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ADDED MASS AND DAMPING COEFFICIENTS FOR SHIPS HEAVING IN SMOOTH WATER

by

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Supervisor, Professor M. A. Abkowitz

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ABSTRACT

Theoretical predictions of the added mass and damping coefficient curves for ships performing small vertical oscillations in calm water are determined. Calculations are made for the following ships:

1. Series 60 Block Coefficient 0.70
2. Series 60 Block Coefficient 0.60
3. Golovato's Surface Ship (Weinblum) Model

A method for the inversion of a power series type transform which conformally maps a unit circle into cylinders and ship-like forms is shown. Using this method, the transform coefficients corresponding to the sections of the ships considered are obtained. From these transform coefficients, plots of the sectional added mass and damping coefficients against frequency of vibration are derived using Porter's analytical solution to the problem. Comparisons are made between these curves and Grim's predictions.

Finally, by virtue of strip theory, the two-dimensional values for each cross section are integrated over the length of the ship to obtain the ship's added mass and damping coefficients at each frequency considered. No correction is made for three-dimensional effects. The resultant curves for each ship are compared with the corresponding published experimental results.

Thesis Supervisor: Martin A. Abkowitz

Title: Professor of Naval Architecture

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I. INTRODUCTION

The naval architect is confronted with the problem of predicting the forces that will act on a given ship moving in a given seaway in order to design for these forces. It has long been recognized that the solution to this complex problem can only be attained by breaking it down into a number of sub-problems, each one simple enough to hopefully permit an analytical solution.

One such problem mentioned in the preceding paragraph is that of a rigid ship made to oscillate vertically by an externally-applied harmonic force in water that is initially still. The linearized force equation of motion in this case is of the form:

$$m(1+k_s)\ddot{y} + b_s\dot{y} + c_s y = F_o \sin(\omega t+\alpha) \tag{1}$$

This is a second order linear differential equation with constant coefficients whose solution is well known. However, k_s, the added mass coefficient of the ship, and b_s, the ship's damping coefficient, are usually not known. The problem is thus reduced to that of determining these coefficients for any particular ship at all frequencies of interest. The product mk_s is called the added mass of the ship and accounts for the component of the hydrodynamic force acting in phase with the heave acceleration. The other component of the hydrodynamic force is the damping force which is the product $b_s\dot{y}$ and acts in phase with velocity.

Efforts to study the behavior of this complex hydrodynamic force led to a number of experiments. In particular, experimental results have

been made available for Series 60 Block 0.70 by Gerritsma and Beukelman [1], [2], for Series 60 Block 0.60 by Gerritsma [3], and for a ship model with mathematically-defined lines* by Golovato [4]. Calculations made in this work are for these ships.

In parallel with these experimental studies, considerable effort has been done to theoretically predict these quantities. In this field, the problem is further simplified from a three-dimensional ship form to that of an infinitely long cylinder with a ship-like cross section. With the acceptance of the validity of "strip theory", work along these lines for two-dimensional forms gained even greater importance. The cylinders of interest are those mapped by conformal transformation of a unit circle by

$$z = \zeta + \sum_{n=0}^{N} a_{2n+1} \zeta^{-(2n+1)} . \qquad (2)$$

The coefficients of this transform determine the shape of the section. Lewis [5] initiated the idea of fitting ship sections with forms from a two-parameter family of more or less ship-like forms which he generated by using this transform with chosen values of a_1 and a_3. Landweber and Macagno [6] extended the generality of the sections generated by using a third coefficient, a_5, for a three-parameter family of forms thereby making possible a wider variation of forms.

* This model shall be referred to as the Weinblum Model in this paper.

The work most often referred to, at present, when theoretical predictions for the added mass and damping coefficients of two-dimensional forms are desired, is that due to Grim [7]. Grim generated a comprehensive set of curves from which these quantities can be estimated for a given ship section using the beam-draft ratio and section area coefficient as the entering arguments. This is equivalent to approximating the section by a Lewis form, i.e., by a shape defined by a_1 and a_3 only. Since a number of shapes can be defined by just one set of beam-draft ratio and area coefficient, it appears that two parameters would in general be insufficient to describe a particular ship section. This ambiguity leads one to question the correctness of Grim's predictions. Now available is Porter's [8] analysis to the hydrodynamic problem which, within the limits of linearized potential theory, is exact. Since Porter's solution does not restrict the number of transform coefficients, we have in fact something that promises to give more accurate predictions. The author seeks an answer by using both methods.

Porter's analysis assumes that these transform coefficients are known. Hence the need for a method to determine these coefficients for a given cylinder shape is called for. The first known method of inverting this conformal transformation is that due to Plant [9]. This was further improved by Porter. While this method proved satisfactory in many cases, it was evident that a more accurate and less time-consuming method would be desirable.

In aeronautics, the problem of conformally mapping an arbitrary airfoil shape into a circle [redacted] has been solved by

Theodorsen [10]. His exact analysis requires the evaluation of a non-linear definite integral for numerical results. Naiman [11] reported a procedure for the numerical evaluation of this integral. The author devised a method of determining the transform coefficients for an arbitrary ship section by an application of Theodorsen's method. The procedure is outlined in this paper.

II. PROCEDURE

2.1 Problem Statement

Consider a rigid ship floating in smooth water. The depth of the water is infinite and its lateral dimensions are likewise infinite. We now impress upon the ship a vertical simple harmonic force so that the ship oscillates up and down in simple harmonic motion with a small amplitude. Assume that steady-state conditions have been attained so that the amplitude of the outgoing waves generated by the ship motion remains constant with time at any point on the water surface.

Required for any given frequency are:

a. The distribution of the added mass and damping coefficients along the length of the ship.

b. The ship's added mass and damping coefficients.

2.2 Solution to the Problem

Since the problem as formulated is not solved, we approximate a solution by "strip theory". We divide the ship into a convenient number of stations with end stations at the forward and after perpendiculars. To be definite, we will call the station at the forward perpendicular "Station 1" and number succeeding stations consecutively up to the last station at the after perpendicular. Since in a later process we

will integrate over the length using an arbitrarily chosen method of integration, namely, Simpson's First Rule, the number of stations must be odd and the station spacings must be equal.

We now proceed to solve for the sectional added mass and damping coefficients. The conditions stated in the problem statement in Section 2.1 apply except that we replace the ship with an infinitely-long cylinder whose lower half cross-section is that of the ship's section up to its free-floating draft at a particular station. The axis of the cylinder is on the free surface of the water. It is clear that we need to consider as many cylinders as there are stations. Two-dimensional conditions and potential flow are implied.

We define an added mass coefficient, k_2k_4, as the ratio of the added mass of the cylinder to the mass of the fluid displaced by a circular cylinder of equal beam. For a cylinder with a half-beam, b, at the free surface, Porter showed that

$$k_2k_4 = \frac{4}{\pi} \frac{M_oB + N_oA}{A^2 + B^2} \tag{3}$$

and the corresponding damping coefficient, referred to the same circular cylinder, is

$$c = \frac{2\pi}{A^2 + B^2} \; . \tag{4}$$

For a given section, A, B, M_o, and N_o are functions of only the non-dimensional frequency, $\delta = \omega^2 b/g$, in which ω is the circular frequency of oscillation and g is the acceleration of gravity. A numerical procedure to evaluate these functions is outlined in detail in Porter's paper [8]. Since the coefficients of the transform (2) determine the shape of the given section, we note that the added mass and damping

coefficients are functions of the frequency and transform coefficients only. It is clear that the number of these transform coefficients fixes the value of N in (2). Thus we have the solution to the problem provided we know these "a's".

The determination of these transform coefficients constitutes a separate problem. Grim infers fitting with two parameters. Landweber and Macagno propose three parameters. We introduce the use of the method described in Appendix A-1 which gives us a more accurate fit by a proper choice of N. The theoretical value of N for any given shape at present is still undetermined. We therefore arbitrarily set a reasonable criterion for our "best fit", and hence for our selection of N.

Knowing the beam-draft ratio and the area coefficient of the cylinder, we can solve for unique values of a_1 and a_3 (see Appendix A-2). By applying these values to equations (3) and (4), we obtain Grim's predictions. We note therefore that Grim's method is a special case of Porter's general solution.

We now have two sets of values for the sectional added mass and damping coefficients at any frequency for each ship's station, namely, those derived from our "best fit" and those due to the method of Grim. We compare these two sets of results which are the predicted curves for the longitudinal distribution of the added mass and damping coefficients.

On the basis of "strip theory", the ship's added mass, mk_s, and damping coefficient, b_s, can be obtained by integrating over the

ship's length the corresponding two-dimensional quantities. For each station at each frequency, we first multiply the k_2k_4 value obtained by ρS where ρ is the density of the water and $S = \frac{\pi}{2} b^2$. Likewise, we multiply the c value by $\rho S\omega$. We then integrate the resulting values over the length of the ship using Simpson's First Rule to obtain the ship's added mass and damping coefficient at each frequency. We compare these curves with the published experimental results.

2.3 Numerical Calculations

Numerical results were obtained for the following ships:

1. Series 60 Block 0.70
2. Series 60 Block 0.60
3. Weinblum Model

For uniformity, twenty one stations and twenty one waterlines from the keel to the designer's waterline were used for each ship. The offsets used for the Series 60 ships are those punched in computer cards available in the Department of Naval Architecture and Marine Engineering Library. The offsets used for the Weinblum Model are those generated by the IBM 7094 computer at the Computation Center, M.I.T. using the mathematical definition for the model's lines. All offsets are normalized with respect to the maximum half-beam and given up to the third decimal place.

The section-fitting method allows one to choose the number of transform coefficients to use. In these calculations, no more than

five coefficients were used solely for the reason of saving on computer time. In each case where the improvement in the fit was not substantial, a lesser number of coefficients was used. The method was programmed by the author thereby allowing calculations to be done by the IBM 7094 digital computer. The transform coefficients selected for each station of the three ships are shown in Tables I, II, and III.

In solving for Grim's a_1 and a_3, the parameter

$$\lambda = \frac{\text{draft}}{\text{half-beam}}$$

was used instead of the beam-draft ratio. This obviously did not change the end results. The other parameter used was the section area coefficient,

$$\sigma = \frac{\text{section area}}{2 \text{ x half-beam x draft}} .$$

For the Series 60 ships, the values of σ were derived from the published ones. In the case of the Weinblum Model, the section area coefficients were obtained by analytical integration. A computer program of Professor Porter which takes λ and σ as input data was used to calculate a_1 and a_3. The values of λ, σ, a_1, and a_3 are presented in Tables IV, V, and VI.

Numerical calculations for the sectional added mass and damping coefficients were likewise done by the IBM 7094 digital computer using Professor Porter's program.

The data for the models used by the experimenters were also used in calculating the ships' added mass and damping coefficients in order that the resultant curves may be compared with the published experimental results.

TABLE I

Transform Coefficients for Series 60 Block 0.70 Ship

Station	a_1	a_3	a_5	a_7	a_9
1	----	----	----	----	----
2	-.58924	-.03769	.00647	.00548	----
3	-.28865	-.05512	.00595	----	----
4	-.10604	-.06690	.01227	.00152	.00771
5	.00286	-.08005	.01031	-.00186	.00698
6	.06121	-.09564	.00662	-.00458	.00526
7	.08877	-.11214	.00370	----	----
8	.09604	-.12545	-.00249	-.00091	.00358
9	.10543	-.13671	-.00928	.00533	----
10	.10543	-.13671	-.00928	.00533	----
11	.10543	-.13671	-.00928	.00533	----
12	.10413	-.13545	-.00808	.00402	----
13	.10047	-.12610	-.00391	----	----
14	.09419	-.10551	.00486	-.00525	----
15	.08197	-.08109	.01288	-.00260	----
16	.05187	-.04343	.02686	----	----
17	-.00480	.00432	.04346	----	----
18	-.10876	.06381	.06217	.00847	----
19	-.31290	.11763	.09148	.02231	.00311
20	-.70043	.12650	.09670	.05817	.02740
21	----	----	----	----	----

TABLE II

Transform Coefficients for Series 60 Block 0.60 Ship

Station	a_1	a_3	a_5	a_7	a_9
1	----	----	----	----	----
2	-.77186	-.02877	.00656	.00522	----
3	-.55189	-.03135	.00364	----	----
4	-.34603	-.02787	.00069	.00619	----
5	-.18343	-.03758	.00410	.00206	.00658
6	-.06696	-.05339	.00617	-.00165	.00744
7	.01148	-.06992	.00582	-.00620	.00469
8	.05821	-.08784	.00412	-.00838	.00381
9	.08669	-.10264	.00265	-.00641	----
10	.09729	-.11366	.00043	-.00475	----
11	.10190	-.12402	-.00566	----	----
12	.09832	-.11822	-.00126	-.00488	----
13	.09603	-.10342	.00293	-.00601	----
14	.08899	-.07995	.00919	-.00679	----
15	.07168	-.05008	.01944	----	----
16	.03123	-.00606	.03972	-.00038	.00687
17	-.02575	.04386	.05626	.00212	.00430
18	-.13091	.09183	.07244	.01083	----
19	-.32985	.11966	.09513	.03366	----
20	-.64328	.09866	.09761	.05619	.02954
21	-.05123	.16732	.00725	.01821	----

TABLE III

Transform Coefficients for Weinblum Model

Station	a_1	a_3	a_5	a_7	a_9
1	----	----	----	----	----
2	-.58026	-.04867	-.00944	.00639	----
3	-.34470	-.07217	-.01064	.00440	----
4	-.19658	-.08429	-.01010	-.00024	.00360
5	-.09651	-.09191	-.00877	-.00340	.00450
6	-.02668	-.09738	-.00712	-.00564	.00476
7	.02263	-.10174	-.00552	-.00700	.00460
8	.05703	-.10531	-.00421	-.00760	.00424
9	.07986	-.10808	-.00330	-.00771	.00383
10	.09295	-.10988	-.00280	-.00760	.00352
11	.09724	-.11051	-.00264	-.00752	.00339

TABLE IV

Grim Parameters for Series 60 Block 0.70 Ship

Station	λ	σ	a_1	a_3
1	---	---	---	---
2	3.941	.801	-.59129	-.00659
3	1.814	.827	-.28221	-.02443
4	1.198	.855	-.08598	-.04381
5	.955	.888	.02167	-.06583
6	.848	.922	.07488	-.08731
7	.810	.956	.09368	-.10908
8	.800	.976	.09752	-.12231
9	.800	.985	.09686	-.12829
10	.800	.986	.09678	-.12896
11	.800	.986	.09678	-.12896
12	.800	.985	.09686	-.12829
13	.800	.980	.09723	-.12496
14	.800	.963	.09847	-.11374
15	.811	.929	.09508	-.09138
16	.844	.978	.07416	-.12412
17	.920	.808	.04131	-.01441
18	1.080	.723	-.03982	.04002
19*	1.465	.619	-.20811	.10280
20	2.857	.493	-.55146	.14533
21	---	---	---	---

*See text paragraph ~~3.6~~ 3.3 for alternate values.

