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RESEARCHES
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KIYOSHI TAKATSU, DIRECTOR.

NO. 203

SUDDEN SHORT-CIRCUIT OF ALTERNATOR

BY

SADATOSHI BEKKU

June, 1927.

ELECTROTECHNICAL LABORATORY,
MINISTRY OF COMMUNICATIONS,
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Researches of the Electrotechnical Laboratory No. 203

TABLE OF ERRATA

Pages	Row	Incorrect	Correct
3	4	M_{ca}	M_{cv}
"	3 from the bottom	$\cos\theta\, I_a$	$\cos\theta\, i_a$
10	3	$(R_a+pS_\iota)i_{a1}$	$(R_a+pS_a)i_{a2}$
"	"	$M^2p(p+j\omega)i_{a1}$	$M^2p(p+j\omega)i_{a2}$
"	3 from the bottom	mumber	numbers
11	5	$(j\omega-\alpha_2{}')$	$(j\omega-\alpha_1{}')$
12	6	i_{a1}	i_{a2}
"	"	Y_{n2}	Y_{a2}
"	"	∂Z_{a1}	∂Z_{a2}
13	2	reaceance	reactance
"	2 from the bottom	crcuiuit	circuit
14	7	$-(R_a+pS_a)\{R_u+pS_u)^2+\omega^2S_u^2\}$	$-(R_a+pS_a)\{(R_u+pS_u)^2+\omega^2S_u^2\}$
17	3	$R_a+(p-j\omega)S_u$	$R_u+(p-j\omega)S_u$
"	12	R_u	R_a
18	11	calculale	calculable
"	12	$R_u+j\omega S_a$	$R_u+j\omega S_u$
19	6	$\omega^2M^2(S_\iota-S_a)^2$	$\omega^2M^2(S_a-L_2)$
"	"	$(R_2-R_a)^2+\omega^2(L_2-S_u)^2$	$(R_2-R_a)^2+\omega^2(L_2-S_\iota)^2$
20	2	indentical	identical
21	3	excition	excitation
22	4	0.878+0.878+0.095=0.973	0.878+0.095=0.973
"	"	$S_u=0.93$	$S_u=0.973$
23	Fig. 9	short-circuit currrent in amp.	short-circuit current in amp.
24	12	$S_u=0.0380$	$S_a=0.0380$
"	"	$R_u=0.154$	$R_a=0.154$
"	3 from the bottom	$(j\omega-\alpha_1{}')Z_{a1}{}'(\alpha_1)$	$(j\omega-\alpha_1{}')Z_{a1}{}'(\alpha_1{}')$
29	4 from the bottom	$\theta_s=103.4°$	$\theta_2=103.4°$
37	2 from the bottom	$R_u\{(2S_a+S_0)-M^2\}$	$R_u\{(2S_a+S_{a0})S_\iota-M^2\}$
"	"	$S_u\{(2S_a+S_0)S_u-2M^2\}$	$S_u\{(2S_a+S_{a0})S_u-2M^2\}$
"	1 from the bottom	$3R_\iota+S_u$	$3R_aS_u$
"	"	$(2S_a+S_0)S_u-2M^2$	$(2S_a+S_{a0})S_u-2M^2$
	Fig. 39	$R_u=70\,8$ ohms	$R_u=70.8$ ohms

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SUDDEN SHORT-CIRCUIT OF ALTERNATOR

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SYNOPSIS

This paper consists of following themes:–

1. The solution of the symmetrical three phase alternator (three phase stator with cylindrical three phase rotor) for sudden short-circuit can be obtained by ordinary method of solving the simultaneous differential equations. The author applied the linear transformation of the method of symmetrical co-ordinates, and the fundamental differential equation was thus made very simple. Heaviside's expansion theorem was thus made applicable for this case, and the actual computation became very simple.

2. The rigorous mathematical solution of the ordinary salient pole alternator for sudden short-circuit is almost impossible. The author, however, verified from experiments that such machine can be treated as the ideal symmetrical alternator (which is called the equivalent symmetrical alternator) of appropriate electrical constants without causing any serious error. The conception of the equivalent symmetrical alternator is very convenient for the study of transients and

the constants of the equivalent symmetrical alternator can be determined from several steady-state measurements.

3. The three phase short-circuit current may be considered to consist of damped D. C. component tending to null and the damped A. C. component tending to the sustained short-circuit current. The damped A. C. component may be considered as the constant induced voltage divided by the variable positive phase sequence impedance. This variable or transient positive phase impedance is comparatively small, thus causing heavy current, at the beginning of short-circuit and tends to the synchronous impedance. From calculation and test, the author verified that the transient positive phase sequence impedance just after the occurrence of short-circuit is practically equal to the sustained negative phase sequence impedance of the alternator. And the negative phase sequence impedance of the alternator is practically constant for the sudden application of negative phase sequence voltage.

4. From the calculation and test, the author verified that the maximum instantaneous value of the short-circuit current is practically the same for the three phase short-circuit as well as the line to line short-circuit, which fact was already published by several authors.

As the results, the author believes that the short-circuit current of the ordinary alternator is made possible to calculate by using several measured data, viz. open circuit test, short-circuit test, stator resistance, negative phase sequence impedance.

Thus the station engineers need not oblige to approve without any consideration the magnitude of the transient impedance of the alternator as supplied by the machine maker.

June, 1927.

CONTENTS

SUDDEN SHORT-CIRCUIT OF ALTERNATOR

BY

SADATOSHI BEKKU

1. Introduction.

The sudden short-circuit of alternator has been one of the most widely discussed topics in the electrical power engineering in connection with the abnormal mechanical stress due to heavy current and with the rupturing capacity of the oil circuit breaker.

This problem has been discussed by numerous authors and there seems nothing to remain to admit further mathematical discussion.

The author, in his previous research,[(1)] verified from test that the method of symmetrical co-ordinates, which is strictly applicable for symmetrical machine, is also applicable with sufficient accuracy to the commercial salient pole machine. Which fact stimulated the author to imagine the possibility of considering the equivalent symmetrical machine for the sudden short-circuit of the commercial alternator and the author attemped to draw any relation between the transient reactance and the sustained negative phase sequence impedance of the commercial alternator.

The author applied the linear transformation of the method of symmetrical co-ordinates proposed by C. L. Fortescue to the fundamental differential equations of the symmetrical three phase machine and obtained directly the sudden short-circuit current by using Heaviside's expansion theorem.

The numerical computation and test were carried out by Mr. M. Urushibata. On publishing the result, the author wishes to express his thanks to Mr.

(1) S. Bekku, M. Doté and M. Urushibata:— On the Method of Measurement of Zero and Negative Phase Sequence Impedance of the Three Phase Alternator. Researches of the Electrotechnical Laboratory No. 170. Tokyo, Japan.

Urushibata as well as to Mr. C. L. Fortescue, for the kind suggestion given to the author while he was staying in East Pittsburgh in 1925.

2. Fundamental Equation and its Transformation.

Consider a symmetrical three phase machine, with Y connected stator and Y connected rotor, as shown in Fig. 1.

We shall assume that the air gap is uniform and there is no slot and tooth. The mutual inductance between stator and rotor winding varies sinusoidally and there is no magnetic saturation.

Let be

- v with suffix, instantaneous value of potential at respective terminal.
- i with suffix, instantaneous value of current in respective branch.
- L_a self inductance of each phase a, b, c.
- R_a resistance of each phase a, b, c.
- M_a mutual inductance between any two of a, b, c.
- L_u self inductance of each phase, u, v, w.
- R_u resistance of each phase, u, v, w.
- M_u mutual inductance between any two of u, v, w.
- M_{au} mutual inductance between a and u, and so on.

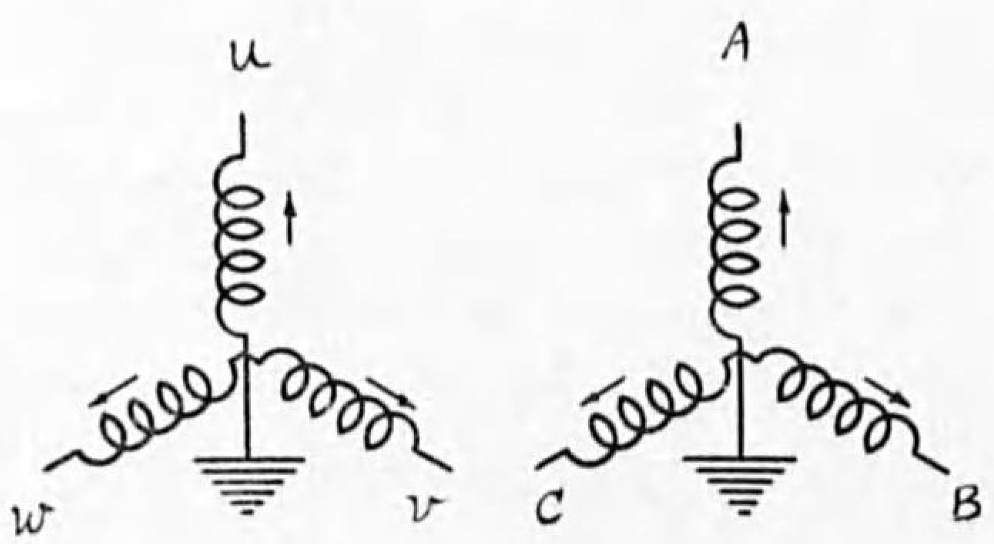


FIG. 1.

Then from symmetry we have

$$M_{au}=M_{bv}=M_{cw}=M'\cos\theta$$

$$M_{bw}=M_{cu}=M_{av}=M'\cos\left(\theta+\frac{2\pi}{3}\right)$$

$$M_{cv}=M_{aw}=M_{bu}=M'\cos\left(\theta+\frac{4\pi}{3}\right)$$

where $\theta=\omega t+\varphi$

Writing p for $\frac{d}{dt}$, we have following equations for the circuit shown in Fig. 1.

$$\left.\begin{aligned}
v_a&=-(R_a+pL_a)i_a-pM_a(i_b+i_c)\\
&\quad-pM'\left\{\cos\theta\, i_u+\cos\left(\theta+\frac{2\pi}{3}\right)i_v+\cos\left(\theta+\frac{4\pi}{3}\right)i_w\right\}\\
v_b&=-(R_a+pL_a)i_b-pM_a(i_c+i_a)\\
&\quad-pM'\left\{\cos\left(\theta+\frac{4\pi}{3}\right)i_u+\cos\theta\, i_v+\cos\left(\theta+\frac{2\pi}{3}\right)i_w\right\}\\
v_c&=-(R_a+pL_a)i_c-pM_a(i_a+i_b)\\
&\quad-pM'\left\{\cos\left(\theta+\frac{2\pi}{3}\right)i_u+\cos\left(\theta+\frac{4\pi}{3}\right)i_v+\cos\theta\, i_w\right\}\\
v_u&=-(R_u+pL_u)i_u-pM_u(i_v+i_w)\\
&\quad-pM'\left\{\cos\theta\, I_a+\cos\left(\theta+\frac{4\pi}{3}\right)i_b+\cos\left(\theta+\frac{2\pi}{3}\right)i_c\right\}\\
v_v&=-(R_u+pL_u)i_v-pM_u(i_w+i_u)\\
&\quad-pM'\left\{\cos\left(\theta+\frac{2\pi}{3}\right)i_a+\cos\theta\, i_b+\cos\left(\theta+\frac{4\pi}{3}\right)i_c\right\}
\end{aligned}\right\}\quad\cdots(1)$$

$$v_w = -(R_u + pL_u) i_w - pM_u (i_u + i_v) - pM' \left\{ \cos\left(\theta + \frac{4\pi}{3}\right) i_a + \cos\left(\theta + \frac{2\pi}{3}\right) i_b + \cos\theta\, i_c \right\}$$

Let

$$\left.\begin{aligned}
i_{a0} &= \frac{1}{3}(i_a + i_b + i_c), \qquad i_{a1} = \frac{1}{3}(i_a + a i_b + a^2 i_c), \\
i_{a2} &= \frac{1}{3}(i_a + a^2 i_b + a i_c), \\
i_{u0} &= \frac{1}{3}(i_u + i_v + i_w), \qquad i_{u1} = \frac{1}{3}(i_u + a i_v + a^2 i_w), \\
i_{u2} &= \frac{1}{3}(i_u + a^2 i_v + a i_w), \\
v_{a0} &= \frac{1}{3}(v_a + v_b + v_c), \quad \text{etc.}
\end{aligned}\right\} \quad \cdots (2)$$

where $\quad a = \varepsilon^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

(1) becomes by the linear transformation (2) as following

$$\left.\begin{aligned}
v_{a0} &= -\left\{R_a + p(L_a + 2M_a)\right\} i_{a0} \\
v_{a1} &= -\left\{R_a + p(L_a - M_a)\right\} i_{a1} - Mp\varepsilon^{j\theta} i_{u1} \\
v_{a2} &= -\left\{R_a + p(L_a - M_a)\right\} i_{a2} - Mp\varepsilon^{-j\theta} i_{u2} \\
v_{u0} &= -\left\{R_u + p(L_u + 2M_u)\right\} i_{u0}
\end{aligned}\right\} \quad \cdots (3)$$

$$\begin{aligned}
v_{u1} &\doteq -\left\{R_u + p(L_u - M_u)\right\} i_{u1} - Mp\varepsilon^{-j\theta} i_{a1} \\
v_{u2} &= -\left\{R_u + p(L_u - M_u)\right\} i_{u2} - Mp\varepsilon^{j\theta} i_{a2}
\end{aligned}$$

where $\quad M = \frac{3}{2} M'$

By putting $L_a + 2M_a = S_{a0}$, $L_u + 2M_u = S_{u0}$, $L_a - M_a = S_a$, $L_u - M_u = S_u$, (3) can be written as

$$\left.\begin{aligned}
v_{a0} &= -(R_a + pS_{a0}) i_{a0} \\
v_{a1} &= -(R_a + pS_a) i_{a1} - Mp\varepsilon^{j\theta} i_{u1} \\
v_{a2} &= -(R_a + pS_a) i_{a2} - Mp\varepsilon^{-j\theta} i_{u2} \\
v_{u0} &= -(R_u + pS_{u0}) i_{u0} \\
v_{u1} &= -(R_u + pS_u) i_{u1} - Mp\varepsilon^{-j\theta} i_{a1} \\
v_{u2} &= -(R_u + pS_u) i_{u2} - Mp\varepsilon^{j\theta} i_{u2}
\end{aligned}\right\} \quad \cdots (3')$$

(3′) is called the fundamental equation of the symmetrical three phase machine. According to the circuit connected to a, b, c or u, v, w we have certain constraint between v and i. The fundamental equation admits easy solution in some cases and not in others. (3) or (3′) was already published by Fortescue in a slightly different form.(1)

3. Problem of Sudden Short-Circuit and Heaviside's Expansion Theorem.

The transient current when any E. M. F. is suddenly applied in one branch of any network originally in equilibrium is obtained in a very easy manner by

(1) C. L. Fortescue:—Trans. A. I. E. E. 1918. p. 1065. (114).

using Heaviside's expansion theorem. The convenience of Heaviside's expansion theorem was already discussed by Carson[1], Cohen[2], Berg[3] and other authors and the theorem is fully appreciated by many physists and engineers.

The transient current when any abrupt change occurs in the circuit or especially when a part of circuit is short-circuited was treated by E. Berg in a very ingeneous manner by using the principle of superposition and Heaviside's expansion theorem.

Suppose an alternator with open armature; its field being excited by a continuous E. M. F. The current, when the armature is suddenly short-circuited, is obtained by superposing following two current distributions.

(1) Current distribution before the occurrence of short-circuit.

(2) Current distribution when the E. M. F. equal and opposite to that just before the occurrence of short-circuit is suddenly applied, without changing the circuit condition of field winding except to nullify the continuous E. M. F.

In the first current distribution there is no armature current, therefore to get the sudden short-circuit armature current it is sufficient to consider the second distribution only. Thus the problem of sudden short-circuit of alternator is reduced to the problem of sudden application of E. M. F. and it is very evident that Heaviside's expansion theorem is directly applicable.

Let the E. M. F. to be applied be $E\cos\theta$, where $\theta=\omega t+\varphi$, and let the current be obtained in the symbolic form as following

$$i=\frac{EY(p)\cos\theta}{Z(p)}$$

where $Y(p)$ and $Z(p)$ rational integral function of p.

If the circuit be originally in equilibrium, the transient current is given by Heaviside's expansion theorem as following.

$$i=\text{Real part of } E\varepsilon^{j\varphi}\left\{\frac{Y(j\omega)}{Z(j\omega)}\varepsilon^{j\omega t}-\sum_{r=1}^{r=n}\frac{Y(\lambda_r)}{(j\omega-\lambda_r)\left.\frac{\partial Z}{\partial p}\right|_{(p=\lambda_r)}}\varepsilon^{\lambda_r t}\right\}$$

(1) J. R. Carson :—Phys. Rev., Sept., 1917.
(2) L. Cohen :—Frank. Inst., J., Dec., 1922.
(3) E. J. Berg :—Frank. Inst., J., Nov., 1924.

λ_r is the root of $Z(p)=0$, and $Z(p)$ is considered to be of nth degree in p and $Z(p)=0$ is assumed to possess no multiple roots.

4. Calculation of Three Phase Short-Circuit Current.

Let the excitation of the symmetrical three phase alternator be as shown in Fig. 2. Then the short-circuit current is equal to the transient current when the symmetrical three phase voltage is suddenly applied on the circuit as shown in Fig. 3. The voltage to be impressed should be of opposite sign as that appears on the terminal in Fig. 2.

We have following relations when the armature is open-circuited as shown in Fig. 2.

$$i_u=0, \qquad i_v=0, \qquad i_w=0$$

$$v_u-v_v=E, \qquad v_u-v_w=E$$

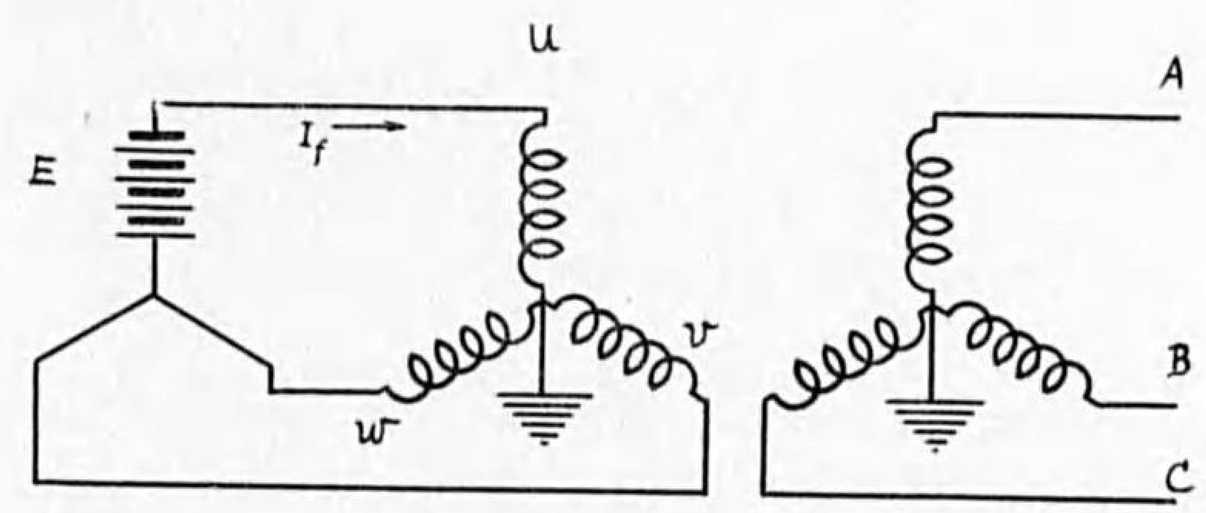


FIG. 2.

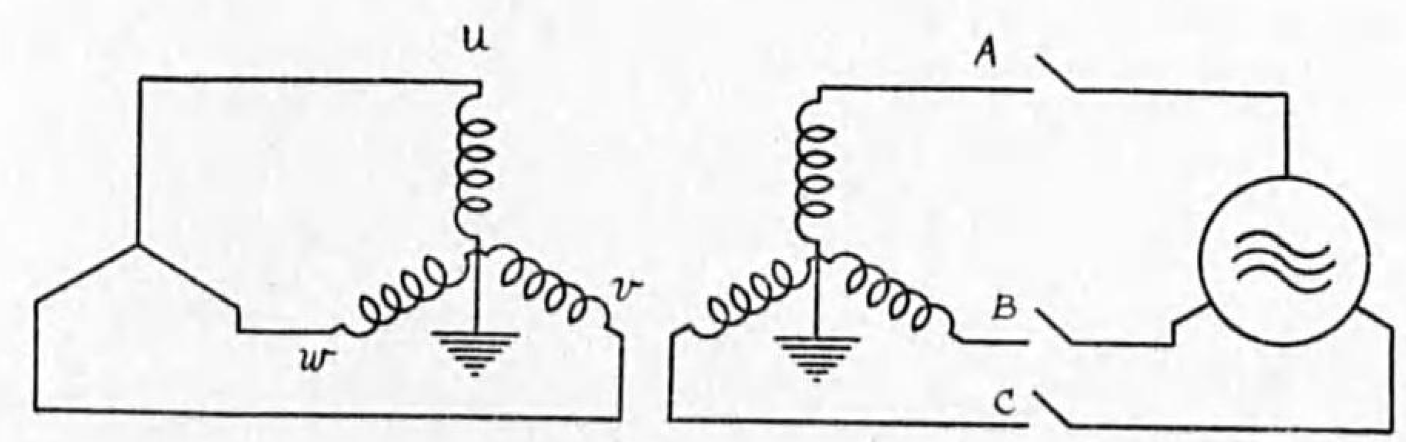


FIG. 3.

Transforming these relations according to (2), we have

$$i_{a0}=0, \quad i_{a1}=0, \quad i_{a2}=0$$

$$(1-a^2)v_{u1}+(1-a)v_{u2}=E$$

$$(1-a)v_{u1}+(1-a^2)v_{u2}=E$$

From these $v_{u1}=v_{u2}=\frac{E}{3}$

Putting these relations into (3′), we get

$$\left.\begin{aligned} i_{u1}&=-\frac{E}{3(R_u+pS_u)} \\ i_{u2}&=-\frac{E}{3(R_u+pS_u)} \end{aligned}\right\} \quad . \; . \; (4)$$

If the steady state is considered, we have by putting $p=0$,

$$i_{u1}=-\frac{E}{3R_u}, \qquad i_{u2}=-\frac{E}{3R_u} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (5)$$

Putting (5) into (3′),

$$\left.\begin{aligned} v_{a1}&=\frac{Mp\varepsilon^{j\theta}E}{3R_u}=\frac{j\omega ME\varepsilon^{j\theta}}{3R_u} \\ v_{a2}&=\frac{Mp\varepsilon^{-j\theta}E}{3R_u}=-\frac{j\omega ME\varepsilon^{-j\theta}}{3R_u} \\ v_{a0}&=0 \\ v_a&=v_{a1}+v_{a2}=-\frac{2\omega ME}{3R_u}\sin(\omega t+\varphi) \end{aligned}\right\} \quad . \; . \; (6)$$

(6) is the potential of a, b, c, in Fig. 2, before the occurrence of short-circuit, transformed according to (2). The short-circuit current is the transient current when $v_{a0}=0$, $v_{a1}=-\frac{j\omega ME\varepsilon^{j\theta}}{3R_u}$, $v_{a2}=\frac{j\omega ME\varepsilon^{-j\theta}}{3R_u}$ is suddenly impressed on a, b, c in Fig. 3.

