	8563
3900	1446	1880	2385	2986	3660	4453	5340	6336	7444	8696
4000	1473	1912	2427	3038	3725	4530	5430	6444	7580	8847
4100	1497	1944	2468	3086	3785	4610	5520	6550	7700	8988
4200	1520	1975	2507	3137	3850	4680	5610	6655	7830	9141
4300	1545	2008	2546	3186	3910	4750	5700	6761	7950	9285
4400	1568	2037	2585	3238	3970	4825	5790	6865	8072	9432
4500	1593	2070	2624	3286	4025	4900	5875	6970	8195	9572
4600	1614	2100	2664	3333	4087	4935	5960	7070	8320	9710
4700	1639	2130	2702	3382	4145	4970	6045	7172	8437	9850
4800	1663	2160	2740	3431	4202	5112	6130	7275	8555	9990
4900	1686	2190	2779	3478	4260	5193	6215	7375	8673	10120
5000	1708	2220	2817	3525	4321	5253	6300	7475	8792	10250
5250	1760	2293	2909	3640	4414	5426	6507	7723	9081	10601
5500	1822	2365	3000	3755	4608	5600	6715	7972	9370	10953
5750	1876	2436	3090	3868	4744	5767	6917	8204	9652	11269
6000	1930	2507	3180	3981	4880	5935	7120	8436	9935	11586
6250	1982	2574	3261	4094	5013	6096	7313	8519	10203	11902
6500	2035	2642	3352	4207	5146	6258	7505	8603	10472	12218
6750	2088	2710	3438	4320	5281	6419	7698	8986	10741	12534
7000	2141	2778	3524	4434	5417	6580	7892	9370	11010	12851
7250	2191	2842	3606	4531	5542	6733	8076	9587	11265	13152
7500	2241	2907	3688	4629	5668	6886	8260	9805	11521	13453
7750	2290	2971	3770	4276	5794	7039	8445	10022	11776	13754
8000	2340	3036	3852	4824	5920	7192	8628	10240	12032	14056
8250	2488	3098	3931	4923	6042	7340	8806	10451	12280	14345
8500	2636	3161	4011	5023	6164	7488	8984	10662	12528	14634
8750	2784	3223	4095	5123	6286	7637	9162	10823	12776	14922
9000	2933	3286	4170	5222	6408	7785	9340	11084	13024	15211
9250	2879	3346	4247	5343	6516	7926	9512	11289	13364	15493
9500	2826	3407	4324	5465	6645	8068	9685	11494	13505	15775
10000	2720	3529	4478	5708	6882	8351	10030	11904	13987	16340

To find the Moment of Stability of a Vessel by Experiments.



W = weight in tons placed on deck at a distance r from the centre-line and h feet above the load-water line, when the vessel is in equilibrium; v = careen angle, d = depth in feet of the centre of gravity of the vessel under meta-centre, Q = moment of stability in foot-tons.

$$Q = W(r \cos. v + h \sin. v),$$

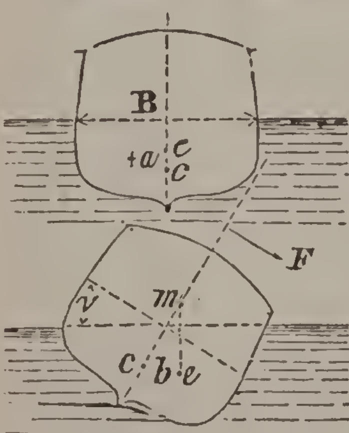
$$d = \frac{Q}{T \sin. v}, \qquad \text{Sin. } v = \frac{Q}{T d}.$$

Example. The weight $W = 15$ tons, the centre of gravity of which is placed at $r = 12$ feet from centre on deck and $h = 8$ feet above the water, which careens the vessel to an angle $v = 2°$. The displacement $T = 4288.8$ tons. Required, the moment of stability Q, and depth centre of gravity d?

$$Q = 15(12 \times \cos. 2° + 8 \times \sin. 2°) = 184.05 \text{ foot-tons},$$

and $d = \frac{184.05}{4288.8 \times 0.0349} = 1.229$ feet, the depth of the centre of gravity of the vessel, under meta-centre.

Moment of Wind on Sails Careening a Vessel in Sailing.



Let F denote the force of wind in tons, acting at right angle to the vessel on the centre of gravity of all the sails, $= l$ feet above the centre of gravity of the displacement. Then the moment of the wind will be— $Q = Fl = Td \sin. v.$

Example. The centre of gravity of all the sails being $l = 35$ feet above the centre of gravity of the displacement of a vessel of $T = 4288.8$ tons. The force of wind on all the sails $F = 7$ tons. The depth of the centre of gravity of the vessel, under meta-centre $d = 1.223$ feet, as found by experiments. Required, the moment Q of the wind, and to what angle the vessel will be careened?

$$Q = 7 \times 35 = 245 \text{ foot-tons},$$

and, $\sin. v = \frac{Q}{T d} = \frac{245}{4288.8 \times 1.223} = 0.04671$

$= \sin. 2° 40' 40''$, the careen.

Tonnage of Vessels.—Old U. S. Measurement.

T = tonnage of vessel. L = length of the vessel in feet, from the fore part of the stem to the after part of the stern-post, measured on the upper deck. B = greatest beam in feet, measured above the main-walls. d = depth of the vessel in feet. For double-decked vessels, half the beam B is taken as the depth d. For single-decked vessels, the depth is taken from the underside of deck plank to the ceiling of the hold.

Example. $L = 186$ feet, $B = 30$ and $d = 15$, for a double-decked vessel. Required, the tonnage?

$$T = \frac{Bd}{95}(L - 0.6B) = \frac{30 \times 15}{95}(180 - 0.6 \times 30) = 767.4 \text{ tons.}$$

Custom-House New Tonnage Law, May 6, 1864.

An Act to regulate the admeasurement of tonnage of ships and vessels of the U. S.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That every ship or vessel built within the United States, or that may be owned by a citizen or citizens thereof, on or after the first day of January, eighteen hundred and sixty-five, shall be measured and registered in the manner hereinafter provided; also every ship or vessel that is now owned by a citizen or citizens of the United States, and shall be remeasured and reregistered upon her arrival after said day at a port of entry in the United States, and prior to her departure therefrom, in the same manner as hereinafter described: *Provided*, That any ship or vessel built within the United States after the passage of this Act may be measured and registered in the manner herein provided.

SEC. 2. *And be it further enacted*, That the register of every vessel shall express her length and breadth, together with her depth, and the height under third or spar deck, which shall be ascertained in the following manner: The tonnage deck, in vessels having three or more decks to the hull, shall be the second deck from below; in all other cases the upper deck of the hull is to be the tonnage deck. The length from the forepart of the outer planking, on the side of the stem, to the after part of the main sternpost of screw steamers, and to the after part of the rudder-post of all other vessels measured on the top of the tonnage deck, shall be accounted the vessel's length. The breadth of the broadest part on the outside of the vessel shall be accounted the vessel's breadth of beam. A measure from the under side of tonnage deck plank, amidships, to the ceiling of the hold (average thickness) shall be accounted the depth of hold. If the vessel has a third deck, then the height from the top of the tonnage-deck plank to the under side of the upper-deck plank shall be accounted as the height under the spar deck. All measurements to be taken in feet and fractions of feet; and all fractions of feet shall be expressed in decimals.

SEC. 3. *And be it further enacted*, That the register tonnage of a vessel shall be her entire internal cubic capacity in tons of one hundred cubic feet each, to be ascertained as follows: Measure the length of the vessel in a straight line along the upper side of the tonnage deck, from the inside of the inner plank (average thickness) at the side of the stem to the inside of the plank on the sterntimbers (average thickness), deducting from this length what is due to the rake of the bow in the thickness of the deck, and what is due to the rake of the stern timber in the thickness of the deck, and also what is due to the rake of the stern timber in one-third of the round of the beam; divide the length so taken into the number of equal parts required by the following table according to the class in such table to which the vessel belongs:

Table of Classes.

Class 1.—Vessels of which the tonnage length according to the above measurement is fifty feet or under, into six equal parts.

Class 2.—Vessels of which the tonnage length according to the above measurement is above fifty feet, and not exceeding one hundred feet long, into eight equal parts.

Class 3.—Vessels of which the tonnage length according to the above measurement is above one hundred feet long, and not exceeding one hundred and fifty feet long, into ten equal parts.

Class 4.—Vessels of which the tonnage length according to the above measurement is above one hundred and fifty feet, and not exceeding two hundred feet long, into twelve equal parts.

Class 5.—Vessels of which the tonnage length according to the above measurement is above two hundred feet, and not exceeding two hundred and fifty feet long, into fourteen equal parts.

Class 6.—Vessels of which the tonnage length according to the above measurement is above two hundred and fifty feet long, into sixteen equal parts.

Then, the hold being sufficiently cleared to admit of the required depths and breadths being properly taken, find the transverse area of such vessel at each point of division of the length as follows:

* Measure the depth at each point of division from a point at a distance of one-third of the round of the beam below such deck, or, in case of a break below a line stretched in continuation thereof, to the upper side of the floor timber, the inside of the limber strake, after deducting the average thickness of the ceiling, which is between the bilge planks and limber strake; then, if the depth at the midship division of the length do not exceed sixteen feet, divide each depth into four equal parts: then measure the inside horizontal breadth, at each of the three points of division, and also at the upper and lower points of the depth, extending each measurement to the average thickness of that part of the ceiling which is between the points of measurement; number these breadths from above (numbering the upper breadth one, and so on down to the lowest breadth); multiply the second and fourth by four, and the third by two; add these products together, and to the sum add the first breadth and the last, or fifth; multiply the quantity thus obtained by one-third of the common interval between the breadths, and the product shall be deemed the transverse area; but if the midship depth exceed sixteen feet, divide each depth into six equal parts, instead of four, and measure as before directed, the horizontal breadths at the five points of division, also at the upper and lower points of the depth; number them from above as before; multiply the second, fourth and sixth by four, and the third and fifth by two; add these products together, and to the sum add the first breadth and the last, or seventh; multiply the quantities thus obtained by one-third of the common interval between the breadths, and the product shall be deemed the transverse area.

Having thus ascertained the transverse area at each point of division of the vessel, as required above, proceed to ascertain the register tonnage of the vessel in the following manner:

Number the areas successively one, two, three, etc., number one being at the extreme limit of the length at the bow, and the last number at the extreme limit of the length at the stern; then whether the length be divided, according to table, into six or sixteen parts, as in classes one and six, or any intermediate number, as in classes two, three, four and five, multiply the second and every even-numbered area by four, and the third and every odd-numbered area (except the first and last) by two; add these products together, and to the sum add the first and last if they yield anything; multiply the quantities thus obtained by one-third of the common interval between the areas, and the product will be the cubical contents of the space under the tonnage deck; divide this product by one hundred, and the quotient, being the tonnage under the tonnage deck, shall be deemed to be the register tonnage of the vessel, subject to the additions hereinafter mentioned.

If there be a break, a poop, or any other permanent closed-in space on the upper decks, on the spar deck available for cargo or stores, or for the berthing or accommodation of passengers or crew, the tonnage of such space shall be ascertained as follows:

Measure the internal mean length of such space in feet, and divide it into an even number of equal parts, of which the distance asunder shall be most nearly equal to those into which the length of the tonnage deck has been divided; measure at the middle of its height the inside breadths—namely, one at each end and at each of the points of division, numbering them successively, one, two, three, etc.; then to the sum of the end breadths, add four times the sum of the even-numbered breadths and twice the sum of the odd-numbered breadths, except the first and last, and multiply the whole sum by one-third of the common interval between the breadths; the product will give the mean horizontal area of such space; then measure the mean height between the plank of the decks, and multiply by it the mean horizontal area; divide the product by one hundred, and the quotient shall be deemed to be the tonnage of such space, and shall be added to the tonnage under the tonnage decks, ascertained as aforesaid.

If a vessel has a third deck, or spar deck, the tonnage of the space between it and the tonnage deck shall be ascertained as follows:

Measure in feet the inside length of the space, at the middle of its height, from the plank at the side of the stem to the plank on the timbers at the stern, and divide the length into the same number of equal parts into which the length of the tonnage deck is divided; measure (also at the middle of its height) the in-

* Chapman's rule, p. 114.

side breadth of the space at each of the points of division, also the breadth of the stem and the breadth at the stern; number them successively one, two, three and so forth, commencing at the stem; multiply the second and all other even-numbered breadths by four, and the third and all the other odd-numbered breadths (except the first and last) by two; to the sum of these products add the first and last breadths, multiply the whole sum by one-third of the common interval between the breadths, and the result will give, in superficial feet, the mean horizontal area of such space; measure the mean height between the plank of the two decks, and multiply by it the mean horizontal area; and the product will be the cubical contents of the space; divide this product by one hundred, and the quotient shall be deemed to be the tonnage of such space, and shall be added to the other tonnage of the vessel, ascertained as aforesaid. And if the vessel has more than three decks, the tonnage of each space between decks, above the tonnage deck, shall be severally ascertained in manner above described and shall be added to the tonnage of the vessel, ascertained as aforesaid.

In ascertaining the tonnage of open vessels the upper edge of the upper strake is to form the boundary line of measurement, and the depth shall be taken from an athwartship line, extending from edge of said strake at each division of the length.

The register of a vessel shall express the number of decks, the tonnage under the tonnage deck, that of the between decks, above the tonnage deck; also that of the poop or other enclosed spaces above the deck, each separately. In every registered United States ship or vessel the number denoting the total registered tonnage shall be deeply carved or otherwise permanently marked on her main beam, and shall be so continued; and if at any time it cease to be so continued, such vessel shall no longer be recognized as a registered United States vessel.

SEC. 4. *And be it further enacted*, That the charge for the measurement of tonnage and certifying the same shall not exceed the sum of one dollar and fifty cents for each transverse section under the tonnage deck; and the sum of three dollars for measuring each between decks above the tonnage deck; and the sum of one dollar and fifty cents for each poop, or closed-in space available for cargo or stores, or for the berthing or accommodation of passengers, or officers and crew, above the upper or spar deck.

SEC. 5. *And be it further enacted*, That the provisions of this act shall not be deemed to apply to any vessel not required by law to be registered, or enrolled, or licensed, and all acts and parts of acts inconsistent with the provisions of this act are hereby repealed.

English Tonnage Measurement.

Divide the length of the upper deck between the after part of the stem and the fore part of the stern-post into 6 *equal parts*, and note the foremost, middle and aftermost points of division. Measure the *depths* at these three points in feet and tenths of a foot, also the *depths* from the under side of the upper deck to the ceiling at the limber strake; or in case of a break in the upper deck, from a line stretched in continuation of the deck. For the *breadths*, divide each depth into 5 equal parts, and measure the inside breadths at the following points, viz.: at .2 and .8 from the upper deck of the foremost and aftermost depths, and at .4 and .8 from the upper deck of the amidship depth. Take the *length*, at half the amidship depth, from the after part of the stem to the fore part of the stern-post.

Then, to twice the amidship depth, add the foremost and aftermost depths for the *sum of the depths;* and add together the foremost upper and lower breadths, 3 times the upper breadth with the lower breadth at the midship, and the upper and twice the lower breadth at the after division for the *sum of the breadths.*

Multiply together the sum of the depths, the sum of the breadths, and the length, and divide the product by 3500, which will give the number of tons, or register.

If the vessel has a poop or half deck, or a break in the upper deck, measure the inside mean length, breadth and height of such part thereof as may be included within the bulkhead; multiply these three measurements together, and divide the product by 92.4. The quotient will be the number of tons to be added to the result as above ascertained.

For Open Vessels.—The depths are to be taken from the upper edge of the upper strake.

For Steam Vessels.—The tonnage due to the engine-room is deducted from the total tonnage computed by the above rule.

To determine this, measure the inside length of the engine-room from the foremast to the aftermost bulkhead; then multiply this length by the midship depth of the vessel, and the product by the inside amidship breadth at .4 of the depth from the deck, and divide the final product by 92.4.

CENTRIPETAL PROPELLER.

THE Centripetal Propeller has, since the year 1851, fought its way through the usual obstructions to success, and is now approved and adopted by the most advanced engineers in Europe and America.

Froude's experiments have revealed to us losses in the propeller of which we have no general mathematical theory. Pending such discovery Nystrom's approximate formulæ may be used.—W. D. M.

The propellers constructed by John Roach for the Pacific Mail Steamship Company are upon the centripetal principle, a full description of which is given in a work entitled "Education and Shipbuilding," published in the year 1866, by H. C. Baird, Philadelphia.

The helicoidal or propelling surface in the common propeller is formed by a straight generatrix at right angle to the axis; whilst in the centripetal propeller that surface is formed by a spiral generatrix constructed in an angle w, Formula 7. In practice this angle can be assumed to be,

$w = 30°$ for the fore-edge, and
$w' = 45°$ for the after-edge of the propeller.

The difference between the angles w and w' makes the pitch expanding from the centre to the periphery.

Having given the spirals a and e, the spirals b, c and d are obtained by dividing the angles into four equal parts, as will be understood by the illustration.

A straight generatrix inclined to the axis will give the same helicoidal surface as that of the curved generatrix at right angles to the axis; but the inclination of the straight generatrix must be according to Formula 8.

The dotted lines $f\ g\ h\ i$ represent a centripetal propeller with straight inclined generatrix. Propellers constructed either as the dotted or drawn lines, or between the two cases, will produce the same propelling effect in the water. When the propeller is constructed between the two cases represented on the drawing, the blades will appear curved in both views.

The length L of the propeller should be from $0{\cdot}2D$ to $0{\cdot}25D$, and the pitch from $1{\cdot}5D$ to $2D$. For very sharp vessels constructed for speed, and when the draft of water is over one-half the beam, the pitch may be made $2{\cdot}5D$.

One quarter of the pitch is set off on the centre line from 0 to 8, and the helix constructed in the ordinary way. The outer edge of the blades should not follow the true helix, but be made slightly concave, as shown in the drawing, which makes the pitch expanding in the direction of the axis.

The mean pitch of the propeller should be calculated by Formula 3, making $r = 0{\cdot}7R$.

Example 1. The diameter of a propeller is 10 feet 6 inches, and the angle $W = 58°$ at the periphery. Required the pitch $P =$ in feet?

$$P = \cot.58° \times 3{\cdot}14 \times 10{\cdot}5 = 20{\cdot}6 \text{ feet.}$$

Example 2. The propeller on Plate XI. is of dimensions $D = 15$ feet, $L = 5$ feet, $W = 57°\ 30'$, the slip is 38 per cent. or $S = 0{\cdot}38$. What power is required to drive it 40 revolutions per minute, $H = ?$

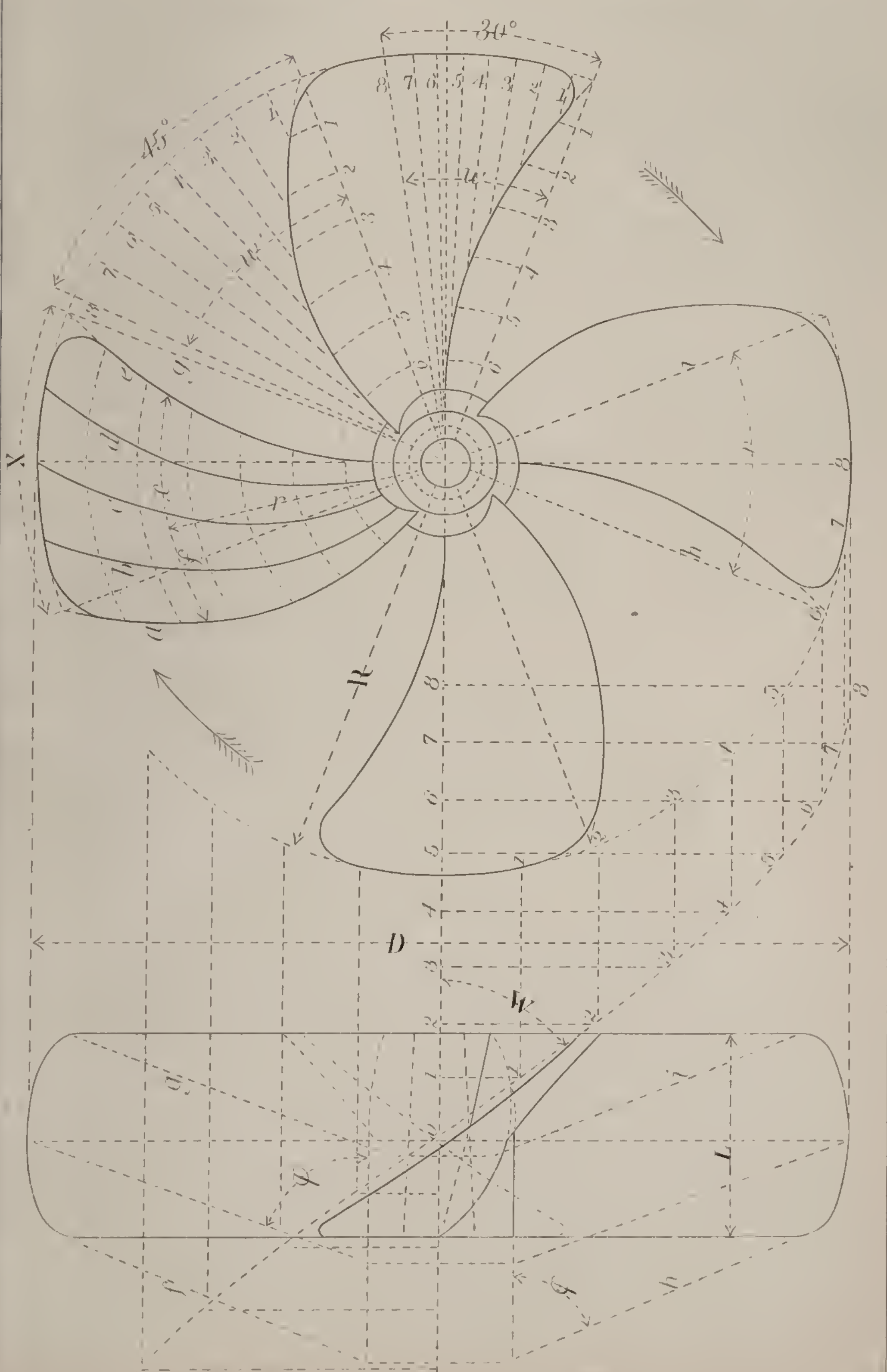
$$H = \frac{15^3 \times 40^3}{480000}\left(5 \times 0{\cdot}38 \times \cos.57°\ 30' + 0{\cdot}11\right) = 514.8\ IP, \text{ nearly.}$$

Example 3. A propeller of diameter $D = 12$ feet, angle $W = 64°$, and length $L = 3$ feet 6 inches, is to be driven by a steam engine of 450 IP, the slip $S = 0{\cdot}28$. How many revolutions will it make per minute, $n = ?$

$$n = \frac{78}{12}\sqrt[3]{\frac{450}{(3{\cdot}5 \times 0{\cdot}28 \times \cos.64° + 0{\cdot}11)}} = 61 \text{ revolutions}$$

per minute.

Centripital Propeller.



Pitch.	*Angles.*	*Areas.*
$P = \pi D \cot. W$ 1	$Cot. W = \frac{P}{\pi D}$, 5	$a = \frac{D^2 L m}{P}$, - - - - - 9
$P = \frac{360 L}{v}$, - 2	$v = \frac{360 L}{P}$, - 6	$A = \frac{D m}{3·5}\left(L + X\right)$, - - 10
$P = \frac{2 \pi r}{\sqrt{x^2 - L^2}}$, 3	$w = \frac{D n^2 S^2}{102·4}$, - 7	$a = \frac{D}{2·75}\left(P + \sqrt{\pi^2 D^2 + P^2}\right)$, 11
$P = \frac{D^2 L m}{a}$, - 4	$Cot. \phi = \frac{w P}{180 D}$, 8	$\bigcirc = \frac{2·5 D^2}{\sqrt{\pi^2 D^2 + P^2}}$, - - - 12

Horsepower and Revolutions.

$$H = \frac{D^2 n^3}{480000}\left(L S \cos. W + 0·11\right), \quad 13$$

$$n = \frac{78}{D}\sqrt[3]{\frac{H}{L S \cos. W + 0·11}}, \quad 14$$

Horsepower of Friction.

$$h = \frac{R L k m n^3}{59{,}400{,}000 \, P}\left(311·7 R^4 + 26·42 R^2 P^2 + P^4\right), \quad - - - 15$$

D = diameter, R = radius, L = length, and P = pitch of the propeller in feet.
W = angle of the blades to the centre line.
v = projecting angle of each blade.
w = centripetal angle for the curved generatrix.
ϕ = angle of inclination of the straight generatrix.
a = projecting area of all the blades.
A = helicoidal surface of the propelling side of all the blades.
a = helicoidal surface of one whole convolution.
○ = acting area at right angles to the axis. All areas in square feet.
x = length of any helix at radius r, and m = number of blades.
X = length of external helix of the blade.
n = number of revolutions per minute.
H = horsepower required to drive the propeller.
h = horsepower required for friction in the water.
k = friction coefficient. See page 448.

The pitch of the propeller is equal to the tabular number opposite the given angle W, multiplied by the diameter.

W	Pitch.	W	Pitch.	W	Pitch.	W	Pitch.	W	Pitch.	W	Pitch.
30	5·45	40	3·74	50	2·63	60	1·81	70	1·14	80	0·55
31	5·23	41	3·62	51	2·54	61	1·74	71	1·11	81	0·50
32	5·03	42	3·50	52	2·45	62	1·67	72	1·02	82	0·44
33	4·85	43	3·27	53	2·37	63	1·60	73	0·96	83	0·37
34	4·66	44	3·20	54	2·28	64	1·53	74	0·90	84	0·33
35	4·50	45	3·14	55	2·20	65	1·46	75	0·84	85	0·27
36	4·33	46	3·09	56	2·12	66	1·40	76	0·78	86	0·22
37	4·17	47	2·93	57	2·04	67	1·33	77	0·72	87	0·16
38	4·02	48	2·83	58	1·96	68	1·27	78	0·67	88	0·11
39	3·88	49	2·73	59	1·89	69	1·20	79	0·61	89	0·06

Approximate Comparative Values of Metals per Pound Avoirdupois.

PRICES OF METALS.

Metal.	Condition.	Value per Pound Avoirdupois.
Vanadium	Cryst. fused	$4,792.40
Rubidium	Wire	3,261.60
Calcium	Electrolytic	2,446.20
Tantalum	Pure	2,446.20
Cerium	Fused globule	2,446.20
Lithium	Globules	2,228.76
Lithium	Wire	2,935.44
Erbium	Fused	1,671.57
Strontium	Electrolytic	1,576.44
Indium	Pure	1,522.08
Ruthium		1,304.64
Columbium	Fused	1,250.28
Rhodium		1,032.84
Barium	Electrolytic	924.12
Thallium		738.39
Osmium		652.32
Palladium		498.30
Iridium		466.59
Uranium		434.88
Gold		299.72
Titanium	Fused	239.80
Tellurium	"	196.20

PRICES OF METALS.

