A COMPUTER SIMULATION ANALYSIS OF A SUGGESTED APPROXIMATE CONFIDENCE INTERVAL FOR SYSTEM MAINTAINABILITY

by

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United States
Naval Postgraduate School

THESIS

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Thesis Advisor: W. M. Woods

- MAR 1971

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
March 1971

ABSTRACT

This paper presents an accuracy analysis of a suggested approximate confidence interval for system maintainability parameters.

Technically, the simulation demonstrates feasible ranges of parameter applicability for a fit of linear combinations of generated gamma variates to the gamma distribution, using the method of moments.

The simulation has application to the classical confidence interval for mean time to repair of a series system, under the assumptions of gamma distributed repair times, and method of moments estimators.

The paper provides no validated conclusions although it does display parameters and ranges of apparent extremely high model validity.

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I. INTRODUCTION

The purpose of this paper is to present a computer simulation in order to investigate the adequacy of a suggested approximate confidence interval for system maintainability parameters. In a technical sense, it will demonstrate feasible ranges of parameter applicability for a fit of linear combinations of generated gamma variates to the gamma distribution, using the method of moments. The application of the simulation is to the classical confidence interval for mean time to repair a system, under the assumptions of series components, gamma distributed repair times, and method of moments estimators. The parameters of the distributions that were investigated were selected so that the maintainability curves would be representative of those curves presently of interest in industry.

II. SUMMARY

A procedure for a fit of linear combinations of generated gamma variates to the gamma distribution, using the method of moments is presented in Chapter III. A simulation of this procedure is proposed in Chapter IV and the results are tabulated in Tables I and II of Chapter V.

III. THE STATISTICAL MODEL

A. SERIES ASSUMPTION

Suppose we are given a system of components which are not necessarily series, where the time to failure of the ith component of the system has the exponential (λ_i) distribution.

Then,

1) for the case where there are k_i identical components in series of type i within the system, set $\lambda_i' = k_i\lambda_i$.

2) for the case where there are h_j identical components in parallel of type j within the system, set $\lambda_j' = \lambda_j^{h_j}$

Therefore, for some parallel-series combination there is obtainable a series system of uniquely different components whose failure ratios are assumed to be exponential.

Thus, the system being described is a series system with k different components and failure rates $\lambda_1, \lambda_2, \cdots, \lambda_i', \lambda_j', \cdots, \lambda_k$ where all the λ_i are in the same units.

B. FAILURE RATE ASSUMPTION

Let X_i denote the time to failure of component i, where X_i has the exponential (λ_i) distribution. Thus,

$$f_{X_i}(x_i; \lambda_i) = \lambda_i e^{-\lambda_i x_i} \tag{3-1}$$

We further assume that the system fails when exactly one component fails or the component which caused the system failure takes the longest time to repair.

C. GAMMA ASSUMPTION

Let T_i denote the time required to repair the ith component and suppose that T_i has the gamma distribution. Thus, $T_i \sim \Gamma\,(t_i;\ a_i,\ b_i)$. Therefore,

$$f_{T_i}\ (t_i;\ a_i,\ b_i) = \frac{a_i^{\ b_i}\ t_i^{\ b_i - 1}\ e^{-a_i t_i}}{\Gamma(b_i)} \qquad (3\text{-}2)$$

is the density equation with

$$E[T_i] = \frac{b_i}{a_i} = \mu_i \qquad (3\text{-}3)$$

which is the mean time to repair (MTTR) for the ith component.

D. SYSTEM MTTR OBTAINED

Let θ denote the mean time to repair for the system. Therefore,

$$\theta = \sum_i P\ [\text{component } i \text{ fails first}]\ \mu_i \qquad (3\text{-}4)$$

and, if the assumption of a series system is valid, then

$$\lambda = \sum_{i=1}^{k} \lambda_i$$

and

$$\theta = \text{MTTR} = \sum_{i=1}^{k} \left(\frac{\lambda_i}{\lambda}\right)\left(\frac{b_i}{a_i}\right) \qquad (3\text{-}5)$$

E. CONFIDENCE LIMIT OBTAINED

An estimated upper confidence limit for system maintainability, denoted by $\hat{\theta}_u$, has been proposed. $\hat{\theta}_u$ was derived by the following procedure:

For any component i, T_{11}, T_{12}, T_{13}, $\cdots$ T_{in_i} is a random sample on T_i, the time to repair component i. Failure data is also available on component i, so that an estimate $\hat{\lambda}_i$ of λ_i is possible.

Thus, θ can be estimated by $\hat{\theta}$ where

$$\hat{\theta} \doteq \sum_{i=1}^{k} \frac{\lambda_i}{\lambda} \bar{T}_i \qquad (3\text{-}6)$$

and

$$\bar{T}_i = \frac{1}{n_i} \sum_j T_{ij}$$

Moreover, because of the large amount of industrial testing, it can be assumed that $\hat{\lambda}_i \doteq \lambda_i$, and for purposes of derivation we shall treat the $\hat{\lambda}_i$ as though they are constants. Thus,

$$\hat{\theta} \doteq \sum_{i=1}^{k} \frac{\lambda_i}{\lambda} \bar{T}_i, \qquad (3\text{-}7)$$

$$E\,[\hat{\theta}] \doteq \sum_{i=1}^{k} \frac{\lambda_i}{\lambda} \left(\frac{b_i}{a_i}\right) \qquad (3\text{-}8)$$

and

$$\text{var}\,(\hat{\theta}) \doteq \sum_{i=1}^{k} \left(\frac{\lambda_i}{\lambda}\right)^2 \left(\frac{b_i}{a_i^2}\right). \qquad (3\text{-}9)$$

