

XIX. *Description of a simple Micrometer for measuring small Angles with the Telescope.* By Mr. Tiberius Cavallo, F. R. S.

Read June 2, 1791.

THE various telescopical micrometers, or machines which have been constructed for the measurement of small angles, may be divided into two classes; namely, those which have not, and those which have, some movement amongst their parts. The micrometers of the former sort consist mostly of fine wires, or hairs, variously disposed, and situated within the telescope, just where the image of the object is formed. In order to determine an angle with those micrometers, a good deal of calculation is generally required. The micrometers of the other sort, of which there is a great variety; some being made with moveable parallel wires, others with prisms, others again with a combination of lenses, and so on; are more or less subject to several inconveniences, the principal of which are the following. 1st, Their motions generally depend upon the action of a screw, and of course the imperfections of its threads, and the greater or less quantity of lost motion, which is observable in moving a screw, especially when small, occasion a considerable error in the mensuration of angles. 2^{dly}, Their complication and bulk renders them difficultly applicable to a variety of telescopes, especially to the

pocket ones. 3dly, They do not measure the angle without some loss of time, which is necessary to turn the screw, or to move some other mechanism. 4thly, and lastly, They are considerably expensive, so that some of them cost even more than a tolerably good telescope.

After having had long in view the construction of a micrometer, which might be in part at least, if not intirely, free from all those objections; and, after various attempts, I at last succeeded with a simple contrivance, which, after repeated trials, has been found to answer the desired end, not only from my own experience, but from that also of several friends, to whom it has been communicated.

This micrometer, in short, consists of a thin and narrow slip of mother of pearl finely divided, and situated in the focus of the eye-glass of a telescope, just where the image of the object is formed. It is immaterial whether the telescope be a refractor or a reflector, provided the eye-glass be a convex lens, and not a concave one, as in the Galilean construction.

The simplest way of fixing it is to stick it upon the diaphragm, which generally stands within the tube, and in the focus of the eye-glass. When thus fixed, if you look through the eye-glass, the divisions of the micrometrical scale will appear very distinct, unless the diaphragm is not exactly in the focus; in which case the micrometrical scale must be placed exactly in the focus of the eye-glass, either by pushing the diaphragm backwards or forwards, when that is practicable; or else the scale may be easily removed from one or the other surface of the diaphragm by the interposition of a circular piece of paper or card, or by a bit of wax. This construction is fully sufficient when the telescope is always to be used by the same person; but when different persons are to use it, then

the diaphragm, which supports the micrometer, must be constructed so as to be easily moved backwards or forwards, though that motion needs not be greater than about a tenth or an eighth of an inch. This is necessary, because the distance of the focus of the same lens appears different to the eyes of different persons, and therefore, whoever is going to use the telescope for the mensuration of any angle, must first of all unscrew the tube, which contains the eye-glass and micrometer, from the rest of the telescope, and, looking through the eye-glass, must place the micrometer where the divisions of it may appear quite distinct to his eye.

In case that any person should not like to see always the micrometer in the field of the telescope, then the micrometrical scale, instead of being fixed to the diaphragm, may be fitted to a circular perforated plate of brass, wood, or even paper, which may be occasionally placed upon the said diaphragm.

I have made several experiments to determine the most useful substance for this micrometer. Glass, which I had successfully applied for a similar purpose to the compound microscope, seemed at first to be the most promising; but it was at last rejected after several trials: for the divisions upon it generally are either too fine to be perceived, or too rough; and though with proper care and attention the divisions may be proportioned to the sight, yet the thickness of the glass itself obstructs in some measure the distinct view of the object. Ivory, horn, and wood, were found useless for the construction of this micrometer, on account of their bending, swelling, and contracting very easily; whereas mother of pearl is a very steady substance, the divisions upon it may be marked
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very easily, and, when it is made as thin as common writing paper, it has a very useful degree of transparency.

Tab. VIII. fig. 1. exhibits this micrometer scale, but shews it four times larger than the real size of one, which I have adapted to a three-feet achromatic telescope, that magnifies about 84 times. It is something less than the 24th part of an inch broad; its thickness is equal to that of common writing paper; and the length of it is determined by the aperture of the diaphragm, which limits the field of the telescope. The divisions upon it are the 200ths of an inch, which reach from one edge of the scale to about the middle of it, excepting every fifth and tenth division, which are longer. The divided edge of it passes through the center of the field of view, though this is not a necessary precaution in the construction of this micrometer. Two divisions of the above described scale in my telescope are very nearly equal to one minute; and as a quarter of one of those divisions may be very well distinguished by estimation, therefore an angle of one-eighth part of a minute, or of $7''\frac{1}{2}$, may be measured with it.

