AMERICAN COMPREHENSIVE ARITHMETIC

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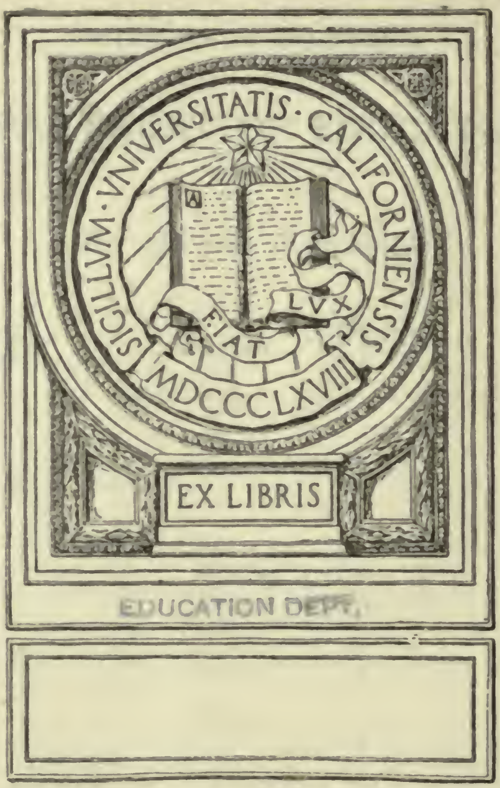
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AMERICAN

COMPREHENSIVE ARITHMETIC

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E-P 6

PREFACE

In this work, the divisions of arithmetic are presented in their natural order. The fundamental operations upon integers, common fractions, decimals, denominate numbers, and numbers expressed by letters, are followed by their applications to business and to various employments. The introduction of the chapter on literal quantities is somewhat of an innovation, but is in accord with the views of leading teachers, and is based upon sound principles. The elementary parts of algebra should be studied before the advanced parts of arithmetic, because they are more easily comprehended, and because they afford valuable assistance in difficult problems. For the same reasons, the solution of many classes of problems should be taught by algebraic processes, before they are taught by the intricate methods required by analysis. The fact, too, that most pupils must leave school quite early, is a strong reason for introducing into arithmetic the elements of algebra and the conclusions of geometry.

From the beginning, the pupil is taught to develop observation and thought power. Instead of being required to memorize a large number of rules and directions, he is encouraged to fix well in mind the result to be obtained, to consider carefully the means at his command, and to employ those means according to his best judgment.

This book is intended to complete the course in arithmetic required by district and city schools.

M. A. BAILEY.

CONTENTS

NOTATION AND NUMERATION

TERMS

A *number* answers the question, How many?

Numeration is the process of reading numbers.

Notation is the process of writing numbers.

ILLUSTRATION

How many apples?

6.

6, *a number, or integer.*

Read, *Six.*

Written, 6.

NUMERATION

In reading an integer, there are three steps:

The number is pointed off from right to left into periods of three figures each.

The periods are named from right to left to learn the name of the left-hand period.

The periods are read and named, but the period, 000, is not read and units' period is not named.

Pointing off into periods of three figures each

Beginning at the right, point off into periods:

1. 3600028371.
2. 23600028371.
3. 423600028371.
4. 378002000028706954.
5. 407008000900056207.
6. 530087009600057008301.

7. How many figures may there be in the left-hand period?

8. How many figures must there be in each of the other periods?

Ex. **1**. 3,600,028,371. Ex. **2**. 23,600,028,371. Ex. **3**. 423,600,028,371.

Naming the periods

a. 1,023,438,019,000,320,678,735.
b. 21,023,438,019,000,320,678,735.
c. 321,023,438,019,000,320,678,735.

From right to left, the periods are: *units, thousands, millions, billions, trillions, quadrillions, quintillions, sextillions* . . . .; from left to right: . . . . *sextillions, quintillions, quadrillions, trillions, billions, millions, thousands, units.*

Higher periods after sextillions are: *septillions*, *octillions*, *nonillions*, *decillions*, *undecillions*, *duodecillions*, ...

9. Memorize the names of the periods from units to sextillions; from sextillions to units.

10. In *a*, beginning at the right, point to and name each period; then beginning at the left, point to and name each period.

11. In the same way, point to and name the periods in *b* and *c*.

12. In the same way, point to and name the periods in examples 1 to 6.

Ex. **10**. Units, thousands, millions, billions, trillions, quadrillions, quintillions, sextillions; sextillions, quintillions, quadrillions, trillions, billions, millions, thousands, units.

Reading the periods

Each period is made up of three orders, *units*, *tens*, and *hundreds.*

Ill. 368 equals 3 *hundreds*, 6 *tens*, 8 *units.*

In units' order, except when 1 is in tens' order, 1, 2, 3, 4, 5, 6, 7, 8, 9 are read *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine.*

In tens' order, 2, 3, 4, 5, 6, 7, 8, 9 are read *twenty*, *thirty*, *forty*, *fifty*, *sixty*, *seventy*, *eighty*, *ninety.* 1 in tens' order is read with the units' figure. Thus: 10, 11, 12, 13, 14, 15, 16, 17, 18, 19; *ten*, *eleven*, *twelve*, *thirteen*, *fourteen*, *fifteen*, *sixteen*, *seventeen*, *eighteen*, *nineteen.*

In hundreds' order, 1, 2, 3, 4, 5, 6, 7, 8, 9 are read *one hundred*, *two hundred*, *three hundred*, *four hundred*, *five hundred*, *six hundred*, *seven hundred*, *eight hundred*, *nine hundred.*

0 in any order is never read.

Reading the periods

d. 1,703,100,306,016,429.
e. 25,603,007,075,018,208.
f. 436,000,056,000,001,200.

13. In *d*, read units' period; thousands' period; millions' period.

14. In *d*, read billions' period; trillions' period; quadrillions'.

15. In *e*, read quadrillions' period; trillions' period; billions' period; etc.

16. In *f*, read each period in succession, beginning at the left.

Ex. **13**. Units' period, 429, *four hundred twenty-nine*. 4 in hundreds' order is read *four hundred;* 2, in tens' order, *twenty;* 9, in units' order, *nine*.

Thousands' period, 016, *sixteen*. 0 is not read; 1 in tens' order is read with 6, the figure in units' order, *sixteen*.

Millions' period, 306, *three hundred six*. 3 in hundreds' order is read *three hundred;* 0 is not read; 6, in units' order, *six*.

The process as a whole

17. Read 37000016328.

Ans. 37 *billion*, 16 *thousand*, 328. Pointing off into periods of three figures each, 37,000,016,328; numerating to learn the name of the highest period, units, thousands, millions, billions; reading and naming the periods, 37 *billion*, 16 *thousand*, 328.

NOTE.—The plural form of the period's name should be used in numerating, but the singular form, in reading. The word "and" should not be used.

Read:

18. 3678; 33678; 235678.
19. 2000; 20000; 200000.
20. 8008; 80008; 800008.
21. 7017; 70017; 700017.
22. 6010; 70107; 701070.
23. 3124567; 1234567890.
24. 3000045; 3000000452.
25. 20305070; 3005007001.
26. 10000001; 2000000002.
27. 88000050; 9120000878.
28. 6180700623074078.
29. 58097060078039007.
30. 136840050000200003.
31. 2780010204009086981.

NOTATION — ARABIC

In writing an integer there are two steps:

The left-hand period is written with one, two, or three figures.

The other periods are then written in succession with three figures each.

Writing the left-hand period

The Arabic notation employs two devices:

The number in each order is represented by one of the symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

The name of the order is represented by relative position.

Three *hundred* eight,

3 *hundred* 0 *ten* 8,

308

Write as a left-hand period:

32. Seven; seventy; seven hundred; nine hundred sixty-eight.

33. Twenty; sixteen; thirty-six; forty-five; twelve; five.

34. Ninety; four hundred; five hundred sixty; one hundred one; nineteen; seventy-five.

35. Nine hundred ninety-nine; two; sixteen; eighty; forty-five; thirteen.

Ex. 32. 7; 70; 700; . . . .

Writing the other periods

The same devices are employed as before, but each period must contain three figures. *See examples 7 and 8.*

Write as a full period:

36. Zero; eight; eighteen; ninety; fifty-six; forty.

37. Thirty; one; six; seventy-five; one hundred; three.

38. Five hundred six; nine; seventeen; sixty-eight; four; seven; seventy; twenty.

39. Sixteen; thirty-six; forty-five; five; fourteen; ninety-nine.

Ex. 36. 000; 008; 018; . . . .

The process as a whole

40. Write thirty-six trillion, one hundred seventy-six million, six.

Ans. 36,000,176,000,006. We think 36 *trillion*, and write 36,; we think no *billion*, and write 000, (the work now appears 36,000,); we think one hundred seventy-six *million*, and write 176, (36,000,176,); we think no *thousand*, and write 000, (36,000,176,000,); we think *six*, and write 006. (36,000,176,000,006.)

Write:

41. Two billion, seventeen thousand, one hundred twenty-six.

42. 307 quadrillion, 20 billion, four hundred seventy-seven.

43. 300 sextillion, 4 trillion, 30 million, 98 thousand, sixty.

44. 65 quadrillion, 700 billion, 99 million, 999 thousand, 999.

45. Nineteen million, 75 thousand, seven hundred twenty-four.

46. Five million, 7 hundred 20 thousand, 6 hundred thirty.

47. 30 quintillion, 300 trillion, 475 million, 4 thousand, 16.

48. 555 sextillion, 505 million, five hundred thousand, 500.

49. 826 quadrillion, 469 billion, 8 million, 95 thousand, 18.

Numeration table

50. Memorize the following table both by orders and by periods.

By Orders		By Periods	
10 units	= 1 ten.	1000 units	= 1 thousand.
10 tens	= 1 hundred.	1000 thou.	= 1 million.
10 hund.	= 1 thousand.	1000 mil.	= 1 billion.
10 thou.	= 1 ten thousand.	1000 bil.	= 1 trillion.
10 t. thou.	= 1 hundred thousand.	1000 tril.	= 1 quadrillion.
10 h. thou.	= 1 million.	1000 quad.	= 1 quintillion.
. . .	. . . .	. . .	. . . .

hun. tens. units.	hun. tens. units.	hun. tens. units.	hun. tens. units.	hun. tens. units.	hun. tens. units.	hun. tens. units.	hun. tens. units.
368,	496,	743,	978,	432,	389,	986,	725.
sex-tillions,	quin-tillions,	quad-rillions,	tril-lions,	bil-lions,	mil-lions,	thou-sands,	u-nits.

NUMBERS NAMED — TO A THOUSAND

In naming numbers to a thousand, it is thought best to consider individuals as collected into *groups of ten*, ten of these groups as collected into a higher group, and ten of these higher groups as collected into a still higher group. The individuals are called *units;* the groups, *orders.* Thus:

Ten *units* are considered a group of *ten;* ten *tens*, a group of a *hundred;* ten *hundreds*, a group of a *thousand.*

The individuals forming a group of ten are given independent names, and these names are repeated with the names of the orders to form larger numbers. Thus:

One, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*,
one *ten*, one *ten* one, one *ten* two, ... one *ten* eight, one *ten* nine,
two *ten*, two *ten* one, two *ten* two, ... two *ten* eight, two *ten* nine,
. . . .
nine *ten*, nine *ten* one, nine *ten* two, ... nine *ten* eight, nine *ten* nine,
one *hundred*, one *hundred* one, ... one *hundred* nine *ten* nine,
. . . .
nine *hundred*, nine *hundred* one, ... nine *hundred* nine *ten* nine.

NOTE. — One ten, one ten one, one ten two, ... two ten, two ten one, ... have been abbreviated to *ten*, *eleven*, *twelve*, ... *twenty*, *twenty-one*, ...

From the above, it will be seen that numbers to a thousand may be expressed by only ten different symbols, if the number in each order is expressed by a figure, and if the names of orders are expressed by relative position. Thus:

. . . .
90, 91, 92, 93, 94, 95, 96, 97, 98, 99,
100, 101, 102, 103, 104, 105, 106, 107, 108, ... 197, 198, 199,
. . . .
900, 901, 902, 903, 904, 905, 906, 907, 908, ... 997, 998, 999.

The Arabic notation affords these devices, and therefore harmonizes with the plan of naming numbers. The oversight of these devices by the Romans and most other nations of antiquity, accounts for their laborious methods of computation.

NUMBERS NAMED — ABOVE A THOUSAND

When the number of individuals is larger than a thousand, it is thought best to consider the individuals as collected into *groups of a thousand*, a thousand of these groups as collected into a higher group, a thousand of these higher groups as collected into a still higher group, and so on. The individuals are called *units;* the groups, *periods.* Thus:

A thousand *units* are considered a group of a *thousand;* a thousand *thousands*, a group of a *million;* a thousand *millions*, a group of a *billion;* and so on.

The individuals forming a group of a thousand are named as on the preceding page, and these names are repeated with the names of the periods to form larger numbers. Thus:

1 thousand, 1 thou. 1, ... 1 thou. 999, ... 999 thou. 999,
1 million, 1 mil. 1, ... 1 mil. 999 thou. 999, ... 999 mil. 999 thou. 999,
. . . .

From the above, it will be seen that numbers larger than a thousand may be expressed by only ten different symbols, if the number in each period except the left-hand is written with three figures, and if the names of periods are expressed by relative position. Thus:

1,000; 1,001; 1,002; 1,003; . . . . . 1,999; 999,999;
1,000,000; 1,000,001; 1,000,002; . . . . 1,999,999; 999,999,999;
. . . .

NOTE. — The people of the United States and of France follow the foregoing plan indefinitely; the English, only to a million.

When the number of individuals is larger than a million, the English and most European nations think it best to consider the individuals as collected into *groups of a million*, a million of these groups as collected into a higher group, a million of these higher groups as collected into a still higher group, and so on. The individuals are called *units;* the groups, *periods.* Thus:

A million *units* are considered a group of a *million;* a million *millions*, a group of a *billion;* a million *billions*, a group of a *trillion;* and so on.

NOTATION — UNITED STATES MONEY

Since 10 mills make 1 cent, 10 cents make 1 dime, and 10 dimes make 1 dollar, the devices already explained may be employed in writing United States money if it is agreed that the orders from the left shall be *dollars, dimes, cents, mills.* Thus:

U. S. Five *dollars* no *dimes* eight *cents* three *mills*
U. S. 5 *dollars* 0 *dimes* 8 *cents* 3 *mills*
U. S. 5083

This plan is modified, however, by writing the symbol, $, *before,* and a period, *after,* the number of dollars. Thus:

$5.083.

Write:

51. 8 dollars 19 cents 2 mills.
52. 16 dollars 16 cents 4 mills.
53. 18 dollars 4 cents 6 mills.
54. 425 dollars sixty-two cents.
55. 521 dollars 38 cents.
56. 600 dollars four cents.
57. 151 dollars ninety cents.
58. 225 dollars eight mills.
59. 15 dollars 15 cents 5 mills.
60. 27 dollars eleven cents.

Ex. **51.** $8.192. 19 cents equals 1 dime 9 cents. The word "dime" is rarely used in writing or reading.

Read:

61. $1.846; $14.18; $95.95.
62. $91.12; $84.98; $41.05.
63. $82.07; $46.04; $37.07.
64. $86.06; $39.09; $27.64.
65. $51.22; $49.87; $91.14.
66. $43.065; $18.03; $15.92.
67. $91.164; $37.33; $41.25.
68. $126.003; $5.05; $23.50.
69. $8971.04; $3.07; $94.01.
70. $6128.97; $9.21; $79.49.

Ex. **61.** 1 dollar 84 cents 6 mills. The natural reading would be, 1 dollar 8 dimes 4 cents 6 mills, but it is customary to read dimes and cents *together* as cents.

NOTATION — ROMAN

The Roman Notation uses seven capital letters, viz.: I, 1; V, 5; X, 10; L, 50; C, 100; D, 500; M, 1000.

Since it does not use an independent symbol for each of the first ten numbers, it is out of harmony with the plan of naming them, and is rarely used except for ornamental purposes.

Independent names are given to *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*, *ten;* the Roman notation uses independent symbols for *one*, *five*, *ten*, *fifty*, *one hundred*, *five hundred*, and *one thousand.*

The Roman notation employs these devices:

Repeating a letter repeats its value.	III, 3; CC, 200; XXX, 30.
When a letter is placed after one of greater value, its value is to be added to that of the greater.	XI, 11; LXXX, 80.
When a letter is placed before one of greater value, its value is to be subtracted from that of the greater.	IX, 9; CD, 400.
When a letter is placed between two letters, each of greater value, its value is to be subtracted from the sum of the other two.	XIV, 14; DXL, 540.
A bar placed over a letter multiplies its value by 1000.	$\overline{\text{X}}$, 10,000; $\overline{\text{CM}}$III, 900,003.

Express by the Arabic:

71. II, XX, CCC, MMM.

72. LI, CV, DC, LXX.

73. IV, XL, XC, CD, CM.

74. XIX, CXL, DXC, MCD.

75. $\overline{\text{D}}$, $\overline{\text{M}}$, $\overline{\text{CD}}$CCCXXXIX.

Express by the Roman:

76. 1, 2, 3, 4, 5, 6, 7.

77. 8, 9, 10, 11, 12, 13.

78. 28, 83, 95, 101, 190.

79. 256, 379, 400, 568.

80. 1492, 1520, 1876, 1896.

ADDITION

TERMS

The whole is equal to the sum of all its parts. Thus:

9 cents = 4 cents + 5 cents.

If the whole is wanting, this becomes

what = 4 cents + 5 cents?

It means "what is the sum of 4 cents and 5 cents?"

Addition is the process of uniting two or more numbers into one. The numbers to be united are *addends;* the result. the *sum,* or *amount.*

The sign of addition is +.

The sign of equality is =.

4
3 } *addends.*
2
9, *sum.*

6 + 4 = 10,
read,
6 plus *4* equals *10.*

COUNTING BY ONES

Addition may be performed by counting by ones.

1. In this way, find the sum of 4, 3, and 2.

Counting 4, then 3, then 2, and making a mark at each count, we have ////, ///, //; counting these together, we have one, two, three, four, five, six, seven, eight, nine.

2. Counting by ones, find the sum of 5 and 6.

3. Counting by ones, find the sum of 8, 9, and 7.

4. Would you care to find the sum of 968 and 754 in this way? Why not?

COMMON METHOD

There are forty-five combinations of the first nine numbers taken two and two. They should be memorized as wholes.

5. *Memorize the following:*

1　　2　　2 3　　3 4　　3 4 5　　4 5 6
1, *2*; 1, *3*; 2, 1, *4*; 2, 1, *5*; 3, 2, 1, *6*; 3, 2, 1, *7*;

4 5 6 7　　5 6 7 8　　5 6 7 8 9
4, 3, 2, 1, *8*; 4, 3, 2, 1, *9*; 5, 4, 3, 2, 1, *10*;

6 7 8 9　　6 7 8 9　　7 8 9
5, 4, 3, 2, *11*; 6, 5, 4, 3, *12*; 6, 5, 4, *13*;

7 8 9　　8 9　　8 9　　9　　9
7, 6, 5, *14*; 7, 6, *15*; 8, 7, *16*; 8, *17*; 9, *18*.

Thus: 4 5 6 7 over 4, 3, 2, 1, are different ways of expressing *8*.

Call the sums:

	8	3	9	2	7	9	4	8	6	9	7	1	7	9	6	9	5
6.	5,	2,	3,	2,	8,	7,	4,	1,	4,	2,	7,	1,	6,	1,	6,	4,	4.
	9	7	8	6	8	1	9	6	9	3	9	4	8	2	1	7	8
7.	8,	9,	7,	9,	8,	4,	9,	6,	2,	9,	5,	9,	3,	8,	5,	4,	9.
	5	3	8	2	8	3	4	6	5	6	7	9	5	4	7	5	2
8.	5,	3,	4,	5,	3,	1,	2,	2,	3,	3,	5,	6,	1,	3,	2,	2,	4.
	1	9	6	2	1	5	7	4	8	9	6	2	7	1	7	9	9
9.	6,	8,	5,	6,	7,	8,	1,	1,	6,	4,	7,	1,	6,	2,	7,	5,	3.
	8	2	9	3	8	9	8	6	7	5	1	4	5	9	7	6	9
10.	7,	9,	3,	7,	5,	9,	2,	1,	3,	9,	8,	7,	1,	6,	5,	8,	8.
	2	9	8	3	5	9	4	8	6	9	3	1	7	2	6	9	8
11.	3,	2,	4,	5,	6,	7,	6,	1,	5,	3,	6,	3,	6,	7,	6,	4,	4.
	5	7	4	8	9	6	1	7	8	7	9	4	8	3	9	7	9
12.	7,	1,	5,	6,	4,	6,	9,	6,	8,	7,	5,	8,	1,	8,	7,	8,	2.

Ex. **6.** 13, 5, 12, 4, 15, ...

Steps in adding

Is the sum of the units more than 9:

13. When 6 is added to 24, 23, 27, 25, 28, 36, 41, 32?

14. When 5 is added to 12, 35, 38, 27, 43, 54, 19, 21?

Ex. 13. Yes, no, yes, yes, ...

Declare the tens of the sum:

15. When 36 is increased by 9, 2, 8, 4, 6, 3, 1, 5, 7.

16. When 24 is increased by 8, 7, 2, 9, 1, 3, 6, 5, 4.

17. When 68 is increased by 5, 9, 2, 8, 4, 3, 7, 1, 6.

18. When 79 is increased by 8, 3, 1, 4, 9, 5, 6, 7, 2.

19. When 27 is increased by 5, 9, 2, 8, 4, 3, 6, 1, 7.

20. When 35 is increased by 3, 1, 7, 2, 5, 4, 9, 6, 8.

Ex. 15. Forty, thirty, forty, forty, ...

Declare the units of the sum:

21. When 9 is added to 15, 29, 37, 46, 53, 22, 41, 68.

22. When 8 is added to 94, 38, 75, 26, 52, 63, 89, 71.

23. When 7 is added to 35, 48, 84, 76, 92, 53, 69, 21.

24. When 6 is added to 25, 38, 47, 56, 63, 72, 51, 49.

25. When 5 is added to 17, 23, 31, 45, 56, 62, 34, 49.

26. When 4 is added to 14, 58, 75, 66, 82, 91, 27, 19.

27. When 3 is added to 18, 16, 27, 39, 45, 64, 23, 12.

Ex. 21. 4, 8, 6, 5, 2, ...

28. Declare the sums in examples 13 to 27 inclusive.

Ex. 13. 30, 29, 33, 31, ...

29. Count from 1 to 100 by 9's; by 8's; by 7's; by 6's; by 5's; by 4's; by 3's; by 2's.

Ex. 29. By 9's; 1, 10, 19, 28, 37, ...

A single order

	51.	52.	53.	54.	55.	56.	57.	58.	59.	60.	61.	62.	63.	64.	65.
30.	8,	6,	9,	3,	4,	7,	8,	4,	9,	5,	3,	8,	7,	6,	8.
31.	6,	4,	8,	2,	5,	6,	4,	6,	9,	8,	8,	6,	4,	9,	8.
32.	5,	6,	9,	9,	8,	4,	7,	6,	8,	8,	7,	9,	4,	8,	3.
33.	6,	7,	8,	9,	4,	6,	8,	9,	8,	7,	6,	5,	4,	3,	2.
34.	4,	4,	5,	5,	6,	6,	7,	7,	8,	8,	9,	9,	8,	8,	7.
35.	3,	6,	9,	6,	3,	2,	4,	8,	4,	2,	5,	7,	9,	8,	6.
36.	6,	7,	9,	8,	8,	4,	7,	6,	9,	8,	3,	4,	7,	7,	9.
37.	9,	4,	3,	2,	8,	7,	6,	3,	9,	4,	8,	6,	3,	3,	5.
38.	6,	8,	3,	9,	7,	4,	2,	5,	8,	7,	7,	4,	7,	6,	8.
39.	8,	8,	4,	4,	6,	6,	5,	8,	9,	7,	6,	3,	7,	4,	8.
40.	8,	9,	7,	6,	4,	3,	9,	8,	2,	6,	3,	8,	9,	4,	3.
41.	6,	6,	4,	3,	3,	4,	8,	7,	6,	9,	4,	6,	8,	7,	6.
42.	5,	5,	6,	3,	7,	2,	8,	1,	9,	4,	7,	9,	3,	8,	7.
43.	2,	2,	8,	6,	5,	4,	7,	9,	8,	6,	3,	6,	5,	7,	4.
44.	8,	8,	6,	5,	3,	2,	9,	4,	7,	6,	3,	8,	7,	6,	3.
45.	8,	6,	5,	3,	9,	8,	6,	6,	4,	8,	8,	5,	7,	2,	3.
46.	2,	9,	1,	3,	8,	2,	5,	3,	7,	6,	4,	7,	5,	5,	6.
47.	2,	3,	4,	5,	6,	7,	8,	9,	8,	7,	6,	5,	4,	3,	2.
48.	4,	5,	5,	4,	7,	8,	6,	7,	5,	3,	2,	9,	7,	6,	6.
49.	1,	1,	3,	5,	6,	2,	3,	4,	7,	1,	1,	8,	2,	5,	4.
50.	7,	6,	3,	2,	9,	8,	8,	4,	3,	8,	7,	6,	3,	1,	9.

Ex. **30**. 8, 14, 23, 26, 30, 37, ... Do not say "8 and 6 are 14, and 9 are 23."

Ex. **31**. By this method, we add one figure at a time, but when the sum of two adjacent addends is 10, it is customary to add the two at once; we say 10, 20, 25, 35, 41, ...

More than one order

66. Add and explain: 368, 496, 709.

368	
496	*23*
709	*17*
	15
1573	

The sum of the units is 23 units, or 2 tens and 3 units; we write 3 in units' column and carry 2 to tens' column.

The sum of the tens is 17 tens, or 1 hundred and 7 tens; we write 7 in tens' column and carry 1 to hundreds' column; etc.

To prove, add in the opposite direction. *See p. 79.*

Note. — For convenience in proving, the sum of each column should be written by itself as soon as it is obtained.

67. Add and explain: 468, 963, 735, 843, 987, 638, 475, 327.

68. Add and explain: 745, 908, 722, 496, 783, 954, 738, 873.

69.	**70.**	**71.**	**72.**	**73.**	**74.**
777	398	666	999	995	672
787	846	694	888	881	547
898	946	784	765	654	857
798	775	996	877	343	351
879	238	886	895	637	893
878	876	259	788	448	752
788	439	448	399	549	695
988	777	679	757	870	452
899	989	876	499	626	537
789	646	947	787	765	794
787	988	589	696	693	686
877	568	487	789	247	964
889	859	444	789	692	415
787	363	668	947	631	213
978	496	843	787	786	429
475	132	246	987	325	737
139	547	745	579	417	217
824	724	993	813	629	425

COUNTING BY COMBINATIONS 10–19

The combinations of three or more digits whose sums are 10 to 19 should be learned so that the results can be recognized and called at a glance.

Thus: $\begin{matrix}3\\2\\8\end{matrix}$ should be recognized as $\begin{matrix}5\\8\end{matrix}$, *13*; $\begin{matrix}4\\2\\6\\8\end{matrix}$ as $\begin{matrix}6\\9\end{matrix}$, *15*; etc.

Declare the sums rapidly:

	8	1	8	8	8	7	5	5	4	4	1
	9	7	8	7	3	3	4	5	4	7	1
75.	2	9	2	2	3	1	3	9	5	8	8

	7	8	4	5	7	8	5	4	9	7	9
	3	5	4	3	4	2	3	6	8	3	5
76.	6	3	3	2	1	2	5	9	1	4	4

	6	5	8	9	7	9	6	7	7	6	9
	3	5	4	3	3	1	3	4	5	3	3
77.	2	4	1	1	2	2	3	4	4	1	2

	4	5	5	5	6	7	7	4	5	4	6
	6	4	5	2	1	3	2	5	1	4	3
	6	2	4	5	3	7	3	2	2	1	5
78.	3	2	5	1	4	2	2	7	2	2	2

	5	6	6	6	4	2	5	4	1	2	3
	2	1	3	1	4	7	3	1	8	4	6
	1	1	4	1	1	1	1	1	1	1	4
79.	3	2	2	4	2	1	3	4	6	3	4

	7	6	6	9	3	1	3	4	1	4	2
	1	3	1	1	3	8	4	5	8	4	6
	1	1	1	4	2	2	2	8	4	2	4
80.	4	2	7	5	2	7	5	1	4	4	4

Ex. **75**. 19, 17, 18, 17, ...

Ex. **78**. 19, 13, 19, 13, ...

Steps in adding

Is the sum of the units more than 9:

81. When 15 is added to 72, 68, 47, 31, 45, 23, 36, 39?

82. When 18 is added to 94, 47, 39, 28, 55, 63, 92, 61?

Ex. **81**. No, yes, yes, no, yes, ...

Declare the tens of the sum:

83. When 46 is increased by 13, 16, 17, 12, 18, 14, 19.

84. When 38 is increased by 15, 19, 17, 12, 14, 16, 13.

85. When 92 is increased by 17, 19, 18, 14, 16, 12, 15.

86. When 29 is increased by 11, 15, 18, 16, 19, 17, 12.

87. When 37 is increased by 17, 14, 16, 19, 13, 14, 11.

88. When 55 is increased by 12, 19, 17, 16, 13, 14, 18.

Ex. **83**. Fifty, sixty, sixty, fifty, sixty, sixty, ...

Declare the units of the sum:

89. When 17 is added to 71, 85, 96, 84, 23, 72, 99, 87.

90. When 16 is added to 15, 29, 37, 46, 53, 22, 41, 74.

91. When 13 is added to 17, 19, 26, 28, 35, 41, 63, 94.

92. When 14 is added to 68, 94, 85, 96, 47, 53, 72, 91.

93. When 18 is added to 29, 37, 46, 55, 83, 92, 24, 38.

94. When 12 is added to 24, 69, 78, 37, 43, 82, 91, 65.

Ex. **89**. 8, 2, 3, 1, ...

95. Declare the sums in examples 81 to 94, inclusive.

Ex. **81**. 87, 83, 62, 46, ...

96. Count from 1 to 200 by 19's; by 18's; by 17's; by 16's; by 15's; by 14's; by 13's; by 12's; by 11's.

Ex. **96**. By 19's. 1, 20, 39, 58, 77, 96, ...

Addends less than 20

	121.	122.	123.	124.	125.	126.	127.	128.	129.	130.	131.
97.	13,	14,	15,	16,	17,	18,	12,	14,	11,	19,	18.
98.	12,	16,	13,	18,	11,	17,	15,	16,	13,	18,	11.
99.	15,	17,	12,	13,	18,	11,	16,	15,	12,	14,	17.
100.	19,	13,	14,	17,	14,	19,	13,	17,	15,	12,	14.
101.	16,	11,	15,	14,	17,	12,	14,	13,	17,	11,	13.
102.	17,	14,	18,	19,	15,	13,	17,	19,	16,	15,	12.
103.	18,	19,	17,	12,	13,	14,	19,	18,	14,	17,	19.
104.	19,	16,	14,	13,	12,	18,	15,	16,	17,	19,	16.
105.	11,	19,	15,	13,	12,	16,	17,	13,	19,	18,	16.
106.	16,	18,	14,	12,	10,	11,	13,	15,	17,	19,	16.
107.	19,	15,	13,	12,	11,	18,	16,	14,	13,	18,	17.
108.	16,	16,	15,	14,	19,	17,	16,	17,	18,	14,	13.
109.	13,	16,	19,	19,	17,	17,	15,	15,	11,	11,	17.
110.	18,	18,	16,	16,	13,	19,	17,	11,	13,	14,	14.
111.	17,	17,	16,	14,	15,	13,	19,	17,	15,	11,	12.
112.	15,	14,	12,	10,	11,	12,	13,	14,	17,	19,	18.
113.	15,	12,	16,	16,	15,	14,	18,	19,	17,	16,	15.
114.	11,	13,	15,	17,	19,	17,	15,	13,	11,	12,	14.
115.	16,	17,	19,	15,	11,	13,	15,	14,	13,	17,	16.
116.	17,	17,	16,	15,	14,	12,	11,	19,	16,	15,	13.
117.	14,	13,	19,	16,	17,	12,	16,	15,	13,	19,	18.
118.	12,	11,	18,	15,	14,	13,	17,	19,	16,	16,	17.
119.	19,	18,	17,	16,	15,	11,	16,	15,	12,	11,	16.
120.	17,	15,	14,	13,	12,	16,	15,	11,	17,	19,	16.

Ex. 97. 13, 27, 42, 58, 75, 93, 105, ...

Addends greater than 19

132. By combinations 10 to 19, add: 24, 72, 98, 69, 34, 73, 28, 43, 52, 89, 73, 58.

24	
72	
98	
69	
34	
73	
28	*63*
43	*71*
52	
89	
73	
58	
713	

We see $\begin{smallmatrix}3\\8\end{smallmatrix}$ as 11, and say 11; we see $\begin{smallmatrix}8\\2\\9\end{smallmatrix}$ as 14, and say 25; $\begin{smallmatrix}4\\8\\8\end{smallmatrix}$ as 15, and say 40; $\begin{smallmatrix}8\\9\end{smallmatrix}$ as 17, and say 57; $\begin{smallmatrix}4\\2\end{smallmatrix}$ as 6, and say 63.

We see the 6 to carry and $\begin{smallmatrix}7\\5\end{smallmatrix}$ as 18, and say 18; we see $\begin{smallmatrix}4\\5\\8\end{smallmatrix}$ as 17, and say 35; $\begin{smallmatrix}6\\8\\7\\2\end{smallmatrix}$ as 18, and say 53; $\begin{smallmatrix}2\\7\\9\end{smallmatrix}$ as 18, and say 71.

NOTE.—The explanation may be given as on p. 20; the sum of the units is 63 units, or 6 tens and 3 units; etc.

133.	134.	135.	136.	137.	138.	139.
625	198	269	597	896	938	364
419	344	475	487	288	544	416
887	765	628	676	144	629	862
164	137	849	345	727	887	424
333	582	137	799	419	368	561
548	991	966	288	589	299	704
727	377	294	376	476	886	861
949	866	888	467	665	743	492
194	454	376	156	647	592	563
487	643	789	165	925	447	316
798	249	649	246	194	918	542
386	818	555	254	886	925	956
666	726	228	334	827	759	599
977	638	494	343	889	967	321
293	247	347	628	929	898	642
586	393	325	627	588	484	494

Practice for any method

140.	141.	142.	143.	144.
63954	611043	545982	662347	845954
87445	626915	606819	257938	358346
13294	537454	146608	689477	214324
51687	442014	889478	893298	843567
91532	780894	395777	478469	413578
72927	122993	865092	328947	495219
51470	726915	323459	886539	254183
82044	484471	478949	445328	145482
94514	705606	338948	768894	562531
12592	560247	667848	325537	214644
88416	827922	769949	892468	523671
97654	439706	876329	729848	954751
92214	364399	448869	849889	552169
82405	671361	732964	946677	773713
96185	225163	985263	833849	631526
35391	954670	868897	486695	652847
69039	258520	402986	329875	156879
71623	184109	399478	684498	235659
89555	194020	456984	725578	921982
66526	184059	737842	398765	567399
68507	680637	698325	438942	936783
21661	457351	447983	668937	356773
92717	721295	892567	865432	692683
98329	665260	443846	598428	578756
31786	539891	329847	697646	578325
97394	382579	666744	393948	565731
86929	426869	889847	872896	757785
31993	389495	653894	498695	567320
51395	708040	555333	213151	315625
34575	943511	773311	431512	567921
11057	443225	773221	772202	521673

PROBLEMS

First form of analysis

145. If there are 96 apples in one pile and 88 apples in another, how many are there in both?

96
88
184

In both, there are the sum of 96 apples and 88 apples, or 184 apples.

To write what each term represents is of service.

146. A man paid $45 for a horse, and $95 more for a carriage than for the horse. How much did he pay for both?

$45, horse
95, increase
140, carriage
185, both

The carriage cost the sum of $45 and $95, or $140; both cost the sum of $45 and $140, or $185.

A drawing is often of service.

147. One village is 37 miles west and another 48 miles east of Denver. How far apart are the villages?

37, A to D
48, D to B
85, A to B

Since it is 37 miles from A to D, and 48 miles from D to B, from A to B it is the sum, or 85 miles.

148. A man paid $50 for a horse, $125 for a carriage, $12 for a harness, and $15 for a saddle. How much did he pay in all?

149. The village A is 36 miles east of Chicago, and B is 43 miles east of A. How far is it from Chicago to B?

150. A grocer sells 18 pounds of coffee, 16 pounds more of tea than of coffee, and 96 pounds more of sugar than of tea. How many pounds in all does he sell?

151. A has $65; B has $29 more than A; C has as much as A and B and $13 more; and D has as much as A, B, and C together. How much do all possess?

152. How many strokes are made in a day by a clock which strikes the hours?

153. Mr. Brown owes three bills, one for $ 1987, another for $ 1849, and a third for $ 2789. How much does he owe in all?

154. A man bought 4 horses, paying for them $ 3985, $ 5025, $ 2789, and $ 6898; he sold them for $ 2399 above cost. How much did he receive for them?

155. A man paid $ 3486 for a lot, $ 2878 for a house, $ 1695 for furniture, and $ 387 for repairs; he disposed of his entire property for $ 285 above cost. How much did he receive?

156. A nurseryman sold 862 pear trees, 965 more apple trees than pear trees, 688 peach trees, 466 more plum trees than apple trees, and 568 ornamental trees. Find the entire number of trees sold.

157. The village A is six hundred sixty-five miles east of Chicago, and B is eight hundred eighty miles west of Chicago. What is the distance from A to B?

158. The Duke of Wellington's army at Waterloo consisted of 26,661 English infantry, 8735 English cavalry, 6877 English artillery, and 33,413 allies. How large was his army?

159. North America has an area of 9,349,741 square miles; and South America, 6,887,794. What is the area of the American continent?

160. Mt. Etna is 10,874 feet above the level of the sea; Mt. Blanc is 4870 feet higher than Mt. Etna; Mt. Everest is 13,258 feet higher than Mt. Blanc. What is the height of Mt. Everest?

161. A train travels 719 miles one day, 698 miles a day for the next three days, 692 miles a day for two days, and 718 miles the seventh day. How many miles does it travel during the week?

162. A farmer raised on one farm ten hundred thirty-eight bushels of wheat and three hundred two bushels of barley; on a second farm one thousand one bushels of wheat and five hundred two bushels of oats. What was his entire crop of grain?

SUBTRACTION

TERMS

$$9 \text{ cents} = 4 \text{ cents} + 5 \text{ cents.}$$

If the second addend is wanting, this becomes

$$9 \text{ cents} = 4 \text{ cents} + \text{what.}$$

This is commonly written

$$\textbf{what} = \mathbf{9} \textbf{ cents} - \mathbf{4} \textbf{ cents ?}$$

It means "what is the other of two numbers when one of them is 4 cents and their sum 9 cents, or what is the difference between 9 cents and 4 cents?"

Subtraction is the process of finding the other of two numbers, when one of them and their sum are given.

The sum is the *minuend;* the number given, the *subtrahend;* the number required, the *difference*, or the *remainder*.

The sign of subtraction is $-$.

One of two numbers is 4, and their sum, 9. Find the other.

9, minuend.
4, subtrahend.
5, remainder.

$9 - 4 = 5$,
read,
9 minus $4 = 5$.

COUNTING BY ONES

Subtraction may be performed by counting by ones.

1. In this way, subtract 5 from 8.

Counting 8 and making a mark at each count, ////////; counting 5 and crossing a mark at each count, XXXXX///; counting what is left, we have 3.

2. Counting by ones, find the difference between 9 and 12.

3. Counting by ones, find the remainder when 11 is subtracted from 15.

4. Would you care to subtract 986 from 2384 in this way? Why not?

COMMON METHOD

If the forty-five combinations in addition have been mastered, the results in the following examples may be called rapidly. In addition, we have two addends to name the sum; in subtraction, the sum and one addend to name the other.

5. *Given the sum and one addend, name the other:*

1   1 2   1 3 2   1 3 2 4   2 5 4 3
?, *2*; ?, ?, *3*; ?, ?, ?, *4*; ?, ?, ?, ?, *5*; ?, ?, ?, ?, *6*;

3 6 5 4 2   3 2 7 5 4 6   4 7 5 3 2 8 6
?, ?, ?, ?, ?, *7*; ?, ?, ?, ?, ?, ?, *8*; ?, ?, ?, ?, ?, ?, ?, *9*;

7 2 9 3 5 4 8 6 1   2 6 3 7 4 9 5 8
?, ?, ?, ?, ?, ?, ?, ?, ?, *10*; ?, ?, ?, ?, ?, ?, ?, ?, *11*;

3 8 4 9 5 7 6   4 7 5 9 6 8   9 8 7
?, ?, ?, ?, ?, ?, ?, *12*; ?, ?, ?, ?, ?, ?, *13*; ?, ?, ?, *16*;

5 8 9 7 6   6 8 9 7   9 8   9
?, ?, ?, ?, ?, *14*; ?, ?, ?, ?, *15*; ?, ?, *17*; ?, *18*.

Thus: 1; 2, 1; 3, 1, 2; . . . .

Declare the remainders:

	18	14	12	10	12	11	17	15	16	12
6.	9	5	3	4	5	3	9	8	8	4
	11	10	17	13	16	15	10	10	11	13
7.	8	2	8	6	9	7	9	1	7	7
	14	13	16	13	11	11	15	13	11	9
8.	6	8	7	5	5	9	9	4	4	6
	13	14	15	14	12	11	9	9	7	4
9.	9	8	6	9	8	2	4	3	3	3
	10	14	10	11	10	12	10	12	10	8
10.	8	7	6	6	7	9	5	6	3	1

Ex. 6. 9, 9, 9, 6, 7, ... Do not say, "9 from 18 leaves 9."

SUBTRAHEND GREATER THAN NINE

Method A

Adding 10 to any order of the minuend and 1 to the next higher order of the subtrahend, cannot affect the remainder.

11. From 501 subtract 398 and explain.

501
398
103

In full. 8 units from 1 unit we cannot take; we add 10 units to 1 unit making 11 units, and 1 ten to 9 tens making 10 tens; 8 units from 11 units leaves 3 units; we write 3 in units' column. 10 tens from 0 tens we cannot take; we add 10 tens to 0 tens making 10 tens, and 1 hundred to 3 hundreds making 4 hundreds; 10 tens from 10 tens leaves 0 tens; we write zero in tens' column; etc.

Abbreviated. 3, 0, 1. We look upon 1 as 11 and say 3; we look upon 9 as 10, and 0 as 10, and say 0; we look upon 3 as 4 and say 1.

Note.—It is a great loss of time to say, "8 from 11 leaves 3." While these words are being said, the process is delayed.

Method B

Taking 1 from any order of the minuend and adding its equivalent to any other order of the minuend, cannot affect the remainder.

11. From 501 subtract 398 and explain.

501
398
103

In full. 8 units from 1 unit we cannot take; we take 1 hundred from 5 hundreds leaving 4 hundreds; 1 hundred, or 10 tens, with 0 tens, 10 tens; we take 1 ten from 10 tens leaving 9 tens; 1 ten, or 10 units, with 1 unit, 11 units; 8 units from 11 units leaves 3 units; we write 3 in units' column.

9 tens from 9 tens leaves 0 tens; we write 0 in tens' column; etc.

Abbreviated. 3, 0, 1. We look upon 1 as 11 and say 3; we look upon 0 as 9 and say 0; we look upon 5 as 4 and say 1.

Note.—Study only one method.

To prove, add the subtrahend and the remainder. The sum should be the minuend. *See p. 79.*

Subtract and prove, explaining in full:

12.	13.	14.	15.	16.	17.
120	353	400	2000	8000	9000
118	168	235	1234	7001	8023

18.	19.	20.	21.	22.	23.
320	452	500	2001	7000	8306
116	348	163	1009	6006	7029

Subtract and prove, abbreviating:

24.	25.	26.	27.	28.	29.
500	728	514	3000	8003	9750
206	599	219	1865	7004	7045

30.	31.	32.	33.	34.	35.
782	633	708	9028	8705	9454
694	238	297	8139	7706	7541

36.	37.	38.	39.	40.	41.
613	785	308	5106	7006	7755
45	98	99	987	707	794

42.	43.	44.	45.	46.	47.
906	555	989	4205	7153	8743
807	499	893	3986	4154	7139

48.	49.	50.	51.	52.	53.
841	736	906	8753	2107	5212
693	463	809	5645	1009	3271

Ex. **24**. *Ans.* 294. Say 4, 9, 2.

For mental and written work

For mental practice, name each figure of the remainder without writing it, and without stating the full answer after it is obtained.

54.	55.	56.	57.
90543001	6398580044	1094600105	75000807063
76392132	1296548980	653864126	52983878156

58.	59.	60.	61.
68686868	7899984343	7205080901	10340000900
16868698	1549800022	6789890993	9876543211

62.	63.	64.	65.
72980132	9453011342	3900080040	19876547001
68997654	3568054321	1777397786	7687589422

66.	67.	68.	69.
74840005	6365433309	1000008000	18470020305
69751126	1350000677	897329443	17885332416

70.	71.	72.	73.
63800402	6999214205	9080706050	97688888823
58911387	5503436661	4398616361	88888888888

74.	75.	76.	77.
54208075	3990911238	7000000700	78432234256
46677666	1089832624	6667893921	7584476899

78.	79.	80.	81.
90040708	6504003311	6501030405	10080090040
89753291	1009876555	5798398763	9865732986

Ex. **54**. For mental work, say 9, 6, 8, 0, 5, ...

PROBLEMS

Second form of ana.ysis

82. From a pile of 184 apples, 96 apples were taken away. How many remained?

184
96
88

There remained the difference between 184 apples and 96 apples, or 88 apples.

To write what each term represents is of service.

83. A man bought a horse for $60 and a cow for $24 less. What was the cost of the cow?

$60, horse
24, amt. less
36, cow

Since the cow cost $24 less than the horse, she cost the difference between $60 and $24, or $36.

A drawing is often of service.

84. One village is 48 miles west and another 37 miles west of Denver. How far apart are the villages?

48, A to D
37, B to D
11, A to B

Since it is 48 miles from A to D, and 37 miles from B to D, from A to B it is the difference, or 11 miles.

85. Which number is nearer to 912; 424 or 1200? By how many?

86. A house was sold for $2387, or for $987 more than a farm. What was the selling price of the farm?

87. The sum of two numbers is 986, and one of them is 298. What is the other?

88. The difference between two numbers is 999, and the greater is 1200. What is the smaller?

89. A man bought a horse for $ 270 and sold him for $ 48 less. What was the selling price?

90. Counting with the hands of a watch, how many minute spaces are there from the point at VI. to the point at VIII.? Draw a diagram.

91. From a farm containing 1100 acres, the owner sold 894 acres. How many acres were left in the original farm?

92. On Monday, Mr. Willow deposited in bank $ 325; on Tuesday, $ 87; on Wednesday, $ 87; on Thursday he drew out $ 255, and on Friday, $ 44. How much did he leave on deposit?

93. The area of Texas is 265,780 square miles; Maine, 33,040 square miles; New York, 49,170 square miles; Pennsylvania, 45,215 square miles; Virginia, 42,450 square miles; Massachusetts, 8315 square miles. By how much does the area of Texas exceed the combined area of the other five states?

94. A steamer sails for a port distant 2150 miles; it sails 195 miles on Monday, 182 miles on Tuesday, 194 miles on Wednesday, 188 miles on Thursday, 198 miles on Friday, and 186 miles on Saturday. How far is it still from its destination?

95. Three persons buy some land for $ 25,850; the first pays $ 9885, the second, $ 1025 more than the first. How much does the third pay?

96. A man wishes to plant 450 trees; in one day he plants 46 trees, his son plants 38 trees, and each of three men who are helping him plants 54 trees. How many trees remain to be planted?

97. A gentleman willed $ 98 more to each of two sons than to each daughter, and to the widow $ 536 less than to his five children; each son received $ 866. What was the value of his estate?

98. Two cities, A and B, are 896 miles apart; a train starts from A and runs 258 miles toward B; another train runs 188 miles from B toward A. How far apart are the two trains?

MULTIPLICATION

TERMS

8 cents = 4 cents + 4 cents.

This is commonly written

8 cents = 4 cents × 2.

If the sum is wanting, this becomes

what = 4 cents × 2 ?

It means "what is the sum when 4 cents is taken two times as an addend?"

Multiplication is the process of finding the sum when the same number is used several times as an addend.

The number used as an addend is the *multiplicand;* the number showing how many times, the *multiplier;* both terms, *factors;* the result, the *product.*

The sign of multiplication is ×.

The multiplier may precede or follow the multiplicand. In the former case, '×' is read *times;* in the latter, *multiplied by.*

Find the sum when 8 is used 5 times as an addend.

8 + 8 + 8 + 8 + 8.

8, multiplicand.
5, multiplier.
40, product.

2 × 4 cents,
read
2 times 4 cents.

4 cents × 2,
read
4 cents multiplied by 2.

ADDITION METHOD

Multiplication may be performed by addition.

1. In this way, multiply 6 by 5.

6 × 5 = 6 + 6 + 6 + 6 + 6; adding, we obtain 30.

2. By the addition method, multiply 4 by 3; multiply 3 by 4.

COMMON METHOD

All combinations whose products are less than 100, and the multiplication table to '12 times 12,' should be memorized.

3. *Memorize the following:*

9 6 10 5 7 11 12 8 6 5 13
2, 3, *18*; 2, 4, *20*; 3, *21*; 2, *22*; 2, 3, 4, *24*; 5, *25*; 2, *26*;

9 14 7 15 10 6 16 8 11 17 7
3, *27*; 2, 4, *28*; 2, 3, 5, *30*; 2, 4, *32*; 3, *33*; 2, *34*; 5, *35*;

18 12 9 6 19 13 20 10 8 21 14 7
2, 3, 4, 6, *36*; 2, *38*; 3, *39*; 2, 4, 5, *40*; 2, 3, 6, *42*;

22 11 15 9 23 24 16 12 8 7 25 10
2, 4, *44*; 3, 5, *45*; 2, *46*; 2, 3, 4, 6, *48*; 7, *49*; 2, 5, *50*;

17 26 13 27 18 9 11 28 14 8 19
3, *51*; 2, 4, *52*; 2, 3, 6, *54*; 5, *55*; 2, 4, 7, *56*; 3, *57*;

29 30 20 15 12 10 31 21 9 32 16 8
2, *58*; 2, 3, 4, 5, 6, *60*; 2, *62*; 3, 7, *63*; 2, 4, 8, *64*;

13 33 22 11 34 17 23 35 14 10
5, *65*; 2, 3, 6, *66*; 2, 4, *68*; 3, *69*; 2, 5, 7, *70*;

36 24 18 12 8 37 25 15 38 19 11
2, 3, 4, 6, 9, *72*; 2, *74*; 3, 5, *75*; 2, 4, *76*; 7, *77*;

39 13 26 40 20 16 10 27 9 41
2, 6, 3, *78*; 2, 4, 5, 8, *80*; 3, 9, *81*; 2, *82*;

42 28 21 14 12 17 43 29 44 22 11
2, 3, 4, 6, 7, *84*; 5, *85*; 2, *86*; 3, *87*; 2, 4, 8, *88*;

45 30 18 15 10 13 46 23 31 47 19
2, 3, 5, 6, 9, *90*; 7, *91*; 2, 4, *92*; 3, *93*; 2, *94*; 5, *95*;

48 32 24 16 12 49 14 33 11 50 25 20 10
2, 3, 4, 6, 8, *96*; 2, 7, *98*; 3, 9, *99*; 2, 4, 5, 10, *100*.

Thus: 9 *twos*, 6 *threes*, 18; 10 *twos*, 5 *fours*, 20; . . . .

Declare the products rapidly:

4. 12, 5, 9, 4, 10, 6, 11, 2, 7, 3, 8, by *12.*

5. 4, 8, 11, 2, 7, 5, 10, 12, 3, 9, 6, by *11.*

6. 8, 4, 6, 12, 2, 5, 10, 7, 11, 3, 9, by *9.*

7. 6, 10, 2, 9, 12, 7, 4, 8, 3, 11, 5, by *8.*

8. 2, 7, 9, 5, 3, 11, 12, 8, 6, 10, 4, by *7.*

9. 9, 5, 12, 8, 6, 3, 2, 10, 7, 4, 11, by *6.*

10. 11, 4, 2, 9, 10, 12, 7, 5, 8, 6, 3, by *5.*

11. 12, 10, 8, 6, 4, 2, 11, 9, 7, 5, 3, by *4.*

Ex. **4**. 144, 60, 108, . . . Do not say, "12 times 12 are 144."

Declare the products rapidly:

12. 13×7, 13×3, 13×5, 13×2, 13×6, 13×4, 14×6, 14×7, 14×3, 14×5, 14×2, 14×4.

13. 15×4, 15×2, 15×5, 15×3, 15×6, 16×4, 16×6, 16×3, 16×2, 16×5, 17×3, 17×5, 17×2, 17×4.

14. 18×3, 18×5, 18×2, 18×4, 19×3, 19×4, 19×2, 19×5, 21×3, 21×2, 21×4.

15. 22×2, 22×4, 22×3, 23×4, 23×2, 24×2, 24×4, 24×3, 25×2, 25×4, 25×3.

Ex. **12**. 91, 39, 65, . . . Do not say, "13 times 7 are 91."

Declare the products rapidly:

16. Of *4*, by 18, 2, 22, 5, 19. 3, 15, 21, 4, 24, 9, 16, 8, 20, 17, 6, 23, 10, 13, 7, 11, 25, 14, 12.

17. Of *3*, by 30, 21, 17, 4, 9, 12, 20, 32, 29. 3, 23, 16, 2, 24, 5, 11, 6, 22, 25, 7, 14, 26, 15, 27, 10, 28, 8, 13, 18, 31, 19.

18. Of *2*, by 50, 4, 9, 12, 46. 36. 24, 11, 10, 20, 35, 45, 44, 34, 22, 21, 9. 33, 7, 43, 32, 6, 42, 31, 19, 5, 30.

MULTIPLIER LESS THAN THIRTEEN

19. Multiply 608 by 12 and explain.

608
12
7296

In FULL. 12 × 8 units are 96 units, or 9 tens and 6 units; we write 6 in units' column and carry 9 tens.

12 times 0 tens are 0 tens, with 9 tens, 9 tens; we write 9 in tens' column.

12 times 6 hundreds are 72 hundreds, or 7 thousands and 2 hundreds; we write 2 in hundreds' column and 7 in thousands' column.

ABBREVIATED. 96, 9, 72.

NOTE.—It is a great loss of time to say "12 times 8 is 96; 12 times 0 is 0, and 9 is 9." While these words are being said, the process is delayed.

To prove, go over the work a second time. *See p. 79.*

Multiply and prove, explaining in full:

20.	**21.**	**22.**	**23.**	**24.**	**25.**
533	234	944	7648	7651	7954
3	4	2	2	8	8

26.	**27.**	**28.**	**29.**	**30.**	**31.**
798	837	988	763	5494	6879
6	9	7	5	11	12

Multiply and prove, abbreviating:

32.	**33.**	**34.**	**35.**	**36.**	**37.**
368	486	832	8497	2899	3312
7	9	8	12	11	12

38.	**39.**	**40.**	**41.**	**42.**	**43.**
437	902	350	2699	1848	4052
6	7	8	7	9	5

44.	**45.**	**46.**	**47.**	**48.**	**49.**
614	894	605	3424	1827	7320
7	8	12	9	2	11

Ex. **32.** *Ans.* 2576. Say 56, 47, 25.

For mental and written work

For mental practice, name each partial product without writing it, and without stating the full answer after it is obtained.

50.	51.	52.	53.
90876514	3203164032	1543827906	25196078934
9	8	8	7

54.	55.	56.	57.
50247638	8223401523	5721694328	21398765439
5	12	6	8

58.	59.	60.	61.
25306801	1643525431	1704008906	52768439807
11	9	12	11

62.	63.	64.	65.
44667788	9364531046	9843086374	95280976493
12	4	9	11

66.	67.	68.	69.
39694781	9780365415	5943240826	96672884732
8	7	5	12

70.	71.	72.	73.
26982654	4932784609	8269000365	29060809072
8	9	7	8

74.	75.	76.	77.
92074931	4693278427	8327936002	78495080329
9	12	8	6

Ex. **50.** For mental work, say 36, 12, 46, 58, ...

MULTIPLIER GREATER THAN TWELVE

78. Multiply 368 by 624 and explain.

```
   368
   624
  ----
  1472
  736
 2208
------
229632
```

368 × 4 units	= 1472	units
368 × 2 tens	= 736	tens
368 × 6 hund.	= 2208	hund.
368 × 624	= 229632	units

We place the right-hand figure of each partial product under that figure of the multiplier which produces it.

79. Multiply 96 by 365.

```
   96
  365
 ----
 2190
 3285
-----
35040
```

If the lower number contains the more figures, we may use the upper number as the multiplier.

80. Multiply 9386 by 5006.

```
    9386
    5006
   -----
   56316
 46930
--------
46986316
```

If there are ciphers within the multiplier, we are careful to place the right-hand figure of each partial product under the figure of the multiplier which produces it.

81. Multiply 638 by 100.

```
  638
  100
-----
63800
```

If the multiplier is 10, we annex one cipher to the multiplicand; if 100, two ciphers; and so on.

82. Multiply 4300 by 230.

```
  4300
   230
  ----
   129
   86
------
989000
```

If one or both terms end with a cipher, we neglect the ciphers in multiplying, and annex to the product as many ciphers as have been neglected.

Perform the indicated operation:

83. 7703 × 834.
84. 3769 × 235.
85. 7777 × 864.
86. 5896 × 338.
87. 7832 × 985.
88. 8965 × 801.
89. 8299 × 624.
90. 9476 × 876.
91. 8972 × 911.
92. 7009 × 861.
93. 9866 × 706.
94. 9006 × 807.
95. 9763 × 491.
96. 8008 × 909.
97. 3452 × 953.
98. 9047 × 162.
99. 4210 × 444.
100. 2875 × 523.
101. 9532 × 231.
102. 5231 × 215.
103. 3261 × 635.

Perform the indicated operation:

104. 7894 × 3400.
105. 8261 × 7000.
106. 3689 × 5900.
107. 2875 × 8200.
108. 2863 × 1000.
109. 3961 × 7200.
110. 3250 × 6120.
111. 5100 × 7400.
112. 6200 × 2000.

Perform the indicated operation:

113. 686 × 876004.
114. 984 × 100000.
115. 876 × 900008.
116. 666 × 693024.
117. 801 × 597636.
118. 943 × 787878.
119. 496 × 840039.
120. 776 × 880000.
121. 998 × 432122.
122. 259 × 820606.
123. 88888 × 4983.
124. 94689 × 2686.
125. 87634 × 6902.
126. 76894 × 8075.
127. 68977 × 9004.
128. 70007 × 7001.
129. 68947 × 8064.
130. 47899 × 7006.
131. 89724 × 4100.
132. 90876 × 7008.
133. 2334 × 45572.
134. 2345 × 18542.
135. 9347 × 52100.
136. 4741 × 47050.
137. 4361 × 49472.
138. 4282 × 53000.
139. 1125 × 97777.
140. 2207 × 39003.
141. 5903 × 39400.
142. 8345 × 12345.

PROBLEMS

Third form of analysis

143. If there are 88 books in each of 12 boxes, how many books are there in all?

88
12
1056

Since there are 88 books in 1 box, in 12 boxes there are 12 times 88 books, or 1056 books.

To write definitely after the multiplicand and the product what they represent, is of service.

144. At $ 125 each, how much will 56 horses cost?

$ 125, cost of one
56
750
625
7000, cost of all

Since 1 horse costs $ 125, 56 horses will cost 56 times $ 125, or $ 7000.

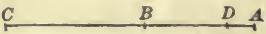
145. A man bought a horse for $ 40 and a cow for twice the cost of the horse. What was the cost of both?

$ 40, cost of horse
2
80, cost of cow
120, cost of both

Since the cow cost twice $ 40, or $ 80, they both cost the sum, or $ 120.

A drawing is often of service.

146. Three villages are in a straight line; A is 27 miles east of Denver; B is 3 times as far west of Denver; and C is west of Denver by twice the distance from A to B. How far is it from A to C?



27, A to D
81, B to D
108, A to B
216, C to D
243, A to C

Since it is 27 miles from A to D, from B to D it is 3 times 27 miles, or 81 miles; since it is 27 miles from A to D, and 81 miles from B to D, from A to B it is the sum, or 108 miles; etc.

147. Mr. White bought 216 tons of coal at $14 a ton. How much did the coal cost him?

148. A sewing machine makes 419 stitches a minute. How many stitches will it make in 98,000 minutes?

149. A man bought 17 plows at $8 apiece, 14 cultivators at $13 apiece, and 134 shovels at $2 apiece. What was his entire bill?

150. In a book of 589 printed pages, there are 32 lines to the page and 12 letters to the line. How many letters does the book contain?

151. Light moves 185,172 miles in a second, and passes from the sun to the earth in 493 seconds. What is the distance from the sun to the earth?

152. It is estimated that the Mississippi river deposits 137,139,200 cubic yards of solid matter in the Gulf of Mexico every year. How many cubic yards have been deposited in 512 years?

153. A man has 194 horses; the average worth of 36 of them is $86 a head; of 82, $97 a head; and of the remainder, $78 a head. What is the value of the whole herd?

154. Mr. Brown sells 296 acres of land at $116 an acre, and invests the proceeds in 9 city lots at $3295 each. How much money has he left?

155. A took a railway journey of 93 miles; B traveled 9 times as far; C, 12 times as far as A and B together; D, 13 times as far as C less B. How many miles did all of them travel?

156. A man bought 7 sheep at $11 a head; twice as many cows at 3 times the price of a sheep; 4 times as many horses as cows at 5 times the cost of a cow; and enough steers to make 100 animals in all at the difference between the price of a sheep and of a cow. How much did he pay in all?

DIVISION

TERMS

$$8 \text{ cents} = 4 \text{ cents} \times 2.$$

If the multiplicand is wanting, this becomes Equation I; if the multiplier is wanting, Equation II.

$$8 \text{ cents} = \text{what} \times 2. \qquad \text{(I)}$$

$$8 \text{ cents} = 4 \text{ cents} \times \text{what}. \qquad \text{(II)}$$

These are commonly written:

$$\textbf{what} = \textbf{8 cents} \div \textbf{2 ?} \qquad \text{(I)}$$

$$\textbf{what} = \textbf{8 cents} \div \textbf{4 cents ?} \qquad \text{(II)}$$

Equation I means "what is the other of two numbers when one of them is 2, and their product, 8 cents?" Equation II means "what is the other of two numbers when one of them is 4 cents and their product, 8 cents?"

Division is the process of finding the other of two numbers when one of them and their product are given.

The product is the *dividend;* the number given, the *divisor;* the number required, the *quotient.*

If the required number was the *multiplicand*, division becomes the process of finding one of the equal parts into which a number may be separated.

If the required number was the *multiplier*, division becomes the process of finding how many times one number contains another.

One of two numbers is 2, and their product, 8. What is the other?

Ans. 4, because 2 × 4 = 8. 8, dividend; 2, divisor; 4, quotient.

What is one of the two equal parts into which $8 may be separated?

Ans. $4, because $8 = $4 × 2, or $4 + $4.

How many times does $8 contain $4?

Ans. 2 times, because $8 = 2 × $4.

Division is expressed in four ways:

By writing the dividend above and the divisor below a horizontal line. — $\frac{8}{2}$, *fractional method.*

By writing the sign '÷' between the terms. — *8 ÷ 2, common method.*

By writing the sign ':' between the terms. — *8 : 2, ratio method.*

By writing the dividend at the right and the divisor at the left of a curved line. — *2)8, working method.*

ADDITION METHOD

Division may be performed by addition.

1. In this way, divide 8 cents by 2.

8 cents ÷ 2 calls for the other of two numbers when one of them is 2, and their product, 8 cents; or it calls for the number that will produce 8 cents when taken 2 times as an addend. *1 cent* taken 2 times as an addend produces 1 cent + 1 cent, or 2 cents; *2 cents,* 2 times as an addend, 2 cents + 2 cents, or 4 cents; *3 cents,* 2 times as an addend, 3 cents + 3 cents, or 6 cents; *4 cents,* 2 times as an addend, 4 cents + 4 cents, or 8 cents. Therefore, *8 cents ÷ 2 = 4 cents.*

2. In this way, divide 8 cents by 4 cents.

8 cents ÷ 4 cents calls for the other of two numbers when one of them is 4 cents, and their product, 8 cents; or it calls for the number of times that 4 cents must be used as an addend to produce 8 cents. *4 cents* used as an addend once produces 4 cents; *4 cents,* as an addend 2 times, 4 cents + 4 cents, or 8 cents. Therefore, *8 cents ÷ 4 cents = 2.*

3. By the addition method, divide 15 quarts by 3; explain in full.

4. By the addition method, divide 15 quarts by 5 quarts; explain in full.

5. By the addition method, divide 16 quarts by 4 quarts; explain in full.

COMMON METHOD

If the combinations in multiplication have been mastered, the results in the following examples may be called rapidly. In multiplication, we have two factors to name their product; in division, the product and one factor to name the other.

6. *Given the product and one factor, name the other:*

9	14 7	15 10 6	16 8	11	17	7
?, *27*;	?, ?, *28*;	?, ?, ?, *30*;	?, ?, *32*;	?, *33*;	?, *34*;	?, *35*;

18 12 9 6	19	13	20 10 8	21 14 7
?, ?, ?, ?, *36*;	?, *38*;	?, *39*;	?, ?, ?, *40*;	?, ?, ?, *42*;

22 11	15 9	23	24 16 12 8	7	25 10
?, ?, *44*;	?, ?, *45*;	?, *46*;	?, ?, ?, ?, *48*;	?, *49*;	?, ?, *50*;

17	26 13	27 18 9	11	28 14 8	19
?, *51*;	?, ?, *52*;	?, ?, ?, *54*;	?, *55*;	?, ?, ?, *56*;	?, *57*;

29	30 20 15 12 10	31	21 9	32 16 8
?, *58*;	?, ?, ?, ?, ?, *60*;	?, *62*;	?, ?, *63*;	?, ?, ?, *64*;

13	33 22 11	34 17	23	35 14 10
?, *65*;	?, ?, ?, *66*;	?, ?, *68*;	?, *69*;	?, ?, ?, *70*;

36 24 18 12 9	37	25 15	38 19
?, ?, ?, ?, ?, *72*;	?, *74*;	?, ?, *75*;	?, ?, *76*;

11	39 13	40 20 16 10	27 9	41
?, *77*;	?, ?, *78*;	?, ?. ?, ?, *80*;	?, ?, *81*;	?, *82*;

42 28 21 14 12	17	43	29	44 22 11
?, ?, ?, ?, ?, *84*;	?, *85*;	?, *86*;	?, *87*;	?, ?, ?, *88*;

45 30 18 15 10	13	46 23	31	47	19
?, ?, ?, ?, ?, *90*;	?, *91*;	?, ?, *92*;	?, *93*;	?, *94*;	?, *95*;

48 32 24 16 12	49 14	33 11	50 25 20 10
?, ?, ?, ?, ?, *96*;	?, ?, *98*;	?, ?, *99*;	?, ?, ?, ?, *100*.

Thus: 3; 2, 4; 2, 3, 5; . . . .

Declare the quotients rapidly:

7. 144, 48, 96, 36, 120, 72, 24, 132, 84, 108, 60, ÷ *12*.

8. 99, 27, 54, 90, 108, 18, 63, 81, 36, 72, 45, ÷ *9*.

9. 72, 48, 24, 96, 16, 80, 40, 32, 88, 56, 64, ÷ *8*.

10. 70, 14, 49, 63, 77, 56, 28, 84, 21, 42, 35, ÷ *7*.

11. 24, 42, 72, 54, 36, 60, 12, 66, 30, 18, 48, ÷ *6*.

12. 50, 10, 20, 30, 40, 60, 45, 55, 35, 15, 25, ÷ *5*.

13. 44, 28, 20, 48, 12, 32, 40, 36, 16, 24, 8, ÷ *4*.

14. 27, 18, 6, 36, 24, 9, 33, 21, 12, 30, 15, ÷ *3*.

Ex. **7**. 12, 4, 8, 3, 10, ...

Declare the quotients rapidly:

15. 98 ÷ 49, 98 ÷ 14, 96 ÷ 24, 96 ÷ 16, 95 ÷ 19, 94 ÷ 47, 92 ÷ 23, 91 ÷ 13, 90 ÷ 18, 90 ÷ 30, 88 ÷ 22.

16. 87 ÷ 29, 86 ÷ 43, 84 ÷ 28, 84 ÷ 14, 82 ÷ 41, 80 ÷ 16, 78 ÷ 13, 76 ÷ 19, 75 ÷ 15, 74 ÷ 37, 72 ÷ 24, 72 ÷ 36, 72 ÷ 18.

17. 70 ÷ 14, 68 ÷ 17, 66 ÷ 22, 65 ÷ 13, 64 ÷ 16, 63 ÷ 21, 62 ÷ 31, 60 ÷ 15, 58 ÷ 29, 57 ÷ 19, 56 ÷ 14, 54 ÷ 18.

18. 52 ÷ 13, 51 ÷ 17, 48 ÷ 16, 48 ÷ 24, 46 ÷ 23, 45 ÷ 15, 42 ÷ 14, 39 ÷ 13, 38 ÷ 19, 34 ÷ 17, 32 ÷ 16, 28 ÷ 14.

Ex. **15**. 2, 7, 4, 6, ...

Declare the quotients rapidly:

19. 24 : 12, 34 : 17, 48 : 8, 27 : 9, 35 : 7, 39 : 13, 45 : 9, 26 : 13, 42 : 14, 60 : 12, 44 : 11, 33 : 11.

20. 42 : 21, 32 : 16, 42 : 21, 45 : 15, 84 : 12, 84 : 21, 84 : 42, 96 : 12, 36 : 18, 65 : 5, 66 : 11, 68 : 4.

21. 72 : 18, 38 : 19, 30 : 15, 75 : 5, 75 : 15, 90 : 18, 68 : 17, 95 : 19, 51 : 17, 48 : 16, 84 : 6, 98 : 14.

Ex. **19**. 2, 2, 6, 3, 5, ...

The remainder

The dividend is not always the product of the divisor and an integer.

In this case, the largest integral quotient is found, and the remainder, obtained by subtracting the product from the dividend, is left undivided. The remainder is usually written above and the divisor below a horizontal line to show that the division has not been performed. Thus:

In $9 \div 2$, the largest integral quotient is 4, because $2 \times 4 = 8$, and 8 from 9 leaves 1, a remainder smaller than the divisor. Therefore $9 \div 2 = 4$ with a remainder, 1, which is left undivided; or, $9 \div 2 = 4\frac{1}{2}$; read, $9 \div 2 = 4$ and $1 \div 2$, or 4 and one half.

Find the value of:

22. $19 \div 3$, $17 \div 4$, $20 \div 3$, $21 \div 2$, $11 \div 3$, $17 \div 2$, $26 \div 5$, $29 \div 3$, $33 \div 4$, $27 \div 6$, $43 \div 7$, $31 \div 8$, $15 \div 7$, $23 \div 7$.

23. $16 \div 6$, $27 \div 4$, $37 \div 4$, $29 \div 3$, $39 \div 2$, $38 \div 3$, $19 \div 8$, $43 \div 8$, $26 \div 3$, $35 \div 6$, $37 \div 3$, $29 \div 7$, $41 \div 5$, $27 \div 5$.

24. $22 \div 5$, $38 \div 8$, $46 \div 5$, $36 \div 7$, $48 \div 7$, $41 \div 6$, $49 \div 6$, $33 \div 7$, $42 \div 5$, $36 \div 7$, $51 \div 5$, $47 \div 9$, $53 \div 7$, $42 \div 9$.

25. $21 \div 6$, $26 \div 7$, $34 \div 4$, $36 \div 5$, $49 \div 3$, $51 \div 2$, $43 \div 3$, $46 \div 7$, $61 \div 3$, $22 \div 7$, $46 \div 3$, $28 \div 8$, $39 \div 7$, $51 \div 7$.

Ex. **22.** 6, 1; 4, 1; 6, 2; 10, 1; 3, 2; . . . .

Find the value of:

26. 119, 17, 111, 113, 14, 117, 86, 118, 90, 109, 91, $\div$ *12.*

27. 65, 97, 45, 110, 78, 56, 32, 75, 23, 85, 98, $\div$ *12.*

28. 107, 68, 73, 95, 25, 59, 64, 28, 35, 102, 80, $\div$ *11.*

29. 20, 50, 40, 70, 100, 60, 30, 90, 85, 63, 79, $\div$ *11.*

30. 89, 58, 26, 67, 39, 75, 16, 32, 48, 83, 60, $\div$ *9.*

31. 15, 34, 55, 70, 80, 20, 42, 64, 86, 82, 75, $\div$ *9.*

Ex. **26.** 9, 11; 1, 5; 9, 3; 9, 5; 1, 2; . . . .

Equal parts

A whole may be separated into equal parts.

When the whole is separated into two equal parts, each part is a *half;* into three equal parts, each part is a *third;* into four equal parts, each part is a *quarter*, or a *fourth;* and so on.

Since $8 = 4 + 4$, one *half* of 8 is 4. This is written $\frac{1}{2}$ of $8 = 4$.
Since $6 = 2 + 2 + 2$, one *third* of 6 is 2. This is written $\frac{1}{3}$ of $6 = 2$.

