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Approximating the Poisson Scan and (A-0) Acoustic Detection model with a random search formula

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THESIS

APPROXIMATING THE POISSON SCAN AND ($\lambda - \sigma$) ACOUSTIC DETECTION MODEL WITH THE RANDOM SEARCH FORMULA

by

Kangmin KIM

December 2009

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APPROXIMATING THE POISSON SCAN AND $(\lambda-\sigma)$ ACOUSTIC DETECTION MODEL WITH A RANDOM SEARCH FORMULA

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requirements for the degree of

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ABSTRACT

In this study, the author develops a MATLAB simulation of area search with acoustic sensors modeled by the Poisson Scan model and the Lambda-Sigma $(\lambda-\sigma)$ model. Detection time results are compared to those given by the much simpler Random Search formula. Random Search was found to closely approximate the more complex models if detection range was selected correctly. Guidelines for selecting the Random Search detection range were developed.

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EXECUTIVE SUMMARY

In this study, the author develops a MATLAB simulation of area search with acoustic sensors modeled by the Poisson Scan and the Lambda-Sigma $(\lambda - \sigma)$ models.

Both the Poisson Scan model and Lambda-Sigma $(\lambda - \sigma)$ model simulation results are found to be approximately exponentially distributed, which is consistent with the Random Search model. Thus, Random Search with the proper deterministic detection range $\tilde{R}$ can be used to closely approximate simulation results obtained using the Poisson Scan and Lambda-Sigma $(\lambda - \sigma)$ models.

Both the Poisson Scan and Lambda-Sigma $(\lambda - \sigma)$ detection time results vary with λ, σ, and $R(50)$. The author constructs a regression model based on the simulation results and finds an approximate linear relationship between $R(50)$ and the best-fit $\tilde{R}$ for reasonable λ and σ. The gradient of the regression line depends on the values of λ and σ. Thus, it is possible to estimate the best-fit $\tilde{R}$ from problem parameters λ, σ, and $R(50)$ and then use Random Search, instead of Monte Carlo simulation, to predict the effectiveness of area search.

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I. INTRODUCTION

A. BACKGROUND

Random Search is one of the most well-known and used models for area search. However, because of its simplicity, it also has inherent limitations. For example, it assumes that the searcher and target search areas are identical and that the searcher uses a perfect cookie-cutter sensor. Captain Gi Young Kim[1], ROK Air Force, studied these issues previously and suggested generalizations to the Random Search formula for cases where the searcher and target areas are not identical. However, there are remaining issues to address, particularly how well the Random Search model can approximate probabilistic sensors, such as those modeled by the Poisson Scan and Lambda-Sigma $(\lambda-\sigma)$ models.

B. RESEARCH QUESTIONS

The primary research questions are the following:

1. When using the Poisson Scan model and the Lambda-Sigma $(\lambda-\sigma)$ model, will detection times be approximately exponentially distributed, as predicted by the Random Search model?

2. Can a non-cookie cutter sensor be replaced with a cookie-cutter sensor with the same sweep width and maintain approximately the same search performance?

3. Are there significant differences in the detection results generated by simulations using the Poisson Scan and Lambda-Sigma $(\lambda-\sigma)$ detection models?

[1] Gi Young Kim, “Development and Testing of a New Area Search Model with Partially Overlapping Target and Searcher Patrol Areas,” Master’s thesis, Naval Postgraduate School, Monterey, California, 2008.

C. THESIS ORGANIZATION

The thesis is organized as follows: Chapter II reviews the Poisson Scan and Lambda-Sigma $(\lambda - \sigma)$ acoustic detection models; Chapters III and IV present a MATLAB simulations of these two models; Chapter V develops the best-fit $\tilde{R}$, which is the equivalent cookie-cutter detection range for the Poisson Scan model and Lambda-Sigma $(\lambda - \sigma)$ model; Chapter VI summarizes all results and recommends future studies.

II. OVERVIEW OF ACOUSTIC DETECTION MODELS

A. BACKGROUND

In this chapter, we review the Passive Sonar Equation (PSE) and use it to develop two models for passive acoustic detection — the Poisson Scan model and the Lambda-Sigma $(\lambda - \sigma)$ model.

B. THE PASSIVE SONAR EQUATION

The passive sonar equation[2] is an audit of energy flow between a source and a sonar receiver. All units are in decibels (dB).

The Passive Sonar Equation (PSE) is

$$L_S - N_W - DNL - DT = SE,$$

where

$L_S =$ signal level of source,

$N_W =$ propagation loss between the source and receiver,

$DNL =$ detection noise level at the receiver,

$DT =$ detection threshold = signal level at receiver required to achieve a P_d of 0.5,

$SE =$ signal excess.

The left side of Passive Sonar Equation is the signal available at the receiver, and right side is the signal required for a probability of detection (P_d) of 0.5.

[2] The references for this section are the unpublished lecture notes of Professor James N. Eagle, "Acoustic Detection Models," 2009.

1. Figure of Merit(FOM) and $R(50)$

Figure of merit (FOM) and $R(50)$ are used as measures of effectiveness(MOE) for sonar detection performance. FOM is the maximum propagation loss a sonar system can absorb in dB and still produce a 0.5 probability of detection. $R(50)$ is the maximum range between the source and receiver resulting in a 0.5 probability of detection.

$FOM = L_S - DNL - DT$

= the transmission loss between source and receiver resulting in P_d=0.5.

$R(50)$ = the maximum range where $L_S = FOM$.

2. Acoustic Modeling Assumptions

- Mean $SE = \overline{SE} = FOM - N_W$. That is, the Passive Sonar Equation is used to calculate mean signal excess at the receiver.
- $SE \sim N(\overline{SE}, \sigma^2)$, where σ is typically 3-9dB. We assume the signal excess is a normally distributed random variable with the mean given by the Passive Sonar Equation, and σ is specified.
- Detection occurs if and only if $SE \geq 0dB$. This is a threshold crossing model. Detection occurs when the random SE is nonnegative and at no other times. And probability of detection (called here instantaneous probability detection, IPD) is $P(SE \geq 0)$.

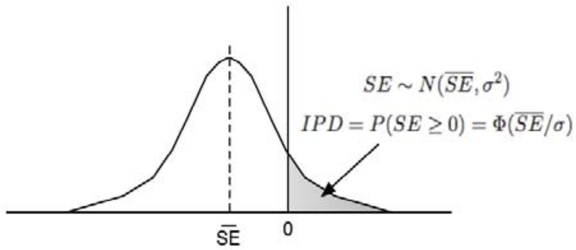

Figure 1. IPD when $\overline{SE} \leq 0$

3. Implications

$$IPD(\overline{SE}) = P(SE \geq 0), \text{ where } SE \sim N(\overline{SE}, \sigma^2)$$
$$= \Phi(\overline{SE}/\sigma).$$

where Φ is the cumulative normal distribution function. This relationship between $\overline{SE}$ and the probability of detection is illustrated in Figure 1.

C. THE FIXED SCAN AND POISSON SCAN MODELS

1. Fixed Scan Model

The Fixed Scan model[3] assumes that probabilistically independent detection opportunities occur at regularly spaced time intervals. It is attractive analytically, since there is a simple formula for calculating the probability of detection over any time interval $[0, t]$. Let S be the set of detection opportunity times τ within the interval, and let

$$IPD(\tau) = \Phi(\overline{SE}(\tau)/\sigma).$$

[3] Alan R. Washburn, *Search and Detection,* 4th ed. 3-4p.

be the detection probability for a scan containing time τ, where the signal excess $\overline{SE}(t)$ is acknowledged to depend on time. Then the cumulative detection probability over the interval is

$$P(T \leq t) = F_T(t) = 1 - \prod_{\tau \in s}(1 - IPD(\tau)).$$

In other words, there will be a detection at $[0, t]$ unless detection fails at every one of the independent scans within the interval. However, the Fixed Scan model suffers from its dependence on an arbitrarily selected origin of time. The Poisson Scan model was developed, in part, to address this issue.

2. Poisson Scan Assumptions

- Independent detection opportunities occur at Poisson times with rate λ(units : 1/time).
- Probability of detection at time t, given that a detection opportunity exists at t, is $IPD(t) = \Phi(\overline{SE}(t)/\sigma)$.

3. Development

From the Poisson Scan assumption, the instantaneous detection rate at time t is

$$\gamma(t) = \lambda\Phi(\overline{SE}(t)/\sigma).$$

Then it follows from the properties of the non-homogeneous Poisson Process that

$$P(T \leq t) = F_T(t) = 1 - \exp\left(-\lambda \int_{s=0}^{t} \Phi(\overline{SE}(t)/\sigma) ds\right).$$

In this model, $1/\lambda$ is the mean time between independent detection opportunities (glimpses). Probabilistic independence is assumed for each glimpse, with λ specifying the average glimpse rate. By varying λ, the sonar can be made as effective (λ larger) or ineffective (λ smaller) as desired, subject to sensor or operational constraints. The sonar system can only call detection at the discrete glimpse times, and otherwise is assumed to be processing previously received data.

D. THE LAMBDA-SIGMA $(\lambda - \sigma)$ MODEL

In the Poisson Scan model, detection can occur at time t only if a detection opportunity occurs at t, and an independent draw from $SE \sim N(\overline{SE}, \sigma^2)$ is greater than or equal to 0.

A potentially more realistic model would assume that $SE(t)$ is a random, continuous function of time t (that is, a continuous - time stochastic process), and that detection occurs at any time t where $SE(t) \geq 0$.

1. $(\lambda - \sigma)$ Model Assumptions and Development

Let

$$SE(t) = \overline{SE}(t) - X(t),$$

where $\overline{SE}(t)$ is the mean signal excess at time t, which is computed(as in the Poisson Scan model) from the PSE as

$$\overline{SE}(t) = FOM(t) - N_W(t).$$

$X(t)$ is a 0-mean stochastic process. More specifically, $X(t)$ is a $(\lambda - \sigma)$ jump process, described below.

Each random sample path of $X(t)$ is a step function, where the duration of each step (the time between jumps) is exponentially distributed with mean $1/\lambda$, and the height of each step (the value of $X(t)$) is normally distributed with mean 0 and variance σ^2.