When a telescope magnifies more, the divisions of the micrometer must be more minute; and I find, that when the focus of the eye-glass of the telescope is shorter than half an inch, the micrometer may be divided with the 500ths of an inch; by means of which, and the telescope magnifying about 200 times, one may easily and accurately measure an angle smaller than half a second.

On the other hand, when the telescope does not magnify above 30 times, the divisions need not be so minute: for instance, in one of DOLLOND's pocket telescopes, which when drawn out for use, is about 14 inches long, a micrometer with
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the hundredths of an inch is quite sufficient, and one of its divisions is equal to little less than three minutes, so that an angle of a minute may be measured by it.

In looking through a telescope, furnished with such a micrometer, the field of view appears divided by the micrometer scale, the breadth of which occupies about one-seventh part of the aperture, and as the scale is semitransparent, that part of the object, which happens to be behind it, may be discerned sufficiently well to ascertain the division, and even the quarter of a division, with which its borders coincide. Fig. 2. shews the appearance of the field of my telescope with the micrometer, when directed to the title page of the Philosophical Transactions, wherein one may observe that the thickness of the letter C is equal to three-fourths of a division, the diameter of the O is equal to three divisions, and so on.

At first view one is apt to imagine that it is difficult to count the divisions which may happen to cover or to measure an object; but upon trial it will be found, that this is readily performed; and even people, who have never been used to observe with the telescope, soon learn to measure very quickly and accurately with this micrometer; for, since every fifth and tenth division is longer than the rest, one soon acquires the habit of saying five, ten, fifteen, and then, by adding the other divisions less than five, completes the reckoning. Even with a telescope, which has no stand, if the object end of it be rested against a steady place, and the other end be held by the hand near the eye of the observer, an object may be measured with accuracy sufficient for several purposes, as for the estimation of small distances, for determining the height of a house, &c.

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After having constructed and adapted this micrometer to the telescope, it is then necessary to ascertain the value of the divisions. It is hardly necessary to mention in this place, that though those divisions measure the chords of the angles, and not the angles or arches themselves, and the chords are not as the arches, yet it has been shewn by all the trigonometrical writers, that in small angles the chords, arches, sines, and tangents, follow the same proportion so very nearly, that the very minute difference may be safely neglected: so that if one division of this micrometer is equal to one minute, we may safely conclude, that two divisions are equal to two minutes, three divisions to three minutes, and so on. There are various methods of ascertaining the value of the divisions of such a micrometer, they being the very same that are used for ascertaining the value of the divisions in other micrometers. Such are the passage of an equatorial star over a certain number of divisions in a certain time, the measuring of the diameter of the sun, by computation from the focal distance of the object, and other lenses of the telescope, the last of which, however, is subject to several inaccuracies; but as they are well known to astronomical persons, and have been described in many books, need not be farther noticed in this Paper. However, for the sake of workmen and other persons not conversant in astronomy, I shall describe an easy and accurate method of ascertaining the value of the divisions of the micrometer.

Mark upon a wall, or other place, the length of six inches, which may be done by making two dots or lines six inches asunder, or by fixing a six-inch ruler upon a stand; then place the telescope before it so that the ruler or six-inch length may be at right angles with the direction of the telescope, and just 57 feet $3\frac{1}{2}$ inches distant from the object-glass of the telescope: this done, look through the telescope at the ruler or other extension

extension of six inches, and observe how many divisions of the micrometer are equal to it, and that same number of divisions is equal to half a degree, or $30'$; and this is all that needs be done for the required determination; the reason of which is, because an extension of six inches subtends an angle of $30'$ at the distance of 57 feet $3\frac{1}{2}$ inches, as may be easily calculated by the rules of plane trigonometry.

In one of DOLLOND'S 14-inch pocket telescopes, if the divisions of the micrometer be the hundredths of an inch, $11\frac{1}{2}$ of those divisions will be found equal to $30'$, or 23 to a degree.

