SCHOOL ARITHMETIC
GRAMMAR SCHOOL BOOK

CAJORI

GRAMMAR SCHOOL BOOK

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GRAMMAR SCHOOL BOOK

BY

FLORIAN CAJORI

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PREFACE

This Grammar School Arithmetic is noteworthy for the topics which it omits, as well as for the new topics which it introduces. A dominant feature is the emphasis upon vocational problems. These problems are of wide range, drawn from life on a farm, in the home, in the carpenter's shop, the manufacturing establishment, and the clerk's office in the city. It is hoped that these vocational problems will be of genuine value to the pupil after he leaves the school and enters into one or another of the various fields of activity.

A new feature in the treatment is the study of curves representing empirical data, such as variations in temperature, of prices of different commodities, of stock quotations, or of agricultural and other scientific data. No other topic in elementary mathematics has sprung into such prominence in modern life. Monthly or yearly fluctuations in stock quotations are shown to the business man graphically; changing sanitary conditions are made the plainer to the physician by curves; changes in the cost of living are exhibited by lines; government publications resort to this geometric device for conveying to the practical farmer the result of scientific experiment; graphic representation is also frequently used as an effective device in advertising. That this topic has been so long neglected in our public schools is the harder to realize when we consider that it is easily comprehended by children.

Simplifications in the technique and the language of arithmetic have been made. Several old but useless technical terms have been omitted. A *circle* is defined as a *line* instead of as an *area*. In higher mathematics, as well as in recent secondary texts, it is regarded as a line. Economy of effort demands that there be uniformity throughout.

The book contains a comprehensive exposition of arithmetic, including the elementary parts. Emphasis is placed on fundamentals. Frequent reviews serve to fix the facts and processes which the pupil has discovered.

Again the author has the pleasure of acknowledging generous help received from several experienced teachers of elementary mathematics, particularly from Miss Letitia R. Odell of the North Side High School in Denver, and from Mrs. L. D. Coffin, Mrs. I. J. Lewis, Miss Minnie L. McCall, and Miss Edna Kinder of the public schools in Colorado Springs.

FLORIAN CAJORI.

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GRAMMAR SCHOOL BOOK

PART ONE

ARITHMETIC

1. Arithmetic has been studied and taught from the earliest times. The science of numbers was taught in the ancient schools of philosophy and there it was studied by those who were interested not in the practical problems of business, but in the science of numbers. Merchants and surveyors, however, used arithmetic to calculate the cost of goods and the area of land. When printed books began to appear the science of numbers and the art of computation were printed together and so they continue together at the present time. **Arithmetic is defined as the science of numbers and the art of computation.**

At the present time arithmetic is considered one of the most important studies in the elementary school. It possesses a definite, well-organized body of knowledge that can be studied and mastered in the elementary school. This knowledge is necessary to an understanding of many sciences and to the solution of most of the practical problems of life.

In the study of arithmetic we need to give considerable attention to the fundamental processes — addition, subtraction, multiplication, and division — so that the operations, wherever they are found, may be performed with accuracy and reasonable speed. We shall want to understand the theory of arithmetic so that we may know the reason for what we do and so that we may be able to explain it to others.

NOTATION AND NUMERATION

The Arabic Notation

2. In the study of arithmetic we find that numbers may be expressed in three ways, — by words, by figures, and by letters. In expressing numbers by figures or letters we employ what is called a scale. By scale is meant the relation that exists between the unit of one order and the unit of the next higher order. A uniform scale is one in which the relation is always the same between one order of units and the next higher order. The Arabic notation has a uniform scale since it always takes ten units of one order to make one unit of the next higher order. United States money and the metric system also have uniform scales. In many tables of weights and measures we have what is called a varying scale.

The Arabic notation is so called because about 1200 A.D. the Europeans received it from the Arabs. But the Arabs, in turn, had received it from the Hindus, and were, therefore, not the inventors of it.

For the reading of large numbers the figures are marked off into periods of three. This is done by beginning at the figure in *ones*' place and proceeding toward the left. In decimal fractions not more than two or three figures are ordinarily used; hence the formation of periods is superfluous. The names of the periods of integral numbers and the three figures in each period may be shown as follows:

Number Chart

ORDERS OF INTEGERS												ORDERS OF DECIMALS		
BILLIONS			MILLIONS			THOUSANDS			UNITS OR ONES			THOUSANDTHS		
Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units or Ones	Tenths	Hundredths	Thousandths
6	7	8	2	1	3	5	0	1	7	3	5	.4	3	8
4th Period			3d Period			2d Period			1st Period			1st Period		

The number 678,213,501,735.438, printed in the number chart is read: six hundred seventy-eight billion, two hundred thirteen million, five hundred one thousand, seven hundred thirty-five *and* four hundred thirty-eight thousandths.

For reading numbers of this kind, the word *and* is used to indicate the place of the decimal point.

A shorter and better way of reading the decimal fraction is: Point, four, three, eight.

Notation is the process of writing numbers by means of characters.

Numeration is the process of reading numbers expressed by characters.

Oral Exercise

3. Read the following populations:

1. Florida, 752,619 Rhode Island, 542,610
2. Hawaii, 191,909 Alaska, 64,356
3. Pennsylvania, 7,665,111
4. Illinois, 5,638,591
5. New York (state), 9,113,614
6. United States, 93,402,151

Written Exercise

4. Write in figures:

1. Eight thousand, two hundred three.
2. Twenty thousand, eight hundred fifteen.
3. Seven hundred forty-three thousand.
4. Two hundred five thousand, six hundred ten, *point* five.
5. Nine hundred seventy thousand, five hundred eighty, *point* four, five.
6. Eight hundred sixty-four thousand, twenty, *point* nine, five.

7. One million, one hundred forty-five thousand, one hundred one.

8. Nine million, twenty-five thousand, four hundred thirty-six

9. Twenty-five million, two hundred forty-four thousand, sixty.

10. One hundred one million, one hundred thousand, four hundred.

11. Eighty-seven million, five hundred forty-six.

12. Five billion, three million, four thousand, nine.

Oral Exercise

5. Read the numbers and tell the order or kind of unit each figure stands for :

1.	34.345	125.06	65.075
2.	345.45	1,125.006	650.75
3.	3,454.5	11,125.6	6,507.5
4.	34,454.555	111,125.66	66,507.57
5.	334,454.55	1,111,125.666	666,507.577
6.	3,445,334.5	2,345,678.6	6,666,507.5

Written Exercise

6. Write in the decimal notation :

1.	$\frac{2}{10}$	$\frac{11}{100}$	$\frac{1}{100}$	$\frac{111}{1000}$	$\frac{11}{1000}$
2.	$\frac{4}{10}$	$\frac{22}{100}$	$\frac{7}{100}$	$\frac{123}{1000}$	$\frac{9}{1000}$
3.	$\frac{7}{10}$	$\frac{40}{100}$	$\frac{9}{100}$	$\frac{205}{1000}$	$\frac{78}{1000}$
4.	$\frac{9}{10}$	$\frac{28}{100}$	$\frac{8}{100}$	$\frac{250}{1000}$	$\frac{40}{1000}$

The Roman Notation

7. The Romans used letters to represent numbers. Their notation is still used in numbering chapters of books and upon watches and clocks and inscriptions on monuments.

The Roman notation uses seven letters, namely:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

1. When the letter is followed by the same letter or one of less value, the values of the letters are to be added; as VI for 6, XV for 15, XX for 20, CX for 110.

2. When a letter is followed by one of greater value, the value of the smaller is to be subtracted from that of the greater; as IV for 4, IX for 9, XC for 90.

3. When a letter is placed between two letters of greater value, the smaller is to be subtracted from the sum of the other two; as, XIV for 14, XIX for 19, MCM for 1900.

4. A bar placed over a letter multiplies its value by one thousand; as $\overline{\text{V}}$ for 5000, $\overline{\text{C}}$ for 100,000.

Oral Exercise

8. Read the following:

1. XVI, XXXII, XLI, XCII, LXXXVI.
2. CLX, CCLXX, DC, DCCL, MDCCC, ID.
3. $\overline{\text{D}}$, $\overline{\text{D}}$CCCL, MC, CM, LXIX, MCD.

Written Exercise

9. Change from Arabic to Roman symbols:

1. 3, 7, 11, 16, 19, 27, 41, 54.
2. 66, 73, 89, 91, 98, 109.
3. 215, 294, 415, 516, 708.
4. 1009, 1492, 1530, 1783, 1915.

Comparison of the Arabic and the Roman Notations

10. Can the operations of addition, subtraction, multiplication, and division be performed with numbers written in the Roman notation as easily as with those written in the Arabic notation? Try, without using Arabic figures, to multiply MDCCLXVI by $\overline{\mathrm{D}}$CCLXVII. Can you do it?

There are two principles that make computation in the Arabic notation easier. They are:

(*a*) The principle of **place value**:

Thus, in 765, the 7 stands for hundreds, because it is in hundreds' place; the 6 stands for tens, because it is in tens' place; and so on.

(*b*) The principle of **uniformity of the scale**:

According to this principle all units are *the same* multiple of the units immediately below.

Thus, 1 thousand = 10 hundreds; 1 hundred = 10 tens; 1 ten = 10 ones.

It is the uniformity of scale, rather than the selection of the particular scale of 10, which is of importance. In some respects a scale of 12 would be preferable to the scale of 10.

Written Exercise

11. 1. America was discovered by Columbus in MCDXCII. How many years was this before the Declaration of Independence in MDCCLXXVI? Use Arabic figures.

2. How many years have elapsed between MDCCLXXVI and MCMXII?

3. Read the following dates: MDCCCLVII–MCMVII. Write down the years and find the time between the dates.

4. Write in Arabic Notation DCXC, MCM, MCMXX.

5. Write 5 dates of local interest in Roman notation.

REVIEW OF FUNDAMENTAL OPERATIONS

Addition

12.

1.	2.	3.	4.	5.
9	98	987	9876	98765
9	88	777	6666	55555

6.	7.	8.
987654	9876543	98765432
444444	3333333	22222222

Drill by Decades

13.

1.	8	18	28	38	48	58
	9	9	9	9	9	9
2.	68	78	88	98	108	118
	9	9	9	9	9	9
3.	9	9	9	9	9	9
	8	18	28	48	58	38
4.	9	19	29	39	49	59
	8	8	8	8	8	8
5.	8	8	8	8	8	8
	69	79	89	99	109	119

6.	9	19	29	39	49	59
	7	7	7	7	7	7
7.	7	7	7	7	7	7
	58	68	78	88	98	108
8.	6	16	26	36	46	56
	7	7	7	7	7	7
9.	7	7	7	7	7	7
	66	76	86	96	106	116
10.	80	180	280	380	480	780
	9	90	90	90	90	90
11.	70	170	270	370	470	970
	90	90	90	90	90	90

Oral Exercise

14. Give the answers:

1.	13	14	15	16	17	18	19
	19	17	18	15	16	15	12
2.	23	24	25	26	27	28	29
	18	19	17	19	16	17	15
3.	33	34	35	36	37	38	39
	19	18	16	17	19	15	14

4.	43	44	45	46	47	48	49
	19	17	18	15	16	15	12
5.	53	54	55	56	57	58	59
	18	19	17	19	16	17	15

Oral Exercise

15. Add at sight:

1.	2.	3.	4.	5.	6.
5	5	5	9	9	9
4	4	4	5	5	5
9	9	9	4	4	4
7	7	7	3	3	3
6	6	6	6	6	6
8	8	8	8	8	8
3	3	3	7	7	7
17	87	117	28	98	108

Oral Exercise

16. Add rapidly:

1.	2.	3.	4.	5.	6.
9	99	8	88	7	97
7	7	6	6	5	5
9	9	8	8	9	9
6	6	9	9	7	7
8	8	7	7	8	8
5	5	5	5	6	6
4	4	4	4	4	4
3	3	3	3	3	3

Exercise for Speed and Accuracy

17. Develop speed in adding columns of figures. Seventy figures a minute should be a minimum rate.

In adding, try to make the process easier by catching the tens, that is, by seizing upon the groups of digits whose sums are ten. Groups like 8 and 2, 6 and 4, or 5, 2, and 3, should be taken as a whole.

Make sure of the accuracy of your results by careful checking. How are the results of addition checked?

Written Exercise

18. Add:

1.	2.	3.
7,869	45,690	23,457
851	10,976	90,909
9,708	24,864	77,851
4,512	80,645	73,258
10,456	10,909	47,842
87,890	4,595	53,260

4.	5.	6.
69.86 T.	160.9 A.	1,678.55
37.2 T.	737.8 A.	8,986.4
45.86 T.	495.2 A.	7,943.35
79.43 T.	746.6 A.	7,645.45
86.72 T.	800.4 A.	8,743.65
45.75 T.	945.6 A.	16,743.5
78.63 T.	123.4 A.	23,456.35
79.77 T.	742.7 A.	9,807.8

In adding, place figures representing the same units in the same column; that is, tens under tens, ones under ones, tenths under tenths, and so on. All decimal points should be in the same column.

7. 756.56 + 8,674.6 + 307.05 + 1,034.65 + 987.655.

8. 750.5 + 1256.8 + 764.88 + 278.65 + 761.75.

9. 123.4 + 876.45 + 678.25 + 755.25 + 9,864.33.

10. 1,009.9 + 800.35 + 789.75 + 999.99 + 777.77.

11.	12.	13.
30,300.00	3,490.67	8,423.85
30,450.00	4,935.84	6,729.31
31,950.00	8,298.56	4,634.74
47,920.00	4,070.63	9,843.79
58,883.16	865.78	7,697.28
87,934.27	5,473.29	4,279.54

14.	15.	16.
7,436	3,650.45	345.73
4,587	7,008.09	4,386.7
3,645	9,877.75	4,567.8
7,465	896.50	10,986.46
13,409	1,237.65	7,699.54
987	5,687.9	3,789.01
34,533	4,786.8	4,560.
50,498	1,098.75	19,864.67
96,792	2,379.95	79,043.75
88,097	4,789.8	89,438.25

17.	18.	19.
3,159	192,837	9,865.5
59,645	273,645	4,376
738,473	384,957	37,654.75
472,332	475,869	3,210.8
677,898	586,970	19,753.96
463,212	49,988	1,246.1
15,783	161,102	28,101.5
346,074	249,207	1,876.705
771,982	432,193	45,439
59,124	764,175	8,756.805
963,457	433,091	34,324.9
47,655	902,186	9,777.15
684,325	23,073	45,678
49,788	234,984	9,182.45
738,793	95,265	67,364
402,348	786,876	5,546.95
15,862	668,797	3,728.05

COMPOUND NUMBERS

19. Simple numbers express one or more units of the same name or denomination; as, 4, 6, and 9, or 10 feet and 15 feet, or 5 pounds and 9 pounds.

Compound numbers express one or more units of different names or denominations of the same measure; as 4 hours 25 minutes; 99 yards 2 feet $5\frac{1}{4}$ inches. Compound numbers are often described as "denominate" numbers.

A number is **denominate** when it expresses defined units of measure. **Units of measure** are defined or established by custom and law, and are described as **standard** or **derived**.

Standard units in the English system of measures are the pound, yard, day, etc. In Metric system the standard units are meter, liter, and kilogram.

Derived units are such as foot, inch, hour, minute, ounce, etc., in the English system; and centimeter, gram, etc., in the metric system.

Measurement is applied usually to the quantity of things; that is, to the length, weight, volume, temperature, curvature, money value, etc. These measures are classified as follows: (1) extension, (2) weight, (3) volume or capacity, (4) angles or arcs,

(5) time or duration, (6) temperature, (7) value or money.

Other measures which are rapidly coming into common use are those applied to measurement of energy or power, as electricity and light. Such measures as volts, amperes, and watt-hours are common. The meter registers in watt-hours, for which there is a charge of about 8 cents per watt-hour. Incandescent lamps are marked 16 or 32 candle power, or 40 or 60 watts, and so on. The power of an engine is estimated in horse-power as 3 h.p., or 60 h.p.

There are other systems of measure which may be regarded as defined, used in counting certain materials as units, dozen, gross, of anything, as brooms, eggs, pencils, pens, etc.; and in the paper trade one meets such terms as sheets, quire, ream, each of which has a definite denotation.

There are two systems of measurement in wide use. In the United States and Great Britain the English system is employed, while in the majority of the other nations the metric system is commonly employed. In scientific work everywhere the metric measures are necessary.

In the United States the standard units are defined in the terms of the metric units by Congressional act. The original metric units of measure, as the liter, meter, and kilogram, are preserved in the International Bureau of Weights and Measures near Paris. Duplicates of these are kept in the United States Bureau of Standards at Washington, D.C.

Tables of Denominate Numbers, both English and Metric, are given beginning on page 426.

Reduction of Denominate Numbers

20. Reduction of denominate numbers is either *descending* or *ascending*.

Reduction descending is to reduce a number to units of smaller name or denomination; as 2 hours may be reduced to 120 minutes, or this to seconds; or 1 meter to 10 decimeters, etc.

Reduction ascending is to reduce a number to the next higher unit; as, 15 feet to 5 yards, or 70 min. to 1 hr. 10 min., or 10 centimeters to 1 decimeter.

Reduction changes the form but not the value.

Written Exercise

21. Reduce

(1) 1 mi. to yards.

(2) 68 pt. to quarts.

(3) 7 hr. to seconds.

(4) 250 cwt. to pounds.

(5) 1 mi. to feet.

(6) 4 bbl. to gallons.

(7) 7,920 ft. to miles.

(8) 5,480 lb. to tons.

Written Problems

22. 1. A grocer buys potatoes at 75¢ a bushel and sells at 20¢ a half peck. How much does he gain on each bushel?

2. A water company charges a rate of $1.00 per hundred cubic feet of water. The meter reads at end of month 120 cu. ft. How many gallons were used, and what was the cost per gallon?

3. A train runs from New York City to Chicago in 22 hr. The distance is 1,062 miles. (*a*) What is the rate of speed per hour? per minute?

(*b*) The railroad fare is $ 20.00. What is the cost per mile?

4. A dealer buys typewriter paper at 80¢ a ream and sells it at the rate of 10 sheets for 5¢. What is his gain per ream?

5. A resident bought 16,540 lb. of coal at the rate of $ 4.50 a ton. How many tons were there?

Addition of Compound Numbers

23. Compound numbers are added in exactly the same way as are simple numbers. 1. Add, 1,775, 3,123, 8,510, 608, 1,492.

PROCESS

thousands	hundreds	tens	units
1	7	7	5
3	1	2	3
8	5	1	0
	6	0	8
1	4	9	2
13	23	19	18
15	5	0	8

15,508 *Ans.*

EXPLANATION. — Arrange the exercise for addition as indicated. Explain the process. Write the answer.

2. Add:

weeks	days	hours	minutes
4	3	11	40
1	17	2	35
6	8	23	17
2	13	18	50
13	41	54	142

Reduced to lower terms the answer is

19	1	8	22

Explanation. — When the total of any column contains more units than are needed to make a unit of the next higher denomination, this total is reduced to units of the higher denomination and these units added to that column. The remainder, if any, is placed under its own, the right-hand column.

In this example the sum of the first column is 142 min., or 2 hr. 22 min. 22 is placed under the minutes' column, and the quotient, 2, carried and added to the next higher denomination, or hour. The sum of this column is thus 56 hr., or 2 da. 8 hr. The same order is followed with this as in the former step. 2 da. is carried and added to the third column. The sum of this column is 43 da., or 6 wk. 1 da. The remainder is placed under the day column and the quotient, 6 wk., added to the next higher denomination, the sum of which is 19. The answer is 19 wk. 1 da. 8 hr. 22 min.

Written Exercise

24. Add:

	yd.	ft.	in.
1.	3	1	9
	4		11
	1	2	4
	7	1	5

	yr.	mo.	wk.	da.
2.	4	6	3	12
	1	10	1	5
	16	3	2	27
	5	8		9

	gal.	qt.	pt.		hr.	min.	sec.
3.	14	1	1	4.	19	48	32
	89	3			8	57	16
	7	2	1		2	21	55
	156	1	1		7	16	27

	°	′	″		gr.	doz.	units
5.	14	59	47	6.	29	11	6
	123	12	35		6	9	4
	26	45	54		18	10	11
	37		12		5		3
		30	26		9	4	10

Study Exercise

25. The unit of length in the metric system is the **meter.** There are about 39.37 in. in a meter. It is about 3.37 in. longer than the yard.

Two sets of prefixes are used in the metric system: one set is Latin, the other, Greek.

All measurements less than any metric unit are made by **decimal** parts of the unit, and expressed by the **Latin prefixes.**

All measurements greater than any unit are made by multiplying the unit by 10 or a multiple of 10, and are expressed by the **Greek prefixes.**

Latin Prefixes	Greek Prefixes
milli means .001	*deka* means 10
centi means .01	*hekto* means 100
deci means .1	*kilo* means 1000

These prefixes and the names *meter*, *liter*, *gram* should be memorized.

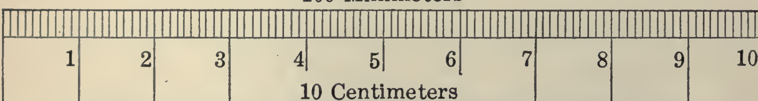
Learn the table.

METRIC MEASURES OF LENGTH

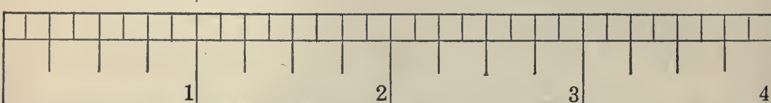
10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 meter (m.)
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 kilometer (Km.)

The abbreviations of the Greek prefixes are capital letters and the abbreviations of the Latin prefixes are small letters.

100 Millimeters



One Decimeter



Inches

Oral Exercise

26. 1. How many decimeters in a meter? How many millimeters?

2. About how many centimeters make an inch?

3. About how many inches in a decimeter?

4. One mile is nearly 1.6 kilometers. How many hektometers in a mile? How many dekameters? meters? decimeters? centimeters?

5. Tell from the illustration on page 22 the approximate number of centimeters in a foot.

6. Explain the following reductions:

$$\begin{aligned}.523475 \text{ Km.} &= 5.23475 \text{ Hm.} = 52.3475 \text{ Dm.}\\ &= 523.475 \text{ m.} = 5{,}234.75 \text{ dm.}\\ &= 52{,}347.5 \text{ cm.} = 523{,}475 \text{ mm.}\end{aligned}$$

7. Change 378,912 mm. to the successive higher units.

8. How much longer is a meter than a yard?

9. In the international Marathon races held some years ago the metric system was used. Which metric unit is nearest in length to a mile? About what part of a mile is a kilometer (see Ex. 4)?

Written Exercise

27. 1. In a Marathon race 40 Km. were run in 2 hr. 28 min. Over how many miles was this race?

2. What was the average time per kilometer?

3. What was the average time per mile?

4. Reduce 72,831 mm. to centimeters; to decimeters; to meters; to dekameters; to hectometers; to kilometers.

5. Reduce 3.45891 Km. to hectometers; to dekameters; to meters; to decimeters; to centimeters; to millimeters.

6. What is the distance in meters between two places, if they are 4,567 feet apart?

Oral Exercise

28. What does each figure stand for?

1. 3,475.126 m.?

EXPLANATION

the 3 stands for	3,000 m., or	? Km.
the 4 stands for	400 m., or	? Hm.
the 7 stands for	70 m., or	? Dm.
the 5 stands for	5 m.	
the 1 stands for	.1 m., or	? Dm.
the 2 stands for	.02 m., or	? Cm.
the 6 stands for	.006 m., or	? Mm.

2. 3,450.987 m. 3. 1,234.567 m. 4. 987.654 m.

Written Exercise

29. Copy and add:

1. 234.05 m. + 7,056.03 m. + 198.658 m.

2. .098 m. + 2.375 m. + 9.876 m. + 7.658 m.

3. .005 m. + .009 m. + 987.5 m. + 87.05 m.

4.	5.	6.
17.25 m.	1,795.00 m.	6,785.075 m.
9.045 m.	675.85 m.	5,908.846 m.

7. 17.45 Dm.	8. 1,965.4 Hm.	9. 134.67 Km.
7.34 Dm.	344.9 Hm.	45.97 Km.

10. 345.6 cm.	11. 4,567 mm.	12. 789.89 dm.
189.7 cm.	2,996 mm.	558.98 dm.

Written Problems

30. 1. The diameter of a circle is 2 in. How much longer is the perimeter of a square drawn around it, than is the length of the circle?

2. The diameter of a bicycle wheel is 2.5 ft. How far will it travel in one revolution of the wheel? in 12 revolutions?

3. How many times must the wheels revolve to travel a distance of 645 ft.?

4. Which makes a greater number of revolutions, the front wheels of a carriage or the hind wheels? Why?

5. If the diameters of two wheels are in the ratio of 3 : 2, in what ratio are their circular lengths?

6. If the large wheels of a locomotive have a diameter just twice that of the small wheels, how many revolutions will the large wheels make in the time that the small ones revolve 125 times?

7. The hour hand of a clock is 5 in. long, the minute hand is 7 in. How much farther does the

outer end of the minute hand move in one revolution than does the hour hand in one revolution?

8. How far does the end of an 8-in. minute hand move in 30 minutes? in 15 minutes?

9. How far does the end of a 10-in. minute hand move in the time that the hour hand makes one complete revolution?

10. How far does the end of a 7-in. hand move while it sweeps through an angle of 90 degrees?

11. A circular athletic track has a diameter of 420 ft. How many times must you run around the track to make a mile?

SUBTRACTION

31. How are the results of subtraction checked?

EXPLANATION. — In subtracting 37 from 56, it is usually best to subtract first 30 and then 7.

Say, 26, 19.

In checking this answer, add to 19, first 30, then 7.

Say, 49, 56.

Subtract and check:

1.	55	78	95	80	91	85
	26	46	46	37	43	59
2.	102	93	97	103	51	57
	45	27	38	75	17	38
3.	125	126	130	115	145	133
	87	97	83	47	77	56
4.	346	749	800	236	783	700
	37	25	47	56	53	124

5. Subtract 7 in. from 10 ft.

6. Subtract 67 sq. in. from 1 sq. ft.

7. From 37 cu. yd. take 27 cu. ft.

8. From 3 hr. subtract 49 min.

9. Take 50 sq. in. from 1 sq. ft.

10. Take 70 sq. in. from 2 sq. ft.

11. From 40 cu. yd. take 70 cu. ft.

12. From 6 min. subtract 80 sec.

13. From 5 lb. take 13 oz.

14. From 10 lb. take 90 oz.

15. Take 31 hr. from 2 days.

Written Exercise

In subtracting mixed numbers, as in whole numbers, it is advantageous to arrange the numbers so that units of the same order fall under each other. To do this it is necessary to write the decimal points under each other in a column.

32. Subtract and check:

1.	\$312.70 169.38	2.	\$14,965.34 6,709.65
3.	\$7,096.41 6,198.98	4.	\$7,465.57 1,986.39

Written Exercise

33. Subtract and check:

	Minuend	Subtrahend	Remainder
1.	798.60	89.8	?
2.	980.09	799.81	?
3.	1,001.01	796.52	?
4.	787.45	785.97	?
5.	7,906.9	6,897	?

	Minuend	Subtrahend	Remainder
6.	796.01	?	516.93
7.	860.96	?	678.4
8.	7,986.3	?	3,907.44
9.	9,768	?	7,890.65
10.	10,123.4	?	9,234.04

Subtraction of Compound Numbers

34. In compound numbers the numbers to be added or subtracted must be of the same unit of denomination.

As in addition, arrange units of the same denomination under each other, the less under the greater quantity. Taking the right-hand column, subtract the units in the subtrahend from those in the minuend, and place the remainders under the proper unit as in simple subtraction. If the units of a minuend are fewer than those in the subtrahend, reduce a unit from the next higher denomination and add to the minuend. Proceed in subtraction as before.

1. From 25 da. 9 hr. 35 min., take 4 da. 11 hr. 45 min.

Process

da.	hr.	min.
25	9	35
4	11	45
20	21	50

Explanation. — 45 min. cannot be taken from 35 min. Take 1 hr. from the 9 hr. and reduce it to 60 min. Add to 35 min., and from the sum, 95 min., subtract 45 min. = 50 min. 11 hr. cannot be subtracted from the

8 hr. remainder. Take 1 da. from the 25 da., reduce to hours and add to 8 hr., which equals 32 hr. 11 hr. from 32 hr. = 21 hr. From the minuend, 24 da., subtract 4 da., which gives 20 da. The answer is 20 da. 21 hr. 50 min.

If the Austrian method of subtraction is followed, the oral statements must conform.

In subtracting one date from another treat the number of the year as so many years, the number of the month as so many months, and day of the month as so many days. Count 30 days to the month.

Subtraction — Denominate Numbers

1.	376 bu.	7 pk.	2.	365 da.	18 hr.	46 min.	
	209 bu.	3 pk.		175 da.	20 hr.	55 min.	
3.	13 wk.	5 da.	4.	17 cu. ft.	240 cu. in.		
	5 wk.	6 da.		10 cu. ft.	560 cu. in.		
5.	75 sq. ft.	100 sq. in.	6.	300°	41′	45″	
	35 sq. ft.	120 sq. in.		250°	40′	55″	
7.	376 A.	80 sq. rd.	8.	146 sq. yd.	3 sq. ft.		
	150 A.	90 sq. rd.		87 sq. yd.	7 sq. ft.		

Written Problems

35. 1. Find the difference between April 12, 1861, and March 10, 1915.

2. A man who was born Nov. 15, 1842, died Jan. 30, 1909. What was his age at death?

3. A gave B his note July 5, 1908, and paid it Dec. 20, 1911. How long did the note run?

4. If you have a brother 2 yr. 4 mo. 15 da. younger than yourself, what was the date of his birth?

5. The distance between two towns X and Y is 65.7 miles farther than the distance between A and B, which is exactly 134 mi. 76 yd. 2 ft. What is the distance between X and Y?

6. Find the difference of longitude between New York City and Denver, Colorado. Between Boston and San Diego, California. Consult geography for longitudes.

Find the difference in time between each of the two sets of places.

Subtraction — English Money

36. £ stands for a *pound*, English money.

A pound equals $4.8665.

s. stands for an English coin, called the *shilling.*

20 shillings are equal to 1 pound.

12 pence (*d.*) are one shilling.

1.	£120	14*s.*		2.	£769	13*s.*	
	£ 83	19*s.*			£536	16*s.*	
3.	£12	16*s.*	4*d.*	4.	£12		4*d.*
	£ 4	9*s.*	8*d.*		£ 7	5*s.*	3*d.*
5.	£5	5*s.*	5*d.*	6.	£10		
	£4	4*s.*	6*d.*		£ 6	4*s.*	4*d.*

Written Problems

37. 1. The sum of two numbers is 1,309 and the lesser is 598. What is the greater number.

2. A merchant sold goods for $ 2,975.75. If his profit was $ 798.45, what did the goods cost him?

3. A man borrowed $ 2,755 and has paid back $ 1,985.50. How much does he still owe?

4. What must be added to the sum of $ 787 and $ 695.45, to make $ 1,998.35?

5. A first-class passage from Liverpool, England, to New York City costs about £67 10*s.* 6*d.* How much money will be left out of £70 after paying for a passage?

6. Find the cost of 15 tons 880 lb. at £9 10*s.* per ton.

7. Find the cost of 25 yd. at 4*s.* 8*d.* per yard.

8. Find the value of 51 bu. 3 pk. at 3*s.* 9$\frac{1}{4}$*d.* per peck.

MULTIPLICATION

Oral Exercise

38. Since $13 = 10 + 3$, if you desire to multiply a number by 13, you may first multiply it by 10, then by 3, and add the two partial products.

Thus,

$$13 \times 23 = 10 \times 23 + 3 \times 23 = 230 + 69 = 299$$

Multiply in this way:

1. 12×19	2. 13×18	3. 14×17
4. 15×16	5. 14×21	6. 16×21
7. 15×34	8. 13×22	9. 14×22
10. 15×22	11. 13×33	12. 15×44
13. 21×22	14. 21×32	15. 21×41

Study Exercise

39. The multiplication of mixed numbers and decimals is performed in the same manner as the multiplication of whole numbers. It is, however, necessary to point off the answer to separate the integer from the decimal.

1. Multiply 1.55 by 8.

PROCESS

$$\begin{array}{r} 1.56 \\ 8 \\ \hline 12.48 \end{array}\ \textit{Ans.}$$

EXPLANATION. — The process may be more clearly understood if fractions are used.

$1.56 = 1\frac{56}{100} = \frac{156}{100}$.

$\frac{156}{100} \times 8 = \frac{1248}{100} = 12\frac{48}{100} = 12.48$ *Ans.*

In this example 1.56 is a mixed number. It is to be multiplied by a whole number. The multiplication is performed the same as in whole numbers. The point is so placed in the answer as to separate the .48 from the integer, 12. It will be seen that there are as many decimal places in the answer as there are decimal places in the multiplicand and the multiplier together. The decimal point in the answer then falls directly under the decimal point in the multiplicand.

To point off the answer after multiplying a mixed number by a whole number, point off as many places in the product as there are decimal places in the mixed number.

2. Multiply 2.37 by 3.75.

PROCESS

$$\begin{array}{r} 2.37 \\ 3.75 \\ \hline 1185 \\ 1659 \\ 711 \\ \hline 8.8875 \end{array}$$

EXPLANATION. — Use fractions instead of decimals. Reduce, therefore, the mixed decimals to mixed numbers.

$$2.37 \times 3.75 = 2\tfrac{37}{100} \times 3\tfrac{75}{100}$$

$$= \frac{237}{100} \times \frac{375}{100} = \frac{237 \times 375}{100 \times 100}$$

$$= \frac{88875}{10000} = 8.8875.$$

In processes where there are decimal places in both the multiplier and the multiplicand point off as many places in the answer as there are places in both the multiplier and the multiplicand.

The general rule for multiplying and pointing off decimals is: Multiply the numbers as in whole numbers and then point off as many decimal places in the product as there are decimal places in the multiplier and the multiplicand together.

Written Exercise

How many decimal places will there be in the answer?

40. Multiply, using pencil only when necessary:

1. 12×12	2. 12×1.2	3. $12 \times .12$
4. $.12 \times 12$	5. 1.1×1.2	6. $13 \times .13$
7. 1.3×1.3	8. $.13 \times .13$	9. 14×14
10. 14×1.4	11. 1.4×1.4	12. $1.4 \times .14$
13. 15×1.5	14. $15 \times .15$	15. 1.5×1.5
16. $.15 \times .15$	17. 15×16	18. $1.5 \times .16$
19. 1.5×1.6	20. $.16 \times .16$	21. 12×2.3
22. 1.2×2.3	23. $12 \times .23$	24. $.12 \times .23$
25. 15×24	26. 1.5×2.4	27. $.15 \times 24$
28. $.15 \times 2.4$	29. 14×22	30. $.14 \times .22$
31. 1.4×2.2	32. $.14 \times 2.2$	33. 13×33
34. $.13 \times 3.3$	35. 1.3×3.3	36. $.13 \times 33$

Study Exercise

41. The process of multiplication of decimals by 10, 100, etc., and by .1, .01, .001, etc., may be shortened by properly locating the decimal point.

1. Multiply 28.6 by 10.

PROCESS	EXPLANATION
28.6	$28.6 \times 10 = 286$
10	This same answer may be obtained by
286.0	moving the decimal point one place to the right.

To multiply a decimal by 10, 100, 1000, etc., it is only necessary to move the decimal point as many places to the right as there are ciphers in multiplier.

2. Multiply 462.5 by .01.

PROCESS	
	EXPLANATION. — In this exercise it is
462.5	necessary to find $\frac{1}{100}$ of the multiplicand.
.01	The answer may be obtained by moving
4.625	point two places to the left.

In multiplying a number by .01 or .001, etc., move the decimal place to the left as many places as there are in the multiplier.

Explain how this method of moving the decimal point to the right or the left comes under the general rule for the multiplication of decimals.

Oral Exercise

How many decimal places will there be in the answer?

42. Multiply:

1.	1.1	.12	.65	1.78	10.05
					× 10
2.	15	15.5	52.3	6.28	10.05
					× .1
3.	1.1	7.9	.93	5.8	.098
					× 100
4.	7.5	75	7.86	100.9	63.81
					× .01
5.	1.789		3.46	4.5	.0035
					× 1000
6.	3456		75	8.6	879.1
					× .001
7.	1.4	.15	16	.017	18
					× 11

$11 \times 16 = 176$. Notice that $7 = 1 + 6$. If a number of two digits is multiplied by 11, the product is easily found by inserting the sum of the two digits between them. If this sum exceeds 9, we must "carry." Thus, $11 \times 29 = 319$.

Written Exercise

43. Multiply:

1.	1.4	23	2.4	.25	2.6
					× 1.1
2.	32	3.2	43	.45	4.6
					× .11
3.	51	5.2	53	62	6.2
					× .011
4.	6.1	.52	.55	4.8	.044
					× 110
5.	80	90	60	.7	.06
					× 12
6.	40	70	50	.8	.9
					× 1.2
7.	12	1.3	.15	40	70
					× .12
8.	.14	1.8	24	27	6.4
					× .2
9.	17	1.9	23	3.2	25
					× .03
10.	.24	.76	1.28	6.44	.78
					× .5 or $\frac{1}{2}$

11.	.4	8	12	18	36	$\times .05$
12.	484	600	720	844	960	$\times 25$
13.	12	36	96		120	$\times 2.5$ or $\frac{10}{4}$
14.	16	32	40	56	36	$\times .75$ or $\frac{3}{4}$

Written Exercise

44. Multiply:

1. 176.8 by 3.18
2. 13.08 by 4.07
3. 617.6 by 951
4. 76.89 by 450
5. .7609 by 3,400
6. 12.93 by 44.5
7. 9.031 by 30.7
8. 17.65 by 300.4
9. 3,765 by 40.5
10. 17,230 by 1.23
11. 1,928.3 by .124
12. 28.374 by 10.19

Written Problems

45. 1. If 1 gallon of water weighs 8.355 pounds, what is the weight of 13.44 gallons?

2. One cubic foot of water at boiling temperature weighs 59.76 lb. At freezing temperature it weighs 62.42 lb. What is the difference in weight between 200 cu. ft. of water at freezing and at boiling?

3. Iron is about 7.15 times heavier than water. If 1 cu. ft. of water weighs 62.42 lb., what is the weight of 1 cu. ft. of iron?

4. What is the cost of 7.25 meters of silk at $ 4.75 a meter?

5. If 1 cubic centimeter of water weighs 1 gram, and aluminum is 2.58 times heavier than water, what is the weight of 1 cubic centimeter of aluminum?

6. If lead is 11.3 times heavier than water, and copper is 8.9 times heavier than water, what is the weight of a piece of lead the same size as a piece of copper weighing 89 grams?

DIVISION

Study Exercise

46. In division of decimals the process is the same as in division of whole numbers. Care must be taken to point off the answer correctly.

1. Divide 19.787 by 1.23.

PROCESS

$$19.787 \div 1.23 = 1978.7 \div 123.$$

$$\begin{array}{r} 16.08^{+} \\ 123\overline{)1978.7} \\ \underline{123} \\ 748 \\ \underline{738} \\ 10\ 70 \\ \underline{9\ 84} \\ .86 \end{array}$$

EXPLANATION. — This process depends upon the principle — multiplying the dividend and divisor by the same number does not alter the value of the quotient. By what number have both the dividend and the divisor been multiplied? Why?

> To divide a number by a divisor that is a decimal or a mixed decimal, multiply both the dividend and the divisor by 10 or a power of 10 that will make the divisor an integer. Then divide as in whole numbers and point off as many decimal places in the quotient as there are decimal places in the new dividend.

2. Divide 863,100 by 7,000.

PROCESS

$$863100 \div 7000 = 863.1 \div 7$$

$$7\overline{)863.1}\quad 123.3$$

EXPLANATION. — This process depends upon the principle — dividing the dividend and divisor by the same number does not alter the value of the quotient.

3. Divide 484 by 2500

PROCESS

$$484 \div 2500 = 4.84 \div 25$$

Divide. Explain the process.

4. Divide 4,876.3 by 2.47 and carry the answer out to four decimal places.

PROCESS

$$\begin{array}{r} 1970.1619^{+} \\ 2_{\times}47\overline{)487630.0000} \\ 247 \\ \hline 2406 \\ 2223 \\ \hline 1733 \\ 1729 \\ \hline 400 \\ 247 \\ \hline 1530 \\ 1482 \\ \hline 480 \\ 247 \\ \hline 2330 \\ 2223 \\ \hline \end{array}$$

EXPLANATION. — Multiply both dividend and divisor by 100 by moving the decimal two places to the right and then removing all decimals from the divisor. Cross out the old decimal points. Insert the new point. Annex four ciphers to the dividend. Indicate the first partial dividend and the point above the right-hand figure in the partial dividend where the first figure in the quotient is to be placed. Divide as in whole numbers. Insert the point in the answer to separate the decimal from the integral part of the answer.

Written Exercise

47. 1. Divide 7,602 by 60.

2. Divide 9,120 by 120.

3. Divide 6,800 by 400.

4. Divide 19,362 by 90,000.

5. Divide 11,580 by 600.

6. Divide 41,248 by 8,000.

Divide, carrying the answers out to thousandths.

7. $18\overline{)112.68}$
8. $27\overline{)6.561}$
9. $48\overline{)33.6}$
10. $24\overline{)40.32}$
11. $42\overline{)19.152}$
12. $21.875\overline{)1{,}575}$
13. $31\overline{)76.88}$
14. $225\overline{)73.35}$
15. $22.5\overline{)18}$
16. $39\overline{)171.99}$
17. $.225\overline{)14.65}$
18. $19.8\overline{)1{,}574.1}$
19. $175.46 \div 1.25$
20. $786.5 \div .375$
21. $88.654 \div 6.7$
22. $569.8 \div 17.8$
23. $9{,}006.75 \div 98.7$
24. $19{,}009.7 \div 7.654$

Written Problems

48. 1. Given the dividend 20.34, the divisor .48, find the quotient to 2 decimal places.

2. Given the divisor 1.56, dividend 176.5, find the quotient to 3 decimal places.

3. A rectangular garden, 289,990 sq. ft. in area, has a length of 1,234 ft. How wide is it?

4. The circumference of a circle is 3.1416 times its diameter. When the circumference is 100 ft., find the diameter to 3 decimal places.

5. If one pound sterling, English money, is equal to $4.87, how many pounds will a traveler receive in exchange for $1,461?

6. How many marks will an American traveling in Germany receive for $64.26, if 1 mark is valued at 23.8 cents?

7. How many francs will an American receive in France for $400, if 1 franc is worth 19.3¢?

8. If it requires 26 days to send mail from New York to Calcutta, a distance of 11,120 miles, how far does it travel a day?

9. Is the rate of travel in Ex. 8 greater or less than that between New York and San Francisco, a distance of 3,250 miles, covered in 105 hours? What is the difference in miles per day?

10. Which is more densely populated, Alabama with an area of 52,250 sq. mi. and a population of 1,900,000, or California with an area of 158,360 sq. mi. and a population of 1,600,000?

11. The village water system cost $70,000. How many $1000 bonds had to be issued?

Information on Railroads

49. Consider the table.

Railroad Employees in the United States	Number	Number per 100 Miles	Average Daily Wages
General officers	6,407	3	$11.93
Other officers	7,549	3	5.99
General office clerks	65,700	29	2.30
Station agents	35,649	16	2.05
Other station men	152,929	67	1.78
Enginemen	65,298	29	4.30
Firemen	69,384	31	2.54
Conductors	48,869	22	3.67
Other trainmen	134,257	59	2.54
Machinists	55,244	24	2.89
Carpenters	70,394	31	2.40
Other shopmen	221,656	97	2.06
Section foremen	41,391	18	1.90
Other trackmen	367,277	162	1.46
Switchtenders	53,414	23	1.87
Dispatchers	39,193	17	2.26
Employees — floating equipment	9,139	4	2.27
Other employees and laborers	228,324	100	1.92

Problems based on the Table

50. 1. Find the total number of employees.

2. If the population of the United States is 93,000,000, find the approximate ratio of people engaged in railroad work to the entire population.

3. Find the total number of employees per 100 mi. of line.

4. Find the average daily wages paid to the various classes of employees.

5. How much more was paid in daily wages to all the general officers than to all the station agents?

6. How much was paid to firemen in wages during the year (360 da.)?

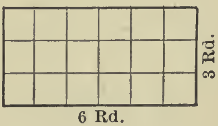
7. What was the annual salary of a general officer?

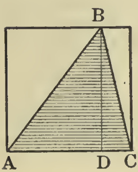
AREAS

Exercise for Study

51. The perimeters and areas of rectangles, triangles, and circles, the lengths of circles, as well as the volumes of rectangular solids must frequently be considered in the solutions of problems.

1. Explain from the figure of the rectangle that the product of the length and width of a rectangle equals its area.





2. Explain from the figure of the triangle that one half the product of the base and height of a triangle equals its area.

3. Explain the difference between the height or altitude of a triangle and its slant height AB or BC.

Oral Exercise

52. Give the areas of the following triangles, naming in each case the unit of area. Give answers quickly.

DIMENSIONS

	Base	Altitude		Base	Altitude		Base	Altitude
1.	2.5 ft.	10 ft.	**2.**	15 in.	8 in.	**3.**	1.7 yd.	2 yd.
4.	10 ft.	6 in.	**5.**	1 yd.	7 ft.	**6.**	1.2 ft.	12 ft.
7.	1.4 rd.	1.1 rd.	**8.**	.11 in.	.26 in.	**9.**	14 in.	12 in.
10.	.27 in.	1 ft.	**11.**	120 ft.	120 ft.	**12.**	.12 ft.	.12 ft.

Oral Exercise

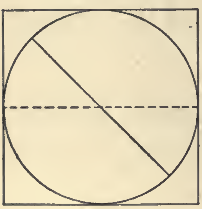
53. Find the altitude of each of the following triangles:

DIMENSIONS

	Area	Base		Area	Base		Area	Base
1.	144 sq. in.	24 in.	**2.**	1.21 sq. ft.	2.2 ft.	**3.**	.14 sq. ft.	.7 ft.
4.	.08 sq. ft.	.2 ft.	**5.**	60 sq. yd.	8 yd.	**6.**	400 sq. yd.	40 yd.
7.	.01 sq. ft.	.2 ft.	**8.**	1 A.	5 rd.	**9.**	10 A.	80 rd.

Exercise for Study

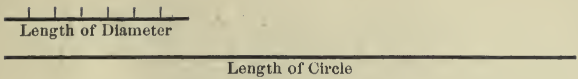
54. 1. How does the length of a circle compare with the perimeter of the square about the circle? If the side of the square is 1 in., its perimeter is



——, and the length of the circle must be less than ——.

2. How does the side of the square compare with the diameter of the circle? The length of a circle is less than how many times the diameter?

3. Which is larger, the area of the square or the area included by the circle? If the sides of the square are $\frac{1}{2}$ in., the area included by the circle must be less than what?



4. The shorter of these two lines is the diameter of a 25¢ piece. If you roll this coin along a straight path, once around, you obtain a line the length of the longer line. From these you can find out, approximately, how many times the length of the diameter is the length of the circle. Apply the shorter line to the longer. The shorter line, the diameter, is contained how many times in the longer line?

> **To find the length of a circle,** multiply the diameter by $3\frac{1}{7}$, or, if a more accurate answer is desired, multiply the diameter by 3.1416.

Written Exercise

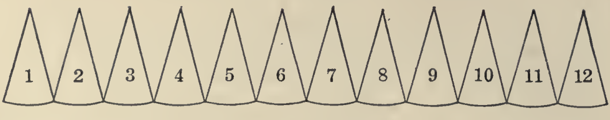
55. Find the lengths of the circles having diameters as follows, using 3.1416:

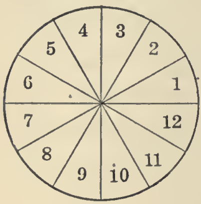
1. 5 ft.	2. 7 ft.	3. 3.5 ft.
4. 2.6 in	5. .4 ft.	6. 3.2 yd.

Area of a Circle

56. The area of a circle is the area of the surface within the circle.

If a circular disk of paper is divided into a large number of equal parts as shown in the circle, it





may be separated into figures that are nearly triangles. The sum of the bases of the triangles is equal to the length of the circle; their height is the radius of the circle.

The area of the circle is the product of its length and its radius, divided by 2.

Written Exercise

57. 1. Find the length and the area of a circle whose radius is 8 in.

PROCESS AND EXPLANATION

The diameter is 16 in.

The length of the circle $= 3.1416 \times 16$ in. $= 50.2656$ in.

The area of the circle $= \dfrac{50.2656 \times \overset{4}{\cancel{8}}}{\cancel{2}} = 201.0624$ sq. in.

Find the areas of circles having the following diameters:

2. 30 ft. 3. 40 ft. 4. 70 in.

5. 10 yd. 6. 200 ft. 7. 5 m.

Written Problems

58. 1. Draw a figure to represent a 16-in. square and circumscribe it with a circle. What is the area of each? What is the difference in their areas? Draw to scale of 1″ to 8″.

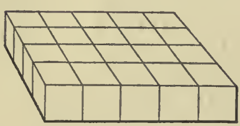
2. A chimney has as its inside dimensions 18 in. by 30 in. What is the diameter of a circular flue with the same area?

3. A gas main is 14 in. in diameter. What is the area of the cross section of the same?

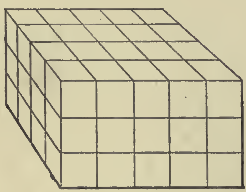
4. A circular plot of ground is 64 ft. in diameter. What is the plot worth at the rate of $5.00 per sq. ft.?

Volume of a Rectangular Solid

59. A rectangular solid has the general shape of a box.



A rectangular solid.
Height one unit



A rectangular solid.
Height three units

In the first figure, the height is 1 in. What is the length and breadth? How many cubic inches in the first solid?

In the second figure, state the length, breadth, and height, in inches. How many cubic inches in the second solid? What is the easiest way of determining the number of cubic inches?

> **The volume of a rectangular solid** is found by multiplying together its length, breadth, and height.

Written Exercise

60. Find the volumes of the rectangular solids having the dimensions given here. In each case name the unit of volume:

	Length	Breadth	Height		Length	Breadth	Height
1.	10 in.	2 in.	3 in.	2.	20 in.	8 in.	7 in.
3.	5.8 ft.	7 ft.	13 ft.	4.	4 cm.	3 cm.	1.5 cm.
5.	7 m.	5.6 m.	1.5 m.	6.	3 yd.	2.6 yd.	1.2 yd.

Written Problems

61. 1. Find the volume of a box 4.5 ft. by 3.5 ft. by 2 ft.

2. The volume of a box is 90 cu. ft. If it is 6 ft. long and 5 ft. wide, what is its height?

3. Find the volume of a box 1.05 in. long, .8 in. wide, and 2.5 in. high.

4. The volume of a box is .48 cu. in. It is .8 in. long and .6 in. wide. How high is it?

5. Measure the length, width, and height of the class room. How many cubic feet of air does it

contain? How many pupils are regularly in the room? How many cubic feet of air for each pupil?

6. How many cubic feet of dirt must be removed in excavating for a basement 34 ft. wide, 80 ft. long, and 14 ft. deep? How many cubic yards? Suppose that the dirt is removed at the cost of 2¢ per cubic foot; what will be the cost of excavation?

7. How many cubic feet of dirt will be removed in excavating for a sewer on a level road $1\frac{1}{2}$ mi. long, and the trench 18 in. wide and 8 ft. deep? At 40¢ per cubic yard, what will the whole cost be?

8. Take the measurements of the class room, as in Example 5; how many bushels of wheat would it contain?

PROPERTIES OF NUMBERS

Factors

62. A **factor** of a whole number is an integer that is an exact divisor of that number.

A **prime number** is an integer that has no factors except itself and 1.

1. Name the 8 prime numbers between 1 and 20.

2. Which of these prime numbers is even? Are all other prime numbers odd? Why?

3. Name the 4 prime numbers between 20 and 40.

4. How many prime numbers between 40 and 50?

5. Name the prime factors of 9, 10, 12, 16, 18, 24, 27.

Divisibility of Numbers

63. A whole number is exactly divisible

(*a*) by 2, if the digit in ones' place is so divisible, or is 0.

(*b*) by 3, if the sum of its digits is so divisible.

(*c*) by 4, if the number made up of the *two* right-hand digits is so divisible.

(*d*) by 5, if it ends in 0 or 5.

(*e*) by 6, if it is divisible by 3 and is even.

(*f*) by 8, if the number made up of the *three* right-hand digits is so divisible.

(*g*) by 9, if the sum of its digits is so divisible.

(*h*) by 10, if it ends in 0.

(*i*) by 25, if the number made up of the two right-hand digits is so divisible.

1. Apply these tests to each of the following numbers: 144, 950, 762, 1782, 1350, 4335, 4781, 8888.

Least Common Multiple

Study Exercise

64. A number which is exactly divisible by another number is a **multiple** of that number.

When a number is exactly divisible by each of two or more numbers, it is a **common multiple** of those numbers.

1. Which of the numbers 8, 12, 15, 72, 76 have 144 as a multiple?

2. Why is 72 a common multiple of 3, 6, 12, 24, 36?

3. Name several common multiples of 2 and 3.

4. Name several common multiples of 8 and 12.

The least number that is exactly divisible by each of two or more numbers is their **least common multiple** (l. c. m.).

5. Why is 12 the least common multiple of 2, 3, and 4?

6. Why is 36 not the *least* common multiple of 2, 3, 4, and 12?

7. Name the l. c. m. of 5, 15, 20; also of 3, 4, 6.

Oral Exercise

65. Give the l.c.m. of the following sets of numbers:

1. 2, 3, 4	2. 2, 5, 6	3. 2, 6, 10
4. 3, 4, 5	5. 3, 6, 8	6. 6, 8, 12
7. 4, 5, 6	8. 4, 6, 8	9. 3, 6, 15
10. 4, 6, 3	11. 4, 6, 10	12. 3, 9, 15

Study Exercise

66. In the addition or subtraction of dissimilar fractions it is necessary to change them so that they become similar. Thus $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}$. When the common denominator is the least that can be found, it is called the **least common denominator** (l. c. d.).

It is readily seen that the **common denominator of fractions** written in their lowest terms must be a multiple of the given denominators, and that the least common denominator is the least common multiple of the given denominators.

Oral Exercise

67. Reduce to fractions having the l. c. d. Add.

1. $\frac{1}{2}, \frac{1}{4}$	**2.** $\frac{3}{4}, \frac{1}{3}$	**3.** $\frac{7}{9}, \frac{1}{3}$	**4.** $\frac{1}{4}, \frac{4}{5}$
5. $\frac{1}{2}, \frac{5}{8}$	**6.** $\frac{1}{5}, \frac{5}{6}$	**7.** $\frac{3}{8}, \frac{5}{12}$	**8.** $\frac{5}{6}, \frac{1}{8}$
9. $\frac{1}{2}, \frac{2}{3}$	**10.** $\frac{4}{5}, \frac{2}{3}$	**11.** $\frac{9}{10}, \frac{2}{3}$	**12.** $\frac{3}{4}, \frac{5}{6}$
13. $\frac{1}{2}, \frac{6}{7}$	**14.** $\frac{7}{6}, \frac{1}{12}$	**15.** $\frac{6}{5}, \frac{1}{7}$	**16.** $\frac{5}{6}, \frac{7}{2}$
17. $\frac{1}{2}, \frac{3}{5}$	**18.** $\frac{1}{6}, \frac{1}{3}$	**19.** $\frac{7}{12}, \frac{2}{3}$	**20.** $\frac{1}{8}, \frac{2}{15}$
21. $\frac{2}{4}, \frac{3}{7}$	**22.** $\frac{1}{6}, \frac{1}{13}$	**23.** $\frac{1}{6}, \frac{4}{15}$	**24.** $\frac{4}{5}, \frac{13}{8}$
25. $\frac{1}{7}, \frac{2}{3}$	**26.** $\frac{7}{8}, \frac{1}{3}$	**27.** $\frac{11}{15}, \frac{2}{3}$	**28.** $\frac{1}{2}, \frac{5}{6}$
29. $\frac{5}{4}, \frac{11}{8}$	**30.** $\frac{5}{6}, \frac{5}{12}$	**31.** $\frac{11}{15}, \frac{1}{6}$	**32.** $\frac{1}{5}, \frac{8}{15}$

Subtract the smaller fraction from the larger.

Study Exercise

68. 1. Find the l. c. m. of 144, 180, and 216.

PROCESS

$$
\begin{aligned}
144 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \\
180 &= 2 \cdot 2 \qquad\quad \cdot 3 \cdot 3 \qquad \cdot 5 \\
216 &= 2 \cdot 2 \cdot 2 \qquad \cdot 3 \cdot 3 \cdot 3 \\
\hline
\text{l. c. m.} &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 = 2160
\end{aligned}
$$

EXPLANATION. — Find the prime factors of each number.

Then, a *common multiple* of 144, 180, 216 must contain the factors $2 \cdot 2 \cdot 2 \cdot 2$ (the greatest number of 2's found in any of the given numbers).

It must contain the factors $3 \cdot 3 \cdot 3$ (the greatest number of 3's found in any of the given numbers).

It must also contain the factor 5 (found in 180).

Hence a common multiple of 144, 180, 216 must contain altogether the factors $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5$, the product of which is 2160.

This must be the *least* common multiple, because we have used no factors which are not absolutely required in a common multiple.

Hence, to find the l. c. m. of two or more numbers:

Find prime factors of each number. Take each prime factor the greatest number of times it occurs in any number.

Written Exercise

69. Find the l. c. m. of the following groups of numbers:

1. 16, 32, 36, 48
2. 24, 72, 124, 144
3. 15, 24, 50, 60
4. 36, 54, 72, 124

FRACTIONS

Review

70. One or more of the equal parts of a unit is called a **fraction.** Does this definition apply to decimal fractions?

The **denominator** shows into how many equal parts a unit is divided; the **numerator** shows how many of the parts have been taken.

Into how many parts is the unit divided in each of the following fractions:

$$\frac{3}{4}, \frac{7}{8}, \frac{9}{10}, \frac{4}{15}, \frac{7}{20}, \frac{9}{25}, \frac{6}{100}?$$

.7, .9, .19, .29, .80, .75, .756?

If the numerator is less than the denominator, the fraction is called a **proper fraction,** otherwise it is called an **improper fraction.**

Classify the following fractions into proper and improper fractions:

$$\frac{4}{5}, \frac{7}{8}, \frac{11}{10}, \frac{21}{20}, \frac{49}{50}, \frac{12}{100}, \frac{120}{100}.$$

A **unit fraction** is one whose numerator is 1. What is the unit fraction in each of the following:

$$\frac{9}{10}, \frac{25}{100}, \frac{11}{12}, \frac{49}{50}, \frac{11}{10}?$$

Similar fractions are fractions having the same denominator.

Classify the following into groups of similar fractions:

$$\frac{3}{4},\ \frac{7}{8},\ \frac{9}{10},\ \frac{1}{4},\ \frac{5}{8},\ \frac{3}{10}.$$

A fraction may also be considered as an **indicated division**, or as the **ratio** of two numbers.

Write the following as fractions:

$$3:9,\ 9:5,\ 4:8,\ 8:16,$$
$$5:10,\ 10 \div 5,\ 5 \div 10,\ 4 \div 8.$$

Study Exercise

71. When the denominators of fractions are different, it is often necessary first to find a common denominator so that the fractions may be reduced to *similar* fractions before they can be added. This is evident from the fact that we cannot add numbers except by reducing them to a common class.