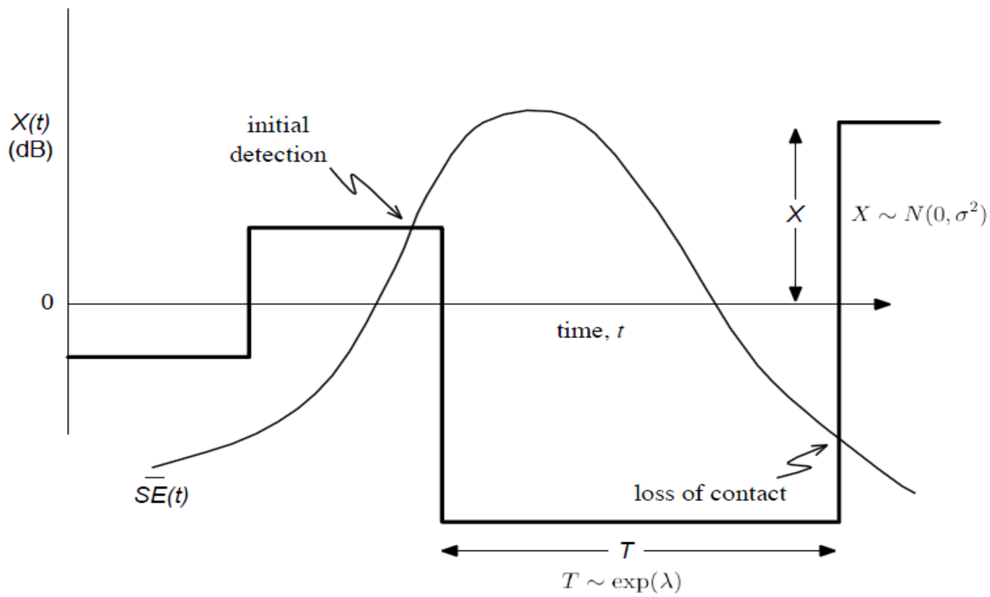

Figure 2. $(\lambda - \sigma)$ Detection Process Example

As illustrated in Figure 2, detection occurs whenever $\overline{SE}(t) \geq X(t)$. $X(t)$ can be thought of as the time-varying sum of all random components of FOM and N_W.

2. Computing P_d when $\overline{SE}$ is Constant

Assume $\overline{SE}(t) = \overline{SE} = k,\ t \in [0,\ \tau]$. We will develop an expression for the probability of initial detection occurring between time 0 and t, which we call $F_T(t)$.

$$F_T(t) = P(X(t) \leq k\ ,\ \text{for some } t \in [0,\ \tau])$$

$$= 1 - P(X(t) > k,\ \text{for all } t \in [0,\ \tau])$$

$$= 1 - \sum_{i=0}^{\infty} P(X(t) > k,\ \text{for all } t \in [0,\ \tau] | i \text{ jumps occur in } [0,\ \tau])$$

$$\times P(i \text{ jumps occur in } [0,\ \tau])$$

$$= 1 - \sum_{i=0}^{\infty} (1 - \Phi(k/\sigma))^{i+1} \frac{(\lambda t)^i}{i!} e^{-\lambda t}$$

$$= 1 - [1 - \Phi(k/\sigma)] e^{-\lambda t} \sum_{i=0}^{\infty} \frac{([1-\Phi(k/\sigma)]\lambda t)^i}{i!}$$

$$= 1 - [1 - \Phi(k/\sigma)] e^{-\lambda t} e^{[1-\Phi(k/\sigma)]\lambda t},\ \text{since } \sum_{i=0}^{\infty} \frac{x^i}{i!} = e^x$$

$$= 1 - [1 - \Phi(k/\sigma)] e^{-\lambda[\Phi(k/\sigma)]t}$$

$$= 1 - [1 - \Phi(\overline{SE}/\sigma)] e^{-\lambda[\Phi(k/\sigma)]t}.$$

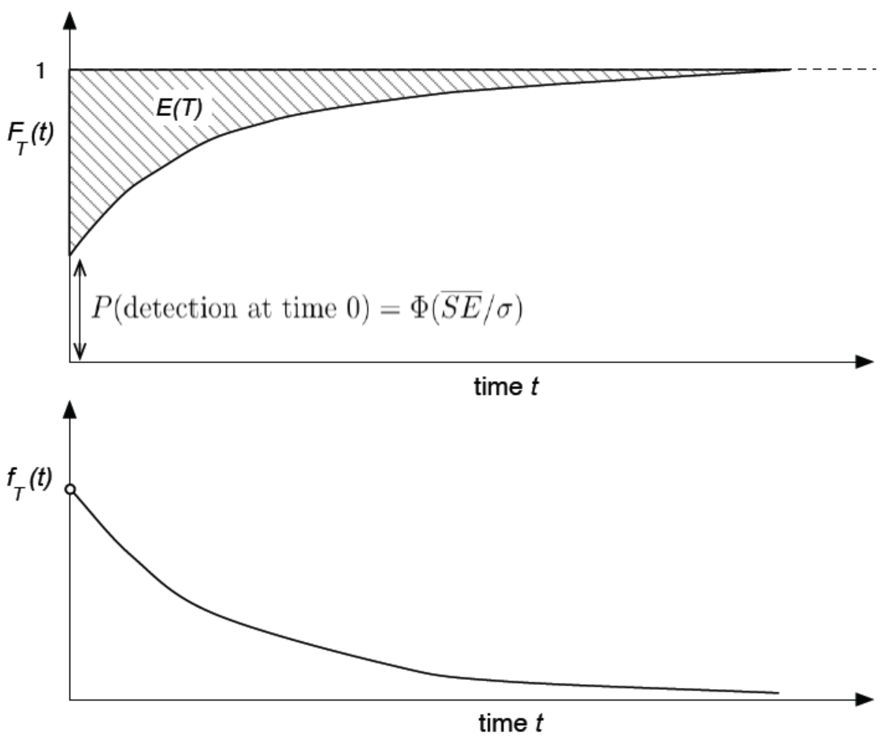

Figure 3. $(\lambda - \sigma)$ *PDF* and *CDF* Functions for Time of Initial Detection T when $\overline{SE}$ is Constant

At time 0, the probability of detection is $\Phi(\overline{SE}/\sigma)$.

The mean time to detection, $E(T)$, and the density function, $f_T(t)$, for time of initial detection are

$$E(T) = \frac{1 - \Phi(\overline{SE}/\sigma)}{\lambda\Phi(\overline{SE}/\sigma)},$$

$$f_T(t) = \lambda\Phi(\overline{SE}/\sigma)[1 - \Phi(\overline{SE}/\sigma)]e^{-\lambda\Phi(\overline{SE}/\sigma)t}, \text{ for } t > 0.$$

Also, $P(t = 0) = \Phi(\overline{SE}/\sigma)$.

Note that this probability distribution has a discrete portion $(T = 0)$ with units of probability (unitless) and a continuous portion $(T > 0)$ with units of probability/time (or, 1/time).

III. SIMULATION OF POISSON SCAN MODEL

A. DESCRIPTION OF POISSON SCAN SIMULATION MODEL

1. Characteristics of the Searcher

The searcher is not allowed to search for a target outside the search area. In addition, the searcher’s initial position is uniformly distributed inside of the search area. After that, the searcher selects his course randomly, independent of the target’s movement. The course change event is determined by Poisson process with rate λ_s. Independent scan opportunities also occur according to a Poisson process, but with rate λ_l. Also, it is assumed that signal transmission loss[4] follows either the spherical spreading law $(N_w = 20\log_{10} r)$ or the cylindrical spreading law $(N_w = 10\log_{10} r)$. The searcher speed was fixed at 15 $nm/hour$.

2. Characteristics of the Target

Like the searcher, the target is not allowed to move outside of the search area and has an initial position uniformly distributed over the search area. The logic of the target movement is the same as that of the searcher; that is, the target has its own, independent Poisson process with course change rate λ_t. The target speed was fixed at 5 $nm/hour$.

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[4] Robert J. Urick, *Principles of Underwater Sound*, 3rd ed. 101p.

B. COMPUTER ALGORITHM

1. Input [Units]

- Number of simulation replications, $N_{reps} = 500$.
- Maximum simulation time, $t_{max} = 150[hour]$.
- The length of search area in X direction, $l_x = 150[nm]$.
- The length of search area in Y direction, $l_y = 150[nm]$.
- Searcher speed, V $[nm/hour]$.
- Target speed, U $[nm/hour]$.
- Searcher's scan rate, λ_l $[Glimpses/hour]$.
- Searcher's course change rate, λ_s $[1/hour]$.
- Target's course change rate, λ_t $[1/hour]$.
- The unit time of simulation, $\Delta t\,[hour]$.
- The size of search area, $A_s = l_x \times l_y$.
- Figure of Merit, FOM $[dB]$.
- Transmission loss, $N_w = 20\log(Distance)$ $[dB]$.
- Signal Excess, $SE = FOM - N_w$.
- Variance of Signal Excess, σ $[dB]$.

2. Functioning of the Poisson Scan Simulation

When a new replication begins, the initial positions of the searcher and the target are chosen from a uniform distribution over the search area. The initial headings are also drawn uniformly between 0 and 2π radians. The subsequent course changes for the searcher and target occur according to Poisson processes with rates λ_s and λ_t, respectively. According to

Captain Gi Young Kim's thesis[5], the recommended value for the course change rate is $\lambda = 2V/\sqrt{l_x \times l_y}$, which implies that on average, two course change events occur during the time required for the searcher to go from edge to edge in the search area.

When the searcher or target encounters an area boundary, a random reflection occurs. After a reflection, the new course in radians is $Uniform\_Random(C_\perp - 0.5, C_\perp + 0.5)$, where $C_\perp$ is the perpendicular course from the reflection boundary. This scheme was recommended and tested by Captain Gi Young Kim[6] to achieve an approximate uniform distribution of the searcher and target tracks in the search area.

In the Poisson Scan model, there are two ways to simulate detection times. One uses the instantaneous probability detection at specific time t, which follows the Poisson process with rate λ. This is because in the Poisson Scan model, detection opportunities occur according to a Poisson process with rate λ. So, the mean time between independent detection opportunities is $1/\lambda$. The distance between the searcher and the target is calculated at specific time t. Then, the mean signal excess($\overline{SE}$) is determined by using the Passive Sonar Equation (PSE) allowing the calculation of $IPD(\overline{SE})$. The next step is generating a standard uniform random number and comparing it with the $IPD(\overline{SE})$. If $Uniform\ Random(0,1) < IPD(\overline{SE})$, then a detection occurs at that specific time t.

[5] Gi Young Kim, "Development and Testing of a New Area Search Model with Partially Overlapping Target and Searcher Patrol Areas," Master's thesis, Naval Postgraduate School, Monterey, California, 2008.

[6] Gi Young Kim, KOREA AIR FORCE CAPTAIN, graduated from the Naval Postgraduate School in 2008.

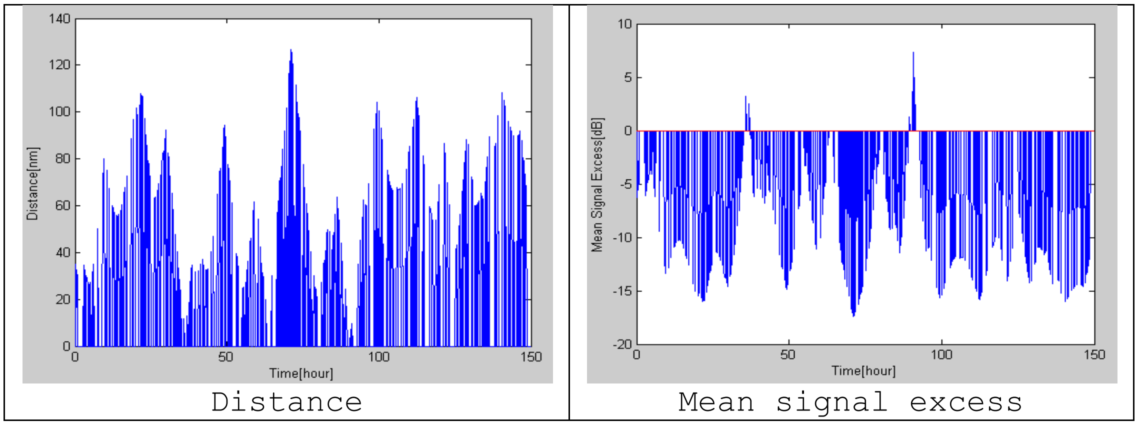

Figure 4. Simulation when calculating distance and $\overline{SE}$ at specific time t

The other way to simulate detections is to check for a detection at each Δt. The distance between the searcher and the target is calculated at every time step Δt, and the mean signal excess is computed.

