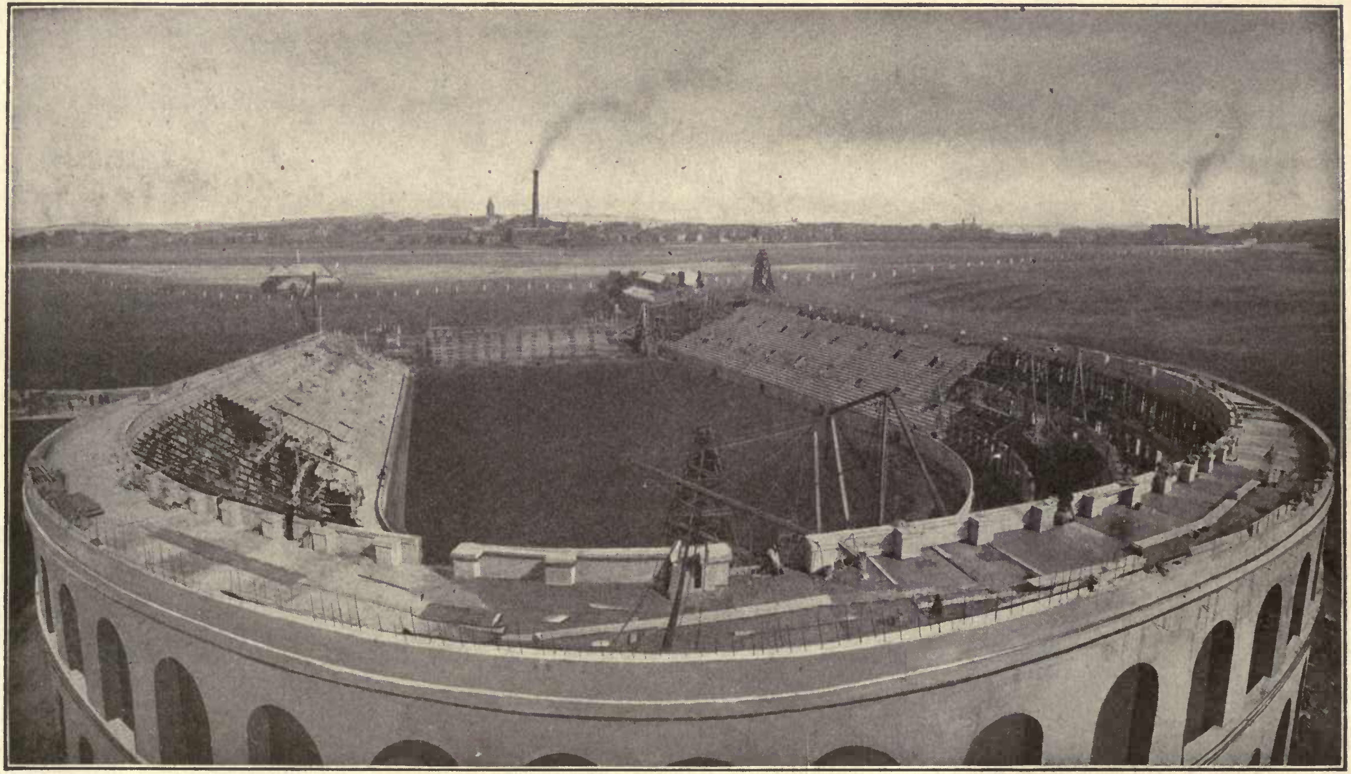
REINFORCED CONCRETE

WEBB — GIBSON

AMERICAN SCHOOL *of* CORRESPONDENCE

CHICAGO ILLINOIS



STADIUM AT HARVARD UNIVERSITY, CAMBRIDGE, MASS.

Built entirely of Reinforced Concrete.

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Reinforced Concrete

A Treatise

ON CEMENT, CONCRETE, AND CONCRETE STEEL, AND THEIR APPLICATIONS TO MODERN STRUCTURAL WORK

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ILLUSTRATED

CHICAGO
AMERICAN SCHOOL OF CORRESPONDENCE
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Foreword

IN recent years, such marvelous advances have been made in the engineering and scientific fields, and so rapid has been the evolution of mechanical and constructive processes and methods, that a distinct need has been created for a series of *practical working guides*, of convenient size and low cost, embodying the accumulated results of experience and the most approved modern practice along a great variety of lines. To fill this acknowledged need, is the special purpose of the series of handbooks to which this volume belongs.

¶ In the preparation of this series, it has been the aim of the publishers to lay special stress on the *practical* side of each subject, as distinguished from mere theoretical or academic discussion. Each volume is written by a well-known expert of acknowledged authority in his special line, and is based on a most careful study of practical needs and up-to-date methods as developed under the conditions of actual practice in the field, the shop, the mill, the power house, the drafting room, the engine room, etc.

¶ These volumes are especially adapted for purposes of self-instruction and home study. The utmost care has been used to bring the treatment of each subject within the range of the common understanding, so that the work will appeal not only to the technically trained expert, but also to the beginner and the self-taught practical man who wishes to keep abreast of modern progress. The language is simple and clear; heavy technical terms and the formulæ of the higher mathematics have been avoided, yet without sacrificing any of the requirements of practical instruction; the arrangement of matter is such as to carry the reader along by easy steps to complete mastery of each subject; frequent examples for practice are given, to enable the reader to test his knowledge and make it a permanent possession; and the illustrations are selected with the greatest care to supplement and make clear the references in the text.

The method adopted in the preparation of these volumes is that which the American School of Correspondence has developed and employed so successfully for many years. It is not an experiment, but has stood the severest of all tests—that of practical use—which has demonstrated it to be the best method yet devised for the education of the busy working man.

For purposes of ready reference and timely information when needed, it is believed that this series of handbooks will be found to meet every requirement.

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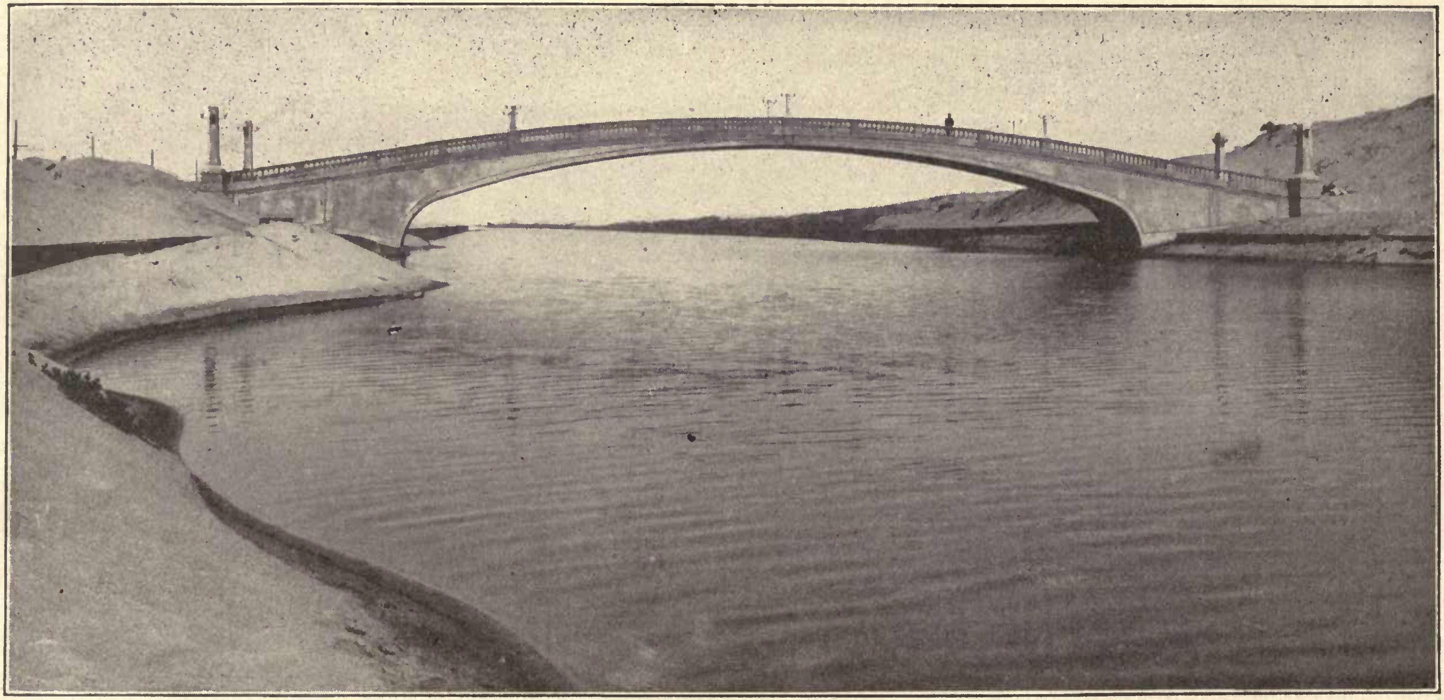


BRIDGE OF REINFORCED CONCRETE, AT PLAYA DEL REY, NEAR LOS ANGELES, CAL.

Extreme Length, 205 feet 8 inches; Span, 146 feet; Width, 19 feet; Spring, 18 feet; Height above water, 20 feet.

REINFORCED CONCRETE

PART I

CEMENT

The discussion of cementing materials will here be confined to Portland cement. A treatise on masonry will usually include a discussion of the various forms of lime and other cementing materials. These are cheaper and sometimes justifiably economical in large masses of masonry. There is hardly an exception to the general statement that Portland cement is the only form of cementing material which should be used in reinforced concrete.

Characteristics of Portland Cement. The value of cement as a building material depends on the following general qualities. When mixed with water and allowed to set, it should harden in a few hours and should develop a considerable proportion of its ultimate strength in a few days. It must also have the characteristic of permanency so that no material change in form or volume will take place on account of inherent qualities or as the result of exterior agencies. There always is a slight shrinkage of the volume of cement and concrete during the process of setting and hardening, but with any good quality of cement this shrinkage is not so great as to be objectionable. Another very essential quality is that the cement shall not lose its strength with age. Although some long-time tests of cement have apparently indicated a slight decrease in the strength of cement after the first year, the decrease is so slight that it need not affect the design of concrete, even assuming the accuracy of the general statement.

CEMENT TESTING

The thorough testing of cement, as it is done for the largest public works, should properly be done in a professional testing laboratory. A text-book of several hundred pages has recently been written on this subject. The ultimate analysis and testing of cement, both chemically and physically, is beyond the province of the ordinary engineer. But the ordinary engineer does have frequent occasion to obtain cement in small quantities when testing in professional laboratories is inconvenient or unduly expensive. Fortunately it is possible to make some simple tests without elaborate apparatus which will at least show whether the cement is radically defective and unfit for use. It is unfortunately true that an occasional barrel of even the best brand of cement will prove to be very inferior to the standard output of that brand. This practically means that in any important work, using a large quantity of cement, it is not sufficient to choose a brand, as the result of preliminary favorable tests, and then accept all shipments without further test. Several barrels in every carload should be sampled for testing. It is not too much to prescribe that *every* barrel should be tested by at least a few of the simpler forms of testing given below. The following methods of testing are condensed from the progress report of the Committee on Uniform Tests of Cement, as selected by the American Society of Civil Engineers. The statements may therefore be considered as having the highest authority obtainable on this subject.

Sampling. The number of samples that should be taken depends on the importance of the work but it is chiefly important that the sample should represent a fair average of the contents. The sample should be passed through a sieve having twenty meshes per linear inch, in order to break up lumps and remove any foreign material. If several small amounts are taken from different parts of the package, this also insures that the samples will be mixed so that the result will be a fair average. When it is only desired to determine the average characteristic of a shipment, the samples taken from different parts of the shipment may be mixed, but it will give a better idea of the uniformity of the product to analyze the different samples separately. Cement should be taken from a barrel by boring a hole through the center of one of the staves, midway between the heads, or through the head. A portion of the cement can then be withdrawn, even from the center, by means of a sampling iron similar to that used by sugar inspectors.

Chemical Analysis. Ordinarily, it is impracticable for an engineer to make a chemical analysis of cement which will furnish reliable information regarding its desirability, but the engineer should understand something regarding the desirable chemical constituents of the cement. It should be realized that the fineness of the grinding and the thoroughness of the burning may have a far greater influence on the value of the cement than slight variations from the recognized standard proportions of the various chemical constituents. Too high a proportion of lime will cause failure in the test for soundness or constancy of volume, although a cement may fail on such a test owing to improper preparation of the raw material or defective burning. On the other hand, if the cement is made from very finely ground material and is thoroughly burned, it may contain a considerable excess of lime and still prove perfectly sound. The permissible amount of magnesia in Portland cement is the subject of considerable controversy. Some authorities say that anything in excess of 8 per cent is harmful, others declare that the amount should not exceed 4 per cent or 5 per cent. The proportion of sulphuric-anhydride should not exceed 1.75 per cent. It may be considered that the other tests of cement are a far more reliable indication of its quality than any small variation in the chemical constituents from the proportions usually considered standard.

Specific Gravity. The specific gravity of cement is lowered by *under-burning, adulteration,* and *hydration,* but the adulteration must be in considerable quantities to affect the results. Since the differences in specific gravity are usually very small, great care must be exercised in making the tests. When properly made, the tests afford a quick check for under-burning or adulteration. The determination of specific gravity is conveniently made with Le Chatelier's apparatus. This consists of a flask D, Fig. 1, of 120-cu. cm. (7.32-cu. in.) capacity, the neck of which is about 20 cm. (7.87 in.) long; in the middle of this neck is a ball C, above and below which are two marks F and E; the volume between these marks is 20 cu. cm. (1.22 cu. in.). The neck has a diameter of about 9 mm. (0.35 in.), and is graduated into tenths of cu. cm. above the mark F. Benzine (62° Baumé naphtha), or kerosene free from water, should be used in making the determination.

The specific gravity may be determined in two ways:

First. The flask is filled with either of these liquids to the lower mark E, and 64 gr. (2.25 oz.) of powder, previously dried at 100° Cent. (212° Fahr.) and cooled to the temperature of the liquid, is gradually introduced through the funnel B (the stem of which extends into the flask to the top of the bulb C) until the proper mark F is reached. The difference in weight between the cement remaining and the original quantity (64 gr.) is the weight which has displaced 20 cu. cm.

Second. The whole quantity of powder is introduced, and the

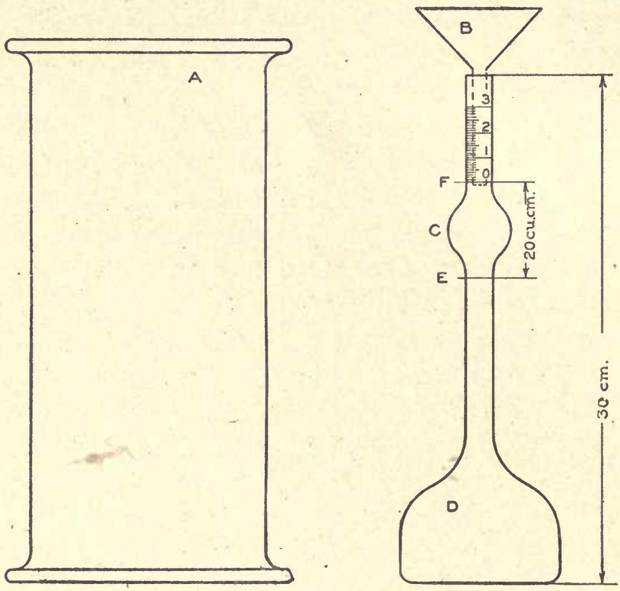


Fig. 1. Le Chatelier's Apparatus for Determining Specific Gravity.

level of the liquid rises to some division of the graduated neck. This reading plus 20 cu. cm. is the volume displaced by 64 gr. of the powder. The specific gravity is then obtained from the formula:

$$\text{Specific Gravity} = \frac{\text{Weight of cement}}{\text{Displaced volume}}.$$

The flask during the operation is kept in water in a jar A in order to avoid variation in the temperature of the liquid. The results should agree within 0.01. The specific gravity of cement thoroughly dried at 100° Cent. should not be less than 3.10.

Fineness. It is generally accepted that the coarser materials in cement are practically inert, and it is only the extremely fine powder that possesses adhesive cementing qualities. The more finely cement is pulverized, all other conditions being the same, the more sand it will carry and produce a mortar of a given strength. The degree of pulverization which the cement receives at the place of manufacture is ascertained by measuring the residue retained on certain sieves. Those known as No. 100 and No. 200 sieves are recommended for this purpose. The sieve should be circular, about 20 cm. (7.87 inches) in diameter, 6 cm. (2.36 inches) high, and provided with a pan 5 cm. (1.97 inches) deep, and a cover. The wire cloth should be woven from brass wire having the following diameters: No. 100, 0.0045 inches; No. 200, 0.0024 inches. This cloth should be mounted on the frame without distortion. The mesh should be regular in spacing and be within the following limits:

No. 100, 96 to 100 meshes to the linear inch.
No. 200, 188 to 200 meshes to the linear inch.

50 grams (1.76 oz.) or 100 gr. (3.52 oz.) should be used for the test and dried at a temperature of 100° Cent. or 212° Fahr., prior to sieving.

The thoroughly dried and coarsely screened sample is weighed and placed on the No. 200 sieve, which, with pan and cover attached, is held in one hand in a slightly inclined position, and moved forward and backward, at the same time striking the side gently with the palm of the other hand, at the rate of about 200 strokes per minute. The operation is continued until not more than $\frac{1}{10}$ of 1 per cent passes through after one minute of continuous sieving. The residue is weighed, then placed on the No. 100 sieve and the operation repeated. The work may be expedited by placing in the sieve a small quantity of large shot. The results should be reported to the nearest tenth of 1 per cent.

It shall leave by weight a residue of not more than 8 per cent on the No. 100, and not more than 25 per cent on the No. 200 sieve.

Normal Consistency. The use of a proper percentage of water in making the pastes, cement and water, from which pats, tests of setting, and briquettes are made, is exceedingly important, and affects vitally the results obtained. The determination consists in measuring the amount of water required to reduce the cement to a given state of plasticity, or to what is usually designated the normal consistency.

Various methods have been proposed for making this determination, none of which has been found entirely satisfactory. The Committee recommends the following:

The apparatus for this test consists of a frame K, Fig. 2, bearing a movable rod L, with the cap A at one end, and at the other the cylinder B, 1 cm. (0.39 in.) in diameter, the cap, rod, and cylinder weighing 300 gr. (10.58 oz.). The rod, which can be held in any

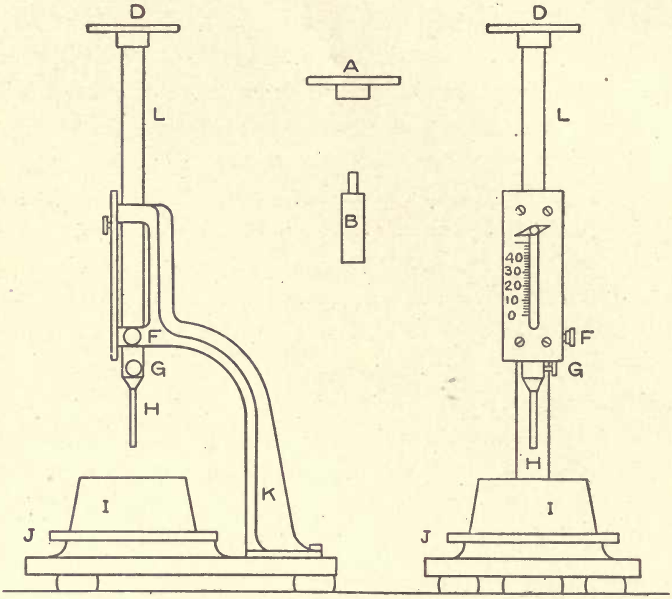


Fig. 2. Apparatus for Testing Normal Consistency of Cement.

desired position by a screw F, carries an indicator, which moves over a scale (graduated to centimeters) attached to the frame K. The paste is held by a conical, hard-rubber ring I, 7 cm. (2.76 in.) in diameter at the base, 4 cm. (1.57 in.) high, resting on a glass plate J about 10 cm. (3.94 in. square).

In making the determination, the same quantity of cement as will be subsequently used for each batch in making the briquettes (but not less than 500 grams) is kneaded into a paste, as described later in paragraph on "Mixing," and quickly formed into a ball with the hands, completing the operation by tossing it six times from one hand to the other, maintained 6 inches apart; the ball is then pressed into the rubber ring, through the larger opening, smoothed off, and placed (on its large end) on a glass plate and the smaller end smoothed off with a trowel; the paste confined in the ring, resting on the plate, is placed under the rod bearing the cylinder, which is brought in contact with the surface and quickly released.

The paste is of normal consistency when the cylinder penetrates to a point in the mass 10 mm. (0.39 in.) below the top of the ring. Great care must be taken to fill the ring exactly to the top. The trial pastes are made with varying percentages of water until the correct consistency is obtained. The Committee has recommended, as normal, a paste the consistency of which is rather wet, because it believes that variations in the amount of compression to which the briquette is subjected in moulding are likely to be less with such a paste. Having determined in this manner the proper percentage of water required to produce a paste of normal consistency, the proper percentage required for the mortars is obtained from an empirical formula. The Committee hopes to devise a formula. The subject proves to be a very difficult one, and, although the Committee has given it much study, it is not yet prepared to make a definite recommendation.

Note. The Committee on Standard Specifications for Cement inserts the following table for temporary use to be replaced by one to be devised by the Committee of the American Society of Civil Engineers.

TABLE I

Percentage of Water for Standard Sand Mortars

Percentage of Water for Neat Cement	One Cement Three Standard Ottawa Sand	Percentage of Water for Neat Cement	One Cement Three Standard Ottawa Sand	Percentage of Water for Neat Cement	One Cement Three Standard Ottawa Sand
15	8.0	23	9.3	31	10.7
16	8.2	24	9.5	32	10.8
17	8.3	25	9.7	33	11.0
18	8.5	26	9.8	34	11.2
19	8.7	27	10.0	35	11.5
20	8.8	28	10.2	36	11.5
21	9.0	29	10.3	37	11.7
22	9.2	30	10.5	38	11.8

	1 to 1	1 to 2	1 to 3	1 to 4	1 to 5
Cement.....	500	333	250	200	167
Sand	500	666	750	800	833

Time of Setting. The object of this test is to determine the time which elapsed from the moment water is added until the paste ceases to be fluid and plastic (called the "initial set"), and also the time required for it to acquire a certain degree of hardness (called the "final" or "hard set"). The former of these is the more important, since, with the commencement of setting, the process of crystallization or hardening is said to begin. As a disturbance of this process may produce a loss of strength, it is desirable to complete the operation of mixing and moulding or incorporating the mortar into the work before the cement begins to set. It is usual to measure arbitrarily the beginning and end of the setting by the penetration of weighted wires of given diameters.

For this purpose the Vicat Needle, which has already been described, should be used. In making the test, a paste of normal consistency is moulded and placed under the rod L, Fig. 2, as described in a previous paragraph. This rod bears the cap D at one end and the needle H, 1 mm. (0.039 in.) in diameter, at the other, and weighs 300 gr. (10.58 oz.). The needle is then carefully brought in contact with the surface of the paste and quickly released. The setting is said to have commenced when the needle ceases to pass a point 5 mm. (0.20 in.) above the upper surface of the glass plate, and is said to have terminated the moment the needle does not sink visibly into the mass.

The test pieces should be stored in moist air during the test; this is accomplished by placing them on a rack over water contained in a pan and covered with a damp cloth, the cloth to be kept away from them by means of a wire screen; or they may be stored in a moist box or closet. Care should be taken to keep the needle clean, as the collection of cement on the sides of the needle retards the penetration, while cement on the point reduces the area and tends to increase the penetration. The determination of the time of setting is only approximate, being materially affected by the temperature of the mixing water, the temperature and humidity of the air during the test, the percentage of water used, and the amount of moulding the paste receives.

The following approximate method, not requiring the use of apparatus, is sometimes used, although not referred to by the Committee. Spread cement paste of the proper consistency on a piece of glass, having the cement cake about three inches in diameter and about one inch thick at the center, thinning towards the edges. When the cake is hard enough to bear a gentle pressure of the finger nail, the cement has begun to set, and when it is not indented by a considerable pressure of the thumb nail, it is said to have set.

The Committee recommends that it shall develop initial set in not less than thirty minutes, but must develop hard set in not less than one hour, nor more than ten hours.

Standard Sand. The Committee recognizes the grave objections to the standard quartz now generally used, especially on account of its high percentage of voids, the difficulty of compacting in the moulds, and its lack of uniformity; it has spent much time in investi-

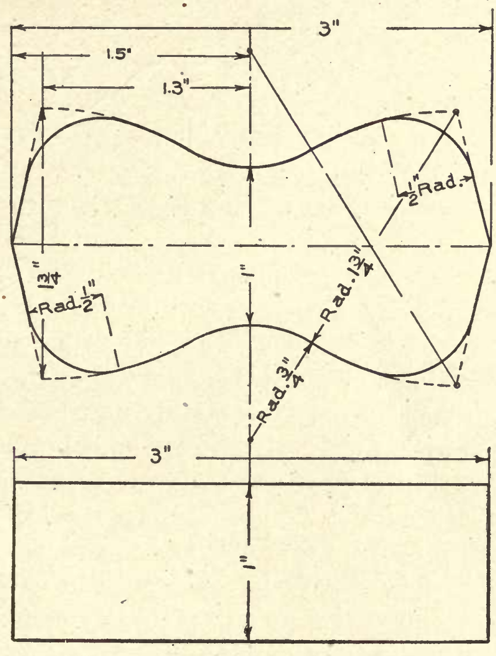


Fig. 3. Form of Briquette.

gating the various natural sands which appeared to be available and suitable for use. For the present, the Committee recommends the natural sand from Ottawa, Ill., screened to pass a sieve having 20 meshes per linear inch and retained on a sieve having 30 meshes per linear inch; the wires to have diameters of 0.0165 and 0.0112 inches, respectively, i.e., half the width of the opening in each case. Sand having passed the No. 20 sieve shall be considered standard when not more than one per cent passes a No. 30 sieve after one minute continuous sifting of a 500-gram sample.

Form of Briquette. While the form of the briquette recommended by a former Committee of the Society is not wholly satisfactory, this Committee is not prepared to suggest any change, other than rounding off the corners by curves of ½-inch radius, Fig. 3.

Moulds. The moulds should be made of brass, bronze, or some equally non-corrodible material, having sufficient metal in the sides to prevent spreading during moulding.

Gang moulds, which permit moulding a number of briquettes at one time, are preferred by many to single moulds; since the greater

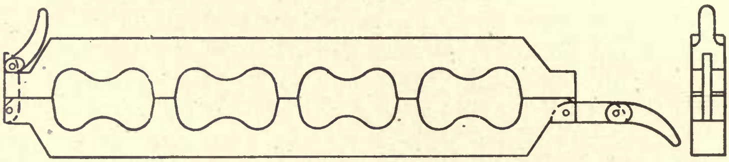


Fig. 4. Gang Moulds.

quantity of mortar that can be mixed tends to produce greater uniformity in the results. The type shown in Fig. 4 is recommended. The moulds should be wiped with an oily cloth before using.

Mixing. All proportions should be stated by weight; the quantity of water to be used should be stated as a percentage of the dry material. The metric system is recommended because of the convenient relation of the gram and the cubic centimeter. The temperature of the room and the mixing water should be as near 21° Cent. (70° Fahr.) as it is practicable to maintain it. The sand and cement should be thoroughly mixed dry. The mixing should be done on some non-absorbing surface, preferably plate glass. If the mixing must be done on an absorbing surface it should be thoroughly dampened prior to use. The quantity of material to be mixed at one time depends on the number of test pieces to be made; about 1000 gr. (35.28 oz.) makes a convenient quantity to mix, especially by hand methods.

The material is weighed and placed on the mixing table, and a crater formed in the center, into which the proper percentage of clean water is poured; the material on the outer edge is turned into the crater by the aid of a trowel. As soon as the water has been absorbed, which should not require more than one minute, the operation is completed by vigorously kneading with the hands for an additional $1\frac{1}{2}$ minutes, the process being similar to that used in kneading dough. A sand-glass affords a convenient guide for the time of kneading. During the operation of mixing the hands should be protected by gloves, preferably of rubber.

Moulding. Having worked the paste or mortar to the proper consistency, it is at once placed in the moulds by hand. The moulds should be filled at once, the material pressed in firmly with the fingers and smoothed off with a trowel without ramming; the material should be heaped up on the upper surface of the mould, and, in smoothing off, the trowel should be drawn over the mould in such a manner as to exert a moderate pressure on the excess material. The mould should be turned over and the operation repeated. A check upon the uniformity of the mixing and moulding is afforded by weighing the briquettes just prior to immersion, or upon removal from the moist closet. Briquettes which vary in weight more than 3 per cent from the average should not be tested.

Storage of the Test Pieces. During the first 24 hours after moulding, the test pieces should be kept in moist air to prevent them from drying out. A moist closet or chamber is so easily devised that the use of the damp cloth should be abandoned if possible. Covering the test pieces with a damp cloth is objectionable, as commonly used, because the cloth may dry out unequally, and, in consequence, the test pieces are not all maintained under the same condition. Where a moist closet is not available, a cloth may be used and kept uniformly wet by immersing the ends in water. It should be kept from direct contact with the test pieces by means of a wire screen or some similar arrangement.

A moist closet consists of a soapstone or slate box, or a metal-lined wooden box: the metal lining being covered with felt and this felt kept wet. The bottom of the box is so constructed as to hold water, and the sides are provided with cleats for holding glass shelves on which to place the briquettes. Care should be taken to keep the air in the closet uniformly moist. After 24 hours in moist air the test pieces for longer periods of time should be immersed in water maintained as near 21° Cent. (70° Fahr.) as practicable; they may be stored in tanks or pans, which should be of non-corrodible material.

Tensile strength. The tests may be made on any standard machine. A solid metal clip, as shown in Fig. 5, is recommended. This clip is to be used without cushioning at the points of contact with the test specimen. The bearing at each point of contact should be ¼-inch wide, and the distance between the center of contact on the same clip should be 1¼ inches. Test pieces should be broken as soon as they are removed from the water. Care should be observed in centering the briquettes in the testing machine, as cross-strains, produced by improper centering, tend to lower the breaking strength. The load should not be applied too suddenly, as it may produce vibration, the shock from which often breaks the briquette before the ultimate strength is reached. Care must be taken that the clips and the sides of the briquette be clean and free from grains of sand or dirt, which would prevent a good bearing. The load should be applied at the rate of 600 lbs. per minute. The average of the briquettes of each sample tested should be taken as the test, excluding any results which are manifestly faulty.

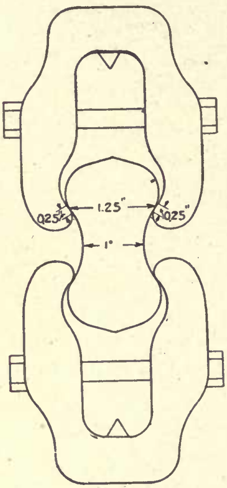


Fig. 5. Metal Clip for Testing Tensile Strength.

The minimum requirements for tensile strength for briquettes one inch square in section shall be within the following limits, and shall show no retrogression in strength within the periods specified:

MINIMUM STRENGTH OF BRIQUETTES

Age	Strength
NEAT CEMENT	
24 hours in moist air	150-200 lbs.
7 days (1 day in moist air, 6 days in water)	450-550 "
28 days (1 day in moist air, 27 days in water)	550-650 "
ONE PART CEMENT, THREE PARTS SAND	
7 days (1 day in moist air, 6 days in water)	150-200 "
28 days (1 day in moist air, 27 days in water)	200-300 "

Constancy of Volume. The object is to develop those qualities which tend to destroy the strength and durability of a cement. As it is highly essential to determine such qualities at once, tests of this character are for the most part made in a very short time, and are known, therefore, as accelerated tests. Failure is revealed by cracking, checking, swelling, or disintegration, or all of these phenomena. A cement which remains perfectly sound is said to be of constant volume.

Methods. Tests for constancy of volume are divided into two classes:

(1) Normal tests, or those made in either air or water maintained at about 21° Cent. (70° Fahr.).

(2) Accelerated tests, or those made in air, steam, or water at a temperature of 45° Cent. (115° Fahr.) and upward. The test pieces should be allowed to remain 24 hours in moist air before immersion in water or steam, or preservation in air. For these tests, pats, about $7\frac{1}{2}$ cm. (2.95 in.) in diameter, $1\frac{1}{4}$ cm. (0.49 in.) thick at the center, and tapering to a thin edge, should be made, upon a clean glass plate [about 10 cm. (3.94 in.) square], from cement paste of normal consistency.

Normal Test. A pat is immersed in water maintained as near 21° Cent. (70° Fahr.) as possible for 28 days, and observed at intervals. A similar pat is maintained in air at ordinary temperature and observed at intervals.

Accelerated Test. A pat is exposed in any convenient way in an atmosphere of steam, above boiling water, in a loosely closed vessel, for 3 hours.

To pass these tests satisfactorily, the pats should remain firm and hard, and show no signs of cracking, distortion, or disintegration. Should the pat leave the plate, distortion may be detected best with a straight-edge applied to the surface which was in contact with the plate. In the present state of our knowledge it cannot be said that cement should necessarily be condemned simply for failure to pass the accelerated tests; nor can a cement be considered entirely satisfactory, simply because it has passed these tests.

Testing Machines. There are many varieties of testing machines on the market. Many engineers have constructed "home-made" machines which serve their purpose with sufficient accuracy. One very common type of machine is illustrated in Fig. 6. B is a reservoir containing shot which falls through the pipe I which is closed with a valve at the bottom. The briquette is carefully placed between the clips, as shown in the figure, and the wheel P is turned until the indicators are in line. The hook lever Y is moved so that a screw worm is engaged with its gear. Then open the automatic valve J so as to allow the shot to run into the cup F. By means of a small valve the flow of shot into the cup may be regulated. Better results will be obtained by allowing the shot to run slowly into the cup. The crank is then turned with just sufficient speed

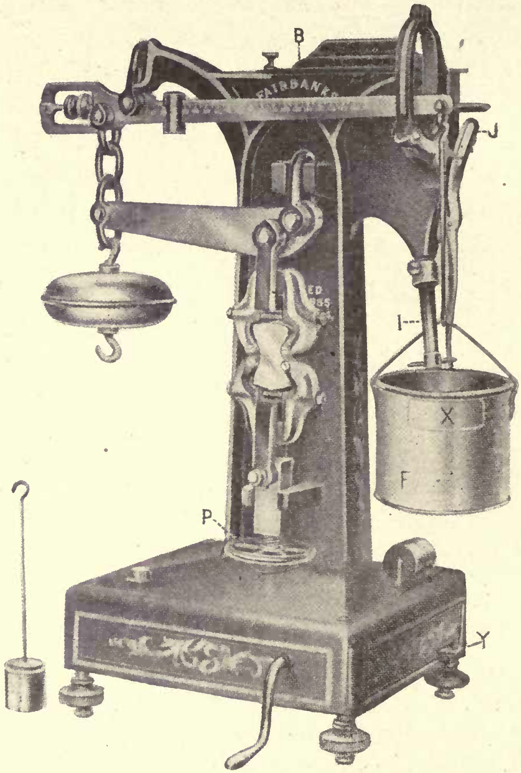


Fig. 6. Cement Testing Machine.

so that the scale beam is held in position until the briquette is broken. Upon the breaking of the briquette, the scale beam falls and automatically closes the valve J. The weight of the shot in the cup F then indicates, according to some definite ratio, the stress required to break the briquette.

