NBS

Technical Note No. 228

A TABULATION OF AIRY FUNCTIONS

H. T. Dougherty and M. E. Johnson

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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Issued September 18, 1964

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For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington D.C. 20402
Price: 20 cents

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A tabulation is provided for Wait's formulation of the Airy function and its first derivative. The argument covers the range of real values from -6.0 to +6.0 in intervals of 0.1. As an aid for their application the tabulations are also graphed.

1. INTRODUCTION

A series of recent papers has indicated a renewed interest in Airy Functions and Airy Integrals [Fock, 1945; Rice, 1954; Wait and Conda 1958 and 1959; Wait, 1959; Spies and Wait, 1961] specifically for the diffraction of radio waves by convex surfaces. There is also a variety of formulations for these Airy Functions and Integrals whose histories and interrelations are described thoroughly by Logan [1959] in an unpublished report. For three of the formulations, that of Furry, $h(-t)$, of Miller, $Ai(t)$ and $Bi(t)$, and of Smirnov, $U_{1,2}(-t, 1)$, there are extensive tabulations available [Furry and Arnold, 1945; Miller, 1946; Smirnov, 1960]. Extensive tabulations are not available however for the formulation, $W(t)$, of Fock or of Wait. The purpose of this report is to provide a tabulation of Wait's formulation of the Airy Functions and Integrals for real values of the argument between -6.0 and +6.0 at intervals of 0.1. As an aid for their application, the tabulations are also graphed.

The Airy functions $u(t)$ and $v(t)$ are linearly independent solutions of the differential equation:

$$f''(t) - tf(t) = 0, \tag{1}$$

which arises, for example, in certain cases of the diffraction of radio waves by convex surfaces. For the formulation of Spies and Wait [1961], the Airy integrals are given by

$$W_1(t) = \frac{1}{\sqrt{\pi}} \int_{C_1} \exp\left(tz - \frac{z^3}{3}\right) dz = u(t) - jv(t) \qquad (2)$$

and

$$W_2(t) = \frac{1}{\sqrt{\pi}} \int_{C_2} \exp\left(tz - \frac{z^3}{3}\right) dz = u(t) + jv(t), \qquad (3)$$

where the contour of integration, C_1, for z extends from $\infty . e^{+j 2\pi/3}$ to zero and from zero to infinity along the real axis. The contour C_2 is the complex conjugate of C_1. These integral representations differ from those of Fock [1945]. The corresponding formulations of these integrals by Fock are given by the complex conjugates of (2) and (3). This is a result of Wait's choice of the $\exp(j\omega t)$ time dependency in the wave equation as contrasted to Fock's choice of $\exp(-j\omega t)$.

2. COMPUTATION OF THE TABLES

Note that for real values of t, $W_2(t) = W_1^*(t)$ where * indicates "complex conjugate of." Because of this relationship $W_2(t)$ may be obtained from the tabulations of $W_1(t)$ which are given in terms of

$$W_1(t) = |W(t)| \exp[-j\theta(t)] \qquad (4)$$

and

$$u(t) = |W(t)| \cos\theta(t), \quad v(t) = |W(t)| \sin\theta(t). \qquad (5)$$

Rather than recomputing u(t) and v(t) directly from their series representation, as given by Spies and Wait [1961] for example, the authors have simply transformed the tabulations given by Furry and Arnold [1945]. There, the real and imaginary components, $R[h_2(-t)]$ and $I[h_2(-t)]$, of the modified function are tabulated. Note that

$$|W(t)| = \frac{1}{\alpha}\sqrt{R^2[h_2(-t)] + I^2[h_2(-t)]} \tag{6}$$

and

$$\theta(t) = \frac{2\pi}{3} - \arctan\left(\frac{I[h_2(-t)]}{R[h_2(-t)]}\right), \tag{7}$$

where $\alpha = 0.8536\ 6721\ 8838\ 952$. The determination of u(t) and v(t) from (5) and (6) and Furry's tabulation of $h_2(-t)$ was carried out by a double precision program on an IBM 7090 computer. The accuracy of the input was checked by means of the Wronskian for the modified Hankel function of the second kind [Furry and Arnold, 1945]:

$$I[h_2(-t)]\ R[h'_2(-t)] - I[h'_2(-t)]\ R[h_2(-t)] = 0.7287\ 4772\ 0520, \tag{8}$$

where the prime indicates differentiation with respect to the argument. Similarly, the accuracy of the output was checked by means of the Airy function Wronskian [Spies and Wait, 1961] given by

$$u'(t)\ v(t) - u(t)\ v'(t) = \text{Unity}. \tag{9}$$

This procedure was intended to maintain the accuracy within a unit in the eighth significant figure.

3. APPLICATION

Recent examples of the application of this formulation of the Airy integrals are given by Spies and Wait [1961] for VLF ionospheric propagation and by Dougherty and Maloney [1964] for VHF propagation over irregular terrain. For such applications, integral expressions for the electromagnetic fields are often encountered which are readily evaluated by numerical integration. The numerical integration involving tabulated values of the Airy functions are generally carried out by means of computer programs. However in some instances sufficient accuracy is achievable even by graphical integration techniques. For example, the integrals

$$I_n = \frac{e^{j\pi/12}}{\sqrt{\pi}} \int_0^\infty \left[- te^{-j2\pi/3} \right]^n \frac{W_1(t)\, v(t)}{|W(t)|^2}\, dt$$

$$+ \frac{e^{-j\pi/4}}{\sqrt{\pi}} \int_0^\infty [-jt]^n \frac{W_2(t)\, v(t)}{|W(t)|^2}\, dt \qquad (10)$$

have been evaluated for $n = 0, 1, 2, 3$ by computer programs based upon the series expansions of the Airy functions [Wait, 1959; Dougherty and Maloney, 1964]. Graphical integration, which involves "counting squares" under the curve obtained by a point-by-point plot of the integral, will also provide an evaluation of (10). This is feasible due to the rapidity with which $v(t)$ decreases to very small values requiring plotting only for $t < 6.0$. The graphical evaluation of (10) from tabulations of $u(t)$ and $v(t)$ for t equal to multiples of 0.5 and for $n = 0, 1, 2, 3$ has provided values which agree with the computer evaluation within 2% or less. J. R. Wait reported similar experience with the graphical

integration technique at the August 1963 VLF Symposium at the National Bureau of Standards, Boulder, Colorado.

4. TABULATION FOR REAL VALUES OF THE ARGUMENT

Table I is a tabulation of the values of $u(t)$, $v(t)$, $|W(t)|$, and $\theta(t)$ for t in the range from -6.0 to +6.0 at intervals of 0.1. For larger values of (t) see asymptotic expressions given by Spies and Wait [1961]. Table II is a similar tabulation for their first derivatives with respect to t, $u'(t)$, $v'(t)$, etc. Figure 1 is a plot of $v(t)$ versus $u(t)$ for the tabulated values. The Airy function $v(t)$ is plotted versus t in Figure 2. Figures 3 and 4 are plots of $\log_{10}|u(t)|$ and $\log_{10}|W(t)|$, respectively. The corresponding plots for the first derivative of the Airy functions are presented in Figures 5, 6, 7, and 8.