In the case of Fig. 3, we have following relations.

$$v_u-v_v=0, \qquad v_u-v_w=0, \qquad i_u+i_v+i_w=0.$$

From these we have $v_{u0}=0$, $v_{u1}=0$, $v_{u2}=0$, and (3′) becames as following

$$\left.\begin{aligned} v_{a1}&=-\frac{j\omega ME\varepsilon^{j\theta}}{3R_u}=-(R_a+pS_a)i_{a1}-Mp\varepsilon^{j\theta}i_{u1} \\ v_{a2}&=\frac{j\omega ME\varepsilon^{-j\theta}}{3R_u}=-(R_a+pS_a)i_{a2}-Mp\varepsilon^{-j\theta}i_{u2} \end{aligned}\right\} \quad . \; . \; (7)$$

$$\left.\begin{aligned} 0&=-(R_u+pS_u)i_{u1}-Mp\varepsilon^{-j\theta}i_{a1} \\ 0&=-(R_u+pS_u)i_{u2}-Mp\varepsilon^{j\theta}i_{a2} \end{aligned}\right\} \quad . \; . \; (8)$$

From (8)

$$\left.\begin{aligned} i_{u1}&=-\frac{Mp\varepsilon^{-j\theta}i_{a1}}{R_u+pS_u} \\ &=-M\varepsilon^{-j\theta}\frac{(p-j\omega)i_{a1}}{R_u+(p-j\omega)S_u} \\ i_{u2}&=-\frac{Mp\varepsilon^{j\theta}i_{a2}}{R_u+pS_u} \\ &=-M\varepsilon^{j\theta}\frac{(p+j\omega)i_{a2}}{R_u+(p+j\omega)S_u} \end{aligned}\right\} \quad . \; . \; (9)^*$$

* Refer A. R. Forsyth:—A Treatise on Differential Equations, 5th Edition, p. 58, § 33.

Putting (9) into (7), we have

$$\left.\begin{aligned} v_{a1} &= -(R_a+pS_a)i_{a1}+\frac{M^2p(p-j\omega)i_{a1}}{R_u+(p-j\omega)S_u} \\ v_{a2} &= -(R_a+pS_a)i_{a1}+\frac{M^2p(p+j\omega)i_{a1}}{R_u+(p+j\omega)S_u} \end{aligned}\right\} \quad . \; . \; (10)$$

The symbolic solution of (10) becomes as following

$$\left.\begin{aligned} i_{a1} &= \frac{\{R_u+(p-j\omega)S_u\}v_{a1}}{M^2p(p-j\omega)-(R_a+pS_a)\{R_u+(p-j\omega)S_u\}} \\ &= -\frac{j\omega ME}{3R_u}\,\frac{Y_{a1}(p)}{Z_{a1}(p)}\,\varepsilon^{j\theta} \\ i_{a2} &= \frac{\{R_u+(p+j\omega)S_u\}v_{a2}}{M^2p(p+j\omega)-(R_a+pS_a)\{R_u+(p+j\omega)S_u\}} \\ &= \frac{j\omega ME}{3R_u}\,\frac{Y_{a2}(p)}{Z_{a2}(p)}\,\varepsilon^{-j\theta} \end{aligned}\right\} \quad . \; . \; (11)$$

where $\qquad Y_{a1}=R_u+(p-i\omega)S_u, \qquad Y_{a2}=R_u+(p+j\omega)S_u,$

$$Z_{a1}=M^2p(p-j\omega)-(R_a+pS_a)\{R_u+(p-j\omega)S_u\}$$

$$Z_{a2}=M^2p(p+j\omega)-(R_a+pS_a)\{R_u+(p+j\omega)S_u\}$$

Let the two roots of $Z_{a1}(p)=0$ and $Z_{a2}(p)=0$ be a_1, a_1' and a_2, a_2' respectively. Then a_1 and a_2, a_1' and a_2' are conjugate complex mumber, and we get by applying the expansion theorem on (11)

$$i_{a1}=-\frac{j\omega ME\varepsilon^{j\varphi}}{3R_u}\left\{\frac{Y_{a1}(j\omega)}{Z_{a1}(j\omega)}\varepsilon^{j\omega t}-\frac{Y_{a1}(a_1)\varepsilon^{\alpha_1 t}}{(j\omega-a_1)\dfrac{\partial Z_{a1}}{\partial p}_{(p=\alpha_1)}}\right.$$

$$\left.\begin{aligned} &\left.-\frac{Y_{a1}(a_1')\varepsilon^{\alpha_1' t}}{(j\omega-a_1')\dfrac{\partial Z_{a1}}{\partial p}_{(p=\alpha_1')}}\right\} \\ i_{a2} &= \frac{j\omega ME\varepsilon^{-j\varphi}}{3R_u}\left\{\frac{Y_{a2}(-j\omega)}{Z_{a2}(-j\omega)}\varepsilon^{-j\omega t}+\frac{Y_{a2}(a_2)\varepsilon^{\alpha_2 t}}{(j\omega+a_2)\dfrac{\partial Z_{a2}}{\partial p}_{(p=\alpha_2)}}\right. \\ &\left.+\frac{Y_{a2}(a_2')\varepsilon^{\alpha_2 t}}{(j\omega+a_2')\dfrac{\partial Z_{a2}}{\partial p}_{(p=\alpha_2')}}\right\} \end{aligned}\right\} \quad . \; . \; (12)$$

$\dfrac{Y_{a1}(j\omega)}{Z_{a1}(j\omega)}$ and $\dfrac{Y_{a2}(-j\omega)}{Z_{a1}(-j\omega)}$, $\quad \dfrac{Y_{a1}(a_1)}{(j\omega-a_1)\dfrac{\partial Z_{a1}}{\partial p}_{(p=\alpha_1)}}$ and

$-\dfrac{Y_{a2}(a_2)}{(j\omega+a_2)\dfrac{\partial Z_{a2}}{\partial p}_{(p=\alpha_2)}}$, $\quad \dfrac{Y_{a1}(a_1')}{(j\omega-a_2')\dfrac{\partial Z_{a1}}{\partial p}_{(p=\alpha_1')}}$ and

$-\dfrac{Y_{a2}(a_2')}{(j\omega+a_2')\dfrac{\partial Z_{a2}}{\partial p}_{(p=\alpha_2')}}$ are conjugate.

We know $i_a=i_{a1}+i_{a2}$ etc. and we see that i_a, i_b, i_c are real though i_{a1}, i_{a2} are complex numbers. The complete solution is thus obtained. If the short-circuit occurrs at the instant when $v_a=0$, we should have $\varphi=0$.

Since i_{a1} and i_{a2} are conjugate, we need not calculate both i_{a1} and i_{a2}, one of them will be sufficient. i_a is twice as large as the real part of i_{a1}.

From (12) we see that the transient term consists of two damped A. C. components, the actual calculation shows that the one is nearly a damped D. C. component and the other is of the frequency quite near to the machine frequency.

Now consider the case when the symmetrical three phase voltage of opposite phase sequence is suddenly impressed on the armature of the symmetrical three phase alternator running with zero excitation. Then this is the case shown in Fig. 3, and since the voltage is of opposite phase sequence we should have

$$\left.\begin{aligned} v_{a1} &= v\varepsilon^{-j\theta} \\ v_{a2} &= v\varepsilon^{j\theta} \end{aligned}\right\} \quad \cdot \cdot \quad (13)$$

And the current will become as following

$$\left.\begin{aligned} i_{a1} &= v\varepsilon^{-j\varphi}\left\{\frac{Y_{a1}(-j\omega)}{Z_{a1}(-j\omega)}\varepsilon^{-j\omega t}+\frac{Y_{a1}(a_1)\varepsilon^{\alpha_1 t}}{(j\omega+a_1)\dfrac{\partial Z_{a1}}{\partial p}_{(p=\alpha_1)}}\right. \\ &\quad \left.+\frac{Y_{a1}(a_1')\varepsilon^{\alpha_1' t}}{(j\omega+a_1')\dfrac{\partial Z_{a1}}{\partial p}_{(p=\alpha_1')}}\right\} \\ i_{a1} &= v\varepsilon^{j\varphi}\left\{\frac{Y_{a2}(j\omega)}{Z_{a2}(j\omega)}\varepsilon^{j\omega t}-\frac{Y_{a2}(a_2)\varepsilon^{\alpha_2 t}}{(j\omega-a_2)\dfrac{\partial Z_{a1}}{\partial p}_{(p=\alpha_2)}}\right. \\ &\quad \left.-\frac{Y_{a2}(a_2')\varepsilon^{\alpha_2' t}}{(j\omega-a_2')\dfrac{\partial Z_{a2}}{\partial p}_{(p=\alpha_2')}}\right\} \end{aligned}\right\} \quad \cdot \cdot \quad (14)$$

Actual calculation shows that the initial magnitude of the damped A. C. component is very small and thus the A. C. component in the current when the three phase voltage of opposite phase sequence is suddenly impressed does not damp practically, in other words, the transient negative phase sequence impedance is practically constant.

From (12) we see that $-\dfrac{Z_{a1}(j\omega)}{Y_{a1}(j\omega)}=R_a+j\omega S_a$ is the symbolic impedance of the armature winding in the steady state and is generally konwn as the synchronous impedance or positive phase sequence impedance Z_1. From (12) we see that $-\dfrac{Z_{a2}(j\omega)}{Y_{a2}(j\omega)}=R_a+j\omega S_a+\dfrac{2\omega^2M^2}{R_u+j2\omega S_u}$ is the symbolic impedance of the armature winding to the three phase current of opposite or negative phase sequence in the steady state, which is generally known as the negative phase sequence impedance Z_2.

We can easily infer that the negative phase sequence impedance possesses less reaceance component and greater resistance component when compared with the positive phase sequence impedance.

$$\left.\begin{aligned} Z_1 &= R_a+j\omega S_a \\ Z_2 &= R_a+j\omega S_a+\frac{2\omega^2M^2}{R_u+j2\omega S_u} \end{aligned}\right\} \quad \cdot \cdot \quad (15)$$

5. Calculation of Line Short-Circuit Current.

Similar to the case of three phase short-circuit, the single phase short-circuit current can be obtained by impressing an E. M. F. opposite to that just before the occurrence of short-circuit between b and c as shown in Fig. 4. The voltage before the occurrence of short-circuit is obtained from (6)

$$v_b=a^2v_{a1}+av_{a2}$$

$$v_c=av_{a1}+a^2v_{a2}$$

$$v_{b-c}=v_b-v_c=(a^2-a)(v_{a1}-v_{a2})=\frac{2\omega ME}{\sqrt{3}\,R_u}\cos(\omega t+\varphi) \quad \cdot \cdot \quad (16)$$

Therefore the voltage to be impressed in the case of Fig. 4 is $-\dfrac{2\omega ME}{\sqrt{3}\,R_u}\cos(\omega t+\varphi)$. The crcuiuit u, v, w in Fig. 4 is exactly the same as before, so that it is evident that (8), (9) and (10) hold good.

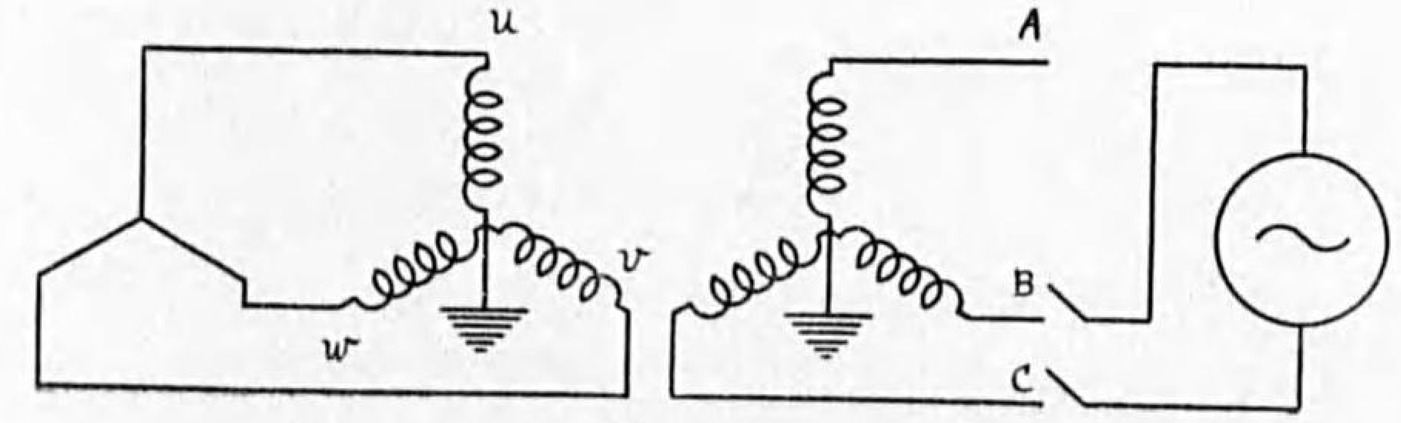


Fig. 4.

We have following conditions in the stator circuit

$$v_{b-c}=(a^2-a)(v_{a1}-v_{a2})=-\frac{2\omega ME}{\sqrt{3}R_u}\cos(\omega t+\varphi)$$

$$i_a=0, \qquad i_c=-i_b \qquad i_{a0}=0, \qquad i_{a2}=-i_{a1}$$

We get from (10)

$$v_{a1}-v_{a2}=\left\{-2(R_a+pS_a)+\frac{M^2p(p-j\omega)}{R_u+(p-j\omega)S_u}+\frac{M^2p(p+j\omega)}{R_u+(p+j\omega)S_u}\right\}i_{a1}$$

and thence

$$\left.\begin{aligned}
i_{a1}&=\frac{\{(R_u+pS_u)^2+\omega^2S_u^2\}(v_{a1}-v_{a2})}{2[M^2p\{p(R_u+pS_u)+\omega^2S_u\}-(R_a+pS_a)\{R_u+pS_u)^2+\omega^2S_u^2\}]}\\
&=\frac{Y(p)(v_{a1}-v_{a2})}{2Z(p)}\\
\text{where}&\\
Y(p)&=(R_u+pS_u)^2+\omega^2S_u^2\\
Z(p)&=S_u(M^2-S_aS_u)p^3+(M^2R_u-R_aS_u^2-2R_uS_uS_a)p^2\\
&\quad+(\omega^2M^2S_u-2R_uR_aS_u-S_aR_u^2-\omega^2S_aS_u^2)p\\
&\quad-(R_aR_u^2+\omega^2S_u^2R_a)\\
i_b&=a^2i_{a1}+ai_{a2}\\
&=(a^2-a)i_1\\
&=\frac{Y(p)(a^2-a)(v_{a1}-v_{a2})}{2Z(p)}
\end{aligned}\right\} \quad . \quad (17)$$

Since $(a^2-a)(v_{a1}-v_{a2})=-\frac{2\omega ME}{\sqrt{3}R_u}\cos(\omega t+\varphi)$, the symbolic solution of i_b is obtained as following

$$i_b=-\frac{2\omega ME}{\sqrt{3}R_u}\frac{Y(p)}{2Z(p)}\cos(\omega t+\varphi)$$

$$=-\frac{2\omega ME}{\sqrt{3}R_u}\text{ Real part of }\frac{\varepsilon^{j\varphi}Y(p)\varepsilon^{j\omega t}}{2Z(p)} \quad . \ . \ . \ . \ . \quad (18)$$

$\frac{\varepsilon^{j\varphi}Y(p)\varepsilon^{j\omega t}}{2Z(p)}$ is easily expanded by the general theorem.

$Z(p)=0$ is a cubic equation in p with real coefficients, so that there are one real and two complex (conjugate with each other) roots or the three roots may be all real. In the first case the transient consists of a damped D. C. component and a damped A. C. component.

Since v_{b-c} is $\frac{2\omega ME}{\sqrt{3}R_u}\cos(\omega t+\varphi)$ before the occurrence of short-circuit, $-\frac{Y(p)}{2Z(p)_{(p=j\omega)}}$ is the symbolic impedance referred to the voltage before the short-circuit v_{b-c} and the sustained short-circuit current i_b.

$$-\frac{Y(p)}{2Z(p)_{(p=j\omega)}}=\left\{2(R_a+pS_a)-\frac{M^2p(p-j\omega)}{R_u+(p-j\omega)S_u}-\frac{M^2p(p+j\omega)}{R_u+(p+j\omega)S_u}\right\}_{(p=j\omega)}$$

$$=2(R_a+j\omega S_a)+\frac{2\omega^2M^2}{R_u+j2\omega S_u}$$

From (16) we find that

$$-\frac{Y(j\omega)}{2Z(j\omega)}=Z_1+Z_2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (19)$$

where Z_1 and Z_2 are positive and negative phase sequence impedance in the steady state.

6. Calculation of Line to Neutral Short-Circuit Current.

The current when one terminal is suddenly short-circuited to the neutral is obtained by impressing a voltage between the terminal and the neutral as shown in Fig. 5, the voltage to be impressed should be opposite to that just before the occurrence of the short-circuit.

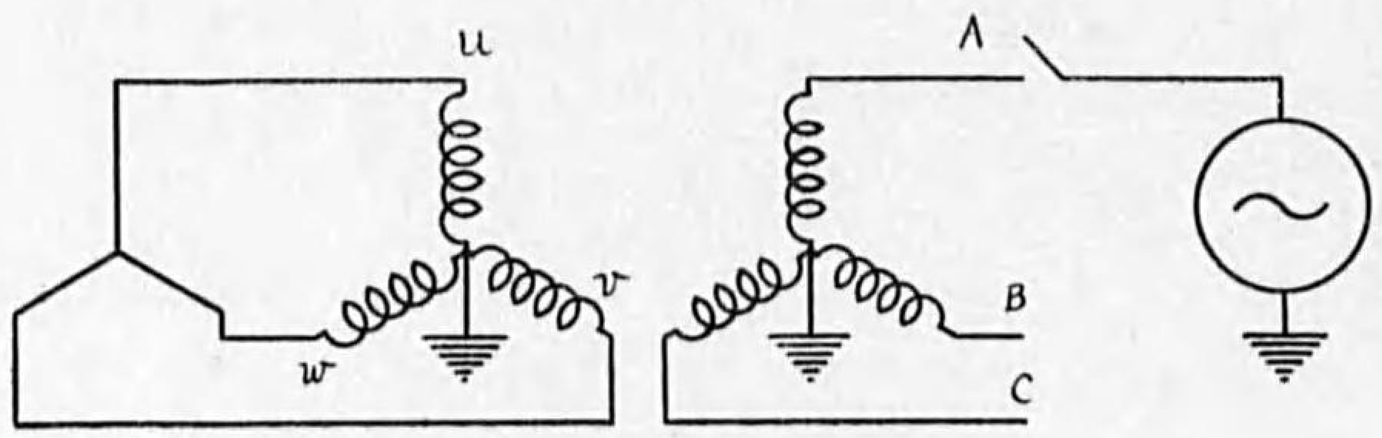


FIG. 5.

v_a, the potential of a, is given by (6) as $-\dfrac{2\omega ME}{3R_u}\sin(\omega t+\varphi)$, so that the potential to be impressed in Fig. 5 should be $\dfrac{2\omega ME}{3R_u}\sin(\omega t+\varphi)$. It is evident that (8), (9) and (10) will hold good as in the previous case.

We have following conditions

$$i_b=0,\quad i_c=0 \quad \text{thence} \quad i_{a0}=i_{a1}=i_{a2}=\frac{i_a}{3} \qquad \cdots\cdots\cdots (20)$$

From the first of (3′) and (10), we get

$$v_{a0}=-(R_a+pS_{a0})i_{a0}$$

$$v_{a1}=-(R_a+pS_a)i_{a1}+\frac{M^2p(p-j\omega)i_{a1}}{R_u+(p-j\omega)S_u}$$

$$v_{a2}=-(R_a+pS_a)i_{a2}+\frac{M^2p(p+j\omega)i_{a2}}{R_u+(p+j\omega)S_u}$$

Adding these three equations together and using the condition (20), we get

$$v_{a0}+v_{a1}+v_{a2}=v_a=\Big\{-(R_a+pS_{a0})-2(R_a+pS_a)$$

$$+\frac{M^2p(p-j\omega)}{R_a+(p-j\omega)S_u}+\frac{M^2p(p+j\omega)}{R_u+(p+j\omega)S_u}\Big\}\frac{i_a}{3} \qquad \cdots\cdots (21)$$

The symbolic solution of i_a and its expansion is easily obtained from (21).

The impedance operator on the right hand side of (21), if the sign is reversed and p is put $j\omega$, will be the symbolic impedance connecting v_a and $i_a/3$.

And we have

$$R_a+j\omega S_{a0}+2(R_a+j\omega S_a)+\frac{2\omega^2M^2}{R_u+j2\omega S_u}=Z_0+Z_1+Z_2 \qquad \cdots (22)$$

(22) is well-known in the sustained state.

7. Measurement of Constants and the Equivalent Symmetrical Alternator.

With the ideal alternator with three phase stator and rotor winding, R_a and R_u can be measured with D. C. in a very simple manner.

With the connection as shown in Fig. 2, let the exciting current in phase u be i_f, we have

$$i_f=-\frac{2E}{3R_u}$$

$$v_{b-c}=\sqrt{3}\,\omega Mi_f\cos(\omega t+\varphi)$$

Thus M is obtained from the relation between the maximum value of line voltage and the exciting current, provided the frequency be given.

Keeping the rotor stationary we can consider the symmetrical machine as a symmetrical three phase transformer. If the stator or rotor winding be excited with the three phase voltage, there will appear the induced three phase voltage in the other winding. We can get M from the relation between the exciting current in one winding and the induced voltage in the other, provided the frequency be given. In this case we need not bother with the factor $\sqrt{2}$ to get the maximum value from the effective value, as the current and voltage are treated both in effective value.

$R_a+j\omega S_a$ is the symbolic synchronous impedance of the armature circuit, so that it is obtained from the open-circuit saturation curve and the short-circuit curve using u, v, w as the field winding. Similarly $R_u+j\omega S_u$ is obtained by using a, b, c as the field winding. Using the value of R_a or R_u already obtained, we can easily calculate S_a or S_u provided the frequency be given.

$R_a+j\omega S_a$ or $R_u+j\omega S_u$ is also obtained by measuring the effective impedance of one phase against the symmetrical three phase voltage applied on the stator or rotor winding keeping the other winding open at the terminal. In this case, if the rotor core be entirely free from eddy current, the value of $R_a+j\omega S_a$ or $R_u+j\omega S_u$ is not affected by the motion of the rotor.

This method is the only one to obtain the effective resistance of the stator or rotor winding.

As is already shown in (15), the value of Z_1 and Z_2, used in the calculation of sustained state, are as following

$$Z_1=R_a+j\omega S_a$$

$$Z_2=R_a+j\omega S_a+\frac{2\omega^2M^2}{R_u+j2\omega S_u}$$

Therefore if Z_1, Z_2 and M be known, $R_u+j\omega S_u$ can be calculated, the actual measurement of the synchronous impedance of the rotor circuit is not absolutely necessary.

