VII. *The Newtonian Constant of Gravitation as Affected by Temperature.*

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Received October 19,—Read December 2, 1915.

Contents.

I. Introduction.

1. This paper deals with the possible existence of a temperature coefficient in the law of gravitation and gives an account of experiments made to discover this coefficient. The apparatus used is of the Cavendish torsion-balance type, and the range of temperatures is from 15° C. to 250° C. The result of a prolonged research is shown in the summary.

The accumulation of negative results in the experimental study of gravitation is remarkable. In consequence of the indifference of the gravitative force to changes of conditions (other than those given by the simple law $f = \mathrm{G}Mm/d^2$), none of the many theories of gravitation so far propounded has received general acceptance for lack of data wherewith to test them. Some recent theories, which consider the possibility of temperature effect, are :—

N. MOROZOV* advances a wave theory on which the attraction of masses would vary with temperature.

G. MIE,† a theory of matter which includes among its corollaries a temperature coefficient of $1/10^{13}$ per 1° C. to the "so-called Newtonian constant."

N. BOHR,‡ in a paper on the constitution of the atom, assumes that gravitation like radio-activity is unaffected by all physical and chemical agencies, and hence the writer ascribes both these properties of the atom to its nucleus; whereas other physical properties are due to the electrons in the outer regions of the atom. But it should be noted that, at least as regards temperature, the data for gravitation are deficient. It is not easy to conduct experiments on a short range of one or two hundred degrees (see later) which is a small portion of the thousands of degrees of known measured temperatures. So it is necessary to speak with diffidence on this special branch of the subject of the constitution of the atom.

2. Determinations of the Newtonian constant have hitherto been made at ordinary temperatures only, special care being taken to maintain uniformity in temperature throughout the apparatus used; otherwise convection in the air or strains in the movable system might produce grave errors. This is shown repeatedly in the well-known researches by Prof. C. V. BOYS§ and Prof. J. H. POYNTING.‖ The necessity of providing a steady temperature about the delicate parts of the apparatus has hitherto been considered an insuperable bar to any direct experiment to discover a temperature effect for G. In fact, in 1905, shortly before the present research began, the late Prof. POYNTING and Mr. P. PHILLIPS¶ wrote as follows :—"The difficulties of exact determination (of the Newtonian constant) at ordinary temperatures are not yet overcome and at very high or low temperatures they would be so much increased that the research seems at present hopeless."

3. Yet indirect investigations have been made. POYNTING and PHILLIPS** counterpoised a mass of 208 gr. on a balance and varied its temperature between 100° C. and −186° C. They came to the conclusion that the resulting change in weight, if any, is less than $1/10^9$ for 1° C. for the range 100° C. to 0° C. and $1/10^{10}$ for 1° C., for the range 0° C. to −186° C.

Another balance experiment on change in weight with temperature by SOUTHERNS†† led to a somewhat similar result.

* 'Jurn. Russik Chimičisk Obščestva,' 40, 2, pp. 23 to 35, 1908.

† 'Ann. der Phys.,' 1913, No. 1.

‡ 'Phil. Mag.,' July 26, 1913.

§ 'Phil. Trans.,' 1895.

‖ 'Phil. Trans.,' 1891.

¶ See 'Roy. Soc. Proc.,' A, vol. 76, p. 445, 1905.

** *Loc. cit.*

†† 'Roy. Soc. Proc.,' A, vol. 78, 1906.

4. In looking for a method to continue and extend these researches it should be observed that a weight of, say, 1 gr. can be determined by a first-rate balance to $1/10^8$, under favourable conditions,* whereas in a gravitation apparatus (*e.g.* that of C. V. Boys) the attraction of one mass to another cannot be found to be better than $1/10^5$ at the utmost. Thus, apart from other reasons, it would be futile on the latter type of apparatus to look for a temperature effect (at least between 100° C. and −186° C.) on the small mass m, since the above negative results have established the result with the greatest possible accuracy.

But in these balance experiments of Poynting and Phillips the large mass M (in this case the earth) is unchanged in temperature. Now M is incomparably larger than m and may have a preponderating influence, whereby change of *its temperature alone* would affect the mutual attraction. In the paper quoted Poynting and Phillips suggested (though without any *a priori* grounds) the feasibility of some such expression as the following :—

$$f = \mathrm{G}\left[1+\kappa\frac{(\mathrm{M}t+mt')}{\mathrm{M}+m}\right]\frac{\mathrm{M}m}{d^2}, \qquad \text{I.}$$

where κ is a temperature coefficient and t, t' are increments in temperature of M and m.

When M/m is very great, this reduces to

$$f = \mathrm{G}\left[1+\kappa t\right]\frac{\mathrm{M}m}{d^2}, \qquad \text{II.}$$

so that, on the above supposition, the mutual attraction would be influenced by change in temperature of the large mass only.

Admitting the possibility involved in II., we must abandon weight experiments, and proceed to the use of a full gravitation apparatus, having both masses (M and m) under control as regards temperature.

It is supposed by some that Kepler's 3rd law establishes the constancy of G, but I have tried to show† that this is false, and that the common practice of obtaining the masses and densities of heavenly bodies (sun, earth, planets, &c.) by assuming the invariability of G is at fault.

5. The above appears to me one case in which Kepler's laws have been strained beyond their legitimate use; another case was pointed out long ago by M. Vicaire.‡ He showed that when one of two attracting gravitative masses, M, is very large compared with the other, m, the fact that the acceleration of the latter is independent of its mass follows in all cases where the law of attraction is of the form

$$\mathrm{M}^k f\left(\frac{m}{\mathrm{M}}\right)\frac{1}{d^2}$$

* See Poynting's paper, 'Phil. Trans.,' 1891.

† See 'Nature,' 7 Oct., 1915.

‡ 'Comptes Rendus,' vol. 78, pp. 790 to 794.

k being any number and f any function expansible by TAYLOR's theorem which vanishes with m/M. For on expanding by MACLAURIN's theorem we have

$$\text{acceleration} = M^{k-1}f'(0)\,\frac{1}{d^2}.$$

This is independent of m and therefore satisfies the experimental facts. There is an application of VICAIRE's principle to the present subject; for k might vary according to the temperature of the large attracting body. It need only be constant as long as all the physical conditions (other than mass) are constant.

II. INDIRECT EXPERIMENTAL EVIDENCE.

1. It was pointed out by POYNTING and PHILLIPS (see paper quoted) that as regards small changes in temperature near, say, 15° C., there can be no great variation in weight with temperature as shown from various common experiments of great precision: (1) pendulum experiments give an appreciably constant value of gravity (g) regardless of the small temperature differences occurring at different times; (2) the value found for the expansibility of a liquid, say, mercury, is appreciably the same whether the dilatometer method or weight thermometer method be used. But the temperature range is very small, and we must not conclude from this or from the experiments of POYNTING and PHILLIPS on weight, that there would be no temperature influence on weight if the range of temperature were great.

2. A survey of previous researches on gravitation affords some slight information as to temperature effect:—

(1) The temperature of mountain masses and of superficial shells of the earth's surface to a depth of, say, half-a-mile, is well above ordinary laboratory temperatures. Hence values of G or of earth's mean density, Δ, obtained by these "earth" methods might differ from those obtained by laboratory methods. The most accurate earth methods, say those of MENDENHALL, PRESTON, VON STERNECK, give a rounded average for Δ of 5·4 c.g.s., whereas the best laboratory methods, say, by BOYS, BRAUN, POYNTING, RICHARZ, and KRIGAR-MENZEL, give a mean of 5·51. As the numbers stand they show a *plus* temperature coefficient for G (for G varies inversely as Δ). But inasmuch as the differences between the various "earth" results are much greater than between the numbers 5·4 and 5·51, no sure inference can be drawn.

(2) It has been pointed out by Prof. W. M. HICKS that BAILY's results for Δ show a fall in value as temperature rises. This again indicates a *plus* temperature coefficient for G.

(3) From CORNU's researches* the mean value of Δ from winter work was 5·50, from summer work 5·56. But in the absence of recorded temperatures we can deduce nothing. The apparatus in a laboratory may have a higher temperature in winter than in summer.

* 'Comptes Rendus,' vols. 76 and 86.

(4) Prof. C. V. Boys kindly gave me access to the note-books compiled by him in his research on Δ at the Clarendon Laboratory. I found there six complete experiments in which temperature was systematically recorded. Giving all the experiments equal weight, we find three whose value for Δ is below 5·52, the average being 5·517, and mean temperature 15°·9 C.; while the other four which have Δ above 5·52 average 5·528, with mean temperature 13°·8 C. This would show a *plus* temperature coefficient for G of 1/500 for 2° C. rise in temperature. In fig. 1 a graph is shown of these results.

(5) Von Sterneck* showed that the temperature gradient at a given depth in a mine varies from one mine to another, and that the mean density Δ for the whole earth, deduced from his pendulum experiments in the mines, increases as gradient increases. This again indicates a *plus* temperature coefficient. When

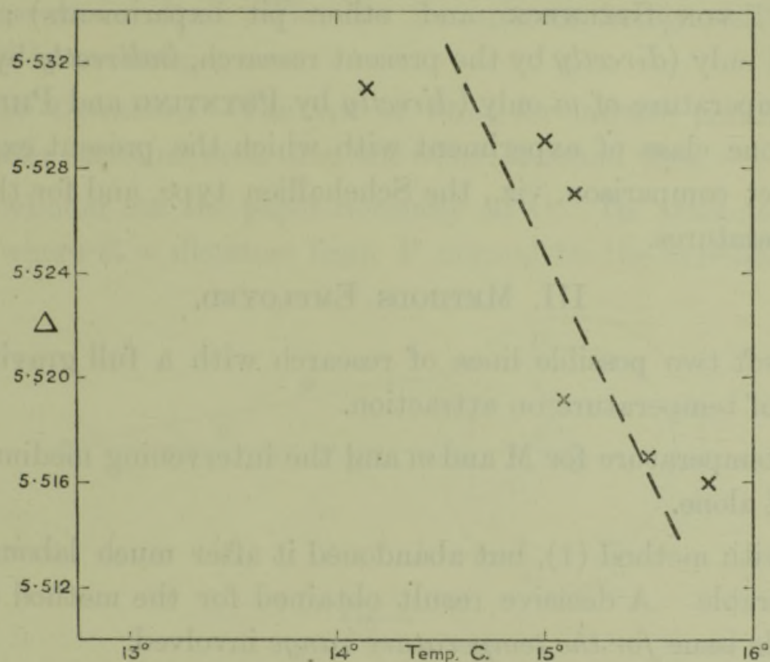


Fig. 1. Shows the relation Δ/θ in C. V. Boys' experiments.

the pendulum is swinging in a mine the strata immediately above and below would have an influence on the pendulum period out of proportion to their masses, on account of their proximity to the pendulum, consequently their temperatures would influence the results for Δ, supposing the existence of a temperature effect. Let us suppose a *plus* temperature coefficient. The strata below would always be at higher temperature than those above and would attract more strongly than the latter. In the case of high gradient this difference in attraction would be greater than when the gradient is low, and the result of the experiments would be an apparent value of Δ greater for high gradients than for low.

3. Of the five sources of evidence above, one is useless from uncertainty as to temperature; but in the other four, in all of which the direction of temperature difference is known, an apparent *plus* temperature coefficient for G is found. There

* 'Akad. Wiss. Wien.,' 1899.

is only a 1/16 chance that this accordance is mere coincidence. Yet the evidence is slender in view of the uncertainties of such delicate measurements which may involve spurious effects of this order of magnitude.

In the case of the results in BOYS' research we have data quite definite, though small, as to temperature, and they are outside the range of experimental error. On the whole we are led to expect that in a *full* gravitative experiment where both M and m are involved we should find a *plus* temperature coefficient of the order 1/1000 for 1° C.

After §§ 3 to 5 (pp. 350 and 351) we see that this result need not be at variance with the numerical figures obtained by POYNTING and PHILLIPS.

In this subject there are three classes of work, the results from which should at present be kept separate, viz. :—(1) Change in temperature of both M and m (*indirectly* by BOYS, BAILY, VON STERNECK and other pit experiments) ; (2) change in temperature of M only (*directly* by the present research, *indirectly* by MENDENHALL) ; (3) change in temperature of m only (*directly* by POYNTING and PHILLIPS).

There is only one class of experiment with which the present experiments can be brought into direct comparison, viz., the Schehallien type, and for this class we have no assigned temperatures.

III. METHODS EMPLOYED.

1. There are left two possible lines of research with a full gravitative apparatus on the influence of temperature on attraction.

(1) Changing temperature for M and m and the intervening medium ; (2) changing temperature of M alone.

I commenced with method (1), but abandoned it after much labour, the difficulties appearing insuperable. A decisive result obtained for the method (2) should go far to settle the whole issue *for the temperature range* involved.

As to the actual type of apparatus used there are three standard forms (a) the torsion balance ; (b) the weight balance ; (c) the pendulum.

The torsion balance was chosen as combining great sensitiveness with accuracy. Under favourable conditions the results agree by this method to 1/10,000 so that it affords unparalleled refinement ; but there are two great troubles attending this extreme delicacy, viz. :—

(1) The law, for maximum load on the fibre short of breaking, is

$$\text{sensitiveness} \propto 1/(\text{diam. of fibre})^2,$$

since carrying power varies as $(\text{diam.})^2$, and stiffness to torsion varies as $1/(\text{diam.})^4$. Thus for attainment of the best results one employs a small factor of safety in loading the fibre and breakages easily occur. Quartz fibres (as used chiefly here) often stand a load for many hours and then break. Hence all fibres before use must be subjected to careful load-time tests. Even after taking this trouble a small shock during installation may cause breakage.

(2) Trouble attends the use, as in these experiments, of a vacuum. The beam of a torsion balance has five degrees of freedom, and each mass suspended from the beam end has two effective degrees of freedom. In a vacuum, where damping is small, the beam system often gets beyond control.