Metal.	Condition.	Value per Pound Avoirdupois.
Chromium	Fused	$196.20
Platinum	"	122.31
Manganese	"	108.72
Molybdenum		54.34
Magnesium	Wire and tape.	45.30
Potassium	Globules	22.65
Silver		18.60
Aluminium	Bar	16.30
Cobalt	Cubes	12.68
Nickel	"	3.80
Cadmium		3.26
Sodium		3.26
Bismuth	Crude	1.95
Mercury		1.00
Antimony		36
Tin		25
Copper		22
Arsenic		15
Zinc		10
Lead		6
Iron		1½

COUNTING SECONDS.

WHEN the occurrence of a distant sound is not anticipated, we are unprepared to record the exact moment, and before an appropriate time-keeper can be procured an uncertain time has elapsed.

With some practice seconds can be counted in the mind with tolerable correctness without the aid of a time-keeper; which practice has been of great service to the author in astronomical observations—practice counting seconds by the aid of an oscillating second pendulum or by the second-hand on a watch until the counting agree with the time-keeper, without attention to the pendulum or second-hand. With good practice the counting should not differ more than one second per minute.

When an unexpected distant sound is heard and its cause observed, we can always be ready to count seconds and thus determine the distance.

In astronomical observations at sea it is customary to keep a watch in the hand or to station an assistant at the chronometer to note the time when the observer says "stop;" but there are known cases when the captain has taken his observations without the aid of a watch or assistant, and walked slowly and comfortably to his cabin and noted the time of his observations from the chronometer, with no little amusement to other observers, who naturally supposed that the captain's observations could not be very correct, but to their surprise were found to be as correct as their observations with ordinary precautions. The captain counted in his mind the seconds, and deducted the sum from the time observed on the chronometer.

The practice of counting seconds correctly is of great utility and service for estimating various movements. When the action is of very short duration, say less than 3 seconds, it is best to count half seconds or even four times per second, and a short time may be determined with a correctness within a quarter of a second.

SOUND.

Velocity of Sound through Air.

v = velocity in feet per second.
t = temperature of the air, Fahr. scale.
D = distance in feet the sound travels in the timo T.

$$v = 1089.42\sqrt{1 + 0.00208(t - 32)}.$$

Velocity of sound in water is about 4 times that in air, and 8 times that through solids.

Intensity of sound is inversely as the square of the distance.

$$D = 1089.42T\sqrt{1 + 0.00208(t - 32)},$$

$$T = \frac{D}{v}.$$

Example. A ship at sea was seen to fire a cannon. and 6.5 seconds afterward the report was heard; the temperature in the air was 60°. Required, the distanco to the ship.

$$D = 1089.42 \times 6.5\sqrt{1 + .00208(60° - 32)} = 7284 \text{ feet, or } 1.38 \text{ miles.}$$

Descriptions of Sound.	Audible at a distance of Feet.	Miles.
A powerful human voice in the open air, no wind,	460	0.087
Report of a musket, ...	16,000	3.02
Drum, ...	10,500	2
Music, strong brass band, ...	15,840	3
Cannonading, very strong, ...	575,000	90
In a barely observable breeze a strong human voice with the wind can be heard.	15,840	3

Distance in Feet which Sound Travels in Air at Different Temperatures.

Time Sec.	Temperature of the Air, Fahrenheit Scale. 0°	10°	20°	32°	40°	50°	60°	70°	80°	90°	100°
1	1000	1064.2	1075.7	1089.4	1098.5	1109	1120	1131	1142	1153	1164
2	1985	2128	2151	2179	2197	2219	2241	2262	2285	2306	2328
3	2978	3193	3227	3268	3295	3328	3361	3393	3427	3459	3492
4	3971	4257	4303	4358	4394	4438	4482	4524	4570	4613	4656
5	4964	5321	5378	5447	5492	5548	5603	5655	5712	5766	5821
6	5956	6385	6454	6536	6591	6657	6723	6786	6855	6919	6984
7	6949	7449	7530	7626	7689	7767	7844	7917	7997	8072	8148
8	7962	8514	8606	8715	8788	8876	8964	9049	9140	9225	9312
9	8934	9578	9681	9805	9886	9986	10085	10180	10282	10379	10476
10	9927	10642	10757	10894	10985	11096	11306	11311	11425	11532	11640
11	10920	11706	11833	11983	12083	12205	12326	12442	12567	12685	12804
12	11912	12770	12908	13073	13182	13315	13447	13573	13710	13838	13968
13	12905	13835	13984	14162	14280	14424	14567	14704	14852	14991	15132
14	13898	14899	15060	15252	15379	15534	15688	15835	15995	16145	16296
15	14891	15963	16135	16341	16477	16644	16809	16966	17137	17298	17460
16	15883	17027	17211	17430	17576	17753	17929	18097	18280	18451	18624
17	16876	17091	18287	18520	18674	18863	19050	19228	19422	19604	19788
18	17889	19156	19363	19609	19773	19972	20170	20360	20565	20757	20952
19	18861	20220	20438	20699	20871	21082	21291	21491	21707	21911	22116
20	19854	21284	21514	21788	21970	22192	22412	22622	22850	23064	23280

MUSICAL VIBRATIONS.

MUSICAL vibration is the most accurate measurement of small intervals of time.

C = first term, or vibrations of the fundamental note.

c = the last term, or vibrations of the octave above the fundamental note. $2C = c$.

n = number of double vibrations of any note in the musical scale whose number of terms, from C inclusive, is a.

a = number of terms between C and n inclusive.

r = ratio of vibrations between each note or term.

Each term multiplied by the ratio r gives the next following term, when the progression is increasing.

$$\text{Ratio } r = \sqrt[a-1]{\frac{c}{C}}. \quad \text{Vibrations } n = C r^{a-1}.$$

In the application of these formulas to the division of the octave into the chromatic scale of thirteen notes, we can assume any arbitrary number of vibrations of the fundamental note, say $C = 32$, and the octave will then vibrate $c = 64$. Making $a = 13$, we find the

$$\text{Ratio } r = \sqrt[a-1]{\frac{c}{C}} = \sqrt[13-1]{\frac{64}{32}} = \sqrt[12]{2}.$$

$$\text{Log. } r = \frac{\text{log. } 2}{12} = \frac{0.30102999566}{12} = 0.02508583 = \text{log. } 1.059462 \text{ the ratio.}$$

The proportionate vibration of any note whose number, from C inclusive, is a, will be

$$n = C r^{a-1} = 32 \times 1.059462^{a-1}.$$

The number a includes also the half-notes or sharps.

The harmonic intonation of the diatonic scale is established as follows:

	C	D	E	F	G	A	B	c
	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Harmonic,	32	36	40	$42\frac{2}{3}$	48	$53\frac{1}{3}$	60	64
Tempered,	32	35.9188	40.3175	42.7149	47.9458	53.8174	60.408	64
Difference,	0.000	−0.0812	+0.3175	+0.0483	−0.0542	+0.4841	+0.408	0.000

The actual number of double vibrations, per second, of the standard concert pitch now generally used was established by a Congress of Philosophers, which met in Stuttgart in the year 1834; namely, $C' = 264$, $A' = 440$, and $C'' = 528$. With this data the following table is calculated.

The last column, "Prop. length of waves," shows the proportionate diameters of bells for the corresponding note, when the sound-bow is of a certain proportion to the diameter in all the bells. This column also shows how to divide the bridge on a *guitar*, when the whole length of the string being 1 or the unit.

Ringing Bells.

D = diameter of the bell in inches.

S = thickness of the sound-bow in inches.

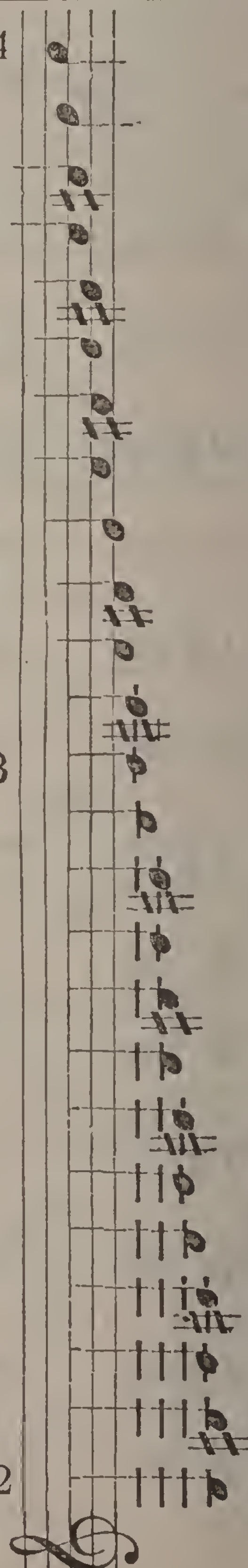
n = double vibration per second, corresponding to the pitch of tone.

W = weight of the bell in pounds avoirdupois.

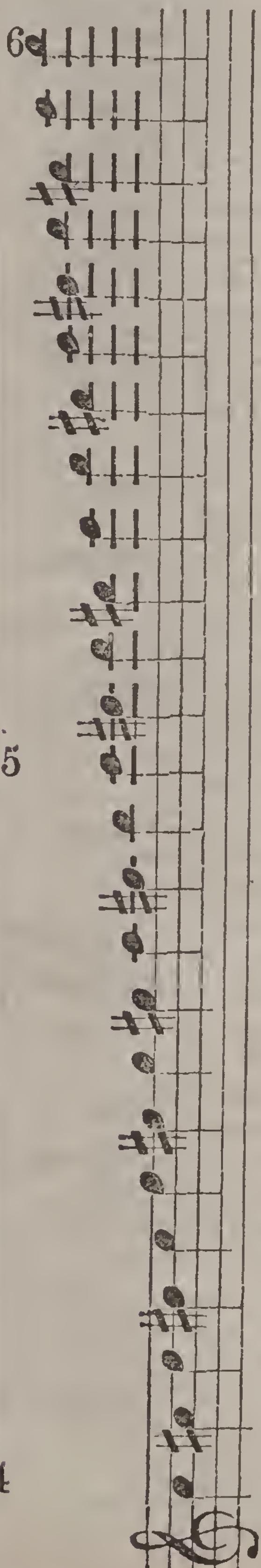
$$\text{Diameter, } D = \frac{240000\,k}{n}. \qquad k = \frac{S}{D}.$$

$$\text{Weight, } W = 0.3\,D^2S \text{ to } 0.35\,D^2S.$$

Stuttgart Harmonic Scale.			Difference.		Tempered Geometric Scale.	
a Terms.	Keynote.	Double Vibrations. n'	Ratio of Pitch. $\frac{n}{n'}$	In Vibration. $n - n'$	Double Vibrations. n	Prop. Length of waves. $\frac{264}{n}$
13	C	528	1.00000	0.000	528	0.50000
		33.			29.635	
12	B	495	1.00680	+3.365	498.365	0.52973
		27.5			27.970	
11	A♯	*467.5*	1.00616	+2.895	470.395	0.56123
		27.5			26.402	
10	A	440	1.00908	+3.993	443.993	0.59461
		22			24.919	
9	G♯	*418*	1.00257	+1.074	419.074	0.62996
		22			23.519	
8	G	396	0.99887	−0.447	395.553	0.66742
		22			22.201	
7	F♯	374	0.99827	−0.648	373.352	0.70711
		22			20.954	
6	F	352	1.00113	+0.398	352.398	0.74915
		22			19.787	
5	E	330	1.00794	+2.611	332.611	0.79370
		16.5			18.660	
4	D♯	*313.5*	1.00144	+0.451	313.951	0.84090
		16.5			17.621	
3	D	297	0.99774	−0.670	296.330	0.89090
		16.5			16.638	
2	C♯	*280.5*	0.99714	−0.808	279.692	0.94388
		16.5			15.692	
1	C	264	1.00000	0.000	264	1.00000
		16.5			14.817	

	Keynote.	Vibrations. n.		Keynote.	Vibrations. n.
2	C	132.	4	C	528.
	B	124.591		B	498.365
	A♯	117.598		A♯	470.395
	A	110.998		A	443.993
	G♯	104.768		G♯	419.097
	G	98.888		G	395.552
	F♯	93.338		F♯	373.352
	F	88.099		F	352.398
	E	83.152		E	332.611
	D♯	78.488		D♯	313.951
	D	74.082		D	296.330
	C♯	69.923		C♯	279.602
1	C	66.	3	C	264.
	B	62.295		B	249.182
	A♯	58.799		A♯	235.197
	A	55.499		A	221.997
	G♯	52.384		G♯	209.537
	G	49.444		G	197.776
	F♯	46.669		F♯	186.676
	F	44.049		F	176.199
	E	41.576		E	166.305
	D♯	39.244		D♯	156.975
	D	37.041		D	148.165
	C♯	34.961		C♯	139.846
0	C	33.	2	C	132.



	Keynote.	Vibrations. n.		Keynote. 3 Octaves above.	Vibrations. n.
6	C	2112.	8	C	8448.
	B	1993.46		B	7973.84
	A♯	1881.58		A♯	7526.32
	A	1775.97		A	7103.88
	G♯	1676.30		G♯	6705.20
	G	1582.21		G	6328.84
	F♯	1493.41		F♯	5973.64
	F	1409.59		F	5638.36
	E	1330.44		E	5321.76
	D♯	1255.80		D♯	5023.20
	D	1185.32		D	4741.28
	C♯	1118.77		C♯	4475.08
5	C	1056.	7	C	4224.
	B	996.730		B	3986.92
	A♯	940.780		A♯	3763.12
	A	887.986		A	3551.94
	G♯	838.148		G♯	3352.59
	G	791.106		G	3164.42
	F♯	746.704		F♯	2986.82
	F	704.796		F	2819.18
	E	665.222		E	2660.89
	D♯	627.902		D♯	2511.61
	D	592.660		D	2370.64
	C♯	559.384		C♯	2237.54
4	C	528.	6	C	2112.

NAME OF ELEMENTS.	Sym-bol.	Old eqvlt.	New eqvlt.	Sp. gr.	REMARKS ON THE ELEMENTS.
Aluminium	Al.	13.7	27.5	2.50	Light metal. Like zinc.
Antimony	Sb.	129.0	122.	6.70	White metal used in types.
Arsenicum	As.	75.	75.	5.80	Metal, steel-gray lustre.
Barium	Ba.	68.5	137.2	4.70	White metal, fuses at red heat.
Bismuth	Bi.	210.30	210.	9.80	Hard brittle reddish metal.
Boron	Bo.	10.9	11.	2.00	Combination with potassium.
Bromine	Br.	80.	80.	3.187	Deep red volatile liquid.
Cadmium	Cd.	56.	111.6	8.60	Very soft and ductile metal.
Cæsium	Cs.	132.4	132.15	......	Two strong blue lines in spectr.
Calcium	Ca.	20.	39.9	1.57	Light yellow malleable metal.
Carbon	C.	6.	12.	3.52	Diamond. Graphite. Coal.
Cerium	Ce.	46.	141.3	5.5	Little known and less used.
Chlorine	Cl.	35.5	35.5	2.44	Gas, greenish-yellow color.
Chromium	Cr.	26.3	52.4	6.8	Dark-gray metal, strong affinity
Cobalt	Co.	29.5	58.6	8.9	Reddish-gray, magnetic metal.
Copper (Cuprum)	Cu.	31.7	63.3	8.9	Yellowish-red ductile metal.
Didymium	D.	48.	147?	......	Little known and less used.
Erbium	E.	......	170.6	......	Classed as a metal.
Fluorine	F.	19.	19.1	1.31	Found in fluor spar.
Gallium	Ga.	......	69.9	5.956	Silver-white metal.
Glucinum	Gl.	4.7	9.25	2.1	Its salt has a sweet taste.
Gold (Aurum)	Au.	196.44	196.2	19.34	Standard of value.
Hydrogen	H.	1.	1.	0.0692	Lightest of gases.
Indium	In.	74.	113.4	7.2	Dark-blue lines in spectrum.
Iodine	I.	127.	127.	4.94	Metallic bluish solid.
Iridium	Ir.	98.6	196.7	22.40	Hard white metal.
Iron (Ferrum)	Fe.	28.	55.9	7.8	The most useful metal.
Lanthanum	La.	46.	139.	......	Little known and less used.
Lead (Plumbum)	Pb.	103.6	206.4	11.44	Soft and malleable metal.
Lithium	Li.	7.	7.022	0.593	White metal, burns brilliantly.
Magnesium	Mg.	12.16	24.	1.7	Burns brilliantly.
Manganese	Mn.	27.40	54.8	8.	Grayish-white metal.
Mercury	Hg.	100.	200.	13.59	White liquid metal.
Molybdenum	Mo.	48.	95.8	8.6	White brittle metal.
Nickel	Ni.	29.5	58.6	8.8	White, hard, ductile metal.
Niobium	Nb.	48.8	94.	......	Not generally known.
Nitrogen	N.	14.	14.044	0.971	Gas without color or taste.
Osmium	Os.	99.4	198.6	22.48	White and brittle metal.
Oxygen	O.	8.	16.	1.1087	Gas, supports life and combus'n.
Palladium	Pd.	53.2	106.2	11.5	Hard ductile white metal.
Phosphorus	P.	31.	31.	1.83	Translucent solid easily ignited.
Platinum	Pt.	98.6	196.7	21.5	Heaviest of all metals.
Potassium (Kalium).	K.	39.	39.137	0.855	Brittle metal, melts at 130°.
Rhodium	Rh.	52.2	104.2	11.	White, hard metal.
Rubidium	Rb.	85.36	85.2	1.52	Metal little known.
Ruthenium	Ru.	52.11	103.5	8.6	Most infusible of metals.
Selenium	Se.	39.7	78.	4.8	A semi-metallic solid.
Silicon	Si.	14.	28.	2.49	Flint, quartz, glass, and clay.
Silver (Argentum)	Ag.	108.	108.	10.5	Metal of standard value.
Sodium (Natrium)	Na.	23.	23.043	0.972	Bluish-white and soft metal.
Strontium	Sr.	43.8	87.2	2.54	White metal like barium.
Sulphur	S.	16.	32.	2.	Brimstone, widely used.
Tantalum	Ta.	68.8	182.	10.7	Little used.
Tellurium	Te.	64.5	128.	6.6	Lustre of metal like sulphur.
Thallium	Tl.	204.	203.6	11.8	Green line in spectrum.
Thorium	Th.	59.5	233.9	7.7	Not used in the arts.
Tin (Stannum)	Sn.	59.	117.8	7.3	White and malleable metal.
Titanium	Ti.	25.	48.	5.28	Its oxide used for painting.
Tungsten (Wolfram).	W.	92.	184.	19.13	An iron-gray metal.
Uranium	U.	60.	240.	10.15	A steel-white metal.
Vanadium	V.	68.5	51.2	5.5	A metal little used.
Yttrium	Y.	32.2	89.6	......	Found in Sweden in 1843.
Zinc	Zn.	32.6	64.9	7.	A bluish-white metal.
Zirconium	Zr.	44.8	90.	4.15	In nature as silicate.

Solids and Salts.	FORMULAS.	COMMERCIAL NAMES AND USE.
Aluminium sulphate........	$Al_2(SO_4)_3$.	For preparing salts of aluminium.
Ammonium chloride........	NH_4Cl.	Sal ammoniac, for soldering.
Arsenious acid................	As_2O_3.	White arsenic, poisonous.
Barium oxide....................	BaO.	Baryta, a gray powder.
Barium sulphate..............	$BaSO_4$.	Heavy Spar. Fr. adult. wt. lead.
Calcium oxide.................	CaO.	Quick or caustic lime.
Camphor............................	$C_{10}H_{16}O$.	Used for making celluloid.
Carbolic acid.....................	C_6H_6O.	Used as a disinfectant.
Carbonate of lime............	CaO,CO_2.	Common limestone, marble.
Chloride of lime..............	$CaCl_2O_2$.	Bleaching powder.
Chloride of sodium..........	$ClNa$.	Common salt.
Copper sulphate...............	$CuSO_4$.	Blue stone or vitriol.
Copper pyrites.................	$Cu_2S.Fe_2S_2$.	Pyramidal and tetrahedral crystals.
Cuprous oxide..................	Cu_2O.	Red oxide of copper.
Gold chloride.....................	$AuCl_3$.	Used in photography.
Gold mercury....................	Au_2Hg.	Gold amalgam.
Gun-cotton	$C_6H_7(NO_2)_3O_5$.	Used as an explosive.
Hydrogen sodium carb'te.	$HNaCO_3$.	Baking powder, artificial yeast.
Hydrogen potass. carb'te..	$HKCO_3$.	Yeast for raising bread.
Iron, ferric oxide.	Fe_2O_3.	Red hematite, iron ore.
Iron, ferric hydrate.........	$Fe_2H_6O_6$.	Yellow ochre, iron ore.
Iron, magnetic oxide........	Fe_3O_4.	Loadstone, iron ore.
Iron, bisulphide...............	FeS_2.	Pyrites, cube crystals.
Iron, ferrous sulphate......	$FeSO_4 + 7H_2O$.	Green vitriol, copperas.
Indigo blue.......................	C_8H_5NO.	Used in dyeing.
Lead chromate.................	PbO,CrO_3.	Chrome-yellow.
Lead protoxide.................	PbO.	Litharge, dryer for oils.
Lead chloride and oxide...	$(PbCl_2.7PbO)$.	Pigment, Turner's yellow.
Lead carbonate................	PbO,CO_2.	White lead, paint.
Lead sequi-oxide.............	Pb_3O_4.	Minium, red lead.
Lead sulphide..................	PbS.	Galena, lead ore.
Lapis lazuli......................	$2AlPO_4.MgH_2O_2$.	Blue precious stone.
Malachite..........................	$CuCO_3.CuH_2O_2$.	Green precious stone.
Manganese binoxide........	MnO_2.	For making chlorine and oxygen.
Mercury chloride.............	$HgCl_2$.	Corrosive sublimate.
Mercury sulphide............	HgS.	Cinnabar, ore of mercury.
Oxalic acid	$C_2H_2O_4$.	A powerful poison.
Paraffin.............................	$C_{27}H_{54}$.	For making candles.
Potassium carbonate........	K_2CO_3.	Used for making glass.
Potassium chlorate...........	$KClO_3$.	For making oxygen in medicine.
Potassium chromate.........	K_2CrO_4.	Used for bleaching. Calico printing.
Potassium cyanide...........	KCN.	Used in photography.
Potassium hyd. tartrate ...	$HKC_4H_4O_6$	Cream of tartar.
Potassium nitrate............	KNO_3.	Saltpetre, prismatic crystals.
Saccharose......	$C_{12}H_{22}O_{11}$.	Cane-sugar, gum-arabic.
Silver chloride.................	$AgCl$.	Horn-silver, in photography.
Silver nitrate	$AgNO_3$.	Lunar caustic.
Silver cyanide	$AgCN$.	Used in electro-plating.
Sodium borate..................	$Na_2B_4O_7.10H_2O$.	Borax, used as a flux.
Sodium nitrate.................	$NaNO_3$.	Soda saltpetre, cubic crystals.
Sodium carbonate............	Na_2CO_3.	Soda, used for making soap.
Sodium oxide...................	NaO.	Soda, oxide of natrium.
Stannous chloride............	$SnCl_2$.	Tin-salt, used in dyeing.
Stannic oxide...................	SnO_2.	Tin-stone, cassiterite.
Starch................................	$C_6H_{10}O_5$.	Used in washing.
Stearic acid......................	$C_{18}H_{36}O_2$.	Solid fat, candles.
Strychnine......	$C_{21}H_{22}N_2O_2$.	Strong poison.
Sulphate of soda	$NaO,SO_3 + 10H_2O$.	Glauber salt, colorless prisms.
Sulphate of lime...............	$Ca,SO_4 + 2H_2O$.	Alabaster, gypsum, plaster Paris.
Tannic acid.......................	$C_{27}H_{22}O_{17}$.	For tanning leather.
Zinc chloride.....................	$ZnCl_2$.	For preserving timber.
Zinc sulphate...................	$ZnSO_4$.	White vitriol, used in medicine.

Equal proportions of different atoms may be formed into different orders and make different substances, as cane-sugar and gum-arabic.

Liquids.	FORMULAS.	COMMERCIAL NAMES OR USE.
Water	H_2O.	The most abundant liquid.
Alcohol. Ethyl	C_2H_6O.	Spirit of wine, intoxicating.
Methyl alcohol	CH_4O.	Wood spirit.
Ether	$(C_2H_5)_2O$.	Used as a solvent, anæsthetic.
Chloroform	$CHCl_3$.	Used as an anæsthetic.
Glycerine	$(C_3H_5)H_3O_3$.	Much used in the arts.
Nitro-glycerine	$C_3H_5N_3O_9$.	The most powerful explosive.
Oil of turpentine	$C_{10}H_{16}$.	Spirit of turpentine.
Benzol	C_6H_6.	Constituent of coal-tar.
Nitro-benzol	$C_6H_5(NO_2)$.	Forms the main portion of aniline.
Aniline	C_6H_7N.	For aniline colors.
Carbon bisulphide	CS_2.	A solvent for India-rubber
Nitric acid	HNO_3.	Aqua-fortis, oxidizing agent.
Sulphuric acid	H_2SO_4.	Oil of vitriol, much used.
Hydrochloric acid	HCl.	Muriatic acid.
Nitro-muriatic acid	$HNO_3 + 2HCl$.	Aqua-regia, dissolves gold.
Citric acid	$C_6H_8O_7$.	Juice of lemons.
Oxalic acid	$C_2H_2O_4 2H_2O$.	A powerful poison.
Quinic acid	$C_7H_{12}O_6$.	From Peruvian bark.
Quinine	$C_{20}H_{24}N_2O_2$.	For chills and fever.
Gases.		
Atmospheric air	$4N + O$.	Not chemically combined.
Nitrous oxide	N_2O.	Laughing-gas.
Nitric oxide	NO.	Extinguishes fire.
Carbonic acid	CO_2.	Perfectly consumed coal.
Carbonic oxide	CO.	Suffocating, poisonous.
Carburetted hydrogen	CH_4.	Marsh-gas, fire-damp.
Olefiant gas	C_2H_4.	Illuminating gas.
Cyanogen	NC.	Produces blue color.
Ammonia	NH_3.	Hartshorn, volatile alkali.
Cyanhydric acid	HCN.	Prussic acid, poisonous.
Hydrogen sulphide	H_2S.	Used as a reagent in laboratory.
Sulphurous anhydride	SO_2.	Used for bleaching straw.

