Neutron Lifetime and Axial Coupling Connection

Andrzej Czarnecki
Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2E1

William J. Marciano
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973, USA

Alberto Sirlin
Department of Physics, New York University, 726 Broadway, New York, New York 10003, USA

(Received 22 February 2018; published 16 May 2018)

Experimental studies of neutron decay, $n \to pe\bar{\nu}$, exhibit two anomalies. The first is a 8.6(2.1) s, roughly 4σ difference between the average beam measured neutron lifetime, $\tau_n^{\text{beam}} = 888.0(2.0)$ s, and the more precise average trapped ultracold neutron determination, $\tau_n^{\text{trap}} = 879.4(6)$ s. The second is a 5σ difference between the pre2002 average axial coupling, g_A, as measured in neutron decay asymmetries $g_A^{\text{pre2002}} = 1.2637(21)$, and the more recent, post2002, average $g_A^{\text{post2002}} = 1.2755(11)$, where, following the UCNA Collaboration division, experiments are classified by the date of their most recent result. In this Letter, we correlate those τ_n and g_A values using a (slightly) updated relation $\tau_n(1 + 3g_A^2) = 5172.0(1.1)$ s. Consistency with that relation and better precision suggest $\tau_n^{\text{favored}} = 879.4(6)$ s and $g_A^{\text{favored}} = 1.2755(11)$ as preferred values for those parameters. Comparisons of g_A^{favored} with recent lattice QCD and muonic hydrogen capture results are made. A general constraint on exotic neutron decay branching ratios, $< 0.27\%$, is discussed and applied to a recently proposed solution to the neutron lifetime puzzle.

DOI: 10.1103/PhysRevLett.120.202002

The neutron lifetime, τ_n, and its axial-current coupling, $g_A = G_A/G_V$, are important weak interaction parameters used in nuclear, particle, and astrophysics, as well as cosmology [1–6]. Employed together, they can determine the quark mixing matrix element V_{ud}, at a level that could eventually become competitive with the current superallowed Fermi transition nuclear beta decay method for determining V_{ud} [7] and constraining "New Physics" via Cabibbo-Kobayashi-Maskawa (CKM) unitarity $|V_{\text{ud}}|^2 + |V_{\text{us}}|^2 + |V_{\text{ub}}|^2 = 1$. Neutron decays have the advantage of no nuclear physics uncertainties [8].

On its own, g_A provides necessary input for the Goldberger-Treiman relation, the Bjorken sum rule, solar and reactor neutrino fluxes, neutrino-nucleon quasielastic scattering cross sections, muon capture rates, and various other weak interaction phenomena. An area of particular importance is the dependence of primordial nucleosynthesis and cosmic microwave background anisotropies on τ_n and g_A [9,10].

Despite their central role in weak interaction phenomenology, τ_n and g_A values have changed, sometimes dramatically, with time. Indeed, the accepted τ_n has decreased over the Particle Data Group (PDG) lifespan from about $1000 \text{ s} \to 932 \text{ s} \to 917 \text{ s} \to 896 \text{ s} \to 886 \text{ s}$ while over a similar time span, g_A has increased from roughly $1.20 \to 1.23 \to 1.25 \to 1.26 \to 1.27$. The correlated movement with time of τ_n and g_A is nicely illustrated in the introduction figures of Ref. [7]. As we shall argue in this Letter, further change in both quantities appears to be in progress. Although the most precise τ_n and g_A experimental measurements have generally been carried out independently of one another, prevailing values at a given time were known to be correlated through the relationship $\tau_n(1 + 3g_A^2) = \text{constant}$, with the constant determined by the standard model (SM) neutron decay rate prediction. Thus, τ_n and g_A experimental values can be expected to move together. Here, we review and update (very slightly) the origin, uncertainty, and status of that constant, by updating the inputs, checking the analysis, and assigning an uncertainty to the theory prediction.

