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INTERNAL REPORT

ELASTIC DISTORTION CORRECTION FOR THE BURNETT APPARATUS

BY

John E. Miller

BRANCH Fundamental Research

PROJECT NO. 4330

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ELASTIC DISTORTION CORRECTION FOR THE BURNETT APPARATUS

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ABSTRACT

General equations for correcting elastic distortion are given. These equations are used to compute pressure expansion coefficients and jacket pressure ratios for the Helium Research Center Burnett apparatus. A method for computing distortion at pressures above the elastic limit is described.

INTRODUCTION

In the isothermal Burnett method, compressibility factors are determined from pressure measurements only. Therefore, anything that affects the accuracy of the pressure measurements should be corrected, when possible. One of the corrections is that for elastic distortion due to internal pressure. The pressure expansion coefficient can be computed for a cylinder if the inner and outer radii, Poisson's ratio, and Young's Modulus are known. Several equations for computing expansion

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[1] Research Chemist, Helium Research Center, Helium Activity, Bureau of Mines, Amarillo, Texas.

Work on manuscript completed May 1965.

coefficients[2/] (2), (3), (4), and (5), are available, but the one

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2/ Underlined numbers in parentheses are bibliography references.

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given by Love (4) is used for all calculations in this report.

$$\frac{\Delta V}{V_0} = \left[E(b^2 - a^2) \right]^{-1} \left[3(1 - 2\mu)(a^2 P_g - b^2 P_j) + 2(1 + \mu) b^2 (P_g - P_j) \right] \ldots (1)$$

See page 13 for definition of terms in equation (1).

An alternate method is to eliminate elastic distortion with pressure jackets. Although the same equation (1) is used to compute the ratio of P_j/P_g to make $\Delta V = 0$, the computed jacket pressure ratio is independent of Young's Modulus and is constant for the temperature range of 0 to 150° F. Values of $\Delta V/V_0$ in (1) will vary slightly with temperature because Young's Modulus is a function of temperature.

GENERAL EQUATIONS FOR THE BURNETT METHOD

The first general relationship is that for pressure ratios:

$$\frac{P_r}{P_{r+1}} = \frac{(V_1 + V_2)_{P_{r+1}}}{(V_1)_{P_r}} \frac{Z_r}{Z_{r+1}} , \quad r = \text{No. of expansions that have been made} \qquad (2)$$

Equation (2) can be rewritten as follows:

$$\frac{P_r}{P_{r+1}} = \frac{(V_1 + V_2)_{P=0}}{(V_1)_{P=0}} \frac{(1 + \alpha P_{r+1})}{(1 + \beta P_r)} \frac{Z_r}{Z_{r+1}} \qquad (3)$$

The volume ratio, N_0, is defined as:

$$N_0 = \frac{(V_1 + V_2)_{P=0}}{(V_1)_{P=0}} \qquad (4)$$

Substituting (4) into (3) gives

$$\frac{P_r}{P_{r+1}} = N_0 \frac{(1 + \alpha P_{r+1})}{(1 + \beta P_r)} \frac{Z_r}{Z_{r+1}} \qquad (5)$$

There are three possible values for α and β:

$$\alpha = \beta = 0 \qquad (6)$$

$$\alpha = \beta \neq 0 \qquad (7)$$

$$\alpha \neq \beta \neq 0 \qquad (8)$$

The relationship given by (6) can be realized by using pressure jackets; (7) will apply when V_1, V_2, and the associated values and tubing have the same expansion coefficient; (8) is the usual case when pressure jackets are not used.

Equation (5) implies that extrapolation of pressure ratios to $P = 0$ will give N_0 as the intercept, with or without a distortion correction.

$$\lim_{P_r \to 0} \frac{P_r}{P_{r+1}} = N_0 \text{ because } \frac{Z_r}{Z_{r+1}} \to 1 \text{ and } \frac{(1 + \alpha P_{r+1})}{(1 + \beta P_r)} \to 1 \qquad (9)$$

Normally, it would be preferable to make the distortion correction before obtaining N_0.

$$\lim_{P_r \to 0} \frac{P_r (1 + \beta P_r)}{P_{r+1}(1 + \alpha P_r)} = N_0 \qquad (10)$$

After N_0 has been determined, the compressibility factors at the measured pressures can be computed. The exact procedure depends on which of the relationships (6), (7), or (8) is applicable.

When (6) is the case, it can be shown that

$$Z_r = \frac{N_0^{\,r} P_r}{\lim\limits_{P_r \to 0} N_0^{\,r} P_r} = \frac{N_0^{\,r} P_r}{\dfrac{P_0}{Z_0}} \qquad (11)$$

When (7) applies, it can be shown that

$$\lim_{P_r \to 0} N_0^{\,r} P_r (1 + \alpha P_r) = \frac{P_0}{Z_0} (1 + \alpha P_0) \qquad (12)$$

$$Z_r = \frac{N_0^{\,r} P_r (1 + \alpha P_r)}{\lim\limits_{P_r \to 0} N_0^{\,r} P_r (1 + \alpha P_r)} \qquad (13)$$

The most complicated case is when equation (8) is applicable.

