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大阪高等工業學校

學 術 報 告

昭 和 二 年

REPORTS

OF

OSAKA TECHNOLOGICAL COLLEGE

OSAKA, JAPAN

1927

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昭和二年七月　大阪高等工業學校

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STUDY OF WATER REACTION - TURBINES

BY

JIRO TANIDE

OSAKA, JAPAN
1927

PREFACE

It is already well known that at the inlet edge of runner the high-speed Francis-turbine has usually the coefficient of absolute velocity of the entering water less than that of the low-speed one and consequently the former may have the so-called degree of reaction greater than the latter. But since a thorough study applicable for all reaction turbines seems to the author to be still lacking, this paper is intended to give the characteristics of these turbines, treating these as " turbines with positive reaction head ". The positive value of reaction head is only one common property for all sorts of reaction turbines.

The first chapter is intended as introduction to give the fundamental equations and the definitions and to explain the process of study. The relations between the degree of reaction, the coefficient of circumferential velocity, those of the several velocities, the velocity angles, etc. are discussed in general in the second chapter, and as the special case the discussion for the state of normal exit is given in the third chapter. In the last chapter the author has added the changing degree of reaction of a turbine regulated by the speed governor, and has given a set of examples illustrating most of computations which must be made before others in the design of reaction turbines.

JIRO TANIDE.

Kyoto, January, Taisho 14.

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CHAPTER I. INTRODUCTION

SECTION I. WATER FLOW THROUGH TURBINE CHANNELS

The sectional view on a meridian plane of a turbine fully filled with the flowing water is illustrated in Fig. 1, and the velocity diagrams at the inlet and outlet edges of the runner are given in Fig. 2. In Fig. 1 $\overline{OO}$ is the head level, $\overline{UU}$ the under level, g_1g_2 the guide channel, 12 the runner channel, 34 the draft tube. Let H be the available head in metres which is taken as the difference in level of water between $\overline{OO}$ and $\overline{UU}$ for an open flume system, as shown in the figure, neglecting the velocity w_0 at the head race. For an enclosed casing system, however, the available head may be taken as the difference in level of water between the tank and the tail race minus the resistance head in the penstock plus the head due to the velocity of approach at the tank.

h_{g1} : the height of the centre of the guide inlet
h_{g2} : that of the guide outlet
h : that of the inlet edge of the runner
h_2 : that of the outlet edge of the runner
h'_1 : that of the runner inlet
h'_2 : that of the runner outlet
h_3 : that of the inlet of the draft tube
} above the level $\overline{UU}$ in metres,

h_4 : the depth of the outlet of the draft tube under the level $\overline{UU}$ in metres, The following pressures are above the atmospheric pressure in kgs. per square metre:

- p_{g1} : the pressure of water at the guide inlet
- p_{g2} : that at the guide outlet
- p : that at the inlet edge of the runner
- p_2 : that at the outlet edge of the runner
- p'_1 : that at the runner inlet
- p'_2 : that at the runner outlet
- p_3 : that at the inlet of the draft tube
- p_4 : that at the outlet of the draft tube

The following velocities are in metres per second,

- w_{g1} : the velocity of water at the guide inlet
- w_{g2} : that at the guide outlet
- w_3 : that at the inlet of the draft tube
- w_4 : that at the outlet of the draft tube
- w : the absolute velocity of water at the inlet edge of the runner
- w_2 : that at the outlet edge of the runner
- w'_1 : that at the runner inlet
- w'_2 : that at the runner outlet
- v : the relative velocity of water at the inlet edge of the runner
- v_2 : that at the outlet edge of the runner
- v'_1 : that at the runner inlet
- v'_2 : that at the runner outlet
- u : the circumferential velocity at the centre of inlet edge of the runner
- u_2 : that at the outlet edge of the runner
- u'_1 : that at the runner inlet
- u'_2 : that at the runner outlet

- α : the angle of w at the inlet edge of the runner, i.e. the angle included by w and u,
- α_2 : that of w_2 at the outlet edge of the runner,
- β : the angle of v at the inlet edge of the runner, i.e. the angle included by v and $-u$,
- β_2 : that of v_2 at the outlet edge of the runner.

Other notations which are not denoted in these figures are

- ξ_{g1} : the resistance coefficient in the passage between the head race and the guide inlet for an open flume system or between the casing entrance and the guide inlet for an enclosed casing system,
- ξ_{g2} : that between the guide inlet and the guide outlet,
- ξ_e : that between the guide outlet and the inlet edge of the runner,
- ξ_1 : that between the inlet edge and the inlet of the runner,
- ξ_{12} : that between the inlet and the outlet of the runner,
- ξ_2 : that between the outlet and the outlet edge of the runner,
- ξ_3 : that between the outlet edge of the runner and the inlet of the draft tube,
- ξ_4 : that between the inlet and the outlet of the draft tube,
- γ : the heaviness of water in kgs. per cubic metre,
- g : the gravitic acceleration in metres per second per second.

If the water flows through all channels of a turbine in the state of permanency, the following equations are established.

$$H-\xi_{g1}H = \frac{w_{g1}^2}{2g}+\frac{p_{g1}}{\gamma}+h_{g1}, \text{ assuming } w_0=0 \quad (1)$$

$$\frac{w_{g1}^2}{2g}+\frac{p_{g1}}{\gamma}+h_{g1}-\xi_{g2}H = \frac{w_{g2}^2}{2g}+\frac{p_{g2}}{\gamma}+h_{g2} \quad (2)$$

$$\frac{w_{g2}^2}{2g}+\frac{p_{g2}}{\gamma}+h_{g2}-\xi_e H = \frac{w^2}{2g}+\frac{p}{\gamma}+h \quad (3)$$

$$\frac{v^2}{2g}-\frac{u^2}{2g}+\frac{p}{\gamma}+h-\xi_1 H = \frac{v_1'^2}{2g}-\frac{u_1'^2}{2g}+\frac{p'_1}{\gamma}+h'_1 \quad (4)$$

$$\frac{v_1'^2}{2g}-\frac{u_1'^2}{2g}+\frac{p'_1}{\gamma}+h'_1-\xi_{12} H = \frac{v_2'^2}{2g}-\frac{u_2'^2}{2g}+\frac{p'_2}{\gamma}+h'_2 \quad (5)$$

$$\frac{v_2'^2}{2g}-\frac{u_2'^2}{2g}+\frac{p'_2}{\gamma}+h'_2-\xi_2 H = \frac{v_2^2}{2g}-\frac{u_2^2}{2g}+\frac{p_2}{\gamma}+h_2 \quad (6)$$

$$\frac{w_2^2}{2g}+\frac{p_2}{\gamma}+h_2-\xi_3 H = \frac{w_3^2}{2g}+\frac{p_3}{\gamma}+h_3 \quad (7)$$

$$\frac{w_3^2}{2g}+\frac{p_3}{\gamma}+h_3-\xi_4 H = \frac{w_4^2}{2g}+\frac{p_4}{\gamma}-h_4 \text{ where } \frac{p_4}{\gamma}=h_4 \quad (8)$$

If the sum of the left sides of the equations (1) to (8) is equated to that of their right sides, the following equation is reduced.

$$H-(\xi_{g1}+\xi_{g2}+\xi_e+\xi_1+\xi_{12}+\xi_2+\xi_3+\xi_4)H = \frac{w^2-w_2^2}{2g}+\frac{v_2^2-v^2}{2g}+\frac{u^2-u_2^2}{2g}+\frac{w_4^2}{2g} \quad (9)$$

put $\xi = \xi_{g1}+\xi_{g2}+\xi_e+\xi_1+{}_{12}+\xi_2+\xi_3+\xi_4$

or $\xi = \xi_g+\xi_r+\xi_d$

where $\xi_g = \xi_{g1}+{}_{g2}+\xi_e$

: the resistance coefficient in the passage between the head race and the inlet edge of the runner for an open flume system or between the casing entrance and the inlet edge of the runner for an enclosed casing system,

$\xi_r = \xi_1+\xi_{12}+\xi_2$

: the resistance coefficient in the passage between the inlet and outlet edges of the runner,

$\xi_d = \xi_3+\xi_4$

: the resistance coefficient in the passage between the outlet edges of the runner and the outlet of the draft tube.

Then the equation (9) reduces to

$$H-\xi H-\frac{w_4^2}{2g} = \frac{u^2-w_2^2}{2g}+\frac{v_2^2-v^2}{2g}+\frac{u^2-u_2^2}{2g} \quad (9)'$$

The left side of the equation (9)' may become the effective head.

Let ηH: the effective head,

then $$\eta H = H-\xi H-\frac{w_4^2}{2g}$$

$$\left.\begin{aligned}\eta &= (1-\xi)-\frac{w_4^2}{2gH}\\ \text{or}\quad \eta &= (1-\xi)-k_4^2\end{aligned}\right\} \quad (10)$$

where η = the hydraulic efficiency

k_4 = the velocity coefficient of w_4 or $w_4 = k_4\sqrt{2gH}$

The equation (9) reduces to

$$\left.\begin{aligned}\eta &= \frac{w^2-w_2^2}{2gH}+\frac{v_2^2-v^2}{2gH}+\frac{u^2-u_2^2}{2gH}\\ \text{or}\quad \eta &= (k^2-k_2^2)+(\psi_2^2-\psi^2)+(\phi^2-\phi_2^2)\end{aligned}\right\} \quad (11)$$

where k, k_2, ψ, ψ_2, ϕ and ϕ_2 are the velocity coefficients of w, w_2, v, v_2, u and u_2 respectively, in respect to H, thus we have

$$w = k\sqrt{2gH},\qquad v = \psi\sqrt{2gH},\qquad u = \phi\sqrt{2gH},$$

$$w_2 = k_2\sqrt{2gH},\qquad v_2 = \psi_2\sqrt{2gH},\qquad u_2 = \phi_2\sqrt{2gH},$$

From the equation (7) and (8) we have

$$\left.\begin{aligned}\frac{p_2}{\gamma} &= -\left[h_2+\left(\frac{w_2^2}{2g}-\frac{w_4^2}{2g}-\xi_d H\right)\right]\\ \text{or}\quad \frac{p_2}{\gamma H} &= -\left[\frac{h_2}{H}+\left(k_2^2-k_4^2-\xi_d\right)\right]\end{aligned}\right\} \quad (12)$$

The equation (12) gives the relation between the head due to the back pressure at the outlet edge of the runner and the several heads in the passage of the draft tube. In this equation

$\frac{p_2}{\gamma}$ is the head due to the back pressure, and

h_2 may be called " the principal effect of the draft tube", which means to utilize the suction head only, and

$\left(\frac{w_2^2}{2g}-\frac{w_4^2}{2g}-\xi_d H\right)$ or $(k_2^2-k_4^2-\xi_d)\,H$ may be called " the secondary effect of the draft tube."

If $(k_2^2-k_4^2-\xi_d)$ is positive, the head due to the back pressure becomes less as $(k_2^2-k_4^2-\xi_d)\,H$ than $(-h_2)$, and the potential head at the outlet edge of runner becomes less as $(k_2^2-k_4^2-\xi_d)\,H$ than zero, or $(p_2/\gamma+h_2)=-(k_2^2-k_4^2-\xi_d)\,H$. If $(k_2^2-k_4^2-\xi_d)$ is negative, vice versa. In these cases the draft tube has both effects.

If $(k_2^2-k_4^2-\xi_d)$ is zero, the head due to the back pressure becomes $(-h_2)$, and the potential head at the outlet edge of the runner may be zero or $(p_2/\gamma+h_2)=0$, which is the same as that of the turbines placed in the tail race without the draft tube. In this case the draft tube has only the principal effect.

Then we have

$$\frac{p_2}{\gamma}+h_2 = -(k_2^2-k_4^2-\xi_d)H \text{ for the turbines with draft tube.} \quad (13)$$

$\frac{p_2}{\gamma}+h_2 = 0$ for the turbines with the draft tube neglecting the secondary effect, and for the turbines placed in the tail race without the draft tube. (14)

SECTION 2. DEGREE OF REACTION

The difference in potential heads between the inlet and outlet edges of the runner

$$\left(\frac{p}{\gamma}+h\right)-\left(\frac{p_2}{\gamma}+h_2\right) \quad \text{or} \quad \frac{p-p_2}{\gamma}+(h-h_2)$$

is called " the reaction head." The ratio of the reaction head to the available head

$$\frac{p-p_2}{\gamma H}+\frac{h-h_2}{H}$$

is called " the degree of reaction " in respect to H. From the equations (1), (2), (3), (7) and (8) we have

$$\frac{p-p_2}{\gamma}+(h-h_2) = (1-\xi_g)H+\left(\frac{w_2^2}{2g}-\frac{w_4^2}{2g}-\xi_d H\right)-\frac{w^2}{2g} \quad (15)$$

Put $\quad \mathcal{T} = (1-\xi_g)+(k_2^2-k_4^2-\xi_d) \quad (16)$

Then the equation (15) reduces to

$$\frac{p-p_2}{\gamma}+(h-h_2) = \mathcal{T}H-k^2H$$

or

$$\left.\begin{aligned} &\frac{p-p_2}{\gamma H}+\frac{h-h_2}{H} = \mathcal{T}-k^2 \\ &\text{where } \mathcal{T} = (1-\xi_g)+(k_2^2-k_4^2-\xi_d) \end{aligned}\right\} \quad (17)$$

Since for the turbines with the draft tube neglecting the secondary effect and for the turbines placed in the tail race without the draft tube

$$(14) \qquad \frac{p_2}{\gamma}+h_2 = 0 \quad \text{and} \quad k_2^2-k_4^2-\xi_d = 0,$$

the equatian (17) becomes

$$\left.\begin{aligned} &\text{if } \frac{p_2}{\gamma}+h_2 = 0, \quad \frac{p}{\gamma H}+\frac{h}{H} = \mathcal{T}-k^2 \\ &\text{where } \mathcal{T} = 1-\xi_g \end{aligned}\right\} \quad (18)$$

Let R : " the degree of reaction " in respect to H,

$$\left.\begin{aligned} &\text{then} \quad R = \frac{p-p_2}{\gamma H}+\frac{h-h_2}{H} \\ &\text{or} \quad R = \mathcal{T}-k^2 \text{ where } \mathcal{T} = (1-\xi_g)+(k_2^2-k_4^2-\xi_d) \end{aligned}\right\} \quad (19)$$

Now by the value of R the turbines fully filled with water are classified as

if $R>0$ or $k<\sqrt{\mathcal{T}}$, reaction turbines,
if $R=0$ or $k=\sqrt{\mathcal{T}}$, limit turbines or the limit case of reaction turbines,
if $R<0$ or $k>\sqrt{\mathcal{T}}$, suction turbines (" Saugstrahlturbinen ").[1]

Since in (19) $\mathcal{T}$ and k^2 are positive, R must be less than $\mathcal{T}$ when R is positive, thus

$$0<R<\mathcal{T} \quad \text{for reaction turbines} \quad (20)$$

(20) is the condition under which reaction turbines may exist and this is called " the condition (20) ".

SECTION 3. VELOCITY-COEFFICIENT DIAGRAMS

The velocity coefficient is proportional to the magnitude of velocity for a turbine, since the velocity coefficient $=1/\sqrt{2gH}\times$ the magnitude of velocity, where H is given for a turbine and $1/\sqrt{2gH}$ is taken as constant. If the velocity coefficient is given with the direction of the corresponding velocity, the velocity coefficient may become a vector, its magnitude is proportional to that of the corresponding velocity and its direction is the same as that of the velocity.

1. see R. Thomann, Die Wasserturbinen u. Turbinenpumpen (1921), Seite 74, Gleichung (49).

A diagram drawn with the vectors of velocity coefficients is similar with the corresponding velocity diagram and may be called " the velocity coefficient diagram." For this reason, this diagram may be used in the discussion of this paper instead of the velocity diagram. Fig. 3 and Fig. 4 show the velocity-coefficient diagrams at the inlet and outlet edges of the runner respectively, which are similar with the corresponding velocity diagrams in Fig. 2.

For a turbine with the normal exit the water discharges from the runner in the normal direction and the absolute velocity has no tangential component, as illustrated in Fig. 6. Fig. 6 shows the velocity-coefficient diagram at the outlet edge of runner, in which α_2 is $\pi/2$ and k_{2l} has no tangential component. In this case some notations are written with the suffix " l," as k_{2l} and K_{2l} instead of k_2 and K_2 respectively. Fig. 5 shows the velocity-coefficient diagram at the inlet edge of runner corresponding to that in Fig. 6, the point of diagram is indicated with K_l instead of K_1 as in Fig. 6.

SECTION 4. TYPES OF TURBINES

The water turbines are usually classified by the vane angle at the runner inlet, which is not equal to the anlge of the relative velocity, excepting the state of entrance without shock. In this paper, however, β, the angle of the relative velocity v, is considered as the point of view in the classification of turbines fully filled with water, as

the group I, turbines with $\beta > \pi/2$

the group II, turbines with $\beta = \pi/2$

the group III, turbines with $\beta < \pi/2$

Besides water turbines may be classified by the value of ϕ, the coefficient of the circumferential velocity u at the inlet edge of runner, as

the case 1, turbines with $\phi < \sqrt{\mathcal{T}}$

the case 2, turbines with $\phi = \sqrt{\mathcal{T}}$

the case 3, turbines with $\phi > \sqrt{\mathcal{T}}$

By the combination of groups and cases nine types may be imagined as

	case 1	case 2	case 3
group I	type I_1 $\beta > \pi/2$, $\phi < \sqrt{\mathcal{T}}$	type I_2 $\beta > \pi/2$, $\phi = \sqrt{\mathcal{T}}$	type I_3 $\beta > \pi/2$, $\phi > \sqrt{\mathcal{T}}$
group II	type II_1 $\beta = \pi/2$, $\phi < \sqrt{\mathcal{T}}$	type II_2 $\beta = \pi/2$, $\phi = \sqrt{\mathcal{T}}$	type II_3 $\beta = \pi/2$, $\phi > \sqrt{\mathcal{T}}$
group III	type III_1 $\beta < \pi/2$, $\phi < \sqrt{\mathcal{T}}$	type III_2 $\beta < \pi/2$, $\phi = \sqrt{\mathcal{T}}$	type III_3 $\beta < \pi/2$, $\phi > \sqrt{\mathcal{T}}$

It is evident that there are no more types. Many turbines may exist with the different values of β although ϕ is taken at one value in a type, thus each type may include the numerous turbines of all sorts (reaction, limit and suction turbines) with the various values of ϕ and β. In order that these types are used to classify reaction turbines, it is important to determine whether every type is existent or non-existent as reaction turbines. If all turbines in a type do not satisfy the condition (20) and are not existent as reaction turbines, this type does not become a type of reaction turbines. If there are such types, these must be eliminated from the types of reaction turbines, and others may remain as the types of reaction turbines. This principle is applied to the discussion of i) the general case in the chapter II and to that of ii) the special case in the chapter III.

i) In general the degree of reaction "R" may become a function of $\mathcal{T}$, ϕ, β and α or k_a. k_a is the velocity ceofficient of w_a, the normal component of the absolute velocity at the inlet edge of runner, and k_a is fixed by a certain value of α for the given values of ϕ and β. α is taken in the range between 0 and π for all types, β and ϕ are taken at the arbitrary values within their ranges for every types, and $\mathcal{T}$ is given at the reasonable value. Every type is inspected whether the value of "R" is positive or negative for all values of α between 0 and π. In Fig. 7

the velocity diagram OCK with α, k and ψ is for 1st. turbine, the velocity diagram OCK' with α', k' and ψ' is for 2nd. turbine, the velocity diagram OCK'' with α'', k'' and ψ'' is for 3rd. turbine and so on. These diagrams have the common side $\overline{OC}$ or ϕ, the common angle β and the points of diagram K, K', K'' on a line $\overline{CK}$, thus these turbines have the same values of β and ϕ, but have the different values of α corresponding to the position of the point of diagram. These turbines may be included in a turbine series with the given values of β and ϕ, and every turbine must have the point of diagram on the line $\overline{CK}$, thus the line $\overline{CK}$ may be said to correspond to one turbine series if β and ϕ are given. But since $\overline{CK}$ is a line determined by the arbitrary values of β and ϕ within their ranges for a type, the line $\overline{CK}$ may be to represent this type. If in a type the value of "R" is not positive at any position of K, there are no reaction turbines existent, and this type may be non-existent as reaction turbines.

ii) In this case "R" becomes a function of τ, ϕ, η_t and α or k_a, in which η_t is the hydraulic efficiency in the state of normal exit. In Fig. 8 the line $\overline{LL'}$ is drawn parpendicular to $\overline{OC}$ at the distance ϕ_t' from the origin O, and this distance is determined by the values of η_t and ϕ, as will be seen in the chapter III.[1] For the given values of η_t and ϕ, therefore, the point of diagram K_t is always on the line $\overline{LL'}$, and
the velocity diagram OCK with α, β, k, & ψ is for 1st. turbine,
the velocity diagram OCK' with α', β', k', & ψ' is for 2nd. turbine,
the velocity diagram OCK'' with α'', β'', k'', & ψ'' is for 3rd. turbine and so on. The point K_t takes one position for a value of α corresponding to a turbine, and the line $\overline{LL'}$ may correspond to a turbine series with the given values of η_t and ϕ in a type. But since $\overline{LL'}$ is the line determined by the arbitrary values of η_t and ϕ within their ranges for a type, the line $\overline{LL'}$ may be to represent this type. If a type has not the positive value of "R" at any position of K_t, no reaction turbines may exist, and this type may be non-existent as reaction turbines.

1. see Camerer, Vorlesungen ueber Wasserkraftmaschinen, Seite 259-279.

CHAPTER II. GENERAL CHARACTERISTICS OF REACTION TURBINES

Section 5. Characteristics of "R" for "k_a" and Classification of Reaction Turbines

a) Equation of "R"

Fig. 9 and Fig. 10 illustrate respectively the velocity diagram and the velocity-coefficient diagram at the inlet edge of the runner.

Let w_a : the normal component of the absolute velocity w at the inlet edge of the runner in metres per second,
k_a : the velocity coefficient of w_a referred to H,
u' : the tangential component of w in metres per second,
ϕ' : the velocity coefficient of u' referred to H,
u'' : the tangential component of v in metres per second,
ϕ'' : the velocity coefficient of u'' referred to H, then we have

$$w_a = w \sin\alpha, \quad u' = w_a \operatorname{ctg}\alpha, \quad u'' = w_a \operatorname{ctg}\beta,$$

or

$$k_a = k \sin\alpha, \quad \phi' = k_a \operatorname{ctg}\alpha, \quad \phi'' = k_a \operatorname{ctg}\beta.$$

Then the equation (19) $R = \tau - k^2$ reduces to

$$R = \tau - \frac{k_a^2}{\sin^2\alpha} \tag{21}$$

In Fig. 10, $\phi' = \phi - \phi''$

or

$$\operatorname{ctg}\alpha = \frac{\phi}{k_a} - \operatorname{ctg}\beta$$

then we have

$$\operatorname{ctg}^2\alpha = \frac{\phi^2}{k_a^2} - 2\frac{\phi}{k_a}\operatorname{ctg}\beta + \operatorname{ctg}^2\beta$$

if 1 is added to both sides of the above equation, then we have

$$\frac{1}{\sin^2\alpha} = \frac{\phi^2}{k_a^2} - 2\frac{\phi}{k_a}\operatorname{ctg}\beta + (1+\operatorname{ctg}^2\beta)$$

then the equation (21) reduces to

$$R = \mathcal{T} - \phi^2 + 2(\phi \operatorname{ctg}\beta)k_a - (1+\operatorname{ctg}^2\beta)k_a^2 \qquad (22)$$

(22) is the equation of "R" as a function of k_a, β, ϕ and $\mathcal{T}$. The value of $\mathcal{T}$ is reasonably taken for every type, and β and ϕ are taken at the arbitrary values in their ranges for every type. And since a turbine has one value of k_a, as mentioned in the chapter I, the values of R for all turbines of every type may be determined by the equation (22). When the values of $\mathcal{T}$, ϕ, and β are given, (22) becomes the equation of R for a turbine series, and R becomes a function of k_a only.

In general the value of $\mathcal{T}$ may be taken as about .92 to .96, excepting the special case in the type III_3, and the value of ϕ may be usually taken at about .43[1] to .98[2] for Francis turbines, and above about .95 for the modern axial flow turbines.

b) Inspection of Value of "R"

The equation (22) reduces to

$$R = \mathcal{T} - \{(\phi - \operatorname{ctg}\beta\ k_a)^2 + k_a^2\} \qquad (23)$$

In the equation (23)

$$\{(\phi - \operatorname{ctg}\beta\ k_a)^2 + k_a^2\} > 0 \qquad \text{always.}$$

To satisfy the condition (20) $\mathcal{T} > R > 0$, $\{(\phi - \operatorname{ctg}\beta k_a)^2 + k_a^2\}$ must be less than $\mathcal{T}$. Hence the condition (20) becomes

$$\{(\phi - \operatorname{ctg}\beta\ k_a)^2 + k_a^2\} < \mathcal{T} \qquad \text{the condition} \quad (24)$$

The condition (24) must be satisfied for the existence of reaction turbines.

For the group I, $\pi > \beta > \pi/2$ or $0 > \operatorname{ctg}\beta > -\infty$

the condition (24) reduces to

$$\left[\{\phi + \operatorname{ctg}(\pi-\beta) \cdot k_a\}^2 + k_a^2\right] < \mathcal{T} \qquad (24)_I$$

where $0 < (\pi-\beta) < \pi/2$, $+\infty > \operatorname{ctg}(\pi-\beta) > 0$

The type I_1, $\phi < \sqrt{\mathcal{T}}$ or $\phi^2 < \mathcal{T}$

1. see the example 14 and 15 in the chapter III,
1 & 2. see V. Gelpke, Turbinen u. Turbinenanlagen (1906), Seite 68,
J. Orten-Böving, Water turbine plant (1910), page 12.

Since the values of k_a and β are selected so that the condition $(24)_I$ may be satisfied, I_1 may exist as the type of reaction turbines.

The type I_2, $\phi = \sqrt{\mathcal{T}}$ or $\phi^2 = \mathcal{T}$ and

The type I_3, $\phi > \sqrt{\mathcal{T}}$ or $\phi^2 > \mathcal{T}$

Since $\{\phi + \operatorname{ctg}(\pi-\beta) \cdot k_a\}^2$ is larger than $\mathcal{T}$, the condition $(24)_I$ may not be satisfied. Hence I_2 and I_3 do not exist as the types of reaction turbines.

For the group II, $\beta = \pi/2$ or $\operatorname{ctg}\beta = 0$

the condition (24) reduces to

$$(\phi^2 + k_a^2) < \mathcal{T} \qquad (24)_{II}$$

The type II_1, $\phi < \sqrt{\mathcal{T}}$ or $\phi^2 < \mathcal{T}$

By taking the value of k_a less than $\sqrt{(\mathcal{T} - \phi^2)}$, the condition $(24)_{II}$ may be satisfied. Hence II_1 may exist as the type of reaction turbines.

The type II_2, $\phi = \sqrt{\mathcal{T}}$ or $\phi^2 = \mathcal{T}$

Since $(\phi^2 + k_a^2)$ becomes equal to $(\mathcal{T} + k_a^2)$ which is larger than $\mathcal{T}$, the condition $(24)_{II}$ may not be satisfied. Hence II_2 does not exist as the type of reaction turbines.

The type II_3, $\phi > \sqrt{\mathcal{T}}$ or $\phi^2 > \mathcal{T}$

Since $(\phi^2 + k_a^2) > \mathcal{T}$, the condition $(24)_{II}$ may not be satisfied. Hence II_3 does not exist as the type of reaction turbines.

For the group III, $0 < \beta < \pi/2$ or $\infty > \operatorname{ctg}\beta > 0$.

Since the values of k_a and β are easily selected so that $\{(\phi - \operatorname{ctg}\beta\ k_a)^2 + k_a^2\}$ is small enough to satisfy the condition (24) in all cases: $\phi \lesseqgtr \sqrt{\mathcal{T}}$, III_1, III_2 and III_3 exist as the types of reaction turbines.

By the above inspection, there are no reaction turbines in the types I_2, I_3, II_2 and II_3, reaction turbines may exist in the types I_1, II_1, III_1, III_2 and III_3, and accordingly reaction turbines are classified into five types as

	case 1 $\phi < \sqrt{\tau}$	case 2 $\phi = \sqrt{\tau}$	caie 3 $\phi > \sqrt{\tau}$
group I $\beta > \pi/2$	type I_1	non-existent	non-existent
group II $\beta = \pi/2$	type II_1	non-existent	non-existent
group III $\beta < \pi/2$	type III_1	type III_2	type III_3

c) Particular Values of "R" and "k_a"

(22) $$R = \tau - \phi^2 + 2\phi \operatorname{ctg}\beta \cdot k_a - (1+\operatorname{ctg}^2\beta)k_a^2$$

If in (22) τ, ϕ and β are given, the particular values of R may be found.

i) The maximum value of R.

From the equation (22)

$$\frac{dR}{dk_a} = 2\phi \operatorname{ctg}\beta - 2(1+\operatorname{ctg}^2\beta)k_a$$

$$\frac{d^2R}{dk^2} = -2(1+\operatorname{ctg}^2\beta) \quad \text{negative,}$$

hence if $\frac{dR}{dk_a} = 0$, R becomes maximum, then we have

$$R_{max} = \tau - \phi^2 \sin^2\beta, \quad \text{if } k_a = \phi \sin\beta \cdot \cos\beta \tag{25}$$

ii) The value of R for $k_a = 0$.

From the equation (22)

$$R = \tau - \phi^2, \quad \text{if } k_a = 0 \tag{26}$$

iii) $R = 0$.

When $R = 0$, the equation (22) becomes

$$(1+\operatorname{ctg}^2\beta)k_a^2 - 2\phi \operatorname{ctg}\beta \cdot k_a - (\tau - \phi^2) = 0,$$

then $$k_a = \phi \sin\beta \cdot \cos\beta \pm \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta}$$

hence we have

$$R = 0, \quad \text{if } k_a = \phi \sin\beta \cdot \cos\beta \pm \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta} \tag{27}$$

iv) The limit values of k_a with the restriction of reaction.

R has one maximum value by (25) and becomes zero for two values of k_a in (27). In order that the value of R becomes positive, $R_{max.}$ must be positive and the value of k_a must be taken in the range between two values of k_a in (27), which become the limit values of k_a.

Let $k_{a_{lt_r}}$: the upper limit value of k_a with the restriction of reaction,

$k'_{a\,lt_r}$: the lower limit,

then $$k_{a_{lt_r}} = \phi \sin\beta \cos\beta + \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta}$$

$$k'_{a\,lt_r} = \phi \sin\beta \cos\beta - \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta}$$

For I_1, $\phi \sin\beta \cdot \cos\beta < 0$,

but $\phi \sin\beta \cdot \cos\beta = -\sin\beta\sqrt{\phi^2 - \phi^2\sin^2\beta}$ and $\phi^2 < \tau$

hence $k_{a_{lt_r}} > 0$.

For II_1,
for III_1, III_2 and III_3 } $\phi \sin\beta \cdot \cos\beta \geqq 0$,

hence $k_{a_{lt_r}} > 0$.

Then $k_{a_{lt_r}} > 0$ always for five types.

For I_1
For II_1 } $\phi \sin\beta \cdot \cos\beta \leq 0$,

hence $k'_{a\,lt_r} < 0$.

For III_1, $\phi \sin\beta \cdot \cos\beta > 0$,

but $\phi \sin\beta \cdot \cos\beta = +\sin\beta\sqrt{\phi^2 - \phi^2 \sin^2\beta}$ and $\phi^2 < \tau$

hence $k'_{a\,lt_r} < 0$.

For III_2, $\phi \sin\beta \cos\beta > 0$,

but $\phi^2 = \tau$

hence $k'_{a\,lt_r} = 0$.

For III_3, $\phi \sin\beta \cos\beta > 0$,

but $\phi^2 > \tau$

hence $k'_{a\,lt_r} > 0$.

Then we have

$$\left.\begin{array}{l}\text{for } I_1,\ II_1 \text{ and } III_1 \\ \text{for } III_2 \\ \text{for } III_3\end{array}\right\} k_a{}'_{II_r} \lesseqgtr 0$$

But since k_a is always taken at the positive value, it is no need to limit the value of k_a at $k_a{}'_{II_r}$ for the types I_1, II_1, III_1 and III_2. Hence we have

$$\left.\begin{array}{l} k_{a_{II_r}} = \phi \sin\beta \cos\beta + \sin\beta\sqrt{\mathcal{T} - \phi^2 \sin^2\beta} \\ k_a{}'_{II_r} = \phi \sin\beta \cos\beta - \sin\beta\sqrt{\mathcal{T} - \phi^2 \sin^2\beta} \quad \text{for only } III_3 \end{array}\right\} \qquad (28)$$

For the types I_1, II_1, III_1 and III_2 the value of k_a must be less than $k_{a_{II_r}}$, and for the type III_3 k_a must be less than $k_{a_{II_r}}$ and greater than $k_a{}'_{II_r}$ with the restriction of reaction.

SECTION 6. "k_aR" CURVES

If $\mathcal{T}$, ϕ and β are given, (22) becomes the equation of parabola in respect to "k_a" and "R." This parabola is called the "k_aR" curve.

The equation (22) reduces to

$$R = (\mathcal{T} - \phi^2) + 2\frac{\cos\beta}{\sin\beta}\phi\, k_a - \frac{1}{\sin^2\beta}\, k_a^2. \qquad (22)'$$

Further the equation (22)' reduces to

$$-\left[R - (\mathcal{T} - \phi^2 \sin^2\beta)\right] = \frac{1}{\sin^2\beta}\left[k_a - \phi \sin\beta \cos\beta\right]^2 \qquad (29)$$

From (29) we have

$$k_a = \phi \sin\beta \cdot \cos\beta \pm \sin\beta\sqrt{(\mathcal{T} - \phi^2 \sin^2\beta) - R} \qquad (30)$$

In Fig. 11, the "k_aR" curve is illustrated in a coordinates with $\overline{Ok_a}$ as the axis of abscissa and with $\overline{OR}$ as that of ordinate.

O' : the vertex of curve or parabola,

$\overline{MO'}$: the symmetrical axis of curve, which is parallel to OR,

M : the intersection of $\overline{MO'}$ with the axis $\overline{Ok_a}$,

B : the intersection of curve with the axis $\overline{Ok_a}$ in the right side of MO',

U : the intersection of curve with the axis $\overline{Ok_a}$ in the left side of $\overline{MO'}$

T : the intersection of curve with the axis OR.

From the equation (30) we have

if $R = 0$, $k_a = \phi \sin\beta \cos\beta \pm \sin\beta\sqrt{\mathcal{T} - \phi^2 \sin^2\beta}$, thus we have

$$\left.\begin{array}{l} \overline{OB} = \phi \sin\beta \cos\beta + \sin\beta\sqrt{\mathcal{T} - \phi^2 \sin^2\beta} = k_{a_{II_r}} \\ \overline{OU} = \phi \sin\beta \cos\beta - \sin\beta\sqrt{\mathcal{T} - \phi^2 \sin^2\beta} = k_a{}'_{II_r} \quad \text{for only } III_3 \end{array}\right\} \qquad (28)$$

$$\overline{UB} = \overline{OB} - \overline{OU}$$

hence

$$\left.\begin{array}{l} \overline{UB} = 2\sin\beta\sqrt{\mathcal{T} - \phi^2 \sin^2\beta} \\ \overline{UM} = \overline{MB} = \sin\beta\sqrt{\mathcal{T} - \phi^2 \sin^2\beta} \end{array}\right\} \qquad (31)$$

In the equation (22)',

if $k_a = 0$, $R = (\mathcal{T} - \phi^2)$, thus we have

$$\overline{OT} = (\mathcal{T} - \phi^2) \qquad (26)$$

By the equation (25)

$$\overline{OM} = \phi \sin\beta \cos\beta \quad \text{and} \quad \overline{MO'} = \mathcal{T} - \phi^2 \sin^2\beta = R_{max.} \qquad (25)$$

If in the equation (25) the value of β is changed, the positions of the vertex O' and the intersection M may change.

For the group I, $\beta > \pi/2$, and $\phi \sin\beta \cdot \cos\beta < 0$. Hence O' and M are situated in the negative side of k_a.

For the group II, $\beta = \pi/2$, and $\phi \sin\beta \cos\beta = 0$. Hence O' and M drop on the axis $\overline{OR}$.

For the group III, $\beta < \pi/2$, and $\phi \sin\beta \cos\beta > 0$. Hence O' and M are situated in the positive side of k_a.

If in the equation (26) the value of ϕ is changed, the position of the intersection T may change.

For the case 1, $\phi < \sqrt{\mathcal{T}}$, or $\mathcal{T} - \phi^2 > 0$. Hence T is situated in the positive side of R.

For the case 2, $\phi = \sqrt{\mathcal{T}}$, or $\mathcal{T} - \phi^2 = 0$. Hence T coincides with the origin O.

For the case 3, $\phi > \sqrt{\mathcal{T}}$, or $\mathcal{T} - \phi^2 < 0$. Hence T is situated in the negative side of R.

Fig. 12 to Fig. 20 illustrate the "k_aR" curves for nine types. Every curve has the proper characteristics with regard to the axes $\overline{OR}$ and $\overline{Ok_a}$. For the sake of abbreviation, $(UO'B)$, the portion of curve in the positive side of R, is called "the symmetrical portion,"

$(O'U)$, the left half of the symmetrical portion, "the U-side,"
$(O'B)$, the right half of the symmetrical portion, "the B-side," and the portion of curve in the positive side of R and k_a "the R-portion." When the R-portion exists, the condition (20) is satisfied.

For the group I, the symmetrical axis $\overline{MO'}$ is situated in the negative side of k_a. Hence the R-portion exists only in the case 1, and accordingly the reaction turbines may exist only in the case 1.

Type I_1, (Fig. 12). The R-portion is less than the B-side, and the symmetrical portion is largely lost in the positive side of k_a.

Type I_2, (Fig. 13). The R-portion is just lost, and the point B coincides with the origin O.

Type I_3, (Fig. 14). The R-portion disappears, and the point B is apart from the origin O in the left side. Since the types I_2 and I_3 have no R-portions, these may not exist as the types of reaction turbines.

For the group II, the symmetrical axis $\overline{MO'}$ coincides with the axis $\overline{OR}$. Hence the R-portion exists only in the case 1, and accordingly the reaction turbines may exist only in the case 1.

Type II_1, (Fig. 15). The R-portion becomes the B-side, and the point M coincides with the origin O.

Type II_2, (Fig. 16). The R-portion is just lost, and the vertex O' coincides with the origin O.

Type II_3, (Fig 17). The R-portion disappears, and the vertex O' falls below the origin O along the axis $\overline{OR}$ Since the types II_2 and II_3 have no R-portions, these may not exist as the types of reaction turbines.

For the group III, the symmetrical axis $\overline{MO'}$ is situated in the positive side of k_a. The R-portion exists in all cases, and accordingly the the reaction turbines may exist in all cases.

Type III_1, (Fig. 18). The R-portion is less than the symmetrical portion and is greater than the B-side.

Type III_2, (Fig. 19). The R-portion becomes just the symmetrical portion, and the point U coincides with the origin O.

Type III_3, (Fig. 20). The R-portion becomes the symmetrical portion, and the point U is apart from the origin O in the right side. Since the types III_1, III_2 and III_3 have the R-portion, these may exist as the types of reaction turbines.

According to the characteristics of the "k_aR" curve above explained, it is also evident that the reaction turbines may exist in five types I_1, II_1, III_1, III_2 and III_3.

Further the ratio of the R-portion to the symmetrical portion is the smallest for I_1, it increases step by step as in the order of I_1, II_1, III_1, and III_2, and it becomes unity for III_2 and III_3.

The example 1. The "k_aR" curve for a turbine series in the type I_1, with $\tau = .93$, $\phi = .56$ and $\beta = 145°$.

(25), $R_{max.} = \overline{MO'} = \tau - \phi^2 \sin^2\beta$, if $k_{a(Rmax.)} = \overline{OM} = \phi \sin\beta \cdot \cos\beta$

$\overline{MO'} = .93 - (.56 \times \sin 145°)^2 = \underline{.827}$

$\overline{OM} = .56 \times \sin 145° \times \cos 145° = \underline{-.263}$

(26), if $k_a = 0$. $R_{(k_a:0)} = \overline{OT} = (\tau - \phi^2)$

$\overline{OT} = .93 - .3136 = \underline{.616}$

(27), if $R = 0$, $k_{a(R:0)} = \overline{OB}$ and $\overline{OU}$

$= \phi \sin\beta \cdot \cos\beta \pm \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta}$

$\overline{OB} = .56 \times \sin 145° \times \cos 145° + \sin 145° \sqrt{.93 - (.56 \times \sin 145°)^2}$

$= -.26311 + .52155 = \underline{.258} = k_{a_{II_r}}$

$\overline{OU} = -.26311 - .52155 = \underline{-.785}$

(31), $\overline{UM} = \overline{MB} = \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta}$

$= \sin 145° \times \sqrt{.93 - (.56 \times \sin 145°)^2} = \underline{.522}$

Fig. 12 shows the "k_aR" curve of this example, and the values of k_a and R at the particular points are denoted in the brackets.

The example 2. The "k_aR" curve for a turbine series in the type II_1, with $\tau = .94$, $\phi = .640$ and $\beta = 90°$.

(25), $R_{max.} = \overline{MO'} = \tau - \phi^2 \sin^2\beta$, if $k_{a(Rmax)} = \overline{OM} = \phi \sin\beta \cdot \cos\beta$

$\overline{MO'} = .94 - (.640 \times \sin 90°)^2 = \underline{.53}$

$\overline{OM} = .640 \times \sin 90° \times \cos 90° = \underline{0}$

(26), if $k_a = 0$, $R_{(k_a:0)} = \overline{OT} = \tau - \phi^2$

$\overline{OT} = .94 - (.640)^2 = \underline{.53}$

(27), if $R = 0$, $k_{a_{(R:0)}} = \overline{OB}$ and $\overline{OU}$

$$= \phi \sin\beta\cos\beta \pm \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta}$$

$$\overline{OB} = .640 \times \sin 90° \times \cos 90° + \sin 90° \times \sqrt{.94-(.640 \times \sin 90°)^2}$$

$$= 0 + \sqrt{.53} = \underline{.728} = k_{a_{II_r}}$$

$$\overline{OU} = 0 - \sqrt{.53} = \underline{-.728}$$

(31), $\overline{UM} = \overline{MB} = \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta}$

$$= \sin 90° \times \sqrt{.94-(.640 \times \sin 90°)^2} = \underline{.728}$$

Fig. 15 shows the "$k_a R$" curve of this example, and the values of k_a and R at the particular points are denoted in the brackets.

The example 3. The "$k_a R$" curve for a turbine series in the type III_1, with $\tau = .95$, $\phi = .80$, and $\beta = 40°$.

(25), $R_{max.} = \overline{MO'} = \tau - \phi^2 \sin^2\beta$, if $k_{a_{(Rmax)}} = \overline{OM} = \phi \sin\beta \cos\beta$

$$\overline{MO'} = .95 - (.8 \times \sin 40°)^2 = \underline{.686}$$

$$\overline{OM} = .8 \times .64279 \times .76604 = \underline{.394}$$

(26), if $k_a = 0$, $R_{(k_a=0)} = \overline{OT} = \tau - \phi^2$

$$\overline{OT} = .95 - (.8)^2 = \underline{.31}$$

(27), if $R = 0$, $k_{a_{(R:0)}} = \overline{OB}$ and $\overline{OU}$

$$= \phi \sin\beta \cos\beta \pm \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta}$$

$$OB = .8 \times \sin 40° \times \cos 40° + \sin 40° \times \sqrt{.95-(.8 \times .64279)^2}$$

$$= .39392 + .53222 = \underline{.926} = k_{a_{II_r}}$$

$$\overline{OU} = .39392 - .53222 = \underline{-.138}$$

(31), $\overline{UM} = \overline{MB} = \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta}$

$$= \sin 40° \times \sqrt{.95-(.8 \times \sin 40°)^2} = \underline{.532}$$

Fig. 18 shows the "$k_a R$" curve of this example, and the values of k_a and R at the particular points are denoted in the brackets.

The example 4. The "$k_a R$" curve for a turbine series in the type III_2, with $\tau = .95$, $\phi = .975$ and $\beta = 25°$.

(25), $R_{max.} = \overline{MO'} = \tau - \phi^2 \sin^2\beta$, if $k_{a_{(Rmax)}} = \overline{OM} = \phi \sin\beta \cos\beta$

$$\overline{MO'} = \tau \cos^2\beta \quad \text{for} \quad \tau = \phi^2$$

$$= .95 \times (.90631)^2 = \underline{.780}$$

$$\overline{OM} = .975 \times .42262 \times .90631 = \underline{.373}$$

(26), if $k_a = 0$, $R_{(k_a:0)} = \overline{OT} = \tau - \phi^2$

$$\overline{OT} = .95 - (.975)^2 = .95 - .95 = \underline{0}$$

(27), if $R = 0$, $k_{a_{(R:0)}} = \overline{OB}$ and $\overline{OU}$

$$= \phi \sin\beta \cos\beta \pm \phi \sin\beta \cos\beta, \text{ for } \phi = \sqrt{\tau}$$

$$\overline{OB} = 2\phi \sin\beta \cos\beta = 2 \times .975 \times .42262 \times .90631 = .747 = k_{a_{II_r}}$$

$$\overline{OU} = \underline{0}$$

(31), $\overline{UM} = \overline{MB} = \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta} = \phi \sin\beta \cos\beta$ for $\phi = \sqrt{\tau}$

$$= .975 \times .42262 \times .90631 = \underline{.373}$$

Fig. 19 shows the "$k_a R$" curve of this example, and the values of k_a and R at the particular points are denoted in the brackets.

The example 5. The "$k_a R$" curve for a turbine series in the type III_3, with $\tau = .96$, $\phi = 1.6$, and $\beta = 12°$.

(25), $R_{max.} = \overline{MO'} = \tau - \phi^2 \sin^2\beta$, if $k_{a_{(Rmax)}} = \overline{OM} = \phi \sin\beta \cos\beta$

$$\overline{MO'} = .96 - (1.6 \times .20791)^2 = \underline{.849}$$

$$\overline{OM} = 1.6 \times .20791 \times .97815 = \underline{.325}$$

(26), if $k_a = 0$, $R_{(k_a:0)} = \overline{OT} = \tau - \phi^2$

$$\overline{OT} = .96 - (1.6)^2 = \underline{-1.6}$$

(27), if $R = 0$, $k_{a_{(R:0)}} = \overline{OB}$ and $\overline{OU}$

$$= \phi \sin\beta \cos\beta \pm \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta}$$

$$\overline{OB} = 1.6 \times 20791 \times .97815 + .20791 \times \sqrt{.96-(1.6 \times .20791)^2}$$

$$= .325387 + .191609 = \underline{.517} = k_{a_{II_r}}$$

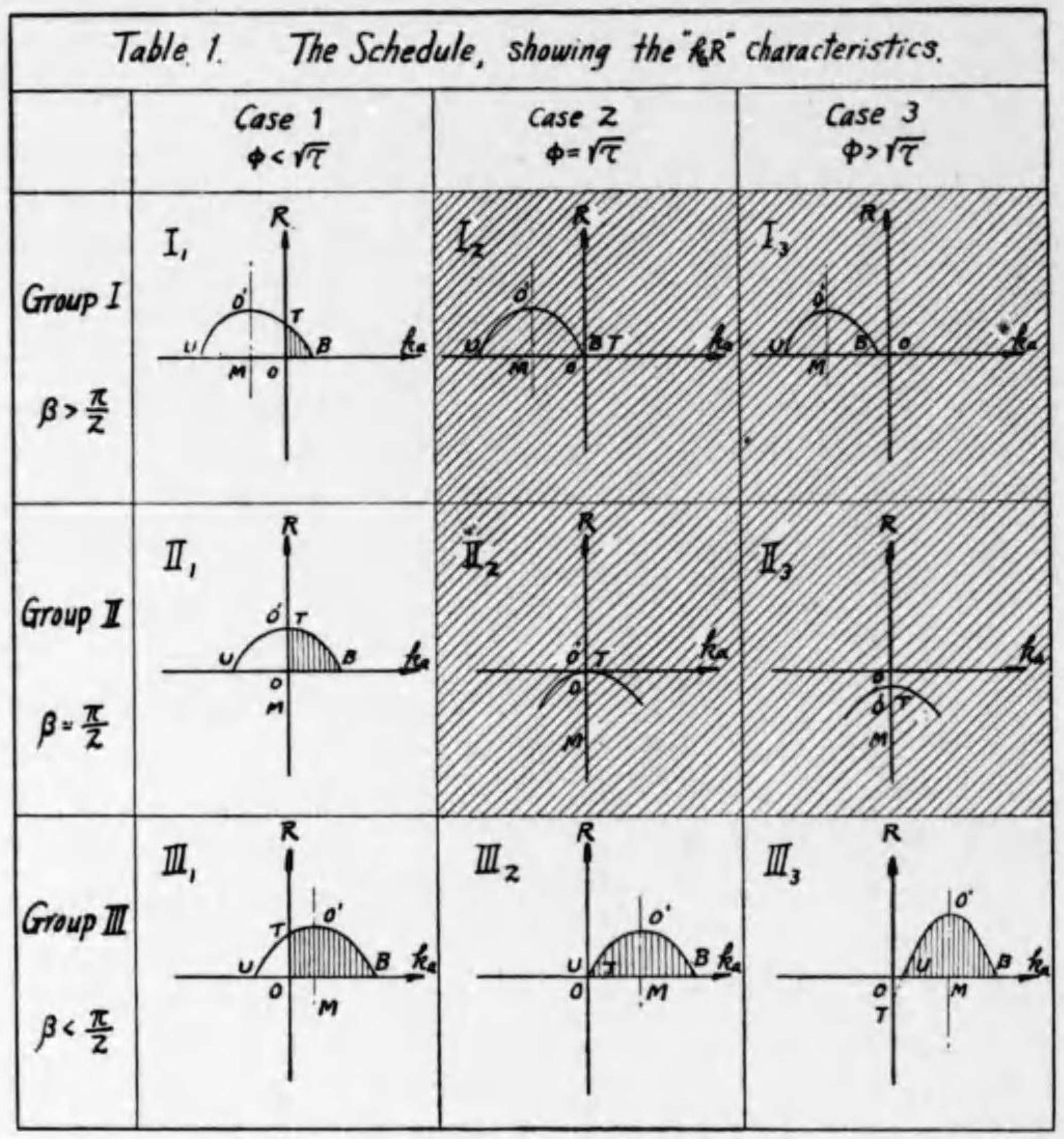
$$\overline{OU} = .325387 - .191609 = \underline{.134} = k'_{a_{II_r}}$$

(31), $\overline{UM} = \overline{MB} = \sin\beta\sqrt{\tau - \phi^2 \cdot \sin^2\beta}$

$$= .20791 \times \sqrt{.96-(1.6 \times .20791)^2} = \underline{.192}$$

Fig. 20 shows the "$k_a R$" curve of this example, and the values of k_a and R at the particular points are denoted in the brackets.

