THE

LADY'S AND GENTLEMAN'S DIARY,

FOR THE YEAR OF OUR LORD

1857,

Being the first after Bissextile.

DESIGNED PRINCIPALLY FOR THE AMUSEMENT AND INSTRUCTION OF

STUDENTS IN MATHEMATICS:

COMPRISING

MANY USEFUL AND ENTERTAINING PARTICULARS,

INTERESTING TO ALL PERSONS ENGAGED IN THAT DELIGHTFUL PURSUIT.

THE ONE HUNDRED AND FIFTY-FOURTH ANNUAL NUMBER.

LONDON:

PRINTED FOR

THE COMPANY OF STATIONERS,

AND SOLD BY J. GREENHILL, AT THEIR HALL, LUDGATE ST.

J. E. ADLARD, PRINTER, BARTHOLOMEW CLOSE.

[*Price* 1*s*. 4*d*. *stitched.*]

CHRONOLOGICAL NOTES, &c. IN 1857.

Dominical Letter	D	Sundays after Epiphany		4	Easter Day	Apr.	12
Golden Number	15	„ „ Trinity		24	Rogation Sunday	May	17
Epact	4	Septuages. Sund.	Feb.	8	Ascension Day	May	21
Solar Cycle	18	Shrove Sunday	Feb.	22	Whit Sunday	May	31
Number of Direction	22	Lent begins	Feb.	25	Trinity Sunday	June	7
Roman Indiction	15	Ist Sund. in Lent	Mar.	1	Advent Sunday	Nov.	29
Julian Period	6570	Midlent Sunday	Mar.	22	Jew. year 5618 begs.	Sep.	19
Year of the Dionysian	186	Good Friday	Apr.	10	Mahn. „ 1274 „	Aug.	22

ECLIPSES, &c.

This year there will be only Two eclipses, both of the SUN.

I. *March* 25*th*.—*A total eclipse of the* SUN, but not visible to this country. It will be visible to New Zealand, New Guinea, Mexico, the south-western parts of the United States of North America, the more eastern parts of Australia, and a large surface of the Pacific Ocean, extending on both sides of the equator.

II. *September* 18*th*.—*An annular eclipse of the* SUN, also invisible to Great Britain. This eclipse will be visible throughout Lapland, Finland, Russia, Turkey, Arabia, Persia, Hindoostan, China, Australia, and also in the North Pacific and Indian Oceans.

MERCURY will be visible in the mornings, before the Sun rises, near the eastern horizon, about *February* 25, *June* 25, and *October* 16; and in the evenings, soon after Sunset, near the western horizon, about *January* 15, *May* 7, *September* 4, and *December* 29.

VENUS will be an *Evening Star* until *May* 9; and afterwards a *Morning Star* to the end of the year.

JUPITER will be an *Evening Star* until *April* 11; then a *Morning Star* until *November* 2; and afterwards an *Evening Star* to the end of the year.

MARS will be in *conjunction* with the SUN on *June* 7, and will therefore be unfavorable for observation throughout the year.

SATURN'S RING is visible. The planet will be in *opposition* to the Sun on *January* 1, in *conjunction* on *July* 10, and at the end of the year another *opposition* will be approaching. Therefore the most favorable times for telescopic observations of this planet and his beautiful *Luminous Rings* will be during the months of *January*, *February*, *March*, *October*, *November*, and *December*. The major and minor axes of the rings will appear nearly in the proportion of 5 to 2.

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N° 154. JANUARY, 31 DAYS.

SUN ENTERS

19d. 13h. 16m.

First Quarter .... 3d, 14m. past Noon.
Full Moon .... 10th, 8m. past 9 Morn.
Last Quarter .... 18th, 50m. past 4 Morn.
New Moon .... 25th, 26m. past 11 Aftern.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	Th	**Circumcision**	8h 9′	3h 59′	23° s 0′	10h a 15′	5
2	F		8 8	4 0	22 54	11 40	6
3	S	♄ sets 8 6 morn.	8 8	4 2	49	morn.	7
4	D	**2d Sun. aft. Christmas**	8 8	4 3	42	1 4	8
5	M		8 8	4 4	36	2 30	9
6	Tu	**Epiphany**: 12th day	8 7	4 5	29	4 0	10
7	W		8 7	4 6	21	5 29	11
8	Th	*Lucian*	8 7	4 8	13	6 50	12
9	F	♃ sets 10 58 aftern.	8 6	4 9	5	7 56	13
10	S	[Term beg.	8 5	4 11	21 56	rises	F
11	D	**1st S. af. Epiph.**: Hil.	8 5	4 12	46	5 a 19	15
12	M	Plough Monday	8 4	4 13	37	6 41	16
13	Tu	*Hilary:* Camb. T. beg.	8 3	4 15	27	7 59	17
14	W	Oxford Term begins	8 3	4 16	16	9 14	18
15	Th		8 2	4 18	5	10 26	19
16	F	♂ sets 7 33 aftern.	8 1	4 20	20 54	11 36	20
17	S	[*Prisca*	8 0	4 21	42	morn.	21
18	D	**2d Sun. aft. Epiph.**:	7 59	4 23	30	0 48	22
19	M		7 58	4 24	17	2 0	23
20	Tu	*Fabian*	7 57	4 26	4	3 14	24
21	W	*Agnes*	7 56	4 28	19 51	4 29	25
22	Th	*Vincent*	7 55	4 30	37	5 41	26
23	F	♀ sets 8 36 aftern.	7 53	4 31	23	6 45	27
24	S		7 52	4 33	9	7 36	28
25	D	**3d S. af. Epiph.: Con.**	7 51	4 35	18 54	sets	N
26	M	[**St. Paul**	7 49	4 36	39	5 a 5	1
27	Tu		7 48	4 38	24	6 34	2
28	W		7 47	4 40	8	8 1	3
29	Th	☿ sets 5 19 aftern.	7 45	4 42	17 52	9 26	4
30	F	**K. Chas. I. mart.** 1649	7 44	4 44	35	10 52	5
31	S	Hilary Term ends	7 42	4 45	19	morn.	6

Day.	Length of D.	Day Inc.	D. breaks	Tw. ends	Sun East.	Cl. bef. Sun.	☉ Semidiameter
1	7h. 51m.	0 6	6 m 2	6 a 5	4 m 45	3′ 58″	16′ 18″
6	58	13	2	10	49	6 15	18
11	8 7	22	1	16	54	8 19	18
16	19	34	5 59	22	59	10 8	18
21	32	47	55	28	5 5	11 41	17
26	47	1 2	50	36	10	12 54	17

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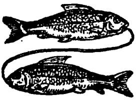
SUN ENTERS



18d. 3h. 53m.

FEBRUARY, 28 DAYS. 1857

First Quarter ...... 1st, 20m. past 8 Aftern.
Full Moon .... 8th, 53m. past 11 Aftern.
Last Quarter ...... 17th, 19m. past 2 Morn.
New Moon .... 24th, 58m. past 11 Morn.

D. M.	D. W.	SUNDAYS, HOLIDAYS, &c. *Planets rise and set.*	*Sun rises*	*Sun sets*	*Sun's declin.*	☾ *rises & sets*	☾'s *age*
1	D	4th Sun. af. Epiphany	7h 41′	4h 47′	17° s 2′	0h m 18′	7
2	M	Purif.: Candl.-Day	7 39	4 49	16 44	1 47	8
3	Tu	*Blase*	7 38	4 51	27	3 14	9
4	W		7 36	4 53	9	4 37	10
5	Th	*Agatha*	7 34	4 54	15 51	5 48	11
6	F	♄ sets 5 43 morn.	7 33	4 56	32	6 41	12
7	S		7 31	4 58	13	7 17	13
8	D	Septuagesima Sunday	7 29	5 0	14 55	rises	F
9	M		7 27	5 2	35	5 a 39	15
10	Tu	Qu. Victoria mar. 1840	7 26	5 4	16	6 56	16
11	W		7 24	5 6	13 56	8 8	17
12	Th	♃ sets 9 20 aftern.	7 22	5 7	36	9 20	18
13	F		7 20	5 9	16	10 31	19
14	S	*Valentine*	7 18	5 11	12 56	11 43	20
15	D	Sexagesima Sunday	7 16	5 13	35	morn.	21
16	M		7 14	5 15	14	0 56	22
17	Tu		7 12	5 17	11 54	2 10	23
18	W	♂ sets 7 51 aftern.	7 10	5 18	32	3 24	24
19	Th	♀ sets 9 47 aftern.	7 8	5 20	11	4 32	25
20	F		7 6	5 22	10 50	5 28	26
21	S	[Term div. noon	7 4	5 24	28	6 9	27
22	D	Shrove Sunday: Cam.	7 2	5 26	6	6 39	28
23	M	[Matthias	7 0	5 27	9 44	7 1	29
24	Tu	Shrove Tuesday: St.	6 58	5 29	22	sets	N
25	W	Lent beg.: Ash Wed.	6 56	5 31	0	7 a 3	1
26	Th	☿ rises 6 0 morn.	6 54	5 33	8 37	8 31	2
27	F		6 52	5 34	15	10 0	3
28	S		6 49	5 36	7 52	11 32	4

Day.	Length of D.	Day Inc.	D. breaks	Tw. ends	Sun East.	Cl. bef. Sun.	⊙ Semidiameter
1	9h. 6m.	1 21	5 m 43	6 a 45	5 m 17	13′ 55″	16′ 16″
6	24	39	36	52	23	14 23	15
11	42	57	29	7 0	29	14 31	14
16	10 0	2 15	20	9	34	14 20	13
21	19	35	11	17	40	13 51	12
26	39	54	1	26	45	13 7	11

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N° 154. MARCH, 31 DAYS.

First Quarter	3d, 30m. past	4	Morn.
Full Moon	10th, 17m. past	4	Aftern.
Last Quarter	18th, 3m. past	9	Aftern.
New Moon	25th, 28m. past	10	Aftern.

SUN ENTERS

20d. 3h. 45m.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	D	1st S. in Lent: *David*	6h 47′	5h 38′	7° s 29′	morn.	5
2	M	*Chad:* Least twilight	6 45	5 40	6	1h 1	6
3	Tu		6 43	5 42	6 43	2 28	7
4	W	Ember Week	6 41	5 43	20	3 42	8
5	Th		6 38	5 45	5 57	4 40	9
6	F	♄ sets 3 51 morn.	6 36	5 47	34	5 20	10
7	S	*Perpetua*	6 34	5 48	11	5 47	11
8	D	2d Sunday in Lent	6 32	5 50	4 47	6 7	12
9	M		6 30	5 52	24	6 21	13
10	Tu		6 27	5 54	0	rises	F
11	W		6 25	5 55	3 37	7 a 5	15
12	Th	*Gregory*	6 23	5 57	13	8 15	16
13	F		6 20	5 59	2 50	9 27	17
14	S	♃ sets 7 59 aftern.	6 18	6 1	26	10 40	18
15	D	3d Sunday in Lent	6 16	6 2	2	11 54	19
16	M		6 14	6 4	1 39	morn.	20
17	Tu	*St. Patrick*	6 11	6 6	15	1 8	21
18	W	Prs. Louisa b. 1848	6 9	6 7	0 51	2 18	22
19	Th	Equal day and night	6 7	6 9	27	3 18	23
20	F	♂ sets 8 4 aftern.	6 5	6 11	0 s 4	4 5	24
21	S	*Benedict*	6 2	6 12	0 N 20	4 39	25
22	D	4th, or Midl. Sunday	6 0	6 14	44	5 4	26
23	M		5 58	6 16	1 7	5 22	27
24	Tu		5 55	6 17	31	5 37	28
25	W	Annunc. or Lady-Day	5 53	6 19	55	sets	N
26	Th	♀ sets 10 51 aftern.	5 51	6 21	2 18	7 a 29	1
27	F		5 49	6 23	42	9 3	2
28	S		5 46	6 24	3 5	10 37	3
29	D	5th Sunday in Lent	5 44	6 26	28	morn.	4
30	M	☿ rises 5 35 morn.	5 42	6 27	52	0 10	5
31	Tu		5 39	6 29	4 15	1 32	6

Day.	Length of D.	Day Inc.	D.breaks	Tw. ends	Sun East.	Cl. bef. Sun.	☉ Semidiameter
1	10h. 51m.	3 6	4 m 54	7 a 31	5 m 49	12′ 33″	16′ 10″
6	11 10	25	43	40	54	11 27	9
11	30	45	31	49	59	10 10	7
16	50	4 5	19	58	6 4	8 46	6
21	12 10	25	6	8 8	8	7 17	5
26	30	45	3 53	18	13	5 45	3

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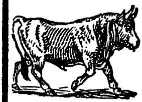
SUN ENTERS



19d. 15h. 50m.

APRIL, 30 DAYS. 1857

First Quarter ...... 1st, 34m. past 1 Aftern.
Full Moon .... 9th, 28m. past 9 Morn.
Last Quarter......... 17th, 0m. past Noon.
New Moon .... 24th, 14m. past 7 Morn.
First Quarter......... 30th, 17m. past Midnight.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	W		5h 37′	6h 31′	4° N 38′	2h m 35′	7
2	Th	[*Bp. Chich.*	5 35	6 32	5 1	3 22	8
3	F	Cam. Term ends: *Rich.*	5 33	6 34	24	3 53	9
4	S	Oxf. T. ends: *St. Am-*	5 30	6 36	47	4 13	10
5	D	**Palm Sunday** [*brose*	5 28	6 38	6 10	4 29	11
6	M	Old Lady-day	5 26	6 39	33	4 41	12
7	Tu	**Pr. Leopold b.** 1853	5 24	6 41	55	4 52	13
8	W	♄ sets 1 44 morn.	5 22	6 42	7 18	5 2	14
9	Th	*Maundy Thursday*	5 19	6 44	40	rises	F
10	F	**Good Friday**	5 17	6 46	8 2	8 a 28	16
11	S		5 15	6 47	24	9 42	17
12	D	**Easter Sunday**	5 13	6 49	46	10 55	18
13	M	**Easter Monday**	5 10	6 51	9 8	morn.	19
14	Tu	**Easter Tuesday**	5 8	6 52	30	0 7	20
15	W	Easter Term begins	5 6	6 54	51	1 10	21
16	Th		5 4	6 56	10 12	2 1	22
17	F	♃ rises 5 1 morn.	5 2	6 57	33	2 38	23
18	S		5 0	6 59	54	3 6	24
19	D	**1st, or Low Sunday**:	4 58	7 1	11 15	3 26	25
20	M	[*Alphege*	4 56	7 2	36	3 42	26
21	Tu	♂ sets 8 14 aftern.	4 53	7 4	56	3 55	27
22	W	Oxf. and Camb. T. beg.	4 51	7 6	12 17	4 8	28
23	Th	*St. George*	4 49	7 7	37	4 21	29
24	F	[1843: **Ds. Gl. b.** 1776	4 47	7 9	56	sets	N
25	S	**St. Mark: Ps. Al. b.**	4 45	7 11	13 16	9 a 36	1
26	D	**2d Sunday aft. Easter**	4 43	7 12	35	11 7	2
27	M		4 41	7 14	55	morn.	3
28	Tu	♀ sets 9 30 aftern.	4 39	7 16	14 14	0 23	4
29	W	☿ sets 9 15 aftern.	4 37	7 17	32	1 17	5
30	Th		4 36	7 19	51	1 56	6

Day.	Length of D.	Day Inc.	D. breaks	Tw. ends	Sun East.	Cl. bef. Sun.	☉ Semidiameter
1	12h. 54m	5 9	3 m 37	8 a 31	6 m 19	3′ 55″	16′ 2″
6	13 13	28	23	43	23	2 25	0
11	32	47	8	55	28	1 1	15 59
16	52	6 7	2 52	9 7	33	0 aft. 16	58
21	14 11	26	36	21	37	1 23	56
26	29	44	19	36	42	2 19	55

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No 154. MAY, 31 DAYS.

Full Moon .... 9th, 11m. past 2 Morn.
Last Quarter ...... 16th, 10m. past 11 Aftern.
New Moon .... 23d, 48m. past 2 Aftern.
First Quarter ...... 30th, 12m. past 1 Aftern.

SUN ENTERS

20d. 15h. 56m.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	F	St. Phil. & James: Pr.	4h 34′	7h 20′	15° N 9′	2h m 19′	7
2	S	[Arthur b. 1850	4 32	7 22	27	2 37	8
3	D	3d S. aft. East.: *Inv.*	4 30	7 24	45	2 50	9
4	M	*[of Cross*	4 28	7 25	16 2	3 1	10
5	Tu	♄ sets 0 6 morn.	4 26	7 27	19	3 11	11
6	W	*John Evan. à P. Lat.*	4 24	7 28	36	3 20	12
7	Th		4 23	7 30	53	3 30	13
8	F	Easter Term ends	4 21	7 32	17 9	3 42	14
9	S	♃ rises 3 44 morn.	4 19	7 33	25	rises	F
10	D	4th Sund. aft. Easter	4 18	7 35	41	9 a 56	16
11	M		4 16	7 36	57	11 3	17
12	Tu		4 15	7 38	18 12	11 58	18
13	W	Old May-day	4 13	7 39	27	morn.	19
14	Th		4 11	7 41	41	0 39	20
15	F	♂ sets 8 18 aftern.	4 10	7 42	55	1 8	21
16	S		4 8	7 44	19 9	1 31	22
17	D	Rogation Sunday	4 7	7 45	23	1 47	23
18	M		4 6	7 47	36	2 1	24
19	Tu	*Dunstan*	4 4	7 48	49	2 13	25
20	W	♀ rises 3 16 morn.	4 3	7 50	20 2	2 26	26
21	Th	Ascension: Holy Th.	4 2	7 51	14	2 40	27
22	F	Trinity Term begins	4 1	7 52	26	2 57	28
23	S	Nights all twilight	3 59	7 54	38	sets	N
24	D	S. af. Asc.: Q. Vict. b.	3 58	7 55	49	9 a 59	1
25	M	Ps. Hel. b. 1846 [1819	3 57	7 56	21 0	11 6	2
26	Tu	*Aug. 1st Ap. Can.* [*Bede*	3 56	7 58	10	11 51	3
27	W	K. Han. b. 1819: *Ven.*	3 55	7 59	21	morn.	4
28	Th	☿ sets 8 22 aftern.	3 54	8 0	30	0 20	5
29	F	K. Chas. II. rest. 1660	3 53	8 1	40	0 42	6
30	S	Oxford Term ends	3 52	8 2	49	0 56	7
31	D	Whit S.: Cam. T. d. m.	3 51	8 3	57	1 9	8

Day.	Length of D.	Day Inc.	D. breaks	Tw. ends	Sun East.	Cl. aft. Sun.	☉ Semidiameter
1	14h. 47m.	7 2	2 m 2	9 a 52	6 m 46	3′ 3″	15′ 54″
6	15 4	19	1 44	10 9	51	3 34	53
11	20	35	24	29	56	3 51	52
16	35	50	0 59	53	7 0	3 53	51
21	49	8 4	25	11 27	4	3 41	50
26	16 2	17	No real night.		8	3 15	49

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SUN ENTERS

21d. 0h. 26m.

JUNE, 30 DAYS. 1857

Full Moon .... 7th, 23m. past 5 Aftern.
Last Quarter .... 15th, 9m. past 7 Morn.
New Moon .... 21st, 3m. past 10 Aftern.
First Quarter .... 29th, 20m. past 4 Morn.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	M	**Whit Monday:** *Nico-*	3h 50'	8h 5'	22° N 6'	1h m 19'	9
2	Tu	**Whit Tuesday** [*mede*	3 50	8 6	14	1 29	10
3	W	**Emb. Week:** Oxf. T. b.	3 49	8 7	21	1 38	11
4	Th		3 48	8 8	28	1 50	12
5	F	*Boniface*	3 47	8 9	35	2 4	13
6	S	♄ sets 10 10 aftern.	3 47	8 10	41	2 22	14
7	D	**Trinity Sunday**	3 46	8 11	47	rises	F
8	M		3 46	8 12	53	9 a 53	16
9	Tu		3 46	8 12	58	10 38	17
10	W	♃ rises 1 53 morn.	3 45	8 13	23 3	11 12	18
11	Th	**St. Barnabas:** *Corpus*	3 45	8 14	7	11 36	19
12	F	Trin. T. ends [*Christi*	3 45	8 14	11	11 53	20
13	S		3 44	8 15	14	morn.	21
14	D	**1st Sund. aft. Trinity**	3 44	8 16	17	0 8	22
15	M		3 44	8 16	20	0 20	23
16	Tu		3 44	8 17	22	0 31	24
17	W	*St. Alban*	3 44	8 17	24	0 45	25
18	Th	♂ rises 3 27 morn.	3 44	8 17	26	1 0	26
19	F		3 44	8 18	27	1 20	27
20	S	**Queen Victoria acc.**	3 44	8 18	27	1 48	28
21	D	**2d S. a. Tr.: Q. V. pro.**	3 45	8 18	28	sets	N
22	M	♀ rises 1 50 morn.	3 45	8 19	27	9 a 42	1
23	Tu		3 45	8 19	27	10 19	2
24	W	**Nat. Jn. Bapt.: Mids.**	3 45	8 19	26	10 44	3
25	Th	[**Day**	3 46	8 19	24	11 1	4
26	F	☿ rises 2 41 morn.	3 46	8 19	22	11 14	5
27	S	[**cor.** 1838	3 47	8 19	20	11 25	6
28	D	**3d S. af. Tr.: Qu. Vic.**	3 47	8 19	18	11 36	7
29	M	**St. Peter**	3 48	8 18	14	11 45	8
30	Tu		3 48	8 18	11	11 56	9

Day.	Length of D.	Day Inc.	D. breaks / Tw. ends	Sun East.	Cl. aft. Sun.	☉ Semidiameter
1	16h. 14m.	8 29		7 m 13	2' 29"	15' 48"
6	23	38	No real Night	16	1 40	47
11	29	44	but constant	19	0 43	47
16	33	.48	Day, or Twi-	21	0 bef. 19	46
21	34	49	light.	22	1 25	46
26	33	0 dec. 1		23	2 29	46

PRINTED FOR THE COMPANY OF STATIONERS.

N° 154. **JULY, 31 DAYS.**

SUN ENTERS

22d. 11h. 19m.

Full Moon .... 7th, 44m. past 6 Morn.
Last Quarter .... 14th, 56m. past Noon.
New Moon .... 21st, 12m. past 6 Morn.
First Quarter .... 28th, 14m. past 9 Aftern.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	W		$3^h49'$	$8^h18'$	23°N 7′	morn.	10
2	Th	*Visit. B. V. M.*	3 50	8 18	3	0^h 10′	11
3	F	Dog-days begin	3 50	8 17	22 58	0 26	12
4	S	*Trs. St. Martin*	3 51	8 17	53	0 50	13
5	D	4th Sun. aft. Trinity	3 52	8 16	48	1 23	14
6	M	Old Mids.-day	3 53	8 16	42	2 8	15
7	Tu	*Th. à Becket:* Oxf. Act:	3 54	8 15	35	rises	F
8	W	[Camb. Com.	3 55	8 15	29	9 a 41	17
9	Th	♄ sets 8 14 aftern.	3 56	8 14	22	9 59	18
10	F	Cambridge Term ends	3 57	8 13	14	10 15	19
11	S	Oxford Term ends	3 58	8 13	6	10 28	20
12	D	5th Sun. aft. Trinity	3 59	8 12	21 58	10 40	21
13	M		4 0	8 11	50	10 52	22
14	Tu	♃ rises 11 48 aftern.	4 1	8 10	41	11 5	23
15	W	*St. Swithin*	4 2	8 9	31	11 22	24
16	Th		4 3	8 8	22	11 46	25
17	F		4 5	8 7	11	morn.	26
18	S	♂ rises 2 59 morn.	4 6	8 6	1	0 19	27
19	D	6th Sund. aft. Trinity	4 7	8 5	20 50	1 11	28
20	M	*Margaret*	4 8	8 4	39	2 20	29
21	Tu		4 10	8 2	28	sets	N
22	W	*Magdalene*	4 11	8 1	16	9 a 5	1
23	Th		4 12	8 0	4	9 19	2
24	F	[1797	4 14	7 59	19 51	9 32	3
25	S	St. Jas.: Ds. Cam. b.	4 15	7 57	38	9 42	4
26	D	7th Sun. aft. Tr.: *St.*	4 17	7 56	25	9 52	5
27	M	[*Anne*	4 18	7 54	12	10 3	6
28	Tu	♀ rises 1 0 morn.	4 20	7 53	18 58	10 15	7
29	W	☿ sets 8 18 aftern.	4 21	7 51	44	10 29	8
30	Th		4 22	7 50	29	10 50	9
31	F		4 24	7 48	15	11 18	10

Day.	Length of D.	Day dec.	D. breaks	Tw. ends	Sun East.	Cl. bef. Sun.	☉ Semidiameter
1	16h. 29m.	0 5			7 m 23	3′ 30″	15′ 46″
6	23	11	No real Night.		22	4 24	46
11	15	19			21	5 9	46
16	5	29			18	5 43	46
21	15 53	41	0 m 8	12 a 4	15	6 5	47
26	39	55	1 1	11 12	11	6 13	47

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SUN ENTERS

22d. 17h. 51m.

AUGUST, 31 DAYS. 1857

Full Moon .... 5th, 28m. past 6 Aftern.
Last Quarter .... 12th, 41m. past 5 Aftern.
New Moon .... 19th, 26m. past 4 Aftern.
First Quarter .... 27th, 5m. past 3 Aftern.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	S	Lammas-day	4h 25′	7h 47′	18°N 0′	11h a 59′	11
2	D	**8th Sund. aft. Trinity**	4 27	7 45	17 44	morn.	12
3	M		4 28	7 44	29	0 55	13
4	Tu	♄ rises 2 35 morn.	4 30	7 42	13	2 7	14
5	W	[*Transfig.*	4 31	7 40	16 57	rises	F
6	Th	**Pr. Alfred born** 1844:	4 33	7 38	40	8 a 21	16
7	F	*Name of Jesus*	4 35	7 37	24	8 35	17
8	S	(Shooting stars)	4 36	7 35	7	8 47	18
9	D	**9th Sund. aft. Trinity**	4 38	7 33	15 49	8 58	19
10	M	*St. Lawrence*	4 39	7 31	32	9 11	20
11	Tu	Dog-days end	4 41	7 29	14	9 27	21
12	W		4 42	7 27	14 56	9 49	22
13	Th	♃ rises 9 58 aftern.	4 44	7 25	38	10 18	23
14	F		4 46	7 23	19	11 2	24
15	S	*Assumption B. V. M.*	4 47	7 21	1	morn.	25
16	D	**10th Sun. aft. Trinity**	4 49	7 19	13 42	0 3	26
17	M	**Ds. Kent born** 1786	4 50	7 17	23	1 18	27
18	Tu		4 52	7 15	3	2 45	28
19	W		4 53	7 13	12 44	sets	N
20	Th	♂ rises 2 45 morn.	4 55	7 11	24	7 a 38	1
21	F	♀ rises 1 8 morn.	4 57	7 9	4	7 49	2
22	S		4 58	7 7	11 44	8 0	3
23	D	**11th Sun. aft. Trinity**	5 0	7 5	24	8 9	4
24	M	**St. Bartholomew**	5 1	7 3	3	8 20	5
25	Tu		5 3	7 1	10 42	8 34	6
26	W	**Pr. Albert born** 1819	5 5	6 59	22	8 51	7
27	Th	☿ sets 7 33 aftern.	5 6	6 57	1	9 16	8
28	F	*St. Augustine*	5 8	6 54	9 39	9 52	9
29	S	*St. John Bapt. beh.*	5 9	6 52	18	10 40	10
30	D	**12th Sun. aft. Trinity**	5 11	6 50	8 57	11 46	11
31	M		5 13	6 48	35	morn.	12

Day.	Length of D.	Day dec.	D. breaks	Tw. ends	Sun fast.	Cl. bef. Sun.	☉ Semidiameter
1	15h. 21m.	1 13	1 m 32	10 a 40	7 m 6	6′ 2″	15′ 48″
6	5	29	51	20	1	5 36	49
11	14 48	46	2 8	1	6 55	4 56	49
16	31	2 3	24	9 44	49	4 2	50
21	13	21	39	27	42	2 55	51
26	13 54	40	52	11	35	1 37	52

N° 154. SEPTEMBER, 30 DAYS.

Full Moon .... 4th, 7m. past 5 Morn.
Last Quarter ...... 10th, 50m. past 10 Aftern.
New Moon .... 18th, 33m. past 5 Morn.
First Quarter ...... 26th, 59m. past 8 Morn.

SUN ENTERS

22d. 14h. 33m.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	Tu	*Giles*	5h 14'	6h 46'	8° N 13'	1h m 2'	13
2	W	London bt. 1666, o.s.	5 16	6 43	7 51	2 24	14
3	Th		5 17	6 41	29	3 50	15
4	F	♄ rises 0 52 morn.	5 19	6 39	7	rises	F
5	S	Old Bartholomew	5 21	6 37	6 45	7 a 6	17
6	D	**13th Sun. aft. Trinity**	5 22	6 34	23	7 19	18
7	M	*Enurchus*	5 24	6 32	0	7 33	19
8	Tu	*Nativity B. V. M.*	5 25	6 30	5 38	7 53	20
9	W		5 27	6 28	15	8 19	21
10	Th	♃ rises 8 10 aftern.	5 29	6 25	4 52	8 59	22
11	F	♂ rises 2 39 morn.	5 30	6 23	29	9 54	23
12	S		5 32	6 21	7	11 6	24
13	D	**14th Sun. aft. Trinity**	5 33	6 18	3 44	morn.	25
14	M	*Holy Cross*	5 35	6 16	20	0 28	26
15	Tu		5 37	6 14	2 57	1 51	27
16	W	**Ember Week**	5 38	6 12	34	3 14	28
17	Th	*Lambert*	5 40	6 9	11	4 32	29
18	F	Geo. I. and II. landed	5 41	6 7	1 48	sets	N
19	S		5 43	6 5	24	6 a 17	1
20	D	**15th Sun. aft. Trinity**	5 44	6 2	1	6 27	2
21	M	**St. Matthew**	5 46	6 0	0 38	6 40	3
22	Tu		5 48	5 57	0 N 14	6 56	4
23	W	♀ rises 2 16 morn.	5 49	5 55	0 S 9	7 17	5
24	Th	☿ sets 5 52 aftern.	5 51	5 53	33	7 47	6
25	F	Equal day and night	5 53	5 51	56	8 30	7
26	S	*St. Cyprian*	5 54	5 48	1 20	9 27	8
27	D	**16th Sun. aft. Trinity**	5 56	5 46	43	10 37	9
28	M		5 57	5 44	2 6	11 56	10
29	Tu	**Michaelmas-Day**	5 59	5 42	30	morn.	11
30	W	*St. Jerome*	6 1	5 39	53	1 20	12

Day.	Length of D.	Day dec.	D. breaks	Tw. ends	Sun East.	Cl. aft. Sun.	☉ Semidiameter
1	13h. 31m.	3 3	3 m 7	8 a 53	6 m 26	0' 10"	15' 54"
6	12	22	19	38	19	1 48	55
11	12 53	41	30	23	11	3 31	56
16	33	4 1	40	9	3	5 16	57
21	14	20	50	7 56	5 55	7 1	59
26	11 54	40	4 0	43	47	8 44	16 0

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SUN ENTERS

22d. 22h. 49m.