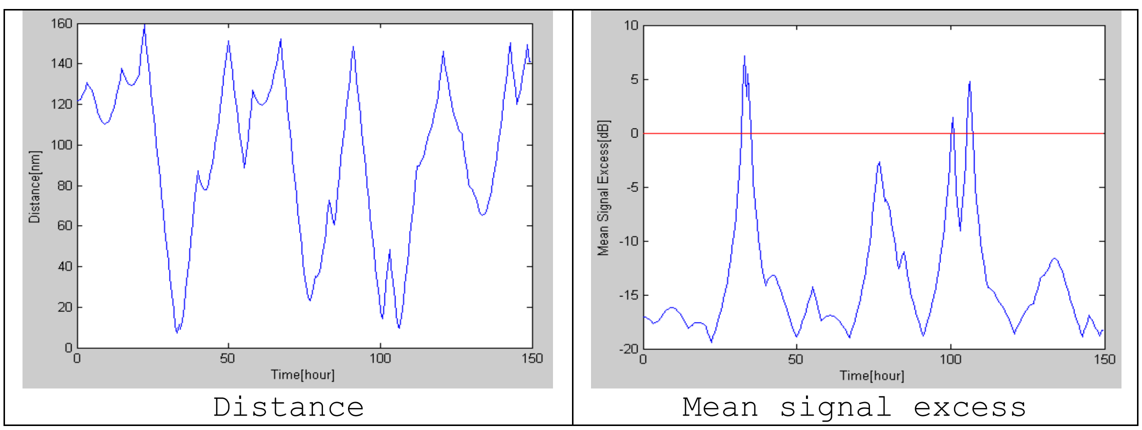

Figure 5. Simulation when calculating distance and $\overline{SE}$ at every Δt

After that, we compute the instantaneous probability of detection $IPD(\overline{SE}) = \Phi(\overline{SE}(t)/\sigma)$ and detection rate $\gamma(t) = \lambda\Phi(\overline{SE}(t)/\sigma)$. At each time step Δt, if $Uniform\,Random(0,1) < \gamma(t)\Delta t$, then a detection occurs.

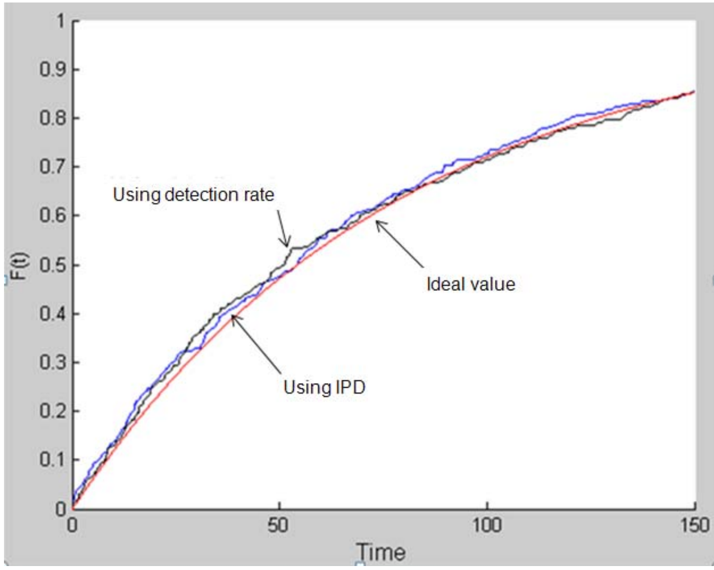

Figure 6. $F_T(t)$ for different simulations

As illustrated in the simulation results of Figure 6, the results of these two simulation methods are very similar, and theoretically should be identical.

For this thesis we used the Δt simulation method. In order to approximate a continuous simulation, Δt should be small. However, too small a Δt results in too many calculations. In the next section, we answer the question, "How small should Δt be to produce accurate results without creating excessively long simulation runs?"

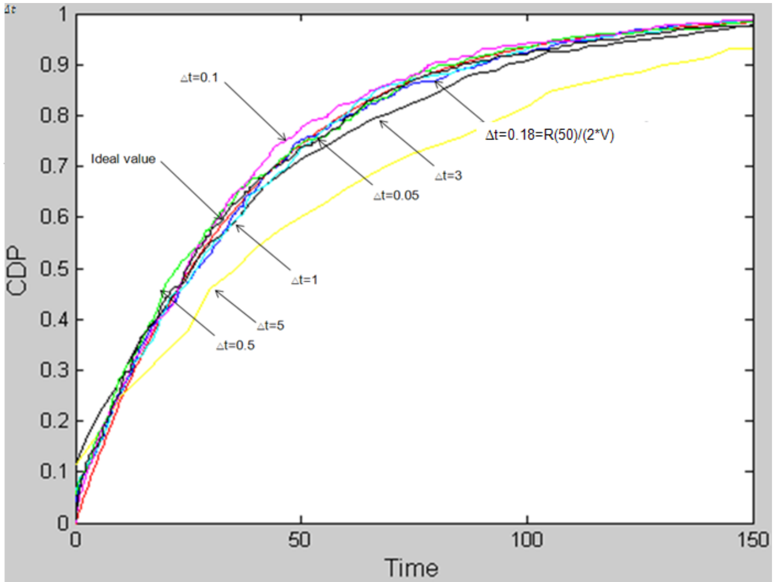

Figure 7. $F_T(t)$ for various values of Δt

3. Output

Extensive simulation experimentation showed that $\Delta t = \frac{R(50)}{2V}$ was the largest Δt resulting in reliable and repeatable results. Thus we move the searcher half the distance of the $R(50)$ at each time step. Typical results are shown in Figure 7.

In order to find best-fit $\tilde{R}$, which is the equivalent range of a cookie-cutter sensor to the Poisson Scan Model, the author experimented with various values of λ, σ, and FOM.