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TABLE I

t	u(t)	v(t)	|W(t)|	θ(t)
-6.0	-0.26001610	0.58339462	0.63871564	245.97771
-5.9	-0.39495534	0.50536697	0.64139340	231.99160
-5.8	-0.50672720	0.39765342	0.64412786	218.12300
-5.7	-0.58926690	0.26696727	0.64692118	204.37290
-5.6	-0.63838864	0.12111301	0.64977566	190.74231
-5.5	-0.65193237	-0.03151696	0.65269375	177.23226
-5.4	-0.62978312	-0.18244682	0.65567798	163.84382
-5.3	-0.57377180	-0.32359323	0.65873110	150.57807
-5.2	-0.48747274	-0.44768699	0.66185596	137.43615
-5.1	-0.37591820	-0.54862054	0.66505563	124.41919
-5.0	-0.24525291	-0.62170769	0.66833334	111.52838
-4.9	-0.10235310	-0.66384840	0.67169254	98.76494
-4.8	0.04556587	-0.67359747	0.67513688	86.13010
-4.7	0.19133098	-0.65114190	0.67867026	73.62515
-4.6	0.32816231	-0.59819604	0.68229686	61.25142
-4.5	0.44997757	-0.51782732	0.68602110	49.01027
-4.4	0.55163833	-0.41422823	0.68984772	36.90309
-4.3	0.62913083	-0.29245100	0.69378181	24.93133
-4.2	0.67967819	-0.15812196	0.69782878	13.09648
-4.1	0.70178491	-0.01715200	0.70199448	1.40007
-4.0	0.69521791	0.12454242	0.70628519	349.84368
-3.9	0.66093133	0.26129497	0.71070760	338.42896
-3.8	0.60094422	0.38791212	0.71526901	327.15759
-3.7	0.51818151	0.49985514	0.71997724	316.03131
-3.6	0.41628969	0.59337763	0.72484075	305.05195
-3.5	0.29943806	0.66561637	0.72986870	294.22136
-3.4	0.17211623	0.71463652	0.73507099	283.54148
-3.3	0.03893726	0.73943395	0.74045843	270.01431
-3.2	-0.09554546	0.73989920	0.74604273	262.64194
-3.1	-0.22700109	0.71674883	0.75183666	252.42652
-3.0	-0.35145921	0.67143086	0.75785419	242.37028
-2.9	-0.46541978	0.60601100	0.76411054	232.47554
-2.8	-0.56593368	0.52304686	0.77062245	222.74472
-2.7	-0.65065382	0.42545646	0.77740826	213.18030
-2.6	-0.71785848	0.31638732	0.78448819	203.78490
-2.5	-0.76644888	0.19909100	0.79188453	194.56122
-2.4	-0.79592443	0.07680776	0.79962187	185.51206
-2.3	-0.80633929	-0.04733574	0.80772751	176.64034
-2.2	-0.79824406	-0.17041324	0.81623174	167.94911
-2.1	-0.77261679	-0.28976875	0.82516824	159.44153
-2.0	-0.73078731	-0.40306918	0.83457465	151.12087

t	u(t)	v(t)	|W(t)|	θ(t)
-2.0	-0.73078731	-0.40306918	0.83457465	151.12087
-1.9	-0.67435821	-0.50833986	0.84449299	142.99055
-1.8	-0.60512608	-0.60398413	0.85497041	135.05412
-1.7	-0.52500553	-0.68878803	0.86605991	127.31525
-1.6	-0.43595842	-0.76191229	0.87782121	119.77776
-1.5	-0.33992982	-0.82287335	0.89032177	112.44557
-1.4	-0.23879217	-0.87151588	0.90363800	105.32278
-1.3	-0.13429820	-0.90797848	0.91785671	98.41357
-1.2	-0.02804264	-0.93265524	0.93307673	91.72223
-1.1	0.07856683	-0.94615450	0.94941091	85.25317
-1.0	0.18433057	-0.94925694	0.96698837	79.01084
-0.9	0.28827003	-0.94287445	0.98595732	72.99976
-0.8	0.38963412	-0.92801066	1.0064882	67.22444
-0.7	0.48789985	-0.90572461	1.0287776	61.68935
-0.6	0.58276837	-0.87709794	1.0530525	56.39884
-0.5	0.67415753	-0.84320609	1.0795762	51.35708
-0.4	0.76219226	-0.80509394	1.1086538	46.56799
-0.3	0.84719366	-0.76375584	1.1406402	42.03507
-0.2	0.92966805	-0.72011996	1.1759487	37.76133
-0.1	1.0102969	-0.67503668	1.2150615	33.74914
-0.0	1.0899290	-0.62927084	1.2585416	30.00000
0.1	1.1695743	-0.58349735	1.3070475	26.51449
0.2	1.2504027	-0.53829983	1.3613499	23.29194
0.3	1.3337459	-0.49417163	1.4223515	20.33038
0.4	1.4211056	-0.45151906	1.4911105	17.62630
0.5	1.5141666	-0.41066622	1.5688681	15.17450
0.6	1.6148177	-0.37186092	1.6570808	12.96803
0.7	1.7251801	-0.33528163	1.7574584	10.99811
0.8	1.8476451	-0.30104475	1.8720098	9.25413
0.9	1.9849228	-0.26921235	2.0030961	7.72382
1.0	2.1401026	-0.23979956	2.1534955	6.39336
1.1	2.3167287	-0.21278206	2.3264798	5.24767
1.2	2.5188938	-0.18810301	2.5259075	4.27075
1.3	2.7513520	-0.16567953	2.7563359	3.44605
1.4	3.0196582	-0.14540865	3.0231571	2.75690
1.5	3.3303370	-0.12717266	3.3327643	2.18685
1.6	3.6910893	-0.11084379	3.6927532	1.72009
1.7	4.1110427	-0.09628818	4.1121702	1.34173
1.8	4.6010586	-0.08336952	4.6018138	1.03807
1.9	5.1741062	-0.07195173	5.1746065	0.79672
2.0	5.8457211	-0.06190141	5.8460488	0.60670