OCTOBER, 31 DAYS. 1857

Full Moon .... 3d, 8m. past 3 Aftern.
Last Quarter ...... 10th, 53m. past 5 Morn.
New Moon .... 17th, 38m. past 9 Aftern.
First Quarter ...... 26th, 5m. past 2 Morn.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise		☉ set		☉ decl.		☾ r. & s.		☾ a
1	Th	*Remigius*	6h	2'	5h	37'	3° s	16'	2h m	45'	13
2	F		6	4	5	35		40	4	11	14
3	S	♄ rises 11 8 aftern.	6	6	5	32	4	3	rises		F
4	D	17th Sun. aft. Trinity	6	7	5	30		26	5 a	39	16
5	M		6	9	5	28		49	5	57	17
6	Tu	*Faith*	6	11	5	26	5	12	6	21	18
7	W		6	12	5	23		35	6	55	19
8	Th	♃ rises 6 17 aftern.	6	14	5	21		58	7	47	20
9	F	*St. Denys*	6	16	5	19	6	21	8	55	21
10	S	Oxf. and Cam. T. beg.	6	17	5	17		44	10	15	22
11	D	18th Sun. aft. Tr.: Old	6	19	5	15	7	7	11	39	23
12	M	Least twilight [M.-d.	6	21	5	12		29	morn.		24
13	Tu	*Trs. K. Edw. Conf.*	6	23	5	10		52	1	0	25
14	W		6	24	5	8	8	14	2	19	26
15	Th		6	26	5	6		37	3	35	27
16	F	♂ rises 2 27 morn.	6	28	5	4		59	4	47	28
17	S	*Etheldreda*	6	29	5	1	9	21	sets		N
18	D	19th S. af. Trin.: St.	6	31	4	59		43	4 a	48	1
19	M	[Luke	6	33	4	57	10	4	5	3	2
20	Tu	♀ rises 3 33 morn.	6	35	4	55		26	5	21	3
21	W		6	36	4	53		48	5	49	4
22	Th		6	38	4	51	11	9	6	25	5
23	F	☿ rises 5 2 morn.	6	40	4	49		30	7	16	6
24	S		6	42	4	47		51	8	21	7
25	D	20th Sun. aft. Trin.:	6	43	4	45	12	12	9	36	8
26	M	[*Crispin*	6	45	4	43		32	10	54	9
27	Tu		6	47	4	41		53	morn.		10
28	W	St. Simon & St. Jude	6	49	4	39	13	13	0	16	11
29	Th		6	50	4	37		33	1	39	12
30	F		6	52	4	35		53	3	4	13
31	S		6	54	4	34	14	12	4	31	14

Day.	Length of D.		Day dec.		D. breaks	Tw. ends	Sun East.	Cl. aft. Sun.		☉ Semidiameter	
1	11h.	34m.	5	0	4 m 9	7 a 31	5 m 39	10'	22"	16'	1"
6		15		19	18	19	32	11	54		3
11	10	55		39	26	7	24	13	15		4
16		36		58	35	6 56	17	14	24		6
21		17	6	17	43	46	10	15	18		7
26	9	58		36	51	37	3	15	56		8

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NOVEMBER, 30 DAYS.

Full Moon.........	1st, 57m. past Midnight.
Last Quarter	8th, 14m. past 4 Aftern.
New Moon	16th, 54m. past 3 Aftern.
First Quarter......	24th, 33m. past 5 Aftern.

SUN ENTERS

21d. 19h. 28m.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r. & s.	☾ a
1	D	21st S. a. Tr.: All Sts.	6h 56′	4h 32′	14° s 31′	6h m 2′	15
2	M	*All Souls:* Mich. T. b.	6 58	4 30	50	rises	F
3	Tu		6 59	4 28	15 9	4 a 50	17
4	W	K. Will. III. landed	7 1	4 26	28	5 36	18
5	Th	**Gunpowder Plot,** 1605	7 3	4 25	46	6 39	19
6	F	*Leonard*	7 5	4 23	16 4	7 58	20
7	S	♄ rises 8 56 aftern.	7 6	4 21	22	9 24	21
8	D	**22d Sun. aft. Trinity**	7 8	4 20	40	10 49	22
9	M	**P. Wales b.** 1841: Ld.	7 10	4 18	57	morn.	23
10	Tu	[Mayor's d.	7 12	4 16	17 14	0 9	24
11	W	*St. Martin:* (Shooting	7 14	4 15	31	1 24	25
12	Th	Cam. T. div. m. [stars)	7 15	4 13	47	2 38	26
13	F	*Britius*	7 17	4 12	18 3	3 50	27
14	S	♃ sets 6 12 morn.	7 19	4 11	19	5 2	28
15	D	**23d Sund. aft. Trin.:**	7 21	4 9	34	6 15	29
16	M	[*Machutus*	7 22	4 8	49	sets	N
17	Tu	*Hugh Bp. Lincoln*	7 24	4 6	19 4	3 a 53	1
18	W		7 26	4 5	18	4 26	2
19	Th	♂ rises 2 9 morn.	7 27	4 4	32	5 11	3
20	F	*Ed. King and Martyr*	7 29	4 3	46	6 12	4
21	S	**Prs. Royal b.** 1840	7 31	4 2	20 0	7 23	5
22	D	**24th Sun. af. Tr.:** *St.*	7 32	4 0	13	8 39	6
23	M	*St. Clement* [*Cecilia*	7 34	3 59	25	9 57	7
24	Tu		7 36	3 58	37	11 17	8
25	W	Michael. Term ends:	7 37	3 57	49	morn.	9
26	Th	[*Catherine*	7 39	3 56	21 1	0 37	10
27	F	♀ rises 5 30 morn.	7 40	3 55	12	2 0	11
28	S	☿ sets 4 1 aftern.	7 42	3 55	22	3 25	12
29	D	**Advent Sunday**	7 43	3 54	33	4 58	13
30	M	**St. Andrew**	7 45	3 53	42	6 34	14

Day.	Length of D.	Day dec.	D. breaks	Tw. ends	Sun East.	Cl. aft. Sun.	☉ Semidiameter
1	9h. 36m.	6 58	5 m 1	6 a 26	4 m 56	16′ 17″	16′ 10″
6	18	7 16	8	19	51	16 13	11
11	1	33	16	13	46	15 48	12
16	8 45	49	23	7	42	15 1	13
21	31	8 3	30	2	39	13 53	14
26	18	16	36	5 59	37	12 26	15

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SUN ENTERS	DECEMBER, 31 DAYS. 1857
 21d. 8h. 16m.	Full Moon .... 1st, 57m. past 10 Morn. Last Quarter ...... 8th, 38m. past 6 Morn. New Moon......... 16th, 1m. past 11 Morn. First Quarter ...... 24th, 36m. past 6 Morn. Full Moon .... 30th, 33m. past 9 Aftern.

D.	W.	SUNDAYS, HOLIDAYS, &c.	☉ rise	☉ set	☉ decl.	☾ r.&s.	☾ a
1	Tu		7h 46′	3h 53′	21° s 52′	rises	F
2	W		7 48	3 52	22 1	4h a 18′	16
3	Th	♄ rises 7 11 aftern.	7 49	3 51	10	5 33	17
4	F		7 50	3 51	18	7 0	18
5	S	[*Nicholas*	7 52	3 50	25	8 28	19
6	D	2d Sunday in Advent:	7 53	3 50	33	9 53	20
7	M		7 54	3 50	40	11 12	21
8	Tu	*Conception B. V. M.*	7 55	3 49	46	morn.	22
9	W		7 56	3 49	52	0 27	23
10	Th		7 57	3 49	57	1 39	24
11	F	♃ sets 4 8 morn.	7 58	3 49	23 3	2 51	25
12	S	[*Lucy*	7 59	3 49	7	4 5	26
13	D	3d Sunday in Advent:	8 0	3 49	11	5 17	27
14	M		8 1	3 49	15	6 30	28
15	Tu	[*O! Sap.*	8 2	3 49	18	7 41	29
16	W	Em. Week: Cam. T. e.:	8 3	3 49	21	sets	N
17	Th	Oxford Term ends	8 4	3 49	23	4 a 5	1
18	F		8 4	3 49	25	5 14	2
19	S	♂ rises 1 48 morn.	8 5	3 50	26	6 29	3
20	D	4th Sunday in Advent	8 6	3 50	27	7 47	4
21	M	St. Thos.: Short. day	8 6	3 51	28	9 5	5
22	Tu		8 7	3 51	28	10 23	6
23	W	♀ rises 6 49 morn.	8 7	3 52	27	11 41	7
24	Th		8 8	3 52	26	morn.	8
25	F	Christmas-day	8 8	3 53	24	1 3	9
26	S	St. Stephen	8 8	3 54	22	2 29	10
27	D	1st S. af. Christ.: St.	8 8	3 55	20	4 0	11
28	M	Innocents [Jn. Evan.	8 9	3 55	17	5 33	12
29	Tu		8 9	3 56	14	7 1	13
30	W	☿ sets 5 32 aftern.	8 9	3 57	10	rises	F
31	Th	*Silvester*	8 9	3 58	6	4 a 24	15

Day.	Length of D.	Day dec.	D. breaks	Tw. ends	Sun East.	Cl. aft. Sun.	☉ Semidiameter
1	8h. 6m.	8 28	5 m 42	5 a 56	4 m 35	10′ 41″	16′ 16″
6	7 57	37	48	55	34	8 40	16
11	50	44	52	55	35	6 26	17
16	46	48	56	56	36	4 2	17
21	44	50	59	58	38	1 33	18
26	45	0 inc. 1	6 1	6 1	41	0 bef. 57	18

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POETICAL ANSWERS TO THE PRIZE ENIGMA.

Answer.—Needle.

1. *Acrostic.* *By* Mrs. Baker, *Vauxhall, London.*

D iana, friend and patroness of song,
I n thy Parnassian bowers I've sported long;
A nd though I never fill'd the Laureate's throne,
R emembrance points to many favours shown.
I feel thy soothing influence o'er my soul,
A ttractive as the *needle* to the pole.

2. *The North Pole.* *By* Cantab, m.a., *of Sevenoaks.*

When Ross had sought the pole*
(For which he came from far),
The *needle*-vertical
Stood perpendicular!
What sight could more refresh
The anxious sailor's mind?—
Sublimer truth by far
Than worldlings ever find.

So, in the pole celestial,
Brave *Ursa Minor* points
(Although an object bestial),
With out-stretch'd tail and joints.
To one fix'd point that never
Or moves or swerves aside;—
Bright pole-star! mayst thou ever
True-hearted sailors guide.

3. *By* Mr. G. H. Butler, *Dalston, London.*

True as the *needle* to the pole,
Our Hope from year to year,
Unfolds his sweet and tuneful scroll.
Long may that name appear,
And deck Diaria's honour'd page
For many a year to come;

Aye! even till, in ripe old age,
His Master calls him home,
And bids him take his happy place
Amidst the glorious band,
Who (monuments of saving grace)
In that blest presence stand.

4. *To Miss Winifred Waverton.* *By* Mrs. Ann Towns, *London.*

Dear Winny, allow me, I fain would inquire
What has come of a lady we all so admire.—
Is she plying her *needle*, and making fine shirts,
Or netting, or knitting, or running gown skirts;
Or has she got married, and broke the old tie
That so long hath endeared her to fair Lady Di?
Should such be the case I gladly would know
If she has had pity on Johnny Densho.

The Laureate bard hath touched the mystic string,
From Dia's page those classic stanzas spring,
That raise a *needle*, homely though it seem,
As high as Homer's heroes, or the dream
That fell on Atreus' son, and bade destroy
The ramparts, towers, and lofty domes of Troy.

* *Vide* Sir John Ross's Voyage to the North Pole.

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5. *By* Mr. ROBERT CLEMITSON, *of Morpeth.*

Such talents rare shine forth in Lady Di,
To gain the prize 'tis *needle*ss me to try!
My muse, howe'er, on humble wing essays
To court her smile—to win her generous praise!
And should this effort—made with pure delight,
On doubtful pinion soaring in its flight—
Fair Dia's cheering approbation gain,
'Twill add new vigour to my future strain!

6. *The Seamstress.* *By* Mr. JOSEPH FURNISS, *Hurley Cottage, Lois Weedon,*

Stitching away from morn till night,
Aching all over, and dimming the sight,
How is the seamstress tired outright!
Fair ones of wealth, who enjoy your ease,
Or adorn and embellish the art when you please,
Pity such "slaves of the *needle*" as these!

7. *By* Mr. JAMES HERDSON, *Tobermory, Mull, N.B.*

I can fancy friend Hope on "the Bank" has an eye,
With his *needle* and *thread*, both so sleek and so sly;
And I'm sure that would be a "material" prize,
If he should in his efforts that end realise;
At least one thing's gained in this Sebastopol feat,—
He has fairly bombarded old "*Thread-needle* Street."

8. *By* Mr. JOSEPH HUTCHINSON, *near Halifax.*

Though changes come as seasons roll,
  And cares increase with years;
True as the *needle* to the pole,
  Our Laureate bard appears.

9. *By* Mr. THOMAS BOWMAN, *Richmond, Yorkshire.*

The fair may be wounded in using the *needle,*
  To form some superb decoration of art;
But the wound she receives is but simple and feeble,
  Compared with that wound she inflicts on the heart.

10. *By* Mr. JAMES LUGG, *Grampound, Cornwall.*

Old classics say the Queen of Lydia threw
  Round great Alcides' heart so strong a chain,
That he, turn'd spinster, from his distaff drew
  The silken twine amidst her menial train.
Twine such as chaste Penelope employ'd,
  When on her scarf she wrought proud Ilion's story,
In honour of her lord, and to avoid
  Base, heartless wretches, dead to truth and glory.
And Dia's Laureate, rich in mystic lore,
  Presents a polish'd *needle* to the fair;
Convinced that no embroider'd robe of yore,
  In taste and skill would now with theirs compare.

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11. *By* Noah Wilmot, *S——s, near Newcastle-upon-Tyne.*

Hope points his *needle* very fine,
May he long *so* in Diary shine.

12. *To the Editor.* *By* Octogenarius.

Friend Hope this year his *needles* sends,
To your female Diarian friends ;
May they work garlands for his brow,
For he deserves them you'll allow.

13. *By* Clericus.

To discover Hope's prize from his mystical lay,
Is " like seeking a *needle* in a bottle of hay."

14. *To the Rev. John Hope.* *By* Mr. John Standring, *Epworth, near Bawtry.*

Your *needle* was so sharp and bright,
That it almost escaped my sight.

15. *By* Mr. James Hewitt, *Hexham.*

To err is human—lovely woman erred,
And drew her hapless lord from Eden's bower ;
Rather than lose his " helpmeet" he preferred*
To feel the vengeance of Almighty power.
'Twas his to show forgiveness is divine—
Towards the fair, frail partner of his fate ;
And share her doom, nor murmur, nor repine
O'er deeds irrevocable, when too late.
In loving gratitude, through life she plies
Her nimble *needle*, and her willing shears,
To clothe, t' adorn, to captivate his eyes,
To future hope, while each the other cheers.
Their reconcilement full let Di attest,
Where mutual efforts still increase her fame,—
Where each in turn by blessing is more blest,
And love and emulation fan the flame.

16. *A Character.* *By* Mary.

Attend, Diarians, to my strain; your aid ye muses lend—
Inspire my pen, in praise of one I'm proud to call my friend.
Firm and unbending he is found, obeying duty's call;
But gentleness and kindness mark his intercourse with all.
His ever-polished, sparkling wit, like *needle*, sharp and bright,
Ne'er, like the needle, gives a wound—its point imparts delight.
When with the friend his mind approves, he yields its varied store,
Discourses on philosophy, or charms with classic lore.
When on his Master's work intent, with earnest, holy zeal,
God's faithful minister he stands, his message to reveal.

---

* *Vide* 'Paradise Lost,' book ix.

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GENERAL ANSWERS TO THE ENIGMAS.

1. Guard.	4. The Past.	7. Horn.	10. (Prize.)
2. Dock.	5. Basin.	8. Crack.	Needle.
3. Cheese.	6. Stall.	9. Bay.	

1. *"The Past."* *By the* Rev. JOHN HOPE, *Stapleton.*

The past, a phrase with vast importance fraught;
In it what countless years are lost for ever!
It to recall how many a wretch has sought,
But has that object once been gained?—ah, never!

The past should *guard* us in our present course;
To its examples we should look with care;
*Do*th it not *c*hec*k* us with a giant's force,
When bent on ill, and seem to say, "beware?"

The past should not be *che*at*e*d of it*s* du*e*;
The past has claims engraved on plates of brass;
The *bas*e *in* heart may them forego, but few,
On thinking of them, do not cry "alas!"

The past in public records has a *stall*,
"Rich with the spoils of time," whose Keeper seems
On all mankind as purchasers to call,
And rouse them from their "fatal waking dreams."

The past, though silent, seems to sound its *horn*,
Nay *crack* its whip to wake the ling'ring band,
Who rest unthinkingly from eve to morn,
And almost through the day inactive stand!

The past has crowned with never-dying *bay*s
Full many a poet,—graceful round their heads
Its verdant leaves their genius still displays,
Though now they're resting in "their lowly beds."

The past the *needle*'s matchless feats of skill
Holds forth to view, when even princely fair,
As time progressed, felt pleasure to fulfil
Their tasks of rivalry and genius rare.

The past the trophies both of peace and war
Brings to remembrance; on its page we find
A host of splendid cities, famed afar,
In ruins sunk, and long to dust consigned!

Where now is Nineveh? where sea-girt Tyre?
Where far-famed Carthage, rival long of Rome?
Gone, gone for ever are their martial fire,
Their threat'ning bulwarks, and each regal dome!

The past, the later past, how strange to say!
The proud Sennacherib's household gods has shown,
Has glanced on Nineveh a fervid ray,
To read its ancient annals long unknown.

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O Layard, gifted with a genius rare,
  Thou by the past hast gained a fair renown ;
The spoils of Nineveh thy name shall bear,
  And still thy memory with honour crown.

Diarians ! soon the future joins the past ;
  How silently the past each moment gains !
The future cometh, but, alas ! how fast
  It quits its station, and no longer reigns !

This shows the value of the present time,
  Which well improved is progress towards heaven ;
It paves the way to happiness sublime,
  That blest reward which to the just is given.

2. *To Miss Helen Ogden.* *By* Mrs. Baker, *Vauxhall.*

  Dear Helen, how shall I express
To thee my ardent thankfulness ?
*Ho*w *r*ecompe*n*ce, while life shall last,
Dia's memorials of *the past ?*
My sister's death ! heart-rending theme
Of those she held in high esteem.
Butler and Hope, in mournful lays,
Have strewn her tomb with living *bay*s ;
And Towns (dear friend) and Furniss too,
To the*s*e, *to all*, my thanks are due.
  A *guard*ian angel comforts me,
*B*lest symp*a*thy, *s*o k*in*d, so free ;
She bids each *needle*ss murmur cease,
And *c*alms my *rack*ing thoughts to peace ;
While thy inspiring, soothing strain
*Chee*rs and r*e*vives my soul again.
  Yes, Helen, friendship has a charm,
The world's unkindness to disarm.
*Do*th fi*ck*le fortune frown on me,
Still that sweet charm I find in thee ;
And should she on my pathway shine,
Thy spirit shall rejoice with mine ;
This shall my grateful thoughts engage
Whene'er we meet on Dia's page,
Until we gain that happy shore
Where pain and parting are no more.

3 *Address to May.* *By* Miss Helen Ogden, *Shaw.*

O come, lovely May ! sweet, smiling, and gay ;
  Since nat*ur*e thy a*d*vent doth greet,
With merriest strain ov'er mountain and plain,
  And gives thee a welcome complete.

Not one of thy class, as onward they pass,
Affords more delight to the eye;
Nor with thee compare, thy *ch*arm*s* *e*v*e*r rar*e*
Are destin'd them all to outvie.

From th' *stall* and th' fold abroad we behold
The sportive young *lambkins* at play;
E'en insects rejoice at the sound of thy voice,
And sport 'neath thy sunniest ray.

And sweetly each song of the warbling throng
Re-echoes through woodland and dell,
As ambitious to raise their anthems of praise
To perfection in melody's swell.

Each opening spray, in splendid array,
Puts forth its rich vesture of green;
The *dock* in the glade, the wheat's springing blade,
Are in beautiful liv'ry seen.

While numberless flow'rs enrich the gay bow'rs,
With beauty enamel the ground;
And choicest perfume from each opening bloom
Sheds sweetest of incense around.

The soft purling rill no longer is still,
In winters ungenial chain;
The huntsman's shrill *horn* sounds not with *the* morn,
Its windings have *pass*'d o'er *t*he plain.

Delightful thy sway, but transient its stay,
As youth's happy season of prime;
Its promises fair with thine may compare,
Unblighted by sorrow and time.

For such is our life, its *crack*lings and strife,
Too often will hold us at *bay*;
Its prospects of joy are damp'd by th' alloy,
That awaits its meridian day.

But hope to the soul, like *needle* to pole,
Preserves its mysterious pow'r;
Like thee, ever bright, presents to the sight
The bud of some opening flow'r.

Then come, lovely May, enchanting and gay,
Dispense through the breadth of the land
Such favours, that we in future may see
The gifts of thy bountiful hand.

. "*The Past.*" (Respectfully inscribed to Mr. James Lugg, the author of the 4th Enigma.) *By* Mr. G. H. Butler, *Dalston*.

The Past! oh, what mingled emotions of rapture
And pain are produced by that truth-telling word;
*Ho*w it whispe*r*s of hours of u*n*sanctified leisure,
Of talents misused or of warnings unheard;

How it bids us to *guard* against every temptation,
Though *bea*uteous in form it appears to the e*y*e,
And (sharp as a *needle*) to use circumspection,
Lest the time to oppose it for ever pass by.
Then it *c*annot but b*r*ing b*ack* those hours of enjoyment
(So sweet to remem*b*er) with m*a*ny a friend,
Such a*s* R*i*chardso*n*, Dia's own dear "Highland Lassie,"
Whose name we shall love till exi*st*ence sh*all* end.
*Do*es it not, too, *c*all on us to see*k* for true pleasure,
Where pleasure that's lasting can only be found,
In those sacred, refined, and all-*h*allowed *e*njoym*e*nt*s*
That alon*e* in the paths of religion abound?

If such a course, dear sir, be *yours* and *mine*,
We'll* "hail life's exit with a shout divine."

5. *My Birthday. By* Mr. Joseph Hutchinson, *near Halifax.*

Though time is ever moving day and night,
With equal speed—unvaried in progression;
Yet there are periods when we view its flight
With more than common feelings and impression.

Seasons in life when we may challenge thought,
And as a faithful *guard* demand inspection;
And such a one to me this day has brought,
Wherein to view *the past* with deep reflection.

Yes, early years, when *dock*s and daisies pleased,
And tart or *cheese*-cake childish sorrows banish'd,
When fond affection painful troubles eased,
And cares and tears like meteors came and vanish'd.

And boyish days, when every happy morn,
The brimful *basin* heartily enjoying,
In spite of tempting *stall* or hunter's *horn*,
Away we went to school our lessons plying.

Then came the teens, that trying time of youth,
How favour'd still—kind Providence restraining
From many evils—by the force of truth,
And principles infus'd in early training.

And in the prime of life, and past its line,
These pleasing reminiscences pursuing,
Though *c*ares may *rack* and energies decline,
Mercies are still my favour'd pathway strewing—

That call for grateful feelings *b*y the w*ay*,
And due acknowledgment on this occasion;
Which here the muse would tender on the day
That adds another year to life's probation.

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* The last line of Mr. Lugg's enigma.

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And while its faults and follies are deplor'd—
A *needful* exercise—with true contrition—
Be all its blessings in the memory stor'd,
For future thought and thankful recognition.

6. *The Island of Tyree, N.B.* *By* Mr. James Herdson, *Tobermory, Mull, N.B.*

Of fairy lands no doubt you've read,
Plac'd far, far from the sea;
But did you ever know, or tread
The Island of Tyree?

'Tis thus,—if you have never heard;—
It grows nor briar nor *thorn;*
It has no *needle*-furze to *guard*,
No wild rose to adorn.

Could I its worth ju*st all* pourtray,
That's far, far in the west;
Not what the isle was *yesterday*,
But what of old possess'd!

Here's *dock, bay, basin,* but no bowers;
It in green plains excels;
Its fields embroider'd are with flowers,
Its shores inpav'd with shells.

This fruitful island, 'midst the seas,
Much milk and butter yields;
With bountiful supplies of *cheese,*
And corn, from ample fields.

Think not I'm *crack*ing here a joke,
Thus far, far in the sea;
One half the *praise* I have no*t* spoke
Of the Island of Tyree.

7. *Song of the Needle.* *By* Mr. George Starmer, *of Heyford.*

I come from fires of Etnaan glow,
To *guard* and guide on his way
The mariner bol*d*, as he's r*ock*ed to and fro,
On the waves of the turbulent sea.

I'm plied by the orphan's delicate hands,
When the weather is ready to freeze,
Who has for a meal no sumptuous viands,
But a pittance of bread and *cheese*.

I once was in Eden, for so 'tis implied,
But why should I mention *the past;*
The first parents used me their shame thus to hide,
*B*eing c*a*ught in transgres*sing* at last.

There's Snip, the tailor, he knows my worth,
His sleeve I glittering adorn,
Ere the ox from his *stall* is again brought forth
Or the bird-boy is pealing his *horn*.

After all, I have little to boast of or *crack*,
*B*ut the tale I've displ*ay*ed is a true one;
And 'tis *needle*ss to add, that the coat on the back
Of the poor, should be changed for a new one.

8. *The Christmas Eve Party.* *By* Mr. Joseph Furniss, *Hurley Cottage, Lois Weedon.*

Oh, what a snug party was old neighbour Tite's,
On the eve before Chri*st*mas, that night of *all* nights,

Where the old folks, and young folks, and all the folks there,
Partook with such welcome of old English cheer.
Where the yule-log blazed up with its *crack*ing and flame,
And the laughter rang out from each light-hearted dame;
Where the fun and the "forfeits" were "ruling the roast,"
And the kiss 'neath the bush which the young ones loved most.
For there hung the misletoe bough in its pride,
And the laurel and *bay* intertwined by its side.
The *basin* of punch—or, more proper, the "bowl"—
Was made by the host, who—a kind-hearted soul—
Would have his guests merry, and merry they got:
And who, in their places, that season would not?
The ale in the *horn*, too, went round to each guest,
Who were willing to taste just a drop of the "best;"
But the ladies drank wine with their hostess, Dame Tite,
And how many kind wishes were wished her that night!
So the old folks, and young folks, and all the folks there
Chatted on of *the past* and the coming new year;
Of the changes and chances gone by and to come,
Very *chee*ring to many—un*s*uit*e*d to some;
Till at length an impression on young Master Tite
Was indelibly fixed by a certain Miss White—
Who, conversing, unconscious of giving a smart,
Darted sharp-pointed *needle*s and pins in his heart.
To finish the thing which her brother began,
Miss Tite fell in love with a handsome young man;
Who declared in his song that for "Annie he'd dee,"
And she thought his glance said, "So I would, love, for thee!"
Miss Dorothy *Dock*well, however, looked sly,
For the handsome young man was a mote in her eye;
He had *guard*ed her there, and she thought it but right
To keep a sharp eye on her rival, Miss Tite.
But twelve o'clock struck, when they all rose to start,
While a feeling of friendship pervaded each heart;
With a hearty farewell, and kind wishes expressed
For a happy to-morrow, departed each guest.

9. *By* Mary.

How different were the days of "royal mail"
From this progressive age of steam and rail!
Then, seated by the *guard*, or on the box,
We travelled, undismayed by railway shocks;
And, as the varied landscape met our view,
There the coarse *dock*, and there bright clover grew.
(But, by the train, we through the country fly;
The objects vanish as they meet the eye.)
And when at noon we stopped, and coachman dined,
If for a meal so early disinclined,

We just could take, our hunger to appease,
A *basin* of warm soup, or bread and *cheese*.
(But food the railways furnish for the mind,
A book-*stall* at each station now we find.)
The time for dinner *past*, the seats regained,
The coachman scarce the prancing steeds restrained,
The guard his *horn* then sounded—no delays—
Crack went the whip, and off the gallant *bays*;
With steady pace they hastened to the goal,
True to their work as *needle* to the pole.

10. *By the* CAWKLEY'S LADDIE.

I love the sweet, the *chee*ring spring,
When first the lark begins to sing;
When busy bees take early wing,
Among the op'ning flow'rs;
To sow and *guard* the tender seeds,
Expel intruding *dock*s and weeds;
With pan or *basin* damp the beds,
That thirst for quick'ning show'rs.

I love the summer's prime, when trees
Wave their gay foliage in the breeze;
When from the *stall* the herds at ease
Stray in *the past*ures green;
When Flora's nymphs rich tints display,
And sweet perfumes around them play;
When eglantine, and rose, and *bay*,
With beauty gilds the scene.

I love the autumn—season's king;
Pomona then does treasures bring,
In mellow fruit that cluster'd cling
Till *crack*ling branches bend;
When waving grain the fields adorn,
Or when their golden locks are s*horn*,
And reapers, singing harvest home,
Their tuneful voices blend.

I love the winter time—for why?
Because it brings our friend—the Di;
While maidens fair the *needle*s ply,
Around the cheerful hearth;
Likewise it brings the merry time,
The cheerful, joyous Christmas chime,
The carol, song, and pantomime,
And scenes of festive mirth.

ANSWERS TO THE REBUSES AND CHARADES.

1. Grass-hopper.	5. Chatter-ton.	9. Cart-ridge.
2. Red-breast.	6. Og-den.	10. Run-a-gate.
3. Smother, mother, throe.	7. Can-robert.	11. Pan-ace-a.
4. Diary, raid, aid, Ida.	8. Wood, woo, wo.	12. Cockle, clock, lock, Locke.

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1. *To the Editor. By* Miss HELEN OGDEN, *Shaw.*

O deem me not rude, dear sir, if now I
Return you my thanks through the pages of Di,
For placing my poor insignificant name
Conspicuously high on the pages of fame.
No *grasshopper*'s note in summer's bright day,
Or *red-breast*'s sweet song when winter bears sway,
More pleasure could give, I dare not say pride,
That feeling I'll *smother*, lest haply you chide,
And *Diary* refuse in future to pay
Her wonted respect, or allow me t' stray
With *Chatterton*, young, unfortunate bard;
For surely it shows a kindly regard,
That *Ogden* by you should worthy be deem'd,
To take her position by one so esteem'd.
Did secret misgivings occur to your mind
That honour like fortune perchance might be blind,
That one worthy name alone could not shed
A halo of fame around my poor head;
But order'd a hero beside me to stand,
Canrobert, renown'd through the breadth of the land.
Greater honour, indeed, could any one crave;
A poet of fame, a warrior brave,
A trio complete on Dia's fam'd pages,
Together we shine with her worthiest sages.
Descend, oh! ye Nine, and deign to inspire
My simple *wood* notes with poetical fire,
I'll *cartridges* leave, with munitions of war,
To those who delight in Bellona's red car.
Let *runagates* roam *panaceas* to find;
Thy lays more congenial far to my mind,
Rehearse at my leisure, and annually bring
To her pages an humble, but cheerful off'ring.
On *cockle* perchance might venture my lay,
Could I but her favour with kindness repay.
Accept then my thanks, transmit my regards
With acknowledgments due to all kindred bards.

2. *To Mr. Hutchinson. By the* Rev. JOHN HOPE, *Stapleton.*

All hail, my dear sir, will you come to the Border,
When *redbreasts* are brooding, and *grasshoppers* sing,
And verdure increasing, the days in their order,
As *mothers* a lovelier progeny bring?

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Pray make up your mind—it will yield you a pleasure
  The mountains to view which around us arise,
And gaze on the valleys extended in measure,
  So well calculated the mind to surprise.
I'll show you my garden : it then will be blooming
  With primroses, crocuses, lilies in flow'r ;
Whilst other bright gems will the air be perfuming,
  Some near to my arbour, a sweet little bow'r.
When wearied with flowers we will turn to the *Diary,*
  On *Chatterton's* fate for a moment reflect;
Regarding the *raids* of the North make inquiry,
  And speak of Miss *Ogden* with highest respect.
Then canvass the war ; but, ah ! war is distressing;
  It often its thousands on thousands destroys ;
Canrobert, the brave, scarcely thinks it a blessing,
  And little as we all its horrors enjoys.
O *who would,* intent on the death of a brother,
  His *cartridge* explode with so sad a design ?
Suppose him a *runagate,* yet a kind mother,
  Deprived of her son, for his loss must repine.
There's no *panacéa,* alas ! for contention ;
  We'll leave it for subjects connected with peace ;
And of them at dinner make praiseworthy mention,
  With hopes that their progress may ever increase.
When warned by the *clock* that deep midnight is nearing,
  And supper is over, to rest we'll retire ;
Then roused by the rays of bright Phœbus appearing,
  Our seats we'll resume by the snug parlour fire.
But whilst we're the bounties of Heaven enjoying,
  We will not forget the GREAT GIVER of all,
But proper occasions in worship employing,
  With gratitude humbly for blessings we'll call.