Table 1 illustrates schematically the principal characteristics of "$k_a R$" curves for nine types.



SECTION 7. CHARACTERISTICS OF "R" FOR "α" AND CLASSIFICATION OF REACTION TURBINES

a) Equation of "R"

In Fig. 10

$$k = \phi \frac{\sin\beta}{\sin(\alpha+\beta)}$$

then the equation (19) reduces to

$$R = \tau - \phi^2 \frac{\sin^2\beta}{\sin^2(\alpha+\beta)} \qquad (32)$$

(32) is the equation of "R" as a function of α, β, ϕ and τ. The value of τ is reasonably taken for every type, and β and ϕ are taken at the arbitrary values in their rages for every type. And since a turbine has one value of α, as mentioned in the chapter I, the values of R for all turbines of every type may be determined by the equation (32).

b) Inspection of Value of "R"

In the equation (32) $R = \tau - \phi^2 \frac{\sin^2\beta}{\sin^2(\alpha+\beta)}$

$$\phi^2 \frac{\sin^2\beta}{\sin^2(\alpha+\beta)} > 0 \qquad \text{always.}$$

To satisfy the condition (20) $\tau > R > 0$, $\phi^2 \frac{\sin^2\beta}{\sin^2(\alpha+\beta)}$ must be less than τ. Then (20) reduces to $\phi^2 \frac{\sin^2\beta}{\sin^2(\alpha+\beta)} < \tau$.

But $\quad 0 < \beta < \pi, \quad 0 < (\alpha+\beta) < \pi$ and $\phi > 0$,

or $\quad \sin\beta > 0, \quad \sin(\alpha+\beta) > 0$.

Hence the above condition reduces again to

$$\phi \frac{\sin\beta}{\sin(\alpha+\beta)} < \sqrt{\tau} \qquad \text{the condition (33)}$$

The condition (33) must be satisfied for the existence of reaction turbines.

For the group I, $\pi>\beta>\pi/2$

Since $\pi/2<\beta<\pi$, $\pi/2<(\alpha+\beta)<\pi$ and $\beta<(\alpha+\beta)$, $\sin\beta>\sin(\alpha+\beta)$

Hence $$1<\frac{\sin\beta}{\sin(\alpha+\beta)}$$

The type I_1, $\phi<\sqrt{\tau}$. The values of α and β are selected so that the condltion (33) may be satisfied.

The type I_2, $\phi=\sqrt{\tau}$,
The type I_3, $\phi>\sqrt{\tau}$, } The condition (33) is not satisfied.

For the group II, $\beta=\pi/2$

The condition (33) becomes

$$\phi\frac{1}{\cos\alpha}<\sqrt{\tau} \qquad \text{the condiction} \qquad (33)_{II}$$

$$\text{where} \quad 0>\frac{1}{\cos\alpha}>1$$

The type II_1, $\phi<\sqrt{\tau}$. By taking the value of α less than $\cos^{-1}\dfrac{\phi}{\sqrt{\tau}}$ the condition (33) may be satisfied,

The type II_2, $\phi=\sqrt{\tau}$,
The type II_3, $\phi>\sqrt{\tau}$. } The condition (33) is not satisfied.

For the group III, $\beta<\pi/2$

Since the values of α and β are easily selected so that

$$1>\frac{\sin\beta}{\sin(\alpha+\beta)},$$

the condition (33) may be satisfied for the types III_1, III_2 and III_3.

Thus is again proved the principle of Section 5, b) that the reaction turbines may exist in five types I_1, II_1, III_1, III_2 and III_3, and are classified into these five.

c) Limit Value of "$(\alpha+\beta)$" with Restriction of Reaction

$$(32) \qquad R=\tau-\phi^2\frac{\sin^2\beta}{\sin^2(\alpha+\beta)}$$

If in (32) $R=0$, $\sin^2(\alpha+\beta)=\dfrac{\phi^2}{\tau}\sin^2\beta$ or $\sin(\alpha+\beta)=\dfrac{\phi}{\sqrt{\tau}}\sin\beta$,

$$\text{where} \quad \frac{\phi}{\sqrt{\tau}}\sin\beta<1, \text{ since } 0<\sin(\alpha+\beta)<1$$

Let (β) : the value of $(\alpha+\beta)$ when $R=0$,

then from the above equation

$$(\beta)=\sin^{-1}\left\{\frac{\phi}{\sqrt{\tau}}\sin\beta\right\} \qquad (34)$$

There are two roots of (β) for $0<(\alpha+\beta)<\pi$

Let $(\beta)'$: the root of (β) less than $\pi/2$ and

$(\beta)''$: another root greater than $\pi/2$.

If $(\beta)'<(\alpha+\beta)<(\beta)''$,

$\sin(\alpha+\beta)>\dfrac{\phi}{\sqrt{\tau}}\sin\beta$ or $\sqrt{\tau}>\phi\dfrac{\sin\beta}{\sin(\alpha+\beta)}$, since $\sqrt{\tau}$ is taken as positive.

$$\text{or} \quad \tau>\phi^2\frac{\sin^2\beta}{\sin^2(\alpha+\beta)}$$

Then we have $\quad 0<R<\tau$

which is the condition (20). Hence (20) becomes

$$(\beta)'<(\alpha+\beta)<(\beta)'' \qquad \text{the condition (35).}$$

The conditien (35) must be satisfied for the existence of reaction turbines. And $(\beta)'$ and $(\beta)''$ may become respectively the lower and upper limits of $(\alpha+\beta)$ with the restriction of reaction.

Further from the condition (35) we have

$$\beta<(\beta)'' \text{ and } \beta \gtreqless (\beta)' \text{ for the reaction turbines} \qquad (35)'$$

The tables 2, 3 and 4 show the values of $(\beta)'$ and $(\beta)''$ compared to β for all types.

TABLE 2. Group I, $\pi/2<\beta<\pi$

types	sin(β)	(β)'	(β)''
I_1 $\phi<\sqrt{\tau}$	$\sin(\beta)<\sin\beta$	$(\beta)'<\beta$ but $(\beta)'<(\pi-\beta)$	$(\beta)''>\beta$
I_2 $\phi=\sqrt{\tau}$	$\sin(\beta)=\sin\beta$	$(\beta)'<\beta$ but $(\beta)'=(\pi-\beta)$	$(\beta)''=\beta$ [1]
I_3 $\phi>\sqrt{\tau}$	$\sin(\beta)>\sin\beta$	$(\beta)'<\beta$ but $(\beta)'>(\pi-\beta)$	$(\beta)''<\beta$ [2]

TABLE 3, Group II, $\beta=\pi/2$

types	sin(β)	(β)'	(β)''
II_1 $\phi<\sqrt{\tau}$	$\sin(\beta)<1$	$(\beta)'<\pi/2$	$(\beta)''>\pi/2$
II_2 $\phi=\sqrt{\tau}$	$\sin(\beta)=1$	$(\beta)'=\beta=\pi/2$	$(\beta)''=\beta=\pi/2$ [3]
II_3 $\phi>\sqrt{\tau}$	$\sin(\beta)>1$ irrational		

TABLE 4. Group III, $0<\beta<\pi/2$

types	sin(β)	(β)'	(β)''
III_1 $\phi<\sqrt{\tau}$	$\sin(\beta)<\sin\beta$	$(\beta)'<\beta$	$(\beta)''>\beta$ but $(\beta)''>(\pi-\beta)$
III_2 $\phi=\sqrt{\tau}$	$\sin(\beta)=\sin\beta$	$(\beta)'=\beta$	$(\beta)''>\beta$ but $(\beta)''=(\pi-\beta)$
III_3 $\phi>\sqrt{\tau}$	$\sin(\beta)>\sin\beta$	$(\beta)'>\beta$	$(\beta)''>\beta$ but $(\beta)''<(\pi-\beta)$

1. 2. and 3. By the conditon (35)' these cases are not allowable for the reaction turbines.

d) Limit Values of "α" with Restriction of Reaction

The condition (35) is written as

$$(\beta)'-\beta < \alpha < (\beta)''-\beta \qquad (35)$$

(35) gives the range of α with the restriction of reaction. By the tables 2, 3 and 4, it is observed that

$$\left[(\beta)'-\beta\right] \begin{matrix} < \\ = \\ > \end{matrix} 0 \quad \begin{matrix} \text{for } I_1,\ II_1 \text{ and } III_1 \\ \text{for } III_2 \\ \text{for } III_3 \end{matrix}$$

and

$$\left[(\beta)''-\beta\right] > 0 \quad \text{for five types.}$$

But

$$0<\alpha<\pi$$

Hence (35) becomes as

$$0<\alpha<(\beta)''-\beta \text{ for } I_1,\ II_1,\ III_1 \text{ and } III_2$$
$$(\beta)'-\beta<\alpha<(\beta)''-\beta \text{ for } III_3$$

Let α_{lt_r} : the upper limit value, α'_{lt_r} : the lower limit value } of α restricted by the reaction,

then we have

$$\left.\begin{aligned} \alpha_{lt_r} &= (\beta)''-\beta \\ \alpha'_{lt_r} &= (\beta)'-\beta \quad \text{for only } III_3 \end{aligned}\right\} \qquad (36)$$

For the types I_1, II_1, III_1 and III_2 the value of α must be less than α_{lt_r}, and for the type III_3 this must be less than α_{lt_r} and greater than α'_{lt_r}.

Fig. 21 to Fig. 25 illustrate the mutual relation of τ, ϕ, β, $(\beta)'$, $(\beta)''$ and α_{lt_r} for five types. In these figures "the $\sqrt{\tau}$ - circle" has $\sqrt{\tau}$ as radius and has the point O as centre. The diagram OCA is the velocity diagram at the inlet edge of the runner in the limit of reaction or in $R=0$. In this limit $\overline{OA}=\sqrt{\tau}$. And if the angle of absolute velocity is greater than the angle COA, the absolute velocity may become greater than $\sqrt{\tau}$. Hence the upper limit of α is the angle COA, or $\alpha_{lt_r}=\angle COA$.

Now the line $\overline{CA}$ is prolonged and intersected with the $\sqrt{\tau}$ - circle at the point B. And the lines $\overline{AD}$ and $\overline{BE}$ are drawn parallel to $\overline{OC}$. Then we have

$\angle OAD = \angle COA = \alpha_{lt_r}$ and $\angle DAF = \angle OCA = \beta$,

$\angle OAF = \angle OAD + \angle DAF = \alpha_{\text{II}_r} + \beta$ but $\alpha_{\text{II}_r} + \beta = (\beta)''$ by (36)

hence $\angle OAF = (\beta)''$.

$\angle OAC = \pi - \angle OAF = \pi - (\beta)''$, but $\pi - (\beta)'' = (\beta)'$

hence $\angle OAC = (\beta)'$.

$\angle OBA = \angle OAC = (\beta)'$ and $\angle ABE = \angle OCA = \beta$.

$\angle OBE = \angle ABE - \angle ABO = \beta - (\beta)'$

For only the type III_3 OCB becomes also the velocity diagram at the limit : $R=0$, but this is the lower limit. Since the lower limit of α is the angle COB, we have $\angle OBE = \angle COB = \alpha'_{\text{II}_r}$.

e) Maximum Value of "R"

$$R = \tau - \phi^2 \frac{\sin^2\beta}{\sin^2(\alpha+\beta)}, \tag{32}$$

If in (32) τ, ϕ and β are given,

$$\frac{dR}{d\alpha} = 2\phi^2 \sin^2\beta \frac{\cos(\alpha+\beta)}{\sin^3(\alpha+\beta)}$$

$$\frac{d^2R}{d\alpha^2} = -2\phi^2 \sin^2\beta \frac{\sin^2(\alpha+\beta)+3\cos^2(\alpha+\beta)}{\sin^4(\alpha+\beta)} \quad \text{negative.}$$

Hence R becomes maximum, if $\frac{dR}{d\alpha} = 0$ or $\frac{\cos(\alpha+\beta)}{\sin^3(\alpha+\beta)} = 0$, in which the denominator is taken at the finite value.

Then we have

$$R_{\max.} = \tau - \phi^2 \sin^2\beta, \quad \text{if} \quad (\alpha+\beta) = \pi/2 \tag{37}$$

f) Values of "R" for $\alpha=0$ and $\alpha=\pi/2$

Let $R_{(\alpha:0)}$: the value of R when $\alpha = 0$,

$R_{(\alpha:\frac{\pi}{2})}$: the value of R when $\alpha = \pi/2$,

then from the equation (32) we have

$$R_{(\alpha:0)} = \tau - \phi^2 \tag{38}$$

$$R_{(\alpha:\frac{\pi}{2})} = \tau - \phi^2 \operatorname{tg}^2\beta \tag{39}$$

(38) corresponds to (26), which is the equation of R for $k_a=0$.

From the condition (35) we have

$$\left.\begin{aligned} &\text{for } R > 0, \quad (\beta)' < (\alpha+\beta) < (\beta)'' \\ &\text{for } R = 0, \ (\alpha+\beta) = (\beta)' \text{ and } (\alpha+\beta) = (\beta)'' \\ &\text{for } R < 0, \ (\alpha+\beta) < (\beta)' \text{ and } (\alpha+\beta) > (\beta)'' \end{aligned}\right\} \tag{40}$$

i) relation between the sign of $R_{(\alpha=0)}$ and β

For the case 1, $\phi < \sqrt{\tau}$, $\tau - \phi^2 > 0$.

$R_{(\alpha=0)} > 0$, from (38).

$(\beta)' < \beta < (\beta)''$, from (40) and for $\alpha = 0$.

For the case 2, $\phi = \sqrt{\tau}$, $\tau - \phi^2 = 0$.

$R_{(\alpha:0)} = 0$, from (38).

$(\beta)' = \beta$ and $\beta = (\beta)''$, from (40) and for $\alpha = 0$.

The latter value of β is not applicable for the reaction turbines by the condition (35)'.

For the case 3, $\phi > \sqrt{\tau}$, $\tau - \phi^2 < 0$.

$R_{(\alpha:0)} < 0$, from (38).

$\beta < (\beta)'$ and $\beta > (\beta)''$, from (40) and for $\alpha = 0$.

The latter value of β is not applicable for the reaction turbines by the condition (35)'.

ii) The relation between the sign of $R_{(\alpha:\frac{\pi}{2})}$ and β

Generally $0 < (\alpha+\beta) < \pi$

But since $\beta > \pi/2$ for the group I and $\beta = \pi/2$ for the group II, $(\alpha = \pi/2)$ does not occur for the groups I and II. Hence this item is considered only for the group III.

The group III, $\beta < \pi/2$

For $R>0$ the condition (35) or (40) reduces to

$$(\beta)' < (\pi/2+\beta) < (\beta)'' \quad \text{where } (\beta)' < \pi/2$$

Further the aboves reduce to

$$\pi/2 < (\beta)'' - \beta \quad \text{or} \quad \pi/2 > (\beta)' + \beta \tag{41}$$

But $(\beta)' + \beta = \{(\beta)' - \beta\} + 2\beta$. Then (41) becomes

$$\beta < \frac{\pi}{4} - \frac{(\beta)' - \beta}{2} \tag{42}$$

For $R = 0$, (40) reduces to

$$(\beta)' = (\pi/2+\beta) \text{ and } (\pi/2+\beta) = (\beta)'', \quad \text{where } (\beta)' < \pi/2$$

Further the aboves reduce to

$$\pi/2 = (\beta)''-\beta \quad \text{or} \quad \pi/2 = (\beta)'+\beta \tag{43}$$

$$\beta = \frac{\pi}{4} - \frac{(\beta)'-\beta}{2} \tag{44}$$

For $R<0$, (40) reduces to

$$(\pi/2+\beta) < (\beta)' \quad \text{and} \quad (\beta)'' < (\pi/2+\beta), \quad \text{where} \quad (\beta)'<\pi/2$$

Further the aboves reduce to

$$\pi/2 > (\beta)''-\beta \quad \text{or} \quad \pi/2 < (\beta)'+\beta \tag{45}$$

$$\beta > \frac{\pi}{4} - \frac{(\beta)'-\beta}{2} \tag{46}$$

The tables 5, 6 and 7 show the relations between the sign of R and β, when $\alpha = 0$ and $\alpha = \pi/2$.

TABLE 5. Group I, $\pi/2<\beta<\pi$.

$\alpha = 0$, $(\alpha+\beta) = \beta$, $R_{(\alpha:0)} = \tau-\phi^2$		
case 1	case 2	case 3
$\phi < \sqrt{\tau}$ $(\beta)''-\beta > 0$ $(\beta)'+\beta < \pi$	$\phi = \sqrt{\tau}$ $(\beta)''-\beta = 0$[1] $(\beta)'+\beta = \pi$[1]	$\phi > \sqrt{\tau}$ $(\beta)''-\beta < 0$[1] $(\beta)'+\beta > \pi$[1]
R ⊕	$R = 0$	R ⊖

TABLE 6. Group II, $\beta=\pi/2$.

$\alpha = 0$, $(\alpha+\beta) = \beta = \pi/2$, $R_{(\alpha:0)} = \tau-\phi^2$		
case 1	case 2	case 3
$\phi < \sqrt{\tau}$ $(\beta)' < \pi/2$ $(\beta)'' > \pi/2$	$\phi = \sqrt{\tau}$ $(\beta)' = \pi/2$[1] $(\beta)'' = \pi/2$[1]	$\phi > \sqrt{\tau}$ $(\beta)' > \pi/2$[1] $(\beta)'' < \pi/2$[1]
R ⊕	$R = 0$	R ⊖

1. These cases do not occur in reaction turbines by the condition (35)'.

TABLE 7. Group III, $0<\beta<\pi/2$

$\alpha = 0$, $(\alpha+\beta) = \beta$, $R_{(\alpha:0)} = \tau-\phi^2$		
case 1	case 2	case 3
$\phi < \sqrt{\tau}$ $(\beta)'-\beta < 0$ $(\beta)''+\beta > \pi$	$\phi = \sqrt{\tau}$ $(\beta)'-\beta = 0$ $(\beta)''+\beta = \pi$	$\phi > \sqrt{\tau}$ $(\beta)'-\beta > 0$ $(\beta)''+\beta < \pi$
R ⊕	$R = 0$	R ⊖

$\alpha = \pi/2$, $(\alpha+\beta) = \pi/2+\beta$, $R_{(\alpha:\frac{\pi}{2})} = \tau-\phi^2 tg^2\beta$				
		case 1 $\phi < \sqrt{\tau}$ $(\beta)' < \beta$	case 2 $\phi = \sqrt{\tau}$ $(\beta)' = \beta$	case 3 $\phi > \sqrt{\tau}$ $(\beta)' > \beta$
		$\beta < \frac{\pi}{4} - \frac{(\beta)'-\beta}{2}$	$\beta < \frac{\pi}{4}$	$\beta < \frac{\pi}{4} - \frac{(\beta)'-\beta}{2}$
$(\beta)''-\beta > \frac{\pi}{2}$	$(\beta)'+\beta < \frac{\pi}{2}$	R ⊕	R ⊕	R ⊕
		$\beta = \frac{\pi}{4} - \frac{(\beta)'-\beta}{2}$	$\beta = \frac{\pi}{4}$	$\beta = \frac{\pi}{4} - \frac{(\beta)'-\beta}{2}$
$(\beta)''-\beta = \frac{\pi}{2}$	$(\beta)'+\beta = \frac{\pi}{2}$	$R = 0$	$R = 0$	$R = 0$
		$\beta > \frac{\pi}{4} - \frac{(\beta)'-\beta}{2}$	$\beta > \frac{\pi}{4}$	$\beta > \frac{\pi}{4} - \frac{(\beta)'-\beta}{2}$
$(\beta)''-\beta < \frac{\pi}{2}$	$(\beta)'+\beta > \frac{\pi}{2}$	R ⊖	R ⊖	R ⊖

SECTION 8. "αR" CURVES

The equation (32) $\tau - R = \phi^2 \dfrac{\sin^2\beta}{\sin^2(\alpha+\beta)}$

If τ, ϕ and β are given, (32) may become the equation of curve in the coordinates with $\overline{O\alpha}$ as the axis of abscissa and with $\overline{OR}$ as that of ordinate. This curve is called the "αR" curve. To plot the "αR" curve the auxiliary curves may be used which are

the auxiliary curve (i)	$y'' = \sin x'$, where $x' = (\alpha+\beta)$	a sine curve,
the auxiliary curve (ii)	$y''^2 = x''$,	a parabola with 1 as parameter,
the auxiliary curve (iii)	$x''y' = \phi^2 \sin^2\beta$	a rectangular hyperbola,
the auxiliary curve (iv)	$y' = \phi^2 \sin^2\beta \dfrac{1}{\sin^2 x'}$.	

The equation of (iv) becomes $y' = \phi^2 \dfrac{\sin^2\beta}{\sin^2(\alpha+\beta)}$,

and by the equation (32) we have $y' = \tau - R$. Hence the auxiliary curve (iv) may become the $(\alpha+\beta)$ $(\tau - R)$ curve.

In Fig. 26 there are five rectangular coordinates, and
the auxiliary curve (i) is plotted in the coordinates (x', y''),
the auxiliary aurve (ii) ,, ,, in the coordinates (x'', y''),
the auxiliary aurve (iii) ,, ,, in the coordinates (x'', y'),
the auxiliary aurve (iv) ,, ,, in the coordinates (x', y'),
and the "αR" curve ,, ,, in the coordinates (α, R).

For the sake of abbreviation, the auxiliary curves (i), (ii), (iii) and (iv) are called the curves (i), (ii), (iii) and (iv) respectively and the coordinates (x', y''), (x'', y''), (x'', y') and (x', y') are called (i), (ii), (iii) and (iv) respectively. The axes of abscissas of (i) and (ii) are taken at a straigth line $\overline{O_I O_{II}}$, and those of (iii) and (iv) at a straight line $\overline{O_{III} O_{IV}}$. The axes of ordinates of (i) and (iv) are taken at a straight line $\overline{O_I O_{IV}}$, and those of (ii) and (iii) at a straight line $\overline{O_{II} O_{III}}$.

In (i), $y'' = \sin x'$ or $y'' = \sin(\alpha+\beta)$
A point P_1 has $(\alpha+\beta)$ as the abscissa and
$\sin(\alpha+\beta)$ as the ordinate.

In (ii), $y''^2 = x''$, and a point P_2 has the same ordinate as P_1.
Hence P_2 has $\sin^2(\alpha+\beta)$ as the abscissa and
$\sin(\alpha+\beta)$ as the ordinate.

In (iii), $x''y' = \phi^2 \sin^2\beta$ or $y' = \phi^2 \dfrac{\sin^2\beta}{x''}$, and a point P_3 has the same abscissa as P_2.
Hence P_3 has $\sin^2(\alpha+\beta)$ as the abscissa and
$\phi^2 \dfrac{\sin^2\beta}{\sin^2(\alpha+\beta)}$ as the ordinate.

In (iv) $y' = \phi^2 \sin^2\beta \dfrac{1}{\sin^2 x'}$, and a point P has the same abscissa as P_1 and has the same ordinate as P_3.
Hence P has $(\alpha+\beta)$ as the abscissa and
$\phi^2 \dfrac{\sin^2\beta}{\sin^2(\alpha+\beta)}$ or $(\tau - R)$ as the ordinate.

A straight line $\overline{OB}$ is drawn parallel to the axis $\overline{O_{IV} x'}$ at the distance of τ, and let a point P' be the intersection of $\overline{P_1 P}$ with $\overline{OB}$.

Then $\overline{PP'} = \tau - \phi^2 \dfrac{\sin^2\beta}{\sin^2(\alpha+\beta)}$

$= R$, by the equation (32).

The "αR" curve.

In the coordinates (α, R) the line $\overline{OB}$ is taken as the axis of abscissa and the axis $\overline{OR}$ is drawn parallel to $\overline{O_{IV} y'}$ at the distance β, but $\overline{OR}$ is in the opposite direction against $\overline{O_{IV} y'}$.
Then a point P may have α as the abscissa and
R as the ordinate.
Other points are similarly plotted, and a curve is determined, then this curve (iv) may become the "αR" curve in the coordinates (α, R).

Fig. 27 to Fig. 35 illustrate the "αR" curves for nine types. The characteristics of the "αR" curve in these figures are similar as that of the

"$k_a R$" curves. The "R-portion" disappears in the "αR" curves of the types I_2, I_3, II_2 and II_3, hence no reaction turbines exist in these types. Since the "R-portion" appears in five types I_1, II_1, III_1, III_2 and III_3, the reaction turbines may exist in these types. Further the particular points with regard to α, β and R are denoted with the notations $(\beta)'$, $(\beta)''$, $(\tau-\phi^2)$, $(\tau-\phi^2 \operatorname{tg}^2\beta)$ etc, in these figures.

The example 6. The "αR" curve of a turbine series in the type I_1 with $\tau=.93$, $\phi=.56$ and $\beta=145°$, which are taken at the same values as those of the example 1.

(34), $(\beta) = \sin^{-1}\left(\frac{\phi}{\sqrt{\tau}}\sin\beta\right)$

$(\beta)' = \sin^{-1}\left(\frac{.56\times.57358}{.96437}\right) = \sin^{-1}.33307 = \underline{19°27.3'}$

$(\beta)'' = \pi-(\beta)' = 180°-19°27.3' = \underline{160°32.7'}$

(37), $R_{max.} = \tau-\phi^2\sin^2\beta$, if $(\alpha+\beta)=\pi/2$

$\alpha_{(Rmax)} = \pi/2-\beta = 90°-145° = \underline{-55°}$

$R_{max.} = \tau-\phi^2\sin^2\beta = \underline{.827}$ by the example 1.

(38), if $\alpha=o$, $R_{(\alpha:o)} = \tau-\phi^2$

$R_{(\alpha:o)} = \tau-\phi^2 = \underline{.616}$ by the example 1.

(40), if $R=0$, $(\alpha+\beta)=(\beta)$ or $\alpha_{(R:0)} = (\beta)''-\beta$ and $(\beta)'-\beta$

$\overline{OB} = (\beta)''-\beta = 160°32.7'-145° = \underline{15°32.7'} = \alpha_{R_r}$

$\overline{OU} = (\beta)'-\beta = 19°27.3' - 145° = \underline{-125°32.7'}$

Fig. 27 shows the "αR" curve of this example, and the values of R and α at the particular points are denoted in the brackets.

The example 7. The "αR" curve of a turbine series in the type II_1 with $\tau=.94$, $\phi=.640$ and $\beta=90°$, which are taken at the same values as those of the example 2.

(34), $(\beta) = \sin^{-1}\left(\frac{\phi}{\sqrt{\tau}}\sin\beta\right) = \sin^{-1}\frac{\phi}{\sqrt{\tau}}$, for $\sin\beta=1$

$(\beta)' = \sin^{-1}\left(\frac{.640}{.96954}\right) = \sin^{-1}.66043 = \underline{41°20'}$

$(\beta)'' = \pi-(\beta)' = 180°-41°20' = \underline{138°40'}$

(37), if $(\alpha+\beta)=\pi/2$, $R_{max.} = \tau-\phi^2\sin^2\beta = \tau-\phi^2$, for $\sin\beta=1$

$\alpha_{(Rmax)} = \pi/2-\beta = 90°-90° = \underline{0}$

$R_{max.} = \tau-\phi^2 = \underline{.53}$, by the example 2.

(38), if $\alpha=0$, $R_{(\alpha:0)} = \tau-\phi^2$

$R_{(\alpha:0)} = \tau-\phi^2 = R_{max.} = \underline{.53}$ as the above

(40), if $R=0$, $(\alpha+\beta)=(\beta)$ or $\alpha_{(R:0)} = (\beta)''-\beta$ and $(\beta)'-\beta$

$\overline{OB} = (\beta)''-\beta = 138°40'-90° = \underline{48°40'} = \alpha_{R_r}$

$\overline{OU} = (\beta)'-\beta = 41°20' - 90° = \underline{-48°40'}$

Fig. 30 shows the "αR" curve of this example, and the values of R and α at the particular points are denoted in the brackets.

The example 8. The "αR" curve of a turbine series in the type III_1 with $\tau=.95$, $\phi=.80$ and $\beta=40°$, which are taken at the same values as those of the example 3.

(34), $(\beta) = \sin^{-1}\left(\frac{\phi}{\sqrt{\tau}}\sin\beta\right)$

$(\beta)' = \sin^{-1}\left(\frac{.8\times.64279}{.97468}\right) = \sin^{-1}.52759 = \underline{31°50.5'}$

$(\beta)'' = \pi-(\beta)' = 180°-31°50.5' = \underline{148°9.5'}$

(37), if $(\alpha+\beta)=\pi/2$, $R_{max.} = \tau-\phi^2\sin^2\beta$

$\alpha_{(Rmax)} = \pi/2-\beta = 90°-40° = \underline{50°}$

$R_{max.} = \tau-\phi^2\sin^2\beta = \underline{.686}$ by the example 3.

(38), if $\alpha=0$, $R_{(\alpha:0)} = \tau-\phi^2$

$R_{(\alpha:0)} = \tau-\phi^2 = ,31$ by the example 3.

(40) if $R=0$, $(\alpha+\beta)=(\beta)$ or $\alpha_{(R:0)} = (\beta)''-\beta$ and $(\beta)'-\beta$

$\overline{OB} = (\beta)''-\beta = 148°9.5'-40° = \underline{108°9.5'} = \alpha_{R_r}$

$\overline{OU} = (\beta)'-\beta = 31°50.5'-40° = \underline{-8°9.5'}$

Fig. 33 shows the "αR" curve of this example, and the values of R and α at the particular points are denoted in the brackets.

The eaxmple 9. The "αR" curve of a turbine series in the type III_3 with $\tau=.95$, $\phi=.975$ and $\beta=25°$, which are taken at the same values as those of the example 4.

(34), $(\beta) = \sin^{-1}\left(\frac{\phi}{\sqrt{\tau}}\sin\beta\right) = \sin^{-1}\sin\beta = \beta$, for $\phi = \sqrt{\tau}$

$(\beta)' = \beta = \underline{25^\circ}$

$(\beta)'' = \pi-(\beta)' = 180^\circ-25^\circ = \underline{155^\circ}$

(37), if $(\alpha+\beta) = \pi/2$, $R_{max.} = \tau - \phi^2 \sin^2\beta = \tau \cos^2\beta$, for $\tau = \phi^2$

$\alpha_{(R max)} = \pi/2 - \beta = 90^\circ-25^\circ = \underline{65^\circ}$

$R_{max.} = \tau\cos^2\beta = \underline{.780}$ by the example 4.

(38), if $\alpha = 0$, $R_{(\alpha:0)} = \tau-\phi^2$

$R_{(\alpha:0)} = \tau-\phi^2 = \underline{0}$

(40), if $R = 0$, $(\alpha+\beta) = (\beta)$ or $\alpha_{(R:0)} = (\beta)''-\beta$ and $(\beta)'-\beta$

$\overline{OB} = (\beta)''-\beta = 155^\circ-25^\circ = \underline{130^\circ} = \alpha_{n_r}$

$\overline{OU} = (\beta)'-\beta = 25^\circ-25^\circ = \underline{0^\circ}$

Fig. 34 shows the "αR" curve of this example, and the values of R and α at the particular points are denoted in the brackets.

The example 10. The "αR" curve of a turbine series in the type III_3 with $\tau = .96$, $\phi = 1.6$ and $\beta = 12^\circ$, which are taken at the same values as those of the example 5.

(34), $(\beta) = \sin^{-1}\left(\frac{\phi}{\sqrt{\tau}}\sin\beta\right)$

$(\beta)' = \sin^{-1}\left(\frac{1.6\times.20791}{.97980}\right) = \sin^{-1}.33951 = \underline{19^\circ 50.8'}$

$(\beta)'' = \pi-(\beta)' = 180^\circ-19^\circ 50.8' = \underline{160^\circ 9.2'}$

(37), if $(\alpha+\beta) = \pi/2$, $R_{max.} = \tau-\phi^2\sin^2\beta$

$\alpha_{(R max)} = \pi/2-\beta = 90^\circ-12^\circ = \underline{78^\circ}$

$R_{max} = \tau-\phi^2\sin^2\beta = \underline{.850}$ by the example 5.

(38), if $\alpha=0$, $R_{(\alpha:0)} = \tau-\phi^2$

$R_{(\alpha:0)} = \tau-\phi^2 = \underline{-1.6}$ by the example 5.

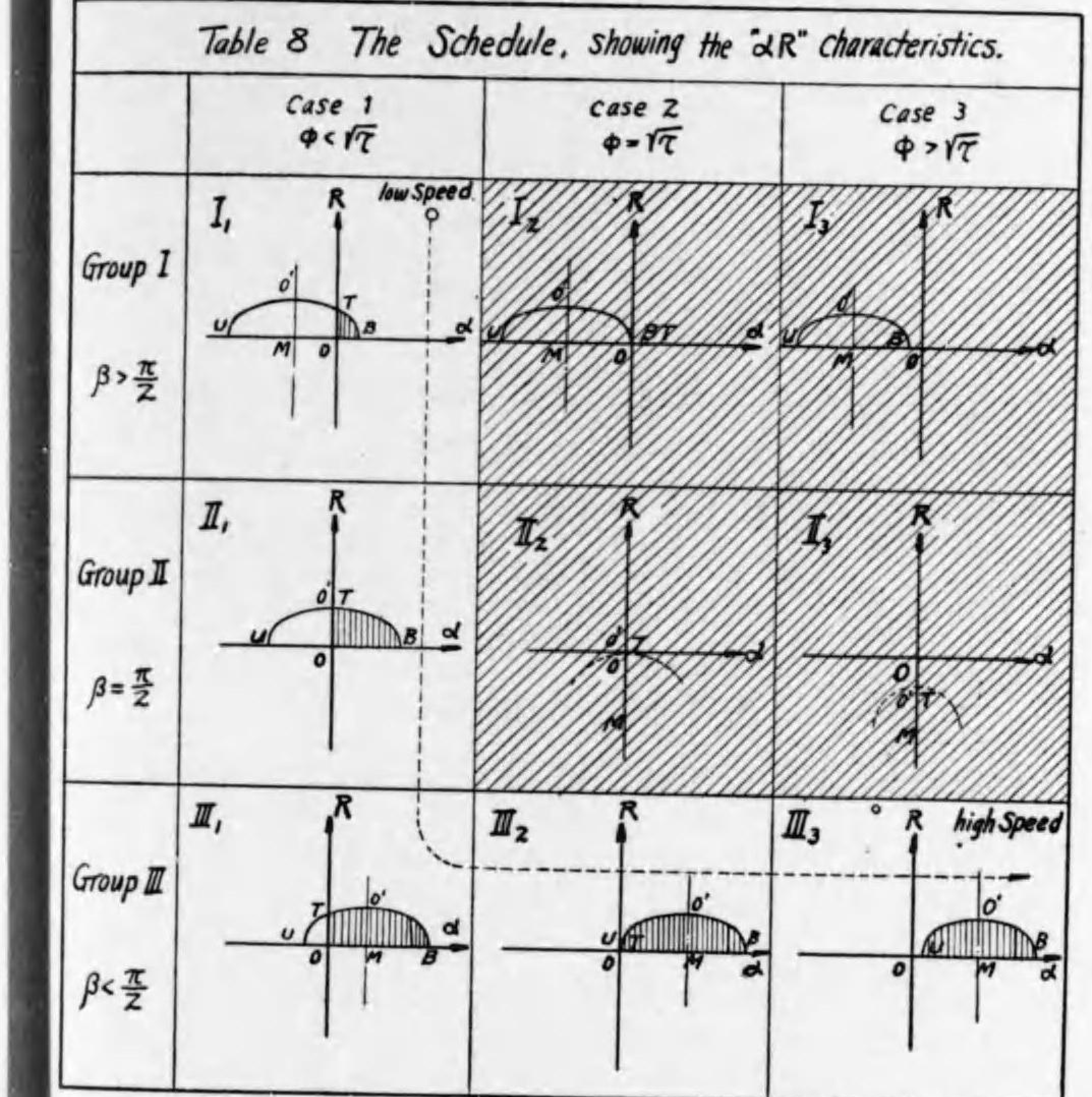
(40), if $R = 0$, $(\alpha+\beta) = (\beta)$ or $\alpha_{(R:0)} = (\beta)''-\beta$ and $(\beta)'-\beta$

$\overline{OB} = (\beta)''-\beta = 160^\circ 9.2'-12^\circ = \underline{148^\circ 9.2'} = \alpha_{I_{R_r}}$

$\overline{OU} = (\beta)'-\beta = 19^\circ 50.8'-12^\circ = \underline{7^\circ 50.8'} = \alpha'_{R_r}$

Fig. 35 shows the "αR" curve of this example, and the values of R and α at the particular points are denoted in the brackets.

The table 8 illustrates schematically the principal characteristics of the "αR" curves for nine types.



CHAPTER III. CHARACTERISTICS OF REACTION TURBINES WITH NORMAL EXIT

SECTION 9. RELATION BETWEEN "α" AND "β" IN STATE OF NORMAL EXIT

The equation (11) reduces to

$$u \cdot w \cdot \cos\alpha - u_2 \cdot w_2 \cdot \cos\alpha_2 = \eta \cdot g \cdot H \tag{47}$$

(47) is the well known fundamental equation of water turbines. Let η_l be the hydraulic efficiency of a turbine with the normal exit, then for the turbines with the normal exit (47) reduces to

$$\left.\begin{aligned} &u\,w\,\cos\alpha = \eta_l\,g\,H, \text{ since } \alpha_2 = \pi/2 \\ \text{or} \quad &\phi\,k\,\cos\alpha = \frac{\eta_l}{2} \end{aligned}\right\} \tag{48}$$

Fig. 36 shows the velocity diagrams in the state of normal exit. In the diagram at the inlet edge of runner

$$\frac{\phi}{k} = \frac{\sin(\alpha+\beta)}{\sin\beta} \tag{49}$$

If k is eliminated from the equation (48) and (49), we have

$$\phi = \sqrt{\frac{\eta_l}{2}}\sqrt{1+\frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta}} \tag{50}$$

(50) is the equation of ϕ as a function of α, β and η_l. (50) becomes

$$\mathrm{tg}\alpha = \left(\frac{2\phi^2}{\eta_l} - 1\right) \cdot \mathrm{tg}\beta \tag{51}$$

The equation (51) may give the relation between α and β for the turbines with normal exit.

SECTION 10. CLASSIFICATION OF REACTION TURBINES BY VALUE OF "φ" IN STATE OF NORMAL EXIT

The reaction turbines are usually classified by the value of β. For the turbines with the normal exit this classification may become as that by the value of ϕ with regard to η_l.

If in the equation (48) $\quad \phi\,k\,\cos\alpha = \dfrac{\eta_l}{2}$

α is equall to $\pi/2$, the hydraulic efficiency may become 0, and if α is greater than $\pi/2$, ϕ may change the direction. Hence $\alpha < \pi/2$ for the turbines with normal exit.

In the equation (50) $\quad \phi = \sqrt{\dfrac{\eta_l}{2}}\sqrt{1+\dfrac{\mathrm{tg}\alpha}{\mathrm{tg}\beta}}$

$$\alpha < \pi/2, \quad 0 < \beta < \pi, \quad 0 < (\alpha+\beta) < \pi$$

or $\quad \cos\alpha > 0, \quad \sin\beta > 0, \quad \sin(\alpha+\beta) > 0.$

then $\quad \dfrac{\sin(\alpha+\beta)}{\sin\beta\,\cos\alpha} > 0, \quad$ but $\quad \dfrac{\sin(\alpha+\beta)}{\sin\beta\,\cos\alpha} = 1+\dfrac{\mathrm{tg}\alpha}{\mathrm{tg}\beta},$

hence $\quad 1+\dfrac{\mathrm{tg}\alpha}{\mathrm{tg}\beta} > 0$.

If in the equation (50) η_l is given, the value of ϕ may change as β changes, as

$$\left.\begin{aligned} &\text{For the group I,} && \frac{\pi}{2} < \beta < \pi, && \mathrm{tg}\beta < 0, && \frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta} < 0, && \phi < \sqrt{\frac{\eta_l}{2}} \\ &\text{For the group II,} && \beta = \frac{\pi}{2} && \mathrm{tg}\beta = \infty, && \frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta} = 0, && \phi = \sqrt{\frac{\eta_l}{2}} \\ &\text{For the group III,} && 0 < \beta < \frac{\pi}{2}, && \mathrm{tg}\beta > 0, && \frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta} > 0, && \phi > \sqrt{\frac{\eta_l}{2}} \end{aligned}\right\} \tag{52}$$

(52) is also well known, which gives the relation between ϕ and η_l for the turbines with the normal exit.

From the equations (16) and (10)

for the normal exit $\quad \tau = (1-\xi_{g_l})+(k_{2_l}^2-k_{4_l}^2-\xi_{d_l})$ (53)

and $\quad \eta_l = 1-\xi_l-k_{4_l}^2$ (54)

or $\quad \eta_l = 1-(\xi_{g_l}+\xi_{r_l}+\xi_{d_l})-k_{4_l}^2$

then $\quad \eta_l = \tau-(\xi_{r_l}+\xi_{2_l}^2)$ (55)

where the suffix "l" is used to indicate the values of some notations for the normal exit.

Hence $\quad \eta_l < \tau$ and $\sqrt{\frac{\eta_l}{2}} < \sqrt{\tau}$ always (56)

The following table gives the values of "ϕ" compared with $\sqrt{\eta_l/2}$ and $\sqrt{\tau}$ for nine types.

	case 1	case 2	case 3
group I $\beta > \pi/2$	type I_1 $\sqrt{\frac{\eta_l}{2}} > \phi < \sqrt{\tau}$ low speed	type I_2 $\sqrt{\frac{\eta_l}{2}} > \phi = \sqrt{\tau}$ impossible	type I_3 $\sqrt{\frac{\eta_l}{2}} > \phi > \sqrt{\tau}$ impossible
group II $\beta = \pi/2$	type II_1 $\sqrt{\frac{\eta_l}{2}} = \phi < \sqrt{\tau}$	type II_2 $\sqrt{\frac{\eta_l}{2}} = \phi = \sqrt{\tau}$ impossible	type II_3 $\sqrt{\frac{\eta_l}{2}} = \phi > \sqrt{\tau}$ impossible
group III $\beta < \pi/2$	type III_1 $\sqrt{\frac{\eta_l}{2}} < \phi < \sqrt{\tau}$	type III_2 $\sqrt{\frac{\eta_l}{2}} < \phi = \sqrt{\tau}$	type III_3 $\sqrt{\frac{\eta_l}{2}} < \phi > \sqrt{\tau}$ →high speed

In the above table the values of "ϕ" are not rational for the types I_2, I_3, II_2 and II_3, and accordingly there are no turbines existent in these types. For this reason, the reaction turbines with the normal exit

may also be classified into five types I_1, II_1, III_1, III_2 and III_3 by the value of "ϕ", as by the restriction of reaction in the general case. This is of course necessary, since the turbines with the normal exit are included in every type of the general case.

The example 11. The values of ϕ for nine types in the normal exit with $\tau = .94$ and $\eta_l = .82$.

$$\sqrt{\tau} = \sqrt{.94} = .96954 \quad \text{or} \quad \underline{.97} \quad \text{say}$$

$$\sqrt{\frac{\eta_l}{2}} = \sqrt{.41} = .64031 \quad \text{or} \quad \underline{.64} \quad \text{say}$$

The values of ϕ for nine types are given in the following table.

	case 1	case 2	case 3
group I	I_1 $\phi < .64$	I_2 $.64 > \phi = .97$ impossible	I_3 $.64 > \phi > .97$ impossible
group II	II_1 $\phi = .64$	II_2 $.64 = \phi = .97$ impossible	II_3 $.64 = \phi > .97$ impossible
group III	III_1 $.64 < \phi < .97$	III_2 $\phi = .97$	III_3 $\phi > .97$

SECTION 11. LIMIT VALUES OF "α", "β" AND "k_a" WITH RESTRICTION OF REACTION IN STATE OF NORMAL EXIT

a) Limit Values of "α"

The equation (48) becomes

$$\cos\alpha = \frac{\eta_l}{2\phi k} \tag{57}$$

where α is taken at (0 to $\pi/2$) for the normal exit.

If in (57) ϕ and η_l are given, α may increase according as k increases. For the restriction of reaction, however, k can not increase without limit. Since the limit value of k is $\sqrt{\tau}$, α may have also the limit value, which corresponds to $\sqrt{\tau}$.

Let α_{lt} : the limit value of α restricted by the reaction in the state of normal exit.

then (57) reduces to

$$\cos\alpha_{lt} = \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} \tag{58}$$

$$\sin\alpha_{lt} = \frac{\sqrt{4\mathcal{T}\phi^2-\eta_t^2}}{2\phi\sqrt{\mathcal{T}}} \tag{59}$$

$$\left.\begin{aligned} \mathrm{tg}\alpha_{lt} &= \frac{\sqrt{4\mathcal{T}\phi^2-\eta_t^2}}{\eta_t} \\ \alpha &< \alpha_{lt} \end{aligned}\right\} \tag{60}$$

The value of α must be less than α_{lt} for the restriction of reaction in the state of normal exit.

b) Limit Values of "β"

Let β_{lt} : the value of β corresponding to α_{lt},

then (51) becomes $\quad \mathrm{tg}\beta_{lt} = \frac{\eta_t}{2\phi^2-\eta_t}\,\mathrm{tg}\alpha_{lt}$

the above reduces to

$$\mathrm{tg}\beta_{lt} = \frac{\sqrt{4\mathcal{T}\phi^2-\eta_t^2}}{2\phi^2-\eta_t}$$

α must be less than α_{lt} by the restriction of reaction, and β must hold the relation of

$$(51) \qquad \mathrm{tg}\beta = \frac{\eta_t}{2\phi^2-\eta_t}\,\mathrm{tg}\alpha$$

against α.

For the group I, $\quad \beta>\frac{\pi}{2},\ \phi<\sqrt{\frac{\eta_t}{2}}$ by (52)

then $\qquad \frac{\eta_t}{2\phi^2-\eta_t} < 0$ in (51)

According as α decreases, $\mathrm{tg}\alpha$ decreases, $\mathrm{tg}\beta$ increases and β increases. In order that α is less than α_{lt}, β must be larger than β_{lt}. Hence β_{lt} may become the limit value of β.

For the group III, $\beta<\frac{\pi}{2},\ \phi>\sqrt{\frac{\eta_t}{2}}$ by (52)

then $\qquad \frac{\eta_t}{2\phi^2-\eta_t} > 0$ in (51)

According as α decreases, also β decreases. In order that α is less than α_{lt}, β must also be less than β_{lt}. Hence β_{lt} may be the limit value of β.