The relations, $\frac{1}{3} = \frac{2}{6} = \frac{3}{9}$ and $\frac{12}{15} = \frac{4}{5}$, illustrate an important principle of Fractions.

> **A PRINCIPLE OF FRACTIONS**
>
> The value of a fraction is not changed, when the numerator and denominator are multiplied, or divided by the same number.

Explain and illustrate how to reduce $\frac{2}{3}$ and $\frac{3}{4}$ to similar fractions.

How are similar fractions added?

FRACTIONS

Review

72. Perform the operations indicated:

1. $\frac{1}{3}+\frac{1}{4}+\frac{4}{5}+\frac{5}{6}+\frac{5}{12}+\frac{29}{60}+\frac{19}{20}=?$

2. $\frac{2}{3}+\frac{3}{2}+\frac{4}{3}+\frac{3}{4}+\frac{5}{4}+\frac{4}{5}+\frac{6}{5}+\frac{5}{6}=?$

3. $20\frac{2}{3}+\frac{7}{18}$

4. $200\frac{7}{16}+187\frac{1}{4}+395\frac{5}{8}$

5. $8\frac{7}{10}+12\frac{11}{12}$

6. $\frac{25}{24}+\frac{41}{36}+\frac{45}{48}$

7. $27\frac{9}{14}+85\frac{8}{21}$

8. $9\frac{17}{21}+8\frac{29}{35}+\frac{13}{42}$

9. $16\frac{19}{36}+42\frac{31}{40}$

10. $40\frac{1}{11}+41\frac{21}{22}+8\frac{28}{33}$

11. $789\frac{5}{6}+897\frac{8}{9}$

12. $97\frac{1}{3}+85\frac{4}{9}+19\frac{14}{15}$

13. $40\frac{7}{8}-27\frac{5}{6}$

14. $70\frac{1}{3}-18\frac{5}{7}+109\frac{1}{6}$

15. $37\frac{11}{12}-11\frac{5}{36}$

16. $100\frac{12}{17}+236\frac{19}{34}+109\frac{1}{2}$

17. $39\frac{5}{6}+\frac{1}{39}$

18. $55\frac{13}{14}+58\frac{2}{21}-10\frac{1}{2}$

19. $12\frac{5}{114}+10\frac{51}{76}$

20. $6\frac{10}{11}+1\frac{42}{55}-3\frac{7}{10}$

21. $3009\frac{45}{56}-399\frac{29}{42}$

22. $40\frac{7}{25}-15\frac{9}{50}+3\frac{89}{150}$

23. $104\frac{7}{12}-85\frac{7}{8}$

24. $144\frac{1}{3}+278\frac{2}{3}-164.55$

25. $903.5-36\frac{1}{7}$

26. $276\frac{3}{4}-167\frac{5}{6}+167\frac{7}{8}$

27. $574\frac{5}{8}-372.25$

28. $64\frac{5}{6}+128\frac{1}{4}-23.6$

29. $793\frac{11}{12}-436.6$

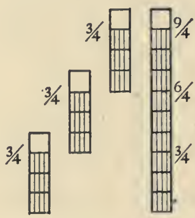
30. $76\frac{1}{4}+67\frac{3}{4}-35.2$

Multiplication of Fractions

73. There are two ways of multiplying a fraction by a whole number. One way is shown in the example, $3\times\frac{3}{4}$. We know that 3 times three fourths gives 9 fourths, or $\frac{9}{4}$. Here the unit fraction in the product is $\frac{1}{4}$, the same as in the given fraction. In the act of multiplication, the size of the unit fraction was not changed, but the number of the unit fractions was increased from 3 to 9.

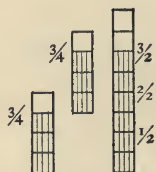
The second way of multiplying a fraction by an integer may be shown in the example, $2 \times \frac{3}{4}$. Instead of increasing the number of units by taking 2 times 3 fourths, increase the size of the units by dividing 4 by 2. This gives the answer $\frac{3}{2}$. Thus, the number of unit fractions is 3 in the answer as well as in the given fraction, but the unit fraction is now changed from $\frac{1}{4}$ to $\frac{1}{2}$. In the example $3 \times \frac{3}{4}$ this method is not applicable because 4 is not exactly divisible by 3.

Explain the same two rules of multiplication by illustration thus:



$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4}$

That is, $3 \times \frac{3}{4} = \frac{9}{4}$



$\frac{3}{4} + \frac{3}{4} = \frac{3}{2}$

That is, $2 \times \frac{3}{4} = \frac{3}{2}$

A fraction is multiplied by an integer, by either multiplying its numerator or dividing its denominator by the integer. In this process use cancelation when possible.

Since $\frac{2}{3} \times 7$ yields the same result as $7 \times \frac{2}{3}$, the multiplication of an integer by a fraction can be effected by the process just explained.

The process may be simplified when cancelation is possible. Thus, $\frac{4}{\cancel{5}} \times \overset{2}{\cancel{10}} = 8$.

Oral Exercise

74. Give answers rapidly:

1. $5 \times \frac{7}{10}$	2. $\frac{3}{5} \times 7$	3. $\frac{1}{10} \times 9$	4. $\frac{4}{3} \times 6$
5. $\frac{5}{6} \times 36$	6. $\frac{13}{14} \times 56$	7. $6 \times \frac{2}{3}$	8. $\frac{4}{5} \times 15$
9. $9 \times \frac{2}{3}$	10. $\frac{11}{12} \times 24$	11. $12 \times \frac{11}{8}$	12. $\frac{13}{15} \times 75$
13. $7 \times \frac{1}{2}$	14. $\frac{5}{6} \times 12$	15. $\frac{7}{8} \times 16$	16. $\frac{13}{12} \times 11$
17. $12 \times \frac{12}{5}$	18. $\frac{14}{16} \times 48$	19. $8 \times \frac{3}{4}$	20. $\frac{6}{7} \times 7$
21. $3 \times \frac{7}{8}$	22. $11 \times \frac{11}{12}$	23. $13 \times \frac{11}{12}$	24. $\frac{1}{12} \times 60$
25. $9 \times \frac{2}{3}$	26. $\frac{7}{8} \times 4$	27. $10 \times \frac{4}{3}$	28. $11 \times \frac{3}{11}$
29. $13 \times \frac{13}{14}$	30. $\frac{11}{18} \times 36$	31. $12 \times \frac{7}{8}$	32. $20 \times \frac{4}{5}$

Study Exercise

75. Mr. James works $\frac{1}{3}$ of a day, 24 hours. During $\frac{7}{8}$ of his working time his young son is helping him. During what part of the day does his son help?

EXPLANATION. — Here we must find $\frac{7}{8}$ of $\frac{1}{3}$ of a day. We know that $\frac{1}{3}$ of a day is 8 hr., and that $\frac{7}{8}$ of 8 hr. is 7 hr., or $\frac{7}{24}$ of a day. Without reducing to hours, we see that we can get the answer, $\frac{7}{24}$ of a day, by multiplying the numerators together and the denominators together.

In multiplying a fraction by a fraction, multiply the numerators together for a new numerator, and the denominators together for a new denominator. Cancel factors common to the numerator and denominator of the product.

Oral Exercise

76. Answer quickly:

1. $\frac{1}{4} \times \frac{2}{3}$	**2.** $\frac{4}{3} \times \frac{3}{5}$	**3.** $1\frac{1}{3} \times 1\frac{1}{3}$	**4.** $5 \times \frac{23}{15}$
5. $.01 \times \frac{1}{3}$	**6.** $\frac{5}{4} \times \frac{6}{5}$	**7.** $1\frac{1}{2} \times \frac{3}{4}$	**8.** $8 \times \frac{7}{16}$
9. $\frac{1}{5} \times \frac{1}{4}$	**10.** $\frac{4}{3} \times \frac{7}{8}$	**11.** $\frac{5}{6} \times .5$	**12.** $\frac{8}{3} \times 1\frac{1}{2}$
13. $\frac{6}{5} \times \frac{3}{4}$	**14.** $\frac{7}{9} \times \frac{12}{7}$	**15.** $\frac{7}{8} \times \frac{11}{14}$	**16.** $\frac{1}{3} \times .3$

Oral Exercise

77. Some prefer to work *all cases* of multiplication of common fractions by the rule for multiplying a fraction by a fraction. An example like $7 \times \frac{23}{21}$ is taken as $\frac{7}{1} \times \frac{23}{21}$, whereupon the numerators are multiplied together and the denominators are multiplied together. The integer 7 is given the fractional form $\frac{7}{1}$, by considering it as 7 *ones*.

1. $\frac{7}{5}$ of $\frac{2}{3}$ **2.** $\frac{4}{15}$ of $\frac{30}{4}$ **3.** $\frac{3}{12}$ of $\frac{4}{5}$ **4.** $\frac{9}{8}$ of $\frac{8}{9}$

5. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{4}{3}$, $\frac{8}{9}$, $\frac{7}{8}$, $1\frac{5}{6}$
$\times 1\frac{1}{2}$

6. $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{3}$, $2\frac{1}{3}$, $\frac{5}{6}$, $\frac{9}{16}$
$\times 1\frac{1}{3}$

7. $\frac{5}{4}$, $\frac{4}{5}$, $\frac{3}{10}$, $\frac{7}{15}$, $\frac{16}{3}$, $3\frac{1}{2}$
$\times 1\frac{1}{4}$

8. $1\frac{1}{2}$, $1\frac{1}{3}$, $2\frac{1}{2}$, $2\frac{1}{3}$, $1\frac{3}{4}$, $3\frac{1}{2}$
$\times 2\frac{1}{2}$

Oral Problems

78. 1. A man in the southern part of the country gets $1\frac{1}{2}$ T. of alfalfa an acre for each cutting and has 5 cuttings a season. How many tons per acre does he obtain in a season?

2. If he sells the alfalfa at $8 per ton, how much is the season's crop from an acre worth?

3. If $\frac{1}{3}$ of the gross earnings is expended for wages and for irrigation water, what is the net profit per acre in a season?

4. How much is the net profit on $5\frac{1}{2}$ acres?

5. Another ranchman claims to get 2 T. per acre at each of the five cuttings. How many tons does he obtain from $2\frac{1}{5}$ A.?

6. He expects to be able to sell the alfalfa at $12 a ton. What is his gross income on $2\frac{1}{2}$ acres?

7. What is the net profit, if one-third of the amount is deducted from $264 to cover expenses?

79. Written Exercise

1. Find $\frac{3}{4}$ of $\frac{8}{15}$ of $\frac{25}{33}$.

Process and Explanation. The pupil should form the habit of canceling all factors common to the numerator and denominator, *before* he carries out the multiplication.

Proceed thus,

$$\frac{\cancel{3} \times \overset{2}{\cancel{8}} \times \overset{5}{\cancel{25}}}{\cancel{4} \times \underset{\cancel{5}}{\cancel{15}} \times 33} = \frac{10}{33}$$

2. $\frac{12}{13} \times \frac{11}{16} \times \frac{143}{121}$

3. $\frac{7}{24} \times \frac{8}{21} \times \frac{5}{4} \times .3$

4. $\frac{5}{6}$ of $\frac{7}{10}$ of $\frac{12}{14}$ of $\frac{2}{3}$

5. $.3 \times 1.5 \times 7\frac{1}{3} \times 4$

6. $\dfrac{18 \times 22 \times 72 \times 42}{14 \times 36 \times 121 \times 16}$

7. $\dfrac{16 \times 27 \times 12 \times 49}{14 \times 21 \times 36 \times 9}$

8. $\frac{121}{16} \times \frac{48}{36} \times \frac{32}{11}$

9. $\frac{64}{66} \times \frac{22}{81} \times \frac{63}{16}$

Written Problems

80. 1. Find the area of a floor $14\frac{1}{2}$ ft. by $13\frac{1}{3}$ ft.

2. How many square feet of surface in a ceiling 50 ft. 6 in. by 25 ft. 6 in.?

3. How many square feet in 1 acre?

4. If on an irrigated farm $30\frac{1}{2}$ gallons of water to the square foot are available annually, how much water is this per acre?

5. If the ranchman has 80 A. that can be irrigated, how much water does he use annually?

6. If the water costs him $62\frac{1}{2}$ cents an acre, annually, how much does he pay for 80 acres?

7. How many square feet must a rug contain, to completely cover a hallway, 10 ft. 6 in. by 5 ft.?

8. An aëroplane flies for 2 hr. 10 min. at an average speed of 42 mi. an hour. How far does it travel?

9. Hay is worth \$25 a ton and oats 45¢ a bushel. How much will it cost to keep 2 horses for 1 month, if each horse is fed 150 lb. of hay and 11 bu. of oats?

10. It takes light 7.47 min. to travel from the sun to the earth. If light travels 186,600 mi. per second, how far is it from the sun to the earth?

11. A wind of 10 miles per hour exerts a pressure of $\frac{1}{8}$ of a pound on an area of 1 sq. ft. What is the pressure against a wall $20\frac{1}{3}$ ft. by $6\frac{1}{2}$ ft.?

12. A hurricane of 100 miles an hour produces a pressure of $49\frac{1}{5}$ lb. against a surface of 1 sq. ft. How much is the pressure against a house 35 ft. by $20\frac{1}{5}$ ft.?

13. A tank measures 7 ft. by 3 ft. by $2\frac{3}{4}$ ft. How many pounds of water will it hold if 1 cu. ft. of water weighs 62.4 lb.?

14. If this tank had been filled with salt water, weighing 64.3 lb. per cubic foot, how much more would it have weighed?

15. How many cubic feet of air in a room $24\frac{1}{2}$ ft. by 16 ft. by $9\frac{1}{2}$ ft.? Find its weight in pounds, if 1 cu. ft. of air weighs 1.28 oz.

Division of Fractions

Review

81. $\frac{8}{9} \div 4 = \frac{2}{9}$, $\frac{4}{5} \div 3 = \frac{4}{15}$.

There are two ways of dividing a fraction by an integer:

(1) by dividing the numerator of the fraction;

(2) by multiplying the denominator of the fraction.

In the first way the number of the unit fractions is diminished. In the second way the size of the unit fractions is diminished.

1. What check do we use in division?

2. If the divisor is 3 and the quotient is $\frac{1}{6}$, what is the dividend?

3. If the quotient is $\frac{3}{7}$ and the divisor is 5, find the dividend.

4. What dividend, divided by 7, yields $\frac{2}{15}$?

5. Divisor $= 6$, quotient $= \frac{5}{12}$, dividend $=$?

6. How many half dollars does it take to pay a debt of \$5? $\$5 \div \$\frac{1}{2} = ?$

7. How many quarters will pay a debt of $\$1\frac{1}{2}$? $\$\frac{3}{2} \div \$\frac{1}{4} = ?$

8. How does $5 \div \frac{1}{2}$ compare with $5 \times \frac{2}{1}$?

9. How does $\frac{3}{2} \div \frac{1}{4}$ compare with $\frac{3}{2} \times \frac{4}{1}$?

To divide by a fraction, invert the divisor and multiply as in multiplication of fractions.

10. In the multiplication of proper fractions, is the product larger than either factor? Illustrate.

11. In the division of a proper fraction by another, is the quotient smaller than the dividend? Illustrate.

12. In the division of a proper fraction by an improper fraction, is the quotient smaller than the dividend? Illustrate.

When the dividend or the divisor is an integer, it may seem easier to write it in the form of a fraction. Then all examples can be worked by one and the same rule. Thus, for $\frac{4}{5} \div 6$ write $\frac{4}{5} \div \frac{6}{1}$, for $3 \div \frac{1}{2}$ write $\frac{3}{1} \div \frac{1}{2}$.

Oral Exercise

82. Give answers rapidly:

1. $\frac{1}{2} \div 5$
2. $\frac{5}{4} \div 5$
3. $\frac{11}{12} \div 121$
4. $\frac{15}{16} \div 5$
5. $\frac{1}{3} \div \frac{1}{2}$
6. $\frac{4}{3} \div 1\frac{1}{2}$
7. $\frac{11}{12} \div \frac{3}{4}$
8. $\frac{1}{3} \div 10$
9. $\frac{4}{5} \div 7$
10. $\frac{13}{14} \div 26$
11. $\frac{11}{10} \div 22$
12. $\frac{1}{2} \div \frac{1}{3}$
13. $1\frac{1}{2} \div \frac{4}{3}$
14. $\frac{5}{12} \div \frac{5}{3}$
15. $\frac{2}{3} \div 11$
16. $\frac{4}{5} \div 8$
17. $\frac{7}{9} \div 14$
18. $\frac{5}{3} \div 10$
19. $\frac{3}{4} \div \frac{1}{3}$
20. $\frac{3}{5} \div \frac{2}{3}$
21. $\frac{7}{16} \div \frac{7}{8}$
22. $\frac{4}{5} \div 2$
23. $\frac{8}{5} \div 4$
24. $\frac{4}{5} \div 16$
25. $\frac{14}{11} \div 5$
26. $\frac{4}{5} \div \frac{1}{5}$
27. $\frac{7}{8} \div 1\frac{1}{4}$
28. $\frac{7}{8} \div \frac{7}{16}$
29. $\frac{8}{9} \div \frac{4}{5}$
30. $\frac{2}{3} \div \frac{7}{9}$

Oral Problems

83. 1. How many quarts of milk does it take to fill a can 4 times, if the can holds $6\frac{2}{3}$ quarts?

2. A can holding $10\frac{2}{3}$ quarts can be filled by pouring into it the contents of a jar 16 times. How much does the jar hold?

3. How many times can I fill a jug holding 1 quart from a supply of $11\frac{1}{2}$ gallons?

4. If $1\frac{1}{2}$ yards of lace can be made in $8\frac{1}{2}$ hours, how long will it take to make 1 yard?

5. If $2\frac{1}{2}$ yards of lace can be made in 10 hours, how much can be made in 1 hour?

6. If $2\frac{1}{2}$ doz. eggs cost $\$\frac{3}{4}$, what is the cost per dozen?

7. If $1\frac{1}{2}$ doz. eggs cost $\$\frac{1}{4}$, how many dozen can be bought for \$1?

8. How many books can be bought for $\$5\frac{3}{5}$, at $\$\frac{4}{5}$ each?

9. If $1\frac{1}{2}$ acres of land yield $8\frac{1}{4}$ tons of hay, what is the yield of 1 acre? What part of an acre yields 1 ton?

10. Change $\frac{2}{3}$ of an inch to a fraction of a foot.

11. Change $1\frac{1}{6}$ of a foot to inches.

12. A school girl made 6 badges of equal length from $\frac{8}{9}$ of a yard of ribbon. How long was each badge?

13. How is the value of a fraction changed by multiplying its denominator by 5? By dividing its numerator by 5?

14. How many $\frac{3}{4}$-lb. packages can be filled from 6 lb. of tea?

15. How many days will it take a boy to earn \$6, if he earns $\$\frac{3}{4}$ a day?

16. At $\$3\frac{1}{2}$ a day, what is due a man for working $1\frac{1}{2}$ days?

17. What is the ratio of 6 inches to 12 inches?

18. What is the ratio of $\frac{1}{2}$ of an inch to $\frac{1}{3}$ of an inch?

Written Exercise

84. **1.** Divide $2349\frac{3}{4}$ by 7.

FIRST PROCESS

$$7\overline{)2349\frac{3}{4}} = 335\frac{19}{28}$$

SECOND PROCESS

$$.7\overline{)2349.75} = 335.68^-$$

EXPLANATION. — In dividing 2349 by 7, we get the remainder 4. To this 4 add $\frac{3}{4}$, and we have $4\frac{3}{4}$, or $\frac{19}{4}$.

Then $\frac{19}{4} \div 7 = \frac{19}{28}$. Write this in the quotient.

2. Divide $8\frac{2}{3}$ by $9\frac{3}{4}$.

FIRST PROCESS

$8\frac{2}{3} = \frac{26}{3}$, $9\frac{3}{4} = \frac{39}{4}$.

Then,

$$\frac{26}{3} \div \frac{39}{4} = \frac{\overset{2}{\cancel{26}} \times 4}{3 \times \underset{3}{\cancel{39}}} = \frac{8}{9}.$$

SECOND PROCESS

$8\frac{2}{3} = 8.666 \cdots$, $9\frac{3}{4} = 9.75$

$$\begin{array}{r} .88+ \\ 975\overline{)866.66\cdots} \\ 7800 \\ \hline 8666 \\ 7800 \\ \hline 866 \end{array}$$

3. Divide $7\frac{4}{5}$ by $8\frac{2}{3}$. Change first to improper fractions.

4. Divide $1725\frac{3}{4}$ by $878\frac{2}{5}$. Change first to decimal fractions.

5. $205\frac{3}{4} \div 199.7$. **6.** $89\frac{5}{6} \div 19\frac{7}{8}$. **7.** $1000 \div 33\frac{1}{3}$.

8. $968.69 \div 73\frac{4}{5}$. **9.** $18\frac{2}{3} \div 19\frac{1}{5}$. **10.** $1000 \div .33\frac{1}{3}$.

Written Problems

85. 1. If $9\frac{1}{2}$ T. of coal cost \$54.62, what is the cost per ton?

2. At \$1.75 a yard, how many yards can be bought for \$21?

3. Find the value of 8816 lb. of oats at 47¢ per bushel, each bushel containing 32 lb.

4. How many cords of wood in a pile containing 17,600 cu. ft.? What is it worth at \$$4\frac{2}{3}$ per cord?

5. A man contracts a debt of \$900, which he promises to pay in monthly installments of \$$12\frac{1}{2}$. After paying \$337.50, how many more payments does he have to make?

6. If $20\frac{3}{4}$ yd. of cloth cost \$60.50, what is the cost of the cloth per yard?

7. How many days' labor at \$3.75 per day will pay for 5 lb. of tea at 90¢ per pound, and two suits of clothes at \$26 and \$$23\frac{1}{4}$ respectively?

8. A load of hay weighs 2763 lb. How much is the hay worth at \$$17\frac{1}{2}$ a ton?

9. A ham weighs 15 lb. 4 oz. and sells at 21¢ a pound; a side of bacon weighs 11 lb. 8 oz. and sells at 23¢ a pound. Find the cost of both.

10. A manufacturer ships 7 machines weighing 2 T. 670 lb. each. How much freight does he pay at 25¢ a hundred pounds?

11. A bought 4 doz. hats at \$16.35 per dozen. Sold them at \$2.00 each. What was total cost? Total sale? What gain?

12. 46 yd. carpet at \$$3\frac{3}{4}$ per yard. What was the cost?

13. From a bolt of cloth containing 48 yd. four pieces are cut. One is $3\frac{1}{4}$ yd.; another, $4\frac{2}{3}$ yd.; another, $12\frac{1}{2}$ yd.; and the fourth, $5\frac{7}{9}$ yd. How much remains in the bolt?

It cost $33\frac{2}{3}$¢ per yard, and was sold at 45¢ per yard. What will be the profit on bolt? How much money is still invested in the unsold part?

14. Bought 36 school desks at \$$3\frac{3}{4}$ each. What did they cost? The freight charges were at the rate of $16\frac{1}{2}$¢ per desk, and the expense of having them placed in class room was $9\frac{3}{4}$¢ each. What was the final cost of the desks?

15. A purchaser bought $8\frac{1}{2}$ lb. beef at 21¢ a pound; $1\frac{1}{4}$ lb. suet at $18\frac{1}{4}$¢ a pound; a ham weighing $10\frac{1}{2}$ lb. at $21\frac{3}{4}$¢ a pound. What was the total amount? How much change would there be from a \$20.00 bill?

COMMON AND DECIMAL FRACTIONS

Study Exercise

86. 1. Change $\frac{34}{45}$ to a three place decimal fraction.

Since $34 = 34.000$, divide the numerator by the denominator. In this case, the answer may be expressed as $.755\frac{5}{9}$, or more simply as $.755^{+}$.

2. Change .135 to a common fraction.

Write the decimal fraction in the *form* of a common fraction, thus $\frac{135}{1000}$. Then cancel factors common to the numerator and denominator. In this case you obtain $\frac{27}{200}$.

3. Change the improper fraction $\frac{147}{35}$ to a mixed number.

Dividing 147 by 35 gives the quotient 4 and the remainder 7. Hence we obtain $4\frac{7}{35}$, or $4\frac{1}{5}$.

Written Exercise

87. 1. Reduce to an integer or to a mixed number:

$\frac{786}{55}$, $\frac{1081}{23}$, $\frac{960}{50}$, $\frac{1234}{105}$, $\frac{909}{91}$.

2. Change to common fractions:

.275, .365, .325, .1375, .3375.

3. Change to four-place decimal fractions:

$\frac{3}{7}$, $\frac{11}{14}$, $\frac{53}{300}$, $\frac{37}{75}$, $\frac{11}{72}$, $\frac{64}{113}$.

If a common fraction in its lowest terms has a denominator which contains prime factors other than 2 and 5, the fraction cannot be expressed exactly as a decimal fraction. For example, $\frac{25}{99}$ is in its lowest terms and the denominator has the prime factors 3 and 11. From this we infer that the division of 25 by 99 does not come out exact. We obtain .252525 .... The digits 25 are continually recurring, as the division is carried on farther.

Written Problems

88. 1. A dealer in carpets sold at different times $25\frac{1}{4}$ yd., $76\frac{3}{4}$ yd., $125\frac{1}{8}$ yd., $250\frac{7}{8}$ yd., $75\frac{1}{2}$ yd. of carpet, at a profit of 35¢ per yard. What was his gain?

2. A merchant bought 10 pieces of silk at $1 per yard, and sold them at $1.45 per yard. The pieces contained $40\frac{1}{4}$, $42\frac{1}{2}$, $44\frac{3}{4}$, $45\frac{1}{4}$, $39\frac{3}{4}$, $40\frac{1}{2}$, $41\frac{1}{8}$, $43\frac{5}{8}$, $42\frac{1}{4}$, $40\frac{3}{4}$ yd. Compute his profit.

3. A man worked $8\frac{1}{2}$ hr. on Monday, $10\frac{1}{3}$ hr. Tuesday, $9\frac{2}{3}$ hr. Wednesday, $6\frac{3}{4}$ hr. Thursday, $5\frac{1}{2}$ hr. Friday, and $8\frac{3}{4}$ hr. Saturday. How much did he earn in the week, if he received 60¢ an hour?

4. The maximum temperatures during a week in July are as follows: $85\frac{1}{3}°$, $90\frac{1}{2}°$, $80\frac{2}{3}°$, $79\frac{1}{2}°$, $87\frac{1}{3}°$, $91\frac{2}{3}°$, $92\frac{3}{4}°$. Find the average of these.

5. The daily cash receipts in a small store are during one week as follows: $\$100\frac{1}{2}$, $\$153\frac{3}{4}$, $\$212\frac{1}{4}$, $\$159\frac{1}{2}$, $\$143\frac{3}{4}$, $\$169\frac{3}{4}$. Find the total receipts for the week.

THE PRICES OF WHEAT IN A CHICAGO MARKET

WHEAT	OPEN	HIGH	LOW	CLOSE
September	102	$102\frac{5}{8}$	$100\frac{5}{8}$	$100\frac{7}{8}$
December	98	$98\frac{3}{8}$	$96\frac{1}{8}$	$96\frac{3}{8}$
May	$101\frac{3}{8}$	$101\frac{3}{4}$	$99\frac{1}{2}$	$99\frac{7}{8}$

By September wheat is meant wheat to be delivered in September. The words *open*, *close* refer to the prices at the opening and at the close of the day; the words *high*, *low* refer to the highest and the lowest price of sale during the day.

6. What was the difference between the opening and the closing price of May wheat? December wheat?

7. A speculator bought 1000 bu. of May wheat at the lowest price and then sold it at the closing price. What was his profit?

First find the profit on 1 bu., then on 1000 bu.

THE PRICE OF OATS

8.

OATS	OPEN	HIGH	LOW	CLOSE
September	$39\frac{1}{4}$	$39\frac{3}{8}$	38	$38\frac{1}{8}$
December	$38\frac{7}{8}$	39	$37\frac{3}{4}$	38
May	$40\frac{1}{8}$	$41\frac{1}{4}$	$40\frac{1}{8}$	$40\frac{1}{4}$

What was the difference, in cents per bushel, between the opening and the closing price of each of the three kinds of oats?

9. A man bought 2000 bu. of May oats at the highest price and sold it at $40\frac{3}{4}$¢ a bushel. How much did he lose?

10. A speculator bought 1000 bu. of September oats at the lowest price for that day and sold it a week later for $39\frac{5}{8}$¢ a bushel. Find his profit.

11. What is the difference in the value of 3000 bu. of corn, if the price per bushel drops from $66\frac{3}{4}$¢ to $65\frac{7}{8}$¢?

12. A farmer sold 500 bu. of corn at $58\frac{1}{4}$¢ a bushel. The price dropped to $57\frac{1}{2}$¢ the following day. How much less would he have received if he had held it another day?

13. A farmer sold 800 bu. of wheat at $102\frac{5}{8}$¢ a bushel, 700 bu. of corn at $58\frac{1}{4}$¢ a bushel, and 900 bu. of oats at $40\frac{1}{8}$¢ a bushel. How much did he receive from his sale?

14. A 5-cent piece weighs $73\frac{1}{6}$ grains and a 25-cent piece weighs $96\frac{9}{20}$ grains. How much more do 72 5-cent pieces weigh than 50 25-cent pieces?

15. Bertha used $5\frac{2}{3}$ yards of linen to make a skirt and $1\frac{3}{4}$ yards to make a waist. What was the total number of yards used? How many more yards did she use for the skirt than for the waist?

16. A father gives one son $\$1\frac{3}{4}$ and another $\$2\frac{1}{4}$. How much does he give to both? How much more did the second son receive than the first?

17. I earned $\$5\frac{1}{2}$ last week and $\$3\frac{5}{8}$ this week. How much do I need to make $10?

Drill Exercise

89. Memorize and use the fractional parts of a dollar.

$\$\frac{1}{2} = 50¢$	$\$\frac{1}{5} = 20¢$	$\$\frac{1}{10} = 10¢$
$\$\frac{1}{3} = 33\frac{1}{3}¢$	$\$\frac{1}{6} = 16\frac{2}{3}¢$	$\$\frac{1}{12} = 8\frac{1}{3}¢$
$\$\frac{1}{4} = 25¢$	$\$\frac{1}{8} = 12\frac{1}{2}¢$	$\$\frac{1}{16} = 6\frac{1}{4}¢$
The important multiples are:		
$\$\frac{2}{3} = 66\frac{2}{3}¢$	$\$\frac{3}{5} = 60¢$	$\$\frac{3}{8} = 37\frac{1}{2}¢$
$\$\frac{3}{4} = 75¢$	$\$\frac{4}{5} = 80¢$	$\$\frac{5}{8} = 62\frac{1}{2}¢$
$\$\frac{2}{5} = 40¢$	$\$\frac{5}{6} = 83\frac{1}{3}¢$	$\$\frac{7}{8} = 87\frac{1}{2}¢$

Study Exercise

90. As prices of articles are often simple fractional parts of one dollar, these relations may be used to shorten computation. As, for example:

1. Find the cost of 96 yd. of calico at $12\frac{1}{2}¢$ per yard.

PROCESS

$12\frac{1}{2}¢ = \$\frac{1}{8}$

$96 \times \$\frac{1}{8} = \$12.$ *Ans.*

EXPLANATION. — It is easier to multiply $\$\frac{1}{8}$ than $12\frac{1}{2}$ cents.

2. Find the cost of 72 bu. of corn at $37\frac{1}{2}$¢ a bushel.

PROCESS

$$37\tfrac{1}{2}¢ = \$\tfrac{3}{8};\ 72 \times \$\tfrac{3}{8} = \$27.\ \ \textit{Ans.}$$

Oral Exercise

91. Find the cost, using pencil only when necessary.

1. 244 bu. @ 50¢.
2. 1456 yd. @ $12\frac{1}{2}$¢.
3. 753 yd. @ $33\frac{1}{3}$¢.
4. 720 lb. @ 25¢.
5. 456 doz. @ $16\frac{2}{3}$¢.
6. 655 lb. @ 20¢.
7. 1250 lb. @ 10¢.
8. 1728 doz. @ $8\frac{1}{3}$¢.
9. 128 yd. @ $6\frac{1}{4}$¢.
10. 300 lb. @ $66\frac{2}{3}$¢.
11. 400 yd. @ 75¢.
12. 350 lb. @ 40¢.
13. 125 doz. @ 80¢.
14. 120 yd. @ $83\frac{1}{3}$¢.
15. 160 bu. @ $37\frac{1}{2}$¢.
16. 888 lb. @ $37\frac{1}{2}$¢.

REVIEW

Oral Exercise

92. Add from dictation. Keep time record.

1.	2.	3.	4.
$ 0.14	$ 1.50	$ 1.00	$ 25.00
.11	.15	2.00	10.00
.13	.10	1.00	5.00
.12	.25	.50	3.00
.10	.20	.20	.25
.11	.03	.14	.10
.04	.27	.16	1.65

Oral Exercise

93. Subtract from dictation.

1. $ 1.64 + ? = $ 5.00
2. $ 8.75 + ? = $ 10.00
3. $ 2.85 + ? = $ 20.00
4. $ 18.67 + ? = $ 50.00
5. $ 3.21 + ? = $ 10.00
6. $ 14.67 + ? = $ 20 00
7. $ 163.50 + ? = $ 200.00

8.	$	98.45 + ? =	$	500.00
9.	$	.73 + ? =	$	2.00
10.	$	.89 + ? =	$	10.00
11.	$	1645.00 + ? =	$	2500.00
12.	$	764.28 + ? =	$	1000.00

Written Exercise

94. Add. Test by adding from top downward. Time your addition both ways. Practice until the time with perfect accuracy is made as short as possible.

1.	2.	3.
		378
	598	549
325	431	492
586	236	626
423	328	873
218	461	297
497	543	576
605	394	234

4.	5.	6.
6,239	3,785	13,654
7,985	7,394	47,396
4,762	1,562	23,243
2,399	5,703	89,472
6,273	4,180	59,368
7,395	3,926	20,645
8,290	7,989	72,137
4,107	5,735	39,599
2,364	3,967	18,364

7.	8.	9.
27,635		
651,443	27.36	123.076
168,005	132.44	5,738.192
273,542	91.68	289.107
189,739	4.98	13,726.0021
264,997	2.13	463.9207
953,268	214.75	8,257.1032
43,721	344.62	5,273.6443
8,754	86.97	993.865
274,646	136.05	8,698.2019

Written Exercise

95. Subtract. Keep time record.

1.	2.	3.
6,943	13,721	368,723
4,285	9,834	69,847

4.	5.	6.
163,872	9,803,001	7,382,039
125,983	3,924,273	1,890,243

7.	8.	9.
7,364,100	14,060,082	30,071,811
1,992,871	9,170,290	8,799,825

Written Exercise

96. Multiply and test, keeping your time accurately.

1. 371×862
2. $813 \times 4{,}226$
3. $374 \times \$46.15$
4. 418×291
5. $1{,}739 \times 8{,}645$
6. $186 \times \$87.33$

7. 781×165 8. $7{,}030 \times 4{,}682$
9. $635 \times \$99$ 10. 343×766
11. $456 \times 37{,}652$ 12. $215 \times \$678.33$

Written Exercise

97. Divide, test, and time yourself in each step.

1. $35\overline{)\,321}$ $\overline{783}$ $\overline{1{,}372}$ $\overline{8{,}647}$ $\overline{37{,}256}$
2. $47\overline{)\,762}$ $\overline{819}$ $\overline{4{,}728}$ $\overline{9{,}163}$ $\overline{86{,}920}$
3. $64\overline{)2{,}869}$ $\overline{1{,}338}$ $\overline{76{,}492}$ $\overline{18{,}639}$ $\overline{702{,}138}$
4. $86\overline{)7{,}926}$ $\overline{8{,}167}$ $\overline{35{,}268}$ $\overline{60{,}932}$ $\overline{427{,}837}$
5. $35\overline{)17{,}041}$ $47\overline{)90{,}030}$ $64\overline{)653{,}793}$ $86\overline{)519{,}006}$

Written Exercise

98. Reduce:

1. 214 mi. to yards; (*b*) to feet.
2. 6784 lb. to Tons.
3. 1963854 sec. to hours.
4. \$475 to dimes; (*b*) to cents.
5. 132 m. to cm.; (*b*) to millimeters.
6. 29 km. to m.; (*b*) to miles.

Written Exercise

99. Add:

1. $\frac{1}{4}, \frac{3}{4}, \frac{2}{3}, \frac{1}{6}$. 2. $\frac{5}{8}, \frac{4}{7}, \frac{11}{14}, \frac{3}{28}$. 3. $\frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{2}{3}, \frac{1}{6}$.
4. $12\frac{2}{3}, 16\frac{1}{8}, 33\frac{5}{6}$. 5. $37\frac{1}{2}, 66\frac{2}{3}, 4\frac{7}{10}, 19\frac{1}{5}$.

Oral Exercise

100. Give products quickly:

1. $4 \times \frac{3}{5}$	2. $2 \times \frac{1}{8}$	3. $7 \times \frac{5}{9}$
4. $6 \times \frac{4}{7}$	5. $12 \times \frac{7}{9}$	6. $11 \times \frac{3}{11}$
7. $18 \times \frac{5}{8}$	8. $15 \times \frac{2}{5}$	9. $16 \times \frac{5}{6}$
10. $12 \times \frac{7}{8}$	11. $17 \times \frac{3}{8}$	12. $9 \times \frac{5}{16}$
13. $\frac{3}{4}$ of $\frac{5}{6}$	14. $\frac{8}{9}$ of $\frac{10}{11}$	15. $\frac{2}{3}$ of $\frac{6}{7}$
16. $\frac{4}{7}$ of $\frac{7}{8}$	17. $\frac{1}{6}$ of 72	18. $\frac{2}{5}$ of 40
19. $\frac{4}{9}$ of 118	20. $\frac{7}{10}$ of 50	21. $\frac{4}{5}$ of 75
22. $\frac{6}{7}$ of 141	23. $\frac{12}{13}$ of $\frac{6}{7}$	24. $\frac{7}{20}$ of 120
25. $20 \times 1\frac{3}{4}$	26. $16 \times 2\frac{1}{8}$	27. $12 \times 4\frac{5}{6}$
28. $14 \times 1\frac{2}{7}$	29. $28 \times 2\frac{7}{8}$	30. $3\frac{1}{4} \times 4\frac{1}{2}$
31. $1\frac{5}{8} \times 4\frac{2}{5}$	32. $2\frac{1}{3} \times 3\frac{1}{5}$	33. $94\frac{1}{8} \times 17\frac{3}{11}$
34. $33\frac{2}{3} \times 14\frac{7}{8}$	35. $7\frac{8}{9} \times 41$	36. $4\frac{3}{5} \times 18$
37. $5\frac{7}{8}$ of 48	38. $47\frac{2}{3} \times 89$	39. $9\frac{1}{10} \times 62\frac{5}{8}$

Oral Exercise

101. Divide:

1. $\frac{4}{5} \div 3$	2. $\frac{2}{3} \div 4$	3. $\frac{3}{4} \div 6$
4. $\frac{11}{12} \div 7$	5. $\frac{9}{10} \div 16$	6. $\frac{8}{15} \div 9$
7. $\frac{7}{9} \div 6$	8. $\frac{4}{7} \div 8$	9. $\frac{21}{23} \div 5$
10. $\frac{7}{13} \div 6$	11. $12 \div \frac{1}{4}$	12. $6 \div \frac{2}{3}$
13. $18 \div \frac{4}{5}$	14. $20 \div \frac{5}{7}$	15. $84 \div \frac{7}{10}$
16. $33\frac{1}{3} \div 3$	17. $2\frac{1}{2} \div 3$	18. $7\frac{2}{3} \div 5$
19. $37\frac{1}{2} \div 6$	20. $28\frac{4}{5} \div 9$	21. $2\frac{1}{2} \div 3\frac{3}{4}$
22. $4\frac{2}{3} \div 1\frac{5}{6}$	23. $7\frac{1}{5} \div 3\frac{2}{5}$	24. $16\frac{2}{3} \div 8\frac{7}{10}$
25. $19 \div 9\frac{1}{10}$	26. $14\frac{5}{6} \div 8\frac{6}{7}$	27. $71 \div 3\frac{16}{19}$

Written Problems

102. 1. Balance the following bank account:

Deposits: \$473.25, \$1,376.40, \$165.75, \$98.70, \$237.05, \$1,463.19, \$468.21.

Withdrawals by check:

\$161.48, \$435.00, \$2,128.14, \$14.37, \$5.84.

2. Paid for coal, \$68.75, wood, \$3.60, Monarch range, \$51.75, rug, \$47.20, Morris chair, \$27.85, dining table, \$41.00, dining chairs, $\frac{1}{2}$ doz. @ \$5.25 each, dining arm chair, \$9.45. Credit at bank was \$869.73. What was the balance after drawing check for above total?

3. (*a*) What fraction of amount in bank was bill? What fraction of total amount remained in bank?

(*b*) Deposits made after above transaction: \$375.13, \$148.68, \$263.99, \$67.44.

Withdrawals by check: \$37.64, \$4.90, \$124.00. What is the balance?

4. A Delicatessen dealer sells bread at 5¢ a loaf. What must he pay for it to make 20¢ on every dollar's worth sold?

5. Robert Burns buys $3\frac{1}{2}$ lb. of sugar for 25¢. What does it cost per pound?

6. A steamship travels $19\frac{1}{2}$ mi. an hour. How far will it travel in $14\frac{3}{4}$ hr. at the same rate?

7. An aëroplane travels at the rate of $1\frac{2}{5}$ mi. in a minute. How far will it go in 45 min. at the same rate?

8. An aëroplane traveled for $1\frac{1}{2}$ hr. at the rate of $62\frac{1}{2}$ mi. an hour. It then increased its speed $\frac{1}{8}$ and traveled at this rate for $1\frac{3}{4}$ hr. What was the entire distance traveled?

9. A stockman sells 4 steers weighing each, 1,143, 997, 1,236, 1,098 pounds at $7.25 per hundredweight. The cost of feeding for two years was about $22.75 for each steer a year. He had paid for them $30.00 each. What was his gain or loss?

10. A dealer sells sheep at $5.25 each, and gains $\frac{1}{5}$ of the cost. What was the cost?

11. What is the cost of 4 chickens @ 21¢ a pound, weighing respectively $2\frac{1}{2}$ pounds, $3\frac{1}{8}$ pounds, $2\frac{3}{4}$ pounds, and $3\frac{5}{16}$ pounds; and 4 turkeys at $24\frac{1}{2}$¢ a pound, weighing $14\frac{1}{8}$ pounds, $12\frac{3}{4}$ pounds, $13\frac{5}{8}$ pounds, and $16\frac{11}{16}$ pounds respectively?

12. A merchant pays $864.22 for goods which he sells at a gain of $33\frac{1}{3}\%$ of the cost. For how much does he sell them?

Oral Exercise

103. 1. What is the l. c. d. of $\frac{1}{12}$, $\frac{7}{24}$, $\frac{11}{36}$?

2. What is the l. c. d. of $\frac{7}{9}$, $\frac{14}{27}$, $\frac{5}{6}$?

3. Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{7}{8}$, $\frac{11}{16}$.

4. Add $\frac{2}{3}$, $\frac{7}{9}$, $\frac{5}{12}$.

5. Find $\frac{2}{5}$ of 20, 15, 25, 55, 65, 75.

6. Find $\frac{3}{4}$ of 20, 24, 28, 48, 60, 64, 72.

7. Find $\frac{4}{3}$ of 6, 12, 18, 36, 42, 72, 90.

8. Divide by $\frac{2}{3}$: 4, 12, 16, 18, 24, 64.

9. Divide by $\frac{3}{4}$: 6, 15, 21, 33, 45, 48.

10. Divide by $1\frac{1}{4}$: 10, 15, 20, 45, 55, 95.

11. Find $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$, $4\frac{1}{5}$.

12. Divide by $1\frac{1}{3}$: $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{3}$, $4\frac{1}{2}$, $4\frac{1}{6}$.

13. What does the expression $\frac{7}{12}$ mean?

Written Problems

104. 1. The average rate of speed of an automobile is 25 mi. an hour. How far will it travel in $3\frac{2}{5}$ hr.?

2. At the average rate of 25 mi. an hour, how long will it take an automobile to travel 70 mi.?

3. A man wishes to take a train that leaves in $2\frac{1}{2}$ hr. If the station is 75 mi. distant, how fast will he have to travel to reach it within the required time?

4. What is the cost of a bushel of apples, if $12\frac{1}{2}$ bu. cost \$10?

5. If $3\frac{3}{4}$ yd. cost \$9, what is the cost of 1 yd.?

6. At $87\frac{1}{2}$¢ a yard, what is the cost of 32 yd.?

7. Find the cost of 64 lb. at $62\frac{1}{2}$¢ each.

8. If $\frac{3}{5}$ of a farm is worth \$5,100, what is the value of the entire farm?

9. After selling $\frac{5}{6}$ of a farm a man has 126 acres left. How many acres were there in the entire farm?

10. If one turn of a screw advances it $\frac{3}{10}$ of an inch, how far will $11\frac{1}{2}$ turns advance it?

11. If 7 turns of a screw advance it $2\frac{4}{5}$ in., how far will $8\frac{1}{2}$ turns advance it?

12. A piece of flannel containing $30\frac{3}{4}$ yd. shrinks $1\frac{2}{5}$ yd. in dyeing. What is its length after dyeing?

13. A cistern $\frac{3}{4}$ full has $\frac{2}{5}$ of its contents drawn out. What part of the entire capacity of the cistern was drawn out? How many gallons were drawn out, if the entire capacity is 5,000 gallons?

14. I bought $66\frac{2}{3}$ yards of flannel at \$.$47\frac{1}{2}$ per yard. How much did I pay for it?

15. John walks at the rate of $21\frac{1}{2}$ mi. a day for $13\frac{1}{3}$ days. How far does he walk in that time?

16. A wheel travels $15\frac{2}{5}$ ft. during every revolution. How far does it travel in $24\frac{1}{2}$ revolutions?

17. If a yard of silk costs \$ $2\frac{1}{4}$, how many yards can be bought for \$ $18\frac{2}{5}$?

18. At \$ $33\frac{1}{3}$ per acre, how many acres of land can be purchased for \$ 4,000?

19. What is the average weight of 16 men whose united weight is $2{,}325\frac{1}{3}$ lb.?

20. What is the price of hay, when $5\frac{7}{8}$ tons are worth \$ 85.19?

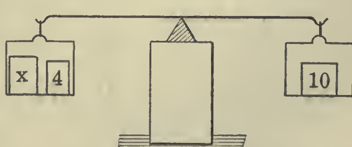
THE SOLUTION OF PROBLEMS

Study Exercise

105. The practical value of arithmetic lies in the help that it gives in the solution of problems. These problems are found in every occupation and in every industry. In the solution of these problems, it is necessary to be accurate in the fundamental operations. It will also be helpful if one is familiar with the kind of arithmetical problems that arise in business and the methods by which these problems may be most easily solved. It will be found that most of these problems may be solved in more than one way. Study, therefore, the problem in order to discover the best method of solution. A knowledge of the use of the simple equation, of analysis, of ratio and proportion, of variation, and of answers that are approximately correct will be helpful in the solution of many of these problems.

Simple Equations

106. 1. The scales balance; how much do the left-hand packages, marked 4 lb. and x lb., weigh together?



Equation: $x + 4 = 10.$

2. If we take away 4 lb. on the left, how many pounds must we take away on the right, to make the two sides balance again?

Equation: $x + 4 - 4 = 10 - 4$

$$x = 6$$

The letter x in a problem stands for the number which we desire to find.

A housekeeper has on hand 4 lb. of meat, but needs 10 lb. for the next meal. How many pounds must she buy?

Let x be the number of pounds she must buy. Then,

$$\left\{\begin{array}{c}\textit{The number}\\ \textit{of pounds}\\ \textit{she must buy}\end{array}\right\} \textit{added to} \left\{\begin{array}{c}\textit{the number}\\ \textit{of pounds}\\ \textit{she has}\end{array}\right\} \textit{equals } 10 \textit{ lb.}$$

The equation is $x \quad + \quad 4 \quad = \quad 10$

The latter way of writing is much more brief. We shall see, as we go on, that some problems can be solved more easily by the use of equations.

3. What does x stand for in the equation $34 + x = 40$?

PROCESS	EXPLANATION
$34 + x = 40$	
$34 - 34 + x = 40 - 34$	To find the number x, take away 34 from both members of the equation.
$x = 6$	
Check: $34 + 6 = 40$	Since $34 + 6 = 40$, the answer is correct.
$40 = 40$	

4. An **equation** is an equality of numbers.

Any change made on one side of the equation must be made on the other side also.

Thus, if one side is divided by a number, the other side must be divided by the same number.

If one side is multiplied by a number, the other side must be multiplied by the same number.

5. If $5 \times x = 15$, what is the value of x?

PROCESS

$$\frac{\cancel{5} \times x}{\cancel{5}} = \frac{\overset{3}{\cancel{15}}}{\cancel{5}}$$

or $x = 3.$

EXPLANATION

We can obtain the value of x by dividing both sides of the equation by 5.

It is customary to write, for brevity, $5x$, instead of $5 \times x$. We shall understand that $5x$ means "five times x."

6. If $$\frac{x}{2} = 75,$$

we may obtain the value of x by multiplying both sides by 2. We have, then

$$\frac{\cancel{2}x}{\cancel{2}} = 2 \times 75$$

or $x = 150$

Written Exercise

107. Find the value of x:

1. $25 + x = 37$
2. $63 + x = 75$
3. $x + 13 = 44$
4. $x + 25 = 39$
5. $37 + x = 55$
6. $45 = 11 + x$
7. $36 = 13 + x$
8. $x + 11 = 73$

Written Exercise

108. Solve the equation:

1. $x - 20 = 51.$

PROCESS	EXPLANATION
$x - 20 = 51$	
$x - 20 + 20 = 51 + 20$	To find x, add 20 to both sides.
$x = 71$	
Check:	Since $71 - 20 = 51$, the answer is correct.
$71 - 20 = 51$	
$51 = 51$	

In each equation find the value of x or y:

2. $x - 30 = 51$ 3. $73 = y - 26$ 4. $x - 13 = 54$

5. $54 = x - 37$ 6. $y - 37 = 53$ 7. $y - 101 = 13$

Written Problems

109. Solve, using the equation:

1. What number added to 24 gives the sum 53?

2. What number diminished by 74 gives the remainder 43?

3. If a number be added to itself, the sum is 60. What is the number?

4. If a number be added to twice itself the sum is 60. What is the number?

5. If a number be subtracted from three times itself the remainder is 25. What is the number?

6. If four times a number less the number is 90, what is the number?

7. If five times a number exceeds three times the number by 20, what is the number?

8. If twice a number be added to one half the number the answer is 25. What is the number?

Written Exercise

110. Find the value of x in each of the following:

1. $4x = 144$
2. $5x = 135$
3. $11x = 121$
4. $\frac{x}{2} = 144$
5. $\frac{x}{7} = 12$
6. $\frac{x}{5} = 17$

Written Problems

111. 1. John said to Jane: "I can tell what number you are thinking of, if you just tell me what you get when you double that number and then add 5." Jane says, "I get 47." How did John find the number?

EXPLANATION AND PROCESS

Let the number be	x
Doubling it gives	$2x$
Adding 5 to this gives	$2x + 5$
Putting $2x + 5$ equal to 47 gives the equation	$2x + 5 = 47$
Subtract 5 from both sides and you have	$2x = 42$
Divide both sides by 2, and you obtain	$x = 21$

Hence the number thought of by Jane is 21.

2. What number, when multiplied by 7, gives 133?

3. What number, when diminished by 49, gives 74?

4. What number, when doubled, and the result increased by 6, gives 378?

5. If a certain number is divided by 2, and 15 is added to the result, the sum is 97. Find the number.

6. A farmer finds that after 3 years his flock of sheep has doubled. He buys 21 sheep more, and then has 325 sheep. How many had he in the beginning?

7. Mary is 3 years older than Lucy. Their combined ages are 47 yr. How old is Lucy?

8. George's bank deposit is now twice what it was last year. If he takes out $10, he has $75 left. What was his deposit last year?

Written Exercise

112. Solve:

1. $2x + 6 = 13$	2. $3x + 10 = 23$
3. $2x + 1 = 5$	4. $4x + 2 = 10$
5. $14 - x = 11$	6. $5 - 2y = 3$
7. $2x - 6 = 13$	8. $2x - 10 = 23$
9. $2x - 1 = 5$	10. $5x + 7 = 12$
11. $50 - y = 30$	12. $15 - 3y = 9$

13. $x+7=24$
14. $13+x=27$
15. $3x+1=10$
16. $6+2x=12$
17. $70-x=32$
18. $17-2y=11$
19. $x-7=24$
20. $18+x=27$
21. $3x-2=10$
22. $10+3x=16$
23. $45=68-x$
24. $12=24-3y$

113. Solve $5x-6.5=8.5$.

PROCESS	EXPLANATION
$5x=15$	Adding 6.5 to both sides.
$\frac{5x}{5}=\frac{15}{5}$	Dividing both sides by 5.
or $x=3$ *Ans.*	

Oral Exercise

114. Solve the following equations, using pencil only where necessary:

1. $10x=1$
2. $3.5x=7$
3. $7x=\frac{7}{2}$
4. $11x=132$
5. $\frac{1}{3}x=4$
6. $\frac{1}{2}x=5\frac{1}{2}$
7. $.2x=2.2$
8. $.4x=24$
9. $\frac{1}{10}x=1$
10. $\frac{2}{3}x=6$
11. $11x=3.3$
12. $4.5x=9$
13. $1.1x=6.6$
14. $12x=1{,}728$
15. $\frac{1}{4}x=5$
16. $\frac{1}{3}x=\frac{2}{3}$
17. $3x=3.3$
18. $.4x=2.4$
19. $.1x=1$
20. $\frac{2}{5}x=4$

21. $12x = 1.2$
22. $1.5x = 4.5$
23. $14x = 42$
24. $13x = 143$
25. $\frac{1}{5}x = 7$
26. $\frac{1}{4}x = \frac{1}{5}$
27. $.3x = 3.3$
28. $.4x = .24$
29. $\frac{1}{100}x = 2$
30. $\frac{3}{4}x = 6$
31. $13x = 2.6$
32. $11x = 121$
33. $13x = 52$
34. $10x = 100$
35. $\frac{1}{6}x = 7$
36. $\frac{1}{2}x = .1$
37. $4x = 24$
38. $4x = .24$
39. $.01x = 2$
40. $\frac{3}{5}x = \frac{1}{2}$

Written Exercise

115. Find the value of x:

1. $\frac{3}{5}x = \frac{1}{2}$.

PROCESS

$$\frac{3x}{5} = \frac{1}{2}$$
$$3x = \frac{5}{2}$$
$$x = \frac{5}{6} \quad \textit{Ans.}$$

EXPLANATION

Multiply both sides by 5 and obtain $3x = \frac{5}{2}$.

Then divide both sides by 3. $x = \frac{5}{6}$. *Ans.*

Check: $\frac{3}{5} \times \frac{5}{6} = \frac{1}{2}$.

2. $\frac{4}{3}x = 8$
3. $\frac{6}{5}x = 1$
4. $\frac{1}{2}x = 3$
5. $\frac{4}{5}x = 5$
6. $\frac{4}{5}x = \frac{4}{5}$
7. $12 = 3x$
8. $12 = \frac{3}{2}x$
9. $\frac{3}{2}x = \frac{1}{3}$
10. $\frac{7}{8}x = 1$
11. $\frac{3}{2}x = 3$
12. $\frac{4}{6}x = 5$
13. $\frac{7}{9}x = \frac{7}{9}$
14. $15 = 3x$
15. $\frac{3}{4} = \frac{5}{3}x$
16. $\frac{5}{3}x = 10$
17. $\frac{8}{9}x = 1$
18. $\frac{5}{2}x = 3$
19. $\frac{4}{7}x = 5$

20. $\frac{2}{3}x=3$ **21.** $\frac{4}{5}=8x$ **22.** $18=\frac{3}{2}x$

23. $\frac{5}{4}x=\frac{2}{3}$ **24.** $\frac{8}{9}x=2$ **25.** $\frac{7}{2}x=3$

26. $\frac{5}{8}x=5$ **27.** $\frac{5}{3}x=3$ **28.** $12=\frac{1}{2}x$

29. $14=\frac{7}{2}x$ **30.** $\frac{2}{3}x=\frac{1}{5}$ **31.** $\frac{4}{3}x=15$

Instead of the letter x any other letter may be used. Sometimes the letter y is used. The frequent use of the letter x is merely a matter of custom. Sometimes it is easier to use some other letter.

Written Problems

116. 1. A flood washed away 32 acres or exactly $\frac{1}{5}$ of a farm along the Mississippi River. How many acres were there in the farm before the flood?

Suggestion: Since A. stands for acres, it is more natural to use A than to use x for our unknown quantity. The equation then is $\frac{1}{5}A=32$, which gives $A=160$. In other words, his farm originally comprised 160 acres.

2. During one month, a man was idle 2 days, or $\frac{1}{13}$ of the number of working days in that month. How many working days were there in that month?

Suggestion: Use d for days.

3. A magician can tell without looking how many marbles you have, if you only tell him what you get by adding 13 to five times the number of marbles. If this number is 113, how does the magician ascertain the number of marbles?

4. Mary refused to tell John how old she was, but finally told him that twice her age, increased by 20 years, made 42 years. How did John find her age?

5. A farmer had 100 chickens. A coyote killed the same number of them on each of three nightly raids, and only 85 chickens remained. How many did the coyote kill each night?

6. Find the dividend x, when the divisor is 6, and the quotient is 9.

7. Find the divisor x, when the dividend is 24 and the quotient is 3.

8. Find the quotient x, when the dividend is 1690 and the divisor is 13.

Written Exercise

117. Solve, using pencil only when necessary:

1. $5x+6=16$
2. $3x+7=19$
3. $10+2x=20$
4. $5=2x-5$
5. $x+\frac{1}{10}x=11$
6. $\frac{1}{5}x+x=6$
7. $\frac{1}{3}x+x=8$
8. $\frac{1}{2}x+\frac{1}{4}x=3$
9. $x+\frac{7}{100}x=1.07$
10. $y+\frac{6}{100}y=1.06$
11. $\frac{3x}{10}=5.4$
12. $\frac{5x}{8}=105$

Study Exercise

118. 1. Solve the equation $2.7\,x + 17.5 = 25$.

PROCESS AND EXPLANATION

Subtracting 17.5 from both sides, the equation becomes $$2.5\,x = 7.5.$$

Dividing both sides by 2.5, we obtain

$$\frac{2.5\,x}{2.5} = \frac{7.5}{2.5}, \text{ or } x = 3.$$

Check:
$$2.5 \times 3 + 17.5 = 25,$$
$$7.5 + 17.5 = 25.$$

2. Solve: $\frac{4}{3}\,x + 110\frac{1}{3} = 123\frac{2}{3}$.

PROCESS AND EXPLANATION

Subtracting $110\frac{1}{3}$ from both sides,

$$\frac{4}{3}\,x = 123\frac{2}{3} - 110\frac{1}{3},$$
$$\frac{4}{3}\,x = 13\frac{1}{3}.$$

Multiplying both sides by 3,

$$4\,x = 40.$$

Then $$x = 10.$$

Written Exercise

119. Solve:

1. $5\,x - 120 = 45$
2. $6\,x - 37 = 53$
3. $1.3\,x + 40.1 = 44$
4. $12.5\,x + 31.5 = 44$
5. $32 = 15.9 + 2.3\,x$
6. $13\,x + 7\,x = 260$

7. $14y - 3y = .55$ 8. $12.5x - 37.5 = 62.5$

9. $82 + 1.05x + 75.7$ 10. $43.5 = 90.8 - 4.3x$

11. $436 = 1,200 - 4x$ 12. $1325 - 5x = 960$

13. $x + \frac{7}{100}x = 321$ 14. $x + \frac{5}{100}x = 168$

Written Problems

120. 1. A village has gained 10% in population during the last 5 years and now has 1,353 inhabitants. Find its population 5 years ago.