TABLE V

Grim Parameters for Series 60 Block 0.60 Ship

Station	λ	σ	a_1	a_3
1	---	---	---	---
2	7.921	.822	-.76856	-.00934
3	3.509	.823	-.54729	-.01642
4	2.046	.807	-.33923	-.01218
5	1.426	.827	-.17107	-.02583
6	1.114	.857	-.05155	-.04572
7	.951	.896	.02322	-.07057
8	.862	.926	.06739	-.09020
9	.817	.953	.08983	-.10719
10	.800	.967	.09818	-.11634
11	.800	.977	.09746	-.12288
12	.800	.973	.09775	-.12026
13	.800	.955	.09912	-.10792
14	.805	.922	.09876	-.08670
15	.820	.864	.09418	-.04961
16	.853	.781	.07961	.00267
17	.932	.693	.03703	.05863
18	1.103	.600	-.05505	.11930
19	1.495	.489	-.23533	.18555
20	2.597	.346	-.54751	.23302
21	1.076	.582	-.04138	.13109

TABLE VI

Grim Parameters for Weinblum Model

Station	λ	σ	a_1	a_3
1	---	---	---	---
2	4.211	.905	-.58706	-.04723
3	2.222	.917	-.35199	-.07202
4	1.569	.927	-.20218	-.08672
5	1.250	.937	-.10043	-.09613
6	1.067	.944	-.02896	-.10241
7	.952	.951	.02179	-.10670
8	.879	.956	.05728	-.10960
9	.833	.959	.08077	-.11149
10	.808	.961	.09420	-.11254
11	.800	.962	.09857	-.11289

III. RESULTS

3.1 General

It has been stated in Chapter II that the offsets used for the Series 60 ships are those that have been previously punched in computer cards. These offsets are tabulated in Appendix B (Tables XXI and XXII). On the other hand, the section area coefficients used are those published in David Taylor Model Basin Report No. 1712. It is to be made clear that the numerical results presented in this paper are based on these data unless otherwise specified. This clarification is necessary because it was discovered late in the process of this investigation that there were differences in some of the offsets used and those in the above-mentioned publication.

3.2 Selected Fits

The resulting fits due to the selected transform coefficients are shown for all stations on the body plan of each ship in Figures 1, 2, and 3. The solid lines are the actual ship sections and the corresponding fits are in broken lines. Since the Weinblum Model is symmetrical about the midship section, only half of the body plan is shown.

3.3 Grim Fits

The fits due to Grim's a_1 and a_3 are shown (broken lines) for

representative sections (solid lines) of Series 60 Block 0.70 ship only. Figure 4 shows the two-parameter fit for Station 19 which is a vee-shaped section and Figure 5 for Station 11, the midship section. Since the offsets for Station 19 were seen to be different from the published ones, the area coefficient of this section was recalculated by integration (using Simpson's Rule) of the offsets used. Based on this section area coefficient ($\sigma = .506$), a new set of a_1 and a_3 ($a_1 = -.22185$; $a_3 = .17562$) was derived for this station to determine the Grim fit shown in Figure 5.

3.4 Fits to a Bulbous Section

A special type of form found on the forebody of many ships is bulbous. Such a shape is certainly of great interest in an investigation of this kind. Since none of the three ships considered has a section of this form, the section at the forward perpendicular of the Mariner was chosen for study. A five-coefficient fit (dotted line) and a Grim fit (broken line) are both shown with the actual section (solid line) in Figure 6. The corresponding transform coefficients and Grim parameters are listed in Tables VII and VIII, respectively.

3.5 Longitudinal Distribution of Added Mass and Damping

The calculated sectional added mass and damping coefficients are presented in tabular form for each station of the three ships at different values of a non-dimensional frequency,

$$\delta_r = \frac{\omega^2 b_m}{g}$$

in which b_m is the maximum half-beam of the ship. The sectional added mass coefficients due to the selected fits are shown in Tables IX, X, and XI and those due to the Grim Fits in Tables XII, XIII, and XIV. The sectional damping coefficients derived from the selected fits are shown in Tables XV, XVI, and XVII and those due to the Grim Fits in Tables XVIII, XIX, and XX. Values of $k_2 k_4$ and c are plotted against δ_r for typical sections of Series 60 Block 0.70 ship in order to have a comparison between the two methods. Stations 11 and 19 were again chosen as the representative sections. Figure 7 shows the sectional added mass coefficient curves and Figure 8 the sectional damping coefficient curves. Curves derived from the Grim fits are labelled "Grim" and those due to the selected fits are labelled "Porter". The values of $k_2 k_4$ and c for the Station 19 Grim curves are based on the new set of a_1 and a_3 calculated for this station. The numerical computations for all added mass and damping constants used Porter's program, simulating Grim's results for two-parameter fits and used in the general way for forms using more coefficients.

3.6 Ship's Added Mass and Damping

The theoretical predictions for the ship's added mass and damping coefficients are plotted against frequency on the same graph

with the corresponding experimental curve for comparison. These curves are labelled in the same manner as that in the preceding section. The experimental curves were plotted by using points picked off the published ones.

In order to conform with the values obtained by experiment, the results due to the selected fits and the Grim fits for the Series 60 ships were made dimensional. The Block 0.70 ship's added mass, mk_s(in kg-sec^2/m), and damping coefficient, b_s (in kg-sec/m), are plotted against the circular frequency, ω(in radians/sec), and shown in Figures 9 and 12, respectively. The corresponding quantities for the Block 0.60 ship are shown in the same manner in Figures 10 and 13. Since the experimental results for the Block 0.60 ship are available only for several values of forward speed and none for pure heaving motion, the curves given for the lowest speed (Froude number, Fn = 0.15) were used.

In the case of the Weinblum Model, the predicted curves are presented in non-dimensional form to agree with the published results. Figure 11 shows the predicted and experimental curves for the ship's added mass coefficient, k_s, plotted as a function of the non-dimensional frequency

$$\delta' = \omega \sqrt{\frac{2b_m}{g}} .$$

Likewise, the corresponding values of the dimensionless ship's damping coefficient,

$$b_s' = \frac{b_s}{\Delta\sqrt{gL}} ,$$

where Δ is the displacement and L the length of the ship, are plotted against δ' in Figure 14.

