PROCEEDINGS

OF

THE ROYAL SOCIETY

OF

EDINBURGH.

VOL. XIX.

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OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. XIX. 1891–92.

The 109th Session.

GENERAL STATUTORY MEETING.

Monday, 23rd November 1891.

The following Council were elected:—

President.

Sir DOUGLAS MACLAGAN, M.D., F.R.C.P.E.

Vice-Presidents.

Rev. Professor Flint, D.D.
Professor Chrystal, LL.D.
Thomas Muir, Esq., LL.D.
Sir Arthur Mitchell, K.C.B., LL.D.
A. Forbes Irvine, Esq. of Drum, LL.D.
Sir William Turner, M.B., F.R.S.

General Secretary—Professor P. G. Tait.

Secretaries to Ordinary Meetings.

Professor Crum Brown, F.R.S.
John Murray, Esq., LL.D.

Treasurer—Adam Gillies Smith, Esq., C.A.

Curator of Library and Museum—Alexander Buchan, Esq., M.A., LL.D.

Ordinary Members of Council.

Professor W. H. Perkin, Ph.D., F.R.S.
A. Beatson Bell, Esq., Advocate.
The Rt. Hon. Lord Kingsburgh, C.B., LL.D., F.R.S.
Alexander Bruce, M.A., M.D.
Dr R. H. Traquair, F.R.S.
Dr Byrom Bramwell, F.R.C.P.E.
Professor Copeland, Astronomer-Royal for Scotland.
Hon. Lord Maclaren, LL.D., F.R.A.S.
Professor Rutherford, F.R.C.P.E., F.R.S.
Stair Agnew, Esq., C.B.
Rev. J. Sutherland Black, M.A.
Robert Kidston, Esq., F.G.S.

By a Resolution of the Society (19th January 1880), the following Hon. Vice-Presidents, having filled the office of President, are also Members of the Council:—

His Grace the DUKE of ARGYLL, K.G., K.T., LL.D., D.C.L.
The Right Hon. LORD MONCREIFF of Tulliebole, LL.D.
Sir WILLIAM THOMSON, LL.D., D.C.L., P.R.S., Foreign Associate of the Institute of France.

PROFESSOR SIR DOUGLAS MACLAGAN, President, in the Chair.

Chairman's Opening Address.

(Read December 7, 1891.)

It is in conformity with time honoured custom, and in the performance of duty, that I offer a short address on the occasion of the opening of the Society's 109th Session.

I confess that I have little to say which can be expected to attract the notice of the Fellows, or to bear to be recorded in our proceedings; but I am glad to be able to state that alike as regards Fellowships, Finances, and the work of the past Session, the Society is in a prosperous condition.

We have added to our number 18 Ordinary Fellows who have passed the triple ordeal of the Council and the ballot of the Society; and we have good reason to believe that not a few of these will prove valuable pioneers in the domains of Science and Literature, and interesting contributors to our Proceedings.

Among the Fellows elected this year, there were two Lords of Session, three Professors, eight Doctors of Science or Medicine, and the Editor of the leading Newspaper of Scotland.

We have had during the past year 76 papers read before the Society, which may be classified thus:—18 in the department of Natural Philosophy, 3 in Astronomy, 7 in Mathematics, 12 in Chemistry, 1 in Geology, 6 in Zoology, 4 in Botany, 8 in Physiology, 3 in Anatomy, 6 in Meteorology, 7 in Physical Geography, and 1 in Philology.

It thus appears that few of the leading departments of Science have been unrepresented at one or more of our meetings; and we have thus the satisfaction of knowing that the Royal Society maintains its character for scientific catholicity, and that its meetings are open to all who have anything to say on the special scientific subjects to which they have devoted their attention in the way of research.

I have more than once, when occupying this chair, expressed my regret that while we rejoice in the activity of the cultivators of Physical and Natural Science, we have so few contributors to

Literature, which, equally with Science, is an object of the institution of the Royal Society. I have to repeat this expression of regret. Of the 76 papers read before us, only one was of a purely literary character, and this was, as has been the case for some time, contributed by my ever active minded and ready penned friend, Emeritus-Professor Blackie. This paper was on the bi-stratification of the Greek language, as it is at present spoken and written in Greece. There are two phases of Greek, the Romaic used in conversation by all classes, and the neo-classical used in the newspapers, the courts of law, the parliament, and in the great majority of literary works. The interest of Professor Blackie's paper consisted in recalling a similar state of matters that existed in our own country in the last century and beginning of this, where the Scottish language, or dialect, was generally used in conversation, but the English language or dialect was as generally used for literary composition. It is a satisfaction to me personally that I lived long enough back to hear the Scottish language used constantly, especially by my maternal relatives, in ordinary family conversation, whilst they could put on their English, as they would their best dinner dress, when they had strangers among them. I hope that it may yet, on account of its pithiness and national character, maintain some ground among us as a spoken language, notwithstanding the sneers of some, alas ! even of our own countrymen, who treat it *naso adunco* as vulgar, because it is the language of our peasantry. Such persons I venture to call imperfectly educated, because they know not the language, perhaps some of them never heard the names, of our poets, Blind Harry the historian of Wallace, John Barbour the historian of the Bruce, King James I. the royal author of "The King's Quhair" and other poems, Dunbar the writer of "The Thistle and the Rose," Gawain Douglas, the Episcopal son of Archibald Bell-the-cat,

> "More pleased that in a barbarous age
> He gave to Scotland Virgil's page,
> Than that beneath his rule he held
> The bishopric of fair Dunkeld ;"

our Scottish Satirist David Lindsay, and our prose writers Lindsay of Pitscottie, and the clerical historian Calderwood.

The Scottish dialect in this country, and the Romaic in Greece may be destined to disappear (*absit omen*), but if our Scotch should perish as a spoken tongue, it will hold its own as a classical language, until the author of the Gentle Shepherd, the tender addresser of the Field Mouse and the Daisy and the rollicking narrator of the adventures of Tam O'Shanter, with the creator of Edie Ochiltree and Caleb Balderston, have become merely names belonging to a remote antiquity.

Professor Blackie has often insisted in this room that the best way of learning Greek is to reside for several months at Athens and learn it conversationally. It must be a great satisfaction to him to know that one young man at least is now going out to Athens to put to a practical test the method of instruction in the classical, but still living, language which he has so persistently advocated. There is a remarkable confirmation of this view in the case of John Basingstoke, who flourished in the first half of the 13th century, dying in 1252, and which has not been noticed by Professor Blackie. Basingstoke while in Athens became acquainted with a remarkable girl, the daughter of the Athenian Archbishop, and by conversing and reading with her he declared he learned more Greek in three months than he had done in the University of Paris in four years. This young lady, though not conversant with modern sanitary science and the relation between the violation of its rules and disease, could foretell the occurrence of pestilence; and though living long previous to the foundation of Ben Nevis Observatory, could predict storms; and though she had not studied the works of Kepler, Newton, or La Place, could foretell eclipses; while she proved herself to be a seismologist of the first order by foretelling the occurrence of earthquakes with unerring certainty. I give these statements on the authority of Matthew Paris in his *Chronica Majora*, though they make a rather heavy demand on our faith as to the accomplishments of this phenomenal damsel. I will only express the hope, for the credit of her escutcheon, that her father the Archbishop had been legitimately married before he was consecrated.

It is not incumbent on, nor would it be proper for me, to offer any commentaries on, or criticism of, the papers which were read before us during last Session, but I venture to make three exceptions, including Professor Blackie's.

It was the privilege of the Society to hear from His Serene Highness the Prince of Monaco some account of his new vessel the "Princesse Alice," which has been specially constructed and equipped for deep sea investigation. It was a great disappointment to the Society that from the delay of the outfit of this ship on the Thames, the Fellows had not the pleasure of inspecting the vessel personally. We have plenty of records of the outfit of ships for scientific exploration, but we have not had the pleasure of inspecting one which was specially ordered and constructed for scientific purposes; and we all earnestly hope that in the course of the ensuing year this privilege may be afforded us. In the meantime we desire to record our satisfaction that a man of His Serene Highness' rank, himself every inch a sailor, with his large means, should devote himself to pure science, and bring with it in his intercourse with us, those unpretending and genial manners which made us, when in his company, entirely forget the Prince in the Scientist.

With the Prince of Monaco we had the pleasure of hearing an interesting paper by the Baron de Guerne, the President of the Zoological Society of France, who laid before us some results of the deep sea exploration by the Prince of Monaco in his sailing vessel the "Hirondelle," with admirable illustrations of species, many of them new to science, and it will be a pleasant addition to his Highness' return visit should he be accompanied by the learned Baron if the "Princesse Alice" should find her way to the Firth of Forth. Where the "Princesse Alice" at present is I know not, whether braving the fierce waves of the Atlantic, or safe in some quiet French harbour. We can all earnestly utter the aspiration of Horace—

"Sic te Diva potens Cypri
Sic fratres Helenae, lucida sidera
Ventorumque regat pater"

that our wishes may be fulfilled.

The other paper read last Session which I desire to notice in an exceptional way, is our colleague Mr John Aitken's description of his instrument for counting the dust particles in the air, which seems to me to be of first-rate importance, and calls for a more than passing notice. By it my mind is carried back to the time now nearly half a century past, when so much attention was directed to this subject in connection with the keen and sometimes acrimonious discussion on spontaneous generation. True, then it was to the organised and living particles in the air that attention was directed, but it gave a great impetus to investigation of the whole question of aerial dust. The laborious work of Tissandier in Paris, of Angus Smith in this country, and of Fodor in Austro-Hungary, have given us some important knowledge of the amount and nature of those dust particles, but how rough even the most careful gravo-metric methods are when compared with that of Aitken! How simple the manipulation, especially with the portable apparatus, which he described in his last communication to the Society! One wonders whether the optically pure air of Tyndall would stand this test. As one interested in hygienic investigations, I should like to see it applied not only to enumerating the particles, but in showing how much is organic, and how much inorganic. Suppose the normal air of a room were examined, and then a sample of the same air previously passed say through a red-hot tube to destroy organic matter, would this give us a means of determining the organic particles of the atmosphere? It might be difficult of execution, but the results would be important. On the vexed question of ventilation I think it can throw much light. Indeed, by some of his experiments in this room and elsewhere, Mr Aitken has shown that with sufficient means of air renewal his instrument can readily detect rates of air purification. One cannot but feel that with this new aid many important points will be elucidated.

It is my duty to make mention of those of our number who have passed away since the opening of last Session. These were fourteen Ordinary, and two Honorary Fellows. I would in no sense have it supposed that a short passing notice here, is to be regarded as anything else than a suggestion to others who may have special knowledge, to give us better records of our deceased colleagues at the ordinary meetings of the Society. But in truth such suggestions are in great measure unnecessary now, as in a large proportion of those whom I have to name this has been anticipated by interesting memoirs read to the Society during last session.

Of Professor COSMO INNES BURTON, the gifted son of a gifted father, I would say no more than that an interesting notice, by Mr William Marshall of this young scientist who has been prematurely taken away, was read to the Society on the 20th July last.

Of the late Professor CAMPBELL SWINTON, an admirable biography was on 6th April last communicated to the Society, by our Ex-President Lord Moncreiff, in which we were reminded of the great abilities of the deceased, his varied acquirements, his eloquence, so often displayed on the occasion of great University ceremonials, when he had as Dean of the Faculty of Law to present candidates for the degree of LL.D., a duty in the performance of which he was unrivalled, and his earnestness in fulfilling all his duties, whether in the University or in the Church, or as a country gentleman. A most interesting notice of Mr Swinton, regarding him chiefly from his position as a churchman, has appeared in the Edrom Parish Magazine, from so competent a pen as that of the Hon. Lord Low, with all of which I cordially concur, and which may be regarded as an admirable appendix to Lord Moncreiff's excellent biography. I must content myself with referring to those interesting memoirs, but I may be permitted to say a word for myself on the score of old friendship. In Archibald Campbell Swinton I always found one whom no difference in opinions, political or ecclesiastical, could prevent from giving his personal regard and esteem, who gave an ungrudging confidence to his medical advisers, and who when shattered by illness and by the severe accident which hastened his death, retained the cheery spirit and quiet resignation of a Christian gentleman.

Of Mr DUNCAN MATTHEWS an elaborate notice by Professor M'Intosh was read to the Society on 5th January last. There is something touching in the remembrance that I had last year to record my esteem of his eminent relative, Dr Matthews Duncan, and to have now to notice the death of the cousin, and to lament that one so young, and animated with such an ardent and disinterested love of biological science, should not have lived to continue the career on which he had entered with such remarkable success.

Dr EDWARD SANG was born in Fife, studied at Edinburgh University under Sir John Leslie, and eventually became assistant to that eminent Professor. He afterwards held the appointment of Professor of Mechanical Philosophy at Constantinople, but returned to Edinburgh on the outbreak of the Crimean War. With the exception of the time spent at Constantinople he resided in Edinburgh, and was engaged in scientific work and in teaching mathematics. The great work of his life was the preparation of a series of exhaustive tables of logarithms. He was Secretary for many years to the Royal Scottish Society of Arts. He received the degree of LL.D. from the University of Edinburgh, was admitted to this Society in 1849, and was adjudged the Makdougall-Brisbane prize in June 1886. He died at the age of 86, and will be remembered with affection and respect by a large number of the Fellows of this Society, and by more than one generation of students to whom he taught mathematics.

ADOLF PAUL SCHULTZE was born on 8th October 1840 at Crimmitschau, in Saxony, studied engineering in the Polytechnic at Chemnitz, and came to England in 1864. His special study was the microscope considered as an optical instrument, and he made substantial contributions to optical science. He died on 3rd January 1891.

ROBERT WILLIAM MYLNE was descended from a family that for several generations acted as the King's master-masons for Scotland. An ancestor of his was the architect for the North Bridge of Edinburgh. The deceased was engineer to the Limerick Water Company, and the New London Water Company. He was frequently called on to give evidence before the Committees of the House of Commons on questions relating to the water supply of towns. He was also an eminent geologist, and his geological map of London was long a standard authority. He died in July 1890, at the age of 74.

Dr PATRICK JAMES STIRLING was born in Dunblane in 1809. He studied at St Andrews University, and was a favourite pupil of Dr Chalmers, then Professor of Moral Philosophy and Political Economy, in whose class he carried off all the highest distinctions. He afterwards attended the Law Classes in the University of Edinburgh; and he managed in a class in which Lord Mure, Lord Jerviswoode, and Principal James David Forbes were his fellow-students, to gain the very high distinction of the Gold Medal in the class of Civil Law. Notwithstanding his being for many years the leading Solicitor in Western Perthshire, he found time to publish a work on "The Philosophy of Trade," and another on "The Gold Discoveries." He translated Frederick Bastiat's works entitled

"The Harmonies," and "The Sophisms." For his literary labours he received the degree of LL.D. from the University of St Andrews. He died 23rd March 1891.

Dr James Sanderson was born in Dunbar in 1812 and received his medical education at the University of Edinburgh. In 1832 he was appointed to the Medical Staff of the Madras Service, and was placed on the retired list in 1863. He was elected a Fellow of this Society in 1863, and for many years held the office of Treasurer to the Scottish Meteorological Society. He died in April 1891.

Sir John Hawkshaw was born and educated as an engineer at Leeds. When only twenty years of age he undertook the management of the Bolivar Copper Mines in South America. On his return to England he was successively engineer to the Manchester and Bolton Canal and Railway Company, and to the Lancashire and Yorkshire Railway. The Severn Tunnel is considered his greatest achievement in engineering, from the difficulties that had to be overcome. He proposed to connect England with the Continent by means of a Submarine Tunnel. He was knighted in 1873, and was President of the meeting of the British Association at Bristol in 1875. He died in his 81st year.

Mr John Turnbull of Abbey St Bathans was born in 1820. He was educated at the University of Edinburgh, and at the age of 21 passed as a Writer to the Signet. As an Archæologist and Naturalist, his attainments were far above the average; and he had so refined a taste, and so deft a hand as an artist, that had he not been a country gentleman and a professional man of law, he might readily have gained for himself an honourable place among the limners of his native land. He spent a year in Egypt and Syria, and brought home a portfolio of sketches of great artistic merit. He died on 21st June last, deeply regretted by all who knew him, and especially by the people of Berwickshire, of which County he had for many years been Convener. Here is another of those for whom on the score of friendship and regard, I desire to add to my estimate of him as a worthy Fellow of the Royal Society, a sincere tribute of my personal regret.