$$\frac{P_0}{P_1} = \frac{N_0 (1 + \alpha P_1) Z_0}{(1 + \beta P_0) Z_1} \qquad (14)$$

$$\frac{P_0}{P_2} = N_0^{\,2} \frac{(1 + \alpha P_1)(1 + \alpha P_2) Z_0}{(1 + \beta P_0)(1 + \beta P_1) Z_2} \qquad (15)$$

$$\frac{P_0}{P_3} = N_0^{\,3} \frac{(1 + \alpha P_1)(1 + \alpha P_2)(1 + \alpha P_3) Z_0}{(1 + \beta P_0)(1 + \beta P_1)(1 + \beta P_2) Z_3} \qquad (16)$$

for $r \geq 2$, the general equation is given with sufficient accuracy by:

$$\frac{P_0}{P_r} = \frac{N_0^{\,r} \left[1 + \alpha \sum_1^r P_r \right] Z_0}{(1 + \beta P_0) \left[1 + \beta \sum_1^{r-1} P_r \right]} \qquad (17)$$

After equation (18), equations (14), and (17) can be rewritten:

$$N_0^{\,0}\ P_0\ (1 + \beta P_0) = \frac{P_0}{Z_0}\ (1 + \beta P_0)\ Z_0 \tag{18}$$

$$N_0^{\,1}\ P_1\ (1 + \alpha P_1) = \frac{P_0}{Z_0}\ (1 + \beta P_0)\ Z_1 \tag{19}$$

$$N_0^{\,r}\ P_r \left[1 + (\alpha - \beta)\sum_1^{r-1} P_r\right]\left[1 + \alpha P_r\right] = \frac{P_0}{Z_0}\ (1 + \beta P_0) Z_r \tag{20}$$

Plotting the quantities given by the left side of equation (18), (19), and (20) versus P_r and extrapolating to $P_r = 0$ will give as the intercept:

$$\frac{P_0}{Z_0} = (1 + \beta P_0) \tag{21}$$

Dividing the left sides of equations (18), (19), and (20), by the intercept will give Z_r for the respective P_r's.

$$Z_0 \text{ at } P_0 = \frac{N_0^{\,0}\ P_0\ (1 + \beta P_0)}{\frac{P_0}{Z_0}\ (1 + \beta P_0)} \tag{22}$$

$$Z_1 \text{ at } P_1 = \frac{N_0\ P_1\ (1 + \alpha P_1)}{\frac{P_0}{Z_0}\ (1 + \beta P_0)} \tag{23}$$

$$Z_r \text{ at } P_r \text{ for } r \geqq 2 = \frac{N_0^{\,r}\ P_r \left[1 + \alpha - \beta)\sum_1^{r-1} P_r\right]\left[1 + \alpha P_r\right]}{\frac{P_0}{Z_0}\ (1 + \beta P_0)} \tag{24}$$

Replacing the products of expansion terms as in equations (15) and (16) by a summation as in equations (17) and (24) will cause an error of about 1 part per million. Also, if $\alpha = \beta$, then equation (24) reduces to equation (13).

EXAMPLE DISTORTION CALCULATION FOR $\alpha = \beta \neq 0$

The following example calculations is given to illustrate use of equation (13). Assumptions are:

$$N_0 = 2$$

$$Z = 1 + BP$$

$$B = 5 \times 10^{-5} \text{ psi}^{-1}$$

$$\alpha = 1.5 \times 10^{-7} \text{ psi}^{-1}$$

$$P_0 = 100{,}000 \text{ psia}$$

The calculations are summarized in table 1. The pressures under column P_c are those that would be observed if there had not been any distortion. Pressures under column P_m would be observed if $\alpha = 1.5 \times 10^{-7}$ psia^{-1}; they are computed from equation (25).

$$\alpha Z_c P_m^2 + P_m - P_c = 0 \qquad (25)$$

The low pressure values of $P_m (1 + \alpha P_m)$ are identical to the corresponding values of P_c, so extrapolation of the appropriate functions of Pm $(1 + \alpha P_m)$ will give:

$N_0 = 2$ from (10)

$B = 5 \times 10^{-5}$ psia^{-1} from (12)

$P_0/Z_0 = 16{,}666.6667$ psia, from (12)

TABLE 1. - Pressures computed for a Burnett isotherm when $\alpha = \beta \neq 0$

r 1/	P_c	Z_c	P_m	$(1 + \alpha P_m)$	$P_m(1 + \alpha P_m)$	N_0^r	$N_o^r\,[P_m(1 + \alpha P_m)]$	$1 + BP_m$
0	100,000.00000	6.000000000	92,327.98832	1.013849198	93,606.65691	1	5.616399	5.616399
1	14,285.71428	1.714285714	14,233.61819	1.002135043	14,264.00758	2	1.711681	1.711681
2	5,263.15789	1.263157894	5,257.91975	1.000788688	5,262.06661	4	1.262896	1.262896
3	2,325.58139	1.116279070	2,324.67651	1.000348701	2,325.48713	8	1.116234	1.116234
4	1,098.90110	1.054945055	1,098.71008	1.000164806	1,098.89115	16	1.054936	1.054936
5	534.75936	1.026737968	534.71533	1.000080207	534.75822	32	1.026736	1.026736
6	263.85224	1.013192612	263,84166	1.000039576	263.85210	64	1.013192	1.013192
7	131.06160	1.006553080	131.05901	1.000019659	131.06159	128	1.006553	1.006553
8	65.31679	1.003265840	65.31615	1.000009797	65.31679	256	1.003266	1.003266
9	32.60515	1.001630258	32.60499	1.000004891	32.60515	512	1.001630	1.001630

1/ r = number of expansions that have been made

The compressibility factors at P_m are then given by equation (13), as can be seen by computing the Z's from the known value of B.

