Baryon as a Quantum Hall Droplet and the Quark-Hadron Duality

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We show that the recent proposal to describe the $N_f = 1$ baryon in the large number of the color limit as a quantum Hall droplet can be understood as a chiral bag in a $(1+2)$-dimensional strip using the Cheshire Cat principle. For a small bag radius, the bag reduces to a vortex line which is the smile of the cat with flowing gapless quarks all spinning in the same direction. The disk enclosed by the smile is described by a topological field theory due to the Callan-Harvey anomaly outflow. The chiral bag naturally carries the unit baryon number and spin $\frac{1}{2}N_c$. The generalization to arbitrary N_f is discussed.

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Introduction.—In the large number of the color limit, 't Hooft suggested that QCD is dominated by planar diagrams, with infinitely many weakly interacting mesons and glueballs [1]. Witten argued that, in this limit, baryons are heavy solitons made out of the interacting mesons. The coupling of the mesons is weak and of order $1/N_c$, while the coupling of the baryons is strong and of order N_c [2].

Chiral solitons made solely of nonlinearly interacting pions are a prototype of these solitons, an idea put forth decades ago by Skyrme [3] well before the advent of QCD. Chiral solitons are topologically protected in $1+3$ dimensions, and their quantum numbers emerge through semiclassical quantization. However, their masses and "charges" depend sensitively on the truncated chiral effective action, and somehow less through the more elaborate chiral holographic constructions [4].

Recently, Komargodski [5] pointed at the peculiar character of the QCD baryons for $N_f = 1$, where the chiral effective theory is dominated by the axial U(1) anomaly for the η' meson, and where the soliton construction no longer applies since, for instance, the standard topological charge cannot be identified. He noted that the effective theory has a conserved topological current $J_{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma\lambda}\partial^\lambda \eta'/2\pi$. These currents are carried by $(1+2)$-dimensional charged sheets with the η' field undergoing a 2π jump across the sheet.

Remarkably, when these sheets are finite dimensional with a boundary, Komargodski noted that they can carry massless edge excitations with baryon quantum numbers. They are identified with fast spinning baryons. These sheets are described by a topological field theory through a level-rank duality argument [6], much like in the fractional quantum Hall (FQH) effect [7]. The baryons are analogous to the gapless edge excitations in quantum Hall (QH) droplets. Arguments were put forth for their generalization to arbitrary N_f.

In sum, QCD with $N_f = 1$ admits baryons for any number of colors N_c, which behave as solitons in the large N_c limit. But these solitons cannot be topological since the effective field in this limit is solely the η' with trivial homotopy. So what are these baryons? Komargodski suggested that these baryons emerge from patches of domain walls bridging different θ vacua. In this Letter, we will show that these baryons are chiral bags with quarks trapped in a patch of domain wall that leaks a baryon number, with a topological cloud that acts like a QH droplet. Our construction will unravel a new relationship between a bag model of a baryon and a soliton for any N_f, and it will reveal a new relationship between topological field theory and quark-hadron continuity. These are new paradigms in hadron physics, shared in concept by condensed matter physics, and potentially useful in addressing compact-star physics, as we argue below.

We now suggest that these baryonic QH droplets can be understood using the Cheshire Cat principle (CCP) [8]. More specifically, we show that a chiral bag with a single quark species of charge e (electric charge or fermion number) confined to a $(1+2)$-dimensional annulus leaks the most quantum numbers. For all purposes, the bag radius is immaterial thanks to the CCP. In particular, when the bag radius is shrunk to zero, only the smile of the cat is left with spinning gapless quarks running luminally, explaining the edge modes and their spin [5].

A current transverse to the smile is shown to appear, embodying the Callan-Harvey anomaly outflow [9]. This transverse current is shown to be analogous to the Hall current typical of the QH effect through the emergence of an effective U(1) gauge field. This U(1) gauge field lives in the disk enclosed by the Cheshire Cat smile and is described by a purely topological field theory in $1+2$ dimensions. The quantum numbers of this baryon as a QH droplet follow readily from the chiral bag construction. The generalization to many species is discussed.

Bag in a domain wall.—Consider a $(1+2)$-dimensional chiral bag in the form of an annulus of radius R lying in the xy plane and clouded by an η' field with a monodromy of 2π or a U(1) winding number of 1. We will refer to x as the radial direction and to y as the tangential direction, as illustrated in Fig. 1. The bag consists of free two-dimensional quarks, say, of charge e, and subject to a chiral bag boundary condition along the radial x direction. We now suggest that this $(1+2)$-dimensional U(1) chiral bag in the limit of the zero bag radius is the pancake baryon suggested by Komargodski thanks to the CCP. Note that in the limit of zero bag radius, the chiral bag reduces to a vortex line.