Proportion of Compounds by Weight or Volume.

NAMES.	*Carbon.* *C.*	*Hydrogen.* *H.*	*Oxygen.* *O.*	*Nitrogen.* *N.*
Olive oil, by weight	772	133	95	
Spermaceti oil, "	780	118	102	
Castor oil, "	740	103	157	
Linseed oil, "	760	113	127	
Alcohol, "	527	129	344	
Sugar, "	432	68	500	
Atmosp. air	......	......	230	770
" air by volume	......	......	210	790
Water, fresh, by weight	......	1	8	
" " " volume	......	2	1	
India-rubber by weight	853	147		

To Transform Atomic Formulas into Weights.

Rule. Multiply together the equivalent (equiv.) and the exponent (exp.) of each substance, and the product is the proportion in the compound by weight. Divide each weight by its specific gravity, gives the proportions by bulk or volume.

Example 1. The chemical formula for common alcohol is C_2H_6O. Required its proportioned parts by weight in 1000?

Equiv. Exp.

$$\left.\begin{array}{lll} \text{Carbon} & C_2 & = 12 \times 2 = 24 \\ \text{Hydrogen} & H_6 & = \ \ 1 \times 6 = \ \ 6 \\ \text{Oxygen} & O & = 16 \times 1 = 16 \end{array}\right\} \times 21{\cdot}74 \left\{\begin{array}{r} 521{\cdot}76 \\ 130{\cdot}44 \\ 347{\cdot}84 \end{array}\right\} \text{by weight.}$$

$$1000 : 46 = 21{\cdot}74 \qquad 1000{\cdot}04$$

NITRO-GLYCERINE, $C_3H_5N_3O_9$.

Nitro-glycerine is an oily liquid of the above composition, which is highly explosive under peculiar circumstances, but can be set fire to and burned like alcohol without explosion. It explodes by concussion or pressure of about 2000 pounds to the square inch, or by the corresponding temperature of about 600° Fahr. suddenly applied.

Small portions of nitro-glycerine, say half an ounce each, placed (any number) within a few feet of one another, if one of them is exploded, all the rest will explode **instantaneously.** Therefore, when a charge is to be exploded, care must be taken that no more of it is in the neighborhood.

It may appear strange that nitro-glycerine can be so dangerous to handle, when it requires the enormous pressure of 2000 pounds to the square inch to explode it; but the fluid may be squeezed between surfaces of only one 10,000th part of one square inch, when the pressure need be only 3 ounces to explode it.

The charge of nitro-glycerine in blasting is exploded by a percussion cap placed on the end of a fuse and dipped into the liquid. The fuse explodes the fulminate in the cap, the concussion of which explodes the charge. On account of the action of nitro-glycerine being instantaneous, no tamping is required in the blast-hole, except water or loose sand, but even that is not necessary. This explosive is therefore entirely unfit for use in firearms, which would be blown to pieces without discharge through the muzzle.

The many and very serious accidents which have happened by unexpected explosions of nitro-glycerine have caused it to be forbidden transportation on railroads and steamboats, for which a new form of the explosive has been invented, which consists in mixing sawdust and some other solid substances with nitro-glycerine, to the form of a moist brown powder, of nearly the same specific gravity as that of water.

Dynamite.

This powder is called dynamite, and is now manufactured by the nitro-glycerine inventor, Alfred Nobel, in Hamburg, and also by the Giant Powder Company, in San Francisco, California.

The strength and instantaneous action of dynamite are precisely the same as those of nitro-glycerine, but it is much safer to handle, it is said—more so than common gunpowder. The dynamite powder is made up into cartridges of different sizes to suit the blast-hole, and is exploded by percussion caps like nitro-glycerine, and requires no tamping. It has been employed with great success in blasting immense masses of rock in the Andes, Peru.

The price of dynamite is higher than that of gunpowder per weight, but its execution per price is much greater. The blast-holes for dynamite need be only one-half the size of those for gunpowder, with equal execution.

The instantaneous action of dynamite makes it far superior to gunpowder in blasting, but it is unfit for use in firearms.

Any number of cartridges of dynamite placed in a deep blast-hole with tampings of sand between, if one of them is exploded, all the rest will explode simultaneously. Small cartridges are made for the percussion cap, and called primers, by which the principal charge is exploded. Should a charge fail to explode, put in a new fuse and primer.

Blasting under Water.

For this purpose the cartridges should be made of strong oiled paper and perfectly water-tight, to save the dynamite from moisture. The cartridges should also be ballasted, so as to sink easy in water, which can be done by placing a lead ball in the bottom and pack the dynamite on the top, after which the cartridge is hermetically sealed with some varnish insoluble in water. The cartridges (any number) are guided into the blast-hole through a tube, and finally the primer with the fuse, by which the whole charge is exploded.

Dynamite is insoluble in water, but will not explode if moist with water. It freezes to a snowy mass at 40° Fahr., but its explosive quality is not impaired thereby. At 212° the nitrogen evaporates and spoils the powder.

CEMENT, CONCRETE AND MORTAR.

Roman Cement. *Parker's analysis.*

One part of common clay to $2\frac{1}{8}$ parts of chalk, set very quick.

Concrete. Eight parts of pebble or pieces of brick about the size of an egg, to 4 parts of scrap river-sand, and 1 part of good lime, mixed with water and grouted in, makes a good concrete.

Lime Mortar. One part of river-sand to two parts of powdered lime, mixed with fresh water.

Hydraulic Mortar. One part of pounded brick powder to two parts of powdered lime mixed with fresh water. This mortar must be laid very thick between the bricks, and the latter well soaked in water before laid.

No. 1. **Hydraulic Concrete,** by Treussart.
30 parts of hydraulic lime, measured in bulk before slacked.
30 " sand.
20 " gravel.
40 " broken stone, a hard limestone.
This concrete diminishes one-fifth in volume after manipulation.
The mortar is made first, and then mixed with gravel and stone.

No. 2. **Another Concrete,** by Treussart.
33 volumes hydraulic lime unslacked.
45 " Puzzolano (Pozzulano).
22 " sand.
60 " broken stone and gravel.

No. 1 should be put in place immediately after mixed, whilst No. 2 ought to rest 12 hours.

Asphalte Composition for street pavement, by Colonel Emy.
$2\frac{1}{2}$ pints (wine measure) of pure mineral pitch.
11 lbs. of Gangeac bitumen.
17 pints of powdered stone-dust, wood-ashes or minion.

Cements for Cast Iron.

Two ounces sal-ammoniac, one ounce sulphur and sixteen ounces of borings or filings of cast iron, to be mixed well in a mortar and kept dry. When required for use, take one part of this powder to twenty parts of clear iron borings or filings, mixed thoroughly in a mortar, make the mixture into a stiff paste with a little water and then it is ready for use. A little fine grindstone sand improves the cement.

Or, one ounce of sal-ammoniac to one hundred weight of iron borings. No heat allowed to it.

The cubic contents of the joint in inches, divided by 5, is the weight of dry borings in pounds Avoir. required to make cement to fill the joint nearly.

Cement for Stone and Brick Work.

Two parts ashes, three of clay and one of sand, mixed with oil, will resist weather equal to marble.

Brown Mortar.

One part Thomaston lime, two of sand and a small quantity of hair.

Hydraulic Mortar.

Three parts of lime, four Puzzolano, one smithy ashes, two of sand and four parts of rolled stone or shingle.

Crushing Weight in Pounds per Square Inch

on Portland cement, mixed with different proportions of sand, and at different age of the mixture in months.

Age in months.	Parts of sand to one of cement.						
	0	1	2	3	4	5	6
3	3800	2490	1900	1500	1200	950	780
6	5280	3550	2750	2190	1800	1500	1200
9	5980	4150	3350	2700	2280	1800	1440
12	6160	5150	3850	3010	2450	2050	1600

About $\frac{1}{3}$ of this weight should be depended upon in practice.

Some iron filings in a very weak solution of sal-ammoniac, mixed with Portland cement, increases its strength to double or more.

BRICKS.

Dimensions.

Common brick, 8 × 4½ × 2½ inches = 90 cubic inches.
Front brick, 8¼ × 4½ × 2½ " = 92.8 " "

When laid in a wall with cement, it occupies a space of—

Common brick, 8¼ × 4½ × 2¾ inches = 102 cubic inches.
Front brick, 8½ × 4¾ × 2¾ " = 111 " "

Weight and Bulk of Bricks.

Tons.	Pounds.	Cub. ft.	Number of bricks. by itself. C. brick.	F. brick.	in wall with cement. C. brick.	F. brick.
1	2240	22.4	448	416.6	381	347
0.04464	100	**1**	20	18.6	17	15½
2.23	5000	50.00	**1000**	930	850	772
2.4	5376	53.76	1075	**1000**	914	834
2.62	5872	58.72	1130	1100	**1000**	913
2.88	6451	64.51	1240	1200	1100	**1000**

One perch of stone is 24.75 cubic feet.

Acids for Soldering or Tinning.

TIN. One part of muriatic acid, with as much zinc as it will dissolve, then add two parts of water and some sal-ammoniac.

BRASS and COPPER. One pound of muriatic acid, four ounces of zinc and five ounces of sal-ammoniac.

ZINC. One pound of muriatic acid, two ounces of sal-ammoniac with all the zinc it will dissolve, then add three pints of water.

IRON. One pound of muriatic acid, six ounces sperm tallow and four ounces of sal-ammoniac.

GOLD and SILVER. One pound muriatic acid, eight ounces sperm tallow and eight ounces of sal-ammoniac.

Silvering Metals.

Ten parts of nitrate of silver, ten parts common salt, thirty parts cream of tartar. Moisten the powder with water when ready to apply.

Glues.

Rice glue. Rice flour mixed in cold water and boiled in china or clay pot; stir it well during the boiling. This makes an excellent white glue.

Houseblose glue. Dissolve the houseblose in strong alcohol, and apply it hot on the articles to be glued. This makes a very strong glue which is not soluble in water or moisture.

Barrel Measure.

A barrel of flour weighs 196 pounds.
A barrel of pork, 200 pounds.
A barrel of rice, 600 pounds.
A barrel of powder, 25 pounds.
A firkin of butter, 56 pounds.
A tub of butter, 84 pounds.

14 pounds, . . . 1 stone.
28 pounds, . . . 1 quarter.
4 quarters, . . . 1 cwt.

Bushel Measure.

The following are sold by weight per bushel:

Wheat, beans and clover-seed, 60 pounds to the bushel.
Corn, rye and flax-seed, 56 pounds.
Buckwheat, 52 pounds.
Barley, 48 pounds.
Oats, 35 pounds.
Bran, 20 pounds.
Timothy-seed, 45 pounds.
Coarse salt, 85 pounds.

Acre.

A square of 208.75 feet each way is one acre.
A circle of 235.5 feet in diameter is one acre.

VALUES OF FOOD.

COMPARATIVE value of various foods as productive of dynamic work when digested in the stomach:

Cabbage	1.
Carrots	1.2
Egg, white of	1.4
Milk	1.5
Apples	1.5
Ale	1.8
Fish	1.9
Potatoes	2.4
Porter	2.6
Veal	2.8
Mackerel	3.8
Ham, lean	4.
Bread-crumbs	5.1
Egg, hard-boiled	5.4
Egg, yolk	7.9
Sugar	8.
Isinglass	8.4
Rice	8.9
Pea meal	9.
Wheat flour	9.1
Arrowroot	9.3
Oatmeal	9.3
Cheese	10.4
Cocoa	16.3
Butter	17.3
Fat of beef	21.6
Cod-liver oil	21.7

Butter and cheese obtainable from 100 pounds of milk:

Pure butter	3 lbs.
Good cheese	7.8 "
Common butter	3.5 "
Common cheese	11.7 "
Skim-milk cheese	13.5 "

Good cream produces about ¼ of its weight of butter.

Cheese made from good milk contains 32 to 33 per cent. of water; that from skim-milk, about 60 per cent.

Unless food is thoroughly deprived of its living animalcules before it enters the stomach, its full nourishment will not be realized. The most effectual mode of destroying the living principle is by application of heat by steaming, boiling, roasting, or smoking.

An ox, to replace the daily loss of muscular fibre, requires from 20 to 24 ounces of dry gluten or vegetable albumen daily. This would be supplied by

120 lbs.	turnips	or 17 lbs.	clover hay.
115 "	wheat straw	or 12 lbs.	peas.
75 "	carrots	or 12 "	barley.
67 "	potatoes	or 10 "	oats.
20 "	meadow hay	or 5 "	beans.

Value of stock food compared with 10 pounds of good hay:

Clover hay	8 to 10
Green clover	45 to 50
Wheat straw	40 to 50
Barley straw	20 to 40
Oat straw	20 to 40
Pea straw	10 to 15
Potatoes	20 to 25
Carrots (red)	25 to 30
" (white)	40 to 45
Rye	54
Wheat	46
Oats	59
Peas and beans mixed	45
Buckwheat	64
Indian corn	57
Acorns	68
Wheat bran	105
Rye	109
Wheat, pear and oat chaff	167
Rye and barley mixed	179

Time required for the full amount of cream to rise to the surface of new milk at different temperatures:

Hours.	Temperature, Fahr.
10 to 12	77°
18 to 20	68°
24	55°
36	50°

An average good cow yields about one gallon of milk per day; the very best yields two gallons, and the poorest only half a gallon, per day.

Percentage of alcohol in 100 parts of the following liquors (Prof. Brande):

Scotch whiskey	54.53
Irish "	53.9
Rum	53.68
Gin	51.6
Brandy	53.39
Burgundy	14.57
Cape Muscal	18.80
Champagne (still)	13.80
" (sparkling)	12.61
Cider	5.2 to 9.8
Constantia	19.75
Gooseberry wine	11.48
Currant wine	20.50
Port wine	22.90
Madeira wine	22.27
Teneriffe wine	19.79
Sherry wine	19.17
Claret wine	15.1
Elder wine	8.79
Ale	6.87
Porter	4.2
Malaga wine	17.26
Rhenish wine	12.8
Small beer	1.28

Proportion of Starch in Vegetables.

	Per cent.
Arrowroot	82.0
Rice	79.1
Rye meal	69.5
Barley flour	69.4
Wheaten flour	66.3
Indian corn meal	64.7
Oat meal	58.4
Peas	55.5
Wheaten bread	47.4
Potatoes	18.8
Parsnips	9.6
Carrots	8.4
Turnips	5.1

Water in Various Foods.

Beer and ale	91
Buttermilk	88
Skim milk	88
New milk	86
Skim cheese	44
Cheese	36
Cream	66
White of egg	78
Yolk of egg	78
Fat beef	51
Fat mutton	53
Fat pork	39
Indian meal	14
Lean beef	72
Lean mutton	72
Oat meal	15
Ox liver	74
Parsnips	82
Pea meal	15
Potatoes	75
Poultry	74
Pure butter and fats	15
Rice	13
Rye meal	15
Sugar	5
Veal	63
White fish	78

Sugar in Various Products.

Raw sugar	95.0
Treacle	77.0
Buttermilk	6.4
Carrot	6.1
Parsnips	5.8
Oat meal	5.4
Skim milk	5.4
New milk	5.2
Barley meal	4.9
Wheat flour	4.2
Rye meal	3.7
Wheaten bread	3.6
Potatoes	3.2
Turnips	3.1
Peas	2.0
Indian meal and rice	0.4

Percentage of Nutritive Elements in Food.

	Per cent.
Raw cucumbers	2
" melons	3
Boiled turnips	4½
Milk	7
Cabbage	7½
Currants	10
Whipped eggs	13
Beets	14
Apples	15
Peaches	20
Boiled cod-fish	21
Broiled venison	22
Potatoes	22½
Fried veal	24
Roast pork	24
Roast poultry	26
Raw beef	26
Raw grapes	27
Raw plums	29
Broiled mutton	30
Oatmeal porridge	75
Rye bread	79
Boiled beans	87
Boiled rice	88
Barley bread	88
Wheat bread	90
Baked corn bread	91
Boiled barley	92
Butter	92
Boiled peas	93
Raw oils	95

Yield of Vegetables in Pounds per Acre.

Hops	442
Wheat	1260
Barley	1600
Oats	1840
Peas	1920
Beans	2000
Plums	2000
Cherries	2000
Onions	2800
Hay	4000
Pears	5000
Grass	7000
Carrot	6800
Potatoes	7500
Apples	8000
Turnips	8400
Cabbage	10900
Parsnips	11200
Mangel-wurzel	22000

Fertilizing Properties of Manures.

Peruvian guano	1000
Human, mixed	69
Horse	48
Swine	44
Farm-yard	30
Cow	26

FIRE-ASSAY OF SILVER AND GOLD ORES.

From actual practice by the author in California and South America.

Assay Composition.

Gold or silver ores,	400	grains.
Litharge (oxide of lead),	500	"
Carbonate of soda,	240	"
Borax,	110	"
Charcoal,	20	"
Total.	1270	"

All the ingredients to be well powdered and mixed before placed in the crucible. Should the ore contain much sulphur, stick a 3-inch nail in the assay. The more galena in the ore, the less litharge is required. Smelt the assay, cupel the lead and weigh the remaining button of precious metal.

Should the button be pure silver, multiply the weight in grains by 100, and the product is the value of silver in dollars per ton of ore; if pure gold, multiply by 1500, and the product is the value in dollars per ton of ore.

When the button contains both gold and silver, the latter metal must be dissolved in nitric acid, for which the alloy must contain at least 3 silver to 1 of gold, otherwise the acid will not dissolve it. In case the alloyed button does not contain sufficient silver, it is necessary to add what is required, and melt it into one button by blowpipe and charcoal. Hammer the button to a thin leaf and boil it in nitric acid; when all the silver is dissolved, the pure gold remains solid. Wash the gold in clean water, dry and weigh it.

Suppose the alloyed button to weigh 2.156 grains, and its color being between that of gold and silver, so as to suspect too little of the latter metal; then add, say, 2 grains of pure silver, and dissolve the button, weigh the remaining gold, which, for example, may be 1.162 grains. Then $2.156 - 1.162 = 1.994$ grains of silver in the assay.

Silver, 1.994 + 100 =	199.40	dollars	per ton.
Gold, 1.162 + 1500 =	1743	"	"
Value of the ore, =	1942.40	"	"

About one per cent. of the precious metal is lost in the cupelling.
This rule is sufficiently correct for practical purposes.

North American Standard.

Pure { Gold, 387 ounces, 8000 dollars.
Silver, 99 ounces, 128 dollars.

Peruvian Standard.

Pure { Gold, 1 ounce, 24.29 pesos = 19.43 soles.
Silver, 1 libra, 25.66 pesos = 20.53 soles.

One peso = 4 francs; one sole = 5 francs.

Assay Table I.—North and South American Measures.

The table will answer for any system of assaying weights.

Percentage of metal in the ore.	Value of Metal per ton of Ore. Gold.	Silver.	Value of Metal per quintal of Ore. Gold.	Silver.	Silver per cajon.
Per ct.	Dollars.	Dollars.	Soles.	Soles.	Marcs.
0.1	602.924	39.709	31.090	2.053	12
0.2	1205.85	79.418	62.179	4.106	24
0.3	1808.77	119.127	93.269	6.158	36
0.4	2411.69	158.836	124.359	8.211	48
0.5	3014.62	198.545	155.448	10.264	60
0.6	3617.54	238.254	186.538	12.317	72
0.7	4220.45	277.963	217.627	14.370	84
0.8	4823.39	317.672	248.717	16.422	96
0.9	5426.31	357.381	279.807	18.473	108
1 per cent.	6029.24	397.090	310.896	20.528	120
	North American.		South American.		

Suppose the assay to be 112 grammes, and the cupelled button weighs 0.657 of a gramme of silver, then $0.657 \times 100 : 112 = 0.586$ per cent.

$$\text{See Table} \left\{ \begin{array}{l} 0.5 \quad = 10.264 \\ 0.08 \;\; = \;\; 1.642 \\ \underline{0.006 = \;\; 0.012} \\ 0.586 = 11.918 \end{array} \right\} \text{Soles per quintal.}$$

$$\text{See Table} \left\{ \begin{array}{l} 0.5 \quad = 198.545 \\ 0.08 \;\; = \;\; 31.767 \\ \underline{0.006 = \;\;\; 2.382} \\ 0.586 = 232.694 \end{array} \right\} \text{Dollars per ton of ore.}$$

Table II. For Gold and Silver.

Weights.				Value in dollars.		Bulk.			
Avoirdupois.		Troy.				Gold.		Silver.	
Tons.	Pounds.	Ounces.	Grains.	Gold.	Silver.	Cub. ft.	Cub. in.	Cub. ft.	Cub. in.
1	2000	29166.6	14 millions.	602924	39709	1.6643	2875.91	3.060	5287.48
0.0005	1	14.5833	7000	301.46	18.854	— —	1.43795	— —	2.64284
— —	0.06857	1	480	20.6718	1.2929	— —	0.09859	— —	0.18129
— —	0.0002	0.00283	1	0.04306	0.00269	— —	0.00020	— —	0.00038
— —	0.00332	0.04837	23.2202	1	1.000	— —	0.0524	— —	— —
— —	0.05304	0.77346	371.264	1.0000	1	— —	— —	— —	0.1401
0.60085	1201.7	17524.8	8411909	362267	— —	1	1728	1.0000	1728
— —	0.69543	10.1416	4867.99	209 645	— —	0.00058	1	0.00058	1.00000
0.32679	653 577	9531.34	4575043	— —	12976.4	1.0000	1728	1	1728
— —	0.378227	5.51581	2647.59	— —	7.5095	0.00058	1.0000	0.00058	1

Table III. Gold, Silver and Platinum.

Weight in grains per square inch of sheet, thickness by Birmingham gauge for those metals, and in inches.

Bir. G. No.	Thick. inches.	Gold. grains.	Silver. grains.	Platin. grains.	Bir. G. No.	Thick. inches.	Gold. grains.	Silver. grains.	Platin. grains.
1	0.004	20.68	11.52	25.50	19	0.063	339.5	184.7	397.5
2	0.005	26.93	14.40	31.26	20	0.069	371.8	201.9	435.0
3	0.006	32.19	17.28	38.00	21	0.075	404.0	220.0	471.8
4	0.008	42.80	23.52	50.43	22	0.081	436.1	237.2	509.2
5	0.010	53.85	29.28	58.14	23	0.087	468.8	255.0	548.6
6	0.012	64.46	35.04	75.45	24	0.093	500.0	272.6	586.0
7	0.014	75.42	40.80	87.88	25	0.099	533.3	290.0	625.0
8	0.016	86.58	46.56	100.1	26	0.105	566.6	307.8	663.0
9	0.018	97.07	52.00	113.2	27	0.111	596.1	325.5	697.3
10	0.022	118.9	64.32	138.5	28	0.117	630.0	342 6	735.6
11	0.025	134.6	72.96	157.7	29	0.124	673.3	363 5	783.1
12	0.029	156 3	84 96	182.3	30	0.130	701.5	380.3	817.0
13	0.033	178.2	96.48	207.4	31	0.136	730.0	398.4	855.0
14	0.038	204.6	111.3	239.5	32	0.142	769.5	416.3	892.0
15	0.043	231.5	125.8	270.5	33	0.148	798.5	433.3	932.0
16	0.048	258.8	140.8	302.6	34	0.152	837.6	451.6	970.0
17	0.053	285.6	155.3	323.8	35	0.160	865.7	470.5	1007
18	0.058	312.6	170.0	365.3	36	0.166	894.0	486.0	1047

California Rule for Silver and Gold.

It is an established custom in California to allow one per cent. for base metal in all gold and silver bars from the mines. The fineness is always stamped in parts of 1000; that is, if a gold bar is stamped 900 fine, it is understood to contain—

900 parts of pure gold,
90 parts of pure silver,
10 parts of base metal,

in 1000 parts of the bar.

To Find the Value of Gold and Silver Bars.

Example 1. Required, the value of the pure gold in a bar weighing 989 ounces and stamped 797 fine?

From table $\left\{ \begin{array}{l} 790 \text{ fine} = 16.33.07 \\ 7 \text{ fine} = \underline{\quad .14.47} \end{array} \right\}$ dollars.

Required value of the bar, $989 \times 16.47.54 = 16294.17$ dollars.

Example 2. A gold bar weighing 366 ounces has been assayed **and stamped** to 860 fine. Required, its total value?

Metals.	Bul.	Fine.	Ounces.	per Ounce.	Value.
Gold,	366 ×	860 =	314.76 ×	20.67.18 =	$6506.65.57.
Silver,	366 ×	130 =	47.58 ×	1.27.29 =	60.57.00.
Base metal,	366 ×	10 =	3.66	no value.	
Total amount		1000 =	366		Answer, $6567.22.

The last two figures in the columns of Table IV. are decimals of a cent.

The fineness of gold is also expressed in *carats*, 24 for pure gold; that is, a piece of gold 18 carats fine is $18 \times 1000 : 24 = 750$ fine.

Table IV.—Value of Gold and Silver, per ounce Troy, of Different Fineness.

