DAMPING OF THE OSCILLATIONS IN THE DISCHARGE OF A LEYDEN JAR

M.A. Thesis

Miss Harriet Brooks
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Damping of the Oscillations in the Discharge of a Leyden jar.

The method employed in the investigation of this subject depends on the partial demagnetisation of a magnetised steel needle when placed inside a solenoid thro' which a Leyden jar discharge is passed.

The action of a rapidly alternating current on a magnetised steel needle has been investigated by Professor Rutherford (Trans. Roy. Soc. June 1896), who has shewn that the method can be employed as a simple means of comparing the intensities of high frequency currents and also as a means of determining the damping. Erskine (Wied. Annal. Oct. 1897 Vol. 62) has employed such magnetised steel needles for measurements of the resistance of metals and electrolytes for rapidly alternating currents and also for the determination of specific inductive capacities.

The object of this investigation was to examine in detail the damping of the electrical oscillations in Leyden jar circuits under varying conditions of spark length, capacity and pressure.

The appearance of an air-break in the circuit connecting the outer and inner coatings of the jar when the discharge was passing, was examined by Feddersen by means of a rotating mirror in which the spark was reflected. He found that the image consisted of a series of bright and dark bands rapidly decreasing in breadth and intensity. and when a large resistance was placed in the circuit the image became a broad band of light gradually fading away in intensity.

This method was used by Trowbridge (Phil. Mag. Vol. 30 1893) who photographed the Vol. 43, 1897. image of the electric spark drawn out by the rotating mirror and measured the distances between the successive oscillations shewn by the dark bands on the photograph. In this way, he was enabled to make a comparison of the damping in a few cases but so complicated are the arrangements for these photographic experiments that no great range of observations could be made, and this is very necessary in order to make any reliable generalisations from so variable a phenomonon as the spark under consideration.

If two small oppositely wound solenoids A and B are placed in series in the discharge circuit of a Leyden jar and two exactly similar steel needle both magnetised to saturation, be placed one in each, with their north poles facing in the same direction, then on the passage of a discharge it will be found that the reduction of the magnetic moment is not the same in the two cases. Let $\alpha_1\ \alpha_2\ \alpha_3$ etc be the half-oscillations of the discharge in one direction and $\beta_1\ \beta_2\ \beta_3$ etc the half-oscillations in the other direction. Suppose that α_1 tends to magnetise the needle in A still further, no effect is produced since it is magnetised to saturation, β_1 demagnetises the surface layer, α_2 tends to remove the effects of β_1 and so on. In the solenoid B α_1 demagnetises the needle, β_1 tends to remagnetise it, α_2 to undo the effects of β_1 and so on. Since the maximum value of the current α_1 is greater than that of β_1 the needle in B will be more demagnetised than that in A. If however the number of turns per centimetre on the solenoid A is increased until the effects on the two needles are exactly the same, then assuming that the value of the current decreases in geometrical progression the maximum value of the magnetic force due to the oscillation β_1 acting on A is equal to the maximum value due to α_1 acting on B. Then if γ_1 γ_2 are the maximum values of the first and second half-oscillations respectively and n_1 and n_2 the number of turns on A and B then since

$$4\pi n_1 \gamma_2 = 4\pi n_2 \gamma_1$$

$$\therefore \frac{\gamma_2}{\gamma_1} = \frac{n_2}{n_1}$$

The ratio of the second to the first half-oscillation is therefore known and the damping determined.

The method of two solenoids was not adopted in practice but one theoretically equivalent was employed in which it was necessary to use only one detector needle. The accuracy of the experiments depends to a very great extent upon the nature of this needle. Professor Rutherford (Trans. Roy Soc. 1896) has shewn that the effect on a magnetised steel needle, differs in any given circuit, according to the length of the needle, the hardness of the steel and the thickness of the wire used in its construction. An examination of a needle after it has been partially demagnetised by a discharge shows a surface layer magnetised in a direction opposite to the internal magnetisation. Since a Leyden jar in general gives several complete oscillations before it is greatly damped down, it would be expected that the surface layer of a uniformly magnetised steel needle would be either completely demagnetised or shew evidence of several oscillations in opposite directions. The effect may be explained when the demagnetising force of the ends of a needle on itself is taken into account. The first half-oscillation that tends to demagnetise the needle has the demagnetising force of the ends of the needle ~~in opposition~~ assisting it, while the return oscillation has it in opposition. The return oscillation will not therefore be able to entirely remagnetise the surface layer already affected but a thin layer will be left in the interior, this layer is added to with every oscillation until the final effect will be that the surface of the needle will be magnetised in the opposite direction to the interior. The shorter the needle, the greater is the demagnetising force of the ends upon it. The length found by experiments with a number of needles of different lengths — to be most suitable for the conditions of the investigation was about one and one half centimetres.