The Right Honourable John Inglis of Glencorse, President of the Court of Session, and Lord Justice General of Scotland, was the son of the Rev. Dr Inglis, and was born in Edinburgh in the year 1810.

He went through the High School Curriculum in this city, and continued his studies at the University of Glasgow, from which, with a Snell Exhibition, he proceeded to Balliol College, Oxford, graduating there as B.A. in 1834, as M.A. in 1837, and as D.C.L. in 1859. He was called to the Scottish Bar in 1835, and about nine years after was appointed an Advocate-Depute. On the return of the Conservative party to power in 1852, and during Lord Derby's short administration, Mr Inglis filled the office of Solicitor General, and afterwards of Lord Advocate. The latter office he ceased to hold on the fall of Lord Derby's Government, but when the Conservatives returned to power in 1858 he was reappointed Lord Advocate, and sat in the House of Commons for the borough of Stamford. It was in this year that he succeeded in carrying through Parliament the Scottish Universities Act, by which he procured a new Constitution for the University of his native city, and it was only a natural thing that when the vacancy occurred, he was elected Chancellor of the University of Edinburgh. He had previously been called on to fill the office of Dean of the Faculty of Advocates. In 1858 he continued in Parliament till he was raised to the Bench as Lord Justice Clerk. In 1867 he was promoted to the Presidency of the First Division of the Court of Session, and became Lord Justice General of Scotland. He adorned the bar with great gifts of eloquence and legal acumen. As a judge it has been said of him that his decisions were based upon accurate and exhaustive knowledge of the law in all its details, and were expressed with a clearness of statement that made his judgments models of legal and scholarly argument, whilst Counsel received from him unbounded courtesy. He died on the 20th of August last.

Under any circumstances it would be little short of an impertinence on my part to offer any detailed character of Lord Justice General Inglis in his forensic and judicial relations.

Fortunately for me this has been admirably done in a faithful and discriminating biography by my learned friend Sheriff Æneas Mackay, in the number for October last of Blackwood's Magazine. I may be permitted again, on the score of longstanding acquaintance, to record my esteem of John Inglis as a personal friend. I may be held entitled to do this, when I state that our acquaintance began as far back as October 1819 when we together entered as "Gaits" Carson's Class at the High School. Of course when Inglis was at Glasgow and Oxford we saw nothing of one another, but when he returned to Edinburgh we were able, as members of Mackay's Class Club, to renew our interrupted but not forgotten friendship, which lasted to the end of his life. John Inglis was by many persons said to be cold in his friendships. I know not how this may have been to others, but it was not so to me. I cannot easily forget that on the occasion of many kind friends presenting to me my bust, by the hands of the University Chancellor, after he had performed the ceremony with all the dignity which so characterised him, he, *sotto voce*, grasped my hand, with the warm expression of "Dear old boy!"

I can add nothing to what has already been published of Lord Justice General Inglis, than that it will not be easy for Scotland to find one who will discharge the functions of that high office with a similar combination of judicial learning, clearness of decision, and dignity of manner.

Thomas Miller, Emeritus-Rector of Perth Academy, was born in 1807, at Ardoch, in Perthshire. He studied at St Andrews University, where he graduated as M.A. After graduation, a vacancy having occurred in the Mathematical Mastership of Madras College, St Andrews, he was unanimously elected to that post. During the four years that he held this appointment, he more than tripled the number of students. He was subsequently elected to the office of Rector of Perth Academy, in which capacity he taught the classes of mathematics, natural philosophy and chemistry. The excellence of his teaching was testified by the approving Reports of Professor Kelland, and of Mr Fearon, the examiner for London and Middlesex, who said that Dr Miller's teaching was better than any he saw in England. He was foremost in the foundation of a Mechanics Institute. He also published a treatise on the Differential Calculus, which obtained for him the degree of LL.D. from two Scottish Universities. Dr Miller continued to hold the position of Rector of the Perth Academy for the long period of forty-four years; and on his retiring from the Rectorship in 1881, his numerous friends and pupils presented him with his portrait. He was Vice-President of the Literary and Antiquarian Society of Perthshire, and was Chairman, Vice-President, or member of various Trusts and Societies in Perthshire, founded for benevolent purposes. He died on 9th September 1891.

DAVID DAVIDSON was the son of the Rev. Dr Davidson, of Muirhouse. Nearly fifty years ago, when in his 35th year, he went to Canada as Manager of the Bank of British North America. He filled this office till 1855, when he was appointed General Manager of the Bank of Montreal. While there, with the assistance of his uncle the late Lord Cockburn, he was instrumental in selecting in Scotland qualified instructors to introduce a high standard of education in the public school then established in Montreal. In 1863 Mr Davidson accepted the General Managership of the Bank of Scotland. This position he occupied with much success till 1879, when owing to failing health he resigned. After his retirement he accepted the Chairmanship of the North British and Mercantile Insurance Company. He was well known as one interested in many charitable works. He died on 30th October last. I may here again be permitted to offer a tribute of regard to the memory of a personal friend.

JAMES RUSSELL LOWELL, one of our Honorary Fellows, was born on 22nd February 1819 at Cambridge, Massachusetts, and was of Puritan origin. It is said that he was the heir of several generations of men of high thinking and pious ways. He studied at Harvard University, and took the degree of LL.B. On leaving College he did not seriously attempt to practise the profession of law, literature having entirely absorbed him. In 1844 he published a volume of poems of such excellence as to show that America possessed a new poet. In 1848 he published the "Biglow Papers." This work was followed by his poems entitled "The Vision of Sir Launfal," "The Present Crisis," and "A Fable for Critics." In 1855 he was appointed to the Chair of Modern Languages and Belles Lettres at Harvard. Whilst holding this Chair he published his "Fireside Travels" and "Under the Willows." He also edited the "Atlantic Monthly and North American Review." His strength as a critic was revealed in his works "My Study Windows," and "Among my Books." From his Chair at Cambridge, Lowell was called to be Minister at Madrid, and in 1880 he was appointed representative of the United States in England. No foreign representative was ever more respected and admired.

It is not permissible to me to say anything critical as to the writings of one who has been the subject of a full article in the Edinburgh Review. Many there are who will read that article, and the more serious writings to which it refers, but Lowell will be better known to thousands who, after mastering the Yankee dialect, have laughed till their sides were sore, over the poetical effusions of Mr Hosea Biglow, the recommendatory epistles of the Rev. Homer Wilbur, M.A., and the experience of Mr Bird-of-Freedom Sawin, translated from Bird-of-Freedom's prose into Mr Biglow's verse.

America has reason to be proud of her literature in Lowell, and this Society in having enrolled him in the list of Honorary Fellows.

Lowell died on 12th August 1891.

Wilhelm Eduard Weber, another of our Honorary Fellows, was born at Wittenberg in 1804. He studied at the University of Halle, where he became Professor-Extraordinary of Physics in 1828. In 1831 he was called to Göttingen to succeed Mayer in the Chair of Physics, and remained there till 1837. In that year the British Sovereign King William IV. died, and Queen Victoria being excluded from the throne of Hanover by the Salic Law, her uncle Ernest Augustus Duke of Cumberland became King of Hanover. This prince considered the narrow liberties then enjoyed by his subjects to be excessive, suspended the constitution, and thereby called forth vigorous protests from Weber and other Professors. As a punishment, seven of them, including Weber, were ejected from their Chairs. Weber was, however, in 1843 appointed Professor of Physics at Leipzig, and in 1849 he returned to his former position in Göttingen.

His first contribution to Science, published in conjunction with his brother Ernest, was "*Die Wellenlehre*," which is regarded as a classical work. After writing various papers on acoustics, he published in 1833, in conjunction with his brother Edward Frederick, an investigation into the mechanism of walking. But his most memorable researches were in the departments of magnetism and electricity. These are contained in the "*Beobachtungen des Magnetischen Vereins*" and in his "*Elektrodynamische Maasbestimmungen*," published in 1864. In this series of papers he showed how methods of absolute measurement could be extended into the region of electricity.

He died on the 23rd of June last.

I stated in my last address to the Society that I ventured to express my regret that we had so few literary papers presented to us. Only one followed upon this, and of that I have expressed my recognition. May I throw out a hint that in the city of David Hume, Dugald Stewart, Brown and Hamilton, it might not be unreasonable that we should look for papers on subjects connected with philosophy in some of its various departments? I feel confident that such communications would be welcomed by the Royal Society; but whether they come or not, I express the confident hope that the Session now opening may be as prosperous as those that have preceded it.

Note on a Theorem regarding a Series of Convergents to the Roots of a Number. By Thomas Muir, LL.D.

(Read January 4, 1892.)

If the positive integral powers of $\sqrt{n}+1$ be taken, and the expansion of each be separated into two parts, rational and irrational, thus—

$$
\begin{aligned}
(\sqrt{n}+1)^1 &= \sqrt{n} + 1, \\
(\sqrt{n}+1)^2 &= 2\sqrt{n} + (n+1), \\
(\sqrt{n}+1)^3 &= (n+3)\sqrt{n} + (3n+1), \\
(\sqrt{n}+1)^4 &= (4n+4)\sqrt{n} + (n^2+6n+1), \\
(\sqrt{n}+1)^5 &= (n^2+10n+5)\sqrt{n} + (5n^2+10n+1), \\
&\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}
$$

then the ratio of the rational portion to the coefficient of $\sqrt{n}$ in the other portion is approximately equal to $\sqrt{n}$, the convergence being perfect when the power of the binomial is infinite. This is the simplest case of a theorem discovered by the late Dr Sang, and enunciated by him as the result of a process of induction in his paper "On the Extension of Brouncker's Method to the Comparison of several Magnitudes" (*Proc. Roy. Soc. Edin.*, vol. xviii. p. 341, 1890–91).

It seems desirable to have some further investigation into this curious proposition, and to try if possible to bring it into relationship with the already known mode of obtaining convergents to $\sqrt{n}$, or, failing this, to show that the two modes are perfectly independent.

In the first place, a proof of the theorem is wanted. To obtain this we must get a suitable expression for the r^{th} convergent, that is to say, for the rational portion and the coefficient of $\sqrt{n}$ in the irrational portion of the expansion of $(\sqrt{n}+1)^r$. Now manifestly

$$(\sqrt{n}+1)^r = \frac{(\sqrt{n}+1)^r + (\sqrt{n}-1)^r}{2} + \frac{(\sqrt{n}+1)^r - (\sqrt{n}-1)^r}{2};$$

and in this when r is odd the first fraction contains $\sqrt{n}$ as a factor and the other is rational, while when r is even the second fraction contains the factor $\sqrt{n}$ and the first is rational. Consequently if we write

$$(\sqrt{n}+1)^{2s-1} = \frac{(\sqrt{n}+1)^{2s-1}+(\sqrt{n}-1)^{2s-1}}{2\sqrt{n}} \cdot \sqrt{n} + \frac{(\sqrt{n}+1)^{2s-1}-(\sqrt{n}-1)^{2s-1}}{2}$$

and

$$(\sqrt{n}+1)^{2s} = \frac{(\sqrt{n}+1)^{2s} - (\sqrt{n}-1)^{2s}}{2\sqrt{n}} \cdot \sqrt{n} + \frac{(\sqrt{n}+1)^{2s}+(\sqrt{n}-1)^{2s}}{2},$$

we have the desired expressions for the numerators and denominators of the $(2s-1)^{\text{th}}$ and $2s^{\text{th}}$ convergents; and all that remains to be shown is that for $s=\infty$ either convergent is equal to $\sqrt{n}$. Taking the $(2s-1)^{\text{th}}$ convergent, viz.,

$$\frac{(\sqrt{n}+1)^{2s-1}-(\sqrt{n}+1)^{2s-1}}{2} \div \frac{(\sqrt{n}+1)^{2s-1}+(\sqrt{n}-1)^{2s-1}}{2\sqrt{n}},$$

we see that in order to prove that its limiting value is $\sqrt{n}$ we have merely to show that

$$\mathcal{L}\,\frac{(\sqrt{n}+1)^{2s-1}-(\sqrt{n}-1)^{2s-1}}{(\sqrt{n}+1)^{2s-1}+(\sqrt{n}-1)^{2s-1}} = 1,$$

i.e.,
$$\mathop{\mathcal{L}}_{s=\infty} \frac{1-\left(\frac{\sqrt{n}-1}{\sqrt{n}+1}\right)^{2s-1}}{1+\left(\frac{\sqrt{n}-1}{\sqrt{n}+1}\right)^{2s-1}} = 1,$$

which is evidently the case; and taking the $2s^{\text{th}}$ convergent we have to establish the equally simple result

$$\mathop{\mathcal{L}}_{s=\infty} \frac{1+\left(\frac{\sqrt{n}-1}{\sqrt{n}+1}\right)^{2s}}{1-\left(\frac{\sqrt{n}-1}{\sqrt{n}+1}\right)^{2s}} = 1:$$

where the signs preceding the vanishing portions of the numerators and denominators bring out the fact that the convergents are alternately less and greater than $\sqrt{n}$.

The next point to settle is, whether the convergents obtained in this manner, viz.,

$$\frac{1}{1},\ \frac{n+1}{2},\ \frac{3n+1}{n+3},\ \frac{n^2+6n+1}{4n+4},\ \frac{5n^2+10n+1}{n^2+10n+5},\ \ldots\ldots$$

be not also obtainable in the ordinary way from some continued fraction. To do this a quite different mode of expressing the numerators and denominators of the convergents must be sought for. Utilising the facts that each denominator is the sum of the immediately preceding numerator and denominator, and that each numerator is the sum of the immediately preceding numerator and n times the corresponding denominator, we readily see that the numerators are expressible in the form

$$\begin{vmatrix} n & 1 \\ -1 & 1 \end{vmatrix},\quad \begin{vmatrix} n & n & 1 \\ -1 & 1 & 1 \\ . & -1 & 1 \end{vmatrix},\quad \begin{vmatrix} n & n & n & 1 \\ -1 & 1 & n & 1 \\ . & -1 & 1 & 1 \\ . & . & -1 & 1 \end{vmatrix},\ \ldots\ldots\ldots$$

and the denominators in the form

$$\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix},\quad \begin{vmatrix} 1 & n & 1 \\ -1 & 1 & 1 \\ . & -1 & 1 \end{vmatrix},\quad \begin{vmatrix} 1 & n & n & 1 \\ -1 & 1 & n & 1 \\ . & -1 & 1 & 1 \\ . & . & -1 & 1 \end{vmatrix},\ \ldots\ldots\ldots$$

But, by subtracting the second row of each of these determinants from the first row, the third row from the second, and so on, each series is transformed into a series of *continuants*, viz., the first series into

$$n+1,\ \begin{vmatrix} n+1 & n-1 \\ -1 & 2 \end{vmatrix},\quad \begin{vmatrix} n+1 & n-1 & . \\ -1 & 2 & n-1 \\ . & -1 & 2 \end{vmatrix},\ \ldots\ldots$$

and the second into

$$2,\ \begin{vmatrix} 2 & n-1 \\ -1 & 2 \end{vmatrix},\quad \begin{vmatrix} 2 & n-1 & . \\ -1 & 2 & n-1 \\ . & -1 & 2 \end{vmatrix},\ \ldots\ldots;$$

consequently, a connection with continued fractions is inevitable. For perfect agreement, however, with the convergents of a continued fraction, each continuant of the first series ought to be of a higher order than the corresponding continuant of the second series, it being necessary, in fact, that the latter should be the first principal minor of the former. This remaining obstacle is readily overcome on trial, the first series being further transformable into

$$\begin{vmatrix} 1 & n-1 \\ -1 & 2 \end{vmatrix}, \quad \begin{vmatrix} 1 & n-1 & . \\ -1 & 2 & n-1 \\ . & -1 & 2 \end{vmatrix}, \quad \begin{vmatrix} 1 & n-1 & . & . \\ -1 & 2 & n-1 & . \\ . & -1 & 2 & n-1 \\ . & . & -1 & 2 \end{vmatrix}, \ldots\ldots;$$

whence it follows that the continued fraction

$$1 + \frac{n-1}{2\,+}\,\frac{n-1}{2\,+}\,\frac{n-1}{2\,+}\cdots$$

furnishes exactly the same series of convergents as the new process. The fact that $\sqrt{n}$ is equal to this continued fraction is included in the already known theorem

$$\sqrt{n} = m + \frac{n-m^2}{2m\,+}\,\frac{n-m^2}{2m\,+}\,\frac{n-m^2}{2m\,+}\cdots$$

a particular case of which, viz., $n = 18$, $m = 4$ dates back to 1613, seven years before the birth of Brounker.*

