Bismillah-i-Rahman-i-Raheem

Quadratic Recurrence

Golden Ratio

Quadratic Recurrence

Recurrence means repeating itself continuously,

Any recurrence associated with quadratic equation (Preferable with real solution) is known as quadratic Recurrence.

For example consider

$$X = 3$$

$$X^2 = 3X \quad \text{(multiply both sides by X)}$$

Taking square root of both side

$$X = \sqrt{3(X)} \quad \text{........ (1)}$$

Replace (X) by (1)

$$X = \sqrt{3\sqrt{3(X)}}$$

Again replace (X) by (1)

$$X = \sqrt{3\sqrt{3\sqrt{3(X)}}}$$

Continue this process until X is eliminated from right Side.

$$X = \sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{3(\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..}}}}}}$$

Now we have learnt about recurrence lets

Discuss what is golden ratio?

Consider human arm.

Measure length of arm

Now measure length from knee to tail of fingers .This is longer part of arm

Interesting fact

$$\text{Golden ratio} = \frac{\text{length of human arm}}{\text{Longer part of arm}}$$

Its exact value is 1.618033-----

For example

Length of my arm is 27.5 inch (approximately)

Length of longer part of my arm is about

17 inch and their ratio is

$\frac{27.5}{17} = 1.61764$----

Which is very close to 1.618-----

Mathematically the positive solution of quadratic equation;

$$X^2 = X+1 \text{ ......(2)}$$

Is golden ratio

Now we discover a non-periodic continued of (GR)

Multiply (2) by X

$$X(X^2) = X(X + 1)$$

Or

$$X^3 = X^2 + X$$

$$X^3 = (X + 1) + X \qquad \text{using (2)}$$

$$X^3 = 2X + 1 \text{ .... (3)}$$

Again multiply (3) by X

$X(X^3) = X\,(2X + 1)$

$X^4 = 2X^2 + X$

$X^4 = 2X + 2 + X$ using (2)

$X^4 = 3X + 2$ ………….. (4)

Lastly multiply (4) by X

$X(X^4) = X\,(3X + 2)$

$X^5 = 3X^2 + 2X$

$X^5 = 3X + 3 + 2X$ using (2)

$X^5 = 5X + 3$ …………. (5)

Continue the process for next power of X

Now by (2)

$$X^2 = X + 1$$

Take $\sqrt[2]{\quad}$ of both side

$$X = \sqrt[2]{1 + (X)}$$

$$X = \sqrt[2]{1 + \sqrt[3]{(X^3)}} \quad \text{rewriting}$$

$$X = \sqrt[2]{1 + \sqrt[3]{1 + 2(X)}} \quad \text{using (3)}$$

$$X = \sqrt[2]{1 + \sqrt[3]{1 + 2\sqrt[4]{(X^4)}}} \quad \text{rewriting}$$

$$X = \sqrt[2]{1 + \sqrt[3]{1 + 2\sqrt[4]{2 + 3X}}}$$

$$X = \sqrt[2]{1 + \sqrt[3]{1 + 2\sqrt[4]{2 + 3(\sqrt[5]{X^5})}}} \quad \text{continuing}$$

$$X = \sqrt[2]{1 + \sqrt[3]{1 + 2\sqrt[4]{2 + 3\sqrt[5]{3 + 5X}}}}$$

We can trace the next pattern of this sequence

Sequence is

(1 , 1) , (1 , 1+1) , (2 , 2+1), (3 , 3+2)

And so on

Now after (3 , 5) it will be (5 , 5+3) = (5 , 8)

$X^6 = 8X + 5$ by tracing

$$X = \sqrt[2]{1 + \sqrt[3]{1 + 2\sqrt[4]{2 + 3\sqrt[5]{3 + 5(\sqrt[6]{X^6})}}}}$$

further continue

$$X = \sqrt[2]{1 + \sqrt[3]{1 + 2\sqrt[4]{2 + 3\sqrt[5]{3 + 5\sqrt[6]{5 + 8X}}}}}$$

$$X = \sqrt[2]{1 + \sqrt[3]{1 + 2\sqrt[4]{2 + 3\sqrt[5]{3 + 5\sqrt[6]{5 + 8(\ldots\ldots\ldots\ldots..}}}}}$$

This Beautiful sequence is embedded in human arm

God is the creator and how brilliant he creates…

Written by : Mathemadict