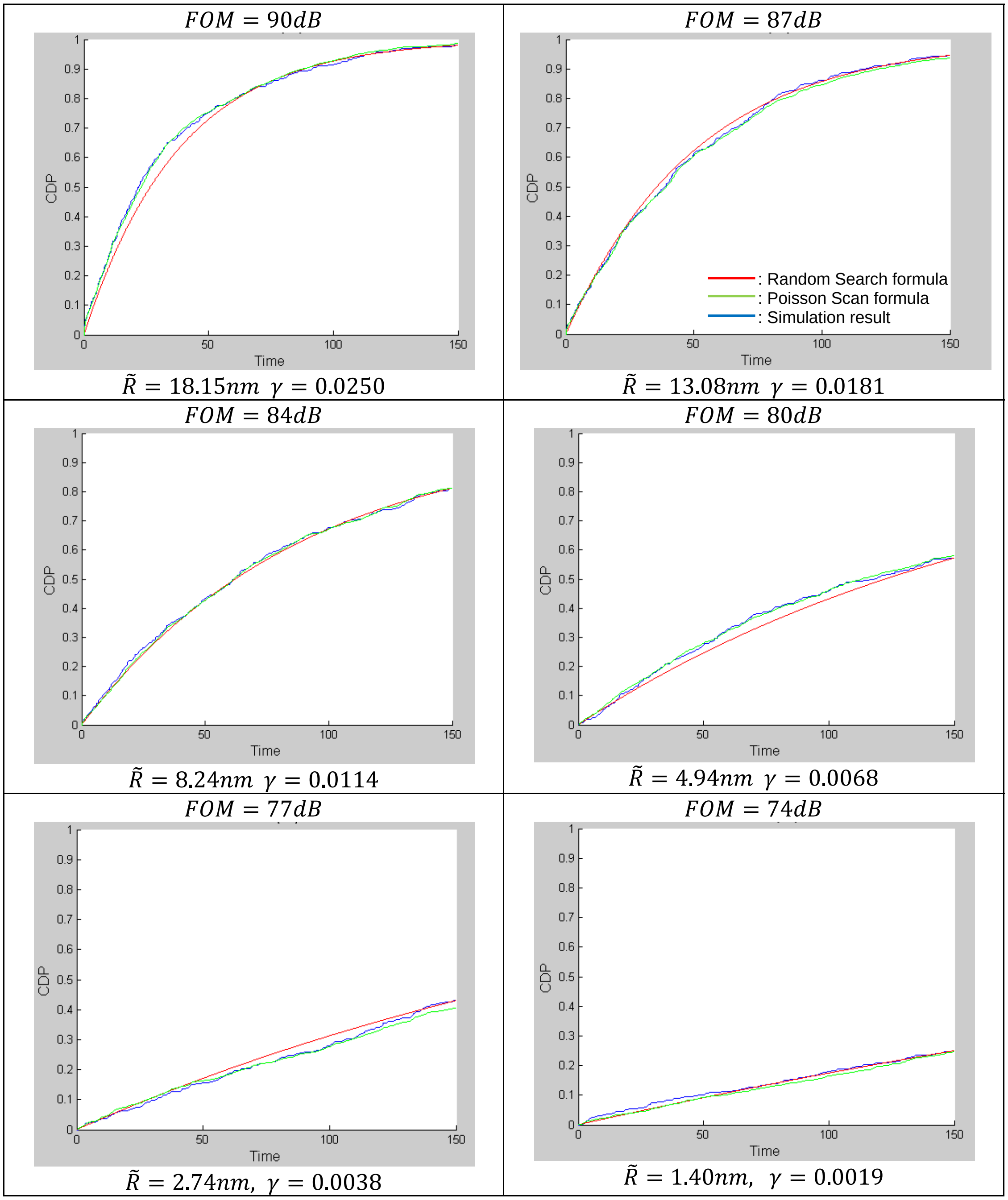

Figure 8. $F_T(t)$ for various FOM in Poisson Scan model

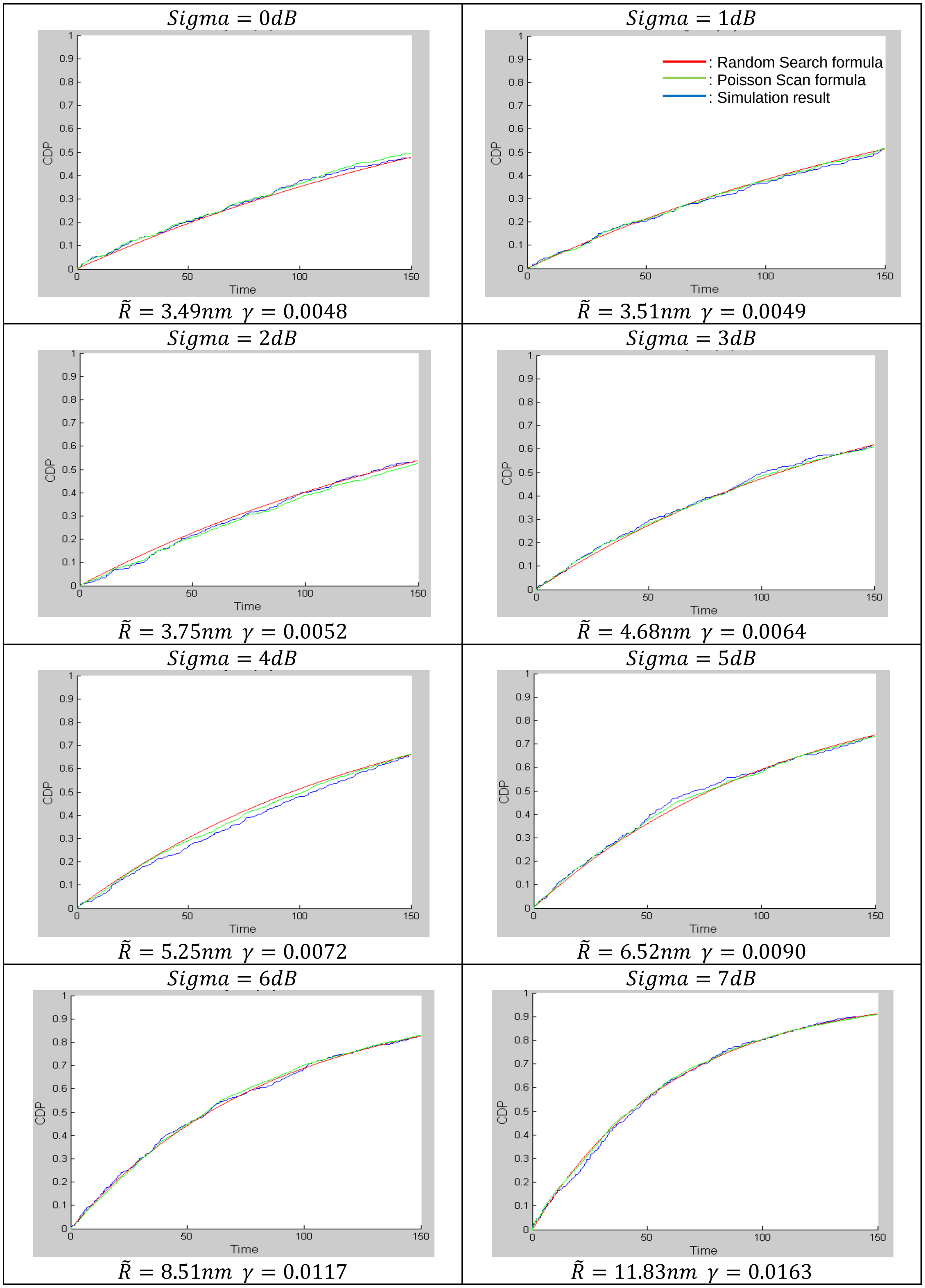

Figure 9. $F_T(t)$ for various *Sigma* in Poisson Scan model

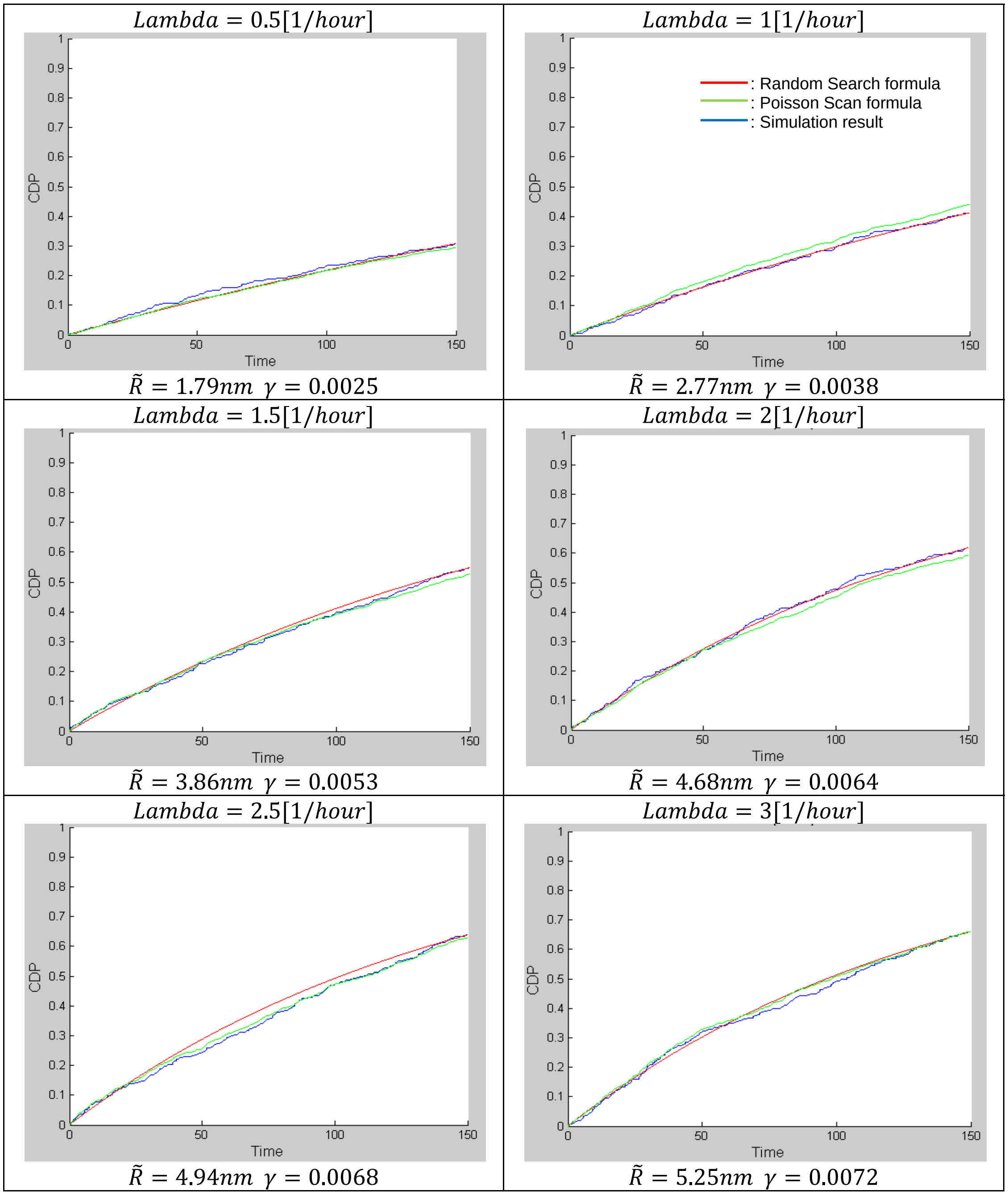

Figure 10. $F_T(t)$ for various $Lambda$ in Poisson Scan model

The Green line represents the Poisson Scan model formula.

$$P(T \le t) = F_T(t) = 1 - \exp\left(-\lambda \int_{s=0}^{t} \Phi(\overline{SE}(s)/\sigma) ds\right).$$

The red line represents the formula resulting from a random search with a cookie-cutter sensor that has best-fit $\tilde{R}$ range.

$$P(T \le t) = F_T(t) = 1 - \exp\left(-2\tilde{R}\tilde{V}t/A_s\right),$$

$$\tilde{R} = \frac{-\ln\left(1 - F_T(t_{max})\right) * A_s}{2\tilde{V}t_{max}},$$

where $\tilde{V}$ is the approximate mean relative speed between target and searcher determined as follows:

$$\tilde{V} = \frac{1}{\pi}\int_0^{\pi} \sqrt{V^2 + U^2 - 2VU\cos\theta}\, d\theta.$$

The blue line represents the results of the simulation.

The results show that for the model parameters examined here, the Poisson Scan model gives detection times that are approximately exponentially distributed, as does the Random Search model. The conclusion is that, the Random Search model using the proper detection range $\tilde{R}$ can closely approximate detection results given by the Poisson Scan model.

A potentially more accurate procedure for determining $\tilde{R}$ would be to plot $-\ln(1 - F_T(t))$ versus t and to solve for the best-fit slope γ. Then,

$$\tilde{R} = \frac{A_s \gamma}{2\tilde{V}}.$$

IV. SIMULATION OF LAMBDA-SIGMA $(\lambda - \sigma)$ MODEL

A. DESCRIPTION OF LAMBDA-SIGMA $(\lambda - \sigma)$ SIMULATION MODEL

The Lambda-Sigma $(\lambda - \sigma)$ detection model[7] is distinguished from the Poisson Scan model by allowing detection at any time during the scan, rather than only at specific scan times. This model assume that $SE(t)$ is a random, continuous function of time t. Then,

$$SE(t) = \overline{SE}(t) - X(t),$$

where $X(t)$ is a 0-mean stochastic process where the duration of each step is exponentially distributed with mean $1/\lambda$, and the height of each step is normally distributed with mean 0 and variance σ^2. Detection occurs at any time t where $SE(t)$>0.

[7] The references for this section are the unpublished lecture notes of Professor James N. Eagle, "Acoustic Detection Models," 2009.

B. COMPUTER ALGORITHM

1. Input [Units]

- Number of simulation replications, $N_{reps} = 500$.
- Maximum simulation time, $t_{max} = 150[hour]$.
- The length of search area in X direction, $l_x = 150[nm]$.
- The length of search area in Y direction, $l_y = 150[nm]$.
- Searcher speed, $V = 15\,[nm/hour]$.
- Target speed, $U = 5\,[nm/hour]$.
- Searcher's course change rate, $\lambda_s = 2V/\sqrt{l_x \times l_y}\ [1/hour]$.
- Target's course change rate, $\lambda_t = 2U/\sqrt{l_x \times l_y}\ [1/hour]$.
- The unit time of simulation, $\Delta t = R(50)/2 \times V\ [hour]$.
- The size of Search area, $A_s = l_x \times l_y$.
- Figure of Merit, $FOM\ [dB]$.
- Transmission loss, $N_w = 20\log(Distance)\ [dB]$.
- Signal Excess, $SE = FOM - N_w$.
- Jump rate of stochastic process, $\lambda_l\ [times/hour]$.
- Standard deviation of jump in stochastic process, $\sigma[dB]$.

2. Functioning of the Program

With the Lambda-Sigma $(\lambda - \sigma)$ model, the duration of each step is exponentially distributed with mean $1/\lambda_l$, and the height of each step is normally distributed with mean 0 and variance σ^2.