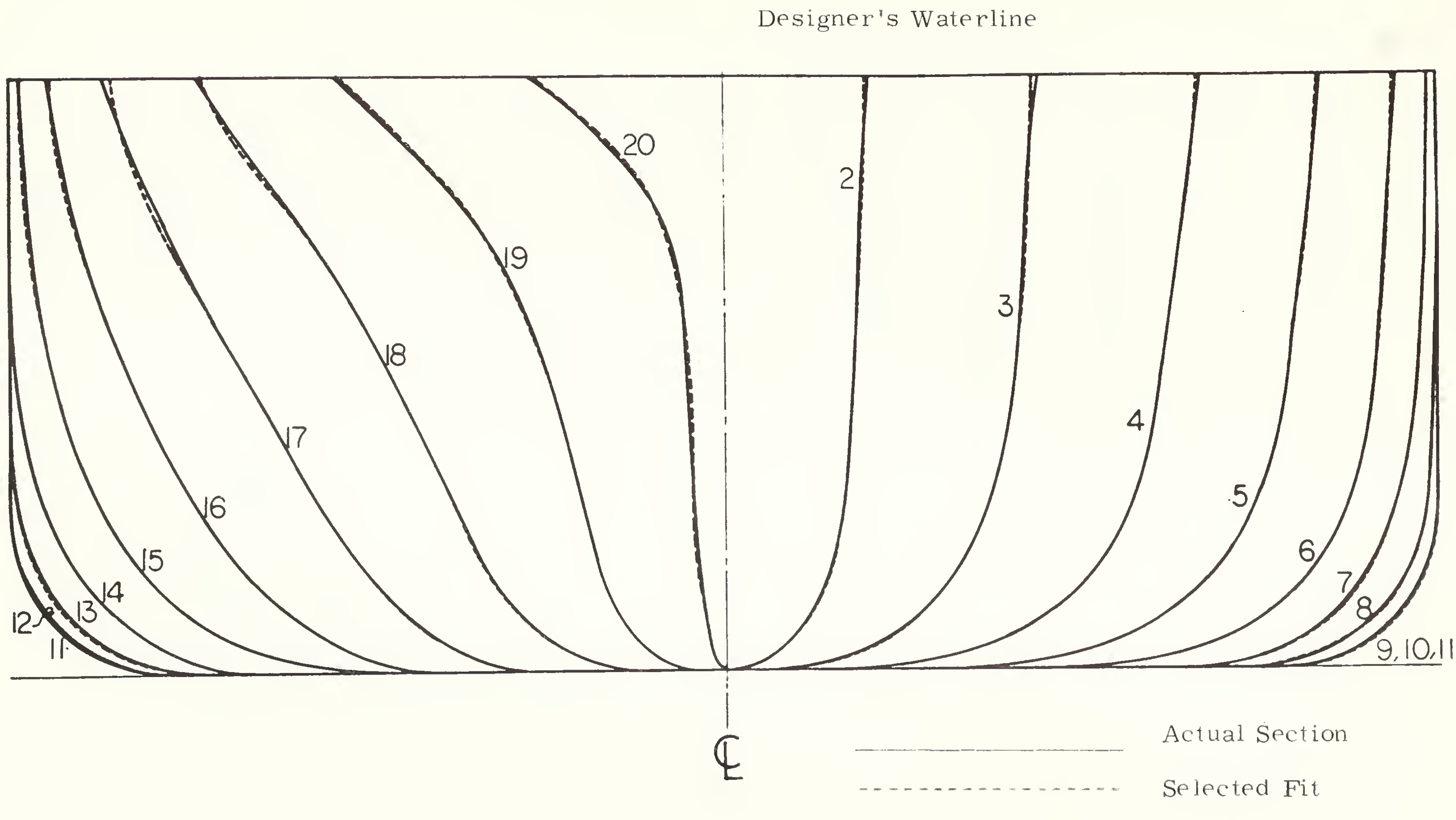


Figure 1. Selected Fits to Sections of Series 60 Block 0.70 Ship

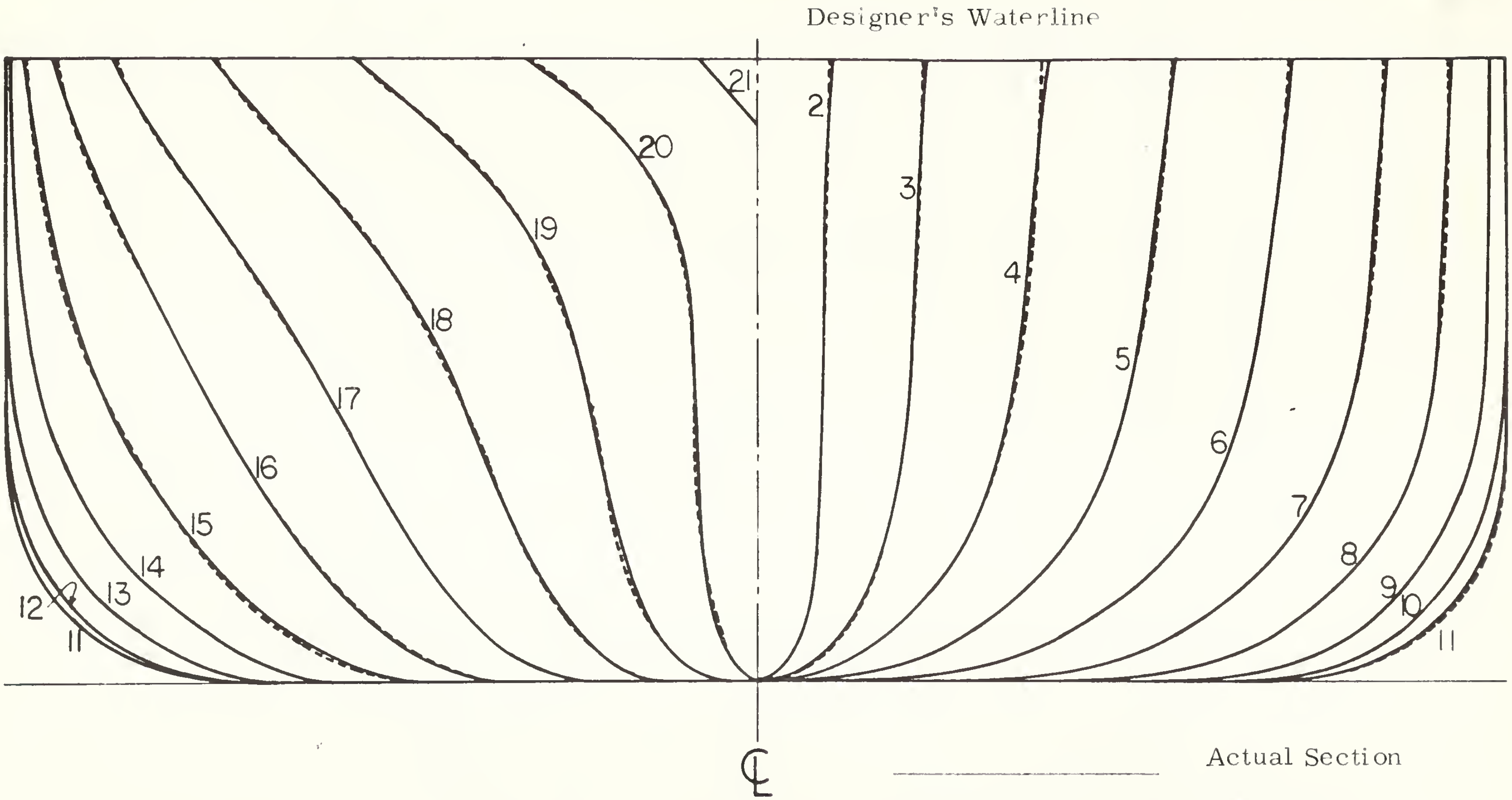


Figure 2. Selected Fits to Sections of Series 60 Block 0.60 Ship

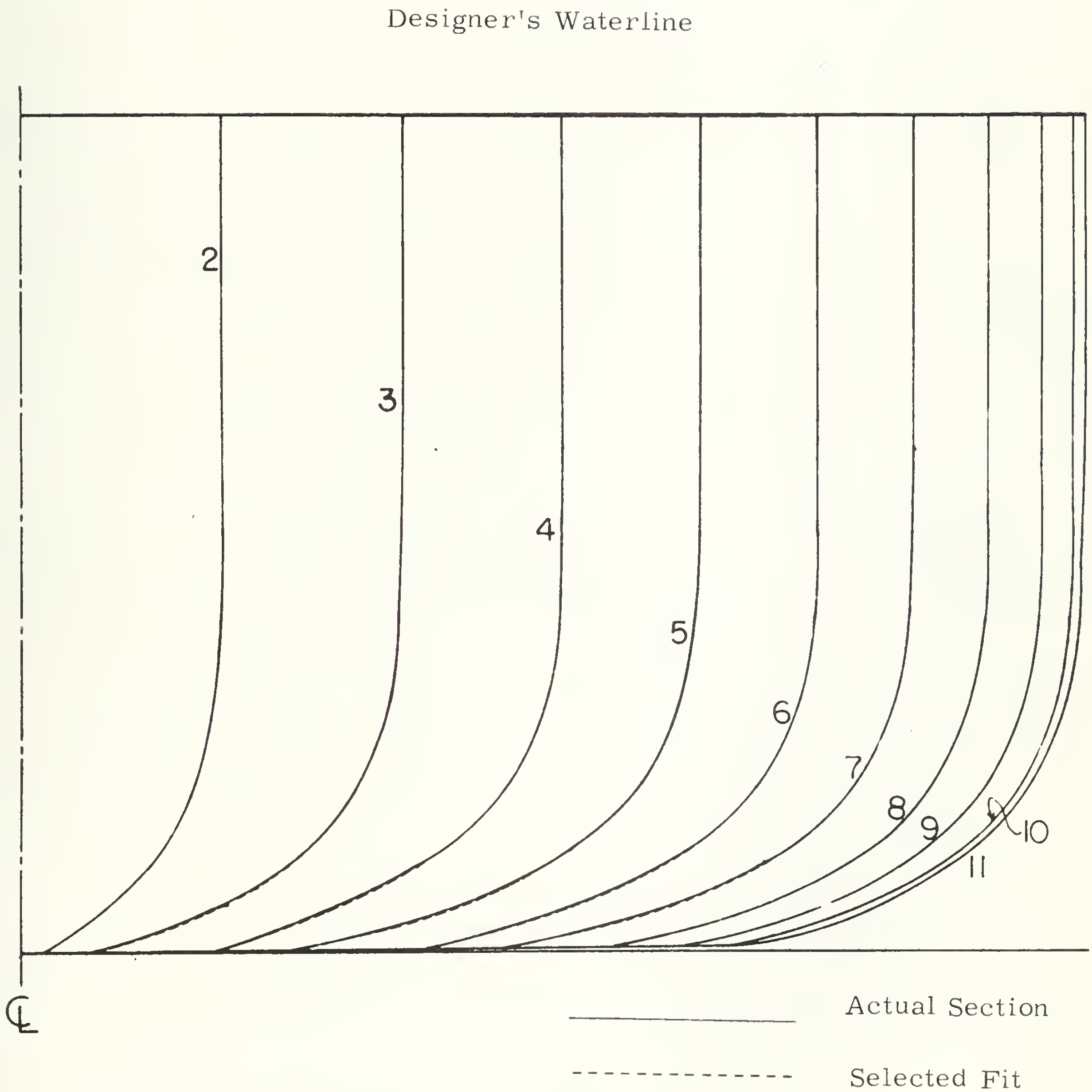


Figure 3. Selected Fits to Sections of Weinblum Model

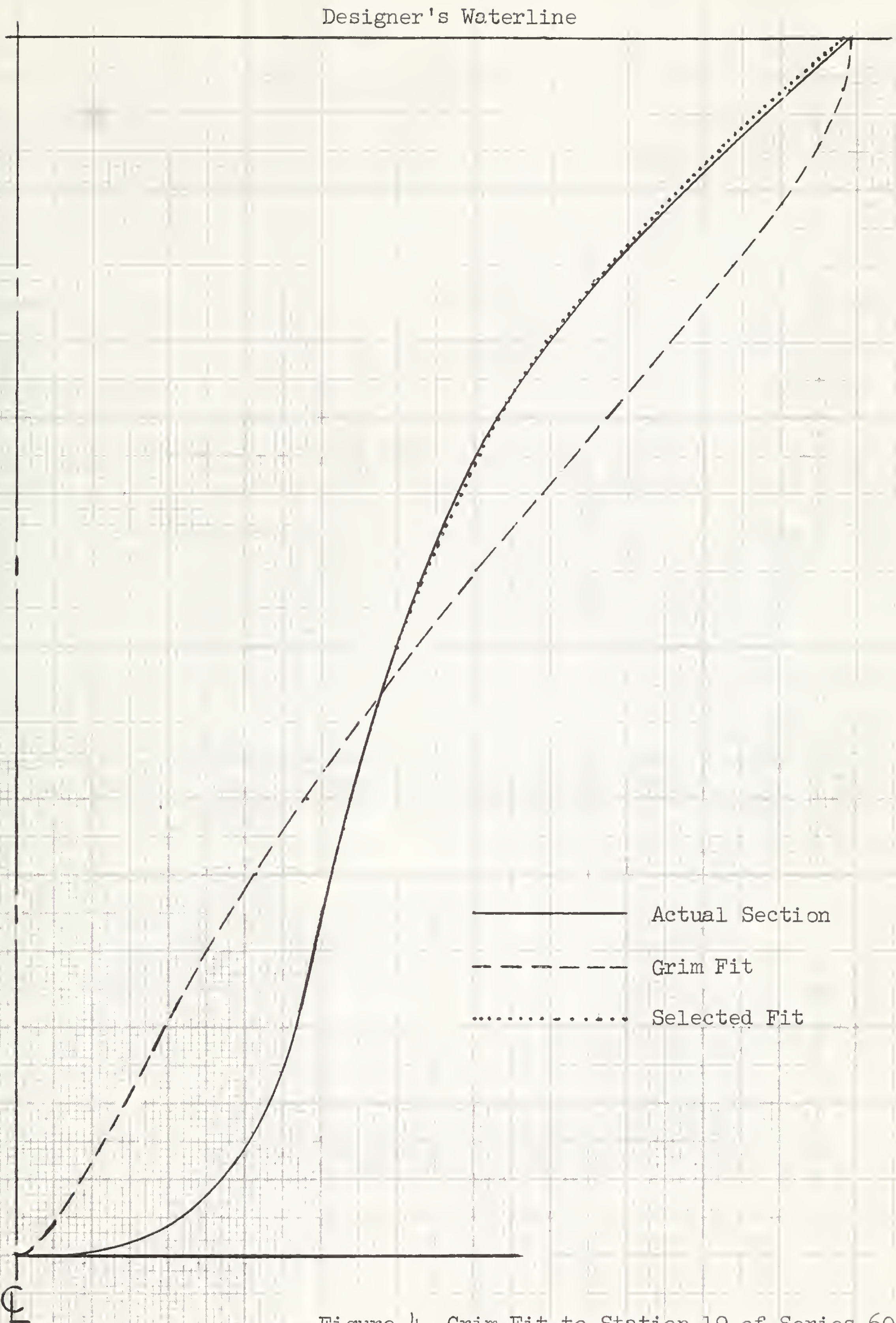


Figure 4. Grim Fit to Station 19 of Series 60 Block 0.70 Ship (Selected fit is also shown in Fig. 1)

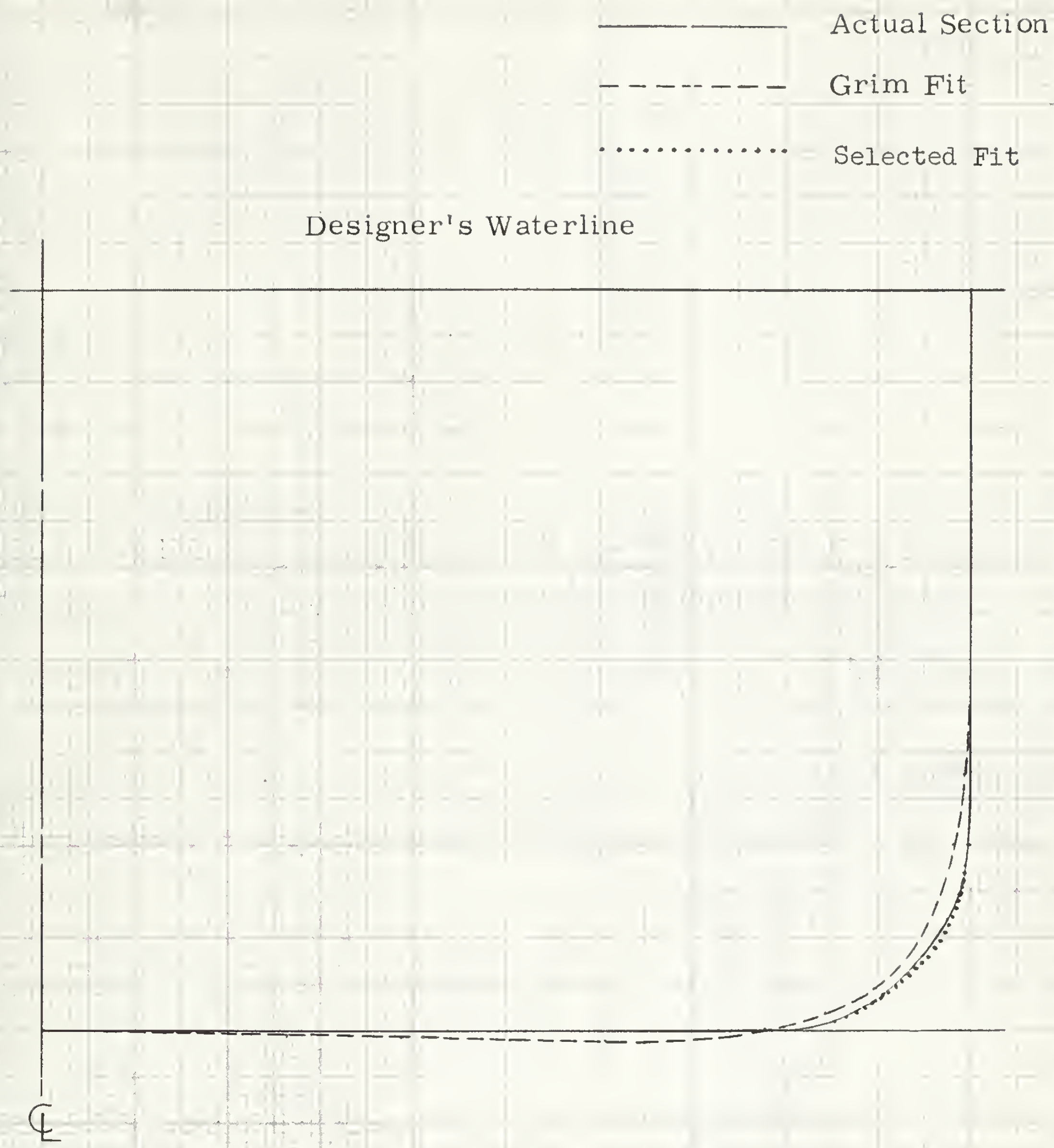


Figure 5. Grim Fit to Midship Section of Series 60 Block 0.70 Ship (Selected fit is also shown in Figure 1)

Figure 6. Fits to a Bulbous Section (Section I of the Mariner)

TABLE VII

Transform Coefficients for Station 1 of the Mariner

a_1	a_3	a_5	a_7	a_9
-.85868	-.11785	-.1892	.00633	-.00054

TABLE VIII

Grim Parameters for Station 1 of the Mariner

λ	σ	a_1	a_3
133.333	7.407	-.85971	-.12730

TABLE IX

Sectional Added Mass Coefficients (k_2k_4) for Series 60 Block 0.70 Ship due to Selected Fits

δ_r / Station	.100	.200	.400	.600	.800	1.000	1.250	1.500	2.000	3.000
1	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
2	1.908	1.336	.879	.711	.652	.645	.699	.707	.791	.918
3	1.750	1.236	.851	.722	.686	.691	.723	.765	.847	.961
4	1.632	1.160	.821	.713	.684	.691	.721	.758	.829	.928
5	1.575	1.132	.828	.741	.727	.743	.779	.820	.892	.987
6	1.556	1.130	.850	.780	.780	.806	.850	.895	.971	1.062
7	1.564	1.146	.880	.822	.831	.864	.915	.963	1.041	1.129
8	1.582	1.168	.912	.864	.881	.921	.978	1.029	1.107	1.193
9	1.609	1.196	.946	.904	.928	.973	1.033	1.086	1.165	1.249
10	1.609	1.196	.946	.904	.928	.973	1.033	1.086	1.165	1.249
11	1.609	1.196	.946	.904	.928	.973	1.033	1.086	1.165	1.249
12	1.606	1.193	.942	.900	.924	.969	1.028	1.082	1.161	1.244
13	1.588	1.174	.919	.872	.891	.932	.990	1.042	1.121	1.207
14	1.553	1.136	.870	.811	.819	.851	.901	.950	1.027	1.116
15	1.525	1.103	.822	.745	.737	.757	.795	.836	.907	.998
16	1.506	1.075	.773	.675	.647	.648	.669	.696	.753	.838
17	1.518	1.073	.747	.627	.578	.561	.563	.576	.613	.682
18	1.582	1.116	.757	.610	.538	.501	.481	.476	.487	.528
19	1.746	1.247	.839	.657	.557	.497	.453	.428	.409	.413
20	2.242	1.690	1.197	.951	.801	.701	.614	.555	.481	.415
21	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

TABLE X

Sectional Added Mass Coefficients (k_2k_4) for

Series 60 Block 0.60 Ship due to Selected Fits

δ_r Station	.200	.400	.600	.800	1.000	1.250	1.500	2.000
1	-----	-----	-----	-----	-----	-----	-----	-----
2	1.424	.936	.754	.692	.684	.710	.752	.841
3	1.304	.855	.690	.634	.628	.654	.694	.779
4	1.219	.808	.657	.602	.592	.610	.642	.714
5	1.170	.801	.670	.626	.621	.641	.672	.739
6	1.140	.806	.698	.668	.673	.700	.735	.804
7	1.129	.823	.734	.719	.734	.770	.811	.885
8	1.130	.846	.774	.772	.797	.842	.887	.946
9	1.140	.871	.810	.817	.850	.900	.949	1.027
10	1.152	.891	.838	.851	.888	.942	.993	1.072
11	1.172	.916	.869	.887	.929	.986	1.038	1.117
12	1.161	.903	.853	.870	.910	.966	1.018	1.097
13	1.136	.869	.810	.817	.850	.900	.949	1.027
14	1.105	.826	.752	.747	.768	.808	.851	.924
15	1.077	.782	.690	.666	.672	.696	.727	.787
16	1.061	.747	.635	.592	.579	.584	.599	.638
17	1.084	.748	.617	.556	.528	.517	.518	.537
18	1.144	.778	.625	.546	.503	.476	.464	.464
19	1.269	.858	.673	.571	.509	.463	.463	.413
20	1.553	1.073	.840	.701	.610	.535	.486	.428
21	2.859	2.285	1.963	1.745	1.583	1.428	1.309	1.133