A most important point to be considered is the thickness of the wire. The detector did not consist of a single wire but of several of the same length made up in the form of a compound magnet and the several wires insulated from one another by paraffin wax to prevent eddy currents. With a rapidly alternating current each effect lasts for so short a time that the needle does not become demagnetised to any depth and the outer layer then acts as a metallic screen for the interior. In such a rapidly alternating field as the Leyden jar circuit under consideration where the alternations are of the order of 10^6 per second, the demagnetisation is confined to a very thin layer on the surface. Thick wires are affected to a less depth than thin ones. Erskine (Wied. Annal. Band 62 1897) has found that the screening effect on the needle in the solenoid B where the first third and fifth half-oscillations demagnetise the needle is less than on that in A where the even oscillations are in opposition to the original magnetisation, therefore the ratio $\frac{\gamma_2}{\gamma_1}$, i.e. the damping will appear greater than it really is. The effect decreases steadily with the decrease in the diameter of the wire and with very fine wires it is almost negligible. The detector used in all the experiments was composed of 55 hard steel wires .0015 cms in diameter, very carefully insulated and made up into a compound magnet 1.5 cms in length. With such a needle, in a circuit where from theoretical considerations the damping could be shewn to be very small, a damping of about 2% could be measured, so that any correction due to magnetic shielding in such a case must have been trifling.

The following arrangement which is theoretically equivalent to that of the two solenoids described above, was adopted in practice.

A strip of brass was taken and bent into an almost complete circle (see Fig. A) which was fixed on a block of ebonite. At the centre of the circle an ebonite tube projected which served as the axis of a metal arm which pressed against the circumference of the circle and could be moved about it. The detector was fixed in the end of a glass tube which could be easily slipped in and out of the central ebonite tube. A scale dividing the circle into 120 divisions was placed about its circumference and the whole fixed in position before a mirror magnetometer. The circle was placed in series with the discharge circuit, one wire

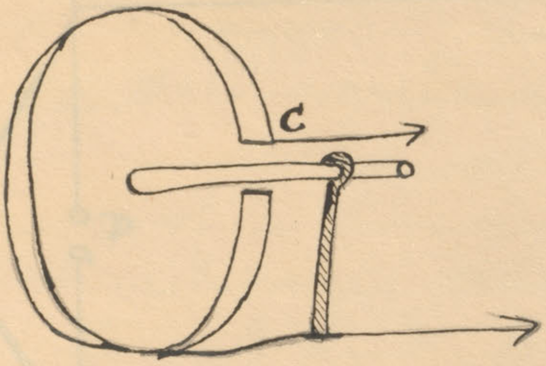


Fig. A.

of the jar E.

The magnetic field due to the wires leading out from the circle was found ~~to~~ to affect the results, to prevent this, the wires were carried out for some distance parallel to the needle and enclosed in a glass tube which was in turn surrounded by a brass cylinder connected to earth, so that all

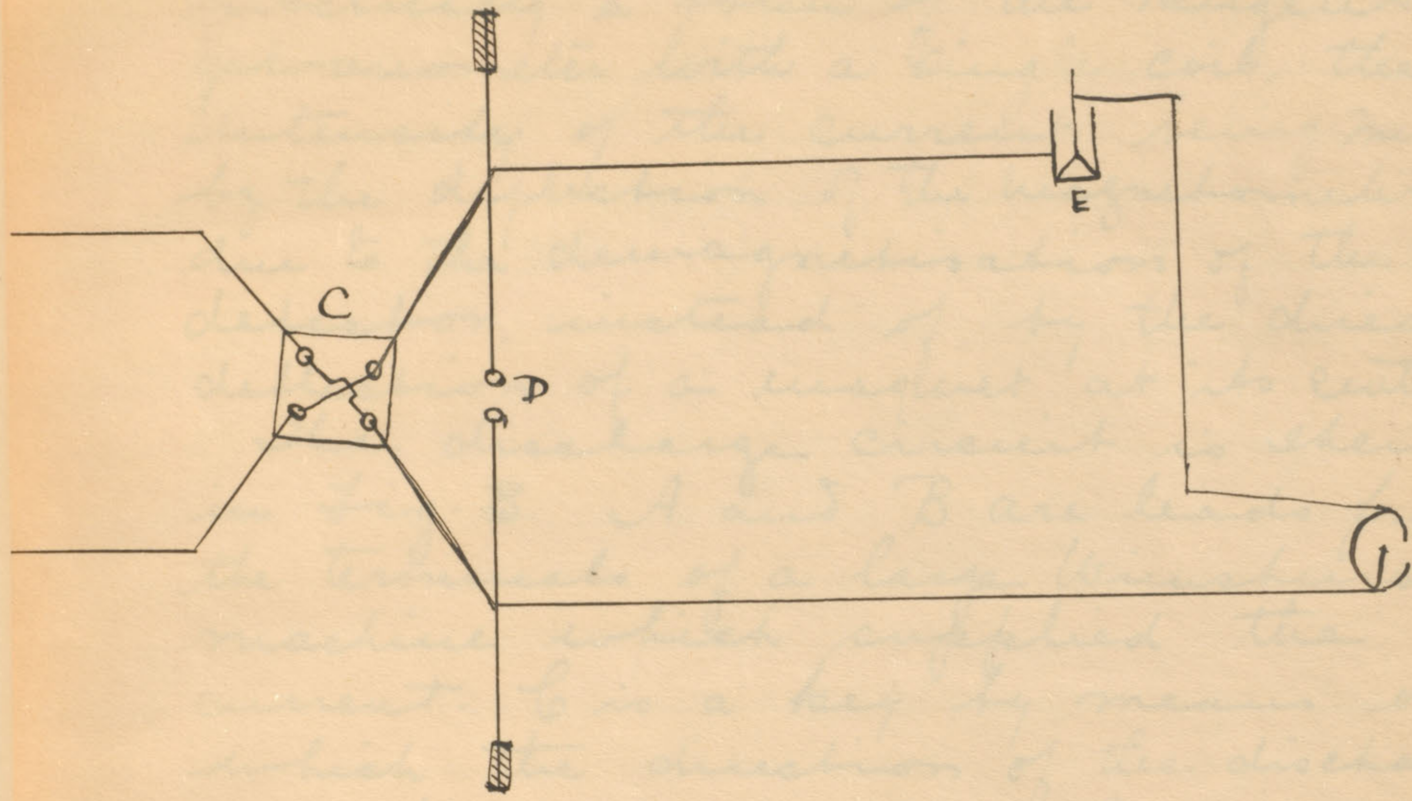


Fig B.

of the jar E.