Suppose we get $Z_2=R_2+j\omega L_2$ from some measurement in the sustained state then

$$R_2+j\omega L_2=R_a+j\omega S_a+\frac{2\omega^2M^2}{R_u+j2\omega S_u} \quad \ldots\ldots (23)$$

from which

$$(R_2-R_a)+j\omega(L_2-S_a)=\frac{2\omega^2M^2}{R_u+j2\omega S_u}$$

or

$$\frac{R_2-R_a-j\omega(L_2-S_a)}{(R_2-R_a)^2+\omega^2(L_2-S_a)^2}=\frac{R_u+j2\omega S_u}{2\omega^2M^2}$$

So that

$$\left.\begin{aligned} R_u &= \frac{2\omega^2M^2(R_2-R_a)}{(R_2-R_a)^2+\omega^2(L_2-S_a)^2} \\ S_u &= \frac{\omega^2M^2(S_a-S_a)^2}{(R_2-R_a)^2+\omega^2(L_2-S_a)^2} \end{aligned}\right\} \quad \ldots (24)$$

R_2, R_a, S_a, L_2, M and ω are all known, so that R_u and S_u are calculated from (24).

Strictly speaking, the alternator with the amortisseur winding should be treated as an alternator with three phase stator and m phase rotor. However, such an alternator is similar to one with three phase rotor winding in one point that both machines possess symmetrical polyphase rotor winding, and both are the so-called sine wave alternator. Thus we do not expect much error by considering that the alternator with m phase rotor winding or amortisseur winding is equivalent to the ideal alternator with three phase rotor windings. It is evident that Z_2 of the alternator with amortisseur winding is measurable, so that R_u and S_u of the ideal alternator (with three phase stator and three phase rotor windings) equivalent to any given three phase alternator with amortisseur winding can be calculated.

Ordinary three phase alternator with salient pole and single field winding is not the symmetrical three phase alternator. Experiments in the sustained state showed that the ordinary salient pole alternator can be considered as the symmetrical alternator without much error, provided proper precautions such as the elimination of higher harmonics in the measurement be taken.

When the frequency is given the characteristic of the symmetrical three phase alternator is uniquely determined by seven constants R_a, L_a, M_a, R_u, L_u, M_u and M'.

Two symmetrical three phase alternators having identical Z_0, Z_1, Z_2 and indentical open-circuit saturation curve are identical, because there are seven conditions of identity in total, since Z consists of resistance and reactance.

We can consider a symmetrical three phase alternator which is exactly equal to any given salient pole alternator with respect to Z_0, Z_1, Z_2 and open-circuit saturation curve, then this symmetrical three phase alternator is unique. This unique symmetrical three phase alternator may be called the equivalent symmetrical alternator.

As stated above, so far the sustained state is concerned, ordinary alternator is practically equivalent to its equivalent symmetrical alternator.

If it be admitted without great error that the ordinary alternator is equivalent to its equivalent symmetrical alternator, the study of transients will become very simple. The validity of this proposition should be verified only with experiments.

8. Constants of Machines used in the Experiment.

Following two machines C and B were used in the experiment.

Machine C.

This is an induction motor of wound rotor and was used in the previous research[1] also, the rating is as following 10 HP, 200 V, 50~, 960 R. P. M. To keep the rotor circuit symmetry, the exciter is connected with balancing impedance Z_v, Z_w as shown in Fig. 6. Z_v and Z_w are flat coils and the impedance of which was made equal to the impedance between the brushes of the exciter measured with

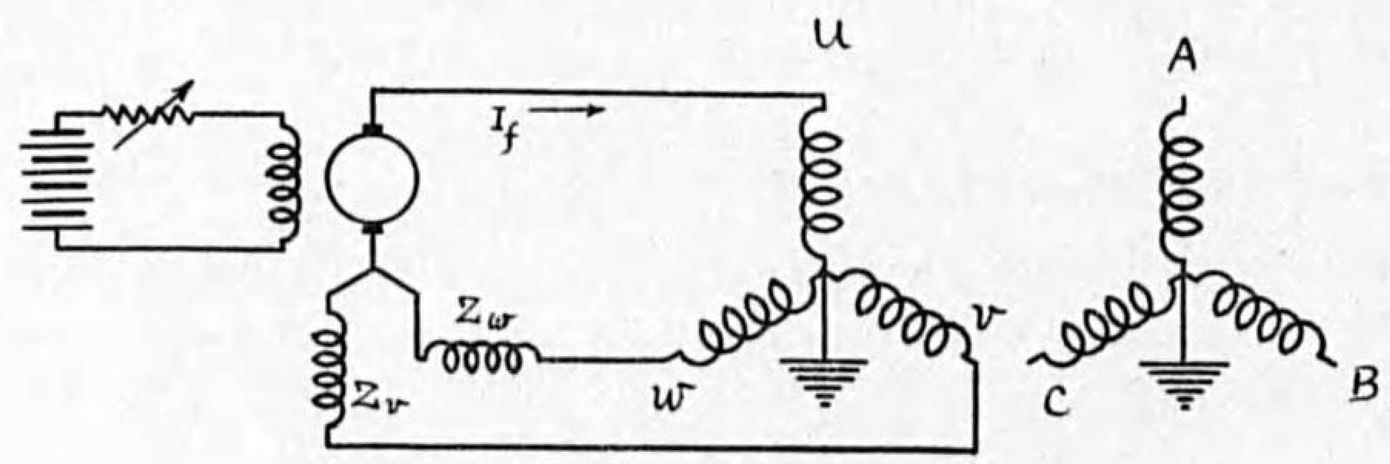


Fig. 6.

(1) loc. cit. Researches E. T. Lab. No. 170.

50 cycles, and they are placed in the rectangular position to avoid the mutual induction between them. The behavior of exciter brush was beyond the scope of the imitation in Z_v and Z_w. The excition of the alternator was controlled by adjusting the exciter field current.

The relation between the exciter current i_f and the voltage between b, c (effective value) was gotten as Fig. 7. From the relation $v_{b-c}=\sqrt{3}\,\omega M i_f \cos(\omega t+\varphi)$ (care should be taken to transform the effective value into the maximum), we got from Fig. 6 as $M = 0.00983$ henries at 150 volts. D. C. resistance of the armature was $R_a=0.154$ ohms per phase. From the synchronous impedance obtained from the short-circuit test we got $S_a=0.0380$ henries.

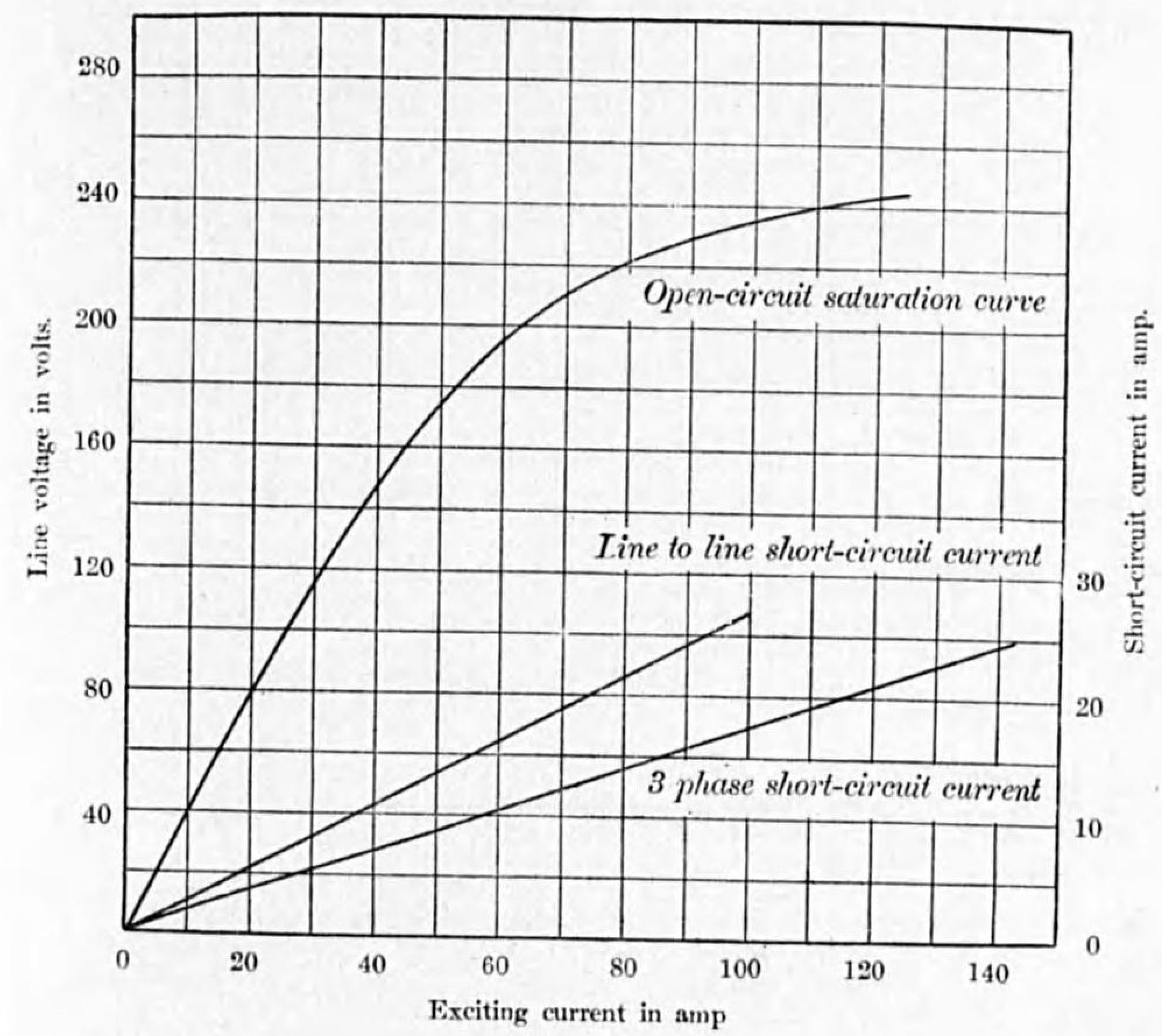


Fig. 7.

Characteristic curve of 10 HP induction motor.

Using a, b, c as the field circuit we got the synchronous impedance of u, v, w as 0.878 ohms. The impedance between the terminals of the exciter was found as 0.095 ohms, so that the total synchronous impedance of the rotor circuit was taken as $0.878+0.878+0.095=0.973$ ohms, then $S_u=0.93\div(2\times\pi\times 50)=0.00310$ henries. The resistance of the exciter brush predominates in the rotor resistance and the total rotor resistance varies with current as Fig. 8.

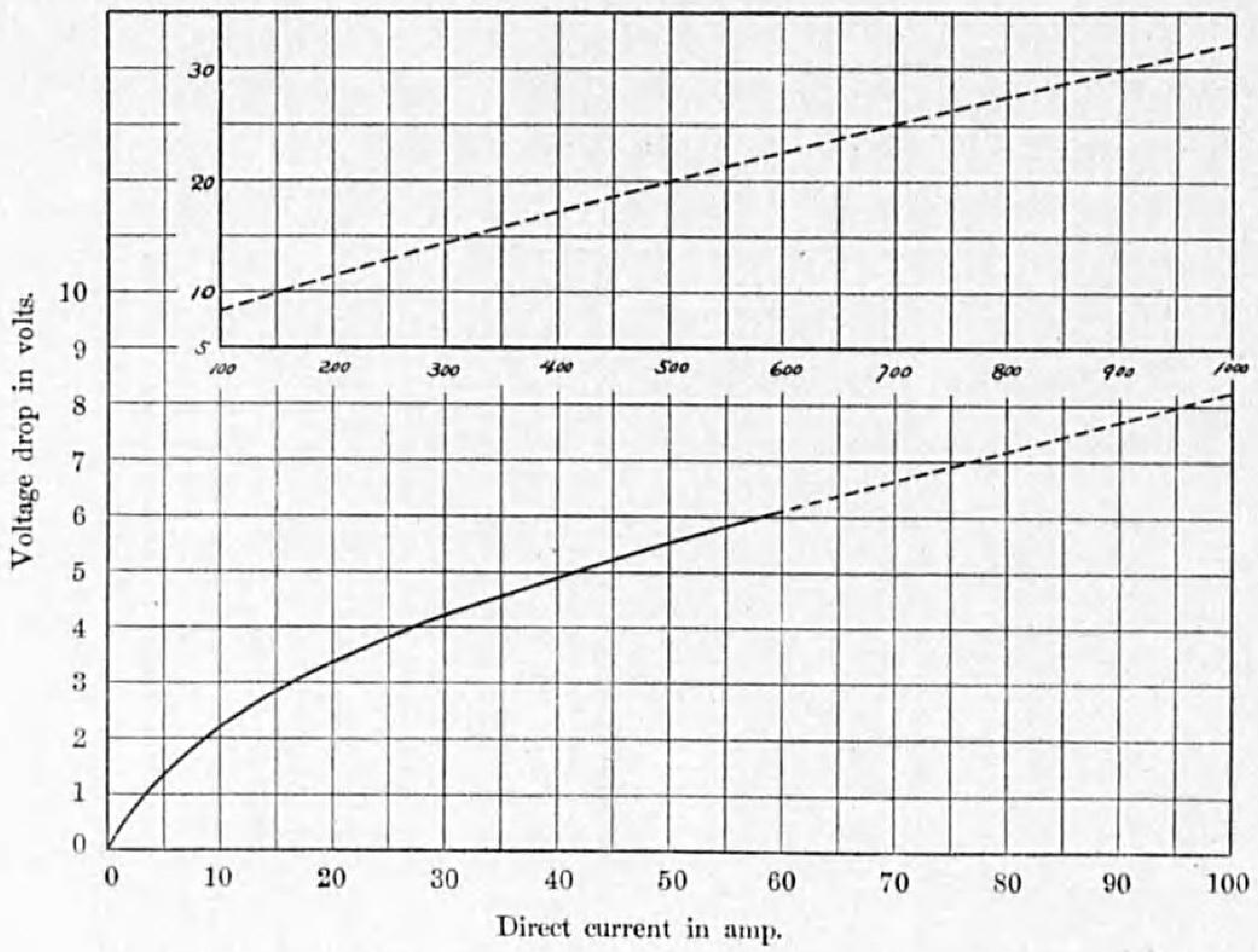


Fig. 8.

Variation of field circuit resistance.

Z_2 of this machine was measured as $2.3/\underline{60°}$ ohms. From Z_1, Z_2 and M we got $R_u=0.191$ ohms, $S_u=0.00303$ henries which check well with the values measured independently.

Machine B was also used in the previous research[1] and it is an ordinary

(1) loc. cit. Researches E. T. Lob. No. 170.

salient pole three phase alternator without amortisseur winding. The excitation was controlled by adjusting the field rheostat, keeping the excitation voltage constant. The open circuit saturation curve was obtained as Fig. 9, from which we got $M=0.153$ henries. From the short-circuit test we got $R_a=0.0535$ ohms and $S_a=0.00232$ henries.

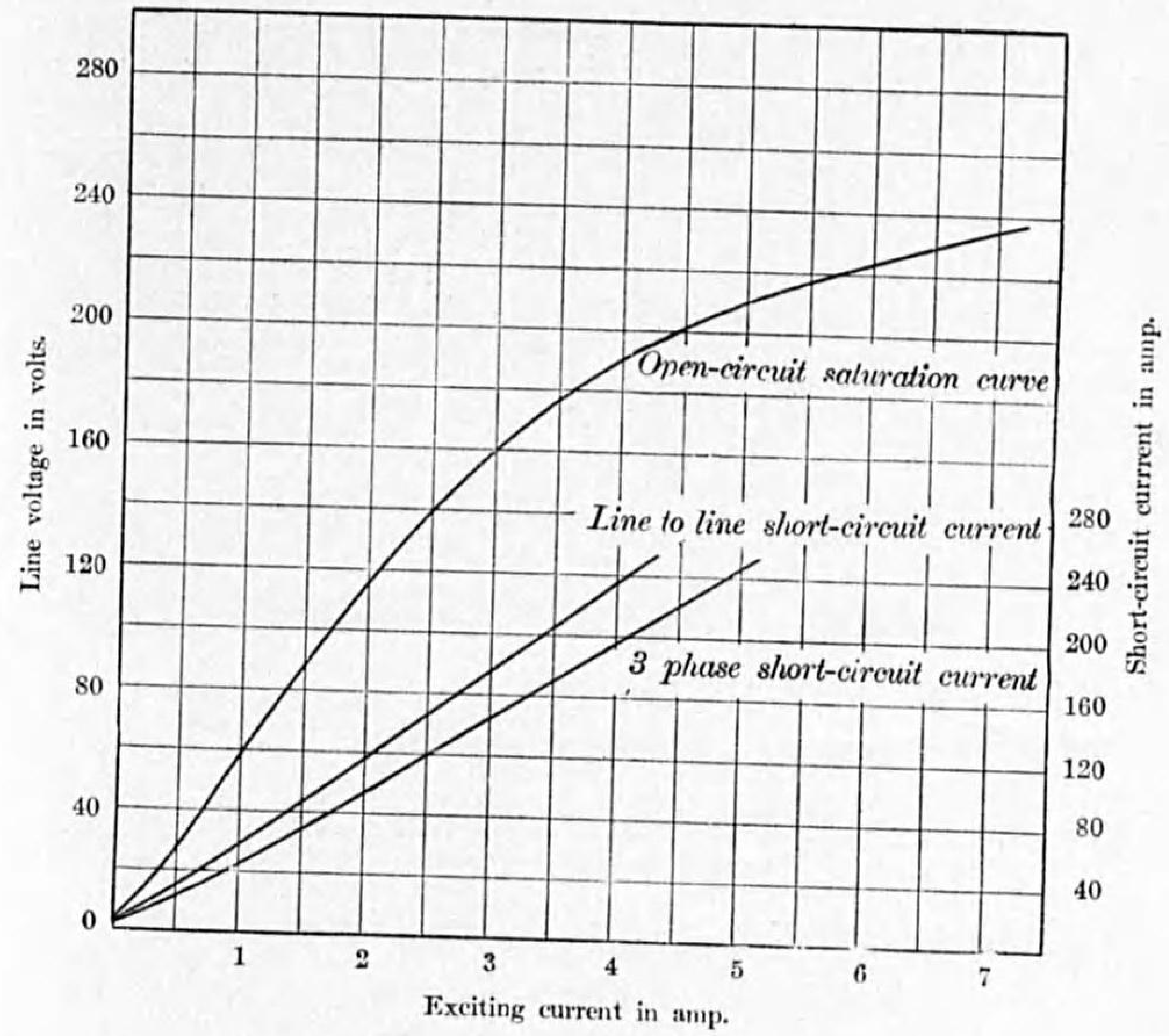


Fig. 9.

Characteristic curves of 35 kVA alternator.

Z_2 is affected to some extent by the change of the field resistance, however, we used the figure $Z_2=0.27/\underline{71°}$ ohms (which corresponds to 10 volts of the open circuit voltage) and obtained $R_u=708$ ohms, $S_u=15.5$ henries, the actually measured D. C. resistance of the field circuit was about 830 ohms.

9. Calculated Value of the Short-Circuit Current.

a. Three phase short-circuit current.

The damping and frequency of the transient in the three phase short-circuit current is obtained from (11) as the root of the equation $Z_{a1}(p)=0$. If arranged in the order of powers of p, we get

$$(S_aS_u-M^2)p^2+\left\{R_aS_u+R_uS_a-j\omega(S_aS_u-M^2)\right\}p+R_a(R_u-j\omega S_u)=0.$$

Since this is a quadratic equation, the roots a, a' can be obtained as complex numbers.

1. Following constants are used in the calculation for machine C. The value R_u is taken from Fig. 8, to correspond to the rotor current actually measured oscillographic in case of sudden short-circuit.

$$M=0.00983, \quad S_u=0.0380, \quad R_u=0.154$$

$$S_a=0.00310, \quad R_a=0.04, \quad \omega=100\pi$$

From these,

$$a_1=-73.05+j\,309.9 \qquad (49.3 \text{ cycles})$$

$$a_1'=-22.03+j\,4.25 \qquad (0.7 \text{ cycles})$$

$$\frac{Y_{a1}(a_1)}{(j\omega-a_1)Z'_{a1}(a_1)}=-0.05976-j0.3877$$

$$\frac{Y_{a1}(a_1')}{(j\omega-a_1')Z'_{a1}(a_1)}=0.05867+j0.4715$$

Putting these values in (12) and remembering that $i_a=i_{a1}+i_{a2}$, $v_a=-\frac{2\omega ME}{3R_u}\sin(\omega t+\varphi)=E_a\sin(\omega t+\varphi)$, we got finally

$$i_a=E_a\left\{0.0838\sin(\omega t+\varphi+\theta_1)-0.392\,\varepsilon^{at}\sin(2\pi\times 49.3t+\varphi+\theta_2)\right.$$
$$\left.+0.475\,\varepsilon^{ct}\sin(2\pi\times 0.7t+\varphi+\theta_3)\right\} \quad . \; . \; . \; . \; . \; . \; . \quad (25)$$

where $a=-73.0$, $c=-22.0$

$\theta_1=-89.3°$, $\theta_2=81.2°$, $\theta_3=82.9°$

The current in phase b or c is gotten by replacing φ with $\varphi+240°$ or $\varphi+120°$.

The calculated value of (25) with $\varphi=\pi$ is shown in Fig. 10.

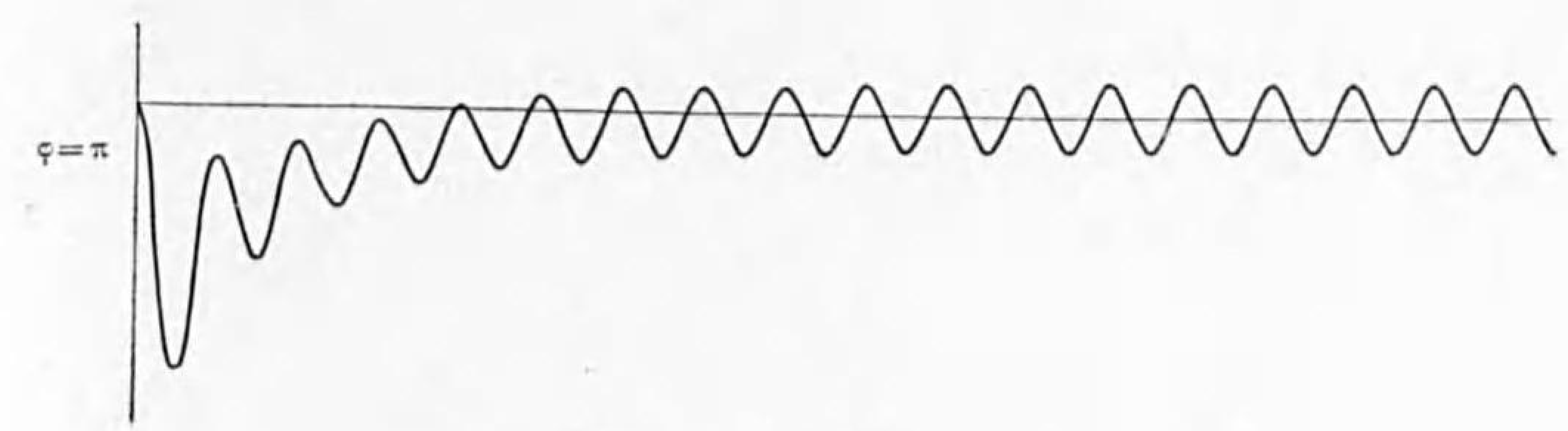


FIG. 10.