To come to details, the large attracting masses outside the vacuum will be denoted as before by letters M, M, and the small ones inside the vacuum by letters m, m. The masses used are of various forms: (a) spheres; (b) cylinders, with axes vertical; (c) the approximately cylindrical form of vertical chains. The spherical form is the most sensitive, yet the cylinder has two advantages. It is compact for some heating purposes; also the law of force for cylinders involves distance to the first power approximately, so that error in position is not so serious as for spheres where the square of distance is involved. The law for spheres being well known, that for cylinders only will be proved :—

2. *The Law for Cylinders.*—The law of force for infinite parallel cylinders, one having much greater sectional area than the other, is found thus :—

Let the small cylinder cut the paper normally at O. Its attraction on unit mass at P $= 2\mathrm{G}m_1/\mathrm{R}$, where R = distance from P normal to the cylinder, and m_1 = mass

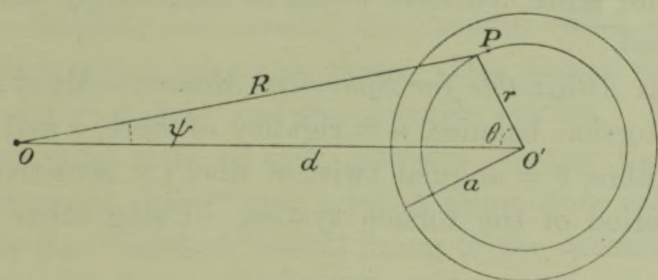


Fig. 2.

of unit length of the cylinder. Let the second parallel cylinder of large section have axis at O′ and have mass per unit length m_2. Consider element of cross-section ds at P. Its mass $= \frac{m_2\, ds}{\pi a^2}$, where a = radius of cylinder.

The total attraction per unit length is

$$\mathrm{F} = \frac{2\mathrm{G}m_1 m_2}{\pi a^2}\iint \frac{\cos\psi \,.\, ds}{\mathrm{R}},$$

the integral to be taken over the whole cross-section of the cylinder. Using the symbols in the figure

$$\mathrm{R}\cos\psi = d - r\cos\theta$$

$$\mathrm{R}^2 = d^2 + r^2 - 2d \,.\, r\cos\theta$$

$$ds = r \,.\, dr \,.\, d\theta,$$

therefore

$$\frac{\cos\psi \,.\, ds}{R} = \frac{(d-r\cos\theta)\ r\,.\,dr\,d\theta}{d^2+r^2-2d\,.\,r\cos\theta}$$

$$= \frac{1}{2d}\left(\frac{d^2-r^2}{d^2+r^2-2d\,.\,r\cos\theta}+1\right) r\,.\,dr\,.\,d\theta,$$

therefore

$$F = 2\,\frac{2Gm_1m_2}{a^2}\int_0^a \frac{r\,dr}{d}$$

$$= 2Gm_1m_2/d \qquad \text{I.}$$

for finite cylinders (M and m) whose length (l) is great *cf.* with the distance between the axes, we have approximately

$$F = 2G\,\frac{M}{l}\cdot\frac{m}{l}\cdot l/d$$

$$= 2GMm/ld \qquad \text{II.}$$

hence increase in length of the cylinder, for constant mass, decreases sensitiveness, since l increases faster than d decreases.

The exact solution for finite cylinders would be exceedingly difficult and for present purposes is not required.

3. *The Sensitiveness Attainable for Spherical Masses.*—Many factors are involved. Let a = arm of the torsion balance, n = rigidity of fibre, r and l equal respectively radius and length of fibre, θ = angular twist of fibre ($\propto$ sensitiveness), K = moment of inertia, and T = period of the torsion system. Using other letters as before, we have

$$\text{couple} = \frac{2GMma}{d^2} = \frac{\pi n r^4\theta}{2l},$$

therefore

$$\theta = \frac{4GMma}{d^2}\cdot\frac{l}{\pi n r^4}; \qquad \text{I.}$$

but $T^2 = 8\pi^2 K\dfrac{l}{\pi n r^4}$, therefore

$$\theta = \frac{4GMmaT^2}{d^2\,.\,8\pi^2 K} \qquad \text{II.}$$

When M, m are close together, m being small, we have

$$M \simeq \tfrac{1}{6}\pi\, d^3$$

therefore

$$\theta = \frac{4GMmaT^2}{(6M/\pi)^{2/3}\,.\,8\pi^2K} = \frac{CM^{1/3}maT^2}{K}$$

where

$$C = \text{constant},$$

but $K \simeq ma^2$, therefore

$$\theta = CM^{1/3}\,T^2/a. \qquad \text{III.}$$

This is the point established by C. V. Boys that for a given convenient value of T, θ varies as $1/a$ and is independent of m. Hence we make the beam short for sensitiveness. As to the value of T, we find the system is easily worked if the period does not exceed 5 mins.; but when it approaches 30 mins., the control is insufficient. It is well to aim at a value of T about 4 mins.

In my experience the best quartz fibres are of about 15μ diam. Taking this as the size of the fibre and knowing its length we arrive at m. Hence for a given value of T we obtain the length of the beam. Equation III shows that sensitiveness $\propto M^{1/3} \propto RD^{1/3}$, where R, D are the radius and density of the large sphere.

We see, then, that radius rather than density of M should be large. Thus copper would do almost as well as lead, provided it can be obtained equally free from magnetic impurities. I used lead, partly for its greater effect, partly for purity, and partly for economy, but for high temperature work where lead would melt, copper or other material of high melting point would be used.

IV. Precautions.

1. Apart from gas effects, there are two kinds of force to be avoided, viz.:—

(*a*) *Electrostatic.*—Entirely surrounding the vacuum vessel, except at the window, are metal tubes through which tap water flows. This system forms a perfect earthed screen between any external field and the movable system. But it is possible that in a vacuum charges may arise on gas particles leaving the solid surfaces after occlusion; or some internal charge may arise in some other way. Any such charge should be removed by the "earthed" metal lining to the vacuum vessel.

Again, charges arising from contact of different metals were avoided by having all the materials composing the beam system (including the lining sheath of the tube mentioned above) made of the *same* metal. Thus, in the final arrangement, the balls, m, m, the wires carrying them, the beam frame, and the mirror case were of the purest silver. The only foreign materials in the whole system were the small beam mirror and a minute amount of Margot's solder (see p. 364) used to fix the mirror case to the beam wire.

(*b*) *Magnetic.*—Impurities of iron, nickel and cobalt were avoided in the materials composing M, m and all parts connected to them. After working any of these with a tool, or after handling, the surfaces were dipped in nitric acid and well washed.

Iron screws, clamps, &c., were not used on parts of the apparatus adjoining M, m. If the internal, moving, system were entirely unmagnetic then magnetism of the outer parts would not matter; but suppose the inner system is slightly magnetic, it would respond to any magnetic influence, say, from M. When temperature is changed this response would change also, for the permeability of M would change with temperature. Thus we should have an apparent change in gravitative attraction, and this spurious effect would be hard to disentangle from the true gravitative temperature effect as they would both be cyclical.

2. We now come to the action of the gas remanent in the vacuum tube. The mechanical effects which this produces on the delicate torsion system are very complex. There are four distinct influences at work, viz., convection, radiometric pressure, discharge of occluded gases, damping :—

(*a*) *Convection.*—This causes great trouble in the ordinary gravitation experiments; here, where temperature is greatly changed, it would make measurements at normal pressures useless. Hence we use a vacuum. In the highest attainable vacuum, convection would quite vanish. We even find that for a poor vacuum of about 1 mm. it produces little disturbance in a properly screened apparatus.

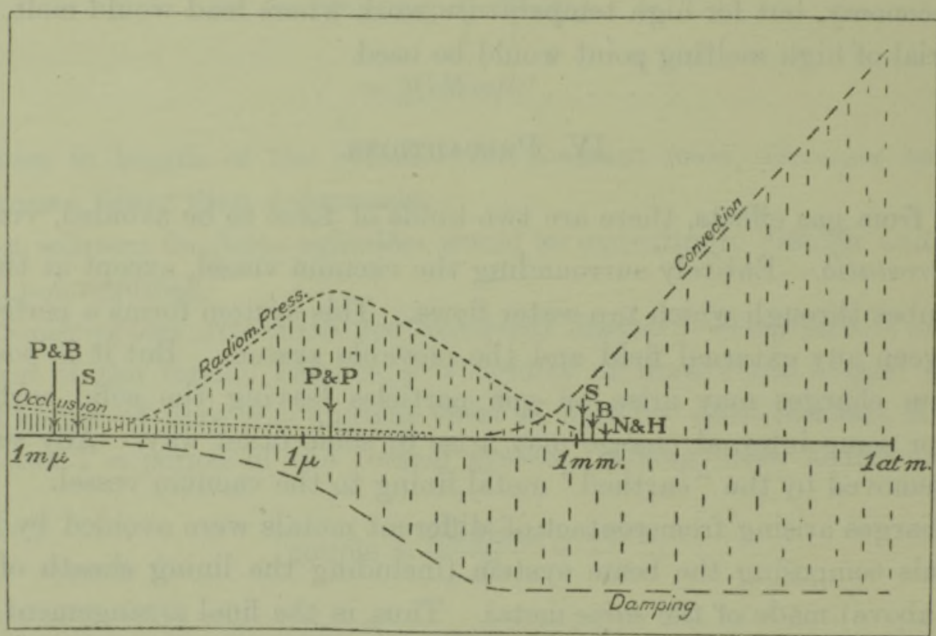


Fig. 3. Gas troubles depending on pressure in a vacuum.

(*b*) *Radiometric pressure* is a maximum at a pressure of 30μ of mercury ($\mu = 1/1000$ mm.). It has been detected at a pressure of $0{\cdot}015\mu$, but it is negligible for lower pressures and probably for pressure higher than 1 mm. with due precautions.

(*c*) The evolution of *occluded gas* from the wall of the vacuum vessel and from the torsion system should be negligible, even in a high vacuum, after long continued exhaustion with heat, as in these experiments.

(*d*) In a high vacuum the *damping* of the torsion system of great period is perceptible, though slight. But it is essential that the tremors continually received by the torsion system from the ground should be *quickly* damped out if accurate telescope readings are to be taken. The present research has been carried out in the heart of Nottingham, with a trunk railway system a quarter of a mile away, so that even in the dead of night it has been impossible to use high vacuum, except for the particular case when m, m have the form of chains. But a low vacuum of 1 mm., or greater, acts well, especially with the work performed between 9 to 5 at night.

It is important to attain the best conditions in the action of all these gas factors (which appear to be mutually incompatible).

The accompanying graph shows gas pressure as abscissæ on a logarithmic scale, while the ordinates indicate trouble due to any factor at any pressure.

The proportions are conjectural, depending on the conditions of any particular experiment. Convection is roughly proportional to pressure down to a vanishing quantity below 1 mm. Damping, unlike convection, is nearly independent of pressure to about 100μ. It is represented below the axis of abscissæ as being a blessing rather than an evil in these experiments. I have worked in these experiments chiefly at about 14 mm. and about $20m\mu$ (*i.e.*, 20×10^{-6} mm.). It will be seen that these two are favourable regions. POYNTING and BARLOW (P. and B.) used a very high vacuum in their research on recoil from light. BRAUN (B.) in the determination of the Newtonian constant had pressure of 4 mm., while NICHOLS and HALL (N. and H.) worked at 16 mm. in their research on radiation pressure.

As distinct from the above we have POYNTING and PHILLIPS (P. and P.) working in their weight experiments at 16μ which is unfavourable, as being in the thick of the radiometer region, and although convection which they specially feared would at that pressure be negligible deocclusion for the temperature 100° C. might be considerable.

3. *Radiation Pressure.*—This is yet another factor which must be borne in mind. The practice is to maintain the lamp for illuminating the telescope scale steadily alight before and throughout the experiment and to keep it practically constant in position. No other lamp comes within view of the interior of the apparatus. Moreover, for the elimination of this and of the gas actions, the walls of the vacuum are maintained uniform in temperature. Taking these precautions it is expected that the radiation pressure on the mirror of the suspended system will be constant throughout and will thus introduce no error.

Besides the many precautions mentioned, there are others incidental to the use rather than the design of the apparatus and will therefore appear in the detailed account following.

V. EARLY EXPERIMENTS.

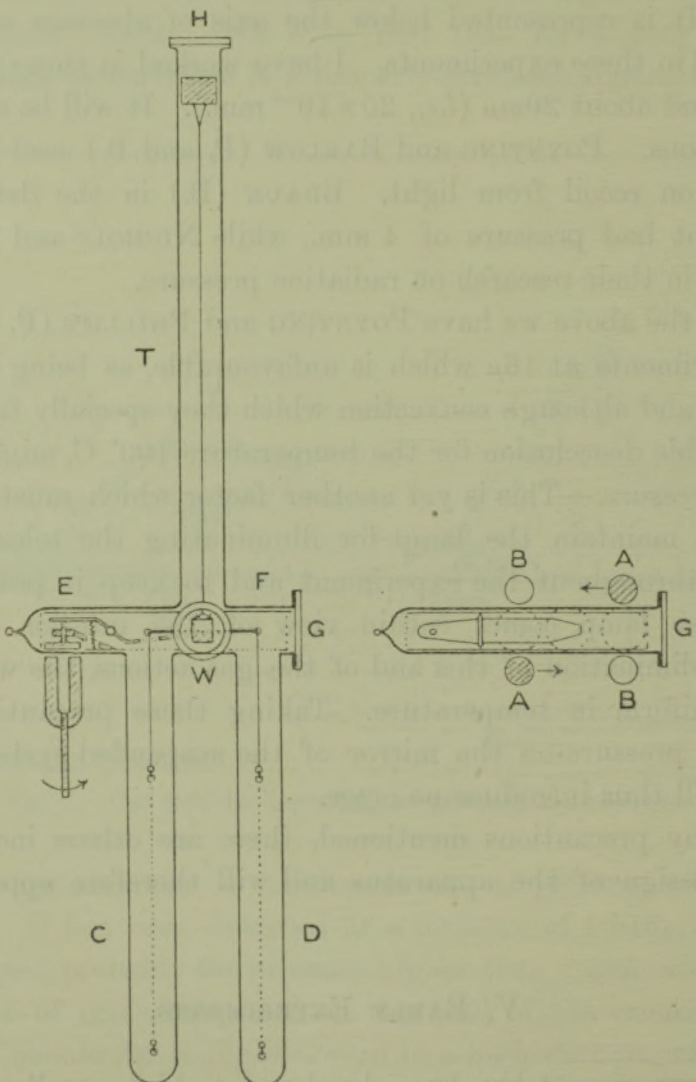
The Cavendish experiment has been developed to high excellence, but there is no previous investigation on the temperature effect to indicate the best way to avoid the many troubles sure to beset the investigation. The early methods, briefly mentioned below, failed for reasons given in each case, but they provided useful experience :—

(1) The first apparatus used was made of brass of special purity. The large masses were rods of lead coated with gold and the small ones were of purest gold wire. These were all hung together in the brass vacuum vessel. This form was

abandoned on account of the difficulty of maintaining a high vacuum in a metal vessel.