3. *The Winter of* 1855. *By* Mr. JOSEPH HUTCHINSON, *near Halifax.*

Thou'rt come again, old Winter, cold and drear,
    Thy visit never fails ;
Though sometimes varied by a better cheer
    Than now prevails.
For though I bid thee welcome as a guest,
    Whose presence is esteem'd ;
Thou'rt view'd by many with an aching breast,
    A cruel despot deem'd.
Binding with stronger cords and tighter bands,
    In poverty, the poor ;
Lessening the means of labour for their hands,
    With pinching calls for more.

And cold and comfortless indeed art thou,
Where want is felt or fear'd;
And thousands 'neath its painful pressure bow,
Unaided and uncheer'd.
Yet there are others upon whom thou teem'st
Favours that joy impart;
Who "love thee all unlovely as thou seem'st,
And dreaded as thou art."
And thou hast charms that often meet the eye,
Thy chilling blasts among;
The snow-rob'd landscape and the starlit sky,
And fireside evenings long.
And tho' no chirp of *grasshopper* is heard,
The pretty *redbreast* comes;
Thy herald true—a *mother*'s favorite bird,
To pick our threshold crumbs.
Thou bring'st the *Diary* too, to cheer the gloom,
Of *Chatterton*'s sad fate;
Where *Ogden* twines the bay ere long to bloom,
On *Canrobert* the great.
But, oh! what painful scenes that name recalls!
Of *wo* and bl*ood* and blaze,
Of shot and shell and showers of *cartridge* balls,
And hapless *runaways*.
O for a *panacea* for those ills,
A universal peace;
When corn, not *cockle*, every garner fills,
And wars and want shall cease.
Haste, happy day! when Britain's sons no more,
To check ambition roam;
But welcome thee, old Winter, as of yore,
With joy, in peace at home.

4. *To Absent Diarian Friends.* *By* Mrs. Baker, *Vauxhall.*

The *cockle* weed, and snowdrop fair,
Proclaim the joyous spring,
While on the budding spray we hear
The little *redbreast* sing.

Returning summer proudly decks
The *woods* and meadows gay,
The *chatter*ing *grasshopper* delights
To wan*ton* through the day.

All nature smiles—but, oh! how dull
The *Diary* now appears! [names,
We miss those dear, those treasur'd
The pride of former years.

Long as a *mother*'s memory holds
Her wonted place in mine,
Thy wreaths of fame, Lavinius, still
With hers I intertwine.

Fain would I hail in Dia's page
Thy ever-pleasing strain,
Priz'd by a sister now no more;
And shall I hope in vain?

And thou, sweet bard of Selby, come,
Resume thy tuneful lays,
And bid our drooping hearts again
Re-echo former days.

Ogden and Hope and Hughes await
The *runagate*'s return;
No longer then thy talent hide
In dark oblivion's urn.

His *cartridge* now *Canrobert* drops,
And should dread warfare cease,
Our *panacea* then will be
The olive branch of peace.

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5. *A Winter Evening*. *By* Mr. JAMES LUGG, *Grampound, Cornwall.*

Though winter is come, clad with ice, storm, and flood,
And earth, sea, and sky wear an aspect of gloom ;
The *grasshopper*'s chirp is not heard in the *wood*,
Nor *redbreast*'s sweet song on the hawthorn or broom ;
Though *cockle* no longer is found in the corn,
And tales of distress by *Canrobert* are told,
How ruin and death on war's *cartridge* are borne,
And nature's best feelings lie *smother*'d and cold ;
Yet we have the *Diary*, and *Ogden*'s sweet strains,
And *Chatterton*, bright, but unfortunate boy,
Whose fate draws a tear from the Muses' loved swains,—
Whose works leave a charm which time will not destroy.
How pleasant to sit with a friend by the fire,
When grim, shiv'ring winter is howling around ;
And thence to a niche in Di's favour aspire,
Where *runagate* wretches will never be found.
But still there is something Diarians should prize,
Above all that art, song, or science bestows:
A meetness in time, for a flight to the skies ;
The true *panacea* for all earthly woes.

. *To Mr. George Starmer*. *By* Mr. JAMES HERDSON, *Tobermory.*

Dearest sir, I'm obliged for the anxious inquiry
You made after me in the 'fifty-six *Diary ;*
In the last, to Miss *Ogden*, my thanks were most due,
And now I must render my best thanks to you:
Though then, as a sparrow, I only could chatter,—
I'm no more than a *redbreast* to you,—but no matter.
Ten winters and summers I've trac'd this rude scene ;
No *runagate* then, nor *grasshopper* I've been.
But I think, like *Canrobert*, I'll give up command,
And let some one else take first *cartridge* in hand.
And now I will thank you, on Di's page of fame,
For placing poor *Chatterton*'s long-slighted name;
By Walpole neglected, or *smother*'d, or slighted—
There was no *panacea* for worth ill requited.
The rose or the *cockle* in *wood* or in field
May all their fair beauty and full fragrance yield ;
Still confirming this truth,—many a flower, sweet and fair,
Spread their fragrance around on the wild desert air.

7. *The Storm on the Moorland Heath*. *By* Mr. JAMES BARTHRAM, *of Scarborough.*

The north wind blew keen on the dreary moor,
And drifted the snow at the cotter's door,
In *smother*ing flakes it down *did* show'r,
And cover'd the dre*ary* waste.
Poor *cock* robin *fle*w to the window sill,
And plum'd his *red breast* with his slender bill,
Then twitter'd and pip'd, for 'twas cold and *c*hill,
*An*d *r*oug*h* *be*at the *s*urly blas*t*.

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The moping owl cry'd in the ivy tree,
And p*artridge*s cluster'd upon the lea,
The *runagate* thought of the home which he
When wild and young had despis'd.
Though nature around was sad and drear,
And winter was frowning with looks austere,
Set o'er a snug ingle with comrades dear,
We cheer'ly ourselves amus'd.

Though we could not boast of *Ogden*'s fire,
Or *Chatterton*'s wit and keen satire,
We screw'd up our pipes, and tun'd our lyre,
And cadences rise and fall.
We sang of the seasons when all was gay,
When fair maidens romp and ted the hay,
When *grasshopper*'s chatter throughout the day,
And nature is lovely all.

We sang of the fair maiden's h*ope*s *an*d loves,
We sang of the ch*ace a*nd shady groves,
We sang of the green*wood*s and gay alcoves,
We sang of the cottage hearth.
Although we were merry and cheerful there,
And felt not the tempest that rag'd elsewhere,
We pitied the wanderer that must bear
The storm on the moorland heath.

8. *Farewell to Miss Winifred Waverton. By* Mr. JAMES HEWITT, *Hexham, Northumberland.*

"Winny," witty, winsome Winny,
Shall we, must we, say farewell;
Can no love nor friendship win ye
From corroding sorrow's cell?

Shall no " satire " now alarm us,
Nor suitors woo the coy "old maid?"
Nature then may cease to charm us,
Unobserved may bloom and fade!

Grasshopper in vain may chirp,
Warblers cheer the summer day,
Redbreast strive in vain to stir up
Pleasure with his winter lay.

Why in sighing sadness *smother*
Wit, our *Diary* ill can spare?
While each loving sister, brother,
Longs in vain to see you there.

I would rather risk, to see it,
Chatterton's untimely fate,
Than to lose thy strains,—albeit
Ogden charms me soon and late.

Canrobert may chase grim Russia
From our lovely island home;
Austria cheat us; naughty Prussia
From the path of duty roam;

Walls of *wood*, with gun and *cartridge*,
Served by private *Runagate*,
Scatter foes like whirring partridge,—
Themes like these belong the State.

Dia still, our loved *panacea*,
" Cheers the *cockle*s of our hearts;"
Patronised, Dei gratia,
By Victoria, Queen of Arts.

Baker, Long, cum multis aliis,
Rouse us by their heavenly fire;
May they live still long to rally us,
Still to strike the mystic lyre.

Come then, Winny, winsome Winny,
Happier still we'll be with you;
Should I plead in vain to win ye,
Peace with thee and thine,—adieu.

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9. *By* Eboracensis, *York*.

First, Hope tells, in mystic yet elegant rhyme,
How the *grasshopper* skips in his brief summer time;
Next, we think we develop the e'er pleasing form
Of sweet robin *redbreast* in wintry storm.
Now Clericus warns that to *sport* may do ill,
And refers us to Shakspeare's magical skill;
Anon we are told of an annual friend,
The *Diary*,—which lore and amusement can blend.
Next, our thoughts are recall'd to the fortunes, so hard,
Of *Chatterton*,—wayward, yet talented bard.
Next, Farn to a lady a tribute would pay;
And *Ogden* well merits the meed of a lay.
To a warrior we pass, from the name of a fair,
And I doubt not the warrior is French *Canrobert*.
Next, we're told of one Roger, who's lost in a *wood;*
And of *cartridge*, which, doubtless, for warfare is good;
Of the *runagate* doom'd here and thither to roam,
And ne'er destin'd to meet with the sweets of a home.
As onward in haste thus in order we press,
How shall we the mind to the next one address?
For Hutchinson speaks of what never is found;
Yet have I,—*panacea* for every wound.
Last, he tells of the *cockle*, to farmers a pest,
Yet as shell-fish oft eaten with middling zest.

10. *By* Clio, *of Hexham*.

How sweet is the rural retreat,
When nature her beauty displays,
And the *grasshopper* chirps at our feet,
To teach us the duty of praise.

When the *redbreast* carols his song,
And melody wakes through the grove,
And *smoo*thly *the* b*r*ook flows along
Through pastures where flocks and herds rove.

E'en when hoary winter appears,
And the beauties of summer are flown,
The *Diary* annually cheers
The home where its merits are known.

There *Chatterton*'s name we unfold,
And *Ogden*'s, to Dia still dear,
And General *Canrobert*'s the bold;
These each in succession appear.

When the *wood*lands their verdure have shed,
And are dreary and bleak to the sight,
Our *car*es *to* ab*ridge* it is read
With feelings of pure delight.

The *runagate* may not indeed
Be able its beauties to prize,
But its votaries all are agreed
It still has new charms in their eyes.

The true *panacea* they find
In its pages for every ill,
Where un*locke*d are the stores of the mind,
With consummate talent and skill.

11. *By* Mr. Thomas Edwards, *Lois Weedon*.

Tho' I last year from Dia's school the idle truant played,
Yet will I try once more to solve each rebus and charade.

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Then Mr. Hope the *grasshopper* first to our notice brings,
And Mr. Lugg, Di's Cornish bard, he of the *redbreast* sings;
While Clericus his real name still *smothers* from our view,
J. Hewitt writes on *Diary*, proves he's to his colours true;
For the fate of Thomas *Chatterton*, Starmer does sympathise;
And Mr. Farn of Brighton veils Miss *Ogden* in disguise;
Mr. Butler, he on *Canrobert* a brief charade does write;
J. Barthram's *wood*, and *woo*, and *wo*, they soon were brought to light;
T. Bowman fires a *cartridge*, 'tis a blank one though I see;
Some think R. T.'s is *runaway*—it *runagate* may be;
With Du Barry, Parr, or Frampton, no more we need engage,
For the real *panacea* now is found on Dia's page;
For which we thank friend Hutchinson,—but no longer must I parley;
So may wheat instead of thistles grow, and *cockles* 'stead of barley.

LIST OF POETICAL ANSWERS.

Aitkin, John, North Muskham, ans. Enigmas.
Albion, of London, ans. all.
Amicus, of Canterbury, ans. all.
Angus, J. C., Horse-le-hope, Shotley Bridge, Durham, ans. all.
Awmack, Mrs., Harom, ans. Prize Enigma.
Baker, Mrs., 25, Vauxhall Street, Vauxhall, ans. all.
Barthram, James, Scarborough, ans. all.
Bowman, Thomas, Richmond, Yorkshire, ans. Prize Enigma.
Bridget, ans. Enigmas.
Briggs, George, Chaddesden, near Derby, ans. all.
Burdon, Henry, Sutton-on-the-Forest, near York, ans. all.
Burns, William, Saville Row Academy, Newcastle-upon-Tyne, ans. all.
Butler, G. H., Shrubland Road, Dalston, London, ans. Enigmas.
Cantab, M.A., of Sevenoaks, ans. Prize Enigma.
Carr, M. R., Carr's Villa, Carr's Hill, near Gateshead-on-Tyne, ans. all.
Catherine, of Farndon, ans. Prize Enigma.
Cawkley's Laddie, ans. all.
Clemitson, Robert, Morpeth, ans. Prize Enigma.
Clericus, ans. all.
Clio, of Hexham, ans. all.
Code, P., Dean Prior, near Ashburton, Devon, ans. all.
Craiggy, Master Colin, of Crawcrook, ans. Prize Enigma.
Dawe, Miss M. N., Landulph, Cornwall, ans. all.
Dawson, Thomas, Long Benton, Northumberland, ans. all.
Densham, Miss E. A., La Folié, Millbrook, Jersey, ans. all.
Dodgson, John, Kirby Mills, near Kirbymoorside, Yorkshire, ans. all.
Douglas, M., Ingo, Northumberland, ans. Enigmas.
Dowson, Thomas, Wombleton, near Kirbymoorside, Yorkshire, ans. all.
Dunsho, John, Epworth, near Bawtry, ans. all.
Eboracensis, of York, ans. all.
Eddy, E. A., St. Just, near Cape Cornwall, ans. Enigmas.
Eddy, Eliza H., St. Just, Cornwall, ans. Prize Enigma.
Eddy, William H., Truthwall, St. Just, Penwith, Cornwall, ans. all.
Edwards, Thomas, Lois Weedon, ans. all.
Ego, of Durham, ans. Enigmas 3, 5, 7, 8, and Rebuses and Charades 4, 7, 8.

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Elliott, John, West Croft, Stanhope, ans. all.
Farn, William Henry, of Brighton, ans. all.
Fenna, John, Alpraham, ans. all.
Furniss, Joseph, Hurley Cottage, Lois Weedon, ans. all.
Grey, John, Castle Eden, Ferry Hill, ans. Prize Enigma.
Grice, George, jun., Wold Newton, near Malton, Yorkshire, ans. all.
Grice, James, Wold Newton, near Malton, Yorkshire, ans. Enigmas.
Hattam, Miss M., St. Just, West of Cornwall, ans. Enigmas.
Hattam, Thomas, jun., Eddystone Lighthouse, English Channel, ans. all.
Herdson, James, Tobermory, Mull, N.B., ans. all.
Hewitt, James, Hexham, Northumberland, ans. all.
Hewitt, John, Commercial Academy, 16, Saville Row, Newcastle-upon-Tyne, ans. all.
Hills, Ann, Little Houghton, Alnwick, ans. all.
Hindle, Thomas, Tarleton, near Chorley, Lancashire, ans. all.
Hope, the Rev. John, Stapleton Rectory, Carlisle, Cumberland, ans. all.
Hutchinson, Joseph, near Halifax, ans. all.
Jackson, Thomas, Felling School, Gateshead, ans. all.
Jane, of Ryedale, ans. Enigmas.
Karcel, R. R., Howport, ans. all.
Langley, Robert, Heywood, ans. Prize Enigma.
Lavinius, of Margate, ans. all.
Lawry, Miss Mary, St. Just, near Cape Cornwall, ans. Prize Enigma.
Levy, W. H., Shalbourne, near Hungerford, ans. Prize Enigma.
Lugg, James, Grampound, Cornwall, ans. all.
Mary ———s, Miss, West of Cornwall, ans. Enigmas.
Mary, ans. all.
Mentor, of Worcester, ans. Enigmas.
Mulcaster, James, jun., Allendale, Northumberland, ans. all.
Mulcaster, John Wallis, Allendale, Northumberland, ans. all.
Nemo, ans. all.
Nimrod, of Wombleton, ans. all.
Nodwons, J., Murah, ans. all.
Oats, William, Tregeseal, St. Just, Cornwall, ans. all.
Octogenarius, of Hickling, Nottinghamshire, ans. all.
Ogden, Miss Helen, Shaw, ans. all.
Perrett, John, Marton, near Kirbymoorside, Yorkshire, ans. all.
Pigg, Edward, Bishopwearmouth, Sunderland, ans. all.
Priestley, Sarah Frances, Beadlam, near Helmsley, Yorkshire, ans. all.
R. Y. C., of Guernsey, ans. all.
Rutter, Matthew, 65, Lawrence Street, Sunderland, ans. all.
Ryley, Robert, jun., Mickleover, Derbyshire, ans. all.
Standring, John, Epworth, near Bawtry, ans. all.
Starmer, George, Heyford, Northamptonshire, ans. all.
T. D. H., Kirby Mills, Kirbymoorside, Yorkshire, ans. Prize Enigma.
Towns, Mrs. Ann, ans. Prize Enigma.
White, John, Manningham, near Bradford, ans. Prize Enigma.
White, John, Holly Terrace, Birmingham, ans. all.
White, Thomas, Allendale, ans. Prize Enigma.
Whittle, John, Blackburn, ans. Prize Enigma.
Wilkinson, T. T., Burnley, Lancashire, ans. all.
Wilmot, Noah, S——s, near Newcastle-upon-Tyne, ans. Prize Enigma.
Wray, James J., Madeley Wood, Ironbridge, Salop, ans. all.

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NEW ENIGMAS.

I. Enigma (1391); *by* Mr. W. H. Farn, *Brighton*.

"God save the Queen!" and may her wise command
Prove the Palladium of her native land;
May her ripe counsels bring enduring peace,
And cause war's terrors and its woes to cease.
But whether war or peace, alarm or cheer,
To me is little, her I'm ever near,
Now at her elbow, now behind her back,
Now at her side, when she my aid may lack;
Till, like the moon, which shines a month at most,
My oscillations ended, I am lost;
Lost, till at length my royal mistress deign
To grant me place and honour once again.
When Joan of Arc did Charles VII dare
To make crown'd king;—most surely I was there.
Once when the Sphinx bright Thebes did oppress,
And gave a riddle Œdipus could guess,
Lo! scores of youths, unskilled its sense to hit,
Were masticated for their want of wit;
So I, by cruel circumstances beaten,
Am, like those Thebans, destined to be eaten.
Thousands will eat me off the tempting shelf,
And, what is odder still, I eat myself.
One closing hint, to kill the marvel, hear:—
When "Dia's" Queen leads in the opening year,
Helen, whose flowing and delightful verse,
Charms one to read, remember, or rehearse;
Helen, who writes for fame, and not for pelf,
Let one word serve her, I am just—*herself*.

II. Enigma (1392); *by* Mr. James Barthram, *Scarborough*.

While Chaos in confusion lay, th' Almighty spake,
And bade the crumbling atoms of the mass to wake;
Then darkness vanish'd, then forth shone the light,
And day was first distinguish'd from the night.
At His command the elements divide,
And gathering waters form the rolling tide;
Rocks, hills, and mountains, at the word arise,
And azure tints bedeck th' ethereal skies;
Then, with this strange congestion, I was found,
Diffusing life and vigour all around.
O'er rugged mountain tops I take my way,
And through the ocean's foaming depths I stray;
I penetrate the convict's gloomy cell,
In earth's deep caverns too I'm known to dwell.
View, next, yon upstart, vain, in spruce array,
Who with affected mien does me display;

Vain of his looks, his figure, or his pelf,
With pompous strut he gives me to himself;
But with the great and wise, observe me there,
Noble and graceful then do I appear.
  Now! listen to yon sweet melodious choir,
Whose notes harmonious souls with rapture fire;
I'm plaintive, brisk, lively, and sad as well,
Sometimes I joy, sometimes I sorrow tell.
  No doubt that long ere this my name's transpir'd,
But if another hint is yet requir'd;
I'm found both hot and cold, I'm damp and dry,
I'm sweet and foul, I'm pris'ner bound and free;
I am essential to all life on earth,
Also at times the fearful cause of death.

III. ENIGMA (1393); *by* MR. JAMES LUGG, *Grampound, Cornwall.*

  Time journeys on, unheeded by the mass
Of busy idlers, though there cannot pass
A year, a day, nor hour, but I am seen,
Like a despotic, but familiar queen,
Extending my dominion over all
The earth and seas with undisputed thrall.
Indeed, all things are subject to my sway,
Both time and place my sov'reign pow'r obey;
The silver moon's majestic ride through space,
When met by me, incurs a slight disgrace;
And yonder sun, whose peerless beauty streams
O'er distant worlds, must yield me all his beams;
Yea, ev'ry creature will confess my pow'r,
And bow, resign'd, in its appointed hour.
I sometimes pains and penalties bestow,
At other times excruciating woe;
And then again, if rightly understood,
My visits bring incalculable good:
As if a man, chain'd down by sin, for years,
Lash'd with the snaky scourge of doubts and fears,
Should fly to me for help, I'll freely give
More blessed counsel how the wretch may live;
I'll aid him in the path direct to heav'n,
And make his passage thither smooth and ev'n.
Poets are beings privileged to deal
With all things that pertain to human weal;
And use elisions when they deem it meet
To prune their wild unmanageable feet:
A noble pile, thus served, leaves me remain,
With nations throng'd, and eager to obtain
An easier lift into the gilded sphere
Where av'rice acts as coach and charioteer;

And yet with all my wealth, dominion, fame,
My mystic nature and extended name,
When counted right, and honestly set forth,
A few halfpence are all that I am worth.
Ladies and gents, may you my presence greet
With kindly welcome, and a hearty treat;
Bend to my stern behest with peace and joy,
And bless my sire when worlds are fled away.

IV. Enigma (1394); *by* Mr. George Starmer, *of Heyford.*

Diarian bards, for mystic lays renowned,
May I adventure on your classic ground?
If I am welcome, hear my artless tale,
And let your wonted candidness prevail.
Both trade and commerce greatly I assist,
Without my aid they scarcely could exist;
But vain credulity in some is such
That they in me place confidence too much;
Be not then too confiding, lest I fail,
For some have cause at my misdeeds to rail.
Now change the scene to yonder river's side,
Where dimpling eddies grace the purling tide;
There you will find me clad in lovely green,
And drooping willows too—a charming scene.
Diarians, doubtless you have heard or read
Of that ill-fated ship the "Birkenhead,"
Nought could avail the hapless crew to save
Themselves from me and from a watery grave;
A few, indeed, attained me on the shore,
But numbers sank, alas! to rise no more!
Avert your eyes from this heart-rending sight
To view sweet spring in all her beauty bright,
When little lambkins at their harmless play,
Each other chace upon the meadows gay;
On me they run, on me they nimbly skip,
Like prancing nags beneath the spur and whip;
When tired, on me they sometimes seek repose,
Pleased with the primrose that upon me blows,
The mountain daisy, and the varied round
Of nature's gems with which I'm mostly crowned.
But now, Diarian friends, my tale is told,
You'll doubtless soon my well-known name unfold.

V. Enigma (1395); *by* Mr. G. H. Butler, *Dalston, London.*

Diarian ladies! loved and honour'd few,
To every tender feeling nobly true,
Your kind attention let me humbly ask,
While your unveiling powers I strive to task.
I have existed from the earliest time,
And have been used and prized in every clime;

The Prophet Samuel—when, at God's command,
He took the stalwart son of Kish in hand,
And taught the hosts of Israel to sing,
In one united voice, "God save the King"—
Invoked my aid,—and, from that solemn hour,
E'en to these days, whene'er to regal power,
The heir succeeds and mounts the vacant seat
(While myriad voices their new ruler greet),
I still am found, and occupy a place
Where I can almost touch the monarch's face;
But more than once does Sacred Writ proclaim
My value, and make mention of my name,
How I have helped existence to sustain,
When famine's prospects filled the heart with pain.
I'm highly favour'd by the young and fair,
Touch their white hands, their locks of jet black hair;
I'm sweet, I'm nauseous, I am thick and clear,
I'm hot and cold, I'm cheap and very dear;
The engineer regards me as his friend,
While to the artist I assistance lend;
Sir Joshua Reynolds (whose time-honour'd name
Is handed down to never-dying fame)
Made use of me to give his wondrous art
The power to charm and captivate the heart;
I can float safely on "the mighty deep,"
Calm as the infant in its gentle sleep;
Talk of the ship that rides upon the wave,
Talk of the manly swimmer bold and brave,
Their buoyant powers are nought compared to mine,
And all their boasted prowess I outshine.
I'm found *at home* when evening shades prevail,
And songs are heard and many a jovial tale;
I've helped to guide the traveller on his way,
And chase the gloom of many a foggy day;
In short I am a universal friend,
I'll say no more, 'tis time that I should end.

VI. ENIGMA (1396); *by* MR. JOSEPH HUTCHINSON, *near Halifax.*

Ladies and gents, to you I make no bow,
'Tis *vice versâ*,—yours the homage now;
Or rather when together we appear
For other purposes than we do here.
I have a being, and am justly prized
Where arts prevail and men are civiliz'd;
Though varied is my character and sphere—
Rude with the peasant—polished with the peer.
In rural life where nature's beauties charm,
I share the busy labours of the farm,
And when the stores that autumn's bounty yields
Are borne in triumph from the fruitful fields,

And "harvest home" is hail'd with heart-felt joy,
No one more actively engaged than I.
But change the scene from rustic mirth and glee,
To civic pomp and hospitality,
Where tables groan with viands rich and rare,
For city gourmand, and you'll find me there.
In music too my properties are known,
As village choristers will freely own,
And others, who a higher station fill,
Where harmony is sought with taste and skill.
But not confin'd to earth—I'm seen on high,
When murky clouds obscure the azure sky,
And warring elements, in strife and storm,
The great Jehovah's purposes perform.
When Hercules one great achievement wrought,
'Tis more than probable my aid was sought;
And such my powers, 'tis said, in ancient days,
That I could islands from the ocean raise.
In India's wilds—the jungle or the brake—
Where lurk the adder and the poisonous snake,
If in your wanderings you should ever stray,
Beware of me, or you may dearly pay.
But more than all avoid the paths of vice,
And let true wisdom be your early choice,
Lest when, at last, you hear the trumpet sound,
You may be cast where I am often found.

VII. Enigma (1397); *by* Mr. James Herdson, *Tobermory, Mull, N.B.*

Ladies and gents, I come not here to sing
That I'm no hidden, no mysterious thing;
But frankly own my nature's to disguise,
Thus I approach you with your open eyes.
So far you're warn'd,—then be not much alarm'd,
For to be warn'd, 'tis said, is to be arm'd.
I cannot tell you when I first began
To give protection, in this world, to man;
In various scenes of life I act my part,—
Sometimes, 'tis said, erect, with tempting art.
No colours of the rainbow do I lack,
And more, I'm virgin white, or mournful black.
Fierce in the field of battle—field of sport;
Or borne in regal state before the court.
Not always seen by vulgar eyes 'tmay be,—
Now on your head,—now dandl'd on your knee.
I male and female with my graces aid,
By day and night,—or husband, wife, or maid.
But I would whisper this in honest truth;—
I'm more caress'd by age than sprightly youth.
Yes, yon young urchin dreads my very face,
To be by me adorn'd is dire disgrace!

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But not to these extraneous scenes confin'd,
Near your domestic hearths I'm oft combin'd;
Of various textures, as the mind of man
Or female chooses in my changing plan.
The vegetable kingdom to my aid,
Long under tribute has with skill been laid;
And animated nature, too, has lent
Her vast resources with the same intent;
You'll find, e'en too, the min'ral kingdom has
Given me in copper, silver, gold, and brass.
A strange anomaly—a symbol, I,
Of knowledge, folly, death, and liberty!
These frank admissions sure your eyes have ope'd,
But take this postscript,—ere the curtain's dropp'd;
I'm bound to tell the truth, then 'tis my doom
T' attend you from the cradle to the tomb.
Ladies, to you chief patronage I owe;
Then tell my name,—obeisantly I bow.

VIII. Enigma (1398); *by* Cantab, M.A., *of Sevenoaks.*

'Mid regions mild or stern, or cold or hot,
Where, in "the wide, wide world," where am I not
I move in air, I dwell in verdant plains,
Glitter in golden mines where Mammon reigns;
I hide in houses and in cellars keep,
Then, heedless, stand expos'd on mountains steep.
I love the animation of the road,
But gardens are my favorite abode;
I am primeval, yet I'm modern too,
Mankind despise me all,—the unfilial crew!
Like modest virtue, oft I seek the shade;
I hate the mistress, when I love the maid;
The servants hate me, though they are my friends;
With one and all, my soft good-nature blends,
Save some prim master, or some mistress sour,
These I detest,—from these in haste I scour.
I dwell where nothing else could being have,—
Start not, ye fair! I live within the grave!
I soar aloft in air, I seek the clouds,
I solitude affect, I flee from crowds,
I bask, like serpents, 'neath the glaring sun,
And, like the humble, oft am trampled on!
In polish'd Europe learnedly I sleep,
'Mong Asiatics stealthily I creep,
In North America I seek the plain,
In Africa triumphantly I reign.
Reader! I've told thee I am very old;
Now list—I'm often worth my weight in gold:
Despise me not, O man! nor vile names call,—
"Ashes to ashes" is the lot of all!

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IX. Enigma (1399); *by the* Rev. John Hope, *Stapleton.*

When nature first assumed a robe of green,
I then arose amid the pleasing scene;
No human eye beheld me, for I grew
To age mature ere man my person knew;
The savage beast, the fowl upon the wing,
In summer, autumn, and refreshing spring,
My presence courted; for I seemed a friend
Disposed a kind assistance still to lend.
I'm yet existent, ladies,—see I stand
An object dear in this your golden land;
I'm in the valley, on the verdant plain,
On sunny banks a cherished seat obtain;
Near to the hall, the castle too beside,
I dignity maintain and native pride;
Nay, I am witnessed on the craggy steep,
O er which the rudest blasts of Boreas sweep!
I now must tell that once there was a time,
When wondrous skill was mine, and art sublime;
I grew to fame, and gifted with a voice,
I made the hearts of mortal men rejoice;
My honours thickened, I was sought afar
To guide the councils both of peace and war:
But, ah! delusive!—whilst upon my tongue
The seeming truth of soft persuasion hung,
The future proved—a fact that's fully known—
That in my conduct dark deceit was shown!
Yes, dark deceit!—alas! what ills I've done!
No tear was mine when lost a darling son!
But these my deeds of darkness passed away,
Expelled by learning's more enlightened sway;
Behold and wonder how I safety gave,
When bent on blood a crowd was seen to rave;
The hapless victim saw no refuge nigh,
Around the woods, above the clouded sky;
To me he fled—say, was he then relieved?
Yes, in my arms the hopeless I received;
Nor unrewarded:—for that gen'rous deed
I gained of honour due the cheering meed.
Though hard of heart—a truth which all allow,—
Of stubborn temper, seldom found to bow,
Yet I'm a kind of shield, a bulwark sure,
On which relying, you can rest secure.
In me advantages so num'rous crowd,
That, if inclined, I might of them be proud;
Britannia loves me, still of me she boasts;
I've spread her glories on remotest coasts:
Have I a crown? O why the question ask?
Enough I've said—'tis now your pleasing task,
With wonted skill to penetrate the veil
Which slightly shades the subject of my tale.

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X. PRIZE ENIGMA (1400); *by* Miss HELEN OGDEN, *Shaw*.