It may be noted in passing that in obtaining, as we have done, a pair of unlike expressions for A and for B in the identity

$$(\sqrt{n}+1)^r = \mathrm{A}\sqrt{n} + \mathrm{B},$$

the equating of the members of each pair furnishes us with the evaluation of two continuants; that is to say, we have

$$\begin{vmatrix} 1 & n-1 & . & . & \ldots\ldots \\ -1 & 2 & n-1 & . & \ldots\ldots \\ . & -1 & 2 & n-1 & \ldots\ldots \\ . & . & -1 & 2 & \ldots\ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots\ldots \end{vmatrix}_r = \frac{(\sqrt{n}+1)^r + (-1)^r(\sqrt{n}-1)^r}{2},$$

* See Libri, *Hist. des Sciences Math. en Italie*, iv. pp. 87–98.

and

$$\left|\begin{array}{ccccc} 2 & n-1 & . & . & \cdots\cdots \\ -1 & 2 & n-1 & . & \cdots\cdots \\ . & -1 & 2 & n-1 & \cdots\cdots \\ . & . & -1 & 2 & \cdots\cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{array}\right|_r = \frac{(\sqrt{n}+1)^{r+1}+(-1)^r(\sqrt{n}-1)^{r+}}{2\sqrt{n}} .$$

In conclusion, and in a line or two, it may be pointed out how the expressions for the coefficients corresponding to A and B in the higher cases may be found. Taking the case of $(n^{\frac{2}{3}}+n^{\frac{1}{3}}+1)^r$, and denoting the rational integral functions which these coefficients are of n by ϕ, χ, ψ, we have

$$(n^{\frac{2}{3}}+n^{\frac{1}{3}}+1)^r = \phi(n).n^{\frac{2}{3}} + \chi(n).n^{\frac{1}{3}} + \psi(n) ;$$

and if in this we write $an^{\frac{1}{3}}$ for $n^{\frac{1}{3}}$, and multiply both sides by a, a being one of the prime cube roots of unity, there results

$$a(a^2n^{\frac{2}{3}}+an^{\frac{1}{3}}+1)^r = \phi(n).n^{\frac{2}{3}} + \chi(n).a^2n^{\frac{1}{3}} + \psi(n).a ;$$

also, by writing $a^2n^{\frac{1}{3}}$ for $n^{\frac{1}{3}}$, and multiplying both sides by a^2, we obtain

$$a^2(an^{\frac{2}{3}}+a^2n^{\frac{1}{3}}+1)^r = \phi(n).n^{\frac{2}{3}} + \chi(n).an^{\frac{1}{3}} + \psi(n).a^2 .$$

Now from these three by addition $\chi(n)$ and $\psi(n)$ are eliminated, and we determine

$$\phi(n) = \frac{(n^{\frac{2}{3}}+n^{\frac{1}{3}}+1)^r + a(a^2n^{\frac{2}{3}}+an^{\frac{1}{3}}+1)^r + a^2(an^{\frac{2}{3}}+a^2n^{\frac{1}{3}}+1)^r}{3n^{\frac{2}{3}}} .$$

Similarly

$$\chi(n) = \frac{(n^{\frac{2}{3}}+n^{\frac{1}{3}}+1)^r + a^2(a^2n^{\frac{2}{3}}+an^{\frac{1}{3}}+1)^r + a(an^{\frac{2}{3}}+a^2n^{\frac{1}{3}}+1)^r}{3n^{\frac{1}{3}}} ,$$

and

$$\psi(n) = \frac{(n^{\frac{2}{3}}+n^{\frac{1}{3}}+1)^r + (a^2n^{\frac{2}{3}}+an^{\frac{1}{3}}+1)^r + (an^{\frac{2}{3}}+a^2n^{\frac{1}{3}}+1)^r}{3} .$$

A Preliminary Communication on the Electrical Resistance of Various Urines. By Dawson Turner, M.D., F.R.C.P. Edin., M.R.C.P. London.

(Read December 21, 1891.)

The object of this enquiry was to ascertain and compare the electrical resistances of various kinds of urine, both in states of health and of disease.

The measurements were made by means of a Wheatstone's bridge, with alternating currents, and a telephone according to Kohlrausch's method.

The fluids were held in a U-shaped electrolysis tube (a drawing of which accompanies this paper), of an internal diameter of about 1·55 centimetres, and containing a delicate thermometer. Two disc shaped platinum electrodes, each with a diameter of 1·4 c'm., were arranged so as just to rest upon the surfaces of the urine, with a very thin film of the fluid upon their upper surfaces.

The electrodes were connected to the bridge by stout copper wires.

An endeavour was made to maintain the temperature during the experiments at 65° Fahrenheit; and where this was not the case, a correction for temperature, from the result of some observations conducted for that purpose, has been made.

The amount of urine used in each experiment was 21 c'c's.

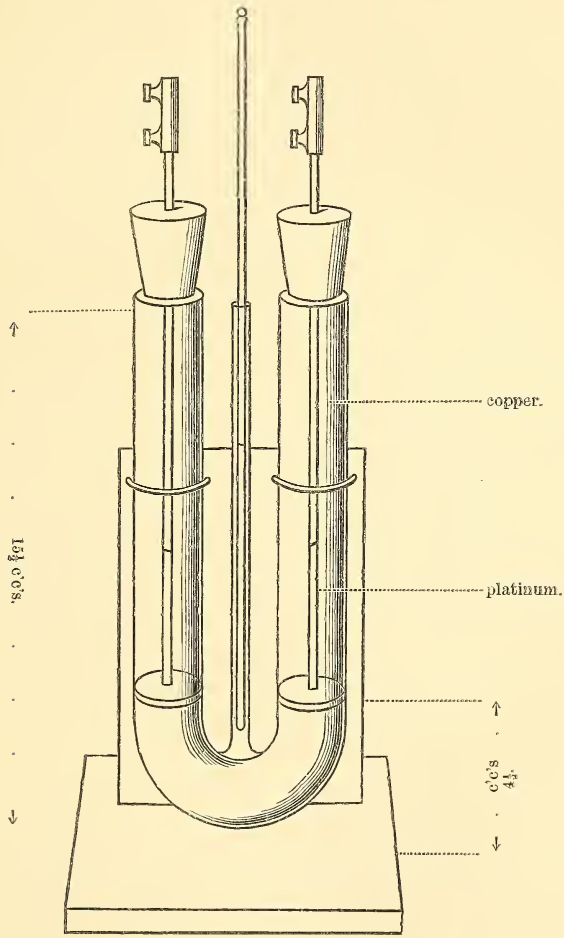
I made an ordinary physical and chemical examination of each urine whose electrical resistance was tested, taking the specific gravity, reaction, colour, amount of urea, in certain cases also of the chlorides, and, if present, of sugar and of albumen.

About three hundred experiments under these conditions have been made.

With regard to the specific resistances, which together with the observed resistances are given in the Table, I found that the resistance of a saturated solution of zinc sulphate amounted under the above conditions (*i.e.*, those under which all the urines were tested) to 125 ohms.; if we take, then, the specific resistance of such a solution to be 21·5 ohms. as given in Lupton's tables, we find that the numerical ratio of the specific to the measured resistance is 21·5/125

= ·17. The specific resistances of the various urines may therefore be approximately found by multiplying the observed resistances by ·17.

The specific resistance of a normal urine amounts on the average, from my observations, to about 40 ohms.



RESULTS.

Put *very briefly*, I find that the resistance of a urine varies as a rule inversely with the amount of the solids it holds in solution ;

when its specific gravity is high, and when it holds in solution much urea and especially salts, its resistance is low, and, *vice versa*, where the specific gravity is low the resistance is high.

To this general rule—" that the resistance varies inversely with the specific gravity"—there are certain exceptions, which can be arranged accordingly as they occur in acute or in chronic diseases.

Amongst the former, acute croupous pneumonia, amongst the latter diabetes mellitus, are the most notable.

That the urine of a case of acute croupous pneumonia should offer a higher electrical resistance than would be predicated from its specific gravity is easily understood, when the great diminution of the chlorides in the urine of a case of this disease is remembered; but the increased resistance of a diabetic urine affords at first sight a question of a more interesting nature.

The field of inquiry, if we include all diseases, being a very wide one, has been restricted in this paper mainly to the consideration of the resistances of the urines of people in apparent good health, and of those suffering from the more *chronic* diseases, excluding also local surgical affections of the urinary organs.

Of all the diseases of this nature, the resistances of the urines of which I have been able to test, that of diabetes mellitus alone affords a tolerably constant exception to the rule already laid down. In this disease the specific gravity of the urine is high, and the electrical resistance offered by it is also high, and sometimes very high.

According to various authorities (Professor V. Jaksch, page 256; MacMunn, pages 36 and 115, &c.), the urea and chlorides shew a greatly increased excretion in this disease: by this they must mean a total increase, and not one relative to the amount of water passed.

From some experiments made by means of artificial solutions of urea, sodium chloride, sugar, &c., to ascertain the influence exercised by the various constituents of the urine upon the electrical resistance, it would appear that this is almost wholly dependent upon the amount of the salts (electrolytes) present; if these be absent, the resistance is determined mainly by the amount of the urea.

A diabetic urine offers (so far as the experiments at present made go) no exception to this rule; its increased resistance is dependent upon the diminished amount of the salts relative to the amount of water, while the sp. gr. is raised by the sugar. Further experiments in these directions are, however, desirable.

For all that, the resistance usually diminishes with the amount of sugar passed and the return to a more normal condition, and may be utilised as a test of the patient's progress.

What influence, if any, other chronic diseases, and what influence medicines and diet, may have in causing exceptions to the rule, "that the resistance varies inversely to the specific gravity," I am at present unable to say, for the results so far obtained by me in them are not in sufficient accordance to permit of any conclusion being drawn.

I feel myself, however, in a position to state, that the healthier the condition of the kidneys, *other things being equal*, the lower will be the electrical resistance of the urine, and *vice versa*, where the kidneys are failing, the resistance will be increased.

This would, *a priori*, have been expected, and is subject to the general rule laid down.

Further, where in a urine of average or more than average specific gravity the resistance is high, the presence of grape sugar may be suspected.

I append, for purposes of comparison, a short table of the observed and specific resistances of some of the typical normal and diabetic urines examined by me; and if permitted, I hope in a future communication to be able to consider the electrical resistance, or its reciprocal, the conductivity, in other and more acute diseases.

In conclusion, I desire to express my great indebtedness to Dr J. O. Affleck, physician to the Royal Infirmary, not only for many of the specimens of urine obtained from patients under his care, but also and particularly for the kind encouragement he has continually extended to me in prosecuting this inquiry. My grateful thanks are also due to Dr Milne Murray for advice and assistance in arranging some of the electrical details.

Table of observed and of specific Electrical Resistances in Various Urines.

No.	Disease.	Sp. gr.	Reaction.	Temperature.	Urea in grs. per ounce.	Albumen.	Sugar in grs. per ounce.	Observed resistance (Ohms.).	Specific resistance (Ohms.).
1	0	1026	Acid	65°	11·	0	0	185	31·45
2	0	1024½	,,	,,	11·7	0	0	185	31·45
3	0	1027	,,	,,	10·29	0	0	195	33·15
4	0	1027	Neutral	,,	12·16	0	0	215	36·55
5	0	1027	Acid	,,	13·1	0	0	227	38·59
6	0	1017	Neutral	,,	7·4	0	0	258	43·86
7	0	1012	Acid	,,	4·21	0	0	370	62·9
8	0	1009	,,	,,	5·85	0	0	435	73·9
9	0	1010	,,	,,	1·5	0	0	700	119
10	Exoph. goitre	1005	,,	,,	1·4	0	0	900	153
11	Diabetes Mellitus	1034	,,	,,	4·68	0	25	920	156·4
12	,,	1034	,,	,,	6·08	0	27	820	139·4
13	,,	1034½	,,	,,	5·6	0	30	775	131·75
14	,,	1034	,,	,,	6·08	0	27	760	129·2
15	,,	1040	,,	,,	8·1	0	28	732	124·44
16	,,	1025	,,	,,	6·08	Trace	14	540	91·8
17	,,	1036	,,	,,	13·1	0	17	320	54·4
18	Artificial diabetes*	1040	,,	,,	10·76	0	30	220	37·4

* 30 grs. of grape sugar per oz. added to a normal urine.

Note on a Problem of Elimination connected with Glissettes of an Ellipse or Hyperbola. By Thomas Muir, LL.D.

(Read January 19, 1891.)

The problem occurs in a paper of Professor Tait's ("Glissettes of an Ellipse and of a Hyperbola," *Proc. Roy. Soc. Edin.*, xvii. pp. 2–4). An ellipse whose semi-axes are a and b is considered as moving so as always to be in contact with both axes of coordinates, and the glissette in question is the curve traced out during this motion by a point whose coordinates with respect to the axes of the ellipse are p and q. Professor Tait states, and it is readily seen, that if the current coordinates of the point be x and y, and θ be the variable angle made by one of the axes of the ellipse with the axis of x, we have the equations

$$\left.\begin{array}{l} x=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}+p\cos\theta-q\sin\theta \\ y=\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}+p\sin\theta+q\cos\theta \end{array}\right\}$$

or

$$\left.\begin{array}{ll} (x-p\cos\theta+q\sin\theta)^2=a^2\cos^2\theta+b^2\sin^2\theta \quad . \quad . \quad . \quad . & (1) \\ (y-p\sin\theta-q\cos\theta)^2=a^2\sin^2\theta+b^2\cos^2\theta \quad . \quad . \quad . \quad . & (2) \end{array}\right\}$$

and that to obtain the x-and-y equation of the glissette it remains to eliminate θ between these two equations.

Before attempting the elimination I first satisfied myself that the resultant must be, as affirmed by Professor Cayley, of the 8th degree.

From (1) and (2) by addition there is obtained

$$2(qx-py)\sin\theta=2(px+qy)\cos\theta+a^2-p^2+b^2-q^2-x^2-y^2 \quad . \quad (S),$$

and, on squaring both sides of this, and putting for shortness' sake

$$\begin{aligned} a^2-p^2&=A, \\ b^2-q^2&=B, \\ x^2+y^2&=r^2, \end{aligned}$$

there results the quadratic in $\cos\theta$

$$4(p^2+q^2)r^2\cos^2\theta + 4(A+B-r^2)(px+qy)\cos\theta \\ +(A+B-r^2)^2-4(qx-py)^2=0 \quad . \quad . \quad . \quad . \quad (\alpha).$$

Again, if in (1) and (2) the terms involving the first power of $\sin\theta$ be alone retained on the left-hand side, $\sin\theta$ may be eliminated between the two equations by division, and thus a second quadratic in $\cos\theta$ obtained, viz.,

$$\{(A-B+2p^2)qx+(B-A+2q^2)py\}\cos^2\theta \\ +\{(A+B)pq-pqr^2-2(p^2+q^2)xy\}\cos\theta \\ +\{px^2y+qxy^2-Aqx-Bpy\}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta).$$

This, it may be noted, is the equation also arrived at by using (S) to eliminate $\sin\theta$ from either (1) or (2).