Dividing by 2 is finding a *half*, dividing by 3 is finding a *third*, dividing by 4 is finding a *fourth*, and so on. Thus:

8 cents ÷ 2 = 4 cents, because 8 cents = 4 cents × 2.
$\frac{1}{2}$ of 8 cents = 4 cents, because 8 cents = 4 cents + 4 cents.

Find the value of one part:

32. When 12¢ is separated into 4 equal parts; 18¢ into 3 equal parts.

33. When 20¢ is separated into 5 equal parts; 28¢ into 4 equal parts.

34. When 60¢ is separated into 6 equal parts; 72¢ into 8 equal parts.

35. When 45¢ is separated into 9 equal parts; 50¢ into 5 equal parts.

36. When 35¢ is separated into 7 equal parts; 45¢ into 9 equal parts.

Ex. **32.** 3¢. 3¢ + 3¢ + 3¢ + 3¢ = 12¢.

Find the value of:

37. $\frac{1}{2}$ of \$6; $\frac{1}{3}$ of \$12.

38. $\frac{1}{4}$ of \$20; $\frac{1}{5}$ of \$30.

39. $\frac{1}{6}$ of \$12; $\frac{1}{7}$ of \$56.

40. $\frac{1}{5}$ of \$45; $\frac{1}{8}$ of \$72.

41. $\frac{1}{9}$ of \$81; $\frac{1}{4}$ of \$56.

42. $\frac{1}{8}$ of 96¢; $\frac{1}{9}$ of 72¢.

43. $\frac{1}{4}$ of 80¢; $\frac{1}{11}$ of 66¢.

44. $\frac{1}{5}$ of 60¢; $\frac{1}{13}$ of 91¢.

45. $\frac{1}{7}$ of 84¢; $\frac{1}{8}$ of 64¢.

46. $\frac{1}{9}$ of 90¢; $\frac{1}{8}$ of 56¢.

Ex. **37.** \$3. $\frac{1}{2}$ of \$6 = \$6 ÷ 2, or \$3.

NOTE. — The examples on pp. 47 and 48 should be reviewed as follows: Ex. 7, p. 47. $\frac{1}{12}$ of 144 is 12; of 48, 4; . . . . Ex. 26, p. 48. $\frac{1}{12}$ of 119 is $9\frac{11}{12}$; . . . .

Times contained

A whole may contain a part an exact number of times.

A whole contains its half two times; its third, three times; its fourth, four times; and so on. 12 cents contains 6 cents 2 times; 4 cents, 3 times; and so on. Thus:

$8 = 4 + 4$; or $8 = \frac{1}{2}$ of $8 + \frac{1}{2}$ of 8.
$6 = 2 + 2 + 2$; or $6 = \frac{1}{3}$ of $6 + \frac{1}{3}$ of $6 + \frac{1}{3}$ of 6.

Dividing a number of cents by 4 cents is finding how many times the number contains 4 cents; dividing a number of eggs by 5 eggs is finding how many times the number contains 5 eggs; and so on. Thus:

8 cents ÷ 4 cents = 2, because 8 cents = 4 cents × 2.
8 cents contains 4 cents 2 times, because 8 cents = 4 cents × 2.

How many times does:

47. 26 days contain 13 days? 24 days contain 12 days?

48. 20 hours contain 2 hours? 16 hours contain 4 hours?

49. 18 gallons contain 2 gallons? 96 gallons contain 6 gallons?

50. 25 quarts contain 5 quarts? 60 quarts contain 4 quarts?

Ex. **47**. 2 times. 26 days = 13 days × 2.

How many times is:

51. 2 pints contained in 18 pints? 3 pints in 12 pints?

52. 4 pecks contained in 20 pecks? 5 pecks in 30 pecks?

53. 3 pounds contained in 15 pounds? 6 pounds in 72 pounds?

54. 5 ounces contained in 30 ounces? 7 ounces in 42 ounces?

Ex. **51**. 9 times. 18 pints = 2 pints × 9.

NOTE. — The examples on pp. 47 and 48 should be reviewed as follows: Ex. 7, p. 47. 144 contains 12, 12 times; . . . . Ex. 26, p. 48. 119 contains 12, 9 times with 11 remaining.

SHORT DIVISION

55. Divide 8609 by 12 and explain.

$$12\overline{)8609}$$
$$717\tfrac{5}{12}$$

IN FULL. 86 hundreds ÷ 12 = 7 hundreds and 2 hundreds remaining; we write 7 in hundreds' column.

2 hundreds and 0 tens = 20 tens; 20 tens ÷ 12 = 1 ten and 8 tens remaining; we write 1 in tens' column.

8 tens and 9 units = 89 units; 89 units ÷ 12 = 7 units and 5 units remaining; we write 7 in units' column and 12 under 5, with a line between, to show that 5 is still to be divided by 12.

ABBREVIATED. 7, 1, 7, $\frac{5}{12}$.

NOTE.—It is a great loss of time to say, "86 divided by 12 is 7 with 2 remaining." While these words are being said, the process is delayed.

To prove, multiply the divisor by the quotient and add the remainder. The result should be the dividend. *See p. 79.*

Divide and prove, explaining in full:

56.	57.	58.	59.	60.
2)864	3)981	9)648	8)1238	12)9754
61.	**62.**	**63.**	**64.**	**65.**
9)369	11)858	7)504	6)3255	4)2367

Divide and prove, abbreviating:

66.	67.	68.	69.	70.
9)495	6)558	8)696	7)3728	5)4163
71.	**72.**	**73.**	**74.**	**75.**
12)864	9)718	7)623	8)3001	12)7000
76.	**77.**	**78.**	**79.**	**80.**
11)979	5)425	4)916	3)7070	2)5007

Ex. 66. *Ans.* 55. Say 5, 5.

For mental and written work

For mental practice, name each figure of the quotient without writing it, and without stating the full answer after it is obtained.

81. 12)567024	**82.** 9)3063205	**83.** 11)30670508	**84.** 9)1023456789
85. 11)781605	**86.** 7)3643036	**87.** 9)20345607	**88.** 8)3321456648
89. 9)612036	**90.** 5)3245321	**91.** 8)72867408	**92.** 7)1111111111
93. 8)921608	**94.** 6)3215750	**95.** 7)30000005	**96.** 6)7340962416
97. 7)333333	**98.** 3)2222667	**99.** 6)55555554	**100.** 5)9765432120
101. 6)100308	**102.** 6)5307943	**103.** 5)98765043	**104.** 4)9998887772
105. 5)666665	**106.** 4)5202415	**107.** 4)12345672	**108.** 3)9012506370
109. 4)999904	**110.** 9)1396735	**111.** 3)12345672	**112.** 2)9012506370
113. 3)444444	**114.** 7)5803214	**115.** 2)56708914	**116.** 12)9876954312
117. 2)973514	**118.** 8)5905309	**119.** 12)63178908	**120.** 11)5443322344

Ex. **81.** For mental work, say 4, 7, 2, 5, 2.

LONG DIVISION

121. Divide 8609 by 12 and explain.

$$\begin{array}{r} 717\frac{5}{12} \\ 12)\overline{8609} \\ \underline{84} \\ 20 \\ \underline{12} \\ 89 \\ \underline{84} \\ 5 \end{array}$$

86 hundreds ÷ 12 = 7 hundreds and 2 hundreds remaining; we write 7 in hundreds' column.

2 hundreds and 0 tens = 20 tens; 20 tens ÷ 12 = 1 ten and 8 tens remaining; we write 1 in tens' column.

8 tens and 9 units = 89 units; 89 units ÷ 12 = 7 units and 5 units remaining; we write 7 in units' column and 12 under 5 with a line between, to show that 5 is still to be divided by 12.

122. Divide 34056 by 17, prove, and explain.

$$\begin{array}{r} 2003\frac{5}{17} \\ 17)\overline{34056} \\ \underline{34} \\ 0056 \\ \underline{51} \\ 5 \end{array}$$

PROOF.

$$\begin{array}{r} 2003\frac{5}{17} \\ \underline{17}\phantom{\frac{5}{17}} \\ 5\phantom{\frac{5}{17}} \\ 14021\phantom{\frac{5}{17}} \\ \underline{2003}\phantom{0\frac{5}{17}} \\ 34056\phantom{\frac{5}{17}} \end{array}$$

34 thousands ÷ 17 = 2 thousands and 0 thousands remaining; we write 2 in thousands' column.

0 hundreds ÷ 17 = 0 hundreds; we write 0 in hundreds' column.

5 tens ÷ 17 = 0 tens and 5 tens remaining; we write 0 in tens' column.

5 tens and 6 units = 56 units; 56 units ÷ 17 = 3 units and 5 units remaining; we write 3 in units' column and 17 under 5 with a line between, to show that 5 is still to be divided by 17.

PROOF. Multiplying the divisor by the quotient and adding the remainder, we obtain the dividend.

Divide, prove, explain:

123. 7649 ÷ 21; 4179 ÷ 22.

124. 3174 ÷ 23; 8754 ÷ 24.

125. 2800 ÷ 13; 6200 ÷ 13.

126. 9963 ÷ 14; 3528 ÷ 14.

127. 8072 ÷ 15; 6123 ÷ 15.

128. 30000 ÷ 16; 5875 ÷ 16.

129. 85038 ÷ 17; 6283 ÷ 17.

130. 72004 ÷ 18; 5796 ÷ 18.

131. 38794 ÷ 19; 4986 ÷ 19.

132. 79498 ÷ 25; 3899 ÷ 23.

133. Divide 896 by 112.

```
     8
  ----
112)896
    896
    ---
```

To find the quotient figure when the first figure of the dividend is larger than the first figure of the divisor, we use only the first figure of each term.

Approximately, $896 \div 112 = 8 \div 1$, or 8. $8 \times 112 = 896$.

134. Divide 2992 by 374.

```
      8
   ----
374)2992
    2992
    ----
```

To find the quotient figure when the first figure of the dividend is smaller than the first figure of the divisor, we use only the first two figures of the dividend and the first figure of the divisor.

Approximately, $2992 \div 374 = 29 \div 3$, or 9. $9 \times 374 = 3366$. Since 9 is too large, we try 8.

135. Divide 3808 by 476.

```
      8
   ----
476)3808
    3808
    ----
```

To determine whether the quotient figure is too large, it is rarely necessary to use more than the first two figures of the divisor.

Approximately, $3808 \div 476 = 38 \div 4$, or 9. $9 \times 47 = 423$. Since 9 is too large, we try 8.

Perform the indicated operation:

136. $730 \div 365$.	**141.** $8300 \div 426$.	**146.** $16748 \div 4187$.
137. $980 \div 245$.	**142.** $8502 \div 321$.	**147.** $10380 \div 2076$.
138. $845 \div 169$.	**143.** $7321 \div 375$.	**148.** $47472 \div 5934$.
139. $984 \div 123$.	**144.** $6325 \div 523$.	**149.** $35315 \div 7063$.
140. $981 \div 109$.	**145.** $8053 \div 437$.	**150.** $61263 \div 6807$.

151. Divide 78264648 by 98076.

```
98076)78264648(798
      686532
       961144
       882684
        784608
        784608
```

$78 \div 9 = 8$, $8 \times 98 = 784$. Since 8 is too large, we try 7.

Since the product of 98076 by 7 must contain six figures, we place 2, the right-hand figure of the product, under the sixth figure of the dividend.

NOTE. — It is sometimes more convenient to place the quotient at the right.

152. Divide 7384 by 100.

$$100)\underline{7384} \quad 73\tfrac{84}{100}$$

If the divisor is 10, we cut off one figure from the right of the dividend; if 100, two figures; and so on.

153. Divide 3218738 by 92000.

```
92.000×)3218.738×(34 90738/92000
        276
         458
         368
          90
```

If the divisor ends with ciphers, we cut off the ciphers, and the same number of figures from the right of the divisor.

We divide the parts left and prefix the remainder to the part cut off, to find the true remainder.

NOTE. — A cross may be placed after units, and a point before the last figure cut off.

154. Prove the answer of the last example.

```
  34 90738/92000
  92000
  90738
  68
 306
 3218738
```

Multiplying the quotient by the divisor and adding the remainder, we obtain the dividend.

Find the value of:

155. $1468953 \div 47963$.

156. $9547964 \div 54328$.

157. $8197385 \div 38900$.

158. $4178967 \div 46000$.

Perform the indicated operation:

159. $105056 \div 34.$

160. $126499 \div 29.$

161. $172929 \div 59.$

162. $209576 \div 68.$

163. $381951 \div 93.$

164. $195600 \div 76.$

165. $336777 \div 87.$

166. $1344455 \div 35.$

167. $1332678 \div 45.$

168. $1570688 \div 64.$

169. $3334932 \div 73.$

170. $2546650 \div 62.$

171. $7920792 \div 88.$

172. $6141408 \div 76.$

Perform the indicated operation:

173. $476204 \div 298.$

174. $978383 \div 487.$

175. $776984 \div 964.$

176. $328372 \div 878.$

177. $375624 \div 888.$

178. $389961 \div 117.$

179. $904321 \div 229.$

180. $3032575 \div 97825.$

181. $8964200 \div 44821.$

182. $9777680 \div 44444.$

183. $7642539 \div 78788.$

184. $9916620 \div 94444.$

185. $2561733 \div 10809.$

186. $3549780 \div 78884.$

Perform the indicated operation:

187. $468797 \div 1000.$

188. $775936 \div 2000.$

189. $589476 \div 2200.$

190. $987654 \div 4600.$

191. $234869 \div 9100.$

192. $489763 \div 8000.$

193. $940849 \div 7000.$

194. $9285564 \div 29292.$

195. $8609250 \div 68866.$

196. $8940008 \div 41000.$

197. $9486534 \div 19900.$

198. $4394988 \div 14000.$

199. $4333333 \div 34000.$

200. $2340899 \div 20000.$

PROBLEMS

Fourth and fifth forms of analysis

201. If 48 oranges cost 96¢, how much will 1 orange cost?

```
      2
  48)96
     96
```

Since 48 oranges cost 96¢, 1 orange will cost $\frac{1}{48}$ of 96¢, or 2¢.

PROOF. *48 × 2¢ = 96¢.*

202. At 2¢ each, how many oranges can be bought for 96¢.

```
  2)96
    48
```

Since 1 orange costs 2¢, as many oranges can be bought for 96¢, as 2¢ is contained times in 96¢, or 48 oranges.

PROOF. *2¢ × 48 = 96¢.*

To write what each term represents is of service.

203. If 17 acres of land cost $204, how much will 1 acre cost?

```
   $ 12, cost 1 acre
 17)204, cost all
    17
     34
     34
```

Since 17 acres cost $204, 1 acre will cost $\frac{1}{17}$ of $204, or $12.

PROOF. *17 × $12 = $204.*

A drawing is often of service.

204. Three villages are in a straight line; B is 3 times as far west as A is east of Denver; C is twice as far west of B as B is west of Denver; from B to C is 18 miles. How far is it from A to C?


```
18, B to C
 9, B to D
 3, D to A
30, A to C
```

Since 2 times BD = 18 miles, BD = 18 miles ÷ 2, or 9 miles. Since 3 times DA = 9 miles, DA = 9 miles ÷ 3, or 3 miles. Distance $AC = DA + BD + BC$, or 3 miles + 9 miles + 18 miles, or 30 miles.

205. If the cost of constructing 459 miles of railway is $596,700, what is the cost per mile?

206. If 1,500,000 people occupy a territory of 25,000 square miles, what is the average population on each square mile?

207. If 324 acres of land produce 20,736 bushels of corn, what is the average yield per acre?

208. The salary of the President of the United States is $50,000. How much is that a day, counting 365 days to the year?

209. My front fence is 5 rods long and cost $130. How much did it cost per rod?

210. Six men owning a mine, sold it to 11 others for $33,000. How much did each of the original owners receive, and how much did each of the new owners pay?

211. If a man pays $16 rent per month, in how many years of 12 months each will he pay $1152 rent?

212. In how many days will a cooper make 1356 barrels, if he makes 12 barrels each day?

213. A miller packed 26,950 pounds of flour into sacks containing 49 pounds each. How many sacks did he fill?

214. There are 3 feet in a yard and 1760 yards in a mile. How many miles are there in 63,360 feet?

215. Into how many farms of 160 acres each can 900 acres of land be divided, and how many acres will remain?

216. After dividing 900 acres of land into farms of 160 acres each, a man sold what was left for $200. How much did he receive per acre?

217. How many pages are there in a book containing 87,024 words, if each page contains 37 lines, and each line contains 14 words?

218. Mr. Gray leases a house for $27 a month. If the expenses are $8 a month, in how many months will he gain $6916 from the house?

219. A man had \$386, which, lacking \$5, was 17 times as much as he had 10 years ago. How much was he worth 10 years ago?

220. The area of Kansas is 82,080 square miles; of Rhode Island, 1250 square miles. The area of Kansas is approximately how many times that of Rhode Island?

221. If I subtract the product of 375 and 25 from the product of 675 and 39, and divide the remainder by 75, what is the quotient?

222. A man sells 2 houses at \$3315 apiece, and with the proceeds buys land at \$65 an acre. How many acres does he buy?

223. A merchant paid \$817 for 19 stoves, and afterwards sold them for \$51 each. How much did the selling price of all exceed the cost?

224. How many tons of hay at \$13 a ton, the cost of shipping being \$2 a ton, can a man buy in exchange for 16 horses at \$153 a head, the cost of delivery being \$3 for each horse, if he pays for both shipping and delivery?

225. A merchant receives as much for 65 pounds of butter as for 15 yards of cloth at 78 cents a yard. What is the price of the butter per pound?

226. A boy makes a journey of 16 hours by rail, traveling 32 miles an hour; he returns on a bicycle at the rate of 8 miles an hour. How long is he in returning?

227. John and James run a race of 720 feet; John runs 16 feet in a second and beats James by 3 seconds. How many feet does James run in a second?

OPERATIONS COMBINED

USE OF THE SIGNS

The whole is equal to the sum of all its parts; or the whole, diminished by all its parts, becomes nothing.

The signs '+' and '−' perform the double office of connecting parts and denoting operations. Whatever is included between the signs '+' or '−' is a *term*. The terms to the left of the sign '=' form the *left-hand member;* the terms to the right, the *right-hand member;* both members, an *equation*.

ILLUSTRATIONS

$20 = 10 + 6 + 4$,
$20 - 10 - 6 - 4 = 0$,
are *equations.*

In the first,
20, *left member;*
10 + 6 + 4, *right member.*
In the second,
20 − 10 − 6 − 4, *left;*
0, *right member.*
In both,
20, 10, 6, 4, *terms.*

A simple term may be reduced to a compound term by expressing it as a sum, a difference, a product, or a quotient.

The *simple term*, 10, equals any one of the *compound terms*, $(8 + 2)$, $(12 - 2)$, 5×2, or $30 \div 3$.

When a compound term is a sum or a difference, its components are connected by the signs '+' or '−', and must therefore be bound together by a parenthesis, by brackets, or by a vinculum.

$6 - 2 + 5$, as three *simple terms;* or $(6-2+5)$, as one *compound term.*

When a compound term is a product or a quotient, its components are not connected by the signs '+' or '−', and a parenthesis is not needed.

5×2, $30 \div 3$;
not
(5×2), $(30 \div 3)$.

In a compound term, if '÷' is followed by '÷' or '×', a parenthesis must be used to avoid a double meaning.

$24 \div 6 \times 2$ would equal
$(24 \div 6) \times 2$,
or $24 \div (6 \times 2)$.

A parenthesis indicates that what it embraces is to be regarded as a single quantity.

In simplifying expressions, compound terms must first be reduced to simple terms.

Read, explain, and reduce to simple terms:

1. $6 \div 3,\ 3 \times 2,\ (6 + 2)$.

2. $(6 - 2),\ (8 \div 2) \div 2,\ 8 \div (2 \div 2)$.

3. $12 \div (4 + 2),\ 12 \div (4 - 2)$.

4. $3 \times (8 + 6),\ 3 \times (8 - 6),\ 3(8 - 6)$.

5. $(8 + 6)(8 - 6),\ (8 + 6) \div (8 - 6)$.

6. $(8 + 6) \div (5 + 2),\ 6 \times (8 \div 2)$.

7. $(8 - 5 + 7) \div 2,\ (8 - 2) \times 3$.

8. $24 \div (6 \div 2),\ 24 \div (6 \times 2)$.

Ex. **2**. $(8 \div 2) \div 2$, the expression, $8 \div 2$, divided by 2; it means divide the quotient of 8 and 2, by 2; its value is 2. $8 \div (2 \div 2)$, 8 divided by the expression, $2 \div 2$; it means divide 8 by the quotient of $2 \div 2$; its value is 8.

Ex. **4**. $3(8 - 6)$, 3 times the expression, $8 - 6$; the sign '×' is usually omitted before a parenthesis; $3(8 - 6)$ means 3 times the difference between 8 and 6; its value is 6.

Write as an equation:

9. That 3 times the sum of 8 and 6, multiplied by the difference between 12 and 9, is equal to 126.

10. That 50, diminished by the product of 6 and 3, is equal to 64 divided by the quotient of 8 and 4.

11. That the difference between 8 and 5, increased by 7, is equal to the product of 3 and 8, diminished by the quotient of 28 and 2.

Find the value of:

12. $9 + 16 \div 8 - 6 \div 2$.

13. $8 - 25 \div 5 + 4 \times 2$.

14. $7 + 6 \times 5 - 8 \div 4$.

15. $9 - (6 - 2) + 2(8 - 5)$.

16. $2 + (9 - 2) \div (10 - 3)$.

17. $9 + (6 + 2) - 2(8 - 5)$.

18. $10 + 12 \div (4 + 2) - 16 \div 2$.

19. $10 - 12 \div (4 - 2) + 16 \times 2$.

20. $9 - 8 \div 2 + (7 - 5 + 4) \div 2$.

21. $10 - 2 \times 3 + 5 \times 6 - 8 \div 2$.

22. $3(9 - 6 + 5) - 2(8 - 5 + 7)$.

23. $(8 + 7 - 5)(8 - 7 + 5) - 2$.

Ex. **12**. *Ans.* 8. Reducing compound terms to simple terms, $9 + 2 - 3$; uniting, 8. Ex. **22**. *Ans.* 4. To simple terms, $24 - 20$; uniting, 4.

ANALYSIS

In solving a problem by analysis, there are three steps:

The meanings and the relations of the given and the required terms must be discovered.

Each relation must be expressed by an equation in such a way that the required term shall form a single member and shall not be subjected to addition, subtraction, multiplication, or division.

The operations suggested in the relations must be performed.

Terms and relations

In most problems, the given and the required terms may be recognized and understood at once, and the relations may be ascertained from experience or from a knowledge of general truths.

State the given terms, the required terms, and the relations:

24. After losing 5¢ a boy had 4¢ left. How much had he at first?

Given terms: amount lost, 5¢; amount left, 4¢. *Required term:* amount at first. *Relation:* amount at first = amount lost + amount left.

25. After losing a certain sum a boy had 4¢ left. If he had 9¢ at first, how much did he lose?

Given terms: amount left, 4¢; amount at first, 9¢. *Required term:* amount lost. *Relation:* amount lost = amount at first − amount left.

26. At 4¢ each, how much will 2 apples cost?

Given terms: cost 1 apple, 4¢; number apples, 2. *Required term:* cost 2 apples. *Relation:* cost 2 apples = 2 × cost 1 apple.

27. If 2 apples cost 8¢, how much will 1 apple cost?

Given terms: number apples, 2; cost all apples, 8¢. *Required term:* cost 1 apple. *Relation:* cost 1 apple = $\frac{1}{2}$ cost all apples.

28. At 4¢ each, how many apples can be bought for 8¢?

Given terms: cost 1 apple, 4¢; cost all apples, 8¢. *Required term:* number apples. *Relation:* number apples = number times cost of all contains cost of 1.

In some problems, there are more relations than one.

29. State the relations. If 6 apples cost 12¢, how much will 5 apples cost?

Relations: cost 1 apple = $\frac{1}{6}$ of 12¢; cost 5 apples = 5 times cost 1 apple.

In some problems, relations must be ascertained from a knowledge of how objects are constructed, or from a knowledge of the sciences.

30. State the principles of construction, the required term, and the relation. How many minute spaces has the hour hand of a watch passed since 5 o'clock when the minute hand is at 3?

Principles: there are 60 minute spaces on a dial; the minute hand passes 60 spaces while the hour hand passes 5 spaces; etc. *Given term:* minute hand has passed 15 minute spaces since 5. *Required term:* number minute spaces passed by hour hand since 5. *Relation:* number spaces passed by hour hand = $\frac{1}{12}$ number spaces passed by minute hand.

In some problems, the relations must be ascertained from a knowledge of business usage.

The gain equals the selling price minus the cost; the loss equals the cost minus the selling price.

A person (principal) may give pay (commission) to another (agent) for buying or selling articles for him. Then:

The entire cost equals the buying price plus the commission; the proceeds equal the selling price minus the commission.

31. State the relation. If an orange sells for 6¢ at a gain of 2¢, how much is the cost?

Relation: cost = selling price − gain.

32. State the relation. If an agent buys an article for $50 and charges a commission of $1, how much does it cost the principal?

33. State the relation. If an agent sells an article for $50 and charges a commission of $1, how much does the principal receive?

Write the relations; do not solve the problems.

34. In jumping, A beats B by 2 feet. If A jumps 10 feet, how far does B jump?

35. In jumping, A beats B by 2 feet. If B jumps 10 feet, how far does A jump?

36. If 3 men can do a piece of work in 6 days, in how many days can 1 man do it?

37. If 1 man can do a piece of work in 18 days, in how many days can 3 men do it?

38. With his present force, a contractor can do a piece of work in 18 days. By what number must he multiply his force to finish the contract in 2 days?

39. If it takes a man 10 minutes to saw a log into 3 pieces, how long will it take him to saw it into 4 pieces?

40. When the hands of a watch are opposite to each other, how many minute spaces are there between them?

41. Three boys bought a top for 10¢; the first gave 2¢, and the second, 4¢. How much did the third give?

42. A and B travel in the same direction, A at the rate of 5 miles per hour, and B at the rate of 7 miles per hour. How many miles does B gain in 9 hours?

43. Conditions as in Ex. 42, if A has a start of 4 hours, in how many hours will B overtake him?

44. If they start from the same place and at the same time and travel in opposite directions, in how many hours will they be 36 miles apart?

45. By selling a watch for $90, a man would gain $20; at what price must he sell it to gain $25?

46. If $4.20 is paid for 3 days' work, how much will be paid for 10 days' work?

The process as a whole

The relation will often suggest a better method than the set forms of analysis. *See pages 26, 33, 42, 57.*

47. If 3 apples cost 6¢, how much will 12 apples cost?

SET FORM. — Since 3 apples cost 6¢, 1 apple will cost $\frac{1}{3}$ of 6¢ or 2¢; since 1 apple costs 2¢, 12 apples will cost 12 times 2¢, or 24¢.

BETTER FORM. — The cost of 12 apples is 4 times the cost of 3 apples, or 4 times 6¢, or 24¢.

Good judgment should be used in selecting the forms.

48. What is the profit on buying 6 cows at $26 each and selling them at $28 each?

GOOD JUDGMENT. — The profit on 1 cow is the difference between $28 and $26, or $2; the profit on 6 cows is 6 times $2, or $12.

POOR JUDGMENT. — Since 1 cow costs $26, 6 cows will cost 6 times $26, or $156; since 1 cow sells for $28, 6 cows will sell for 6 times $28, or $168; the profit is the difference, or $12.

In written work, write what each term represents.

49. Through an agent, I sell 8 horses at $58 each, commission $1 per head, and buy with the proceeds cows at $26 each, commission $1 per head. How much should the agent remit?

$58, sell. p. 1 horse
1, commission
57, proceeds 1 horse
8
456, entire proceeds

$26, cost 1 cow
1, commission
27, entire cost 1 cow
27) 456 (16$\frac{24}{27}$
24, amount to remit

Be sure to prove every answer.

50. Prove in Ex. 49, that the agent should remit $24.

$27, cost 1 cow
16
432, entire cost cows
24, amount remitted
456, amount to invest in cows

Since the entire cost of the cows plus the amount remitted, is the proceeds from the sale of the horses, the answer is correct.

51. In an election, Mr. Jones received 3689 votes, but was defeated by 216 votes. How many votes did his opponent receive?

52. In an election, Mr. Brown received 3905 votes and defeated his opponent by 216 votes. How many votes did his opponent receive?

53. I bought a horse for $15,284, paying $2684 cash and the balance in monthly payments of $1575 each. How many monthly payments did I make?

54. A farm house is worth $2450; the farm is worth 12 times as much, less $600; and the stock is worth twice as much as the house. How much are the house, stock, and farm worth?

55. In still water, a crew can row 10 miles per hour; the current runs 2 miles per hour. How many miles can they row down stream in 1 hour? State the relation.

56. How many miles can they row up stream in 1 hour? State the relation.

57. In how many hours can they row 48 miles down stream and return? State the relations.

58. A crew can row down stream 12 miles per hour and up stream 8 miles per hour. What is the rate of the current? State the relations.

59. In how many hours could the crew in example 58 row 30 miles in still water? State the relations.

60. How much will 5 barrels of potatoes cost if 13 barrels of apples cost $39, and 6 barrels of apples cost as much as 9 barrels of potatoes? State the relations.

61. If 5 horses eat 14 bushels of oats in 2 weeks, how long would it take them at the same rate to eat 56 bushels?

62. E owes a debt of $365. How many sheep must he sell at $15, commission $1 each, to discharge the debt? How much money will he have left?

63. A man owed $2896; he paid $499 at one time, and all but $375 a second time. How much did he pay the second time?

64. If 37 horses cost $1295, how much will 48 horses cost? State the relations.

65. How much will 126 barrels of beans cost if 9 barrels cost $22? State the relation.

66. If an orchard is sold for $375 at a loss of $28, what was the cost? State the relation.

67. If a stock of goods costing $4376 is sold at a gain of $1094, what is the selling price? State the relation.

68. B sells a house through an agent and receives $1975. If the agent's commission is $97, what is the selling price?

69. If a hound runs 78 rods while a hare runs 64 rods, how far will the hound run while the hare runs 1856 rods? State the relations.

70. Suppose a body falls 16 feet the first second, 48 feet the next, 80 feet the next, and so on, constantly increasing, how far will it fall in 5 seconds? State the relations.

71. Three men can do a piece of work in 5 days. In what time can 1 man and 8 boys do it, if 1 man does the work of 2 boys? State the relations.

72. Conditions as in Ex. 71, how many boys would be required to do the work in one day? State the relations.

73. A has 7 loaves of bread; B, 5; C, none. The three eat all of the bread, each the same amount. C pays to A and B 12¢. How much should each receive?

74. A liveryman makes an annual profit of $125 from each horse; his income each year is $2125; his horses cost $87 per head. How much did he pay for the horses? State the relations.

FACTORING

TERMS

$$8 = 4 \times 2.$$

If both multiplicand and multiplier are wanting, this becomes

8 = what × what?

It means, "what are the numbers whose product is 8?"

Factoring is the process of finding numbers whose product is given. The numbers required are *factors* or *measures;* the product, a *multiple.*

What are the factors of 30?

Ans. 2 and *15*, *3* and *10*, *5* and *6*, or *30* and *1*; $30 = 2 \times 15$, 3×10, 5×6, or 30×1.

Every integer is the product of itself and one.

If a number has no set of integral factors besides itself and one, it is a *prime number.* If it has another set besides itself and one, it is a *composite number.*

7, *a prime number;* 30, *a composite number.*

Numbers are *prime to each other* when their greatest common factor is *one.*

4, 8, 9, are *prime to each other.*

Numbers are *severally prime* when each is prime to each of the others.

4, 9, 25, are *severally prime.*

FROM THE COMBINATIONS

If the combinations in multiplication are known, the factors of all numbers less than 100 may be called rapidly. *See pp. 36, 46.*

State sets of two factors for:

1. 99, 98, 96, 95, 94, 93, 92, 91, 90, 88, 87, 86, 85, 84, 82, 81, 80, 78, 77, 76, 75, 74, 72, 70, 69, 68, 66, 65, 64, 63, 62, 60, 58, 57, 56, 55, 54, 52, 51, 50, 49, 48, 46, 45, 44, 42, 40.

Ex. 1. *Ans.* $96 = 2 \times 48$, 3×32, 4×24, 6×16, 8×12; . . . .

BY INSPECTION

Whether one of the factors of a number is 2, 3, 4, 5, 8, 9, 11, or a product of two or more factors severally prime, may be found by the following principles. *See p. 80.*

	Illustrations
A number is divisible by 2, when the number denoted by its last digit is divisible by 2, or is 0.	2772*6* is divisible by 2, because *6* is divisible by 2.
A number is divisible by 5, when the number denoted by its last digit is divisible by 5, or is 0.	2872*5* is divisible by 5, because *5* is divisible by 5.
A number is divisible by 4, when the number denoted by its last two digits is divisible by 4.	277*12* is divisible by 4, because *12* is divisible by 4.
A number is divisible by 8, when the number denoted by its last three digits is divisible by 8.	27*816* is divisible by 8, because *816* is divisible by 8.
A number is divisible by 3, when the sum of its digits is divisible by 3.	27810 is divisible by 3, because 18, the sum of its digits, is divisible by 3.
A number is divisible by 9, when the sum of its digits is divisible by 9.	27810 is divisible by 9, because 18, the sum of its digits, is divisible by 9.
A number is divisible by 11, when the difference between the sum of its digits in the odd places and the sum of its digits in the even places, is divisible by 11, or is 0.	1639 is divisible by 11, because 11, the difference between 15, the sum of its digits in the odd places, and 4, the sum of its digits in the even places, is divisible by 11.
A number is divisible by the product of any number of its factors which are severally prime to each other.	27720 is divisible by *7*, *8*, *9*, and is therefore divisible by $7 \times 8 \times 9$.

Of the numbers 2, 3, 4, 5, 8, 9, 11, use those which are severally prime, and form combinations:

2. Of 2 and one other.

3. Of 3 and one other.

4. Of 4 and one other.

5. Of 5 and one other.

6. Of 8 and one other.

Ex. **2.** 2×3, 2×5, 2×9, 2×11.

7. Of 9 and one other.

8. Of 2, 3, and one other.

9. Of 2, 5, and one other.

10. Of 5, 8, and one other.

11. Of 5, 8, 9, and one other.

Ex. **8.** $2 \times 3 \times 5$, $2 \times 3 \times 11$.

State why 27720 is divisible:

12. By 2; 3; 4; 5.

13. By 7; 8; 9; 11.

14. By 2×3; 2×5; 2×9.

15. By 2×11; 3×4; 3×5.

Ex. **13.** By 7, by trial.

16. By 3×8; 3×11; 4×5.

17. By 4×9; 4×11; 5×9.

18. By $5 \times 8 \times 9$; $8 \times 9 \times 11$.

19. By 3×7; $5 \times 8 \times 9 \times 11$.

Ex. **16.** 3 and 8 are severally prime.

By inspection tell why.

20. 2448 is divisible by 72.

21. 6930 is divisible by 55.

22. 4788 is divisible by 63.

23. 8184 is divisible by 88.

24. 20934 is divisible by 18.

25. 14630 is divisible by 77.

26. 30144 is divisible by 24.

27. 98000 is divisible by 35.

Ex. **20.** 2448 is divisible by 8 and 9, which are severally prime, and therefore by 8×9, or 72.

Find all the factors of:

28. 12540, less than 100.

29. 27720, less than 100.

30. 17622, less than 100.

31. 26585, less than 100.

Ex. **28.** 2, 3, 4, 5, 11; 6, 10, 22, 12, 15, 33, 20, 44, 55; 30, 66 . . . .

NOTE.—It is best to test for 2, 3, 4, 5, 7, 8, 9, 11; and then to combine those factors which are severally prime.

Finding prime numbers

Whether an integer is a prime number is found by trial. In the trial, it is necessary actually to divide only by 7 and by prime numbers greater than 11; divisibility by 2, 3, 5, 11, and by all composite numbers may be tested by inspection.

32. Is 397 a prime number?

Ans. Yes. 397 is not divisible by 2, 3, 4, 5, 6, *7*, 8, 9, 10, 11, 12, *13*, 14, 15, 16, *17*, 18, *19*, nor 20. $397 \div 20 = 19 +$. It is not divisible by a number larger than 20, for the quotient would then be a number that has already been tried.

NOTE.—In this example, to test divisibility by 7, 13, 17, and 19, the division must be performed.

33. Make a list of all the prime numbers to 100. How many are there?

34. Is 431 a prime number? 323? 131? 523? 601? 319?

35. Find the prime number next greater than 401.

Expressing by factors

36. Express 2772 by factors.

$$\begin{array}{r} 12\overline{)2772} \\ 11\overline{)231} \\ 21 \end{array}$$

$2772 = 12 \times 11 \times 21.$

37. Express by prime factors.

$2772 = 12 \times 11 \times 21.$

In prime factors,

$$2772 = 2 \times 2 \times 3 \times 11 \times 3 \times 7$$
$$= 2^2 \times 3^2 \times 11 \times 7.$$

NOTE. — A number written over a factor shows how many times the factor is used.

Express by factors:

38. 72; 630; 2808.

39. 56; 836; 7425.

40. 48; 840; 1232.

41. 45; 945; 5929.

Express by prime factors:

42. 120; 220; 5616.

43. 108; 144; 1445.

44. 156; 160; 7392.

45. 135; 288; 3025.

NOTE. — To express by prime factors, it is best to reduce to factors as most convenient, and then to reduce all composite to prime factors. Do not form the habit of always dividing by 2 and by 3; try 12, 11, 9, 8, 7, 5, 4, 3, 2, in order.

THE FOUR OPERATIONS

It is often convenient to add or subtract numbers each of which is expressed by two factors. We add or subtract the factors not common and retain the common factor.

46. Add 16×7 and 22×7.

$$\begin{array}{r} 16 \times 7 \\ 22 \times 7 \\ \hline 38 \times 7 \end{array}$$

16×7 is 16 *sevens*; 22×7 is 22 *sevens*. The sum is 38 *sevens*, or 38×7.

47. From 38×7 subtract 16×7.

$$\begin{array}{r} 38 \times 7 \\ 16 \times 7 \\ \hline 22 \times 7 \end{array}$$

38×7 is 38 *sevens*; 16×7 is 16 *sevens*. The difference is 22 *sevens*, or 22×7.

It is often convenient to multiply or divide numbers expressed by factors. We use each factor of the multiplier or the divisor, together with only one factor of the other term.

48. Multiply 2×3 by 5.

$$\begin{array}{r} 2 \times 3 \\ 5 \\ \hline 10 \times 3 \end{array} \qquad \begin{array}{r} 3 \times 2 \\ 5 \\ \hline 15 \times 2 \end{array}$$

2×3 is 2 *threes*, or 3 *twos*; 5 times 2 *threes* is 10 *threes*, or 10×3.
5×3 *twos* is 15 *twos*, or 15×2.

49. Multiply $2 \times 3 \times 5$ by 4×6.

$$\begin{array}{r} 2 \times \ 3 \times \ 5 \\ 4 \times \ 6 \\ \hline 2 \times 12 \times 30 \end{array}$$

$(2 \times 3 \times 5) \times (4 \times 6)$
$= 2 \times (3 \times 4) \times (5 \times 6)$, or
$2 \times 12 \times 30$.

50. Divide 4×6 by 2.

$$\frac{\overset{2}{\not{4}} \times 6}{\not{2}} = 2 \times 6$$

4 *sixes* $\div 2 = 2$ *sixes*; or 6 *fours* $\div 2 = 3$ *fours*.

51. Divide $2 \times 12 \times 30$ by 4×6.

$$\frac{2 \times \overset{3}{\not{12}} \times \overset{5}{\not{30}}}{\not{4} \times \not{6}} = 2 \times 3 \times 5$$

$12 \div 4 = 3$; $30 \div 6 = 5$; the result is $2 \times 3 \times 5$.

NOTE.—This method is called cancellation. *See pp. 90, 92.*

Written work

Add:

52. 89×15, 16×15, 13×15.

53. 7×12, 8×12, 9×12.

Subtract:

54. 75×15 from 89×15.

55. 16×17 from 73×17.

Multiply:

56. $2 \times 3 \times 4$ by 6; by 8.

57. $8 \times 9 \times 7$ by 3; by 7.

Divide:

58. $12 \times 15 \times 21$ by 3; by 6.

59. $21 \times 35 \times 42$ by 7; by 3.

Express the quotients by factors:

60. $(18 \times 9 \times 16) \div (2 \times 3 \times 3)$.

61. $(16 \times 18 \times 12) \div (4 \times 9 \times 2)$.

62. $(72 \times 24 \times 36) \div (18 \times 3 \times 3)$.

63. $(75 \times 18 \times 21) \div (35 \times 9 \times 5)$.

64. $(56 \times 49 \times 63) \div (14 \times 7 \times 9)$.

65. $(48 \times 96 \times 98) \div (16 \times 24 \times 49)$.

66. $(36 \times 24 \times 90) \div (18 \times 18 \times 12)$.

67. $(18 \times 12 \times 30) \div (5 \times 6 \times 3)$.

68. $(94 \times 78 \times 65) \div (47 \times 13 \times 39)$.

69. $(57 \times 91 \times 77) \div (19 \times 13 \times 11)$.

Mental work

Divide, declaring the results by factors:

70. $85 \times 6 \times 7$, by 17.

71. $95 \times 8 \times 3$, by 24.

72. $72 \times 8 \times 7$, by 56.

73. $90 \times 3 \times 7$, by 54.

74. $75 \times 4 \times 8$, by 50.

75. $18 \times 8 \times 4$, by 36.

76. $16 \times 7 \times 8$, by 64.

77. $19 \times 9 \times 4$, by 38.

78. $17 \times 19 \times 18$, by 34.

79. $23 \times 12 \times 15$, by 69.

80. $26 \times 27 \times 28$, by 63.

81. $96 \times 35 \times 17$, by 48.

82. $80 \times 9 \times 11$, by 88.

83. $62 \times 8 \times 63$, by 93.

84. $22 \times 3 \times 33$, by 66.

85. $18 \times 4 \times 62$, by 93.

Ex. **73**. *Ans.* 5×7. The factors of 54 are 18 and 3; $90 \div 18 = 5$; $3 \div 3 = 1$.

NOTE. — For practical illustrations of Exs. 70 to 85, *see Notes, pp. 88, 89.*

GREATEST COMMON DIVISOR

The greatest common divisor of numbers less than 100 may be known from the combinations. *See pp. 36 and 46.*

Find, from the combinations, the G. C. D. of:

86. 40, 96.	**89.** 30, 48.	**92.** 48, 80.
87. 24, 56.	**90.** 64, 72.	**93.** 46, 94.
88. 35, 91.	**91.** 56, 60.	**94.** 49, 77.

Ex. **86**. 8. $40 \div 8 = 5$; $96 \div 8 = 12$; 5 and 12 are prime to each other.

Smaller numbers may be found which have the same G. C. D.

I. *The G. C. D. of two numbers is the G. C. D of the smaller, and of the remainder found by dividing the greater by the smaller.*

95. In Exs. 86 to 94, find smaller numbers with same G. C. D.

Ex. **86**. *Ans.* 40, 16. Since $96 = 40 + 40 + 16$, the G. C. D. of 40 and 96 is the G. C. D. of 40, and $40 + 40 + 16$, or of 40 and 16.

96. Making use of the first expedient, solve examples 86 to 94.

Ex. **86**. *Ans.* 8. The G. C. D. of 40 and 96 is the G. C. D. of 40 and 16, or 8.

II. *One of several numbers may be divided by a factor prime to any other, without affecting the G. C. D.*

97. In Exs. 86 to 94, find smaller numbers with same G. C. D.

Ex. **86**. *Ans.* 8, 96. Since 40 contains 5, which is prime to 96, 5 may be canceled from 40 without affecting the G. C. D.

98. Making use of the second expedient, solve examples 86 to 94.

Ex. **86**. *Ans.* 8. The G. C. D. of 40 and 96 is the G. C. D. of 8 and 96, or 8.

Find the G. C. D of:

99. 36, 40, 48, 72.	**102.** 55, 77, 88, 99.
100. 49, 56, 70, 77.	**103.** 45, 75, 90, 60.
101. 60, 84, 96, 72.	**104.** 42, 56, 70, 77.

III. *The G. C. D. of two or more numbers is the product of all the common factors that may be used as successive divisors until the quotients are prime to each other.*

Find the G. C. D of:

105. 288, 432, 720.
106. 495, 660, 990.
107. 475, 855, 760.
108. 1728, 1296, 1872.
109. 945, 1260, 2625.
110. 1680, 4200, 5040.
111. 1875, 3750, 5000.
112. 2850, 3800, 4750.

Ex. **105**. By expedient III.

$$\begin{array}{r|rrr} 12 & 288 & 432 & 720 \\ \hline 12 & 24 & 36 & 60 \\ \hline & 2 & 3 & 5 \end{array}$$

Since 12 and 12 are component factors of the G. C. D., and the quotients are prime to each other, 12×12, or 144, is the G. C. D.

NOTE. — In finding common factors by inspection, it is best to try 12, 11, 9, 8, 7, 5, 4, 3, 2, in order.

If no common factor is seen by inspection, smaller numbers should be found which have the same G. C. D. *See p. 74.*

Find the G. C. D. of:

113. 153, 374.
114. 625, 1728.
115. 1144, 1365.
116. 1177, 2675.
117. 143, 1755, 2912.
118. 495, 1452, 9317.
119. 1152, 1728, 3375.
120. 875, 448, 567.

Ex. **113**. By expedient I.

```
153)374(2
    306
     68)153(2
        136
         17
```

The G. C. D. of 153 and 374 is the G. C. D. of 153 and 68, or the G. C. D. of 68 and 17, or 17.

Ex. **113**. By expedient II.

```
153  374
 51
 17
```

The G. C. D. of 153 and 374 is the G. C. D. of 51 and 374, or the G. C. D. of 17 and 374, or 17.

LEAST COMMON MULTIPLE

Multiples of small numbers are easily found from the combinations. *See pp. 36, 46.*

I. *The least common multiple of two or more numbers is the product of all their prime factors, each taken the greatest number of times it is found in any one of them.*

121. Write all the multiples of 2, to 36; all the multiples of 3, to 36.

122. Make a list of all the multiples of 2, less than 36, that exactly contain 3; of all the multiples of 3, less than 36, that exactly contain 2.

123. What is the least common multiple of 2 and 3? Is it exactly contained in each of the common multiples? Why?

124. Answer examples 121 and 122, with reference to 4 and 6, instead of 2 and 3.

125. What is the least common multiple of 4 and 6? Is it exactly contained in each of the common multiples? Why?

126. Why is 2×3 the least common multiple of 2 and 3, while 4×6 is not the least common multiple of 4 and 6?

Ans. 2×3 is the L. C. M. of 2 and 3 because 2 and 3 are prime to each other; 4×6 is not the L. C. M. of 4 and 6 because 4 and 6 are not prime.

Find, from the combinations, the L. C. M. of:

127. 10, 18.	**131.** 16, 20.	**135.** 21, 14.
128. 14, 12.	**132.** 12, 16.	**136.** 12, 18.
129. 10, 12.	**133.** 24, 16.	**137.** 24, 32.
130. 10, 14.	**134.** 25, 20.	**138.** 27, 18.

139. Taking the product of prime factors, solve Exs. 127 to 138.

Ex. **136.**

$$12 = 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$
$$2 \times 2 \times 3 \times 3 = 36, \text{ L. C. M.}$$

The L. C. M. must contain 12, or $2 \times 2 \times 3$, and we retain these factors. The L. C. M. must contain 18, or $2 \times 3 \times 3$. We already have 2 and one of the 3's; we retain the other. Then $2 \times 2 \times 3 \times 3$ is the L. C. M.

The product of the prime factors may be found more easily by the following principles:

II. *To find the L. C. M of two numbers, divide one of them by their G. C. D and multiply the quotient by the other.*

III. *To find the L. C. M. of more than two numbers, find the L. C. M of two of them, then of the result and a third, and so on.*

IV. *If one of the numbers exactly contains another, the smaller may be neglected.*

140. In Exs. 127 to 138, find the L. C. M. by principle II.

Ex. **136.**

12, 18

$12 \times 3 = 36$, *L. C. M.*

The G. C. D. of 12 and 18 is 6; $18 \div 6$ is 3; 12×3 is 36, L. C. M.

As before, the L. C. M. must contain 12, or $2 \times 2 \times 3$. Instead of retaining these factors, we retain 12 itself. The L. C. M. must contain 18, or $2 \times 3 \times 3$. In 12, we already have 2 *once* and 3 *once*, or 6, the G. C. D. of 12 and 18, and we simply retain the other 3, or $18 \div$ G. C. D.

NOTE. — By retaining 12, instead of $2 \times 2 \times 3$, we are saved the effort of first separating 12 into its factors, and later of multiplying them together.

Find mentally the L. C. M. of:

141. 60, 72.	**145.** 16, 20.	**149.** 21, 45.
142. 25, 30.	**146.** 20, 45.	**150.** 16, 30.
143. 24, 27.	**147.** 24, 36.	**151.** 24, 30.
144. 49, 63.	**148.** 42, 56.	**152.** 60, 75.

Ex. **141.** 60×6, or 360. The G. C. D. of 60 and 72 is 12; $72 \div 12 = 6$; $60 \times 6 = 360$, L. C. M.

By these principles, find mentally the L. C. M. of:

153. 2, 3, 4, 5, 6, 7.	**155.** 10, 12, 15, 6, 5, 4.
154. 5, 8, 9, 6, 12, 10.	**156.** 18, 8, 9, 36, 24, 12.

Ex. **153.** *Ans.* 420. We neglect 2 and 3 because they are contained in 6. The L. C. M. of 4 and 5 is 20; of 20 and 6, 60; of 60 and 7, 420.

Another method of finding the L.C.M. is to divide by prime divisors that are common to any two of the numbers.

157. Find the L.C.M. of 63, 108, 28, 42.

3	*63*	*108*	*28*	*42*
3	*21*	*36*	*28*	*14*
2	*7*	*12*	*28*	*14*
2	*7*	*6*	*14*	*7*
7	*7*	*3*	*7*	*7*
	1	*3*	*1*	*1*

$3^3 \times 2^2 \times 7 = 756$, *L. C. M.*

Since 3 is contained in 63 and 108, 3 is used *once.* Since 3 is contained in 21 and 36, 3 is used a *second* time. Since 2 is contained in 12 and 28, 2 is used *once.* Since 2 is contained in 6 and 14, 2 is used a *second* time. Since 7 is contained in 7 and 7, 7 is used *once.* Since 3 is among the quotients, 3 is used a *third time.*

If no two numbers appear to have a common factor, it is necessary to find their G.C.D.

158. Find the L.C.M. of 592, 703, 171.

37	*592*	*703*	*171*
19	*16*	*19*	*171*
	16	*1*	*9*

$37 \times 19 \times 16 \times 9 = 101{,}232$, *L. C. M.*

The G.C.D. of 592 and 703 is 37. The G.C.D. of 19 and 171 is 19.

159. Solve Ex. 157 again. Divide by 4; then select such divisors as you please. Why is the answer, 1512, incorrect?

Ans. Because the greatest number of times 2 is used in any number is *twice*, viz., in 108 and 28. Dividing by 4, took 2 *twice* out of 108 and 28, but left 2 *once* in 42, making *three* times that it appears by this process.

NOTE.—It is unsafe to divide by other than prime numbers.

Find the L.C.M. of:

160. 30, 32, 45, 48, 64, 75.

161. 125, 245, 147, 225.

162. 36, 72, 128, 126, 243.

163. 169, 221, 204.

164. 266, 285, 209.

165. 708, 1062, 1475.

Casting out 9's

Addition, subtraction, multiplication, and division may be proved by casting out 9's.

In addition, the sum of the excesses of 9's in the several addends, should equal the excess of 9's in the sum.

5678	8
4934	2
3627	0
14,239	1

We add the digits of the first addend, 5, 11, 18, 26; then the digits of the sum, 26, and write 8. This is the excess of the 9's, for 5678 ÷ 9 gives 8 for a remainder. We proceed in the same way with the other addends: 4, 13, 16, 20; 2: 3, 9, 11, 18; 9; 0 (a sum of 9, or any multiple of 9, we count as 0). We proceed in the same way with the excesses, 8, 10; 1.

We proceed in the same way with the sum, 1, 5, 7, 10, 19; 10; 1.

The sum of the excesses of 9's in the several addends is 1; the excess of 9's in the sum is 1; the answer is probably correct.

In subtraction, the excess of 9's in the minuend, should equal the excess of 9's in the subtrahend and the remainder.

7683	6
3997	1
3686	5

Minuend, 7, 13, 21, 24; 6.

Subtrahend, 3, 12, 21, 28; 10; 1: remainder, 3, 9, 17, 23; 5: their sum, 6.

In multiplication, the excess of 9's in the product of the excesses of the factors, should equal the excess in the answer.

968	5
468	0
453,024	0

Multiplicand, 9, 15, 23; 5: multiplier, 4, 10, 18; 9; 0: their product, 0.

Product, 4, 9, 12, 14, 18; 9; 0.

In division, the excess of 9's in the product of the excesses of the quotient and divisor, plus the excess in the remainder, should equal the excess in the dividend.

488)472,871(968 $\frac{487}{488}$

5	1
2	1
10	2

Quotient, 9, 15, 23; 5: divisor, 4, 12, 20; 2: their product, 10; 1: the remainder, 4, 12, 19; 10; 1; their sum, 2.

Dividend, 4, 11, 13, 21, 28, 29; 11; 2.

RELATIONS

166. When 13,825, or 13,820 + 5, is divided by 2, why is the remainder the same as when its last digit is divided by 2? Give the rule for the divisibility of a number by 2.

The first part, 13,820, or 1382 *tens*, is divisible by 2 because *ten* is divisible by 2. Since the first part is divisible by 2, the divisibility of the number depends upon the last digit.

167. When 13,825, or 13,800 + 25, is divided by 4, why is the remainder the same as when the number denoted by its last two digits is divided by 4? Give the rule.

168. When 13,825, or 13,000 + 825, is divided by 8, why is the remainder the same as when the number denoted by its last three digits is divided by 8? Give the rule.

169. What is the remainder when 1 with any number of ciphers is divided by 9?

170. What is the remainder when 2, 3, 4, 5, 6, 7, or 8, times 1 with any number of ciphers, is divided by 9?

171. When 13,825, or 10,000 + 3000 + 800 + 20 + 5, is divided by 9, why is the remainder the same as when the sum of its digits is divided by 9?

When 10,000 is divided by 9 the remainder is 1; when 3000 is divided by 9 the remainder is 3; etc. *See Ex. 170.*

172. If a number divided by 9 gives the same remainder as the sum of its digits divided by 9, what is the rule for the divisibility of a number by 9?

173. Show that 1 with any odd number of ciphers lacks 1 of being a multiple of 11.

174. Show that 1 with any even number of ciphers exceeds by 1 a multiple of 11.

175. How much does 2, 3, 4, 5, 6, 7, 8, 9, times 1 with any odd number of ciphers, lack of being a multiple of 11?

176. How much does 2, 3, 4, 5, 6, 7, 8, 9, times 1 with any even number of ciphers, exceed a multiple of 11?

177. When 75,316, or (70,000+300+6)+(5000+10), is divided by 11, why is the remainder the same as when the sum of the digits in the odd places minus the sum of the digits in the even places, is divided by 11?

70,000+300+6 exceeds a multiple of 11 by 7+3+6 (Ex. 176); 5000+10 lacks 5 + 1 of being a multiple of 11 (Ex. 175); 75,316 exceeds a multiple of 11 by (7 + 3 + 6)−(5 + 1).

178. If a number divided by 11, gives the same remainder as the difference between the sum of its digits in the odd places and the sum of its digits in the even places, divided by 11, what is the rule for the divisibility of a number by 11?

179. A contractor is to build houses 24, 36, 48, and 60 feet long, and 16, 32, 32, and 48 feet wide. What length of clapboard can be used most conveniently for the sides? for the ends? State the relations.

Relation: number of feet in length of clapboard for the sides = the largest number that is exactly contained in 24, 36, 48, and 60, or their G.C.D. . . .

180. A lady wishes to buy a piece of cloth which she can cut without waste into an exact number of pieces either 3, 4, or 5 yards long, as she may decide later. What is the smallest number of yards the piece can contain?

181. A real estate agent wishes to divide 3 pieces of land 325, 675, and 950 feet wide, into town lots of equal width. What is the largest possible width for each lot?

182. Ropes 48, 52, and 56 feet long are to be cut into the longest possible equal lengths. How long must each piece be?

183. A, B, and C start together around a circular track. A goes once around in 6 minutes; B, in 8 minutes; C, in 9 minutes. What is the least number of minutes before they will be together again at the starting point? State terms and relations.

Given terms: number of minutes passed when A is at the starting point is 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, . . . ; B, 8, 16, 24, 32, 40, 48, 56, 64, 72, . . . ; C, 9, 18, 27, 36, 45, 54, 63, 72. . . .

Relation: number of minutes before they are again together = least number that will exactly contain 6, 8, and 9, or their L. C. M.

184. After how many minutes will A and B first be together at the starting point? A and C? B and C?

185. D, E, and F start together around a circular track 5280 feet in length. D rides 1760 feet per minute; E, 1320; and F, 1056. How many times must D ride around the track before they are all together again at the starting point? State the relations.

Relations: number of minutes D goes once around = 5280 ÷ 1760; number minutes B = 5280 ÷ 1320; number minutes C = 5280 ÷ 1056. The minutes when they are first together = the L. C. M. of the minutes each makes the circuit; number times A goes around = L. C. M. ÷ number minutes A makes the circuit.

186. By counting eggs, 4, 6, or 10 at a time, a farmer had none left over in each case. What is the least number he could have had? State the relations.

187. By counting eggs 4, 6, or 10 at a time, a farmer had 3 left over in each case. What is the least number he could have had? State the relation.

188. By counting eggs 4, 6, or 10 at a time, a farmer had 3 eggs left over in each case; counting 11 at a time, he had none left. What is the least number he could have had? State the relations.

Relations: possible numbers = 3 + common multiples of 4, 6, and 10; the least number = the least of these results divisible by 11.

Solution: the L. C. M. of 4, 6, and 10 is 60; 63, 123, 183, 243, 303, 363, . . . are numbers representing in order 3 + common multiples of 4, 6, and 10; 363 is the least of these which contains 11.

COMMON FRACTIONS

FIRST CONCEPTION

AN EXPRESSION OF DIVISION

Division may be expressed by writing the dividend above, and the divisor below, a horizontal line. Such an expression is a *common fraction;* the dividend is the *numerator;* the divisor, the *denominator.*

The numerator, or the denominator, or both, may contain fractions; such an expression is a *complex fraction.*

We sometimes speak of a fraction of a fraction; a *compound fraction.*

An integer plus a fraction is a *mixed number.* The plus sign is usually omitted.

State the terms, and the meaning of the fractions : $\frac{3}{4}$; $\frac{\frac{2}{3}}{\frac{3}{4}}$.

ILLUSTRATIONS

$\frac{4}{5}$, *common fraction.*

4, numerator.

5, denominator.

Read, $4 \div 5$.

$\frac{\frac{2}{3}}{4}$, $\frac{4}{\frac{5}{6}}$, $\frac{\frac{7}{8}}{\frac{8}{9}}$ *complex fractions.*

$\frac{2}{3}$ of $\frac{5}{6}$, *a compound fraction.*

$5 + \frac{2}{3}$, or $5\frac{2}{3}$, *a mixed number.*

$\frac{3}{4}$; 3 is the numerator; 4, the denominator; it means $3 \div 4$.

$\frac{2}{3}$ is the numerator; $\frac{3}{4}$, the denominator; it means $\frac{2}{3} \div \frac{3}{4}$.

NOTE. — The pupil should read the first part of p. 45, the whole of p. 49, and the explanation of Ex. 55 on p. 51.

EQUAL PARTS OF A UNIT

A unit may be divided into two or more equal parts, and one or more of these parts may be considered.

The number showing into how many parts the unit is divided, is written below a horizontal line, and is the *denominator.*

The number showing how many parts are considered, is written above the line, and is the *numerator.*

The whole expression is a *common fraction.*

According to this conception, is $\frac{8}{5}$ a fraction? No. It is called an *improper fraction, i.e.,* not properly a fraction.

According to this conception, is $\frac{\frac{2}{3}}{\frac{3}{4}}$ a fraction? No. It is called a *complex fraction.*

Illustrations

AB is divided into 8 equal parts; AC contains 5 of them; AC = 5 eighths of AB; expressed, AC = $\frac{5}{8}$ of AB.

$\frac{5}{8}$, *common fraction.*

8, denominator.

5, numerator.

It means that a unit is divided into 8 equal parts, and that 5 of these parts are considered.

$\frac{1}{2}$, read, *one half;* $\frac{3}{4}$, read, *three quarters,* or *three fourths.*

It is impossible to divide a unit into 5 equal parts and then consider 8 of them.

It is impossible to divide a unit into $\frac{3}{4}$ equal parts.

Define by each conception:

1. A common fraction.
2. The numerator.
3. The denominator.
4. A complex fraction.
5. A compound fraction.
6. A mixed number.

NOTE. — Sometimes deductions are made from the first conception; and sometimes, from the second. For illustrations of the first, *see pp. 87 and 110;* of the second, *pp. 101, 102.*

CHANGE OF FORM

To lower terms

Dividing both numerator and denominator by the same number does not change the value of a fraction.

$AB = \frac{4}{6}$, or $\frac{2}{3}$, of AC;

$\therefore \frac{4}{6} = \frac{2}{3}$.

$\frac{2}{3}$ is obtained from $\frac{4}{6}$ by dividing both terms by 2.

By this principle, we reduce fractions to their simplest forms.

Which fraction is the more readily comprehended, $\frac{38}{57}$ or $\frac{2}{3}$? Why?

$\frac{2}{3}$, because the terms are smaller.

7. Reduce $\frac{144}{216}$ to lowest terms.

$$\frac{144}{216} = \frac{12}{18} = \frac{2}{3}.$$

By inspection, we find common factors. Dividing both terms by 12, $\frac{144}{216} = \frac{12}{18}$; by 6, $\frac{12}{18} = \frac{2}{3}$.

8. Reduce $\frac{5567}{6739}$ to lowest terms.

$$\frac{5567}{6739} = \frac{19}{23}.$$

When no common factor is found by inspection, we find the G. C. D.; here it is 293.

Reduce to lowest terms:

9. $\frac{12}{18}$; $\frac{9}{27}$.
10. $\frac{8}{16}$; $\frac{10}{20}$.
11. $\frac{19}{38}$; $\frac{11}{44}$.
12. $\frac{14}{42}$; $\frac{18}{54}$.
13. $\frac{20}{60}$; $\frac{40}{68}$.
14. $\frac{14}{16}$; $\frac{24}{93}$.
15. $\frac{24}{28}$; $\frac{17}{51}$.
16. $\frac{21}{42}$; $\frac{15}{45}$.
17. $\frac{11}{22}$; $\frac{26}{39}$.
18. $\frac{14}{28}$; $\frac{42}{70}$.
19. $\frac{30}{60}$; $\frac{45}{60}$.
20. $\frac{12}{84}$; $\frac{28}{84}$.

Reduce to lowest terms:

21. $\frac{4879}{6601}$; $\frac{625}{5625}$.
22. $\frac{945}{2520}$; $\frac{1210}{1573}$.
23. $\frac{4752}{5544}$; $\frac{6912}{8640}$.
24. $\frac{1843}{2425}$; $\frac{323}{342}$.
25. $\frac{2461}{4173}$; $\frac{213}{324}$.
26. $\frac{2436}{3654}$; $\frac{389}{491}$.
27. $\frac{13464}{15048}$; $\frac{1125}{2550}$.
28. $\frac{3411}{1469}$; $\frac{9659}{21547}$.
29. $\frac{2772}{36036}$; $\frac{33264}{44352}$.

To higher terms

Multiplying both numerator and denominator by the same number does not change the value of a fraction.

By this principle, we prepare fractions for addition and subtraction.

$AB = \frac{2}{3}$, or $\frac{4}{6}$, of AC;

$\therefore \frac{2}{3} = \frac{4}{6}$.

$\frac{4}{6}$ is obtained from $\frac{2}{3}$ by multiplying both terms by 2.

30. Change $\frac{2}{3}$ to 12ths.

$$\frac{2}{3} = \frac{8}{12}.$$

To make the denominator 12, we must multiply 3 by 4; multiplying both terms by 4, $\frac{2}{3} = \frac{8}{12}$.

Change:

31. $\frac{5}{6}$ to 12ths.
32. $\frac{7}{8}$ to 40ths.
33. $\frac{5}{7}$ to 35ths.
34. $\frac{3}{8}$ to 48ths.
35. $\frac{7}{12}$ to 24ths.
36. $\frac{5}{13}$ to 39ths.
37. $\frac{7}{11}$ to 77ths.
38. $\frac{5}{14}$ to 28ths.
39. $\frac{117}{193}$ to 1351sts.
40. $\frac{185}{273}$ to 6825ths.
41. $\frac{3}{119}$ to 2856ths.
42. $\frac{87}{253}$ to 2277ths.

43. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to equivalent fractions having L. C. D.

$$\frac{8}{12},\ \frac{9}{12},\ \frac{10}{12}.$$

The L. C. D. is 12; $12 \div 3 = 4$; we multiply both terms of $\frac{2}{3}$ by 4; $12 \div 4 = 3$; we multiply both terms of $\frac{3}{4}$ by 3; $12 \div 6 = 2$; we multiply both terms of $\frac{5}{6}$ by 2.

Reduce to equivalent fractions having L. C. D:

44. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.
45. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$.
46. $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{30}$.
47. $\frac{5}{7}$, $\frac{3}{14}$, $\frac{8}{21}$.
48. $\frac{2}{3}$, $\frac{5}{39}$, $\frac{3}{52}$.
49. $\frac{5}{9}$, $\frac{11}{18}$, $\frac{7}{36}$.
50. $\frac{1}{8}$, $\frac{5}{9}$, $\frac{7}{24}$.
51. $\frac{8}{9}$, $\frac{7}{8}$, $\frac{5}{72}$.
52. $\frac{4}{5}$, $\frac{7}{50}$, $\frac{11}{75}$.
53. $\frac{3}{7}$, $\frac{8}{77}$, $\frac{5}{11}$.
54. $\frac{3}{4}$, $\frac{19}{29}$, $\frac{105}{116}$.
55. $\frac{1}{2}$, $\frac{16}{17}$, $\frac{153}{119}$.
56. $\frac{5}{7}$, $\frac{19}{21}$, $\frac{307}{924}$.
57. $\frac{5}{6}$, $\frac{13}{57}$, $\frac{153}{741}$.
58. $\frac{3}{8}$, $\frac{27}{68}$, $\frac{213}{493}$.

To whole or mixed numbers

A fraction is an expression of division. It means that the numerator is to be divided by the denominator.

By this principle, we change to a form that can be more readily comprehended.

$\frac{8}{3} = 8 \div 3 = 2\frac{2}{3}$.

Or, since there are 3 thirds in 1, in 8 thirds there are as many 1's as 3 is contained times in 8, or $2\frac{2}{3}$.

59. Reduce $\frac{45}{13}$ to a whole or mixed number.

$$\frac{45}{13} = 3\frac{6}{13}.$$

$\frac{45}{13} = 45 \div 13$. Performing the operation, we obtain $3\frac{6}{13}$.

Reduce to a whole or mixed number:

60. $\frac{96}{7}$; $\frac{84}{9}$.	**63.** $\frac{45}{13}$; $\frac{52}{18}$.	**66.** $\frac{1451}{68}$.	**69.** $\frac{15467}{89}$.
61. $\frac{39}{14}$; $\frac{56}{13}$.	**64.** $\frac{64}{14}$; $\frac{86}{17}$.	**67.** $\frac{2897}{45}$.	**70.** $\frac{21671}{142}$.
62. $\frac{95}{19}$; $\frac{84}{18}$.	**65.** $\frac{90}{17}$; $\frac{45}{11}$.	**68.** $\frac{3698}{67}$.	**71.** $\frac{31692}{392}$.

Whole or mixed numbers to common fractions

A mixed number is an integer plus a fraction. If the integer is reduced to an equivalent fraction having the denominator of the fraction, the two parts may be united.

By this principle, we prepare fractions for multiplication and division.

$$2\frac{2}{3} = 2 + \frac{2}{3} = \frac{6}{3} + \frac{2}{3} = \frac{8}{3}.$$

Or, since there are 3 thirds in 1, in 2 there are 2 times 3 thirds, or 6 thirds; with 2 thirds, 8 thirds.

72. Reduce $4\frac{5}{17}$ to an improper fraction.

$$4\frac{5}{17} = \frac{73}{17}.$$

$4\frac{5}{17} = \frac{68}{17} + \frac{5}{17} = \frac{73}{17}$.

Reduce to an improper fraction:

73. $5\frac{1}{3}$; $4\frac{5}{7}$.	**75.** $5\frac{11}{13}$; $6\frac{9}{11}$.	**77.** $35\frac{18}{19}$; $74\frac{7}{11}$.
74. $7\frac{1}{2}$; $8\frac{3}{4}$.	**76.** $3\frac{11}{16}$; $4\frac{3}{17}$.	**78.** $87\frac{13}{14}$; $86\frac{15}{16}$.

ADDITION

Before fractions can be added, they must be reduced to equivalent fractions having a common denominator.

The least common denominator should always be found.

$$\frac{2}{3} = \frac{8}{12} \qquad \frac{3}{4} = \frac{9}{12}$$

79. Find the sum of $\frac{2}{3}$ and $\frac{3}{4}$.

BEGUN

$\frac{2}{3}$	8
$\frac{3}{4}$	9
	$\frac{\ }{12}$

$\frac{2}{3} = \frac{8}{12}$; $\frac{3}{4} = \frac{9}{12}$; the sum of the 12ths is 17 twelfths, or 1 unit and 5 twelfths; we write $\frac{5}{12}$ in fractions' column, and carry 1 to units' column.

COMPLETED

$\frac{2}{3}$	8
$\frac{3}{4}$	9
$1\frac{5}{12}$	$\frac{17}{12}$

NOTE.—When the fractions are ready for addition, the work appears as at the left; when finished, as at the right. In practice, it should not be begun in one place and finished in another.

80. Find the sum of $48\frac{78}{95}$, $32\frac{32}{57}$, $76\frac{15}{19}$.

$48\frac{78}{95}$	234	
$32\frac{32}{57}$	160	95×3
$76\frac{15}{19}$	225	
$158\frac{49}{285}$	$\frac{619}{285}$	

$\frac{78}{95} = \frac{234}{285}$; $\frac{32}{57} = \frac{160}{285}$; $\frac{15}{19} = \frac{225}{285}$. The sum of the 285ths is 619 285ths, or 2 units and 49 285ths; we write 49 285ths in fractions' column, and carry 2 to units' column. *See p. 20.*

NOTE.—It is convenient to express the L. C. D. by its factors. Thus: the L. C. D. is 95×3; 95 is contained in this 3 times; 57, or 19×3, is contained 5 times; 19 is contained 5×3, or 15 times. *See p. 73.*

Find the value of:

81. $\frac{5}{8} + \frac{2}{3}$; $\frac{5}{16} + \frac{3}{4} + \frac{7}{8}$.

82. $\frac{3}{4} + \frac{5}{6}$; $\frac{9}{10} + \frac{8}{9} + \frac{3}{5}$.

83. $\frac{5}{9} + \frac{4}{7}$; $\frac{11}{12} + \frac{3}{8} + \frac{4}{9}$.

84. $98\frac{1}{4} + 25\frac{2}{9} + 5\frac{1}{2}$.

85. $65\frac{2}{5} + 91\frac{5}{6} + 8\frac{4}{9}$.

86. $27\frac{7}{9} + 18\frac{3}{8} + 7\frac{5}{6}$.

SUBTRACTION

Before fractions can be subtracted, they must be reduced to equivalent fractions having a common denominator.

The least common denominator should always be found.

$$\frac{3}{4}=\frac{9}{12}$$
$$\frac{2}{3}=\frac{8}{12}$$

87. From $\frac{3}{4}$ subtract $\frac{2}{3}$.

BEGUN

$\frac{3}{4}$	9
$\frac{2}{3}$	8
	$\overline{12}$

$\frac{3}{4}=\frac{9}{12}$; $\frac{2}{3}=\frac{8}{12}$; $\frac{8}{12}$ from $\frac{9}{12}$ leaves $\frac{1}{12}$; we write $\frac{1}{12}$ in fractions' column.

COMPLETED

$\frac{3}{4}$	9
$\frac{2}{3}$	8
$\frac{1}{12}$	$\frac{1}{12}$

NOTE. — When the fractions are ready for subtraction, the work appears as at the left; when finished, as at the right. In practice, it should not be begun in one place and finished in another.

88. From $300\frac{2}{51}$ subtract $200\frac{7}{68}$.

	204	
$300\frac{2}{51}$	8	
	212	
$200\frac{7}{68}$	21	51×4
$99\frac{191}{204}$	$\frac{191}{204}$	

$\frac{2}{51}=\frac{8}{204}$; $\frac{7}{68}=\frac{21}{204}$; 21 204ths from 8 204ths we cannot take; we add 204 204ths to 8 204ths making 212 204ths, and 1 to 0 units making 1 unit; 21 204ths from 212 204ths leaves 191 204ths; we write 191 204ths in fractions' column; 201 units from 300 units leaves 99 units. *See p. 30.*

NOTE. — It is convenient to express the L. C. D. by its factors. Thus: the L. C. D. is 51×4; 51 is contained in this 4 times; 68, or 17×4, is contained 3 times. *See p. 73.*

Find the value of:

89. $\frac{4}{5}-\frac{7}{10}$; $\frac{7}{8}-\frac{5}{6}$; $\frac{17}{25}-\frac{4}{7}$.