The essence of the CCP lies in the fact that the charge e of the chiral bag leaks through an anomaly. This leakage is best described by noting that, in the presence of the η' cloud along the x direction, the Dirac spectrum in the bag undergoes a spectral flow. Since the discussion is about leakage of charge along the x direction and flow of charge along the y direction, the shape of the bag as an annulus is topologically equivalent to an infinite strip along the y direction with periodic boundary condition, and the U(1) chiral boundary condition along the x direction as illustrated in Fig. 2.

For a single quark species, the chiral bag model on the strip is described by

$$\begin{aligned} &(i\partial_t + i\sigma_2\partial_x - i\sigma_3\partial_y)q(t,x,y) = 0, \qquad |x| < R, \\ &(e^{-i\sigma_2\theta(t,x)} - \sigma_3\epsilon(x))q(t,x,y) = 0, \qquad |x| = R, \end{aligned} \tag{1}$$

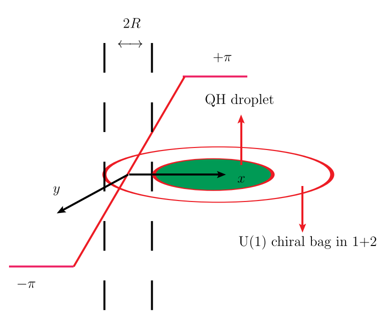

FIG. 1. $(1+2)$-dimensional chiral bag surrounding a QH droplet. The bag is an annulus of width $2R$ clouded by an η' with a monodromy of 2π. In the limit of the zero bag radius, the chiral bag reduces to a vortex string with unit baryon number.

with $\epsilon(x) = x/|x|$ being the outside normal to the bag, and $(\gamma^0, \gamma^1, \gamma^2) = (\sigma_1, i\sigma_3, i\sigma_2)$. The η' field acts only at the boundary through the chiral angle $\theta = \eta'/f_\eta$, which is in general time dependent but y independent. f_η is the η' decay constant. Throughout, the reference to chirality in $1+2$ dimensions will be a slight abuse of language for a discrete parity transformation $x_1, x_2 \to -x_1, x_2$, and $q \to \sigma_2 q$ with the mass term $\bar{q}q = q^+\sigma_1 q \to -\bar{q}q$ breaking parity. It becomes chirality in $1+1$ dimensions only under dimensional reduction. The anomaly in $1+2$ dimensions is the parity anomaly [10].

With this in mind, the spectral flow is seen by considering the case of a static boundary condition for the η' field. In this case, the mode solution to Eq. (1) is of the form

$$q_n(t,x,y) = e^{-iE_n t + ik_y y}\varphi_n(x), \tag{2}$$

with E_n following from the transcendental equation

$$\tan\left(2R\sqrt{E_n^2 - k_y^2}\right) = \frac{1 + t_+ t_-}{t_-\sqrt{\frac{E_n+k_y}{E_n-k_y}} - t_+\sqrt{\frac{E_n-k_y}{E_n+k_y}}} \tag{3}$$

with $t_\pm = \tan[\theta(\pm R)/2)]$. Note that the spectrum is now twisted through $t_\pm$.

For the special case of $1+1$ dimensions with $k_y = 0$, the twist is manifest as Eq. (3) simplifies to

$$\tan(2RE_n) = \tan\left(\frac{\pi}{2} + \frac{\Delta\theta}{2}\right) \tag{4}$$

with $\Delta\theta = [\theta(+R) - \theta(-R)]$ as the jump of the η' field across the chiral bag. The twisted spectrum is now

$$E_n = \frac{(2n+1)\pi}{4R} + \frac{\Delta\theta}{4R}, \tag{5}$$

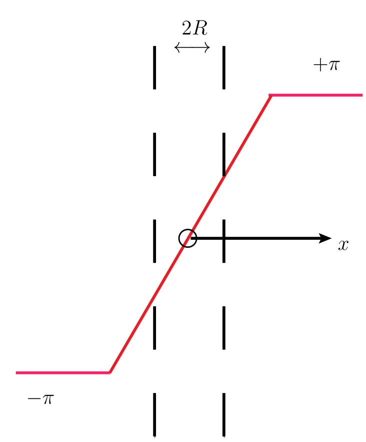

FIG. 2. $(1+2)$-dimensional chiral bag as an infinite strip in the y direction (out of the page) with periodic boundary condition and U(1) chiral boundary condition along the x direction.