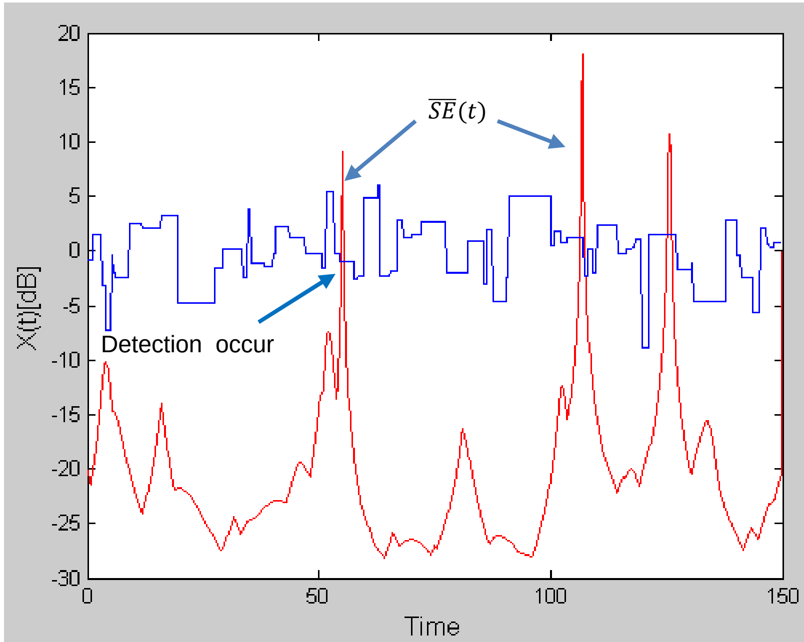

Figure 11. Example of $(\lambda-\sigma)$ model simulation

In Figure 11, the blue line represents one realization of the 0-mean stochastic process $X(t)$, and the red line represents mean signal excess. At each time step Δt, if $\overline{SE}(t) \geq X(t)$, then a detection occurs. In the Figure 11, the first detection occurs at 55 hours.

3. Output

In order to find the best-fit $\tilde{R}$, which is the equivalent range of a cookie-cutter sensor to that of the Lambda-Sigma $(\lambda-\sigma)$ model, the author experimented with various values of model parameters λ, σ, and FOM.