Currently, there are two competing values for τ_n and two for g_A (see Table I). Although the values in each set are generally averaged by the PDG with errors increased by a scale factor based on the χ^2, we keep them separate. The average beam measurements $\tau_n^{\text{beam}} = 888.0(2.0)$ s differ by about 4σ from the newer, more precise, ultracold trapped

TABLE I. Input data used for the $\tau_n^{\rm trap}$, $\tau_n^{\rm beam}$, $g_A^{\rm post2002}$ and $g_A^{\rm pre2002}$ averages. Values and methodology were based on PDG2016 but with updates from [12–15]. The error in $\tau_n^{\rm trap}$ average was scaled by a factor of 1.5 in accordance with PDG protocol. Statistical and systematic uncertainties were added in quadrature and kept to two significant figures before averaging. Averages have not been sanctioned by the PDG.

$\tau_n^{\rm trap}$	Source
881.5(0.92) s	[12]
877.7(0.76) s	[13]
878.3(1.9) s	[14]
880.2(1.2) s	[16]
882.5(2.1) s	[17]
880.7(1.8) s	[18]
878.5(0.76) s	[19]
882.6(2.7) s	[20]
879.4(6) s	Average (includes scale factor $S = 1.5$)
$\tau_n^{\rm beam}$	Source
887.7(2.2) s	[21]
889.2(4.9) s	[22]
888.0(2.0) s	Average
$g_A^{\rm post2002}$	Source
1.2772(20)	[15]
1.2748^{+13}_{-14}	[23]
1.2750(160)	[24]
1.2755(11)	Average
$g_A^{\rm pre2002}$	Source
1.2686(47)	[25]
1.2660(40)	[26]
1.2594(38)	[27]
1.2620(50)	[28]
1.2637(21)	Average

neutron average $\tau_n^{\rm trap} = 879.4(6)$ s. That difference is sometimes referred to as the neutron lifetime puzzle, enigma, or problem. Similarly, an earlier set of g_A measurements labeled pre2002 averages to $g_A^{\rm pre2002} = 1.2637(21)$, while determinations completed after 2002, labeled post2002, average to $g_A^{\rm post2002} = 1.2755(11)$, a 5σ difference, even more pronounced than the neutron lifetime problem. A notable difference [1,11] between pre and post 2002 experiments, is that the earlier efforts required larger corrections to the measured asymmetries. As a result, those corrections and their systematic uncertainties may have been more difficult to properly estimate. The two g_A values are generally PDG averaged, and the uncertainty is increased by a scale factor of approximately 2, primarily due to pre2002 χ^2 contributions. Here, we keep the method dependent τ_n as well as the pre and post 2002 g_A values separate, and we argue in favor of the more recent values in both cases because of their better precision and, more important, their remarkable consistency with our evaluation of the constant in the $\tau_n - g_A$ relation previewed above. On that basis, we will argue that, within the SM, $\tau_n^{\rm favored} = 879.4(6)$ s and $g_A^{\rm favored} = 1.2755(11)$ currently represent our recommended "favored values." They may be the final word, within errors.

Relating τ_n and g_A begins with a very precise SM prediction for the total (radiative inclusive) neutron decay rate. That inverse lifetime formula includes Fermi function final state electron-proton Coulomb interactions, electroweak radiative corrections (normalized relative to the muon lifetime [29]), and a number of smaller effects including proton recoil, finite nuclear size etc. Overall, those corrections are rather large, $> +7\%$. A very detailed analysis of those corrections was given in the classic study by Wilkinson [30]. Later, that relationship was checked, updated, and refined in [31] where higher order $\mathcal{O}(\alpha^2)$ contributions were properly included. The radiative corrections uncertainty was reduced in [32].

In the SM, the inverse lifetime equation relating τ_n and g_A is given by [31]

$$\frac{1}{\tau_n} = \frac{G_\mu^2 |V_{\rm ud}|^2}{2\pi^3} m_e^5 (1 + 3g_A^2)(1 + {\rm RC}) f, \qquad (1)$$

where G_μ is the Fermi constant determined from the muon lifetime [33–42], $G_\mu = 1.1663787(6) \times 10^{-5}$ GeV^{-2}, $V_{\rm ud}$ is the CKM mixing element generally obtained from superallowed nuclear beta decays [7,43], RC represents electroweak radiative corrections [44–50], which were most recently evaluated [32] to be $+0.03886(38)$, and f is a phase space factor [30]. The electroweak radiative corrections in Eq. (1) have been factorized to be the same for vector and axial-vector contributions [31]. That prescription defines g_A as determined by the neutron lifetime. Expressing the polarized neutron spin-electron correlation coefficient, $A_0(g_A) = 2g_A(1 - g_A)/(1 + 3g_A^2)$, in terms of that g_A will, therefore, induce small $\mathcal{O}(0.1\%)$ radiative corrections [51] along with the $\mathcal{O}(1\%)$ residual Coulomb, recoil, and weak magnetism corrections to the measured asymmetry that must be corrected for before extracting g_A [5,30].