90. $\frac{5}{6}-\frac{3}{8}$; $\frac{8}{9}-\frac{5}{6}$; $\frac{11}{12}-\frac{3}{4}$.

91. $\frac{7}{9}-\frac{3}{4}$; $\frac{5}{8}-\frac{3}{7}$; $\frac{12}{13}-\frac{7}{8}$.

92. $5\frac{1}{2}-3\frac{2}{3}$; $68\frac{5}{39}-27\frac{14}{65}$.

93. $6\frac{2}{5}-4\frac{7}{8}$; $75\frac{7}{85}-29\frac{23}{34}$.

94. $4\frac{2}{3}-2\frac{5}{6}$; $67\frac{3}{85}-30\frac{23}{75}$.

MULTIPLICATION

Common method

Multiply the numerators for a new numerator and the denominators for a new denominator, canceling when possible.

Mixed numbers should be reduced to improper fractions.

$\frac{3}{4}$ of AB = AC;
$\frac{2}{3}$ of $\frac{3}{4}$ of AB = $\frac{2}{3}$ AC = AD;
$\frac{1}{2}$ of AB = AD.
$\therefore$ $\frac{2}{3}$ of $\frac{3}{4}$ of AB = $\frac{1}{2}$ of AB,
or $\frac{2}{3}$ of $\frac{3}{4} = \frac{1}{2}$.

95. Multiply $\frac{7}{8}$ by 4.

$$\frac{7}{\cancel{8}_{2}} \times \cancel{4} = \frac{7}{2} = 3\tfrac{1}{2}$$

Divide both 4 and 8 by 4; *i.e.*, cancel 4 from both 4 and 8. *See p. 72.*

96. Multiply $\frac{36}{49}$ by $5\frac{1}{4}$ by $2\frac{1}{3}$.

$$\frac{\overset{9}{\cancel{36}}}{\underset{7}{\cancel{49}}} \times \frac{\overset{3}{\cancel{21}}}{\cancel{4}} \times \frac{\cancel{7}}{\cancel{3}} = 9.$$

$5\frac{1}{4} = \frac{21}{4}$; $2\frac{1}{3} = \frac{7}{3}$; cancel 7 from 21 and 49; 7 from 7 and 7; 4 from 36 and 4; 3 from 3 and 3.

Find the value of:

97. $\frac{2}{3} \times 6$; $8 \times \frac{3}{4}$; $9 \times \frac{2}{3}$.

98. $\frac{3}{4} \times \frac{6}{12}$; $7 \times \frac{8}{9}$; $9 \times \frac{7}{18}$.

99. $\frac{7}{8} \times 16$; $18 \times \frac{5}{9}$; $72 \times \frac{3}{4}$.

100. $84 \times \frac{7}{12}$; $\frac{5}{6} \times 96$; $\frac{7}{8} \times 48$.

101. $96 \times \frac{3}{16}$; $\frac{11}{12} \times 84$; $\frac{9}{16} \times 64$.

102. $\frac{5}{6}$ of $\frac{6}{25}$; $\frac{3}{4}$ of $\frac{7}{9}$; $\frac{5}{12}$ of 16.

103. $\frac{4}{5}$ of 25; $\frac{2}{3}$ of 24; $\frac{3}{4}$ of 16.

104. $\frac{2}{3}$ of 18; $\frac{5}{7} \times 91$; $\frac{3}{5} \times 65$.

105. $\frac{5}{7}$ of 21; $\frac{9}{11}$ of 44; $\frac{7}{13}$ of 65.

106. $\frac{2}{5}$ of 20; $\frac{9}{13}$ of 91; $\frac{8}{11}$ of 55.

Find the value of:

107. $\frac{19}{36} \times \frac{18}{57} \times \frac{85}{87} \times \frac{9}{17}$.

108. $5\frac{1}{2} \times 6\frac{2}{3} \times 6\frac{3}{4} \times 8\frac{2}{5}$.

109. $\frac{19}{28} \times \frac{14}{57} \times \frac{15}{16} \times \frac{32}{45}$.

110. $\frac{3}{4} \times \frac{31}{40} \times 1\frac{1}{13} \times \frac{52}{93} \times \frac{20}{21}$.

111. $\frac{132}{143} \times \frac{27}{44} \times \frac{16}{17} \times \frac{13}{27} \times \frac{11}{12}$.

112. $\frac{41}{49} \times \frac{7}{82} \times \frac{15}{31} \times 12\frac{2}{5}$.

113. $15\frac{2}{3} \times 4\frac{4}{9} \times 8\frac{1}{10} \times \frac{1}{94}$.

114. $55\frac{1}{8} \times \frac{16}{77} \times \frac{121}{144} \times \frac{12}{21}$.

115. $\frac{5}{6}$ of $\frac{64}{65}$ of $\frac{9}{10}$ of $\frac{11}{12}$ of $\frac{13}{22}$.

116. $1\frac{1}{20} \times 5\frac{2}{3} \times \frac{9}{19} \times \frac{38}{63} \times 1\frac{3}{17}$.

DIVISION

European method

Divide the numerators for a new numerator and the denominators for a new denominator, changing to equivalent fractions with their least common denominator, if necessary.

Mixed numbers should be reduced to improper fractions.

$$\frac{9}{8} \div \frac{3}{4} = \frac{9\ \textit{8ths}}{6\ \textit{8ths}} = \frac{3}{2}.$$

Dividing the numerators and the denominators gives the same result.

$$\frac{9 \div 3}{8 \div 4} = \frac{3}{2}.$$

117. Divide $\frac{3}{4}$ by $\frac{5}{6}$ mentally.

$$\frac{3}{4} \div \frac{5}{6} = \frac{9}{10}.$$

When $\frac{3}{4}$ is reduced to 12ths, the numerator becomes 9; when $\frac{5}{6}$ is reduced to 12ths, the numerator becomes 10; $9 \div 10 = \frac{9}{10}$. The quotient of the denominators, $12 \div 12$, is 1.

NOTE. — When two fractions have the same denominator, since the quotient of the denominators is always 1, it is necessary to think of the numerators only.

118. Divide $9\frac{1}{8}$ by $10\frac{3}{4}$ mentally.

$$9\frac{1}{8} \div 10\frac{3}{4} = \frac{73}{86}.$$

When reduced to 8ths, the numerator of $9\frac{1}{8}$ becomes 73; of $10\frac{3}{4}$, 86; the answer is $\frac{73}{86}$.

Find the value mentally:

119. $\frac{1}{2} \div \frac{2}{3}$.

120. $\frac{3}{4} \div \frac{7}{8}$.

121. $\frac{7}{8} \div \frac{3}{4}$.

122. $\frac{3}{4} \div \frac{8}{9}$.

123. $\frac{3}{4} \div \frac{9}{8}$.

124. $\frac{7}{8} \div \frac{3}{5}$.

125. $\frac{4}{5} \div \frac{2}{3}$.

126. $\frac{3}{4} \div \frac{2}{3}$.

127. $\frac{5}{6} \div \frac{3}{4}$.

128. $\frac{1}{2} \div \frac{3}{7}$.

129. $\frac{2}{3} \div \frac{4}{5}$.

130. $16 \div \frac{2}{3}$.

131. $18 \div \frac{3}{4}$.

132. $27 \div \frac{3}{5}$.

133. $63 \div \frac{7}{9}$.

134. $20 \div \frac{4}{5}$.

135. $25 \div \frac{5}{7}$.

136. $49 \div \frac{7}{8}$.

137. $84 \div \frac{7}{9}$.

138. $48 \div \frac{3}{5}$.

139. $54 \div \frac{2}{9}$.

140. $77 \div \frac{7}{8}$.

141. $5\frac{1}{2} \div 3\frac{1}{8}$.

142. $2\frac{1}{6} \div 1\frac{1}{2}$.

143. $2\frac{1}{6} \div 6\frac{1}{2}$.

144. $7\frac{3}{5} \div 8\frac{1}{5}$.

145. $7\frac{1}{5} \div 8\frac{3}{4}$.

146. $6\frac{2}{3} \div 8\frac{2}{3}$.

147. $9\frac{5}{7} \div 6\frac{3}{7}$.

148. $4\frac{2}{3} \div 6\frac{1}{6}$.

149. $8\frac{2}{5} \div 6\frac{1}{5}$.

150. $9\frac{3}{5} \div 3\frac{1}{5}$.

151. $7\frac{4}{7} \div 2\frac{1}{7}$.

Common method

$\frac{3}{4} \div \frac{2}{3} = \frac{9}{12} \div \frac{8}{12} = \frac{9}{8}.$

Invert the divisor and proceed as in multiplication.

Inverting the divisor and proceeding as in multiplication gives the same result.

$\frac{3}{4} \times \frac{3}{2} = \frac{9}{8}.$

152. Divide $\frac{7}{8}$ by 4.

$$\frac{7}{8} \times \frac{1}{4} = \frac{7}{32}.$$

153. Divide 4 by $\frac{7}{8}$.

$$4 \times \frac{8}{7} = \frac{32}{7} = 4\frac{4}{7}.$$

154. Divide $\frac{7}{8}$ by $\frac{8}{9}$.

$$\frac{7}{8} \times \frac{9}{8} = \frac{63}{64}.$$

155. Divide $16\frac{2}{3}$ by $3\frac{3}{4}$.

$$\frac{\overset{10}{\not{50}}}{3} \times \frac{4}{\underset{3}{\not{15}}} = \frac{40}{9} = 4\frac{4}{9}.$$

Divide, inverting the divisor:

156. $\frac{16}{25}$ by 18.

157. $\frac{12}{19}$ by 16.

158. $\frac{7}{45}$ by 21.

159. 36 by $\frac{9}{11}$.

160. 25 by $\frac{5}{6}$.

161. 48 by $\frac{16}{20}$.

162. $\frac{21}{80}$ by $\frac{4}{5}$.

163. $\frac{7}{16}$ by $\frac{28}{40}$.

164. $\frac{14}{33}$ by $\frac{7}{11}$.

Divide, inverting the divisor:

165. $14\frac{10}{11}$ by 41.

166. $17\frac{15}{19}$ by 13.

167. $18\frac{13}{16}$ by 33.

168. $16\frac{12}{19}$ by 79.

169. $19\frac{13}{17}$ by 56.

170. 113 by $13\frac{5}{17}$.

171. 594 by $18\frac{9}{16}$.

172. 190 by $26\frac{7}{18}$.

173. 850 by $38\frac{7}{11}$.

174. 221 by $58\frac{3}{19}$.

175. $42\frac{2}{9}$ by $36\frac{8}{12}$.

176. $57\frac{3}{8}$ by $12\frac{3}{16}$.

177. $1\frac{229}{275}$ by $\frac{168}{605}$.

178. $2\frac{14}{425}$ by $\frac{312}{935}$.

179. $2\frac{46}{625}$ by $\frac{81}{125}$.

SPECIAL METHODS

180. Multiply 475 by $36\frac{2}{3}$.

$$
\begin{array}{rl}
475 & \\
36\frac{2}{3} & 950 \\
\hline
316\frac{2}{3} & \\
2850 & \\
1425 & \\
\hline
17416\frac{2}{3} &
\end{array}
$$

We first multiply 475 by $\frac{2}{3}$; 2 times 475 = 950; $950 \div 3 = 316\frac{2}{3}$. It is well to write 950 to the right as we multiply, and to divide by 3 without writing the 3.

We then multiply 475 by 36 in the usual way.

181. Multiply $638\frac{3}{4}$ by $27\frac{2}{3}$.

$$
\begin{array}{rl}
638\frac{3}{4} & 1276 \\
27\frac{2}{3} & 81 \\
\hline
\frac{1}{2} & 6 \\
425\frac{1}{3} & 4 \\
20\frac{1}{4} & 3 \\
4466 & \\
1276 & \\
\hline
17672\frac{1}{12} & \frac{13}{12}
\end{array}
$$

We first multiply $\frac{3}{4}$ by $\frac{2}{3}$; then, 638 by $\frac{2}{3}$; then, $\frac{3}{4}$ by 27; then, 638 by 27.

182. Divide $55\frac{2}{3}$ by 4.

$$
\begin{array}{r}
4\,)\,55\frac{2}{3} \\
\hline
13\frac{11}{12}
\end{array}
$$

5 tens ÷ 4 = 1 ten and 1 ten remaining; we write 1 in tens' column.

1 ten and 5 units = 15 units; 15 units ÷ 4 = 3 units and 3 units remaining, we write 3 in units' column.

3 units and 2 thirds = $\frac{11}{3}$; $\frac{11}{3} \div 4 = \frac{11}{12}$. *See p. 51.*

183. Divide 275 by $3\frac{2}{3}$.

$$
\begin{array}{r}
3\frac{2}{3}\,)\,275 \\
11\,)\,825 \\
\hline
75
\end{array}
$$

Multiplying both dividend and divisor by the same number does not affect the quotient.

We multiply both dividend and divisor by something that will make the divisor an integer. Here, we multiply both terms by 3.

THE FOUR OPERATIONS

Add:

184.	185.	186.	187.
$23\frac{3}{95}$	$14\frac{5}{18}$	$3\frac{3}{22}$	$13\frac{7}{16}$
$14\frac{5}{76}$	$16\frac{5}{36}$	$4\frac{5}{88}$	$18\frac{9}{80}$
$13\frac{2}{57}$	$17\frac{11}{54}$	$6\frac{13}{33}$	$8\frac{11}{48}$
$12\frac{1}{19}$	$12\frac{7}{90}$	$7\frac{3}{11}$	$12\frac{13}{32}$

Subtract:

188.	189.	190.	191.
$200\frac{3}{52}$	$263\frac{171}{256}$	$734\frac{1}{55}$	$325\frac{7}{48}$
$118\frac{19}{39}$	$189\frac{511}{512}$	$673\frac{13}{66}$	$266\frac{11}{60}$

192.	193.	194.	195.
$145\frac{31}{64}$	$164\frac{211}{488}$	$585\frac{24}{25}$	$424\frac{44}{45}$
$122\frac{23}{56}$	$142\frac{211}{322}$	$425\frac{59}{60}$	$222\frac{36}{55}$

Multiply:

196.	197.	198.	199.
$324\frac{2}{3}$	425	$625\frac{5}{6}$	$738\frac{1}{7}$
48	$75\frac{2}{5}$	$28\frac{3}{5}$	$231\frac{2}{3}$

200.	201.	202.	203.
$687\frac{5}{9}$	$584\frac{7}{8}$	$938\frac{3}{4}$	$865\frac{4}{7}$
$27\frac{1}{3}$	$48\frac{3}{4}$	$69\frac{1}{2}$	$36\frac{8}{9}$

Divide:

204.	205.	206.	207.
$4)\underline{723\frac{1}{5}}$	$3)\underline{9345\frac{2}{3}}$	$4)\underline{67851\frac{1}{2}}$	$7)\underline{9286\frac{3}{4}}$

208.	209.	210.	211.
$4\frac{2}{5})\underline{935}$	$62\frac{1}{2})\underline{3863\frac{1}{3}}$	$428\frac{4}{7})\underline{3285}$	$266\frac{2}{3})\underline{3586}$

COMPLEX FRACTIONS

To simplify complex fractions, it is often best to multiply both terms by the least common denominator of the several fractions.

212. Simplify $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}-\frac{1}{6}}$.

This $=\dfrac{6+4}{9-2}=\dfrac{10}{7}$.

The L.C.D. of the several fractions is 12; $12 \times \frac{1}{2} = 6$; $12 \times \frac{1}{3} = 4$; $12 \times \frac{3}{4} = 9$; $12 \times \frac{1}{6} = 2$.

213. Simplify $\dfrac{8}{\frac{2}{3}+\frac{3}{5}}$.

This $=\dfrac{120}{10+9}=\dfrac{120}{19}$.

The numerator of the complex fraction is $\frac{8}{1}$; the L. C. D. of $\frac{8}{1}$, $\frac{2}{3}$, and $\frac{3}{5}$ is 15.

214. Simplify $\dfrac{\frac{2}{3}\times\frac{3}{4}}{\frac{2}{3}-\frac{1}{2}}$.

This $=\dfrac{\frac{1}{2}}{\frac{2}{3}-\frac{1}{2}}=\dfrac{3}{4-3}$.

Before applying this principle, compound terms must be simplified. $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$.

215. Simplify $\dfrac{\frac{11}{16}-\frac{3}{8}}{\frac{3}{29}-\frac{4}{87}}$.

This $=\dfrac{\frac{5}{16}}{\frac{5}{87}}=\dfrac{87}{16}$.

When the L.C.D. is large, it is often better to perform the indicated operations.

216. Simplify $\dfrac{\frac{3}{4}\div\frac{2}{3}}{\frac{5}{4}\div\frac{7}{9}}\div\dfrac{\frac{1}{2}}{\frac{5}{7}}$.

This $=\frac{3}{4}\times\frac{3}{2}\times\frac{4}{5}\times\frac{7}{9}\times 2\times\frac{5}{7}=1$.

If a fraction is used an odd number of times as a divisor, we invert; if an even number of times, we do not invert.

NOTE. — In Ex. 214, the L. C. D. of the simple fractions is 12. If we had multiplied both $\frac{2}{3}$ and $\frac{3}{4}$ by 12, the numerator would have been multiplied by 12×12, or 144. *See p. 72.*

It should not be forgotten that complex fractions may always be solved by performing the indicated operations.

217. Solve examples 212, 213, 216 by performing the indicated operations.

218. Solve again example 216, as on page 95, and explain why $\frac{5}{4}$ is inverted; $\frac{7}{9}$, not inverted; $\frac{1}{2}$, inverted; $\frac{5}{7}$, not inverted.

$\frac{5}{4}$ is used once as a divisor (it is in the denominator of the complex fraction), invert; $\frac{7}{9}$, *twice* (preceded by '÷,' and in the denominator of the complex fraction), do not invert; $\frac{1}{2}$, *once* (the second complex fraction is preceded by '÷'), invert; $\frac{5}{7}$, *twice* (the whole complex fraction is preceded by '÷,' and $\frac{5}{7}$ is in the denominator), do not invert.

Using expedients of page 95, simplify:

219. $\dfrac{\frac{2}{3}}{\frac{3}{4}}$; $\dfrac{\frac{1}{2}}{\frac{4}{5}}$.

220. $\dfrac{\frac{5}{6}}{\frac{7}{8}}$; $\dfrac{\frac{5}{6}}{\frac{3}{4}}$.

221. $\dfrac{5\frac{1}{2}}{8\frac{1}{4}}$; $\dfrac{3\frac{1}{3}}{6\frac{1}{4}}$.

222. $\dfrac{7\frac{1}{3}}{9\frac{1}{6}}$; $\dfrac{6\frac{3}{4}}{9\frac{3}{4}}$.

---

223. $\dfrac{\frac{3}{8}}{1\frac{1}{4}} \times 3\frac{1}{7}$.

224. $\dfrac{1\frac{3}{4}}{2\frac{4}{5}} \times 1\frac{3}{5}$.

225. $\dfrac{2\frac{5}{14}}{2\frac{2}{21}} \times \frac{8}{9}$.

226. $\dfrac{5\frac{1}{3}}{9\frac{1}{5}} \div \frac{15}{23}$.

---

227. $\dfrac{\frac{1}{2} - \frac{7}{20}}{\frac{2}{4} - \frac{2}{5}}$.

228. $\dfrac{2\frac{1}{2} - \frac{5}{8}}{3\frac{1}{8} - 2\frac{3}{4}}$.

229. $\dfrac{5\frac{2}{5} \times 8\frac{1}{3}}{1\frac{4}{7} \div \frac{11}{35}}$.

230. $\dfrac{3\frac{3}{4} \div 6\frac{2}{3}}{4\frac{3}{8} \div 23\frac{1}{3}} \times \dfrac{\frac{2}{3}}{\frac{1}{5}}$.

231. $\dfrac{6\frac{2}{3} \text{ of } 3\frac{7}{20} - 2\frac{2}{3}}{7 - 2\frac{2}{5} \div \frac{2}{5}}$.

232. $\dfrac{2\frac{1}{3} - \frac{7}{8} \times \frac{16}{21} - \frac{5}{4}}{3\frac{2}{3} \times \frac{3}{22} \div 1\frac{1}{5}}$.

MISCELLANEOUS

233. Analyze and read $\frac{5}{8}$ by the first conception; by the second.

234. Give the meaning of $\frac{5}{8}$ by the first conception; by the second.

235. Why is $\frac{7}{4}$ not a fraction by the second conception? Why is it a fraction by the first conception?

236. Reduce $\frac{432}{1728}$ to lowest terms; state the principle, and the object of the reduction.

237. Reduce $\frac{344}{387}$ to lowest terms, and explain how we proceed when no common factors can be found by inspection.

238. Change $\frac{5}{6}$ to 24ths; state the principle. Why do we reduce fractions to higher terms?

239. Change $\frac{27}{24}$ to a whole or a mixed number; explain. What is the object of this reduction?

240. Reduce $6\frac{3}{4}$ to an improper fraction; explain in two ways.

241. Why are whole and mixed numbers reduced to fractions?

242. Add and explain: $22\frac{2}{3}$, $16\frac{3}{4}$; subtract and explain; multiply and explain; divide and explain; find the product of the sum and the difference.

243. Multiply $632\frac{2}{3}$ by $24\frac{3}{4}$; (*a*) by the common method; (*b*) by the special method. Which do you prefer?

244. Divide $\frac{2}{3}$ by $\frac{3}{4}$; (*a*) by the European method; (*b*) by the common method. Which do you prefer for mental work?

245. Divide $157\frac{2}{3}$ by 5; (*a*) by the common method; (*b*) by the special method. Which do you prefer?

246. Simplify $\dfrac{\frac{2}{3} - \frac{1}{4}}{\frac{3}{4} + \frac{1}{3}}$; (*a*) by performing the indicated operations; (*b*) by the expedient on page 95. Which do you prefer?

247. Simplify $\dfrac{\frac{3}{4} \times 2\frac{1}{2}}{\frac{5}{6} \div 1\frac{1}{3}} \div \dfrac{\frac{3}{2}}{\frac{4}{5} \div 2\frac{1}{3}}$; (*a*) by performing the indicated operations; (*b*) by the expedient. Which do you prefer?

ANALYSIS

The pupil will derive great benefit from writing out the relations.

State the relations and analyze:

248. If 1 yard of cloth costs 12¢, how much will $5\frac{1}{2}$ yards cost?

Relation: cost $5\frac{1}{2}$ yards = $5\frac{1}{2}$ times cost 1 yard.

Analysis: since 1 yard costs 12¢, $5\frac{1}{2}$ yards will cost $5\frac{1}{2}$ times 12¢, or 66¢.

NOTE. — Some prefer two *relations:* cost 5 yards = 5 times cost 1 yard; cost $\frac{1}{2}$ yard = $\frac{1}{2}$ cost 1 yard. *Analysis:* the cost of 5 yards is 5 times 12 cents, or 60 cents; the cost of $\frac{1}{2}$ yard is $\frac{1}{2}$ of 12 cents, or 6 cents; the cost of $5\frac{1}{2}$ yards is the sum, or 66 cents.

249. If $\frac{3}{4}$ of a yard of cloth costs 12¢, how much will 1 yard cost?

Relation: cost 1 yard = $\frac{4}{3}$ cost $\frac{3}{4}$ yard.

Analysis: since $\frac{3}{4}$ of a yard costs 12¢, 1 yard will cost $\frac{4}{3}$ of 12¢, or 16¢.

NOTE. — Some prefer two *relations:* cost of $\frac{1}{4}$ of a yard = $\frac{1}{3}$ cost of $\frac{3}{4}$ yard; cost of $\frac{4}{4}$ = 4 times cost of $\frac{1}{4}$. *Analysis:* since $\frac{3}{4}$ of a yard costs 12 cents, $\frac{1}{4}$ of a yard will cost $\frac{1}{3}$ of 12 cents, or 4 cents; $\frac{4}{4}$, or 1 yard, will cost 4 times 4 cents, or 16 cents.

250. 15 is $\frac{3}{4}$ of what number?

Relation: the required number is $\frac{4}{3}$ of the given number.

Analysis: since $\frac{3}{4}$ of a number is 15, the number is $\frac{4}{3}$ of 15, or 20.

NOTE. — Some prefer two *relations:* $\frac{1}{4}$ of the number = $\frac{1}{3}$ of $\frac{3}{4}$ the number; $\frac{4}{4}$, or the number, = 4 × $\frac{1}{4}$ the number. *Analysis:* since $\frac{3}{4}$ of a number is 15, $\frac{1}{4}$ of the number is $\frac{1}{3}$ of 15, or 5; $\frac{4}{4}$, or the number, is 4 times 5, or 20.

251. If 1 bushel of potatoes costs 36¢, how much will $2\frac{3}{4}$ bushels cost?

252. If $2\frac{1}{2}$ bushels of apples cost 90¢, how much will $1\frac{1}{2}$ bushels cost?

253. 20 is $\frac{5}{6}$ of what number? 30 is $\frac{6}{5}$ of what number? 45 is $\frac{5}{9}$ of what number?

254. A watch was sold for $12. The selling price was $\frac{3}{4}$ of the cost. What was the cost?

255. A man lost $\frac{3}{16}$ of his money and afterwards found $\frac{1}{3}$ of what he had lost. What part of the original amount had he then?

256. A man bequeathed $\frac{1}{2}$ of his estate to his wife, $\frac{1}{3}$ of the remainder to his daughter, and the part remaining to his son. What part of the estate did the son receive?

257. In a certain school, $\frac{2}{5}$ of the scholars belong to the fourth class, $\frac{1}{6}$ to the third, $\frac{1}{7}$ to the second, and the remainder to the first class. What part of the school belongs to the first class?

258. Of a farm of 160 acres, $\frac{2}{5}$ is used for grazing, $\frac{3}{8}$ for corn, $\frac{3}{16}$ for wheat, and the rest for oats. How many acres of oats are there?

259. If $1\frac{7}{8}$ yards of cloth are required for each coat, how many yards will be required for 17 coats?

260. How many coats may be made from $33\frac{3}{4}$ yards of cloth, if $1\frac{7}{8}$ yards are required for each coat?

261. Of a farm of 160 acres, 48 acres are in wheat, 36 acres in oats; the remainder is pasture. What part of the whole is used for pasture?

262. If my whole farm is planted with corn, wheat, and oats, what part of the whole is in corn, if I have twice as many acres of corn as of wheat, and three times as many acres of oats as of corn?

263. A grocer sells 18 bunches of radishes at $4\frac{1}{2}$¢ a bunch, 12 quarts of peas at $2\frac{1}{4}$¢ a quart, and takes his pay in eggs at $13\frac{1}{2}$¢ a dozen. How many dozen eggs does he receive?

264. Henry is 18 years old, or $\frac{3}{4}$ as old as Albert. How old is Albert?

265. A farmer sold a quantity of rye for $24, which was only $\frac{4}{5}$ of its value. How much did he lose?

266. If $\frac{2}{3}$ of a ship is worth $12,000, how much is $\frac{5}{6}$ of it worth?

267. Jane spent 30 cents, or $\frac{5}{6}$ of her money, for a book; with the remainder she bought apples at 2¢ apiece. How many apples did she buy?

268. If 1 yard of cloth costs $\$5\frac{3}{4}$, how much will $12\frac{1}{2}$ yards cost?

269. If 4 yards of cloth cost $\$9\frac{1}{2}$, how much will $\frac{7}{8}$ of a yard cost?

270. A lady gave $\frac{1}{3}$ of all her money for a dress, and $\frac{1}{4}$ of it for a shawl. What part remained?

271. A farmer, having lost 24 sheep, had only $\frac{7}{9}$ of his flock remaining. How many sheep had he at first?

272. If 6 apples cost 7¢, how many apples can be bought for 14¢?

273. B owned $\frac{7}{8}$ of a ship, and sold $\frac{2}{3}$ of his share. What part of the whole ship did he still own?

274. $\frac{2}{3}$ of 6 is $\frac{2}{5}$ of what number? $\frac{3}{5}$ of 10 is $\frac{3}{8}$ of what number?

275. If 12 apples cost $2\frac{1}{2}$¢, how many apples can be bought for 25¢?

276. A coat cost $20, and $\frac{4}{5}$ of the cost of the coat is $\frac{2}{7}$ of the price of the suit. What is the price of the suit?

277. A farmer has $\frac{1}{4}$ of his cattle in one field, $\frac{1}{3}$ in a second, and the remainder, or 15 head of cattle, in a third. How many cattle are in the herd?

278. If $\frac{2}{3}$ of a yard of silk costs $\frac{3}{4}$ of a dollar, how much will $\frac{8}{9}$ of a yard cost?

279. A is $\frac{2}{3}$ as old as B; B is $\frac{3}{4}$ as old as C; C is 60 years old. How old is A?

280. If the difference between $\frac{3}{4}$ and $\frac{2}{3}$ of my age is 5 years, how old am I?

281. If 7 men can do a piece of work in $4\frac{2}{7}$ days, how long will it take 6 men to do the same work?

282. If I give $18\frac{3}{4}$ bushels of potatoes at 60 cents a bushel for cloth at $22\frac{1}{2}$ cents a yard, how many yards of cloth do I receive?

283. If a man can do a piece of work in 9 days by working $7\frac{1}{3}$ hours per day, in how many days of $8\frac{1}{4}$ hours each, can he do the same work?

PROOFS

I. *Multiplying the numerator multiplies a fraction.*

Multiplying the numerator multiplies the number of equal parts that are taken, without affecting the size of the parts, and thus multiplies the fraction.

II. *Multiplying the denominator divides a fraction.*

Multiplying the denominator multiplies the number of equal parts into which the unit is divided, thereby dividing the size of each part, without affecting the number of parts taken, and thus divides the fraction.

III. *Dividing the numerator divides a fraction.*

Dividing the numerator divides the number of equal parts that are taken, without affecting the size of the parts, and thus divides the fraction.

IV. *Dividing the denominator multiplies a fraction.*

Dividing the denominator divides the number of equal parts into which the unit is divided, thereby multiplying the size of each part, without affecting the number of parts taken, and thus multiplies the fraction.

V. *Multiplying both numerator and denominator by the same number does not change the value of a fraction.*

Since multiplying the numerator multiplies the fraction, and multiplying the denominator divides the fraction, multiplying both terms by the same number, first multiplies and then divides the fraction by the same number, and does not, therefore, change the value of the fraction.

VI. *Dividing both numerator and denominator by the same number does not change the value of a fraction.*

Since dividing the numerator divides the fraction, and dividing the denominator multiplies the fraction, dividing both terms by the same number, first divides and then multiplies the fraction by the same number, and does not, therefore, change the value of the fraction.

NOTE. — The pupil should distinguish carefully the difference between *proofs* and *illustrations*. *See pp. 85, 86, 90, 91.*

VII. *To multiply fractions, multiply the numerators for a new numerator and the denominators for a new denominator.*

To prove that $$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5}.$$

$$\frac{2}{3} \times 4 = \frac{2 \times 4}{3}.$$ *(Multiplying the numerator multiplies the fraction.)*

Since 4 is $5 \times \frac{4}{5}$, multiplying by 4 is multiplying by a number 5 times too large; the product is 5 times too large, and must be divided by 5.

$$\frac{2 \times 4}{3} \div 5 = \frac{2 \times 4}{3 \times 5}.$$ *(Multiplying the denominator divides the fraction.)*

$$\therefore \frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5}.$$

VIII. *To divide fractions, divide the numerators for a new numerator and the denominators for a new denominator.*

To prove that $$\frac{8}{9} \div \frac{2}{3} = \frac{8 \div 2}{9 \div 3}.$$

$$\frac{8}{9} \div 2 = \frac{8 \div 2}{9}.$$ *(Dividing the numerator divides the fraction.)*

Since 2 is 3 times $\frac{2}{3}$, dividing by 2 is dividing by a number 3 times too large; the quotient is 3 times too small, and must be multiplied by 3.

$$\frac{8 \div 2}{9} \times 3 = \frac{8 \div 2}{9 \div 3}.$$ *(Dividing the denominator multiplies the fraction.)*

$$\therefore \frac{8}{9} \div \frac{2}{3} = \frac{8 \div 2}{9 \div 3}.$$

IX. *To divide fractions, invert the divisor and proceed as in multiplication.*

To prove that $$\frac{8}{9} \div \frac{2}{3} = \frac{8}{9} \times \frac{3}{2}.$$

$$\frac{8}{9} \div 2 = \frac{8}{9 \times 2}.$$ *(Multiplying the denominator divides the fraction.)*

Since 2 is 3 times $\frac{2}{3}$, dividing by 2 is dividing by a number 3 times too large; the quotient is 3 times too small, and must be multiplied by 3.

$$\frac{8}{9 \times 2} \times 3 = \frac{8 \times 3}{9 \times 2}.$$ *(Multiplying the numerator multiplies the fraction.)*

$$\therefore \frac{8}{9} \div \frac{2}{3} = \frac{8}{9} \times \frac{3}{2}.$$

DECIMALS

TERMS AND RELATIONS

A common fraction may have for its denominator 10, 100, 1000, ... a *decimal fraction.*

ILLUSTRATIONS

$\frac{378}{1000}$, a *decimal fraction.*

A decimal fraction may be resolved into a series of decimal fractions whose denominators are 10, 100, 1000, ... and whose numerators are less than ten.

Since $378 = 300 + 70 + 8$,

$$\frac{378}{1000} = \frac{300}{1000} + \frac{70}{1000} + \frac{8}{1000}$$
$$= \frac{3}{10} + \frac{7}{100} + \frac{8}{1000}$$

Since . . . . 10 thousandths make 1 hundredth, 10 hundredths make 1 tenth, ..., this series of fractions may be expressed by the devices of the Arabic notation. *See p. 10.*

$$\cdots \frac{10}{1000} = \frac{1}{100},\ \frac{10}{100} = \frac{1}{10}, \cdots$$

By the Arabic notation, the names of the orders are expressed by relative position. Thus, 378 means 3 *hundreds* 7 *tens* 8 *units.* If 378 is to mean 3 *tenths* 7 *hundredths* 8 *thousandths,* an additional device is necessary. This device consists in placing before *tenths* a period, called the *decimal point.*

Write $\frac{378}{1000}$, using the devices of the Arabic notation.

$$\frac{378}{1000} = \frac{3}{10} + \frac{7}{100} + \frac{8}{1000},$$

3 tenths 7 hundredths 8 thousandths, decimal 378,

.378

A decimal fraction whose denominator is expressed by a decimal point, is a *decimal.*

.378, *a decimal.*

Write the steps to show:

1. That $\frac{37}{100}=.37$

2. That $\frac{37}{1000}=.037$

3. That $\frac{379}{10000}=.0379$

4. That $\frac{37}{10000}=.0037$

Ex. **2.** $37=000+30+7$; $\frac{37}{1000}=\frac{000}{1000}+\frac{30}{1000}+\frac{7}{1000}=\frac{0}{10}+\frac{3}{100}+\frac{7}{1000}$ $=0$ *tenths* 3 *hundredths* 7 *thousandths* = decimal 037 = .037.

Write the steps to show:

5. That $.93=\frac{93}{100}$.

6. That $.093=\frac{93}{1000}$.

7. That $.0973=\frac{973}{10000}$.

8. That $.0097=\frac{97}{10000}$.

Ex. **6.** $.093=0$ *tenths* 9 *hundredths* 3 *thousandths* $=\frac{0}{10}+\frac{9}{100}+\frac{3}{1000}$ $=\frac{90}{1000}+\frac{3}{1000}=\frac{93}{1000}$.

9. State the numerator of each decimal in examples 1 to 8, and observe that it is the decimal regarded as an integer.

Ans. Ex. 4. In .0037, or $\frac{37}{10000}$, the numerator is 37, or it is the decimal .0037 regarded as an integer.

10. State the denominator of each decimal in examples 1 to 8, and observe that it may be ascertained by numerating from the decimal point to the right-hand figure.

Ans. Ex. 4. In .0037, or $\frac{37}{10000}$, the denominator is 10,000. Numerating .0037; tenths, hundredths, thousandths, ten-thousandths.

11. Show that the decimal orders are: tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, ...

Ans. $\frac{1}{10}=\frac{10}{100}$; $\frac{1}{100}=\frac{10}{1000}$; $\frac{1}{1000}=\frac{10}{10000}$; $\frac{1}{10000}=\frac{10}{100000}$; ...

12. Name the decimal orders to quadrillionths; give the table of decimal orders to quadrillionths. *See p. 11.*

13. Show that the decimal periods are: thousandths, millionths, billionths, trillionths, quadrillionths, ... *See p. 11.*

Ans. $\frac{1}{1000}=\frac{1000}{1000000}$; $\frac{1}{1000000}=\frac{1000}{1000000000}$; . . . .

NUMERATION

Read a decimal as a common fraction; first its numerator, then its denominator in the ordinal form.

Read the numerator, the denominator, the fraction:

14. .5; .06; .007

15. .16; .027; .0039

16. .368; .0407; .00362

17. .0002006; .00000213

18. .003080654

19. .0123005678937286

Ex. **18**. The numerator is 3,080,654; the denominator, 1 billion; the fraction, 3,080,654 billionths.

NOTE.—In Ex. 19, we may numerate by periods; thousandths, millionths, billionths, trillionths, quadrillionths, *ten-quadrillionths*.

A mixed decimal is made up of an integer and a decimal.

Analyze and read:

20. 175.68; 400.006; .406

21. 3000.003; 20.586; 1.7

22. 1.0003; 62.832546

23. 23.0708; 35682.3

24. 7583.64; 700.0075

25. 328.00062; 10.0001

Ex. **20**. 175 is the integer; .68, the decimal; read, 175 *and* 68 hundredths.

NOTE.—In reading mixed decimals, the word "and" should be used at the decimal point, but nowhere else. *See p. 9, Note.* Without this understanding, *four hundred and six thousandths* might be written either 400.006, or .406.

A complex decimal is made up of a decimal and a common fraction. A common fraction can not occupy a decimal order, but is of the same denomination as the order which it follows.

Read:

26. $.06\frac{1}{2}$; $.001\frac{2}{3}$

27. $.0008\frac{1}{2}$; $.000\frac{1}{2}$

28. $.00\frac{1}{2}$; $.0\frac{1}{2}$; $.\frac{1}{2}$

29. $20.086\frac{2}{3}$; $5.0123\frac{3}{4}$

Ex. **26**. *Ans.* $6\frac{1}{2}$ hundredths; $1\frac{2}{3}$ thousandths.

Ex. **28**. *Ans.* $.\frac{1}{2}$ is an improper expression.

NOTATION

Write the numerator as in common fractions, and place the decimal point so as to make the name of the last order the name of the denominator.

30. Write 638 millionths.

.000638 We write the numerator as in common fractions, 638; looking at 8 we think *millionths* because this is the name of the denominator; at 3, *hundred-thousandths;* at 6, *ten-thousandths;* supplying 0 (it now appears 0638), we think thousandths; supplying 0 (00638), we think hundredths; supplying 0 (000638), we think tenths; writing the decimal point, we have .000638.

Write decimally:

31. Five hundred millionths.

32. Seventeen, and 7 tenths.

33. 638 hundred-millionths.

34. Twenty-three trillionths.

35. 85 thousand 2 billionths.

36. 7007 ten-thousandths.

37. 360 hundred-thousandths.

38. Six hundred four thousandths.

39. 9431, and 906532 ten-millionths.

40. Two thirds ten-thousandths.

41. 2 billion, and two billionths.

42. 83 thousand, and 5 thousandths.

43. 99 thousand 999 ten-millionths.

44. 1 million 6 thousand 4 billionths.

Read:

45. 75.06432

46. $.000000\frac{2}{3}$

47. 35.02767

48. 999.9999

49. 9.0999999

50. 200.0016

51. 1400.014567

52. 62.00634010

53. .09622635

54. 304.00672

55. .98637208

56. .83000008

57. 9200.0929

58. 9000.0009372

ADDITION

Write units of the same order in the same column.

59. Add 36.03, 7.864

36.03
7.864
43.894

For explanation, *see p. 20.* The sum of the thousandths is 4 thousandths; etc.

SUBTRACTION

Write units of the same order in the same column.

60. From 38.03 subtract 25.128

38.03
25.128
12.902

For explanation, *see p. 30.* 8 thousandths from 0 thousandths we cannot take; etc.

MULTIPLICATION

Multiply as in integers, and point off as many decimal places in the product as there are decimal places in both multiplicand and multiplier.

61. Multiply .473 by .23, and explain.

.473
.23
1419
946
.10879

To multiply fractions, we multiply the numerators for a new numerator, and the denominators for a new denominator. *See p. 90.*

The numerators are 473 and 23; we multiply them as in integers. The denominators are 1000 and 100; their product is 100,000; we point off 3 + 2, or 5 decimal places.

62. Multiply 378.6954 by 1000.

378.6954
1000
378695.4

Moving the decimal point one place to the right multiplies by 10; two places, by 100; etc. *See p. 40, Ex. 81.*

DIVISION

Divide as in integers, and point off as many decimal places in the quotient as those in the dividend exceed those in the divisor.

63. Divide .0414 by .23 and explain.

$$\begin{array}{r} .23).0414(.18 \\ \underline{23} \\ 184 \\ \underline{184} \end{array}$$

To divide fractions, we divide the numerators for a new numerator and the denominators for a new denominator. *See p. 91.*

The numerators are 414 and 23; we divide them as in integers. The denominators are 10,000 and 100; their quotient is 100; we point off 4 − 2, or 2 decimal places.

64. Divide 426 by .08.

$$\begin{array}{r} .08)\underline{426.00} \\ 5325 \end{array}$$

Before dividing, it is necessary to annex ciphers to the dividend until the number of decimal places equals or exceeds those in the divisor.

Since there are two places in the divisor and none in the dividend, we annex two ciphers to the dividend.

65. Divide 896 by 432 true to two decimal places.

$$\begin{array}{r} 432)896.00(2.07\frac{11}{27} \\ \underline{864} \\ 3200 \\ \underline{3024} \\ 176 \end{array}$$

We annex two ciphers to make the places in the dividend exceed those in the divisor by two.

$\frac{176}{432}$ should be reduced to lowest terms.

NOTE.—If the exact quotient is not required, it is customary to write '+' instead of the common fraction.

66. Divide 268 by 10,000.

$$\begin{array}{r} 10000)\underline{268} \\ .0268 \end{array}$$

Moving the decimal point one place to the left divides by 10; two places, by 100; etc. *See p. 55, Ex. 152.*

67. Divide .06 by 6000.

$$\begin{array}{r} 6.000_{\times})\underline{.000_{\times}06} \\ .00001 \end{array}$$

If the divisor ends with ciphers, we move the decimal point in both dividend and divisor. *See p. 55, Ex. 153.*

THE FOUR OPERATIONS

Add.

68. $988.058, $75.896, $75, $68.299, $86.43, $45.256.

69. $77.948, $89.23, $57.637, $88.009, $6.783, $.086, $55.

70. $235.06, $578.085, $735.88, $967.08201, $3.6872, $20.98, $66, $8.7963, $48.23415, $9996.6, $8763.

Subtract:

71. 35.1789 from 103.45

72. .463268 from 13.603

73. 21.9875 from 86.013

74. 178.369 from 10,000

75. .166 from 55.03

76. 8.954 from 9.862

77. .88875 from 100.02

78. 88.86 from 856.9

Multiply:

79. 3.876 by .3815

80. .4081 by 1001

81. .6375 by .5082

82. 700.87 by .642

83. 23.478 by 3300

84. 2.0087 by 2500

85. 6.0961 by 1600

86. 365.08 by 68.092

Divide:

87. .00072 by .016

88. 63.904 by 4.86

89. .97230 by .313

90. 27013 by 176

91. 17.181 by 567

92. 5.0000 by 789

93. 1.4511 by .214

94. .0005 by 5000

95. 5000 by .0005

96. .1265 by 7.91

97. .9614 by .907

98. .9396 by 67.12

99. 6800 by .0034

100. .0006 by 3400

NOTE. — The pupil should turn to p. 54 and solve examples 141 to 145, carrying out each quotient two or three decimal places.

COMMON FRACTIONS TO DECIMALS

101. Reduce $\frac{5}{8}$ to a decimal.

$$8\overline{)5.000}$$
$$.625$$

$\frac{5}{8}$ means $5 \div 8$; performing the indicated operation, we obtain .625.

102. Reduce $\frac{2}{7}$ to a decimal.

$$7\overline{)2.000000}$$
$$.285714\tfrac{2}{7}$$

Our last remainder is 2, and the next dividend is 20, what we had at first. Hence, the quotient will repeat the figures 2, 8, 5, 7, 1, 4, and will not come out exact. It is, therefore, impossible to reduce $\frac{2}{7}$ to a decimal *exactly*.

I. Result exact.

$$7\overline{)2.000}$$
$$.285\tfrac{5}{7}$$

If the exact result is required, it is customary to carry out the division a few places, and to write the remainder as in I.

II. Result approximate.

$$7\overline{)2.000}$$
$$.285+$$

If an approximation is desired, in place of the remainder, we write '+' as in II.

III. Result as a repetend.

$$7\overline{)2.000000}$$
$$.\dot{2}8571\dot{4}$$

If we desire to show the figures which repeat, we carry out the division until the quotient begins to repeat, and place dots over the first and last figures of the repetend, as in III. The result is a *circulating decimal;* it is read: decimal, repetend, 2, 8, 5, 7, 1, 4, repetend.

103. What fractions cannot be reduced to decimals exactly?

Ans. Those fractions which have prime factors other than 2 or 5 in their denominators. The factors of 10 are 2 and 5. Hence, if a fraction has any prime factor other than 2 or 5 in its denominator, the reduction will not be exact.

104. Can $\frac{1}{256}$ be reduced to a decimal exactly? Why?

105. If $\frac{1}{256}$ is reduced to a decimal exactly, how many decimal places will there be in the result?

Ans. 8. $\frac{1}{256} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = (.5)^8$.

DECIMALS TO COMMON FRACTIONS

106. Reduce .625 to a common fraction.

$.625 = \frac{625}{1000} = \frac{5}{8}.$ We write the decimal as a common fraction, and reduce to lowest terms.

107. Reduce $.83\frac{1}{3}$ to a common fraction.

$$.83\tfrac{1}{3} = \frac{83\frac{1}{3}}{100} = \frac{250}{300} = \frac{5}{6}.$$

We first reduce the complex fraction, $\frac{83\frac{1}{3}}{100}$, to a simple fraction.

Reduce to common fractions in lowest terms:

108. .032; .0016

109. .075; .0128

110. .025; .1125

111. .005; .3125

112. .004; .1728

113. .225; .4375

114. .024; .1625

115. $.12\frac{1}{2}$; $.33\frac{1}{4}$

116. $.87\frac{1}{2}$; $.37\frac{5}{6}$

117. $.83\frac{4}{5}$; $.62\frac{1}{2}$

118. $.125\frac{1}{7}$; .0375

119. .0875; $.000\frac{2}{3}$

120. $.086\frac{2}{3}$; $.216\frac{2}{3}$

121. .1875; $.080\frac{1}{3}$

Reduce to decimals exactly:

122. $\frac{1}{2}$, $\frac{1}{4}$

123. $\frac{3}{4}$, $\frac{1}{5}$

124. $\frac{2}{5}$, $\frac{3}{5}$

125. $\frac{1}{8}$, $\frac{7}{8}$

126. $\frac{3}{16}$, $\frac{4}{25}$

127. $\frac{15}{128}$, $\frac{12}{125}$

128. $.75\frac{1}{4}$

129. $.87\frac{3}{8}$

130. $.00\frac{1}{16}$

131. Can $\frac{11}{18}$ be reduced to a decimal exactly? Why?

132. Reduce $\frac{11}{13}$ to a decimal, writing the divisor under the remainder after the third decimal place of the quotient.

133. Reduce $\frac{11}{13}$ to a decimal, writing '+' after the fourth decimal place of the quotient.

134. Reduce $\frac{11}{13}$ to a circulating decimal, writing dots over the first and last figures of the repetend.

COMPLEX DECIMALS

If a complex decimal is to be subjected to any operation, it is best first to reduce to a simple decimal.

135. Prepare $36.00\frac{1}{4}$, $7.1\frac{3}{8}$ for the operations.

$$36.00\tfrac{1}{4} = 36.0025;\quad 7.1\tfrac{3}{8} = 7.1375$$

When a complex decimal cannot be reduced to a simple decimal, it is generally sufficient to carry out to three or four decimal places and to substitute '+' for the common fraction.

136. Prepare $16.2\frac{1}{3}$, $48.32\frac{5}{6}$ for the operations.

$$16.2\tfrac{1}{3} = 16.233+;\quad 48.32\tfrac{5}{6} = 48.328+$$

If absolute accuracy is required, the common fraction cannot be neglected.

137. Add exactly $16.2\frac{1}{3}$, $48.32\frac{5}{6}$.

$$\begin{aligned} 16.2\tfrac{1}{3} &= 16.23\tfrac{1}{3} \\ 48.32\tfrac{5}{6} &= \underline{48.32\tfrac{5}{6}} \\ & \ 64.56\tfrac{1}{6} \end{aligned}$$

Since $\frac{1}{3}$ is *tenths* and $\frac{5}{6}$ is *hundredths*, it is necessary to reduce $\frac{1}{3}$ to hundredths.

In division, it is frequently possible to simplify by multiplying both dividend and divisor by some number.

138. Divide 60 by $2.8\frac{4}{7}$.

$$\begin{aligned} &2.8\tfrac{4}{7})60(\\ &2.0_{\times}0)\underline{42.0_{\times}} \\ &\qquad\quad 21 \end{aligned}$$

We multiply both terms by 7, and proceed as in Ex. 67.

139. Divide $.365\frac{5}{6}$ by $3.8\frac{2}{3}$.

$$\begin{aligned} &3.8\tfrac{2}{3}).365\tfrac{5}{6}(\\ &23.2)2.195(\end{aligned}$$

We multiply both terms by 6, and proceed as usual.

CIRCULATING DECIMALS

If a circulating decimal is to be subjected to any operation, it is best first to reduce to a common fraction.

To reduce a circulating decimal to a common fraction, for the numerator, write the repetend, and for the denominator as many 9's as there are figures in the repetend.

140. Reduce $.\dot{1}\dot{5}$ and $.\dot{2}0\dot{7}$ to common fractions and illustrate the rule.

$$.\dot{1}\dot{5} = \frac{15}{99} = \frac{5}{33}$$

$$.\dot{1}\dot{5} \times 100 = 15.1515 \ldots$$
$$.\dot{1}\dot{5} \times 1 = .1515 \ldots$$
$$.\dot{1}\dot{5} \times 99 = 15$$
$$.\dot{1}\dot{5} = \tfrac{15}{99}$$

$$.\dot{2}0\dot{7} = \frac{207}{999} = \frac{23}{111}$$

$$.\dot{2}0\dot{7} \times 1000 = 207.207 \ldots$$
$$.\dot{2}0\dot{7} \times 1 = .207 \ldots$$
$$.\dot{2}0\dot{7} \times 999 = 207$$
$$.\dot{2}0\dot{7} = \tfrac{207}{999}$$

141. Reduce $.03\dot{1}\dot{5}$ to a common fraction.

$$.03\dot{1}\dot{5} = .03\tfrac{15}{99} = .03\tfrac{5}{33}$$
$$= \frac{\frac{104}{33}}{100} = \frac{104}{3300} = \frac{26}{825}$$

By the rule, $.03\dot{1}\dot{5} = .03\frac{15}{99}$.

142. Reduce $\dot{1}.\dot{5}$ to a common fraction.

$$\dot{1}.\dot{5} = .\dot{1}\dot{5} \times 10$$
$$\dot{1}.\dot{5} = \frac{15}{99} \times 10 = \frac{150}{99}$$

$$\dot{1}.\dot{5} \times 100 = 151.515 \ldots$$
$$\dot{1}.\dot{5} \times 1 = 1.515 \ldots$$
$$\dot{1}.\dot{5} \times 99 = 150$$
$$\dot{1}.\dot{5} = \tfrac{150}{99} = 1\tfrac{51}{99}$$

Find the value of:

143. $.3 + .\dot{1}\dot{2} + 1.\dot{5}$

144. $23.\dot{3}\dot{6} - 16.\dot{4}$

145. $.\dot{2}\dot{5} \times 1.\dot{6}$

146. $.\dot{6}2\dot{5} + 2.7$

147. $6 + .6 + .\dot{6}$

148. $9.16 - .\dot{7}\dot{3}$

149. $5.\dot{0}0\dot{3} \times 6$

150. $15.\dot{3}2\dot{4} + 81$

SPECIAL CASES

The decimal and the per cent equivalents of a few common fractions are used so frequently that they should be memorized.

151. *Memorize the decimal and the per cent equivalents:*

$\frac{1}{2}$, .50, 50%.	$\frac{2}{5}$, .40, 40%	$\frac{3}{8}$, .37$\frac{1}{2}$, 37$\frac{1}{2}$%.
$\frac{1}{3}$, .33$\frac{1}{3}$, 33$\frac{1}{3}$%.	$\frac{3}{5}$, .60, 60%.	$\frac{5}{8}$, .62$\frac{1}{2}$, 62$\frac{1}{2}$%.
$\frac{2}{3}$, .66$\frac{2}{3}$, 66$\frac{2}{3}$%.	$\frac{4}{5}$, .80, 80%.	$\frac{7}{8}$, .87$\frac{1}{2}$, 87$\frac{1}{2}$%.
$\frac{1}{4}$, .25, 25%.	$\frac{1}{6}$, .16$\frac{2}{3}$, 16$\frac{2}{3}$%.	$\frac{1}{9}$, .11$\frac{1}{9}$, 11$\frac{1}{9}$%.
$\frac{3}{4}$, .75, 75%.	$\frac{5}{6}$, .83$\frac{1}{3}$, 83$\frac{1}{3}$%.	$\frac{1}{12}$, .08$\frac{1}{3}$, 8$\frac{1}{3}$%.
$\frac{1}{5}$, .20, 20%.	$\frac{1}{8}$, .12$\frac{1}{2}$, 12$\frac{1}{2}$%.	$\frac{1}{16}$, .06$\frac{1}{4}$, 6$\frac{1}{4}$%.

Thus: $\frac{1}{2}$, 50 ***hundredths***, 50 ***per cent.***

NOTE. — When the denominator of a decimal is 100, it may be expressed by the sign %, read, *per cent.*

State rapidly the decimal equivalents:

152. $\frac{1}{5}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{2}$.
155. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{1}{12}$.
158. $\frac{1}{6}$, $\frac{7}{8}$, $\frac{1}{12}$, $\frac{2}{3}$.
153. $\frac{4}{5}$, $\frac{1}{6}$, $\frac{3}{5}$, $\frac{1}{8}$.
156. $\frac{3}{5}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{3}{4}$.
159. $\frac{4}{5}$, $\frac{2}{5}$, $\frac{1}{16}$, $\frac{1}{8}$.
154. $\frac{5}{8}$, $\frac{1}{9}$, $\frac{1}{12}$, $\frac{1}{16}$.
157. $\frac{2}{5}$, $\frac{3}{8}$, $\frac{7}{8}$, $\frac{1}{16}$.
160. $\frac{5}{8}$, $\frac{5}{6}$, $\frac{3}{5}$, $\frac{4}{5}$.

Ex. **152**. .20, .33$\frac{1}{3}$, .25, .50.

State rapidly the per cent equivalents:

161. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{1}{5}$.
164. $\frac{1}{16}$, $\frac{1}{9}$, $\frac{7}{8}$, $\frac{3}{4}$.
167. $\frac{3}{8}$, $\frac{2}{5}$, $\frac{5}{8}$, $\frac{1}{3}$.
162. $\frac{3}{5}$, $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{2}$.
165. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{1}{12}$.
168. $\frac{1}{16}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.
163. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{12}$, $\frac{5}{8}$.
166. $\frac{5}{6}$, $\frac{3}{5}$, $\frac{1}{6}$, $\frac{1}{4}$.
169. $\frac{4}{5}$, $\frac{1}{12}$, $\frac{5}{6}$, $\frac{1}{5}$.

Ex. **161**. 75%, 83$\frac{1}{3}$%, 87$\frac{1}{2}$%, 20%.

State rapidly the fractional equivalents:

170. 87$\frac{1}{2}$%, 66$\frac{2}{3}$%, 33$\frac{1}{3}$%.
173. 37$\frac{1}{2}$%, 40%, 8$\frac{1}{3}$%.
171. 16$\frac{2}{3}$%, 60%, 25%.
174. 62$\frac{1}{2}$%, 16$\frac{2}{3}$%, 20%.
172. 62$\frac{1}{2}$%, 20%, 12$\frac{1}{2}$%.
175. 66$\frac{2}{3}$%, 50%, 11$\frac{1}{9}$%.

Ex. **170**. $\frac{7}{8}$, $\frac{2}{3}$, $\frac{1}{3}$.

In multiplication and division, it is easier to use the common fractions than to use the decimal or the per cent equivalents.

Find mentally the value of:

176. $24 \times \frac{1}{6}$, $24 \times .16\frac{2}{3}$, $24 \times 16\frac{2}{3}\%$.

177. $36 \times \frac{2}{3}$, $36 \times .66\frac{2}{3}$, $36 \times 66\frac{2}{3}\%$.

178. $64 \times \frac{3}{8}$, $64 \times .37\frac{1}{2}$, $64 \times 37\frac{1}{2}\%$.

179. $84 \times 83\frac{1}{3}\%$.

180. $76 \times 75\%$.

181. $81 \times 11\frac{1}{9}\%$.

Find mentally the value of:

182. $72 \div \frac{3}{8}$, $72 \div .37\frac{1}{2}$, $72 \div 37\frac{1}{2}\%$.

183. $60 \div \frac{5}{6}$, $60 \div .83\frac{1}{3}$, $60 \div 83\frac{1}{3}\%$.

184. $84 \div \frac{2}{3}$, $84 \div .66\frac{2}{3}$, $48 \div 66\frac{2}{3}\%$.

185. $49 \div 87\frac{1}{2}\%$.

186. $65 \div 83\frac{1}{3}\%$.

187. $64 \div 80\%$.

What is the cost of:

188. 48 articles at $6\frac{1}{4}$¢? $8\frac{1}{3}$¢? $12\frac{1}{2}$¢? $16\frac{2}{3}$¢? $33\frac{1}{3}$¢? $66\frac{2}{3}$¢?

189. 72 articles at $87\frac{1}{2}$¢? 75¢? $83\frac{1}{3}$¢? $37\frac{1}{2}$¢? $12\frac{1}{2}$¢? 25¢?

190. 36 articles at \$$1.33\frac{1}{3}$? \$1.25? \$1.75? \$$1.66\frac{2}{3}$? \$$2.16\frac{2}{3}$?

Ex. **190.** \$48; \$$1.33\frac{1}{3}$ = \$$1\frac{1}{3}$; 36 × \$$1\frac{1}{3}$ = \$48.

How many articles can be bought for:

191. \$24 @ $37\frac{1}{2}$¢? 75¢? $66\frac{2}{3}$¢? 40¢? $12\frac{1}{2}$¢? 80¢? $33\frac{1}{3}$¢?

192. \$30 @ $83\frac{1}{3}$¢? $62\frac{1}{2}$¢? 25¢? $37\frac{1}{2}$¢? 60¢? $66\frac{2}{3}$¢? 50¢?

193. \$40 @ \$1.25? \$$1.33\frac{1}{3}$? \$$1.66\frac{2}{3}$? \$2.50? \$$3.33\frac{1}{3}$?

Ex. **193.** 32; \$1.25 = \$$\frac{5}{4}$; \$40 ÷ \$$\frac{5}{4}$ = 32.

194. What is the cost of cloth per yard, when \$1 is the cost of 2 yards? 3 yards? 4 yards? 5 yards? 6 yards? 8 yards? 9 yards? 10 yards? 12 yards? 16 yards?

195. How many yards of cloth can be bought for \$1 at 50¢ a yard? $33\frac{1}{3}$¢? 25¢? 20¢? $16\frac{2}{3}$¢? $12\frac{1}{2}$¢? $11\frac{1}{9}$¢? $8\frac{1}{3}$¢? $6\frac{1}{4}$¢? 10¢? $9\frac{1}{11}$¢?

In multiplication and division, it is easier to use the common fractions than to use the equal (aliquot) parts of 100.

State the rule for:

196. Multiplying by 20.

197. Dividing by 20.

198. Multiplying by 25.

199. Dividing by 25.

200. Multiplying by 50.

201. Dividing by 50.

202. Multiplying by $33\frac{1}{3}$.

203. Dividing by $33\frac{1}{3}$.

204. Multiplying by $12\frac{1}{2}$.

205. Dividing by $12\frac{1}{2}$.

Ex. **198.** Multiply by 100 and divide by 4; since $25 = \frac{100}{4}$.
Ex. **199.** Multiply by 4 and divide by 100; since $25 = \frac{100}{4}$.

Find the value of:

206. 3683×25.

207. $3876 \div 25$.

208. $4773 \times 33\frac{1}{3}$.

209. $4873 \div 33\frac{1}{3}$.

210. 4686×50.

211. $4686 \div 50$.

212. 7265×20.

213. $7265 \div 20$.

214. $9678 \times 16\frac{2}{3}$.

215. $9678 \div 16\frac{2}{3}$.

216. $6272 \times 12\frac{1}{2}$.

217. $6272 \div 12\frac{1}{2}$.

When the price of articles is given by the 100 or 1000, it is easier to reduce the number of articles to hundreds or thousands.

218. How much will 13625 bricks cost at $4.75 per M (1000)?

SUGGESTION.—There are 13.625 M. at $4.75 per M.

219. How much will 1875 pounds of beef cost at $7.25 per C (100)?

220. How much will 1975 pounds of hay cost at $6.75 per ton (2000 pounds)?

221. How much will 2146 feet of lumber cost at $22.50 per M?

222. How much will 875 pounds of binder twine cost at $6.25 per C?

ANALYSIS

In answers to problems involving U. S. money, the mills are usually dropped, but 5 or more are reckoned as an additional cent.

223. How much will $13\frac{1}{4}$ yards of cloth cost at \$1.25 a yard? $13\frac{1}{2}$ yards?

Ans. \$16.56(\$16.5625); \$16.88(\$16.875).

Before multiplying, it is often wise to determine the lowest order required in the product, and to neglect unnecessary orders in the multiplicand.

224. In $\$1.23456789 \times 52$, what decimal orders may be neglected in the multiplicand? Find the product true to cents.

The product must be true to cents, or to two decimal places. It is well to find one more decimal order than is required, because the figure in the last order will not always be exact. The highest order in the multiplier is *tens*; *tens* × *ten-thousandths* = *thousandths*; all to the right of *ten-thousandths* may be neglected.

225. In $\$20.123456789 \times 3200$, what decimal places may be neglected in the multiplicand? Find the product true to cents; to mills.

226. Find the value of $\$(1.06)^{10} \times 200$, true to cents.

We will find 3 decimal places in the answer. The highest order in the multiplier is *hundreds*; *hundreds* × *hundred-thousandths* = *thousandths*; all to the right of *hundred-thousandths* may be neglected; *i.e.*, we must retain 5 decimal places.

$(1.06)^{10} = 1.06 \times 1.06 \times 1.06 \times \ldots$ where 1.06 is taken 10 times as a factor. 1.06×1.06, or $(1.06)^2 = 1.1236$; 1.1236×1.1236, or $(1.06)^4 = 1.26247$; 1.26247×1.26247, or $(1.06)^8 = 1.59383$; 1.59383×1.1236, or $(1.06)^8 \times (1.06)^2$, or $(1.06)^{10}$, $= 1.79083$, $\$1.79083 \times 200 = \$358.17$. The pupil should perform these multiplications.

227. How many decimal places are there in the exact value of $(1.06)^{10}$?

228. If \$1 amounts to $\$(1.06)^5$ in 5 years, how much will \$234.63 amount to in the same time?

229. Find the cost of $5\frac{1}{2}$ cords of wood at $3.25 a cord; of $7\frac{1}{4}$ tons of hay at $4.75 a ton; of $53\frac{3}{4}$ bushels of corn at 23¢ a bushel.

230. The distance around a wheel is 3.1416 times its diameter. How many times will a wheel 4 feet in diameter turn around in going a mile (5280 feet)?

231. A wheel 3 feet in diameter turns 286 times in going a certain distance. What is the distance?

232. If a man's income is $2000 a year (365 days), and his average expenses are $3.68 a day, in how many days will he save $75? Prove the answer.

233. What is the average expense per day of a newspaper that costs $1.50 per year?

234. At an average expense of 5¢ per day, how much does a man spend for tobacco in 20 years?

235. There are 2150.42 cubic inches in a bushel and 231 cubic inches in a gallon. How many bushels are there in 86.378 gallons?

236. When pork sells in the market for $4.25 per hundred pounds, how many pounds can be bought for $340?

237. From a granary containing 287 bushels of corn, .36 of the whole was sold at 23¢ per bushel. How much was received from the sale?

238. A man worked 221 days at the rate of $1.15 per day, and took his pay in wheat at $62\frac{1}{2}$¢ per bushel. How many bushels did he receive?

239. A farmer raised 6768 bushels of corn from 160 acres of land, at an average cost of $6.25 per acre. How much will he gain if he sells the whole at 18¢ per bushel?

240. On a certain day the sales at a lumber yard were: 5280 feet rough lumber at $14.50 per M; 3060 feet dressed lumber at $17.25 per M; 4327 feet yellow pine flooring at $42.50 per M; 224 pounds lime at $6\frac{1}{4}$¢ per pound. To what did the sales amount?

DENOMINATE NUMBERS

ENGLISH SYSTEM

Substances may be measured in various ways to determine how much they possess of certain attributes, such as length, surface, volume, capacity, weight, value, ...

A denominate number answers the question, How much? *See p. 7.* Thus:

How much length? How much surface? How much volume? How much capacity? How much weight? How much value? The answers; 4 feet, 96 square feet, 64 cubic feet, 50 quarts, 2 tons, $50, are *denominate numbers.*

In naming units of measure, it is thought best to consider several smaller units as forming a unit of larger measure; several of this larger unit as forming a unit of still larger measure; . . . . Thus:

12 *inches* make 1 *foot;* 3 *feet* make 1 *yard;* 5½ *yards* make 1 *rod.* Each of these denominations is a unit of measure.

It is difficult to conceive of a system of measures more unscientific and more confusing than the English. Not only do the same denominations occur in different tables with different values, but there are four different tables for weight, two for capacity, and a multitude of miscellaneous units. The same number of each unit does not make one of the next higher; the names of the smaller and the larger units in the same table have nothing in common; the reduction from one denomination to another in the same or different tables, involves much labor. *Compare with metric system, p. 136.*

LENGTH

The length of a line is the number of linear units which it contains.

Thus: the line, A, contains the linear unit, M, 3 times; its length is 3 units. If M is 1 inch, A is 3 inches; if M is 1 foot, A is 3 feet; . . . .

The units of length are: *inch*, in.; *foot*, ft.; *yard*, yd.; *rod*, rd.; *mile*, mi.; *chain*, ch.; *link*, li. The surveyors' chain is 4 rd., or 66 ft., or 792 in., long, divided into 100 links of 7.92 in. each; the engineers' chain is 100 ft. long, divided into 100 li. of 1 ft. each.

Long Measure		Surveyors' Long Measure	
12 in.	= 1 ft.	7.92 in.	= 1 li.
3 ft.	= 1 yd.	25 li.	= 1 rd.
5½ yd. / 16½ ft.	= 1 rd.	4 rd. / 66 ft.	= 1 ch.
320 rd.	= 1 mi.	80 ch.	= 1 mi.

NOTE.—Other units are: *hand*, 4 in.; *fathom*, 6 ft.; *furlong*, 40 rd.; *league*, 3 mi.; . . . .

1. What instruments for measuring length have you seen? What does a carpenter use? a tailor? a dry-goods clerk?

2. Draw a line 12 units long. If each unit is an inch, what unit is the whole line? Draw a line 5½ units long. If each unit is a yard, what unit is the whole line?

3. Draw a square 6 units long and separate into 36 equal squares. If each linear unit is a mile, what is the length in rods of one side of each small square? of the large square?

4. In example 3, separate one of the smaller squares (a section) into 4 equal squares. How long is a quarter-section?

5. Draw a line an inch long; separate into 2 equal parts; each half into 2 equal parts; each quarter into 2 equal parts. What is the length of AB? of AC? of AE? of AH? of AF?

SURFACE

The area of a surface is the number of square units which it contains.

Thus: the surface, *A*, contains the square unit, *M*, 6 times; its area is 6 square units. If one side of *M* is 1 in., *A* is 6 square inches; if one side of *M* is 1 ft., *A* is 6 square feet; . . . .

The tables are formed by squaring the units of long measure: 12 in. = 1 ft., 144 sq. in. = 1 sq. ft.; 3 ft. = 1 yd., 9 sq. ft. = 1 sq. yd.; $5\frac{1}{2}$ yd., or $16\frac{1}{2}$ ft. = 1 rd.; $30\frac{1}{4}$ sq. yd., or $272\frac{1}{4}$ sq. ft. = 1 sq. rd. Other units are: *acre*, A.; *township*, Tp.

Square Measure

144 sq. in.	= 1 sq. ft.
9 sq. ft.	= 1 sq. yd.
$30\frac{1}{4}$ sq. yd. / $272\frac{1}{4}$ sq. ft.	= 1 sq. rd.
160 sq. rd.	= 1 A.

Surveyors' Square Measure

625 sq. li.	= 1 sq. rd.
16 sq. rd.	= 1 sq. ch.
10 sq. ch.	= 1 A.
640 A.	= 1 sq. mi.
36 sq. mi.	= 1 Tp.

Note.—The surveyors' chain was made 4 rd. long, because a tenth of an acre is 16 sq. rd., or a square whose side is 4 rd.

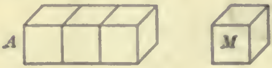
6. Separate a square 3 units long into 9 equal squares. If each linear unit is a foot, what unit is the large square?

7. Draw a square $5\frac{1}{2}$ units long, making a dot at the end of each unit. Connect the dots by lines parallel to the sides. How many whole squares are there? How many half squares are there? What part of one of the 25 equal squares is the small square in one corner? If each linear unit is a yard, what is the area of the large square?

8. Draw a rectangle 4 units long and 2 units wide; separate into strips, and separate one of the strips into squares. What is the relation between the number of squares in each strip and the number of linear units in the length? between the number of strips and the number of linear units in the width? What is the area of the rectangle?

VOLUME

The volume of a solid is the number of cubic units which it contains.



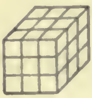
Thus: the solid, *A*, contains the cubic unit, *M*, 3 times; its volume is 3 cubic units. If one side of *M* is 1 in., *A* is 3 cubic inches; if 1 ft., *A* is 3 cubic feet; . . . .

The tables are formed by cubing the units in long measure: 12 in. = 1 ft.; 1728 cu. in. = 1 cu. ft.; . . . . A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high, is called a *cord*, cd.

Cubic Measure	Wood Measure
1728 cu. in. = 1 cu. ft.	16 cu. ft. = 1 cd. ft.
27 cu. ft. = 1 cu. yd.	8 cd. ft. = 1 cd.

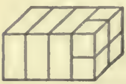
Note. — A perch of stone or masonry is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high, and contains $24\frac{3}{4}$ cu. ft.

9. Draw a cube whose length is 3 linear units, and separate into small cubes. If each linear unit is a foot, how long is one side of the cube? What is the area of each face? What is the entire surface of the cube? If each linear unit is 1 yd., answer the above questions.



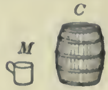
10. In example 9, if each linear unit is a foot, what unit is the large cube? each small cube? What is the volume of the large cube in cu. ft? in cu. yd.? What is the volume of each small cube in cu. ft.? How many times does the large cube contain the small cube?

11. Draw a prism 4 units long, 2 units wide, and 2 units high; separate into layers, and separate one of the layers into cubes. What is the relation between the number of cubes in each layer, and the product of the linear units in the width and the height? between the number of layers and the number of linear units in the length? What is the volume of the prism?



CAPACITY

The capacity of a vessel is the number of volume units which it contains.



Thus: C will hold the measure, M, 20 times; its capacity is 20 volume units. If M is a pint, the capacity of C is 20 pints; if a quart, C is 20 quarts; if a gallon, C is 20 gallons; . . . .

There are two sets of measures, one for liquids, and one for dry commodities. The units are: *gill*, gi.; *pint*, pt.; *quart*, qt.; *gallon*, gal.; *barrel*, bbl.; *bushel*, bu.

Liquid Measure	Dry Measure
4 gi. = 1 pt.	2 pt. = 1 qt.
2 pt. = 1 qt.	8 qt. = 1 pk.
4 qt. = 1 gal.	4 pk. = 1 bu.

12. Name an article that is sold by the gal.; by the barrel. Name an article that is sold by the dry qt.; by the peck; by the bushel.

13. Draw a rectangle, $11\frac{1}{4}$ in. long and $5\frac{3}{4}$ in. wide; draw the flap *BEFC*. Cut out the rectangle *AEFD*, roll *AD* to *BC* and paste *BEFC*; a cylinder holding a liquid quart will be formed.

14. In the same way prepare a cylinder holding a dry quart, making the dimensions of the rectangle $12\frac{1}{8}$ in. and $5\frac{3}{4}$ in. Notice that the dry quart is larger than the liquid quart.

15. Since a cube is a volume unit, capacity may also be expressed by cubic measure. A bu. is 2150.42 cu. in.; a gal. is 231 cu. in. How many cu. in. in a dry qt.? in a liquid qt.?

16. Capacity may be determined by weighing, and in many states the number of pounds of the different grains to be reckoned a bu. is fixed by law. How many bu. of corn are there in 224 lb., if a bu. = 56 lb.?