with the level E_{-1} crossing zero at the magic angle $\Delta\theta = \pi$ and requiring a vacuum redefinition. This redefinition implies that the charge Q in the chiral bag fractionalizes with the result [11]

$$\Delta Q = \frac{e\Delta\theta}{2\pi}, \tag{6}$$

as the rest of the charge is now located in the U(1) topological charge carried by the outside η' field, i.e.,

$$Q = \frac{e}{2\pi}\{[\pi - \theta(+R)] + [\theta(-R) - (-\pi)]\} + \Delta Q = e. \tag{7}$$

At the magic angle, half of the charge is in and half is out. The in charge is solely carried by the crossing state

$$q_{-1} = \frac{1}{\sqrt{4R}}\begin{pmatrix} -1 \\ +1 \end{pmatrix} \tag{8}$$

with $\sigma_1 q_{-1} = -q_{-1}$. The spin can be read by embedding Eq. (8) in a (1 + 3)-dimensional spinor $\Psi^T = (q_{-1}^T, 0)$,

$$S^x = \int_{-R}^{+R} dx \frac{1}{2}\bar{\Psi}\gamma^5\gamma^x\Psi = 2Rq_{-1}^\dagger \frac{-1}{2}\sigma_1 q_{-1} = \frac{1}{2}, \tag{9}$$

with here $(1+3)$-γ matrices. If the monodromy is flipped, $2\pi \to -2\pi$, the charge and the spin are flipped.

The explicit description of the present chiral bag model with space-time dependent boundaries is involved for general R and finite k_y, but around the magic angle the spectrum becomes CP symmetric with gapless modes of energy $E_{-1} \sim k_y$ running in the y direction with fixed spin. For this choice, the physics becomes more transparent thanks to the CCP, with the emergence of low-dimensional anomalies and bosonization as we now detail.

Anomaly outflow.—When viewed in $1+1$ dimensions, the preceding result is the consequence of an exact bosonization which captures the essence of the CCP—namely, that the bag radius R is immaterial (the smile of the Cheshire Cat). More specifically, Eq. (6) is the first of the two standard Abelian bosonization relations in $1+1$ dimensions:

$$\rho^{1+1} = eq^\dagger q \to \frac{e\partial_x\theta}{2\pi} \equiv \frac{e}{2\pi}\frac{\partial_x\eta'}{f_\eta},$$
$$j_x^{1+1} = eq^\dagger\sigma_3 q \to \frac{e\partial_t\theta}{2\pi} \equiv \frac{e}{2\pi}\frac{\partial_t\eta'}{f_\eta}. \tag{10}$$

These observations are now important for the $(1+2)$-dimensional chiral bag and its mapping on the baryon as a QH droplet.

When the bag radius is increasingly small, the chiral bag is more like a vortex line. At the magic angle, a gapless mode with half fermion number (the other half is sitting on the wall) and momentum k_y flows along the y direction. More importantly, the vortex line carries a charge per unit length ρ and is leaking radially a current j_x, as given by Eq. (10), irrespective of how small R is. An observer along the vortex line will see e-charge increasing or decreasing and would conclude that his or her tangential current j_y is anomalous and not conserved. In other words,

$$\partial^t\rho + \partial^y j_y = \frac{e^2}{2\pi}E_y, \tag{11}$$

as if an emergent U(1) effective electric field E_y were acting on his or her vacuum. However, this increase or decrease is caused by the leaking current in the radial direction noted earlier so that $j_x = j_y$ in magnitude, leading to the identification of the emergent electric field E_y as

$$\frac{e^2}{2\pi}E_y = \frac{j_x^{1+1}}{L_y} = \frac{e\partial_t\theta}{2\pi L_y} \tag{12}$$

after using Eqs. (10) and (11) with L_y as the y length of the chiral bag as a strip. We note that the emergent U(1) gauge field is $A_y \sim \theta/e$. A close reading of Eq. (12) shows that $j_x \sim E_y$, which is reminiscent of the Hall current, hence the immediate analogy of the present chiral bag construction with the QH effect. This is the Callan-Harvey mechanism for anomaly outflow [9], now realized for a proposed baryon. It is a physical realization of the descent equation between anomalies in even and odd dimensions (see Refs. [12,13] and the references therein).