Finen. in 1000.	Gold.	Silver.	Finen. in 1000.	Gold.	Silver.	Fineness in 1000.	Gold.	Silver.
	$ cts.	$ cts.		$ cts.	$ cts.		$ cts.	$ cts.
1	0 2.07	0 00.13	290	5 99.48	0 37.49	650	13 43.67	0 84.04
2	0 4.13	0 00.26	300	6 20.16	0 38.79	660	13 64.34	0 85.33
3	0 6.20	0 00.39	310	6 40.83	0 40.08	670	13 85.01	0 86.63
4	0 8.27	0 00.52	320	6 61.50	0 41.37	680	14 05.68	0 87.62
5	0 10.33	0 00.65	330	6 82.17	0 42.67	690	14 26.36	0 89.21
6	0 12.40	0 00.77	340	7 02.84	0 43.96	700	14 47.03	0 90.51
7	0 14.47	0 00.90	350	7 23.51	0 45.25	710	14 67.70	0 91.80
8	0 16.54	0 01.03	360	7 44.19	0 46.55	720	14 88.37	0 93.09
9	0 18.60	0 01.16	370	7 64.86	0 47.84	730	15 09.04	0 94.51
10	0 20.67	0 01.29	380	7 85.53	0 49.13	740	15 29.72	0 95.68
20	0 41.34	0 02.59	390	8 06.20	0 50.42	750	15 50.39	0 96.97
30	0 62.02	0 03.88	400	8 26.87	0 51.72	760	15 71.06	0 98.26
40	0 82.69	0 05.17	410	8 47.55	0 53.01	770	15 91.73	0 99.56
50	1 03.36	0 06.46	420	8 68.22	0 54.30	780	16 12.40	1 00.85
60	1 24.03	0 07.76	430	8 88.89	0 55.60	790	16 33.07	1 02.14
70	1 44.70	0 09.05	440	9 09.56	0 56.89	800	16 53.75	1 03.43
80	1 65.37	0 10.34	450	9 30.23	0 58.18	810	16 74.42	1 04.73
90	1 86.05	0 11.64	460	9 50.90	0 59.47	820	16 95.09	1 06.02
100	2 06.72	0 12.93	470	9 71.58	0 60.77	830	17 15.76	1 07.31
110	2 27.39	0 14.22	480	9 92.25	0 62.06	840	17 36.43	1 08.61
120	2 48.06	0 15.52	490	10 12.92	0 63.35	850	17 57.11	1 09.90
130	2 68.73	0 16.81	500	10 33.59	0 64.65	860	17 77.78	1 11.19
140	2 89.41	0 18.10	510	10 54.26	0 65.94	870	17 98.45	1 12.48
150	3 10.08	0 19.39	520	10 74.94	0 67.23	880	18 19.12	1 13.78
160	3 30.75	0 20.69	530	10 95.61	0 68.53	890	18 39.79	1 15.07
170	3 52.42	0 21.98	540	11 16.28	0 69.82	900	18 60.46	1 16.36
180	3 72.09	0 23.27	550	11 36.95	0 71.11	910	18 81.14	1 17.66
190	3 92.76	0 24.57	560	11 57.62	0 72.14	920	19 01.81	1 18.95
200	4 13.44	0 25.86	570	11 78.29	0 73.69	930	19 22.48	1 20.24
210	4 34.11	0 27.15	580	11 98.97	0 74.99	940	19 43.15	1 21.54
220	4 54.78	0 28.44	590	12 19.64	0 76.28	950	19 63.82	1 22.83
230	4 75.45	0 29.74	600	12 40.31	0 77.58	960	19 84.50	1 24.12
240	4 96.12	0 31.03	610	12 60.98	0 78.87	970	20 05.17	1 25.41
250	5 16.80	0 32.32	620	12 81.65	0 80.16	980	20 25.84	1 26.71
260	5 37.47	0 33.62	630	13 02.33	0 81.45	990	20 46.51	1 28.00
270	5 58.14	0 34.91	640	13 23.00	0 82.75	1000	20 67.18	1 29.29
280	5 78.81	0 36.20						

To Refine Silver.

Dissolve the impure silver in nitric acid, add chloride of sodium (salt) sufficient to precipitate all the silver in form of chloride; then all the impurities will remain in solution.

Filter, wash and dry the chloride of silver. Fuse in a crucible two weights of carbonate of potash, add gradually one weight of chloride of silver, raise the heat, and the pure silver will melt and collect on the bottom.

Tests for Metals in Solution with Acids.

The reagents are placed in the liquid, which precipitates the metal in solution.

REAGENTS.	PRECIPITATES.	SOLUTIONS.
Sulphate of iron,	Gold, as brown powder,	Gold in aqua-regia.
Oxalic acid,	Gold in large flakes,	
Potash or soda,	Gold, yellow,	
Potash or soda,	Silver, dark olive,	Silver in nitric acid.
Plate of copper,	Metallic silver,	
Muriatic acid,	White crude silver,	
Common salt,	White crude silver,	
Tincture of nutgall,	Brown silver,	
Potash or soda,	Blue cobalt,	Cobalt in nitric acid.
Ferro-prussiate of potash,	Green "	
Carbonate of potash,	Red "	
Pure water,	White bismuth,	Bismuth in nitric acid.
Gallic acid,	Greenish yellow,	
Potash or soda,	White bismuth,	
Sulphate of soda,	White lead,	Lead in nitric acid.
Sulphuric acid,		
Infusion of nutgall,		
Plate of iron or zinc,	Metallic copper,	Copper in nitric acid.
Potash,	Green copper,	
Ammonia,	Azure-blue copper,	
Infusion of nutgall,	Brown copper,	
Pure water,	White antimony,	Antimony in 4 muriatic acid, 1 nitric acid.
Plate of iron,	Black antimony,	
Plate of copper,	Metallic mercury,	Mercury in muriatic or nitric acid.
" iron,	Dark powder,	
Gallic acid,	Orange yellow,	
Infusion of nutgall,	Black iron,	Iron in muriatic acid.
Ferro-prussiate of potash,	Blue iron,	
Ammonia,	Dark-red iron,	

Acid Test for Strength and Quality of Iron and Steel.

This is a subject well worthy of attention by workers in iron and steel. The sample to be tested is filed smooth, or polished on all sides, and placed in dilute nitric or sulphuric acid for 12 to 24 hours; then wash the sample and dry it. The action of the acid has revealed the structure of the sample, from which its quality can be decided with great precision.

The best steel presents a frosty appearance; ordinary steel, honeycombed. Iron presents a fibrous structure in the direction in which it has been worked; the best iron shows the finest fibres. Should the iron be uneven, or made from a pile of different kinds of iron, all are exposed by the action of the acid. Hammered blooms show slag and iron; gray cast iron shows crystals of graphic carbon; other cast irons show different figures, all with marked characteristics.

Iron Pyrites, Sulphurets.

There are two kinds of iron pyrites—namely, *protosulphuret* and *bisulphuret*, of which the latter is generally richest in gold. All iron pyrites are slightly magnetic, but the gold seems to destroy the magnetism. The protosulphuret acts sensibly on the magnetic needle, whilst the bisulphuret does not, and may therefore be distinguished for gold.

The presence of arseniuret of iron in sulphurets indicates richness in gold.

Roasting of Sulphurets.

When sulphurets contain magnesia, lime or arsenic, sufficient salt should be added to chlorinate those substances, which then evaporate and go out through the chimney. The amount of those impurities should be ascertained beforehand. The salt should be well mixed with the ore before put into the furnace. Ten pounds of salt contain six pounds of chlorine and four pounds of sodium.

	Ten pounds of	*Pounds required.* Chlorine.	Salt.
Those impurities are very injurious to chlorination of the gold in the vat.	Magnesia, . . .	3.58	6
	Calcium, . . .	5.78	9.65
	Arsenic, . . .	10.64	17.6

Chlorination of Gold in Roasted Sulphurets.

Free gold is attacked and dissolved by chlorine gas, and forms two chlorides, namely,

Au. 844 parts of gold.	Au. 648.5 parts of gold.
Cl. 156 " chlorine.	Cl₃. 351.5 " chlorine.
Au. Cl. 1000 protochloride of gold.	Au. Cl₃. 1000 terchloride of gold.

Gold-bearing sulphurets are roasted for the purpose of obtaining the gold free for the action of chlorine gas. The combination is very slow, and requires the gold to be very fine for the prompt formation of chloride. In some ores, the gold is too coarse for chlorination, when it must be extracted by amalgamation.

Composition for Making Chlorine Gas.

For each ton of roasted ore in the vat are required 14 pounds of salt, 10 pounds of peroxide of manganese and 5 quarts of sulphuric acid. The composition should be constantly stirred in the gasometer, and kept to a uniform temperature of about 180° Fahr. The chlorine gas thus formed is led into the vat containing the ore.

On account of chlorine gas being much heavier than air, the gasometer ought to be placed at a considerable height above the vat, to facilitate the chlorination of the gold. In California they place the gasometer below the vat, which is decidedly wrong.

Chloride of gold is soluble in water, and can be washed out from the vat simply by pouring water on the top of the ore and running it into another vessel, where the gold is precipitated with sulphate of iron.

Chloride of silver is not soluble in water, and remains in the ore in the vat. There is always some silver in gold sulphurets.

Quartz Mills.

Each stamp, weighing about 800 pounds, lifted one foot 60 times per minute, can crush one ton of quartz per 24 hours with a dynamic effect of two horse-power. This is the average performance. The custom-mill in Grass Valley, California, crushes quartz for about four dollars per ton.

The stamps are generally divided into sets of four or five, working in one mortar, and called a *battery*. The shoes and dies in the battery are made of chilled cast iron.

Most of the gold is collected by amalgamation in the battery. The pulp from the battery contains much gold, which is often allowed to run away, but generally the sulphuret in the pulp is concentrated and roasted for chlorination; the rest of the pulp is ground in pans and the gold amalgamated.

Amalgams.

GOLD. One weight of mercury amalgamates with two weights of gold.

SILVER. 10 silver to 19 mercury.
7 " " 20 "

TIN. 1 tin to 3 mercury, for looking-glasses.
1 tin, 1 lead, 2 bismuth, 10 mercury, for glass-globes.
1 tin, 1 zinc, 3 mercury, for rubbers in electric machines.

OPTICS.

Optics is that branch of philosophy which treats of the properties and motion of light.

Mirrors.

Example 1. Fig. 307. Before a *concave mirror* of $r = 6$ feet radius, is placed an object $O = 1$, at $d = 1·75$ feet from the vertex. Required the size of the image $I = ?$

$$\text{image } I = \frac{O\ r}{r - 2\ d} = \frac{1 \times 6}{6 - 2 \times 1·75} = 2·4$$

Example 2. Fig. 308. Before a *concave mirror* of $r = 5·25$ feet radius, is placed an object $O = 1$, at $D = 4·5$ feet from the vertex. Required the size of the inverted image $I = ?$

$$\text{image } I = \frac{O\ r}{2\ D - r} = \frac{1 \times 5·25}{2 \times 4·5 - 5·25} = 1·4$$

Example 3. Fig. 309. Before a *convex mirror* of $r = 1·8$ feet radius, is placed an object $O = 1$, at $D = 3·15$ feet from the vertex. Required the size of the image $I = ?$, and the distance in the mirror $d = ?$

$$\text{image } I = \frac{1 \times 1·8}{2 \times 3·15 + 1·8} = 0·222 \quad \text{distance } d = \frac{3·15 \times 1·8}{2 \times 3·15 + 1·8} = 0·699 \text{ ft.}$$

Example 4. Fig. 310. A *parabolic mirror* is $h = 1·31$ feet high, and $d = 2·15$ feet in diameter. Required the focal distance $f = ?$ from the vertex.

$$\text{focal distance } f = \frac{d^2}{16\ h} = \frac{2·15^2 \times 12}{16 \times 1·31} = 2·646 \text{ inches.}$$

Optical Lenses.

Example 5. Fig. 316. A double *convex lens*, of *crown glass*, having its radii $R = r = 6$ inches. Required its principal focal distance $f = ?$
For crown glass the index of refraction is $m = 1·52$. See table.

$$f = \frac{6}{2(1·52 - 1)} = 5·768 \text{ inches.}$$

Microscope.

Letters denote.

$\mathfrak{p}$ = magnifying power of a lens.

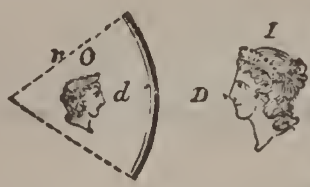
$\mathfrak{D}$ = limit of distinct vision.

$\mathfrak{a}$ = limit of distinct sight, which for *long-sighted* eyes is about 10 or 12 inches, and *near-sighted* 6 to 8 inches. For common eyes take $\mathfrak{a}$ = 10 inches.

$\mathfrak{d}$ = limit distance of the object from the optical centre at distinct vision.

Example 6. Fig. 322. Required the magnifying power of a single microscope with principal focal distance, $f = 4·3$ inches?

$$\text{Mag. power } \mathfrak{p} = \frac{\mathfrak{a} + f}{f} = \frac{10 + 4·3}{4·3} = 3·325 \text{ times.}$$

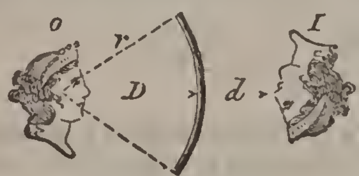


307 *Spherical Concave Mirror.*

r = radius, and $f = \frac{1}{2} r$, focal distance of the mirror.

$$I = \frac{O\,r}{r - 2\,d}. \quad D = \frac{d\,r}{r - 2\,d}.$$

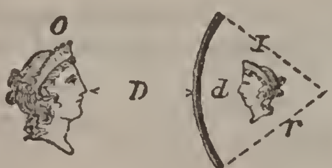
The image disappears when $d = f = \frac{1}{2} r$.



308 *Spherical Concave Mirror.*

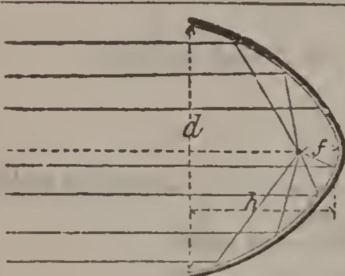
$$I = \frac{O\,r}{2\,D - r}. \quad d = \frac{D\,r}{2\,D - r}.$$

When the object is beyond the focal distance the image will be inverted.



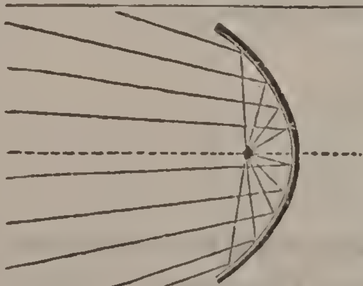
309 *Spherical Convex Mirror.*

$$I = \frac{O\,r}{2\,D + r} \quad d = \frac{D\,r}{2\,D + r}.$$



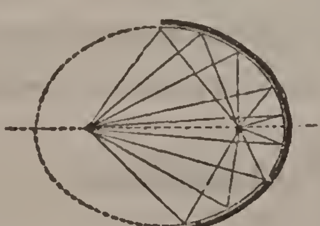
310 *Parabolic Concave Mirror.*

$$f = \frac{d^2}{16\,h}. \quad h = \frac{d^2}{16\,f}.$$



311 *Hyperbolic Concave Mirror.*

Heat, Light, or Sound emanating from the foci of a hyperbola will be reflected divergently, from the concave surface.



312 *Eliptic Concave Mirror.*

Emanating rays from either of the two foci in an elipse, will be reflected by the convex surface to the other foci.

Astronomical Telescopes and Opera Glasses.

Example 7. Fig. 325. The principal focal distance $f = 0{\cdot}65$ inches of the ocular or eye-lens. $F = 58$ inches the principal focal distance of the objective-lens. Required the magnifying power of the telescope $I = ?$

$$\text{image } I = \frac{O\ F}{f} = \frac{1 \times 58}{0{\cdot}65} = 89{\cdot}23 \text{ times the object.}$$

The telescope is to be used at the limited distance $D = 1380$ feet and $D = \infty$. Required the proper lengths $l = ?$ and micrometrical motion of the ocular or eye-lens? when the limit of distinct sight $a = 10$in. $F = 58 : 12 = 4{\cdot}833$ feet. $f = 0{\cdot}65 : 12 = 0{\cdot}05416$ feet.

$$l = \frac{1380 \times 4{\cdot}833}{1380 - 4{\cdot}833} + \frac{10 \times 0{\cdot}05416}{10 + 0{\cdot}05416} = \begin{array}{l} 4{\cdot}89035 \\ 0{\cdot}05386 \end{array}$$

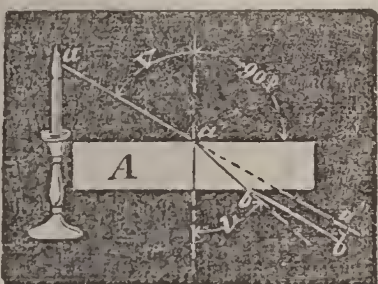
When $D = 1380$ feet, the length $l = 4{\cdot}94421$ feet.

When $D = \infty$, $l = 4{\cdot}8333 + 0{\cdot}05386 = 4{\cdot}88719$ „

$$\text{Micrometrical motion of eye lens} \begin{cases} 0{\cdot}05702 & \text{„} \\ 0{\cdot}68424 & \text{inches.} \\ \frac{11}{15} & \text{nearly.} \end{cases}$$

Table of Refractive Indices.

Substances.	Index. m.	Substances.	Index. m.
Cromate of Lead	2·97 / 2·50	Quartz	1.54
Realgar	2·55	Muriatic Acid	1.40
Diamond	2·45	Water	1.33
Glass, flint	1·57	Ice	1.30
Glass, crown	1·52	Hydrogen	1.000138
Oil of Cassia	1·63	Oxygen	1.000272
Oil of Olives	1·47	Atmospheric air	1.000294

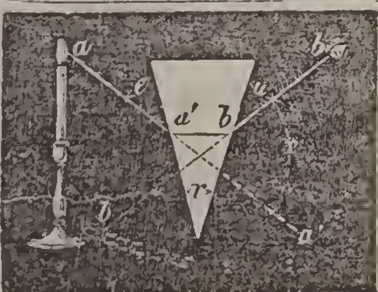


314 *Prism.*

A ray of light $a\,a'$ falling upon a transparent medium A (say a glass prism) will be transmitted through in the direction $a\,b$, and delivered in the direction $b\,b'$, parallel $a\,a'\,a''$.

V = angle of incidence, v = angle of refraction.

$$\text{Index of refraction } m = \frac{\sin.\ V}{\sin.\ v}.$$



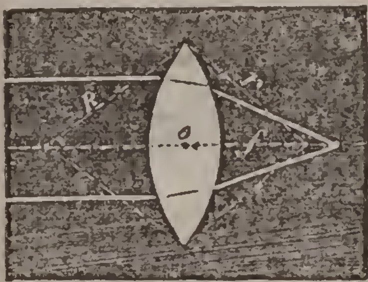
315 Given the direction of the incident ray $a\,a'$, angles e and r,—to find the angles u and z,—or the direction of the rays $b\,b'$.

$$\cos.\ z = \frac{\cos.\ e}{m}, \quad \cos.\ u = m \cos.(180 - z - r).$$

$$x = 180 - (e + r + u).$$

When $e = u$, the angle x is smallest.

An eye in b' will see the candle in the direction $b'\ b\ b''$.

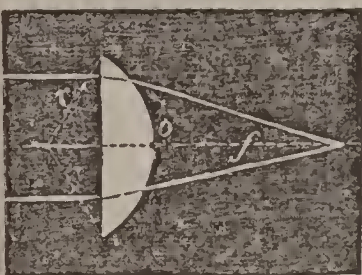


316 *Double Convex Lens.*

$$f = +\frac{R\,r}{(m-1)(R+r)}.$$ the principal focal distance.

$$f = \frac{r}{2(m-1)}.$$ when $R = r$

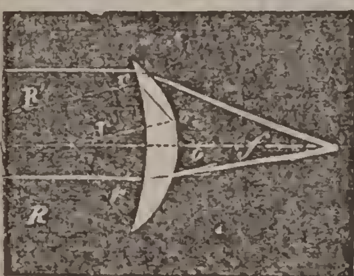
o = optical centre of the lens.



317 *Plano Convex Lens.*

$$f = +\frac{r}{m-1}.$$

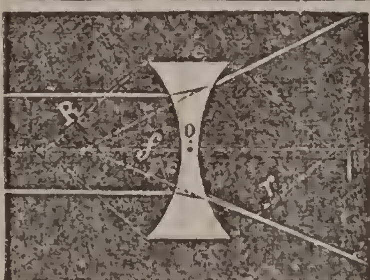
The optical centre is in the convex surface.



318 *Convex-concave Lens (Meniscus.)*

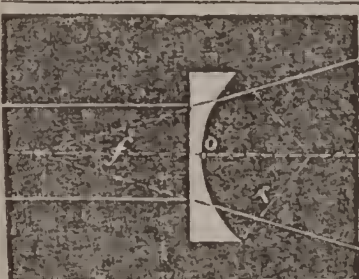
$$f = +\frac{R\,r}{(m-1)(R-r)}.$$

Draw the radii R' and r' parallel to one another.—Draw $n\ o$, then o is the optical centre.



319 *Double Concave Lens.*

$$f = -\frac{R\,r}{(m-1)(R+r)}.$$



320 *Plano Concave Lens.*

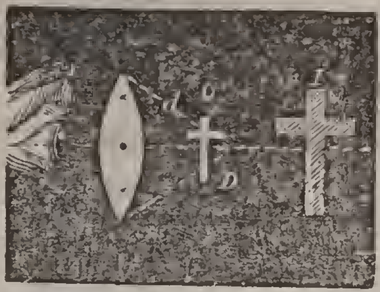
$$f = -\frac{r}{m-1}.$$

The optical centre is in the concave surface.

321 *Concavo-convex Lens.*

$$f = -\frac{R\,r}{(m-1)(R-r)}.$$

Draw R and r' parallel to one another. Draw $n\ o$, then o is the optical centre.

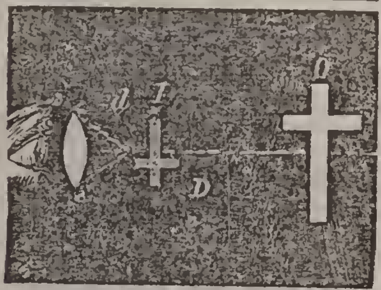


322

Single Microscope.

$$I : O = f : f - d,\quad I = \frac{O f}{f - d},\quad D = \frac{d f}{f - d},$$

$$\mathfrak{p} = \frac{\mathfrak{a} + f}{f},\quad \mathfrak{D} = \frac{\mathfrak{d} f}{f - \mathfrak{d}},\quad \mathfrak{d} = \frac{\mathfrak{a} f}{\mathfrak{a} + f}.$$



323

When the object O is beyond the focal distance the image I will be inverted.

$$I : O = f : D - f,\quad I = \frac{O f}{d - f},\quad d = \frac{D f}{D - f}.$$

324 *Diminishing Power of a Double Concave Lens.*

$$I : O = f : f + D \qquad I = \frac{O f}{f + D},$$

$$D = \frac{f\,(O - I)}{I}. \qquad d = \frac{D f}{D + f}.$$

325 *Astronomical Telescope.*

$$I : O = F : f,\qquad I = \frac{O F}{f},\qquad d = \frac{D f}{F}.$$

$$l = \frac{D F}{D - F} \overset{*}{\pm} \frac{\mathfrak{a} f}{\mathfrak{a} + f}.\qquad \left(\text{if } D = \infty,\quad l = F + \frac{\mathfrak{a} f}{\mathfrak{a} + f}\right)$$

* + for astronomical telescope, — for opera-glasses.

326 *Opera Glass.*

Formulas are the same as for Astronomical Telescope.

ELECTRO-DYNAMICS.

The Conversion of Heat into Electrical Power.

The direct conversion of heat into electrical energy has already had a partially successful, but not economical, solution in Clamond's stoves.

According to Cabanellas, a Clamond's stove consisting of 6000 elements, and burning 22 pounds of coke per hour, will give a current of 7 ampères, and 218 volts difference of potential.

Cabanellas also states that the amount of light obtained was equal to about 560 standard English candles.

This would give us nearly 26 candles per pound of coke. As we shall presently see, this is a result much less economical of fuel than can be obtained by the use of an engine and dynamo under very unfavorable circumstances. The liability to derangement and the first cost of Clamond's Pile have prevented it from becoming commercially successful.

The ohm, volt, and ampère are the practical British Association units used by electricians.

The legal ohm is the resistance of a column of mercury 1 square millimetre in cross section and 106 centimetres in length, at the temperature of melting ice.

Ohm's law is

$$\text{Intensity of current} = \frac{\text{Diff. of Potential}}{\text{Resistance}}, \text{ or } I = \frac{E}{R},$$

from which we at once see that the resistance equals the ratio between the electro-motive force lost in the circuit and the intensity of the current.

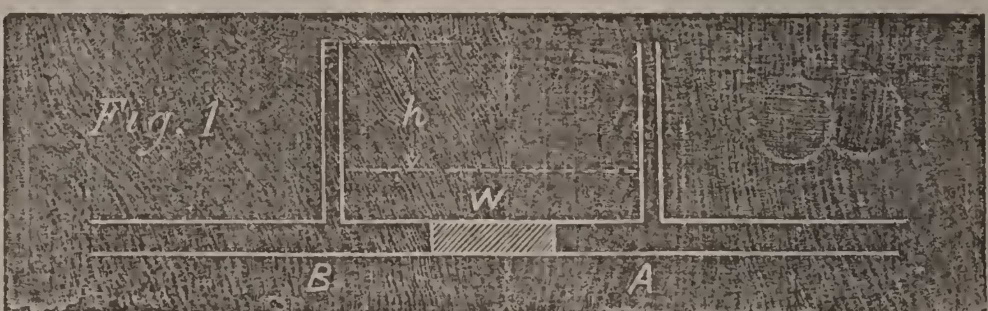
This is a constant for any solid so long as its form and temperature are not changed.

The volt and ampère are more difficult to define, and perhaps can have their meaning made clear by making use of analogous hydraulic formulæ.

This is only a case of analogy. It cannot be said that electricity is a fluid, or that there is a current of electricity, or that it flows one way or the other.

The volt may be said to represent the pressure or head of the assumed current of electricity, and the ampère to represent the intensity or weight of the current passing in one second. Lord Rayleigh has carefully determined the weight of silver precipitated from a solution of nitrate of silver by one ampère. It is 0·06708 grammes per minute, or 4·0248 grammes per hour.

Referring to Fig. 1, if W equals the weight of water that passes the



point A in a pipe in one second, and h the loss of head, we have, for the work done in one second, $W\,h$ foot-pounds.

Again, if I represents the intensity of a current passing the point A in a second, and E the difference of potential in volts between B and A, we have for the work done in one second $I\,E$ volt-ampères, or watts. If we divide $W\,h$ by 550 foot-pounds, we obtain the horse-power.

In the case of the pipe, if it were level, the loss of work would be due to friction and transformed into heat.

$$\text{Thus } h = \frac{v^2}{2g} \text{ and } W h = \frac{W v^2}{2g} = \frac{M v^2}{2}.$$

Joule has shown us by experiment that the heating of a wire conductor is proportional to $I^2 R = I E = \frac{E^2}{R}$, or, using the analogous hydraulic formula, the heating is, $W^2 \times \frac{h}{W} = W h = \frac{h^2}{\frac{h}{W}}$.

Again, $\frac{I E}{9 \cdot 81}$ = work per second in kilogrammetres, but an English horse-power equals 76·01 kilogrammetres per second, and, therefore, one horse-power $= \frac{I E}{745 \cdot 9}$. Returning again to the ohm, we have $R = \frac{E}{I}$. That is, the resistance is the loss of electro-motive force per second and per unit of intensity which an electrical current experiences when passing along a conductor. If this conductor is the standard quicksilver column, $R = 1$ ohm.

In an analogous manner we would say of a horizontal pipe conveying water that the resistance is the loss of head per pound and per second when passing through the pipe.

The resistance of the various materials used as conductors for the electrical current has been repeatedly and carefully determined.

By the electro-motive force in volts is meant something similar to the head of water in feet or its pressure in pounds.

By the intensity of a current in ampères is meant something similar to the weight of water passing in pounds per second.

By the resistance in ohms is meant something similar to the loss of head of water per pound and per second.

The dynamo electric machine is the newest and the most perfect of machines for the transformation of energy from one form to another. Like the turbine, its efficiency has been proved so great as to preclude all hope of further increase of practical value. Its cost may be reduced by improved processes of the machine-shop: we cannot do more.