O say not, ladies, I am out of place,
In venturing here my varied worth to trace,
Or claim the favour of your kind regard,
And at your hands receive a just award;
For where amidst life's ever busy round,
A more important servant can be found.
When first or where my usefulness was known,
The page of history has not clearly shown,
Though sacred records here and there unfold
Some faint allusions I was known of old,
To our first parents, ere the mandate high
Warn'd them, alas! from Paradise to fly.
And unto her whose silent pray'r, preferr'd
To Him who rules on high, was quickly heard;
The vow perform'd, maternal grief suppress'd,
And with parental love her offspring bless'd.
At Joppa, too, when later years disclos'd
The pious deeds of one in death repos'd;
And though my name was there not blazoned forth
With works of purest charity and worth,
'Tis not unfair to say that without me
Those works of love could not completed be.
And such my value in this present age,
That few competitors with me can wage:
Where can you turn, and not admiring see
What matchless fabrics daily spring from me?
'Tis I enhance the beauty of the lawn,
Where taste with judgment frequently is drawn:
As lily fair I often meet your view,
Or glow in tints that mark the rainbow's hue;
Am often coarse, yet beautifully fine,
In gold and silver too am known to shine;
Both strength and firmness are to me assign'd,
Yet such my nature you must bear in mind,
In all my movements I for aid depend
Upon a constant and unvarying friend,
Whose leading pow'rs are of importance found,
As we together ply our wonted round.
Let not proud man regard me with disdain,
For not unfrequently 'tis mine to gain
His mark'd attention, when the list'ning ear
Is bent on subjects not profoundly clear.
Me to preserve in one unbroken strain,
Sometimes requires great effort to attain.
While you, whose gifted off'rings all engage
Our yearly thoughts on Dia's honour'd page,
Each heart warm tribute fraught with skill and grace,
Can boast the favour of my fond embrace.
And may I still, as time his course prolongs,
Unite in truest bonds your pleasing songs.

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NEW CHARADES, REBUSES, &c.

1. Charade; *by the* Rev. John Hope, *Stapleton.*

By fair poetic licence see my first
At times connected with the sweets of May,
When num'rous flow'rets into beauty burst,
And breathe their fragrance on the face of day.

My next in heav'n itself is said to dwell,
Yet we on earth admire its varied hues,
As on it over heathy moor and fell,
My whole its widely devious course pursues.

Which, strange to say, avoids the cultured plains,
More fond of nature in its native pride;
Where free and unmolested it remains,
As heather blooming on the mountain side.

2. Rebus; *by* Mrs. Baker, *Vauxhall.*

A planet of some magnitude,
Without a telescope I'm viewed,
And travellers by land and sea
Discover wonders vast in me.
Transpose me, then you will reveal
Something which joy or grief can feel.
Now from this whole subtract a male:
From what remains you will not fail
To own the progress I have made
In every clime, in every trade.

3. Charade; *by* Cantab, M.A., *of Sevenoaks.*

My first will never own
To have been in a minority;
But (proud!) hath never known
The least inferiority.

Reader! if thou art reckon'd
A mortal, like myself;
Then, O! thou wilt my second,
When laid upon the shelf!

Go, seek in summer regions
My gay and beauteous whole;
'Mong fair and feather'd legions,
Whose joys no bars control.

4. Charade; *by* Clericus.

My first is an article woven for dress,
But ladies of fashion reject it, I guess;
And yet there's a class of recluses who find
My quality suited to their sober mind.
An insect's my next, that dwells in a mountain,
Compared with the Alps—like the sea to a fountain.
My whole will be found midst the conflicts of war,
And the terror of criminals placed at the bar.

5. Rebus; *by* Clio, *of Hexham*

Lively, sparkling, heavy, dull,
I may be each perchance.
Transpose—a light and tiny thing,
I in the sunbeam dance.

6. Rebus; *by* Mr. W. H. Farn, *Brighton.*

My proper place, of course, should be
Upon the wild and stormy sea;
When suffering from misfortune, I,
Like a blind Cyclops, lose mine eye,
My proper place is then on land,
Above, below, you understand,
For I (a feature all my own)
Quite secret am, and yet well known.

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7. Charade; *by* Mr. G. H. Butler, *Dalston, London.*

My first is a favorite, delicate dish,
Which oft on our tables with pleasure we find;
You may guess, if you can, whether flesh, fowl, or fish,
Or a savoury dainty of some other kind.

My second, fair ladies, you very much prize,
And from girlhood you fervently hope to obtain;
And, though small it may be, it has charms in your eyes,
And you prize it more dearly *because* it is plain.

My third you'd be puzzled to know how to move,
Except you'd recourse to machinery's aid,
And, even thus help'd, it a labour might prove
That, perchance, in the end would be sorely repaid.

My whole is a gallant commander whose name
For valorous actions has long been renown'd,
And who safely the prayers of each freeman may claim
That his brows may with victory's laurels be crown'd.

8. Charade; *by* Mr. James Herdson, *Tobermory.*

My first a tanner is, you'll find,
Of skins, light, dark, or fainter;
But to this craft he's lately join'd
The trade of portrait-painter.

Soon as appear night's shades and glooms,
My second takes the warning;
Is absent till Aurora comes,
And ushers in the morning.

On each day of the month has been
My whole; known as a high-day;
And yet it never once was seen,
Or known, on a Good-Friday.

9. Rebus; *by the same.*

My first's the heart of honest trade,
When 'tis judiciously displayed;
But when 'tis of its head bereft,
It then becomes a public theft.

10. Charade; *by* Mr. J. W. Mulcaster, *Allendale.*

If a squeeze you combine with a sign,
A metal you'll have, not found in a mine.

11. Rebus; *by* Mr. James J. Wray, *Madeley Wood, Salop.*

My whole is found in heathen lands;
Behead, curtail—made by men's hands,
A lifeless thing appears to view,
The offspring of a creed untrue,
A constant tenant of my whole,
Where Ganges' sacred waters roll.

12. Charade; *by* Mr. Joseph Hutchinson, *near Halifax.*

While daily papers told the tale of war,
To do my first at home we felt perplex'd;
But little knew the dire effects afar,
That issued from close conflict for my next.

The clash of arms, the dying and the dead—
And in the camp too oft my scanty third.
More awful still, when far and wide was spread
My reckless whole—as often seen and heard!

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ANSWERS TO THE QUERIES.

I. QUERY; *by the* Rev. JOHN HOPE, *Stapleton.*

Our English grammarians, in declining substantives, omit the vocative case: what is the reason? Is the vocative a nominative?

Answered by the Rev. JOHN HOPE, *the Proposer.*

I think it must be admitted that the position of nouns in a sentence, as affected by other parts of speech, is the criterion of a case, as well as difference of termination. Greek grammarians, seeing only one form for both dative and ablative, have generally admitted only a dative—as great an absurdity as contending that there is no difference between giving and taking away! It is a well-known fact that Lindley Murray had published his grammar some years before he would admit an objective case of nouns. He saw the objective case of pronouns, because it assumes a form different from the nominative. He advanced strong reasons, as he thought, and also brought forward high authorities, to prove that there were only two cases in English, the nominative and possessive! He was, however, at last convinced, gave way to three, and made his apology: but the admission of a vocative was with him, though a Latin scholar, out of the question. It is easy to see that we have all the cases of the Latin and Greek, but our objective does well enough for three of them. Now I contend for a vocative so often distinguished by the interjection *O*; and as a teacher I regularly make the boys attend to it, notwithstanding Murray, and Lennie, the abridger of Murray. How much better to say *Nom.* man, *Poss.* man's, *Voc.* O man, *Obj.* man!—It is true the vocative has the same form as the nominative, but what is there gained by calling it so? It has no verb, nor can have; the pronoun supplies its place for the verb. The non-admission of a vocative is supposed to simplify the language; but such simplifying is of a very dubious character; for what case is more distinctly marked than the vocative?

An answer agreeing with the preceding was also given by Miss Mary ——, West of Cornwall.

Again by MR. JAMES HEWITT, *Hexham, Northumberland.*

As we have no changes in English nouns to express any case except the possessive, we have properly no vocative case. With regard to pronouns, we have *three* cases: nominative, possessive, and objective. Verbs of calling, or naming, and also the verb *to be*, admit a nominative both before and after them; and in all vocative sentences we shall find one of these verbs either expressed or understood. All our vocative sentences may be resolved into nominatives; but, whether or not, our English substantives have no *peculiar form* for vocative sentences, and, consequently, the English language has no vocative case.

Answers to this effect were likewise given by the Rev. Thomas Brady, Senica Falls, Senica County, State of New York; Mr. William Burns, Saville Row Academy, Newcastle-upon-Tyne; Eboracensis, York; Ego, Durham; Mr. Thomas Hattam, Eddystone Lighthouse; Mr. Thomas Jackson, Felling School, Gateshead; Mr. James Lugg, Grampound, Cornwall; Mr. John Wallis Mulcaster, Allendale; and Mr. J. White, Holly Terrace, Birmingham.

II. QUERY; *by the* Rev. JOHN HOPE, *Stapleton.*

The infliction of capital punishment was sanctioned by the law of Moses; is that sanction sufficient cause for its continuance under the Gospel? or is capital punishment consistent with the doctrines of the Gospel?

Answered by the Rev. JOHN HOPE, *the Proposer.*

In attempting to answer this query, I may observe that Cain, the first murderer, was permitted to live, nay that sevenfold vengeance was to fall upon the person who should kill him. This made Lamech, who had committed a double murder, boast that he would be "seventy and seven times avenged." We have, indeed, no recorded instance of capital punishment previously to the Flood. After the Flood it was said (Gen. ix, 6),—

שֹׁפֵךְ דַּם הָאָדָם בָּאָדָם דָּמוֹ יִשָּׁפֵךְ,

well translated thus: "Whoso sheddeth man's blood, by man shall his blood be shed." Now it may be asked, is this a command or a prediction? It has neither the form nor the imperativeness of a command. If it be a command, to whom is it addressed? It is addressed to no authorities, or rulers. The verb being in the future tense, it seems to me that the passage is merely predictive, and similar to that passage in the New Testament, where Christ says, "He that taketh the sword shall perish with the sword," where nothing more than a strong probability is intimated.

Under the law there was enough of capital punishment; but the law "was the ministration of condemnation;" it made nothing "perfect;" the inspired writers admit that there was something "unprofitable" in it. It was, in fact, only a transition state preparatory to "a better scene of things." We are under the Gospel, and if in the Gospel there is no express abolition of capital punishment, yet the general tendency of Gospel doctrines is against it. Have we not done away with many things which the law sanctioned, and which are as little annulled in the New Testament as death-punishment for murder? Yes; we have abolished slavery; we no longer burn witches; we never thought of stoning a person to death for gathering sticks on a Sunday; we have, indeed, gone far in opposition to the law, having reduced our long, long list of capital crimes to one only—that for murder.* So far we have done well, and advanced in the right direction. Now, why, I ask, should this remain as a blot in our statute-book, and a stain upon the fair face of Christendom? Believe me, it is unchristian; devise some other punishment, and let the criminal live to repent. Why repay murder with murder? And what a horrid and brutalizing spectacle is a public execution, where thousands are assembled to see a fellow-creature launched into eternity! Does it deter others, or diminish the crime which it punishes? No more than hanging for stealing formerly rendered property more secure.

Again by Mr. J. White, *Birmingham.*

The extensive prescription of capital punishment by the Mosaic Law may be accounted for by the peculiar circumstances of the people. They were a nation of newly emancipated slaves, and were by nature, perhaps, more than commonly intractable, and if we may judge by the laws enjoined on them, which Hume well remarks "are a safe index to the manners and disposition of any people," we must infer that they had imbibed all the degenerating influences of slavery among heathens.

Their wanderings and isolation did not admit of penal settlements or remedial punishments; hence wilful offences evinced an incorrigibleness which rendered death the only means of ridding the community of such transgressors. It appears that Moses understood the true end of punishment, which is not to gratify the antipathy of society against crime, nor moral vengeance, which belongs

* A very few other crimes still remain capital, but for some time past the extreme punishment has been inflicted only in cases of murder; the others have all been commuted.

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to God alone, but prevention. "All the people shall hear and fear, and do no more so presumptuously."

Then, as capital punishment, as carried out by Moses, was a necessity, on account of their isolation and wanderings, we are led to the conclusion that the sanction of it by Moses is not sufficient cause for its continuance and recognition under the Gospel, as the necessity is removed, and remedial punishments are easily to be adopted.

Similar answers were given by Ego, Eboracensis, and Messrs. Brady, Hattam, Herdson, Hewitt, and Lugg.

III. Query; *by* Mr. J. White, *Birmingham.*

What circumstances gave rise to the several emblematical symbols of the rose for England, the shamrock for Ireland, the thistle for Scotland, and the leek for Wales?

Answered by Mr. J. White, *the Proposer.*

The intestine wars which so long devastated England were carried on under the symbols of the White and the Red Rose. The partisans of the house of Lancaster chose the Red Rose as their mark of distinction, and those of York the White, hence they were called the Wars of the Roses.

These civil commotions continued till the union of the Roses, in the marriage of Henry VII with the Princess Elizabeth, daughter of Edward IV, 1486; since which happy event the rose has continued to be the emblem of England.

The Shamrock for Ireland. When Patric M'Alpine (St. Patrick) landed near Wicklow, to convert the Irish, in A.D. 432, the Pagan inhabitants were ready to stone him, when he requested to be heard, and endeavoured to explain God to them as the Trinity in Unity, but they could not understand him, till, plucking a trefoil or shamrock from the ground, he said, "Is it not as possible for the Father, Son, and Holy Ghost as for these three leaves to grow upon a single stalk?" Then says Brand, "the Irish were convinced, and became converts to Christianity; and in memory of which event, the Irish have ever since worn the shamrock as a badge of honour."

The Thistle for Scotland. When the Danes invaded Scotland, it was deemed unwarlike to attack an enemy at night, instead of a pitched battle by day; but on one occasion the invaders resolved to avail themselves of a stratagem, and in order to prevent their tramp from being heard, they marched bare-footed. They had thus neared the Scotch force unobserved, when a Dane unluckily stepped with his foot upon a superbly prickled thistle, and uttered a cry of pain, which discovered the assailants to the Scotch, who ran to their arms, and defeated the foe with great slaughter. The thistle was afterwards adopted as the insignia of Scotland.

The Leek for Wales. Upon the 1st March, King Cadwallo met a Saxon army in the field. In order to distinguish his men from their enemies, he proceeded to an adjoining field of leeks, and ordered one to be placed in each of their hats; and having gained a decisive victory over the Saxons, the leek became the future badge of honour among the Welsh.

Answers essentially the same as the above were likewise given by Messrs. Burns, Hattam, and Jackson.

Second Answer by Mr. James Herdson, *Tobermory.*

This query opens a wide field of inquiry. It would be an herculean task, if it could be accomplished, to trace the cause and origin of all emblems. The olive has been the emblem of peace, perhaps since Noah's flood. Each clan in the Highlands of Scotland has some separate plant, flower, or shrub, for its badge, besides its distinctive tartan.

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The Rose. The houses of York and Lancaster adopted, respectively, the red and white rose. The houses eventually becoming united, the red and white rose was adopted, which is the present badge of England.

The Shamrock. The *trifolium repens* (white trefoil, or Dutch clover) is commonly said to be the shamrock of Ireland, and worn by the Irish as the badge of their country. But the *oxalis acetosella* (wood sorrel) is now supposed to be the true shamrock of the Irish.

The Thistle. The *onopordium acanthium* (cotton thistle) is cultivated in Scotland as the "Scotch thistle;" but it is doubtful whether this national badge has any existing type, as the representations of the Scotch thistle in ancient wood engravings, coins, and armorial bearings, differ more from each other than any known species of thistles. The thistle was not the acknowledged badge or symbol of Scotland until the latter part of the fifteenth century. It afterwards became a badge of an order of knighthood, viz., "knights of the Thistle."

The Leek. St. David, the tutelar saint of Wales, lived in the fifth and sixth centuries of the Christian era, and it is recorded that he died at the age of 146 years. He is said, in the days of the memorable Prince Arthur, to have gained a victory over the Saxons; his soldiers, during the conflict, for distinction, and as a military colour, wearing *leeks* in their caps. In memory of this fight, the Welsh still wear the leek on St. David's day, the 1st of March.

It was thus answered by Mr. James Hewitt, of Hexham.

IV. Query; *by* Mr. James Lugg, *Grampound, Cornwall.*

Whether is the postmaster, or the schoolmaster, in a populous neighbourhood, the more important functionary?

Answered by Mr. James Lugg, *the Proposer.*

Weighing the importance of the two functionaries in the query in an even balance, that of the postmaster practically preponderates; for we know not how business, which is the life and soul of every great nation, could be conducted with such expedition and accuracy as at present, without the aid of the postmaster, nor how the present mass of multifarious and important intelligence could be transmitted with such ease and certainty without his assistance. When we consider the great number of youths and children, who are to become the future adult population of the neighbourhood, and who are under the especial training of the schoolmaster, we must admit his importance to be great, and that he renders a very essential service to society; but then the ultimate effects of his services are developed in futurity, while the postmaster's are present and visible; and since the present moment produces a greater impression than any time future, the postmaster's importance appears to be greater than that of the schoolmaster.

Again by the Rev. John Hope, *Stapleton.*

I think it will be generally admitted that the schoolmaster is more important. He teaches the young, and forms in a great measure the character of the rising generation. He lays the foundation of knowledge, and often extends that knowledge to a high degree of advancement. Lord Brougham's expression of "the schoolmaster being abroad," speaks much, in this enlightened age, for the importance of that functionary; for his being abroad has caused that enlightenment. There was a time when, in this country, there were so few schoolmasters and so little learning that the postmaster did not exist; for letters were "few and far between," since many even of the highest ranks could scarcely do more than write their names. The postmaster therefore has arisen from the efforts of the schoolmaster; and he is now no doubt a person of considerable importance. The number of letters passing through the post, in Great Britain alone, is now reckoned by hundreds of millions! But who qualified people to write so many letters? I answer, as before intimated, the schoolmaster; the office of the schoolmaster, in consequence, claims precedence in importance to that of the postmaster.

Third Answer by Mr. James Hewitt, *Hexham, Northumberland.*

Were the schoolmaster's functions suspended, the postmaster would soon be pretty much in the predicament of Othello--his occupation would be gone. It is true the schoolmaster is by too many regarded as a necessary evil, like the doctor and lawyer, but the time will come when the pedagogue shall take his proper place in society, as the greatest benefactor to his species. Again, the office of schoolmaster requires one fit to be entrusted with the development of the better part of humanity, while the postmaster might be a mere piece of business mechanism; lastly, the schoolmaster made the postmaster and the necessity for his office, and the creator must be essentially more important, in every sense, than the creature.

Similar answers to this query were also given by Eboracensis, Ego, and Messrs. Burns, Hattam, Jackson, Mulcaster, and White.

V. Query; *by* Mr. James Herdson, *Tobermory.*

Is the phrase a "broken heart," only a metaphor, or mere figure of speech; or may it be a reality?

Answered by Mr. James Herdson, *the Proposer.*

According to anatomical and physiological writers, the muscles of an animal of the higher degrees of organization, such as man, are divided into two classes, the one set comprising those which are concerned in carrying on the functions most essential to life, viz., the circulation, respiration, and digestion, which act independently of the will, and are therefore called *involuntary* muscles; the other, which are organs of motion, and subject, in a certain degree, to the control of the will, are termed *voluntary* muscles. Each set act in consequence of the application to them of some stimulus; and their action is only uniform or natural when their appropriate stimuli are applied. A variety of circumstances influence the action both of voluntary and involuntary muscles, and render it irregular; when influenced by any of these, the action of the involuntary muscles becomes sensible and painful, and the voluntary muscles cease to be under the control of the will, and act not only without its stimulus, but often against its consent. Both voluntary and involuntary muscles, and the organs of secretion, are also very much influenced by emotions of the mind. Under the influence of hope, or joy, the heart beats vigorously, causing what is termed a "light heart;" while under the depressing passions its action is slow and laborious, and accompanied with such oppression as to have given origin to the phrase "a heavy heart." These mental emotions, either directly or indirectly, through the altered and unhealthy secretions, occasion in many persons spasmodic contractions of some muscular organs, which are so violent as to produce alarming and often fatal diseases, and so powerful are the effects of excessive joy in some instances, that the heart "bursting" is not a mere figure of speech; and of grief in other instances, that the heart "breaking" is not a metaphor, but a reality.

A similar answer was given by Mr. James Hewitt.

Again by Mr. James Lugg, *Grampound, Cornwall.*

The heart, like every other organ of the human frame, is liable to disease; and medical gentlemen assert that instances have been known of the heart actually bursting, and also of some of the vessels immediately connected with it; and, therefore, the phrase in the query may be a reality. But the common acceptation of the term "a broken heart," is nothing more than a simple and beautiful metaphor for that ill-treatment, or intense grief, by whatever cause produced, which induces atrophy, and ultimately ends in death.

Answers to the same effect were given by Eboracensis, and Messrs. Burns, Hattam, Jackson, Mulcaster, and White.

VI. QUERY; *by* MR. JAMES HEWITT, *Hexham, Northumberland.*

Which of the stars is most probably the central sun?

Answered by MR. JAMES HEWITT, *the Proposer.*

In the present state of our knowledge, any attempt to answer this query must be considered as speculative. M. Maedler (or Mädler), the successor of M. Struve as director of the observatory at Dorpat, seems to have investigated this subject with more enthusiasm and perseverance than any other astronomer; and first fixed upon Aldebaran, in the constellation Taurus, as the central sun. A more rigid examination, however, proved that this star did not fulfil the requisite conditions; its own proper motion being greater than the surrounding stars; demonstrating its comparative proximity to our own system. He next directed his attention to Alcyone, the principal star in the group of the Pleiades, and after considerable research concludes that "it now occupies the centre of gravity, and is at present the sun about which the universe of stars composing our astral system are all revolving." In this group, taking the brilliant star Alcyone as the centre, the telescope shows fourteen conspicuous stars besides smaller ones.

Assuming Alcyone as the grand centre of the millions of stars composing our astral system, and the direction of the sun's motion, as determined by Argelander and Struve, the consequent movements of all the stars in every quarter of the heavens have been investigated, and where the swiftest motions should be found, they actually exist, either confirming the truth of the theory, or exhibiting the most remarkable coincidences.

Sir John Herschel and others, however, consider the conclusions of Dr. Maedler as premature, and that we require much additional data before pronouncing any decided opinion.

It was similarly answered by Cantab, Eboracensis, Mr. Wm. Stevenson, and Messrs. Burns, Hope, Jackson, Lugg, Mulcaster, and White.

I. QUERY; *by* MR. JAMES LUGG, *Grampound, Cornwall.*

What was the origin and import of the term "Hue and Cry?"

II. QUERY; *by* MR. J. WHITE, *Holly Terrace, Birmingham.*

Can the terms "Case" and "Declension" be legitimately applied to the English language?

III. QUERY; *by* MR. JAMES HERDSON, *Tobermory.*

Whence was it that money obtained the very common but cant term of "*tin*?"

IV. QUERY; *by the* REV. JOHN HOPE, *Stapleton.*

What conclusions can be drawn from (Gen. i, 2)—"And the earth was without form and void, and darkness was upon the face of the deep?"

V. QUERY; *by* MR. WM. GIBSON, *Whittonstall.*

What is the cause of the strong winds which attend showers, especially when they come from the north?

VI. QUERY; *by* CANTAB, M.A., *Sevenoaks.*

What simple rules are there for an amateur astronomer to know where in the heavens to look for *Mercury, Venus, Jupiter,* and *Saturn*?

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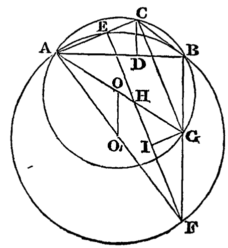
ANSWERS TO THE MATHEMATICAL QUESTIONS PROPOSED LAST YEAR.

I. QUEST. (1899); *by* Mr. EDWARD RUTTER, *Sunderland.*

Let ABC be a plane triangle, CD the perpendicular on AB; on AC or BC set off CE = CD; then will the distance between the centres of the circles ABC and AEB be equal to $\frac{1}{2}$AC or $\frac{1}{2}$BC, according as the point E is taken on AC or BC. Required proof.

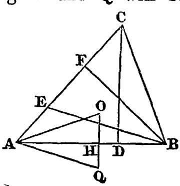
Answered by Mr. RUTTER, *the Proposer;* Mr. J. WHITE, *of Birmingham;* Mr. J. F. LIGHT, *Stoke Newington, London; and* Mr. JOHN SMITH, *Radcliffe Terrace, Northumberland.*

Let ABC be the plane △, draw EF at right angles to AC to meet the perpendicular to AB from B in the point F; also draw GC parallel to EF, and join AG, AF. Then the angles AEF, ACG, ABF are right angles; hence ACBG are on a circle; also the points AEBF are on a circle; and, because AB is the common chord, OO_1 the line joining the centres of these circles is parallel to BF. Draw GI parallel to AC; and the ∠EFB = ∠CGB = ∠CAB; but the ∠AGC = ∠ABC, and the ∠ACB = ∠FGA; also GI = CE = CD. Hence the triangles ABC, FGH are similar and equal, and AC = FG. Therefore, since the centre O bisects AG, we have $OO_1 = \frac{1}{2}FG = \frac{1}{2}AC$.



Second Solution, by Dr. RUTHERFORD, *of the Royal Military Academy, Woolwich.*

Let O and Q be the centres of the circles described about the triangles ACB and AEB; then the line joining O and Q will be perpendicular to AB and bisect it in H. Draw AO, AQ, BE, and also BF perpendicular to AC; then the angle AOH at the centre of the circle described about the triangle ACB is equal to ACB, and, for a similar reason, the angle AQO = BEC; consequently the triangles AOQ and BEC are equiangular and similar; and therefore the triangles on each side of the perpendiculars AH and BF are likewise similar, as well as the triangles ACD and ABF; therefore



$$AH : OQ :: BF : CE \text{ or } CD :: AB : AC.$$

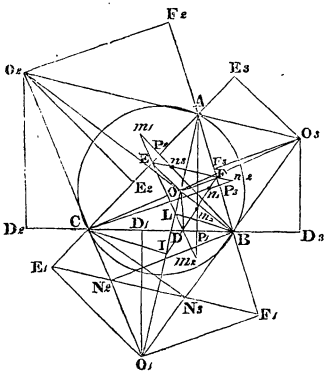
But AH = $\frac{1}{2}$AB; therefore also OQ = $\frac{1}{2}$AC. In a similar manner it is proved that if CD be set off on the side BC, the distance OQ will be equal to $\frac{1}{2}$BC.

It was nearly thus answered by Amicus, and Messrs. Brooks, Dexter, Eland, Hewitt, Hindle, Knight, Levy, M'Cormick, M'Namara, Mawson, Milbourn, Miller, Mulcaster, Ryan, Smith, Traynor, Turnbull, Watson, White, and Younger.

II. QUEST. (1900); *by* Mr. W. H. Levy, *Shalbourne*.

Let the three lines which join the points of contact of the inscribed circle with the sides of a triangle cut the three perpendiculars from the angles upon the opposite sides, in six points: then will the distances of these points from the angles, two and two, be equal to the radii of the three escribed circles of the triangle.

Answered by Mr. W. H. Levy, *the Proposer*.



Let ABC be the triangle; O, O_1, O_2, and O_3 the centres of the inscribed and escribed circles, and D, D_1, D_2, D_3 their points of contact with the side BC; AP_1, BP_2, CP_3 the perpendiculars from the angles upon the opposite sides; also E and F the points of contact of the inscribed circle in the sides AC, AB.

Let DF cut AP_1 in m_3, CP_3 in n_1 and AO produced in I; and join CI. Then since DF is parallel to O_1O_3, $\angle OO_1O_3 = \angle OID = \angle OCB$; therefore the points O, C, I, D are in a circle; whence $\angle CIO = \angle CDO = \angle OAO_3$, and by similar triangles IO : AO : : CO : OO_3. Therefore, by composition and similar triangles,

$$IO : IA :: CO : CO_3 :: OD : Am_3 :: OD : O_3D_3.$$
$$\therefore\ Am_3 = O_3D_3 = r_3.$$

A like process gives $Cn_1 = O_1F_1 = r_1$; and, in a similar manner, if DE cut BP_2 and AP_1 in m_1 and m_2, we find $Bm_1 = r_1$ and $Am_2 = r_2$; also, if EF cut BP_2 and CP_3 in n_3 and n_2, we find $Bn_3 = r_3$ and $Cn_2 = r_2$.

To this I add the following neat properties, which may interest the readers of the 'Diary:'

1. The three lines joining the points D_2, F_2; D_3, E_3; E_1, F_1 will meet, two and two, in three points t_1, t_2, t_3, in AP_1, BP_2, CP_3, produced respectively; so that $At_1 = r_1$, $Bt_2 = r_2$, and $Ct_3 = r_3$.

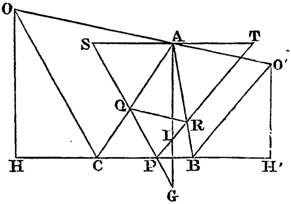
2. If the points D_1, E_1; D_1, F_1; E_2, D_2; E_2, F_2; F_3, D_3; F_3, E_3 be joined, they will meet, two and two, in three points s_1, s_2, s_3 in AP_1, BP_2, CP_3 respectively, so that $As_1 = Bs_2 = Cs_3 = (r)$ the radius of the inscribed circle; and further, these lines will also meet,

two and two, in the same six points $m_1, m_2, m_3, n_1, n_2, n_3$ as in the question.

3. It follows, from what precedes, that CI, and similarly BL, is perpendicular to AO_1. In the same manner it may be shown, if E_1F_1 intersect BO_1 and CO_1 in N_3 and N_2, that BN_2 is perpendicular to CO_1, and CN_3 is perpendicular to BO_1, &c.

Again, by Dr. RUTHERFORD; *and similarly by* AMICUS, *and* Messrs. DEXTER, HEWITT, MILLER, RUTTER, *and* WATSON.

Let ABC be the triangle, AG a perpendicular to BC, and PQR the triangle formed by joining the points of contact of the inscribed circle. Let O and O′ be the centres of two escribed circles which touch BC produced in H and H′. Draw OO′ which passes through A, and also CO, BO′. Through A draw SAT parallel to BC, meeting PQ and PR produced in S and T, and let the perpendicular AG intersect PQ and PR in G and I. Then, since P, Q, R are the points of contact of the inscribed circle with the sides of the triangle, we have BP = BR, CP = CQ, AR = AQ; therefore AR = AT, AQ = AS, and consequently AS = AT. Again, since CO and BO′ bisect the angles ACH and ABH′, it is manifest that PQ and PR are respectively parallel to CO and BO′; therefore the triangles COH and GAS are similar, as also the triangles BO′H′ and ATI. But the lines CH, BH′, AQ, AR, AS, AT are all equal, and therefore the triangle AGS is equal to the triangle COH, and ATI to BOH′; therefore AG = OH, and AI = O′H′. In a similar manner the other properties are demonstrated.

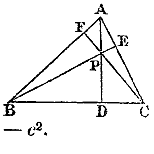


Mr. Thomas Hindle, Tarleton, Lancashire, also gave a good analytical solution.

III. QUEST. (1901); *by* Mr. STEPHEN WATSON, *Haydonbridge.*

Let AD, BE, CF be the perpendiculars from the angles of a triangle ABC to the opposite sides, mutually intersecting in P; then the sum of the squares of the radii of the *sixteen* inscribed and escribed circles of the triangles ABC, APB, BPC, CPA is equal to *ten* times the square of the diameter of the circle circumscribing the triangle ABC.

Answered by Mr. STEPHEN WATSON, *the Proposer; and similarly by* Messrs. MILLER *and* RYAN.



Since the angle APE = C $\therefore$ $AP^2 = \dfrac{AE^2}{\sin^2 C}$

$$= \frac{c^2 \cos^2 A}{\sin^2 C} = \frac{a^2(1 - \sin^2 A)}{\sin^2 A} = D^2 - a^2.$$

Similarly, $BP^2 = D^2 - b^2$, $CP^2 = D^2 - c^2$.