TABLE XI

Sectional Added Mass Coefficients ($k_2 k_4$) for Weinblum Model due to Selected Fits

δ_r	.200	.400	.600	.800	1.000	1.250	1.500	2.000	4.000	6.250
Station										
1	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
2	1.462	1.011	.854	.809	.814	.852	.901	.999	1.202	1.269
3	1.362	.967	.842	.815	.831	.875	.927	1.021	1.203	1.260
4	1.290	.934	.829	.814	.835	.882	.933	1.023	1.190	1.242
5	1.242	.915	.825	.818	.845	.894	.945	1.032	1.187	1.235
6	1.208	.902	.825	.824	.854	.905	.956	1.040	1.187	1.232
7	1.184	.895	.826	.831	.863	.915	.966	1.049	1.189	1.232
8	1.167	.891	.829	.837	.871	.924	.975	1.056	1.191	1.232
9	1.157	.889	.831	.842	.878	.932	.982	1.062	1.194	1.234
10	1.151	.888	.833	.846	.883	.937	.987	1.066	1.196	1.235
11	1.149	.888	.834	.847	.884	.938	.989	1.068	1.197	1.236

TABLE XII

Sectional Added Mass Coefficients (k_2k_4) for

Series 60 Block 0.70 Ship due to Grim Fits

δ_r / Station	.100	.200	.400	.600	.800	1.000	1.250	1.500	2.000	3.000
1	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
2	1.858	1.282	.807	.617	.539	.515	.532	.551	.624	.749
3	1.730	1.212	.811	.664	.610	.600	.617	.648	.719	.832
4	1.634	1.160	.814	.698	.663	.665	.691	.726	.797	.900
5	1.581	1.135	.826	.734	.715	.729	.763	.803	.876	.974
6	1.559	1.131	.847	.772	.768	.791	.833	.877	.953	1.046
7	1.566	1.147	.879	.820	.828	.861	.911	.960	1.037	1.127
8	1.578	1.163	.905	.854	.870	.908	.963	1.014	1.093	1.179
9	1.587	1.173	.918	.871	.891	.932	.989	1.041	1.120	1.205
10	1.588	1.174	.920	.873	.893	.935	.992	1.044	1.123	1.207
11	1.588	1.174	.920	.873	.893	.935	.992	1.044	1.123	1.207
12	1.587	1.173	.918	.871	.891	.932	.989	1.041	1.120	1.205
13	1.582	1.167	.910	.862	.879	.919	.975	1.026	1.105	1.190
14	1.567	1.150	.887	.831	.842	.877	.929	.978	1.056	1.144
15	1.547	1.126	.849	.779	.778	.803	.847	.892	.967	1.059
16	1.602	1.181	.916	.864	.880	.919	.975	1.027	1.107	1.194
17	1.545	1.100	.780	.668	.630	.625	.639	.664	.720	.810
18	1.590	1.124	.769	.631	.570	.545	.540	.550	.587	.665
19	1.697	1.195	.790	.616	.526	.479	.451	.443	.455	.513
20	1.945	1.386	.894	.659	.525	.443	.383	.351	.331	.364
21	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

TABLE XIII

Sectional Added Mass Coefficients (k_2k_4) for

Series 60 Block 0.60 Ship due to Grim Fits

δ_r / Station	.200	.400	.600	.800	1.000	1.250	1.500	2.000
1	-----	-----	-----	-----	-----	-----	-----	-----
2	1.354	.846	.643	.561	.539	.551	.584	.667
3	1.290	.828	.649	.579	.562	.576	.608	.685
4	1.217	.799	.639	.577	.560	.572	.599	.666
5	1.175	.802	.668	.621	.614	.632	.662	.729
6	1.148	.812	.701	.669	.673	.699	.734	.804
7	1.139	.832	.742	.726	.742	.778	.819	.893
8	1.138	.853	.779	.775	.800	.843	.888	.965
9	1.147	.877	.816	.822	.854	.904	.952	1.030
10	1.154	.892	.838	.850	.886	.939	.989	1.067
11	1.164	.906	.856	.872	.911	.966	1.017	1.095
12	1.160	.900	.848	.863	.901	.955	1.005	1.084
13	1.142	.876	.816	.824	.856	.906	.954	1.032
14	1.119	.841	.769	.766	.789	.831	.875	.949
15	1.094	.800	.709	.688	.697	.725	.759	.825
16	1.086	.772	.660	.619	.609	.619	.639	.688
17	1.109	.772	.640	.580	.555	.547	.553	.581
18	1.167	.799	.644	.566	.523	.498	.490	.497
19	1.291	.876	.688	.583	.521	.474	.449	.431
20	1.575	1.086	.845	.700	.605	.527	.476	.421
21	2.823	2.248	1.925	1.706	1.544	1.389	1.269	1.093

TABLE XIV

Sectional Added Mass Coefficients $(k_2 k_4)$ for

Weinblum Model due to Grim Fits

δ_r	.200	.400	.600	.800	1.000	1.250	1.500	2.000	4.000	6.250
Station										
1	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
2	1.434	.982	.825	.778	.782	.818	.867	.963	1.166	1.234
3	1.345	.949	.822	.793	.806	.848	.898	.991	1.173	1.231
4	1.283	.927	.822	.805	.825	.871	.921	1.010	1.176	1.229
5	1.240	.912	.822	.814	.839	.887	.938	1.023	1.179	1.228
6	1.208	.902	.824	.822	.851	.900	.950	1.033	1.180	1.226
7	1.185	.895	.825	.828	.859	.910	.959	1.041	1.181	1.225
8	1.168	.891	.827	.833	.866	.917	.966	1.046	1.182	1.224
9	1.157	.888	.828	.836	.870	.922	.971	1.050	1.183	1.224
10	1.151	.886	.829	.838	.873	.924	.974	1.052	1.183	1.223
11	1.149	.885	.829	.839	.874	.925	.974	1.052	1.183	1.223

TABLE XV

Sectional Damping Coefficients (c) for

Series 60 Block 0.70 Ship due to Selected Fits

δ_r	.100	.200	.400	.600	.800	1.000	1.250	1.500	2.000	3.000
Station										
1	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
2	2.078	1.760	1.301	.979	.743	.567	.408	.296	.160	.053
3	1.944	1.598	1.142	.839	.624	.467	.329	.233	.121	.037
4	1.850	1.497	1.056	.775	.579	.438	.314	.228	.126	.045
5	1.776	1.410	.969	.694	.506	.374	.261	.184	.097	.032
6	1.726	1.350	.901	.627	.443	.317	.211	.143	.069	.019
7	1.700	1.316	.860	.584	.402	.279	.179	.117	.052	.012
8	1.685	1.292	.828	.548	.367	.248	.153	.096	.039	.008
9	1.677	1.279	.808	.526	.345	.227	.136	.082	.031	.005
10	1.677	1.279	.808	.526	.345	.227	.136	.082	.031	.005
11	1.677	1.279	.808	.526	.345	.227	.136	.082	.031	.005
12	1.677	1.280	.809	.527	.346	.228	.137	.082	.031	.005
13	1.683	1.290	.823	.543	.361	.242	.148	.091	.036	.006
14	1.699	1.315	.861	.586	.404	.281	.181	.118	.052	.012
15	1.724	1.354	.918	.651	.471	.345	.237	.166	.085	.026
16	1.765	1.414	1.000	.745	.569	.442	.329	.249	.149	.063
17	1.819	1.488	1.097	.854	.683	.557	.440	.353	.238	.123
18	1.897	1.539	1.227	.997	.833	.709	.591	.501	.373	.232
19	2.011	1.741	1.405	1.187	1.028	.906	.786	.692	.555	.390
20	2.200	1.998	1.722	1.531	1.384	1.267	1.148	1.051	.901	.704
21	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

TABLE XVI

Sectional Damping Coefficients (c) for

Series 60 Block 0.60 Ship due to Selected Fits

δ_r	.200	.400	.600	.800	1.000	1.250	1.500	2.000
Station								
1	-----	-----	-----	-----	-----	-----	-----	-----
2	1.824	1.355	1.018	.770	.586	.419	.302	.162
3	1.741	1.282	.961	.727	.553	.395	.284	.150
4	1.663	1.224	.927	.712	.552	.405	.300	.170
5	1.572	1.140	.857	.656	.508	.373	.278	.160
6	1.484	1.050	.774	.581	.442	.318	.232	.129
7	1.413	.972	.697	.509	.376	.261	.184	.095
8	1.358	.909	.634	.449	.321	.213	.144	.068
9	1.323	.868	.591	.408	.284	.182	.118	.051
10	1.303	.842	.564	.382	.261	.163	.103	.043
11	1.291	.825	.545	.363	.243	.149	.092	.036
12	1.295	.832	.552	.370	.250	.154	.095	.038
13	1.316	.862	.586	.404	.281	.180	.117	.051
14	1.347	.908	.640	.459	.333	.226	.156	.077
15	1.395	.977	.720	.543	.417	.305	.227	.131
16	1.456	1.063	.821	.653	.529	.416	.333	.223
17	1.526	1.154	.923	.761	.639	.525	.439	.320
18	1.619	1.265	1.042	.883	.762	.646	.557	.430
19	1.753	1.421	1.205	1.047	.926	.807	.714	.577
20	1.950	1.657	1.454	1.300	1.178	1.056	.957	.808
21	2.313	2.165	2.050	1.954	1.871	1.781	1.702	1.571

TABLE XVII

Sectional Damping Coefficients (c) for

Weinblum Model due to Selected Fits

δ_r	.200	.400	.600	.800	1.000	1.250	1.500	2.000	4.000	6.250
Station										
1	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
2	1.736	1.258	.923	.680	.502	.344	.236	.112	.007	.000
3	1.612	1.134	.814	.587	.425	.285	.191	.087	.004	.000
4	1.526	1.052	.744	.531	.380	.251	.167	.075	.004	.000
5	1.459	.989	.690	.486	.344	.224	.147	.065	.003	.000
6	1.407	.941	.649	.452	.317	.204	.133	.057	.003	.000
7	1.368	.904	.618	.426	.296	.189	.122	.052	.002	.000
8	1.339	.877	.594	.407	.281	.178	.113	.048	.002	.000
9	1.319	.858	.578	.394	.270	.170	.108	.045	.002	.000
10	1.307	.847	.569	.386	.264	.165	.105	.043	.002	.000
11	1.303	.843	.565	.383	.262	.164	.103	.043	.002	.000

TABLE XVIII

Sectional Damping Coefficients (c) for Series 60 Block 0.70 Ship due to Grim Fits

δ_r / Station	.100	.200	.400	.600	.800	1.000	1.250	1.500	2.000	3.000
1	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
2	2.105	1.806	1.371	1.061	.828	.650	.484	.363	.208	.075
3	1.969	1.640	1.206	.914	.702	.545	.400	.296	.167	.059
4	1.859	1.512	1.078	.799	.602	.458	.330	.240	.131	.045
5	1.782	1.421	.983	.709	.521	.387	.270	.191	.099	.031
6	1.733	1.360	.915	.642	.458	.330	.221	.151	.073	.020
7	1.702	1.318	.863	.587	.405	.281	.181	.118	.052	.012
8	1.688	1.297	.835	.556	.375	.254	.158	.100	.041	.008
9	1.683	1.289	.823	.543	.362	.243	.149	.092	.037	.007
10	1.682	1.288	.822	.542	.361	.242	.148	.091	.036	.006
11	1.682	1.288	.822	.542	.361	.242	.148	.091	.036	.006
12	1.683	1.289	.823	.543	.362	.243	.149	.092	.037	.007
13	1.686	1.294	.830	.550	.369	.249	.154	.096	.039	.007
14	1.695	1.308	.851	.575	.393	.271	.172	.111	.048	.010
15	1.716	1.340	.896	.624	.442	.316	.211	.143	.068	.018
16	1.703	1.312	.845	.563	.379	.256	.159	.099	.040	.008
17	1.802	1.459	1.050	.794	.614	.481	.361	.274	.164	.067
18	1.875	1.555	1.166	.918	.739	.605	.478	.382	.253	.123
19	1.985	1.695	1.328	1.086	.908	.769	.634	.529	.378	.209
20	2.154	1.916	1.583	1.345	1.161	1.012	.861	.738	.553	.328
21	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

TABLE XIX

Sectional Damping Coefficients (c) for

Series 60 Block 0.60 Ship due to Grim Fits

δ_r	.200	.400	.600	.800	1.000	1.250	1.500	2.000
Station								
1	-----	-----	-----	-----	-----	-----	-----	-----
2	1.869	1.422	1.094	.847	.658	.482	.355	.196
3	1.774	1.331	1.019	.787	.612	.449	.332	.185
4	1.682	1.252	.960	.747	.585	.436	.327	.190
5	1.579	1.150	.868	.666	.517	.380	.282	.161
6	1.489	1.057	.780	.587	.446	.320	.233	.127
7	1.415	.974	.699	.510	.377	.261	.183	.093
8	1.361	.914	.639	.454	.326	.218	.147	.070
9	1.324	.869	.593	.410	.286	.185	.121	.054
10	1.305	.846	.569	.388	.266	.168	.107	.046
11	1.296	.834	.555	.374	.253	.157	.099	.041
12	1.300	.839	.561	.379	.258	.162	.102	.043
13	1.316	.862	.587	.405	.282	.182	.119	.053
14	1.343	.902	.632	.450	.325	.218	.149	.072
15	1.389	.967	.706	.527	.399	.285	.207	.114
16	1.448	1.050	.802	.629	.500	.382	.297	.185
17	1.519	1.140	.904	.735	.608	.488	.398	.273
18	1.612	1.252	1.023	.858	.732	.609	.514	.378
19	1.747	1.409	1.187	1.023	.895	.768	.666	.516
20	1.942	1.640	1.428	1.265	1.133	.997	.885	.711
21	2.309	2.158	2.040	1.940	1.855	1.762	1.681	1.545

TABLE XX

Sectional Damping Coefficients (c) for Weinblum Model due to Grim Fits

δ_r	.200	.400	.600	.800	1.000	1.250	1.500	2.000	4.000	6.250
Station										
1	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
2	1.739	1.261	.928	.686	.508	.350	.242	.117	.007	.000
3	1.617	1.141	.823	.598	.436	.295	.200	.094	.006	.000
4	1.528	1.056	.749	.536	.385	.257	.172	.079	.005	.000
5	1.460	.992	.694	.491	.349	.230	.152	.069	.004	.000
6	1.410	.945	.654	.458	.323	.210	.138	.061	.004	.000
7	1.371	.909	.624	.433	.303	.196	.128	.056	.003	.000
8	1.343	.884	.602	.416	.289	.186	.120	.053	.003	.000
9	1.324	.866	.588	.404	.280	.179	.116	.050	.003	.000
10	1.313	.856	.579	.397	.274	.175	.113	.049	.003	.000
11	1.310	.853	.576	.395	.273	.174	.112	.048	.003	.000