Writing now the equations (α) and (β), for shortness' sake, in the forms

$$\left.\begin{array}{l} L_2\cos^2\theta + M_3\cos\theta + N_4 = 0 \\ \lambda_1\cos^2\theta + \mu_2\cos\theta + \nu_3 = 0 \end{array}\right\} ,$$

where the suffixes indicate the degrees of the coefficients as regards x and y, the resultant desired is known to be

$$\begin{vmatrix} . & L_2 & M_3 & N_4 \\ L_2 & M_3 & N_4 & . \\ . & \lambda_1 & \mu_2 & \nu_3 \\ \lambda_1 & \mu_2 & \nu_3 & . \end{vmatrix} = 0 ,$$

or

$$\begin{vmatrix} L_2\mu_2-\lambda_1M_3 & L_2\nu_3-\lambda_1N_4 \\ L_2\nu_3-\lambda_1N_4 & M_3\nu_3-\mu_2N_4 \end{vmatrix} = 0 .$$

This equation is seen to be of the 10^{th} degree, and the difficulty—not by any means a small one—has to be attacked of finding the extraneous quadratic factor. After many trials a fortunate observation led to the discovery that the factor is

$$(qx-py)^2,$$

and then a lengthy and inelegant transformation brought about its ejection and the final equation

$0 =$

	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x	x^0
y^0	Ω		-4Θ		2χ		$-4\sigma^2\Theta$		$\sigma^4\Omega$
y^1		$-16pq(a^2-b^2)$		$32pq(a^2-b^2)\rho$		$-16pq(a^2-b^2)\Lambda$			
y^2			4Γ		-4Ψ		4Δ		$-4\sigma^2\Theta$
y^3				$-16pq(a^2-b^2)$				$16pq(a^2-b^2)\Lambda$	
y^4					2Φ		-4Ψ		2χ
y^5						$16pq(a^2-b^2)$		$-32pq(a^2-b^2)\rho$	
y^6							4Γ		-4Θ
y^7								$16pq(a^2-b^2)$	
y^8									Ω

where

$$\begin{aligned}\Omega &= a^4+b^4+p^4+q^4-2a^2b^2-2a^2p^2+2a^2q^2+2b^2p^2-2b^2q^2+2p^2q^2,\\ &= (p^2+q^2)^2-2(p^2-q^2)(a^2-b^2)+(a^2-b^2)^2,\\ &= (a^2-b^2-p^2+q^2)^2+(2pq)^2;\end{aligned}$$

$$\begin{aligned}\Gamma &= a^4+b^4+p^4+q^4-2a^2b^2+2a^2p^2-2a^2q^2-2b^2p^2+2b^2q^2+2p^2q^2,\\ &= (p^2+q^2)^2+2(p^2-q^2)(a^2-b^2)+(a^2-b^2)^2,\\ &= (a^2-b^2+p^2-q^2)+(2pq)^2,\\ &= \Omega+4(p^2-q^2)(a^2-b^2);\end{aligned}$$

$$\begin{aligned}\Phi &= 3a^4+3b^4+3p^4+3q^4-6a^2b^2+10a^2p^2-10a^2q^2-10b^2p^2+10b^2q^2+6p^2q^2,\\ &= 3(p^2+q^2)^2+10(p^2-q^2)(a^2-b^2)+3(a^2-b^2)^2,\\ &= 3\Gamma+4(p^2-q^2)(a^2-b^2);\ \text{and } \therefore\ \Phi+\Omega=4\mathrm{T};\end{aligned}$$

$$\begin{aligned}\Theta &= \left.\begin{aligned}&a^6+b^6+p^6+q^6-a^4b^2-a^4p^2+a^4q^2-b^4a^2+b^4p^2-b^4q^2\\ &\quad -p^4a^2+p^4b^2+3p^4q^2+q^4a^2-q^4b^2+3q^4p^2\end{aligned}\right\},\\ &= (p^2+q^2)^3-(p^4-q^4)(a^2-b^2)-(a^4-b^4)(p^2-q^2)+(a^4-b^4)(a^2-b^2);\end{aligned}$$

$$\begin{aligned}\Psi &= \left.\begin{aligned}&3a^6+3b^6+3p^6+3q^6-3a^4b^2+5a^4p^2-5a^4q^2-3b^4a^2-5b^4p^2+5b^4q^2\\ &\quad +5p^4a^2-5p^4b^2+9p^4q^2-5q^4a^2+5q^4b^2+9q^4p^2\end{aligned}\right\},\\ &= 3(p^2+q^2)^3+5(p^4-q^4)(a^2-b^2)+5(a^4-b^4)(p^2-q^2)+3(a^4-b^4)(a^2-b^2),\\ &= 3\Theta+8(p^2-q^2)(a^2-b^2)\rho;\end{aligned}$$

$$\rho = a^2+b^2+p^2+q^2;$$

$$\chi = \left.\begin{aligned}&3a^8+3b^8+3p^8+3q^8-4a^6p^2-4b^6q^2-4p^6a^2-4q^6b^2+12p^6q^2+12q^6p^2\\ &\quad -6a^4b^4+2a^4p^4+2a^4q^4+2b^4p^4+2b^4q^4+18p^4q^4\\ &\quad +4a^4b^2q^2-4a^4p^2q^2+4b^4a^2p^2-4b^4p^2q-4p^4b^2q^2\\ &\quad -8p^4a^2q^2-4q^4a^2p^2-8q^4b^2p^2\\ &\quad +16a^2b^2p^2q^2;\end{aligned}\right\}$$

$$\Lambda = a^4 + b^4 + p^4 + q^4 + 2a^2b^2 - 2a^2p^2 + 2a^2q^2 + 2b^2p^2 - 2b^2q^2 + 2p^2q^2 ,$$
$$= (p^2 + q^2)^2 - 2(p^2 - q^2)(a^2 - b^2) + (a^2 + b^2)^2 ,$$
$$= \Omega + (2ab)^2 ;$$

$$\Delta = \left.\begin{array}{l} 3a^8 + 3b^8 + 3p^8 + 3q^8 - 4a^6q^2 - 4b^6p^2 - 4p^6b^2 - 4q^6a^2 + 12p^6q^2 + 12q^6p^2 \\ \quad - 6a^4b^4 - 6a^4p^4 - 6a^4q^4 - 6b^4p^4 - 6b^4q^4 + 18p^4q^4 \\ \quad + 4a^4b^2p^2 + 12a^4p^2q^2 + 4b^4a^2q^2 + 12b^4p^2q^2 - 4p^4a^2q^2 \\ \quad - 8p^4b^2q^2 - 4q^4b^2p^2 - 8q^4a^2p^2 \\ \quad - 48a^2b^2p^2q^2 ; \end{array}\right\}$$

$$\sigma = a^2 + b^2 - p^2 - q^2 .$$

A complicated result like this, dependent upon lengthy calculations, is more of a hindrance than a help, unless it can be perfectly relied upon.

One test as to its accuracy is suggested by the constitution of the original equations (α) and (β), which on examination will be seen to be symmetrical with respect to the interchange

$$\left\{\begin{array}{cc} a & b \\ p & q \\ x & y \end{array}\right\} .$$

Now a glance at Ω, Γ, Φ, Θ, Ψ, ρ, χ, Λ, Δ, σ will be sufficient to show that they are all symmetrical with respect to the interchange

$$\left\{\begin{array}{cc} a & b \\ p & q \end{array}\right\} ,$$

and this being ascertained it is at once manifest that the resultant is, as it ought to be, symmetrical with respect to the same interchange as the original equations.

Next, there is the test of its agreement with the results otherwise obtainable for special cases,—a test which if satisfied is additionally instructive in that it makes evident the mode in which the degenerate forms of the equations arise.

Taking in the first place the case where $p = q = 0$. We have

$$\Omega = (a^2 - b^2)^2 = \Gamma = \tfrac{1}{3}\Phi, \qquad \chi = 3(a^4 - b^4)^2 = \Delta,$$
$$\Theta = (a^4 - b^4)(a^2 - b^2) = \tfrac{1}{3}\Psi, \quad \Lambda = (a^2 + b^2)^2,$$
$$\rho = a^2 + b^2 = \sigma,$$

and the equation becomes

$$(a^2 - b^2)^2\{(x^2+y^2) - (a^2+b^2)\}^4 = 0,$$
$$i.e., \qquad x^2+y^2 = a^2+b^2,$$

as it should be.

Secondly, for the case where the tracing point is a *focus* of the ellipse, that is, where $p^2 = a^2 - b^2$ and $q = 0$, we find

$$\Omega = 0, \qquad \chi = 8b^4(a^2-b^2)^2$$
$$\Gamma = 4(a^2-b^2)^2 = \tfrac{1}{4}\Phi, \qquad \Lambda = 4a^2b^2,$$
$$\Theta = 0, \qquad \sigma = 2b^2\,;$$
$$\Psi = 16a^2(a^2-b^2)^2,$$
$$\rho = 2a^2,$$

and the equation takes the form

$$16(a^2-b^2)^2(x^2+y^2)\{(x^2+y^2)x^2y^2 - 4a^2x^2y^2 + b^4(x^2+y^2)\} = 0,$$
$$i.e., \qquad (x^2+y^2)(x^2y^2+b^4) = 4a^2x^2y^2\,;$$

which is readily seen to be correct, because the original equations then become

$$\left.\begin{array}{l} 2px\cos\theta + b^2 - x^2 = 0 \\ 2py\sin\theta + b^2 - y^2 = 0 \end{array}\right\}$$

and thus give

$$\left(\frac{x^2-b^2}{2px}\right)^2 + \left(\frac{y^2-b^2}{2py}\right)^2 = 1,$$

$$i.e., \qquad x^2 - 2b^2 + \frac{b^4}{x^2} + y^2 - 2b^2 + \frac{b^4}{y^2} = 4p^2 = 4a^2 - 4b^2,$$

$$i.e., \qquad (x^2+y^2)\left(1 + \frac{b^4}{x^2y^2}\right) = 4a^2,$$

$$i.e., \qquad (x^2+y^2)(x^2y^2+b^4) = 4a^2x^2y^2.$$

Thirdly, there is the very interesting case where the tracing point is an extremity of an axis; say, where $p = 0$, $q = b$. In this case we find

$$\Omega = a^4, \qquad \rho = a^2 + 2b^2,$$
$$\Gamma = (a^2-2b^2)^2, \qquad \sigma = a^2,$$
$$\Phi = 3a^4 - 16a^2b^2 + 16b^4, \qquad \chi = 3a^8,$$
$$\Theta = a^6, \qquad \Lambda = a^4 + 4a^2b^2,$$
$$\Psi = 3a^6 - 8a^4b^2 - 8a^2b^4 + 16b^6, \qquad \Delta = 3a^8 - 4a^6b^2 - 12a^4b^4\,;$$

and the equation takes at the outset the rather awkward form

$$
\begin{aligned}
&a^4x^8 + 4(a^2 - 2b^2)^2x^6y^2 + 2(3a^4 - 16a^2b^2 + 16b^4)x^4y^4 + 4(a^2 - 2b^2)^2x^2y^6 + a^4y^8 \\
&- 4a^6x^6 - 4(3a^6 - 8a^4b^2 - 8a^2b^4 + 16b^6)x^4y^2 - 4(3a^6 - 8a^4b^2 - 8a^2b^4 + 16b^6)x^2y^4 - 4a^6y^6 \\
&+ 6a^8x^4 + 4(3a^8 - 4a^6b^2 - 12a^4b^4)x^2y^2 + 6a^8y^4 \\
&- 4a^{10}x^2 - 4a^{10}y^2 \\
&+ a^{12} = 0 .
\end{aligned}
$$

A suspicion attaches to the coefficient of x^2y^2, which, judging from the forms of the other coefficients, we should consider to be incorrect in not having a term in a^2b^6. This suspicion is intensified on proceeding with the simplification of the equation. In the first line we observe that the terms of the expansion of $a^4(x^2+y^2)^4$ occur, and following up this observation we transform the equation into

$$
\begin{aligned}
&a^4(x^2+y^2)^4 + 16b^2(b^2-a^2)x^6y^2 + 32b^2(b^2-a^2)x^4y^4 + 16b^2(b^2-a^2)x^2y^6 \\
&- 4a^6(x^2+y^2)^3 + 32b^2(a^4+a^2b^2-2b^4)x^4y^2 + 32b^2(a^4+a^2b^2-2b^4)x^2y^4 \\
&+ 6a^8(x^2+y^2)^2 - 16a^2b^2(a^4+3a^2b^2)x^2y^2 \\
&- 4a^{10}(x^2+y^2) \\
&+ a^{12} = 0 .
\end{aligned}
$$

But the sum of the first terms of all the lines here is evidently $a^4(x^2+y^2-a^2)^4$, consequently we make the further transformation

$$
\begin{aligned}
a^4(x^2+y^2-a^2)^4 &+ 16b^2(b^2-a^2)x^2y^2(x^2+y^2)^2 \\
&+ 32b^2(a^2+2b^2)(a^2-b^2)x^2y^2(x^2+y^2) \\
&- 16a^2b^2(a^4+3a^2b^2)x^2y^2 = 0 .
\end{aligned}
$$

Now, were it not for the absence of the term above referred to, a most important simplification could have been effected. This is best seen by actually supplying such a term, viz., $-4b^4$ in the bracketed portion of the coefficient of x^2y^2. We then obtain

$$
a^4(x^2+y^2-a^2)^4 - 16b^2(a^2-b^2)x^2y^2\begin{pmatrix} (x^2+y^2)^2 \\ -2(a^2+2b^2)(x^2+y^2) \\ +a^2(a^2+4b^2) \end{pmatrix} = 64a^2b^6x^2y^2 ;
$$

and since the quadratic expression here in x^2+y^2 resolves into two factors $(x^2+y^2)-a^2$ and $(x^2+y^2)-(a^2+4b^2)$, the former factor, $x^2+y^2-a^2$, becomes a factor of the whole of the left-hand side, and would with a more complete-looking coefficient of x^2y^2, have disappeared altogether and left for this particular case of the glissette an equation of the 6th degree. As a matter of fact the equation takes the form

$$a^4(x^2+y^2-a^2)^4 - 16b^2(a^2-b^2)x^2y^2(x^2+y^2-a^2)(x^2+y^2-a^2-4b^2) = 64a^2b^6x^2y^2.$$

The question therefore is whether a separate investigation of the equation for the case $p=0$, $q=b$ would confirm or remove the suspicion raised in regard to the coefficient of x^2y^2. When $p=0$ and $q=b$ the original equations become

$$\left.\begin{array}{l} x^2 - 2bx\sin\theta = a^2\cos^2\theta \\ y^2 - 2by\cos\theta = a^2\sin^2\theta \end{array}\right\}$$

From these we can derive two simple equations in $\sin\theta$, $\cos\theta$, viz. :

$$\left.\begin{array}{r} 2bx\sin\theta - 2by\cos\theta = -\mathrm{P} \\ (a^2x\mathrm{P} - 4b^2xy^2)\sin\theta + (a^2y\mathrm{P} - 4b^2x^2y)\cos\theta = 0 \end{array}\right\}$$

where P stands for $x^2+y^2-a^2$. Solving for $\sin\theta$, $\cos\theta$ we obtain

$$\left\{\frac{\mathrm{P}(a^2\mathrm{P}-4b^2y^2)}{4by(a^2\mathrm{P}-2b^2r^2)}\right\}^2 + \left\{\frac{\mathrm{P}(4b^2x^2-a^2\mathrm{P})}{4bx(a^2\mathrm{P}-2b^2r^2)}\right\}^2 = 1,$$

which readily leads to

$$a^4\mathrm{P}^4r^2 - 16a^2b^2x^2y^2\mathrm{P}^2(\mathrm{P}+a^2) + 16b^4\mathrm{P}^2x^2y^2r^2 = -64a^2b^4x^2y^2\mathrm{P}r^2 + 64b^6x^2y^2r^4,$$

so that, on striking out the factor x^2+y^2, we have the equation in the form

$$a^4\mathrm{P}^4 - 16b^2x^2y^2(a^2-b^2)\mathrm{P}^2 = -64b^4x^2y^2\{a^2\mathrm{P} - b^2(\mathrm{P}+a^2)\},$$

or

$$= -64b^4x^2y^2(a^2-b^2)\mathrm{P} + 64a^2b^6x^2y^2,$$

or

$$a^4\mathrm{P}^4 - 16b^2x^2y^2(a^2-b^2)\mathrm{P}(\mathrm{P}-4b^2) = 64a^2b^6x^2y^2;$$

and this agrees to the letter with the suspected result obtained by specializing from the general equation.

There thus, I think, can be little doubt that the calculation of the coefficients in the general equation has been accurately performed.

On the Foundations of the Kinetic Theory of Gases. V. By Professor Tait.

(*Abstract.*)

(Read January 18 and February 15, 1892.)

The first instalment of this part of my paper deals mainly with the theory of the behaviour of *mixtures* of CO_2 and N, for which some remarkable experimental results were given by Andrews about 1874. His full paper, so far as he had drawn it up for press, was published posthumously in the *Phil. Trans.* for 1886, and is reprinted in his *Scientific Papers*, No. L. One special reason for the introduction of this question at the present stage of my work was the desire to attempt a correction of Amagat's numbers, for the (very small) admixture of air with his CO_2. The virial equation for a mixture is formed on the same general principle as that I employed for a single gas. There are, of course, more undetermined constants :—and, unfortunately, the data for their determination are barely adequate. The general results, however, agree in character with those described by Andrews :—the particular phenomenon which is most closely studied being the increase of volume, at constant pressure, when the gases (originally separated by the liquefaction of one) were allowed to diffuse into one another.