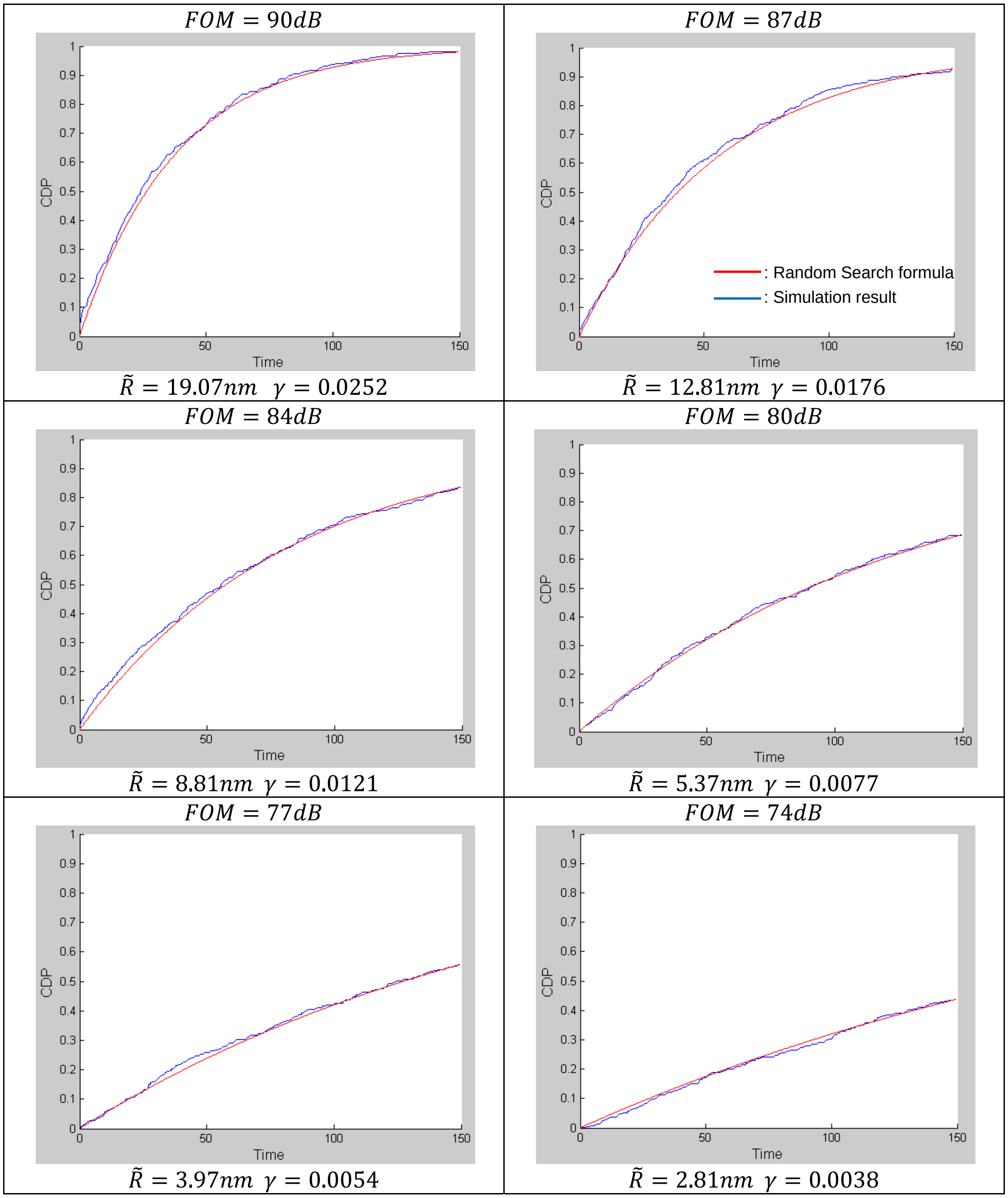

Figure 12. $F_T(t)$ for various FOM in $(\lambda - \sigma)$ model

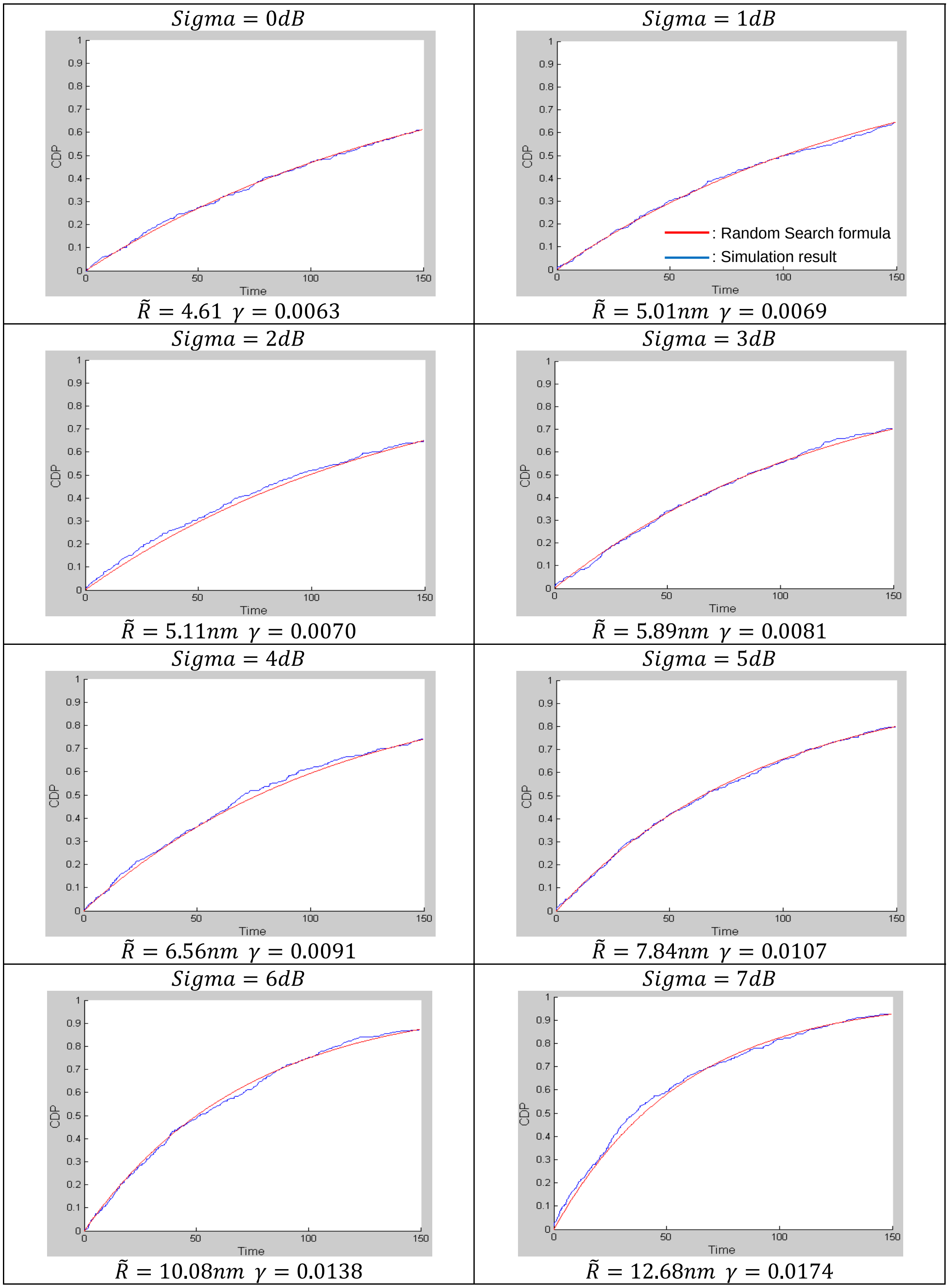

Figure 13. $F_T(t)$ for various $Sigma$ in $(\lambda - \sigma)$ model

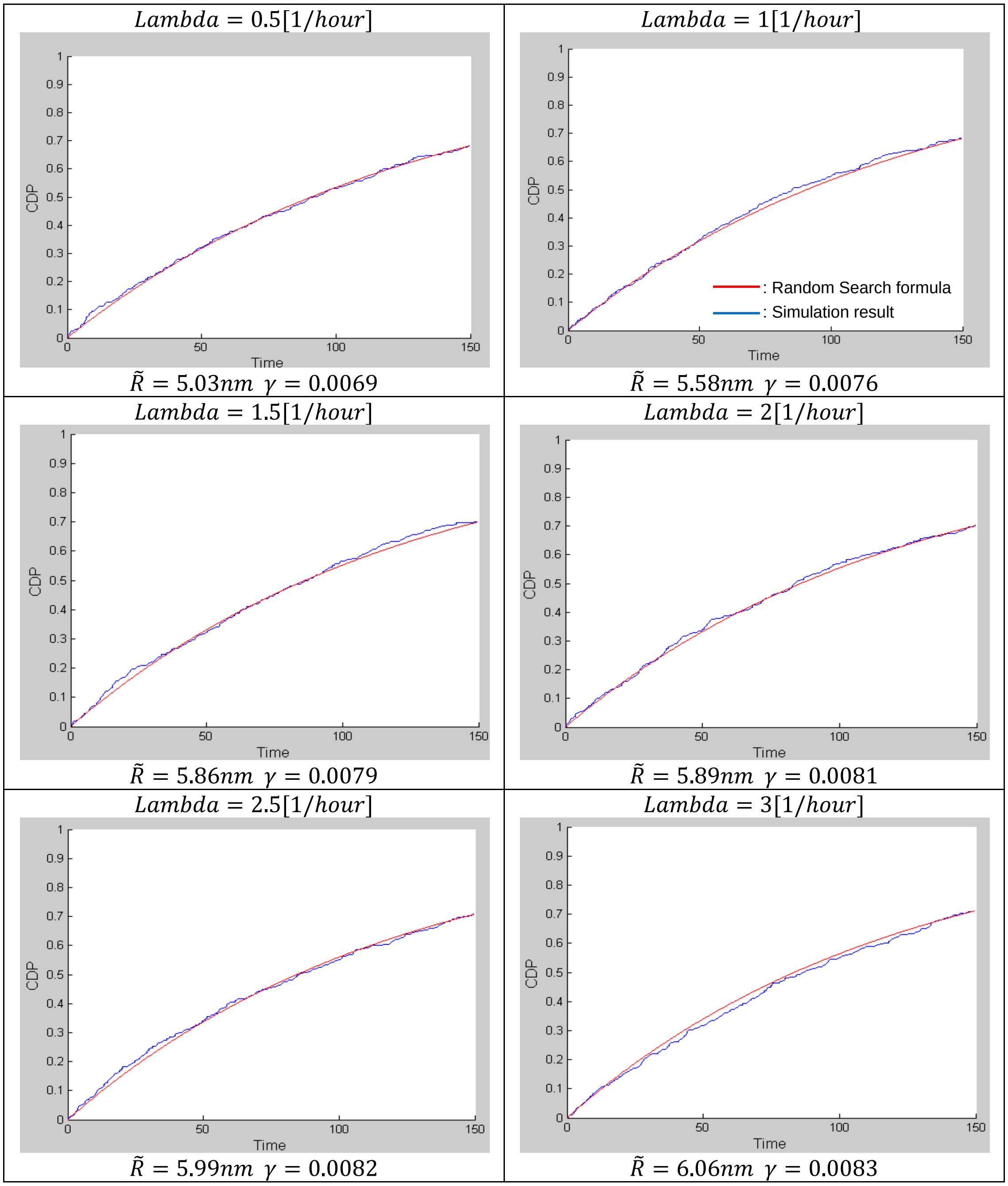

Figure 14. $F_T(t)$ for various *Lambda* in $(\lambda - \sigma)$ model

The red lines in Figures 12–14 show Random Search results with a cookie-cutter sensor having best-fit $\tilde{R}$ range. The blue line represents the results of the simulation. As was the case with the Poisson Scan model, simulation results using the Lambda-Sigma $(\lambda - \sigma)$ model showed an exponential time to initial detection. Thus, Random Search with the correct deterministic detection range $\tilde{R}$ can be used to closely approximate simulation results obtained using the stochastic Lambda-Sigma $(\lambda - \sigma)$ model. The task that remains is to estimate from problem parameters the appropriate value of $\tilde{R}$.

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V. ESTIMATION OF BEST-FIT $\tilde{R}$

A. LATERAL RANGE CURVE

1. Description of the Lateral Range Curve

In order to estimate the best-fit $\tilde{R}$, the author simulated lateral range curves for sensors with the same parameters as the Poisson Scan model and Lambda-Sigma $(\lambda-\sigma)$ model. Sweep width W is defined to be the area underneath the lateral range curve.

$$W = \int_{-\infty}^{\infty} l(x)dx,$$

where $l(x)$ is the lateral range curve. In the cookie-cutter sensor with detection range R,

$$l(x) = \begin{cases} 1, x \in [-R, R] \\ 0, \text{otherwise} \end{cases},$$

therefore, $R = W/2$.

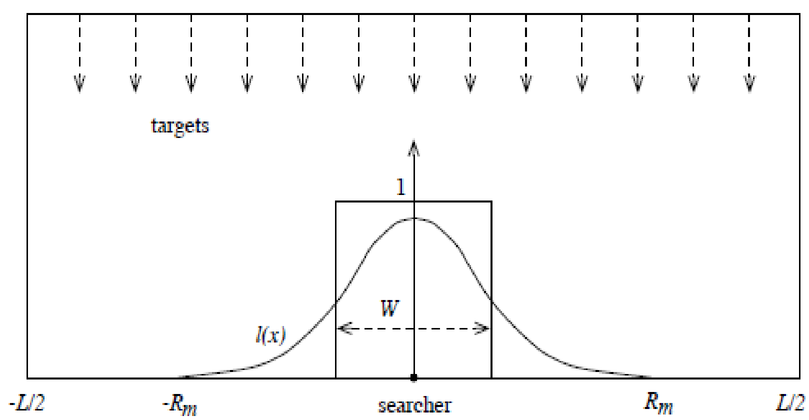

Figure 15. Sweep Width Interpretation

2. Computer Algorithm of Lateral Range Curve

In the lateral range curve simulation geometry, shown in Figure 15, target lateral ranges are uniformly distributed over $-L/2$ to $L/2$ and target tracks are straight. Except for this characteristic, input elements in each lateral range curve simulation are the same for both the Poisson Scan model and the Lambda-Sigma $(\lambda - \sigma)$ model. The lateral range curve is determined by the ratio of the number of detections that occur to number of replications at each lateral range from the searcher.

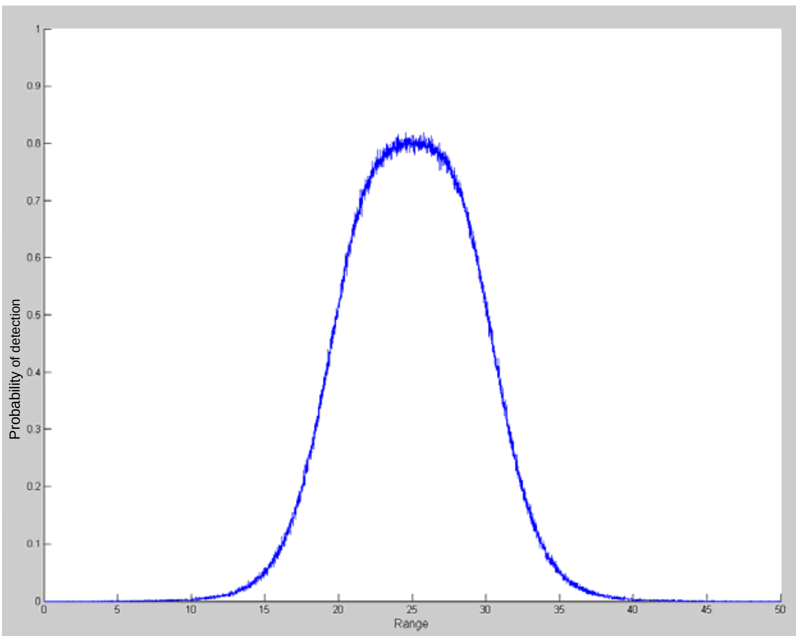

Figure 16. Simulation result of lateral range curve

Then, the area underneath the lateral range curve is sweep width, which can be computed numerically.

3. Output of Lateral Range Curve

The following tables compare the half-sweep width computed from lateral range curve simulations to the best-fit $\tilde{R}$ obtained from Poisson Scan and Lambda-Sigma $(\lambda-\sigma)$ simulations, for various FOM, λ, and σ values and assuming spherical spreading.

$FOM[dB]$	74	77	80	84	87	90
Poisson Scan model $[nm]$	1.40	2.74	4.94	8.24	13.08	18.15
$(\lambda-\sigma)$ model$[nm]$	2.80	3.97	5.37	8.81	12.81	19.07
Lateral range half sweep width of P-S model $[nm]$	2.14	3.75	6.31	10.26	16.15	28.42
Lateral range half sweep width of $(\lambda-\sigma)$ model $[nm]$	3.07	4.49	6.65	11.31	16.87	25.63

Table 1. Comparison of $\tilde{R}$ between Poisson Scan model, $(\lambda-\sigma)$ model and half sweep width of each model when $\sigma = 3dB,\ \lambda = 2\frac{1}{hour}, V = 15\frac{nm}{hour}, U = 5\frac{nm}{hour}$

$Sigma[dB]$	0	1	2	3	4	5	6	7
Poisson Scan model $[nm]$	3.49	3.51	3.75	4.68	5.25	6.52	8.51	11.83
$(\lambda-\sigma)$ model$[nm]$	4.61	5.01	5.11	5.89	6.56	7.84	10.08	12.68
Lateral range half sweep width of P-S model $[nm]$	3.68	4.29	5.12	6.31	8.07	10.62	14.56	20.78
Lateral range half sweep width of $(\lambda-\sigma)$ model $[nm]$	5.30	5.55	5.98	6.64	7.70	9.36	11.86	15.42

Table 2. Comparison of $\tilde{R}$ between Poisson Scan model, $(\lambda-\sigma)$ model and half sweep width of each model when $FOM = 80dB,\ \lambda = 2\frac{1}{hour}, V = 15\frac{nm}{hour}, U = 5\frac{nm}{hour}$

$\lambda[1/hour]$	0.5	1	1.5	2	2.5	3
Poisson Scan model[nm]	1.79	2.77	3.86	4.68	4.94	5.25
$(\lambda-\sigma)$ model[nm]	5.03	5.58	5.86	5.89	5.99	6.06
Lateral range half sweep width of P-S model[nm]	0.89	2.98	4.89	6.32	7.40	8.23
Lateral range half sweep width of $(\lambda-\sigma)$ model[nm]	5.73	6.07	6.36	6.64	6.84	7.02

Table 3. Comparison of $\tilde{R}$ between Poisson Scan model, $(\lambda-\sigma)$ model and half sweep width of each model when $FOM = 80dB,\ \sigma = 3dB,\ V = 15\frac{nm}{hour}, U = 5\frac{nm}{hour}$

These results showed that:

- The best fit $\tilde{R}$ is strongly dependent on FOM, λ, and σ for both the Poisson Scan and Lambda-Sigma $(\lambda-\sigma)$ models.
- The Lambda-Sigma $(\lambda-\sigma)$ best fit $\tilde{R}$ somewhat exceeds the Poisson Scan best fit $\tilde{R}$, and this might be due to the Lambda-Sigma $(\lambda-\sigma)$ model starting with a positive probability of detection at time 0.
- The best fit $\tilde{R}$ generally increases with increases in FOM, λ, and σ for both the Poisson Scan and Lambda-Sigma $(\lambda-\sigma)$ models
- The lateral range curve model produces half-sweep width values which can significantly exceed the best fit $\tilde{R}$ values produces by the Poisson Scan and Lambda-Sigma $(\lambda-\sigma)$ models

It is not clear why the lateral range procedure produced detection range estimates not matching well with those of the Poisson Scan and Lambda-Sigma $(\lambda-\sigma)$ models; but it is possible that the lateral range assumptions (infinite, straight-line paths with uniformly distributed closest points of approach) were not well enough met in the area search simulation.