For these reasons, β_{lt} may become the limit for both groups, i. e. the lower or upper limit for the group I or III.

$$\left.\begin{aligned} \mathrm{tg}\beta_{lt} &= \frac{\sqrt{4\mathcal{T}\phi^2-\eta_t^2}}{2\phi^2-\eta_t} \\ \beta &> \beta_{lt} \quad \text{for the group I,} \\ \beta &< \beta_{lt} \quad \text{for the group III.} \end{aligned}\right\} \tag{61}$$

The example 12. The limit values of α and β for the type III_2.

In this case, $\phi^2=\mathcal{T}$, $\beta<\pi/2$, $\mathrm{tg}\beta>0$.

$$(61) \quad \mathrm{tg}\beta_{lt} = \frac{\sqrt{4\mathcal{T}\phi^2-\eta_t^2}}{2\phi^2-\eta_t} = \frac{\sqrt{4\mathcal{T}^2-\eta_t^2}}{2\mathcal{T}-\eta_t} = \sqrt{\frac{2\mathcal{T}+\eta_t}{2\mathcal{T}-\eta_t}}$$

$$(60) \quad \mathrm{tg}\alpha_{lt} = \frac{\sqrt{4\mathcal{T}\phi^2-\eta_t^2}}{\eta_t} = \frac{\sqrt{4\mathcal{T}^2-\eta_t^2}}{\eta_t} = \left(\frac{2\mathcal{T}}{\eta_t}-1\right)\sqrt{\frac{2\mathcal{T}+\eta_t}{2\mathcal{T}-\eta_t}}$$

If $\mathcal{T}=.95$ and $\eta_t=.82$,

$$\mathrm{tg}\beta_{lt} = \sqrt{\frac{1.9+.82}{1.9-.82}} = \underline{1.58698}$$

$$\beta_{lt} = \underline{57^\circ 47'}$$

$$\frac{2\mathcal{T}}{\eta_t}-1 = \frac{.95}{.41}-1 = \underline{1.30244}$$

$$\mathrm{tg}\alpha_{lt} = 1.30244\times\mathrm{tg}\beta_{lt} = 1.30244\times 1.58698 = \underline{2.06695}$$

$$\alpha_{lt} = \underline{64^\circ 26'}$$

For check, $\alpha_{lt} = (\beta)''-\beta_{lt} = \pi-(\beta)'-\beta_{lt} = \pi-2\beta_{lt}$

for $(\beta)'=\beta$ in the type III_2.

$$\alpha_{lt} = 180^\circ-2\times 57^\circ 47' = \underline{64^\circ 26'}$$

c) Limit Values of "k_a"

In Fig. 36. $k_a = k \sin\alpha$

by the equation (48) the above reduces to

$$k_a = \frac{\eta_l}{2\phi}\,\mathrm{tg}\alpha \tag{62}$$

If in (62) ϕ and η_l are given, k_a may decrease according as α decreases. In order that α may be less than α_{lt}, k_a must be also less than the particular value of k_a, which corresponds to α_{lt}. Hence this value may become the limit of k_a, which is denoted with the notation "$k_{a_{lt}}$".

In the limit, (62) becomes

$$k_{a_{lt}} = \frac{\eta_l}{2\phi}\,\mathrm{tg}\alpha_{lt},$$

by the equation (60) the above reduces to

$$\left.\begin{aligned} k_{a_{lt}} &= \sqrt{\tau - \frac{\eta_l^2}{4\phi^2}} \\ k_a &< k_{a_{lt}} \end{aligned}\right\} \tag{63}$$

SECTION 12. GRAPHICAL SOLUTION OF LIMIT VALUES OF "α" AND "β" WITH RESTRICTION OF REACTION IN STATE OF NORMAL EXIT

Fig. 37 shows the velocity diagrams in the state of normal exit.

Let ϕ_l' : the circumferential component of k,

ϕ_l'' : the circumferential component of ψ,

$= \phi - \phi_l'$

then. $k_a = \phi_l'\,\mathrm{tg}\alpha, \quad k_a = \phi_l''\,\mathrm{tg}\beta = (\phi - \phi_l')\cdot\mathrm{tg}\beta,$

hence

$$\frac{\phi}{\phi_l'} - 1 = \frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta} \tag{64}$$

But (51) $\mathrm{tg}\alpha = \left(\frac{2\phi^2}{\eta_l} - 1\right)\mathrm{tg}\beta$

hence we have

$$\phi_l' = \frac{\eta_l}{2\phi} \tag{65}$$

If in (65) ϕ and η_l are given, ϕ_l' may be fixed.

In Fig. 38 a line $\overline{LL'}$ is drawn parallel to the axis $\overline{Ok_a}$ at the distance ϕ_l', and $\overline{LL'}$ may intersect with the $\sqrt{\tau}$-circle at the point L and with the axis $\overline{O\phi}$ at the point L'. OCK_l is the velocity diagram at the inlet edge of runner for a turbine, and OCK_l' is that for another turbine which has ϕ and η_l at the same values as those of the former in the state of normal exit. However the velocity angles and the velocity coefficients of the second turbine may be different from those of the first, i. e.

for the first turbine α, β, k and ψ } and for the second turbine α', β', k' and ψ' }

But the circumferential component of k' becomes equal to that of k, because ϕ and η_l are taken at the same values for both turbines. Hence the point K_l' of the second turbine may drop on the line $\overline{LL'}$, in which the point K_l of the first is situated. In the state of normal exit a lot of turbines with the same values of ϕ and η_l but with the various values of α, β, k, and ψ may be imagined. For the similar reason as the second turbine, all points of these turbines may drop on the line $\overline{LL'}$.[1] And a turbine series with the same values of ϕ and η_l may consist of these turbines.

According as the value of α increases, the point K_l may approach to the intersection L along the line $\overline{LL'}$ and the value of k may increase. For the restriction of reaction, however, α can not increases without limit, and the value of k is limited at $\sqrt{\tau}$. Hence α has also the limit value, when k becomes $\sqrt{\tau}$. In the limit the point K_l coincides with the intersection L, and the diagram becomes OCL which corresponds to the limit turbines. Then we have

$\triangledown OCL$ = the velocity diagram of the limit turbine,

$\angle COL = \alpha_{lt}$

$\angle OCL = \beta_{lt}$

In the velocity diagram OCL,

1. see Camerer, Vorlesungen über Wasserkraftmaschinen, (1914), seite 259-279.

$$\cos\alpha_{1L} = \frac{\overline{OL'}}{\overline{OL}} = \frac{\phi_i'}{\sqrt{\tau}} = \frac{\eta_l}{2\phi\sqrt{\tau}}$$

which is the same as the equation (58).

$$\overline{L'C} = \overline{OC} - \overline{OL'} = \phi - \phi_i' = \phi - \frac{\eta_l}{2\phi} = \frac{2\phi^2 - \eta_l}{2\phi}$$

$$\overline{L'L} = \sqrt{\overline{OL}^2 - \overline{OL'}^2} = \sqrt{\tau - \phi_i'^2} = \frac{\sqrt{4\tau\phi^2 - \eta_l^2}}{2\phi}$$

$$\operatorname{tg}\beta_{1L} = \frac{\overline{LL'}}{\overline{L'C}} = \frac{\sqrt{4\tau\phi^2 - \eta_l^2}}{2\phi} \cdot \frac{2\phi}{2\phi^2 - \eta_l} = \frac{\sqrt{4\tau\phi^2 - \eta_l^2}}{2\phi^2 - \eta_l}$$

the above is the same as the equation (61).

Fig. 39 to Fig. 43 show the velocity diagrams at the inlet edges of runner for five types I_1, II_1, III_1, III_2 and III_3 in the state of normal exit.

The example 13. The values of ϕ_i' for five types with

$$\eta_l = .82 \quad \text{and} \quad \tau = .93 \quad \text{to} \quad .96$$

For I_1 $\begin{cases} \tau = .93 \\ \phi = .56 \end{cases}$ $\sqrt{\tau} = \underline{.964}$ $\quad \frac{\eta_l}{2\phi} = \frac{.41}{.56} = \underline{.732}$

For II_1 $\begin{cases} \tau = .94 \\ \phi = \sqrt{\frac{\eta_l}{2}} = .640, \end{cases}$ $\sqrt{\tau} = \underline{.970}$, $\quad \frac{\eta_l}{2\phi} = \sqrt{\frac{\eta_l}{2}} = \sqrt{.41} = \underline{.640}$

For III_1 $\begin{cases} \tau = .95 \\ \phi = .8 \end{cases}$ $\sqrt{\tau} = \underline{.975}$ $\quad \frac{\eta_l}{2\phi} = \frac{.41}{.8} = \underline{.513}$

For III_2 $\begin{cases} \tau = .95 \\ \phi = \sqrt{\tau} = \underline{.975}, \end{cases}$ $\sqrt{\tau} = \underline{.975}$ $\quad \frac{\eta_l}{2\phi} = \frac{.41}{.975} = \underline{.421}$

For III_3 $\begin{cases} \tau = .96 \\ \phi = 1.6 \end{cases}$ $\sqrt{\tau} = \underline{.980}$ $\quad \frac{\eta_l}{2\phi} = \frac{.41}{1.6} = \underline{.256}$

Fig. 39 to Fig. 43 show the diagrams of this example, and the values of ϕ, $\sqrt{\tau}$ and $(\eta_l/2\phi)$ are denoted in the bracktes.

SECTION 13. CHARACTERISTICS OF "R" FOR "k_a" IN STATE OF NORMAL EXIT

a) Equation of "R"

In Fig. 37 $\qquad k^2 = \phi_i'^2 + k_a^2$

but $\qquad$ (65) $\qquad \phi_i' = \frac{\eta_l}{2\phi}$

hence the former equation reduces to

$$k^2 = \left(\frac{\eta_l}{2\phi}\right)^2 + k_a^2 \qquad (66)$$

The equation (19) reduces to

$$R = \tau - \left(\frac{\eta_l}{2\phi}\right)^2 - k_a^2 \qquad (67)$$

(67) is the equation of R as a function of k_a, ϕ, η_l and τ. The value of τ is reasonably taken for every type, η_l is taken at the practical value, and ϕ is taken at the arbitrary value in its range for every type. And since a turbine has one value of k_a as mentioned in the chapter I, the values of R for all turbines of every type in the state of the normal exit may be determined by the equation (67).

b) Particular Values of "R"

$$(67) \qquad R = \tau - \left(\frac{\eta_l}{2\phi}\right)^2 - k_a^2$$

If in (67) τ, η_l and ϕ are given,

$$\frac{dR}{dk_a} = -2\,k_a \quad \text{and} \quad \frac{d^2R}{dk^2} = -2$$

hence if $dR/dk_a = 0$ or $k_a = 0$, R becomes maximum.

Then we have $\quad R_{max} = \tau - \left(\frac{\eta_l}{2\phi}\right)^2$, if $k_a = 0$ $\qquad$ (68)

By the equation (67)

$$R = 0, \ \text{if} \ k_a = \sqrt{\tau - \left(\frac{\eta_l}{2\phi}\right)^2} \qquad (69)$$

By the equations (68), (67) and (69), it is observed that
if k_a is zero, R may become maximum,
according as k_a increases, R may decrease,
if k_a becomes $\sqrt{\tau-(\eta_t/2\phi)^2}$, R may become zero,
and after k_a has increased above $\sqrt{\tau-(\eta_t/2\phi)^2}$, R may become negative and any reaction turbine may not exist. Thus $\sqrt{\tau-(\eta_t/2\phi)^2}$ may become the limit value of k_a. Then we have

$$k_{a_{lt}} = \sqrt{\tau-\left(\frac{\eta_t}{2\phi}\right)^2} \qquad (70)$$

(70) is the same as the equation (63).

If in the equation (68)

$R_{max} \gtreqless 0$, $\tau-(\eta_t/2\phi)^2 \gtreqless 0$ or $\phi \gtreqless (\eta_t/2\sqrt{\tau})$, R may not become positive for any positive value of k_a. In order that R is positive, therefore, R_{max} must be positive. Then we have

$$\phi > \frac{\eta_t}{2\sqrt{\tau}} \quad \text{for the reaction turbines with the normal exit.} \qquad (71)$$

Further it is of course necessary that R_{max} must be the rational value.

The example 14. The smallest value of ϕ, with which the reaction turbines may exist, having $\eta_t = .82$ and $\tau = .92$.

$$\frac{\eta_t}{2\sqrt{\tau}} = \frac{.41}{\sqrt{.92}} = \frac{.41}{.95917} = \underline{0.427}$$

(71) $\phi > 0.427$ or $\underline{0.43}$ say, for the reaction turbines with the normal exit, η_t : $\underline{.82}$ and τ : $\underline{.92}$.

Now the existence of reaction turbine is inspected by the value of $\{\tau-(\eta_t/2\phi)^2\}$

For the type II_1, $\phi = \sqrt{(\eta_t/2)}$, then $\tau-\left(\frac{\eta_t}{2\phi}\right)^2$ becomes $\tau-\frac{\eta_t}{2}$.

For the type III_2, $\phi = \sqrt{\tau}$, then $\tau-\left(\frac{\eta_t}{2\phi}\right)^2$ becomes $\tau-\frac{\eta_t^2}{4\tau}$.

But (56) $\tau > \eta_t$

hence

$$\frac{\eta_t^2}{4\tau} < \frac{\eta_t}{2} \qquad (72)$$

then

$$\tau-\frac{\eta_t^2}{4\tau} > \tau-\frac{\eta_t}{2} \qquad (73)$$

In the table 9 the values of $\{\tau-(\eta_t/2\phi)^2\}$ or R_{max} for nine types are compared to $(\tau-\eta_t/2)$ and $(\tau-\eta_t^2/4\tau)$.

TABLE 9

(73) $\tau-\frac{\eta_t}{2} < \tau-\frac{\eta_t^2}{4\tau}$	case 1 $\phi < \sqrt{\tau}$ $\tau-\left(\frac{\eta_t}{2\phi}\right)^2 < \tau-\frac{\eta_t^2}{4\tau}$	case 2 $\phi = \sqrt{\tau}$ $\tau-\left(\frac{\eta_t}{2\phi}\right)^2 = \tau-\frac{\eta_t^2}{4\tau}$	case 3 $\phi > \sqrt{\tau}$ $\tau-\left(\frac{\eta_t}{2\phi}\right)^2 > \tau-\frac{\eta_t^2}{4\tau}$
group I $\phi < \sqrt{\frac{\eta_t}{2}}$ $\tau-\frac{\eta_t}{2} > \tau-\left(\frac{\eta_t}{2\phi}\right)^2$	type I_1 $\tau-\frac{\eta_t}{2} > \tau-\left(\frac{\eta_t}{2\phi}\right)^2 < \tau-\frac{\eta_t^2}{4\tau}$ the smallest value	type I_2 $\tau-\frac{\eta_t}{2} > \tau-\left(\frac{\eta_t}{2\phi}\right)^2 = \tau-\frac{\eta_t^2}{4\tau}$ or $2\tau < \eta_t$ impossible by (73)	type I_3 $\tau-\frac{\eta_t}{2} > \tau-\left(\frac{\eta_t}{2\phi}\right)^2 > \tau-\frac{\eta_t^2}{4\tau}$ or $2\tau < \eta_t$ impossible by (73)
group II $\phi = \sqrt{\frac{\eta_t}{2}}$ $\tau-\frac{\eta_t}{2} = \tau-\left(\frac{\eta_t}{2\phi}\right)^2$	type II_1 $\tau-\frac{\eta_t}{2} = \tau-\left(\frac{\eta_t}{2\phi}\right)^2 < \tau-\frac{\eta_t^2}{4\tau}$	type II_2 $\tau-\frac{\eta_t}{2} = \tau-\left(\frac{\eta_t}{2\phi}\right)^2 = \tau-\frac{\eta_t^2}{4\tau}$ or $2\tau = \eta_t$ impossible by (73)	type II_3 $\tau-\frac{\eta_t}{2} = \tau-\left(\frac{\eta_t}{2\phi}\right)^2 > \tau-\frac{\eta_t^2}{4\tau}$ or $2\tau < \eta_t$ impossible by (73)
group III $\phi > \sqrt{\frac{\eta_t}{2}}$ $\tau-\frac{\eta_t}{2} < \tau-\left(\frac{\eta_t}{2\phi}\right)^2$	type III_1 $\tau-\frac{\eta_t}{2} < \tau-\left(\frac{\eta_t}{2\phi}\right)^2 < \tau-\frac{\eta_t^2}{4\tau}$	type III_2 $\tau-\frac{\eta_t}{2} < \tau-\left(\frac{\eta_t}{2\phi}\right)^2 = \tau-\frac{\eta_t^2}{4\tau}$	type III_3 $\tau-\frac{\eta_t}{2} < \tau-\left(\frac{\eta_t}{2\phi}\right)^2 > \tau-\frac{\eta_t^2}{4\tau}$ the largest value

Since in the table 9 the values of $R_{\max.}$ or $\{\tau-(\eta_l/2\phi)^2\}$ for the types I_2, I_3, II_2 and II_3 are not rational, there are no reaction turbines in these types.

Assuming that τ and η_l are taken at the same values for all types, the values of $k_{a_{1t}}$ or $\sqrt{\tau-(\eta_l/2\phi)^2}$ is the smallest for the type I_1, it increases step by step as in the order of I_1, II_1, III_1, III_2 and III_3, and it becomes the largest for the type III_3.

SECTION 14. "k_aR" CURVES IN STATE OF NORMAL EXIT

The equation (67) $R=\tau-\left(\frac{\eta_l}{2\phi}\right)^2-k_a^2$ becomes

$$k_a^2=-\left[R-\left\{\tau-\left(\frac{\eta_l}{2\phi}\right)^2\right\}\right] \tag{74}$$

If τ, η_l and ϕ are given, (67) or (74) may become the equation of the parabola with 1 as parameter in regard to k_a and R. This parabola is called the "k_aR" curve in the state of normal exit.

Fig. 44 shows the "k_aR" curve is a coordinates with $\overline{Ok_a}$ as the axis of abscissa and with $\overline{OR}$ as that of ordinate. T is the vertex of the parabola, which is the intersection of curve with $\overline{OR}$. The symmetrical axis of curve coincides with $\overline{OR}$. B is the intersection of curve with $\overline{Ok_a}$.

Since the parameter of the parabola is always 1 and does not depend on the values of τ, ϕ and η_l, all curves may become the equal parabolas in the state of the normal exit. By the value of $\tau-(\eta_l/2\phi)^2$, however, the position of T may be changed for every curve.

Let $\overline{OT_{(\phi:\infty)}}=\tau$, and a line is drawn parallel to $\overline{Ok_a}$ through the point $T_{(\phi:\infty)}$. This line is called "τ-line."

Then $$\overline{T_{(\phi:\infty)}T}=\overline{T_{(\phi:\infty)}O}-\overline{TO}=\tau-\left\{\tau-\left(\frac{\eta_l}{2\phi}\right)^2\right\}=\left(\frac{\eta_l}{2\phi}\right)^2.$$

If η_l is given, the length $\overline{T_{(\phi:\infty)}T}$ may increase as ϕ decreases. If η_l is given and $T_{(\phi:\infty)}$ is assumed as the fixed point, the vertex T and all other points of curve fall evenly, according as ϕ decreases. And all

curves with ϕ at the different values do not coincide with each other, as shown in Fig. 51.

Fig. 45 to Fig. 49 show the "k_aR" curves of turbine series for five types. A point on curve may correspond to a turbine with the normal exit and gives the values of k_a and R.

For the type I_1, $\beta>\frac{\pi}{2}$ and $\phi<\sqrt{\frac{\eta_l}{2}}$ or $\left(\frac{\eta_l}{2\phi}\right)^2>\frac{\eta_l}{2}$

In Fig. 45,
for this type,

$$\overline{T_{(\phi:\infty)}T}=\left(\frac{\eta_l}{2\phi}\right)^2,\ \overline{OT}=\tau-\left(\frac{\eta_l}{2\phi}\right)^2,\ \overline{OB}=\sqrt{\tau-\left(\frac{\eta_l}{2\phi}\right)^2}$$

for the type II_1,

$$\overline{T_{(\phi:\infty)}T_{II_1}}=\frac{\eta_l}{2},\quad \overline{OT}_{II_1}=\tau-\frac{\eta_l}{2},\quad \overline{OB}_{II_1}=\sqrt{\tau-\frac{\eta_l}{2}}$$

where T_{II_1}: the vertex of the curve for II_1.

If η_l and τ are taken at the same values for both types,
$\overline{OT}<\overline{OT}_{II_1}$ & $OB<OB_{II_1}$ or $R_{\max.}<(R_{\max.})_{II_1}$ & $k_{a_{1t}}<(k_{a_{1t}})_{II_1}$
Hence all curves of this type may not exist in the outside of the II_1 "k_aR" curve or the curve of II_1, which may become the boundary and the space $\overline{OT_{II_1}B_{II_1}}$ may become the territory for the "k_aR" curves of I_1.

For the type II_1, $\beta=\frac{\pi}{2}$ and $\phi=\sqrt{\frac{\eta_l}{2}}$ or $\left(\frac{\eta_l}{2\phi}\right)^2=\frac{\eta_l}{2}$

In Fig. 46,

$$\overline{T_{(\phi:\infty)}T_{II_1}}=\frac{\eta_l}{2},\ \overline{OT_{II_1}}=\tau-\frac{\eta_l}{2}\ \text{and}\ \overline{OB_{II_1}}=\sqrt{\tau-\frac{\eta_l}{2}} \tag{75}$$

If η_l and τ are taken at the same values for all turbines of this type, all curves may coincide altogether and become only one, which is called the II_1 "k_aR" curve.

For the type III_1,

$$\beta<\frac{\pi}{2}\ \text{and}\ \sqrt{\frac{\eta_l}{2}}<\phi<\sqrt{\tau}\ \text{or}\ \frac{\eta_l}{2}>\left(\frac{\eta_l}{2\phi}\right)^2>\frac{\eta_l^2}{4\tau}$$

In Fig. 47,

for this type

$$\overline{T_{(\phi:\infty)}T} = \left(\frac{\eta_l}{2\phi}\right)^2, \quad \overline{OT} = \tau - \left(\frac{\eta_l}{2\phi}\right)^2, \quad \overline{OB} = \sqrt{\tau - \left(\frac{\eta_l}{2\phi}\right)^2}$$

for the type II_1,

$$\overline{T_{(\phi:\infty)}T_{II_1}} = \frac{\eta_l}{2}, \quad \overline{OT_{II_2}} = \tau - \frac{\eta_l}{2}, \quad \overline{OB_{II_1}} = \sqrt{\tau - \frac{\eta_l}{2}}$$

for the type III_2,

$$\overline{T_{(\phi:\infty)}T_{III_2}} = \frac{\eta_l^2}{4\tau}, \quad \overline{OT_{III_1}} = \tau - \frac{\eta_l^2}{4\tau}, \quad \overline{OB_{III_2}} = \sqrt{\tau - \frac{\eta_l^2}{4\tau}}$$

If η_l and τ are taken at the same values for these types,

$$\overline{OT_{II_1}} < \overline{OT} < \overline{OT_{III_2}} \quad \text{and} \quad \overline{OB_{II_1}} < \overline{OB} < \overline{OB_{III_2}}$$

or $\quad (R_{max.})_{II_1} < R_{max.} < (R_{max.})_{III_2}$ and $(k_{a_{1t}})_{II_1} < k_{a_{1t}} < (k_{a_{1t}})_{III_2}$

Hence all curves of this type may not exist in the inside of the II_1 "k_aR" curve nor in the outside of the III_2 "k_aR", and the space $\overline{T_{III_2}B_{III_2}B_{II_1}T_{II_1}}$ may become the territory for "k_aR" curves of III_1.

For the type III_2, $\beta < \frac{\pi}{2}$ and $\phi = \sqrt{\tau}$ or $\left(\frac{\eta_l}{2\phi}\right)^2 = \frac{\eta_l^2}{4\tau}$

In Fig. 48,

$$\overline{T_{(\phi:\infty)}T_{III_2}} = \frac{\eta_l^2}{4\tau}, \overline{OT_{III_2}} = \tau - \frac{\eta_l^2}{4\tau} \text{ and } \overline{OB_{III_2}} = \sqrt{\tau - \frac{\eta_l^2}{4\tau}} \quad (76)$$

If η_l and τ are taken at the same values for all turbines of this type, all curves may coincide altogether and become only one, which is called the III_2 "k_aR" curve.

For the type III_3, $\beta < \frac{\pi}{2}$ and $\phi > \sqrt{\tau}$ or $\left(\frac{\eta_l}{2\phi}\right)^2 < \frac{\eta_l^2}{4\tau}$

In Fig. 49,

for this type,

$$\overline{T_{(\phi:\infty)}T} = \left(\frac{\eta_l}{2\phi}\right)^2, \quad \overline{OT} = \tau - \left(\frac{\eta_l}{2\phi}\right)^2, \quad \overline{OB} = \sqrt{\tau - \left(\frac{\eta_l}{2\phi}\right)^2}$$

for the type III_2,

$$\overline{T_{(\phi:\infty)}T_{III_2}} = \frac{\eta_l^2}{4\tau}, \quad \overline{OT_{III_2}} = \tau - \frac{\eta_l^2}{4\tau}, \quad \overline{OB_{III_2}} = \sqrt{\tau - \frac{\eta_l^2}{4\tau}}$$

If η_l and τ are taken at the same values for both types,

$\overline{OT} > \overline{OT_{III_2}}$ & $\overline{OB} > \overline{OB_{III_2}}$ or $R_{max.} > (R_{max.})_{III_2}$ & $k_{a_{1t}} > (k_{a_{1t}})_{III_2}$.

Hence all curves of this type may exist in the outside of the III_2 "k_aR."

If $\phi = \infty$, $\overline{OT_{(\phi:\infty)}} = \tau$ and $\overline{OB_{(\phi:\infty)}} = \sqrt{\tau}$

where $\overline{OT_{(\phi:\infty)}}$: the length of $\overline{OT}$ for $\phi : \infty$,

and $\overline{OB_{(\phi:\infty)}}$: that of $\overline{OB}$.

The curve $T_{(\phi:\infty)}P_{(\phi:\infty)}$ is called the limit "k_aR" curve. Any turbine has never ϕ at ∞. By taking ϕ as about (2.5 to 3), the curve may be very near the limit curve. And it is evident, any curve is never existent in the outside of the limit curve.

The example 15. The values of $R_{max.}$ and $k_{a_{1t}}$ for five types with $\eta_l = .82$ and $\tau = .93$ to $.96$.

The type I_1, $\tau = .93$ and $\phi = .56$,

(68) $$R_{max.} = \tau - \left(\frac{\eta_l}{2\phi}\right)^2 = .93 - \left(\frac{.14}{.56}\right)^2 = \underline{.394}$$

(70) $$k_{a_{1t}} = \sqrt{\tau - \left(\frac{\eta_l}{2\phi}\right)^2} = \sqrt{.93 - \left(\frac{.41}{.56}\right)^2} = \underline{.628}$$

The type II_1, $\tau = .94$ and $\phi = \sqrt{\frac{\eta_l}{2}} = \sqrt{.41} = \underline{.640}$

(68) $$(R_{max.})_{II_1} = \tau - \frac{\eta_l}{2} = .94 - .41 = \underline{.53}$$

(70) $$(k_{a_{1t}})_{II_1} = \sqrt{\tau - \frac{\eta_l}{2}} = \sqrt{.53} = \underline{.728}$$

The type III_1, $\tau = .95$ and $\phi = .80$,

(68) $$R_{max.} = \tau - \left(\frac{\eta_l}{2\phi}\right)^2 = .95 - \left(\frac{.41}{.80}\right)^2 = \underline{.687}$$

(70) $$k_{a_{1t}} = \sqrt{\tau - \left(\frac{\eta_l}{2\phi}\right)^2} = \sqrt{.95 - \left(\frac{.41}{.80}\right)^2} = \underline{.829}$$

The type III_2, $\tau = .95$ and $\phi = \sqrt{\tau} = \sqrt{.95} = \underline{.975}$

(68) $$(R_{max.})_{III_2} = \tau - \frac{\eta_l^2}{4\tau} = .95 - \frac{(.41)^2}{.95} = .773$$

(70) $$(k_{a_{lt}})_{III_2} = \sqrt{\tau - \frac{\eta_l^2}{4\tau}} = \sqrt{.95 - \frac{.1681}{.95}} = .879$$

The type III_3, $\tau = .96$ and $\phi = 1.6$

(68) $$R_{max.} = \tau - \left(\frac{\eta_l}{2\phi}\right)^2 = .96 - \left(\frac{.41}{1.6}\right)^2 = .894$$

(70) $$k_{a_{lt}} = \sqrt{\tau - \left(\frac{\eta_l}{2\phi}\right)^2} = \sqrt{.96 - \left(\frac{.41}{1.6}\right)^2} = .946$$

Fig. 45 to Fig. 49 show the "k_aR" curves of this example. In these figures the values of $R_{max.}$, $k_{a_{lt}}$, τ and $(\eta_l/2\phi)^2$ are denoted in the brackets.

On the relations between the "k_aR" curves in the state of the normal exit and in the general case.

If for the types I_1, III_1, III_2 and III_3 τ and ϕ are taken as the constants but β changes, there are plotted many "k_aR" curves of the general case, every one of which has only one point corresponding to a turbine with normal exit and with η_l at a given value. Hence these points correspond to some turbines of a series in the state of normal exit having τ, ϕ and η_l at the given values, and accordingly the locus of such points may become a "k_aR" curve in the state of normal exit. To be a little more definite, this is explained by the preceding examples.

e. g. On the relation between the "k_aR" curves in Fig. 49 and that in Fig. 20.

Fig. 20 for the example 5, the type III_3, (page 21) with $\tau = .96$, $\phi = 1.6$ and $\beta = 12°$

Fig. 49 for the example 15, the type III_3, (page 54) with $\tau = .96$, $\phi = 1.6$ and $\eta_l = .82$

Fig. 43 for the example 13, the type III_3, (page 46) with $\tau = .96$, $\phi = 1.6$ and $\eta_l = .82$

For these examples τ and ϕ are taken at the same values : .96 and 1.6 respectively.

In Fig. 43 $\overline{OCK_l}$ is a velocity diagram at the inlet edge of the runner and $\overline{L'K_l}$ becomes the value of k_a for $\beta = 12°$. At the point K_l $\tau = .96$, $\phi = 1.6$ and $\beta = 12°$, hence a turbine for K_l may be one of the turbine series in the example 5. In Fig. 20 a point P_l on the "k_aR" curve has the length of $\overline{L'K_l}$ (Fig. 43) as the abscissa, then P_l may correspond to only one turbine with the normal exit and with $\eta_l = 82$. when $\beta = 12°$.

Further in Fig. 43 $\overline{L'K_l}$ becomes the value of k_a for $\beta = 12°$ in the state of normal exit. Since at K_l $\tau = .96$, $\phi = 1.6$ and $\eta_l = .82$ in the state of normal exit, the turbine for K_l may be one of the turbine series in the example 15. In Fig. 49 a point $P_{(\beta:12°)}$ on the "k_aR" curve has the length of $\overline{L'K_l}$ (Fig. 43) as the abscissa, then $P_{(\beta:12°)}$ may correspond to only one turbine with $\beta = 12°$ when $\eta_l = .82$ in the state of normal exit.

For this reason, the point P_l in Fig. 20 must have the same values of k_a and R as those of the point $P_{(\beta:12°)}$ in Fig. 49, or these points coincide with each other in a coordinates (k_a, R) and correspond to one turbine.

When in the example 5 the value of β changes, a lot of curves with $\tau = .96$ and $\phi = 1.6$ may be plotted in Fig. 20 according to the characteristics of "k_aR" curve for III_3, excepting that in the limit β and k_a become zero and the curve coincides with the axis $\overline{OR}$. Every curve corresponds to one turbine series and has only one point for the turbine with the normal exit and with $\eta_l = .82$. The locus of these points becomes the "k_aR" curve in Fig. 49.

Similarly the relation between the "k_aR" curves in Fig. 45 and Fig. 12, Fig. 47 and Fig. 18 or Fig. 48 and Fig. 19 may be as that between the curves in Fig. 49 and Fig. 20.

For the type II_2, however, the "k_aR" curves in the both cases coincide with each other taking τ and ϕ at the same values, since (29) and (74) reduce to

$$-\left[R-(\tau-\phi^2)\right] = k_a^2 \qquad \text{for II}_2 \text{ in the general and special cases.} \qquad (77)$$

Fig. 50 shows the "k_aR" curve with another parabola to find graphically the value of $(\eta_t/2\phi)$. A point O is taken as the origin of the coordinates (k_a, R) and the point O' is that of the coordinates $(\phi_t', \phi_t'^2)$, at the distance τ from O. The axes $\overline{OR}$ and $\overline{O'\phi_t'^2}$ are taken on a line but in the opposite direction to each other, and the axes $\overline{Ok_a}$ and $\overline{O'\phi_t'}$ are parallel. The curve TB is the "k_aR" curve, the curve TC is a parabola with 1 as the parameter and has the point T as the vertex, and thus the parabola TC may be as the inverted view of the "k_aR" curve. A point C is the intersection of the parabola TC with the axes $\overline{O'\phi_t'}$, then $\overline{O'T} = (\overline{O'C})^2$, since the curve TC is a parabola with 1 as the parameter,

but $$\overline{O'T} = \overline{OO'} - \overline{OT} = \tau - \left[\tau - \left(\frac{\eta_t}{2\phi}\right)^2\right] = \left(\frac{\eta_t}{2\phi}\right)^2,$$

hence $$(\overline{O'C})^2 = \left(\frac{\eta_t}{2\phi}\right)^2 \quad \text{or} \quad \overline{O'C} = \frac{\eta_t}{2\phi}$$

By the above idea, $(\eta_t/2\phi)$ may be graphically found, if a "k_aR" curve is given. Then η_t may be computed for the given value of ϕ, or vice versa.

Fig. 51 shows the "k_aR" curves with the curve CT for five turbine series of the example 15. If a lot of "k_aR" curves with the curve CT is plotted at the intervals as small as possible, a "k_aR" chart with ϕ_t' may be made. To omit the trouble of calculation this chart may be applicable to the design of reaction turbine.

SECTION 15 CHARACTERISTICS OF "R" FOR "α" IN STATE OF NORMAL EXIT

a) Equation of "R"

The equation (48) becomes

$$k = \frac{\eta_t}{2\phi} \cdot \frac{1}{\cos\alpha} \qquad (78)$$

Then the equation (19) reduces to

$$R = \tau - \left(\frac{\eta_t}{2\phi}\right)^2 \frac{1}{\cos^2\alpha} \qquad (79)$$

(79) is the equation of "R" as a function of α, η_t, ϕ and τ. The values of τ, η_t and ϕ are taken as mentioned in Section 13. And since a turbine has one value of α as explained in the chapter I, the values of R for all turbines with the normal exit may be determined by the equation (79).

b) Particular Values of "R"

If in the equation (79)

$$R = \tau - \left(\frac{\eta_t}{2\phi}\right)^2 \frac{1}{\cos^2\alpha}$$

τ, ϕ and η_t are given,

$$\frac{dR}{d\alpha} = -2\left(\frac{\eta_t}{2\phi}\right)^2 \frac{\sin\alpha}{\cos^3\alpha}, \quad \frac{d^2R}{d\alpha^2} = -2\left(\frac{\eta_t}{2\phi}\right)^2 \frac{\cos^2\alpha + 3\sin^2\alpha}{\cos^4\alpha} \quad \text{negative,}$$

hence if $dR/d\alpha = 0$, R becomes maximum, Thus we have

$$R_{max.} = \tau - \left(\frac{\eta_t}{2\phi}\right)^2 \quad \text{if } \alpha = 0 \qquad (80)$$

(80) corresponds to the equation (68).

By the equatian (79),

$$R = 0 \qquad \text{if } \alpha = \cos^{-1}\frac{\eta_t}{2\phi\sqrt{\tau}} \qquad (81)$$

(81) corresponds to the equation (69).

By the equations (80), (79) and (81) it is observed that
if α is zero, R may be maximum,
according as α increase, R may decrease,
if α becomes $\cos^{-1}(\eta_t/2\phi\sqrt{\mathcal{T}})$, R may become zero,
and after α has increased above $\cos^{-1}(\eta_t/2\phi\sqrt{\mathcal{T}})$, R may be negative and any reaction turbine may not exist.

Hence $\cos^{-1}(\eta_t/2\phi\sqrt{\mathcal{T}})$ becomes the limt value of α, or $\alpha_{1t} = \cos^{-1}(\eta_t/2\phi\sqrt{\mathcal{T}})$ which is the same as the equation (58) and $\alpha < \alpha_{1t}$ by the restriction of reaction,

$$\text{or} \qquad \text{cas}\ \alpha > \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} \qquad \text{where } 0 < \alpha < \frac{\pi}{2} \tag{82}$$

If in the equation (80) $R_{max.} \lesseqgtr 0$ or $\phi \lesseqgtr (\eta_t/2\sqrt{\mathcal{T}})$, R may not become positive for any positive value of α between 0 and $\pi/2$, In order that R is positive, therefore, $R_{max.}$ must be positive. Then $\phi > (\eta_t/2\sqrt{\mathcal{T}})$ or $1 > (\eta_t/2\phi\sqrt{\mathcal{T}})$ for the reaction turbines with the normal exit. Further it is of course necessary that the value of $(\eta_t/2\phi\sqrt{\mathcal{T}})$ must be rational.

Now the value of $(\eta_t/2\phi\sqrt{\mathcal{T}})$ or $\cos\alpha_{1t}$ is inspected.

For the type II_1 $\quad \phi = \sqrt{\eta_t/2}$, then $\cos\alpha_{1t} = \sqrt{\eta_t/2\mathcal{T}}$

For the type III_2, $\quad \phi = \sqrt{\mathcal{T}}$, then $\text{ccs}\alpha_{1t} = (\eta_t/2\mathcal{T})$

But (56) $\quad \mathcal{T} > \eta_t$

$$\text{hence} \qquad \frac{\eta_t}{2\mathcal{T}} < \sqrt{\frac{\eta_t}{2\mathcal{T}}} \tag{83}$$

In the table 10 the values of $(\eta_t/2\phi\sqrt{\mathcal{T}})$ for nine types are compared to $\sqrt{\eta_t/2\mathcal{T}}$ and $(\eta_t/2\mathcal{T})$.

TABLE 10

(83) $\frac{\eta_t}{2\mathcal{T}} < \sqrt{\frac{\eta_t}{2\mathcal{T}}}$	case 1 $\phi < \sqrt{\mathcal{T}}$, $\frac{\eta_t}{2\mathcal{T}} < \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}}$	case 2 $\phi = \sqrt{\mathcal{T}}$, $\frac{\eta_t}{2\mathcal{T}} = \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}}$	case 3 $\phi > \sqrt{\mathcal{T}}$, $\frac{\eta_t}{2\mathcal{T}} > \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}}$
group I $\phi < \sqrt{\frac{\eta_t}{2}}$, $\frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} > \sqrt{\frac{\eta_t}{2\mathcal{T}}}$	type I_1 $\frac{\eta_t}{2\mathcal{T}} < \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} > \sqrt{\frac{\eta_t}{2\mathcal{T}}}$ the largest value	type I_2 $\frac{\eta_t}{2\mathcal{T}} = \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} > \sqrt{\frac{\eta_t}{2\mathcal{T}}}$ or $2\mathcal{T} < \eta_t$ impossible by (83)	type I_3 $\frac{\eta_t}{2\mathcal{T}} > \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} > \sqrt{\frac{\eta_t}{2\mathcal{T}}}$ or $2\mathcal{T} < \eta_t$ impossible by (83)
group II $\phi = \sqrt{\frac{\eta_t}{2}}$, $\frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} = \sqrt{\frac{\eta_t}{2\mathcal{T}}}$	type II_1 $\frac{\eta_t}{2\mathcal{T}} < \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} = \sqrt{\frac{\eta_t}{2\mathcal{T}}}$	type II_2 $\frac{\eta_t}{2\mathcal{T}} = \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} = \sqrt{\frac{\eta_t}{2\mathcal{T}}}$ or $2\mathcal{T} = \eta_t$ impossible by (83)	type II_3 $\frac{\eta_t}{2\mathcal{T}} > \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} = \sqrt{\frac{\eta_t}{2\mathcal{T}}}$ or $2\mathcal{T} < \eta_t$ impossible by (83)
group III $\phi > \sqrt{\frac{\eta_t}{2}}$, $\frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} < \sqrt{\frac{\eta_t}{2\mathcal{T}}}$	type III_1 $\frac{\eta_t}{2\mathcal{T}} < \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} < \sqrt{\frac{\eta_t}{2\mathcal{T}}}$	type III_2 $\frac{\eta_t}{2\mathcal{T}} = \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} < \sqrt{\frac{\eta_t}{2\mathcal{T}}}$	type III_3 $\frac{\eta_t}{2\mathcal{T}} > \frac{\eta_t}{2\phi\sqrt{\mathcal{T}}} < \sqrt{\frac{\eta_t}{2\mathcal{T}}}$ the smallest value

Since in the table 10 the value of $(\eta_t/2\phi\sqrt{\mathcal{T}})$ for the types I_2, J_3, II_2 and II_3 are not rational, there are no reaction turbines in these types.

Assuming that $\mathcal{T}$ and η_t are taken at the same values for all types, the value of $(\eta_t/2\phi\sqrt{\mathcal{T}})$ is the largest for I_1, it decreases step by step as in the order of I_1, II_1, III_1, III_2 and III_3, and it becomes the smallest for III_3. The value of α_{1t} is vice versa.

SECTION 16. "αR" CURVES IN STATE OF NORMAL EXIT

The equation (79)

$$R = \tau - \left(\frac{\eta_t}{2\phi}\right)^2 \frac{1}{\cos^2\alpha} \quad \text{or} \quad \tau - R = \left(\frac{\eta_t}{2\phi}\right)^2 \frac{1}{\cos^2\alpha}$$

If τ, η_t and ϕ are given, (79) may become the equation of a curve in the coordinates with $\overline{O\alpha}$ as the axis of abscissa and with $\overline{OR}$ as that of ordinate. This curve is called the "αR" curve in the state of the normal exit. To plot the "αR" curve the auxiliary curves may be used, as in the general case.

The auxiliary curve (i) is $y'' = \cos x'$ a cosine curve, where $x' = \alpha$

the auxiliary curve (ii) is $y''^2 = x''$ a parabola with 1 as the parameter,

the auxiliary curve (iii) is $x''y' = \left(\frac{\eta_t}{2\phi}\right)^2$ a rectangular hyperbola,

the auxiliary curve (iv) is $y' = \left(\frac{\eta_t}{2\phi}\right)^2 \frac{1}{\cos^2 x'}$

where $y' = \tau - R$ by (79).

In the Fig. 52 there are five coordinates, and
the curve (i) is plotted in the coordinates (x', y'') or (i),
the curve (ii) ,, ,, in the coordinates (x'', y'') or (ii),
the curve (iii) ,, ,, in the coordinates (x'', y') or (iii),
the curve (iv) ,, ,, in the coordinates (x', y') or (iv),
and the "αR" curve is plotted in the coordinates (α, R).
The axes of abscissas of (i) and (ii) are taken at a straight line $\overline{O_{I}\,O_{II}}$, and those of (iii) and (iv) at a straight line $\overline{O_{III}\,O_{IV}}$. The axes of ordinates of (i), (iv) and (α, R) are taken at a straight line $\overline{O_{I}\,O_{IV}O}$, and those of (ii) and (iii) at a straight line $\overline{O_{II}\,O_{III}}$.

In (i), $y'' = \cos x'$ or $y'' = \cos\alpha$,
a point P_1 has α as the abscissa and
$\cos\alpha$ as the ordinate.

In (ii), $y''^2 = x''$,
a point P_2 has the same ordinate as P_1,
hence P_2 has $\cos^2\alpha$ as the abscissa and
$\cos\alpha$ as the ordinate.

In (iii), $x''y' = \left(\frac{\eta_t}{2\phi}\right)^2$ or $y' = \left(\frac{\eta_t}{2\phi}\right)^2 \frac{1}{x''}$

a point P_3 has the same abscissa as P_2,
hence P_3 has $\cos^2\alpha$ as the abscissa and

$\left(\frac{\eta_t}{2\phi}\right)^2 \frac{1}{\cos^2\alpha}$ as the ordinate.

In (iv), $y' = \left(\frac{\eta_t}{2\phi}\right)^2 \frac{1}{\cos^2 x'}$

a point P has the same abscissa as P_1 and has the same ordinate as P_3,
hence p has α as the abscissa and

$\left(\frac{\eta_t}{2\phi}\right)^2 \frac{1}{\cos^2\alpha}$ or $(\tau - R)$ as the ordinate.

A straight line $\overline{OB}$ is drawn parallel to the axis of abscissa in (iv) at the distance τ, and let a point P' be the intersection of $\overline{OB}$ with the line $\overline{P_1P}$.

Then $\overline{PP'} = \tau - \left(\frac{\eta_t}{2\phi}\right)^2 \frac{1}{\cos^2\alpha}$,

$= R$ by the equation (79).

The "αR" curve.

In the coordinates (α, R) the line $\overline{OB}$ is taken as the axis of abscissa and the line $\overline{OO_{IV}}$ as that of ordinate. Then a point P may have α as the abscissa and R as the ordinate. Thus the curve (iv) may become the "αR" curve in the coordinates (α,R).

Fig. 53 shows the "αR" curves for five types. A point on a curve may correspond to a turbine with the normal exit and gives the values of α and R.

For the type I_1, $\beta > \frac{\pi}{2}$ and $\phi < \sqrt{\frac{\eta_t}{2}}$,

The value of ϕ may be arbitrarily taken between $(\eta_t/2\sqrt{\tau})$[1] and $\sqrt{\eta_t/2}$. By the various value of ϕ a lot of curves may be plotted, although τ and η_t are taken at the same values for every turbine series.

For the type II_1, $\beta = \frac{\pi}{2}$ and $\phi = \sqrt{\frac{\eta_t}{2}}$,

$$\left.\begin{aligned} &\text{by (80),} \quad \overline{OT}_{II_1} \text{ or } (R_{max.})_{II_1} = \tau - \frac{\eta_t}{2} \\ &\text{by (81),} \quad \overline{OB}_{II_1} \text{ or } (\alpha_{1t})_{II_1} = \cos^{-1}\sqrt{\frac{\eta_t}{2\tau}} \end{aligned}\right\} \quad (84)$$

If τ and η_t are given, all curves may coincide altogether, and they become only one curve, which is called the II_1 "αR" curve.

For the type III_1, $\beta < \frac{\pi}{2}$ and $\sqrt{\frac{\eta_t}{2}} < \phi < \sqrt{\tau}$,

By the various value of ϕ there are many curves plotted with τ and η_t at the given values, as often mentioned.

For the type III_2, $\beta < \frac{\pi}{2}$ and $\phi = \sqrt{\tau}$

$$\left.\begin{aligned} &\text{by (80),} \quad \overline{OT}_{III_2} \text{ or } (R_{max.})_{III_2} = \tau - \frac{\eta_t^2}{4\tau} \\ &\text{by (81),} \quad \overline{OB}_{III_2} \text{ or } (\alpha_{1t})_{III_2} = \cos^{-1}\frac{\eta_t}{2\tau} \end{aligned}\right\} \quad (85)$$

If τ and η_t are given, one curve may exist, which is called the III_2 "αR" curve, as in the type II_1.

For the type III_3, $\beta < \frac{\pi}{2}$ and $\phi > \sqrt{\tau}$

By the various value of ϕ, there are many curves plotted with τ and η_t at the given values, as often written.

1. See the equation (71) and the example 14 in the Section 13.

Assuming that τ and η_t are taken at the same values for all turbine series, the "αR" curves of I_1, III_1 and III_3 may be plotted in the territories under the II_1 "αR" curve, between the II_1 and III_2 "αR" curves and between the III_2 and the limit "αR" curves respectively, as written in regard to the "$k_a R$" curves.

The example 16. The values os $R_{max.}$ and α_{1t} for five types with $\eta_t = .82$ and $\tau = .93$ to $.96$.