When this value has been once ascertained, any other angle measured by any other number of divisions is determined by the rule of three. Thus, suppose that the diameter of the sun, seen through the same telescope, be found equal to 12 divisions, say as $11\frac{1}{2}$ divisions are to 30 minutes, so are 12 divisions to $\left(\frac{12' \times 30'}{11,5}\right) 31',3$, which is the required diameter of the sun.

Notwithstanding the facility of this calculation, a scale may be made answering to the divisions of a micrometer, which will shew the angle corresponding to any number of divisions to mere inspection. Thus, for the above-mentioned small telescope the scale is represented in fig. 3. AB is a line drawn at pleasure; it is then divided into 23 equal parts, and those divisions, which represent the divisions of the micrometer that are equal to one degree, are marked on one side of it. The line then is divided again into 60 equal parts, which are marked on the other side of it; and these divisions represent the minutes which correspond to the divisions of the micrometer: thus the figure shews, that six divisions of the micrometer are equal to $15\frac{1}{2}$ minutes, $11\frac{1}{2}$ divisions are nearly equal to 29

minutes, &c. What has been said of minutes may be said of seconds also, when the scale is to be applied to a large telescope.

Thus far this micrometer, and its general use, have been sufficiently described, and mathematical persons may easily apply it to the various purposes to which micrometers have been found subservient. But as the simplicity, cheapness, and at the same time the accuracy of this contrivance, may render the use of it much more general than that of any other micrometer; and I may venture to say, that it will be found very useful in the army, and amongst sea-faring people, for the determination of distances, heights, &c.; I shall therefore subjoin some practical rules to render this micrometer useful to persons unacquainted with trigonometry and the use of logarithms.

Problem I. The angle, not exceeding one degree, which is subtended by an extension of one foot being given, to find its distance from the place of observation.

N. B. This extension of one foot, or any other which may be mentioned hereafter, must be perpendicular to the direction of the telescope through which it is observed. The distances are reckoned from the object-glass of the telescope, and the answers obtained by the rules of this problem, though not exactly true, are however so little different from the truth, that the difference seldom amounts to more than two or three inches, which may be safely neglected.

Rule 1. If the angle be expressed in minutes, say, as the given angle is to 60, so is 687,55 to a fourth proportional, which gives the answer in inches.

2. If the angle be expressed in seconds, say, as the given angle is to 3600, so is 687,55 to a fourth proportional, which expresses the answer in inches.

3. If the angle be expressed in minutes and seconds, turn it all into seconds, and proceed as above.

Example. At what distance is a globe of one foot in diameter when it subtends an angle of two seconds?

$2 : 3600 :: 687,55 : \frac{3600 \times 687,55}{2} = 1237590$ inches, or $103132\frac{1}{2}$ feet, which is the answer required.

This calculation may be shortened; for since two of the three proportionals are fixed, their product in the first case is 41253, and in the other two cases is 2475180; so that in the first case, *viz.* when the angle is expressed in minutes, you need only divide 41253 by the given angle; and in the other two cases, *viz.* when the angle is expressed in seconds, divide 2475180 by the given angle, and the quotient in either case is the answer in inches.

Problem II. The angle, not exceeding one degree, which is subtended by any known extension, being given, to find its distance from the place of observation.

Rule. Proceed as if the extension were of one foot by Problem I. and call the answer B; then, if the extension in question be expressed in inches, say, as 12 inches are to that extension, so is B to a fourth proportional, which is the answer in inches; but if the extension in question be expressed in feet, then you need only multiply it by B, and the product is the answer in inches.

Example. At what distance is a man, six feet high, when he appears to subtend an angle of 30''.

By Problem I. if the man were one foot high, the distance would be 82506 inches; but as he is six feet high, therefore multiply 82506 by 6, and the product gives the required distance, which is 495036 inches, or 41253 feet.

For greater conveniency, especially in travelling, or in such circumstances in which one has not the opportunity of making even the easy calculations required in those problems, I have calculated the following two tables; the first of which shews the distance answering to any angle from one minute to one degree, which is subtended by an extension of one foot; and the second table shews the distance answering to any angle from one minute to one degree, which is subtended by a man, the height of which has been called an extension of six feet; because, at a mean, such is the height of a man when dressed with hat and shoes on. These tables may be transcribed on a card, and may be had always ready with a pocket telescope furnished with a micrometer. Their use is evidently to ascertain distances without any calculation; and they are calculated only to minutes, because with a pocket telescope and micrometer it is not possible to measure an angle more accurately than to a minute.