Now, if S, S_1, S_2, S_3 be the sum of the squares of the radii of the inscribed and escribed circles of the triangles ABC, APB, BPC, CPA respectively, then ('Diary,' Quest. 1699)

$$S = 4D^2 - a^2 - b^2 - c^2 \text{..............} (1);$$

and, since the circles circumscribing the four triangles just named are equal. we have, by putting in (1), first $D^2 - a^2$, $D^2 - b^2$ for a^2 and b^2; next $D^2 - b^2$, $D^2 - c^2$ for b^2 and c^2; and, lastly, $D^2 - a^2$, $D^2 - c^2$ or a^2 and c^2,

$$S_1 = 2D^2 + a^2 + b^2 - c^2,$$
$$S_2 = 2D^2 - a^2 + b^2 + c^2,$$
$$\text{and } S_3 = 2D^2 + a^2 - b^2 + c^2,$$
$$\therefore S + S_1 + S_2 + S_3 = 10D^2, \text{ as required.}$$

By the aid of the above values of AP^2, BP^2, CP^2, we easily find that,

Cor. 1. The sum of the squares of the sides of the four triangles above $= 6D^2$.

Cor. 2. The sum of the squares of the twelve perpendiculars of the same triangles $= 3D^2$. This is found by Eucl. vi, c.

Cor. 3. Twice the sum of the squares of the twelve lines drawn from the angles to the middle of the opposite sides of the same four triangles $= 9D^2$.

Cor. 4. The sum of the squares of the 48 lines, drawn as in Quest. 1841, in the same four triangles $= 30D^2$.

Again, by Mr. T. T. Wilkinson, *Burnley.*

Let ABC be the triangle, and P the intersection of the perpendiculars. Then, by Quest. 1699, 'Diary,' 1843, p. 57, we have

$$r^2 + r_1^2 + r_2^2 + r_3^2 + a^2 + b^2 + c^2 = 4D^2.$$
$$r'^2 + r'^2_1 + r'^2_2 + r'^2_3 + AP^2 + PB^2 + c^2 = 4D^2.$$
$$r''^2 + r''^2_1 + r''^2_2 + r''^2_3 + PC^2 + PB^2 + a^2 = 4D^2.$$
$$r'''^2 + r'''^2_1 + r'''^2_2 + r'''^2_3 + PC^2 + AP^2 + b^2 = 4D^2.$$
$$\therefore \Sigma(r^2) = 16D^2 - (2a^2 + 2b^2 + 2c^2 + 2AP^2 + 2PB^2 + 2PC^2)$$
$$= 16D^2 - 6D^2 = 10D^2, \text{ by Quest. 1706, 'Diary,' 1844.}$$

Thus also were the answers by Messrs. Bills, Brooks, Buttery, Dale, Hewitt, Hindle, Levy, Milbourn, Rutherford, and Rutter.

IV. QUEST. (1902); *by* Mr. William Mawson, *Witton-le-Wear.*

There are two purses, one containing 3 sovereigns and 2 shillings, the other 2 sovereigns and 3 shillings; a coin is taken from each of them and put in the other; the first is then given to A and the other to B, and A takes a coin from his purse and finds it to be a sovereign. Required the probable value of their expectations.

Answered by Mr. William Mawson, *the Proposer.*

A's purse contained at first £3 2*s*. and B's £2 3*s*., and the coins transferred from A's and B's may be £ and £, or £ and *s.*, or *s.* and £, or *s.* and *s.*; for which the respective probabilities are

$$P_1=\frac{3}{5}\cdot\frac{2}{5}=\frac{6}{25},\ P_2=\frac{3}{5}\cdot\frac{3}{5}=\frac{9}{25},\ P_3=\frac{2}{5}\cdot\frac{2}{5}=\frac{4}{25},\ P_4=\frac{2}{5}\cdot\frac{3}{5}=\frac{6}{25};$$

corresponding to these cases, A's purse will now contain £3 2*s.*, £2 3*s.*, £4 1*s.*, £3 2*s.*, and B's £2 3*s.*, £3 2*s.*, £1 4*s.*, and £2 3*s.* and the probabilities of drawing a sovereign from A's purse respectively become $p_1=\frac{3}{5},\ p_2=\frac{2}{5},\ p_3=\frac{4}{5},\ p_4=\frac{3}{5}.$

Therefore

$$\Sigma(Pp)=\frac{6}{25}\cdot\frac{3}{5}+\frac{9}{25}\cdot\frac{2}{5}+\frac{4}{25}\cdot\frac{4}{5}\times\frac{6}{25}\cdot\frac{3}{5}=\frac{70}{125};$$

and the probabilities of the hypotheses are

$$Q_1=\frac{P_1p_1}{\Sigma(Pp)}=\frac{18}{70},\quad Q_2=\frac{P_2p_2}{\Sigma(Pp)}=\frac{18}{70},\quad Q_3=\frac{P_3p_3}{\Sigma(Pp)}=\frac{16}{70},$$

$$\text{and}\quad Q_4=\frac{P_4p_4}{\Sigma(Pp)}=\frac{18}{70}.$$

Hence A's expectation $=Q_1\cdot(62s.)+Q_2\cdot(43s.)+Q_3\cdot(81s.)+Q_4\cdot(62s.)=\frac{4302s.}{70}=$ £3 1*s.* 5*d.* $\frac{34}{70}$, and B's expectation (the remainder of £5 5*s.*) = £2 3*s.* 6*d.* $\frac{36}{70}$.

Similar answers were given by Messrs. Buttery, Levy, M'Namara, Miller, Ryan, Traynor, and White.

Again, by Mr. Stephen Watson, *Haydonbridge, Northumberland.*

In this question we have four cases to consider:

(A) When the coin taken from each purse and put into the other is a sovereign.

(B) When that from the first purse is a sovereign and from the other a shilling.

(C) The reverse of (B); and

(D) When both are shillings.

The probabilities of (A), (B), (C), (D) are respectively

$$\frac{3}{5}\cdot\frac{2}{5}=\frac{6}{25},\quad \frac{3}{5}\cdot\frac{3}{5}=\frac{9}{25},\quad \frac{2}{5}\cdot\frac{2}{5}=\frac{4}{25},\ \text{and}\ \frac{2}{5}\cdot\frac{}{5}=\frac{6}{25};$$

and the first purse will now contain respectively, in the four cases, 3 sov. and 2*s.*, 2 sov. and 3*s.*, 4 sov. and 1*s.*, and 3 sov. and 2*s.*; hence the probability of A drawing a sovereign in each of the cases is

$$\frac{3}{5}\cdot\frac{6}{25}=\frac{18}{125},\quad \frac{2}{5}\cdot\frac{9}{25}=\frac{18}{125},\quad \frac{4}{5}\cdot\frac{4}{25}=\frac{16}{125},\quad \frac{3}{5}\cdot\frac{6}{25}=\frac{18}{125}.$$

Consequently, out of 125 trials, the probability is that the condition of

A drawing a sovereign will be fulfilled in 70, of which 18 belong to each of the cases (A), (B), (D), and 16 to (C); hence

$$\frac{18(£3\ 2s. + £2\ 3s. + £3\ 2s.) + 16 \times £4\ 1s.}{70} = £3\ 1s.\ 5d.\tfrac{17}{35}$$

= A's expectation, which includes the sovereign he is supposed absolutely to draw, and B's expectation

$$= £5\ 5s. - £3\ 1s.\ 5d.\tfrac{17}{35} = £2\ 3s.\ 6d.\tfrac{18}{35}.$$

V. QUEST. (1903); *by* Mr. JOHN BUTTERY, *Chatham.*

Show that the length of the path of a projectile, between the body leaving the horizontal plane and returning to it, will be the greatest possible for a given velocity, when the angle (θ) of elevation satisfies the equation

$$2\sec\theta = \epsilon^{\operatorname{cosec}\theta} + \epsilon^{-\operatorname{cosec}\theta}.$$

Answered by Mr. JOHN BUTTERY, *the Proposer.*

The point of projection being the origin, the equation to the path of the projectile is

$$y = x\tan\theta - \frac{g\sec^2\theta}{2v^2}x^2 \ldots\ldots\ldots\ldots\ldots (1).$$

Let $\qquad p = \dfrac{dy}{dx}$; then $s = \displaystyle\int_x \sqrt{1+p^2} = \int_p \sqrt{1+p^2}\cdot\frac{dx}{dp}$.

From (1) $\qquad p = \tan\theta - \dfrac{g\sec^2\theta}{v^2}x \ldots\ldots\ldots\ldots\ldots (2)$;

$$\therefore \frac{dp}{dx} = -\frac{g\sec^2\theta}{v^2} \quad \therefore \frac{dx}{dp} = -\frac{v^2}{g}\cos^2\theta.$$

Whence $\qquad s = -\dfrac{v^2}{g}\cos^2\theta.\displaystyle\int_p \sqrt{1+p^2}$

$$= -\frac{v^2}{2g}\cos^2\theta.\{p\sqrt{1+p^2} + \log_\epsilon(\sqrt{1+p^2}+p) + c\}.$$

Now, the limits of p for the whole length of the path are $p = \tan\theta$ and $p = -\tan\theta$;

$$\therefore \text{ length of path} = \frac{v^2}{2g}\cos^2\theta\left\{2\tan\theta.\sec\theta + \log_\epsilon\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}\right\}$$

$$= \frac{v^2}{g}\{\sin\theta + \cos^2\theta\log_\epsilon(\sec\theta+\tan\theta)\}$$

a maximum.

Differentiating with respect to θ,

$$\cos\theta - 2\cos\theta\sin\theta\log_\epsilon(\sec\theta+\tan\theta) + \cos^2\theta.\sec\theta = 0;$$

$$\therefore \log_\epsilon(\sec\theta+\tan\theta) = \operatorname{cosec}\theta$$

$$\sec\theta + \tan\theta = \epsilon^{\operatorname{cosec}\theta}.$$

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But $\sec^2\theta - \tan^2\theta = 1$, $\therefore \sec\theta - \tan\theta = \varepsilon^{-\operatorname{cosec}\theta}$

Whence $2\sec\theta = \varepsilon^{\operatorname{cosec}\theta} + \varepsilon^{-\operatorname{cosec}\theta}$.

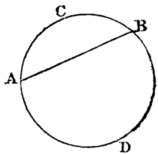
It was thus answered by Amicus, and Messrs. Brooks, Dale, Dexter, Hindle, Levy, M'Cormick, M'Namara, Milbourn, Miller, Rutherford, Rutter, Turnbull, Watson, and Younger.

VI. QUEST. (1904) ; *by the* EDITOR.

In a dark room, two persons each of them draw a chord at random across a circular slate: what is the chance that they will intersect?

Solution by Dr. RUTHERFORD.

Let AB be any chord in the circle ACBD, and let the arc ACB be denoted by x, the whole circumference being represented by *unity*. Then the chance that the second chord has one extremity in ACB is x, and the chance that its other extremity is also in the arc ACB is also x; therefore the chance that the second chord is in the segment ACB is x^2. Again, the chance that one extremity of the second chord falls in the circumference ADB is $1-x$; and the chance that its other extremity falls also in ADB is $1-x$; therefore the chance of non-intersection, in this case, is $(1-x)^2$, and the sum of these chances is $2x^2-2x+1$.



Now, the extremity of the chord AB may be in any part of the circumference; and the chance of non-intersection with the chord AB is therefore $\int(2x^2-2x+1)dx$, between the limits $x=0$ and $x=1$; consequently, $\int_0^1(2x^2-2x+1)dx=\frac{2}{3}$; that is, it is two chances out of three that the chords do not intersect, and therefore *one* chance in *three* that they do intersect.

We may arrive at the same result in a different manner. Thus the chance of one extremity of the second chord being in ACB and the other in ADB is $x(1-x)$; and the chance of the first extremity being in ADB and the other in ACB is $(1-x)x$; and their sum is $2x(1-x)$; hence $\int_0^1(2x-2x^2)dx=\frac{1}{3}$; that is, the chance of the second chord intersecting the first is as 1 to 3.

Cor. 1. If the first chord is given in position, the chance of the second not intersecting it is to the chance of its intersecting it as the sum of the squares of the two arcs to twice their rectangle.

Cor. 2. If the first chord is a diameter, the chances of intersection and non-intersection are equal.

It was similarly answered by Mr. Thomas Hindle, Tarleton, Lancashire.

Second Solution, by Mr. W. J. MILLER, B.A., *Eltham, Kent.*

Let us consider a regular polygon of $(2n+1)$ sides, and consequently $(2n+1)(n-1)$ diagonals, inscribed in the circle, and ascertain what is the probability that "if two persons draw each a diagonal at random, they will intersect." Here, $(n-1)$ *different* cases are possible: the 1st diagonal may cut off either 1, 2, 3, 4......or $(n-1)$ angles of the polygon; and, as these cases are equally probable, the probability of each is $\frac{1}{n-1}$. Also the probability that the 2d diagonal will intersect the 1st, in each of these cases, is

$$\frac{2n-2}{(n-1)(2n+1)}, \quad \frac{2(2n-3)}{(n-1)(2n+1)} \cdots\cdots\cdots\cdots \frac{(n-1)(2n-n)}{(n-1)(2n+1)}.$$

The whole probability that the diagonals will intersect is, therefore,

$$\frac{1}{(n-1)^2(2n+1)} \cdot \left\{ (2n-2) + 2(2n-3) + \ldots (n-1)(2n-n) \right\}$$

or
$$\frac{n(2n-1)}{3(2n+1)(n-1)} = \frac{2n^2-n}{6n^2-3n-3}.$$

If we suppose $n=\infty$, the polygon merges into a circle, the diagonals become chords, and the above fraction has for its limit $\frac{1}{3}$, the probability required.

VII. QUEST. (1905); *by* Dr. RUTHERFORD.

Let a circle be described through the focus to cut a conic in four points whose distances from the focus are r_1, r_2, r_3, r_4; then, if p be the semi-parameter of the conic,

$$\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\frac{1}{r_4}=\frac{2}{p}.$$

Answered by Dr. RUTHERFORD, *the Proposer.*

Let $\alpha\beta$ be the coordinates of the centre of the circle referred to rectangular coordinates, the focus being the origin, and the principal diameter of the conic the axis of x. Let $r'\theta'$ be the polar coordinates of the centre of the circle; then, since the circle passes through the focus, its equation will be

$$r = 2r'\cos(\theta-\theta') = 2r'\cos\theta\cos\theta' + 2r'\sin\theta\sin\theta';$$

but $\alpha = r'\cos\theta'$, and $\beta = r'\sin\theta'$; therefore we have

$$r = 2(\alpha\cos\theta + \beta\sin\theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1).$$

Again, the polar equation to the conic is

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

From (1), $2\beta\sin\theta = r - 2\alpha\cos\theta$, or $4\beta^2\sin^2\theta = (r-2\alpha\cos\theta)^2$; and adding $4\beta^2\cos^2\theta$ to both members, we have

$$4\beta^2 = r^2 - 4\alpha r\cos\theta + 4(\alpha^2+\beta^2)\cos^2\theta \quad \ldots\ldots\ldots (3).$$

From (2), $1+e\cos\theta=\frac{a}{r}(1-e^2)$; therefore

$$e\cos\theta=\frac{a}{r}(1-e^2)-1 \;\ldots\ldots\ldots\ldots\; (4).$$

Eliminating $\cos\theta$ from (3) and (4), the first two terms of the resulting equation in $\frac{1}{r}$ are

$$\frac{1}{r^4}-\frac{2}{a(1-e^2)}\cdot\frac{1}{r^3};$$

hence, by the theory of equations, we have

$$\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\frac{1}{r_4}=\frac{2}{a(1-e^2)}=\frac{2a}{b^2};$$

but the parameter of the conic is $2p=\frac{2b^2}{a}$; therefore

$$\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\frac{1}{r_4}=\frac{2}{p}.$$

The same demonstration applies to the other conics which has been here applied to the ellipse.

It was answered in like manner by Amicus, W. P. H., and Messrs. Brooks, Buttery, Collins, Dale, Hindle, Levy, M'Cormick, M'Namara, Mawson, Milbourn, Miller, Ryan, Traynor, Watson, and Younger.

Mr. Collins adds the following neat corollory:

Cor. If a circle, whose centre is the focus F of a conic and whose radius is equal to the parameter, cut the conic in G and G', then the circle passing through FGG' will touch the conic at G and G'.

VIII. QUEST. (1906); *by* Mr. STEPHEN FENWICK.

Show that the equation

$$\sin^{-1}\mu x+\sin^{-1}\mu' y=a$$

represents an ellipse, the principal semi-diameters (a, b) of which are determined by the relations

$$\frac{1}{ab}=\frac{\mu\mu'}{\sin a};\ \frac{1}{a^2}+\frac{1}{b^2}=\frac{\mu^2+\mu'^2}{\sin^2 a}.$$

Answered by Mr. STEPHEN WATSON, *Haydonbridge, Northumberland.*

Put $\sin^{-1}\mu x=m$, $\sin^{-1}\mu' y=n$; then $\sin m=\mu x$, $\sin n=\mu' y$, and $\cos\alpha=\cos(m+n)=\cos m\cos n-\sin m\sin n$,

$$\therefore (\cos\alpha+\sin m\sin n)^2=(1-\sin^2 m)(1-\sin^2 n),$$

$$\therefore \sin^2 m+\sin^2 n+2\cos\alpha\sin m\sin n-\sin^2\alpha=0;$$

that is $$\mu^2x^2+\mu'^2y^2+2\cos\alpha\,\mu\mu' xy-\sin^2\alpha=0\ldots\ldots(1).$$

Now, if A, B, C, and $-$P denote the coefficients of this equation taken in order, then, since $C^2 - 4AB = 4\mu^2\mu'^2(\cos^2\alpha - 1) < 0$, the curve is an ellipse, referred to its centre as origin.

Making first $y = 0$ and then $x = 0$, in (1), we have

$$\frac{1}{x^2}=\frac{\mu^2}{\sin^2\alpha},\quad \frac{1}{y^2}=\frac{\mu'^2}{\sin^2\alpha}\quad \therefore \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{x^2}+\frac{1}{y^2}=\frac{\mu^2+\mu'^2}{\sin^2\alpha};$$

also $$\frac{1}{ab}=\frac{\sqrt{AB-\frac{1}{4}C^2}}{P}=\frac{\mu\mu'\sqrt{1-\cos^2\alpha}}{\sin^2\alpha}=\frac{\mu\mu'}{\sin\alpha}.$$

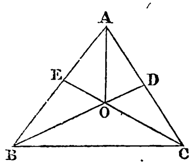
It was similarly answered by Messrs. Brooks, Collins, Dale, Hindle, M'Cormick, M'Namara, Miller, Rutherford, Ryan, Traynor, Younger, and W. P. H.

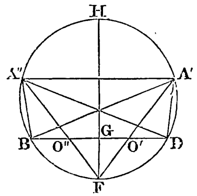
IX. QUEST. (1907); *by* Mr. J. W. Elliott, *Greatham.*

Let ABC be a plane triangle, and BD, CE the bisectors of the angles B, C; then, if BD = CE, AB = AC.

Solution by Mr. T. T. Wilkinson, F.R.A.S., *Burnley.*

Let ABC be the triangle, and O th intersection of the bisectors BD, CE. Then in the triangles ABD, AEC we have EC = BD, the angle BAC common, and the bisectors AO common and equal in both. Whence the triangles are equal in all respects. For if (fig. 2) on BD we describe a circle capable of containing the angle BAC, and on the common base BD so place the triangles BAD, EAC that the point A may occupy the positions A′ and A″ on the circle, the bisectors A′O′ = A″O″ (= AO) will evidently cut BD in O′ and O″, and will intersect on the circumference at F, making BF = FD. It also follows that ∠ A′FH = ∠ A″FH, and consequently that the arc A′H = A″H. The arcs A′D, A″B, and consequently the chords A′D, A″B, are therefore equal, and by adding equals to equals we find the arc A′B = A″D, and hence the chord A′B = A″D. But A′B = AB, and A″D = AC, and consequently AB = AC.





A similar answer was given by Mr. Edward Rutter, of Sunderland.

Mr. Elliott, the proposer, adds the following:

The property holds when BD, CE are perpendiculars from B, C. For AB . CE = 2△ABC = AC . BD; therefore AB = AC.

It also holds when D, E are the points of bisection of AB, AC; for then

$$AB^2 + BC^2 = 2BD^2 + 2AD^2,$$

$$AC^2 + BC^2 = 2CE^2 + 2AE^2;$$

from which $AB^2 - AC^2 = 2AD^2 - 2AE^2 = \frac{1}{2}AC^2 - \frac{1}{2}AB^2$; and $\frac{3}{2}AB^2 = \frac{3}{2}AC^2$; $\therefore$ AB = AC.

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Analytical Solution, by Mr. C. H. Brooks, *Newcastle-upon-Tyne; and in like manner by* Messrs. Bills, Levy, *and* Rutherford.

The squares of the bisectors of the angles of a triangle are

$$CE^2 = ab - \frac{abc^2}{(a+b)^2} \quad \text{and} \quad BD^2 = ac - \frac{acb^2}{(a+c)^2}.$$

Equating these, $\quad b - \dfrac{bc^2}{(a+b)^2} = c - \dfrac{b^2c}{(a+c)^2}.$

Hence $\quad (b-c)(a+b)^2(a+c)^2 = bc(\overline{a+c}|^2c - \overline{a+b}|^2b)$

$$= bc(a^2c + 2ac^2 + c^3 - a^2b - 2ab^2 - b^3)$$
$$= bc\{a^2(c-b) + 2a(c^2-b^2) + c^3 - b^3\}$$
$$= (b-c)bc\{-a^2 - 2a(b+c) - c^2 - bc - b^2\}$$

Now it is evident that the two sides of the above equation can only be equal when each is equal to 0, or when $b = c$.

X. QUEST. (1908); *by* Mr. John Joshua Robinson, *Portsea.*

Compare the area of the curve which is the locus of the intersection of two normals to an ellipse, at right angles to each other, with that of the locus of the intersection of a normal to the ellipse and a perpendicular let fall upon it from the centre.

Answered by Mr. W. J. Miller, B.A., *Eltham, and* Mr. Thomas Milbourn, *Darlington.*

If we denote by A_1 and A_2 the respective areas of the two curves, we have, by 'Diary,' Quest. 1853,

$$A_1 = \pi \,.\, (a-b)^2.$$

Again, because the normal at any point of an ellipse is perpendicular to the diameter conjugate to that drawn to the same point of the curve, the second locus is the same as that which forms the subject of Quest. 1365 of the 'Gentleman's Diary' (see No. 98, for 1838, pp. 30-1 and 78—86); and

$$A_2 = \pi \,.\, \frac{(a-b)^2}{2};$$

$$\therefore\ A_1 = 2A_2.$$

It was also fully discussed by Amicus, and Messrs. Brooks, Buttery, Collins, Dale, Farmar, Hindle, Levy, M'Cormick, M'Namara, Mawson, Milbourn, Rutherford, Rutter, Ryan, Traynor, Turnbull, Watson, and Younger.

XI. QUEST. (1909); *by* Sum.

If $f\left(x, \frac{1}{x}\right)$ be a symmetrical function of x and $\frac{1}{x}$, prove that

$$\int_0^\infty \frac{1}{x} f\left(x, \frac{1}{x}\right) dx = 2\int_0^1 \frac{1}{x} f\left(x, \frac{1}{x}\right) dx.$$

Answered by Sum, *the Proposer, and* Dr. Rutherford.

By a property of a definite integral we have

$$\int_0^\infty \frac{1}{x} f\left(x, \frac{1}{x}\right) dx = \int_0^1 \frac{1}{x} f\left(x, \ _{x}\right) dx + \int_1^\infty \frac{1}{x} f\left(x, \frac{1}{x}\right) dx.$$

Let $$x = \frac{1}{y} \quad \therefore \frac{dx}{dy} = -y^{-2}$$

$$\therefore \int_1^\infty \frac{1}{x} f\left(x, \frac{1}{x}\right) dx = -\int_1^0 y f\left(\frac{1}{y}, y\right) y^{-2} dy$$

$$= -\int_1^0 \frac{1}{y} f\left(y, \frac{1}{y}\right) dy,$$

since $f\left(x, \frac{1}{x}\right)$ is a symmetrical function of $x, \frac{1}{x}$,

$$= -\int_1^0 \frac{1}{x} f\left(x, \frac{1}{x}\right) dx = \int_0^1 \frac{1}{x} f\left(x, \frac{1}{x}\right) dx$$

$$\therefore \int_0^\infty \frac{1}{x} f\left(x, \frac{1}{x}\right) dx = 2\int_0^1 \frac{1}{x} f\left(x, \frac{1}{x}\right) dx.$$

It was answered in like manner by Messrs. Brooks, Buttery, Collins, M'Namara, Ryan, Traynor, and Younger.

XII. QUEST. (1910); *by* Dr. Rutherford.

The integer parts of $(\sqrt{3}+1)^{2m+1}$ and $(\sqrt{3}+1)^{2m}+1$ are always divisible by 2^{m+1}; and if S_n, S'_n denote the sum of n terms of the two series of quotients (q and q') obtained by giving to m the consecutive values 0, 1, 2, 3, &c.; then will $S_n = q'_{n+1} - 1$, $S'_n = \frac{1}{2}(q_n + 1)$.

Answered by Mr. John Buttery, Mr. Matthew Collins, *and* Dr. Rutherford, *the Proposer.*

The integer part of $(\sqrt{3}+1)^{2m+1}$ is $(\sqrt{3}+1)^{2m+1} - (\sqrt{3}-1)^{2m+1}$, since it is a whole number, and the negative term is less than 1.

Now $(\sqrt{3}+1)^{2m+1} - (\sqrt{3}-1)^{2m+1}$
$= (\sqrt{3}+1)(4+2\sqrt{3})^m - (\sqrt{3}-1)(4-2\sqrt{3})^m$
$= 2^m(\sqrt{3}+1)(2+\sqrt{3})^m - 2^m(\sqrt{3}-1)(2-\sqrt{3})^m$
$= 2^m\{(2+\sqrt{3})^m + (2-\sqrt{3})^m\} + 2^m\sqrt{3}\{(2+\sqrt{3})^m - (2-\sqrt{3})^m\}$
$= 2^m\{$twice the odd terms of $(2+\sqrt{3})^m\} + 2^m\sqrt{3}\,\{$twice the even terms of $(2+\sqrt{3})^m\}$,
which is therefore evidently divisible by 2^{m+1}.

Again, the integer part of $(\sqrt{3}+1)^{2m}+1$ is $(\sqrt{3}+1)^{2m} + (\sqrt{3}-1)^{2m}$, since it is a whole number, and $(\sqrt{3}-1)^{2m}$ is less than 1.

And $(\sqrt{3}+1)^{2m} + (\sqrt{3}-1)^{2m} = (4+2\sqrt{3})^m + (4-2\sqrt{3})^m$
$= 2^m\{(2+\sqrt{3})^m + (2-\sqrt{3})^m\}$
$= 2^m\{$twice the odd terms of $(2+\sqrt{3})^m\}$, which is also divisible by 2^{m+1}.

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Now if q'_{m+1}, q_{m+1} ...... &c., be the corresponding quotients, we have

$$(\sqrt{3}+1)^{2m} + (\sqrt{3}-1)^{2m} = 2^{m+1}q'_{m+1} \quad \text{.........} \quad (1)$$
$$(\sqrt{3}+1)^{2m+1} - (\sqrt{3}-1)^{2m+1} = 2^{m+1}q_{m+1} \quad \text{.........} \quad (2)$$
$$(\sqrt{3}+1)^{2m+2} + (\sqrt{3}-1)^{2m+2} = 2^{m+2}q'_{m+2} \quad \text{.........} \quad (3)$$
$$(\sqrt{3}+1)^{2m+3} - (\sqrt{3}-1)^{2m+3} = 2^{m+2}q_{m+2} \quad \text{.........} \quad (4).$$

We will now show that, generally, *twice the sum of any two consecutive of these integers is equal to the succeeding one;* for from (1), (2), (3),

$$(3-1)(\sqrt{3}+1)^{2m} + (3-1)(\sqrt{3}-1)^{2m} + 2(\sqrt{3}+1)^{2m+1} - 2(\sqrt{3}-1)^{2m+1} = (\sqrt{3}+1)^{2m+2} + (\sqrt{3}-1)^{2m+2}.$$

And from (2), (3), (4),

$$(3-1)(\sqrt{3}+1)^{2m+1} - (3-1)(\sqrt{3}-1)^{2m+1} + 2(\sqrt{3}+1)^{2m+2} + 2(\sqrt{3}-1)^{2m+2} = (\sqrt{3}+1)^{2m+3} - (\sqrt{3}-1)^{2m+3}.$$

Whence, also,

$$q'_{m+1} + q_{m+1} = q'_{m+2}. \qquad \text{And } q_{m+1} + 2q'_{m+2} = q_{m+2}.$$

Now, by giving to m the consecutive values 0, 1, 2, 3, &c., we have

$$\begin{array}{llll}
q_1 = q'_2 - q'_1 & \qquad & 2q'_1 = q_1 + 1 \\
q_2 = q'_3 - q'_2 & & 2q'_2 = q_2 - q_1 \\
q_3 = q'_4 - q'_3 & & 2q'_3 = q_3 - q_2 \\
\ldots \quad \ldots \quad \ldots & & \ldots \quad \ldots \quad \ldots \\
q_{n-1} = q'_n - q'_{n-1} & & 2q'_{n-1} = q_{n-1} - q_{n-2} \\
q_n = q'_{n+1} - q'_n & & 2q'_n = q_n - q_{n-1} \\
\therefore \text{Adding } S_n = q'_{n+1} - q'_1 & & \text{Adding } 2S'_n = q_n + 1 \\
\qquad\qquad = q'_{n+1} - 1. & & \therefore S'_n = \tfrac{1}{2}(q_n + 1).
\end{array}$$

Cor. Each of the series q_1, q_2, q_3, &c., q'_1, q'_2, q'_3, &c., possesses the property $Q_n = 4Q_{n-1} - Q_{n-2}$.

Analogous answers were given by Messrs. Hindle, M'Namara, Watson, and Younger.

XIII. QUEST. (1911); *by* Mr. MATTHEW COLLINS, *Kilkenny College.*

Required a complete list of all the positive integral values of A less than 100 such that $x^2 + y^2$ and $x^2 + Ay^2$ can be both square numbers, using the same values of x and y in both.

First Solution, by Mr. C. H. BROOKS, C.E., *Newcastle-upon-Tyne.*

Let $\dfrac{x}{y} = v$; then $v^2 + 1 = \square$ and $v^2 + A = \square$.

Assume $v^2 + 1 = \overline{v+n}|^2$ and $v^2 + A = (v - pn)^2$;

$$\text{from which } v = \frac{1-n^2}{2n} = \frac{p^2n^2 - A}{2pn};$$

$\therefore n^2 = \dfrac{A+p}{p^2+p}$ .... (1) and $n^2p^2 = \dfrac{Ap+p^2}{p+1}$ .... (2)

Let p be negative; then

$n^2 = \dfrac{A-p}{p^2-p}$ .... (3) and $n^2p^2 = \dfrac{p^2-Ap}{1-p}$ .... (4)

Let $n^2p^2 = m^2$, then

$A = (p^2+p)n^2 - p$ ...... (5) $A = \dfrac{p+1}{p}m^2 - p$ .... (6)

$A = (p^2-p)n^2 + p$ ...... (7) $A = \dfrac{p-1}{p}m^2 + p$ .... (8).

In all these n^2 and m^2 may be any square numbers, whole or fractional, which will render A a whole number; n, however, must not $= 1$, and m must not $= p$.

The expressions (5) and (7) may be written $A = m^2 + \dfrac{m^2}{p} - p$ and $A = m^2 - \dfrac{m^2}{p} + p$; but if p be one factor of m^2, $\dfrac{m^2}{p}$ is the other; hence A = any square number $\pm$ difference of any two of its factors, the two factors not being equal.