The type I_1, $\tau = .93$ and $\phi = .56$

(80) $R_{max.} = \tau - \left(\frac{\eta_t}{2\phi}\right)^2 = .93 - \left(\frac{.41}{.56}\right)^2 = \underline{.394}$

(58) $\alpha_{1t} = \cos^{-1}\left(\frac{\eta_t}{2\phi\sqrt{\tau}}\right) = \cos^{-1}\left(\frac{.41}{.56\times\sqrt{.93}}\right) = \cos^{-1} .75920 = \underline{40°36'}$

The type II_1, $\tau = .94$ and $\phi = \sqrt{.41} = \underline{.640}$

(84) $(R_{max.})_{II_1} = \tau - \frac{\eta_t}{2} = .94 - .41 = \underline{.53}$

(84) $(\alpha_{1t})_{II_1} = \cos^{-1}\sqrt{\frac{\eta_t}{2\tau}} = \cos^{-1}\sqrt{\frac{.41}{.94}} = \cos^{-1} .66043 = \underline{48°40'}$

The type III_1, $\tau = .95$ and $\phi = .80$

(80) $R_{max.} = \tau - \left(\frac{\eta_t}{2\phi}\right)^2 = .95 - \left(\frac{.41}{.80}\right)^2 = \underline{.687}$

(58) $\alpha_{1t} = \cos^{-1}\sqrt{\frac{\eta_t}{2\phi\sqrt{\tau}}} = \cos^{-1}\left(\frac{.41}{.8\times\sqrt{.96}}\right) = \cos^{-1} .52581 = \underline{58°17'}$

The type III_2, $\tau = .95$ and $\phi = \sqrt{\tau} = \sqrt{.95} = \underline{.975}$

(85) $(R_{max.})_{III_2} = \tau - \frac{\eta_t^2}{4\tau} = .95 - \frac{(.41)^2}{.95} = \underline{.773}$

(85) $(\alpha_{1t})_{III_2} = \cos^{-1}\frac{\eta_t}{2\tau} = \cos^{-1}\left(\frac{.41}{.95}\right) = \cos^{-1} .43158 = \underline{64°26'}$

The tppe III_3, $\tau = .96$ and $\phi = 1.6$,

(80) $R_{max.} = \tau - \left(\frac{\eta_t}{2\phi}\right)^2 = .96 - \left(\frac{.41}{1.6}\right)^2 = \underline{.894}$

(58) $\alpha_{1t} = \cos^{-1}\left(\frac{\eta_t}{2\phi\sqrt{\tau}}\right) = \cos^{-1}\left(\frac{.41}{1.6\times\sqrt{.96}}\right) = \cos^{-1} .26153$

$= \underline{74°50'}$

Fig. 53 shows the "αR" curves of this example, and the values of α and R at the particular points are denoted in the brackets and in the table.

The relation between the "αR" cruves in the state of the normal exit and those in the general case is similar as the relation between the "$k_a R$" curves, excepting the curves of the type II_1. For the type II_1 the "αR" curves in both cases coincide with each other, since (32) and (79) reduce to

$$R = \tau - \frac{\phi^2}{\cos^2\alpha} \quad \text{for } II_1 \text{ in the general and the special cases,} \quad (86)$$

which corresponds to the equation (77).

CHAPTER IV.
CHANGING DEGREE OF REACTION OF A TURBINE REGULATED BY SPEED GOVERNOR

According as the water admission is adjusted by the speed regulation, k and τ vary. Hence R of a turbine does not take one value during the regulation. This is discussed in this chapter, considering that the regulation is operated by the means of the usual movable vane; i.e. Fink's regulator.

SECTION 17. CHANGE OF VELOCITY DIAGRAMS BY REGULATION WITH CONSTANT "ϕ"

a) Velocity Diagrams

Fig. 54 and Fig. 55 show the velocity diagrams at the inlet and outlet edges of runner for a turbine in a state of running.

At the inlet edge (Fig. 54)

let k : the coefficient of the absolute velocity of water,

k_a : the normal component of k,

ϕ' : the circumferential component of k,

ψ : the coefficient of the relative velocity of water,

α' : the angle of k,

β' : the angle of ψ, and

ϕ : the coefficient of the circumferential velocity of runner.

At the outlet edge (Fig. 55)

let k_2 : the coefficient of the absolute velocity of water,

k_{2a} : the normal component of k_2,

ψ_2 : the coefficient of the relative velocity of water,

α_2 : the angle of k_2.

β_2 : the angle of ψ_2 which is taken as equal to the vane angle,

ϕ_2 : the coefficient of the circumferential velocity of runner.

ϕ is assumed as constant during the regulation. Since ϕ_2 is proportional to ϕ, ϕ_2 becomes also to be assumed as constant. β_2 may be taken as constant during the regulation, because water discharges usually in the direction of vane angle from the runner outlet. Hence β_2 may be taken as equal to the vane angle at the outlet. At the inlet edge, however, β' may not coincide with the vane angle β, excepting the state of entrance without shock. α' corresponds to β', and in the state of entrance without shock α' becomes α which corresponds to β, as shown with the diagram $OC(K)$.

In the permanent flow k_{2a} is always proportional to k_a, for instance $k_{2a(1/1)}$, $k_{2a(3/4)}$, $k_{2a(1/2)}$, $k_{2a(1/4)}$ and $k_{2a(0)}$ are proportional to $k_{a(1/1)}$, $k_{a(3/4)}$, $k_{a(1/2)}$, $k_{a(1/4)}$ and $k_{a(0)}$ respectively, in which the indexes (1/1), (3/4), (1/2), (1/4) and (0) are used for the full, 3/4, 1/2, 1/4 and zero admissions respectively.

b) Exit Line

In Fig. 55 the point K_2 is determined by the angle of the relative velocity and the magnitude of k_{2a}. But this angle is taken at the constant β_2 during the regulation. Hence K_2 is always on a line $\overline{C_2K_{2l}}$ which includes the angle β_2 with $\overline{O_2C_2}$, as shown in Fig. 56. The meanings of the notations used in this figure are given in the following table.

admis-sions	velocity coefficients			angles of absolute velocity	points	velocity diagrams
	normal compo-nents	absolute	relative			
1/1	$k_{2a(1/1)}$	$k_{2(1/1)}$	$\psi_{2(1/1)}$	$\alpha_{2(1/1)}$	$K_{2(1/1)}$	$O_2C_2K_{2(1/1)}$
3/4	$k_{2a(3/4)}$	$k_{2(3/4)}$	$\psi_{2(3/4)}$	$\alpha_{2(3/4)}$	$K_{2(3/4)}$	$O_2C_2K_{2(3/4)}$
1/2	$k_{2a(1/2)}$	$k_{2(1/2)}$	$\psi_{2(1/2)}$	$\alpha_{2(1/2)}$	$K_{2(1/2)}$	$O_2C_2K_{2(1/2)}$
1/4	$k_{2a(1/4)}$	$k_{2(1/4)}$	$\psi_{2(1/4)}$	$\alpha_{2(1/4)}$	$K_{2(1/4)}$	$O_2C_2K_{2(1/4)}$

During the regulation the point K_2 moves along the line $\overline{C_2K_{2l}}$, which is called "the exit line."

c) Entrance Curve and Entrance Parabola

If in Fig. 54 $\overline{OC}$ is taken as the axis of ϕ' and $\overline{ON}$ as that of k_a, the position of K depends on the values of ϕ' and k_a. According as k_a and ϕ' vary by the regulation, the point K displaces. An actual curve traced by the successive positions of K is called "the entrance curve." In theory, however, the curve is usually assumed as a parabola.[1]

Let F : the normal intersectional area of the effective opening at the inlet edge of runner,

F_2 : that at the outlet edge.

then $$k_a = \frac{F_2}{F} \sin\beta_2 \cdot \psi_2,$$

or $$k_a = c\,\psi_2 \tag{87}$$

where $$c = \frac{F_2}{F} \sin\beta_2 \tag{88}$$

: a constant for a turbine

$$\left.\begin{array}{l} \psi_{2(1/1)} = 1.333\ \phi_2 \text{ for Frsncis turbines}^{2} \\ \phi_2 < \psi_{2(1/1)} < 1.333\ \phi_2 \text{ for the high-speed axial-flows} \end{array}\right\} \tag{89}$$

In the diagram at the inlet edge

$$k^2+\phi^2-\psi^2 = 2\phi\,\phi' \tag{90}$$

or $$0 = c^2(k^2+\phi^2-\psi^2)-2c^2\phi\,\phi'$$

by (87) $$k_a^2 = c^2\psi_2^2$$

$$+)\quad k_a^2 = c^2(k^2+\psi_2^2-\psi^2+\phi^2)-2c^2\phi\,\phi' \tag{91}$$

But the equatian (11) becomes

$$(\eta+k_2^2)+\phi_2^2 = k^2+\psi_2^2-\psi^2+\phi^2$$

1. see Camerer: Vorlesungen über Wasserkraftmaschinen (1914), Abb. 405 u. 407, seite 278 u. 279.
Honold u. Albrecht: Francis-Turbinen (1908), Seite 20 u. 21.

2. see Honold n. Albrecht: Francis-Turbinen (1908), Seite 21, 26, 29, 36 u. 37 (Tabellen).

hence the equation (91) reduces to

$$k_a^2 = c^2\left\{(\eta+k_2^2)+\phi_2^2\right\}-2c^2\phi\,\phi' \tag{92}$$

In (92) η and k_2 are not constant during the regulation.[1] In theory, however, $(\eta+k_2^2)$ is assumed as constant.[2] If $(\eta+k_2^2)$, ϕ and ϕ_2 are assumed as constant, (92) becomes the equation of parabola which is called " the entrance parabola ".

In the equation (92),

$$\text{if } k_a=0, \quad k_{(k_a:0)} = \phi'_{(k_a:0)} = \frac{(\eta+k_2^2)+\phi_2^2}{2\phi} \tag{93}$$

$$\text{if } \phi'=0, \quad k_{(\phi':0)} = k_{a(\phi':0)} = c\sqrt{(\eta+k_2^2)+\phi_2^2} \tag{94}$$

In Fig. 57 XKY is the entrance parabola, and a point X is the intersection of the parabola with the axis $\overline{O\phi'}$ and a point Y is that with the axis $\overline{Ok_a}$.

Let $\chi = \overline{OX}$, and $\upsilon = \overline{OY}$,

then by (93)
$$\chi = \frac{(\eta+k_2^2)+\phi_2^2}{2\phi} \tag{95}$$

by (94)
$$\upsilon = c\sqrt{(\eta+k_2^2)+\phi_2^2} \tag{96}$$

In the states near the entrance without shock the entrance curve coincides almost with the entrance parabola. According as the running state is further regulated from the entrance without shock, the former curve deviates more from the latter. Sometimes the former deviates much from the latter at the small admission, but not so much at the large admission.[3]

Fig. 58 shows the positions of K, assuming that K moves on the entrance parabola XKY by the regulation. The meanings of the notations are given in the following table.

1. see Camerer, Vorlesungen über Wasserkraftmaschinen, Seite 369—372.
2. see Honold u. Albrecht, Francis-Turbinen, Seite 20 u. 21.
3. see Z.V.D.I., Band 55, Nr. 23, 1911, Seite 1025, and Camerer, Vorlesungen über Wasserkraftmaschinen, Seite 364, Abb. 475 u. 476.

admissions	velocity coefficients			velocity angles		points	velocity diagrams
	normal components	absolute	relative	absolute	relative		
1/1	$k_{a(1/1)}$	$k_{(1/1)}$	$\psi_{(1/1)}$	$\alpha'_{(1/1)}$	$\beta'_{(1/1)}$	$K_{(1/1)}$	$OCK_{(1/1)}$
3/4	$k_{a(3/4)}$	$k_{(3/4)}$	$\psi_{(3/4)}$	$\alpha'_{(3/4)}$	$\beta'_{(3/4)}$	$K_{(3/4)}$	$OCK_{(3/4)}$
1/2	$k_{a(1/2)}$	$k_{(1/2)}$	$\psi_{(1/2)}$	$\alpha'_{(1/2)}$	$\beta'_{(1/2)}$	$K_{(1/2)}$	$OCK_{(1/2)}$
1/4	$k_{a(1/4)}$	$k_{(1/4)}$	$\psi_{(1/4)}$	$\alpha'_{(1/4)}$	$\beta'_{(1/4)}$	$K_{(1/4)}$	$OCK_{(1/4)}$

SECTION 18 CHARACTERISTCS OF "R" CHANGED BY REGULATION WITH CONSTANT "ϕ"

a) Equation of "R"

From the equation (92) we have

$$\phi'^2+k_a^2 = \frac{1}{4\phi^2}\left\{(\eta+k_2^2)+\phi_2^2\right\}^2+\left\{1-\frac{(\eta+k_2^2)+\phi_2^2}{2c^2\phi^2}\right\}k_a^2+\frac{1}{4c^4\phi^2}k_a^4 \tag{97}$$

Then the equation (19) reduces to

$$R = \tau-\frac{1}{4\phi^2}\left\{(\eta+k_2^2)+\phi_2^2\right\}^2+\left\{\frac{(\eta+k_2^2)+\phi_2^2}{2c^2\phi^2}-1\right\}k_a^2-\frac{1}{4c^4\phi^2}k_a^4 \tag{98}$$

(98) is the equation of R as a function of k_a and τ during the speed regulation, assuming ϕ, ϕ_2 and $(\eta+k_2^2)$ as constant or that K moves on the entrance parabola. In theory, however, τ may be also assumed as constant, and the error by the assumption is easily corrected, as will be seen in the pages 76 and 77.

b) Particular Values of "R"

If in (98) τ, ϕ, ϕ_2 and $(\eta+k_2^2)$ are given,

$$\frac{dR}{dk_a} = 2\left\{\frac{(\eta+k_2^2)+\phi_2^2}{2c^2\phi^2}-1\right\}k_a-\frac{1}{c^4\phi^2}k_a^3,$$

$$\frac{d^2R}{dk_a^2} = 2\left\{\frac{(\eta+k_2^2)+\phi_2^2}{2c^2\phi^2}-1\right\}-\frac{3}{c^4\phi^2}k_a^2,$$

put $\dfrac{dR}{dk_a} = 0$, that is $\begin{cases} k_a = \pm c\sqrt{\{(\eta+k_2^2)+\phi_2^2\}-2c^2\phi^2}, \\ k_a = 0. \end{cases}$

But since $k_a>0$, two of the above values are taken.

Hence $\dfrac{dR}{dk_a} = 0$ becomes $\begin{cases} k_a = +c\sqrt{\{(\eta+k_2^2)+\phi_2^2\}-2c^2\phi^2} \\ k_a = 0 \end{cases}$ (99)

if $k_a = c\sqrt{\{(\eta+k_2^2)+\phi_2^2\}-2c^2\phi^2}$,

$$\frac{d^2R}{dk_a^2} = -\frac{2}{c^2\phi^2}\left[\{(\eta+k_2^2)+\phi_2^2\}-2c^2\phi^2\right]$$

if $k_a = 0$, $\dfrac{d^2R}{dk_a^2} = +\dfrac{1}{c^2\phi^2}\left[\{(\eta+k_2^2)+\phi_2^2\}-2c^2\phi^2\right]$

put $\kappa = (\eta+k_2^2)+\phi_2^2-2c^2\phi^2$

or $\kappa = (\eta+k_2^2)+\left[\left(\dfrac{D_2}{D}\right)^2-2c^2\right]\phi^2$,

where D : the diameter at the centre of the inlet edge,

D_2 : that of the outlet edge.

According as the type of turbine becomes a high speed or the value of ϕ increases, $(\eta+k_2^2)$ and (D_2/D) increase but c decreases. Hence the value of κ increases according as that of ϕ increases.

e. g. The value of κ of a turbine with $\phi=.50$, $\phi_2=.25$, $\eta+k_2^2=.80$ and $c=.70$

In practice .50 is the smallest value of ϕ, although .43 may be taken as the theoretical limit, as shown in the example 14, and others are also taken at the limit values to make that of κ as small as possible.

$\kappa=(\eta+k_2^2)+\phi_2^2-2c^2\phi^2$ becomes

$\kappa_{(\phi;.50)} = .80+.25^2-2\times.70^2\times.50^2= .6175 = \underline{0.6}$ say. $\underline{0.6}$ may be the smallest value of κ. Hence κ may be the positive value above $\underline{0.6}$ for all reaction turbines.

The case $\kappa > 0$ or $(\eta+k_2^2)+\phi_2^2-2c^2\phi^2 > 0$,

If $k_a = c\sqrt{\{(\eta+k_2^2)+\phi_2^2\}-2c^2\phi^2}$, (d^2R/dk_a^2) becomes negative and R the maximum. Then we hove

$$\left.\begin{aligned} &\text{if } k_a = c\sqrt{\{(\eta+k_2^2)+\phi_2^2\}-2c^2\phi^2}, \\ &\qquad R_{max.} = \tau - c^2\left[\{(\eta+k_2^2)+\phi_2^2\}-c^2\phi^2\right] \\ &\qquad \text{assuming } (\eta+k_2^2)+\phi_2^2-2c^2\phi^2 > 0, \end{aligned}\right\} \quad (100)$$

if $k_a = 0$, (d^2R/dk_a^2) becomes positive and R the minimum. Then we have

$$\left.\begin{aligned} &\text{if } k_a=0,\ R_{min.} = \tau - \frac{1}{4\phi^2}\left\{(\eta+k_2^2)+\phi_2^2\right\}^2 = \tau - \chi^2 \\ &\qquad \text{assuming } (\eta+k_2^2)+\phi_2^2-2c^2\phi^2 > 0 \end{aligned}\right\}$$

The case $\kappa < 0$ or $(\eta+k_2^2)+\phi_2^2-2c^2\phi^2 < 0$.

In (99) $k_a = c\sqrt{\{(\eta+k_2^2)+\phi_2^2\}-2c^2\phi^2}$ becomes imaginary, and $(dR/dk_a)=0$ becomes only $k_a=0$. If $k_a=0$, (d^2R/dk^2) becomes negative and R the maximum. Then we hawe

$$\left.\begin{aligned} &\text{if } k_a = 0,\ R_{max.} = \tau - \frac{1}{4\phi^2}\left\{(\eta+k_2^2)+\phi_2^2\right\}^2 = \tau - \chi^2 \\ &\qquad \text{assuming } (\eta+k_2^2)+\phi_2^2-2c^2\phi^2 < 0, \end{aligned}\right\}$$

this case occurs scarcely, as above written.

Let $R_{(k_a;0)}$: the value of R, when k_a is zero. Then

$$\left.\begin{aligned} &\text{if } k_a = 0,\ R_{(k_a:0)} = \tau - \frac{1}{4\phi^2}\left\{(\eta+k_2^2)+\phi_2^2\right\}^2 = \tau - \chi^2 \\ &\text{where if } (\eta+k_2^2)+\phi_2^2-2c^2\phi^2 \gtreqless 0,\ R_{(k_a:0)} \text{ is } \begin{cases}\text{minimum}\\ \text{maximum}\end{cases} \end{aligned}\right\} \quad (101)$$

Let $R_{(\phi':0)}$: the value of R, when $\phi' = 0$.

Then by (94) and (98), or (19) we have

$$\text{if } \phi' = 0 \text{ or } k_a = Y,\ R_{(\phi':0)} = \tau - c^2\left\{(\eta+k_2^2)+\phi_2^2\right\} = \tau - Y^2 \quad (102)$$

From the equation (98)

if $R = 0$,

$$\left.\begin{aligned} k_a^2 = {} & c^2\left[\{(\eta+k_2^2)+\phi_2^2\}-2c^2\phi^2\right] \\ & \pm c^2\sqrt{\left[\{(\eta+k_2^2)+\phi_2^2\}-2c^2\phi^2\right]^2+\left[4\tau\phi^2-\{(\eta+k_2^2)+\phi_2^2\}^2\right]} \end{aligned}\right\} \quad (103)$$

or

$$R = 0, \quad \text{if} \quad k_n^2 = \iota^2\Big[\{(\eta+k_2^2)+\phi_2^2\}-2\iota^2\phi^2\Big] \pm 2\iota^2\phi\sqrt{\mathcal{T}-\iota^2\Big[\{(\eta+k_2^2)+\phi_2^2\}-\iota^2\phi^2\Big]} \qquad (103)'$$

In (103) or (103)′ there are four roots of k_n. But since k_n must be taken as the positive value, only two positive roots are taken as the values of k_n.

In theory the change of R behaves according to the equation (98), when k_n is varied by the speed regulation. By taking ϕ as constant and by assuming $\Big[\{(\eta+k_2^2)+\phi_2^2\}-2\iota^2\phi^2\Big]$ positive, the behaviour of the changing R is observed from the equations (102), (100), (103) and (101).

When k_n is $\iota\sqrt{(\eta+k_2^2)+\phi_2^2}$, R takes $\Big[\mathcal{T}-\iota^2\{(\eta+k_2^2)+\phi_2^2\}\Big]$ or $(\mathcal{T}-\Upsilon^2)$, according as k_n is diminished less than $\iota\sqrt{(\eta+k_2^2)+\phi_2^2}$, R increases, when k_n becomes $\iota\sqrt{\{(\eta+k_2^2)+\phi_2^2\}-2\iota^2\phi^2}$, R becomes the maximum, according as k_n is decreased less than $\iota\sqrt{\{(\eta+k_2^2)+\phi_2^2\}-2\iota^2\phi^2}$, R decreases also,
when k_n becomes as the smaller positive root of k_n in (103) or (103)′, R may be zero, (for Francis turbines, however, this case occurs scarcely.) and when at last k_n becomes to be zero, R is the minimum.

Since the value of k_n at the (1/1) admission is usually less than that of k_n for $R_{max.}$, the degree of reaction may diminish by closing the opening of the movable guide vanes.

SECTION 19. "k_nR" CURVE BY REGULATION WITH CONSTANT "ϕ"

Assuming $\mathcal{T}$, ϕ, ϕ_2 and $(\eta+k_2^2)$ constant during the regulation, (98) may become the equation of a curve in the coordinates with $\overline{Ok_n}$ as the axis of abscissa and with $\overline{OR}$ as that of ordinate. This curve is called the "k_nR" curve by the regulation. The "k_nR" curve is concisely to illustrate the behaviour of changing of R, and accordingly this is an important curve to be applied to the solution of the several questions which occurs during the regulation. The "k_nR" curve may be plotted by computation or by the graphical means, and for the latter there are two methods : A) and B).

The method A)

The equation (98) reduces to

$$\Big[k_n^2-\iota^2\Big\{(\eta+k_2^2+\phi_2^2)-2\iota^2\phi^2\Big\}\Big]^2 = 4\iota^4\phi^2\Big[\mathcal{T}-\iota^2\Big\{(\eta+k_2^2+\phi_2^2)-\iota^2\phi^2\Big\}-R\Big] \qquad (104)$$

If $\mathcal{T}$, ϕ, ϕ_2 and $(\eta+k_2^2)$ are given, (104) becomes the equation of parabola for k_n^2 and R, which is called the "k_n^2R" parabola.

Fig. 59 shows the "k_n^2R" parabola (P_Y P_M P_1 $P_\mathcal{T}$) in the coordinates with $\overline{O_1x'}$ or $\overline{O_1k_n^2}$ as the axis of abscissa and with $\overline{O_1y'}$ or $\overline{O_1R}$ as that of ordinate. P_M is the vertex of parabola which has

$$\left.\begin{aligned} x'_M &= \iota^2\Big\{(\eta+k_2^2+\phi_2^2)-2\iota^2\phi^2\Big\} \\ y'_M &= \mathcal{T}-\iota^2\Big\{(\eta+k_2^2+\phi_2^2)-\iota^2\phi^2\Big\} \end{aligned}\right\} \qquad (105)$$

as abscissa and ordinate respectively. The symmetrical axis of parabola $\overline{P_MK_M}$ is parallel to the axis $\overline{O_1y'}$. $P_\mathcal{T}$ is the intersection of parabola with the axis $\overline{O_1x'}$. Then

P_M becomes the point corresponding to $R_{max.}$,

$P_\mathcal{T}$ becomes the point where $R=0$,

and P_Y is the point corresponding to $k=\Upsilon$ or $\phi'=0$.

For the point P_M, by (105) or (100)

$$\left.\begin{aligned} k^2_{n(Rmax.)} &= \iota^2\Big\{(\eta+k_2^2+\phi_2^2)-2\iota^2\phi^2\Big\} \\ R_{max.} &= \mathcal{T}-\iota^2\Big\{(\eta+k_2^2+\phi_2^2)-\iota^2\phi^2\Big\} \end{aligned}\right\} \qquad (106)$$

For the point $P_\mathcal{T}$, by (103)′

$$\left.\begin{aligned} k^2_{n(\mathcal{T})} &= \iota^2\Big\{(\eta+k_2^2+\phi_2^2)-2\iota^2\phi^2\Big\} \\ &\quad \pm 2\iota^2\phi\sqrt{\mathcal{T}-\iota^2\Big\{(\eta+k_2^2+\phi_2^2)-\iota^2\phi^2\Big\}}, \\ R &= 0, \end{aligned}\right\} \qquad (107)$$

where $k_{n(\mathcal{T})}$ is the value of k_n, if $R=0$

For the point P_Y, by (102) and (96)

$$\left.\begin{aligned} k^2_{n(\phi'=0)} &= \Upsilon^2 = \iota^2(\eta+k_2^2+\phi_2^2) \\ R_{(\phi'=0)} &= \mathcal{T}-\iota^2(\eta+k_2^2+\phi_2^2) \end{aligned}\right\} \qquad (108)$$

At first the points P_M, $P_{\mathcal{T}}$ and P_Y are plotted in the coordinates (x', y'), then the "k_a^2R" parabola is drawn by the usual method. The "k_a^2R" parabola and others are used as the auxiliary curves to plot the "k_aR" curve.

In Fig. 60 the method of plotting the "k_aR" curve is shown.

The auxiliary curve (i), $(x'-x'_M)^2 = 4c^4\phi^2(y'_M-y')$ (109)

$$\text{where } \left.\begin{array}{l} x' = k_a^2 \\ y' = R \end{array}\right\} \text{ and } \left.\begin{array}{l} x'_M = c^2\left\{(\eta+k_2^2+\phi_2^2)-2c^2\phi^2\right\} \\ y'_M = \mathcal{T}-c^2\left\{(\eta+k_2^2+\phi_2^2)-c^2\phi^2\right\} \end{array}\right\} \quad (105)$$

The equation (109) is reduced to (104), hence the auxiliary curve (i) is the "k_a^2R" parabola.

The auxiliary curve (ii), $y''^2 = x'$, a parabola with 1 as parameter, (110)

where $x' = k_a^2$ and $y'' = \sqrt{x'} = k_a$

The auxiliary straight line (iii), $y'' = x''$ (111)

where $y'' = k_a$ and $x'' = k_a$

The "k_aR" curve (iv), in which

$$\left.\begin{array}{l} \text{the abscissa, } k_a = x'' \text{ in (iii)} \\ \text{the ordinate, } R = y' \text{ in (i)} \end{array}\right\} \quad (112)$$

The relative positions of the axes of four coordinates are that the axes $\overline{O_Ix'}$ and $\overline{Ok_a}$ are taken at a straight line $\overline{O_IO}$, the axes $\overline{O_{II}x'}$ and $\overline{O_{III}x''}$ at a straight line $\overline{O_{II}O_{III}}$, the axes $\overline{O_Iy'}$ and $\overline{O_{II}y''}$ at a straight line $\overline{O_I\,O_{II}}$, and the axes $\overline{OR}$ and $\overline{O_{III}y''}$ at a straight line $\overline{O\,O_{III}}$.

In (i) a point P_1 is on the "k_a^2R" parabola, hence P_1 has k_a^2 as abscissa and R as ordinate.

In (ii) a point P_2 has the same abscissa as that of P_1, hence P_2 has k_a^2 as abscissa and k_a as ordinate.

In (iii) a point P_3 has the same ordinate as that of P_2, hence P_3 has k_a as abscissa and as ordinate.

In (iv) a point P has the same abscissa as that of P_3 and the same ordinate as that of P_1, hence P has k_a as abscissa and R as ordinate.

Similarly the successive points of (iv) are plotted from the "k_a^2R" parabola in (i) through (ii) and (iii), then the "k_aR" curve consists of these

points. By the "k_aR" curve the behaviour of the change of R may be observed.

When $k_a = \Upsilon$ or $c\sqrt{\eta+k_2^2+\phi_2^2}$, $R = \mathcal{T}-c^2(\eta+k_2^2+\phi_2^2)$, this point is indicated with P_Y,

according as k_a decreases, R increases and the curve ascends,

when k_a becomes $c\sqrt{(\eta+k_2^2+\phi_2^2)-2c^2\phi^2}$, R is the maximum, this point is indicated with P_M,

according as k_a decreases less than the above value, R decreases also and the curve descends,

when k_a becomes the value in the equation (107), R is zero, this point is indicated with $P_{\mathcal{T}}$. The existence of the point $P_{\mathcal{T}}$ is taken as the meaning of that R becomes zero before the admission is shut off. In this case "χ" is greater than $\sqrt{\mathcal{T}}$. For the usual reaction turbines, however, such case occurs scarcely, and accordingly the point $P_{\mathcal{T}}$ does probably not appear on the curve.

The method B)

The "k_aR" curve may be plotted according to the equation (92) or the entrance parabola.

In Fig. 61, the axes $\overline{O_{II}k^2}$ and $\overline{O_Ik_a}$ are taken at a line $\overline{O_{II}O_I}$, the axis $\overline{O_{III}P_{\mathcal{T}3}}$ and the $\mathcal{T}$-line in (iv) at a horizontal line, the axes $\overline{O_{II}k}$ and $\overline{O_{III}P_{\mathcal{T}4}}$ at a line $\overline{O_{II}O_{III}}$, the axes $\overline{O_I\phi'}$ and $\overline{OR}$ at a line $\overline{O_IO}$, and the axis $\overline{Ok_a}$ at a line $\overline{P_{\mathcal{T}4}Ok_a}$ at the distance $\mathcal{T}$ from the $\mathcal{T}$-line.

In (i) the entrance parabola is plotted, and P_1 is the point on the parabola (i), then

$k = \overline{O_IP_1}$, which is the radius at P_1 with O_1 as centre.

In (ii) the parabola with 1 as parameter is plotted, and P_2 is the point on the parabola (ii) and has k or the radius $\overline{O_IP_1}$ in (i) as ordinate, as shown with the arrow head, and has k^2 as abscissa.

In (iii) a point P_3 is on the axis $\overline{O_{III}P_{\mathcal{T}3}}$ and has k^2 as abscissa which is the same as that of P_2.

let $\overline{O_{III}P_{\tau 3}} = \tau$

then $\overline{P_3P_{\tau 3}} = \overline{O_{III}P_{\tau 3}} - \overline{O_{III}P_3} = \tau - k^2 = R$ by (19)

A point P_4 is on the axis $\overline{O_{III}P_{\tau 4}}$ and has k^2 as ordinate which is equal to the abscissa of P_3, since P_3 and P_4 are on a circular arc with O_{III} as centre. Then we have

$$\overline{O_{III}P_{\tau 4}} = \overline{O_{III}P_{\tau 3}} \text{ and } \overline{O_{III}P_4} = \overline{O_{III}P_3}$$

hence $\overline{P_4P_{\tau 4}} = \overline{P_3P_{\tau 4}} = R.$

The "k_aR" curve.

In (iv) a point P has the same abscissa as P_1 and has $\overline{P_4P_{\tau 4}}$ as ordinate, as shown with the arrow heads. Hence P has k_a as abscissa and R as ordinate, and accordingly (iv) becomes the coordinates (k_a, R). Similarly the successive points in (iv) are plotted from the entrance parabola in (i) through (ii) and (iii), and the "k_aR" curve consists of these points.

The method B) may be also applied to plot the "k_aR" curve from the entrance curve, although τ is variable during the regulation. In this case the approximate curve is drawn by the above method taking τ as constant, and then this curve is corrected by the varying value of τ.

In Fig. 62 the method of correcting the approximate curve is shown. $P'_{(1/1)}P_NP'$ is the approximate or the "k_aR'" curve which is drawn from the entrance curve by the above method, assuming τ' is constant. τ' is taken as the value of τ in the normal state of running. $\overline{\tau' T' T_N \tau'}$ is the "τ'-line" which is a straight line parallel to the axis $\overline{Ok_a}$ at the distance τ'. $\tau T T_N \tau$ is the "τ-curve," on which a point has the distance

$$\tau = (1-\xi_g)+(k_2^2-k_4^2-\xi_d)$$

from the axis $\overline{Ok_a}$. By the regulation the resistance coefficient ξ_g and the secondary effect of draft tube $(k_2^2-k_4^2-\xi_d)$ are changed, and τ may be diminished according to the decrease of k_a[1].

Let $\Delta\tau = \tau' - \tau$

1. see Z.V.D.I., 1911, Seite 1024-1026, Camerer, Beiträge zur Berechung der Zentripetal Turbinen.

$R' = \tau' - k^2$: the degree of reaction in respect to τ',

or $k^2 = \tau' - R'$

then $\overline{K_aT'} - \overline{K_aT} = \Delta\tau$ and $\overline{K_aP'} = R'$

The equation (19), $R = \tau - k^2$ reduces to

$$R = \tau - (\tau' - R') = R' - (\tau' - \tau) = \overline{K_aP'} - \Delta\tau$$

Let $\overline{K_aP} = \overline{K_aR'} - \Delta\tau$, then $\overline{K_aP} = R.$

Hence P becomes the correct point, and accordingly P has k_a and R as abscissa and ordinate respectively. The actual "k_aR" curve consists of the successive points which are corrected by the above method.

SECTION 20. EXAMPLES ON "R" CHANGED BY REGULATION WITH CONSTANT "φ"

The example 17.

The changing value of R by the regulation for a Francis turbine of the type I_1 with

$$\phi = .56,\ \phi_2 = .30,\ F_2/F = .78,\ \beta_2 = 40^\circ \text{ and } \tau' = .93.$$

(88), $c = \frac{F_2}{F}\sin\beta_2 = .78\times.643 = \underline{.50}$

$k_{2l} = \phi_2 \operatorname{tg}\beta_2 = 0.3\times.839 = .25173$

$k_{2l}^2 = .25173^2 = \underline{0.06}$ and take $\eta_l = \underline{.82}$

if $(\eta + k_2^2)$ is taken at the value of $(\eta_l + k_{2l}^2)$,

$\eta + k_2^2 = .82 + .06 = \underline{.88}$

(95), $\chi = \phi'_{(k_a:0)} = \frac{(\eta+k_2^2)+\phi_2^2}{2\phi} = \frac{.88+.3^2}{2\times.56} = \underline{.866}$

(96), $y = k_{a(\phi':0)} = c\sqrt{(\eta+k_2^2)+\phi^2} = .5\times\sqrt{.88+.3^2} = .5+\sqrt{.97}$

$= \underline{.492}$

(89), $\psi_{2(1/1)} = \frac{4}{3}\phi_2 = 1.333\times.3 = \underline{.40}$

(87), $k_{a(1/1)} = c\,\psi_{2(1/1)} = .50\times.40 = \underline{.20}$

$k_{a_l} = (F_2/F)k_{2_l} = .78\times.25173 = \underline{.196}$

The values os R at the particular points are computed below.

(101), if $k_a = 0$, $R_{(k_a:0)} = \tau' - \chi^2$ where τ is assumed constant and is taken as τ'.

$$= .93-\left(\frac{.97}{1.12}\right)^2 = \underline{.180}$$

(102), if $k_a = \Upsilon$, $R_{(k_a:\Upsilon)} = \tau' - \Upsilon^2 = .93 - .5^2 \times .97 + \underline{.688}$

(100), if $k_a = c\sqrt{\left\{(\eta+k_2^2)+\phi_2^2\right\}-2c^2\phi^2}$,

$$R_{max.} = \tau' - c\left[\left\{(\eta+k_2^2)+\phi_2^2\right\} - c^2\phi^2\right]$$

$$k_{a(Rmax.)} = .5\times\sqrt{.97-2\times.5^2\times.56^2} = \underline{.451}$$

$$R_{max.} = .93-.5^2\times\left[.97-.5^2\times.56^2\right] = \underline{.707}$$

The equation (104) reduces to

$$R = R_{max.} - \left[(\chi - c^2\phi) - \frac{k_a^2}{2c^2\phi}\right]^2 \tag{113}$$

(113),
$$R = R_{max.} - \left[(\chi - .5^2\times.56) - \frac{k_a^2}{2\times.5^2\times.56}\right]^2$$

$$R = R_{max.} - \left[.72607 - \frac{k_a^2}{.28}\right]^2 \tag{114}$$

$k_{a(1/1)} = \underline{.20}$, as above computed,

$k_{a(3/4)} = (3/4)\ k_{a(1/1)} = \underline{.15}$

$k_{a(1/2)} = (1/2)\ k_{a(1/1)} = \underline{.10}$

$k_{a(1/4)} = (1/4)\ k_{a(1/1)} = \underline{.05}$

admis-sions	k_a	k_a^2	$\frac{k_a^2}{.28}$
(1/1)	.20	.0400	.142857
(3/4)	.15	.0225	.080357
(1/2)	.10	.0100	.035714
(1/4)	.05	.0025	.008928

If the above values are substituted in (114)

$R_{(1/1)} = R_{max.} - .58321^2 = .7071 - .34014 = \underline{.367}$

$R_{(3/4)} = R_{max.} - .64571^2 = .7071 - .41695 = \underline{.290}$

$R_{(1/2)} = R_{max.} - .69036^2 = .7071 - .47659 = \underline{.231}$

$R_{(1/4)} = R_{max.} - .71714^2 = .7071 - .51429 = \underline{.193}$

Fig. 64 shows the velocity diagrams of this example, assuming that the point K moves on the entrance parabola (XY) during the regulation.

Fig. 65 shows the "k_aR" curve, corresponding to Fig. 64, assuming that the point K moves on the entrance parabola and τ' is taken at $\underline{.93}$ during the regulation.

Fig. 66 shows the graphical method to plot the same curve as that in Fig. 65. The values of R in this figure are almost fit to the computed ones.

The example 18.

The changing value of R by the regulation for a Francis turbine of the type II_4 with

$$\phi = .64,\ \phi_2 = .45,\ F_2/F = .80,\ \beta_2 = 30^\circ \text{ and } \tau' = .94$$

(88), $c = (F_2/F)\sin\beta_2 = .8\times.5 = \underline{.40}$

$k_{2t} = \phi_2\ \mathrm{tg}\beta_2 = .45\times.57735 = .25981$

$k_{2t}^2 = .25981^2 = \underline{.067}$

$\eta_t = 2\phi^2 = 2\times.64^2 = \underline{.819}$

if $(\eta+k_2^2)$ is taken at the value of $(\eta_t+k_{2t}^2)$,

$\eta+k_2^2 = .819+.067 = \underline{.886}$

(95). $\chi = \phi'_{(k_a:0)} = \frac{\eta+k_2^2+\phi_2^2}{2\phi} = \frac{.886+.45^2}{2\times.64} = \underline{8.51}$

(96), $\Upsilon = k_{a(\phi':0)} = c\sqrt{\eta+k_2^2+\phi_2^2} = .40\times\sqrt{1.0887} = \underline{.417}$

(89), $\psi_{x(1/1)} = 1.333\ \phi_2 = 1.333\times.45 = \underline{.60}$

(87), $k_{a(1/1)} = c\ \psi_{x(1/1)} = .4\times.6 = \underline{.24}$

$k_{at} = (F_2/F)\,k_{2t} = .8\times.25981 = \underline{.208}$

(101), if $k_a = 0$, $R_{(k_a:0)} = \tau' - \chi^2 = .94 - .85055^2 = \underline{.217}$

(102), if $k_a = \Upsilon$, $R_{(k_a:\Upsilon)} = \tau' - \Upsilon^2 = .94 - .4^2\times1.0887 = \underline{.766}$

(100), if $k_a = c\sqrt{(\eta+k_2^2+\phi_2^2)-2c^2\phi^2}$,

$$R_{max.} = \tau' - c^2\left[(\eta+k_2^2+\phi_2^2) - c^2\phi^2\right]$$

$$k_{a(R_{max.})} = .4\times\sqrt{1.0887-2\times.4^2\times.64^2} = \underline{.391}$$

$$R_{max.} = .94-.4^2\times\left[1.0887-.06554\right] = \underline{.776}$$

(113), $$R = R_{max.}-\left[(\chi-c^2\phi)-\frac{k_a^2}{2c^2\phi}\right]^2$$

$$R = R_{max.}-\left[(\chi-.4^2\times.64)-\frac{k_a^2}{2\times.4^2\times.64}\right]^2$$

$$R = R_{max.}-\left[.74815-4.88281\ k_a^2\right]^2 \qquad (115)$$

$k_{a(1/1)} = \underline{.24}$ $\qquad$ $k_{a(3/4)} = (3/4)\ k_{a(1/1)} = \underline{.18}$

$k_{a(1/2)} = (1/2)\ k_{a(1/1)} = .12$ and $k_{a(1/4)} = (1/3)\ k_{a(1/1)} = \underline{.06}$

admissions	k_a	k_a^2	$4.88281\ k_a^2$
(1/1)	.24	.0576	.28125
(3/4)	.18	.0324	.15820
(1/2)	.12	.0144	.07031
(1/4)	.06	.0036	.01758

If the above values are substituted in (115),

$$R_{(1/1)} = R_{max.}-.46690^2 = .77629-.21799 = \underline{.558}$$

$$R_{(3/4)} = R_{max.}-.58994^2 = .77629-.34803 = \underline{.428}$$

$$R_{(1/2)} = R_{max.}-.67783^2 = .77629-.45946 = \underline{.317}$$

$$R_{(1/4)} = R_{max.}-.73057^2 = .77629-.53373 = \underline{.243}$$

Fig. 67 shows the velocity diagrams of this example, assuming that the point K moves on the entrance parabola (XY) during the regulation. Fig. 68 shows the "k_aR" curve corresponding to Fig. 67, assuming that the point K moves on the entrance parabola and $\mathcal{T}'$ takes at $\underline{.94}$ during the regulation.

The example 19.

The changing value of R by the regulation for a Francis turbine of the type III_1 with

$\phi = .80$, $\phi_2 = .60$, $F_2/F = .83$, $\beta_2 = 25°$ and $\mathcal{T}' = .95$.

(88), $c = (F_2/F)\ \sin\beta_2 = .83\times.42262 = \underline{.35}$

$k_{2t} = \phi_2\ \text{tg}\beta_2 = .60\times\text{tg}25° = .6\times.46631 = .27979$

$k_{2t}^2 = .27979^2 = \underline{.07827}$ and take $\eta_t = \underline{.82}$

if $(\eta+k_2^2)$ is taken at the value of $(\eta_t+k_{2t}^2)$,

$\eta+k_2^2 = .82+.07827 = \underline{0.9}$

(95), $$\chi = \frac{\eta+k_2^2+\phi_2^2}{2\phi} = \frac{.9+.6^2}{2\times.8} = \underline{.788}$$

(96), $$\mathcal{Y} = c\sqrt{\eta+k_2^2+\phi_2^2} = .35\times\sqrt{.9+.6^2} = \underline{.393}$$

(89), $\psi_{2(1/1)} = 1.333\,\phi_2 = 1.333\times.6 = \underline{.8}$

(87), $k_{a(1/1)} = c\,\psi_{2(1/1)} = .35\times.8 = \underline{.28}$

$k_{at} = (F_2/F)\,k_{2t} = .83\times.27979 = \underline{.232}$

(101), if $k_a = 0$, $R_{(k_a:0)} = \mathcal{T}'-\chi^2 = .95-.7875^2 = \underline{.330}$

(102), if $k_a = \mathcal{Y}$, $R_{(k_a:\mathcal{Y})} = \mathcal{T}'-\mathcal{Y}^2 = .95-.35^2\times(.9+.6^2) = \underline{.796}$

(100), if $k_a = c\sqrt{(\eta+k_2^2+\phi_2^2)-2c^2\phi^2}$,

$$R_{max.} = \mathcal{T}'-c^2\left[(\eta+k_2^2+\phi_2^2)-c^2\phi^2\right]$$

$$k_{a(R_{max.})} = .35\times\sqrt{(.9+.6^2)-2\times.35^2\times.8^2} = \underline{.368}$$

$$R_{max.} = .95-.35^2\times\left[1.26-.0784\right] = \underline{.805}$$

(113), $$R = R_{max.}-\left[(\chi-c^2\phi)-\frac{k_a^2}{2c^2\phi}\right]^2$$

$$R = R_{max.}-\left[(.7875-.35^2\times.8)-\frac{k_a^2}{1.6\times.36^2}\right]^2$$

$$R = R_{max.}-\left[.6895-\frac{k_a^2}{.4\times.49}\right]^2 \qquad (116)$$

$k_{a(1/1)} = \underline{.28}$ $\qquad$ $k_{a(3/4)} = (3/4)\ k_{a(1/1)} = \underline{.21}$

$k_{a(1/2)} = (1/2)\ k_{a(1/1)} = \underline{.14}$ and $k_{a(1/4)} = (1/4)\ k_{a(1/1)} = \underline{.07}$

admissions	k_a	k_a^2	$\frac{1}{.4\times.49}k_a^2$
(1/1)	.28	.0784	.400
(3/4)	.21	.0441	.225
(1/2)	.14	.0196	.100
(1/4)	.07	.0049	.025

If the above values are substituted in (116),

$$R_{(1/1)} = R_{max.} - .2895^2 = .80525 - .08381 = \underline{.721}$$
$$R_{(3/4)} = R_{max.} - .4645^2 = .80525 - .21576 = \underline{.589}$$
$$R_{(1/2)} = R_{max.} - .5895^2 = .80525 - .34751 = \underline{.458}$$
$$P_{(1/4)} = R_{max.} - .6645^2 = .80525 - .44156 = \underline{.364}$$

Fig. 69 shows the velocity diagrams of this example assuming that the point K moves on the entrance parabola (XY) during the regulation. Fig. 70 shows the "$k_a R$" curve corresponding to Fig. 69, assuming that the point K moves on the entrance parabola and τ' takes at .95 during the regulation.

The example 20.

The changing values of R by the regulation for an axial flow turbine of the type III_2 with

$$\phi = .98,\ \phi_2 = \phi,\ \beta_2 = 15^\circ \text{ and } F_2/F = .966$$

(88), $c = (F_2/F)\sin\beta_2 = (F_2/F)\sin 15^\circ = .966 \times .25882 = \underline{.25}$

$k_{2t} = \phi_2 \operatorname{tg}\beta_2 = .98 \times \operatorname{tg}15^\circ = .98 \times .26795 = \underline{.25723}$

$k_{2t}^2 = .25723^2 = \underline{.06617}$ and take $\eta_t = \underline{.82}$

if $(\eta + k_2^2)$ is taken at the value $(\eta_t + k_{2t}^2)$,

$\eta + k_2^2 = .82 + .06617 = \underline{.89}$

For the type III_2 $\tau' = \phi^2 = .98^2 = \underline{.9604}$ take $\tau' = \underline{.96}$

(95), $\chi = \dfrac{\eta + k_2^2 + \phi_2^2}{2\phi} = \dfrac{.89 + .98^2}{2 \times .98} = \underline{.944}$

(96). $\Upsilon = c\sqrt{\eta + k_2^2 + \phi_2^2} = .25 \times \sqrt{.89 + .98^2} = \underline{.340}$

(89). $\psi_{2(1/1)} = \underline{1.15}\ \phi_2 = 1.15 \times .98 = \underline{1.127}$

(87). $k_{a(1/1)} = c\,\psi_{2(1/1)} = .25 \times 1.127 = \underline{0.28}$

$k_{a_t} = (F_2/F)\,k_{2t} = .966 \times .25723 = \underline{.248}$

(101), if $k_a = 0$, $R_{(k_a:0)} = \tau' - \chi^2 = .96 - .944^2 = \underline{.069}$

(102), if $k_a = \Upsilon$, $R = \tau' - \Upsilon^2 = .96 - .25^2 \times (.89 + .98^2) = \underline{.844}$

(100), if $k_a = c\sqrt{(\eta + k_2^2 + \phi_2^2) - 2c^2\phi^2}$

$$R_{max.} = \tau' - c^2\left[(\eta + k_2^2 + \phi_2^2) - c^2\phi^2\right]$$

$$k_{a(Rmax)} = .25 \times \sqrt{(1.8504) - 2 \times \frac{.98^2}{16}} = \underline{.329}$$

$$R_{max.} = .96 - \frac{1}{16} \times \left[(1.8504) - .06003\right] = \underline{.848}$$

(113),

$$R = R_{max.} - \left[(\chi - c^2\phi) - \frac{k_a^2}{2c^2\phi}\right]^2$$

$$R = R_{max.} - \left[\left(.94408 - \frac{.98}{16}\right) - \frac{8}{.98}k_a^2\right]^2$$

$$R = R_{max.} - \left[.88283 - \frac{4}{.49}k_a^2\right]^2 \qquad (117)$$

$k_{a(1/1)} = \underline{.28}$ $\qquad k_{a(3/4)} = (3/4)\,k_{a(1/1)} = \underline{.21}$

$k_{a(1/2)} = (1/2)\,k_{a(1/1)} = \underline{.14}$ and $k_{a(1/4)} = (1/4)\,k_{a(1/1)} = \underline{.07}$

admis-sions	k_a	k_a^2	$\frac{4}{.49}k_a^2$
(1/1)	.28	.0784	.64
(3/4)	.21	.0441	.36
(1/2)	.14	.0196	.16
(1/4)	.07	.0049	.04

If the above values are substituted in (117),

$$R_{(1/1)} = R_{max.} - .24283^2 = .84810 - .05897 = \underline{.789}$$
$$R_{(3/4)} = R_{max.} - .52283^2 = .84810 - .27335 = \underline{.575}$$
$$R_{(1/2)} = R_{max.} - .72283^2 = .84810 - .52249 = \underline{.326}$$
$$R_{(1/4)} = R_{max.} - .84283^2 = .84810 - .71037 = \underline{.138}$$

Fig. 71 shows the velocity diagrams of this example, assuming that the point K moves on the entrance parabola (XY) during the regulation. Fig. 72 shows the "$k_a R$" curve corresponding to Fig. 71, assuming that the point K moves on the entrance parabola and τ' takes at $\underline{.96}$ during the regulation.