We shall fit a two parameter gamma to the density of $\hat{\theta}$ and obtain the upper confidence limit from this fitted distribution. Thus we are assuming $\hat{\theta} \sim \Gamma(a,b)$ and use will be made of the method of moments to make the fit. Through use of formulas (3-8) and (3-9)

$$E\,[\hat{\theta}] \doteq \sum_{i=1}^{k} \frac{\lambda_i}{\lambda} \left(\frac{b_i}{a_i}\right) = \frac{b}{a} \qquad (3\text{-}10)$$

and

$$\text{var}\,(\hat{\theta}) \doteq \sum_{i=1}^{k} \left(\frac{\lambda_i}{\lambda}\right)^2 \left(\frac{b_i}{a_i^2}\right) = \frac{b}{a^2} \qquad (3\text{-}11)$$

The parameters a and b can be solved for, as follows:

$$a = \frac{\sum_{i=1}^{k} \left(\frac{\lambda_i}{\lambda}\right) \left(\frac{b_i}{a_i}\right)}{\sum_{i=1}^{k} \left(\frac{\lambda_i}{\lambda}\right)^2 \left(\frac{b_i}{a_i^2}\right)} \qquad (3\text{-}12)$$

and

$$b = \frac{\left[\sum_{i=1}^{k} \frac{\lambda_i}{\lambda} \frac{b_i}{a_i}\right]^2}{\sum_{i=1}^{k} \left(\frac{\lambda_i}{\lambda}\right)^2 \frac{b_i}{a_i^2}} \qquad (3\text{-}13)$$

For the available data, this then becomes:

$$\hat{b} = \frac{\left[\sum_{i=1}^{k} \frac{\hat{\lambda}_i}{\hat{\lambda}} \bar{T}_i\right]^2}{\sum_{i=1}^{k} \left(\frac{\hat{\lambda}_i}{\hat{\lambda}}\right)^2 S_i^2} \qquad (3\text{-}14)$$

where,

$$S_i = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (T_j - \bar{T})^2$$

If b is an integer, $2\hat{\theta}a$ has the $\chi^2_{(2b)}$ distribution; therefore, define $\overline{(2b)}$ as the integer closest to 2b and the following approximate confidence interval can be formed:

$$1 - \alpha = P\,[2\hat{\theta}a > \chi^2_{(1-\alpha)(2b)}] \qquad (3\text{-}15)$$

$$= P\,[\frac{2\hat{\theta}a}{(2b)} > \frac{\chi^2_{(1-\alpha)(2b)}}{(2b)}]$$

$$= P\,[\frac{b}{a} < \frac{\hat{\theta}\ (2b)}{\chi^2_{(1-\alpha)\ (2b)}}]$$

$$\doteq P\,[\frac{b}{a} < \frac{\hat{\theta}\ (2\hat{b})}{\chi^2_{(1-\alpha)\ (2\hat{b})}}] \qquad (3\text{-}16)$$

where the approximate equality compensates for the use of estimators.

Finally, the choice of $\alpha = .20$ results in the following:

$$.80 \doteq P\,[\frac{b}{a} < \frac{\hat{\theta}\ (2\hat{b})}{\chi^2_{(.80)(2\hat{b})}}]. \qquad (3\text{-}17)$$

Thus, the desired suggested approximate 80% confidence limit for system maintainability is $\hat{\theta}(2\hat{b})/\chi^2_{(.8)(2\hat{b})}$.

IV. THE SIMULATION PROCEDURE

As explained in Chapter III, the system to be simulated consists of k components in logical series with system maintainability (θ) expressed as:

$$\theta = \sum_{i=1}^{k} \left(\frac{\lambda_i}{\lambda}\right) \left(\frac{b_i}{a_i}\right) \qquad (4\text{-}1)$$

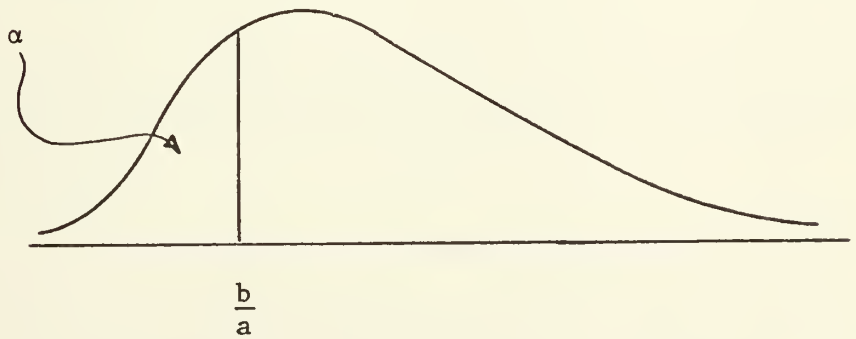
where b_i/a_i is the true maintainability of the ith component of the system.

Denote an upper confidence limit for θ by $\hat{\theta}_u$. If $\hat{\theta}_u$ is in fact the exact 100 $(1-\alpha)$% upper confidence limit for θ, then

$$P \left[\frac{b}{a} < \frac{\hat{\theta}\ (2\hat{b})}{\chi^2_{(1-\alpha)(2\hat{b})}}\right] \doteq 1 - \alpha$$

holds to a reasonably close approximation.

In fact, b/a should then be the αth percentile point of the simulated distribution of $\hat{\theta}_u$.



The choice of α = .20 defined $\hat{\theta}_{u(2)}$ as the 20th percentile point of the distribution of $\hat{\theta}_u$.

In order to investigate this distribution, a computer was utilized to generate the required gamma variates. Then, 500 values of $\hat{\theta}_u$ were computer and ordered such that

$$\hat{\theta}_{u1} < \hat{\theta}_{u2} < \hat{\theta}_{u3} < \cdots < \hat{\theta}_{u500} \qquad (4\text{-}2)$$

Since it was desired to display the 80% upper confidence limit for θ ($\hat{\theta}_{u(.2)}$), which implies 80% of the ordered values be greater than $\hat{\theta}_{u(.2)}$, the 20th percentile point of the $\hat{\theta}_u$ distribution was found. This 20th percentile point is the 100th ordered value in formula (4-2) above and, if the procedure is correct, should approximately equal $\frac{b}{a}$.