One reason for this rapid perfecting lies in the apparent obscurity of electrical phenomena, which has had the effect of repelling all but subtle and acute minds from their study. The right end of the thread once seized by such minds, they have followed the clue with such rapidity and thoroughness of apprehension as to leave nothing more for us to accomplish.

The recent experiments of the Franklin Institute upon the dynamos of Weston and Edison have set the seal of absolute measurement, with as great exactitude as we can hope to reach, upon the ability of these machines to transform mechanical work into electrical work.

Of the five dynamo electric machines which successfully withstood the severe conditions of the code, Weston's mammoth incandescent lamp machine, of a rated capacity of 125 ampères and 160 volts, returned as an average of four tests, in the form of electrical energy, 96·56 per cent. of the mechanical power used to drive it: 89·37 per cent. of the mechanical power was available as electrical energy in the external circuit.

Of the total mechanical power applied, about 1 per cent. was lost in friction of the armature shaft and resistance of the air to its rapid revolution.

Two and one half per cent. only remains to be accounted for, and was presumably lost in the form of heat and eddy currents.

Every precaution was taken to avoid results which would not appear in every-day use, and all of the machines were run under full load for ten hours before the measurements began, and so were at as high a temperature as would be reached in actual practice with the same atmospheric temperature. The performance of this particular machine only exceeded the least efficient of the machines tested by 2·1 per cent. total efficiency.

(*Continued on page* 595.)

The New British Gauge.

(Legal standard in England from Mar. 1, 1884. Superseding all other Gauges.)

Gauge Number.	Differences.	DIAMETER. Inches.	DIAMETER. Centimeters.	Area of Cross Section. Cm².	PURE COPPER WIRE (Soft Drawn). Resistance. Ohms per meter.	PURE COPPER WIRE (Soft Drawn). Conductivity. Meters per ohm.	Weight of Wire. Density 8.90 (Copper) Grammes per Meter.
7/0	......	.500	1.270	1.267	.000135	7402.1	1127.4
6/0	36	.464	1.178	1.090	.000157	6370.	970.2
5/0	32	.432	1.097	.945	.000181	5521.	840.8
4/0	32	.400	1.016	.811	.000211	4736.	721.3
3/0	28	.372	.945	.701	.000244	4098.	624.2
2/0	24	.348	.884	.613	.000279	3584.	545.9
0	24	.324	.832	.532	.000322	3107.	473.2
1	24	.300	.762	.546	.000375	2666.	406.1
2	24	.276	.701	.386	.000444	2253.	343.2
3	24	.252	.640	.322	.000532	1881.	286.5
4	20	.232	.589	.273	.000628	1592.	242.5
5	20	.212	.538	.228	.000751	1331.	202.7
6	20	.192	.488	.187	.000916	1092.	166.3
7	16	.176	.447	.157	.00109	917.8	139.8
8	16	.160	.406	.130	.00132	757.2	115.3
9	16	.144	.366	.105	.00163	614.9	93.7
10	16	.128	.325	.0829	.00206	484.6	73.8
11	12	.116	.295	.0682	.00251	398.3	60.7
12	12	.104	.264	.0548	.00312	320.3	48.8
13	12	.092	.234	.0429	.00398	250.6	38.2
14	12	.080	.203	.0324	.00528	189.5	28.9
15	8	.072	.183	.0263	.00651	153.5	23.4
16	8	.064	.163	.0208	.00824	121.3	18.5
17	8	.056	.142	.0159	.0108	92.7	14.1
18	8	.048	.122	.0117	.0147	68.2	10.4
19	8	.040	.1016	.00811	.0211	47.4	7.19
20	4	.036	.0914	.00659	.0260	38.4	5.84
21	4	.032	.0813	.00519	.0330	30.3	4.62
22	4	.028	.0711	.00397	.0431	23.2	3.54
23	4	.024	.0610	.00292	.0587	17.05	2.60
24	2	.022	.0559	.00245	.0698	14.32	2.18
25	2	.020	.0508	.00203	.0845	11.84	1.80
26	2	.018	.0457	.00164	.104	9.59	1.46
27	1.6	.0164	.0417	.00136	.125	7.97	1.21
28	1.6	.0148	.0376	.00111	.154	6.48	.988
29	1.2	.0136	.0345	.000937	.183	5.46	.834
30	1.2	.0124	.0315	.000779	.220	4.55	.693
31	.8	.0116	.0295	.000682	.251	3.98	.607
32	.8	.0108	.0274	.000591	.290	3.45	.526
33	.8	.0100	.0254	.000507	.338	2.96	.451
34	.8	.0092	.0234	.000429	.398	2.51	.382
35	.8	.0084	.0213	.000358	.478	2.09	.318
36	.8	.0076	.0193	.000293	.585	1.71	.260
37	.8	.0068	.0173	.000234	.730	1.37	.208
38	.8	.0060	.0152	.000182	.943	1.06	.162
39	.8	.0052	.0132	.000137	1.248	.801	.122
40	.4	.0048	.0122	.000117	1.466	.682	.1038
41	.4	.0044	.0112	.0000982	1.742	.574	.0874
42	.4	.0040	.0102	.0000811	2.109	.474	.0721
43	.4	.0036	.00914	.0000656	2.611	.383	.0584
44	.4	.0032	.00813	.0000519	3.300	.303	.0462
45	.4	.0028	.00711	.0000397	4.310	.232	.0353
46	.4	.0024	.00610	.0000292	5.848	.171	.0260
47	.4	.0020	.00508	.0000203	8.475	.118	.0180
48	.4	.0016	.00406	.0000129	13.23	.076	.0115
49	.4	.0012	.00305	.00000730	23.42	.043	.00650
50	.2	.0010	.00254	.00000507	33.78	.030	.00451

ELECTRICITY.

Electro-Chemical Order of *Simple Substances.*

ELECTRO-POSITIVE.

Potassium.
Sodium.
Lithium.
Barium.
Strontium.
Calcium.
Magnesium.
Aluminium.
Uranium.
Manganese.
Zinc.
Iron.
Nickel.
Cobalt.
Cadmium.
Lead.
Tin.
Bismuth.
Copper.
Silver.
Mercury.
Palladium.
Platinum.
Gold.
Hydrogen.
Silicon.
Titanium.
Tellurium.
Antimony.
Carbon.
Boron.
Tungsten.
Molybdenum.
Vanadium.
Chromium.
Arsenicum.
Phosphorus.
Iodine.
Bromine.
Chlorine.
Fluorine.
Nitrogen.
Selenium.
Sulphur.
Oxygen.

ELECTRO-NEGATIVE.

In chemical formulas the electro-positive substance is placed first, and the negative last.

Order of Compounds.

ELECTRO-POSITIVE.

Fur.
Smooth glass.
Woollen cloth.
Feathers.
Wood.
Paper.
Silk.
Lac.
Rough glass.
Sulphur.
Cotton.

ELECTRO-NEGATIVE.

The substances are arranged in their order of positive and negative electricity. The substance is positive to either one below it, and negative to any one above. The exciting fluid to be diluted sulphuric acid. Other fluids cause some difference in the order, depending upon the different chemical affinity between the fluid and the substances in the galvanic couple.

Oxygen, being the substance most electro-negative, combines with the most electro-positive substance in the couple, and the force liberated by the oxidation, or that which kept the oxidated substance solid, forms the electricity. No electricity can be formed without the consumption of some force or substance.

Order of Conducting Power *for Electricity.*

Metals, best conductors.
Well-burnt charcoal.
Plumbago.
Concentrated acids.
Powdered charcoal.
Diluted acids.
Saline solutions.
Metallic ores.
Animal fluids.
Sea water.
Spring water.
Rain water.
Ice above 13° Fahr.
Snow.
Living vegetables.
Living animals.
Steam.
Salts soluble in water.
Rarefied air.
Vapor of alcohol.
Moist earth and stones.
Powdered glass.
Flower of sulphur.
Dry metallic oxides.
Oils, the heaviest the best.
Ashes.
Transparent crystals.
Ice below 13° Fahr.
Phosphorus.
Lime.
Dry chalk.
Caoutchouc.
Camphor.
Silicious stones.
Dry marble.
Porcelain.
Baked wood.
Dry gases and air.
Leather.
Parchment.
Dry paper.
Feathers.
Hair.
Wool.
Dyed silk.
Bleached silk.
Raw silk.
Diamond.
Mica.
All vitrifications.
Glass.
Jet.
Wax.
Sulphur.
Resins.
Amber.
Shellac.
Gutta-percha, *the worst conductor of all.*

The substances are set up in their order of conducting power for electricity. The conducting power of substances for heat appears to be in the same proportion as that for electricity. (See page 511.) The poor conductors for electricity are called *insulators*, and employed between good conductors to stop the flow or passage of the electric fluid.

Velocity of electricity through the best conductors is equal to that of light through planetary space—about 200,000 miles per second. When the conductor is insulated with a solid non-conducting substance, like gutta-percha, and immersed in water as a submarine cable, the velocity may be reduced to only 20,000 miles per second, or less.

GEOGRAPHY.

The *Earth* on which we live is a round ball or sphere, with a mean diameter of 7914 statute miles. The whole surface of the earth is 196,800,000 square miles, of which only one-fourth or nearly 50,000,000 square miles is land, and about 150,000,000 square miles water.

Table of Area and Population of the Whole Earth, 1883.

Divisions of the Earth.	Area in Square Miles.	Population.	Proportion to Square Mile.
America, ...	14,491,000	100,466,000	7
Europe, ...	3,760,000	327,743,400	87
Asia, ...	16,313,000	795,591,000	49
Africa, ...	10,936,000	205,823,260	20
Oceanica, ...	4,500,000	31,619,000	7
Total, ...	50,000,000	1,461,242,660	30

About $\frac{1}{30}$th of the whole population are born every year, and nearly an equal number die in the same time; making about one born and one dead per second.

The annual increase of population per 1000 is about 6 in Europe and 19 in America. Europe loses and America gains by emigration. The annual increase of population in the whole world is about 6 per 1000.

The Earth is not a perfect sphere, it is flatted at the Poles. The following are her true dimensions in statute miles of 5280 feet.

Dimensions of the Earth.

Diameter, . .	7898.8809	miles at the Poles.
	7911.92	miles mean, or in 45° lat.
	7924.911	miles at the Equator.
Difference, . .	26.0302	miles Poles and Equator.
Flatted, . . .	13.015	miles at each Pole.
Circumference,	24802.486	miles round the Poles.
	24851.640	miles mean, or in 45° lat.
	24884.22	miles round the Equator.

To Find the Radius of the Earth in Any Given Latitude.

$R = 3955.96(1 + 0.00164 \cos.2l)$, statute miles.

Population of Countries and Cities 1880.

Br. Amer.,	6,000,000	**Russia, . . .**	100,372,562	Guanajoato,	64,000
Montreal, . .	140,600	St.Petersb'rg,	861,900	**Cuba, ...**	1,409,860
Quebec, . . .	62,400	Moscow, . . .	601,969	Havana, . . .	220,000
Toronto, . . .	86,400	**England,**	24,608,391	St. Jago Cuba,	105,000
St. John, N. B.	26,120	London, . . .	3,814,571	**Porto Rico,**	661,494
Halifax, N. S.	36,000	**Scotland,**	3,734,441	**C. America,**	2,750,000
Ottawa, Ont.	27,400	Edinburgh, .	228,190	Whites, . .	150,000
U. S. Amer.,	50,155,783	Glasgow, . . .	511,532	Indians, . .	1,500,000
N. Y. & Brk.,	1,772,962	**Ireland, . .**	5,159,839	Negroes, . .	40,000
Philadelphia,	847,170	Dublin, . . .	249,486	Mixed, . . .	1,100,000
Chicago, . . .	503,185	**France, . . .**	37,672,048	**Guatemala,**	1,215,310
St. Louis, . . .	350,518	Paris, ...	2,225,900	Guatemala, A.	57,728
Baltimore, . .	332,313	**Germ. Emp.**	45,194,177	**St. Salvador**	600,000
Boston, ...	262,839	Berlin, ...	1,122,360	St. Salvador, A.	21,000
Cincinnati, .	255,139	**Austria, . . .**	26,096,860	**Nicaragua,**	400,000
San Francisco	233,959	Vienna, . . .	1,003,857	Managua, . .	11,000
Washington,	147,293	**Hungary,**	11,644,574	**Honduras,**	351,700
Buffalo, . . .	155,134	Pest-Buda . .	359,821	Comayugua,	9,000
Newark, . . .	136,508	**Holland, . .**	4,060,580	**Costa Rica,**	185,000
Louisville, . .	123,758	Amsterdam,	326,196	San Jose, . . .	30,000
Cleveland, . .	160,146	**Bavaria, . .**	5,284,778	**S. America,**	32,000,000
Pittsburg, . .	156,389	Munich, . . .	230,023	Wild Indians,	3,600,000
Jersey City, .	120,722	**Switzerl'd,**	2,846,102	Whites, . .	10,000,000
Detroit, . . .	116,340	Berne, ...	30,000	Negroes, . .	650,000
Milwaukee, .	115,587	**Belgium, . .**	5,536,664	Mixed, . . .	15,000,000
Providence, .	104,857	Brussels, . . .	170,345	**U. S. Colom.**	2,950,017
Albany, . . .	90,758	**Spain, ...**	16,623,384	Bogota, ...	60,000
Rochester, . .	89,366	Madrid, . . .	397,690	Panama, . . .	25,000
Alleghany, .	78,682	**Italy, ...**	28,459,628	**Venezuela,**	2,075,245
Richmond, .	63,600	Rome, ...	289,321	Caraccas, . .	50,000
New Haven,	62,882	**Greece, . . .**	1,679,775	**Equador,**	1,066,137
Charleston, .	49,984	Athens, . . .	63,374	Quito, ...	28,000
Troy, ...	56,747	**Turkey, E.,**	21,000,000	**Guiana, . . .**	222,000
Syracuse, . .	51,792	Const'tinople,	1,150,000	Georgetown,	28,000
Indianapolis,	75,056	**Turkey, A.,**	16,463,000	**Brazil, . . .**	9,930,478
Worcester, .	58,291	Smyrna, . . .	160,000	Rio Janeiro,	430,000
Lowell, ...	59,475	**Arabia, . . .**	9,000,000	Bahia, ...	155,000
Memphis, . .	33,592	Mecca, ...	62,000	Slaves, . . .	1,400,000
Cambridge, .	52,669	**Persia, . . .**	11,299,500	**Peru, ...**	3,374,000
Hartford, . .	42,015	Teheran, . . .	70,000	Lima, ...	130,000
Scranton, . .	45,850	**Afgh'nist'n**	5,000,000	Callao, ...	40,000
Reading, . . .	43,278	Cabul, ...	65,000	**Bolivia, . .**	2,000,000
Kansas City,	55,785	**Beloochs'n,**	450,000	La Paz, ...	23,000
Mobile, ...	29,132	Kelat, ...	17,000	**Chili, ...**	2,136,720
Portland, . . .	33,810	**Turkistan,**	7,000,000	Santiago, . .	129,807
Wilmington,	42,478	Bokhara, . . .	120,000	Valparaiso, .	97,737
Toledo, ...	50,137	**India, ...**	252,541,210	**Argentine,**	1,737,923
Columbus, . .	51,647	Bombay, . . .	753,000	Buenos Ayres	248,710
Lawrence, . .	39,151	Calcutta, . . .	683,458	**Paraguay,**	293,844
Utica, ...	33,914	**China, ...**	360,279,897	Asuncion,	16,000
Savannah, . .	30,709	Peking, . . .	2,000,000	**Uruguay,**	438,245
Nashville, . .	43,350	Canton, . . .	1,200,000	Montevideo,	130,000
Alaska, . . .	32,500	Hong-Kong,	50,000	**Patagonia,**	1,200,000
Sweden, . .	4,567,300	**Japan, . . .**	36,357,368	Antonio, . . .	?
Stockholm, .	175,000	Yeddo, ...	2,000,000	**Australia,**	2,271,245
Gotheborg, .	76,761	Miaco, ...	500,000	Melbourne,	65,860
Norköping, .	28,000	**Barbary, . .**	2,890,000	Wellington,	5,000
Malmö, . . .	37,000	Tunis, ...	140,000	**Jamaica, . .**	580,804
Gefle, ...	19,000	**Egypt, . . .**	5,252,000	Kingston, . .	40,000
Norway,	1,818,853	Cairo, ...	349,883	**Hayti, ...**	572,000
Christiania, .	119,407	Jerusalem, .	26,000	Pt. au Prince,	22,000
Bergen, . . .	39,271	**Mexico, . . .**	9,300,000	**Sandwich I.**	75,000
Denmark,	1,969,039	Mexico City,	225,000	Honolulu, . .	14,000
Copenhagen,	234,850	Puebla, . . .	80,000	**W. Indies,**	4,000,000

Latitude and Longitude of Places (from Greenwich.)

America Atl. Coast.	Latitude. D. M. S.		Longitude. D. M. S.	
Quebec	46.49.	N	71.16.	W.
Halifax	44.38.	"	63.35.	"
Chicago	42.00.	"	87.35.	"
Boston	42.21.	"	71.04.	"
New York	40.42.	"	74.00.42	"
Philadelphia	39.57.	"	75.10.	"
Cincinnati	39.05.	"	84.30.	"
St. Louis	38.35.	"	89.36.	"
Washington	38.53.	"	77.00.18	"
Charleston	32.42.	"	79.54.	"
New Orleans	29.57.30	"	90.00.	"
Georgetown, Br.	32.22.12	"	46.37.06	"
Nassau	25.05.12	"	77.21.12	"
Port-au-Prince	19.46.24	"	72.11.12	"
Porto Rico	18.29.	"	66.07.06	"
Kingston, Jam.	17.58.	"	76.46.	"
Havana	23.09.	"	82.22.	"
Vera Cruz	19.12.	"	96.09.	"
Mexico, City	19.26.	"	99.05.	"
Colon, N G.	9.22.	"	79.55.	"
Para	1.28.	S.	48.29.	"
Rio Janeiro	22.56.	"	43.09.	"
Buenos Ayres	34.36.	"	58.22.	"
Cape Horn	55.59.	"	67.16.	"
Pac. Coast.				
Valparaiso	33.02.	"	71.41.	"
Callao	12.04.	"	79.13.	"
Lima*	12.02.34	"	79.06.	"
Cuzco*	13.31.45	"	74.15.50	"
Payta	5.05.	"	81.10.	"
Guayaquil	2.13.	"	79.53.	"
Panama	8.57.	N.	79.31.	"
Acapulco	16.55.	"	99.48.	"
San Francisco	37.47.	"	122.21.	"
Alaska	58.		158	
Behring's Strait	67°.		170	
China, Ind.				
Peking	39.54.	"	116.28.	E.
Canton	23.07.	"	113.14.	"
Hongkong	22.15.	"	114.12.	"
Honolulu	21.19.	"	157.52.	"
Jeddo	35.40	"	139.43.	"
Owyhee, S. Isl.	20.23.	"	155.54.	W.
Calcutta	22.34.	"	88.20.	E.
Batavia	6.08.	"	106.50.	"
Sydney	34.00.	S.	151.23.	"
Melbourne	37.48.36	"	144.57.45	"
Wellington	41.14	"	174.44.	"
Africa.				
Cp. of G. Hope.	34.22.	"	18.30.	"
Morocco	39.34.	N.	2.23.	"
Algiers	36.47.	"	3.04.	"
England.				
London, Tower	51.31.	"	0.05.	W.
Greenwich	51.28.38	"	0 0 0	
Liverpool	53.22.	"	2.52.	"
Glasgow	55.52.	"	4.16.	"
Edinburgh	55.57.	"	3.12.	"
Bristol	51.27.	"	2.35.	"
Dover	51.08.	"	1.19.	E.
Dublin	53.23.	"	6.20	W.

France.	Latitude. D. M. S.		Longitude. D. M. S.	
Paris, Obs.	48.50.13	N.	0.09.21	E.
Cherbourg	49.38.	"	1.37.	"
Marseilles	43.18.	"	5.22.	"
Calais	50.58.	"	1.51.	"
Brussels	50.51.	"	4.22.	"
Antwerp	51.13.	"	4.24.	"
Italy.				
Turin	45.04.06	"	7.42.	"
Florence	43.46.	"	11.16.	"
Leghorn	43.32.	"	10.18.	"
Rome	41.54.	"	12.27.	"
Malta	35.54.	"	14.30.	"
Naples	40.50.	"	14.16.	"
Palermo	38.08.	"	13.22.	"
Venice	45.26.	"	12.21.	"
Austria.				
Vienna	48.13.	"	16.23.	"
Trieste	45.39.	"	13.46.	"
Pesth	47.28.	"	19.13.	"
Germany.				
Berlin	52.31.	"	13.24.	"
Hamburg	53.33.	"	9.56.	"
Cologne	50.56.	"	6.58.	"
Amsterdam	52.22.	"	4.51.	"
Bremen	53.05.	"	8.49.	"
Berne	46.57.	"	7.25.	"
Turkey.				
Constantinople	41.01.	"	28.59.	"
Ragusa	42.38.	"	18.07.	"
Salonica	40.39.	"	22.57.	"
Athens	37.58.	"	23.44.	"
Smyrna	38.26.	"	27.07.	"
Cairo	30.03.	"	31.18.	"
Jerusalem, Pal.	31.48.	"	37.20.	"
Russia.				
St. Petersburg	59.56.	"	30.19.	"
Moscow	55.46.	"	35.33.	"
Nish Novgorod	56.20.	"	43.43.	"
Cazan	55.48.	"	48.50.	"
Archangel	64.32.	"	40.14.	"
Jecatherinburg	56.50.	"	60.21.	"
Astracan	46.21.	"	47.46.	"
Odessa	46.27.	"	30.42.	"
Warsaw	52.13.05	"	21.02.9	"
Sweden.				
Stockholm	59.21.	"	18.04.	"
Gothenburg	57.42.	"	11.57.	"
Wisby, Gotland	57.39.	"	18.17.	"
Christiania	59.55.	"	10.52.	"
Bergen	60.24.	"	5.20.	"
Ystad	55.25.	"	13.50.	"
Haparanda	65.49.	"	24.11.	"
Copenhagen	55.41.	"	12.34.	"
Spain.				
Madrid	40.25.	"	3.42.	W.
Barcelona	41.23.	"	2.11.	E.
Gibraltar	36.06.	"	5.20.	W.
Carthagena	37.36.	"	1.01.	"
Lisbon	38.42.	"	9.09.	"
Oporto	41.11.	"	8.38.	"
Terra, Island	27.47.	"	17.56.	"

* Measured by the author.

Difference of Longitude in Time Between Places.

	San Francisco.			New York.			London. Greenwich.			St. Petersburg.			Canton, China.		
	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.
Amsterdam	8	29	19	5	15	32	0	19	32	1	41	44	7	13	24
Antwerp	8	27	17	5	13	30	0	17	36	1	43	40	7	15	20
Batavia	8	42	50	11	56	37	7	07	20	5	6	4	0	25	36
Berlin	9	3	22	5	49	35	0	53	35	1	7	41	6	39	21
Boston	3	25	33	0	11	46	4	44	14	6	45	30	11	42	50
Buenos Ayres . . .	4	16	19	1	2	32	3	53	28	5	54	44	11	26	24
Canton	8	17	17	11	31	4	7	32	56	5	31	40	0	0	0
Calcutta	9	56	53	10	49	20	5	53	20	3	52	4	1	39	36
Cairo	10	14	59	7	1	12	2	5	12	0	3	56	5	27	44
Copenhagen	9	0	3	5	46	16	0	50	16	1	11	0	6	42	40
Constantinople . . .	10	5	43	6	51	56	0	55	56	0	5	20	5	37	0
Dublin	7	44	25	4	30	38	0	25	22	2	26	38	8	18	18
Florence	8	54	51	5	41	4	0	45	4	1	16	12	6	47	52
Gibraltar	7	47	56	4	34	32	0	21	28	2	22	44	7	54	24
Gothenburg	8	57	38	5	43	51	0	47	48	1	13	28	6	45	08
Halifax	3	54	27	0	41	40	4	14	20	6	15	36	11	47	16
Hamburg.	8	49	39	5	35	52	0	39	52	1	21	24	6	53	04
Jecatherinburg . . .	11	48	57	7	25	20	4	1	16	2	0	0	3	31	40
Jerusalem	10	39	7	7	25	20	2	23	20	0	22	4	5	09	36
Lima	1	43	2	0	12	24	5	08	24	7	9	40	11	18	40
London	8	9	50	4	56	3	0	0	24	2	1	40	7	33	20
Lisbon	7	33	11	4	19	24	0	36	36	2	37	52	8	08	32
Madrid	7	55	39	4	41	52	0	14	8	2	15	24	7	47	04
Melbourne	4	2	24	7	16	11	9	39	51	11	41	31	2	7	0
Naples	9	6	51	5	53	4	0	57	4	1	04	12	6	35	52
New Orleans	2	9	37	1	04	10	6	0	10	8	01	26	10	26	54
New York	3	13	47	0	0	0	4	56	3	6	57	19	11	31	01
Paris	8	19	7	5	5	20	0	9	20	1	51	50	7	23	36
Peking	8	4	21	12	41	52	7	45	52	5	44	36	0	12	56
Quebec	3	2	36	0	11	11	4	44	49	6	46	5	11	19	53
Rome	8	59	35	5	45	48	0	49	48	1	11	28	6	43	08
San Francisco	0	0	0	3	13	47	8	9	47	10	11	3	8	17	17
St. Louis	2	8	46	1	5	1	6	1	1	8	2	17	10	26	03
St. Petersburg	10	11	3	6	57	16	2	1	16	0	0	0	5	31	40
Stockholm	9	22	11	6	8	24	1	12	24	0	58	52	6	20	32
Turin	8	40	38	5	26	51	0	30	48	1	20	28	7	02	08
Washington	3	1	46	0	12	1	5	8	1	7	9	17	11	19	53
Wellington, N. Z. . . .	5	30	22	8	44	09	11	38	56	10	19	48	2	46	52

To Reduce Longitude in Degrees into Time, and vice versa.

RULE 1. Divide the number of degrees by 15, and the quotient is the corresponding hours. Should the degrees be less than 15, multiply them by 4, and the product will be minutes in time. The minutes of degrees multiplied by 4 will be seconds in time. The seconds of degrees divided by 15 will be seconds in time.

RULE 2. The time in hours, minutes and seconds, multiplied by 15, will be the corresponding angle in degrees, minutes and seconds. The trigonometrical table contains the conversion of time and angle.

Example 1. Required, the difference in time between Philadelphia and Paris?