Since Part IV. of this paper was printed, M. Amagat has published (*Comptes Rendus*, October 12, 1891) additional data of a most valuable character bearing on the isothermals of CO_2 :—especially the very important isothermal of 32° C. ; and he has given the pressure of the saturated vapour at 0°, 10°, 20°, and 30° C. I have endeavoured to utilize these, as far as possible, not only for my present main object :—the examination of the relation between temperature and kinetic energy :—but also, incidentally, for the determination of the latent heat of the saturated vapour at various temperatures, and the relative densities of the liquid and vapour when in equilibrium. These data have also enabled me to obtain more exact approximations to the values of the

constants in my formula, and thence to improve my determinations of the critical temperature, pressure, and volume.

In § 71 of Part IV. I arrived at the conclusion that "in a liquid the temperature is no longer measured by E [the part of the kinetic energy which is independent of the molecular forces], but by E + c, where c is a quantity whose value increases steadily, as the temperature is lowered, from the value zero at the critical point." For numerical data to test this conclusion, I study a cycle formed from the critical isothermal and any lower one, and two lines of equal volume, corresponding to those of the liquid and the saturated vapour when in equilibrium at that lower temperature. The change of energy in passing from one of these limits of volume to the other is found to be less for the critical isothermal than for any lower one. Thus the mean specific heat at constant volume, for the range of temperature employed, is less in the vapour than in the liquid. But from the equation, which is found to satisfy very closely the data for the isothermals of the gas for some 70 degrees above the critical point and of the vapour for 30 degrees below that point, it appears that the specific heat at constant volume is sensibly constant within these limits. [At 100° C. and upwards, it appears that $\frac{dp}{dt}$ falls off; so that $\frac{d^2p}{dt^2}$ is negative, and the specific heat at constant volume is therefore, even in the gas, greater for smaller volumes. But this does not seriously affect the above statement.] Hence, at any volume less than the critical volume, more heat is required to raise the temperature 1 degree when the substance is wholly liquid than when it is gaseous. This completely justifies the statement quoted above, provided that we assume the properties of the liquid and gas to merge continuously into one another at the critical temperature; but, unfortunately, the data are not sufficient to give more than very rough estimates of the value of the quantity c there spoken of.

I am at present engaged in endeavouring to obtain more exact values of the constants in my equation, in order to improve my estimates. Thus the numbers which follow may have to undergo some modifications, but there seems to be no reason for thinking that these are likely to be serious.

If v_1, v_2 be the respective volumes of the saturated vapour, and of the liquid, at absolute temperature t, we know that the latent heat is expressed by the formula—

$$\lambda = t\frac{dp}{dt}(v_1 - v_2).$$

From Amagat's data I find for the values of this quantity, and for the ratio of the densities of the liquid and vapour:—

Temperature C.	λ	v_1/v_2
0°	4·369	9·023
10	3·788	6·200
20	2·882	3·823
30	1·460	1·906

Taking the density of CO_2 at 0° C. and 1 atm. as 0·002, it is easy to see that the values of λ must be multiplied by

$$\frac{500}{62{\cdot}5}\cdot\frac{2117}{1390}, \quad = 12{\cdot}2 \text{ nearly},$$

to reduce them to ordinary heat units. Thus the latent heat at 0° C. is about 53, while at 30° C. it is only 17·8.

In the following table P represents the gain of energy from the liquid state to that of saturated vapour, at the indicated temperature:—*i.e.*,

$$P = \left(t\frac{dp}{dt} - p\right)(v_1 - v_2);$$

while

$$Q = \int_{v_2}^{v_1} \left(t\frac{dp}{dt} - p\right)dv$$

is the corresponding gain of energy, in the critical isothermal, between the same limits of volume.

Temperature C.	P	Q
0°	3·747	3·577
10	3·244	3·113
20	2·459	2·409
30	1·233	1·203

The difference, P – Q, is (when multiplied, as above by 12·2) nearly equal to the excess of the heat required to raise the temperature of the liquid (at constant volume) to the critical point, over that required to raise the temperature of the vapour, from saturation, through the same range, the volume remaining unaltered.

It appears that CO_2, when passing through the range of volume, spoken of in § 69 of Part IV. of my paper, has about half the density of water.

The Lesser Rorqual (*Balænoptera rostrata*) in the Scottish Seas, with Observations on its Anatomy. By Professor Sir Wm. Turner, LL.D., D.C.L., F.R.S.

(Read February 15, 1892.)

It is outside the scope of the present paper to enter into the geographical distribution generally of the Lesser Rorqual or Pike Whale (*Balænoptera rostrata*), and indeed there is no reason why I should do so, as it has so recently been considered by Professor Van Beneden, in his *Histoire naturelle des Balénoptères.** For a number of years it has been my custom to collect information regarding the Cetacea captured or stranded on the Scottish coasts, and when possible to obtain the animals or their skeletons. I have accumulated, therefore, a considerable body of information regarding the whales frequenting the Scottish seas, some of which I have already published. In the present communication it is my intention to record some facts regarding *Balænoptera rostrata*, and I may say that when a newspaper paragraph relates the capture of a whalebone whale, and, when measurements are given, if the length is under 30 feet, the animal is most probably the Lesser Rorqual.

It is difficult to fix with precision the first notice of the capture of this animal on the Scottish coast, as it is only within the last thirty or forty years that the different species of *Balænoptera* have been properly discriminated from each other. The two whalebone whales described by Sir Robert Sibbald in his classical treatise *Phalainologia nova*,† were much too long to be of this species. The one caught near Abercorn in September 1692 was 78 feet long. In my memoir on the Longniddry whale,‡ I identified it and Sibbald's specimen as in all probability the same species, *i.e.*, *Balænoptera sibbaldii*. The other whale, caught at Burntisland in 1690, was 46 feet long, and from its sharp beak was probably an immature specimen of *Balænoptera musculus*. For a similar reason the whale, 46 feet long, which Dr John Walker described as stranded

* Brussels, 1887. † Edinburgh, 1692.

‡ *Trans. Roy. Soc. Edin.*, 1870, vol. xxvi.

at Burntisland in June 1761, and that which Mr Patrick Neill saw stranded at Alloa in 1808,* which was 43 feet long, were also in all likelihood immature examples of *B. musculus.*

Dr Scoresby, in his famous work *On the Arctic Regions*,† described a whalebone whale 17½ feet long, killed in Scalpa Bay, Orkney, in November 1808, which there can be little doubt was *Balænoptera rostrata.* A Finner whale 14 feet long, caught in the stake-nets near Largo, Firth of Forth,‡ in May 1832, was in all probability another example. But the specimen which one can cite as the first definitely determined Scottish example was a young female captured in February 1834 in the stake-nets at Queensferry. It was 9 ft. 11 inches long, and was anatomised by Dr Knox and Mr Frederick Knox.§ When their museum was dispersed, the skeleton was procured for the Anatomical Museum of the University of Edinburgh. Dr Knox referred the animal to the species *rostrata* of Fabricius, though, in the printed catalogue of his museum, it is named *Balæna minimus borealis.* This specimen was contrasted by Knox with the Great Rorqual, taken at North Berwick in 1831, also in his collection, which was called by Knox *Balæna maxima borealis*, and the skeleton of which is now suspended in the Museum of Science and Art, Edinburgh. A number of years ago I identified this skeleton, which it had been customary to regard as *Balænoptera musculus*, as *B. sibbaldii.* From the difference in the number of ribs and dorsal vertebræ in the Lesser as compared with the Greater Rorqual, Knox showed conclusively that the former could not possibly be the young of the latter, but must be a distinct species. In September 1857, a male, 14 ft. 5 inches long, was found near the Bell Rock and taken into Leith. Its characters were described by Dr James M'Bain.|| Mr E. R. Alston refers (Scottish Mammalia, p. 18) to one caught in 1858 in the Firth of Forth.

* *Memoirs of Wernerian Soc.*, vol. i. p. 201.

† Vol. i. p. 485, Edinburgh, 1820. Scoresby figured this specimen from a drawing by Jas. Watson, Esq., in the possession of Dr Traill.

‡ *Edinburgh Advertiser*, May 22, 1832. Loudon's *Magazine of Natural History*, vol. v. p. 570, 1832.

§ For references to this specimen, see Jardine's *Naturalist's Library*, volume on Whales, plate vii.; F. Knox in *Trans. New Zealand Inst.*, vol. 2, plate 2 *a*; Knox's *Catalogue of Anatomical Preparations of Whale;* R. Knox in *Proc. Linn. Soc.*, Oct. 1857, and *Journal l'Institut*, 1834.

|| *Proc. Roy. Phys. Soc.*, 1858, vol. i.

In 1866, a large specimen of *Balænoptera rostrata* was cast ashore on the island of Islay, the skull and scapula of which are in the University Museum, Cambridge.* In July 1870, a young female, 14½ feet long, was stranded alive at Aberdeen, and was dissected by Professor Struthers. The skeleton and various soft parts were preserved. Dr Struthers also records another female, 16 feet long, captured at Bervie in April 1877.†

In 1870, I received through Dr Millen Coughtrey, at that time one of my pupils, the two halves of the mandible of a *B. rostrata*, which had been stranded a short time previously at Hillswick, Shetland. In October 1872, a male *Balænoptera*, said to be about 25 feet long, was harpooned at Stornoway by Mr Mackenzie, manager to Mr Methuen's fish-curing establishment. I am indebted to Mr Methuen for the baleen wreath of one side of the palate, the blades of which were yellowish white, and were fringed with whitish hairs. The blades in the middle of the wreath averaged about 8 inches in length, and about 2½ inches in width at the base. The length of the entire wreath was 3 feet 2 inches, and 311 transverse rows of plates were counted in it. The animal was a specimen of *Balænoptera rostrata*. The blubber was said to be very thin, ranging from 1½ to 3 inches. In November 1879, a fisherman at Stromness, Orkney, telegraphed to me that a specimen of "Rorqualus rostratus," 16 feet long, had just been caught there.

Between the years 1870 and 1888 several specimens of *B. rostrata* were captured in the Firth of Forth, portions of each of which I secured for the Anatomical Museum of the University. In September 1870, a female, 18 feet long, was stranded at Sea Mills, a little to the west of Burntisland, and the skull and baleen were presented to the Museum by Mr George Prentice of Newbigging. I was able to inject the vessels of the baleen papillæ in this animal, and I gave an account of my observations in my memoir on the Longniddry whale.‡ In September 1871, a specimen, said to be about 30 feet long, and

* Mr W. Evans, *Proc. Roy. Phys. Soc.*, Session 1890–91, vol. xi. p. 159, refers to a specimen described in the *Scottish Naturalist* in July 1869, by Mr Robert Walker, as stranded near Arbroath: it was 13 feet long.

† See his Memoir on the Anatomy of *Megaptera longimana*, Edinburgh, 1889, and in *Jour. of Anat. and Phys.*, vol. xxii., 1888. I am indebted to Dr Struthers for a note on the Bervie specimen.

‡ *Trans. Roy. Soc. Edin.*, vol. xxvi. p. 216.

believed to be a male, was stranded amongst the rocks opposite the Old Harbour, Dunbar. I secured the skull, the sternum, the hyoid apparatus, the cervical and some other vertebræ, and a first rib, for the Museum, together with some of the baleen plates. The skeleton was fully ossified, for the epiphysial plates were fused with the vertebral bodies. The skull is one of the largest examples of this species in any museum ; and when Professor van Beneden saw it some years ago, he thought at first, from its magnitude, that it was *B. musculus*, but when I showed him the baleen plates, he agreed with me that it was *B. rostrata.** The largest plates in my possession are $8\frac{3}{4}$ inches long and 3 inches wide at the base ; they have the distinctive yellowish-white colour. In April 1872, a young female, $15\frac{1}{2}$ feet long, was captured in the salmon-nets near Anstruther. It was towed into the harbour, and was obtained by the late Dr Woodcock, who presented me with a few blades of the yellowish-white baleen, which are $3\frac{1}{4}$ inches long by $1\frac{1}{4}$ in width at the base. The skull is now in the Museum of Science and Art, Edinburgh. In July 1879, a young male, 18 feet long, was stranded at Elie, and the skull and some other bones were acquired for the University Museum. In October 1888 a very young specimen, the sex of which was not ascertained, was stranded at Alloa, and the skull, with some of the cervical vertebræ, was purchased for the Museum. Each half of the baleen wreath is 2 feet $4\frac{1}{2}$ inches long, the largest plates are $5\frac{1}{2}$ by $1\frac{1}{2}$ inches, and the colour is yellowish-white ; 274 rows of plates were counted in one half, and 270 in the other.

Granton Specimen.—In January 1888 a female *B. rostrata* was seen floundering in the shallows at the entrance to the disused stone quarry at Granton. A boat was launched, ropes were attached to the animal, and after a severe struggle it was hauled ashore. I visited it, as it lay on the beach, along with Mr John Murray, LL.D., a few hours after its capture. Through the courtesy of Mr John Howkins, C.E., a steam tug was obtained, and the whale was towed to Granton Harbour, put on a railway truck, and conveyed to the Marine Station, where it remained on exhibition for some time. I had an opportunity, therefore, of examining the external characters of a full-grown and perfectly fresh specimen of *Balænoptera*

* Van Beneden briefly refers to this specimen as "une tête de grande taille," in his *Histoire Naturelle des Balénoptères*, p. 42, Brussels, 1887.

rostrata, and of making a number of measurements under more favourable conditions than often falls to the lot of the cetologist. I propose, therefore, to give an account of the appearance of this animal. One of my pupils, Mr Edmund Frost, who is an accomplished photographer, took under my direction several negatives. Mr Murray has most liberally presented the skeleton to the Anatomical Museum and through the courtesy of Mr Robert Irvine, F.R.S.E., I was enabled to have the skeleton cleaned by having it buried for four years in a sand-heap at the Granton Oil Works.

General Form.—The head was flattened on the dorsum, and tapered forwards on each side to a pointed beak. The two blow-holes were situated on the dorsum a little anterior to the eyes and the angle of the mouth. They were separated from each other by a mesial depressed septum, 11 inches long, which bifurcated in its posterior half. Immediately in front of the blow-holes, a low mesial ridge, triangular in shape, and with the base behind, extended forward, and gradually subsided near the tip of the beak. This ridge was not due to a subjacent ridge of bone, but was formed of fat and coarse fibrous tissue, covered by integument; at its base it was 5 inches thick in the mesial plane, but in the lateral slope it diminished to $1\frac{1}{2}$ inch in thickness.

The eye was placed immediately above the angle of the mouth, and occupied a definite slit in the integument. An inch and a half above the eye-slit was an antero-posterior depression in the integument, 6 inches long, half an inch in depth at its middle, and tapering at its two ends. A similar but shallower depression, $4\frac{1}{2}$ inches long, was situated $1\frac{1}{4}$ inch below the eye-slit. The external meatus of the ear was a minute opening almost in a direct line with the eye-slit, and 15 inches behind it.

From the dorsum of the head the middle line of the back gradually sloped upwards and backwards until the highest point of the curve of the back was reached, about midway in the length of the animal. The deeply falcate dorsal fin was situated on the curve of the back as it sloped downwards and backwards to the tail. The general form of the body behind the flipper was convex at the sides, and the greatest girth was at a point about 3 feet behind the axilla. The body then tapered backwards, and in the plane of the

FIG. 1.—*Balænoptera rostrata*, stranded at Granton. This and figs. 2 and 3 are from photographs by Mr Edmund Frost.

dorsal fin it became laterally compressed. Behind this fin the sides of the body were still more flattened, and it was "keeled" both in the dorsal and ventral mesial lines. The antero-posterior diameter of the base of the dorsal fin was 19 inches, and its vertical diameter in a straight line was 12 inches. The girth at the root of the tail was only 2 feet 10 inches. The tail itself possessed two large lateral lobes, separated at the posterior border by a mesial notch $9\frac{3}{4}$ inches deep.

The flipper projected backwards and outwards from the side of the body, in a line with the angle of the mouth, and some distance behind it. Its anterior or radial border had a continuous convex curve from the shoulder-joint to the tip. The posterior border formed a convex curve from the axilla backwards, which reached its greatest projection about half-way to the tip, where the flipper had the greatest breadth. Some inches from the tip the posterior border was concave. The tip was attenuated. The dorsal surface was a little convex, the under surface was a little concave and some of the flutings of the belly reached it.