Sand. Specifications for concrete usually state that the sand shall be clean, coarse, and sharp; free from clay, loam, sticks, organic matter, or other impurities. A mixture of coarse and fine grains, with the coarse grains predominating, is found very satisfactory as it makes a denser and stronger concrete with a less amount of cement than when coarse-grained sand is used with the same proportion of cement. The small grains of sand fill the voids caused by the coarse grains so that there is not as great a volume of voids to be filled by the cement. The sharpness of sand can be determined approximately by rubbing a few grains in the hand or by crushing it near the ear and noting if a grating sound is produced; but an examination through a small lens is better.

Experiments have shown that round grains of sand have less voids than angular ones, and that water-worn sands have from 3 per cent to 5 per cent less voids than corresponding sharp grains. In many parts of the country where it is impossible, except at a great expense, to obtain the sharp sand, the rounded grain is used with very good results. Laboratory tests made under conditions as nearly as possible identical show that the rounded-grain sand gives as good results as the sharp sand. In consequence of such tests, the requirement that sand shall be *sharp* is now considered useless by many engineers, especially when it leads to additional cost.

In all specifications for concrete work is found the clause that "the sand shall be clean." This requirement is sometimes questioned as experimenters have found that a small percentage of clay or loam often gives better results than when clean sand is used. "Lean" mortar may be improved by a small percentage of clay or loam, or by using dirty sand, for the fine material increases the density. In rich mortars this fine material is not needed, as the cement furnishes all the fine material necessary, and if clay or loam or dirty sand were used it might prove detrimental. Whether it is really a benefit or not depends chiefly upon the richness of the concrete and the coarseness of the sand. Some idea of the cleanliness of sand may be obtained by placing it in the palm of one hand and rubbing it with the fingers of the other. If the sand is dirty, it will badly discolor the palm of the hand. When it is found necessary to use dirty sand the strength of the concrete should be tested.

Sand containing loam or earthy material is cleansed by washing with water, either in a machine specially designed for the purpose, or by agitating the sand with water in boxes provided with holes to permit the dirty water to flow away.

Very fine sand may be used alone, but it makes a weaker concrete than either coarse sand or coarse and fine sand mixed. A mortar consisting of very fine sand and cement will not be so dense as one of coarse sand and the same cement, although when measured or weighed dry, each contain the same proportion of voids and solid matter. In a unit measure of fine sand there are more grains than in a unit measure of coarse sand, and therefore more points of contact. More water is required in gauging a mixture of fine sand and cement than in a mixture of coarse sand and the same cement. The water forms a film and separates the grains, thus producing a larger volume having less density.

The screenings of broken stone are sometimes used instead of sand. Tests frequently show a stronger concrete when screenings are used than when sand is used. This is perhaps due to the variable sizes of the screenings, which would have a less percentage of voids.

Stone. The stone used in concrete should be hard and durable, such as trap, granite, lime stone, sand stone or a conglomerate. Lime stone should not be used as a fireproofing material as heat will calcinate it. Trap rock and gravel are perhaps the best stone for fireproof purposes. Crushed stone should have all the dust removed by a $\frac{1}{4}$-inch screen, although it may be replaced again as a part of the sand. If the product from the crusher is shown by frequent sampling to be uniform, the dust may be retained in place of a corresponding amount of sand.

The maximum size of stone usually permitted in plain concrete is $2\frac{1}{2}$ inches, and in reinforced concrete $\frac{3}{4}$ inch, although in some reinforced concrete structures 1 inch stone is permitted. Sometimes specifications state that the stone to be used shall be screened to a practically uniform size, while other specifications state that the stone shall be of graduated sizes so that the smaller shall fit into the voids between the larger so that less mortar is required. A single size of broken stone has a greater tendency to form arches while being rammed into place, than stone of graded sizes. The graded stone makes a denser, stronger, and more economical concrete. Usually in graded stone for reinforced concrete the stones vary in size from $\frac{1}{4}$ inch to $\frac{3}{4}$ or 1 inch and in plain concrete from $\frac{1}{4}$ inch to $2\frac{1}{2}$ inches.

Gravel. When gravel is used instead of stone, or is mixed with stone, it should be composed of clean pebbles free from clay or other materials. A film of dirt on the gravel lessens the strength of the concrete. Graded round gravel contains a smaller percentage of voids than angular stones and makes a dense concrete which compares very well with stone concrete. The greater density of the gravel concrete tends to overcome the slight difference in strength due to the varying character of the surfaces of the particles of the gravel and the broken stone. Sometimes it is economical to mix a small percentage of gravel with broken stone.

Cinders. Cinders for concrete should be free from coal or soot. Usually a better mixture can be obtained by screening the fine stuff from the cinders and then mixing in a larger proportion of sand, than by using unscreened material, although if the fine stuff is uniformly distributed through the mass, it may be used without screening and a less proportion of sand used.

As shown later the strength of cinder concrete is far less than that of stone concrete and on this account it cannot be used where high compressive values are necessary. But on account of its very low cost compared with broken stone, especially under some conditions, it is used quite commonly for roofs, etc., on which the loads are comparatively small.

One possible objection to the use of cinders lies in the fact that they frequently contain sulphur and other chemicals which may produce corrosion of the reinforcing steel. In any structure where the strength of the concrete is a matter of importance, cinders should not be used without a thorough inspection and even then the unit compressive values allowed should be at a very low figure.

Proportions of Concrete. When large and important structures are to be built, or when the concrete is to be water tight, it pays from an economical standpoint to make a thorough study of the material of the aggregates and their relative proportions. The proportions below will serve as a guide for various classes of work.

A rich mixture, proportions 1:2:4, that is 1 barrel (4 bags) packed Portland cement (as it comes from the manufacturer), 2 barrels (7.6 cubic feet) loose sand, and 4 barrels (15.2 cubic feet) loose stone, is used in arches, reinforced concrete floors, beams and columns for heavy loads, engine and machine foundations subject to vibrations, tanks, and for water-tight work.

A medium mixture, proportions 1 : 2½ : 5, that is, 1 barrel (4 bags) packed Portland cement, 2½ barrels (9.5 cubic feet) loose sand, and 5 barrels (19 cubic feet) loose gravel or stone, may be used in arches, thin walls, floors, beams, sewers, sidewalks, foundations, and machine foundations.

An ordinary mixture, proportions 1 : 3 : 6, that is, 1 barrel (4 bags) packed Portland cement, 3 barrels (11.4 cubic feet) loose sand, and 6 barrels (22.8 cubic feet) loose gravel or broken stone, may be used for retaining walls, abutments, piers, floor slabs, and beams.

A lean mixture, proportions 1 : 4 : 8, that is, 1 barrel (4 bags) packed Portland cement, 4 barrels (15.2 cubic feet) loose sand, and 8 barrels (30.4 cubic feet) loose gravel or broken stone, may be used in large foundations supporting stationary loads, backing for stone masonry, or where it is subject to a plain compressive load.

These proportions must not be taken as always being the most economical to use, but they represent average practice. Cement is the most expensive ingredient; therefore a reduction of the quantity of cement, by adjusting the proportions of the aggregate so as to produce a concrete with the same density, strength, and impermeability, is of great importance. By careful proportioning and workmanship water-tight concrete has been made of a 1 : 3 : 6 mixture. In floor construction where the span is very short and it is specified that the slab must be at least 4 inches thick, while with a high grade concrete a 3-inch slab would carry the load, it is certainly more economical to use a leaner concrete.

The method often used in determining the voids in stone and in sand, by finding the quantity of water that can be poured into the voids of a unit measure of stone or sand and then taking that amount of sand or cement as the amount required to fill the voids in the stone or sand, is not satisfactory. The greatest inaccuracy of this method is due to the difference in compactness of the materials under varied methods of handling, and to the fact that the actual volume of voids in a coarse material may not correspond to the quantity of sand required to fill the voids. The grains of sand separate the stone and with most aggregates a portion of the sand is too coarse to get in the voids of the coarser material. That is, in a mass of crusher-run broken stone many of the individual voids are so small that the larger grain of natural bank sand will not fit into them, but will get between the stones and increase the bulk of the mass. This increase in bulk means that more sand is required than the actual volume of voids in the coarse material.

An accurate and simple method to determine the proportions of concrete is by trial batches. The apparatus consists of a scale and a cylinder which may be a piece of wrought iron pipe 10 inches to 12 inches in diameter capped at one end. Measure and weigh the cement, sand, stone, and water and mix on a piece of sheet steel, the mixture having a consistency the same as to be used in the work. The mixture is placed in the cylinder, carefully tamped, and the height to which the pipe is filled is noted. The pipe should be weighed before and after being filled so as to check the weight of the material. The cylinder is then emptied and cleaned. Mix up another batch using the same amount of cement and water, slightly varying the ratio of the sand and stone but having the same total weight as before. Note the height in the cylinder, which will be a guide to other batches to be tried. Several trials are made until a mixture is found that gives the least height in the cylinder, and at the same time works well while mixing, all the stones being covered with mortar, and which makes a good appearance. This method gives very good results, but it does not indicate the changes in the physical sizes of the sand and stone so as to secure the most economical composition as would be shown in a thorough mechanical analysis.

There has been much concrete work done where the proportions were selected without any reference to voids, which has given much better results in practice than might be expected. The proportion of cement to the aggregate depends upon the nature of the construction and the required degree of strength, or water-tightness, as well as upon the character of the inert materials. Both strength and imperviousness increase with the proportion of cement to the aggregate. Richer mixtures are necessary for loaded columns, beams in building construction and arches, for thin walls subject to water pressure, and for foundations laid under water. The actual measurements of materials as actually mixed and used usually show leaner mixtures than the nominal proportions specified. This is largely due to the heaping of the measuring boxes.

TABLE II

Proportions of Cement, Sand, and Stone in Actual Structures

Structure	Proportions	Reference
C. B. & Q. R. R.		
Reinforced Concrete Culverts	1:3:6	Engr. Cont., Oct. 3, '06
Phila. Rapid Transit Co.		
Floor Elevated Roadway....	1:3:6	" " Sept. 26, '06
Subway { Walls...........	1:2.5:5	
Subway { Floors..........	1:3:6	
C. P. R. R.		
Arch Rings..................	1:3:5	
Piers and Abutments........	1:4:7	Cement Era, Aug. '06
Hudson River Tunnel Caisson	1:2:4	Eng. Record, Sept. 29, '06
Stand Pipe at Attleboro, Mass. Height, 106 feet.	1:2:4	" " " 29, '06
C.C.& St.L.R.R.,Danville Arch		
Footings......................	1:4:8 or 1:9.5	" " March 3, '06
Arch Rings...................	1:2:4	
Abutments, Piers............	1:3:6 or 1:6.5	
N. Y. C. & H. R. R. R.		
Ossining Tunnel { Footing..........	1:4:7.5	" " " 3, '06
Ossining Tunnel { Walls...........	1:3:6	
Ossining Tunnel { Coping..........	1:2:4	
American Oak Leather Co.		
Factory at Cincinnati, Ohio.	1:2:4	" " " 3, '03
Harvard University Stadium..	1:3:6	
New York Subway		
Roofs and Sidewalks........	1:2:4	
Tunnel Arches..............	1:2.5:5	
Wet Foundation 2′ th. or less	1:2:4	
" " exceeding 2′	1:2.5:5	
Boston Subway...............	1:2.5:4	
P. & R. R. R.		
Arches.......................	1:2:4	" " Oct. 13, '06
Piers and Abutments........	1:3:6	
Brooklyn Navy Yd. Laboratory		
Columns......................	1:2:3 Trap rock	Eng. News, March 23, '05
Beams and Slabs............	1:3:5 " "	
Roof Slab.....................	1:3:5 Cinder	
Southern Railway		
Arches........................	1:2:4	
Piers and Abutments........	1:2.5:5	

Methods of Mixing Concrete. The method of mixing concrete is immaterial, if a homogeneous mass is secured of a uniform consistency, containing the cement, sand, and stone in the correct proportions. The value of the concrete depends greatly upon the thoroughness of the mixing. The color of the mass must be uniform, every grain of sand and piece of the stone should have cement adhering to every point of its surface.

TABLE III

Barrels of Portland Cement Per Cubic Yard of Mortar

(Voids in Sand Being 35 per cent and 1 Bbl. Cement Yielding 3.65 Cubic Feet of Cement Paste.)

PROPORTION OF CEMENT TO SAND	1:1	1:1.5	1:2	1:2.5	1:3	1:4
	Bbls.	Bbls.	Bbls.	Bbls.	Bbls.	Bbls.
Bbl. specified to be 3.5 cu. ft........	4.22	3.49	2.97	2.57	2.28	1.76
" " " 3.8 "	4.09	3.33	2.81	2.45	2.16	1.62
" " " 4.0 "	4.00	3.24	2.73	2.36	2.08	1.54
" " " 4.4 "	3.81	3.07	2.57	2.27	2.00	1 40
Cu. yds. sand per cu. yd. mortar...	0.6	0.7	0.8	0.9	1.0	1.0

TABLE IV

Barrels of Portland Cement Per Cubic Yard of Mortar

(Voids in Sand Being 45 per cent and 1 Bbl. Cement Yielding 3.4 Cubic Feet of Cement Paste.)

PROPORTION OF CEMENT TO SAND	1:1	1:1.5	1:2	1:2.5	1:3	1:4
	Bbls.	Bbls.	Bbls.	Bbls.	Bbls.	Bbls.
Bbl. specified to be 3.5 cu. ft........	4.62	3.80	3.25	2.84	2.35	1.76
" " " 3.8 "	4.32	3.61	3.10	2.72	2.16	1.62
" " " 4 0 "	4.19	3.46	3.00	2.64	2.05	1.54
" " " 4.4 "	3.94	3.34	2.90	2.57	1.86	1.40
Cu. yds. sand per cu. yds. mortar..	0.6	0.8	0:9	1.0	1.0	1.0

TABLE V

Ingredients in 1 Cubic Yard of Concrete

(Sand Voids, 40 per cent; Stone Voids, 45 per cent; Portland Cement Barrel Yielding 3.65 cu. ft. Paste. Barrel specified to be 3.8 cu. ft.)

PROPORTIONS BY VOLUMI	1:2:4	1:2:5	1:2:6	1:2.5:5	1:2.5:6	1:3:4
Bbls. cement per. cu. yd. concrete..	1.46	1.30	1.18	1.13	1.00	1.25
Cu. yds. sand " " ..	0.41	0.36	0.33	0.40	0.35	0.53
" stone " " ..	0.82	0.90	1.00	0.80	0.84	0.71
Proportions by volum	1:3:5	1:3:6	1:3:7	1:4:7	1:4:8	1:4:9
Bbls. cement per cu. yd. concrete..	1.13	1.05	0.96	0.82	0.77	0.73
Cu. yds. sand " " ..	0.48	0.44	0.40	0.46	0.43	0.41
" stone " " ..	0.80	0.88	0.93	0.80	0.86	0.92

This table is to be used when cement is measured packed in the barrel, for the ordinary barrel holds 3.8 cu. ft.

TABLE VI

Ingredients in 1 Cubic Yard of Concrete

(Sand Voids, 40 per cent; Stone Voids, 45 per cent; Portland Cement Barrel Yielding 3.65 cu. ft. of Paste. Barrel specified to be 4.4 cu. ft.)

Proportions by Volume	1:2:4	1:2:5	1:2:6	1:2.5:5	1:2.5:6	1:3:4
Bbls. cement per cu. yd. concrete...	1.30	1.16	1.00	1.07	0.96	1.08
Cu. yds. sand " " ...	0.42	0.38	0.33	0.44	0.40	0.53
" stone " " ...	0.84	0.95	1.00	0.88	0.95	0.71
Proportions by volume.............	1:3:5	1:3:6	1:3:7	1:4:7	1:4:8	1:4:9
Bbls. cement per cu. yd. concrete...	0.96	0.90	0.82	0.75	0.68	0.64
Cu. yds. sand " " ...	0.47	0.44	0.40	0.49	0.44	0.42
" stone " " ...	0.78	0.88	0.93	0.86	0.88	0.95

This table is to be used when the cement is measured loose, after dumping it into a box, for under such conditions a barrel of cement yields 4.4 cu. ft. of loose cement.

[Tables II to VI have been taken from Gillette's Handbook of Cost Data.]

The two methods used in mixing concrete are by hand and by machinery. Good concrete may be made by either method. Concrete mixed by either method should be carefully watched by a good foreman. If a large quantity of concrete is required it is cheaper to mix it by machinery. On small jobs where the cost of erecting the plant and the interest and depreciation, divided by the number of cubic yards to be made, is a large item, or if frequent moving is required, it is very often cheaper to mix the concrete by hand. The relative cost of the two methods usually depends upon circumstances, and must be worked out in each individual case.

Hand Mixing. The placing and handling of materials and arranging the plant is varied by different engineers and contractors. In general the mixing of concrete is a simple operation but should be carefully watched by an inspector. He should see:

(1) That the exact amount of stone and sand are measured out.
(2) That the cement and sand are thoroughly mixed.
(3) That the mass is thoroughly mixed.
(4) That the proper amount of water is used.
(5) That care is taken in dumping the concrete in place.
(6) That it is thoroughly rammed.

The mixing platform, which is usually 10 to 20 feet square, is made of 1-inch or 2-inch plank planed on one side and well nailed to stringers, and should be placed as near the work as possible, but so situated that the stone can be dumped on one side of it and the sand on the opposite side. A very convenient way to measure the stone and sand is by the means of bottomless boxes. These boxes are of such a size that they hold the proper proportions of stone or sand to mix a batch of a certain amount. Cement is usually measured by the package, that is by the barrel or bag, as they contain a definite amount of cement.

The method used for mixing the concrete has little effect upon the strength of the concrete, . . the mass has been turned a sufficient number of times to thoroughly mix them. One of the following methods is generally used. (Taylor and Thompson's *Concrete.*)

(a) Cement and sand mixed dry and shoveled on the stone or gravel, leveled off, and wet as the mass is turned.

(b) Cement and sand mixed dry, the stone measured and dumped on top of it, leveled off, and wet, as turned with shovels.

(c) Cement and sand mixed into a mortar, the stone placed on top of it and the mass turned.

(d) Cement and sand mixed with water into a mortar which is shoveled on the gravel or stone and the mass turned with shovels.

(e) Stone or gravel, sand, and cement spread in successive layers, mixed slightly and shoveled into a mound, water poured into the center, and the mass turned with shovels.

The quantity of water is regulated by the appearance of the concrete. The best method of wetting the concrete is by measuring the water in pails. This insures a more uniform mixture than by spraying the mass with a hose.

Mixing by Machinery. On large contracts the concrete is generally mixed by machinery. The economy is not only in the mixing itself but in the appliances introduced in handling the raw materials and the mixed concrete. If all materials are delivered to the mixer in wheel-barrows, and if the concrete is conveyed away in wheel-barrows, the cost of making concrete is high, even if machine mixers are used. If the materials are fed from bins by gravity into the mixer, and if the concrete is dumped from the mixer into cars and hauled away, the cost of making the concrete should be very low. On small jobs the cost of maintaining and operating the mixer will usually exceed the saving in hand labor and will render the expense with the machine greater than without it.

The design of a plant for handling the material and concrete, and the selection of a mixer, depend upon local conditions, the amount of concrete to be mixed per day, and the total amount required on the contract. It is very evident that on large jobs it pays to invest a large sum in machinery to reduce the number of men and horses, but if not over 50 cubic yards are to be deposited per day the cost of the machinery is a big item and hand labor is generally cheaper. The interest on the plant must be charged against the number of cubic yards of concrete; that is, the interest on the plant for a year must be charged to the number of cubic yards of concrete laid in a year. The depreciation of the plant is found by taking the cost of the entire plant when new, and then appraising it after the contract is finished, and dividing the difference by the total cubic yards of concrete laid. This will give the depreciation per cubic yard of concrete manufactured.

Concrete Mixers. The best concrete mixer is the one that turns out the maximum of thoroughly mixed concrete at the minimum of cost for power, interest, and maintenance. The type of mixer with a complicated motion gives better and quicker results than one with a simpler motion. There are two general classes of concrete mixers; *continuous* mixers and *batch* mixers. A *continuous* mixer is one into which the materials are fed constantly and from which the concrete is discharged constantly. *Batch* mixers are constructed to receive the cement with its proportionate amount of sand and stone all at one charge, and when mixed it is discharged in a mass. A very distinct line cannot be drawn between these two classes, for many of these mixers are adapted to either continuous or batch mixing. Generally batch mixers are preferred, as it is a very difficult matter to feed the mixers uniformly unless the materials are mechanically measured.

Continuous mixers usually consist of a long screw or pug mill, that pushes the materials along a drum until they are discharged in a continuous stream of concrete. Where the mixers are fed with automatic measuring devices the concrete is not regular as there is no reciprocating motion of the materials. In a paper recently read before the Association of American Portland Cement Manufacturers by S. B. Newberry, he states: "For the preparation of concrete for blocks in which thorough mixing and use of an exact and uniform proportion of water are necessary, continuous mixing machines are unsuitable, and batch mixers, in which a measured batch of the material is mixed the required time, and then discharged, are the only type which will be found effective."

There are three general types of concrete mixers: *gravity* mixers, *rotary* mixers, and *paddle* mixers.

Gravity mixers are the oldest type of concrete mixers. They require no power, the materials being mixed by striking obstructions which throw them together in their descent through the machine. Their construction is very simple. Fig. 7 illustrates a portable gravity mixer. This mixer, as will be seen by the figure, is a steel trough or shoot in which are contained mixing members consisting of pins or blades. The mixer is portable and requires no skilled labor to operate it. There is nothing to get out of order or cause delays. It is adapted for both large and small jobs. In the former case, it is usually fed by measure and by this method will produce concrete as fast as the materials can be fed to their respective bins and the mixed concrete can be taken from the discharge end of the mixer. On very small jobs, the best way to operate is to measure the batch in layers of stone, sand, and cement respectively and feed to the mixer by men with shovels.

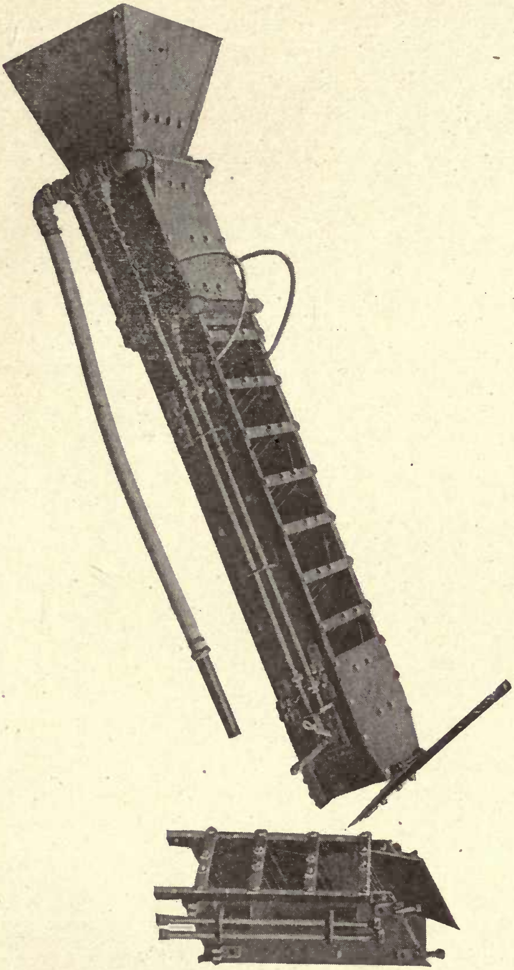


Fig. 7. Portable Gravity Mixer.

There are two spray pipes placed on the mixer: for feeding by hand one spray only would be used; the other spray is only intended for use when operating with the measure and feeder, and a large amount of water is required. These sprays are operated by handles which control two gate valves and regulate the quantity of water which flows from the spray pipes.

These mixers are made in two styles, sectional and non-sectional. The sectional can be made either 4, 6, or 8 feet long. The non-sectional are in one length of 6, 8, or 10 feet. Both are constructed of $\frac{1}{8}$-inch steel. To operate this mixer, the materials must be raised to a platform, as shown in Fig. 8.

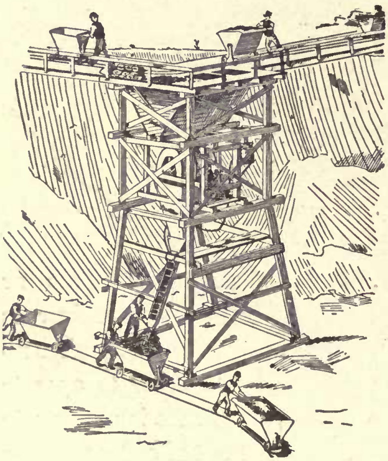


Fig. 8. Operation of Portable Gravity Mixer.

Rotary mixers, Fig. 9, generally consist of a cubical box made of steel and mounted on a wooden frame. This steel box is supported by a hollow shaft through two diagonally opposite corners and the water is supplied through openings in the hollow shaft. Materials are dropped in at the side of the mixer through a hinged door. The machine is then revolved several times, usually about 15 times, the door is opened, and the concrete is dumped out into carts or cars. There are no paddles or blades of any kind inside the box to assist in the mixing. This mixer is not expensive itself, but the erection of the frame and the hoisting of the stone and sand often render it less economical than some of the more expensive devices.

Rotating mixers which contain reflectors or blades, Fig. 10, are usually mounted on a suitable frame by the manufacturers. The rotating of the drum tumbles the material and it is thrown against the mixing blades which cut it and throw it from side to side. Many of these machines can be filled and dumped while running, either by tilting or by their shutes. Fig. 10 illustrates the Smith mixer and Fig. 11 gives a sectional view of the drum and shows the arrangement of the blades. This mixer is furnished on skids with driving pulley. The concrete is discharged by tilting the drum, which is done by power.

Fig. 12 represents a Ransome mixer which is a batch mixer. The concrete is discharged after it is mixed, without tilting the body of the mixer. It revolves continuously even while the concrete is

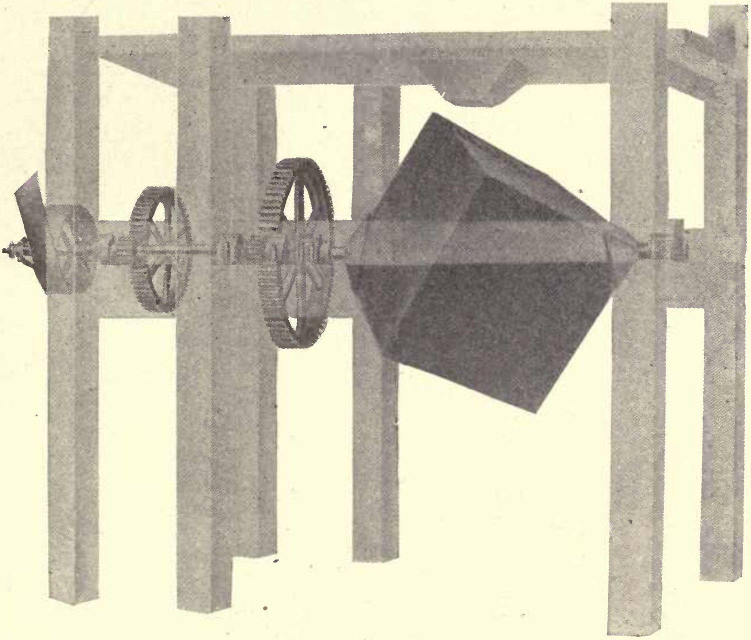


Fig. 9. Rotary Mixer with Cubical Box.

being discharged. Riveted to the inside of the drum is a number of steel scoops or blades. "These scoops pick up the material in the bottom of the mixer, and, as the mixer revolves, carry the material upward until it slides out of the scoops" and therefore assists in mixing the materials.

Fig. 13 represents a McKelvey batch mixer. In this mixer, the lever on the drum operates the discharge. The drum is fed and discharged while in motion and does not change its direction or its position in either feeding or discharging. The inside of the drum is provided with blades to assist in the mixing of the concrete.

Paddle mixers may be either continuous or of the batch type. Mixing paddles, on two shafts, revolve in opposite directions and the

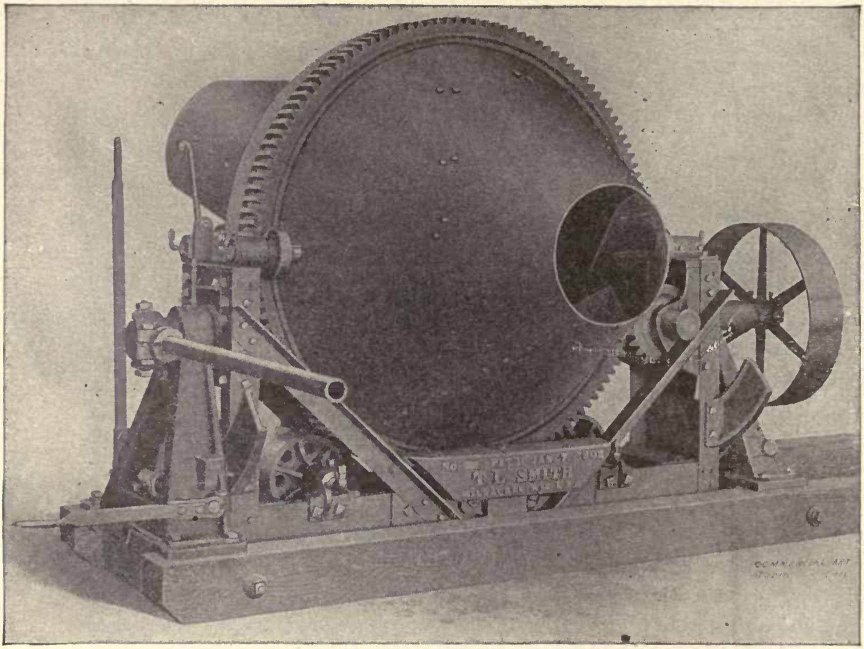


Fig. 10. Rotary Mixer Mounted on Frame.

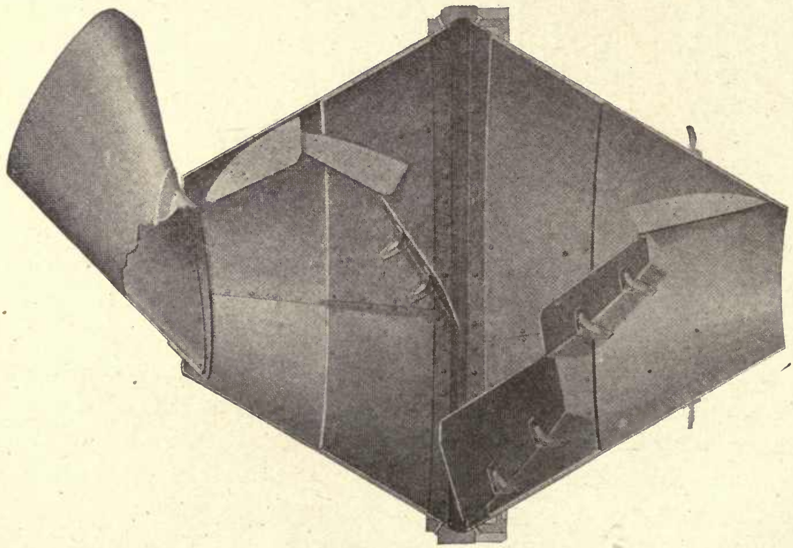


Fig. 11. Cross Section of Drum (front half cut away), Showing Blades and Lining.

concrete falls through a trap door in the bottom of the machine. In the continuous type the materials should be put in at the upper end

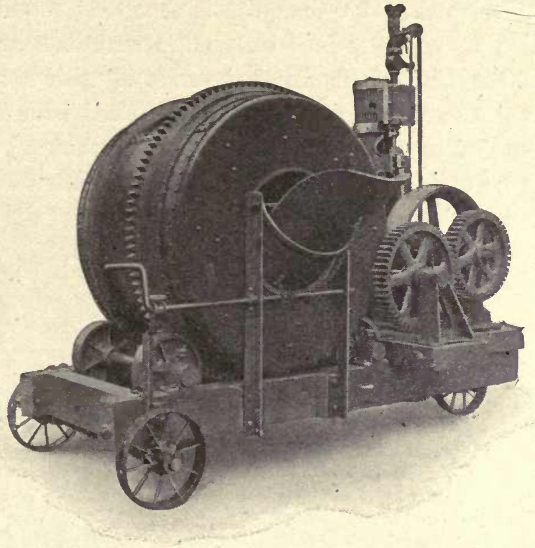


Fig. 12. Ransome Batch Mixer.

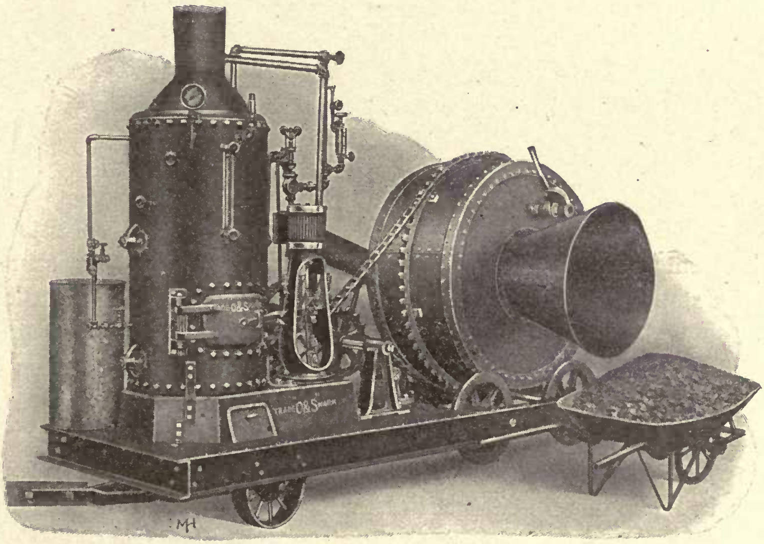


Fig. 13. McKelvey Batch Mixer.

so as to be partially mixed dry. The water is supplied near the middle of the mixer. Fig. 14 represents a type of the paddle mixer.

Automatic Measures for Concrete Materials. Mechanical measuring machinery for concrete materials have not been very extensively developed. One difficulty is that they require the constant attention of an attendant unless the materials are perfectly uniform. If the machine is adjusted for sand with a certain percentage of moisture and then is suddenly supplied with sand having greater or less moisture, the adjustment must be changed or the mixture will not be uniform. If the attendant does not watch the condition of the materials very closely, the proportions of the ingredients will vary greatly from what they should.