The magnetic field due to the wires leading out from the circle was found to affect the results, to prevent this, the wires were carried out for some distance parallel to the needle and enclosed in a glass tube which was in turn surrounded by a brass cylinder connected to earth, so that all being connected to one extremity of the circle and the other to the movable arm so that any desired portion of the circumference might be included in the circuit (Fig A) This arrangement is practically a form of the tangent galvanometer with a single coil, the intensity of the current being measured by the deflection of the magnetometer due to the demagnetisation of the detector, instead of by the direct deflection of a magnet at its centre.

The discharge circuit is shewn in Fig. B. A and B are leads from the terminals of a large Wimshurst machine which supplied the current. C is a key by means of which the direction of the discharge could be reversed and at D is an ~~ab~~ air-break whose terminals were brass-balls 2.1 cms in diameter and whose distance could be varied at will. E is the Leyden jar to the outer coating of which a wire is led from one terminal of the air break, while the other terminal is connected through the circle to the inner coating of the jar E.

The magnetic field due to the wires leading out from the circle was found to affect the results, to prevent this, the wires were carried out for some distance parallel to the needle and enclosed in a glass tube which was in turn surrounded by a brass cylinder connected to earth, so that all respectively, then since the current at any time γ is given by

$$\gamma = \frac{C V_0}{\sqrt{LC}} \cdot \varepsilon^{-\frac{R}{2L} t} \sin \frac{t}{\sqrt{LC}}$$

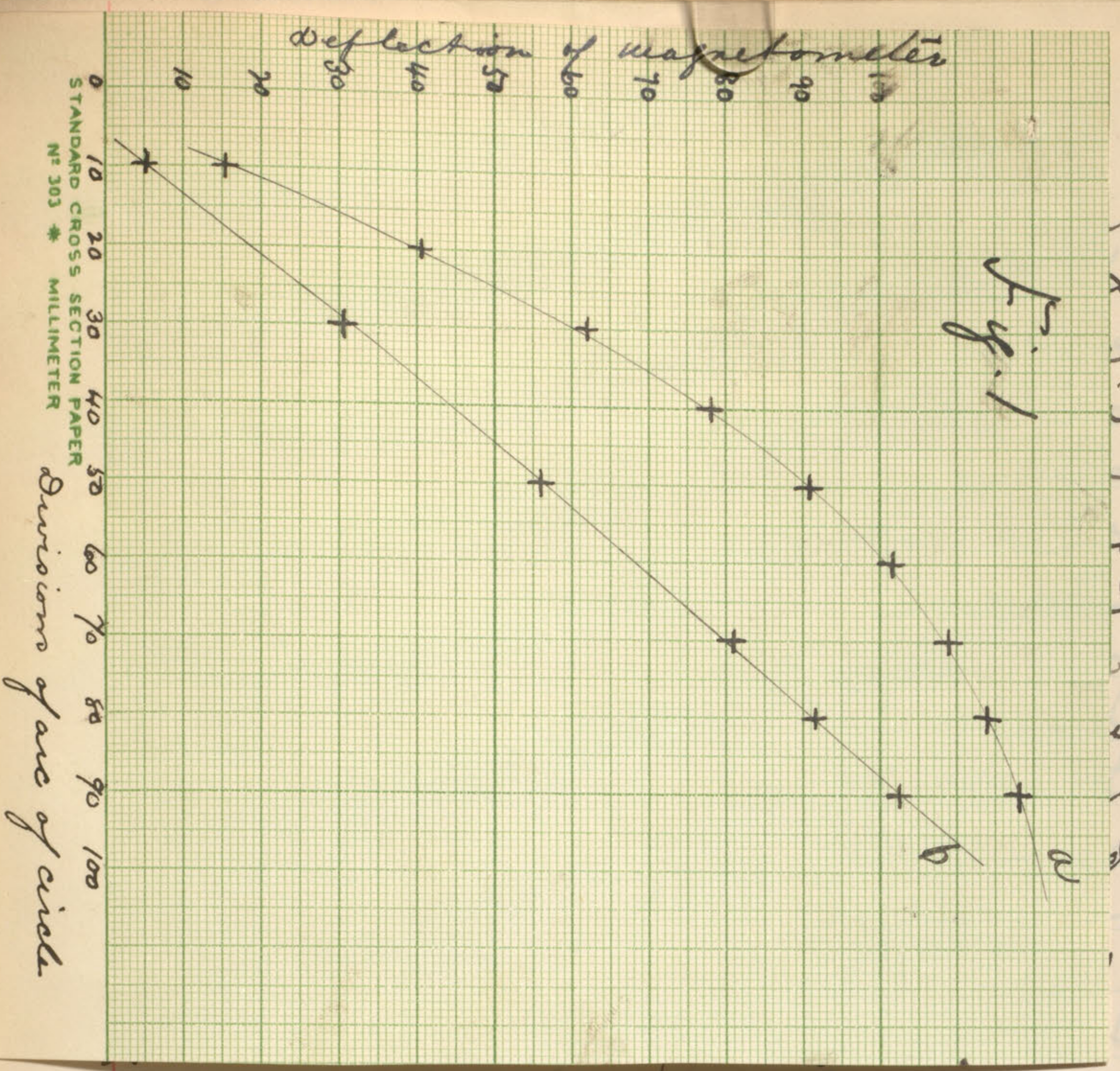
then $\gamma_1 = p C V_0 \, \varepsilon^{-\frac{R}{2L} \cdot \frac{T}{2}}$ where $p = \frac{1}{\sqrt{LC}}$