Employing masses [7] (with highly correlated uncertainties due to atomic mass units to MeV translation) $m_n = 939.5654133(58)$ MeV, $m_p = 938.2720813(58)$ MeV, and $m_e = 0.5109989461(31)$ MeV leads to $f = 1.6887(1)$ [30,31], where we have redone the numerical evaluation of Wilkinson's perturbative analysis and employed a conservative error consistent with his assessment [52]. Using the above input parameters, but keeping $V_{\rm ud}$, τ_n and g_A arbitrary, produces the SM master formula

$$|V_{\rm ud}|^2 \tau_n (1 + 3g_A^2) = 4908.6(1.9)\ {\rm s}, \qquad (2)$$

where the uncertainty comes primarily from the RC. That formula can be used to determine $V_{\rm ud}$ from independent experimental measurements of τ_n and g_A. Future experiments optimistically hope to eventually reach $\pm 0.01\%$ sensitivity for those input parameters. At that level, the RC theory uncertainty will be dominant.

Our intention is to correlate τ_n and g_A, rather than determine $V_{\rm ud}$. To that end, we employ the superallowed $0^+ \to 0^+$ nuclear transitions current best value $V_{\rm ud} = 0.97420(10)(18)_{\rm RC}$, a value consistent with CKM unitarity [7], where the first error (10) results from experiment, nuclear structure, and nucleus dependent radiative corrections, while the second error $(18)_{\rm RC}$ represents universal radiative corrections common to both neutron and nuclear beta decays. Importantly, the RC error in $|V_{\rm ud}|^2$ and in Eq. (1) are anticorrelated and effectively cancel. For that reason, one finds the following very precise relation between τ_n and g_A

$$\tau_n(1 + 3g_A^2) = 5172.0(1.1)\ \mathrm{s}, \tag{3}$$

where the uncertainty stems primarily from nuclear and experimental uncertainties in $V_{\rm ud}$. That connection allows one to translate between τ_n and g_A with high precision and thereby test their mutual consistency.

In that way, lifetime and axial-charge measurements can be directly compared or, for some purposes, even averaged. Toward that end, it is useful to divide the lifetime averages into trap, which includes bottle and magnetic confinement trap experiments, and beam measurements, the two areas of disagreement. Similarly, following the classification introduced by the UCNA Collaboration [15], asymmetry values of g_A naturally separate into pre2002 and post2002, where 2002 represents the year when larger values of g_A, seen earlier, were confirmed with improved errors [11,53]. Experiments are arranged by the year of their last result (see Table I). The post2002 measurements of g_A tended to have larger central values and better controlled systematics. That approach leads to the following direct and indirect averages, connected by arrows representing the relationship in Eq. (3),

$$\tau_n^{\rm trap} = 879.4(6)\ \mathrm{s} \to g_A = 1.2756(5), \tag{4}$$

$$\tau_n^{\rm beam} = 888.0(2.0)\ \mathrm{s} \to g_A = 1.2681(17), \tag{5}$$

$$g_A^{\rm post2002} = 1.2755(11) \to \tau_n = 879.5(1.3)\ \mathrm{s}, \tag{6}$$

$$g_A^{\rm pre2002} = 1.2637(21) \to \tau_n = 893.1(2.4)\ \mathrm{s}. \tag{7}$$

One notices that $\tau_n^{\rm trap}$ and $g_A^{\rm post2002}$ provide the most precise direct and indirect lifetimes, respectively, and they are remarkably consistent. Those features are illustrated in Fig. 6 of Ref. [15]. That agreement is exactly the type of consistency one expects of the true parameters. On the other hand, the beam and pre2002 g_A determined lifetimes disagree with those more precise values and are not particularly consistent with one another. Because of their better precision and relationship consistency, we refer to the trap lifetime and post2002 g_A as our favored values,