WEIGHT

For weight, there are four sets of measures: troy, for the precious metals; apothecaries', for dry medicines; apothecaries' fluid, for liquid medicines; avoirdupois, for the coarser articles of merchandise.

The troy units are: *grain*, gr.; *pennyweight*, pwt.; *ounce*, oz.; *pound*, lb. The apothecaries': *grain*, gr.; *scruple*, ℈; *dram*, ʒ; *ounce*, ℥; *pound*, ℔; *minim*, ♏; *pint*, O.; *gallon*, Cong. The avoirdupois: *ounce*, oz.; *pound*, lb.; *hundredweight*, cwt.; *ton*, T.

TROY

24 gr.	= 1 pwt.
20 pwt.	= 1 oz.
12 oz.	= 1 lb.

AVOIRDUPOIS

16 oz.	= 1 lb.
100 lb.	= 1 cwt.
20 cwt.	= 1 T.

APOTHECARIES'

20 gr.	= 1 ℈
3 ℈	= 1 ʒ
8 ʒ	= 1 ℥
12 ℥	= 1 ℔

APOTHECARIES' FLUID

60 ♏	= 1 f ʒ
8 f ʒ	= 1 f ℥
16 f ℥	= 1 O.
8 O.	= 1 Cong.

NOTE.—The English, or long ton of 2240 lb., is used at the U. S. Custom House.

17. The grain is the same for all measures; 5760 gr. = 1 troy or apothecaries' pound; 7000 gr. = 1 avoirdupois pound. How many gr. make a troy or apothecaries' oz.? an avoirdupois oz.?

18. 60 drops make a teaspoonful; 8 teaspoonfuls make an ounce. 30 drops are what part of a teaspoonful? How many doses of 10 drops to a dose, are there in an ounce of medicine?

19. A pint of water weighs an avoirdupois pound (nearly). Why do 16 oz. instead of 12, make a pint apothecaries'?

20. In writing prescriptions, physicians employ the small letters of the Roman notation, placing the symbols first, and writing j for final i. Read: ℥ij ʒiij ℈ij gr. xij.

TIME AND ARC

A *day*, da., is the time of the revolution of the earth upon its axis; a *month*, mo., the time of the revolution of the moon around the earth; a *year*, yr., the time of the revolution of the earth around the sun. The other units are: *second*, sec.; *minute*, min.; *hour*, hr.; *week*, wk.; *century*, cen.

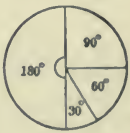
Circular measure is used in measuring arcs and angles. The units are: *second*, ″; *minute*, ′; *degree*, °; *sign*, S; *circumference*, C. The sign is rarely used.

TIME

60 sec.	=	1 min.
60 min.	=	1 hr.
24 hr.	=	1 da.
7 da.	=	1 wk.
52 wk. 1 da.	=	1 yr.
12 mo.	=	1 yr.
365 da.	=	1 yr.
366 da.	=	1 leap year.
100 yr.	=	1 cen.

CIRCULAR MEASURE

$60'' = 1'$.
$60' = 1^\circ$.
$360^\circ = 1$ C.



NOTE.—$30^\circ = 1$ sign; 12 signs = 1 circumference; $90^\circ = 1$ right angle.

21. How many degrees are there in a right angle? in the circumference of a circle? What is the angular space about a point?

22. Name the months and give the number of days in each. January is the first month; name and give the ordinal numeral for each month.

23. A year lacks 674 sec. of being $365\frac{1}{4}$ days. The fourth of a day is disregarded for 3 years and a whole day is added to February every year that is divisible by 4. How much of an error does this plan make in 400 yr.?

24. How is this error of 3 da. in 400 yr. corrected? Is 1900 a leap year? is 2000?

Ans. The addition of 1 day to centennial years is omitted unless the centennial year is divisible by 400.

VALUE

The units in U. S. and Canada money are: *mill*, m. (Latin, *mille*, 1000); *cent*, ¢ (Latin, *centum*, 100); *dime*, d. (Latin, *decem*, 10); *dollar*, $; *eagle*, E.

The units in English money are: *farthing*, far. or qr.; *penny*, plural, *pence*, d.; *shilling*, s.; *pound*, £ ($4.8665).

The French units are: *centime, decime, franc* (19.3 cents).

The German units are: *pfennig, mark* (23.85 cents).

United States Money

10 m.	= 1¢
10¢	= 1 d.
10 d.	= $1
$10	= 1 E.

English Money

4 far.	= 1 d.
12 d.	= 1 s.
20 s.	= £1

French Money

10 centimes	= 1 decime
10 decimes	= 1 franc

German Money

100 pfennigs = 1 mark

Note. — The English use also *crown* (5 shillings); *florin* (2 shillings); *sovereign* (£1).

The pupil should thoroughly memorize the following equivalents:

24 sheets	= 1 quire	12 units	= 1 dozen
20 quires	= 1 ream	12 dozen	= 1 gross
480 sheets	= 1 ream	12 gross	= 1 great gross
231 cu. in.	= 1 gal.	7000 gr.	= 1 lb. avoir.
2150.4 cu. in.	= 1 bu.	60 ♏	= 1 teaspoonful
4 bu. (approx.)	= 5 cu. ft.	48 lb.	= 1 bu. barley
7½ gal. (approx.)	= 1 cu. ft.	56 lb.	= 1 bu. corn
24 hours	= 360°	32 lb.	= 1 bu. oats
5760 gr.	= 1 lb. troy	56 lb.	= 1 bu. rye
5760 gr.	= 1 lb. apoth.	60 lb.	= 1 bu. wheat
62½ lb.	= weight 1 cu. ft. water	60 lb.	= 1 bu. potatoes

EXERCISES

U. S. Money

How many:

25. m. make $1?

26. d. make $1?

27. ¢ make $1?

28. ¢ make 1 E.?

29. d. make 1 E.?

30. nickels make $1?

31. quarters make $1?

32. m. make 1 d.?

33. m. make 1 E.?

34. m. make 1 ¢?

35. ¢ make 1 d.?

36. $ make 1 E.?

37. halves make $1?

38. ¢ make 1 quarter?

Foreign Money

How many:

39. far. make £1?

40. s. make £1?

41. s. make 1 crown?

42. s. make 1 florin?

43. far. make 1 s.?

44. centimes make 1 franc?

45. cents make 1 franc?

46. d. make 1 s.?

47. d. make 1 crown?

48. d. make £1?

49. far. make 1 d.?

50. dollars make £1?

51. pfennigs make 1 mark?

52. cents make 1 mark?

Troy and Avoirdupois Weights

How many:

53. gr. make 1 lb. (troy)?

54. gr. make 1 oz. (troy)?

55. pwt. make 1 lb.?

56. pwt. make 1 oz.?

57. oz. make 1 lb. (troy)?

58. gr. make 1 lb. (avoir.)?

59. gr. make 1 oz. (avoir.)?

60. gr. make 1 ℔ (apoth.)?

61. oz. make 1 ℔ (apoth.)?

62. cwt. make 1 T.?

63. lb. make 1 T.?

64. gr. make 1 T.?

65. lb. make 1 long T.?

66. oz. make 1 lb. (avoir.)?

Apothecaries' Weight

How many:

67. ♏ make 1 fʒ?
68. fʒ make 1 f℥?
69. fʒ make 1 O.?
70. f℥ make 1 Cong.?
71. fʒ make 1 Cong.?
72. drops make 1 fʒ?
73. drops make 1 teaspoonful?
74. gr. make 1 ℔?
75. gr. make 1 ℥?
76. ℈ make 1 ℔?
77. ʒ make 1 ℔?
78. ʒ make 1 ℥?
79. drops make 1 f℥?
80. teaspoonfuls make 1 f℥?

Long Measure

How many:

81. in. make 1 yd.?
82. in. make 1 rd.?
83. ft. make 1 rd.?
84. ft. make 1 mi.?
85. rd. make 1 mi.?
86. in. make 1 hand?
87. ft. make 1 fathom?
88. li. make 1 ch.?
89. li. make 1 rd.?
90. ch. make 1 rd.?
91. ch. make 1 mi.?
92. in. make 1 li.?
93. rd. make 1 furlong?
94. mi. make 1 league?

Square Measure

How many:

95. sq. in. make 1 sq. ft.?
96. sq. in. make 1 sq. yd.?
97. sq. ft. make 1 sq. rd.?
98. sq. rd. make 1 A.?
99. sq. ft. make 1 sq. yd.?
100. A. make 1 section?
101. A. make 1 quarter section?
102. sq. ch. make 1 A.?
103. sq. li. make 1 sq. ch.?
104. sq. li. make 1 sq. rd.?
105. A. make 1 sq. mi.?
106. sq. mi. make 1 Tp.?
107. A. make 1 half section?
108. sections make 1 Tp.

Cubic and Wood Measures

How many:

109. cu. in. make 1 cu. ft.?
110. cu. in. make 1 cu. yd.?
111. cu. ft. make 1 cu. yd.?
112. cu. in. make 1 cd. ft.?
113. cu. in. make 3 cu. ft.?
114. cu. ft. make 1 perch?
115. cu. in. make 1 perch?
116. cu. ft. make 1 cd.?
117. cu. ft. make 1 cd. ft.?
118. cu. ft. make 3 cu. yd.?
119. cu. ft. make 2 cu. yd.?
120. cu. ft. make 3 cd.?
121. cd. ft. make 4 cd.?
122. cu. ft. make 4 cd.?

Liquid and Dry Measures

How many:

123. pt. make 1 qt. (dry)?
124. qt. make 1 pk.?
125. pt. make 1 bu.?
126. pt. make 1 pk.?
127. pk. make 1 bu.?
128. lb. in 1 bu. corn?
129. lb. in 1 bu. oats?
130. pt. make 1 qt. (liquid)?
131. qt. make 1 gal.?
132. qt. make 1 bbl. of $31\frac{1}{2}$ gal.?
133. pt. make 1 bbl. of 45 gal.?
134. qt. make 1 bbl. of 40 gal.?
135. lb. in 1 bu. wheat?
136. lb. in 1 bu. potatoes?

Miscellaneous

How many:

137. da. make 1 leap year?
138. degrees make 1 C.?
139. cu. in. make 1 gal.?
140. cu. in. make 1 bu.?
141. degrees make 1 S.?
142. da. in year 1900?
143. da. in year 1904?
144. units make 1 dozen?
145. units make 1 score?
146. sheets make 1 quire?
147. sheets make 1 ream?
148. units make 1 gross?
149. da. in year 2000?
150. da. in year 2004?

TO LOWER DENOMINATIONS

151. Reduce 2 T. 4 cwt. 3 lb. 6 oz. to oz.

T.	cwt.	lb.	oz.
2	*4*	*3*	*6*

20
44 cwt.
100
4403 lb.
16
70454 oz.

1 T. is 20 cwt.; 2 T., 2 times 20 cwt., or, with 4 cwt., 44 cwt.

1 cwt. is 100 lb.; 44 cwt., 44 times 100 lb., or, with 3 lb., 4403 lb., etc.

Ans. 70,454 oz.

152. Reduce .875 bu. to integers of lower denominations.

.875 bu.
4
3.500 pk.
8
4.0 qt.

1 bu. is 4 pk.; .875 bu., .875 times 4 pk., or 3.5 pk.

1 pk. is 8 qt.; .5 pk., .5 times 8 qt., or 4 qt.

Ans. 3 pk. 4 qt.

153. Reduce $\frac{5}{9}$ ℔ apothecaries' to integers of lower denominations.

$\frac{5}{9}$ ℔ $= \frac{5}{9} \times 12$, *or* $6\frac{2}{3}$ ℥.

$\frac{2}{3}$ ℥ $= \frac{2}{3} \times 8$, *or* $5\frac{1}{3}$ ʒ.

$\frac{1}{3}$ ʒ $= \frac{1}{3} \times 3$, *or* 1 ℈.

1 ℔ is 12 ℥; $\frac{5}{9}$ ℔ is $\frac{5}{9}$ times 12 ℥, or $6\frac{2}{3}$ ℥. 1 ℥ is 8 ʒ; $\frac{2}{3}$ ℥ is $\frac{2}{3}$ times 8 ʒ, or $5\frac{1}{3}$ ʒ. 1 ʒ is 3 ℈; $\frac{1}{3}$ ʒ is $\frac{1}{3}$ times 3 ℈, or 1 ℈.

Ans. 6 ℥ 5 ʒ 1 ℈.

Reduce:

154. 3 bu. 2 pk. 1 pt. to pt.

155. £8 3 s. 5 d. 2 far. to far.

156. ℥ iij, ʒ vij to drops. *See p. 124.*

157. 1 ℔ 8 ℥ 7 ʒ 2 ℈ 19 gr. to gr.

Reduce to integers of lower denominations:

158. .628 T.
159. .345 mi.
160. $\frac{11}{20}$ A.
161. $\frac{5}{6}$ hr.
162. .125 yr.
163. .385 A.
164. $\frac{5}{12}$ cu. yd.
165. $\frac{5}{9}$ lb. troy.
166. £.1416.
167. .3285 bu.
168. $\frac{8}{9}$ mi.
169. $\frac{7}{18}$ T.
170. .7854 ℔.
171. .2825 C.
172. $\frac{7}{8}$ gal.
173. $\frac{5}{6}$ Tp.

TO HIGHER DENOMINATIONS

174. Reduce 2736 oz. avoirdupois to integers of higher denominations.

16)2736 oz.
100)171 lb.
1 cwt. 71 lb.

16 oz. are 1 lb.; 2736 oz., as many lb. as 16 is contained times in 2736, or 171 lb.; etc.

Ans. 1 cwt. 71 lb.

175. Reduce 4 cwt. 3 lb. 8 oz. to the fraction of a T.

$$8 \text{ oz.} = \frac{8}{16}, \text{ or } \tfrac{1}{2} \text{ lb.}$$

$$3\tfrac{1}{2} \text{ lb.} = \frac{7}{2} \div 100, \text{ or } \frac{7}{200} \text{ cwt.}$$

$$4\tfrac{7}{200} \text{ cwt.} = \frac{807}{200} \div 20, \text{ or } \frac{807}{4000} \text{ T.}$$

Or

4 cwt. 3 lb. 8 oz.	= 6456 oz.
1 T.	= 32000 oz.
The fraction	= $\frac{6456}{32000}$ T.
	= $\frac{807}{4000}$ T.

176. Reduce 7 ℥ 3 ʒ 2 ℈ to the decimal of a ℔.

3)2 ℈
8)3.666+ ʒ
12)7.458+ ℥
.621+ ℔

3 ℈ are 1 ʒ; 2 ℈, as many ʒ as 3 is contained times in 2, or, with 3 ʒ, 3.666+ ʒ; etc.

Ans. .621+ ℔.

Reduce to integers of higher denominations:

177. 8269 gi.
178. 3846 gr. troy.
179. 8637 gr. apothecaries'.
180. 6732 in. surveyors'.
181. 32844 s.
182. 46381 in.
183. 47351 sq. in.
184. 48394 cu. in.

Reduce to common fractions of the highest denomination:

185. 8 ℥ 7 ʒ 1 ℈ 10 gr.
186. 1 A. 9 sq. ch. 2000 sq. li.
187. 1 lb. 9 oz. 2 pwt. 12 gr.
188. 5 ch. 60 li. 3.96 in.
189. 2 cd. 7 cd. ft. 12 cu. ft.
190. 2 T. 15 cwt. 60 lb. 4 oz.

Reduce to decimals of the highest denomination:

191. 3 gal. 2 qt. 1 pt.
192. 5 bu. 3 pk. 7 pt.
193. £6 8s. 6d.
194. 7.16 T. 6 cwt. 5 oz.
195. 40° 20′ 15″.
196. 5 yr. 4 mo. 3 wk. 5 da.

FROM TABLE TO TABLE

197. Reduce 288 lb. avoirdupois to lb. troy.

$$\frac{288 \times 7000}{5760} = 350$$

7000 gr. = 1 lb. avoirdupois.
5760 gr. = 1 lb. troy.

198. Reduce 87 ℔ 6 ℥ apothecaries' to lb. avoirdupois.

$$\frac{87.5 \times 5760}{7000} = 72$$

7000 gr. = 1 lb. avoirdupois.
5760 gr. = 1 lb. apothecaries'.

199. Reduce 10 bbl. to bu., approximately.

$$10 \times \frac{63}{2} \times \frac{2}{15} \times \frac{4}{5} = 33.6$$

$31\frac{1}{2}$ gal. = 1 bbl.; $7\frac{1}{2}$ gal. = 1 cu. ft.; 5 cu. ft. = 4 bu.

200. Reduce 10 bbl. to bu., true to 1 decimal place.

$$10 \times \frac{63}{2} \times \frac{231}{2150.4} = 33.8$$

$31\frac{1}{2}$ gal. = 1 bbl.; 231 cu. in. = 1 gal.; 2150.4 cu. in. = 1 bu.

201. Reduce 11° 13′ 47″ to time.

	min.	sec.
$11° =$	44	
$13' =$		52
$47'' =$		$3\frac{2}{15}$
	44	$55\frac{2}{15}$

$360° = 24$ hr.
$1° = \frac{24}{360}$ or $\frac{1}{15}$ hr., or 4 min.
$1' = \frac{4}{60}$ or $\frac{1}{15}$ min., or 4 sec.
$1'' = \frac{4}{60}$ or $\frac{1}{15}$ sec.

202. Reduce 6 hr. 3 min. to arc.

6 hr. =	90°	
3 min. =		45′
	90°	45′

24 hr. = 360°.
1 hr. = $\frac{360}{24}$ or 15°.
1 min. = $\frac{15}{60}$ or $\frac{1}{4}$°, or 15′.

Reduce and explain:

203. 630 lb. avoir. to apoth.

204. 6 lb. 9 oz. troy to avoir.

205. 30 cu. ft. to bu. approx.

206. 10 bu. to bbl. approx.

207. 10 bu. to bbl., to 1 dec. place.

208. 5 bu. to gal. approximately.

209. 5° 5′ 5″ to time; 6° 6′ to time.

210. 5 hr. 5 min. 5 sec. to arc.

ADDITION

211. Add: 4 rd. 4 yd. 2 ft. 7 in., 3 rd. 5 yd. 1 ft. 8 in.

rd.	yd.	ft.	in.
4	*4*	*2*	*7*
3	*5*	*1*	*8*
8	*$4\frac{1}{2}$*	*1*	*3*
		1	*6*
8	*4*	*2*	*9*

Since $\frac{1}{2}$ yd. = 1 ft. 6 in., to avoid fractions in the answer, we may proceed as above.

The sum of the in. is 15 in., or 1 ft. 3 in.; we write 3 in in. column and carry 1 to ft. column.

The sum of the ft. is 4 ft., or 1 yd. 1 ft.; we write 1 in ft. column and carry 1 to yd. column. *See p. 20.*

212. To Dec. 5, 1883, add 165 da.

Dec. 170, Mar. 80,
Jan. 139, Apr. 49,
Feb. 108, May 19.

Dec. 5 + 165 = Dec. 170. Since there are 31 da. in Dec., Dec. 170 = Jan. 139; . . . .

213. To Apr. 30, add 3 mo. 3 da.

July 30, July 33, Aug. 2.

Apr. 30 + 3 mo. = July 30. July 30 + 3 da. = July 33, or Aug. 2.

214. Add: 5 T. 4 cwt. 16 lb. 5 oz., 16 T. 17 cwt. 13 oz. 75 lb., 3 T. 7 cwt. 12 oz.

215. Add: 5 rd. 3 yd. 2 ft. 3 in., 13 rd. 5 yd. 2 ft. 9 in., 7 rd. 2 yd. 5 ft.

216. Add: 3 lb. 3 oz. 7 pwt. 22 gr., 16 lb. 11 oz. 19 pwt. 3 gr., 7 lb. 14 gr.

217. Add: 20 yr. 5 mo. 6 da. 4 hr., 30 yr. 8 mo. 7 da. 16 hr., 9 mo. 7 da. 7 hr.

218. Add: $\frac{5}{9}$ mi. and $\frac{5}{9}$ rd.; first, reduce each fraction to integers of lower denominations.

219. Add $\frac{5}{9}$ mi. and $\frac{5}{9}$ rd.; first, reduce each fraction to the decimal of a rod.

220. Add $\frac{3}{4}$ lb. and $\frac{2}{5}$ pwt; first, reduce each fraction to grains.

221. To July 23, add 93 da. To Aug. 5, add 100 da.

222. To July 23, add 3 mo. 3 da. To Mar. 20, add 200 da.

SUBTRACTION

223. Subtract 20 gal. 2 qt. 1 pt. from 30 gal.

gal.	qt.	pt.
30	*0*	*0*
20	*2*	*1*
9	*1*	*1*

1 pt. from 0 pt. we cannot take; we add 2 pt. to 0 pt., making 2 pt., and 1 qt. to 2 qt., making 3 qt. 1 pt. from 2 pt. leaves 1 pt.; we write 1 in pt. column. *See p. 30.*

224. Find the exact number of days from July 20, 1897, to Nov. 19, 1897.

11, July	*31, Oct.*
31, Aug.	*19, Nov.*
30, Sept.	*122, Ans.*

In July there are 11 da. left; in Aug., 31; in Sept., 30; etc.

NOTE.—This method is used in Equation of Payments. *See p. 231.*

225. Find the time from July 20, 1897, to Nov. 19, 1897.

1897	*11*	*19*
1897	*7*	*20*
	3	*29*

Nov. 19, 1897, is the 1897th yr., the 11 mo., and the 19th da.

NOTE.—This is the method commonly used in Partial Payments. *See p. 223.*

226. Subtract 5 lb. 6 oz. 3 pwt. 9 gr. from 8 lb. 4 gr.; 4 mi. 100 rd. 4 yd. from 5 mi.

227. Subtract 5 rd. 4 yd. 6 in. from 16 rd. 1 ft. 1 in.; 3 A. 142 sq. rd. from 5 A.

228. From $\frac{5}{7}$ da., subtract $4\frac{4}{9}$ hr. Give the answer (*a*) as a common fraction of a day; (*b*) as a decimal of a day; (*c*) in hr., min., and sec.

229. Find the exact number of days from Sept. 3, 1899, to Apr. 4, 1900.

230. Find the time from Sept. 3, 1899, to Apr. 4, 1900, as practiced in partial payments.

231. How many days' difference by the methods in examples 229 and 230?

MULTIPLICATION AND DIVISION

232. Multiply 2 T. 3 cwt. 46 lb. by 8.

T.	cwt.	lb.
2	*3*	*46*
		8
17	*7*	*68*

8 times 46 lb. are 368 lb., or 3 cwt. and 68 lb.; we write 68 in lb. column, and carry 3 cwt. *See p. 38.*

233. Divide 17 T. 7 cwt. 68 lb. by 8.

	T.	cwt.	lb.
8)	*17*	*7*	*68*
	2	*3*	*46*

17 T ÷ 8 = 2 T. and 1 T. remaining; we write 2 in T. column.

1 T. and 7 cwt. = 27 cwt.; 27 cwt. ÷ 8 = 3 cwt. and 3 cwt. remaining; etc. *See p. 51.*

234. Divide 19 rd. 5 yd. 10 in. by 2 rd. 2 yd. 2 ft. 2 in.

19 rd. 5 yd. 10 in. = 3952 in.
2 rd. 2 yd. 2 ft. 2 in. = 494 in.
494)3952(8

We reduce both dividend and divisor to the same denomination.

Multiply:

235. 10° 36′ 48″ by 8.

236. 2 bu. 3 pk. 4 qt. by 9.

237. 3 gal. 1 qt. 1 pt. by 5.

238. 6 rd. 4 yd. 2 ft. 3 in. by 6.

239. 4 da. 8 hr. 25 min. 40 sec. by 7.

240. 4 sq. yd. 2 sq. ft. 56 sq. in. by 4.

Divide:

241. 3 da. 6 hr. 2 min. by 4.

242. 3 mi. 6 ft. 10 in. by 6.

243. 15 T. 16 cwt. 4 lb. by 9.

244. 2 bu. 3 pk. by 5 pk. 1 qt.

245. 5 yr. 2 mo. 2 da. by 12.

246. 20 bbl. 3 gal. by 3 qt. 1 pt.

247. 3 rd. 5 yd. 2 ft. by 6 yd. 1 ft.

248. £6 6s. 6d. by 3d. 3 far.

METRIC SYSTEM — LAWS

The principal unit of each table is derived from the *meter*. The other units are formed by prefixing to the principal unit the Latin sub-multiples: *milli*, 1000; *centi*, 100; *deci*, 10, and the Greek multiples: *deca*, 10; *hecto*, 100; *kilo*, 1000; *myria*, 10,000. The abbreviation of each sub-multiple is the small form of its first letter; of each multiple, the capital form.

Sub-Multiples and Multiples

10 milli *units* = 1 centi *unit*	10 deca *units* = 1 hecto *unit*
10 centi *units* = 1 deci *unit*	10 hecto *units* = 1 kilo *unit*
10 deci *units* = 1 *unit*	10 kilo *units* = 1 myria *unit*

10 units = 1 deca *unit*

249. Substitute *meter* for *unit* in the table above, and write the new table thus formed; compare with long measure, p. 137.

250. In the same way, substitute *are*, and compare with the table of land measure, p. 138.

251. Substitute *stere*, *liter*, *gram*, and compare with tables for wood measure, capacity, weight, pp. 139, 140, 141.

252. Give the sub-multiples for 100, 10, 1000. State the meaning of deci, milli, centi. Read m, d, c.

253. Give the multiples for 100, 10, 10000, 1000. State the meaning of deca, kilo, hecto, myria. Read K, M, D, H.

254. How many *milli* make 1 unit? How many centi? How many deci?

255. How many units make 1 deca? 1 hecto? 1 kilo? 1 myria?

256. How many m make 1 M? How many c make 1 H?

Ans. 10,000,000 m = 1 M; m = 1000; M, 10,000. Since one is a sub-multiple and the other a multiple, we multiply 1000 by 10,000.

257. How many D make 1 M? How many m make 1 d?

Ans. 1000 D = 1 M; D = 10; M, 10,000. Since both are multiples, we divide 10,000 by 10.

LENGTH

The principal unit of long measure is the *meter*, m. It is designed to be 1 ten-millionth of the distance from the equator to the pole; its equivalent is 39.37 in. The other units are formed by prefixing to the principal unit the sub-multiples and multiples, as explained on p. 136. Approximately, 1 m = 1.1 yd.; 1 Km = $\frac{5}{8}$ mi.

Long Measure

10 mm = 1 cm	10 Dm = 1 Hm
10 cm = 1 dm	10 Hm = 1 Km
10 dm = 1 m	10 Km = 1 Mm
10 m = 1 Dm	

258. State the principal unit of long measure; its abbreviation; how obtained; its English equivalent; approximate equivalents for 1 m, 1 Km.

259. Explain how the table of long measure is made up from the table of sub-multiples and multiples. *See Ex. 249.*

260. To illustrate *long measure*, prepare a strip of paper 39$\frac{3}{8}$ in. long; divide it into 10 equal parts (dm); divide each dm into 10 equal parts (cm); divide the first cm into 10 equal parts (mm).

261. To illustrate long measure, place 5 nickels (5-cent pieces) in a row; the row will be nearly 1 decimeter long, since the diameter of each nickel is nearly 2 centimeters. Place them in a pile; the pile will be nearly 1 centimeter high, since each nickel is nearly 2 millimeters thick.

262. How tall are you? How long is your arm? What is the length of your forefinger? What is the thickness of your thumb nail? What is the width of your thumb nail?

263. Prove that 1 Km = $\frac{5}{8}$ mi. (nearly). Equivalents: 1 m = 39.37 in., 1 mi. = 5280 ft.

264. What is the distance around the earth in meters? in inches? in miles?

SURFACE

Square measure is formed by squaring long measure. Thus: 10 mm = 1 cm; 100 sq mm = 1 sq cm; . . . .

The principal unit of land measure is the *are*, a; it is a square, 10 m by 10 m. Approximately, 1 are = $\frac{1}{40}$ of an acre.

SQUARE MEASURE	LAND MEASURE
100 sq mm = 1 sq cm	10 ma = 1 ca
100 sq cm = 1 sq dm	10 ca = 1 da
100 sq dm = 1 sq m	10 da = 1 a
100 sq m = 1 sq Dm	10 a = 1 Da
100 sq Dm = 1 sq Hm	10 Da = 1 Ha
100 sq Hm = 1 sq Km	10 Ha = 1 Ka
100 sq Km = 1 sq Mm	10 Ka = 1 Ma

NOTE. — The final vowel of each sub-multiple and multiple is dropped before *are*; millare, centare, ...; not *milliare*, *centiare*, ...

The table of land measure is often given: 100 ca = 1 a; 100 a = 1 Ha.

265. State the principal unit of land measure; its abbreviation; how obtained; its approximate equivalent.

266. Read: 50.23 sq mm; 238.6 sq Mm; 560 a; 378.5 Ha 50.23 sq Dm.

267. How many sq mm make 1 sq Km? How many sq Dm make 1 sq Mm?

268. Is the multiple in land measure 10 or 100? In the abbreviated form, why does it appear to be 100?

269. How could you illustrate land measure on the school grounds?

270. How much land would you like for a flower garden? What is the area of this floor? How much land is needed for a good farm?

271. Prove that 1 are is nearly $\frac{1}{40}$ of an acre. Equivalents: 1 a = 10 m × 10 m; 1 m = 39.37 in.

272. An emigrant exchanges his farm of 4 hectares, at 12 francs an are, for land in America at $ 5 an acre. How many acres does he secure?

VOLUME

Cubic measure is formed by cubing long measure. Thus: 10 mm = 1 cm; 1000 cu mm = 1 cu cm; . . . .

The principal unit of wood measure is the *stere*, s; it is a cube, 1 m × 1 m × 1 m. Approximately, 1 stere = ¼ of a cord.

Cubic Measure	Wood Measure
1000 cu mm = 1 cu cm	10 ms = 1 cs
1000 cu cm = 1 cu dm	10 cs = 1 ds
1000 cu dm = 1 cu m	10 ds = 1 s
1000 cu m = 1 cu Dm	10 s = 1 Ds
1000 cu Dm = 1 cu Hm	10 Ds = 1 Hs
1000 cu Hm = 1 cu Km	10 Hs = 1 Ks
1000 cu Km = 1 cu Mm	10 Ks = 1 Ms

Note. — The table of wood measure is often given, 10 ds = 1 s, the other units being omitted.

273. Explain how the table of cubic measure is made up from the table of long measure.

274. Explain how the table of wood measure is made up from the table of sub-multiples and multiples. *See Ex. 251.*

275. State the principal unit of wood measure, its abbreviation; how obtained; its approximate equivalent.

276. How many cu dm make 1 cu Hm? cu Dm make 1 cu Mm?

277. How could you illustrate a stere on the school grounds?

278. What is the contents of this room in cu m? How much earth will make a good load for two horses? Approximately, how many steres are there in a cord of wood?

279. Prove that 1 stere is nearly ¼ of a cord. Equivalents: 1 stere = 1 m × 1 m × 1 m; 1 m = 39.37 in.

280. Which is the cheaper, to buy wood at $ 3 a cord or at $ 1 a stere?

281. What is the cost of excavating 40 cu Dm of earth at 12¢ a cu m?

CAPACITY

The principal unit of capacity is the *liter*, l; it is a cube, 1 dm by 1 dm by 1 dm; its equivalent is .908 qt. dry, or 1.05 qt. liquid. Approximately, 1 l = 1 qt.

Table of Capacity

10 ml = 1 cl	10 Dl = 1 Hl
10 cl = 1 dl	10 Hl = 1 Kl
10 dl = 1 l	10 Kl = 1 Ml
10 l = 1 Dl	

282. State the principal unit of capacity; its abbreviation; how obtained; its exact equivalent; its approximate equivalent.

283. How much milk do you need for a cup of coffee? How much milk per day would you use for a family of six? How much will a teacup hold? How much will a tablespoon hold?

284. How many liters in 23.645 cu m? How many cu m in 385,623 l?

285. Prove that 1 liter is .908 qt. dry. Equivalents: 1 liter = 1 dm × 1 dm × 1 dm; 1 m = 39.37 in; 1 bu. = 2150.4 cu. in.

286. Prove that 1 liter is 1.05 qt. liquid. Equivalents: 1 liter = 1 dm × 1 dm × 1 dm; 1 m = 39.37 in.; 1 gal. = 231 cu. in.

287. Counting a liter as a quart, what is the capacity in cu m of a bin that will hold 256 bu.?

288. If the bin in Ex. 287 is filled with wheat, what is the value of the wheat at 22¢ a Dl?

289. To illustrate measures of capacity, draw a rectangle, as *ABCD*, whose length is 29.16 cm, and breadth, 14.78 cm. Draw the flap *BEFC*. Cut out *AEFD*, roll over *AD* to *BC* and paste *BEFC*; a cylinder holding a liter will be formed. Compare with the liquid and dry quarts. *See Exs. 14 and 15.*

WEIGHT

The principal unit of weight is the *gram*, g; it is the weight of a cu cm of pure water at its maximum density; its exact equivalent is 15.432 gr. Approximately, 1 Kg. = $2\frac{1}{5}$ lb. avoirdupois.

Table of Weight

10 mg = 1 cg	10 Dg = 1 Hg
10 cg = 1 dg	10 Hg = 1 Kg
10 dg = 1 g	10 Kg = 1 Mg
10 g = 1 Dg	

Note. — There are two additional units: 10 myriagrams = 1 *quintal*, Q; 10 quintals = 1 *tonneau*, T.

290. State the principal unit of weight; its abbreviation; how obtained; its exact equivalent; the approximate equivalent of 1 Kg in avoirdupois pounds.

291. How many dg in 3 Hg? How many cg in 63 Dg? How many mg in 46 Kg?

292. How many grams in 1273.14 gr.? How many Hg? How many grains in 9 Dg 8 g?

293. To illustrate *weights*, procure a piece of tin foil that weighs as much as a nickel (5-cent piece); cut the foil into five equal parts. Each part will weigh a gram, since a nickel weighs 5 grams. Procure a stone that weighs 2 lb. 3 oz.; it will weigh 1 Kg.

294. What is your weight? How much beefsteak would you buy for breakfast for three? What is the weight of a nickel? How heavy a letter of the first class will go for 2¢? What does an average horse weigh?

295. Prove that 1 cu m of water weighs nearly 1 long ton.

296. Prove that 1 Kg is nearly $2\frac{1}{5}$ lb. av.; nearly $2\frac{2}{3}$ lb. troy.

297. Prove that the weight of 1 gram is 15.432 gr. Relations: 1 g = weight of 1 cu cm of water; 1 m = 39.37 in.; 1 cu. ft. water weighs $62\frac{1}{2}$ lb.

APPROXIMATIONS — MENTAL EXERCISES

The approximate equivalents given in the several tables should be memorized, and should be used except when exact results are required.

Reduce approximately:

298. 5 bu. to l.

299. 10 g to gr.

300. 30 gr. to g.

301. 10 bu. to l.

302. 12 l to gal.

303. 22 lb. to Kg.

304. 60 gal to l.

305. 84 l to gal.

306. 64 l to bu.

307. 15 gal. to l.

308. 256 l to bu.

309. 9 cords to s.

Reduce approximately:

310. 16 Km to mi.

311. 16 Dl to bu.

312. 75 l to qt.

313. 32 l to bu.

314. 40 l to gal.

315. 10 mi. to Km.

316. 42 l to gal.

317. 60 qt. to l.

318. 25 l to cu. in.

319. 10 l to cu. in.

320. 400 a to acres.

321. 72 s to cords.

Reduce approximately:

322. 9 Kg to lb.

323. 8 acres to a.

324. 60 m to ft.

325. 4 m to in.

326. 3 m to yd.

327. 1 T. of water to cu. ft.

328. 6 tons of water to cu m.

329. 10 tons of water to cu m.

330. 10 cu m of water to tons.

331. 7500 sq. miles to sq Km.

EXACT RESULTS — WRITTEN EXERCISES

The exact equivalents of the principal units should be memorized; the exact equivalents of the other units should be computed from the principal units.

332. Reduce 24 Km to mi.

$$\frac{24 \times 1000 \times 39.37}{12 \times 5280} = 14.9+$$

24 Km = 14.9+ mi.

The principal unit of long measure is the meter, 39.37 in.; 1 Km = 1000 m; 1 mi. = 5280 ft.

333. Reduce 5 Ha to sq. dm.

$$5 \times 100 \times 100 \times 100 = 5{,}000{,}000$$

5 Ha = 5,000,000 sq dm.

The principal unit of land measure is the are, 100 sq m; 1 Ha = 100 a; 1 sq m = 100 sq dm.

334. Reduce 5 cu Dm to cubic yd.

$$\frac{5 \times 1000 \times 39.37^3}{1728 \times 27} = 6539.7+$$

5 cu Dm = 6539.7+ cu. yd.

The principal unit of cubic measure is 1 cu m, $(39.37)^3$ cu. in.; 1728 cu. in. = 1 cu. ft.; 27 cu. ft. = 1 cu. yd.

335. Reduce 365 cu m to Dl.

$$\frac{365 \times 1000}{10} = 36{,}500.$$

365 cu m = 36,500 Dl.

The principal unit of capacity is the liter, 1 cu dm. 1 cu m = 1000 cu dm.

336. What is the capacity, in cu. ft., of a cistern that will hold 5 tonneaux of water? Find the exact answer to two decimal places; the approximate answer.

337. 6.4 Dl of potatoes to the are is equivalent to how many bushels to the acre? Find the exact answer to two decimal places; the approximate answer.

338. If a bushel of oats weighs 32 lb., how many Kg will a Hl of oats weigh? Find the exact answer to two decimal places; the approximate answer.

MISCELLANEOUS

339. Change 20 mi. 70 rd. 5 ft. to feet; change 106,760 ft. to integers of higher denominations.

340. Express $\frac{3}{8}$ rd. in yards and feet; express 3 yd. 2 ft. as a fraction of a rod.

341. Reduce .3975 of a mi. to integers of lower denominations; reduce 127 rd. 3 ft. 3.6 in. to the decimal of a mile.

342. Reduce 72 lb. avoirdupois to lb. troy; reduce 87.5 lb. apothecaries' to lb. avoirdupois.

343. Reduce 157.5 gal. to bu. approximately; reduce 16.8 bu. to gal. approximately.

344. Reduce 157.5 gal. to bu., exact to 1 decimal place; reduce 16.9 bu. to gal. exactly.

345. Reduce $38° 50' 30''$ of arc to time; reduce 2 hr. 35 min. 22 sec. of time to arc.

346. Find the sum of 18 gal. 3 qt., and 6 gal. 3 qt. 1 pt.; find the difference between 25 gal. 2 qt. 1 pt., and 18 gal. 3 qt.

347. To Mar. 3, add 182 da.; from Sept. 1, subtract 182 da.

348. To Mar. 3, add 6 mo. 2 da.; from Sept. 5, subtract 6 mo. 2 da.

349. Find the time from the discovery of America, Oct. 21, 1492, to the Declaration of Independence, July 4, 1776.

350. Find the exact number of days from the fourth of July to Christmas.

351. How many cu cm are there in 9 l? l in 9000 cu cm?

352. Give the weight in Kg of 5 cu m of water; give the number of cu m in 5000 Kg of water.

353. How much does a grocer gain by buying 3 bu. of chestnuts at $ 3 a bu. dry measure, and selling for 5¢ a half pint liquid measure?

354. How much does an apothecary gain by buying 50 lb. of medicine at 20¢ a lb. avoirdupois weight, and retailing it at 10¢ an ounce apothecaries' weight?

LITERAL QUANTITIES

NOTATION AND NUMERATION

A number may be considered under conditions that are directly opposed.

One of the conditions is regarded as positive, and is represented by '+'; the opposite is regarded as negative, and is represented by '−.' The sign '+' has, therefore, an arbitrary signification; the sign '−' denotes the opposite of '+' in the same position.

A number may be represented by a letter, and this letter may be subjected to the various operations.

The same quantity, *base*, may be used several times as an addend. The expression is abbreviated by writing the base and, before it, a number, *coefficient*, denoting how many times the base is used as an addend. If both base and coefficient are numbers, the sign '×' is necessary.

Illustrations

6° may be regarded as 6° above zero, or 6° below zero; 6 mi. may be regarded as 6 mi. north, or 6 mi. south; 6 may be regarded as 6 to be added, or 6 to be subtracted.

In $+6°$, '+' has the arbitrary signification *above zero;* in $-6°$, '−' means the opposite, or *below zero.* In $8+6$, '+' has the arbitrary signification *add;* in $8-6$, '−' means the opposite, or *subtract.*

Take a number, x; multiply by 4, $4x$; add 8, $4x+8$; divide by 2, $2x+4$; subtract 2 times the number, 4.

$$a+a+a+a=4a;$$
$$2+2+2+2=4\times 2.$$

In the former, a is the *base* and 4 the *coefficient;* in the latter, 2 is the *base* and 4 the *coefficient.*

The same quantity, *base*, may be used several times as a factor. The expression is abbreviated by writing the base and, over it, a number, *exponent*, denoting how many times the base is used as a factor.

$aaaa = a^4$; $2 \times 2 \times 2 \times 2 = 2^4$. In the former, a is the *base* and 4 the *exponent;* in the latter, 2 is the *base* and 4 the *exponent*.

1. What does $+4$ mean? -4?

Since '$+$' has an arbitrary signification, we may assume it to mean any condition which has an opposite. Thus: $+4$ may mean 4 to the right, 4 up, 4 to the north, ...

Since '$-$' means the opposite of '$+$,' if $+4$ means 4 to the right, -4 means 4 to the left; . . . . To find the meaning of a '$-$' sign, we must inquire the meaning of the '$+$' sign in the same position.

2. Write a, expressing both coefficient and exponent.

Ans. $+1a^{+1}$. When either coefficient or exponent is omitted, '$+1$' is always understood.

3. If '$+6$ mi.' means 6 mi. east, what does '-8 mi.' mean?

4. Analyze, explain the meaning of, and read, $3a^5$.

$3a^5$ is a term; 3, the coefficient; a, the base; 5, the exponent. It means $a^5 + a^5 + a^5$, or that a is taken 5 times as a factor, and that the result is taken 3 times as an addend. It is read, 3, a to the fifth power.

5. Write in simplest form that 2 is used 6 times as an addend; that a is used 6 times as an addend.

6. Write in simplest form that 2 is used 6 times as a factor; that a is used 6 times as a factor.

7. Write in simplest form that a is used 6 times as a factor, and that the result is used 5 times as an addend.

8. Analyze, explain the meaning of, and read, $4x^3$.

9. What is the meaning of $+5$? of -5? What is the meaning of '$-\$6$,' if '$+\$6$' means '*worth* \$6'?

10. After losing 3¢, a boy had 4¢ left. How much had he at first? Analyze.

11. After losing a¢, a boy had b¢ left. How much had he at first? Analyze. *Ans.* $(a+b)$¢.

12. After losing a certain sum, a boy had 4¢ left. If he had 7¢ at first, how much did he lose? Analyze.

13. After losing a certain sum, a boy had b¢ left. If he had a¢ at first, how much did he lose? Analyze. *Ans.* $(a-b)$¢.

14. At 2¢ each, how much will 5 apples cost? x apples?

15. At b¢ each, how much will x apples cost? *Ans.* bx cents.

16. If 4 apples cost 8¢, how much will 1 apple cost? If x apples cost 8¢, how much will 1 apple cost?

17. If x apples cost b¢, how much will 1 apple cost? *Ans.* $\frac{b}{x}$¢.

18. At 4¢ each, how many apples can be bought for 8¢? for x¢? Analyze.

19. At x¢ each, how many apples can be bought for a¢?

20. If a boy had x marbles, and lost y of them, how many had he left? *Ans.* $(x-y)$ marbles.

21. If a dog runs b ft. in 1 minute, how far will he run in 6 min.? in 8 min.? in x min.?

22. When eggs sell at x¢ a dozen, what is the selling price of each egg? *Ans.* $\frac{x}{12}$¢.

23. In Ex. 22, what is the selling price of 3 eggs?

24. If eggs cost a¢ each, how much will 1 egg cost if the price is increased 1¢? *Ans.* $(a+1)$¢.

25. If 3 eggs sell for 6¢, what is the selling price of 4 eggs? of 5 eggs? of x eggs? *Ans.* $2x$ cents.

26. How many horses at x dollars each, must a man sell to pay for b cows at c dollars each? *Ans.* $\frac{bc}{x}$ horses.

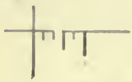
27. The product of the sum and difference of 4 and 2 is 4^2-2^2. Make a similar statement with letters instead of numbers.

ADDITION

I. *To add when the signs are alike, write the sum and use the common sign; to add when the signs are unlike, write the difference and use the sign of the greater.*

Let us take	$+3$	-3	$+3$	-3
	$+2$	-2	-2	$+2$
To prove the sums	$+5$	-5	$+1$	-1

In these examples, let us assume that + means *to the right*, and − *to the left*.



To + 3 add + 2. + 3 means 3 to the right, one, two, three; + 2 means 2 to the right, one, two; counting, we have 5 to the right, or + 5.

To − 3 add + 2. − 3 means 3 to the left, one, two, three; + 2 means 2 to the right, one, two; counting, we have 1 to the left, or − 1.

In like manner, the other results may be proved to be − 5, and + 1.

Examining these results, we see that there are two cases; where the signs are alike, and where the signs are unlike; and that the results are $+5$, -5, $+1$, and -1.

By diagram, find the sum of:

	5	-3	-8	$+7$	-4	-5
28.	-6	-2	$+4$	$+5$	-3	$+3$

II. *To add quantities having a common factor, add the factors not common and retain the common factor.*

Let us take	$6a$
	$2a$
To prove the sum	$8a$

$6a + 2a$ means $6 \times a + 2 \times a$, or that a is taken 6 times as an addend, and then 2 times more as an addend; or $6 + 2$, or 8, times as an addend, or $8a$.

Hence, the rule.

NOTE. — The pupil should compare this with the same principle on p. 72.

29. Add $+17$ and -20.

$$\begin{array}{r} +17 \\ -20 \\ \hline -3 \end{array}$$

30. Add -12 and -8.

$$\begin{array}{r} -12 \\ -8 \\ \hline -20 \end{array}$$

31. Add $+6, -8, -9, +7$.

$$\begin{array}{r} +6 \\ -8 \\ -9 \\ +7 \\ \hline -4 \end{array} \qquad \begin{array}{r} +13 \\ -17 \\ \hline -4 \end{array}$$

The sum of the positive addends is $+13$; the sum of the negative addends is -17; the sum of $+13$ and -17 is -4.

32. Add $3a-2b+c$, $-a+3b-2c$, $2a-3b-4c$.

$$\begin{array}{r} 3a-2b+c \\ -a+3b-2c \\ 2a-3b-4c \\ \hline 4a-2b-5c \end{array}$$

The sum of the a's is $4a$; the sum of the b's is $-2b$; the sum of the c's is $-5c$.

NOTE. — Add the columns from the left in their order.

Add:

	$+36$	-25	-37	$+82$	$+37$
33.	-47	$+39$	-45	-38	$+39$
	-64	-56	-54	$+63$	-56
34.	$+38$	-32	$+72$	-84	-60

Add:

35. $-8, -7, +6, +5, -4, -3, +8, +9, -10, +6$.

36. $5, +6, -7, -5, +8, +9, -7, -8, +12, -3$.

Add:

37. $7a+4b+2c$, $6a-3b-2c$, $-a+4b-c$, $a-b+c$.

38. $3a^2-2b^2-c^3$, $-a^2+8b^2+2c^3$, $a^2-4b^2-c^3$, $2a-3b$.

39. $2x^3-2x^2+3x$, $-4x^3+3x^2+x$, $3x^3-x^2-3x$, $-3x^2$.

40. $4a^2b-3ab^2+bc^2$, $2a^2b+4ab^2-3bc^2$, $-3a^2b-ab^2+bc^2$.

SUBTRACTION

III. *To subtract, change the sign of the subtrahend and proceed as in addition.*

Let us take	$+2$	-2	-2	$+2$
	$+3$	-3	$+3$	-3
To prove the remainders	-1	$+1$	-5	$+5$

In these examples, let us assume that + means to the right, and − to the left.

From + 2 subtract + 3. + 2 means 2 to the right, one, two. If we were to add + 3, we would count 3 to the right, but since subtraction is the opposite of addition, we must count 3 to the opposite of the right, or 3 to the left, one, two, three; counting, we have 1 to the left, or − 1.

From − 2 subtract − 3. − 2 means 2 to the left, one, two. If we were to add −3, we would count 3 to the left, but since subtraction is the opposite of addition, we must count 3 to the opposite of the left, or 3 to the right, one, two, three; counting, we have 1 to the right, or + 1.

In like manner, the other results may be proved to be − 5 and + 5.

By diagram, subtract the lower number from the upper.

	2	6	3	-4	-8	$+3$
41.	3	-2	8	-6	-3	-5

IV. *To subtract quantities having a common factor, subtract the factors not common, and retain the common factor.*

Let us take	$8a$
	$6a$
To prove the remainder	$2a$

$8a - 6a$ means $8 \times a - 6 \times a$, or that a is taken 8 times as an addend, and then 6 times less as an addend, or $8 - 6$, or 2, times as an addend, or $2a$.

Hence, the rule.

Note.—The pupil should compare this with the same principle on p. 72.

42. From -12 subtract -19.

$$\begin{array}{r} -12 \\ -19 \\ \hline +\ 7 \end{array}$$

43. From 16 subtract 20.

$$\begin{array}{r} 16 \\ 20 \\ \hline -4 \end{array}$$

44. From $a - 2b - 3c + 5d$ subtract $-a + 3b + 2c - 2d$.

$$\begin{array}{r} a - 2b - 3c + 5d \\ -a + 3b + 2c - 2d \\ \hline 2a - 5b - 5c + 7d \end{array}$$

a means $+1a$; $-a$ means $-1a$; changing the sign of the subtrahend, *mentally*, and adding, we get $2a$, etc.

Subtract:

	17	-72	$+75$	-72	$+81$	-86
45.	18	-80	-85	$+83$	-71	$+73$

	-50	-35	-56	$+35$	-62	-35
46.	-80	$+40$	-48	$+60$	$+35$	$+62$

Subtract:

	$-4a$	$-9a$	$+6a$	$-8a$	$+7a$	$-10a$
47.	$+5a$	$-8a$	$-7a$	$+6a$	$+3a$	$-\ 7a$

	$-5c^2$	$+16d^3$	$+18e$	$-14m^2$	$+11x^2$	$-16n^3$
48.	$+4c^2$	$+\ 4d^3$	$-17e$	$-16m^2$	$-\ 9x^2$	$+14n^3$

Subtract:

49. $4a - 6b + 10c - 12d$ from $2b - 2a + 6c + 2d$.

50. $2x^2 + 5xy + xz + 2y^2$ from $4x^2 - 6xy + 2xz + 2y^2$.

51. $4x^3 - 2x^2y + 3xy^2 + 2y^3$ from $3x^3 + 3x^2y - 4xy^2 - 3y^3$.

52. $3x^4y + 9x^2y^2 + 6xy^3 - 8$ from $4x^4y + 8x^2y^2 - 6xy^3 - 2$.

53. $-a^2b^2 - 3a^2bc - 2ab^2c + bc^2$ from $a^2b^2 + 2a^2bc + 4ab^2c + 3bc^2$.

MULTIPLICATION

V. *The product of like signs is +; the product of unlike signs is −.*

Let us take $+3 \times +2, \; -3 \times -2, \; +3 \times -2, \; -3 \times +2.$

To prove the products $+6, \; +6, \; -6, \; -6.$

$+3 \times +2$ means that $+3$ is taken 2 times as an addend, or $+3+3$, or $+6$.

-3×-2 means that -3 is taken 2 times as a subtrahend, or $-(-3) -(-3)$, or $+3+3$, or $+6$.

$+3 \times -2$ means that $+3$ is taken 2 times as a subtrahend, or $-(+3) -(+3)$, or $-3-3$, or -6.

$-3 \times +2$ means that -3 is taken 2 times as an addend, or $(-3)+(-3)$, or $-3-3$, or -6.

Hence, the rule.

Declare the products:

54. $+5 \times +6; \; -5 \times -6; \; -5 \times +6; \; +5 \times -6.$

55. $-6 \times -7; \; -6 \times +7; \; +6 \times -7; \; +6 \times +7.$

VI. *To multiply when the bases are the same, write the common base, and over it, the exponent of the multiplicand plus the exponent of the multiplier.*

Let us take $a^2 \times a^3$

To prove the product a^5

$$\begin{aligned} a^2 &= a \times a \\ a^3 &= a \times a \times a \\ \hline a^3 \times a^2 &= a \times a \times a \times a \times a \\ &= a^5 \end{aligned}$$

Analyzing the product, we see that the base, a, is the common base; that the exponent, 5, is the exponent of the multiplicand plus the exponent of the multiplier.

Declare the products:

56. $a^4 \times a^5; \; x^4 \times x^2; \; y^3 \times y^3; \; b^4 \times b; \; -a^3 \times -a^2; \; -a^3 \times +a.$

57. $a^6 \times a; \; a^2 \times a^2; \; x^4 \times x^3; \; y^2 \times y; \; +x^2 \times -x; \; +x^2 \times -x^2.$

58. Multiply $-8a$ by $-3a^3$.

$$\begin{array}{r} -\ 8a \\ -\ 3a^3 \\ \hline +24a^4 \end{array}$$

The product of like signs is +.

To multiply when the bases are the same, write the common base, and over it, the exponent of the multiplicand plus the exponent of the multiplier.

59. Multiply $a^2-2ab+b^2$ by $a+b$.

$$\begin{array}{llll} a^2 - 2ab & + b^2 & & \\ \quad\ a & + b & & \\ \hline a^3 - 2a^2b & + ab^2 & & \\ \quad\ a^2b & - 2ab^2 & + b^3 & \\ \hline a^3 - a^2b & - ab^2 & + b^3 & \end{array}$$

Beginning at the left, we multiply by a; $a \times a^2 = a^3$; $a \times -2ab = -2a^2b$; $a \times b^2 = ab^2$.

Then we multiply by b; $b \times a^2 = a^2b$; $b \times -2ab = -2ab^2$; $b \times b^2 = b^3$.

Multiply:

60. $ab \times ab$; $a^2b \times ab^2$; $a^3b \times a^2b^2$; $a^2b^3 \times a^3b^2$.

61. $-3a \times -6a$; $-3b \times 2b$; $a^2 \times -2a^2$.

62. $-3xyz \times 2x^2$; $2x \times 3xy^2$; $-3xy \times 3x^2y^3$.

63. $2ab \times 3ab$; $-2a^2b \times 3ab^2$; $2a^2b^2 \times 3ab$.

64. $-6xyz \times 2$; $-2 \times -6xyz$; $-2xyz \times 3xy^2z$.

Multiply

65.	66.	67.	68.
$a+b$	$a-b$	$a+b$	$a^2+2ab+b^2$
$a+b$	$a-b$	$a-b$	$a^2-2ab+b^2$

69.	70.	71.	72.
$3a+4b$	x^2y+xy^2	a^2-b^2	x^2+xy+y^2
$3a-4b$	x^2y-xy^2	a^2+b^2	x^2-xy+y^2

73.	74.	75.	76.
$(a+b)^3$	$(a+b)^4$	$(a+b)^5$	$(a+b-c)^3$

DIVISION

VII. *The quotient of like signs is +; the quotient of unlike signs is −.*

Let us take $\frac{+6}{+3}$, $\frac{-6}{-3}$, $\frac{+6}{-3}$, $\frac{-6}{+3}$.

To prove the quotients $+2$, $+2$, -2, -2.

$$+3 \times +2 = +6$$
$$-3 \times +2 = -6$$
$$-3 \times -2 = +6$$
$$+3 \times -2 = -6$$

Since the product divided by either factor is equal to the other factor,

$$\frac{+6}{+3} = +2; \quad \frac{-6}{-3} = +2; \quad \frac{+6}{-3} = -2; \quad \frac{-6}{+3} = -2.$$

Hence, the rule.

Declare the quotients:

77. $+6 \div +2$; $-6 \div -2$; $+6 \div -2$; $-6 \div +2$.

78. $+8 \div -2$; $-8 \div +2$; $-8 \div -2$; $+8 \div +2$.

VIII. *To divide when the bases are the same, write the common base, and over it, the exponent of the dividend minus the exponent of the divisor.*

Let us take $a^5 \div a^3$

To prove the quotient a^2

$$\frac{a^5}{a^3} = \frac{a \times a \times a \times a \times a}{a \times a \times a}$$
$$= a \times a = a^2$$

Analyzing the quotient, we see that the base, a, is the common base; that the exponent, 2, is the exponent of the dividend minus the exponent of the divisor.

Declare the quotients:

79. $a^6 \div a^2$; $a^7 \div a^3$; $a^5 \div a$; $a^2 \div a^2$; $-a^3 \div -a^2$.

80. $x^2 \div x$; $x^6 \div x^6$; $x^4 \div x^2$; $x^5 \div x^4$; $+x^2 \div -x^2$.

81. Divide $9c^4$ by $-3c^2$.

$9c^4 \div -3c^2 = -3c^2$. The quotient of unlike signs is $-$.

To divide when the bases are the same, write the common base, and over it, the exponent of the dividend minus the exponent of the divisor.

82. Divide $a^4 + b^4 - 2a^2b^2$ by $b^2 + a^2 - 2ab$.

$$
\begin{array}{l}
a^2 - 2ab + b^2)\,a^4 - 2a^2b^2 + b^4\,(a^2 + 2ab + b^2 \\
\qquad\qquad\qquad \underline{a^4 - 2a^3b + a^2b^2} \\
\qquad\qquad\qquad\quad +2a^3b - 3a^2b^2 + b^4 \\
\qquad\qquad\qquad\quad \underline{+2a^3b - 4a^2b^2 + 2ab^3} \\
\qquad\qquad\qquad\qquad\qquad a^2b^2 - 2ab^3 + b^4 \\
\qquad\qquad\qquad\qquad\qquad \underline{a^2b^2 - 2ab^3 + b^4}
\end{array}
$$

Before dividing, it is necessary to arrange the terms according to the descending powers of some letter.

a^2 is contained in a^4, a^2; $a^2 \times a^2$ is a^4; $a^2 \times -2ab$ is $-2a^3b$; $a^2 \times b^2$ is a^2b^2; subtracting the terms in the order of the descending powers of a, $+2a^3b - 3a^2b^2 + b^4$; etc.

NOTE. — $a^4 + a^3b + a^2b^2 + ab^3 + b^4$ is arranged according to the descending powers of a. The exponents decrease in order from the largest.

Divide:

83. $-a^3 \div -a$; $-6a^2b \div +ab$; $-4ab^2 \div ab$.

84. $-x^6 \div x^3$; $-6x^2 \div -3x^2$; $-12x^2y \div 2xy$.

85. $-x^5 \div -x^5$; $-4ab \div 2ab$; $6x^3y \div 2x$.

86. $x^4 \div -x^2$; $6a^3b^3 \div -2ab^2$; $8x^4y^2 \div -2xy$.

87. $x^3 \div -x^3$; $4a^2b^2 \div a^2b^2$; $6xy^4 \div y^3$.

Divide:

88. $x^2 + 2xy + y^2$ by $x + y$; $x^2 - 2xy + y^2$ by $x - y$.

89. $x^3 - y^3$ by $x - y$; $x^3 + y^3$ by $x + y$.

90. $x^5 + y^5$ by $x + y$; $25x^2 - 20xy + 4y^2$ by $5x - 2y$.

91. $x^4 + x^2y^2 + y^4$ by $x^2 - xy + y^2$; $x^5 - y^5$ by $x - y$.

92. $6x^2 - 5xy - 6y^2$ by $2x - 3y$; $x^{12} - y^{12}$ by $x^3 - y^3$.

FINDING FACTORS

Factor:

93. $3a^2b + 3ab^2$; $6x^3 - 12x^2 + 24x - 6$.

94. $12a^2b^3 - 12a^3b^2$; $14x^2y - 28xy^2 + 7xy + 14$.

95. $24a^3b^3 - 36a^2b^2$; $12x^2y^2 - 10xy^2 + 8xy - 4x^2y$.

96. $20a^4b^4 - 30a^3$; $10x^3y - 20x^2y^2 + 10xy^3 - 5y^4$.

97. $12a^3b^2 + 9a^4b^4$; $18x^4y + 30x^3y^2 - 24x^2y^3 + 12xy^4$.

Ex. **93.** $3ab(a+b)$. $3ab$ is contained in $3a^2b$, a times; in $3ab^2$, b times.

GREATEST COMMON DIVISOR

Find the G. C. D.:

98. $7xy$, $14y^2$, $21y^3$.

99. $12x^2$, $6x^3$, $18x^4$.

100. $9x^2$, $12x^3$, $18x^4$.

101. $4y^3$, $16y^4$, $22y^5$.

102. $12x^2$, $10x^3$, 18.

103. $15a^2b$, $20a^3b^3$, $40a^2b^2$.

104. $6a^4b^2$, $18a^3b^3$, $2a^4b$.

105. $5a^4b$, $10a^3b^2$, a^2b.

106. $9a^2b^2$, $18a^3b^3$, $21a^2b^3$.

107. $16a^2b$, $18a^4$, $12a^3b^3$.

Ex. **98.** *Ans.* $7y$. Use principle III, p. 75.

LEAST COMMON MULTIPLE

Find the L. C. M.:

108. $2a^2b$, $3a^4b^4$, $4ab^2$.

109. $5a^2b^3$, $6a^4b$, $8ab^4$.

110. $3a^3b$, $6a^4b^2$, $9a^2b^4$.

111. $8a^3b$, $10a^5b^6$, $12a^5b^7$.

112. $6a^2b$, $5a^3b^4$, $15a^4b^6$.

113. $4x^3y$, $6x^4$, $3y^2$.

114. $2x^2y^2$, $5x^3y$, $3x$.

115. x^3, $2y^2$, $3y$.

116. x^2y^2, $6y^3$, $7x^4$.

117. $8x^4y^3$, $12x^5y^2$, $16xy^5$.

Ex. **108.** $12a^4b^4$. Use principle I, p. 76.

REDUCTION

118. Reduce $\frac{16a^3b^2}{24ab^3}$ to lowest terms.

$$\frac{16a^3b^2}{24ab^3}=\frac{2a^2}{3b}.$$

Dividing both terms by $8ab^2$, we obtain $\frac{2a^2}{3b}$. *See p. 85.*

119. Reduce $\frac{2a}{3b}, \frac{4}{9b^2}, \frac{2}{15ab}$ to fractions having the same denominator.

$$\frac{30a^2b}{45ab^2}, \frac{20a}{45ab^2}, \frac{6b}{45ab^2}$$

The L. C. D. is $45ab^2$. $3b$ is contained in the L. C. D., $15ab$; $15ab \times 2a = 30a^2b$. *See p. 86.*

ADDITION AND SUBTRACTION

120. Simplify $\frac{a}{c}+\frac{c}{a}$.

$$\frac{a}{c}+\frac{c}{a}=\frac{a^2+c^2}{ac}.$$

We reduce to equivalent fractions having the L. C. D., and unite the numerators. *See pp. 88, 89.*

121. Simplify $\frac{4x-5}{3}-\frac{2x-3}{2}$.

$$\frac{2(4x-5)-3(2x-3)}{6}$$

$$=\frac{8x-10-6x+9}{6}=\frac{2x-1}{6}.$$

In removing $-3(2x-3)$ from the parenthesis, we say, -3 times $2x = -6x$; -3 times $-3=+9$, because $2x-3$ is to be multiplied by -3.

MULTIPLICATION AND DIVISION

122. Simplify $\frac{6a^2}{25b^2}\times\frac{5b}{3a}$.

$$\frac{6a^2}{25b^2}\times\frac{5b}{3a}=\frac{2a}{5b}.$$

We multiply the numerators for a new numerator and the denominators for a new denominator, canceling when possible. *See p. 90.*

123. Simplify $\frac{8}{9x^3y^2}\div\frac{16}{27x^4y^2}$.

$$\frac{8}{9x^3y^2}\times\frac{27x^4y^2}{16}=\frac{3x}{2}.$$

We invert the divisor, and proceed as in multiplication. *See p. 92.*

EXERCISES

Reduce to lowest terms:

124. $\frac{6\,a^4b^2}{12\,a^2b}$; $\frac{3\,x^2y}{15\,x^3y}$.

125. $\frac{9\,x^3y^8}{24\,x^8y^9}$; $\frac{8\,x^3}{12\,x^2y^3}$.

126. $\frac{75\,a^3b^3c^3}{125\,a^4b^4c^4}$; $\frac{96\,x^2y^3}{144\,x^3y^3}$.

127. $\frac{32\,abc}{128\,a^2b^2c^2}$; $\frac{72\,x^2y^2}{216\,xy}$.

Reduce to fractions having the L. C. D.:

128. $\frac{3\,x}{4}, \frac{2\,x}{5}, \frac{7\,x}{3}, \frac{5\,x}{2}$.

129. $\frac{2}{a}, \frac{3}{ab}, \frac{4}{a^2}, \frac{2}{b^2}$.

130. $\frac{3\,x-2}{3}, \frac{5\,x-4}{5}$.

131. $\frac{7\,x+3}{8\,x}, \frac{8\,x-3}{12\,x}$.

Find the value of:

132. $\frac{3\,x-7}{6}+\frac{6\,x-4}{9}$.

133. $\frac{5\,x-2\,y}{8}+\frac{7\,x-3\,y}{12}$.

134. $\frac{3\,x-7}{3}-\frac{2\,x-3}{2}$.

135. $\frac{5\,x-3\,y}{8}-\frac{3\,x-7\,y}{12}$.

Find the value of:

136. $\frac{25\,x^3}{16\,y^3}\times\frac{32\,y^4}{35\,x^2}$.

137. $\frac{12\,x^2y^3}{55}\times\frac{22}{15\,y^4}$.

138. $\frac{32\,a^2c}{45\,b^2}\div\frac{44\,a^3}{35\,b^3}$.

139. $\frac{27\,a^3}{125\,b^3}\div\frac{9\,a^2}{25\,b^2}$.

EQUATIONS

IX. *To transpose a term from one member of an equation to the other, change its sign.*

ILLUSTRATION. $8 - 6 = 2$; transposing -6 to the right-hand member, $8 = 6 + 2$.

X. *Either of two factors is equal to their product divided by the other.*

ILLUSTRATION. $6 \times 8 = 48$; whence $6 = \frac{48}{8}$, or $8 = \frac{48}{6}$.

XI. *Multiplying or dividing both members of an equation by the same number, cannot affect the equality.*

ILLUSTRATION. $12 = 8 + 4$; multiplying both members by 2, $24 = 16 + 8$; dividing both members by 2, $6 = 4 + 2$.

XII. *Raising both members of an equation to the same power, or depressing both members to the same root, cannot affect the equality.*

ILLUSTRATION. $x^2 = 4$; squaring both members, $x^4 = 16$; extracting the square root of both members, $x = 2$.

Find the value of x:

140. $6x - 12 = 4x + 4$.

141. $6 = 5x + 15 - 8x$.

142. $9x + 10 - 18 = 7x$.

143. $7x - 10 = 3x + 6$.

144. $33x + 8 + 7 = -51$.

145. $-17 - 8 + 30 = -2x$.

146. $15x - 4 + 7x = 20x$.

147. $5x^2 = 45$.

148. $5\sqrt{x} = 25$.

149. $6x^3 = 48$.

150. $6\sqrt[3]{x} = 12$.

151. $3x^4 = 48$.

152. $3\sqrt[4]{x} = 6$.

153. $2x^5 = 64$.

Ex. **140**. Transposing to the left-hand side all terms which contain x, $6x - 4x = 12 + 4$; uniting, $2x = 16$; $x = 8$.

Ex. **147**. $5x^2 = 45$; $x^2 = 9$; extracting square root, $x = 3$.

Ex. **150**. $6\sqrt[3]{x} = 12$; $\sqrt[3]{x} = 2$; cubing, $x = 8$.

NOTE. — To depress a term to the cube root, is to find one of the three equal factors whose product is that term. Thus, the cube root of 8, written $\sqrt[3]{8}$, is 2, because $2 \times 2 \times 2 = 8$.

PROBLEMS

The solution of problems by the *literal method* differs from the *analysis method* (*see p. 62*) in that the relations are stated directly, the required terms are represented by letters, and these letters are subjected to addition, subtraction, multiplication, and division.

In explaining a problem:

After every statement give a reason, unless the reason adds nothing to clearness.

After the equation is formed, do not explain the solution, but declare the result.

In the proof, state the first relation and show how it is met; state the second relation and show how it is met; and so on.

154. The sum of two numbers is 32, and one of them is 3 times the other. What are the numbers?

Relations: 32 = the sum of two numbers; the larger = 3 times the smaller.

First Solution

Let $x =$ *the smaller*, 8
$3x =$ *the larger*, 24
$4x = 32$
$x = 8$

Let $x =$ the smaller; then $3x$ must equal the larger, because the larger is 3 times the smaller. $4x = 32$, because their sum is 32. Whence $x = 8$, the smaller; and $3x = 24$, the larger.

Second Solution

Let $x =$ *the larger*, 24
$32 - x =$ *the smaller*, 8
$x = 3(32 - x)$
$x = 96 - 3x$
$4x = 96$
$x = 24$

PROOF

1. $24 + 8 = 32$
2. $24 = 3 \times 8$

Let $x =$ the larger; then $32 - x$ must equal the smaller, because the sum of the two is 32. $x = 3(32 - x)$, because the larger is 3 times the smaller. Whence, $x = 24$, the larger; and $32 - x = 8$, the smaller.

Proof. — The first relation is, the sum of the numbers is 32; $24 + 8 = 32$. The second relation is, the larger is 3 times the smaller; $24 = 3 \times 8$.

Note. — There is no rule as to which of the required terms shall be represented by x, nor as to which of the relations shall be used first.

155. There are 54 children in a schoolroom, and twice as many boys as girls. How many boys are there?

156. There are 135 books on three shelves; on the second shelf there are twice as many as on the first, and on the third, three times as many as on the second. How many books are there on each shelf?

157. A man's property is worth $3600. His barn is worth twice as much as his house, and his land is worth as much as his house and barn together. What is the value of each?

158. A farmer has 208 animals, consisting of horses, sheep, and cows. What is the number of each, provided that the number of sheep equals three times the number of cows, and the number of cows equals three times the number of horses?

159. A is 5 times as old as B; and C, 3 times as old as B; the sum of their ages is 81 years. What is the age of each?

160. The sum of $264 was raised by 4 persons, A, B, C, and D; B contributed twice as much as A; C, 4 times as much as A and B together; and D, one half as much as B and C together. How much did each contribute?

161. A certain fish is 6 ft. 6 in. in length; its tail is twice as long as its head, and its body is as long as its tail and head together. What is the length of its body?

162. Divide 96 into 3 parts, such that the first part shall be three times the second, and the third twice the sum of the other two.

163. A certain number is expressed by three digits whose sum is 12. The digit in hundreds' place is twice the digit in units' place, and the digit in tens' place is three times the sum of the other two. What is the number?

164. A boy has 30 pieces of money, nickels, dimes, and quarters; the number of quarters is six times the number of dimes; the number of nickels is a third more than the number of quarters. How much money has he?

165. Had the cost of a horse been three times as much and \$70 more, it would have been \$445. What was the cost?

Relation: 3 times cost + \$70 = \$445.

$$\text{Let } x = \textit{cost}, 125$$
$$3x + 70 = \textit{supposed cost}$$
$$445 = \textit{supposed cost}$$
$$\therefore 3x + 70 = 445$$
$$3x = 445 - 70$$
$$3x = 375$$
$$x = 125$$

PROOF

1. $125 \times 3 + 70 = 445$

166. If 63 is subtracted from a number, three times the remainder will be twice the sum of the original number and 16. What is the number?

Relation: $3 \times (\text{No.} - 63) = 2 \times (\text{No.} + 16)$.

$$\text{Let } x = \textit{the No.}, 221$$
$$x - 63 = \textit{the rem.}$$
$$3(x - 63) = \textit{three times rem.}$$
$$2(x + 16) = \textit{twice sum}$$
$$\therefore 3(x - 63) = 2(x + 16)$$
$$3x - 189 = 2x + 32$$
$$x = 221$$

PROOF

1. $3(221 - 63) = 2(221 + 16)$

167. One number is three times another; if I take the smaller from 24 and the greater from 46, the remainders are equal. What are the numbers?

168. If to twice a certain number I add 18, I obtain 120. Find the number.

169. Anna is four years younger than Mary; if three times Anna's age is taken from five times Mary's, the remainder will be 62 years. What is the age of each?

170. Find a number whose excess over 50 is equal to twice what it lacks of being 113.

171. Divide 60 into two parts such that one part may exceed the other by 24.

172. The joint ages of a father and son are 70 years; if the age of the son were doubled, the result would be five years more than his father's age. What is the age of each?

173. The difference between two numbers is 28; and if four times the less is added to the greater, the sum is 43. What are the numbers?

174. The sum of two numbers is 34; the larger increased by 56, is 9 times the other. Find the numbers.

175. Divide 180 into two such parts that one of them diminished by 35, shall be equal to the other diminished by 15.

176. After 30 gallons had been drawn out of one of two equal casks and 82 gallons out of the other, there remained five times as much in one cask as in the other. What was the capacity of each cask if both were full at first?

177. Divide 41 into three such parts that the second shall be 4 more than the first, and the third 3 less than the second.

178. A man borrowed as much money as he had, and then spent \$4; he then borrowed as much as he had left, and spent \$3; again he borrowed as much as he had left, and spent \$2; he then had nothing left. How much had he at first?

179. An estate valued at \$4800 was divided in such a manner that the wife's share plus \$400, was equal to three times the share of the children. What was the wife's share?