The ventral surface, as is characteristic of the Rorquals, was fluted from the region of the lower jaw backwards nearly to the umbilicus. The highest furrow was on a level with the angle of the mouth, and passed directly backwards for only 10 inches. Below it the furrows gradually increased in length, and extended both further forwards in relation to the side of the lower jaw, and backwards below the root of the flipper to the belly. Those which were nearest the ventral mesial line reached as far forwards as the symphysis menti. The furrow which extended furthest back was the twentieth in order below the articulation of the flipper, but it was not the lowest, for five furrows, gradually diminishing in their extent backwards, intervened between it and the ventral mesial line. The average breadth of the ridges between the furrows immediately below the angle of the mouth was about 1 inch, but further back some were as wide as 2 inches; below the lower jaw they averaged $1\frac{1}{2}$ inch in width.

Seventeen inches behind the end of the twentieth furrow, but in the middle line of the belly, was the umbilical depression, 8 inches long; and 3 feet further back was the genital furrow, in the hinder part of which the orifice of the vagina was seen. The anal orifice was 10 inches behind the vagina. Two inches on each side of the vagina was the mammary furrow, from which the end of the nipple projected; about 2 inches external to this furrow was an antero-posterior accessory furrow in the skin about 8 inches long. The distance from the anal orifice to the mesial notch in the tail was 7 feet 2 inches.

Mouth and Baleen.—The baleen wreath projected downwards from the palatal mucous membrane to the dorsum of the tongue. Each half of the wreath consisted of rows of plates and bristles arranged transversely. In one half 287 rows were counted, in the other, 290. The outermost segment of each row was the large baleen plate; the innermost was little more than a thick bristle. Between these two segments short, narrow plates were situated. At each end of the wreath the plates were also short, and so reduced in size as to be only thick bristles. In the middle of the wreath the plates of the outermost segment of each row were the largest, and measured $8\frac{1}{4}$ inches long by 3 inches in breadth at the base. Each of these plates was triangular, the base being attached to the palatal mucous membrane, the inner edge being fringed with bristles, and directed from above obliquely downwards and outwards; the outer edge being vertical, and with very few bristles projecting from it. The length of each half of the wreath was 3 feet $11\frac{1}{2}$ inches. The plates were all yellowish white, and the bristles projecting from them were similar in colour. The collective series of plates and bristles in each wreath formed an inclined plane sloping from the roof of the mouth downwards and outwards, so that objects taken into the mouth would be collected in the interval between the opposite halves of the wreath, and be entangled by the bristles, whilst the water would filter outwards in the intervals between the rows of plates. Each half of the baleen wreath was curved, and at the anterior ends the short plates and bristles of the wreaths of opposite sides came close together immediately below the point of the beak, whilst posteriorly they began to curve inwards almost opposite the angle of the mouth, so as to be only a short distance asunder immediately above the passage into the pharynx. Nearer the middle of the roof of the mouth they were separated by a wide interval, which narrowed as it was traced forwards and backwards; and in it the mucous membrane covering the

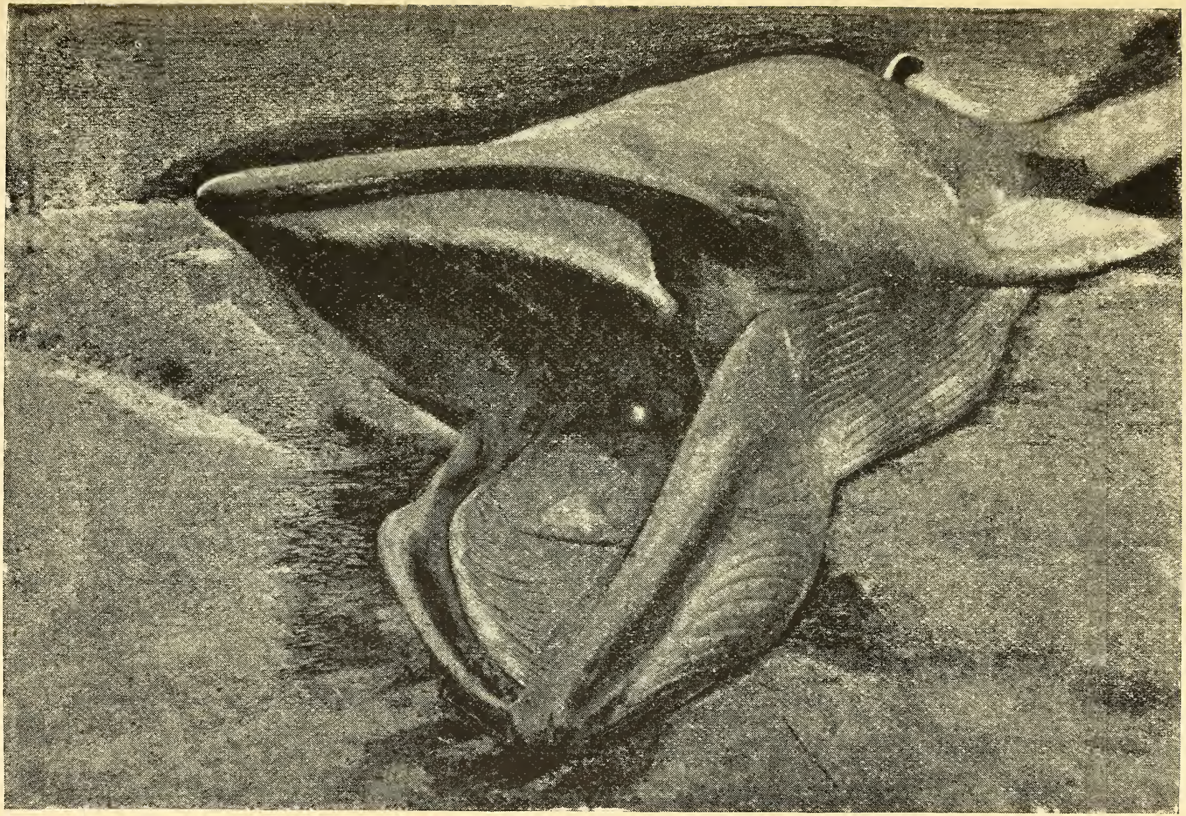


Fig. 2.—The mouth and tongue. The lightish spot at the root of the tongue is the bucco-pharyngeal opening.

hard palate was seen, with the strong mesial longitudinal keel marking the site of the vomer and inner borders of the two superior maxillæ. In the interval between the posterior ends of the two halves of the baleen wreath, the mucous membrane showed rows of shallow ridges and furrows, which converged to the opening into the pharynx.

The two halves of the lower jaw came to a point at the symphysis, and they enclosed within their curvature the massive tongue. This

Fig. 3.—The root of the tongue and a large part of the roof of the mouth. The convergence of the posterior ends of the baleen wreaths in front of the bucco-pharyngeal opening is shown.

organ occupied the floor of the mouth, and was sunk below the level of the lower jaw, whilst the fluted integument below it bulged both laterally and inferiorly, so as to give an almost pouch-like character to the infra-oral region. The tongue itself was covered by a coarse, rugose mucous membrane. The ridges and furrows partly ran in a longitudinal direction and partly in curves from the sides towards the middle of the upper surface. The angle of the mouth was smooth and rounded where the integument turned in to become continuous with the oral mucous membrane. The inner surface of the lower jaw sloped downwards and inwards to the side of the tongue, so that the upper jaw with its baleen wreath could fit within the curvature of the mandible. No hairs were seen projecting from the integument covering either the upper or lower jaws.

The soft palate, as had been previously noticed by Drs Carte and Macalister * in this species, and as I had also shown in *B. sibbaldii*, had no uvula. A little behind and below the posterior end of the baleen wreath was the bucco-pharyngeal opening, down which the closed hand with the forearm could without much difficulty be forced, for, as is the case with mucous canals when empty, the walls had fallen together when the tube was not transmitting an object. When the passage was opened up by the introduction of the fist, the opening into the mouth was seen to be circular. My assistant, Mr James Simpson, extracted from the gullet some stones, the largest of which was flattened, and measured 5 by 3¼ inches in diameter. They were impacted some distance down, and on making inquiries I learned that as the whale was floundering in the shallows, stones had been thrown at it, some of which had entered the mouth and been partially swallowed. The curving inwards of the two halves of the baleen wreath at their posterior ends would have the effect of directing the food and other objects collected between them towards and into the bucco-pharyngeal opening.

It will have been seen from the statements as to the rows of plates counted in the baleen wreaths in the different examples which I have referred to in this paper, that the number is not uniform in this species. The maximum was reached in the wreaths belonging to the specimens taken at Stornoway and Granton, whilst the young animal caught at Alloa had the smallest number.

* *Phil. Trans.*, 1867.

Colour.—The integument of the dorsum of the head, of the back of the body, of the dorsal fin, of the dorsum of the tail, of the side of the body for about half-way down, and the integument covering the upper and outer surface of the lower jaw, and that of the under surface of the flipper, possessed a rich, shining black lustre; but a thin greyish band passed for several inches horizontally backwards behind the blow-holes. The skin of the ridges and furrows below the mouth, that of the sides of the body below the level of the flippers, and the corresponding part of the ventral surface, was white. Immediately behind the furrows the white tint of the belly was blood-stained. As one passed backwards blackish spots were interspersed amidst the white surface, so that it assumed a slaty grey colour, which was also distinctly seen on the under surface and sides behind the plane of the dorsal fin. The skin of the flipper was a rich black on the antero-superior surface in its posterior half, but nearer its junction with the side of the body it was crossed obliquely by a broad white patch, which is one of the most characteristic colour-markings of the species. One of my pupils, Mr Robert Gray, who has had great practical experience in the whale fishing, told me that the white patch on the front of the flipper enabled the whaling seamen readily to recognise this cetacean when swimming at the surface. The presence of this white patch on the otherwise black surface of the flipper was figured by John Hunter,* and was described by Lacepède† in the *Baleinoptère museau pointu*, which we now know as *Balænoptera rostrata*. Eschricht has figured it,‡ but in his specimen it occupied a relatively larger part of the surface than in the Granton specimen, in which animal also it was partly interspersed with black patches. Messrs Carte and Macalister refer to it as a pinkish-white band in their specimen.

The following are some of the most important measurements of the Granton specimen:—

* *Phil. Trans.*, vol. lxxvii. p. 48.
† *Hist. Nat. des Cétacées*, p. 140, l'an xii. de la Republique.
‡ *Die Nordischen Wallthiere*, tab. viii., 1849.

TABLE I.

	Feet.	Inches.
Extreme length in straight line, . . .	28	4
Tip of beak to anterior border of dorsal fin, .	18	2
Posterior border of dorsal fin to notch of tail, .	7	0
Base of dorsal fin, ...	1	7
Angle of mouth to tip of lower jaw, . .	5	6
,, ,, ,, upper jaw, . .	5	0½
Tip of upper jaw to blow-hole, . . .	3	9
Greatest breadth across mouth, . . .	3	2
Axilla to tip of flipper (straight line), . .	2	7¾
Anterior border of flipper along curve, . .	4	4
Greatest breadth of flipper, ...	1	1
Antero-post. diameter of tail, . . .	1	11
Breadth of tail in straight line, . . .	7	4
,, ,, along posterior border, .	7	7

Abdominal Viscera.—As the accounts which have been given by previous writers of the Stomach in *Balænoptera rostrata* vary materially from each other, I took the opportunity of securing the stomach of the Granton specimen and of submitting it to anatomical examination.

The stomach, along with 14 inches of the œsophagus and the commencement of the duodenum, was carefully removed. After removal they were inflated, and the exterior of the organ examined. Its extreme transverse diameter in a straight line was 5 feet 9 inches, and its extreme antero-posterior diameter, also in a straight line, was 4 feet 2 inches. It was a compound stomach, and an examination was made of the number and nature of the compartments.

The 1st compartment was directed obliquely backwards and to the left, and tapered behind so as to terminate in a narrow, rounded, blind end, in relation to which the spleen was situated. It measured 38 inches from the place where the œsophagus joined it to the splenic end, its greatest transverse diameter about the middle of the ventral surface was 3 feet 3 inches, and its circumference was 6 feet 6 inches. The œsophagus was continuous with its anterior surface, and it was observed that at the place of junction the gullet was constricted in comparison with its calibre a few inches before it joined the stomach. This compartment was separated from the 2nd compartment by a deep constriction visible on the ventral and dorsal surfaces of the stomach, but scarcely marked on the anterior or œsophageal aspect. When cut into, the œsophagus was seen to open into the 1st compartment about 5 inches to the left of the large opening of communication between the 1st and 2nd compartments. The wall of the 1st compartment had a muscular coat, the fasciculi of which, distinctly marked, ran for the most part transversely around the wall. The mucous membrane lining this compartment was relatively smooth as compared with that lining the 2nd compartment, and the epithelium covering its free surface could be stripped off as a relatively thick and horny layer continuous with the horny epithelium which covered the rugose mucous membrane of the œsophagus. When examined microscopically, it was seen to resemble the œsophageal epithelium, and to consist of stratified squamous cells.

A strong septum separated the 1st and 2nd compartments, the anterior free edge of which was crescentic, and left a large opening, about 1 foot in diameter, between the two compartments. The boundary of this opening, opposite the crescentic edge of the septum, was formed by the anterior wall of the stomach about 5 inches to the right of the orifice of the œsophagus, and at this boundary the mucous linings of the two compartments, which differed greatly in their appearance, became continuous with each other. From the size of this opening I believe that it is usually patent, and that the food can pass from the 1st into the 2nd compartment, or regurgitate from the 2nd into the 1st.

The 2nd compartment, somewhat cylindriform in shape, was placed immediately to the right of the 1st. It passed anteroposteriorly and almost parallel to it for 3 feet 4 inches, and then bent to the right for an additional 2 feet 4 inches; where the bend took place, a constriction on the surface was seen. Before the compartment was opened into, one could see through the wall that the mucous lining was elevated into reticulated folds, a character which was of course much more distinctly recognised when a window was made in the wall. Some of the folds of the network were relatively thick and strong, and the intervals between them were occupied by a finer meshwork of folds. Projecting into the compartment from the wall, where a constriction on the exterior marked the change in the direction of this compartment, was a broad fold, the free edge of which was crescentic. I am not disposed to regard it as a septum dividing this compartment into two, but only as a fold corresponding with the change of direction in the compartment, for the mucous lining had a similar reticulated

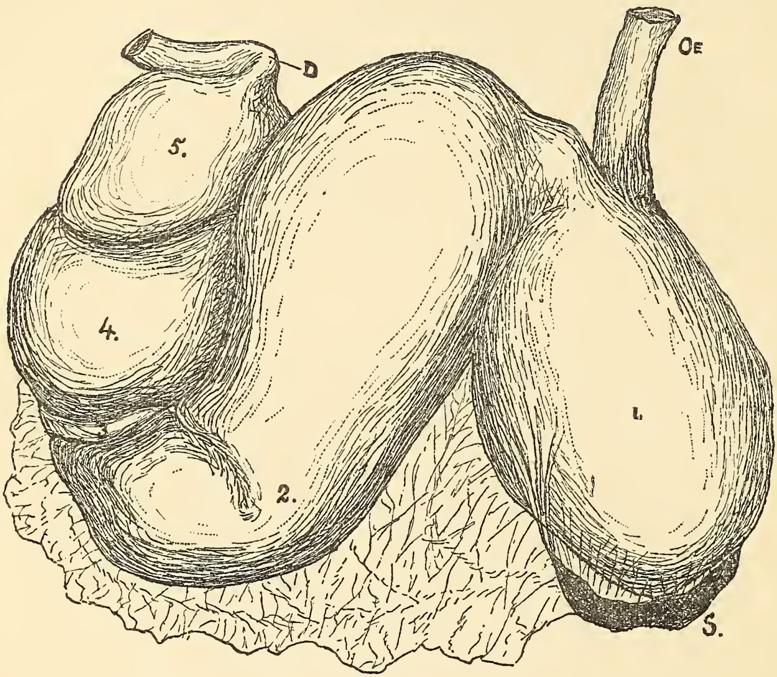


FIG. 4.—Stomach of *B. rostrata*, from a pen-and-ink drawing by H. G. Melville, M.B. OE, Œsophagus; D, duodenum; S, spleen; 1–5, compartments of stomach.

character on both aspects of the fold, and the aperture of communication between the two parts of this compartment was about a foot and a half in diameter. When vertical sections through the mucous membrane were examined microscopically, numerous tubular glands were seen in which the granular peptic cells characteristic of a true digestive chamber were contained.