$$\tau_n^{\rm favored} = 879.4(6)\ \mathrm{s}, \tag{8}$$

$$g_A^{\rm favored} = 1.2755(11). \tag{9}$$

These favored experimental averages, in conjunction with the indirect determination of g_A in Eq. (4), provide standards for comparison with future lifetime and asymmetry measurements which will aim at the long term goal of 0.01% precision in τ_n and g_A. Our current favored values in Eqs. (8) and (9) should be compared with our updates of the 2016 PDG averages based on recent results [12–15] in Table I,

$$\tau_n^{\rm update16} = 879.7(8)\ \mathrm{s} \quad (\text{with scale factor } S = 2), \tag{10}$$

$$g_A^{\rm update16} = 1.2731(23) \quad (\text{with } S = 2.3). \tag{11}$$

Those updates are consistent with our preferred values in Eqs. (8), (9), but they have larger errors due to scale factors that represent inconsistencies in the experiments averaged. They are useful as a conservative perspective on the current τ_n and g_A situation.

Regarding our neglect of $\tau_n^{\rm beam}$ and $g_A^{\rm pre2002}$ in deriving our favored values, we make the following observations. $\tau_n^{\rm beam}$ differs from $\tau_n^{\rm trap}$ by about 4σ and $g_A^{\rm pre2002}$ differs from $g_A^{\rm post2002}$ by 5σ. So, a case can be made that one should not continue to include in averaging outlying values based on older techniques when a significant disagreement arises. Indeed, the history of τ_n and g_A experimental shifts indicate that they come in pairs as new technological methods emerge. In this case, the 2002 confirmation [11,53] of a relatively large g_A with small errors may be viewed as the harbinger of a shorter lifetime, which several years later began to be directly observed in trapped lifetime experiments.

One might ask whether theory or some other weak interaction phenomenon can be used to determine g_A (and τ_n indirectly)? On the theory side, there is the promise of lattice QCD [54,55]. The lattice approach is, in principle, an ideally suited first principles method for computing a relatively pure, strong interaction effect such as g_A. However, early lattice attempts to compute g_A generally obtained smaller than expected values with large systematic uncertainties. Recently, the situation has been improving. Indeed, a recent study [56] found the preliminary result, $g_A^{\rm lattice} = 1.285(17)$, in good agreement with our "favored" value at about the $\pm 1\%$ level. How much further the lattice precision can improve remains to be seen. Fortunately, the current uncertainty is statistics dominated; so, long dedicated lattice running can potentially reduce the error. Perhaps our suggestion of a favored value with small uncertainty may help to motivate a heroic effort.

An alternative independent experimental g_A determination using muon capture in Muonic Hydrogen was recently examined [57]. Using theory and experimental input for other parameters, the measured capture rate gave $g_A = 1.276(11)$ i.e. somewhat better than 1% agreement with our favored value. It was suggested in that study that a future factor of 3 improvement in the measured capture rate combined with a better lattice determination of the axial charge radius could provide a g_A determination at the level of $\pm 0.2 - 0.3\%$. That would be a nice check on our favored values, but it would appear difficult to improve that approach much further.

To illustrate an application of the favored values, we end our discussion by deriving a general constraint on possible exotic (beyond the standard model) neutron decays and applying it to an interesting scenario recently put forward by Fornal and Grinstein (FG) [58] in an effort to solve the neutron lifetime puzzle. Those authors suggest that the BR = branching ratio for radiative inclusive $n \to pe\bar{\nu}(\gamma)$ could be 0.99 rather than 1 due to a speculated 1% exotic neutron branching ratio into dark particle decay modes (e.g., $n \to \text{dark}\, n + \text{scalar}$) without protons and electrons. In that case, beam experiments that only detect decays with final state protons or electrons would actually measure a partial lifetime $\tau_n^{\text{full}}/\text{BR}$ with $\text{BR} < 1$, while trapped neutron experiments that count the number of neutrons as a function of time measure the full inclusive lifetime τ_n^{full}. Although throughout this Letter, we tacitly conclude that the beam lifetime is an outlier whose value will shift in future, more precise, follow-up experiments, and eventually agree with our favored trapped $\tau_n^{\text{trap}} = 879.4(6)$ s, addressing the Fornal-Grinstein solution is an instructive exercise that we will use to conclude this Letter.