The primary measure of accuracy for the simulation will be the value of

$$\frac{|\hat{\theta}_{u100} - \frac{b}{a}|}{b/a} \qquad (4\text{-}3)$$

which is an expression relating the estimated value of system maintainability ($\hat{\theta}_{u100}$) to the gamma value for MTTR (b/a). Thus, the accuracy of the simulation is presented in the notion of relative error.

Also, the statistic

$$\frac{[\#\theta_{uj} \geq b/a]}{500} \qquad (4\text{-}4)$$

will be computed in order to display the relative error between the true value of system maintainability (b/a) and the number of generated estimates of this value ($\hat{\theta}_{uj}$).

The analysis for this proposed method was conducted using different combinations of values of the gamma input parameters, varying λ_i values, one of two values for k, and n_i values of 10, 20, 50, and 100.

A. THE SIMULATION PROGRAM EXPLAINED

Available in Appendix E is a flow chart for the computer simulation. An explanation of the blocks on that diagram follow:

1) Dimension the computer matrix as required to include the possible values for the following input parameters:

$$k$$

$$\lambda_1, \lambda_2, \cdots, \lambda_k$$

$$(a_1, b_1), (a_2, b_2), \cdots, (a_k, b_k)$$

$$n_1, n_2, \cdots, n_i$$

2) Generate the following n_i random repair times according to the gamma distribution with parameters (a_i, b_i). Utilize the IBM Scientific Subroutine Package GAMMA to get:

$$T_{11}, T_{12}, T_{13}, \cdots, T_{1n_1}$$

$$T_{21}, T_{22}, T_{23}, \cdots, T_{2n_2}$$

-

-

-

$$T_{k1}, T_{k2}, T_{k3}, \cdots, T_{kn_k}$$

3) For each row above, compute the mean value $(\overline{T}_i)$ and the variance (S_i^2) to get the pairs:

$$\overline{T}_1, S_1^2; \overline{T}_2, S_2^2; \cdots; \overline{T}_k, S_k^2$$

4) Utilize formula (3-14) to compute the method of moments estimate of the unknown parameter b, denoted by $\hat{b}$.

5) Compute the integer closest to $2\hat{b}$ for use with the $\chi^2_{(2\hat{b})}$ distribution.

6) Utilize formula (3-6) to compute the estimate of system maintainability θ, denoted by $\hat{\theta}$.

7) Utilize formula (3-16) and the χ^2 formula (see Appendix C) to compute the approximate upper confidence limit on system maintainability. Denote this value as $\hat{\theta}_u$.

8) Repeat step 2 through step 7 a total of 500 times.

9) Utilize the IBM SSP SHSORT to order the 500 values of $\hat{\theta}_u$ from smallest to largest.

10) Pick out the 100th value of the above ordering ($\hat{\theta}_{u100}$) and print out this value as the 20th percentile point of the distribution.

11) Compute the value of b/a and then utilize formula (4-3) to compute the primary measure of accuracy for the simulation. Print out this value.

12) Utilize formula (4-4) to compute another measure of simulation accuracy. Print out this value.

V. RESULTS

For the procedure as presented in Chapter III, the value of system maintainability will follow the gamma distribution. The parameters of the distribution were chosen so that the maintainability of each component was at a preselected value position on the curve or would be representative of those curves currently obtainable in industrial applications. (See Appendix A.) The following cases were simulated:

CASE #	INPUT PARAMETERS
I	$k = 15$
	$a_i = 10$, $b_i = 1$
	$\lambda_i = .005$
	$n_i = 10, 20, 50, 100$
II	$k = 15$
	$a_i = 10$, $b_i = 2$
	$\lambda_i = .005$
	$n_i = 10, 20, 50, 100$
III	$k = 15$
	$a_i = 5$, $b_i = 1$
	$\lambda_i = .005$
	$n_i = 10, 20, 50, 100$
IV	$k = 15$
	$a_i = 30$, $b_i = 1$, $i = 1, 2, \cdots, 10$
	$a_i = 10$, $b_i = 1$, $i = 11, 12, \cdots, k$
	$\lambda_i = .005$
	$n_i = 10, 20, 50, 100$

CASE #	INPUT PARAMETERS
V	$k = 15$
	$a_i = 30,\ b_i = 1,\ i = 1, \cdots, 10$
	$a_i = 10,\ b_i = 1,\ i = 11, \cdots, k$
	$\lambda_i = .01$
	$n_i = 10, 20, 50, 100$
VI	$k = 30$
	$a_i = 30,\ b_i = 1,\ i = 1, \cdots, 10$
	$a_i = 10,\ b_i = 1,\ i = 11, \cdots, k$
	$\lambda_i = .005$
	$n_i = 10, 20, 50, 100$
VII	$k = 30$
	$a_i = 30,\ b_i = 1,\ i = 1, \cdots, 10$
	$a_i = 10,\ b_i = 1,\ i = 11, \cdots, k$
	$\lambda_i = .01$
	$n_i = 10, 20, 50, 100$
VIII	$k = 15$
	$a_i = 5,\ b_i = 1$
	$\lambda_i = .05$
	$n_i = 10, 20, 50, 100$

Each case is specified by the relevent input parameters and the varying number of random repair times generated (n_i). The cases differ most significantly in their values for the parameters of the Gamma distribution. However, the value of λ_i is varied as is the value of k, the number of components in series. Of course, all of the cases were run over an identical range for n_i.

The results are compiled in the following tables. The column headings are self-explanatory and their use has been discussed earlier in this report.