Emergent effective action.—The anomaly outflow to the outside disk formed by the chiral bag as an annulus can be captured in a $(1+2)$-dimensional effective action describing the outside of the bag. Indeed, since the leaking and radial current to the chiral bag is j_x, its extension in $1+2$ dimensions defines the variation of the effective action S_{1+2} with respect to the emergent U(1) gauge field A_x as

$$\frac{\delta S_{1+2}}{\delta A_x} = \frac{j_x^{1+1}}{L_y}. \tag{13}$$

Inserting Eq. (12) into Eq. (13) and solving gives

$$S_{1+2} = \int_{1+2}\frac{e^2}{2\pi}A_x E_y = \frac{e^2}{4\pi}\int_{1+2} AdA, \tag{14}$$

where covariance was subsumed in the 3-form. This is the topological field theory describing the FQH droplet outside the bag illustrated in Fig. 1. One of the chief purposes of the emergent U(1) field A_μ in the $(1+2)$-dimensional droplet is to enforce the anomaly outflow—hence its topological rather than dynamical character. This emergent gauge field outside the bag is the dual of the U(1) gauge field inside the bag that mediates the e-charge.

In Eq. (14), the Chern-Simons coupling or flux attachment factor is $\kappa = e^2/2\pi$. A coupling of a charge e to the emergent U(1) field in $1+2$ dimensions amounts to a flux attachment of e/κ. The exchange of any pair of particles will generate a statistical phase $e^2/2\kappa = \pi$ through the Aharonov-Bohm interaction. A charged boson coupled to the emergent U(1) gauge field in $1+2$ dimensions transmutes to a fermion, and vice versa.

The generalization of these results to many quark species—say, of different colors N_c—requires the use of non-Abelian bosonization, but the CCP still holds [8,12]. However, in our case, this is not needed. Indeed, ordinary quarks carry baryon or fermion number $1/N_c$ (instead of the integer $e^2 \to 1$ discussed here), and hence a fraction π/N_c of the fermion statistics. This statistics is readily enforced through a flux attachment factor $\kappa = N_c/2\pi$, leading to the emergent Abelian Chern-Simons contribution

$$\frac{N_c}{4\pi}\int_{1+2} AdA. \tag{15}$$

This is the QH droplet suggested by Komargodski for baryons made of N_c quarks and $N_f = 1$, where the negative of Eq. (15) was argued to follow from the level-rank duality $\mathrm{SU}(N_c)_{-1} \leftrightarrow \mathrm{U}(1)_{N_c}$. The emergent U(1) gauge field outside the bag is the dual of the $\mathrm{SU}(N_c)$ gauge field inside the bag.

For vanishingly small radius, the Cheshire Cat smile reduces to a vortex line with running gapless quarks all spinning in concert (recall the magic angle), naturally explaining the large spin $\frac{1}{2}N_c$. The baryon number is still 1 and is now lodged in the η' field through the 2π monodromy. Antibaryons follow from a -2π monodromy, with the outflow turning into an inflow.

For arbitrary N_f, the spin and statistics arguments do not change, as they are solely fixed by N_c. However, the leaking flavor currents lead to a $\mathrm{U}(N_f)$ flavor-valued emergent gauge field $\mathbb{A}_\mu$. Again, the CCP applies *mutatis mutandis*. In particular, the emergent non-Abelian Chern-Simons action (15) is now

$$\frac{N_c}{4\pi}\int_{1+2} \mathrm{Tr}\left(\mathbb{A}d\mathbb{A} + \frac{2}{3}\mathbb{A}^3\right). \tag{16}$$

Tale of two hotels.—In Ref. [14] a chiral bag model was constructed to prevent the charge from leaking from the bag following the CCP. In other words a boundary term was added to the chiral bag to seal the leaking charge. This boundary term can be readily obtained by noting that, for y-independent fields, Eq. (14) describes the outside of the bag as a line segment in $1+1$ dimensions with

$$\frac{e}{2\pi}\int_{1+1} Ad\theta = \frac{e}{2\pi}\int_{1+1} \theta F - \frac{e}{2\pi}\int_B A_0\theta \tag{17}$$

after an integration by parts, clearly showing the leaking of the e-charge through the boundary. To seal the leak, the inside of the bag has to be supplemented by the opposite boundary term,

$$\frac{e}{2\pi}\int_B nA\frac{\eta'}{f_\eta} \equiv -\frac{e}{2\pi}\int_B \epsilon^{\mu\nu} n_\nu A_\mu \frac{\eta'}{f_\eta}, \tag{18}$$

with n^ν being the spatial normal to the bag boundary, after enforcing covariance on the 2-form. This is exactly the surface term suggested in the Cheshire Cat construction in Ref. [12] [see Eq. (8.24)] and used in Ref. [15]. The present arguments illustrate the subtle relationship between the chiral bag in Ref. [14] and the present chiral bag for the baryon as a FQH droplet. In the former, the e-charge is absolutely confined, while in the latter, the e-charge is allowed to flow transversely, with both making use of a Chern-Simons term. This is the tale of two hotels: the infinite hotel in our world for the confined anomaly, and the finite hotel in the other world for the flowing anomaly.

