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KIYOSHI TAKATSU, DIRECTOR.

NO. 277

FURTHER RESEARCHES ON ALTERNATING CURRENT BRIDGES WITH PERFECT EARTHING DEVICE

By

Kazukiyo Ogawa

February, 1930.

ELECTROTECHNICAL LABORATORY,
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TOKYO, JAPAN.

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SYNOPSIS

Air condensers of special construction which give pure capacities free from losses when used in alternating current bridge with perfect earthing device, are described. They may be profitably used for the measurement of the capacity and the leakage conductance of a condenser and the effective resistance and the inductance of an inductance coil at any frequency. They can also be used in the determination of the effective resistance and the residual capacity of a resistance coil in terms of capacity and frequency.

A new accurate frequency bridge composed of resistances and condensers is also described.

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145-9

CONTENTS

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I. INTRODUCTION

The alternating current bridge with perfect earthing device has been investigated analytically by the writer by means of the theory of equivalent networks, and the results were published in the Research No. 254 of this laboratory.(20) The writer proposed in that paper a few bridges composed of resistances and condensers but of no inductances for the measurement of the capacity and the leakage conductance of a condenser and the effective resistance and the inductance of an inductance coil at any frequency. Those bridges make use of no-loss condensers and resistances with known residual capacities and therefore several researches have been devoted ever since to this direction. Air condensers of special construction which are quite free from losses, have at first been developed, and then the effective resistance and the residual capacity of a resistance coil could have been determined by the use of them.

A new accurate frequency bridge composed of resistances and condensers have also been thought of in this connection.

The present paper describes the results of these researches.

II. NO-LOSS AIR CONDENSERS

Air condensers free from losses have been investigated by many authorities. Quartz, amberite, etc. are usually used as the solid insulator to support the two sets of plates of the condenser and their amount is reduced to the minimum and they are placed in a weak electric field. The standard air condenser constructed by

E. Giebe on this principle is known to be a practically perfect condenser.(5)(12)

Such a condenser, however, requires a special precaution in construction and maintenance so that it is hardly possible for general use. The writer has constructed air condensers practically free from losses on the quite different principle.

Suppose a condenser formed of two sets of plates entirely surrounded by an earthed metallic case. The distribution of capacity in this condenser is obviously represented as shown in Fig. 1 where $C_0 C_1 C_2$ are three capacities accompanied with their respective leakage conductances. Now if we construct the condenser in such a way that the solid insulator to support the two sets of plates never lie immediately between them but necessarily between each set of plates and the earthed metallic case, the capacity C_0 in Fig. 1 would become absolutely pure with no leakage conductance, although the remaining two capacities $C_1 C_2$ are still accompanied with leakage conductances. The pure capacity C_0 thus obtained can be used most profitably in an arm of alternating current bridge with perfect earthing device where the earth capacities $C_1 C_2$ do not give rise to the balance condition of the bridge.

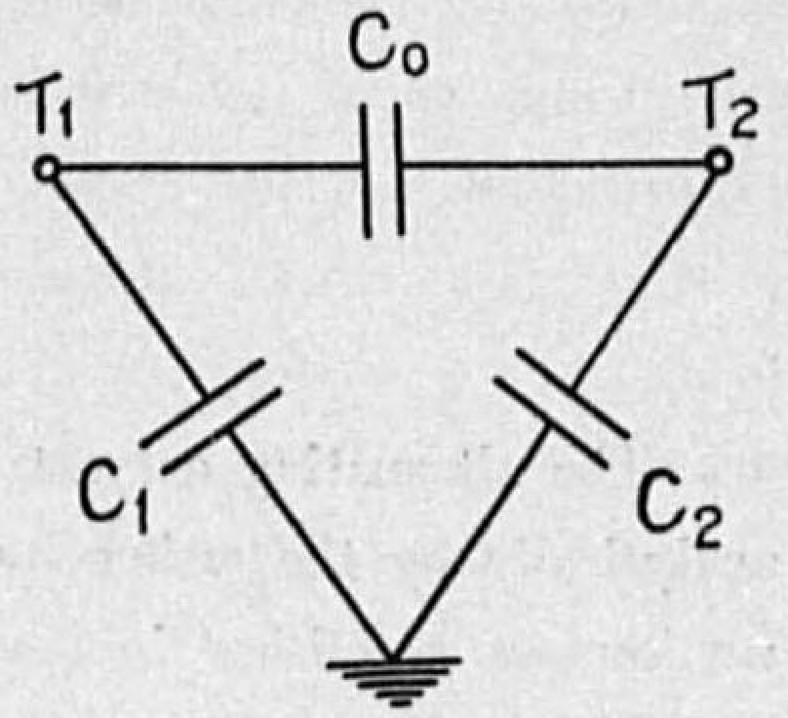

FIG. 1.

No-loss air condensers of the above principle can be constructed in various forms, fixed or variable. The writer has constructed the five kinds of them which are shown in the photographs at the end of this paper.

Type *A* is a fixed condensers of about 5000 micro-microfarads with square aluminium plates 77 in number and each 1 mm. thick and 130 mm. square with an air space of 2 mm. between successive plates. The solid insulators to support each set of plates were fixed to upper and lower metallic plates connected to earth and each was surrounded by an earthed metallic case.

Type *B* and *C* are fixed condensers of about 5000 and 1000 micro-microfarads respectively with the arrangement of subdividing the capacity in ten steps by means of plugs. The plates are of aluminium and circular 150 mm. diameter and 1 mm. thick and 61 in number for type *B* and 21 for type *C*, the air space between two successive plates being 2 mm. and 3 mm. respectively. The mode of mounting the two sets of plates is similar to that of type *A*.

Type *D* is a variable condenser of about 100 to 600 micro-microfarads. The vernier is provided for close reading and it is possible to read to about 0.5 micro-microfarad. The fixed plates are semi-circular 140 mm. diameter and 23 in number while the moving plates are of a sector of a circle 106 mm. diameter, the angle subtended at the center being 150° and 22 in number. They are both of aluminium, each 1 mm. thick and with 2 mm. air space between successive plates. They are so proportioned that the change of capacity is, over a wide range, nearly proportional to the angle turned through by the moving plates. The mode of mounting the fixed and the moving plates is similar to that of type *A*. In connecting the condenser of this type in alternating current bridge, the moving plates must be connected to the detector terminal of the bridge, the fixed plates being connected to the oscillator terminal, in order to avoid the undesirable effect of the hand of the observer.

Type *E* is a variable condenser of about 10 to 130 micro-microfarads. It consists of two sets of fixed plates and one set of moving plates, the former being connected in an arm of alternating current bridge, while the latter to earth. When the moving plates are passed through between two sets of fixed plates, the capacity between two sets of fixed plates is continuously diminished until the minimum capacity is attained when the moving plates are laid entirely between two sets of fixed plates. The moving plates and that set of fixed plates which is to be connected to the detector terminal of alternating current bridge are semi-circular while the other set of fixed plates which is to be connected to the oscillator terminal of alternating current bridge is made circular with a circular hole in the center through which the shaft of the moving plates passes. All the plates are of aluminium 1 mm. thick and about 105 mm. diameter. The number of fixed plates are 12 and 13 while that of moving plates 24. The air space between two successive plates is 2 mm., making the air space between two successive fixed plates 5 mm. The change of capacity between two sets of fixed plates is made, over a wide range, nearly proportional to the angle turned through by the moving plates. A vernier is provided as in type *D* and it is possible to read to about 0.1 micro-microfarad. The advantage of the condenser of this construction is that the minimum capacity can be made very small so that it is particularly fitted to be connected in parallel with the resistance of small residual capacity.

The condensers above described are all expected to be absolutely free from losses, although we can not verify it by means of alternating current bridges whatever, as we are only able to compare any two impedances by means of alternating current bridges and we have no standard to compare with for the above purpose. It is possible, however, to confirm that the condenser constructed on the above principle has always the same loss-angle (which is expected to be zero). This can be done in the following way.

