

Algebraic Fractions

$$\begin{aligned} 1 \quad & \frac{x^2 - 8x + 15}{x^2 - 9} \times \frac{2x^2 + 6x}{(x - 5)^2} \\ &= \frac{(x-5)(x-3)}{(x-3)(x+3)} \times \frac{2x(x+3)}{(x-5)(x-5)} \\ &= \frac{\cancel{(x-5)} \cancel{(x-3)}}{\cancel{(x-3)} \cancel{(x+3)}} \times \frac{2x \cancel{(x+3)}}{\cancel{(x-5)}(x-5)} \\ &= \frac{2x}{x-5} \end{aligned}$$

$$\begin{aligned} 2 \quad & \frac{2x^2 + 9x - 5}{x^2 + 2x - 15} \\ &= \frac{(2x-1)(x+5)}{(x-3)(x+5)} \\ &= \frac{(2x-1)\cancel{(x+5)}}{(x-3)\cancel{(x+5)}} \\ &= \frac{2x-1}{x-3} \end{aligned}$$

$$\begin{aligned} 3 \quad (a) \quad & \frac{3x^2 - x - 2}{x^2 - 1} \\ &= \frac{(x-1)(3x+2)}{(x-1)(x+1)} \\ &= \frac{\cancel{(x-1)}(3x+2)}{\cancel{(x-1)}(x+1)} \\ &= \frac{3x+2}{x+1} \end{aligned}$$

$$\begin{aligned}
3 \quad (b) \quad & \frac{3x^2 - x - 2}{x^2 - 1} - \frac{1}{x(x+1)} \\
&= \frac{3x+2}{x+1} - \frac{1}{x(x+1)} \quad [\text{use the answer above}] \\
&= \frac{x(3x+2) - 1}{x(x+1)} \\
&= \frac{3x^2 + 2x - 1}{x(x+1)} \\
&= \frac{(3x-1)(x+1)}{x(x+1)} \\
&= \frac{(3x-1)\cancel{(x+1)}}{x\cancel{(x+1)}} \\
&= \frac{3x-1}{x}
\end{aligned}$$

$$\begin{aligned}
4 \quad & \frac{x+1}{3x^2-3} - \frac{1}{3x+1} \\
&= \frac{x+1}{3(x^2-1)} - \frac{1}{3x+1} \\
&= \frac{x+1}{3(x-1)(x+1)} - \frac{1}{3x+1} \\
&= \frac{\cancel{x+1} \cdot 1}{3(x-1)\cancel{(x+1)}} - \frac{1}{3x+1} \\
&= \frac{1(3x+1) - 1(3(x-1))}{3(x-1)(3x+1)} \\
&= \frac{4}{3(x-1)(3x+1)}
\end{aligned}$$

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$$\begin{aligned}
& \frac{2x^2 + 3x}{(2x + 3)(x - 2)} - \frac{6}{x^2 - x - 2} \\
&= \frac{x(2x + 3)}{(2x + 3)(x - 2)} - \frac{6}{(x - 2)(x + 1)} \\
&= \frac{\cancel{x(2x + 3)}}{\cancel{(2x + 3)}(x - 2)} - \frac{6}{(x - 2)(x + 1)} \\
&= \frac{x(x + 1) - 6}{(x - 2)(x + 1)} \\
&= \frac{(x - 2)(x + 3)}{(x - 2)(x + 1)} \\
&= \frac{\cancel{(x - 2)}(x + 3)}{\cancel{(x - 2)}(x + 1)} \\
&= \frac{x + 3}{x + 1}
\end{aligned}$$