& $\gamma_2 = p C V_0 \, \varepsilon^{-\frac{R}{2L} \cdot \frac{3T}{2}}$

$\therefore$ the damping $\gamma_2/\gamma_1 = \varepsilon^{-\frac{R}{2L} \cdot \frac{T}{2}}$

The observation were made in the following way. The needle was magnetised to saturation in a solenoid which had a fixed iron core extending through part of its length, against this core the needle was pressed so that it was thoroughly magnetised and could be replaced in exactly the same position every time it was to be remagnetised. The needle was then placed in position at the centre of the circle and the deflection compensated by a neighboring magnet. If the arm of the circle was at when a discharge passed no effect was produced on the needle but if any part of the arc was included there was a deflection due to the partial demagnetisation of the needle. The deflection was noted, the needle removed, completely demagnetised and then magnetised again to exactly the same amount as before and replaced in its position at the centre of the circle. The direction of the discharge was reversed and the arm of the circle moved until the deflections in the two cases were the same. When this is the case, the ratios of the maximum values of the first and second half-oscillations is given by the ratio of the arcs traversed by the discharge.

The amount of demagnetisation of the needle is approximately proportional to the magnetic force measured by the length of the arc of the circle thro' which the discharge passes until the needle is more than half-demagnetised, after which the magnetisation falls off more ~~rapidly~~ slowly and the curve representing the relation becomes concave with respect to the axis along which the arcs are measured. (Fig. 1a) The circuit was always so arranged that the magnetic moment of the needle was not reduced to more than half its value by the passage of the discharge and the measurements were thus always made on the straight part of the curve. When the current was very large the part of the circuit containing the needle had to be shunted. Repeated experiments with and without the shunt when the current was just at its limiting value for use without it, shewed that it made no difference in the final relations of the observations.



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of the circle as before and the discharge passed in the direction in which the second half-oscillation demagnetised the needle, the deflection was read and then without removing the needle or changing the arc of the circle, the discharge was reversed and the increased deflection noted. By means of the curve (Fig 1b) shewing the relation between the arc and the deflection, the arcs corresponding to these deflections were determined and the damping found as before much more accurately than by the first method where it was necessary to make a number of trials to find the exact arc which would give the required deflection.

The first discharge which passes through the circuit causes a reduction of the magnetisation, the amount depending on the arc ~~and~~, the period of oscillation and the resistance. A few

It was found rather troublesome in practice to adjust the arcs to give exactly the same deflection for the two directions of the discharge and a slight change was made in the method of making the observations which simplified them. The observations made in this way could be reduced by means of a calibration curve to the exact equivalent of those obtained in the manner described before.

The needle was magnetised to saturation and placed at the centre of the circle as before and the discharge passed in the direction in which the second half-oscillation demagnetised the needle, the deflection was read and then without removing the needle or changing the arc of the circle, the discharge was reversed and the increased deflection noted. By means of the curve (Fig 16) shewing the relation between the arc and the deflection, the arcs corresponding to these deflections were determined and the damping found as before much more accurately than by the first method where it was necessary to make a number of trials to find the exact arc which would give the required deflection.

The first discharge which passes through the circuit causes a reduction of the magnetisation, the amount depending on the arc ~~and~~, the period of oscillation and the resistance. A few successive discharges in the same direction slightly increase the amount but the needle soon reaches a steady state so that the passage of any further number of discharges have no appreciable effect. Precautions were taken to have a sufficient number of discharges in one direction that the magnetisation might be reduced to a steady state before the direction of the current and the same number of discharges took place in the reversed direction.

The damping increases steadily with the length of the spark, all the other conditions of the circuit remaining the same - The amount of this increase for a given range of spark lengths was found to differ according to the amount of moisture in the atmosphere, the greater the moisture the more uniform was the damping. The following table gives the means of a large number of results on sparks varying from 1 to 13 mms in a moderately dry atmosphere.

Spark length in mms.	$\frac{\gamma_2}{\gamma_1}$
1	.905
3	.88
5	.885
7	.865
9	.86
11	.845
13	.845

With Smaller capacities the increase in the damping with increase of spark length was much more rapid. The Following values were obtained

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With smaller capacities the increase in the damping with increase of spark length was much more rapid. The following values were obtained with a capacity of 725 C.G.S. units

Spark length	γ_2/γ_1
2 mms	.875
3 "	.84
5 "	.765
7 "	.68
9 "	.59
11 "	.515

The curves shewing the relation between the spark length and damping in these two cases are given in Fig. 2.