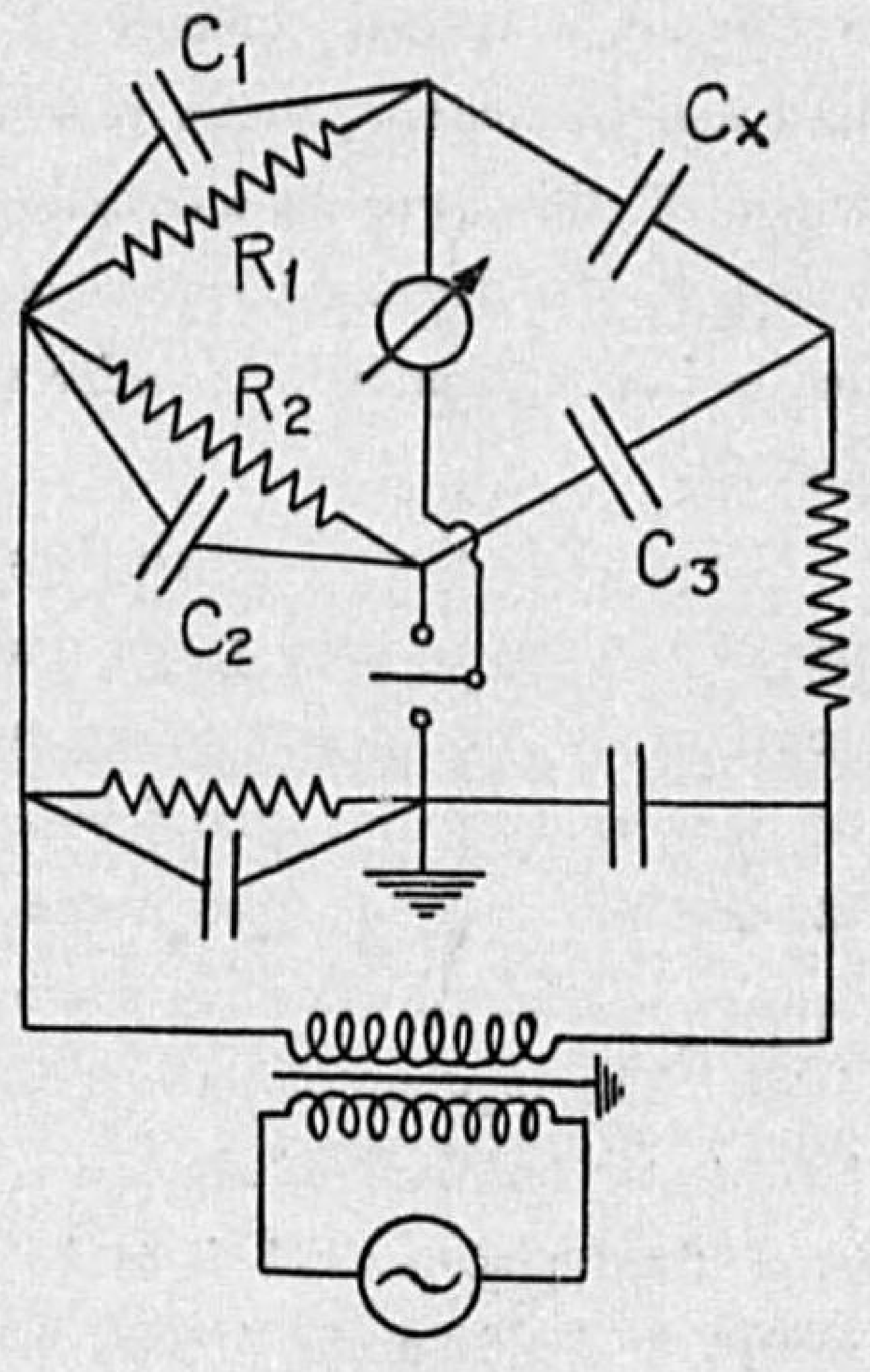

Fig. 2.

Firstly, the loss-angles of two condensers of nearly equal capacity can easily be compared by the substitution method preferably in the bridge shown in Fig. 2, where $R_1 R_2$ are two fixed resistances of nearly equal magnitude, and $C_1\ C_2\ C_3$ are all adjustable condensers. Two condensers, the loss-angles of which are to be compared, are connected in the place of C_x one after the other, the bridge being balanced at each time by the adjustment of the condensers C_2 and C_3. The difference of the loss-angles to be compared is then readily obtained from the resistance R_2 and the difference of the two capacities of the condenser C_2 at the two balances of the bridge. Thus, let $\delta_1\delta_2$ be loss-angles of the two condensers to be compared, and C_2C_2' be the corresponding two capacities of the condenser C_2. Then evidently we have

$$\delta_2-\delta_1=\omega R_2(C_2'-C_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where ω is 2π times the frequency.

Secondly, the loss-angles of two condensers of unequal capacity can, of course, not be compared directly by the above method, as the loss-angle of the condenser C_3 can not be assumed to be equal in the two balances of the bridge. But we can easily compare, by the above method, the resultant loss-angle of any number of condensers connected in parallel, with that of a single condenser of equal capacity, and consequently by the repeated application of the above method, we are able to compare the loss-angles of any two condensers.

The writer has made these measurements for the various types of condensers described before and confirmed experimentally that they have nearly equal loss-angles, not differing more than a few parts in 100000 at the frequency of 1000 cycles per second. These results have, however, been attained after the following precautions had been thoroughly paid.

Firstly, the sharp edges of the plates were all round off to prevent the silent discharge therefrom. This effect had been found considerable especially when the air is damp, even at such low voltages as used in ordinary alternating current bridges.

Secondly, the air space between two successive plates was kept not less than 2 mm. The writer had actually experienced that 1.5 mm. air space was far inferior to 2 mm. air space.

The air condenser described before is also considered to offer the same capacity up to the fairly high frequencies because the air is only used as the dielectric between two sets of plates and no appreciable losses are produced in it.

The use of the no-loss air condenser described before is wide; it may be used with advantage in the bridges proposed in the writer's previous paper for the measurement of the capacity and the leakage conductance of a condenser and the effective resistance and the inductance of an inductance coil at any frequency. It may also be used for the determination of the effective resistance and the residual capacity of a resistance coil, as will be fully described subsequently.

Finally it is to be added that the no-loss air condenser designed by the writer is so convenient for fine adjustment, that the writer is now principally making adjustment of condensers rather than resistances in the balance of the bridge except at very low frequencies. This is especially recommended in the case of high frequency bridges where the adjustment of resistances accompanies the trouble of changing the residual capacity at the same time.

III. CALIBRATION OF CAPACITY OF NO-LOSS AIR CONDENSERS

The capacity of no-loss air condenser described in the previous article may be calibrated from the ratio of two resistances in a bridge such as shown in Fig. 2. It requires, however, the exact knowledge of the ratio of two resistances which is often troublesome. The following method is used by the writer for the purpose.

The method is based on the same principle as comparing two lengths by subtracting the smaller length from the longer one as many times as is feasible and then comparing the remainder with the smaller length by the same process and so on. Thus, for example, let the standard capacity be 10000 micro-microfarads and the capacity to be calibrated be 1050 micro-microfarads. Then subtract at first 1050 micro-microfarads nine times from 10000 micro-microfarads, and the remainder is 550 micro-microfarads. Next subtract the remainder 550 micro-microfarads from 1050 micro-microfarads, and the remainder is 500 micro-microfarads. Finally subtract the second remainder 500 micro-microfarads from the first remainder 550 micro-microfarads and the third remainder is 50 micro-microfarads. Now if the third remainder 50 micro-microfarads could be measured by some supplementary method, it is evident that we can readily determine the capacity to be calibrated with reference to the standard capacity. The small capacity such as 50 micro-microfarads can be best measured by the substitution method in a variable air condenser which is so constructed that the change of capacity is, over a wide range, proportional to the angle turned through by the moving plates. The calibration of such a condenser will be described in a later part of this article.

Now considering the general case, let the standard capacity be C_s and the capacity to be calibrated be C_x. Assume that the capacity C_x is smaller than the capacity C_s. Then subtract the capacity C_x from the capacity C_s by n_1 times until the remainder C_{r1} becomes smaller than C_x. Then we have

$$C_s - n_1 C_x = C_{r1} \quad \dots\dots\dots\dots\dots\dots \quad (2)$$

Next subtract the remainder C_{r1} from C_x by n_2 times until the second remainder C_{r2} becomes smaller than the first remainder C_{r1}. Then we have

$$C_x - n_2 C_{r1} = C_{r2} \quad \dots\dots\dots\dots\dots\dots \quad (3)$$

Next subtract the second remainder C_{r2} from the first remainder C_{r1} by n_3 times until the third remainder C_{r3} becomes smaller than the second remainder C_{r2}. Then we have

$$C_{r1} - n_3 C_{r2} = C_{r3} \quad \dots\dots\dots\dots\dots\dots \quad (4)$$

Proceeding in this way, we finally come down to a small capacity as the final remainder which can be measured by the substitution method in a variable air condenser. Let this remainder be the mth remainder and denoted by C_{rm}. If $m=1$, we have the single equation (2) from which we can readily determine the capacity C_x. If $m=2$, we have the two equations (2) and (3) from which we have

$$C_s - \left(n_1 + \frac{1}{n_2}\right) C_x = -\frac{C_{r2}}{n_2} \quad \dots\dots\dots\dots \quad (5)$$

which may be used for the determination of the capacity C_x. It is to be noticed that the equation (5) may be obtained from the equation (2) by substituting $n_1 + \frac{1}{n_2}$ for n_1 and $-\frac{C_{r2}}{n_2}$ for C_{r1}. If $m=3$, we have the three equations (2) (3) (4) from which we can easily get the equation which is to be used for the determination of the capacity C_x by eliminating the first and the second remainders $C_{r1} C_{r2}$. That is, we have

$$C_s - \left(n_1 + \cfrac{1}{n_2 + \cfrac{1}{n_3}}\right) C_x = \frac{C_{r3}}{n_3\left(n_2 + \frac{1}{n_3}\right)} \quad \dots\dots \quad (6)$$

It is easily seen that the equation (6) may be obtained from the equation (5) by substituting $n_2 + \frac{1}{n_3}$ for n_2 and $-\frac{C_{r3}}{n_3}$ for C_{r2}. Similarly, if $m=4$, we have

$$C_s - \left(n_1 + \cfrac{1}{n_2 + \cfrac{1}{n_3 + \cfrac{1}{n_4}}}\right) C_x = -\frac{C_{r4}}{n_4\left(n_3 + \frac{1}{n_4}\right)\left(n_2 + \cfrac{1}{n_3 + \cfrac{1}{n_4}}\right)} \quad . \quad (7)$$

which may be obtained from the equation (6) by substituting $n_3+\frac{1}{n_4}$ for n_3 and $-\frac{C_{r4}}{n_4}$ for C_{r3}.