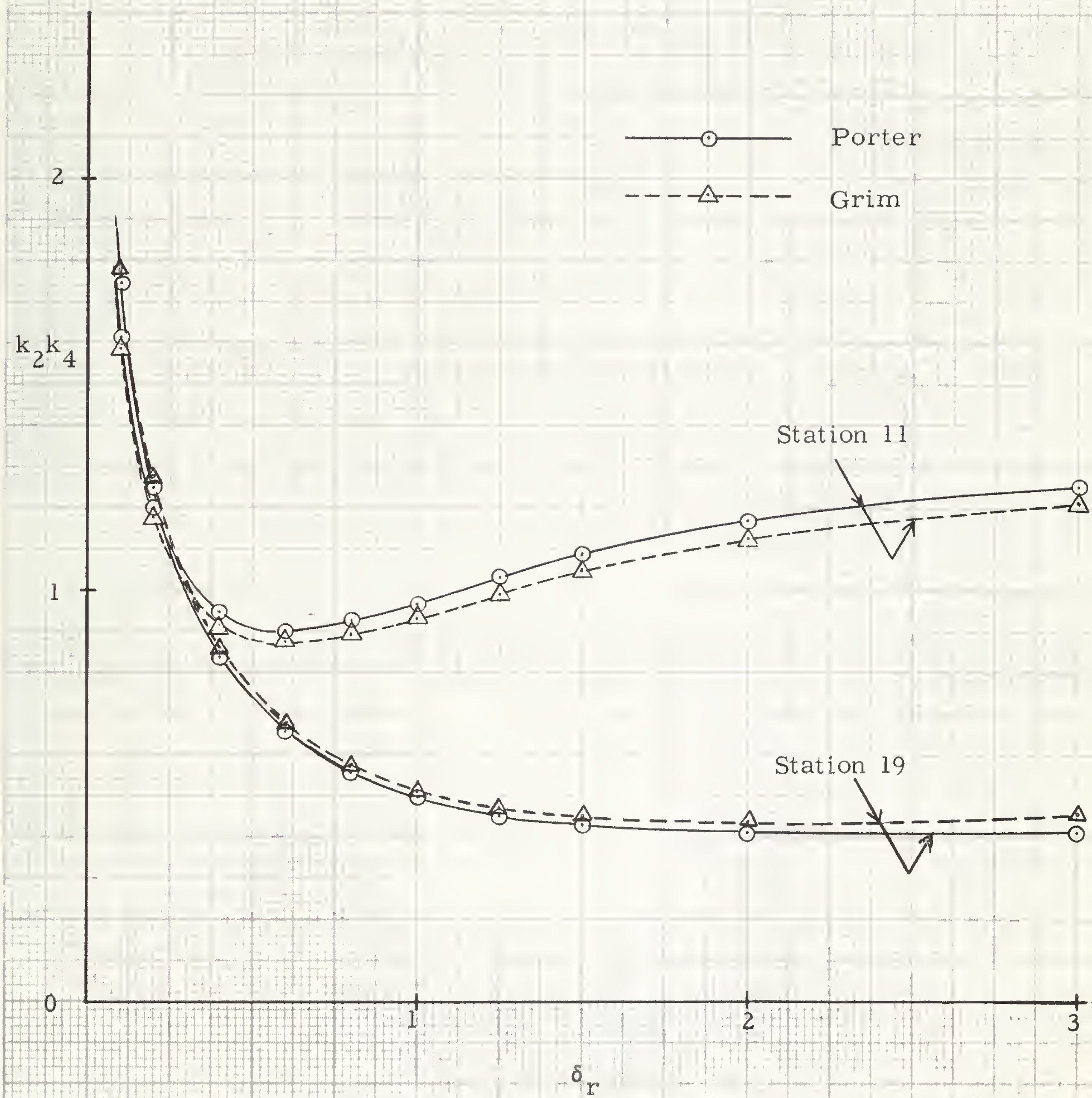


Figure 7. Comparison of Sectional Added Mass Coefficient Curves for Typical Sections of Series 60 Block 0.70 Ship

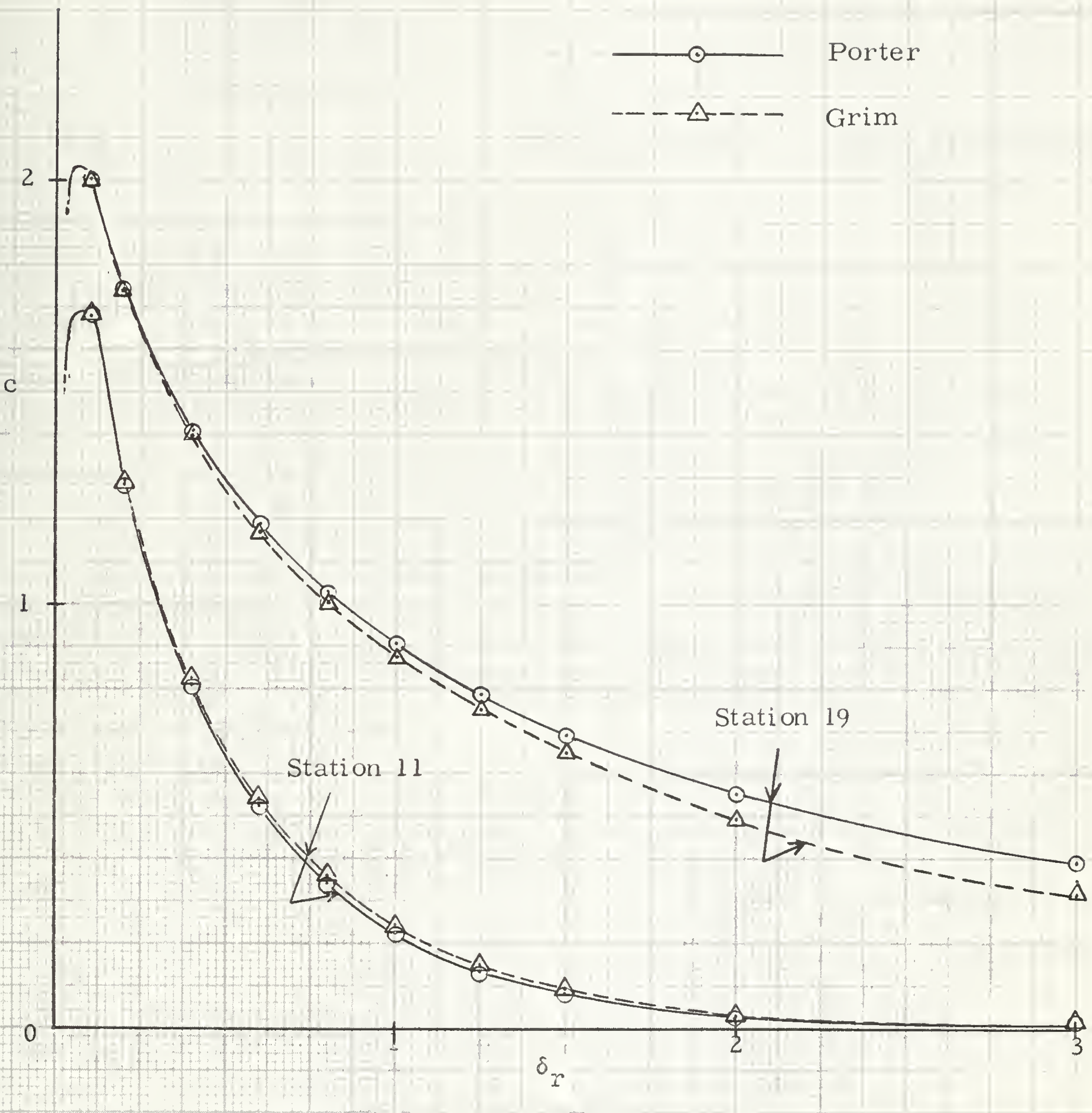


Figure 8. Comparison of Sectional Damping Coefficient Curves for Typical Sections of Series 60 Block 0.70 Ship

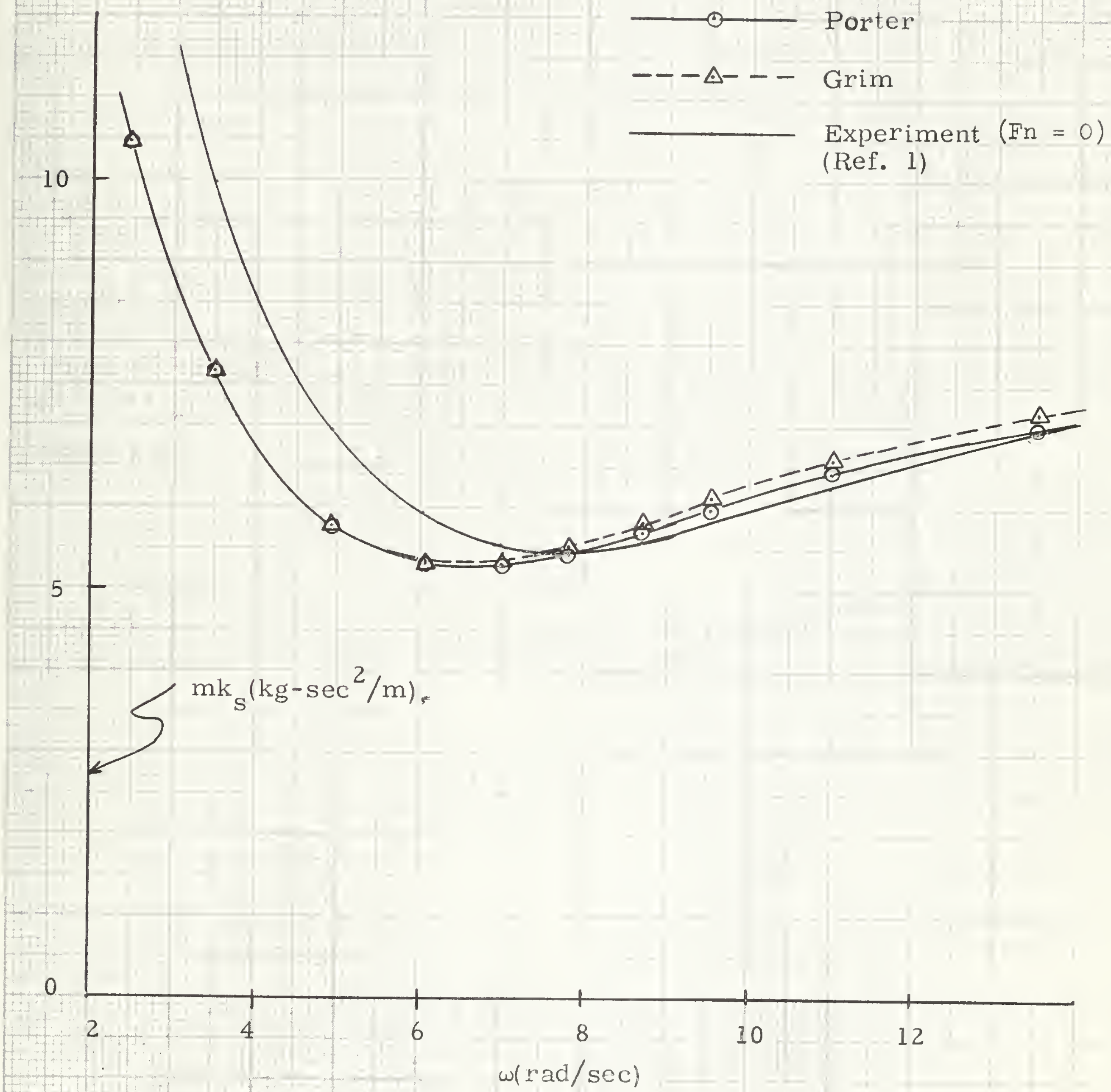


Figure 9. Comparison of Predicted Values of Added Mass for Series 60 Block 0.70 Ship with Experimental Results

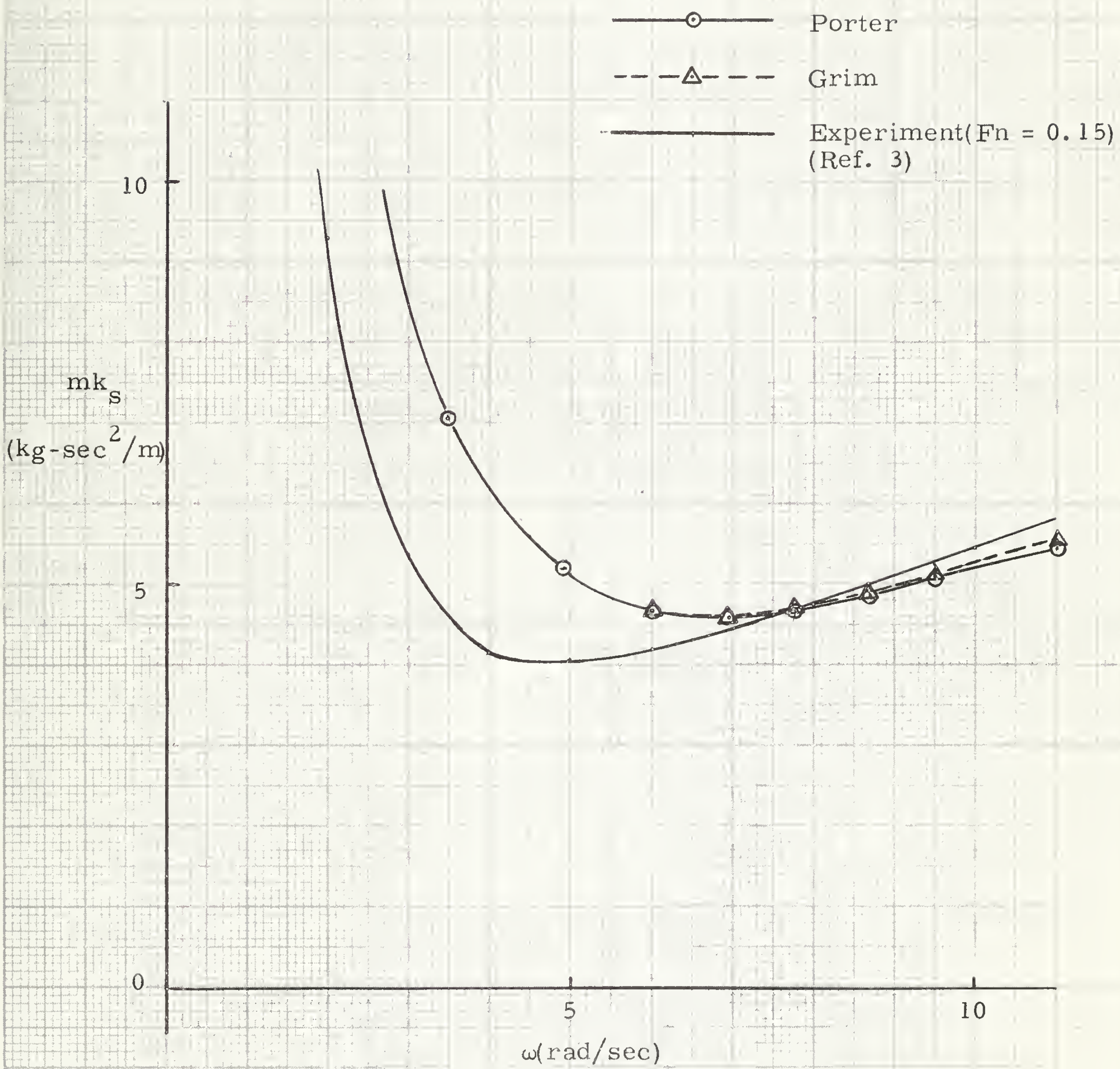


Figure 10. Comparison of Predicted Values of Added Mass for Series 60 Block 0.60 Ship with Experimental Results