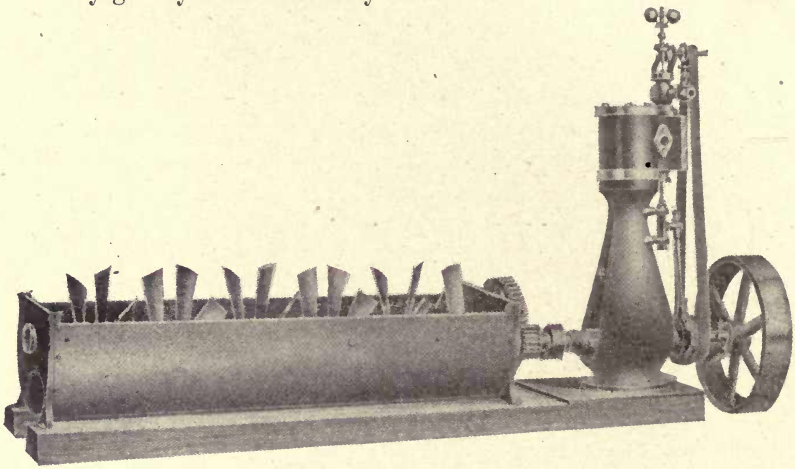


Fig. 14. Paddle Mixer.

"The *Trump* measuring device, shown in Fig. 15, consists of a horizontal revolving table on which rests the material to be measured and a stationary knife set above the table and pivoted on a vertical shaft outside the circumference. The knife can be adjusted to extend a proper distance into the material and peel off, at each revolution of the table, a certain amount which falls into the shoot. The material peeled off is replaced from the supply contained in a bottomless storage cylinder somewhat smaller in diameter than the table and revolving with it. The depth of the cut of the knife is adjusted by swinging the knife around on its pivot, so that it extends a greater or less distance into the material. The swing is controlled by a screw attached to an arm cast as part of the knife. A micrometer scale with pointer indicates the position of the knife. When it is desired to measure off and mix three materials, the machines are made with three tables set one above the other and mounted on the same spindle so that they revolve together. Each table has its own storage cylinder above it, the cylinders being placed one within the other as shown by Fig. 16."

Wetness of Concrete. The plasticity of concrete may be divided into three classes: *dry, medium,* and *very wet.*

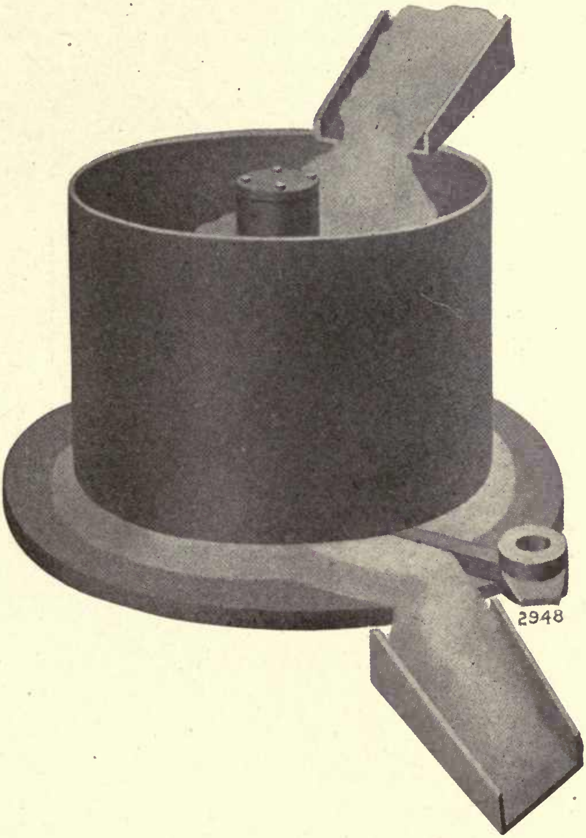


Fig. 15. Trump Measuring Device.

Dry concrete is used in foundations which may be subjected to severe compression a few weeks after being placed. It should not be placed in layers of more than 8 inches and should be thoroughly rammed. In a dry mixture the water will just flush to the surface only when it is thoroughly tamped. A dry mixture sets and will support a load much sooner than if a wetter mixture is used, and generally is only used where the load is to be applied soon after the concrete is placed. This mixture requires more than ordinary care in ramming as pockets are apt to be formed, and one argument against it is the difficulty of getting a uniform product.

Medium concrete will quake when rammed and has a con-

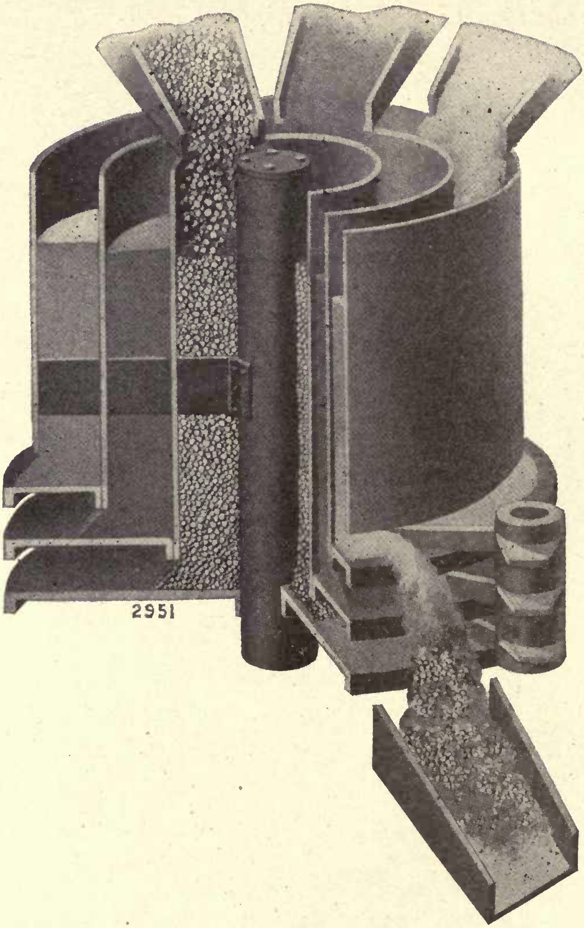


Fig. 16. Interior View of Trump Concrete Mixer.

sistency of liver or jelly. It is adapted for mass concrete, such as retaining walls, piers, foundations, arches, abutments, and sometimes for reinforced concrete.

A *Very Wet* mixture of concrete will run off a shovel unless it is handled very quickly. An ordinary rammer will sink into it of its own weight. It is suitable for reinforced concrete, such as thin walls, floors, columns, tanks, and conduits.

Within the last few years there has been a marked change in the amount of water used in mixing concrete. The dry mixture has been superseded by a medium or very wet mixture, often so wet as to require no ramming whatever. Experiments have shown that *dry mixtures* give better results in *short time tests* and *wet mixtures* in *long time tests.* In some experiments made on dry, medium, and wet mixtures it was found that the medium mixture was the most dense, wet next, and dry least. This experimenter concluded that the medium mixture is the most desirable, since it will not quake in handling, but will quake under heavy ramming. He found medium 1 per cent denser than wet and 9 per cent denser than dry concrete; he considers thorough ramming important.

Concrete is often used so wet that it will not only quake but flow freely, and after setting it appears to be very dense and hard, but some engineers think that the tendency is to use far too much rather than too little water, but that thorough ramming is desirable. In thin walls very wet concrete can be more easily pushed from the surface so that the mortar can get against the forms and give a smooth surface. It has also been found essential that the concrete should be wet enough so as to flow under and around the steel reinforcement so as to secure a good bond between the steel and concrete.

Following are the specifications (1903) of the American Railway Engineering and Maintenance of Way Association:

"The concrete shall be of such consistency that when dumped in place it will not require tamping; it shall be spaded down and tamped sufficiently to level off and will then quake freely like jelly, and be wet enough on top to require the use of rubber boots by workman."

Transporting and Depositing Concrete. Concrete is usually deposited in layers of 6 inches to 12 inches in thickness. In handling and transporting concrete care must be taken to prevent the separation of the stone from the mortar. The usual method of transporting concrete is by wheel-barrows, although it is often handled by cars and carts, and on small jobs it is sometimes carried in buckets. A very common practice is to dump it from a height of several feet into a trench. Many engineers object to this process as they claim that the heavy and light portions separate while falling and the concrete is therefore not uniform through its mass, and they insist that it must be gently slid into place. A wet mixture is much easier to handle than a dry mixture, as the stone will not so readily separate from the mass. A very wet mixture has been deposited from the top of forms 43 feet high and the structure was found to be waterproof. On the other hand, the stones in a dry mixture will separate from the mortar on the slightest provocation. Where it is necessary to drop a dry mixture several feet, it should be done by means of a chute or pipe.

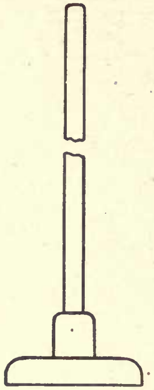


Fig. 17 Rammer for Dry Concrete. (Shoe 6 inches square.)

Ramming Concrete. Immediately after concrete is placed, it should be rammed or puddled, care being taken to force out the air-bubbles. The amount of ramming necessary depends upon how much water is used in mixing the concrete. If a very wet mixture is used, there is danger of too much ramming, which results in wedging the stones together and forcing the cement and sand to the surface. The chief object in ramming a very wet mixture is simply to expel the bubbles of air.

The style of rammer ordinarily used depends on whether a dry, medium, or very wet mixture is used. A rammer for dry concrete is shown in Fig. 17; and one for wet concrete, in Fig. 18. In very thin walls, where a wet mixture is used, often the tamping or puddling is done with a part of a reinforcing bar. A common spade is often employed for the face of work, being used to push back stones that may have separated from the mass, and also to work the finer portions of the mass to the face, the method being to work the spade up and down the face until it is thoroughly filled. Care must be taken not to pry with the spade, as this will spring the forms unless they are very strong.

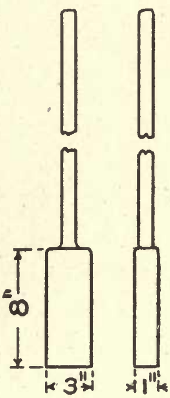


Fig. 18. Rammer for Wet Concrete.

Bonding Old and New Concrete. To secure a water-tight joint between old and new concrete, requires a great deal of care. Where the strain is chiefly compressive, as in foundations, the surface of the concrete laid on the previous day should be washed with clean water, no other precautions being necessary. In walls and floors, or where a tensile stress is apt to be applied, the joint should be thoroughly washed and soaked, and then painted with neat cement or a mixture of one part cement and one part sand, made into a very thin mortar.

In the construction of tanks or any other work that is to be water-tight, in which the concrete is not placed in one continuous operation, one or more square or V-shaped joints are necessary. These joints are formed by a piece of timber, say 4 inches by 6 inches, being imbedded in the surface of the last concrete laid each day. On the following morning, when the timber is removed, the joint is washed and coated with neat cement or 1:1 mortar. The joints may be either horizontal or vertical. The bond between old and new concrete may be aided by roughening the surface after ramming or before placing the new concrete.

Effects of Freezing of Concrete. Many experiments have been made to determine the effect of freezing of concrete before it has a chance to set. From these and from practical experience, it is now generally accepted that the ultimate effect of freezing of Portland cement concrete is to produce only a surface injury. The setting and hardening of Portland cement concrete is retarded, and the strength at short periods is lowered, by freezing; but the ultimate strength appears to be only slightly, if at all, affected. A thin scale about $\frac{1}{16}$ inch in depth is apt to scale off from granolithic or concrete pavements which have been frozen, leaving a rough instead of a troweled wearing surface; and the effect upon concrete walls is often similar; but there appears to be no other injury. Concrete should not be laid in freezing weather, if it can be avoided, as this involves additional expense and requires greater precautions to be taken; but with proper care, Portland cement concrete can be laid at almost any temperature.

The heating of the material hastens the setting of the cement, and also keeps it above the freezing point for a longer period. Salt lowers the freezing point of water, and when used in moderate quantities does not appear to affect the ultimate strength of the concrete. Authorities differ on the amount of salt that may be used;

but from four to ten pounds to each barrel of cement will not decrease the strength of the concrete.

Finish. To give a satisfactory finish to exposed surfaces of concrete is rather a difficult problem. Usually, when the forms are taken down, the surface of the concrete shows the joints, knots, and grain of the wood. It has more the appearance of a piece of rough carpentry work than of finished masonry. Some special treatment is therefore necessary. Plastering is not usually successful, although there are cases where a mixture of equal parts of cement and sand have apparently been successful. Where finished rough, it did not show hair-cracks; but when finished smooth, it did show them. In constructing the Harvard University Stadium, care was taken, after the concrete was placed in the forms, to force the stones back from the face and permit the mortar to cover every stone. When the forms were removed, the surface was picked with a tool as shown in Fig. 19. A pneumatic tool has also been adopted for this purpose.

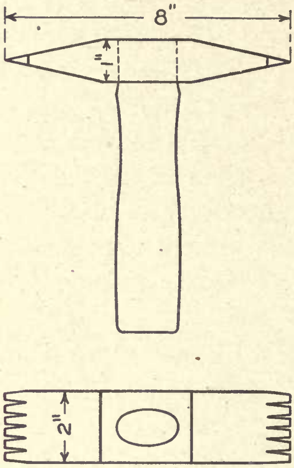
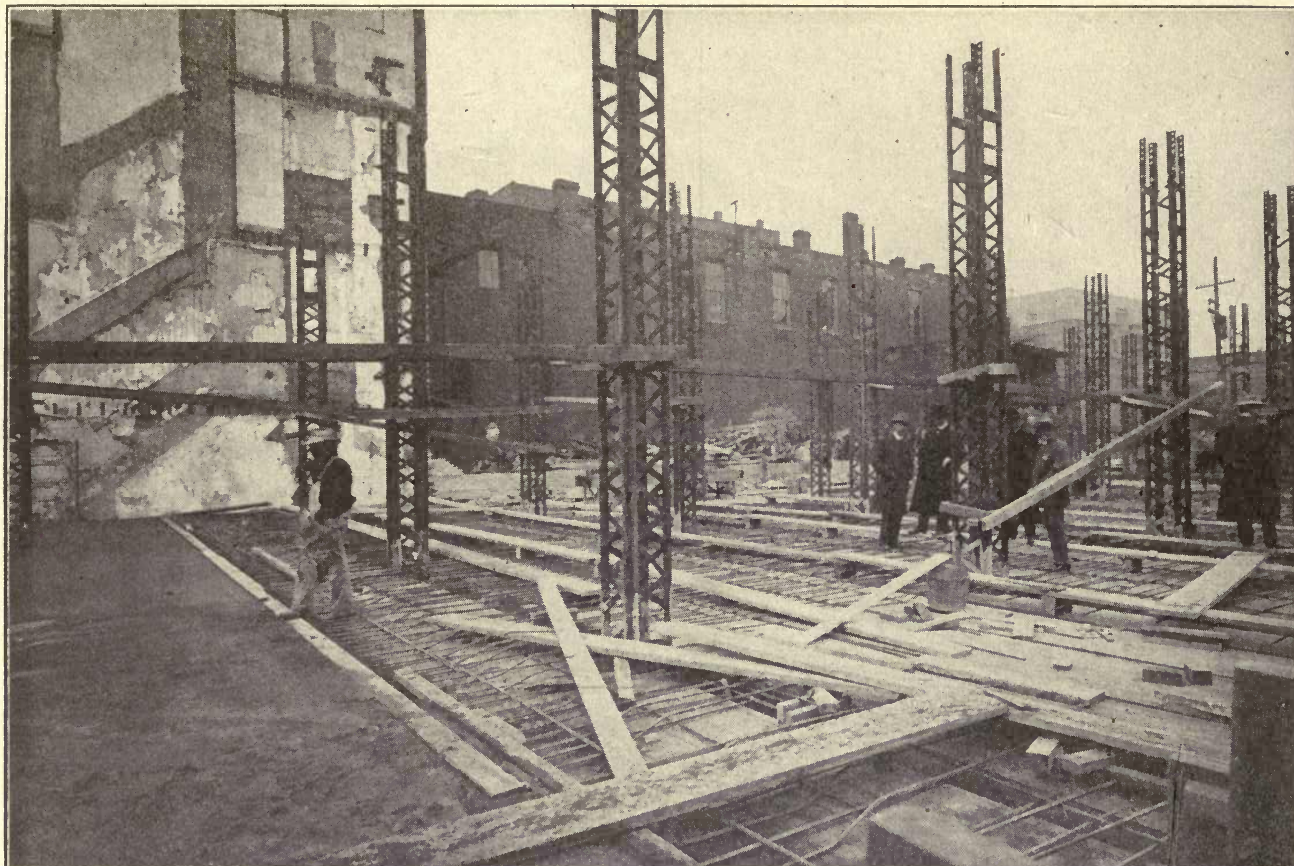


Fig. 19. Pick for Facing Concrete.

The number of square feet to be picked per day, depends on the hardness of the concrete. If the picking is performed by hand, it is done by a common laborer; and he is expected to cover, on an average, about 50 square feet per day of ten hours. With a pneumatic tool, a man would cover from 400 to 500 square feet per day.

Several concrete bridges in Philadelphia have been finished according to the following specifications; and their appearance is very satisfactory:

"Granolithic surfacing, where required, shall be composed of 1 part cement, 2 parts coarse sand or gravel, and 2 parts granolithic grit, made into a stiff mortar. Granolithic grit shall be granite or trap rock, crushed to pass a ¼-inch sieve, and screened of dust. For vertical surfaces, the mixture shall be deposited against the face forms to a minimum thickness of 1 inch, by skilled workmen, as the placing of the concrete proceeds; and it thus forms a part of the body of the work. Care must be taken to prevent the occurrence of air-spaces or voids in the surface. The face shall be removed



MANUFACTURERS' FURNITURE EXCHANGE BUILDING, CHICAGO, ILL., SHOWING METHOD OF REINFORCED CONCRETE CONSTRUCTION.

Column Centers are 14 feet and 16 feet 9 inches. Columns Consist of 4 Angles Latticed, Filled in with Concrete, with the Angles Acting as Reinforcement for Same. Floor Construction, 14-Foot-Span Slabs on Reinforced Concrete Beams. Live Loads Figured on: First Floor, 150 lbs.; Second to Eighth Floors, 100 lbs.; Roof, 40 lbs. Test was made over an Area of 12 by 15 feet, with a Total Load of 55,780 lbs. plus Dead Load, making a Load of 310 lbs. per Square Foot plus Dead Load. The Deflection was Less than $\frac{1}{16}$ Inch.

as soon as the concrete has sufficiently hardened; and any voids that may appear shall be filled with the mixture. The surface shall then be immediately washed with water until the grit is exposed and rinsed clean, and shall be protected from the sun and kept moist for three days. For bridge-seat courses and other horizontal surfaces, the granolithic mixture shall be deposited on the concrete to at least a thickness of 1½-inches, immediately after the concrete has been tamped and before it has set, and shall be troweled to an even surface, and, after it has set sufficiently hard, shall be washed until the grit is exposed."

A very satisfactory finish for a ten-span reinforced concrete viaduct on the Utica & Mohawk Railway, was produced in the following manner: For a hard wall, the surface was wet, and a thin 1:2 mortar was applied with a brush. The surface was then thoroughly rubbed with a piece of grindstone or carborundum, removing all broad marks and filling all pores, and producing a lather on the surface of the concrete; and before this had time to dry, it was gone over with a brush dipped in water, producing a smooth, even, and uniform color. For a green wall, when the forms were removed in less than seven days, the surface was wet, and a thin grout of pure cement was applied with a brush; the surface was then rubbed with a piece of grindstone or carborundum, and finished in the same manner as above described. This method has been used by other railroad companies also, on similar work; and the results have been found exceedingly satisfactory.

The following method has been adopted by the New York Central Railroad for giving a good finish to exposed concrete surfaces: The forms of 2-inch tongued-and-grooved pine were coated with soft soap, all openings in the joints of the forms being filled with hard soap. The concrete was then deposited, and, as it progressed, was drawn back from the face with a square-pointed shovel, and 1:2 mortar poured in along the forms. When the forms were removed, and while the concrete was green, the surface was rubbed, with a circular motion, with pieces of white firebrick or brick composed of one part cement and one part sand. The surface was then dampened and painted with a 1:1 grout, rubbed in, and finished with a wooden float, leaving a smooth and hard surface when dry.

Floors and walks are often finished with a 1-inch coat of cement and sand, or of cement, sand, and grit, which is usually mixed in the proportions of 1 part cement and 1 part sand, or of 1 part cement, 1 part sand, and 1 part grit. (See Fig. 20.) This finishing coat must be put on before the concrete of the base sets. The cement and sand must be thoroughly mixed while dry, so as to have a uniform color.

In office buildings, and generally in factory buildings, a wooden floor is laid over the concrete. Wooden stringers are first laid on

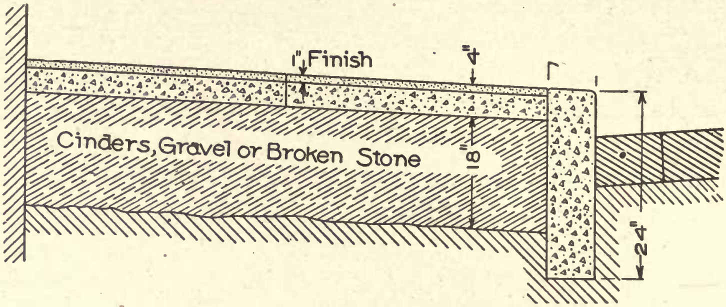


Fig. 20. Concrete Sidewalk and Curb.

the concrete, about 2 to 2½ feet apart. The stringers are 2 inches thick and 3 inches wide on top, with sloping edges. The space between the stringers is filled with cinder concrete, as shown in Fig. 21, usually mixed 1 : 4 : 8. When the concrete has set, the flooring is nailed to the stringers.

The following method of placing mortar facing has been found very satisfactory, and has been adopted very extensively in the last

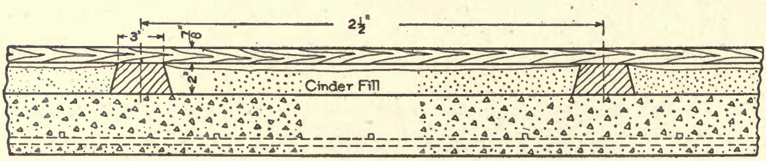


Fig. 21. Cinder Fill Between Stringers.

few years: A sheet-iron plate 6 or 8 inches wide and about 5 or 6 feet long, has riveted across it on one side angles of ¾-inch size or such other size as may be necessary to give the desired thickness of mortar facing, these angles being spaced about two feet apart. In operation, the ribs of the angles are placed against the forms; and the space between the plate and forms is filled with mortar, which is mixed in small batches, and thoroughly tamped. The concrete back filling is then placed; the mold is withdrawn; and the facing and back filling are rammed together. The mortar facing is mixed in the proportion of one part cement, to 1, 2, or 3 parts sand; usually a 1:2 mixture is employed, mixed wet and in small batches as used. As mortar facing shows the roughness of the forms more readily than concrete does, care is required in constructing, to secure a smooth finish. When

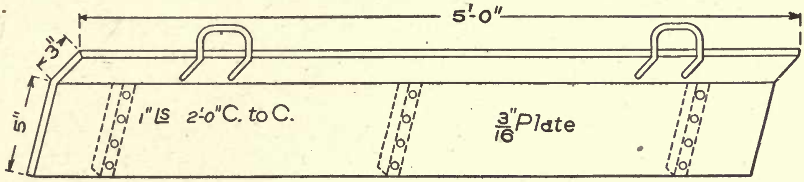


Fig. 22. Mold for Mortar Facing.

the forms are removed, the face may be treated either in the manner already described, or according to the following method taken from the "Proceedings" of the American Railway Engineering and Maintenance of Way Association:

"After the forms are removed, any small cavities or openings in the concrete shall be filled with mortar if necessary. Any ridges due to cracks or joints in the lumber shall be rubbed down; the entire face shall be washed with a thin grout of the consistency of whitewash, mixed in the proportion of 1 part cement to 2 parts of sand. The wash shall be applied with a brush."

Concrete surfaces may be finished to represent ashlar masonry. The process is similar to stone-dressing; and any of the forms of finish employed for cut stone can be used for concrete. Very often, when the surface is finished to represent ashlar masonry, vertical and horizontal three-sided pieces of wood are fastened to the forms to make V-shaped depressions in the concrete, as shown in Fig. 23.

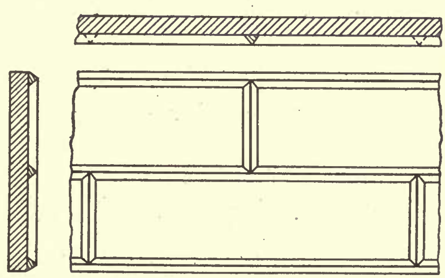


Fig. 23. Concrete Molding.

A facing of stone or brick is frequently used for reinforced concrete, and is a very satisfactory solution of the problem of finish. The same care is required with a stone or brick facing as if the entire structure were stone or brick. The Ingalls Building at Cincinnati, Ohio, 16 stories, is veneered on the outside with marble to a height of three stories, and with brick and terra-cotta above the third story. Exclusive of the facing, the wall is 8 inches thick.

Water-tightness of Concrete. Water-tight concrete, or concrete made water-tight by some kind of waterproof coating, is frequently required, either for inclosing a space which must be kept dry, or for storing water or other liquids.

It is generally considered that in monolithic construction, a wet mixture, a rich concrete, and an aggregate proportioned for great density, are essential for water-tightness. With the wet mixtures of concrete now generally used in engineering work, concrete possesses far greater density, and is correspondingly less porous, than with the older, dryer mixtures. At the same time, in the large masses of actual work, it is difficult to produce concrete of such close texture as to prevent undesirable seepage at all points. Many efforts have been made to secure water-tightness of concrete in a practical manner—some with success, but others with unsatisfactory results. There are now a great many special preparations being advertised for making concrete water-tight.

It has frequently been observed that when concrete was green, there was a considerable seepage through it, and that in a short time absolutely all seepage stopped. Some experiments have been made to render porous concrete impermeable, by forcing water through a rich concrete under pressure. In these experiments, a mixture of 1 part Portland cement to 4 parts crushed gravel was used. The concrete tested was 6 inches thick. The flow through the concrete on the first day of the experiment, under a pressure of 36 pounds per square inch, was taken as 100 per cent. On the forty-sixth day, under a pressure of 48 pounds per square inch, the flow amounted to only 0.7 per cent.

While the pressure was constant, the rate of seepage of the water decreased with the lapse of time, showing a marked tendency of the seepage passages to become closed. The experimenter is of the opinion that the water, under pressure, *dissolves* some of the material and then deposits it in stalactitic form near the exterior surface of the concrete, where the water escapes under much reduced pressure. Others, however, think it quite possible that fine material carried *in suspension* by the water aids in producing the result.

For cistern work, two coats of Portland cement grout—1 part cement, 1 part sand—applied on the inside, have been found sufficient. About one inch of rich mortar has usually been found effective under high pressure. A coating of asphalt, or of asphalt with tarred or asbestos felt, laid in alternate layers between layers of concrete, has been used successfully. Coal-tar pitch and tarred felt, laid in alternate layers, have been used extensively and successfully in New York City for waterproofing.

Mortar may be made practically non-absorbent by the addition of alum and potash soap. One per cent by weight of powdered alum is added to the dry cement and sand, and thoroughly mixed; and about one per cent of any potash soap (ordinary soft soap) is dissolved in the water used in the mortar. A solution consisting of 1 pound of concentrated lye, 5 pounds of alum, and 2 gallons of water, applied while the concrete is green and until it lathers freely, has been successfully used. Coating the surface with boiled linseed oil until the oil ceases to be absorbed, is another method that has been used with success.

A reinforced concrete water-tank, 10 feet inside diameter and 43 feet high, designed and constructed by W. B. Fuller at Little Falls, N. J., has some remarkable features. It is 15 inches thick at the bottom and 10 inches thick at the top. The tank was built in eight hours, and is a perfect monolith, all concrete being dropped from the top, or 43 feet at the beginning of the work. The concrete was mixed very wet, the mixture being 1 part cement, 3 parts sand, and 7 parts broken stone. No plastering or waterproofing of any kind was used, but the tank was found to be absolutely water-tight, although the mixture used has not generally been found or considered water-tight.

At Attleboro, Mass., a large reinforced concrete standpipe, 50 feet in diameter, 106 feet high from the inside of the bottom to the top of the cornice, and with a capacity of 1,500,000 gallons, has been constructed, and is in the service of the water works of that city. The walls of the standpipe are 18 inches thick at the bottom, and 8 inches thick at the top. A mixture of 1 part cement, 2 parts sand, and 4 parts broken stone, the stone varying from ¼ inch to 1½ inches, was used. The forms were constructed, and the concrete placed, in sections of 7 feet. When the walls of the tank had been completed, there was some leakage at the bottom with a head of water of 100 feet. The inside walls were then thoroughly cleaned and picked, and four coats of plaster applied. The first coat contained 2 per cent of lime to 1 part of cement and 1 part of sand; the remaining three coats were composed of 1 part sand to 1 part cement. Each coat was floated until a hard, dense surface was produced; then it was scratched to receive the succeeding coat.

On filling the standpipe after the four coats of plaster had been applied, the standpipe was found to be not absolutely water-tight. The water was drawn out; and four coats of a solution of castile soap, and one of alum, were applied alternately; and, under a 100-foot head, only a few leaks then appeared. Practically no leakage occurred at the joints; but in several instances a mixture somewhat wetter than usual was used, with the result that the spading and ramming served to drive the stone to the bottom of the batch being placed, and, as a consequence, in these places porous spots occurred. The joints were obtained by inserting beveled tonguing pieces, and by thoroughly washing the joint and covering it with a layer of thin grout before placing additional concrete.

In the construction of the filter plant at Lancaster, Pa., in 1905, a pure-water basin and several circular tanks were constructed of reinforced concrete. The pure-water basin is 100 feet wide by 200 feet long and 14 feet deep, with buttresses spaced 12 feet 6 inches center to center. The walls at the bottom are 15 inches thick, and 12 inches thick at the top. Four circular tanks are 50 feet in diameter and 10 feet high, and eight tanks are 10 feet in diameter and 10 feet high. The walls are 10 inches thick at the bottom, and 6 inches at the top. A wet mixture of 1 part cement, 3 parts sand, and 5 parts stone, was used. No waterproofing material was used, and the basin and tanks are water-tight.

Forms. In actual construction work, the cost of forms is a large item of expense, and offers the best field for the exercise of ingenuity. For economical work, the design should consist of a repetition of identical units; and the forms should be so devised that it will require a minimum of nailing to hold them, and of labor to make and handle them. Forms are constructed of the cheaper grades of lumber. To secure a smooth surface, the planks are planed on the side on which the concrete will be placed. Green lumber is preferable to dry, as it is less affected by wet concrete. If the surface of the planks that are placed next to the concrete are well oiled, the planks can be taken down much easier, and, if they are kept from the sun, can be used several times.

Crude oil is an excellent and cheap material for greasing forms, and can be applied with a white-wash brush. The oil should be applied every time the forms are used. The object is to fill the pores of the wood, rather than to cover it with a film of grease. Thin soft soap, or a paste made from soap and water, is also sometimes used.

In constructing a factory building of two or three stories, usually the same set of forms are used for the different floors; but when the building is more than four stories high, two or more sets of forms are specified, so as always to have one set of forms ready to move.

The forms should be so tight as to prevent the water and thin mortar from running through, and thus carrying off the cement. This is accomplished by means of tongued-and-grooved or beveled-edge boards; but it is often possible to use square lumber if it is thoroughly wet so as to swell it before the concrete is placed. The beveled-edge boards are often preferred to tongue-and-grooved boards, as the edges tend to crush as the boards swell, and this prevents buckling.

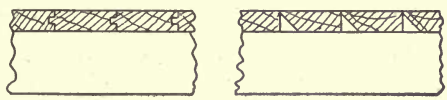


Fig. 24. Tongued and Grooved Edge. Beveled Edge

Lumber for forms may be made of 1-inch, 1½-inch, or 2-inch plank. The spacing of studs depends in part upon the thickness of concrete to be supported, and upon the thickness of the boards on which the concrete is placed. The size of the studding depends upon the height of the wall and the amount of bracing used. Except in very heavy or high walls, 2 by 4-inch or 2 by 6-inch studs are used. For ordinary floors with 1-inch plank, the supports should be placed about 2 feet apart; with 1½-inch plank, about 3 feet apart; and 2-inch plank, 4 feet apart.

The length of time required for concrete to set depends upon the weather, the consistency of the concrete, and the strain which is to come on it. In good drying weather, and for very light work, it is often possible to remove the forms in 12 to 24 hours after placing the concrete, if there is no load placed on it. The setting of concrete is greatly retarded by cold or wet weather. Forms for concrete arches and beams must be left in place longer than in wall work, because of the tendency to fail by rupture across the arch or beam. In small, circular arches, like sewers, the forms may be removed in 18 to 24 hours if the concrete is mixed dry; but if wet concrete is used, in 24 to 48 hours. Forms for large arch culverts and arch bridges are seldom taken down in less than 14 days; and it is often specified that they must not be struck for 28 days after placing the last concrete. In ordinary floor construction, consisting of slabs, girders, and beams, the forms are usually left in place at least a week.

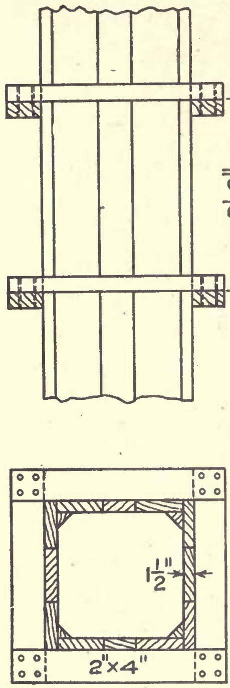


Fig. 25. Forms for Columns.

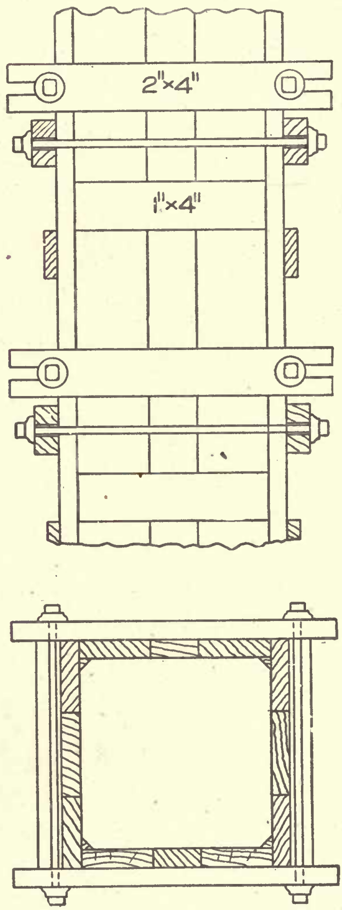


Fig. 26. Forms for Columns.