(2) Next I tried, as the vacuum case, a glass tube 160 cm. long, 2·5 cm. bore. The small masses were wires, and the large ones, now outside the vessel, were also stout wires. This particular form of suspension, if in high vacuum, retains any vibrations it may receive with great persistency, so the apparatus was set aside.

(3) The third form was a glass vessel provided with a window, W, and two other flanged openings, G, H (see elevation fig. 4, and plan section fig. 5). It was very



Figs. 4 and 5. Show the third form of apparatus used. The reason for having twin tubes is to provide for change in temperature of both large and small masses. The window is not shown in fig. 5.

sensitive; in one experiment the period was 33 mins., and the scale movement was 560 mm. The beam is shown carrying the small masses, *m*, *m* in chain form. The whole suspended system was of purest aluminium. The large masses, M, M, as shown in fig. 5, are in front or behind tubes, C, D, according to requirement.

One peculiarity of this form of suspension is that it never of itself comes to rest. For when the beam touches one tube at the end of a swing, it receives, by the tremors in the apparatus, enough energy to send it to the other end of its swing. This perpetual motion was counteracted by an arrestment (shown in the left side tube) worked by a magnet outside the vacuum. By this device the system was brought under control.

This form of apparatus was abandoned on account of an unexpected form of attraction which caused the masses, m, m, to rush to the walls of the vacuum vessel and cling there indefinitely. These forces are very strong and existed despite the fact that there was a complete "earthed" system of aluminium tubing lining the vacuum vessel and that the suspension was "earthed." After separate investigation this effect was attributed to radiometric-pressure, which cannot be avoided in this form of vacuum vessel.

In the final form of apparatus, to be next described, there are two distinct sets: (a) where the attracting masses are cylindrical; (b) where these are spherical. But as the latter type is more important, the following description applies to it throughout.

VI. Final Experiments.

1. *General Description.*—In this form one may be said to have at last attained some mastery over the investigation, so that details will be given. A glass tube, AB, (figs. 6 and 7) is 1200 mm. long, 50 mm. bore. It is supported near the top by two strong glass tubes, C, D, issuing from opposite sides of the main tube (see fig. 6). The top and bottom of the main tube are closed and there is a window, W, in front. Tube C, finishes with a platinum wire sealed in for the purpose of earthing the inside of AB; while tube D is the connection to the pumping system (see fig. 6).

2. *The Support for the Vacuum Tube.*—The glass tubes, C, D, rest on gimbals, CD, forming one axle, while a forked frame-work, V, of brass (fig. 8) carries the perpendicular axle, and itself is supported by being screwed to a stout beam. This beam, loaded by about 40 kilos. of lead weights, is carried by steel springs hung from the main scaffolding, and there are castor oil dash-pots to damp any chance vibrations which may be received from the ground.

The lower end of tube AB, socketted into a copper sleeve, *Cu* (fig. 8) is controlled by four setting screws mounted on a horizontal ring, *ab*. By this means AB can be set and held accurately vertical.

3. *The Torsion-head System.*—A brass frame-work (fig. 6) fits firmly by three pairs of brass springs into the upper part of tube, AB. These six spring points are tipped with solder to avoid the well-known danger of hard metal scratching the inner surface of the glass tube. There are two magnets in the frame-work; the lower one, m_1, carries the suspension by pin, p, which turns freely, but without shake, in the two thoroughfare bearings shown. The upper magnet, m_2, is mounted on a screw so that

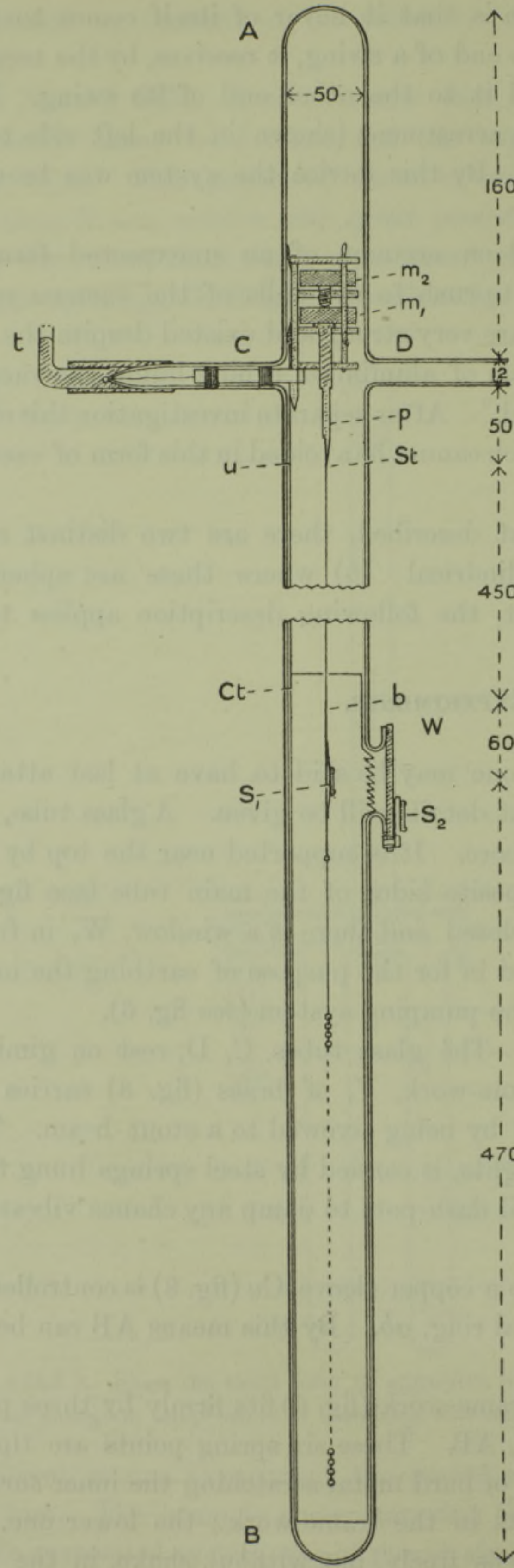


Fig. 6. Shows *m*, *m* in the form of chains.

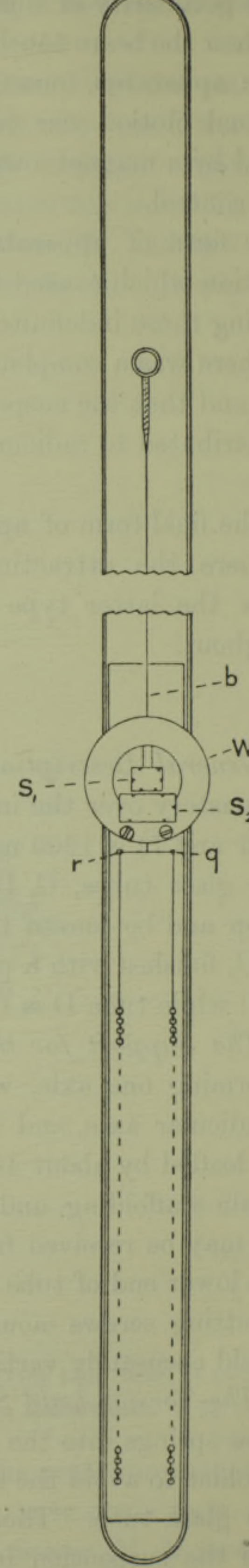


Fig. 7.

on turning one way on a vertical axis, it rises; on turning the other way, it falls, and in so doing clamps m_1 by pressing hard on it. The horse-shoe electro-magnet, E (figs. 9 and 10), can be raised or lowered to operate m_1 or m_2. By this torsion-head arrangement the suspension can be set and clamped, wherever desired, *in vacuo*.

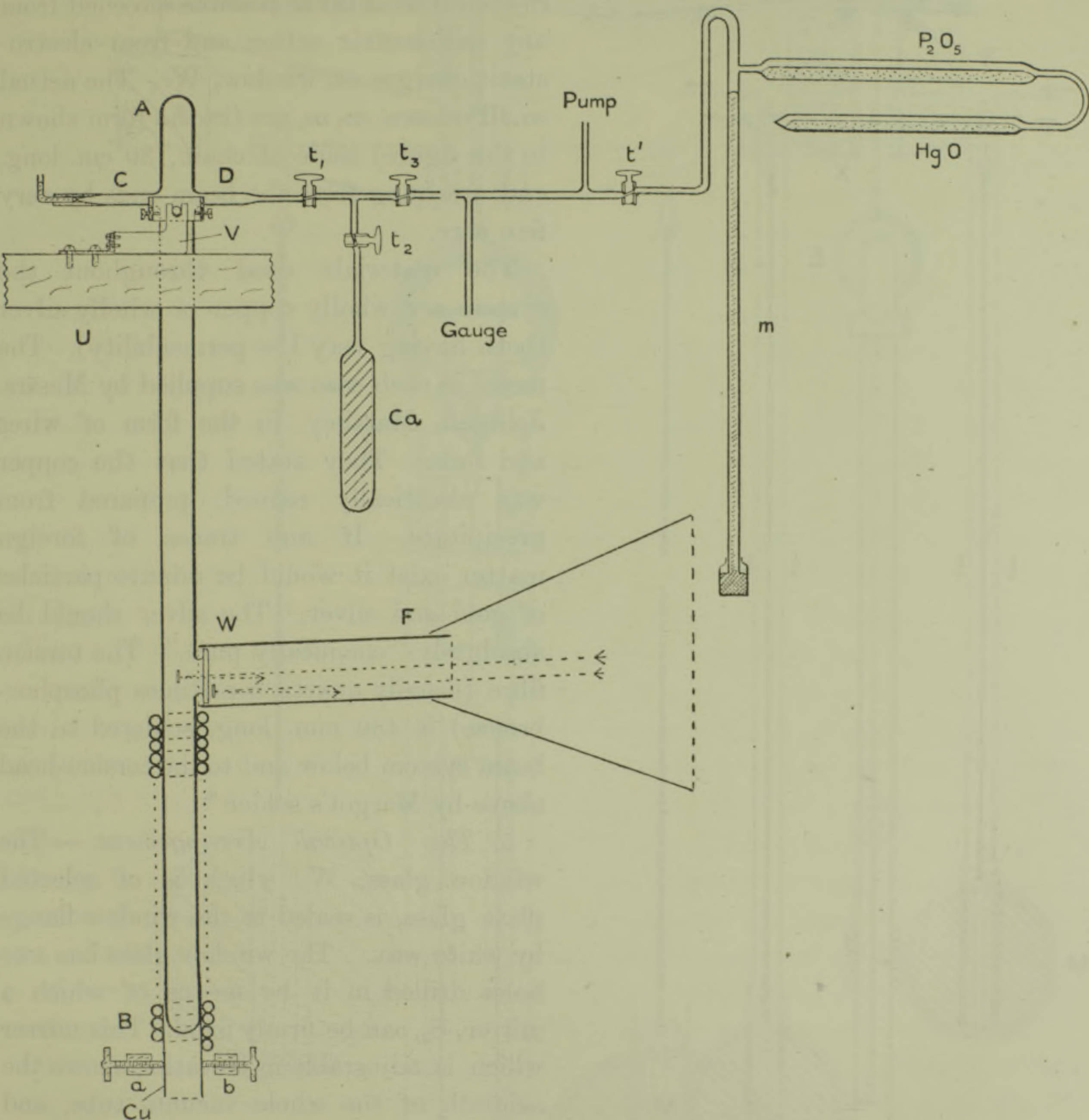


Fig. 8. Shows the tubing for evacuating and washing out with oxygen; also the tube and funnel mounted on the window. The weight of this tube and funnel is carried by a wire attached to the main scaffolding. Part of the water-jacket, but no lagging, is shown at the lower end of tube AB.

4. *The Small Mass* (m, m) *System.*—The suspension is seen in the core of tube, AB (figs. 6, 7, 9, 10). The beam and the vertical wire, b, attached to its centre are of thin wire. The mirror, S_1, is enclosed in a case of thin foil and is so attached to the vertical wire that a fine inclination up and down can be made at will. The long copper or

silver tube, Ct, inside AB, has a rosette hole (see fig. 6) cut in it at the window, W. By opening the rosette carefully it is possible to see mirror, S_1, whilst exposing little of the inside of the tube. Thus mirror, S_1 should be as far as possible screened from any radiometric action and from electrostatic charges on window, W. The actual small masses, m, m, are (in the form shown in the figure) made of chain, 30 cm. long, and are hung from the beam ends by very fine wire.