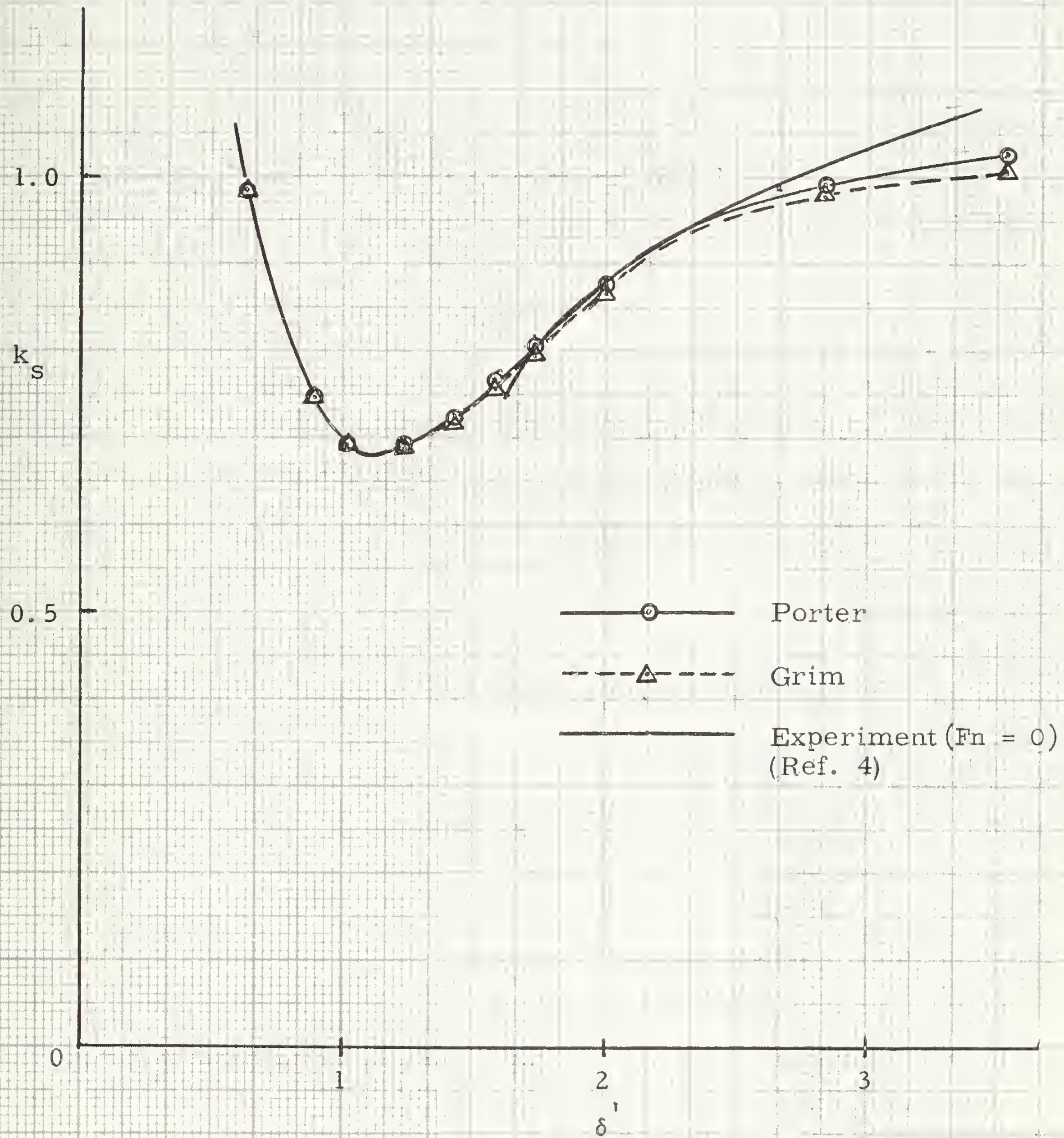


Figure 11. Comparison of Predicted Values of Added Mass Coefficient for Weinblum Model with Experimental Results

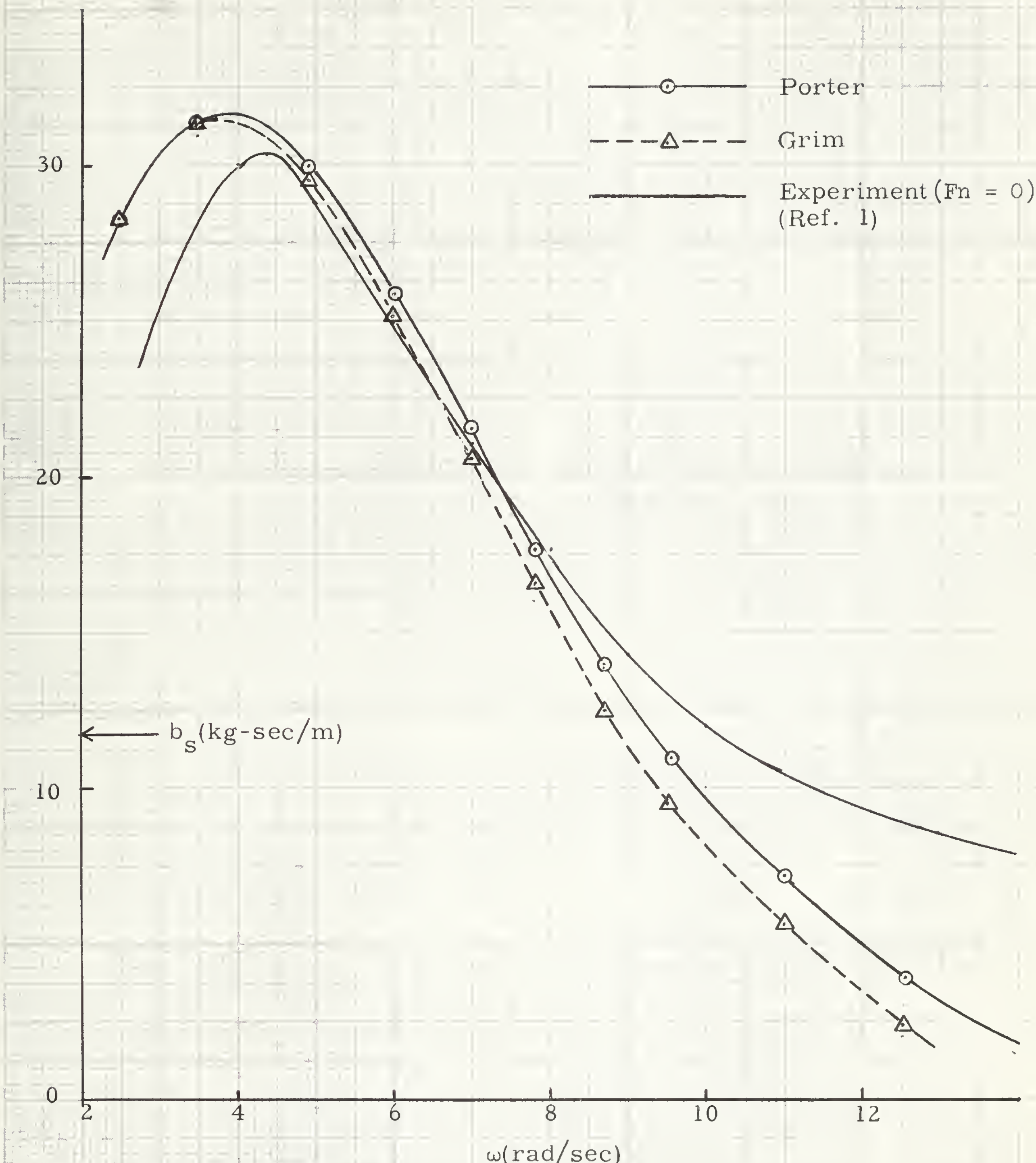


Figure 12. Comparison of Predicted Values of Damping Coefficient for Series 60 Block 0.70 Ship with Experimental Results

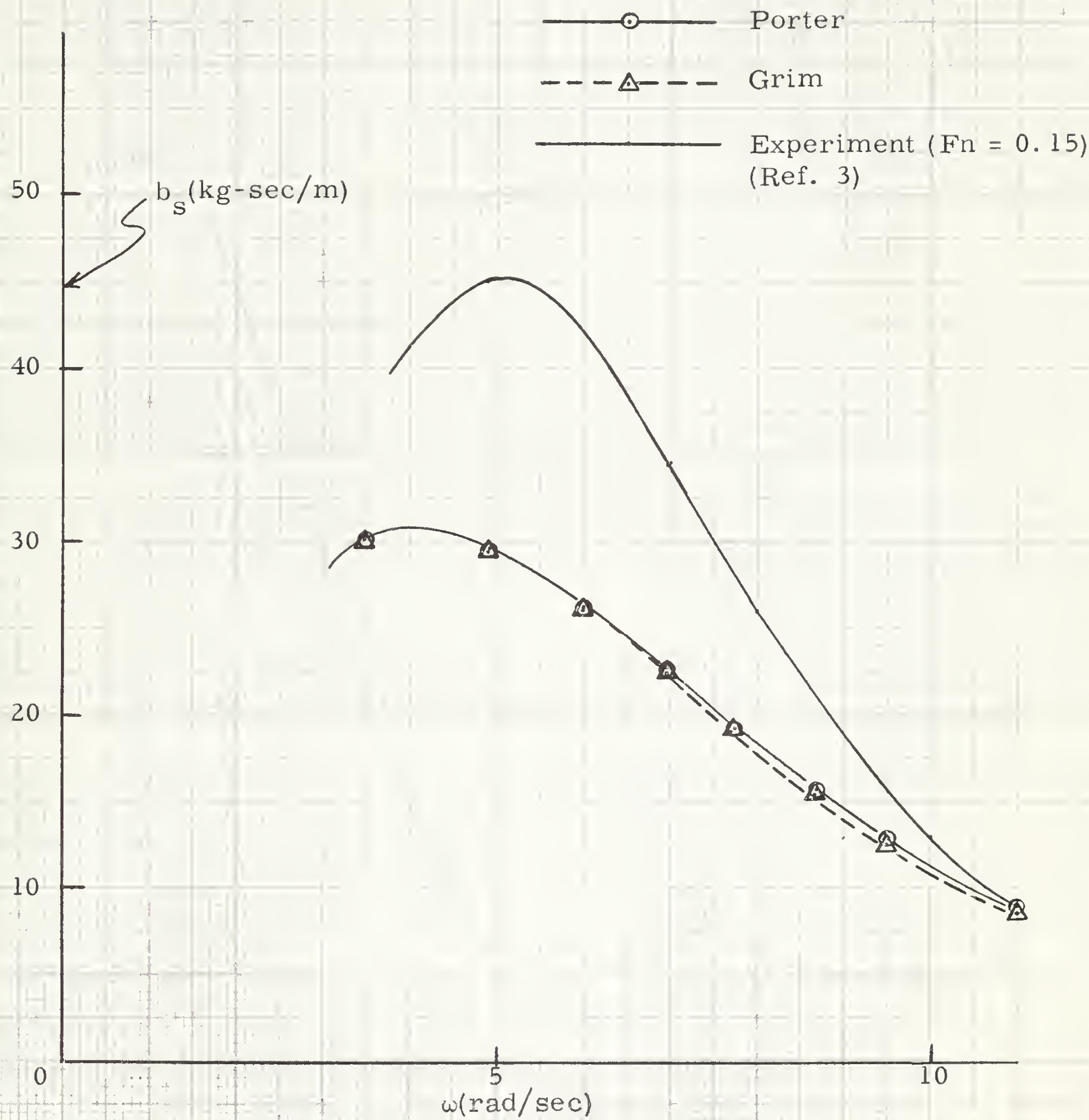


Figure 13. Comparison of Predicted Values of Damping Coefficient for Series 60 Block 0.60 Ship with Experimental Results

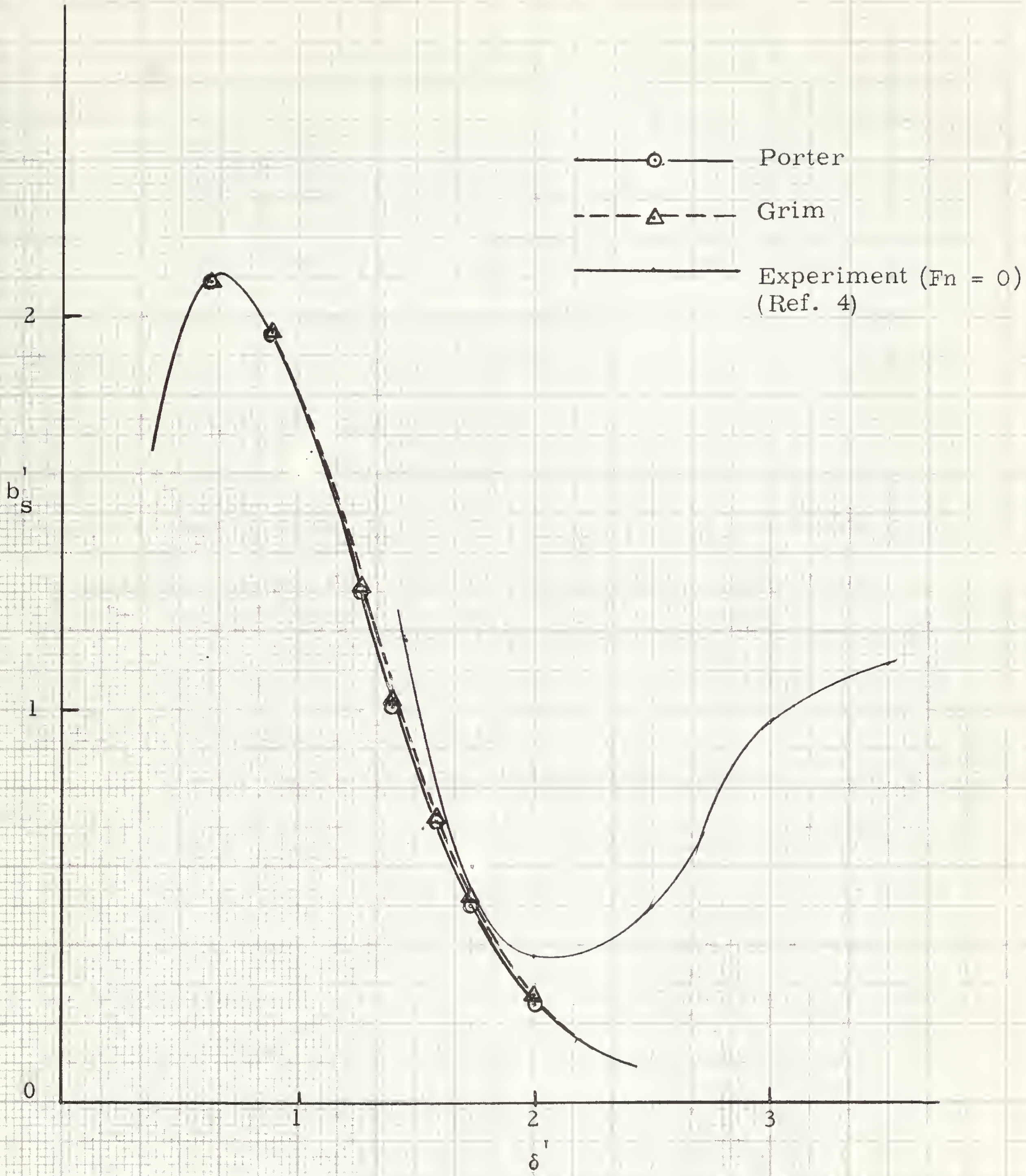


Figure 14. Comparison of Predicted Values of Damping Coefficient for Weinblum Model with Experimental Results