The presence of a small 3rd compartment between the 2nd and the large 4th compartment was so faintly indicated on the surface by a constriction that it was not until this portion of the stomach was opened into, that I could feel quite certain about it. When the opening was made it was seen that the 2nd compartment communicated by an orifice, through which all the digits could be passed, with this 3rd compartment. One boundary of this orifice was the dorsal wall of the stomach, and the other was the free crescentic edge of the septum between compartments 2 and 3.

The 3rd compartment extended across the stomach from the dorsal to the ventral wall; its superficial area, corresponding to that of the right end of the 2nd compartment, was about 12 inches in greatest diameter; but in the direction of the longitudinal axis of the stomach, the diameter of the compartment was apparently not more than from 1 to 2 inches. Its aperture of communication with the 4th compartment was next the ventral wall, and readily admitted the fingers and thumb; and as the opening into the 2nd compartment was next the dorsal wall, the food passing through the 3rd compartment had to traverse its greater diameter before it could reach the 4th.

The 4th compartment was inclined forwards towards the diaphragm. It was 2 feet 4 inches in its long diameter, which curved from left to right, and formed the posterior border of this part of the stomach. It was much wider about the middle of its length than at its two ends, and its greatest girth was 4 feet 1 inch. When opened into, the mucous lining was seen to be smooth. It was separated from the dilatation situated to its right by a deep constriction externally, corresponding to which, in the interior of the organ, was a broad septum attached around its periphery to the wall of the stomach. The aperture of communication between the 4th compartment and the succeeding dilatation was not, therefore, at the periphery of the septum, as was the case with all the preceding septal openings, but was to the right side of the centre of the septum. The opening was not more than 1 inch in diameter and was valvular, and the flap next the dilatation, which was crescentic in shape, overlapped that which lay next the 4th compartment. From the small size of this opening as compared with the other septal apertures, the passage of the food into the dilatation must have been much slower than between the other compartments of the stomach, so that the retention of the food in these compartments sufficiently long to ensure its complete mingling with the gastric juices was provided for.

The dilatation referred to in the previous paragraph was situated at the extreme right of the stomach, and was directed towards the diaphragm. Its general form was globular; its length was 18 inches, and its greatest girth 3 feet 6 inches. A short, funnel-shaped tube proceeded from its right aspect, which ended in the cylindriform intestine. The mucous lining of this dilatation was smooth.

There was no valvular fold at the place where the funnel arose from the dilatation, but at the origin of the cylindriform intestine a projecting valvular fold was situated, which might, on the supposition that the dilatation was the last compartment of the stomach, represent the pyloric valve.

Descriptions of the stomach of the Lesser Rorqual have been written by John Hunter,* Eschricht,† Drs Carte and Macalister,‡ and Mr J. B. Perrin.§ Their attention was specially directed to the number and general arrangement of the compartments, but their descriptions do not tally. Eschricht, whose observations were apparently limited to fœtuses at different ages, stated that the stomach of this whale had only three compartments. Perrin described four, whilst Hunter, Carte and Macalister recognised five compartments. All these observers agreed in the presence of the compartments numbered as 1 and 2 in my figure. Hunter, Carte and Macalister described the compartment numbered 3, but this was not recognised either by Eschricht or Perrin. Compartment 4 is also described by the above anatomists, though the two last named regard it as the 3rd compartment.

The question now to be considered is if the dilated cavity succeeding 4 is a compartment of the stomach or the dilated commencement of the duodenum. Hunter, Carte and Macalister make it the 5th compartment of the stomach. Perrin holds a similar view, but calls it the 4th compartment, whilst Eschricht does not definitely refer to it. From the fact that the valve-like opening into it from the 4th compartment is small, and near the middle of the intervening septum, and not at the edge, one would at first be inclined to regard the small valve-like opening as a pylorus, and the

* *Op. cit.* † *Die Nordischen Wallthiere.*

‡ *Op. cit.* § *Proc. Zool. Soc.*, December 6, 1870.

dilatation consequently as a pouch-like commencement of the duodenum. It is well known that in the common Porpoise and in *Lagenorynchus albirostris* the duodenum begins in a subglobular dilatation, into which the biliary and pancreatic ducts open. I have figured a corresponding dilatation in the stomachs of *Delphinus delphis* and *Grampus griseus*,* which also receives the ducts of the liver and pancreas. If these ducts in *B. rostrata* opened into it, obviously it would be the duodenum; but, unfortunately, in taking my specimen out of the abdomen the biliary and pancreatic ducts were not dissected out and preserved, so that I am unable to say from personal observation where they opened. On this point, however, Drs Carte and Macalister have made a definite statement, for not only do they call the dilatation a 5th compartment, but they say that it communicated with the duodenum by a small pylorus, and that the conjoined hepatico-pancreatic ducts, after running obliquely for 2 inches between the coats of the intestine, opened in a valve-like manner into the interior of the duodenum about $6\frac{5}{8}$ inches below the pyloric orifice. Relying on the accuracy of this observation, the dilatation should be regarded as a 5th compartment of the stomach.

Something more is, however, required in the study of the Cetacean stomach than the determination of the number and arrangement of the compartments: the structure of the mucous membrane has to be determined. In a memoir on Sowerby's whale, published in 1885,† I showed, from the examination of its stomach and that of the dried stomach of Hyperoodon, that in these animals, and presumably in the Ziphioid whales generally, the 1st compartment of the stomach is not a mere paunch-like receptacle lined by a prolongation of the œsophageal epithelium, as in the Dolphins. It is, on the other hand, a true digestive chamber, the mucous membrane lining which is abundantly provided with gastric glands, like those of the 2nd compartment in the stomach of the Dolphins, with which it is therefore homologous. Shortly after the publication of that paper, Professor Max Weber came to the same conclusion from the study of the stomach of Hyperoodon.‡ More recently I have

* *Jour. of Anat. and Phys.*, vols. xxiii. p. 479, and xxvi. p. 260.

† *Ibid.*, vol. xx.

‡ *Studien über Säugethiere*, Jena, 1886.

obtained confirmatory evidence from the examination of a second specimen of Sowerby's whale and a fresh stomach of Hyperoodon.*

One has now to consider the question if the stomach of *B. rostrata* resembles the type of stomach in the Delphinidæ or in the Ziphioid whales, or if it is of a different type from either.

Observations on the structure of the mucous lining of the stomach in the Rorquals are as yet few in number. Eschricht states † that in *B. rostrata* and *Megaptera boops*, both the œsophagus and 1st compartments are lined by a thick epithelium which terminates abruptly at the entrance to the 2nd compartment. Professor Max Weber ‡ examined the stomach of a fœtus of *Balænoptera sibbaldii*, 227 cm. long. He recognised three compartments. The mucous lining of the 1st was smooth and without glands, but the fœtus was too young to show a horny epithelium. The mucous lining of the 2nd compartment was elevated into convoluted folds, and contained branched tubular glands in which distinct pepsin cells were recognised. The 3rd compartment had a mucous lining elevated into longitudinal folds; it contained tubular glands in which no pepsin cells were seen. From this examination Weber came to the conclusion that the Rorquals have the same structural type of stomach as is found in the Cetacea generally, to which type, however, the Ziphioids, as has already been explained, form a remarkable exception. My examination of the stomach of the adult *B. rostrata* corroborates the view that in the type of structure the Rorquals resemble the Delphinidæ. If I were to employ in the description of the Rorqual's stomach the method which I followed in my account of the stomach in the Dolphins,§ I should say that the 1st compartment, as in the Dolphins, is a large œsophageal paunch lined by a prolongation of the horny squamous epithelium. The 2nd is the cardiac compartment, lined by a succulent reticulated mucous membrane, abundantly provided with peptic glands, so that it is a true digestive chamber. The last chamber, next the duodenum, is the pyloric compartment, whilst between the cardiac and pyloric chambers are two intermediate compartments, of which the small

* *Jour. of Anat. and Phys.*, vol. xxiii. p. 466.
† *Die Nordischen Wallthiere*, p. 98.
‡ *Morphologisches Jahrbuch*, May 1888, p. 646.
§ *Jour. of Anat. and Phys.*, vol. xxiii. p. 466.

3rd chamber is not unlike in its position and relations the 3rd compartment in the Narwhal. The mode of description employed by Max Weber has, however, the merit of greater simplicity. Like myself, he recognises the œsophageal and cardiac chambers as structurally and functionally distinct compartments; but he groups all the compartments between the cardiac chamber and the duodenum, whatever may be their number, under the name of the "pylorial" division of the stomach. He regards this division as characterised in structure by the possession of mucous glands similar to those present in the pyloric part of the stomach of the Carnivora.*

The stomach of the Granton *B. rostrata* contained about 2 gallons of a greenish fluid, in which numerous nematoid worms and broken pieces of the bristles of the baleen plates were suspended. No definite light, therefore, as to the food of this whale was obtained from this dissection, but from the scanty references to this subject in cetological literature it would appear that the Lesser Rorqual lives, in part at least, on fish. John Hunter found the bones of fish, and in particular he refers to dog-fish, as present in the stomach of his specimen. Dr M'Bain stated that the stomach of the Rorqual found near the Bell Rock contained a considerable quantity of pultaceous matter, "with bodies of the vertebræ and bones of the herring intermingled throughout the mass." Messrs Carte and Macalister found the crystalline lens of a small fish in the stomach of their specimen. Professor Flower says the remains of numerous fish of considerable size—his informant thought cod-fish—were got in the stomach of a specimen caught at Overstrand, Cromer, on the coast of Norfolk, in 1860.† In Mr Perrin's specimen small pebbles were found in the stomach. As the animal is apparently a fish-eater, it is possible that the nematoid worms found in the stomach of the Granton specimen had belonged to the fish on which the animal had fed.‡

* Messrs Sims Woodhead and R. W. Gray have given an account (*Jour. of Anat. and Phys.*, Jan. 1890) of the structure of the mucous membrane of the stomach of the Narwhal, and have found in intermediate compartment 3 in the stomach of that animal glands adjoining compartment 2 which have the structure of peptic cardiac glands, others adjoining compartment 4 like mucous pyloric glands. This compartment is therefore histologically, as well as topographically, intermediate.

† *Proc. Zool. Soc.*, May 24, 1864.

‡ The species *Balænoptera borealis* was found by Mr R. Collett during the month of July to have its stomach and intestines filled with the Copepod

The Great Omentum formed a very noticeable object. It was not attached to the left border of the 1st compartment, but depended from its right border, and was prolonged on to the left border of the 2nd compartment, and thence backwards to its posterior border, from which it was continued as far as the 4th compartment, but did not reach the 5th compartment. The great blood-vessels ran along the line of attachment of the omentum to the different compartments. The ventral surface of the stomach was covered by peritoneum, but the dorsal surface was in a large measure without a serous covering, and the pancreas was in apposition with the dorsal surface of the 4th compartment and the dilatation next to it.

The Spleen was attached by a short gastro-splenic omentum to the blind end of the 1st compartment, and it was curved so as to be adapted to it. The length of the spleen along the convexity was 19 inches; its greatest breadth was 4 inches at its middle, from which it tapered to each end. The Intestine was measured after its removal from the cavity of the abdomen, and its length may be stated approximately as about 140 feet.

As the animal was an adult female, I took the opportunity of examining the Uterus and its appendages. The vagina admitted the closed hand and forearm. Its mucous membrane was folded both longitudinally and transversely; three transverse folds were of large size, and the highest apparently represented the boundary of an os uteri. The uterus possessed a body which divided into two cornua. The corpus uteri was 29 inches long and from 5 to 7 inches broad. Its mucous membrane was folded longitudinally, and the biggest folds projected for nearly 2 inches. Twelve of these folds were counted in the circumference of one part of the canal, but in proximity to the os they increased in number and diminished in depth. A well-defined septum, the lower edge of which was free and sickle-shaped, separated from each other the openings of the two cornua where they arose from the corpus uteri. The uterine cornua measured each 29 inches along the curvature, and were from 4 to 5 inches broad. Each Fallopian tube was 18 inches long,

(*Calanus finmarchicus*); he refers also to observations by Captain Bull, who found that the stomach contained the so-called "Kril," *Euphausia inermis*, a Thysanopod Crustacean about 1½ inch long, which forms also the chief nourishment of *Balænoptera sibbaldii*.—*Proc. Zool. Soc.*, London, April 20, 1886.

and terminated at its outer end in a large trumpet-shaped mouth. The uterine horns tapered to a narrow end, where the tubes arose from them.

When the cornua were opened into, the cavity was seen to be dilated, and the mucous membrane was folded longitudinally; seven to nine folds were present in the circumference of the horn, and the biggest fold projected as much as $1\frac{1}{2}$ inches into the cavity. As the folds approached the uterine end of the Fallopian tube they diminished greatly in depth, but increased somewhat in number, and were continuous with similar mucous folds in the Fallopian tube. For about 9 inches from the tip of the cornu, the Fallopian tube readily admitted a thick porcupine quill; it then rapidly dilated so that two fingers could be passed into it; but when it formed the trumpet-shaped mouth, it was so widely dilated that both fists could be accommodated in it. The membrane lining this greatly dilated mouth was longitudinally folded. These folds were prolonged into the tube proper by one extremity, but by the other they formed a series of membranous fimbriated processes, which varied in length from 2 to 10 inches; some of the processes were narrow bands, but others were triangular flaps of membrane. External to these fimbriæ, but continuous with their bases, were three large somewhat triangular flaps, which formed, as it were, an outer circle of broad fimbriæ, each of which measured about 10 inches at the base and varied in the extent of projection from 8 to 10 inches. The surface of these flaps next the mouth of the tube was traversed by longitudinal folds of delicate translucent membrane. As one traced the longitudinal folds outwards from the tuba proper, the membrane forming them became thinner and more translucent; it lost the appearance to the naked eye of a mucous membrane, and approximated in its aspect to the serous membrane, which formed the opposite surface. When examined microscopically, however, some elongated tubular glands were seen to open on the free surface, so that it was structurally a mucous and not a serous membrane. The arteries and veins, both in the uterine walls and in the broad ligament, were of large size, and the arteries were very tortuous.

Each Ovary was connected to the posterior aspect of the broad ligament by a fold of peritoneum. It measured 7 inches long by 2 inches in breadth, and was indented with deep furrows. Notwithstanding the length of the ovary, there can be no doubt from the size of the trumpet-shaped mouth of the tube, and from the remarkable arrangement of the fimbriæ, that the ovary could be completely embraced by them; whilst the longitudinal arrangement of the folds would direct the ovum, after its escape from the ovary, into the tube and cornu of the uterus. Projecting from the surface of the right ovary was a swelling about the size of a small walnut, which on being cut into was seen to be due to a corpus luteum. About 1 inch from it was a second and smaller swelling about the size of an almond, which on section was seen to contain a much-shrivelled corpus luteum obviously on the point of disappearing. From the presence of the large corpus luteum on the right ovary, from the size of the cavities of the cornua, and the somewhat flaccid and dilated condition of the uterine cavities generally, I was led to think that the animal had been pregnant at a recent period. No trace of fœtal membranes was, however, to be seen in the uterine cavity; nor could I recognise with a simple lens an arrangement of recesses or crypts on the free surface of the mucous lining such as I described a number of years ago both in *Orca gladiator* * and in the Narwhal.† On making vertical sections through the mucous membrane, and examining them with the compound microscope, distinct evidence was obtained of the presence both of a glandular layer in the mucosa and of a more or less defined layer between the divided gland tubes and the free surface. In this more superficial layer depressions were seen irregular in shape and arrangement, which seemed as if they were crypts or recesses in process of disintegration, in which the villi growing from the surface of the chorion might at one time have been lodged. The microscopic examination strengthened the opinion I had formed on the general grounds above referred to, that the animal had been recently delivered of a young one, and that the uterus was in process of involution.

Heart.—The Heart, as is usual in the Cetacea, was wide in its transverse and short in its antero-posterior diameter. Its greatest width at the base of the ventricles was 20 inches, but antero-posteriorly it measured only 12 inches. The apex of the heart was

* *Trans. Roy. Soc. Edin.*, vol. xxvi., April 1871.

† *Proc. Roy. Soc. Edin.*, 1876.

blunt, and there was no appearance of a cleft separating the right from the left ventricle.

The right auricle formed a large chamber divided into a sinus venosus and an appendage. The muscular wall of the interior of the appendage was elevated into ridges, many of which were attached to the wall only by their opposite ends. The inner wall of the sinus venosus was smooth, and the great mouths of the anterior and posterior venæ cavæ and the smaller mouth of the coronary sinus opened into it.