PROCESS

$$x + \frac{1}{10}x = 1,353$$
$$\frac{11}{10}x = 1,353$$
$$11x = 13,530$$
$$x = 1,230$$

EXPLANATION

Let x be the number of inhabitants 5 years ago, then
$\frac{1}{10}x$ is the increase in 5 years, and
$x + \frac{1}{10}x$ is the population now, which we know to be 1,353.

2. Find the population of a city five years ago which now numbers 7,644 inhabitants and is $\frac{1}{5}$ larger than it was then.

3. Mr. Roe's yearly salary is now 25% higher than it was three years ago. If it is 1,400 now, what was it three years ago?

4. James thinks of a number, then doubles it, adds 15, and obtains 63. Of what number does he think?

5. A father is 44 years old, which is 18 years more than twice the age of his son. Find the son's age.

6. The area of the United States exceeds 50 times that of Alaska by 25,200 square miles. Find the area of Alaska, if that of the United States is 3,000,500 sq. mi.

7. Florida and Pennsylvania have together an area of 99,000 sq. mi. The area of Florida exceeds that of Pennsylvania by 9,000 sq. mi. Find the area of each.

8. During a summer vacation a boy earned \$25, which was $\frac{2}{5}$ of the sum his older brother earned. How much did the latter earn?

9. Mary and her brother are attending college. Her annual expenses are $\frac{6}{5}$ of her brother's. Both together spend \$660. How much does each spend?

SUGGESTION: Let x be her brother's expense, then hers is $\frac{6x}{5}$; both together spend $x + \frac{6x}{5}$ or $\frac{11x}{5}$. The equation is $\frac{11x}{5} = 660$.

10. Mr. Jones and Mr. Brown together invest \$45,000 in a farm. Mr. Jones contributes $\frac{4}{5}$ as much as Mr. Brown. How much does each invest?

11. If the annual income from this investment is \$720, how much of this goes to Mr. Jones and how much to Mr. Brown?

12. A carpenter saws a 14-foot board into 2 parts, one of which is $\frac{2}{5}$ as long as the other part. How long is each part?

13. A wagon loaded with potatoes weighed 2100 lb. The potatoes weighed 500 lb. more than the wagon. Find the weight of the potatoes.

14. A house is lighted by twice as many 16-candle power lamps as 10-candle power lamps. The total illumination from these lamps is 294 candle power. How many lamps of each kind?

15. A pleasure resort is reached from a city by electric cars and also by steam cars. During a holiday 700 more passengers traveled on the electric cars than on the steam cars. How many traveled on each, if the total number of passengers was 3,500?

16. It is predicted that next year there will be used 196,000 automobiles, or $\frac{7}{4}$ as many as are used this year in the United States. How many are used this year?

17. Among these 196,000 machines there will be 6 times as many high-speed cars and 33 times as many pleasure cars as commercial carriages. How many of each kind will there be?

18. An equal number of \$1,200 automobiles and \$1,500 automobiles were recently sold by a firm. The sales aggregated \$54,000. How many automobiles of each price were sold?

19. A boy can buy an ordinary bicycle for \$20. He can buy four bicycles with coaster brakes and puncture-proof tires and 5 ordinary bicycles for

\$ 208. What is the price of one with coaster brakes and puncture-proof tires?

20. A garage has a floor space of 6,270 sq. ft. Its width is 60 ft. What is its length?

21. In this garage the washing stand is 26 ft. by 16 ft., the repair shop is 24 ft. by 20 ft. Each automobile is allowed a space 19 ft. by 8 ft., and as much room is taken up by approaches as is assigned to the automobiles when in place. Moreover, 206 sq. ft. are taken up by a storage room. How many automobiles can this garage accommodate?

22. Of the 19 branch Post Offices in New York City, 8 have 34 daily mail deliveries by electric mail wagons. All the offices together have 382 deliveries in a day. What is the average number of deliveries per day of the 11 other branch offices?

23. What sum increased by .1 of itself yields \$ 60,500?

24. What sum increased by .06 of itself amounts to \$ 344.50?

25. What number decreased by $\frac{1}{2}$ of itself gives 384.20?

26. If $\frac{4}{5}$ of the number of seats in church are occupied, and there are 196 persons present, how many seats are there?

27. A ball team won 13 games, which is 5 more than $\frac{1}{3}$ of the games it played. How many games did it play?

28. On an excursion $\frac{3}{5}$ of the persons were women. How many excursionists were there, if the number of men was 476?

29. A silver dollar weighs 412.5 grains, $\frac{9}{10}$ of which is pure silver. How many dollars must a silversmith melt to extract 3,341.25 grains of pure silver?

30. A man spends $ 50 more than $\frac{2}{3}$ of his yearly salary, and has $ 450 left. Determine his yearly salary.

31. If $\frac{1}{3}$ and $\frac{2}{5}$ of a farm are together worth $ 1650, what is the whole farm worth?

32. If $\frac{1}{5}$ and $\frac{2}{3}$ of another farm are worth $ 2,600, what is $\frac{1}{4}$ of the remainder worth?

33. At what rate of simple interest will $ 350 in 5 years amount to $ 446.25?

34. The population of a county increased during one year from 12,500 to 12,915. Find the rate of increase.

Analysis

121. 1. If 25 men reap 15 acres in one day, how many acres will 24 men reap in 2 days?

Frequently time is saved by merely indicating the intermediate operations, and finally canceling as much as possible.

It is usually more convenient to arrange the statement of the analysis in the form of steps as shown in the following:

ANALYSIS

If 25 men reap in 1 da.	15 A.
Then 1 man reaps in 1 da.	$\frac{15}{25}$ A.
and 24 men reap in 1 da.	$\frac{24 \times 15}{25}$ A.
and 24 men reap in 2 da.	$\frac{24 \times \overset{3}{\cancel{15}} \times 2}{\underset{5}{\cancel{25}}}$ A.
	$= 28.8$ A. *Ans.*

2. A man's wages are at the rate of $ 900 per year of 313 working days. What are his earnings for 65 days' work?

3. If a train travels 60 mi. an hour, how long will it take to go 13 mi.?

4. When eggs are at 35¢ a dozen, how many eggs must be given in payment for a debt of $ 2.10?

5. If 16 men can do a piece of work in 27 days of 10 hours each, in how many days of 8 hours each can 12 men do the same work?

6. If 68 men can do a piece of work in 48 days of 9 hours each, how many men will be required to do this work in 36 days of 8 hours?

7. The scale of a map is 3 in. to the mile. How many acres are there in an estate which occupies 36 sq. in. on the map?

8. If the area of a certain rectangular garden is 640 sq. rd., what is the area of another rectangular garden 2.4 times longer, but only $\frac{1}{2}$ as wide?

9. A contractor plans to build 4 mi. of railway in 184 da. He finds that 77 men, working 52 da., construct one mile of track. How many men must he employ?

10. A housekeeper takes 3 half pints of milk each week day and 1 pint on Sunday. Her bill for the week comes to $32\frac{1}{2}$¢. What is the price of milk per quart?

11. If $900 yield $29.7 interest in six years, what sum will yield $400 interest in nine years, at the same rate of interest?

12. If 10 chickens average 640 eggs in 3 months, how many chickens will be needed to get 1,000 eggs in 4 months?

13. A store of provisions would last 2,100 men for 15 da.; how long will it last 2,800 men?

14. If 3 geese are worth 2 ducks, and 1 duck is worth 2 chickens, find the value of a goose when a chicken is worth 48¢.

Ratio

122. A common fraction like $\frac{3}{4}$ may be considered in two different ways:

(1) As indicating that 3 out of the 4 equal parts of a unit are taken.

(2) As an indicated division, 3 divided by 4.

Choose whichever point of view is best suited to the conditions.

In dealing with problems involving units which have been subdivided into equal parts, the first view is preferable.

If we desire to reduce the given fraction to a decimal, or wish to tell which of two numbers is the larger and to indicate the exact relation between the two, then it is better to look upon the fraction as an indicated division.

In dealing with the relations of numbers, the word **ratio** is commonly used. Thus,

A common fraction is often called a **ratio**.

My bank account is to that of Mr. Hart in the ratio of "three to four," or $\frac{3}{4}$. This means that I have the smaller sum in the bank; more than that, it shows that for every $3 that I have deposited, he has $4 deposited.

Comparison

1. Find the ratio of E to A, D to A, C to A, B to A.

2. State the ratio of F to B, E to B, D to B, C to B, A to B.

3. Find the ratio of each to C.

In this drawing the ratio of F to A is $\frac{1}{6}$.

Study Exercise

123. One boat is 200 ft. long, another is 300 ft. long. We can find the ratio of the length of the first to that of the second by writing $\frac{200}{300}$, which can be simplified to $\frac{2}{3}$.

> **To find the ratio of two numbers,** take the first number as the numerator, the second as the denominator; cancel common factors.

1. Harry's number of chickens is to John's in the ratio $\frac{1}{2}$.

If Harry has 20 chickens, how many has John?

EXPLANATION

There are two ways of writing ratios. For example, the ratio of 4 to 5 may be written as a fraction $\frac{4}{5}$, or it may be written 4 : 5. We read 4 : 5 by saying 4 to 5 or 4 is to 5.

2. What is the ratio of 2 ft. to 3 in.?

PROCESS AND EXPLANATION

To compare the two, change the feet to inches. The ratio of 24 in. to 3 in. is $\frac{24}{3}$ or $\frac{8}{1}$.

Written Exercise

124. What is the ratio? (Use pencil only when necessary.)

1. 1 ft. to 1 yd.
2. 1 in. to 1 ft.
3. 1 lb. to 1 oz.
4. 1 mi. to 1 ft.

5. 1 sq. ft. to 1 sq. yd.

6. 1 sq. in. to 1 sq. ft.

7. 1 da. to 1 hr.

8. 1 dime to 1 quarter.

9. Find the ratio of 6 bu. to 3 pk.

10. Express, in the simplest form, the ratio of 200 to 600, 75 to 125, 72 to 144.

Written Problems

125. 1. One boy can get a lesson in 1 hr. 10 min., another boy can get the same lesson in 1 hr. 20 min. Find the ratio.

2. The length of a book cover is 7 in. Its width is $5\frac{1}{4}$ in. What is the ratio of the length to the width?

3. The area of the United States is 3,624,122 sq. mi. The area of the state of Texas is 265,896 sq. mi. What is the ratio of the area of Texas to the area of the United States?

Proportion

126. A **proportion** expresses the equality of two common fractions.

The fraction $\frac{4}{8}$ is equal to the fraction $\frac{1}{2}$. We write $\frac{4}{8}=\frac{1}{2}$ and call this a proportion. Another way of writing it is $4:8=1:2$. We read this proportion by saying *four eighths* equals *one half*, or 4 is to 8 as 1 is to 2.

1. If $\frac{x}{8}=\frac{3}{4}$, find x.

PROCESS

$$\frac{\cancel{8}\,x}{\cancel{8}}=\frac{\overset{2}{\cancel{8}}\times 3}{\cancel{4}}$$

$$x=6$$

EXPLANATION. — Multiply both sides of this equation by 8.

Check: Write 6 in place of x, and the equation becomes $\frac{6}{8}=\frac{3}{4}$.

2. If $x:3=6:9$, find x.

PROCESS

$$\frac{x}{3}=\frac{6}{9}$$

$$\frac{\cancel{3}\,x}{\cancel{3}}=\frac{\cancel{3}\times\overset{2}{\cancel{6}}}{\underset{\cancel{3}}{\cancel{9}}}$$

$$x=2$$

EXPLANATION. — Write the proportion in the fractional form, multiplying both sides by 3.

Check: In the given proportion write 2 in place of x and the proportion becomes $\frac{2}{3}=\frac{6}{9}$.

We see that proportions are special kinds of equations.

Written Exercise

127. Find x in the following proportions:

1. $\frac{x}{12}=\frac{3}{4}$

2. $\frac{x}{15}=\frac{2}{5}$

3. $\frac{x}{20}=\frac{4}{5}$

4. $\frac{x}{17}=\frac{12}{51}$

5. $\frac{13}{29}=\frac{x}{116}$

6. $\frac{7}{x}=\frac{19}{133}$

7. $\frac{x}{102}=\frac{16}{17}$

8. $\frac{3}{4}=\frac{x}{144}$

9. $\frac{x}{16}=\frac{24}{128}$

10. $\frac{1}{11}=\frac{x}{176}$

11. $x:5=15:32$

12. $11:2=77:x$

13. $15:28=4:x$

14. $7:9=x:126$

15. $15:16=x:4$

16. $11:12=x:72$

17. $66:70=x:50$

18. $4:5=23:x$

Written Exercise

128. 1. Separate 24 into two parts having the ratio $\frac{3}{5}$, or $3:5$.

PROCESS

$$\begin{aligned} 3x+5x&=24\\ 8x&=24\\ x&=3\\ 3x&=9, \text{ one of the required numbers}\\ 5x&=15, \text{ the other number.}\end{aligned}$$

EXPLANATION. — Let one of the required numbers be $3x$ and the other number $5x$.

We see that the $3x$ and $5x$ are in the ratio $3:5$.

The sum of the two numbers must be 24.

Separate each into two numbers having the given ratio:

2. 50, $\frac{3}{7}$

3. 200, $\frac{2}{3}$

4. .75, $\frac{3}{4}$

5. 144, $1:11$

6. 700, $\frac{7}{13}$

7. 1,000, $\frac{4}{5}$

8. 179, $2:3$

9. 600, $\frac{11}{19}$

10. 2,500, $\frac{1}{8}$

Written Exercise

129. 1. Solve 3 yd. : x yd. = \$ 12 : \$ 20

Explanation. — In solving, we can write the terms as abstract numbers. For we have $\frac{3 \text{ yd.}}{x \text{ yd.}} = \frac{3}{x}$, and $\frac{\$12}{\$20} = \frac{12}{20}$. We need, therefore, only solve the proportion $\frac{3}{x} = \frac{12}{20}$, which gives $x = 5$. *Ans.*

Solve the following proportions:

2. $\frac{10 \text{ bu.}}{x \text{ bu.}} = \frac{\$7.25}{\$58}$.

3. $\frac{\$25}{\$76} = \frac{8 \text{ T}}{x \text{ T}}$.

4. $\frac{5 \text{ yr.}}{7\frac{1}{2} \text{ yr.}} = \frac{\$6000}{\$x}$.

5. $\frac{5 \text{ lb.}}{13 \text{ lb.}} = \frac{x \text{ ft.}}{16.5 \text{ ft.}}$.

Variation

130. 1. If 7 yd. of silk cost \$ 17.50, what will 9 yd. cost?

Process and Explanation

The *greater* the number of yards, the *greater* is the cost.

That is, the cost "varies in direct ratio" with the quantity.

Hence we have $\frac{7 \text{ yd.}}{9 \text{ yd.}} = \frac{\$17.50}{\$x}$.

The answer is $x = \$22.50$.

2. If certain provisions last 15 men 8 days, how long will the same amount of provisions last 20 men?

Process and Explanation

The greater the number of men, the less is the time.

That is, the proportion is *not* $\frac{15 \text{ men}}{20 \text{ men}} = \frac{8 \text{ da.}}{x \text{ da.}}$, but it is $\frac{15 \text{ men}}{20 \text{ men}} = \frac{x \text{ da.}}{8 \text{ da.}}$. We say that $\frac{x \text{ da.}}{8 \text{ da.}}$ "is the inverse ratio" of $\frac{8 \text{ da.}}{x \text{ da.}}$. Solving $\frac{15}{20} = \frac{x}{8}$, we obtain $x = 6$.

The answer is 6 *days*.

> If a quantity varies in *direct ratio* with a second, the first is sometimes said to be *directly proportional* to the second.
>
> If a quantity varies in *inverse ratio* to a second, the first is sometimes said to be *inversely proportional* to the second.

Problems 1 and 2 in this exercise indicate the importance of observing carefully how quantities change. In our problems notice the following:

(*a*) If one quantity decreases when the other increases, then the first varies in direct ratio to the second; that is, the first is "directly proportional" to the second.

(*b*) If one quantity decreases when the other increases, then the first varies in inverse ratio to the second; that is, the first is "inversely proportional" to the second.

Oral Exercise

131. State which of the following vary in direct ratio and which in inverse ratio:

1. The price of shoes, the price of leather.

2. The number of sheep, the amount of mutton obtainable.

3. The number of workmen, the time to do a given amount of work.

4. The number of workmen, the amount of work done in a given time.

5. The length of a circle, the diameter.

6. The height of a pole, the length of shadow.

7. The price of goods, the amount bought with a given sum.

8. The velocity of a train, the distance covered in a given time.

9. The velocity of a train, the time required for a given distance.

10. The amount of lumber, the length of a board fence.

Written Problems

132. 1. The ratio of a certain number to 105 is the same as the ratio of 12 to 35. Find the number.

2. In a year of 365 days the number of cloudy days was to the number of days in the year, as 33 is to 73. How many days were cloudy?

3. If the number of clear days is to the number of cloudy days, as 43 : 30, and there were 215 clear days, how many days were cloudy?

4. For \$6 I can buy 20 neckties; how many ties can I get for \$21?

SUGGESTION: This problem can be worked by unitary analysis, and by proportion. The costs are in the same ratio as the number of articles purchased. This gives the proportion, $6 : 21 = 20 : x$.

5. If a train goes $3\frac{1}{2}$ miles in 4 minutes, how far will it go in one hour?

6. If an aviator flies 20 mi. in 23 minutes, how far will he fly in 1 hr., at the same rate?

7. Another aviator travels 20 mi. in 23 minutes and 10 seconds. How far will he fly at this rate in 30 minutes?

8. Mary has just enough money to buy 14 yd. of cloth at 30¢ a yard. How many yards can she buy at 35¢ a yard?

9. If the interest on \$475 is \$50, how much is the interest on \$876 at the same rate and for the same time?

10. A certain piece of work can be done by 8 men in 15 days, but the work must be completed in 10 days. How many men must be employed?

11. If a train travels 213 mi. in 5 hr., how far will it travel at the same rate in 7 hr.?

12. If a train travels from Omaha to Chicago in 15 hr. at the rate of 40 mi. an hour, how long would it take if the train traveled at the rate of 60 mi. an hour?

13. A garrison of 250 soldiers consumed 45 barrels of flour in 9 weeks; how long would the same amount of flour last a garrison of 150 men?

14. A garrison of 250 soldiers consumed 45 barrels of flour in 9 weeks. How many barrels of flour would 150 soldiers consume in the same time?

15. In the same time, how many soldiers would consume 63 barrels of flour?

16. If the interest on a certain sum of money is $237 for 2 years, what is the interest on the same sum, at the same rate, for 7 years?

17. In a shipment of 500 doz. bananas, 5 bananas out of every 4 doz. were rotten. How many bananas were rotten? How many dozens were good?

18. A street was paved at a cost of $3,600. The cost was borne by the property owners in proportion to the frontage owned. A owned 100 ft.; B, 250 ft.; C, 350 ft. How much did each pay?

Oral Exercise

133. 1. If any one term in a proportion is an unknown number, how may it be found?

2. Is a proportion an equation? Can it be solved like other equations?

Find the value of x in each of the following:

3. $\frac{x}{3}=\frac{7}{1}$ 4. $\frac{x}{7}=\frac{12}{1}$ 5. $2:3=4:x$

6. $\frac{x}{7}=\frac{105}{3}$ 7. $\frac{144}{x}=\frac{12}{1}$ 8. $1:5=x:25$

9. $\frac{15}{y}=\frac{3}{1}$ 10. $\frac{12}{y}=\frac{3}{2}$ 11. $2:1=x:10$

Written Exercise

134. Solve:

1. $\frac{x}{144}=\frac{1}{2}$ 2. $\frac{y}{64}=\frac{3}{2}$ 3. $\frac{x}{72}=\frac{4}{9}$

4. $\frac{3.2}{2.4}=\frac{y}{36}$ 5. $\frac{3}{2}=\frac{8}{x}$ 6. $\frac{2}{7}=\frac{x}{49}$

7. $3:7.5=y:3.4$ 8. $7.1:6.4=7.5:y$

9. $6.5:x=7.3:10.5$

Written Problems

135. 1. An automobile passed 5 mile posts in 8 minutes and 6 seconds. How many miles an hour was it moving?

2. If a distance can be traversed in 1 hour and 12 minutes, when the rate is 40 mi. an hour, in what time can the same distance be traversed, when the rate is 44 mi. an hour?

3. Separate 150 into parts proportional to 6 and 7.

4. Separate 250 into three parts proportional to 1, 2, and 3.

5. Two partners are dividing their annual profit of $ 5,000 in the ratio of 3 : 4. What does each get?

6. Three partners are dividing a profit of $ 10,000 in the ratio of 2 : 3 : 4. What is the share of each?

7. If a railroad track has a rise of $1\frac{1}{2}$ in. in 100 ft., what rise has it in one mile?

8. If an aëroplane travels 86.5 miles in 78 minutes, how far does it travel in an hour?

9. If the oats of a certain field will last 15 horses 6 months, how many months would it last 21 horses?

10. A boy saves $ 12.50 every 6 months. How many months will it take him to save $ 18?

11. If 5 bags of flour last a boarding house 7 weeks, how many weeks will 9 bags last?

12. If $ 1,200 yield $ 240 interest in a certain time, what interest will $ 950 yield in the same time and at the same rate?

13. If 12 men can do a piece of work in 24 da., in how many days can 19 men do the same piece of work?

14. My property is assessed at $ 3,500 and I pay $ 45.45 a year for taxes. What is the amount of taxes on my neighbor's property which is assessed at $ 4,370?

15. An automobile passed 5 mile posts in 9 minutes and 30 seconds; what was its speed per hour?

16. If $25\frac{3}{4}$ gal. of oil can be extracted from $\frac{3}{4}$ T. of cotton seed, how many gallons of oil can be extracted from $2\frac{1}{3}$ T.?

17. If the yearly interest on a sum of money is $376.75, what is the interest on the same sum for 216 days? [Take 1 yr. = 365 da.]

18. A city's assessed valuation is $5,700,000; the taxes to be raised on this valuation are $83,000. How much are the taxes of a man whose property is assessed at $5,100?

19. A and B are in partnership. A has invested twice as much money as B. At the end of a year they have a profit of $1,250, which is divided in the ratio of their investments. How much does each get?

20. A and B enter into a partnership. A contributes $2,500 and B $4,500. Find the share of each in a profit of $1,200.

SUGGESTION: The ratio of $2,500 to $4,500 is the same as the ratio of 25 to 45, or of 5 to 9. Hence let one man's profit be $5x$, and the other man's $9x$.

21. A, B, and C enter into a partnership. A puts in $1,000, B $1,500, C $2,500. At the end of two years their joint profit is $1,250. How much does each get?

SUGGESTION: The amounts \$1,000, \$1,500, \$2,500 are to each other as 10 : 15 : 25, or as 2 : 3 : 5. Hence let the profits be $2x$, $3x$, and $5x$, respectively.

22. A, B, C invest \$250, \$350, \$550, respectively. They make \$250. What is the share of each?

23. A invests \$1,500 for two years. B invests \$2,500 for one year. Their joint profit is \$750. How much does each get?

SUGGESTION: A's capital of \$1,500 yields in 2 years as much as \$3,000 yield in 1 year. Then solve as if A had put in \$3,000, and B \$2,500, each for one year.

24. A invested \$3,000 in a dry goods business and 3 years later B invested \$4,000 in the same. At the close of the sixth year the business was sold for \$10,000. This sum was divided according to the amount and time of the investments. How much does each partner receive?

Approximation

136. Make approximate computations, and ascertain which of the answers to the following examples are grossly in error:

1. How many times greater were the receipts of the post office department in 1908, when it aggregated \$183,585,006, than in 1850, when it was \$5,499,985.

SUGGESTION: Use 183,600,000 or 184,000,000, and 5,500,000.

$$184{,}000{,}000 \div 5{,}500{,}000 = 1840 \div 55 = ?$$

2. In 1850 the United States produced 110,526 tons of cane sugar; in 1880, 223,719 tons. What per cent is the former amount of the latter?

3. The amount of sugar consumed in 1900 was 2,219,847 tons; in 1910 it was 35% higher. How many tons were consumed in 1910?

4. During the last fifty years there have been recorded 2,793 earthquakes for France and 10,306 for Greece. How many times greater was the number in Greece? What fraction of the former number is the latter?

5. During 50 years the United States has had 5,404 earthquakes; Mexico has had 5,586. What fraction of the latter number is the former?

6. The average rise and fall of the tide is 18 ft. 2 in. in Eastport, Maine, and 1 ft. 1 in. in Galveston, Texas. What fraction is the latter of the former?

7. The earth is 92.8 million miles and the planet Neptune is 2,791.6 million miles from the sun. How many times farther is Neptune?

REVIEW

Written Problems

137. 1. A dealer sells a piano for \$ 270, thereby gaining $\frac{1}{5}$ of the cost of the piano. What did the piano cost?

2. A dealer sells $\frac{1}{2}$ of his coal, and then $\frac{1}{3}$ of it, and has $32\frac{1}{2}$ tons left. How many tons had he originally?

3. How long will it take a person to earn \$ 55.25, if he earns \$ 3.25 a day?

4. Find the cost of a carload of coal weighing 37,900 lb., at \$ 5.25 per ton.

5. Find the cost of 45 tubs of butter weighing 56 lb. each, at $35\frac{1}{2}$ ¢ a pound.

6. A man owns $\frac{3}{5}$ of a vineyard and sells $\frac{2}{3}$ of his share for \$ 1,580. What is the value of the vineyard?

7. A farmer bought a horse and harness. The harness cost $\frac{1}{6}$ as much as the horse, and both cost \$ 350. What was the cost of each?

8. A man receiving a salary of \$ 1,250 a year pays, on an average, \$ 4.50 a week for his board, \$ 12 a month for clothing, and \$ 400 for all other expenses. How much of his salary does he save?

9. In buying property, a man gives his check for \$2,465.40, which is $\frac{3}{5}$ of all the money he has in the bank. How much money has he in the bank?

10. What is the freight charge on 17,550 lb. of iron, at \$1.75 per ton?

11. A dealer bought 9 gross of lead pencils at \$3.75 per gross, and sold them at 5¢ each. What was his gain?

12. A bar of iron 30 ft. long is cut into rods, each $3\frac{2}{3}$ ft. long. How many rods will it make, and what is the length of the remaining piece?

13. If a grocer buys 4 bushels of cranberries at \$2.60 a bushel, and sells them at $8\frac{1}{2}$¢ a quart, does he gain or lose, and how much?

14. How many bushels of beans, at 7¢ a quart, can be bought for \$20.16?

15. How many cubic yards of earth will be needed to raise the grade of part of a road 250 ft. long and 75 ft. wide, if the grade is to be raised 2 ft. 6 in.?

16. Two persons have \$4,560, of which the second has $\frac{2}{5}$ as much as the first. How much has each?

17. The crop of buckwheat from an acre, on an average, is 25 bu. If $\frac{1}{4}$ is destroyed by frost, how many bushels may be expected from 125 acres?

18. A man has $37,580.50 in a bank. He buys 27 cows at $38.50 a head, and 15 horses at $150.75 a head. How much money will he have left, if he pays for the cows and horses out of the sum in the bank?

19. How many loads of earth must be removed in digging a cellar 28 ft. long, 20 ft. 6 in. wide, and 7 ft. 6 in. deep, if each load is a cubic yard of earth? What is the cost of excavating, at $1.25 a cubic yard?

20. If the rim of a bicycle is 12 ft. in length, how many revolutions will the bicycle make in going a distance of $3\frac{1}{4}$ miles?

21. A man leaves $\frac{7}{9}$ of his money to his wife, and the remainder to his daughter. The daughter receives $1,765 less than her mother. What is the share of each?

22. A grain drill sows a strip of ground 5 ft. 6 in. wide. How many times must it pass lengthwise along a field 400 yd. wide and 660 yd. long, to seed the entire field?

23. After selling $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{7}{20}$ of a barrel of sugar, a grocer has 75 lb. left. How many pounds of sugar did the barrel contain?

24. A flock of 300 sheep was bought for $1,050. What was the gain or loss, if 15 of the sheep died and the remainder of the flock was sold at $3.85 per head?

25. Mr. Brown fails in business and he can pay his creditors \$.75 on the dollar. The excess of his liabilities over his resources is \$3,500. Mr. Gregg is given \$1,500 in payment for the amount owed him. How much did Mr. Brown originally owe Mr. Gregg? What was Mr. Brown's full indebtedness to all his creditors?

26. The living expenses of a family of 5 persons is \$375 for 15 weeks. At this rate, what will be the living expenses for 1 year of a family of 6 persons?

27. A farmer raises 510 bushels of apples, Baldwins and Greenings, in the ratio of 8 to 9. How many bushels were there of each kind?

28. The daily receipts in a shop are, per week, \$45.75, \$80.25, \$50.55, \$35.70, \$76.15, \$95.45. What is the average daily receipt?

29. What sum remains out of a yearly income of \$2,250, after a weekly expenditure throughout the year of \$25.35?

30. The expenses of a picnic party of 45 persons were \$38.35; 18 paid \$.75 each, 16 paid \$.85 each. What sum did each of the others pay?

31. Seven men can do a piece of work in $18\frac{1}{2}$ days. How long will it take two men to do the same work?

32. Sugar costing $4\frac{7}{8}$¢ a pound is sold for $6\frac{1}{2}$¢ a pound. What is the profit on 764 lb.?

33. Find the cost of making a cement sidewalk, 25 yd. by 2 yd., at $10\frac{1}{2}$¢ a square foot.

34. A railroad train runs 1 mi. in 1 min. 15 sec. Find its speed in miles per hour.

35. In the construction of an electric line between two towns, 15 miles apart, the road having a double track the whole distance, rails were used which weighed 80 lb. to the yard. How many tons of rails were used in building this road?

36. A box car 40 ft. long, 8 ft. 6 in. wide, 8 ft. high is loaded with corn. How many bushels of corn will it hold, if $1\frac{1}{4}$ cu. ft. are allowed per bushel?

37. Two men rent a pasture for $ 159; one man puts 25 cattle in the pasture for 30 da., and the other 35 cattle for 24 da. How much rent should each pay?

38. A father wishes to divide $ 4,389 among three children, so that the oldest shall receive twice as much as the second oldest, and the second oldest twice as much as the youngest. Find the share of each.

PART TWO

PERCENTAGE AND ITS APPLICATIONS

Review

138. 1. A stock raiser had 600 cattle and sold 30 of them. How many did he sell out of *every hundred?*

2. \$50 out of every \$200 a merchant takes in is clear profit. How much clear profit has he out of *every hundred?* How many *hundredths* or *per cent* does he clear?

The term *per cent* means per hundred or *hundredths,* and is indicated by the symbol %.

5 % is another way of writing .05, $\frac{5}{100}$, 5 hundredths, 5 per cent, or five per cent.

3. Express in three different ways:

$$6\ \%,\ .07,\ \tfrac{8}{100},\ 12\tfrac{1}{2}\ \%,\ .25,\ \frac{33\frac{1}{3}}{100}.$$

4. Express 5 % as a common fraction in its lowest terms.

SUGGESTION: $5\ \% = \frac{5}{100} = \frac{1}{20}$.

Oral Exercise

139. Express as common fractions, integers, or mixed numbers; then as decimals:

	(a)	(b)	(c)	(d)	(e)	(f)
1.	3%	5%	8%	7%	10%	12%
2.	16%	15%	18%	20%	25%	34%
3.	50%	75%	80%	85%	90%	105%
4.	110%	115%	120%	134%	150%	165%
5.	170%	175%	180%	190%	225%	350%
6.	400%	640%	1,000%	$\frac{1}{4}$%	$\frac{1}{2}$%	$\frac{3}{4}$%
7.	$\frac{1}{3}$%	$\frac{2}{3}$%	$\frac{1}{5}$%	$\frac{2}{5}$%	$\frac{3}{5}$%	$\frac{3}{8}$%
8.	$\frac{5}{8}$%	$\frac{7}{8}$%	$\frac{1}{20}$%	$\frac{1}{10}$%	$16\frac{2}{3}$%	$33\frac{1}{3}$%
9.	$66\frac{2}{3}$%	$37\frac{1}{2}$%	$12\frac{1}{2}$%	$8\frac{1}{3}$%	$87\frac{1}{2}$%	$62\frac{1}{2}$%
10.	$40\frac{1}{3}$%	$108\frac{1}{2}$%	$116\frac{2}{3}$%	$133\frac{1}{3}$%	$166\frac{2}{3}$%	

Oral Exercise

140. Express as per cents; then as decimals:

	(a)	(b)	(c)	(d)	(e)	(f)
1.	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{2}{2}$	$\frac{2}{4}$	$\frac{2}{3}$
2.	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{40}$
3.	$\frac{7}{8}$	$\frac{5}{16}$	$\frac{7}{10}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{17}{16}$
4.	$\frac{6}{50}$	$\frac{9}{10}$	$\frac{11}{16}$	$\frac{4}{3}$	$\frac{9}{8}$	$\frac{15}{16}$

Oral Exercise

141. Express as per cents; then as fractions:

	(a)	(b)	(c)	(d)	(e)	(f)
1.	.12	.16	.25	.50	.75	.90
2.	$.16\frac{2}{3}$	$.33\frac{1}{3}$	$.66\frac{2}{3}$	$.37\frac{1}{2}$	$.87\frac{1}{2}$	$.90\frac{1}{4}$
3.	$.05\frac{1}{2}$	$.08\frac{3}{4}$	$.02\frac{1}{2}$	1.12	1.25	1.35
4.	1.50	1.85	1.30	2.50	3.25	4.75
5.	5.35	6.80	$1.37\frac{1}{2}$	$3.66\frac{2}{3}$	$10.16\frac{2}{3}$	20.75

Fundamental Relations

142. The **rate** or **rate per cent** is the number of hundredths taken.

The **base** is the number of which some per cent is to be found.

The **percentage** is the result obtained by finding a given per cent of the base.

Thus, in 6 % of 500 = 30, we see that 6 is the number of hundredths taken, or the rate; 500 is the base; 30 is the percentage.

Since 8 % of 600, or .08 × 600, = 48, we see that the percentage is computed on the following principle:

> **Percentage is equal to the base multiplied by the rate.** Here percentage is the product, the base and rate are factors.

Given the product of two factors and one of the factors, how is the other factor found?

The rate (one factor) is equal to the percentage (the product) divided by the base (the other factor).

The base (one factor) is equal to the percentage (the product) divided by the rate (the other factor).

$$\boxed{\begin{array}{c} \textit{Percentage} = \textit{base} \times \textit{rate}, \text{ or } p = b \times r; \\ \textit{Rate} = \dfrac{\textit{percentage}}{\textit{base}}, \text{ or } r = \dfrac{p}{b}; \\ \textit{Base} = \dfrac{\textit{percentage}}{\textit{rate}}, \text{ or } b = \dfrac{p}{r}. \end{array}}$$

In these formulas b is written for *base*, r for *rate* (expressed decimally) and p for *percentage*.

Study Exercise

143. Another way of obtaining the last two formulas from the first is by the method of equations.

When we know the base and the percentage, and are required to find the rate, then we look upon r as our unknown quantity x, and upon b and p as given numbers. To solve for r, we divide both sides of $p = b \times r$ by b, then

$$\frac{p}{\cancel{b}} = \frac{\cancel{b} \times r}{b} = r, \text{ or, changing sides, } r = \frac{p}{b}.$$

When we know the rate and the percentage, and are required to find the base, then we look upon b as our unknown quantity (x) and upon p and r as given numbers. To solve for b, we divide both sides of $p = b \times r$ by r, then

$$\frac{p}{r} = \frac{b \times \cancel{r}}{\cancel{r}} = b, \text{ or, changing sides, } b = \frac{p}{r}.$$

It is important that this reasoning be mastered. When the process is understood it is easy to derive the second and third formulas from the first formula. The first one is easily remembered, and the two others, if forgotten, can be easily obtained from the first by a student who understands the equation. The use of equations makes some parts of arithmetic much easier to understand and to remember.

Oral Exercise

144. Find the percentage:

1. 6 % of \$ 400	2. 7 % of \$ 110	3. 8 % of \$ 120
4. 9 % of 500 T.	5. 1 % of 730 yd.	6. 2 % of 370 lb.
7. 3 % of 800 A.	8. 6 % of 210 yd.	9. 7 % of \$ 310

Written Exercise

145. 1. Find 25 % of 800.

SUGGESTION: $25\,\% = \frac{1}{4}$.

It is easier to multiply by $\frac{1}{4}$ than by .25.

$\frac{1}{4} \times 800 = 200$, the answer.

Whenever it is more convenient, express the rate as a common fraction in its lowest terms.

Find:

2. 10% of 480
3. 25% of 400
4. 50% of 520
5. $33\frac{1}{3}$% of $330
6. 75% of 800 lb.
7. $12\frac{1}{2}$% of 160 lb.
8. $66\frac{2}{3}$% of $900
9. 25% of 700 lb.
10. 50% of 345 da.

Oral Exercise

146. 48 is 6% of what number?

PROCESS AND EXPLANATION

Here 48 is the percentage, 6% is the rate, and we are to find the base. In our equation $p = br$, we know that $p = 48$ and $r = .06$; we must find b. Dividing both sides of the equation by r, we obtain $b = \frac{p}{r}$. Writing for p and r their values, we obtain $b = \frac{48}{.06}$, or $b = \frac{4800}{6} = 800$.

147. Find the base when

1. 60 is 6%
2. 35 is 5%
3. 63 is 7%
4. 72 is 8%
5. 88 is 8%
6. 20 is 5%
7. 200 is 5%
8. 33 is 3%
9. 50 is 2%
10. 150 is 15%
11. 200 is 4%
12. 84 is 7%

Oral Exercise

148. Using the rate expressed as a common fraction, find the base when

1. 50 is 25 %	2. 50 is $12\frac{1}{2}$ %	3. 120 is $33\frac{1}{3}$ %
4. 267 is $66\frac{2}{3}$ %	5. 450 is 50 %	6. 180 is 75 %
7. 88 is $12\frac{1}{2}$ %	8. 84 is 25 %	9. 64 is $66\frac{2}{3}$ %

Written Exercise

149. Find the rate.

1. What per cent of \$ 500 is \$ 35?

Process and Explanation

Here \$ 500 is the base, \$ 35 is the percentage; we must find the rate. In our equation $p = br$, we know that $b = 500$, $p = 35$; we are to find r. Dividing both sides of the equation by b, we get $r = \frac{p}{b}$. Writing for p and b their values, we obtain $r = \frac{35}{500} = \frac{7}{100} = .07$. Hence the rate is 7 %.

What per cent of

2. 500 is 50?	3. 660 is 6.6?
4. 700 is 49?	5. 350 is 21?
6. 1,000 is 100?	7. \$ 3,000 is \$ 30?
8. 4,000 lb. is 80 lb.?	9. 900 A. is 72 A.?
10. 300 mi. is 100 mi.?	11. 400 bu. is 100 bu.?
12. 800 doz. is 600 doz.?	13. 400 is 500?

Drill Device

150. Find the missing part:

	Base	Rate %	Percentage		Base	Rate %	Percentage
1.	360	5	p	**17.**	b	$66\frac{2}{3}$	10
2.	700	r	49	**18.**	60	80	p
3.	b	11	121	**19.**	b	$33\frac{1}{3}$	100
4.	1000	r	150	**20.**	65	r	39
5.	3000	$6\frac{1}{3}$	p	**21.**	b	50	500
6.	b	4	160	**22.**	240	$62\frac{1}{2}$	p.
7.	50	6	p	**23.**	b	10	48
8.	350	r	35	**24.**	b	75	60
9.	240	$12\frac{1}{2}$	p	**25.**	40	60	p
10.	b	$37\frac{1}{2}$	120	**26.**	b	40	24
11.	500	r	130	**27.**	b	$12\frac{1}{2}$	100
12.	b	5	6	**28.**	60	40	p
13.	1500	r	900	**29.**	b	75	600
14.	18	$66\frac{2}{3}$	p	**30.**	96	r	72
15.	999	$66\frac{2}{3}$	p	**31.**	b	$33\frac{1}{3}$	30
16.	25	r	10	**32.**	4	r	$\frac{3}{5}$

Written Exercise

151. 1. Find $27\frac{1}{3}\%$ of 714.45.

PROCESS

$$\begin{array}{r} 714.45 \\ .27\frac{1}{3} \\ \hline 23815 \\ 500115 \\ 142890 \\ \hline 195.2830 \end{array}$$

2. Find 95% of $618.15.

3. How much is $131\frac{1}{4}\%$ of 24.68?

4. Compute $213\frac{1}{5}\%$ of $31.60.

5. What is $\frac{1}{2}\%$ of $2,557.37?

6. The base is \$375, the rate is $3\frac{1}{2}\%$. What is the percentage?

7. The rate is $4\frac{1}{2}\%$, the percentage is 90. What is the base?

8. The percentage is 180, the base is \$9,000. What is the rate?

Written Exercise

152. Find the missing term:

	Base	Rate	Percentage		Base	Rate	Percentage
1.	163.78	r	87.65	**11.**	975.75	6 %	p
2.	b	$13\frac{1}{5}\%$	127.65	**12.**	b	$3\frac{1}{2}\%$	\$15.93
3.	b	$37\frac{1}{2}\%$	864.15	**13.**	\$900.50	r	\$3186
4.	1960.35	r	965.35	**14.**	\$704.32	r	\$2.64
5.	\$450	6 %	p	**15.**	575.30 bu.	$\frac{3}{4}$	p
6.	\$1275.50	$\frac{1}{8}\%$	p	**16.**	945.34 sq. ft.	$24\frac{1}{8}\%$	p
7.	b	$\frac{1}{2}\%$	50	**17.**	\$6254	$62\frac{1}{2}\%$	p
8.	b	$\frac{3}{8}\%$	\$875	**18.**	\$3127	r	\$18.54
9.	\$100	r	\$.50	**19.**	\$93.81	r	\$37.08
10.	\$1000	r	\$5.00	**20.**	b	$12\frac{1}{2}\%$	\$250

Written Problems

153. 1. After a severe winter storm it was found that 300 sheep of a flock of 2000 had been killed. What per cent of the original flock perished?

2. In a large collection of pennies 7 %, or 21, bore the date of 1909. How many pennies were there in the collection?

3. In the transmission of electric power in a certain plant it was found that 20 % of the power, or 120 kilowatts, were lost in transmission. How many kilowatts of power were generated?

4. At the top of Pikes Peak the temperature of boiling water is 28° F. lower than at sea level, where it is 212° F. About how many per cent lower is it on top of the mountain than at sea-level?

5. A man weighing 160 lb. at the beginning of a vacation gains 5 % in weight during vacation. How much did he weigh at the end of his vacation?

6. The Sault Ste. Marie Canal which connects Lake Superior and Lake Huron is $28\frac{4}{7}$ % deeper than the Welland Canal, connecting Lake Ontario and Lake Erie. How deep is the former canal, if the latter is 14 ft. deep?

7. The Miami and Erie Canal at Cincinnati has 30.8 % fewer locks than the Ohio Canal which connects Cleveland and Portsmouth and has 150 locks. How many locks has the former canal?

8. What per cent of the length of the Suez Canal (90 mi. long) is the Manchester ship canal ($35\frac{1}{2}$ mi. long), connecting Manchester and Liverpool?

9. The bottom of the Suez Canal is 108 ft. wide. What per cent is this of the width of the Kaiser Wilhelm Canal (72 ft. wide), connecting the Baltic and North Seas?

10. Find how many per cent heavier steel is than iron, if iron weighs 468 lb. per cubic foot and steel 492 lb.

11. One year the Bell Telephone Co. had 4,889 exchanges and branch offices; the year following it had 5,108. What was the rate per cent of increase over the number for the first year?

12. The number of newspapers published in the United States a few years ago was 22,500, of which 1600 were weeklies and 2,700 monthlies. What per cent of the total were weeklies? What per cent monthlies?

13. A few years ago the state of New York, with a population of 9,100,000, had 1900 newspapers. What per cent of the population was this number of newspapers?

14. Maine has enrolled in its public schools 135,000 pupils, which is 18.4% of its population. Find the population of Maine.

15. One year the total value of telephone apparatus manufactured in the United States was $10\frac{1}{2}$ million dollars; 5 years later it was 16 million dollars. What was the rate per cent of growth?

16. In the United States there were sent in 1870, 9,200,000 telegraphic messages. In 1907 this number was 878% larger. How many telegraphic messages were sent in 1907?

17. The railroads of the United States carried one year 874,000,000 passengers. Of these, 11,839 were killed. Find the rate per cent of those who were killed. In round numbers, how many of every million passengers were killed?

18. In Ex. 17 compute the ratio of the number killed to the total number of passengers.

19. The average amount of money in circulation in the United States was $5 per capita in 1800 and $34.81 per capita in 1908. What per cent is the latter amount of the former amount?

20. The public debt of the United States, per capita, was $15.63 in 1800, and $10.76 in 1908. By how many per cent of the original amount, per capita, has the debt diminished?

21. The population, per square mile, was 6.41 in 1800 and 29.71 in 1910. Find the rate per cent of increase.

22. The annual copper production in the United States is 398,000 T. The production in 1850 was less than 17 % of this amount. Find it in tons.

23. The exports of merchandise, per individual, were, in 1800, $13.37, and in 1910, $22.04. What per cent is the latter sum of the former?

24. The area of the United States in 1800 was 27 % of the area of the continental United States at present. What was it then, if now it is 3,027,000 sq. mi.?

25. The population in the continental United States was 5,300,000 in 1800 and 92,000,000 in 1910. What per cent is the former of the latter?

26. In 1880 the manufacture of steel was 1,250,000 tons. What was it in 1910, if it had *increased* 1,770%?

27. Between 1800 and 1910 the number of United States post offices increased from 903 to 62,800. The population increased from 5,300,000 to 92,000,000. Has the per cent of post offices to the population increased or decreased?

APPLICATION OF PERCENTAGE

Introduction

154. The subject of Percentage finds wide application in business. The term **per cent** is used extensively, but the words **percentage** and **base** are often displaced by other terms designating more specifically the nature of the transaction. Thus, in place of percentage we shall encounter the terms **profit, loss, commission, brokerage, discount, premium, taxes, interest, and duty. Statistics** of all kinds, as school attendance, population increase or decrease, production, etc., are often expressed in terms of percentage. In place of base we speak of the **principal, cost,** etc.

In these applications few new principles are involved. As soon as we are able to say what takes the place of *base*, what of *percentage*, then we simply apply the knowledge of percentage already acquired. It is helpful to see that these topics, which may first seem independent of each other, are really closely related and are merely different applications of the fundamental processes of percentage. The equation $p = b \times r$ covers nearly all cases. Thus, we shall find that nearly all the new problems are actually old ones that spring up under some new guise.

Study Exercise

155. In this exercise, notice what is taken as the base, what as the percentage.

Topic	Base	Percentage
Profit and loss	Buying price	The profit or the loss
Commission	Value of goods bought or sold	Commission or brokerage
Taxation	Value of property taxed	Taxes
Insurance	Sum for which property is insured	Premium
Commercial discount	Price of article	Deduction from price
Bank discount	Face of the note	Discount (interest) for unit time
Interest	Sum bearing interest	Interest for unit time
Tariff	*Ad valorem*, or invoiced cost of goods as 15% on cost. Specific, quantity of importation	Duties

Topic	Base	Percentage
	regardless of value as $1.50 per gallon, $5.00 per ton.	
Statistics	Population Industrial Educational Social, etc	Rate of gain or loss, etc., for unit of time, group, etc., expressed in per cent.

PROFIT AND LOSS

Study Exercise

156. We have seen that in problems under this head, the buying price of goods is taken as the base, the gain or loss is taken as the percentage.

Merchants often adopt some private marks to indicate the cost and the selling price of goods. The following illustrates one of the methods employed. Some word of ten different letters, like *IMPORTANCE*, may be taken as the *key*, thus

I	M	P	O	R	T	A	N	C	E
1	2	3	4	5	6	7	8	9	0

By this device the cost and selling price can be read by salesmen who know the key, but not by others.

To make it more difficult to detect the key, some letter, say k, is used, when a digit is repeated in a number. We call k the *repeater*.

If the purchasing price is \$2.45, and the selling price is \$3.55, then the goods may be marked $\dfrac{mor}{prk}$, where the upper letters indicate the cost and the lower indicate the selling price.

Written Problems

157. 1. Goods are marked $\frac{mre}{pmr}$. What will be the gain? What per cent of the cost will the profit be?

2. Compute the profit, and the rate per cent of the profit, on goods marked thus: $\frac{rek}{aek}$.

3. How must the selling price of goods be marked, to allow a profit of 50%, if the cost is *mkt*?

4. If the selling price is marked *pre*, and the profit is $1.50, find the cost and the rate per cent of the profit.

5. What per cent of the cost *pke* is the selling price *oke*? What per cent of the cost is the gain?

6. The first cost of certain goods is $2.50; freight charges are 10% of this. How should the goods be marked, in order that a profit of 20% on the total cost may be realized?

7. If the first cost is $1.00, freight charges 20% of the first cost, and the profit 50% of the total cost, what is the selling price?

8. The selling price on an article is 35¢. What is the cost, if there is a 25% gain on the cost?

PROCESS USING DECIMALS. EXPLANATION

Let the cost be x cents, then the profit is the *cost multiplied by* .25, or $.25\,x$. Adding the profit to the cost, we get $.25\,x + 1\,x$, which, combined, is $1.25\,x$. But the profit plus the cost gives the selling price. This yields the equation

$$1.25\,x = 35.$$

Hence $x = \dfrac{35}{1.25} = 28.$ The cost is 28¢.

In this example it is easier to use common fractions. Take $25\,\% = \frac{1}{4}$.

PROCESS USING FRACTIONS

Then the profit is	$\frac{1}{4}\,x$ cents
The cost is	$1\,x$ cents
The profit plus the cost is	$\frac{5}{4}\,x$ cents
Hence we obtain	$\frac{5}{4}\,x = 35.$
Multiplying both sides by 4,	$5\,x = 35 \times 4.$
Dividing both sides by 5,	$x = \dfrac{\overset{7}{\cancel{35}} \times 4}{\cancel{5}} = 28.$

9. What is the cost, when the selling price is \$ 2.20 and the rate of profit is 10 % ?

10. Eggs retail at 35¢ a dozen. At what price must they be bought wholesale, that the grocer may realize 40 % on the cost ?

11. If salt water weighs 3 % more than fresh water, how much would the fresh water in a tub weigh which will hold exactly 309 lb. of salt water? How much salt is there in 309 lb. of salt water?

12. A merchant is forced to sell damaged articles at a loss of $33\frac{1}{3}\%$. What was the cost of articles which he sells for \$1.60?

13. A patient lost 10% of his weight during an illness, his weight being reduced to 135 lb. How much did he weigh before his illness?

14. A general lost 5% of his men in a battle and had 4750 men left. How many men did he have at first? How many did he lose?

15. What is the cost of an automobile that sells at \$1300, yielding the agent a rate of profit of 30%?

16. At what price must hats costing \$65.25 be sold to yield a profit of 35%?

17. 400 pairs of children's shoes cost \$700. What price must be put on each pair to gain 60%?

18. What is the cost of merchandise which yields 35% profit, when sold for \$137.45?

19. A farmer bought a cow and then sold it at a profit of $11\frac{1}{9}\%$, the selling price being \$50. Find the cost.

20. Make a list of the manufactured articles made in the neighboring manufacturing centers. Use the prices listed in the manufacturers' catalogues and the discounts quoted there. From these facts make a series of problems in trade and discount. Solve these problems.

Written Exercise

158. Find the cost. Use business fractions when possible.

	Selling Price	Rate Per Cent on Cost	Profit or Loss
1.	$75.35	$12\frac{1}{2}$%	Profit
2.	$125.75	$66\frac{2}{3}$%	Profit
3.	$435.60	$16\frac{2}{3}$%	Loss
4.	$67.50	$83\frac{1}{3}$%	Profit
5.	$1,274.40	25%	Loss
6.	$760.50	$8\frac{1}{3}$%	Profit
7.	$675.60	$37\frac{1}{2}$%	Loss
8.	$1,760.25	4%	Loss
9.	$87.55	75%	Profit
10.	$196.45	60%	Loss
11.	$768.00	$14\frac{2}{7}$%	Loss
12.	$7.63	50%	Profit
13.	$786.50	15%	Loss
14.	$6,000	$3\frac{1}{3}$%	Loss

COMMISSION AND BROKERAGE

Introduction

159. Fruit, farm products, and other articles of produce, when sent from the country to the city, are usually bought or sold through *agents*. Some varieties of manufactured articles are bought and sold in the same way.

An agent usually charges a per cent of the amount bought or sold as his fee. If the goods which he sells are actually sent to him, so that he has possession of them, he is usually called a **commission merchant**. If he arranges for the purchase or sale of goods without having them shipped to him, he is usually called a **broker**. The one who sends the merchandise to be sold is the **shipper** or **consignor**.

Oral Problems

160. 1. An agent collected a bill of $450 and received 6% of this sum for his services. What was the amount of his commission?

2. A commission merchant buys $110 worth of turkeys for a dealer and receives $5\frac{1}{2}$% for his services. How much commission does he receive?

3. An agent receives \$15 a week, plus 5% of his sales. If he sold \$350 worth of goods during a week, how much was his income for that week?

4. A commission merchant bought 20 barrels of Baldwin apples at \$3.50 a barrel for a grocer and was allowed 7% commission. How much was the commission?

5. How much do I make as an agent if I sell 25 acres at \$100 each and receive 3% for my services?

6. I buy through a broker 1000 bu. of wheat at 80¢ each and pay $\frac{1}{5}$¢ per bushel brokerage. How much does the wheat cost me?

7. An agent received \$15 for selling goods at 5% commission. What was the amount of his sale?

8. For selling 1000 bu. of wheat at \$1 each a broker receives \$10. Find the rate per cent charged by him.

9. I collect a bill for my employer and send him the sum, less 3% commission. My commission is \$9.00. How much do I remit to my employer?

Written Problems

161. 1. An agent collected 60% of an account of \$950, charging 5% commission. How large was his commission and what sum did he pay over?

2. How much does a broker receive for selling 1,800 bales of cotton at 20¢ a bale?

3. What are the net proceeds of 100 boxes of Florida oranges at $3.25 a box, commission $2\frac{1}{2}\%$?

4. A commission merchant, named William Fay, sells apples for George Bacon and sends him the following statement:

OMAHA, NEB., *March 16*, 19*11*

SALE OF MERCHANDISE FOR ACCOUNT OF
George Bacon, Lincoln, Neb.

By William Fay

March	*7*	*200 bbl. Ben Davis Apples*	*3.00*			*600*	*00*
"	*11*	*150 bbl. Russet Apples*	*2.75*			*412*	*50*
		CHARGES					
"	*11*	*Freight and drayage*		*20*	*50*		
"	*16*	*Commission 5%*		*50*	*63*		
"	*16*	*Net proceeds remitted*		*941*	*37*		
				1012	*50*	*1012*	*50*

Name the amount sold, also the selling price, date of sale, the charges for transportation, the pay which Fay received for his services, and the sum received by George Bacon from Fay for the apples.

5. A commission merchant sells for a certain person 150 doz. eggs at 30¢ a dozen and 150 lb. of beef at $19.50, commission 4%. He paid $46 freight. Make out a statement like the one in the preceding example, to be sent by the commission merchant to his employer.

6. An agent sells six automobiles in seven months, at $1,675 each and receives a commission of 12%. How much did he make per month?

7. A dealer in agricultural implements sold one season 7 harvesting machines @ $120, 8 mowing machines @ $90. The cost of advertising and delivery was $48.75. What was his net profit, if he was allowed 15% commission?

8. (*a*) Make a statement for goods sold on commission. State commission, incidental expenses, as packing, insurance, freight and drayage, net proceeds remitted, and gross receipts.

(*b*) Make a series of problems in commission, with goods sold thus in local market.

(*c*) Solve these problems, showing total gross sale, commission, net proceeds, etc.

Written Exercise

162. Find the commission paid for selling the following merchandise. Find the net proceeds of the sale.

	Article	Selling Price	% Commission	Freight, etc.
1.	200 bbl. flour	$6.10	4%	$15
2.	160 bx. lemons	$4.15	$3\frac{1}{2}$%	$11.50
3.	5,600 lb. cotton	$10\frac{1}{3}$¢	$2\frac{1}{2}$%	$47.50
4.	270 geographies	67¢	25%	$9.75
5.	2,000 bu. corn	63¢	$\frac{1}{2}$%	$46.35
6.	2,000 gross screws	37¢	2%	$13

Written Exercise

163. What is the rate of commission paid to an agent when he receives:

1. \$68 on \$1,700.
2. \$18.75 on \$375.
3. \$119.47+ on \$796.50.
4. 7.12+ on \$5,700.
5. \$400 on \$80,000.
6. \$90 on \$27,000.

Written Problems

164. 1. A library of 2,000 books was sold at auction at the average price of 45¢ a volume. What were the net receipts, if the cost of advertising was \$11.35, the cost of storage \$9.75, and the commission $2\frac{1}{2}\%$ of the selling price?

2. A broker buys potatoes at 1% commission. How much did he buy, if his commission was \$13.50?

3. How many bushels of corn at 90¢ a bushel did a broker buy at 3% commission, if his commission was \$18.90?

4. A boy earns \$24 a month collecting electric light bills. If his commission is $\frac{3}{4}\%$, how much does he collect in a month? How many bills does he collect, if they average \$4 each?

5. A land agent receives \$30,000 with which to buy land. After deducting his commission of 5% on the amount sent, how many acres of land can he buy at \$100 per acre?

The present tendency in business is to charge commission on the whole amount of money sent, as in the last example. The older practice is to charge only on the amount actually expended upon the purchase, as in the following example.

6. I send an agent \$ 103 with orders to buy rubber overshoes, after deducting his commission of 3 % on what he buys. What amount will be spent for the rubbers?

7. How many pounds of coffee at 30¢ per pound can be purchased for \$ 625, if the agent is allowed 4 % on the amount of money sent him?

8. I sent my agent \$ 1,122.45 with which to buy corn at 55¢ a bushel after deducting his commission of 2 % on the amount sent. How much corn does he buy?

9. A commission merchant sells 250 doz. eggs at 24¢ a dozen and charges 6 % commission. How much money does he send to his employer, if the charges for storage are \$ 1.75? Make out a statement as in Ex. 4.

10. My agent sells my four bungalows for \$ 2,700 each and sends me a check for \$ 10,260. What rate of commission is he charging?

11. A dealer in New Orleans buys through a broker in St. Louis 4,800 bbl. of flour at \$ 3.90 a barrel, commission 2 %. The freight to New

Orleans cost \$805.25, the cartage \$97.50. Find the total cost and the cost per barrel in the warehouse in New Orleans.

12. My agent sells for me 550 bunches of celery at \$.75 each and charges 10% commission. How large a check will he send me?

13. I sent my agent a check for \$195.30 and asked him to buy oysters at \$1.50 a barrel. Out of the sum remitted he kept his commission of 5% on what he bought. How much did he spend on oysters? How many barrels did he buy?

14. A merchant sends a broker \$546 to buy beans. After deducting his commission of 4%, how many bushels of beans can the broker buy at \$2.10 each?