In carrying out the above process in practice, the bridge shown in Fig. 2 may be conveniently employed. The resistances $R_1 R_2$ are made nearly equal and the condensers C_3 and C_x (the latter will be called C_4 hereafter in order to avoid the confusion) are made adjustable continuously from the capacity equal to the standard capacity C_s down to a small capacity in the range of a variable air condenser by the combination of the various types of no-loss air condensers described in the previous article. The standard condenser C_s is at first connected in place of the condenser C_4 and the bridge is balanced by the adjustment of the condensers C_3 and C_2. The standard condenser C_s is then cut off and the condenser C_4 and the condenser C_x which is to be calibrated are connected, and the bridge is balanced by the adjustment of the condensers C_4 and C_2, the condenser C_3 being kept absolutely unchanged. By the above process, the condenser C_4 is obviously adjusted to represent the difference of the capacities C_s and C_x. Next, disconnect the condenser C_x and balance the bridge by the adjustment of the condensers C_3 and C_2, the condenser C_4 being kept absolutely unchanged and then connect the condenser C_x and balance the bridge by the adjustment of the condensers C_4 and C_2, the condenser C_3 being kept absolutely unchanged. After the above process, we evidently get the capacity C_s-2C_x in the condenser C_4. Hence proceeding in this way, we can obviously carry out the measurement previously described.

In the above measurements, it is often necessary to have a fixed condenser of about 1000 micro-microfarads connected in parallel with the condenser C_4 in order to remove the practical difficulty in balancing the bridge when the capacity of the condenser C_4 has become sufficiently small.

The variable air condenser, such as Type D and E described in the previous article, may be calibrated by the similar process. The condenser has, however, to be tested at first in what range the change of capacity is proportional to the angle turned through by the moving plates. This can be done easily by dividing the whole scale division of the condenser into a number of equal sections and comparing the change of capacity in each section by the substitution method.

After the condenser has been tested for the range in which the change of capacity is proportional to the angle turned through by the moving plates, the change of capacity in this range is calibrated in the following way.

The variable air condenser which is to be calibrated is connected in the arm of the condenser C_4 in the bridge described before and never cut off during the measurement. The standard condenser is at first connected in place of the condenser C_4 as before and the variable air condenser to be calibrated is set in its minimum position in the range of proportionality and the bridge is balanced by the adjustment of the condensers C_3 and C_2. Then the standard condenser is cut off and the condenser C_4 is connected and the variable air condenser to be calibrated is set in its maximum position in the range of proportionality and the bridge is balanced by the adjustment of the condensers C_4 and C_2, the condenser C_3 being kept absolutely unchanged. Next, the variable air condenser to be calibrated is again set in its minimum position and the bridge is balanced by the adjustment of the condensers C_3 and C_2, the condenser C_4 being kept absolutely unchanged; and then the variable air condenser to be calibrated is set in its maximum position and the bridge is balanced by the adjustment of the condensers C_4 and C_2, the condenser C_3 being kept absolutely unchanged. Proceeding in this way, we finally get the capacity of the condenser C_4 smaller than the difference of the maximum and the minimum capacities of the variable air condenser to be calibrated in the range of proportionality. Then the variable air condenser to be calibrated is set in its minimum position as before and the bridge is balanced by the adjustment of the condensers C_3 and C_2, the condenser C_4 being kept absolutely unchanged; and then the condenser C_4 is cut off and the bridge is balanced by the adjustment of the condenser C_2 and the variable air condenser to be calibrated. Let the scale reading of the variable air condenser in this balance be θ_r and those of the maximum and the minimum capacities in the range of proportionality be $\theta_{max.}$ and $\theta_{min.}$ respectively. Then we can easily show that the change of capacity per scale division of the variable air condenser to be calibrated is given by

$$\gamma=\frac{C_s}{n(\theta_{max.}-\theta_{min.})+(\theta_r-\theta_{min.})} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where C_s is the capacity of the standard condenser and n is one half of the number of balances of the bridge, excepting the last two balances.

IV. RESIDUAL CAPACITY OF RESISTANCE COILS

The residual capacity of a resistance coil as we use throughout this paper is the same as defined in the writer's previous paper. It is not simply the capacity distributing around the resistance coil, but has a definite meaning. A resistance coil, whatever the mode of its winding may be, is a two-terminal network, and in the case of the steady state of sinusoidal alternating current of a definite frequency, it can be represented by the equivalent network shown in Fig. 3 where R is the direct current resistance of the coil and $y_0\, y_1\, y_2$ are admittances due to the distributed inductances and capacities in the coil and the frequency of the alternating current. The residual capacity of the coil is defined to be that capacity which is corresponding to the imaginary component of the admittance y_0. We shall now show that the residual capacity thus defined can be made practically independent of the frequency and at the same time the real component of the admittance y_0 can be made practically negligible compared with the direct current resistance R up to fairly high frequencies by the suitable design of the coil.

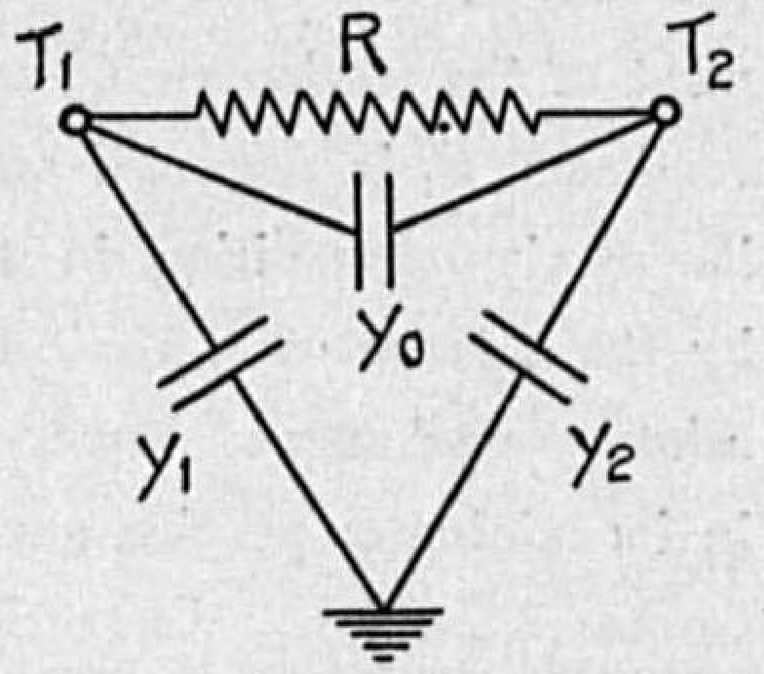

FIG. 3.

Let us now first consider the effect of distributed inductances. The distributed inductance in a resistance coil, however complicated the distribution is, can be represented as an inductance l in series with the direct current resistance R. If the power loss due to this inductance l is also to be considered, it may be represented as an effective resistance r in series with the inductance l. The increase of effective resistance of the resistance coil due to skin effect in the wire may be included in this effective resistance r. Thus the whole impedance of the resistance coil can be denoted, in symbolic vector notations, as

$$Z=R+r+j\omega l \qquad (9)$$

where ω is 2π times the frequency.

Now if we transform the formula (9) as

$$Z=\frac{1}{\dfrac{R+r-j\omega l}{(R+r)^2+(\omega l)^2}} \qquad (10)$$

we easily see that the admittance y_0 in the equivalent network of Fig. 3 is represented by

$$y_0=\frac{R+r-j\omega l}{(R+r)^2+(\omega l)^2}-\frac{1}{R} \qquad (11)$$

If r and ωl are sufficiently small compared with R, the formula (11) can be approximately written as

$$y_0 \doteqdot -(r+j\omega l)\frac{1}{R^2} \qquad (12)$$

which shows that the residual capacity due to the distributed inductance l is independent of frequency and is given by $-\frac{l}{R^2}$ approximately provided that the distributed inductance is small. The real component of the admittance y_0 is also negligible in that case. Hence we see that the distributed inductance should be made as small as possible in order to get a resistance coil of constant resistance and residual capacity. The distributed inductance can be reduced considerably by winding the coil in such a way that the two wire carrying the current in opposite directions lie close together.

Next, let us consider the effect of capacities around the resistance coil. For brevity, suppose that a single capacity c is located between two points ab of the coil, as shown in Fig. 4. Let the leakage conductance of this capacity be g and the resistance between the two points ab of the coil be R'. Then the impedance of the coil as seen from the two terminals T_1T_2 is evidently

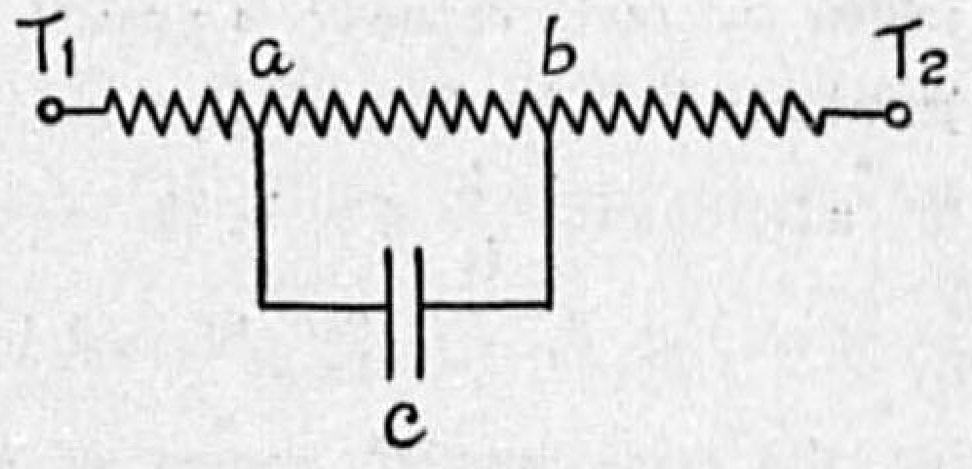

FIG. 4.

$$Z=R-R'+\frac{1}{\dfrac{1}{R'}+g+j\omega c} \qquad (13)$$

Now if g and ωc are sufficiently small compared with $\frac{1}{R}$, the formula (13) can be approximately written as

$$Z \doteqdot R-(g+j\omega c)R'^2 \quad \dots\dots\dots\dots\dots \quad (14)$$

Whence the admittance y_0 of the equivalent network of Fig. 3 is given approximately by

$$y_0 \doteqdot (g+j\omega c)\left(\frac{R'}{R}\right)^2 \quad \dots\dots\dots\dots\dots \quad (15)$$

showing that the residual capacity is independent of frequency and is given by $c\left(\frac{R'}{R}\right)^2$ approximately provided that the distributed capacity is small. The real component of the admittance y_0 can also be made negligible by the use of the good insulating materials. Further, it is very important to wind the resistance coil in such a way that the distributed capacity is confined between the neighbouring parts of the whole length of the resistance wire, thus making the ratio $\frac{R'}{R}$ small and reducing the admittance y_0.