TABLE I

RELEVENT STATISTICS OF COMPUTER SIMULATION FOR CASES I - IV

CASE NO.	n_i	20th PERCENTILE	b/a	$\frac{\lvert\hat{\theta}_u - b/a\rvert}{b/a}$	$\frac{[\#\hat{\theta}_{uj} \geq b/a]}{500}$
I	10	.09570	.10000	.04304	.61200
	20	.09787	.10000	.02129	.66400
	50	.09917	.10000	.00832	.72600
	100	.09999	.10000	.00009	.80000
II	10	.19575	.20000	.02125	.69400
	20	.19710	.20000	.01451	.69800
	50	.20051	.20000	.00253	.82400
	100	.20132	.20000	.00661	.89400
III	10	.19649	.20000	.01754	.71800
	20	.20071	.20000	.00356	.81600
	50	.20283	.20000	.01414	.89800
	100	.20456	.20000	.02282	.94600
IV	10	.05225	.05556	.05947	.60800
	20	.05378	.05556	.03188	.65000
	50	.05491	.05556	.01165	.70400
	100	.05538	.05556	.00318	.75800

TABLE II

RELEVENT STATISTICS OF COMPUTER SIMULATION FOR CASES V - VIII

CASE NO.	n_i	20th PERCENTILE	b/a	$\frac{\lvert\hat{\theta}_u - b/a\rvert}{b/a}$	$\frac{[\#\hat{\theta}_{uj} \geq b/a]}{500}$
V	10	.05225	.05556	.05947	.60800
	20	.05378	.05556	.03188	.65000
	50	.05491	.05556	.01165	.70400
	100	.05538	.05556	.00318	.75800
VI	10	.07552	.07778	.02907	.64200
	20	.07612	.07778	.02130	.63800
	50	.07751	.07778	.00785	.71400
	100	.07777	.07778	.00009	.80000
VII	10	.07552	.07778	.02907	.64200
	20	.07612	.07778	.02130	.63800
	50	.07717	.07778	.00785	.71400
	100	.07777	.07778	.00009	.80000
VIII	10	.19649	.20000	.01754	.71800
	20	.20071	.20000	.00356	.81600
	50	.20283	.20000	.01414	.89800
	100	.20456	.20000	.02282	.94600

APPENDIX A

THE GAMMA DISTRIBUTION

Use of the gamma function to describe maintenance-time functions has been described in the literature.

The gamma distribution is a two-parameter function.

$$f_{T_i}(t_i; a_i, b_i) = \frac{a_i^{b_i} t_i^{b_i - 1} e^{-a_i t_i}}{\Gamma(b_i)}$$

The parameters are denoted by the letters a and b. Of the two, b is considered to be the more critical because it controls the shape of the curve; while a merely determines the scale of the axes.

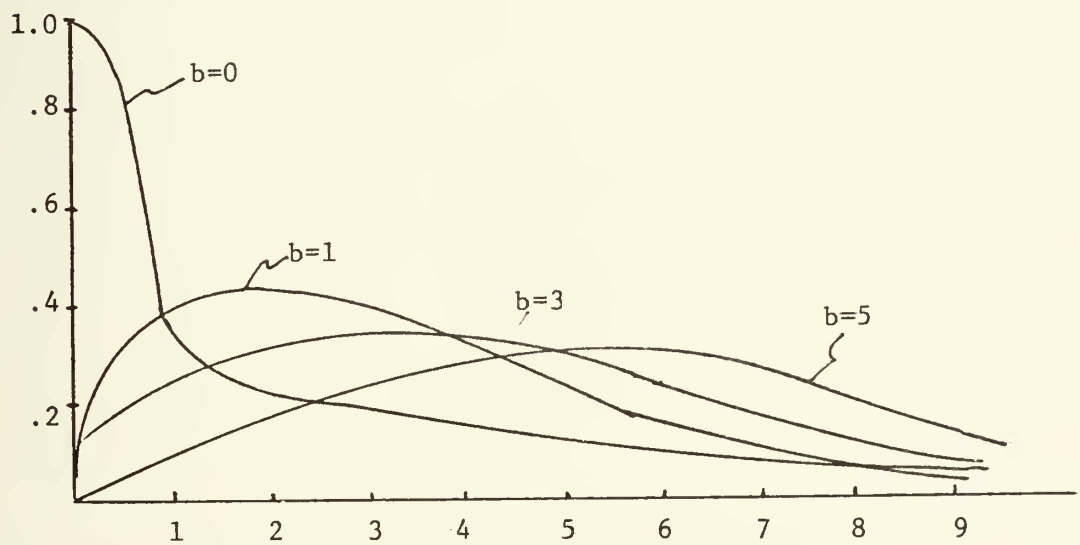


Figure 1. Gamma density function; $a = 1$, $b = 0, 1, 3, 5$

For the distribution as presented, the parameters a and b are restricted by the inequalities $a > 0$, $b \geq 0$. Also, both a and b will remain as whole numbers.

The b parameter is seen to vary as a function of the repair concept. When the repair concept being described involves relatively simple actions (e.g., black box replacement, etc.) and very few long downtime tasks, the b parameters will be a small number. As the downtime density involves more time-consuming actions, the b parameter will take on a larger value.

Throughout the literature it has been shown that the gamma distribution could be used to describe a family of distribution curves varying from the negative exponential to the normal. Thus, by suitable choice of the parameters a and b, the downtime of a wide variety of reasonably complex equipments can be described.

APPENDIX B

THE EXPONENTIAL DISTRIBUTION

One of the most widely used distributions in fields of Reliability/ Maintainability is the one-parameter exponential function.