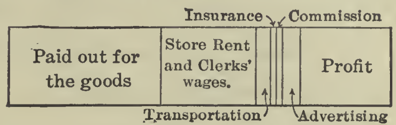
15. If an agent charges 7% for collecting debts and in one week has collected \$375.75, what is his commission?

16. This same agent finds that his commission from this same source, during the week following, is \$39.48. How much money did he collect?

17. 25 tons of California prunes were sold to net grower $8\frac{1}{2}$¢ per pound. Commission was $2\frac{1}{4}$%, and insurance $\frac{1}{6}$%. What were the gross receipts from the sale of the prunes?

Graphic Exercise

165. A merchant finds that the total gross receipts in his store, for the past year, have been $11,200. That he may be better able to see the various sources of expense and his profit, he draws this figure. The entire rectangle represents the gross receipts. Which has been his heaviest expense? Which his smallest? About how much of the gross receipts is clear profit? Measure to the nearest $\frac{1}{16}$ of an inch and compute the following:



1. How much did the merchandise sold last year cost the merchant?

2. How much did he pay out for store rent and wages?

3. Find the cost of transportation.

4. What did he pay out for insurance? For commission?

5. Find the expense of advertising.

6. Compute his net profit.

7. What rate of commission did he pay on the goods purchased?

8. What per cent of the gross receipts was clear profit?

9. What per cent of the gross receipts was the cost of the goods (omitting commission)?

10. What per cent of the gross receipts went out for store rent and wages?

11. How much greater would his clear profit have been, had he paid no insurance?

12. If the store rent and wages had been reduced $\frac{1}{7}$, what would the profit have been, supposing the sales unchanged?

TRADE DISCOUNT

Introduction

166. 1. In business we speak of two kinds of discount, *trade discount* and *bank discount.* The two terms mean different things. *Bank discount* is what a bank charges for paying a note before it is due. *Trade* or *commercial discount* is a reduction made by merchants or manufacturers from catalogue prices. Frequently there are allowed several trade discounts, forming a *discount series,* as for instance, "25 %, 10 %, 5 %." In this case compute the first discount of 25 %, taking the list price of the goods as the base; then compute the second discount of 10 % on the remainder after deducting the first discount; finally compute the 5 % discount on the second remainder as a base.

2. Why do merchants use discount series, rather than a single discount on the catalogue prices? The reason is that the discount series enables a wholesale house more easily to adjust itself to changes in the cost of production and to fluctuations in the market. If catalogue prices were adhered to; it would be necessary to print a new catalogue every time there is a change in price. Instead of following this slow and expensive pro-

cess, the house publishes prices in the catalogue frequently higher than the actual selling price and then prints lists of discounts for its customers, whenever the prices change. The price obtained after deducting all discounts is called the *net price.*

3. In order to meet the new conditions of trade, a firm announces several commercial discounts on pianos. Thus, a piano listed at $300 may be sold at 30% and 10% off. Find the net price.

PROCESS AND EXPLANATION

30% of $300 is $90. This reduces the price to $210. The second discount of 10% is on the $210, and amounts to $21. Deducting $21 from $210 leaves $189, which is the net price on the piano.

4. Show that the net price on the piano in Ex. 3 would have been the same had the discounts been taken in the opposite order of, first 10%, and then 30% of the remainder.

NOTE.—*Cash* discounts are sometimes allowed for cash payments. Invoices may read "*Terms 30 days net,* or 3% cash." *Time discounts* for time payments as "5% 10 days" etc., are not uncommon.

Written Exercise

167. Find the net price:

	LIST PRICE	TRADE DISCOUNT		LIST PRICE	TRADE DISCOUNT
1.	$10	25%	**2.**	$21	$33\frac{1}{3}$%
3.	$6	$16\frac{2}{3}$%	**4.**	$150	20%
5.	$450	20%	**6.**	$640	$12\frac{1}{2}$%

Written Exercise

168. Find the selling price or net price on goods listed and discounted as follows:

1. \$100, 20 %, 10 %
2. \$200, 20 %, 10 %
3. \$300, 10 %, 10 %
4. \$400, 25 %, 10 %
5. \$500, 20 %, 10 %
6. \$100, 30 %, 10 %
7. \$100, 10 %, 5 %
8. \$300, $33\frac{1}{3}$ %, 10 %
9. \$300, 10 %, 5 %
10. \$600, $33\frac{1}{3}$ %, 25 %

Written Problems

169. 1. The list price of 12 doz. knives is \$35, but this price is subject to a discount of 20 % and 10 %. Find the net price.

2. Which is the lower net price, \$100 discounted at 20 % and 10 %, or \$100 discounted at 30 %?

3. What is the difference in net price between \$100 discounted at 20 % and 10 %, and \$100 discounted at 25 %?

4. Rather than sell at a discount of 40 %, hardware listed at \$100, a merchant allows a discount series of 20 % and 20 %. How much does he save thereby?

5. An automobile is listed at \$2,000, but a discount series of 20 % and 5 % is allowed, with an additional discount of 2 % for payment within 10 days. What is the selling price if you can pay within 10 days? If payment must be deferred?

6. A publisher offers a certain book at $2.65, subject to a discount of 20%. How many copies can be bought for $99.64?

7. The list price of silk is $3.70 a yard, but it is subject to 20% and 15% discount. Find the net price per yard. How many yards can be bought for $148.68?

8. A jobber bought merchandise at a discount of 25% and 10% below the list price and sold the same at 30% and 5% discount below the same list price. Did he gain or lose?

9. I can buy from one firm a piano listed at $300, less a discount series 10% and 7%. From another firm I can buy a piano of the same make, listed at $300 less the discount series 12% and 5%. What is the net price in each case?

10. What single rate of discount on the gross price is equivalent to a discount series of 20%, 15%?

Process and Explanation

Take a list price of $1, or 100¢. After the first discount there remain 80¢. Subtracting from 80¢ 15% of 80¢ (or 12¢) we obtain 68¢ as the second remainder. But 100¢ − 68¢ = 32¢; that is, the reduction is 32¢ on 100¢, or 32%.

11. What single rate of discount is equivalent to a discount series of 35% and 25%?

12. Which is a better offer, a single discount of 45 %, or a discount series of 25 % and 25 % ?

13. A merchant buys hats at $ 1 each and marks them at a price that would yield him a profit of 50 % on the cost. To stimulate sales he later advertises them at 20 % off the marked price. For how much can these hats be bought now?

14. How much more would his profit have been, if he had sold the hats at 30 % above their cost?

15. Find the profit on goods, costing $ 500, that were marked 40 % above cost and were sold at 15 % off the marked price.

16. Which is more profitable to a merchant, to sell goods that cost him $ 1 at 20 % off a marked price that is 50 % above cost, or to sell at 10 % off a marked price that is 40 % above cost?

17. Find the per cent of profit *on the cost,* when goods costing $ 1 are marked 50 % above cost and then sold 10 % below the marked price.

18. Find out what discounts are allowed by merchants where you live and construct problems based on these discounts.

19. Obtain catalogues of local manufacturing companies and the latest price list of goods issued by the companies. From these catalogues and price lists make an order for goods, estimate the cost, and deduct the discount.

Written Exercise

170. Find the selling price of the following:

	Cost	Marked Price above Cost	Discount on Marked Price	Selling Price
1.	\$200	50 %	10 %	——
2.	\$300	40 %	15 %	——
3.	\$500	30 %	$12\frac{1}{2}$ %	——
4.	\$600	$33\frac{1}{3}$ %	5 %	——
5.	\$12,000	$12\frac{1}{2}$ %	3 %	——
6.	\$4.50	$62\frac{2}{3}$ %	30 %	——

SIMPLE INTEREST

Introduction

171. Interest is money paid for the use of money.

In computing interest one must consider the *time* during which the money has been borrowed. Except for this element of time, problems in interest are simply problems in ordinary percentage.

The **principal** is the sum of money on which interest is paid.

The **rate of interest** is the per cent, the number of hundredths, of the principal paid for its use for *one year*.

Find the interest on \$400 at 5 % for 3 yr.

PROCESS AND EXPLANATION

Interest for 1 yr. $= .05 \times \$400 = \$20$.

Interest for 3 yr. $= 3 \times \$20 = \$60$.

In this example the principal is \$400, the interest is \$60.

The *amount* is $\$400 + \$60 = \$460$.

The *amount* is the sum of the principal and the interest.

$$\boxed{\textit{Interest} = \textit{Principal} \times \textit{Rate} \times \textit{Time}}$$

Interest for Years and Months

172. Find the interest.

	Principal	Rate	Time		Principal	Rate	Time
1.	$100	6 %	2 yr.	11.	$150	4 %	3 yr.
2.	$200	5 %	3 yr.	12.	$250	2 %	10 yr.
3.	$300	4 %	5 yr.	13.	$700	6 %	6 mo.
4.	$500	6 %	$2\frac{1}{2}$ yr.	14.	$400	7 %	2 yr.
5.	$700	5 %	4 yr.	15.	$300	4 %	10 yr.
6.	$800	3 %	$3\frac{1}{4}$ yr.	16.	$900	5 %	5 yr.
7.	$900	7 %	2 yr.	17.	$1,200	4 %	8 yr.
8.	$1,000	8 %	3 yr.	18.	$1,100	5 %	2 yr.
9.	$50	5 %	2 yr.	19.	$600	6 %	6 mo.
10.	$5	6 %	2 yr.	20.	$400	6 %	1 mo.

173. Find the approximate interest:

	Principal	Rate	Time		Principal	Rate	Time
1.	$ 295	6 %	2 yr.	8.	$ 703	7 %	2 yr.
2.	$ 198	5 %	2 yr. 1 mo.	9.	$ 205	7 %	20 yr.
3.	$ 990	4 %	3 yr. 1 mo.	10.	$ 199	6 %	15 yr.
4.	$ 154	6 %	10 yr.	11.	$ 497	4 %	6 yr. 4 mo.
5.	$ 305	5 %	3 yr.	12.	$ 563	5 %	2 yr.
6.	$ 507	5 %	2 yr. 11 mo.	13.	$ 400	10 %	15 da.
7.	$ 601	7 %	4 yr.	14.	$ 709	3 %	2 yr. 10 mo.

Written Problems

174. Find the amount:

1. \$300 at 5 %, 2 yr.
2. \$400 at 6 %, 4 yr.
3. \$500 at 3 %, 6 yr.
4. \$700 at 7 %, 2 yr.
5. \$1,000 at $6\frac{1}{2}$ %, 1 yr.
6. \$2,000 at $4\frac{1}{2}$ %, 5 yr.
7. \$1,200 at 4 %, 3 yr.
8. \$2,000 at 5 %, $5\frac{1}{2}$ yr.
9. \$3,000 at 6 %, 1 yr.
10. \$200 at $5\frac{1}{2}$ %, 7 yr.
11. \$300 at 7 %, 3 yr.
12. \$500 at 5 %, 5 yr.

Written Exercises

175. 1. Find the interest on \$423 at 4 % for 3 yr. 8 mo.

PROCESS AND EXPLANATION

$$
\begin{array}{r l}
\$\,423 & = \text{principal} \\
.04 & = \text{rate} \\
\hline
\$\,16.92 & = \text{interest for 1 yr.} \\
3 & \\
\hline
\$\,50.76 & = \text{interest for 3 yr.} \\
11.28 & = \tfrac{2}{3} \text{ of int. for 1 yr.} \\
\hline
\$\,62.04 & = \text{int. for 3 yr. 8 mo.}
\end{array}
$$

In ordinary calculations of interest a month is taken as 30 days, a year as 12 months, or 360 days.

2. Find the interest on \$937.50 at $3\frac{1}{2}$ % for 2 yr. 6 mo.

SUGGESTION

It is generally accurate enough to express intermediate results to the nearest mill and final results to the nearest cent.

3. What is the interest at 4 % on $ 627.33 for 2 yr. 4 mo.?

4. Compute the interest on $ 1,065 at 5 % for 3 yr. 3 mo.

5. Find the interest on $3,143 at 5 % for 60 days.

6. A man borrowed $ 467 on Feb. 15 at 5 %. He paid the principal and interest on March 15 of the following year. How much did he pay?

Bank Checks

176. We often pay money to a person or to a firm by writing a *check*, which is an order on the bank where we keep our money to pay the sum of money named in the check.

No. 74 *Boston, Mass., June 2, 1912*

First National Bank

Pay to the order of ------ *William Howe* ------ $ $36\frac{25}{100}$

Thirty-six and $\frac{25}{100}$ ~~~~~~~~~~ *Dollars*

Robert Sherman

CHECK

Robert Sherman pays William Howe $ 36.25. Sherman writes out a check, as shown here. The check is payable to the **order** of William Howe.

Howe must therefore order it paid, which he can do by writing his name across the back. This is called a **blank indorsement**, because it does not state to whom the $ 36.25 is made payable; it is payable to William Howe himself or to any one else who may come in possession of it.

William Howe	*Pay to the order of* *Donald Bascom* *William Howe*
BLANK INDORSEMENT	FULL INDORSEMENT

If William Howe wishes to use the check in paying some other person, for instance, Donald Bascom, he may write on the back of the check "Pay to the order of Donald Bascom. William Howe." This is called a **full indorsement,** for no one but Donald Bascom can collect the $ 36.25, unless he should indorse it in favor of some one else.

Certified Checks are used to assure prompt honoring and payment of check. A certified check is guaranteed by the bank upon which it is drawn. This check is certified by the cashier of the bank upon which the check is drawn. The bank sets apart the amount of money named in the certified check, and will not pay out any part of it for any other purpose.

Written Problems

177. 1. John Andrews owes Henry Hunt $375, with interest at 6% for 1 yr. and 6 mo. Andrews writes out a check for the principal and interest together. He makes the check payable to the order of Henry Hunt. Andrews keeps his money in the Second National Bank of Chicago. Make out the check, naming the present year and date.

2. Albert Salisbury owes me $56 and interest on this sum for 5 years and 6 months at 5%. Make out the check that he draws to pay me the entire amount.

3. Robert Hunter's electric light bill for January is $5.80. If he pays before the 15th of February, he is allowed a discount of 10%. Hunter pays by check on Feb. 11. Make out his check on the Corn Exchange Bank of Athens, Ohio, payable to the order of the Athens Electric Co.

4. Compute the interest on $2,300 at 4% for 2 yr. 9 mo. and write the check on the First National Bank of Chicago which John Mills signs this day and makes payable to Harry Parsons, the amount of the check being the principal plus the interest.

5. Find the interest on $63.25 at 8% for 5 yr. 3 mo., and write out a check for the interest, payable to some member of your class.

6. John Cary built a house costing \$ 7,560. He paid the contractor, William Steele, the sum of \$ 5,500, and promised to pay the balance in 8 months, with interest at 7 % per annum. John Cary keeps his money in the Exchange National Bank of Omaha, Neb. Write out the check which John Cary sends this day to William Steele in payment of the principal and the interest for 8 months.

7. Make up a similar problem and write out a check to some person whom you know.

Bank Drafts

178. 1. When a debt is to be paid to a person or firm at a distance, **bank drafts** are used more than bank checks.

2. Suppose that John Watt of San Diego, Cal., wants to send \$1,500 to Tiffany & Co. in New York. He sends the following bank draft.

San Diego, Cal., July 24, 1915

First National Bank

Pay to the order of-------John Watt--------\$ 1500 $\frac{00}{100}$

Fifteen hundred and $\frac{00}{100}$ ~~~~~~~~~~~~~~~~~~Dollars

To the Mercantile National Bank
New York, N. Y.

W. C. Hay,
Cashier.

3. John Watt has his money in the First National Bank in San Diego. The above draft makes the $1,500 payable to himself or his order. To make it payable to Tiffany & Co., he makes on the back of the draft the following full indorsement:

Pay to the order of
Tiffany & Co.
John Watt.

This draft is sent to Tiffany & Co. who collects the money at a bank in New York City.

4. Banks usually charge from 0.1 % to $\frac{1}{4}$ % on the face of the draft to pay for their trouble.

5. If John Watt was charged 0.1 % for the above draft, how much did the draft cost him?

Written Exercise

179. Find the costs of drafts for the following amounts at the rates stated:

1. $2,500, $\frac{1}{10}$ %	**2.** $5,700, 0.1 %
3. $3,800, 0.2 %	**4.** $1,550, $\frac{1}{5}$ %
5. $100, $\frac{1}{4}$ %	**6.** $200, $\frac{1}{4}$ %

Interest for Years, Months, and Days

180. 1. In computing interest it is customary to regard a year as 12 months and a month as 30 days. The computation of interest is thereby simplified.

2. Find the interest on $800 at 6 % for 3 yr. 5 mo. and 23 da.; find also the amount.

PROCESS AND EXPLANATION

$$
\begin{array}{rll}
6\,\%\text{ of }\$800 = & \$48.00, & \text{int. for 1 yr.} \\
3 \times \$48 = & \$144.00, & \text{int. for 3 yr.} \\
\frac{5}{12}\text{ of }\$48 = & 20.00, & \text{int. for 5 mo.} \\
\frac{23}{30}\text{ of }\frac{1}{12}\text{ of }\$48 = & 3.07, & \text{int. for 23 da.} \\
\hline
 & \$167.07, & \text{int. for 3 yr. 5 mo. 23 da.} \\
 & 800.00 & \\
\hline
 & \$967.07, & \text{amount.}
\end{array}
$$

This method of finding interest is called the Aliquot Parts Method.

3. Find in the same way the interest on $400 at 6 % for 2 yr. 4 mo. 15 da.

Written Exercise

181. Find the interest and the amount:

4. $600 at 6 % 2 yr. 3 mo. 15 da.

5. $300 at 4 % 4 yr. 6 mo. 10 da.

6. $500 at 6 % 5 yr. 6 mo. 20 da.

7. $400 at 5 % 6 yr. 3 mo. 3 da.

8. $700 at 3 % 6 mo. 15 da.

9. $150 at 6 % 4 yr. 1 mo. 20 da.

10. $780 at 2 % 10 yr. 6 mo. 10 da.

11. \$1,000 at 4 % 5 yr. 4 mo. 20 da.

12. \$1,200 at 5 % 2 yr. 5 mo. 25 da.

13. \$725 at 3 % 3 yr. 7 mo. 26 da.

Study Exercise

182. Many business men prefer the Six Per Cent Method of computing interest. It is evident that, if the rate for 1 yr. or 12 mo. is 6 % = .06
then for 2 mo. it is 1 % = .01
and for 6 da. it is .1 % = .001

Thus the interest on \$853
for 60 da. is .01 times \$853 = \$8.53
for 6 da. is .001 times \$853 = \$ .85
for 10 da. is $\frac{1}{6}$ of \$8.53 = \$1.42

Oral Exercise

183. Find the interest at 6 % by the six per cent method, on:

1. \$500 for 2 mo.
2. \$760 for 60 da.
3. \$732 for 6 da.
4. \$38.60 for 60 da.
5. \$5,000 for 6 da.
6. \$660 for 10 da.
7. \$660 for 6 da.
8. \$660 for 12 da.
9. \$840 for 30 da.
10. \$960 for 30 da.

Study Exercise

184. 1. Find the interest on \$385 at 6 % for 2 yr. 5 mo. 12 da.

PROCESS AND EXPLANATION

Interest on \$385 for 1 yr. is	\$23.10	
2 yr. is		\$46.20
2 mo. is	\$3.85	
4 mo. is		7.70
1 mo. is $\frac{1}{2}$ of \$3.85		1.925
12 da. is 2 × \$.385		.770
Total		\$56.60
		Ans.

2. Find the interest on \$576 at 6 % for 3 yr. 8 mo. 18 da.

Write out the process and explanation.

Written Exercise

185. Find the interest at 6 % on:

1. \$185 for 66 da.
2. \$795 for 30 da.
3. \$701 for 90 da.
4. \$475 for 2 mo. 6 da.
5. \$895 for 4 mo. 12 da.
6. \$486 for 5 mo. 10 da.
7. \$63 for 10 mo. 18 da.
8. \$173 for 5 yr. 24 da.
9. \$875 for 10 yr. 2 mo. 6 da.
10. \$675 for 8 yr. 7 mo. 12 da.
11. \$605 for 72 da.
12. \$709 for 3 yr. 2 mo. 18 da.

Study Exercise

186. If required to compute interest at 5 %, we may first compute the interest at 6 % and then take $\frac{5}{6}$ of the result. A short process of taking $\frac{5}{6}$ of the result is to subtract from the result $\frac{1}{6}$ of itself. To find the interest at 4 % subtract from the 6 % interest $\frac{1}{3}$ of itself. Why?

Written Exercise

187. Find the interest at 5 %:

1. $600 for 60 da.
2. $600 for 6 da.
3. $120 for 2 mo.
4. $360 for 60 da.
5. $180 for 12 da.
6. $240 for 60 da.
7. $1,000 for 2 mo. 12 da.
8. $600 for 10 da.

Written Exercise

188. 1. Find the interest at 4 % on $673 for 5 yr. 2 mo. 18 da.

2. Find the interest at 3 % on $793 for 2 yr. 4 mo. 12 da.

3. Find the interest at 5 % on $6740 for 1 yr. 8 mo. 6 da.

4. Find the interest at 7 % on $755 for 3 yr. 6 mo. 6 da.

5. Find the interest at 6 % on $896 for 10 yr. 4 mo. 24 da.

6. Find the interest at 5 % on $699 for 1 yr. 2 mo. 6 da.

Study Exercise

189. How to find the difference in time between two dates.

If the year is taken at 360 days and each month at 30 days, it is customary to find the difference in time between two given dates by the method shown in the following example.

1. What is the time from May 13 to Oct. 7?

PROCESS

Oct. 7 is	10 mo.	7 da.	
May. 13 is	5	13	
Subtracting,	4 mo.	24 da.	= 144 da.

Written Exercise

190. Find the time between:

1. Jan. 29 and Dec. 24.
2. Feb. 6 and Nov. 16.
3. May 7 and July 31.
4. Jan. 16 and April 27.
5. Feb. 28 and Aug. 17.
6. April 25 and Sept. 18.

Study Exercise

191. How to use the Table of Days.

Sometimes it is preferred, in the computation of interest, to use the *exact* number of days between two given dates. In fact, **exact interest** is computed on all obligations by the United States government and on foreign securities.

For the computation of exact interest bankers often use the following table.

TABLE OF DAYS BETWEEN GIVEN DATES

	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	333	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

This table gives the exact number of days from any day of one month to the *same* day of another month. Thus, from March 14 to Sept. 14 are 184 da., as is seen by looking opposite the first month (March) and under the second (September). From March 14 to Sept. 29 are 184 da. plus 15 da., or 199 da.

If Feb. 29 of a leap year intervenes between the given dates, then add 1 da.

Written Exercise

192. Using this table, find the exact number of days between the following dates:

1. March 10 and Dec. 26.
2. Oct. 2 and Dec. 7.

3. Aug. 24 and Nov. 13.
4. Aug. 25, 1913 and March 10, 1914.
5. Feb. 10, 1912 and Dec. 27.
6. Feb. 6, 1913 and Nov. 20.
7. Jan. 10, 1912 and July 27.
8. Jan. 13, 1913 and Aug. 14.

Written Exercise

193. Using the Table of Days, compute the interest by the six per cent method on:

1. \$545 at 6% from March 1 to Sept. 3.
2. \$674 at 6% from April 5 to June 10.
3. \$744 at 6% from May 25 to Sept. 22.
4. \$801 at 6% from May 26 to Aug. 24.
5. \$790 at 6% from Dec. 3, 1913 to Sept. 10, 1914.
6. \$240 at 5% from Feb. 24, 1914 to Jan. 26, 1915.
7. \$1,250 at 3% from March 11 to Dec. 28.
8. \$12,750 at 4% from July 4, 1912 to Nov. 23, 1915.
9. \$1,050 at 7% from May 14, 1913 to Jan. 26, 1915.

Interest Tables

194. In banks and insurance offices where there is much computation of interest, the work is reduced somewhat by the use of interest tables.

There are several kinds of such tables. The part of an interest table printed below gives an idea of the general plan.

INTEREST TABLE AT 6%

TIME	$1	$2	$3	$4	$5	$6	$7	$8	$9
1 da.	.00017	.00033	.0005	.00067	.00083	.001	.00117	.00133	.0015
2 da.	.00033	.00067	.001	.00133	.00167	.002	.00233	.00267	.003
3 da.	.0005	.001	.0015	.002	.0025	.003	.0035	.004	.0045
4 da.	.00067	.00133	.002	.00267	.00333	.004	.00467	.00533	.006
5 da.	.00083	.00167	.0025	.00333	.00417	.005	.00583	.00667	.0075
6 da.	.001	.002	.003	.004	.005	.006	.007	.008	.009
7 da.	.00117	.00233	.0035	.00467	.00583	.007	.00817	.00933	.0105
8 da.	.00133	.00267	.004	.00533	.00667	.008	.00933	.01067	.012
9 da.	.0015	.003	.0045	.006	.0075	.009	.0105	.012	.0135
10 da.	.00167	.00333	.005	.00667	.00833	.01	.01167	.01333	.015
20 da.	.00333	.00667	.01	.01333	.01667	.02	.02333	.02667	.03
1 mo.	.005	.01	.015	.02	.025	.03	.035	.04	.045
2 mo.	.01	.02	.03	.04	.05	.06	.07	.08	.09
3 mo.	.015	.03	.045	.06	.075	.09	.105	.12	.135
4 mo.	.02	.04	.06	.08	.10	.12	.14	.16	.18
5 mo.	.025	.05	.075	.10	.125	.15	.175	.20	.225
6 mo.	.03	.06	.09	.12	.15	.18	.21	.24	.27
7 mo.	.035	.07	.105	.14	.175	.21	.245	.28	.315
8 mo.	.04	.08	.12	.16	.20	.24	.28	.32	.36
9 mo.	.045	.09	.135	.18	.225	.27	.315	.36	.405
10 mo.	.05	.10	.15	.20	.25	.30	.35	.40	.45
11 mo.	.055	.11	.165	.22	.275	.33	.385	.44	.495
1 yr.	.06	.12	.18	.24	.30	.36	.42	.48	.54
2 yr.	.12	.24	.36	.48	.60	.72	.84	.96	1.08

Study Exercise

195. **1.** Find the interest on $1,940 for 5 mo. 7 da. at 6%.

PROCESS AND EXPLANATION

From the Interest Table obtain: the rate on $1.00 for 5 mo. 7 da.

Int. on $1.00 for 5 mo.	= .025	
Int. on $1.00 for 7 da.	= .00117	
Int. on $1.00 for 5 mo. 7 da.	= .02617	
Int. on $1000 for 5 mo. 7 da.	= .02617 × 1000 =	$26.17
Int. on 900 for 5 mo. 7 da.	= .02617 × 900 =	23.55
Int. on 40 for 5 mo. 7 da.	= .02617 × 40 =	10.46
Int. on $1940 for 5 mo. 7 da.	=	$50.18

In practice this process is much abbreviated. Solve the problem using the table and see how briefly it can be done.

2. Find the interest on $87 at 7% for 2 yr. 7 mo. 9 da.

Process and Explanation

Interest on:

$8 @ 6% for 2 yr. = .96;	on $80 = 10 × .96 or	$9.60
$7 @ 6% for 2 yr. = .84		.84
$8 @ 6% for 7 mo. = .28;	on $80 = 10 × .28 or	$2.80
$7 @ 6% for 7 mo. = .24		.24
$8 @ 6% for 9 da. = .0125;	on $80 = 10 × .012 =	.12
$7 @ 6% for 9 da. = .0105		.01
Interest on $87 @ 6% for 2 yr. 7 mo. 9 da. =		$13.61

Int. on $87 at 7% = $\frac{1}{6}$ of $13.61 × 7 or $2.27 × 7 or $15.89.

Written Exercise

196. Using the Interest Table, find the interest:

1. \$95 at 6% for 1 yr. 9 mo. 8 da.
2. \$130 at 6% for 2 yr. 7 mo. 10 da.
3. \$109 at 6% for 1 yr. 10 mo. 19 da.
4. \$9,000 at 6% for 3 yr. 11 mo. 7 da.

Written Exercise

197. Find the interest:

1. \$1,760 at $3\frac{1}{2}$% for 2 yr. 6 mo. 10 da.
2. \$1,900 at 4% for 10 yr. 10 mo. 20 da.
3. \$2,000 at 7% for 1 yr. 5 mo. 13 da.
4. \$765 at 5% for 7 yr. 20 da.
5. \$5,000 at 6% for $3\frac{1}{2}$ yr.

POSTAL SAVINGS AND SAVINGS BANK SYSTEMS

Review

198. The postal savings system provides facilities for depositing savings at 2 % interest. Interest is allowed for each full year that the money remains on deposit, beginning with the first day of the month next following the one in which it is deposited. Deposits may be made by any person of the age of ten years or more.

An account may be opened with $ 1 or more, but less amounts may be saved for deposit by the purchase of postal savings cards and stamps. Postal savings certificates are issued in amounts of $ 1, $ 2, $ 5, $ 10, $ 20, $ 50, and $ 100.

No person is permitted to deposit more than $ 100 in any one calendar month nor to have a total balance to his credit at one time of more than $ 500, exclusive of interest.

Postal savings certificates may, under certain conditions, be exchanged for United States registered or coupon bonds. These bonds bear $2\frac{1}{2}$ % interest, payable semiannually. They are redeemable at the pleasure of the United States government after one year from date of issue, and will be redeemed 20 years from date of issue, both principal and interest due paid in United States gold coin. Depositors who have savings certificates in amounts of $ 20, $ 40, $ 60, $ 80, and $ 100, or multiples of $ 100 to the total value of $ 500 may exchange these certificates under dates of Jan. 1 and July 1 for bonds of this description if any such are at the time available.

Banks and Trust Companies maintain savings departments paying interest on deposits. In these institutions a savings account may be started with a small amount of money. The interest on these deposits is generally computed semiannually. For this reason savings bank accounts are treated under Compound Interest, page 237.

Written Problems

199. 1. Each of four members of a family has money in postal savings. A has $ 264, B has $ 306, C has $ 494, D has $ 86. How much interest do all four get yearly from this source?

2. A man has $ 360 in postal savings. How many years must this sum remain on deposit before the (simple) interest aggregates to not less than $ 100?

3. A clerk deposits in a postal savings bank $ 2.50 each month, beginning with January. How much has he on deposit on Feb. 1 of the following year?

Written Exercise

200. Compute the interest on the following postal savings deposits:

1. $393 deposited April 1, 1912, withdrawn May 28, 1915.

2. $489 deposited March 31, 1913, withdrawn Dec. 7, 1915.

3. $81 deposited June 14, 1912, withdrawn March 4, 1913.

4. $407 deposited Aug. 17, 1911, withdrawn Feb. 14, 1915.

5. A boy deposits in postal savings bank as follows: Nov. 2, 1914, $25; Dec. 4, 1914, $50; Dec. 26, $25. What interest will be due Jan. 1, 1915? Suppose that under date of January the first he exchange his postal savings certificates for United States bonds, what will be the interest due 6 Jan. 1, 1915? By which investment will he gain? How much?

6. Two members of the same family have each postal savings certificates to the value of $500. They exchange these for government bonds under date of July 1, 1913. What will be the total amount of interest due Jan. 1, 1915?

PROMISSORY NOTES

Study Exercise

201. When a man wishes to buy something and does not have the money to pay cash, the seller may agree to accept a written note expressing a promise to pay. Such a note is called a *promissory note.* Promissory notes are also given when one party borrows money from another.

For example, Frederick Davis borrows from Herbert Johnson $65.25 and promises to repay this amount in six months with interest at 6% *per annum.* Frederick Davis signs the following note:

ORDINARY PROMISSORY NOTE

$ $65\frac{25}{100}$	*Columbus, Ohio, July 10, 1913*
Six months after date, for value received, I promise to pay to ....Herbert Johnson........or order, Sixty-five and $\frac{25}{100}$~~~~~~~~~~~~~~~~~~~~~Dollars with interest at 6% per annum.	
	Frederick Davis.

The $65.25 is called the *face* of the note.

Frederick Davis is the *maker* of the note.

Herbert Johnson is the *payee.*

This note is due on Jan. 10, 1914; this is called the *date of maturity.*

When the note is drawn for a time that is expressed in months, the date of maturity is found by counting by months. If the note is given for a number of days, the date of maturity is found by counting by days.

Draw a note for sixty days and find the date of maturity. Draw a note for ninety days and find the date of maturity. Find the date of maturity of a note that is given for one year and six months from date.

A note may be made to read *On demand, Value received.* Such a note is called a **demand note.** This form of note is due and payable on demand of the holder. Examine some of the United States currency, as one dollar and two dollar bills. Do you find any of these *demand notes?* Such currency has been made *legal tender* by legislation.

Draw a demand note.

In the note of Frederick Davis to Herbert Johnson, you will see that it is to be paid *with interest at* 6%. This is called an **interest bearing note.** If it did not contain these words *with interest,* it would be a **non-interest bearing note.** If a note not bearing interest should not be paid at maturity, it becomes an interest bearing note from that date until paid. In this note the interest is specified as

6 %. If no interest were given in a note, the interest would be at the rate made legal by the state.

What is the legal rate of interest in your state?

This legal rate of interest must be observed when there is no contract between the parties to accept another rate. In some states the rate settled in contract cannot be higher than 6 %, though in other states it may be unrestricted. States in which great financial interests are located, as New York, allow any rate which the parties may agree to fix, as in the case of *call loans* by banks. The minimum amount to be borrowed on this basis, however, is about $ 5,000. Usually the rate of interest is regulated by law. To charge a higher than legal rate of interest is illegal and punishable by law.

What is a higher than legal rate usually called?

States enact different penalties for illegal interest, as forfeiting the amount of excess interest to forfeiting of interest and principal.

In all states there is a statute of limitation; and also a provision for payment of a note that happens to become due on a Sunday or legal holiday. Some states require payment on the day just preceding and other states on the day next following such holiday.

Every note that is expected to hold in law must specify *value received*, and should also state definitely where it is made, the date, the amount to be paid, and whether bearing interest or not bearing interest. Without *value received* the holder of the note may in case of contest have to demonstrate that the maker of the note actually received value for the amount of money specified.

Notes are *negotiable* or *non-negotiable.* A negotiable note may be transferred by one person to another. If a negotiable note is payable *to bearer,* it requires no *indorsement.* If a note is made payable "to the order of" the payee, it may be transferred by indorsement to another. The common ways of writing the indorsement on the back of the note are:

(1)

Herbert Johnson

INDORSEMENT IN BLANK

(2)

Pay to the order of the
Corn Exchange Bank
Herbert Johnson

INDORSEMENT IN FULL

(3)

Pay to John Smith
Herbert Johnson

RESTRICTIVE INDORSEMENT

(4)

Without Recourse
Herbert Johnson

WITHOUT RECOURSE INDORSEMENT

Explain the meaning of each form of indorsement.

If a note should not be paid when it matures, which form of indorsement makes the payee liable for the payment of note?

Which form of indorsement allows the note to be transferred again, or leaves it negotiable?

A note unpaid at maturity and indorsed by one or more persons may be *protested*, and if such protest is legally made it makes the indorsers responsible for the amount due, both principal and interest. If the protest is not legally executed, it does not make the indorsers liable for payment. Such protest must be presented to the indorsers within the period of time prescribed by the state law, usually not later than the date of maturity of the note.

A *joint and several* note is one which is signed by two or more parties, and which holds these parties jointly and individually responsible for payment. By omitting *and severally* from such a note, two signers or makers are made responsible for only one half of the sum due when it matures.

Written Problems

202. 1. On Jan. 10, 1914, Frederick Davis pays the above note with interest. How much does he pay? Who held the note before payment?

2. Write out the check which Frederick Davis sent to Herbert Johnson, in full payment of the above promissory note.

3. William Gray buys from Christopher Moulton an automobile for $1,000 and pays $600 cash. For the balance, Gray gives Moulton a promissory note, in which Gray agrees to pay the principal and interest, at 5% per annum, after 9 months. Write the promissory note, also the check for $600.

4. Compute the interest on this note. Find the amount of the note.

5. The day on which the note comes due, William Gray pays the note and interest by check. Write the check.

6. James Moore buys household furniture of Tucker, Mills & Co., and gives a note for $75.80, for 6 mo., with interest at 8% per annum. Write the promissory note and compute the amount James Moore has to pay after six months.

7. John Barr buys a cottage from Arthur Berg for $1,550. He pays $1,000 in cash and promises to pay the balance, with interest at 7%, after 3 years. Write the promissory note which John Barr signed, and compute the interest he paid at the time when the note became due.

8. Suppose that John Barr gives a mortgage on the property for $550 at 7% to Arthur Berg. How much interest will he pay each year on this mortgage?

9. Make up a similar problem and write the promissory note.

10.

$ 750	*Boston, Mass., March 1, 1915*
.......Three years.......after date I promise to pay	
..................W. Bain..................or bearer	
Seven hundred fifty~~~~~~~~~~~~~~~~~~~~Dollars	
with interest at six per cent. Value Received.	
	Henry Jones.

Is this note negotiable? What shows you this? How could it be made non-negotiable?

Name the maker, the payee, and the holder of the note. What is the face of the note? What is the date of maturity?

Compute the interest and write the check which Henry Jones sends to W. Bain.

11. A 60-day note for $ 695, with interest at 7 %, was made July 5, 1915. On what date did the note mature? How much was due at maturity?

12. Find the amount at maturity of a 90-day note for $ 1,375.75, bearing interest at 5 %.

13. Write a promissory note, having the face value of $ 576.45, and payable 90 days from the date of the note, with interest at 7 %. Suppose that you are the maker of the note and your father is the payee. What is the amount of the note on the date of maturity?

MORTGAGES

Study Exercise

203. A **mortgage** is an instrument by which a person may convey real or personal property or some interest in such property, as a security for a debt or a loan of money.

The mortgage is given by the mortgagor, who receives the money or other value equivalent, to the mortgagee, who lends the money or assumes the debt on the property as security.

Written Exercise

204. 1. A person buys a house for \$7,500. He pays cash \$2,000. For the balance he gives a mortgage bearing interest at $5\frac{1}{2}\%$. What will be the annual interest on mortgage?

2. A farmer owns a farm of 1,000 acres valued at \$150 per acre. He wishes to borrow enough money from a local bank to move the crops. The bank demands security to the amount of \$2.00 in real estate to \$1.00 loaned. The farmer needs \$7,500, which he borrows from the bank on these terms. The interest is to be at the rate of 6%.

The period of time is one year. How many acres of land must he mortgage? What per cent of his land does he mortgage? What will be the amount of interest to be paid at maturity?

3. What will be the semiannual interest due on a mortgage for $ 30,500 at $6\frac{1}{2}$%?

4. Mr. Lyon sells a house to Mr. Sears and receives $ 2000 in cash and a mortgage for $ 1500 bearing 6% interest. What is the yearly interest on the mortgage?

5. If Mr. Lyon invests the $ 2000 in stock that pays annually $4\frac{1}{2}$% interest, what will be his income from the mortgage and the stock investments?

6. If Mr. Sears pays taxes to the amount of $ 48.50 on his property and interest on the mortgage at 6%, how much is he paying annually in interest and taxes?

BANK DISCOUNT

Introduction

205. In borrowing money from a bank, the borrower gives his note for the sum; the banker deducts a discount from the face value of the note and pays to the borrower the remainder. This discount is called **bank discount**; the note is said to be **discounted**.

Thus, if I give a note for $1,000, the banker deducts, let us say, $10 as the 6% interest on the money for 60 days, and gives me $1,000 – $10 or $990. The banker takes his interest in advance. For that reason no mention of interest is made in the promissory note. I sign the following note:

> *$1000.00* *St. Paul, Minn., July 1, 1912*
>
> *Sixty days after date---I---promise to pay to the order of-----First National Bank of St. Paul-----*
>
> *One thousand~~~~~~~~~~~~~~~~~~~~Dollars*
>
> *at St. Paul, Minn. Value received.*
>
> *No. 45. Due Sept. 1, 1912.* *L. M. Max.*

Note payable to the bank.

The banker may require me to find some person who **indorses** the note by writing his name across the back of the note. If I fail to pay, the indorser becomes responsible for its payment.

The sum left over after discounting the note, that is, after subtracting the bank discount from the face of the note, is called the **proceeds**. In the above example the proceeds are $990.

Notice that the bank discount is computed as so many per cent of the **face** of the note. Some states allow three **days of grace** for the payment of notes. Are days of grace allowed in the state in which you live?

Rates of discount, like rates of interest, are understood to be per annum, unless otherwise specified.

A man borrows money from a bank and signs a 60-day note to the bank for $700. If the note is discounted at 7%, how much money does the bank pay the man for his note?

PROCESS AND EXPLANATION

Discount for 1 yr. $= .07 \times \$700 = \$49$.
Discount for 60 da., or $\frac{1}{6}$ yr., $= \frac{1}{6} \times \$49 = \$8.17$.
Proceeds $= \$700 - \$8.17 = \$691.83$.

Written Exercise

206. Find the bank discount:

1. $525, 2 mo., 6%
2. $600, 4 mo., 6%
3. $783, 60 da., 6%
4. $850, 30 da., 6%

5. $1,200, 30 da., 5% 6. $1,800, 60 da., 7%
7. $240, 3 mo., 6% 8. $500, 2 mo., 5%

Written Exercise

207. Find the bank discount and the proceeds of each note:

1. $200, 60 da., 6% 2. $300, 60 da., 3%
3. $300, 60 da., 5% 4. $120, 60 da., 5%
5. $1,200, 30 da., 5% 6. $600, 3 mo., 6%
7. $400, 2 mo., 6% 8. $300, 4 mo., 6%

Written Problems

208. 1. For money I am borrowing to-day, I give the Exchange National Bank of Colorado Springs, Colo., a 60-day note for $5,800. Write out the note which I must sign. If the bank discounts the note at 8%, find the discount and the proceeds. What is the date of maturity of this note?

2. Make out a 90-day note for $560, dated to-day, payable to some bank which you know. Discount the note at 7%. Give the date of maturity and the proceeds.

3. On the first of the present month George Ray's bank book indicated a balance of $1,753, in the First National Bank. Since then he has drawn on this amount by checks for $13.50, $164.75, $7.50, $18.25. He needs money for some new enterprise. He decides to leave $300 in the bank

for current expenses. To the rest he adds the proceeds of a 60-day note for $1,500, signed by him and made payable to the bank. If the bank discounts the note at 6%, how much money has Ray available for his new enterprise? Write out the note he must sign.

4. Henry Lane has made the following deposits in the Second National Bank of Hartford, Conn.: $37.50, $380, $763, $475, $1,200. He has drawn out by check the following amounts: $194, $72, $64.50, $174, $6.55, $72.80, $83.25, $45.80, $42.60. He takes out all he has in the bank, except $500, and borrows from the bank enough more so that the face of his note to the bank and the cash withdrawn aggregate $3,000. How much is the face of the note? If it is a 90-day note, find the bank discount at 6%, also the proceeds. Write the note.

5. John Snow borrows money from a bank and signs a 90-day note for $400. I indorse the note. He was unfortunate in his business ventures and I had to pay the note to the bank, 18 months after the maturity of the note. I had to pay the face of the note and 6% interest for the 18 months. How much did I pay altogether? How much did the bank pay to John Snow, if the note was discounted at 6%? Altogether how much interest did the bank get on the $400?

6. I wish to get \$1,188 from a bank. For what sum must I make out a 60-day note to obtain this sum, bank discount being at the rate of 6 % per annum?

PROCESS AND EXPLANATION

Let the required sum be $\$x$.

The bank discount for 60 da. is 1 % of $\$x$, or $\frac{\$x}{100}$.

Subtracting $\frac{\$x}{100}$ from $\$x$, leaves $\frac{\$99x}{100}$, the proceeds.

The proceeds must be \$1188.

Hence, we have the equation $\frac{99x}{100} = 1188$.

Multiplying both sides by 100, we get

$$99x = 118,800.$$

Dividing both sides by 99, we get $x = 1200$.

The face of the note must be \$1,200.

7. John Halifax needs \$1,980. For what sum must he make out a 60-day note to obtain the \$1,980 as proceeds, if the bank discounts the note at 6 %?

8. Mr. S. Cox makes out a 90-day note, which is discounted by the bank at 2 % for the 90 days. What must be the face of the note, to yield \$1,078 as proceeds?

9. Oscar Allen issued a note to a bank which secured for him, after it was discounted at 3 % for the term of 4 months, the sum of $11.64. Find the face of the note. Write the note with the proper dates, the signature of the maker, and the name of the bank.

DISCOUNTING NOTES

Written Problems

209. I sell Mr. Evans goods aggregating $ 1,895. Mr. Evans is not able to pay cash. He gives me a note, promising to pay this sum in 60 da. with interest at 5 %. Being in immediate need of money, I go to a bank, indorse the note by writing my name across the back, and sell it to the bank for the amount of the note at maturity less the bank discount of 6 %.

1. Find the amount at maturity of the note described above.

2. What was the bank discount at 6 % on that amount? How much cash does the bank pay me for the note?

3. Harry Jones owes Mr. Cary $ 450. He gives him a 90-day note, with interest at 7 %, dated April 5. Thirty days later Mr. Cary needs the money and sells the note to a bank, where it is discounted at 6 %. How much cash does Mr. Cary receive from the bank?

PROCESS AND EXPLANATION

Interest on $450, 7%, 90 da.	= $ 7.88
Face of note	= $ 450
Amount at maturity	= $ 457.88
Bank discount, 60 da., 6%	= $ 4.58
Proceeds	= $ 453.30

Interest is computed on the *face* of the note; discount on the *amount* of the note at maturity.

Drill Exercise

210. Find the discount and the proceeds on the following notes:

	FACE OF THE NOTE	DATE OF THE NOTE	WHEN DUE	INTEREST RATE	DATE OF DISCOUNT	DISCOUNT RATE
1.	$ 200	Oct. 7, '13	Oct. 7, '14	5%	July 9, '14	6%
2.	$ 300	Dec. 1, '13	June 1, '14	6%	April 2, '14	6%
3.	$ 250	Jan. 17, '14	July 17, '15	6%	Jan. 17, '15	6%
4.	$ 450	Feb. 10, '14	Feb. 10, '16	5%	Feb. 10, '14	8%
5.	$ 650	Mch. 1, '14	Dec. 1, '15	6%	Mch. 1, '15	6%
6.	$ 760	Sept. 7, '15	Sept. 7, '16	6%	Dec. 7, '15	5%
7.	$ 820	Aug. 6, '13	Aug. 6, '15	5%	Aug. 6, '14	6%
8.	$1000	June 1, '13	Dec. 1, '14	6%	June 1, '13	6%
9.	$1200	July 1, '15	Jan. 1, '16	6%	Oct. 1, '15	5%
10.	$ 280	Sept. 1, '15	Mch. 1, '17	7%	Sept. 1, '16	8%

Written Problems

211. 1. Find the date of maturity, the amount at maturity, the bank discount, and the proceeds of a 60-day note for $1,250, with interest at 4%, dated March 7, and discounted at a bank 30 days later, at 6%.

2. Find the proceeds of the following note:

NEW ORLEANS, July 15, 1913.

Ninety days after date I promise to pay to the order of ASA ALLEN, Fifteen Hundred Dollars, with interest at 5%.

HARRY LOWE.

Discounted Aug. 15, 1913, at 6%, at the Exchange Bank.

3. On March 12, I discounted at the bank, at 6%, a note for $840, dated Feb. 28 and due June 12 of the same year. What were my proceeds?

4. A note of $980, bearing 6% interest, was given March 7, 1915, payable in 6 months. It was discounted at the bank on May 16 at 8%. For what length of time was it discounted? Find the discount and the proceeds.

5. George Beecroft gives John Morrow a note for $1,560 payable in 16 months with interest at 7%. Two months later, John Morrow has the note discounted at a bank at $7\frac{1}{2}$%. Find the discount and the proceeds. Write out the promissory note which George Beecroft signed, using appropriate dates.

6. Find the date of maturity, the amount at maturity, the bank discount, and the proceeds of a note for $1,989, dated April 6, 1915, and due in 6 months with interest at 5 %, and discounted after 3 months at $5\frac{1}{2}$ %.

7. J. Roe has the following note discounted at his bank on June 7, at 6 % :

San Francisco, June 7, 1914.

After 90 days I promise to pay J. Roe the sum of Three Thousand Dollars, with interest at 7 %.

Horace Hand.

Before this Roe had $ 375 to his credit in the bank. He issues checks now to the following amounts: $ 1,285.50, $ 65.25, $ 876.40, $ 375.25. How much does he still have in the bank?

TAXES

Introduction

212. Taxes are sums of money levied for the support of government and for other public purposes. There are different kinds of taxes.

A **property tax** is a tax levied on real estate or personal property.

A **poll tax** is a tax sometimes levied on each male inhabitant who has attained his majority.

City, county, and state governments are supported by property taxes, poll taxes, and license fees. These taxes serve also for the support of public schools and charities, for fire protection, for sewers, and for paving streets.

The United States government meets its expenses chiefly from different kinds of taxes, namely, from the taxes or **duties** levied on goods imported from other countries, the taxes or **excise duties** on tobacco, liquors, etc., produced in the United States, and taxes on large annual incomes.

The rate of taxation is generally expressed as a certain *number of mills* on each *dollar*, or a certain *number of cents* on each *hundred dollars* of assessed valuation.

Taxes are usually not computed on the full value of the property. The value used is called the *assessed value.*

My house and lot are assessed at \$2,320. The tax rate is 26 mills on the dollar. What is my tax?

Mr. Fay's home is worth \$7,200, but is assessed at only $\frac{1}{2}$ of its value. What are his taxes at the rate of 46 mills on the dollar?

Written Exercise

213. Find the tax on the following assessed values:

1. \$3,450 @ 3 mills	2. \$7,860 @ 12 mills
3. \$9,600 @ $8\frac{1}{2}$ mills	4. \$10,000 @ $20\frac{6}{10}$ mills
5. \$7,400 @ $7\frac{3}{4}$ mills	6. \$6,980 @ $8\frac{7}{10}$ mills

Written Problems

214. 1. In a city where taxes may be paid in two equal installments prior to March 1 and August 1, a certain property is assessed at \$17,800, the tax rate for the year being 11 mills. How much must be paid prior to March 1?

2. A man's real estate is assessed at \$7,500, his personal property at \$450. What are his taxes for the year, if the rate is 17 mills?

3. A certain property is assessed at \$8,200, the tax rate being 20 mills. The first half of the taxes should have been paid prior to March 1, but

was not paid until May 1 of the same year. How much was paid on May 1, if, as a penalty for delay in payment, there was an extra charge of 1% a month on the sum due?

4. Taxes on $1,375, at the rate of 12 mills, should have been paid in two equal installments before March 1 and August 1, 1909, but were not paid until August 1, 1910. From March 1 to August 1 interest was charged at the rate of 1% a month. After August 1, interest was charged at 15% per annum. How much was paid August 1, 1910?

5. The tax rate per $100 being $1.432, find the yearly taxes on $7,100.

6. A town with an assessed valuation of $17,540,000 must raise $95,700 taxes. What will be the tax rate?

7. A property owner's tax is $37.35 on an assessment of $1,800. Find the rate of taxation.

8. Ascertain the rate of taxation in your own town or county, and construct problems, as if you were the assessor.

Written Exercises

215. Find the tax rate in mills on a dollar:

	Valuation	Tax		Valuation	Tax
1.	$170	$ 1.70	2.	$1,200	$18
3.	$1,550	$18.60	4.	$720	$21.60
5.	$6,500	$84.50	6.	$6,350	$254.

DUTIES AND CUSTOMS

Introduction

The expenses of the United States government are annually more than $700,250,000. The government supports the army and the navy, constructing new battleships, new armament and equipment of coast fortifications and inland forts and stations; pays a very large amount of money annually for pensions to disabled soldiers or their families; supports a vast army of employees, as the Chief Executive and his Cabinet, the Legislature, foreign ministry and consulates, clerical and other assistance, forest rangers, scientific bureaus, the United States Courts, and so on through a long list, whose total cost should convince any one of the magnitude of the work performed by the United States government.

The money for the expenses of the government is not raised by direct taxation of the population of the country. It is raised by **indirect taxation**. Indirect taxation includes among other things *customs* or *duties* on imported merchandise, *revenues* on tobacco and liquors, *income tax*, and *special tax* on certain business instruments, drugs, liquors, etc., at certain times, especially during war.

The duties or customs levied on imported merchandise are collected at entry ports or stations, as New York, Seattle, San Francisco, Philadelphia, Boston, etc., or boundary stations between Canada and the United States or Mexico and the United States. The persons responsible for these collections are called *customs officers*.

Duties are of two forms: *ad valorem* and *specific*.

216. An **ad valorem duty** is a certain per cent of the cost of the goods. A **specific duty** is a certain amount per yard, ton, dozen, etc., without regard to the cost. Certain imported goods are assessed both ad valorem and specific duties. Some articles, for instance hides, are admitted free of duty. A schedule of duties on imported goods, as fixed in this country by the United States Congress, is called a *tariff*.

The importer submits to the Customs officials an *invoice* or itemized statement of quantity, quality, cost of goods, and the names of buyer and seller, incidental expenses, as packing, freight and drayage, insurance, etc. The quantities and cost of goods are given in the system of measures and value of the country from which the goods are exported. That is, English measures and money values are employed for exports from Great Britain; the metric system of measurement and the cost in *francs* for exports from France, in *marks* from Germany, in *yen* from Japan, etc.

The net invoice value is determined by deducting for *tare, i.e.* estimated weight of barrels, boxes, and other packing, in which goods are transported, for *leakage* or *breakage, i.e.* the loss by leakage or evaporation of liquids in bottles, casks, etc., for commission paid by buyer, insurance, and other incidental expenses; and it may be fixed at a certain percentage of the market value of the goods imported. The net invoice is taken as the *base*, the ad valorem or specific duty as the *per centum.*

217. **Table of Tariff Rates**

Hosiery, valued above $1, 50% ad valorem
Crockery, 40%, ad valorem
Ornamental feathers, 60%, ad valorem
Hats and bonnets, 40%, ad valorem
Gloves, $2.50 per dozen pairs
Iron, steel, or wire, 15%, ad valorem
Wire rope, 30%, ad valorem

Steel rails, free	Honey, 10¢ a gallon
Coal, free	Cider, 2¢ a gallon
Straw, 50¢ a ton	Oats, 6¢ per bushel

Bicycles, 25%, ad valorem

Written Exercise

Find the duty. (Use Table of Rates.)

1. 400 pairs of gloves.
2. 100 T. of coal.
3. Wire rope, $300.
4. 20 T. of straw.

5. 300 bu. oats.

6. 500 bicycles. $30

7. 125 gal. honey.

8. $325, crockery.

9. 1 bbl. cider.

10. $487 ornamental feathers.

Written Problems

218. 1. A merchant imports 1,200 doz. pieces of hosiery, worth $1.75 a doz., and $175 worth of ornamental feathers. How much duty must he pay?

2. How much cheaper must the owners of a foreign steel plant bid on steel chains which can be manufactured in the United States for $7,500, in order to compete with American prices, if the duty on steel chains is 20 % ad valorem?

3. A foreign merchant has $375 worth of bonnets and $3,600 worth of gloves (200 doz.). He sells this merchandise to a merchant in the United States for $5,800, less the duties thereon. Find his profit.

4. A few years ago the internal revenue collected by the United States on fermented liquors was in round numbers 69 million dollars. If the tax per barrel is $1.60, find the approximate number of barrels of liquor on which revenue was collected.

5. Cigars valued at $45 a thousand are subject to an internal revenue tax of $40. What rate per cent of the value is this tax?

6. The internal revenue on distilled spirits is \$1.10 per gallon. Upon how many gallons, in round numbers, was revenue collected in 1899, when the total revenue from this source was \$99,283,000?

7. A few years ago the total revenue tax on tobacco in the United States was \$51,841,000, which showed an increase of \$3,388,000 over the tax of the preceding year. How much was the tax of the preceding year? What was the rate per cent of increase on the tax of the preceding year?

8. In 1905 the total value of manufactured tobacco was \$331,118,000. What rate per cent of increase is this over the amount in 1860, which was 31 million dollars?

9. In North Carolina there are 162,000 acres given to the raising of tobacco. If the crop amounts to 100,875,000 lb. and is valued at the farm at \$11,096,000, what is the crop per acre? The value of the crop per acre? The farm value of a pound of tobacco?

INCOME TAX

Study Exercise

219. The Federal income tax, enacted into law in 1913, requires the yearly payment, into the treasury of the United States, of taxes on incomes according to the following rates:

Unmarried persons pay 1% on the part of their income which is over \$3,000. Married persons pay 1% on the part of their joint income which is over \$4,000. The following *additional taxes*, or surtaxes, are collected on incomes over \$20,000.

1% on that part of income over \$20,000 and not above \$50,000.
2% on that part of income over \$50,000 and not above \$75,000.
3% on that part of income over \$75,000 and not above \$100,000.
4% on that part of income over \$100,000 and not above \$250,000.
5% on that part of income over \$250,000 and not above \$500,000.
6% on that part of income over \$500,000.

There is also a graduated tax on large incomes. In this manner the larger incomes are made to pay a proportionately larger tax. Full information concerning the income tax can usually be obtained from the president of the local bank.

1. Adolph Brace and his wife have a gross income jointly as follows: Salary, \$4,000; interest

on notes, $ 900; rent received, $ 1,500. Expenses to be deducted from gross income are: Taxes, $ 300; house repairs, $ 200; interest paid on mortgage, $ 250. Find the income tax on the net income.

PROCESS AND EXPLANATION

Gross income	$ 6,400
Deduction	750
Net income	$ 5,650
Exemption	4,000
Taxable income	$ 1,650

1 % on $ 1,650 is $ 16.50, *Ans.*

2. Find the income tax of an unmarried man on a net income of $ 150,000.

PROCESS AND EXPLANATION

1 % on $ 150,000 – $ 3,000, or $ 147,000 is	$ 1,470
1 % on $ 50,000 – $ 20,000, or $ 30,000 is	300
2 % on $ 75,000 – $ 50,000, or $ 25,000 is	500
3 % on $ 100,000 – $ 75,000, or $ 25,000 is	750
4 % on $ 150,000 – $ 100,000, or $ 50,000 is	2,000
Total income tax	$ 5,020

Oral Exercise

220. Find the Federal income tax of a man whose exemption is $ 3,000 on a net income of

1. $ 4,000
2. $ 4,350
3. $ 3,826
4. $ 8,700
5. $ 10,000
6. $ 9,860

7. $15,000	8. $19,750	9. $23,000
10. $25,000	11. $30,000	12. $3,000

Written Exercise

221. Find the Federal income tax of a man and wife whose joint net income per year is as follows:

1. $30,000	2. $55,000	3. $65,000
4. $85,000	5. $92,700	6. $83,500
7. $69,750	8. $100,000	9. $130,000
10. $250,000	11. $99,000	12. $101,000

FIRE INSURANCE

Introduction

Fire insurance is insurance against loss or damage of property by fire. No prudent man will allow his property to be without insurance. When a person insures his property, he makes a written agreement, or contract, with the insurance company. This agreement is called a **policy**. The amount of indemnity or protection is called the **face of the policy** which is treated as the **base.** The consideration or price paid for the insurance is called the **premium**, which corresponds to the **percentage.**

There are insurances against other kinds of losses. Thus, there is a **marine insurance** against loss of property by sea, an **accident insurance** against personal injury or loss of life by accident, a **liability insurance** against loss by the insolvency of debtors, and so on.

Rates of insurance are expressed in per cent or in cents per $100, or in dollars per $1000.