The example 21.

The changing values of R by the regulation for an axial flow turbine of the type III_3 with

$$\phi=\phi_2=1.6,\ F_2/F=.98,\ \beta_2=10^\circ \text{ and } \tau'=.96.$$

(88), $c = (F_2/F)\sin\beta_2 = (F_2/F)\sin 10^\circ = .98\times.17365 = \underline{.17}$

$k_{2l} = \phi_2 \operatorname{tg}\beta_2 = 1.6\times\operatorname{tg}10^\circ = 1.6\times.17633 = \underline{.28213}$

$k_{2l}^2 = .28213^2 = \underline{.08}$ and take $\eta_l = \underline{.82}$

if $(\eta+k_2^2)$ is taken at the value of $(\eta_l+k_{2l}^2)$,

$\eta+k_2^2 = .82+.08 = \underline{.90}$

(95), $\chi = \dfrac{\eta+k_2^2+\phi_2^2}{2\phi} = \dfrac{.9+1.6^2}{2\times1.6} = \underline{1.081}$

(96), $\Upsilon = c\sqrt{\eta+k_2^2+\phi_2^2} = .17\times\sqrt{.9+1.6^2} = \underline{.316}$

(89), $\psi_{2(1/1)} = \underline{1.12}\ \phi_2 = 1.12\times1.6 = \underline{1.792}$

(87), $k_{a(1/1)} = c\ \psi_{2(1/1)} = .17\times1.792 = \underline{.30}$

$k_{al} = (F_2/F)k_{2l} = .98\times.282 = \underline{.277}$

(101), if $k_a = 0$, $R_{(k_a:0)} = \tau'-\chi^2 = .96-1.08125^2 = \underline{-.209}$

(102), if $k_a=\Upsilon$, $R_{(k_a:\Upsilon)} = \tau'-\Upsilon^2 = .96-.17^2\times(.90+1.6^2) = \underline{.860}$

(103), if $R = 0$,

$$k_{a(R:0)}^2 = c^2\Big[(\eta+k_2^2+\phi_2^2)-2c^2\phi^2\Big]\pm2c^2\phi\sqrt{\tau'-c^2\Big[(\eta+k_2^2+\phi_2^2)-c^2\phi^2\Big]}$$

$$= .17^2\times\Big[3.46-2\times.17^2\times1.6^2\Big]$$

$$\pm2\times.17^2\times1.6\sqrt{.96-.17^2\times\Big[3.46-.17^2\times1.6^2\Big]}$$

$$= .09573 \pm .08585$$

$$= .00988 \text{ and } .18158$$

$k_{a(R:0)} = .099$ and $.426$, of which the latter is greater than Υ : .316

Hence take $k_{a(R:0)} = \underline{.099}$

(100), if $k_a = c\sqrt{(\eta+k_2^2+\phi_2^2)-2c^2\phi^2}$,

$$R_{max.} = \tau'-c^2\Big[(\eta+k_2^2+\phi_2^2)-c^2\phi^2\Big]$$

$k_{a(Rmax)} = .17\times\sqrt{(3.46)-2\times.17^2\times1.6^2} = \underline{.309}$

$R_{max.} = .96-.17^2\times\Big[(3.46)-.17^2\times1.6^2\Big] = \underline{.862}$

The equation (98) reduces to

$$R = \tau'-\chi^2+\left\{\frac{\chi}{c^2\phi}-1\right\}k_a^2-\frac{1}{4c^4\phi^2}k_a^4$$

$$R = .96-1.08125^2+\left\{\frac{1.08125}{.17^2\times1.6}-1\right\}k_a^2-\frac{1}{.4\times.17^4\times1.6^2}k_a^4$$

$$R = -.20910+22.38343\ k_a^2-116.92419\ k_a^4 \qquad (118)$$

$k_{a(1/1)} = \underline{.30}$ $\qquad k_{a(3/4)} = (3/4)\,k_{a(1/1)} = \underline{.225}$

$k_{a(1/2)} = (1/2)k_{a(1/1)} = \underline{.15}$ and $k_{a(1/4)} = (1/4)\,k_{a(1/1)} = \underline{.075}$

if $k_{a(1/1)} = .3$, $k_{a(1/1)}^2 = .3^2 = .09$ and $k_{a(1/1)}^4 = .3^4 = .0081$

(118), $R_{(1/1)} = -.20910+22.38343\times.09-116.92419\times.0081 = \underline{.858}$

if $k_{a(3/4)} = .225$, $k_{a(3/4)}^2 = .05063$ and $k_{a(3/4)}^4 = .00256$

(118), $R_{(3/4)} = -.20910+22.38343\times.05063-116.92419\times.00256$
$= \underline{.624}$

if $k_{a(1/2)} = .15$, $k_{a(1/2)}^2 = .0225$ and $k_{a(1/2)}^4 = .00050$

(118), $R_{(3/4)} = -.20910+22.38343\times.0225-116.92419\times.00050$
$= \underline{.235}$

if $k_{a(1/4)} = .075$, $k_{a(1/4)}^2 = .00563$ and $k_{a(1/4)}^4 = .00003$

(118), $R_{(1/4)} = -.20910+22.38343\times.00563-116.92419\times.00003$
$= \underline{-.087}$

Fig. 73 shows the velocity diagrams of this example, assuming that the point K moves on the entrance parabola (XY) during the regulation. Fig. 74 shows the "k_aR" curve corresponding to Fig. 73, assuming that the point K moves on the entrance parabola and τ' takes at $\underline{.96}$ during the regulation.

It is seen in the above examples that using the reasonable coefficients the value of $\mathcal{Y}$ in (96) is determined to be less than $\sqrt{\tau}$, and $\mathcal{Y}$ is usually less than $\mathcal{X}$. In (95), however, the value of $\mathcal{X}$ tends to become greater than $\sqrt{\tau}$, especially it is inevitable in the high-speed, axial-flow turbine of the type III_3, as illustrated in the example 21, in this case a value of k becomes as equal to $\sqrt{\tau}$ and R may become zero before the gate-opening is entirely shut off. At the small partial admission the actual value of k is less than that in the entrance parabola, and in the states near the zero admission the former diminishes remarkably, thus it seems as if R does not become zero before the entire closing. But since τ diminishes also by closing the gate opening and this is considerable after the (1/4) admission, it might not be neglected to take care of this point on the turbine design of the type III_3.

SUMMARY

A) General Remarks

1) The head "τH" will have to be not only that required to rotate the runner and to overcome the hydraulic resistance in the runner passage, but also enough to produce the velocity of discharging water at the outlet-edge of the runner. The reaction head is the remainder of "τH" reduced by "k^2H" which is the impulse head at the inlet edge. Then the degree of reaction

$$R = \tau - k^2, \tag{19}$$

in which τ is taken as 0.92 to 0.96, excepting the particular case of the type III_3. The existence of reaction turbine necessitates the degree of reaction satisfying the condition

$$0 < R < \tau, \tag{20}$$

which means the restriction of reaction.

2) The equation (19) reduces to

$$\left.\begin{aligned} &(22)\quad R = \tau - \phi^2 + 2(\phi\ \mathrm{ctg}\beta)\,k_a - (1+\mathrm{ctg}^2\beta)\,k_a^2 \\ &(32)\quad R = \tau - \phi^2 \frac{\sin^2\beta}{\sin^2(\alpha+\beta)} \end{aligned}\right\}\ \text{for the general case}$$

$$\left.\begin{aligned} &(67)\quad R = \tau - \left(\frac{\eta_l}{2\phi}\right)^2 - k_a^2 \\ &(79)\quad R = \tau - \left(\frac{\eta_l}{2\phi}\right)^2 \frac{1}{\cos^2\alpha} \end{aligned}\right\}\ \text{for the special case}$$

In general (22) and (32) are the equations of R which express the characteristics of R in regard to other coefficients, and in the state of normal exit (67) and (79) are such equations. For the existence of reaction turbines the value of R must of course satisfy the condition (20), although R is expressed in the several equations.

3) By combining the ranges of β with those of ϕ the author imagines nine types of turbines fully filled with the flowing water as

ranges of ϕ / ranges of β	case 1 $\phi < \sqrt{\tau}$	case 2 $\phi = \sqrt{\tau}$	case 3 $\phi > \sqrt{\tau}$
group I $\pi > \beta > \frac{\pi}{2}$	type I_1 $\begin{cases}\beta > \frac{\pi}{2} \\ \phi < \sqrt{\tau}\end{cases}$	type I_2 $\begin{cases}\beta > \frac{\pi}{2} \\ \phi = \sqrt{\tau}\end{cases}$	type I_3 $\begin{cases}\beta > \frac{\pi}{2} \\ \phi > \sqrt{\tau}\end{cases}$
group II $\beta = \frac{\pi}{2}$	type II_1 $\begin{cases}\beta = \frac{\pi}{2} \\ \phi < \sqrt{\tau}\end{cases}$	type II_2 $\begin{cases}\beta = \frac{\pi}{2} \\ \phi = \sqrt{\tau}\end{cases}$	type II_3 $\begin{cases}\beta = \frac{\pi}{2} \\ \phi > \sqrt{\tau}\end{cases}$
group III $0 < \beta < \frac{\pi}{2}$	type III_1 $\begin{cases}\beta < \frac{\pi}{2} \\ \phi < \sqrt{\tau}\end{cases}$	type III_2 $\begin{cases}\beta < \frac{\pi}{2} \\ \phi = \sqrt{\tau}\end{cases}$	type III_3 $\begin{cases}\beta < \frac{\pi}{2} \\ \phi > \sqrt{\tau}\end{cases}$

It is evident that there are no more types to be imagined

B) General Case

4) In general the value of R in (22) or (32) does not satisfy the condition (20) for the types I_2, I_3, II_2 and II_3, and accordingly there are no reaction turbines existent in these types. Hence the reaction turbines may be classified into five types I_1, II_1, III_1, III_2 and III_3.

5) The limit values of k_a and α with the restriction of reaction are

$$(28)\quad \begin{cases} k_{a_{II_r}} = \phi \sin\beta \cos\beta + \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta} \\ k'_{a_{II_r}} = \phi \sin\beta \cos\beta - \sin\beta\sqrt{\tau - \phi^2 \sin^2\beta} \quad \text{for only } III_3 \end{cases}$$

$$(36)\quad \begin{cases} \alpha_{a_{II_r}} = (\beta)'' - \beta \\ \alpha'_{a_{II_r}} = (\beta)' - \beta \quad \text{for only } III_3 \end{cases}$$

where $(\beta)''$: the root of $\sin^{-1} (\phi \sin\beta / \sqrt{\tau})$ in the range ($\pi/2$ to π)

$(\beta)'$: that in the range (0 to $\pi/2$),

then for reaction turbines the values of k_a and α must be as

$$\begin{cases} k_a < k_{a_{II_r}} \\ k'_{a_{II_r}} < k_a < k_{a_{II_r}} \quad \text{for only } III_3 \end{cases}$$

$$\begin{cases} \alpha < \alpha_{II_r} \\ \alpha'_{II_r} < \alpha < \alpha_{II_r} \quad \text{for only } III_3 \end{cases}$$

Further the value of β may be restricted as

$$\beta < (\beta)'' \text{ and } \beta \neq (\beta)''.$$

6) For the turbine series with τ, ϕ and β as the given values the "k_aR" and "αR" curves illustrate concisely the characteristics of R in respect to k_a and α. The tables 1 and 8 illustrate schematically these principal characteristics for nine types, and it is seen that the "R-portion" disappears in the types I_2, I_3, II_2 and II_3, and accordingly no reaction turbines exist in these types. Further the particular characteristics of curves in regard to the maximum value of R, the limits of k_a and α, $(\beta)'$ and $(\beta)''$, etc. are shown in Fig. 12 to Fig. 20 and Fig. 27 to Fig. 35.

C) Special Case

7) In the case of the normal exit the value of ϕ becomes as irrational for the types I_2, I_3, I_2 and II_3, and consequently the reaction turbines with the normal exit may also be classified into five types I_1, II_1, III_1, III_2 and III_3 as in the general case. This is of course necessary, because these turbines must be included within every one of types in the general case. In this case the values of ϕ are

the types	the values of ϕ
I_1	$\frac{\eta_t}{2\sqrt{\tau}} < \phi < \sqrt{\frac{\eta_t}{2}}$
II_1	$\phi = \sqrt{\frac{\eta_t}{2}}$
III_1	$\sqrt{\frac{\eta_t}{2}} < \phi < \sqrt{\tau}$
III_2	$\phi = \sqrt{\tau}$
III_3	$\sqrt{\tau} < \phi <$ about 0.30

If in the above $\tau = 0.92$ and $\eta_l = 0.82$, the lower limit of ϕ is computed as 0.43 which may be the smallest value of ϕ for all reaction turbines with the normal exit, as already mentioned. In practice 0.30 may be the almost upper limit of ϕ for III_3, and consequently this may be the nearly greatest value of ϕ for all reaction turbines with the normal exit.

8) The value of R in (67) and (79) becomes irrational for the types I_2, I_3, II_2 and II_3 in the state of the normal exit, thus it is again proved that any reaction turbines does not exist for these four.

9) For the turbines with the normal exit the limit values of k_a, α and β are respectively

$$(63) \quad k_{a_{lt}} = \sqrt{\tau - \frac{\eta_l^2}{4\phi^2}}, \qquad \text{then } k_a < k_{a_{lt}}.$$

$$\left.\begin{aligned} (60) \quad \alpha_{lt} &= \mathrm{tg}^{-1}\left(\frac{\sqrt{4\tau\phi^2-\eta_l^2}}{\eta_l}\right) \text{ or} \\ (58) \quad \alpha_{lt} &= \cos^{-1}\left(\frac{\eta_l}{2\phi\sqrt{\tau}}\right) \end{aligned}\right\} \alpha < \alpha_{lt}$$

$$(61) \quad \beta_{lt} = \mathrm{tg}^{-1}\left(\frac{\sqrt{4\tau\phi^2-\eta_l^2}}{2\phi^2-\eta_l}\right), \begin{cases} \beta > \beta_{lt} \text{ for } I_1 \\ \beta < \beta_{lt} \text{ for } III_1,\ III_2 \text{ and } III_3. \end{cases}$$

Assuming that τ and η_l are taken at the same values for all turbines, the values of $k_{a_{lt}}$ and α_{lt} are the smallest for I_1, these increase according to the order of I_1, II_1, III_1, III_2 and III_3, and these become the greatest for III_3.

10) For the turbine series with τ, ϕ and η_l as the given values, the "k_aR" curve which is a parabola with 1 as parameter and "αR" curve illustrate concisely the characteristics of R in regard to k_a and α respectively. It is seen in Fig. 45 to Fig. 49 and Fig. 53 that if τ and η_l are taken at the same values for all series, the curve with the smaller value of ϕ is situated at the lower position and that with the larger ϕ at the higher one, and the II_1, III_2 and limit curves become the boundary lines of the territories, in each of which many curves of the type I_1, III_1 or III_3 may exist.

If τ and ϕ are taken at the same values for the special and general cases, the "k_aR" or "αR" curve of the special case may become the locus of the points, any one of which corresponds to a turbine with normal exit on the "k_aR" or "αR" curve of the general case. All points on a "k_aR" or "αR" curve of the special case correspond to the turbines with the same value of η_l but with the different values of β. For the type II_2, however, these curves in both cases coincide with each other.

D) Changing Degree of Reaction by Speed Regulation

11) The equation of R changed by the speed regulation is

$$(98) \quad R = \tau - \frac{1}{4\phi^2}\left\{(\eta+k_2^2)+\phi_2^2\right\}^2 + \left\{\frac{(\eta+k_2^2)+\phi_2^2}{2c^2\phi^2} - 1\right\}k_a^2 - \frac{1}{4c^4\phi^2}k_a^4$$

In (98) τ and $(\eta+k_2^2)$ are almost constant in the states near the normal running, but vary remarkably at the small partial admissions which are less than (1/4). In theory, however, they are assumed as constant, then R may become a function of k_a with constants ϕ and ϕ_2. If the coefficient of (k_a^2) is assumed as positive, R has one maximum for a particular value of k_a and has one minimum for $k_a = 0$. Since the value of k_a corresponding to $R_{max.}$ is usually greater than that at the (1/1) admission, the degree of reaction may decrease by closing the gate opening.

12) If in the entrance parabola χ is greater than $\sqrt{\tau}$, R may become zero before the gate opening is entirely shut off. For the types I_1, II_1, III_1 and III_2 this might not occur by the good design, but for the type III_3 is perhaps inevitable.

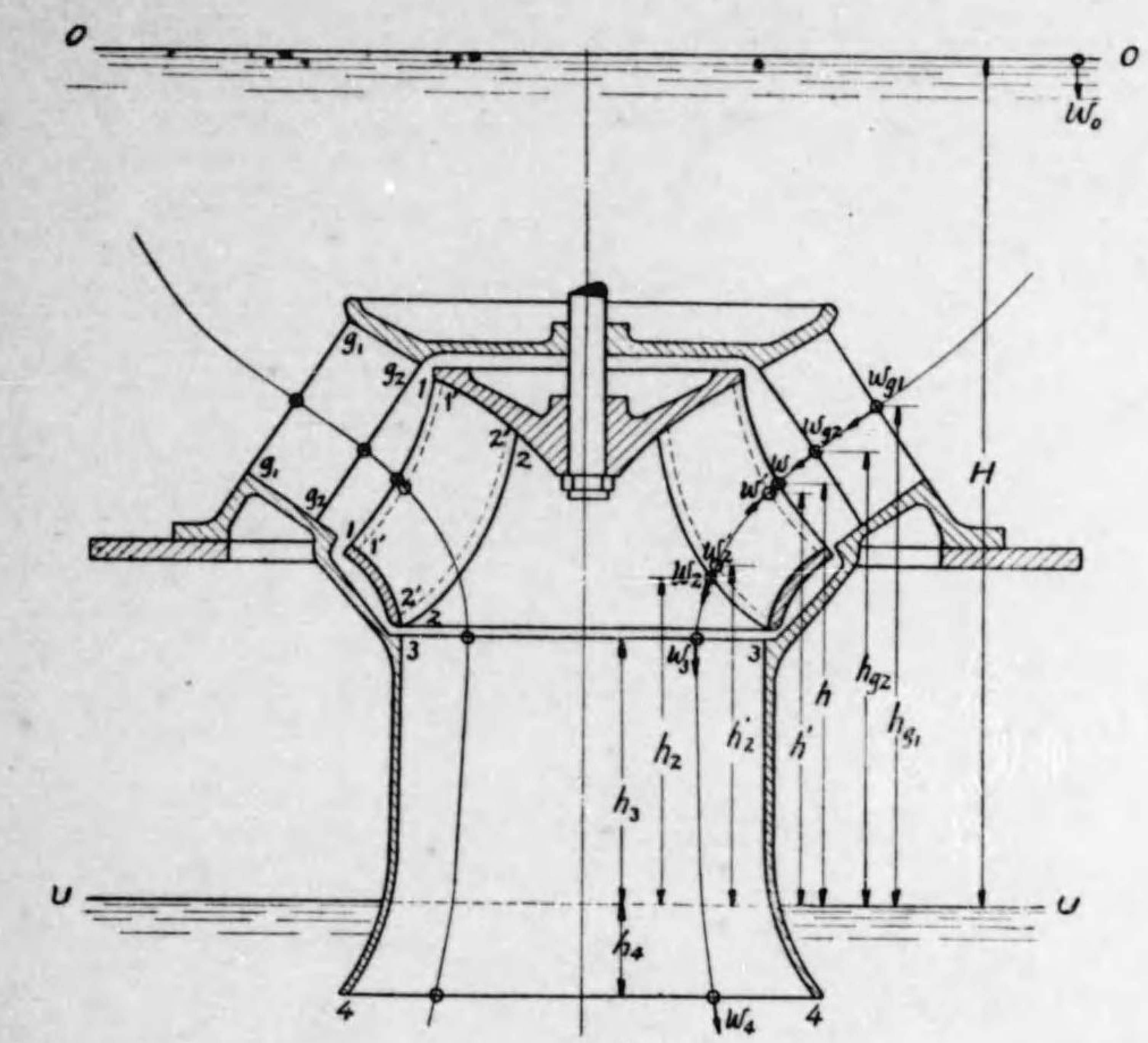
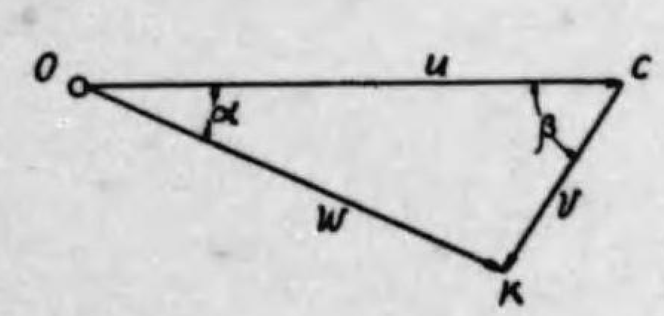
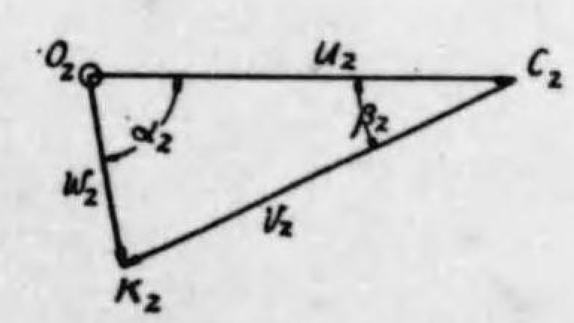


Fig. 1



Velocity Diagram at Inlet Edge of Runner $\overline{11}$



Velocity Diagram at Outlet Edge of Runner $\overline{22}$

Fig. 2

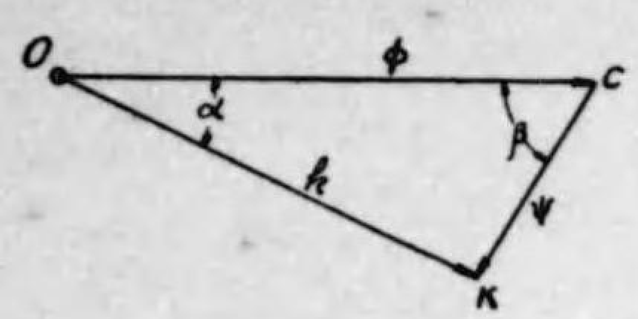


Fig. 3 Velocity-Coefficient Diagram at Inlet Edge of Runner

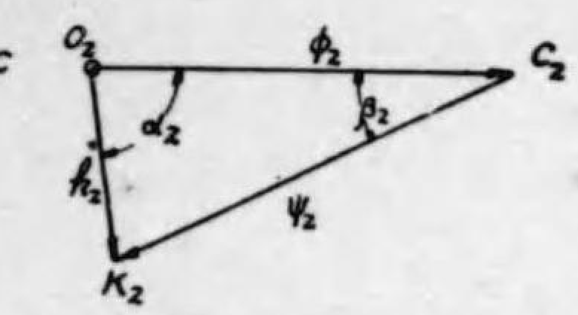


Fig. 4 Velocity-Coefficient Diagram at Outlet Edge of Runner

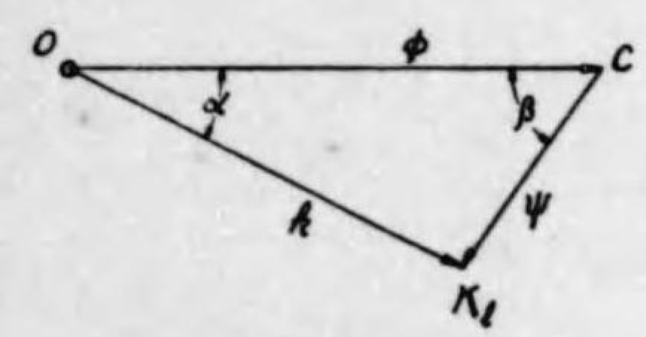


Fig. 5 Velocity-Coefficient Diagram with Normal Exit at Inlet Edge of Runner.

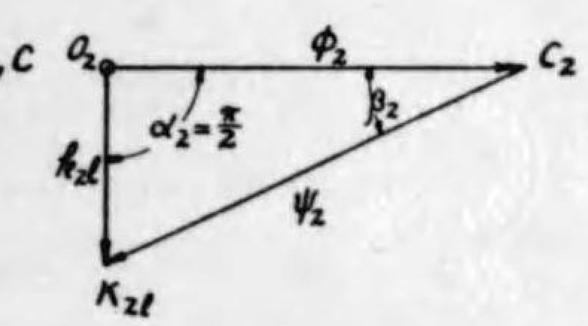


Fig. 6 Velocity-Coefficient Diagram with Normal Exit at Outlet Edge of Runner.