EXAMPLE DISTORTION CALCULATION WHEN $\alpha \neq \beta \neq 0$

Pressures in the following example have been taken from Canfield (1); they are for helium at 0° C and have been converted to psia. According to Canfield, $\alpha = 1.402 \times 10^{-7}$ psia^{-1}, $\beta = 1.415 \times 10^{-7}$ psia^{-1}, and $N_0 = 1.49952$. Canfield makes the distortion correction to N_0, which should be equivalent to using equations 18 - 24.

The error introduced in equation 20 by replacing the product of the distortion terms by a summation can be evaluated from the data in table 2. If r = 11:

$$\left[1 + (\alpha - \beta)\sum_{r=1}^{r=10} P_r\right]\left[1 + \alpha P_{11}\right] = (0.9999830)(1.000010) = 0.9999930$$

while
$$\frac{(1 + \alpha P_1)(1 + \alpha P_2)\ldots(1 + \alpha P_{11})}{(1 - \beta P_1)(1 + \beta P_2)\ldots(1 + \beta P_{10})} = \frac{1.00184433}{1.00185233} = 0.9999920$$

The difference in the two expressions is about 1 part per million; the compressibility factor at P_{11} would be in error by the same amount.

DISTORTION CORRECTIONS FOR THE HELIUM RESEARCH CENTER BURNETT APPARATUS

The distortion coefficients and jacket pressure ratios have been computed from Love's equation (4):

$$\frac{\Delta V}{V_0} = \left[E(b^2 - a^2)\right]^{-1}\left[3(1 - 2\mu)(a^2 P_g - b^2 P_j) + 2(1 + \mu)b^2 (P_g - P_j)\right] \quad (1)$$

TABLE 2. - Example calculation of distortion correction when $\alpha \neq \beta \neq 0$

r	P_r	N_0^r	$(1 + \beta P_r)$	$(1 + \alpha P_r)$	$\sum_{r=1}^{r-1} P_r$	$\left[1 + (\alpha - \beta)\sum_{r=1}^{r-1} P_r)\right]$
0	7646.755	1.000000	1.001082	--	--	--
1	4705.042	1.49952	1.000666	1.000660	--	--
2	2978.306	2.248560	1.000421	1.000418	4705.0	0.9999939
3	1919.606	3.371761	1.000272	1.000269	7683.3	.9999900
4	1251.674	5.056023	1.000177	1.000175	9602.9	.9999875
5	822.387	7.581608	1.000116	1.000115	10854.6	.9999859
6	543.044	11.368772	1.000077	1.000076	11677.0	.9999848
7	359.760	17.047702	1.000051	1.000050	12220.1	.9999841
8	238.861	25.563370	1.000034	1.000033	12579.8	.9999836
9	158.834	38.332784	1.000022	1.000022	12818.7	.9999833
10	105.722	57.480776	1.000015	1.000015	12977.5	.9999831
11	70.420	86.193574	1.000010	1.000010	13083.3	.9999830

TABLE 2. - Example calculation of distortion correction when $\alpha \neq \beta \neq 0$ (Con.)

r	$Y^{1/}$	$N_0^r\ Y^{2/}$	$Z_r^{\ 3/}$	Z_r given[4/] by Canfield (1)
0	7655.029	7655.029	1.26454	1.26434
1	4708.147	7059.961	1.16624	1.16605
2	2979.533	6699.659	1.10672	1.10654
3	1920.103	6474.128	1.06947	1.06929
4	1251.877	6329.519	1.04558	1.04541
5	822.470	6235.645	1.03007	1.02991
6	543.077	6174.119	1.01991	1.01974
7	359.772	6133.286	1.01317	1.01300
8	238.865	6106.194	1.00869	1.00852
9	158.835	6088.588	1.00578	1.00561
10	105.722	6076.983	1.00386	1.00370
11	70.420	6069.751	1.00267	1.00250

1/ For r = 0, $Y = P_0\ (1 + \beta P_0)$; r = 1, $Y = P_1\ (1 + \alpha P_1)$; for $r \geq 2$,

$$Y = P_r\ [1 + (\alpha - \beta)\sum_{r=1}^{r-1} P_r]\ [1 + \alpha P_r].$$

2. The average straight line through Y's for r = 5 to 11 gives 6053.5904 as the intercept at $P_r = 0$. This intercept is the quantity given by (21);

$$\frac{P_0\ (1 + \beta P_0)}{Z_0}$$

3. $Z_r = Y/6053.5904$.