222. 100 persons in a city have property valued at $300,000. They pay a fire insurance company every year $\frac{3}{10}$ % of this sum. What is the sum paid to the company annually? [$\frac{3}{10}\% = .003$.]

Suppose the loss on this property for a year is \$ 800. How much of the sum paid to the company remains on hand as a surplus?

A man insured his house against fire for \$ 4,000 and paid the insurance company $\frac{3}{10}$% every year. After 25 years the house was destroyed by fire. How much did the man pay the company a year? How much in 25 years? Did he gain or lose by paying insurance on his house, if the interest on the money which he paid in would have yielded him \$ 200? Did the insurance company gain or lose?

Another man insured his house for \$ 5,000, paid every year $\frac{3}{5}$%, and after 30 years had had no fire. How much did he pay every year? How much in 30 years? Did the insurance company gain or lose?

Written Problems

223. 1. What is the premium on a \$ 6,500 policy at $\frac{4}{5}$%?

2. What is the cost per annum of \$ 8,700 insurance at $\frac{3}{4}$%?

3. A farmer insures a barn for \$ 560 at $\frac{1}{2}$%. What is the premium for a period of three years?

4. A \$ 5,000 house is insured for $\frac{2}{3}$ its value, at 50¢ per \$ 100. What is the annual premium?

5. A frame house with a shingle roof is insured for \$ 2,600 at 85¢ per \$ 100 for 3 years. Find the premium.

6. A brick dwelling, insured for $2,600, at a rate of 60¢ per $100. How much less is the premium for three years on this building than on the house in Example 5?

Written Exercise

224. State the premium in each of the following cases:

	Face of Policy	Rate		Face of Policy	Rate
1.	$5,800	$\frac{3}{10}$%	2.	$6,750	$\frac{3}{5}$%
3.	$6,890	$\frac{9}{10}$%	4.	$8,910	$3 per $1,000

	Face of Policy	Rate
5.	$1,250	70¢ per $100
6.	$17,600	$2.50 per $1,000

Written Exercise

225. Find the face of the policy in each case:

	Premium	Rate		Premium	Rate
1.	$2.2	$\frac{2}{5}$%	2.	$158	$\frac{2}{3}$%
3.	$6	$1\frac{1}{5}$%	4.	$6.3	$\frac{9}{10}$%
5.	$226	$\frac{1}{3}$%	6.	$67.56	1%

Written Exercise

226. Find the rate of insurance in each case:

	Face of Policy	Premium		Face of Policy	Premium
1.	$7,650	$38.25	2.	$8,950	$29.83
3.	$5,750	$69	4.	$9,850	$49.25
5.	$10,500	$35	6.	$1,250	$8.33

Written Problems

227. 1. If the rate of insurance for three years is only twice the annual rate, how much will be saved in 15 years by insuring a \$7,000 house in 3-year periods, the annual rate being $\frac{3}{10}$ %?

2. A building worth \$7,500 was insured for \$5,000 at a rate of 25¢ on \$100. How much does the owner lose and how much the insurance company, if the building is damaged to the extent of \$5,500?

3. A brick business block is insured for \$17,000 at 50¢ per \$100. Its contents is insured for \$7,500 at 75¢ per 100. What is the amount of the yearly premium?

4. If two annual premiums paid in advance count as three years' insurance, what will it cost to insure for 3 years a \$7,500 frame store dwelling at 40¢ per \$100 and the stock, \$12,500, at 50¢ per \$100?

5. A merchant insures his newly arrived stock for \$12,000 for 45 days at 20 % of the yearly rate, which was $\frac{9}{10}$ %. How much premium does he pay?

6. A ship worth \$75,000 is insured in one company for \$50,000 and in another company for \$25,000. If the ship is damaged \$35,000, how much must each company pay, if it is specified in the contract that each company pays only such part of a partial loss as the amount insured is of the full valuation?

7. A steamer valued at \$ 40,000 is insured at \$ 30,000 and is damaged \$ 6,000. According to the contract, the company is obliged to pay only a part of the \$ 6,000, namely, that part of the \$ 6,000 that \$ 30,000 is of \$ 40,000. How much must the company pay?

8. Under the same kind of contract, how much must an insurance company pay, if the damage is \$ 12,000, the face of the policy \$ 25,000, the full value of the steamer \$ 35,000?

9. A man starts on a week's travel and takes out, before starting, an accident insurance policy for \$ 5,000 and pays \$ 2 for it. What is the rate per cent of the insurance?

10. A liability company agrees to pay all expenses arising from injuries sustained by employees of a manufacturing establishment and charges for this service a premium of $\frac{2}{3}$ % of the monthly pay roll. What is the premium for a month when the pay roll amounts to \$ 6,000? How much was the pay roll during a month when the premium paid was \$ 45?

11. A growing crop was insured at $3\frac{1}{2}$ %. What was the face of the policy, if the premium amounted to \$ 122.50?

12. The premium paid for insuring a cargo of wheat, for $\frac{2}{3}$ of its full value, at 2 %, was \$ 72. Find the full value of the cargo.

13. A tank of oil, holding 2,600 gal., worth 23¢ a gallon, was insured at $4\frac{1}{2}\%$. Compute the premium.

14. A lawyer insured his library, worth $5,000, at $\frac{2}{3}$ its value, and paid a premium of $22.22. Find the rate per cent.

15. Find out the rates of insurance in the town where you live, also the values of some of the buildings. Construct problems of your own.

LIFE INSURANCE

Introduction

228. The aim of life insurance is to provide a certain sum for those dependent upon the person insured, in case of his death, or to secure the sum for the person himself later in life. There are many kinds of insurance policies. The following are the most important:

Ordinary Life Policy. — The person insured pays premiums during his life, and the insurance company guarantees to pay a stated sum of money at his death.

Limited Life Policy. — The person insured pays premiums during a limited or fixed number of years (10, 15, or 20 years) only, while the company guarantees to pay a certain sum, at his death.

Endowment Policy. — The person insured pays premiums for a certain number of years and the company guarantees to pay him a stated sum of money after a specified number of years, or, in case of prior death, to pay the stated sum to the person to whose benefit the insurance is made.

Term Policy. — The insured pays premiums for a fixed time, and the company agrees to pay a stated sum in case of death within the term of insurance.

Unless the insured dies during the term, no part of the face of the policy is paid.

The **rate** of life insurance is expressed as a certain sum for every $1,000 of insurance carried. The rate is higher for older persons because the risk of death is greater. For the same age of the insured, the rate depends also upon the kind of policy selected. The rate is lower in a term policy than in others, for the reason that the company may never be called upon to pay the insurance. Again, the rate is higher for a limited life policy than for an ordinary life policy, because in the former the insured pays premiums for a specified number of years only, while in the latter he pays premiums up to the time of his death.

Written Exercise

229. Find the premium for each policy:

	Face of Policy	Rate on $1000		Face of Policy	Rate on $1000
1.	$2,000	$16.91	2.	$3,000	$19.30
3.	$4,000	$21.06	4.	$2,000	$44.80
5.	$2,100	$33.30	6.	$10,000	$29.97

Written Exercise

230. Find the rate per $1,000 for each policy:

	Face of Policy	Premium		Face of Policy	Premium
1.	$3,000	$67.20	2.	$10,000	$292.90
3.	$2,000	$70.72	4.	$6,000	$603
5.	$8,000	$808.96	6.	$7,000	$707.37

Written Exercise

231. Find the face of each policy:

	Premium	Rate on $1000		Premium	Rate on $1000
1.	$46.60	$23.30	2.	$102.12	$34.04
3.	$134.40	$44.80	4.	$153.60	$25.60
5.	$454.60	$30.30	6.	$14.33	$28.66

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Written Exercise

232. Find approximately the premium for each policy:

	Face of Policy	Rate on $1000		Face of Policy	Rate on $1000
1.	$25,000	$19.85	2.	$6,500	$25.66
3.	$2,000	$47.68	4.	$10,000	$67.36
5.	$500	$41.87	6.	$1,500	$52.27
7.	$30,000	$46.06	8.	$7,900	$29.98
9.	$4,000	$68.76	10.	$7,000	$16.91
11.	$6,000	$66.18	12.	$12,000	$17.79

Written Problems

233. 1. A man, 25 years old, takes out an ordinary life policy for $6,000 and pays an annual premium of $20.70 on every $1,000. He dies at the age of 65, after having paid 40 premiums. What is the total sum paid in premiums? How much less is this than the face of the policy? Explain how the insurance company can realize a profit on this policy.

2. Another man, 45 years old, takes out an ordinary life policy for $ 6,000, but has to pay a premium of $ 38.80 on every $ 1,000 annually, because he is older than the first man. He dies at the age of 69, after having paid 24 premiums. How much is the face of the policy in excess of the total premiums?

3. A person 20 years old takes out a policy for $ 2,000, in which the payment of premiums is limited to 10 years, so that after 10 years the policy becomes a paid-up policy, that is, a policy payable at the death of the insured. The rate for such a policy is comparatively high, $ 42.70 per $ 1,000. Find the total cost of this protection.

If the insured dies at the age of 63, the insurance company has the use of the total premiums for a period of 33 years and of part of the premiums for even a longer time. It is through the income derived from this source that the company makes its profit.

4. George Bancroft, aged 35, takes out an endowment policy for $ 10,000 to be paid to himself after 20 years, or if he should die before that time, to his wife, if she survives him, otherwise to his executors. It is agreed that this policy shall participate annually in the surplus earnings of the company. The rate of insurance on this policy is $ 50.80 on every $ 1,000.

(*a*) How much is the yearly premium?

(*b*) If 20 premiums are paid, how much do they exceed the face of the policy?

(*c*) Suppose the insured dies before the eleventh payment is due. How much will his widow or the executors receive above his total payments, if there are no surplus earnings?

(*d*) Suppose the insured survives the endowment period and the surplus earnings on this policy are $ 3,100, how much will he receive?

5. A man obtained $ 5,000 insurance against accident for 8 months and was charged 75 % of the yearly premium of $ 11.45 on each $ 1,000. How much did he pay for the insurance? Why is the rate of insurance low in this case?

Exercise based on a Table

TABLE FOR REFERENCE

234. *Annual Premium Rates for an Insurance of $ 1000*

AGE	ORDINARY LIFE	20-PAYMENT LIFE	10-PAYMENT LIFE	15-YEAR ENDOWMENT	20-YEAR TERM
20	$18.50	$26.95	$42.75	$65.60	$13.00
30	23.55	32.40	50.90	67.27	15.40
40	32.25	40.65	62.90	69.76	21.50
50	47.95	54.80	80.80	76.20	37.70

Solve to find x. Use the reference table.

	Age	Kind of Policy	Face of Policy	Annual Premium
1.	50	Ordinary Life	$ 7,500	x
2.	30	15-Year Endow.	12,500	x
3.	20	20-Payment	9,500	x
4.	40	10-Payment	8,000	x
5.	50	10-Payment	8,000	x
6.	30	15-Year Endow.	x	$ 336.35
7.	20	10-Payment	x	320.62
8.	20	Ordinary Life	x	203.50
9.	x	Ordinary Life	9,000	290.25
10.	x	20-Year Term	10,000	377.00
11.	x	20-Payment	13,000	528.45
12.	30	x	6,000	403.62
13.	20	x	4,000	74.00

Written Problems

235. 1. A man 40 years old took out a 15-year endowment policy for $ 7,000. He received yearly dividends from the surplus fund of the company as follows: $ 12.50, $ 10.75, $ 13.65, $ 14.85, $ 12.50, $ 14, $ 16, $ 15.25, $ 13.50, $ 14.60, $ 15, $ 13, $ 12.75, $ 14, $ 15. At the end of the 15 years he is paid the face value of his policy. How much money did he receive from the company altogether? How much does this sum exceed the total premiums that he paid?

2. A man 30 years old takes out an ordinary life policy of $ 3,500. If he is killed in a railroad wreck two years later, how much does the company lose?

3. A man of twenty takes out a 20-year term policy for $ 2,000. He dies at the age of 65. What profit does the company make?

4. Obtain from the local agent of a life insurance company a description of a $ 5,000 20-year endowment policy for a man 25 years old. Find out the annual premium on this policy.

5. Obtain a description of an ordinary life policy of $ 5,000 for a man 25 years old. Compare the premium on this policy with the premium to be paid on the policy in problem 4. Why is this premium less than the premium on the endowment policy?

COMPOUND INTEREST

Study Exercise

236. 1. Interest on both the principal and the overdue interest added to the principal is called *compound interest.*

2. The collection of compound interest on money lent or due cannot be enforced, as the laws of the different states generally prohibit it. The usual custom is to charge simple interest only. If the interest is not paid when it is due, interest on that unpaid interest cannot be legally collected.

3. But if the interest is reinvested as soon as it is paid in, so that it too bears interest, then the mode of increase is the same as in compound interest. Problems involving the result of reinvestment of interest, as soon as it is paid in, arise sometimes, especially with life insurance companies and loan associations.

4. Most savings banks allow compound interest on their deposits.

5. Find the compound interest, compounded semiannually, on $200 at 4% per annum for 2 years.

PROCESS AND EXPLANATION

\$ 200	first principal
.02	rate for $\frac{1}{2}$ yr.
4	first interest
200	
204	amt. at the end of $\frac{1}{2}$ year
.02	
4.08	second interest
204	
208.08	amt. at the end of 1 year
.02	
4.16	third interest
208.08	
212.24	amt. at the end of $1\frac{1}{2}$ years
.02	
4.24	fourth interest
212.24	
216.48	amt. at the end of 2 years.

If we subtract from the last amount the original principal, we obtain the compound interest. \$ 216.48 – \$ 200 = \$ 16.48, the compound interest. The simple interest is \$ 16. Hence, in this instance, the compound interest exceeds the simple interest by 48 ¢.

Written Exercise

237. 1. Find the amount of \$ 5,000 at 5 % for three years, the interest being compounded annually.

2. Of \$ 4,000, 3 years, 3 % semiannually, compounded semiannually.

3. Of $1,000, 4 years, 4 % annually, compounded semiannually.

4. Of $1,000, 4 years, 4 % annually, compounded annually.

Compound Interest Tables

238. Business men save time by computing the amount in compound interest, by the use of tables.

COMPOUND INTEREST TABLE GIVING THE AMOUNT ON $1.

YR.	2 %	2½ %	3 %	3½ %	4 %	4½ %	5 %
1	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500	1.05000
2	1.04040	1.05063	1.06090	1.07123	1.08160	1.09203	1.10250
3	1.06121	1.07689	1.09273	1.10872	1.12486	1.14117	1.15763
4	1.08243	1.10381	1.12551	1.14752	1.16986	1.19252	1.21551
5	1.10408	1.13141	1.15927	1.18769	1.21665	1.24618	1.27628
6	1.12616	1.15969	1.19405	1.22926	1.26532	1.30226	1.34010
7	1.14869	1.18869	1.22987	1.27228	1.31593	1.36086	1.40710
8	1.17166	1.21840	1.26677	1.31681	1.36857	1.42210	1.47746
9	1.19509	1.24886	1.30477	1.36290	1.42331	1.48610	1.55133
10	1.21899	1.28009	1.34392	1.41060	1.48024	1.55297	1.62889

Written Exercise

239. Using the above table, find the amount at 4 %, interest compounded annually.

1. $200, 4 yr.
2. $300, 5 yr.
3. $600, 3 yr.
4. $250, 3 yr.
5. $750, 7 yr.
6. $1,000, 10 yr.
7. $960, 9 yr.
8. $275, 6 yr.
9. $385, 8 yr.

Written Problems

240. 1. A young man has \$2,000 invested at 6%. He wants to know what this will amount to in 5 years, if the interest is reinvested at the end of each year, at the same rate.

2. Using the compound interest table, find the amount of \$400 at 5% in 4 years, compounded semiannually.

Process and Explanation

When interest is compounded semiannually, take one half the rate for twice the time. That is, take in this example, $2\frac{1}{2}\%$ for 8 years.

\$1.17166 = amt. of \$1 for 8 yr. at $2\frac{1}{2}\%$.

400 × \$1.17166 = \$468.66 = amt. of \$400 for 8 yr. at $2\frac{1}{2}\%$.

3. Find the amount of \$950 at 6% for 60 yr., compounded annually.

4. The population of a town increased every year 2% on the population of the preceding year. Three years ago it was 12,000, about what is it now?

Suggestion: Increase in population is approximately found by this method employed in solving compound interest problems.

5. A certain town increases, partly by immigration, every year $2\frac{1}{2}\%$ over that of the year

before. If the population is now 35,000, what will it be in 10 years? Give the answer to the nearest hundred.

6. Can a town of 20,000 inhabitants, increasing every year at the rate of 2 %, have a population of 30,000 in 5 years? Answer by making a rough computation only.

7. A school enrolls for the year 1912–1913, 2,760 pupils. The enrollment increased 1913–1914, 10 %; and the registration of the first term 1914–1915 showed increase of 12 % above the enrollment of 1913–1914. What was the enrollment of 1913–1914? That of the first term, 1914–1915?

8. A school system has a total daily attendance for one month of 37,843. The total enrollment was 38,594. What was the percentage of attendance?

9. The 1900 census reports the population of a certain city as 285,692, which is thirty-seven per cent more than the census report of the population for 1890, and 29 % less than the population given in the census of 1910. What was the population in 1890? That of 1910? What was the average rate of increase per year?

10. Obtain from the local bank a statement of the method by which interest on savings accounts is computed. Is this method a compound interest method?

Written Exercise

241. Using the compound interest table, find the amount at 5%, interest compounded semiannually.

1. \$400, 2 yr.
2. \$800, 5 yr.
3. \$575, 4 yr.
4. \$425, 3 yr.
5. \$675, 3 yr.
6. \$700, $3\frac{1}{2}$ yr.

EXPLANATION. — The amount, due to compound interest, becomes large, for long periods of time. Thus, \$1 at 6%, compounded annually for 60 years, amounts to \$32.96. The amount in simple interest is only \$4.60, nearly one-seventh of the compound interest.

SAVINGS ACCOUNTS

Study Exercise

242. A savings bank is a bank established and maintained for the purpose of receiving deposits of money and paying interest on the deposits. National banks, state banks, and many trust companies have savings departments. Savings banks and savings departments in banks are generally under the control of the government.

Savings banks pay interest on a sum of money which has been on deposit for an entire *interest term*. The lengths of such interest terms vary in different banks. Usually there are two terms per year, one extending from Jan. 1 to July 1, the other from July 1 to Jan. 1. Sometimes there are four terms, namely the periods of time between the following successive dates: Jan. 1, April 1, July 1, Oct. 1.

Written Problems

243. 1. William Murray deposits money in a savings bank which pays interest on Jan. 1 and July 1, at 4% per annum. He is provided with a bank book which shows the following entries.

Page of Bank Book

Date	Deposited		Drawn Out		Interest		Balance	
1913 Jan. 1	200	00					200	00
May 10	100	00					300	00
July 1					4	00	304	00
Aug. 6			50	00			254	00
1914 Jan. 1					5	08	259	08
March 6	75	00					334	08
July 1					5	18	339	26
Sept. 29			25	00			314	26
1915 Jan. 1					6	29	320	55
July 1					6	41	326	96

When did each of the deposits begin to draw interest? On what sum was each interest computed? Why? Does this bank pay compound interest? Explain.

2. John Smith deposited in a savings bank $195 on May 28, 1914, $75 on Jan. 1, 1915, and $35 on May 31, 1915. He took out $25 on Oct. 23, 1914, and $45 on June 16, 1915. Interest at 4% per annum is due Jan. 1 and July 1. Make the proper entries in his bank book and find the balance due July 1, 1915.

3. Find the balance due Jan. 1, 1916, on the following account, with interest at $3\frac{1}{2}$%, due Jan. 1 and July 1. Deposits: June 30, 1914, $400; Oct. 15, 1914, $100. Withdrawals: July 1, 1914, $50; Nov. 8, 1915, $35.

4. A savings bank pays interest on deposits on Jan. 1, April 1, July 1, and Oct. 1, at 4 % per annum. John Brown made the following deposits: April 1, 1914, $250, Sept. 1, 1914, $175. He withdrew $50 on Oct. 10, 1914. Find the balance due Jan. 1, 1915. Write out the statements which appear in his bank book.

PARTNERSHIPS AND CORPORATIONS

Study Exercise

244. When two or more persons wish to engage in business requiring a comparatively small sum of money, they may form a **partnership.** We have had some problems on partnership on pages 119–120. In an ordinary partnership or firm each partner is liable, together with the others, for the whole indebtedness of the firm.

In recent years the old-time partnership has been largely supplanted by a **stock company** or **corporation.** A corporation generally uses more capital, contributed by many, perhaps several thousand, individuals. A corporation is organized under a state law and has a **board of directors** or **officers,** elected by the members, to conduct the business. The state law gives it certain powers and privileges, but also subjects it to certain limitations. A corporation, through its officers and directors, can legally transact business like an individual.

The persons contributing the money, or **capital,** to start the business, constitute the **members** of the corporation.

The **capital** or **capital stock** is divided into **shares,** usually of $100 each. A member of the corporation owns one or more shares of stock and is called a **stockholder.** As a rule the shares of the corporation can be bought and sold.

John Mills has $500 to invest. He knows that the New York Coal and Coke Company is a prosperous company and he decides to buy some of its shares. He can buy five $100 shares for $500. He pays the money and receives in return the **certificate** on page 238 which shows that he is the owner of five shares and is a stockholder in the company.

After all expenses are paid, the earnings of the company are divided among the stockholders. The sum paid is called a **dividend.** A dividend is a certain per cent on the **face value** or **par value** of each share.

Sometimes the capital stock is divided into two kinds, **preferred** and **common.** On preferred stock a dividend is usually **guaranteed,** and this must be paid before any dividend is paid on the common stock. Usually no dividend is guaranteed on common stock.

When a business is prosperous and pays large dividends, the stock usually sells **above par.** It is then said to be at a premium. If a company is not able to pay large dividends, its stock usually sells **below par** and is said to be at a discount.

When stock is quoted at 97 or 105, it means that a share whose par value is $100 can be bought for $97 or $105.

In promotion and development companies, as mining, stock-raising, stocks are often issued with liability to assessment. **Assessment** is a tax determined by a corporation and levied upon the stockholders. Assessments may be made for the purpose of improvement, development, or meeting losses incurred in the course of business. The statement of an assessment is usually made in the terms of percentage, as a certain per cent of the par value of the capital stock of the corporation.

INCORPORATED UNDER THE LAWS OF THE STATE OF NEW YORK

No. 76 *5 Shares*

New York Coal & Coke Company

Capital Stock $300,000 $225,000 Common $75,000 Preferred

COMMON

Stock Certificate

This certifies that John Mills *is the owner of* Five *shares of one hundred dollars each of the Capital Stock of the*

New York Coal & Coke Company

transferable only on the books of the Company in accordance with the rules and regulations of said company on surrender of this certificate.

New York, April 12, 1914.

Robert F. Lyon, *Secretary.* John G. Lee, *President.*

245. 1. Examine this stock certificate.

What is the entire capital stock?

2. How much of New York Coal and Coke Company stock is common stock?

3. How much of New York Coal and Coke Company stock is preferred stock?

4. Of which kind of stock is this certificate?

5. What is meant by the statement that the stock is transferable only on the books of the company?

6. Would it be possible for John Mills to sell these five shares to anybody except the coal and coke company?

Written Problems

246. 1. The New York Coal and Coke Company declared a dividend of 6% on the preferred stock. What sum was paid out as a dividend on the \$75,000 of preferred stock?

2. On the common stock the company declared a dividend of 5%. What sum was necessary to pay this?

SUGGESTION: First find 10%, then take half of that.

3. How large a check was sent to John Mills?

4. The New York Coal and Coke Company declared a 6% dividend on the preferred stock. The total sum divided among all the stockholders was

$10,125. Find the per cent of dividend paid on the common stock.

PROCESS AND EXPLANATION

Dividend on preferred stock = .06 × $75,000 = $4,500.

Dividend on common stock = $10,125 − $4,500 = $5,625.

Per cent of dividend on common stock =

$225,000)$5,625 = .025 = $2\frac{1}{2}$ %.

5. How much money must the company take from its treasury to pay a dividend of 6 % on the preferred and $4\frac{1}{2}$ % on the common stock?

6. In Ex. 2, how large a check does a stockholder receive who owns 250 shares of the common stock and 130 shares of the preferred stock?

7. A corporation with a capital of $100,000 loses $2,500. What per cent of his stock must each stockholder be assessed, to meet this loss?

8. John Mills learns that shares of the New York Coal and Coke Company sell at $118\frac{3}{4}$ and decides to sell. How much does he realize from the sale of his five shares, after paying his broker $\frac{1}{8}$ % on the par value for services in effecting the sale?

9. How much was realized from the sale of 125 shares of Illinois Central Railway stock at $112\frac{1}{2}$?

Study Exercise

247. *Newspaper clipping showing sales and range of prices.*

	SALES	OPEN	HIGH	LOW	CLOSE
N. Y. Central . . .	2,700	$114\frac{1}{4}$	$114\frac{1}{4}$	$113\frac{1}{4}$	114
Nevada Copper . .	700	$18\frac{1}{8}$	$18\frac{1}{8}$	$17\frac{3}{4}$	18
North. Pacific . .	4,600	$117\frac{1}{4}$	$118\frac{3}{4}$	$116\frac{1}{2}$	$118\frac{3}{4}$
Pacific Mail . . .	100	$25\frac{1}{8}$	$25\frac{1}{8}$	25	$25\frac{1}{8}$
Pullman	100	$156\frac{1}{4}$	$156\frac{1}{4}$	$156\frac{1}{4}$	$156\frac{1}{4}$
Union Pacific . . .	200	$162\frac{5}{8}$	$162\frac{3}{4}$	$159\frac{3}{4}$	$161\frac{7}{8}$
U. S. Steel	121,100	$70\frac{1}{2}$	$70\frac{5}{8}$	$68\frac{7}{8}$	70
" p'f'd. . .	1,100	$115\frac{5}{8}$	$115\frac{7}{8}$	$115\frac{3}{8}$	$115\frac{1}{2}$

1. The table given here shows the transactions in a few stocks during one day. The U. S. Steel Co. has two kinds of stock listed, the second being preferred stock (p'f'd). Of this preferred stock, 1,100 shares were sold that day, the price at the opening being $115\frac{5}{8}$ a share. The highest price paid was \$ $115\frac{7}{8}$; the lowest was $115\frac{3}{8}$. The price at the close of the day was \$ $115\frac{1}{2}$. The preferred stock of this company is seen to be "above par," while its common stock is "below par."

Written Exercise

248. 1. Find the cost, at the opening price, of 250 shares of each stock in the table. The cost includes a brokerage of $\frac{1}{8}$ % of the par value.

Process and Explanation

Total cost per share = selling price + brokerage.

1 share N. Y. Central stock costs $\$114\frac{1}{4} + \$\frac{1}{8} = \$114\frac{3}{8}$.

250 shares N. Y. Central stock cost 250 times $\$114\frac{3}{8} = \$28,593.75$.

2. Find the difference in cost of 500 shares of Northern Pacific stock at the high and the low price given in the table.

3. I bought 1000 shares of Nevada copper stock at the lowest price in the table and sold the same at the highest. I had to pay the usual brokerage of $\frac{1}{8}\%$ both for buying and for selling. Did I gain or lose, and how much?

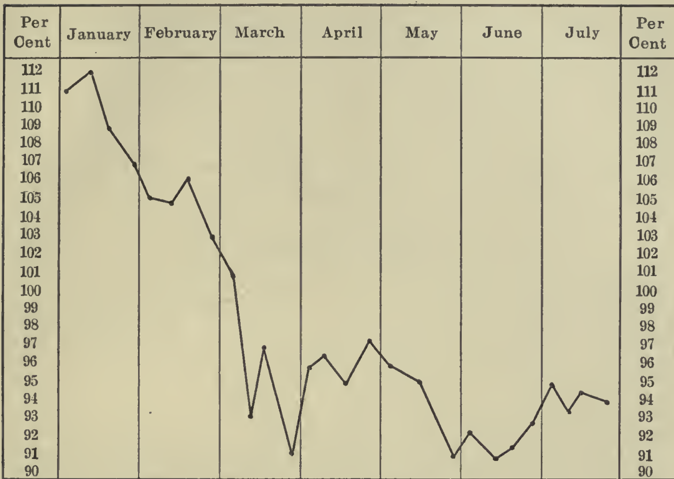
4. A man sells 1000 shares of Union Pacific stock at the opening price. Allowing for $\frac{1}{8}\%$ brokerage, how much does he realize from the sale?

5. Compute the profit in buying 2000 shares U. S. Steel at the lowest figure and selling them at the closing figure, when $\frac{1}{8}\%$ brokerage is paid for buying and for selling.

6. From these quotations make up problems of your own.

Study Exercise

249. The graph shows the fluctuations in the quotations of railroad stock from week to week during the first seven months of a year of financial uncertainty.



1. Tell from this graph what the quotations of railroad stock were on an average, from week to week. When were they lowest? When highest?

2. If a speculator bought stock the second week in February and sold it the last week in April, did he gain or lose?

3. During May, June, and July Industrial Stock fluctuated from week to week as follows:

For May: 88, 87, 87, 82 per cent.
For June: 83, 81, 82, 84 per cent.
For July: 85.5, 84, 86, 85 per cent.

4. Prepare a graph like the one above, to show graphically the fluctuations in these stock quotations.

BONDS

Study Exercise

250. Corporations, like individuals, sometimes borrow money. The same is true of cities and states. This is usually done by the issuing of **bonds.**

A promise to pay a certain sum of money, signed by the officers of a corporation under its corporate seal is called a *bond*. To insure the payment of the money, the bonds issued by corporations are secured by a **mortgage,** which is an agreement that the owners of the bonds may sell the property of the corporation, if the bonds and the interest on them are not paid by the corporation.

Care must be taken not to confuse stocks and bonds, or stockholders and bond owners. The stockholders are members of a corporation and the **owners** of the stock; bond owners are **lenders** to the corporation. Bonds bear a fixed rate of interest; the income of stocks depends upon the earnings of the corporation, after the interest on the bonds issued by the corporation, and other running expenses, have been paid.

Bonds issued by corporations are called **corporate** bonds; those issued by the United States, by states, cities, or counties are called **government** bonds. Government bonds are not secured by mortgage.

Bonds may be bought or sold in the same way as stocks. The income on bonds, and brokers' fees, are computed on the **par value** of the bond.

Coupon bonds are bonds with interest certificates attached, which may be cut off at specified dates and presented for payment to bearer at a bank or wherever the bonds are issued.

Registered bonds are recorded in the books of the company that issues them, with the name of the owner. The owner cannot transfer them unless he notify the company and have the name of the party receiving them recorded in the company's books.

Written Problems

251. 1. The New York Coal and Coke Company, referred to on page 236, decides to enlarge its business by the purchase of more delivery wagons and other equipment and by the enlargement of its yards for the storage of coal. To effect these extensions of its business the company needs $50,000. Lacking the necessary funds, it issues bonds. Each bond has a face value of $500. How many bonds are issued?

2. Each bond is issued for the period of 10 yr., with interest at 5%, payable semiannually. John

Smith lends the company \$ 500 and receives one of these bonds in return. How much interest does John Smith receive every six months? How much in the ten years before the company pays back the money?

3. How much interest must the company pay on the 100 bonds every six months? How much during the ten years?

4. When John Smith collects the interest on his bond, he tears off from his bond a *coupon*, which reads as follows:

INTEREST COUPON

The New York Coal & Coke Co.

will pay to bearer at the Second National Bank, New York, N. Y.

Twelve and $\frac{50}{100}$ Dollars

on the 15 day of May, 1914, being six months' interest on coupon bond No. 37.

Robert F. Lyon,
Secretary.

5. When and where can he collect his interest?

6. The next coupon is like this, except that the date is six months later. Just how does the next coupon read?

7. How many such coupons did the bond originally have?

8. How many does John Smith tear off every year?

9. If the bonds of the New York Coal and Coke Co. sell at $5\frac{1}{8}$% above their par value, how much could John Smith realize from the sale of his bond, after paying $\frac{1}{8}$% brokerage?

10. James Haywood has 5% bonds which yield him $80 annually; what is their par value?

11. He has $4\frac{1}{2}$% bonds having a face value of $8,000. What interest do they yield him a year?

12. The semiannual coupon of a city 6% bond is $150. What is the face of the bond?

13. A government bond of $5,000 yields $100 annually. What is the rate of interest?

14. A corporation fails in business and is able to pay only 80¢ on the dollar on its bonds. I hold bonds with a face value of $6,000. How much do I lose?

Written Problems

252. 1. George Cort owns five shares of common stock of the New York Coal and Coke Co.; John Smith owns one of the $500 bonds issued by that company. One year the earnings of the company are $16,150. Out of this sum the company has still to pay the annual interest on the bonds; the balance is divided among the stockholders, the preferred stock to receive an annual dividend of 6%.

Who received the larger amount during the year, George Cort or John Smith?

OUTLINE OF PROCESS

(*a*) Find the annual interest on all the bonds.

(*b*) Find the total dividend on the preferred stock.

(*c*) Subtract the sum of these two from $16,150, and you get the amount to be paid in dividends on the common stock.

(*d*) Find the annual rate of the dividend on the common stock.

(*e*) Then find the amount paid George Cort in dividends.

(*f*) Find the interest received by John Smith.

(*g*) Compare the two amounts.

2. When United States 4 % bonds are quoted at $116\frac{7}{8}$, what rate of interest will the purchaser of such bonds receive on his money? (Take into account $\frac{1}{8}$ % brokerage.)

3. What is the cost of 25 bonds ($1,000 each) of Brooklyn Rapid Transit bonds at the price of $89\frac{1}{4}$, including brokerage?

4. I gave my broker orders to sell $20,000 Missouri Pacific 4 % bonds at 95 and buy $10,000 Union Pacific 4 % bonds at $105\frac{3}{4}$. My broker charged me $\frac{1}{8}$ % brokerage in each of the two transactions. What balance was placed to my credit?

5. A man purchased some stock at $108 a share, which yielded an annual dividend of $6\frac{1}{2}$%. What was the rate of income on the investment? (Remember the $\frac{1}{8}$% brokerage.)

6. I receive $270 a year interest on bonds whose face value is $6,000. What is the rate of interest?

7. Which yields a greater rate of income, a 4% bond bought at par value, or a 3% bond bought at 70?

8. If a man has money to invest, which will he prefer, a stock paying regularly a 6% annual dividend, which he can purchase at 125, or a 5% bond at 98, the two investments being considered equally safe?

9. A speculator bought 500 shares of railroad stock at $83\frac{7}{8}$ and sold it the same day for $89\frac{1}{8}$. How much did he gain? How much did his broker receive?

10. Money in a Savings Bank pays me 3% annual interest. I take it out, and buy fifty $100 bonds at $110\frac{7}{8}$. The bonds bear 4% interest. Is my annual income increased or decreased? How much?

11. How much interest will be received each year on ten U. S. 3's? What rate of interest does this investment bring if the bonds were bought at 102?

Written Problems

253. Bond Transactions, July 21.

Sales	Price
10,000 N. Y. City 4's	$101\frac{3}{4}$
6,000 C. B. & Q. 4's	$97\frac{3}{4}$
1,000 Colorado Southern $4\frac{1}{2}$'s	$95\frac{3}{4}$
10,000 Manhattan 4's	$93\frac{7}{8}$
10,000 Texas Pac. 5's	109
3,000 West Shore 4's	$97\frac{5}{8}$
1,000 Wisconsin Cent. 4's	$90\frac{7}{8}$

The adjoined list shows newspaper quotations of New York bond transactions. The first line means that New York City bonds bearing 4% interest were sold, the total face value of the sales being \$10,000, and the price \$$101\frac{3}{4}$ per \$100 of face or par value.

1. Find the proceeds from each of these sales, allowing the usual brokerage.

2. Find the brokerage at $\frac{1}{8}$% on each of the above sales.

3. On August, 1914, a city borrowed \$350,000 to erect a new high school building, and issued $4\frac{1}{2}$% 15-year coupon bonds. These bonds had each a face value of \$1,000. They were turned over to brokers for sale. The credit of the city being considered good, the bonds sold above par, at $102\frac{3}{8}$. How much did the city get after the bonds were all sold?

4. How much did the brokers earn in the preceding transaction?

5. How much did a man pay who bought seven of these \$1,000 bonds?

6. If interest is payable semiannually, how much is the interest named on each coupon? What date is named on the last coupon of each bond?

7. Which is the better investment, 4% bonds at $106\frac{1}{2}$ or 5% bonds at $115\frac{7}{8}$, the security being regarded equally good in both?

REVIEW

Oral Exercise

254. **1.** Explain the formulas $p = b \times r$, $b = \frac{p}{r}$, $r = \frac{p}{b}$.

2. How are the second and third formulas derived from the first?

3. In computing profit and loss, what is taken as the base? What as the percentage?

4. In taxation, what is the percentage? What is taken as the base?

5. In commercial discount what sum is taken as the base? What one as the percentage?

6. Make similar statements relating to commission and insurance.

7. State the difference between simple interest and compound interest.

8. What is meant by exact interest?

9. What is the 5% interest of \$300 for 3 yr. 4 mo.?

Written Problems

255. **1.** A house worth \$4,000 was insured for $\frac{3}{4}$ of its value, at $1\frac{1}{3}\%$. What was the premium?

2. If a company with a capital stock of \$100,000 loses \$5,000, what % of the capital stock is the loss?

3. At what rate per cent will \$200 yield \$36 interest in 3 yr.?

4. A book costing \$6 was sold for \$4. What per cent of the cost was lost?

5. If boots that cost \$3 a pair are sold for \$4, what part of the cost is gained?

6. A horse was sold for \$50 less than it cost. If the loss was 20 % of the cost, find the cost.

7. In New England there are 15 motor vehicles for every 1,000 population. At this rate, how many are there in a town of 4,200 inhabitants?

8. For the entire United States there are said to be 11 cars for every 1,000 of population. If the total number of cars is 1,100,000, what is the estimated population?

9. A merchant gained \$4,761 in one year on goods sold at a profit of 23 %. What was the cost of the goods?

10. A stock of goods was sold at \$375, which was a loss of 7 %. What did the goods cost?

11. A real estate man sold two houses at \$5,000 each and lost 20 % on the first, but gained 20 % on the second. How much did he gain or lose on the two transactions?

12. A lawyer collected 75 % of a debt of \$1,760, and charged 5 % commission on the sum collected. How much did the creditor receive?

13. An agent was paid $355 for collecting rents. How much did he collect, his commission being 5%?

14. If the rate of taxation is $6\frac{3}{4}$ mills on a dollar, what is the assessed value of a residence on which the taxes are $43.20?

15. What is the duty, at $2\frac{1}{2}$¢ per pound, on 72 boxes of fruit, each weighing $23\frac{1}{2}$ lb., the tare being 3 lb. a box?

16. A cargo worth $1,275 was insured for $\frac{6}{7}$ of its value, at $3\frac{1}{4}$%. In case of shipwreck, what would be the total loss to the owner, including the premium?

17. A man's life is insured for $5,000 at an annual cost of $160. What was the annual premium per $1,000?

18. Find the interest of $1,243 from April 1, 1913 to June 25, 1914, at 6%.

19. What is the interest on $292.50 for 3 yr. 8 mo. 24 da., at 5%?

20. What is the interest on $785 from March 20, 1912 to August 7, 1914, at 6%?

21. Find the interest on $850 from Feb. 3, 1910 to April 6, 1915, at 4%.

22. Write a promissory note for $950, payable to the order of John Hay after 90 days with interest at 7%, and signed by Harvey Noble. Name the payee; the maker of the note. How must the note be indorsed for transferring it to bearer? For transferring it to W. P. Jones or order?

23. In what time will $400 produce $77 interest at $5\frac{1}{2}$%?

24. How long must $940 be at simple interest at 6% to amount to $1,109.20?

25. In what time will a sum of money at 6% simple interest be doubled?

~~yields 720~~ ~~earns $~~ interest yields 720 at 6% in 4 yr.

26. What principal will produce $720 in 4 yr.?

27. Find the compound interest on $400 for 3 yr. at 4%.

28. What must be paid for 275 $100-shares of Telegraph stock at 12% premium, brokerage $\frac{1}{8}$%?

29. I bought 150 shares of railroad stock at 108 and later sold them at $112\frac{1}{2}$, paying brokerage in each case $\frac{1}{8}$%. What was my gain?

30. What is the rate per cent of income realized from 6% bonds bought at 85?

31. A grain dealer paid $175 for insuring a cargo of wheat at $1\frac{1}{4}$%. For how much was it insured?

32. A man paid $5,960 for a farm and sold it for $6,500. What was the gain per cent?

33. When the entire budget of a city is $75,000 and 625 men pay $1 apiece poll tax, what must be the rate of taxation per $1,000, if the total assessed valuation is $3,700,000?

APPROXIMATIONS

Oral Exercise

256. Estimating the cost.

State the cost of the following, giving only approximate answers:

1. 8 yd. @ \$2.98.

PROCESS AND EXPLANATION

\$2.98 is nearly \$3,

$8 \times \$3 = 24$, appr. ans.

2. 15 bu. at $49\frac{1}{7}$¢.

PROCESS AND EXPLANATION

$49\frac{1}{7}$¢ is nearly $\$\frac{1}{2}$,

$15 \times \$\frac{1}{2} = \$7.50$, appr. ans.

3. $17\frac{1}{2}$ yd. @ 99¢
4. $17\frac{1}{2}$ yd. @ \$1.99
5. $18\frac{3}{4}$ yd. @ \$1.95
6. $47\frac{1}{2}$ yd. @ 33¢
7. 79 lb. @ 49¢
8. $32\frac{1}{2}$ lb. @ $12\frac{1}{2}$¢
9. $47\frac{1}{2}$ lb. @ $16\frac{2}{3}$¢
10. 100 lb. @ $14\frac{1}{7}$¢
11. 300 bu. @ $97\frac{3}{4}$¢
12. 2000 bu. @ $99\frac{1}{4}$¢
13. 1950 ft. @ $9\frac{2}{3}$¢
14. $4458\frac{1}{3}$ ft. @ 11¢
15. 30 A. @ \$497.50
16. 60 A. @ \$305.90
17. $76\frac{3}{4}$ A. @ \$25
18. 79 books @ $12\frac{1}{3}$¢
19. 41 books @ $51\frac{1}{4}$¢
20. 119 books @ $40\frac{1}{2}$¢

257. Find the interest, approximately:

1. \$209 at 5% 2 yr. 5 da.
2. \$1,300 at 6% 2 yr. 3 da.
3. \$400 at 6% 3 yr. 11 mo.
4. \$700 at 5% 4 yr. 11 mo. 18 da.
5. \$698 at 5% 2 yr.

258. Find the rate per cent approximately:

1. Base 1,200, percentage 59.
2. Base 1,799, percentage 108.
3. Base 3,009, percentage 150.
4. Base 798, percentage 56.
5. Base 12,000, percentage 239.

PART THREE

VOCATIONAL ARITHMETIC

ACCOUNTS, BILLS, AND STATEMENTS

Introduction

259. 1. A *cash account* shows the cash received and the cash paid out; also the amount of cash on hand. The cash received is entered on the left or *debit* side; the cash paid out is entered on the right or *credit* side. Debit and credit are expressions used to indicate value received and value delivered.

2. CASH

1915					1915				
March	1	On hand	$305	50	March	2	House rent	$60	
	2	Salary	250			7	Books and stationery	15	65
	25	Interest	70	60		28	Household expenses	175	75
						31	Balance	374	70
			626	10				626	10
April	1	On hand	374	70					

3. Examine this cash account and tell what sums were received and what sums were expended. How much cash was on hand at the beginning of March? How much at the end?

Keeping Accounts

260. 1. Continue this cash account for April with the following items: April 2, received salary, $250; April 2, paid house rent, $60; April 5, paid doctor's bill, $72.75; April 17, paid for bicycle, $35.50; April 20, paid for clothes, $32.50; April 22, received interest, $35; April 30, household expenses, $140.65.

2. Write out a cash account of your receipts and expenses during the past month, and find the balance at the end of the month.

3. A college student receives from home $100 on Oct. 1. He pays $25 tuition and $5 laboratory fees on Oct. 2; $12.50 for books on Oct. 3; $15 room rent on Oct. 4. On Oct. 15 he sells some old books for $2.50. On Oct. 31 he pays $21.50 for board. His incidental expenses are $17.75. Make out his cash account for October.

4. Close the following account and find the amount on hand:

Cash

July	1	On hand	189	45	July	1	Rent	95	
	1	Salary	225			6	Bicycle	40	
	15	Interest	44	80		7	Servant	25	
	17	Interest	75	50		9	Grocery	39	46
						10	Meat	15	75
						11	Dairy	12	45
						14	Medicine	15	35
						19	Doctor	85	63

5. Write the cash account for a boy, for the month of December. Dec. 1, cash on hand, $ 7.85; Dec. 5, bought a pair of skates, $ 1.25; Dec. 7, bought a geography, $ .95; Dec. 9, received for work done in store, $ 1.00; Dec. 11, received for splitting kindling, $ .35; Dec. 22, bought Christmas presents, $ 1.80; Dec. 23, bought more Christmas presents, $ 2.35; Dec. 25, gift from Uncle John, $ 1.00; Dec. 27, paid for sharpening skates, $ .15; Dec. 28, bought a sled, $ 1.15. How much is the balance on hand?

6. Continue the boy's account for January and find the cash on hand at the close of the month. Jan. 2, bought stationery, 25¢; Jan. 7, bought cap, 75¢; Jan. 13, received for work in store, $ 1.25; Jan. 15, paid for sharpening skates, 15¢; Jan. 18, bought knife, 55¢; Jan. 19, received for shoveling snow, 25¢; Jan. 21, bought paper and ink, 35¢; Jan. 24, bought pair gloves, $ .75; Jan. 27, paid for ticket to a lecture, 25¢.

BILLS AND STATEMENTS

Review of Bills

261. 1. A written statement of goods sold or of services rendered is called a **bill**. When a bill is paid, the person receiving the amount writes at the bottom Paid or Received payment, and signs his name, or the name of the firm which he represents. This shows that the bill has been paid. When the bill is receipted in this way, it is called a **receipted bill**. If, by some oversight, those who sold the goods should attempt to collect the bill a second time, the purchaser shows the receipted bill, as evidence that the bill has been paid. Receipted bills should be kept on file for one year.

2. MR. F. M. RORER

To THE RUSTIC HOME DAIRY CO. DR.

For the Month of Feb. 1912
Telephone 442 Route No. 6

39 qt. Bottled Milk	@ 13 qt. for $1	3	00
8 hf. Pt. Standard Cream	@ 40¢ a pt.	1	60
6 hf. Pt. X Cream	@ 50¢ a pt.	1	50
2 hf. Pt. XX Cream	@ 60¢ a pt.		60
2 qt. Buttermilk	@ 5¢		10
		6	80

3.

GIDDENS BROS.

15TH AND CURTIS STREETS,

DENVER, COLO., Jan. 31, 1912.

SOLD TO MRS. C. A. CHAPIN,

1747 ARAPAHOE ST., DENVER.

Jan.	11	1 doz. napkins, 2.67	1 doz. towels, 2.19	4	86		
		$10\frac{5}{9}$ yd. sheeting .23, 2.43	$6\frac{1}{3}$ yd. casing .17, 1.08	3	51		
	18	6 yd. sheeting .23, 1.38	2 towels, 70	2	08		
						10	45
		Paid Feb. 17, 1912. Giddens Bros. per L. C.					

Making out a Bill

262. 1. On April 7, 1912, Mr. John Halifax purchases from the Murray Drug Co. in Nashville, Tenn., the following articles: 1 lb. vaseline \$.35, 1 bottle listerine \$.68, 5 lb. plaster of Paris @ \$.10, 1 bottle tooth powder \$.25, 6 bottles Bethesda water @ \$.50. Write out the bill which The Murray Drug Co. sends Mr. Halifax on April 30 and the signature attached by the drug company when the bill is paid on May 13.

2. Make out a bill that some merchant in your town sends to one of his customers.

3. John Green has been working for George Brown $6\frac{1}{2}$ da. at \$2.50 a day and has furnished 40 lbs. of nails @ $3\frac{1}{2}$¢, and 695 ft. of lumber at

$22.75 per M. Make out the bill that Green sent to Brown, inserting appropriate dates and places.

4. Orr & Co., dealers in china and glassware, sell Mrs. Anna Smith a dinner set, 130 pieces, for $19.86, also 1 doz. extra plates, $2.75, 1 doz. extra fruit saucers, $1.25. The expense of crating is $2.50, of carting $.75. Make out a receipted bill.

5. Name each of the parties in the following bill. Has it been paid?

St. Louis, Mo., May 1, 1915.

Mr. J. C. White,
1119 Wood Ave.,

To F. C. COLEMAN, M.D., Dr.,
207–209 Market St.

To professional services rendered, Jan. 1 to May 1, Received payment May 18, 1915. Thanks. F. C. Coleman.	$45	50

6. M. Jones gave 32 music lessons to Alma Ehrich at $1.50 a lesson. M. Jones sends a bill to Mr. F. L. Ehrich, Alma's father. Make out this bill and receipt it as if you were M. Jones.

The Monthly Statement

263. 1. The following is a model form of **statement** sent by a business firm to a customer:

2.

MILWAUKEE, WIS., July 1, 1915

MR. WILLIAM DROWN,
Madison, Wis.

In account with MORLEY & CO.

1915		Dr.				
March	1	To acct. rendered	$425	75		
	17	To mdse.	270			
May	5	To mdse.	750		$1445	75
		Cr.				
March	7	By cash	400			
April	1	By cash	250			
June	20	By cash	750		400	00
		Balance			45	75

By whom is this statement sent out? To whom? The part marked Dr. indicates that William Drown is *Debtor* to Morley & Co. on a previous account and for merchandise received.

The part marked Cr. indicates that William Drown is *Creditor* for three cash payments.

The Balance shows that William Drown still owes Morley & Co. $ 45.75.

Making out a Statement

264. 1. John Hay owes Forbes Bros. on Aug. 1, $ 300. On Aug. 2 Hay receives merchandise costing $ 155.75; on Aug. 28 he receives merchandise again, costing $ 260.70. On Sept. 1 he pays Forbes Bros. $ 625.50. On Sept. 10, Forbes Bros. send him a statement. Make out this statement.

2. The Board of Education, Butte, Montana, has an account with John W. Graham Company, Spokane, Washington. Nov. 1, the account rendered shows an indebtedness of \$ 437.50. During the month of November the Board purchases the following items: 325 Tarr and McMurry New Geography, First book, at 65 ¢ a copy; 135 doz. Second book, at \$ 1.10 a copy; 500 lb. newspaper at 5 ¢ a pound; 400 ream foolscap at 80 ¢ a ream. A check is sent to J. W. Graham Co., Nov. 15, for \$ 437.50. Make out statement of account under date of Dec. 1.

3. George Lincoln owes his grocer, J. A. Stevens & Co., \$ 53.35, May 1. In the course of the month his purchases are: 6 doz. eggs at 24 ¢ a dozen; 2 bushels of potatoes at \$ 1.05 a bushel; fruit and vegetables to the amount of \$ 7.40; 6 lb. of butter at 30 ¢ a pound; 20 lb. granulated sugar at $5\frac{1}{2}$ ¢ a pound; 5 lb. bacon at 24 ¢ a pound; a 14 lb. ham at 21 ¢ a pound. He pays cash on account, \$ 45.00. Make out statement of account under date of June 1.

4. Make statements to be sent out by local business houses. In this statement give attention to the following items: The address and date, the name and address of the purchaser or debtor, the name of the business house, the correctness of the statement.

Balancing an Account

265. *Second National Bank*, in acct. with John Brown.

1915					1915		
March 4	Balance	73	45	March 5	By check	23	75
April 2	To deposit	25	50	March 12	By check	19	65
April 7	To deposit	73	45	April 1	By check	25	50
May 3	To deposit	115	63	April 5	By check	9	78
June 25	To deposit	209	75	April 11	By check	35	75
July 7	To deposit	88	65	May 17	By check	79	85
July 30	To deposit	67	25	July 20	By check	146	65
Aug. 1	Balance						

This statement is prepared by the Second National Bank.

On the left-hand side of this account is shown how much John Brown has deposited in the bank. On the right are shown the amounts withdrawn from the bank by John Brown. Formerly the left side was marked at the head **Dr.**, the right side **Cr.** These designations are now usually omitted. Upon subtracting the sum of the amounts withdrawn from the sum of the amounts on deposit, one obtains the balance of the account. The process is the same as in Cash Accounts.

1. Find the balance in this account and check.

2. Rule a sheet of paper so that it may be used for the making of a bank account. Insert four deposits items and two withdrawals and balance the account.

3. What happens if the account is overdrawn?

4. Secure a check book from the local bank. Find out how a person can keep his own account in the check book.

266. Find the balance in each of the following accounts and check:

1.

375	40	296	45
678	50	475	37
379	67	509	75
430	75	409	63
675	15	708	15
45	10	87	45
75	05	7	77
123	40	17	89
175	23		

2.

175	67	73	40
257	76	209	97
314	25	306	45
712	63	596	68
100	00	104	63
371	46	275	40
574	13	379	46
615	45	193	25
217	45	70	70

3.

315	75	293	36
1136	95	239	35
746	64	745	30
647	46	109	95
317	16	347	63
550	62	745	00
100	05	316	45
700	63	407	60
875	45	397	65
305	10	432	10

4.

185	74	100	00
87	23	75	00
206	48	250	00
16	24	25	00
527	39	500	00
846	48	1000	00
27	50		
30	35		
25	75		
544	99		

BUSINESS APPLICATIONS

Making Change

267. In business, change is made by the *method of adding*. Thus, if $2 is given in payment for $1.78, the clerk will say "One dollar seventy-eight cents, eighty, ninety, two dollars," and lay down each time the coin or coins that make the sum named.

Acting as a clerk, make the change and repeat the amounts, in each of the following cases. The first figure is the cost of the purchase, the second figure is the amount given in payment.

1. $1.55, $2	**2.** $1.60, $2.
3. $1.76, $2	**4.** $1.34, $1.50
5. $1.03, $1.50	**6.** $2.13, $2.50
7. $2.45, $3	**8.** $3.73, $4.
9. $4.67, $5	**10.** $3.76, $5.
11. $1.25, $1.50	**12.** $.63, $1
13. $1.98, $5	**14.** $2.75, $3
15. $3.15, $5	**16.** $4.09, $5
17. $5.98, $10	**18.** $2.01, $3
19. $6.18, $10	**20.** $.79, $1

21. $2.88, $5
22. $6.79, $7
23. $.91, $1
24. $3.67, $4
25. $2.51, $3
26. $1.97, $5
27. $3.17, $4
28. $4.63, $5
29. $2.57, $3
30. $5.91, $10.

Using the Telephone

268. In telephoning, 20–5 means a charge of 20¢ for the first 3 minutes and 5¢ for each additional minute. 40–10, or 60–15 has a corresponding meaning.

Find the cost of telephoning:

1. For 7 minutes at 20–5.
2. For 5 minutes at 40–10.
3. For 7 minutes at 60–15.
4. For 6 minutes at 100–25.
5. For 4 minutes at 100–25.
6. For 12 minutes at 40–10.
7. For 5 minutes at 60–10.

Cost of Money Orders

269. 1. The fees charged by the United States post office for making out money orders that are payable at the post office in the United States specified in the order are as follows:

Sums Over	Not Over	Fee	Sums Over	Not Over	Fee
	$ 2.50	3¢	$ 30.00	$ 40.00	15¢
$ 2.50	$ 5.00	5¢	$ 40.00	$ 50.00	18¢
$ 5.00	$ 10.00	8¢	$ 50.00	$ 60.00	20¢
$ 10.00	$ 20.00	10¢	$ 60.00	$ 75.00	25¢
$ 20.00	$ 30.00	12¢	$ 75.00	$ 100.00	30¢

2. William Bartlett sends a money order to John Hay for $ 67.50. How much must he pay to the clerk at the money order window?

He must pay the face of the order, $ 67.50, and a fee of 25¢ in addition.

3. How much must be paid for a postal money order of $ 45.75?

4. Harry Jones buys three money orders for $ 15.73, $ 27.60, and $ 11.45. How much must he pay for the three orders? If he could have used one order for the entire amount, how much would he have been able to save by so doing?

5. Mr. Emery goes to the money order department at a post office and has the following money orders cashed: $ 7.68, $ 11.73, $ 15.39. He also purchases money orders for the payment of amounts, as follows: $ 2.52, $ 15.45, $ 44.63. How much did the sum paid to the clerk exceed the sum received?

6. By what methods may money be paid to a person at a distance?

Oral Exercise

270. State rapidly the total cost of postal money orders for:

1. \$2.68	2. \$4.21	3. \$6.37
4. \$10.41	5. \$25.36	6. \$33.43
7. \$45.67	8. \$40.45	9. \$76.41
10. \$67.05	11. \$55.36	12. \$44.54

Telegrams

271. Telegrams are classified by the telegraph companies of the United States as (1) Day Message, (2) Night Message, (3) Day Letter, and (4) Night Letter.

The rate for day message is based upon minimum of ten words, and zone, as Chicago to San Francisco, \$1.00, New York to Philadelphia, 25¢. Night messages are sent at reduced rates, but are not delivered until the morning of the next following business day. The charge for night messages is less than the charge for sending day messages because the telegraph company is able to send the messages when the wires are less busy.

Oral Exercise

272. What is the cost of a telegram containing 15 words @ 25–2?

Process and Explanation

@ 25–2 means a cost of 25¢ for the first 10 words and 2¢ for each additional word. 15 words cost $25¢ + 5 \times 2¢ = 35¢$.

The rate for the 50-word night letter is the same as the rate for the 10-word day letter. It frequently happens that the telegram contains more or less words than are specified on the rate cards. In this case each additional ten words or less over the 50 words in the night letter costs $\frac{1}{5}$ as much as the first fifty words cost. No reduction from the price of a 50-word night letter for a letter containing a less number of words.

The rate for a day letter of 50 words is $1\frac{1}{2}$ times the rate of the 50-word night letter. Each additional 10 words or less to the 50-word day letter is charged at $\frac{1}{5}$ the rate of the 50-word day letter. No reduction is made from the regular rate for a day letter of less than 50 words.

Oral Exercise

273. State rapidly the cost of the following telegrams:

1. 14 words @ 50–3
2. 13 words @ 60–4
3. 18 words @ 25–2
4. 17 words @ 50–3
5. 16 words @ 75–5
6. 15 words @ 25–1
7. 20 words @ 25–2
8. 25 words @ 50–3
9. 19 words @ 60–4
10. 23 words @ 25–2

Oral Exercise

274 State the cost of the following night letters:

1. 40 words, when standard day rate for 10 words is 25 ¢.

2. 55 words, when standard day rate for 10 words is 40 ¢.

3. 61 words, when standard day rate for 10 words is 50 ¢.

4. 75 words, when standard day rate for 10 words is 75 ¢.

State the cost of the following day letters:

5. 50 words, when standard day rate for 10 words is 25 ¢.

6. 60 words, when standard day rate for 10 words is 35 ¢.

7. 65 words, when standard day rate for 10 words is 40 ¢.

8. 70 words, when standard day rate for 10 words is 75 ¢.

9. If the cable rate is $ 1.75 a word, what will be the cost of the following message: William J. Sloan, London, Option on stock for twenty-four hours. Brown and Seeman.

When messages are likely to be very lengthy and expensive and such occur with great frequency, codes may be employed or even abbreviations when possible. Many firms have code words associated with their catalogued articles, which purchasers may use in telegraphing orders.

If codified the message might read "infanti." What would be the amount saved by codified message over the former message?