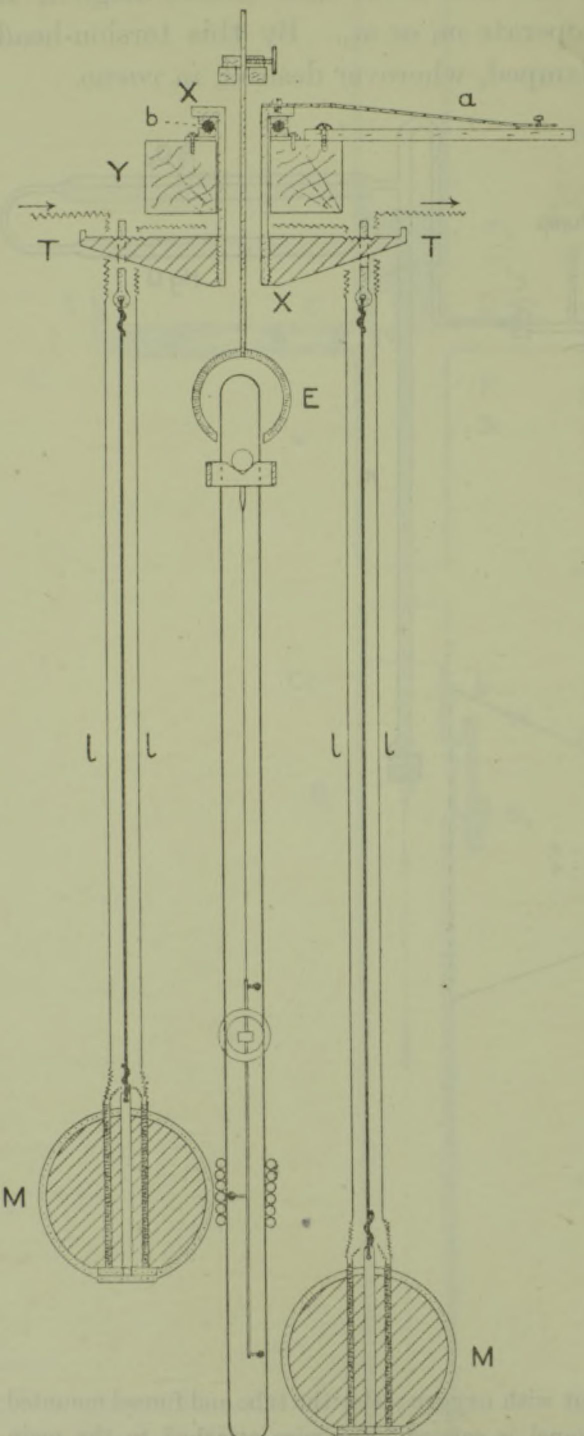


Fig. 9. Shows the final form as used with rigid suspension. (No wrappings shown.) ($\frac{1}{10}$ full size.)

The materials used throughout the system are wholly copper or wholly silver (both having very low permeability). The metal in each case was supplied by Messrs. Johnson, Matthey, in the form of wires and foils. They stated that the copper was electrically refined, prepared from precipitate. If any traces of foreign matter exist it would be minute particles of gold and silver. The silver should be absolutely "chemically pure." The torsion fibre (usually quartz, sometimes phosphor-bronze) is 480 mm. long, soldered to the beam system below and to the torsion-head above by Margot's solder.*

5. *The Optical Arrangement.*—The window glass, W, which is of selected plate glass, is sealed to the window flange by white wax. The window glass has two holes drilled in it by means of which a mirror, S_2, can be firmly fixed. This mirror which is adjustable in elevation shows the azimuth of the whole vacuum tube, and therefore that of the torsion-head, whereas mirror, S_1, shows the azimuth of the torsion beam. Thus the difference between the two telescope readings given by these two mirrors shows at any time the exact amount of torsion on the fibre with elimination of any error due to movement of tube or telescope.

* See "Sealing Metals," by P. E. SHAW, 'Proc. Phys. Soc.,' Feb., 1912.

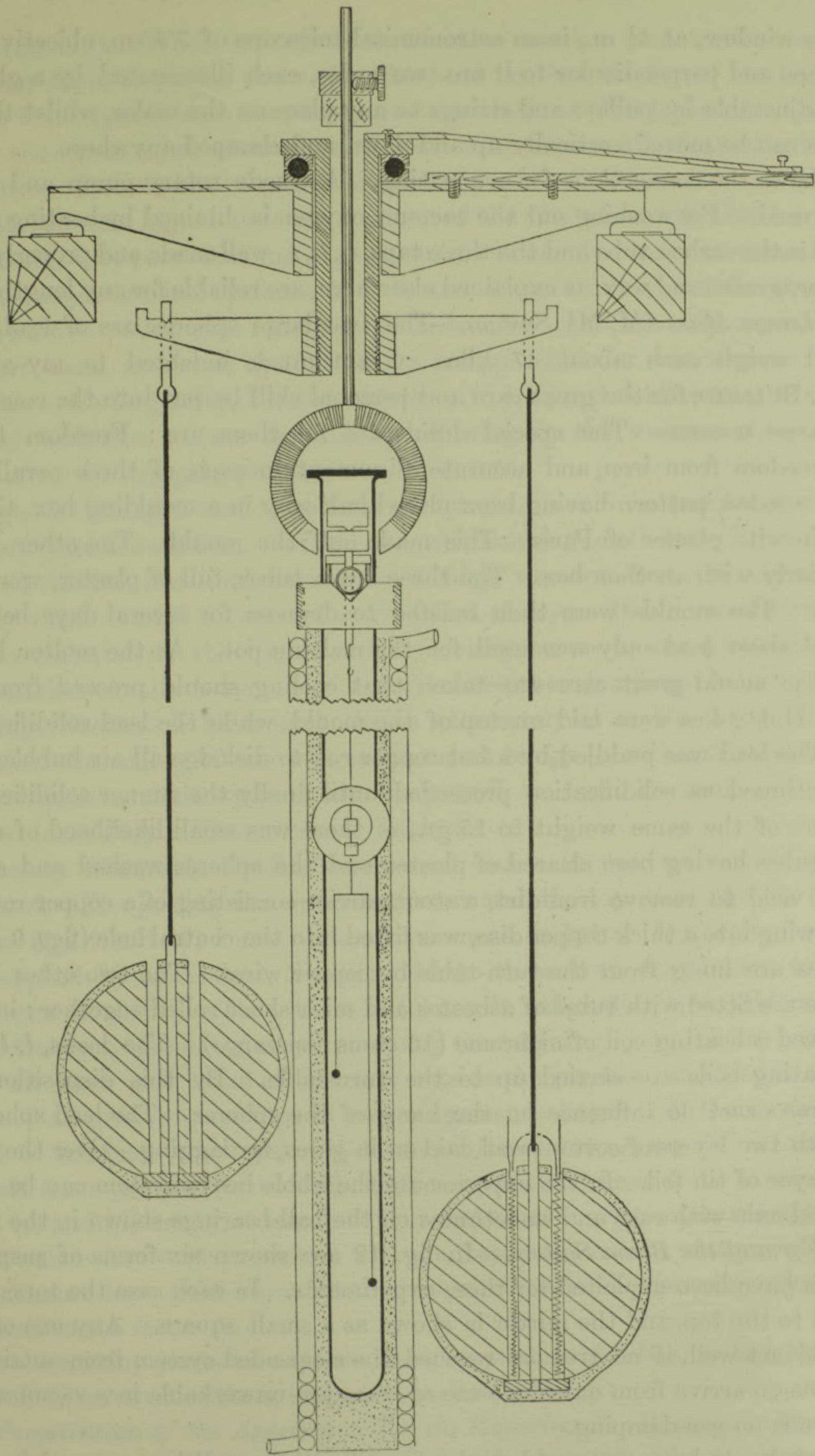


Fig. 10. Shows the final apparatus ($\frac{1}{6}$ full size) with limp suspension. This is the best arrangement. For clearness no lagging outside the water jacket is shown. The large amount cut out half way up the diagram will be appreciated by comparison with fig. 9, which has identical essential parts.

Facing the window, at $4\frac{1}{2}$ m., is an astronomical telescope of 7·5 cm. objective. Over the telescope and perpendicular to it are two scales, each illuminated by a glow lamp which is adjustable by pulleys and strings to any place on the scales, whilst the scales themselves can be moved vertically up and down and clamped anywhere.

Fig. 8 shows the connections for evacuating. A Gaede rotary pump and McLeod gauge are used. For washing out the vacuum, oxygen is obtained by heating mercuric oxide. Ca is the carbon tube and the three taps, t_1, t_2, t_3, well-made and properly treated with rubber, grease and wax, as explained elsewhere, are reliable for any length of time.

6. *The Large Mass* (M, M) *System.*—The two large spheres are of lead, 20 cm. diam., and weigh each about 47 kilos. I am much indebted to my colleague, Prof. C. H. Bulleid, for the great care and personal skill he put into the construction of these large masses. The special desiderata for them are: Freedom from air bubbles, freedom from iron, and accurate alignment in each, of three parallel brass tubes. A wooden pattern having been placed half-way in a moulding box, the latter was filled in with plaster of Paris. This made half the mould. The other half was made similarly with another box. The three brass tubes, full of plaster, were placed in position. The moulds were then roasted to dryness for several days before use. Commercial sheet lead only was used for the melting pot. As the molten lead was run into the mould great care was taken that cooling should proceed from below upwards. Hot cokes were laid on top of the mould, while the lead solidified at the bottom. The lead was puddled by a hot copper rod to dislodge all air bubbles. This process continued as solidification proceeded until finally the runner solidified. The spheres were of the same weight to 15 gr., so there was small likelihood of enclosed air. The tubes having been cleared of plaster and the spheres washed and swabbed with nitric acid to remove iron dirt, a stout carrier consisting of a copper rod, 1 cm. diam., screwing into a thick copper disc, was fitted into the central hole (figs. 9 and 10). The spheres are hung from the turn-table by copper wires. The two other holes in each sphere are fitted with tubes of asbestos and mica sheet rolled together; into each is then placed a heating coil of nichrome (16 ohms resistance). The leads, l, l (fig. 9) for the heating coils are carried up to the turn-table. By this disposition these leading wires exert no influence on the hang of the spheres. The lead spheres are covered with two layers of cotton-wool, laid on in gores, for lagging. Over the cotton-wool is a layer of tin foil. In the experiments the whole heavy system can be rotated on a vertical axis with ease and smoothness on the ball-bearings shown in the figures.

7. *The Form of the Beam System.*—In fig. 12 are shown six forms of suspension, all of which have been exploited for these experiments. In each case the torsion fibre is attached to the top, and the mirror is shown as a small square. Any one of these forms would act well, if no tremors reached the suspended system from outside, but as vibrations do arrive from outside, some of them are unworkable in a vacuum vessel where there is no gas-damping.

It is a novelty to have a suspended system of limiting sensitiveness used in a high vacuum, so that the technical difficulties are new. To explain the action, consider form 3 (fig. 12). When a horizontal vibration reaches the torsion head of the suspending fibre, it passes down the fibre and reaches the top of the beam system, which at first experiences a simple horizontal vibration. Next, the vibration is carried down to the wires, m, m, each of which is set vibrating on a horizontal axis about its mass-centre. Soon we have a certain amount of pendular motion of two unequal periods. When at last the high frequency tremors of the fine wires and fibre have died out, there remain, very persistent in a high vacuum, the low frequency movements of m, m.

All the unsymmetrical systems, 3, 4, 5, and 6, are troublesome for these reasons, whereas forms 1 and 2, give relatively little trouble. But the unsymmetrical form is, for a short beam, as shown by C. V. Boys, indispensable for sensitiveness. Working in the heart of a large city, I failed with Nos. 3, 4, 5, and 6 after long trial, and only succeeded with No. 7 (see fig. 10) by allowing in the vacuum chamber a small amount of air for damping purposes.

It might be thought that some form of damping would be possible so as to render any beam system workable. Of the known damping methods (1) gas-friction and (2) liquid-friction are inadmissible in a high vacuum, but (3) electro-magnetic damping which I tried in several ways, failed always. It is impossible to have any magnetic material on the beam, and it is risky to have, say, a closed copper wire circuit carried on the beam. But there is one other damping method possible, viz., (4) rolling friction. I have used chains extensively as in No. 2 (fig. 12) and have found that the rolling of one link on the next brings in rolling friction to damp out tremors. Such a chain system acts very well in high vacuum, even in a very disturbed laboratory. (See results in Table I.)

8. *Sealing Materials Used.*—The use of waxes in vacuum vessels is now well understood. For joining the optical window and the top glass plate to the vacuum tube (see figs. 9 and 10) I use (a) Faraday cement, or, better (b) a white "vapour-free" wax (supplied by Lilliendahl, Neudietendorf); these both seem much tougher than sealing wax or shellac. Then there is (c) a soft red sealing wax sold commercially. I never trust any ground glass joint, or any mercury-trap joint, or any platinum seal, but in all cases melt some of this wax outside the junction, and also on the top and bottom of all ground glass taps to ensure against leakage. (d) Ramsay's tap grease is used to lubricate the taps. At moderate temperature, say 60° C., vapour comes off freely from (a) and (b), so such high temperatures have to be avoided for these seals. (c) and (d) are by no means vapour-free at ordinary temperatures, but the only way in which they come in contact with the vacuum is on the vacuum taps; and the small amount here used soon becomes vapour-free in the vacuum, without serious detriment to the latter.

There are many joints in a Gaede mercury pump which become leaky periodically. It is well to serve all of them with some melted wax (c).

9. *Preparation of the Apparatus for an Experiment.*—Suppose in this instance a copper internal system is to be used. The long glass tube (fig. 6) as received from

the glass blowers is open at top, A, and has the two side tubes welded on and the window flange prepared. First the inside is given a thick coat of silver up to about 20 cm. above the window flange. This coating reduces radiation during the experiment and enables one to thoroughly "earth" that part of the vacuum surrounding the suspended system. The central portion, DW, of the main tube is wrapped externally with asbestos paper, and over that eureka wire is wound to act as a heating coil. The brass tube already mentioned, with platinum wire at one end, is inserted in tube C, and the glass sealing on tubes C and D is completed. A mercury seal t outside the platinum seal is shown. The copper inner sheath is lowered into the main tube and the rosette at the window opened; the strap, U, which carries the sheath, is fitted to the brass tube in C to keep the sheath in position. The optically-true window is now sealed on to its flange.

The torsion system is next prepared. From a frame holding some hundred quartz fibres of 60 cm. length and suitable diam. (*i.e.*, from 21μ to 15μ diam.), four or five have been chosen several days before and all have been attached to a horizontal bar and loaded with a weight about 25 per cent. more than the actual load used in the experiment. It is impossible to predict to within 50 per cent. what a quartz fibre of known diameter will carry. A fibre may sustain a load for several hours or even a day and then break without external shock being applied. Hence every fibre is given a three or four days' test before use. The finest fibre which has stood the test is fixed by Margot's solder to the torsion head above and to the beam system below, which in the present case we will suppose has form 2, fig. 12. The whole suspended system, except the small mirror, is of the purest obtainable metal, say copper. After the beam system has been put together, but before the fibre is attached, the whole of it, except the mirror, is immersed in nitric acid, and then thoroughly washed, to remove any trace of iron which may have become attached by tools or by the hands during construction.

It is important to see at this stage that the mirror is properly inclined to the vertical as no adjustment of it can be performed later in the vacuum.