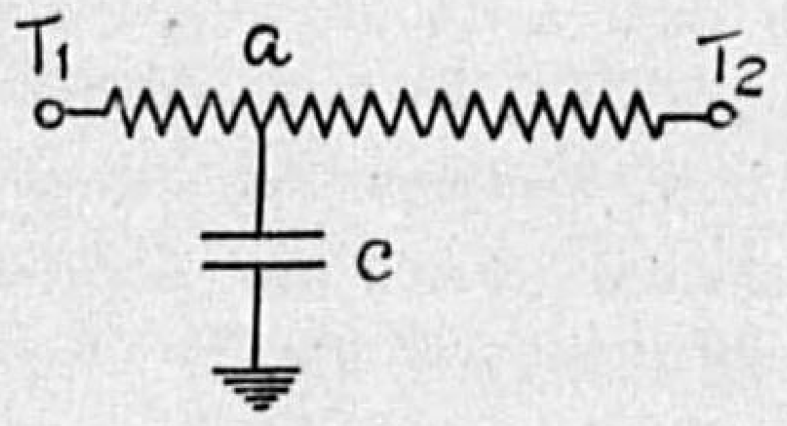

Fig. 5.

Finally consider the earth capacity effects of the coil. For brevity, suppose that a single capacity c is located between a point of the coil and the earth as shown in Fig. 5. Let the leakage conductance of this capacity be g and the resistances between the point a at which the earth capacity is present and the two terminals T_1T_2 be R_1 and R_2 respectively. Then the admittance $\frac{1}{R}+y_0$ in the equivalent network of Fig. 3, is easily obtained by transforming the star-connected system of admittances $\frac{1}{R_1}$, $\frac{1}{R_2}$ and $g+j\omega c$ into its equivalent delta-connected system. That is, we have

$$Z=\frac{1}{\frac{1}{R}+y_0}=R+(g+j\omega c)R_1R_2 \quad \dots\dots\dots\dots \quad (16)$$

Whence the admittance y_0 is given by

$$y_0 \doteqdot -(g+j\omega c)\frac{R_1R_2}{R^2} \quad \dots\dots\dots\dots\dots \quad (17)$$

approximately. Hence the residual capacity is also independent of frequency and is given by $-c\frac{R_1R_2}{R^2}$ approximately provided that the earth capacity is small. The real component of the admittance y_0 is also negligible in that case.

The actual distribution of inductance and capacity in a resistance coil is never so simple as treated above, but if they are small, it is obvious that their effects may be considered independently; and hence we can generally say that it is possible to make a resistance coil of constant resistance and residual capacity by reducing the distributed inductance and capacity independently. The subdivided bifilar winding, Chaperon's winding or Curtis and Grover's winding might be used the most advantageously for the purpose. The insulating material surrounding the resistance wire should, of course, be so good that the dielectric constant is not appreciably varied with frequency and the dielectric loss is not considerable.

In the above discussions, we have also been given a means of getting a pure resistance with no residual capacity by counter-balancing the effects of the distributed inductance and capacity in the coil and the capacity to the earth, as the effects of the distributed inductance and the capacity to the earth are opposite to that of the distributed capacity in the coil. However, so far for the use in alternating current bridges with perfect earthing device, it is more convenient to make the residual capacity of a resistance coil a little negative so that when it is connected in parallel with a variable air condenser, it may become possible to make the total shunt capacity vary continuously from a certain positive value down to a small negative value, passing through zero. Such a combination of resistance and capacity is very often useful in alternating current bridges with perfect earthing device.

V. DETERMINATION OF EFFECTIVE RESISTANCE AND RESIDUAL CAPACITY OF RESISTANCE COILS

The effective resistance and the residual capacity of a resistance coil can be best determined by the use of no-loss air condensers described before. The method is based on the assumption that the condensers described before are absolutely free from losses and their capacities do not vary with frequency. The effective resistance and the residual capacity of a resistance coil are then determined in terms of capacity and frequency.

The two alternating current bridges shown in Figs. 6 and 7 are successively made use of for this purpose. The first bridge is at first balanced and then it is transformed into the second bridge and it is again balanced.

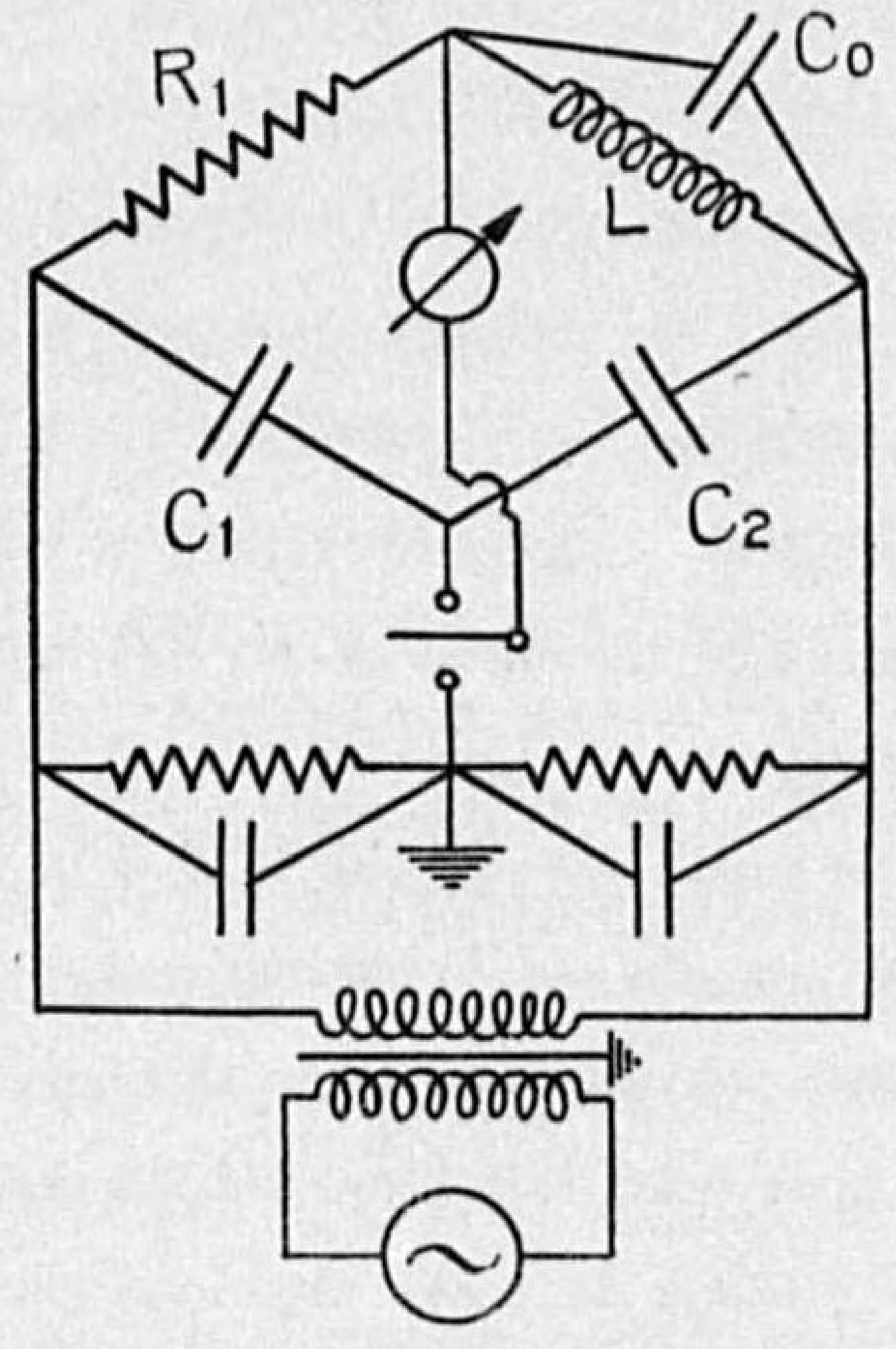

FIG. 6.