A very small percentage of this damping is to be attributed to the resistance of the leads, for if R = resistance of the leads for an alternating current of frequency 10^6 per", then the damping due to this resistance is given by

$$\gamma_2/\gamma_1 = \varepsilon^{-\frac{R}{2L}\cdot\frac{T}{2}}$$

Now $R = \sqrt{\frac{1}{2} p \mu l R_0}$ where l = length of wire and R_0 its resistance for direct currents. The circuit measured 125 cms by 145 cms therefore l = 540 cms, the diameter of the wire was .07 cms

then $$R = \sqrt{\frac{1}{2} \times 2\pi \times 10^6 \times 540 \times \frac{540 \times 1640}{\pi \times (.035)^2}} \times \frac{1}{10^9}$$

$$= .58 \text{ ohms}$$

Then damping due to R

$$= \varepsilon^{-\frac{.58}{2\times10^4} \times \frac{1}{2\times10^6} \times \frac{1}{10^{-9}}}$$

$$= \varepsilon^{-.0145}$$

$$= .9855$$

which is a small damping. The damping in all the cases investigated is quite large compared with this so that the expenditure of energy to which the damping is due must take place in the spark-gap. The dissipation of energy due to the excitation of electrical waves is very small ~~and may~~ in a Leyden jar circuit and may be neglected.

The ohmic resistance corresponding to the absorption of energy by the spark gap was deduced by inserting in the circuit a known electrolytic resistance. If R is the resistance of the air-break and leads then since

$$p_2/p_1 = \varepsilon^{-\frac{R}{2L}\cdot\frac{T}{2}} = \rho_1 \text{ say}$$

$$\text{then } \log \rho_1 = -\frac{R}{2L}\cdot\frac{T}{2}$$

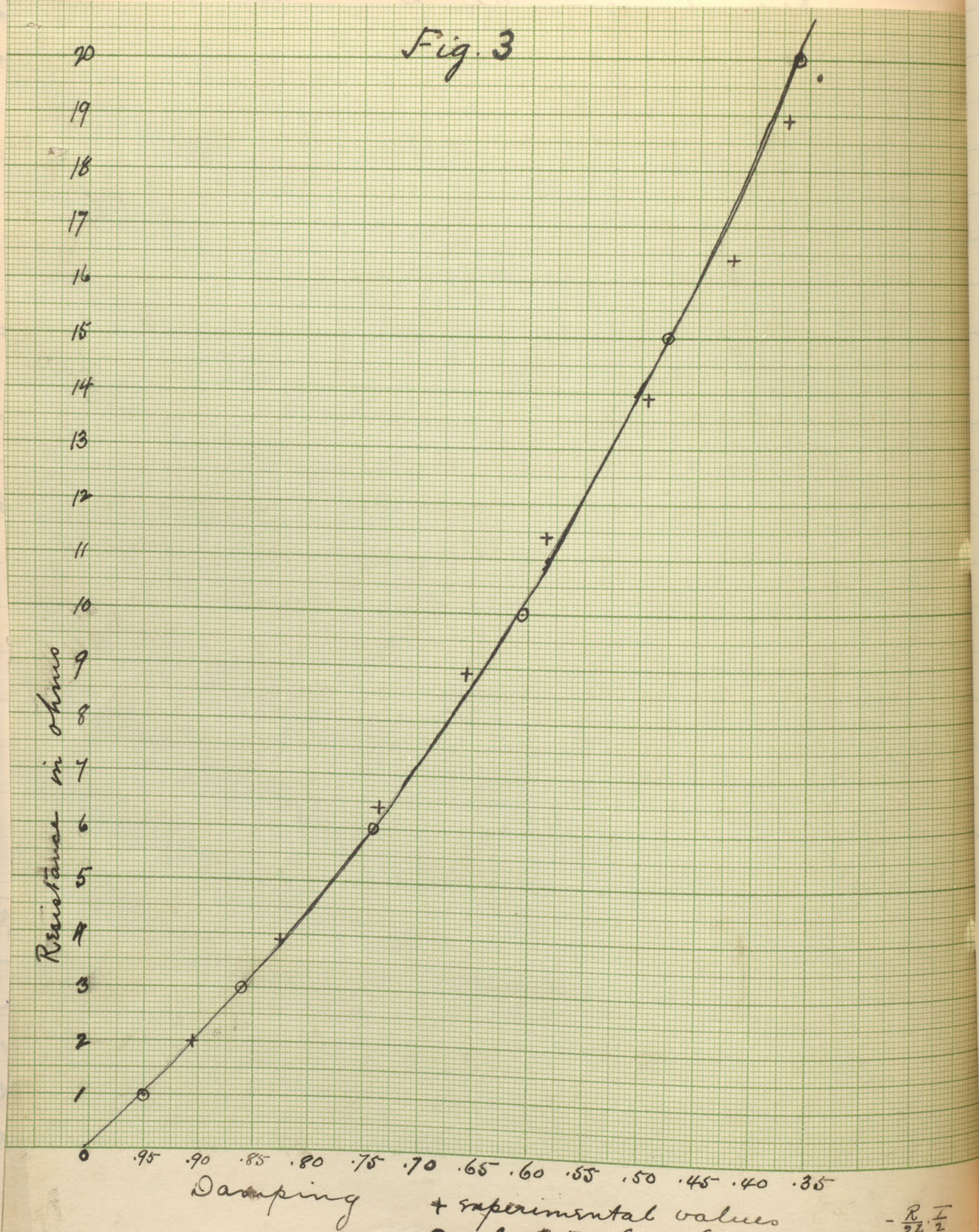
and if R + r is the total resistance when the known resistance, consisting in the present instance of a solution of zinc sulphate with zinc electrodes, is inserted, the damping will be given by $\rho_2 = \varepsilon^{-\frac{R+r}{2L}\cdot\frac{T}{2}}$

