Monday, December 8th, 1851.

WILLIAM HAMILTON DRUMMOND, D. D.,
in the Chair.

The following antiquities, found in the lake of Cloonfree, were presented to the Museum of the Royal Irish Academy by Alonzo Lawder, Esq., of Cloonfinlough, Strokestown, through Robert Callwell, Esq.:

1. A horse-shoe, made of iron.
2. A fragment of iron, probably part of the hilt of a sword.
3. An iron spike, for butt-end of a spear.
4. A bone spear-head.
5. A bone pin.
6. An amber bead.
7. A bronze tweezer.
8. Ditto, broken, but of different matter.
9. A bronze pin, with ornamented head, having a cross and arrow-shaped device carved on two sides of it.
10. A very long bronze pin, with ornamented spike, head, and ring; a peculiarly fine specimen.
11. A small iron pin, with head bound with bronze wire, and small circular disc pendent.
12. An amber bead.
13. A buckle.
14. A bore's tusk.

The Secretary, in the absence of Sir W. R. Hamilton, read the following remarks on the connexion of Quaternions with continued fractions and quadratic equations.

1. If we write

$$u_x = \frac{b_1}{a_1 +} \frac{b_2}{a_2 +} \cdots \frac{b_x}{a_x} = \frac{N_x}{D_x},$$

it is known (see Sir J. F. W. Herschel's Treatise on Finite Differences) that the numerator and denominator of the resultant fraction satisfy two equations in differences, which are of one common form, namely,

$$N_{x+1} = N_x a_{x+1} + N_{x-1} b_{x+1},$$
$$D_{x+1} = D_x a_{x+1} + D_{x-1} b_{x+1}.$$

And by the nature of the reasoning employed, it will be found that these equations in differences, thus written, hold good for quaternions, as well as for ordinary fractions.

2. Supposing a and b to be two constant quaternions, these equations in differences are satisfied by supposing

$$N_x = Cq_1^x + C'q_2^x,$$
$$D_x = Eq_1^x + E'q_2^x,$$
$$C + C' = 0, \; Cq_1 + C'q_2 = b,$$
$$E + E' = 1, \; Eq_1 + E'q_2 = a;$$

C, C', E, E' being four constant quaternions, determined by the four last conditions, after finding two other and unequal quaternions, q_1 and q_2, which are among the roots of the quadratic equation,

$$q^2 = qa + b.$$

3. By pursuing this track it is found, with little or no difficulty, that

$$2u_x^{-1} + q_1^{-1} + q_2^{-1} = \frac{q_1^x + q_2^x}{q_1^x - q_2^x} \; \frac{q_1 - q_2}{b};$$

where

$$u_x = \left(\frac{b}{a+}\right)^x 0; \; \frac{q_1 - q_2}{b} = q_1^{-1} - q_2^{-1},$$

q_1, q_2, being still supposed to be two unequal roots of the lately written quadratic equation in quaternions,

$$q^2 = qa + b.$$

4. Let the continued fraction in quaternions be

$$u_x = \left(\frac{j}{i+}\right)^x 0;$$

then the quadratic equation becomes

$$q^2 = qi + j:$$

and two unequal roots of it are the following:

$$q_1 = \tfrac{1}{2}(1 + i + j - k),$$
$$q_2 = \tfrac{1}{2}(-1 + i - j - k).$$

Substitution and reduction give hence these two expressions:

$$\left(\frac{j}{i+}\right)^{2n} 0 = \frac{\sin\frac{2n\pi}{3}}{i\sin\frac{2n\pi}{3} - k\sin\frac{(2n-1)\pi}{3}};$$

$$\frac{2 \div \left(\frac{j}{i+}\right)^{2n-1} 0}{i - k} = 1 - \frac{\sin\frac{(2n-1)\pi}{3}}{\sin\frac{2(n-1)\pi}{3} + j\sin\frac{2n\pi}{3}};$$

which may easily be verified by assigning particular values to n. No importance is attached by the writer to these particular results: they are merely offered as examples.

5. It may have appeared strange that Sir William R. Hamilton should have spoken of *two* unequal quaternions, as being *among* the roots, or *two of the roots*, of **a** *quadratic equation* in quaternions. Yet it was one of the earliest results of that calculus, respecting which he made (in November, 1843) his earliest communication to the Academy, that *such* a quadratic equation (if of the above-written form) has generally *six roots:* whereof, however, *two only* are *real quaternions*, while the other four may, by a very natural and analogical extension of received language, be called *imaginary quaternions*. But the theory of such *imaginary*, or *partially* imaginary quaternions, in short, the theory of what Sir William R. Hamilton has ventured to name "*Biquaternions*," in a paper already published, appears to him to deserve to be the subject of a separate communication to the Academy.

The Rev. Samuel Haughton communicated a short account of an Aurora, visible in Dublin on the night of October 2, 1851. This Aurora passed the zenith; its crown, and the point of the horizon at which the streamers were vertical, being situated in the magnetic meridian. The transverse arcs were sensibly portions of great circles to a distance of about 45° from the horizon, and intersected the magnetic meridian at right angles. In the neighbourhood of the crown of the Aurora the transverse arcs were not great circles, and presented opposite curvatures at the different sides of the crown. At 8.30, P.M. the streamers to the west of magnetic north were red, the streamers to the east being colourless, or perhaps slightly yellowish. At 9, P.M. the bearing of the north pole-star was taken with a Kater's compass and another. The readings were 30° W. and 31° W.; assuming the mean of these, and subtracting the variation in Dublin, 26° 30′, this observation would appear to indicate a westerly deflection of 4° produced by the Aurora. The air at the time of observation was saturated with moisture; barom. 29·15 in.; dry bulb therm. 49°; wet bulb therm. 49°. The streamers seemed to intersect the transverse arcs at right angles, and to follow the deviations of the latter from great circles in the neighbourhood of the crown of the Aurora. The distance of the latter from zenith was not measured, but it appeared about the same as the distance of the north point of the Aurora from the meridian.

The Rev. Charles Graves, D. D., communicated a notice, extracted by Mr. Charles P. Mac Donnell from the Catalogue of MSS. in the Library of Cambray:

"Catalogue descriptif et raisonné des manuscrits de la Bibliothéque de Cambrai, par A. le Glay. Cambrai, in 8°. 1831. pp. 122.

"MS. 619. Canones Hibernici, in fol. vel. b. C. M. MS. a 2 colonnes, écriture minuscule du 8 Siecle. À la fin du