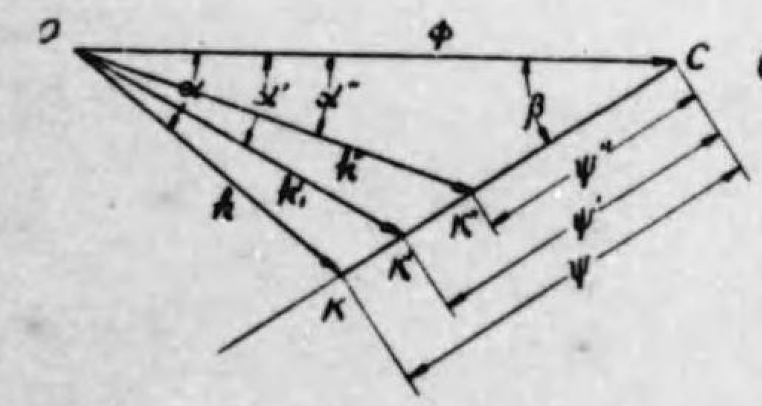


Fig. 7

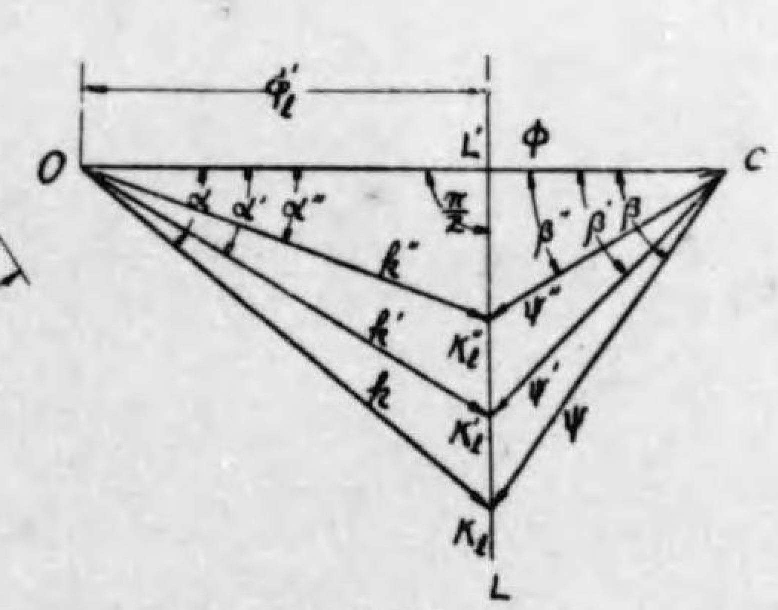


Fig. 8

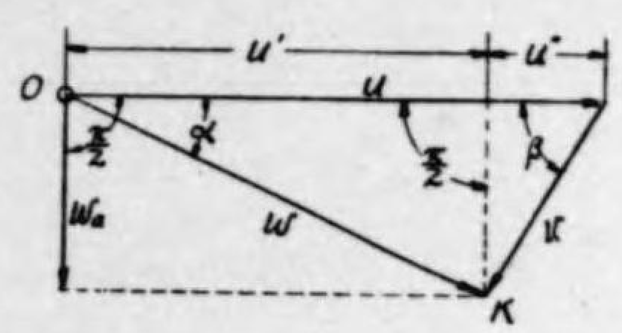


Fig. 9

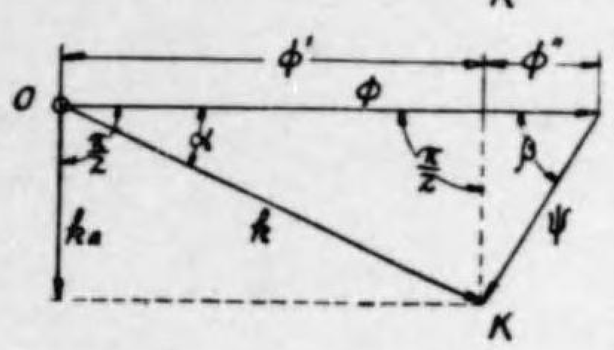


Fig. 10

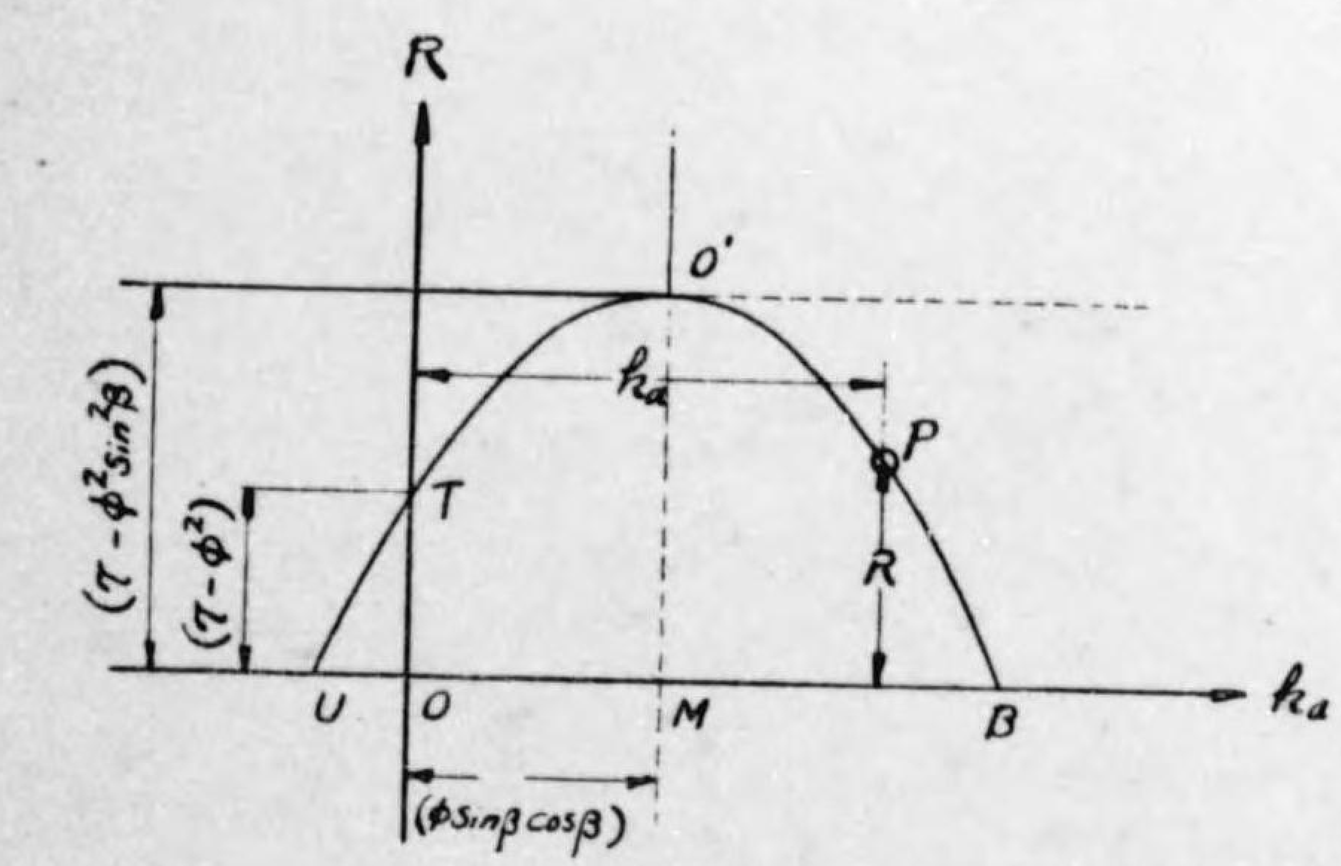


Fig. 11

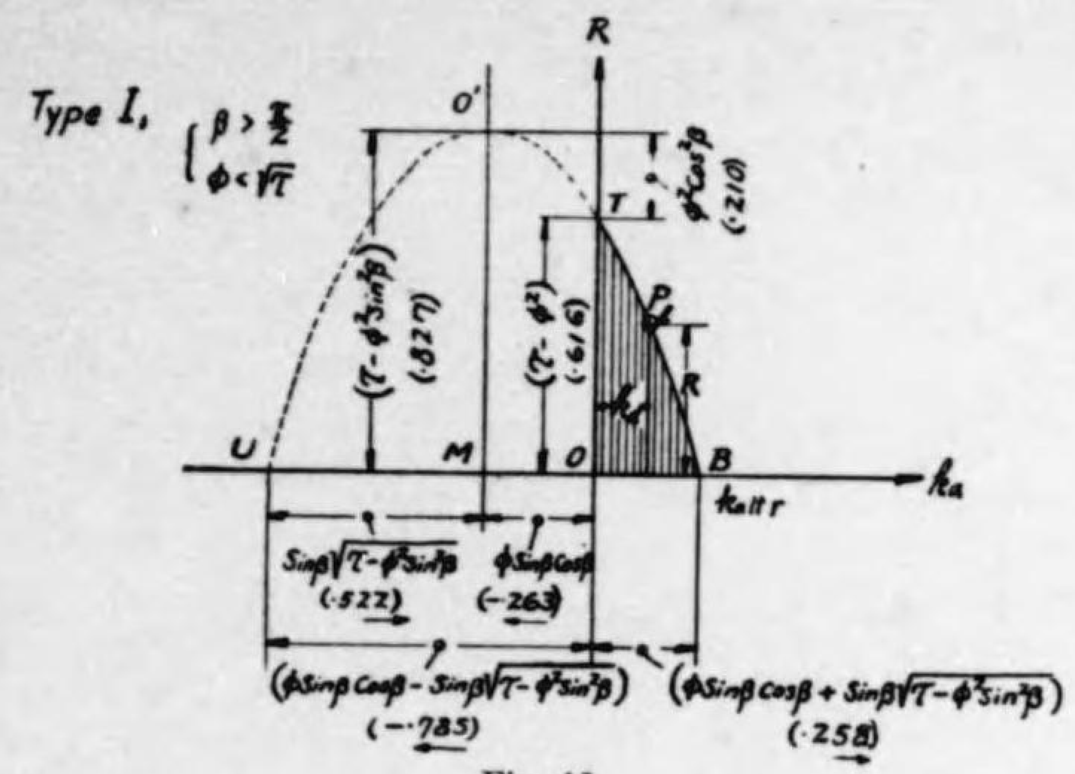


Fig. 12

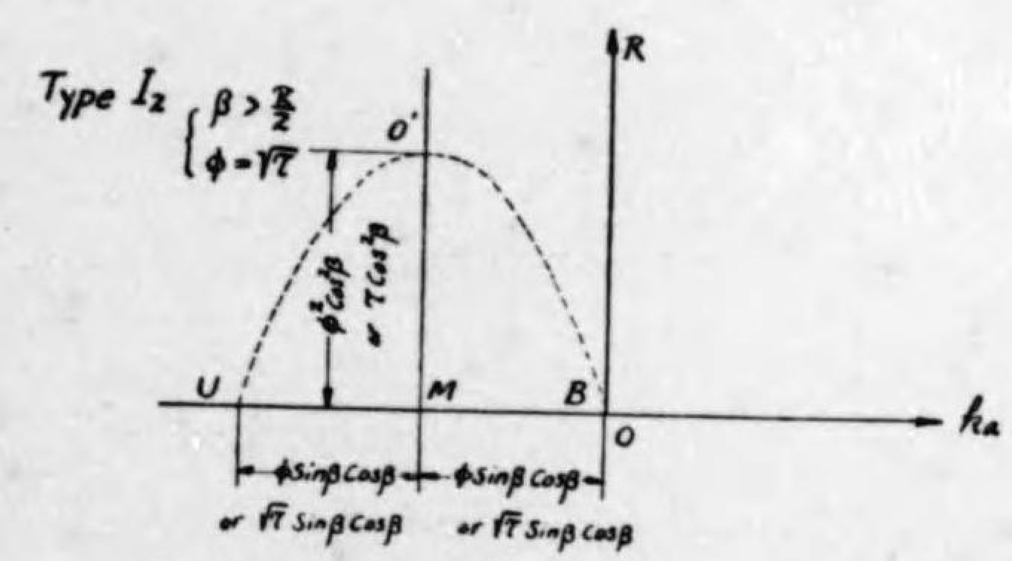


Fig. 13

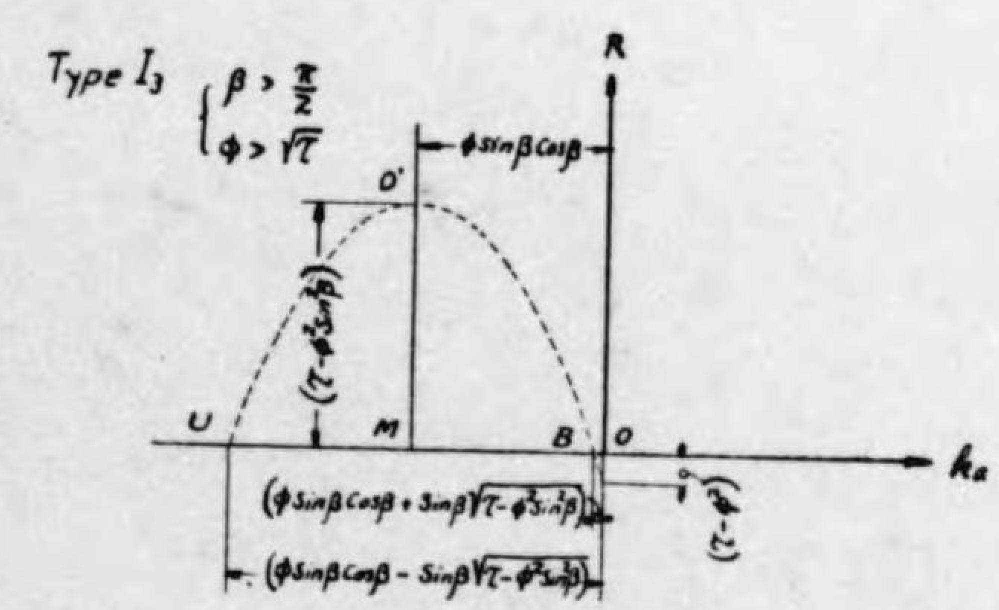


Fig. 14

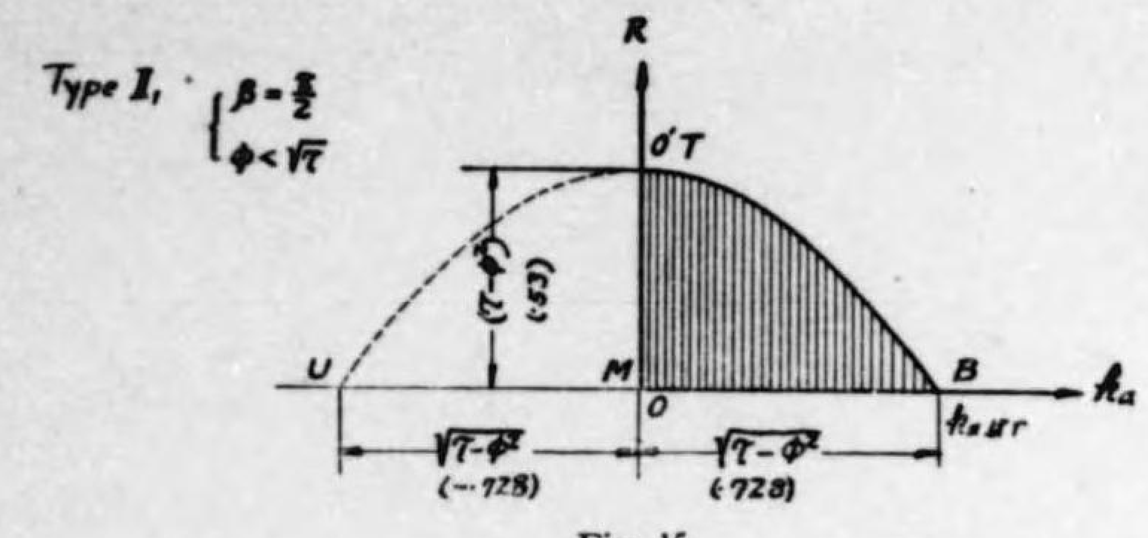


Fig. 15

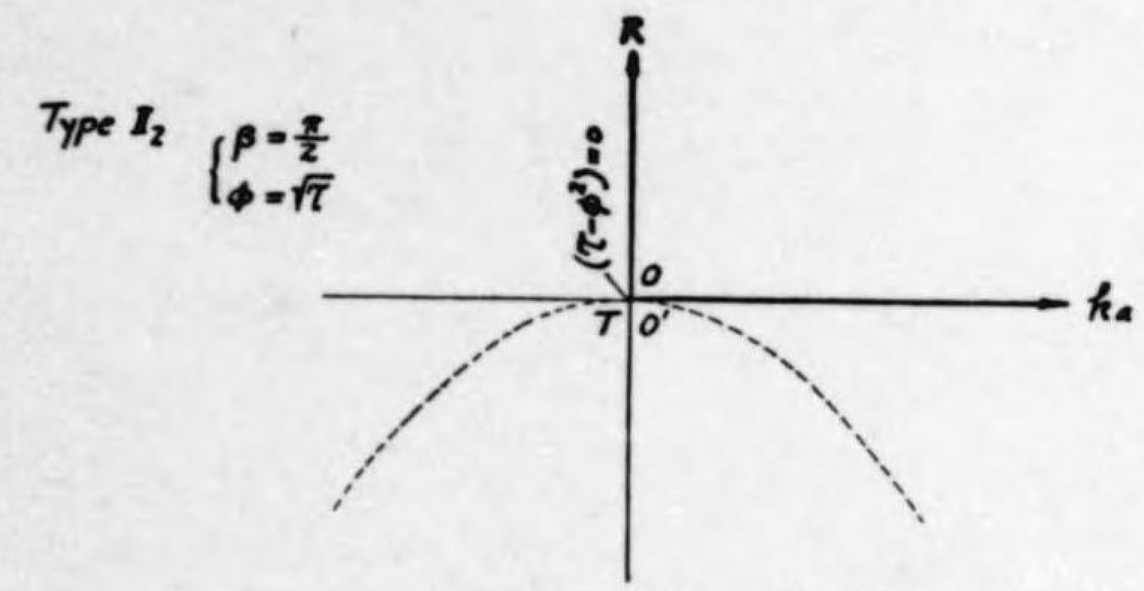


Fig. 16

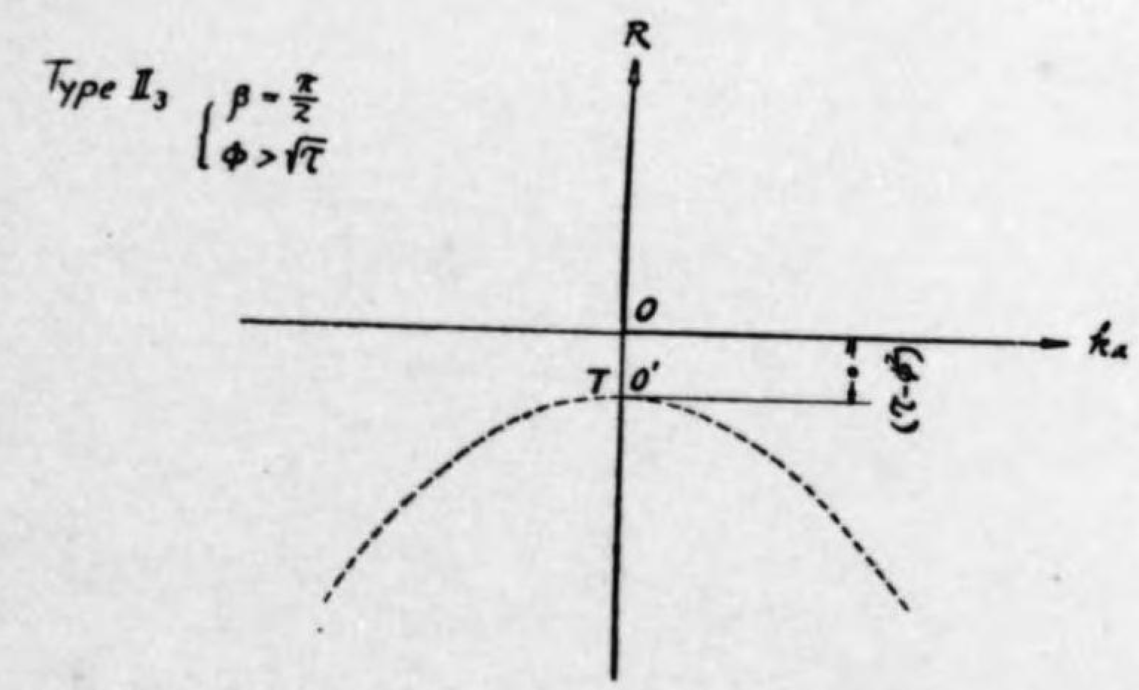


Fig. 17

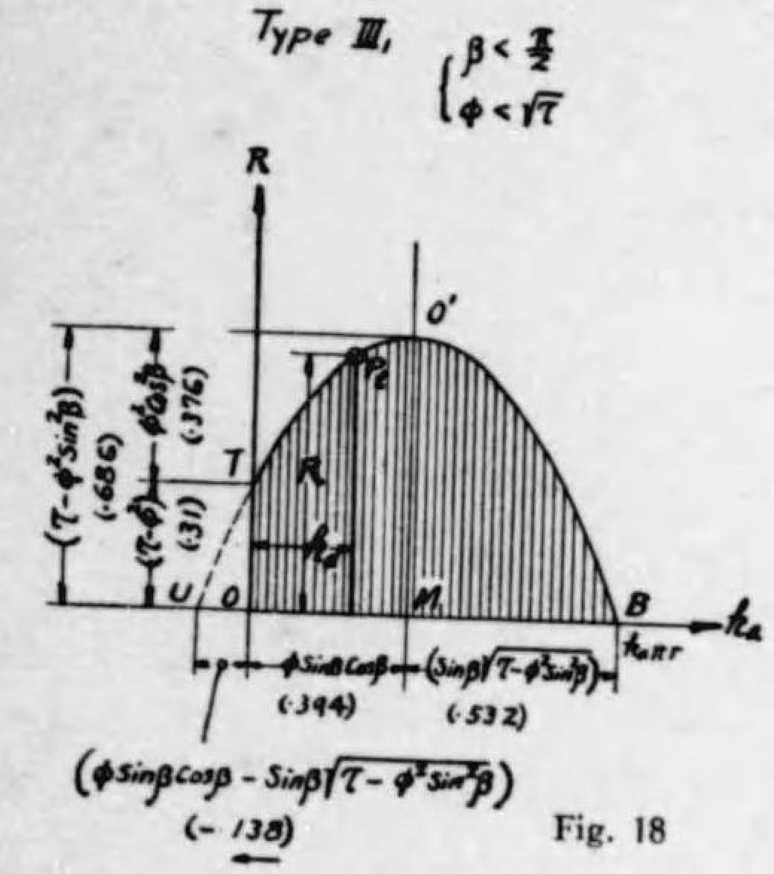


Fig. 18

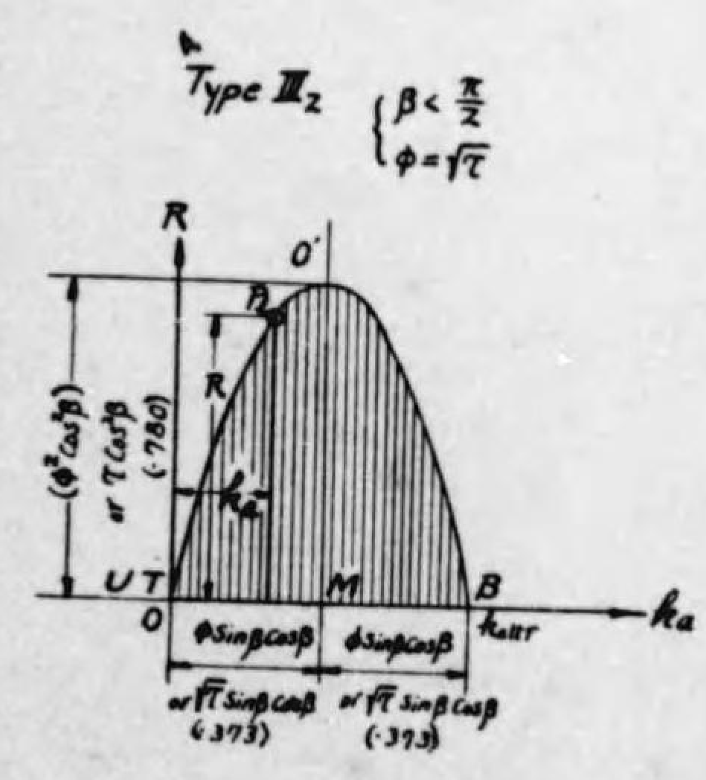


Fig. 19

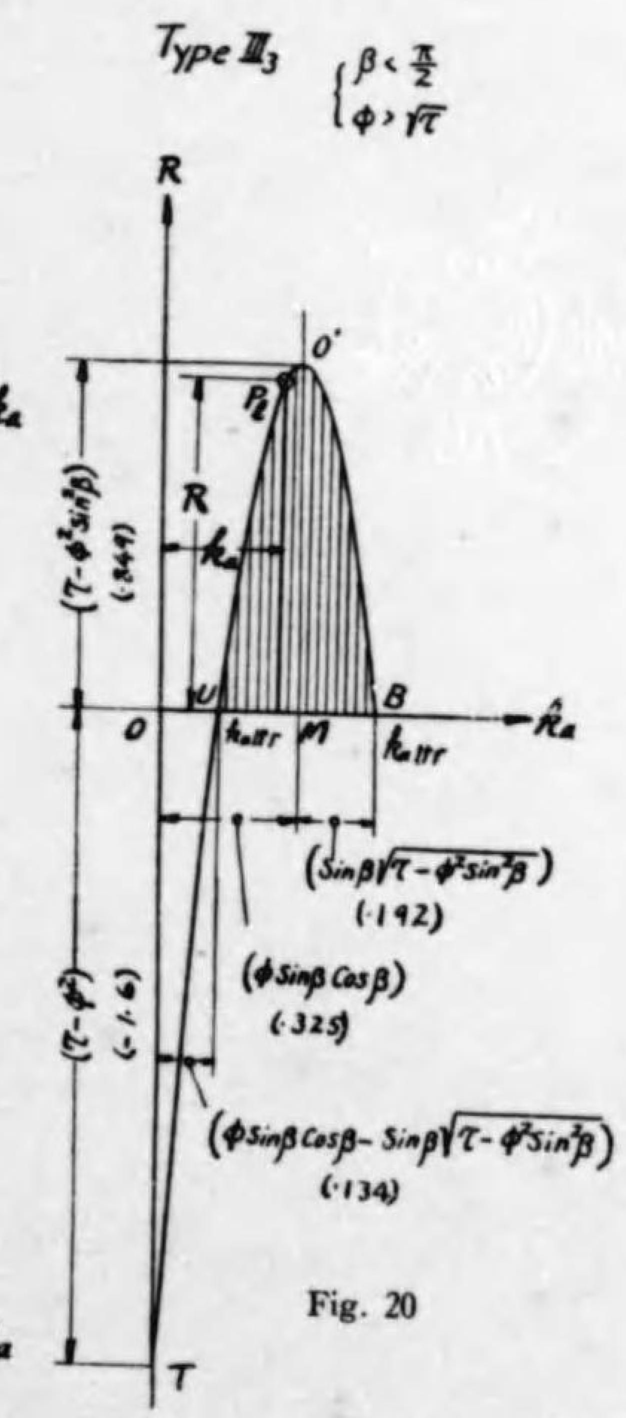


Fig. 20

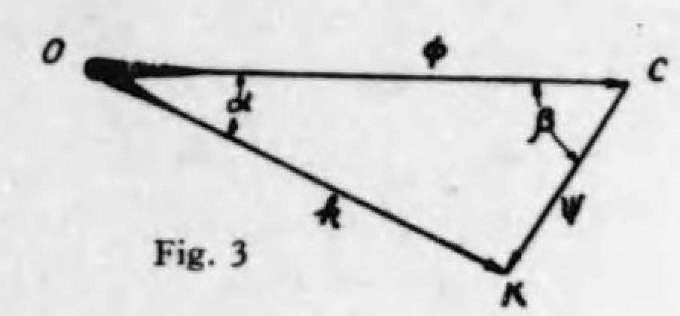


Fig. 3

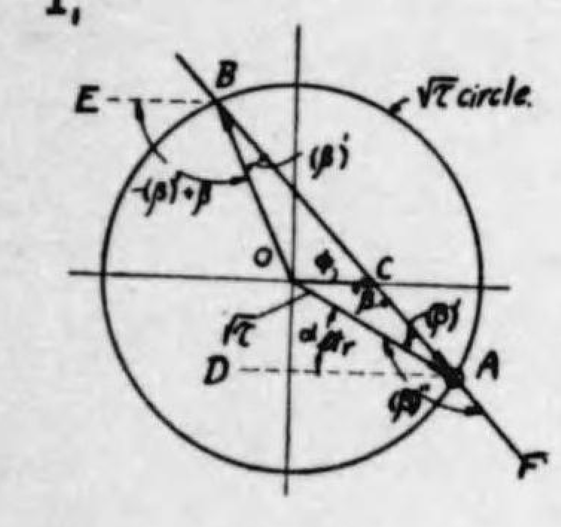


Fig. 21

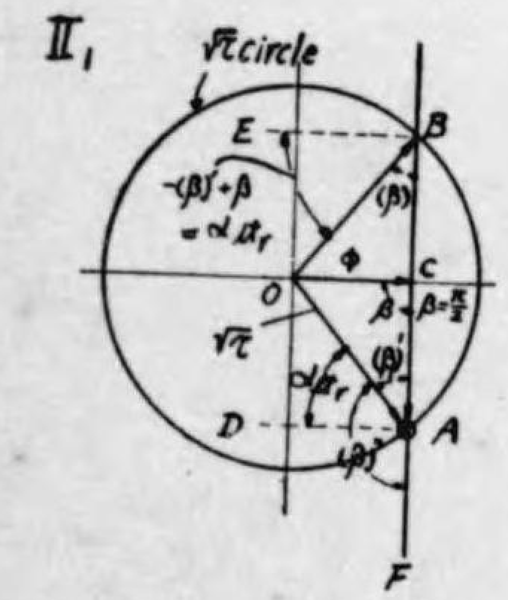


Fig. 22

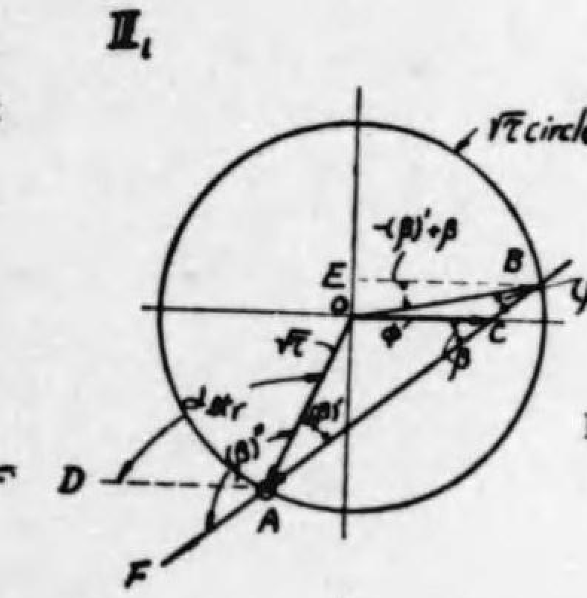


Fig. 23

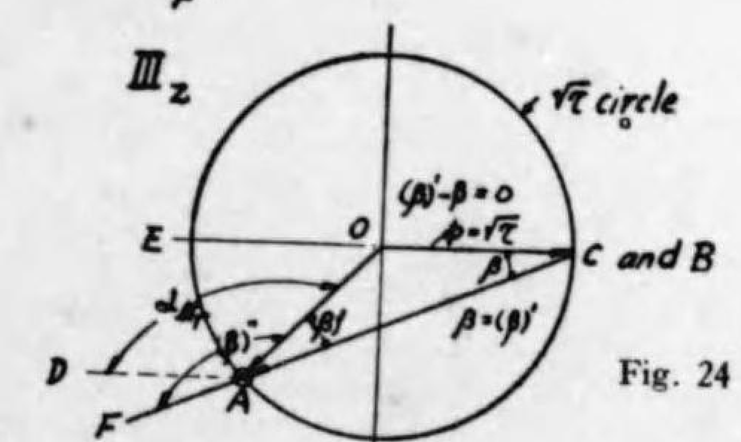


Fig. 24

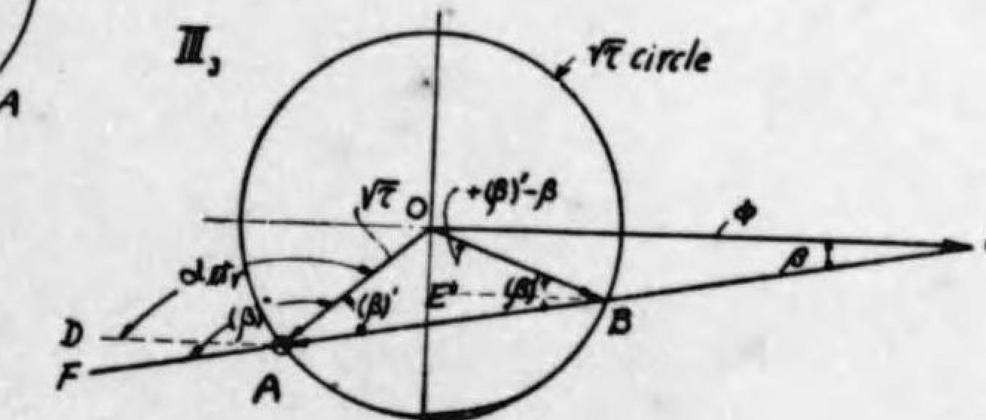


Fig. 25

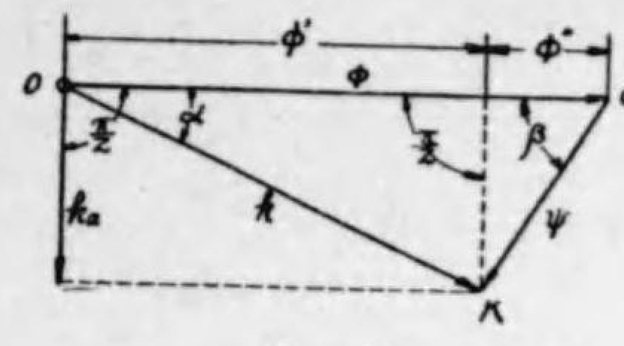
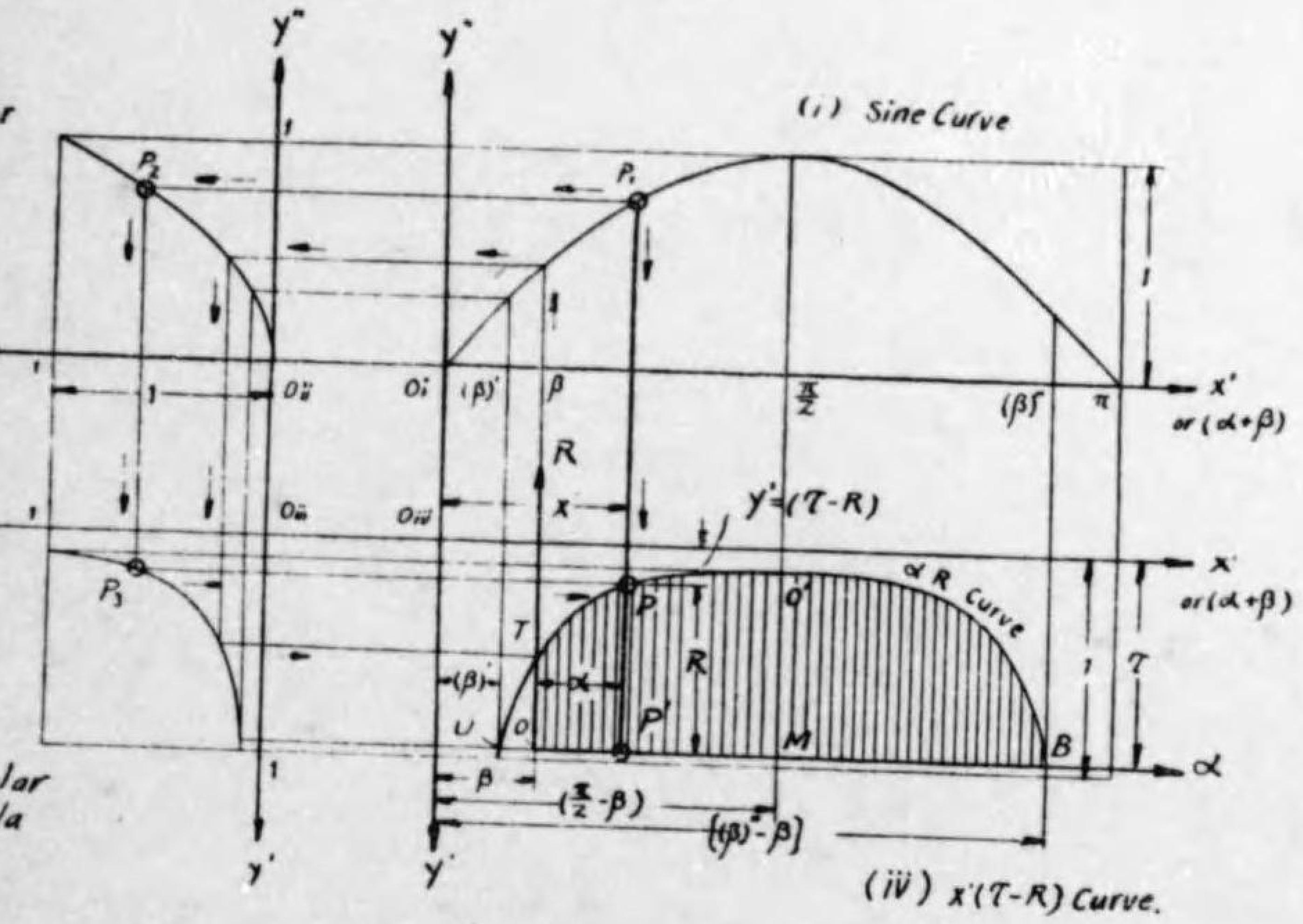


Fig. 10


(ii) Parabola
With 1 as parameter
(i) Sine Curve
(iii) Rectangular
Hyperbola
(iv) x'(T-R) Curve.
αR Curve
Fig. 26

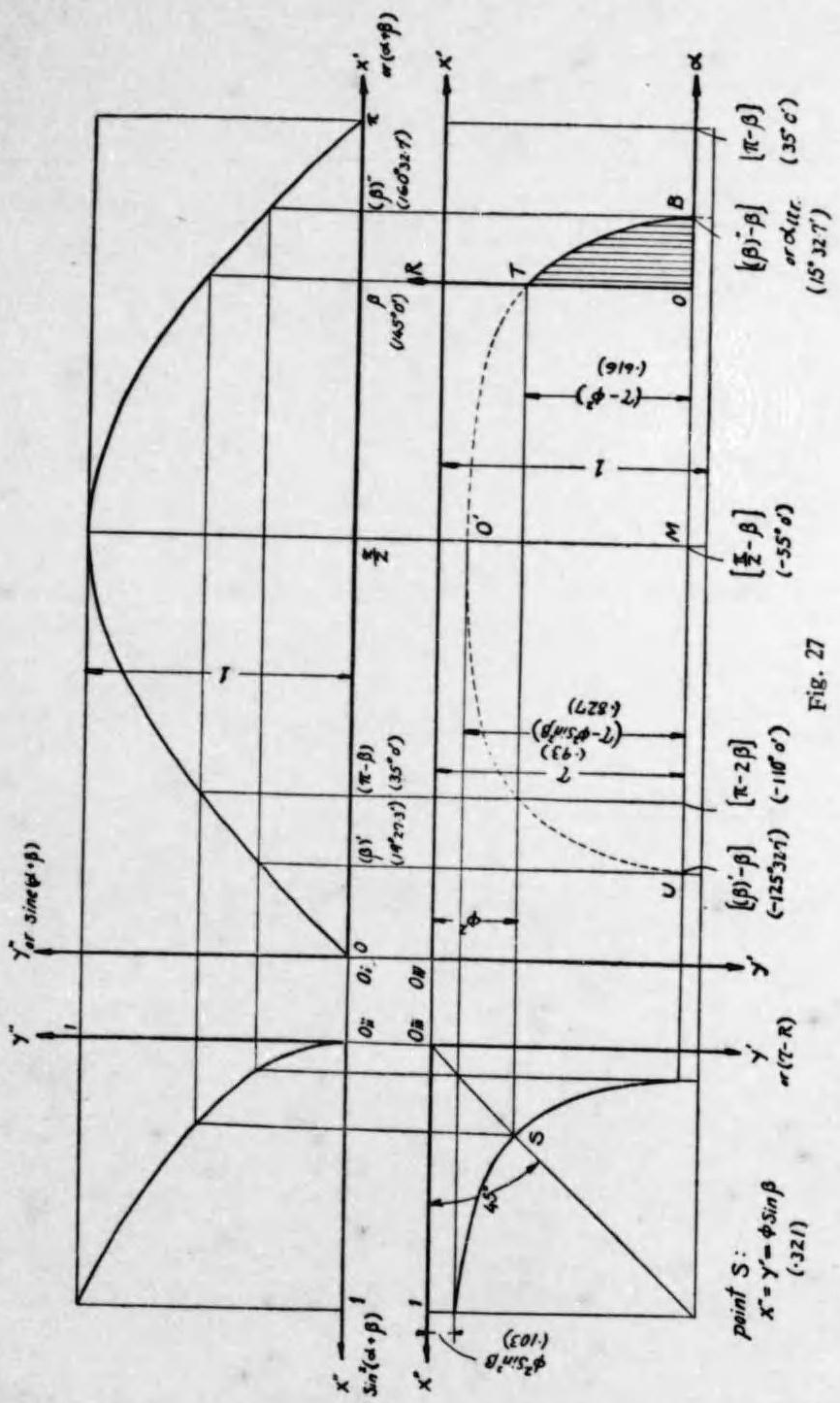


Fig. 27

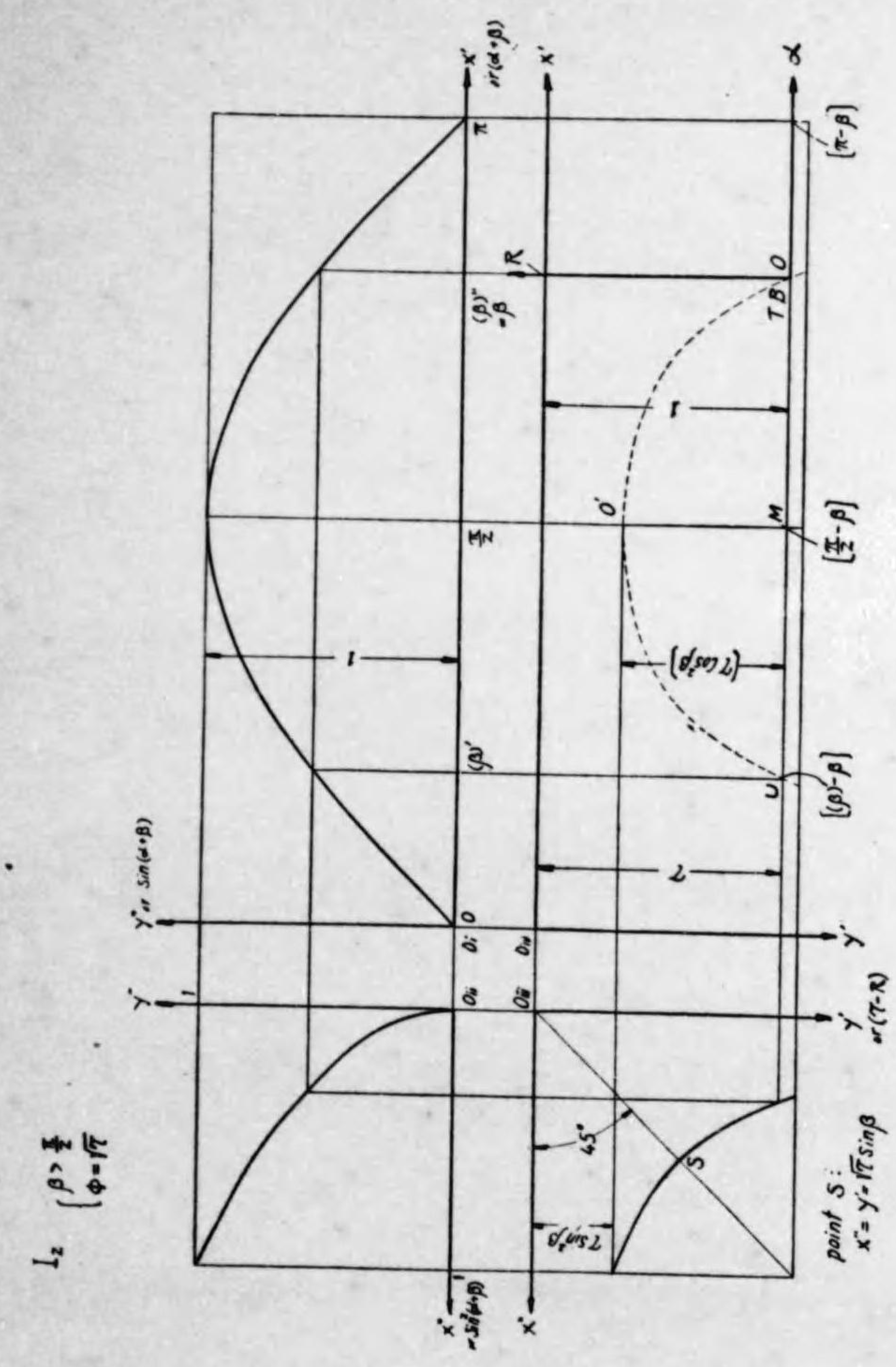


Fig. 28

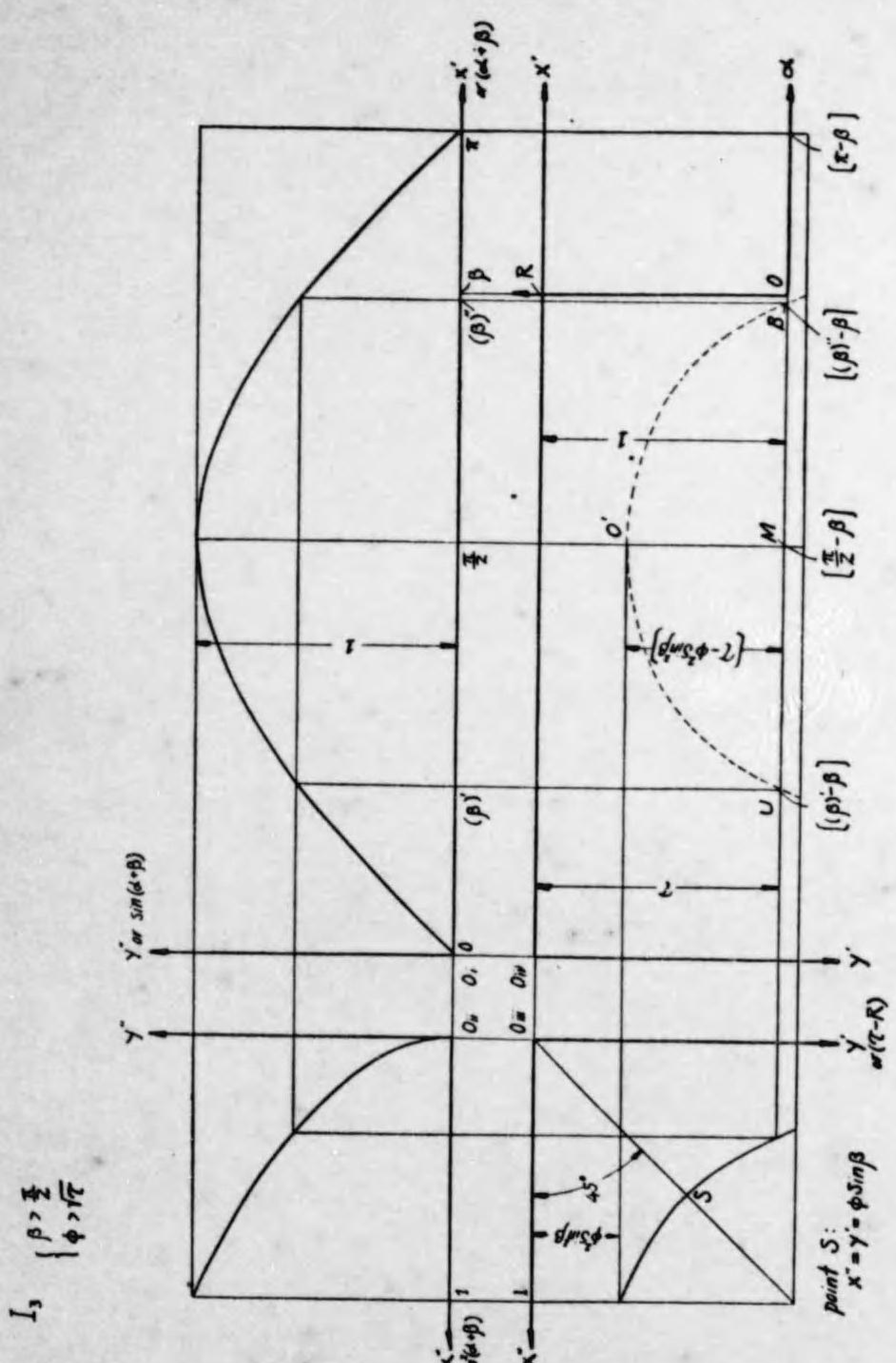


Fig. 29

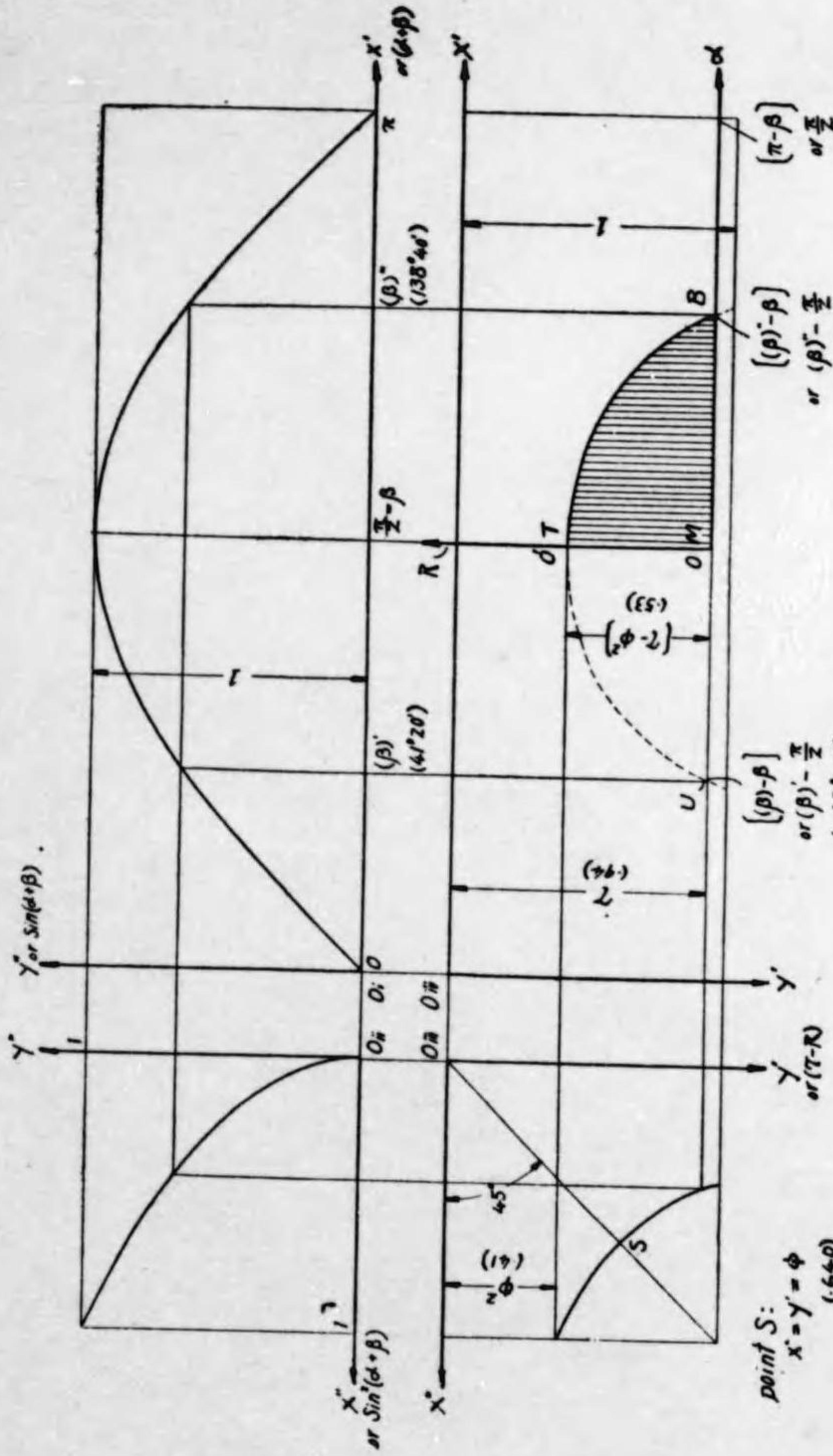


Fig. 30

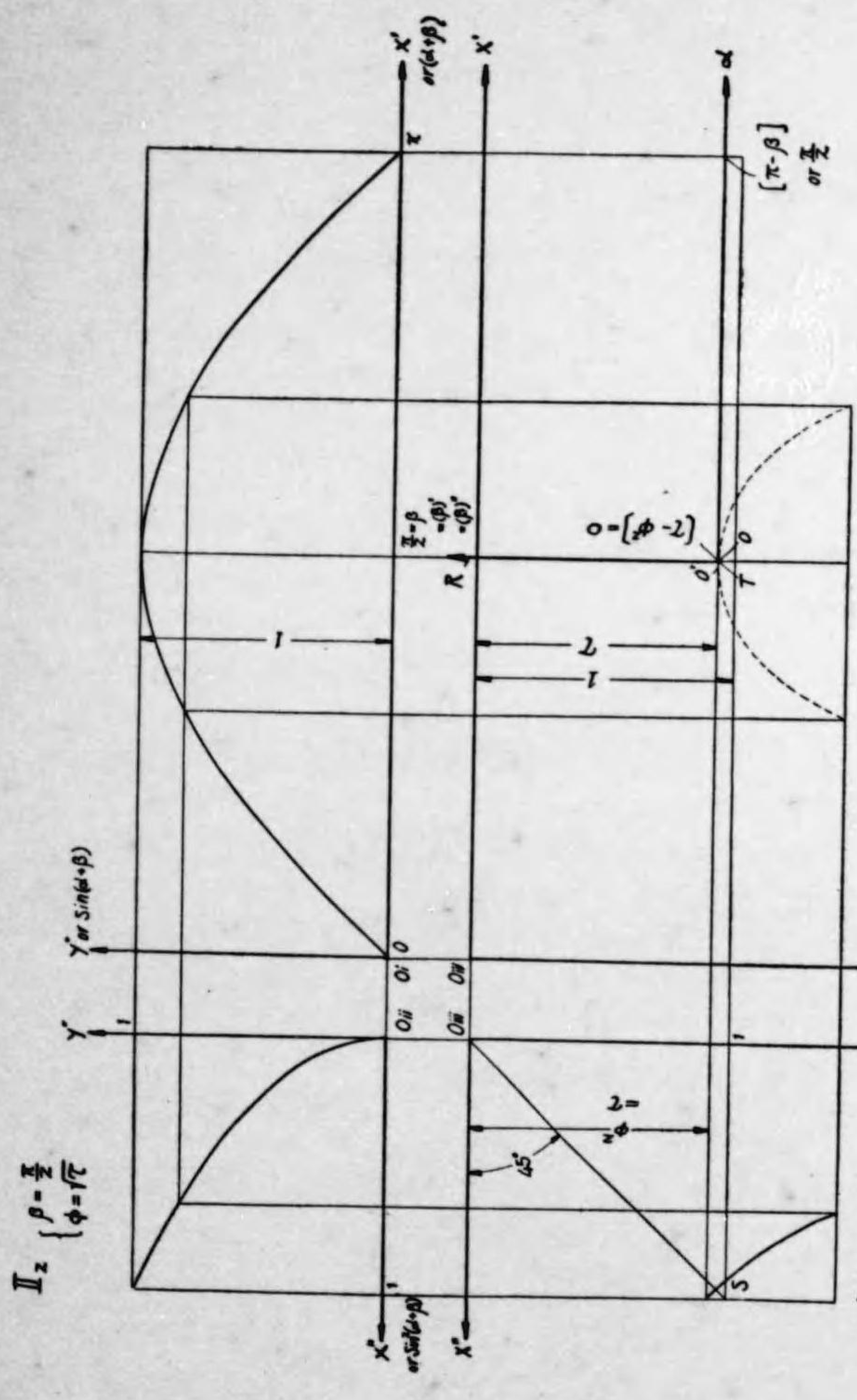


Fig. 31

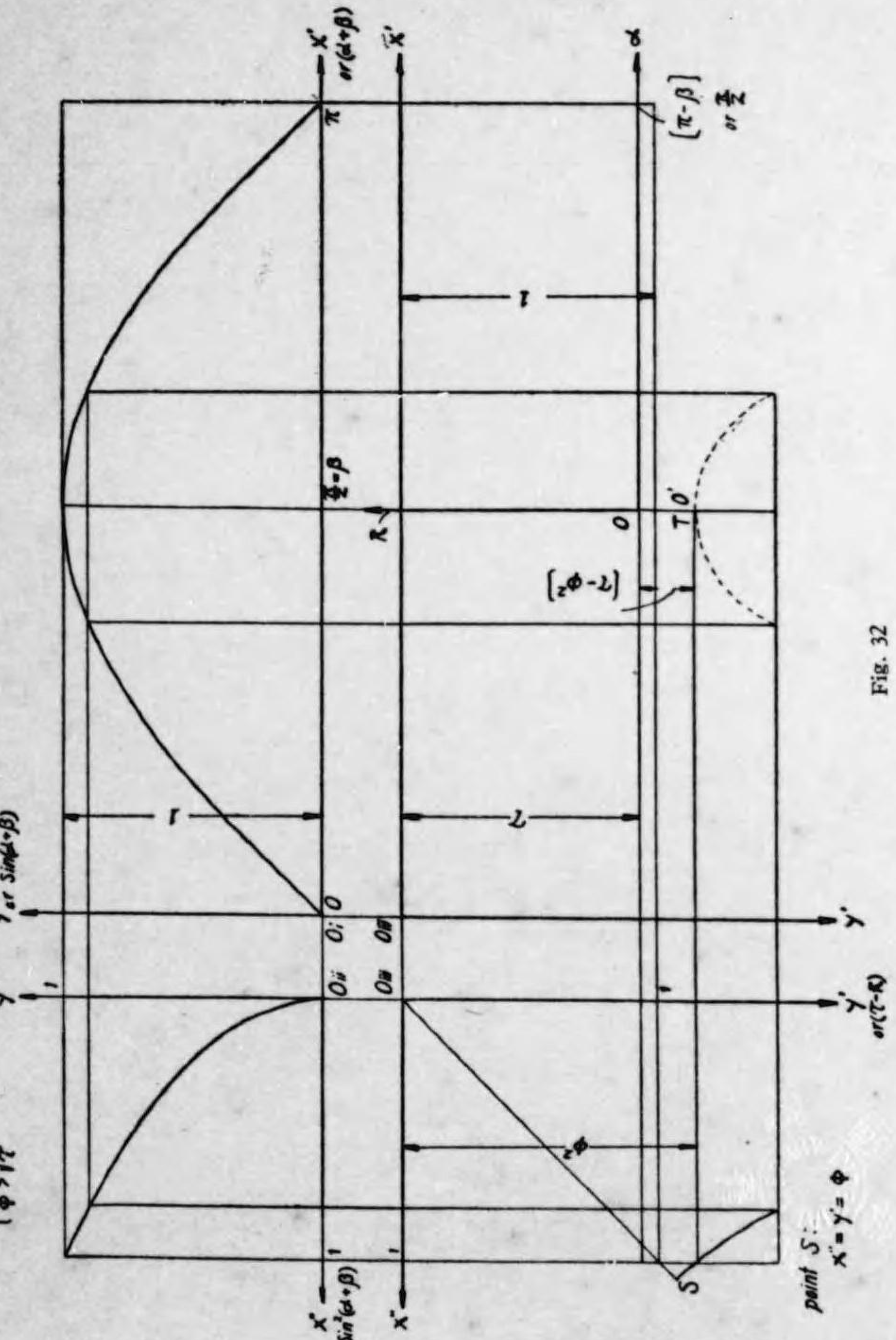


Fig. 32

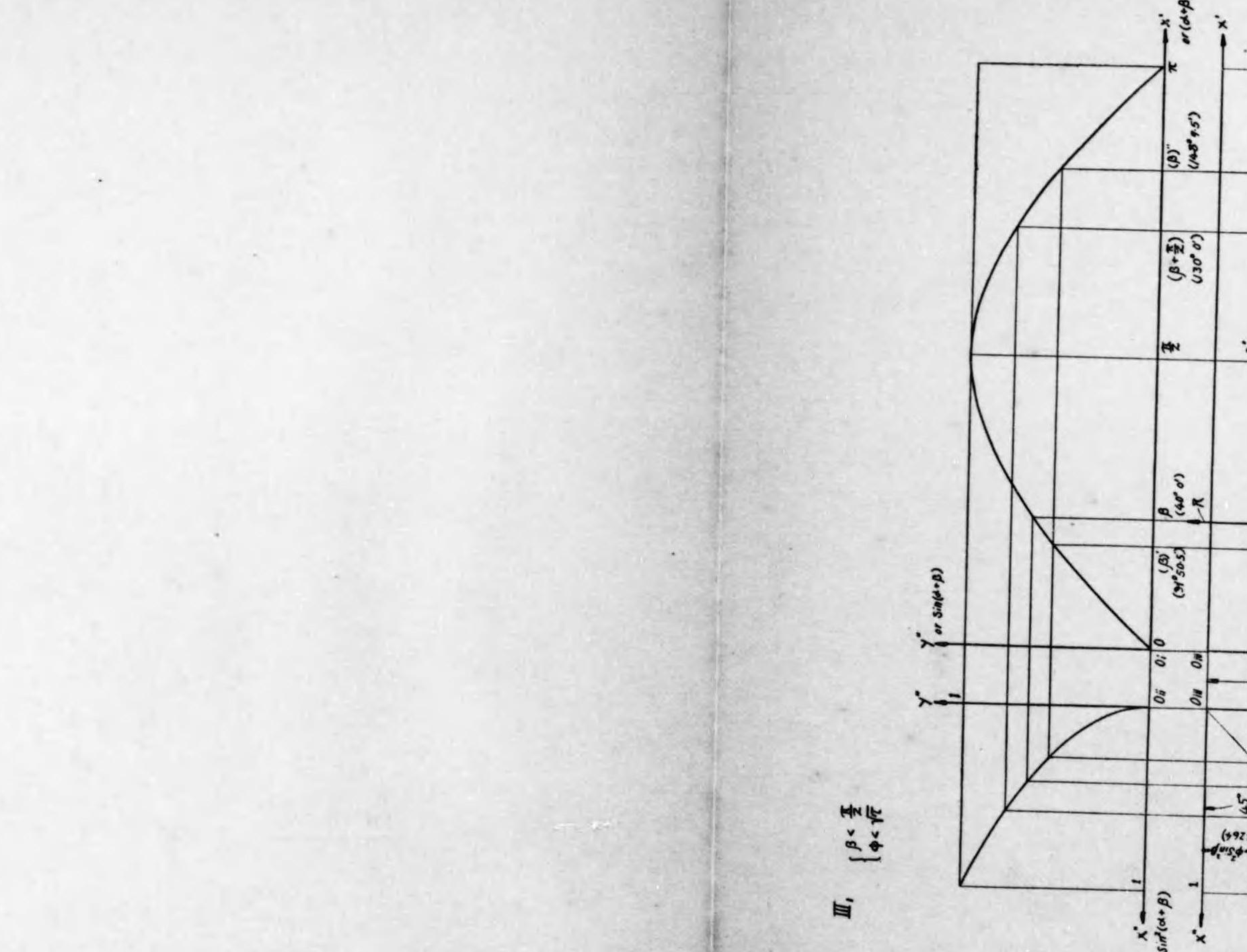


Fig. 33

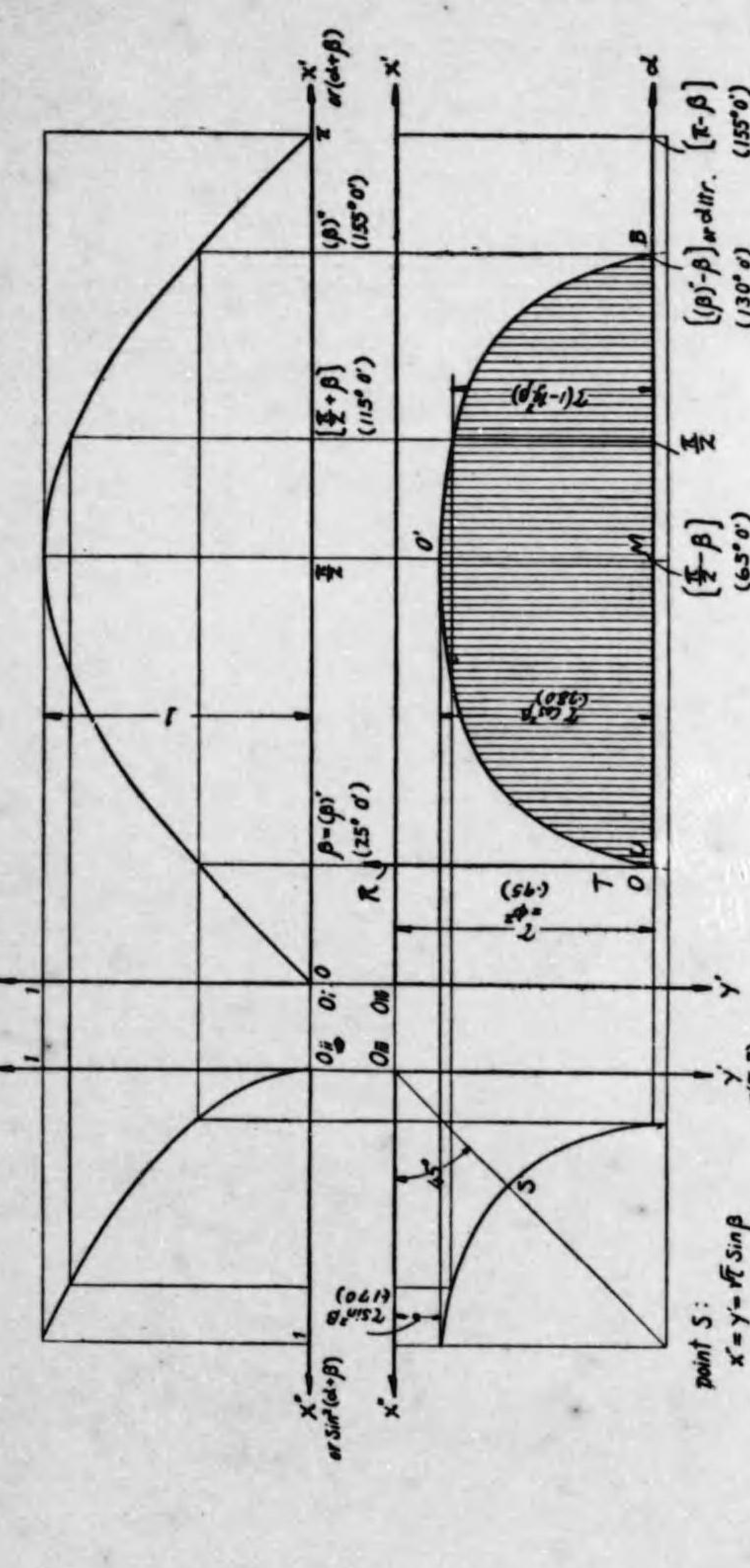


Fig. 34

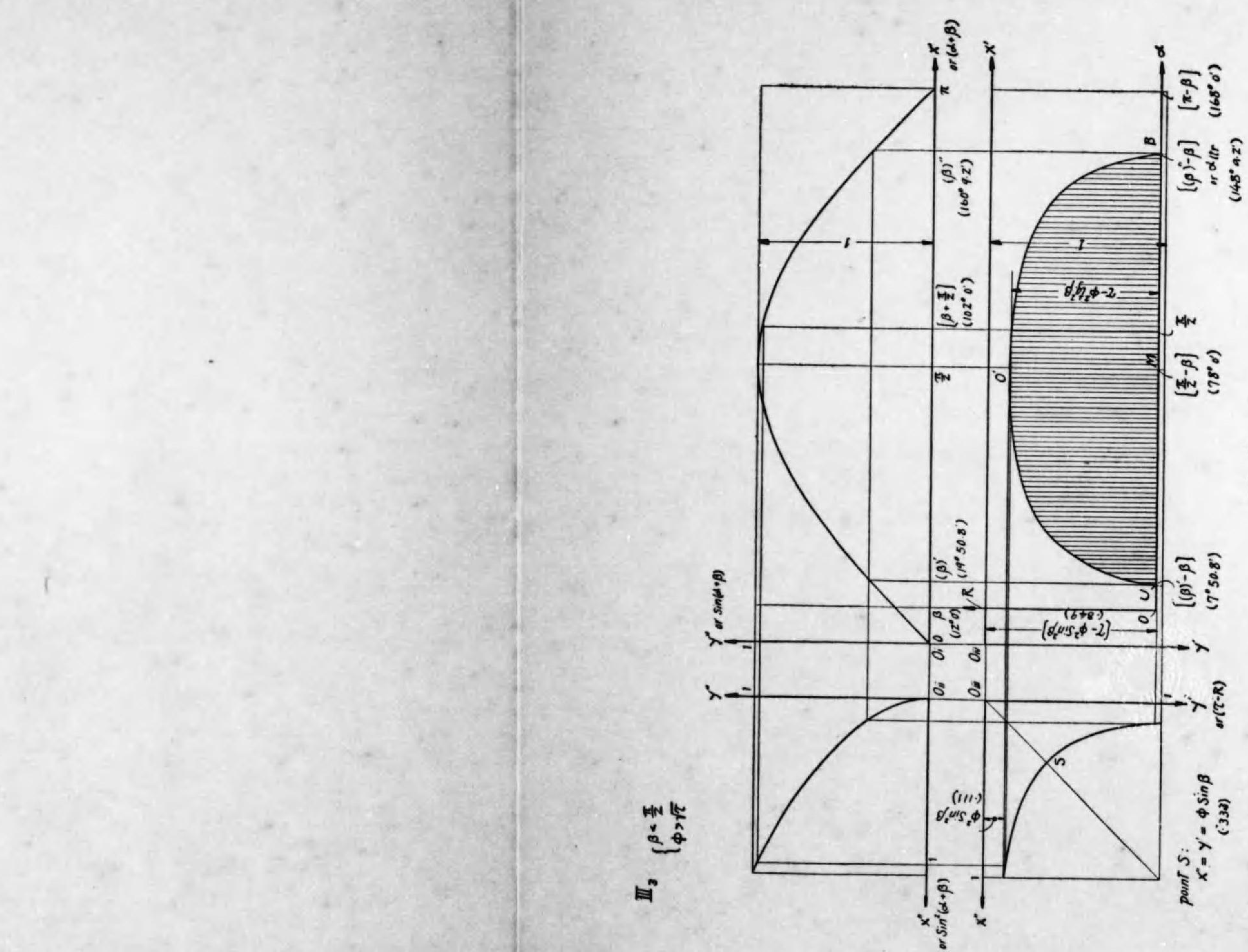
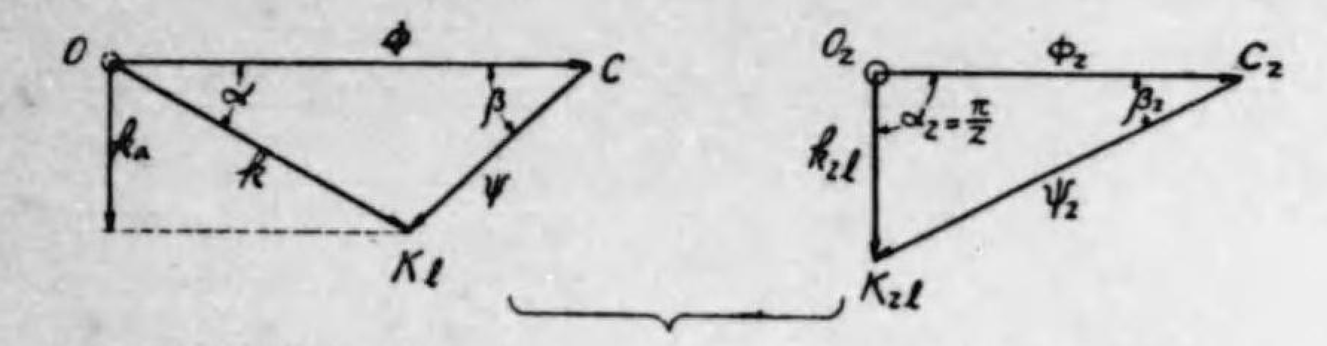