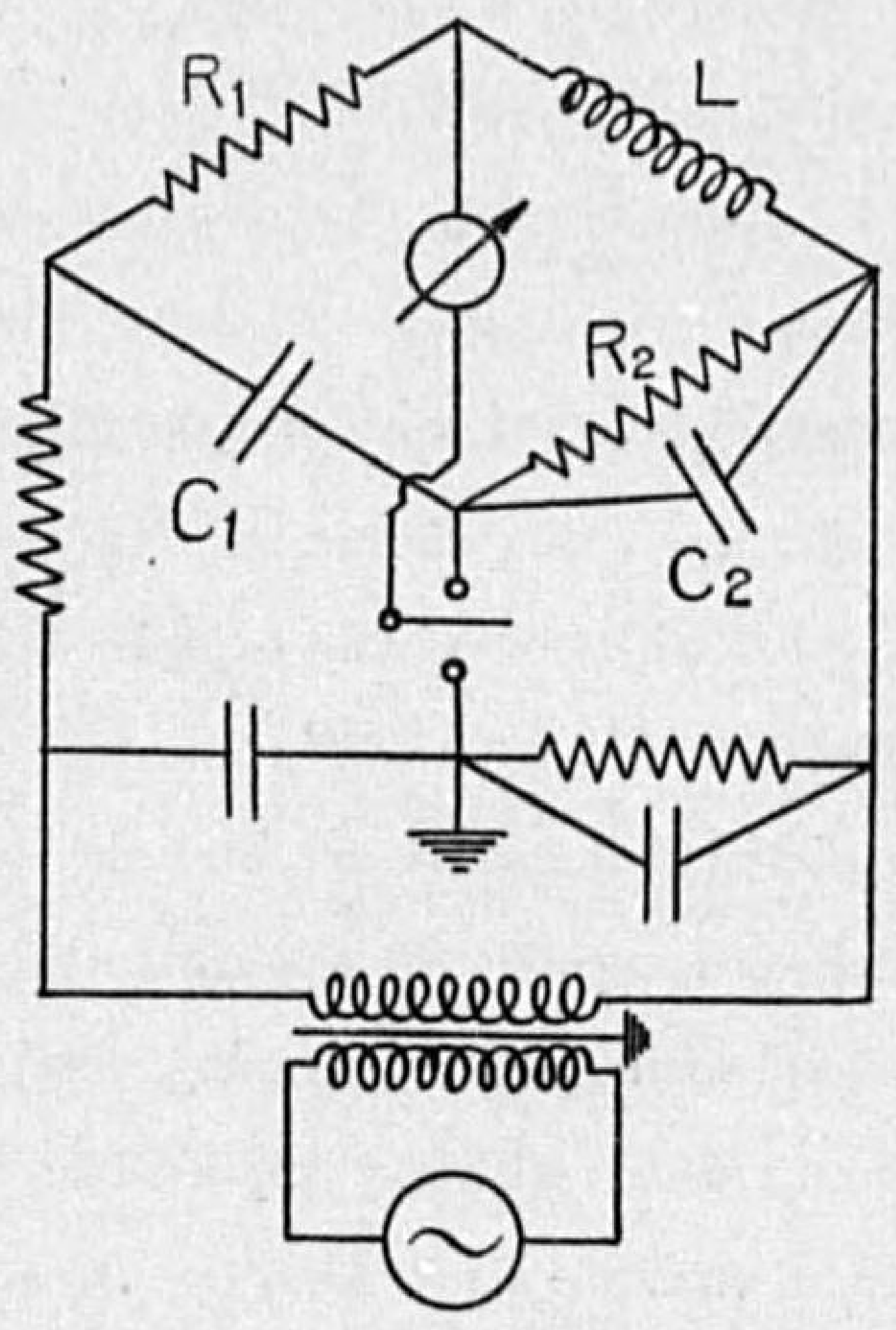

FIG. 7.

The first bridge is a kind of resonance bridge where the inductance coil L is put in resonance with the condenser C_0, the bridge being balanced by the adjustment of two condensers C_0 and C_2. These condensers should be all of no-loss air condensers described previously and the inductance coil should be of an air core type. The resistance R_1 is that resistance, the effective resistance and the residual capacity of which are to be determined.

The second bridge is the same as used by the writer for the measurement of the effective resistance and the inductance of an inductance coil at any frequency. The inductance coil L and the resistance R_1 in this bridge are the same as in the first bridge, the former being not allowed even to displace from its initial position so that the inductance may not vary during the measurement. The condensers C_1 and C_2 are no-loss air condensers, by the adjustment of which the bridge is to be balanced.

These two bridges should be so connected up that they may be transformed to each other with a slight change of connections, every apparatus in the bridges being not displaced or dislocated as far as possible.

The method of measurement is as follows:

The first bridge is balanced by the adjustment of the condensers C_0 and C_2 as described before, the frequency of the source being kept constant. The bridge is then immediately transformed to the second bridge and it is balanced by the adjustment of the condensers C_1 and C_2 at the same frequency as in the first bridge.

The balance condition of the first bridge is evidently

$$C_2\left(\frac{1}{R_1}+j\omega c_1\right)=C_1\left(\frac{1}{Z_0}+j\omega C_0\right) \quad \cdots\cdots\cdots \quad (18)$$

where Z_0 is the impedance of the inductance coil L and c_1 is the residual capacity of the resistance R_1. The balance condition of the second bridge is

$$Z_0=\frac{j\omega C_1'}{\left(\frac{1}{R_1}+j\omega c_1\right)\left\{\frac{1}{R_2}+j\omega(C_2'+c_2)\right\}} \quad \cdots\cdots\cdots \quad (19)$$

where the capacities C_1C_2 are attached with dashes in order to distinguish them from those in the first bridge and c_1c_2 are residual capacities of the resistances R_1R_2 respectively.

Eliminating the impedance Z_0 from the above two equations (18) and (19), we have

$$\left(\frac{1}{R_1}+j\omega c_1\right)\left\{\frac{1}{R_2}+j\omega\left(C_2'-C_1'\frac{C_2}{C_1}+c_2\right)\right\}=\omega^2 C_0 C_1' \quad . \quad . \quad . \quad (20)$$

Or separating the real and imaginary components, we have

$$\frac{1}{R_1}\frac{1}{R_2}=\omega^2\left\{C_0 C_1'+c_1\left(C_2'-C_1'\frac{C_2}{C_1}+c_2\right)\right\} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

and

$$\frac{R_1}{R_2}c_1+c_2=C_1'\frac{C_2}{C_1}-C_2' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Now the resistances $R_1 R_2$ can be easily compared by the supplementary bridge shown in Fig. 8 where $R_1 R_2$ are the two resistances to be compared and $C_1 C_2 C_3 C_4$ are all no-loss air condensers described previously. The bridge is balanced by the adjustment of the condensers C_2 and C_4. The balance condition of the bridge is evidently

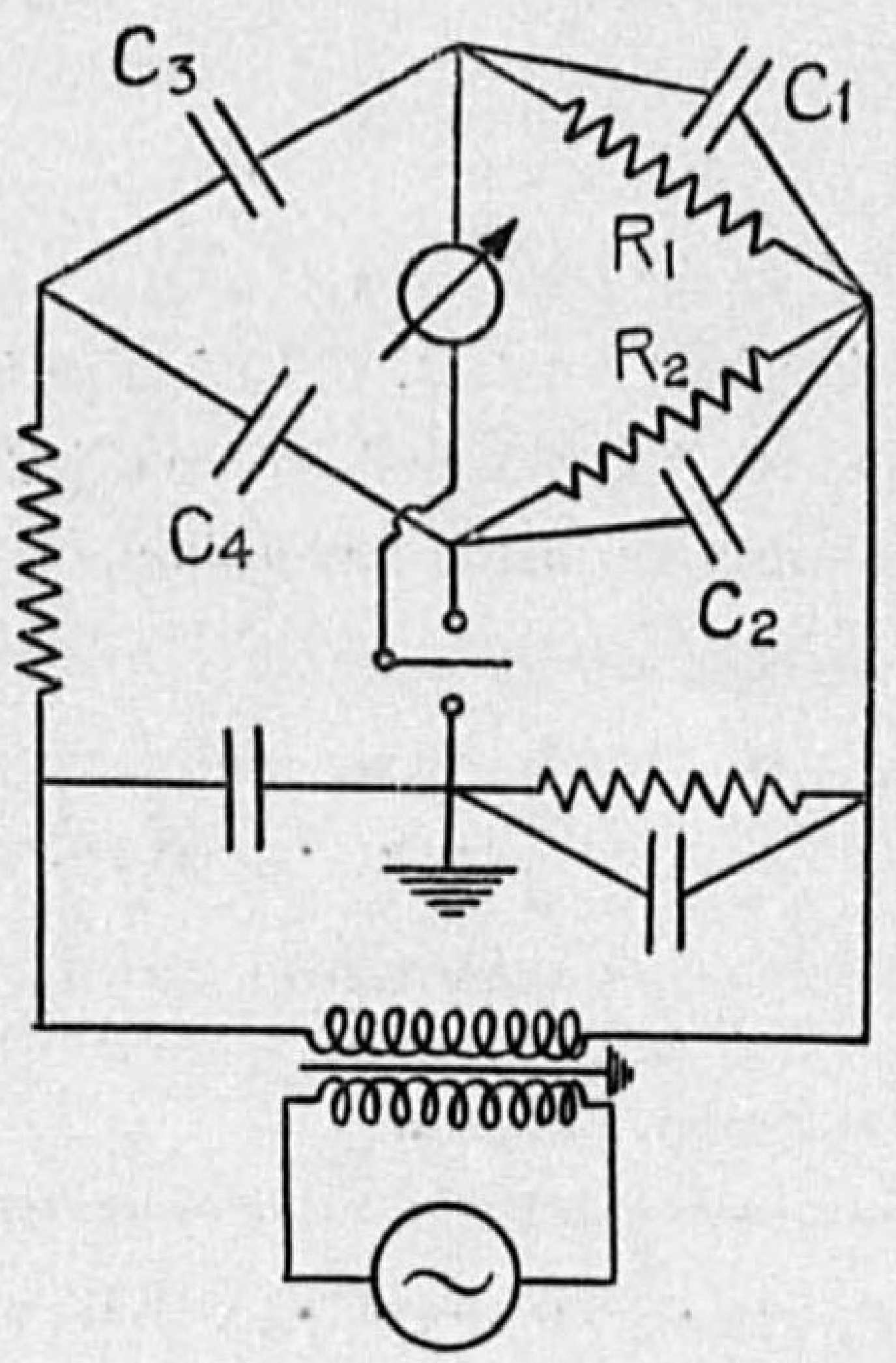

Fig. 8.

$$C_3\left\{\frac{1}{R_2}+j\omega(C_2+c_2)\right\}$$
$$=C_4\left\{\frac{1}{R_1}+j\omega(C_1+c_1)\right\} \quad . \quad . \quad (23)$$

where $c_1 c_2$ are the residual capacities of the resistances $R_1 R_2$ respectively.