3 phase short-circuit current of 10 HP induction motor..

2. With machine B two cases with different R_u were carried out.

In the first place using the constants $M=0.153$, $S_a=0.00232$, $R_a=0.0535$, $S_u=15.5$ and $R_u=708$, we got i_a as following

$$i_a=E_a\left\{1.37\sin(\omega t+\varphi+\theta_1)+2.90\,\varepsilon^{at}\sin(2\pi\times 47.1t+\varphi+\theta_2)\right.$$
$$\left.+4.24\,\varepsilon^{ct}\sin(2\pi\times 2.9t+\varphi+\theta_3)\right\} \quad . \; . \; . \; . \; . \; . \; . \quad (26)$$

where $a=-136,\quad c=-62.1$

$$\theta_1=-85.8^\circ,\qquad \theta_2=-101.1^\circ,\qquad \theta_3=83.8^\circ$$

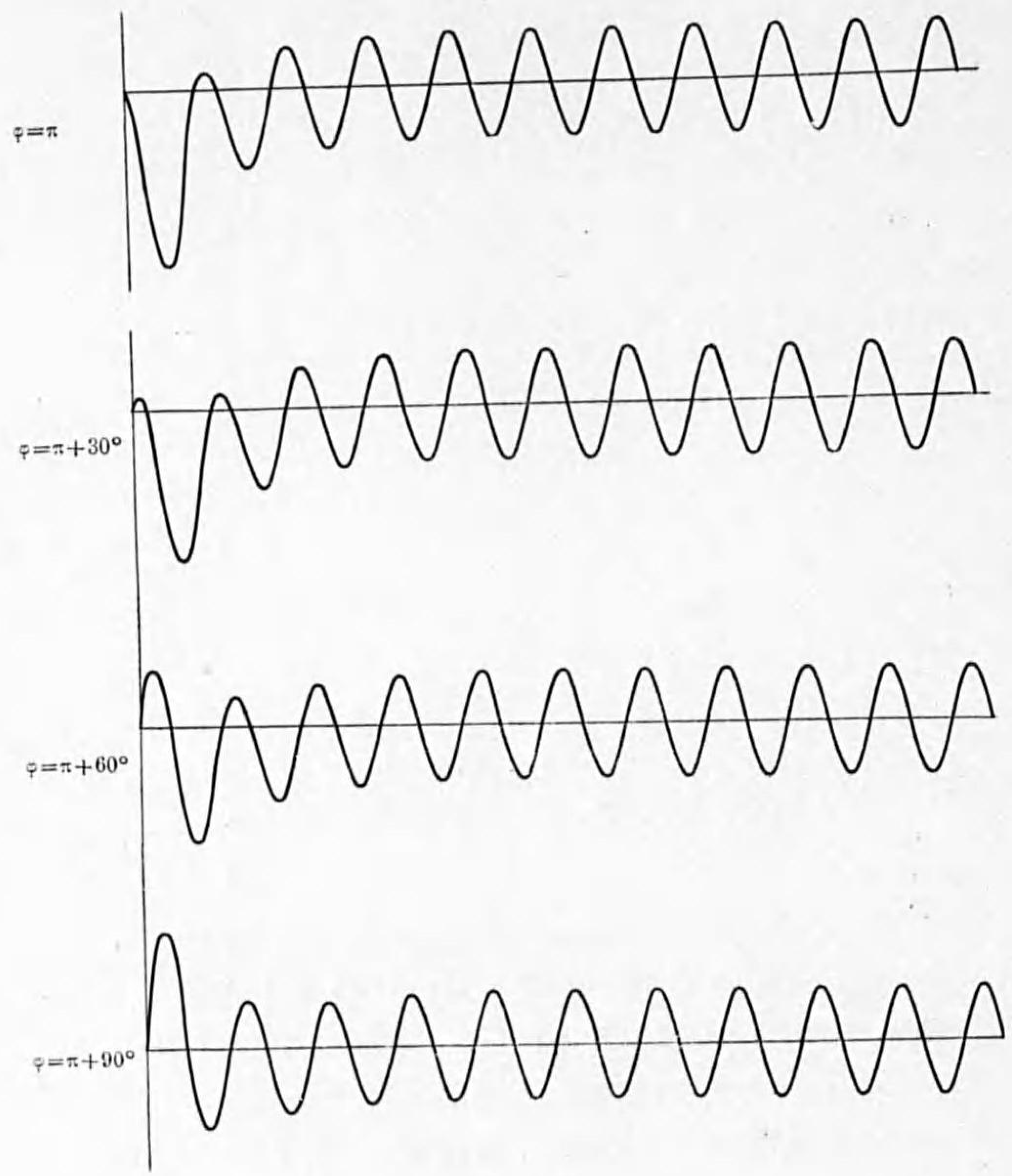


FIG. 11.

3 phase short-circuit current of 35 kVA alternator, with R_u =708 ohms.

With $R_u=70.8$ and other constants as before, we got

$$i_a=E_a\Big\{1.37\sin(\omega t+\varphi+\theta_1)+3.93\,\varepsilon^{at}\sin(2\pi\times 0.3t+\varphi+\theta_2)$$
$$+2.57\,\varepsilon^{ct}\sin(2\pi\times 49.7t+\varphi+\theta_3)\Big\}\quad\ldots\ldots\quad(27)$$

where $a=-66.7,\quad c=-12.9$

$$\theta_1=-85.8^\circ,\qquad \theta_2=100.4^\circ,\qquad \theta_2=-75.9^\circ$$

(26) with $\varphi=\pi$, $\varphi=\pi+30^\circ$, $\varphi=\pi+60^\circ$, $\varphi=\pi+90^\circ$ and (27) with $\varphi=\pi$ are shown in Fig. 11 and Fig. 12. It is very remarkable that the mode of damping is very much affected by R_u.

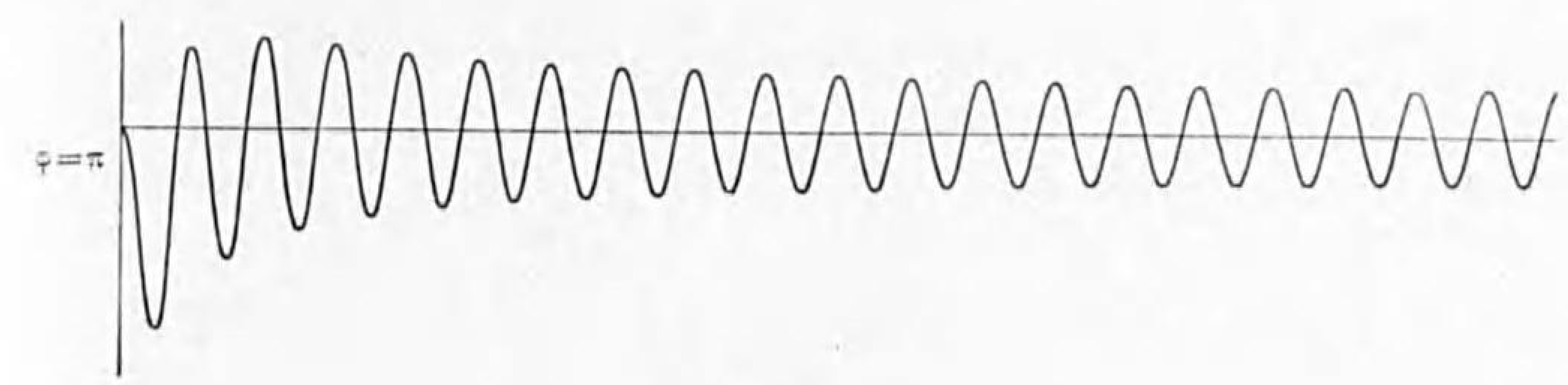


FIG. 12.

3 phase short-circuit current of 35 kVA alternator, with R_u =70.8 ohm.

3. For the next place, though the short-circuit test was not carried out, the calculation was worked out for machine A, which was used in the previous research.(1) Machine A is a salient-pole three phase alternator with amortisseur winding and its rating is 10 kVA, 100 V. 50∼, 1500 R. P. M. Constants were worked out from the measured value $Z_1=1.23\underline{/88.8^\circ}$, $Z_2=0.11\underline{/70^\circ}$ as following $S_a=0.00392$, $R_a=0.0268$, $S_u=40.3$, $R_u=243$ and $M=0.380$ each in ohm and henry.

(1) loc. cit.

We got for the three phase short-circuit current

$$i_a=E_a\{0.813\sin(\omega t+\varphi+\theta_1)+10.89\,\varepsilon^{at}\sin(2\pi\times 2.9t+\varphi+\theta_2)$$

$$+10.25\,\varepsilon^{ct}\sin(2\pi\times 47.1t+\varphi+\theta_3)\} \quad .\ .\ .\ .\ .\ .\ . \quad (28)$$

where $a=-82.3,\qquad c=-71.3$

$\theta_1=-89.1°,\qquad \theta_2=-93.4°,\qquad \theta_3=-85.5°$

(28) with $\varphi=\pi$ is shown in Fig. 13.

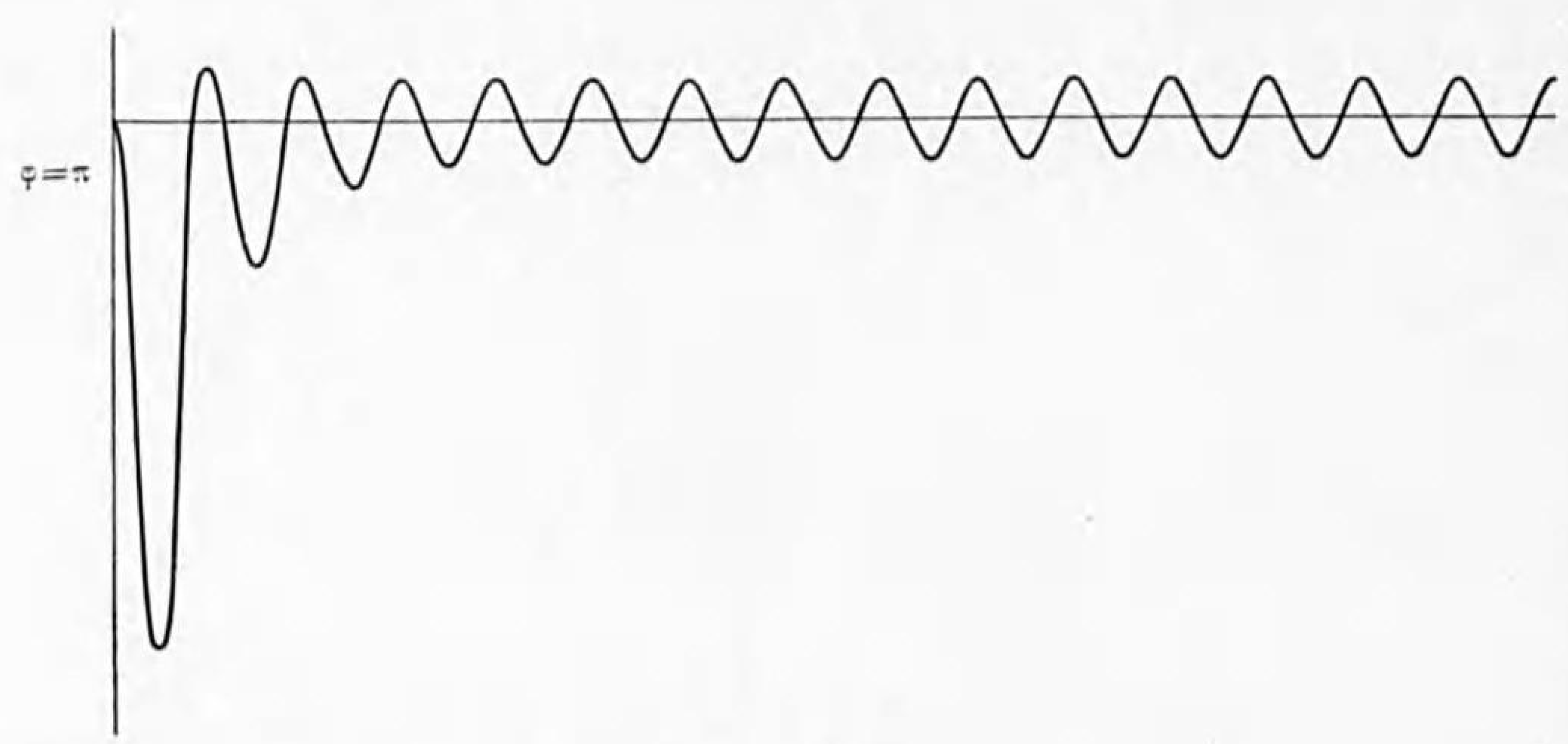


Fig. 13.

3 phase short-circuit current of 10 kVA alternator.

a'. Sudden application of the three phase voltage of the opposite phase sequence.

Putting the same constants as the case a in (14) we got following results, the Y voltage of phase a being considered as $E_a\sin(\omega t+\varphi)$.

(1') Machine C.

$$i_a=E_a\{0.471\sin(\omega t+\varphi+\theta_1)+0.0457\,\varepsilon^{at}\sin(2\pi\times 49.3t+\varphi+\theta_2)$$

$$+0.463\,\varepsilon^{ct}\sin(2\pi\times 0.7t+\varphi+\theta_3)\} \quad .\ .\ .\ .\ .\ .\ . \quad (25')$$

where $a=-73.0,\qquad c=-22.0$

$\theta_1=-75.7°,\qquad \theta_2=12.1°,\qquad \theta_3=105.1°$

(2') Machine B, $R_a=708$

$$i_a=E_a\{3.70\sin(\omega t+\varphi+\theta_1)+0.612\,\varepsilon^{at}\sin(2\pi\times 47.1t+\varphi+\theta_2)$$

$$+3.79\,\varepsilon^{ct}\sin(2\pi\times 2.9t+\varphi+\theta_3)\} \quad .\ .\ .\ .\ .\ .\ .\ . \quad (26')$$

where $a=-136,\qquad c=-62.1$

$\theta_1=-71.0°,\qquad \theta_2=14.9°,\qquad \theta_3=118.6°$

Machine B, $R_a=70.8$

$$i_a=E_a\{3.84\sin(\omega t+\varphi+\theta_1)+3.88\,\varepsilon^{at}\sin(2\pi\times 0.3t+\varphi+\theta_2)$$

$$+0.053\,\varepsilon^{ct}\sin(2\pi\times 49.7t+\varphi+\theta_3)\}$$

where $a=-66.7,\qquad c=-12.9$

$\theta_1=-77.3°,\qquad \theta_3=103.4°,\qquad \theta_3=-20.6°$

(3') Machine A.

$$i_a=E_a\{9.09\sin(\omega t+\varphi+\theta_1)+1.208\,\varepsilon^{at}\sin(2\pi\times 47.1t+\varphi+\theta_2)$$

$$+9.78\,\varepsilon^{ct}\sin(2\pi\times 2.9t+\varphi+\theta_3)\}$$

where $a=-82.3,\quad c=-71.3$

$\theta_1=-70.0°,\quad \theta_2=-11.2°,\quad \theta_3=116.1°$

(25′) with $\varphi=\pi$ is shown in Fig. 14.

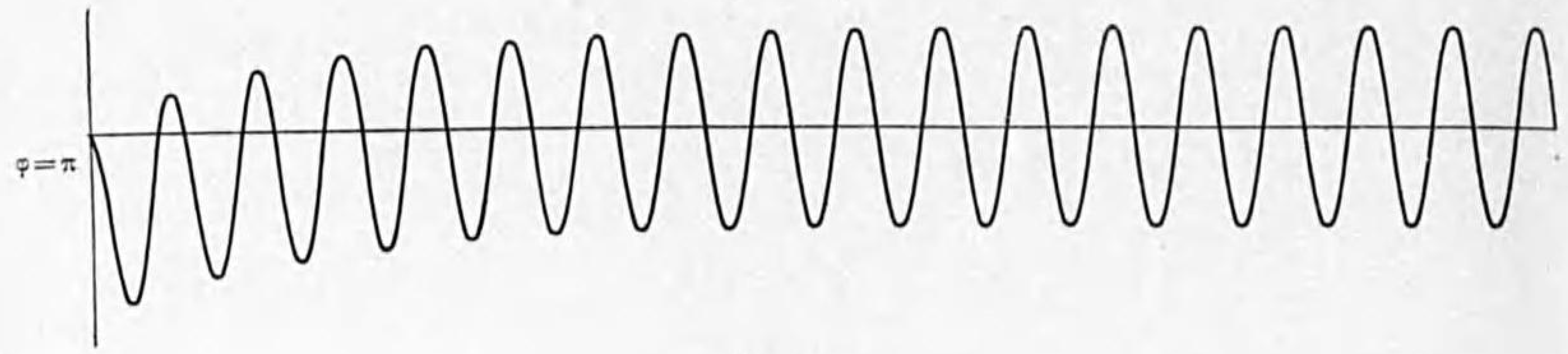


FIG. 14.

Sudden application of 3 phase voltage of negative phase sequence on 10 HP induc.ion motor.

From the results we see that the A. C. component is practically constant throughout the transient period.

b. Line to Line Short-Circuit.

Machine C.

Using the constants as before we got three roots of $Z(p)=0$ as following $(\omega=100\,\pi)$

$$\alpha=-22.87$$

$$\beta=-42.56+j310.7 \qquad \text{(49.5 cycles)}$$

$$\gamma=-42.56-j310.7$$

From these

$$\frac{Y(\alpha)}{(j\omega-\alpha)Z'(\alpha)}=-0.03470+j0.4766$$

$$\frac{Y(\beta)}{(j\omega-\beta)Z'(\beta)}=0.007719-j0.3335$$

$$\frac{Y(\gamma)}{(j\omega-\gamma)Z'(\gamma)}=0.02273-j0.00085$$

Putting these values in (18) we got following results, considering the voltage between b and c as $E_{bc}\cos(\omega t+\varphi)$,

$$i_b=\frac{E_{bc}}{2}\Big\{0.142\cos(\omega t+\varphi+\theta_1)+0.478\,\varepsilon^{at}\cos(\varphi+\theta_2)$$
$$+0.334\,\varepsilon^{bt}\cos(2\pi\times49.5t+\varphi+\theta_3)$$
$$+0.0228\,\varepsilon^{bt}\cos(2\pi\times49.5t-\varphi+\theta_4)\Big\} \quad \cdots\cdots \quad (29)$$

where $a=-22.9,\quad b=-42.6$

$\theta_1=-87.9°,\quad \theta_2=94.2°,\quad \theta_3=-88.7°,\quad \theta_4=2.1°$

Machine B, $R_a=708$

$$i_b=\frac{E_{bc}}{2}\Big\{2.01\cos(\omega t+\varphi+\theta_1)+4.19\,\varepsilon^{at}\cos(\varphi+\theta_2)$$
$$+2.08\,\varepsilon^{bt}\cos(2\pi\times48.0t+\varphi+\theta_3)$$
$$+0.297\,\varepsilon^{bt}\cos(2\pi\times48.0t-\varphi+\theta_4)\Big\} \quad \cdots\cdots \quad (30)$$

where $a=-67.7,\quad b=-88.1$

$\theta_1=-81.8°,\quad \theta_2=102.2°,\quad \theta_3=-81.7°,\quad \theta_4=8.2°$

Machine B, $R_a=70.8$

$$i_b=\frac{E_{bc}}{2}\Big\{2.03\cos(\omega t+\varphi+\theta_1)+3.90\,\varepsilon^{at}\cos(\varphi+\theta_2)$$
$$+1.88\,\varepsilon^{bt}\cos(2\pi\times49.9t+\varphi+\theta_3)$$
$$+0.026\,\varepsilon^{bt}\cos(2\pi\times49.9t-\varphi+\theta_4)\Big\} \quad \cdots\cdots \quad (31)$$

where $a=-66.8, \quad b=-8.7$

$\theta_1=-83.6°, \quad \theta_2=102°, \quad \theta_3=-72.6°, \quad \theta_4=22.6°$

Machine A.

$$i_b=\frac{E_{bc}}{2}\Big\{1.50\cos(\omega t+\varphi+\theta_1)+10.3\,\varepsilon^{at}\cos(\varphi+\theta_2)$$

$$+8.55\,\varepsilon^{bt}\cos(2\pi\times 48.4t+\varphi+\theta_3)$$

$$+0.53\,\varepsilon^{bt}\cos(2\pi\times 48.4t-\varphi+\theta_4)\Big\} \quad \ldots\ldots\ldots \quad (32)$$

where $a=-85.9, \quad b=-36.8$

$\theta_1=-87.2°, \quad \theta_2=105.3°, \quad \theta_3=-75.1°, \quad \theta_4=26.8°$

(29)-(32) with $\varphi=\frac{\pi}{2}$ are shown in Figs. 15-18.

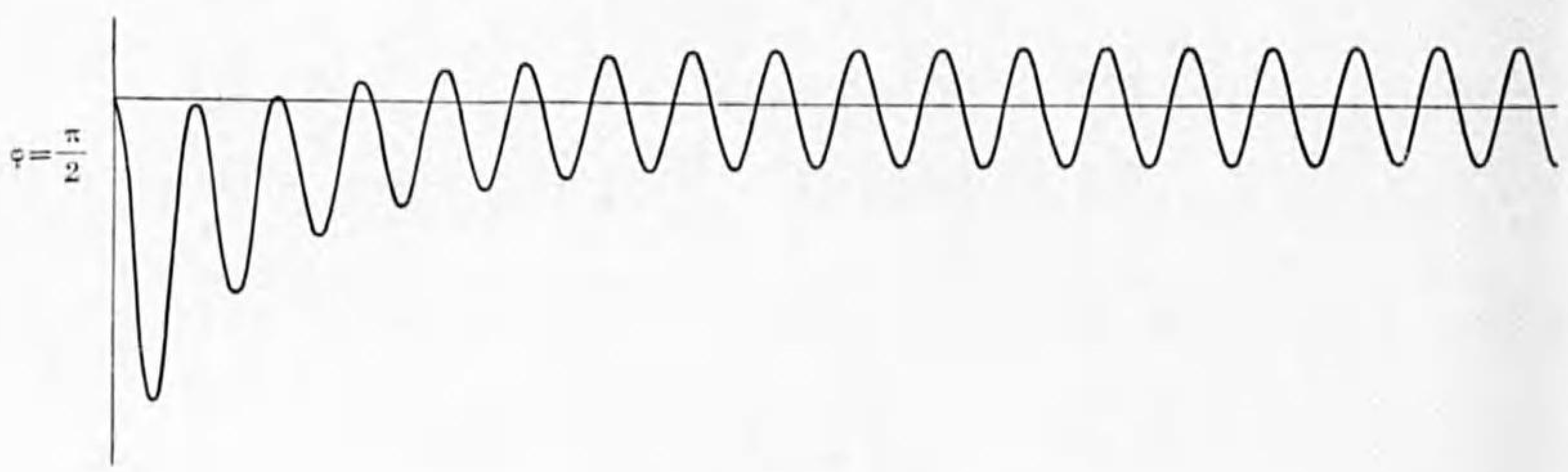


FIG. 15.