By taking various values of n^2 and m^2 we obtain forty one values, viz.:

A = 1, 7, 10, 11, 17, 20, 22, 23, 24, 27, 30, 31, 34, 41, 42, 45, 49, 50, 52, 57, 58, 59, 60, 61, 68, 71, 72, 74, 76, 77, 79, 82, 85, 86, 90, 92, 93, 94, 97, 99, 100.

Second Solution, by Mr. Stephen Watson, *of Haydonbridge, Northumberland.*

$$\text{Put } x^2 + y^2 = (x-ny)^2 \quad \therefore \frac{x}{y} = \frac{n^2-1}{2n},$$

$$\text{and } x^2 + Ay^2 = (x-tny)^2 \quad \therefore \frac{x}{y} = \frac{2t^2n^2-A}{2nt};$$

$$\text{hence } n^2 - 1 = \frac{t^2n^2-A}{t} \quad \therefore A = t(t-1)n^2 + t.$$

Now by taking $t = 2, 3, 4$, &c., we get $A = 2n^2+2$, $6n^2+3$, $12n^2+4$, &c., the last becomes, by substituting $\frac{1}{2}m$ for n, $3m^2+4$; hence by taking n any whole number except 1, and m any but 2, which renders $n = 1$, we get the following values of A, viz.,

(10, 20, 34, 52, 74), (27, 57, 99), (7, 31, 79).

And proceeding in this way for higher values of t, and also taking t negative; always being careful when $t(t-1)$ consists of a square p^2, and another factor q, to put $\frac{m^2}{p^2}$ for n^2, and then take all values of m except p; we shall obtain the whole of the values of A required, as follows, viz.,

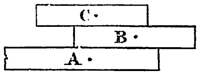
1, 7, 10, 11, 17, 20, 22, 23, 24, 27, 30, 31, 34, 41, 42, 45, 49, 50, 52, 57, 58, 59, 60, 61, 68, 71, 72, 74, 76, 77, 79, 82, 85, 86, 90, 92, 93, 94, 97, 99.

It was similarly answered by Messrs. Rutter and Ryan.

XIV. QUEST. (1912); *by* Mr. SEPTIMUS TEBAY, *St. John's College, Cambridge.*

A plank is cut at random into three lengths or rectangular pieces. If they be simultaneously placed upon one another at random (like three bricks in a wall), the chance that they will not fall down is $\frac{7}{2}-\frac{\pi^2}{3}$.

Answered by Mr. SEPTIMUS TEBAY, *the Proposer; and in like manner by* Mr. C. H. BROOKS, *Newcastle-upon-Tyne, and* Messrs. BUTTERY, HINDLE, *and* WATSON.



Let the annexed figure represent any position of the pieces when placed upon one another, A, B, C their centres of gravity, and x, x', x'' their respective lengths.

Then the probability that the centre of gravity C falls upon the piece B is

$$\frac{x'}{x'+x''};$$

for $x'+x''$ is evidently the space through which C can move so as to remain upon the piece B. In like manner the probability that the *centre of gravity* of B and C falls upon A is

$$\frac{x}{x+x'}.$$

Hence the probability that the three pieces are in equilibrium is

$$\frac{xx'}{(x+x')(x'+x'')}.*$$

Let the straight line AB represent the length of the plank, C, D the points of division, C′, D′ points indefinitely near to C, D.

Let $AB=a$, $AC=x$, $AD=y$, $CC'=\delta x$, $DD'=\delta y$. Then, since it is indifferent in what order the pieces are taken, if we put

---

* These conditions evidently assume the pieces B and C to be placed on A *simultaneously*.—ED.

$x' = y - x$, $x'' = a - y$, the probability that the pieces are in equilibrium is

$$\frac{x(y-x)}{y(a-x)}.$$

Also the probability that the points of division fall within the spaces δx, δy is

$$\frac{\delta x}{a} \cdot \frac{\delta y}{a}.$$

Hence, if P be the chance required,

$$\mathrm{P} = \frac{2}{a^2}\int_0^a \int_x^a \frac{x(y-x)}{y(a-x)}\,dy\,dx.$$

The integral is doubled, because the above limits assume that the point D must lie in BC; but when the plank is once cut, either piece is liable to be cut the next time.

Integrating with respect to y, we have

$$\mathrm{P} = \frac{2}{a^2}\int_0^a \frac{x}{a-x}\left(a - x + x\log\frac{x}{a}\right)dx$$

$$= 1 + \frac{2}{a^2}\int_0^a \frac{x^2}{a-x}\log\frac{x}{a}\,dx$$

$$= 1 + 2\int_0^1 \frac{u^2}{1-u}\log u\,du \quad \left(\text{putting } \frac{x}{a} = u\right)$$

$$= 1 - 2\int_0^1 (1+u)\log u\,du + 2\int_0^1 \frac{\log u}{1-u}\,du.$$

Now $$\int (1+u)\log u\,du = \left(u + \frac{u^2}{2}\right)\log u - \left(u + \frac{u^2}{4}\right)$$

In taking the limits of this integral, the expression $\left(u + \frac{u^2}{2}\right)\log u$ assumes the indeterminate form $0 \times \infty$, when $u = 0$.

To find the value of this, let $u = \frac{1}{z}$. Then generally,

$$u^n \log u = -\frac{\log z}{z^n} = -\frac{\infty}{\infty} \text{ when } z = \infty.$$

$$\therefore \text{ Limit of } \frac{\log z}{z^n} = \text{Limit of } \frac{1}{nz^n} = 0 \text{ when } z = \infty.$$

$$\therefore \text{ Limit of } u^n \log u = 0 \text{ when } u = 0.$$

$$\therefore \int_0^1 (1+u)\log u\,du = -\frac{5}{4}.$$

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$$\therefore\ P = \frac{7}{2} + 2\int_0^1 \frac{\log u}{1-u}\,du$$
$$= \frac{7}{2} + 2\int_0^1 \frac{\log(1-u)}{u}\,du.$$
$$= \frac{7}{2} - 2\int_0^1 \frac{du}{u}\left(u + \frac{u^2}{2} + \frac{u^3}{3} + \&c.\right)$$
$$= \frac{7}{2} - 2\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \&c.\right)$$
$$= \frac{7}{2} - \frac{\pi^2}{3}.$$

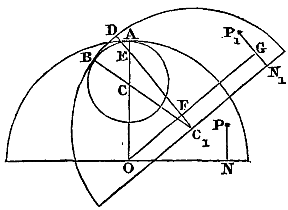
[NOTE.—If the plank be supposed to be cut horizontally, the three pieces being of the same length, the probability of equilibrium for successive piling upwards will be $\frac{7}{32}$; and when simultaneously placed, as supposed in the foregoing question, the probability will be $\frac{1}{4}$.—ED.]

XV. OR PRIZE QUEST. (1913); *by* MR. C. H. BROOKS, C.E., *Newcastle-upon-Tyne.*

Find the time of a small oscillation of a thin hemispherical basin enveloping the earth, the hemisphere's radius being equal to the earth's diameter.

Solution by PETRARCH.

Perhaps it may be allowed to substitute for this the following more general problem: *A thin hemispherical shell is placed over a sphere and acted on by a force at a distance δ below the centre of the sphere, and varying as the m^{th} power of the distance; find the time of oscillation of the hemisphere.*



Let AC the radius of sphere $= r_1$, AO the radius of shell $= r$, the $\angle$ ACB $= \alpha_1$, $\angle DC_1B = \alpha$, $\therefore\ r\alpha = r_1\alpha_1$ and $\theta = \alpha_1 - \alpha$, where θ is the inclination of the axis of the shell to its original position; P a particle of the shell, which is removed to P_1 by the oscillation; ON $= x$, NP $= y$, and z the distance of P from the plane of the paper; $x_1 y_1 z_1$ the coordinates of P_1 with reference to the same axes.

Now, $CE = \overline{r - r_1}\,.\,\dfrac{\sin\alpha}{\sin\theta}$ $\therefore$ $OF = \overline{r - r_1}\,.\,\overline{\sin\theta + \sin\alpha}$; $N_1G = FC_1 =$ the difference of the perpendiculars from C on C_1N_1 and OF $= \overline{r - r_1}\,.\,\overline{\cos\alpha - \cos\theta}$; $\therefore$ $GP_1 = y - \overline{r - r_1}\,.\,\overline{\cos\alpha - \cos\theta}$ and the coordinates of P_1 are

$x_1 = OG\cos\theta - P_1G\sin\theta$, $y_1 = OG\sin\theta + P_1G\cos\theta$, and $z_1 = z$;

$$\therefore\ x_1 = x\cos\theta - y\sin\theta + \overline{r-r_1}\,.\sin\alpha_1\,;$$
$$y_1 = x\sin\theta + y\cos\theta + \overline{r-r_1}\,.\,\overline{1-\cos\alpha_1}.$$

Let $p=$ the distance of the centre of force from P_1, and $f\,.\,p^m$ its value, which call P; $\therefore$ $Pdp = f\,.\,p^m dp$.

The following general theorem appears in the 'Cambridge Philosophical Transactions' for 1843: that if $V = \dfrac{mds^2 + m_1 ds_1{}^2 \&c.}{d\theta^2}$ and $U = \dfrac{mPdp + m_1 P_1 dp_1 \&c.}{d\theta}$; then the time of oscillation $= \pi\,.\sqrt{\dfrac{V_0}{-U_1}}$, θ being any independent variable upon which all the rest depend. We will first find V_0 and U_1 for the particle P, and then integrate for their values over the whole hemisphere.

We have, first, $p^2 = x_1{}^2 + \overline{y_1 + \delta + r_1 - r}|^2 + z^2$, which, by substitution and a little reduction, $= r^2 + \overline{r-r_1}|^2 + 2x\,.\,\overline{r-r_1}\,.\sin\dfrac{r_1\theta}{r-r_1}$

$- 2y\,.\,\overline{r-r_1}\,.\cos\dfrac{r_1\theta}{r-r_1} + 2\delta\,.\,(x\sin\theta + y\cos\theta) - 2\delta\,.\,\overline{r-r_1}$

$\cos\dfrac{r\theta}{r-r_1} + \delta^2$, and, expanding as far as θ^2, we have $p^2 = r^2 + \overline{r-r_1}|^2$

$- 2y\,.\,\overline{r-r_1} + 2\delta\,.\,\overline{y + r_1 - r} + \delta^2 + 2x\,.\,\overline{r_1 + \delta}\,.\,\theta$

$+ \left(\dfrac{r_1{}^2 y}{r-} + \dfrac{r^2\delta}{r-r_1} - y\delta\right).\,\theta^2 = A + B\theta + C\theta^2$ by substitution.

$$\therefore\ \frac{Pdp}{d\theta} = f\,.\,p^m.\,\frac{B + 2C\theta}{2p}\,;$$

let $m - 1 = 2n$,

$$\therefore\ \frac{Pdp}{d\theta} = f\,.\,(A^n + nA^{n-1}B\theta)\,.\,\frac{B + 2C\theta}{2}\,;$$

$$\therefore \text{ the coefficient of } \theta, \text{ or } -U_1 = f.\left(A^n C + \frac{n\,.\,A^{n-1}B^2}{2}\right).$$

Let
$$A = r^2 + \overline{\delta + r_1 - r}|^2 + 2\,.\,\overline{\delta + r_1 - r}\,.\,y = D + Ey,$$
$$B = 2\,.\,\overline{r_1 + \delta}\,.\,x = Fx,$$
$$C = \frac{r^2\delta}{r-r_1} + \left(\frac{r_1{}^2}{r-r_1} - \delta\right).y = G + Hy,$$

where D, E, F, G, H are constants.

Also, let $\gamma =$ the angle OP makes with the plane of xz, $\beta =$ the angle its projection on that plane makes with the axis of x, l the length of that projection; $\therefore$ $l = r\cos\gamma$, $x = l\cos\beta = r\cos\beta\cos\gamma$,

$y = r\sin\gamma$, and $z = r\cos\gamma\sin\beta$; and therefore an element of the surface $= l d\beta \times r d\gamma = r^2\cos\gamma\, d\gamma\, d\beta$. Hence for the whole surface of the hemisphere,

$$-\mathrm{U}_1 = fr^2\iint\cos\gamma\, d\gamma\, d\beta(\overline{\mathrm{D}+\mathrm{E}y}\big|^n.\overline{\mathrm{G}+\mathrm{H}y} + \frac{n}{2}.\overline{\mathrm{D}+\mathrm{E}y}\big|^{n-1}.\mathrm{F}^2x^2)$$

$$= fr^2\iint\cos\gamma\, d\gamma\, d\beta(\overline{\mathrm{D}+\mathrm{E}r\sin\gamma}\big|^n.\overline{\mathrm{G}+\mathrm{H}r\sin\gamma}$$

$$+\frac{n}{2}.\overline{\mathrm{D}+\mathrm{E}r\sin\gamma}\big|^{n-1}.\mathrm{F}^2r^2\cos^2\beta\cos^2\gamma),$$

which, integrated from $\beta = 0$ to 2π, becomes

$$\frac{-\mathrm{U}_1}{\pi fr^2} = \int\cos\gamma d\gamma.(2.\overline{\mathrm{D}+\mathrm{E}r\sin\gamma}\big|^n.\overline{\mathrm{G}+\mathrm{H}r\sin\gamma}$$

$$+\frac{n}{2}\overline{\mathrm{D}+\mathrm{E}r\sin\gamma}\big|^{n-1}\mathrm{F}^2r^2\cos^2\gamma).$$

An easy way of integrating this is by putting $\overline{\mathrm{D}+\mathrm{E}r\sin\gamma}\big| = v$; after which, by substituting the values of the capital letters and reducing, which has some complication but no difficulty, we ultimately find

$$\frac{-\mathrm{U}_1}{\pi fr^3} = \frac{\delta.\overline{r_1+\delta}\big|^{2n+3}}{2.\overline{n+1}.\overline{n+2}.\overline{r-r_1}.\overline{\delta+r_1-r}\big|^3}.(\overline{2n+3}.\overline{r_1+\delta}-\overline{2n+4}.r)$$

$$+\frac{\delta.(r^2+\overline{\delta+r_1-r}\big|^2)^{n+1}}{2.\overline{n+1}.\overline{n+2}.\overline{r-r_1}.\overline{\delta+r_1-r}\big|^3}.(r^2-\overline{2n+3}.\overline{\delta+r_1-r}\big|^2)$$

$$-\frac{\overline{r_1+\delta}\big|^2.(r^2+\overline{\delta+r_1-r}\big|^2)^n}{\delta+r_1-r}.$$

Again, to find $\mathrm{V}_0 = \dfrac{mds^2+m_1ds_1{}^2\ \&c.}{d\theta^2}$; $\dfrac{ds^2}{d\theta^2} = \left(\dfrac{dx_1}{d\theta}\right)^2 + \left(\dfrac{dy_1}{d\theta}\right)^2$

which will be found $= x^2+y^2 = r^2(\cos^2\beta\cos^2\gamma+\sin^2\gamma)$.

$$\therefore \mathrm{V}_0 = r^4\iint\cos\gamma\, d\gamma\, d\beta.(\cos^2\beta\cos^2\gamma+\sin^2\gamma) = \frac{4\pi r^4}{3};$$

$\therefore$ Time of oscillation $= \pi.\sqrt{\dfrac{\mathrm{V}_0}{-\mathrm{U}_1}}$ is completely known.

If the force be in the centre of the sphere,

$$\delta = 0, \text{ and } \therefore -\mathrm{U}_1 = \frac{\pi fr^3r_1{}^2.(r^2+\overline{r-r_1}\big|^2)^n}{r-r_1};$$

if also $r = 2r_1$, then $-U_1 = 8.5^n \pi f . r_1^{2n+4}$; if also the force vary as $\frac{1}{\text{dist}^2}$, $-U_1 = \frac{8\pi f . r_1}{5\sqrt{5}}$, and

The time of oscillation $= \pi . \sqrt{\frac{8r_1^3 5\sqrt{5}}{3f}} = \pi \sqrt{\frac{8r_1 5\sqrt{5}}{3g}}$,

which is that required in the proposed problem; g denoting the force at the earth's surface, and the result is about five and a half hours.

If the force be in the point occupied by the centre of the shell, $\delta + r_1 - r = 0$; let $\delta + r_1 - r = z$ and expand $-U_1$ in *ascending* powers of z, when it will be found that $\frac{-U_1}{\pi f r^2} = \frac{4n}{3} . r^{2n+2} + \frac{r^{2n+3}}{r - r_1}$.

If the force be as $\frac{1}{\text{dist}^2}$ and $r = 2r_1$, we have $-U_1 = 0$, or the time of oscillation is infinite, or there is no oscillation.

If r and r_1 be small compared with the radius of the earth, which may be assumed to be $\delta + r_1 - r$ or z, then expanding U_1 in *descending* powers of z, we find $-U_1 = \pi g r^3 . \frac{r + r_1}{r - r_1}$.

It may also be remarked that if the sum of the numerators of the first two terms of U_1 be expanded according to the powers of any quantity, as $\overline{r, r_1}$ or $\overline{\delta \text{ or } \delta + r_1 - r}$, every term in the expansion has the factor $\overline{n+1} . \overline{n+2}$, which is necessary to remark, for otherwise it might be supposed that U_1 would be infinite if $n + 1$ or $n + 2$ vanished.

Want of space precludes the insertion of the solutions by Mr. C. H. Brooks, the proposer, Mr. Stephen Fenwick, and Dr. Rutherford. Those of Messrs. Buttery, Fleming, and Mawson were also deserving of special notice.

LIST OF MATHEMATICAL ANSWERS.

Addam, Joseph, Wisbeach, ans. 9, 13.
Amicus, Jersey, ans. 1, 2, 5, 7, 8, 10.
Atkinson, John, Newborough, ans. 1.
Bills, Samuel, Hawton, near Newark-upon-Trent, ans. 3, 9, 13.
Brooks, C. H., C.E., 5, Elswick Villas, Newcastle-upon-Tyne, ans. all the questions.
Buttery, John, Mathematical Master, H.M. Dockyard, Chatham, ans. all the questions.
Buttery, Thomas, Thurcaston, Leicestershire, ans. 13.

Collins, Matthew, B.A., Kilkenny College, ans. 1 to 13.
Dale, James, Leadhills, ans. 1, 3, 4, 5, 7, 8, 10.
Dawson, Thomas, Long Benton, ans. 4.
Dexter, Thomas, Long Whatton, Loughbow, ans. 1, 2, 3, 5, 7, 11.
Eland, Thomas J., 19, Hill Street, Bridge Street, Bolton, ans. 1:
Elliott, J. W., Greatham, Stockton-on-Tees, ans. 9.
Farmar, William, 50, York Street, Dover, ans. 10.
Fenwick, Stephen, of the Royal Military Academy, Woolwich, ans. Prize.
Fleming, Thomas E., Newcastle-upon-Tyne, ans. Prize.
Hattam, Thomas, Eddystone Lighthouse, near Plymouth, ans. 2, 9.
Hewitt, John, Commercial Academy, 16, Saville Row, Newcastle-upon-Tyne, ans. 1, 2, 3, 9.
Hindle, Thomas, Tarleton, near Chorley, Lancashire, ans. 1 to 14.
Knight, J., Lancaster, ans. 1.
Levy, W. H., Shalbourne, near Hungerford, ans. 1 to 10, 12.
Light, J. F., 1, Spencer Road, Stoke Newington, London, ans. 1, 4, 9, 13.
M'Cormick, Edward, C.E., Grosmount, near Hereford, ans. 1, 2, 3, 4, 5, 8, 9, 10, 12, 13.
M'Namara, T., Ballina, Mayo, Ireland, ans. 1 to 13.
Mawson, William, Witton-le-Wear, Durham, ans. 1, 4, 7, 8, 9, 10, 13, Prize.
Milbourn, Thomas, Witton Park, Darlington, ans. 1, 2, 3, 5, 7, 8, 9, 10.
Miller, W. J., B.A., Eltham, Kent, ans. 1 to 10.
Mulcaster, James, jun., Allendale, Northumberland, ans. 1, 4, 7, 9, 12, 13, Prize.
Mulcaster, John Wallis, Allendale, Northumberland, ans. 1, 4, 7, 9, 12, 13, Prize.
Octogenarius, of Hickling, Nottinghamshire, ans. 13.
Petrarch, ans. Prize.
Robinson, John Joshua, H.M. Dockyard, Portsea, ans. 10.
Rutherford, Dr., of the Royal Military Academy, Woolwich, ans. all the questions.
Rutter, Edward, 65, Lawrence Street, Sunderland, ans. 1 to 13, Prize.
Ryan, Lawrence, Assistant Leighlinbridge National School, Ireland, ans. 1 to 13.
Smith, James, Manchester, ans. 1.
Smith, John, Radcliffe Terrace, Northumberland, ans. 1, 2, 3 7, 9.
Sum, ans. 11.
Tebay, Septimus, St. John's College, Cambridge, ans. 14.
Traynor, James, C.E., Carrickmacross, Ireland, ans. 1 to 13.
Turnbull, John, Bedlington, ans. 1, 2, 3, 5, 10, 13.
W. P. H., Harleston, Norfolk, ans. 1, 2, 3, 5, 7, 8, 9.
Waite, John, Whitehaven, ans. 9, 13.
Watson, Stephen, Grammar School, Haydonbridge, Northumberland, ans. 1 to 14.
White, J., Holly Terrace, Birmingham, ans. 1, 4, 9.
White, John, Manningham, near Bradford, Yorkshire, ans. 1.
Wilkinson, T. T., Burnley, Lancashire, ans. 3, 9.
Younger, Samuel, ans. 1, 2, 4 to 8, 10 to 13.

We regret to have to record the deaths of four talented contributors, viz., Mr. James Hann, Professor Charles Gill, Mr. John Laws, and Mr. Henry Buckley.

Mr. James Hann, whose genius and intimate friendship we held in high esteem, was a self-taught mathematician of extraordinary perseverance and talent, and attained a considerable eminence in the scientific world by his writings in the 'Diaries' and other periodicals, and more especially by the publication of some valuable works on mechanics and pure mathematics, admirably adapted to the wants of engineers and practical men. He died on the 16th of August, 1856, aged 58 years.

Professor Charles Gill was a mathematician of uncommon originality and ingenuity, and his solutions to many of the more difficult Diary questions were always neat and systematic. His "Application of the Angular Analysis to Indeterminate Problems of the Second Degree" published by John Wiley, 161, Broadway, New York, is a very characteristic and ingenious work. He died at New York on the 25th of October, 1855.

Mr. John Laws, of Newcastle-upon-Tyne, also conspicuous as a highly talented mathematical contributor to the 'Diaries' for many years, died on the 7th of January, 1855, aged 49 years.

Mr. Henry Buckley, of Wood House, Delph, near Manchester, was a pupil of the late Mr. John Butterworth. His communications, under the signature of "Geometricus," established him as one of the ablest geometers of the day. He was about to prepare for publication a work on Geometrical Analysis, in which Porisms were to form a principal feature, but death put a period to his labours on the 15th of July, 1856, in the 47th year of his age.

Many of our readers will be glad to know that the 'Mathematician,' in three volumes, may now be had of Messrs. Edward and F. N. Spon, 16, Bucklersbury, London, at a moderate price. Sets can also be completed.

A useful little volume, entitled "The Measures, Weights, and Moneys of all Nations, and an Analysis of the Christian, Hebrew, and Mahometan Calendars," by W. S. B. Woolhouse, is just published by John Weale, 59, High Holborn, London. "The Elements of the Differential Calculus," by the same author, before published, forms another volume of Weale's instructive series.

Mr. Matthew Collins, of Kilkenny College, Ireland, is about to publish a Tract on a new and general method of solving Diophantine Equations, a short abstract of which is given at the end of the present Diary. The complete Tract may be obtained on application, by letter, to Mr. Collins, the author.

We have to thank the Rev, Thomas P. Kirkman, M.A., for copies of his able and interesting memoirs "On the Enumeration of x — edra having Triedral Summits, and an (x — 1)-gonal Base," and "On the Representation of Polyedra."

We have likewise to convey our thanks to the Rev. John Peat, M.A., for copies of two editions of his grand and beautiful poem, entitled 'Thoughts on a Plurality of Worlds' (Rivingtons, Waterloo Place, London, 1856); also to Dr. Dodd, of North Shields, for his interesting and instructive pamphlet, 'Ten Letters on Self-Education' (Hamilton, Adams, and Co., London, 1856); and to Mr. T. T. Wilkinson, of Burnley, for his paper on "The Ancient Geometrical Analysis, illustrated from the Writings of the Lancashire Geometers," from the 'Transactions of the Historic Society of Lancashire and Cheshire.'

*** Our correspondents will please to bear in mind, that the arranging of the matter for the printer is greatly facilitated when they obligingly write out their contributions intended for insertion on one side of the paper only, or so that each distinct answer or subject may admit of an easy separation, without the necessity of having it re-written.—ED.

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NEW MATHEMATICAL QUESTIONS.

I. QUEST. (1914); *by* Mr. Robert Ambler, *Grammar School, Stevenage.*

Given the sum of the three sides, the line bisecting the vertical angle, and the area a given space, or a maximum, to construct the triangle.

II. QUEST. (1915); *by* Mr. Daniel White, *of the Educational Institute, Newcastle-upon-Tyne.*

Given AB + BC and AB + AC, together with the angle C, it is required to construct the triangle ABC geometrically.

III. QUEST. (1916); *by* Mr. Stephen Watson, *Haydonbridge.*

Let AB be the transverse diameter of an ellipse; CD an ordinate, upon which, as a diameter, a circle is described; and AP a tangent to the circle, touching it in P. Determine CD when the point P is on the ellipse.

IV. QUEST. (1917); *by* Mr. T. M'Namara, *Ballina, Ireland.*

From the centre of an ellipse let a tangent be drawn to a circle described on an ordinate to the axis major. Find the equation and area of the curve described by the point of contact.

V. QUEST. (1918); *by* Mr. B. Hooke, *London.*

A party of n persons have all different sums of money. The nth person having the largest sum, gives to all the others as much as each had, thereby doubling their respective sums; then the $(n-1)$th person gives to all the others as much as each had, and so on, until each person has made a similar distribution of his money. After which it is found that each had the same sum. Find general expressions for the snms held by each person throughout the several distributions.

VI. QUEST. (1919); *by* Mr. Thomas Hindle, *Tarleton, Lancashire.*

From any point P let two tangents be drawn to a parabola, and from M, the foot of the ordinate PM, as centre, let a circle be described cutting off, from the directrix, a segment CC′ equal to the chord of contact TT′, and meeting the axis produced in A. Then if another circle be described on AF as diameter, through the focus F, and cutting the directrix in B, B′, the area of the triangle ABB′ will be equal to that of the triangle PTT′.

VII. QUEST. (1920); *by* W. P. H., *Harleston, Norfolk.*

On the side BC, (or BC produced) of a plane triangle ABC let points D, E be taken so that BD=CE but measured in opposite directions; similarly on the other sides CA, AB let there be taken CF=AG and AH=BK, so that BD, CF, AH are proportional to the corresponding sides BC, CA, AB. Let AD be drawn to intersect BG, CK in P, Q; BF to intersect CK, AE in R, S; and CH to intersect AE, BG in T, V: then the hexagon PQRSTV will be double the mean proportional between the triangles PRT, QSV.

VIII. QUEST. (1921); *by* Mr. C. H. Brooks, C.E., *Newcastle-upon-Tyne.*

Two coins each one inch in diameter are thrown horizontally into a circular box two inches in diameter; find the probability of only one of them resting on the box, supposing every possible position of each coin to be equally probable.

IX. QUEST. (1922); *by* $\alpha\delta\alpha$, *Southampton.*

From a given paraboloid it is required to cut off a segment by a plane perpendicular to the axis, such that the attraction of the mass of the segment upon a particle in its focus shall be zero.

X. QUEST. (1923); *by* Mr. THOMAS DOBSON, B.A.

A cube rocks, without sliding, on a fixed sphere, and is acted upon by the gravity of the earth; find the time of a small oscillation, and the condition of stable equilibrium.

XI. QUEST. (1924); *by* PETRARCH.

The fraction $\frac{992}{9791}$ when expanded as a decimal gives in succession all the even numbers of two places of figures to 88 places of decimals.

XII. QUEST. (1925); *by* Mr. MATTHEW COLLINS, *Kilkenny College.*

A uniform chain, 1000 miles long, being suspended vertically, with its lower end just touching the earth's surface, required the velocity of the upper end when it arrives at the earth, taking into account the variation of the force of gravity along the chain, but abstracting from the earth's motion and the friction of the atmosphere.

XIII. QUEST. (1926); *by* Mr. W. H. LEVY, *Shalbourn.*

Twenty-eight persons play at dice, each throwing three times with three dice, for a stake of £14. The seventh player having thrown 40, it is required to determine the value of his chance of winning.

XIV. QUEST. (1927); *by* Dr. RUTHERFORD, *of the Royal Military Academy, Woolwich.*

Two equal rods AB, AC are freely moveable about a joint at A, and are placed vertically in a given position, with their extremities B, C on a smooth horizontal plane; the extremity B is constrained to move uniformly, whilst the extremity C moves freely along the plane. Determine the path of the joint A, the position when there is no pressure upon it, and also the pressure on the joint when the rods strike the plane.

XV. or PRIZE QUEST. (1928); *by* PETRARCH.

A rod CC' of a given length has its two ends in the curve of an ellipse and moved round, having a tracing point P, at the distances c and c' from its ends, tracing a curve. Show that the area contained between the curve and the ellipse $= \pi cc'$, and is therefore independent of the ellipse.

The several prizes are allotted as follows: 1st, For answering the Prize Enigma, to Mrs. Ann Towns, London, and Mr. Joseph Furniss, Lois Weedon, each *ten* Diaries. 2dly, For the general answers to the enigmas, to the Rev. John Hope, Stapleton, Carlisle, and Mr. Joseph Hutchinson, near Halifax, each *ten* Diaries. 3dly, For the answers to the rebuses and charades, to Mr. James Herdson, Tobermory, and Mr. William Henry Farn, Brighton, each *eight* Diaries. 4thly, For answering the Prize Question, to Petrarch and Mr. Stephen Fenwick, of the Royal Military Academy, Woolwich, each *twelve* Diaries. 5thly, For the general mathematical answers, to Dr. Rutherford, of the Royal Military Academy, Woolwich, and Mr. Stephen Watson, Haydonbridge, Northumberland, each *ten* Diaries. They will please to send (or write, post-paid) for their respective prizes to Mr. Joseph Greenhill, Stationers' Hall, London.

All letters must, as usual, be directed "TO THE EDITOR OF THE LADY'S AND GENTLEMAN'S DIARY, STATIONERS' HALL, LONDON." They must likewise be post-paid, and arrive *before* May 1st, 1857.

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MATHEMATICAL PAPERS.

ON EQUATIONS OF THE FIFTH DEGREE.

By JAMES COCKLE, M.A., F.R.A.S., F.C.P.S., *Barrister-at-Law.*

(*The subject resumed from p.* 87 *of the* '*Diary*' *for* 1856.)

31. Waring, at p. 47 of his 'Miscellanea Analytica' (Camb., 1762), assuming that the root of a quintic deprived of its second term is a sum of four quintic surds, states that on calculation he found that the biquadratic, whose roots are the quantities under the radical signs, necessarily involves irrational quantities, and consequently that the given equation cannot be solved by this process.

32. At p. 120 of his 'Meditationes Algebraicæ' (Camb., 1770) Waring repeats his statement. And he remarks that such a sum would be the root of an equation of 625 dimensions, which perchance may admit of the given equation as a divisor. He subjoins (p. 121) a second and (p. 122) a third demonstration that a sum of four quintic surds cannot be the general root of an imperfect quintic. And he adds that his last argument is applicable to biquadratics if their root be represented as a sum of three quartic surds.

