VOLUME OF CONE BY USING INTEGRATION:-

Let us consider a right circular cone of radius $r$ and the height $h$. The volume of cone is obtained by the formula,

$$V = \int_a^b \pi y^2 \, dx$$

Here equation of the slant height i.e a straight line passing through origin is given by $y = mx$ and $m = \frac{dy}{dx}$ i.e $m = \frac{r}{h}$

Hence, $y = \frac{rx}{h}$.

$$V = \int_a^b \pi \left(\frac{rx}{h}\right)^2 \, dx$$

$$V = \pi \int_0^h \left(\frac{rx}{h}\right)^2 \, dx$$

$$V = \frac{\pi r^2}{h^2} \int_0^h x^2 \, dx$$

$$V = \frac{\pi r^2}{h^2} \left[\frac{x^3}{3}\right]_0^h$$

$$V = \frac{\pi r^2}{h^2} \left[\frac{h^3}{3}\right]$$

$$V = \frac{1}{3} \pi r^2 h \text{ cubic units.}$$