Line to line short-circuit current of 10 HP induction motor.

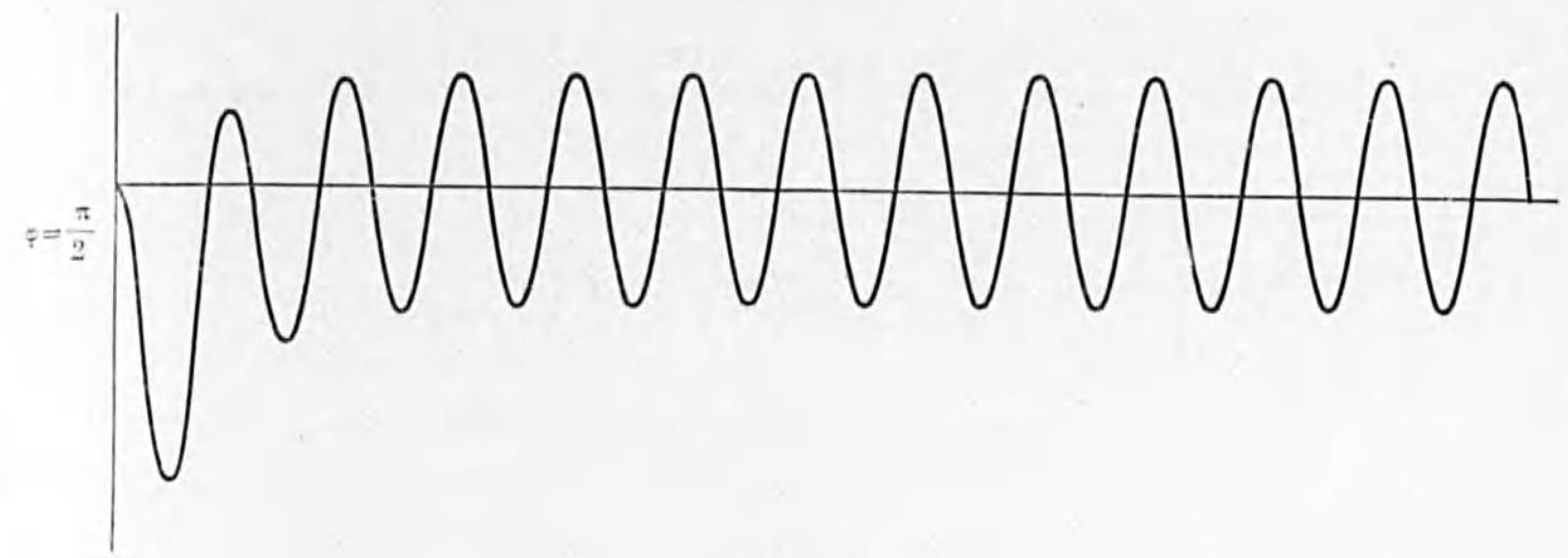


FIG. 16.

Line to line short-circuit current of 35 kVA alternator, R_a = 708 ohm.

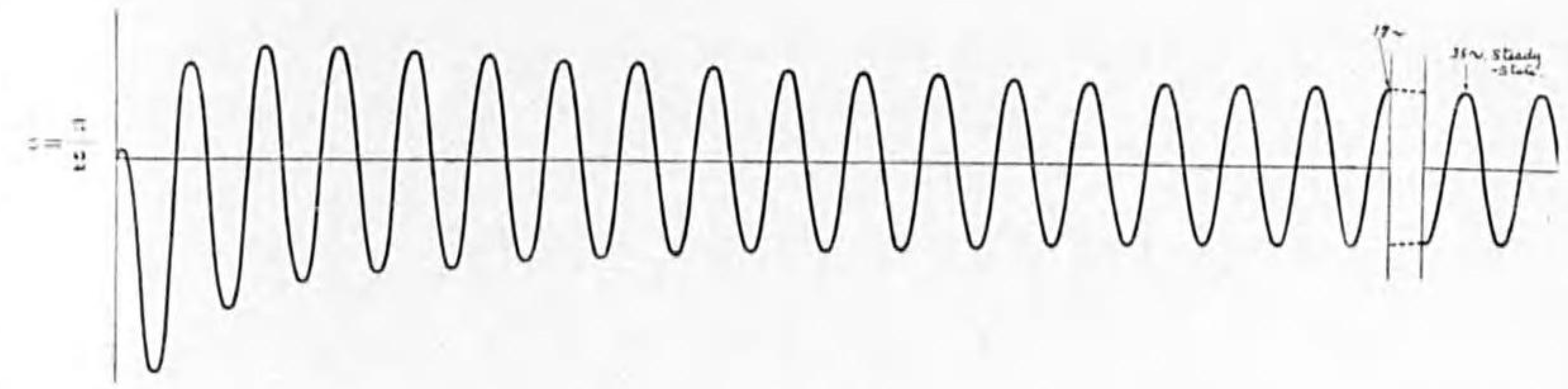


FIG. 17.

Line to line short-circuit current of 35 kVA alternator, R_a = 70.8 ohm.

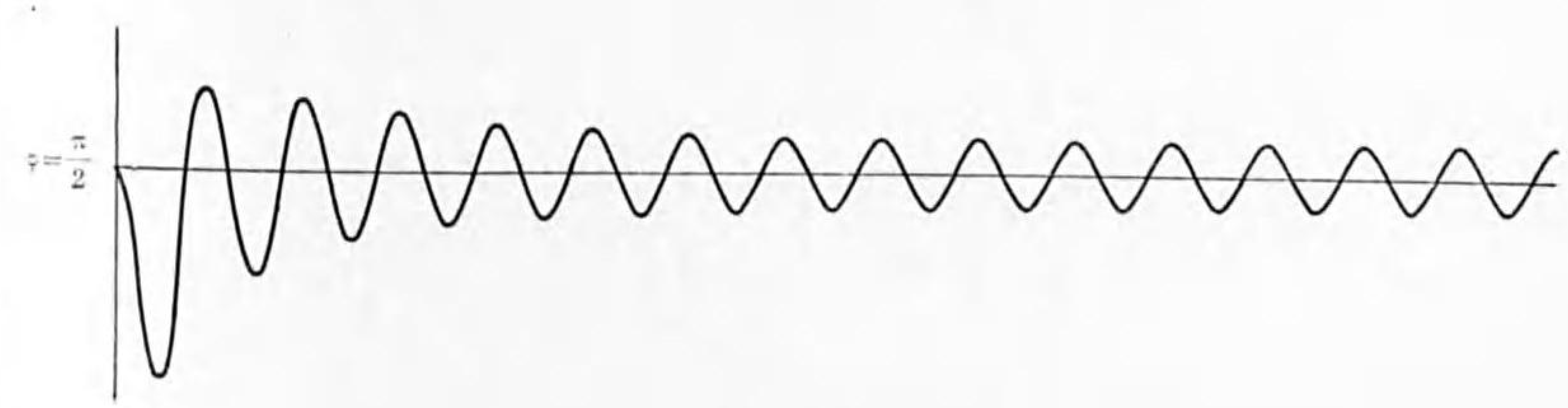


FIG. 18.

Line to line short-circuit current of 10 kVA alternator.

c. Line to Neutral Short-Circuit.

As the machines B and C possessed no neutral lead, the calculation was worked out only for machine A.

On measuring $Z_0=\sqrt{R_a^2+\omega^2 S_{a0}^2}$ with A. C. of 50 cycles, we got 0.083 ohms. Using the D. C. value of R_a we got $S_{a0}=0.000257$ henries.

From (21) we got

$$i_a=3E_a\{0.706\sin(\omega t+\varphi+\theta_1)+3.61\varepsilon^{\alpha_2 t}\sin(\varphi+\theta_2)$$

$$+2.83\varepsilon^{\beta_2 t}\sin(2\pi\times 48.8t+\varphi+\theta_3)$$

$$-0.132\varepsilon^{\beta_2 t}\sin(2\pi\times 48.8t-\varphi+\theta_4)\} \quad \ldots\ldots \quad (33)$$

where v_a being considered as $E_a\sin(\omega t+\varphi)$, and

$$\alpha_2=-91.5, \qquad \beta_2=-28.0$$

$$\theta_1=-86.3°, \qquad \theta_2=106.2°, \qquad \theta_3=-72.5°, \qquad \theta_4=29.5°$$

(33) with $\varphi=\pi$ is shown in Fig. 19.

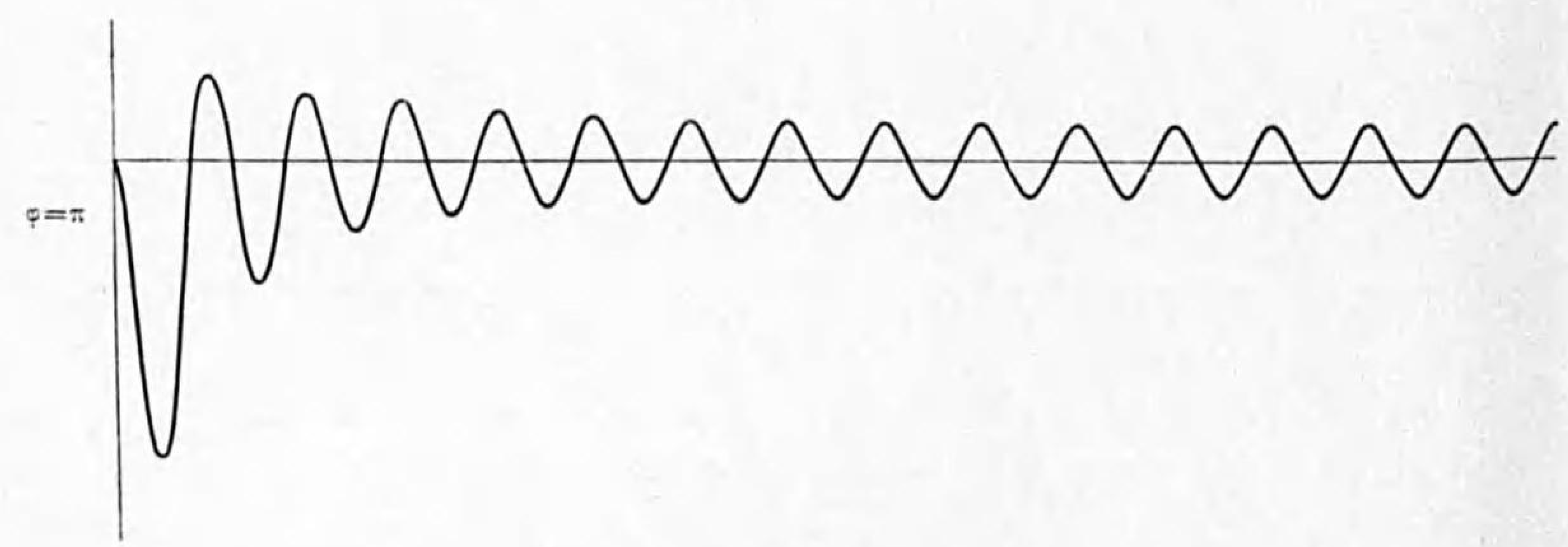


FIG. 19.

One line short-circuit current of 10 kVA alternator.

10. Comparison of Test and Calculated Results.

The actual tests were carried out with machine B and C, no test was performed with machine A.

Ordinary oscillograph was employed, however, no special device was used to close the short-circuiting switch at the particular instant.

Among numerous oscillograms taken at random we could select various short-circuit at different phase. Typical ones are shown in Figs. 20—28. To facilitate the comparison of test and calculated results, the maximum points were connected with a curve, considering the maximum value of the sustained current as unity, full line shows the test results and dotted line shows the calculated results in Figs. 29—34. The calculated results for machine C show some discrepancies, which may be attributed to the inconstancy of the collector brush resistance, as shown in Fig. 8.

Fig. 35 and Fig. 36 show the relation of the ratio of the maximum possible short-circuit current to the maximum value of the sustained short-circuit and the open-circuit voltage. Machine B possessed no exclusive exciter so that the results show the effect of the magnetic saturation as well as the effect of change of R_a. If we consider the magnetic saturation as the reduction of the magnitude of M, the maximum short-circuit current should be diminished as the increase of open-circuit voltage, however, the reduction in R_a means the diminution of damping and thus the maximum short-circuit current tends to increase as the decrease of R_a. On the other hand, with machine C, the collector brush resistance complicated the phenomena, and the author is very sorry that the pure effect of saturation was not observed in this test.

11. Transient Value of Z_1 and Z_2.

Let Z_1 be the ratio of open circuit Y voltage to the A. C. component of the three phase short-circuit current, then Z_1 is smallest at the moment of short-circuit $t=0$ and increases gradually tending to the synchronous impedance. In the sustained state the ratio of the line voltage to the line to line short-circuit current is Z_1+Z_2, where Z_1 the synchronous impedance and Z_2 the negative phase sequence impedance.

We shall extend this principle to transient case and Z_1+Z_2 be the ratio of the line voltage to the A. C. component of the line to line short-circuit current, further we shall consider that Z_1 in Z_1+Z_2 is identical with the transient Z_1 of the three phase short-circuit. Then the simple subtraction will give Z_2 as a function of time, this proposition is rigorously true for the sustained state or $t=\infty$. Figs. 37—40 show the variation of Z_1, Z_1+Z_2 and Z_2 with time for the test results and the calculated results for machine B and C. From these curves we infer that Z_2 is practically constant; which agrees with the case of the sudden application of three phase voltage of negative phase sequence, and Z_1 is equal to Z_2 at the instant of short-circuit $t=0$.

If we admit $Z_1=Z_2$ at $t=0$, the ratio of the A. C. component of three phase short-circuit to that of line to line short-circuit should be 1 : 0.86.

The maximum current occurs after the elapse of half period and we know from our preceding study the damping of A. C. component is greater for the three phase short-circuit than for the line to line short-circuit and the actual maximum current (A. C. component together with D. C. component) is nearly equal for three phase as well as for line to line short-circuit. The test and calculation checked well with the generally approved fact.

12. Approximate Formulas for Sudden Short-Circuit Current.

Mr. M. Urushibata proposed following approximate formulas for the various sudden short-circuit. The damping factors in the formulas are obtained from the general equation by admitting several approximations and they are formulated to conform with the rigorous results at $t=0$ and $t=\infty$. The results worked out with these formulas checked very well with those calculated from the expansion formulas.

1. Three phase short-circuit.

$$i_a=E_a\left\{\left(\frac{1}{Z_1}+\frac{(Z_1-Z_2)\varepsilon^{-\alpha t}}{Z_1Z_2}\right)\cos(\omega t+\theta)-\frac{\varepsilon^{-\beta t}\cos\theta}{Z_2}\right\} \quad . \quad . \quad . \quad (34)$$

Open circuit Y voltage of phase a being considered as $E_a\sin(\omega t+\theta)$ and

$$\alpha=\frac{R_u S_a}{S_a S_u-M^2}$$

$$\beta=\frac{R_a S_u}{S_a S_u-M^2}$$

2. Line to line short-circuit.

$$i_b=E_{bc}\left\{\left(\frac{1}{Z_1+Z_2}+\frac{(Z_1-Z_2)\varepsilon^{-\alpha t}}{2Z_2(Z_1+Z_2)}\right)\cos(\omega t+\theta)-\frac{\varepsilon^{-\beta t}\cos\theta}{2Z_2}\right\} \quad . \quad (35)$$

Open circuit voltage between a and b being considered as $E_{bc}\sin(\omega t+\theta)$ and

$$\alpha=\frac{R_u(2S_aS_u-M^2)}{2S_u(S_aS_u-M^2)}$$

$$\beta=\frac{R_aS_u}{S_aS_u-M^2}$$

3. Line to neutral short-circuit.

$$i_0=3E_a\left\{\left(\frac{1}{Z_0+Z_1+Z_2}+\frac{(Z_1-Z_2)\varepsilon^{-\alpha t}}{(Z_0+Z_1+Z_2)(Z_0+2Z_2)}\right)\cos(\omega t+\theta)\right.$$

$$\left.-\frac{\varepsilon^{-\beta t}\cos\theta}{Z_0+2Z_2}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

Open circuit Y voltage of phase a being considered as $E_a\sin(\omega t+\theta)$ and

$$\alpha=\frac{R_u\{(2S_a+S_0)-M^2\}}{S_u\{(2S_a+S_0)S_u-2M^2\}}$$

$$\beta=\frac{3R_a+S_u}{(2S_a+S_0)S_u-2M^2}$$

Z_0, Z_1 and Z_2 in these formulas should be the reactance part of the zero, positive and negative phase sequence impedances actually measured in the sustained state.

13. Conclusive Remarks.

The themes of this paper can be summarized as following.

1. The solution for sudden short-circuit of the symmetrical three phase alternator is obtained in a very convenient expansion, which is effected by the application of Fortescue's linear transformation.

2. Ordinary alternator with salient pole and single exciting winding can be represented by the equivalent symmetrical alternator for various short-circuit. The constant of the equivalent symmetrical alternator is easily determined from several measurements in the sustained state.

3. The actual fact that the A. C. component of the three phase short-circuit current decreases gradually tending to the sustained short-circuit current can be interpretted as that the positive phase sequence impedance increases gradually tending to the synchronous impedance. The magnitude of the positive phase sequence impedance at the instant of short-circuit or $t=0$, is equal to the negative phase sequence impedance in the sustained state. The negative phase sequence impedance is practically constant throughout the transient phenomena.

4. The maximum current is approximately the same for three phase as well as line to line short-circuit. The damping of the A. C. component, however, is greater for three phase short-circuit than for single phase.

Tokyo, June, 1927.

Fig. 20.
3 phase short-circuit current of 10 HP induction motor
$\varphi \doteqdot \pi$ $V_{ab}=150$ volts

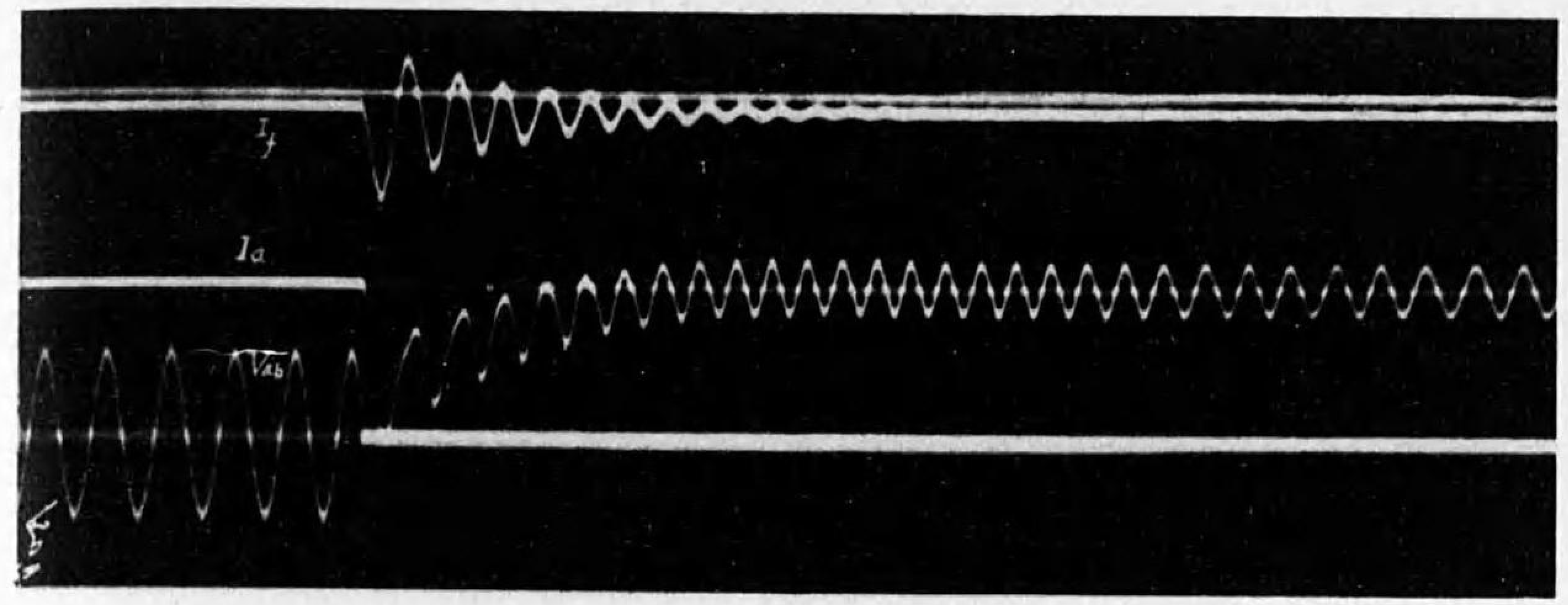


Fig. 21.
3 phase short-circuit current of 35 kVA alternator
$\varphi \doteqdot \pi$ $V_{ab}=10$ volts

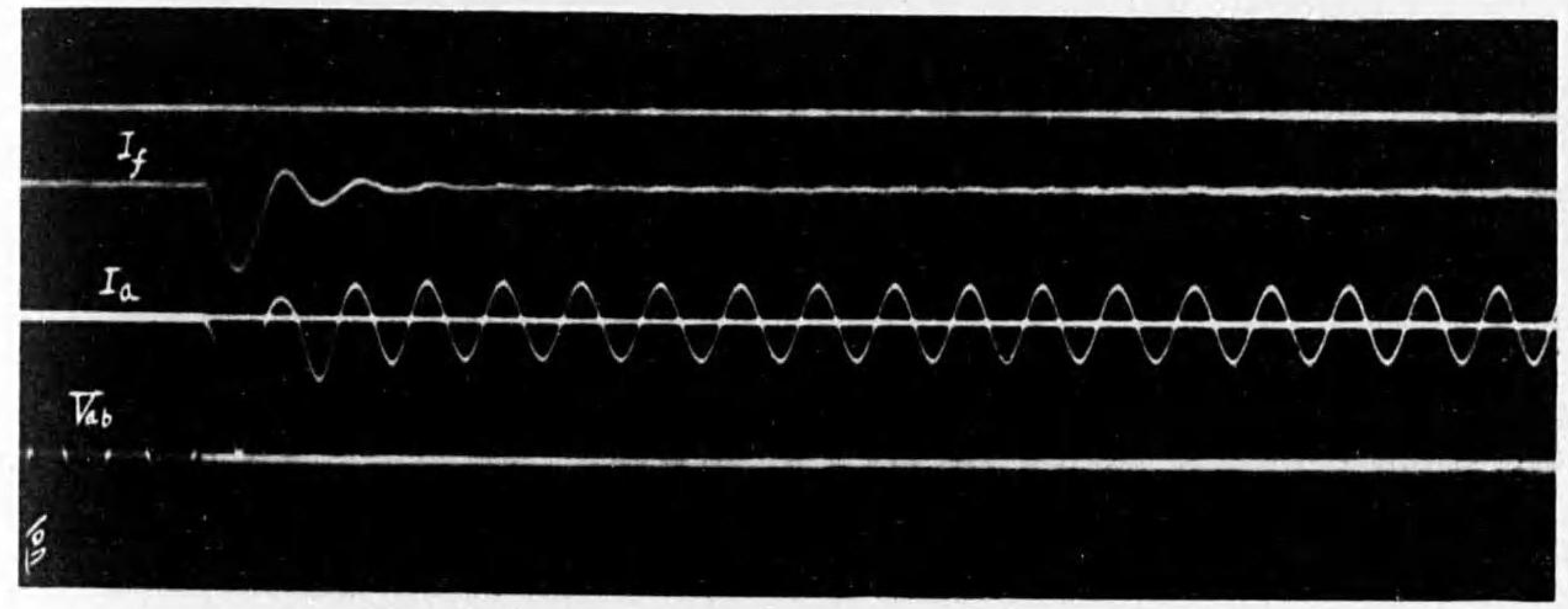


Fig. 22.
3 phase short-circuit current of 35 kVA alternator
$\varphi \doteqdot 30°$ $V_{ab}=10$ volts