IV. DISCUSSION OF RESULTS

4.1 Fits to Sections

It is evident from Figures 1, 2, 3, and 6 that the section-fitting method introduced in this paper gives very close fits to all the forms considered. We note that the accuracy of the fit is greater for sections that are somewhat elliptical as in the case of the sections of the Weinblum Model and the sections in the middle body of the Series 60 ships. However, the loss of accuracy is small for fits to more general shapes such as those at the bow and stern of the Series 60 ships. The inaccuracies may be largely attributed to the nature of the numerical procedures employed and possibly due to the limited number of transform coefficients used.

Figures 4, 5, and 6 show that the fits due to the method of Grim are not as close as those due to the new procedure used in this paper. We observe in Figure 4 that the slope at the waterline of the actual section is inclined at an angle of aboutforty-five degrees from the waterline while that of the Grim fit is vertical. Since it is known that Grim's method is restricted to Lewis forms, this result is to be expected. On the other hand, the slope of the corresponding selected fit, as shown in Figure 1, is more or less the same as that of the actual section near the waterline and fits as well as at any other point.*

Comparing Figures 4 and 5, we note that a two-parameter fit to the midship section of Series 60 Block 0.70 ship is relatively a better fit than a two-parameter fit to Station 19 of the same ship.

*The slope at the waterline of the selected fit is also infinite but for infinitesimal draft.

This is again to be anticipated since a closer approximation with a Lewis shape can be done to the midship section than to Station 19.

The fits to the Mariner's bulbous bow section in Figure 6 are quite interesting. It is to be noted that while the Grim fit is not as close a fit as that of the corresponding selected fit, it nevertheless is surprisingly a good one. This result is rather unexpected since such a form has heretofore been considered to be beyond the limits of even the three-parameter family of forms of Landweber and Macagno. It seems, therefore, that more work need be done in this field. Since any further investigation is beyond the scope of the present work, we leave this worthy endeavor to future research.

4.2 Sectional Added Mass and Damping

It has been pointed out that Porter's solution to the problem was used to calculate the values for the sectional added mass and damping coefficients from the selected transform coefficients and Grim's parameters. We bring up the question as to how many transform coefficients or parameters are necessary to adequately describe the shape of the cylinder considered. It had been noted that two-parameters may give a reasonable approximation to the form provided the actual section is somewhat of a Lewis shape. However, as may be seen from Tables I, II, and III, at least three transform coefficients were selected for all the sections considered. This is due to the fact that relatively much better fits are obtained by using more than two "a's". In cases where two transform coefficients might

have been considered to give reasonably good fits, it was observed that those "a's" did not differ much from Grim's a_1 and a_3. The Weinblum Model bore this conclusion out rather convincingly.

As we might therefore have anticipated, Figure 7 and 8 show that the values of the added mass and damping coefficients due to the two methods for the midship section (Station 11) of Series 60 Block 0.70 ship are not very different. On the other hand, bearing in mind the fact that Grim's a_1 and a_3 do not very well fit Station 19, one would expect a pronounced difference in the curves for this section. However, such an expectation is not fully realized as shown by the plots of these values. It seems that a more thorough investigation is necessary and we will not make any conclusions that might just be premature.

4.3 Ship's Added Mass and Damping

Comparing the curves of the predicted values of the ship's added mass and damping due to the two methods, we note that the difference is small for every case. We also observe that these theoretical curves are in good agreement with the corresponding experimental results. This is especially so for the Series 60 Block 0.70 ship as may be seen in Figures 9 and 12. Since the experiment was performed quite recently, it is reasonable to assume that the values obtained are more accurate than those for the other two ships due to improved techniques and better instrumentation. These encouraging results indeed reaffirm the correctness and practicability

of Porter's solution.

A comparative study of Grim's method and its results is not quite reassuring. We have noted from the Grim fits that two parameters may give a very different form from the actual section. Hence one has reason to feel unsafe when using the procedure. Nevertheless, the predicted curves are very similar to those due to the selected fits, at least for these three ships considered.

On the other hand, we have seen how closely we were able to approximate the ship form by properly selecting the transform coefficients of the ship's sections. Hence we are more assured that we are making calculations for the correct ship by this more general application of Porter's solution.

V. CONCLUSIONS

In this chapter, we summarize the conclusions stated or implied in the discussion of results (Chapter IV).

We conclude that accurate fits to ship sections can be obtained by a procedure such as that introduced in this paper. We further conclude that in general, two parameters are not sufficient to adequately describe the shape of the section.

The conclusion that one is more assured of correct results in calculating the added mass and damping coefficients of a heaving ship by a more general application of Porter's solution than that associated to Grim's procedure is certainly valid.

VI. RECOMMENDATIONS

The results of this study show that more work needs to be done in certain specific aspects of this subject.

The study on a bulbous section done in this work is just a brief beginning of what may be done in an investigation of the hydrodynamic properties of this important ship form. It is significant, however, in that a good fit to the section was obtained even with just two parameters. This seems to indicate that good fits could be obtained to a wide variety of bulbous forms. It would certainly be interesting to see how the added mass and damping coefficients of such forms would behave with respect to change of shape as well as to the frequency of oscillation. It is therefore recommended that further research along these lines be conducted.

Likewise, a more detailed study on shapes with inclined slopes at the waterline is called for. To establish the effect of such slopes on the sectional added mass and damping coefficients would certainly be an important contribution.

The effect of the goodness of fit on the estimation of the hydrodynamic forces in sway motion and roll motion remains to be studied. Similar procedures as those used in this paper could be used in applying Porter's solution.

NOMENCLATURE

a_{2n+1}	transform coefficient; $n = 0, 1 \ldots N$
b	half-beam of section
b_s	damping coefficient of the ship
b_s'	dimensionless damping coefficient of the ship
b_m	maximum half-beam of the ship
c	sectional damping coefficient
c_s	buoyant force coefficient of the ship
g	acceleration of gravity
$k_2 k_4$	sectional damping coefficient
k_s	added mass coefficient of the ship
m	mass of the ship
t	time
y	heave displacement
$\dot{y}$	heave velocity
$\ddot{y}$	heave acceleration
z	a complex variable
F_o	amplitude of externally-applied vertical harmonic force
Fn	Froude number
L	length of the ship
S	submerged area of a circular cylinder with half-beam, b

α	phase angle
δ	a non-dimensional frequency
δ_r	a non-dimensional frequency
δ'	a non-dimensional frequency
ω	circular frequency of oscillation
ρ	mass density of water
λ	draft to half-beam ratio
σ	section area coefficient
Δ	displacement of the ship
ζ	a complex variable

REFERENCES

1. Gerritsma, J. and Beukelman, W., "The Distribution of the Hydrodynamic Forces on a Heaving and Pitching Shipmodel, with Zero Speed in Still Water," Shipbuilding Laboratory, Technological University, Delft, Publication No. 124, February 1965.

2. Gerritsma, J. and Beukelman, W., "Distribution of Damping and Added Mass along the Length of a Shipmodel," International Shipbuilding Progress, Vol. 10, No. 103, March 1963, pp. 73 - 84.

3. Gerritsma, J., "Experimental Determination of Damping Added Mass and Added Mass Moment of Inertia of a Shipmodel," International Shipbuilding Progress, Vol. 4, No. 38, October 1957, pp. 505 - 519.

4. Golovato, P., "A Study of the Forces and Moments on a Heaving Surface Ship," DTMB Report 1074, September 1957.

5. Lewis, F. M., "The Inertia of the Water Surrounding a Vibrating Ship," Transactions of SNAME, Vol. 37, 1929, pp. 1 - 20.

6. Landweber, L. and Macagno, M., "Added Mass of a Three-parameter Family of Two-dimensional Forms Oscillating in a Free Surface," Journal of Ship Research, Vol. 2, No. 4, 1959, pp. 36 - 48.

7. Grim, O., "A Method for a More Precise Computation of Heaving and Pitching Motions in Both Smooth Water and in Waves," Proc. of Third Symposium on Naval Hydrodynamics, Office of Naval Research, Department of the Navy, ACR-55, 1960, pp. 483 - 524.

8. Porter, W. R., "Pressure Distributions, Added-Mass and Damping Coefficients for Cylinders Oscillating in a Free Surface," Institute of Engineering Research, University of California, Report, July 1960.

9. Plant, J. B., "An Application of Linear Programming to the Problem of Inverting a Conformal Transformation," M.I.T., Department of Naval Architecture and Marine Engineering, January 1964, (unpublished document).

10. Theodorsen, T., "Theory of Wing Sections of Arbitrary Shape," National Advisory Committee for Aeronautics, Report No. 411, 1932.

11. Naiman, I., "Numerical Evaluation by Harmonic Analysis of the ϵ-Function of the Theodorsen Arbitrary-Airfoil Potential Theory," National Advisory Committee for Aeronautics, Wartime Report No. 153, September 1945.

VII. APPENDICES

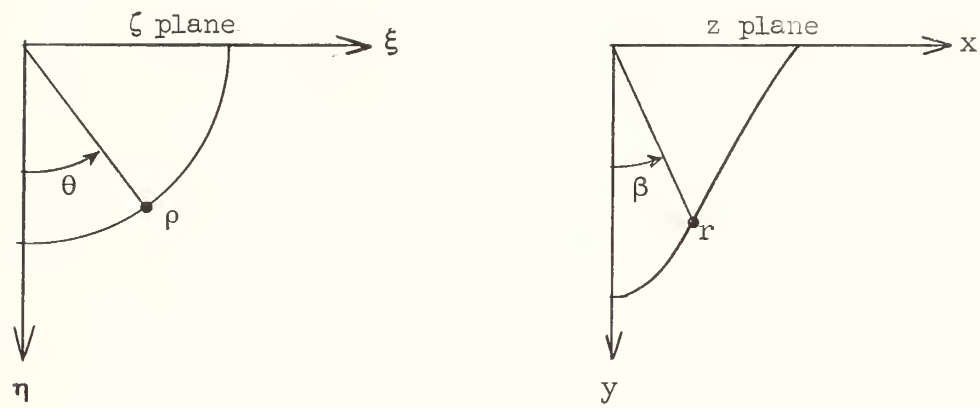
Appendix A	Details of Procedure	
	A-1	A Method of Inverting a Conformal Transformation
	A-2	Procedure to Derive Grim's a_1 and a_3
Appendix B	Original Data	

APPENDIX A - DETAILS OF PROCEDURE

A-1 A Method of Inverting a Conformal Transformation

A-1.1 The Problem

Consider the two complex planes shown below



where $\zeta = i\rho e^{-i\theta}$ and $z = x + iy = ire^{-i\beta}$. Given M points on the ship section as shown in the z plane, determine the coefficients of the transform

$$z = \zeta + \sum_{n=0}^{N} a_{2n+1} \zeta^{-(2n+1)}$$

which will conformally map the unit circle* in the ζ plane into the ship section in the z plane.

*Only the quadrant $0 \leq \theta \leq \pi/2$ is of interest.

A-1.2 Solution

We separate the transform into its real and imaginary parts. For the j^{th} point of the M known offsets, we have

$$x_j = \rho \sin \theta_j + \sum_{n=0}^{N} (-1)^n \frac{a_{2n+1}}{\rho^{2n+1}} \sin (2n+1)\theta_j \quad (5)$$

$$y_j = \rho \cos \theta_j + \sum_{n=0}^{N} (-1)^{n+1} \frac{a_{2n+1}}{\rho^{2n+1}} \cos(2n+1)\theta_j \quad (6)$$

The $\frac{\text{half-beam}}{\text{draft}}$ ratio, H, is a known constant and the following relationship

$$H = \frac{\rho + \sum_{n=0}^{N} (a_{2n+1}/\rho^{2n+1})}{\rho + \sum_{n=0}^{N} (-1)^{n+1} (a_{2n+1}/\rho^{2n+1})} \quad (7)$$

can be derived. Hence we have from (5), (6), and (7) a set of 2 M + 1 simultaneous equations. The unknowns are ρ, a_i, and θ_j where $i = 1, 3, \ldots 2N + 1$ and $j = 1, 2, \ldots M$.