Longitude of Philadelphia, 75° 10′ *W.*
" " Paris, . . 2 20 *E.*

Difference in longitude 77° 30′ divided by 15 will be 5*h* 10*m*, the difference in time. When it is 12 o'clock in Philadelphia, it is 5*h* 10*m* o'clock in Paris.

Example 2. A vessel sails from New York to Liverpool; after she has been at sea about one week, her difference in time with New York is found to be 2*h* 7*m* 45*s*. Required, her longitude from New York?

15(2*h* 7 45) = 31° 58′ 15″ from New York.

The time is ahead in the east, from where the sun rises. The time is behind in the west, toward sunset.

Distances by Sea

between the most important places on the earth: in Geographical miles, 60 per degree.

* Round Cape of Good Hope.
† Round Cape Horn.

From San Francisco 1980 to Sandwich Islands.
From Canton, 4500 " "

	Batavia	Cape Good Hope	Constantinople	Rome	Gibraltar	Lisbon	Liverpool	London	Paris	Antwerp	Hamburg	Gottenburg	Copenhagen	Stockholm	St. Petersburg	New York	New Orleans	Cape Horn	San Francisco	Canton
San Francisco																				5820
Cape Horn																			6100	9780
New Orleans																		7620	13800†	17400†
New York																	2129	7200	13380†	16980†
St. Petersburg																4516	6076	8837	15936†	15796*
Stockholm															377	4311	5871	8632	15731†	15591*
Copenhagen														495	700	3816	5376	8137	15236†	15096*
Gottenburg													139	634	839	3727	5287	8047	15147†	15007*
Hamburg												360	488	983	1188	3576	5136	7896	14076†	14836*
Antwerp											348	582	721	1216	1421	3348	4908	7668	13848†	14628*
Paris										518	726	897	986	1481	1686	3290	4850	7610	13790	14570*
London									410	258	451	617	706	1201	1407	3240	4800	7560	13740	14520*
Liverpool								710	760	818	1256	1197	1286	1781	1986	3000	4560	7320	13500†	14280*
Lisbon							900	1060	1110	1168	1370	1547	1636	2131	2336	2980	4300	6460	1254†	13020*
Gibraltar						330	1200	1400	1380	1438	1680	1817	1906	2405	2606	3220	4500	6460	12640	13080*
Rome					1018	1348	2218	2418	2398	2456	2698	2835	2924	3419	3624	4238	5518	7578	13658	14091*
Constantinople				1150	2040	2340	3240	3440	3490	3548	3720	3927	4016	4511	4716	5260	6540	8460	14680†	15000*
Cape Good Hope			7320	6418	5400	5340	6600	6840	6840	6838	7080	7277	7366	7861	8066	6900	7380	3600	9780†	7680
Batavia		5180	12500*	11598*	10580*	10520*	11780*	12020*	12060*	12018*	12260*	12457*	12544*	13039*	13244*	12080*	12560*	9100	7860	2580
Port Jackson	3860	6120	13560*	12538*	11520*	11460*	12720*	12960*	1301*	13068*	13296*	41347*	13536*	14031*	14236*	12300†	12720	5280	6480	4620

Distances in Statute Miles.

Nearest travelling distances between Cities in Europe.

	Copenhagen	Stockholm	St. Petersburg	Moscow	Odessa	Constantinople	Warsaw	Triest	Rome	Munich	Vienna	Turin	Berne	Berlin	Hamburg	Antwerp	Lisbon	Madrid	Paris	Liverpool	London
Liverpool																					212
Paris																				509	297
Madrid																			625	1002	790
Lisbon																		265	890	1267	1055
Antwerp																	1142	877	252	544	332
Hamburg																391	1560	1295	670	936	724
Berlin															175	509	1515	1250	625	984	772
Berne														608	605	541	1150	885	442	951	739
Turin													193	803	901	736	895	630	445	954	742
Vienna												662	574	500	679	751	1724	1459	790	1295	1083
Munich											270	508	281	454	600	513	1431	1166	551	1045	833
Rome										507	745	328	521	963	1109	1022	1223	958	773	1373	1161
Triest									407	342	338	456	584	796	942	855	1351	1086	1107	1399	1187
Warsaw								798	1069	563	460	1034	836	536	711	1045	2051	1786	1297	1589	1377
Constantinople							1200	1000	1050	1679	1409	1456	1584	1909	2084	2160	2350	2086	2412	2704	2492
Odessa						750	805	1400	1907	1369	1099	1856	2000	1336	1511	1836	2750	2485	1900	2450	2238
Moscow					910	1660	792	2751	3158	2683	2413	3191	2964	2489	2664	3173	4004	3739	3114	3717	3505
St. Petersburg				403	1200	1950	733	1506	1774	1271	1162	1742	1544	969	1144	1478	2484	2219	1594	1953	1741
Stockholm			440	835	1953	2775	1148	1633	1928	1291	1370	1799	1606	866	691	1082	2251	1986	1334	1626	1413
Copenhagen		435	875	1270	1772	2340	967	1198	1493	856	935	1364	1171	431	256	647	1816	1551	899	1191	979
Lubeck	234	669	1109	1504	1514	2123	709	748	1146	639	718	849	643	214	39	352	1599	1334	710	896	634

Sailing distances

between the most important seaports in the United States, in Geographical miles, 60 per Degree.

	Halifax.	Boston.	Nantucket light.	Sandy Hook.	New York.	Cape Henlopen.	Philadelphia.	Cape Henry.	Baltimore.	Washington.	Norfolk.	Richmond.	Cape Hatteras.	Charleston.	Savannah.	Key West.	Havana.	New Orleans.	Vera Cruz.
Boston.	390																		
Nantucket light.	360	107																	
Sandy Hook.	648	284	193																
New York.	666	302	211	18															
Cape Henlopen.	650	380	274	131	149														
Philadelphia.	753	483	377	234	252	103													
Cape Henry.	765	509	403	258	276	135	238												
Baltimore.	917	661	555	410	428	287	390	152											
Washington.	923	667	561	416	434	293	396	158	174										
Norfolk.	795	539	433	288	306	165	268	80	172	178									
Richmond.	865	609	503	358	375	235	338	100	242	248	90								
Cape Hatteras.	786	624	426	322	340	218	321	114	266	272	144	214							
Charleston.	1065	905	707	603	621	499	602	395	547	553	425	495	281						
Savannah.	1162	1000	802	698	716	594	697	490	642	648	520	590	376	95					
Key West.	1930	1768	1570	1166	1484	1362	1465	1258	1410	1406	2288	1348	768	622	542				
Havana.	1900	1738	1540	1436	1454	1332	1435	1228	1380	1376	1258	1328	738	592	562	85			
New Orleans.	2685	2413	2215	2111	2129	2007	2110	1903	2055	2051	1933	2003	1413	1267	1187	640	640		
Vera Cruz.	2800	2638	2440	2336	2354	2232	2335	2128	2280	2276	1158	2228	1638	1492	1412	870	823	850	
Porto Bello	2635	2738	2640	2536	2554	2432	2535	2328	2480	2476	2358	2428	1338	1692	1612	1070	1200	1370	1407

Distances in Statute Miles

between the most important places in the United States by Railroads and Rivers. The distances are given by the shortest routes.

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*Mississippi River

																				Montreal.
																			Boston.	326
																		Albany.	200	253
																	New York.	144	236	396
																Philadelphia.	87	231	323	483
															Baltimore.	98	185	329	421	581
														Washington.	39	137	224	368	460	610
													Richmond.	131	170	268	355	499	591	741
												Wilmington.	247	378	417	515	602	746	838	988
											Charleston.	262	509	640	679	777	864	308	1100	1250
										Savannah.	112	374	621	732	791	889	976	1088	1212	1462
									Atlanta.	292	308	619	866	977	1036	1134	1221	1333	1487	1754
								Montgomery.	175	467	483	794	1041	1152	1211	1309	1396	1508	1632	1929
							New Orleans.	496	671	963	979	1290	1537	1648	1707	1805	1892	2004	2128	2425
						St. Louis.	1201*	726	901	1018	1034	1392	1121	990	951	967	1293	1143	1490	1069
					Chicago.	310	1618*	1493	1318	1611	1441	1937	1011	880	841	865	1048	908	1073	918
				Cincinnati.	273	697	1548*	1148	973	1266	1436	1109	862	731	692	663	847	693	993	813
			Pittsburgh.	310	344	1174	2025*	1625	1450	1169	1057	795	533	402	294	353	470	545	693	595
		Harrisburg.	245	684	1094	904	1790*	1294	1119	874	762	500	253	122	82	106	194	289	430	489
	Buffalo.	664	220	468	543	1165	2016*	1937	1441	1446	1334	1072	825	694	655	557	470	325	525	375
Cleveland.	183	395	150	255	360	511	1803*	1403	1327	1235	1123	861	614	483	444	608	636	508	708	558

ASTRONOMY.

The matter constituting the heavenly bodies has probably been evenly distributed in space from the beginning; the force of attraction gradually formed bodies, which accumulated into groups or nebulæ, each of which finally became a planetary system with the largest body in the centre, now seen as stars. This operation of forming or creating bodies and planetary systems is still and will for ever be continued in parts of the infinite space. Each star has a planetary system, and astronomers have even been able to observe some planets of the star Sirius.

The magnitude of this operation, with the enormous dimensions, even within a single group, can hardly be conceived by any human mind; for the long row of figures expressing a number of conceivable units of length or weight does not bring the *real magnitude* within conception.

The Sun.

The sun is a dark body surrounded by a luminous substance in which spots are frequently seen. The spots are caused by meteors or other heavenly bodies falling into it.

Mean distance from the earth to the sun, 95,000,000 miles, or 11,992 diameters of the earth.

Inclination of the ecliptic to the equinoctial, 23° 28′ 40″.

The sun subtends an angle from the earth of 32′ 3″.

Horizontal parallax of the sun, 8.6″.

The Moon.

Distance from the Earth to the Moon, 273,000 miles, = 30 diameters of Earth, or about 0.25 diameters of the Sun, or the diameter of the Sun is twice the diameter of the Moon's orbit around the Earth.

Diameter of the Moon, 2160 miles, or about 0.2729 of the diameter of the Earth.

Volume of the Moon, 0.02024 of that of the Earth.

Density of the Moon, 0.5657 of that of the Earth.

Mass of the Moon, 0.0114 of that of the Earth.

Inclination of the Moon's orbit to the ecliptic, 5° 8′ 48″.

The Moon subtends an angle from the Earth of 31′ 7″.

Mean sidereal revolution of the Moon, 27.32166 solar days.

Mean synodical revolution of the Moon, 29.5305887 solar days.

The Moon passes the meridian in periods of 24.814 hours, or 48m. 50s. later every day.

Moon's Age is the number of days from the last new moon.

Epact of the Year is the Moon's age on the 1st of January of each year. See Almanac for the 19th Century.

The sum of the epact of the year and that of the month is the moon's age on the first of the month.

Epact of the Month.

Jan.	Feb.	March	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
0	2	1	2	3	4	5	6	8	8	10	10

To Find the Moon's Age on Any Given Day.

Add together the epacts of the year and month and the date of the month; the sum will be the moon's age. If it exceeds 30, reject that much, and the remainder is the moon's age.

Almanac for the 19th Century.

Yrs.	Days.	Dom. letter.	Epact.	Yrs.	Days.	Dom. letter.	Epact.	Yrs.	Days.	Dom. letter.	Epact.
1800	Saturd.*	FE	4	1834	Saturd.	E	20	1868	Sunday*	ED	6
1801	Sunday.	D	15	1835	Sunday.	D	1	1869	Monday.	C	17
1802	Monday.	C	26	1836	Tuesd.*	CB	12	1870	Tuesday.	B	28
1803	Tuesday.	B	7	1837	Wednes.	A	23	1871	Wednes.	A	9
1804	Thursd.*	AG	18	1838	Thursd.	G	4	1872	Friday.*	GF	20
1805	Friday.	F	29	1839	Friday.	F	15	1873	Saturd.	E	1
1806	Saturd.	E	11	1840	Sunday*	ED	26	1874	Sunday.	D	12
1807	Sunday.	D	22	1841	Monday.	C	7	1875	Monday.	C	23
1808	Tuesd.*	CB	3	1842	Tuesday.	B	18	1876	Wedns.*	BA	4
1809	Wednes.	A	14	1843	Wednes.	A	29	1877	Thursd.	G	15
1810	Thursd.	G	25	1844	Friday.*	GF	11	1878	Friday.	F	26
1811	Friday.	F	6	1845	Saturd.	E	22	1879	Saturd.	E	7
1812	Sunday.*	ED	17	1846	Sunday.	D	3	1880	Monday*	DC	18
1813	Monday.	C	28	1847	Monday.	C	14	1881	Tuesday.	B	29
1814	Tuesday.	B	9	1848	Wedns.*	BA	25	1882	Wednes.	A	11
1815	Wednes.	A	20	1849	Thursd.	G	6	1883	Thursd.	G	22
1816	Friday.*	GF	1	1850	Friday.	F	17	1884	Saturd.*	FE	3
1817	Saturd.	E	12	1851	Saturd.	E	28	1885	Sunday.	D	14
1818	Sunday.	D	23	1852	Mond.*	DC	9	1886	Monday.	C	25
1819	Monday.	C	4	1853	Tuesday.	B	20	1887	Tuesday.	B	6
1820	Wedns.*	BA	15	1854	Wednes.	A	1	1888	Thursd.*	AG	17
1821	Thursd.	G	26	1855	Thursd.	G	12	1889	Friday.	F	28
1822	Friday.	F	7	1856	Saturd.*	FE	23	1890	Saturd.	E	9
1823	Saturd.	E	18	1857	Sunday.	D	4	1891	Sunday.	D	20
1824	Monda.*	DC	29	1858	Monday.	C	15	1892	Tuesd.*	CB	1
1825	Tuesday.	B	11	1859	Tuesday.	B	26	1893	Wednes.	A	12
1826	Wednes.	A	22	1860	Thurs.*	AG	7	1894	Thursd.	G	23
1827	Thursd.	G	3	1861	Friday.	F	18	1895	Friday.	F	4
1828	Saturd.*	FE	14	1862	Saturd.	E	29	1896	Sunday*	ED	15
1829	Sunday.	D	25	1863	Sunday.	D	11	1897	Monday.	C	26
1830	Monday.	C	6	1864	Tuesd.*	CB	22	1898	Tuesday.	B	7
1831	Tuesday.	B	17	1865	Wednes.	A	3	1899	Wednes.	A	18
1832	Thursd.*	AG	28	1866	Thursd.	G	14	1900	Friday.*	GF	29
1833	Friday.	F	9	1867	Friday.	F	25	1901	Saturd.	E	11

The day of the week opposite the year in the almanac falls on the dates in this table.

February, March, November.	February,* August.	May.	January, October.	January,* April, July.	September, December.	June.
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

In leap years take January,* February.*

Example 1. On what day of the week will the 4th of July fall in the year 1880? See table 1880 = Monday, which answers to the 5th in the date table, consequently the 4th of July is on Sunday.

Example 2. It is known to be Saturday in the middle of August, 1875. Required, the date of that day?

The year 1875 = Monday (see almanac), then August the 16th falls on Monday, and Saturday on the 14th.

Example 3. A gentleman was born on the 8th of February, 1824. Required, the day of the week?

1824 = Monday, which fell on the 9th. The gentleman was consequently born on a Sunday.

CHRONOLOGY.

Our unit of time, *year*, is the period in which the earth makes one revolution around the sun, in reference to a fixed star.

The unit *day* is the period in which the earth makes one revolution around its axis, in reference to the sun; but as the earth moves in an ellipse in which the sun is in one focus, the apparent solar day is not a constant period—that one hundred solar days in the winter are about half an hour longer than one hundred solar days in summer, for which a mean day is assumed in reference to an imaginary sun which falls in with the real sun about the 15th of April and 24th of December, when the mean time and apparent time are alike. The mean solar day is divided into twenty-four hours, of which the clock indicates twelve hours twice.

Sidereal Time.

The Sidereal Day is the interval of time in which any fixed star passes the meridian. The sidereal day is only 23*h*. 56*m*. 4.09*s*. mean solar time, or the fixed stars pass the meridian, rise or set, 3*m*. 55.909*s*. earlier every day.

A Sidereal Clock in an astronomical observatory marks twenty-four hours in the interval of time in which any fixed star passes the meridian. The Right Ascension of the heavenly bodies is the time when the body passes the meridian by the sidereal clock. The dial of a sidereal clock is divided into twenty-four hours, and the hours are numbered from one up to twenty-four.

Years.

The tropical year, or the periodical return of seasons, is 365.24224 days = 365*d*. 5*h*. 48*m*. 49.536*s*. mean solar time.

The civil year is 365 days, or nearly one-fourth of a day too short, for which one day is added every four years, called leap year. But this addition makes one day too much in every 128.866 years, which error is corrected every fourth century which can be divided by 4 without a remainder.

Leap years of the Christian era are those that can be divided by 4 without a remainder.

In some countries these important corrections are not properly attended to, as in Russia and Greece, where the dates are now twelve days behind our Gregorian reckoning.

The wild Indians of South America reckon their time by new moons, when all their festivals are celebrated.

Dates.

The civil date of the month commences at midnight. The astronomical date commences at noon. The mariner's date (sea account) commences twelve hours before the civil date, and twenty-four hours before the astronomical date, or the mariner's date is one day ahead of the astronomical date.

Cycle of the Sun is the period of twenty-eight years, at which the days of the week return to the same days of the month.

Lunar Cycle or Golden Number is the period of nineteen years, at which the changes of the moon fall on the same days of the month.

To Find the Golden Number.

Add 1 to the given year, divide the sum by 19, and the remainder will be the golden number. If 0 remains, the golden number is 19.

The age of the Julian period on the 1st of January, 1872, is 2,404,794 days.

Creation of the World.

Creation of the world, 4000 before Christ. Julius Africanus says 5508; Samaritan Pentateuch, 4700; Septuagint, 5872; Josephus, 4658; Talmudists, 5344; and others give different times; but the Chinese tradition and history claim an antiquity of 100,000 years before Christ. From geological formations, and from the working of rivers like that of Niagara, and the Danube cutting through the Alps at the Iron Gate, it can be estimated at millions of years.

Chronological Notes.

Before Christ. B. C.	YEARS.	After Christ. A. D.	YEARS.
Deluge, 2348 (Hales),	3154	Beginning of Christian Era,	0
Tower of Babel finished,	2247	Christ crucified,	37
Chinese Monarchy,	2203	Destruction of Jerusalem,	69
Egyptian Pyramids,	2090	Arabic numbers introduced,	991
Moses born,	1567	Mohammedan Era,	622
Troy destroyed,	1180	New Style in England,	1752
The compass discovered,	1111	America discovered,	1492
Foundation of Rome,	753	Pizarro conquered Inca, Peru,	1530
Maps and Geometry introduced,	605	Lutheran religion,	1527
Money coined at Rome,	576	New South Wales discovered,	1606
Hannibal crossed the Alps,	219	Australia discovered,	1622
Time first measured by water,	155	American great Republic,	1775
Carthage destroyed,	146	Slaves free in West Indies,	1834
Cæsar invaded Britain,	51	Slaves free in Russia,	1861
Cæsarean era,	48	Slaves free in America,	1866

ASTRONOMICAL ALMANAC.

From the English Nautical Almanac.

The following tables show the sun's right ascension and declination; also, the equation of time at Greenwich, apparent noon, for the year 1873:

In leap years . . . . at 6 h. A. M.
First year after leap year . . . . . . at noon.
Second year after leap year. . . . . at 6 h. P. M.
In the year before leap year, at midnight following the date.

By the aid of the tables of correction the data can be found for any time and for any meridian.

Example 1. Required, the sun's R. A. on the 10th of April, 1874, at Greenwich, apparent noon?

1874 is the second year after leap year, when the tabular data is for 6 o'clock in the evening. The daily variation of the sun's R. A. is 3 m. 40 s. for the 10th of April, which for 6 hours will be 55 s.

The sun's R. A. on the 10th of April, .	1 h. 16 m. 24 s.
Correction for 6 hours, subtract . . .	55 s.
The required R. A. will be . . .	1 h. 15 m. 29 s.

Example 2. Required, the sun's declination on the 20th of September, in the leap year 1876, at 3 o'clock P. M., in longitude 75°, or 5 hours west of Greenwich?

The tabular data for leap years is at 6 o'clock A. M. 3 P. M. and 5 hours difference in longitude make 14 hours of correction.

The daily variation in the sun's declination is 23′.

$$\left.\begin{array}{l} 20' = 11'\,40'' \\ \ \ 3 \ = \ \ 1\ \ 45 \end{array}\right\}$$ See table of correction, page 502.

Correction, 13′ 25″ for 14 hours.

Sun's declination 20th September, . . .	0° 58′
Subtract correction, ...	13′ 25″
The required declination, ...	0° 44′ 35″

In leap years take the tabular data one day earlier in January and February.

Date	January. Rt. As. h. m. s.	Declin. ° ′	Eq. tm. m. s.	February. Rt. As. h. m. s.	Declin. ° ′	Eq. tm. m. s.	March. Rt. As. h. m. s.	Declin. ° ′	Eq. tm. m. s.	Date
1	18 48 49	S 22 59	+ 3 59	21 0 59	S 16 59	+13 54	22 49 58	S 7 16	+12 30	1
2	18 53 14	S 22 54	+ 4 27	21 5 3	S 16 42	+14 2	22 53 42	S 7 3	+12 18	2
3	18 57 38	S 22 48	+ 4 55	21 9 6	S 16 24	+14 8	22 57 26	S 6 40	+12 5	3
4	19 2 2	S 22 41	+ 5 23	21 13 8	S 16 6	+14 14	23 1 9	S 6 17	+11 52	4
5	19 6 26	S 22 35	+ 5 49	21 17 9	S 15 48	+14 18	23 4 52	S 5 54	+11 38	5
6	19 10 49	S 22 27	+ 6 16	21 21 10	S 15 30	+14 22	23 8 35	S 5 31	+11 24	6
7	19 15 11	S 22 20	+ 6 42	21 25 9	S 15 11	+14 25	23 12 17	S 5 8	+11 10	7
8	19 19 33	S 22 12	+ 7 7	21 29 8	S 14 52	+14 28	23 15 58	S 4 44	+10 55	8
9	19 23 55	S 22 3	+ 7 32	21 33 6	S 14 33	+14 29	23 19 39	S 4 21	+10 39	9
10	19 28 16	S 21 54	+ 7 56	21 37 3	S 14 13	+14 30	23 23 20	S 3 57	+10 24	10
11	19 32 36	S 21 45	+ 8 20	21 41 0	S 13 53	+14 30	23 27 1	S 3 34	+10 7	11
12	19 36 56	S 21 35	+ 8 43	21 44 56	S 13 33	+14 29	23 30 41	S 3 10	+ 9 51	12
13	19 41 15	S 21 25	+ 9 6	21 48 51	S 13 13	+14 27	23 34 21	S 2 47	+ 9 34	13
14	19 45 33	S 21 14	+ 9 27	21 52 45	S 12 53	+14 25	23 38 0	S 2 23	+ 9 17	14
15	19 49 51	S 21 3	+ 9 48	21 56 38	S 12 32	+14 22	23 41 39	S 1 59	+ 9 0	15
16	19 54 8	S 20 52	+10 9	22 0 31	S 12 12	+14 18	23 45 19	S 1 36	+ 8 43	16
17	19 58 24	S 20 40	+10 29	22 4 23	S 11 51	+14 13	23 48 57	S 1 12	+ 8 25	17
18	20 2 40	S 20 28	+10 48	22 8 14	S 11 30	+14 8	23 52 36	S 0 48	+ 8 8	18
19	20 6 55	S 20 15	+11 6	22 12 5	S 11 8	+14 2	23 56 15	S 0 24	+ 7 50	19
20	20 11 9	S 20 2	+11 24	22 15 55	S 10 47	+13 56	23 59 53	S 0 1	+ 7 32	20
21	20 15 23	S 19 49	+11 40	22 19 44	S 10 25	+13 49	0 3 32	N 0 23	+ 7 13	21
22	20 19 35	S 19 35	+11 57	22 23 33	S 10 3	+13 41	0 7 10	N 0 47	+ 6 55	22
23	20 23 47	S 19 21	+12 12	22 27 31	S 9 41	+13 32	0 10 48	N 1 10	+ 6 37	23
24	20 27 59	S 19 7	+12 27	22 31 9	S 9 19	+13 23	0 14 26	N 1 34	+ 6 18	24
25	20 32 9	S 18 52	+12 40	22 34 56	S 8 57	+13 14	0 18 4	N 1 57	+ 6 0	25
26	20 36 19	S 18 37	+12 53	22 38 42	S 8 34	+13 4	0 21 42	N 2 21	+ 5 42	26
27	20 40 27	S 18 21	+13 6	22 42 28	S 8 12	+12 53	0 25 20	N 2 44	+ 5 23	27
28	20 44 35	S 18 6	+13 17	22 46 13	S 7 49	+12 42	0 28 59	N 3 8	+ 5 5	28
29	20 48 43	S 17 49	+13 28				0 32 37	N 3 31	+ 4 47	29
30	20 52 49	S 17 33	+13 37				0 36 15	N 3 55	+ 4 28	30
31	20 56 54	S 17 16	+13 46				0 39 53	N 4 18	+ 4 10	31