t	u(t)	v(t)	|W(t)|	θ(t)
2.0	5.8457211	-0.06190141	5.8460488	0.60670
2.1	6.6345669	-0.05308960	6.6347793	0.45847
2.2	7.5631253	-0.04539326	7.5632616	0.34389
2.3	8.6585462	-0.03869619	8.6586326	0.25607
2.4	9.9536942	-0.03288982	9.9537485	0.18933
2.5	11.488444	-0.02787347	11.488478	0.13902
2.6	13.311283	-0.02355463	13.311304	0.10139
2.7	15.481298	-0.01984888	15.481311	0.07347
2.8	18.070646	-0.01667968	18.070653	0.05289
2.9	21.167625	-0.01397812	21.167630	0.03784
3.0	24.880517	-0.01168249	24.880520	0.02691
3.1	29.342372	-0.00973786	29.342374	0.01902
3.2	34.717010	-0.00809557	34.717011	0.01337
3.3	41.206544	-0.00671279	41.206544	0.00934
3.4	49.060830	-0.00555193	49.060830	0.00649
3.5	58.589360	-0.00458019	58.589360	0.00449
3.6	70.176259	-0.00376908	70.176259	0.00308
3.7	84.299229	-0.00309394	84.299229	0.00211
3.8	101.55350	-0.00253353	101.55350	0.00144
3.9	122.68224	-0.00206961	122.68224	0.00097
4.0	148.61506	-0.00168660	148.61506	0.00066
4.1	180.51713	-0.00137122	180.51713	0.00044
4.2	219.85164	-0.00111220	219.85164	0.00029
4.3	268.45959	-0.00090002	268.45959	0.00020
4.4	328.66176	-0.00072666	328.66176	0.00013
4.5	403.38936	-0.00058535	403.38936	0.00009
4.6	496.35159	-0.00047028	496.35159	0.00006
4.7	612.25098	-0.00037728	612.25098	0.00004
4.8	757.06070	-0.00030189	757.06070	0.00003
4.9	938.38201	-0.00024104	938.38201	0.00002
5.0	1165.9060	-0.00019203	1165.9060	0.00001
5.1	1452.0113	-0.00015266	1452.0113	0.00000
5.2	1812.5381	-0.00012110	1812.5381	0.00000
5.3	2267.7937	-0.00009587	2267.7937	0.00000
5.4	2843.8598	-0.00007573	2843.8598	0.00000
5.5	3574.2950	-0.00005970	3574.2950	0.00000
5.6	4502.3568	-0.00004697	4502.3568	0.00000
5.7	5683.9044	-0.00003687	5683.9044	0.00000
5.8	7191.1997	-0.00002889	7191.1997	0.00000
5.9	9117.8891	-0.00002259	9117.8891	0.00000
6.0	11585.548	-0.00001762	11585.548	0.00000

TABLE II

t	u'(t)	v'(t)	|W'(t)|	θ'(t)
-6.0	-1.4408255	-0.61315469	1.5658662	156.94745
-5.9	-1.2451125	-0.93874220	1.5593402	142.98584
-5.8	-0.98017057	-1.20426100	1.5527327	129.14279
-5.7	-0.66362273	-1.39636967	1.5460411	115.41935
-5.6	-0.31520893	-1.50664347	1.5392632	101.81658
-5.5	0.04450912	-1.53174968	1.5323962	88.33559
-5.4	0.39539008	-1.47330453	1.5254374	74.97754
-5.3	0.71883096	-1.33745010	1.5183842	61.74359
-5.2	0.99870504	-1.13420236	1.5112335	48.63495
-5.1	1.2220805	-0.87663086	1.5039822	35.65289
-5.0	1.3796989	-0.57993416	1.4966271	22.79867
-4.9	1.4662075	-0.26047526	1.4891648	10.07364
-4.8	1.4801580	0.06516445	1.4815917	357.47917
-4.7	1.4237931	0.38106017	1.4739042	345.01668
-4.6	1.3026551	0.67270634	1.4660982	332.68765
-4.5	1.1250558	0.92763593	1.4581697	320.49359
-4.4	0.90145110	1.13587737	1.4501142	308.43610
-4.3	0.64376354	1.29024149	1.4419273	296.51682
-4.2	0.36469463	1.38644107	1.4336042	284.73745
-4.1	0.07706401	1.42305452	1.4251396	273.09977
-4.0	-0.20679320	1.40135266	1.4165283	261.60563
-3.9	-0.47554647	1.32501219	1.4077648	250.25696
-3.8	-0.71929000	1.19974311	1.3988429	239.05577
-3.7	-0.92985342	1.03285813	1.3897565	228.00418
-3.6	-1.1010001	0.83281220	1.3804990	217.10439
-3.5	-1.2285166	0.60873763	1.3710633	206.35871
-3.4	-1.3102007	0.36999805	1.3614420	195.76958
-3.3	-1.3457624	0.12577974	1.3516276	185.33956
-3.2	-1.3366509	-0.11526471	1.3416116	175.07136
-3.1	-1.2858258	-0.34531025	1.3313855	164.96782
-3.0	-1.1974897	-0.55758521	1.3209401	155.03198
-2.9	-1.0767981	-0.74652710	1.3102659	145.26705
-2.8	-0.92956412	-0.90787032	1.2993529	135.67644
-2.7	-0.76196966	-1.03867070	1.2881904	126.26381
-2.6	-0.58029637	-1.13727371	1.2767675	117.03305
-2.5	-0.39068455	-1.20323513	1.2650728	107.98836
-2.4	-0.19892669	-1.23720397	1.2530943	99.13426
-2.3	-0.01029982	-1.24077734	1.2408200	90.47561
-2.2	0.17056179	-1.21633728	1.2282376	82.01771
-2.1	0.33975394	-1.16687849	1.2153345	73.76631
-2.0	0.49415156	-1.09583558	1.2020987	65.72769

t	u'(t)	v'(t)	|W'(t)|	θ'(t)
-2.0	0.49415156	-1.09583558	1.2020987	65.72769
-1.9	0.63142341	-1.00691634	1.1885184	57.90875
-1.8	0.75001628	-0.90394726	1.1745829	50.31709
-1.7	0.84911411	-0.79073518	1.1602831	42.96113
-1.6	0.92857712	-0.67094856	1.1456122	35.85021
-1.5	0.98886637	-0.54801963	1.1305672	28.99478
-1.4	1.0309588	-0.42506832	1.1151498	22.40655
-1.3	1.0562579	-0.30484754	1.0993693	16.09872
-1.2	1.0665034	-0.18970852	1.0832446	10.08622
-1.1	1.0636844	-0.08158455	1.0668086	4.38600
-1.0	1.0499584	0.01800913	1.0501128	359.01735
-0.9	1.0275788	0.10796193	1.0332347	354.00226
-0.8	0.99883197	0.18754333	1.0162862	349.36581
-0.7	0.96598578	0.25637000	0.99942689	345.13651
-0.6	0.93124929	0.31436702	0.98287937	341.34658
-0.5	0.89674415	0.36172535	0.96695156	338.03202
-0.4	0.86448753	0.39885779	0.95206419	335.23232
-0.3	0.83638591	0.42635513	0.93878649	332.98939
-0.2	0.81423923	0.44494382	0.92787959	331.34546
-0.1	0.79975486	0.45544638	0.92034735	330.33925
-0.0	0.79457042	0.45874545	0.91749090	330.00000
0.1	0.80028529	0.45575180	0.92095942	330.33907
0.2	0.81850019	0.44737703	0.93278550	331.33981
0.3	0.85086475	0.43451060	0.95539013	332.94799
0.4	0.89913335	0.41800139	0.99154725	335.06659
0.5	0.96522973	0.39864353	1.0443108	337.55919
0.6	1.0513212	0.37716623	1.1169291	340.26433
0.7	1.1599042	0.35422701	1.2127880	343.01768
0.8	1.2939035	0.33040819	1.3354235	345.67521
0.9	1.4567867	0.30621593	1.4886220	348.12927
1.0	1.6526996	0.28208150	1.6765995	350.31414
1.1	1.8866258	0.25836423	1.9042345	352.20213
1.2	2.1645762	0.23535595	2.1773338	353.79457
1.3	2.4938174	0.21328621	2.5029216	355.11162
1.4	2.8831463	0.19232824	2.8895541	356.18359
1.5	3.3432241	0.17260512	3.3476768	357.04454
1.6	3.8869831	0.15419623	3.8900404	357.72828
1.7	4.5301250	0.13714342	4.5322004	358.26598
1.8	5.2917319	0.12145701	5.2931255	358.68517
1.9	6.1950201	0.10712141	6.1959462	359.00937
2.0	7.2682696	0.09410025	7.2688787	359.25825