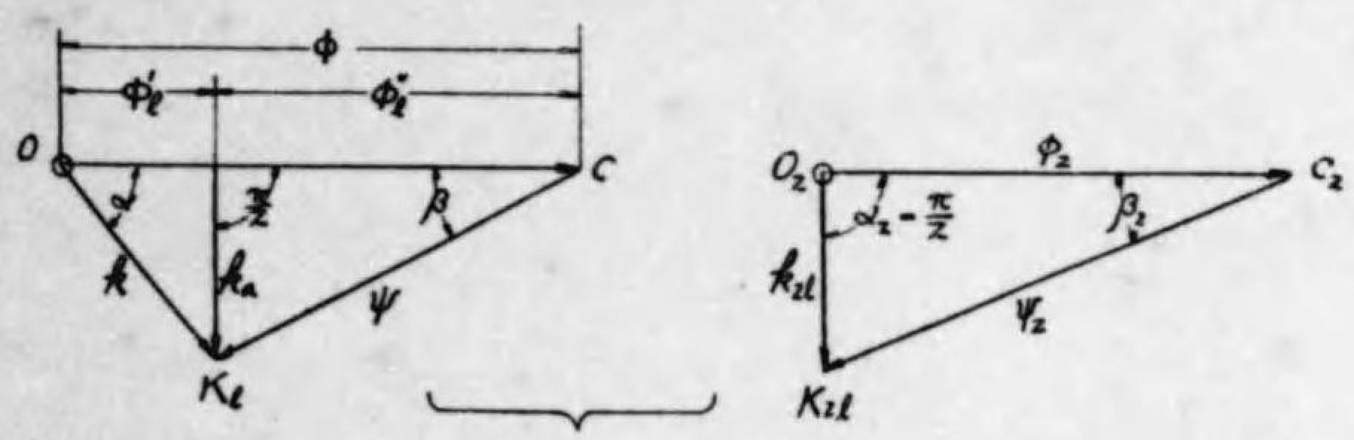
Fig. 35



Velocity-Coefficient Diagram at Inlet Edge of Runner

Velocity-Coefficient Diagram at Outlet Edge of Runner

Fig. 36



Velocity-Coefficient Diagram at Inlet Edge of Runner

Velocity-Coefficient Diagram at Outlet Edge of Runner

Fig. 37

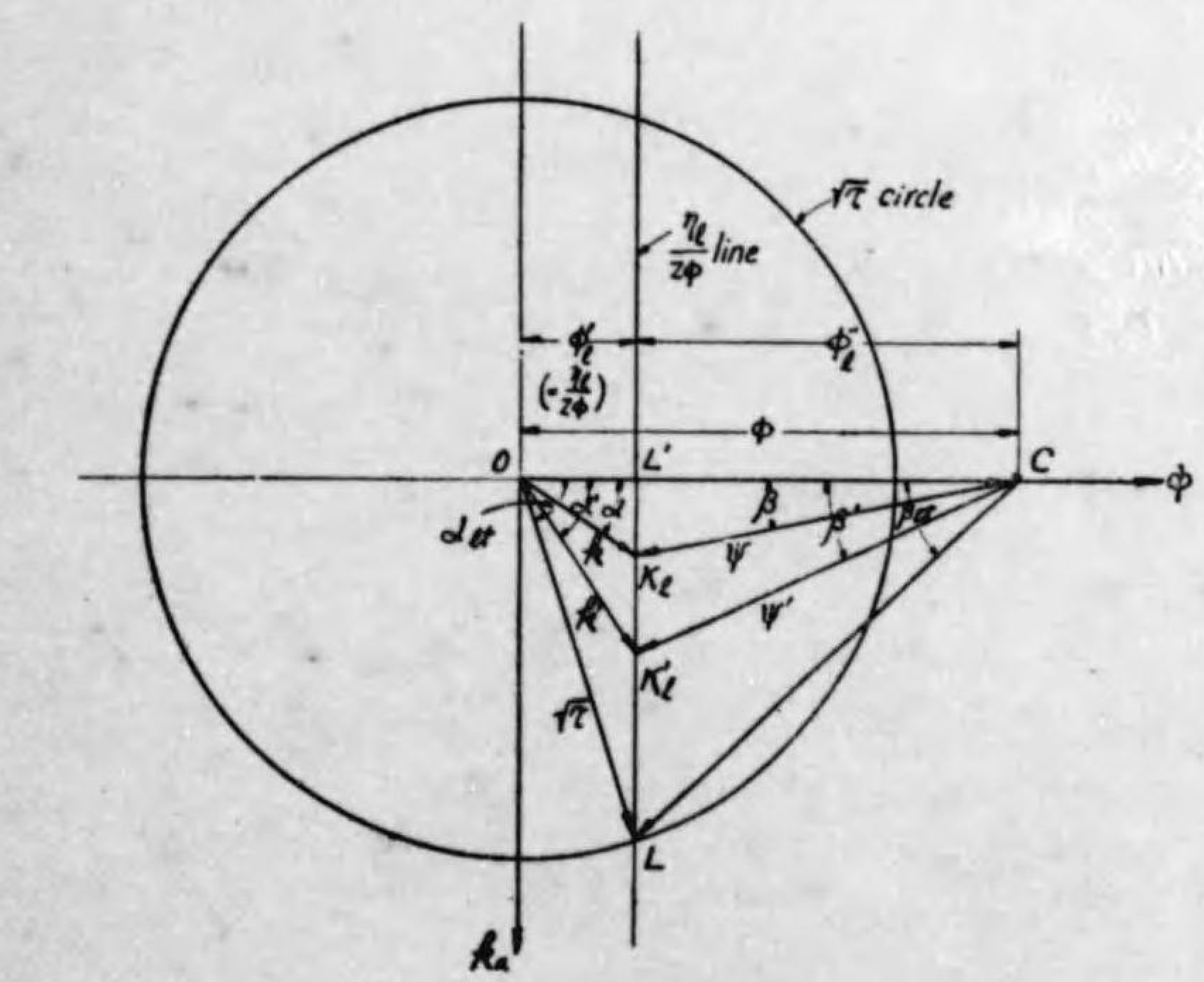


Fig. 38

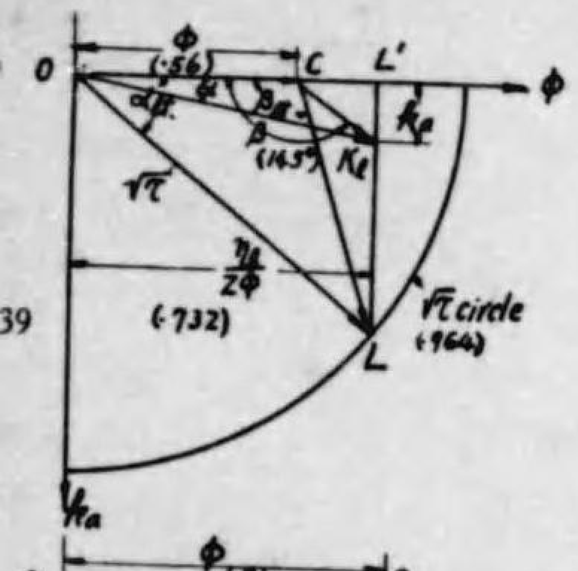


Fig. 39

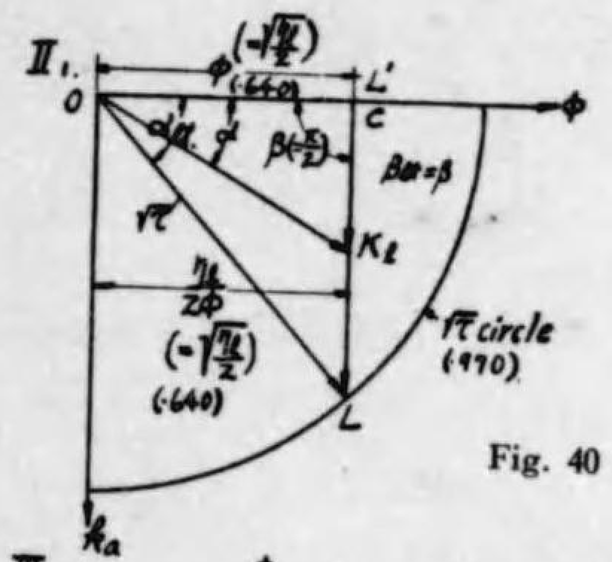


Fig. 40

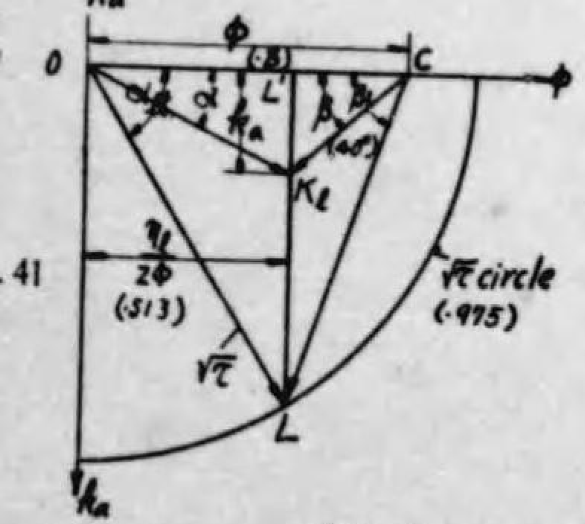


Fig. 41

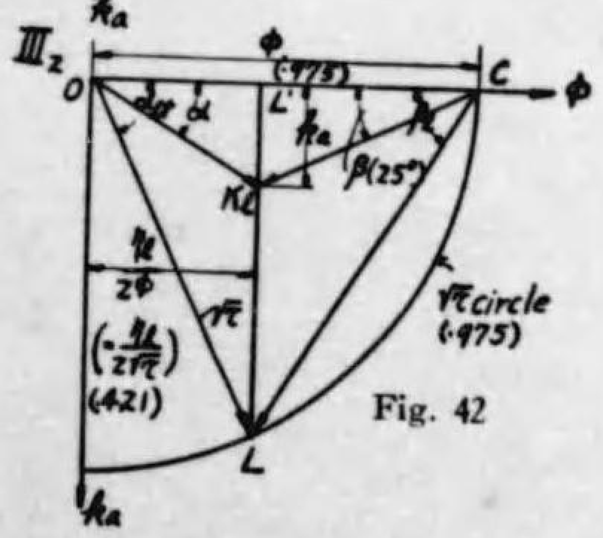


Fig. 42

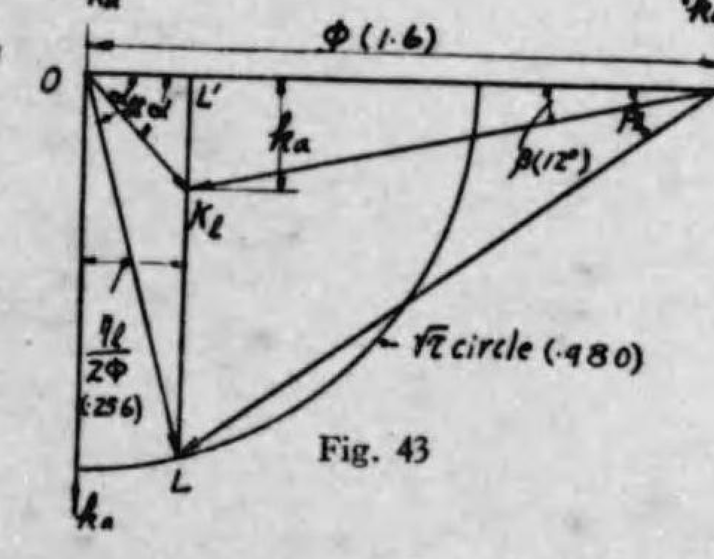


Fig. 43

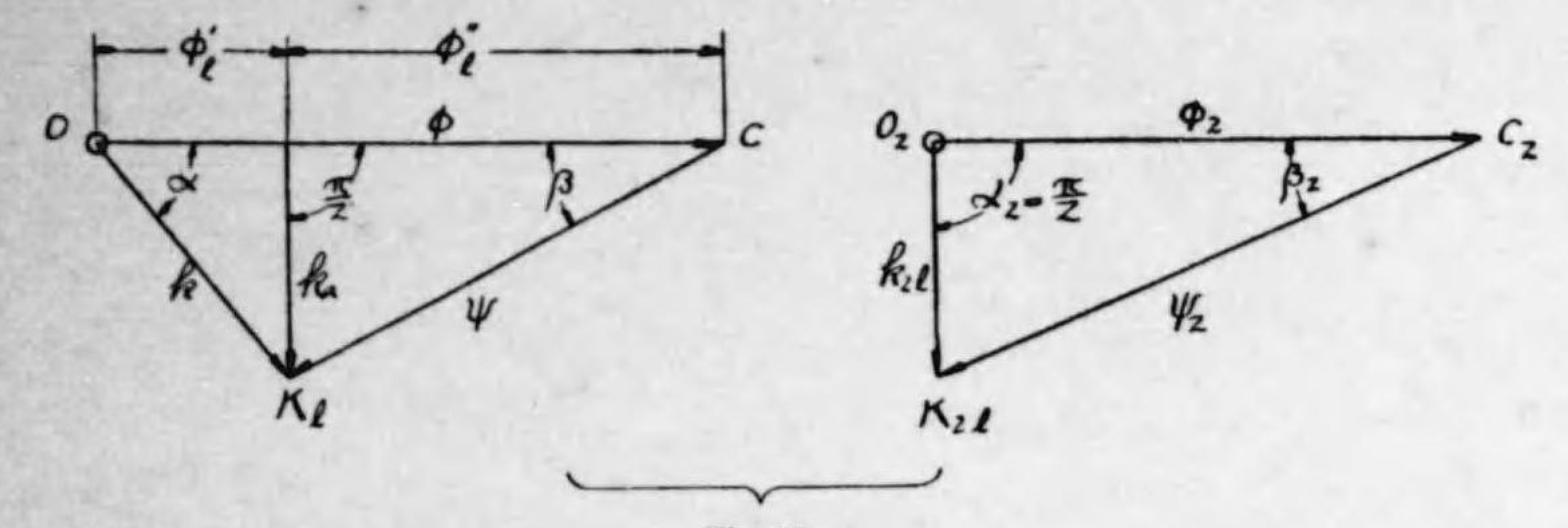


Fig. 37

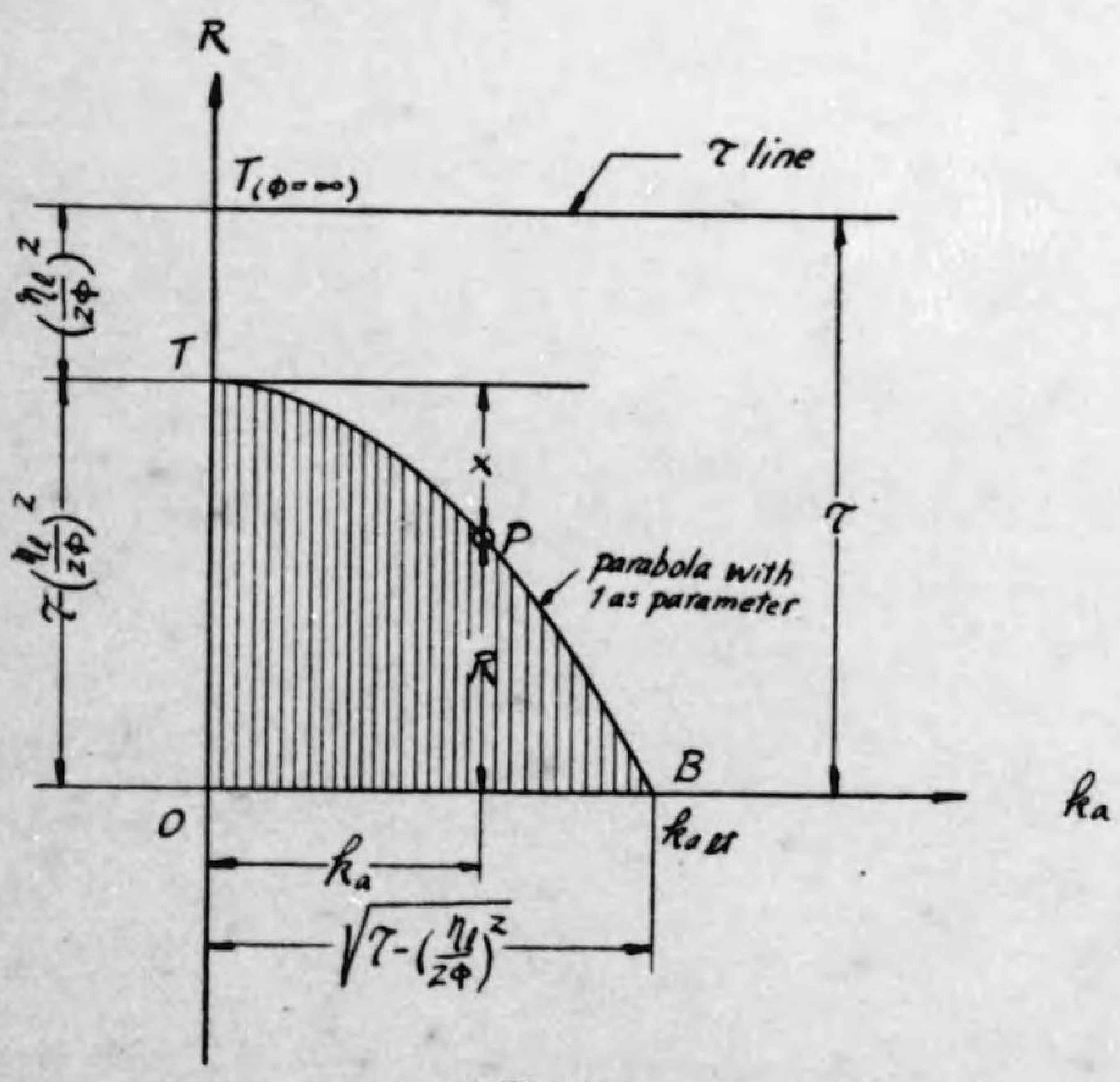


Fig. 44

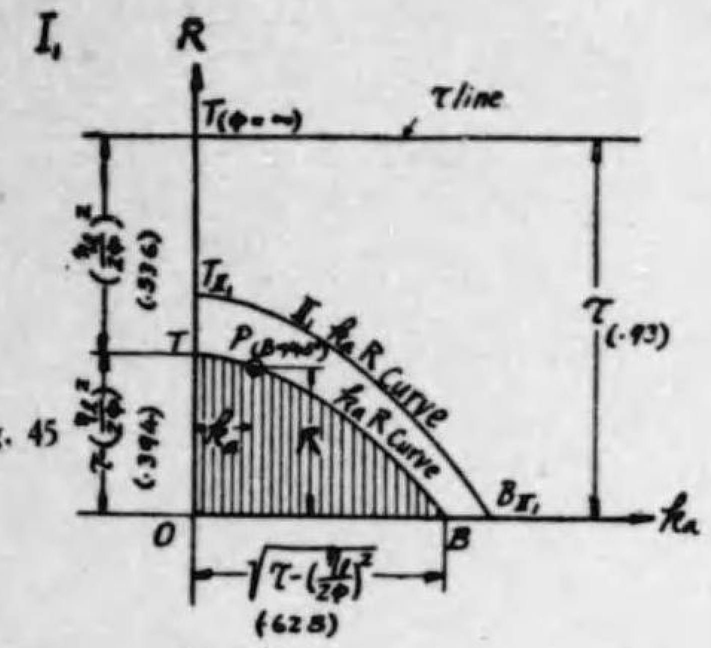


Fig. 45

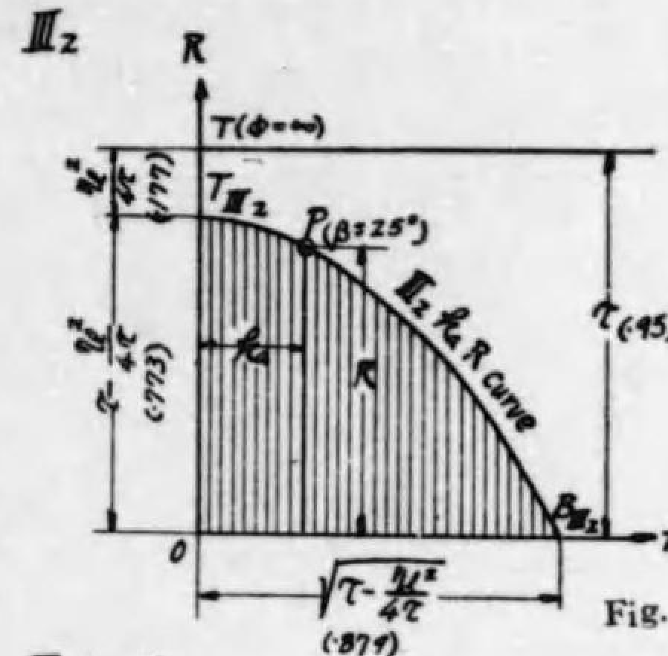


Fig. 48

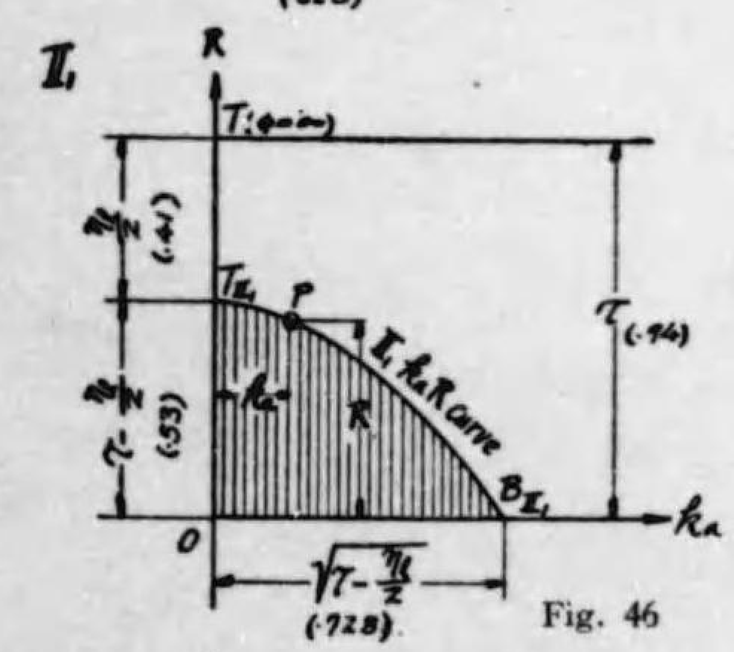


Fig. 46

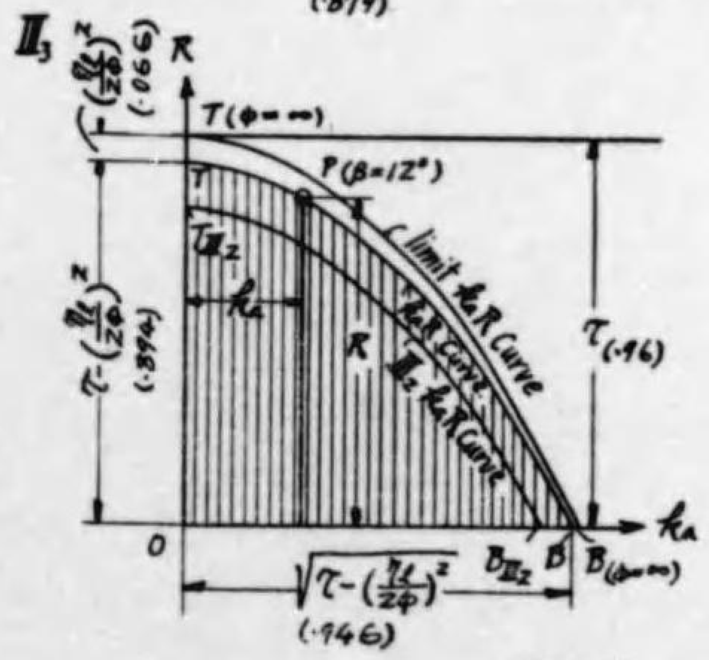


Fig. 49

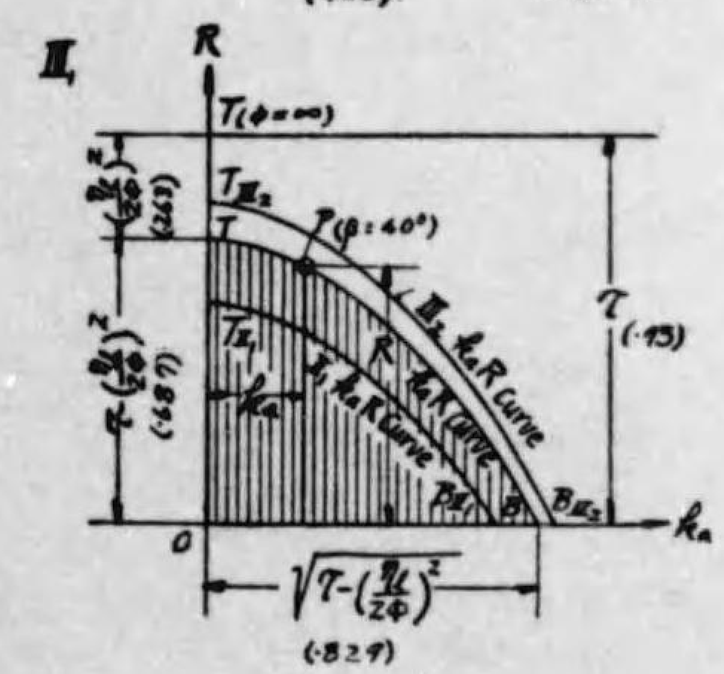


Fig. 47

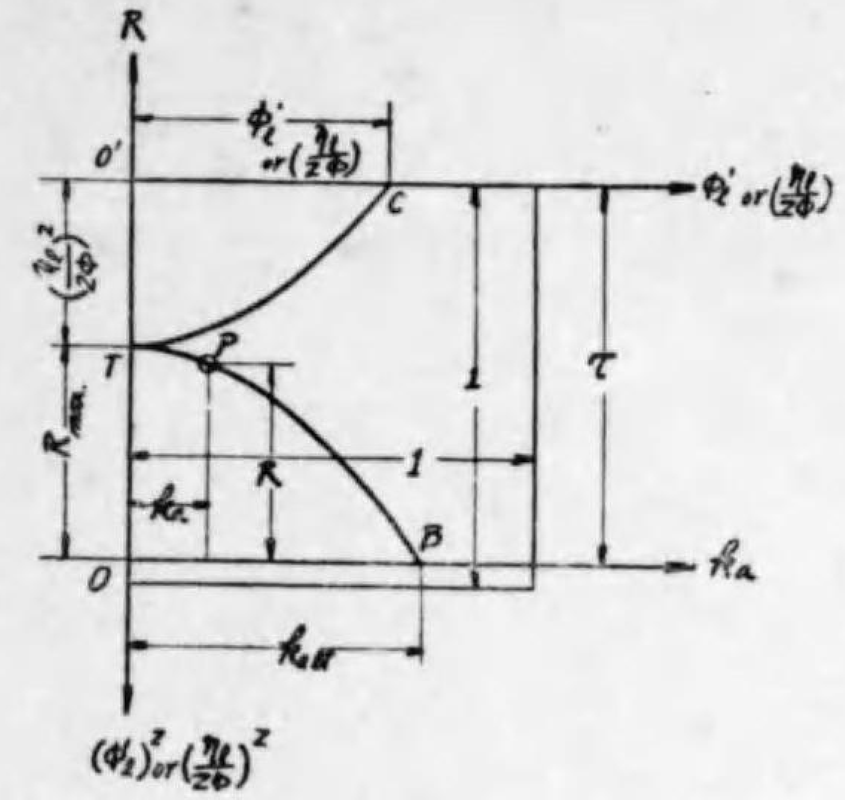


Fig. 50

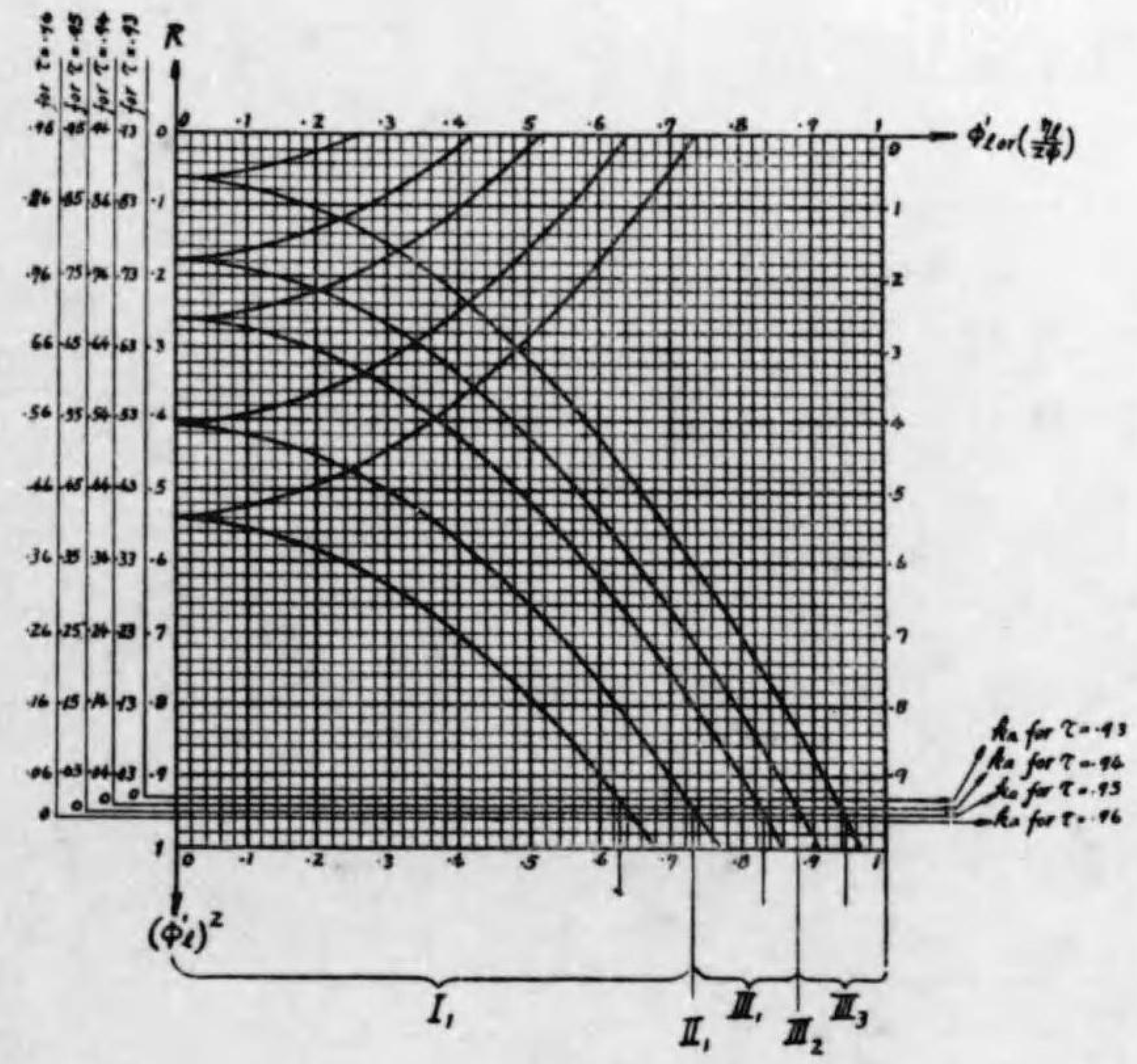


Fig. 51

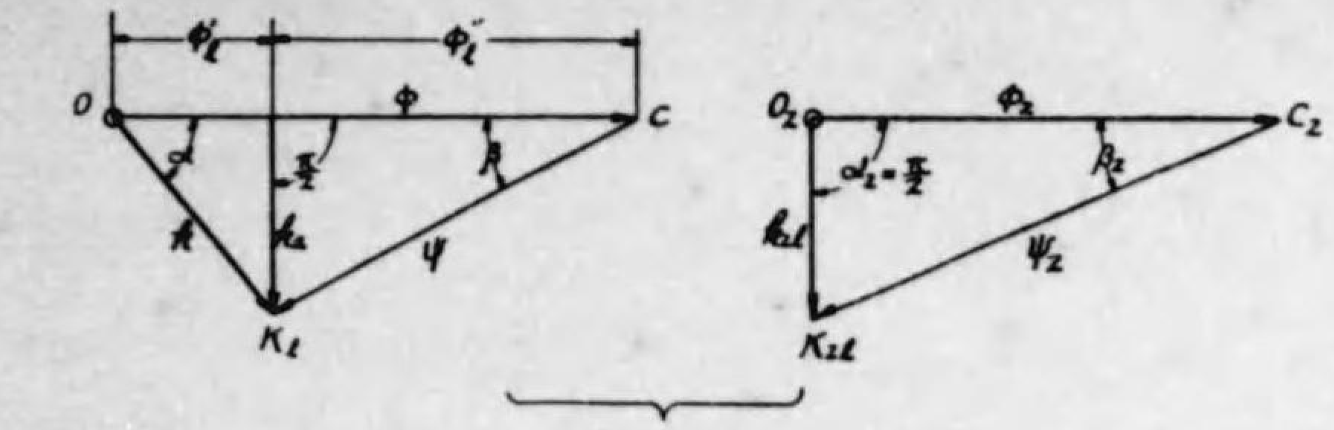


Fig. 37

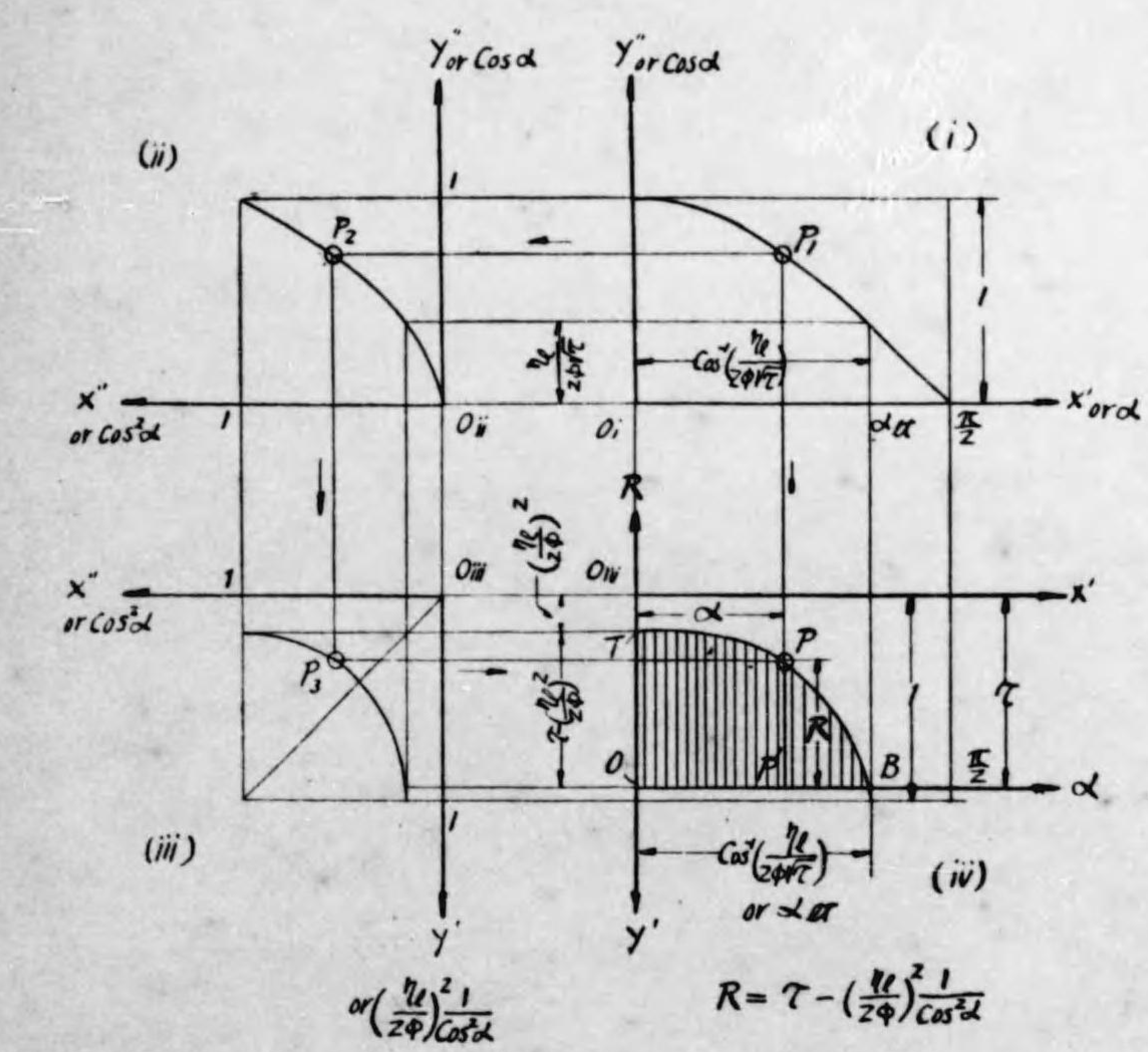
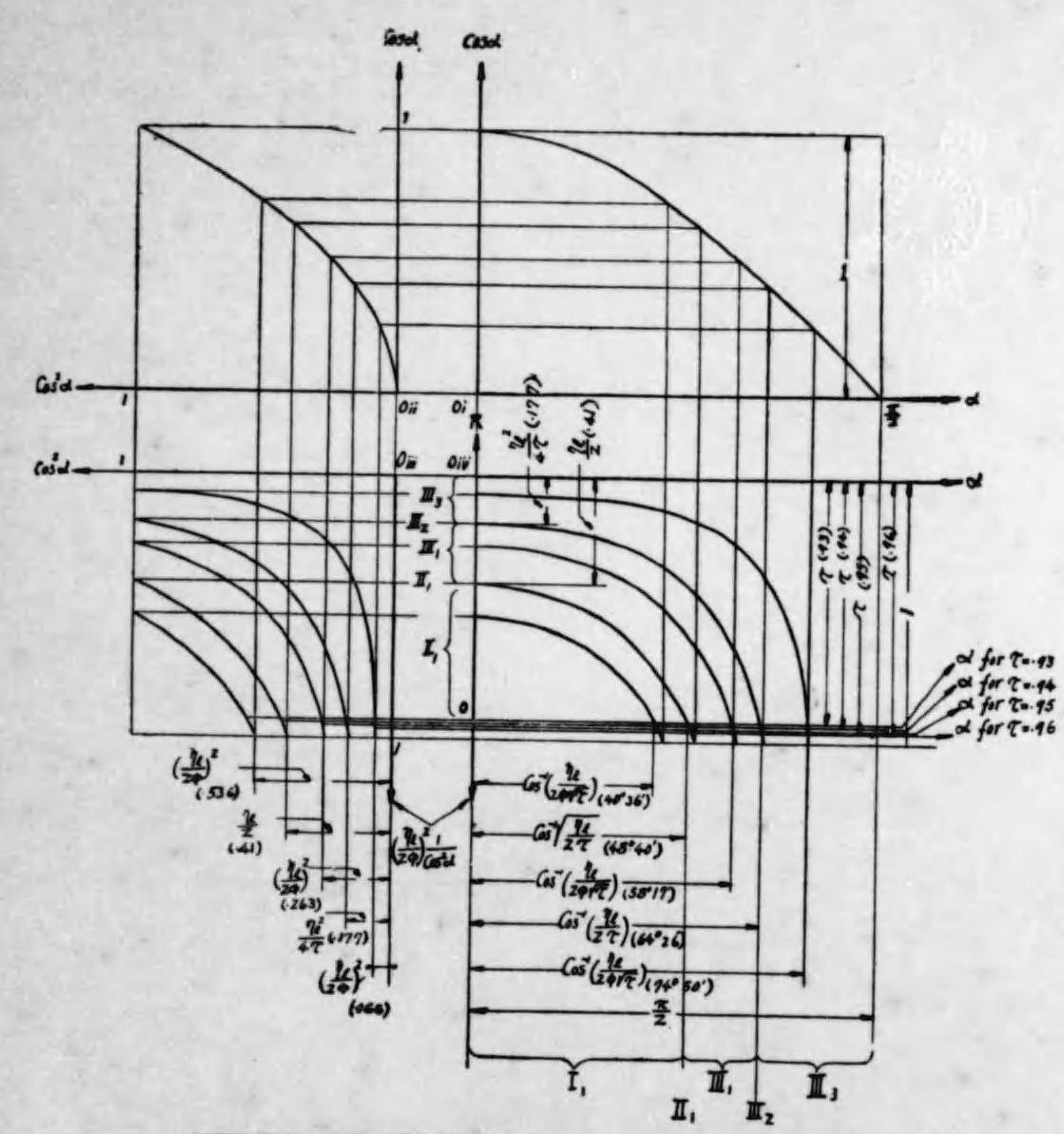


Fig. 52



Examples with $\eta_t > .82$	τ	Φ	$\frac{\eta_t}{2\Phi}$	$\left(\frac{\eta_t}{2\Phi}\right)^2$	$\tau-\left(\frac{\eta_t}{2\Phi}\right)^2$ or R_{max}	$\cos^{-1}\left(\frac{\eta_t}{2\Phi\sqrt{\tau}}\right)$ or α_{s}
I_1	.93	.56	.732	.536	.394	48° 36'
II_1	.94	$\sqrt{\frac{\eta_t}{2}}=.640$	$\sqrt{\frac{\eta_t}{2}}=.640$	$\frac{\eta_t}{2}=.41$	$\tau-\frac{\eta_t}{2}=.53$	$\cos^{-1}\sqrt{\frac{\eta_t}{2\tau}}=48°\ 40'$
III_1	.95	.80	.513	.263	.687	58° 17'
III_2	.95	$\sqrt{\tau}=.975$	$\frac{\eta_t}{2\sqrt{\tau}}=.421$	$\frac{\eta_t^2}{4\tau}=.177$	$\tau-\frac{\eta_t^2}{4\tau}=.773$	$\cos^{-1}\left(\frac{\eta_t}{2\tau}\right)=64°\ 26'$
III_3	.96	1.16	.256	.066	.894	74° 50'

Fig. 53

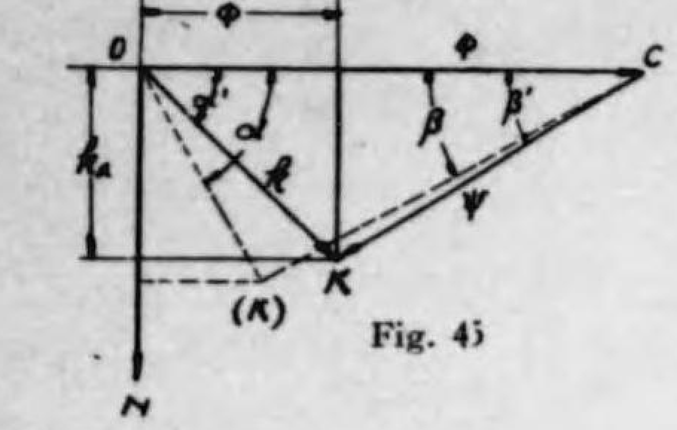


Fig. 45

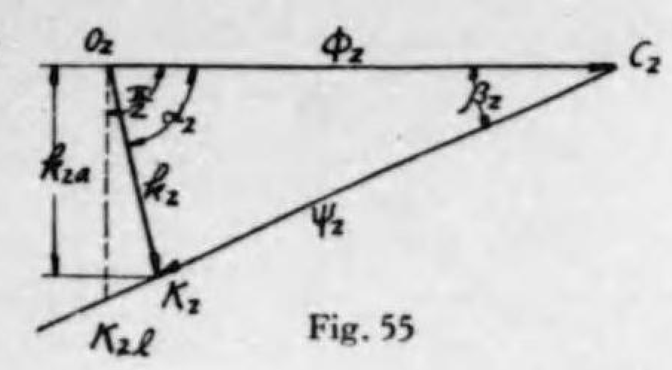


Fig. 55

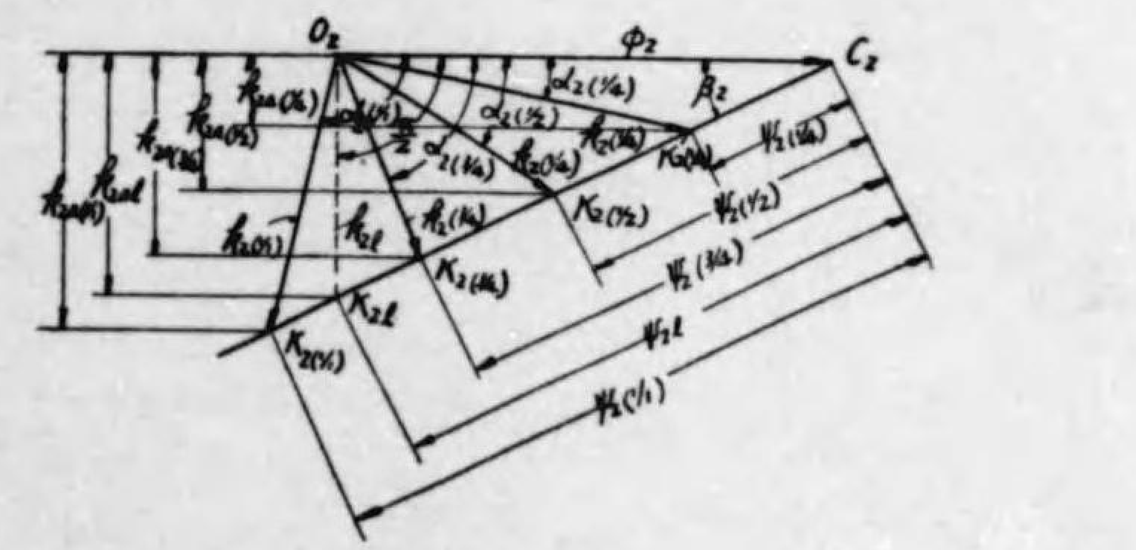


Fig. 56

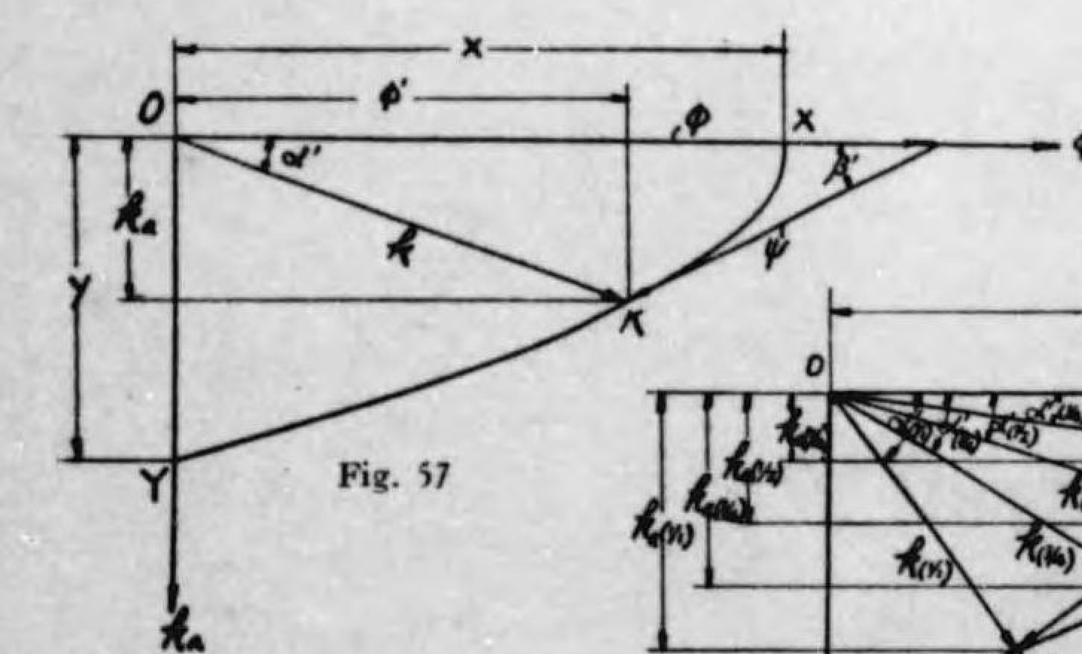


Fig. 57

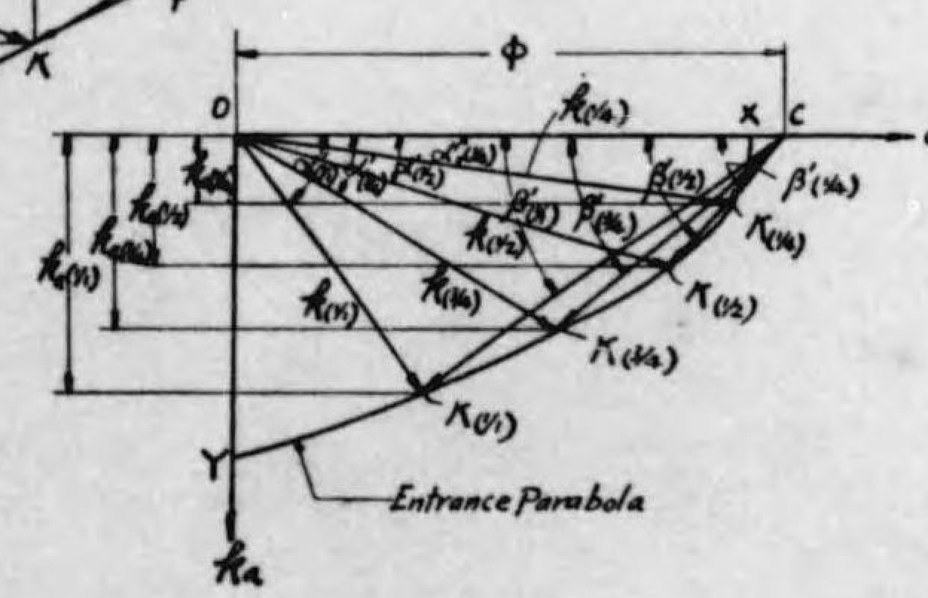
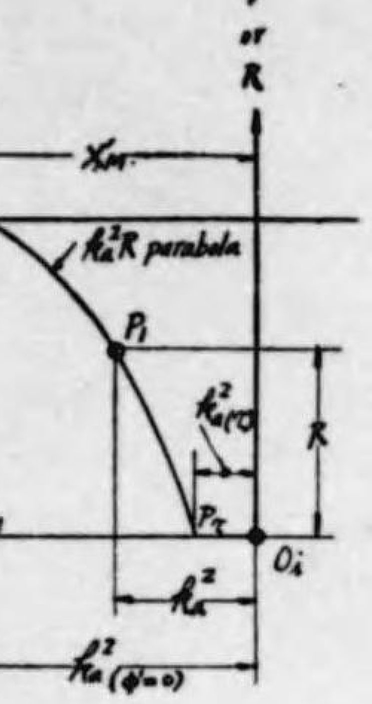
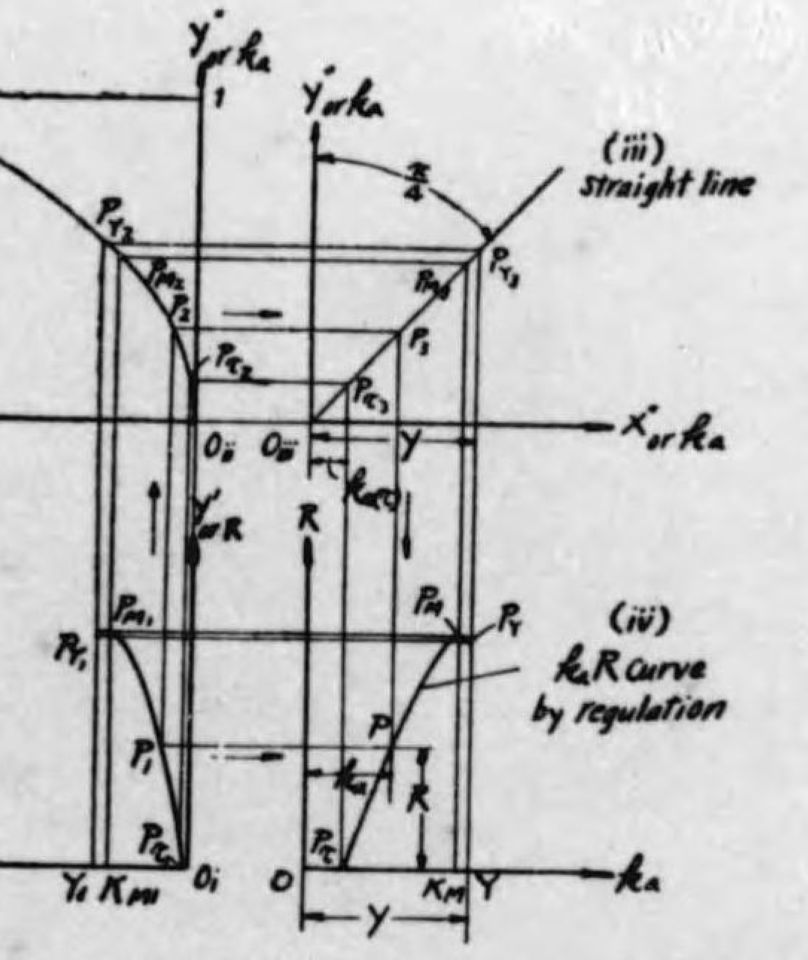