The function is defined by:

$$f(X_i, \lambda_i) = \lambda_i e^{-\lambda_i X_i} \qquad X \geq 0$$

$$= 0 \qquad \text{elsewhere}$$

where λ_i is the failure rate associated with component i.

$$F(X_i) = P(X \leq x_i) = \lambda_i \int_0^{X_i} e^{-\lambda_i t_i} dt_i = 1-e^{-\lambda_i X_i} \qquad X_i > 0$$

$$= 0 \qquad X_i \leq 0$$

is the cumulative distribution function, and

$$E[X_i] = 1/\lambda_i .$$

Thus, the expected value equals the reciprocal of the parameter λ_i.

$$\text{var}\ (X_i) = 1/\lambda_i^{\ 2}$$

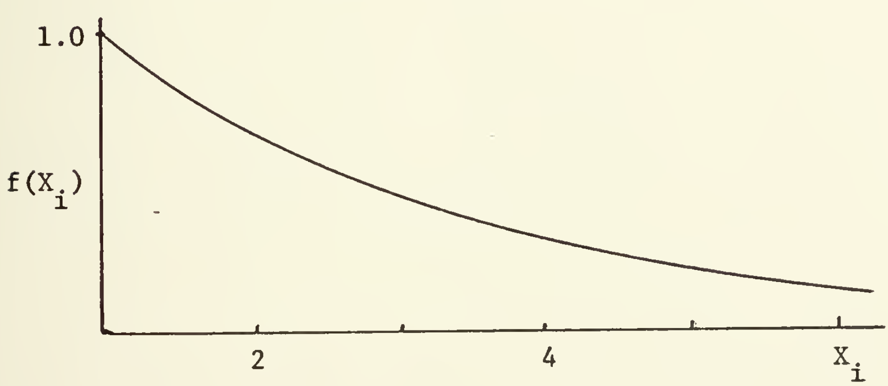


Figure 2. The exponential distribution

NOTES:

1) The exponential distribution is a special case of both the gamma and weibull distributions.

2) It is the distribution which is expected when the mechanisms are so complex that many deteriorations with different failure rates are operable.

3) When parts have an exponential failure distribution, the equipment consisting of these parts also has an exponential distribution function.

APPENDIX C

THE CHI-SQUARE DISTRIBUTION

A quite useful case of the gamma distribution occurs when a = 1/2 and b = n/2, where n is a positive integer. Then, a one-parameter family of distributions is obtainable with density function:

$$f(z) = \frac{z^{(n/2 - 1)} e^{-z/2}}{2^{n/2} \Gamma(n/2)} \qquad z > 0$$

A random variable z having the above density is said to have the chi-square distribution with n degrees of freedom (denoted by $\chi^2_{(n)}$).

In Figure 3 below, the density function for n = 1, 2, and n > 2 is shown.

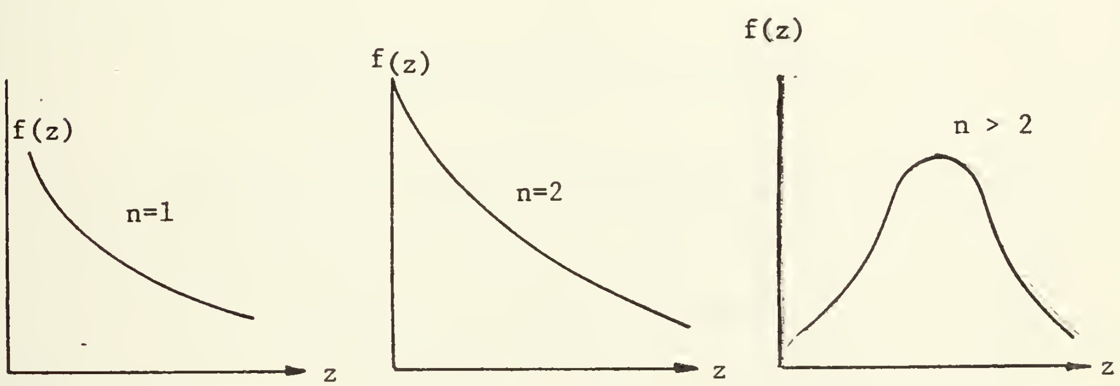


Figure 3

with E[z] = n, var (z) = 2n.

Our interest in the chi-square distribution is based upon its many important applications in statistical inference.

The chi-square distribution is tabled for degrees of freedom up to 30. Thus we may find in the table that value, denoted by $\chi^2_{(\alpha)(n)}$ and satisfying $P(Z \leq \chi^2_\alpha) = \alpha$, $0 < \alpha < 1$.