Thus, if one wants to measure the extension of a street, let a foot ruler be placed at the end of the street; measure the angular appearance of it, which suppose to be $36'$, and in the table you will have the required distance against $36'$, which is $95\frac{1}{2}$ feet. Thus also a man, who appears to be $49'$ high, is at the distance of 421 feet.

T. CAVALLO.

Wells-street,
May 26, 1793.

Angles

Angles subtended by an extension of one foot at different distances.

Angles.	Distances in feet.	Angles.	Distances in feet.
Min. 1	3437,7	Min. 31	110,9
2	1718,9	32	107,4
3	1145,9	33	104,2
4	859,4	34	101,1
5	687,5	35	98,2
6	572,9	36	95,5
7	491,1	37	92,9
8	429,7	38	90,4
9	382,0	39	88,1
10	343,7	40	85,9
11	312,5	41	83,8
12	286,5	42	81,8
13	264,4	43	79,9
14	245,5	44	78,1
15	229,2	45	76,4
16	214,8	46	74,7
17	202,2	47	73,1
18	191,0	48	71,6
19	180,9	49	70,1
20	171,8	50	68,7
21	162,7	51	67,4
22	156,2	52	66,1
23	149,4	53	64,8
24	143,2	54	63,6
25	137,5	55	62,5
26	132,2	56	61,4
27	127,3	57	60,3
28	122,7	58	59,2
29	118,5	59	58,2
30	114,6	60	57,3

Angles subtended by an extension of six feet at different distances.

Angles.	Distances in feet.	Angles.	Distances in feet.
Min. 1	20626,8	Min. 31	665,4
2	10313.	32	644,5
3	6875,4	33	625.
4	5156,5	34	606,6
5	4125,2	35	589,3
6	3437,7	36	572,9
7	2946,6	37	557,5
8	2578,2	38	542,8
9	2291,8	39	528,9
10	2062,6	40	515,6
11	1875,2	41	503,1
12	1718,8	42	491,1
13	1586,7	43	479,7
14	1473,3	44	468,8
15	1375.	45	458,4
16	1289,1	46	448,4
17	1213,3	47	438,9
18	1145,9	48	429,7
19	1085,6	49	421.
20	1031,4	50	412,5
21	982,2	51	404,4
22	937,6	52	396,7
23	896,8	53	389,2
24	859,4	54	381,9
25	825.	55	375.
26	793,3	56	368,3
27	763,9	57	361,9
28	736,6	58	355,6
29	711,3	59	349,6
30	687,5	60	343,7



Fig. 2.

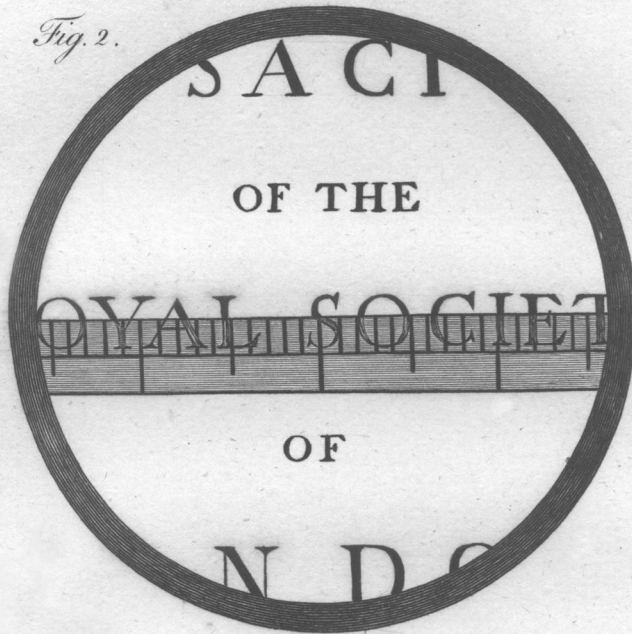


Fig. 1.

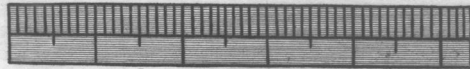


Fig. 3.