We begin by generalizing our analysis to the case where the BR for $n \to pe\bar{\nu}(\gamma)$ can be < 1 due to exotic decays, such as $n \to$ dark particles. In that case, Eqs. (1), (2), and (3) are modified by replacing τ_n with $\tau_n^{\text{full}}/\text{BR}$, where $\tau_n^{\text{full}} = 1/(\text{total nentron decay rate})$. That replacement leads, via Eq. (3), to (assuming V_{ud} extracted from superallowed beta decays and CKM unitarity agreement are negligibly affected by the exotic new physics)

$$\text{BR} = \tau_n^{\text{full}}(1 + 3g_A^2)/5172.0(1.1)\ \text{s}. \tag{12}$$

Accepting $\tau_n^{\text{trap}} = 879.4(6)$ s as the full lifetime in Eq. (12) and expanding BR in g_A about $g_A = 1.2755$, the directly measured axial coupling post2002 central value, leads to

$$\text{BR} = 0.9999(7) + 1.30(g_A - 1.2755) + \ldots \tag{13}$$

That formula demonstrates the closeness of BR to 1 for $g_A^{\text{favored}} = 1.2755(11)$. It suggests a degree of tension between the recent determinations of g_A^{post2002} and the Fornal-Grinstein solution to the neutron lifetime puzzle. In fact, phrased as a one sided 95% C.L. bound, it requires

$$\begin{aligned} 1 - \text{BR} &= \text{Total exotic neutron decay branching ratio} \\ &< 0.27\% \quad \text{for } g_A = 1.2755(11). \end{aligned} \tag{14}$$

That bound implies that satisfying more than 2.4 s of the 8.6 s lifetime puzzle difference has less than a 5% chance of being realized. One can overcome such a likelihood restriction by assuming a smaller g_A in Eq. (13). For example, $g_A = 1.268$ leads to $\text{BR} = 0.99$, which corresponds to about a 9 s lifetime difference. Any axial coupling roughly in the range $1.268 < g_A < 1.272$ could account for a good part of the puzzle. Unfortunately, there would be a price to pay for a smaller g_A in that range. Those values are in disagreement with the most recent $g_A^{\text{post2002}} = 1.2755(11)$ by 3 or more σ. Thus, the lifetime puzzle would be replaced by a g_A inconsistency.

The Fornal-Grinstein scenario will be tested by new measurements of τ_n, both beam and trap, to see if the current puzzle survives and needs a solution. If so, the next step will be more precise determinations of g_A, via neutron decay asymmetries or perhaps lattice gauge theories. Will g_A revert back to a smaller value? Updates of g_A in the past have almost always led to larger values, but the past is not always a good predictor for the future.

A scenario similar to that of Fornal and Grinstein was envisioned by K. Green and D. Thompson [59] for the rare decay $n \to \text{hydrogen} + \bar{\nu}$. They used the different effects of that decay on beam and trap lifetimes to obtain a bound of $< 3\%$ for that branching ratio (to be compared with the 4×10^{-6} prediction [60–63]). Our general analysis employing τ_n^{trap} and g_A^{post2002} in Eq. (12) can be used to reduce that bound by an order of magnitude to $< 0.27\%$.

Our 0.27% bound in Eq. (14) also applies to neutron oscillations into mirror or dark neutrons [64–66], exotic phenomena proposed to explain the neutron lifetime puzzle.

Future expected order of magnitude improvements in τ_n^{trap} and asymmetry measurements, should improve the sensitivity of our bound in Eq. (14) to roughly 3×10^{-4} for the exotic phenomena described above.

The work of A. C. was supported by the Natural Sciences and Engineering Research Council of Canada. The work of W. J. M. was supported by the U.S. Department of Energy under Grant No. DE-SC0012704. The work of A. S. was supported in part by the National Science Foundation under Grant No. PHY-1620039.