Next the torsion system, *i.e.*, torsion head, fibre and beam system, are lowered with the utmost care to prevent breakage of the fibre, into the vacuum tube till the torsion head rests socketted in a hole prepared in the brass tube, C (fig. 6). Several asbestos paper discs are placed on the top of the torsion head frame to keep the latter cool in the next stage. The top of the main glass tube is next sealed. This sealing of a 5 cm. tube requires two operators, one on each side with a blowpipe. Care is taken to keep heat from the torsion head just below. In later experiments a glass plate was waxed on the top (see figs. 9 and 10). The vacuum tube and contents is now cautiously removed and set on its gimbals in the supporting frame. The adjusting screws below are so set that the suspended system swings free of the tube walls so as to be ready for the experiment. The vacuum side tube, D, is sealed to the carbon tube, Ca, and this to the pump (fig. 8).

Preparation of the Vacuum.—A heating coil of eureka wire is pushed up over the lower section, WB (fig. 6), and another put on the upper section, AC, of the main tube. The middle section, as stated, is already wired. Also the whole of the glass tubing connecting to the pump and McLeod gauge, and the oxygen plant are similarly wired. The large carbon tube has a special heating coil of several layers designed to attain a high temperature. The whole system of heating coils is put in series with a lighting circuit and a current of 3·5 amperes is developed. The injector pump and Gaede pumps are started. The temperature attained by various parts is arranged according to requirement. The lower part of the vacuum tube, WB, attains a temperature of 130° C. The upper tube containing the soldered joints is not raised much above 100° C, while the sealed-on plates are kept as a rule below 60° C. Any small condensation of vapour on the cool window can be carefully removed by placing a carbon glow-lamp in front of it for a short time. The carbon tube is raised to 340° C., as shown by a platinum thermometer placed between it and the heating coil. The double process of heating and exhausting proceeds for five days (say 50 working hours), during which time the McLeod gauge should show steadily improving vacuum. At night, when the evacuation is stopped, the taps have to be shut in such order that the rapidly cooling main tube shall not receive vapour from the hot carbon. On the last day of exhaustion the vacuum would be, say, 10μ, when the temperature is full everywhere. Then the vacuum is washed out, once or twice, with oxygen to remove traces of the less absorbable gases, nitrogen, helium, argon. Finally on cutting off the heat the pressure will drop to the smallest readable by the McLeod gauge, say, $0{\cdot}05\mu$. The carbon will then be about 200° C. The taps are closed till, when temperature everywhere is normal, the taps t_1 and t_2 are opened, and all three taps have warm siegelwachs run over them. The glass between tap t_3 and the pump is sealed off.

Evacuation being finished, the apparatus has to be prepared for the experiment.

Adjustments.—First, the torsion head is turned so that the beam mirror, S_1 (figs. 6 and 7), exactly faces the window; the electro-magnet, E, is lowered until its poles are level with the torsion head magnet, and by rotation of E the torsion head is brought to the correct azimuth. By raising E to the level of the clamping magnet and again suitably rotating the former, the inside magnet is brought down to bear hard on the torsion head and thus fix it. Under this condition the mirror, S_2, fixed to the window will exactly stand for the position of the top of the fibre. Rotation of one always accompanies that of the other, both being rigidly attached to the main tube.

Next, the loose heating coils are stripped off and the whole vacuum system, including the connecting tubes wrapped in several layers of cotton wool. Before, however, the cotton wool is put on two platinum thermometer wires are wrapped on the glass of the vacuum tube, one above, and one below the window, and compensating copper wires are arranged.

In providing a water-jacket to screen the vacuum tube from the hot masses, M, M, it must be borne in mind that temperature should be uniform radially rather than longitudinally in the long tube; for radiometric pressure, not convection, is likely to be the main gas trouble. The plan adopted is to have water tubes wound over the whole vacuum tube up to the level of the top of the torsion fibre. Similar metal water tubes are wound over the tube covering the window. In fig. 8, the covering tube, but not the water tubes, are shown. There is a space of about 6 cm. near the window on the long tube which cannot be wrapped with metal tube, so rubber tube is used, with a specially large amount of closely-packed cotton-wool, in this region. Thus we have several layers of cotton wool both within and outside the water helix; while the hot spheres, M, M, are themselves covered doubly in cotton-wool and then in tin-foil.

Water from the tap passes to the window region and branches into four parts, viz.: (*a*) the upper tube; (*b*) the lower tube; (*c*) the window cover; and (*d*) the window region. Each section thus receives water at one and the same temperature.

After leaving the water-jacket, the water passes into vessels where its temperature can be watched. A Page thermostat was attached, but proved unnecessary.

Next, the whole system of M, M, with the turn-table above (see figs. 9 and 10) is set co-axial with the torsion fibre, *i.e.*, until masses, M, M, clear the wrapped tubes equally all round; M, M being so far apart as to just clear the water-jacket everywhere. The heating coils of nichrome inserted in insulating tubes in M, M (see fig. 9) are connected in series with a 200-volt circuit, the leads being taken right up the copper suspending wires to the turn-table to avoid any hampering of the free motion of the heavy mass system.

The last adjustment is to bring the beam to rest. The period of the beam may be anything from 2 mins. to 10 mins. The angle of rotation after the above violent adjustments is sure to be considerable and in high vacuum the swings are practically undamped.

There are three ways of reducing the swing:—

(1) By moving the vacuum tube to one side and so bringing it against the beam system. Sometimes this succeeds, but great judgment and timing are required or the amplitude will be increased instead of decreased by the contact.

(2) Unclamp the torsion head and rotate the magnet, E, and so time the movement of the head that the torsion of the fibre is always acting against the motion of the beam. The head must be left clamped finally.

(3) Gravitational damping by moving large masses, M, M, to always oppose the motion of the beam system. This method is too weak to be effective, except as a finishing touch to reduce the swing when it is already small, say, 5 degrees of arc.

The process of damping the swing may take many hours, but it must be done so that finally the mirror when at rest would face the centre of the scale (of 500 mm.) which is placed about 5 m. away and immediately over the large reading telescope.

The apparatus is now ready, but it is found that no accurate work can be performed for several days after the rough handling incidental to the adjustments. Since the fibre has a factor of safety of only 50 per cent. or so, great care is taken to avoid any jolts to the apparatus; the movements given to the beam are due not to jolts, but to the occasional contact of the torsion system with the vacuum tube.

A drift of zero is found to occur, perhaps due to small elastic after-effect following the recent heating. I have found that in some unsymmetrical systems *in vacuo*, the zero never comes to rest, since the violent tremors set up by recurrent outside vibrations cause either constant strains in the fibre, or constant unbalanced pendular oscillations. However, under favourable conditions, readings can commence in three days.

The next step is to ascertain the best angle at which the turn-table must be set for

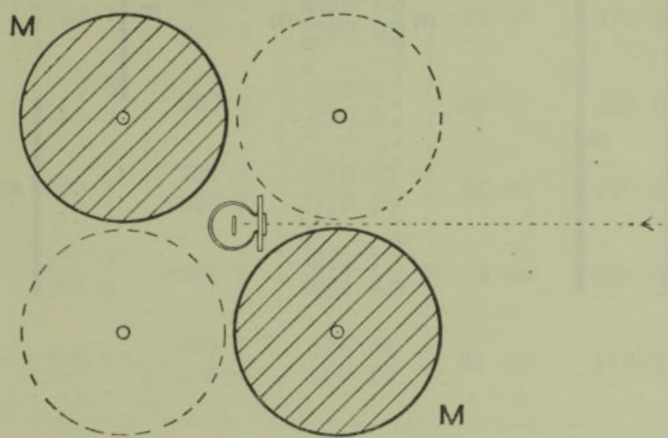


Fig. 11. This plan section shows the disposition of the large masses in the A and B positions.

maximum deflection of the beam in the A and B positions (see fig. 11). For spherical masses the formula given by C. V. Boys* is very useful, viz.:—

$$\cos^2\theta + \frac{a^2+c^2}{ac}\cos\theta - 3 = 0,$$

where θ is the angle for maximum attraction,

c is distance between centre of beam and that of large mass;
a is half the length of the beam.

Determine θ in terms of a and c. But as it is most important both for sensitiveness and accuracy to have the exact positions of maximum attraction, the matter is settled by trial, and when the best positions on the dial of the swing table are found stops are placed so that in the coming experiment the same angle is used in every case.

Before commencing actual readings, it must be decided whether or not the carbon tube is to be used cooled by liquid air. In the earlier experiments, liquid air, boiled under reduced pressure to give a temperature of $-200°$ C., was used. Later this was given up, as the very exhausted carbon at ordinary temperature acted well enough.

* 'Proc. Roy. Soc.,' 46, 1889.

Supposing the carbon is not to be cooled, it is given layer after layer of cotton wool and is otherwise screened from heat rising from below.

10. *Description of Experiments.*—At the commencement of readings, set the masses M, M, at position A (shaded in fig. 11). Watch the beam mirror with the telescope

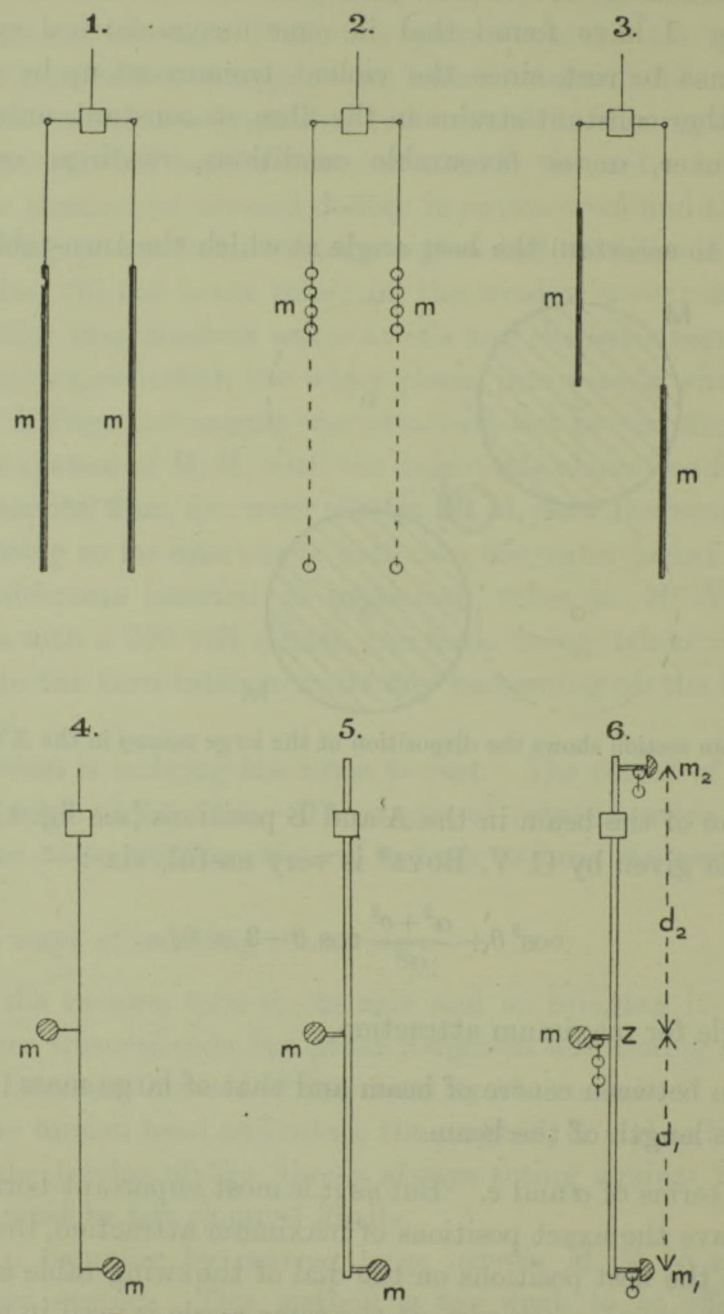


Fig. 12. Various forms of suspension used.

and set down the extreme readings as the beam swings. When three such readings have been taken, rotate M, M to position B, and proceed as before. For every position, A or B, the tube mirror reading (S_2, fig. 6) is taken by the telescope.

In Table I. are shown the results of an experiment with cylindrical masses.

TABLE I.—June 23, 1914. Silver Chains (m, m.). Lead Cylinders (M, M).
Vacuum Pressure, 1·0μ. Oscillation Period, 252 minutes.

Position.	θ_L.	θ_T.	L.	R.	Mean.	T.R.	Corrected mean.	Range.	Mean range.
I. Steady Cold.									
	° C.	° C.							
A	17	17·0	191·8 192·1	263·8	227·8 227·9	*94·5*	227·85		
								57·48	
B			132·7	208·1 207·8	170·4 170·25	94·45	170·37		
								57·63	
A			193·8 194·0	262·0	227·9 228·0	94·45	228·0		
								57·68	57·57
B			133·7	207·0 206·7	170·35 170·20	94·45	170·32		
								57·48	
A			187·8 188·0	267·7	227·7 227·8	94·45	227·8		
								57·58	
B			136·8	203·7 203·4	170·25 170·10	94·45	170·22		
II. Steady Hot (after heating for five hours).									
B	210	18·5	133·4	213·5 213·3	173·45 173·35	94·7	173·20		
								57·70	
A			194·9 195·3	267·2	231·05 231·25	94·95	230·90		
								57·65	
B	212		139·0	208·6 208·4	173·80 173·70	95·2	173·25		
								57·87	
A		18·5	193·3 193·6	269·4	231·35 231·50	95·0	231·12		
								57·65	57·67
B			137·6	210·1 209·8	173·85 173·70	95·0	173·47		
								57·60	
A			196·9 197·0	265·8	231·35 231·40	95·0	231·07		
								57·72	
B			139·0	209·0 208·8	174·0 173·9	95·3	173·35		
								57·52	
A	212	18·5	193·6 193·9	269·2	231·4 231·55	95·3	230·87		

In and after column 4, all readings are expressed in millimetres.

TABLE I. (continued).

Position.	θ_L.	θ_T.	L.	R.	Mean.	T.R.	Corrected mean.	Range.	Mean range.
III. Falling Temperature.									
	° C.	° C.							
B	177	18·5	137·3	210·2 209·8	173·75 173·55	95·70	172·65		
								57·85	
A			194·0 194·0	269·3	231·65 231·65	95·75	230·50		
								57·50	57·65
B	160		138·7	209·6 209·4	174·15 174·05	95·80	173·0		
								57·65	
A	152		198·0 198·2	265·3	231·65 231·75	95·80	230·65		

In and after column 4, all readings are expressed in millimetres.