This tale is highly relevant for nuclear and astrophysical processes involving hadron-quark continuity [15]. For instance, the role of the η' for the color charge conservation is responsible for the Cheshire Cat mechanism for the tiny flavor singlet axial charge for the proton $g_A^{(0)}$. Furthermore since the η' is expected to become light at high density, it could have a strong impact on the stiffness of the equation of state in compact-star matter required for the observed massive $\gtrsim 2\ M_\odot$ stars.

Conclusions and discussions.—QCD in the large number of colors and $N_f = 1$ does not admit a representation of baryons as chiral solitons since $\pi_3[U(1)] = 0$. In this limit, Komargodski suggested that baryons are edge excitations of a $(1+2)$-dimensional QH droplet, and he concluded that these baryons are heavy and highly spinning.

We have shown that the nature of these baryons follows from an anomaly outflow (inflow for antibaryons) in a $(1+2)$-dimensional chiral bag model as an annulus of shrinking size thanks to the CCP. The outflow from the bag is captured by an emergent U(1) gauge field and described by a topological field theory. The normalization of the latter is fixed by the quark fractional statistics. The emergence of a QH description in the outside of the bag is an illustration of the Callan-Harvey mechanism for the parity anomaly in $1+2$ dimensions.

When the bag is shrunk to zero size, the baryonic charge 1 is lodged in the 2π monodromy. The chiral bag reduces to a vortex line (the smile of the Cheshire Cat), with running gapless modes of fixed spin as edge excitations carrying net spin $\frac{1}{2}N_c$. In this limit, the baryon is mostly the outside of the bag as a QHD droplet. Its size is fixed by the overall size of the droplet, a balance between the boundary and bulk tensions of the sheet as suggested in Ref. [5]. For $N_f = 1$ and various N_c, the present description of baryons can be tested using current lattice simulations.

These observations generalize to arbitrary N_f, but admittedly they also lead to more questions. Since these pancake baryons are highly spinning at large N_c, do they really correspond to the large spin states in the hadron spectrum as suggested in Ref. [5]? It is quite conceivable that high spinning skyrmion states which are usually ignored as spurious are less stable than deformed skyrmions, and they may even centrifuge to pancakes or strings. While we do not have answers to these interesting questions,we do think that with more work answers can be reached. In fact skyrmions simulated on a crystal lattice, a description known to be reliable at high density and in the large N_c limit, indicate the formation of sheets of stability with fractionalized baryon charges on the surface of sheets resembling FQH droplets [16]. These are open problems that need to be solved.

These facts prompt us to ask about the relationship of these highly spinning baryons, with the lowest spinning skyrmions in the spin-isospin tower $J = I = 1/2, \ldots, N_c/2$. As both descriptions rely on QCD in the large number of colors, a dynamical relation may be at work that selects one from the other. Also, domains of various forms and shapes made of η', or even the lighter π^0, are likely to form at a few times the nuclear matter density—say, in the crust of neutron stars or deeper—making the baryons as QH droplets potential candidates. To answer quantitatively these questions requires a more detailed dynamical description of the bag model as a QH droplet. In particular, the thickness and tension of the droplet need to be considered and modeled, including the effects of the Dirac sea in the bag contribution.

The present interplay between the QH effect and QCD baryons is much in line with the recent suggestion between quantum magnetism and QCD confinement [17], showing the intricate interplay between concepts of particle physics and condensed matter physics at strong coupling. More insights can be achieved by perhaps using holography since, for instance, baryons and the QH states find common ground for an explanation [4,18].

Finally and more speculatively, axion quark nuggets are suggested as candidates for dark matter [19]. In the cosmic QCD phase transition, axion domain walls are argued to form copiously and decay, trapping antimatter in the form of $(1+3)$-dimensional nuggets. It is tempting to suggest that breaking cosmic axion domain walls can also result in $(1+2)$-dimensional pancakes much like the ones discussed here, trapping topological fields instead, with confined hypothetical quark fields circling the boundary. Both the axion (boundary) and the topological fields (disk) are topologically stable and carry energy but are so far invisible, a good combination for dark matter. Conversely, $(1+3)$-dimensional η' or even neutral π^0 domain walls instead of axions can be used to trap few quarks in the more standard baryon configuration with low spin, or in the superconducting diquark phase in QCD matter at moderately high density, with tangible consequences for the neutron star equation of state. We hope to return to these and some other issues next.

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