In constructing columns, the forms are usually erected complete, the full height of the column, and concrete is dumped in at the top. The concrete must be mixed very wet, as it cannot be rammed very thoroughly at the bottom, and care must be taken not to displace the steel. Sometimes the forms are constructed in short sections, and the concrete is placed and rammed as the forms are built. The ends of the bottom of the forms for the girders and beams, are usually supported by the column forms. To give a beveled edge to the corner of the columns, a triangular strip is fastened in the corner of the forms.

Fig. 25 shows the common way, or some modification of it, of constructing forms for column. The plank may be 1 inch, 1½ inches, or 2 inches thick; and the cleats are usually 1 by 4 inches and 2 by 4 inches. The spacing of the cleats depends on the size of the columns and the thickness of the vertical plank.

Fig. 26 shows column forms similar to those used in constructing the Harvard stadium. The planks forming each side of the column are fastened together by cleats, and then the four sides are fastened together by slotted cleats and steel tie-rods. These forms can be quickly and easily removed.

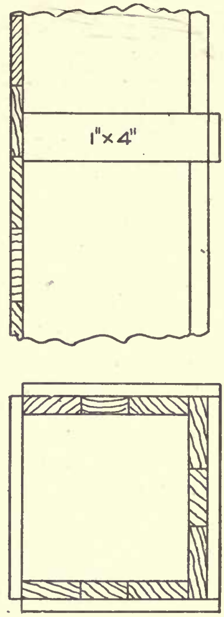


Fig. 27. Forms for Columns.

Fig. 27 shows column forms in which the concrete is placed and rammed as the forms are constructed. Three sides are erected to the full height, and the steel is then placed. The fourth side is built up with horizontal boards as the concrete is placed and rammed.

A very common style of forms for beam and slab construction is shown in Fig. 28. The size of the different members of the forms

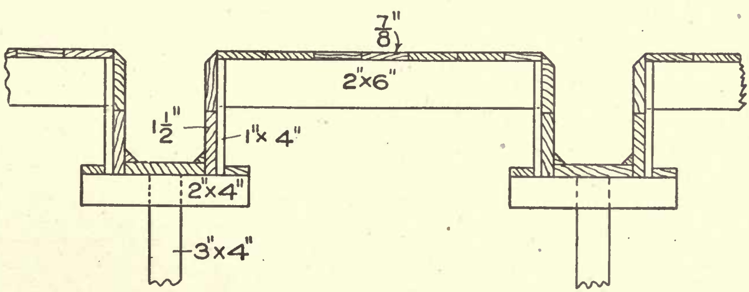


Fig. 28. Forms for Beams and Slabs.

depends upon the size of the beams, the thickness of the slabs, and the relative spacing of some of the members. If the beam is

10 by 20 inches, and the slab is 4 inches thick, then 1-inch plank supported by 2 by 6-inch timbers spaced 2 feet apart, will support the slab. The sides and bottom of the beams are enclosed by 1½-inch or 2-inch plank supported by 3 by 4-inch posts spaced 4 feet apart.

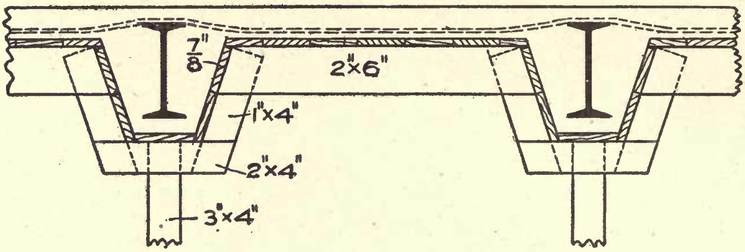


Fig. 29. Forms for Reinforced Concrete Slab Supported by I-beams.

In Fig. 29 is shown the forms for a reinforced concrete slab, with I-beam construction. These forms are constructed similarly to those just described.

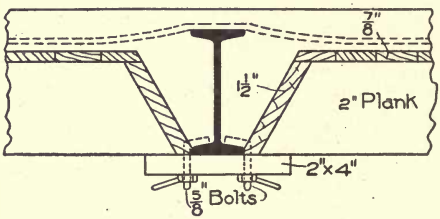


Fig. 30. Form for Reinforced Concrete Slab between I-beams.

A slab construction, supported on I-beams, the bottom of which is not covered with concrete, may have forms constructed as shown in Fig. 30. This method of constructing forms was designed by Mr. William F. Kearns (Taylor & Thompson, "Plain and Reinforced Concrete").

The construction of forms for a slab that is supported on the top of I-beams, is a comparatively simple process, as shown in Fig. 31. In any form of I-beam and slab construction, the forms can be constructed to carry the combined weight of the concrete and forms. When the bottom of the I-beam is to be covered with concrete, it is not so easily done as when the haunch rests on the bottom flange (Fig. 30) or is a flat plate (Fig. 31).

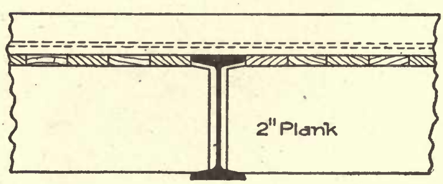


Fig. 31. Form for Floor Slab on I-beams.

Forms for conduits and sewers must be strong enough not to give way under, or to become deformed, while the concrete is being placed and rammed, and must be rigid enough not to warp from being alternately wet and dry. They must be constructed so that they can readily be put up and taken down and can be used several times on the same job. The forms must give a smooth and even finish to the interior of the sewer or conduit. This has been accomplished on several jobs by covering the forms with light-weight sheet iron.

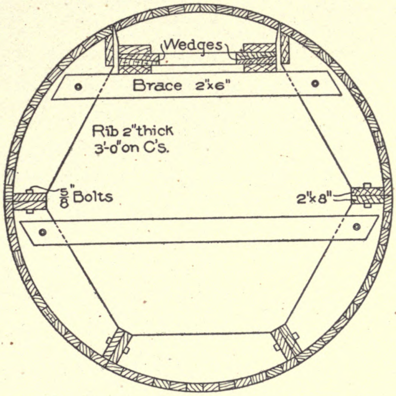
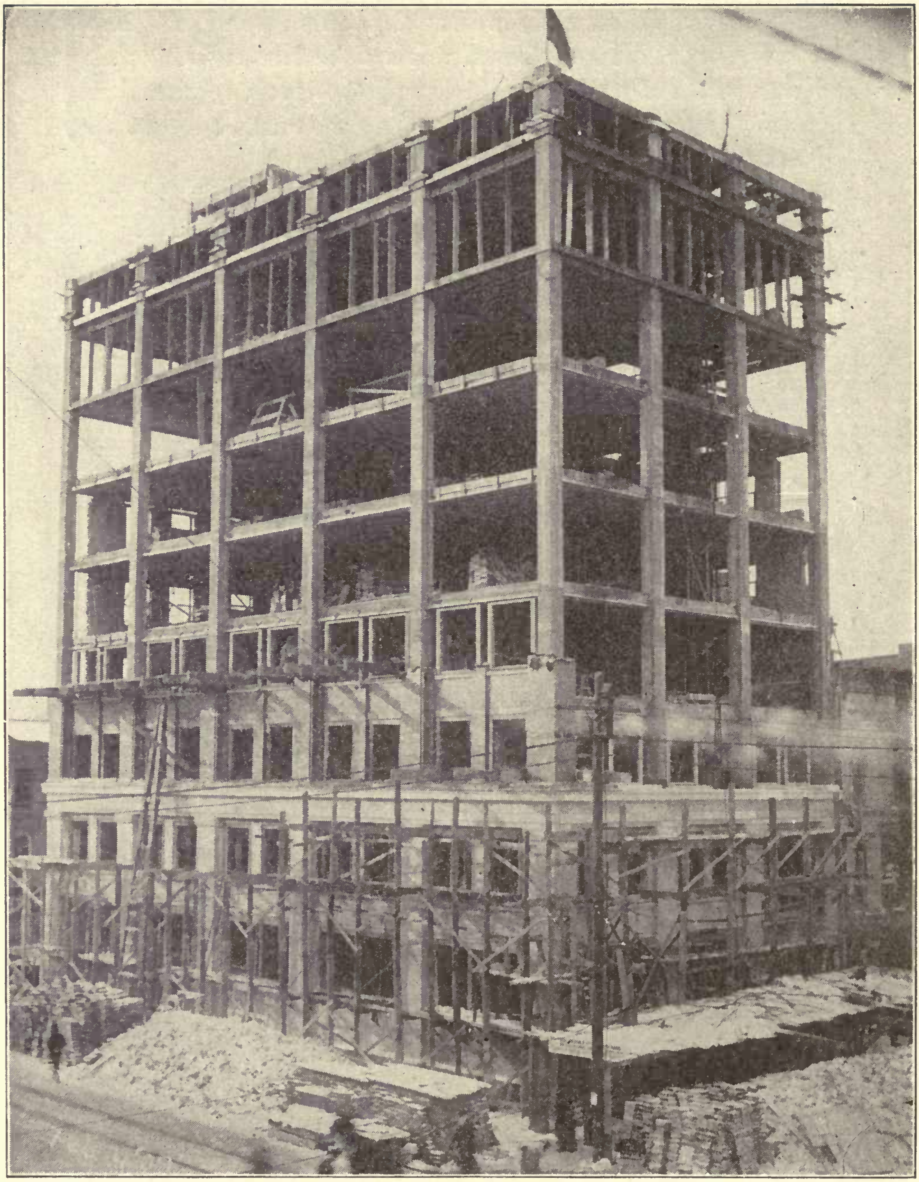


Fig. 32. Center for 8-ft. Sewer.

These forms are usually built in lengths of 16 feet, with one center at each end, and with three to five (depending on the size of the sewer or conduit) intermediate centers in the lengths of 15 feet. The segmental ribs are bolted together. The plank for these forms are made of 2 by 4-inch material, surfaced on the outer side, with the edge beveled to the radius of the conduit. The segmental ribs are bolted together, and are held in place by wooden ties 2 by 4 inches or 2 by 6 inches.



KALAMAZOO NATIONAL BANK, KALAMAZOO, MICH.

J. C. Llewellyn, Architect, Chicago, Ill.

Reinforced Concrete Floors; Steel Columns Reinforced with Concrete. First Two Stories, Buff Bedford Stone; Upper Stories, Pressed Brick. Built in 1907.

REINFORCED CONCRETE

PART II

GENERAL THEORY OF FLEXURE IN REINFORCED CONCRETE

Introduction. The theory of flexure in reinforced concrete is exceptionally complicated. A multitude of simple rules, formulæ, and tables for designing reinforced concrete work have been proposed, some of which are sufficiently accurate and applicable *under certain conditions.* But the effect of these various conditions should be thoroughly understood. Reinforced concrete should not be designed by "rule-of-thumb" engineers. It is hardly too strong a statement to say that a man is criminally careless and negligent when he attempts to design a structure on which the safety and lives of people will depend, without thoroughly understanding the theory on which any formula he may use is based. The applicability of all formulæ is so dependent on the quality of the steel and of the concrete, and on many of the details of the design, that a blind application of a formula is very unsafe. Although the greatest pains will be taken to make the following demonstration as clear and plain as possible, it will be necessary to employ symbols, and to work out several algebraic formulæ on which the rules for designing will be based. The full significance of many of the terms mentioned below may not be fully understood until several subsequent paragraphs have been studied:

b = Breadth of concrete beam;

d = Depth from compression face to center of gravity of the steel;

A = Area of the steel;

p = Ratio of area of steel to area of concrete above the center of gravity of the steel, generally referred to as "percentage of reinforcement,"

$$= \frac{A}{b\,d};$$

E_s = Modulus of elasticity of steel;

E_c = *Initial* modulus of elasticity of concrete;

$r = \frac{E_s}{E_c}$ = Ratio of the moduli;

s = Tensile stress per unit of area in steel;

c = Compressive stress per unit of area in concrete at the outer fiber of the beam; this may vary from zero to c';
c' = Ultimate compressive stress per unit of area in concrete — the stress at which failure might be expected;
ϵ_s = Deformation per unit of length in the steel;
ϵ_c = " " " " " in outer fiber of concrete;
ϵ'_c = " " " " " in outer fiber of concrete when crushing is imminent;
ϵ''_c = Deformation per unit of length in outer fiber of concrete under a certain condition (described later);

$q = \frac{\epsilon_c}{\epsilon_c''}$ = Ratio of deformations;

k = Ratio of depth from compressive face to the neutral axis to the total effective depth d;
x = Distance from compressive face to center of gravity of compressive stresses;
ΣX = Summation of horizontal compressive stresses;
M = Resisting moment of a section.

Statics of Plain Homogeneous Beams. As a preliminary to the theory of the use of reinforced concrete in beams, a very brief discussion will be given of the statics of an ordinary homogeneous beam. Let A B represent a beam carrying a uniformly distributed load W; then the beam is subjected to transverse stresses. Let us imagine that *one-half* of the beam is a "free body" in space and is acted on by exactly the same external forces; we shall also assume the forces C and T (acting on the exposed section), which are just such forces as are required to keep that half of the beam in equilibrium.

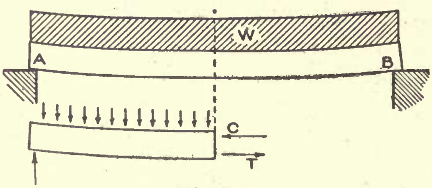


Fig. 33.

These forces, and their direction, are represented in the lower diagram by arrows. The load W is represented by the series of small, equal, and equally spaced vertical arrows pointing downward. The reaction of the abutment *against the beam* is an *upward* force, shown at the left. The forces acting on a *section* at the center are the equivalent of the two equal forces C and T.

The force C, acting at the top of the section, must act toward the left, and there is therefore compression in that part of the section. Similarly, the force T is a force acting toward the right, and the fibers of the lower part of the beam are in tension. For our present purpose we may consider that the forces C and T are in each case the resultant of the forces acting on a very large number of "fibers." The stress in the outer fibers is of course greatest. At the center of the height there is neither tension nor compression. This is called the "neutral axis."

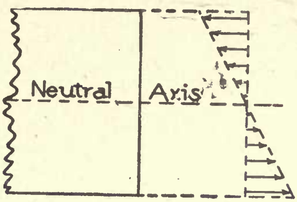


Fig. 34.

Let us consider for simplicity a very narrow portion of the beam, having the full length and depth, but so narrow that it includes only one set of fibers, one above the other, as shown in Fig. 35. In the case of a plain, rectangular, homogeneous beam, the stresses in the fibers would be as given in Fig. 34; the neutral axis would be at the center of the height, and the stress at the bottom and the top would be equal but opposite. If the section were at the center of the beam, with a uniformly distributed load (as indicated in Fig. 33), the "shear" would be zero. These general principles have already been explained in "Strength of Materials," sections 57-60.

A beam *may* be constructed of plain concrete; but its strength will be very small, since the tensile strength of concrete is comparatively insignificant. Reinforced concrete utilizes the great tensile strength of steel, in combination with the compressive strength of concrete. It should be realized that the essential qualities are *compression* and *tension*, and that (other things being equal) the cheapest method of obtaining the necessary compression and tension is the most economical.

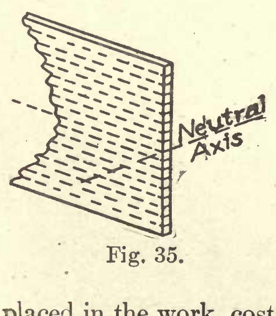


Fig. 35.

Economy of Concrete for Compression. The ultimate compressive strength of concrete is generally 2,000 pounds or over per square inch. With a factor of safety of four, a working stress of 500 pounds per square inch may be considered allowable. We may estimate that the concrete costs twenty cents per cubic foot, or $5.40 per cubic yard. On the other hand, we may estimate that the steel, placed in the work, costs about three cents per pound. It will weigh 480 pounds per cubic foot; therefore the steel costs $14.40 per cubic foot, or 72 times as much as an equal *volume* of concrete or an equal *cross-section* per unit of length. But the steel can safely withstand a compressive stress of 16,000 pounds per square inch, which is 32 times the safe working load on concrete. Since, however, a given volume of steel costs 72 times an equal volume of concrete, the cost of a given compressive resistance in steel is $\frac{72}{32}$ (or 2.25) times the cost of that resistance in concrete. Of course, the above assumed unit prices of concrete and steel will vary with circumstances. The advantage of concrete over steel for compression may be somewhat greater or less than the ratio given above, but the advantage is almost invariably with the concrete. There are many other advantages in addition, which will be discussed later.

Economy of Steel for Tension. The ultimate tensile strength of ordinary concrete is rarely more than 200 pounds per square inch. With a factor of safety of four, this would allow a working stress of only 50 pounds per square inch. This is generally too small for practical use, and certainly too small for economical use. On the other hand, steel may be used with a working stress of 16,000 pounds per square inch, which is 320 times that allowable for concrete. Using the same unit values for the cost of steel and concrete as given in the previous section, even if steel costs 72 times as much as an equal volume of concrete, its real tensile value economically is $\frac{320}{72}$ (or 4.44) times as great. Any reasonable variation from the above unit values cannot alter the essential truths of the economy of steel for tension and of concrete for compression. In a reinforced concrete beam, the steel is placed in the tension side of the beam. Usually it is placed from one to two inches from the outer face, with the double purpose of protecting the steel from corrosion or fire, and also to better insure the union of the concrete and the steel. But the concrete below the steel is not considered in the numerical calculations. Even the concrete which is between the steel and the neutral axis (whose position will be discussed later), is chiefly useful in transmitting the tension in the steel to the concrete. Although such concrete is theoretically subject to tension, and does actually contribute its share of the tension when the stresses in the beam are small, the proportion of the necessary tension which the concrete can furnish when the beam is heavily loaded, is so very small that it is usually ignored, especially since such a policy is on the side of safety, and also since it greatly simplifies the theoretical calculations and yet makes very little difference in the final result. We may therefore consider that in a unit section of the beam, as in Fig. 36, the concrete above the neutral axis is subject to compression, and that the tension is furnished entirely by the steel.

Elasticity of Concrete in Compression. In computing the transverse stresses in a wooden beam or steel I-beam, it is assumed that the modulus of elasticity is uniform for all stresses within the elastic limit. Experimental tests have shown this to be so nearly true that it is accepted as a mechanical law. This means that if a force of 1,000 pounds is required to stretch a bar .001 of an inch, it will require 2,000 pounds to stretch it .002 of an inch. Similar tests have been made with concrete, to determine the law of its elasticity. Unfortunately, concrete is not so uniform in its behavior as steel. The results of tests are somewhat contradictory. Many engineers have argued that the elasticity is so nearly uniform that it may be considered to be such within the limits of practical use. But all experimenters who have tested concrete by measuring the proportional compression produced by various pressures, agree that the additional shortening produced by an additional pressure of say 100 pounds per square inch, is greater at higher pressures than at low pressures.

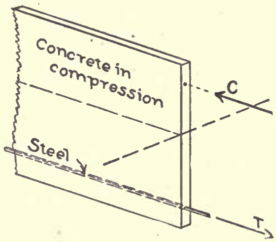


Fig. 36. Transmission of Tension in Steel to Concrete.

A test of this sort may be made substantially as follows: A square or circular column of concrete at least one foot long is placed in a testing machine. A very delicate micrometer mechanism is fastened to the concrete by pointed screws of hardened steel. These points are originally at a known distance apart—say 8 inches. When the concrete is compressed, the distance between these points will be slightly less. A very delicate mechanism will permit this distance to be measured as closely as the ten-thousandth part of an inch or, to about $\frac{1}{100,000}$ of the length. Suppose that the various pressures per square inch, and the proportionate compressions, are as given in the following tabular form:

Pressure per Square Inch	Proportionate Compression
200 pounds	.00010 of total length
400 "	.00020 " " "
600 "	.00032 " " "
800 "	.00045 " " "
1,000 "	.00058 " " "
1,200 "	.00062 " " "
1,400 "	.00090 " " "
1,600 "	.00112 " " "

We may plot these pressures and compressions as in Fig. 37, using any convenient scale for each. For example, for a pressure of 800 pounds per square inch, we select the vertical line which is at the

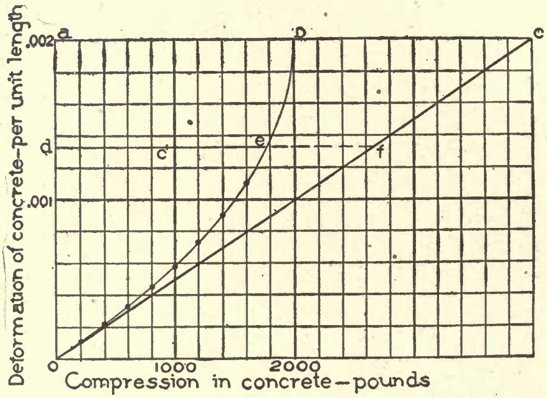


Fig. 37.

horizontal distance from the origin *O* of 800, according to the scale adopted. Scaling off on this vertical line the ordinate .00045, according to the scale adopted for compressions, we have the position of one point of the curve. The other points are obtained similarly. Although the points thus obtained from the testing of a single block of concrete would not be considered sufficient to establish the law of the elasticity of concrete in compression, a study of the curves which may be drawn through the series of points obtained for each of a large number of blocks, shows that these curves will average very closely to parabolas that are tangent to the initial modulus of elasticity, which is here represented in the diagram by a straight line running diagonally across the figure.

It is generally considered that the axis of the parabola will be a horizontal line when the curve is plotted according to this method. The position of the vertex of the parabola cannot be considered as definitely settled. Professor Talbot has computed the curve as if the vertex were at the point of the ultimate compression of the concrete, although he conceded that the vertex might be in an imaginary position corresponding to a compression in the concrete higher than that which the concrete could really endure. Mr. A. L. Johnson, another noted authority, bases his computation of formulæ on the assumption that the ultimate compressive strength of the concrete is two-thirds of the value which would be required to produce that amount of compression, in case the initial modulus of elasticity was the true value for all compressions. In other words, looking at Fig. 37, if *o c* is a line representing the initial modulus of elasticity, then, if the elasticity were uniform throughout, it would require a force of about 2,340 pounds (or *d f*) to produce a proportionate compression of .00132 of the length (represented by *o d*). Actually that compression will be produced when the pressure equals *d e*, which is $\frac{2}{3}$ of *d f*. It should not be forgotten that the above numerical values are given merely for illustrative purposes. They would, if true, represent a rather weak concrete. The following theory is therefore based on the assumption that the stress-strain curve is represented by the parabolic curve *o e* (see Fig. 37); and that the ultimate stress per square inch in the concrete c' is represented by *d e*, which is $\frac{2}{3}$ of the compressive stress that would be required to produce that proportionate compression if the modulus of elasticity of the concrete were uniformly maintained at the value it has for very low pressures.

Theoretical Assumptions. The theory of reinforced concrete beams is based on the usual assumptions that,

(*a*) The loads are applied at right angles to the axis of the beam. The usual vertical gravity loads supported by a horizontal beam, fulfil this condition.

(*b*) There is no resistance to free horizontal motion. This condition is seldom if ever exactly fulfilled in practice. The more rigidly the beam is held at the ends, the greater will be its strength above that computed by the simple theory. Under ordinary conditions the added strength is quite indeterminate; and is not allowed for, except in the appreciation that it adds indefinitely to the safety.

(*c*) The concrete and steel stretch together without breaking the bond between them. This is absolutely essential.

(*d*) Any section of the beam which is plane before bending is plane after bending.

In Fig. 38, is shown, in a very exaggerated form, the essential meaning of assumption *d*. The section *a b c d* in the unstrained condition, is changed to the plane *a′ b′ c′ d′* when the load is applied. The compression at the top = *a a′* = *b b′*. The neutral axis is unchanged. The concrete at the bottom is stretched an amount = *c c′* = *d d′*, while the stretch in the steel equals *g g′*. The compression in the concrete between the neutral axis and the top is proportional to the distance from the neutral axis.

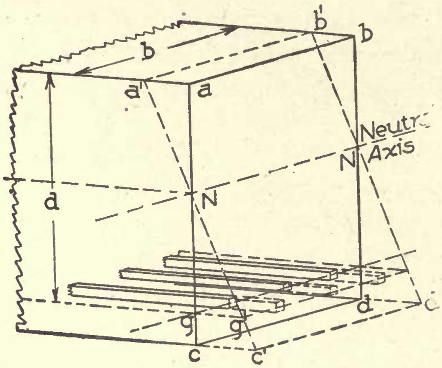


Fig. 38.

In Fig. 39*a*, is given a side view of the beam, with special reference to the deformation of the fibers. Since the fibers between the neutral axis and the compressive face are compressed proportionally, then, if *a a′* represents the lineal compression of the outer fiber, the shaded lines represent, at the same scale, the compression of the intermediate fibers.

In Fig. 39*b*, *m n* indicates the stress there would be in the outer fiber if the initial modulus of elasticity applied to all stresses. But since the force required to produce the compression *a a′* is *proportionately* so much less than that required for the lesser compressions, the actual pressure in pounds on the outer fiber may be represented by a line *v n*, and the pressure on the intermediate fibers by the ordinates to the curve *v N*.

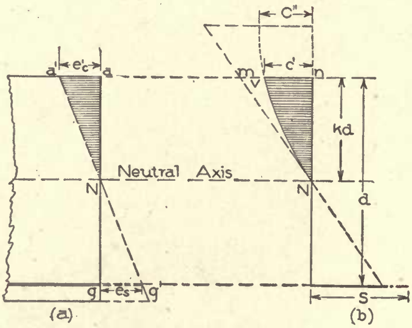


Fig. 39.

In Fig. 40, *a* and *b*, are shown a pair of figures corresponding with those of Fig. 39, except that the compressive deformation of the concrete in the outer fiber $a\ a'$ is only *one-half* of the value in Fig. 39. But it will require about three-fourths as much pressure to produce one-half as much compression. In Fig. 40, $v'\ n'$ is therefore three-fourths of $v\ n$ in Fig. 39. The student should note that k' here differs slightly from k, which means that the position of the neutral axis varies with the conditions.

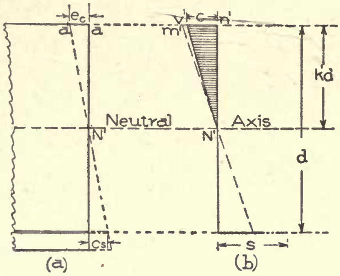


Fig. 40.

Summation of the Compressive Forces. The summation of the compressive forces is evidently indicated by the area of the shaded portion in Fig. 41. The curve $v\ N$ is a portion of a parabola. The area of the shaded portion between the curve $v\ N$ and the straight line $v\ N$, equals one-third of the area of the triangle $m N v$. The area of the triangle $v n N = \frac{1}{2}\ c\ kd$. Therefore, for the total shaded area, we have

$$\text{Area} = \tfrac{1}{2}\, c\, kd + \tfrac{1}{3}\,(c_o - c)\, \tfrac{1}{2}\, kd,$$
$$= \tfrac{1}{2}\, kd\,(c + \tfrac{1}{3}\, c_o - \tfrac{1}{3}\, c),$$
$$= \tfrac{1}{2}\, kd\,(\tfrac{2}{3}\, c + \tfrac{1}{3}\, c_o).$$

But in this case, $c_o = E_c\ \epsilon_c$; therefore

$$\text{Area} = \tfrac{1}{2}\, kd\,(\tfrac{2}{3}\, c + \tfrac{1}{3}\, E_c \epsilon_c \quad \text{.......} \quad (1)$$

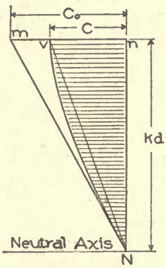


Fig. 41.

In Fig. 42 has been redrawn the parabola of Fig 37, in which o is the vertex of the parabola. Here c'' is the force which would produce a compression of ϵ_c'' provided the concrete could endure such a pressure without rupture. If the initial modulus of elasticity applied to all stresses, the required force would be the line $E_c\ \epsilon_c''$. And $c'' = \frac{1}{2}\ E_c\ \epsilon_c''$.

It is one of the well-known properties of the parabola that abscissas are proportional to the squares of the ordinates, or that (in this case),

$$k\,l : m\,n :: \overline{o\,k}^2 : \overline{o\,m}^2$$

Transforming to the symbols, we have

$$(c'' - c) : c'' :: (\epsilon_c'' - \epsilon_c)^2 : \epsilon_c''^2 ;$$

$$(c'' - c) = c'' \frac{(\epsilon_c'' - \epsilon_c)^2}{\epsilon_c''^2}$$

$$c'' - c = c''(1-q)^2, \text{ since } \frac{\epsilon_c}{\epsilon_c''} = q.$$

$$c = c''\left\{1-(1-q)^2\right\};$$

$$= c''(2q - q^2);$$

$$= \tfrac{1}{2} E_c\, \epsilon_c''(2q - 2q^2), \text{ since } c'' = \tfrac{1}{2} E_c\, \epsilon_c''; \text{ and also, since } \epsilon_c'' = \frac{\epsilon_c}{q}$$

$$= E_c\, \epsilon_c (1 - \tfrac{1}{2}q) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Substituting this value of c in Equation (1), we have:

$$\text{Area} = \tfrac{1}{2} k d \left\{\tfrac{2}{3} E_c\, \epsilon_c (1 - \tfrac{1}{2} q) + \tfrac{1}{3} E_c\, \epsilon_c\right\}$$

$$= \tfrac{1}{2} k d \left\{E_c\, \epsilon_c (1 - \tfrac{1}{3} q)\right\}.$$

The summation of the horizontal forces (ΣX) within the shaded area, is evidently expressed by the above "area" multiplied by the breadth of the beam "b." Therefore,

$$\Sigma X = \tfrac{1}{2}(1 - \tfrac{1}{3}q)\, E_c\, \epsilon_c\, b\, kd \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In order to avoid the complication resulting from the attempt to develop formulæ which are applicable to all kinds of assumptions, it will be at once assumed, as previously referred to, that the ultimate

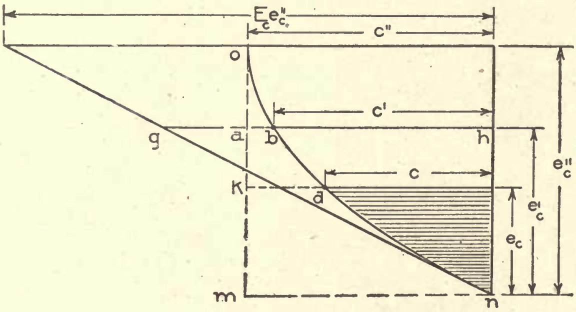


Fig. 42.

compressive strength of the concrete is $\frac{2}{3}$ of the value which would be required to produce that amount of compression in case the initial modulus of elasticity was the true value for all compressions. The practical result of this assumption is that we should always use *ultimate* values for the unit stresses in the steel and the concrete, and also that q will have the constant value of $\frac{2}{3}$.

The proof that q will equal $\frac{2}{3}$ under these conditions, is perhaps determined most easily by computing the ratio of $b\,h$ to $g\,h$ (see Fig. 42) when $o\,a$ is *assumed* to be $\frac{1}{3}$ of $o\,m$. In this case, from the properties of the parabola, $a\,b = \frac{1}{9}\,m\,n$; $c' = \frac{8}{9}\,m\,n = \frac{8}{9}\,c'' = \frac{4}{9}\,E_c\,\epsilon_c''$.

But when $o\,a = \frac{1}{3}$ of om, $g\,h = \frac{2}{3}\,E_c\epsilon_c = \frac{6}{9}\,E_c\epsilon_c''$.

Therefore $c' = \frac{2}{3}\,g\,h$. But when $o\,a = \frac{1}{3}$ of $o\,m$, $\frac{\epsilon_c'}{\epsilon_c''} = \frac{2}{3}$.

Therefore when $c' = \frac{2}{3}\,gh$, $q = \frac{2}{3}$.

It has already been shown that $c'' = \frac{1}{2}\,E_c\,\epsilon_c''$, and also that $\epsilon_c'' = \frac{\epsilon_c}{q}$. Therefore $\frac{1}{2}\,E_c\,\epsilon_c = c''q$. It has also been shown that $c' = \frac{8}{9}\,c''$, or that $c'' = \frac{9}{8}c'$. Therefore $\frac{1}{2}\,E_c\,\epsilon_c = \frac{9}{8}\,c'q$.

Substituting this value in Equation 3, we have for the summation of the compressive forces above the neutral axis under such conditions:

$$\Sigma X = \tfrac{9}{8}(1 - \tfrac{1}{3}\,q)\,q\,c'b\,kd \quad \ldots\ldots (4)$$

Substituting the further condition that $q = \frac{2}{3}$, we have

$$\Sigma X = \tfrac{7}{12}\,c'b\,kd \quad \ldots\ldots (5)$$

Center of Gravity of Compressive Forces. This is also called the *centroid of compression.* The theoretical determination of this center of gravity is virtually the same as the determination of the center of gravity of the shaded area shown in Figs. 40 and 41. The general method of determining this center of gravity requires the use of differential calculus, and is a very long and tedious calculation. But the final result may be reduced to a surprisingly simple form, as expressed in the following equation:

$$x = kd\,\frac{4 - q}{12 - 4q}.$$

Assuming, as explained above, the value of $q = \frac{2}{3}$, this reduces to

$$x = .357\,kd \quad \ldots\ldots (6)$$

When q equals zero, the value of x equals $.333\,kd$; and, at the other extreme when $q = 1$, $x = .375\,kd$.

There is, therefore, a very small range of inaccuracy in adopting the value of $q = \frac{2}{3}$ for *all* computations.

Position of the Neutral Axis. According to one of the fundamental laws of mechanics, the sum of the horizontal tensile forces must be equal and opposite to the sum of the compressive forces. Ignoring the very small amount of tension furnished by the concrete below the neutral axis, the tension in the steel $= As = pbds =$ the total compression in the concrete. Therefore,

$$p\,b\,d\,s = \tfrac{1}{2}\,(1 - \tfrac{1}{3}\,q)\,E_c\,\epsilon_c\,k\,b\,d.$$

But $s = E_s\,\epsilon_s$; therefore,

$$p\,E_s\,\epsilon_s = \tfrac{1}{2}\,(1 - \tfrac{1}{3}q)\,E_c\,\epsilon_c k.$$

But $\frac{E_s}{E_c} = r$, and by proportional triangles, as shown in Fig. 40.

$$\frac{\epsilon_c}{kd} = \frac{\epsilon_s}{d - kd};\ \text{or}\ \epsilon_c = \epsilon_s\,\frac{k}{1-k}.$$

Making these substitutions, we have:

$$p\,r = \tfrac{1}{2}\,(1 - \tfrac{1}{3}q)\,\frac{k^2}{1-k}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Solving this quadratic for k, we have:

$$k = \sqrt{\frac{2\,p\,r}{(1-\frac{1}{3}q)} + \frac{p^2r^2}{(1-\frac{1}{3}q)^2}} - \frac{p\,r}{(1-\frac{1}{3}q)}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Equation 8 is a perfectly general equation, which depends for its accuracy only on the assumption that the law of compressive stress to compressive strain is represented by a parabola. The equation shows that k, the ratio determining the position of the neutral axis, depends on three variables—namely, the percentage of the steel (p), the ratio of the moduli of elasticities (r), and the ratio of the deformations in the concrete (q). These must all be determined more or less accurately before we can know the position of the neutral axis.

