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RESEARCH AND EXPERIMENTS DEPARTMENT

DENSITY OF BOMBING OF INDUSTRIAL TARGETS

1. SUMMARY. This note presents a simple method for estimating the density of bombs of a given weight and type required to do specified damage to industrial targets. It is a natural development of the method used successfully in analysing damage to cities. Thus this note is largely concerned with showing how the fundamental concept of the Mean Area of Effectiveness of a bomb, which is usually applied only to fairly densely built-up areas, can be adapted to a scattered group of buildings such as an industrial target presents. While the method is subject to certain limitations, which are discussed, it gives good and conservative estimates of the average damage which a given bomb load may be expected to cause to a given industrial complex. The load required to reach a specified level of average damage is then at once obtained. It is proposed in a subsequent note to give refinements of the method, appropriate to exceptional and highly specialised targets.

2. DAMAGE BY DIRECT HITS AND BY NEAR MISSES. Experience has shown that direct hits by bombs of a given calibre on industrial buildings of a given type do visible structural damage whose extent is reasonably constant in area. Near misses are effective only if they fall within a certain critical distance of the building; if, however, a bomb falls within this distance of a building, the area of the building which is damaged is again fairly constant, although of course smaller than the area of building damaged by a direct hit. We shall write

d_1 for the area of damage caused by a direct hit, and
d_2 for the area of damage caused by an (effective) near miss, from a bomb of the given calibre.

In addition, we shall write

A_1 for the area of some one given building, which is being specifically examined, and
A_2 for the area surrounding this building within which a near miss will be effective against the building.
A is the whole target area.

All these areas are measured in the same unit: say, in acres.

It is not assumed that the building of area A_1 which is being specifically examined is at or near an aiming point of the attack. On the contrary, we shall make the simplest possible assumption, that bombing over the whole target area A is random. This is usually a conservative assumption; and is reasonably realistic, if the number of aiming points for the attack is large.

3. EXPECTATION OF DAMAGE FROM ONE BOMB. We begin by examining the damage to be expected to the building of area A_1 from one bomb dropped in the target area. Since we assume this bomb to fall at random, it is as likely to fall on one point of the target area A as on another. Therefore the probability that it will fall at one of the points making up the area A_1, of all the points making up the area A, is A_1/A. Similarly, the probability that the bomb will fall in the near-miss area A_2 is A_2/A. In the first case, it does damage d_1; in the second, d_2. Therefore the expected damage is

$$(d_1 \times A_1/A) + (d_2 \times A_2/A) = (d_1 + d_2A_2/A_1) \times A_1/A.$$

As an abbreviation, we shall write M for the area $d_1 + d_2A_2/A_1$.

With this notation, therefore, the area of damage which it is expected that one bomb will do, to the building under consideration, is

$$M \times A_1/A.$$

We shall see in Section 6 below that the proportion of a building which is damaged is more important than the mere extent of the area of damage. We therefore remark here that the proportion of the building of area A_1 which, it is expected, will be damaged by one bomb is therefore

$$M/A.$$

(over)

4. EXPECTATION OF DAMAGE FROM SUCCESSIVE BOMBS. Suppose that the target area A is attacked by a number of bombs of the same calibre; we shall write

B for the number of bombs dropped in the target area.

Since each of these bombs is expected to damage an area $M \times A_1/A$ of the building under consideration, it might seem that the total area of damage to the building which may be expected is

$$B \times M \times A_1/A.$$

For light bombing, this is in fact a good approximation. But when bombing is heavy, this estimate is in error, for a fairly simple reason: that it counts, among the damage done by one bomb, the expected destruction of parts of the building which have already been destroyed by another. In fact, the estimate of damage area $B \times M \times A_1/A$ includes no reduction for the overlap of damage areas, on the occasions on which the damage areas of two or more bombs overlap. When bombing is heavy, however, these occasions are frequent; and this crude estimate is then seriously in error.