4. Canfield's Z's are lower by a constant percent (about 0.016%) which indicates that he probably used a slightly different value of P_0/Z_0.

where $\triangle V$ = change in volume due to pressure

V_0 = internal volume of a closed cylinder at zero pressure

E = Young's Modulus

a = inner radius

b = outer radius

μ = Poisson's ratio

P_g = gas pressure or internal pressure

P_j = jacket pressure or external pressure

A cross-section drawing of one of the jacketed pressure cells is shown in figure 1. The two jacketed-cells are constructed of free-machining 300 stainless steel; both are identical in size and construction. For the gas cylinder part of the cells (V_1 and V_2):

a = 0.500 inch

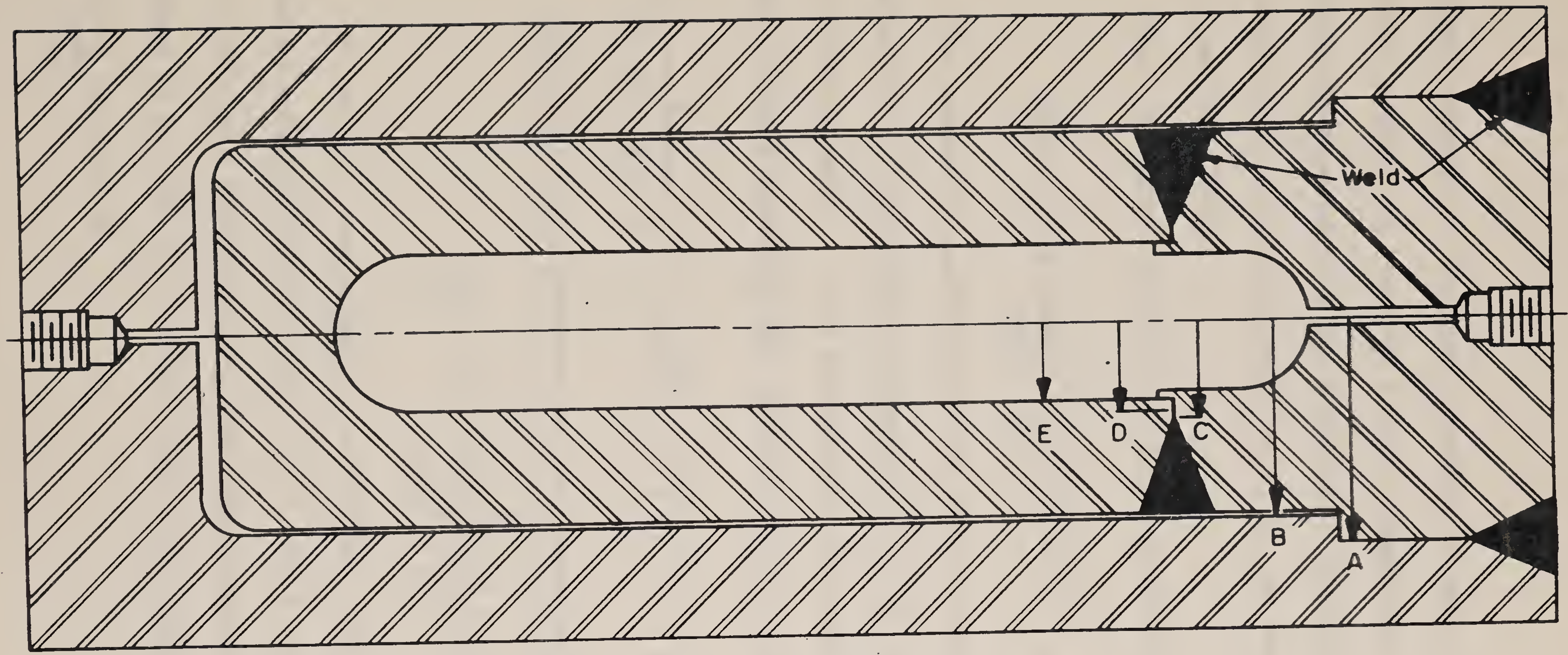
b = 1.250 inch

E = 29×10^6 psi

μ = 0.305

The internal volume of each gas cylinder is about 76 cc. The total volume that is not jacketed is estimated to be 2.8 cc; when gas is in V_1 only, the unjacketed volume is estimated to be 1.4 cc. It is assumed that this volume has the same expansion coefficient as 1/8 inch tube (a = .028" and b = 0.0625", E and μ same as for the jacketed cells).

As was previously mentioned, the distortion can be eliminated by using pressure jackets; equation (1) is used to compute the ratio of



Dimensions at Point	Inside Radius	Outside Radius
A	1.4375"	2.000"
B	1.2656"	2.000"
C	0.625"	1.250"
D	0.5625"	1.250"
E	0.500"	1.250"

FIGURE 1. - Cross Section of one of the Jacketed Bombs

external pressure to internal pressure to make $\Delta V = 0$. These jacket pressure ratios are constant for the temperature range of 0 to 150° F and are independent of Young's Modulus. Young's Modulus is a function of temperature; for stainless steels, it can change 5 percent between 32 °F and 150 °F (6).