From the equation (23), we get readily

$$\frac{R_1}{R_2}=\frac{C_4}{C_3} \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and

$$\frac{C_1+c_1}{C_2+c_2}=\frac{C_3}{C_4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Transforming the equation (25) and substituting the equation (24), we have

$$\frac{R_1}{R_2}c_1-c_2=C_2-C_1\frac{C_4}{C_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Hence we can determine the residual capacities $c_1 c_2$ from the equations (22) (24) (26) and then the effective resistances $R_1 R_2$ from the equations (21) and (24).

The above method can be carried out at audio-frequency as well as at radio-frequency. However, the following precautions must be thoroughly paid.

Firstly, the inductance coil L should be at a considerable distance from the oscillator and the detector as well as the other apparatus of the bridge, so that it may not interfere with them by electromagnetic induction. The leading wires to the inductance coil may be of any length whatsoever, since its effect is eliminated together with the impedance of the coil. If the connection of the inductance coil to the bridge is reversed and the balance of the bridge is not changed, it is sure that the electromagnetic effect of the inductance coil is of negligible importance.

Secondly, the modulation error in the detector as already noticed by J. G. Ferguson and B. W. Bartlett (19) must be carefully eliminated. The heterodyne detector or simply the telephone receiver, if overloaded, is known to cause an appreciable amount of modulation. Hence if the higher harmonics are much present in the bridge circuit, they will cause the fundamental as the modulation product even when the bridge is actually balanced for the fundamental. Under such conditions, the fundamental in the detector can be eliminated only by unbalancing the bridge slightly. This effect can be well eliminated by the use of anti-resonant circuits or wave filters.

The above method is also applicable to the determination of capacity in terms of resistance and frequency. Since the effective resistance of a resistance coil with good insulation can be assumed to be independent of the frequency at least in the range of audio-frequencies, the effective resistance determined by the above method at audio-frequency ought to agree with the direct current resistance; and consequently if we know the direct current resistance, we can determine the unit of capacity in terms of resistance and frequency. The following modification of the above method will be most conveniently employed for this purpose.

A fixed condenser, the capacity of which is to be determined, is connected in the place of the condenser C_0 in the bridge of Fig. 6 and a small adjustable air condenser is connected in parallel with it. The resistance R_1 should be non-inductive as nearly as possible. The bridge is balanced by the adjustment of the condenser C_2 and the small adjustable air condenser connected in parallel with the fixed condenser. The bridge is then transformed into the bridge of Fig. 7 when the fixed condenser is connected in the place of the condenser C_1 but the small adjustable air condenser connected in parallel with it, is left connected in parallel with the inductance coil with that setting at which the first bridge is balanced. The resistances R_1R_2 in the second bridge should be non-inductive as nearly as possible. The second bridge is balanced by the adjustment of the resistance R_2 and the condenser C_2. Then it is obvious from the equations (21) and (22) that we have in this case

$$\frac{1}{R_1}\frac{1}{R_2}\fallingdotseq\omega^2C^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (27)$$

approximately where C is the capacity of the fixed condenser to be determined. The capacity C is readily obtained from the equation (27). That is, we have

$$C\fallingdotseq\frac{1}{\omega\sqrt{R_1R_2}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (28)$$

In carrying out the above method, it is of fundamental importance to ascertain whether the effect of the residual capacities of the resistances R_1R_2 is negligible so that the equation (28) may be used without introducting errors. If it is not the case, the exact equation (21) must be employed which requires the knowledge of the residual capacities of the resistances R_1R_2.

VI. A NEW FREQUENCY BRIDGE COMPOSED OF RESISTANCES AND CONDENSERS

Although there are proposed a number of frequency bridges, most of them make use of inductance coils. The frequency bridge which does not make use of inductance coils is, so far as the writer knows, that developed by C. Robinson(13) in the Research Section of the Post Office Engineering Department in England. The writer has thought of a new frequency bridge composed of resistances and condensers which will be described here.

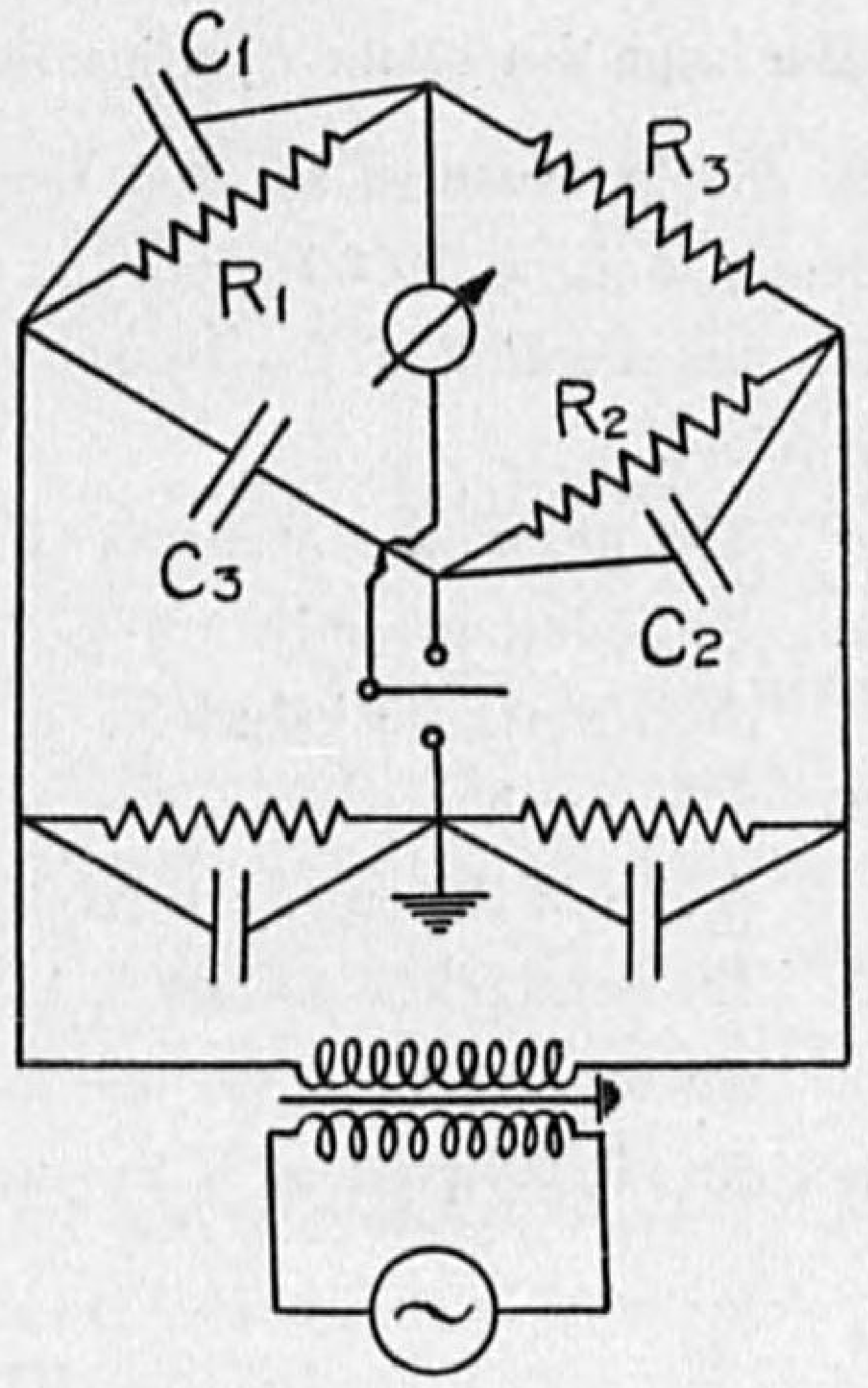

FIG. 9.

The connection of the bridge is shown in Fig. 9 where $R_1R_2R_3$ are resistances with known residual capacities and $C_1C_2C_3$ are no-loss air condensers described in Art. II. The bridge is balanced by the adjustment of the condensers $C_1C_2C_3$. The balance condition is evidently

$$j\omega C_3\left(\frac{1}{R_3}+j\omega c_3\right)=\left\{\frac{1}{R_1}+j\omega(C_1+c_1)\right\}\left\{\frac{1}{R_2}+j\omega(C_2+c_2)\right\} \quad . \quad (29)$$

where $c_1c_2c_3$ are residual capacities of the resistances $R_1R_2R_3$ respectively. Separating the real and imaginary components of the equation (29), we have

$$\frac{1}{R_1R_2}=\omega^2\{(C_1+c_1)(C_2+c_2)-c_3\mathrm{C}_3\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (30)$$

and $$\frac{C_2+c_2}{R_1}+\frac{C_1+c_1}{R_2}=\frac{C_3}{R_3} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (31)$$

The equation (30) is used for the determination of the frequency. That is, from the equation (30), we have

$$\omega=\frac{1}{\sqrt{R_1R_2\{(C_1+c_1)(C_2+c_2)-c_3C_3\}}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (32)$$

The inspection of the equation (32) shows that the frequency higher than $\frac{10^4}{2\pi}$ can be readily measured by this frequency bridge, provided that the resistances are available up to 10000 ohms and the condensers up to 20000 micro-microfarads. For the measurement of the lower frequency, the larger resistances or condensers are required.