B. $R(50)$

1. Description of $R(50)$

Another way to estimate the best-fit $\tilde{R}$ is to use $R(50)$, which is the maximum range between the searcher and target resulting in an instantaneous detection probability of 0.5. In other words, $R(50)$ is the maximum range where mean signal excess $\overline{SE}$ is zero. For this analysis, the author assumes that signal transmission loss follows a mixture of spherical spreading $(N_w = 20\log_{10} r)$ and cylindrical spreading $(N_w = 10\log_{10} r)$. Specifically, we use $N_w = 15\log_{10} r$. The author also assumes that the source level of the target L_S is in the range 84 to 95dB, detection noise level at the receiver DNL is 45 dB, and detection threshold DT is -15 dB. Therefore, by the passive sonar equation, FOM is in the range 54 to 65 dB.

2. Compute $R(50)$

With the author's assumptions, the FOM and $R(50)$ can be calculated.

$$\begin{aligned} \overline{SE} &= FOM - N_w \\ &= FOM - 15\log_{10} r. \end{aligned}$$

Setting $\overline{SE} = 0dB$, $P_d = 0.5$, and $1nm = 1852m$, we determine that

$$R(50) = \frac{10^{\frac{FOM}{15}}}{1852}.$$

This value can be compared to the best-fit $\tilde{R}$ values obtained from the Poisson Scan and Lambda-Sigma $(\lambda - \sigma)$ simulations.

3. Output of $R(50)$

For acoustic detection models, a reasonable λ is 1 to $2hr^{-1}$ and a reasonable σ is 2 to $4dB$. The author used regression to examine how well the computed $R(50)$ values estimated the best-fit $\tilde{R}$ values obtained from the Poisson Scan and Lambda-Sigma $(\lambda - \sigma)$ simulations.

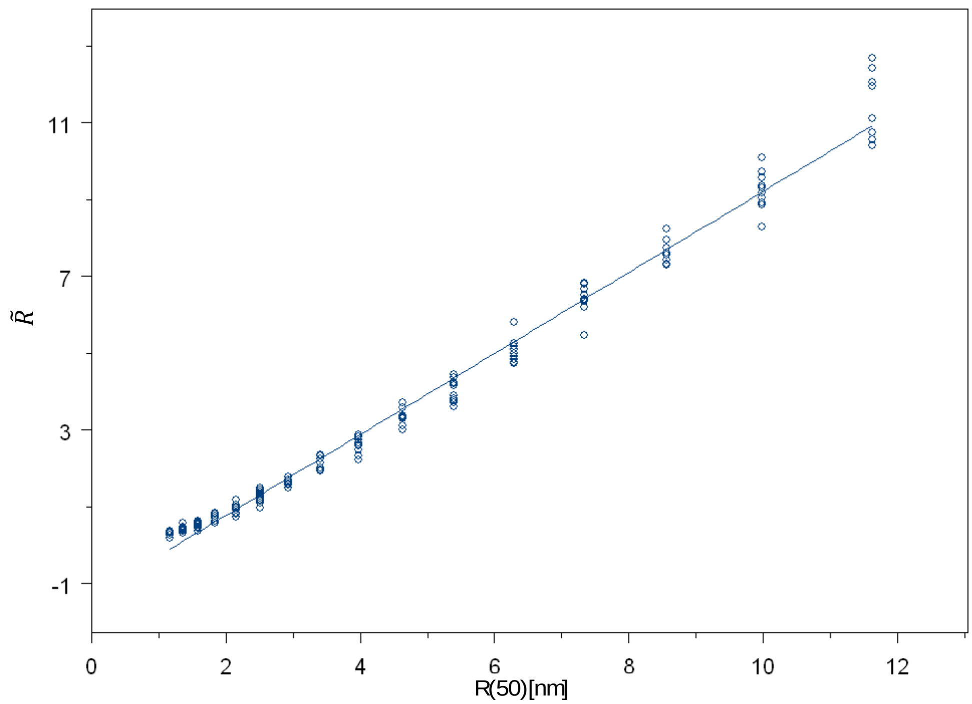

Figure 17. Regression result of Poisson Scan model

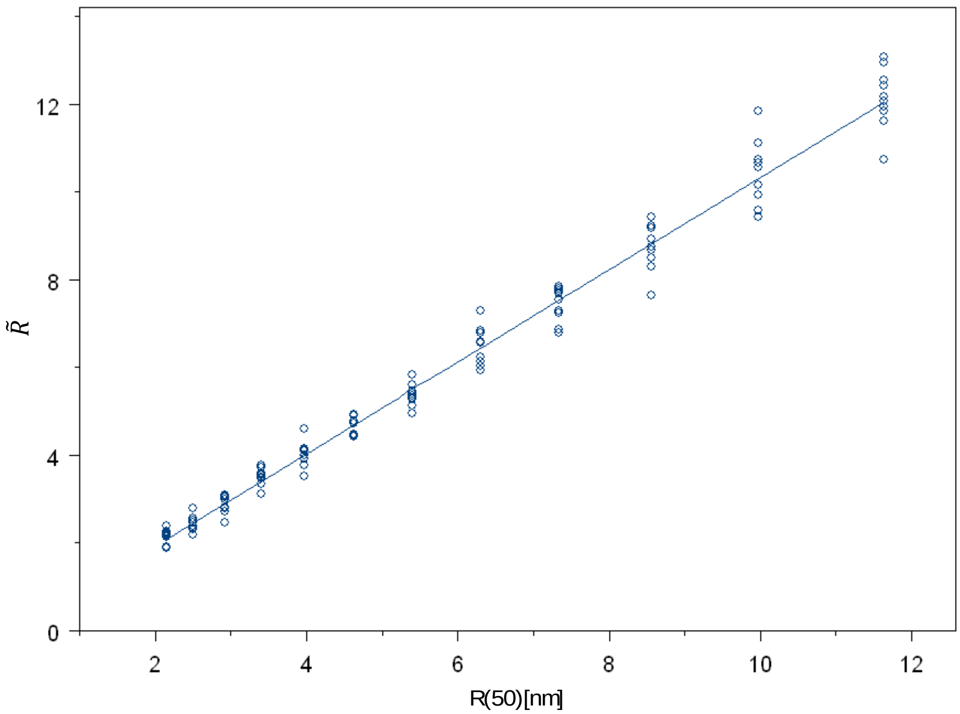

Figure 18. Regression result of Lambda-Sigma $(\lambda - \sigma)$ model

As shown in Figures 17 and 18, the relationship between $R(50)$ and best-fit $\tilde{R}$ is approximately linear. In addition, the slope of each regression line strongly depends on the values of λ, σ, and the model being used.

σ \ λ	$1\ [hr^{-1}]$	$2\ [hr^{-1}]$
$2dB$	$\tilde{R} = 0.85 \times R(50) - 1.71$	$\tilde{R} = 1.10 \times R(50) - 1.67$
$3dB$	$\tilde{R} = 1.06 \times R(50) - 2.17$	$\tilde{R} = 1.44 \times R(50) - 2.41$
$4dB$	$\tilde{R} = 1.49 \times R(50) - 3.05$	$\tilde{R} = 2.13 \times R(50) - 3.79$

Table 4. Regression result of Poisson Scan model at each λ and σ

σ \ λ	$1[hr^{-1}]$	$2[hr^{-1}]$
$2dB$	$\tilde{R} = 1.05 \times R(50) - 0.16$	$\tilde{R} = 1.14 \times R(50) - 0.23$
$3dB$	$\tilde{R} = 1.26 \times R(50) - 0.44$	$\tilde{R} = 1.50 \times R(50) - 0.93$
$4dB$	$\tilde{R} = 1.65 \times R(50) - 1.09$	$\tilde{R} = 2.15 \times R(50) - 2.27$

Table 5. Regression result of Lambda-Sigma ($\lambda - \sigma$) model at each λ and σ

As illustrated in Tables 4 and 5, it is possible to estimate the best-fit $\tilde{R}$ from problem parameters $R(50)$, λ, and σ. Then this best-fit $\tilde{R}$ can be used with the Random Search formula below to further estimate the area search probability of detection by time t.

$$P(T \leq t) = F_T(t) = 1 - e^{\frac{-2\tilde{R}\tilde{V}t}{A}}.$$

VI. CONCLUSIONS

A. CONCLUSIONS AND RECOMMENDATIONS

There are two primary contributions of this thesis. The first is the demonstration that initial detection times for area search simulations using both the Poisson Scan and Lambda-Sigma $(\lambda - \sigma)$ acoustic detection models are approximately exponentially distributed, allowing the simulation results to be closely approximated by the venerable Random Search formula. And the second contribution is the observation that the best-fit cookie-cutter detection range used in the Random Search formula can be accurately predicted using the simulation model parameters λ, σ, and the $R(50)$ detection range.

In this thesis, it is assumed that acoustic signal transmission loss follows either spherical spreading or cylindrical spreading. A potentially more realistic model could be developed by using actual propagation loss data.