Column 1 shows the position of masses, M, M. The next two columns have the temperatures of the masses, M, M, and of the vacuum tube, respectively. Columns 4 and 5 give the extreme left and right scale readings, as the beam swings. Column 6 shows the arithmetic mean of the two preceding column readings, while column 8 shows this value corrected for change, if any, in the tube reading (column 7). The rest is obvious, heat being applied as already explained. There are three sections in the experiment, I., steady cold; II., steady hot; and III., falling hot. By combining I. and II. we obtain as temperature effect

$$\alpha = +0{\cdot}8 \times 10^{-5} \text{ per } 1^\circ \text{ C.}$$

By combining I. and III., we find a like result.

It will be observed that the vacuum is high; the pressure $1{\cdot}0\ \mu$ is calculated from the damping of the oscillations.

Only this one example for cylinders will be quoted in full, for, though the readings are steady, the subsequent work with spheres is more sensitive and more reliable in general.

In Table II. are entered the results of an experiment with spheres. Here we have seven sections, the last half, hot and cold, being taken ten days after the first. It will be seen that the interval of rest makes no appreciable difference in the result. Column 2 shows the time, column 3 the reading of the tube (reference) mirror. The fourth column has the extreme scale readings right and left as they occur, and the

Table II.—July 27, 1915. Silver Balls (m, m). Lead Spheres (M, M). Vacuum Pressure, 14 mm. Oscillation Period, 280 minutes.

Position.	T.	T.R.	R.	Amplitude.	Z.	Range.	Mean range.	θ_L.	θ_{LT}.
								° C.	° C.
A	4.11	345·0	37·5	230·3				17	13·0
			267·8	155·3	175·0				
			112·5	104·3	174·9				
			216·8		(174·95)				
						200·75			
B		345·35	493·0	195·5					
			297·5	131·4	376·1				
			428·9	88·4	376·0				
			340·5		(375·7)				
						200·85	200·85		
A		345·1	53·3	204·0					
			257·3	137·8	175·0				
			119·5	92·7	174·9				
			212·2		(174·85)				
						200·95			
B		345·4	425·9	83·0				17	
			342·9	55·9	376·3				
			398·8	37·8	376·2				
			361·0		(375·8)				
Heat at 8.2. Stop Heat at 10.2.									
B	10.23	345·45	241·8	225·0				206	12·3
			466·8	151·6	376·2				
			315·2	102·1	376·2				
			417·3		(375·75)				
						201·05			
A		345·0	19·4	259·8				198	
			279·2	174·8	174·7				
			104·4	117·6	174·7				
			222·0		(174·7)				
						201·25			
B		345·35	476·9	168·3				191	
			308·6	113·2	376·3				
			421·8	76·1	376·3				
			345·7		(375·95)				
						201·25			
A	10.55	345·0	52·7	204·0				183	
			256·7	137·1	174·7				
			119·6	92·2	174·7				
			211·8		(174·7)				
						201·3	201·25		

Readings in columns 3, 4, 5, 6, 7, 8 are in millimetres.

TABLE II (continued).

Position.	T.	T.R.	R.	Amplitude.	Z.	Range.	Mean range.	θ_L.	θ_{LT}.
Heat at 8.2. Stop Heat at 10.2 (continued).								° C.	° C.
B	11.6	345·3	482·1 305·1 424·1 344·1	177·0 119·0 80·0	376·3 376·3 (376·0)			174	
						201·35			
A	11.22	345·0	40·4 264·7 114·0 215·6	224·3 150·7 101·6	174·6 174·7 (174·65)			163	
						201·25			
B		345·3	492·9 297·9 428·8 340·8	195·0 130·9 88·0	376·2 376·2 (375·9)				
						201·15			
A	11.50	344·95	50·3 258·4 118·3 212·7	208·1 140·1 94·4	174·7 174·7 (174·75)			148	12·4
Pause.									
B	3.21	345·2	483·9 303·5 425·0 343·3	180·4 121·5 81·7	376·1 376·1 (375·9)				
						200·8			
A		344·85	45·7 262·1 116·4 214·1	216·4 145·7 97·7	175·0 174·9 (175·1)			71	
						200·8	200·85		
B		345·15	464·8 316·5 416·0 349·2	198·3 99·5 66·8	376·1 376·0 (375·9)				
						201·0			
A	3.54	344·8	52·0 257·5 118·9 212·3	205·2 138·6 93·4	174·7 174·7 (174·9)			66	

Readings in columns 3, 4, 5, 6, 7, 8 are in millimetres.

TABLE II. (continued).

Position.	T.	T.R.	R.	Amplitude.	Z.	Range.	Mean range.	θ_L.	θ_{LT}.
				Pause.					
								°C.	°C.
B	7.2	345·0	467·2	152·9				43	
			314·3	103·1	375·9				
			417·4	69·4	375·9				
			348·0		(375·9)				
						201·1			
A		344·7	53·0	203·3					
			256·3	136·8	174·5				
			119·5	92·1	174·5				
			211·6		(174·8)				
						200·85			
B	7.24	344·95	477·0	169·4				41	
			307·6	114·0	375·7				
			421·6	76·7	375·7		200·95		
			344·9		(375·75)				
						200·9			
A		344·7	269·0	158·2					
			110·8	106·7	174·5				
			217·5	71·8	174·6				
			145·7	48·2	174·5				
			193·9		(174·85)				
						201·0			
B	7.52	344·95	461·2	142·7				40	
			318·5	95·8	375·8				
			414·3		(375·85)				
				Pause for 9 days. August 4, 1915.					
A	9.50	342·7	61·5	184·9				17	
			246·4	124·2	172·1				
			122·2	83·5	172·1				
			205·7		(172·1)				
						200·85			
B		343·0	473·0	166·8					
			306·2	112·1	373·3				
			418·3	75·5	373·2				
			342·8		(372·95)				
						200·7			
A	10.19	342·7	47·9	208·1					
			256·0	140·1	172·3				
			115·9	94·1	172.2		200·80		
			210·0		(172·25)				
						200·7			
B		343·0	469·6	161·3					
			308·3	108·7	373·2				
			417·0	73·1	373·3				
			343·9		(372·95)				
						200·8			
A	10.45	342·75	35·0	229·6				17	13·5
			264·6	154·5	172·2				
			110·1	103·9	172·2				
			214·0		(172·15)				

Readings in columns 3, 4, 5, 6, 7, 8 are in millimetres.

TABLE II (continued).

Position.	T.	T.R.	R.	Amplitude.	Z.	Range.	Mean range.	θ_{L}.	θ_{LT}.
Start Heat at 11.5. Stop Heat at 1.5.									
								° C.	° C.
A	2.5	343·0	46·8 257·0 115·2 210·7	210·2 141·8 95·5	172·3 172·3 (172·0)			180	13·1
						201·2			
B		343·2	480·8 302·0 422·0 341·0	178·8 120·0 81·0	373·8 373·6 (373·2)		201·2	173	
						201·2			
A	2·27	342·95	51·2 253·7 117·5 209·0	202·5 136·2 91·5	172·3 172·2 (172·0)			166	

Readings in columns 3, 4, 5, 6, 7, 8 are in millimetres.

fifth column shows the amplitude of swing. The next column has the rest position (Z) calculated from the formulæ

$$Z = c - \frac{(c-b)^2}{(a-b)+(c-b)} = b + \frac{(c-b)^2}{(c-b)+(c-d)}$$

where a, b, c, d, are the readings in order in column 4. These formulæ are based on the supposition that damping is so small that the successive amplitudes may be considered to be in geometric progression. The numbers in brackets in column 6 denote the mean value of Z corrected for the change, if any, in the reading of the reference mirror. The seventh column shows the range, *i.e.*, the corrected scale reading when passing from A to B position or *vice versa*. The eighth column has the temperature of the lead spheres as given by mercury thermometers. The last column shows the temperature of the outside of the vacuum tube below the window, as given by a platinum thermometer. The change in temperature varies up and down without much connection with the temperature of masses, M, M. Likewise the change of temperature in the water leaving the water-jacket amounts to 0·25° C., due mostly to change in temperature of the tap water. These small changes sometimes up, sometimes down, cannot influence the general result. As a rule the water temperature is very steady for long periods. After a set of readings at low temperature, the thermometers are removed from the lead spheres and the nichrome heating coils are substituted. When, after say two hours, the lead spheres are at a high temperature, the thermometers are again substituted for the coils, so that for the set of hot readings all the conditions are exactly the same as for the cold readings,

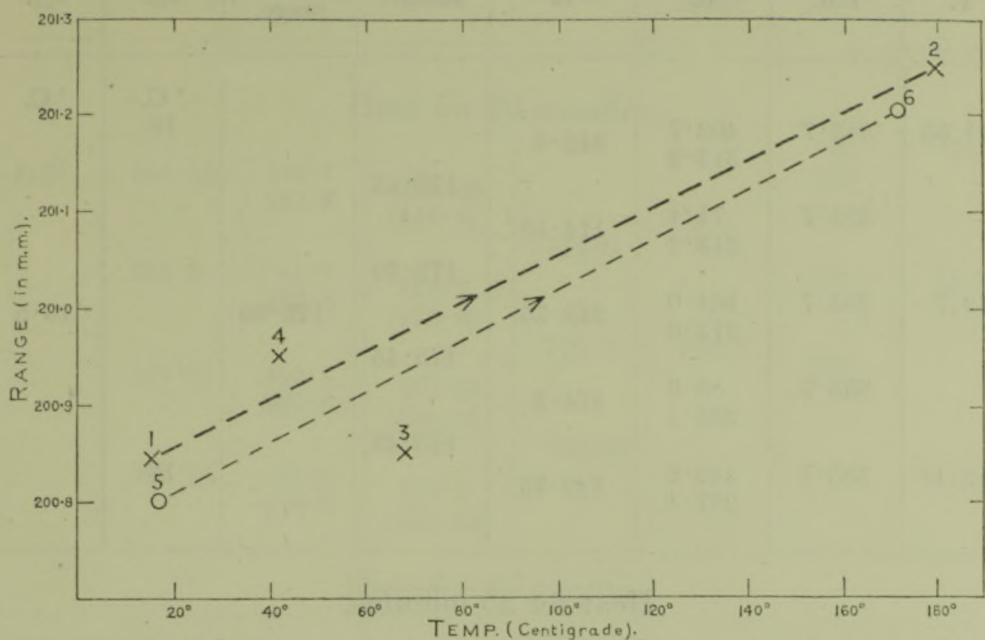


Fig. 13. Graph for Table II. The crosses show the first series of readings, the numbers being in order. The circles show the second series.

except as regards the raised temperature in masses M, M. A graph is given for the result. The effect is

$$+1\cdot2/10^5 \text{ per } 1^\circ \text{ C.}$$

Table III. is another instance. It differs in some details from the preceding table. In particular, the fourth column consists of only two entries at each A, B position. These are two consecutive extreme scale readings. Call them a, b. Then if Z be the rest position, we have, supposing geometrical progression in the amplitudes,

$$Z = \frac{ad + b}{1 + d}$$

where d = decrement (*i.e.*, ratio of one swing to the preceding one).

The value of d is found independently before or after the experiment. The method used here has several advantages over the usual method of taking three or more extreme scale readings, viz. :—

(1) The experiment is shortened. It may be very tedious when, as here, each half-swing takes 160 secs., and the whole series lasts 6 to 8 hours.

(2) It is important when the large spheres are hot that they should remain as short a time as possible at any one part of the vacuum tube. We reduce the time of rest at any place by this process to about two-thirds of the least time possible by any other means.

TABLE III.—August 21, 1915. Silver Balls (mm.). Lead Spheres (M, M). Vacuum Pressure 14 mm. Oscillation Period 280 minutes.

Position.	T.	T.R.	R.	Z.	Range.	Mean range.	θ_L.	θ_{LT}.	θ_{HT}.
							° C.	° C.	° C.
B	11.55	353·7	402·7 313·9	349·6			18		
					175·45				
A		353·7	71·4 243·2	174·15					
					175·60				
B	12.7	353·7	464·0 273·0	349·75		175·50		13·5	13·5
					175·45				
A		353·7	85·6 233·9	174·3					
					175·45				
B	12.18	353·7	442·6 287·4	349·75			18		
Heat for 35 minutes.									
B	1.37	354·15	407·7 311·0	349·9 (349·45)			88	13·5	
					175·55				
A		354·15	75·4 240·8	174·35 (173·9)			86		
					175·75	175·65			
B	1.47	354·15	436·9 291·8	350·1 (349·65)			84		
					175·70				
A		354·2	80·9 237·15	174·35 (173·95)			83		13·5
Heat for 30 minutes.									
B	3.36	354·6	410·4 311·0	350·95 (350·05)			216		
					176·2				
A		354·6	77·3 240·5	174·85 (173·95)			210		13·6
					175·9	175·95			
B		354·5	443·0 288·6	350·65 (349·85)			206	13·5	
					175·75				
A	3.53	354·5	83·4 236·3	174·9 (174·1)			202		

Readings in columns 3, 4, 5, 6, 7 are in millimetres.

TABLE III. (continued).

Position.	T.	T.R.	R.	Z.	Range.	Mean range.	θ_{L}.	θ_{LT}.	θ_{HT}.
Heat for 90 minutes.									
							° C.	° C.	° C.
B	4.27	354·55	436·7 292·8	350·65 (349·8)			215		13·8
					176·0				
A		354·6	81·7 273·3	174·7 (173·8)			211	13·5	
					176·3	176·1			
B		354·55	440·1 291·0	350·95 (350·1)			206		
					176·05				
A		354·55	84·0 236·0	174·9 (174·05)					
Heat for 30 minutes.									
B	5.30	354·6	415·0 307·5	350·7 (349·8)			233		
					175·95				
A		354·6	76·9 240·5	174·75 (173·85)			227		
					175·95	175·9			
B		354·5	438·5 291·5	350·6 (349·8)			222		
					175·7				
A		354·5	81·8 237·4	174·9 (174·1)			216		
Heat for 40 minutes.									
A	6.48	354·1	75·9 240·9	174·6 (174·2)			246	13·6	
					175·8				
B		354·1	436·4 292·6	350·4 (350·0)			242		
					175·95				
A		354·05	72·6 242·8	174·4 (174·05)		175·95	237	13·6	
					175·95				
B		354·0	452·7 281·5	350·3 (350·0)			231		
					176·0				
A		354·0	88·0 232·3	174·3 (174·0)			224		

Readings in columns 3, 4, 5, 6, 7 are in millimetres.