A simple device enables us to correct this error, and this device has the advantage that it is equivalent to making some allowance for the changes in the direct hit area A_1 and the near-miss area A_2 which earlier hits and near misses effect. Namely: we have shown that the expected area of damage by one bomb to the building under consideration is $M \times A_1/A$. Therefore the area of this building which is expected to remain undamaged after one bomb has fallen is

$$A_1 - (M \times A_1/A) = A_1(1 - M/A);$$

that is, a proportion

$$1 - M/A$$

of the building of area A_1. Of this undamaged area, in turn, a proportion

$$1 - M/A$$

will be expected to remain undamaged when the second bomb has fallen; so that only a proportion

$$(1 - M/A)(1 - M/A) = (1 - M/A)^2$$

of the original building of area A_1 will be expected to remain undamaged by both bombs - that is, to have been damaged by neither. Proceeding in this way, step by step, we see that only a proportion

$$(1 - M/A)^B$$

of the building will be expected to remain undamaged by all B bombs dropped in the attack. Hence the expected proportion of damage to this building is the remaining proportion of the building,

$$1 - (1 - M/A)^B;$$

and the expected area of the building destroyed is

$$A_1\{1 - (1 - M/A)^B\}.$$

These results can be exhibited in a somewhat simpler form. Namely, if M/A is small (as in practice it always is) a conservative approximation of high accuracy to the proportion

$$1 - (1 - M/A)^B$$

is

$$1 - e^{-MB/A},$$

where e is a constant (the base of Napierian logarithms). Now B is the total number of bombs dropped on the target area A; so that if we write

D for the density of bombing per unit area,

D is precisely B/A. Our result therefore is, that the expected proportion of damage to the building which is being specifically examined is

$$1 - e^{-DM},$$

where D is the density of bombing per unit area, and M is the area defined in Section 3, and measured in the same units.

5. MEAN AREA OF EFFECTIVENESS. Those familiar with the analysis of bombing of more densely built-up areas will observe that the result obtained is analagous to the proportion of damage there to be expected,

$$1 - e^{-DM'},$$

where D is also the density of bombing, and M' is the Mean Area of Effectiveness of a bomb of the given calibre. (In that analysis, it is more usual to exhibit both density and area of effectiveness in terms of tons, in place of single bombs. Since, however, only the product DM' appears in the formula, this does not affect the result significantly). Thus, in our result, the area M defined in Section 3 certainly plays the part of the Mean Area of Effectiveness of bombs of the given calibre. It is illuminating to observe that, in fact, this area is the Mean Area of Effectiveness against targets of the type under examination, when damage is caused in the two distinct ways set out in Section 2. This may be shown, starting from any of the alternative (and equivalent) definitions of the Mean Area of Effectiveness. Let us begin from the standard definition: the Mean Area of Effectiveness of a bomb, for damage of a particular type and degree to objects (buildings) of a particular class, is the area within which it is expected that all objects (buildings) of the class will suffer damage of the type specified to a degree equal to, or greater than, that specified, as the result of an attack by one bomb. It follows from this definition that if A_1/A is the proportion of a target area occupied by buildings of the class under consideration, and M' is the Mean Area of Effectiveness of a bomb for the particular type and degree of damage, then the expected area of damage is $M' \times A_1/A$. We saw in Section 3 that the probability of a direct hit on the building under examination is A_1/A, and that the area of damage is then d_1; the probability of an effective near miss is A_2/A, and the area of damage is d_2. Hence the expected area of damage by a single bomb is

$$(d_1A_1 + d_2A_2)/A$$

to a building occupying a proportion A_1/A of the target area. Therefore the Mean Area of Effectiveness is

$$(d_1A_1 + d_2A_2)/A \div A_1/A = d_1 + d_2A_2/A_1;$$

and this is precisely the value of M defined in Section 3. Our result is therefore a natural extension of that used in analysing damage to cities: the proportion of damage to be expected, whether in a city or on a single industrial building, is

$$1 - e^{-DM}$$

where D is the density of bombing per unit area, and M is the Mean Area of Effectiveness; provided that the latter is calculated appropriately for the target under consideration.

6. COMPUTATION FOR SPECIFIED PROPORTION OF DAMAGE. Experience has shown that an industrial building in effect ceases to be productively useful for a considerable time when the visible structural damage done to it reaches about one-third of its area. Hence the density of bombing D required to destroy the productive capacity of the building under consideration in this way is such that

$$1 - e^{-DM} = 1/3;$$

i.e. $e^{-DM} = 2/3,$

i.e. $e^{DM} = 3/2 = 1.5;$

whence $DM = \log_e 1.5 = 0.4055,$

so that $D = 0.4055/M;$

the density D being per unit area in which M is measured, say acres.