On the other hand, if it were necessary to work out equation 8, as well as many others, for every computation in reinforced concrete, the calculations would be impracticably tedious. Fortunately the extreme range in k for any one ratio of moduli of elasticities, is only a few per cent, even when q varies from 0 to 1. We shall therefore simplify the calculations by using the constant value $q = \frac{2}{3}$, as explained above.

Substituting $q = \frac{2}{3}$ in Equation 8, we have

$$k = \sqrt{\frac{18}{7}\,p\,r + \frac{81}{49}\,p^2\,r^2} - \frac{9}{7}\,p\,r\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

The various values for the ratio of the moduli of elasticity (r) are discussed in the succeeding section. The values of k for various values of r and p, and for the uniform value of $q = \frac{2}{3}$, have been computed in the following tabular form. Four values have been chosen for r, in conjunction with nine values of p, varying by 0.2 per cent and covering the entire practicable range of p, on the basis of which values k has been worked out in the tabular form. Usually the value of k can be determined directly from the table. By interpolating between two values in the table, any required value within the limits of ordinary practice can be determined with all necessary accuracy.

TABLE VII

Values of k for Various Values of r and p

	p								
r	.020	.018	.016	.014	.012	.010	.008	.006	.004
10	.505	.487	.468	.446	.422	.395	.361	.323	.274
12	.536	.517	.497	.475	.450	.422	.388	.348	.295
20	.623	.604	.583	.561	.535	.505	.465	.422	.362
40	.736	.718	.700	.678	.654	.623	.584	.536	.467

Ratio of Moduli. Theoretically there is an indefinite number of values of r, the ratio of the moduli of elasticity of the steel and the concrete. The modulus for steel is fairly constant at about 29,000,000 or 30,000,000. The value of the *initial* modulus for concrete varies according to the quality of the concrete, from 1,500,000 to 3,000,000 for stone concrete. An average value for cinder concrete is about 750,000. Some experimental values for stone concrete have fallen somewhat lower than 1,500,000, while others have reached 4,000,000 and even more. We may probably use the following values with the constant value of 29,000,000 for the steel.

TABLE VIII

Modulus of Elasticity of Some Grades of Concrete

Kind of Concrete	Mixture	E_c	r
Cinder..................	1:2:5	750,000	40
Broken Stone...........	1:6:12	1,450,000	20
" "	1:3:6	2,400,000	12
" "	1:2:4	2,900,000	10

The value given above for 1:6:12 concrete is mentioned only because the value $r = 20$ is sometimes used with the weaker grades of concrete, and the value of approximately 1,450,000 for the elasticity of such concrete has been found by experimenters. The use of such a lean concrete is hardly to be recommended, because of its unreliability. Considering the variability in cinder concrete, the even value of $r = 40$ is justifiable rather than the precise value 38.67.

Percentage of Steel. The previous calculations have been made as if the percentage of the steel might be varied almost indefinitely. While there is considerable freedom of choice, there are limitations beyond which it is useless to pass; and there is always a most economical percentage, depending on the conditions. We have already determined that

$$\frac{\epsilon_c}{\epsilon_s} = \frac{k}{1-k}.$$

But $\epsilon_c = \frac{c}{E_c(1-\frac{1}{2}q)}$; and

$\epsilon_s = \frac{s}{E_s}$; therefore,

$$\frac{\epsilon_c}{\epsilon_s} = \frac{c\ E_s}{sE_c(1-\frac{1}{2}q)} = \frac{c\,r}{s(1-\frac{1}{2}q)} = \frac{1}{1-k}.$$

Solving for k, we have:

$$k = \frac{c\,r}{c\,r + s(1-\frac{1}{2}q)}.$$

Using as before the value of $q = \frac{2}{3}$, the equation becomes:

$$k = \frac{c\,r}{c\,r + .667\,s}.$$

Using the same value of q in equation **7**, and solving for p, we have:

$$p\,r = \frac{7\,k^2}{18(1-k)}.$$

Substituting the above value of k in this equation, we have, after considerable reduction:

$$p = \frac{7\,c\ c\,r}{12\,s\,(c\,r + .667s)} \qquad \textbf{(10)}$$

The above equation shows that we cannot select the percentage of steel at random, since it evidently depends on the selected stresses for the steel and concrete, and also on the ratio of their moduli. For example, consider a high-grade concrete (1:2:5) whose modulus of elasticity is considered to be 2,800,000, and which has a limiting compressive stress of 2,700 pounds (c'), which we may consider in conjunction with the limiting stress of 55,000 pounds in the steel. The values of c, s, and r are therefore 27,000, 55,000, and 10.37 respectively. Substituting these values in equation **10**, we compute $p = .012$.

Example. What percentage of steel would be required for ordinary stone concrete, with $r=12$, $c=500$, and $s=16{,}000$? Ans. 0.66 per cent.

Resisting Moment. The moment which resists the action of the external forces is evidently measured by the product of the distance from the center of gravity of the steel to the centroid of compression of the concrete, times the total compression of the concrete, or, otherwise, times the tension in the steel. The compression in the concrete and the tension in the steel are equal, and it is therefore only a matter of convenience to express this product in terms of the tension in the steel. Therefore, adopting the notation already mentioned, we may write the formula:

$$M = A\,s\,(d - x) \quad \text{(11)}$$

But since the computations are frequently made in terms of the dimensions of the concrete and of the percentage of the reinforcing steel, it may be more convenient to write the equation:

$$M = p\,b\,d\,s\,(d - x) \quad \text{(12)}$$

Example 1. What is the resisting moment of a concrete beam made of ordinary 1:3:6 concrete which is 7 inches wide, 10 inches deep to the reinforcement, and which uses 1 per cent of reinforcement?

Answer. We shall assume that the steel is to be used with a working stress of 16,000 pounds per square inch, which means that our resulting resisting moment may be considered as the resisting moment for a working load. If we were to use, say, 55,000 pounds per square inch for the stress in the steel, which should be considered an ultimate value, we should obtain a proportionately larger value for the resisting moment, but it would in that case represent the ultimate resisting moment. The concrete is supposed to have a ratio for the moduli of elasticity (r) equals 12. With 1% reinforcement (or $p = .01$), the value of k is .422. According to equation **6.**

$$x = .357\,kd = .357 \times .422\,d = .151\,d.$$

Therefore,

$$d - x = .849\,d.$$

Therefore,

$$M = .01 \times 7 \times 10 \times 16{,}000 \times .849 \times 10 = 95{,}088 \text{ inch-pounds.}$$

Example 2. What will be the ultimate resisting moment of a 5-inch slab made of a high quality of concrete (1:2:4) using the most economical percentage of steel?

Answer. For this quality of concrete, $r = 10$; the ultimate compressive strength of the concrete is 2,700; and the ultimate tension in the steel is assumed at 55,000. Substituting these values in Equation **10**, we find that the economical percentage of steel is 1.21. Interpolating this value of p in Table VII, considering that $r = 10$, we have $k = .424$. Substituting this value of k in Equation **6** we find that $x = .151\, d$. In the case of the 5-inch slab, we shall assume that the center of gravity of the steel is placed 1 inch from the bottom of the slab. Therefore $d = 4$ inches. For a slab of indefinite width, we shall assume that $b = 12$ inches. Therefore our computed value for the ultimate resisting moment, gives the moment of a strip of the slab one foot wide, and the computed amount of the steel is the amount of steel per foot of width of the slab.

Substituting these various values in Equation **12**, we find as the value of the ultimate resisting moment:

$$M_o = .0121 \times 12 \times 4 \times 55{,}000 \times .849 \times 4 = 108{,}482 \text{ inch-pounds.}$$

The area of steel required for each foot of width is:

$$A = .0121 \times 12 \times 4 = .5808 \text{ square inch.}$$

This equals .0484 square inch per inch of width. Since a $\frac{1}{2}$-inch square bar has an area of .25 square inch, we may provide the reinforcement by using $\frac{1}{2}$-inch square bars spaced $\frac{.25}{.0484} = 5.17$ inches, or, say, $5\frac{1}{4}$ inches.

Example 3. A very instructive comparison may be made by considering a 5-inch slab with $d = 4$ inches, but made of 1:3:6 concrete. In this case we call $r = 12$; $c = 2{,}000$; and s (as before) $=$ 55,000. By the same method as before, we obtain $p = .0084$; $k = .395$; and therefore $x = .141\, d$. Substituting these values in Equation **12**, we have:

$$M_o = .0084 \times 12 \times 4 \times 55{,}000 \times .859 \times 4 = 76{,}197 \text{ inch-pounds.}$$

The area of steel per foot of width is:

$$A = .0084 \times 12 \times 4 = .4032 \text{ square inch.}$$

This would require $\frac{1}{2}$-inch square bars spaced 7.33 inches. Although the amount of steel required in this slab is considerably less than was required in the previous case, the ultimate moment of the slab is also very much less. In fact the reduction of strength is very nearly in proportion to the reduction in the amount of steel. Therefore, it must be observed that, although the percentage of steel used with high-grade concrete is considerably higher, the thickness of the concrete will be considerably less; and in spite of the fact that the *percentage* of steel may be higher, its absolute amount for a slab of equal strength may be approximately the same.

Example 4. Another instructive principle may be learned by determining the required thickness of a slab made of 1:3:6 concrete, which shall have the same ultimate strength as the high-grade concrete mentioned in example 2. In other words, its ultimate moment per foot of width must equal 108,482 inch-pounds. The values of r, c, and s are the same as in example 3, and therefore the value of p must be the same as in example 3; therefore $p = .0084$. Since r and p are the same as in example 3, k again equals .395, and therefore $x = .141\ d$. We therefore have from Equation **12:**

$$M_o = 108{,}482 = .0084 \times 12 \times d \times 55{,}000 \times .859 \times d.$$

Solving this equation for d, we find $d^2 = 22.78$; and $d = 4.77$. The area of the steel $A = p\,b\,d = .0084 \times 12 \times 4.77 = .481$. This is considerably less than the area of steel per foot of width as computed in example 2, for a slab of equal strength. On the other hand, the slab of 1:3:6 concrete will require about 15 per cent more concrete. It will also weigh about 10 pounds per square foot more than the thinner slab, which will reduce by that amount the permissible live load. The determination of the relative economy of the two kinds of concrete will therefore depend somewhat on the relative price of the concrete and the steel. The difference in the total cost of the two methods is usually not large; and abnormal variation in the price of cement or steel may be sufficient to turn the scale one way or the other.

Determination of Values for Frequent Use. The above methods of calculation may be somewhat simplified by the determination, once for all, of constants which are in frequent use. For example, a very large amount of work is being done using 1:3:6 concrete. Sometimes Engineers will use the formulæ developed on the basis of 1:3:6 concrete, even when it is known that a richer mixture will be used. Although such a practice is not economical, the error is on the side of safety; and it makes some allowance for the fact that a mixture which is nominally richer *may* not have any greater strength than the values used for the 1:3:6 mixture, on account of defective workmanship or inferior cement or sand. Some of the constants for use with 1:3:6 mixture and 1:2:4 mixture will now be worked out.

For the 1:3:6 mixture, $r = 12$; $c = 2{,}000$; and we shall assume $s = 55{,}000$. On the basis of such values, the economical *percentage* of steel is .84 per cent. Under these conditions, k will always be .395; and x will equal .141 d. Therefore the term $(d - x)$ will always equal .859 d, or say, .86 d, which is close enough for a working value. Since the above values for c and s represent the ultimate values, the resulting moment is the ultimate moment, which we will call M. Therefore, for 1:3:6 concrete, we have the constant values:

$$\left.\begin{aligned} M_o &= .0084 \times b\,d \times 55{,}000 \times .86d \\ &= 397\,b\,d^2 \\ A &= .0084\,b\,d \\ (d\text{-}x) &= .86d \end{aligned}\right\} \quad \ldots\ldots\ldots (13)$$

Similarly we can compute a corresponding value for 1:2:4 concrete, using the values previously allowed for this grade:

$$\left.\begin{aligned} M_o &= .565\,b\,d^2 \\ A &= .0121\,b\,d \\ (d\text{-}x) &= .86d \end{aligned}\right\} \quad \ldots\ldots\ldots (14)$$

Numerical Examples. 1. A flooring with a live load capacity of 150 pounds per square foot, is to be constructed on I-beams spaced 6 feet from center to center, using 1:3:6 concrete. What thickness of slab will be required, and how much steel must be used?

Answer. Using the approximate estimate, based on experience, that such a slab will weigh *about* 50 pounds per square foot, we can compute the ultimate load by multiplying the live load, 150, by four, and the dead load, 50, by two, and obtain a total ultimate load of 700 pounds per square foot. A strip 1 foot wide and 6 feet long (between the beams) will therefore carry a total load of $700 \times 6 = 4{,}200$ pounds. Considering this as a simple beam, we have:

$$M_o = \frac{W_o\,l}{8} = \frac{4{,}200 \times 6 \times 12}{8} = 37{,}800 \text{ inch-pounds.}$$

Placing this numerical value of $M_o = 397\,b\,d^2$, as in Equation **13**, we have $37{,}800 = 397\,b\,d^2$. In this case, $b = 12$ inches. Substituting this value of b, we solve for d^2, and obtain $d^2 = 7.93$, and $d = 2.82$ inches. Allowing an extra inch below the steel, this will allow us to use a 4-inch slab. Theoretically we could make it a little less. Practically this figure should be chosen. The required steel, from

Equation 13, equals .0084 bd. Taking $b = 1$, we have the required steel per *inch* of width of the slab = .0084 × 2.82 = .0237 square inch. If we use ½-inch square bars which have a cross-sectional area of .25 square inch, we may space the bars $\frac{.25}{.0237} = 10$ inches. This reinforcement could also be accomplished by using ⅜-inch square bars, which have an area of .1406. The spacing may therefore be $\frac{.1406}{.0237} = 6.0$ inches. As referred to later, there should also be a few bars laid perpendicular to the main reinforcing bars, or parallel with the I-beams, so as to prevent shrinkage. The required amount of this steel is not readily calculable. Since the I-beams are 6 feet apart, if we place two lines of ⅜-inch square bars spaced 2 feet apart, parallel with the I-beams, there will then be reinforcing steel in a direction parallel with the I-beams at distances apart not greater than 2 feet, since the I-beams themselves will prevent shrinkage immediately around them.

Table for Slab Computation. The necessity of very frequently computing the required thicknesses of slabs, renders very useful a table such as is shown in Table IX, which has been worked out on the basis of 1:3:6 concrete, and computed by solving Equation 13 for various thicknesses d, and for various spans L varying by single feet. It should be noted that the loads as given are *ultimate* loads *per square foot*, and that they therefore include the weight of the slab itself, which must be multiplied by its factor of safety, which is usually considered as 2.

For example, in the above numerical case, we computed that there would be a total load of 700 pounds on a span of 6 feet. In the column headed 6, we find 794 on the same line as the value of 3.0 in the column d. This shows that 3.0 is somewhat excessive for the value of d. We computed its precise value to be 2.82. On the same line, we find under "Spacing of Bars," that ⅜-inch square bars spaced 5½ inches will be sufficient. In the above more precise calculation, we found that the bars could be spaced 6 inches apart, as was to be expected, since the computed ultimate load is considerably less than the nearest value found in the table.

Example 1. What is the ultimate load that will be carried by a 5-inch slab on a span of 10 feet using 1:3:6 concrete?

TABLE IX

Ultimate Load on Slabs of "Average" Concrete (1:3:6) in Pounds per Square Foot
Weight of Slab Included

Effective Thickness of Slab d	Area of Steel in 12-In. Width	Spacing of Bars		Span in Feet (L)											
		$\frac{3}{8}$-In. Sq.	$\frac{1}{2}$-In. Sq.	4	5	6	7	8	9	10	11	12	13	14	15
2.5	.252	$6\frac{3}{4}$ in.	12 in.	1,241	794	551	405	310	245	198	.....	.....	.....	.....	.....
3.0	.302	$5\frac{1}{2}$ "	10 "	1,786	1,143	794	583	446	353	286	236	198	.....	.....	.....
3.5	.353	$4\frac{3}{4}$ "	$8\frac{1}{2}$ "	2,432	1,556	1,080	793	608	480	389	322	270	230	198	.....
4.0	.403	$4\frac{1}{4}$ "	$7\frac{1}{2}$ "	3,176	2,033	1,411	1,037	794	627	508	420	353	300	259	226
4.5	.454	$3\frac{3}{4}$ "	$6\frac{3}{4}$ "	4,020	2,573	1,786	1,312	1,005	794	643	531	446	380	328	286
5.0	.504	$3\frac{3}{8}$ "	6 "	4,962	3,176	2,206	1,620	1,241	980	794	656	551	470	405	353
5.5	.554	3 "	$5\frac{1}{2}$ "	6,005	3,843	2,669	1,960	1,501	1,186	960	794	667	569	490	427
6.0	.605	.....	5 "	.....	4,573	3,176	2,334	1,787	1,412	1,142	945	793	677	583	508
7.0	.706	.....	$4\frac{1}{4}$ "	.....	.....	4,323	3,176	2,432	1,921	1,556	1,286	1,080	921	794	692
8.0	.806	.....	$3\frac{3}{4}$ "	.....	.....	.....	4,148	3,176	2,509	2,033	1,680	1,410	1,203	1,037	904

Answer. The 5 inches here represent the total thickness, and we shall assume that the effective thickness (d) is 1-inch less. Therefore $d = 4$ inches. On the line opposite $d = 4$ in Table IX, and under the column L = 10, we have 508, which gives the ultimate load per square foot. A 5-inch slab will weigh approximately 60 pounds per square foot, allowing 12 pounds per square foot per inch of thickness. Using a factor of 2, we have 120 pounds, which, subtracting from 508, leaves 388 pounds; dividing this by 4, we have 97 pounds per square foot as the allowable working load. Such a load is heavier than that required for residences or apartment houses. It would do for an office building.

Example 2. The floor of a factory is to be loaded with a live load of 300 pounds per square foot, the slab to be supported on beams spaced 8 feet apart. What must be the thickness of the floor slab?

Answer. With 1,200 pounds per square foot ultimate load for the live load alone, we notice in Table IX, under $L = 8$, that 1,241 is opposite to $d = 5$. This shows that it would require a slab nearly 6 inches thick to support the live load alone. We shall therefore add another half-inch as an estimated allowance for the weight of the slab and, assuming that a $6\frac{1}{2}$-inch slab having a weight of 78 pounds per square foot will do the work, we multiply 300 by 4, and 78 by 2, and have 1,356 pounds per square foot as the ultimate load to be carried. Under $L = 8$, in Table IX, we find that 1,356 comes between 1,241 and 1,501, showing that a slab with an effective thickness d of about $5\frac{1}{4}$ inches will have this ultimate carrying capacity. The total thickness of the slab should therefore be about $6\frac{1}{4}$ inches. The table also shows that $\frac{1}{2}$-inch bars spaced about $5\frac{3}{4}$ inches apart will serve for the reinforcement. We might also provide the reinforcement by $\frac{3}{8}$-inch square bars spaced a little over 3 inches apart; but it would probably be better policy to use the half-inch bars, especially since the $\frac{3}{8}$-inch bars will cost somewhat more per pound.

Practical Methods of Spacing Slab Bars. It is too much to expect of workmen that bars will be accurately spaced when their distance apart is expressed in fractions of an inch. But it is a comparatively simple matter to require the workmen to space the bars evenly, provided it is accurately computed how many bars should be laid in a given width of slab. For example, in the above case, a panel of the flooring which is, say, 20 feet wide, should have a definite number of bars, 20 feet = 240 inches, and 240 ÷ 5.75 = 41.7. We will call this 42, and instruct the workmen to distribute 42 bars equally in the panel 20 feet wide. The workmen can do this without even using a foot-rule, and can adjust them to an even spacing with sufficient accuracy for the purpose.

Table for Computation of Simple Beams. In Table X has been computed for convenience, the ultimate total load on rectangular beams made of average concrete (1:3:6) and with a width of 1 inch. For other widths, multiply by the width of the beam. Since $M_0 = \frac{1}{8} W_0 l$; and since by Equation 13, for this grade of concrete, $M_0 = 397\, b d^2$; and since for a computation of beams 1 inch wide, $b = 1$, we may write $\frac{1}{8} W_0 l = 397\, d^2$. For l we shall substitute $12\, L$. Making this substitution and solving for W_0, we have $W_0 = 265\, d^2 \div L$. Since $b = 1$, A, the area of steel per inch of width of the beam $= .0084\, d$.

Example. What is the ultimate total load on a simple beam having a depth of 16 inches to the reinforcement, 12 inches wide and having a span of 20 feet?

Answer. Looking in Table X, under $L = 20$, and opposite $d = 16$, we find that a beam 1 inch wide will sustain a total load of 3,392 pounds. For a width of 12 inches, the total ultimate load will be 12 × 3,392 = 40,704 pounds. At 144 pounds per cubic foot, the beam will weigh 3,840 pounds. Using a factor of 2 on this, we shall have 7,680 pounds, which, subtracted from 40,704, gives 33,024. Dividing this by 4, we have 8,256 lbs. as the allowable live load on such a beam.

Resistance to the Slipping of the Steel in the Concrete. The previous discussion has considered merely the tension and compression in the upper and lower sides of the beam. A plain, simple beam resting freely on two end supports, has neither tension nor compression in the fibers at the ends of the beam. The horizontal tension and compression, found at or near the center of the beam, entirely disappear by the time the end of the beam is reached. This is done by transferring the tensile stress in the steel at the bottom of the beam, to the compression, fibers in the top of the beam, by means of the intermediate concrete. This is, in fact, the main use of the concrete in the lower part of the beam.

TABLE X

Ultimate Total Load on Rectangular Beams of Average Concrete (1:3:6), One Inch Wide

For other widths, multiply by width of beam. Formulæ: $W_o = 265\, d^2 \div L$; $A = .0084\, d$. Ultimate compression in concrete 2,000 pounds per sq. in.; ultimate tension in steel 55,000 pounds per sq. in.

Effective Depth of Beam, d	Area of Steel per Inch of Width	Span in Feet (L)																	Twice Dead Load per Foot of Beam
		4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
4	.0336	1,060	848	707	606	530	471	424	385	353	326	303	283	265	249	236	223	212	10
5	.0420	1,656	1,324	1,104	946	828	736	662	602	552	510	473	441	414	390	368	349	331	12
6	.0504	2,385	1,908	1,590	1,363	1,192	1,060	954	867	795	734	681	636	596	561	530	502	477	15
7	.0588	3,246	2,596	2,164	1,855	1,623	1,443	1,298	1,180	1,082	999	927	865	812	764	721	683	649	17
8	.0672	4,240	3,392	2,827	2,423	2,120	1,884	1,696	1,542	1,413	1,305	1,211	1,131	1,060	998	942	893	848	20
9	.0756	5,366	4,292	3,577	3,066	2,683	2,385	2,146	1,951	1,789	1,651	1,533	1,431	1,341	1,263	1,192	1,130	1,073	22
10	.0840	6,625	5,300	4,417	3,786	3,312	2,944	2,650	2,409	2,208	2,038	1,893	1,767	1,656	1,559	1,472	1,395	1,325	24
11	.0924	8,016	6,412	5,344	4,581	4,008	3,563	3,206	2,915	2,672	2,466	2,290	2,137	2,004	1,886	1,781	1,688	1,603	26
12	.1008	9,540	7,632	6,360	5,451	4,770	4,240	3,816	3,469	3,180	2,935	2,726	2,544	2,385	2,245	2,120	2,008	1,908	28
13	.1092	11,196	8,957	7,464	6,398	5,598	4,976	4,478	4,071	3,732	3,445	3,199	2,986	2,799	2,634	2,488	2,357	2,239	30
14	.1176	12,985	10,388	8,657	7,420	6,492	5,771	5,194	4,722	4,328	3,995	3,710	3,463	3,246	3,055	2,886	2,734	2,597	32
15	.1260	14,906	11,924	9,937	8,518	7,453	6,625	5,962	5,420	4,969	4,586	4,259	3,975	3,726	3,508	3,312	3,138	2,981	34
16	.1344	16,960	13,568	11,307	9,691	8,480	7,538	6,784	6,167	5,653	5,218	4,845	4,523	4,240	3,991	3,769	3,571	3,392	36
17	.1428	19,146	15,317	12,764	10,941	9,573	8,509	7,658	6,962	6,382	5,891	5,470	5,106	4,786	4,505	4,255	4,031	3,929	38
18	.1512	21,465	17,172	14,310	12,266	10,732	9,540	8,586	7,805	7,155	6,605	6,133	5,724	5,366	5,051	4,770	4,519	4,293	40
19	.1596	23,916	19,133	15,944	13,666	11,958	10,629	9,566	8,697	7,972	7,359	6,833	6,378	5,979	5,627	5,315	5,035	4,783	42
20	.1680	26,500	21,200	17,667	15,143	13,250	11,778	10,600	9,636	8,833	8,154	7,571	7,067	6,625	6,235	5,889	5,579	5,300	44

For values in the lower left-hand corner of the table, possible failure by diagonal shear must be very carefully tested and provided for.

It is therefore necessary that the bond between the concrete and the steel shall be sufficiently great to withstand the tendency to slip. The required strength of this bond is evidently equal to the difference in the tension in the steel per unit of length. For example, suppose that we are considering a bar 1-inch square in the middle of the length of a beam. Suppose that the bar is under an actual tension of 15,000 pounds per square inch. Since the bar is 1-inch square, the actual total tension is 15,000 pounds. Suppose that, at a point 1 inch beyond, the moment in the beam is so reduced that the tension in the bar is 14,900 pounds instead of 15,000 pounds. This means that the difference of pull (100 pounds) has been taken up by the concrete. The surface of the bar for that length of one inch, is four square inches. This will require an adhesion of 25 pounds per square inch between the steel and the concrete, in order to take up this difference of tension The adhesion between concrete and plain bars is usually considerably greater than this and there is therefore but little question about the bond in the center of the beam. But near the ends of the beam, the change in tension in the bar is far more rapid, and it then becomes questionable whether the bond is sufficient.

Although there is no intention to argue the merits of any form of patented bar, this discussion would not be complete without a statement of the arguments in favor of *deformed* bars, or bars with *a mechanical bond,* instead of plain bars. The deformed bars have a variety of shapes; and since they are not prismatic, it is evident that, apart from adhesion, they cannot be drawn through the concrete without splitting or crushing the concrete immediately around the bars. The choice of form is chiefly a matter of designing a form which will furnish the greatest resistance, and which at the same time is not unduly expensive to manufacture. Of course, the deformed bars are necessarily somewhat more expensive than the plain bars. The main line of argument of those engineers who defend the use of plain bars, may be summed up in the assertion that the plain bars are "good enough," and that, since they are less expensive than deformed bars, the added expense is useless. The arguments in favor of a mechanical bond, and against the use of plain bars, are based on three assertions:

First: It is claimed that tests have apparently verified the assertion that the mere soaking of the concrete in water for several months is sufficient to reduce the adhesion from $\frac{1}{2}$ to $\frac{2}{3}$. If this contention is true, the adhesion of bars in concrete which is likely to be perpetually soaked in water, is unreliable.

Second: Microscopical examination of the surface of steel, and of concrete which has been moulded around the steel, shows that the adhesion depends chiefly on the roughness of the steel, and that the cement actually enters into the microscopical indentations in the surface of the metal. Since a stress in the metal even within the elastic limit necessarily reduces its cross-section somewhat, the so-called adhesion will be more and more reduced as the stress in the metal becomes greater. This view of the case has been verified by recent experiments by Professor Talbot, who used bars made of tool steel in many of his tests. These bars were exceptionally smooth; and concrete beams reinforced with these bars failed generally on account of the slipping of the bars. Special tests to determine the bond resistance, showed that it was far lower than the bond resistance of ordinary plain bars.

Third: There is evidence to show that long-continued vibration, such as is experienced in many kinds of factory buildings, etc., will destroy the adhesion during a period of years. Some failures of buildings and structures which were erected several years ago, and which were long considered perfectly satisfactory, can hardly be explained on any other hypothesis. Owing to the fact that there are comparatively few reinforced concrete structures which have been built for a very long period of years, positive information as to the durability and permanency of adhesion is lacking. It must be conceded, however, that comparative tests of the bond between concrete and steel when the bars are plain and when they are deformed (the tests being made within a few weeks or months after the concrete is made), have comparatively little value as an indication of what that bond will be under some of the adverse circumstances mentioned above, which are perpetually occurring in practice. Non-partisan tests have shown that, even under conditions which are most favorable to the plain bars, the deformed bars have an actual hold in the concrete which is from 50 to 100 per cent greater than that of plain bars. It is unquestionable that age will increase rather than diminish the relative inferiority of plain bars.

Computation of the Bond Required in Bars. From Equation 11 we have the formula that the resisting moment at any point in the beam equals the area of the steel, times the unit tensile stress in the steel, times the distance from the steel to the centroid of compression of the steel, which is the distance $d - x$. We may compute the moment in the beam at two points at a unit distance apart. The area of the steel is the same in each equation, and $d - x$ is substantially the same in each case; and therefore the *difference* of moment, divided by $(d - x)$, will evidently equal the *difference* in the unit stress in the steel, times the area of the steel. To express this in an equation, we may say, denoting the difference in the moment by dM, and the difference in the unit stress in the steel by ds:

$$\frac{dM}{(d - x)} = A \times ds.$$

But $A \times ds$ is evidently equal to the actual difference in tension in the steel, measured in pounds. It is the amount of tension which must be transferred to the concrete in that unit length of the beam. But the computations of the difference of moments at two sections that are only a unit distance apart, is a comparatively tedious operation, which, fortunately, is unnecessary. Theoretical mechanics teaches us that the difference in the moment at two consecutive sections of the beam is measured by the *total vertical shear* in the beam at that point. The shear is very easily and readily computable; and therefore the required amount of tension to be transferred from the steel to the concrete can readily be computed. A numerical illustration may be given as follows: Suppose that we have a beam which, with its load, weighs 20,000 pounds, on a span of 20 feet. Using ultimate values, for which we multiply the loading by 4, we have an ultimate loading of 80,000 pounds. Therefore,

$$M_o = \frac{W_o\, l}{8} = \frac{80{,}000 \times 240}{8} = 2{,}400{,}000.$$

Using the constants previously chosen for 1:3:6 concrete, and therefore utilizing Equation 13, we have this moment equal to 397 bd^2. Therefore $bd^2 = 6{,}045$.

If we assume $b = 15$ inches; $d = 20.1$ inches; then $d - x = .86d = 17.3$ inches. The area of steel equals

$$A = .0084\, b\, d = 2.53 \text{ square inches.}$$

We know from the laws of mechanics, that the moment diagram for a beam which is uniformly loaded is a parabola, and that the ordinate to this curve at a point one inch from the abutment will, in the above case, equal $(\frac{119}{120})^2$ of the ordinate at the abutment. This ordinate is measured by the maximum moment at the center, multiplied by the factor $(\frac{119}{120})^2 = \frac{14,161}{14,400} = .9834$; therefore the actual moment at a point one inch from the abutment = (1.00 − .9834) = .0166 of the moment at the center. But .0166 × 2,400,000 = 39,840.

But our ultimate loading being 80,000 pounds, we know that the shear at a point in the middle of this one-inch length equals the shear at the abutment, minus the load on this first $\frac{1}{2}$ inch, which is $\frac{1}{240}$ of 40,000 (or 167) pounds. The shear at this point is therefore 40,000 − 167 (or 39,833) pounds. This agrees with the above value 39,840 as closely as the decimals used in our calculations will permit.

The value of $d - x$ is somewhat larger when the moment is very small than when it is at its ultimate value. But the difference is comparatively small, is on the safe side, and it need not make any material difference in our calculations. Therefore, dividing 39,840 by 17.3, we have 2,303 pounds as the difference in tension in the steel in the last inch at the abutment. Of course this does not literally mean the last inch in the length of the beam, since, if the net span were 20 feet, the actual length of the beam would be considerably greater. The area of the steel as computed above is 2.53 square inches. Assuming that this is furnished by five $\frac{3}{4}$-inch square bars, the surfaces of these five bars per inch of length equals 15 square inches. Dividing 2,303 by 15, we have 153 pounds per square inch as the required adhesion between the steel and the concrete. While this is not greater than the adhesion usually found between concrete and steel, it is somewhat risky to depend on this; and therefore the bars are usually bent so that they run diagonally upward, and thus furnish a very great increase in the strength of the beam, which prevents the beam from failing at the ends. Tests have shown that beams which are reinforced by bars only running through the lower part of the beam without being turned up, or without using any stirrups, will usually fail at the ends, long before the transverse moment, which they possess at their center, has been fully developed.

Distribution of Vertical Shears. Beams which are tested to destruction frequently fail at the ends of the beams, long before the transverse strength at the center has been fully developed. Even if the bond between the steel and the concrete is amply strong for the requirements, the beam may fail on account of the shearing or diagonal stresses in the concrete between the steel and the neutral axis. The student must accept without proof some of the following statements regarding the distribution of the shear.

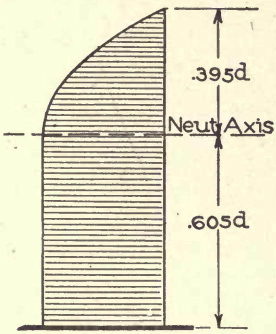


Fig. 43.

The intensity of the shear at various points in the height of the beam, may be represented by the diagram in Fig. 43. If we ignore the tension in the concrete due to transverse bending, the shear will be uniform between the steel and the neutral axis. Above the neutral axis, the shear will diminish toward the top of the beam, the curve being parabolic.

If the distribution of the shear were uniform throughout the section, we might say that the shear per square inch would equal $V \div bd$. It may be proved that v, the intensity of the vertical shear per square inch, is

$$v = \frac{V}{b\,(d - x)} \quad \text{..........} \quad \textbf{(15)}$$

In the above case, the ultimate total shear V in the last inch at the end of the beam, is 39,840 pounds. Then,

$$v = \frac{39{,}840}{15 \times 17.3} = 153.5 \text{ pounds per square inch.}$$

The agreement of this numerical value of the unit intensity of the vertical shear with the required bond between the concrete and the steel, is due to the accidental agreement of the width of the beam (15 inches) with the superficial area of the bars per inch of length of the beam (15 square inches). If other bars of the same *cross-sectional* area, but with greater or less superficial surface, had been selected for the reinforcement, even this accidental agreement would not have been found.

The actual strength of concrete in shear is usually far greater than this. The failure of beams, which fail at the ends when loaded with loads far within their capacity for transverse strength, is generally due to the *secondary stresses.* The computation of these stresses is a complicated problem in Mechanics; but it may be proved that if we ignore the tension in the concrete due to bending stresses, the diagonal tension per unit of area equals the vertical shear per unit of area (v). But concrete which may stand a shearing stress of 1,000 pounds per square inch will probably fail under a direct tension of 200 pounds per square inch. The diagonal stress has the nature of a direct tension. In the above case the beam probably would not fail by this method of failure, since concrete can usually stand a tension up to 200 pounds per square inch; but such beams, when they are not diagonally reinforced, frequently fail in that way before their ultimate loads are reached.