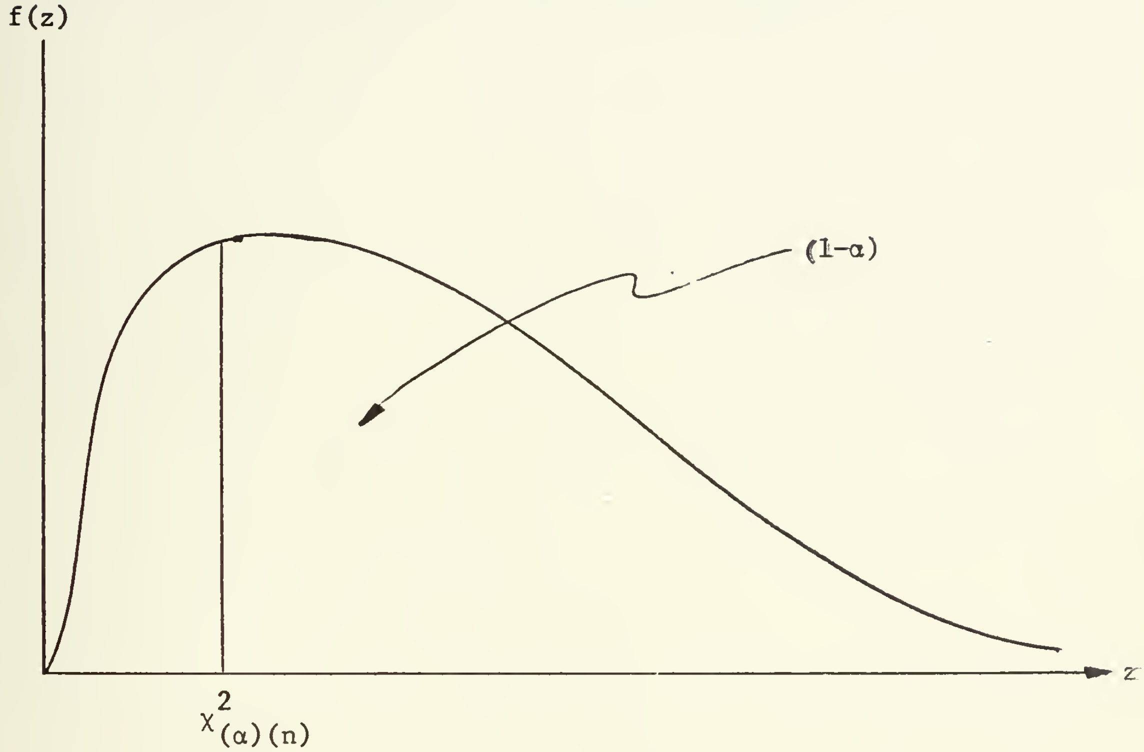


Figure 4. The chi-square distribution

For the case where the degrees of freedom exceed 30 ($n > 30$) the chi-square distribution can accurately be approximated by the normal distribution as indicated in the following theorem:

Theorem: Suppose that the random variable Y has distribution χ^2_n. Then for sufficiently large n, the random variable $\sqrt{2Y}$ has approximately the distribution $N(\sqrt{2n - 1}, 1)$. Therefore,

$$P(Y \le t) = P(\sqrt{2Y} \le \sqrt{2t})$$

$$= P(\sqrt{2Y} - \sqrt{2n - 1} \le \sqrt{2t} - \sqrt{2n - 1})$$

$$\doteq \Phi(\sqrt{2t} - \sqrt{2n - 1})$$

where Φ values are obtained from the normal tables.

APPENDIX D

THE METHOD OF MOMENTS

The oldest general method for generating estimates of unknown parameters, given a sample, is known as the method of moments. It is a comparatively simple method and it generates "reasonable" estimators.

In general, the method proceeds as follows:

Given a random variable X, with distribution function F_X where this distribution is indexed by the unknown parameter λ. Assume the first moment of X (its mean value) is dependent in some simple way upon λ, like $\mu_X = g(\lambda)$. Then, given a sample of n values of X, define the first sample moment as $\bar{X}$. The method of moments now specifies that $\bar{X}$ be equated to $g(\lambda)$ and finally to solve for $\hat{\lambda}$. This resulting value for $\hat{\lambda}$ is the method of moments estimator of λ.

For the two dimensional case, such as X being distributed according to the gamma distribution with unknown parameters α and β, there is required a multi-dimensional parameter space $\Omega = E_k$ which in the gamma case has $k = 2$. Then, this space can be defined such that $\Omega = \{(\alpha, \beta); \alpha > 0, \beta > 0\} \subset E_2$ and further, it can be shown that

$$\mu_X = \alpha\beta \text{ and } \mu_X^2 = \alpha\beta^2 + \alpha^2\beta^2$$

where μ_X^2 is the second moment with respect to X. Then, as in the one dimensional case, a random sample is required which can be analyzed to give:

$$\bar{X} = \alpha\beta$$

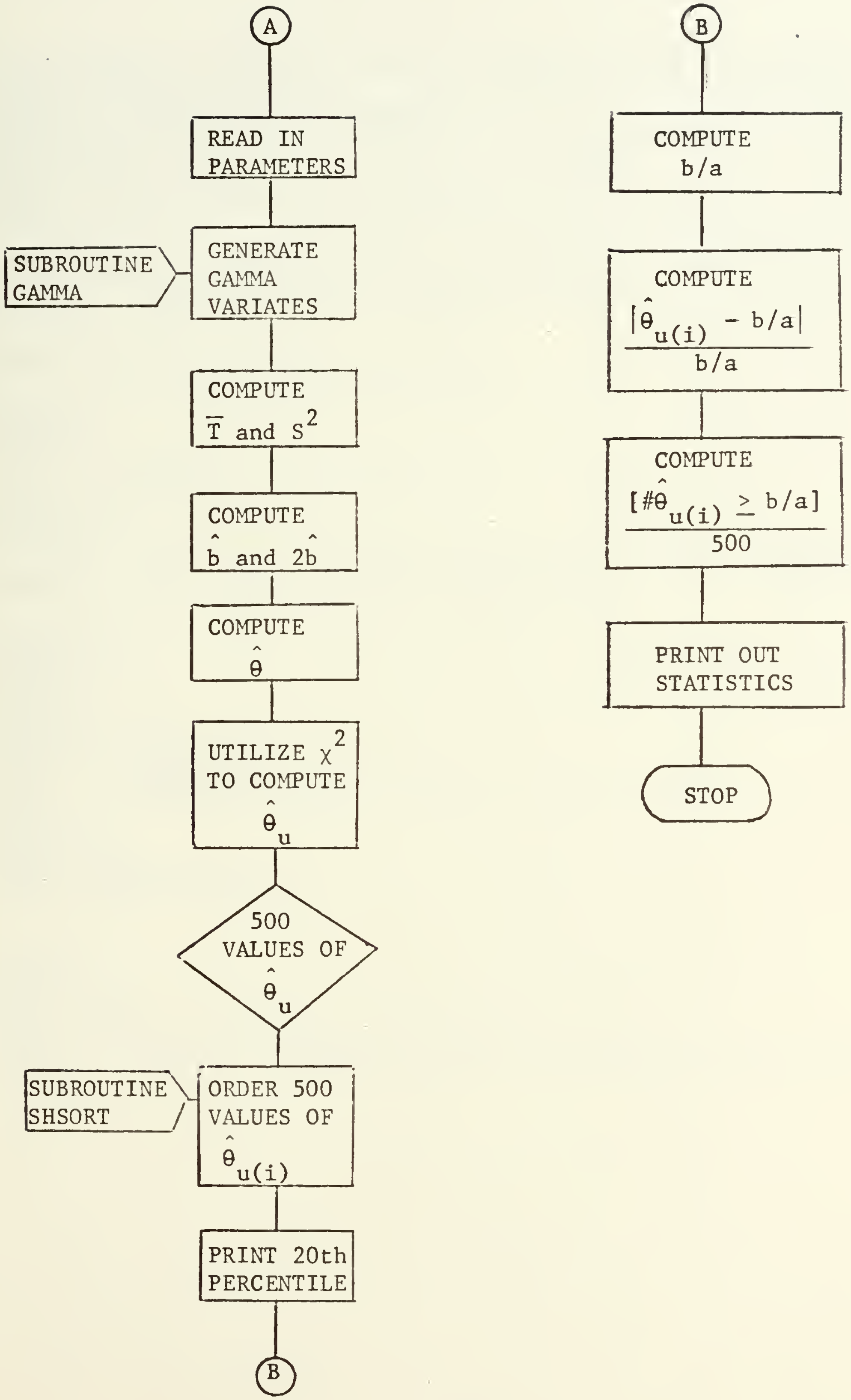
$$S_X^2 + \bar{X}^2 = \alpha\beta^2 + \alpha^2\beta^2$$