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APPENDIX

A. MATLAB CODE

1. Poisson Scan Model(I)

```
Nreps=500; %number of simulation replications
tmax=150; %max. simulation time (hr)
lx=150; %search area length in x direction (nm)
ly=150; %search area length in y direction (nm)
V=15; %searcher speed (nm/hr)
U=5; %target speed (nm/hr)
lams=(2*V)/sqrt(lx*ly); %searcher course change rate (1/hr)
lamt=(2*U)/sqrt(lx*ly); %searcher course change rate (1/hr)
laml=2; %searcher looking rate (1/hr)
sig=3; %signal excess variance (dB)
FOM=60; %figure of merit (dB)
R50=10^(FOM/15)/1852; %R(50) (nm)
dt=R50/(2*V); %delta t (hours)
Xs=zeros(1,tmax/dt+1); %initialize x-position to zero(searcher)
Ys=Xs; %initialize y-position to zero(searcher)
Cs=Xs; %initialize searcher course to zero
Xt=Xs; %initialize x-position to zero(target)
Yt=Xs; %initialize y-position to zero(target)
Ct=Xs; %initialize target course to zero
T=0:dt:tmax; %simulation time vector
A=lx*ly; %search area
CumDet=zeros(1, tmax/dt+1); %initialize cumulative detection state
for n=1:Nreps %main simulation loop
    xs=rand*lx; %initial searcher and target x and y positions
    ys=rand*ly;
    xt=rand*lx;
    yt=rand*ly;
    cs=rand*2*pi; %initial searcher course
    ct=rand*2*pi; %initial target course
    t=0; %set simulation time to 0
    tindex=1; %initialize time index to 1
    Xs(tindex)=xs; %save initial searcher x position
    Ys(tindex)=ys; %save initial searcher y position
    Cs(tindex)=cs; %save initial searcher course
    Xt(tindex)=xt; %save initial target x position
    Yt(tindex)=yt; %save initial target y position
    Ct(tindex)=ct; %save initial target course
    Distance=zeros(1, tmax/dt+1); %initialize distance between target and
searcher
    SE=zeros(1, tmax/dt+1); %initialize signal excess
    Gamma=zeros(1, tmax/dt+1); %initialize instantaneous probability of
detection
    Detection=zeros(1, tmax/dt+1); % initialize detection vector
    for t=1:tmax/dt %inner loop
```

```
        tindex = tindex+1; %update simulation time index
        if rand<lams*dt; cs=rand*2*pi; end
        if Xs(tindex-1)<0; cs=(rand-0.5); end
        if Xs(tindex-1)>lx; cs=pi+(rand-0.5); end
        if Ys(tindex-1)<0; cs=pi/2+(rand-0.5); end
        if Ys(tindex-1)>ly; cs=-pi/2+(rand-0.5); end
        if rand<lamt*dt; ct=rand*2*pi; end
        if Xt(tindex-1)<0; ct=(rand-0.5); end
        if Xt(tindex-1)>(lx); ct=pi+(rand-0.5); end
        if Yt(tindex-1)<0; ct=pi/2+(rand-0.5); end
        if Yt(tindex-1)>(ly); ct=-pi/2+(rand-0.5); end
        Xs(tindex) = Xs(tindex-1)+V*dt*cos(cs); %Update x and y positions
        Ys(tindex) = Ys(tindex-1)+V*dt*sin(cs);
        Cs(tindex)=cs;
        Xt(tindex) = Xt(tindex-1)+U*dt*cos(ct); %Update x and y positions
        Yt(tindex) = Yt(tindex-1)+U*dt*sin(ct);
        Ct(tindex)=ct;

Distance(tindex-1)=sqrt((Xs(tindex-1)-Xt(tindex-1)).^2+(Ys(tindex-1)-Yt
(tindex-1)).^2);
        SE(tindex-1)=FOM-15*log10(Distance(tindex-1)*1852);
        Gamma(tindex-1)=laml*normcdf(SE(tindex-1)/sig);
        if rand <= Gamma(tindex-1)*dt;
           Detection(t:(tmax/dt+1))=1;
        end
    end %inner loop (time increasing from 0 to tmax)
    CumDet = CumDet + Detection;
end %outer loop (simulation replications)
Probability=CumDet/Nreps;
plot(T,Probability,'b-'), axis([0,tmax,0,1])
xlabel('Time', 'Fontsize', 12), ylabel('CDP', 'FontSize', 12)
```

2. Poisson Scan Model(II)

```
Nreps=500; %number of simulation replications
tmax=150; %max. simulation time (hr)
lx=150; %search area length in x direction (nm)
ly=150; %search area length in y direction (nm)
V=15; %searcher speed (nm/hr)
U=5; %target speed (nm/hr)
lams=(2*V)/sqrt(lx*ly); %searcher course change rate (1/hr)
lamt=(2*U)/sqrt(lx*ly); %searcher course change rate (1/hr)
laml=2; %searcher looking rate (1/hr)
sig=3; %signal excess variance (dB)
FOM=60; %figure of merit (dB)
R50=10^(FOM/15)/1852; %R(50) (nm)
dt=R50/(2*V); %delta t (hours)
Xs=zeros(1,tmax/dt+1); %initialize x-position to zero(searcher)
Ys=Xs; %initialize y-position to zero(searcher)
Cs=Xs; %initialize searcher course to zero
Xt=Xs; %initialize x-position to zero(target)
Yt=Xs; %initialize y-position to zero(target)
Ct=Xs; %initialize target course to zero
T=0:dt:tmax; %simulation time vector
A=lx*ly; %search area
CumDet=zeros(1, tmax/dt+1); %initialize cumulative detection state
for n=1:Nreps %main simulation loop
    xs=rand*lx; %initial searcher and target x and y positions
    ys=rand*ly;
    xt=rand*lx;
    yt=rand*ly;
    cs=rand*2*pi; %initial searcher course
    ct=rand*2*pi; %initial target course
    t=0; %set simulation time to 0
    tindex=1; %initialize time index to 1
    Xs(tindex)=xs; %save initial searcher x position
    Ys(tindex)=ys; %save initial searcher y position
    Cs(tindex)=cs; %save initial searcher course
    Xt(tindex)=xt; %save initial target x position
    Yt(tindex)=yt; %save initial target y position
    Ct(tindex)=ct; %save initial target course
    Distance=zeros(1, tmax/dt+1); %initialize distance between target and
searcher
    SE=zeros(1, tmax/dt+1); %initialize signal excess
    Gamma=zeros(1, tmax/dt+1); %initialize instantaneous probability of
detection
    Detection=zeros(1, tmax/dt+1); % initialize detection vector
    for t=1:tmax/dt %inner loop
        tindex = tindex+1; %update simulation time index
        if rand<lams*dt; cs=rand*2*pi; end
        if Xs(tindex-1)<0; cs=(rand-0.5); end
        if Xs(tindex-1)>lx; cs=pi+(rand-0.5); end
        if Ys(tindex-1)<0; cs=pi/2+(rand-0.5); end
```

```
        if Ys(tindex-1)>ly; cs=-pi/2+(rand-0.5); end
        if rand<lamt*dt; ct=rand*2*pi; end
        if Xt(tindex-1)<0; ct=(rand-0.5); end
        if Xt(tindex-1)>(lx); ct=pi+(rand-0.5); end
        if Yt(tindex-1)<0; ct=pi/2+(rand-0.5); end
        if Yt(tindex-1)>(ly); ct=-pi/2+(rand-0.5); end
        Xs(tindex) = Xs(tindex-1)+V*dt*cos(cs); %Update x and y positions
        Ys(tindex) = Ys(tindex-1)+V*dt*sin(cs);
        Cs(tindex)=cs;
        Xt(tindex) = Xt(tindex-1)+U*dt*cos(ct); %Update x and y positions
        Yt(tindex) = Yt(tindex-1)+U*dt*sin(ct);
        Ct(tindex)=ct;
        if rand<laml*dt

Distance(tindex-1)=sqrt((Xs(tindex-1)-Xt(tindex-1)).^2+(Ys(tindex-1)-Yt
(tindex-1)).^2);
            SE(tindex-1)=FOM-20*log10(Distance(tindex-1)*1852);
            Gamma(tindex-1)=normcdf(SE(tindex-1)/sig);
            if rand <= Gamma(tindex-1);
                Detection(t:(tmax/dt+1))=1;
            end
        end
    end %inner loop (time increasing from 0 to tmax)
    CumDet = CumDet + Detection;
end %outer loop (simulation replications)
Probability=CumDet/Nreps;
plot(T,Probability,'b-'), axis([0,tmax,0,1])
xlabel('Time', 'Fontsize', 12), ylabel('CDP', 'FontSize', 12)
```

3. Lambda-Sigma ($\lambda-\sigma$) Model

```
Nreps=500; %number of simulation replications
tmax=150; %max. simulation time (hr)
lx=150; %search area length in x direction (nm)
ly=150; %search area length in y direction (nm)
V=15; %searcher speed (nm/hr)
U=5; %target speed (nm/hr)
lams=(2*V)/sqrt(lx*ly); %searcher course change rate (1/hr)
lamt=(2*U)/sqrt(lx*ly); %searcher course change rate (1/hr)
lamda=2; %duration of each step (1/hr)
sig=3; %height of each step (dB)
FOM=60; % figure of merit (dB)
R50=10^(FOM/15)/1852; %R(50) (nm)
dt=R50/(2*V); %delta t (hours)
Xs=zeros(1,tmax/dt+1); %initialize x-position to zero(searcher)
Ys=Xs; %initialize y-position to zero(searcher)
Cs=Xs; %initialize searcher course to zero
Xt=Xs; %initialize x-position to zero(target)
Yt=Xs; %initialize y-position to zero(target)
Ct=Xs; %initialize target course to zero
SP=Xs; %initialize step function
T=0:dt:tmax; %simulation time vector
A=lx*ly; %search area
CumDet=zeros(1, tmax/dt+1); %initialize cumulative detection state
for n=1:Nreps %main simulation loop
    xs=rand*lx; %initial searcher and target x and y positions
    ys=rand*ly;
    xt=rand*lx;
    yt=rand*ly;
    cs=rand*2*pi; %initial searcher course
    ct=rand*2*pi; %initial target course
    sp=randn*sig; %initial step
    t=0; %set simulation time to 0
    tindex=1; %initialize time index to 1
    Xs(tindex)=xs; %save initial searcher x position
    Ys(tindex)=ys; %save initial searcher y position
    Cs(tindex)=cs; %save initial searcher course
    Xt(tindex)=xt; %save initial target x position
    Yt(tindex)=yt; %save initial target y position
    Ct(tindex)=ct; %save initial target course
    SP(tindex)=sp; %save initial step
    Distance=zeros(1, tmax/dt+1); %initialize distance between target and
searcher
    SE=zeros(1, tmax/dt+1); %initialize signal excess
    Detection=zeros(1, tmax/dt+1); %initialize detection vector
    for t=1:tmax/dt %inner loop
        tindex = tindex+1; %update simulation time index
        SP(tindex)=SP(tindex-1);
        if rand<lamda*dt; SP(tindex)=randn*sig; end
        if rand<lams*dt; cs=rand*2*pi; end
```

```
        if Xs(tindex-1)<0; cs=(rand-0.5); end
        if Xs(tindex-1)>lx; cs=pi+(rand-0.5); end
        if Ys(tindex-1)<0; cs=pi/2+(rand-0.5); end
        if Ys(tindex-1)>ly; cs=-pi/2+(rand-0.5); end
        if rand<lamt*dt; ct=rand*2*pi; end
        if Xt(tindex-1)<0; ct=(rand-0.5); end
        if Xt(tindex-1)>(lx); ct=pi+(rand-0.5); end
        if Yt(tindex-1)<0; ct=pi/2+(rand-0.5); end
        if Yt(tindex-1)>(ly); ct=-pi/2+(rand-0.5); end
        Xs(tindex) = Xs(tindex-1)+V*dt*cos(cs); %Update x and y positions
        Ys(tindex) = Ys(tindex-1)+V*dt*sin(cs);
        Cs(tindex)=cs;
        Xt(tindex) = Xt(tindex-1)+U*dt*cos(ct); %Update x and y positions
        Yt(tindex) = Yt(tindex-1)+U*dt*sin(ct);
        Ct(tindex)=ct;

Distance(tindex-1)=sqrt((Xs(tindex-1)-Xt(tindex-1)).^2+(Ys(tindex-1)-Yt
(tindex-1)).^2);
        SE(tindex-1)=FOM-15*log10(Distance(tindex-1)*1852);
        if SP(tindex-1) <= SE(tindex-1);
           Detection(t:(tmax/dt+1))=1;
        end
    end %inner loop (time increasing from 0 to tmax)
    CumDet = CumDet + Detection;
end %outer loop (simulation replications)
Probability=CumDet/Nreps;
plot(T,Probability,'b-'), axis([0,tmax,0,1])
xlabel('Time', 'Fontsize', 12), ylabel('CDP', 'FontSize', 12)
```

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