[1] H. Abele, Prog. Part. Nucl. Phys. **60**, 1 (2008).
[2] D. Dubbers and M. G. Schmidt, Rev. Mod. Phys. **83**, 1111 (2011).
[3] F. E. Wietfeldt and G. L. Greene, Rev. Mod. Phys. **83**, 1173 (2011).
[4] N. Severijns and O. Naviliat-Cuncic, Annu. Rev. Nucl. Part. Sci. **61**, 23 (2011).
[5] A. N. Ivanov, M. Pitschmann, and N. I. Troitskaya, Phys. Rev. D **88**, 073002 (2013).
[6] G. L. Greene and P. Geltenbort, Sci. Am. **314**, 36 (2016).
[7] C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C **40**, 100001 (2016).

[8] H. Abele, E. Barberio, D. Dubbers, F. Gluck, J. C. Hardy, W. J. Marciano, A. Serebrov, and N. Severijns, Eur. Phys. J. C **33**, 1 (2004).

[9] G. J. Mathews, T. Kajino, and T. Shima, Phys. Rev. D **71**, 021302 (2005).

[10] C. Pitrou, A. Coc, J.-P. Uzan, and E. Vangioni, arXiv:1801.08023.

[11] H. Abele, M. A. Hoffman, S. Baessler, D. Dubbers, F. Gluck, U. Muller, V. Nesvizhevsky, J. Reich, and O. Zimmer, Phys. Rev. Lett. **88**, 211801 (2002).

[12] A. P. Serebrov *et al.*, arXiv:1712.05663.

[13] R. W. Pattie, Jr. *et al.*, arXiv:1707.01817.

[14] V. F. Ezhov *et al.*, arXiv:1412.7434.

[15] M. A. P. Brown *et al.* (UCNA), Phys. Rev. C **97**, 035505 (2018).

[16] S. Arzumanov, L. Bondarenko, S. Chernyavsky, P. Geltenbort, V. Morozov, V. V. Nesvizhevsky, Yu. Panin, and A. Strepetov, Phys. Lett. B **745**, 79 (2015).

[17] A. Steyerl, J. M. Pendlebury, C. Kaufman, S. S. Malik, and A. M. Desai, Phys. Rev. C **85**, 065503 (2012).

[18] A. Pichlmaier, V. Varlamov, K. Schreckenbach, and P. Geltenbort, Phys. Lett. B **693**, 221 (2010).

[19] A. Serebrov *et al.*, Phys. Lett. B **605**, 72 (2005).

[20] W. Mampe, L. N. Bondarenko, V. I. Morozov, Yu. N. Panin, and A. I. Fomin, Pis'ma Zh. Eksp. Teor. Fiz. **57**, 77 (1993) [JETP Lett. **57**, 82 (1993)].

[21] A. T. Yue, M. S. Dewey, D. M. Gilliam, G. L. Greene, A. B. Laptev, J. S. Nico, W. M. Snow, and F. E. Wietfeldt, Phys. Rev. Lett. **111**, 222501 (2013).

[22] J. Byrne and P. G. Dawber, Europhys. Lett. **33**, 187 (1996).

[23] D. Mund, B. Markisch, M. Deissenroth, J. Krempel, M. Schumann, H. Abele, A. Petoukhov, and T. Soldner, Phys. Rev. Lett. **110**, 172502 (2013).

[24] M. Schumann, T. Soldner, M. Deissenroth, F. Gluck, J. Krempel, M. Kreuz, B. Markisch, D. Mund, A. Petoukhov, and H. Abele, Phys. Rev. Lett. **99**, 191803 (2007).

[25] Yu. A. Mostovoi *et al.*, Yad. Fiz. **64**, 2040 (2001) [Phys. At. Nucl. **64**, 1955 (2001)].

[26] P. Liaud, K. Schreckenbach, R. Kossakowski, H. Nastoll, A. Bussiere, J. P. Guillaud, and L. Beck, Nucl. Phys. **A612**, 53 (1997).

[27] B. Erozolimsky, I. Kuznetsov, I. Stepanenko, and Yu. A. Mostovoi, Phys. Lett. B **412**, 240 (1997).

[28] P. Bopp, D. Dubbers, L. Hornig, E. Klemt, J. Last, H. Schutze, S. J. Freedman, and O. Scharpf, Phys. Rev. Lett. **56**, 919 (1986); **57**, 1192(E) (1986).