The procedure for computing the density of attack required is therefore straightforward. First, the mean area of effectiveness M is calculated for each building of primary (or primary and secondary) importance, as in Section 3: using the values of d_1, d_2, A_1, and A_2 of Section 2, appropriate to that building and the calibre of bomb under consideration. Of the values of M thus obtained for the various buildings, that alone is then considered which is smaller than all others. This value is substituted in the expression

$$D = 0.4055/M.$$

Then the value of D obtained is that which may be expected to do one-third visible structural damage to the least vulnerable of the buildings: and therefore a fortiori to all the buildings. The procedure is repeated for each bomb size in turn; and the most efficient bomb load, per ton, follows immediately.

7. LEVEL OF EXPECTATION OF SPECIFIED DAMAGE. We have discussed throughout the expected damage; and it is natural to ask, with what assurance we may expect this damage, from any one such attack. For the computation of Section 6, this question is readily answered. Namely, the proportion of damage there specified is the average proportion of damage to be expected, for a large number of buildings and over a large number of attacks. Since this proportion of damage is an average, in general one-half of such buildings will suffer greater damage than one-third, and one-half will suffer smaller damage, over a large number of attacks. Hence our level of expectation is 50%, in any one attack, for the least vulnerable building: but is higher for other buildings in the target area.

It would, of course, be possible, by choosing a density D for which the average proportion of damage

$$1 - e^{-DM}$$

is somewhat larger than one-third, to stipulate a higher expectation than 50% of buildings each of which has at least one-third of itself damaged. Indeed, it can be shown that, in theory, we need only increase the average proportion of damage to about 37%, in order to have an overwhelmingly high expectation that the bulk of buildings will have each a proportion of damage of at least one-third. Unhappily, for practical reasons, this gain in assurance would be illusory. In the first place, the Mean Area of Effectiveness which we use is merely a mean; and the fluctuations of actual damage areas about this mean sharply limit any assurance above 50%. In the second place, the damage proportion of one-third does not ensure that a building is rendered useless; it is again a mean, the fluctuations about which reach beyond any new attainable damage level such as 37%. Finally, the number of buildings on a given target is always small, and the distribution of damage among them therefore necessarily exhibits wide fluctuation from the normal distribution of expected damage among a large number of buildings: these fluctuations growing larger, the larger the size of the individual bombs used. In short, the foregoing results are subject to limitations, particularly for the larger bombs. Some of these limitations can be removed, for highly specialised targets, by refinements of the formulae. But it must be stressed that, in the main, the effect of these limitations is inconsiderable, so long as we are content to calculate bomb density for an average level of one-third damage. Their effect is largely to make it difficult to specify a level of expectation above the average level. For this reason, the formulae are recommended in their most effective form, that set out in Section 6.

8. EXAMPLE OF THE METHOD OF CALCULATION. Table 1 shows the calculations for a typical target. Buildings 2 and 4 are normal, fairly low shed buildings (height 16 ft. to eaves) with north light roofs. The figures entered in cols. 5, 6 and 7 for these buildings are average values obtained from British experience and from a study of the effects of identified British and U.S. bombs on enemy buildings of a similar type. Buildings 1 and 3 are of Zeiss-Dywidag construction, their roofs consisting of a thin, curved, reinforced concrete slab poured in situ. No direct data are so far available on the behaviour of this type of construction; the figures given in cols. 5, 6 and 7 have therefore been estimated from a theoretical consideration of the effect of blast on this type of building. The calculations for the other columns in the table follow directly the method given in Sections 5 and 6. The Mean Area of Effectiveness calculated in col. 9 is obtained from the expression

$$M = d_1 + d_2 A_2 / A_1$$

M, d_1, d_2, A_1 and A_2 being here measured, for convenience, in square feet. In col. 10, M is converted into acres; and in col. 11 the number of bombs, D, per acre required to cause a proportion of visible structural damage of one-third is calculated from the expression

$$D = 0.4055/M.$$

In col. 12 this density of bombing is converted into short tons per acre.

Table 2 summarises these calculations, and gives the bomb density required effectively to destroy the productive capacity of this target, i.e. to cause an average of at least one-third visible structural damage to the least vulnerable building.