Methods of Guarding against Failure by Shear or Diagonal Tension. The failure of a beam by actual shear is almost unknown. The failures usually ascribed to shear are generally caused by diagonal tension. A solution of the very simple equation (**15**) will indicate the intensity of the vertical shear.

The relation of crushing strength to shearing strength is expressed by the equation:

$$\text{Unit shearing strength } z = \frac{c'}{2 \tan \theta'}$$

in which z is the unit shearing strength, and θ is the angle of rupture under direct compression. This angle is usually considered to be 60°; for such a value the shearing strength would equal $c' \div 3.464$. When $\theta = 45°$, the shearing strength would equal *one-half* of the crushing strength, and this agrees very closely with the results of tests made by Professor Spofford. But the shearing strength is considered to be a far less reliable quantity than the crushing strength; and therefore dependence is not placed on shear, even for *ultimate* loading, to a greater value than about one-half of the above value; or,

$$\text{Unit shearing strength } z = c' \div 6.928.$$

Usually the unit intensity of the vertical shear (even for ultimate loads) is less than this. But this ignores the assistance furnished by the bars. Actual failure would require that the bars must crush the concrete under them. When, as is usual, there are bars passing obliquely through the section, a considerable portion of the shear is carried by direct tension in the bars.

It seems impracticable to develop a rational formula for the amount of assistance furnished by these diagonal bars, unless we make assumptions which are doubtful and which therefore vitiate the reliability of the whole calculation. Therefore the "rules" which have been suggested for a prevention of this form of failure are wholly empirical. Mr. E. L. Ransome uses a rule for spacing vertical "stirrups," made of wires or ¼-inch rods, as follows:

The first stirrup is placed at a distance from the end of the beam equal to one-fourth the depth of the beam; the second is at a distance of one-half the depth beyond the first stirrup; the third, three-fourths of the depth beyond the second; and the fourth, a distance equal to the depth of the beam beyond the third. This empirical rule agrees with the theory, in the respect that the stirrups are closer at the ends

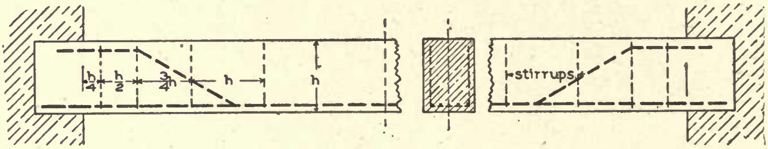


Fig. 44.

of the beam, where the shear is greatest. The four stirrups extend for a distance from the end equal to 2½ times the depth of the beam. Usually this is a sufficient distance; but some "systems" use stirrups throughout the length of the beam. On very short beams, the shear changes so rapidly that at 2½ times the depth from the end of the beam the shear is not generally so great as to produce dangerous stresses. With a very long beam, the change in the shear is correspondingly more gradual; and it is possible that stirrups or some other device must be used for a greater actual distance from the end, although for a less proportional distance.

When the diagonal reinforcement is accomplished by bending up the bars at an angle of about 45°, the bending should be done so that there is at all sections a sufficient area of steel in the lower part of the bar to withstand the transverse moment at that section. As fast as the bars can be spared from the bottom of the beam, they may be turned up diagonally so that there are at every section of the beam one or more bars which would be cut diagonally by such a section. On this account it is far better to use a larger number of bars, than a smaller number of the same area. For example, if it were required that there shall be 2.25 square inches of steel for the section at the middle of the beam, it would be far better to use nine ½-inch bars than four ¾-inch bars. In either case, the steel has the same area and the same weight. The nine ½-inch bars give a much better distribution of the metal. The superficial area of the nine ½-inch bars is 18 square inches per linear inch of the beam, while the area of the four ¾-inch bars is only 12 square inches per inch of length. But an even greater advantage is furnished by the fact that we have nine bars instead of four, which may be bent upward (and bent more easily than the ¾-inch bars) as fast as they can be spared from the bottom of the beam. In this way the shear near the end of the beam may be much more effectually and easily provided for.

Since the shear is greatest at the ends of the beam, more bars should be reserved for turning up near the ends. For example, in the

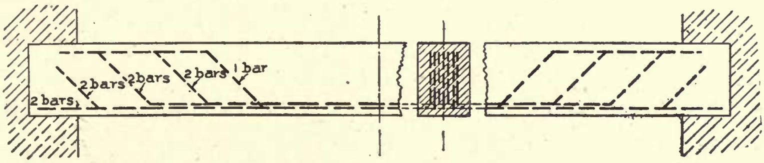


Fig 45.

above case of the nine bars, one or two bars might be turned up at about the quarter-points of the beam. One or two more might be turned up at a distance equal to, or a little less than, the depth of the beam from the quarter-points toward the abutments. Others would be turned up at intermediate points; at the abutments there should be at least two, or perhaps three, diagonal bars, to take up the maximum shear near the abutments. This is illustrated, although without definite calculations, in Fig. 45.

Detailed Design of a Plain Beam. This will be illustrated by a numerical example. A beam having a span of 18 feet supports one side of a 6-inch slab 8 feet wide which carries a live load of 200 pounds per square foot. In addition, a special piece of machinery, weighing 2,400 pounds, is located on the slab so near the middle of the beam that we shall consider it to be a concentrated load at the center of the beam. The floor area carried by the beam is 18 feet by 4 feet = 72 square feet. Adding 3 inches to the 6 inches thickness of the slab as an allowance for the weight of the beam, we have 9 × 12 = 108 pounds per square foot for the dead weight of the floor. With a factor of 2 for dead load, this equals 216. Using a factor of 4 on the live load (200), we have 800 pounds per square foot. Then the ultimate load on the beam, due to these sources, is (216 + 800) 72 = 73,152 pounds. So far as its effect on moment is concerned, the concentrated load of 2,400 pounds at the center would have the same effect as 4,800 pounds uniformly distributed. As it is a piece of vibrating machinery, we shall use a factor of *six* (6), and thus have an ultimate effect of 6 × 4,800 = 28,800 pounds. Adding this to 73,152, we have 101,952 pounds as the equivalent, ultimate, uniformly distributed load. Then

$$M_o = \tfrac{1}{8} W_o\, l = \tfrac{1}{8} \times 101{,}952 \times 216 = 2{,}752{,}704.$$

In order to reduce as much as possible the size and weight of this beam, we shall use 1:2:4 concrete, and therefore apply Equation **14:**

$$2{,}752{,}704 = 565\, b\, d^2;$$
$$bd^2 = 4{,}872.$$

If $b =$ 16 inches; $d^2 = 304.5$, and $d =$ 17.5 inches.

A still better combination would be a deeper and narrower beam with $b =$ 12 inches, and $d =$ 20.15 inches. With this combination, the required area of the steel will equal

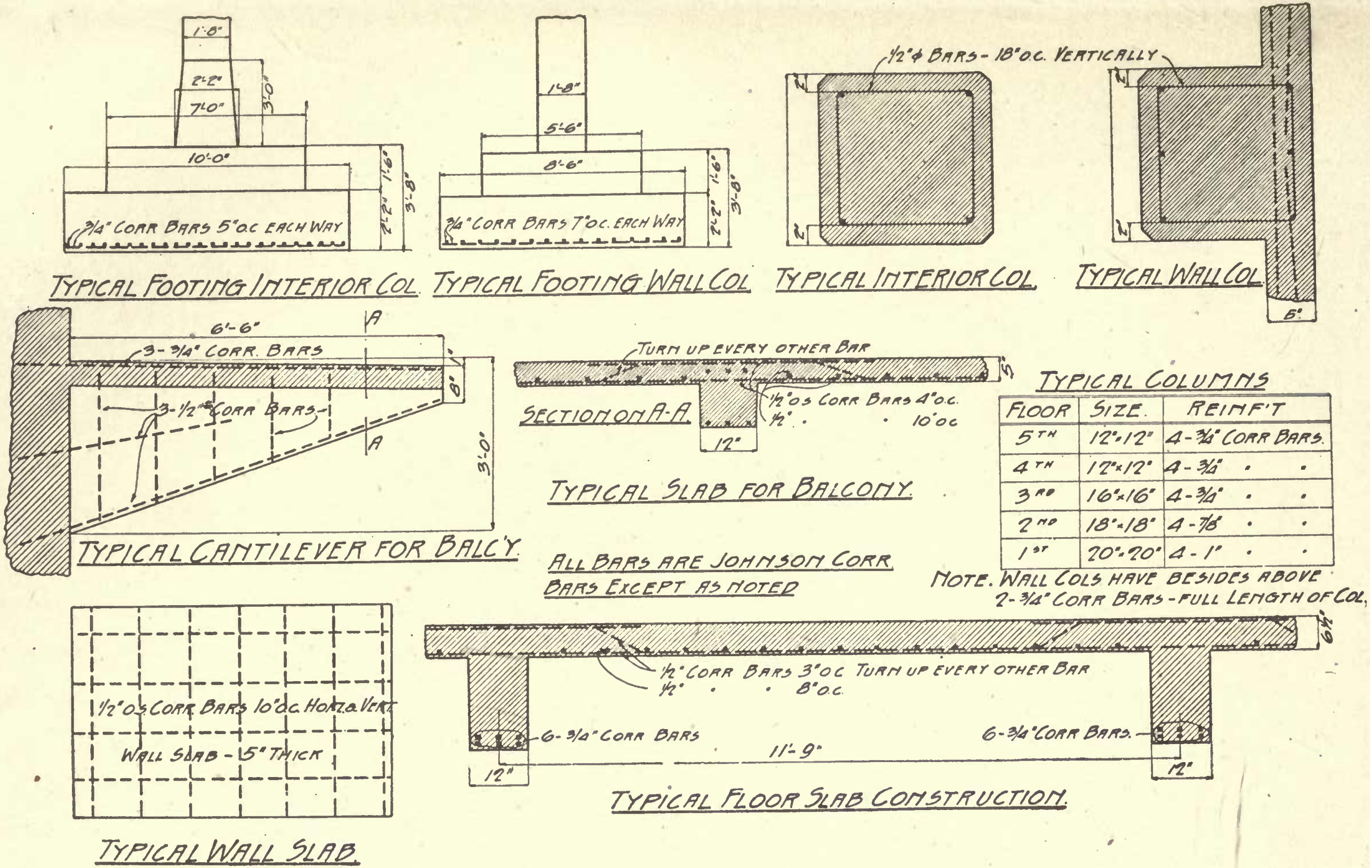
$$A = .0121\, bd = .0121 \times 12 \times 20.15 = 2.93 \text{ square inches.}$$

This can be supplied by eight bars $\frac{5}{8}$ inch square.

The total ultimate load as determined above, is 101,952 pounds. One-half of this gives the maximum shear at the ends, or 50,976 pounds. Applying Equation **15,** we have, since $d - x = .85\, d = 17$ inches

$$v = \frac{V}{b\,(d - x)} = \frac{50{,}796}{12 \times 17} = 249 \text{ pounds per square inch.}$$

As already discussed in previous cases, the ends of the beam must be reinforced against diagonal tension, since the above value of v is too great, even as an ultimate value, for such stress. Therefore the ends of the beam must be reinforced by turning the bars up, or by the use of stirrups. The beam must therefore be reinforced about as



TYPICAL COLUMNS

FLOOR	SIZE	REINF'T
5TH	12"×12"	4-3/4" CORR BARS.
4TH	12"×12"	4-3/4" · ·
3RD	16"×16"	4-3/4" · ·
2ND	18"×18"	4-7/8" · ·
1ST	20"×20"	4-1" · ·

NOTE. WALL COLS HAVE BESIDES ABOVE 2-3/4" CORR BARS - FULL LENGTH OF COL.

DETAILS OF WAREHOUSE BUILT OF REINFORCED CONCRETE.

Reproduced by Courtesy of Expanded Metal & Corrugated Bar Company.

shown in Fig. 46. Although the concentrated center load in this case is comparatively too small to require any change in the design, it should not be forgotten that a concentrated load *may* cause the shear to change so rapidly that it might require special provision for it in the center of the beam, where there is ordinarily no reinforcement which will assist shearing stresses.

Effect of Quality of Steel. There is one very radical difference between the behavior of a concrete-steel structure and that of a structure composed entirely of steel, such as a truss bridge. A truss bridge may be overloaded with a load which momentarily passes the

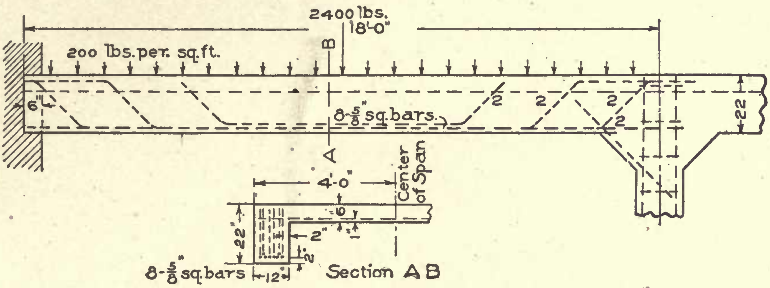


Fig. 46. Reinforced Beam.

elastic limit, and yet the bridge will not necessarily fail nor cause the truss to be so injured that it is useless and must be immediately replaced. The truss might sag a little, but no immediate failure is imminent. On this account, the factor of safety on truss bridges is usually computed on the basis of the ultimate strength.

A concrete-steel structure acts very differently. As has already been explained, the intimate union of the concrete and the steel at *all* points along the length of the bar (and not merely at the ends), is an absolute essential for stability. If the elastic limit of the steel has been exceeded owing to an overload, then the union between the concrete and the steel has unquestionably been destroyed, provided that union depends on mere adhesion. Even if that union is assisted by a mechanical bond, the distortion of the steel has broken that bond to some extent, although it will still require a very considerable force to pull the bar through the concrete. It is therefore necessary that the elastic limit of the steel should be considered the virtual ultimate so far as the strength of the steel is concerned. It is accordingly considered advisable, as already explained, to multiply all working loads by the desired factor of safety (usually taken as 4), and then to proportion the steel and concrete so that such an ultimate load will produce crushing in the upper fiber of the concrete, and at the same time will stress the steel to its elastic limit. On this basis, economy in the use of steel requires that the elastic limit should be made as high as possible.

The manufacture of steel of very high elastic limit requires the use of a comparatively large proportion of carbon, which may make the steel objectionably brittle. The steel for this purpose must therefore avoid the two extremes—on the one hand, of being brittle; and on the other, of being so soft that its elastic limit is very low.

Several years ago, bridge engineers thought that a great economy in bridge construction was possible by using *very* high carbon steel, which has not only a high elastic limit but also a correspondingly high ultimate tensile strength. But the construction of such bridges requires that the material shall be punched, forged, and otherwise handled in a way that will very severely test its strength and perhaps cause failure on account of its brittleness. The stresses in a concrete-steel structure are very different. The steel is never punched; the individual bars are never subjected to transverse bending *after* being placed in the concrete. The direct shearing stresses are insignificant. The main use, and almost the only use, of the steel, is to withstand a direct tension; and on this account a considerably harder steel may be used than is usually considered advisable for steel trusses.

If the structure is to be subject to excessive impact, a somewhat softer steel will be advisable; but even in such a case, it should be remembered that the mere weight of the structure will make the effect of the shock far less than it would be on a skeleton structure of plain steel. The steel ordinarily used in bridge work, generally has an elastic limit of from 30,000 to 35,000. If we use even 33,000 pounds as the value for s on the basis of ultimate loading, we shall find that the required percentage of steel is very high. On the other hand, if we use a grade of steel in which the carbon is somewhat higher, having an ultimate strength of about 90,000 to 100,000 pounds per square inch, and an elastic limit of 55,000 pounds per square inch, the required percentage of steel is much lower.

A study of Equation **10** will show that for any one kind of concrete the percentage of steel increases even *faster* than the value of s diminishes—which means, for example, that if s is diminished 50 per cent, p is *more* than doubled. Notwithstanding this incontrovertible fact, some engineers insist on using a low percentage of soft steel, apparently ignoring the fact that the elastic limit of the steel will be reached, and the structure will fail, long before the full strength of the concrete has been developed. There is, of course, no harm in using soft steel, provided a sufficient percentage of steel is used; but it should be remembered that formulæ developed on the basis of high elastic limit (or a high value of s) *must not* be used for soft steel. It will not even be correct to say that, because the ultimate *breaking* strength of soft steel is 60,000 pounds, we may employ formulæ with $s = 55{,}000$. Such formulæ are derived on the basis that the concrete reaches its ultimate compression (say 2,000 pounds) when the stress in the steel is 55,000. But since the soft steel cannot exceed 30,000 pounds without virtual failure, on account of the rupture of the bond between the steel and the concrete, the stress in the concrete will *never* reach 2,000 pounds, nor can it approach relatively as near 2,000 pounds as the steel approaches to 30,000 pounds.

All general equations previous to Equation **13** are perfectly general, except that in some cases q is limited to the value $\frac{2}{3}$. The later equations have, for simplicity, been worked on the uniform basis of steel having an elastic limit, which is its virtual ultimate, of 55,000 pounds, and a modulus of elasticity of 29,000,000. The subsequent tables have also further limited the concrete to that with an ultimate compression (c') of 2,000 pounds, and an initial modulus of elasticity (E_c) of 2,400,000. Other equations, similar to **13** and **14**—and other tables, similar to IX to XIV—may be similarly computed for other ultimate tensions in steel and other grades of concrete; but the engineer should be scrupulously careful about using any equations or tables *except for the grades of steel and concrete for which they have been computed.* When other grades of steel and concrete are to be used, the equations must be suitably modified. This can readily be done by deriving equations, similar to Equations **13**, **14**, and the later equations, from the general equations **1** to **12**.

Slabs on I-Beams. There are still many engineers who will not adopt reinforced concrete for the skeleton structure of buildings, but who construct the frames of their buildings of steel, using steel I-beams for floor girders and beams, and then connect the beams with concrete floor slabs. These are usually computed on the basis of transverse beams which are free at the ends, instead of considering them as "continuous beams," which will add about 50 per cent to their strength. Since it would be necessary to move the reinforcing steel from the lower part to the upper part of the slab when passing over the floor beams, in order to develop the additional strength which is theoretically possible with continuous beams, and since this is not usually done, it is by far the safest practice to consider all floor slabs as being "free-ended." The additional strength which they undoubtedly have to some extent because they are continuous over the beams, merely adds indefinitely to the factor of safety. Usually the requirement that the I-beams shall be "fireproofed," by surrounding the beam itself with a layer of concrete such that the outer surface is at least

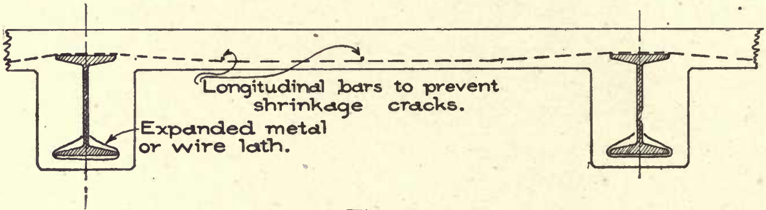


Fig. 47.

2 inches from the nearest point of the steel beam, results in having a shoulder of concrete under the end of each slab, which quite materially adds to its structural strength. But usually no allowance is made; nor is there any reduction in the thickness of the slab on account of this added strength. In this case also, the factor of safety is again indefinitely increased. The fireproofing around the beam must usually be kept in place by wrapping a small sheet of expanded metal or wire lath around the lower part of the beam before the concrete is placed.

Slabs Reinforced in Both Directions. When the floor beams of a floor are spaced nearly equally in both directions, so as to form, between the beams, panels which are nearly square, a material saving can be made in the thickness of the slab by reinforcing it with bars running in both directions. The theoretical computation of the strength of such slabs is exceedingly complicated. It is usually considered that such slabs have twice the strength of a slab supported only on two sides and reinforced with bars in but one direction. The usual method of computing such slabs is to compute the slab thickness, and the spacing and size of the reinforcing steel for a slab which is to carry *one-half* of the actual load. Strictly speaking, the slab should be thicker by the thickness of one set of reinforcing bars.

Reinforcement against Temperature Cracks. The modulus of elasticity of ordinary concrete is approximately 2,400,000 pounds per square inch, while its ultimate tensional strength is about 200 pounds per square inch. Therefore a pull of about $\frac{1}{12{,}000}$ of the length would nearly, if not quite, rupture the concrete. The coefficient of expansion of concrete has been found to be almost identical with that of steel, or .0000065 for each degree Fahrenheit. Therefore, if a block of concrete were held at the ends with absolute rigidity, while its temperature were lowered about 12 degrees, the stress developed in the concrete would be very nearly, if not quite, at the rupture point. Fortunately the ends will not usually be held with such rigidity; but nevertheless it does generally happen that, unless the entire mass of concrete is permitted to expand and contract freely so that the temperature stresses are small, the stresses will usually localize themselves at the weak point of the cross-section, wherever it may be, and will there develop a crack, provided the concrete is not reinforced with steel. If, however, steel is well distributed throughout the cross-section of the concrete, it will prevent the concentration of the stresses at local points, and will distribute it uniformly throughout the mass.

Reinforced concrete structures are usually provided with bars running in all directions, so that temperature cracks are prevented by the presence of such bars, and it is generally unnecessary to make any special provision against such cracks. The most common exception to this statement occurs in floor slabs, which structurally require bars in only one direction. It is found that cracks parallel with the bars which reinforce the slab will be prevented if a few bars are laid perpendicularly to the direction of the main reinforcing bars. Usually ½-inch or ⅜-inch bars, spaced about 2 feet apart, will be sufficient to prevent such cracks.

Retaining walls, the balustrades of bridges, and other similar structures, which may not need any bars for purely structural reasons, should be provided with such bars in order to prevent temperature cracks. A theoretical determination of the amount of such reinforcing steel is practically impossible since it depends on assumptions which are themselves very doubtful. It is usually conceded that if there is placed in the concrete an amount of steel whose cross-sectional area equals about $\frac{1}{3}$ of 1 per cent of the area of the concrete, the structure will be proof against such cracks. Fortunately, this amount of steel is so small that any great refinement in its determination is of little importance. Also, since such bars have their value in tying the structure together and thus adding somewhat to its strength and ability to resist disintegration owing to vibrations, the bars are usually worth what they cost.

TANKS

Design. The extreme durability of reinforced concrete tanks, and their immunity from deterioration by rust, which so quickly destroys steel tanks, have resulted in the construction of a large and increasing number of tanks in reinforced concrete. Such tanks must be designed to withstand the bursting pressure of the water. If they are very high compared with their diameter, it is even possible that failure might result from excessive wind pressure.

The method of designing one of these tanks may best be considered from an example. Suppose that it is required to design a reinforced concrete tank with a capacity of 50,000 gallons, which shall have an inside diameter of 18 feet. At 7.48 gallons per cubic foot, a capacity of 50,000 gallons will require 6,684 cubic feet. If the inside diameter of the tank is to be 18 feet, then the 18-foot circle will contain an area of 254.5 square feet. The depth of the water in the tank will therefore be 26.26 feet. The lowest foot of the tank will therefore be subjected to a bursting pressure due to 25.76 vertical feet of water. Since the water pressure per square foot increases $62\frac{1}{2}$ pounds for each foot of depth, we shall have a total pressure of 1,610 pounds per square foot on the lowest foot of the tank. Since the diameter is 18 feet, the bursting pressure it must resist on *each* side is one-half of $18 \times 1{,}610 = \frac{1}{2} \times 28{,}980 = 14{,}490$ pounds. If we allow a working stress of 15,000 pounds per square inch, this will require .966 square inch of metal in the lower foot. Since the bursting pressure is strictly proportional to the depth of the water, we need only divide this number proportionally to the depth to obtain the bursting pressure at other depths. For example, the ring one foot high, at one-half the depth of the tank, should have .483 square inch of metal; and that at one-third of the depth, should have .322 square inch of metal. The actual bars required for the lowest foot may be figured as follows: .966 square inch per foot equals .0805 square inch per inch; $\frac{3}{4}$-inch square bars, having an area .5625 square inch, will furnish the required strength when spaced 7 inches apart. At one-half the height, the required metal per linear inch of height is half of the above, or .040. This *could* be provided by using $\frac{3}{4}$-inch bars spaced 14 inches apart; but this is not so good a distribution of metal as to use $\frac{5}{8}$-inch square bars having an area of .39 square inch, and to space the bars nearly 10 inches apart. It would give a still better distribution of metal, to use $\frac{1}{2}$-inch bars spaced 6 inches apart at this point, although the $\frac{1}{2}$-inch bars are a little more expensive per pound, and, if they are spaced very closely, will add slightly to the cost of placing the steel. The size and spacing of bars for other points in the height can be similarly determined.

A circle 18 feet in diameter has a circumference of somewhat over 56 feet. Assuming as a preliminary figure that the tank is to be 10 inches thick at the bottom, the mean diameter of the base ring would be 18.83 feet, which would give a circumference of over 59 feet. Allowing a lap of 3 feet on the bars, this would require that the bars should be about 62 feet long. Although it is possible to have bars rolled of this length, they are very difficult to handle, and require to be transported on the railroads on *two* flat cars. It is therefore preferable to use bars of slightly more than half this length, and to make two joints in each band.

The bands which are used for ordinary wooden tanks are usually fastened at the ends by turn-buckles. Some such method is necessary for the bands of concrete tanks, provided the bands are made of plain bars. Deformed bars have a great advantage in such work, owing to the fact that, if the bars are over-lapped from 18 inches to 3 feet, according to their size, and are then wired together, it will require a greater force than the strength of the bar to pull the joints apart after they are once thoroughly incased in the concrete and the concrete has hardened.

Test for Overturning. Since the computed depth of the water is over 26 feet, we must calculate that the tank will be, say, 28 feet high. Its outer diameter will be approximately 20 feet. The total area exposed to the surface of the wind, will be 560 square feet. We may assume that the wind has an average pressure of 50 pounds per square foot; but owing to the circular form of the tank, we shall assume that its effective pressure is only one-half of this; and therefore we may figure that the total overturning pressure of the wind equals 560 × 25 = 14,000 pounds. If this is considered to be applied at a point 14 feet above the ground, we have an overturning moment of 196,000 foot-pounds, or 2,352,000 inch-pounds. Using a factor of 4 on this, we may consider this as an ultimate moment of 9,408,000 inch-pounds.

Although it is not strictly accurate to consider the moment of inertia of this circular section of the tank as it would be done if it were a strictly homogeneous material, since the neutral axis, instead of being at the center of the section, will be nearer to the compression side of the section, our simplest method of making such a calculation is to assume that the simple theory applies, and then to use a generous factor of safety. The effect of shifting the neutral axis from the center toward the compression side, will be to increase the unit compression on the concrete, and reduce the unit tension in the steel; but, as will be seen, it is generally necessary to make the concrete so thick that its unit compressive stress is at a very safe figure, while the reduction of the unit tension in the steel is merely on the side of safety.

Applying the usual theory, we have, for the moment of inertia of a ring section, $.049\ (d_1^4 - d^4)$. Let us assume as a preliminary figure that the wall of the tank is 10 inches thick at the bottom. Its outside diameter is therefore 18 feet + twice 10 inches, or 236 inches. The moment of inertia $I = .049\ (236^4 - 216^4) = 45{,}337{,}842$ biquadratic inches. Calling c the unit compression, we have, as the ultimate moment due to wind pressure:

$$M = \frac{c'\ I}{\frac{1}{2}\,d'} = \frac{c' \times 45{,}337{,}842}{\frac{1}{2}\,d_1} = 9{,}408{,}000 \text{ inch-pounds,}$$

in which $\frac{1}{2}\,d_1 = 118$ inches.

Solving the above equation for c, we have c equals a fraction less than 25 pounds per square inch. This pressure is so utterly insignificant, that, even if we double or treble it to allow for the shifting of the neutral axis from the center, and also double or treble the allowance made for wind pressure, although the pressure chosen is usually considered ample, we shall still find that there is practically no danger that the tank will fail owing to a crushing of the concrete due to wind pressure.

The above method of computation has its value in estimating the amount of steel required for vertical reinforcement. On the basis of 25 pounds per square inch, a sector with an average width of 1 inch and a diametral thickness of 10 inches would sustain a compression of about 250 pounds. Since we have been figuring ultimate stresses, we shall figure an ultimate tension of, say, 55,000 pounds per square inch in the steel. This tension would therefore require $\frac{250}{55,000} =$ .0045 square inch of metal per inch of width. Even if $\frac{1}{4}$-inch bars were used for the vertical reinforcement, they would need to be spaced only about 14 inches apart. This, however, is on the basis that the neutral axis is at the center of the section, which is known to be inaccurate.

A theoretical demonstration of the position of the neutral axis for such a section, is so exceedingly complicated that it will not be considered here. The theoretical amount of steel required is always less than that computed by the above approximate method, but the necessity for preventing cracks, which would cause leakage, would demand more vertical reinforcement than would be required by wind pressure alone.

Practical Details of the Above Design. It was assumed as an approximate figure, that the thickness of the concrete side wall at the base of the tank should be 10 inches. The calculations have shown that, so far as wind pressure is concerned, such a thickness is very much greater than is required for this purpose; but it will not do to reduce the thickness in accordance with the apparent requirements for wind pressure. Although the thickness at the bottom might be reduced below 10 inches, it probably would not be wise to do so. It may, however, be tapered slightly towards the top, so that at the top the thickness will not be greater than 6 inches, or perhaps even 5 inches. The vertical bars in the lower part of the side wall must be bent so as to run into the base slab of the tank. This will bind the side wall to the bottom. The necessity for reinforcement in the bottom of the tank depends very largely upon the nature of the foundation, and also to some extent on the necessity for providing against temperature cracks, as has been discussed in a previous section. Even if the tank is placed on a firm and absolutely unyielding foundation, some reinforcement should be used in the bottom, in order to prevent cracks which might produce leakage. These bars should run from a point near the center, and be bent upward at least 2 or 3 feet into the vertical wall. Sometimes a gridiron of bars running in both directions is used for this purpose. This method is really preferable to the radial method. The methods of making tanks water-tight have already been discussed.

RETAINING WALLS

Essential Principles. The economy of a retaining wall of reinforced concrete lies in the fact that by adopting a skeleton form of construction and utilizing the tensional and transverse strength which may be obtained from reinforced concrete, a wall may be built, of which the volume of concrete is, in some cases, not more than one-third the volume of a retaining wall of plain concrete which would answer the same purpose. Although the cost of reinforced concrete per cubic foot will be somewhat greater than that of plain concrete, it sometimes happens that such walls can be constructed for one-half the cost of plain concrete walls. The general outline of a reinforced concrete retaining wall is similar to the letter **L**, the base of which is a base-plate made as wide as (and generally a little wider than) the width usually considered necessary for a plain concrete wall. As a general rule, the width of the base should be about one-half the height. The face of the wall is made of a comparatively thin plate whose thickness is governed by certain principles, as explained later. At intervals of 10 feet, more or less, the base-plate and the face are connected by *buttresses*. These buttresses are very strongly fastened by tie-bars to both the base-plate and the face-plate. The stress in the buttresses is almost exclusively tension. The pressure of the earth tends to force the face-plate outward; and therefore the face-plate must be designed on the basis of a vertical slab subjected to transverse stresses which are maximum at the bottom and which reduce to zero at the top.

If the wall is "surcharged" (which means that the earth at the top of the wall is not level, but runs back at a slope), then the face-plate will have transverse stresses even at the top. The base-plate is held down by the pressure of the superimposed earth. The buttresses must transmit the bursting pressure on the face of the wall backward and downward to the base-plate. The base-plate must therefore be designed by the same method as a horizontal slab carry-

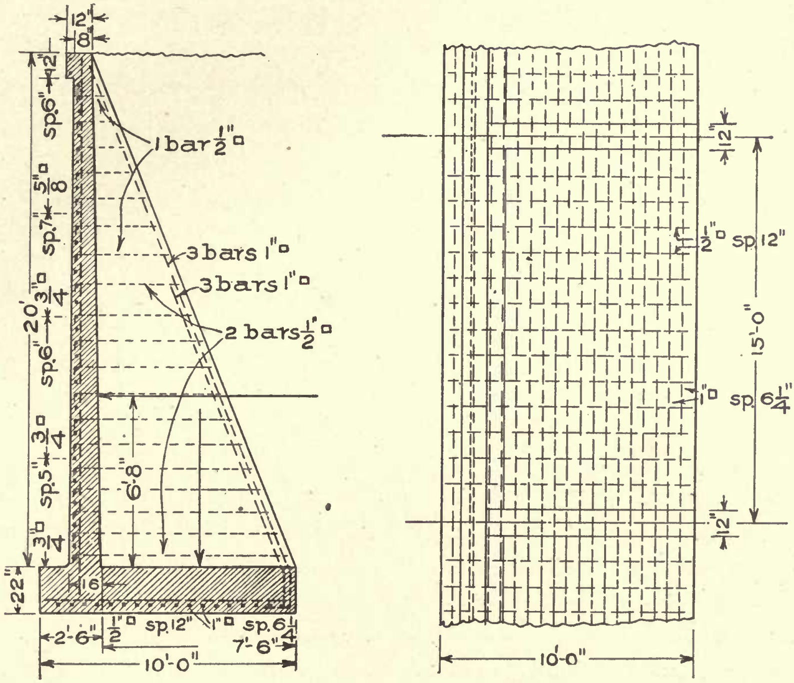


Fig. 48.

ing a load equal and opposite to the upward pull in each buttress. If the base-plate extends in front of the face of the wall, thus forming an extended toe, as is frequently done with considerable economy and advantage, even that toe must be designed to withstand transverse bending at the wall line, and also shearing at that point. The application of these principles can best be understood by an illustration.

Numerical Example. Assume that it is required to design a retaining wall to withstand an ordinary earthwork pressure of 20 feet, the earth being level on top. We are at once confronted with the determination of the actual lateral pressure of the earthwork. Unfortunately, this is an exceedingly uncertain quantity, depending upon the nature of the soil, upon its angle of repose, and particularly upon its condition whether wet or dry. The *angle of repose* is the largest angle with the horizontal at which the material will stand without sliding down. A moment's consideration will show that this angle depends very largely on the condition of the material, whether wet or dry, etc. On this account any great refinement in these calculations is utterly useless.

Assuming that the back face of the wall is vertical, or practically so; that the upper surface of the earth is horizontal; and that the angle of repose of the material is 30°, the total pressure of the wall equals $\frac{1}{6}\ w\ h^2$, in which h is the total height of the wall, and w is the weight per unit volume of the earth. If the angle of repose is steeper than this, the pressure will be less. If the angle of repose is less than this, the fraction $\frac{1}{6}$ will be larger, but the unit weight of the material will *probably* be smaller. Assuming the weight at the somewhat excessive figure of 96 pounds per cubic foot, we can then say, as an ordinary rule, that the total pressure of the earth on a vertical strip of the wall one foot wide will equal $16\ h^2$, in which h is the height of the wall in feet. The average pressure, therefore, equals $16\ h$; and the maximum pressure at a depth of h feet equals $32\ h$. Applying this figure to our numerical example, we have a total pressure on a vertical strip one foot wide, of $16 \times 20^2 = 6{,}400$ pounds. The pressure at a depth of 20 feet $= 32 \times 20 = 640$ pounds.