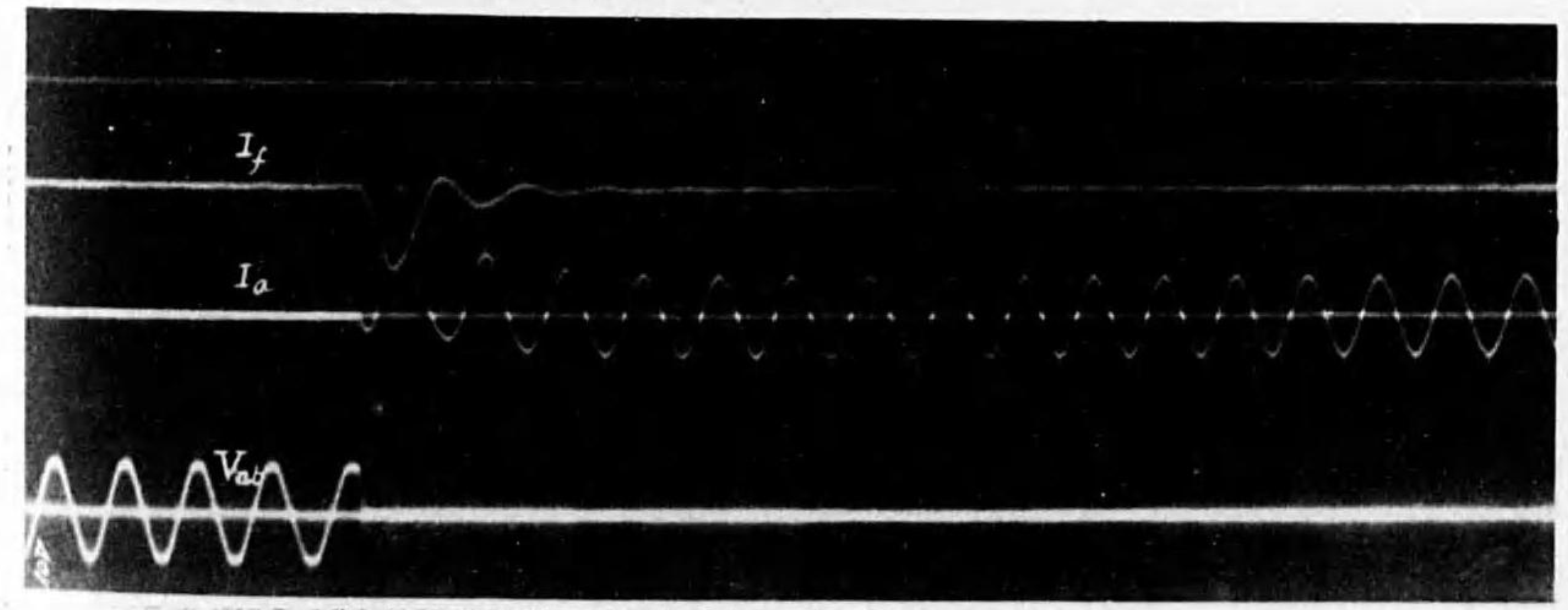


Fig. 23.
3 phase short-circuit current of 35 kVA alternator
$\varphi \doteqdot 60^\circ$ $V_{ab}=10$ volts

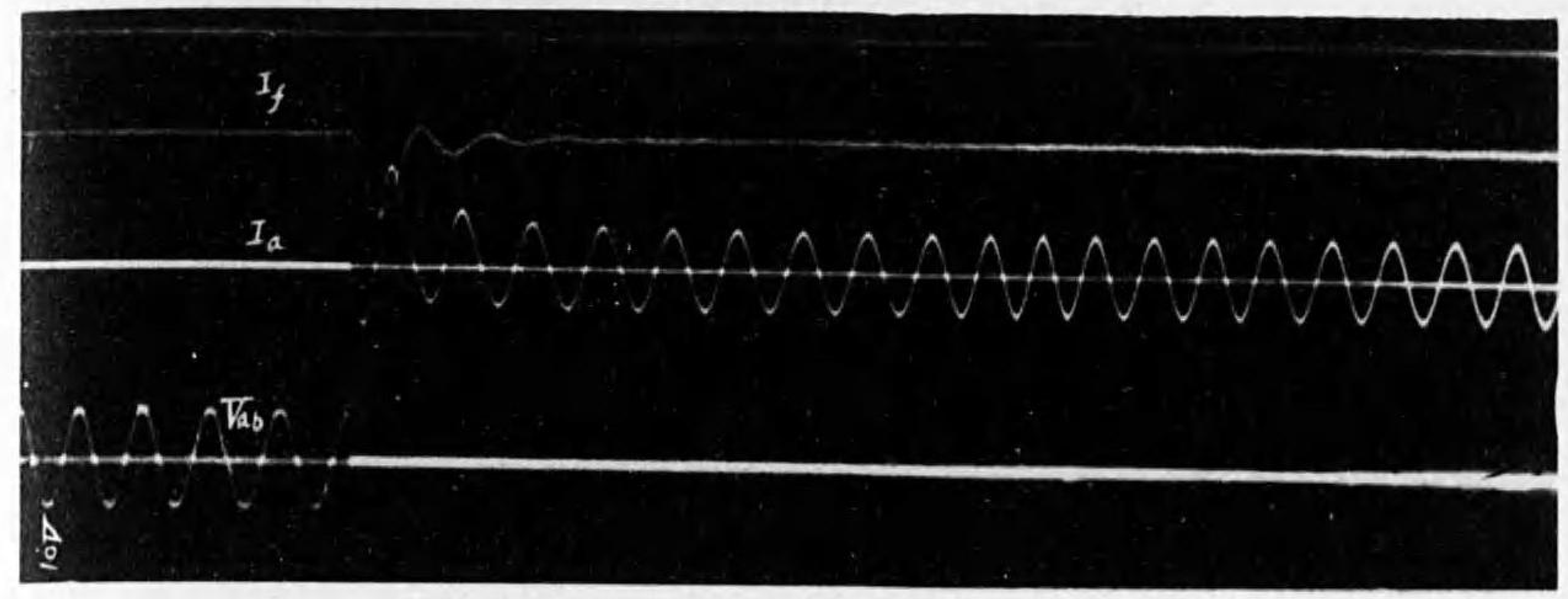


Fig. 24.
3 phase short-circuit current of 35 kVA alternator
$\varphi \doteqdot 90^\circ$ $V_{ab}=10$ volts

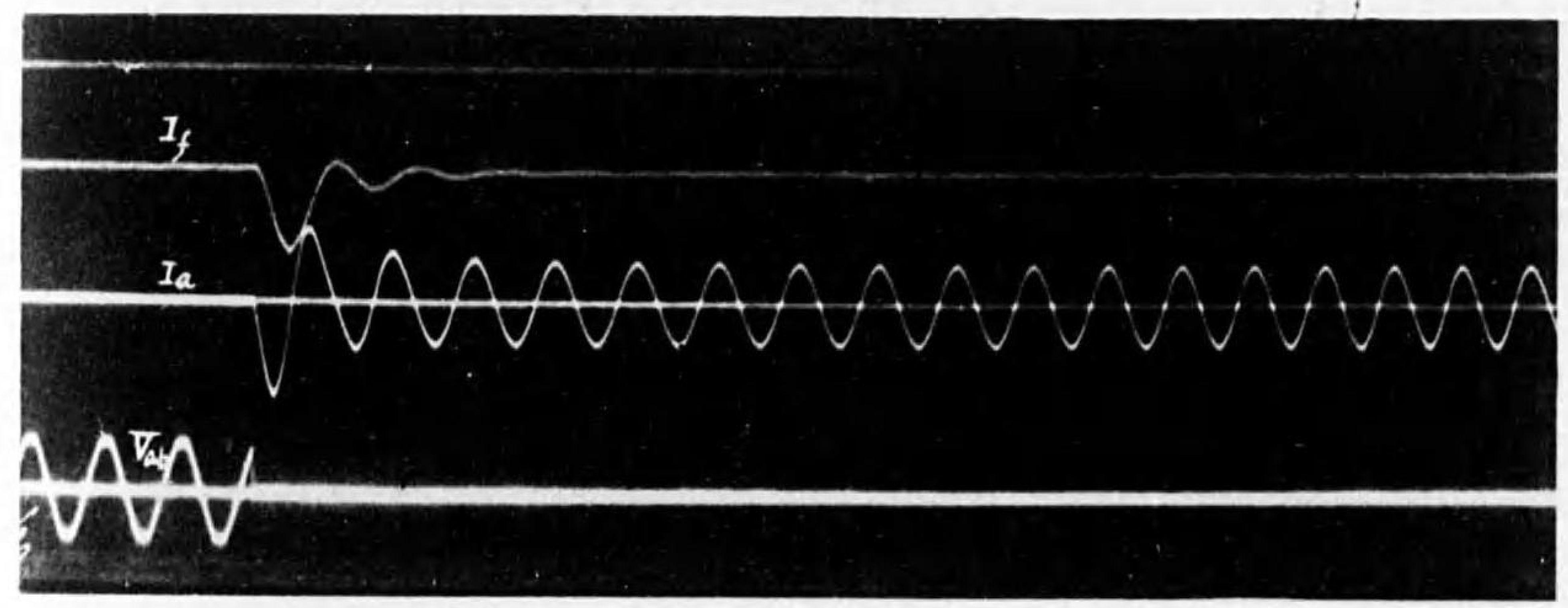


Fig. 25.
3 phase short-circuit current of 35 kVA alternator
$\varphi \doteqdot 0^\circ$ $V_{ab}=100$ volts

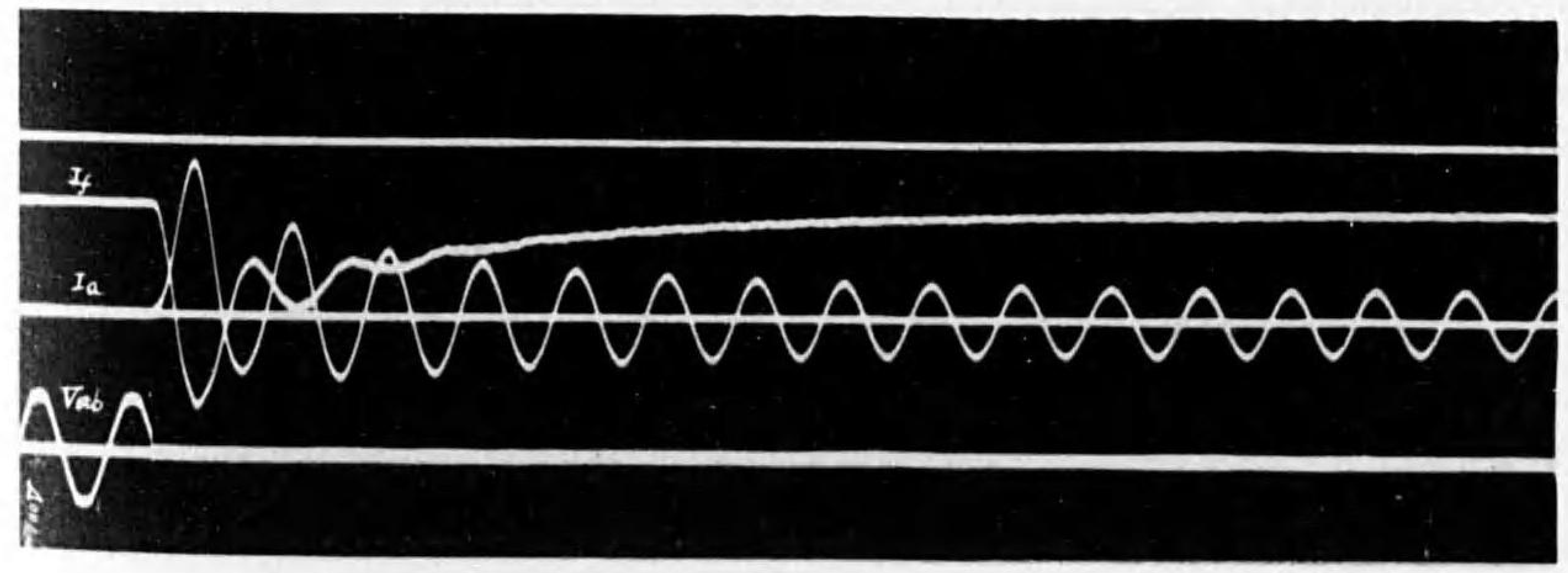


Fig. 26.
Line to line short-circuit current of 10 HP induction motor
$\varphi \doteqdot 90°$ $V_{bc}=150$ volts

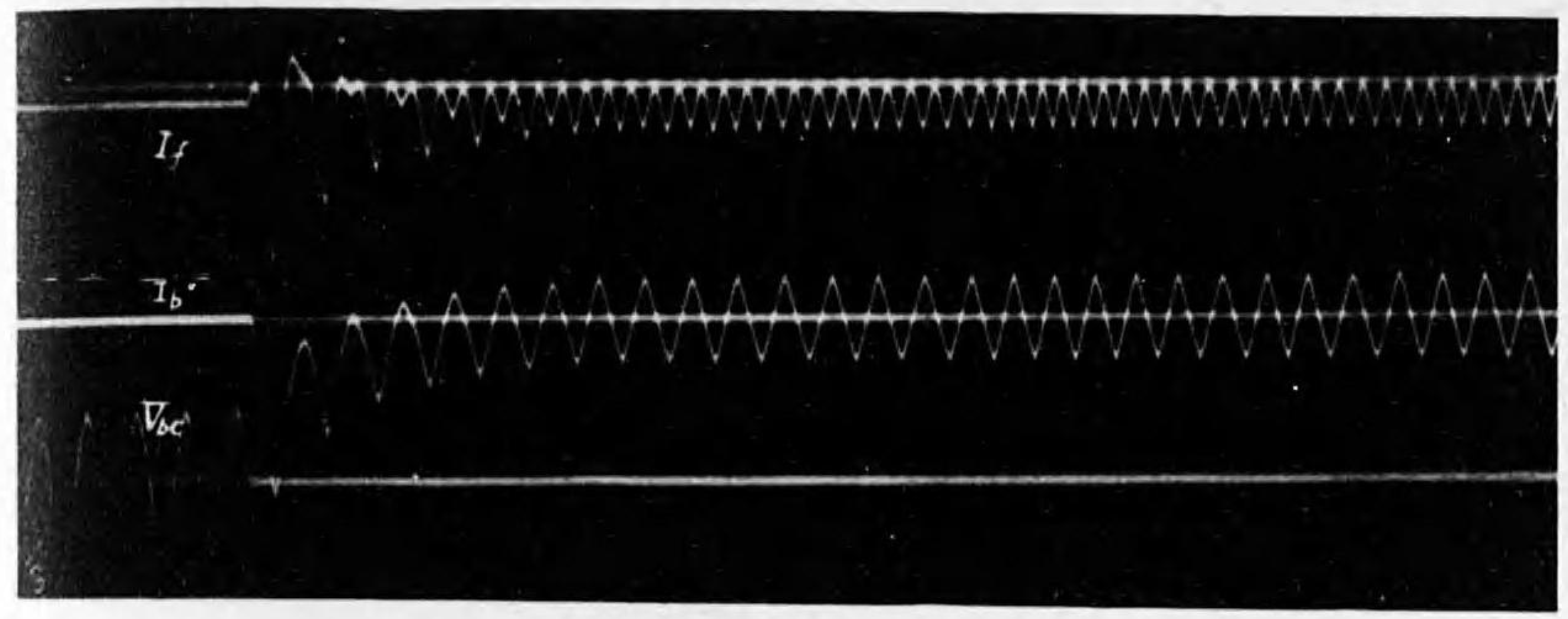


Fig. 27.
Line to line short-circuit current of 35 kVA alternator
$\varphi \doteqdot 90°$ $V_{bc}=10$ volts

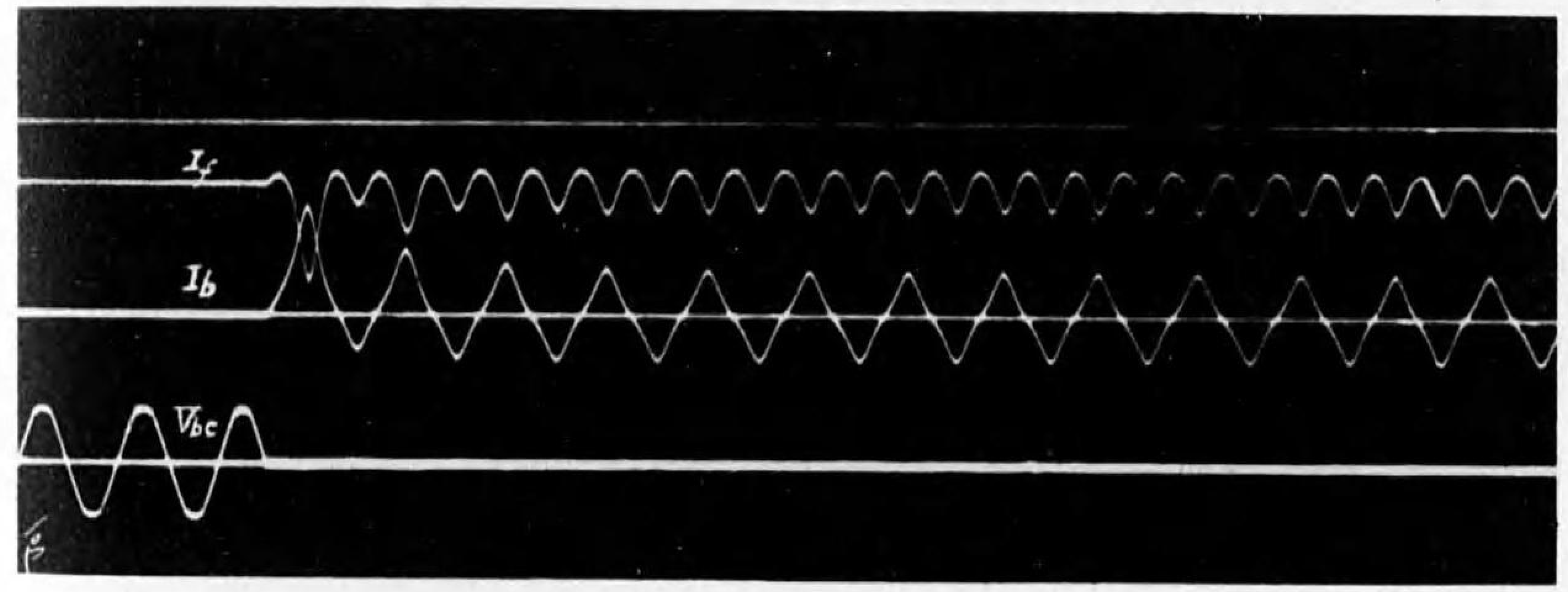
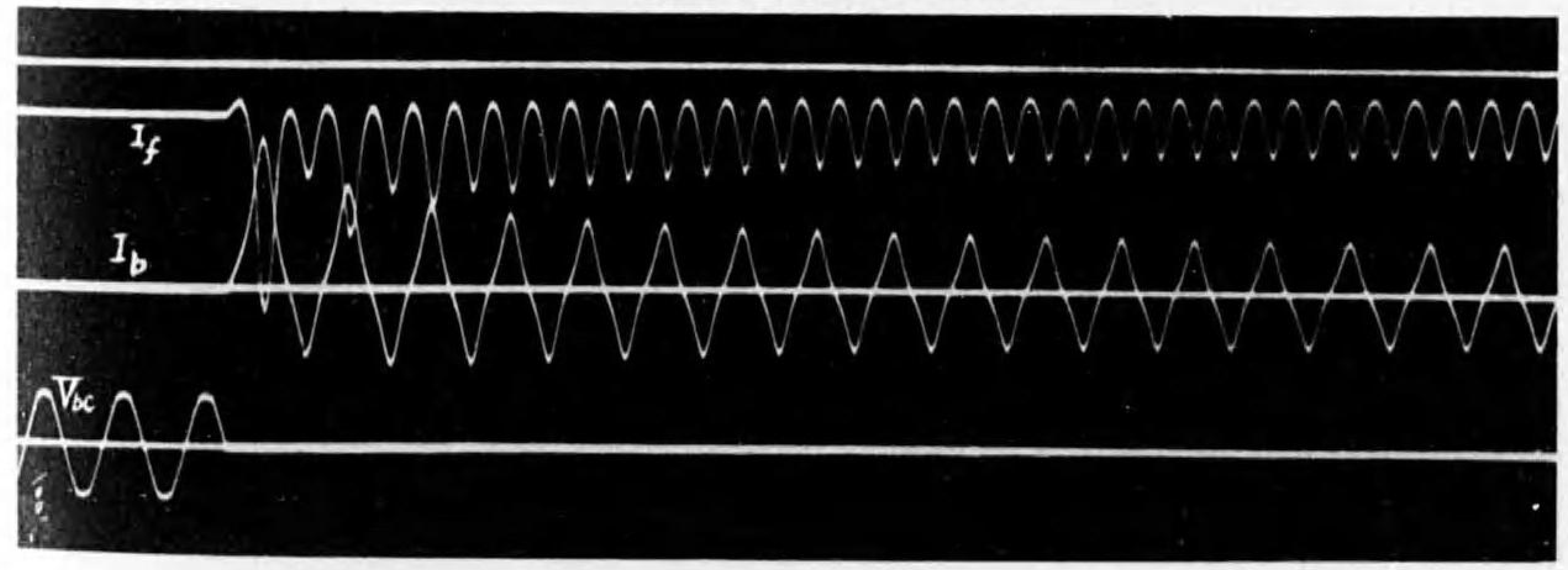


Fig. 28.
Line to line short-circuit current of 35 kVA alternator
$\varphi \doteqdot 90°$ $V_{bc}=100$ volts