When the jacket pressure is zero, equation (1) reduces to

$$\frac{\Delta V}{V_0} = \left[\frac{3(1 - 2\mu)\, a^2 + 2(1 + \mu)\, b^2}{E(b^2 - a^2)}\right] P_g = \alpha P_g \qquad (26)$$

The expansion coefficients and jacket pressure ratios for possible combinations of the three volumes (V_1, V_2, and the unjacketed volume) are given in table 3. For comparison, the calculations were repeated using the Lamé equation (8), which is for long open-end cylinders.

$$\frac{\Delta V}{V_0} = \frac{2}{E\,(b^2 - a^2)} \left[(1 - \mu)(a^2 P_g - b^2 P_j) + (1 + \mu) b^2\,(P_g - P_j)\right] \qquad (27)$$

When the jacket pressure is zero, equation (27) reduces to

$$\frac{\Delta V}{V_0} = \frac{2\,[(1 - \mu)a^2 + (1 + \mu)b^2]\, P_g}{E(b^2 - a^2)} \qquad (28)$$

In table 3, the expansion coefficient for V_1 and/or V_2 was computed by substituting $a = 0.5''$, $b = 1.25''$, $E = 29 \times 10^6$ psi, and $\mu = 0.305$ into equation (26). The expansion coefficient for V_3 is assumed to be that for 1/8 inch tube. The expansion coefficient for $V_1 + \frac{V_3}{2}$ is taken as

TABLE 3. - Summary of distortion coefficients and jacket pressure ratios

Section of Apparatus	Estimated Int. vol., cc	Love's eq. (1) $(\frac{\Delta V}{V_0})_{70° F}$ 1/	Love's eq. (1) $(P_j/P_g)_{\Delta V=0}$ 2/	Lamé eq. (27) $(\frac{\Delta V}{V_0})_{70° F}$ 3/	Lamé eq. (27) $(P_j/P_g)_{\Delta V=0}$ 4/
V_1	76.0	$1.148 \times 10^{-7}P$	0.74000	$1.163 \times 10^{-7}P$	0.70810
$V_1 + V_2$	152.0	$1.148 \times 10^{-7}P$	.74000	$1.163 \times 10^{-7}P$	.70810
V_3 5/	2.8	$1.227 \times 10^{-7}P$	--	$1.246 \times 10^{-7}P$	--
$V_1 + \frac{V_3}{2}$	77.4	$1.149 \times 10^{-7}P$	.75457	$1.164 \times 10^{-7}P$	.72208
$V_1 + V_2 + V_3$	154.8	$1.149 \times 10^{-7}P$	.75457	$1.164 \times 10^{-7}P$	.72208

1/ Computed from equation (26), P is in psi.

2/ Computed from equation (1).

3/ Computed from equation (28). P is in psi.

4/ Computed from equation (27).

5/ V_3 is the total unjacketed volume; about 1/2 of V_3 is associated with V_1.

$$\frac{76.0}{77.4}(1.148 \times 10^{-7}P) + \frac{1.4}{77.4}(1.227 \times 10^{-7} P) = 1.149 \times 10^{-7} P \quad (29)$$

while that for $V_1 + V_2 + V_3$ is

$$\frac{152}{154.8}(1.148 \times 10^{-7} P) + \frac{2.8}{154.8}(1.227 \times 10^{-7} P) = 1.149 \times 10^{-7} P \quad (30)$$

The temperature dependence of Young's Modulus for annealed 303 stainless steel is not readily available at this time, but it is possible that there could be about 5 percent variation in α between 0° and 150 °F.

The jacket pressure ratios are computed by setting $\Delta V = 0$ in equation (1). If the unjacketed volume (V_3) was zero, the proper ratio of P_j/P_g would be 0.74000. Setting the jacket pressure ratio at 0.75457 will keep the total volume constant by making the decrease in volume of $V_1 + V_2$ equal to the increase in the unjacketed volume.

The jacket pressure is measured with a Heise gage: 0 to 10,000 psi with 10 psi divisions. It is possible to estimate to within 5 psi. Using equation (1), it can be shown that changing the jacket pressure by 5 psi will cause the gas pressure to change by about 1 part per million. This is less than the sensitivity of the Ruska piston gage (5 to 50 parts per million).

BORE DEFORMATION AT PRESSURES ABOVE THE LIMIT OF PERFECT ELASTICITY

A recent paper (7) by N. L. Svennson gives a theory of stress-strain relationships applicable to large plastic strains in a cylinder. The author (Svennson) claims the theory is exact if the true stress-logarithmic

strain relationship is known exactly for the cylinder material. This theory may be used to estimate permanent bore deformations due to internal pressures greater than P_y, where P_y is the pressure required to initiate plastic deformation at the bore of a cylinder. Several recent papers (2), (7) give:

$$P_y = \frac{\sigma_{y.01\%}}{\sqrt{3}} \left(\frac{b^2 - a^2}{b^2}\right) \tag{31}$$

where $\sigma_{y.01\%}$ is the yield strength at 0.01% offset in a simple tensile test.

The initial portion of a stress-strain curve for a particular metal can be constructed if one knows Young's Modulus and yield strength at two offsets. The true stress-log strain curve may be computed from the relationships:

$$\sigma = \sigma' \ (1 + \epsilon') \tag{32}$$

$$\epsilon = \ln \ (1 + \epsilon') \tag{33}$$

where:

σ is true stress

σ' is engineering stress = load/original area

ϵ is log (or natural) strain

ϵ' is engineering strain = $\Delta L/L_o$

For small values of ϵ', there is little difference between the engineering stress - engineering strain curve and the true stress-log strain curve.