180. If \$300 is subtracted from B's income, five times the remainder will be three times, the sum of \$3100 and the original income. What is his income?

181. A merchant began business with a certain capital; the first year he doubled it; the second year he gained a sum equal to the original capital plus \$100; the third year he lost as much as he had gained the first year, and then had \$3100. What was his original capital?

182. A lady bought two pieces of cloth; the longer lacked 9 yards of being three times the length of the shorter. She paid \$2 per yard for the longer, and \$3 for the shorter, and the shorter piece cost as much as the longer. How many yards were there in each piece?

EQUATIONS WITH FRACTIONS

183. Simplify $\frac{6x}{5}+\frac{2}{5}=-\frac{2x}{3}+6.$

$$18x+6=-10x+90$$
$$18x+10x=90-6$$
$$28x=84$$
$$x=3$$

Multiplying both members of an equation by the same number does not affect the equality.

We multiply both members by the L. C. D., 15. $\frac{6x}{5}\times 15=18x$; $\frac{2}{5}\times 15=6$; $-\frac{2x}{3}\times 15=-10x$; $6\times 15=90$. We then simplify as in the preceding case.

184. Simplify $\frac{3-x}{2}-\frac{x-5}{3}=\frac{13}{2}.$

$$3(3-x)-2(x-5)=39$$
$$9-3x-2x+10=39$$
$$-3x-2x=39-9-10$$
$$-5x=20$$
$$x=-4$$

We multiply both members by the L. C. D., 6. $\frac{3-x}{2}\times 6=3(3-x)$; $-\frac{x-5}{3}\times 6=-2(x-5)$; $\frac{13}{2}\times 6=39$.

Find x;

185. $\frac{2x}{3}-\frac{3x-4}{5}=1.$

186. $\frac{3x}{4}-\frac{5x-4}{6}=\frac{1}{3}.$

187. $\frac{4x}{5}-\frac{3x+2}{4}=-\frac{1}{4}.$

188. $\frac{4x}{5}+\frac{3x+2}{4}=\frac{33}{4}.$

189. $\frac{4x}{5}+\frac{3x-2}{4}=\frac{29}{4}.$

190. $\frac{5x+1}{2}-\frac{3x-2}{3}=\frac{10x+1}{6}.$

191. $\frac{3x-2}{3}+\frac{4x+5}{4}=\frac{25x}{12}.$

192. $\frac{x-1}{5}-\frac{x+1}{6}=\frac{x-9}{10}.$

193. $\frac{x+1}{6}-\frac{x-1}{5}=\frac{2x-17}{5}.$

194. $\frac{x-2}{3}+\frac{x-3}{4}=\frac{5x+3}{12}.$

PROBLEMS

195. Three times the number of hours before noon is equal to $\frac{3}{5}$ of the number since midnight. What is the time?

Relations: 3 times hr. before noon $= \frac{3}{5}$ hr. since midnight; hr. since midnight + hr. before noon = 12.

Let $x =$ *no. hr. since mid.*, 10

$12 - x =$ *no. hr. before noon*, 2

$$3(12 - x) = \frac{3x}{5}$$
$$15(12 - x) = 3x$$
$$180 - 15x = 3x$$
$$180 = 18x$$
$$x = 10$$

PROOF

1. $3 \times 2 = \frac{3}{5} \times 10$
2. $10 + 2 = 12$

196. Divide 56 into two such parts that $\frac{5}{6}$ of the less, diminished by $\frac{1}{4}$ of the greater, may equal 12.

Relations: the larger number + the smaller number = 56; $\frac{5}{6}$ of the less $- \frac{1}{4}$ of the greater = 12.

Let $x =$ *the smaller*, 24

$56 - x =$ *the larger*, 32

$$\frac{5x}{6} - \frac{56 - x}{4} = 12$$
$$10x - 3(56 - x) = 144$$
$$10x - 168 + 3x = 144$$
$$13x = 312$$
$$x = 24$$

PROOF

1. $32 + 24 = 56$
2. $\frac{5}{6}$ of $24 - \frac{1}{4}$ of $32 = 12$.

197. In a school of 495 pupils there are $\frac{7}{8}$ as many boys as girls. How many girls are there in the school?

198. John and James together have \$98; if James has $\frac{3}{4}$ as much as John, how much has each?

199. Mr. Drake, who owns $\frac{2}{9}$ of a tract of land, has 14 acres less than Mr. Brown, who owns $\frac{3}{10}$ of it. How many acres does the whole tract contain?

200. In a certain orchard there are 40 more apple trees than peach trees, and $\frac{13}{28}$ of the whole number are peach trees. How many peach trees are there?

201. Jane's age is $\frac{3}{4}$ of Ann's, and the sum of their ages equals 21 years. What is the age of each?

202. Find a number whose $\frac{3}{8}$ part exceeds its $\frac{4}{11}$ part by 3.

203. A man has $\frac{5}{8}$ as many horses as cows, and the cows are 15 more in number than the horses. How many horses has he?

204. The width of a room is $\frac{8}{11}$ of its length; if the width had been 4 feet more and the length 2 feet less, the room would have been square. Find its dimensions.

205. I bought a number of apples at the rate of 3 for 1 cent; sold one third of them at 2 for a cent, and the remainder at 5 for 3 cents, gaining 7 cents. How many did I buy?

206. A man sold a horse for $\frac{1}{2}$ of its cost, plus $75, and thereby gained $15. How much did the horse cost?

207. Find three consecutive numbers such that if they are divided by 7, 10, and 17 respectively, the sum of the quotients will be 15.

208. A man bought a horse and carriage for $280; if $\frac{1}{2}$ the price of the horse is subtracted from $\frac{3}{5}$ the price of the carriage, the remainder will be the same as if 1.9 times the price of the carriage is subtracted from 2 times the price of the horse. What was the price of each?

209. Divide $115 between two men so that $\frac{3}{4}$ of what the first receives shall be equal to $\frac{2}{5}$ of what the second receives.

210. B's expenses are $\frac{3}{5}$ of A's, plus $30; and A's expenses plus $\frac{1}{2}$ of B's, amount to $795. How much does each spend?

211. A father divided $1.43 among his three sons so that the first had $\frac{6}{7}$ as much as the second; and the third, $\frac{3}{8}$ as much as the first and the second together. How much did each son receive?

212. A man paid $806 for four horses; for the second he gave $\frac{1}{3}$ more than for the first; for the third, $\frac{1}{3}$ more than for the second; and for the fourth as much as for the first and the third together. What was the cost of the fourth horse?

213. A farm of 263 acres was divided among four heirs, so that A had $\frac{10}{13}$ as much as B; C, as much as A and B; and D, $\frac{1}{5}$ as much as A and C. What was the share of each?

TWO UNKNOWN QUANTITIES

If we have two equations with two unknown quantities, the first step is to get one equation with one unknown quantity. This step may be taken by addition and subtraction, by substitution, or by comparison.

By Addition and Subtraction

214. $\begin{cases} 3x - 4y = 10 & (1) \\ 2x + 3y = 18 & (2) \end{cases}$, find x and y.

$$6x - 8y = 20 \quad (3)$$
$$6x + 9y = 54 \quad (4)$$
$$17y = 34 \quad (5)$$
$$y = 2$$
$$3x - 8 = 10$$
$$3x = 18$$
$$x = 6$$

PROOF

1. $18 - 8 = 10$
2. $12 + 6 = 18$

We may eliminate x by multiplying (1) by 2, and (2) by 3, and subtracting. Multiplying (1) by 2, we obtain (3); multiplying (2) by 3, we obtain (4).

Subtracting (3) from (4), we obtain (5); whence, $y = 2$.

Substituting this value of y in (1), we obtain $3x - 8 = 10$; whence, $x = 6$.

By Substitution

$$3x - 4y = 10 \quad (1)$$
$$2x + 3y = 18 \quad (2)$$
$$3x = 10 + 4y$$
$$x = \frac{10 + 4y}{3}$$
$$\frac{2(10 + 4y)}{3} + 3y = 18 \quad (3)$$
$$2(10 + 4y) + 9y = 54$$
$$20 + 8y + 9y = 54$$
$$17y = 34$$
$$y = 2$$
$$x = 6$$

We may eliminate x by finding the value of x in terms of y in (1), and substituting this value in (2). From (1), $3x = 10 + 4y$, and $x = \frac{10 + 4y}{3}$; substituting this value of x in (2), we obtain (3).

Clearing (3) of fractions, and proceeding as usual, we obtain $y = 2$.

Substituting and proceeding as before, we obtain $x = 6$.

By Comparison

$$3x - 4y = 10 \quad (1)$$
$$2x + 3y = 18 \quad (2)$$

$$x = \frac{10 + 4y}{3}$$
$$x = \frac{18 - 3y}{2}$$

$$\frac{10 + 4y}{3} = \frac{18 - 3y}{2} \quad (3)$$
$$2(10 + 4y) = 3(18 - 3y)$$
$$20 + 8y = 54 - 9y$$
$$17y = 34$$
$$y = 2$$
$$x = 6$$

We may eliminate x by finding the value of x in terms of y in (1) and (2), and placing these values equal to each other. From (1), $x = \frac{10 + 4y}{3}$; from (2), $x = \frac{18 - 3y}{2}$; placing these values equal to each other, we obtain (3).

Clearing of fractions and proceeding as usual, we find that $y = 2$, and $x = 6$.

Find the values of x and y:

215. $\begin{cases} 2x + 3y = 8, \\ x + y = 3. \end{cases}$

216. $\begin{cases} 3x - y = 3, \\ 4x + y = 11. \end{cases}$

217. $\begin{cases} 2x + 3y = 10, \\ x + 2y = 7. \end{cases}$

218. $\begin{cases} x - y = 4, \\ 2x + y = 14. \end{cases}$

219. $\begin{cases} 15x + 8y = 1, \\ 10x - 7y = -24. \end{cases}$

220. $\begin{cases} \frac{2x}{3} - \frac{3y}{4} = 5, \\ \frac{x}{2} + \frac{y}{4} = 7. \end{cases}$

221. $\begin{cases} 3x + y = 13, \\ x - 3y = -9. \end{cases}$

222. $\begin{cases} x + y = 7, \\ x - y = 1. \end{cases}$

223. $\begin{cases} 3x + 2y = 8, \\ 2x - y = 3. \end{cases}$

224. $\begin{cases} 5x - y = 4, \\ 3x + y = 4. \end{cases}$

225. $\begin{cases} 7x - 3y = 21, \\ 2y - 5x = -16. \end{cases}$

226. $\begin{cases} \frac{5x}{3} + \frac{2y}{7} = 94, \\ \frac{7x}{2} - \frac{5y}{3} = 7. \end{cases}$

Note. — In examples 220 and 226, the first step is to clear of fractions.

227. A certain number expressed by two digits, is equal to 4 times the sum of those digits; if 27 is added to the number, the digits will be reversed. Find the number.

Relations: units' digit + 10 times tens' digit = number; 4 (units' digit + tens' digit) = number; number + 27 = tens' digit + 10 times units' digit.

Let $x = \textit{units' digit},\ 6$

$y = \textit{tens' digit},\ 3$

$x + 10y = \textit{number}$

$4x + 4y = \textit{number}$

$\therefore x + 10y = 4x + 4y \quad (1)$

$x + 10y + 27 = y + 10x \quad (2)$

$x = 6$

$y = 3$

PROOF

1. $4(3 + 6) = 36$
2. $36 + 27 = 63$

Let x = units' digit and let y = tens' digit; then $x + 10y$ = the number, because a number is equal to its units' digit + 10 times its tens' digit + · · · ·

$4x + 4y$ = the number, because the number is equal to 4 times the sum of its digits; $x + 10y = 4x + 4y$, because things equal to the same thing are equal to each other; etc.

228. The sum of two numbers is 5 times their difference; twice the greater, increased by 4 times the less, is 56. Find the numbers.

229. A farmer received $8.75 for 6 bu. of potatoes and 5 bu. of apples. What was the price per bushel of each, if, at the same rate, 7 bu. of potatoes and 3 bu. of apples were sold for $8.65?

230. A man, having $2.50 to divide among a certain number of boys and girls, found that if he gave each of them 10¢, he would be 30¢ out of pocket; so he gave each of the boys 8¢ and each of the girls 9¢, and had 10¢ left. How many were there of each?

231. A certain fraction becomes $\frac{7}{12}$ when 3 is added to each of its terms, but becomes $\frac{1}{6}$ when 3 is subtracted from each of its terms. Find the fraction.

232. If A gives B $10, B has twice as much as A has left; if B gives A $30, A will have twice as much as B has left. How much has each?

PROPORTION

SIMPLE PROPORTION — TERMS

Division may be expressed by writing the dividend *before* and the divisor *after* a colon. Such an expression is a *ratio*. *See p. 45.*

The dividend is the *antecedent;* the divisor, the *consequent.*

The colon is read '*is to.*'

Two fractions may be equal; in the same way, two ratios may be equal, a *proportion.*

The sign of equality is often abbreviated by writing the extremities of the sign '=,' thus making '::' read '*as.*'

The first and last terms of a proportion are *extremes;* the second and third, *means.*

The means of a proportion may be equal; then each is a *mean proportional* between the extremes.

ILLUSTRATIONS.

3 : 4, *ratio.*
Meaning, $3 \div 4$.

3, *antecedent.*
4, *consequent.*
3 *is to* 4.

$\frac{3}{4} = \frac{6}{8}$.
$3 : 4 = 6 : 8$,
a *proportion.*

$3 : 4 :: 6 : 8$,
3 *is to* 4 *as* 6 *is to* 8.

3 and 8 are *extremes*,
4 and 6, *means.*

$4 : 6 = 6 : 9$.
6, a *mean proportional* between 4 and 9.

NOTE. — All problems in proportion may be solved by analysis. The pupil is advised to master *simple proportion*, but to use analysis for the solution of problems which fall under the other divisions of proportion.

I. *In a proportion, the product of the extremes is equal to the product of the means.*

Proof. By definition, $\frac{\textit{1st antecedent}}{\textit{1st consequent}} = \frac{\textit{2d antecedent}}{\textit{2d consequent}}$; clearing of fractions, *1st antecedent* × *2d consequent* = *1st consequent* × *2d antecedent.*

Illustration. $3 : 4 : : 9 : 12$, $3 \times 12 = 4 \times 9$.

II. *If three terms of a proportion are given, the other term may be found.*

Proof. Since the product of the extremes = the product of the means, either extreme = the product of the means divided by the other extreme; either mean = the product of the extremes divided by the other mean.

Illustration. If $3 : 4 = 9 : x$, $x = \frac{4 \times 9}{3}$; if $3 : 4 = x : 12$, $x = \frac{3 \times 12}{4}$.

III. *The mean proportional between two quantities is the square root of their product.*

Proof. To find the mean proportional between two numbers, as 4 and 9, we may form a proportion whose extremes are 4 and 9, and whose means are x and x. Thus, $4 : x = x : 9$; whence $x^2 = 4 \times 9$, and $x = \sqrt{4 \times 9}$.

1. Write a fraction whose value is $\frac{1}{2}$; a ratio whose value is $\frac{1}{2}$.

2. Write a proportion each of whose ratios is equal to $\frac{2}{3}$; each of whose ratios is equal to $\frac{3}{4}$.

3. Write an equation which is an equality of two fractions; write the same equation as a proportion.

4. Is $4 : 8 : : 5 : 6$ a proportion? Why not? What is the test of a proportion?

5. Find the mean proportional between 9 and 16; 3 and 12; 4 and 25.

6. Define ratio; antecedent; consequent; proportion; means; extremes.

PROBLEMS

The method of solving a problem by proportion differs from the analysis method, in that each relation is expressed as a proportion whose third term has the same denomination as the answer, and whose fourth term is the required term. *See p. 62.*

7. If 2 apples cost 8¢, how much will 3 apples cost?

Relation: 2 apples : 3 apples = cost 2 apples : cost 3 apples.

$2 : 3 = 8 : x$

$x = \frac{3 \times 8}{2} = 12$

Cost 3 apples = 12¢.

Since the answer is to be *cents*, we make 8¢ the third term, and write the ratio *8¢ : x*.

Will 3 apples cost more or less than 2 apples? More; then we make the second term of the other ratio greater than the first, and we have *2 apples : 3 apples = 8¢ : x;* whence $x = 12$¢, the cost of 3 apples.

8. If 3 apples cost 12¢, how many apples can be bought for 8¢?

Relation: 12¢ : 8¢ = apples for 12¢ : apples for 8¢.

$12 : 8 = 3 : x$

$x = \frac{8 \times 3}{12} = 2$

No. apples = 2.

Since the answer is to be *apples*, we make 3 apples the third term, and write the ratio *3 apples : x*.

Will 8¢ buy more or less apples than 12¢? Less; then we make the second term of the other ratio less than the first, and we have *12¢ : 8¢ = 3 apples : x;* whence $x = 2$, the number of apples.

9. If 3 men can do a piece of work in 12 days, how many days will 2 men require?

Relation: 2 men : 3 men = days for 3 men : days for 2 men.

$2 : 3 = 12 : x$

$x = \frac{3 \times 12}{2} = 18$

No. days = 18.

Since the answer is to be *days*, we make 12 days the third term, and write the ratio *12 days : x*.

Will 2 men require more or less days than 3 men? More; then we make the second term of the other ratio greater than the first, and we have *2 men : 3 men = 12 days : x;* whence $x = 18$, the number of days.

NOTE. — Ask about the term *in the question*, in the denomination of the answer.

10. How much will 54 bu. of potatoes cost if 16 bu. cost $5.60?

11. How many eggs can be bought for 70¢ at the rate of 2 for 5¢?

12. If 16 men can dig a trench in 24 days, how many men will be required to dig it in 32 days?

13. If 16 men can dig a trench in 24 days, how many days will 12 men require?

14. If $150 gains $12 in 8 months, in what time will it gain $17?

15. If a man travels 32 miles in 4 hours, how many miles can he travel in 5 hours at the same rate?

16. A flagstaff 82 ft. high casts a shadow 62 ft. long. Under the same conditions, what must be the height of a steeple which casts a shadow 93 ft. long?

17. If it costs $12 to carpet a room with carpet 4 ft. wide, how much will it cost if the carpet is 3 ft. wide, provided there is no waste and the cost per linear yard is the same? *See Ex. 31.*

18. How many bushels of wheat can be bought for $102.12, if 24 bu. can be bought for $13.32?

19. If a man gains $1500 from his business in 1 yr. 6 mo., how much will he gain in 3 yr. 9 mo. at the same rate?

20. A garrison of 600 men has provisions for 80 days; how many men must leave to make the provisions hold out 20 days longer?

21. A's rate of working is to B's as 3 : 5. How long will it take B to do what A does in 48 days?

22. If 40 men can build a wall in 6 days, at the same rate how long will it take 16 men to do ½ the same work?

23. If 3 men can do a piece of work in 51 days, how many men must be added to the number to do the work in 17 days?

24. James can do 2⅓ times as much work in a given time as John. How long will it take James to do what John does in 38 hours?

COMPOUND PROPORTION

A compound fraction is a fraction of a fraction; in the same way, a *compound ratio* is a ratio of a ratio.

To find the value of a compound fraction, we multiply the numerators for a new numerator and the denominators for a new denominator; to find the value of a compound ratio, we multiply the antecedents for a new antecedent and the consequents for a new consequent.

A compound proportion is a proportion having one or both of its ratios compound.

25. Find the value of the compound ratio, $\left.\begin{matrix}2:3\\3:5\end{matrix}\right\}$; of $\left.\begin{matrix}3:4\\8:9\end{matrix}\right\}$; of $\left.\begin{matrix}6:12\\3:2\end{matrix}\right\}$; of $\left.\begin{matrix}7:5\\10:21\end{matrix}\right\}$.

26. In the compound proportion, $\left.\begin{matrix}4:7\\9:6\\14:18\end{matrix}\right\} = 2:x$, find the value of x.

Ans. $x = \dfrac{7 \times 6 \times 18 \times 2}{4 \times 9 \times 14} = 3.$ The product of the extremes is equal to the product of the means.

27. If 2 men in 14 da. of 10 hr. each earn \$280, how many hr. per da. must 3 men work to earn \$120 in 5 da.?

$$\left.\begin{matrix}3:2\\280:120\\5:14\end{matrix}\right\} = 10:x$$

$$x = \frac{2 \times 120 \times 14 \times 10}{3 \times 280 \times 5} = 8 \text{ hr. per da.}$$

Since the answer is to be *hr. per da.*, we make 10 hr. the third term, and write the ratio *10 hr. : x.*

Will 3 men require more or less *hr. per da.* than 2 men? Less; then we make the second term of the other ratio less than the first, and we have *3 men : 2 men.*

Will \$120 require more or less *hr. per da.* than \$280? Less; then we make the second term of the other ratio less than the first, and we have *\$280 : \$120.*

Will 5 days require more or less *hr. per da.* than 14 days? More; then we make the second term of the other ratio greater than the first, and we have *5 days : 14 days.*

NOTE 1. — We compare each set of ratios with the second ratio separately. We ask about the term *in the question*, in the denomination of the answer.

NOTE 2. — Examples in Compound Proportion are found on p. 177.

ANALYSIS METHOD

It is recommended that Analysis be substituted for Simple Proportion.

28. If 2 apples cost 8¢, how much will 3 apples cost? *See Ex. 7.*

Relation: 3 apples will cost $\frac{3}{2}$ as much as 2 apples.

$\frac{3}{2} \times 8 = 12$

Cost 3 apples = 12¢.

Since 3 apples will cost $\frac{3}{2}$ as much as 2 apples, 3 apples will cost $\frac{3}{2}$ of 8¢, or 12¢.

29. If 3 apples cost 12¢, how many apples can be bought for 8¢? *See Ex. 8.*

Relation: 8¢ will buy $\frac{8}{12}$ as many apples as 12¢.

$\frac{8}{12} \times 3 = 2$

No. apples = 2.

Since 8¢ will buy $\frac{8}{12}$ as many apples as 12¢, 8¢ will buy $\frac{8}{12}$ of 3 apples, or 2 apples.

30. If 3 men can do a piece of work in 12 days, how many days will 2 men require? *See Ex. 9.*

Relation: 2 men will require $\frac{3}{2}$ as many days as 3 men.

$\frac{3}{2} \times 12 = 18$

No. days = 18.

Since 2 men will require $\frac{3}{2}$ as many days as 3 men, 2 men will require $\frac{3}{2}$ of 12 days, or 18 days.

31. If it costs $12 to carpet a room with carpet 4 ft. wide, how much will it cost if the carpet is 3 ft. wide, provided there is no waste and the cost per linear yard is the same? *See Ex. 17.*

Necessary knowledge: carpets are sold by the linear yard; the less the width of the carpet, the greater the length.

Relation: for the room, carpet 3 ft. wide will cost $\frac{4}{3}$ as much as carpet 4 ft. wide.

$\frac{4}{3} \times 12 = 16$

Cost = $16.

Since, for the room, carpet 3 ft. wide will cost $\frac{4}{3}$ as much as carpet 4 ft. wide, the 3 ft. carpet will cost $\frac{4}{3}$ of $12, or $16.

NOTE.—The pupil should solve the examples on p. 173 by this method.

It is recommended that Analysis be substituted for Compound Proportion.

32. If 2 men in 14 da. of 10 hr. each earn $280, how many hr. per day must 3 men work to earn $120 in 5 days?

$$\frac{2}{3} \times \frac{120}{280} \times \frac{14}{5} \times 10 = 8$$

Ans. 8 hr. per da.

Relations: 3 men will require $\frac{2}{3}$ as many *hr. per day* as 2 men; to earn $120 will require $\frac{120}{280}$ as many *hr. per day* as to earn $280; 5 days will require $\frac{14}{5}$ as many *hr. per day* as 14 days.

Since the first set require 10 hr. per day, the second will require $\frac{2}{3} \times \frac{120}{280} \times \frac{14}{5} \times 10$ hr. per day, or 8 hr. per day.

NOTE. — In such problems, the relation is more plainly seen, if we ask about the term *in the question*, in the denomination of the answer. Thus: will 3 men require more or less *hr. per day* than 2 men? Less, $\frac{2}{3}$ as many; will $120 require more or less *hr. per day* than $280? Less, $\frac{120}{280}$ as many ....

33. If 5 compositors, in 16 days, of 8 hours each, can compose 20 sheets of 24 pages in each sheet, 50 lines in a page, and 40 letters in a line, in how many days, of 4 hours each, will 10 compositors compose a volume to be printed in the same type, containing 40 sheets, 16 pages in a sheet, 60 lines in a page, and 50 letters in a line?

$$\frac{8}{4} \times \frac{5}{10} \times \frac{40}{20} \times \frac{16}{24} \times \frac{60}{50} \times \frac{50}{40} \times 16 = 32$$

Ans. 32 days.

Relations: 4 hr. per day will require $\frac{8}{4}$ as many *days* as 8 hr. per day; 10 compositors will require $\frac{5}{10}$ as many *days* as 5 compositors; 40 sheets will require $\frac{40}{20}$ as many *days* as 20 sheets; 16 pages will require $\frac{16}{24}$ as many *days* as 24 pages; 60 lines will require $\frac{60}{50}$ as many *days* as 50 lines; 50 letters will require $\frac{50}{40}$ as many *days* as 40 letters.

Since the first task requires 16 days, the second will require $\frac{8}{4} \times \frac{5}{10} \times \frac{40}{20} \times \frac{16}{24} \times \frac{60}{50} \times \frac{50}{40} \times 16$ days, or 32 days.

34. If 9 men can perform a certain labor in 17 days, how long will it take 3 men to do twice as much?

35. If a man working 6 hours a day, 6 days in a week, and 42 weeks in a year, earns $1323, how much will he earn if he works 8 hours a day, 5 days in a week, and 50 weeks in a year?

36. If 25 men can build a wall 200 ft. long in 40 days, how many days will 4 men require to build a similar wall 600 ft. long?

37. If 12 iron bars 4 ft. long, 3 in. wide, and $2\frac{1}{2}$ in. thick, weigh 1050 lb., what is the weight of 26 iron bars 6 ft. long, 4 in. wide, and 2 in. thick?

38. If 36 men can dig a trench 150 ft. long, 6 ft. wide, and 4 ft. 6 in. deep, in 24 days, how many days will 32 men require for a trench 210 ft. long, 5 ft. wide, and 4 ft. deep?

39. If 5 men in 6 days of 8 hours each can mow 16 acres, how many days of 6 hours each must 10 men work to mow 24 acres?

40. What is the weight of a block of stone 10 ft. 6 in. long, 6 ft. 8 in. wide, 5 ft. 3 in. thick, if a block of the same stone 8 ft. long, 3 ft. 6 in. wide, and 3 ft. thick, weighs 12,600 lb.?

41. If 20 men build 240 rd. of fence in 24 da. of $9\frac{1}{3}$ hr., how many hours a day will 16 men have to work to build 300 rd. of fence in 50 da.?

42. If a bin 16 ft. long, 6 ft. wide, and 4 ft. high, holds 308 bu. of grain, what must be the height of a bin 24 ft. long, 5 ft. 4 in. wide, to hold 462 bu.?

43. If there is no waste in either case, how much will it cost to carpet a room with carpet 3 ft. wide at 90¢ per yard, if it costs $25 to carpet the same room with carpet 4 ft. wide at $1.25 per yard?

44. In the reprint of a book consisting of 810 pages, 50 lines, instead of 40, are contained in a page, and 72 letters, instead of 60, in a line. Of how many pages will the new edition consist?

45. If 15 men cut 480 steres of wood in 10 days of 8 hours each, how many boys will it take to cut 1152 steres, only $\frac{2}{3}$ as hard, in 16 days of 6 hours each, provided that, while working, a boy can do only $\frac{3}{4}$ as much as a man, and that $\frac{1}{3}$ of the boys are idle at a time, throughout the work?

It is recommended that Analysis be substituted for Partitive and for Conjoined Proportion.

46. Divide 360 into three parts proportional to 3, 4, and 5.

$\frac{3}{12}$ *of 360 = 90, first*

$\frac{4}{12}$ *of 360 = 120, second*

$\frac{5}{12}$ *of 360 = 150, third*

PROOF

1. *90 + 120 + 150 = 360*
2. *90 : 120 = 3 : 4*
3. *90 : 150 = 3 : 5*
4. *120 : 150 = 4 : 5*

Relations: first given part = $\frac{3}{12}$ of the sum of the given parts; second given part = $\frac{4}{12}$ of the sum of the given parts; third given part = $\frac{5}{12}$ of the sum of the given parts.

Therefore, first required part = $\frac{3}{12}$ of the sum of the required parts, or $\frac{3}{12}$ of 360, or 90; second required part = $\frac{4}{12}$ of the sum of the required parts, or $\frac{4}{12}$ of 360, or 120; etc.

47. A, B, and C go into partnership. A puts in $960; B, $510; and C, $1440. If they gain $727.50, how much should each receive?

$$A's = \frac{960}{2910} \times 727.50 = 240$$

$$B's = \frac{510}{2910} \times 727.50 = 127.50$$

$$C's = \frac{1440}{2910} \times 727.50 = 360$$

Relations: A's investment = $\frac{960}{2910}$ of the whole investment; B's = $\frac{510}{2910}$ of the whole; C's = $\frac{1440}{2910}$ of the whole.

Therefore, A's gain = $\frac{960}{2910}$ of the whole gain, or $\frac{960}{2910}$ of $727.50, or $240; . . . .

48. If 10 barrels of apples are worth 7 cords of wood, and 14 cords of wood are worth 5 tons of hay, how many barrels of apples are worth 50 tons of hay?

$$50 \times \frac{14}{5} \times \frac{10}{7} = 200$$

Ans. 200 bbl. apples.

Relations: 1 ton of hay is worth $\frac{14}{5}$ cords of wood; 1 cord of wood is worth $\frac{10}{7}$ bbl. of apples.

Therefore, 1 ton is worth $\frac{14}{5} \times \frac{10}{7}$ bbl., and 50 tons are worth $50 \times \frac{14}{5} \times \frac{10}{7}$ bbl., or 200 bbl.

49. Divide $510 into three parts which shall be to each other as 2, 3, and 5.

50. An insolvent debtor fails for \$3780, and is able to pay only \$1550. If A's claim is \$378, how much will he receive?

51. Divide \$873 among A, B, and C, so that for every \$2 that A receives, B shall receive \$4, and C, \$3.

52. A and B buy goods to the amount of \$600, of which A pays \$250, and B, \$350. If they lose \$150, what will be the loss of each?

53. A bankrupt owes A \$600; B, \$800; C, \$1000; D, \$1200; but his property is worth only \$1440. How much should each of his creditors receive?

54. Divide a man's estate of \$29,000 so that his wife shall receive \$7 for every \$5 received by each of his two sons, and every \$4 received by each of his three daughters.

55. A, B, and C engage in business, A putting in \$982; B, \$365; and C, \$843. If their profit in one year is \$1460, what is each one's share?

56. A, B, and C plant 1200 acres of corn, A planting 2 times as many acres as B; and B, 3 times as many acres as C. They sell the entire crop, amounting to 45 bu. to the acre, at 22¢ per bu. What is each man's share of the profit?

57. If 10 lb. of cheese are equal in value to 7 lb. of butter, and 14 lb. of butter to 5 bu. of corn, and 12 bu. of corn to 8 bu. of rye, how many pounds of cheese are equal in value to 4 bu. of rye?

58. If 15 bu. of wheat are worth 18 bu. of rye, and 5 bu. of rye are worth 8 bu. of corn, and 9 bu. of corn are worth 12 bu. of oats, and 16 bu. of oats are worth 20 lb. of coffee, how many pounds of coffee should be exchanged for 20 bu. of wheat?

SOLUTION OF PROBLEMS

METHODS OF PROCEDURE

There are three methods of solving problems: *analysis, the literal method*, and *proportion*. In each, the relations between the given terms and the required terms are expressed by equations.

By analysis, the required term must form one member of an equation, and the given terms the other; no operation is performed upon the required term.

By the literal method, no attempt is made to place the required term by itself, but the equation is stated naturally, and the operations are performed upon the terms without discrimination.

By proportion, the equation is stated as an equality of two ratios, the antecedent of the second ratio being of the same denomination as the answer, and the consequent, the required term.

ILLUSTRATIONS

At 4¢ each, how many apples can be purchased for 8¢?

ANALYSIS

Relation: number of apples = the number of times cost of 1 apple is contained times in cost of all.

Solution: since 1 apple costs 4¢, as many apples can be bought for 8¢, as 4¢ is contained times in 8¢, or 2 apples.

LITERAL METHOD

Relation: cost of all = cost of 1 apple × no. of apples.

Solution: let x = no. of apples; $4x$ = cost of all; $4x = 8$; $x = 2$, no. of apples.

PROPORTION

Relation: cost of 1 : cost of all = 1 apple : all apples.

Solution: $4 : 8 = 1 : x$; $x = \frac{8 \times 1}{4} = 2$, no. apples.

PROBLEMS OF PURSUIT

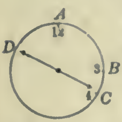
Analysis proceeds indirectly, introducing a special method for each special case; the literal method proceeds directly, expressing the required term by a letter, which is used as a number. Both methods should be mastered; the former will give power for an indirect, and the latter for a direct, attack upon a problem.

1. At what time between 3 and 4 o'clock are the hands of a watch opposite to each other?

Necessary knowledge: at 3 o'clock the min. hand is at 12, and the hr. hand at 3; when the hands are opposite to each other, they are 30 min. spaces apart; while the min. hand advances 60 spaces, the hr. hand advances 5 spaces.

Analysis

Relation: the min. hand will advance as many min. spaces as the number of spaces it gains in 1 min. is contained times in the number of spaces to be gained.



The min. hand has advanced from A to D, or 45 spaces + B to C; because from A to B is 15 spaces, and from C to D is 30 spaces. The hr. hand has advanced from B to C; therefore, the min. hand has gained 45 spaces.

Since the min. hand advances 60 spaces while the hr. hand advances 5 spaces, the min. hand gains 55 spaces in 60 min., or $\frac{55}{60}$, or $\frac{11}{12}$ of a space in 1 min. Since it gains $\frac{11}{12}$ of a space in 1 min., it will take as many min. to gain 45 spaces, as $\frac{11}{12}$ is contained times in 45, or $49\frac{1}{11}$ min.

Literal Method

Relations: spaces passed by min. hand = 12 times spaces passed by hr. hand; spaces passed by min. hand = spaces passed by hr. hand + spaces gained by min. hand.

Let $x =$ *sp. pas. hr. h.*, $4\frac{1}{11}$

$12x =$ *sp. pas. min. h.*, $49\frac{1}{11}$

$x + 45 =$ *sp. pas. min. h.*

$$\therefore\ 12x = x + 45$$

$$12x - x = 45$$

$$11x = 45$$

$$x = 4\frac{1}{11}$$

Proof

1. $49\frac{1}{11} = 12 \times 4\frac{1}{11}$.
2. $49\frac{1}{11} = 45 + 4\frac{1}{11}$.

2. How many minute spaces does the minute hand of a watch gain on the hour hand in 1 minute?

3. If the three hands of a watch all turn on the same point, how many minute spaces does the second hand gain on the hour hand in 1 minute?

4. When the hands of a watch are first 20 min. spaces apart between 5 and 6 o'clock, how many spaces has the min. hand gained on the hr. hand since 5? Draw a diagram.

5. When the hands are at right angles between 2 and 3 o'clock, how many spaces has the minute hand gained since 2? Draw a diagram.

6. At what time between 5 and 6 o'clock are the hands of a watch first 20 minute spaces apart?

7. At what time between 2 and 3 o'clock are the hands of a watch at right angles?

8. Between 4 and 5 o'clock, when the hour hand is as much after 4 as the minute hand is before 10, how many minute spaces have the hour and minute hands together passed since 4 o'clock? How many spaces do they together pass in 1 minute?

9. In Ex. 8, what is the time? Solve both with and without the use of x.

10. At what time between 8 and 9 o'clock are the hands of a watch together?

11. A and B start from the same point and travel in the same direction. If A travels 6 miles an hour, and B 4 miles an hour, how far apart are they after 6 hours?

12. In how many hours will A overtake B, if the latter has 5 hr. the start?

13. If they travel in opposite directions, how far apart are they at the end of 6 hours?

14. If they are 60 miles apart and travel toward each other, how far will A travel before they meet?

15. Two men, A and B, 26 miles apart, set out toward each other, B 30 minutes after A; A travels 3 mi. an hr., and B 4 mi. an hr. How far will each have traveled when they meet?

16. A fox has 60 of its leaps the start of a hound. While the fox makes 5 leaps the hound makes 4; 3 leaps of the fox cover the same distance as 2 leaps of the hound. How many leaps must the hound make to catch the fox?

Analysis

Relation: the hound must make as many leaps as the distance (in fox leaps) he gains in 1 leap is contained times in the distance (in fox leaps) to be gained.

Solution: the distance the hound goes in 1 leap = the length of $\frac{3}{2}$ fox leaps, because 2 leaps of the hound cover the same distance as 3 leaps of the fox. The distance the fox goes during the same period is $\frac{5}{4}$ fox leaps, because the fox makes 5 leaps while the hound makes 4. Therefore, in 1 leap, the hound gains $(\frac{3}{2} - \frac{5}{4})$ fox leaps, or $\frac{1}{4}$ of a fox leap.

It would take the hound as many leaps to gain 60 fox leaps, as $\frac{1}{4}$ is contained times in 60, or 240 leaps.

Literal Method

Relations: distance hound runs = length of 1 leap × no. of leaps; distance hound runs = distance fox runs + start of fox.

Let $5x =$ *no. lp. fox*, 300

then $4x =$ *no. lp. hound*, 240

Let $3a =$ *length 1 h. lp.*

then $2a =$ *length 1 f. lp.*

$12ax =$ *distance h. runs*

$10ax + 120a =$ *distance h. runs*

$$\therefore 12ax = 10ax + 120a$$

$$2ax = 120a$$

$$x = 60$$

17. A fox pursued by a hound makes 3 leaps while the hound makes 2; but the latter in 3 leaps goes as far as the former in 7. Find the length of 1 hound leap in terms of fox leaps.

18. Find the distance in terms of fox leaps that the hound gains in one leap.

19. If the fox has 60 of her own leaps the start, how many times will the hound leap before he catches the fox?

20. If the fox has 60 of the hound leaps the start, how many times will the fox leap before she is overtaken?

21. A hare is pursued by a hound. The hare makes 5 leaps while the hound makes 3 leaps; 2 leaps of the hound cover the same distance as 5 leaps of the hare. If the hare has 50 of her leaps the start, in how many leaps will the hound overtake her?

BUYING AND SELLING

22. By selling eggs at 6¢ each, I shall lose 24¢; by selling at 10¢ each, I shall gain 24¢. How many eggs have I?

ANALYSIS

Relation: I have as many eggs as the gain on 1 egg is contained times in the entire gain.

Solution: since the difference in selling price on 1 egg is 4¢, the difference in gain on 1 egg must be 4¢.

Since I lose 24¢ in one case, and gain 24¢ in the other, the difference in gain on all the eggs is 48¢.

Since the difference in gain on 1 egg is 4¢, I must have as many eggs to gain 48¢, as 4¢ is contained times in 48¢, or 12 eggs.

LITERAL METHOD

Relations: cost of all = 1st sell. price of all + 24¢; cost of all = 2d sell. price of all − 24¢.

Let $x =$ *no. eggs*, 12

$6x + 24 =$ *cost all*

$10x - 24 =$ *cost all*

$\therefore 6x + 24 = 10x - 24$

$48 = 4x$

$x = 12$

PROOF

1. $12 \times 6 + 24 = 96.$
2. $12 \times 10 - 24 = 96.$

23. If I gain 2¢ apiece by selling eggs at 72¢ a dozen, how much apiece do I gain by selling them at 60¢ a dozen?

24. If I gain 2¢ apiece by selling eggs at 72¢ a dozen, how much apiece do I lose by selling them at 24¢ a dozen?

25. If I sell eggs at 96¢ a dozen, I gain on all 48¢ more than if I sell them at 72¢ a dozen. How many eggs have I?

26. If I sell eggs at 24¢ a dozen, I lose 30¢ on all; if I sell them at 60¢ a dozen, I gain 15¢ on all. How many eggs have I?

27. If I sell eggs at 12¢ a dozen, I lose 3¢ apiece. How much a dozen must I charge to gain 3¢ apiece?

28. I sell 8 eggs for a certain price. Had I sold 2 more for the same money, the price of each egg would have been diminished 1¢. For how much did I sell each egg?

29. B bought apples at 2 for a cent and the same number at 3 for a cent; he sold them all at 5 for 2¢, and thereby lost 2¢. How many did he buy?

LABOR PROBLEMS

30. A agreed to work 24 days for \$2 a day and his board, and to pay 50¢ a day for board when idle; at the end of the time he received \$38. How many days was he idle?

Analysis

Relation: he was idle as many days as the amount lost for each idle day is contained times in the entire amount lost.

Solution: on each idle day he lost \$2 that he might have earned, and 50¢ for board, or \$2.50 in all.

If he had worked the whole time, he would have received \$48; he lost through idleness, \$48 – \$38, or \$10.

Since he lost \$2.50 for 1 idle day, he must have been idle as many days as \$2.50 is contained times in \$10, or 4 days.

Literal Method

Relation: amount received for labor – amount paid for board = \$38.

Let $x =$ *no. da. idle*, 4

then $24 - x =$ *no. da. work*, 20

$48 - 2x =$ *amt. for labor*

$\frac{x}{2} =$ *amt. for board*

$$48 - 2x - \frac{x}{2} = 38$$

$$96 - 4x - x = 76$$

$$20 = 5x$$

$$x = 4$$

Proof

1. $20 \times \$2 - 4 \times \$.50 = \$38.$

31. A agrees to work for 40 days at \$1.50 a day and his board, and to pay 50¢ a day for board when idle. How much does he lose each idle day?

32. If he receives \$20 at the end of the time, how many days was he idle?

33. A agrees to work 30 days at \$3 a day, and to forfeit \$1 a day for every day he is idle. How much does he lose each idle day?

34. If he receives \$60 at the end of the time, how many days did he work?

35. A agrees to work 30 da. at \$3 a day and his board, and to pay \$1 a day for his board when idle; at the end of the time he receives \$40. How many days was he idle?

INVOLVING A PART

36. If A can do a piece of work in 5 days, and B in 3 days, in how many days can they do the work together?

Relation: both can do the work in as many days, as the part they can do in 1 day is contained times in the whole.

Solution: in 1 day, A can do $\frac{1}{5}$ of it; B, $\frac{1}{3}$ of it; both, the sum of $\frac{1}{5}$ and $\frac{1}{3}$, or $\frac{8}{15}$ of it. It will take them as many days to do the whole as $\frac{8}{15}$ is contained times in $\frac{15}{15}$, or $1\frac{7}{8}$ days.

37. A pastures 5 cows, and B, 4 cows. If the whole expense is \$18, how much should each pay?

Relation: A's expense will be 5 times the cost of pasturing 1 cow; B's expense, 4 times the cost of pasturing 1 cow.

Since the expense for 9 cows is \$18, the expense for 1 cow is $\frac{1}{9}$ of \$18, or \$2. A's expense is $5 \times$ \$2, or \$10; B's, $4 \times$ \$2, or \$8.

38. A crew row down stream 8 mi. an hr. and up stream 6 mi. an hr. How far down stream can they row and return in 7 hr.?

Analysis

Relation: they can go as many miles, as the number of hours required to go and return 1 mile are contained times in 7 hr.

Solution: to row 1 mi. down stream requires $\frac{1}{8}$ hr.; up stream, $\frac{1}{6}$ hr.; both ways, $\frac{1}{8}+\frac{1}{6}$, or $\frac{7}{24}$ hr. They can go and return as many mi. in 7 hr. as $\frac{7}{24}$ is contained times in 7, or 24 mi.

Literal Method

Relation: no. hours down + no. hours up = 7.

Let x = *no. miles*, 24

$\frac{x}{8}$ = *no. hr. down*

$\frac{x}{6}$ = *no. hr. up*

$$\therefore \frac{x}{8}+\frac{x}{6}=7$$

$$3x+4x=168$$

$$x=24$$

39. A can do a piece of work in 4 days, and B in 5 days. What part of the work can A do in 1 day?

40. What part can both do in 1 day? How much more can A do in 1 day than B? How many days will it take them both to do it?

41. C and D together can do a piece of work in 6 days; C alone, in 8 days. How many days will it take D alone?

42. A, B, and C can do a piece of work in 20 days; A and B, in 40 days; A and C, in 30 days. In how many days can each alone do it?

43. Two pipes can fill a reservoir in 8 days; with the help of a third pipe, they can fill it in 3 days. How many days will it take the third alone to fill it?

44. A and B can do a piece of work in 6 hr. After A has worked alone for 3 hr., B commences and, working alone, finishes the work in $10\frac{1}{2}$ hr. In how many hours can A do the work alone?

45. A can ride on a bicycle 12 miles an hour, and return on the cars 30 miles an hour. What part of an hour does it take him to ride 1 mile on his bicycle?

46. How many miles can he ride on his bicycle and return by the cars, in 7 hours?

47. If a steamer sails 9 mi. an hr. down stream, and 5 mi. an hr. up stream, how far can it sail down stream and return in 28 hr.?

48. A puts 8 cows into a pasture for 5 months; B, 10 calves for 8 months; B pays $16. How much should A pay, if 4 cows eat as much as 5 calves?

49. A and B rent 32 A. of land for $63. A agrees to take 12 A. of timber, and B, 20 A. of meadow land. How much should each pay, if 3 A. of timber rent for the same as 4 A. of meadow?

50. Henry has 8 marbles, James 10, and Walter none; they divide the marbles equally among them, and Walter pays 6¢ to Henry and James. How much should each receive?

51. A, B, and C enter into partnership. A puts in $6000 for 4 mo.; B, $8000 for 3 mo.; and C, $4000 for 6 mo. If their profits amount to $5040, what is each man's share?

52. A, B, C, and D plant 5464 acres of corn. A puts in $1\frac{3}{4}$ times as many acres as B; C, $2\frac{4}{5}$ times as many; and D, $5\frac{5}{6}$ times as many. If they market their crop for $43,712, what is each man's share of the profits?

A PART MODIFIED

53. What number increased by $\frac{2}{3}$ of itself becomes 15?

LITERAL MODIFIED

Relation: required number + $\frac{2}{3}$ of itself = 15.

Solution: a number increased by $\frac{2}{3}$ of itself becomes $\frac{5}{3}$ of itself. Since $\frac{5}{3}$ times the number is 15, the number is $15 \div \frac{5}{3}$, or 9.

NOTE. $\frac{5}{3} \times$ No. = 15; No. = $15 \div \frac{5}{3}$, because either of two factors is equal to their product divided by the other.

Analysis: since $\frac{5}{3}$ of the number is 15, $\frac{1}{3}$ of the number is $\frac{1}{5}$ of 15, or 3; $\frac{3}{3}$, or the number, is 3 times 3, or 9. *See p. 98, Ex. 250, Note.*

LITERAL METHOD

Relation: required number + $\frac{2}{3}$ of itself = 15.

$$\text{Let } x = \textit{number}$$
$$\frac{2x}{3} = \textit{increase}$$
$$\frac{5x}{3} = \textit{no. increased}$$
$$\frac{5x}{3} = 15$$
$$5x = 45$$
$$x = 9$$

PROOF. $9 + \frac{2}{3}$ of $9 = 15$

54. By selling a watch for $40, a man lost $\frac{1}{3}$. What was the cost?

Business usage: the gain or loss is always some part of the cost.

Relation: cost − $\frac{1}{3}$ of the cost = selling price.

Solution: the cost diminished by $\frac{1}{3}$ of itself becomes $\frac{2}{3}$ of itself. Since $\frac{2}{3}$ times the cost is $40, the cost is $40 ÷ $\frac{2}{3}$, or $60.

55. An agent sold an article for $100 on a commission of $\frac{1}{10}$. What were the proceeds?

Business usage: when an agent buys, his commission is some part of the buying price; when an agent sells, his commission is some part of the selling price.

Relation: proceeds = selling price − commission.

Solution: since an agent sold on commission at $\frac{1}{10}$, his commission was $\frac{1}{10}$ of $100, or $10; the proceeds were $100 − $10, or $90.

56. An agent bought an article for $100 on a commission of $\frac{1}{10}$. What was the entire cost?

Relation: entire cost = buying price + commission.

Solution: since an agent bought on commission at $\frac{1}{10}$, his commission was $\frac{1}{10}$ of $100, or $10; the entire cost was $100 + $10, or $110.

57. What number increased by 5 times itself becomes 30? by 4 times itself, 20?

58. What number increased by $\frac{1}{3}$ of itself becomes 20? by $\frac{1}{2}$ of itself, 30?

59. What number diminished by $\frac{1}{4}$ of itself becomes 30? by $\frac{1}{5}$ of itself, 20?

60. A horse cost \$60 and was sold at a gain of $\frac{1}{5}$. What was the selling price?

61. A horse cost \$60 and was sold at a loss of $\frac{1}{4}$. What was the selling price?

62. By selling a horse for \$40, I lose $\frac{1}{5}$. How much did he cost?

63. By selling a horse for \$36, I gain $\frac{1}{8}$. How much did he cost?

64. By selling a horse for \$40, I gain $\frac{1}{4}$. By how much must I increase my price to gain $\frac{1}{2}$?

65. If my gain was $\frac{2}{3}$, or \$40, what was the selling price? the cost?

66. If my loss was $\frac{1}{10}$, or \$20, what was the cost? the selling price?

67. If I buy at \$3 and sell at \$4, what part do I gain?

68. If I sell $\frac{5}{9}$ of an article for what the whole cost, what part of the whole do I gain?

69. An article was sold at a gain of $\frac{1}{10}$; if it had cost \$120 more, the same selling price would have entailed a loss of $\frac{1}{10}$. Find the cost.

70. A man sold an article for 40¢, and thereby gained $\frac{1}{3}$ as much as if he had sold it for 60¢. What was the cost?

71. An agent sold flour for \$200 at a commission of $\frac{1}{20}$. What was the commission? the proceeds?

72. An agent bought flour for \$200 at a commission of $\frac{1}{50}$. What was the commission? the entire cost to his employer?

73. An agent sold flour at a commission of $\frac{1}{10}$, and received as commission, \$25. What were the proceeds?

PERCENTAGE

TERMS AND RELATIONS

In many cases, it has become customary to use the term *per cent*, written %, in place of hundredths.

The number on which the per cent is computed is the *base;* the product is the *percentage;* the base + the percentage, the *amount;* the base − the percentage, the *difference.*

% means hundredths. *See p. 114.*

ILLUSTRATION

.06 of 200 = 12
Or,
6% of 200 = 12

200, *base*
12, *percentage*
212, *amount*
188, *difference*
6% means *.06*

A per cent expression may be changed to a fraction or an integer, and conversely.

Reduce:

1. 16% to a decimal.
2. $33\frac{1}{3}$% to a decimal.
3. $33\frac{1}{3}$% to a fraction.
4. 400% to an integer.
5. .16 to per cent.
6. $.33\frac{1}{3}$ to per cent.
7. $\frac{1}{3}$ to per cent.
8. 4 to per cent.

Ex. **3.** $33\frac{1}{3}\% = .33\frac{1}{3} = \frac{1}{3}$.
Ex. **4.** $400\% = \frac{400}{100} = 4$.

Ex. **7.** $\frac{1}{3} = .33\frac{1}{3} = 33\frac{1}{3}\%$
Ex. **8.** $4 = \frac{400}{100} = 400\%$

9. State rapidly the per cent equivalents: $\frac{7}{8}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{12}$, $\frac{5}{8}$, $\frac{5}{6}$, $\frac{1}{5}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{4}{5}$, $\frac{1}{16}$, $\frac{1}{9}$. *See p. 114.*

10. State rapidly the fractional equivalents: $62\frac{1}{2}$%, $87\frac{1}{2}$%, $37\frac{1}{2}$%, $33\frac{1}{3}$%, $16\frac{2}{3}$%, 25%, 40%, 75%, 60%, 20%, 80%, $8\frac{1}{3}$%, $83\frac{1}{3}$%, $12\frac{1}{2}$%, 50%. *See p. 114.*

CASE I — DIRECT

11. What is 6% of 50?

50
.06
3.00

6% of 50 is .06 of 50, or 3.

12. What is $16\frac{2}{3}\%$ of 486?

486
$.16\frac{2}{3}$
81

$16\frac{2}{3}\%$ of 486 is $\frac{1}{6}$ of 486, or 81.

Note. — A per cent expression must be reduced to a decimal or to a common fraction, before it can be used as a multiplier.

13. A farmer having 360 sheep, lost 5%. How many had he left?

360 = no. sheep
.05
18.00 = no. lost
342 = no. left

He lost .05 of 360 sheep, or 18 sheep; he had left the difference, or 342 sheep.

14. What is 16% of 5250? 18% of 3825? $66\frac{2}{3}\%$ of 3483?

15. A man wills 17% of his property to his son, and 18% to his daughter. If his property is worth $ 8200, how much will each receive?

16. A has $ 625; B has 86% as much as A; C, 78% as much as B. How much has C?

17. A lady buys 55 yards of muslin at 8¢ a yard, and 27 yards of cloth at 65¢ a yard. If she pays 66% of the bill, how much does she still owe?

18. A man's salary is $ 75 a month; if he spends 65% of his salary each month, how much will he have at the end of 6 months?

19. A gentleman gave $ 11.20 to his children; his sons received 65% of the money, and his four daughters the remainder. How much did each daughter receive?

20. A man has a library of 1600 volumes; 14% are biography; 62%, history, and $83\frac{1}{3}\%$ of the remainder, fiction. How many volumes of fiction are there in his library?

CASE II — INDIRECT

21. 3 is 6% of what number?

$$3 = 6\% \times No.$$

$$No. = \frac{3}{.06} = 50$$

The equation is readily formed by substituting '=' for 'is'; '×' for 'of,' and 'No.' for 'what number.'

Since either factor is equal to the product divided by the other factor, No. = 3 ÷ .06, or 50.

22. Of what number is 81, $16\frac{2}{3}\%$?

$$16\tfrac{2}{3}\% \; No. = 81$$

$$No. = 81 \div \frac{1}{6} = 486$$

The sign of multiplication may be omitted.

Since either factor is equal to the product divided by the other factor, No. = 81 ÷ $\frac{1}{6}$, or 486.

NOTE. — A per cent expression must be reduced to a decimal or to a common fraction, before it can be used as a divisor.

23. A farmer lost 18 sheep, or 5% of his flock. How many had he at first?

$$5\% \; F = no. \; lost$$
$$18 = no. \; lost$$
$$\therefore 5\% \; F = 18$$
$$F = \frac{18}{.05} = 360$$

PROOF

360 = no. at first
.05
18 = no. lost

Of what number:

24. Is 12, 50%?
25. Is 368, 23%?
26. Is 57, 15%?
27. Is 522, 18%?
28. Is 10.5, $12\frac{1}{2}\%$?
29. Is 40.5, $7\frac{1}{2}\%$?
30. Is 4578, 84%?
31. Is 7735, 85%?

32. What is a man's income if $31\frac{1}{4}\%$ is $600?

33. A general lost $16\frac{1}{2}\%$ of his army; 315 killed, 110 prisoners, and 70 deserters. How many men had he at first?

34. A man paid $750 for a house, which was 24% of what he paid for 160 A. of land. What was the cost of the land per acre?

35. Mr. Turner earns $85 a month; his salary for the year is 68% of his brother's salary for 8 months. How much does his brother receive a month?

CASE III — INDIRECT

36. 3 is what % of 50?

$$3 = R \times 50$$

$$R = \frac{3}{50} = .06 = 6\%$$

The equation is readily formed by substituting '=' for 'is'; 'R,' for %, and '×' for 'of.'

37. 81 is what % of 486?

$$81 = R \times 486$$

$$R = \frac{81}{486} = \frac{1}{6} = 16\tfrac{2}{3}\%$$

The $\frac{1}{6}$ must be reduced to % because the answer is to be expressed in %.

38. A farmer having 360 sheep, lost 18. What % did he lose?

$$18 = R \times 360$$

$$R = \frac{18}{360} = \frac{1}{20} = 5\%$$

Relation: the number lost is some % of the whole number.

What %:

39. Of 45 is 15?

40. Of 72 is 18?

41. Of 78 is 63?

42. Of 1311 is 437?

43. Of 2288 is 286?

44. Of 2700 is 300?

45. A man worth $250,000 lost $27,500. What % remained?

46. Mr. Brown works 6 days each week for 44 weeks of the year. What per cent of the time is he idle, counting 300 working days to the year?

47. The population of a certain town is 57,500; of these, 3450 are Irish; 22,540, Spanish; and the remainder, English. What per cent of the population is English?

48. A man having $1275, spent $210 for a carriage, $112 for a horse, and $86 for a harness. What per cent of his money remained?

49. A rectangular field 75 feet long, is $\frac{16}{25}$ as wide. The breadth is what per cent of the length?

50. The product of two numbers is 11,250; the first is 125. What per cent of the first is the second?

CASE IV — INDIRECT

51. What number increased by 6% of itself, becomes 53?

$100\%\ N =$ *the no.*
$6\%\ N =$ *the increase*
$106\%\ N =$ *the no. incr.*
$53 =$ *the no. incr.*
$\therefore 106\%\ N = 53$
$$N = \frac{53}{1.06} = 50$$

PROOF

50 = the no.
.06
3 = the increase
53 = the no. incr.

52. What number diminished by $16\frac{2}{3}\%$ of itself, becomes 405?

$100\%\ N =$ *the no.*
$16\frac{2}{3}\%\ N =$ *the decrease*
$83\frac{1}{3}\%\ N =$ *the no. decr.*
$405 =$ *the no. decr.*
$\therefore 83\frac{1}{3}\%\ N = 405$
$$N = 405 \div \tfrac{5}{6} = 486$$

PROOF

486 = the no.
$.83\frac{1}{3}$
81 = the decrease
405 = the no. decr.

53. After losing 5% of his sheep, a farmer had 342 left. How many had he at first?

$100\%\ F =$ *no. at first*
$5\%\ F =$ *no. lost*
$95\%\ F =$ *no. left*
$342 =$ *no. left*
$\therefore 95\%\ F = 342$
$$F = \frac{342}{.95} = 360$$

PROOF

360 = no. at first
.05
18 = no. lost
342 = no. left

What number increased by:

54. $66\frac{2}{3}\%$ of itself becomes 30?

55. 6% of itself becomes 212?

56. 25% of itself becomes 60?

What number diminished by:

57. 25% of itself becomes 30?

58. 6% of itself becomes 188?

59. 10% of itself becomes 81?

MISCELLANEOUS PROBLEMS

60. After 16% of a heap of grain was taken away, there remained 252 bushels. How many bushels were there at first?

61. In 1890, the population of a town was 17,280, which was 35% more than in 1880. What was the population in 1880?

62. My crop of corn this year is $12\frac{1}{2}$% greater than last year, and I have raised, during the two years, 3825 bushels. What was my last year's crop?

63. A young man having received a fortune, deposited 60% of it in bank; he afterward drew 30% of his deposit, and then had $7560 in bank. What was his entire fortune?

64. Mr. Black's property is valued at $15,000, and 85% of his property is 2% more than his brother's property. What is the value of his brother's property?

65. At a forced sale, a bankrupt sold his farm for $6642, which was 18% less than its real value. What was the value of the farm?

66. Into a vessel containing pure vinegar, there were poured $12\frac{1}{2}$ gallons of water, which was $16\frac{2}{3}$% of the mixture. What was the quantity of pure vinegar?

67. A man paid $16.20 for the use of land which cost $360. What % did the owner realize on his investment?

68. From a cask containing 24 gal. 3 qt., 17 gal. $3\frac{3}{5}$ qt. leaked out. What % leaked out?

69. A fruit grower, having sent 2200 baskets of peaches to Philadelphia, found that 11% of them had decayed. If he sold the balance at 75¢ a basket, what sum did he receive for his peaches?

70. On Jan. 1, a man weighed 150 lb. In January he lost $2\frac{1}{2}$% in weight, and in February gained $2\frac{1}{2}$%. What % of his weight Jan. 1 was his weight on the first day of March?

71. How many gallons of water must be mixed with $70\frac{1}{2}$ gallons of wine, so that the mixture may contain 6% of water?

PROFIT AND LOSS

In buying or selling, the gain or loss is always some % of the cost.

If I buy for \$100 and sell for \$110, the gain is \$10, or 10% of the cost.

This principle is established by business usage.

72. A boat was bought for \$9136.50, and sold at a loss of 3%. What was the selling price?

$$\begin{array}{rl} \$9136.50 & = cost \\ .03 & \\ \hline \$274.095 & = loss \\ 9136.50 & \\ \hline \$8862.40 & = selling\ price \end{array}$$

NOTE. — Since our smallest coin is *one cent*, we give the answer to the nearest cent. The loss is \$274.10. Count 5 or more mills as 1¢; neglect less than 5 mills.

73. The profit on the sale of a horse was \$39.20, or 14%. What was the cost?

$$14\%\ C = gain$$
$$\$39.20 = gain$$
$$\therefore 14\%\ C = 39.20$$
$$C = \frac{39.20}{.14} = 280$$
$$Cost = \$280.$$

PROOF

$$\begin{array}{rl} \$280 & = \text{cost} \\ .14 & \\ \hline 1120 & \\ 280 & \\ \hline \$39.20 & = \text{gain} \end{array}$$

74. A house was sold for \$320.32 at a gain of 12%. What was the cost?

$$\begin{array}{rl} 100\%\ C & = cost \\ 12\%\ C & = gain \\ \hline 112\%\ C & = selling\ price \\ 320.32 & = selling\ price \end{array}$$
$$\therefore 112\%\ C = 320.32$$
$$C = \frac{320.32}{1.12} = 286$$
$$Cost = 286.$$

PROOF

$$\begin{array}{rl} \$286 & = \text{cost} \\ .12 & \\ \hline 34.32 & = \text{gain} \\ 286. & \\ \hline \$320.32 & = \text{selling price} \end{array}$$

75. I sold a plow for $16.32, and thereby lost 4%. What was the cost?

76. My sales exceeded the cost by $240; the sales were $560. What was the gain %?

77. I sold silks for $270.90, thereby losing 10%. What selling price would have made the gain $26.50?

78. A merchant bought 540 yards of muslin at 7¢, and sold it at a reduction of $2\frac{1}{2}$%. What was the entire loss?

79. I bought a quantity of wheat at 75¢ a bushel. At what price must it be sold to gain 16%?

80. I bought 75 horses at $56 each, expenses, $8. If I lost 8 of them, at what average price must I sell them to gain 15%?

81. By selling for $2000, I gained 14% of the selling price. Had I gained 14% of the cost, how much would I have received?

82. How shall I mark shoes that cost $2.50, so that I may deduct 20% from the marked price and still make 10%?

83. I sold a carriage at 20% gain, and with the money bought a horse which I sold for $168.30, thereby losing 15% of the selling price of the carriage. What was the cost of the carriage?

84. I began business with $24,000; gained 16% the first year and added it to the capital. What was my capital at the beginning of the second year?

85. 4% of my capital was invested in 18 watches at $180 a dozen; 7% of my capital was invested in clocks at $2.10 each. How many clocks had I?

86. A merchant marked goods at $16\frac{2}{3}$% above cost, and sold the goods at this marked price for $350. What was the cost? What % would he have gained by selling the goods for $375?

87. A man gained $33\frac{1}{3}$% on the sale of a horse, 20% on the sale of a carriage, and 50% on the sale of a buggy. If the selling price of each was $150, what was his entire gain?

88. A man lost 10% of his money, then gained 10% of what he had left, and then had $396. How much had he at first?

COMMISSION

A person may be employed to buy or sell for another. The employer is the *principal*, the employed, the *agent;* the price paid for the service, the *commission;* the amount returned to the principal, the *net proceeds.*

Business Usage

If an agent sells, his commission is some per cent of the sales.

If an agent buys, his commission is some per cent of the purchase.

A farmer takes to a grocer 10 barrels of apples to be sold at $2 a barrel, agreeing to pay 10% commission for selling them. It is just that the grocer should keep 10% of the sales.

A manufacturer sends his agent $1050 to buy wool, after deducting a commission of 5%. If the agent takes 5% of $1050, he will take 5% of what he pays for the wool, and also 5% of what he keeps. It is not just that he should receive 5% of what he keeps, because he performs no labor for it, but he is entitled to 5% of what he pays for the wool.

89. Find the amount of sales, when an agent receives $40 at a commission of 20%.

$$20\%\ S = 40$$
$$S = \frac{40}{.20} = 200$$
Sales = $200.

Since the agent sells, his commission is 20% of sales.

90. Find the amount of sales, when the principal receives $40; agent's commission 20%.

$$100\%\ S = \text{sales}$$
$$20\%\ S = \text{com.}$$
$$80\%\ S = \text{proceeds}$$
$$\$40 = \text{proceeds}$$
$$\therefore 80\%\ S = 40$$
$$S = \frac{40}{.80} = 50$$
Sales = $50.

Proof

$$\begin{array}{rl} \$50 & = \text{sales} \\ .20 & \\ \hline 10 & = \text{com.} \\ 50 & \\ \hline \$40 & = \text{proceeds} \end{array}$$

91. At 3%, what is an agent's commission on $1150 sales?

92. At 3%, what is an agent's commission on $1150 purchase?

93. Find the commission when an agent receives $259.60 to be invested in goods, after deducting his commission of 10%.

94. How many pounds of sugar at 8¢ a pound can an agent buy for $15.99, after deducting his commission of 2½%?

95. Find the rate per cent of commission when $2.40 is paid for a sale of $160.

96. Find the amount of sales when a commission of 2¼% pays the agent $6.48.

97. Find the net proceeds from the sale of 36 barrels of sugar at $4, commission 6¼%.

98. Find the amount of the purchase when an agent receives $351 to invest in sugar after deducting his commission of 8%.

99. My principal instructed me to invest $1220 in wool, and sent me a draft for $1220 plus 2% commission. What was the amount of the draft?

100. My principal instructed me to invest $1244.40 in wool after deducting my commission of 2%. What was my commission?

101. After selling wheat, an agent deducts $56 commission, and sends his principal $2744. What rate of commission does he receive?

102. An agent bought 4000 bushels of oats; his commission at 2¼% was $28; charges for freight and storage, $52. How much per bushel did the oats cost the principal?

103. I sold on a commission of 4½%, 320 barrels of sugar at $18, and 60 barrels of oil, 44 gallons to the barrel, at 15¢ a gallon. How much did I remit to my employer?

104. An agent sells 1200 barrels of apples at $4.50 a barrel, and charges 2½% commission. After deducting his commission of 8% for buying, he invests the net proceeds in flour at $5 a barrel. What is his entire commission?

TAXES

In some states, every male citizen is annually taxed a small amount, *poll tax*, without regard to his property; property owners pay an additional tax. In these states, the entire tax is diminished by the sum of the *poll tax*, and the remainder is divided by the assessed value of all the taxable property, to find the tax on each dollar.

The collector usually receives some % of the amount collected.

105. A tax is $16,020; the taxable property $784,750; the number of polls at $1.25 is 260. A's property is $7800; what is his tax?

$1.25
260
$325 = poll tax
16,020 = whole tax
$15,695 = property tax
784750)15695.00(.02
15695.00

$7800 = A's property
.02
$156.00 = A's property tax
1.25 = A's poll tax
$157.25 = A's whole tax

106. The tax on A's property at 4 mills on a dollar is $615.80. What is the assessed value of his property?

107. The school tax of a certain town is $4782. If the rate of taxation is 3 mills on a dollar, what is the amount of taxable property?

108. A town is to be taxed $13651.48; the taxable property is $865,432; the number of polls at $1 is 670. What is B's tax, whose property is assessed at $9720?

109. The whole amount of taxable property in a certain town is $386,722; there are also 1560 polls at $1.50 each. If the rate of taxation is 5 mills on a dollar, what is the tax?

110. A tax of $14,846 is to be assessed on a certain village; the property is valued at $1,060,000, and there are 2123 polls at $2. What is the assessment on a dollar? What is Mr. Doan's tax, whose property is assessed at $11,600?

TRADE DISCOUNT

Wholesale dealers and manufacturers issue *price lists* of their goods. As the market varies, they change the *rate of discount* instead of changing the fixed price. They frequently offer other discounts and an additional discount for cash.

111. A man sold a bill of goods, list price $100, on 4 mo. at 5, 10, and 6% off for cash. What was the net price for cash?

$100 = list
5 = 1st dis.
$95 = after 1st
9.50 = 2nd dis.
$85.50 = after 2nd
5.13 = cash dis.
$80.37 = net price

Meaning: If the goods are paid for at the end of 4 mo., 5% will be deducted from the list price, and 10% from the remainder. If cash is paid at the time of purchase, an additional discount of 6% will be deducted from the last remainder.

112. A man sold a bill of goods, list price $100, on 4 mo. at 10, 5, and 6% off for cash. What was the net price for cash?

113. If the purchaser decides not to pay cash, is one of the above offers better for him than the other?

114. A merchant offered a bill of goods, list price $100, on 3 mo. at 8, and 4% off for cash. What was the cash price?

115. A merchant offered a bill of goods, list price $100, on 3 mo. at 4%, and 8% off for cash. What was the cash price?

116. If the purchaser had decided not to pay cash, which of the offers would have been to his advantage; that in Ex. 114, or that in Ex. 115?

117. A bill of goods, list price $100, was offered at 45% discount; or at 30% and 15% off. Which offer was better for the purchaser?

118 Which is better for the purchaser, 55% discount, or two successive discounts of 50% and 10%?

INSURANCE

An agreement to pay for loss or damage is *insurance.* The indemnity may be against loss on property, *property insurance;* by fire, *fire insurance;* or by accidents of navigation, *marine insurance.* The indemnity may also be against personal injury, or loss of life, *life insurance.*

The contract or agreement between the insurance company and the person protected is the *policy;* the sum paid for insurance, the *premium.*

The premium is some per cent of the amount insured.

119. A store worth \$20,000 was insured for $\frac{4}{5}$ of its value at $1\frac{1}{4}\%$. What was the premium?

\$16000 = amt. insured
$.01\frac{1}{4}$
\$200.00 = premium

Relation: the premium is some % of the amount insured.

120. Find the premium to be paid for insuring a person's life for \$5000, at an age for which the rate is $2\frac{1}{2}\%$.

121. For what sum should a cargo worth \$60,000 be insured at 4%, so that, in the event of loss, the owner may receive both the value of the cargo and the premium?

122. A man paid \$125 for insuring at $\frac{3}{4}\%$. What was the amount insured?

123. I paid \$53, including cost of policy, \$1.75, for insuring my property to the amount of \$8200. What was the rate?

124. What is the amount of the annual premium at \$35.20 per \$1000, on a life policy of \$6500?

125. I paid \$40.50, at $1\frac{1}{8}\%$, for insuring my house for $\frac{2}{3}$ its value. What is the value of the house?

126. A man 35 years old, takes out a \$6000 life insurance policy payable when he reaches the age of 55. If he pays an annual premium of \$63.10 per \$1000, how much money will he have paid in if he lives till the policy falls due?

DIFFICULT PROBLEMS

Care should be used in deciding what the '% is of.'

127. I mix 20% of rosin with 8 lb. of tallow. How many pounds of rosin in the mixture?

$100\%\ M = mixture$
$20\%\ M = rosin$
$80\%\ M = tallow$
$8\ lb. = tallow$
$\therefore 80\%\ M = 8$
$M = \frac{8}{.80} = 10$
Ans. 2 lb. rosin.

The 20% is of the *mixture* and not of the *rosin*; another form of statement would be "20% of a mixture is rosin."

Generally, it is best to represent what the '% is of' by 100% of that thing.

128. A quantity of sugar was sold at 10 per cent gain. If it had cost $120 more, the same selling price would have entailed a loss of 10 per cent. Find the cost of the sugar.

$100\%\ C = cost$
$100\%\ C + 120 = sup.\ cost$
$10\%\ C + 12 = sup.\ loss$
$90\%\ C + 108 = sell.\ p.$

$100\%\ C = cost$
$10\%\ C = gain$
$110\%\ C = sell.\ p.$

$$\therefore 110\%\ C = 90\%\ C + 108$$
$$110\%\ C - 90\%\ C = 108$$
$$20\%\ C = 108$$
$$C = \frac{108}{.20} = 540$$
$$Cost = \$540.$$

Sometimes it is more convenient to represent what the '% is of' by a number.

129. I mix 20% of rosin and 80% of tallow. What per cent of the weight of the tallow, is the weight of the rosin?

Let $5\ lb. = wt.\ mixture$
$.20$
$1\ lb. = wt.\ rosin$
$4\ lb. = wt.\ tallow$

Since rosin is some % of tallow,
$1 = R \times 4$
$R = \frac{1}{4} = 25\%$

In some problems, there is % of two different bases, and neither base is given.

130. How many per cent above cost must a man mark his goods, in order to take off 10 per cent and still make a profit of $12\frac{1}{2}\%$?

$$\begin{array}{rl} 100\ \%\ C &= \text{cost} \\ 12\frac{1}{2}\%\ C &= \text{gain} \\ \hline 112\frac{1}{2}\%\ C &= \text{sell p.} \end{array} \qquad \begin{array}{rl} 100\%\ M &= \text{marked p.} \\ 10\%\ M &= \text{deduction} \\ \hline 90\%\ M &= \text{sell p.} \end{array}$$

$$\therefore\ 90\%\ M = 112\frac{1}{2}\%\ C$$

$$M = \frac{112\frac{1}{2}\%\ C}{.90} = 125\%\ C$$

Mark. p. is 25% above cost.

In some problems, two equations with two unknown quantities are necessary.