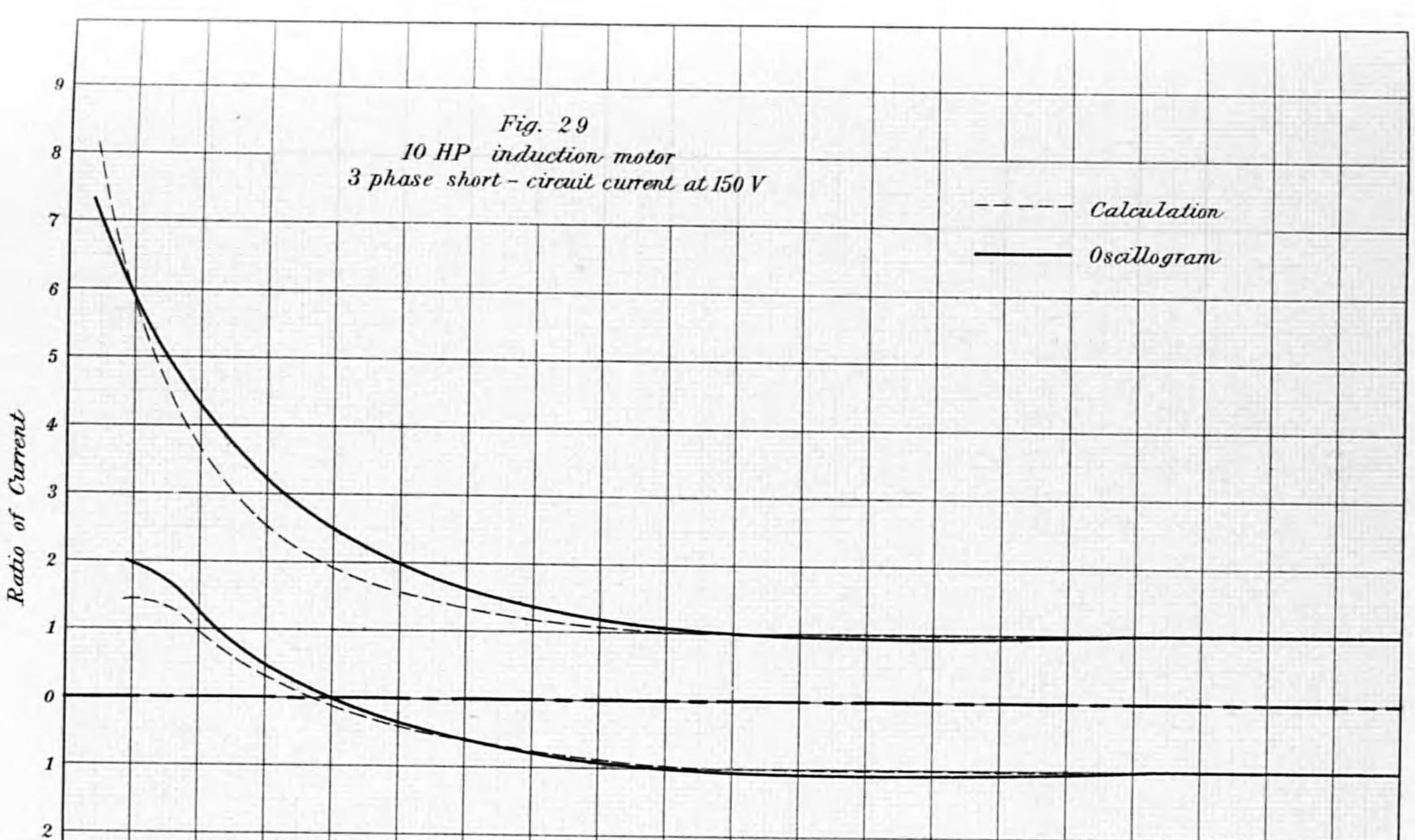


Fig. 29
10 HP induction motor
3 phase short - circuit current at 150 V
Calculation
Oscillogram
Ratio of Current
Time in Sec.
9
8
7
6
5
4
3
2
1
0
1
2
0
0.02
0.06
0.10
0.14
0.18
0.22
0.26
0.30
0.34
0.38

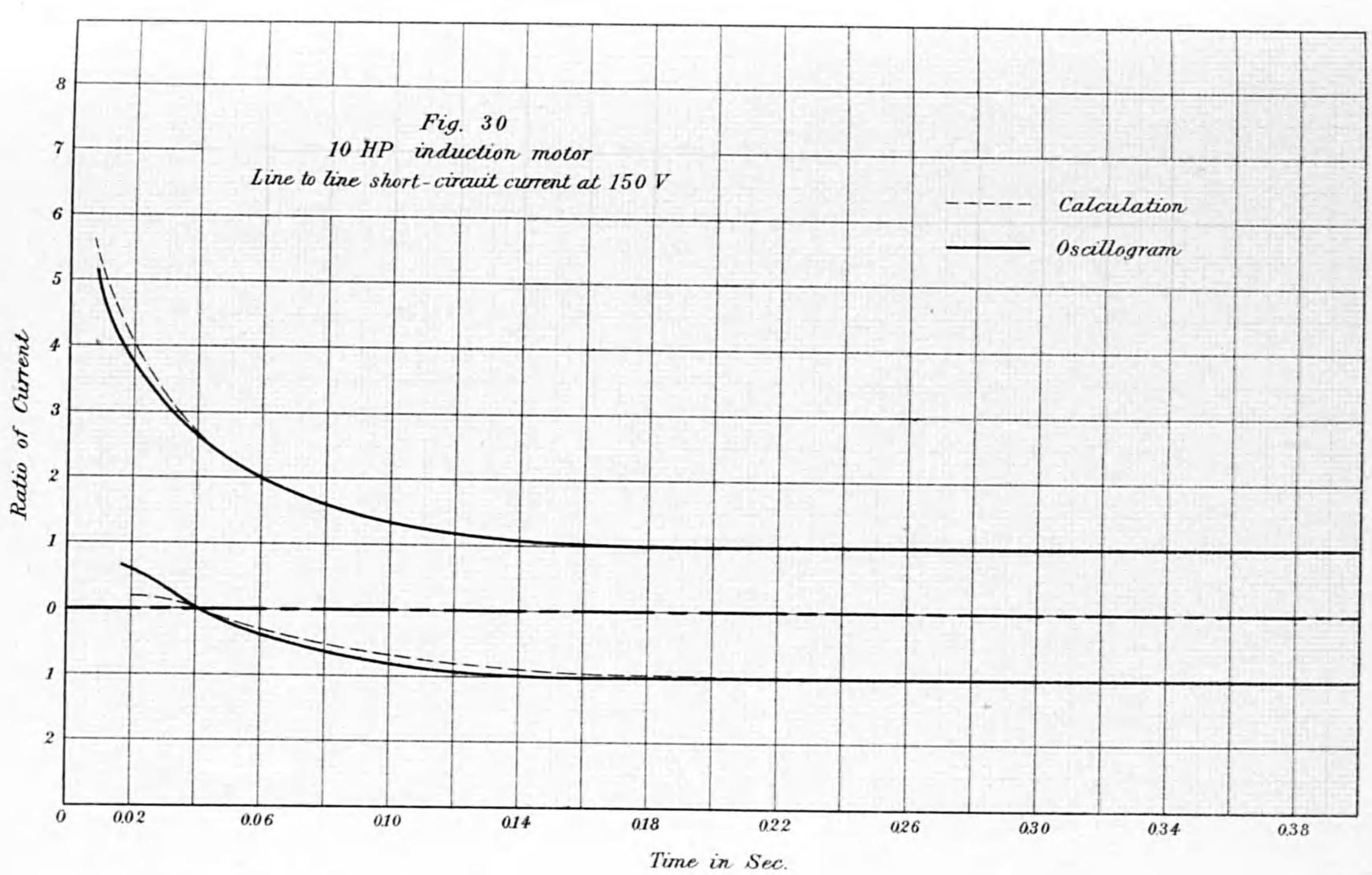


Fig. 30
10 HP induction motor
Line to line short-circuit current at 150 V

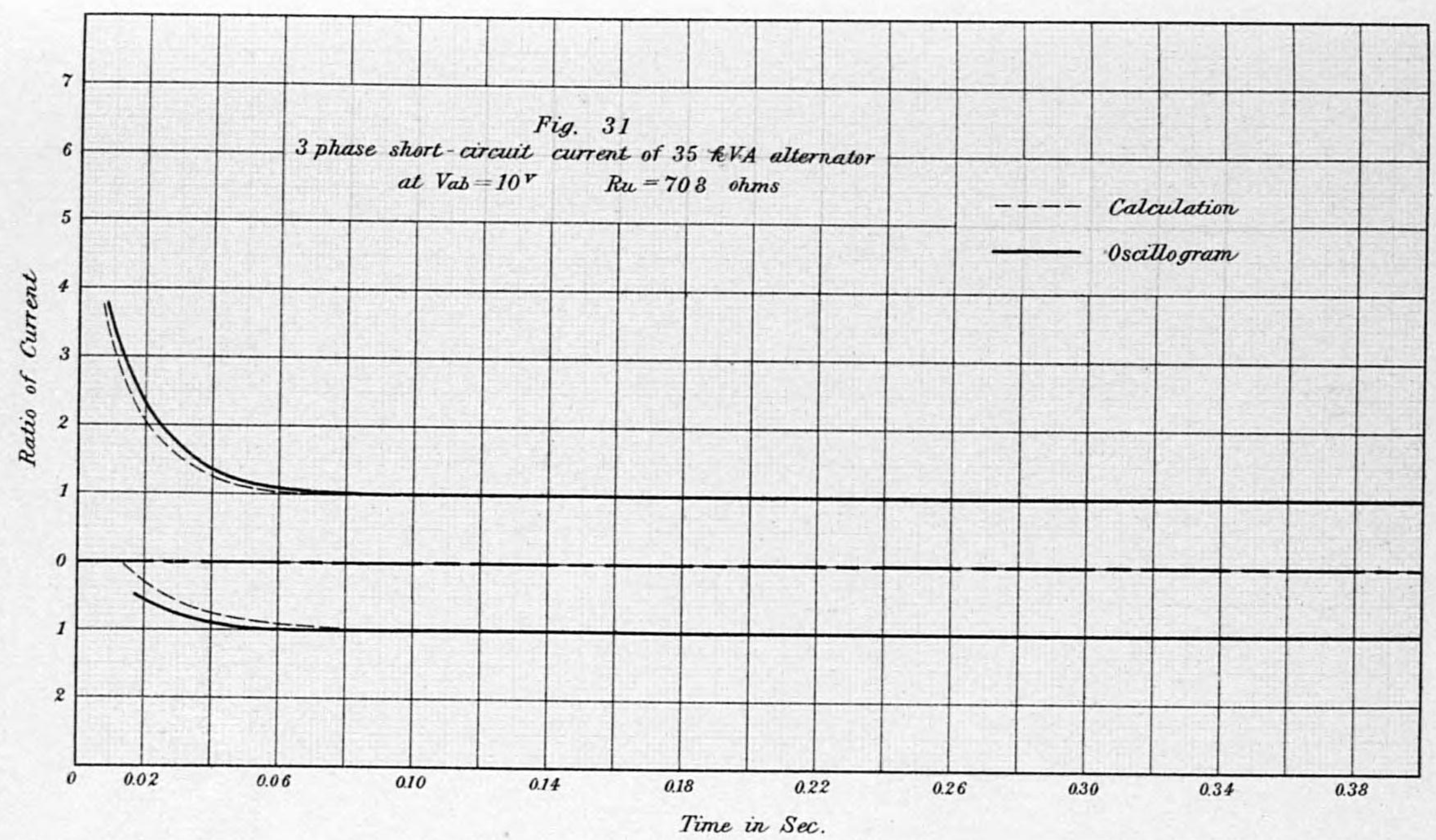


Fig. 31
3 phase short-circuit current of 35 kVA alternator
at $V_{ab} = 10^{v}$ $R_u = 70.8$ ohms

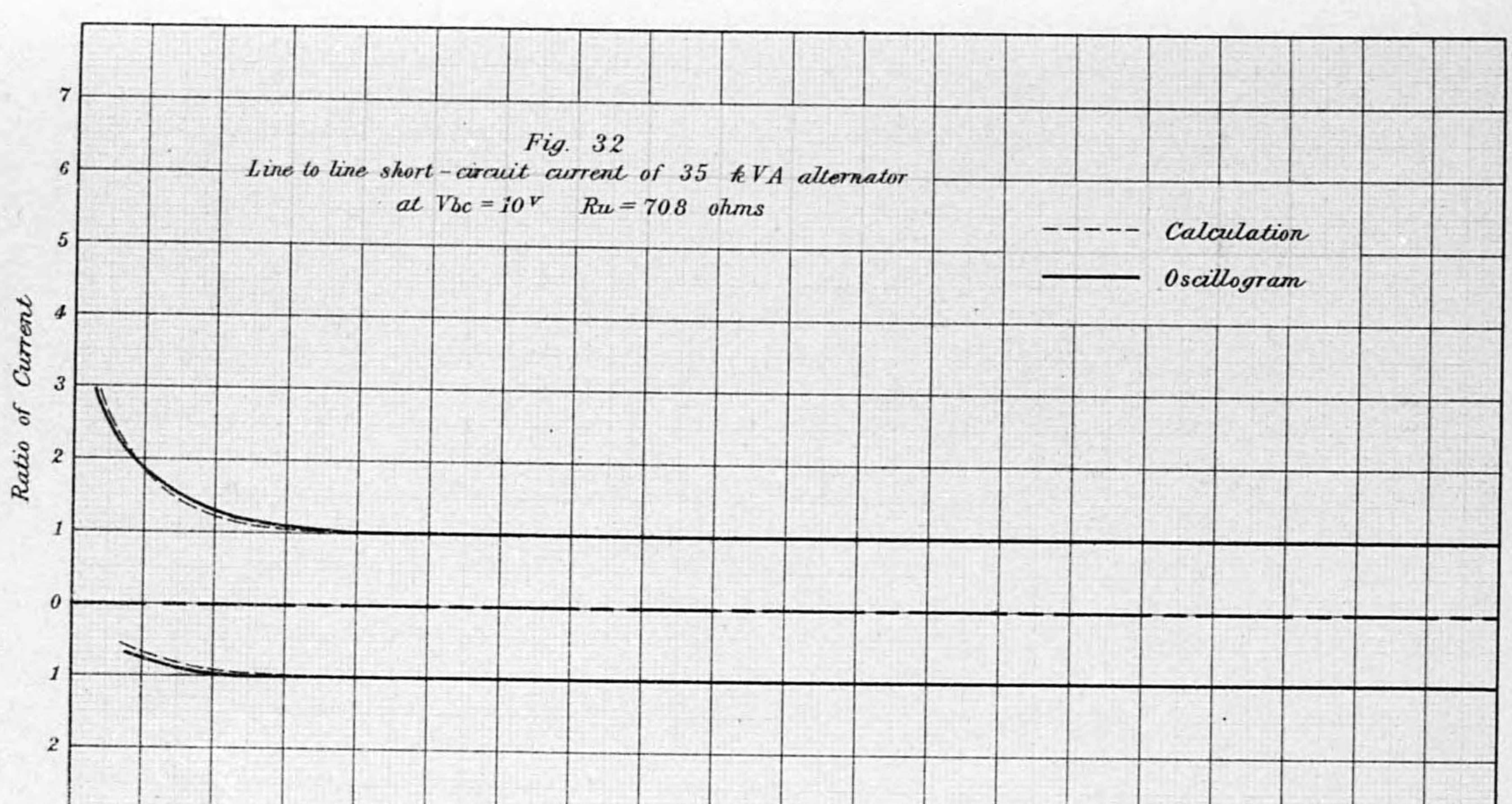


Fig. 32
Line to line short-circuit current of 35 kVA alternator
at Vbc = 10V Ru = 70.8 ohms
Calculation
Oscillogram
Ratio of Current
7
6
5
4
3
2
1
0
1
2
0
0.02
0.06
0.10
0.14
0.18
0.22
0.26
0.30
0.34
0.38
Time in Sec.

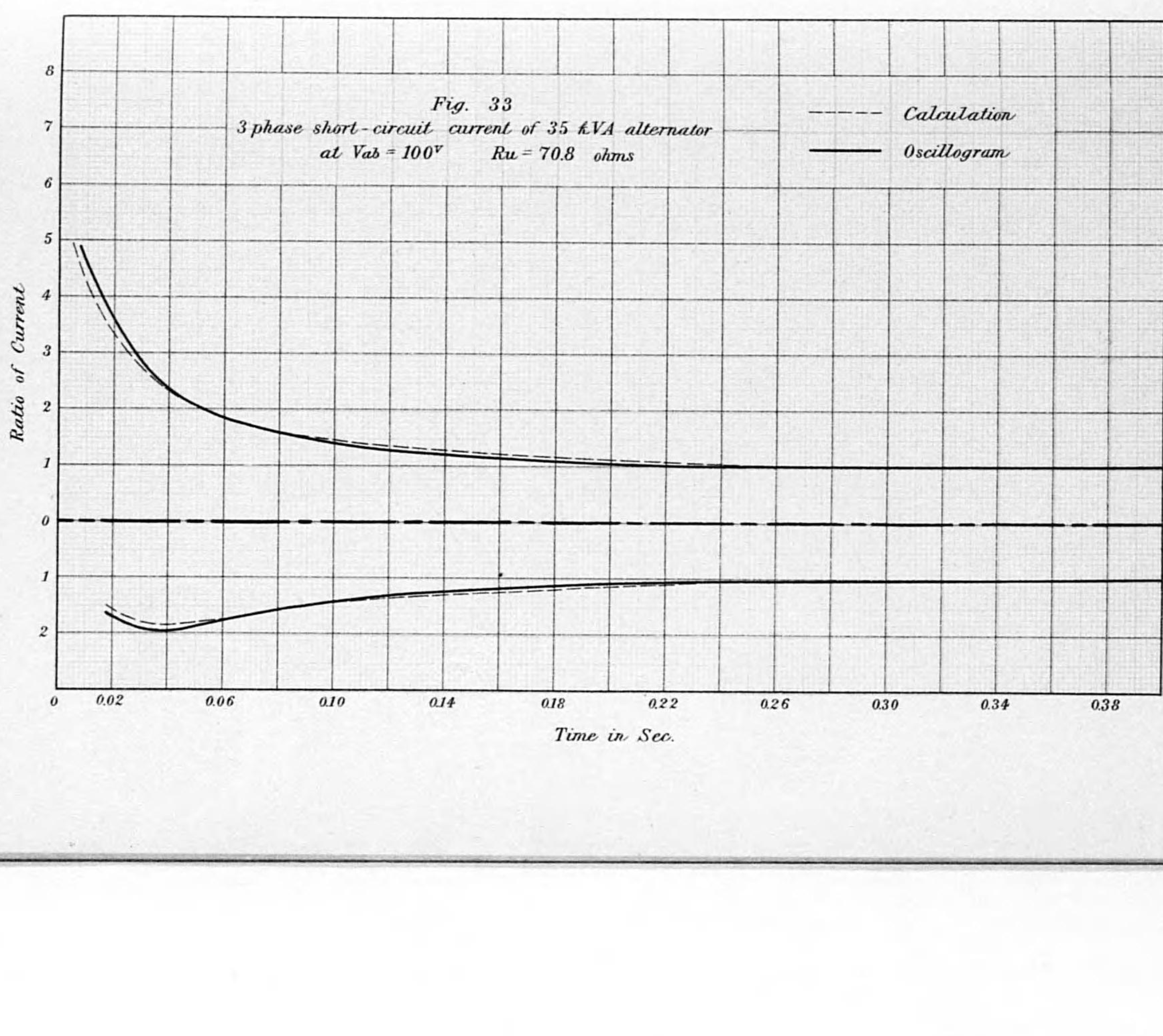


Fig. 33
3 phase short-circuit current of 35 kVA alternator
at Vab = 100V Ru = 70.8 ohms
Calculation
Oscillogram
Ratio of Current
8
7
6
5
4
3
2
1
0
1
2
0
0.02
0.06
0.10
0.14
0.18
0.22
0.26
0.30
0.34
0.38
Time in Sec.

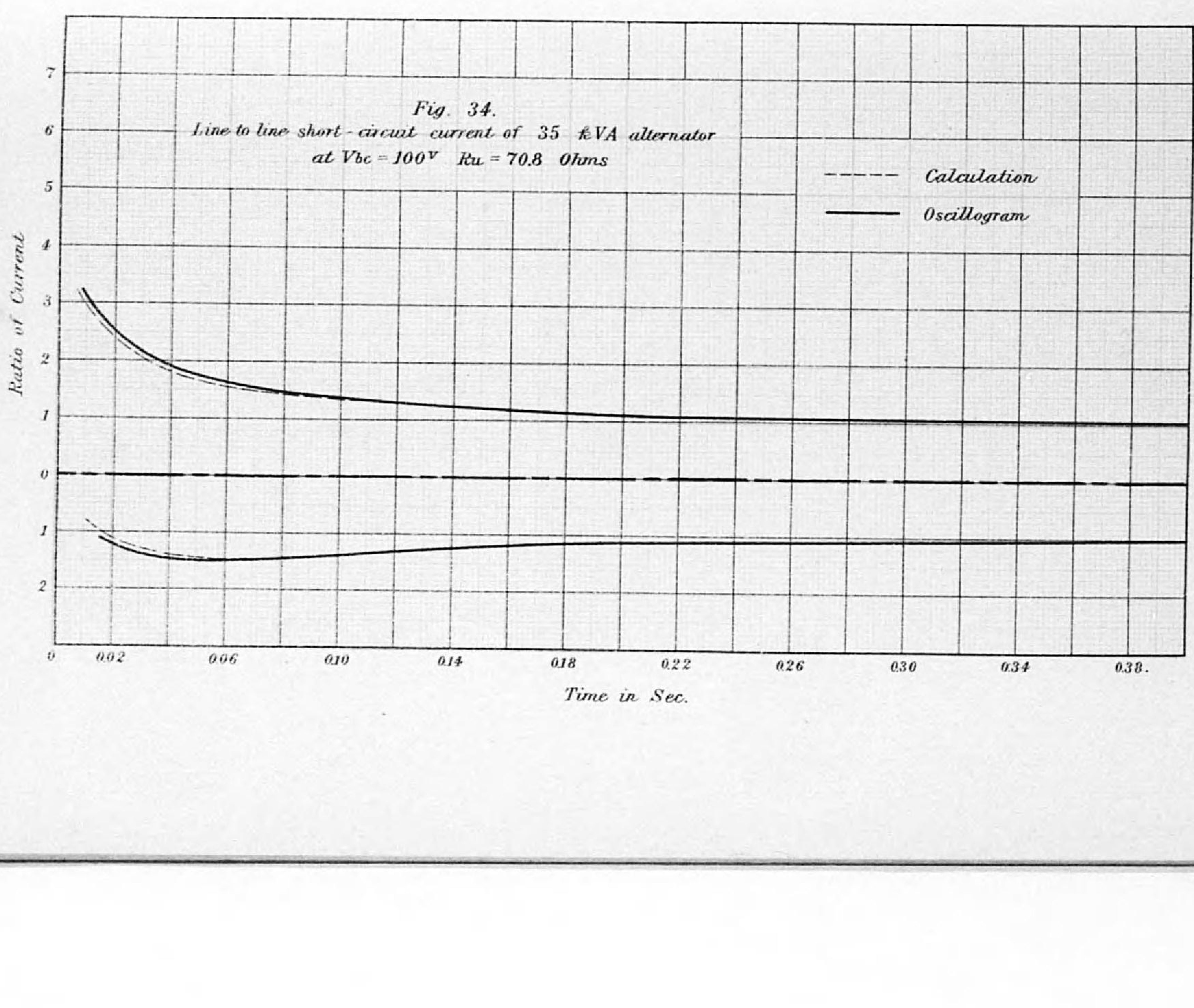
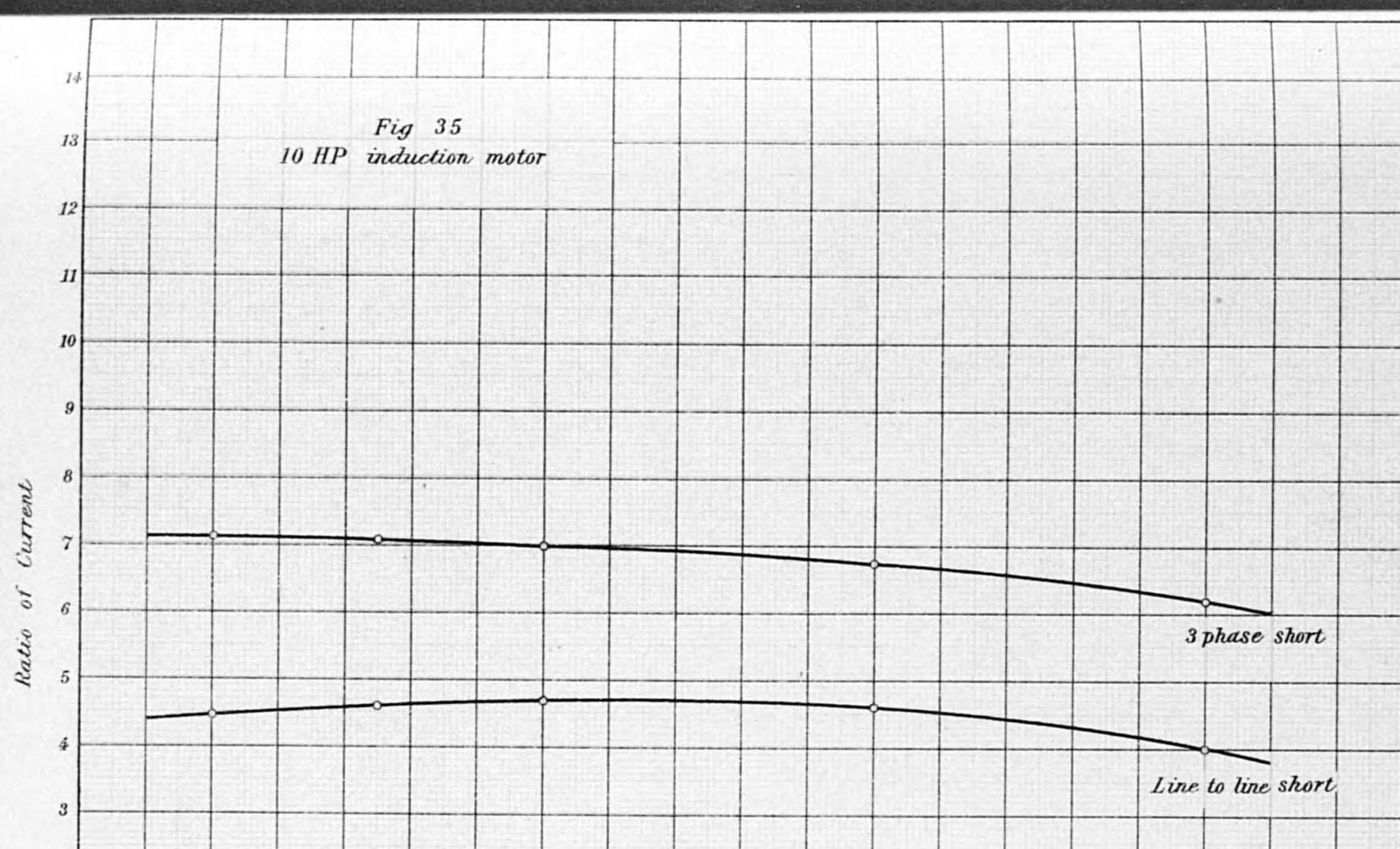