The nonlinear part of the stress-strain curve can be represented by an equation of the form:

$$\sigma = c\epsilon^n \tag{34}$$

Then if σ at two offsets is known, the constants c and n can be found. The 0.2% yield strength will usually be given; it is desirable to have the other offset at some smaller value - 0.01% or 0.002%. The stress-strain curve up to the 0.2% yield stress is given by:

$$\sigma = f(\epsilon) = E\epsilon \quad \text{for } 0 \leqq \epsilon < \epsilon \text{ at the proportional limit}$$

$$= c\epsilon^n \quad \text{for } \epsilon \text{ at prop. limit} \leqq \epsilon \leqq \epsilon \text{ at 0.2\% offset.} \tag{35}$$

Svensson gives the strain equations for a cylinder:

$$X_i = 1 + \epsilon_i' = 1 + \frac{\mu_i}{R_i} \tag{36}$$

$$\text{bore strain} = \epsilon_a = \frac{2}{\sqrt{3}} \ln X_a \tag{37}$$

$$\text{surface strain} = \epsilon_b = \frac{1}{\sqrt{3}} \ln \left[1 - \left(\frac{1 - e^{\sqrt{3}\,\epsilon_a}}{K^2} \right) \right] \tag{38}$$

where K = b/a

Then if a bore strain is assumed, the corresponding ϵ_b can be computed. In equation (36), μ_i is displacement from the original radius, R_i.

Svensson gives the pressure required to cause a given bore strain as:

$$P = \int_{\epsilon_a}^{\epsilon_b} \frac{\sigma \, d\epsilon}{1 - e^{\sqrt{3}\,\epsilon}}, \text{ and for } \sigma = c\epsilon^n: \tag{39}$$

$$P = \frac{c}{\sqrt{3}} \left[\epsilon^n \left\{ -\frac{1}{n} + \frac{\sqrt{3}\,\epsilon}{2(n+1)} - \sum_{m=1}^{\infty} \frac{(-1)^{m-1} B_m (\sqrt{3}\,\epsilon)^{2m}}{(2m!)(2m+n)} \right\} \right]_{\epsilon_a}^{\epsilon_b} \tag{40}$$

where B_m are the Bernoulli numbers:

$B_1 = 1/6$, $B_2 = 1/30$, $B_3 = 1/42$, $B_4 = 1/30$, ...

The stress-strain function (35) has two segments, so (39) must be integrated accordingly. Therefore, when σ is given by equation (35), (40) must be:

$$P = -\frac{E}{\sqrt{3}}\left[\epsilon\left\{-1 + \frac{\sqrt{3}\,\epsilon}{4} - \frac{(\sqrt{3}\,\epsilon)^2}{36} + \frac{(\sqrt{3}\,\epsilon)^4}{3600} - \cdots\right\}\right]_{\epsilon_b}^{\epsilon \text{ at proporational limit}} \quad (41)$$

$$-\frac{c}{\sqrt{3}}\left[\epsilon^n\left\{-\frac{1}{n} + \frac{\sqrt{3}\,\epsilon}{2(n+1)} - \frac{(\sqrt{3}\,\epsilon)^2}{12\,(2+n)} + \frac{(\sqrt{3}\,\epsilon)^4}{720(4+n)} - \frac{(\sqrt{3}\,\epsilon)^6}{30,240(6+n)} + \cdots\right\}\right]_{\epsilon \text{ at proportional limit}}^{\epsilon_a}$$

Example Calculation

An example calculation of the bore deformation curve for the gas cylinder part of the jacketed bombs is given to illustrate the method. The initial information is: material is 303 stainless steel, 0.2% yield stres = 38,000 psi, Young's Modulus = 29×10^6 psi, inner radius = 0.5 inch, outer radius = 1.25 inch. Also, it can be estimated that the 0.01% yield stress is about 28,500 psi.

At the 0.2% yield stress, $\epsilon' = 0.00200 + 38 \times 10^3/29 \times 10^6 = 0.003310$; and at the 0.01% yield stress, $\epsilon' = .0001 + \dfrac{28.5 \times 10^3}{29 \times 10^6} = 0.001083$. Substituting into equations (32) and (33):

σ (0.2% offset) = (38,000)(1.003310) = 38,125 psi, $\epsilon = 0.0033045$

σ (.01% offset) = (28,500)(1.001083) = 28,530 psi, $\epsilon = 0.0010822$

Substituting the true stress-natural strain values into equation (34) and solving for c and n gives:

$$\sigma = 168,100\ \epsilon^{0.2597} \quad (42)$$

Equation (42) intersects the elastic line, $\sigma = 29 \times 10^6 \epsilon$, at (0.00095145; 27,592 psi). This intersection is assumed to be the proportional limit. Equation (42) and the elastic lines are plotted in fig. 2.

Substituting equation (42) and Young's Modulus into equation (41):

$$P = \frac{29 \times 10^6}{\sqrt{3}} \left[\epsilon \left\{ 1 - \frac{\sqrt{3}\,\epsilon}{4} + \frac{(\sqrt{3}\,\epsilon)^2}{36} \right\} \right]_{\epsilon_b}^{0.00095145}$$

$$+ \frac{168,100}{\sqrt{3}} \left[\epsilon^{0.2597} \left\{ \frac{1}{0.2597} - \frac{\sqrt{3}\,\epsilon}{2.5194} + \frac{(\sqrt{3}\,\epsilon)^2}{27.12} \right\} \right]_{0.00095145}^{\epsilon_a} \qquad (43)$$

To use equation (43), a value of ϵ_a is assumed, ϵ_b is computed from equation (38), and the pressure required to cause ϵ_a is given by equation (43).