33. In a paper on the general resolution of algebraical equations ('Phil. Trans.' for 1779, p. 86) Waring states that in 1757 he sent some papers to the Royal Society, which were printed in 1759, and copies of them delivered to several persons; that these papers, somewhat corrected, with the addition of a second part on the properties of curve lines, were published (as his 'Miscellanea'?) in 1762; and that in 1767, 1768, and 1769 he printed, and published in the beginning of 1770, the same papers with additions and emendations under the title of Meditationes Algebraicæ. He afterwards (pp. 101-2) recurs to the publication of his papers, and in concluding says (p. 104) that in 1762 he published some reasons, for which his method could not extend to the general resolution of algebraical equations. These facts throw light on an obscure passage in the preface to his 'Meditationes' where (p. v) Waring apparently claims for his own researches priority to Bezout's first memoir, already discussed (in art. 28).

34. Lagrange's reflections on the algebraic resolution of equations were printed in the Berlin 'Nouveaux Mémoires' for 1770 (published in 1772). The 'Suite,' consisting of the third and fourth sections of his reflections appear in the Berlin 'Nouveaux Mémoires' for 1771 (published in 1773).

35. Lagrange, in the opening of his third section (Berl. 'Nouv. Mém.' for 1771, p. 138) observes that the resolution of equations of

degrees superior to the fourth was one of those problems which up to that time no one had been able to fathom, although there was nothing to demonstrate its impossibility; and that he only knew of two methods (1st, that of Tschirnhausen; 2d, that of Euler and Bezout) which appeared to give any hope of success. He points out that the application of Tschirnhausen's method to quintics gives rise, as the final result of elimination, to an equation of the 24th degree (p. 139); and that in following the method of Euler (from which Bezout's does not essentially differ) we also arrive at a "réduite" of the same degree. Bezout's conclusion, that this equation ought only to involve the difficulties of degrees inferior to the fifth, seems to Lagrange (p. 140) somewhat forced.

36. Lagrange then examines Tschirnhausen's method, and points out how it may be made to give resulting equations of lower degrees than the first aspect of the eliminations would lead us to expect. In solving quintics we may so conduct our operations as to arrive at a final sextic. But if this be incapable of depression the solution is useless. The depression of the resulting equations at which he has arrived seems to Lagrange to be in general almost impossible (p. 150).

37. In applying Tschirnhausen's method to sextics, Lagrange is conducted (pp. 158-9) to a "réduite" of the 15th, and also to one of the 40th degree. The latter, of which the mode of derivation is somewhat different from that of the former, may be resolved into 20 quadratics by means of an equation of the 20th degree (p. 159).

38. Lagrange then investigates the process of Euler and Bezout in a manner which serves (p. 161) to connect this method with that of Tschirnhausen, and to show their analogy and mutual dependence.

39. In applying the process of Euler and Bezout to quintics (pp. 169—173) Lagrange arrives at a "réduite" of the sixth degree, which, he says, is not resolvable unless it be capable of depression to a degree inferior to the fifth. But this depression seems to him to be scarcely possible when regard is had to the forms of the roots of the réduite (p. 176).

40. Adverting to Bezout's suggestion that, in the case of sextics, the final equation can be decomposed into two others by means of a quadratic, Lagrange says that upon this matter he entertains strong doubts (p. 187), reasons for which he (pp. 187-9) assigns.

41. Lagrange (p. 235) remarks that it would be well to apply his own peculiar method of investigation to equations of the fifth and higher degrees, of which the resolution was, up to that time, unknown. But, he adds, this application demanded too great a number of researches and of combinations, the success of which was, moreover, very doubtful, for him to be then able to devote himself to the undertaking, which, however, he hoped to recur to at a future day.

42. Vandermonde, in the Paris 'Mémoires' for 1771 (published in 1774), gave a memoir on the resolution of equations, which had been read (to the French Academy) in November 1770. ('Hist.,' p. 49, 'Mém.,' p. 365.) The 'Réflexions' of Lagrange, as well as their

"Suite," appear to have been read to the Berlin Academy in the course of 1771 (Berl. 'Nouv. Mém.' for 1770, pp. 134 and 215).

43. Speaking of the solution of quintics, Vandermonde (p. 414) remarks that, in dealing with a matter so bristling with difficulties, he had not sufficient faith in conjectures to dare to abandon himself to them. But he adds that, in order to pronounce an opinion upon the possibility of the general solution, it would not perhaps suffice to confine the attention to the forms of the resulting expressions (p. 415) but to examine the relations which may exist among them.

44. I interrupt, for the present, the course of this history, in order that I may offer some remarks on my "method of symmetric products." In a note (†) at p. 178 of the 'Mathematician' for November, 1848, I have given some references connected with the subject. To these may be added the following—'Mech. Mag.,' vol. lii, pp. 226 and 486; 'C. and D. Math. Jour.,' vol. vii, p. 114; and 'Phil. Mag.,' ser. iv, vols. iv (p. 492), v (p. 170), and vii (p. 130).

45. The method may be thus described. Let u, v, w, x, &c., be the roots of an n-ic equation, and let

$$\begin{aligned} L_1 &= au + bv + cw + dx + \&c. \\ L_2 &= \alpha u + \beta v + \gamma w + \delta x + \&c. \\ \&c. &= \&c. \end{aligned}$$

where a, b, &c. α, β, &c. are undetermined. Determine these quantities in such manner that, if possible, the product

$$L_1 L_2 L_3 \ldots L_{n-2} L_{n-1}$$

of $n-1$ linear functions L may be a symmetric function of u, v, w, x, &c.

46. This determination is possible in the case of a cubic. And a transformation of the given cubic to a form which involves the evanescence of the "symmetric product" enables us, by elimination between

$$L_m = 0, \Sigma u = A \text{ and } \Sigma uv = B,$$

to attain a solution.

47. It is also possible in the case of a biquadratic, and the conditions

$$L_m = 0, \text{ and } A' = \Sigma u = 0,$$

conduct us to

$$B' = -(u + w) \text{ and } D' = u^2 w^2,$$

and all the roots may be determined. It is from this system, and not from one which I have before given ('C. and D. Math. Jour.' for May, 1852, p. 116), that the roots will be obtained. In other respects, my discussion of a biquadratic in the 'Mathematical Journal' may be followed, but, the evanescence of C′ not being an independent condition, another must be substituted for it.

48. Quintics do not afford us a symmetric product. In seeking to satisfy the equations of symmetry we are, however, conducted *à priori* to Lagrange's linear functions.

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49. The functions L (which are independently deduced, although identical with Lagrange's) are "critical." They do not vary when all the roots are increased or diminished by equal quantities; nor, consequently, does their product. Such functions possess many interesting properties. For instance, if in the symmetric expression

$$x^2 + y^2 - xy$$

we diminish each of the quantities x and y by a third quantity z, the result is equal to L_1L_2 and is a critical and symmetric function of x, y, and z.

50. The functions of Lagrange seem to fulfil as nearly as is possible the conditions of symmetry. Their product is what I have ('Phil. Mag.' for December, 1853, p. 444) proposed to term "epimetric."

51. Denote by v, w, x, y, z the five roots of a quintic respectively, and by α one of the unreal fifth roots of unity.

52. Assume that

$$v + w + x + y + z = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (17),$$

and

$$v + \alpha w + \alpha^2 x + \alpha^3 y + \alpha^4 z = 0 \quad \ldots\ldots\ldots\ldots \quad (18),$$

and subtract (18) from (17). Then

$$(1-\alpha)w + (1-\alpha^2)x + (1-\alpha^3)y + (1-\alpha^4)z = 0 \ldots \quad (19).$$

53. Divide (19) by $1-\alpha$, and, by means of

$$1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 = 0 \ldots\ldots\ldots\ldots\ldots\ldots \quad (20),$$

modify the result. We may thus obtain

$$w + (1+\alpha)x - \alpha^3(1+\alpha)y - \alpha^4 z = 0 \ldots\ldots\ldots\ldots \quad (21).$$

54. Let

$$a = -(1+\alpha),\ b = \alpha^3(1+\alpha),\ c = \alpha^4,$$

and let ϕ be such that

$$\phi(a) = a^2 + a + 1$$

and

$$\phi(a, b) = 2ab + a + b + 1.$$

55. Then we see that

$$w = ax + by + cz,$$

and

$$-v = (a+1)x + (b+1)y + (c+1)z,$$

and also, after substitutions and reductions, that

$$\left.\begin{array}{l} \tfrac{1}{2}(v^2 + w^2 + x^2 + y^2 + z^2) = \phi(a)x^2 \\ + \phi(b)y^2 + \phi(c)z^2 + \phi(a,b)xy + \phi(a,c)xz + \phi(b,c)yz \end{array}\right\} \ldots (22).$$

56. But, availing ourselves of (20) in the reductions, we find

$$\phi(a) = -\alpha^3(1+\alpha),\ \phi(b) = \alpha^2(1+\alpha),$$

$$\phi(c) = -\alpha(1+\alpha),\ \phi(a,b) = (1+\alpha+2\alpha^2)(1+\alpha),$$

$$\phi(a,c) = -(1+\alpha^4)(1+\alpha), \text{ and } \phi(b,c) = (\alpha^2-\alpha-1)(1+\alpha).$$

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57. We hence arrive at

$$\left.\begin{array}{r}\tfrac{1}{2}(v^2+w^2+x^2+y^2+z^2)\\ =(x^2+Ay^2+Bz^2+Cxy+Dxz+Eyz)\phi(a)\end{array}\right\}\ldots (23),$$

where

$$A=-\alpha^4,\ \ B=\alpha^3,\ \ C=-(\alpha^2+\alpha^3+2\alpha^4),$$
$$D=\alpha+\alpha^2,\ \ E=\alpha^2+\alpha^3-\alpha^4.$$

58. Let

$$F=-\alpha^3-\alpha^4,\ \ H=-\alpha^2-\alpha^4,$$
$$G=\alpha^2,\ \text{and}\ K=\alpha;$$

then

$$F+H=C,\ \ FH=A,\ \ G+K=D,$$
$$GK=B,\ \text{and}\ FK+GH=E.$$

59. It follows that

$$\left.\begin{array}{r}x^2+Ay^2+Bz^2+Cxy+Dxz+Eyz\\ =(x+Fy+Gz)\,(x+Hy+Kz)\end{array}\right\}\ldots (24).$$

60. The three equations of art. 52 are respectively equivalent to those of art. 12 ('Diary' for 1848, p. 86). Let

$$y^5+q_1y^4+q_2y^3+q_3y^2+q_4y+q_5=0\ldots\ (25)$$

be the quintic of which $y_1, y_2, \ldots, y_5$ are the roots; then, since

$$\Sigma\,.\,y_1y_2=\tfrac{1}{2}\{(\Sigma\cdot y)^2-\Sigma\,.\,y^2\}\ldots\ldots\ldots\ (26),$$

we infer that, under the conditions (4) and (5), q_2 may be broken up into factors. In other words, if

$$\Sigma\,.\,y=0\ \text{ and }\ \Sigma\,.\,\alpha^r y_r\doteq L=0,$$

then

$$\Sigma\,.\,y_1y_2=(y_2+Fy_3+Gy_4)\,(y_2+Hy_3+Ky_4)\phi(a)\ldots\ (27).$$

61. But, more than this, the evanescence of L leads to a solution of the quintic.

62. For, let

$$\beta_1=d,\ \ \beta_2=e,\ \ \beta_3=f,$$
$$-B=5P,\ \ -C=5Q,\ \ -D=5R,$$

then, by means of the equations (4) and (5) of p. 67 of the 'Diary' for 1847, we obtain

$$P=ef\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (28),$$
$$Q=d^2f+de^2\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (29),$$
$$R=d^3e+df^3-P^2.\ \ \ldots\ldots\ldots\ldots\ldots\ (30).$$

63. Combining (28) and (29), we find

$$Q=P\frac{d^2e^4}{e^5}+de^2$$

or

$$P(de^2)^2+(de^2-Q)e^5=0\ \ldots\ldots\ldots\ldots\ (31).$$

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64. Combining (28), (29), and (30), we find

$$\frac{R+P^2}{Q}=\frac{d^2e+f^3}{df+e^2}=\frac{d^2e^4+P^3}{Pde^2+e^5}$$

or

$$Q\{(de^2)^2+P^3\}-(R+P^2)(Pde^2+e^5)=0\ \dots\ (32).$$

65. The equations (31) and (32) are of two dimensions in de^2 and of one in e^5; and e and d may, consequently, be determined from those two conditions. The remaining quantity f may then be found from (28).

66. Now, if the roots of a quintic be expressed in the manner of Lagrange and Vandermonde (see Lag., 'Equ.,' 3d ed., pp. 248-50, num. 16, and pp. 270-1, num. 41), and if one of the surds which constitute it be made to vanish, we have an equation of the form of that which I proposed at p. 78 of the 'Diary' for 1846, and which, as I have just shown, admits of complete solution.

67. Hence, if one of the linear factors L of the symmetric or epimetric product vanishes, the quintic may be solved.

68. It is to be observed (compare 'Diary' for 1847, p. 67) that

$$-E=d^5+e^5+f^5+5PQ-10Pde^2\ \dots\dots\dots\dots\ (33)$$

or

$$(de^2)^5-10Pe^{10}(de^2)+e^{15}+(5PQ+E)e^{10}+P^5e^5=0\ \dots\ (34).$$

69. Now, from (31) we find

$$(de^2)^2=-\frac{e^5}{P}(de^2)+\frac{Qe^5}{P};$$

and, consequently,

$$(de^2)^5=T(de^2)+U\ \dots\dots\dots\dots\dots\dots\dots\ (35)$$

where

$$T=\frac{e^{20}}{P^4}+\frac{3Qe^{15}}{P^3}+\frac{Q^2e^{10}}{P^2}\ \dots\dots\dots\dots\ (36)$$

and

$$U=-\frac{Qe^{20}}{P^4}-\frac{2Q^2e^{15}}{P^3}\ \dots\dots\dots\dots\dots\dots\ (37).$$

70. Hence, after substitution in (34), we shall arrive at an equation of the form

$$V(de^2)+W=0\ \dots\ \dots\ \dots\dots\dots\dots\ (38).$$

71. And, multiplying (31) into Q and (32) into P, and subtracting the latter from the former result, we shall obtain

$$M(de^2)+N=0\ \dots\dots\ \dots\dots\dots\dots\ (39),$$

where

$$M=Qe^5+(R+P^2)P^2\ \dots\dots\dots\dots\dots\ (40),$$

$$N=\{(R+P^2)P-Q^2\}e^5-P^4Q\ \dots\ (41).$$

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72. From (38) and (39) we find

$$MW - NV = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42),$$

a relation which furnishes us with a new solvable form of quintics.

73. I beg leave to add the following supplementary remarks on "Approximation," &c. Some observations of Professor De Morgan upon Newton's parallelogram, and other subjects connected with the theory of equations, are abstracted at pp. 308-10 of the 'Philosophical Magazine' for April, 1856. An abstract of a paper by Mr. C. M. Willich on the Geometrical Quadrature of the Circle appears at pp. 148-9 of the same journal, for August, 1855. A solution of a perfect cubic, by Mr. Rotherham, will be found at p. 531 of vol. lxii of the 'Mechanics' Magazine' (for June 9th, 1855, No. 1661). Bombelli gave that geometrical solution of the irreducible case which depends upon the trisection of an angle. Ivory has shown ('Enc. Brit.,' pp. 343-4 of vol. ix, 7th ed., art. "Equations") how the "primitive roots" employed by Gauss may be determined directly. [In art. 81 of p. 86 of the 'Diary" for 1855, *for* "circumf." *read* "semicircumf."]

74. The subject of art. 44 of p. 81 of the 'Diary' for 1854 demands, perhaps, further explanation. Fourier's views extend to the determination of the quadratic, cubic, or higher factors of an equation. His process for quadratic factors, as amended by Murphy, is as follows.

75. Let $$x^n + ax^{n-1} + bx^{n-2} + cx^{n-3} + \&c. = 0 \ldots (43)$$

and $$f(x) = 1 + ax + bx^2 + cx^3 + \&c.$$

Also let $F(x)$ be a rational function of x of dimensions inferior to n. Then (see Murphy's 'Equations,' pp. 95-6), if we make

$$\frac{F(x)}{f(x)} = 1 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots a_mx^m + \&c.,$$

the coefficients a form a recurring series.

76. The "rule" given by Murphy at p. 116 of his 'Equations' is, with obvious slight additions, equivalent to the statement that

$$(a_{m-1}a_{m-3} - a^2_{m-2})x^2 + (a_{m-1}a_{m-2} - a_ma_{m-3})x + (a_ma_{m-2} - a^2_{m-1})$$

is a factor of (43). This factor appears to contain the two greatest roots; conjugate unreal roots being considered as greater than a real root when their product exceeds its square. (Ibid.)

77. In arts. 75 and 76 I have adopted such a notation as will enable the reader to compare the process of Fourier and Murphy with that of Mr. Beecroft, who, in his 'General Method,' &c., has, in a variety of instructive examples, brought to light the value and efficiency of solution by decomposition into quadratic factors.

78. I may add that the quantity $a_{m+1} \div a_n$ converges (if it con-

verge) to the greatest root of (43) (Murphy, p.114); but for information upon the mode in which unreal roots affect and are affected by the processes, as well as upon the whole subject, I would refer the reader to Murphy's 'Equations,' pp. 107 *et seq.*, to Peacock's 'Report,' pp. 347-9, to Mr. Beecroft's 'General Method,' &c., and to Note vi of Lagrange's 'Equations' (3d ed.). As to the transformations necessary for the application of the process in particular cases, see Lagrange, p. 136; Murphy, p. 117; and Beecroft, pp. 1, 13, &c.

79. Weddle's 'New, Simple, and General Method,' &c. (Lond., 1842), and Horner's 'Synthetic Division' must not be lost sight of in treating of the subject of equations.

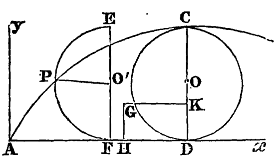
76, Cambridge Terrace, Hyde Park;
3d June, 1856.

(*To be continued.*)

NOTE ON THE CENTRE OF GRAVITY OF THE SEMI-CYCLOID.

By Stephen Fenwick, F.R.A.S., *of the Royal Military Academy, Woolwich.*

Whether the method of investigating the centre of gravity of a semi-cycloid by means of the variable angle at the centre of the generating circle has been noticed, I know not; but the process is so very simple as to deserve especial attention. The integration of the expression for the distance of the centre of gravity of the semi-cycloid from the *axis* of the curve involves, as is well known, a little difficulty. In the following investigation, the distances of the centre of gravity of the semi-cycloid from the base and axis of the curve are both determined with great ease.



Take the base AD of the semi-cycloid for the axis of x, and A the extremity of it for the origin of rectangular coordinates. Then if θ be the angle at the centre O′ of the generating circle EPF, contained by a vertical diameter EF and PO′, the radius of the "generating point" $P(x, y)$,

$$x = r(\theta - \sin\theta),\quad y = r(1 - \cos\theta),$$

r being the radius of the generating circle.

Let A be the area of the semi-cycloid, G its centre of gravity, and

GH, GK perpendiculars, respectively, on its base and axis, then the coordinates of the centre of gravity of the element ydx being x and $\frac{1}{2}y$, it follows that

$$\text{GH} = \frac{\int \frac{1}{2}y \,.\, ydx}{\text{A}} = \frac{r^3 \int (1-\cos\theta)^3 d\theta}{3r^2\pi} \ldots\ldots\ldots\ldots (1),$$

$$\text{AH} = \frac{\int x \,.\, ydx}{\text{A}} = \frac{r^3 \int (\theta - \sin\theta)\,(1-\cos\theta)^2 d\theta}{\frac{3}{2}\, r^2 \pi} \ldots (2);$$

the limits of θ extending from $\theta = 0$ to $\theta = \pi$.

It will be obvious that the centre of gravity of the whole and that of the semi-cycloid are equally distant from the base. This distance is given by (1).

Now,

$$\int (1-\cos\theta)^3 d\theta = \int (1 - 3\cos\theta + 3\cos^2\theta - \cos^3\theta)\, d\theta$$

$$= \int \left(\frac{5}{2} - \frac{15}{4}\cos\theta + \frac{3}{2}\cos 2\theta - \frac{1}{4}\cos 3\theta\right) d\theta$$

$$= \frac{5}{2}\theta - \frac{15}{4}\sin\theta + \frac{3}{4}\sin 2\theta - \frac{1}{12}\sin 3\theta.$$

Hence $$\int_0^\pi (1-\cos\theta)^3 d\theta = \frac{5}{2}\pi.$$

Wherefore, by (1), the distance of the centre of gravity of the semi-cycloid from its base is

$$\text{GH} = \frac{5}{6} r.$$

Again,

$$\int (\theta - \sin\theta)(1-\cos\theta)^2 d\theta = \int \theta (1-\cos\theta)^2 d\theta - \int (1-\cos\theta)^2 \sin\theta d\theta$$

$$= \int \left(\frac{3}{2}\theta + \frac{1}{2}\theta\cos 2\theta - 2\theta\cos\theta\right) d\theta - \frac{1}{3}\left(1 - \cos\theta\right)^3$$

$$= \frac{3}{4}\theta^2 - \frac{1}{3}(1-\cos\theta)^3 + \int \left(\frac{1}{2}\theta\cos 2\theta - 2\theta\cos\theta\right) d\theta.$$

But, by the method of "parts,"

$$\frac{1}{2}\int \theta\cos 2\theta \,.\, d\theta = \frac{1}{4}\theta\sin 2\theta + \frac{1}{8}\cos 2\theta,$$

$$-2\int \theta\cos\theta \,.\, d\theta = -2\theta\sin\theta - 2\cos\theta.$$

Wherefore $$\int_0^\pi (\theta - \sin\theta)\,(1-\cos\theta)^2 d\theta = \frac{3}{4}\pi^2 + \frac{4}{3}.$$

Hence by (2), $$AH = r\left(\frac{\pi}{2} + \frac{8}{9\pi}\right);$$

and therefore $$DH = GK = \pi r - AH = r\left(\frac{\pi}{2} - \frac{8}{9\pi}\right)$$

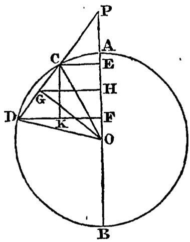
is the distance of the centre of gravity of the semi-cycloid from the axis.

ON THE SURFACE AND VOLUME OF THE SPHERE.

By STEPHEN FENWICK, F.R.A.S., *of the Royal Military Academy, Woolwich.*

There are some formulæ used in the mathematical student's earlier reading which require for their investigation a higher degree of mathematical knowledge than he can be expected to possess at that stage of his progress. Of such are the expressions for the surface and volume of a sphere. Different methods, therefore, have been given for the determination of the surface and volume of a sphere without the aid of the Integral Calculus. The following method seems to me to be more simple and concise than any that has been given.

A sphere is generated by the revolution of a semicircle about its diameter. Let CD be a side of an equilateral polygon inscribed in the circle ADB, such that CD is contained a certain number of times in the semicircle ADB; OG a perpendicular on CD from the centre O; CE, GH, DF perpendiculars on the diameter AB; and CK a perpendicular to DF from C. Then G is obviously the middle of CD, and consequently $EC + FD = 2GH$.



Now, the surface of the conical frustum generated by CD in revolving about AB as a fixed axis is

$$S = CD(EC + FD)\pi.$$

But if α be the inclination of CD to AB,

$$CD \cos\alpha = CK = EF, \text{ and } EC + FD = 2GH = 2GO\cos\alpha;$$

wherefore $$S = 2GO \,.\, EF \,.\, \pi.$$

Hence, as this relation holds for a side CD of the inscribed equilateral polygon, a similar one holds for any other side; and therefore, as the distance of each of the equal sides of the polygon from the centre O is equal to OG, and the sum of all the intercepted parts corresponding to EF is manifestly equal to AB, the surface generated by the perimeter of the inscribed polygon in revolving about the diameter AB is

$$S = 2\mathrm{GO} \, . \, \mathrm{AB} \, . \, \pi.$$

Moreover, this expression for the surface of the polyhedron generated by the revolution of the inscribed polygon about AB is equally true when each side of the polygon is indefinitely diminished, that is, when the number of sides is indefinitely increased; in which case, OG becomes ultimately equal to r, the radius of the semicircle, and the surface of the polyhedron becomes equal to the surface of the sphere generated by the semicircle ADB. Hence the surface of the sphere is

$$S = 2r \, . \, 2r\pi = 4r^2\pi.$$

Volume of the Sphere.

The volume of the sphere is founded on the following theorem

THEOREM. If a plane triangle revolve about one of its sides as a fixed axis, it will generate a solid which is equal in volume to a cone of which the altitude is equal to the perpendicular of the triangle drawn from one extremity of the fixed side, and the base equal to the surface generated by that side on which the perpendicular falls.

Thus let the triangle PDO (preceding figure) revolve about the side PO, then it will generate two cones, having a common circular base, of which DF is the radius; and hence the volume thus generated during a complete revolution is

$$V = \frac{\pi}{3} \, . \, \mathrm{DF}^2(\mathrm{PF} + \mathrm{FO}) = \frac{\pi}{3} \, . \, \mathrm{DF}^2 . \, \mathrm{PO}.$$

But $\mathrm{DF} = \mathrm{DP} \sin \alpha$, and $\mathrm{PO} \sin \alpha = \mathrm{OG}$; wherefore

$$V = \tfrac{1}{3}\mathrm{GO} \times \mathrm{DP} \, . \, \mathrm{DF} \, . \, \pi$$
$$= \tfrac{1}{3}\mathrm{GO} \times \text{surface generated by DP}.$$

This proves the theorem, and enables us very readily to deduce the volume of the sphere generated by the semicircle ADB.

It is manifest that the volume of the solid generated by the triangle COD in revolving about AB is equal to the difference of the volumes of the solids generated by the triangles PDO and PCO, about the same axis of revolution. Hence, by the preceding theorem,

$$\text{Volume generated by COD} = \tfrac{1}{3}\mathrm{GO} \times \text{surface generated by CD}.$$

Reasoning with this as in the case of the surface of the sphere, we get at once

$$\text{Volume of sphere} = \tfrac{1}{3} \, r \times 4r^2\pi = \tfrac{4}{3} \, r^3\pi.$$

It is hence obvious that *the volume of a sphere is equal to that of a cone of which the altitude is equal to the radius of the sphere, and the base a plane area equal to the surface of the sphere.*

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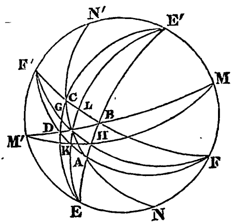
MODERN GEOMETRY.

By Mr. John Joshua Robinson, *of H.M. Dockyard, Portsea.*

(*Continued from the 'Diary' for* 1856, *page* 96.)

(32) *The complete spherical quadrilateral is formed by four great circles of the sphere which intersect each other, two and two, in six points.*

Thus ABCDEF is a complete spherical quadrilateral, the diagonals of which are AC, BD, EF; it is evident that ABCDE′F′ is also a complete spherical quadrilateral, the diagonals of which are AC, BD, E′F′. By completing the circles on the sphere of which the semicircumferences only are shown in the figure, we shall form two other complete spherical quadrilaterals, the summits of which are A′, B′, C′, D′, E, F and E′, F′, A′, B′, C′, D′ respectively; the last four points A′, B′, C′, D′ being diametrically opposite to A, B, C, D.



All the theorems relating to the complete quadrilateral *in plano* have analogous ones on the sphere. Thus Art. (27) may be at once applied to the complete spherical quadrilateral by using the sines of the segments of the arcs instead of the segments themselves.

For

$$\frac{\sin FB}{\sin FM}=\frac{\sin FMB}{\sin FBM},\quad \frac{\sin CD}{\sin BC}=\frac{\sin CBD}{\sin BDC}=\frac{\sin FBM}{\sin BDC},$$

$$\frac{\sin EM}{\sin ED}=\frac{\sin EDM}{\sin EMD}=\frac{\sin BDC}{\sin FMB};$$

whence

$$\frac{\sin FB\,.\sin CD\,.\sin EM}{\sin FM\,.\sin BC\,.\sin ED}=1$$

or $\sin FB\,.\sin CD\,.\sin EM=\sin FM\,.\sin BC\,.\sin ED$... (3).*

In the same way as *in plano*, it may also be shown that

$$\sin FB\,.\sin CD\,.\sin EN=\sin NF\,.\sin BC\,.\sin ED\text{... (4).*}$$

From (3) and (4),

$$\sin EM : \sin EN :: \sin FM : \sin NF.$$

All the other relations on the sphere may be deduced from those *in plano* in precisely the same way as this has been done.

* These are the fundamental theorems in spherical transversals.

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It now only remains to show the application of the general principle to the deduction of other theorems on the sphere. For instance, Theorem (27) becomes

"*If any spherical transversal intersect the four sides and two diagonals of a spherical quadrilateral (formed by great circles of the sphere*) in six points, the sines of these segments are in involution.*"

Art. (31) becomes

"*The middle points of the three diagonals of a complete spherical quadrilateral lie on the same great circle of the sphere.*"

Quest. (1891), in the 'Diary' for last year, is similarly an application of this principle *in plano* to the sphere. Thus it has been shown, in the solution to Quest. (1878), that

If two diagonals of the complete quadrilateral (in plano) *subtend right angles at any point, the third diagonal must also subtend a right angle at that point.*

Whence Quest. (1891). Since two diagonals subtend right angles at the centre of the sphere, so must the third diagonal subtend a right angle at the same point.†

Before proceeding, it may be useful to enunciate one or two theorems which follow at once from elementary principles in the theory of combinations.

(33) 1st. *Since three circles give rise to one radical centre, it follows that n circles, taken three and three together, have* $\frac{n(n-1)(n-2)}{1.2.3}$ *radical centres.*

2d. *Since every pair of circles have two poles of similitude, n circles, taken two and two, will have* $n(n-1)$ *poles of similitude, which, taken three and three, lie on* $\frac{2n(n-1)(n-2)}{3}$ *straight lines, or axes of similitude.*‡

These theorems being premised, I shall go on to the consideration of four circles; but as the investigation of four circles, any how posited, must necessarily lead to complicated results, I have preferred a particular case to the more general one; namely, when four circles have only *four* axes of similitude instead of twelve. This is the celebrated and much studied case which has been so fully discussed by the late Professor Davies and Mr. Weddle. The results obtained by the latter were highly elegant, and appeared to be very complete; but it seems to me that by far the most general way of dealing with the

* Great circles of the sphere are always to be understood.

† This subject was first developed in this country by Davies, in his edition of 'Hutton's Course,' the 'Mathematician,' &c.

‡ These follow at once from elementary principles. The first is apparent enough, and it is well known that the six poles of similitude of three circles lie, three and three, on four right lines; hence the latter portion of the theorem.

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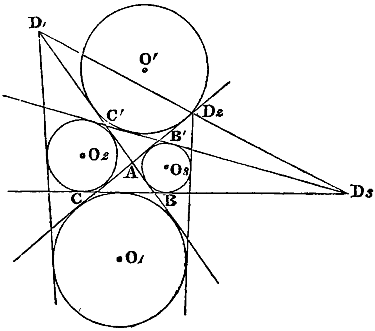
subject is that which I have here adopted, namely, by means of the elementary properties of Radical Axes and Poles of Similitude, from which Diarian geometers may readily obtain nearly all the theorems which were given in the 'Horæ Geometricæ.' It is not my object, however, to go over the ground which has already been so well and carefully ploughed by Mr. Weddle; but I shall merely point out the numerous properties that may be obtained by taking into consideration the fourth axis of similitude (represented by $D_1D_2D_3$ in the annexed diagram). I may also here explain how the principle of *duality* makes its appearance, and that, instead of the three escribed circles, we have in reality four (the centres of which are represented by O_1, O_2, O_3, O_1' in the same figure), and two triangles ACB, AC′B′ which are equal to each other in all respects.

Hence the duality; for we may consider either of these triangles and the circles O_1, O_2, O_3 or O_1', O_2, O_3 respectively.

I was led to consider the case of four circles by the elegant Prize Question proposed by Mr. Wilkinson in the 'Diary' for 1853. At the time I forwarded my solution to that question, I anticipated that it was only a particular case of a much more general theorem, and accordingly I obtained many interesting properties, which I drew up in the form of a paper on the "Centroïd," and forwarded to the Editor.