Fig. 58


y' or R
X_M
P_M
P_Y
k_a^2R parabola
P_1
Y'_M
$k^2_{a(\tau)}$
R
x or k_a^2
Y_1
K_M
P_τ
O_i
k_a^2
$k^2_{a(\phi'=0)}$
Fig. 59


(ii)
parabola with 1 as parameter
(iii)
straight line
(i)
k_a^2R parabola
(iv)
k_aR curve by regulation
x' or k_a^2
X'' or k_a
k_a
R
Y
Fig. 60

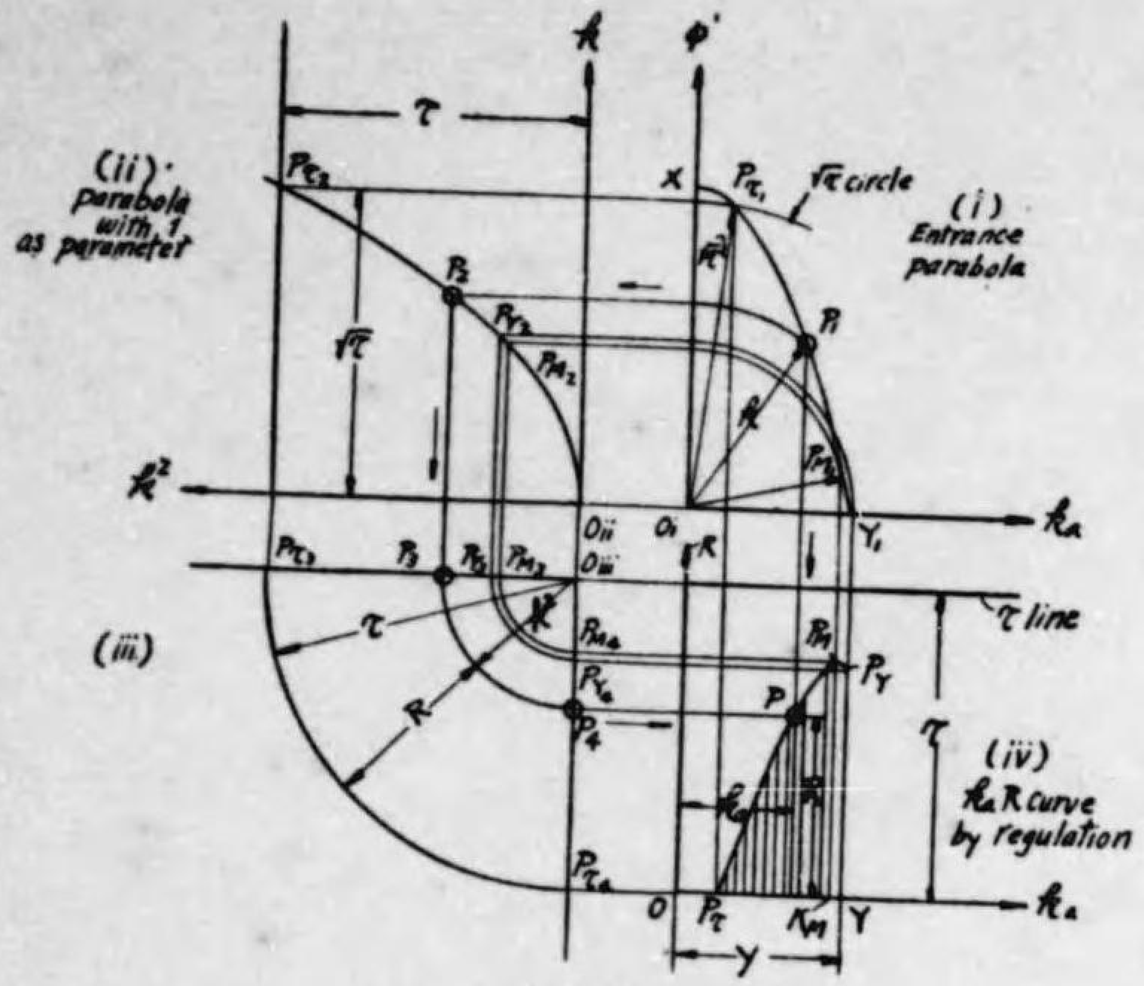


Fig. 61

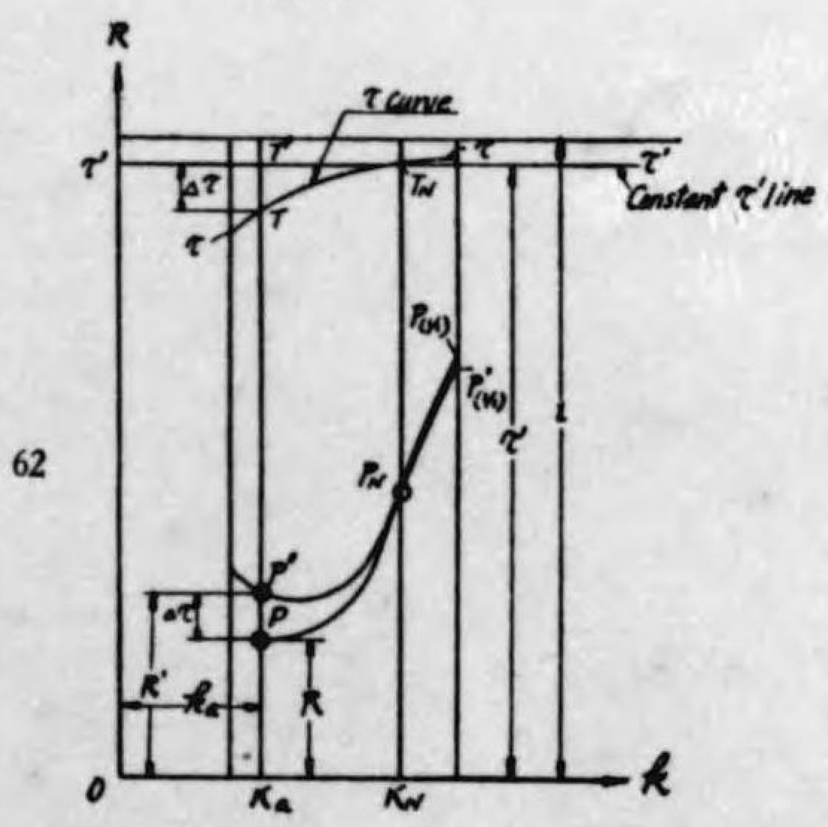


Fig. 62

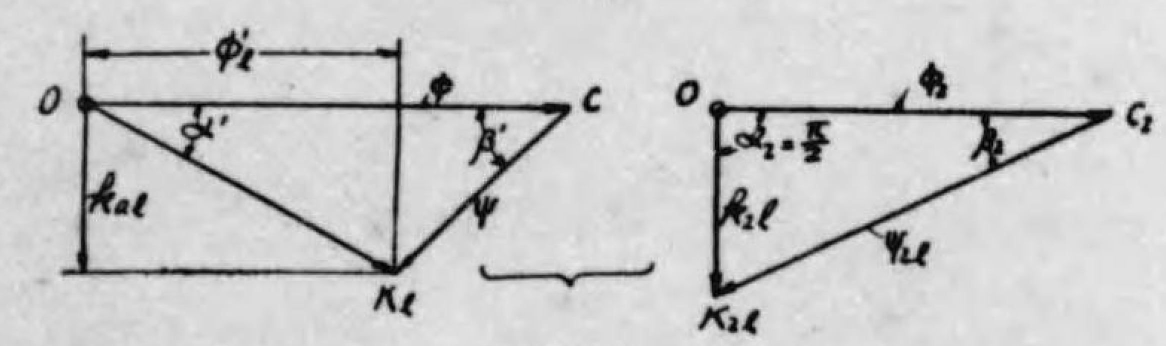


Fig. 63

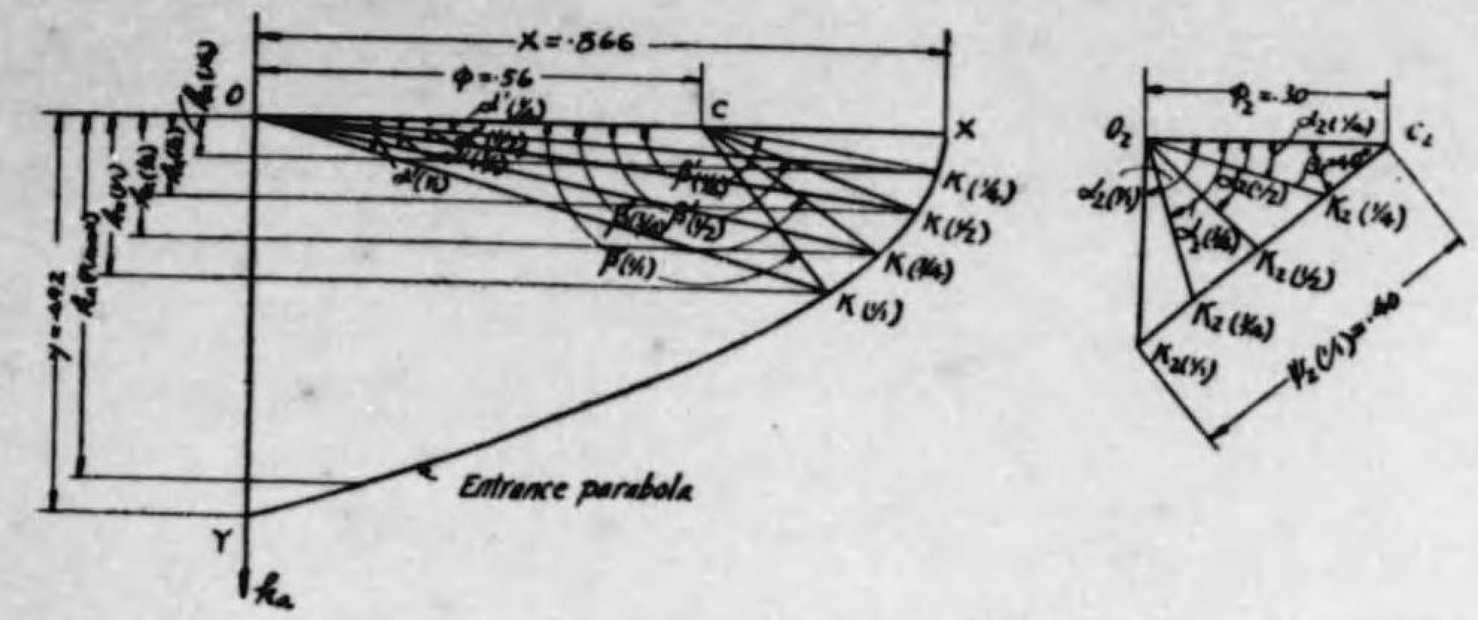
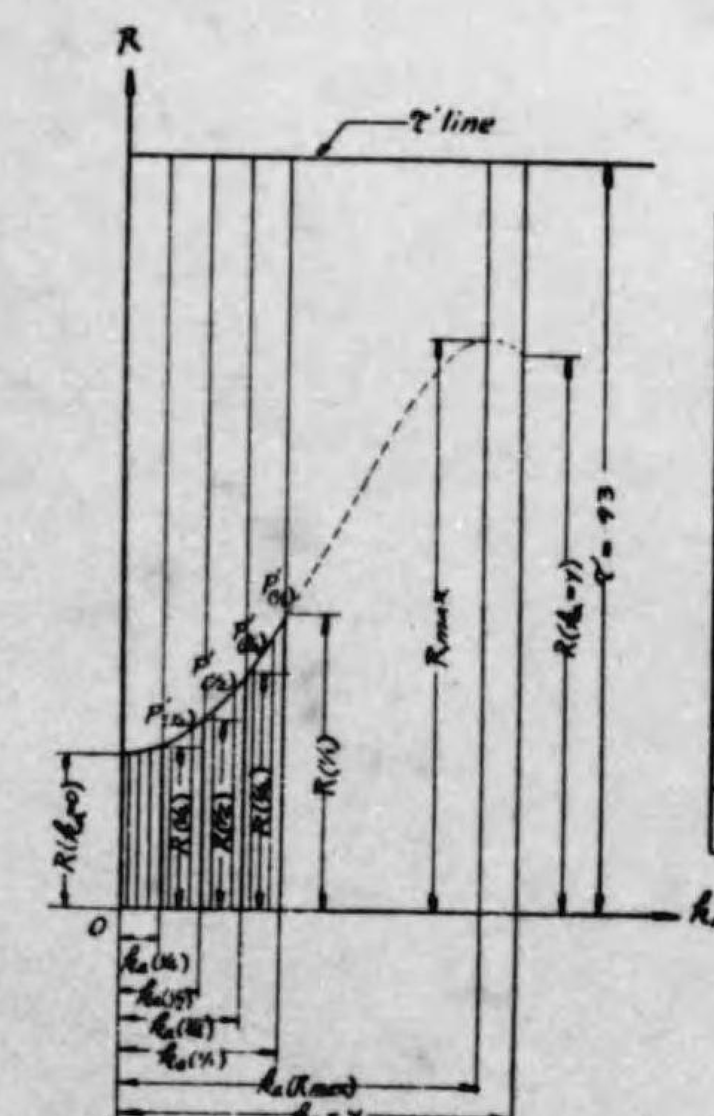


Fig. 64



I, Francis Turbine $\tau' = \cdot 43$			
$\phi = \cdot 56$ $\beta_2 = 40°$ $x = \cdot 866$			
$\phi_2 = \cdot 30$ $\psi_2(1/1) = \cdot 40$ $y = \cdot 492$			
$h_a =$	0	$R(h_a = 0)$	·180
$h_a =$	y	$R(h_a = y)$	·688
$h_a(R_{max})$	·451	R_{max}	·707
$h_a(1/1)$	·20	$R(1/1)$	·367
$h_a(3/4)$	·15	$R(3/4)$	·290
$h_a(1/2)$	·10	$R(1/2)$	·231
$h_a(1/4)$	·05	$R(1/4)$	·193

Fig. 65

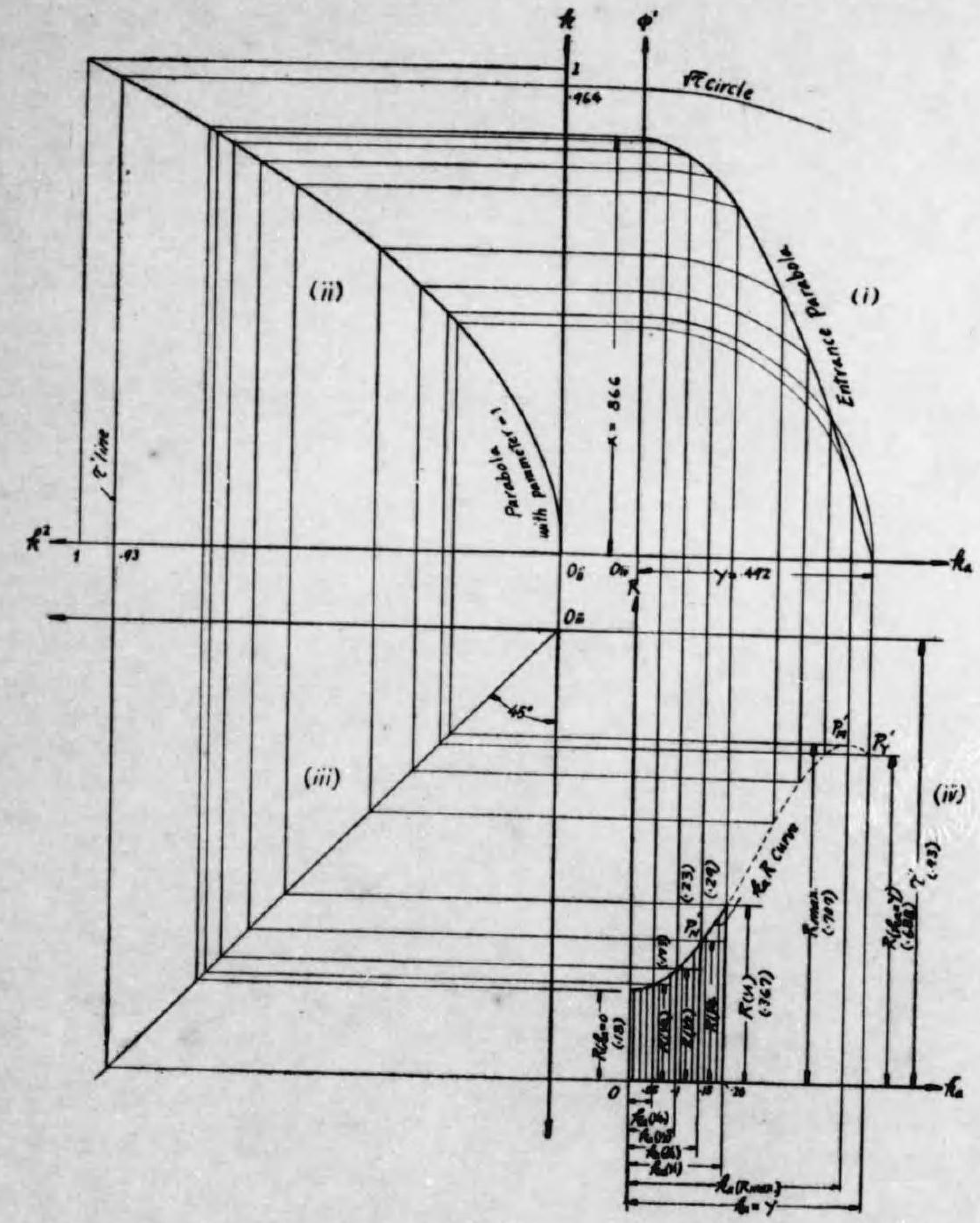


Fig. 66

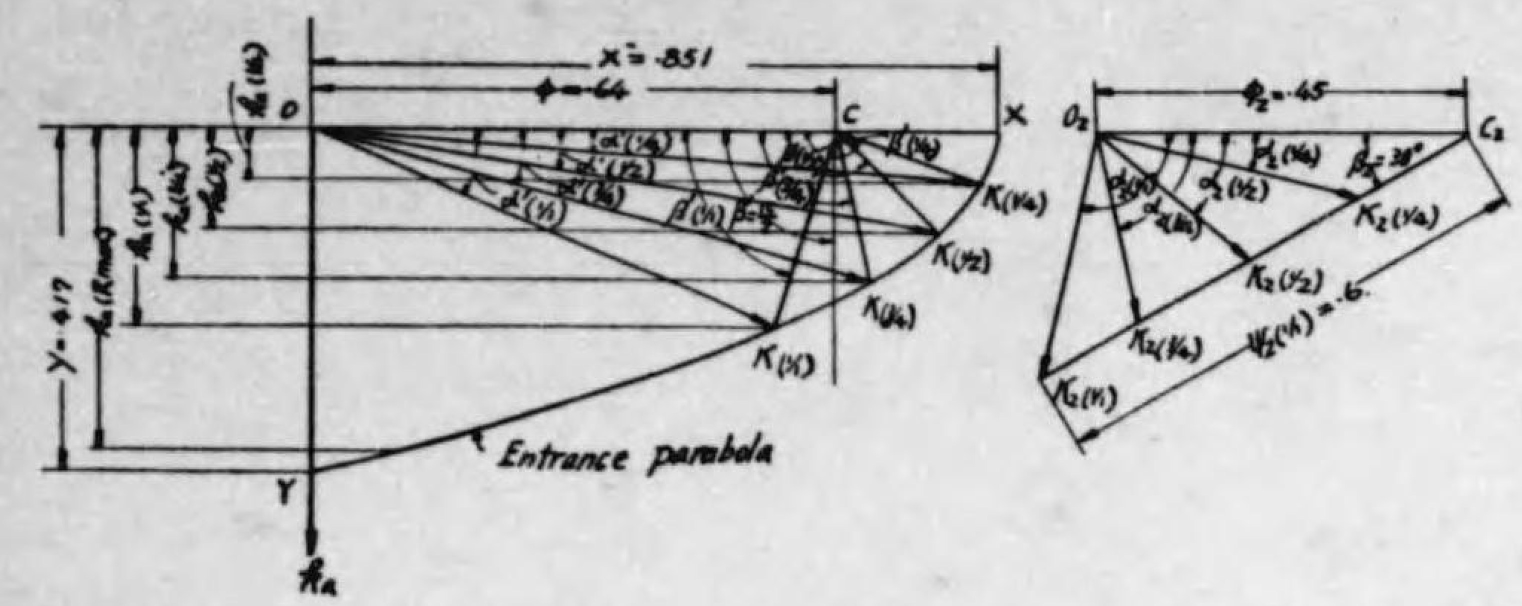
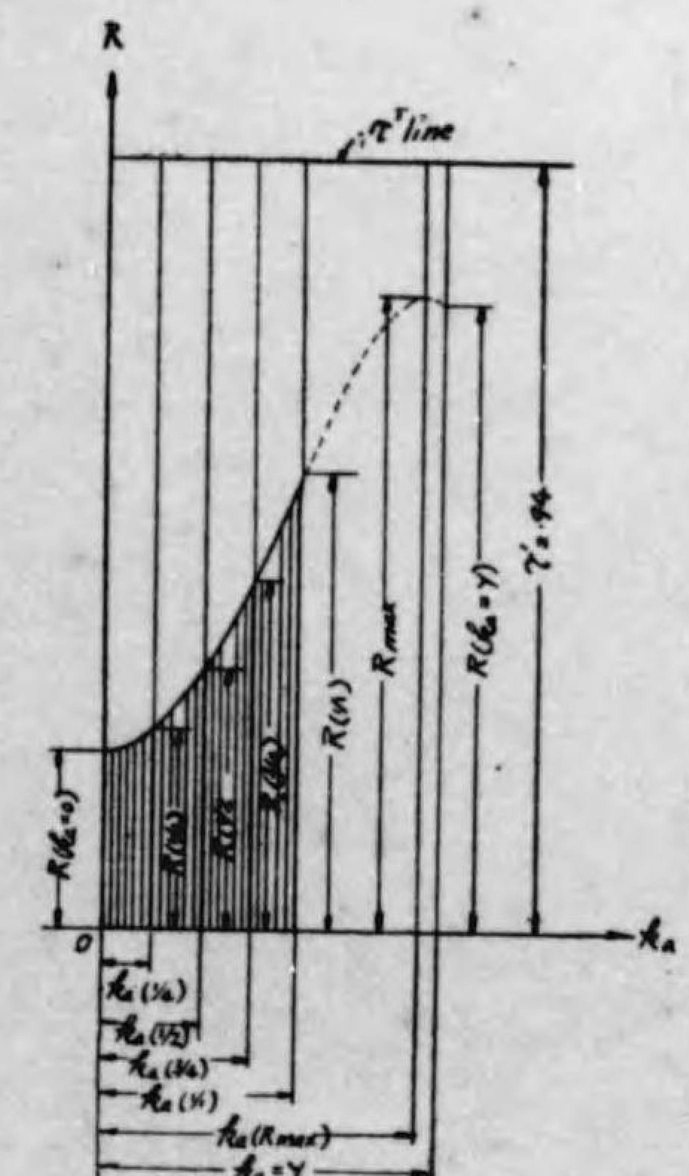


Fig. 67



II₁ Francis Turbine $\tau' = .94$			
$\phi = .64$	$\beta_2 = 30°$	$x = .851$	
$\phi_2 = .45$	$\psi_2(1/1) = .60$	$y = .417$	
$h_a =$	0	$R(h_a = 0)$	·217
$h_a =$	y	$R(h_a = y)$	·766
$h_a(R_{max})$	·391	R_{max}	·776
$h_a(1/1)$	·24	$R(1/1)$	·558
$h_a(3/4)$	·18	$R(3/4)$	·428
$h_a(1/2)$	·12	$R(1/2)$	·317
$h_a(1/4)$	·06	$R(1/4)$	·243

Fig. 68

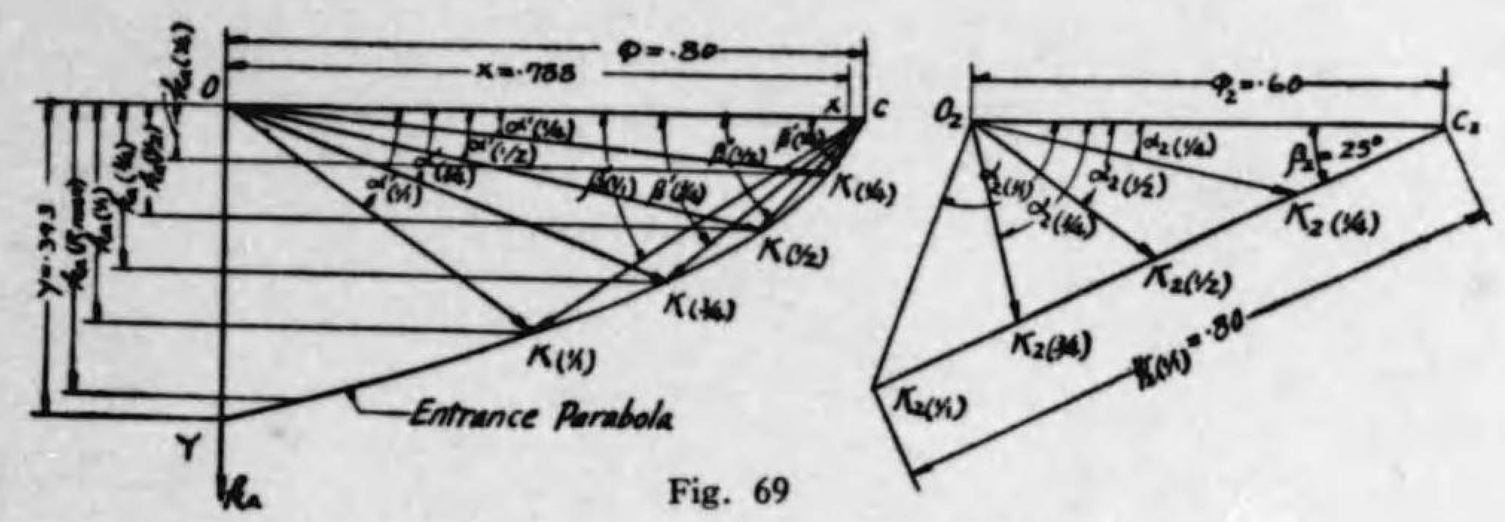
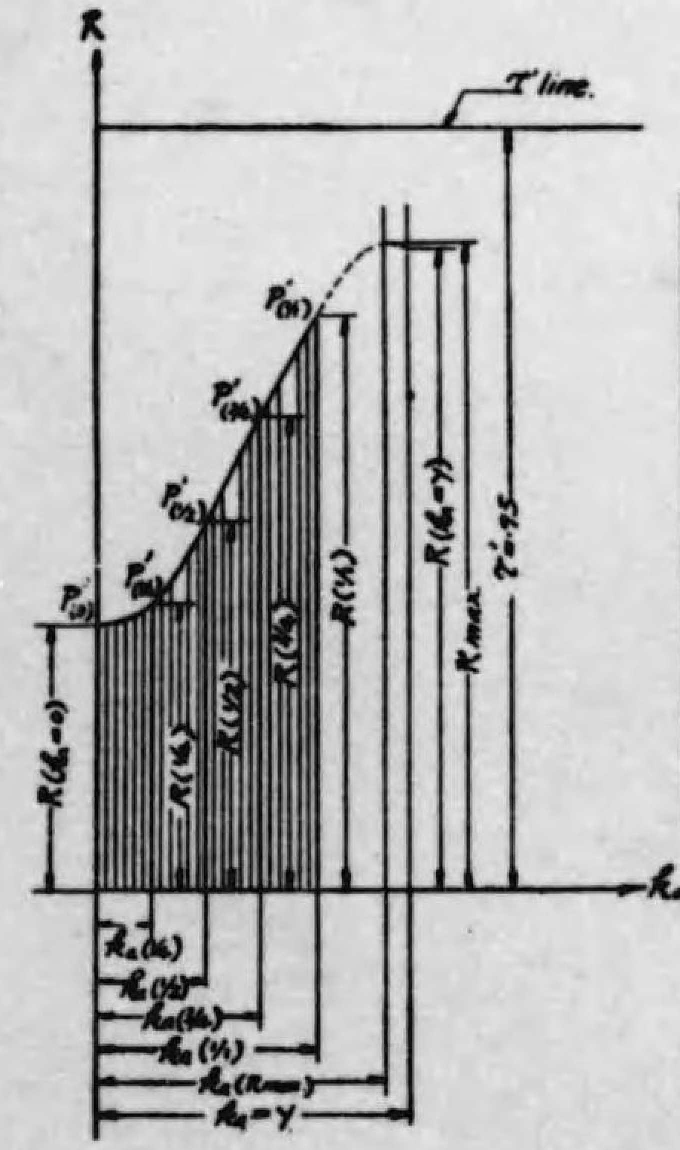


Fig. 69



III, Francis Turbine τ'=·95 φ=·80 β_2=25° x=·788 φ_2=·60 $V_2(1/1)$=·80 y=·393			
h_a=	·0	$R(h_a=0)$	·330
h_a=	y	$R(h_a=y)$	·796
$h_a(R_{max})$	·368	R max.	·805
$h_a(1/1)$	·28	R(1/1)	·721
$h_a(3/4)$	·21	R(3/4)	·589
$h_a(1/2)$	·14	R(1/2)	·458
$h_a(1/4)$	·07	R(1/4)	·364

Fig. 70

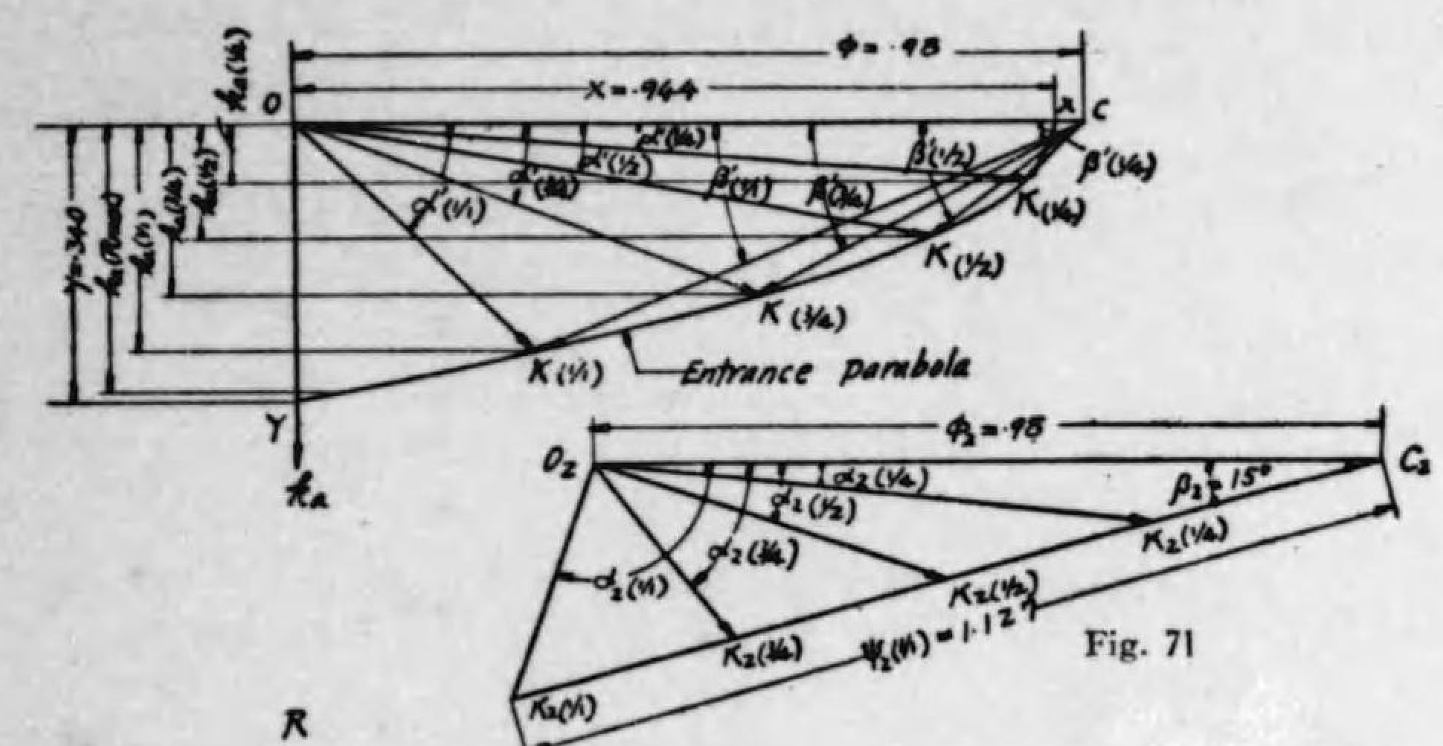
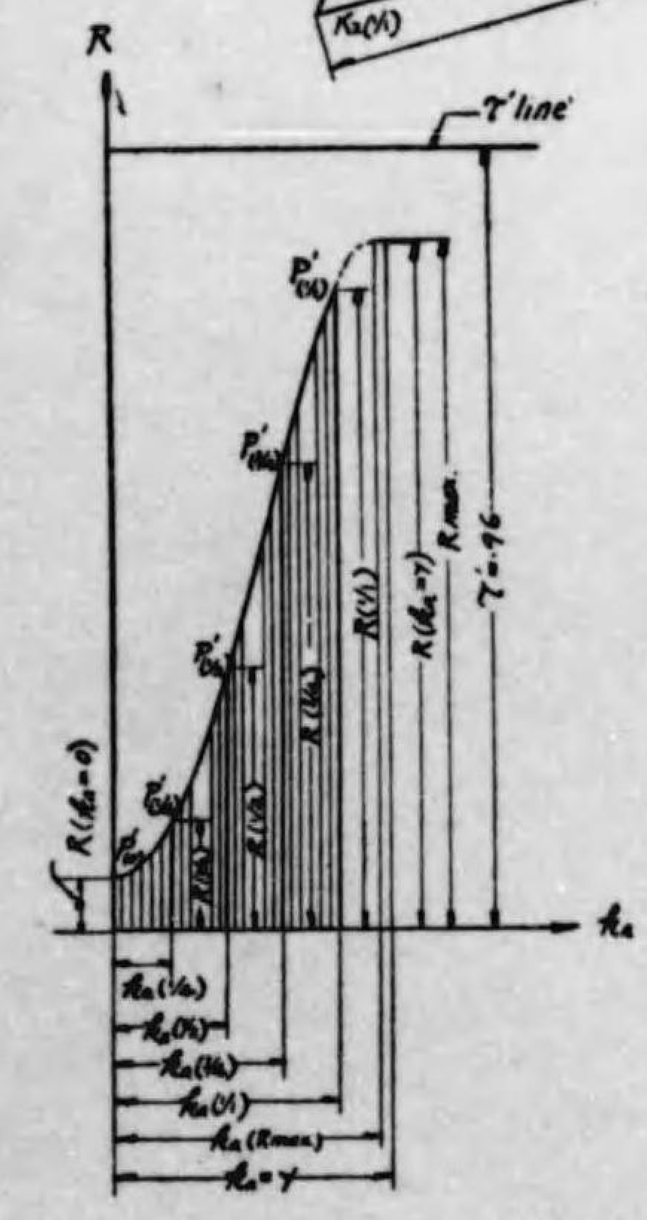


Fig. 71



III₂ Axial Flow Turbine $\tau = \cdot 96$ $\phi = \cdot 98$ $\beta_2 = 15°$ $x = \cdot 944$ $\phi_2 = \phi$ $\psi_2(1/1) = 1{\cdot}127$ $y = \cdot 340$			
$h_a =$	0	$R(h_a=0)$	·069
$h_a =$	y	$R(h_a=y)$	·844
$h_a(R_{max})$	·329	$R_{max.}$	·848
$h_a(1/1)$	·28	$R(1/1)$	·789
$h_a(3/4)$	·21	$R(3/4)$	·575
$h_a(1/2)$	·14	$R(1/2)$	·326
$h_a(1/4)$	·07	$R(1/4)$	·138

Fig. 72

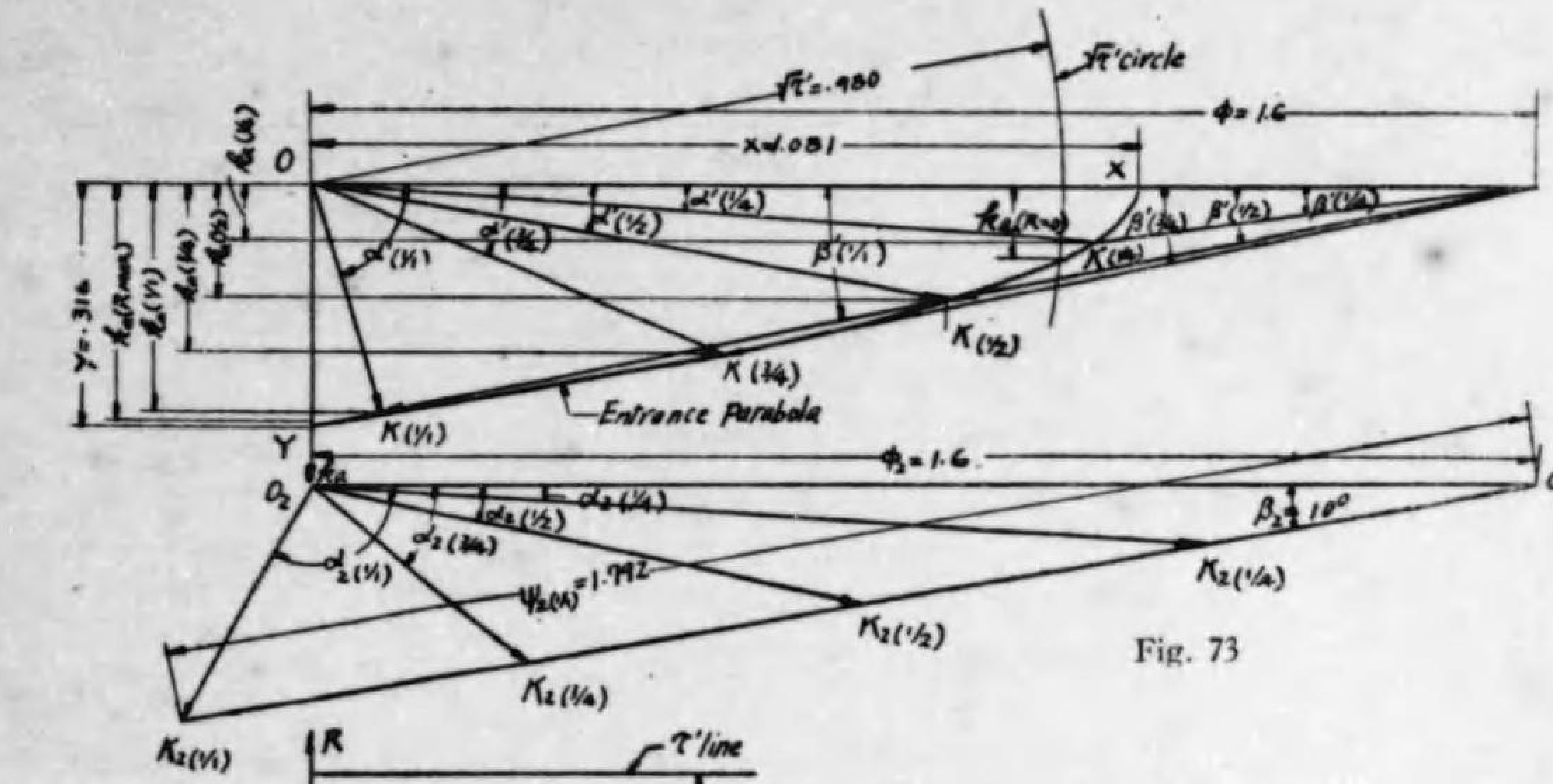
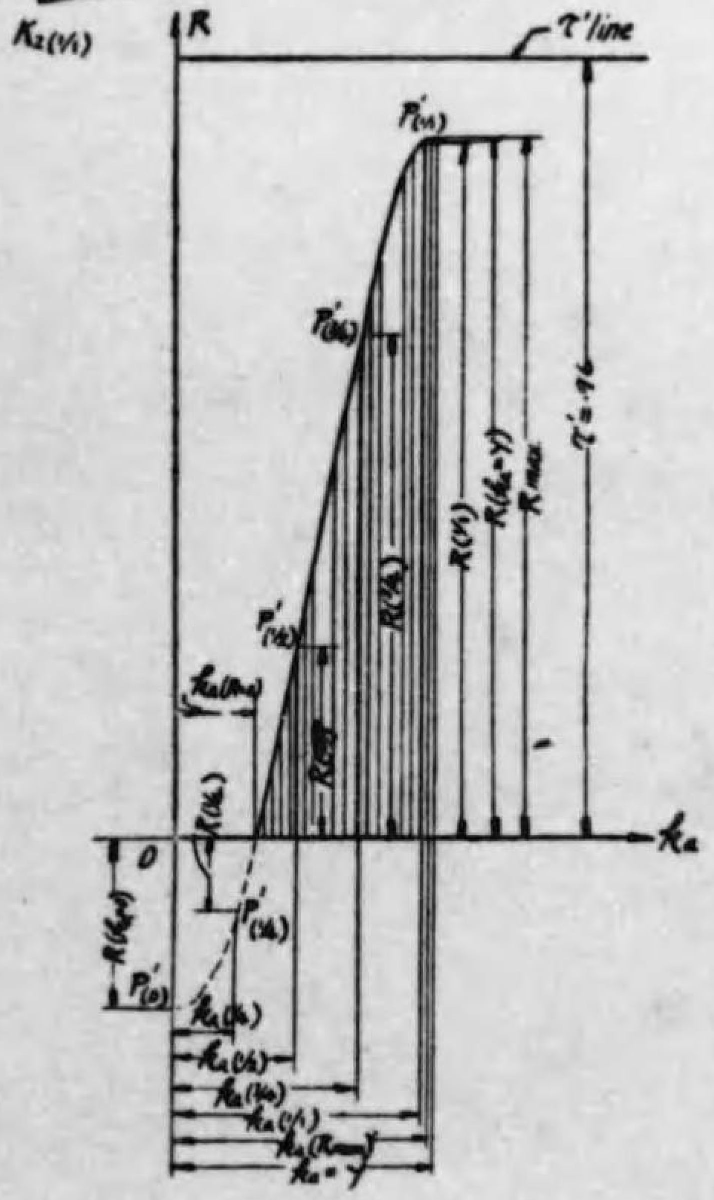


Fig. 73



III₃ Axial Flow Turbine $\tau = .96$			
$\Phi = 1.6$ $\beta_2 = 10°$ $X = 1.081$			
$\Phi_2 = \Phi$ $\Psi_2(1/1) = 1.792$ $Y = .316$			
$k_a =$	0	$R(k_a=0)$	−.209
$k_a =$	Y	$R(k_a=Y)$	.860
$k_a(R=0)$	.099	$R =$	0
$k_a(R_{max})$	.309	$R_{max.}$	.862
$k_a(1/1)$	.300	$R(1/1)$	.858
$k_a(3/4)$	.225	$R(3/4)$	.624
$k_a(1/2)$	.150	$R(1/2)$	.235
$k_a(1/4)$	.075	$R(1/4)$	−.087

Fig. 74

【非賣品】

昭和二年十月六日印刷
昭和二年十月十日發行

編輯兼發行者　大阪高等工業學校

印刷者　大阪市西區土佐堀通四丁目五番地
飯田彌之助

印刷所　大阪市西區土佐堀通四丁目五番地
三有社

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