[29] W. J. Marciano, J. Phys. G **29**, 23 (2003).

[30] D. H. Wilkinson, Nucl. Phys. **A377**, 474 (1982).

[31] A. Czarnecki, W. J. Marciano, and A. Sirlin, Phys. Rev. D **70**, 093006 (2004).

[32] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **96**, 032002 (2006).

[33] T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959).

[34] S. M. Berman, Phys. Rev. **112**, 267 (1958).

[35] S. Berman and A. Sirlin, Ann. Phys. (N.Y.) **20**, 20 (1962).

[36] E. S. Abers, D. A. Dicus, R. E. Norton, and H. R. Quinn, Phys. Rev. **167**, 1461 (1968).

[37] T. van Ritbergen and R. G. Stuart, Phys. Rev. Lett. **82**, 488 (1999).

[38] M. Steinhauser and T. Seidensticker, Phys. Lett. B **467**, 271 (1999).

[39] A. Ferroglia, G. Ossola, and A. Sirlin, Nucl. Phys. **B560**, 23 (1999).

[40] W. J. Marciano, Phys. Rev. D **60**, 093006 (1999).

[41] I. Blokland, A. Czarnecki, M. Ślusarczyk, and F. Tkachov, Phys. Rev. Lett. **93**, 062001 (2004).

[42] A. Pak and A. Czarnecki, Phys. Rev. Lett. **100**, 241807 (2008).

[43] J. Hardy and I. S. Towner, Proc. Sci. CKM2016 (**2016**) 028.

[44] A. Sirlin, Phys. Rev. **164**, 1767 (1967).

[45] A. Sirlin, in Particles, Currents, Symmetries, *Acta Physica Austriaca, Suppl. V*, edited by P. Urban (Springer Verlag, New York, 1968), p. 353.

[46] A. Sirlin, Nucl. Phys. **B71**, 29 (1974).

[47] A. Sirlin, Rev. Mod. Phys. **50**, 573 (1978).

[48] R. Blin-Stoyle and J. M. Freeman, Nucl. Phys. **A150**, 369 (1970).

[49] A. Sirlin, AIP Conf. Proc. **698**, 385 (2004).

[50] A. Sirlin and A. Ferroglia, Rev. Mod. Phys. **85**, 263 (2013).

[51] R. T. Shann, Nuovo Cimento A **5**, 591 (1971).

[52] D. H. Wilkinson, Nucl. Instrum. Methods Phys. Res., Sect. A **404**, 305 (1998).

[53] H. Abele *et al.*, Phys. Lett. B **407**, 212 (1997).

[54] S. Capitani, M. Della Morte, D. Djukanovic, G. M. von Hippel, J. Hua, B. Jäger, P. M. Junnarkar, H. B. Meyer, T. D. Rae, and H. Wittig, arXiv:1705.06186.

[55] E. Shintani and H. Wittig, JPS Conf. Proc. **10**, 062001 (2016).

[56] C. C. Chang *et al.*, arXiv:1710.06523.

[57] R. J. Hill, P. Kammel, W. J. Marciano, and A. Sirlin, arXiv:1708.08462.

[58] B. Fornal and B. Grinstein, Phys. Rev. Lett. **120**, 191801 (2018).

[59] K. Green and D. Thompson, J. Phys. **16**, L75 (1990).

[60] J. N. Bahcall, Phys. Rev. **124**, 495 (1961).

[61] P. K. Kabir, Phys. Lett. **24B**, 601 (1967).

[62] J. McAndrew, S. Paul, R. Engels, P. Fierlinger, E. Gutsmiedl, J. Schön, and W. Schott, Phys. Procedia **51**, 37 (2014).

[63] R. Daudel, M. Jean, and M. Lecoin, J. Phys. Radium **8**, 238 (1947).

[64] Z. Berezhiani and L. Bento, Phys. Rev. Lett. **96**, 081801 (2006).

[65] A. P. Serebrov *et al.*, Phys. Lett. B **663**, 181 (2008).

[66] A. P. Serebrov, R. M. Samoilov, I. A. Mitropolsky, and A. M. Gagarsky, arXiv:1802.06277.