The following extended theorem is also easily established.*

(34) *The circle described through the middle points of the sides of any triangle is tangential to several infinite systems of inscribed and escribed circles to triangles drawn according to a given law.*

Or it may be thus enunciated:

If the radical centres of the inscribed and escribed circles of any triangle be taken, and circles be inscribed and escribed to the triangles formed by joining these radical centres, and the radical centres of the latter system of circles be again taken and circles inscribed and escribed to the triangles thus formed, and so on ad infinitum, *the infinite number of circles thus found, as well as the original system of inscribed and escribed circles, always touch the circle drawn through the middle points of the first triangle.*

* [Mr. Wilkinson first announced his beautiful theorem under this general form of enunciation.—Ed.]

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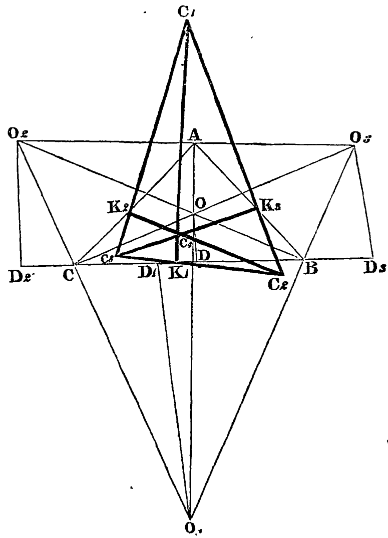
The demonstration may be made to depend on the following

LEMMA. *The radical axes of the circles inscribed and escribed to any triangle intersect each other, two and two, at right angles, in the middle points of the sides of the triangle.*



In the annexed diagram, O, O_1, O_2, O_3 are the centres of the inscribed and escribed circles; D, D_1, D_2, D_3 are the points of contact of those circles with the side CB of the triangle ABC, and K_1, K_2, K_3 the middle points of the sides of that triangle. The radical centres are represented by C_1, C_2, C_3, C_4.

First, taking the pair of circles (O) and (O_1).

Since the radical axis is defined to be the locus of equal tangents to a pair of circles, and D_1D is a common tangent to (O_1) and (O), therefore D_1D will be bisected in the point K_1 where the radical axis of these circles intersects it; that is, $D_1K_1 = K_1D$ and K_1 is the point of bisection of CB.

Otherwise,

$CD_1 = BD$ ('Lady's Diary' for 1835, p. 52, Art. II).

$CK_1 = BK_1 =$ half the base.

$CK_1 - CD_1 = BK_1 - BD$ or $D_1K_1 = K_1D$.

But

$O_1K_1^2 = O_1D_1^2 + K_1D_1^2$ (Eucl. i, 47) $= r_1^2 + K_1D^2$, since $K_1D = D_1K_1$;

and $OK_1^2 = OD^2 + K_1D^2$ (Eucl. i, 47) $= r^2 + K_1D^2$;

whence $O_1K_1^2 - OK_1^2 = r_1^2 - r^2$.

(*a*) That is K_1, or the middle point of the base BC is a point on the radical axis of (O_1) and (O). Now the radical axis of two circles intersects the line joining their centres at right angles; therefore the radical axis of (O_1) and (O) intersects AO_1 at right angles; but AO_1

is at right angles to O_2O_3; whence the radical axis of (O_1) and (O) is parallel to O_2O_3.

Again,

$$BD_2 = CD_3 \text{ ('Lady's Diary' for 1835, p. 51, Art. II)}$$

and $\quad CK_1 = BK_1 = \text{half the base.}$

$$\therefore D_2K_1 = BD_2 - BK_1 \text{ and } K_1D_3 = CD_3 - CK_1 = BD_2 - BK_1$$
$$= (\therefore) K_1D_2.$$

But $\quad O_2K_1^2 = O_2D_2^2 + D_2K_1^2$ (Eucl. i, 47) $= r_2^2 + D_2K_1^2$,

and $\quad O_3K_1^2 = O_3D_3^2 + D_3K_1^2$ (Eucl. i, 47) $= r_3^2 + D_2K_1^2$.

Hence $\quad O_2K_1^2 - O_3K_1^2 = r_2^2 - r_3^2$.

(β) Therefore the radical axis of (O_2) and (O_3) passes through K_1, and, since it is perpendicular to the line joining their centres, and AO_1 is perpendicular to the same line, it follows that the radical axis of (O_2) and (O_3) is parallel to AO_1, and consequently intersects the radical axis of (O_1) and (O) at right angles in K_1.

In the same way it may be shown that the radical axes of (O_1), (O_2); (O), (O_2) and (O_1), (O_3); (O), (O_3) respectively intersect each other at right angles in the middle points of AC and AB.

By producing the radical axes of each pair of circles (O), (O_1), (O_2), (O_3) until they intersect each other, four triangles (to which I propose to give the name of *radical triangles*) will be formed, which will be similar to the triangles $O_1O_2O_3$, &c.

Denoting the triangle which is similar to the triangle $O_1O_2O_3$ by $C_1C_2C_3$, it is at once evident, from what has just been done, that C_4, the fourth radical centre, is the point of intersection of the perpendiculars let fall from the angular points of the triangle $C_1C_2C_3$ on the opposite sides, and further that the feet of these perpendiculars are the middle points of the sides of the triangle ABC. Hence we conclude, since K_1, K_2, K_3 are the feet of the perpendiculars let fall from the angular points of the triangle $C_1C_2C_3$ upon the opposite sides of that triangle, and as these points are likewise the middle points of the triangle ABC, that the circle through K_1, K_2, K_3 passes through the middle points of the sides of the triangle $C_1C_2C_3$ (first solution to Prize Question for 1854). It is also shown in the solution just mentioned that this circle touches the inscribed and escribed circles of the triangles APB, BPC, APC, and ABC (P being the intersection of the perpendiculars from A, B, C on the opposite sides); it is evident that the same circle touches the circles inscribed and escribed to the triangles $C_1C_2C_3$, $C_1C_4C_2$, $C_1C_4C_3$, and $C_2C_3C_4$.

Now, as we have applied the same process to the triangles $C_1C_2C_3$, &c., that we did to the triangles ABC, &c., and have shown that the circles inscribed and escribed to the former series of triangles touch the same circle as do those inscribed and escribed to the latter series of triangles, it is clear, from the method of demonstration which has been employed, that by taking the radical axes of the circles inscribed and escribed to the triangles $C_1C_2C_3$, &c., they will intersect at right angles in the middle points of the sides C_1C_2, C_1C_3, &c. Hence the truth of the proposition.

We may here also remark how the principle of *duality* makes its appearance; for, by omitting the circle (O_1) and taking (O_1') (Fig. 2) in its stead, we shall find that the radical triangles $C_1C_2C_3$, &c., in this case are *similar* and *similarly placed* to the triangles $O_1O_2O_3$, &c.

(35) The ABRIDGED NOTATION may be employed as follows, to demonstrate this proposition:

(1) *To show that the radical axes of the circles inscribed and escribed to any triangle, pass, two and two, through the middle points of the sides of the triangle.*

Let $\alpha=0$, $\beta=0$, $\gamma=0$ represent the sides of the triangle, then is the inscribed circle represented by

$$\alpha^{\frac{1}{2}}\cos\frac{A}{2}+\beta^{\frac{1}{2}}\cos\frac{B}{2}+\gamma^{\frac{1}{2}}\cos\frac{C}{2}=0 \ldots (1)$$ 'Diary' for 1856, p. 94.

In the same way it may be readily shown that the equation of the circle which touches the side

BC externally is $\alpha^{\frac{1}{2}}\cos\frac{A}{2}+\beta^{\frac{1}{2}}\sin\frac{B}{2}+\gamma^{\frac{1}{2}}\sin\frac{C}{2}=0 \ldots\ldots\ldots (2)$

AC ,, ,, $\alpha^{\frac{1}{2}}\sin\frac{A}{2}+\beta^{\frac{1}{2}}\cos\frac{B}{2}+\gamma^{\frac{1}{2}}\sin\frac{C}{2}=0 \ldots\ldots\ldots (3)$

AB ,, ,, $\alpha^{\frac{1}{2}}\sin\frac{A}{2}+\beta^{\frac{1}{2}}\sin\frac{B}{2}+\gamma^{\frac{1}{2}}\cos\frac{C}{2}=0 \ldots\ldots\ldots (4)$

(1) — (2) gives $\beta^{\frac{1}{2}}\left(\cos\frac{B}{2}-\sin\frac{B}{2}\right)+\gamma^{\frac{1}{2}}\left(\cos\frac{C}{2}-\sin\frac{C}{2}\right)=0$;

or $\beta^{\frac{1}{2}}\sqrt{1-\sin B}+\gamma^{\frac{1}{2}}\sqrt{1-\sin C}=0 \ldots\ldots\ldots (5).$

But this is the equation of the radical axis of (1) and (2), where β, γ represent the perpendiculars let fall from any point of it on the sides AC and AB. If K_1 represent the middle point of the base BC, the ratio of the perpendiculars let fall from this point on the other two sides of the triangle is sin B : sin C and the equation of the bisector is therefore $\beta\sin B-\gamma\sin C=0$. But (5) may be put into the form $\beta\sin B-\gamma\sin C-\beta+\gamma=0$. Hence it follows that (5) passes through the point K_1, or the middle point of BC, the coordinates of which are $\beta\sin B-\gamma\sin C=0$ and $\alpha=0$.

Again,

(3) — (4) gives $\beta^{\frac{1}{2}}\left(\cos\frac{B}{2}-\sin\frac{B}{2}\right)-\gamma^{\frac{1}{2}}\left(\cos\frac{C}{2}-\sin\frac{C}{2}\right)=0$;

or, $\beta^{\frac{1}{2}}\sqrt{1-\sin B}-\gamma^{\frac{1}{2}}\sqrt{1-\sin C}=0 \ldots\ldots (6).$

Hence (5) and (6) both pass through the same point K_1, and it is at once evident that the lines which they represent are at right angles to each other, since they are of the forms $\alpha^{\frac{1}{2}}\sqrt{A}-\beta^{\frac{1}{2}}\sqrt{B}$ and $\alpha^{\frac{1}{2}}\sqrt{A}+\beta^{\frac{1}{2}}\sqrt{B}$. It may also be seen from the form under which (5) appears, that the radical axis of (1) and (2) is parallel to the bisector of the exterior angle at A; that is, to the line $O_1 O_2$ (Fig. 3). Because the bisector of the exterior angle at A is represented by the equation $\beta+\gamma=0$, and a line parallel to the latter has for its equation $\beta+\gamma\pm c=0$ (c being a constant). Also (6) is parallel to the interior bisector of the angle A, and therefore perpendicular to (5). Attention must be paid to the signs of β, γ, &c., after squaring. The same property may in like manner be proved for each of the other radical axes.* Hence we conclude, that

One radical centre is the point of intersection of the perpendiculars let fall from the angular points of the radical triangle upon its opposite sides.

Many other properties of this interesting proposition may be established in like manner; thus

(36) The equations of the lines joining the middle points of the sides of the triangle are respectively

$\beta\sin B+\gamma\sin C-\alpha\sin A=0$, which is parallel to α or BC,
$\alpha\sin A-\beta\sin B+\gamma\sin C=0$, „ „ „ β or AC,
$\alpha\sin A+\beta\sin B-\gamma\sin C=0$, „ „ „ γ or AB,

(see Salmon's 'Conics,' 3d edition,) and therefore the equation to the circle through the middle points is,

$$(\alpha\sin A-\beta\sin B+\gamma\sin C)(\alpha\sin A+\beta\sin B-\gamma\sin C)\sin A+$$
$$(\beta\sin B+\gamma\sin C-\alpha\sin A)(\alpha\sin A+\beta\sin B-\gamma\sin C)\sin B+$$
$$(\beta\sin B+\gamma\sin C-\alpha\sin A)(\alpha\sin A-\beta\sin B+\gamma\sin C)\sin C=0,$$

which admits of great simplification; but I have preferred writing the equation in this form, from its analogy to the equation

$$\beta\gamma\sin A+\alpha\gamma\sin B+\alpha\beta\sin C=0.$$

By making $\alpha=0$ in the former equation, we readily see that the foot of the perpendicular, let fall from A, on the base of the triangle, lies on the same circle; for we obtain

$$\frac{\beta}{\gamma}=\frac{\cos^2\frac{C}{2}}{\cos^2\frac{B}{2}}=\frac{1+\cos C}{1+\cos B},$$

$$\text{or,}\quad \beta-\gamma+(\beta\cos B-\gamma\cos C)=0.$$

But $\beta\cos B-\gamma\cos C=0$ is the equation of the perpendicular let fall from A on the base of the triangle, and $\beta-\gamma=0$ is the equation of the bisector of the side BC; so that these two points lie on the same circle, as was proved in the first solution to the Prize Question for 1854.

* The remainder of the demonstration as before.

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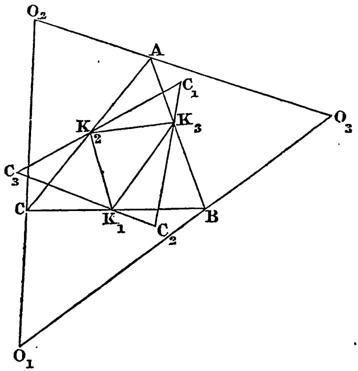
Among other interesting theorems which occur in this figure is the following.

(37) *The radical triangle is equal to one fourth of the escribed triangle.* The last theorem, as well as the escribed triangle of the 'Horæ Geom.,' is a particular case of the following.

(38) *If three right lines be drawn through the middle points of the sides of a triangle parallel to the sides of any escribed* triangle, these three right lines will form a triangle equal to one fourth of the escribed triangle.*



Let ABC be a triangle, and $O_1O_2O_3$ any other triangle escribed to ABC; that is, a triangle on the three sides of which are respectively posited the three vertices of the triangle ABC. Take K_1, K_2, K_3, the middle points of the sides BC, AC, and AB respectively, and through K_1 draw C_3C_2 parallel to O_2O_3; also through K_2 draw C_1C_3 parallel to O_1O_3; lastly, through K_3 draw C_1C_2 parallel to O_1O_2; C_3, C_2, C_1 being the points in which these right lines respectively intersect each other. Then

$$C_1C_2 = \tfrac{1}{2}O_1O_2;\ C_2C_3 = \tfrac{1}{2}O_2O_3;\ C_1C_3 = \tfrac{1}{2}O_1O_3.$$

For join K_1K_2, K_1K_3, &c., and because C_2C_3 is parallel to O_2O_3, by a well-known property K_1K_2 is parallel, and equal to $\tfrac{1}{2}AB$; also C_1C_3 is parallel to O_1O_3; therefore the triangles $C_3K_1K_2$, and ABO_3 are similar; whence

$$C_3K_1 : K_1K_2 :: AO_3 : AB,$$
$$\text{or}\quad C_3K_1 : \tfrac{1}{2}AB :: AO_3 : AB,$$
$$\therefore\quad C_3K_1 = \tfrac{1}{2}AO_3.$$

In the same way it may be shown that the triangles $C_2K_1K_3$, and ACO_2 are similar; hence

$$C_2K_1 : K_1K_3 :: AO_2 : AC,$$
$$C_2K_1 : \tfrac{1}{2}AC :: AO_2 : AC,$$
$$\therefore\quad C_2K_1 = \tfrac{1}{2}AO_2.$$

Consequently, $C_3K_1 + C_2K_1 = \tfrac{1}{2}AO_3 + \tfrac{1}{2}AO_2$,

or, $C_3C_2 = \tfrac{1}{2}O_2O_3$.

Similarly it may be demonstrated that the sides C_1C_2 and C_1C_3 are

* By an *escribed* triangle is here meant any triangle on which are posited the three vertices of another triangle.

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respectively equal to $\frac{1}{2}O_1O_2$ and $\frac{1}{2}O_1O_3$; and since the triangles $C_1C_2C_3$ and $O_1O_2O_3$ are similar, and similar triangles are to each other in the duplicate ratio of their homologous sides, therefore "if three right lines," &c.

LEICESTER COTTAGE, NEAR PORTSEA,
May, 1856.

(*To be continued.*)

ABSTRACT OF A TRACT

On the Possible and Impossible Cases of Duplicate Quadratic Equations in the Diophantine Analysis.

By MATTHEW COLLINS, B.A. *Kilkenny College.*

The author of this tract divides it into three chapters.

Chapter I treats of the possible and impossible cases of the two simultaneous equations of $x^2 + Ay^2 = \square$ and $x^2 - Ay^2 = \square$. It is proved in the original paper from which the present abstract is taken that this is impossible when A is any integer < 20, except 5, 6, 7, 13, 14, or 15. And the demonstrations of the impossibility possess the advantage of being effected, in all the different cases, by one uniform method. This first chapter terminates with a general demonstration of the impossibility whenever A is a prime number, and such that neither $m^2 + 1$ nor $m^2 - 2$ is divisible by A, m being $< \frac{1}{2}A$.

In the cases that are possible, any number of solutions, in integers (x, y) prime to each other, are obtained with facility by means of the following

General Theorem.—The solution of $X^2 + abY^2 = \square = Z^2$ and $X^2 - abY^2 = \square = W^2$ can be obtained from a solution of the two auxiliary equations $ax^2 + by^2 = nz^2$ and $abx^2 - y^2 = \pm nw^2$; for in fact $X = \frac{1}{2}n(z^4 + w^4)$ and $Y = 2xyzw$ will answer.

The difference of the squares of the two auxiliary equations gives $4abx^2y^2 = n^2(z^4 - w^4)$, and

$$\therefore abY^2 = 4abx^2y^2z^2w^2 = n^2z^2w^2(z^4 - w^4);$$

and as $4X^2 = n^2(z^4 + w^4)^2 = n^2(z^4 - w^4)^2 + n^2(2z^2w^2)^2 = n^2(t^2 + v^2)$,

$$\text{where } t = z^4 - w^4 \text{ and } v = 2z^2w^2,$$

$$\text{and } 4abY^2 = 4n^2z^2w^2(z^4 - w^4) = n^2(2tv),$$

$$\therefore 4(X^2 \pm abY^2) = n^2(t \pm v)^2, \text{ which are both squares.}$$

By taking $n = 1$ and also $b = 1$, we can, from one solution of the equations $x^2 + ay^2 = z^2$ and $x^2 - ay^2 = w^2$, derive another solution of the same equations in larger integers; thus new $X = \frac{1}{2}(z^4 + w^4)$ and new $Y = 2xyzw$.

Ex. gr. When $A = 5$, then the auxiliary equations $x^2 + 5y^2 = nz^2$ and $x^2 - 5y^2 = -nw^2$ are obviously fulfilled by taking $n = 1 = y = w$,

$x = 2$ and $z = 3$; hence, by the general theorem, we find

$$X = \tfrac{1}{2}(z^4 + w^4) = \tfrac{1}{2}(3^4 + 1^4) = 41 \text{ and } Y = 2xyzw = 12$$

to fulfil the proposed equations

$$x^2 + 5y^2 = \square = z^2 \text{ and } x^2 - 5y^2 = \square = w^2,$$

giving $z = 49$ and $w = 31$; and from this set of answers we can, according to the above observation, deduce another set in larger integers; in fact, it is evident that

$$\text{new } x = \tfrac{1}{2}(49^4 + 31^4) = 3344161,$$
$$\text{new } y = 2 \times 41 \times 12 \times 49 \times 31 = 1494696,$$

from which we could again find new and very high values of x and y, and thus ascend to very great whole numbers.

When $A = 6$, then $x = 5$ and $y = 2$ give $z = 7$ and $w = 1$;

$$\therefore \text{new } x = \tfrac{1}{2}(7^4 + 1^4) = 1201,$$
$$\text{new } y = 10 \times 2 \times 7 = 140.$$

When $A = 7$, then taking $n = 2$, one obvious solution of the auxiliary equations $x^2 + 7y^2 = 2z^2$ and $x^2 - 7y^2 = 2w^2$ is $x = 5$, $y = 1$, $z = 4$, and $w = 3$; and hence, by the theorem, we find $X = \tfrac{1}{2}n(z^4 + w^4) = 4^4 + 3^4 = 337$ and $Y = 2xyzw = 120$ to fulfil the two proposed equations $x^2 + 7y^2 = \square = z^2$ and $x^2 - 7y^2 = \square = w^2$, giving $z = 463$ and $w = 113$; and thence we find again, according to the above observation,

$$\text{new } x = \tfrac{1}{2}(463^4 + 113^4),$$
$$\text{new } y = 337 \times 240 \times 463 \times 113,$$

from which we could again find values of x and y in integers still larger, &c.

When $A = 13$, then taking $n = 1$, one obvious solution of the auxiliary equations $x^2 + 13y^2 = z^2$ and $x^2 - 13y^2 = -w^2$ is $x = 6$, $y = 5$, giving $z = 19$ and $w = 17$; and hence we find $X = \tfrac{1}{2}(19^4 + 17^4) = 106921$ and $Y = 10 \times 6 \times 19 \times 17 = 19380$ to fulfil the two proposed equations, $x^2 + 13y^2 = \square = z^2$ and $x^2 - 13y^2 = \square = w^2$. These values of x and y give $z = 127729$ and $w = 80929$, from which again we find, according to the foregoing observation,

$$\text{new } x = \tfrac{1}{2}(127729^4 + 80929^4)$$
$$\text{new } y = 2 \times 106921 \times 19380 \times 127729 \times 80929, \text{ \&c.}$$

Finally, it is observed that the solution of $X^2 + abY^2 = \square = Z^2$ and $X^2 - abY^2 = \square = W^2$ can be also derived from a solution of the auxiliary equations $x^2 + y^2 = az^2$ and $x^2 - y^2 = bw^2$; since in fact $X = x^4 + y^4$ and $Y = 2xyzw$ will answer; for then

$$abY^2 = 4abx^2y^2z^2w^2 = 4x^2y^2(az^2)(bw^2) = 4x^2y^2(x^4 - y^4) = 2tv$$
$$\text{where } t = x^4 - y^4 \text{ and } v = 2x^2y^2,$$
$$\text{and } X^2 = (x^4 + y^4)^2 = t^2 + v^2;$$
$$\therefore X^2 \pm abY^2 = (t \pm v)^2, \text{ which are both squares.}$$

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Chapter II treats of the possible and impossible cases of the two simultaneous equations $x^2+y^2=\square$ and $x^2+Ay^2=\square$. In the original paper it is demonstrated by one uniform method, that this is impossible when A is any positive integer <20, except 7, 10, 11, or 17; and it is also proved that the proposed equations will be always possible or solvable whenever A is $=2a^2-8$, or $2a^2-1$, or $2a^2+2$, or $2a^2+9$, or $2a^2+50$, or $3a^2-48$, or $3a^2-3$, or $3a^2+4$, or $3a^2+49$, or $5a^2-4$, or $5a^2+5$, or $5a^2-80$, or $5a^2+81$, or $6a^2-2$, or $6a^2+3$, or $4a^2\pm 3a$, or $\frac{5a^2}{4}$, diminished either by $\frac{1}{4}$ or by $1\frac{1}{4}$, &c. &c. And thus the proposed equations will be possible or soluble whenever A is any of the following integers; viz. 7, 10, 11, 17, 20, 22, 24, 27, 30, 31, 34, 41, 42, 45, 49, 50, 52, 57, 58, 59, 60, 61, 68, 71, 72, 74, 76, 79, 82, 85, 86, 90, 92, 94, 97, 99, 100, 101, 104, 105, 112, 115, 119, 120, 121, 122, &c.

The solutions of the possible cases are inferred with facility from the following

General Theorem.—The values of X and Y in $X^2+Y^2=\square=Z^2$ and $X^2+abY^2=\square=W^2$ can be deduced or inferred from the values of x and y in the auxiliary equations $x^2+ay^2=nz^2$ and $y^2+bx^2=nw^2$; in fact, $X=x^2w^2-y^2z^2$ and $Y=2xyzw$ will answer;

$$\text{for then } X^2+Y^2=(x^2w^2+y^2z^2)^2;$$

and so the first condition is fulfilled.

$$\text{Now } nX=x^2(y^2+bx^2)-y^2(x^2+ay^2)=bx^4-ay^4;$$

$$n^2Y^2=4x^2y^2(x^2+ay^2)(y^2+bx^2)=4x^4y^4(1+ab)+4bx^6y^2+4ax^2y^6;$$

$$\therefore n^2(X^2+abY^2)=(bx^4+2abx^2y^2+ay^4)^2;$$

and hence

$$X^2+abY^2=(bx^4+2abx^2y^2+ay^4)^2\div n^2=\square;$$

and thus these values of X and Y satisfy the second condition also.

If a or b be negative, we obtain a solution of $X^2+Y^2=\square$ and $X^2-abY^2=\square$; but by taking $b=1$ and $n=1$, and interchanging z and w, this general theorem shows that, "from one solution of the proposed equations $x^2+y^2=z^2$ and $x^2+Ay^2=w^2$ we can obtain another solution of the same equations, in larger integers, by only taking new $X=Ay^4-x^4$ and new $Y=2xyzw$." We shall give here only a few instances of the use of this theorem.

When $A=7$, then the proposed equations $x^2+y^2=\square=z^2$ and $x^2+7y^2=\square=w^2$ are obviously fulfilled by $x=3$, $y=4$, $z=5$, and $w=11$; whence for a second solution we have only to take new $x=7\times 4^4-3^4=1711$ and new $y=2xyzw=1320$, giving new $z=2161$ and new $w=3889$; and thence again a third set of answers are

$$\text{new } x=7\times 1320^4-1711^4$$

$$\text{new } y=2\times 1711\times 1320\times 2161\times 3889.$$

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When $A=10$, one solution is obviously $x=3$ and $y=4$, from which new solutions can be obtained as above. When $A=11$, then taking $n=5$, a possible remainder of squares to modulus 11, the auxiliary equations $x^2+y^2=5z^2$ and $x^2+11y^2=5w^2$ are obviously fulfilled by $x=1$, $y=2$, $z=1$, and $w=3$; whence by the general theorem we have $X=x^2z^2-y^2w^2=35$ and $Y=2xyzw=12$, which are the least values of x and y to answer the proposed equations $x^2+y^2=\square=z^2$ and $x^2+11y^2=\square=w^2$, giving $z=37$ and $w=53$; and thence again another set of answers are

$$\text{new } x=11y^4-x^4=1272529;$$

$$\text{new } y=2xyzw=70\times12\times37\times53=1647240,$$

and thence again

$$\text{new } X=11y^4-x^4=11\times1647240^4-1272529^4, \text{ \&c.}$$

When $A=4$, the proposed equations $x^2+y^2=\square$ and $x^2+4y^2=\square$ are proved to be impossible; whence by taking $a=b=-2$ and $n=-1$, it follows from the foregoing general theorem that the auxiliary equations $2y^2-x^2=z^2$ and $2x^2-y^2=w^2$ must be also impossible, *i. e.* there cannot be four square numbers, w^2, x^2, y^2, z^2, in arithmetical progression.

Chapter III treats of the possible and impossible cases of the two simultaneous equations $x^2-y^2=\square$ and $x^2-Ay^2=\square$. In the paper, of which we here present a very short abstract, this is demonstrated to be impossible when A is any integer <13, except 7 or 11; the solutions of the possible cases in integers x, y prime to each other are obtained with facility and generality from the following

General Theorem.—The values of X and Y to fulfil $X^2-Y^2=\square=Z^2$ and $X^2-abY^2=\square=W^2$ can be got from the solution of the auxiliary equations $x^2-ay^2=nz^2$ and $bx^2-y^2=nw^2$; since in fact $X=x^2w^2+y^2z^2$ and $Y=2xyzw$ will answer the purpose, as is easily demonstrated.

By taking $b=1$, and interchanging z and w in this general theorem, we see that the solution of $X^2-Y^2=Z^2$ and $X^2-aY^2=W^2$ can be obtained from the solution of $x^2-y^2=nz^2$ and $x^2-ay^2=nw^2$ merely by taking $X=x^2z^2+y^2w^2$ and $Y=2xyzw$. And then again, by taking $n=1$, this general theorem shows how to find a solution in great integers from a known solution in smaller integers of $x^2-y^2=z^2$ and $x^2-ay^2=w^2$; for then new $X=x^2z^2+y^2w^2=x^4-ay^4$ and new $Y=2xyzw$ in all cases.

Ex. gr. Let $a=7$, so that the two equations to be solved are $x^2-y^2=\square=z^2$ and $x^2-7y^2=\square=w^2$; then taking $n=2$, a possible remainder of square numbers to divisor 7, we see that one obvious solution of the two auxiliary equations $x^2-y^2=2z^2$ and $x^2-7y^2=2w^2$ is $x=3$, $y=1$, $z=2$, and $w=1$; and $\therefore$, by the foregoing, $X=x^2z^2+y^2w^2=37$ and $y=2xyzw=12$, which are the least integers to answer the two proposed equations; they give $z=35$

and $w = 19$; and from this solution we find another, viz.

$$\text{new } X = x^4 - ay^4 = 37^4 - 7 \cdot 12^4 = 1729009,$$
$$\text{new } Y = 2xyzw = 37 \times 24 \times 35 \times 19 = 590520.$$

As another example, let $a = 11$, so that the two equations to be solved are $x^2 - y^2 = \square = z^2$ and $x^2 - 11y^2 = \square = w^2$; then taking $n = 5$, we see that one obvious solution of the two auxiliary equations $x^2 - y^2 = 5z^2$ and $x^2 - 11y^2 = 5w^2$ is $x = 7$, $y = 2$, $z = 3$, and $w = 1$; and, by the foregoing theorem,

$$X = x^2z^2 + y^2w^2 = 21^2 + 2^2 = 445 \text{ and } Y = 2xyzw = 84,$$

which are the least integral values of x and y to fulfil the proposed equations; they give $z = 437$ and $w = 347$; and now from this solution we find another, viz.

$$\text{new } X = x^4 - ay^4 = 445^4 - 11 \cdot 84^4,$$
$$\text{new } Y = 2xyzw = 2 \times 445 \times 84 \times 437 \times 347 = \text{\&c.};$$

and by using these values of X and Y for x and y, we can thence again find X and Y in very great integers, &c. By taking a negative, we could obviously deduce the solution of $X^2 - Y^2 = Z^2$ and $X^2 + abY^2 = W^2$ from a solution of the two auxiliary equations $x^2 + ay^2 = xz^2$ and $bx^2 - y^2 = nw^2$. Finally, we may observe that the two equations $x^2 - y^2 = \square$ and $x^2 - Ay^2 = \square$ will be simultaneously possible whenever A is $= 9 - 2a^2$, or $50 - 2a^2$, or $49 - 3a^2$, or $81 - 5a^2$, or $25 - 6a^2$, or $64 - 7a^2$, or $100 - 11a^2$, or any of the following integers, viz. 7, 11, 18, 19, 22, 32, 36, 37, 42, 46, 48, 56, 57, 61, &c.

General Theorem.—The solution of $X^2 + Y^2 = \square$ and $X^2 + (a+1)Y^2 = \square$ can be obtained from a solution of the two auxiliary equations $x^2 \pm y^2 = nz^2$ and $ay^2 \mp x^2 = nw^2$; in fact $X = x^2z^2 - y^2w^2$ and $Y = 2xyzw$ will answer, as is easily demonstrated.

General Theorem.—The solution of $X^2 - Y^2 = \square$ and $X^2 - (a+1)Y^2 = \square$ can also be obtained from a solution of the two auxiliary equations $x^2 - y^2 = nz^2$, $ax^2 + y^2 = nw^2$, or from a solution of the pair $y^2 + x^2 = nz^2$, $y^2 - ax^2 = nw^2$; for in fact $X = x^2w^2 + y^2z^2$ and $Y = 2xyzw$ will answer, as is also easily demonstrated.

The author states, that it is the demonstrations of the impossible cases that have led to the discovery of the foregoing general theorems for solving the possible cases; and although these demonstrations of the impossible cases are the most interesting and valuable part of the Tract, they are necessarily, on account of their length, omitted in the present brief abstract; but the Tract itself is now published by the author. See notice, page 70.

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