t	u'(t)	v'(t)	|W'(t)|	θ'(t)
2.0	7.2682696	0.09410025	7.2688787	359.25825
2.1	8.5459747	0.08234110	8.5463713	359.44797
2.2	10.070270	0.07177953	10.070526	359.59162
2.3	11.892705	0.06234273	11.892869	359.69966
2.4	14.076448	0.05395264	14.076552	359.78040
2.5	16.699038	0.04652848	16.699102	359.84036
2.6	19.855817	.03998888	19.855857	359.88461
2.7	23.664237	0.03425366	23.664261	359.91707
2.8	28.269239	0.02924510	28.269254	359.94073
2.9	33.850027	0.02488895	33.850036	359.95788
3.0	40.628568	0.02111519	40.628573	359.97023
3.1	48.880315	0.01785850	48.880319	359.97907
3.2	58.947736	0.01505844	58.947738	359.98537
3.3	71.257403	0.01265973	71.257404	359.98983
3.4	86.341650	0.01061207	86.341650	359.99296
3.5	104.86602	0.00887009	104.86602	359.99516
3.6	127.66417	0.00739313	127.66417	359.99669
3.7	155.78225	0.00614499	155.78225	359.99774
3.8	190.53549	0.00509359	190.53549	359.99847
3.9	233.58048	0.00421070	233.58048	359.99897
4.0	287.00757	0.00347160	287.00757	359.99931
4.1	353.45932	0.00285473	353.45932	359.99954
4.2	436.28240	0.00234140	436.28240	359.99970
4.3	539.72298	0.00191549	539.72298	359.99980
4.4	669.17814	0.00156312	669.17814	359.99987
4.5	831.52027	0.00127238	831.52027	359.99992
4.6	1035.5159	0.00103319	1035.5159	359.99995
4.7	1292.3681	0.00083692	1292.3681	359.99997
4.8	1616.4187	0.00067632	1616.4187	359.99998
4.9	2026.0605	0.00054523	2026.0605	359.99998
5.0	2544.9230	0.00043853	2544.9230	359.99999
5.1	3203.4169	0.00035189	3203.4169	360.00000
5.2	4040.7486	0.00028171	4040.7486	360.00000
5.3	5107.5521	0.00022502	5107.5521	360.00000
5.4	6469.3341	0.00017934	6469.3341	360.00000
5.5	8210.9876	0.00014261	8210.9876	360.00000
5.6	10442.720	0.00011315	10442.720	360.00000
5.7	13307.847	0.00008959	13307.847	360.00000
5.8	16993.055	0.00007077	16993.055	360.00000
5.9	21741.942	0.00005580	21741.942	360.00000
6.0	27872.904	0.00004389	27872.904	360.00000

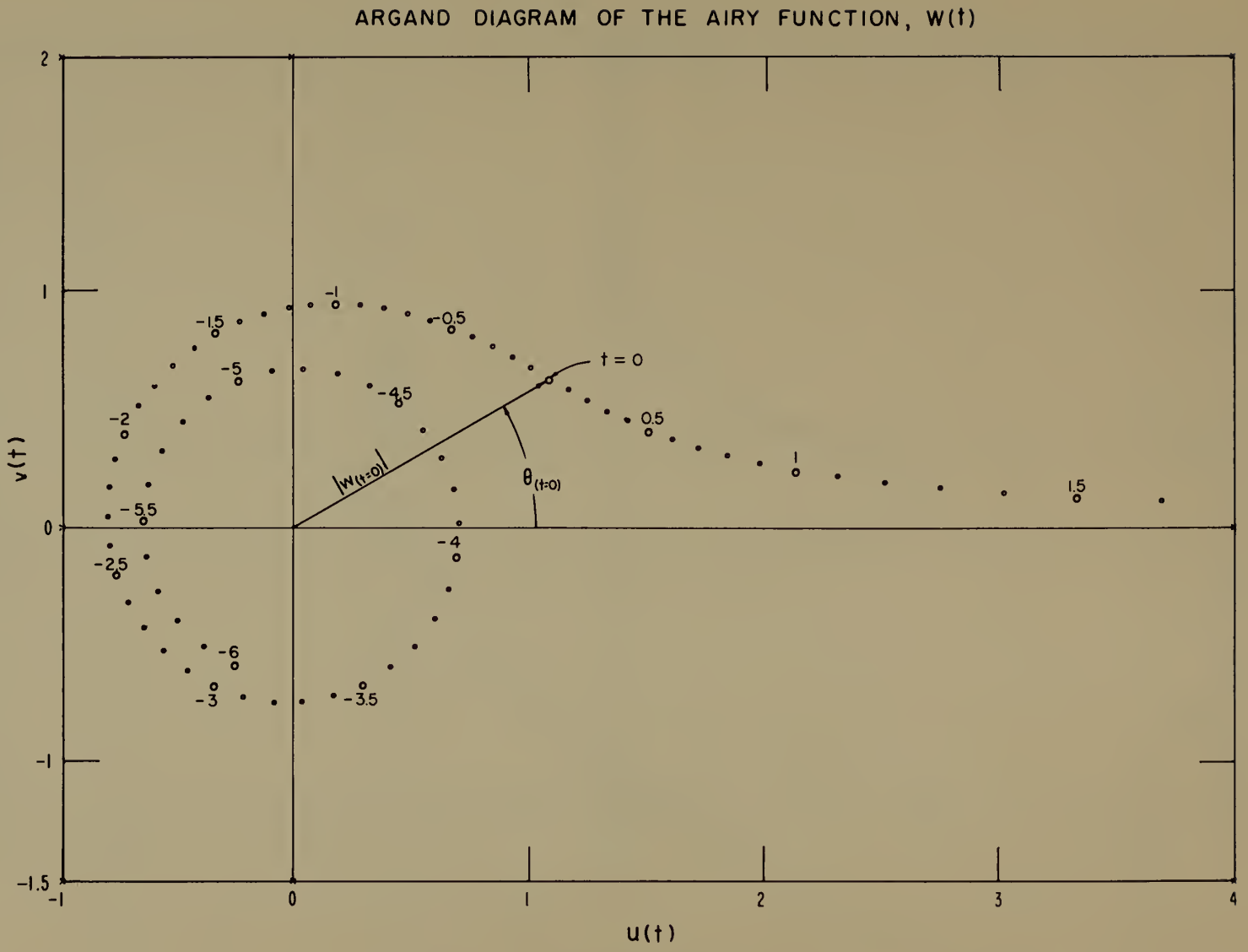
ARGAND DIAGRAM OF THE AIRY FUNCTION, W(t)
v(t)
u(t)
t = 0
|W(t=0)|
θ(t=0)
-1
-0.5
-1.5
-5
-4.5
-2
0.5
1
1.5
-5.5
-4
-2.5
-6
-3
-3.5
2
1
0
-1
-1.5
-1
0
1
2
3
4