It is usual to compute the thickness and reinforcement of a strip one foot wide running horizontally between two buttresses. Practically the strip at the bottom is very strongly reinforced by the base-plate, which runs at right angles to it; but if we design a strip at the bottom of the wall without allowing for its support from the base-plate, and then design all the strips towards the top of the wall in the same proportion, the upper strips will have their proper design, while the lower strip merely has an excess of strength. We shall assume in this case that the buttresses are spaced 15 feet from center to center. Then the load on a horizontal strip of face-plate 12 inches high, 15 feet long, and 19 feet 6 inches from the top, will be $15 \times 19.5 \times 32$, or

9,360 pounds. Multiplying this by 4, we have an ultimate load of 37,440 pounds. The span in inches equals 180. Then,

$$M_o = \frac{37,440 \times 180}{8} = 842,400 \text{ inch-pounds.}$$

Placing this equal to 397 $b\,d^2$, in which $b = 12$ inches, we find that $d^2 = 176.8$, and $d = 13.3$ inches. At one-half the height of the wall, the moment will equal one-half of the above, and the required thickness d would be 9.4 inches. The actual thickness at the bottom, including that required outside of the reinforcement, would therefore make the thickness of the wall about 16 inches at the bottom. At one-half the height, the thickness must be about 12 inches. Using a uniform taper, this would mean a thickness of 8 inches at the top.

The reinforcement at the bottom would equal .0084 × 13.3 = .112 square inch of metal per inch of height. Such reinforcement could be obtained by using ¾-inch bars spaced 5 inches apart. The reinforcement at the center of the height would be .0084 × 9.4 = .079 square inch per inch of width. This could be obtained by using ⅝-inch bars about 5 inches apart, or by using ¾-inch bars about 7 inches apart. The selection and spacing of bars can thus be made for the entire height. While there is no method of making a definite calculation for the steel required in a vertical direction, it may be advisable to use ½-inch bars spaced about 18 inches apart.

Base-Plate. We shall assume that the base-plate has a width of one-half the height of the wall, or is 10 feet wide. If the inner face of the face-plate is 2 feet 6 inches from the toe, the width of the base-plate sustaining the earth pressure is 7 feet 6 inches. The actual pressure on the base-plate is that due to the total weight of the earth. The upward pull on the buttresses is less than this, and is measured by the moment of the horizontal pressure tending to tip the wall over. To resist this overturning tendency, there must be a downward pressure on the plate whose moment equals the moment of the couple tending to turn the wall over. The pressure on the wall on a vertical strip one foot wide, as found above, is 6,400 pounds, which has a lever arm, about the center of the base of the face-plate, of 6 feet 8 inches. The vertical pressure to resist this will be applied at the center of the 7-foot 6-inch base, or 4 feet 5 inches from the center of the face-plate. The total necessary pressure will therefore be $\frac{6,400 \times 6.67}{4.42}$, or 9,653 pounds. This means an average pressure of 1,287 pounds per square foot. Making a similar calculation for this base-plate to that previously made for the face-plate, we find that the thickness $d = 19.1$ inches. This shows that our base-plate should have a total thickness of about 22 inches.

The amount of steel *per inch of width* of the slab equals .0084 $\times$ 19.1 = .160 square inch. This can be provided by $\frac{7}{8}$-inch bars spaced $4\frac{3}{4}$ inches apart, or by 1-inch bars spaced $6\frac{1}{4}$ inches apart. This reinforcement will be uniform across the total width of the base-plate.

Buttresses. The total pressure on a vertical strip one foot wide is 6,400 pounds. For a panel of 15 feet, this equals 96,000 pounds; and its moment about the base of the wall equals 96,000 $\times$ 80 inches = 7,680,000 inch-pounds. If the tie-bars in the buttresses are placed about 3 inches from the face of the buttresses, their distance from the center of the base of the face wall will be about 89 inches. Therefore the tension in the bars in each buttress will equal $\frac{7,680,000}{89}$ = 86,292 pounds.

Since the earth pressures considered above are actual pressures, we must here consider working stresses in the metal. Allowing 15,000 pounds' tension in the steel, it will require 5.75 square inches of steel for the tie-bars of each buttress. Six 1-inch square bars will more than furnish this area. Even these bars need not all be extended to the top of the buttress, since the tension is gradually being transferred to the face-plate.

The width of the buttress is not very definitely fixed. It must have enough volume to contain the bars properly, without crowding them. In this case, for the six 1-inch bars, we shall make the width 12 inches. At the base of the buttresses, these bars should be bent around bars running through the base-plate, so that the lower part of the buttress will be very thoroughly anchored into the base-plate. It is also necessary to tie the buttress to the face-plate. The amount of this tension is definitely calculated for each foot of height, from the total pressure on the face-plate in each panel for that particular foot of height. At a depth of 19.5 feet, we found a bursting pressure of 624 pounds per square foot, or 9,360 pounds on the 15-foot panel. This would therefore be the required bond between the buttress and the face-plate at a depth of 19.5 feet. With a working tension of 15,000 pounds per square inch, such a tension would be furnished by .624 square inch of metal. This equals .05 square inch of metal for each inch of height, and $\frac{1}{2}$-inch bars spaced 5 inches apart will furnish this tension. The amount of this tension varies from the above, to zero at the top of the wall. This tension is usually provided by small bars, such as $\frac{1}{2}$-inch bars, which are bent at a right angle so as to hook over the horizontal bars in the face-plate and run backward to the back of the buttress.

In the design described above, the extension of the toe beyond the face of the wall is so short that there is no danger that the toe will be broken off on account of either shearing or transverse stress. It is usually good policy to place some transverse bars in the base-plate which are perpendicular to the face of the wall, and to have them extend nearly to the point of the toe. No definite calculation can be made of the required number of these bars unless they are required to withstand transverse bending of the toe.

If there is any danger that the subsoil is liable to settle, and thus produce irregular stresses on the base-plate, a large reinforcement in this direction may prove necessary. It is good policy to place at least $\frac{1}{2}$-inch bars every 12 inches through the base-plate, for the prevention of cracks; and this amount should be increased as the uncertainty in the stress in the base-plate increases. Although there are no definite stresses in the top of the wall, it is usual to make the thickness of the face-plate at least 6 inches at the top, and also to place a finishing cornice on top of the wall, somewhat as is shown in Fig. 48.

When the subsoil is very unreliable, it is even possible that there might be a tendency for the front and back of the base-plate to sink, and to break the base-plate by tension of the top. This can be resisted by bars in the upper part of the base-plate which are perpendicular to the wall.

Box Culverts. The permanency of concrete, and particularly reinforced concrete, has caused its adoption in the construction of culverts of all dimensions, from a cross-sectional area of a very few square feet, to that of an arch which might be more properly classified under the more common name "masonry arch." The smaller sizes can be constructed more easily, and with less expense for the forms, by giving them a rectangular cross-section. The question of foundations is solved most easily by making a concrete bottom, as well as side walls and top. The structure then becomes literally a "box." Its design consists in the determination of the external pressure exerted by the earth and of the required thickness of the concrete to withstand the pressure on the flat sides considered as slabs. The most uncertain part of the computation lies in the determination of the actual pressure of the earth. Under the heading, "Retaining Walls," this uncertainty was discussed.

One very simple method is to assume that the earth pressure is equivalent to that of a liquid having a unit weight equal to that

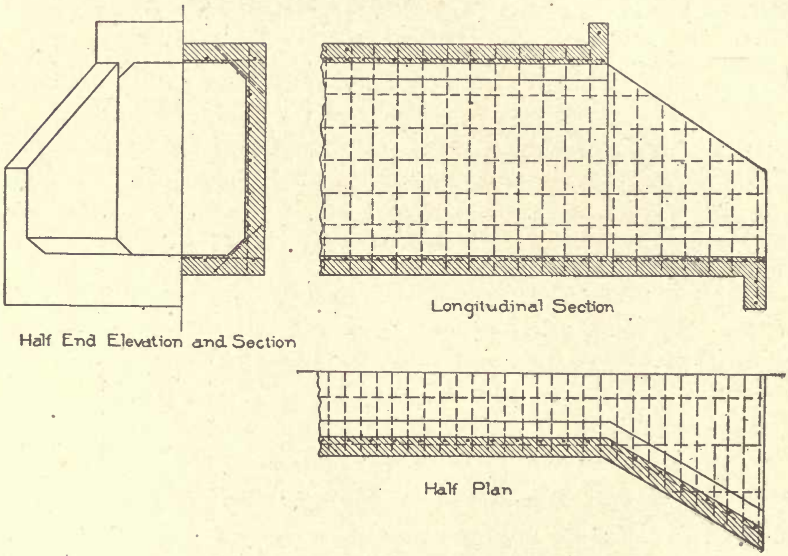


Fig. 49.

of the weight of a cubic foot of the earth, which is nearly 100 pounds. Under almost any circumstances, these figures would be sufficiently large, and perhaps very excessive. Calculations on such a basis are therefore certainly safe. If the pressure is computed on this basis, and a factor of safety of 2 is used, it is equivalent to an actual pressure of only one-half the amount (which is more probable) having a factor of 4. If the depth of the earth is quite large compared with the dimensions of the culvert, we may consider that the upward pressure on the bottom, as well as the lateral pressure on the sides, is practically the same as the downward pressure on the top. If the bottom of the culvert is laid on rock, or on soil which is practically unyielding, there will be no necessity of considering that there is any upward pressure on the bottom slab tending to burst that slab upward. The softer the soil, the greater will be the tendency to transverse bending in the bottom slab.

Since the design of rectangular box culverts is purely an application of the equations for transverse bending, after the external pressures have been determined, no numerical example will here be given. These structures are not only reinforced with bars, considering the sides as slabs, but should also have bars placed across the corners, which will withstand a tendency for the section to collapse in case the pressure on opposite sides is unequal. They must also be reinforced with bars running longitudinally with the culvert. As in the other cases of longitudinal reinforcement, no definite design can be made for its amount. A typical cross-section for such a culvert is shown in Fig. 49. The longitudinal bars are indicated in this figure. They are used to prevent cracks owing to expansion or contraction, and also to resist any tendency to rupture which might be caused by a settling or washing-out of the subsoil for any considerable distance under the length of the culvert.

Arch Culverts. No attempt will here be made to explain the general theory of arches. A stone arch is always designed on the basis that there is no tension in the arch ring. The design is also based on the principle that the line of pressure within the arch ring shall always be such that there is some pressure on every part of each joint, which practically means that the line of pressure shall not at any point be outside the middle third of the arch ring. It is usually a simple matter so to design an arch ring that when the arch is uniformly loaded from end to end, either with a light load or with its maximum load, the line of pressure shall at all points be within the middle third of the arch ring; but when the load on the arch is eccentric—or, in other words, when one portion of the arch is heavily loaded, and the other parts of the arch have no load—the line of pressure may pass near the edge of the arch ring, or even entirely outside of it. This is especially true when the weight of the live load is large compared with the dead weight of the arch. An arch built of stone would certainly fail under such conditions. An arch built of plain concrete would probably also fail under such conditions, although the tensional strength of the concrete would permit a considerable variation of the line of pressure before failure would take place. If the arch is built of reinforced concrete, the tensional strength furnished by the bars will permit a very large variation in the line of pressure before failure will take place. This will permit the use of a very much thinner arch ring than would be safe for either

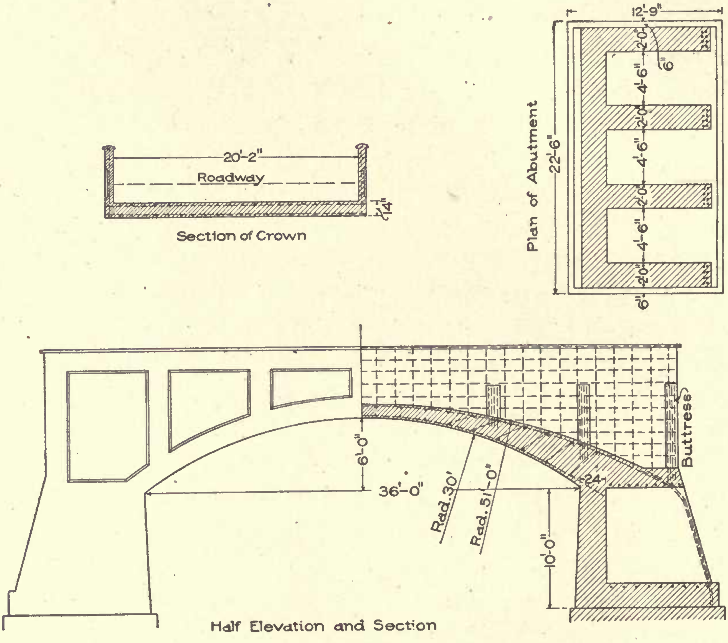


Fig. 50.

stone or plain concrete. Variations in the loading of an arch will cause such a change in the line of pressure, and such a variation in the place where a tendency to bending may occur, that it is usual to place two layers of bars, one slightly within the extrados of the arch, and the other slightly above the intrados. These bars are connected by cross-bars which resist the tendency to shearing. Bars are also placed parallel with the axis of the arch. These are illustrated in Fig. 50.

The design of the arch consists in the determination, according to the theory of elastic arches, of the maximum moment which can occur at any point of the arch, under any probable system of loading. The depth of the arch at that point, and the amount of steel required, can then be computed according to the principles of transverse bending previously laid down. Since it is impracticable to vary the amount of reinforcement at different sections of the arch, it is usual to compute the amount of reinforcement needed at the point where the requirement is the greatest, and to use such steel throughout the entire section of the arch. Almost the only variation from this occurs when additional bars are sometimes run from the abutment for several feet across the arch in order to provide for the very excessive moment that may occur near the abutment in some designs, that moment not being found under any conditions at or near the center of the arch. The amount of reinforcement which should be placed parallel with the axis of the arch is indefinite, as is the case with other forms of longitudinal reinforcement.

The centering for concrete arches is not materially different from that of the centering of any masonry arch, except in the fact that, since the concrete is usually a very wet mixture, the forms must be made with closer joints than would be required for a stone arch.

A very material saving can frequently be made in the amount of concrete in the abutments—especially when the soil is so soft that it cannot easily withstand the thrust of the arch—by connecting the two abutments by a concrete bottom in which are placed sufficient steel tie-rods to take up the thrust of the arch. Frequently there is a very considerable economy in this method, which has the added advantage that the bottom of the culvert will have a smooth surface, which will materially accelerate the flow of the water, and will even permit of a slight reduction in the cross-section of the arch opening. It is also possible to effect some economy in the amount of concrete required for the abutments, by using a skeleton form of construction, having a base-plate and buttresses somewhat similar to the skeleton design already shown for retaining walls (see Fig. 48). In such designs the pressure of the earth on the base-plate assists in furnishing the necessary anchorage for the abutments. The economy which is thus possible—and which is possible only because the structure is made of reinforced concrete—is very considerable.

Footings. When a definite load, such as a weight carried by a column, is to be supported on a subsoil whose bearing power has been estimated at some definite figure, the required area of the footing becomes a perfectly definite quantity, regardless of the method of construction of the footing. But with the area of the footing once determined, it is possible to effect considerable economy in the construction of the footing, by the use of reinforced concrete. An ordinary footing of masonry is usually made in a pyramidal form, although the sides will be stepped off instead of being made sloping. It may be approximately stated, that the depth of the footing below the base of the column, when ordinary masonry is used, must be practically equal to the width of the footing. The offsets in the masonry cannot ordinarily be made any greater than the heights of the various steps. Such a plan requires an excessive amount of masonry.

A footing of reinforced concrete consists essentially of a slab, which is placed no deeper in the ground than is essential to obtain a proper pressure from the subsoil. In the simplest case, the column is placed in the middle of the footing, and thus acts as a concentrated load in the middle of the plate. The mechanics of such a problem are somewhat similar to those of a slab supported on four sides and carrying a concentrated load in the center, with the very important exception, that the resistance, instead of being applied merely at the edges of the slab, is uniformly distributed over the entire surface. Since the column has a considerable area, and the slab merely overlaps the column on all sides, the common method is to consider the overlapping on each side to be an inverted cantilever carrying a uniformly distributed load, which is in this case an upward pressure. The maximum moment evidently occurs immediately below each vertical face of the column. At the extreme outer edge of the slab the moment is evidently zero, and the thickness of the slab may therefore be reduced considerably at the outer edge. The depth of the slab, and the amount of reinforcement, which is of course placed near the bottom, can be determined according to the usual rules for obtaining a moment. This can best be illustrated numerically.

Example. Assume that a load of 252,000 pounds is to be carried by a column, on a soil which consists of hard, firm gravel. Such soil will ordinarily safely carry a load of 7,000 pounds per square foot. On this basis, the area of the footing must be 36 square feet, and therefore a footing 6 feet square will answer the purpose. A concrete column 24 inches square will safely carry such a loading. Placing such a column in the middle of a footing will leave an offset 2 feet broad outside each face of the column. We may consider a section of the footing made by passing a vertical plane through one face of the column. This leaves a block of the footing 6 feet long and 2 feet wide, on which there is an upward pressure of 12 × 7,000 = 84,000 pounds. The center of pressure is 12 inches from the section, and the moment is therefore 12 × 84,000 = 1,008,000 inch-pounds. Multiplying this by 4, we have 4,032,000 inch-pounds as the ultimate moment. Applying Equation **13**, we place this equal to $397\, b\, d^2$, in which b = 72 inches. Solving this for d, we have d = 11.9 inches. A total thickness of 15 inches would therefore answer the purpose. The amount of steel required per inch of width = $.0084\, d = .0084 \times 11.9 = .100$ square inch of steel per inch of width. Therefore ¾-inch bars, spaced 5.6 inches apart, will serve the purpose. A similar reinforcing of bars should be placed perpendicularly to these bars.

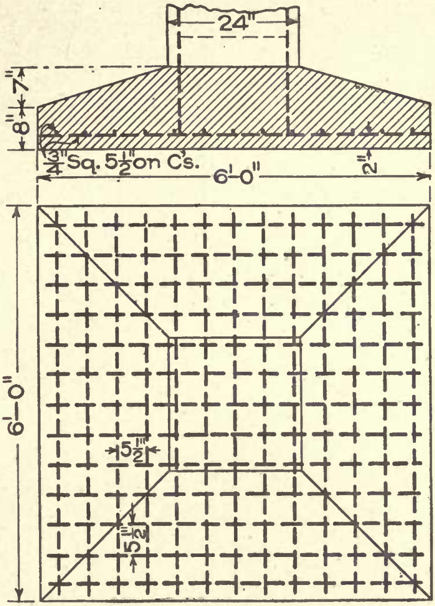


Fig. 51

The above very simple solution would be theoretically accurate in the case of an offset 2 feet wide for the footing of a wall of indefinite length, assuming that the upward pressure was 7,000 pounds per square foot. The development of such a moment uniformly along the section of our square footing, implies a resistance to bending at the outer edges of the slab which will not actually be obtained. The moment will certainly be greater under the edges of the column. On the other hand, we have used bars in both directions. The bars passing under the column in each direction are just such as are required to withstand the moment produced by the pressure on that part of the footing directly in front of each face of the column. It may be considered that the other bars have their function in tying the two systems into one plate whose several parts mutually support one another. If further justification of such a method is needed, it may be said that experience has shown that it practically fulfils its purpose.

When a simple footing supports a single column, the center of pressure of the column must pass vertically through the center of

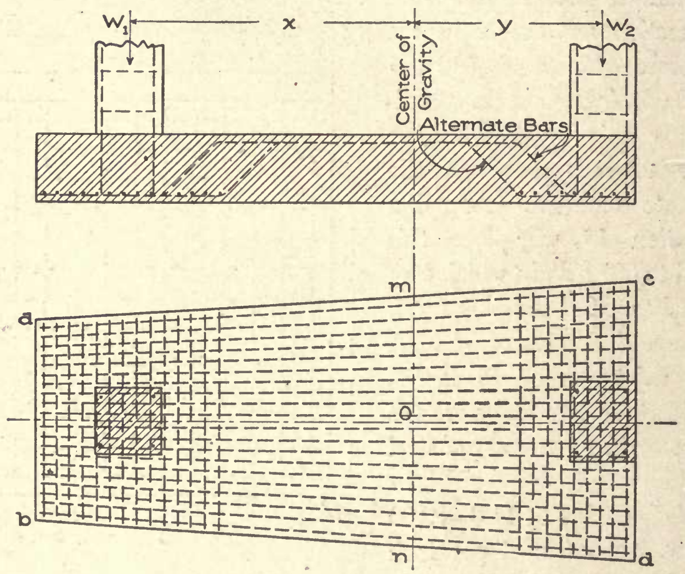
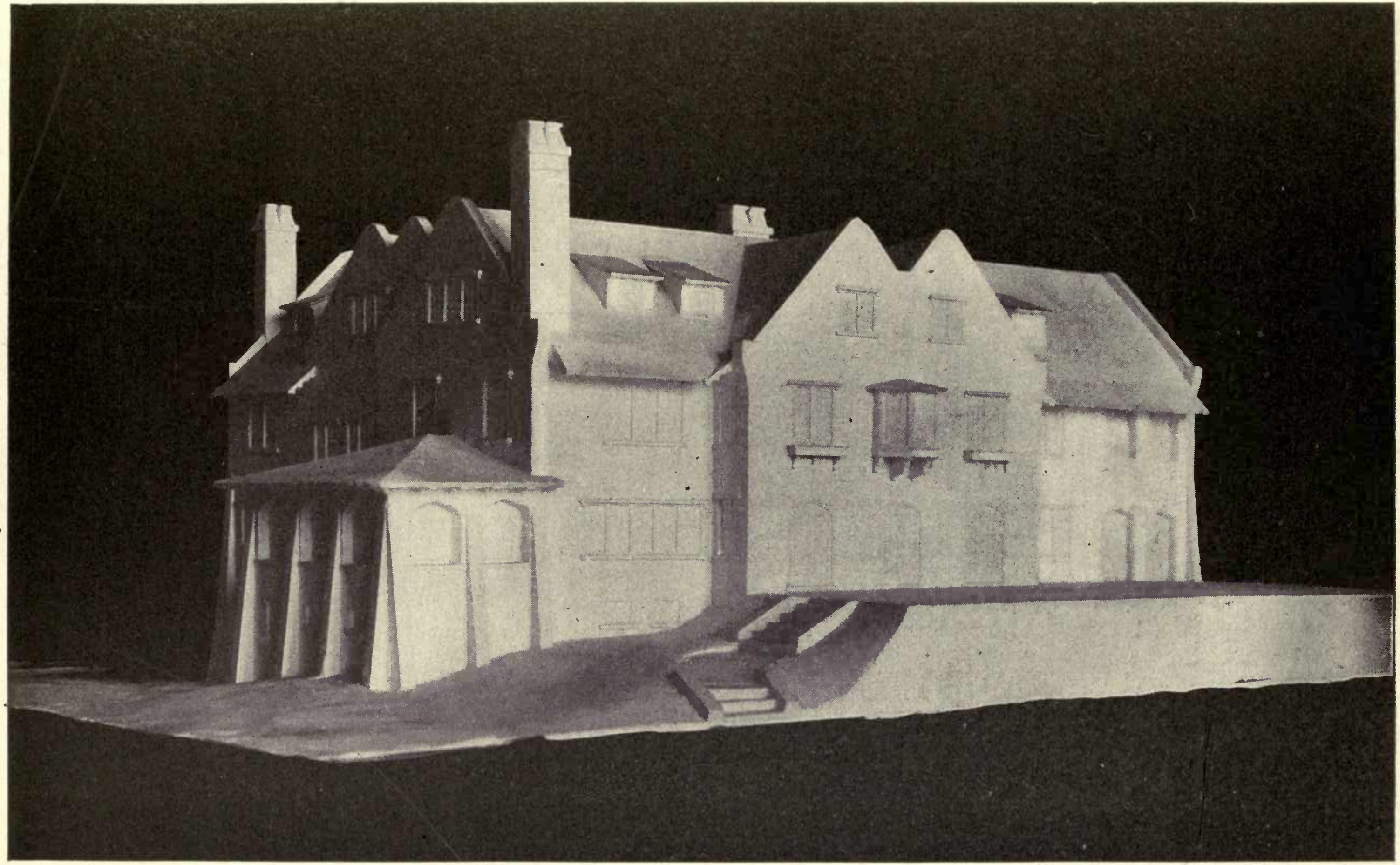


Fig. 52.

gravity of the footing, or there will be dangerous transverse stresses in the column, as discussed later. But it is sometimes necessary to support a column on the edge of a property line when it is not permissible to extend the foundations beyond the property line. In such a case, a simple footing is impracticable. The method of such a solution is indicated in Fig. 52, without numerical computation. The nearest interior column (or even a column on the opposite side of the building, if the building be not too wide) is selected, and a combined footing is constructed under both columns. The weight on both columns is computed. If the weights are equal, the center of



ARCHITECTS' MODEL OF RESIDENCE FOR MR. FRED PABST, ON OCONOMOWOC LAKE, WIS.

Fernekes & Cramer, Architects, Milwaukee, Wis.; Newton Engineering Co., Engineers.

All Walls and Partitions are of Reinforced Concrete. The Exterior Walls will not be Plastered, but are to have a Hammered Finish. Red Tile Roof. Cost, about $40,000. Building is to be Completed in the Fall of 1907.

gravity is half-way between them; if unequal, the center of gravity is on the line joining their centers, and at a distance from them such that (see Fig. 52) $x : y :: W_2 : W_1$. In this case, evidently, W_2 is the greater weight. The area *abcd* must fulfil two conditions:—

(1) The area must equal the total loading ($W_1 + W_2$), divided by the allowable loading per square foot; and

(2) The center of gravity must be located at O.

An analytical solution of the relative and absolute values of *ab* and *cd* which will fulfil the two conditions, is very difficult, and fortunately is practically unnecessary. If x and y are equal, *abcd* is a rectangle. If W_2 is greater than 2 W_1, then y will be less than $\frac{1}{2}x$; and even a triangle with the vertex under the column W_1, would not fulfil the condition. In fact, if W_1 is very small compared with W_2, it might be impracticable to obtain an area sufficiently large to sustain the weight. The proper area can be determined by a few trials, with sufficient accuracy for the purpose.

The footing must be considered as an inverted beam at the section *mn*, where the moment $= W_2y + W_2x$. The width is *mn*; and the required depth and the area of the steel must be computed by the usual methods. The bars will here be in the top of the footing, but will be bent down to the bottom under the columns, as shown in Fig. 52. The cross-bars under each column will be designed, as in the case of the simple footing, to distribute the weight on each column across the width of the footing, and to transfer the weight to the longitudinal bars.

Vertical Walls. Vertical walls which are not intended to carry any weight, are sometimes made of reinforced concrete. They are then called *curtain walls*, and are designed merely to fill in the panels between the posts and girders which form the skeleton frame of the building. When these walls are interior walls, there is no definite stress which can be assigned to them, except by making assumptions that may be more or less unwarranted. When such walls are used for exterior walls of buildings, they must be designed to withstand wind pressure. This wind pressure will usually be exerted as a pressure from the outside tending to force the wall inward; but if the wind is in the contrary direction, it may cause a lower atmospheric pressure on the outside, while the higher pressure of the air within the building will tend to force the wall outward. It is improbable, however, that such a pressure would ever be as great as that tending to force the wall inward. Such walls may be designed as slabs carrying a uniformly distributed load, and supported on all four sides. If the panels are approximately square, they should have bars in both directions, and should be designed by the same method as "slabs reinforced in both directions," as has been previously explained. If the vertical posts are much closer together than the height of the floor, as sometimes occurs, the principal reinforcing bars should be horizontal, and the walls should be designed as slabs having a span equal to the distance between the posts. Some small bars spaced about 2 feet apart should be placed vertically to prevent shrinkage. The pressure of the wind corresponding to the loading of the slab, is usually considered to be 30 pounds per square foot, although the actual wind pressure will very largely depend on local conditions, such as the protection which the building receives from surrounding buildings. A pressure of thirty pounds per square foot is usually sufficient; and a slab designed on this basis will usually be so thin, perhaps only 4 inches, that it is not desirable to make it any thinner. Since designing such walls is such an obvious application of the equations and problems already solved in detail, no numerical illustration will here be given.

Wind Bracing. The practical applications of the principles of reinforced concrete which have already been discussed, have been almost exclusively those required for sustaining vertical loads; but a structure consisting simply of beams, girders, slabs, and columns *may* fall down, like a house of cards, unless it is provided with lateral bracing to withstand wind pressure and any lateral forces tending to turn it over. The necessary provision for such stresses is usually made by placing *brackets* in the angles between posts and girders, as has been illustrated in Fig. 46. These brackets are reinforced with bars which will resist any tensile stress on the brackets. The compressive strength of concrete may be relied on to resist a tendency to crush the brackets by compression. Usually such brackets will occur in pairs at each end of a beam supported on two columns. If we consider that any given moment is to be divided equally between two brackets, then, if we are to have a working tension of 15,000 pounds per square inch in the steel, and a working compression of 500 pounds per square inch in the concrete, the area of the concrete must be 30 times the area of the steel. But since the outer face of the concrete will have practically twice the compression of the concrete at the angle of the beam and column, and since the maximum of 500 pounds per square inch must not be exceeded, we must have twice that area of concrete; or, in other words, the area of the concrete from the point of the angle down to the face must be 60 times the area of the steel.

Although these brackets are frequently put in without any definite design, it is possible to make some sort of computation, especially when a building is directly exposed to wind pressure, by computing the moment of the wind pressure. For example, if a building is 100 feet long and 50 feet high, and is subjected to a wind pressure of 30 pounds per square foot, the total wind pressure will be 50 × 100 × 30 = 150,000 pounds. Considering the center of pressure as applied at half the height, this would give a moment about the base of the building, of 150,000 × 25 = 3,750,000 foot-pounds = 45,000,000 inch-pounds. If this 100-foot building had eight lines of columns with a pair of brackets on each line, and was four stories high, there would be 64 such brackets to resist wind pressure. Each bracket would therefore be required to resist $\frac{1}{64}$ of 45,000,000 inch-pounds, or about 700,000 inch-pounds. We shall assume that the bracket will have a depth of 25 inches, from the intersection of the center lines of the column and the beam to the steel near the face of the bracket. Then, since each bracket must withstand a moment of 700,000 inch-pounds, the stress in the steel will be 700,000 ÷ 25 = 28,000 pounds. If the actual stress in the steel is 15,000 pounds per square inch, this would require 1.87 square inches of steel, which would be more than supplied by four $\frac{3}{4}$-inch square bars. If these brackets were 12 inches wide and 25 inches deep, the area of concrete is 300 square inches, which is 160 times the area of the steel. There is, therefore, an ample amount of concrete to withstand compression, on the part of those brackets which are subject to compression rather than tension. It is probable that the above calculation is excessive on the side of safety, since it is quite improbable that such a broad area would ever be subject to a pressure of 30 pounds per square foot over the whole area. The method of calculation also ignores the fact that the monolithic character of a reinforced concrete structure furnishes a very considerable resistance at the junction of columns and girders, and that they should not by any means be considered as if they were "pin-connected" structures, which would require that the whole of the lateral stiffening should be supplied by these brackets. Nevertheless these brackets must be designed according to some such method.

COLUMNS

Methods of Reinforcement. The laws of mechanics, as well as experimental testing on full-sized columns of various structural materials, show that very short columns, or even those whose length is ten times their smallest diameter, will fail by crushing or shearing of the material. If the columns are very long, say twenty or more times their smallest diameter, they will probably fail by bending, which will produce an actual tension on the convex side of the column. The line of division between long and short columns is practically very uncertain, owing to the fact that the center line of pressure of a column is frequently more or less eccentric because of irregularity of the bearing surface at top or bottom. Such an eccentric action will cause buckling of the column even when its length is not very great. On this account, it is always wise (especially for long columns) to place reinforcing bars within the column. The reinforcing bars consist of longitudinal bars (usually four, and sometimes more with the larger columns), and bands of small bars spaced about 12 or 18 inches apart vertically, which bind together the longitudinal bars. The longitudinal bars are used for the purpose of providing the necessary transverse strength to prevent buckling of the column. As it is practically impossible to develop a satisfactory theory on which to compute the required tensional strength in the convex side of a column of given length, without making assumptions which are themselves of doubtful accuracy, no exact rules for the sizes of the longitudinal bars in a column will be given. The bars ordinarily used vary from $\frac{1}{2}$ inch square to 1 inch square; and the number is usually four, unless the column is very large (400 square inches or larger) or is rectangular rather than square. It has been claimed by many, that longitudinal bars in a column may actually be a source of danger, since the buckling of the bars outward may tend to disintegrate the column. This buckling can be avoided, and the bars made mutually self-supporting, by means of the bands which are placed around the column. These bars are usually $\frac{1}{4}$-inch or $\frac{3}{8}$-inch round or square bars. The specifications of the Prussian Public Works for 1904 require that these horizontal bars shall be spaced a distance not more than 30 times their diameter, which would be $7\frac{1}{2}$ inches for $\frac{1}{4}$-inch bars, and $11\frac{1}{4}$ inches for $\frac{3}{8}$-inch bars. The bands in the column are likewise useful to resist the bursting tendency of the column, especially when it is short. They will also reinforce the column against the tendency to shear, which is the method by which failure usually takes place. The angle between this plane of rupture and a plane perpendicular to the line of stress, is stated to be 60°. If, therefore, the bands are placed at a distance apart equal to the smallest diameter of the column, any probable plane of rupture will intersect one of the bands, even if the angle of rupture is somewhat smaller than 60°.

The unit working pressure permissible in concrete columns is usually computed at from 350 to 500 pounds per square inch. The ultimate compression for transverse stresses for 1:3:6 concrete has been taken at 2,000 pounds per square inch. With a factor of 4, this gives a working pressure of 500 pounds per square inch; but the ultimate stress in a column of plain concrete is generally less than 2,000 pounds per square inch. Tests of a large number of 12 by 12-inch plain concrete columns showed an ultimate compressive strength of approximately 1,000 pounds per square inch; but such columns generally begin to fail by the development of longitudinal cracks. These would be largely prevented by the use of lateral reinforcement or bands. Therefore the use of 500 pounds per square inch, as a working stress for columns which are properly reinforced, may be considered justifiable although not conservative.

Design of Columns. It may be demonstrated by theoretical mechanics, that if a load is jointly supported by two kinds of material with dissimilar elasticities, the proportion of the loading borne by each will be in a ratio depending on their relative areas and moduli of elasticity. The formula for this may be developed as follows:

C = Total unit compression upon concrete and steel in pounds per square inch = Total load divided by the combined area of the concrete and the steel;

c = Unit compression in the concrete, in pounds per square inch;

s = Unit compression in the steel, in pounds per square inch;

p = Ratio of area of steel to total area of column;

$r = \frac{E_s}{E_c}$ = Ratio of the moduli of elasticity;

ϵ_s = Deformation per unit of length in the steel;

ϵ_c = Deformation per unit of length in the concrete;
A_s = Area of steel;
A_c = Area of concrete.