which allows, as shown in Chapter III

$$\hat{\alpha} = \frac{\bar{X}^2}{S_X^2} \qquad \hat{\beta} = \frac{S_X^2}{\bar{X}}$$

as acceptable estimators of the parameters α and β.

APPENDIX E


A
READ IN PARAMETERS
SUBROUTINE GAMMA
GENERATE GAMMA VARIATES
COMPUTE $\bar{T}$ and S^2
COMPUTE $\hat{b}$ and $2\hat{b}$
COMPUTE $\hat{\theta}$
UTILIZE χ^2 TO COMPUTE $\hat{\theta}_u$
500 VALUES OF $\hat{\theta}_u$
SUBROUTINE SHSORT
ORDER 500 VALUES OF $\hat{\theta}_{u(i)}$
PRINT 20th PERCENTILE
B
B
COMPUTE b/a
COMPUTE $\frac{|\hat{\theta}_{u(i)} - b/a|}{b/a}$
COMPUTE $\frac{[\#\hat{\theta}_{u(i)} \geq b/a]}{500}$
PRINT OUT STATISTICS
STOP

```
C      THIS PROGRAM COMPUTES THE 80 PERCENT UPPER CONFIDENCE LIMIT
C      DIMENSION THE MATRIX
       REAL*4 LAMBDA(100),N(100),LAM
       DIMENSION  T(100,100)   ,TB(100),S2(100),TU(500),ALPHA(100),BETA(10
      $0),KEY(500)
C      READ INTO PROGRAM PARAMETERS OF INTEREST
 1235  READ(5,100,END=4321) K,(LAMBDA(I),I=1,K),(ALPHA(J),BETA(J),J=1,K)
       READ(5,105) NI
C
C      PROGRAM WILL LOOP 500 TIMES, GENERATING INDEPENDENT ESTIMATES OF
C      THE 80 PERCENT UPPER CONFIDENCE LIMIT
C
       DO 1234 JJ=1,500
C
C      GENERATE GAMMA TIMES TO REPAIR
C
       DO 1 I=1,K
       N(I)=NI
       NN=N(I)
       DO 1 J = 1,NN
       CALL GAMMA(BETA(I),ALPHA(I),X)
       T(I,J) = X
     1 CONTINUE
C
C      COMPUTE MEAN TIME TO REPAIR FOR EACH K GAMMAS GENERATED AND THE
C      VARIANCE FROM THE METHOD OF MOMENTS FIT
C
       DO 2 I=1,K
       SUM=0.0
       SUMSQ=0.0
       NN=N(I)
       DO 3 J=1,NN
       SUM=SUM + T(I,J)
     3 SUMSQ=SUMSQ + T(I,J)*T(I,J)
       TB(I) = SUM/N(I)
       S2(I) = (SUMSQ-N(I)*TB(I)*TB(I))/(N(I)-1.0)
     2 CONTINUE
       LAM=0.0
       DO 4 I=1,K
       LAM= LAM+LAMBDA(I)
     4 CONTINUE
C
C
       SUM=0.0
       SUMSQ=0.0
       DO 5 I=1,K
       SUM= SUM + (LAMBDA(I)/LAM)*TB(I)
       SUMSQ = SUMSQ + (LAMBDA(I)/LAM)*(LAMBDA(I)/LAM)*(S2(I)*S2(I))
```

```
    5 CONTINUE
C
C     COMPUTE VALUE FOR ESTIMATE OF PARAMETER BETA, CALLED BHAT
C
      BHAT=(SUM*SUM)/SUMSQ
C
C     COMPUTE INTEGER CLOSEST TO TWO BHAT
C
      IH=2.0*BHAT+0.5
      BHAT2 = IH
C
C     COMPUTE ESTIMATE OF MEAN TIME TO REPAIR FOR SYSTEM,CALL IT THETA
C
      THETA = 0.0
      DO 6 I=1,K
      THETA = THETA + (LAMBDA(I)/LAM)*TB(I)
    6 CONTINUE
C
C     UTILIZE CHISQ FORMULATION TO GET STATISTIC FOR CONFIDENCE LIMITS
C
      CHISA=(2.0/(9.0*BHAT2))
      CHISQ=BHAT2*(1.0-CHISA-0.84178*SQRT(CHISA))**3
      TU(JJ)=(THETA*BHAT2)/CHISQ
C
C
 1234 CONTINUE
C
C     NOW THAT THE 500 ESTIMATED VALUES HAVE BEEN GENERATED, SORT THEM
C     INTO ORDER
C
      KK=500
      DO 99 I=1,KK
   99 KEY(I)=I
      CALL SHSORT(TU,KEY,KK)
C
C     PICK OUT AND PRINT THE 20TH PERCENTILE ACCORDING TO THE FOLLOW-
C     ING FORMAT
C
      WRITE(6,222) NI,K,LAMBDA(1)
      WRITE(6,200) TU(100)
C
C
  222 FORMAT('1','NI = 'I5, ' K=',I5, ' LAMBDA=',F10.5,///)
  100 FORMAT(I5/ 15F5.3 / 15F5.3 /(10(F4.0,F4.0)))
  105 FORMAT(I5)
  200 FORMAT (1X,' THE 20TH PERCENTILE IS ',F15.5)
C
      WRITE(6,400)(TU(I),I=1,500)
```

```
  400 FORMAT(' ',10(F8.4,2X))
C
C       COMPUTE THE VALUE OF THE THEORETICAL MEAN, CALL IT BOA
C
        DENOM=0.0
        TOP=0.0
        DO 50 I=1,K
        TOP = TOP  +(LAMBDA(I)/LAM)*(BETA(I)/ALPHA(I))
        DENOM=DENOM+(LAMBDA(I)/LAM)**2 *(BETA(I)/ALPHA(I)**2)
   50 CONTINUE
        A = TOP/DENOM
        B = TOP**2/DENOM
        BOA = B/A
C       COMPUTE THE ACTUAL PERCENTAGE
        C=0.0
        DO 60 I=1,500
        IF (TU(I).GE.BOA) C=C+1.0
   60 CONTINUE
        STAT = C/500.0
C       COMPUTE THE ABSOLUTE VALUE OF THE RELATIVE DIFFERENCE
        E=  ABS(TU(100)-BOA)/BOA
C       PRINT OUT THE ABOVE STATISTICS , ACCORDING TO THE FOLLOWING FORMAT
        WRITE(6,220) BOA,STAT,B,A
  220 FORMAT(1X,' B OVER A IS ',F15.5,5X,' STAT IS ',F15.5,5X,' B IS ',
     $F15.5,5X,' A IS ',F15.5)
        WRITE(6,230) E
  230 FORMAT(1X,' ABS OF VALUE IS ',F15.5)
        IN = 20
        RANGE(1) =  0.1822 + 0.00159
        DO 91 I=2,20
   91 RANGE(I) = RANGE(I-1) + 0.00159
        K=1
        DO 98 I=1,20
        FREQ(I) = 0.0
        DO 97 J=K,500
        IF(RANGE(I).LT.TU(J)) GO TO 96
        FREQ(I)=FREQ(I)+1.0
        GO TO 97
   96 K=J
        GO TO 98
   97 CONTINUE
   98 CONTINUE
        GO TO 1235
 4321 STOP
        END
```

```
      SUBROUTINE GAMMA(B,A,X)
      K=B
      TR=1.0
      DO 5 I=1,K
      R=RN(0)
    5 TR=TR*R