FIGURE 1

GRAPH OF $v(t)$, THE IMAGINARY PART
OF THE AIRY FUNCTION, $W(t)$

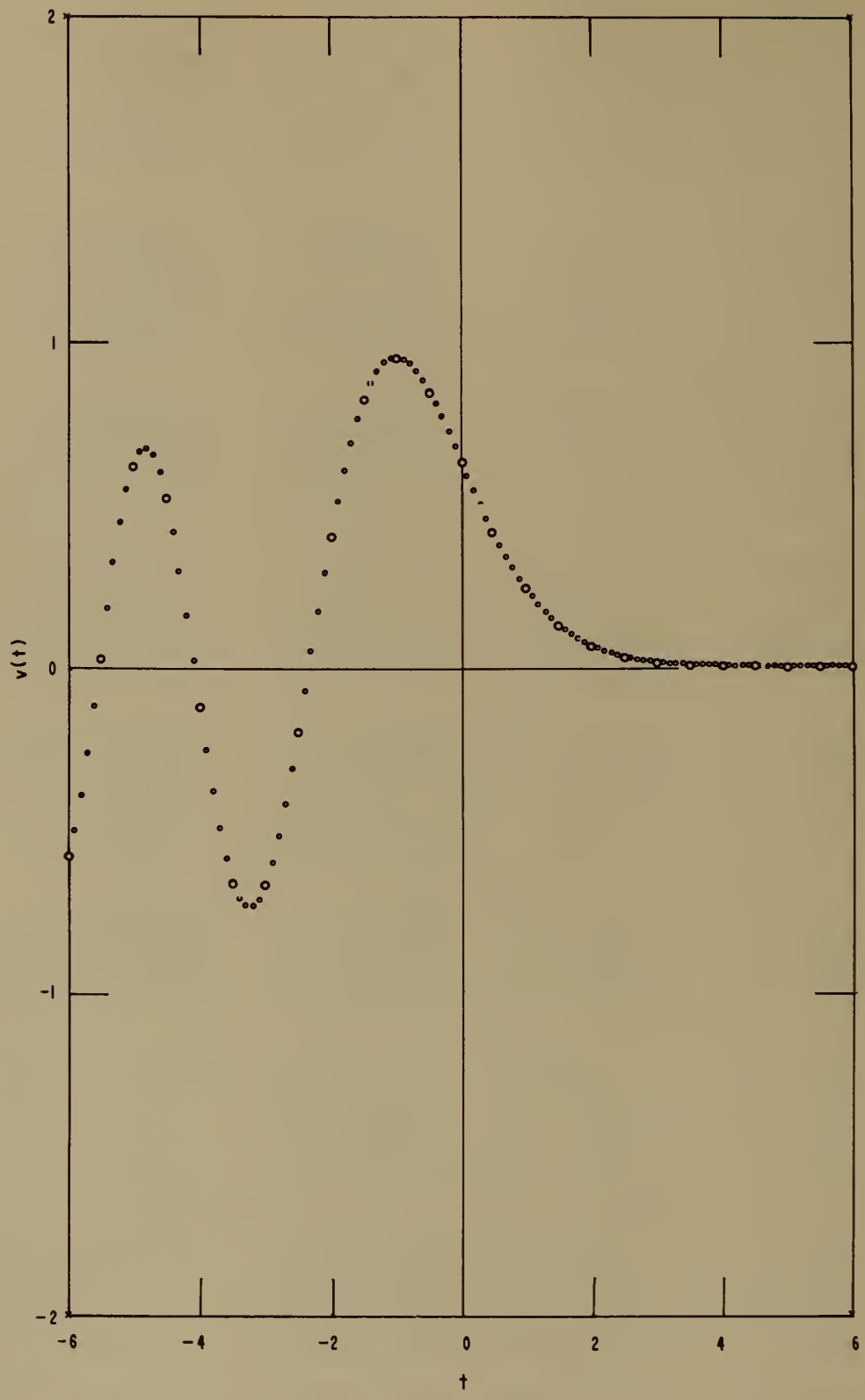

FIGURE 2

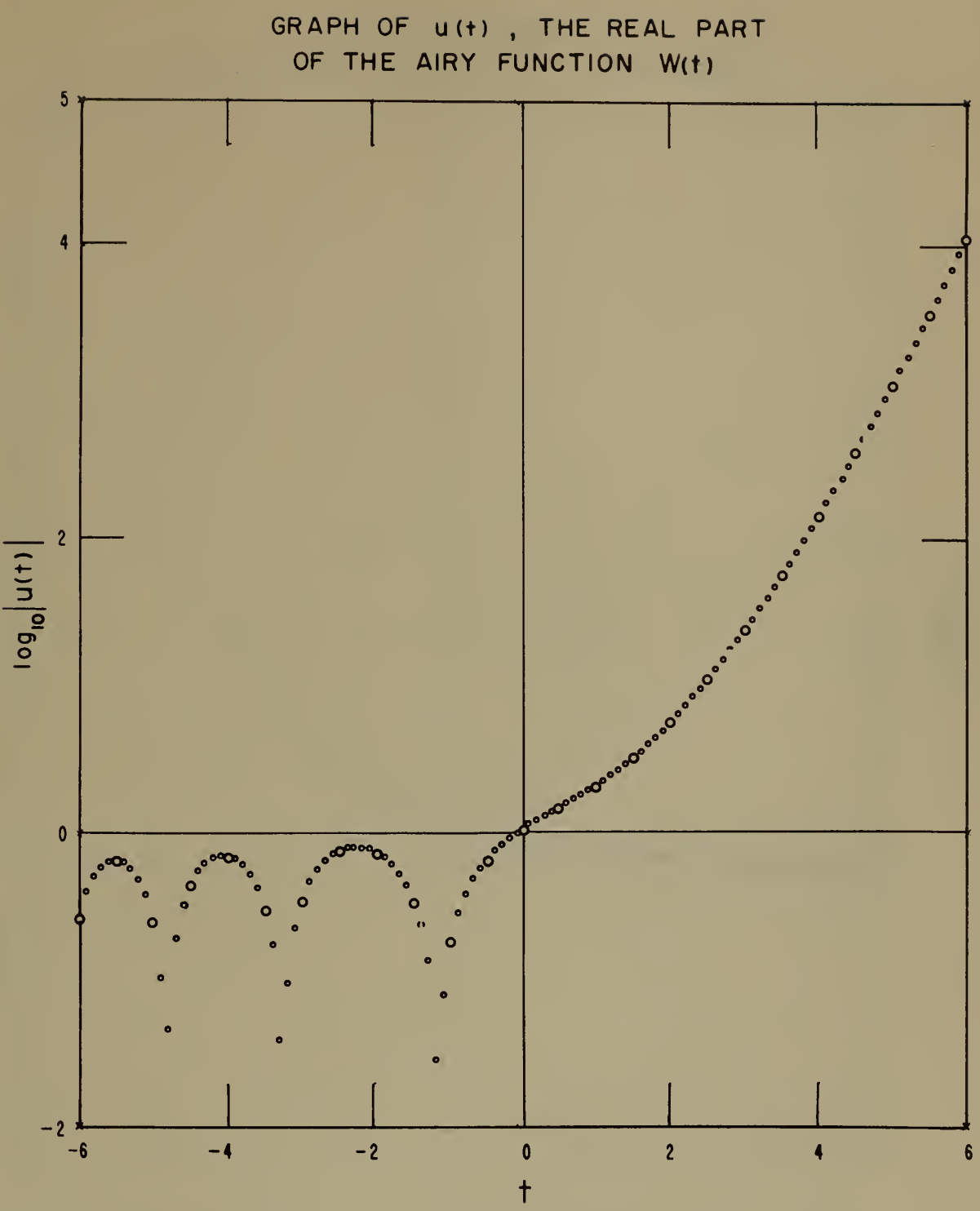
GRAPH OF u(t) , THE REAL PART
OF THE AIRY FUNCTION W(t)
5
4
2
0
-2
$\log_{10}|u(t)|$
-6
-4
-2
0
2
4
6
t