Date	April. Rt. As.	Declin.	Eq. tm.	May. Rt. As.	Declin.	Eq. tm.	June. Rt. As.	Declin.	Eq. tm.	Date
1	0 43 32	N 4 41	+ 3 52	2 34 51	N 15 11	— 3 4	4 37 41	N 22 6	— 2 27	1
2	0 47 10	N 5 4	+ 3 34	2 38 41	N 15 29	— 3 11	4 41 47	N 22 14	— 2 18	2
3	0 50 49	N 5 27	+ 3 16	2 42 31	N 15 57	— 3 18	4 45 53	N 22 22	— 2 8	3
4	0 54 27	N 5 50	+ 2 58	2 46 21	N 16 4	— 3 24	4 50 0	N 22 29	— 1 58	4
5	0 58 6	N 6 13	+ 2 41	2 50 12	N 16 21	— 3 29	4 54 7	N 22 36	— 1 48	5
6	1 1 45	N 6 35	+ 2 23	2 54 4	N 16 38	— 3 34	4 58 14	N 22 42	— 1 37	6
7	1 5 25	N 6 58	+ 2 6	2 57 56	N 16 55	— 3 39	5 2 22	N 22 48	— 1 27	7
8	1 9 4	N 7 20	+ 1 49	3 1 49	N 17 11	— 3 42	5 6 29	N 22 53	— 1 15	8
9	1 12 44	N 7 43	+ 1 32	3 5 42	N 17 27	— 3 46	5 10 37	N 22 58	— 1 4	9
10	1 16 24	N 8 5	+ 1 15	3 9 36	N 17 43	— 3 48	5 14 46	N 23 3	— 0 52	10
11	1 20 4	N 8 27	+ 0 59	3 13 30	N 17 58	— 3 51	5 18 54	N 23 7	— 0 40	11
12	1 23 44	N 8 49	+ 0 43	3 17 25	N 18 13	— 3 52	5 23 3	N 23 11	— 0 28	12
13	1 27 25	N 9 11	+ 0 27	3 21 21	N 18 28	— 3 53	5 27 12	N 23 15	— 0 16	13
14	1 31 6	N 9 32	+ 0 12	3 25 17	N 18 43	— 3 54	5 31 21	N 23 18	— 0 4	14
15	1 34 48	N 9 54	— 0 3	3 29 13	N 18 57	— 3 53	5 35 30	N 23 20	+ 0 9	15
16	1 38 30	N 10 15	— 0 18	3 33 11	N 19 11	— 3 53	5 39 39	N 23 22	+ 0 22	16
17	1 42 12	N 10 36	— 0 32	3 37 9	N 19 24	— 3 51	5 43 49	N 23 24	+ 0 35	17
18	1 45 55	N 10 57	— 0 46	3 41 7	N 19 38	— 3 50	5 47 58	N 23 26	+ 0 48	18
19	1 49 38	N 11 18	— 0 59	3 45 6	N 19 51	— 3 47	5 52 8	N 23 27	+ 1 1	19
20	1 53 21	N 11 38	— 1 12	3 49 6	N 20 3	— 3 44	5 56 17	N 23 27	+ 1 14	20
21	1 57 5	N 11 59	— 1 25	3 53 6	N 20 15	— 3 40	6 0 27	N 23 27	+ 1 27	21
22	2 0 49	N 12 19	— 1 37	3 57 6	N 20 27	— 3 36	6 4 37	N 23 27	+ 1 40	22
23	2 4 34	N 12 39	— 1 49	4 1 8	N 20 39	— 3 32	6 8 46	N 23 26	+ 1 53	23
24	2 8 20	N 12 59	— 2 0	4 5 10	N 20 50	— 3 26	6 12 55	N 23 25	+ 2 6	24
25	2 12 5	N 13 18	— 2 11	4 9 12	N 21 1	— 3 21	6 17 5	N 23 24	+ 2 19	25
26	2 15 52	N 13 38	— 2 21	4 13 15	N 21 11	— 3 15	6 21 15	N 23 22	+ 2 31	26
27	2 19 39	N 13 57	— 2 30	4 17 18	N 21 22	— 3 8	6 25 24	N 23 20	+ 2 44	27
28	2 23 26	N 14 16	— 2 40	4 21 22	N 21 31	— 3 0	6 29 33	N 23 17	+ 2 56	28
29	2 27 14	N 14 34	— 2 48	4 25 26	N 21 41	— 2 53	6 33 42	N 23 14	+ 3 8	29
30	2 31 2	N 14 53	— 2 56	4 29 31	N 21 50	— 2 45	6 37 50	N 23 10	+ 3 20	30
31				4 33 36	N 21 58	— 2 36				31

Date	July. Rt. As. h. m. s.	July. Declin. ° ′	July. Eq. tm. m. s.	August. Rt. As. h. m. s.	August. Declin. ° ′	August. Eq. tm. m. s.	September. Rt. As. h. m. s.	September. Declin. ° ′	September. Eq. tm m. s.	Date
1	6 41 58	N 23 6	+ 3 32	8 46 42	N 17 57	+ 6 2	10 42 40	N 8 10	— 0 12	1
2	6 46 6	N 23 2	+ 3 44	8 50 35	N 17 42	+ 5 58	10 46 18	N 7 49	— 0 31	2
3	6 50 14	N 22 57	+ 3 55	8 54 27	N 17 26	+ 5 54	10 49 55	N 7 27	— 0 50	3
4	6 54 22	N 22 52	+ 4 5	8 58 18	N 17 11	+ 5 49	10 53 32	N 7 4	— 1 10	4
5	6 58 29	N 22 47	+ 4 16	9 2 9	N 16 54	+ 5 43	10 57 8	N 6 42	— 1 30	5
6	7 2 35	N 22 41	+ 4 26	9 5 59	N 16 38	+ 5 36	11 0 45	N 6 20	— 1 50	6
7	7 6 42	N 22 34	+ 4 36	9 9 49	N 16 21	+ 5 29	11 4 21	N 5 57	— 2 10	7
8	7 10 48	N 22 28	+ 4 45	9 13 38	N 16 4	+ 5 22	11 7 57	N 5 35	— 2 30	8
9	7 14 53	N 22 20	+ 4 54	9 17 26	N 15 47	+ 5 14	11 11 33	N 5 12	— 2 51	9
10	7 18 58	N 22 13	+ 5 3	9 21 14	N 15 29	+ 5 5	11 15 9	N 4 49	— 3 12	10
11	7 23 3	N 22 5	+ 5 11	9 25 1	N 15 12	+ 4 55	11 18 45	N 4 27	— 3 33	11
12	7 27 7	N 21 57	+ 5 18	9 28 47	N 14 54	+ 4 46	11 22 20	N 4 4	— 3 54	12
13	7 31 11	N 21 48	+ 5 26	9 32 34	N 14 35	+ 4 35	11 25 56	N 3 41	— 4 15	13
14	7 35 14	N 21 39	+ 5 32	9 36 19	N 14 17	+ 4 24	11 29 31	N 3 18	— 4 36	14
15	7 39 17	N 21 30	+ 5 39	9 40 4	N 13 58	+ 4 13	11 33 6	N 2 54	— 4 57	15
16	7 43 19	N 21 20	+ 5 44	9 43 49	N 13 39	+ 4 1	11 36 42	N 2 31	— 5 18	16
17	7 47 21	N 21 10	+ 5 50	9 47 33	N 13 20	+ 3 48	11 40 17	N 2 8	— 5 39	17
18	7 51 23	N 20 59	+ 5 54	9 51 16	N 13 1	+ 3 35	11 43 53	N 1 45	— 6 0	18
19	7 55 24	N 20 49	+ 5 59	9 54 59	N 12 41	+ 3 22	11 47 28	N 1 22	— 6 21	19
20	7 59 24	N 20 37	+ 6 2	9 58 42	N 12 21	+ 3 8	11 51 4	N 0 58	— 6 42	20
21	8 3 24	N 20 26	+ 6 5	10 2 24	N 12 1	+ 2 54	11 54 39	N 0 35	— 7 3	21
22	8 7 23	N 20 14	+ 6 8	10 6 6	N 11 41	+ 2 39	11 58 15	N 0 11	— 7 24	22
23	8 11 22	N 20 2	+ 6 10	10 9 47	N 11 21	+ 2 24	12 1 51	S 0 12	— 7 45	23
24	8 15 20	N 19 49	+ 6 12	10 13 28	N 11 0	+ 2 8	12 5 26	S 0 35	— 8 5	24
25	8 19 17	N 19 36	+ 6 13	10 17 8	N 10 40	+ 1 52	12 9 3	S 0 59	— 8 26	25
26	8 23 14	N 19 23	+ 6 13	10 20 48	N 10 19	+ 1 35	12 12 39	S 1 22	— 8 46	26
27	8 27 10	N 19 10	+ 6 13	10 24 28	N 9 58	+ 1 18	12 16 15	S 1 46	— 9 6	27
28	8 31 6	N 18 56	+ 6 12	10 28 7	N 9 37	+ 1 1	12 19 52	S 2 9	— 9 26	28
29	8 35 1	N 18 42	+ 6 11	10 31 46	N 9 15	+ 0 43	12 23 29	S 2 32	— 9 46	29
30	8 38 55	N 18 27	+ 6 9	10 35 24	N 8 54	+ 0 25	12 27 6	S 2 56	—10 5	30
31	8 42 49	N 18 13	+ 6 6	10 39 2	N 8 32	+ 0 7				31

Date	October. Rt. As.	October. Declin.	October. Eq. tm.	November. Rt. As.	November. Declin.	November. Eq. tm.	December. Rt. As.	December. Declin.	December. Eq. tm.	Date
1	12 30 43	S 3 19	—10 24	14 27 2	S 14 33	—16 17	16 30 57	S 21 53	—10 40	1
2	12 34 20	S 3 42	—10 43	14 30 58	S 14 52	—16 18	16 35 17	S 22 2	—10 17	2
3	12 37 58	S 4 6	—11 2	14 34 54	S 15 11	—16 18	16 39 37	S 22 10	— 9 53	3
4	12 41 37	S 4 29	—11 20	14 38 52	S 15 30	—16 17	16 43 58	S 22 18	— 9 29	4
5	12 45 15	S 4 52	—11 38	14 42 50	S 15 48	—16 15	16 48 20	S 22 26	— 9 4	5
6	12 48 54	S 5 15	—11 56	14 46 49	S 16 6	—16 13	16 52 42	S 22 33	— 8 38	6
7	12 52 33	S 5 38	—12 13	14 50 49	S 16 24	—16 10	16 57 4	S 22 40	— 8 13	7
8	12 56 13	S 6 1	—12 29	14 54 50	S 16 41	—16 6	17 1 27	S 22 46	— 7 46	8
9	12 59 53	S 6 24	—12 46	14 58 51	S 16 59	—16 1	17 5 51	S 22 52	— 7 19	9
10	13 3 34	S 6 47	—13 2	15 2 54	S 17 16	—15 55	17 10 15	S 22 58	— 6 52	10
11	13 7 15	S 7 9	—13 17	15 6 57	S 17 32	—15 48	17 14 39	S 23 3	— 6 24	11
12	13 10 57	S 7 32	—13 32	15 11 1	S 17 48	—15 40	17 19 4	S 23 7	— 5 56	12
13	13 14 39	S 7 54	—13 46	15 15 6	S 18 4	—15 32	17 23 29	S 23 12	— 5 28	13
14	13 18 22	S 8 17	—14 0	15 19 12	S 18 20	—15 22	17 27 54	S 23 15	— 4 59	14
15	13 22 5	S 8 39	—14 13	15 23 19	S 18 36	—15 12	17 32 20	S 23 18	— 4 30	15
16	13 25 49	S 9 1	—14 26	15 27 27	S 18 51	—15 1	17 36 46	S 23 21	— 4 1	16
17	13 29 34	S 9 23	—14 38	15 31 35	S 19 5	—14 49	17 41 12	S 23 23	— 3 31	17
18	13 33 19	S 9 45	—14 49	15 35 45	S 19 20	—14 36	17 45 38	S 23 25	— 3 1	18
19	13 37 4	S 10 7	—15 0	15 39 55	S 19 34	—14 23	17 50 5	S 23 26	— 2 32	19
20	13 40 51	S 10 28	—15 10	15 44 6	S 19 47	—14 8	17 54 32	S 23 27	— 2 2	20
21	13 44 38	S 10 50	—15 19	15 48 18	S 20 1	—13 53	17 58 58	S 23 27	— 1 31	21
22	13 48 26	S 11 11	—15 28	15 52 30	S 20 14	—13 37	18 3 25	S 23 27	— 1 1	22
23	13 52 14	S 11 32	—15 36	15 56 44	S 20 26	—13 20	18 7 52	S 23 27	— 0 31	23
24	13 56 3	S 11 53	—15 44	16 0 58	S 20 38	—13 3	18 12 18	S 23 26	— 0 1	24
25	13 59 53	S 12 14	—15 51	16 5 13	S 20 50	—12 44	18 16 45	S 23 24	+ 0 29	25
26	14 3 43	S 12 34	—15 57	16 9 29	S 21 2	—12 25	18 21 11	S 23 22	+ 0 58	26
27	14 7 35	S 12 55	—16 2	16 13 45	S 21 13	—12 5	18 25 38	S 23 20	+ 1 28	27
28	14 11 27	S 13 15	—16 7	16 18 2	S 21 23	—11 45	18 30 4	S 23 17	+ 1 58	28
29	14 15 19	S 13 35	—16 10	16 22 20	S 21 34	—11 24	18 34 30	S 23 13	+ 2 27	29
30	14 19 13	S 13 55	—16 14	16 26 38	S 21 43	—11 2	18 38 55	S 23 9	+ 2 56	30
31	14 23 7	S 14 14	—16 16				18 43 21	S 23 5	+ 3 24	31

Corrections in Minutes and Seconds of Right Ascension, Declination and Equation of Time for Hours up to 18.

Hours.	Variations in Seconds for 24 Hours. 5	6	7	8	9	10	15	20	25	30	35	40	45	50	55	Degrees.
	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	
1	0	0	0	0	0	0	1	1	1	1	1	2	2	2	2	15
2	0	1	1	1	1	1	1	2	2	3	3	3	4	4	5	30
3	1	1	1	1	1	1	2	3	3	4	4	5	6	6	7	45
4	1	1	1	1	2	2	3	3	4	5	6	7	7	8	9	60
5	1	1	1	2	2	2	3	4	5	6	7	8	9	10	11	75
6	1	2	2	2	2	3	4	5	6	8	9	10	11	12	14	90
7	1	2	2	2	3	3	4	6	7	9	10	12	13	15	16	105
8	2	2	2	3	3	3	5	7	8	10	12	13	15	17	18	120
9	2	2	3	3	3	4	6	8	9	11	13	15	17	19	21	135
10	2	3	3	3	4	4	6	8	10	13	15	17	19	21	23	150
11	2	3	3	4	4	5	7	9	11	14	16	18	21	23	25	165
12	3	3	4	4	5	5	8	10	13	15	17	20	22	25	27	180
13	3	3	4	4	5	5	8	11	14	16	19	23	24	27	30	195
14	3	4	4	5	5	6	9	12	15	18	20	23	26	29	32	210
15	3	4	4	5	6	6	9	13	16	19	22	25	28	31	34	225
16	3	4	5	5	6	7	10	13	17	20	23	27	30	33	37	240
17	4	4	5	6	6	7	11	14	18	21	25	28	32	35	39	255
18	4	5	5	6	7	8	11	15	19	23	26	30	34	37	41	270

Hours.	Variation in Minutes for 24 Hours. 1	2	3	4	5	6	7	8	9	10	20	Degrees.
	'	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	
1	2	5	7	10	12	15	17	20	22	25	50	15
2	5	10	15	20	25	30	35	40	45	50	1 40	30
3	7	15	22	30	37	45	52	1	1 7	1 15	2 30	45
4	10	20	30	40	50	1	1 10	1 20	1 30	1 40	3 20	60
5	12	25	37	50	1 2	1 15	1 27	1 40	1 52	2 5	4 10	75
6	15	30	45	1	1 15	1 30	1 45	2	2 15	2 30	5	90
7	17	35	52	1 10	1 27	1 45	2 2	2 20	2 37	2 55	5 50	105
8	20	40	1	1 20	1 40	2	2 20	2 40	3	3 20	6 40	120
9	22	45	1 7	1 30	1 52	2 15	2 37	3	3 22	3 45	7 30	135
10	25	50	1 15	1 40	2 5	2 30	2 55	3 20	3 45	4 10	8 20	150
11	27	55	1 22	1 50	2 17	2 45	3 12	3 40	4 7	4 35	9 10	165
12	30	1	1 30	2	2 30	3	3 30	4	4 30	5	10	180
13	32	1 5	1 37	2 10	2 42	3 15	3 47	4 20	4 52	5 25	10 50	195
14	35	1 10	1 45	2 20	2 55	3 30	4 5	4 40	5 15	5 50	11 40	210
15	37	1 15	1 52	2 30	3 7	3 45	4 22	5	5 37	6 15	12 30	225
16	40	1 20	2	2 40	3 20	4	4 40	5 20	6	6 40	13 20	240
17	42	1 25	2 7	2 50	3 32	4 15	4 57	5 40	6 22	7 5	14 10	255
18	45	1 30	2 15	3	3 45	4 30	5 15	6	6 45	7 30	15	270

Explanation of the Sidereal and Solar Time Table.

The Sidereal Time = Mean Solar + Correction.
Mean Solar Time = Sidereal — Correction.

To Find the True Sidereal Time.

The sun's Right Ascension + or — the Equation of time is the Sidereal time a Greenwich, mean noon.

The sign + or — must be used as noted in the Astronomical or Nautical Almana for the given day. For any other meridian or longitude from Greenwich, correct the sun's R. A. and the equation of time, and perform the same operation.

Alt. D.M.	Refr. M. S.	Alt. D.M.	Refr. M. S.	Alt. D. M.	Refr. M. S.	Alt. D. M.	Refr. M. S.	Alt. D.	Refr. M. S.	Alt. D.	Refr. M. S.
0. 0	33. 0	2.30	16.23	6.30	7.52	12.20	4.16	30	1.38	60	0.33
0. 5	32.11	2.35	16. 4	6.40	7.41	12.40	4. 9	31	1.35	61	0.32
0.10	31.22	2.40	15.45	6.50	7.31	13. 0	4. 3	32	1.31	62	0.30
0.15	30.36	2.45	15.27	7. 0	7.21	13.20	3.57	33	1.28	63	0.29
0.20	29.50	2.50	15. 9	7.10	7.12	13.40	3.51	34	1.24	64	0.28
0.25	29. 6	2.55	14.52	7.20	7. 3	14. 0	3.46	35	1.21	65	0.27
0.30	28.23	3. 0	14.35	7.30	6.54	14.20	3.40	36	1.18	66	0.25
0.35	27.41	3. 5	14.19	7.40	6.46	14.40	3.35	37	1.16	67	0.24
0.40	27. 0	3.10	14.03	7.50	6.38	15. 0	3.30	38	1.13	68	0.23
0.45	26.20	3.15	13.48	8. 0	6.30	15.30	3.23	39	1.10	69	0.22
0.50	25.42	3.20	13.33	8.10	6.22	16. 0	3.17	40	1. 8	70	0.21
0.55	25. 5	3.25	13.19	8.20	6.15	16.30	3.11	41	1. 5	71	0.20
1. 0	24.29	3.30	13.05	8.30	6. 8	17. 0	3. 5	42	1. 3	72	0.19
1. 5	23.54	3.40	12.39	8.40	6. 1	17.30	2.59	43	1. 1	73	0.17
1.10	23.20	3.50	12.14	8.50	5.55	18. 0	2.54	44	0.59	74	0.16
1.15	22.47	4. 0	11.50	9. 0	5.49	18.30	2.49	45	0.57	75	0.15
1.20	22.15	4.10	11.28	9.10	5.43	19. 0	2.44	46	0.55	76	0.14
1.25	21.44	4.20	11.07	9.20	5.37	19.30	2.40	47	0.53	77	0.13
1.30	21.15	4.30	10.47	9.30	5.31	20. 0	2.36	48	0.51	78	0.12
1.35	20.46	4.40	10.28	9.40	5.26	20.30	2.32	49	0.50	79	0.11
1.40	20.18	4.50	10.10	9.50	5.20	21. 0	2.28	50	0.48	80	0.10
1.45	19.51	5. 0	9.53	10. 0	5.15	21.30	2.24	51	0.46	81	0. 9
1.50	19.25	5.10	9.37	10.15	5. 8	22. 0	2.20	52	0.45	82	0. 8
1.55	18.59	5.20	9.21	10.30	5. 0	23. 0	2.14	53	0.43	83	0. 7
2. 0	18.35	5.30	9. 7	10.45	4.54	24. 0	2. 7	54	0.41	84	0. 6
2. 5	18.11	5.40	8.53	11. 0	4.47	25. 0	2. 2	55	0.40	85	0. 5
2.10	17.48	5.50	8.39	11.15	4.41	26. 0	1.56	56	0.38	86	0. 4
2.15	17.26	6. 0	8.27	11.30	4.35	27. 0	1.51	57	0.37	87	0. 3
2.20	17. 4	6.10	8.15	11.45	4.29	28. 0	1.47	58	0.36	88	0. 2
2.25	16.44	6.20	8. 3	12. 0	4.23	29. 0	1.43	59	0.34	89	0. 1

Conversion of Sidereal & Mean Solar Times.

Hour. H.	Corr. M. S.	Min. M.	Corr. S.	Min. M.	Corr. S.	Sec. S.	Corr. S.	Sec. S.	Corr. S.
1	0 9.8	1	0.2	31	5.1	1	0.0	31	0.1
2	0 19.7	2	0.3	32	5.2	2	0.0	32	0.1
3	0 29.5	3	0.5	33	5.4	3	0.0	33	0.1
4	0 39.3	4	0.7	34	5.6	4	0.0	34	0.1
5	0 49.1	5	0.8	35	5.7	5	0.0	35	0.1
6	0 59.0	6	1.0	36	5.9	6	0.0	36	0.1
7	1 8.9	7	1.1	37	6.1	7	0.0	37	0.1
8	1 18.7	8	1.3	38	6.2	8	0.0	38	0.1
9	1 28.6	9	1.5	39	6.4	9	0.0	39	0.1
10	1 38.4	10	1.6	40	6.6	10	0.0	40	0.1
11	1 48.2	11	1.8	41	6.7	11	0.0	41	0.1
12	1 58.1	12	2.0	42	6.9	12	0 0	42	0.1
13	2 8.0	13	2.1	43	7.0	13	0.0	43	0.1
14	2 17.8	14	2.3	44	7.2	14	0.0	44	0.1
15	2 27 6	15	2.5	45	7.4	15	0.0	45	0.1
16	2 37.5	16	2.6	46	7.5	16	0.0	46	0.1
17	2 47.3	17	2.8	47	7.7	17	0.0	47	0.1
18	2 57.1	18	2.9	48	7.9	18	0.0	48	0.1
19	3 7.0	19	3.1	49	8.0	19	0.1	49	0.1
20	3 16.9	20	3.3	50	8.2	20	0.1	50	0.1
21	3 26.7	21	3.4	51	8.4	21	0.1	51	0.1
22	3 36.5	22	3.6	52	8.5	22	0.1	52	0.1
23	3 46.4	23	3.8	53	8.7	23	0.1	53	0.1
24	3 56.3	24	3.9	54	8.8	24	0.1	54	0.1
		25	4.1	55	9.0	25	0.1	55	0.2
		26	4.3	56	9.2	26	0.1	56	0.2
		27	4.4	57	9.3	27	0.1	57	0.2
		28	4.6	58	9.5	28	0.1	58	0.2
		29	4.8	59	9.7	29	0.1	59	0.2
		30	4.9	60	9.8	30	0 1	60	0.2

The Sun's Parallax in Altitude.

Altitude. D.	Parallax. S
0	9
10	9
20	8
30	8
40	7
50	6
55	5
60	4
65	4
70	3
75	2
80	2
85	1
90	0

Explanation.

The sun's parallax must be added to the observed altitude.

LATITUDE AND APPARENT TIME

By Altitude of the Heavenly Bodies.

Notation of Letters.

A = meridian altitude above horizon.

D = declination, to be found in the Astronomical Almanac.

l = latitude of the place of observation.

L = angle of apparent time from noon.

a = any altitude of the heavenly body, before or after noon.

When the latitude and declination are of **Same Name.**	*When the latitude and declination are of* **Different Names.**
Latitude, $l = 90 + D - A.$	Latitude, $l = 90 - A - D.$
Altitude, $A = 90 + D - l.$	Altitude, $A = 90 - l - D.$
Declination, $D = A + l - 90.$	Declination, $D = 90 - A - l.$
Apparent Time,	**Apparent Time,**
$\text{Cos.}L = \sin.a \sec.l \sec.D - \tan.l \tan.D.$	$\text{Cos.}L = \sin.a \sec.l \sec.D + \tan.l \tan.D.$

At sea the altitude is observed from the visible horizon of the ocean, from which must be subtracted the dip of horizon. (See table, page 131.)

On land the horizon must be determined by a spirit-level, or more correctly by an artificial horizon of quicksilver, oil, syrup or some similar liquid.

The refraction of light through the atmosphere (see table, page 503) must also be subtracted from the observed altitude.

When the sun or moon is observed, the parallax must be added to the observed altitude.

Latitude.

Example 1. On the 7th of April, 1872, the sun's lower limb was observed to be 51° 42′ 50″ above the horizon at noon, in longitude about 45° west of Greenwich; the observation was made from the deck of a vessel 20 feet above the sea. Required, the sun's true altitude and latitude of observation?

The declination and latitude are both north or of same name.

Dip of horizon for 20 feet,	4′ 24″	Declination Naut. Almanac,	7° 3′ 19″
Refraction, 51°, ...	46	Correct. 45° W. long., add	2 48
☉'s semi-diameter, . . .	16 00	True declination, . . D =	7 6 7
	21 10	Add	90
Sun's parallax, subtract .	6		97 6 7
Correction to be subtracted,	21 4	True altitude, subtract A =	51 21 46
☉'s observed altitude, . . .	51 42 50	The required latitude, l =	45° 44′ 21″
☉'s true altitude, . . .	51° 21′ 46″		

Artificial Horizon.

When the observation is made by a sextant through an artificial horizon, the observed angle must be divided by 2 for the altitude, and there will be no correction for dip of horizon, nor for semi-diameter, as the sun's discs are brought to cover one another. When a regular quicksilver horizon is not at hand, some viscous liquid, like oil or molasses, in an open vessel, may be used in calm weather.

In perfectly calm weather the altitude may be taken in a pool of water, which has been done by the author in South America.

Apparent Time.

Example. On the 8th of February, 1872, the sun's true altitude was found to be $a = 30° 46' 29''$ in the afternoon, the latitude $l = 38° 18' 38''$ N., and declination corrected $D = 15° 5' 10''$ S. Required, the apparent and mean time of observation?

The latitude and declinations are of different names.

$$\text{Cos } L = \sin.30° 46' 29'' \times \sec.\ 38° 18' 38'' \times \sec.15° \ 5' 10'' = 0.675349$$
$$+ \tan.\ 38° 18' 38'' \times \tan.15° \ 5' 10'' = 0.21293$$

Apparent time of obs., $L = 1h.\ 49m.\ 23s. = \cos.\ 27° 20' 34'' = 0.888279$
Equation of time, add 14 26

Mean time, $2h.\ 3m.\ 58s.$

The calculation with logarithms is set up as follows:

Log. sin.30° 46′ 29″	= 9.70898		
" sec.38 18 38	= 0.10531	Tan. =	9.89765
" " 15 5 10	= 0.01523	Tan. =	9.43067
Logarithms, . . .	9.82952		9.32832
Natural numbers, .	0.675349		0.21293
Add for different names,	0.21293		

App. time, $L = 1h.\ 49m.\ 22s. = 27° 20' 34'' = 0.888279$ = cosine for hour angle.

The hour angle in time can be read off directly for the cosine in the trigonometrical tables.

When the observation is made in the forenoon, subtract the cosine hour angle from 12 hours; or it can be read off directly from the tables by calling cosine for sine L.

To Find the Longitude.

Small differences in longitude can be obtained from actual measurement, as explained in Plane Sailing and Traverse Surveying.