Codes are nearly always used in cablegrams. The cost of a cablegram message is very high, and may often reach a charge of two dollars a word. When a word of more then fifteen letters is used, it is counted as two words. The name, address, and signature are charged for in cablegrams. Consequently codes have been devised by nearly all who have to make use of the cable. A code word must not contain more than ten letters, otherwise it counts as two words.

Messages by wireless telegraph are increasingly common to-day. Almost daily the metropolitan papers communicate with persons on ocean liners several hundred miles from the port. One at sea may send messages to coast points from which they are transmitted inland if necessary. The rate is determined on the basis of so much a word, as 25¢ a word within a certain zone. The calculation of cost of messages is identical with that of cost of telegraph messages.

The wireless telephone has recently been invented. It may soon be used for transmitting messages.

Telegraphic Money Orders

275. John Brown desires to remit a sum of money to William Jones in another city, without loss of time. John Brown goes to a telegraph office and pays the money to the telegraph company. The company sends a message, directing the telegraph office in the city where William Jones is to pay the money to William Jones.

The rates for such *telegraph money orders* are as follows:

Twice the cost of a 10-word standard day message, plus 1% of the amount of the order. If the amount of the order is less than $25, the fee is the same as for $25.

State the cost of the following telegraph money orders:

1. $25 when standard day rate for 10 words is 25¢.

2. $100 when standard day rate for 10 words is 40¢.

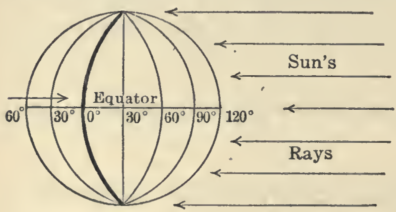
3. $200 when standard day rate for 10 words is 50¢.

4. $250 when standard day rate for 10 words is 75¢.

LONGITUDE AND TIME

Study Exercise

276. 1. In railroad travel, as well as in telegraphing, telephoning, and cablegram messages, it is frequently necessary to consider the difference in time between different localities. The difference in time of sunrise or of sunset will be considered first. This yields the difference in what is called *local time*. Later the subject of *standard time* will be explained.



2. The earth is a huge sphere turning like a top around an axis. It makes one complete revolution from west to east in 24 hr. If one such rotation is measured by 360° of longitude, then we see that

24 hr. correspond to 360°
1 hr. corresponds to $\frac{1}{24}$ of 360°, or 15°
1 min. of time corresponds to $\frac{1}{60}$ of $15° = \frac{1}{4}° = 15'$
1 sec. of time corresponds to $\frac{1}{60}$ of $15' = \frac{1}{4}' = 15''$

> The number of degrees, minutes, and seconds of longitude is fifteen times the corresponding number of hours, minutes, and seconds of time.

Finding the Difference in Time

277. 1. In Denver the sun rises one hour later than in St. Louis. What is the difference in longitude?

2. What is the difference in longitude between two places having 1 minute's difference in the time of sunset?

3. The sun rises at a place A 1 hr. 1 min. 1 sec. later than at another place B that is farther east. What is the difference in longitude?

Find the difference in longitude, when the difference in time is

4. 2 hr. 30 min. 5. 4 hr. 10 min. 6. 1 hr. 45 min.

7. 3 hr. 20 min. 8. 2 hr. 30 min. 9. 65 min.

10. When it is noon at Denver, what is the time 15° west? 30° west? 45° west? 15° east? 45° east? 60° east? 90° east? 165° east? 180° west?

11. If I start from New York and travel until my watch is 2 hr. 30 min. too slow, in what direction and how many degrees of longitude do I go? How many miles?

The Longitude of Places

278. 1. Astronomers of different countries have agreed to measure longitude from a meridian passing through Greenwich, in England, the seat of the Royal Observatory. In our diagram the heavy circle passing from the north pole to the south pole is intended for this *principal meridian.* Its longitude is marked 0°. Longitudes are indicated as so many degrees *east* or *west* of Greenwich.

2. The longitude of most places may be found listed in the Geographical Gazetteer. For practical purposes it may be obtained from the geography textbook. This list is given for convenience.

Albany, N. Y., 74° 2′ W.
Berlin, Germany, 14° 36′ E.
Boston, Mass., 71° 4′ W.
Philadelphia, Pa., 75° 10′ W.
San Francisco, Cal., 122° 26′ 15″ W.
Calcutta, 88° 20′ E.
Nashville, Tenn., 86° 48′ W.
New Orleans, La., 90° 3′ W.
Chicago, Ill., 87° 36′ 41″.
Seattle, Wash., 122° 19′ 59″.
Honolulu, H. T., 157° 52′.

The Difference in Longitude and the Difference in Time

279. 1. Find from the list the difference in longitude between Albany and Boston.

Find the difference in longitude between

2. Albany and Nashville.

3. Albany and New Orleans.

4. Albany and Berlin.

5. Albany and Calcutta.
6. Berlin and Calcutta.
7. Berlin and Nashville.

PROCESS AND EXPLANATION

Let the difference in time be x hr. If x is multiplied by 15, it will give the difference in longitude, expressed in degrees. The difference in longitude is 2° 58′. Expressed in degrees it is 2.967.° Hence

$$15x = 2.967$$
$$x = .197 \text{ hr.}$$
$$= 11.9 \text{ min.}$$

Written Exercise

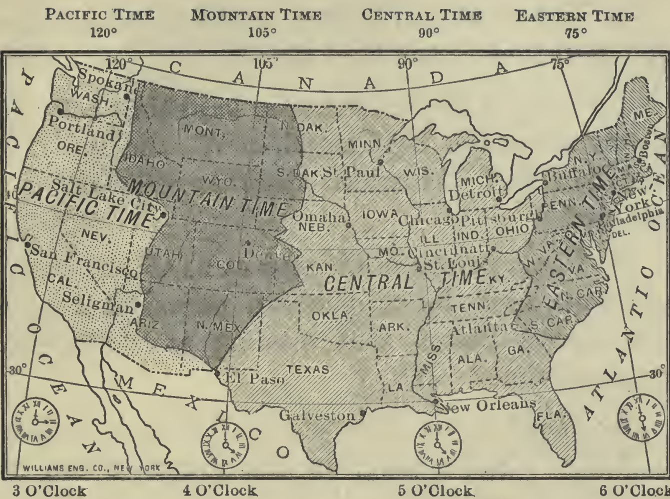
280. Find the difference in the time of sunrise between:

1. New Orleans and Nashville.
2. Calcutta and Berlin.
3. Boston and Calcutta.
4. New Orleans and Berlin.
5. Boston and Albany.
6. ~~New York City~~ Philadelphia and Chicago.
7. Philadelphia and Seattle.
8. Chicago and Honolulu.

STANDARD TIME

Introduction

281. Primarily for the convenience of the railroads, a standard time was established in 1883, by mutual agreement of the railroads. According to the agreement, the continent of the United States, extending roughly from 65° to 125° W. longitude, is divided into four belts. The belt farthest east takes its time from the 75° meridian. Accordingly, all places in that belt have the same standard time. Of the other belts, each has its time set one hour *slower* than the time of the belt immediately east of it. The difference between "Eastern time" and "Pacific time" is, therefore, three hours. When it is 9 A.M. in Boston, it is 6 A.M. in San Francisco, by standard time. The dividing lines between the belts have been changed from time to time and are determined by what is most convenient. By this group of belts, the standard times of all places in the United States differ only by whole hours. The time fixed by the exact longitude of a place is called "local time," to distinguish it from the "standard time" of the railroads.



Finding the Standard Time

282. 1. When it is 10 o'clock in the morning in Boston, by standard time, what is the standard time in Chicago? In Denver? In San Francisco?

2. When, by standard time, it is 12 o'clock, noon, in Wyoming, what is the time in Portland, Maine? In New Orleans? In St. Paul? In Washington, D.C.?

3. What is the difference in standard time between Maine and New Mexico? Between Florida and Minnesota?

4. For places in the state of Maine, which is ahead, the standard time or the local time?

5. For Portland, Oregon, which is ahead, the standard time or the local time?

6. What is the difference in local time between places along the 105° meridian W., and the 120° meridian W.? At which meridian does the sun rise first?

7. Can a person in Chicago at 9:45 P.M. standard time reach by telephone a person in Boston who leaves Boston that evening at 10:30 P.M. standard time? Why?

8. What is the latest standard time in a place in Idaho when a party in San Francisco can be reached by telephone, if that party sails at 11 o'clock A.M., standard time?

9. A telegram is sent from San Francisco to Boston at 3 P.M., standard time. If 45 minutes are lost by delays in transmitting the message, at what time is it received in Boston?

10. A person in St. Louis wishes to send a telegram to a bank in Boston which closes at 3 P.M., standard time. If 45 minutes should be allowed for the transmission of the telegram, what is the latest moment, by standard time in St. Louis, that the telegram can be sent?

11. Up to what time can a man in Boston wire a bank in Denver, if he allows 30 minutes for transmission, the Denver banks closing at 3 P.M.?

12. Great Britain uses Greenwich time. The standard eastern time is the time of the 75° meridian, west. When it is 1 P.M. by clocks in London, what is the standard time in New Haven, Conn.?

13. At what place on the surface of the earth are the 75° and 76° meridian lines, west, the farthest apart from each other? If on the equator they are approximately $69\frac{4}{9}$ *miles apart*, what is the circumference of the earth, measured along the equator?

14. Two places on the equator have a difference in longitude of 2° 15′. How many miles apart are they?

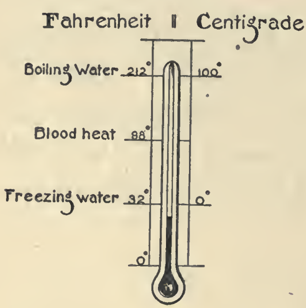
15. A ship traveling along the equator changed its longitude in two days by $3\frac{1}{2}$ degrees. How far did it travel? What was its average rate per hour? If the ship traveled eastward, did a clock on shipboard gain or lose time on account of the ship's motion?

TEMPERATURE

Introduction

283. 1. In the transportation and in the preservation of fruit, meats, eggs, and other food materials, cold storage often plays an important part. This involves the determination of temperatures.

2. In the determination of temperatures two kinds of thermometers are in common use, the *Fahrenheit* and the *Centigrade*. The latter is used mainly in scientific work. The boiling point of water is marked 212° on the Fahrenheit scale and 100° on the Centigrade scale. The freezing point of water is marked 32° on the Fahrenheit scale and 0° on the Centigrade.



Study Exercises

284. 1. How many degrees are there between the freezing point and the boiling point of water on the Fahrenheit scale? How many on the Centigrade scale?

2. If a difference in temperature of 180° F. is equal to a difference of 100° C., then 9° F. are equal to how many degrees C.?

3. If the temperature of a room is 25° C., what is the temperature in Fahrenheit degrees?

PROCESS AND EXPLANATION

Since 5° C. = 9° F., 25° C. = $\frac{9}{5} \cdot 25$° F. = 45° F. Since the freezing point on the Fahrenheit scale is 32°, we must add 32° to 45° and we get 77° F.

4. Designate any temperature Centigrade by $C°$. To what temperature Fahrenheit is this equal?

PROCESS AND EXPLANATION

As in the preceding case, $C°$ must be multiplied by $\frac{9}{5}$, and we get $\frac{9}{5}\,C°$ as the number of degrees Fahrenheit above the freezing point. Since the freezing point is marked 32° on the Fahrenheit scale, add 32° to $\frac{9}{5}\,C°$, and we get the temperature in the Fahrenheit scale. We indicate this by the equation

$$F° = 32° + \tfrac{9}{5}\,C°.$$

Written Exercise

285. Change from degrees C. to degrees F.:

1. 30° C.	2. 25° C.	3. 35° C.
4. 50° C.	5. 70° C.	6. 85° C.

Written Exercise

286. Degrees F. are changed to degrees C. by the following formula:

$$C° = \frac{5}{9}(F° - 32°).$$

1. Change 50° F. to degrees C.

PROCESS AND EXPLANATION

For $F°$ write 50°. Then 50° − 32 = 18°. $\frac{5}{9}$ times 18° = 10°. Hence, 50° F. = 10° C.

Change from degrees F. to degrees C.:

2. 59° F. 3. 68° F. 4. 75° F. 5. 100° F.

Written Problems

287. 1. A railroad car is loaded with 32 rows of boxes of oranges, 12 boxes in each row. If the weight of each box is 72 lb., find the weight of the entire load.

2. If the freight rate between California and the East varies from \$1.12½ to \$1.50 per hundred pounds, when shipped in carload lots, what is the minimum and the maximum freight on the carload of oranges in Ex. 1?

3. A few years ago 32,000 such carloads of oranges and lemons were sent East from California. At \$1.12½ a hundred pounds, what sum did the railroads receive for freight?

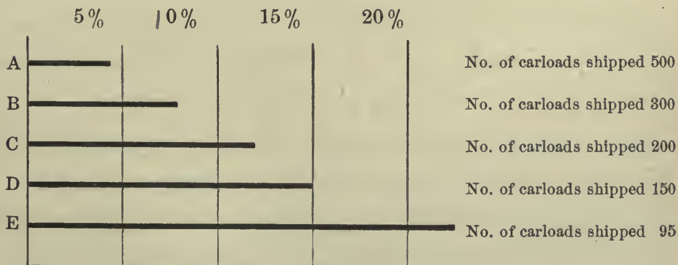
4. A refrigerator car has two ice bunkers, one at each end, each bunker holding from 2 to 2½

tons of ice. If, on an average, each car has to be iced twice before it reaches its destination, how much ice is used for 32,000 carloads? How much is the expense for ice at $.95 a ton?

5. The rate charged by the roads for refrigeration of a standard car of 384 boxes is $ 62.50 from California to Chicago and $ 75 from California to New York. If $\frac{5}{8}$ of the 32,000 carloads went to Chicago and the rest to New York, how much was paid to the railroads for refrigeration?

6. Even in refrigerator cars, oranges will decay, if proper precautions are not taken. In 1905 it was found decay was largely due to cuts or blows, and that 17 % of the oranges were thus injured. If that year yielded a crop of 24,000 carloads, each carload estimated at $ 1,000, what was the threatened loss due to decay?

7. In some packing houses the injuries were more numerous than in others. In 5 such houses the per cent of oranges injured was as indicated in the following chart:



What per cent of all the oranges from these five packing houses were injured?

8. By great care, the amount of injury to oranges in picking and packing has been reduced from 17% to almost nothing. If injured fruit is estimated at only $\frac{1}{3}$ of the regular value, how much is saved by this reduction, on 10 carloads of fruit, valued at \$900 a load?

9. In picking 600 boxes of oranges, which is better economy, to hire laborers each of whom picks 100 boxes of oranges a day at 3¢ a box, but cuts 10% with the clippers and sometimes drops the fruit several feet into a bag, thereby injuring 15% more, or to hire laborers each of whom picks 40 boxes for \$3, without injuring the fruit? Uninjured fruit is worth to the grower \$1.25 a box, injured fruit only half that amount. How much is saved on the 600 boxes by employing the higher-priced laborers?

10. Oranges loaded into a refrigerator car in a hot condition, say 87° F., take 5 days to cool to 40° F. At 40° F. there is no loss due to ripening and decay, under otherwise good conditions. What is the average rate of cooling per hour?

11. In recent years fruit has been cooled before transportation by special devices, from 87° to 40°, in only 20 hr. How many times more rapid is this rate of cooling than that in the previous example?

12. Oranges subjected to this process of pre-cooling can be packed more closely in the refrigerator cars. If an increase of nearly 43 % over the standard load of 384 boxes is effected, how many boxes are packed in one car? How many tons of fruit is this?

13. The temperature best suited for keeping oranges is 45° F. Bananas keep their condition best at a temperature 33 % above that of 45° F. What is the best temperature for bananas, expressed in degrees of the Centigrade scale?

14. A pre-cooling plant reduces the temperature of fruit from 75° F. to 60° F. in 12 hr. Find the average rate of cooling per hour.

Written Problems

288. **1.** The same degree of low temperature is not suited to the economic preservation of every kind of food. Apples freeze at – 5° C., that is, 5° C. below the freezing point of water, while raisins freeze at – 3° C.

How many degrees Centigrade below the temperature of boiling water are – 3° C. and – 5° C.?

2. Tell from the drawing the number of Fahrenheit degrees which are equal to – 3° C. and – 5° C.

3. Meat must be chilled gradually, before it is put into cold storage, or the inside will spoil. Chilling rooms are kept at about 40° F. Later it is put into cooling rooms having a temperature of 30° to 35° F, and finally into cold storage at less than 28° F. Change these temperatures to the Centigrade scale.

4. Meat at blood temperature (98° F.) is put in a chilling room where in 36 hr. it cools to 40° F. What is the average rate of cooling?

5. The U. S. cold storage building at Manila has a storage capacity of 1,200 tons of beef, 200 tons of mutton, 50 tons of butter, 100 tons of potatoes, and 100 tons of bacon. If 1 lb. of beef is worth 15¢, of mutton 12¢, of butter 20¢, of potatoes $\frac{1}{2}$¢, of bacon 18¢, what is the total worth of the provisions that the building can hold?

6. The low temperatures of cold storage are usually obtained by the evaporation of liquid ammonia. A small refrigerating machine will keep the chilling in a packing house containing 12,000 cu. ft. of space at 40° F. How many times greater must be the capacity of a machine that keeps at 40° F. an 8-story building, 100 ft. by 150 ft., each story being 8 ft. high?

TRANSPORTATION

Written Problems

289. 1. If the freight on a steer of average weight (1,250 lb.) from Chicago to New York is \$4 to \$4.40, while the freight on the amount of fresh beef yielded by the animal (700 lb.) is \$3.15, how much cheaper can 21,000 lb. of fresh beef be transported than the live animals necessary to produce that amount of meat?

2. From Kansas City to New York the corresponding difference between live and dead freight is \$2.50 per head. If this difference is diminished $\frac{1}{10}$ when allowance is made for the expense of icing, what is the difference on 275 head?

3. The cost of shipping a live steer from Chicago to Liverpool, including freight, feed, and attendance, is \$13.60 to \$16.70. The average weight of fresh beef yielded by the animal can be shipped for only 48% of that price. Find the difference in price on 150 head.

4. One year the rate of insurance on cattle shipped from New York to England was $\frac{1}{4}$ of 1 per cent. What insurance was paid on 300 head of cattle valued at \$70 each?

FOREIGN MONEY

Study Exercise

290. Table to be memorized.

ENGLISH MONEY

12 pence ($d.$) = 1 shilling ($s.$) = 24.3¢+

20 shillings = 1 pound (£) = $ 4.86+

FRENCH MONEY

100 centimes (c.) = 1 franc (fr.) = 19.3¢

Switzerland and Belgium have monetary units of the same name and value as the French. Spain, Italy, and Greece have monetary units of the same value as the French, but the coins have different names. Thus, in Italian money 1 fr. = 1 lira.

GERMAN MONEY

100 pfennigs (pf.) = 1 mark (M.) = 23.8¢.

Oral Exercise

291. 1. One pound is (nearly) how many dollars?

2. What is the value of 10*s*. in money of the United States?

3. $100 are roughly how many francs?

4. Which is worth more, 1 mark or 1 franc?

5. A German hotel has a rate of 10 M. a day. How much is that in our money?

Oral Exercise

292. About how much are the following sums in our money?

1. £2	2. 9*d*.	3. 15*s*.
4. 10 fr.	5. 1 lira	6. 2 fr.
7. 100 M.	8. 50 pf.	9. £10 2*s*.
10. 1 M. 75 pf.	11. 100.50 fr.	12. 200 lira
13. 1000 fr.	14 300 M.	15. £50

Written Problems

293. 1. A student buys a post-office money order to pay for books that he purchased from a dealer in Berlin. If the office has a rate of 4.18 M. on a dollar, how much will a money order for 50 M. cost, the fee for the money order being 15¢?

2. A man wants to send \$ 75 to Paris by money order. The post office has a rate of 5.18 fr. on a dollar. For how many francs is the money order made out?

3. A certain set of books is quoted at £2 3*s*. 9*d*. How much is this in our money?

4. When exchange between New York and Paris is quoted as 5.22, how much will it cost to buy in New York, without brokerage, a draft for 10,000 francs?

SUGGESTION: Rate of exchange 5.22 means that \$1.00 will buy 5.22 francs.

5. When the rate of exchange between New York and London is quoted as 4.83, what will it cost in New York, without brokerage, to buy a draft for £500?

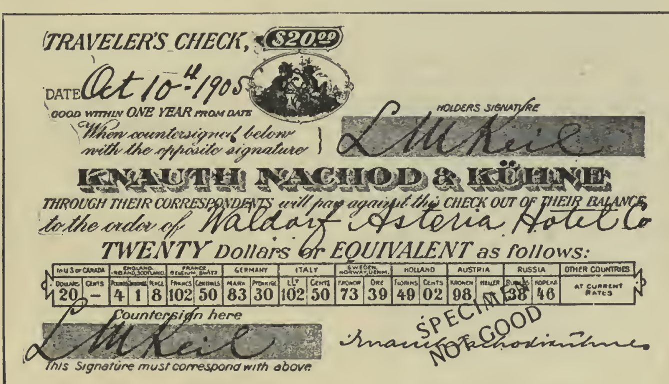
6. A merchant buys a draft at London for £400, paying $\frac{1}{8}\%$ brokerage. On the date that he bought the draft, the rate of exchange was 4.85. What was the cost of the draft?

7. Quotations on rates of exchange between New York and most foreign countries may readily be obtained from bankers. Get if possible the rate of exchange between New York and Berlin. Find the cost of a draft for 100 Reichsmarks.

8. Get if possible the rate of exchange between New York and Amsterdam, and find the cost of a draft for 1,000 guilders.

Traveler's Checks

294. Travelers may secure from banks and express companies traveler's checks, which can be cashed at hotels and banks abroad. They are issued for fixed amounts and state on their face their value in the coinage of the country in which they may be presented. The purchaser of the check signs his name on the face of the check at the time of purchase, and again on the face or back at the time of cashing, for the purpose of identification.



TRAVELER'S CHECK, $20.00

DATE Oct 10th 1905

GOOD WITHIN ONE YEAR FROM DATE

When countersigned below with the opposite signature

HOLDERS SIGNATURE L. M. Keil

KNAUTH, NACHOD & KÜHNE

THROUGH THEIR CORRESPONDENTS will pay against this CHECK OUT OF THEIR BALANCE to the order of Waldorf Astoria Hotel Co

TWENTY Dollars or EQUIVALENT as follows:

In U.S. or Canada		England, Ireland, Scotland			France, Belgium, Switz.		Germany		Italy		Sweden, Norway, Denm.		Holland		Austria		Russia		Other Countries
Dollars	Cents	Pounds	Shillings	Pence	Francs	Centimes	Mark	Pfennige	Lire	Cents	Kronor	Ore	Florins	Cents	Kronen	Heller	Rubles	Kopecks	
20	–	4	1	8	102	50	83	30	102	50	73	39	49	02	98	–	38	46	At current rates

Countersign here L. M. Keil

This Signature must correspond with above

Knauth Nachod & Kühne

SPECIMEN NOT GOOD

295. 1. How much is £20 8*s.* 2*d.* worth, taking 1 pound = $4.866? How much less than this does L. M. Keil receive in England? How much profit is made in issuing this traveler's check?

2. From the data given in this traveler's check, find the approximate value in our money of one Russian ruble and of one Austrian krone.

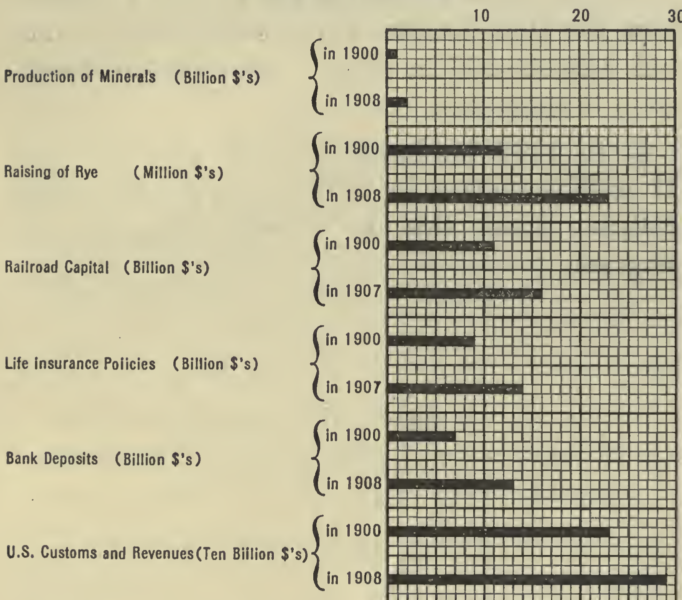
3. Had this traveler's check been made out for \$75, instead of \$100, what would the amount have been in German money?

4. About how much is a franc, expressed in rubles? Expressed in kronen?

5. About how much is one pound expressed in French money? In German money? In Austrian money? In Russian money?

COMMERCE IN THE UNITED STATES

Written Problems



296. 1. Name the value of the minerals mined in the year 1900 and in 1908. Find the ratio of the first amount to the second.

The diagram indicates a production valued at \$1,000,000,000 for 1900, and \$2,000,000,000 for 1908.

These are in the ratio of 1 : 2. All the figures shown in the diagram give the amounts only in round numbers.

2. Name the value of each article given above, for each year, and find the ratio of the value of each item in 1900 to the value at the later date.

3. The values of the exports of cotton products from the United States were $24,000,000 in 1900 and $25,000,000 in 1908. Draw line in the diagram to show this.

4. In 1900 the imports of silk products were 31 million dollars, in 1908 they were 33 million dollars. Show this by drawing lines in the diagram.

HOUSEHOLD ACCOUNTS

Family Budget

297. 1. At the beginning of a new year the head of a family prepares the following budget for the ensuing year:

Food	$180	Advancement:		
Shelter	121		Church	$ 30
Clothes	140		Y. M. C. A.	10
Operating expenses:			Summer school	100
Coal, wood, ice	50		Insurance	140
Gas, laundry	20		Vacation	50
			Doctor	10
			Bank	325

How much is his entire budget?

If his income is only $1,000 a year, by how much a month must he reduce his expenses?

2. A father has prepared the following budget for one year:

Food	$344.93	Advancement	$312.00
Shelter	144.00	Operation	150.50
Clothes	100.00	Incidentals	46.85

What must be the father's monthly wages, if this budget meets all the expenses of his family?

3. The income of a family is $2,510; what is the year's deficit, if the following budget is adhered to?

Food	$617.82	Operation	$274.61
Shelter	231.67	Advancement	784.03
Clothes	520.07	Incidentals	128.50

4. How much must the head of a family earn in one year, to be able to save $800 and meet the following expenses:

Food	$260.00	Advancement:	
Shelter	500.00	Life insurance	$192
Clothes	259.60	Children's allowances	20.50
Operating expenses:		Benevolence	84.00
Fuel	120.00	Incidentals	80.00
Service	72.00	Taxes	95.50
Telephone, light	42.00	Tuition at school	75.00

Written Problems

298. 1. How many yards of casing 45 in. wide must be bought to make one half dozen pillow cases, if each case is to measure 35 in. when finished, with a 3 in. hem, and 1 in. is allowed for shrinkage?

2. How many yards of curtain material 36 in. wide are needed for six pairs of curtains, if each

curtain when finished is to measure $2\frac{1}{4}$ yd. with a 3 in. hem at the bottom and a $1\frac{1}{2}$ in. hem at the top?

3. How many yards of sheeting $2\frac{1}{4}$ yd. wide are required to make 8 sheets if each sheet when finished is to measure $2\frac{3}{4}$ yd. with a $2\frac{1}{2}$ in. hem at one end and a 2 in. hem at the other, and 2 in. is to be allowed for shrinkage?

4. A girl's sailor collar is 41 in. around. How many yards of narrow braid are required for 9 rows around the collar?

5. If 15 yd. of black silk 20 in. wide will make a dress, how many yards of silk 36 in. wide will it take for 7 dresses of like pattern?

6. Which is cheaper to buy for a dress, 11 yd. of plain goods 27 in. wide at 75¢ per yard, or goods of the same quality 40 in. wide at 95¢ per yard?

7. If it takes $1\frac{3}{8}$ yd. of broadcloth at $2.25 per yard and $2\frac{7}{8}$ yd. of ribbon ruching at 25¢ per yard with 50¢ worth of thread and buttons to make a child's coat that might be bought ready-made for $8.50, what is saved by making the coat at home?

8. How long must a skirt be cut to measure 41 in. when finished, if it has a $3\frac{1}{2}$ in. hem and 3 clusters of tucks each composed of seven $\frac{1}{8}$-inch tucks?

Estimated Cost of Materials

299. Secure from the local grocer a price list of the following commodities. Estimate the cost of materials upon those prices.

Baking powder, 1 teaspoonful $\frac{2}{3}$¢
Butter (2 cups = 1 lb.), 35¢ a lb.
Butter, 1 tablespoonful = 1 oz.
Eggs, 30¢ a doz.
Farina (1 pt. = 1 lb.), 10¢ for 3 lb.
Flour (4 cups = 1 qt. = 1 lb.), $1.65 for 50 lb.
Flour, 1 tablespoonful = $\frac{1}{2}$ oz.
Lard, 85¢ for 5 lb.
Lard, 1 tablespoonful = 1 oz.
Milk, $1 for 10 qt.
Nutmeg, 10¢ a doz.
Onions, 6¢ a lb.
Peas, 10¢ a lb.
Salt, 1 teaspoonful $\frac{1}{5}$¢
Sugar (2 cups = 1 lb.), $1 for 20 lb.
Sugar, 1 tablespoonful = 1 oz.
Yeast, 5¢ for 2 cakes

1. Using the cost of material given in this table find the cost of making 4 times the quantity of bread called for in the following recipe:

$2\frac{1}{2}$ qt. flour	2 tablespoonfuls sugar
1 qt. warm water	2 tablespoonfuls lard
1 tablespoonful salt	1 cake compressed yeast

2. Find the cost of each of the two kinds of puddings named below. Find the difference in the cost of the two.

New England Pudding

1 pt. milk
5 eggs
$\frac{1}{2}$ pint flour
1 tablespoonful salt
$\frac{1}{2}$ cup sugar

Farina Pudding

1 pt. water
1 pt. milk
2 eggs
$\frac{1}{2}$ pt. farina
1 tablespoonful salt

3. Compute the cost of cookies made according to the following recipes:

$\frac{1}{2}$ cup butter
3 eggs
2 cups sugar
1 nutmeg

$\frac{1}{2}$ pt. milk
1 nutmeg
3 cups flour
2 teaspoonfuls baking powder

4. Find the difference in the cost of the two following recipes for soup:

Browned Flour Soup

1 tablespoonful butter
$\frac{1}{2}$ cup flour
2 pt. water
1 pt. milk
1 teaspoonful salt

Green Pea Soup

1 pt. shelled peas
3 pt. water
1 small onion
1 tablespoonful butter
1 tablespoonful flour

Written Problems

300. 1. A housekeeper purchases eggs at the rate of 35¢ a dozen from an old lady who keeps hens; fresh eggs sell in a grocery store at 40¢ a dozen. How much does the housekeeper save by this arrangement in 3 months, if she buys 3 doz. a week?

2. In Ex. 1, how much does the producer gain by selling directly to the consumer, if the grocer buys eggs at 32¢?

3. A box of apples bought directly from a fruit-raiser of the neighborhood costs $1.37; a grocer charges $1.60. How much is saved by purchasing 9 boxes directly from the producer?

4. Glass jars filled with honey can be bought at 54¢ each, or at $6.25 for a dozen jars. How much is saved by buying 24 jars by the dozen rather than by single jars?

5. A grocer sells sirup at 48¢ a quart or at $1.75 a gallon. How much more will he receive by selling 13 gallons of sirup by the quart than by the gallon?

6. A woman can buy a 5-pound can of cocoa at $3.20, but needing a smaller quantity, she buys 5 $\frac{1}{4}$-pound cans at 24¢ a can. How much more does she have to pay?

7. A dealer sells lignite coal at $3.75 a ton, or two tons for $7.15; what is the saving in buying 14 tons in 2-ton orders?

8. Two families together buy a 5-pound package of Ceylon tea at $3.25. If each family takes half, how much does each save by buying in the larger quantity, when this tea is sold by the pound at 73¢?

Study Exercise

Secure at the post office the parcel post rates, also the maximum weight and the maximum size of parcels which can be sent by the United States parcel post. Study the rates to see what use can be made of the parcel post especially in sending goods to market.

Compare the parcel post rate with the express rate on the same parcel. Which is the less?

1. How far from a post office can a parcel be sent at first-zone rates?

2. How far can a parcel be sent at second-zone rates? Make similar statements for the other zones.

3. How much postage must a man pay on each of the three following parcels?

7 lb. to be sent 160 mi.
9 lb. to be sent 350 mi.
6 lb. to be sent 500 mi.

4. Which of the following parcels are too large for mailing?

(1) Length 40 in., girth 25 in.
(2) Length 70 in., girth 38 in.
(3) Box 26 in. × 15 in. × 12 in.
(4) Box 30 in. × 12 in. × 8 in.
(5) Box 5 ft. × 10 in. × 8 in.

Written Problems

301. The food substances needed by the human body are contained in varying proportions in meat, milk, wheat, potatoes, etc. Their nutritive values depend partly on their composition and partly on their digestibility. There are three essential constituents of human food, namely *protein*, a substance that is a great body builder and is especially abundant in muscles and bones, *fat*, which is known to the housewife as butter, lard, and tallow, and *carbohydrates*, found mainly in vegetables and known to the housewife as starches and sugars. The average amount of daily food of a laboring man is given in the following table:

Average Amount of Food, Drink, and Oxygen taken into the Human Body Daily	Ounces	%
Proteins	4.23	3
Fat	3.17	
Carbohydrates	11.64	
Salts	1.13	
Water	99.30	
Oxygen	26.66	
Sum		

1. How many ounces of food, drink, and oxygen does a man take, in a day, on an average? Write the sum in the table.

2. Compute the per cent that each substance named in the table is of the entire sum. Compute

only to the nearest integer, and enter the result in the last column of the table.

3. Air is made up, by weight, of 23% oxygen, of 76% nitrogen, and of about 1% other gases. How many ounces of air does a man breathe in a day, when the oxygen part of it amounts to 26.66 ounces?

4. How many pounds of food, water, and air does a man take in a day on an average?

Oral Exercise

302. Study the table.

COMPOSITION OF FOODS

KIND OF FOOD	% WATER	% UNAVAILABLE FOOD	% PROTEINS	% FAT	% CARBOHYDRATES
Beef, lean loin . .	67.0	2.2	19.1	12.1	—
fat loin . . .	54.7	2.8	17.0	26.2	—
Veal	70.7	2.1	19.7	7.3	—
Mutton, leg . . .	62.8	2.5	17.9	17.1	—
Pork, ham	53.9	2.7	14.8	27.5	—
Milk	87.0	1.0	3.2	3.8	5.0
Eggs	73.2	1.8	12.8	11.4	—
Lima beans, dried .	10.4	9.8	12.8	1.4	65.6
Potatoes	75.5	2.5	1.9	0.1	20.0
Cabbage	91.5	1.5	1.2	0.3	5.5
Tomatoes	94.3	0.8	0.7	0.4	3.8
White bread . . .	35.3	4.1	7.1	1.2	52.3
Rye flour	12.9	4.1	5.3	0.8	76.9

1. Which kinds of food contain the largest amount of proteins? Which contain the least?

2. Which kinds of food contain the largest amount of carbohydrates? Which contain the least?

3. Which kinds of food contain a small amount of fat?

4. Of the foods given in this table, which one contains the proteins, fat, and carbohydrates in the ratios nearest to those obtained from the first table? Hence, if you had to live on one kind of food only, which should you select?

5. Why is it a good thing to butter the bread?

6. Considering the larger amount of water in lean beef, which is more economical to buy, lean beef or fat beef?

Written Problems

303. 1. How many ounces of proteins, fat, and carbohydrates in 1 qt. of milk (32 ounces)?

2. How many ounces of water, proteins, and fat in 1 lb. of beef (lean loin)?

3. A boy eats in one day $\frac{1}{2}$ lb. of fat loin beef and 1 lb. of white bread. He drinks 1 qt. of milk. Compute the amount of proteins, fat, and carbohydrates in these three foods.

4. How many ounces of mutton (leg) will supply 4.23 oz. of protein?

5. How much white bread is needed to supply 11.64 oz. of carbohydrates?

6. A physician found that a poor family of three members had in one week for their food:

Potatoes	41 lb.	~~Rice~~ Cabbage	$\frac{1}{2}$ lb.
Rye flour	$2\frac{1}{2}$ lb.	White bread	12 lb.
Lean beef	$1\frac{3}{4}$ lb.	Milk	2 lb.

Calculate the proteins, fat, and carbohydrates contained in these amounts; how much less than the average consumption (No. 301) was this, per individual, daily? Were the food substances in the right proportion?

Written Problems

304. 1. One obtains the breathing space for each individual by dividing the volume of the room by the number of people in the room. What is the breathing space per individual when 32 people are in a hall $40' \times 20' \times 20'$?

2. Physicians tell us that where there is good ventilation each person has a supply of 3,000 cu. ft. of fresh air per hour. Accordingly, how many times per hour should there be a complete change of air in Ex. 1?

3. In the best American school buildings, 200 cu. ft. of breathing space are allowed for each pupil. How many times per hour must the air be changed to afford 3,000 cu. ft. of fresh air for each pupil?

4. How many pupils can be put into a room $40' \times 25' \times 10'$, if each pupil is allowed 200 cu. ft. breathing space?

5. If each pupil is allowed only 160 cu. ft. breathing space, how high must the ceiling be of a room having a floor area $25' \times 25'$, in order to accommodate 35 pupils? How often should the air be changed per hour to allow each child 2400 cu. ft. of fresh air per hour?

6. If in breathing we inhale air that is, by volume, 20.8% oxygen and 79.2% nitrogen, and exhale air that is 16% oxygen, 4.4% carbon dioxide, and 79.6% nitrogen, what per cent of oxygen has been lost? If in one respiration 25 cubic inches are taken in and then given off, how many cubic inches of carbon dioxide are given off by one person in the course of 1 hr., if he breathes 18 times per minute?

AGRICULTURAL PURSUITS

305. 1. **Written Exercise**

1915		Items	Received		Paid	
Sept.	29	Coal for thrashing, $\frac{3}{4}$ T. at $4	$		$	
	30	Household expense				89
		Sold 2 T. hay, at $15				
		Personal expense			2	25
Oct.	2	Received for milk and cream	114	25		
		Bought 1 T. cottonseed meal			35	00
	5	Bought 2 milk pails			2	00
		Bought $\frac{1}{2}$ T. bran			15	50
	7	Sold 1 yearling	19	00		
	10	Sold 20 bu. potatoes, at 60 ¢	12			
	11	Sold 18 doz. eggs, at 27 ¢				
	15	Sold 2 T. hay, at $16				
	17	Personal expense			5	75

Compute and enter the amounts in columns "Received" and "Paid." Add each column.

2. On April 2 a farmer sold 35 bu. of potatoes at 65¢, 5 doz. eggs at 24¢, 1 cow for $49.75; bought garden seeds and paid parcel post on same, $8.75; personal expense $3.75. On April 5 he sold 25 bu. of potatoes at $1; paid for repairing plow, $2.50; sold 3 cows at $48.50 each; bought 2 T. bran at $53.25; paid for mending harness $1.25. Write out an account, as in Ex. 1, and find the total receipts and expenditures.

Written Exercise

306. Several farmers own a creamery and secure men to manage it. At the close of each month a statement is prepared, showing how much milk each farmer brought to the creamery, how much has been realized from the sale of the butter, what the operating expenses have been, etc. For one month six patrons supplied milk as shown in the following table:

Name	Pounds Milk	% of Butter Fat	Pounds Butter Fat	Amount Due (Dollars)
John Burgess	2750	5.02	138.05	
George White	7640	3.75		
Roy Roberts	5780	4.10		
Samuel Palmer	5015	3.95		
James Martin	6714	4.85		
Arthur Jackson	4310	5.15		

1. If the butter fat was sold for $380 and the share of the operating expenses to be borne by the six farmers was $50, find the sum due each.

The % of butter fat which milk contains varies from about 2% to about 6%.

2. One pound of butter fat will make about $1\frac{1}{6}$ lb. of butter. How much butter was obtained from the total amount of butter fat in Ex. 1?

Written Exercise

307. 1. One year a farmer kept an account of 2 of his cows. His memoranda were as follows: Returns from creamery, \$ 128.00, from butter, \$ 15.20, from milk, \$ 9.00, from sale of 2 calves, \$ 30.00. The total expense of keeping the cows was 65 % of the total receipts. What was this expense? What was the net profit?

2. In the fall 15,800 bu. of corn, weighing 56 lb. each, were placed in a crib. If during the winter the corn shrank 20 % of its original weight, how many pounds did it lose in shrinkage?

3. How many miles does a team travel in plowing 8 acres with a 12″ plow? Must it travel farther if the 8 acres are in the form of a rectangle instead of a square?

4. A man digs 8 rows of potatoes, each row 64 rd. long. The rows are 3′ apart and the yield is 120 bu. Find the yield per acre.

5. A farmer has 30 acres of wheat. The cost per acre was, for plowing, \$ .80, seed, \$ .85, working the ground, \$ .90, drilling, \$ .20, harvesting, \$ .75, threshing, \$ 1.50, use of implements, \$ .30. The sale of the wheat yielded \$ 750. If the land is valued at \$ 125 an acre, what per cent of the value of the land was the net profit?

6. Land that had been rotated with corn, oats, and clover produced 75 bu. of corn per A., while

adjoining land that had the same rotation, but had been fertilized with lime, produced 90 bu. per A. The second is better than the first by how many per cent of the first?

7. A farmer gives his sons $\frac{1}{3}$ acre of ground on which to raise vegetables. He agrees to furnish the seed costing \$ 7 and take 20 % of the gross income from the sale of the vegetables, or else to let them pay for the seed and give him \$ 5 as rent. If the vegetables sell for \$ 102, which plan is better for the sons? How much better?

8. A fruit-grower can get on Oct. 1, \$ 2.25 a barrel for his apples and on Dec. 1, \$ 2.75. By keeping them until the latter date there is a loss of 10 % in waste and a cost of 10¢ a barrel for storage. Which is the better plan? What is the difference on 47 bbl.?

9. A farmer needs a water tank which will hold 50 bbl. How high must it be, if the area of its base is 19.6 sq. ft.?

10. The weight of a mile of 10-gage wire is 309 lb. Find the length of wire per pound.

11. 25.8 ft. of 11-gage wire weighs 1 lb. Find the weight of this wire per mile.

12. A good farm fence, hog tight and horse high, is built 58″ high. It costs 55¢ per rod. What is the cost of such a fence per mile?

13. The same fence, made of thicker wire, costs 65¢ per yard. How much more does it cost per mile than the one in Ex. 12?

14. If 9-gage wire is $\frac{5}{32}''$ in diameter, and 11-gage wire is $\frac{1}{8}''$ in diameter, what is the difference in the radii?

15. A 100-acre dairy farm, including buildings, implements, and live stock, is valued at $9,740. The farmer finds the expenses of management for one year to be: (1) 5% interest on the $9,740; (2) a 5% depreciation for repairs and insurance on buildings that are valued at $2,050; (3) a 10% depreciation on teams and tools that are valued at $1,190; (4) taxes $60; (5) help $900; (6) incidentals $200. The farmer's returns for the year are: (1) 12,500 lb. butter @ 25¢; (2) 35 calves @ $10; (3) 10 cows @ $40. Find the farmer's net profit for the year.

16. The value of a New England farm is estimated as follows: 200 acres @ $10, buildings $4,600, live stock $1,645, teams and tools $823. The yearly expenses are as follows: (1) 4% interest on the entire value of the farm, tools, and stock, (2) taxes, $160, (3) help, $500, (4) feeds, $500, (5) fertilizers, $50, (6) incidentals, $200. The yearly returns are: (1) 5,000 lb. butter @ 22¢, (2) 1,000 lb. pork @ $6\frac{1}{2}$¢, (3) 875 doz. eggs @ 20¢, (4) 80 bu. potatoes @ 60¢, (5) 400 bbl. apples @

\$1.50, (6) 700 lb. wool @ 18¢, (7) lambs sold, \$400. Find the farmer's net profit.

17. An 8-room farmhouse is lighted by acetylene gas. The generator cost \$200. The cost of maintenance is \$3.00 a month. If the rate of interest on the cost of the acetylene plant is 8%, how much more expensive is this than buying electric light at 10¢ a kilowatt hour, the average daily consumption being 1.2 kilowatt hours?

Written Exercise

308. 1. Peach trees are sprayed two or three times in the spring with a lime-sulphur mixture to protect the fruit against brown-rot, scab, and the peach worm. If the spraying mixture is in the proportion of

8 lb. of fresh stone lime,
8 lb. of sulphur,
50 gallons of water,

and if sulphur costs $2\frac{1}{2}$¢ a pound, lime \$1.10 a barrel of 220 lb., what is the cost of materials for 300 gallons of the mixture?

2. If each tree is sprayed twice, one gallon of the mixture being required per tree at each application, how many trees can be sprayed with 300 gallons?

3. In an orchard 68 sprayed peach trees yielded 101 bushels of fruit, of which 86% was suitable for market, while 63 unsprayed peach trees in the same orchard yielded 92 bushels, of which only 54 % was suitable for market. How many bushels of peaches from the sprayed trees had to be withheld from market? How many bushels from the unsprayed trees?

4. Tender cucumber and melon vines are sprayed with a mixture of

Copper sulphate (bluestone)	3 lb.
Fresh stone lime	6 lb.
Water	50 gal.

as a protection against leaf blight, downy mildew, and other diseases. What is the cost of the mixture needed for spraying 8 acres, applying 100 gallons per acre, if copper sulphate is bought at 7¢ a pound, and lime at 1¢ a pound?

5. What is the cost of spraying the vines in Ex. 4 a second time, when the vines are hardier, and a stronger solution is used, in the proportion of 4 lb. of copper sulphate, 4 lb. of lime, and 60 gal. of water?

Written Exercise

309. The following table shows the yield of barley in two 3-year rotations of crops compared with continuous cropping. Rotation No. 1 was: Oats on ground plowed early the preceding fall, corn on ground plowed early the preceding fall, barley on disked corn ground. Rotation No. 2 was: Corn on spring-plowed land, oats on spring-plowed land, barley on spring-plowed land.

Station	Average Yield of Barley, Bushels per Acre		
	Rotation No. 1	Rotation No. 2	Continuous Cropping
Judith Basin, Mont.	42.7	39.1	45.2
Dickinson, N. Dak.	45.6	34.4	33.5
Dickinson, N. Dak.	53.8	49.2	39.8
Edgeley, N. Dak.	18.3	10.6	10.2
Edgeley, N. Dak.	31.9	26.0	25.0
Edgeley, N. Dak.	33.1	32.7	27.0
Highmore, S. Dak.	25.0	28.3	30.2
Highmore, S. Dak.	30.0	29.9	29.8
Bellefourche, S. Dak.	47.1	28.1	23.8
North Platte, Nebr.	30.6	40.2	39.0
North Platte, Nebr.	24.9	22.3	19.6
Akron, Colo.	24.9	22.2	19.7
Hays, Kans.	12.3	9.2	5.8
Average			

1. Find the average yield of bushels per acre in Rotation No. 1.

2. Find the average yield of bushels per acre in Rotation No. 2.

3. Find the average yield of bushels per acre from continuous cropping.

4. Which rotation was better for barley, to be preceded by an oat crop or by a corn crop?

5. On the whole, which yielded better results, continuous cropping or rotation of crops?

Written Exercise

310. 1. Concrete is made by mixing cement, sand, gravel, and water. A 1 : 2 : 4 concrete is made of one volume of cement, twice that volume of sand, and 4 times that volume of stone. The volume of the mixed cement is less than the sum of the volumes of the parts, because of the open spaces between the stones, which must be filled in making the concrete. One cu. ft. of concrete contains,

For 1 : 2 : 4 concrete: .22 cu. ft. cement, .44 cu. ft. sand, .88 cu. ft. gravel,

For 1 : $2\frac{1}{2}$: 5 concrete: 19 cu. ft. cement, .47 cu. ft. sand, .95 cu. ft. gravel.

If there are needed for the silo 750 cu. ft. of 1 : 2 : 4 concrete and 185 cu. ft. of 1 : $2\frac{1}{2}$: 5 concrete,

(*a*) Find the number of cu. ft. of cement needed altogether. If 1 barrel of cement holds 4 cu. ft., how many barrels are needed?

(*b*) Find the number of cubic feet of sand needed. Express this in cubic yards.

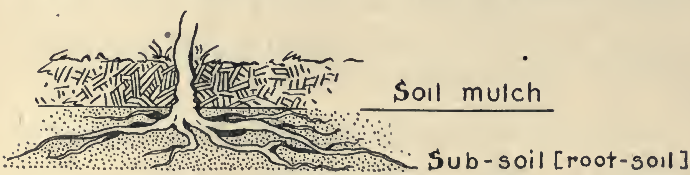
(*c*) Find the number of cubic feet of gravel needed. Express this in cubic yards.

2. The silo has inside and outside a surface of 400 sq. yd. which must be plastered with a 1 : 1 mixture of sand and cement. If 1 cu. ft. of this mixture plasters 15 sq. yd. of surface, how many barrels of cement and how many cubic yards of sand are needed to prepare this mixture?

3. What must be the capacity of a silo to hold feed for 15 cows to last 200 days, if each cow is fed daily 30 lb. (= 1 cu. ft.) of ensilage?

4. How many tons of ensilage in a silo having a capacity of 2,500 cu. ft.? How many cows will it feed for 100 days? (see Ex. 3).

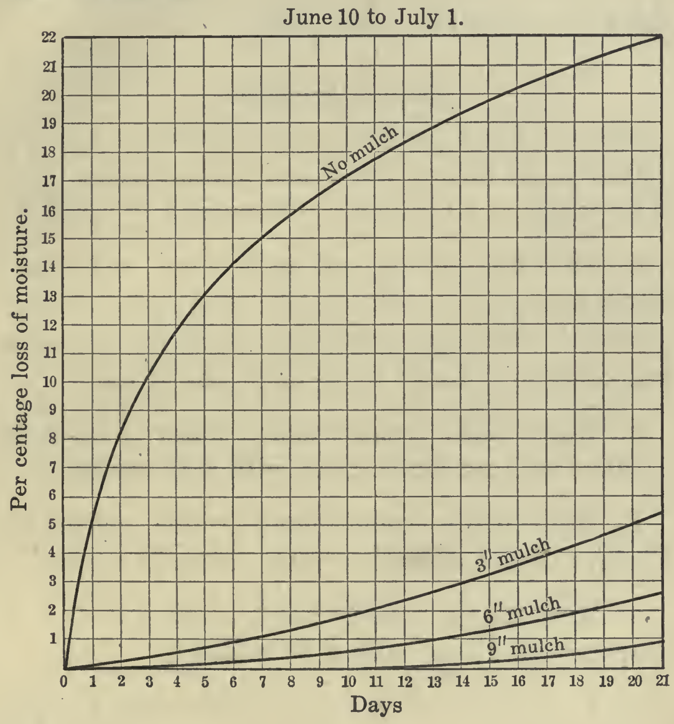
311. In some parts of California, Nevada, and other western regions where there is little rain it is important to prevent the rapid evaporation of moisture from the soil. It has been found by trial that if the soil which holds the roots of vines or of trees in orchards is covered by a layer of loose, dry, granular soil, the damp soil beneath retains its moisture much better than if such a top layer is not present. It is found that if the top soil in orchards is *cultivated right after irrigation or rain*, the moisture remains much longer in the root soil. The water goes down deeper, while the loose earth or *soil mulch* on top becomes dry, and stops rapid



evaporation from the root soil. The thicker the mulch, the less the evaporation from the root soil.

If the soil is cultivated to a depth of 3″, 6″, or 9″, we have a soil mulch that is 3″, 6″, or 9″ deep.

The chart shows what per cent of the water, after drenching the root soil, has evaporated in the time indicated,



(1) When there is no mulch over the root soil,
(2) When a mulch 3″ deep covers the root soil,
(3) When a mulch 6″ deep covers the root soil,
(4) When a mulch 9″ deep covers the root soil.

Thus, after 7 da., the chart shows that 15 % of the moisture in the root soil has evaporated when there is no mulch over it, while only about $1\frac{1}{3}$ % has evaporated when there is a soil mulch of 3″ over it, and only about $\frac{3}{4}$ % has evaporated when there is a soil mulch of 6″ over it.

Written Problems

312. 1. What per cent of the moisture has disappeared, 21 days after a heavy rain, when there is no mulch? 3″ mulch? 6″ mulch? 9″ mulch?

2. To 1,054 lb. of soil in a tank 202 lb. of water were added. No mulch being put over the drenched soil, how many pounds of the moisture evaporate in 2 da.? In 10 da.? In 21 da.?

3. How many times more water wastes in 21 days with no mulch than with a 3″ mulch?

4. How many times more water wastes in 18 days with a 3″ mulch than with a 6″ mulch?

5. How many pounds out of 202 pounds will evaporate in 15 da. with a 3″ mulch?

6. Is there much of any loss of moisture underneath a 9″ mulch during the first three weeks after a thorough wetting?

7. If a cubic foot of water weighs $62\frac{1}{2}$ lb., how much water soaks into a square foot of level earth during a heavy rainfall of "1 inch"? How much will fall on one acre?

8. After a rainfall, the soil was cultivated to a depth of 6″. If there was in the subsoil before the rain five times as much moisture as was brought to it by the rain, and if the amount brought was "one inch," how many pounds of water evaporated from 1 sq. ft. of subsoil in 20 da.?

9. In the previous example, how much more would have evaporated, if there had been no mulch?

10. After irrigation, the subsoil in an orchard contained as much moisture as if 10″ of rain had penetrated the dry soil. What was the weight of water per square inch of surface? Per square yard?

11. In the previous example, how much water per acre evaporated from the subsoil in 20 days, the mulch being 9″?

12. In 32 da., from Sept. 1 to Oct. 3, the evaporation indicated in inches is shown in the following

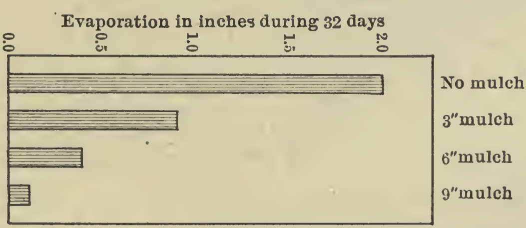


diagram. What was the average daily evaporation, expressed in a fraction of an inch, for no mulch? For the 3″ mulch? For the 6″ mulch?

13. How many times faster is the evaporation in the case of no mulch than for each of the other three cases?

14. In another locality it was found that in 28 days the evaporation from the subsoil was 2.8″ for no mulch, 1.5″ for a 3″ mulch, .6″ for a 6″ mulch, and .6″ for a 9″ mulch. Draw a figure like the one above to show these relations.

Written Problems

313. Every crop of hay or grain removes plant food from the soil, and the soil becomes impoverished thereby. In careful farming, plant food is returned to the soil through fertilizers.

1. Barnyard manure exposed in loose heaps loses from 42 % to 62 % of its value in 6 months. If fresh manure is worth $ 2.25 per ton, what is the least and what the maximum probable loss on 18 tons, in six months, due to exposure?

2. An orchard contains 18 rows of fruit trees, 9 trees in each row. If each tree receives 25 lb. of fertilizing material a year, what is the yearly cost of fertilizing the orchard, at $2.10 a ton?

3. The most important substances in a fertilizer are nitrogen, phosphoric acid, and potash. A commercial fertilizer labeled 2–8–4 contains 2 % of nitrogen, 8 % of phosphoric acid, and 4 % of potash; the remaining 86 % consists of other ingredients. How many pounds of nitrogen in a ton of fertilizer? How many pounds of phosphoric acid? Of potash?

4. A farmer prepares one ton of a 2–8–4 fertilizer from nitrate of soda containing 15 % of nitrogen, acid phosphate containing 16 % of phosphoric acid, and muriate of potash containing 51 % of potash. How much nitrate of soda, phosphoric acid, and muriate of potash must he take?

PROCESS AND EXPLANATION

2 % of 2,000 lb., or 40 lb., must be nitrogen.
8 % of 2,000 lb., or 160 lb., must be phosphoric acid.
4 % of 2,000 lb., or 80 lb., must be potash.

If x pounds is the required amount of nitrate of soda, then must $\frac{15}{100}x = 40$ lb., the amount of nitrogen. This gives $x = \frac{4000}{15} = 267$ lb. of nitrate of soda.

If x pounds is the required amount of acid phosphate, then must $\frac{16}{100}x = 160$ lb., the amount of phosphoric acid. This gives $x = \frac{16000}{16} = 1{,}000$ lb. of acid phosphate.

If x pounds is the required amount of muriate of potash, then must $\frac{51}{100}x = 80$ lb., the amount of potash. This gives $x = \frac{8000}{51} = 157$ lb. of muriate of potash.

Adding together 267, 1,000, and 157 pounds, we get 1,424 lb. As this is less than one ton, we add sand or some other "filler" to make the total weight 2,000 lb.

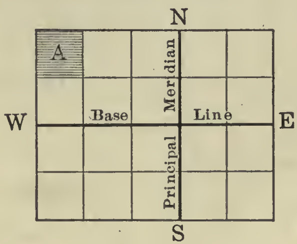
5. Using data given in Ex. 4, compute the amount of nitrate of soda, acid phosphate, and muriate of potash required in the preparation of a 4–8–4 potato fertilizer. How many pounds of sand must be added as a filler?

6. A corn and wheat fertilizer is mixed in the ratio 3–10–3. Find the amount of nitrate of soda, acid phosphate, and muriate of potash required for 100 tons. How much sand is needed as a filler?

AREAS AND POPULATION

Public Lands

314. In the western states land has been marked out by surveyors into divisions in the form of squares. This was done by selecting a north and south line as a **principal meridian** and an east and west line as a **base line**. Other lines are then run north and south at intervals of six miles, and also east and west at the same interval. These lines divide the land into squares, six miles on a side, or six miles square. These large squares are called **townships**. A line of townships extending north and south is called a **range**.



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Thus, the township which is shaded in the above diagram and is marked A, may be described as "township number two north and range 3 west," or T. 2 N., R. 3 W.

This method of measuring

public lands has been established by law. The law also requires townships to be subdivided into smaller squares, called *sections,* and to be numbered as shown in the figure on page 327.

1. In which corner of a township is section 1?

2. How do the section numbers run?

Sections are themselves subdivided into half sections, quarter sections, and sometimes into eighths and sixteenths.

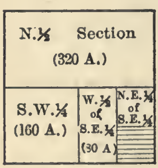
3. Review the table of square measure. (Page 426.)

4. How many square miles in a township?

5. How many acres in two square miles?

6. How many square rods in 10 acres?

7. Review the table of surveyor's measure. (Page 426.)



Section 22

8. How many feet in a chain? How many links?

9. From the following description of farms, find the number of acres owned by each of the three men (see map of Section 22):

Owner	Description	No. of Acres
R. Seldomridge	N. ½ of Section 22 N. E. ¼ of S. E. ¼ of Section 22	——
J. Rareback	S. W. ¼ of Section 22 W. ½ of S. E. ¼ of Section 22	——
A. Southpaw	S. E. ¼ of S. E. ¼ of Section 22	——

10. Is the number of acres in each of the following correct?

John Ball	N. E. $\frac{1}{4}$ of N. W. $\frac{1}{4}$ of Section 35 .	50 A.	
George Brown	W. $\frac{1}{2}$ of S. W. $\frac{1}{4}$ of Section 35 . .	80 A.	
John Carter	S. E. [illegible] of Section 35	320 A.	$\frac{1}{4}$
Horace Mat	S. W. $\frac{1}{4}$ of N. E. $\frac{1}{4}$ of Section 35 . .	40 A.	