The principal advantages of the above frequency bridge would be as follows:—

1. No inductance coils are used in the bridge so that there is no trouble due to electromagnetic induction.
2. The earth capacity effects of the bridge are entirely eliminated if the earthing device is thoroughly balanced.
3. No assumptions are made in the principle except that the capacity of no-loss air condenser and the effective resistance and the residual capacity of resistance coil are all independent of frequency.

VII. CONCLUSION

The no-loss air condensers described in Art. II are very useful in alternating current bridges with perfect earthing device, the accuracy and the use of the bridge being thereby considerably increased. Indeed it might be said that the advantage of the alternating current bridge with perfect earthing device has been fully realized by the introduction of the no-loss air condensers described in Art. II. We are now capable of measuring any impedance, inductive or capacitive, with reference to the standard of capacity and frequency by means of alternathing current bridges.

Thanks are accorded, in conclusion, to may officials and workmen in the workshop of this laboratory for kindness rendered by them in making various types of no-loss air condensers.

The writer also wishes to record his indebtness to Mr. M. Imatani who gave him his unstinted help throughout this work.

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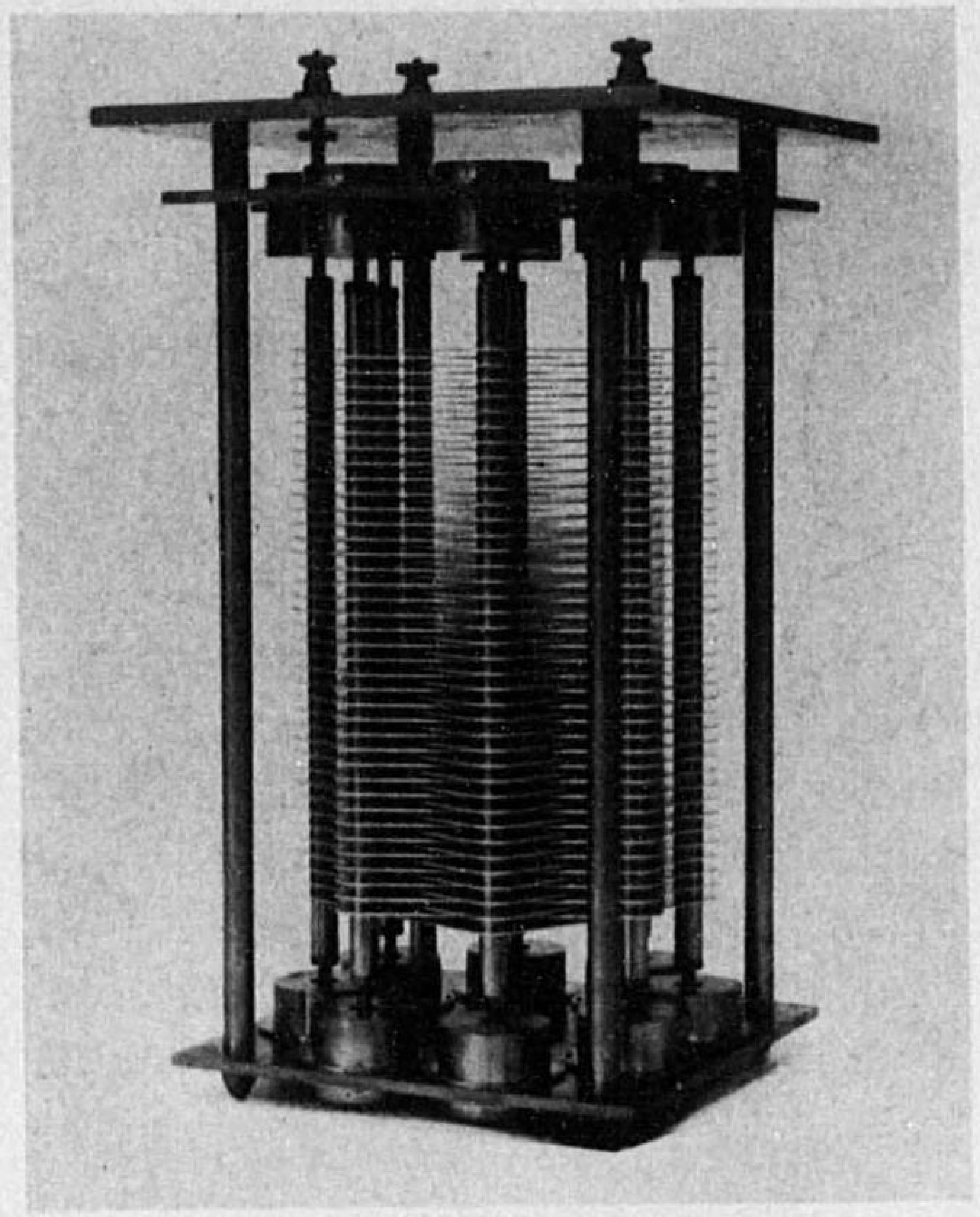
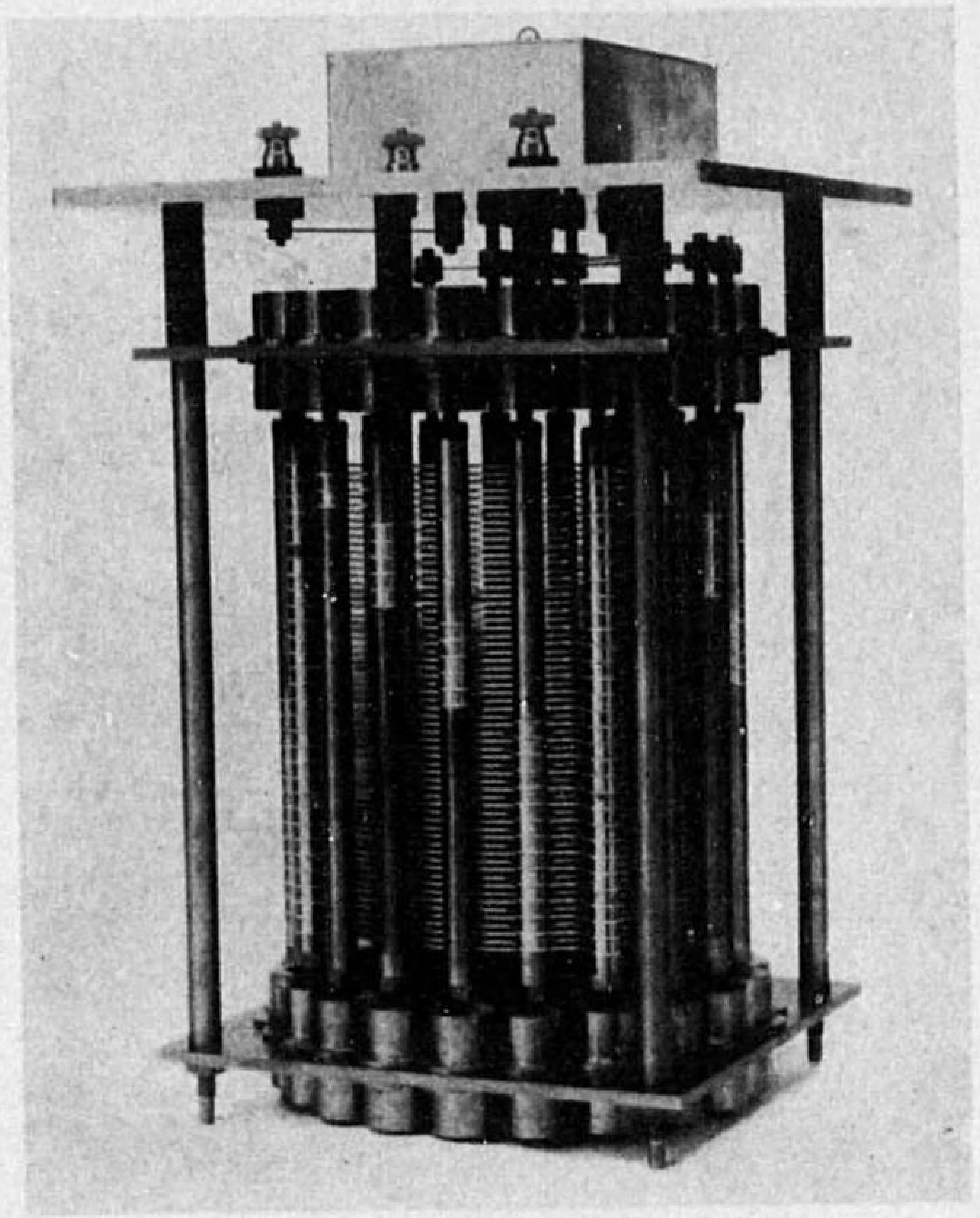
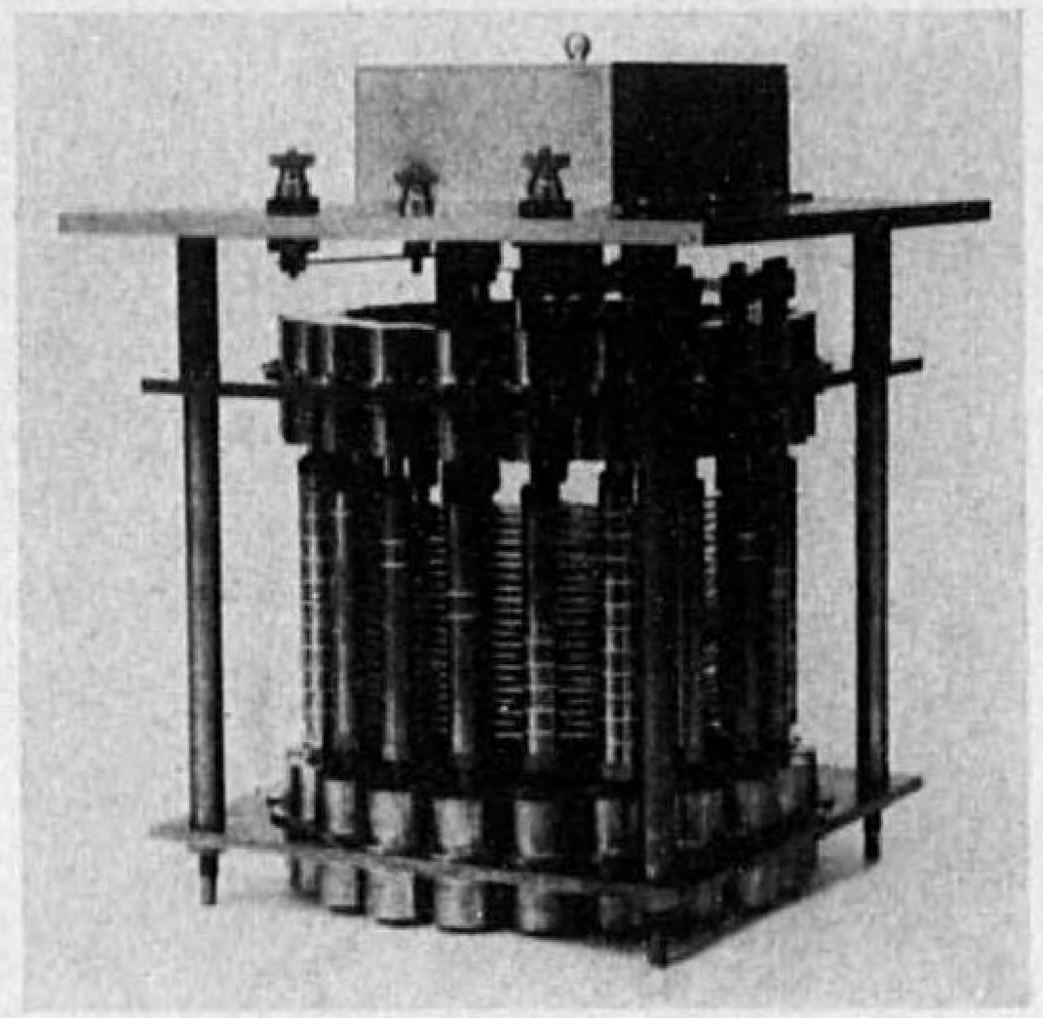