For great distances, it is necessary to know the simultaneous times at the two meridians between which the longitude is required.

At sea, the time at a distant meridian is kept by a chronometer, generally regulated for Greenwich mean time, and the difference in time between the two meridians is the difference in longitude.

Eclipses of Jupiter's Satellites.

The most simple astronomical observations for finding the Washington or Greenwich mean time are of the eclipses of Jupiter's satellites, but, unfortunately, the tables for those eclipses, which are published in the English and American Nautical Almanacs, are not yet reliable, as has been found by the author in using these tables in the interior of South America. The observations of several eclipses in one locality did not give the same difference in longitude. Prof. S. Newcomb says, "The times of the eclipses of Jupiter's satellites differ greatly in accuracy. Those of the first satellite are generally correct within the necessary errors of observation; the errors are larger with each succeeding one as you go out, and in the case of the outer one they are frequently several minutes in error. The labor of constructing tables is so great that no one person can undertake it. It would require many years' labor."

This is a subject well worthy of attention at the National Observatory at Washington. The eclipses of Jupiter's satellites can be observed by an ordinary good spy-glass, even at sea, in calm weather, which would be of great importance over the whole world if accurate tables could be procured. Chronometers could then be corrected with great precision to Greenwich or Washington mean time in any part of the world, and difference of longitude could be determined without the aid of complicated, bulky, and expensive instruments, which few can afford to buy or know how to use.

When the Pacific coast of South America was first surveyed, the longitude was determined by the average time of a dozen chronometers, which gave very incorrect results, as was afterward found, whilst the eclipses of Jupiter's satellites would have given the longitude correct even if the tables erred, because the same eclipses could be observed in regular observatories at the same time, and proper corrections made from compared data.

The Nautical Almanac is prepared some three years in advance, and it therefore requires very accurate data to locate the time of the eclipses with precision.

Elements of the Planetary System.

The principal Planets.	Signs.	Mean distance fr. Sun.	Sidereal period, Days.	Eccent. part, sm. axis	Diameter in miles.	Vel orbt Miles per sec.	Rotation in hours. H. M.	Density.	Mass.	Volume.
Sun.. .	☉	. .	. .	. .	882000	. .	607 48	0.25	355000	1378000
Mercury,	☿	0.3871	87.969	0.2055	3140	30.4	24 05	1.12	0.06966	0.06218
Venus,	♀	0.7233	224.70	0.0068	7800	22.3	23 21	0.92	0.877	0.9531
Earth, . .	⊕	1.	365.25	0.0168	7912	18.9	24 0	1.	1.	1.
Mars, .	♂	1.5236	686.98	0.0933	4100	15.33	24 37	0.95	0.1313	0.1384
Jupiter,	♃	5.2028	4332.6	0.0482	87000	8.31	9 56	0.24	317.5	1322.5
Saturn,	♄	9.5388	10759	0.0561	79160	6 14	10 29	0.14	139.5	996.2
Uranus,	♅	19.182	30687	0.0467	34500	4.33	9 30	0.24	198.	82.47
Neptune,	♆	30.037	60127	0.0087	41500	3.45	. .	0.14	20.	143.5

Position of some Stars of the 1st and 2d Magnitudes, Jan. 1, 1885.

NAME OF STAR.	Mg. td.	Right As'sion.	Ann. Var.	Declination.	Ann. Var.
N. Hemisphere.		h. m. s.	s.	North. ° ′ ″	″
α Andromedæ,	2	0 2 27	+ 3.09	28 27 20	+ 19.9
α Polaris,	2	1 16 37	+ 20.7	88 41 44	+ 19.0
α Arietis,	2	2 0 41	+ 3.36	22 55 05	+ 17.2
α Persi,	2	3 16 07	+ 4.25	49 27 03	+ 13.1
α Aldebaran,	1	4 29 19	+ 3.43	16 16 37	+ 7.61
α Capella,	1	5 8 12	+ 4.42	45 52 46	+ 4.13
β Tauri,	2	5 19 01	+ 3.79	28 30 32	+ 3.42
α^2 Castor,	2.1	7 27 16	+ 3.84	32 8 33	— 7.48
α Procyon,	1	7 33 17	+ 3.14	5 31 8	— 8.97
β Pollux,	1.2	7 38 17	+ 3.68	28 18 10	— 8.35
α Regulus,	1.2	10 2 15	+ 3.20	12 31 42	— 17.4
γ^1 Leonis,	2	10 13 38	+ 3.31	20 25 22	— 18.0
α Great Bear,	2	10 56 37	+ 3.76	62 22 18	— 19.4
γ Great Bear,	2.3	11 47 47	+ 3.19	54 20 02	— 20.0
η Great Bear,	2	13 43 05	+ 2.37	49 53 15	— 18.1
α Arcturus,	1	14 10 25	+ 2.73	19 46 53	— 18.8
α Coronæ,	2	15 29 49	+ 2.54	27 6 08	— 12.3
η Herculis,	3.2	16 38 57	+ 2.26	39 8 29	— 6.69
α^1 Herculis,	Var.	17 9 24	+ 2.73	14 31 20	— 4.39
α Vega,	1	18 33 03	+ 2.03	38 40 38	+ 3.13
α Altair,	1.2	19 45 10	+ 2.93	8 33 55	+ 9.22
S. Hemisphere.				South.	
β Ceti,	2	0 37 49	+ 3.01	18 37 05	+ 19.8
α Achernar,	1	1 33 25	+ 2.23	57 49 17	+ 18.4
β Rigel,	1	5 9 06	+ 2.88	8 20 08	+ 4.45
δ Orionis,	2	5 26 08	+ 3.06	0 23 07	+ 2.96
α Canopus,	1	6 21 24	+ 1.33	52 37 59	— 1.85
α Sirius,	1	6 40 05	+ 2.64	16 33 33	— 4.69
ε Canis Major,	2.1	6 54 06	+ 2.36	28 48 59	— 4.67
ι Argus,	2	9 14 05	+ 1.60	58 47 33	— 14.9
α Hydræ,	2	9 21 56	+ 2.94	8 9 38	— 15.4
α^1 Crucis,	1	12 20 12	+ 3.27	62 27 42	— 19.9
α Spica,	1	13 19 08	+ 3.15	10 33 39	— 18.9
β Centauri,	1	13 55 43	+ 4.16	59 49 03	— 17.6
α^2 Centauri,	1	14 31 49	+ 4.04	60 21 46	— 15.0
β Libræ,	2	15 10 49	+ 3,22	8 57 28	— 13.5
β^1 Scorpii,	2	15 58 45	+ 3.48	19 29 23	— 10.2
α Antares,	1.2	16 22 21	+ 3.67	26 10 33	— 8.37
α Australis,	2	16 36 30	+ 6.28	68 48 52	— 7.32
α Pavonis,	2	20 16 33	+ 4.79	57 6 08	+ 11.1
β Aquarii,	3	21 25 30	+ 3,16	6 4 36	+ 15.6
α Gruis,	2	22 0 59	+ 3.81	47 31 02	+ 17.2
α Fomalhaut,	1.2	22 51 18	+ 3.33	30 13 53	+ 19.0

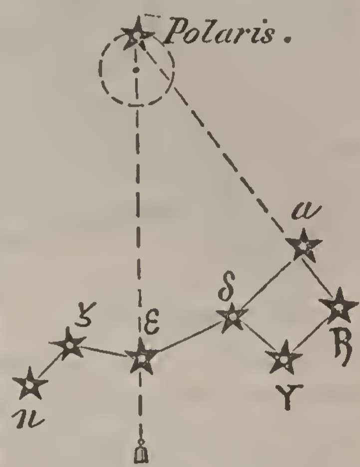
Tide Table.

Tide Table.	h. m.	Rise feet.
Albany, N. Y.,	— 0 40	
Altona. Ger..	+ 3 12	
Amboy, N. Y.,	+ 5 27	
Antwerp, Bel.,	+ 2 18	
Baltimore,	— 1 52	12
Belfast, Ire.,	+ 8 36	
Bergen, Nor.,	— 0 37	
Bordeaux, Fr.,	+ 4 34	
Boston, Mass.,	+ 9 20	12
Boulogne, Fr.,	+ 9 18	19
Buenos Ayres,	+ 6 6	
Bremen,	+ 8 43	
Cadiz, Spain,	— 0 22	
Calais, Fr.,	+ 9 42	19
Calc'tta, Beng.	+ 0 23	
Charleston,	+ 5 44	5
Cherbourg, Fr.	+ 8 23	20
Cape Henry, A.	+ 7 3	4
Cape G. Hope,	+ 0 29	
Cape Horn,	+ 2 9	9
C. Henlopen,	+ 6 41	5
Dublin Bar,	+ 9 5	12
Gibraltar, Sp.,	+ 0 13	
Glasgow, Scot.	— 0 42	
Hamburg,	+ 3 22	
Halifax, N. A.,	+ 5 42	8
Havana, Cuba,	+ 7 41	3
Havre, Fr.,	+ 7 44	12
Hull, Eng.,	+ 4 22	18
Key West, U.S.	+ 7 22	2
Lisbon B. Port.	+ 0 13	
Liverpool,	+ 9 16	25
New York,	+ 6 6	5
New Haven, A.	+ 8 38	17
Newcastle, E.	+ 2 16	
Norfolk, U. S.,	+ 5 14	7
Ostend, Belg.,	+ 10 18	16
Panama,	+ 1 24	24
Philadelphia,	— 0 49	6
Portsmouth, Eng. & U. S.	+ 9 28	10
Providence,	+ 5 25	
Quebec, Can.,	+ 4 31	17
Queenstown.	+ 2 54	
Rio Janeiro,	+ 0 53	6
Rotterdam,	+ 1 38	
Sandy Hook,	+ 4 58	6
Valparaiso,	+ 7 2	5
San Francisco,	+ 8 27	6
Washington,	+ 1 58	

To Find the Meridian or True North by the North Star, *Polaris.*

Polaris is not in the true north, but revolves in a circle of radius 1° 22′ 24″ co-declination on the 1st of January, 1872, which diminishes 19″ every year; that on the 1st of January, 1873, its co-declination will be 1° 22′ 5″.

The position of Polaris is generally traced by the direction of the stars α and β, in the Great Bear, which point nearly to the North Star. See figure.



Polaris passes the meridian, or is true north when the star ϵ in the Great Bear is perpendicular either over or under Polaris. In low latitudes the Polaris is near the horizon, and the star ϵ cannot be seen when under, but must be observed at its upper transit. When the star ϵ is horizontal with Polaris, substract the radius of the circle, and the remainder will be the true north, from which the variation of the compass is ascertained. There is no star near the South Pole from which a similar observation can be made.

Table Showing how Much Earlier any Fixed Star Passes

the Meridian, rises or sets, in number of days or nights up to 100.

Nights.	H. M. S.	Nights.	H. M. S.	Nights.	H. M. S.	Nights.	H. M. S.	Nights.	H. M. S.
1	0 3 55.9	11	0 43 15.0	21	1 22 34.1	31	2 01 53.2	45	2 56 55.0
2	0 7 51.8	12	0 47 10.9	22	1 26 30.0	32	2 05 49.1	50	3 16 35.9
3	0 11 47.7	13	0 51 06.8	23	1 30 25.9	33	2 09 45.0	55	3 36 15.0
4	0 15 43.6	14	0 55 02.7	24	1 34 21.8	34	2 13 40.9	60	3 55 54.5
5	0 19 39.5	15	0 58 58.6	25	1 38 17.7	35	2 17 36.8	65	4 15 34.1
6	0 23 35.5	16	1 02 54.5	26	1 42 13.6	36	2 21 32.7	70	4 35 13.6
7	0 27 31.4	17	1 06 50.5	27	1 46 09.6	37	2 25 28.6	75	4 54 53.2
8	0 31 27.3	18	1 10 46.4	28	1 50 05.5	38	2 29 24.5	80	5 14 32.7
9	0 35 23.2	19	1 14 42.3	29	1 54 01.4	39	2 33 20.4	90	5 53 51.8
10	0 39 19.1	20	1 18 38.2	30	1 57 57.3	40	2 37 16.4	100	6 33 10.9

The preceding table is for regulating a watch, clock or chronometer. The fixed stars set 3 m. 55.909 s. earlier every day, and by observing the time of setting over a sharp, distant object, as a hill, mountain or a house, the time-keeper can be regulated with great precision.

Example. A fixed star is observed to set at 9 h. 35 m. 51 s.

Twenty-five days after, the same star set at	7 h.	66 m.	49 s.
Add the correction for 25 days, . . .	1	38	17.7
Sum, ...	9	35	06.7

The time-keeper has lost 51 — 06.7 = 44.3 seconds in 25 days, or 1.772 seconds per day.

To Find the Time When Any Star or Planet Passes the *Meridian.*

Subtract the sun's right ascension from that of the star, increased by 24 if necessary, and the remainder will be the apparent time when the star passes the meridian. The sun's R. A. must be corrected for longitude from Greenwich, and for time of observation. This is the best mode of finding the meridian and variation of the compass, but the apparent time must be correctly known.

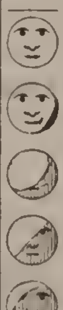
To Find Which Star Passes the Meridian Near a Desired Time.

Add the sun's R. A. to the desired hour, and the sum will be the nearest R. A. of the star passing the meridian at that time. Reject 24 hours if necessary. Find in the table of stars the one which comes nearest to that R. A.

TO APPROXIMATE THE TIME OF HIGH WATER.

On account of the Moon's orbit being an ellipse, in which the Earth is one of the foci, and that the major axis of that ellipse does not point to the Sun, but to a fixed constellation of stars, the actual time of high water and also that of passing the meridian are not equal for equal age of the Moon, but may differ as much as 15 minutes from the average in the accompanying table. Also, the force and direction of winds cause a still greater variation.

Find first the Moon's age for the given day, as described on page 496. Opposite the age in the table is the time of the day when the Moon is south, or passes the meridian, and in the following columns are the times of high water at London Bridge in the morning and afternoon. Add or subtract the time in the tide table, page 506, for any other location, and the sum or difference is the time of high water at that place.



Moon.				High water at London Bridge.		
Quart'r	Face	Age d.	South. h. m.	Age.	Morning. h. m.	Afternoon h. m.
New.		0	12 0 p. m.	0	1 59	2 7
1st quarter.		1	12 49 "	1	2 21	2 36
		2	1 38 "	2	2 50	3 3
		3	2 26 "	3	3 18	3 33
		4	3 26 "	4	3 47	4 2
		5	4 4 "	5	4 16	4 31
		6	4 55 "	6	4 50	5 5
Half.		7	6 42 "	7	5 24	5 45
		8	6 30 "	8	6 7	6 35
2d quarter.		9	7 19 "	9	7 7	7 46
		10	8 8 "	10	8 33	9 25
		11	8 57 "	11	10 14	10 54
		12	9 46 "	12	11 28	11 54
		13	10 34 "	13	Noon.	0 17
		14	11 23 "	14	0 40	1 0
Full.		15	12 12 a. m	15	1 20	1 40
3d quarter.		16	1 1 "	16	2 1	2 22
		17	1 50 "	17	2 42	3 5
		18	2 38 "	18	3 26	3 48
		19	3 27 "	19	4 10	4 34
		20	4 16 "	20	4 55	5 19
		21	5 5 "	21	5 44	6 12
Half.		22	5 54 "	22	6 44	7 21
		23	6 42 "	23	7 59	8 43
4th quarter.		24	7 31 "	24	9 31	10 15
		25	8 20 "	25	10 52	11 23
		26	9 9 "	26	11 49	M'night.
		27	9 58 "	27	0 11	0 29
		28	10 46 "	28	0 48	1 5
		29	11 35 "	29	1 23	1 36
		29½	12 "	29½	1 51	2 7

High Water.

Examples.

Required, the time of H. W. in Philadelphia on the 8th of Feb., 1874?

Epact, year, . 12 pg. 497.
Epact, month, 2 pg. 496.
Date, February, 8

Moon's age, . 22 days.

H.W.Lond., 6h.44m.
Phila. subt., 0 49 pg. 506.

H.W.Phila., 5h.55m.
in the morn., Feb. 8, 1874.

Required, the time of H. W. in Panama on the 7th of October, 1873?

Epact, year, 1 pg. 497.
Epact, month, 8 pg. 496.
Date in October, 7

Moon's age, 16 days.

H.W. Lond., 2h. 1m. a.m.
Panama add 1 24, pg. 506

H.W. 3h. 25m. a.m.
in Panama.

Elements of Jupiter's Satellites.

Order of satellite.	Radius of orbit, that of Jupiter=1	Time in days of one revolution.	Revolutions ar. Jupiter per year.	Mass, that of Jupiter=1	Diameter of satellite miles.
1st.	6.0485	1.7691	206.457	0.000017	
2d.	9.6235	5.5512	65.7952	0.000028	
3d.	15.3502	7.1546	51.0499	0.000088	
4th.	26.9983	16.6888	21.8855	0.000043	

Number of Moons or Satellites to each Planet.

Earth, 1. Jupiter, 4. Saturn, with rings, 8. Uranus, 8 moons.

SOUNDINGS.

To Reduce Soundings to Low Water.

Letters denote—

T = time in hours between high and low water.
t = time in hours from low water to the time when the soundings are taken.
R = vertical rise of tide in feet from high to low water.
r = reduction of the sounding taken at the time, t, in feet.

$$v = \frac{180t}{T}, \qquad \text{and } r = \tfrac{1}{2}R\,(1 \mp cos.v),$$

$$-\,cos.v \text{ when } v < 90$$
$$+\,cos.v \text{ when } v > 90$$

Example. High water at 10 h. 15 m. p. m.
Low water at 3 h. 45 m. "

Time T = 6 h. 30 m. "
The sounding taken at 5 h. 30 m. " was 16 feet 6 inches.

Time t = 1 h. 45 m.
Vertical rise R = 9.75 feet.

Required, the reduction r = ? and true sounding at low water?

$$v = \frac{180 \times 1.75}{6.5} = 48^\circ\ 27', \qquad cos.v = 0.6631.$$

Reduction $r = \frac{1}{2} \times 9.75\,(1 - 0.6631) = 1.6419$ feet.
Sounding taken at 5 h. 30 m. was 16.5 feet.
Reduction subtract r = 1.6419

True sounding at low water, 14.8581 feet.

Reduction for Soundings to Low Water.

This table will answer for any unit of measure of rise.

Rise R.	Time of sounding in hrs. and min. before or after that of high water. 0.30	1	1.30	2	2.30	3	3.30	4	4.30	5	5.30	6	Rise R.
1	0.98	0.94	0.87	0.78	0.67	0.55	0.43	0.31	0.20	0.12	0.05	0.01	1
2	1.97	1.88	1.74	1.56	1.34	1.10	0.86	0.62	0.40	0.24	0.10	0.02	2
3	2.95	2.82	2.61	2.34	2.01	1.65	1.29	0.93	0.60	0.36	0.15	0.03	3
4	3.93	3.76	3.48	3.12	2.68	2.20	1.72	1.25	0.82	0.46	0.20	0.04	4
5	4.92	4.70	4.35	3.90	3.35	2.74	2.15	1.56	1.03	0.58	0.25	0·05	5
6	5.91	5.65	5.22	4.68	4.03	3.30	2.58	1.87	1.23	0.69	0.30	0.06	6
7	6.90	6.59	6.10	5.46	4.70	3.84	3.01	2.18	1.44	0.81	0.35	0.07	7
8	7.88	7.52	6.97	6.24	5.36	4.40	3.44	2.50	1.65	0 93	0.40	0.08	8
9	8.86	8.47	7.84	7.02	6.03	4.94	3.87	2.80	1.85	1.04	0.45	0.09	9
10	9.85	9.41	8.71	7.79	6.71	5.52	4.30	3.12	2.06	1.16	0.50	0.10	10
11	10.9	10.3	9.59	8.59	7.39	6.05	4.74	3.43	2.27	1.28	0.55	0.11	11
12	11.9	11.3	10.5	9.37	8.06	6.60	5.16	3.74	2.47	1.40	0.60	0.12	12
13	12.8	12.2	11.3	10.1	8.72	7.14	5.60	4.05	2.68	1.51	0.65	0.13	13
14	13.8	13.2	12.2	11.0	9.40	7.70	6.02	3.36	2.89	1.62	0.70	0.14	14
15	14.8	14.1	13.0	11 7	10.0	7.25	6.45	3.67	3.09	1.74	0.75	0.15	15
16	15.8	15.0	14.0	12.5	10.7	8.78	6.88	5.00	3.30	1.85	0.80	0.16	16
17	16.8	16.0	14.8	13.3	11.4	9.35	7.31	5.25	3.50	1.97	0.85	0.17	17
18	17.8	17.0	15.7	14.0	12.1	9.90	7.75	5.60	3.70	2 08	0.90	0.18	18
19	18.7	17.9	16.6	14.8	12.8	10.4	8.17	5.91	3.81	2.20	0.95	0.19	19
20	19.7	18.9	17.5	15.6	13.4	11.0	8.60	6.23	4.11	2.32	1.00	0.20	20
21	20.7	19.8	18.3	16.4	14.1	11.5	9.04	6.54	4.32	2.43	1.05	0.21	21
22	21 7	20.7	19.2	17.2	14.8	12.1	9.46	6.85	4.53	2.55	1.10	0.22	22
23	22.7	21.7	20.0	18.0	15.4	12.6	9.90	7.16	4.73	2.67	1.15	0.23	23
24	23.7	22.6	20.9	18.7	16.1	13.2	10.3	7.47	4.94	2.78	1.20	0.24	24
25	24.7	23.5	21.8	19.5	16.8	13.7	10 8	7.78	5.14	2.90	1.25	0.25	25
26	25.6	24.5	22.7	20.3	17.4	14.3	11.2	8.10	5.52	3.01	1.30	0.26	26
27	26.6	25.4	23.5	21.1	18.1	14.9	11.6	8.41	5.55	3.13	1.35	0.27	27
28	27.6	26.4	24.4	21.9	18.8	15.4	12.0	8.72	5.76	3.25	1.40	0.28	28
29	28 6	27.3	25.3	22.7	19.4	16.0	12.5	9.03	5.96	3.36	1.45	0 29	29
30	29.6	28.3	26.2	23.4	20.1	16.5	12.9	9.34	6.17	3.48	1.50	0.30	30
R.	5.45	5.15	4.45	4.15	3.45	3.15	2.45	2.15	1.45	1.15	0.45	0.15	R.
Rise	Time of sounding in hrs. and min. before or after that of low water.												Rise

To Find at what Time the Sun Sets and Rises.

Let v denote the time angle from 6 o'clock to when the sun sets or rises, then—

$$\text{Sin.}v = \tan.l \tan.D.$$

Example. What time does the sun set and rise on the 21st of June, in 60° latitude?

The declination on this day is about 23° 27′.

$$\text{Sin.}v = \tan.60^\circ \times \tan.23^\circ\, 27' = 0.75131 = \sin.3h.\ 14m.\ 48s.$$

The sun rises at 2*h*. 45*m*. 12*s*., and sets at 9*h*. 14*m*. 48*s*.

To Find the Length of Day and Night.

DAY.—Double the time of sunset, is the length of the day.
NIGHT.—Double the time of sunrise, is the length of the night.

Amplitude.

The angle or bearing from east or west to where any heavenly body sets or rises, is called the amplitude of that body, which, denoted by x, will be—

$$\text{Sin.}x = \sec.l \sin.D.$$

The amplitude is used for finding the variation of the compass.

Example. The sun's declination being 18° 25′ south, required, his amplitude in latitude 48° 45′ north?

$$\text{Sin.}x = \sec.48^\circ\, 45' \times \sin.18^\circ\, 25' = \sin.28^\circ\, 38' \text{ south},$$

the amplitude required.

Azimuth.

$l =$ latitude, $D =$ declination, and $a =$ altitude.

$z =$ angle of azimuth, or bearing of the heavenly body from meridian to the pole above horizon.

When the latitude and declination are of

Same Names—

$$m = \frac{l + a - D + 90}{2}$$

Different Names—

$$m = \frac{l + a + D + 90}{2}$$

$$n = \pm\, m \mp (90 - D).$$

Subtract the smallest, and the remainder is n.

$$\text{Cos.}\ \tfrac{1}{2}\, z = \sqrt{\cos.m \cos.n \sec.l \sec.a.}$$

ABCDEFGHIJKLMNO
PQRSTUVWXYZ.
abcdefghijklmnopqrstuvwxyz.

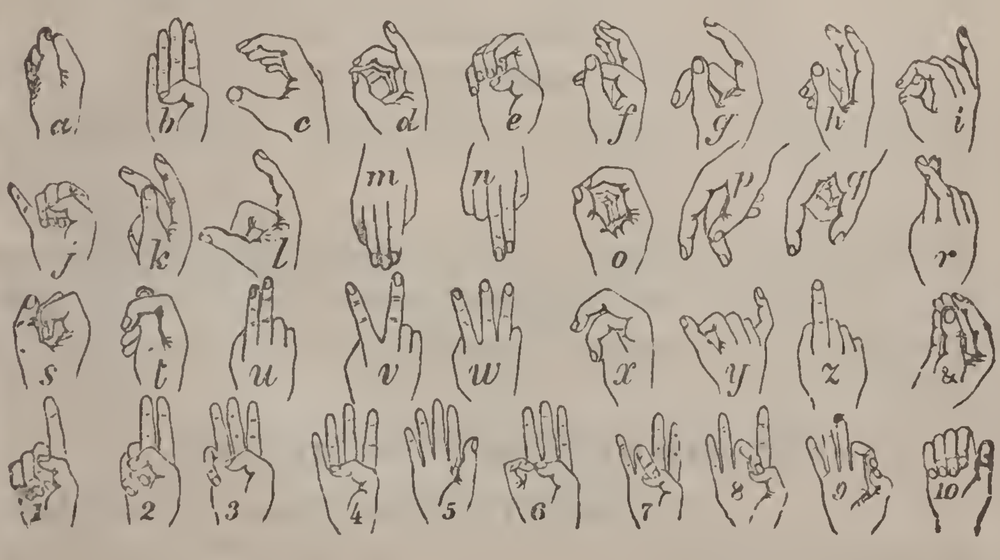
ABCDEFGHIKLM
NOPQRSTUVWX
YZ abcdefghijklmnopqrstuvwxyz.

ABCDEFGHIJKLMNOPQRSTUVWX
YZ abcdefghijklmnopqrsſtuvwxyz

ABCDEFGHIJKLMNOPQRS
TUVWXYZ. 1234567890
abcdefghijklmnopqrstuvwxyz.

DEAF AND DUMB ALPHABET.

Keep your hand horizontal for the letters *a*, *g*, *h*, *r*, *s* and *t*.
For *j* and *z* describe the letter with the finger in the air
For *x* make a motion up and down with the index finger.



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