Written Exercise

315. 1. Draw a figure of Section 35 in the previous example 10, and mark the parts owned by Ball, Brown, Carter and Mat.

2. How many acres in Section 35 are owned by others?

3. How many feet of fence will inclose the entire section?

4. How many feet of fence will inclose Carter's land?

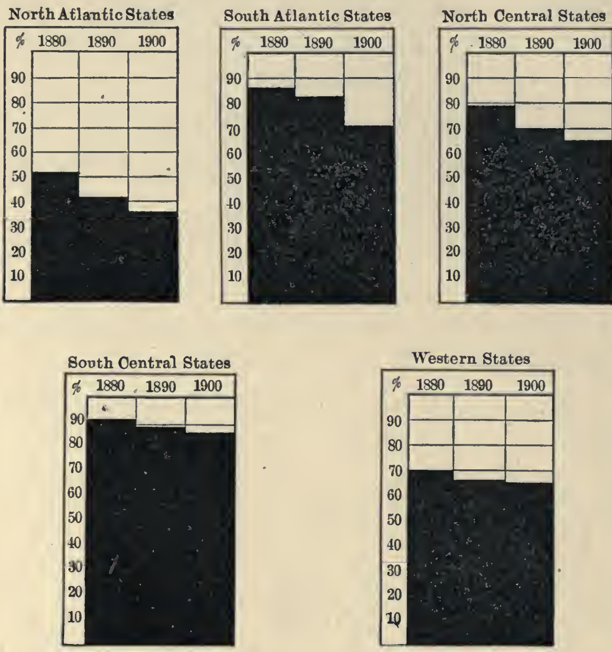
5. Find the value, at $15.50 per acre, of T. 3 N., R. 3 W.

6. What is the cost of E. $\frac{1}{2}$ of S. E. $\frac{1}{4}$ of Section 7, at $35.25 an acre?

7. How many rods of fence will inclose the tract of land named in Ex. 6?

316. Study Exercise

Diagrams showing the increase in the per cent of urban population and the decrease in the per cent of rural population in the five divisions of continental United States, from the census of 1880 to that of 1900. In each diagram, the portion in black represents the rural population; that in outline the urban population.



Oral Exercises

1. State as near as you can the per cent of rural population in the North Atlantic States.

2. Do the same for each of the other groups of states.

3. Find as near as you can the per cent of urban population in the North Atlantic States.

4. Do the same for each of the other groups of states.

5. In which group of states has the decrease in the per cent of rural population been most rapid? In which least rapid?

Written Problems

317. Give answers to the nearest 100,000:

1. In 1880 the population of the North Atlantic States was 14,500,000. Find the rural population, also the urban population.

2. Find the rural population for 1890, if the total population was then 17,400,000.

3. Find the rural population for 1900, if the total population was then 21,000,000.

4. How much greater was the rural population in the North Atlantic States in 1900 than in 1880? Give the answer approximately.

5. In the South Atlantic States the total population was 7,600,000 in 1880 and 10,500,000 in 1900. Find the rural population for both years and also the increase in rural population during the 20 years.

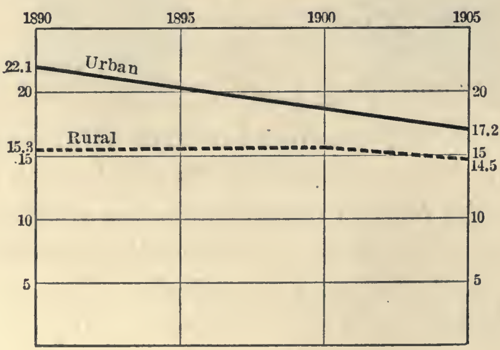
6. In the North Central States the total population was 17,400,000 in 1880 and 26,300,000 in 1900. Find the urban population for each year and the increase in the same for the 20 years.

7. In the South Central States the rural population in 1880 was 8,100,000. Find the total population that year.

8. In the South Central States the rural population in 1900 was 12,750,000. Find the total population then.

9. Between 1890 and 1900, 49.6 % of the total increase of population in the entire United States was in the cities of 25,000 inhabitants, or over. Between 1900 and 1910, 73 % of the total increase of population in the United States was in such cities. What per cent of the total increase during 1900–1910 was in the country? How many times faster has been the growth of the urban population than the rural during 1900–1910? Which population increased faster during 1890–1900, the urban or the rural?

Study Exercise



318. This diagram shows the death rate per thousand in one year, from 1890 to 1905. The solid line represents the urban rate, the dotted line the rural rate. Thus, in 1890 the rural death rate averaged 15.3 persons out of every 1000 persons, and the urban death rate averaged 21.1.

Oral Problems

319. 1. Which is greater, the death rate in the cities or that in the country?

2. Tell as near as you can the rural and urban rate in 1895.

3. Has the death rate in the cities decreased? This is due to purer water and milk and the closer observance of sanitary regulations in general.

4. Has the rural death rate decreased between 1890 and 1900? Between 1900 and 1905? Has the death rate between 1890 and 1905 been falling more rapidly in the cities than in the country?

Written Exercise

320. 1. About how many deaths were there in a city of 25,000 inhabitants in 1890? In 1905? What was the difference in the number?

2. About how many deaths were there in a city of 125,000 inhabitants in 1900? In 1890?

3. About how many more deaths were there in 1905 in a city of 650,000 inhabitants than in a country district of the same population?

4. What was the excess of deaths in a city population of 375,000 over an equal country population in 1890?

5. Find roughly the number of deaths in a city of 450,000 in the year 1892.

BUILDING

Study Exercise

321. Carpentry may, in a general way, be taken to include all the constructive woodwork of a building, though in many localities it includes also joinery, as outside and inside finish, windows and doors, stairs and mantels, etc.

In the carpentry part of construction there are two things to be estimated, the materials and the labor. The materials are all comprised in three divisions; the frame, the covering, and the finish.

All lumber is estimated in board measure, a standard unit of a square foot of board one inch thick. Lumber is sold at the rate of so much per thousand feet; that is, one thousand square feet one inch thick. The process of reducing lumber of thickness greater than one inch is simple. The usual method followed is to multiply the width by the thickness, divide product by 12 and then multiply by the number of feet in length.

The formula is $\frac{W \times T}{12} \times L$.

A piece of lumber $4'' \times 4'' \times 12'$ contains how many feet board measure?

PROCESS

$$\frac{4 \times 4 \times \cancel{12}}{\cancel{12}} = 16$$

A table of board measure may be made for material 1″ thick as follows:

Size in Inches	Length in Feet					
	8	10	12	16	18	20
1 × 2	1.3+	1.7−	2	2.6+	3	3.4−
1 × 3	2	2.5	3	4	4.5	5
1 × 4	?	?	?	?	?	?
1 × 5	?	?	?	?	?	?
1 × 6	?	?	?	?	?	?
1 × 7	?	?	?	?	?	?
1 × 8	?	?	?	?	?	?
1 × 9	?	?	?	?	?	?
1 × 10	?	?	?	?	?	?
1 × 12	?	?	?	?	?	?
1 × 14	?	?	?	?	?	?
1 × 16	?	?	?	?	?	?
1 × 20	?	?	?	?	?	?

Make a table for material 2″, 3″, 4″, 6″, 8″, 10″, and 12″ thick and varying in width.

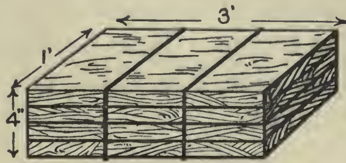
322. A **board foot** is a board 1 ft. long, 1 ft. wide, and 1 in. thick.

1. How many board feet in a board 11 ft. long, 1 ft. wide, and 1 in. thick?

2. How many square feet can be covered with a board 12 ft. long, 6 in. wide ($= 12' \times 6''$)?

3. How many board feet in that board, if it is 2″ thick?

4. The plank in this drawing is 3′ long, 1′ wide, and 4″ thick. Count the number of board feet in it.



5. How many board feet of lumber in a board 18 ft. long, 6 in. wide, and 2 in. thick?

PROCESS AND EXPLANATION

It contains $18 \times \frac{6}{12} \times 2$ ft. B. M. = 18 ft. B. M.

Notice that the answer is *the product of the length and width in feet by the thickness in inches.*

6. How many board feet in a board 5 ft. long, 1 ft. wide, and $\frac{1}{2}$ in. thick?

PROCESS AND EXPLANATION

It is customary to compute the number of board feet in a board that is *less* than 1 inch thick as if it were exactly 1 inch thick. In this example, we have, accordingly, $5 \times 1 \times 1$ ft. B. M. = 5 ft. B. M.

Written Exercise

323. 1. Find the number of feet B. M., when the thickness is 1″:

a. $3'' \times 12'$	*c.* $4'' \times 16'$	*e.* $6'' \times 9'$
b. $12'' \times 10'$	*d.* $9'' \times 16'$	*f.* $3'' \times 20'$

It is usual to write down first the number of pieces of lumber, then the thickness and width of each in inches, then the length in feet, and finally the article. Thus, 6 pieces of hemlock, 12 ft. long, $\frac{1}{2}$ ft. wide, 2 in. thick, is written: 6 pc. $2'' \times 6''$, $12'$ hemlock.

2. Find the number of feet B. M. in the following:

a. 5 pc., $1'' \times 6''$, $16'$	*c.* 20 pc., $1'' \times 8''$, $12'$
b. 40 pc., $2'' \times 4''$, $18'$	*d.* 25 pc., $3'' \times 8''$, $14'$

3. Find the number of board feet to the nearest foot:

a. 50 pc., $2'' \times 4''$, $16'$	*c.* 15 pc., $2'' \times 6''$, $12'$
b. 24 pc., $3'' \times 5''$, $15'$	*d.* 20 pc., $2'' \times 8''$, $20'$

Lumber is sold at so much per 1000 ft. B. M.

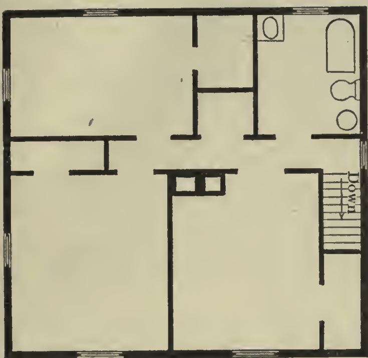
4. Find the cost of the following at $20 per 1000 ft. B. M.:

a. 25 pc., $1'' \times 6''$, $16'$	*d.* 100 pc., $2'' \times 8''$, $15'$
b. 250 pc., $2'' \times 8''$, $12'$	*e.* 50 pc., $3'' \times 4''$, $10'$
c. 100 pc., $2'' \times 4''$, $24'$	*f.* 150 pc., $1'' \times 8''$, $16'$

5. Estimate the cost of the following bill of lumber at $35 per M board measure:

400 pc., $6'' \times 8'' \times 16'$
1000 pc., $2'' \times 4'' \times 8'$
800 pc., $2'' \times 6'' \times 18'$
200 pc., $2'' \times 4'' \times 8'$

THE HOUSE PLAN



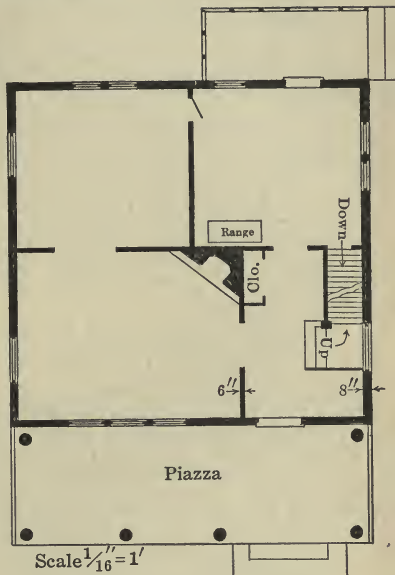
Scale $\frac{1}{16}'' = 1'$

Height from floor to ceiling, first floor, 9′ 6″.

Height from floor to ceiling, second floor, 9′ 6″.

Height from ground to ridge of roof, 32′ 8″.

Height from ground to roof (sides), 22′.



Scale $\frac{1}{16}'' = 1'$

Study Exercise

324. In this plan of a two-story house the scale is $\frac{1}{16}$ inch to one foot. With a ruler reading 16ths of an inch ascertain the following dimensions: the length of the house; the width; the dimensions of the rooms, the halls, of the piazza, and of the kitchen porch.

Put the dimensions in convenient form for future use, as many of the succeeding problems are based upon them.

Written Exercise

325. 1. Estimate the number of square feet in the floor of each of the following rooms: the living room; the dining room; the kitchen; the hall; the three bedrooms, including closet space; the bath room.

2. Estimate the number of square feet of wall and ceiling surface, computing them as solid; that is, making no deductions for openings as doors and windows.

3. Get actual dimensions of a room and draw to scale a plan of the same. Take as scale $\frac{1}{4}$ of an inch to one foot. Locate fixed objects in room exactly as the dimensions require. Mark also openings for doors and windows, and give accurately the scale measurements for the width of these.

Calculate the number of square feet of floor, and the number of square feet of surface of walls and ceilings.

4. Get the dimensions of a building. Draw the plan showing the construction of the framework as nearly as you can. Estimate the number of square feet of surface of outside walls of same.

5. Suppose that you are to build a small house whose dimensions are $14' \times 20'$. The height of the building is to be $12'$ on all sides, the roof square with a rise at the center of $4'$ above the sides. There are one door $30'' \times 78''$ and three windows $36'' \times 50''$.

The floor joists are laid 16 in. from center to center on sills $4'' \times 4''$. The studding is also to be $16''$ from center to center.

Make a floor plan of the building to a scale of $\frac{1}{4}''$ to $1'$.

Make a floor plan showing the position of floor joists.

Make a plan of the front of the building, showing framing plan, with studding and openings for door and one window.

Make a plan of one side of building showing framing plan.

Mark all necessary specifications for framework on your plans, and save for use in later problems.

Study Exercise

326. It is not always practicable to count the actual number of pieces of lumber required by a building plan and shown in the framing plan. When the framing plan is not available contractors or builders calculate the amounts needed in this manner. The dimensions of a building are of course given. The sills are in the common house $6'' \times 6''$ to $6'' \times 10''$; the girders vary from $6'' \times 8''$ to $8'' \times 10''$, while floor joists may be taken as $2'' \times 8''$ to $3'' \times 10''$ or $12''$. Practically all studding of outside and inside walls is $2'' \times 4''$ and will be placed exactly $16''$ from center to center. In certain other parts the studding may be of smaller dimensions as $2'' \times 3''$, or larger where greater stress is to be met, as $4'' \times 4''$. These extra dimensions may be secured by doubling the $2'' \times 4''$ studding.

With these facts in mind, it is necessary to note that the outside dimensions will be taken in linear feet. Allowing for double studding at openings and corners, fireplaces and stairways, it is easy to calculate the number that will be required for a building. A simple method is to take $\frac{3}{4}$ of the total number of linear feet of the outside walls of the building, add to this result one studding for each corner, and one for each opening as doors, windows; for inside walls or partitions $\frac{3}{4}$ of the linear feet with no additions for openings.

A building is, for example, $30' \times 28'$ or 116 total linear feet of outside walls. Taking $\frac{3}{4}$ of 116, the result is 87 studding. Add to this 4 for corners, and assuming that there are 2 doors and 8 windows add 10 more, which gives a total of 101 studding for the outside walls. The inside walls are similarly estimated. This is a shorter method than the reduction of the linear feet to inches, dividing this by $16''$ or the distance from center to center of studding.

The prices for frame materials differ greatly in different parts of the country. The table given is based upon Boston quotations for January, 1915.

$2'' \times 4'' \times 20'$	\$30 per M
$2'' \times 4'' \times 28'$	\$30 per M
$3'' \times 10'' \times 20'$	\$32 per M
$6'' \times 6'' \times 20'$	\$30 per M
$6'' \times 8'' \times 20'$	\$30 per M

$1 extra per M for each two feet over twenty feet in length is charged.

Written Problems

327. **1.** (*a*) What is the quantity in board measure of the following?

174 linear feet of sill	$6'' \times 6''$
128 linear feet of sill	$4'' \times 6''$
635 linear feet of joists	$2'' \times 10''$
340 linear feet of joists	$3'' \times 10''$
2,150 linear feet of studding	$2'' \times 4''$

(*b*) Taking the prices given in above table, what will be the cost of each item? what the cost of the total list?

Lengths are not to exceed 20 feet.

2. (*a*) How many floor joists, 16″ from center to center, will be needed for a room measuring $15' \times 18'$?

(*b*) What will be the B. M. of the same?

(*c*) What would be the cost at $32 per thousand?

3. (*a*) Taking the dimensions of a one-story house $28' \times 30'$, how many studding will be needed for the frame? See page 342

2" X 4", 12'

12'

(*b*) What will be the B. M. of the studding (stock 2" × 4")?

(*c*) At price quoted per thousand, what will be the cost?

4. (*a*) How many linear feet of 6" × 6" stock for sill will be needed for building of the dimensions given in example 3?

(*b*) What will be the amount in board feet?

(*c*) At the quoted price per thousand, what will be the cost for sills?

5. Prepare a statement of the number of pieces of lumber required in above examples 3 and 4, the quantity of each in board measure, price per thousand, extended cost of each specified quantity, and amount of whole bill. Statement should be such as a dealer in lumber would submit to a purchaser.

Study Exercise

328. Siding is computed in same way as flooring and roofing. A general method of procedure is to deduct for openings, as windows, doors, and to add about $\frac{1}{4}$ of the surface in square feet to get the total amount of siding required. The waste in squaring and joining and tongue and groove will account for this extra quantity.

1. Taking the square (a square measures 10' × 10' or 100 sq. ft.) as the unit we may estimate its cost thus:

1 square or 100 feet B. M. of California Redwood 4″ wide, plus $\frac{1}{4}$ of stock or 25 ft. B. M. for waste, making a total of 125 ft. B. M., at \$ 35 per thousand	\$ 4.37$\frac{1}{2}$
Nails 2 lb. 8*d*. at 4 cents per pound	.08
Labor, 2 hours, at \$ 5 per eight-hour day	\$ 1.25
Cost per square of siding, outside walls	\$ 5.70$\frac{1}{2}$

This cost per square will vary with differences in scale of wages, in cost of lumber, and with the kind of surface to be covered with siding; on some walls, straight and unbroken, a carpenter may put as many as 5 to 8 squares a day. But on others 2 squares may be a good day's work.

Written Problems

329. 1. A frame 20′ × 20′ is to be covered with siding 4″ wide. How many feet board measure will it take? Allow for one door and three windows of usual dimensions, and add for waste.

2. Estimate the number of squares in the outside walls of the house plan, page 339.

What amount of 4″ siding will be needed?

At \$ 35 per thousand what will it cost?

Taking the average amount of siding for a day's work as 3 squares, how many days will it take two carpenters to put it all on?

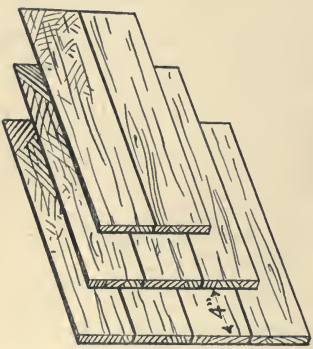
At the regular scale of wages, \$ 5 per day of eight hours, what will the labor of putting on siding cost?

How many pounds of 8*d.* nails, and what will be their cost?

3. Make a statement of amounts of materials and cost, number of hours of labor and cost, of outside walls of this house.

Study Exercise

330. Shingles are packed 250 in a bundle, or 1,000 in four bundles. The average width of the shingles is four inches. If the width is greater throughout the bundle, the number will be correspondingly less. Shingles also are measured as to thickness of butts and marked as 6 to 2″ or 5 to 2″, which means that the combined thickness of 6 butts or 5 butts will equal 2 inches. This is important to the purchaser. The length of shingles is 16″, 18″, or 20″.



The unit of measurement of space to be covered with shingles is taken as 100 sq. ft. and is called a *square*. The number of shingles required to cover a square depends upon the length to which each shingle is exposed to the weather. This varies from 4″ to 7″. On roofs the current practice requires a $4\frac{1}{2}$″ exposure, side walls as high as 6″ or $6\frac{1}{2}$″, according to pattern or design to be worked out.

The number of shingles required to cover a square may be shown in a table.

No. of Shingles	Exposure	Sq. Ft.	No. in Square
1,000	$4\frac{1}{2}''$	125	800
1,000	5 "	139	720
1,000	6 "	167	600
1,000	7 "	194	514

The calculation of number of shingles in square, or the number of square feet 1,000, with different lengths of butt exposure, will cover, may be easily made on the basis of the figures given in this table.

Cost of shingles depends upon quality, average width and thickness, length, and the locality. The price per thousand will with these conditions vary from \$3.50 to \$18. For example, red cedar, 5 to 2″, marked *Extra* may cost \$5 per thousand. A white cedar same quality and dimensions will cost about \$7 per thousand. A white cedar with clear butts and 6 to 2″ will cost less, about \$5.50 or \$6 per thousand. The highest grade of cypress and silver gray cedar shingles will run to \$18 per thousand.

Nails required per thousand shingles are $3\frac{1}{8}$ pounds of 3*d.* or threepenny, or 5 pounds of 4*d.* Nails cost usually 3¢ per pound, or if galvanized 4¢.

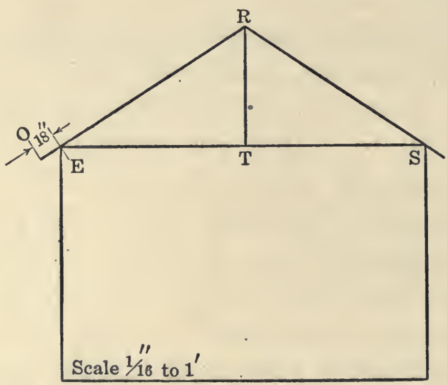
A good shingler will be able in an eight-hour day to lay two and one half squares of plain shingles;

if fancy plan is required, he will lay only one to one and three quarter squares a day. On walls the shingler will lay about $1\frac{1}{2}$ squares; if fancy, only one square.

Written Problems

331. **1.** What will be the cost of one square of shingled roof computed from the following items: shingles cost $ 4.50 per thousand; the weather exposure is to be $4\frac{1}{2}$ inches; nails $4d.$ cost 3¢ per pound; carpenter's wages is $ 5 per day.

Make out a systematic statement of items and cost of each, with total cost per square.



2. Find the length of the line ER, the length of the line marked SE, and the length of the line marked TR. What is the pitch of this roof?

NOTE. — Pitch is the total rise of the rafter OR per foot to width of building; it is determined by dividing TR, the height, by $\frac{1}{2}$ of ES, the width. The eaves marked OE measure generally

18 inches. Pitch is usually specified as a $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$ pitch. The most common, apart from the flat roof, is the $\frac{1}{2}$ pitch. A $\frac{1}{2}$ pitch means $\frac{1}{2}$ of the horizontal distance ET.

3. Compute the length of the rafter RO from the width of the building and the height indicated by TR.

4. If the length of the building is 40 feet, and the overhanging ends and eaves are 18 inches each, how many squares of roof will there be?

5. How many shingles per square, if exposure to weather be $4\frac{1}{2}$ inches?

6. What will be the cost per square for shingling, if shingles cost \$4.50 per thousand, nails four cents per pound of 4*d*., and labor, carpenter, at the rate of \$5 per eight-hour day?

7. (*a*) How many thousand shingles will it take for the entire roof?

(*b*) What will be the cost for entire roof, including labor and materials?

Study Exercise

332. Flooring is estimated very much like roofing. A square is taken as the unit of measurement of materials, labor, and cost. An area $10' \times 10'$ or 100 square feet is called a square. At first it may appear that one square will take 100 feet of flooring, but squaring, joining, tongue and groove, and other waste will require from $\frac{1}{7}$ to $\frac{1}{3}$ extra stock.

A general rule is to add $\frac{1}{6}$ extra for flooring 6″ wide; $\frac{1}{3}$ extra for $2\frac{1}{2}$″ wide, when the lumber is such as pine, hemlock, and fir. Hardwoods such as oak and maple make the extra quantity to be added less.

It must be noticed that flooring listed as $2\frac{3}{4}$″ wide gives an exposure of only 2″, the tongue and groove accounting for the remainder. A 3″ flooring gives a $2\frac{1}{2}$″ exposure, while a 4″ gives about $3\frac{1}{4}$ inch exposure. This must be remembered in computing the board measure of a given number of pieces to a square foot. For example, in a square foot there would be 6 boards of the 2″ flooring, which in board measure is estimated as $2\frac{3}{4}$ inches each.

Written Problems

333. 1. (*a*) A room $10' \times 10'$ is to be covered with fir flooring $4'' \times 1''$. (*a*) How many boards long will be required?

(*b*) What will be the amount in board measure?

(*c*) At \$35 per thousand what will be the cost?

2. How many squares of flooring are there in a one-story building $24' \times 32'$?

3. One square of flooring requires the following: 5 lb. 8*d*. nails at 4¢ per pound; 130′ B. M. of hemlock at \$26 per thousand; labor 2 hours at \$5 per eight-hour day. What will be the cost of a square?

4. Take the number of squares in building of example 2, and find the quantity of flooring, nails, and hours of labor, together with the cost of each item, for entire floor. Make use of data and results gained in example 3.

5. A house, two stories, $30' \times 36'$ inside measurements needs new floors throughout.

(*a*) How many squares of flooring are there?

(*b*) How many feet board measure of 4″ fir will be needed? Allow $\frac{1}{4}$ extra for waste in cutting, joining, etc.

(*c*) What will be the cost of the flooring at $ 30 per thousand?

(*d*) How many pounds of 8*d.* nails, and what their total cost at 4¢ a pound?

(*e*) If the labor be worth $ 5 a day of eight hours, what will it cost for the entire job?

(*f*) What will be the total cost of the new floors?

Study Exercise

334. Lathing is generally included in the work of the plasterer. This is estimated by the square yard or by the 1,000 laths, the cost for the latter being from $2.75 to $3.50 per thousand. A good lather will put on, in an eight-hour day, from 1,500 to 2,000 laths. The following table will be useful in estimating materials and labor for lathing.

Laths are 48″ long, distance of studding 16″ on center. Amount is 100 square yards.

1,500 laths at $4.75 per thousand	$ 7.13
10 lb. nails 3*d*. fine at $3.25 per cwt.	.32½
6 hr. labor at $5.50 per day	4.12½
Cost of 100 square yards	$11.57

The cost of one square yard is 11½ to 12¢.

Written Problems

335. 1. What is the number of square yards to be lathed in a room the dimensions of which are $14' \times 18' \times 9'6''$?

2. (*a*) How many thousand laths will be needed for this room?

(*b*) What will they cost if laths are selling at $4.50 per thousand?

(*c*) How long will it take one man to lath the walls and ceiling?

(*d*) What will be the cost of labor of lathing, if he is paid at the rate of $3.25 per thousand?

(*e*) What will the labor cost, if the lather is paid $5 per day of eight hours?

(*f*) What will be the total cost of materials and labor (compute nails as indicated in the table)?

Study Exercise

336. Plastering is usually estimated in terms of square yards and specified as two-coat or three-coat work. In rough estimating of cost of plastering no deductions for openings as doors, windows, etc., are made unless they amount to more than seven yards. But in modern competitive estimating openings are exactly measured, as also are round corners, beads, etc., which require more time in plastering. A plasterer is supposed to put on twenty to twenty-five square yards of three-coat plastering, or thirty to thirty-five square yards of two-coat, in an eight-hour day. One helper will be needed, but in actual work, one helper serves two plasterers.

At the present time lime plaster is very seldom used. Cement plasters have almost entirely taken its place.

Find out what sort of plaster is obtainable in your locality, what it costs per hundredweight or ton, and something of its composition.

For certain purposes plasterers often add more hair to the mixture for the first or scratch coat, but the fibered plasters require nothing of this sort.

The chief items in estimating the cost of 100 square yards of plastering (solid) run as follows:

Quantity of Material or Labor	Cost
1600 lb. hard wall plaster and white finish at $7.50 per ton	$6.00
2 cu. yd. sand at $1.50 per cu. yd.	3.00
Water estimated	.12
4 da. plasterer at $5.50 per day	22.00
2 da. helper ~~at $3.25 per day~~	~~6.75~~
Cost per 100 square yards	$37.87

The cost per square yard is 38¢

In different localities cost of material and labor may vary from that cited above. In large amounts of plastering, where plasterers buy in large quantities and at lower rates, there will probably appear a lower cost per square yard. The current quotations give from 32¢ a square yard to 67¢ a square yard, depending in part on quality and kind of materials, and in part on scale of wages of localities.

Try to learn what the usual cost per square yard is in your locality. You will also find that other items, as insurance, scaffolding, heating, plastering on brick or tile work, will be included in the estimates, either increasing or reducing the usually quoted cost per square yard.

Written Problems

337. **1.** Taking the number of square yards you found in the room $14' \times 18' \times 9'\ 6''$ (under lathing), employing table compute the amount of material required to plaster it.

2. How many days will it require two plasterers to do the work with the aid of one helper?

3. What will be the cost of labor if the scale of wages is for plasterer \$5.50 and for helper \$3?

4. What will be the cost of the materials?

5. What will be the total cost of labor and materials?

Written Problems

338. 1. Taking the plan of the house on page 339, compute the number of square yards to be plastered, making no deduction for openings.

2. Estimate the amount of materials required for lathing for both floors, in accord with the table given on page 350.

3. Estimate the cost of labor of lathing with the wages at the plasterer's rate of $5.50 per day.

4. Compute the total cost of materials and labor in lathing the house.

5. With the quantity of material and labor estimated for 100 square yards in above table, compute the cost of lathing and plastering the house.

6. Make out an itemized statement of quantities and cost of materials and labor for lathing and plastering the house, such as any interested person would require.

Study Exercise

339. Painting is estimated on the basis of square yard like plastering. But painters make no allowance for openings as windows to compensate in part for the extra work involved in painting moldings, sashes, and the like. Railings are usually estimated as solid, with extra amount added for more tedious work.

It is estimated that a good painter can cover in an eight-hour day 100 yards of outside wall work for the priming, or first coat, and about 80 yards for the second and third coats. Inside painting is more difficult and may not amount to more than 30 or 35 yards a day; except floors, of which 135 yards can be covered.

Painters' wages vary from $4 a day upwards according to locality and the kind of work, as plain painting or decorating, gilding, and so on.

The surface which is ordinarily covered by one pound of mixed paint is four square yards on the priming coat and about six square yards on second and succeeding coats. Putty, in new work, is always taken as 5 pounds for 100 square yards. Painters generally estimate that 100 square yards will require for priming coat about $2\frac{1}{3}$ gallons of white lead and linseed (usually boiled) oil; or for the priming and second coats about 4 gallons of mixed paint; or for the priming and two coats about $6\frac{1}{2}$ gallons.

Shingles are stained in two ways, either by brush or by dipping. If shingles are dipped two thirds of length, 1,000 will take $2\frac{1}{2}$ to $2\frac{3}{4}$ gallons of stain. For brush work two coats will be given to 100 square feet by one gallon, while one coat will be given to 150 square feet by the same quantity. Shingle stain costs from $.55 to $1 or more a gallon.

The following may serve as a statement of the items in making up ten gallons of paint.

1. Priming, or first coat

100 lb. white lead at 8¢ per pound	$8.00
7 gal. boiled linseed oil at 50¢ a gal.	3.50
Mixing $1\frac{1}{2}$ hour at $5 a day	.94
Total cost for 10 gallons	$12.44
Cost for one gallon	1.24
Cost for one pound	$.07\frac{3}{4}$

2. Second and subsequent coats

120 lb. white lead at 8¢ per pound		$9.60
5 gal. linseed oil at 50¢ per gallon		2.50
$\frac{1}{2}$ gal. turpentine at 90¢ per gallon		.45
Color pigments, if other than white is to be secured, $3\frac{1}{2}$ lb. at 25¢ per pound		$.87\frac{1}{2}$
Mixing $1\frac{1}{2}$ hr. at $5 per day		.94
Total cost		$\$14.36\frac{1}{2}$
Cost per gallon	$1.44	
Cost per pound	.09	

3. If zinc is used, the amount of white lead will be reduced by just so many pounds as pounds of zinc are added.

The estimated cost of 100 square yards, materials and labor, is as follows:

The priming coat

$2\frac{1}{2}$ gallons at $1.24 per gallon	$3.12	
Labor at $5 per day	5.00	$8.12

Second and succeeding coats

2 gallons paint at $1.44 per gallon	$2.88	
Labor (80 yd.) at $5 per day	7.50	$10.38
Ditto for third coat		10.38
		$28.88

Cost per square yard $.29

4. This estimate may be somewhat high for certain localities, but one may easily find out the prevailing day's wages of painters and compute cost for 100 sq. yd. accordingly. The prices of lead and oil fluctuate greatly, and the prices given above may be now too high and again too low. Get local prices.

Written Problems

340. 1. The house on page 339 is to be painted three coats with lead and oil of the standard given in tables above. The dimensions are 29′ × 30′, the height on sides from ground to roof 22 feet and on ends from ground to ridge of roof 32′ 8″. Make no deductions for windows.

(*a*) How many square yards of surface are to be painted?

(*b*) How many gallons of paint for the first coat will it take?

(*c*) How many gallons for the other two coats?

(*d*) How many gallons for all?

(*e*) What will the paint for priming coat cost?

(*f*) What will the paint for other coats cost?

(*g*) What will be the total cost for paint?

(*h*) How many days will it take two painters to put on first coat?

(*i*) How many days to put on the other two coats?

(*j*) What will be the labor cost of the first coat?

(*k*) What will be the labor cost of the other two coats?

2. The roof is 33 ft. long and each side measures 21 ft. from edge to ridge.

(*a*) What is the number of *squares* that the roof contains?

(*b*) How many square yards?

(*c*) How many gallons of shingle stain will it take to give two brush coats to roof?

(*d*) What will be the cost of stain at $.75 per gallon?

(*e*) What will be the cost of labor of staining roof two coats? (A man can brush about 150 yards in eight hours.)

(*f*) If the shingles were dipped, how many gallons would be needed? (This is easily computed if you remember the number of shingles per square, and thus estimate the number needed for this roof.)

3. Assemble the items and their respective costs for painting the house and staining the roof, two brush coats, in a regular form of statement, and give total cost of materials and labor.

4. Obtain the dimensions of a building and estimate (*a*) the cost of siding, (*b*) the cost of shingling, (*c*) the cost of painting.

Written Problems

341. 1. The foundations of the house on page 339 are 12″ thick and 8′ deep. How many cubic yards of concrete will be needed for the construction of this wall?

2. If 1 : 2 : 4 concrete is used, how many barrels of cement and how many cubic yards of sand and broken stone are needed? What is the cost of the concrete at $5.15 a cubic yard?

3. The inside surface of the wall is to be plastered smooth with 1 : 2 cement mortar, one volume of cement to two volumes of sand, making the layer $\frac{1}{2}$ in. thick. If the foundation wall incloses a cellar 27′ by 28′, and 8′ deep, how many cubic feet of cement mortar are needed for plastering the exposed surface of the wall?

4. The foundation of the house may be built of rubble, or stones of irregular size most easily obtained from the quarry. How many cubic yards of rubble are needed for foundations, if the wall is to be 9′ deep? What will be its cost, if the rubble is $1.25 per ton (= 22 cu. ft.) and the cost for labor, etc., per cubic yard of wall is $4.75?

5. How many cubic yards of masonry in the foundation, if the walls are $2\frac{1}{2}'$ thick and $8'$ high?

6. How many perches of masonry in the foundation walls of a building 60 ft. long by 36 ft. wide? The wall is $2\frac{1}{4}$ ft. thick and 12 ft. high.

Written Problems

1. The dimensions of a common brick are $8'' \times 4'' \times 2''$. How many cubic inches in one brick?

2. In brickwork, it is customary to allow for $\frac{1}{4}''$ thickness of mortar. How many cubic inches in a brick, if to each of its dimensions the thickness of the mortar is added?

3. How many bricks each $8\frac{1}{4}'' \times 4\frac{1}{4}'' \times 2\frac{1}{4}''$, including the mortar, will be required to make a cubic foot of wall?

4. How many common bricks with mortar $\frac{1}{4}''$ thick will it require to build a wall $30' \times 12\frac{1}{2}' \times 1\frac{1}{2}'$?

5. At $6.75 per thousand, find the cost of bricks required to build a wall 50′ long, 12′ high and 18″ thick.

Written Problems

342. 1. Examine the drawing of the house on page 339. Find the length and width of each room. Find the height of each room.

2. A roll of wall paper is 16 yd. long and is generally 18 in. wide. How many strips each 8 ft. long in one roll?

Parts of a roll of wall paper are not sold. The waste in matching designs in wall paper varies greatly. In these problems plain paper alone is considered.

3. Find the number of rolls of paper needed for the four walls of the living room, if 2 strips are deducted for each door and for every window.

4. Find the cost of these rolls at $45\frac{1}{2}$¢ a roll.

5. If on the dining room ceiling the strips of paper are laid the short way of the room, how many strips are needed?

6. Find the number of rolls and the cost, at 43¢ a roll, of paper for the ceiling of the dining room.

7. Make similar computations for the three bedrooms.

8. If a linoleum is put on the kitchen floor which does not cover all of it, but leaves on all sides an uncovered margin of 1 ft., compute the cost of the linoleum at $ 1.75 a square yard.

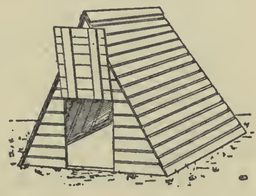
9. How many $6'' \times 6''$ tiles are needed to cover the bathroom floor?

10. If $4'' \times 4''$ tiles are preferred, how many of them will be needed?

11. Would it be possible to cover the entire floor with $5'' \times 5''$ tiles, without using parts of tiles? Why not?

12. What is the cost of kalsomining the ceiling and walls of the kitchen at 8¢ a square yard, no allowance to be made for doors and windows because of the low price per square yard? The ceiling is $9\frac{1}{3}$ ft. high.

13. The approach to the front porch is 5 ft. wide by 14 ft. long. How many bricks, $8'' \times 4''$, will be needed to cover it?



14. The dog-kennel shown in the figure has a floor 3′ long and 3′ wide. The slanting roof measures 4′ on each side, making the kennel about 3.7′ high. Compute the number of square feet of board surface in the exterior, including the floor.

15. The boards used are 1″ thick. How many "board feet" of lumber should be purchased, if 20% extra is allowed for pieces used in strengthening the kennel, and for waste?

16. How many "board feet" are needed, if the boards used are $1\frac{1}{2}''$ thick? 2″ thick?

17. What will be the cost of the lumber needed (allowing 20% extra as in Ex. 2), if 1″ boards are used and the cost of lumber is \$18 per 1,000 board feet? How much if $1\frac{1}{2}''$ boards are used?

18. How many square feet of exterior surface, including the floor, would there be in this kennel, if the length, breadth, and height were doubled?

INDUSTRIAL PURSUITS

Written Problems

343. Time Sheet for Week ending January 25

Name	M.	T.	W.	Th.	F.	S.	Total Hours	Days	Daily Wages	Amount	Daily Average
John Roe	8	10	6	8	9	7			$3.50		
Chas. Seipp	7	9	7	7	10	6			4.25		
Geo. Halifax	6	8	8	8	8	5			5.00		
John Saxe	5	8	9	9	10	4			4.50		
Al. Fox	9	7	10	8	7	3			3.75		
Fred Mack	8	7	10	7	10	0			5.25		
Will. Sell	10	8	9	8	6	2			6.00		

The numbers in the columns headed M., T., W., etc., indicate the hours on the respective days.

Counting 8 hr. to a day, find:

1. The wages which the employer owes each man at the end of the week.

2. The entire amount paid in wages for that week.

3. The average wage of each man per day.

4. Make a table of wages for a 10 hr. day, showing the rate for 1 hr., 2 hr., etc., at $3.50 a day.

344. TIME SHEET FOR WEEK ENDING JULY 17

NAME	RATE AN HOUR	M.	T.	W.	TH.	F.	S.	AMOUNT	
Sam. Foot	75¢	10	9	6	7	6	4		
Louis Sahm	60¢	7	8	7	8	8	4		
Fred Holt	55¢	6	8	5	6	10	6		
John Curr	45¢	8	7	0	10	8	4		
Paul Orr	35¢	10	8	8	9	8	8		

1. Find the total amount each man receives at the end of the week and the total amount the employer pays out in wages.

TIME SHEET FOR WEEK ENDING SEPTEMBER 18

NAME	DAILY WAGE	M.	T.	W.	TH.	F.	S.	AMOUNT	
George Halifax . .	$3.25	8	7	8	9	10	6		
William Sutton . .	4.75	9	7	8	6	9	4		
John Hare . . .	5.00	5	8	9	7	6	9		
Charles Noyes . .	4.00	7	10	9	8	6	4		
Atherton Parsons .	2.50	10	9	9	8	7	8		
Harold Davis . . .	3.75	6	9	10	9	6	4		
Samuel Roe . . .	3.50	7	9	8	10	9	9		

2. Counting 10 hr. to a day, find the wages due each man at the end of the week, the entire amount paid in wages for that week, and the average amount earned by each man per day.

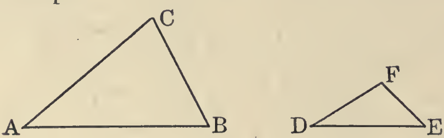
3. In drawing the cash from the bank, what change must be asked for in order that each man may receive in his pay envelope the right amount?

GEOMETRICAL APPLICATION

Similar Triangles

345. Distances which cannot be measured directly may be found by *similar triangles.*

Triangles are said to be *similar* when they have the same shape.



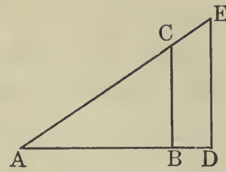
The two triangles drawn here have the same shape, although one is larger than the other. The side AB is just twice as long as the side DE. How does the side CB compare in length with FE? AC with DE?

We have the proportions, $\frac{AB}{DE} = \frac{BC}{EF}$

and $$\frac{AB}{DE} = \frac{AC}{DF}.$$

1. If $AB = 10''$, $DE = 6''$, $AC = 12''$, find DF.

2. If DE is 5 ft., EF 4 ft., and AB 10 ft., how long is BC? State the proportion.



3. If $AB = 12$ ft., $AC = 10$ ft., $DE = 5$ ft., find DF.

4. If $CB = 15$ in., $AC = 20$ in., $FE = 8$ in., find DF.

5. Name the similar triangles in the figure.

6. Which are the bases of the triangles? Which their altitudes?

7. What proportion holds between the bases and the altitudes?

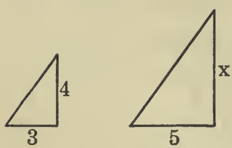
8. If $AB = 10$ ft., $AD = 12$ ft., $BC = 8$ ft., find DE.

9. If $AB = 12$ ft., $AD = 14$ ft., $DE = 10$ ft., find BC.

10. If $DE = 20$ ft., $BC = 18$ ft., $AD = 25$ ft., find AB.

11. If $DE = 25$ ft., $BC = 21$ ft., $AB = 27$ ft., find AD.

12. If $AC = 15$ in., $AE = 18$ in., $AB = 12$ in., find AD.



13. In these two triangles, the ratio $\frac{x}{5}$ is equal to what other ratio? Write the proportion and find x.

14. Practice the drawing of two similar triangles. Vary the positions. Name the similar parts.

Study Exercise

346. The United States Forestry Service computes the height of trees as follows: The first method is somewhat rough and ready, but sufficiently accurate for the purposes. A suitable distance is measured or paced off from the foot of the tree B on a level place to point F. At F an upright stick is placed. The observer lies down with his body pointing directly toward the tree and his feet touching the stick at F. He looks at the top of the tree and an assistant marks the place where his line of vision crosses the stick. Then the following data are secured: the distance from base of tree to upright stick, BF; the length of observer from the point F to his eyes, FE; and the distance from ground to place on stick where line of vision crosses, FS. The height of tree, BT, is to be found.

The arrangement of the terms in a proportion is:

$$EF : FS :: EB : BT.$$

Make a drawing to explain this method of finding the height of trees. Letter the drawing as indicated in the description. What are the similar triangles? Suppose EF equals 5′ 6″; FS equals 9′; and EB equals 5′ 6″ plus FB or 120′ or 125′ 6″. What is the height of the tree, BT?

Substitute the numbers in the proportion as follows:

$5\frac{1}{2} : 9 :: 125\frac{1}{2} : x$. Solve the proportion.

It is advisable to reduce the inches in this problem to fractions of a foot.

When instruments of measure are not at hand, the distance from tree base may be paced off, and a pace is usually made to equal about 3 feet. The height of the stick may be measured by a stick the length of one pace.

Written Problems

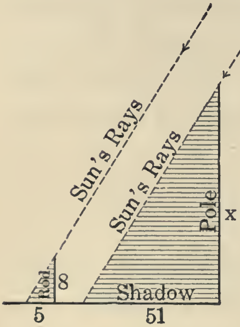
347. 1. An observer is 5′ 8″ to eye level; the distance from a tree to a measuring stick is 165′; and the distance on stick from the ground to place where line of vision crosses is 11′ 6″. What is the height of the tree?

2. A forest ranger measures off 150 paces from a tree, places there a stick across which he sights from a lying down position and finds that the distance from ground to line of vision is 11 ft. The ranger is six feet tall. What is the approximate height of the tree?

3. Some boys wish to ascertain the height of a pole for wireless telegraphy. They take a rod 8 ft. long, hold it on the ground, straight up, and measure the length of its shadow. They also measure the length of the shadow cast by the wireless telegraph pole. From these data, how can they compute the height of the pole?

In the figure, which are the similar triangles?

How many feet long is the shadow cast by the rod? By the pole? What proportion will determine x?



4. A telegraph pole casts a shadow of 30.5 ft. at the same time of the day that a vertical rod, 10 ft. long, casts a shadow of 6 ft. Find the height of the telegraph pole.

5. A boy finds that a poplar tree casts a shadow 48 ft. long on level ground at the same time that a fence post casts a shadow of 5 ft. The post is 4 ft. high. How high is the tree?

6. Find the height of a flagpole in your vicinity by any two of the methods given.

SQUARE ROOT AND ITS APPLICATION

A Problem Proposed

348. A farmer is planning a shed, 20′ wide, with a roof $\frac{1}{2}$ pitch.

SUGGESTION. See note on pitch, page 348.

He wants to find the exact length of the rafter AC.

One way of finding this is by an important property of right triangles, such as the triangle ABC, and by processes called squaring and the extraction of the square root.

Computations of this sort arise frequently in industrial work.

Squares and Square Roots

349. We know that if a table in the form of a square has a side of 4 feet, it has an area of 16 square feet. We call 16 the *square* of 4, and 4 the *square root* of 16.

Often the square of 4 is written 4^2, and the square root of 16 is written $\sqrt{16}$. Thus, we have $4^2 = 16$ and $\sqrt{16} = 4$.

We call the number 16 a *perfect square*, because its square root can be found exactly. But 15 is not a perfect square; its square root is less than 4, but greater than 3; we find $\sqrt{15} = 3.87^+$, and $(3.87)^2$ is nearly equal to 15.

1. Name the squares of each of the following numbers:

8, 9, 10, 11, 12, 13, 15, 20, 60, 90.

2. Which of the following are perfect squares? Factor to find their square roots.

49, 64, 81, 121, 125, 144, 175, 169, 100, 200, 196, 225, 300.

3. Find x in the following:

$5^2 = x$	$14^2 = x$	$\sqrt{625} = x$	$\sqrt{1.69} = x$
$6^2 = x$	$\sqrt{81} = x$	$\sqrt{121} = x$	$\sqrt{400} = x$
$1.1^2 = x$	$\sqrt{100} = x$	$1.2^2 = x$	$\sqrt{900} = x$

4. What are the sides of the squares whose areas are:

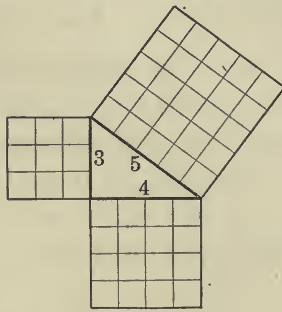
25 sq. in.	.49 sq. ft.	$\frac{1}{16}$ sq. ft.
36 sq. mi.	.81 sq. rd.	$\frac{1}{121}$ sq. yd.
49 sq. ft.	.81 sq. rd.	.25 sq. ft.

5. Commit to memory the following squares:

$2^2 = 4$	$7^2 = 49$	$20^2 = 400$	$60^2 = 3,600$
$3^2 = 9$	$8^2 = 64$	$30^2 = 900$	$70^2 = 4,900$
$4^2 = 16$	$9^2 = 81$	$40^2 = 1,600$	$80^2 = 6,400$
$5^2 = 25$	$10^2 = 100$	$50^2 = 2,500$	$90^2 = 8,100$
$6^2 = 36$			

Solution of the Right Triangle

350. In a **right triangle,** or triangle having one angle a right angle, the longest side has a special name. It is called the **hypotenuse.** Consider the right triangle in the figure, whose sides are 3, 4, and 5 units in length. The side whose length is 5 units is the hypotenuse. Now imagine a square constructed on each of the three sides of the right triangle, as in the figure. Each of the three squares is made up of smaller squares, all equal to each other.



1. How many little squares are there in the large square on the side marked 3?

2. How many little squares are there in the large square on the side marked 4?

3. How many little squares are there in the large square on the side marked 5?

4. How does the number of little squares in the last case compare with the *sum* of the numbers of little squares in the first two cases?

In this right triangle $5^2 = 3^2 + 4^2$.

In the study of geometry it will be shown that this property holds true of every right triangle. That is,

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two other sides.

Note the length of the hypotenuse by h and the lengths of the two other sides by a and b. The equation that expresses the relation is: $h^2 = a^2 + b^2$.

When it is known that the triangle is a right triangle, we can compute one of the sides when the other two are known.

5. In a right triangle $a = 6$, $b = 8$, find h.

PROCESS

$$h^2 = a^2 + b^2.$$
$$h^2 = 6^2 + 8^2$$
$$h^2 = 36 + 64 = 100.$$
$$\sqrt{100} = 10 = h, \text{ the hypotenuse.}$$

6. In a right triangle, $h = 15$, $a = 12$, find b.

PROCESS

$$h^2 = a^2 + b^2.$$
$$225 = 144 + b^2.$$
$$225 - 144 = b^2.$$
$$81 = b^2.$$
$$9 = b.$$

Written Exercise

351. In the following right triangles, find the unknown side:

1. $a = 12,\ b = 16$
2. $a = 24,\ b = 7$
3. $b = 15,\ h = 25$
4. $a = 24,\ b = 32$
5. $h = 35,\ b = 21$
6. $a = 36,\ h = 45$
7. $h = \overset{15}{\cancel{40}},\ b = 9$
8. $h = 61,\ b = 60$
9. $h = 85,\ a = 84$
10. $h = 37,\ a = 12$

1600
81
519
38.+

EXTRACTING THE SQUARE ROOT

Study Exercise

352. If in the right triangle $a=2$, $b=3$, then $h^2=a^2+b^2=4+9=13$, $h^2=13$. As 13 is not a perfect square, the side h is larger than 3 and smaller than 4. The square root of 13 or such a number may be found approximately by a simple method. This method can be used also for finding the square root of numbers that are perfect squares, but are not at once recognized as such.

1. Find the square of 37.

PROCESS

Common Method	Meaning of Common Method	Expressed by Letters
37	$30+7$	$t+u$
37	$30+7$	$t+u$
259	$210+49$	$tu+u^2$
111	$900+210$	t^2+tu
1369	$900+420+49$	$t^2+2tu+u^2$

$$37^2=1369;\quad 37^2=900+420+49;$$
$$(t+u)^2=t^2+2tu+u^2.$$

EXPLANATION. — It is plain that 37 may be broken up into two numbers, $t=30$ and $u=7$. We first found 7×7 or $7^2=u^2$, then $30\times 7=tu$, next $7\times 30=tu$, and finally $30\times 30=t^2$.

In extracting the square root, which is the process of separating a number into two equal factors, reverse the above process.

2. Find the square root of 1,369.

PROCESS

$$
\begin{array}{lrl}
\text{Let} & 1{,}369 = & t^2 + 2\,tu + u^2 \\
\text{then} & \underline{900} = & t^2 \\
2\,t \quad = 60 & 469 = & 2\,tu + u^2, \text{ where } t = 30 \\
2\,t + u = 67 & \underline{469} = & (2\,t + u)u, \text{ where } u = 7
\end{array}
$$

The required root is $30 + 7 = 37$.

Check: $37 \times 37 = 1{,}369$.

EXPLANATION. — $30^2 = 900$, $40^2 = 1{,}600$; hence the answer lies somewhere between 30 and 40. Take $t = 30$. Subtract $t^2 = 900$; the remainder 469 must be equal to $2\,tu + u^2$. Find u by trial; dividing 469 by the trial divisor $2\,t = 60$ gives $u = 7$. Multiplying 67 by 7 gives 469 or $2\,tu + u^2$.

SHORTENED PROCESS

$$
\begin{array}{r|l}
 & 13'69 \,\underline{|\,37} \\
 & 9 \\
\hline
2\,t = 60 & 469 \\
u = \underline{\ \ 7} & \\
67 & \underline{469}
\end{array}
$$

THE SQUARE ROOT OF 7921

$$
\begin{array}{r|l}
 & 79'21 \,\underline{|\,89} \\
 & 64 \\
\hline
2\,t = 160 & 1521 \\
u = \underline{\ \ \ 9} & \\
169 & \underline{1521}
\end{array}
$$

EXPLANATION. — Since $80^2 = 6{,}400$, $90^2 = 8{,}100$, $t = 80$. Then $2\,t = 160$, $1{,}521 \div 160$ suggests $u = 9$.

Observe that, in the shortened process, we separated the digits into periods of two digits. This is found to be a great convenience in practice. The period on the extreme left may have only one digit.

> **To find the square root of a number:**
>
> (1) Point off the number into periods of two figures each, beginning at the right (at the decimal point in a decimal).
>
> (2) Find the largest integer whose square is not greater than the left-hand period.
>
> (3) Continue as indicated in the preceding and in the following examples.

3. Find the square root of 83,745.075.

PROCESS

$$
\begin{array}{r|l}
 & 8'37'45.07'5 \;\underline{|\,289.38^{+}} \\
 & 4 \\
2\,t = 40 & 437 \\
u = \underline{\quad 8} & \\
48 & \underline{384} \\
2\,t = 560 & 5345 \\
u = \underline{\quad 9} & \\
569 & \underline{5121} \\
2\,t = 5780 & 22407 \\
u = \underline{\quad 3} & \\
5783 & \underline{17349} \\
2\,t = 57860 & 505850 \\
u = \underline{\quad 8} & \\
57868 & \underline{462944} \\
 & \quad 42906
\end{array}
$$

EXPLANATION. — The left-hand period is 8. The largest perfect square in 8 is 4; $\sqrt{4} = 2$. Subtract and take down the next period. Then $2t = 4$ hundreds or 40 tens. Use this as a *trial divisor*. Here $437 \div 40$ would seem to give 10, but it is found that even 9 is too large. We find $u = 8$. The *complete divisor* is 48. Proceed as shown here.

If we are required to find the square root of an integer that is not a perfect square, correct to, say, two decimal places, then annex two periods of zeros after the decimal point.

4. Find the square root of 2, to three decimal places.

PROCESS

$$
\begin{array}{r|l}
 & 2.00'00'00 \;\big|\; 1.414^{+} \\
 & 1 \\
\hline
2t = 20 & 100 \\
u = 4 & \\
\hline
24 & 96 \\
\hline
2t = 280 & 400 \\
u = 1 & \\
\hline
281 & 281 \\
\hline
2t = 2820 & 11900 \\
u = 4 & \\
\hline
2824 & 11296 \\
\hline
\end{array}
$$

EXPLANATION. — We see that $\sqrt{2}$ cannot be found with absolute accuracy. But we can approximate to its true value as close as we desire.

Check: $(1.414)^2 = 1.999396$.

Written Exercises

353. Find the square roots, to three decimal places:

1. 3 **2.** 5 **3.** 6 **4.** 7 **5.** 8 **6.** 10

Written Problems

354. **1.** A farmer is planning a shed 20 ft. wide, with a roof $\frac{1}{2}$ pitch. How long must the rafter be? See the problem and figure on page 348.

PROCESS

$$\overline{ER}^2 = 10^2 + 5^2$$
$$= 100 + 25$$
$$= 125$$
$$ER = \sqrt{125}.$$

Extracting the square root, we find $ER = 11.18^+$. Hence the rafter ER must be 11.2 ft. long.

EXPLANATION. — By the property of the right triangle,

$$h^2 = a^2 + b^2.$$

2. A house is to be 36′ wide and the ridge is to be 12′ above the top of the walls. How long must the rafters be, if the projections of the rafters at the eaves are 1′?

3. Two vessels start from the same point; one travels north at the rate of 14 knots, and the other east at the rate of 16 knots. How far apart are they at the end of one hour?

[Carry answer to *one* decimal place.]

4. An iron smokestack, 50′ high, must be anchored by wires to posts 30′ from the bottom. How long must each wire be?

5. A man starts to row across a stream at the rate of 5 miles an hour; the current carries him downstream at the rate of 3 miles an hour. What is the actual rate of progress? Draw a figure.

6. A public square in a city covers an area of 5 acres. How many feet does the square measure on a side?

7. A baseball diamond is 90′ square. How far is it from the home plate to second base? Verify your computation by actual measurement.

8. A wireless telegraph pole 100′ long is to be erected and anchored by wires reaching from a place 15′ from the top of the pole to posts 40′ from the bottom. How long must the wires be?

9. A pole at a wireless telegraph station is 80′ high. From its top to a point 60′ from the foot of the pole is stretched a wire support. Find the length of the wire. Disregard sags in the wire and irregularities in the surface of the earth.

AREAS AND VOLUMES

Review

355. 1. State the rule for finding the area of a rectangle.

2. State the rules for finding the area of a triangle.

3. How is the length of a circle found?

4. How is the area of a circle found?

5. How is the volume of a rectangular solid found?

Written Problems

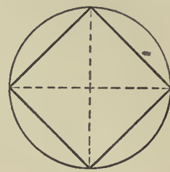
356. 1. The diameter of a drumhead is 2 ft. How many square feet of skin are in the two heads? Use the ratio $3\frac{1}{7}$.

2. A cow is held by a rope 25 ft. long that is tied to a stake. Over how large an area can the cow graze?

3. A big tree is 25.7 ft. around at the base. Compute its diameter.

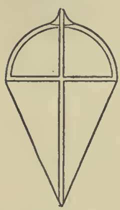
4. What is the diameter and area of the largest circle that can be drawn on a sheet of paper 10 in. square?

5. How much larger is a table 5 ft. square than a round table 5 ft. in diameter?



6. Find the area of a square inscribed in a circle whose diameter is 11 ft.

7. Find the difference in area between a circle whose radius is 10 ft. and the area of an inscribed square.



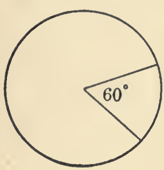
8. From light sticks and a hoop cut in half, a boy made a frame and then pasted paper upon it, to make a kite. How many square inches of paper did he use, if the radius of the hoop was 14 in., and the triangular frame below the semicircle was 15 in. deep?