Fig. 34.

Line to line short-circuit current of 35 kVA alternator at $V_{bc} = 100^V$ $R_u = 70.8$ Ohms

Fig 35
10 HP induction motor



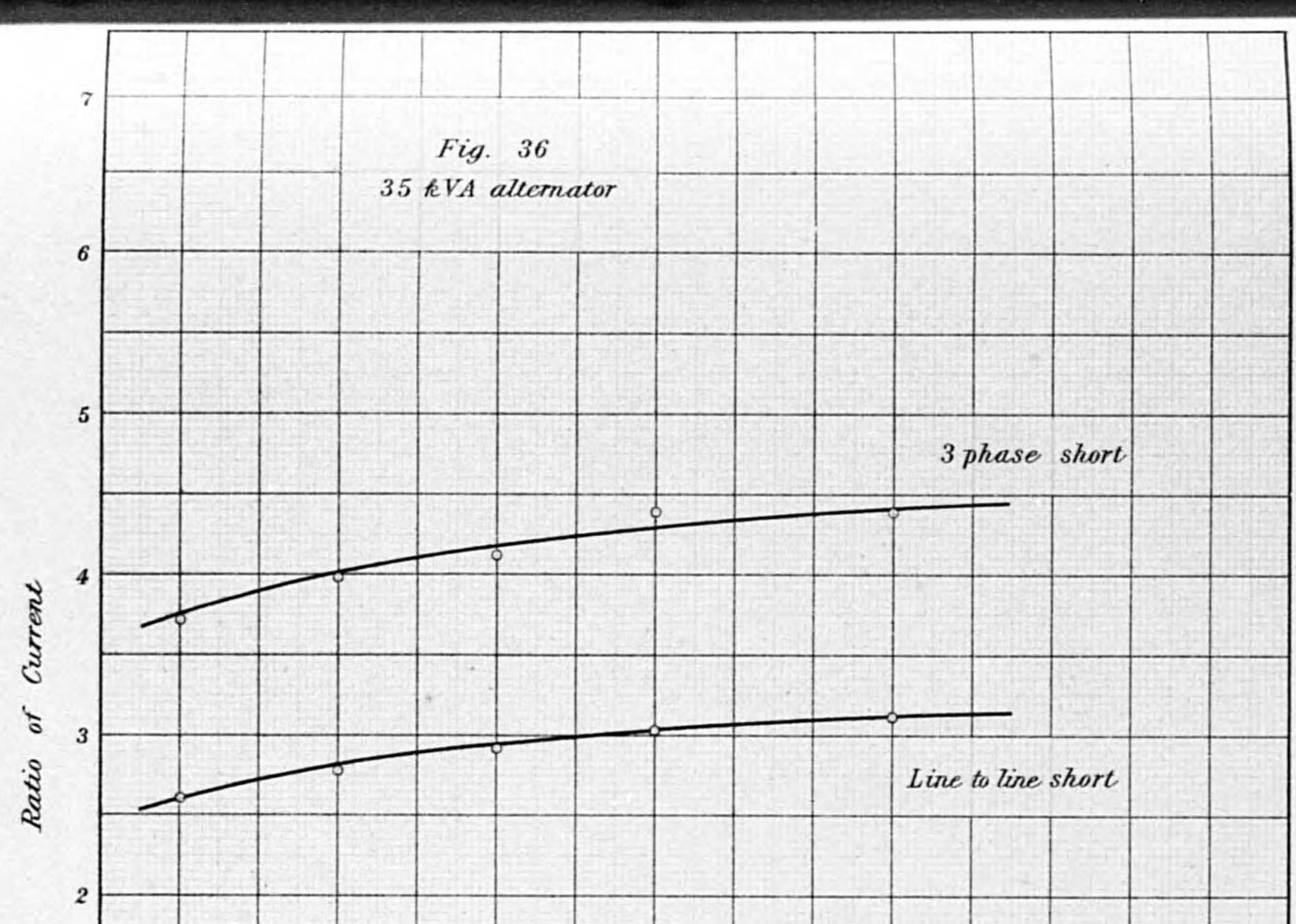


Fig. 36
35 kVA alternator
3 phase short
Line to line short
Ratio of Current
1
2
3
4
5
6
7
10
30
50
70
100
130
Open-circuit voltage in volts

Fig. 37
Impedance of 35 kVA alternator (Calculation)
$Ru = 708$ ohms

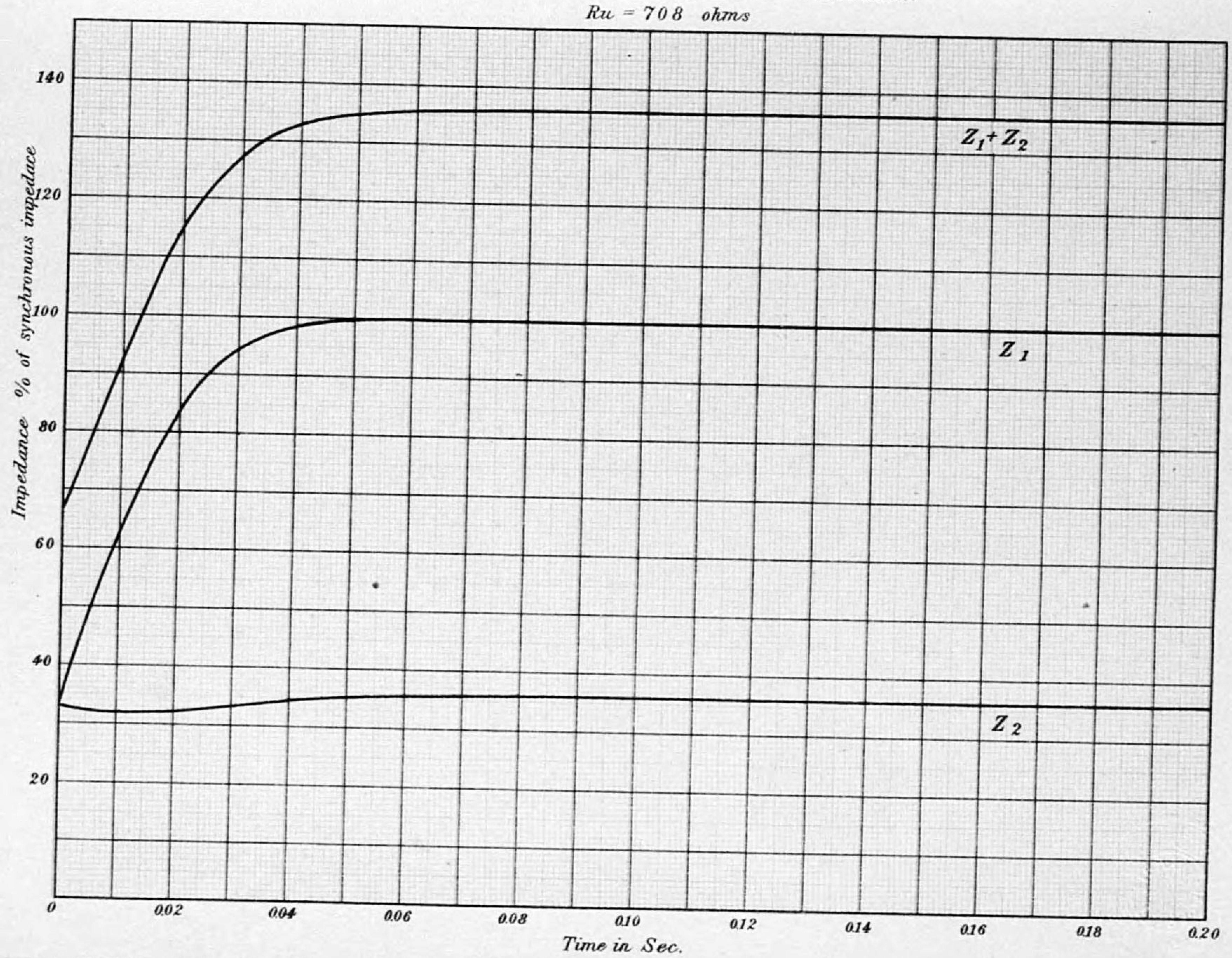


Fig. 38

Impedance of 35 kVA alternator (Oscillogram)

$V_{ab} = 10^{V}$

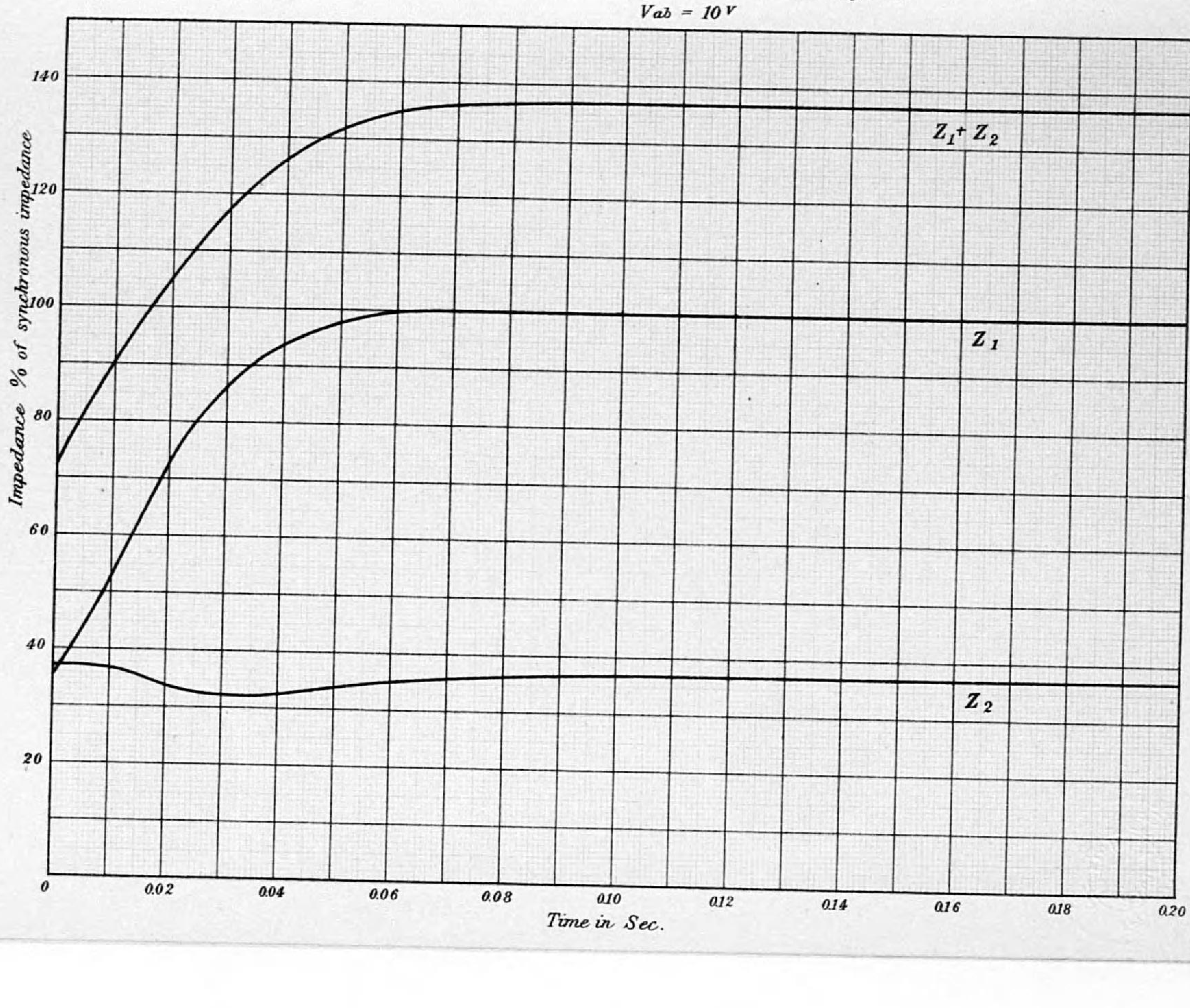


Fig. 39
Impedance of 35 kVA alternator (Calculation)
Ru = 70 8 ohms

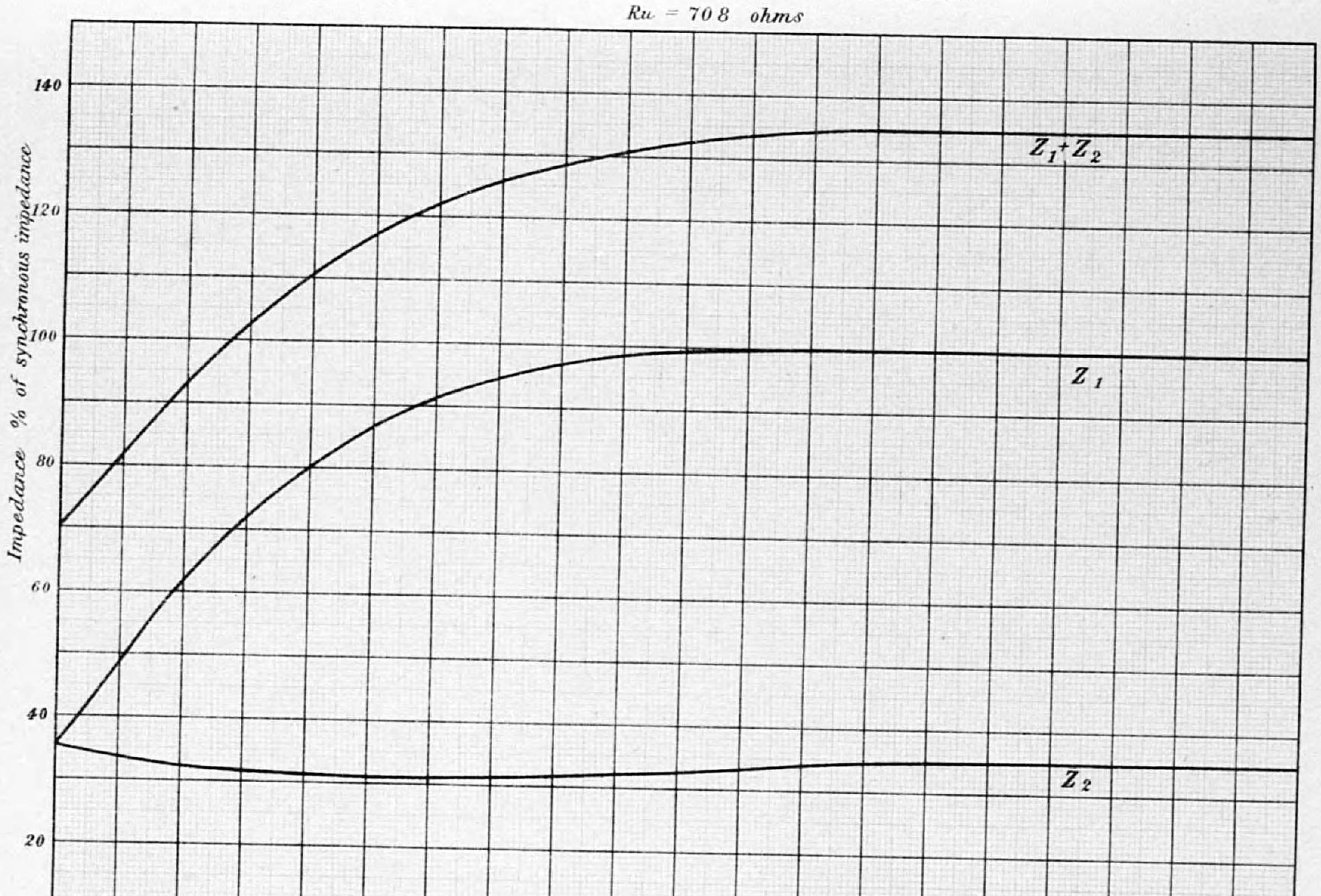
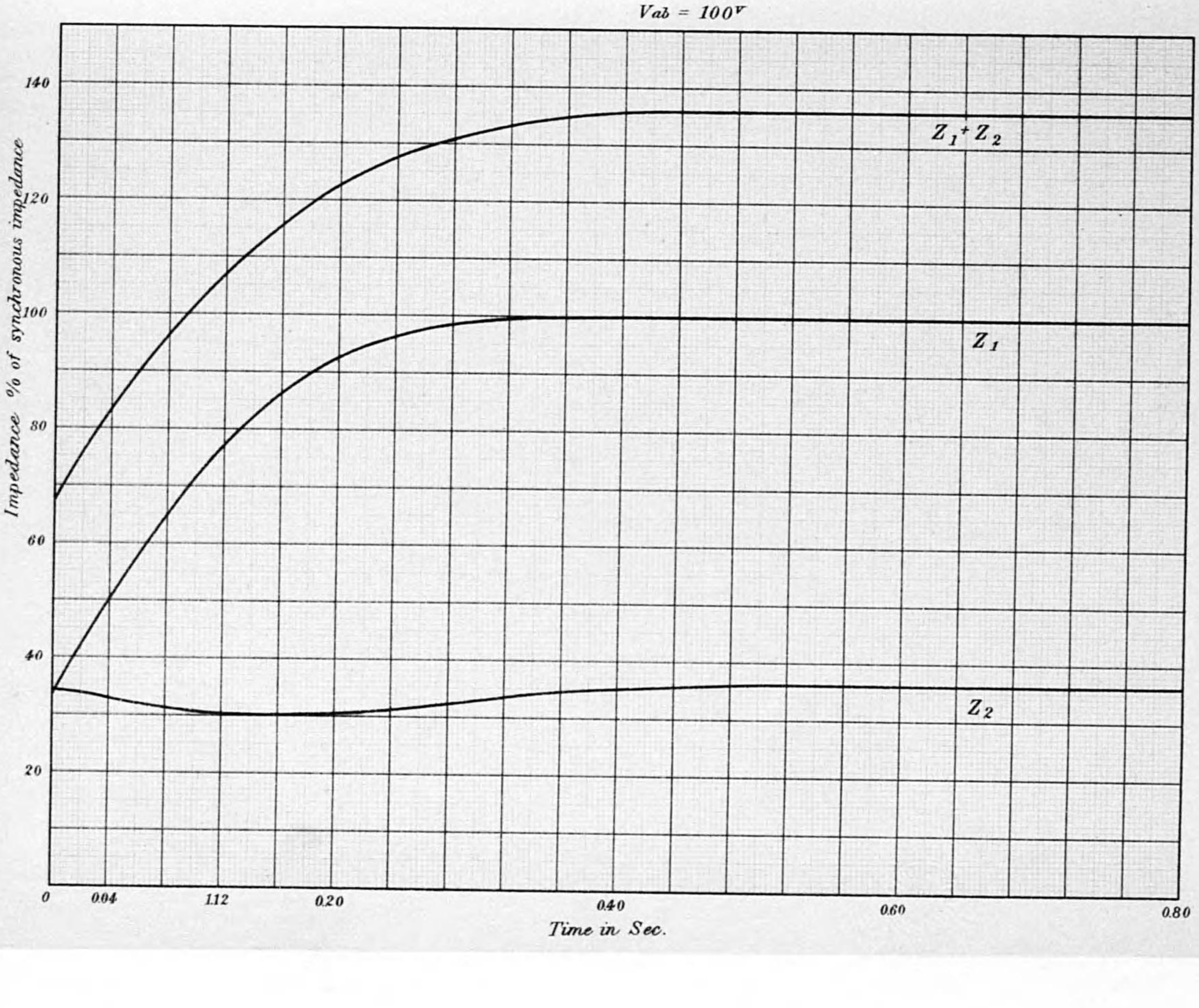


Fig. 40
Impedance of 35 kVA alternator (Oscillogram)
$V_{ab} = 100^V$



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