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$$\begin{aligned}
& \frac{4x - 5}{(2x + 1)(x - 3)} - \frac{2x}{x^2 - 9} \\
&= \frac{4x - 5}{(2x + 1)(x - 3)} - \frac{2x}{(x - 3)(x + 3)} \\
&= \frac{(4x - 5)(x + 3) - 2x(2x + 1)}{(2x + 1)(x - 3)(x + 3)} \\
&= \frac{5x - 15}{(2x + 1)(x - 3)(x + 3)} \\
&= \frac{5(x - 3)}{(2x + 1)(x - 3)(x + 3)} \\
&= \frac{5\cancel{(x - 3)}}{(2x + 1)\cancel{(x - 3)}(x + 3)} \\
&= \frac{5}{(2x + 1)(x + 3)}
\end{aligned}$$

$$\begin{aligned}
7 \quad & \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4} \\
&= \frac{3(x+1)}{(2x-1)(x+4)} - \frac{1}{x+4} \\
&= \frac{3(x+1) - 1(2x-1)}{(2x-1)(x+4)} \\
&= \frac{x+4}{(2x-1)(x+4)} \\
&= \frac{\cancel{x+4} \cdot 1}{(2x-1)\cancel{(x+4)}} \\
&= \frac{1}{2x-1}
\end{aligned}$$

$$\begin{aligned}
8 \quad & \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3} \\
&= \frac{2(x-1)}{(x-3)(x+1)} - \frac{1}{x-3} \\
&= \frac{2(x-1) - 1(x+1)}{(x-3)(x+1)} \\
&= \frac{x-3}{(x-3)(x+1)} \\
&= \frac{\cancel{x-3} \cdot 1}{\cancel{(x-3)}(x+1)} \\
&= \frac{1}{x+1}
\end{aligned}$$

$$\begin{aligned}
9 \quad & \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3} \\
&= \frac{2(x+1)}{(x-3)(x+1)} - \frac{x+1}{x-3} \\
&= \frac{2\cancel{(x+1)}}{(x-3)\cancel{(x+1)}} - \frac{x+1}{x-3} \\
&= \frac{2}{x-3} - \frac{x+1}{x-3} \\
&= \frac{2-(x+1)}{x-3} \\
&= \frac{1-x}{x-3}
\end{aligned}$$

$$\begin{aligned}
10 \quad & \frac{2x+3}{x+2} - \frac{9+2x}{2x^2+3x-2} \\
&= \frac{2x+3}{x+2} - \frac{9+2x}{(2x-1)(x+2)} \\
&= \frac{(2x+3)(2x-1) - (9+2x)}{(2x-1)(x+2)} \\
&= \frac{4x^2+2x-12}{(2x-1)(x+2)} \\
&= \frac{2(2x^2+x-6)}{(2x-1)(x+2)} \\
&= \frac{2(2x-3)(x+2)}{(2x-1)(x+2)} \\
&= \frac{2(2x-3)\cancel{(x+2)}}{(2x-1)\cancel{(x+2)}} \\
&= \frac{4x-6}{2x-1}
\end{aligned}$$

$$\begin{aligned}
11 \quad & \frac{5x+1}{x^2+x-2} - \frac{3}{x+2} \\
&= \frac{5x+1}{(x-1)(x+2)} - \frac{3}{x+2} \\
&= \frac{5x+1-3(x-1)}{(x-1)(x+2)} \\
&= \frac{2x+4}{(x-1)(x+2)} \\
&= \frac{2(x+2)}{(x-1)(x+2)} \\
&= \frac{2\cancel{(x+2)}}{(x-1)\cancel{(x+2)}} \\
&= \frac{2}{x-1}
\end{aligned}$$

$$\begin{aligned}
12 \quad & 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2} \\
&= \frac{1(x+2)^2 - 3(x+2) + 3}{(x+2)^2} \\
&= \frac{x^2 + 4x + 4 - 3(x+2) + 3}{(x+2)^2} \\
&= \frac{x^2 + x + 1}{(x+2)^2}
\end{aligned}$$