The total compressive force in the concrete $= A_c \times c$; and that in the steel $= A_s \times s$.

The sum of these compressions = the total compression; and therefore,

$$C(A_c + A_s) = A_c\, c + A_s\, s.$$

The actual lineal compression of the concrete = that of the steel; therefore,

$$\frac{c}{E_c} = \frac{s}{E_s}.$$

From this equation, since $r = \frac{E_s}{E_c}$, we may write the equation $rc = s$. Solving the above equation for C, we obtain:

$$C = \frac{A_c\, c + A_s\, s}{A_c + A_s}.$$

Substituting the value of $s = rc$, we have:

$$C = c\left(\frac{A_c + A_s\, r}{A_c + A_s}\right) = c\left(\frac{A_s + A_c - A_s + A_s\, r}{A_c + A_s}\right).$$

If p = the ratio of cross-section of steel to the *total* cross-section of the column, we have:

$$p = \frac{A_s}{A_c + A_s}.$$

Substituting this value of $\frac{A_s}{A_c + A_s}$ in the above equation, we may write:

$$C = c\,(1 - p + pr).$$

Solving this equation for p, we obtain:

$$p = \frac{C - c}{c\,(r - 1)} \qquad \textbf{(16)}$$

Example 1. A column is designed to carry a load of 160,000 pounds. If the column is made 18 inches square, and the load per square inch to be carried by the concrete is limited to 400 pounds, what must be the percentage of the steel, and how much steel would be required?

Answer. A column 18 inches square has an area of 324 square inches. Dividing 160,000 by 324, we have 494 pounds per square inch as the total unit compression upon the concrete and the steel, which is C in the above formula. Assume that the concrete is 1:3:6 concrete, and that the ratio of the moduli of elasticity (r) is therefore 12. Substituting these values in Equation **16**, we have:

$$p = \frac{494 - 400}{400\,(12 - 1)} = .0214.$$

Multiplying this ratio by the total area of the column, 324 square inches, we have 6.93 square inches of steel required in the column. This would very nearly be provided by four bars $1\frac{1}{4}$ inches square. Four round bars $1\frac{1}{2}$ inches in diameter would give an excess in area. Either solution would be amply safe under the circumstances, provided the column was properly reinforced with bands.

Example 2. A column 16 inches square is subjected to a load of 115,000 pounds, and is reinforced by four $\frac{7}{8}$-inch square bars beside the bands. What is the actual compressive stress in the concrete per square inch?

Answer. Dividing the total stress (115,000) by the area (256), we have the combined unit stress C = 449 pounds per square inch. By inverting one of the equations above, we can write

$$c = \frac{C}{1 - p + r\,p}.$$

In the above case, the four $\frac{7}{8}$-inch bars have an area of 3.06 square inches; and therefore,

$$p = \frac{3.06}{256} = .012;\ r = 12.$$

Substituting these values in the above equation, we may write:

$$c = \frac{449}{1 - .012 + (.012 \times 12)} = \frac{449}{1.132} = 397 \text{ pounds per square inch.}$$

The net area of the concrete in the above problem is 252.94 square inches. Multiplying this by 397, we have the total load carried by the concrete, which is 100,117 pounds. Subtracting this from 115,000 pounds, the total load, we have 14,885 pounds as the compressive stress carried by the steel. Dividing this by 3.06, the area of the steel, we have 4,864 pounds as the unit compressive stress in the steel. This is practically twelve times the unit compression in the concrete, which is an illustration of the fact that if the compression is shared by the two materials in the ratio of their moduli of elasticity, the unit stresses in the materials will be in the same ratio. This unit stress in the steel is about one-third of the working stress which may properly be placed on the steel. It shows that we cannot economically use the steel in order to reduce the area of the concrete, and that the only object in using steel in the columns is in order to protect the columns against buckling, and also to increase their strength by the use of bands.

It sometimes happens that in a building designed to be structurally of reinforced concrete, the column loads in the columns of the lower story may be so very great that concrete columns of sufficient size would take up more space than it is desired to spare for such a purpose. For example, it might be required to support a load of 320,000 pounds on a column 18 inches square. If the concrete (1:3:6) is limited to a compressive stress of 400 pounds per square inch, we may solve for the area of steel required, precisely as was done in example 1. We would find that the required percentage of steel was 13.4 per cent, and that the required area of the steel was therefore 43.3 square inches. But such an area of steel could carry the entire load of 320,000 pounds without the aid of the concrete, and would have a compressive unit stress of only 7,400 pounds. In such a case, it would be more economical to design a steel column to carry the entire load, and then to surround the column with sufficient concrete to fireproof it thoroughly. Since the stress in the steel and the concrete are divided in proportion to their relative moduli of elasticity, which is usually about 10 or 12, we cannot develop a working stress of, say, 15,000 pounds per square inch in the steel, without at the same time developing a compressive stress of 1,200 to 1,500 pounds in the concrete, which is objectionably high as a working stress.

Effect of Eccentric Loadings of Columns. It is well known that if a load on a column is eccentric, its strength is considerably less than when the resultant line of pressure passes through the axis of the column. The theoretical demonstration of the amount of this eccentricity depends on assumptions which may or may not be found in practice. The following formula is given without proof or demonstration, in Taylor & Thompson's Treatise on Concrete:

Let e = Eccentricity of load;
b = Breadth of column;
f = Average unit pressure;
f' = Total unit pressure of outer fiber nearest to line of vertical pressure.

Then,

$$f' = f\left(1 + \frac{6e}{b}\right) \qquad \textbf{(17)}$$

As an illustration of this formula, if the eccentricity on a 12-inch column were 2 inches, we would have $b = 12$, and $e = 2$. Substituting these values in Equation 17, we would have $f' = 2f$, which means that the maximum pressure would equal twice the average pressure. In the extreme case, where the line of pressure came to the outside of the column, or when $e = \frac{1}{2}b$, we would have that the maximum pressure on the edge of the column would equal four times the average pressure.

Any refinements in such a calculation, however, are frequently overshadowed by the uncertainty of the actual location of the center of pressure. A column which supports two equally loaded beams on each side, is probably loaded more symmetrically than a column which supports merely the end of a beam on one side of it. The best that can be done is arbitrarily to lower the unit stress on a column which is probably loaded somewhat eccentrically.

STRENGTH OF TEE-BEAMS

When concrete beams are laid in conjunction with overlying floor-slabs, the concrete for both the beams and the slabs being laid in one operation, the strength of such beams is very much greater than their strength considered merely as plain beams, even though we compute the depth of the beams to be equal to the total depth from the bottom of the beam to the top of the slab. An explanation of this added strength may be made as follows:

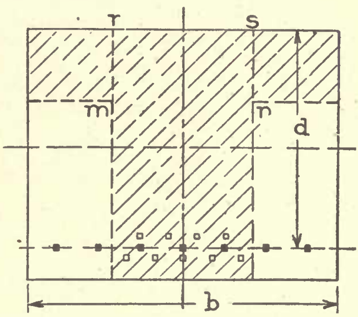


Fig. 53. Tee-Beam in Cross-Section.

If we were to construct a very wide beam with a cross-section such as is illustrated in Fig. 53, there is no hesitation about calculating such strength as that of a plain beam whose width is b, and whose effective depth to the reinforcement is d. Our previous study in plain beams has shown us that the steel in the bottom of the beam takes care of practically all the tension; that the neutral axis of the beam is somewhat above the center of its height; that the only work of the concrete below the neutral axis is to transfer the stress in the steel to the concrete in the top of the beam; and that even in this work it must be assisted somewhat by stirrups or by bending up the steel bars. If, therefore, we cut out from the lower corners of the beam two rectangles, as shown by the unshaded areas, we are saving a very large part of the concrete, with very little loss in the strength of the beam, provided we can fulfil certain conditions. The steel, instead of being distributed uniformly throughout the bottom of the wide beam, is concentrated into the comparatively narrow portion which we shall hereafter call the *rib* of the beam. The concentrated tension in the bottom of this rib must be transferred to the compression area at the top of the beam. We must also design the beam so that the shearing stresses in the plane (mn) immediately below the slab shall not exceed the allowable shearing stress in the concrete. We must also provide that failure shall not occur on account of shearing in the vertical planes $(m\,r$ and $n\,s)$ between the sides of the beam and the flanges. In computing the compression in the fibers in the upper part of the simple beam, it is assumed that all fibers at the same distance above the neutral axis are stressed equally. The same assumption is sometimes made when developing the formula for tee-beams. Such an assumption is substantially true in the case of the simple beam, but is practically untrue (and perhaps dangerously so) in the case of tee-beams with wide flanges. The maximum compression is evidently found immediately above the rib of the beam, while the compressive stress probably diminishes on each side of the rib. Fig. 54 gives a graphical representation of the diminution in intensity of pressure in the flange. When the distance between adjacent beams is comparatively great, there is probably (and in fact usually) a considerable portion of the slab between consecutive beams which is practically unaffected by the compression required for the top of each tee-beam. Since this compression is concentrated above the rib of each tee-beam, the work must be so designed that the *maximum* pressure (instead of the *average* pressure) does not exceed the safe working value.

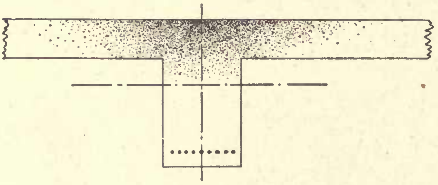


Fig. 54. Graphical Representation of Diminution in Intensity of Pressure in Flange.

Let us consider a tee-beam such as is illustrated in Fig. 55. If we were to insert an excessively large amount of steel in the lower part of the rib, we could probably develop a compression in the flange which would require a very wide flange. But the beam would probably fail by shearing along the horizontal plane immediately under the flange. In order to have the most economical design, which means that the beam shall be equally strong in every respect, or, in other words, that it shall be equally liable to failure in several ways when loaded to its ultimate load, we must obtain a relation between the total compression in the flange and the required shearing strength in the rib immediately under the flange. In the lower part of Fig. 55, is represented one-half of the length of the flange, which is considered to have been separated from the rib. Following the usual

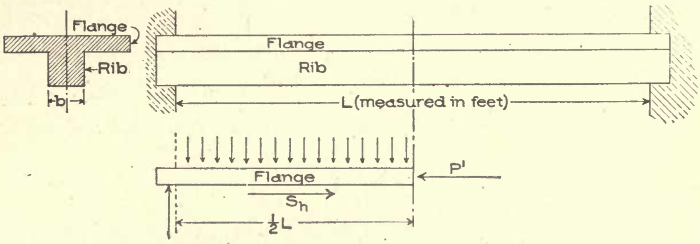


Fig. 55. Tee-Beam.

method of considering this as a free body in space, acted on by external forces and by such internal forces as are necessary to produce equilibrium, we find that it is acted on at the left end by the abutment reaction, which is a vertical force, and also by a vertical load on top. We may consider P' to represent the summation of all compressive forces acting on the flange at the center of the beam. In order to produce equilibrium there must be a shearing force acting on the underside of the flange. We represent this force by S_h. Since these two forces are the only horizontal forces, or forces with horizontal components, which are acting on this free body in space, P' must equal S_h. Let us consider z to represent the ultimate shearing force per unit of area. We know from the laws of mechanics, that, with a uniformly distributed load on the beam, the shearing force is maximum at the ends of the beam, and diminishes uniformly towards the center, where it is zero. Therefore the *average* value of the unit shear for the half-length of the beam, must equal $\frac{1}{2} z$. As before, we represent the width of the rib by b. For convenience in future computations, we shall consider L to represent the length of the beam measured in *feet.* All other dimensions are measured in inches. Therefore the total shearing force along the lower side of the flange, will be:

$$S_h = \tfrac{1}{2} z \times b \times \tfrac{1}{2} L \times 12 = 3\, b\, z\, L \qquad \textbf{(18)}$$

There is also a possibility that a beam may fail in case the flange (or the slab) is too thin; but the slab is always reinforced by bars which are transverse to the beam, and the slab will be placed on both sides of the beam, giving two shearing surfaces. Beams supporting a slab on only one side, should be computed as plain beams. Therefore, if we adopt the rule that the thickness of the slab should be at least one-half the width of the rib, or perhaps permitting the reduction to one-third of the width of the rib on account of the reinforcement which will tend to prevent shearing, we need not pay any further attention to the tendency to shear in vertical planes along the rib. Expressing the above condition algebraically, we should say:

$$t > \tfrac{1}{3} b, \text{ or } b < 3\, t \qquad \textbf{(19)}$$

The summation (P') of the horizontal forces in the flange of the beam, is computed as follows:

It is assumed that the diminution of pressure from the upper fibers downward follows the usual law as already developed for simple beams. It is also assumed that the pressure on the fibers in any horizontal plane through the flange will also vary as the ordinates of a parabola. This is practically the equivalent of saying that the total pressure on the rectangle *mnvs* (see Fig. 56) is *two-thirds* of what it would be if *mnvs* were part of a simple beam, with width b' and effective depth d. We shall first compute the total pressure on the rectangle *mnop,* calling it *two-thirds* of the pressure on *mnop,* if it be a simple beam, and then subtract from it

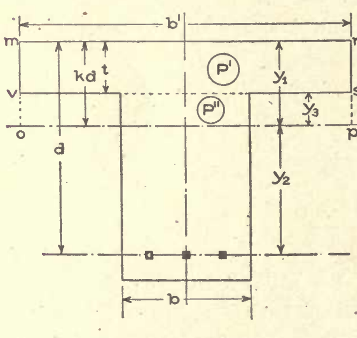


Fig. 56.

the pressure on $vsop$, computed on the same basis. We may apply Equation **5** directly for the rectangle $mnop$, and say that:

$$\text{Pressure on } m\,n\,o\,p = \frac{2}{3} \times \frac{7}{12} \times c'\,b'\,y_1.$$

For $vsop$ we must apply Equation **4**, since, for the fibers in the plane vs, q is not $\frac{2}{3}$, but is $\frac{2}{3}\frac{ps}{pn} = \frac{2}{3}\frac{y_3}{y_1}$. Substituting this value of q in Equation **4**, we have:

$$\text{Pressure on } v\,s\,o\,p = \frac{2}{3} \times \frac{9}{8}\left(1 - \frac{2}{9}\frac{y_3}{y_1}\right)\frac{2}{3}\frac{y_3}{y_1}c'\,b'\,y_3\ ;$$

$$= \frac{1}{2}\left(1 - \frac{2\,y_3}{9\,y_1}\right)\frac{y_3}{y_1}c'\,b'\,y_3\ ;$$

$$P' = c'\,b'\left\{\frac{7}{18}y_1 - \frac{1}{2}\left(1 - \frac{2}{9}\frac{y_3}{y_1}\right)\frac{y_3^2}{y_1}\right\} \quad\ldots\ldots\ldots\ldots\ldots\ldots(\mathbf{20})$$

The pressure (P'') on the rib between the flange and the neutral axis, is computed on the basis that the pressure on all fibers in any one horizontal plane is uniform (as in the case of simple beams), but that q is the same as above, $\frac{2}{3}\frac{y_3}{y_1}$. Applying Equation **4**, we have:

$$P'' = \frac{9}{8}\left(1 - \frac{2}{9}\frac{y_3}{y_1}\right)\frac{2}{3}\frac{y_3}{y_1}c'\,b\,y_3\ ;$$

$$= \frac{3}{4}c'b\left(1 - \frac{2}{9}\frac{y_3}{y_1}\right)\frac{y_3^2}{y_1} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\mathbf{21})$$

It has already been shown in a previous section, that the allowable unit intensity of the shear, even for ultimate loads, equals

$$z = \frac{c'}{6.928}.$$

Substituting this value in Equation **18**, we have:

$$P' = S_h = 3b\frac{c'}{6.928}L = \frac{1}{2.309}b\,c'\,L.$$

For greater convenience in numerical calculation, and especially in view of the uncertainty of the value and the excessive margin allowed, this ratio is placed at the round value:

$$P' = \frac{4}{9}b\,c'\,L.$$

We may then place this value equal to the value of P' in Equation **20,** and solve for b':

$$P' = c'b' \left\{ \frac{7}{18} y_1 - \frac{1}{2}\left(1 - \frac{2}{9}\frac{y_3}{y_1}\right)\frac{y_3^2}{y_1} \right\} = \frac{4}{9}\, b\, c'L\,;$$

$$b' = \frac{bL}{\frac{9}{4}\left\{ \frac{7}{18} y_1 - \frac{1}{2}\left(1 - \frac{2}{9}\frac{y_3}{y_1}\right)\frac{y_3^2}{y_1} \right\}};$$

$$= \frac{bL}{\frac{7}{8} y_1 - \frac{9}{8}\left(1 - \frac{2}{9}\frac{y_3}{y_1}\right)\frac{y_3^2}{y_1}} \qquad \text{(22)}$$

When the neutral axis is at or near the bottom of the slab, it is practically correct to say that:

$$b' = b\,L \div \frac{7}{8}\, y_1\,.$$

If the beams are very deep, and the neutral axis is as far below the slab as the thickness of the slab, such an approximate value would be about 30 per cent too small.

Area of Steel. The required area of steel equals the total compression in the concrete, divided by s. Therefore,

$$A = \frac{P' + P''}{s} = \frac{4}{9}\frac{b\,c'}{s} L + \frac{3}{4}\frac{b\,c'}{s}\left(1 - \frac{2}{9}\frac{y_3}{y_1}\right)\frac{y_3^2}{y_1} \qquad \text{(23)}$$

Moment of Section. The ultimate moment of the cross-section of a simple beam depends only on the dimensions of the cross-section. This would also be true of tee-beams, except for the fact that under some conditions the beam might fail by shearing under the flange; and the above theory provides for those conditions, by determining the pressure (P') as a function of the length of the beam (L). The determination of the precise points of application of the two forces P' and P'', is a very complicated mathematical problem. There is no material error in assuming that P' is applied at the middle of the

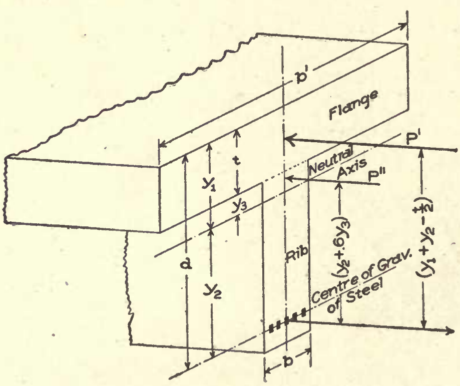


Fig. 57.

slab height, and that P'' is applied at $\frac{3}{5}$ of the height y_3. By taking moments about the center of gravity of the steel, we eliminate the steel tension from the equation, and have the equation:

$$M_o = P'\left(d - \frac{1}{2}t\right) + P''\left(y_2 + .6\,y_3\right);$$

$$= b\left\{\frac{4}{9}c'L\left(d - \frac{1}{2}t\right) + \frac{3}{4}c'\left(1 - \frac{2}{9}\frac{y_3}{y_1}\right)\frac{y_3^2}{y_1}\left(y_2 + .6y_3\right)\right\} \quad (24)$$

Design of Tee-Beams. Although Equations 22, 23, and 24 are the only equations which are essential to design tee-beams, the work is very tedious without the use of tables, since the equations involve unknown quantities which must be assumed first, and then tested whether the dimensions are mutually satisfactory. For any one grade of concrete, k has the same value as already figured for simple beams, and therefore for a beam of any assumed depth (say, d), kd, which in these calculations has been abbreviated to y_1, becomes known; $y_2 = (d - kd)$, and $y_3 = (y_1 - t)$. In any given numerical case, the thickness of the slab (t) is first computed on the basis of the floor load to be carried between beams spaced at a chosen distance apart.

We must then compute the weight of the live load on the panel whose area is the product of the span of the beam and the distance between beams. Adding to this an estimate for the dead weight of the floor, and multiplying the total load by 4, we have the ultimate load on one tee-beam. We then make an estimate of the *probable* required depth (d) of the beam. Knowing the quality of the concrete, we know the ratio k, which determines the position of the neutral axis; and we may then compute y_1, y_2, and y_3 as explained above. We also know the span L and the ultimate compressive strength of the concrete c'. Substituting all of these quantities in Equation 24, b is the only unknown quantity; and therefore we may solve the equation for b, which is the required width of the beam. We must apply two checks. In the first place, b must not be greater than three times the slab thickness (t). Also the breadth b', as computed from Equation 22, must not be greater than the distance between consecutive tee-beams.

Even though these two checks are satisfactory, it is quite possible that a recalculation should be made for a beam of greater or less depth, in order that the breadth b shall bear a more satisfactory proportion to the depth d. Of course, an increase in the depth d will result in a decrease in the computed width b, and *vice versa*.

Having satisfactorily settled on the depth d and the corresponding width b, we can determine the area of the steel from Equation **23.** All of the quantities on the right-hand side of Equation **23** are known, and the area may therefore be computed directly. As in the case of simple beams, the bars should be bent upward at an angle of 45°, as illustrated in Fig. 45. It will add considerably to the shearing strength in the horizontal plane immediately underneath the slab, if the bars which are bent upward are allowed to penetrate the slab, and are then bent so as to run horizontally for the remainder of their length within the slab. Of course this will occur only near the ends of the beams, where the shear immediately under the slab has its maximum value.

Numerical Example. Let us assume that a flooring for a building 20 feet wide is to be made of a 1:3:6 concrete floor-slab supported by concrete beams spaced 8 feet apart from center to center. We shall assume that the floor is to carry a live load of 150 pounds per square foot. An experienced man will know that a 5-inch slab will probably answer the purpose, and that this slab will weigh about 12 pounds per square foot per inch of thickness of the slab, or about 60 pounds per square foot. In this case, we shall obtain the ultimate loading by adopting the frequent practice of multiplying our live load (150) by 4, and our dead load (60) by 2, this giving 720 pounds per square foot ultimate load. With a span of 8 feet, and on a strip 1 foot wide, we have a total ultimate load of $720 \times 8 = 5,760$ pounds. We therefore have, for the ultimate moment:

$$M_o = \frac{W_o\, l}{8} = \frac{5,760 \times 96}{8} = 69,120 \text{ inch-pounds.}$$

Using Equation **13**, which is applicable in this case, we have:

$$397\, b\, d^2 = 69,120;$$

$$b\, d^2 = 174$$

But $b = 12$ inches; therefore $d^2 = 14.5$, and $d = 3.8$ inches. Therefore a 5-inch slab, with the bars 1 inch from the bottom, has a slight excess of thickness. The required area of steel equals $.0084\, b\, d = .0084 \times 12 \times 3.8 = .383$ square inches per foot of width.

This equals .032 square inch per inch, which will require $\frac{1}{2}$-inch bars, to be spaced 8 inches. The student should compare these results with those which may be derived directly from Table XI.

We have figured an ultimate load of 720 pounds per square foot for the floor-slab. In figuring the ultimate load for the tee-beam, we must add something for the dead weight of the beam itself. Of course this depends on the size of the beam, which is still an unknown quantity. It is usually found that the added amount of concrete in the beam underneath the slab is the equivalent of an added inch or two of thickness over the entire area of the slab. At 12 pounds per square foot per inch of thickness, this will add 12 or 15 pounds per square foot to the dead load. Multiplying this by 2 for factor of safety, we have, say, 30 pounds additional, and we may therefore say that the ultimate load per square foot for the beam shall be considered in this case 750 pounds rather than 720. Therefore, on the span of 20 feet, and with 8 feet between the beams, each beam must support an ultimate load of 8 × 20 × 750 = 120,000 pounds. Then,

$$M_o = \frac{W_o\, l}{8} = 120{,}000 \times 240 \div 8 = 3{,}600{,}000 \text{ inch-pounds.}$$

We must substitute this value of M_o in Equation **24**, and obtain the dimensions of the beam. This can be done only by assuming some value for the depth of the beam, and solving for b. We shall commence with the assumption that $d = 15$ inches. Using 1:3:6 concrete, $k = .395$; and kd therefore equals in this case 5.92 inches. This gives us the value $y_1 = 5.92$; and since $y_1 + y_2 = d$, then $y_2 = 9.08$; $y_3 = y_1 - t = 5.92 - 5.00 = 0.92$; $c' = 2{,}000$; L, which is the span in *feet*, = 20. This determines all the quantities in Equation **24** except the value b. Substituting these quantities in Equation **24**, we have:

$$b \left\{ \frac{4}{9} \times 2{,}000 \times 20\,(15 - 2.5) + \frac{3}{4} \times 2{,}000 \times \left(1 - \frac{2}{9}\,\frac{.92}{5.92}\right) \frac{.92^2}{5.92} \left(9.08 + .6 \times .92\right) \right\} = 3{,}600{,}000;$$

$$b \left\{ 222{,}222 + 1{,}500\,(1 - .034)\,.143 \times 9.63 \right\} = 3{,}600{,}000;$$

$$b\,(222{,}222 + 1{,}992) = 3{,}600{,}000;$$

$$b = \frac{3{,}600{,}000}{224{,}214} = 16.0 \text{ inches.}$$

But this trial value of b is greater than three times the thickness of the slab. It is also greater than the depth of the slab to the reinforcement, which shows that it is not an economical design, even if it fulfilled the other condition. We must therefore use a deeper beam. We shall accordingly make another trial with $d = 17$ inches. The student should work this out in detail, the calculation being very similar to that given above; and it will be found that b then = 13.6 inches. This being a suitable width for $d = 17$ inches—or a total depth of, say, 19 inches, or 14 inches under the slab—this combination of breadth and depth will be accepted.

The required area of the steel can now readily be found by a direct application of Equation 23, since all the symbols on the right-hand side of the equation have now become known quantities. Making these substitutions, which the student should work out in detail, we find that the required area equals 4.55 square inches. This can be furnished by six $\frac{7}{8}$-inch square rods (area 4.59 square inches) or by eight $\frac{3}{4}$-inch square rods (area 4.50 square inches). Probably the eight $\frac{3}{4}$-inch rods would be the better choice, in spite of the slight deficiency in area, since it gives a better distribution of the metal, and furnishes a greater number of bars which may be turned up near the ends of the beam.

The student should work out still another combination of values for the above case, on the basis that $d = 19$ inches. He should find in this case that b will be 11.6 inches, but that the amount of steel required will be only 4.00 square inches. Although the amount of concrete will be very nearly the same in these last two solutions, the last method requires less steel, and is therefore more economical.

Shear. The theoretical computation of the shear of a tee-beam is a very complicated problem. Fortunately it is unnecessary to attempt to solve it exactly. The shearing resistance is certainly far greater in the case of a tee-beam than in the case of a plain beam of the same width and total depth and loaded with the same total load. Therefore, if the shearing strength is sufficient, according to the rule, for a plain beam, it is certainly sufficient for the tee-beam. In the above numerical case, the total *ultimate* load on the beam is 120,000 pounds. Therefore the maximum shear (V) at the end of the beam, is 60,000 pounds. With this grade of concrete, $d - x = .86\,d$. For this beam, $d = 17$ inches, and $b = 13.6$ inches. Substituting these values in Equation 15, we have:

$$v = \frac{V}{b\,(d-x)} = \frac{60{,}000}{13.6 \times .86 \times 17} = 302 \text{ pounds per square inch.}$$

Although this is probably a very safe *ultimate* stress for direct shearing, it is 50 per cent in excess of the allowable direct ultimate tension due to the diagonal stresses; and therefore ample reinforcement must be provided. If only two of the $\frac{3}{4}$-inch bars are turned at an angle of 45° at the end, these two bars will have an area of 1.12 square inches, and will have an ultimate tensile strength (at the elastic limit of 55,000 pounds) of 61,600 pounds. This is more than the ultimate total vertical shear at the ends of the beam; and we may therefore consider that the beam is protected against this form of failure.

Tables for Computation of Tee-Beams. The above computation has purposely been worked out in detail in order thoroughly to explain every feature of the solution. If it were necessary to adopt identically the same method for the design of every tee-beam, the work would be very tedious. Fortunately the work may be very greatly simplified by solving Equations **22, 23,** and **24** for some one grade of concrete and for various depths of beams. Such tables are illustrated in Tables XI to XIV inclusive. They are all worked out on the basis of the use of 1:3:6 concrete. Their use may be illustrated as follows:

Assume that a flooring having a span of 18 feet is to be supported by a 4-inch slab and by tee-beams spaced 6 feet apart, the working load being 150 pounds per square foot.

We shall compute, as before, an ultimate floor loading of 725 pounds per square foot, and the ultimate moment on one panel to be supported by one tee-beam of 2,349,000 inch-pounds. As a trial, we shall assume $d = 14$ as the proper depth. In Table XI, opposite $d = 14$, we find ultimate moment $= b\ (5{,}596 + 10{,}667\ L)$. Multiplying 10,667 by 20, and adding 5,596, we have 218,936. Dividing this into 2,349,000, we have $10\frac{3}{4}$ inches as the required width b. This being a proper proportion, it may be adopted. Substituting this value of b in the expression on the same line for "area of steel," we have:

Area of steel $= 10.75\ (.0108 + .0162 \times 20) = 3.60$ square inches of steel.

As a check, $b' = .227\, b\, l' = .227 \times 10.75 \times 20 = 48.8$ inches. But, since the beams are spaced 6 feet (or 72 inches) apart, there is ample width of slab between each beam. The ultimate shear at the end of the beam is 39,150 pounds. Applying Equation **15**, we have in this case $(d - x) = .86\, d = 12.04$ inches. Then,

$$v = \frac{39{,}150}{10.75 \times 12.04} = 303 \text{ pounds per square inch.}$$

We may consider this as the diagonal tension in the end of the beam, which shows that it must be amply reinforced either by stirrups or by some of the reinforcing bars being bent up diagonally at the ends.

TABLE XI

Tee-Beams—1 : 3 : 6 Concrete—4-Inch Slabs

d	Ultimate Moment—M_0	Area of Steel—A	b'
11	$b($ 269 + 8,000$L)$	$b($ .0007 + .0162$L)$	.265 bL
12	$b($ 1289 + 8,889$L)$	$b($ .0030 + .0162$L)$	.248 bL
13	$b($ 3035 + 9,778$L)$	$b($ .0064 + .0162$L)$	.236 bL
14	$b($ 5596 + 10,667$L)$	$b($ .0108 + .0162$L)$	.227 bL
15	$b($ 8868 + 11,556$L)$	$b($ .0157 + .0162$L)$	.221 bL
16	$b($ 12993 + 12,444$L)$	$b($ .0213 + .0162$L)$	.215 bL
17	$b($ 17807 + 13,333$L)$	$b($ .0272 + .0162$L)$	.211 bL
18	$b($ 23502 + 14,222$L)$	$b($ .0335 + .0162$L)$	.207 bL
19	$b($ 29865 + 15,111$L)$	$b($ .0399 + .0162$L)$	.203 bL
20	$b($ 37133 + 16,000$L)$	$b($ .0467 + .0162$L)$	.201 bL
22	$b($ 53884 + 17,778$L)$	$b($ .0608 + .0162$L)$	.196 bL
24	$b($ 73755 + 19,556$L)$	$b($ .0754 + .0162$L)$	.193 bL

TABLE XII

Tee-Beams—1 : 3 : 6 Concrete—5-Inch Slabs

d	Ultimate Moment—M_0	Area of Steel—A	b'
13	$b($ 42 + 9,333$L)$	$b($ .0001 + .0162$L)$	.223 bL
14	$b($ 655 + 10,222$L)$	$b($ .0014 + .0162$L)$	.209 bL
15	$b($ 2,014 + 11,111$L)$	$b($ .0038 + .0162$L)$	.199 bL
16	$b($ 4,130 + 12,000$L)$	$b($ .0072 + .0162$L)$	.191 bL
17	$b($ 7,012 + 12,889$L)$	$b($ .0113 + .0162$L)$	.185 bL
18	$b($ 10,665 + 13,778$L)$	$b($ .0160 + .0162$L)$	.180 bL
19	$b($ 15,093 + 14,667$L)$	$b($ .0211 + .0162$L)$	.175 bL
20	$b($ 20,297 + 15,556$L)$	$b($ .0267 + .0162$L)$	.172 bL
22	$b($ 33,043 + 17,333$L)$	$b($ .0387 + .0162$L)$	.166 bL
24	$b($ 48,909 + 19,111$L)$	$b($ .0517 + .0162$L)$	.162 bL
26	$b($ 67,895 + 20,889$L)$	$b($ .0654 + .0162$L)$	.159 bL
28	$b($ 90,003 + 22,667$L)$	$b($ .0796 + .0162$L)$	.156 bL

TABLE XIII

Tee-Beams—1 : 3 : 6 Concrete—6-Inch Slabs

d	Ultimate Moment—M_0	Area of Steel—A	b'
16	$b($ 237 + 11,556$L)$	$b($ .0004 + .0162$L)$	.181 bL
17	$b($ 1,195 + 12,444$L)$	$b($ .0020 + .0162$L)$	.173 bL
18	$b($ 2,900 + 13,333$L)$	$b($ .0046 + .0162$L)$	.166 bL
19	$b($ 5,362 + 14,222$L)$	$b($ .0079 + .0162$L)$	.160 bL
20	$b($ 8,590 + 15,111$L)$	$b($ .0118 + .0162$L)$	.156 bL
22	$b($ 17,358 + 16,889$L)$	$b($ .0206 + .0162$L)$	.148 bL
24	$b($ 29,228 + 18,667$L)$	$b($ .0320 + .0162$L)$	.143 bL
26	$b($ 44,212 + 20,444$L)$	$b($ .0439 + .0162$L)$	.139 bL
28	$b($ 62,313 + 22,222$L)$	$b($ .0567 + .0162$L)$	.136 bL
30	$b($ 83,536 + 24,000$L)$	$b($ .0701 + .0162$L)$	.134 bL
32	$b($ 107,882 + 25,778$L)$	$b($ .0841 + .0162$L)$	.132 bL

TABLE XIV

Tee-Beams—1:3:6 Concrete—7-Inch Slabs

d	Ultimate Moment—M_0	Area of Steel—A	b'
18	b (28 + 12,889L)	b (.0000 + .0162L)	.161 bL
19	b (592 + 13,778L)	b (.0009 + .0162L)	.153 bL
20	b (1,895 + 14,667L)	b (.0027 + .0162L)	.147 bL
22	b (6,756 + 16,444L)	b (.0086 + .0162L)	.138 bL
24	b (14,672 + 18,222L)	b (.0164 + .0162L)	.131 bL
26	b (25,676 + 20,000L)	b (.0266 + .0162L)	.127 bL
28	b (39,783 + 21,778L)	b (.0373 + .0162L)	.123 bL
30	b (57,003 + 23,556L)	b (.0492 + .0162L)	.120 bL
32	b (77,342 + 25,333L)	b (.0618 + .0162L)	.118 bL
34	b (100,802 + 27,111L)	b (.0750 + .0162L)	.116 bL
36	b (127,383 + 28,889L)	b (.0888 + .0162L)	.114 bL

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