If ϵ_a is known, μ_i can be found from equations (36) and (37). The bore pressure-bore deformation calculations are summarized in table 4.

The bore pressure-bore deformation curve is plotted in fig. 3. Equation (31) gives P_y = 13,800 psi; taking ϵ_a as the proportional limit in equation (43) gives P_y = 13,400 psi. In fig. 3, the curve becomes non-linear at pressures above P_y, and the deformation becomes inelastic (non-reversible).

The permanent deformation at the bore can be estimated by drawing a line that is parallel to the elastic portion of the curve in fig. 3 through a particular point on the non-linear part of the curve. The intersection of this line with the bore displacement axis gives the

TABLE 4. - Summary of calculations for bore deformation at pressures above P_y

ϵ_a	$e^{\sqrt{3}\,\epsilon_a}$	$\epsilon_a^{0.2597}$	ϵ_b	Bore Pressure P, psi	μ_a, in[1/]
0.00095145	1.0016494	0.164149	0.00015234	13,373	0.0004121
.0010	1.0017335	.166285	.00016011	14,040	.0004332
.0012	1.0020807	.174346	.00019218	16,513	.0005198
.0014	1.0024278	.181502	.00022423	18,648	.0006066
.0016	1.0027751	.187872	.00025629	20,488	.0006933
.0020	1.0034701	.199084	.00032046	23,597	.0008667
.0024	1.0041655	.208738	.00038466	26,122	.0010403
.0028	1.0048615	.217264	.00044891	28,189	.0012139
.0032	1.0055580	.224930	.00051319	30,007	.0013876

1/ μ_a is $(\epsilon_a')(R_i)$ = displacement from original radius, R_i. See eq. (36).