FIGURE 3

GRAPH OF THE
MAGNITUDE OF THE AIRY FUNCTION, W(t)

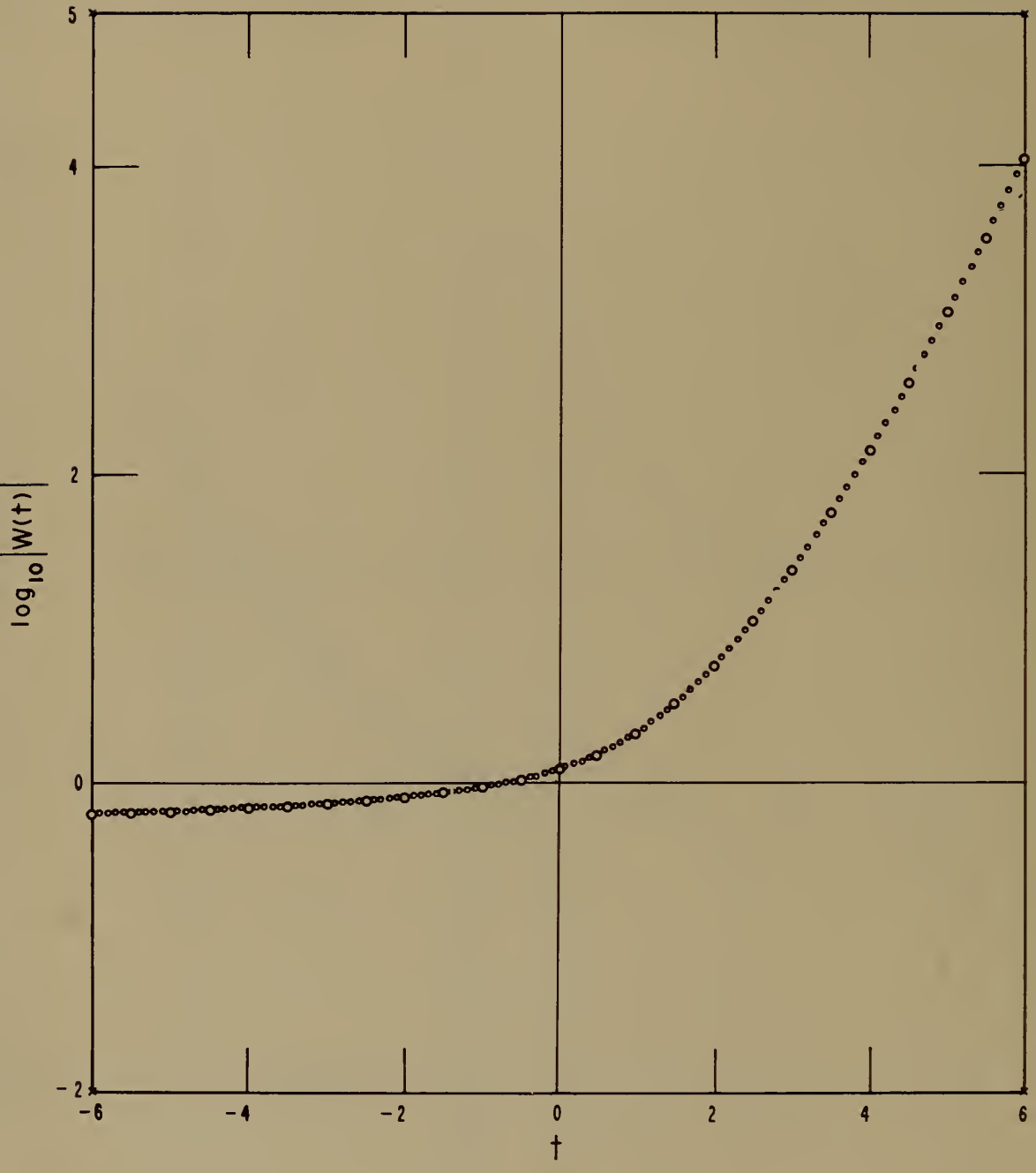

FIGURE 4

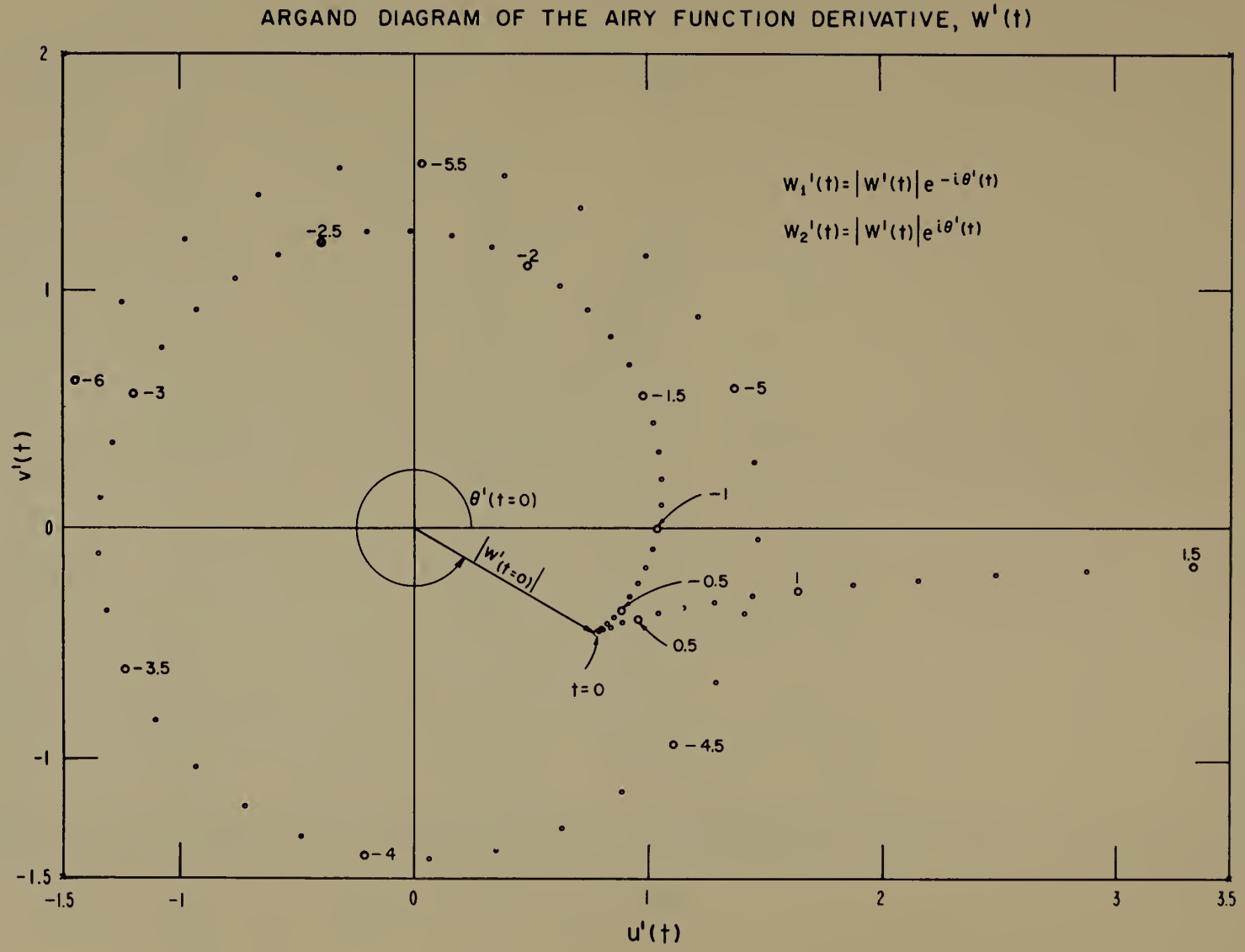
ARGAND DIAGRAM OF THE AIRY FUNCTION DERIVATIVE, W'(t)
$W_1'(t) = |W'(t)| e^{-i\theta'(t)}$
$W_2'(t) = |W'(t)| e^{i\theta'(t)}$
-5.5
-2.5
-2
-6
-3
-1.5
-5
θ'(t=0)
-1
|W'(t=0)|
-0.5
1
1.5
0.5
t=0
-3.5
-4.5
-4
v'(t)
u'(t)
2
1
0
-1
-1.5
-1.5
-1
0
1
2
3
3.5

FIGURE 5

GRAPH OF V'(t), THE IMAGINARY PART
OF THE AIRY FUNCTION DERIVATIVE, W'(t)

