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TEMPERED DISTRIBUTIONS AND FOURIER TRANSFORM ON THE HEISENBERG GROUP

DISTRIBUTIONS TEMPÉRÉES ET TRANSFORMATION DE FOURIER SUR LE GROUPE DE HEISENBERG

ABSTRACT. — We aim at extending the Fourier transform on the Heisenberg group \mathbb{H}^d , to tempered distributions. Our motivation is to provide the reader with a hands-on approach that allows for further investigating Fourier analysis and PDEs on \mathbb{H}^d .

As in the Euclidean setting, the strategy is to show that the Fourier transform is an isomorphism on the Schwartz space, then to define the extension by duality. To achieve it, the Fourier transform of an integrable function is viewed as a uniformly continuous mapping on the set $\tilde{\mathbb{H}}^d = \mathbb{N}^d \times \mathbb{N}^d \times \mathbb{R} \setminus \{0\}$, that may be completed to a larger set $\hat{\mathbb{H}}^d$ for some suitable distance. This viewpoint provides a user friendly description of the range of the Schwartz space on \mathbb{H}^d by the Fourier transform, which makes the extension to the whole set of tempered distributions straightforward.

Keywords: Fourier transform, Heisenberg group, frequency space, tempered distributions, Schwartz space.

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