$$\& \log \rho_2 = -\frac{R+r}{2L}\cdot\frac{T}{2}$$

$$\text{then } \frac{\log \rho_2}{\log \rho_1} = \frac{R+r}{R}$$

R is therefore determined in terms of r the known resistance. The value of R determined from a number of observations in which different values of r were ~~two~~ used, was about two ohms for a spark of 6 mms, from this of course is to be subtracted the resistance of the leads but it is so small in comparison with the total resistance that it is almost negligible.

Fig. 3



+ experimental values
⊙ calculated from formula $p = \varepsilon^{-\frac{R}{2L}\cdot\frac{T}{2}}$

When the resistance of the air-break is known any ~~other~~ unknown resistance r_1 can be determined in a similar manner for if ρ_1 and ρ_2 be the dampings without and with the unknown resistance respectively then $\frac{\log \rho_2}{\log \rho_1} = \frac{R + r_1}{R}$ and r_1 is therefore determined in terms of R.

This gives an accurate and practicable method of determining the values of resistances in rapidly alternating fields.

By inserting a series of increasing electrolytic resistances in the circuit a curve shewing the relation between the damping and the resistance was obtained - This experimental curve corresponded very closely with one found by calculating the values of the damping from the formula $\rho = \epsilon^{-\frac{R+r_0}{2L}\cdot\frac{T}{2}}$, different values being substituted for r_0 and R being determined from one of the points on the experimental curve.

The two curves are shewn in Fig - 3

The damping remained unchanged whatever the diameter of the copper wire employed in the circuit, the small increase of resistance with decrease of diameter was not sufficient to have any appreciable effect. When leads of iron wire however were substituted the oscillations were greatly damped down owing to the circular magnetisation of the iron which prevents the current from sinking to any depth and thus greatly increases the resistance. An iron wire .7 mms in diameter gives a damping of .72 for a spark of 11 mms while the damping for a copper of same diameter in the same circuit is .85. This increase of the damping for iron wires has been frequently ~~been~~ noticed before but by this method an exact measurement of the increase of resistance of an iron wire for any given rapidly alternating current can be obtained in a manner similar to that used to determine any other unknown resistance in the circuit. The damping was found to increase as the diameter of the iron wire decreased.

Diam. of wire in mms	.50	.65	1.2	2
damping	.70	.72	~~[illegible]~~ .79	.82

A very thin coating of copper deposited on the iron wire served to bring the damping back to its value for a copper wire.