We first determine the angles, θ_j. By an application of Theodorsen's method*, the angle in the ζ plane corresponding to any given point in the z plane can be calculated. This is done by using an intermediate z' plane (not shown) where $z' = ae^{\psi+i\gamma}$. The transformation

* The interested reader is referred to Reference 10 for the details of the method.

$$z = z' + \frac{a^2}{z'}$$

maps the ship section in the z plane into a curve in the z' plane which may be expected not to differ greatly from a circle. ψ_j and γ_j can be solved from the following developed relationships:

$$2 \sin^2 \gamma = p + \sqrt{p^2 + (\frac{y}{a})^2}$$

$$2 \sinh^2 \psi = -p + \sqrt{p^2 + (\frac{y}{a})^2}$$

where

$$p = 1 - (\frac{x}{2a})^2 - (\frac{y}{2a})^2 .$$

Next, we wish to map the curve in the z' plane into a circle in the ζ plane using the general transform

$$z' = \zeta e^{\sum_{n=1}^{\infty} (A_n + iB_n) \frac{1}{\zeta^n}} .$$

Theodorsen showed that

$$\epsilon_j = \theta_j - \gamma_j = -\frac{1}{2\pi} \int_0^{2\pi} \psi(\theta) \cot \frac{(\theta - \theta_j)}{2} d\theta.$$

If we know ψ as a function of θ, we can evaluate the integral by Naiman's [11] numerical procedure. Specifying the values of ψ at $2\bar{n}$ equally spaced intervals in the range $0 \leq \theta \leq 2\pi$, the method gives

$$\epsilon = \sum_{k=1}^{\bar{n}} \frac{1}{\bar{n}} \cot \frac{k\pi}{2\bar{n}} (\psi_{-k} - \psi_k)$$

where the summation is for odd values of k only and

$$\psi_k \equiv \psi(\theta + \frac{k\pi}{\bar{n}}).$$

However, we only know ψ as a function of γ. Since γ may be expected to differ but little from θ, we take $\psi(\gamma)$ as a first approximation of $\psi(\theta)$. A second approximation to the dependence between ψ and θ is given by $\psi(\theta+\epsilon)$. This enables us to calculate a better estimate of ϵ. Thus we see that an iteration process is necessary to determine both ψ and ϵ as a function of θ correctly. Knowing $\epsilon(\theta)$, we can get $\epsilon(\gamma)$ and therefore ϵ_j. This enables us to determine the angle,

$$\theta_j = \gamma_j + \epsilon_j$$

for each of the given offsets.

Substitution of these now known angles, θ_j, into (5), (6), and (7) leaves us N + 2 unknowns (ρ and d_i) in the 2M + 1 simultaneous and now linear equations where

$$d_i \equiv \frac{a_{2n+1}}{\rho^{2n+1}}, \quad n = 0, 1 \ldots N.$$

We now solve this set of over-determined equations for these unknowns minimizing the sum of the squares of the residuals (or individual errors) to get $\rho = \rho_o$ and $d_i = d_i'$, $i = 1, 3, \ldots 2N + 1$. Since we have a choice over the value of N, we may choose N to be that value that gives the least total squared error.

We are interested in the values of a_i for $\rho = 1$. Hence it is seen that ρ_o is in fact the scale factor and the desired normalized transform coefficients are

$$a_i = \frac{d_i'}{\rho_o} \qquad i = 1, 3, \ldots 2N + 1$$

This numerical procedure has been programmed by the author in order that the calculations may be done by IBM 7094 digital computer at the Computation Center, M.I.T. This program has been submitted to the Department of Naval Architecture and Marine Engineering Library of this school so that it may be available for general use.

A-2 Procedure to Derive Grim's a_1 and a_3

It is known that Grim's method is restricted to Lewis cylinders only. For a Lewis shape, the following relationships hold:

$$\lambda = \frac{1 - a_1 + a_3}{1 + a_1 + a_3}$$

$$\sigma = \frac{\pi}{4} \frac{1 - a_1^2 - 3a_3^2}{(1+a_3)^2 - a_1^2}$$

Knowing λ and σ, we can solve for unique values of a_1 and a_3.

APPENDIX B - ORIGINAL DATA

The only purpose of this appendix is to include the offsets used in this work for the Series 60 ships. As pointed out in the text, some differences exist between these offsets and those published in DTMB Report No. 1712.

The offsets for the Series 60 Block 0.70 ship are tabulated in Table XXI. Likewise, the offsets for the Series 60 Block 0.60 ship are listed in Table XXII. These values are normalized with respect to the maximum half-beam. The waterlines are numbered from 1 at the designers' waterline down to 21 at the keel. The values given for the waterlines are distances from the DWL and are in error. They should all be multiplied by 0.8 to obtain the correct values.

These offsets have been punched in computer cards and are available for general use at the Department of Naval Architecture and Marine Engineering Library, Massachusetts Institute of Technology.

TABLE XXI

Table of Offsets for Series 60 Block 0.70 Ship

	WL*	STA(1)	STA(2)	STA(3)	STA(4)	STA(5)	STA(6)	STA(7)	STA(8)	STA(9)	STA(10)	STA(11)
1	0.	0.	.203000	.441000	.668000	.838000	.943000	.988000	1.000000	1.000000	1.000000	1.000000
2	.050000	0.	.198000	.435000	.663000	.834000	.940000	.987000	1.000000	1.000000	1.000000	1.000000
3	.100000	0.	.196000	.430000	.657000	.829000	.938000	.985000	1.000000	1.000000	1.000000	1.000000
4	.150000	0.	.195000	.425000	.651000	.824000	.934000	.984000	1.000000	1.000000	1.000000	1.000000
5	.200000	0.	.193000	.421000	.646000	.820000	.930000	.983000	.999000	1.000000	1.000000	1.000000
6	.250000	0.	.190000	.420000	.641000	.815000	.928000	.982000	.998000	1.000000	1.000000	1.000000
7	.300000	0.	.188000	.419000	.635000	.809000	.925000	.981000	.998000	1.000000	1.000000	1.000000
8	.350000	0.	.187000	.416000	.629000	.804000	.922000	.980000	.996000	1.000000	1.000000	1.000000
9	.400000	0.	.184000	.414000	.623000	.800000	.919000	.977000	.996000	1.000000	1.000000	1.000000
10	.450000	0.	.183000	.412000	.618000	.794000	.916000	.975000	.995000	1.000000	1.000000	1.000000
11	.500000	0.	.181000	.407000	.611000	.789000	.913000	.973000	.994000	1.000000	1.000000	1.000000
12	.550000	0.	.179000	.402000	.605000	.782000	.907000	.970000	.992000	1.000000	1.000000	1.000000
13	.600000	0.	.175000	.395000	.597000	.774000	.901000	.964000	.992000	1.000000	1.000000	1.000000
14	.650000	0.	.172000	.386000	.587000	.764000	.893000	.957000	.990000	1.000000	1.000000	1.000000
15	.700000	0.	.168000	.374000	.576000	.750000	.881000	.949000	.988000	1.000000	1.000000	1.000000
16	.750000	0.	.162000	.358000	.559000	.731000	.864000	.936000	.982000	.998000	.998000	.998000
17	.800000	0.	.155000	.339000	.536000	.706000	.839000	.918000	.971000	.991000	.991000	.991000
18	.850000	0.	.142000	.316000	.504000	.674000	.810000	.893000	.952000	.976000	.976000	.976000
19	.900000	0.	.123000	.277000	.458000	.625000	.765000	.859000	.921000	.949000	.949000	.949000
20	.950000	0.	.091000	.223000	.389000	.551000	.696000	.806000	.873000	.908000	.908000	.908000
21	1.000000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

* Values for waterlines (not offsets) should be multiplied by 0.8

TABLE XXI (cont'd)

Table of Offsets for Series 60 Block 0.70 Ship

	WL*	STA(12)	STA(13)	STA(14)	STA(15)	STA(16)	STA(17)	STA(18)	STA(19)	STA(20)	STA(21)	STA(
1	0.	1.000000	1.000000	1.000000	.987000	.948000	.870000	.741000	.546000	.280000	0.	
2	.050000	1.000000	1.000000	1.000000	.984000	.941000	.856000	.715000	.503000	.217000	0.	
3	.100000	1.000000	1.000000	1.000000	.982000	.934000	.839000	.685000	.459000	.179000	0.	
4	.150000	1.000000	1.000000	1.000000	.980000	.926000	.822000	.657000	.418000	.139000	0.	
5	.200000	1.000000	1.000000	1.000000	.976000	.916000	.803000	.625000	.379000	.107000	0.	
6	.250000	1.000000	1.000000	1.000000	.971000	.907000	.783000	.594000	.347000	.084000	0.	
7	.300000	1.000000	1.000000	1.000000	.968000	.897000	.764000	.566000	.321000	.070000	0.	
8	.350000	1.000000	1.000000	.999000	.963000	.884000	.743000	.539000	.298000	.062000	0.	
9	.400000	1.000000	1.000000	.997000	.957000	.869000	.722000	.515000	.280000	.058000	0.	
10	.450000	1.000000	1.000000	.995000	.949000	.852000	.698000	.492000	.263000	.057000	0.	
11	.500000	1.000000	1.000000	.991000	.940000	.835000	.672000	.468000	.248000	.054000	0.	
12	.550000	1.000000	1.000000	.987000	.932000	.817000	.651000	.449000	.237000	.054000	0.	
13	.600000	1.000000	1.000000	.980000	.916000	.796000	.627000	.428000	.225000	.050000	0.	
14	.650000	1.000000	.998000	.971000	.902000	.775000	.605000	.409000	.215000	.049000	0.	
15	.700000	1.000000	.994000	.960000	.885000	.753000	.580000	.391000	.204000	.046000	0.	
16	.750000	.998000	.984000	.943000	.863000	.727000	.555000	.369000	.196000	.045000	0.	
17	.800000	.989000	.970000	.922000	.835000	.696000	.528000	.350000	.186000	.040000	0.	
18	.850000	.973000	.950000	.894000	.802000	.661000	.496000	.326000	.173000	.035000	0.	
19	.900000	.946000	.920000	.856000	.754000	.615000	.453000	.294000	.155000	.029000	0.	
20	.950000	.907000	.870000	.800000	.690000	.551000	.395000	.245000	.127000	.023000	0.	
21	1.000000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	

* Values for waterlines (not offsets) should be multiplied by 0.8

TABLE XXII

Table of Offsets for Series 60 Block 0.60 Ship

	WL*	STA(1)	STA(2)	STA(3)	STA(4)	STA(5)	STA(6)	STA(7)	STA(8)	STA(9)	STA(10)	STA(11)
1	0.	0.	.101000	.228000	.391000	.561000	.718000	.841000	.928000	.979000	1.000000	1.000000
2	.050000	0.	.098000	.223000	.385000	.555000	.711000	.838000	.925000	.979000	1.000000	1.000000
3	.100000	0.	.095000	.220000	.380000	.550000	.708000	.834000	.923000	.979000	1.000000	1.000000
4	.150000	0.	.093000	.218000	.375000	.544000	.701000	.832000	.920000	.978000	1.000000	1.000000
5	.200000	0.	.091000	.215000	.371000	.539000	.695000	.828000	.919000	.978000	1.000000	1.000000
6	.250000	0.	.090000	.214000	.368000	.534000	.691000	.824000	.916000	.977000	1.000000	1.000000
7	.300000	0.	.089000	.213000	.365000	.529000	.686000	.820000	.915000	.977000	1.000000	1.000000
8	.350000	0.	.089000	.211000	.360000	.524000	.680000	.818000	.914000	.976000	1.000000	1.000000
9	.400000	0.	.088000	.209000	.356000	.518000	.675000	.814000	.911000	.975000	1.000000	1.000000
10	.450000	0.	.088000	.208000	.351000	.510000	.668000	.809000	.910000	.974000	.999000	1.000000
11	.500000	0.	.088000	.205000	.346000	.503000	.660000	.803000	.906000	.971000	.997000	1.000000
12	.550000	0.	.086000	.201000	.339000	.494000	.651000	.793000	.900000	.968000	.995000	1.000000
13	.600000	0.	.086000	.198000	.331000	.483000	.640000	.781000	.891000	.963000	.993000	1.000000
14	.650000	0.	.085000	.191000	.320000	.470000	.625000	.766000	.880000	.954000	.988000	1.000000
15	.700000	0.	.084000	.184000	.306000	.453000	.605000	.746000	.863000	.940000	.978000	.996000
16	.750000	0.	.081000	.175000	.289000	.429000	.581000	.723000	.841000	.921000	.965000	.986000
17	.800000	0.	.078000	.164000	.266000	.400000	.549000	.690000	.813000	.897000	.945000	.970000
18	.850000	0.	.074000	.149000	.239000	.361000	.506000	.650000	.775000	.868000	.919000	.946000
19	.900000	0.	.064000	.126000	.196000	.308000	.446000	.591000	.721000	.823000	.879000	.910000
20	.950000	0.	.048000	.095000	.145000	.244000	.375000	.519000	.654000	.759000	.823000	.856000
21	1.000000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

*Values for waterlines (not offsets) should be multiplied by 0.8

TABLE XXII (cont'd)

Table of Offsets for Series 60 Block 0.60 Ship

	WL*	STA(12)	STA(13)	STA(14)	STA(15)	STA(16)	STA(17)	STA(18)	STA(19)	STA(20)	STA(21)	STA(
1	0.	1.000000	1.000000	.994000	.976000	.938000	.858000	.725000	.535000	.308000	.079000	
2	.050000	1.000000	1.000000	.992000	.971000	.925000	.836000	.692000	.489000	.251000	.040000	
3	.100000	1.000000	1.000000	.991000	.965000	.909000	.811000	.655000	.439000	.202000	.005000	
4	.150000	1.000000	1.000000	.991000	.958000	.892000	.785000	.618000	.394000	.166000	0.	
5	.200000	1.000000	1.000000	.989000	.951000	.875000	.756000	.582000	.355000	.139000	0.	
6	.250000	1.000000	1.000000	.986000	.942000	.856000	.728000	.546000	.321000	.116000	0.	
7	.300000	1.000000	1.000000	.984000	.935000	.838000	.699000	.512000	.295000	.101000	0.	
8	.350000	1.000000	1.000000	.980000	.924000	.816000	.669000	.481000	.274000	.092000	0.	
9	.400000	1.000000	.998000	.976000	.912000	.796000	.648000	.456000	.258000	.089000	0.	
10	.450000	1.000000	.996000	.970000	.899000	.775000	.615000	.431000	.245000	.086000	0.	
11	.500000	1.000000	.992000	.962000	.885000	.754000	.591000	.412000	.236000	.085000	0.	
12	.550000	1.000000	.989000	.951000	.868000	.731000	.568000	.394000	.226000	.082000	0.	
13	.600000	1.000000	.982000	.938000	.848000	.708000	.545000	.379000	.218000	.081000	0.	
14	.650000	.996000	.974000	.922000	.825000	.684000	.522000	.362000	.209000	.080000	0.	
15	.700000	.988000	.961000	.901000	.799000	.659000	.500000	.345000	.200000	.078000	0.	
16	.750000	.975000	.944000	.879000	.770000	.630000	.476000	.326000	.189000	.074000	0.	
17	.800000	.956000	.919000	.848000	.736000	.593000	.448000	.306000	.180000	.073000	0.	
18	.850000	.931000	.890000	.311000	.699000	.562000	.419000	.281000	.164000	.066000	0.	
19	.900000	.895000	.848000	.750000	.650000	.520000	.384000	.253000	.146000	.059000	0.	
20	.950000	.840000	.782000	.698000	.581000	.469000	.341000	.216000	.119000	.045000	0.	
21	1.000000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	

* Values for waterlines (not offsets) should be multiplied by 0.8