$$\begin{aligned}
13 \quad & \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2 \\
&= \frac{(4x-1)(2x-1) - 3 - 2[2(x-1)(2x-1)]}{2(x-1)(2x-1)} \\
&= \frac{8x^2 - 6x + 1 - 3 - 4(2x^2 - 3x + 1)}{2(x-1)(2x-1)} \\
&= \frac{6x - 6}{2(x-1)(2x-1)} \\
&= \frac{6(x-1)}{2(x-1)(2x-1)} \\
&= \frac{3 \cancel{6(x-1)}}{2(x-1)(2x-1)} \\
&= \frac{3}{2x-1}
\end{aligned}$$

$$\begin{aligned}
14 \quad & 1 - \frac{2}{x+4} + \frac{x-8}{(x-2)(x+4)} \\
&= \frac{1(x-2)(x+4) - 2(x-2) + (x-8)}{(x-2)(x+4)} \\
&= \frac{x^2 + x - 12}{(x-2)(x+4)} \\
&= \frac{(x-3)(x+4)}{(x-2)(x+4)} \\
&= \frac{(x-3)\cancel{(x+4)}}{(x-2)\cancel{(x+4)}} \\
&= \frac{x-3}{x-2}
\end{aligned}$$

$$15 \quad f(x) = 2x^3 - x^2 + 4x + 15$$

$$f\left(-\frac{3}{2}\right) = 0$$

$\therefore (2x+3)$ is a factor of $2x^3 - x^2 + 4x + 15$.

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$x - 2$	$\begin{array}{r} x^3 + 2x^2 + 4x + 4 \\ \hline x^4 + 0 \cdot x^3 + 0 \cdot x^2 - 4x - 8 \\ x^4 - 2x^3 \\ \hline 2x^3 - 4x - 8 \\ 2x^3 - 4x^2 \\ \hline 4x^2 - 4x - 8 \\ 4x^2 - 8x \\ \hline 4x - 8 \\ 4x - 8 \\ \hline \end{array}$
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$x^2 - 1$	$\begin{array}{r} 2x^2 - 1 \\ \hline 2x^4 + 0 \cdot x^3 - 3x^2 + x + 1 \\ 2x^4 - 2x^2 \\ \hline -x^2 + x + 1 \\ -x^2 + 1 \\ \hline x \end{array}$
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$x^2 + x - 6$	$\begin{array}{r} x^2 + 1 \\ \hline x^4 + x^3 - 5x^2 + 0 \cdot x - 9 \\ x^4 + x^3 - 6x^2 \\ \hline x^2 - 9 \\ x^2 + x - 6 \\ \hline -x - 3 \end{array}$
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$$\frac{(x^4 + x^3 - 5x^2 - 9)}{x^2 + x - 6}$$

$$\equiv x^2 + 1 + \frac{-x - 3}{x^2 + x - 6}$$

$$\equiv x^2 + 1 + \frac{-(x + 3)}{(x - 2)(x + 3)}$$

$$\equiv x^2 + 1 + \frac{-1(x + 3)}{(x - 2)(x + 3)}$$

$$\equiv x^2 + 1 - \frac{1}{x - 2}$$

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$x^2 - 3x + 3$	$x^2 + 4x - 4$
	$x^4 + x^3 - 13x^2 + 26x - 17$
	$x^4 - 3x^3 + 3x^2$
	$4x^3 - 16x^2 + 26x - 17$
	$4x^3 - 12x^2 + 12x$
	$- 4x^2 + 14x - 17$
	$- 4x^2 + 12x - 12$
	$2x - 5$

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$x^2 - 3$	$3x^2 - 4x + 1$
	$3x^4 - 4x^3 - 8x^2 + 16x - 2$
	$3x^4 - 9x^2$
	$- 4x^3 + x^2 + 16x - 2$
	$- 4x^3 + 12x$
	$x^2 + 4x - 2$
	$x^2 - 3$
	$4x + 1$

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$x - 2$	$x^2 - 4x + 3$
	$x^3 - 6x^2 + 11x + 2$
	$x^3 - 2x^2$
	$- 4x^2 + 11x + 2$
	$- 4x^2 + 8x$
	$3x + 2$
	$3x - 6$
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