131. My agent sold my flour at 4% commission. Increasing the proceeds by $168, I bought wheat, paying 2% commission; wheat declining 3%, my loss, including commissions, was $30. What was the selling price of the flour?

$$\begin{array}{rl} 100\%\ S &= \text{selling flour} \\ 4\%\ S &= \text{com.} \\ \hline 96\%\ S &= \text{proceeds} \\ 96\%\ S + 168 &= \text{invst. wheat} \end{array} \qquad \begin{array}{rl} 100\%\ C &= \text{cost wheat} \\ 2\%\ C &= \text{com.} \\ \hline 102\%\ C &= \text{invst. wheat} \\ 5\%\ C &= \text{dec. and com.} \end{array}$$

$$\begin{array}{rll} 4\%\ S + 5\%\ C &= 30 & (1) \\ 96\%\ S - 102\%\ C &= -\ 168 & (2) \\ \hline 96\%\ S + 120\%\ C &= 720 & (3) \\ 96\%\ S - 102\%\ C &= -\ 168 & (2) \\ \hline 222\%\ C &= 888 & (4) \\ C &= 400 & \\ \text{Cost} + \text{Com.} &= \$408. & \\ \text{Sell. p. Flour} &= \$250. & \end{array}$$

Since the loss, including commissions, was $30, we obtain (1).

Since the investment in wheat was $168 more than the proceeds of the flour, we obtain (2).

We solve these equations as on p. 167; 24 × (1) gives (3); (2) from (3) gives (4); . . . .

132. What per cent would a dishonest dealer gain by using a false weight of 15 oz. instead of a pound?

133. What per cent does a customer lose, if his grocer uses a false weight of 15 oz. instead of a pound?

134. The lead ore from a certain mine yields 40% of metal, and of the metal, $\frac{3}{4}$% is silver. How many ounces of silver will be obtained from a ton of ore?

135. A brewery is worth 4% less than a tannery, and the tannery 16% more than a boat; the owner of the boat has traded it for 75% of the brewery, thus losing $103. How much is the tannery worth?

136. 8 lb. of a certain article loses 3 oz. in weight by drying. What per cent of the original weight is water?

137. A man sold an article at 20% gain; had it cost $300 more, he would have lost 20%. What was the cost?

138. How must I mark goods so that I may deduct 10% from the marked price and still make 17%?

139. A merchant marked damaged goods at a certain per cent below cost, but sold them for cost at an advance of $11\frac{1}{9}$% on the marked price. What was the marked price?

140. An agent sold my corn, and, after reserving his commission, invested all the proceeds in corn at the same price; his commission, buying and selling, was 3% each, and his whole charge $12. For how much was the corn sold?

141. A dealer sold wheat, losing 4%; keeping $18 of the proceeds, he gave the remainder to an agent to buy corn, 8% commission; his loss together with the commission was $32. How much did the agent pay for the corn?

STOCKS AND BONDS

TERMS AND RELATIONS

For the prosecution of a business enterprise, individuals sometimes form a stock company. Officers are elected, and a charter (authority to do business as an individual) is obtained from the state.

The engraved part of a certificate states the number of parts (shares) into which the property (stock) is divided, the face value (par value) of each part, etc. The written part states the number of shares bought, the name of the purchaser, etc.

From a study of the above certificate, the pupil should answer the following questions:

Where is the Altar mine situated?

Who was the president of the Altar Mining Co.? the secretary?

Into how many shares is the property divided?

What is the par value of 1 share?

What is the capital stock?

Who is the owner of this certificate?

How can this certificate be transferred?

Usually, shares do not sell for their par value, but for less (at a discount), or for more (at a premium), according to the prospects of the company.

John Fluker bought the stock when it was first issued, at $2 a share, or at 2% of the par value, or at 98% discount.

If the business is successful, the profits (dividends) are divided according to the number of shares.

The profits of the company at the end of the first year were $660,000, and a dividend of 6% was declared.

Brokers buy and sell stock for their employers, charging something (brokerage) both for buying and selling.

After the dividend, James Lyman bought the 50 shares of John Fluker, through a broker, at $110 a share, paying ⅛% brokerage.

1. What is the number of this certificate of stock?
2. To whom was the certificate issued? For how many shares?
3. What was the par value of each share? How do you know?
4. What was the market value of each share at the time of issue? How do you know?
5. If the earnings were $660,000, what dividend might have been paid on each share?
6. How many dollars were paid as dividend on each share?
7. What dividend, in all, did John Fluker receive?
8. What was the par value of each share after the dividend? Does the par value ever change?
9. What was the market value of each share after the dividend? Does the market value change?
10. What was John Fluker's entire gain from the stock?
11. What % brokerage did James Lyman pay? What part of a dollar on each share, if the brokerage is some % of the par value?
12. How much brokerage in all did James Lyman pay? What was the entire cost of the stock to James Lyman?

PROBLEMS

The par value of one share is $ 100, unless otherwise stated.

This understanding saves confusion, and makes it unnecessary to call attention to the par value.

The cost, the selling price, the dividend, the brokerage, each is some % of the par value.

These terms must be some per cent either of the *market* value or of the *par* value. The par value never changes, the market value is constantly changing; hence the former is selected.

All examples in stocks may be analyzed by using $ 1, 1%, or 1 share as the unit. The last is the simplest, because it eliminates % as much as possible.

The student should be able to declare in *dollars* the cost of *one share*, the selling price of *one share*, the dividend on *one share*, and the brokerage of *one share*, however the same is stated.

13. *Cost.* What is the cost of 1 share of 6% stock at 60?

Ans. $ 60. The cost is 60% of $ 100, or $ 60.

14. *Selling price.* What is the selling price of 1 share of 8% stock at 70?

Ans. $ 70. The selling price is 70% of $ 100, or $ 70.

15. *Dividend.* What is the dividend on 1 share of 6% stock?

Ans. $ 6. The dividend is 6% of $ 100, or $ 6.

16. *Brokerage.* What is the brokerage on 1 share of 6% stock at $\frac{1}{8}$%?

Ans. $ $\frac{1}{8}$. The brokerage is $\frac{1}{8}$% of $ 100, or $ $\frac{1}{8}$.

NOTE. — On the New York Stock Exchange, brokerage higher than $\frac{1}{8}$% is not allowed.

How many shares in:

17. $ 600 6% stock?

18. $ 800 3% stock?

19. $ 1200 2% stock?

20. $ 1500 8% stock?

Ex. **17.** 6 shares. The par value of one share is $ 100; $ 600 is the par value of as many shares as $ 100 is contained times in $ 600, or 6 shares.

Brokerage $\frac{1}{8}$%, find the cost of:

21. \$400 stock at 75.

22. \$320 stock at 60.

23. \$800 stock at 80.

24. \$640 stock at 90.

Ex. **21**. \$300.50. The entire cost of 1 share is \$75$\frac{1}{8}$; the cost of 4 shares is 4 times \$75$\frac{1}{8}$, or \$300.50.

Brokerage $\frac{1}{8}$%, find the net proceeds of:

25. \$400 stock sold at 75$\frac{1}{8}$.

26. \$600 stock sold at 90$\frac{1}{8}$.

27. \$800 stock sold at 80.

28. \$240 stock sold at 60.

Ex. **25**. \$300. The net proceeds of 1 share is \$75$\frac{1}{8}$ − \$$\frac{1}{8}$, or \$75; of 4 shares, 4 times \$75, or \$300.

Find the dividend on:

29. \$400 3%'s bought at 80.

30. \$300 6%'s bought at 60.

31. \$1200 4%'s sold at 75.

32. \$1500 5%'s sold at 90.

Ex. **29**. \$12. The dividend on 1 share is \$3; on 4 shares, 4 times \$3, or \$12.

How many shares may be bought for:

33. \$900 at 89$\frac{3}{4}$, brok. $\frac{1}{4}$%?

34. \$600 at 74$\frac{7}{8}$, brok. $\frac{1}{8}$%?

35. \$1200 at 59$\frac{7}{8}$, brok. $\frac{1}{8}$%?

36. \$1600 at 79$\frac{3}{4}$, brok. $\frac{1}{4}$%?

Ex. **33**. 10 shares. The cost of 1 share is \$90; \$900 will buy as many shares as \$90 is contained times in \$900, or 10 shares.

How much stock gives an income of:

37. \$200; stock 5%?

38. \$400; stock 4%?

39. \$600; stock 3%?

40. \$800; stock 8%?

Ex. **37**. \$4000. The income on 1 share is \$5; it will take as many shares to yield \$200 as \$5 is contained times in \$200, or 40 shares. 40 shares = \$4000 stock.

What % will I realize on my investment when:

41. 5%'s are bought at 40?

42. 8%'s are bought at 60?

43. 3%'s are bought at 60?

44. 4%'s are bought at 80?

Ex. **41**. 12$\frac{1}{2}$%. One share costs \$40 and gains \$5; the gain % is \$5 ÷ \$40, or 12$\frac{1}{2}$%.

45. Find the cost of $5000 3% stock at 80, brokerage $\frac{1}{8}$%.

Given terms: no. shares, 50; first cost of 1 share, $80; brokerage on 1 share, $$\frac{1}{8}$.

$$\begin{array}{rl} \$80 & = \text{cost 1 sh.} \\ \frac{1}{8} & = \text{brok. 1 sh.} \\ \hline 80\frac{1}{8} & = \text{entire c. 1 sh.} \\ 50 & \\ \hline \$4006.25 & = \text{entire c. all.} \end{array}$$

Since the first cost of 1 share is $80, and the brokerage $$\frac{1}{8}$, the entire cost is the sum, or $$80\frac{1}{8}$. The cost of 50 shares is 50 times $$80\frac{1}{8}$, or $4006.25.

46. Find the proceeds of $5000 3% stock at 80, brokerage $\frac{1}{8}$%.

Given terms: first sell. p. 1 share, $80; brokerage 1 share, $$\frac{1}{8}$.

$$\begin{array}{rl} \$80 & = \text{sell. p. 1 sh.} \\ \frac{1}{8} & = \text{brok. 1 sh.} \\ \hline 79\frac{7}{8} & = \text{proceeds.} \\ 50 & \\ \hline \$3993.75 & = \text{entire proc.} \end{array}$$

Since the selling price of 1 share is $80, and the brokerage $$\frac{1}{8}$, the proceeds will be the difference, or $$79\frac{7}{8}$. The proceeds of 50 shares will be 50 times $$79\frac{7}{8}$, or $3993.75.

47. How much 3% stock (50) can be bought at 98 for $2456.25, brokerage $\frac{1}{4}$%?

Given terms: first cost 1 share $49; brokerage on 1 share, $$\frac{1}{8}$.

$$\begin{array}{rl} \$49. & = \text{cost 1 share.} \\ .125 & = \text{brokerage.} \\ \hline \$49.125 & = \text{entire cost 1 share.} \end{array}$$

$$49.125)2456.25(50$$
$$\underline{2456\ 25}$$

Since the first cost of 1 share is $49, and the brokerage $$\frac{1}{8}$, the entire cost is the sum, or $$49\frac{1}{8}$. As many shares can be bought, etc.

48. What is the dividend on $2500 3% stock (50)?

Given terms: dividend on 1 share, $1.50; no. of shares, 50.

$$\begin{array}{r} \$1.50 \\ 50 \\ \hline \$75 \end{array}$$

Since the dividend on 1 share is $1.50, on 50 shares it is 50 times $1.50, or $75.

49. How much 5% stock at 80, must be bought to get an annual income of $240?

Given terms: dividend on 1 share, $5; entire income, $240; cost of 1 share, $80.

$$5)\underline{240} \quad 48$$

Since the gain on 1 share is $5, it will take as many shares to gain $240, as $5 is contained times in $240, or 48 shares, or $4800 stock.

50. Find the rate per cent of dividend when $6200 stock yields $310.

Given terms: no. shares, 62; entire income, $310.

$$62)310(5 \quad \underline{310}$$

Since 62 shares yield an income of $310, 1 share will yield $\frac{1}{62}$ of $310, or $5.

Since the dividend on 1 share is $5, the stock is 5% stock.

51. If 3% stock is at 80, what rate % will a person receive on his investment?

Given terms: investment in 1 share, $80; dividend on 1 share, $3. *Relation:* income = R × investment.

$$3 = R \times 80$$
$$R = \frac{3}{80} = 3\tfrac{3}{4}\%$$

Since the income is some % of the investment, the rate is $3 ÷ $80, or $3\frac{3}{4}$%.

52. What must be the price of a 3% stock to equal a 4% stock at 80?

Given terms: income on 1 share first stock, $3; income on 1 share second stock, $4; investment in 1 share second stock, $80. *Relations:* income on 2d = R × invest. in 2d; rate on first investment = rate on second investment.

$$4 = R \times 80$$
$$R = \frac{4}{80} = 5\%$$
$$5\%\ \textit{Invst.} = 3$$

$$\textit{Invst.} = \frac{3}{.05} = 60$$

Cost 1 share 3% stock, $60

53. What must be the price of stock when $8300 stock is bought for $4150?

Given terms: no. shares, 83; cost of all, $4150.

54. What is the cost of 4000 $3\frac{1}{2}\%$ government bonds ($1000) at $91\frac{5}{8}$, brokerage $\frac{1}{8}\%$?

55. I buy 200 shares railroad stock at $102\frac{1}{8}$, and sell them at $104\frac{1}{2}$, brokerage $\frac{1}{4}\%$ in each transaction. How much do I gain?

56. My broker buys for me 360 New York state bonds at $72\frac{1}{2}$, brokerage $\frac{1}{8}\%$. How much do the bonds cost me?

57. What are the proceeds from the sale of 45 shares of Ohio state bonds at 93, brokerage $\frac{1}{8}\%$.

58. Find the proceeds of $1000 New York city $4\frac{1}{2}$'s at $101\frac{1}{4}$, brokerage $\frac{1}{4}\%$.

NOTE. — N. Y. city $4\frac{1}{2}$'s means New York city bonds yielding $4\frac{1}{2}\%$ dividend.

59. What are the proceeds from the sale of 80 shares of Ohio state bonds at 90, brokerage $\frac{1}{8}\%$?

60. If Altar mining stock sells for $83\frac{1}{4}$, what are the proceeds from 150 shares, brokerage $\frac{1}{8}\%$?

61. How many shares of stock at $62\frac{1}{8}$, brokerage $\frac{1}{8}\%$, can be bought for $9337.50?

62. A broker invested $24,062.50 in Union Pacific bonds at 4% discount, brokerage $\frac{1}{4}\%$. How many shares of stock will his principal receive?

63. A pays a debt of $2045 with bank stock at $102\frac{1}{4}$. How much stock will the creditor receive?

64. The Altar Mining Company declares a dividend of $6\frac{1}{3}\%$. How much will a stockholder receive who holds 50 shares?

65. What income will be realized from investing $58,080 in $4\frac{1}{2}\%$ stock at $90\frac{1}{2}$, allowing $\frac{1}{4}\%$ for brokerage?

66. A speculator invested $2400 in mining stock which pays him a dividend of $4\frac{1}{2}\%$. If he agrees to take his dividend in coal at 6¢ a bu., how many bushels should he receive?

67. Stock bought at $93\frac{1}{8}$ and yielding 5%, bears $225 annual income. How many shares are there?

68. A man receives $882 as a 7% dividend on his stock. How many shares does he hold?

69. How much must be invested in U. S. 4's at $121\frac{1}{4}$, brokerage $\frac{1}{8}\%$, that I may pay a debt of $900 from one year's income?

70. If I have to pay an assessment of $610 on 305 shares of stock (50), what is the rate of assessment?

71. I paid $16,605 for stock at 92, brokerage $\frac{1}{4}\%$. If it yields $3600 the first year, what is the rate of dividend?

72. The earnings of a mining company, whose capital is $120,000, amount in one year to $24,000; their expenses are $9000. What rate of dividend can they declare?

73. What per cent on my investment will I realize by buying 6% stock at 115?

74. If I invest $18,000 in U. S. 4's at 102, my annual income is what per cent of my investment?

75. Philadelphia 6's are bought at $81\frac{1}{2}$. What is the rate of investment?

76. How much must a broker pay for Virginia 6's to realize 8% on his investment?

77. What must be the price of $3\frac{1}{2}\%$ stock to equal $4\frac{1}{2}\%$ stock at 90?

78. What must be the market value of 60 shares 4% stock to equal 6% stock at 75?

79. A man's rate of dividend is 4%. What must be the price of stock if he realizes 3% on his investment?

80. What income is derived from $500 6% stock (25)?

81. A sells 60 shares of stock at 80, and invests the proceeds in stock at 75. If the latter yields 4% annually, what is his income?

82. I own 500 shares mining stock bought at 102. If the stock yields 3% annually, what is my income in dollars? What per cent of my investment is my income?

83. What is the better investment for $1500, 6% stock at 75, or 4% stock at 60? What is the difference, in dollars, in the annual income?

84. A stockholder owns 3% railroad stock worth 90, and $2\frac{1}{2}$% mining stock worth 70. If his income from each is $600 a year, in which stock has he the larger investment?

85. How much stock, brokerage $\frac{1}{8}$%, must be sold at $137\frac{1}{8}$ to buy with the proceeds $3600 of stock at $68\frac{1}{4}$, brokerage $\frac{1}{4}$%?

86. A man receives an annual income of $270 from $4\frac{1}{2}$% stock, and $160 from 2% stock. If he sells the former at 85, and half the latter at 95, brokerage $\frac{1}{4}$% in each transaction, how much does he receive?

87. If 4% stock is at 60, what rate per cent will a person receive on his investment?

88. A man invested a certain sum of money in 5% stock at 80, and twice as much in 4% stock. If his income from the former is $300, and from the latter $1\frac{1}{3}$ times as much, what was the price of a share in the latter investment?

89. B's income on 80 shares of stock is $320. If this yields him 5% on his investment, what is his entire stock worth?

90. A man owns 70 shares $2\frac{1}{2}$% stock and 45 shares 4% stock. Which is worth more in market, and by how much, if the rate realized on each investment is 5%?

91. What amount must be invested in 3% stock at 90 to yield an annual income of $225?

92. By selling 60 shares of 4% stock at 90, and investing the proceeds in other stock at 75, my income is increased by $84. What is the entire income from the latter investment?

93. A man owns $8000 5% insurance stock. He exchanges this for mining stock at 80, which increases his income $150. What rate per cent dividend does the mining stock yield?

94. My income on 70 shares of stock is $245. What rate of dividend does the stock pay?

95. I sell $9000 5% stock at 80, and invest the proceeds in 3% stock at 60. I increase or decrease my income, and by how much?

DIFFICULT PROBLEMS

In some problems, it is best to represent a required term by x.

96. Suppose 10% state stock is 20% better in market than 4% railroad stock. If A's income is \$500 from each, how much money has he paid for each, the whole investment bringing $6\frac{2}{333}\%$?

Given terms: income on 1 share state stock, \$10; income on 1 share railroad stock, \$4; whole income on state stock, \$500; whole income on R. R. stock, \$500.

Relations: cost 1 share state stock = 120% cost 1 share R. R. stock. $6\frac{2}{333}\%$ of investment = \$1000.

Let $x =$ *cost 1 share R. R. stock,* \$90
$125\,x =$ *entire cost R. R. stock,* \$11250
$1.2\,x =$ *cost 1 share state stock,* \$108
$60\,x =$ *entire cost state stock,* \$5400
$185\,x =$ *entire investment*
$6\frac{2}{333}\%$ of $185\,x = 1000$
$x = 90$

97. I received a 10% stock dividend, and then had 102 shares (\$50 each) and \$15 of another share. How many shares had I before the dividend?

98. Thomas Reed bought 6% mining stock at $114\frac{3}{4}$, and 4% furnace stock at $112\frac{1}{4}$, brokerage $\frac{1}{4}\%$. The latter cost him \$430 more than the former, but yielded the same income. How much did each cost him?

99. I bought stock at 10% discount, which rose to 5% premium, and sold for cash. Paying a debt of \$33, I invested the balance in stock at 2% premium, which, at par, left me \$11 less than at first. How much money had I at first?

100. W. T. Baird invested a certain sum of money in Philadelphia 6's at $115\frac{7}{8}$, and three times as much in Union Pacific 7's at $89\frac{7}{8}$, brokerage $\frac{1}{8}\%$ in each case. How much was invested in each kind of stock, if the annual income was \$9920 in all?

INTEREST

TERMS AND RELATIONS

Money paid for the use of money is *interest;* the money loaned is the *principal;* the sum of the principal and the interest is the *amount.*

ILLUSTRATION

If $6 is paid for the use of $100, $100 is the *principal;* $6, the *interest;* $106, the *amount.*

1. If $100 is loaned for 3 yr. at 6%, and no payment is made until the end of the third year, how much interest is then due?

There are three conceptions:

That the principal alone bears interest, *simple interest.*

That the principal, and the interest on the principal at the end of each year, bear interest, *annual interest.*

That the principal, the interest on the principal at the end of each year, and all other interest at the end of each year, bear interest, *compound interest.*

Unless otherwise stated, simple interest is always understood.

SIMPLE INTEREST

1 yr.	*2 yr.*	*3 yr.*	*Total.*
$6	*$6*	*$6*	*$18.00*

The interest on $100 is $6 each year.

ANNUAL INTEREST

1 yr.	*2 yr.*	*3 yr.*	*Total.*
$6	*$6*	*$6*	
	.36	*.36*	
		.36	
$6	*$6.36*	*$6.72*	*$19.08*

The interest on $100 is $6 each year. The interest on the first $6 is 36¢ the 2d year, and 36¢ the 3d year. The interest on the second $6 is 36¢ the 3d year.

COMPOUND INTEREST

1 yr.	*2 yr.*	*3 yr.*	*Total.*
$6	*$6*	*$6*	
	.36	*.36*	
		.36	
		.0216	
$6	*$6.36*	*$6.7416*	*$19.1016*

In addition to the annual interest, the first 36¢ gains 2.16¢.

SIMPLE INTEREST

FIRST CONCEPTION: *The principal alone bears interest.*

2. What is the interest of \$1 for 1 yr. at 6%?

3. What is the interest of \$1 for 1 mo. at 6%?

4. What is the interest of \$1 for 1 da. at 6%?

The interest of \$1 for 1 yr. at 6% is .06 of \$1, or 6¢.

Since the interest of \$1 for 12 mo. is 6¢, for 1 mo., it is $\frac{1}{12}$ of 6¢, or 5 m.

Since the interest of \$1 for 30 da. is 5 m., for 1 da., it is $\frac{1}{30}$ of 5 m., or $\frac{1}{6}$ of a mill.

TO BE MEMORIZED: *The interest of \$1 for 1 year at 6% is 6¢; for 1 month, $\frac{1}{2}$ of a cent; for 1 day, $\frac{1}{6}$ of a mill.*

5. What is the interest of \$1 for 5 yr. 5 mo. 16 da. at 6%?

\$.30
.025
.002⅔
.327⅔

The interest of \$1 for 5 yr. at 6% is .30; for 5 mo., \$.025; for 16 days, \$.002⅔; for the whole time, \$.327⅔.

6. What is the amount of \$360 for 5 yr. 5 mo. 16 da. at 6%?

\$360
.327⅔
117.96, interest
360.
477.96, amount

Assume \$1. The interest of \$1 for 5 yr 5 mo. 16 da. at 6% is \$.327⅔; for \$360, 360 times \$.327⅔, or \$117.96. The amount is \$477.96.

Find the interest of \$1 at 6%:

7. For 4 yr. 2 mo. 16 da.

8. For 5 yr. 7 mo. 9 da.

9. For 3 yr. 1 mo. 17 da.

10. For 7 yr. 7 mo. 7 da.

11. For 2 yr. 6 mo. 12 da.

Find the amt. of \$125 at 6%:

12. For 2 yr. 2 mo. 2 da.

13. For 6 yr. 6 mo. 6 da.

14. For 5 yr. 0 mo. 19 da.

15. For 5 yr. 7 mo. 27 da.

16. For 4 yr. 1 mo. 20 da.

If the rate is not 6%, the interest may be found at 6 per cent, and the result modified for the required rate.

17. What is the amount of \$360 for 3 yr. 3 mo. 5 da. at 5%?

\$360
$.195\frac{5}{6}$
70.50 @ 6%.
11.75 @ 1%.
58.75 @ 5%.
360.
\$418.75, amount.

Assume \$1. The interest of \$1 for 3 yr. @ 6% is \$.18; for 3 mo., \$.015; for 5 da., \$.000⅚; for the whole time, \$.195⅚.

The interest of \$360 is 360 times \$.195⅚, or \$70.50; at 5%, ⅙ less, or \$58.75.

18. How would you modify the interest @ 6% to find the interest at 8%? at 10%? at 7%? at 9%?

Ans. At 8%, I would add a third of the interest, because 8 is 6 plus ⅓ of 6; at 10%, I would divide the interest by 6 and multiply by 10.

When the time does not exceed 4 months, a modification of the 6% method is in general use.

Moving the decimal point 2 places to the left in the principal, gives the interest for 60 days at 6%.

PROOF: the interest of \$1 for 60 da. @ 6% is 1¢, or $\frac{1}{100}$ of \$1.

19. What is the interest of \$167.80 for 3 mo. 3 da. @ 10%?

\$1.678	*60*
.839	*30*
.0839	*3*
2.6009 @	*6%*
4.334 @	*10%*

The interest for 60 da. @ 6% is \$1.678; for 30 da., ½ of that, or \$.839; for 3 da., $\frac{1}{10}$ of that, or \$.0839.

Find the interest of \$438.96 for:

20. 3 yr. 10 mo. 17 da. @ 6%.
21. 4 yr. 0 mo. 25 da. @ 9%.
22. 2 yr. 6 mo. 17 da. @ 10%.
23. 6 yr. 1 mo. 18 da. @ 7%.
24. 4 yr. 3 mo. 12 da. @ 4½%.

What is the interest of:

25. \$295.40 for 36 da. @ 5%?
26. \$360.20 for 93 da. @ 7%?
27. \$235.90 for 93 da. @ 9%?
28. \$840.50 for 33 da. @ 8%?
29. \$501.02 for 63 da. @ 6%?

Accurate interest, that is, interest found by counting 365, instead of 360, days to the year is sometimes computed.

30. Find the accurate interest of $625 for 80 days at 6%.

$\frac{80}{365} \times .06 \times 625 = 8.219.$

Accurate Int. $= \$8.219.$

Since the interest for 365 days is .06 of the principal, the interest for 80 days is $\frac{80}{365} \times .06 \times \$625$, or $8.219.

31. Find the interest of $625 for 80 days, counting 360 days to the year, subtract $\frac{1}{73}$ of the result, and observe that accurate interest has been computed.

32. What is the accurate interest of $100 from Jan. 1, 1897, to Jan. 1, 1898, at 6%? the interest as commonly found?

Ans. $6 in each case. The two methods agree for a whole number of years; for a fraction of a year accurate interest is less.

33. Show that accurate interest for any number of days less than a year, may be found by deducting $\frac{1}{73}$ of the interest found by counting 360 days to a year.

Let $R =$ *the int. for 1 yr.*

$\frac{R}{365} =$ *int. 1 da., 365 da. to a yr.*

$\frac{R}{360} =$ *int. 1 da., 360 da. to a yr.*

$\frac{R}{360} \times \frac{360}{365}$, or $\frac{R}{360} \times \frac{72}{73} = \frac{R}{365}.$

That is, the interest found by counting 360 da. to the year multiplied by $\frac{72}{73}$, or diminished by $\frac{1}{73}$, becomes the accurate interest.

Find the accurate interest of:

34. $200 from Dec. 13, 1896, to May 1, 1897, @ 6%.

35. $440 from Jan. 20, 1896, to Apr. 5, 1897, @ 8%.

36. $450 from June 16, 1895, to Nov. 8, 1896, @ 10%.

37. $300 from Dec. 1, 1896, to May 10, 1898, @ 4%.

INDIRECT CASES

As a basis, always assume 1 of the denomination required.

38. What principal will gain $ 76.80 in 3 yr. 2 mo. 12 da. @ 8%?

```
                        .18
                        .01
.256)76.800(300         .002
     768                ----
                        .192
                        .064
                        ----
                        .256
```

Assume $ 1. $ 1 in 3 yr. 2 mo. 12 da. @ 8% will gain $ .256; it will take as many dollars to gain $ 76.80, as $ .256 is contained times in $ 76.80, or $ 300.

39. At what % will $ 300 gain $ 76.80 in 3 yr. 2 mo. 12 da.?

```
300                     .18
.192                    .01
-----                   .002
 57.60  at 6%           ----
  9.60  at 1%           .192
  9.60)76.80(8
       76.80
```

Assume 1%. $ 300 in 3 yr. 2 mo. 12 da. at 1% will gain $ 9.60; it will take as many % to gain $ 76.80, as $ 9.60 is contained times in $ 76.80, or 8%.

40. In what time will $ 300 gain $ 76.80 at 8%?

```
300                 3.2 yr.
.08                  12
---                 -----
24                  2.4 mo.
24)76.80(3.2         30
   72               -----
   --               12 da.
    48
    48
```

Assume 1 yr. $ 300 in 1 yr. @ 8% will gain $ 24; it will take as many years to gain $ 76.80, as $ 24 is contained times in $ 76.80, or 3 yr. 2 mo. 12 da.

41. What principal will amount to $ 376.80 in 3 yr. 2 mo. 12 da. at 8%?

```
                        .18
                        .01
                        .002
1.256)376.800(300       ----
      3768              .192
                        .064
                        ----
                        .256
```

Assume $ 1. $ 1 in 3 yr. 2 mo. 12 da. at 8% will amount to $ 1.256; it will take as many dollars to amount to $ 376.80, as $ 1.256 is contained times in $ 376.80, or $ 300.

Each example should be changed to a form already given.

42. At what % will $ 300 amount to $ 376.80 in 3 yr. 2 mo. 12 da.?

This means, "At what % will $ 300 gain $ 76.80 in 3 yr. 2 mo. 12 da.?" *See Ex. 39.*

43. In what time will $ 300 amount to $ 376.80 at 8%?

This means, "In what time will $ 300 gain $ 76.80 at 8%?" *See Ex. 40.*

44. At what % will a sum triple in 30 years?

This means, "At what % will $ 10 gain $ 20 in 30 yr.?"

45. In what time will a sum quadruple at 6%?

This means, "In what time will $ 10 gain $ 30 @ 6%?"

46. Is it proper to reason thus: "Since $ 1 amounts to $ 1.06, $ 5 will amount to 5 times $ 1.06, or $ 5.30?

Yes. Because $ 5 will amount to 5 times as much as $ 1.

47. Is it proper to reason thus: "If a principal amounts to $ 1.06 in 1 yr., in 2 yr. it will amount to 2 times $ 1.06, or $ 2.12?"

No. The amount is always *once* the principal plus the interest.

48. At what % will a sum double in 10 years?

49. In what time will $ 260 gain $ 29.90 at 5%?

50. In what time will a sum triple at 10 per cent?

51. In what time will $ 260 amount to $ 289.90 at 5%?

52. At what % will $ 260 gain $ 29.90 in 2 yr. 3 mo. 18 da.?

53. At what % will $ 260 amount to $ 289.90 in 2 yr. 3 mo. 18 da.?

54. What principal will gain $ 29.90 in 2 yr. 3 mo. 18 da. @ 5%?

55. What principal will amount to $ 289.90 in 2 yr. 3 mo. 18 da. @ 5%?

MISCELLANEOUS

Find:

56. Interest of $ 1025 for 3 mo. 6 da. @ 9%.
57. Amount of $ 2400 for 6 yr. 5 mo. 20 da. @ 6%.
58. Amount of $ 930 for 10 yr. 3 mo. 24 da. @ 4%.
59. Amount of $ 150 from Apr. 4 to Dec. 18 @ 8%.

What principal:

60. Will produce $ 68.20 interest in 2 yr. 7 mo. @ 4%?
61. Will produce $ 1610 interest in 4 yr. 5 mo. 20 da. @ 6%?
62. Will amount to $ 742 in 6 yr. 18 da. @ 8%?
63. Will amount to $ 1065.75 in 10 mo. 15 da. @ 10%?

At what rate per cent:

64. Will $ 824 amount to $ 957.90 in 3 yr. 3 mo.?
65. Will $ 235 produce $ 84.60 interest in 6 yr.?
66. Will $ 900 amount to $ 1200 in 10 years?
67. Will $ 840 produce $ 70.84 interest in 4 yr. 2 mo. 18 da.?

Find the time in which:

68. $ 286 will gain $ 70.07 @ 7%.
69. $ 800 will amount to $ 1040 @ 6%.
70. $ 340 will produce $ 13.60 interest @ 5%.
71. $ 760 will gain $ 285 interest @ 10%.

What will be the amount of:

72. $ 100 in 3 yr., if the amount in 1 yr. is $ 125?
73. $ 80 in 5 yr., if the amount in 2 yr. is $ 100?
74. $ 160 in 6 yr., if the amount in 2 yr. is $ 192?
75. $ 450 in 10 yr., if the amount in 3 yr. is $ 595?

PARTIAL PAYMENTS — LONG NOTES

A written promise to pay a sum of money on demand or at a stated time, is a *note;* the sum promised, the *principal*, or *face* of the note; the time when a note is due, its *maturity;* the person to whom the money is to be paid, the *payee;* the person who signs the note, the *maker* or *drawer.*

Whenever a payment, called an *indorsement*, is made, the date and amount of the payment are written upon the back of the note.

June 4, 1893, Harold Blake bought a farm of Peter Ford for $1000, paying $475 cash and giving Note No. 1 for the balance. Mr. Blake agrees to pay the principal, with interest at 6% from date, on June 4, 1896. He may delay payment as long as Mr. Ford will consent.

The *drawer* is Harold Blake; the *payee*, Peter Ford. Mr. Blake paid $114.20, Sep. 9, 1894; $8.29, May 15, 1895; $244.38, Aug. 6, 1896.

$525.00. *Emporia, Kans., June 4, 1893.*

Three years after date I promise to pay to the order of ~~~~ Peter Ford ~~~~ Five hundred twenty-five ~~~~ $\frac{00}{100}$ Dollars, with interest at 6%, at The First National Bank of Emporia ~~

Value received.

Harold Blake.

No. 1. Due June 4, 1896.

Sep. 9, 1894, $114.20

May 15, 1895, $8.29

Aug. 6, 1896, $244.38

The United States rule for partial payments is derived from the decision of the Supreme Court, for finding the amount due on a note when payments have been made.

United States Rule

Find the amount of the principal to the time of the first payment; if the payment equals or exceeds the interest, subtract the payment from the amount and treat the remainder as a new principal.

If the payment is less than the interest, find the amount of the same principal to the time when the sum of the payments shall equal or exceed the interest due, and subtract the sum of the payments from the amount.

Proceed in the same manner with the remaining payments until the time of settlement.

76. What was due on note No. 1, Feb. 9, 1898?

1893 6 4		
1894 9 9	*1 3 5*	*$ 114.20*
1895 5 15	*8 6*	*8.29*
1896 8 6	*1 2 21*	*244.38*
1898 2 9	*1 6 3*	
4 8 5	*4 8 5*	

$ 525.00,	*P.*
39.81,	*I. 9/9, '94*
564.81	
114.20	
450.61,	*P.*
*18.48,**	*I. 5/15, '95*
469.09	
33.12,	*I. 8/6, '96*
502.21	
252.67,	*Sum pay.*
249.54,	*P.*
22.58	
$ 272.12,	*Due 2/9, '98*

Note. — By arranging the dates as above, errors in subtracting will be discovered. Each date is subtracted from the next below. The difference between the first and last dates should equal the sum of the differences.

* Since the interest exceeds the payment, we find the interest on $ 450.61 and not on $ 469.09, for 1 yr. 2 mo. 21 da.

77. On note No. 2, how much remained due Sept. 25, 1897?

78. On note No. 3, how much remained due Oct. 9, 1897, counting interest at 6%?

$750.00. BOSTON, MASS., June 17, 1892.

On demand, for value received, I promise to pay John E. Wiley, or order, seven hundred fifty $\frac{00}{100}$ dollars, with interest at 6%.

No. 2. J. J. HILL.

Payments: Mar. 1, 1893, $75.50; June 11, 1893, $165; Sept. 15, 1893, $161; Jan. 21, 1894, $47.25; Mar. 12, 1895, $12.50; Dec. 6, 1895, $98; July 7, 1896, $169.

$300.00. WILMINGTON, DEL., Apr. 30, 1895.

On demand, for value received, I promise to pay G. R. Bell, or order, three hundred $\frac{00}{100}$ dollars, with interest.

No. 3. RUSSELL HIBBS.

Payments: June 27, 1896, $150; Dec. 9, 1896, $150.

$1500.00. CINCINNATI, O., Jan. 10, 1895.

On demand, for value received, I promise to pay Ernest Buckman, or order, one thousand five hundred dollars, with interest at 6%.

L. H. TAYLOR.

No. 4.

Indorsements: Mar. 3, 1895, $250; May 20, 1895, $300; July 1, 1895, $125; Nov. 15, 1895, $75.

$525.00. EMPORIA, KANS., Dec. 15, 1895.

On demand, for value received, I promise to pay James Tyner, or order, five hundred twenty-five dollars, with interest at 8%.

No. 5. W. N. SIMPSON.

Indorsements: Jan. 15, 1896, $75; Apr. 20, 1896, $60; June 15, 1896, $140.50; Aug. 1, 1896, $80; Sept. 25, 1896, $120.

NOTE. — For treatment of No. 4 and No. 5, see next page.

PARTIAL PAYMENTS — SHORT NOTES

When notes, upon which payments have been made, are settled within a year of date, merchants commonly disregard the U. S. rule.

Mercantile Rule

Find the amount of each item to date of settlement.

$ 1728.00. Little Rock, Ark., Jan. 2, 1897.

On demand, for value received, I promise to pay William Chamberlain, or order, one thousand seven hundred and twenty-eight $\frac{00}{100}$ dollars, with interest at 6%.

No. 6. Henry King.

Indorsements: Mar. 1, 1897, $300; May 16, 1897, $150; Sept. 1, 1897, $270; Dec. 11, 1897, $135.

79. How much was due Dec. 13, 1897, on Note No. 6?

Principal,		$1728.00
Interest to Dec. 13, 1897, 345 da.,		99.36
		$1827.36
First payment,	$300.00	
Int. to Dec. 13, 1897, 287 da.,	14.35	
Second payment,	150.00	
Int. to Dec. 13, 1897, 211 da.,	5.28	
Third payment,	270.00	
Int. to Dec. 13, 1897, 103 da.,	4.64	
Fourth payment,	135.00	
Int. to Dec. 13, 1897, 2 da.,	.05	
		879.32
Bal. due Dec. 13, 1897,		$948.04

Owing $1728 Jan. 2, is equivalent to owing Dec. 13, $1728 + the interest of $1728 from Jan. 2 to Dec. 13, or $1827.36.

Paying $300 Mar. 1, is equivalent to paying Dec. 13, $300 + the interest of $300 from Mar. 1 to Dec. 13, or $314.35.

80. On Note No. 4, how much remained due Dec. 12, 1895?

81. On Note No. 5, how much remained due Nov. 1, 1896?

TRUE DISCOUNT

The true present worth of a note, or of a sum of money due in the future, is that sum which, put at interest now, will amount to the given debt at the expiration of the time.

82. $275 is due me in 1 yr. 6 mo. What is the true present worth, interest at 6%? What is the true discount?

1.09)275.00(252.29

$275, face
252.29, true pres. worth
$22.71, discount

The present worth is that sum which, put at interest to-day, will amount to $275 in 1 yr. 6 mo. at 6%.

$1 in 1 yr. 6 mo. at 6% will amount to $1.09; it will take as many dollars to amount to $275, as $1.09 is contained times in $275, or $252.29.

The true discount is $275 – $252.29, or $22.71.

NOTE.—True discount is rarely found on notes or debts running less than four months.

83. Find the true present worth and true discount of Note No. 7, discounted at 10%, Mar. 1, 1896.

$300.00. EMPORIA, KANS., Jan. 2, 1896.

Ninety days after date, value received, I promise to pay to the order of William Clarke, at the First National Bank, Three hundred and $\frac{00}{100}$ Dollars, with interest at 10% after maturity.

No. 7. W. C. STEVENSON.

Find the:

84. True present worth of $412 due in 6 mo., int. @ 6%.
85. True present worth of $324 due in 8 mo., int. @ 12%.
86. True present worth of $321 due in 1 yr. 9 mo., int. @ 7%.

Find the:

87. True discount of $590 due in 2 yr. 8 mo., interest at 9%.
88. True discount of $336 due in 3 yr. 10 mo., interest at 12%.
89. True discount of $427 due in 5 yr. 11 mo., interest at 8%.

BANK DISCOUNT

In discounting notes, bankers use the following rule:

To find the bank discount, compute the interest on the amount due at maturity for (three days more than) the specified time.*

To find the proceeds, subtract the bank discount from the amount due at maturity.

90. Find the bank discount at 10% Jan. 2, 1896, on Note 7; find the bank proceeds; find the date of maturity.

$3.00 60
1.50 30
.15 3
$4.65, D., 6%
7.75, D., 10%
$292.25, Proceeds
Apr. 4, date of maturity

Since the note does not begin to bear interest until 93 days after Jan. 2, the amt. due at maturity is the face of the note, or $300.

The interest of $300 for 93 days at 10% is $7.75.

91. Write Note No. 7, substituting "with interest at 6%" in place of "with interest at 10% after maturity." Find the bank discount and proceeds at 10%, Feb. 2, 1896.

$3.00 60
1.50 30
.15 3
$4.65, Int.
$304.65, amt. at maturity

Since the note now begins to bear interest Jan. 2, the amount at maturity, or the *face value* of the note, is the amount of $300 for 93 days @ 6%, or $304.65.

$3.0465 60
.1015 2
$3.148, D., 6%
5.25, D., 10%
$299.40, Proceeds

Since the note was not discounted until Feb. 2, or 31 days after date, the specified time is 93 da.—31 da., or 62 da.

* This phrase should be stricken out for the following states and territories in which the three days, called *days of grace*, have been abolished by statute: Alas., Cal., Colo., Conn., D. C., Fla., Ida., Ill., Me., Md., Mass., Mont., N. H., N. J., N. Y., N. D., O., Ore., Pa., Utah, Vt., and Wis.

92. Write a bank note for 90 days, with interest after maturity, that will give bank proceeds at 10 %, of $292.25.

Assume $1. Since the bank proceeds of $1 at 10 % for 93 da. are $.974⅙, it will take as many dollars to yield $292.25, as $.974⅙ is contained times in $292.25, or $300.

.01	*60*
.005	*30*
.0005	*3*
.0155	*6 %*
$.025\frac{5}{6}$	*10 %*

$.974\frac{1}{6})292.250(300$

93. Find the date of maturity, and bank discount at 6 %, Jan. 10, 1896, on Note No. 8.

$400.00 MIDDLETOWN, CONN., Jan. 10, 1896.

Sixty days after date, value received, I promise to pay to the order of the Middlesex County National Bank, four hundred and $\frac{00}{100}$ dollars, with interest at 6 % after maturity.

No. 8. JOHN S. CAMP.

94. Find the bank discount at 6 %, Mar. 3, 1896, on Note No. 7.

95. Write Note No. 8, substituting "with interest at 5 %" in place of "with interest at 6 % after maturity," and find the bank discount at 6 %, Feb. 19.

96. Find the true discount at 6 %, of Note No. 8, Jan. 10, 1896. How much does the banker make by collecting bank, instead of true discount?

97. Show that the difference between the bank discount and the true discount of Note No. 8, Jan. 10, 1896, is the interest on the true discount for 60 days.

NOTE. — Do not count 3 da. of grace in either case.

98. What is the face of a note, which, when discounted for 2 mo. 24 da. at 5 %, will yield $224.60 bank proceeds?

99. Write a bank note for 3 mo., maker, John Brown; payee, Henry Short; proceeds at 6 %, $732.92.

SETTLEMENT OF ACCOUNTS—INTEREST METHOD

In settling accounts between wholesale and retail dealers, or where the amounts are large or of long standing, it is customary to take account of interest.

100. Thomas Stone bought merchandise of us as shown on the Dr. side of the following account, and made payments as on the Cr. side. Settlement was made Sep. 15, 1896; how much did Mr. Stone owe at that time, counting interest at 6% ?

Dr. *Thomas Stone* *Cr.*

1896				*1896*			
Mar. 7	*To Mdse*	*500*		*Mar. 9*	*By Cash*	*300*	
Apr. 3	*To Mdse*	*300*		*Apr. 1*	*By Cash*	*200*	
Aug. 2	*To Mdse*	*700*		*July 1*	*By Cash*	*500*	

First debt	*$ 500.00*	First payment	*$ 300.00*
Int. to Sep. 15	*16.00*	Int. to Sep. 15	*9.50*
Second debt	*300.*	Second payment	*200.00*
Int. to Sep. 15	*8.25*	Int. to Sep. 15	*5.57*
Third debt	*700.*	Third payment	*500.00*
Int. to Sep. 15	*5.13*	Int. to Sep. 15	*6.33*
	$ 1529.38		*$ 1021.40*

Bal. due Sep. 15, 1896, $ 507.98.

Owing $500 Mar. 7, is equivalent to owing Sep. 15, $500 + the interest of $500 from Mar. 7 to Sep. 15, or $516.00.

Owing $300 Apr. 3, is equivalent to owing Sep. 15, $300 + the interest of $300 from Apr. 3 to Sep. 15, or $308.25; etc.

The pupil should finish the explanation.

Paying $300 Mar. 9, is equivalent to paying Sep. 15, $300 + the interest of $300 from Mar. 9 to Sep. 15, or $309.50.

Paying $200 Apr. 1, is equivalent to paying Sep. 15, $200 + the interest of $200 from Apr. 1 to Sep. 15, or $205.57 ; etc.

The pupil should finish the explanation.

NOTE.—This method is identical with that used in settling short time notes, when payments are made within a year of date. *See p. 226.*

EQUATED TIME METHOD

101. Solve example 100 by the equated time method.

Dr.		*Cr.*	
500 × 192 =	*96000*	*300 × 190 =*	*57000*
300 × 165 =	*49500*	*200 × 167 =*	*33400*
700 × 44 =	*30800*	*500 × 76 =*	*38000*
1500	*176300*	*1000*	*128400*

1500	*176300*
1000	*128400*
500	*47900*

$ 500.
7.98 Int. $ 47900 1 da.
$ 507.98 Bal. due Sep. 15.

Owing $ 500 Mar. 7, is equivalent to owing Sep. 15, $ 500 + the interest of $ 500 for 192 days, or the interest of $ 96,000 for 1 da.

Owing $ 300 Apr. 3, is equivalent to owing Sep. 15, $ 300 + the interest of $ 300 for 165 days, or the interest of $ 49,500 for 1 da.

. . . .

Paying $ 300 Mar. 9, is equivalent to paying Sep. 15, $ 300 + the interest of $ 300 for 190 days, or the interest of $ 57,000 for 1 da.

Paying $ 200 Apr. 1, is equivalent to paying Sep. 15, $ 200 + the interest of $ 200 for 167 days, or the interest of $ 33,400 for 1 da.

. . . .

Sep. 15, he owes $ 1500 + the interest of $ 176,300 for 1 da., and has paid $ 1000 + the interest of $ 128,400 for 1 da. The bal. due is the difference, or $ 500 + the interest of $ 47,900 for 1 da., or $ 507.98.

102. Find the equated time for the payment of the account in example 100.

By the equated time, is meant the time when the balance of the account is due.

Sep. 15, Mr. Stone owed $ 500 + the interest of $ 47,900 for 1 da., or $ 500 + the interest of $ 500 for as many days as $ 500 is contained times in $ 47,900, or for 96 da. The *equated time*, or the time when he owed just $ 500, was 96 da. before Sep. 15, or June 11.

103. Find the equated time for the payment of the Dr. side in example 100.

The last debt was due Aug. 2. Reasoning as in the previous example, we find that Stone owed, Aug. 2, \$1500 + interest of \$110,300 for 1 da., or \$1500 + interest of \$1500 for 74 da. The *equated time*, or the time when he owed only \$1500, was 74 days before Aug. 2, or May 20.

Aug. 2.	*Dr.*
500 × 148 =	*74000*
300 × 121 =	*36300*
700 × 0 =	*0*
1500	*110300*

1500)110300
74

Dr.	*James Whitman*					*Cr.*
1895				*1895*		
Apr. 3	*To Mdse.*	*600*		*May 1*	*By Cash*	*400*
June 10	*To Mdse.*	*500*		*July 15*	*By Cash*	*300*
Sep. 15	*To Mdse.*	*400*		*Aug. 20*	*By Cash*	*200*

104. Settlement was made on the above account Nov. 1, 1895. How much did Mr. Whitman owe at that time, counting interest at 6 %? Solve by the interest method. Give the explanation in full.

105. Solve example 104 by the equated time method. Give the explanation in full. Which do you prefer; the interest method, or the equated time method?

106. In the above account, find the equated time for the payment of the Dr. side; that is, find the time when all of the Dr. side may be regarded as due.

107. In the above account, find the equated time for the payment of the Cr. side; that is, find the time when all of the Cr. side may be regarded as due.

108. In the above account, find the interest on the sum of the debts from the equated time to the date of settlement, Nov. 1; find the interest on the sum of the payments to the date of settlement; find the difference.

NOTE. — Ex. 108 illustrates another method of solving such problems.

ANNUAL INTEREST

Annual interest is rarely, if ever, computed in actual business.

The principal bears interest, and the interest on the principal at the end of each year bears interest. *See p. 216.*

109. What is the annual interest of $ 100 for 3 yr. 3 mo. 3 da. at 6%?

	yr.	*mo.*	*da.*
$ 6	*2*	*3*	*3*
$ 6	*1*	*3*	*3*
$ 6		*3*	*3*
$ 6	*3*	*9*	*9*

$ 19.55 = int. on prin.
1.36 = int. on int.
20.91 = annual int.

The simple interest of $ 100 for 3 yr. 3 mo. 3 da. at 6% is $ 19.55.

The $ 6, interest on the principal at the end of the first year, bears interest for 2 yr. 3 mo. 3 da.; the $ 6, interest on the principal at the end of the second year, bears interest for 1 yr. 3 mo. 3 da.; the $ 6, interest on the principal at the end of the third year, bears interest for 3 mo. 3 da. The whole is equivalent to the interest of $ 6 for the sum of the intervals, or for 3 yr. 9 mo. 9 da.

The interest of $ 6 for 3 yr. 9 mo. 9 da. at 6% is $ 1.36; the annual interest, $ 20.91.

What is the:

110. Annual interest of $ 420 for 3 yr. 2 mo. 5 da. @ 6%?

111. Annual interest of $ 240 for 1 yr. 6 mo. 12 da. @ 9%?

112. Annual interest of $ 186 for 4 yr. 9 mo. 18 da. @ 6%?

113. Annual interest of $ 252 for 5 yr. 5 mo. 15 da. @ 8%?

114. Find the amount of a note for $ 1500, interest 6%, payable annually, given Sep. 3, 1893, and not paid until July 1, 1897.

115. Find the amount of a note for $ 2000, interest 10%, payable annually, given Jan. 16, 1895, and not paid until April 1, 1900.

116. Find the amount of a note for $ 600, interest 8%, payable annually, given Oct. 10, 1896, and not paid until May 16, 1902.

117. Find the amount of a note for $ 1200, interest 7%, payable annually, given May 4, 1897, and not paid until Feb. 19, 1899.

COMPOUND INTEREST

In all states, both law and business usage are opposed to the collection of compound interest. There are many problems, however, especially in connection with insurance, annuities, and reserve funds, where this computation is necessary.

The principal bears interest, the interest on the principal at the end of each year bears interest, and all other interest at the end of each year bears interest. *See p. 216.*

118. What is the amount at compound interest of $1 for 3 yr. at 6%.

During the first year $1 is on interest; it amounts to $1.06.

During the second year $1.06 is on interest; since $1 amounts to $1.06, $1.06 will amount to 1.06 times $1.06, or $ $(1.06)^2$.

During the third year $ $(1.06)^2$ is on interest; since $1 amounts to $1.06, $ $(1.06)^2$ will amount to $(1.06)^2$ times $1.06, or $ $(1.06)^3$, or $1.191016.

119. What is the amount at compound interest of $1 for 4 yr. at 6%? Explain as in example 118.

AMOUNT OF $1 AT COMPOUND INTEREST

YEARS	5%	6%	10%
3	$(1.05)^3 = 1.157625$	$(1.06)^3 = 1.191016$	$(1.10)^3 = 1.331000$
4			
5			

120. Explain as in example 118 and fill out the blanks in the above diagram.

121. Prove that the amount of $1 for n years at r% is $ $(1+r)^n$.

122. What is the amount at compound interest of $1 for 2000 yr. at 6%? Express the answer as suggested in Ex. 121. How many decimal places are there in the exact answer?

123. Using the table just prepared, find the compound interest of $ 129.36 for 3 yr. 3 mo. 3 da. @ 6%.

$ (1.06)³, or
$ 1.191016, amt. $ 1, 3 yr.
129.36
154.07, amt. $ 129.36
1.0155, amt. $ 1, 3 mo. 3 da.
156.46, amt. whole time
129.36
27.10 int. whole time

$ 1 in 3 yr. at 6% will amount to $ (1.06)³, or $ 1.191016; $ 129.36 will amount to 129.36 times $ 1.191016, or $ 154.07.

At the end of 3 yr., $ 154.07 is on interest for 3 mo. and 3 da. $ 1 in this time will amount to $ 1.0155; $ 154.07 will amount to 154.07 times $ 1.0155, or $ 156.46.

124. In solving example 123, would it not be better to multiply $ 1.0155 by 1.191016 to find the amount of $ 1 for the whole time, and to multiply this product by 129.36?

Ans. No; because the product of the first two is a very long decimal.

125. Without the table, find the amount of $ 2630 for 3 yr. 4 mo. 4 da. at 4%, compound interest.

Using the table on p. 236, find the:

126. Compound int. of $ 365.25 for 4 yr. 5 mo. 5 da. at 5%.

127. Compound int. of $ 762.28 for 3 yr. 3 mo. 3 da. at 10%.

128. Compound int. of $ 625.50 for 5 yr. 6 mo. 4 da. at 9%.

Without the table, find the:

129. Compound int. of $ 72.38 for 3 yr. 7 mo. 2 da. at 7%.

130. Compound int. of $ 65.96 for 3 yr. 2 mo. 18 da. at 8%.

131. Compound int. of $ 85.50 for 1 yr. 9 mo. 16 da. at 6%.

Using the table, find the:

132. Compound amt. of $ 455 for 10 yr. 6 mo. 10 da. at 2½%.

133. Compound amt. of $ 366 for 15 yr. 9 mo. 20 da. at 4½%.

134. Compound amt. of $ 930 for 18 yr. 4 mo. 4 da. at 3½%.

AMOUNT OF $1 AT COMPOUND INTEREST FROM 1 TO 20 YEARS

YEARS	2 %	2½ %	3 %	3½ %	4 %	4½ %
1	1.0200000	1.0250000	1.0300000	1.0350000	1.0400000	1.0450000
2	1.0404000	1.0506250	1.0609000	1.0712250	1.0816000	1.0926250
3	1.0612080	1.0768906	1.0927270	1.1087178	1.1248640	1.1411661
4	1.0824321	1.1038128	1.1255088	1.1475230	1.1698585	1.1925186
5	1.1040808	1.1314082	1.1592740	1.1876863	1.2166529	1.2461819
6	1.1261624	1.1596934	1.1940523	1.2292553	1.2653190	1.3022601
7	1.1486856	1.1886857	1.2298738	1.2722792	1.3159317	1.3608618
8	1.1716593	1.2184029	1.2667700	1.3168090	1.3685690	1.4221006
9	1.1950925	1.2488629	1.3047731	1.3628973	1.4233118	1.4860951
10	1.2189944	1.2800845	1.3439163	1.4105987	1.4802442	1.5529694
11	1.2433743	1.3120866	1.3842338	1.4599697	1.5394540	1.6228530
12	1.2682417	1.3448888	1.4257608	1.5110686	1.6010322	1.6958814
13	1.2936066	1.3785110	1.4685337	1.5639560	1.6650735	1.7721961
14	1.3194787	1.4129738	1.5125897	1.6186945	1.7316764	1.8519449
15	1.3458683	1.4482981	1.5579674	1.6753488	1.8009435	1.9352824
16	1.3727857	1.4845056	1.6047064	1.7339860	1.8729812	2.0223701
17	1.4002414	1.5216182	1.6528476	1.7946755	1.9479005	2.1133768
18	1.4282462	1.5596587	1.7024330	1.8574892	2.0258165	2.2084787
19	1.4568111	1.5986501	1.7535060	1.9225013	2.1068491	2.3078603
20	1.4859474	1.6386164	1.8061112	1.9897888	2.1911231	2.4117140

YEARS	5 %	6 %	7 %	8 %	9 %	10 %
1	1.0500000	1.0600000	1.0700000	1.0800000	1.0900000	1.1000000
2	1.1025000	1.1236000	1.1449000	1.1664000	1.1881000	1.2100000
3	1.1576250	1.1910160	1.2250430	1.2597120	1.2950290	1.3310000
4	1.2155063	1.2624770	1.3107960	1.3604890	1.4115816	1.4641000
5	1.2762816	1.3382256	1.4025517	1.4693281	1.5386240	1.6105100
6	1.3400956	1.4185191	1.5007304	1.5868743	1.6771001	1.7715610
7	1.4071004	1.5036303	1.6057815	1.7138243	1.8280391	1.9487171
8	1.4774554	1.5938481	1.7181862	1.8509302	1.9925626	2.1435888
9	1.5513282	1.6894790	1.8384592	1.9990046	2.1718933	2.3579477
10	1.6288946	1.7908477	1.9671514	2.1589250	2.3673637	2.5937425
11	1.7103394	1.8982986	2.1048520	2.3316390	2.5804264	2.8531167
12	1.7958563	2.0121965	2.2521916	2.5181701	2.8126648	3.1384284
13	1.8856491	2.1329283	2.4098450	2.7196237	3.0658046	3.4522712
14	1.9799316	2.2609040	2.5785342	2.9371936	3.3417270	3.7974983
15	2.0789282	2.3965582	2.7590315	3.1721691	3.6424825	4.1772482
16	2.1828746	2.5403517	2.9521638	3.4259426	3.9703059	4.5949730
17	2.2920183	2.6927728	3.1588152	3.7000181	4.3276334	5.0544703
18	2.4066192	2.8543392	3.3799323	3.9960195	4.7171204	5.5599173
19	2.5269502	3.0255995	3.6165275	4.3157011	5.1416613	6.1159090
20	2.6532977	3.2071355	3.8696845	4.6609571	5.6044108	6.7275000

From the table, the amount for any time may be found.

135. What is the amount of $100 for 90 years at 6% compound interest?

$3.2071355, amt. $1, 20 yr.
3.2071355
$10.2857179, amt. $1, 40 yr.
10.2857179
$105.7959927, amt. $1, 80 yr.
1.7908477
$189.4645099, amt. $1, 90 yr.
$18946.45, amt. $100

$3.20 is on interest at the end of 20 yr. During the next 20 yr., $1 will amount to $3.20; $3.20+ will amount to 3.20+ times $3.20, or $10.28.

$10.28 is on interest at the end of 40 yr. During the next 40 yr., $1 will amount to $10.28; etc.

The pupil should finish the explanation.

If interest is payable at intervals of less than a year, the example should be reduced to an equivalent example, interest payable annually.

136. What is the amount at compound interest, of $100 for 3 yr. 2 mo. 4 da. at 8%, interest payable quarterly?

3 yr. 2 mo. 4 da.
4
12 yr. 8 mo. 16 da.

Paying quarterly is paying each time for a period one fourth of a year. The interest for a period one fourth of a year @ 8% is the same as the interest for a whole year @ 2%; ∴ the interest @ 8%, payable quarterly, is the same as the interest for 4 times 3 yr. 2 mo. 4 da., or for 12 yr. 8 mo. 16 da. @ 2%, payable annually.

The pupil should finish the explanation.

137. Show that the amount for 5 years at 12%, payable semiannually, is the same as the amount for 2 times 5 years, at ½ of 12% payable annually.

138. Find the amount at compound interest, of $100 for 25 yr. 5 mo. 7 da. at 4%.

139. Find the amount at compound interest, of $100 for 8 yr. 3 mo. 2 da. at 12%, interest payable 3 times a year.

INVOLUTION AND EVOLUTION

INVOLUTION — TERMS AND RELATIONS

$$8 = 2 \times 2 \times 2, \text{ or } 8 = 2^3.$$

If the product is wanting, this becomes

what $= 2^3$?

It means "what is the product when 2 is used 3 times as a factor?" *See p. 146.*

Involution is the process of finding the product when the same number is used several times as a factor.

The result is the *power*.

What is the product when 2 is used 3 times as a factor?

Ans. 8; $2 \times 2 \times 2 = 8$.

8 *is the third power of* 2.

1. Read: 5^2; 5^3; 5^4; 5^x.

Ans. 5^2, 5 square, or 5 to the second power; 5^3, 5 cube, or 5 to the third power; 5^4, 5 to the fourth power; 5^x, 5 to the xth power.

2. Write the squares of the integers from 13 to 25, and memorize the results.

3. Write the cubes of the integers from 1 to 10, and memorize the results.

4. Declare the results rapidly: 16^2; 25^2; 13^2; 17^2; 21^2; 22^2; 24^2; 14^2; 15^2; 18^2; 19^2; 20^2; 23^2; 9^3; 7^3; 2^3; 4^3; 6^3; 3^3; 5^3; 8^3.

To raise a factor to any power, write the base, and over it, the product of the exponent by the number denoting the required power.

Illustration: $(4^3)^2 = 4^6$; $(4^3)^2 = 4^3 \times 4^3 = (4 \times 4 \times 4) \times (4 \times 4 \times 4) = 4^6$.

5. Find the value of: $(2^3)^4$; $(2^4)^3$; $(3^2)^4$; $(3^4)^2$.

6. The square of 12 is 144; what is its 6th power? *Ans.* 144^3.

7. The cube of 12 is 1728; what is its 6th power? *Ans.* 1728^2.

EVOLUTION — TERMS AND RELATIONS

$$2^3 = 8.$$

If the base is wanting, this becomes

$$\textbf{(what)}^3 = \textbf{8 ?},\ \textbf{what} = \sqrt[3]{8}\ \textbf{?},\ \textbf{or what} = 8^{\frac{1}{3}}\ \textbf{?}.$$

It means "what number must be taken 3 times as a factor to produce 8 ?"

Evolution is the process of finding the number which must be taken several times as a factor to produce a given product.

The result is the *root*.

What is the number which must be taken 3 times as a factor to produce 8 ?

Ans. 2, *because* $2^3 = 8$.
2 *is the third root of* 8.

8. Read: $\sqrt{9}$; $\sqrt[3]{27}$; $\sqrt[4]{81}$; $\sqrt[x]{81}$.

Ans. $\sqrt{9}$, the square root of 9, or the second root of 9; $\sqrt[3]{27}$, the cube root of 27, or the third root of 27; $\sqrt[4]{81}$, the fourth root of 81; $\sqrt[x]{81}$, the xth root of 81.

9. Read: $9^{\frac{1}{2}}$; $27^{\frac{1}{3}}$; $81^{\frac{1}{4}}$; $81^{\frac{1}{x}}$.

Ans. These are read as in Ex. 8. Also, $9^{\frac{1}{2}}$ may be read, 9 to the $\frac{1}{2}$ power; $27^{\frac{1}{3}}$, 27 to the $\frac{1}{3}$ power; . . . .

10. Declare the results rapidly: $\sqrt{225}$; $\sqrt{400}$; $\sqrt{625}$; $\sqrt{169}$; $\sqrt{324}$; $\sqrt{196}$; $\sqrt{256}$; $\sqrt{289}$; $\sqrt{361}$; $\sqrt{441}$; $\sqrt{484}$; $\sqrt{529}$.

11. Declare the results rapidly: $\sqrt[3]{729}$; $\sqrt[3]{27}$; $\sqrt[3]{125}$; $\sqrt[3]{343}$; $\sqrt[3]{8}$; $\sqrt[3]{64}$; $\sqrt[3]{216}$; $\sqrt[3]{512}$.

A root of a perfect power may be extracted by factoring.

By factoring, find the:

12. Value of $\sqrt{5184}$

13. Value of $\sqrt[3]{1728}$

14. Value of $\sqrt[3]{32768}$

15. Value of $\sqrt[5]{248832}$

16. Value of $\sqrt[4]{4100625}$

17. Value of $\sqrt[5]{7962624}$

Ex. 12. *Ans.* $\sqrt{5184} = \sqrt{9^2 \times 8^2} = 9 \times 8 = 72.$

Ex. 15. *Ans.* $\sqrt[5]{248{,}832} = \sqrt[5]{9^2 \times 8^3 \times 6} = \sqrt[5]{3^5 \times 2^5 \times 2^5} = 3 \times 2 \times 2 = 12.$

SQUARE ROOT

I. *If a number is separated into periods of two figures each, beginning with units' place, the number of periods will be equal to the number of figures in the square root of that number.*

Illustration

$1^2 = 1$	$99^2 = 98'01$	That is, the square of a number of
$9^2 = 81$	$100^2 = 1'00'00$	1 figure has 1 period; of 2 figures, 2
$10^2 = 100$	$999^2 = 99'80'01$	periods; of 3 figures, 3 periods; etc.

II. *If a number is separated into periods of two figures each, beginning at units' place, the square root of the number denoted by the left-hand period, will give the first figure of the root; the square root of the number denoted by the first two periods, will give the first two figures of the root; etc.*

Illustration

$1234^2 = 1'52'27'56$	or	$\sqrt{1'52'27'56} = 1234$
$1^2 = 1$		$\sqrt{1} = 1$, *first figure*
$12^2 = 1'44$		$\sqrt{1'52} = 12+$, *first two fig.*
$123^2 = 1'51'29$		$\sqrt{1'52'27} = 123+$, *first three fig.*

The number denoted by the first period is 1; its square root gives the first figure of the root.

The number denoted by the first two periods is 152; its square root gives the first two figures of the root; etc.

III. $(a+b)^2 = a^2 + 2ab + b^2$, or $a^2 + (2a+b)b$

$$\begin{array}{r} a + b \\ a + b \\ \hline a^2 + ab \\ ab + b^2 \\ \hline a^2 + 2ab + b^2 \end{array}$$

In $a^2 + 2ab + b^2$

b is a factor of the last two terms. $2ab \div b = 2a$; $b^2 \div b = b$. Therefore, $a^2 + 2ab + b^2 =$

$a^2 + (2a+b)b$

18. Extract the square root of 625.

$$(a+b)^2 = a^2 + 2\,ab + b^2$$
$$= a^2 + (2\,a + b)b$$

```
      6'25(25                          6'25(25
      4                                4
 40 | 225         a = 20          45 | 225
  5 |             b =  5             | 225
 45 | 225
```

We separate the number into periods of two figures each, because the square root of the first period will give the first figure of the root, and the square root of the first two periods will give the first two figures of the root.

We raise $a + b$ to the second power to have a perfect square before us, and factor the terms containing b to cause the second term of the root to appear.

To obtain a, the first term of the root in the model, we must extract the square root of a^2. Hence, to obtain the first term of the root in this example, we must extract the square root of what corresponds to a^2, or of 6. $\sqrt{6} = 2$; $a = 20$, because 6 is not 6 *units*, but 6 *hundreds*.

To obtain b, the second term of the root in the model, we must divide $2\,ab$ by $2\,a$. Hence, to obtain the second term of the root in this example, we must divide what corresponds to $2\,ab$, or the greater part of 225, by what corresponds to $2\,a$, or by 40. $225 \div 40 = 5$; $b = 5$.

To obtain the rest of the model, we must multiply what is within the parenthesis, that is, $(2\,a + b)$, by b. Hence, to obtain the rest of this number, we must multiply what corresponds to what is within the parenthesis, by what corresponds to b. $b = 5$; $2\,a + b = 45$; $45 \times 5 = 225$. Since there is no remainder, $\sqrt{625} = 25$.

NOTE. — Instead of writing 40 as at the left, it is customary to write 4, regarding it as 4 tens. The 5 may then be added without an extra line.

Extract the square root and explain the process:

19. 9025. **22.** 1369. **25.** 8281.

20. 9409. **23.** 2304. **26.** 1024.

21. 6889. **24.** 7225. **27.** 5329.

28. Extract the square root of 1′52′27′56.

We separate the number into periods of two figures each, because the square root of the first period will give the first figure of the root; the square root of the first two periods, the first two figures; etc.

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$= a^2 + (2a+b)b$$

FIRST

We extract the square root of 1′52 to obtain the first two figures of the root. The explanation is the same as already given. *The pupil should give it in full. See p. 241.*

```
 1′52′27′56 |12
 1
 ---
22 | 52
   | 44
   ----
      8
```

SECOND

We extract the square root of 1′52′27 to obtain the first three figures of the root. We may regard 152′27 as 152 *hundreds* and 27 *units*. That is, we are again to extract the square root of a number of two periods, the first being 152; the second, 27.

To obtain a, the first term of the root in the model, we must extract the square root of a^2. Hence, to obtain the first term of the root in this example, we must extract the square root of what corresponds to a^2, or of 152. $\sqrt{152} = 12$; $a = 120$, because 152 is not 152 *units* but 152 *hundreds*. *The pupil should finish the explanation.*

```
 1′52′27′56 |123
 1
 ---
 22 | 52
    | 44
    -----
243 | 827
    | 729
    -----
       98
```

THIRD

We extract the square root of 1′52′27′56 to obtain the first four figures of the root. We may regard 15227′56 as 15227 *hundreds* and 56 *units*. That is, we are again to extract the square root of a number of two periods, the first period being 15227; the second, 56.

$\sqrt{15227} = 123$; $a = 1230$, because 15227 is not 15227 *units* but 15227 *hundreds*. *The pupil should finish the explanation.*

```
  1′52′27′56 |1234
  1
  ---
  22 | 52
     | 44
     -----
 243 | 827
     | 729
     ------
2464 | 9856
     | 9856
```

NOTE. — In practice, only the third form should be written.

Find the value of:

29. $\sqrt{98596}$	**31.** $\sqrt{390625}$	**33.** $\sqrt{1079521}$
30. $\sqrt{53361}$	**32.** $\sqrt{522729}$	**34.** $\sqrt{3345241}$

35. Extract the square root of $\frac{25}{36}$.

$$\sqrt{\frac{25}{36}}=\frac{5}{6}$$

We extract the square root of the numerator, and then of the denominator.

36. Extract the square root of .000025.

$$\sqrt{.00'00'25}=.005, \text{ for}$$
$$\sqrt{.000025}=\sqrt{\frac{25}{1000000}}$$
$$=\frac{5}{1000}$$

We point off into periods of two figures each, beginning at the decimal point.

We point off as many decimal places in the root as there are decimal periods in the number.

There are 3 decimal periods in the number, and 3 decimal places in the root.

37. Extract the square root of 2.

$$\sqrt{2}=\sqrt{2.'00'00'00+}=1.414+$$

38. Extract the square root of .00365.

$$\sqrt{.00'36'50}$$

CAUTION. — Be sure to point off, beginning with the decimal point.

39. Extract the square root of $\frac{2}{3}$.

$$\sqrt{\frac{2}{3}}=\sqrt{.66'66'66+}$$

If the denominator is not a perfect square, it is best to reduce the fraction to a decimal.

Find the value of:

40. $(\frac{8}{18})^{\frac{1}{2}}$	**43.** $(\frac{2}{9})^{\frac{1}{2}}$	**46.** $(\frac{3}{7})^{\frac{1}{2}}$
41. $\sqrt{.0625}$	**44.** $\sqrt{6.25}$	**47.** $\sqrt{.0000625}$
42. $\sqrt{.625}$	**45.** $\sqrt{62.5}$	**48.** $\sqrt{.000625}$

CUBE ROOT

I. *If a number is separated into periods of three figures each, beginning with units' place, the number of periods will be equal to the number of figures in the cube root of that number.*

Illustration

$1^3 = 1$ $\quad 99^3 = 970'299$
$9^3 = 729$ $\quad 100^3 = 1'000'000$
$10^3 = 1'000$ $\quad 999^3 = 997'002\,999$

That is, the cube of a number of 1 figure has 1 period; of 2 figures, 2 periods; of 3 figures, 3 periods, etc.

II. *If a number is separated into periods of three figures each, beginning with units' place, the cube root of the number denoted by the left hand period, will give the first figure of the root; the cube root of the number denoted by the first two periods, will give the first two figures of the root, etc.*

$1234^3 = 1'879'080'904$
$1^3 = 1$
$12^3 = 1'728$
$123^3 = 1'860'867$

or $\sqrt[3]{1'879'080'904} = 1234$

$\sqrt[3]{1} = 1$, first figure

$\sqrt[3]{1'879} = 12 +$, first two figures

$\sqrt[3]{1'879'080} = 123 +$, first three figures

The number denoted by the first period is 1; its cube root gives the first figure of the root.

The number denoted by the first two periods is 1879; its cube root gives the first two figures of the root; etc.

III. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$, or $a^3 + (3a^2 + 3ab + b^2)b$

$$\begin{array}{l}
a + b \\
\underline{a + b} \\
a^2 + ab \\
\underline{\quad\;\; ab + b^2} \\
a^2 + 2ab + b^2 \\
\underline{\qquad\quad a + b} \\
a^3 + 2a^2b + \;\; ab^2 \\
\underline{\qquad\; a^2b + 2ab^2 + b^3} \\
a^3 + 3a^2b + 3ab^2 + b^3
\end{array}$$

In $a^3 + 3a^2b + 3ab^2 + b^3$

b is a factor of the last three terms. $3a^2b \div b = 3a^2$; $3ab^2 \div b = 3ab$; $b^3 \div b = b^2$. Therefore,

$$a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + (3a^2 + 3ab + b^2)b$$

49. Extract the cube root of 1728.

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$= a^3 + (3a^2 + 3ab + b^2)b$$

$$\begin{array}{r|l l}
\multicolumn{2}{l}{1'728(12} & \\
\multicolumn{2}{l}{1} & \\
\hline
300 & 728 & a = 10 \\
60 & & b = 2 \\
4 & & \\
\cline{1-1}
364 & \underline{728} &
\end{array}$$

We separate the number into periods of three figures each, because the cube root of the first period will give the first figure of the root, and the cube root of the first two periods will give the first two figures of the root.

We raise $a + b$ to the third power to have a perfect cube before us, and factor the terms containing b to cause the second term of the root to appear.

To obtain a, the first term of the root in the model, we must extract the cube root of a^3. Hence, to obtain the first term of the root in this example, we must extract the cube root of what corresponds to a^3, or of 1. $\sqrt[3]{1} = 1$; $a = 10$, because 1 is not 1 *unit*, but 1 *thousand*.

To obtain b, the second term of the root in the model, we must divide $3a^2b$ by $3a^2$. Hence, to obtain the second term of the root in this example, we must divide what corresponds to $3a^2b$, or the greater part of 728, by what corresponds to $3a^2$, or by 300. $728 \div 300 = 2$; $b = 2$.

To obtain the rest of the model, we must multiply what is within the parenthesis, that is $(3a^2 + 3ab + b^2)$, by b. Hence, to obtain the rest of this number, we must multiply what corresponds to what is within the parenthesis, by what corresponds to b. $3ab = 60$; $b^2 = 4$; $3a^2 + 3ab + b^2 = 364$; $364 \times 2 = 728$. Since there is no remainder, $\sqrt[3]{1728} = 12$.

Extract the cube root and explain the process:

50. 12167. **53.** 405224. **56.** 704969.

51. 39304. **54.** 314432. **57.** 148877.

52. 117649. **55.** 250047. **58.** 166375.

NOTE. — The pupil should observe that the explanation is identical with that for the extraction of square root. In each case, the leading of the formula is followed. Compare with p. 241.

59. Extract the cube root of 1′879′080′904.

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$= a^3 + (3a^2 + 3ab + b^2)b$$

We separate the number into periods of three figures each, because the cube root of the first period will give the first figure of the root; the cube root of the first two periods, the first two figures, etc.

FIRST

We extract the cube root of 1′879 to obtain the first two figures of the root. The explanation is the same as already given. *The pupil should give it in full.* *See p. 245.*

SECOND

We extract the cube root of 1′879′080 to obtain the first three figures of the root. We may regard 1879′080 as 1879 *thousands* and 080 *units*. That is, we are again to extract the cube root of a number of two periods, the first being 1879; the second, 080.

To obtain a, the first term of the root in the model, we must extract the cube root of a^3. Hence, to obtain the first term of the root in this example, we must extract the cube root of what corresponds to a^3, or of 1879. $\sqrt[3]{1879} = 12$; $a = 120$, because 1879 is not 1879 *units*, but 1879 *thousands*. *The pupil should finish the explanation.*

THIRD

We extract the cube root of 1′879′080′904 to obtain the first four figures of the root. We may regard 1879080′904 as 1879080 *thousands* and 904 *units*. That is, we are again to extract the cube root of a number of two periods, the first period being 1879080; the second, 904.

$\sqrt[3]{1'879'080} = 123$; $a = 1230$, because 1879080 is not 1879080 *units* but 1879080 *thousands*. *The pupil should finish the explanation.*

FIRST

1′879′080′904(12

	1	
300	*879*	
60		$a = 10$
4		$b = 2$
364	*728*	
	151	

SECOND

1′879′080′904(123

	1	
300	*879*	
60		$a = 10$
4		$b = 2$
364	*728*	
43200	*151080*	
1080		$a = 120$
9		$b = 3$
44289	*132867*	
	18213	

THIRD

1′879′080′904(1234

	1	
300	*879*	
60		$a = 10$
4		$b = 2$
364	*728*	
43200	*151080*	
1080		$a = 120$
9		$b = 3$
44289	*132867*	
4538700	*18213904*	
14760		$a = 1230$
16		$b = 4$
4553476	*18213904*	

NOTE. — In practice only the third form should be written.

Find the value of:

60. $\sqrt[3]{94818816}$

61. $\sqrt[3]{177504328}$

62. $\sqrt[3]{1067462648}$

63. $\sqrt[3]{1039509197}$

64. Extract the cube root of $\frac{125}{216}$.

$$\sqrt[3]{\frac{125}{216}}=\frac{5}{6}$$

We extract the cube root of the numerator, and then of the denominator.

65. Extract the cube root of .000125.

$$\sqrt[3]{.000'125}=.05, \text{ for}$$
$$\sqrt[3]{.000'125}=\sqrt[3]{\frac{125}{1000000}}$$
$$=\frac{5}{100}$$

We point off into periods of three figures each, beginning at the decimal point.

We point off as many decimal places in the root as there are decimal periods in the number.

There are 2 decimal periods in the number and 2 decimal places in the root.

66. Extract the cube root of 2.

$$\sqrt[3]{2}=\sqrt[3]{2.'000'000'000+}=1.259+$$

67. Extract the cube root of .00365.

$$\sqrt[3]{.003'650}$$

CAUTION. — Be sure to point off, beginning with the decimal point.

68. Extract the cube root of $\frac{2}{3}$.

$$\sqrt[3]{\frac{2}{3}}=\sqrt[3]{.666'666+}$$

If the denominator is not a perfect cube, it is best to reduce the fraction to a decimal.

Find the value of:

69. $\left(\frac{16}{54}\right)^{\frac{1}{3}}$

70. $\sqrt[3]{.001728}$

71. $\sqrt[3]{.0001728}$

72. $\left(\frac{2}{5}\right)^{\frac{1}{3}}$

73. $\sqrt[3]{1.728}$

74. $\sqrt[3]{172.8}$

75. $\left(\frac{3}{7}\right)^{\frac{1}{3}}$

76. $\sqrt[3]{.00001728}$

77. $\sqrt[3]{.000015625}$

ANY ROOT

To extract any root of a number, point off into periods of as many figures each as there are units in the index of the root, raise $a+b$ to the corresponding power, and proceed as the formula indicates.

78. Extract the 4th root of 2847396321.

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$= a^4 + (4a^3 + 6a^2b + 4ab^2 + b^3)b$$

```
                28'4739'6321(231
                16
        32000 | 124739
         7200 |                 a = 20
          720 |                 b = 3
           27 |
        ----- |
        39947 | 119841
       ---------------------
              |  4898632 1
      4866800 0|                a = 230
       317400 |                 b = 1
          920 |
            1 |
      ------- |
      4898632 1| 4898632 1
```

NOTE. — The pupil should supply the explanation.

79. Prepare the formula for the extraction of the fifth root.

80. Prepare the formula for the extraction of the sixth root.

Find the

81. Value of $\sqrt[4]{221533456}$.

82. Value of $\sqrt[5]{7962624}$.

83. Value of $\sqrt[4]{9354951841}$.

84. Value of $\sqrt[6]{2985984}$.

To depress a factor to any root, write the base, and over it the quotient of the exponent by the number denoting the required root.

Illustration: $\sqrt[3]{5^6} = 5^2$; because $5^2 \times 5^2 \times 5^2 = 5^6$.

85. Find the value of: $\sqrt[3]{5^{12}}$; $\sqrt[4]{5^{12}}$; $\sqrt{5^{12}}$; $\sqrt[6]{5^{12}}$.

86. Find the value of: $\sqrt[3]{\sqrt{64}}$; $\sqrt{\sqrt[3]{64}}$; $\sqrt[6]{64}$.

Ans. $\sqrt[3]{\sqrt{64}} = 2$; $\sqrt{\sqrt[3]{64}} = 2$; $\sqrt[6]{64} = 2$; $\sqrt{\sqrt[3]{x}} = \sqrt[3]{\sqrt{x}} = \sqrt[6]{x}$.

87. Solve Ex. 78 by extracting the square root of the square root.

MENSURATION

We may consider that which has no dimension, that which has one dimension, that which has two dimensions, or that which has three dimensions.

NO DIMENSION

That which has no dimension is a *point.*

Note.—Strictly speaking, the illustration is not a point, because however small it may be, it has length, breadth, and thickness.

Illustrations

•

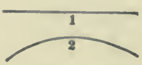
Point.

ONE DIMENSION

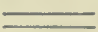
That which has one dimension is a *line.*

A line may extend in the same direction, a *straight* line; or it may constantly change its direction, a *curved* line.

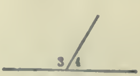
If two straight lines in the same plane are extended, they will meet, or they will not meet. If they do not meet, they are *parallel;* if they meet, they form *angles.*



Lines.
1. *Straight.* 2. *Curved.*



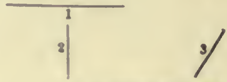
Parallel Lines.



Angles.

ONE DIMENSION

Angles.
1 & 2. *Right.* 3. *Obtuse.*
3 & 4. *Oblique.* 4. *Acute.*



1. *Horizontal Line.*
2. *Vertical Line.*
3. *Oblique Line.*

If two lines meet, the angles will be equal, *right* angles; or not equal, *oblique* angles; the larger is *obtuse*, the smaller *acute*.

A straight line may be parallel to the horizon, a *horizontal* line; perpendicular to the horizon, a *vertical* line; or neither parallel nor perpendicular to the horizon, an *oblique* line.

Draw and define:

1. A point.
2. A line.
3. A straight line.
4. A curved line.
5. Parallel lines.
6. Two perpendicular lines.
7. A vertical line.
8. A horizontal line.
9. An oblique line.
10. An angle.
11. A right angle.
12. An obtuse angle.
13. An acute angle.
14. Two oblique angles.

15. 16. 17. 18.

19. 20. 21. 22.

Draw points and lines situated like the above, and from each point draw a perpendicular to the nearest line.

TWO DIMENSIONS

That which has two dimensions is a *surface.*

If any two points of a surface are connected by a straight line, that line will lie wholly on the surface, a *plane surface,* or *plane;* or it will not lie wholly on the surface, a *curved surface.*

If straight lines inclose a surface, the figure is a *polygon.*

The least number of straight lines which can inclose a plane is three, a *triangle,* (1). The three lines may be equal, an *equilateral* triangle, (2); two of them may be equal, an *isosceles* triangle, (3); or no two of them may be equal, a *scalene* triangle, (4).

A triangle may have one right angle, a *right-angled* triangle, (6); one obtuse angle, an *obtuse-angled* triangle, (7); or three acute angles, an *acute-angled* triangle, (8).

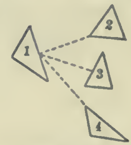
The next number of straight lines which can inclose a plane is four, a *quadrilateral,* (9). The quadrilateral may have both pairs of its opposite sides parallel, a *parallelogram,* (10); one pair parallel, a *trapezoid,* (11); or neither pair parallel, a *trapezium,* (12).

A parallelogram may have its angles right angles, a *rectangle,* (13); or not right angles, a *rhomboid,* (14).

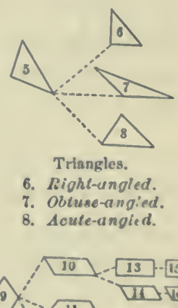
The rectangle may have its sides all equal, a *square,* (15); the rhomboid may have its sides all equal, a *rhombus,* (16).

The face of this page is a plane surface, or a plane.

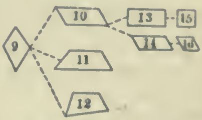
Curved surface.



Triangles.
2. *Equilateral.*
3. *Isosceles.*
4. *Scalene.*



Triangles.
6. *Right-angled.*
7. *Obtuse-angled.*
8. *Acute-angled.*



Quadrilaterals.
10. *Parallelogram.*
11. *Trapezoid.*
12. *Trapezium.*
13. *Rectangle.*
14. *Rhomboid.*
15. *Square.*
16. *Rhombus.*

Five sides may inclose a surface, *pentagon;* six sides, *hexagon;* seven sides, *heptagon;* eight sides, *octagon;* nine sides, *nonagon;* ten sides, *decagon,....*

Regular hexagon.

A polygon may have its sides and angles equal, a *regular polygon;* or its sides and angles not equal, an *irregular polygon.* Hence we may have regular and irregular pentagons, regular and irregular hexagons . . . .

Irregular hexagon.

A regular polygon of an infinite number of sides is a *circle.*

Circle.

Define:

23. A surface.

24. A polygon.

25. A triangle.

26. An equilateral triangle.

27. An isosceles triangle.

28. A scalene triangle.

29. A right-angled triangle.

30. An acute-angled triangle.

31. A regular polygon.

32. A regular pentagon.

33. A regular hexagon.

34. A regular heptagon.

35. An obtuse-angled triangle.

36. A circle.

37. Beginning with "*plane surface,*" (see Note) define: *parallelogram; rectangle; rhomboid; rhombus; square.*

38. Beginning with *quadrilateral,* (see Note) define: *parallelogram; rectangle; square; rhombus.*

NOTE. — A definition may begin with different terms, *e.g.:*

A square is a *plane surface* bounded by two pairs of opposite sides, having each pair parallel, having its angles all right angles, and having its sides all equal. Or,

A square is a *quadrilateral* having each pair of its opposite sides parallel, having its angles all right angles, and having its sides all equal. Or,

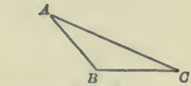
A square is a *parallelogram* having its angles all right angles, and having its sides all equal. Or,

A square is a *rectangle,* having its sides all equal.

That definition is the best which is the shortest, *provided it begins with a term which is understood by the person for whom the definition is given.*

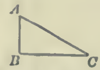
PARTS OF A POLYGON

That part of a polygon on which it is supposed to rest is its *base;* the distance around a polygon, its *perimeter;* the perimeter of a circle, its *circumference.*



BC, Base.
AB+BC+AC, Perimeter.

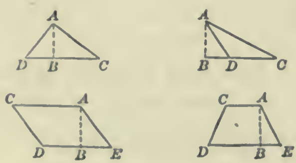
In a right-angled triangle, the side opposite the right angle is the *hypotenuse;* the other sides, *legs.*



AB and BC, Legs.
AC, Hypotenuse.

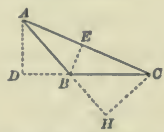
The *altitude* of a triangle, parallelogram, or trapezoid, is a perpendicular to the base from the vertex opposite the base.

AB is the altitude in each of these figures. Observe that the base must sometimes be extended.



AB, Altitude.

Since a triangle may be regarded as resting on any one of its sides, it may have three bases with an altitude corresponding to each.

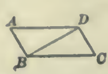


Base, BC; Altitude, AD.
Base, AB; Altitude, CH.
Base, AC; Altitude, BE.

The *apothem* of a regular polygon is the perpendicular from its center to any side.

AB, Apothem.

The *diagonal* of a polygon is a line which joins any two vertices not adjacent.



BD, Diagonal.

PARTS OF A CIRCLE

A circle is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the center.

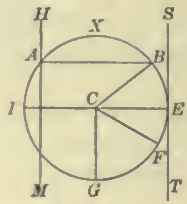
Linear Parts

We may consider the whole of the bounding line, the *circumference;* or a part of it, an *arc.*

A line may cut the circumference in two points, a *secant;* or may touch it at one point, a *tangent.*

A line may be drawn from the center to any point of the circumference, a *radius.*

A line may connect any two points of the circumference, a *chord.* A chord may pass through the center, a *diameter.*



Linear parts.

ABEFGD, Circumference.
DG, Arc.
HM, Secant.
ST, Tangent.
CB, Radius.
AB, Chord.
DE, Diameter.

Surface Parts

We may consider the portion of a circle between a chord and its arc, a *segment;* or the portion between two radii and their arc, a *sector.* A sector may be half of the circle, a *semicircle;* a quarter, a *quadrant;* a sixth, a *sextant.*

Surface parts.

ABX, Segment.
BCF, Sector.
DGE, Semicircle.
EGC, Quadrant.
CFG, Sextant.

Draw and define:

39. A circle.
40. A segment.
41. A sector.
42. A quadrant.
43. A sextant.
44. A semicircle.
45. A radius.
46. A chord.
47. A diameter.
48. A tangent.
49. A secant.
50. An arc.
51. The center.
52. A circumference.

COMPUTATIONS — LINEAR PARTS

I. *The circumference of a circle is equal to twice the radius times 3.1416.*

II. *The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.*

NOTE. — For illustration and explanation, the pupil should turn to p. 272.

53. The radius of a circle is 5 in.; find its circumference.

Relation: $C = 2 \times R \times 3.1416$

$C = 2 \times 5 \times 3.1416 = 31.416$

Circumference $= 31.416$ *in.*

We substitute the value given.

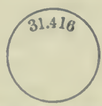
54. The circumference of a circle is 31.416 in.; find its radius.

Relation: $C = 2 \times R \times 3.1416$

$31.416 = 2 \times R \times 3.1416$

$R = \frac{31.416}{2 \times 3.1416} = 5$

Radius $= 5$ *in.*



We substitute the value given.

55. The hypotenuse of a right-angled triangle is 10 in.; its perpendicular 6 in.; find its base.

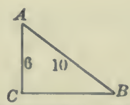
Relation: $\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2$

$100 = 36 + \overline{BC}^2$

$100 - 36 = \overline{BC}^2 = 64$

$\overline{BC} = \sqrt{64} = 8$

Base $= 8$ *in.*



The pupil should always draw the figure.

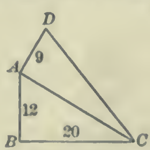
56. A room is 20 ft. × 12 ft. × 9 ft. How far is it from a lower corner to the opposite upper corner?

$\overline{AC}^2 = 144 + 400 = 544$

$\overline{DC}^2 = 81 + 544 = 625$

$\overline{DC} = \sqrt{625} = 25$

Distance $= 25$ *ft.*



AC, Diagonal of floor; AD, Height of room.

Find the circumference:

57. Radius 8 in.

58. Radius 6 in.

59. Diameter 20 ft.

Find the radius:

60. Circumference 41.888 ft.

61. Circumference 502.656 in.

62. Circumference 15.708 m.

Find the hypotenuse:

63. Legs 6 in. and 8 in.

64. Legs 24 in. and 10 in.

65. Legs 32 in. and 24 in.

66. Legs 12 in. and 16 in.

Find the other leg:

67. Hypot. 40 in.; base 32 in.

68. Hypot. 13 in.; base 12 in.

69. Hypot. 30 in.; base 18 in.

70. Hypot. 35 in.; base 21 in.

71. If the foot of a ladder 50 ft. long is put 30 ft. from the base of a wall, how far will it reach? Draw the figure.

72. A ladder 91 ft. long was placed between two buildings. The base being at the same point, the ladder reached a point 84 ft. from the ground on the first building, and 35 ft. from the ground on the other. How far apart were the two buildings? Draw the figure.

73. A room is 30 ft. long, 18 ft. wide, and $13\frac{1}{2}$ ft. high. How far is it from a lower corner to the opposite upper corner? Draw the figure.

74. What is the length of the longest straight rod which, without bending, can be put into a box 5 ft. long, 1 yd. wide, and $\frac{3}{4}$ yd. deep? Draw the figure.

75. Two boats start from the same point and sail, one north 10,560 ft., the other east 7920 ft. How far apart are they at last?

76. Two towers, 94 and 78 ft. high, are situated on opposite banks of a river 30 ft. broad. What is the length of the shortest line connecting the tops of the towers? Draw.

77. Two objects, A and B, are in a straight line south of a flagstaff 16 ft. high. If the lines joining the top of the flagstaff with each object are 20 ft. and 34 ft. respectively, how far apart are A and B? Draw the figure.

COMPUTATION — AREAS

The area of a polygon is the number of square units in its surface. The square unit is one of the denominations in square measure.

Note. — For illustrations, explanations, and proofs, the pupil should turn to p. 121, and to pp. 272, 273, and 274.

III. *The area of a rectangle is equal to the product of its base by its altitude.*

IV. *The area of a parallelogram is equal to the product of its base by its altitude.*

V. *The area of a triangle is equal to one half the product of its base by its altitude.*

VI. *The area of a triangle is equal to the square root of the continued product of the half sum of its sides and the remainders found by subtracting each side from the half sum separately.*

VII. *The area of a trapezoid is equal to one half the product of the sum of its parallel sides by its altitude.*

VIII. *The area of a regular polygon is equal to one half the product of its perimeter by its apothem.*

IX. *The area of a circle is equal to the square of its radius times 3.1416.*

X. *The area of an irregular polygon is equal to the sum of the areas of the triangles into which it may be divided.*

78. The area of a triangle is 12 sq. ft.; its altitude 6 ft. Find its base.

Relation: Area $= \frac{1}{2} B \times A$,

$$12 = \frac{1}{2} B \times 6 = 3B,$$

$$B = \frac{12}{3} = 4,$$

Base $= 4$ *ft.*

We substitute the value given.

79. Find the area of a triangle whose sides are 6 in., 8 in., and 10 in.

Relation: $Area = \sqrt{s(s-a)(s-b)(s-c)}$

$Area = \sqrt{12 \times 6 \times 4 \times 2} = 24$

$Area = 24$ sq. in.

$s = \frac{1}{2}$ sum of the sides.

$a, b, c =$ the sides respectively.

80. The area of a circle is 78.54 sq. in. Find its circumference.

Relation: $Area = R^2 \times 3.1416$

$78.54 = R^2 \times 3.1416$

$R^2 = \frac{78.54}{3.1416} = 25$

$R = 5$ in.

Relation: $C = 2 \times R \times 3.1416$

$C = 2 \times 5 \times 3.1416 = 31.416$

Circumference $= 31.416$ in.

81. The area of an equilateral triangle is 62.352 sq. rd. Find its perimeter.

Relation: $Area = \frac{1}{2} BC \times AD$

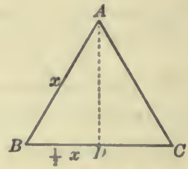
$= \frac{x}{2} \times .866\,x = .433\,x^2$

$62.352 = .433\,x^2$

$x^2 = \frac{62.352}{.433} = 144$

$x = 12$

Perimeter $= 36$ rd.



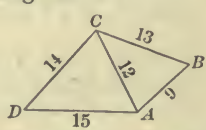
$AD^2 = x^2 - \frac{x^2}{4} = \frac{3\,x^2}{4}$

$AD = \frac{x\sqrt{3}}{2} = .866\,x$

82. Find the area of a trapezium whose sides are 9, 15, 13, and 14 m. respectively, and whose shortest diagonal is 12 m.

Area = area ABC + area ACD.

The pupil should complete the work.



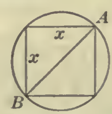
83. The circumference of a circle is 125.664 m. Find the area of an inscribed square.

Relation: $C = 2 \times R \times 3.1416$

$125.664 = 2 \times R \times 3.1416$

$R = \frac{125.664}{2 \times 3.1416} = 20$

$AB = 40$



$x^2 + x^2 = 1600$

$x^2 = 800$

Area $= 800$ sq m

84. Find the base of a rectangle whose area is 48 A. and altitude 48 rd.

85 Find the area of a triangle whose sides are 25 rd., 60 rd. and 65 rd.

86. Find the area of a square whose diagonal is 20 m.

87. Find the diagonal of a square whose area is 625 sq. ft.

88. Find the area of a trapezoid whose parallel sides are 60 rd. and 80 rd., and whose altitude is 30 rd.

89. Find the area of a rhombus, one of whose sides is 12 dm and altitude 8 dm.

90. Find the altitude of a rhombus whose area is 48 sq. rd. and base 48 rd.

91. Find the area of a triangle whose base is 9 m, and altitude 6 m.

92. Find the area of a trapezium whose diagonal is 40 rd., and perpendiculars from the opposite vertices, 16 rd. and 20 rd.

93. Find the area of a circle whose diameter is 20 m.

94. Find the area of a regular octagon, one of whose sides is 8 feet, and whose apothem is 9.656 ft.

95. Find the altitude of a triangle whose area is 48 A. and base 48 rd.

96. Find the circumference of a circle whose area is 392.70 sq. rd.

97. Find the altitude and area of an equilateral triangle, each of whose sides is 20 ft.

98. Find the altitude and area of an isosceles triangle whose base is 40 ft. and whose equal sides are 52 ft.

99. What is the area of a square circumscribed about a circle whose circumference is 314.16 ft.?

100. A circular ring is formed by two circles having the same center. The radius of the inner circle is 8 ft.; the radius of the outer circle is 10 ft. Draw the circular ring, and compute its area. What is the ratio of the circumferences of the bounding circles?

THREE DIMENSIONS

That which has three dimensions is a *solid.*

That part on which a solid rests is its *base;* its other surfaces are *faces;* the union of two faces is an *edge;* the union of three or more edges, a *vertex.*

A solid may have *two* bases equal and parallel polygons, and its faces rectangles, a *prism.* If its bases are triangles, *triangular* prism; squares, *square* prism; . . . . circles, *circular* prism, or *cylinder.*

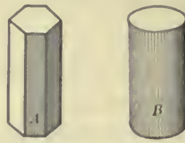
A solid may have *two* bases parallel polygons, and its faces trapezoids, *frustum of a pyramid.* If its bases are triangles, *frustum of a triangular pyramid;* . . . . circles, *frustum of a circular pyramid,* or *frustum of a cone.*

A solid may have one base and its faces triangles, a *pyramid.* If its base is a triangle, *triangular* pyramid; square, *square* pyramid; circle, *circular* pyramid, or *cone.*

A solid bounded by four surfaces is a *tetrahedron;* eight surfaces, an *octahedron;* twenty surfaces, an *icosahedron;* six squares, a *cube;* twelve surfaces, a *dodecahedron;* a curved surface every point of which is equally distant from the center, a *sphere.*

ILLUSTRATIONS

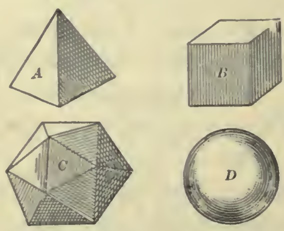
ABCD, Solid.
BCD, Base.
ACD, Face.
AC, Edge.
A, Vertex.



A, Hexagonal prism.
B, Cylinder.

A, Frustum of hexagonal pyramid.
B, Frustum of cone.

A, Hexagonal pyramid.
B, Cone.



A, Tetrahedron. *B, Cube.*
C, Icosahedron. *D, Sphere.*

The *entire surface* of a solid is the number of square units in all its surface.

The *convex surface* of a solid is all its surface but the bases.

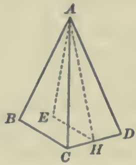
The *volume* of a solid is the number of cubic units in its contents.

The *altitude* of a cone or pyramid is the perpendicular distance from a vertex to the plane of the base; the *altitude* of a cylinder, prism, or frustum of a pyramid or cone is the perpendicular distance between its bases.

The *slant height* of a regular pyramid is the perpendicular distance from its vertex to one of the sides of its base.

Square unit.

Cubic unit.



The entire surface is $ABC + ACD + ABD + BCD$.

The convex surface is $ABC + ACD + ABD$.

The altitude is AE; AE is perpendicular to BDC.

The slant height is AH; the base of the face ACD is the line CD; AH is perpendicular to CD.

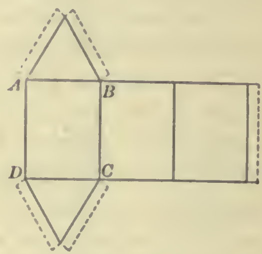
Define:

101. Solid.
102. Base of a solid.
103. Face of a solid.
104. Edge of a solid.
105. Vertex of a solid.
106. Prism.
107. Triangular prism.
108. Square prism.
109. Pentagonal prism.
110. Hexagonal prism.
111. Cylinder.
112. Frustum of a pyramid.
113. Frustum of a cone.
114. Pyramid.
115. Triangular pyramid.
116. Square pyramid.
117. Pentagonal pyramid.
118. Hexagonal pyramid.
119. Cone.
120. Tetrahedron.
121. Octahedron.
122. Icosahedron.
123. Hexahedron.
124. Dodecahedron.
125. Sphere.
126. Altitude of a pyramid.
127. Slant height.
128. Convex surface of a solid.
129. Entire surface of a solid.
130. Volume of a solid.

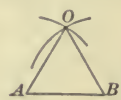
CONSTRUCTIONS

The pupil should make solids from pasteboard. Each form should be cut out from a single piece. It should then be folded and pasted.

131. Construct a triangular prism.

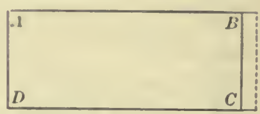


Draw two parallel lines. Lay off on each as many equal distances as the base is to have sides. Connect the points of division. Construct upon two opposite sides, as AB and DC, regular polygons. Prepare flaps for pasting as represented by the dotted lines. Cut entirely through outside lines and partly through inside lines. Fold, and paste the flaps on the inside.



To draw the Equilateral Triangle. With A and B as centers and a radius equal to AB, draw arcs of circles. These arcs will intersect at the vertex of the triangle; draw AO and OB.

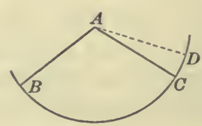
132. Construct a cylinder.



Proceed as in drawing the form for a prism; prepare the flap, roll over until AD coincides with BC, and paste on the inside.

Note. — No base is necessary. Light cardboard or paper should be used.

133. Construct a cone.



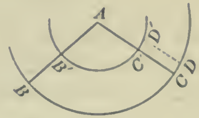
From any point as a center draw an arc of a circle. Lay off any convenient distance, as BC; draw AD for the flap. Cut entirely through the outside lines; roll the form until AB coincides with AC, and paste the flap on the inside.

Note. — No base is necessary. Light cardboard or paper should be used.

134. Construct a square prism. *See Ex. 137.*

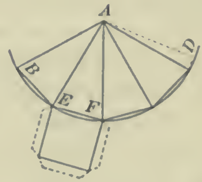
135. Construct an hexagonal prism. *See Ex. 138.*

136. Construct a frustum of a cone.



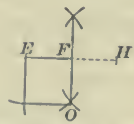
Proceed as in drawing the form for a cone, constructing two arcs from the same center. Cut out $BB'D'D$, roll over until BB' coincides with CC', and paste on the inside.

137. Construct a square pyramid.

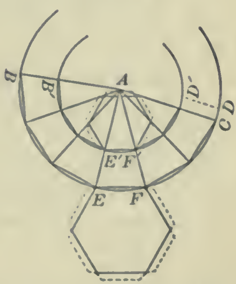


Proceed as in drawing the form for a cone. From B lay off as many equal distances as the base is to have sides. Connect the points of division with A. Construct on one of the sides, as EF, a regular polygon. Prepare flaps for pasting as represented by dotted lines. Cut entirely through outside lines and partly through inside lines. Fold, and paste the flaps on the inside.

To DRAW THE SQUARE. Prolong EF, and lay off FH equal to EF. From E and H as centers, with a radius greater than EF, draw arcs of circles and connect the points of intersection. Lay off FO equal to EF. From O and E as centers, with a radius equal to EF, draw arcs of circles; connect their intersection with O and E.

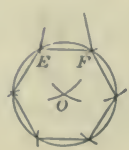


138. Construct a frustum of an hexagonal pyramid.



Proceed as in drawing the form for a frustum of a cone. Lay off on BD and $B'D'$ as for a pyramid. Construct on two sides, as EF and $E'F'$, regular polygons. Prepare flaps, cut and paste as before.

To DRAW THE REGULAR HEXAGON. From E and F as centers, draw arcs of circles with a radius equal to EF. From O, their point of intersection, as a center, with a radius equal to EF, describe a circle. From F, lay off on the circumference, distances equal to EF; connect the points thus found.



COMPUTATIONS — CONVEX SURFACES

XI. *The convex surface of a prism is the product of the perimeter of its base by its altitude.*

XII. *The convex surface of a cylinder is the product of the circumference of its base by its altitude.*

XIII. *The convex surface of a pyramid is half the product of the perimeter of its base by its slant height.*

XIV. *The convex surface of a cone is half the product of the circumference of its base by its slant height.*

XV. *The convex surface of a frustum of a pyramid is half the product of the sum of the perimeters of its two bases by its slant height.*

XVI. *The convex surface of a frustum of a cone is half the product of the sum of the circumferences of its two bases by its slant height.*

XVII. *The surface of a sphere is four times the square of its radius times 3.1416.*

NOTE. — For illustrations, explanations, and proofs, the pupil should turn to p. 261, and to pp. 275 and 276.

139. Find the convex surface of a cylinder, radius of the base 6 in., altitude 8 in.

Relation: $S = Cir. \times Alt.$

$S = 2 \times R \times 3.1416 \times Alt.$ *Conv. surf.* $= 301.5936$ *sq. in.*

$= 2 \times 6 \times 3.1416 \times 8 = 301.5936$

140. Find the convex surface of a square pyramid, one side of the base 4 in., slant height 12 in.

Relation: $S = \frac{1}{2} Per. \times S.\ H.$

$S = \frac{1}{2} \times 16 \times 12 = 96$ *Conv. surf.* $= 96$ *sq. in.*

141. Find the convex surface of a frustum of a cone, radius of upper base 6 in., radius of lower base 8 in., slant height 12 in.

Relation: $S = \frac{1}{2}(C + C') \times S.\ H.$

$S = \frac{1}{2}(2 \times 6 \times 3.1416 + 2 \times 8 \times 3.1416) \times 12$ *Conv. surf.* $= 527.7888$ *sq. in.*

$= 527.7888$

142. Find the radius of a sphere whose surface is 314.16 sq. ft.

Relation: $S = 4 \times R^2 \times 3.1416$

$314.16 = 4 \times R^2 \times 3.1416$

$R^2 = \frac{314.16}{4 \times 3.1416} = 25$

$R = 5$

Radius = 5 ft.

NOTE. — The relation is stated in its natural form, and the given terms are substituted in the relation.

Find the convex surface of:

143. A sq. pyr.; side of base, 6 in.; S. H., 4 in.

144. A cone; area of base 50.2656 sq. ft.; alt., 3 ft.

145. A cyl.; radius of base, 6 in.; alt., 4 in.

Find the surface of:

146. A sphere, whose radius is 6 in.

147. A sphere, whose diameter is 10 in.

148. A cube, whose diagonal is 12 in.

Find the:

149. Radius of a sphere whose surface is 201.0624 sq. in.

150. Circum. of a sphere whose surface is 804.2496 sq. ft.

151. Edge of a cube whose surface is 864 sq. ft.

152. Diagonal of a cube whose surface is 150 sq. ft.

153. Find the convex surface of a frustum of a square pyramid, one side of upper base 4 m, of lower base 8 m, slant height 24 m.

154. Find the convex surface of a frustum of a cone, radius of upper base 6 ft., of lower base 11 ft., altitude 12 ft.

155. What is the approximate area of the earth's surface, its diameter being nearly 8000 mi.?

156. What is the convex surface of a rectangular prism, base 8 ft. × 6 ft., altitude 10 ft.?

157. Find the cost, at 1¢ a sq. ft., of painting a church spire whose base is a pentagon, each side 6 ft., and whose slant height is 50 ft.

COMPUTATIONS — VOLUMES

XVIII. *The volume of a prism is equal to the product of the area of its base by its altitude.*

XIX. *The volume of a cylinder is equal to the product of the area of its base by its altitude.*

XX. *The volume of a pyramid is equal to one third the product of the area of its base by its altitude.*

XXI. *The volume of a cone is equal to one third the product of the area of its base by its altitude.*

XXII. *The volume of a frustum of a pyramid is equal to one third the product of the sum of the areas of its upper base, lower base, and mean proportional base, by its altitude.*

XXIII. *The volume of a frustum of a cone is equal to one third the product of the sum of the areas of its upper base, lower base, and mean proportional base, by its altitude.*

XXIV. *The volume of a sphere is equal to four thirds times the cube of its radius times 3.1416.*

Note. — For illustrations, explanations, and proofs, the pupil should turn to p. 122, p. 261, and pp. 276, 277, and 278.

158. Find the volume of a square prism, each side of the base 4 ft., altitude 12 ft.

Relation: $V = B \times Alt.$ *Volume = 192 cu. ft.*

$V = 16 \times 12 = 192$

159. Find the volume of a triangular pyramid, each side of the base 4 ft., altitude 12 ft.

Relation: $V = \frac{1}{3} B \times Alt.$ *Volume: 27.71 cu. ft.*

$V = \frac{1}{3}\sqrt{6 \times 2 \times 2 \times 2} \times 12 = 27.71$

160. Find the volume of a frustum of a cone, radius of the upper base 6 ft., radius of the lower base 8 ft., altitude 12 ft.

Relation: $V = \frac{1}{3}(B + B' + B'')\ Alt.$

$$36 \times 3.1416 = B$$
$$64 \times 3.1416 = B'$$
$$48 \times 3.1416 = B''$$
$$148 \times 3.1416 = \text{sum areas}$$
$$4$$
$$592 \times 3.1416 = 1859.8272$$
$$\text{Volume} = 1859.8272 \text{ cu. in.}$$

$$B'' = \sqrt{36 \times 3.1416 \times 64 \times 3.1416}$$
$$= 6 \times 8 \times 3.1416$$
$$= 48 \times 3.1416$$

It is better to extract the sq. rt. of the factors before performing the multiplication.

NOTE.—For meaning of *mean proportional*, *see p. 171, III.*

161. The volume of a sphere is 113.0976 cu. in. Find its surface.

Relation: $V = \frac{4}{3} \times R^3 \times 3.1416.$

$$113.0976 = \frac{4}{3} \times R^3 \times 3.1416$$
$$339.2928 = 4 \times R^3 \times 3.1416$$
$$R^3 = \frac{339.2928}{4 \times 3.1416} = 27$$
$$R = 3$$

Relation: $S = 4 \times R^2 \times 3.1416.$

$$S = 4 \times 9 \times 3.1416$$
$$\text{Surface} = 113.0976 \text{ sq. in.}$$

NOTE.—The relation is stated in its natural form, and the given terms are substituted in the relation.

Find the volume of:

162. A tri. prism; each side of base 4 in.; alt. 10 in.

163. A cylinder; radius of base 6 in.; alt. 10 in.

164. A tri. pyr.; each side of base 6 in.; alt. 12 in.

165. A cone; diameter of base 12 in.; alt. 15 in.

Find the required part:

166. The vol. cyl. 3141.6 cu. in.; alt. 10 in.; *D of base?*

167. The vol. cone 188.496 cu m; alt. 8 m; *area of base?*

168. The vol. cone 336 cu m; alt. 12 m; *D of base?*

169. The vol. of a sphere 904.7808 cu. in.; *D?*

170. The vol. of a sphere 33.5104 cu. in.; *S?*

171. The vol. of a cube 110592 cu. in.; *S?*

SIMILARITY

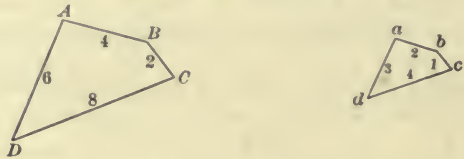
Two figures may be alike in form, *similar figures.* In order that figures may be similar, two conditions must be fulfilled:

For every angle of the one, there must be a corresponding equal angle in the other.

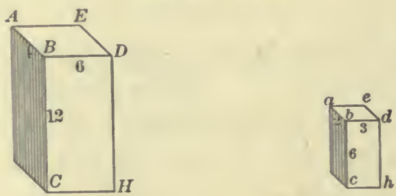
The sides about the equal angles must be proportional.

Similar Triangles

Similar Quadrilaterals



Similar Prisms



Similar Cones

XXV. *In similar figures, linear parts are to each other as homologous linear parts.*

XXVI. *In similar figures, surfaces are to each other as the squares of homologous linear parts.*

XXVII. *In similar figures, volumes are to each other as the cubes of homologous linear parts.*

172. In the similar triangles, find the ratio of AB to ab.

$\overline{AB} : \overline{ab} :: 3 : 1$ — Prin. xxv.

173. Find the ratio of area triangle ABC to triangle abc.

$ABC : abc :: 3^2 : 1^2$
$:: 9 : 1$ — Prin. xxvi.

174. In the similar cones, find the ratio of their surfaces.

$sur.\ A : sur.\ a :: 8^2 : 2^2$
$:: 16 : 1$ — Prin. xxvi.

175. In the similar cones, find the ratio of their volumes.

$vol.\ A : vol.\ a :: 8^3 : 2^3$
$:: 64 : 1$ — Prin. xxvii.

176. If the volume of the large prism is 288 cu. in., what is the volume of the smaller?

$vol.\ 1st : vol.\ 2d :: \overline{BD}^3 : \overline{bd}^3$
$288 : vol.\ 2d :: 6^3 : 3^3$
$$vol.\ 2d = \frac{288 \times 3^3}{6^3} = 36$$

In similar figures, volumes are to each other as the cubes of homologous linear parts.

177. The convex surface of a sphere is 100 sq. ft. What is the convex surface of a sphere whose radius is twice as long?

Long Way

Let x = radius small sphere
$2x$ = radius large sphere
$sur.\ small : sur.\ large :: x^2 : (2x)^2$
$100 : sur.\ large :: x^2 : 4x^2$
$$sur.\ large = \frac{100 \times 4x^2}{x^2} = 400$$

Short Way

Since the radius of the larger is twice the radius of the smaller, the surface of the larger must be the square of 2, or 4 times the smaller, or 400 sq. ft.

178. If a pipe 2 inches in diameter discharges 40 gal. per minute, how much will a pipe 3 inches in diameter discharge?

Section 1st : section 2d : : 2^2 : 3^2, or
40: discharge 2d : : 4 : 9
$$\textit{discharge 2d} = \frac{40 \times 9}{4} = 90$$

The amount of water discharged depends upon the area of a transverse section; this section is a circle.

179. If it costs $1200 to build a house 20 ft. by 30 ft., what will be the approximate cost of a house the same height, 40 ft. by 60 ft.?

Cost 1st : cost 2d : : 20×30 : 40×60
: : 1 : 4
Cost 2d = $4800

Since the heights are the same, the houses are not similar. The cost depends approximately upon the area of the base.

180. A and B buy a grindstone 4 in. thick and 2 ft. in diameter. A uses the stone until the diameter of the part left is 1 ft. If the whole stone cost $2, how much ought each to pay?

B's : whole : : 1^2 : 2^2
: : 1 : 4
B's = $\frac{1}{4}$ of $2 = 50¢
A's = $1.50

The whole and B's part are not similar because they have the same thickness. The cost of each part depends upon the area of the base.

All problems under Similarity may be solved in other ways; in some cases, it is best to neglect the rules on p. 269.

181. Find the radius of a circle having an area equal to the sum of the areas of two circles whose radii are 3 in. and 4 in.

Relation: area of a circle = $R^2 \times 3.1416$.

$$\text{Area 1st} = 9 \times 3.1416$$
$$\text{Area 2d} = 16 \times 3.1416$$
$$\text{Sum} = 25 \times 3.1416$$
$$\text{Sum} = R^2 \times 3.1416$$
$$\therefore R^2 \times 3.1416 = 25 \times 3.1416$$
$$R^2 = 25$$
$$R = 5 \text{ in.}$$

NOTE.—In many cases, it is wise to defer multiplying and dividing as long as possible. *See p. 72. See, also, Ex. 160, p. 267.*

In this example, it would be unwise to multiply 9 by 3.1416.

182. The volume of a cone whose altitude is 8 in., is 50.2656 cu. in. What is the volume of a similar cone whose altitude is 12 in.?

183. The convex surface of a frustum of a pyramid whose altitude is 12 in. is 432 sq. in. What is the altitude of a similar frustum whose convex surface is 3888 sq. in.?

184. The area of a circle is 10 sq. in. What is the area of a circle whose diameter is twice the diameter of the first?

185. Two lead pipes are respectively 1 inch and 2 inches in diameter. The area of a horizontal section of the larger is how many times a similar section of the smaller?

186. How many lead pipes 1 inch in diameter, will discharge as much water as one pipe 4 inches in diameter?

187. A cannon ball weighs 32 lb. What is the weight of a similar ball whose diameter is half the diameter of the first?

188. What is the ratio of the surface of the two balls in Ex. 187?

189. A is 6 ft. tall; his bronze statue is 12 ft. tall. If the length of A's little finger is $2\frac{1}{2}$ in., what is the length of the little finger of the statue?

190. If it costs $1 to paint a statue of A's size, how much will it cost to paint the statue in Ex. 189?

191. If a statue of A's size weighs 500 lb., how much will the statue in Ex. 189 weigh?

192. If a bin 6 ft. deep holds 60 bu., what are the contents of a similar bin 12 ft. deep?

193. The volume of a sphere is 100 cu. ft. What is the volume of a sphere whose surface is 16 times as great?

194. Four pipes, each 2 inches in diameter, empty into a tank. What must be the diameter of a single pipe to carry away all of the water?

195. A and B bought a ball of twine, 8 inches in diameter, for $1; A wound from the outside until the diameter of the part that was left was 4 inches. How much should each pay?

PROOFS AND ILLUSTRATIONS — LINEAR PARTS

I. *The circumference of a circle is equal to twice the radius times 3.1416.*

For proof, the pupil is referred to geometry. We may illustrate by measurement. Take a circle, as the base of a pail; with string and rule, measure its circumference; measure its diameter; divide the circumference by the diameter; the quotient will be 3.1416 approximately.

II. *The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.*

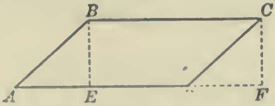
For proof, the pupil is referred to geometry. We may illustrate by measurement. Draw two lines at right angles; from the right angle, lay off 3 in. on one line, 4 in. on the other, and connect the extremities; by measurement, the hypotenuse will be 5 in.; $5^2 = 3^2 + 4^2$.

AREAS

III. *The area of a rectangle is equal to the product of its base by its altitude.*

This rectangle may be regarded as separated into strips by lines parallel to the base. The number of squares in each layer is the same as the number of linear units in the base; the number of squares in each strip multiplied by the number of strips, is the number of squares in the rectangle; ∴ the area of a rectangle is equal to the product of its base and its altitude.

IV. *The area of a parallelogram is equal to the product of its base by its altitude.*

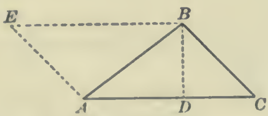


Let $ABCD$ be a parallelogram, and BE its altitude.

To prove that its area $= AD \times BE$.

Draw CF parallel to BE and prolong AD to F. The right triangles ABE and DCF are equal, because $AB = DC$, $BE = CF$, and $AE = DF$. From the whole figure, take away the triangle ABE; the rectangle $EBCF$ remains. From the whole figure, take away the triangle CDF; the parallelogram remains. Therefore, the parallelogram has the same area as the rectangle. The area of the rectangle $= BC \times BE$; $\therefore$ the area of the parallelogram $= AD \times BE$.

V. *The area of a triangle is equal to one half the product of its base by its altitude.*



Let ABC be a triangle, and BD its altitude.

To prove that its area $= \frac{1}{2} AC \times BD$.

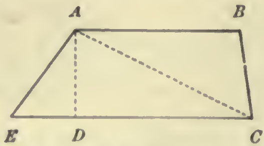
Draw BE parallel to AC, and EA parallel to BC. $EBCA$ is a parallelogram. Triangles ABE and ABC are equal, because $AB = AB$, $EA = BC$, and $EB = AC$. Therefore, triangle ABC is one half of the parallelogram. The area of the parallelogram is $AC \times BD$, therefore, the area of the triangle $ABC = \frac{1}{2} AC \times BD$.

VI. *The area of a triangle is equal to the square root of the continued product of the half sum of its sides and the remainders found by subtracting each side from the half sum separately.*

For proof, the pupil is referred to geometry. We may illustrate by finding the area of a right-angled triangle by each rule. The sides of a right-angled triangle are 3, 4, and 5. By the first rule, the area is $\frac{1}{2} \times 3 \times 4$, or 6; by the second, the half sum is $\frac{3+4+5}{2} = 6$; remainders are $6-3$, $6-4$, $6-5$, or 3, 2, 1; area is $\sqrt{6 \times 3 \times 2 \times 1}$, or 6.

AREAS

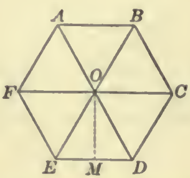
VII. *The area of a trapezoid is equal to one half the product of the sum of its parallel sides by its altitude.*

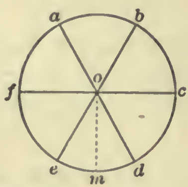


SUGGESTION. Area $AEC = \frac{1}{2} AD \times EC$; area $ABC = \frac{1}{2} AD \times AB$.

VIII. *The area of a regular polygon is equal to one half the product of its perimeter by its apothem.*

IX. *The area of a circle is equal to the square of its radius times 3.1416.*





Let $ABCDEF$ be a regular polygon, and OM its apothem.

To prove that its area $= \frac{1}{2} ABCDEF \times OM$.

From the center of the regular polygon draw lines to the vertices. They divide the regular polygon into equal triangles. The area of the regular polygon is the sum of the areas of the triangles. The area of a triangle $= \frac{1}{2}$ the product of its base by its altitude; the sum of the bases of the triangles is the perimeter of the polygon; the altitude (OM) of one of the triangles is the apothem of the polygon. Therefore, the area of the regular polygon is equal to $\frac{1}{2} ABCDEF \times OM$.

The area of the circle $= \frac{1}{2}\, abcdef \times om$; $abcdef = 2 \times R \times 3.1416$ (I.); $om = R$; $\therefore$ area of the circle $= \frac{1}{2} \times 2 \times R \times 3.1416 \times R = R^2 \times 3.1416$.

X. *The area of an irregular polygon is equal to the sum of the areas of the triangles into which it may be divided.*

The proof is self-evident. The pupil should draw an irregular polygon.

CONVEX SURFACES

XI. *The convex surface of a prism is the product of the perimeter of its base by its altitude.*

XII. *The convex surface of a cylinder is the product of the circumference of its base by its altitude.*

The faces of this prism are rectangles; the sum of the rectangles is the convex surface of the prism; the area of a rectangle is the product of its base by its altitude; ∴ the convex surface of a prism is the product of the perimeter of its base by its altitude.

The cylinder is a variety of the prism; ∴ the convex surface of a cylinder is the product of the circumference of its base by its altitude.

XIII. *The convex surface of a pyramid is half the product of the perimeter of its base by its slant height.*

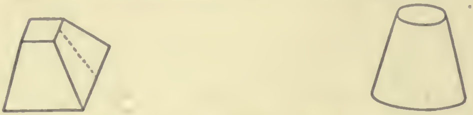
XIV. *The convex surface of a cone is half the product of the circumference of its base by its slant height.*

The faces of this pyramid are triangles; the sum of the triangles is the convex surface of the pyramid; the area of a triangle is half the product of its base by its altitude; the altitude of one of the triangles is the slant height of the pyramid; ∴ the convex surface of a pyramid is one half the product of the perimeter of its base by its slant height.

The cone is a variety of the pyramid; ∴ the convex surface of a cone is one half the product of the circumference of its base by its slant height.

XV. *The convex surface of a frustum of a pyramid is half the product of the sum of the perimeters of its two bases by its slant height.*

XVI. *The convex surface of a frustum of a cone is half the product of the sum of the circumferences of its two bases by its slant height.*



The faces of this frustum are trapezoids; the sum of the trapezoids is the convex surface of the frustum; the area of a trapezoid is the product of half the sum of its parallel sides and its altitude; the altitude of one of the trapezoids is the slant height of the pyramid; therefore, the convex surface of the frustum of a pyramid is half the product of the sum of the perimeters of its two bases by its slant height.

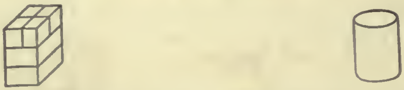
The frustum of a cone is a variety of the frustum of a pyramid; therefore, the convex surface of the frustum of a cone is half the product of the sum of the circumferences of its two bases by its slant height.

XVII. *The surface of a sphere is four times the square of its radius times 3.1416.*

The proof is too difficult for introduction here. See any good geometry.

VOLUMES

XVIII., XIX. *The volume of a prism or cylinder is equal to the product of the area of its base by its altitude.*



This prism may be regarded as separated into layers by planes parallel to the base. The number of cubes in each layer is the same as the number of squares on the surface of the base; the number of cubes in each layer multiplied by the number of layers is the number of cubes in the prism; therefore, the volume of a prism is equal to the product of the area of its base by its altitude. The cylinder is a variety of the prism; therefore, the volume of a cylinder is equal to the product of the area of its base by its altitude.

XX., XXI. *The volume of a pyramid or cone is equal to one third the product of the area of its base by its altitude.*

The proof is too difficult for introduction here, but the rule may be verified by making a hollow pyramid, and a hollow prism with equal bases and altitudes. It will be found that the pyramid will hold one third as much as the prism.

The cone is a variety of the pyramid; therefore, the volume of a cone is equal to one third the product of the area of its base by its altitude.

XXII. *The volume of a frustum of a pyramid is equal to one third the product of the sum of the areas of its upper base, lower base, and mean proportional base, by its altitude.*

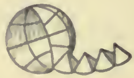
XXIII. *The volume of a frustum of a cone is equal to one third the product of the sum of the areas of its upper base, lower base, and mean proportional base, by its altitude.*

The proof is too difficult for introduction here, but the rule may be verified by making a hollow frustum of a pyramid, and three hollow pyramids; one having the lower base of the frustum, one the upper base, and one the mean proportional base, and all having the same altitude. It will be found that the frustum will hold as much as the three pyramids together.

The frustum of a cone is a variety of the frustum of a pyramid; therefore, the volume of a frustum of a cone is equal to one third the product of the sum of the areas of its upper base, lower base, and mean proportional base by its altitude.

VOLUMES

XXIV. *The volume of a sphere is equal to four thirds times the cube of its radius times 3.1416.*



A sphere may be regarded as made up of equal pyramids. The sum of the bases of the pyramids is the surface of the sphere; the altitude of each pyramid is the radius of the sphere; the sum of the volumes of the pyramids is the volume of the sphere. Therefore,

$$\text{The volume of a sphere} = \text{surface} \times \tfrac{1}{3} R.$$

(Since surface $= 4 \times R^2 \times 3.1416$.)

$$\text{Volume} = 4 \times R^2 \times 3.1416 \times \tfrac{1}{3} \times R, \text{ or } \tfrac{4}{3} \times R^3 \times 3.1416.$$

SIMILARITY

XXV.–XXVII. *The laws of similarity are deduced from many propositions in geometry.*

DIMENSIONS

XXVIII. *There can be only three dimensions.*

By this is meant, that only three lines can be drawn at a given point, each of which is perpendicular to each of the others. This truth is self-evident.

OCCUPATIONS

WITH THE LUMBER DEALER

The classification of lumber is not exact, but the following may prove helpful:

LUMBER — wooden building material.

I. *Boards* — 1 in. thick — less if specified.
   1. Stock boards; boards of uniform width, 12 in. wide.
   2. Fencing; 6 in. wide.
   3. Flooring; matched boards.
   4. Siding or clapboards; $\frac{1}{2}$ in. thick, thicker at one edge.

II. *Dimension Stuff* — more than 1 in. thick.
   1. Scantling; 2 in. to 4 in. thick, 3 in. to 4 in. wide.
   2. Joist; 2 in. thick, any width.
   3. Plank; 2 in. thick, wider than 4 in.
   4. Timber; thicker than 2 in., wider than 4 in.

III. *Foot Stuff* — sold by linear foot.
   1. Battens for covering cracks.
   2. Molding for finishing.

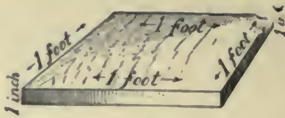
IV. *Laths* — 4 ft. long, $1\frac{1}{2}$ in. wide, 50 to a bundle.

V. *Shingles* — 4 in. wide, 250 to the bunch. They are not of uniform width, but every 4 in. is reckoned as one shingle.

Lumber is sold by the board foot.

A board foot is the equivalent of 1 ft. long, 1 ft. wide, and 1 in. thick.

Inch lumber 12 ft. long contains as many board feet as there are inches in its width.



1 board ft.

Because 12 ft. × $\frac{1}{12}$ ft. × 1 in. = 1 ft. × 1 ft. × 1 in.

How many feet of lumber are there in:

1. 1 8 in., 12 ft. board?
2. 1 8 in., 10 ft. board?
3. 1 8 in., 6 ft. board?
4. 1 8 in., 14 ft. board?
5. 1 8 in., 18 ft. board?
6. 3 6 in., 10 ft. boards?
7. 4 7 in., 16 ft. boards?
8. 1 joist 2 × 4, 12?
9. 1 joist 2 × 4, 16?
10. 1 scantling 3 × 4, 12?
11. 1 plank 2 × 10, 12?
12. 1 plank 2 × 10, 16?
13. 4 timbers 4 × 4, 20?
14. 3 pieces 8 × 8, 24?
15. 5 pieces 2 × 4, 12?
16. 8 3 in., 12 ft. boards?
17. 6 14 ft. fencing?
18. 6 pieces 4 × 6, 20?
19. 10 6 in., 12 ft. siding?
20. 10 scantlings 3 × 4, 16

Ex. **2**. 7. An 8 in. 12 ft. board would contain 8 ft.; $10 = 12 - \frac{1}{6}$ of 12; $8 - \frac{1}{6}$ of $8 = 6\frac{2}{3}$; taking the nearest whole number, 7.

Ex. **7**. 37. 4 7 in. boards = 1 28 in. board. A 28 in. 12 ft. board would contain 28 ft.; $16 = 12 + \frac{1}{3}$ of 12; $28 + \frac{1}{3}$ of $28 = 37\frac{1}{3}$; taking the nearest whole number, 37.

Ex. **9**. 11. This is read, "1 joist 2 by 4, 16." The joist is 2 in. thick, 4 in. wide, 16 ft. long.

Ex. **13**. 107. 4 4 × 4 pieces = 1 64 in. board. A 64 in. 12 ft. board would contain 64 ft.; $20 = 12 + \frac{2}{3}$ of 12; $64 + \frac{2}{3}$ of $64 = 106\frac{2}{3}$; taking the nearest whole number, 107.

NOTE.—Unless otherwise specified, stuff less than an inch thick is counted as an inch thick. *See Ex. 19.*

Dealers sometimes use a card like the following, carried out for a great variety of lengths and dimensions. Usually, as in this table, fractions of a foot are neglected.

Lumber Table

SIZE	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
2 × 6	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
2 × 8	16	19	21	24	27	29	32	35	37	40	43	45	48	51	53
3 × 4															
4 × 6															
2 × 4															
2 × 3															

21. Verify the results in the 1st horizontal line.

22. Verify the results in the 2d horizontal line.

23. Declare the results for the 3d horizontal line.

24. Declare the results for the 4th horizontal line.

25. Declare the results for the 5th horizontal line.

26. Declare the results for the 6th horizontal line.

How many board feet are there in:

27. 4 plk., 2 × 4, 12?

28. 2 timbers, 4 × 6, 40?

29. 9 joists, 2 × 3, 16?

30. 12 bds., 1 × 12, 16?

31. 24 fencing, 1 × 6, 16?

32. 4 piece stuff, 3 × 3, 12?

33. 6 scantlings, 2 × 4, 16?

34. 3 timbers, 6 × 6, 40?

35. 2 timbers, 8 × 8, 60?

36. 3 joists, 2 × 8, 12?

37. 6 plk., $1\frac{1}{8}$ × 12, 12?

38. 6 plk., $1\frac{3}{8}$ × 12, 12?

39. 6 plk., $1\frac{3}{4}$ × 12, 12?

40. 6 plk., $1\frac{7}{8}$ × 12, 12?

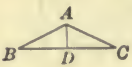
Note.—Count lumber less than 1 in. thick as 1 in.; from $1\frac{1}{8}$ to $1\frac{1}{4}$, as $1\frac{1}{4}$; from $1\frac{3}{8}$ to $1\frac{1}{2}$, as $1\frac{1}{2}$; from $1\frac{5}{8}$ to 2, as 2.

Boards, Planks, Scantling

41. To build 120 ft. of sidewalk, 4 ft. wide, I use 2×4 scantlings, (3 rows running lengthwise). How many feet of scantling will be required? How many feet of boards?

42. Which is the best length of boards in Ex. 41, 14, or 16 ft.? Which is the best length of scantling, 12, 14, or 16 ft.?

43. How many feet of stock boards (12 in. wide) will be required to cover a barn 30 ft. by 40 ft., 16 ft. posts, quarter-pitch roof, making no allowance for doors or windows?



Note. — A roof has a quarter-pitch when the height of the gable, *AD*, is ¼ of the width of the building, *BC*.

44. How many feet of boards, not counting waste, will cover both gables of a building 30 ft. wide, and having a quarter-pitch roof?

Battens

45. How many linear feet of battens, not counting the gables, will be required to cover the cracks in the barn of Ex. 43?

Note. — Battens at the corners are not necessary, but are generally used for appearance.

46. At $ 10 *per* M, what is the cost of the boards in Ex. 43?

Siding

47. If 6 in. clapboards are laid 4 in. to the weather, how many feet of clapboards, no allowance for waste, will be required to cover 1000 sq. ft. of surface?

48. How does your result agree with the rule, "*To the sq. ft. in the surface add $\frac{1}{2}$ the surface*"?

Matched Flooring

49. How many feet of 16 ft. 6 in. matched flooring, no allowance for waste, will be required for a room 16 ft. square?

Note. — The width of one piece of flooring is 6 in., but since this includes the tongue, one piece covers about 5 in. only.

50. How does your result agree with the rule, "*To the sq. ft. in the surface, add $\frac{1}{5}$ the surface*"?

Laths

51. If laths are laid $\frac{3}{8}$ of an in. apart, how many square inches (including the space between laths) does 1 lath cover?

52. How many square yards, no allowance for waste, will 1 bundle of lath cover?

53. How does this agree with the rule, "*One bundle of lath will cover 3 sq. yd.*"?

54. How many bundles of lath will be required for the ceiling and sides of a room $18 \times 20 \times 8$ ft., no allowance for openings?

Shingles

55. If shingles are laid 4 in. to the weather, no allowance for waste, how many square feet will 1 bunch of shingles cover?

56. How does this agree with the rule, "*4 bunches of shingles laid 4 in. to the weather will cover 100 sq. ft.*"?

57. If shingles are laid $4\frac{1}{2}$ in. to the weather, no allowance for waste, how many square feet will 1 bunch cover?

58. How does your result agree with the rule, "*900 shingles laid $4\frac{1}{2}$ in. to the weather will cover 100 sq. ft.*"?

59. How many bunches of shingles, laid 4 in. to the weather, will be required for the barn mentioned in Ex. 43, supposing the rafters to project 1 ft., and the roof to project 16 in. at each end?

Miscellaneous

60. How many linear feet of battens are required for a barn 40 ft. $\times$ 40 ft., the posts 22 ft., the pitch a quarter, the boards being 12 in. wide? Batten the gables.

61. How many board feet are there in 40 pieces 16 ft. siding?

62. How many feet of 6 in. matched flooring will be required for a room 16×20 ft.?

63. If lathing is laid $\frac{3}{8}$ of an inch apart, how many square yards will 4 bundles of lath cover?

64. If shingles are laid 4 in. to the weather, how many square feet will 4 bunches cover?

MEASUREMENT OF LOGS

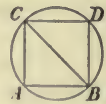
4 board feet.

By the number of board feet in a log, is meant the number of board feet in the largest piece of timber that can be sawed from the log.

A piece 1 in. × 1 in. × 12 ft. (usually written 1 × 1, 12) contains 1 board foot.

A log 12 ft. long contains as many board feet as there are sq. inches on its squared end, or as many board feet as there are sq. inches in half the square of its diameter.

If the log is to be sawed into boards, deduct $\frac{1}{5}$ for waste.



The area of the greatest inscribed square is $\overline{AB}^2$.

$$\overline{AB}^2 + \overline{AC}^2 = \overline{BC}^2$$

(sq. of hyp. = sum of sqs.)

$$2\,\overline{AB}^2 = \overline{BC}^2$$

$$\overline{AB}^2 = \frac{\overline{BC}^2}{2}$$

How many feet of lumber are there in a log:

65. Length 12 ft., D. 8 in.?
66. Length 16 ft., D. 10 in.?
67. Length 20 ft., D. 12 in.?
68. Length 24 ft., D. 10 in.?
69. Length 32 ft., D. 18 in.?
70. Length 16 ft., D. 12 in.?

Ex. **66**. 67 ft. of lumber. $10^2 = 100$; $\frac{1}{2}$ of $100 = 50$; if the log were 12 ft. long, there would be 50 board ft.; $50 + \frac{1}{3}$ of $50 = 67$.

How many feet of boards are there in a log:

71. Length 12 ft., D. 8 in.?
72. Length 18 ft., D. 10 in.?
73. Length 24 ft., D. 12 in.?
74. Length 20 ft., D. 6 in.?
75. Length 36 ft., D. 10 in.?
76. Length 40 ft., D. 16 in.?

Ex. **71**. 26 ft. There would be 32 ft. of lumber; $32 - \frac{1}{5}$ of $32 = 26$.

WITH THE CARPET DEALER

Carpeting is made of various widths, and sold by the linear yard.

To determine the number of yards of carpet for a room, decide whether the breadths shall run lengthwise or crosswise, find the number of breadths required and multiply by the length of a breadth.

NOTE. — Allowance should also be made for waste in matching.

77. How many yards of carpet $\frac{3}{4}$ of a yard wide, will be required for a room 16 ft. long and 13 ft. wide, the breadths running lengthwise, no waste in matching?

Ans. 32 yd. $\frac{3}{4}$ yd. $= 2\frac{1}{4}$ ft.; 13 ft. $\div 2\frac{1}{4}$ ft. $= 5\frac{7}{9}$; $\therefore$ 6 strips will be required; 16 ft. $\times 6 = 96$ ft., or 32 yd.

78. In Ex. 77, how many square yards must be turned under?

79. A floor 18 ft. by 16 ft. is to be covered with carpeting $\frac{3}{4}$ yd. wide. Which will be the more economical, to run the breadths lengthwise or crosswise, if there is a waste in matching of 6 in. on each breadth except the first? Why is there no waste on the first breadth?

80. At 75¢ per yard, how much will it cost to carpet a room 19 ft. by 15 ft. with carpeting $\frac{3}{4}$ yd. wide, the same figure recurring at intervals of 8 in., and the breadths running lengthwise?

81. You can find carpeting of the same grade which pleases you equally well, $\frac{3}{4}$ yd. wide at 75¢ per yd., or 1 yd. wide at $1 per yard, no waste in matching in either case. Your room is 16 ft. by 14 ft.; which way should the breadths run, and which width of carpet ought you to buy to carpet the room with least expense? What would be the least expense?

82. At 95¢ a yard, what will be the cost of carpeting a room 18×20 ft. with carpet 1 yd. wide, if the breadths run lengthwise, and there is a loss in matching of 8 in. to a breadth?

83. What will be the cost of carpeting the room mentioned in Ex. 82, with Brussels carpet $\frac{3}{4}$ yd. wide at $1.25? The breadths are to run crosswise, and there is to be no waste in matching.

WITH THE PAPER HANGER

Wall paper is sold by the *double roll*, 48 ft. × $1\frac{1}{2}$ ft., or by the *single roll*, 24 ft. × $1\frac{1}{2}$ ft. It is matched, cut up into strips, and pasted upon the walls or ceiling.

From the distance around the room in feet, deduct 3 ft. for each opening (door or window). The remainder ÷ $\frac{3}{2}$ *will give the number of strips required for the walls.*

The walls and ceiling of an 8 foot room, 20 ft. × 16 ft., are to be papered; there are four windows and a door.

84. How many strips for the walls will a double roll make? Explain.

Ans. 6 strips. There is a loss in matching, but this need not be considered, because the paper does not extend to the floor, on account of the *base board*, nor to the ceiling, on account of the border.

85. By the *rule*, how many double rolls will be needed for the walls? Explain.

86. If the strips run lengthwise, how many strips for the ceiling will a double roll make? Explain.

87. If the strips run crosswise, how many strips for the ceiling will a double roll make? Explain.

88. How many double rolls will be required for the ceiling, if the strips run lengthwise?

89. How many double rolls will be required for the ceiling, if the strips run crosswise?

90. What will be the cost of papering this room, walls and ceiling, at 36¢ a double roll, with border around the walls at 3¢ per linear foot?

91. Do we ever in practice find the exact number of square feet on the walls, deduct for the doors and windows, and then divide by the number of square feet in a double roll? Why not?

WITH THE MASON

Bricks are usually 8 in. × 4 in. × 2 in.

In estimating amount of work, masons measure the length of walls on the outside, thus counting the corners twice. They consider this just on account of the greater labor of construction. Except by special contract, no allowance is made for windows and doors.

In estimating amount of material, the corners are measured twice, but allowance for windows and doors must be made.

92. How many bricks, increasing the length and thickness $\frac{1}{4}$ in. for mortar, will lay 1 cubic foot?

Ans. $23\frac{3}{11}$. There are $8\frac{1}{4} \times 4 \times 2\frac{1}{4}$ cu. in. in 1 brick; $1728 \times \frac{4}{33} \times \frac{1}{4} \times \frac{4}{9} = 23\frac{3}{11}$, the no. of bricks.

NOTE. — Experience shows that 22 bricks 8 × 4 × 2 will lay 1 cu. ft.

93. How many bricks will be required to build the walls of a house 60 × 40 × 34 ft., outside measure, 2 bricks thick. There are 20 windows 6 × 3 ft.; and 6 doors, 6 × $3\frac{1}{2}$ ft.

200 = ft. around house

$200 \times 34 \times \frac{2}{3} = 4533\frac{1}{3}$ = *cu. ft. walls*

324 = *cu. ft. openings*

$4209\frac{1}{3}$ = *cu. ft. clear*

22

92605 = *no. of bricks*

$20 \times 6 \times 3 \times \frac{2}{3} = 240$ *cu. ft. windows*

$6 \times 6 \times 3\frac{1}{2} \times \frac{2}{3} = 84$ *cu. ft. doors*

324 *cu. ft. openings*

NOTE. — Wall is 2 bricks, or $\frac{2}{3}$ ft. thick.

94. For how many cubic feet would a mason charge for building the walls in Ex. 93?

Ans. $4533\frac{1}{3}$. He would not allow for openings.

95. If the mason laid the brick per M (by the thousand), for how many thousand would he charge?

Ans. 99.733 thousand. $4533\frac{1}{3} \times 22 \div 1000 = 99.733$. He would not allow for openings.

96. How much sand and lime will be required for the building?

NOTE. — $1\frac{1}{2}$ bbl. of lime and [illegible] cu. yd. of sand will lay 1000 bricks.

97. How many masons will be required to build the walls of the house in Ex. 93, in 11 days?

NOTE. — One mason will lay about 1800 bricks per day.

98. How many bricks will be required to build a house 25×80, 36 ft. high, if the walls are 3 bricks thick, and 400 cu. ft. is to be allowed for openings?

99. How much will it cost to lay the brick at $4.50 per M?

100. How many loads of sand will be required? How many barrels of lime? In how many days will six masons do the work?

101. If there are two chimneys 50 ft. high and 2 ft. square, outside measure, how many bricks will be required for the chimneys?

NOTE. — Consider each chimney solid.

102. How many bricks are required for the 4 in. walls of a cistern $12 \times 8 \times 8$ ft.?

103. How many bricks are required for the 4 in. walls of a cistern $24 \times 16 \times 16$ ft.?

104. In Ex. 103 why is the answer not twice as many bricks as in Ex. 102?

105. How many cords of stone are required for the foundation of the house in Ex. 98; the walls to be 10 ft. high and 2 ft. wide?

NOTE. — A cord of stone (128 cu. ft.) will lay 100 cu. ft.

106. For how many perch of stone would the mason charge in building the above foundation?

NOTE. — $24\frac{3}{4}$ cu. ft. make one perch.

107. How much sand and lime will be required for the foundation?

NOTE. — $1\frac{1}{4}$ bbl. of lime and 1 cu. yd. of sand will lay 100 cu. ft. of stone.

108. What would be the cost of plastering a room 40×30, 12 ft. high, at 30¢ a sq. yd.; 8 windows 3×6, 2 doors, 6×4?

NOTE. — Do not deduct for openings.

109. How much lime and sand would be required for two coats?

NOTE. — $3\frac{1}{2}$ bbl. of lime and $1\frac{1}{4}$ cu. yd. of sand will plaster 100 sq. yd., 2 coats.

WITH THE FARMER

Bins

110. How many bushels of wheat will a bin $12 \times 8 \times 6$ ft. contain?

Note. — Grain is sold by *struck measure;* 4 struck bu. = 5 cu. ft.

111. How many bushels of potatoes will a bin $12 \times 8 \times 6$ ft. contain?

Note. — Potatoes are sold by *heaped measure;* 4 heaped bu. = 5 struck bu.

112. How many bushels of shelled corn will a bin $12 \times 8 \times 6$ ft. contain? How much will it weigh?

Note. — Shelled corn is sold by the struck bushel; 56 lb. = 1 bu.

113. How many bushels of shelled corn may be obtained from a bin $12 \times 18 \times 6$ ft., filled with corn in the ear?

Note. — 3800 to 4000 cu. in. corn in ear make 1 bushel shelled corn. For small bins, 7 cu. ft. = 3 bu. shelled corn.

114. How many bushels of oats are there in a conical pile, altitude, 9 ft.; circumference of the base, 20 ft.?

115. A crib whose lower base is 8×10 ft., and whose upper base is 12×15 ft., depth, 12 ft., is filled with corn in the ear. How many bushels of shelled corn may be obtained from the crib?