TABLE III. (continued).

Position.	T.	T.R.	R.	Z.	Range.	Mean range.	θ_L.	θ_{LT}.	θ_{HT}.
				Pause.					
							° C.	° C.	
B	8.55	353·25	442·2 287·1	349·45 (349·9)			143	13·7	
					175·5				
A		353·3	71·7 242·7	174·0 (174·4)			139		
					175·55				
B		353·3	444·1 286·0	349·55 (349·95)		175·65	136		
					175·7				
A		353·35	85·5 233·3	173·9 (174·25)					
					175·8				
B		353·35	445·6 285·3	349·7 (350·05)			130		
				Pause.					
B	11.7	353·5	424·0 299·5	349·55) (349·75)			90		
					175·75				
A		353·5	79·7 237·1	173·8 (174·0)		175·75	88		
					175·75				
B		353·5	439·3 289·2	349·55 (349·75)					

Readings in columns 3, 4, 5, 6, 7 are in millimetres.

There are no temperatures for water shown in this table. Long experience shows that the temperature of the outflow remains sufficiently steady, and the scale readings settle down provided the water has been set running abundantly for an hour or two previously. The rate of flow of water remains constant, say 5 litres/minute, throughout the whole experiment. A graph is given of results in Table III. The temperature effect works out as

$$\alpha = +1{\cdot}3 \times 10^{-5} \text{ per } 1^\circ \text{ C.}$$

The above three examples given are among the most extensive and successful taken.

The process in the three differs in important respects and the conditions differ as to (*a*) vacuum pressure ; (*b*) sensitiveness ; and (*c*) lagging.

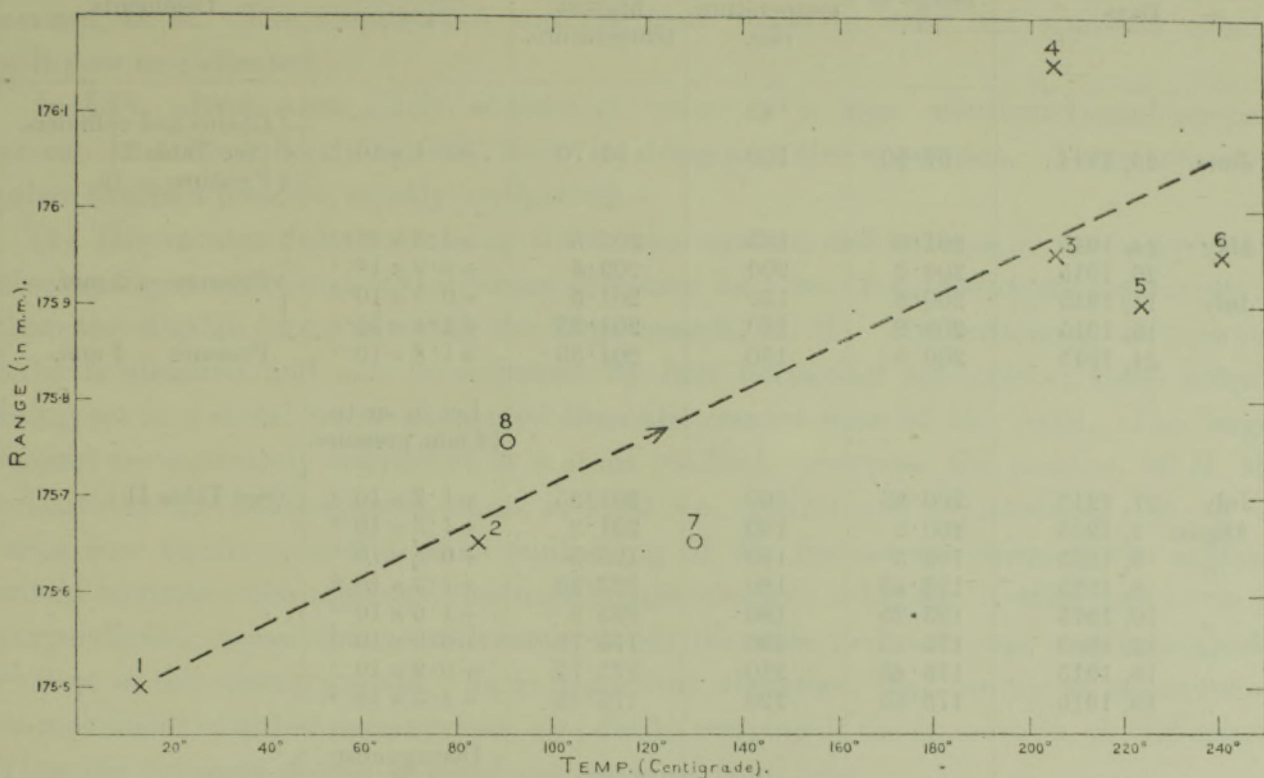


Fig. 14. Graph for Table III. The first three crosses show rising temperatures, the next three are for temperatures high but variable, and the two circles are for falling temperatures.

Out of a total of many scores of experiments eighteen are summarised in Table IV. A great number of experiments have failed because the conditions were unsuitable ; for instance, the apparatus may be changed in the attempt to gain accuracy or to apply some test—but this change produces, sometimes, results which are non-cyclic. Or, a new vacuum pressure may be tried, which is unsuitable, the torsion system getting out of control through convection or tremors. In a *few* cases small *negative* temperature effects have been found, but in every case the results have been non-cyclic ; for instance, a negative effect when temperature rises may be followed by a positive effect when it falls again. The net result for both rise and fall is found invariably to be a positive effect. But it is non-cyclic and is considered worthless. These negative effects are due to strain and change in position of the parts of the apparatus and have always been removed by proper care.

In Table IV. the high value $+1 \cdot 8 \times 10^{-5}$ and the low value $0 \cdot 5 \times 10^{-5}$ occur and are included in finding the arithmetic mean, which might be considered to be vitiated by their presence. But it is noteworthy that the longest and best experiments give results about $+1 \cdot 2 \times 10^{-5}$ which agree with the mean result and so tend to confirm it.

TABLE IV.—Summarised Results of Several Experiments.

Date.	Range at 15°.	Greatest temperature rise.	Range at highest temperature.	Effect for 1° C.	Comments.
		° C.			
June 23, 1914 . .	57·60	150	57·70	$+1\cdot1\times10^{-5}$	Chains and cylinders. (See Table I.) Pressure = 1μ.
May 24, 1915 . .	201·9	185	202·5	$+1\cdot6\times10^{-5}$	Pressure = 2 mm.
" 26, 1915 . .	209·2	200	209·6	$+0\cdot9\times10^{-5}$	
July 14, 1915 . .	200·8	135	201·5	$+0\cdot9\times10^{-5}$	
" 16, 1915 . .	200·8	180	201·35	$+1\cdot4\times10^{-5}$	
" 24, 1915 . .	200·8	150	201·35	$+1\cdot8\times10^{-5}$	Pressure = 4 mm.
				Let in air to 14 mm. pressure.	
July 27, 1915 . .	200·85	160	201·25	$+1\cdot2\times10^{-5}$	(See Table II.)
August 4, 1915 . .	200·8	160	201·2	$+1\cdot3\times10^{-5}$	
" 5, 1915 . .	196·3	140	196·5	$+0\cdot8\times10^{-5}$	
" 8, 1915 . .	193·45	180	193·95	$+1\cdot3\times10^{-5}$	
" 10, 1915 . .	193·25	180	193·9	$+1\cdot6\times10^{-5}$	
" 12, 1915 . .	175·45	190	175·75	$+0\cdot9\times10^{-5}$	
" 18, 1915 . .	175·45	210	173·75	$+0\cdot9\times10^{-5}$	
" 19, 1915 . .	175·55	220	176·15	$+1\cdot5\times10^{-5}$	
				Demagnetise balls.	
August 21, 1915 . .	175·5	200	175·95	$+1\cdot3\times10^{-5}$	(See Table III.)
" 22, 1915 . .	199·5	200	199·7	$+0\cdot5\times10^{-5}$	
" 24, 1915 . .	201·25	210	201·65	$+1\cdot0\times10^{-5}$	
" 25, 1915 . .	190·75	230	191·2	$+1\cdot1\times10^{-5}$	
				Rotate M, M by 180°.	
August 30, 1915 . .	185·7	150	186·2	$+1\cdot7\times10^{-5}$	
Mean effect				$+1\cdot2\times10^{-5}$	

The readings in columns 2, 4 are in millimetres.

It will be observed that the vacua used vary from 1μ (for chains) down to the low vacuum 14 mm., which proved most satisfactory when tried for spheres, and was used always afterwards.

VII. TESTS APPLIED.

We have now proved that there is a temperature effect which repeats itself with as much consistency as can be expected in a delicate apparatus, where the effect observed is only 0·5 mm. of scale reading. But this effect need not be a gravitation/temperature effect, but may be wholly or in part due to systematic errors in the work. The results are, of course, as the tables show, outside the range of observational error; but the delicate torsion system would be very susceptible to spurious effects, due directly or indirectly to the heat in the large masses, M, M. The steps taken and the tests made to weed out spurious effects will now be indicated.

In § IV. above some likely sources of error have been mentioned, and special precautions were taken against them in designing the apparatus. There are some other troubles possible, mostly mechanical.

(1) The vacuum tube containing the torsion system and spheres, m, m, hangs from steel springs, and is steadied in every direction by a set of rubber corks separating it from the wooden frame carrying the large masses, M, M. The bottom of the vacuum tube is steadied and can be adjusted by four horizontal set-screws; these screws being set in a metal frame supported from the cement floor of the room. The large masses are separately supported by a stout scaffold. Suppose the position of M, M, relative to the small spheres, m, m, is ideal, *i.e.*, central and symmetrical. If m, m were now to move in the plane containing M, M, the sensitiveness of the system would increase—the telescope readings would change; whereas, if they moved in a perpendicular sense the sensitiveness would decrease and telescope readings would change in the contrary sense. It is important, therefore, that during an experiment no movement of either mass system, *e.g.*, due to warping of the framework, should occur. Thus the supports should be rigid and screened from heat.

(2) The rubber corks, mentioned above, must be maintained firmly in place.

(3) It is possible that the masses, M, M, might be hung under strains by the copper wires, and when heating occurs this strain might vary. This possibility was tested (and found non-existent) by turning both masses, M, M, by 180° C. on a vertical axis, as in the last experiment in Table IV.

(4) The centre of mass of the spheres, M, M, might be displaced vertically by expansion as temperature rises. This movement would decrease the couple, due to M, M, on the torsion system, supposing the centres of M and m lie exactly in a horizontal line. The masses, M, M, are carried by copper discs (see fig. 9) attached to copper rods which pass through holes in the masses. The radius of each mass is 10 cm. Thus, when temperature rises 200° C. we have the centre of mass rising $2{\cdot}8 \times 200\mu = 560\mu$, due to expansion of the lead; and falling $3{\cdot}2 \times 200\mu = 640\mu$, due to expansion of copper (the coefficients of the two materials being 28×10^{-6} and 16×10^{-6} respectively). The net fall in the centre of mass would then be 80μ. Now the large masses, M, M, are 150 mm. from the small ones, m, m, and the small cosine error resulting from the vertical movement 80μ, will be found negligible. If, however, due to error in setting, M and m are not on a horizontal level, but that the former be 5 mm. above its ideal position (an error of setting which is most unlikely) we should then find that the fall in the lead would give an effect of order 10^{-5}, which is negligible compared with our effect of 2×10^{-3}.

(5) If the temperature of the quartz torsion fibre rises during the experiment, its rigidity will change and the telescope readings will be affected.

In Table III. the indicated temperature change in column 9 is 0·2° C. from beginning to end of the experiment. There is reason to think that this change is almost entirely due to heat getting into the thermometer above the water cooler where it would not affect either the torsion fibre or the lower part of the suspension. But suppose for present purposes we allow a temperature rise of 0·3° C. Fuzed quartz has temperature coefficient -1×10^{-4}; thus the fibre would be *stiffened* by $0\cdot3\times10^{-4}$. Our temperature effect is 20×10^{-4}. So we see that this error may be considered negligible numerically and it acts in *the wrong direction.*

The following list comprises the sources of error mentioned here and in § IV. The first four are due to heat entering the vacuum tube, the last six to the effects of heat on the masses, M, M, and other parts external to the vacuum :—

1. (Temperature change in fibre).
2. (Convection).
3. Radiometric pressure.
4. Radiation pressure.
5. (Electrostatic forces).
6. Magnetic forces.
7. (Movement of base of vacuum tube).
8. (Movement of top of vacuum tube).
9. (Rotation of M, M).
10. (Rise or fall of M, M, due to expansion.)

The numbers of this list enclosed in brackets have already been dealt with and may be considered to have been eliminated by precautions already indicated. Nos. 3 and 4 may be grouped together as they will invariably act in conjunction ; thus we have two errors left to deal with.

No. 6.—The susceptibility of purest lead is -1×10^{-6}; that of iron for weak (earth) fields is, say 10. Thus a trace of iron, 1 part in 1,000,000, would mask any magnetic effect of the lead. Commercial lead has traces of iron varying from 1/300,000 to 1/5000 according to the source.* Suppose the lead used for M, M is of the worst commercial quality. The spheres, M, M, each weigh 50 kilos. so that a rod of iron 20 cm. long weighing 10 gr. should have a greater magnetic influence than the presumed iron impurity in the lead. I placed iron rods of this mass in each sphere, and found the spheres, thus loaded, acted appreciably in the same way, as regards temperature effect, as when unloaded. Thus the temperature effect on the susceptibility is negligible.