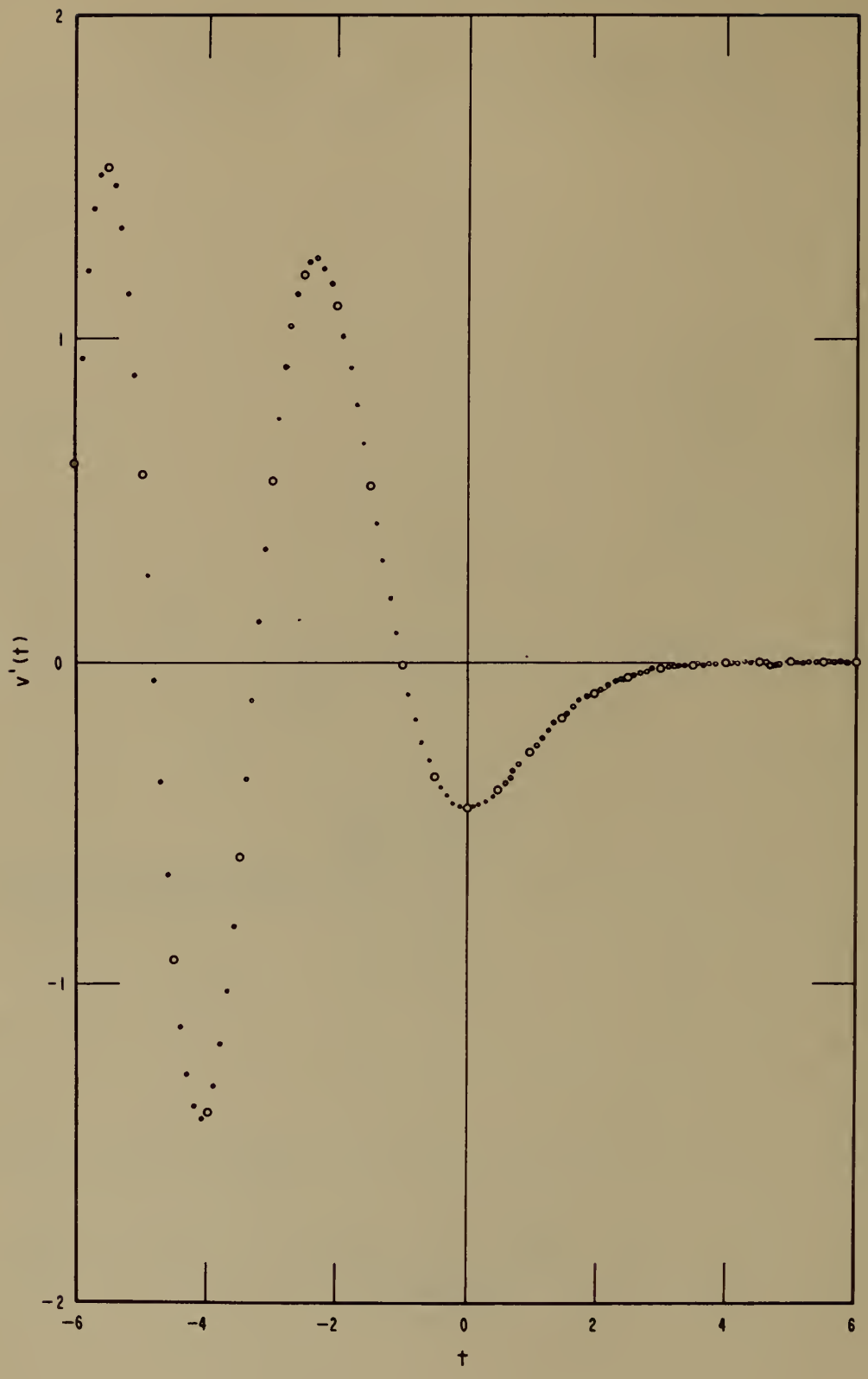

FIGURE 6

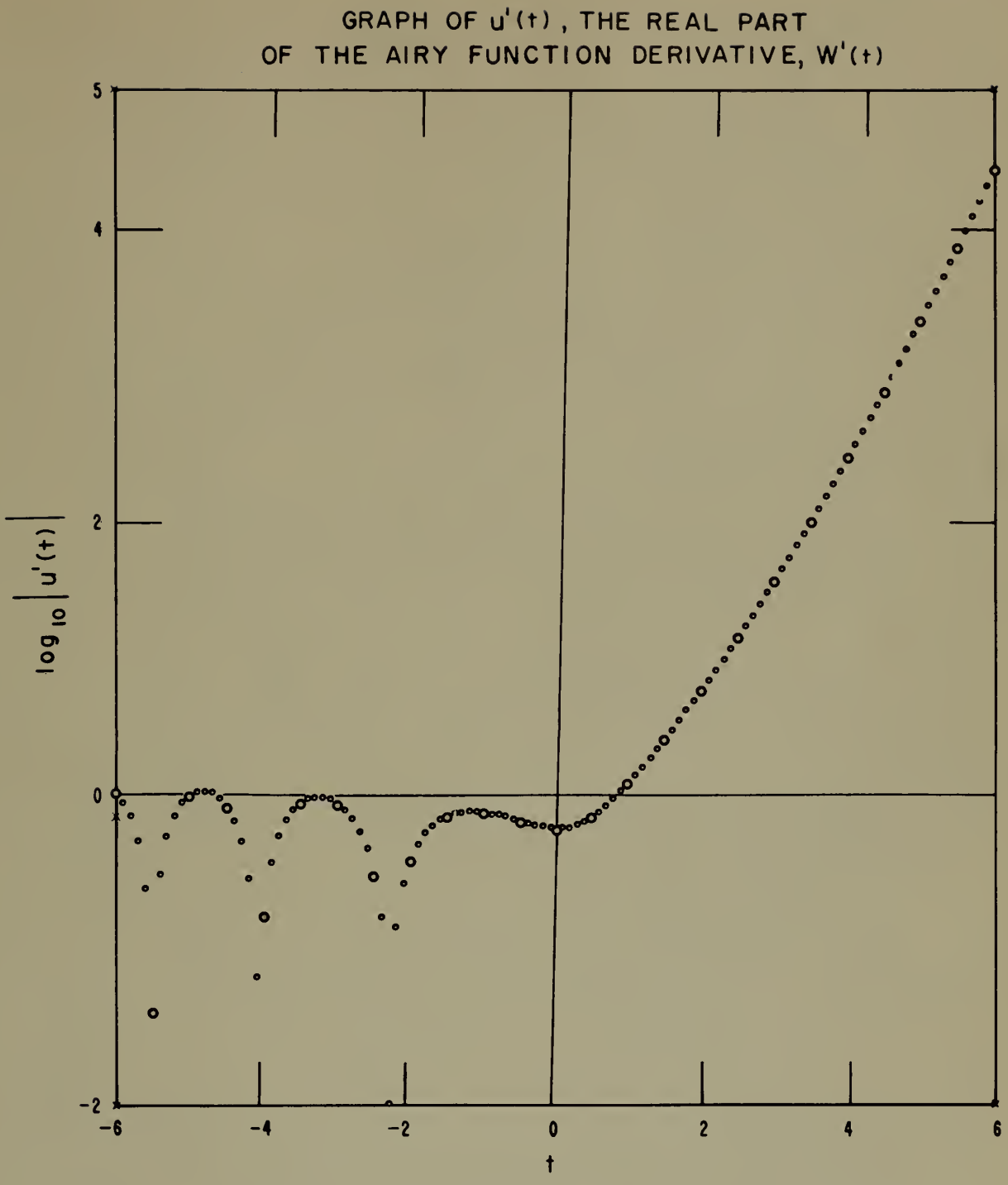
GRAPH OF u'(t), THE REAL PART
OF THE AIRY FUNCTION DERIVATIVE, W'(t)
5
4
2
0
-2
-6
-4
-2
0
2
4
6
t