116. How many bushels of shelled corn may be obtained from 2100 lb. of old corn in the ear? Of new?

Note. — Count 70 lb. of *old* corn, and 75 lb. of *new*, to the bushel.

117. How many bushels of potatoes are there in a conical pile, altitude, 8 ft.; diameter of the base, 8 ft.?

118. How many bushels of oats are there in a bin $10 \times 6 \times 4$ ft., if the bin is $\frac{3}{4}$ full?

119. A crib, whose lower base is 16×18 ft., upper base 24×27 ft., depth 12 ft., is filled with corn in the ear. How many bins, $12 \times 10 \times 6$ ft., will be required to hold the corn after it is shelled?

Wagon Boxes

120. A wagon box is 10 ft. × 3 ft. How many bushels of wheat will it hold for every inch in depth?

121. How many bushels of rye are there in a wagon box 10 × 3 ft., depth of rye, 12 in.?

Ans. 24 bu. rye. In Ex. 120, 2 bu. were found for each inch in depth.

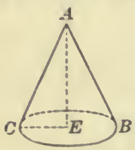
122. What must be the depth of a wagon box 10 × 3 ft. to hold 50 bu. of shelled corn?

Cisterns

123. What is the capacity, in barrels, of a cistern in the shape of two equal frustums of cones with their larger bases placed together; radius of smaller base 3 ft., radius of larger base 6 ft., altitude 9 ft.?

Hay

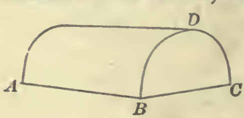
124. How many tons of hay are there in a conical stack, the circumference of base 60 ft., and distance over 40 ft., counting $(7)^3$ cu. ft. to the ton?



Relations: Circumference $= 2 \times CE \times 3.1416$; $AC^2 = CE^2 + AE^2$; vol. $= \frac{1}{3} \times CE^2 \times 3.1416 \times AE$.

Solution. $CE = 9.5$; $CA = 20$; $AE = 17.6$; vol. $= 1663.4$ cu. ft. *Ans.* 5 tons approximately.

125. How many tons of hay are there in a rick 40 by 20 ft., distance over 44 ft., counting 400 cu. ft. to the ton?



$AB = 40$ ft.
$BC = 20$ ft.
$BDC = 44$ ft.

Note. — Of the many different rules in use, the Government Rule is the best.

Government Rule. *To find the cu. ft. in a rick, multiply the length, by the breadth, by half the difference between the width and the distance over.*

Plowing

126. With a 16 in. plow, not counting distance in turning, how far will a team walk in plowing 1 acre?

127. With a 14 in. plow, not counting distance in turning, how far will a team walk in plowing 1 acre?

Yield per Acre

128. If corn is planted in rows 3 ft. 8 in. apart, each way, how many hills are there to an acre, 20 rd. × 8 rd.?

129. If each hill of corn yields 1 ear, and 150 ears make a bushel, what is the yield per acre?

130. How do your results agree with the rule, "*One ear of corn to the hill yields 20 bu. to the acre*"?

Fencing

131. How many rods of fence will inclose an acre in the form of a rectangle 1 rd. × 160 rd.?

132. How many rods of fence will inclose an acre in the form of a square?

133. How many rods of fence will inclose an acre in the form of a circle?

Miscellaneous

134. In an apple orchard, the trees are planted 33 ft. apart each way. How many trees are there in the orchard, if it is 20 rd. long, and contains 2 A.?

135. If each apple tree yields 6 bu., what must be the length of a bin 10 ft. wide, 5 ft. deep, to contain the crop?

136. Approximately, how many barrels of water will a tank 8 × 7 × 3 ft. contain? In what time will it be filled by a pipe which discharges 10 gal. an hour?

MISCELLANEOUS

LONGITUDE AND TIME

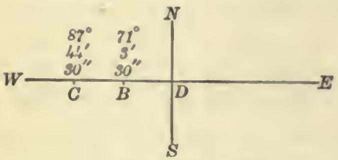
I. *At places on the earth's surface it is earlier farther west.*

When the sun is rising at a place, it has not yet risen at places farther west. Where the sun has not yet risen it is earlier; therefore, it is earlier farther west.

II. *The sun appears to pass through 360° in 24 hours.*

There are 360° in the earth's circumference and 24 hours in a day.

1. The longitude of Boston is 71° 3′ 30″ west, and that of Chicago 87° 44′ 30″ west. What is their true difference in time?



87° 44′ 30″
71° 3 30
16 41

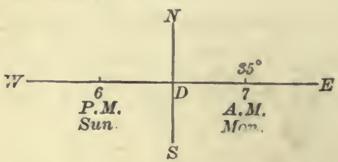
Ans. 1 hr. 6 min. 44 sec.

Meaning. Boston is 71° 3′ 30″ west of the meridian of Greenwich; Chicago, 87° 44′ 30″ west.

Solution. Let *NS* represent the meridian of Greenwich; locate Boston and Chicago on the diagram. The difference in longitude between *B* and *C* is *CD* – *BD*, or 16° 41′.

Since the sun appears to move through 360° in 24 hr., 16° 41′ = 1 hr. 6 min. 44 sec. *See p. 132, Ex. 201.*

2. At a place whose longitude is 35° E., the time is 7 A.M. Monday. What is the longitude of a place where it is 6 P.M. Sunday?



We locate the place 35° E.

Since 6 P.M. Sunday is earlier, the other place must be farther west. We locate it.

From 6 P.M. Sunday to 7 A.M. Monday, is 13 hours; 13 hr. = 195°. *See p. 132, Ex. 202.*

The other place is 195° west of the given place, or 160° west of the prime meridian, or 160° west longitude.

Draw a diagram for each problem.

3. The longitude of St. Petersburg is 30° 19′ east. What is the difference in longitude between St. Petersburg and Boston?

4. What is the true difference in time between Boston and St. Petersburg?

5. At a place whose longitude is 35° 17′ 25″ E., the time is 7.30 A.M. What is the longitude where it is 11.55 P.M.?

6. At a place 37° 25′ 37″ east longitude, the time is 7.30 A.M. What is the time at a place whose longitude is 37° 25′ 37″ W.?

STANDARD TIME. — In 1883, the standard time system was introduced by the railroads and adopted by the people of the United States.

By this system, time meridians 15° apart have been established; viz., the meridians 60°, 75°, 90°, 105°, 120° west from Greenwich. Places within 7½° east or west of a time meridian have the time of the time meridian. Thus, the time between two places differs by *whole* hours, or not at all.

7. What is the difference in standard time between Boston and Chicago?

NOTE. — Boston has the time of the meridian of 75°; Chicago, of the meridian of 90°.

INTERNATIONAL DATE LINE. — In traveling completely around the earth from west to east, one appears to gain a day; in traveling from east to west, to lose a day.

Hence, vessels in sailing around the earth must correct their apparent days. It is customary to make the change at the meridian of 180° longitude. Vessels going east, count the same day twice; vessels going west, skip a day. Thus, if it is Sunday when an east-bound vessel reaches the meridian of 180°, it counts the next day Sunday also; if it is Sunday when a west-bound vessel reaches this meridian, it counts the next day Tuesday. For this reason, the meridian of 180° is called the *International Date Line*.

8. A traveler found that his watch was gaining time. If his watch was accurate, was he traveling east or west?

9. If a man were to travel around the earth from west to east in 101 days, reckoning by local time at the various places, in how many days would he actually make the trip?

PROBLEMS IN PHYSICS

The *specific gravity* of a substance is its weight divided by the weight of an equal volume of water.

Illustration: A certain volume of lead weighs 34 lb.; the same volume of water, 3 lb.; the specific gravity of lead is 34 ÷ 3, or 11.3. That is, lead is 11.3 times as heavy as water.

10. A bar of iron weighs 38.5 lb.; an equal volume of water weighs 5 lb. What is the specific gravity of the iron?

11. What is the weight of 1 cubic foot of the iron?

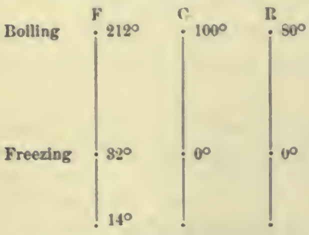
12. A block of wood weighs 15 lb.; the amount of water that it displaces weighs 25 lb. Find its specific gravity.

13. What is the weight of 1 cubic foot of the wood?

14. A rock on the bank of a river weighs 2 T.; but in the water it weighs only 1 T. 4 cwt. What is its specific gravity?

In the Fahrenheit thermometer, the freezing and boiling points are 32° and 212°, respectively; in the Centigrade, 0° and 100°; in the Réaumur, 0° and 80°.

Reduce and explain by diagram, 14° F to C; to R.



180° F = 100° C = 80° R
1° F = $\frac{5}{9}$° C = $\frac{4}{9}$° R
18° F = 10° C = 8° R

When the thermometer reads 14° F, the temperature below freezing is 18° F, 10° C, or 8° R, *i.e.*, 14° F = − 10° C, or − 8° R.

15. The temperature of a room is 68° F. Find the temperature in C; in R.

16. Lead melts at 335° C. Find its melting point in F.

17. Silver melts at 800° R. Find its melting point in C.

18. Alcohol boils at 78° C. Find its boiling point in F and in R.

19. On a certain day the temperature fell from 86° F to 5° C. How many degrees R did it fall?

ACCOUNTS

Bookkeeping is a systematic method of recording business transactions. Many or few books may be used. Accounts may be kept with many or with few things. The underlying principle is simple: *Everything which owes the proprietor is a debtor* (Dr.); *everything the proprietor owes is a creditor* (Cr.).

From this general rule, special rules may be derived:

PERSONS. *Dr. when he owes me, or I get out of his debt; Cr. when I owe him, or he gets out of my debt.*

CASH. *Dr. all cash receipts; Cr. all cash disbursements.*

EXPENSE. *Dr. all outlays from which no direct return is expected.*

PROPRIETOR. *Dr. everything withdrawn; Cr. everything put into the business.*

In the following, only two books are used, — the daybook and the ledger; accounts are kept with only four things, — Cash, Expense, the Proprietor, and Persons. At the time of each transaction, William Harris or his clerk entered it on the daybook, as on p. 296; each day the transactions were posted from the daybook to the ledger, as on p. 297.

20. Enter the following in daybook and ledger.

Jan. 1, 1897. William Harris began a grocery business with $ 2000 cash and $ 2000 mdse. Paid rent for January by bank draft, No. 560, $ 50. Books and stationery, $ 15. Cash sales, $ 5.

Jan. 2. Sold A. D. Russel, on account: 300 lb. sugar @ 6¾¢; 25 lb. coffee @ 30¢; 180 cwt. flour @ $ 2.50. Cash sales, $ 67.50.

Jan. 4. Bought of A. D. Russel, on %: horse and wagon for use in the business, $ 175. Cash sales, $ 25.

Jan. 5. Rec'd from A. D. Russel, check on First Nat'l Bank, for $ 250, to apply on his %. Withdrew for private use $ 100. Cash sales, $ 16.50.

Jan. 6. Sold A. D. Russel 25 lb. pork @ 11¢; 5 gal. molasses @ 25¢; 5 cwt. flour @ $ 2.80, and accepted in payment an order on Geo. Adair, for $ 18, which Adair paid in cash. Cash sales, $ 22.50.

Jan. 7. Paid cash for 1 T. coal for store, $ 5. Cash sales, $ 30.

DAYBOOK *Jan. 1, 1897*

√	*Wm. Harris began a grocery business with*				
√	*Cash,*	2000	00		
√	*Mdse.,*	2000	00	4000	00
√√	*Paid Jan. rent by bank draft No. 560,*	50	00		
√√	*Books and Stationery,*	15	00	65	00
√	*Cash Sales,*			5	00
	2.				
√	*Sold A. D. Russel, on %:*				
	300 lb. sugar, @ .06⅔	20	00		
	25 lb. coffee, .30	7	50		
	180 cwt. flour, 2.50	450	00	477	50
√	*Cash Sales,*			67	50
	4.				
√√	*Bot. of A. D. Russel, on %:*				
	Horse and Wagon for use in the business,			175	00
√	*Cash Sales,*			25	00
	5.				
√√	*Rec'd check from A. D. Russel, on %,*			250	00
√√	*Withdrew for private use,*			100	00
√	*Cash Sales,*			16	50
	6.				
√	*Sold A. D. Russel, for order on Geo. Adair, which Adair has cashed,*				
	25 lb. pork, @ .11	2	75		
	5 gal. molasses, .25	1	25		
	5 cwt. flour, 2.80	14	00	18	00
√	*Cash Sales,*			22	50
	7.				
√√	*Bot. for cash, to use in store, 1 T. coal,*			5	00
√	*Cash Sales,*			30	00
	7.				
√	*Found net gain to date to be*	67	00		
	Carry to Cr. side proprietor's %.				

LEDGER *Dr.* *Wm. Harris* *Cr.*

Jan.	5		296	100	00	Jan.	1		296	4000	00
"	7	**Present Worth**		**3967**	**00**	"	7	Net Gain	296	67	00
				4067	00					4067	00
						Jan.	7	Present Worth		3967	00

Cash

Jan	1		296	2000	00	Jan.	1		296	65	00
"	1		296	5	00	"	5		296	100	00
"	2		296	67	50	"	7		296	5	00
"	4		296	25	00					170	00
"	5		296	250	00	Jan.	7	**Balance**		**2264**	**50**
"	5		296	16	50					2434	50
"	6		296	18	00						
"	6		296	22	50						
"	7		296	30	00						
				2434	50						
				2434	50					2434	50
Jan.	7	Balance		2264	50						

Expense

Jan.	1		296	65	00						
"	4		296	175	00						
"	7		296	5	00						
				245	00						

A. D. Russel

Jan.	2		296	477	50	Jan.	4		296	175	00
						"	5		296	250	00
										425	00

NOTE. — To find the proprietor's worth, find the total resources and liabilities, and take the difference. At this time the ledger shows resources: Cash, $2264.50 and A. D. Russel, $52.50. The inventory of Mdse. found by estimating the value of goods on hand is $1750. These resources aggregate $4067. As there are no liabilities, this would be the proprietor's present worth had he not withdrawn $100. The net gain of $67 is carried to the credit side of proprietor's acct., and then the difference between the sides shows his present worth to be $3967.

Accounts may be balanced, as shown in Cash acct., when they become very large. The small type footings are written with a pencil.

21. Turn to p. 297. How much did Wm Harris invest? How much did he withdraw up to Jan. 7?

22. What was the amount of cash on hand Jan. 7? Can the Cr. side of "Cash" exceed the Dr.?

23. What were the expenses to Jan. 8? How much did A. D. Russel owe Jan. 7?

24. How is the proprietor's worth found? How is the net gain or net loss found?

25. Continue the daybook of p. 296, entering the following transactions.

26. Continue the ledger of p. 297, posting from daybook just completed; balance the accounts as before.

27. Make out and receipt Henry Cook's bill.

28. Make out A. D. Russel's bank check.

Jan. 8. Sold Henry Cook, on %: 25 lb. sugar @ 6¢; 6 cans peaches @ 12½¢; 2 bu. potatoes @ 80¢. Cash sales, $27.30.

Jan. 9. Bot. for cash: 3 bbl. sugar @ $25; 2 bags coffee, 200 lb. @ 20¢; 3 cases canned goods, 6 doz. at $1.20.

Jan. 11. Sold A. D. Russel, on %: 7 jars pickles @ 25¢; 14 doz. clothes-pins @ 5¢; 2 lb. candy @ 10¢. Cash sales, $34.

Jan. 12. Sold A. J. Davis, on %: 20 lb. sugar @ 6½¢; 17 lb. beans @ 5¢; 3 lb. cheese @ 15¢; 15 lb. dried beef @ 18¢. Cash sales, $23.25.

Jan. 13. Bot. office desk and fixtures for $30.

Jan. 15. Sold A. D. Russel, on %: 20 lb. beans @ 5¢; 15 lb. coffee @ 30¢; 14 cans tomatoes @ 10¢. Cash sales, $19.

Jan. 16. Bot. for cash: 4 kegs fish @ $3; 1 keg vinegar, 20 gal. @ 15¢. Cash sales, $18.

Jan. 18. Rec'd from A. D. Russel, on %, check for $100. Sold Henry Cook, on %: 3 gal. vinegar @ 25¢; 20 lb. sugar @ 8½¢; 2 lb. tea @ 65¢. Cash sales, $21.15.

Jan. 19. Withdrew for private use, $25. Cash sales, $34.50.

Jan. 20. Rec'd from Henry Cook, cash in full of his %. Sold Jas. Otis, on %: 50 cwt. flour @ $3; 5 gal. kerosene @ 15¢; 1 doz. cans apples @ 15¢. Cash sales, $30. Found net gain from Jan. 7 to Jan. 21, $100.

FORM OF BILL

Emporia, Kans., Jan. 5, 1897.

Wm Harris, Grocer,

Sold to A. D. Russel,

Terms: Cash. 1018 Market Street.

1897 Jan.	6	25 lb. pork @ 11¢	2	75		
		5 gal. molasses @ 25¢	1	25		
		5 cwt. flour @ $2.80	14	00	18	00

Rec'd Payment,

Wm Harris.

FORM OF BANK DRAFT

The First National Bank of Emporia. No. 560.

Emporia, Kans., Jan. 1, 1897.

Pay to the order of Wm Harris

$50.00 Fifty $\frac{00}{100}$ Dollars.

To First National Bank,
Kansas City, Mo.

Harry Brown,
Cashier.

FORM OF BANK CHECK

No. 98.

Emporia, Kans., Jan. 5, 1897.

The First National Bank.

Pay to the order of Wm Harris

Two hundred fifty $\frac{00}{100}$ Dollars.

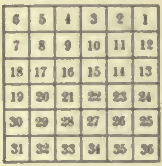
A. D. Russel.

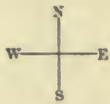
$250.00.

GOVERNMENT DIVISIONS OF LAND

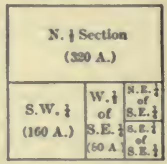
In the western states, the land has been divided by the United States government into squares 6 mi. each way, called *townships;* each township has been divided into squares 1 mi. each way, called *sections;* many sections have been divided into 4 equal squares, called *quarter sections;* and some quarter sections have been divided into 4 equal squares, called *quarter quarter sections.*

A TOWNSHIP





A SECTION



The sections forming a township are numbered from the N. E. corner as shown in the left-hand diagram.

The subdivisions of a section are named as shown in the right-hand diagram. A division smaller than a quarter quarter section is called a *lot.*

29. Draw a north and south line to represent 1 mi. Upon this line construct a square, and divide it into 2 equal parts by an east and west line. Upon the south part, write its name. How many rods long is a half section? How many rods wide? How many acres does it contain?

30. Divide the N. $\frac{1}{2}$ section into 2 equal squares by a north and south line. Upon the N. E. square, write its name. What are the dimensions in rods of a quarter section? How many acres does a quarter section contain?

31. Divide the N. W. $\frac{1}{4}$ section into 2 equal parts by a north and south line. Upon the east part, write its name. What are the dimensions and the area of a half quarter section?

32. Divide the west half of the N. W. quarter into 2 equal squares by an east and west line. Upon each part, write its name.

DEFINITIONS AND INDEX

NOTE. ∠, angle; ⊥, perpendicular; △, triangle; ⊙, circle; ∥, parallel.

TERM	P.	DEFINITION
Cipher	40	Naught; zero.
Circle	252	A plane figure bounded by a curved line every point of which is equally distant from a point within called the center.
Circulating decimal	110	Decimal containing a repetend.
Circumference	253	The boundary line of a ⊙.
Coefficient	145	A number showing how many times a base is used as an addend.
Commission	198	An agent's pay for buying or selling.
Common denominator	88	A denominator common to two or more fractions.
Common factor	72	A factor common to two or more numbers.
Common fraction	83	A fraction whose denominator is written below a horizontal line.
Common multiple	76	A number which will exactly contain each of several numbers.
Complex decimal	105	A decimal ending in a common fraction, as .33⅓.
Complex fraction	83	A fraction having fractions in one or both of its terms.
Composite number	68	A number having another set of factors besides itself and 1.
Compound fraction	83	A fraction of a fraction.
Compound interest	216	A conception of interest by which prin., int. on prin. at end of each yr., and all other int., bear interest.
Compound proportion	174	A proportion having one or both of its ratios compound.
Compound ratio	174	A ratio of a ratio.
Cone	260	A pyramid whose base is a ⊙.
Consequent	170	Second term of a ratio.
Convex surface of a solid	261	All its surface except the bases.
Creditor	295	One to whom something is owing.
Cube	238 260	Product of three equal factors; a solid having its three dimensions equal.
Cube root	244	One of the three equal factors of a number.
Curved line	249	A line which constantly changes its direction.
Curved surface	251	A surface such that no two points lie in same plane.
Cylinder	260	A prism whose base is a ⊙.
Debtor	295	One who owes a debt.
Decagon	252	A polygon of ten sides.
Decimal	103	A fraction written with the aid of a decimal point.

TERM	P.	DEFINITION
Decimal fraction	103	A fraction whose denominator is 10, 100, 1000, ...
Decimal point	103	A period written before tenths in a decimal.
Denominate number	119	A number which answers the question 'how much?'
Denominator	83	The part of a fraction which is a divisor; the number showing into how many parts a unit is divided.
Diagonal	253	A line in a polygon joining two vertices not adjacent.
Diameter	254	A line through the center of a ⊙ joining two points of the circumference.
Difference	28 190	Result obtained in subtraction; base minus percentage.
Discount	201 207	A deduction from the face value of a commercial paper; the amount the market value of a stock is below the par value.
Dividend	44 207	A number to be divided; earnings of stock.
Division	44	Process of finding the other of two numbers when one of them and their product are given.
Divisor	44	A number by which to divide.
Dodecahedron	260	A solid bounded by twelve faces.
Draft	299	A written order directing one person to pay a sum of money to another.
Drawee	223	The person to whose order a draft or note is to be paid.
Drawer	223	The person who signs a draft or note.
Edge	260	The intersection of two faces of a polygon.
Equation	159	An expression of equality.
Equation of payments	231	Process of finding an average time at which several payments may be justly made.
Equilateral triangle	251	A △ whose sides are equal.
Equivalent fractions	87	Fractions equal in value.
Evolution	239	Process of finding a root of a number.
Exponent	146	A number denoting how many times the base is used as a factor.
Extremes	170	First and last terms of a proportion.
Faces of a solid	260	Its bounding planes.
Factor	68	An exact divisor of a number.
Factoring	68	Process of finding numbers whose product is given.

TERM	P.	DEFINITION
Figures	7	Symbols representing numbers; diagrams representing forms.
Fraction	83	An expression of division in which the dividend is written above and the divisor below a horizontal line; one or more of the equal parts of a unit.
Frustum	260	The portion of a cone or a pyramid included between its base and a plane ∥ to the base.
Grace, days of	228	An allowance of three days after a note is due for its payment.
Gram	141	Metric unit of weight.
Greatest common divisor.	74	The greatest number that will exactly divide each of several numbers.
Heptagon	252	A polygon of seven sides.
Hexagon	252	A polygon of six sides.
Horizontal line	250	A straight line ∥ to the horizon.
Hypotenuse	253	The side of a right-angled △ opposite the right angle.
Icosahedron	260	A solid bounded by twenty faces.
Improper fraction	84	A fraction whose value is 1, or more than 1.
Indorsement	223	The signature on the back of a note.
Insurance	202	Agreement to pay for loss or damage.
Integer	7	A whole number.
Interest	216	Money paid for the use of money.
International date line	293	Boundary line between regions where the calendar day is different, coinciding approximately with meridian of 180°.
Involution	238	Process of finding a power of a number.
Irregular polygon	252	A polygon whose angles are unequal.
Isosceles triangle	251	A △ having two of its sides equal.
Least common multiple	77	The least number that will exactly contain each of several numbers.
Least common denominator	76	The least common multiple of several denominators.
Liabilities	297	Debts to be paid.
Line	249	That which has one dimension.
Liter	140	Metric unit of capacity.
Literal quantity	145	A quantity represented by letters.
Long division	53	Division in which processes are written out.
Longitude	292	Distance E. or W. of the prime meridian, as measured on the equator.
Market value	207	Price of stock in the market.
Maturity of a note	223	Date at which a note legally becomes due.

TERM	P.	DEFINITION
Mean proportional	170	One of the means of a proportion in which the means are equal.
Means	170	Second and third terms of a proportion.
Measures of numbers	68	Factors.
Members	60	Parts of equations connected by the sign =.
Mensuration	249	Science of measurement.
Meter	136	Metric unit of linear measure.
Metric system	136	A decimal system of measurement, whose principal unit is the meter.
Minuend	28	Number in subtraction to be diminished.
Minus sign	145	Sign which denotes subtraction, or the opposite of '+.'
Mixed number	83	An integer plus a fraction.
Multiple	68, 76	Number which contains another an integral number of times.
Multiplicand	35	Number to be multiplied.
Multiplication	35	Process of finding the sum when the same number is used several times as an addend.
Multiplier	35	Number by which to multiply.
Negative quantity	145	A quantity preceded by the minus sign.
Net proceeds	198	Sum left after every charge is paid.
Nonagon	252	A polygon of nine sides.
Notation	7	Art of writing numbers by symbols.
Note	223	Written promise to pay a sum of money at a stated time.
Number	7	Unit or collection of units.
Numerator	84	The part of a fraction which is the dividend; the number showing how many parts are taken.
Numeration	7	Process of naming or reading numbers.
Numbers prime to each other	68	Numbers having no common factor greater than 1.
Numbers severally prime	68	Numbers prime, each to each of the others.
Oblique line	250	Line neither ⊥ nor ∥ to the horizon.
Oblique angle	250	An ∠ formed by two lines which meet so as to make the adjacent ∠s unequal.
Obtuse angle	250	An ∠ greater than a rt. ∠.
Obtuse-angled triangle	251	A △ which has one obtuse ∠.
Octagon	252	A polygon of eight sides.
Octahedron	260	A solid bounded by eight faces.
Order	8	One of the three places in a period.
Parallel lines	249	Lines in the same plane which do not meet however far they may be produced.

TERM	P.	DEFINITION
Parallelogram	251	A quadrilateral having both pairs of its opposite sides parallel.
Parenthesis	60	Curved marks inclosing one or more quantities considered as a whole.
Par value	206	Face or nominal value.
Partial payments	223	Payments in installments of notes, etc.
Payee	223	The person to whom a note is payable.
Pentagon	252	A polygon of five sides.
Per cent	190	Hundredths.
Percentage	190	Computations with per cent.
Perimeter	253	Distance around a polygon.
Periods	7	In numeration, groups of 3 figures each; in evolution, groups of any number of figures.
Perpendicular line	250	A line which meets another so as to make the adjacent ∠s equal.
Plane	251	A surface such that a straight line joining any two points in it, lies wholly within that surface.
Plus sign	145	Sign which denotes addition or a positive quantity.
Point	249	That which has no dimension.
Policy	202	A contract between an insurance company and the person insured.
Polygon	251	A plane surface inclosed by straight lines.
Poll tax	200	A tax levied at so much per head.
Positive quantity	145	A quantity preceded by the plus sign.
Power	238	A product obtained by using the same number several times as a factor.
Premium	202	Sum paid for insurance; excess of market value above par value.
Present worth	227	Present value of a debt due at a future time.
Prime meridian	292	The circle passing through the poles of the earth and through Greenwich, Eng.
Prime number	68	A number which has no integral factor besides itself and 1.
Principal	198 216	One who employs an agent; a sum on which interest is reckoned.
Principle	77	A fundamental truth.
Prism	260	A solid having its two bases equal and ∥ polygons, and its faces rectangles.
Problem	26	An example in which the operations are not stated.

TERM	P.	DEFINITION
Proceeds	228	Difference between face value of a note and the discount. See "net proceeds."
Product	35	Result in multiplication.
Proof	79	Operation checking accuracy of a calculation.
Proportion	170	An equality of ratios.
Pyramid	260	A solid having one base, and its faces △s.
Quadrant	254	A sector equal to $\frac{1}{4}$ of a ⊙.
Quadrilateral	251	A plane figure bounded by four straight lines.
Quotient	44	Result in division.
Radius	254	A straight line drawn from the center of a ⊙ to the circumference.
Ratio	170	An expression of division in which the dividend is written before, and the divisor after, a colon.
Rectangle	251	A parallelogram whose ∠s are all rt. ∠s.
Reduction	85	Process of changing the form of an expression without changing its value.
Regular polygon	252	A polygon which is equiangular and equilateral.
Remainder	28 48	The result in subtraction; the part left undivided in division.
Repetend	110	A figure or set of figures in a circulating decimal which repeat.
Rhomboid	251	A parallelogram whose ∠s are not rt. ∠s.
Rhombus	251	A rhomboid whose sides are equal.
Right angle	250	An ∠ formed by two lines meeting so as to make the adjacent angles equal.
Right-angled triangle	251	A △ containing 1 rt. ∠.
Roman notation	15	Method of expressing numbers by means of the characters, I, V, X, L, C, D, M.
Root	239	One of the equal factors of a number.
Scalene triangle	251	A △ which has no two of its sides equal.
Secant	254	A straight line cutting the circumference of a ⊙ in two points.
Sector	254	A portion of a circle between two radii and their included arc.
Segment	254	A portion of a ⊙ between a chord and its arc.
Semicircle	254	A segment which is $\frac{1}{2}$ of a ⊙.
Sextant	254	A sector which is $\frac{1}{6}$ of a ⊙.
Share	206	One of the equal parts into which a capital stock is divided.

TERM	P.	DEFINITION
Short division	51	Division in which the operations are performed mentally.
Similar figures	268	Figures whose ∠s are mutually equal and whose homologous sides are proportional.
Simple interest	216	Interest on the principal alone.
Simple proportion	170	An equality of two simple ratios.
Slant height	261	Distance from vertex of a pyramid to base of one of its faces.
Solid	260	That which has length, breadth, and thickness.
Solution	160	Process by which the answer to a problem is obtained.
Specific gravity	294	The ratio of the weight of a substance to weight of same volume of water.
Sphere	260	A solid bounded by a curved surface, every point of which is equally distant from a point within, called the center.
Square	238, 251	The product of two equal factors; a rectangle whose sides are equal.
Square prism	260	A prism whose base is a square.
Square root	240	One of the two equal factors of a number.
Standard time	293	Railroad time.
Stere	139	Metric unit of wood measure.
Stock	206	Property owned by members of a charter company.
Subtraction	28	Process of finding the other of two numbers when one of them and their sum are given.
Subtrahend	28	A number to be subtracted.
Sum	16	The result in addition.
Surface	251	That which has two dimensions.
Tangent	254	A straight line which touches the circumference of a ⊙ at only one point.
Tax	200	A sum assessed on a person or on property to meet public expenses.
Terms	60	Expressions separated by the signs '+' or '−'; the numerator and denominator of a fraction; nomenclature in the various operations.
Tetrahedron	260	Solid bounded by four equal △s.
Thermometer	294	An instrument for measuring temperature.
Trade discount	201	Discount from the list price of goods.
Transposition	159	Transferring a term from one member of an equation to the other.

TERM	P.	DEFINITION
Trapezium	251	A quadrilateral having neither pair of its opposite sides ‖.
Trapezoid	251	A quadrilateral having one pair only of its opposite sides ‖.
Triangle	251	A plane figure bounded by three straight lines.
Triangular prism	260	A prism whose bases are △.
True discount	227	Principal minus true present worth.
Unit	84	One, a single thing.
Unknown quantities	167	Quantities whose values are sought.
Vertex	260	Union of three or more edges of a solid.
Vertical line	250	A line ⊥ to the horizon.
Vinculum	60	A straight line placed over two or more quantities to show they are to be regarded as a single term.
Volume of a solid	261	The number of cubic units in its contents.

ANSWERS

pp. 25 to 41 | pp. 41 to 41 | pp. 41 to 54 | pp. 54 to 56

Addition

pp. 25–27

140. 2,036,810
141. 16,260,634
142. 19,063,439
143. 19,066,705
144. 16,757,799
148. $202
149. 79 miles
150. 182 pounds
151. $662
152. 156 strokes
153. $6625
154. $21,096
155. $8731
156. 6238 trees
157. 1545 miles
158. 75,686 men
159. 16,237,535 square miles
160. 29,002 feet
161. 4915 miles
162. 2843 bushels

Subtraction

pp. 33–34

85. 1200; by 200
86. $1400
87. 688
88. 201
89. $222
90. 10 spaces
91. 206 acres
92. $200
93. 87,590 square miles
94. 1007 miles
95. $5055
96. 204 trees
97. $7536
98. 450 miles

Multiplication

p. 41

83. 6,424,302
84. 885,715
85. 6,719,328

Multiplication

p. 41

86. 1,992,848
87. 7,714,520
88. 7,180,965
89. 5,178,576
90. 8,300,976
91. 8,173,492
92. 6,034,749
93. 6,965,396
94. 7,267,842
95. 4,793,633
96. 7,279,272
97. 3,289,756
98. 1,465,614
99. 1,869,240
100. 1,503,625
101. 2,201,892
102. 1,124,665
103. 2,070,735
104. 26,839,600
105. 57,827,000
106. 21,765,100
107. 23,575,000
108. 2,863,000
109. 28,519,200
110. 19,890,000
111. 37,740,000
112. 12,400,000
113. 600,938,744
114. 98,400,000
115. 788,407,008
116. 461,553,984
117. 478,706,436
118. 742,968,954
119. 416,659,344
120. 682,880,000
121. 431,257,756
122. 212,536,954
123. 412,928,904
124. 254,334,654
125. 604,849,868
126. 620,919,050
127. 621,068,908
128. 490,119,007
129. 555,988,608
130. 335,580,394
131. 367,868,400
132. 636,859,008
133. 106,365,048
134. 43,480,990

Multiplication

pp. 41–43

135. 486,978,700
136. 223,064,050
137. 215,747,392
138. 226,946,000
139. 109,999,125
140. 86,079,621
141. 232,578,200
142. 103,019,025
147. $3024
148. 41,062,000 stitches
149. $586
150. 226,176 letters
151. 91,289,796 miles
152. 70,215,270,400 cubic yards
153. $16,978
154. $4681
155. 146,289 miles
156. $10,285

Division

pp. 53–54

123. 364; 5: 189; 21
124. 138: 364; 18
125. 215; 5: 476; 12
126. 711; 9: 252
127. 538; 2: 408; 3
128. 1875: 367; 3
129. 5002; 4: 369; 10
130. 4000; 4; 322
131. 2041; 15: 262; 8
132. 3179; 23: 169; 12
136. 2
137. 4
138. 5
139. 8
140. 9
141. 19; 206
142. 26; 156
143. 19; 196

Division

pp. 54–56

144. 12; 49
145. 18; 187
146. 4
147. 5
148. 8
149. 5
150. 9
155. 30[illegible]
156. 175[illegible]
157. 210[illegible]
158. 90[illegible]
159. 3089; 30
160. 4362; 1
161. 2931
162. 2082
163. 4107
164. 2573; 52
165. 3871
166. 38,413
167. 29,615; 3
168. 24,542
169. 45,684
170. 41,075
171. 90,009
172. 80,808
173. 1598
174. 2009
175. 806
176. 374
177. 423
178. 3333
179. 3949
180. 31
181. 200
182. 220
183. 97; 103
184. 105
185. 237
186. 45
187. 468; 797
188. 387; 1936
189. 267; 2076
190. 214; 3254
191. 25; 7369
192. 61; 1763
193. 134; 2849
194. 317
195. 125; 1000
196. 218; 2008

pp. 56 to 66

Division
pp. 56–59

197. 476; 14134
198. 313; 12988
199. 127; 15333
200. 117; 899
205. $1300
206. 60 people
207. 64 bushels
208. $136 [illegible]
209. $26
210. $5500; $3000
211. 6 years
212. 113 days
213. 550 sacks
214. 12 miles
215. 5 farms, 100 acres
216. $2
217. 168 pp.
218. 364 months
219. $23
220. 65 times; 830 acres remaining
221. 226
222. 102 acres
223. $152
224. 160 tons
225. 18 cents
226. 64 hours
227. 15 feet

Operations Combined
pp. 61–66

13. 11
14. 35
15. 11
16. 3
17. 11
18. 4
19. 36
20. 8
21. 30
23. 58
51. 3905 votes
52. 3689 votes
53. 8 payments
54. $36150
55. 12 miles

pp. 66 to 71

Operations Combined
pp. 66–67

56. 8 miles
57. 10 hours
58. 2 miles
59. 3 hours
60. $10
61. 8 weeks
62. 27 sheep, $13 left
63. $2022
64. $1680
65. $308
66. $403
67. $5470
68. $2072
69. 2262 rods
70. 400 feet
71. 3 days
72. 30 boys
73. A, 9¢; B, 3¢
74. $1479

Factoring
pp. 70–71

29. 2, 2, 2, 5, 3, 3, 7, 11; 4, 6, 8, 9, 10, 12, 14, 15, 18, 20, 21, 22, 24, 28, 30, 33, 35, 36, 40, 42, 44, 45, 55, 56, 60, 63, 66, 70, 72, 77, 84, 88, 90, 99
30. 2, 3, 3, 11, 89; 6, 33, 9, 22, 66, 99, 18
31. 5, 13; 65
33. 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97; 26 numbers
34. Yes, no, yes, yes, yes, no

pp. 71 to 81

Factoring
pp. 71–81

35. 409
42. 2, 2, 2, 3, 5; 5, 11, 2, 2; 2, 2, 2, 2, 3, 3, 3, 13
43. 2, 2, 3, 3, 3; 2, 2, 2, 2, 3, 3; 5, 17, 17
44. 2, 2, 3, 13; 2, 2, 2, 2, 2, 5; 2, 2, 2, 2, 2, 3, 7, 11
45. 3, 3, 3, 5; 3, 3, 2, 2, 2, 2, 2; 5, 5, 11, 11
106. 165
107. 95
108. 144
109. 105
110. 840
111. 625
112. 950
113. 17
114. 1
115. 13
116. 107
117. 13
118. 11
119. 9
120. 7
160. 14,400
161. 55,125
162. 217,728
163. 34,476
164. 43,890
165. 53,100
169. 1
170. 2, 3, 4, 5, 6, 7, or 8, times 1
175. 2, 3, 4, 5, 6, 7, 8, or 9, times 1
176. 2, 3, 4, 5, 6, 7, 8, or 9, times 1
179. 12 feet for ends, 16 feet for sides
180. 60 yards
181. 25 feet
182. 4 feet

pp. 82 to 90

Factoring
p. 82

184. A and B, 24 min.; A and C, 18 min.; B and C, 72 min
185. 20 times
186. 60 eggs
187. 63 eggs

Common Fractions
pp. 85–90

21. $\frac{11}{15}$; $\frac{1}{4}$
22. $\frac{3}{8}$; $\frac{12}{13}$
23. $\frac{5}{9}$; $\frac{2}{3}$
24. $\frac{11}{12}$; $\frac{17}{18}$
25. $\frac{33}{35}$; $\frac{71}{108}$
26. $\frac{3}{5}$; $\frac{289}{297}$
27. $\frac{13}{16}$; $\frac{11}{32}$
28. $\frac{2411}{1280}$; $\frac{13}{36}$
29. $\frac{1}{15}$; $\frac{3}{4}$
39. $\frac{819}{1331}$
40. [illegible]
41. $\frac{77}{3888}$
42. $\frac{783}{3277}$
54. $\frac{87}{118}$, $\frac{79}{118}$, $\frac{105}{118}$
55. $\frac{112}{336}$, $\frac{224}{336}$, $\frac{392}{336}$
56. $\frac{560}{924}$, $\frac{836}{924}$, $\frac{397}{924}$
57. $\frac{1335}{1482}$, $\frac{339}{1482}$, $\frac{306}{1482}$
58. $\frac{1472}{3024}$, $\frac{1896}{3024}$, $\frac{1701}{3024}$
66. $23\frac{2}{63}$
67. $64\frac{17}{18}$
68. $55\frac{17}{87}$
69. $396\frac{13}{38}$
70. $152\frac{87}{143}$
71. $80\frac{33}{88}$
77. $\frac{683}{16}$; $\frac{921}{11}$
78. $\frac{1231}{14}$; $\frac{1381}{16}$
84. $128\frac{15}{36}$
85. $165\frac{81}{85}$
86. $53\frac{71}{72}$
92. $1\frac{5}{8}$; $40\frac{173}{195}$
93. $1\frac{21}{40}$; $45\frac{69}{176}$
94. $1\frac{5}{8}$; $36\frac{402}{625}$
107. $\frac{5}{28}$
108. 2079
109. $\frac{1}{6}$
110. $\frac{1}{3}$
111. $\frac{4}{17}$
112. $\frac{3}{7}$

pp. 90 to 94

Common Fractions

pp. 90–94

113. 6
114. 5[illegible]
115. [illegible]
116. 2
165. [illegible]
166. 1[illegible]
167. [illegible]
168. [illegible]
169. [illegible]
170. 8[illegible]
171. 32
172. 7[illegible]
173. 22
174. 3[illegible]
175. 1[illegible]
176. 4[illegible]
177. 6[illegible]
178. 6[illegible]
179. 3[illegible]
184. 62[illegible]
185. 59[illegible]
186. 20[illegible]
187. 52[illegible]
188. 81[illegible]
189. 73[illegible]
190. 60[illegible]
191. 58[illegible]
192. 23[illegible]
193. 21[illegible]
194. 159[illegible]
195. 202[illegible]
196. 15584
197. 32045
198. 17898[illegible]
199. 171003[illegible]
200. 18793[illegible]
201. 28512[illegible]
202. 65243[illegible]
203. 319291[illegible]
204. 180[illegible]
205. 3115[illegible]
206 1696[illegible]
207 13264[illegible]
208. 212[illegible]
209. 61[illegible]
210. 7[illegible]
211. 13[illegible]

pp. 96 to 100

Common Fractions

pp. 96–100

219. [illegible] : [illegible]
220. [illegible]; 1[illegible]
221. [illegible]; [illegible]
222. [illegible]; [illegible]
223. [illegible]
224. 1
225. 1
226. [illegible]
227. 1[illegible]
228. 5
229. 9
230. 10
231. 19[illegible]
232. 1
236. [illegible]
237. [illegible]
238. [illegible]
239. 1[illegible]
242. Sum, 39[illegible]; dif., 5[illegible]; prod., 379[illegible]; quo., 1[illegible]; prod. of sum and dif., 233[illegible]
243. 15658[illegible]
246. [illegible]
247. [illegible]
251. 99¢
252. 54¢
253. 24, 25, 81
254. $16
255. [illegible]
256. [illegible]
257. [illegible]
258. 6 acres
259. 31[illegible] yards
260. 18 coats
261. [illegible]
262. [illegible] in corn
263. 8 dozen
264. 24 years
265. $6
266. $15000
267. 3 apples
268. $71[illegible]
269. $2[illegible]

pp. 100 to 106

Common Fractions

p. 100

270. [illegible]
271. 108 sheep
272. 12 apples
273. [illegible] of whole ship
274. 10; 6
275. 120 apples
276. $56
277. 36 cattle
278. $1
279. A, 30 years
280. 60 years
281. 5 days
282. 50 yards
283. 8 days

Decimals

p. 106

31. .000500
32. 17.7
33 .00000638
34. .000000000023
35. .000085002
36. .7007
37. .00360
38. .604
39. 9431.0906532
40. .0000[illegible]
41. 2,000,000,000-.000,000,002
42. 83000.005
43. .0099999
44. .001006004
55. 98 million 637 thousand 208 hundred-millionths
56. 83 million 8 hundred-millionths
57. 9 thousand 2 hundred, and 929 ten-thousandths

pp. 106 to 111

Decimals

pp. 106–111

58. 9 thousand, and 9 thousand 372 ten-millionths
68. $1338.939
69. $374.693
70. $21423.40466
71. 68.2711
72. 13.139732
73. 64.0255
74. 9821.631
75. 54.864
76. .908
77. 99.13125
78. 768.04
79. 1.478694
80. 408.5081
81. .3239775
82. 449.95854
83. 77477.4
84. 5021.75
85. 9753.76
86. 24859.02736
87. .015
88. 13.1489+
89. 3.1063+
90. 153.482+
91. .0303+
92. .00633+
93. 6.7808+
94. .0000001
95. 10,000,000
96. .01599+
97. 1.05997+
98. .01399+
99. 2,000,000
100. .000000176+
108. [illegible]; [illegible]
109. [illegible]; [illegible]
110. [illegible]; [illegible]
111. [illegible]; [illegible]
112. [illegible]; [illegible]
113. [illegible]; [illegible]
114. [illegible]: [illegible]
115. [illegible]: [illegible]
116. [illegible]: [illegible]
117. [illegible]; [illegible]
118. [illegible]: [illegible]

pp. 111 to 118

Decimals

pp. 111–118

119. $\frac{7}{80}$: $\frac{1}{1600}$
120. $\frac{13}{160}$; $\frac{13}{88}$
121. $\frac{3}{16}$: $\frac{241}{5000}$
126. .1875 ; .16
127. .1171875 ; .096
128. .7525
129. .87375
130. .000625
131. No; 13 is a factor, other than 2 or 5
132. .846$\frac{2}{13}$
133. .8461 +
134. $.\dot{8}4615\dot{3}$
143. 1[illegible]
144. 6[illegible]
145. [illegible]
146. [illegible]
147. 7$\frac{4}{15}$
148. 8[illegible]
149. 30$\frac{2}{11}$
150. $\frac{7}{37}$
218. \$64.71875
219. \$135.94
220. \$6.67
221. \$48.29
222. \$54.69
224. \$64.19
225. \$64,395.06; \$64,395.061
227. 20 decimal places
228. 234.63 × \$$(1.06)^5$, or \$313.99
229. \$64.68
230. 420.16+times
231. 2695.4928 feet
232. 41.66$\frac{2}{3}$ days
233. \$.004 +
234. \$365.25 (5 leap years)
235. 9.278 + bu.
236. 8000 pounds
237. \$23.76
238. 406.64 bu.
239. \$218.24
240. \$327.24

pp. 130 to 131

Denominate Numbers

pp. 130–131

154. 225 pt.
155. 7846 far.
156. 1860 drops
157. 10,079 gr.
158. 12 cwt. 56 lb.
159. 110 rd. 2 yd. 7$\frac{1}{2}$ in.
160. 88 sq. rd.
161. 50 min.
162. 1 mo. 15 da. (30 da. to the mo.)
163. 61 sq. rd. 18 sq. yd. 1 sq. ft. 50$\frac{2}{5}$ sq. in.
164. 11 cu. ft. 432 cu. in.
165. 6 oz. 13 pwt. 8 gr.
166 2 s. 9 d. 3.936 far.
167. 1 pk. 2 qt. 1.024 pt.
168. 284 rods 2 yards 1 foot 4 inches
169. 7 cwt. 77 lb. 12$\frac{4}{5}$ oz.
170. 9 ℥ 3 ʒ 1 ℈ 3.904 gr.
171. 101° 42'
172. 3 qt. 1 pt.
173. 30 sq. mi.
177. 8 bbl. 6 gal. 1 qt. 1 pt. 1 gi.
178. 8 oz. 6 gr.
179. 1 ℔ 5 ℥ 7 ʒ 2 ℈ 17 gr.
180. 8 ch. 2 rd.
181. £1642 4 s
182. 234 rd. 1 yd. 1 ft. 1 in.
183. 1 sq. rod 6 sq. yards 2 sq. feet 83 sq. inches

pp. 131 to 134

Denominate Numbers

pp. 131–134

184. 1 cu. yd. 1 cu. ft. 10 cu. in.
185. [illegible] ℔.
186. $\frac{1}{17600}$ Tp.
187. 1[illegible] ℔.
188. $\frac{1121}{10000}$ mi.
189. 2$\frac{31}{32}$ cd.
190. 2[illegible] T.
191. 3.625 gal.
192. 5.859375 bu.
193. £6.425
194. 7.460015625 T.
195. .112 + C.
196. 5.41071 + yr. by 1st meth.; 5.378 + yr. by 2d meth.
203. 765[illegible] ℔.
204. 5.55[illegible] ℔
205. 24 bu.
206. 2[illegible] bbl.
207. 2.955 + bbl.
208. 46[illegible] gal.
209. 20 min. 20$\frac{1}{4}$ sec.; 24 min. 24 sec.
210. 76° 16' 15"
214. 25 T. 8 cwt. 92 lb. 14 oz.
215. 27 rd. 2 yd. 1 ft.
216. 27 lb. 3 oz. 7 pwt. 15 gr.
217. 51 yr. 10 mo. 21 da. 3 hr.
218. 178 rd. 1 yd. 2 ft. 6 in.
219. 178.33$\frac{1}{3}$ rd.
220. 4329.6 gr.
221. Oct. 24; Nov. 13
222. Oct. 26; Oct. 6
226. 2 lb. 5 oz. 16 pwt. 19 gr.
227. 10 rd. 1 yd. 2 ft. 1 in.; 1 A. 18 sq. rd.

pp. 134 to 139

Denominate Numbers

pp. 134–139

228. [illegible] da.; .5291+ da.; 12 hr. 41 min. 54$\frac{3}{7}$ sec.
229. 213 da.
230. 7 mo. 1 da.
231. 2 da.
235. 84° 54' 24"
236. 25 bu. 3 pk. 4 qt.
237. 16 gal. 3 qt. 1 pt.
238. 41 rd. 1 yd.
239. 30 da. 10 hr. 59 min. 40 sec.
240. 17 sq. yd. 80 sq. in.
241. 19 hr. 30 min. 30 sec.
242. 160 rd. 1 ft. 1$\frac{5}{8}$ in.
243. 1 T. 15 cwt. 11$\frac{1}{8}$ lb.
244. 2$\frac{8}{21}$
245. 5 mo. 5 da. 4 hr.
246. 723[illegible]
247. 3.5
248. 404[illegible]
264. 40,000,000 m; 1,574,800,000 in.; 24,854.79+ mi.
267. 1,000,000,000,000 sq mm; 1,000,000 sq Dm
268. 10
269. Lay off a square 10 m × 10 m
272. 185.28 A.
276. 1,000,000.000 cu dm; 1,000,000,000 cu Dm
277. Lay off a cube 1 m × 1 m × 1 m

pp. 139 to 144

Denominate Numbers
pp. 139–144

280. Wood at \$3 per cd.
281. \$4800
284. 23645 l; 385.623 cu m
287. 8.192 cu m
288. \$180.22
336. 176.57+ cu. ft. ex.; 180 cu. ft. approx.
337. 73.52+ bu. ex.; 80 bu. approx.
338. 41.18+ Kg ex.; 45$\frac{5}{11}$ Kg approx.
339. 106760 ft.; 20 mi. 70 rd. 5 ft.
340. 3 yd. 2 ft.; $\frac{2}{3}$ rd.
341. 127 rd. 1 yd. 3.6 in.; .3975 mi.
342. 87.5 lb. troy; 72 lb. avoir.
343. 16.8 bu.; 157.5 gal.
344. 16.9 bu.; 157.5 gal.
345. 2 hr. 35 min. 22 sec.; 38° 50′ 30″
346. 25 gal. 2 qt. 1 pt.; 6 gal. 3 qt. 1 pt.
347. Sept. 1; Mar. 3
348. Sept. 5; Mar. 3
349. 283 yr. 8 mo. 13 da.
350. 174 da.
351. 9000 cu cm; 91
352. 5000 Kg; 5 cu m
353. \$13.34
354. \$62.92

pp. 149 to 156

Literal Quantities
pp. 149–156

39. x^3-3x^2+x
40. $3a^2b-bc^2$
51. $-x^3+5x^2y-7xy^2-5y^3$
52. $x^4y-x^2y^2-12xy^3+6$
53. $2a^2b^2+5a^2bc+6ab^2c+2bc^2$
56. a^9; x^6; ...
57. a^7; a^4; ...
72. $x^4+x^2y^2+y^4$
75. $a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$
76. $a^3+b^3-c^3+3ab^2+3a^2b+3ac^2-3a^2c-6abc+3bc^2-3b^2c$
79. a^4; a^4; ...
80. x; x^0, or 1; ...
90. $x^4-x^3y+x^2y^2-xy^3+y^4$; $5x-2y$
91. x^2+xy+y^2; $x^4+x^3y+x^2y^2+xy^3+y^4$
92. $3x+2y$; $x^9+x^6y^3+x^3y^6+y^9$
93. $3ab(a+b)$; $6(x^3-2x^2+4x-1)$
94. $12a^2b^2(b-a)$; $7(2x^2y-4xy^2+xy+2)$
95. $12a^2b^2(2ab-3)$; $2xy(6xy-5y+4-2x)$
96. $10a^3(2ab^4-3)$; $5y(2x^3-4x^2y+2xy^2-y^3)$

pp. 156 to 159

Literal Quantities
pp. 156–159

97. $3a^3b^2(4+3ab^2)$; $6xy(3x^3+5x^2y-4xy^2+2y^3)$
103. $5a^2b$
104. $2a^3b$
105. a^2b
106. $3a^2b^2$
107. $2a^2$
113. $12x^4y^2$
114. $30x^3y^2$
115. $6x^3y^2$
116. $42x^4y^3$
117. $48x^5y^5$
124. $\frac{a^2b}{2}$; $\frac{1}{5x}$
125. $\frac{3}{8x^5y}$; $\frac{2x}{3y^3}$
126. $\frac{3}{5abc}$; $\frac{2}{3}$
127. $\frac{1}{4abc}$; $\frac{xy}{3}$
129. $\frac{2ab^2}{a^2b^2}$, $\frac{3ab}{a^2b^2}$, $\frac{4b^2}{a^2b^2}$, $\frac{2a^2}{a^2b^2}$
131. $\frac{21x+9}{24x}$, $\frac{16x-6}{24x}$
134. $-\frac{1}{8}$
135. $\frac{9x+5y}{24}$
137. $\frac{8x^2}{25y}$
139. $\frac{3a}{5b}$
140. $x=8$
141. $x=3$
142. $x=4$
143. $x=4$
144. $x=-2$

pp. 159 to 163

Literal Quantities
159–163

145. $x=-2\frac{1}{4}$
146. $x=2$
147. $x=3$
148. $x=25$
149. $x=2$
150. $x=8$
151. $x=2$
152. $x=16$
153. $x=2$
155. 36 boys
156. 15 on 1st; 30 on 2d; 90 on 3d
157. \$600, house; \$1200, barn; \$1800, land
158. 16 horses, 48 cows, 144 sheep
159. 45 yr., A's age; B's, 9; C's, 27
160. \$12, A's share; \$24, B's; \$144, C's; \$84, D's
161. 39 in.
162. 24, first part; 8, second; 64, third
163. 291
164. \$4
167. 11, the less; 33, the greater
168. 51
169. 21 yr., Anna; 25 yr., Mary
170. 92
171. 18, the less; 42, the greater
172. 25 yr., son; 45, father
173. 3, the less; 31, the greater
174. 9, the less; 25, the greater
175. 80, the less; 100, the greater
176. 95 gal.

pp. 163 to 168

Literal Quantities

pp. 163–168

177. 12, first part; 16, second; 13, third
178 \$3
179. \$3500
180. \$5400
181. \$1500
182. 6 yd.; 9 yd.
190. $x = 6$
191. $x = 7$
192. $x = 8$
193. $x = 8\frac{3}{13}$
194. $x = 10$
197. 264 girls
198. \$56, John; \$42, James
199. 180 acres
200. 260 peach trees
201. 12 yr., Ann; 9 yr., Jane
202. 264
203. 25 horses
204. 16 ft. wide, 22 ft. long
205. 30 apples
206. \$120
207. 49, 50, 51
208. \$140, horse; \$140, carriage
209. \$40, first; \$75, second
210. \$600, A; \$390, B
211. 48 cts., first son; 56 cts., second; 39 cts., third
212. \$325
213. 50 A, A; 65, B; 115, C; 33, D
215. $x = 1,\ y = 2$
216. $x = 2,\ y = 3$
217. $x = -1,\ y = 4$
218. $x = 6,\ y = 2$
219. $x = -1,\ y = 2$
220. $x = 12,\ y = 4$

pp. 168 to 179

Literal Quantities

168–169

221. $x = 3,\ y = 4$
222. $x = 4,\ y = 3$
223. $x = 2,\ y = 1$
224. $x = 1,\ y = 1$
225. $x = 6,\ y = 7$
226. $x = 42,\ y = 84$
228. 8, 12
229. \$1, potatoes; 55 cts., apples
230. 16 girls, 12 boys
231. $\frac{1}{3}$
232. \$50, A; \$70, B

Proportion

pp. 173–179

10. \$18.90
11. 28 eggs
12. 12 men
13. 32 da.
14. $11\frac{1}{3}$ mo.
15. 40 mi.
16. 123 ft.
17. \$16
18. 184 bu.
19. \$3750
20. 120 men
21. $28\frac{4}{9}$ da.
22. $7\frac{1}{2}$ da.
23. 6 men
24. 18 hr.
34. 102 da.
35. \$1750
36. 750 da.
37. 3640 lb.
38. 28 da.
39. 6 da.
40. 55,125 lb.
41. 7 hr.
42. $4\frac{1}{2}$ ft.
43. \$24
44. 540 pp.
45. 24 boys
49. \$102, 1st part; \$153, 2d part; \$255, 3d part
50. \$155

pp. 179 to 184

Proportion

p. 179

51. \$194, A; \$388, B; \$291, C
52. \$62.50, A; \$87.50, B
53. \$240, A; \$320, B; \$400, C; \$480, D
54. \$7000, wife; \$5000, each son; \$4000, each daughter
55. \$654$\frac{3}{4}$, A; \$243$\frac{1}{4}$, B; \$562, C
56. \$7128, A; \$3564, B; \$1188, C
57. 24 lb. cheese
58. 64 lb. coffee

Solution of Problems

pp. 181–184

2. $\frac{11}{12}$ min. space
3. $59\frac{11}{12}$ min. sp.
4. 5 min. sp.
5. 25 sp.
6. $5\frac{5}{11}$ min. past 5
7. $27\frac{3}{11}$ min. past 2
8. 50; $1\frac{1}{11}$ min. sp.
9. $46\frac{2}{3}$ min. past 4
10. $43\frac{7}{11}$ min. past 8
11. 12 mi.
12. 10 hr.
13. 60 mi.
14. 36 mi.
15. 12 mi., A; 14 mi., B
17. 1 hound leap $= \frac{3}{2}$ fox leaps
18 $\frac{1}{2}$ fox leap
19. 72 times
20. 252 times
21. 60 leaps
23. 1 ct.

pp. 184 to 189

Solution of Problems

pp. 184–189

24. 2 cts.
25. 2 doz. eggs
26. 15 eggs
27. 84 cts. per doz.
28. 5 cts.
29. 120 apples
31. \$2
32. 20 da.
33. \$4
34. $22\frac{1}{2}$ da.
35. $12\frac{1}{2}$ da.
39. $\frac{1}{4}$
40. $\frac{9}{20}$; $\frac{1}{20}$; $2\frac{2}{9}$ da.
41. 24 da.
42. 120 da., A; 60 da., B; 40 da., C
43. $4\frac{4}{5}$ da.
44. 10 hr.
45. $\frac{1}{12}$ hr.
46. 60 mi.
47. 90 mi.
48. \$10
49. \$28, A; \$35, B
50. 2 cts., Henry; 4 cts., James
51. \$1680 each
52. \$6720, A; \$3840, B; \$10,752, C; \$22,400, D
57. 5; 4
58. 15; 20
59. 40; 25
60. \$72
61. \$45
62. \$50
63. \$27
64. \$8
65. \$60, cost; \$100, selling p.
66. \$200, cost; \$180, selling p.
67. $\frac{1}{3}$
68. $\frac{3}{8}$
69. \$540
70. 30 ct.
71. \$10, com.; \$190, proceeds

pp. 189 to 195

Solution of Problems

p. 189

72. $4; $204
73. $225

Percentage

pp. 191–195

15. $1394, son; $1476, daughter
16. $419.25
17. $7.46
18. $157.50
19. $.98
20. 320
24. 24
25. 1600
26. 380
27. 2900
28. 84
29. 540
30. 5450
31. 9100
32. $1920
33. 3000 men.
34. 19.53\frac{1}{8}$ per A.
35. $187.50
41. 80$\frac{16}{19}$%
42. 33$\frac{1}{3}$%
43. 12$\frac{1}{2}$%
44. 11$\frac{1}{9}$%
45. 89%
46. 12%
47. 54$\frac{6}{11}$%
48. 68%
49. 64%
50. 72%
54. 18
55. 200
56. 48
57. 40
58. 200
59. 90
60. 300 bu.
61. 12,800
62. 1800 bu.
63. $18,000
64. $12,500
65. $8100

pp. 195 to 201

Percentage

pp. 195–201

66. 62.5 gal.
67. 4$\frac{1}{6}$%
68. 72$\frac{4}{33}$%
69. $1468.50
70. 99$\frac{11}{18}$%
71. 4$\frac{1}{6}$ gal.
75. $17
76. 75%
77. $327.50
78. 95 ct.
79. 87 ct.
80. $72.23
81. $1960.80
82. $3.44
83. $165
84. $27,840
85. 225 clocks
86. $300, cost; 25%, gain
87. $112.50
88. $400
91. $34.50
92. $34.50
93. $23.60
94. 195 lb.
95. 1$\frac{1}{4}$%
96. $288
97. $135
98. $325
99. $1244.40
100. $24.40
101. 2%
102. 30 ct.
103. $5878.98
104. $525
106. $153,950
107. $1,594,000
108. $146.80
109. $4273.61
110. 1 ct. on the dollar; $118
112. $80.37
113. No
114. $88.32
115. $88.32
116. Ex. 114 better by $4
117. First by $4.50

pp. 201 to 213

Percentage

pp. 201–205

118. Both alike
120. $125
121. $62,500
122. $16,666.67
123. 62$\frac{1}{2}$ ct. on $100
124. $228.80
125. $5400
126. $7572
132. 6$\frac{2}{3}$%
133. 6$\frac{1}{4}$%
134. 96 oz.
135. $725
136. 2$\frac{11}{12}$%
137. $600
138. 30% above cost
139. 90% of cost
140. $206
141. $250

Stocks and Bonds

pp. 212–213

54. $3,670,000
55. $375
56. $26,145
57. $4179.38
58. $1010
59. $7190
60. $12,468.75
61. 150 shares
62. 250 shares
63. 20 shares
64. $316.67
65. $2880
66. 1800 bu.
67. 45 shares
68. 126 shares
69. $27,309.38
70. 4%
71. 20%
72. 12$\frac{1}{2}$%
73. 5$\frac{5}{11}$%
74. 3$\frac{11}{17}$%
75. 7$\frac{13}{109}$%
76. $75 per share
77. $70 per share

pp. 213 to 218

Stocks and Bonds

pp. 213–215

78. $3000
79. 133\frac{1}{3}$ per share
80. $30
81. $256
82. $1500, inc.; 2$\frac{14}{17}$%
83. 6% stock; by $20
84. R.R. stock, by $1200.
85. 18 shares
86. $8875
87. 6$\frac{2}{3}$%
88. $96
89. $6400
90. 4% stock; by $100
91. $6750
92. $324
93. 5$\frac{1}{4}$%
94. 3$\frac{1}{3}$%
95. $90 decrease
97. 93 shares
98. $920, mining; $1350, furnace
99. $148.50
100. $34,800 Phil. 6's; $104,400 R.R. 7's

Interest

pp. 217–218

7. $.252$\frac{2}{3}$
8. $.3365
9. $.187$\frac{1}{2}$
10. $.456$\frac{1}{4}$
11. $.152
12. $141.29
13. $173.88
14. $162.90
15. $167.44
16. $156.04
20. $102.20
21. $160.77
22. $111.81
23. $188.46

pp. 218 to 227

Interest

pp. 218–227

24. $84.61
25. $1.48
26. $6.51
27. $5.48
28. $6.16
29. $5.26
34. $4.57
35. $42.43
36. $62.88
37. $17.26
48. 10%
49. 2 yr. 3 mo. 18 da.
50. 20 yr.
51. 2 yr. 3 mo. 18 da.
52. 5%
53. 5%
54. $260
55. $260
56. $24.60
57. $3332
58. $1313.78
59. $158.60
60. $660
61. $6000
62. $500
63. $980
64. 5%
65. 6%
66. 3½%
67. 2%
68. 3 yr. 6 mo.
69. 5 yr.
70. 9 mo. 18 da.
71. 3 yr. 9 mo.
72. $175
73. $130
74. $256
75. $933.33
77. $143.05
78. $26.73
80. $808.11
81. $72.66
83. $297.44 pres. worth; $2.56 true dis.
84. $400

pp. 227 to 235

Interest

pp. 227–235

85. $300
86. $285.97
87. $114.19
88. $105.86
89. $137.18
93. Due March 10; $4, bank dis.
94. $1.60
95. $1.34
96. $3.96, true dis.; banker makes 4¢
97. Int. on $3.96 60 da. is 4¢
98. $227.25.
99. $744.08, face of note
104. $616.18
105. $616.18
106. June 9
107. June 20
108. $36.25, int. on amt. of debts: $20.10, int. on amt. of pay-m'ts; $16.15, dif.
110. $85.51
111. $34.16
112. $59.73
113. $129.86
114. $1874.11
115. $3262.50
116. $918.72
117. $1355.16
119. $$(1.06)^4$, $1.262477
122. $$(1.06)^{2000}$; 4000 decimal places
125. $2999.15
126. $88.27
127 $278.52
128. $381.18
129. $19.94
130. $18.57
131. $9.45
132. $580.12

pp. 235 to 245

Interest

pp. 235–237

133. $733.99
134. $1748.28
138. $271.23
139. $264.19

Involution and Evolution

pp. 239–245

14. 32
16. 45
17. 24
19. 95
20. 97
21. 83
22. 37
23. 48
24. 85
25. 91
26. 32
27. 73
29. 314
30. 231
31. 625
32. 723
33. 1039
34. 1829
38. .060415+
39. .81649+
40. ⅔
41. .25
42. .79056+
43. .4714+
44. 2.5
45. 7.9056+
46. .65465+
47. .0079056+
48. .025
50. 23
51. 34
52. 49
53. 74
54. 68
55. 63
56. 89
57. 53
58. 55

pp. 247 to 256

Involution and Evolution

pp. 247–248

60. 456
61. 562
62. 1022
63. 1013
67. .1539+
68. .8735+
69. ⅔
70. .12
71. .05569+
72. .6057+
73. 1.2
74. 5.569+
75. .7539+
76. .02585+
77. .025
79. $a^5+(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)b$
80. $a^6+(6a^5+15a^4b+20a^3b^2+15a^2b^3+6ab^4+b^5)b$
81. 122
82. 24
83. 311
84. 12

Mensuration

p. 256

57. 50.2656 in.
58. 37.6992 in.
59. 62.832 ft
60. 6⅔ ft.
61. 80 in.
62. 2½ m
63. 10 in.
64. 26 in.
65. 40 in.
66. 20 in.
67. 24 in.
68. 5 in.
69. 24 in.
70. 28 in.
71. 40 ft.
72. 119 ft.

Mensuration

pp. 256–267

73. 37½ ft.
74. 6 ft. 3 in.
75. 13,200 ft.
76. 34 ft.
77. 18 ft.
82. 131.08 sq. m.
84. 160 rd.
85. 4 A. 110 sq. rd.
86. 200 sq. m.
87. 35.35+ ft.
88. 13 A. 20 sq. rd.
89. 96 sq. dm
90. 1 rd.
91. 27 sq m
92. 4½ A.
93. 314.16 sq m
94. 308.992 sq. ft.
95. 1 mi.
96. 70.246+ rd.
97. 173.2 sq. ft., area; 17.32 ft.
98. 960 sq. ft.; 48 ft.
99. 10,000 sq. ft.
100. 113.0976 sq. ft.; 8 : 10
143. 48 sq. in.
144. 62.832 sq. ft.
145. 150.7968 sq. in.
146. 452.3904 sq. ft.
147. 314.16 sq. in.
148. 288 sq. in., entire surface
149. 4 in.
150. 50.2656 in.
151. 12 ft.
152. 8.66+ ft.
153. 576 sq m
154. 673.87 sq. ft.
155. 201,062,400 sq. mi.
156. 280 sq. ft.
157. $7.50
162. 69.28 cu. in.
163. 1130.976 cu. in.
164. 62.352 cu. in.
165. 565.488 cu. in.
166. 20 in.

Mensuration

pp. 267–271

167. 70.686 sq m
168. 10.34+ m
169. 12 m
170. 50.2656 sq. in.
171. 13,824 sq. in.
182. 169.6464 cu. in.
183. 36 in.
184. 40 sq. in.
185. 4 times
186. 16
187. 4 lb.
188. 1 : 4
189. 5 in.
190. $4.
191. 4000 lb.
192. 480 bu.
193. 6400 cu. ft.
194. 4 in.
195. 87½ ct., A; 12½ ct., B

Occupations

pp. 280–281

1. 8 bd. ft.
3. 4 bd. ft.
4. 9 bd. ft.
5. 12 bd. ft.
6. 15 bd. ft.
8. 8 bd. ft.
10. 12 bd. ft.
11. 20 bd. ft.
12. 27 bd. ft.
14. 384 bd. ft.
15. 40 bd. ft.
16. 24 bd. ft.
17. 42 bd. ft.
18. 240 bd. ft.
19. 60 bd. ft.
20. 160 bd. ft.
27. 32 bd. ft.
28. 160 bd. ft.
29. 72 bd. ft.
30. 192 bd. ft.
31. 192 bd. ft.
32. 36 bd. ft.
33. 64 bd. ft.

Occupations

pp. 281–284

34. 360 bd. ft.
35. 640 bd. ft.
36. 48 bd. ft.
37. 90 bd. ft. See Note.
38. 108 bd. ft.
39. 144 bd. ft.
40. 144 bd. ft.
41. 240 bd. ft.; 480 bd. ft.
42. 16 ft.; 12 ft.
43. 2465 bd. ft.
44. 225 bd. ft.
45. 2240 ft.
46. $24.65
47. 1500 bd. ft.
48. Agrees
49. 307 bd. ft.
50. Agrees
51. 90 sq. in.
52. 3 11/18 sq. yd.
53. Agrees nearly
54. 36 bundles, by rule
55. 27⅞ sq. ft.
56. Agrees nearly
57. 31¼ sq. ft.
58. Agrees nearly
59. 61 bunches, by rule; rafters, 17.77 ft. long
60. 3920 ft.
61. 320 bd. ft.
62. 384 bd. ft.
63. 12 sq. yd., by rule; 13⅜ sq. yd., by com.
64. 100 sq. ft., by rule; 111⅑ sq. ft.
65. 32 bd. ft.
67. 120 bd. ft.
68. 100 bd. ft.
69. 432 bd. ft.
70. 96 ft. of boards
72. 60 ft. of boards
73. 115 ft. of boards

Occupations

pp. 284–288

74. 24 ft. of boards
75. 120 ft. of boards
76. 341 ft. of boards
78. ⅜ sq. yd.
79. Crosswise; makes no difference where the figure begins
80. $33.75
81. 1 yd. wide, lengthwise, $26¼
82. $39.06
83. $67.50
85. 6½ double rolls
86. 2 strips
87. 3 strips
88. 5½ double rolls
89. 4½ double rolls
90. $6.48 (strips on ceiling run lengthwise)
91. No; strips must match and be whole
96. 104.18 bbl.; 57.87 cu. yd.
97. 5 masons
98. 157,520 bricks
99. $748.44; charge for the openings
100. 98.45 cu. yd.; 177.21 bbl.; 15.4 da.
101. 8800 bricks
102. 2347 bricks
103. 9387 bricks
104. The number of cu. ft. in walls is 4 times as many

pp. 288 to 289	pp. 289 to 291	pp. 291 to 293	pp. 293 to 294

Occupations

pp. 288–289

105. 42 cd.
106. 169$\frac{33}{35}$ perch
107. 52$\frac{1}{2}$ bbl.; 42 cu. yd.
108. $96
109. 11$\frac{1}{4}$ bbl.; 4 cu. yd.
110. 460$\frac{5}{8}$ bu.
111. 368$\frac{11}{16}$ bu.
112. 460$\frac{5}{8}$ bu.; 25,804.8 lb.
113. 555$\frac{5}{7}$ bu. approximately
114. 76+ bu.
115. 651$\frac{3}{7}$ bu. approximately
116. 30 bu. old; 28 bu. new

Occupations

pp. 289–291

117. 85+ bu.
118. 128 bu.
119. 4+ bins
120. 2 bu.
122. 2 ft. 1 in.
123. 282.7+ bbl.; count 7$\frac{1}{2}$ gal. 1 cu. ft.
125. 24 T.
126. 6$\frac{3}{16}$ mi.
127. 7$\frac{1}{16}$ mi.
128. 3240 hills; each hill the center of a 3 ft. 8 in. square
129. 21$\frac{3}{8}$ bu. per A.
130. Agrees nearly

Occupations

p. 291

131. 322 rd.
132. 50.56+ rd.
133. 44.8 rd.
134. 80 trees; each tree the center of a 2 rd. square
135. 15 ft. long
136. 40 bbl.; 126 hr.

Miscellaneous

p. 293

3. 101° 22′ 30″

Miscellaneous

pp. 293–294

4. 6 hr. 45 min. 30 sec.
5. 78° 27′ 35″ W.
6. 35$\frac{5}{11}$ sec. past 2.30 A.M.
7. 1 hr.
8. West
9. 100 days
10. 7.7
11. 481$\frac{1}{4}$ lb.
12. $\frac{3}{5}$
13. 37$\frac{1}{2}$ lb.
14. 2$\frac{1}{2}$
15. 20° C.; 16° R.
16. 635° F.
17. 1000° C.
18. 172$\frac{2}{5}$° F.; 62$\frac{2}{5}$° R.
19. 20° R.