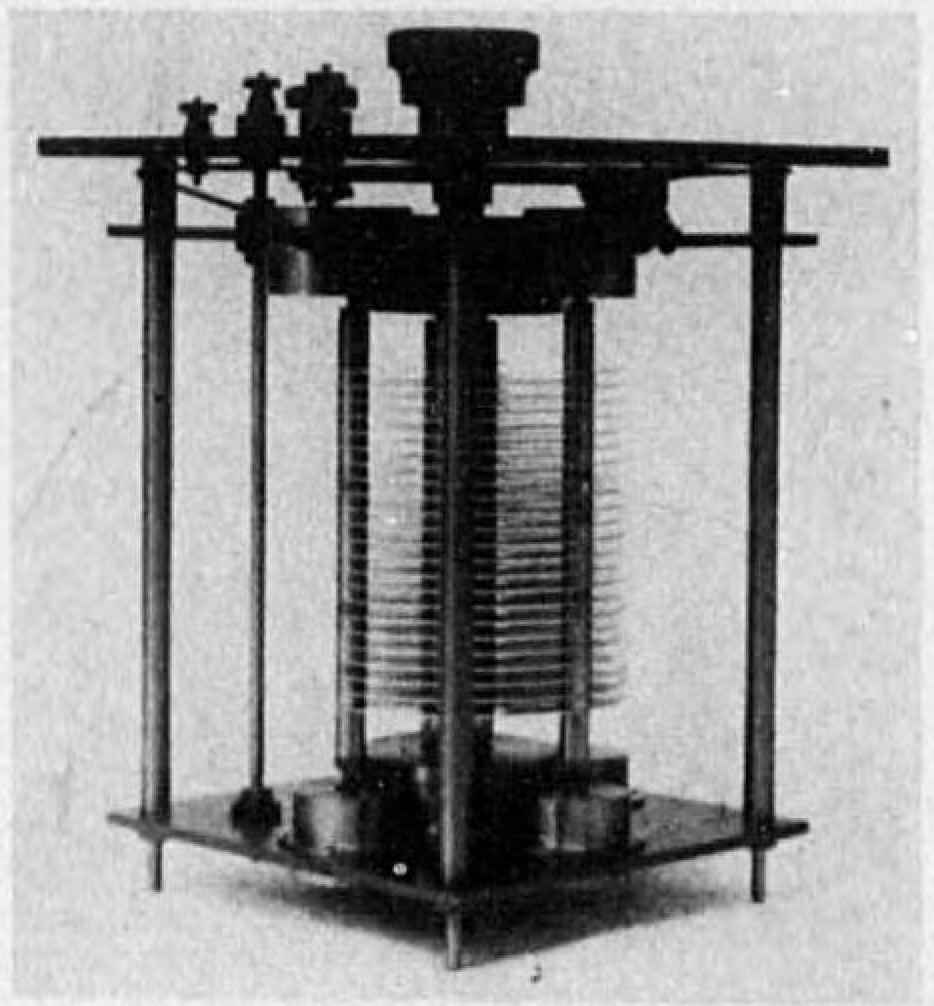
No-loss Air Condenser Type A



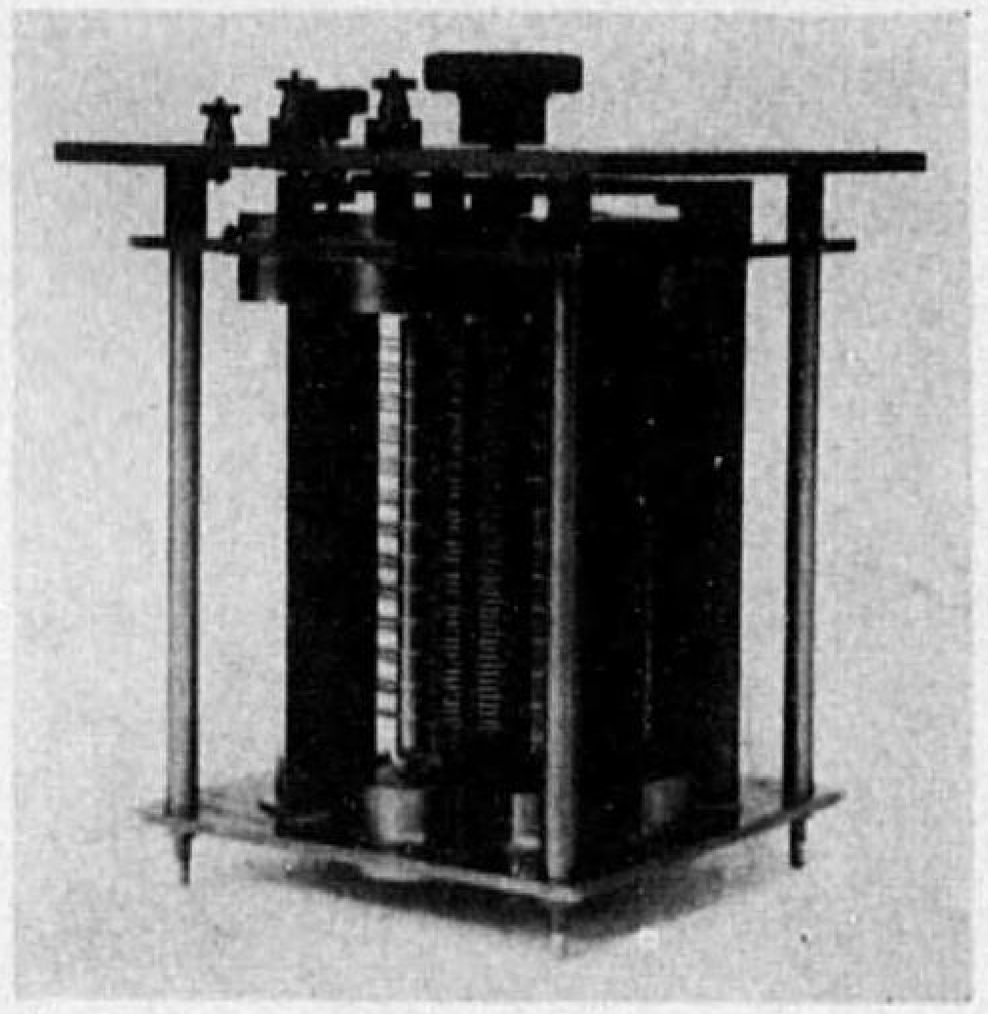
No-loss Air Condenser Type B



No-loss Air Condenser Type C



No-loss Air Condenser Type D



No-loss Air Condenser Type E

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