Instead of the brass balls at the

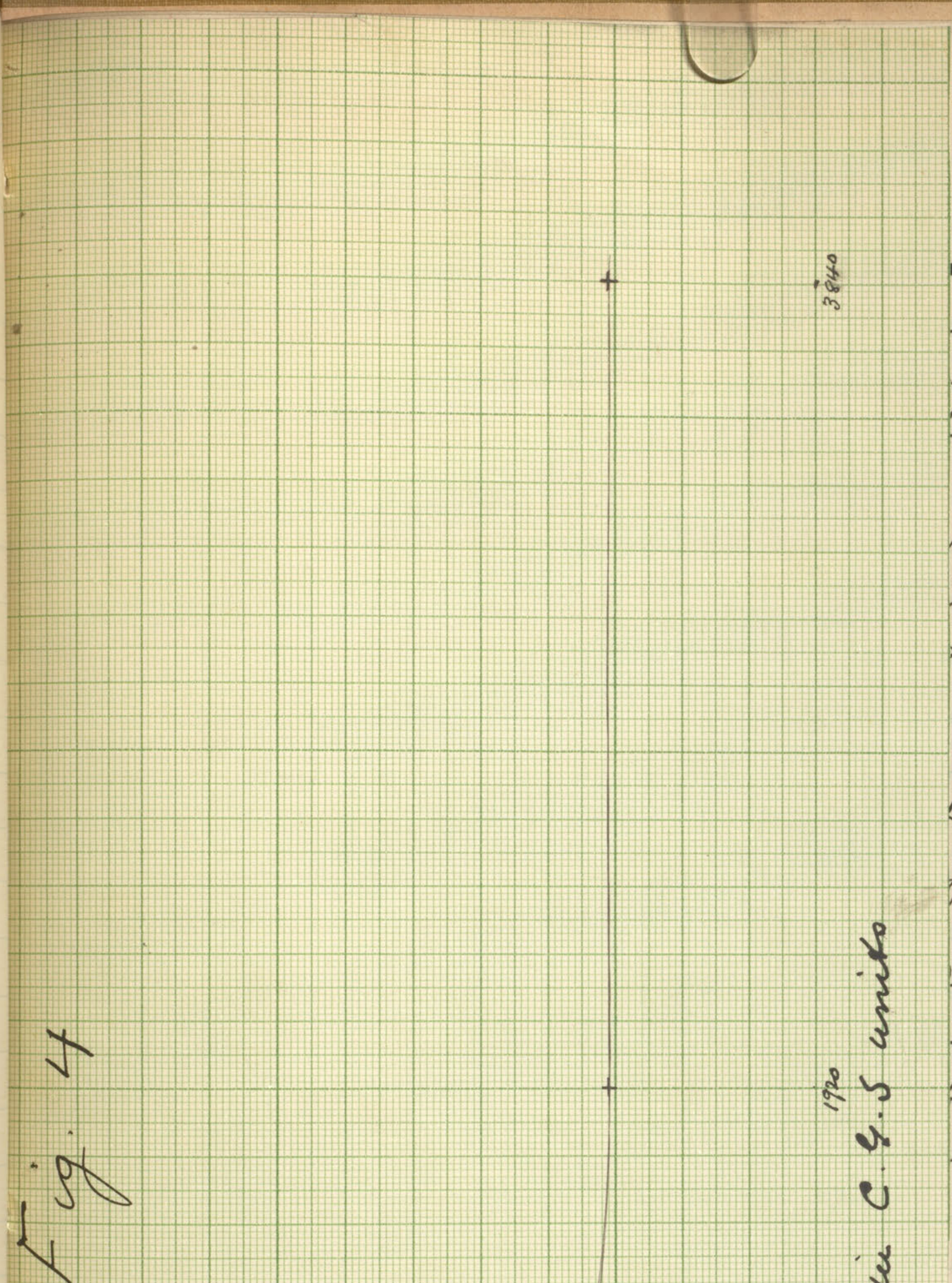


Fig. 4

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length and damping with different
capacities, that the damping reaches a
comparatively steady condition when

air-break, balls made of iron and aluminium were substituted but no difference in the damping was detected. The balls were always kept in a high state of polish as the spark shews a disinclination to pass from any point on the ball which has become roughened or tarnished. It prefers to leap a longer distance from a smooth place.

To investigate the effect of increase of capacity on the damping several jars were connected in parallel and placed in the circuit. The damping was found to be practically the same for one, two, four, eight or twelve jars. The relation for smaller capacities than that of the Leyden jar employed in the first experiments i.e. less than 2500 E.S. units, was obtained by means of an Ebonite condenser whose capacity could be varied as desired by removing or adding to the tin-foil coating. It was found that the damping changed very slightly for capacities over 1000 C.G.S units but below that point the damping increased rapidly with decrease of capacity. See Fig. 4

This points to the same conclusion as that indicated by the curves for spark length and damping with different capacities, that the damping reaches a comparatively steady condition when

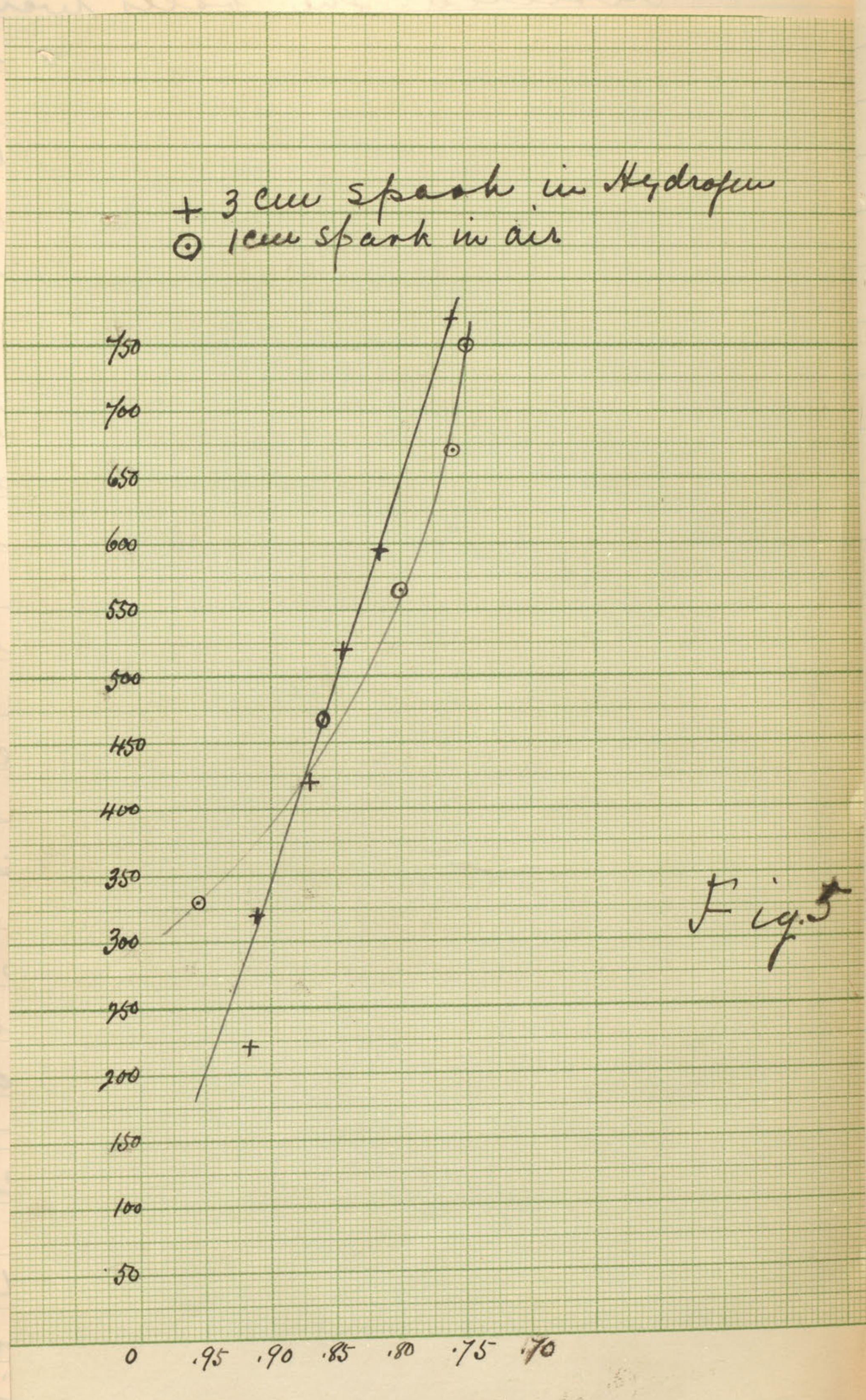
+ 3 cm spark in Hydrogen
⊙ 1 cm spark in air
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
.95
.90
.85
.80
.75
.70