log10 |u'(t)|

FIGURE 7

GRAPH OF THE
MAGNITUDE OF THE AIRY FUNCTION DERIVATIVE, $W'(t)$

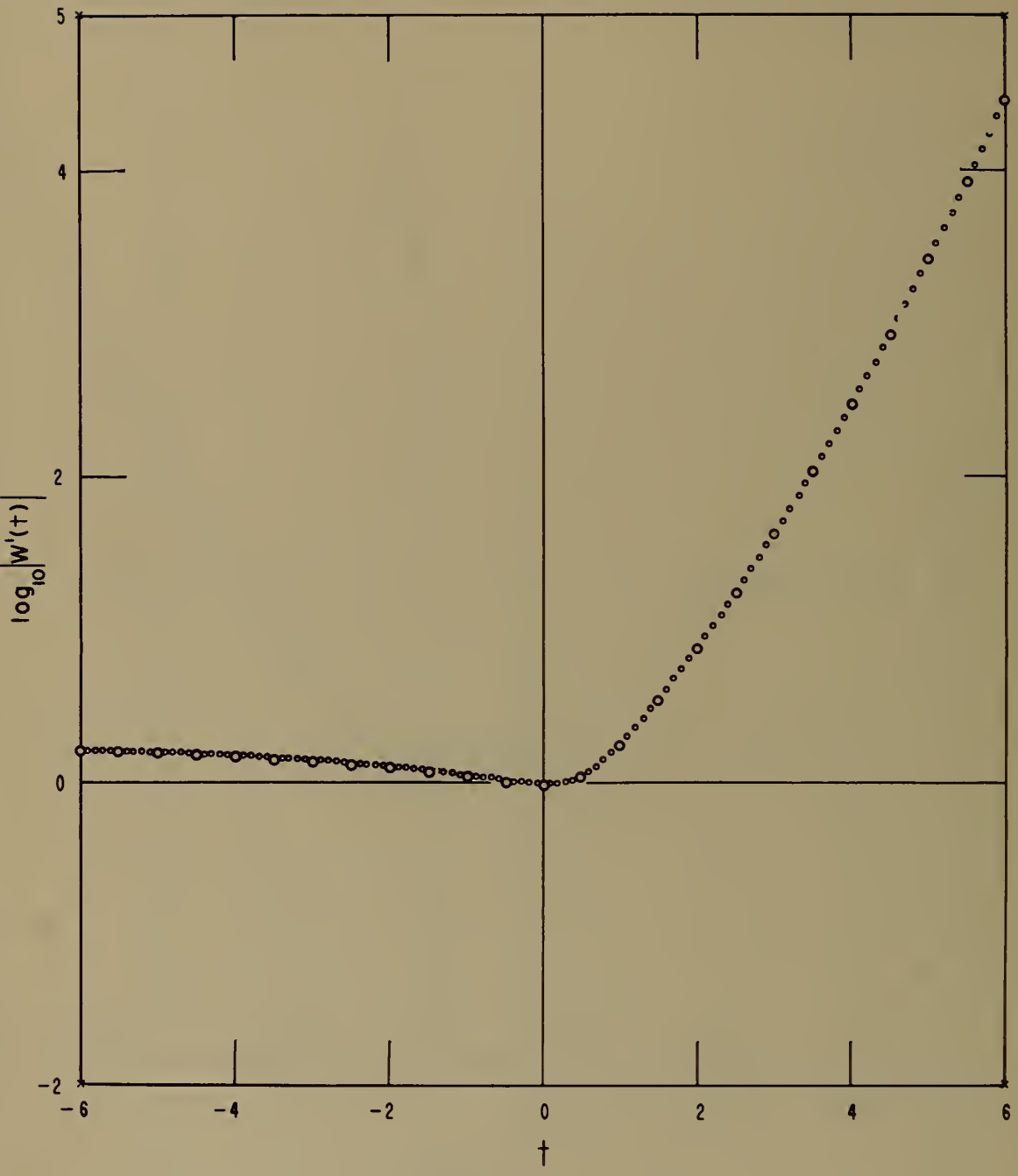

FIGURE 8

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