      X=-ALOG(TR)/A
      RETURN
      END
*                                                                        RN
************************************************************************ RN
*                                                                        RN
* PURPOSE                                                                RN
*     RN COMPUTES UNIFORMLY DISTRIBUTED RANDOM VARIATES OVER THE RANGE   RN
*     (0,1).  THE PERIOD FOR THIS GENERATOR IS 2**32 WHICH IS THE LENGTH RN
*     OF THE FULL SYSTEM/360 WORD GIVING TWICE THE PERIOD OF RANDU.      RN
*                                                                        RN
* USAGE                                                                  RN
*     IN    X = RN(0)  WHERE X WILL BE ASSIGNED THE NEXT RANDOM DEVIATE. RN
*                                                                        RN
* DESCRIPTION OF PARAMETERS                                              RN
*     NO FORMAL PARAMETERS                                               RN
*                                                                        RN
* REMARKS                                                                RN
*     THIS ASSEMBLY LANGUAGE ROUTINE GIVES TWICE THE PERIOD OF EXISTING  RN
*     GENERATORS AND IS EXTREMELY FAST.  11.88 MICROSECONDS WITH 4.23    RN
*     MICROSECONDS FOR THE CALLING SEQUENCE.  IT IS TAKEN DIRECTLY FROM  RN
*     , COMM. ACM 12,12 (DEC. 1969) 695.                                 RN
*                                                                        RN
* METHOD                                                                 RN
*     LEHMER'S MULTIPLICATIVE CONGRUENTIAL SCHEME IS USED.               RN
*     U(K+1) = L*U(K) (MOD P),  X(K+1) = U(K+1)/P  WHERE P = 2**32 AND   RN
*     L = 32781.                                                         RN
*                                                                        RN
RN        CSECT                                                          RN
          USING *,15          SET BASE ADDRESS                           RN
          L      1,SEED       LOAD U(K)                                  RN
          M      0,MULT       SEED=SEED*MULT                             RN
          ST     1,SEED       STORE U(K+1)                               RN
          LD     0,FLOT       FLOAT AND                                  RN
          AD     0,ZERO       NORMALIZE                                  RN
          BR     14           RETURN                                     RN
MULT      DC     F'32781'     2**32                                      RN
ZERO      DC     D'0'                                                    RN
FLOT      DC     X'46000000'                                             RN
SEED      DC     F'1'                                                    RN
          END    RN                                                      RN
```

30

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Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

A Computer Simulation Analysis of a Suggested Approximate Confidence Interval for System Maintainability

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

Master's Thesis; March 1971

5. AUTHOR(S) (First name, middle initial, last name)

Carl Flack Logan

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March 1971	38	2

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

This paper presents an accuracy analysis of a suggested approximate confidence interval for system maintainability parameters.

Technically, the simulation demonstrates feasible ranges of parameter applicability for a fit of linear combinations of generated gamma variates to the gamma distribution, using the method of moments.

The simulation has application to the classical confidence interval for mean time to repair of a series system, under the assumptions of gamma distributed repair times, and method of moments estimators.

The paper provides no validated conclusions although it does display parameters and ranges of apparent extremely high model validity.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
System maintainability						
Maintainability						
Maintainability confidence limit						
Gamma distributed repair times						
Method of moments estimators						

DD FORM 1 NOV 65 1473 (BACK)

S/N 0101-807-6821

Security Classification

A-31409