Fig. 5

the current is large.

Change of pressure at the spark gap had a very noticeable effect on the damping. The terminals of the air-break were enclosed in an air-tight glass bulb which was connected to an air-pump. The air was thoroughly dried before entering the bulb.

The damping for any given spark length decreases as the pressure becomes less. An exactly similar effect was observed when hydrogen and Carbonic acid were introduced into the bulb instead of air. For a given spark length and pressure the damping in hydrogen is less than that in air. In Fig 5 is given a curve for pressure and damping with a spark of 1 cm in air and also for a spark of 3 cms in Hydrogen.

Observations point to the existence of a critical pressure depending on the spark length and the nature of the gas, below which the damping again begins to increase, evidently with great rapidity, for the resistance soon becomes so great that no discharge at all is obtainable. This point has not been very closely investigated as the arrangement employed was not sensitive enough to be sufficiently affected by these very small currents. With a more sensitive arrangement accurate measurements might easily be made.

It has been shewn that a very small part of the damping is due to the resistance of the leads and in a Leyden jar circuit the loss of energy from radiation is also very small so that by far the greater part of the dissipation of energy must take place at the spark gap.

Consider the amount of energy required to dissociate the gas at the terminals into ions sufficient to carry across the current. We will suppose that the ions are the same as are produced by X rays and that the energy required to produce them is independent of the temperature. This will give a maximum expenditure of energy since the energy required to produce the ions probably diminishes with rise of temperature.

Maximum current conveyed over the air-break in the first half-oscillation $= CV_0$ approximately.

Suppose spark gap = 1 cm

then $V_0 = 10^4 \times 4$ volts.

& $C = 25 \times 10^2$ E.S. units for 1 jar

$$\therefore Q = \frac{25 \times 10^2 \times 4 \times 10^4}{300} \text{ E.S. units}$$

$$= \frac{1}{3} \times 10^6 \text{ E.S. units}$$

Now the charge on an ion has been found by ~~Prof. Rutherford~~ J.J. Thomson to be equal to 6×10^{-10} E.S. units and the energy required to produce an ion to be $= 1.9 \times 10^{-10}$ ergs

$\therefore$ energy required for the dissociation of 1 E.S. unit $= \dfrac{1.9 \times 10^{-10}}{6 \times 10^{-10}} = .3$ ergs

∴ energy required to carry across a quantity $Q = \frac{1}{3} \times 10^6 \times \frac{3}{10} = 10^5$ ergs

Now the total E.S. energy of the charge $= \frac{1}{2} C V_0^2$ ~~ergs~~

$$= \frac{1}{2} . 25 \times 10^2 \times \left(\frac{4 \times 10^4}{3 \times 10^2} \right)^2 \text{ ergs}$$

$$= 22 \times 10^6 \text{ ergs}$$

$$\therefore \frac{\text{Ratio of energy required to dissociate sufficient no. of ions}}{\text{total energy of charge}}$$

$= \frac{10^5}{22 \times 10^6} = \frac{1}{220}$ a damping of about $\frac{1}{2}$ per cent.

If $V_0 = 10000$ E.S. for a spark gap of $\frac{1}{10}$ inch, the ratio $= \frac{1}{55}$ a damping of about 2%

Thus we see that the expenditure of energy required to produce a sufficient number of ions to carry the discharge across is not enough to account for the damping. We are therefore driven to the conclusion that a much greater number of ions are produced than are sufficient to convey the discharge. The path of the discharge is very narrow and confined and the ions so numerous and close together that recombination is very rapid. To these recombinations is probably due the high temperature of the spark. This high temperature in turn is favorable to the production of fresh ions and to this fact, the comparatively steady state of the damping in circuits with large capacities may probably be attributed. In such a case the current is so great that a large number of ions are produced, the recombination of these produce a high temperature by means of which fresh ions are formed to take the place of those that have recombined; and the field is thus kept comparatively uniform.

I wish to express my obligation and gratitude to Professor Rutherford for his kind assistance at every stage of the investigation.

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