Figure 2. - Partial stress-strain curve for 303 stainless steel

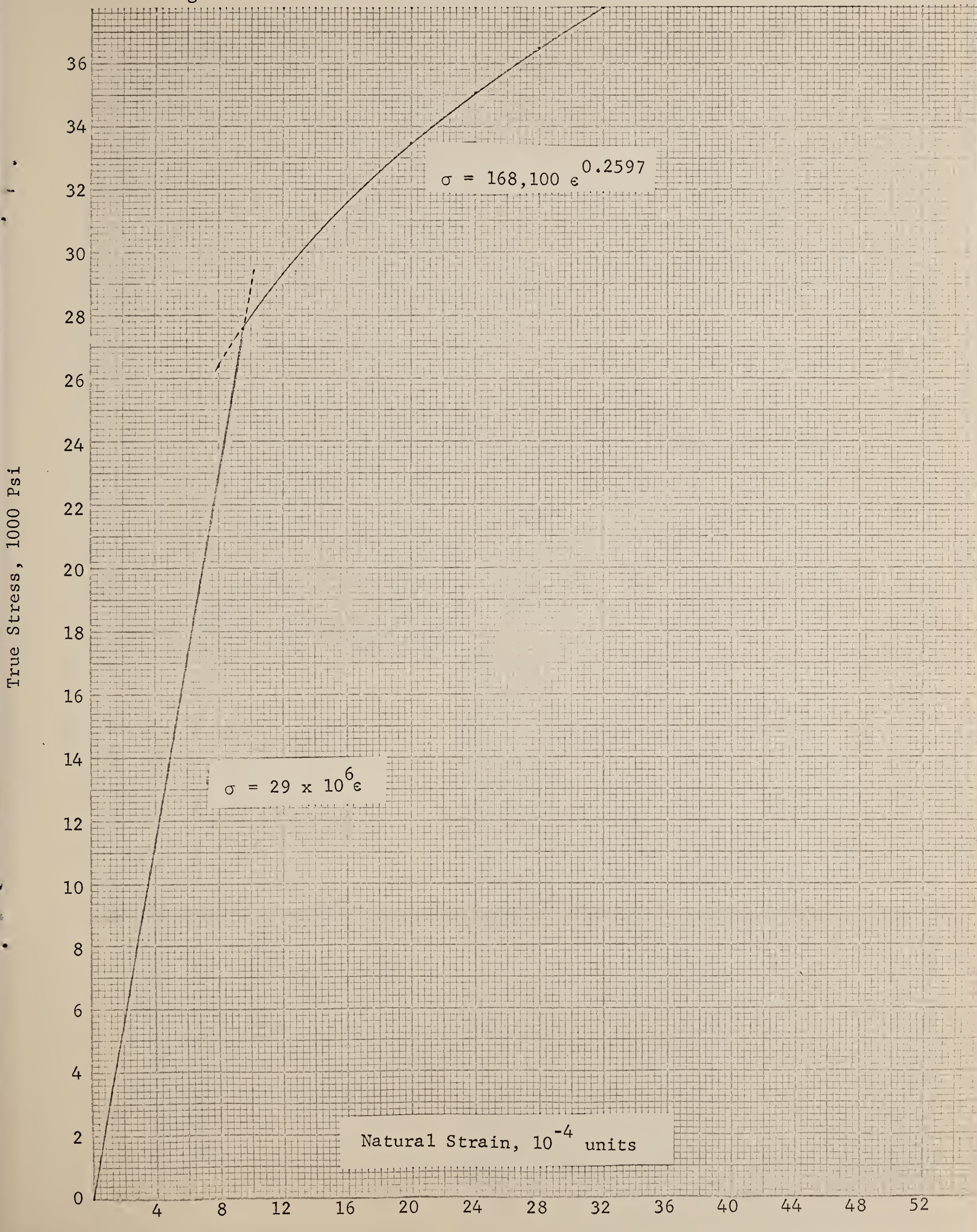
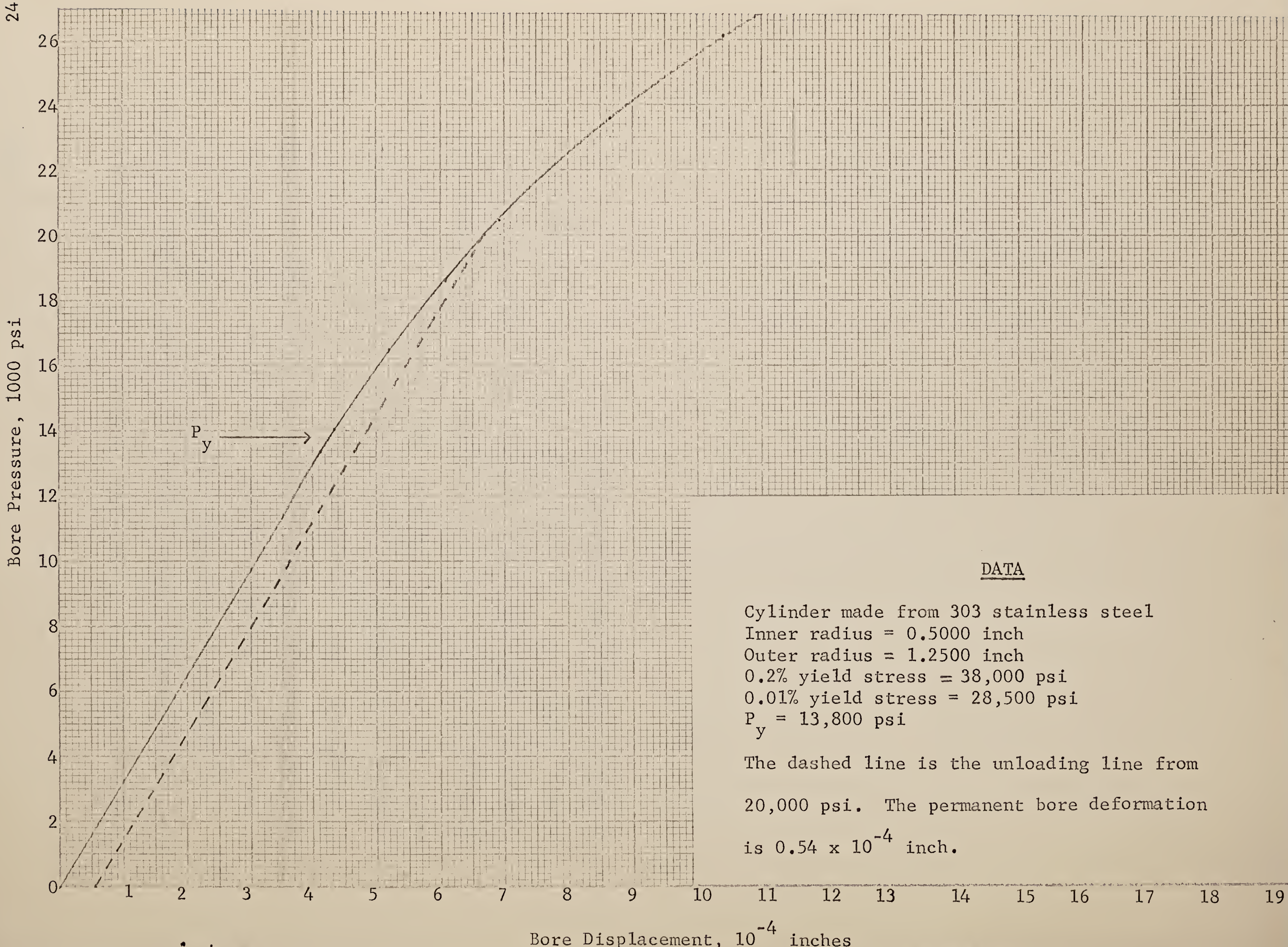


FIGURE 3. - Bore displacement for the gas cylinder in Fig. 1



permanent deformation at the bore. In other words, it is assumed that the bore deformation follows Hooke's Law when the cylinder is unloaded.

For example, the unloading line from 20,000 psi is shown in fig. 3. If the cylinder, initially annealed, is pressured up to 20,000 psi, the bore deformation is given by the solid curve between P = 0 and P = 20,000 psi. When the cylinder is depressured, the bore deformation is given by the dashed line, which intersects the bore deformation axis at 0.54×10^{-4} inch. The bore radius would now be 0.500054 inch. The normal tolerance in making a cylinder on a lathe is about 0.001 inch; so for the curve in fig. 3, the pressure required to cause a permanent deformation as great as the uncertainty in the original radius is about 32,000 psi (by extrapolation).

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