CIRCLES AND ANGLES

Study Exercise

357. Surveyors measure angles in degrees, minutes, and seconds. If a right angle is divided into 90 equal parts, one of these parts is a **degree** (°). Two lines that are perpendicular to each other form four right angles at the point where they cut each other. Hence it takes 360° to measure the four right angles about that point.

1. If a circle has an area of 100 sq. in., what is the area of each of the four parts into which two perpendicular diameters divide the circle?



2. In a circle two radii are drawn, forming an angle of 60°. What part of the whole circle is the area of the "sector" thus formed?

3. Two radii form an angle of 45°. What part of the entire circle is the sector inclosed by them?

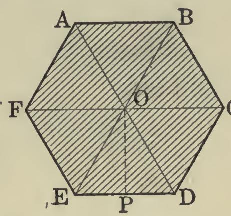
4. A sector with an angle of 45° has an area of 15 sq. in. What is the area of another sector in the same circle having an angle of 18°?

AREAS OF POLYGONS

The Regular Hexagon

358. A polygon having six equal sides and six equal angles, as shown in the figure, is called a **regular hexagon**. It is divided into six equal triangles by lines drawn from the center to the vertices.

1. If the area of the triangle EOD is known, how can the area of the entire hexagon be found?



2. Find the area of the hexagon when that of the triangle EOD is $1\frac{1}{3}$ sq. in., $4\frac{2}{3}$ sq. in., $7\frac{5}{6}$ sq. ft., $4\frac{1}{2}$ sq. in.

3. Find the area of the triangle EOD, if the area of the hexagon is $1\frac{1}{5}$ sq. ft., $1\frac{1}{3}$ sq. in., $3\frac{1}{2}$ sq. in., 1.2 sq. ft., .24 sq. rd.

4. If the area of triangle EOD is .886 sq. in., and its base is 2 in., find the height PO.

5. If each side of the hexagon is 1 in., and the height of each triangle is $.44^+$ in., find the area of the hexagon.

Have you seen the regular hexagon as the form of tiling or of decorative designs?

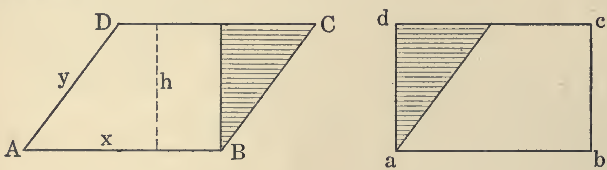
Quadrilaterals

359. A **parallelogram** is a four-sided figure having the opposite sides parallel. If the angles are equal to each other, then the parallelogram is a special kind, ordinarily called a **rectangle**. If the angles are not all equal, the parallelogram is a special kind, called a **rhomboid**.

Parallelogram Rectangle Trapezoid

1. Does the rhomboid $ABCD$ have the same area as the rectangle $abcd$? Why?

2. Frame two rules for finding the area of a rhomboid.

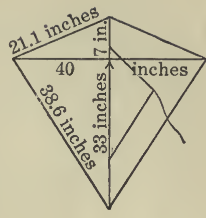


3. Which has the greater perimeter, the rectangle or the rhomboid in these illustrations?

4. A rhomboid has $x = 80$ yd., $y = 50$ yd., $h = 40$ yd. Find its perimeter and area.

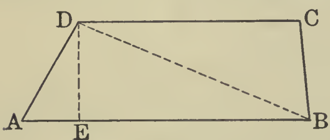
5. A boy makes a kite by fixing two thin sticks in form of a cross, covering the frame with light cloth, and bending the cross arm backward in bow

fashion. How many square inches of cloth does he use? If the upper slanting sides are 21.1 in. each, the lower slanting sides 38.6 each, how long must a strip of colored paper be, to reach all around the kite?



The Trapezoid

360. A four-sided figure having only two sides parallel is called a **trapezoid**. The figure $ABCD$ has the two sides AB and CD parallel and is a trapezoid. Draw the diagonal DB.



1. The line DB divides the trapezoid into how many triangles?

2. Name the base and the altitude of the triangle ABD.

3. How would you find the area of the triangle ABD?

4. The line DC may be taken as the base of the triangle BCD, and the line DE as its altitude.

5. How may the area of the triangle BCD be found?

6. Have the two triangles the same altitude?

7. How may the area of a trapezoid be computed?

> **The area of a trapezoid** is equal to the sum of the areas of the two triangles into which it is cut by a diagonal.

8. If AB is 4″, CD is 3″, and DE 2″, what is the area of the triangle ABD? Of BCD? Of the trapezoid?

Instead of multiplying each of the parallel sides or bases separately by the altitude, it is shorter to multiply the *sum* of the two bases by one half the altitude. Hence the rule:

> **The area of a trapezoid** is one half the product of the sum of the bases and its altitude.

Written Exercise

361. Find the areas of the following trapezoids; the two parallel sides are given first, the altitude last:

1. 4″, 6″, 3″
2. 7′, 5′, 5′
3. 12″, 11″, 6″
4. 7 rd., 8 rd., 8 rd.
5. 2 yd., 1 yd., 2 yd.
6. 3 mi., 1 mi., 1 mi.

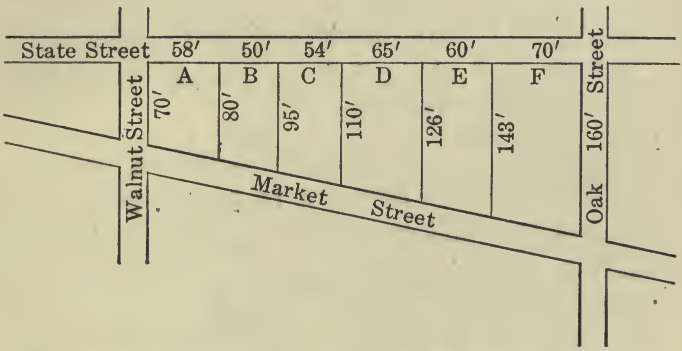
Written Exercise

362. 1. An irregular city block is divided into lots as shown in the figure. Find the area of each of the lots A, B, C, D, E, F.

2. Lot A was sold at a price amounting to \$9 a square foot. Find the price.

3. Lot B sold at \$$97\frac{1}{2}$ per linear foot on State Street. Find the selling price.

4. Lot C sold for \$5,000. How much is this per square foot?



5. Lot E sold for \$7,000. How much is that per linear foot on State Street?

6. The trapezoidal floor of an office contains 315 sq. ft. The parallel sides are 15 ft. and 20 ft. What is the width of the office?

7. Take measurements of your own and make up problems on mensuration.

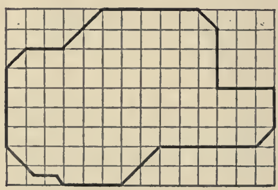
8. Another piece of ground of exactly the same shape and dimensions as the ground bounded by State Street, Walnut, Market and Oak was purchased by a dealer before it was divided up into lots, for \$5,000. He paid out \$450 for surveying the

lots and for other improvements. The smaller lots are better located than the others; he offers all for sale at the same price. What does he ask for each lot, if he aims to make a profit of 30 % on the money expended?

Estimates of Land Areas

363. 1. A ranch of 640 A. is laid out as described in this figure. About how many acres of each kind are there?

2. The area of an irregular piece of land can be found by drawing a map of the land to scale on squared paper, and counting the number of squares covered by the figure. Estimate a part of a square, correct to the nearest one fourth of it. Find the area of the county mapped in the figure, each square being a square mile.



[First, count the squares in each row.
Next, add the numbers thus obtained.]

3. What would be the area of this land, if each square were $\frac{1}{10}$ of a square mile? $\frac{1}{2}$ of a square mile? $\frac{1}{3}$ of a square mile?

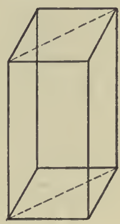
4. What would be the area of this land, if the *sides* of the squares were $\frac{1}{10}$ of a mile each? $\frac{1}{2}$ of a mile? $\frac{1}{3}$ of a mile?

SOLIDS

Study Exercise

364. 1. Explain the formula $v = l \cdot w \cdot h$, when applied to a rectangular solid.

The rectangular solid shown in the figure is sometimes called a **rectangular prism**. If it is cut in two along the dotted lines, each part is called a **triangular prism**, its base being a triangle.



2. How does the volume of this triangular prism compare with that of the rectangular prism?

3. How do the areas of the bases of the two compare?

4. Hence, if you know the area of the triangular base, how can you find the volume of the triangular prism?

5. If b is the area of the base of the prism, explain the formula $v = b \cdot h$.

> It can be shown that the volume of any prism can be found by the rule:
>
> **Volume = Area of Base × Height of Prism.**

6. How much paper is needed to cover the outside of a rectangular solid, $6'' \times 7'' \times 3''$?

7. A triangular prism, 10 ft. high, has a base 6 sq. ft. in area; the sides of the base are 3 ft., 4 ft., and 5 ft. How many sq. ft. of cloth are needed to cover the prism all over?

Written Exercise

365. Find the volumes of prisms having bases and altitudes as follows:

1. 6.4 sq. ft., 5.5 ft.
2. $125\frac{1}{2}$ sq. in., 1.6 in.
3. 3.27 sq. yd., 2.3 yd.
4. $3\frac{1}{3}$ sq. yd., 7.5 ft.
5. 75 sq. in., 2.5 ft.
6. 27 sq. ft., 7.3 yd.

Written Exercise

366. Find the heights of the following prisms:

1. $v = 24.5$ cu. in., $b = 7.3$ sq. in.
2. $v = 27.5$ cu. ft., $b = 3.8$ sq. ft.
3. $v = 30.6$ cu. ft., $b = 35.4$ sq. ft.
4. $v = .8$ cu. in., $b = 1.2$ sq. in.
5. $v = .6$ cu. ft., $b = 144$ sq. in.
6. $v = 72$ cu. ft., $b = .9$ sq. in.

Written Problems

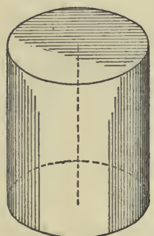
367. 1. A swimming tank is 50 ft. long and 30 ft. broad. When 50 men dive into the tank at the same time, how high will the water rise, if each man displaces on an average 3 cu. ft. of water?

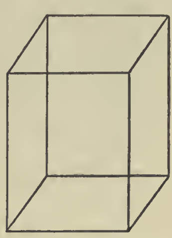
2. In the last example, how many men dive in, if the water rises $1\frac{3}{5}$ inches?

3. A man owns coal land showing a seam of coal 370 yd. long. If its width is estimated at 175 yd. and its thickness at 4 ft. 6 in., how many tons of coal can be mined, if one cubic yard yields one ton?

Cylinders

368. The accompanying illustration shows a solid, called a **cylinder**. Its base is a circle. This circle and the base of the prism are so drawn that the two have the same area. The cylinder and prism have the same altitude.





Imagine the two figures as representing two cans, one with a circular base, the other with a square base.

It is found by trial that the two cans will hold exactly the same amount of water. The two solids must, therefore, have the same volume.

Tell how to find the volume of a cylinder.

1. Express in words, $v = b \cdot h$.

2. Find the volume of a cylinder, 5 in. high, whose base has an area of $3\frac{1}{2}$ sq. in.

3. The cylindrical water tank of a street sprinkler is 10 ft. long and has an internal

diameter of 4 ft. How many cubic feet of water can it hold?

SUGGESTION: Use $3\frac{1}{7}$ in finding the area of the base.

4. This figure shows the surface of a cylinder spread out on a plane. It is the surface of a cylinder $\frac{1}{4}$ in. in diameter and 1 in. high. Explain how the area of this can be found.

5. How does the base line of the rectangle compare with the length of the circular base?

6. If a cylinder has a diameter of 2 in., what is the length of its circular base? What is the area of the base?

SUGGESTION: Use $3\frac{1}{7}$ as the ratio.

7. If the cylinder in Problem 6 is 7 in. high, what is the area of the curved part of the cylinder? what is the volume of the cylinder?

8. A well, 4 ft. in diameter, contains water to a depth of 20 ft. How many cubic feet of water are there in it?

Written Exercise

369. Find the volumes of these cylinders:

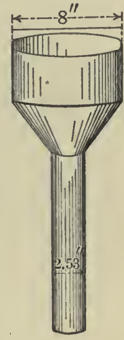
1. Diameter 4 ft., height 7 ft.
2. Diameter 5 in., height 3 in.
3. Diameter 6 in., height 14 in.

4. Diameter 14 in., height 21 in.

5. Find the area of the entire surface of a cylinder 7 in. in diameter and $4\frac{1}{2}$ in. high.

Rain Gauge

370. 1. Many farmers and gardeners keep a record of rainfall. They use a rain gauge of the shape shown here. Across the top it has a diameter of 8 in. Find the area of the cross section at the top.



2. At the bottom the inside diameter is 2.53 in. Find the area across the bottom. How does this area compare with the preceding?

3. A rainfall which measures 10 in. in the lower vessel would measure 1 in. in the upper. If one day the rain fills the lower tube to 8 in., how many actual inches of rainfall were there?

4. After a shower, the reading taken from the small tube was $\frac{3}{4}$ in. What fraction of an inch of rain was there? What was the weight of rain that fell upon 8 sq. ft. of ground, if 1 cu. ft. of water weighs 62.5 lb.?

5. Find the lateral surface of the rain gauge (*a*) the cylinder 4 in. high, (*b*) the frustum of the cone slant height 5 in., (*c*) the lower cylinder 12 in. long.

Written Problems

371. 1. What is the weight of an iron rod, cylindrical in form, $\frac{1}{2}$ in. in diameter and 20 ft. long, if a cubic foot of iron weighs 450 lb.?

2. The pressure of steam in a boiler is 100 lb. to the square inch. Find the pressure on a valve $2\frac{1}{4}$ in. in diameter.

3. What would it cost to paint the outside surface of a cylinder at 1¢ per square foot, the cylinder having a base of 4 ft. diameter, and an altitude of 10 ft.?

4. An irregular piece of metal is sunk into a cylindrical water tank standing on end. When the piece of iron is completely submerged, the surface of the water has risen 2 in. What is the volume of the piece of metal, if the diameter of the water tank is 1 ft.?

5. A bunch of iron wire $\frac{1}{8}$ in. in diameter is wound on a large spool. The wire and the spool weigh 20 lb., the spool alone is known to weigh 2 lb. How long is the wire on the spool, if one cubic foot of iron weighs 450 lb.?

6. The hollow circular steel column, 9″ in external diameter and 6″ in internal diameter and 15′ high, supports a weight of 5 tons. Considering this weight and also the weight of the column itself, what is the pressure exerted by the column

upon its foundation? Steel weighs 490 lb. per cubic foot.

7. Find the length of the tire of a wheel 3.6 ft. in diameter.

8. Water expands, during freezing, nearly $\frac{1}{10}$ of its volume. The more exact amount is .0909 of a unit volume. A waterpipe 100 ft. long and 2 sq. in. in cross section has a volume of 2,400 cu. in. Find the increase in volume during freezing.

9. Which in the long run is the more economical way of building a walk 5 ft. wide, to make it of concrete at 10¢ a sq. ft., or to make it of inch boards resting on two $2'' \times 4''$ stringers, lumber selling at $ 30 per M? Assume that concrete will last five times as long as wood. What is the cost per foot of each walk?

10. Copper expands .00001 of its length for a rise in temperature of 1° F. A copper wire transmitting electric power is supported by towers 500 ft. apart. How much does the wire between two towers stretch when the temperature rises 86°?

11. Ice floats on water. 1 cu. ft. of ice weighs only .9166 times as much as a cubic foot of water at 32° F. If a cu. ft. of water at 32° F. weighs 62.42 lb., what does 10 cu. ft. of ice weigh? How much more than this will 10 cu. ft. of water weigh?

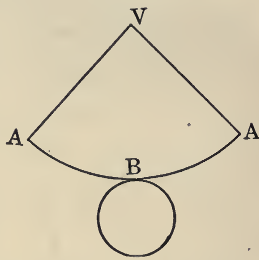
Pyramids and Cones

372. Study the figure of the **triangular pyramid.**

When a pyramid has a rectangle for its base, it is called a **rectangular pyramid.**

Study the figure of the cone.

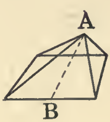
Explain how the development of the cone is made.

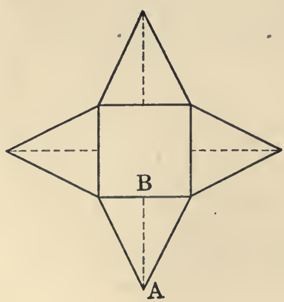


1. How does the curved line ABA compare with the length of the circular base of the cone?

The lateral surface of this cone is part of a circle. Its area is found by multiplying the length ABA by the radius AV, and dividing the product by 2. We shall call AV the *slant height* of the cone.

The lateral surface of a square pyramid is shown in this figure. We call AB the *slant height* of the pyramid.





2. Tell how to find the area of one of the lateral triangles. Of all four lateral triangles.

> **The lateral area of a pyramid** whose lateral triangles are all of the same size and shape, or of a cone on page 398, can be computed by the following rule:
>
> Multiply the perimeter or the length of the circle of the base by the slant height, and divide the product by 2.

Written Problems

373. 3. A pyramid has a square base, 6 in. on a side, and a slant height of 5 in. Find the lateral surface.

PROCESS AND EXPLANATION

The slant height AB is 5 in. The perimeter of the base is 4×6 in. $= 24$ in. Hence the lateral area is $\frac{5 \times 24}{2} = 60$ sq. in.

4. Find the area of the entire surface of the pyramid.

5. How many yards of canvas will be required to make a conical tent, 15 ft. in diameter and 20 ft. in slant height?

6. A mountain peak, formed by volcanic action, has the shape of a cone. The circumference of its

base is 9 mi., and the mountain side is $3\frac{3}{4}$ mi. How many square miles of forest and pasture are there on the side of this mountain?

7. The tower of a building is in the shape of a cone, $20\frac{1}{2}$ ft. across the base and 30.4 ft. in slant height. A workman is making estimates on the painting of the tower. On how many square feet must he figure?

is to contain 8 triangular pieces of canvas

5', h=12.

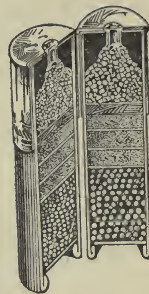
8. An 8-sided tent has its dimensions indicated in the figure. How many square yards of canvas are needed, if 5 % is added for seams?

9. A manhole, on the inside 10 ft. deep and 3.6 ft. in diameter, is built by a circular brick wall, 8″ thick. The foundation is also 8″ thick. How many cubic feet of water will the manhole hold?

10. How many cubic feet of masonry are used in building this manhole, including the foundation?

11. How many square feet of surface were plastered inside?

12. If the cartridge shown in the figure has an inside diameter of .4″ and length of 1.2″, how much space is available for the primer, the powder, the wads, and the shot?



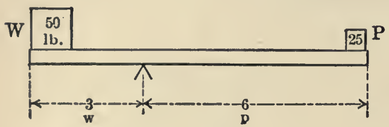
13. If the part of the cylinder holding the shot has an inside length of .42″, a diameter of .4″, and contains 860 shot of the kind shown in the figure, how much space is allowed each particle of shot?

14. If the powder weighs 150 grains while the shot weighs $3\frac{2}{3}$ times as much as the powder, and the rest of the cartridge weighs twice as much as the powder, what is the total weight of the cartridge?

PHYSICAL APPLICATIONS

The Lever

374. A familiar example of a lever is the seesaw board. It will balance when the weight on one end multiplied by its distance from the point of support equals the weight on the other end multiplied by its distance from the point of support.



The point of support is called the *fulcrum*. The figure shows a weight of 50 lb. balanced by a weight of 25 lb. Which weight is nearer the fulcrum? Observe that $50 \times 3 = 25 \times 6$.

If one weight is W and its distance from the fulcrum w, and the other weight is P, and its distance from the fulcrum is p, then we have the equation

$$\mathbf{Ww = Pp.}$$

If three of the four quantities in this equation are known, the fourth may be found. Thus,

1. A weight $W = 250$ lb. is 5″ from the fulcrum, how large a weight P, 15″ from the fulcrum will balance W?

PROCESS AND EXPLANATION

Here $W = 250$ lb.; $w = 5''$, $p = 15''$; P is to be found.

We have the equation $250 \times 5 = 15\,P$

or
$$15\,P = 1{,}250$$
$$P = \frac{1250}{15} = 83\tfrac{1}{3} \text{ lb.}$$

2. How far from the fulcrum must a weight of 20 lb. be, to balance another weight of 72 lb. which is 12″ from the fulcrum?

PROCESS AND ILLUSTRATION

Here $P = 20$ lb., p is unknown, $W = 72$ lb., $w = 12''$.

Hence,
$$20\,p = 72 \times 12$$
$$20\,p = 864$$
$$p = 43.2$$

As w is measured in inches, so p must be inches. Hence the required distance is 43.2″.

Written Exercise

375. Given the distances from the fulcrum, find the power P necessary to raise the weight W:

	w	W	p	P		w	W	p	P
1.	6″	45 lb.	8″	?	4.	2″	500 lb. ?	12″	?
2.	2′	75 lb.	3′	?	5.	3′	76 oz.	$1\frac{1}{2}'$	?
3.	5′	50 oz.	2′	?	6.	3″	800 lb.	1′	?

Written Exercise

376. Find the unknown part:

	w	W	p	P
1.	5″	1,000 lb.	25″	?
2.	?	2,000 lb.	14″	125 lb.
3.	3′	?	8′	72 lb.
4.	12″	3 oz.	?	10 oz.
5.	?	100 lb.	8″	125 lb.
6.	2″	1,700 lb.	2′	?

Written Problems

377. 1. A man with a crowbar 6′ long places one end of it under a stone, 1′ from the fulcrum. If the stone presses down upon the crowbar with a weight of 800 lb., how hard must he press down at the other end to move the stone?

SUGGESTION: [$w = 1'$, $p = 5'$].

2. To move a heavier stone he places the fulcrum 3″ from the lower end of the crowbar. If he weighs 150 lb., how heavy a stone can he move with the crowbar?

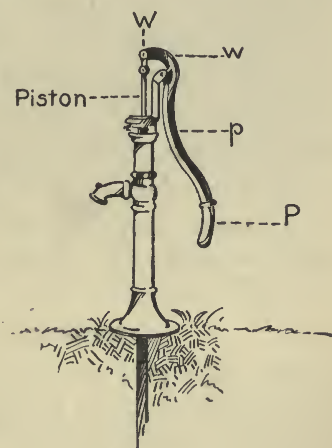
3. A doubletree is attached by a clevis to a plow. One horse is hitched to one end of the doubletree, 18″ from the clevis; another horse is hitched 20″ from the clevis at the other end of the doubletree. The two horses pull evenly. If the first pulls 155 lb., how much does the other pull?

4. If one horse can pull 160 lb., another horse 130 lb., and the first is hitched 16″ from the clevis of a doubletree fastened to a plow, how far from the clevis must the other horse be hitched?

5. A team is hitched to a doubletree 4′ long. At what point must the doubletree be fastened to a plow, so that one horse must pull twice as hard as the other?

Water Pumps and Windmills

378. 1. A water pump is operated on the principle of the lever. If a man exerts a pressure of 24 lb. on the pump handle, and if his hand is 4 times as far from the fulcrum as the rod connecting with the piston of the pump, what is the lifting force which raises the water?



2. If a pump with a 2-inch piston requires for its operation a pressure of 25 lb. at the handle, how much pressure is required at the handle, if the piston is 3-inch, all other conditions being the same?

Explanation. — The amount of water raised, in strokes of equal length, is proportional to the *areas* of the pistons. These areas are to each other as the *squares* of the diameters of the pistons — in this case, as 4 : 9. Hence in a 3-inch piston W is $\frac{9}{4}$ times greater than in a 2-inch piston; consequently, P must be $\frac{9}{4}$ times greater. Then, $\frac{9}{4}$ times 25 lb. is $56\frac{1}{4}$ lb., the pressure which must be exerted at the handle.

3. Two pumps are operated under the same conditions, except that one has a $2\frac{1}{2}$-inch piston, the other a 3-inch piston. If the first requires a pressure exerted at the handle of 20 lb., what pressure must be exerted at the handle of the other pump? Which pump delivers water in larger quantity? How many times larger?

4. The amount of water pumped by a windmill in a given time varies as the cube of the velocity of the wind. How many times more water can a windmill pump when the wind blows 15 miles an hour than when it blows 5 miles an hour? How many times more, when the wind blows 35 miles an hour instead of 5 miles an hour?

Resistance of Roads

379. The pull necessary to move a vehicle on a level road depends largely upon the size of the wheels and the condition of the road. The necessary pull is less for high wheels and, of course, less for a dry road made of small broken stone, than for one over soft ground. Results of experiments are contained in the following table, in which the weight of the vehicle is counted in as part of the load.

TABLE

Pull in Pounds, to move a Load of One Ton in a Vehicle

Kind of Road	Diameter of Wheels		
	50″	30″	26″
Level, dry, gravel road (macadam road)	57	61	70
Timothy and blue grass sod, dry, grass cut	132	145	179
Timothy and blue grass, wet	173	203	288
Plowed ground, dry and cloddy	252	303	374

Problems

380. 1. It is said that a horse walking at $2\frac{1}{2}$ miles an hour can exert a pull of 125 lb., working 8 hr. a day. At this rate, how heavy a load can a horse move on a level macadam road, if the wheels are 50″ in diameter? If they are 26″ in diameter?

2. How heavy a load can 2 horses move, walking $2\frac{1}{2}$ miles an hour, over plowed ground, the wheels being 30″ in diameter?

3. How many horses, each capable of pulling 150 lb., are necessary to move 5 tons over wet timothy and blue grass, the wheels being 50″ in diameter?

4. On hard and smooth roads, made of broken stone, with no dust or loose stones, the traction per ton was 121 lb. with $1\frac{1}{2}$ in. tires, and 98 lb.

with 6-inch tires. What was the difference in traction for loads of $3\frac{1}{3}$ ton each?

5. A country storekeeper pays 1¢ a mile for each 100 lb. of freight hauled over a gravel road from the railroad station. The storekeeper in another county pays 2¢ a mile for the same service over the same distance but over a poorer road. What is the difference on 65 tons of freight paid by the two storekeepers?

GENERAL REVIEW

Written Exercise

381. Copy and add quickly. Note the time required for each process.

Repeat the process from time to time until the column can be added accurately in minimum time.

1.	2.	3.
47,937	475,620	1,926,841
73,291	384,796	1,455,357
46,358	825,437	8,151,200
20,294	594,681	1,994,033
68,432	930,012	513,000
33,969	175,698	32,763
28,376	345,756	4,298,137
82,001	486,943	12,923,641
59,467	923,639	1,246,594
25,899	326,283	6,541,637

4. \$55,143.24 + \$13,210.78 = ——
30,166.48 + 19,443.85 = ——
12,186.75 + 7,867.56 = ——
73,492.80 + 47,140.09 = ——
15,743.55 + 4,946.96 = ——
5,552.46 + 10,351.18 = ——

382. MAKING CHANGE — ORAL

	Purchase Cost	Payment Amount	Change Received		Purchase Cost	Payment Amount	Change Received
1.	$ 0.62	$ 0.75	?	**13.**	$ 2.98	?	$ 7.02
2.	0.73	1.00	?	**14.**	10.61	$ 15.00	?
3.	1.91	5.00	?	**15.**	37.50	?	2.50
4.	14.95	20.00	?	**16.**	158.27	?	1.83
5.	3.68	5.00	?	**17.**	?	2.00	1.73
6.	5.27	10.00	?	**18.**	?	5.00	4.58
7.	0.39	2.00	?	**19.**	0.85	2.00	?
8.	17.65	50.00	?	**20.**	?	10.00	9.64
9.	28.30	40.00	?	**21.**	?	5.00	4.25
10.	63.20	100.00	?	**22.**	12.85	?	7.15
11.	12.75	?	$ 3.25	**23.**	24.17	40.00	?
12.	24.95	?	5.05	**24.**	?	500.00	63.78

Written Exercise

383. Copy, subtract, and check. Keep your time record:

1.	**2.**	**3.**	**4.**
87,365	90,035	273,958	876.14
42,987	29,969	190,069	397.96

5.	**6.**	**7.**
3,687.937	73,900.1006	842.09
1,945.942	8,391.492	7.304

8.	**9.**	**10.**
93,642.00113	1,937.01	$89\frac{7}{10}$
789.27014	8.27659	$46\frac{3}{7}$

11.	12.	13.	14.
$1\frac{5}{7}$	$843\frac{2}{3}$	$16\frac{2}{7}$	$\$1,896.66\frac{2}{3}$
$\frac{9}{16}$	$657\frac{8}{9}$	$9\frac{4}{5}$	$927.87\frac{1}{2}$

15.

4 mi.	189 yd.
3 mi.	246 yd.

16.

27 yd.	1 ft.	7 in.
18 yd.	2 ft.	8 in.

17.

2 da.	23 hr.
	36 hr.

18.

1 da.	18 hr.	48 min.
	23 hr.	57 min.

19.

4 min.	49 sec.
1 min.	55 sec.

20.

$143°$	$50'$	$22''$
$87°$	$58'$	$45''$

Written Exercise

384. Copy and find products; check answers:

1. 40×65
2. 27×22
3. 81×14
4. 346×94
5. 642×138
6. $8,704 \times 209$
7. $100,968 \times 9,806$
8. $47\frac{1}{5} \times 87\frac{2}{3}$
9. $.05\frac{1}{2} \times 1.76\frac{9}{16}$
10. $66\frac{2}{3} \times 987\frac{9}{10}$
11. $2.04\frac{1}{8} \times .793$
12. $14.62 \times 2,763,941$
13. $5.032\frac{1}{8} \times 1,000,469$
14. $121\frac{5}{16} \times 37\frac{2}{7}$
15. $144 \times \$27.50$
16. $48 \times \$236.47$
17. $.08 \times \$15,463.72$
18. $.005 \times \$463.00$
19. $2.019 \times \$4,500.55$
20. $\frac{2}{3} \times \frac{7}{8}$
21. $\frac{5}{7} \times \frac{4}{5}$
22. $3\frac{3}{8} \times 7\frac{5}{11}$.

Written Exercise

385. Reduce to denominations as indicated.

	Number	Denominations	
1.	3 mi. 175 yd.	yards	feet
2.	40 yd. 2 ft.		feet
3.	2 yr. 7 mo.	months	
4.	1 yr. 9 mo.	months	weeks
5.	3 mo. 2 wk.	weeks	days
6.	3 wk. 5 da.		days
7.	4 da. 17 hr.	hours	minutes
8.	3470 min.	hours	
9.	645 da.	months	
10.	5340 sec.	minutes	hours
11.	48 ounces	pounds	
12.	1645′	(°) degrees	
13.	380″	(′) minutes	
14.	6 gallons	quarts	
15.	9 quarts	pints	
16.	3 bushels	pecks	quarts
17.	2 pecks	quarts	
18.	7 pecks	bushels	

Written Exercise

386. Find the quotient. Check the answers:

1. 2,964 ÷ 872

2. $8,749.20 ÷ 89

3. 14.842 ÷ 2.27

4. 4 hr. 37 min. ÷ 9

5. 3 pk. 7qt. ÷ 6

6. 5 mi. 840 yd. ÷ 325

7. 12° 48″ ÷ 15

8. 640 A. ÷ 2

9. 160 A. 80 sq. rd. ÷ 6

10. 20 bbl. 18 gal. ÷ 4

11. 2240 lb. 14 oz. ÷ 6

12. £21 8*s*. ÷ 4

Written Exercise

387. Find the quotients:

1. $21\frac{2}{3} \div 3\frac{5}{8}$	3. $\frac{5}{16} \div \frac{1}{5}$	5. $\frac{2}{7} \div \frac{1}{9}$
2. $\frac{25}{32} \div \frac{3}{5}$	4. $16\frac{18}{30} \div \frac{19}{27}$	6. $10\frac{2}{5} \div 3$

Oral Exercise

388. Express in decimals, then in per cents:

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
1.	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{2}{7}$	$\frac{4}{5}$	$\frac{3}{20}$
2.	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{10}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{13}{20}$
3.	$\frac{5}{6}$	$\frac{1}{8}$	$\frac{1}{7}$	$\frac{1}{5}$	$\frac{5}{12}$	$\frac{27}{40}$
4.	$\frac{11}{12}$	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{2}{5}$	$\frac{7}{12}$	$\frac{8}{75}$
5.	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{7}{8}$	$\frac{3}{5}$	$\frac{1}{20}$	$\frac{9}{50}$

Oral Exercise

389. Express in per cents and then in common fractions:

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
1.	.75	.50	$.33\frac{1}{3}$	1.25	.08
2.	.12	$.66\frac{2}{3}$	$.12\frac{1}{2}$	$1.16\frac{2}{3}$	.03
3.	.25	$.37\frac{1}{2}$	$.62\frac{1}{2}$	$1.33\frac{1}{3}$	$.05\frac{1}{2}$
4.	.40	$.16\frac{2}{3}$	$.48\frac{1}{5}$	$4.66\frac{2}{3}$	$1.07\frac{2}{3}$
5.	.60	$.87\frac{1}{2}$	$.27\frac{2}{5}$	$8.37\frac{1}{2}$	$.006\frac{2}{3}$

Written Exercise

390. Find the percentage, using pencil only when necessary.

	Rate Per Cent	Base (a)	(b)	(c)	(d)	(e)
1.	15	25	40	85	100	500
2.	20	50	70	95	150	463
3.	$16\frac{2}{3}$	185	940	1,073	2,760	9,841
4.	$5\frac{1}{2}$	120	180	197	236	508
5.	$37\frac{1}{2}$	230	97	750	495	2,764
6.	2	1,485	3,926	1,075	14,365	4,397
7.	115	25	250	375	695	2,738
8.	200	370	865	2,073	15,495	13,987
9.	300	155	370	4,690	12,005	43,216
10.	450	275	450	4,000	10,000	100,000

Written Exercise

391. What is the commission?

	Rate Per Cent Commission	Gross Sales		Rate Per Cent Commission	Gross Sales
1.	3	$47,500	**9.**	$37\frac{1}{2}$	$ 26,843
2.	$12\frac{1}{2}$	2,685	**10.**	$2\frac{1}{2}$	136,450
3.	10	23,264	**11.**	$\frac{1}{4}$	2,375,675
4.	$33\frac{1}{3}$	1,580	**12.**	.4	942,760
5.	.4	2,765	**13.**	40	22,500
6.	18	24,320	**14.**	33	16,000
7.	4.8	15,500	**15.**	$12\frac{1}{2}$	25,000
8.	$16\frac{2}{3}$	4,575	**16.**	$\frac{1}{2}$	125,000

Written Exercise

392. Find the missing term.

	Percentage	Base	Rate %		Percentage	Base	Rate %
1.	?	$1,000.00	$4\frac{1}{2}$	**11.**	100	50	?
2.	$300.00	6,000.00	?	**12.**	400	100	?
3.	1.50	?	3	**13.**	1	2	?
4.	45.00	900.00	?	**14.**	$\frac{1}{2}$	1	?
5.	?	2,750.00	$2\frac{1}{2}$	**15.**	.50	2	?
6.	?	27.50	$2\frac{1}{2}$	**16.**	?	100	$\frac{1}{8}$
7.	18.00	?	12	**17.**	?	10	$\frac{1}{8}$
8.	5.00	?	.5	**18.**	?	1	$\frac{1}{8}$
9.	?	200.00	$\frac{1}{4}$	**19.**	.125	1	?
10.	20.00	20.00	?	**20.**	.125	2	?

393. What is the premium on each policy?

	Face	Rate		Face	Rate
1.	$ 2,000	$26\frac{1}{2}$% per $100	**6.**	$ 50,000	$\frac{1}{4}$ of 1%
2.	4,000	$17\frac{3}{4}$% per $100	**7.**	250,000	$\frac{2}{5}$ of 1%
3.	6,500	$2.60 per $100	**8.**	1,000	$18\frac{2}{3}$%
4.	5,000	$32\frac{1}{2}$% per $100	**9.**	2,500	$9\frac{1}{3}$%
5.	10,000	$\frac{2}{5}$%	**10.**	8,760	$12\frac{1}{2}$%

Written Exercise

394. Interest and amount for

1. 1 yr. 6 mo. of $1,500 @ $5\frac{1}{2}$%.
2. 3 yr. 4 mo. of $2,250 @ 6 %.
3. 2 yr. 9 mo. of $1,000 @ 7 %.
4. 4 yr. 5 mo. of $87 @ 8 %.

5. 2 yr. 7 mo. 10 da. of $650 @ 7 %.

6. 5 yr. 1 mo. 27 da. of $3,520 @ $4\frac{1}{2}$ %.

7. 1 yr. 11 mo. 8 da. of $5,450 @ 9 %.

Written Exercise

395. Find the cost of stocks bought as follows. Assume that the par value of one share of each stock is $160.

	Kind of Stock	Number of Shares	Market Price	Broker's Commission
1.	Alaska Gold Mines	500	$36\frac{1}{2}$	$\frac{1}{4}$%
2.	Amalgamated Copper	650	$67\frac{3}{8}$	$\frac{1}{8}$%
3.	C. M. & St. P.	1500	$92\frac{1}{4}$	$\frac{1}{8}$%
4.	Bethlehem Steel	500	$107\frac{5}{8}$	$\frac{1}{8}$%
5.	Pullman Co.	50	156	$\frac{1}{8}$%
6.	So. Pacific	450	$91\frac{1}{2}$	$\frac{1}{8}$%
7.	N.Y. N.H. & H.	500	$61\frac{1}{2}$	$\frac{1}{8}$%

GENERAL EXERCISES AND PROBLEMS

396. 1. Express in words 2,374,560,091,345.

2. Multiply 374,050 by 89.56 and divide the product by 5.763.

3. A man earns $45 a week and saves one half of that sum every fortnight. Find how many weeks it will take him to save as much as he spends in four weeks.

4. The area of a county is 250,783 acres. If the population is 105,200, find to the nearest integer the number of people per square mile.

5. Reduce $\frac{4785}{12892}$ to its lowest terms.

6. What number multiplied by 59.05 will give the same product as 25.7 multiplied by 398?

7. What decimal multiplied by 125 will give the sum of $\frac{3}{4}+\frac{7}{8}+\frac{2}{3}+\frac{7}{12}+\frac{4}{5}$?

8. What number is the same multiple of 6 that 148,995 is of 9?

9. A clock loses 2 minutes every week. At 10 P.M. on Sunday it is 45 seconds fast. When will it be exactly at the right time?

10. How many telegraph poles will be required for wires extending 100 mi., 366 yd., 2 ft., if the poles are 100 ft. apart?

11. A number diminished by $\frac{2}{3}$ of itself, when divided by 809, gives a quotient 327 and a remainder 456. What is the number?

12. A man owns $\frac{7}{25}$ of a ship valued at $125,000, and sells $\frac{5}{14}$ of his share. What fraction of the ship does he still own, and what is its value?

13. The rails of a railway are 10 yd. long at 32° F. and .000644 of their length longer at 92° F. To allow for this increase, find, to two decimal places of a foot, the least distance between two consecutive rails at 32° F.

14. A dealer buys 15 horses at $145 each; he sells 8 at $167 each, and the rest at $193 each; find his total gain, if his expenses were $167.25.

15. Find the area of the walls of a room 20 ft. long, 15 ft. broad, and 10 ft. high. How many double rolls of wall paper, 18 in. wide, will be needed to cover the walls if no allowance is made for openings?

16. How many bricks, $8\frac{1}{4}'' \times 4'' \times 2\frac{1}{4}''$ will be needed for a wall 25 yd. long, 15 ft. high, and 1 ft. 1 in. thick?

17. If 27 horses can be bought for $5,400, how many can be bought for $89,200?

18. A farmer bought 4 horses and 7 cows for $1,225. The prices of a horse and a cow being 7 : 3, how much did he give for each?

19. A railway track rises 1 ft. in 80 ft. the first 12 mi., then 1 in 120 for the next 10 mi., and 1 in 110 for the next 15 mi. Find the average rise.

20. In a class of 90 pupils there are 35 cases of absence in a week of 10 sessions. Find the average attendance.

21. Out of 169 persons committed for trial during one year, 130 were convicted. What per cent was acquitted?

22. A quantity of wheat which cost \$.98 per bushel was sold at a gain of 25 %. How many bushels were sold, if the total profit was \$122.50?

23. A merchant buys goods at 50 % and 10 % off, and sells them at 10 % and 20 % off the list price. What is his gain per cent?

24. An agent is offered a yearly salary of \$1,700 or a commission of 3 % on his sales. If his sales, on an average, are \$5,000 a month, which is the better offer to accept? What is the difference between the two offers for a year?

25. A town has an assessed valuation of \$7,500,000 and decides to levy a tax of \$45,000 for a school building. What must be the additional rate of taxation?

26. A married man must pay 1 % tax on his income in excess of \$4,000. At this rate, what is the tax on a yearly income of \$14,350?

27. What is the duty, at 30 % ad valorem, on an invoice of $ 750 worth of imported blankets?

28. A merchant failing in business has debts to the amount of $ 8,500; his property amounts to $ 6,550. What per cent of his debts can he pay? How much can he pay on a claim of $ 850?

29. Determine the speed in miles an hour of a train which goes a mile in 48 seconds, and the number of revolutions made in a minute by a wheel of the engine 8 feet 2 inches in diameter.

30. Express $\frac{22}{7}$ and $\frac{355}{113}$ as decimal fractions to five places; and prove that their difference is about $\frac{1}{25}$ per cent of either of them.

31. A workman finds that he has to spend 50 % of his income on food, 15 % on clothes, 12 % on rent, and he saves the remainder. If he emigrates to a country where wages are 50 % higher, where food costs $\frac{1}{2}$ as much, clothes 3 times as much, rent $2\frac{1}{2}$ times as much, is he better or worse off? The criterion is the length of time his savings would support him in the country where he is settled.

32. A man invests $\frac{1}{3}$ of his capital in 6 % stock at 150, and the rest in 4 % stock at 112. What is the average return per cent on his capital?

33. How many rectangular tiles, $4\frac{1}{2}$ in. by 6 in., will be required to cover a floor 18 ft. long by 13 ft. 6 in. broad? What will be the cost at 80 ¢ per dozen tiles?

34. When the duty on a commodity is reduced 40 %, the consumption is increased 60 %. By how much per cent is the revenue from that commodity increased or diminished?

35. Milk that was 4.3 % butter fat yielded 375 lb. of butter fat; how many pounds of milk were there? If 1 lb. of butter fat makes $1\frac{1}{6}$ lb. of butter, how many pounds of butter were obtained and what was its value at $27\frac{1}{2}$¢ a pound?

36. A farmer has 20 cows. For a month of 30 days he obtained the following milk sheet (the number of pounds of milk for each day):

1. 2,806	2. 3,095	3. 3,165	4. 2,980
5. 2,875	6. 2,659	7. 3,003	8. 2,900
9. 2,863	10. 2,860	11. 2,599	12. 3,000
13. 3,019	14. 2,916	15. 2,798	16. 2,789
17. 2,860	18. 2,895	19. 2,830	20. 3,000
21. 2,991	22. 2,899	23. 2,861	24. 2,895
25. 2,794	26. 2,805	27. 2,794	28. 2,783
29. 2,659	30. 2,787		

Four milk tests were made which indicated 3.93, 4.12, 3.91, 3.87 per cent of butter fat.

Price of butter fat per pound, 32.4¢; of skimmed milk, 21¢ per 100 lb. The skimmed milk sold was 80 % of the number of pounds of milk furnished.

The amount and cost of feed was as follows:

Corn, 3,600 lb., $.95 for 100 lb.

Bran, 3,000 lb., $1.05 for 100 lb.

Linseed meal, 1,100 lb., $1.65 for 100 lb.

Timothy hay, 6,200 lb., $11.75 a ton.

How many pounds of milk did he get for the month?

37. From the above data determine the average per cent of butter fat.

38. How many pounds of butter fat for the month?

39. Find the total value of the butter fat.

40. Find the total value of the skimmed milk.

41. Find the total monthly income from this milk industry.

42. Compute the expense of feeding the cows during the month.

43. What was the excess of the monthly income over the expense for feed?

44. Compute the average daily amount of milk per cow.

45. Compute the average daily income from the milk of one cow.

46. Compute the average daily cost of feeding a cow.

47. In an experiment station 15 calves were fed on skim milk for 150 days and they gained an average of 3.15 lb. a day. Another lot of 12 calves, fed during the same period of time on buttermilk, gained 2.15 lb. a day. How much more did a calf of the first lot gain, on an average, in 150 days?

48. Two boys are carrying to camp a deer, weighing 110 pounds, slung on a pole which is supported on the shoulders. The distance from A's shoulder to B's is 8 feet, and the deer hangs $2\frac{1}{2}$ feet from B. How much weight does each carry? Do not consider weight of pole.

49. A man and a boy have to carry a bundle of 200 pounds weight on a pole 10 feet long. At what distance from the boy must the load be placed to allow the boy to carry only $\frac{1}{4}$ of the total weight?

50. Federal Reserve Law requires that federal reserve banks hold in reserve 15 % of their total deposits. What was the excess reserve on March 4, 1915, in the following banks?

	City	Per Cent		City	Per Cent
(*a*)	Galveston	49.69	(*f*)	Kansas City	32.13
(*b*)	Pueblo	33.35	(*g*)	Cincinnati	30.60
(*c*)	Des Moines	36.82	(*h*)	Portland	30.42
(*d*)	Minneapolis	39.20	(*i*)	Dubuque	43.14
(*e*)	Seattle	34.83	(*j*)	Forth Worth	32.15

51. The 51 reserve cities held 26.52 per cent of reserves on March 4, 1915, against the 22.63 per cent on Dec. 31, 1914. What was the gain?

52. The U. S. Treasury statement.

WASHINGTON, April 10, 1915 — Receipts and expenditures:

	APRIL 9	MONTH, APRIL 1–9 inc.	FISCAL YEAR
Customs	$576,524	$4,469,972	$162,740,818
Int. rev., ord. . . .	900,193	7,144,529	251,135,778
Corp. and inc. tax .	19,166	575,166	12,250,385
Miscellaneous . . .	49,944	918,772	54,816,425
Total Receipts			
Expenditures . . .	2,304,549	21,061,587	577,163,059
Balance			surplus or deficit?

If the expenditures for Jan. Feb., March, and the first nine days of April are $577,163,059, what is the average per month? What would be the approximate yearly expenditure on this monthly basis? What will be the surplus, or deficit at end of fiscal year at same rate of income and expenditure as given above?

53. The Hoosac tunnel is 25,031 ft. long. About how many miles is this? How many kilometers?

54. The eastern portal of the tunnel has an elevation above sea level of 768 ft. From here the grade rises $.3\frac{1}{2}\%$ for 2,000 ft.; then rises $.5\%$ for 10,000 ft.; at about the middle the track is level for 250 ft.; beyond this it descends at a $.5\%$

grade to the western portal. What is the elevation of the western portal?

Change all the feet to meters; then work the problem anew.

55. The Mont Cenis tunnel through the Alps, connecting France and Italy, was constructed about the same time as the Hoosac tunnel. It is 12,849 meters long. How many kilometers is this? How many meters longer is this than the Hoosac tunnel?

397. TABLES OF ENGLISH MEASURES

Linear Measure

12 inches (in. or ″)	= 1 foot (ft. or ′)
3 feet	= 1 yard (yd.)
16.5 feet	= 1 rod (rd.)
320 rods	= 1 mile (mi.)
6086 feet	= 1 nautical mile (knot)
1 mi.	= 1760 yd. = 5280 ft.
6 ft.	= 1 fathom.

Square Measure

144	square inches (sq. in.)	= 1 square foot (sq. ft.)
9	square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$	square yards	= 1 square rod (sq. rd.)
160	square rods	= 1 acre (A.)
640	acres	= 1 square mile (sq. mi.)
36	square miles	= 1 township
1	acre	= 43,560 sq. ft.

One acre of land, in the form of a square, is nearly 209 ft. on a side.

Surveyor's Linear Measure

100	links	= 1 chain
80	chains	= 1 mile
66	feet	= 1 chain
7.92	inches	= 1 link

Surveyor's Square Measure

10 square chains = 1 acre

Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord of wood (cd.)

1 *board foot* is a board 1′ long, 1′ wide and 1″ thick.

Liquid Measure

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
231 cu. in.	= 1 gal.
1 cu. ft.	= $7\frac{1}{2}$ gal., nearly.

1 cu. ft. of water weighs $62\frac{1}{2}$ lb., nearly.

$32\frac{1}{2}$ gal. = 1 barrel (bbl.) in measuring cisterns.

4.27 cu. ft. = 1 bbl.

Dry Measure

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)
2150.4 cu. in.	= 1 bu., (approximately)

Counting

12 units = 1 dozen (doz.)

12 dozen = 1 gross (gr.)

Avoirdupois Weight

16 ounces (oz.)	= 1 pound (lb.)
100 pounds (lb.)	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)

1 **long** or **gross ton** = 2240 pounds.

1 av. lb. = 7000 gr.; 1 av. oz. = $437\frac{1}{2}$ gr.

Troy Weight

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

1 troy lb. = 5760 gr. = $\frac{5760}{7000}$ av. lb.

1 troy oz. = 480 gr. or about 1.1 av. oz.

Apothecaries' Weight

This is used to some extent in filling prescriptions. The **grain**, **ounce**, and **pound** are the same as in troy weight, but the ounce is divided differently.

20 grains (gr.)	= 1 scruple (sc. or ℈)
3 scruples	= 1 dram (dr. or ʒ)
8 drams	= 1 ounce (oz. or ℥)
12 ounces	= 1 pound (lb. or ℔)

Apothecaries' Liquid Measures

60 drops (gtt.) or minims (♏)	= 1 fluid dram (*f* ʒ)
8 fluid drams	= 1 fluid ounce (*f* ℥)
16 fluid ounces	= 1 pint (O.)
8 pints	= 1 gallon (Cong.)

Measure of Time

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 common year
366 days	= 1 leap year
10 years	= 1 decade
100 years	= 1 century

The *solar year* = 365 da. 5 hr. 48 min. and 46 sec.

Measures of Angles

60 seconds (″)	= 1 minute (′)
60 minutes	= 1 degree (°)
360 degrees	= 4 right angles or 1 circumference

90° of angle = 1 **right angle**; 90° of arc = 1 **quadrant.**

Paper Measure

24 sheets	= 1 quire
20 quires	= 1 ream
2 reams	= 1 bundle
5 bundles	= 1 bale

398. METRIC SYSTEM

Measures of Length

10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters (cm.)	= 1 decimeter (dm.)
10 decimeters (dm.)	= 1 meter (m.)
10 meters (m.)	= 1 dekameter (Dm.)
10 dekameters (Dm.)	= 1 hektometer (Hm.)
10 hektometers (Hm.)	= 1 kilometer (Km.)

Measures of Surface

100 sq. millimeters ($mm.^2$)	= 1 sq. centimeter ($cm.^2$)
100 sq. centimeters	= 1 sq. decimeter ($dcm.^2$)
100 sq. decimeters	= 1 sq. meter ($m.^2$)
100 sq. meters	= 1 sq. dekameter ($Dm.^2$)
100 sq. dekameters	= 1 sq. hektometer ($Hm.^2$)
100 sq. hektometers	= 1 sq. kilometer ($Km^2.$)

Measures of Land

1 sq. meter	= 1 centare (ca.)
100 centares	= 1 are
100 ares	= 1 hectare (Ha.)

Measures of Volume

1000 cubic millimeters ($mm.^3$)	= 1 cubic centimeter ($cm.^3$)
1000 cubic centimeters	= 1 cubic decimeter ($dm.^3$)
1000 cubic decimeters	= 1 cubic meter ($m.^3$) and so on.

Measures of Capacity

1 cubic decimeter	= 1 liter
10 milliliters (ml.)	= 1 centiliter (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 liter (l.)
10 liters	= 1 dekaliter (Dl.)
10 dekaliters	= 1 hektoliter (Hl.)
10 hektoliters	= 1 kiloliter (Kl.)

Measures of Weight (Mass)

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 gram (g.)
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hektogram (Hg.)
10 hektograms	= 1 kilogram (Kg.)
1000 kilograms	= 1 metric ton (T.)

399. TABLE OF EQUIVALENTS

Metric Measures to Common

1 cm.	= .4 in., about
1 m.	= 39.37″ = 1.094 yd.
1 Km.	= .621 mi.
1 m^2	= 1.196 sq. yd.
1 Ha.	= 2.471 A.
1 m^3	= 1.308 cu. yd.
1 l.	= .908 dry quart
1 l.	= 1.057 liquid quart
1 Hl.	= 2.834 bu.
1 Kg.	= 2.205 lb. av
1 T.	= 1.102 tons

Common Measures to Metric

1 in.	= 2.5 cm., about
1 yd.	= .9144 m.
1 mi.	= 1.609 Km.
1 sq. ft.	= 9.29 dm.^{2}
1 A.	= .4047 Ha.
1 cu. yd.	= .765 m.^{3}
1 liquid quart	= .946 l.
1 dry quart	= 1.101 l.
1 gallon	= 3.785 l.
1 bushel	= .352 Hl.
1 av. pound	= .455 Kg.
1 ton	= .907 T.

DEFINITIONS OF COMMON TERMS OF ARITHMETIC

(The numbers in parentheses refer to pages in the text.)

Above Par. At a premium (237).

Accident Insurance. Insurance against personal injury or loss of life by accident (214).

Ad Valorem Duty. A certain per cent of the cost of the goods (207).

Arithmetic. The science of numbers and the art of computation (1).

Bank Discount. Interest paid in advance on a note (193).

Below Par. At a discount (237).

Bill. A written statement of goods sold or of services rendered (261).

Bonds. A promise to pay a certain sum of money, signed by the officers of a corporation under its corporate seal (244).

Broker. An agent who buys or sells goods for a merchant without having them shipped to him (148).

Capital. The property invested in the business (236).

Centigrade Thermometer. A thermometer having the point indicating the freezing state of water at zero, and the distance between that and the point indicating the boiling state of water divided into a hundred divisions (384).

Certified Check. A check that is guaranteed by the bank upon which it is drawn (167).

Check. An order on a bank to pay a sum of money (166).

Commission Merchant. An agent who does the business of a merchant for a certain per cent of the amount of the sales, handling goods owned by other people (148).

Common Multiple. A number that is exactly divisible by each of two or more numbers is a *common multiple* of those numbers (55).

Common Stock. Stock that may share in the net earnings after the dividends on the preferred stock have been paid (237).

Compound Interest. Interest on both the principal and the overdue interest added to the principal (277).

Compound Number. A number which expresses one or more units of different names or denominations of the same measure (16).

Consignor. The firm or person who sends the merchandise to be sold (148).

Corporate Bonds. Bonds issued by corporations (245).

Corporation. An association of individuals recognized under the law as a single person (236).

Coupon Bonds. Bonds with interest certificates attached, which may be detached at specified dates and presented for payment to bearer at a bank or wherever the bonds are issued (245).

Denominator. The number in a fraction below the line that shows into how many equal parts a unit is divided (59).

Dividend. The sum paid on each share of stock from the profits of the business (237).

Duties. Taxes levied on goods imported from other countries (203).

Endowment Policy. A guarantee to pay the insured a stated sum of money after a specified number of years, or, in case of prior death, to the beneficiary named in the policy, in consideration of premiums paid for a certain number of years (220).

Equation. An equality of numbers (91).

Excise Duties. Taxes levied on tobacco, liquors, etc., produced in the United States (203).

Face of the Policy. The amount of indemnity or protection (214).

Face Value. The original price (237).

Factor. An integer that is an exact divisor of that number (54).

Fahrenheit Thermometer. A thermometer so graduated that the freezing of water is at 32 degrees above the zero of its scale, and the boiling point at 212 degrees above (284).

Fire Insurance. An insurance against loss or damage of property by fire (214).

Fraction. One or more of the equal parts of a unit (59).

Government Bonds. Bonds issued by the United States, by states, by cities, or counties (245).

Improper Fraction. A fraction whose numerator is equal to or greater than its denominator (59).

Income Tax. A tax levied by the United States government on incomes above a certain amount (211).

Interest. Money paid for the use of money (163).

Least Common Denominator. The common denominator that is the least that can be found is called the *least common denominator* (56).

Least Common Multiple. The least number that is exactly divisible by each of two or more numbers (55).

Liability Insurance. Insurance against loss by the insolvency of debtors (214).

Limited Life Policy. A guarantee to pay a specified sum at the death of the insured in consideration of premiums paid during a limited number of years by the insured (220).

Marine Insurance. Insurance against loss of property at sea (214).

Mortgage. An instrument by which a person may convey real or personal property or some interest in such property as a security for a debt or a loan of money (191).

Mortgagee. The person or firm who lends the money or assumes the debt on the property (191).

Mortgagor. The person who gives the mortgage (191).

Multiple. A number which is exactly divisible by another number is a *multiple* of that number (55).

Notation. The process of writing numbers by means of characters (5).

Numerator. The number in the fraction above the line that shows how many parts have been taken (5, 59).

Ordinary Life Policy. A guarantee to pay a specified sum at the death of the insured to the beneficiary named in the policy, in consideration of premiums paid during the life of the insured (220).

Parallelogram. A four-sided figure having the opposite sides parallel (386).

Partnership. A formal agreement made between two or more persons for the purpose of conducting a business in common and of sharing in its profits and losses (236).

Par Value. The face value (237).

Payee. The person to whom a check, draft, or note is payable (185).

Percentage. The result obtained by finding a given per cent of the base (129).

Policy. A written agreement or contract with the insurance company (214).

Poll Tax. A tax levied on each male inhabitant who has attained his majority (203).

Preferred Stock. Stock that must pay a certain proportion of the profits before any other dividends can be paid (237).

Premium. The consideration or price paid for the insurance (214).

Prime Number. An integer that has no factors except itself and 1 (54).

Principal. The sum of money on which interest is paid (163).

Proceeds. The sum left over after subtracting the bank discount from the face of the note (194).

Promissory Note. A written promise to pay on demand or at a specified time a specified sum (184).

Proper Fraction. A fraction whose numerator is less than its denominator (59).

Property Tax. A tax levied on real estate or personal property (203).

Proportion. An expression of the relation of two ratios (109).

Rate of Interest. The per cent, the number of hundredths, of the principal paid for its use for one year (163).

Rate Per Cent. The number of hundredths taken (129).

Ratio. The relation which one quantity has to another (107).

Rectangle. A parallelogram whose angles are equal to each other (386).

Registered Bonds. Bonds that are recorded with the name of the owner in the books of the company that issues them. The owner cannot transfer these bonds unless he notifies the company and has the name of party receiving them recorded in the company's books (245).

Regular Hexagon. A polygon having six equal sides and six equal angles (385).

Rhomboid. A parallelogram whose angles are not all equal (386).

Similar Fractions. Fractions having the same denominator (59).

Simple Number. A number which expresses one or more units of the same name or denomination (16).

Specific Duty. Duty charged on the weight or quantity of goods imported without regard to their cost (207).

Stockholder. A number of a corporation owning one or more shares of stock (237).

Tariff. A schedule of duties on imported goods, as in this country, fixed by the United States Congress (207).

Tax. A sum of money levied for the support of the government and for other public purposes (203).

Term Policy. A guarantee to pay a stated sum in case of death within the term of insurance, in consideration of premiums paid for a certain number of years (220).

Trapezoid. A four-sided figure having only two sides parallel (387).

Unit Fraction. A fraction whose numerator is 1 (59).

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