We have so far considered only the large spheres, M, M ; but it is evident that if the small spheres, *m*, *m*, are quiteun magnetised, the susceptibility of M, M would be immaterial. Separate tests were therefore made on the magnetism of *m*, *m*, and these

* See THORPE'S 'Dictionary of Applied Chemistry.'

tests may be considered crucial. Curiously, it was found that though these were the purest silver obtainable from Messrs. Johnson, Matthey and Co., they were both in slight degree permanently magnetised. This is in itself an interesting fact. The torsion system used here is, in this case, a most sensitive magnetometer and when a N. pole of a bar magnet was brought outside the vacuum, slightly above one silver sphere, it repelled it, the corresponding scale reading being 4 mm.; whereas when the N. pole was below, there was a like attraction. This permanent magnetism was removed with some difficulty by the passage of a current through a coil round the vacuum tube, until at last the scale reading was reduced to the small amount of 0·2 mm. for each sphere. Thus the permanent magnetism was reduced to 1/20th of its former amount. Several days later the spheres, m, m, were again tested and were found to remain demagnetised. Probably the permanent magnetism was originally produced when the heating coils round the vacuum tube were excited during production of the vacuum. (See § VI., paragraph 9.)

The above demagnetisation of m, m occurred on date 20th August, 1915, after which, as is seen from Table IV., several full experiments were performed yielding results similar to those found before demagnetisation. From this one may deduce that magnetisation does not influence our gravitation/temperature effect.

Nos. 3 and 4.—These are taken last, their effect being the most difficult of all errors to dissect out from the net result. No calculation can be made of the forces set up due to their action since we do not know the amount or distribution of the supposed irregular heating on the inner face of the vacuum tube. The apparatus was designed and fitted up to avoid these errors to the utmost. There are a great number of layers of cotton wool, paper and flannel both inside and outside the helical water-jackets. Also extra screens were in some cases arranged to keep heat from the vacuum tube. The water passing through the water-jacket was steady in temperature for long intervals; any small rises in temperature due to the source would, in a long series of experiments, pair off against similar falls. The water flowed at a great rate, say, 3 to 6 litres/minute. It is hard to believe that any heat from the thickly-lagged lead spheres could penetrate the water-jacket and cotton wool under these conditions. But one can never be sure, without special tests, that heat will not find some joint or weak spot in the lagging and so reach the vacuum tube somewhere. If the inside of the vacuum tube were warmed irregularly we might have radiometric pressure effects; but even then one would expect no change in the range (see Table II., column 7), but rather a removal of the same range up or down the scale. The special tests applied were:—

(a) The tap water was heated before entering the helix and it was found that a change in telescope readings occurred at first, but that when the temperature became steady, though raised, the readings became normal.

(b) More and yet more wrappings were put over the helixes, so that weak places, if any existed, would be covered up. It will be observed that the numbers in column 2,

Table IV., vary for spheres from 209 to 175. This is due to the fact that as more wrappings are used, the masses, M, M, can be swung through less angle; hence the range is reduced. Thus we have some eight degrees of wrapping indicated in the above table.

(*c*) The outer layer of cotton wool was covered with tin-foil, and several experiments were performed in which the parts of the helix coverings, near the hot spheres, M, M, were sprayed with tap water every two minutes. The experiments on dates 18th August, 1915, and 19th August, 1915, in Table IV., were conducted thus. In this way the helix coverings remain cool always.

Seeing then that similar results are obtained under all these varying conditions as to temperature and conductibility of the surface of the central tube containing the vacuum, it seems permissible to state that the effect observed is not attributable to radiometric pressure or other forces due to the entrance of heat through the walls of the vacuum tube. A further reason why radiometric pressure should be counted out is that like effects have been found for vacuum pressures varying from 2 mm. to 14 mm., so that the corresponding radiometric pressures, if existing, would drop in the ratio about 5 to 1. Yet Table IV. shows no material difference in the results throughout these changes.

In like manner, none of the other possible errors, scheduled above, seem capable of giving the observed temperature effect. So, unless some other error has been overlooked or some agency, at present unknown, comes into play, we must conclude that, at least for lead there is a temperature effect of gravitation. The plotted results in figs. 13 and 14 do not lie on smooth curves. But it will be observed that the scale of ordinates is very open.

Still, provisionally, the foregoing numerical results have been worked out on the basis of a linear relation,

$$f = \mathrm{G}\,(1+a\theta)\,\mathrm{M}m/d^2$$

Where α is a temperature coefficient of amount

$$+\ 1{\cdot}2\times10^{-5} \text{ per } 1^\circ \text{ C.}$$

It will be observed that my results overlap the weight experiments of POYNTING and PHILLIPS (§ I., 3) for range of temperature 20° C. to 100° C. I cannot speak with any certainty as to the effect in my apparatus for this small range. If the effect be linear, the scale movement would be under 0·2 mm., *i.e.*, too small an amount to be sure about. But the effect may increase much faster than temperature, in which case one could not expect to observe any result for a range 20° C. to 100° C. It must also be remembered that (as stated in § II.) these experiments are not strictly comparable with the weight experiments, and so, for various reasons, we see that the two results, which appear to differ considerably, do not necessarily clash. No other investigations on the subject have yet been made.

VIII. Conclusion.

1. *Technical Summary and Suggestions.*—As the experimental work has been long and troublesome, it may not be out of place here to summarise the more important technical difficulties met and overcome. To perform this research it has been found necessary :—

(1) To obtain a sealing material for making joints between a quartz or metal fibre and any metal including aluminium, so that the joints should stand a temperature of 160° C. and considerable load. No wax would do, and no ordinary solder with flux, but the alloy of 88 Zn/12Sn acted perfectly.* In the final experiments a quartz fibre, 15μ diam., was attached thus to brass above and silver below, the load being seven gm. (which is little short of the breaking load). This stood for seven months though subjected to lateral shock and long-continued temperature of 130° C.

(2) To realise a system of a delicate torsion balance in a high vacuum, provided with an optically true window. After long trial a wax of high melting point was found for fastening the two windows, but, of course, no wax is vapour-free when heated. Yet heating is essential for the production of high vacua. The joints containing the wax had, therefore, to be carefully treated in the preparation of the vacuum.

(3) To test the quartz fibres. These fibres, while splendid in torsional qualities, are most uncertain as to tensional strength. Systematic testing was therefore always adopted.

(4) To damp out tremors in a delicate torsion system in a high vacuum while leaving the main (torsional) oscillation free. None of the many known methods of damping is here permissible. But if a chain be suspended from the torsion beam, its links provide rolling friction at every tremor and the damping is excellent. Without this chain method, such measurements as these with high vacua would appear to be impossible under the conditions of the experiment.

(5) To guard a delicate torsion system from external vibrations so as to make it useable even in a large city. The apparatus, heavily loaded with lead, was suspended by steel springs and steadied on all sides by rubber bungs. The vault in which the experiments were made is immediately under a workshop. Yet with this system, continuous readings could be taken even when the lathes and anvil were in use. Again, the movement of the heavy masses, M, M (total weight 100 kilos.) throughout the experiments, caused no trouble.

No doubt these investigations will be repeated and extended. In the general design I cannot suggest any improvement. The final form I used, which in type resembles that originated by Prof. C. V. Boys, worked excellently and was sensitive. I have long felt that the greatest defect, or rather weak spot, in the apparatus lies in

* See Shaw, "Sealing Metals," 'Proc. Phys. Soc.,' January, 1912.

the window, which necessitates a break in the water-jacket system to allow light to enter the window and return to the telescope; and though an elaborate system of water helix and lagging is provided in the window region (not fully shown in any figure) one feels that here there is a joint in the armour through which heat may enter. The ideal vacuum tube would have no front window on the wall of the tube. The light from the scale would enter the top window and pass by two totally reflecting prisms to and from the mirror back to the telescope. The optical difficulty in realising this plan would no doubt be great, but if it were possible we should have the great advantage of a *continuous* water-jacket, without joints, from end to end of the vacuum tube.

Suppose, however, the ideal arrangement could not be made to work, one seems to have only one alternative if the window defect, above indicated, is to be minimised. This alternative is to reduce the bore of the vacuum tube to, say, 36 mm., and the beam to, say, 25 mm. The window would be reduced proportionally and it might then be possible to have a continuous water-jacket with a small slit opening at the window. In reducing the beam as just suggested we should have the further advantage of reducing the period of oscillation to one half. Sensitiveness would be slightly lessened, but spurious heat effects, if any, would be greatly reduced.

2. *General Summary*.—I. It has been found possible :—(*a*) to obtain consistent cyclic readings in a gravitational experiment of the Cavendish type, even though the large masses are maintained for hours above 200° C., while the small masses remain at ordinary temperature; (*b*) to carry on this investigation in the centre of a city at any time by day or by night, in spite of the attendant tremors and the special disadvantage of having the torsion balance in a vacuum.

II. The conclusion reached is that there is a temperature effect of gravitation. *When one large mass attracts a small one, the gravitative force between them increases by about 1/500 as temperature of the large mass rises from, say, 15° C. to 215° C.* At present the result is provisionally stated as being $+1{\cdot}2\times10^{-5}$ per 1° C.; but the readings are not steady enough to justify the statement that there is a *linear* relation for G/θ. It seems possible that time may be a factor in the effect; but the net result has not been shaken by a long series of tests.

III. The above result, though new, is not entirely unsupported by other experiments, for previous gravitation experiments give indirect evidence of a positive temperature coefficient. The weight experiments of POYNTING and PHILLIPS, which yielded negative results, are not strictly comparable with mine.

IV. As a bye-product of these experiments, it was found that silver balls of the highest purity, after being heated to 130° C. and placed in a strong magnetic field, were permanently, though weakly, magnetised, and that the coercivity was considerable. This is probably due to residual iron, see § VII. 6.

V. Several technical troubles overcome during the research are summarised above.

For much general assistance from friends and colleagues, during the progress of this work, I gladly acknowledge my indebtedness. All the readings have been taken by myself; but on many occasions during the preparation of the apparatus, I have received most willing aid in various ways. To the many who have thus assisted me in accomplishing the present work I wish to record my thanks. I am also much indebted to Messrs. Baird and Tatlock, Hatton Garden, E.C., for great help in glass blowing.

This investigation could not have started nor continued without considerable expenditure on apparatus. At the kind suggestion of Prof. W. H. HEATON, the Council of University College, Nottingham, made a generous grant at the outset, eight years ago; while Prof. E. H. BARTON has greatly helped by his unquestioning supply of material.

Finally, I wish to acknowledge my obligation to Prof. C. V. BOYS and the late Prof. J. H. POYNTING for kind advice at the commencement of the work.

[*Notes added April 25, 1916.*—Since the reading of this paper two possible sources of error, in addition to those in the table above (p. 386), have been suggested:—

I. As temperature rises, the air surrounding the spheres, M, M, will decrease in density, so that the total gravitative pull felt by the torsion system, due to the external system, will be reduced.

Let ρ be the mean density of lead and lagging and let ρ' be the mean air density. The effective large mass is really $M(1-\rho'/\rho)$. Both lead and air expand, and we must calculate (a) the mass of cold air displaced by the expanding lead; (b) the mass of air expelled from the field by expansion of air shells round the lead.

Under the first head. Let the sphere have radius r and expand to $(r+\delta r)$, with rise of temperature $\theta°$. The mass of the air shell displaced is, calling β its coefficient of expansion,

$$\begin{aligned} & 4\pi r^2 . (\delta r) . \rho' \\ = {} & 4\pi r^2 (r\beta\theta) \rho' \\ = {} & 1/14 \text{ gm., approximately.} \end{aligned}$$

But M = 50,000 gms., so the proportionate change in the attraction due to this cause would be 1/700,000. This is negligible.

Under the second head. Suppose, as an extreme case, a shell of cold air equal in volume to the lead were removed from the field by expansion; the mass of this would be 1/9000 of the mass M. This also is negligible, since the temperature effect observed is 1/500.

II. With the high temperature of the lead spheres (250° C.) considerable convection currents would be set up round them, even when lagged.

In the region where the spheres are close to the tube, the air velocity might be very much larger than on the outer regions; and, as a consequence, difference of pressure would be called into play, pushing the suspended spheres towards the tube and therefore increasing the actual attraction on the suspended small balls.

Let α be the temperature effect as found in this research, and let dr be the actual displacement accounting for it, on the above supposition, we have

$$G \cdot \frac{Mm}{r^2} \cdot \alpha = 2G \cdot \frac{Mm}{r^3} dr,$$

hence

$$dr = \alpha r/2.$$

As

$$r = 15 \text{ cm.}, \qquad \alpha = 0{\cdot}002,$$

we have

$$dr = 0{\cdot}015 \text{ cm.}$$

If the mass M have weight W, and hang at distance l from the supports, the force required to produce this movement (dr) is

$$W \cdot dr/l = 5{\cdot}5 \text{ gm. weight.}$$

In discussing this hypothesis of greater convection on the inner side of the mass M we should notice that the vacuum tube is surrounded by a water screen at about 11° C., so that one would expect the inner side of the sphere to be colder, not hotter, than elsewhere, and the push on the sphere due to convection differences would be *outwards, not inwards.*

Suppose, however, that through some cause there is greater convection on the inner side of M. By applying BERNOULLI's theorem we can calculate what velocity is required to give the calculated push of 5·5 gm. weight.

Let v_0, v be convection velocities on the outer and inner sides of the sphere ; and let p_0, p be the corresponding pressures ; and let d be the air density. We have

$$\frac{v^2 - v_0^2}{2} = \frac{p_0 - p}{d}.$$

The effective area (*i.e.*, the total resolved area of the sphere on which the pressure difference, $p_0 - p$, acts horizontally) would be not more than 75 cm². Then taking $v_0 = 0$ we find $v = 380$ cm./sec. If, however, we take $v_0 = 100$ cm./sec. we find $v = 360$ cm./sec. It is fair to assume that v_0 lies between 0 and 100 cm./sec.

Thus the upward velocity of a broad column of air on the inner side of the sphere would have to be some 370 cm./sec. to account for the observed effect. This velocity is enormous. Even if the large value of 37 cm./sec. were assumed the error introduced would be only 1 per cent.

In a recent paper by H. A. WILSON* we find it stated that the velocity of a bunsen flame is only 300 cm./sec. So, even supposing there were proved to be excess convection on the inner side, we have no reason to think that it would introduce a calculable effect.]

* 'Phil. Trans.,' A, 1915, vol. 216, p. 71.