The right ventricle formed the right half of the ventricular portion of the heart, both on its dorsal and ventral surfaces. The septum between it and the left ventricle was not obliquely directed as in the human heart, but lay approximately in the vertical mesial plane of the organ. The wall of the right ventricle averaged from $\frac{1}{2}$ to 1 inch in thickness, and the trabeculæ carneæ projecting from it were strong and numerous. The papillary muscle growing from the ventral wall was $2\frac{1}{2}$ inches broad at its base, and was divided into three secondary papillæ before giving origin to the chordæ tendineæ, which distributed themselves almost equally between the left or infundibular cusp, the right cusp, and the intermediate flap, which connected these cusps together. A papillary muscle, about one-third the size of that just described, arose from the dorsal wall in the interval between the right cusp and the dorsal cusp, to both of which and to the flap between them its chordæ tendineæ were attached. The chordæ tendineæ which passed to the dorsal and infundibular cusps did not arise from a single papillary muscle, but from four slender and elongated muscular papillæ attached to the septal wall of the ventricle. The most prominent object, however, in this ventricle was a great, round, moderator band of muscular fibre, which arose by three muscular processes from the septal wall, and passed across the cavity to the ventral wall, where it ended close to the base of the ventral papillary muscle. The length of this band was 5 inches, and its circumference varied from $3\frac{1}{2}$ to $2\frac{1}{2}$ inches. In addition, a slender thread, which seemed to be little more than a fold of endocardium, passed across the cavity. The pulmonary artery arose from a conus arteriosus, and its mouth was guarded by three large semilunar valves.

The left auricle closely resembled the right in the appearance both of the sinus venosus and appendix. Four pulmonary veins without valves opened into it.

The left ventricle formed the left half of the ventricular division of the heart both dorsally and ventrally. Its walls varied in thickness from $1\frac{1}{2}$ to $3\frac{1}{2}$ inches, and it possessed strong trabeculæ carneæ. The papillary muscles were for the most part thick and stunted, and resembled mound-like elevations rather than papillæ, and their chordæ tendineæ were very strong. The left auriculo-ventricular valve was not bicuspid, but consisted of four distinct triangular cusps continuous with each other at their bases around the auriculo-ventricular opening, and in this respect it resembled generally the heart of Sowerby's whale described by me some years ago.* The four cusps were not uniform in size : one (*a*), the largest, was situated at the dorsal (superior) border of the opening; one (*b*) at the dorsal, another (*c*) at the ventral (inferior) border of the left boundary of the opening ; whilst the fourth (*d*) was placed between *c* and the large dorsal cusp (*a*). A stunted papillary muscle arose opposite the interval between *c* and *d* ; another was opposite the interval between *a* and *d* ; a third lay between *b* and *c*, and in each case they distributed their chordæ tendineæ to the cusps between which they were placed. The chordæ tendineæ that passed to the adjoining borders of *a* and *b* did not arise from a thick stunted muscle, but from several slender elongated papillæ situated between their borders. Four short but powerful trabeculæ carneæ passed from the septal wall of the ventricle across the cavity to the adjoining part of the ventral wall, and would, doubtless, act as muscular moderator bands. Between these trabeculæ and the dorsal cusp of the left auriculo-ventricular valve, an elongated and comparatively narrow passage directed the blood into the mouth of the aorta, which was guarded by its semilunar valves.

The three great arteries which arose from the transverse part of the arch of the aorta had been cut across close to their origin. But if I may interpret the arrangement from the dissection which I made and figured many years ago in the heart of *Balænoptera sibbaldii*,† they were in all probability a right brachio-cephalic, a left carotid, and a left subclavian, from the last named of which a left posterior

* *Jour. of Anat. and Phys.*, Oct. 1885.
† *Trans. Roy. Soc. Edin.*, 1870, vol. xxvi.

thoracic artery almost immediately arose. The cord of the ductus arteriosus connected the commencement of the left pulmonary artery with the arch of the aorta almost opposite the origin of the left subclavian.

The Skeleton.—As the bones of the Granton Balænoptera were fully ossified, and as the animal had attained its adult proportions, some general observations which I have made on the skeleton may be referred to. It is not, however, necessary to write a description of the individual bones, as Eschricht, Van Beneden, Gervais, Flower, and other cetologists have already entered with considerable detail into the subject.

The vertebral formula was $C_7D_{11}L_{13}Cd_{19} = 50$. In order to obtain a correct determination of the number of caudal vertebræ, regarding which observers have differed materially in their statements, the terminal caudals were carefully dissected *in situ*, when ten vertebræ were counted between the two flanges of the tail, and in front of the flanges nine vertebræ bore chevron bones. The terminal caudal was a nodule of bone 0·4 inch (11 mm.) broad and 0·27 inch. (7 mm.) in antero-posterior diameter: a circular constriction differentiated it from the vertebra immediately anterior to it, with which it was fused. The distance between the last caudal and the free edge of the median notch of the tail was half an inch (13 mm.). The penultimate vertebra was 0·86 inch (22 mm.) broad by 0·7 inch (18 mm.) in antero-posterior diameter. The third vertebra from the tip of the tail was 1·45 inch (37 mm.) broad by 1·2 inch (30 mm.) in antero-posterior diameter. Had the tail been macerated, the two terminal caudals would in all probability have been lost, and the vertebral formula would then have read $C_7D_{11}L_{13}Cd_{17} = 48$, which is the usual number ascribed to *B. rostrata*, although some observers have given only 46 or 47 vertebræ in this species. Professor Flower counted 50 vertebræ in the male, 24 feet 4 inches long, stranded at Overstrand, near Cromer, in November 1860,* a number corresponding to that in the Granton specimen, and which, doubtless, expresses the correct

* *Proc. Zool. Soc.*, May 24, 1864, in which the following formula is given, $C_7, D_{11}, L_{12}, Cd_{20} = 50$. This skeleton is in the Museum of the Royal College of Surgeons of England, but in Professor Flower's catalogue of the collection the formula is modified as follows, $C_7, D_{11}, L_{12}, Cd_{19} = 49$.

formula when all the vertebræ are ossified, and due care is taken for their preservation. The epiphysial plates were fused with the bodies of their vertebræ in the Granton specimen, but in the lumbar and dorsal regions evidence of their original separation could still be seen.

The skeleton of the young *rostrata* from Queensferry dissected by the brothers Knox, which is now in the Anatomical Museum of the University, and which is preserved as a natural skeleton, had the formula C_7D_{11}. Twelve lumbar vertebræ intervened between the last rib-bearing dorsal and the vertebra to the posterior border of the body of which the 1st chevron bone was articulated. Nine caudal vertebræ carried chevron bones, and behind the last of these ten osseous or cartilaginous nodules represented the terminal vertebræ of the tail: the entire formula is $C_7D_{11}L_{12}Cd_{19} = 49$.*

The 1st, 4th, and 7th *cervical vertebræ* in the Granton specimen were separate bones. The axis and 3rd cervical were partially fused by their bodies, laminæ, and spines. The 5th and 6th were fused at the tips of the inferior transverse processes, but elsewhere they were quite distinct. In the adult Dunbar specimen all the cervicals were separate bones except the axis and 3rd cervical, the bodies of which were fused together. In the young animal caught at Alloa, the axis and 3rd cervical were fused at their spines. It is obvious, therefore, that in *B. rostrata* there is a tendency to fusion between the 2nd and 3rd cervicals. In none of the specimens had the atlas a foramen in its transverse process. In the Granton and Dunbar animals each lamina was perforated by a foramen dorsad to the condylar articular surface for the 1st cervical nerve; but in the Alloa specimen the hole was represented by a deep groove. The height of the atlas in the Granton and Dunbar specimens was 8 and $8\frac{1}{4}$ inches, and in that from Alloa 6 inches. The breadth between the tips of the transverse processes was $13\frac{1}{2}$, $13\frac{1}{4}$, and 8 inches respectively. The axis was as usual the largest vertebra of the neck, its height in the Granton and Dunbar specimens was $8\frac{1}{2}$ inches, in that from Alloa 6 inches; the greatest transverse diameter in

* The brothers Knox in their account of this animal give the vertebræ as follows, $C_7D_{11}LCd_{30} = 48$; but from the appearance of the tail in the dried skeleton it seems as if an additional cartilaginous nodule were present in the caudal series.

these animals was $20\frac{1}{4}$, $19\frac{1}{4}$, and $10\frac{1}{2}$ inches respectively. The massive transverse processes were each perforated by a large oval foramen, the boundary of which was ossified in the adults, and also on the right side of the young Alloa specimen, but on the left side it was completed by cartilage. In the Granton specimen the superior and inferior transverse processes of the 3rd cervical had joined externally so as to complete the boundary of the vertebrarterial foramen; but in the 4th, 5th, and 6th they did not quite meet, so that the osseous boundary was incomplete externally. In the Dunbar specimen the osseous boundary of this foramen externally was incomplete in the 3rd as well as in the 4th, 5th, and 6th cervicals. In the Alloa specimen, and apparently also in Knox's skeleton, the boundaries of these foramina were completed by bars of cartilage. In the 7th cervical, the inferior transverse process was represented by a faint tuberosity on the side of the body; but the superior was long, and the diameter between the tips of these processes was in the Granton specimen $19\frac{1}{2}$ inches, and in that from Dunbar $15\frac{1}{2}$ inches. The neural arches were complete in all the cervicals in the adult animals, except in the 3rd, in which a gap of $1\frac{3}{8}$ inch existed in the Dunbar specimen, whilst in that from Granton a chink admitting the blade of a scalpel separated the right lamina from the spinous process.

The *dorsal vertebræ* in the Granton specimen increased in magnitude from before backwards, both as regards the bodies and spinous processes. The transverse processes of the 1st dorsal were not quite so long as those of the last cervical, but were longer than those of the 2nd, 3rd, and 4th dorsals. A mesial keel appeared on the ventral surface of the body of the 4th dorsal, and was seen also in those which were posterior to it. The relative magnitude of the 1st and last dorsal vertebræ was as follows in inches:—

	1st.	Last.
Height of vertebra, ...	10	19
Antero-post. diam. of body, ...	$2\frac{1}{4}$	6
Vertical diam. of body, ...	$3\frac{3}{4}$	$4\frac{1}{2}$
Height of spine, ...	3	12
Length of transverse process, ...	$7\frac{3}{4}$	$10\frac{1}{2}$

The *ribs* in the Granton specimen articulated with the outer ends of the dorsal transverse processes. The 2nd and 3rd were the only ribs which gave indications of a capitular process projecting inwards towards the side of the body; but it was so feeble as to be little more than a mere tubercle, though it was somewhat longer in the 2nd than in the 3rd rib. The 1st pair were the broadest and shortest ribs; the length along the outer curvature was 2 feet 11 inches. The last rib, again, was the most slender, and its length was 3 feet $5\frac{1}{2}$ inches. The 4th and 5th ribs were the longest, and their corresponding diameters were 5 feet $1\frac{1}{2}$ inches.

The *sternum* had the shape of a Latin cross, characteristic of the species. In the Granton specimen it was $19\frac{1}{2}$ inches long by 9 inches in transverse diameter. In the Dunbar specimen it was $19\frac{1}{4}$ by $10\frac{3}{4}$ inches. In both these adults the posterior limb was long and slender (in the one $11\frac{1}{4}$, in the other $10\frac{3}{4}$ inches); whilst the anterior and lateral limbs were shorter and together rather trefoil-shaped. In the natural skeleton of Knox's young animal, the 1st pair of ribs were seen to be joined to the posterior border of the transverse limbs and the anterior part of the side of the posterior limb. In the same skeleton the 2nd pair of ribs were joined together mesially and ventrally by what seemed to be a dried bar of cartilage, and a similar arrangement connected the ventral ends of the 3rd pair of ribs.

The *lumbar* vertebræ in the Granton specimen were thirteen in number. The keel of the 13th was prominent, and broadened out at its posterior end into a somewhat flattened triangular area, but as there was not distinct evidence of an articulation for a chevron bone I have included it in the lumbar series. It is possible that the 1st chevron bone may have been lost and too small to produce a definite articular area on its vertebra, and yet to have belonged to this bone, which would then be the 1st caudal and not a lumbar vertebra. The bodies of all the lumbar vertebræ were antero-posteriorly elongated and keeled. The spines were long and massive, and the transverse processes broad and flattened. The 6th lumbar had the following dimensions:—Vertical diameter of vertebra, $20\frac{1}{2}$ inches; body, antero-posterior diameter, $6\frac{3}{4}$ inches, vertical diameter, $5\frac{1}{2}$ inches; spine, length, $14\frac{3}{4}$ inches, antero-posterior diameter, $5\frac{1}{2}$ inches; transverse process, length, $9\frac{3}{4}$ inches, breadth, $5\frac{1}{4}$ inches.

The *caudal* vertebræ, nineteen in number in the Granton specimen, consisted of two groups, nine of which had chevron bones, whilst ten without chevrons were included between the flanges of the tail. The greater number of these vertebræ were keeled on the ventral surface of the body, and the keel was grooved antero-posteriorly, so as to be divided into two ridges, which possessed in the vertebræ associated with chevron bones distinct areas for their articulation. The eight anterior caudals possessed laminæ and spines enclosing a neural canal, they diminished in magnitude from before backwards. Transverse processes were present in the five anterior caudals, but in the 5th the process was diminished to a slight ridge. In the 4th and 5th the transverse process was perforated, where it sprang from the side of the body, by a vertical foramen. The height of the 1st caudal vertebræ was 19 inches, its breadth was 17 inches ; the vertical diameter of the body was 7 inches, its transverse diameter was 8 inches.

Nine chevron bones were present, the eight anterior of which consisted of a pair of hæmal arches with a hæmal spine; but the 9th was only ossified in its left half, and was not more than 1 inch in antero-posterior diameter. Except the last chevron, the others had separated from the caudals, and it seemed as if that which articulated with the 2nd caudal was the largest of the series; it measured 9 inches in height and $6\frac{3}{4}$ inches in its greatest antero-posterior diameter. In the natural skeleton of Knox's specimen the 3rd chevron was the longest. They diminished in size from it in one direction to the 1st, which was very small, and in the other to the last, which was also very small.

It is unnecessary to give a description of the *Skull* of *B. rostrata*, as it has been described and figured by several writers on this species. It may, however, for purposes of comparison, and as the crania are from animals of different ages, be useful to arrange in a tabular form some of the principal measurements of the series of crania now in the Anatomical Museum of the University (Table II.). Both Knox's specimen and that caught at Alloa are quite young. If we take the length of the adult animal as 28 to 30 feet, and that of Knox's specimen as 10 feet, the latter had attained about one-third its growth, and was probably not more than a few weeks old.

In removing the lower jaw of the Granton specimen, it was observed that the great fibrous pad which connected the condyle of

TABLE II.

Dimensions of Crania.	Queensferry, 1834 (Knox).		Alloa, 1888.		Elie, 1879.*		Burntisland, 1870.*		Hillswick, 1870.		Dunbar, 1871.*		Granton, 1888.	
	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.
Condylo-premaxillary length in straight line,	2	8	3	4	3	6½	3	8	...		5	9½	5	10
Length of beak,	1	8	2	1	2	3	2	3½	...		3	10¾	3	11½
,, superior maxilla,	1	9½	2	3¾	2	6¼	2	8¼	...		4	4	4	3¼
,, premaxilla,	1	8¾	2	3¾	2	5¾	2	7½	...		4	3½	4	5
From anterior border of foramen magnum over vertex to tip of beak,	...		3	5	3	8	3	9½	...		6	0	6	2½
From anterior border of foramen magnum over vertex to upper border of occiput,	...		0	11¼	1	0½	1	1	...		1	6¾	1	7
Greatest breadth of skull,	1	4	...		1	11	1	11	...		3	3	3	2¾
Breadth at base of beak,	0	10	1	0¾	1	2½	1	2	...		2	0¼	1	11
,, middle of beak,	0	7	0	10¼	0	10	0	10	...		1	4½	1	5
,, middle of orbital borders of frontals,	1	3	1	6	1	7½	1	9	...		3	0¼	2	11¾
Greatest breadth of superior maxilla behind base of beak,	1	2½	...		1	6½	...		...		2	11½	2	10½
,, ,, between outer borders of both premaxillæ,	0	3	0	4¾	0	5¼	0	5	...		0	9½	0	9½
,, ,, ,, inner borders of same,	0	2½	0	4	0	4½	0	4	...		0	7½	0	7
Height from vertex to pterygoids,	...		0	10	1	1	1	0¾	...		1	9	1	9
Length of mandible in straight line,	2	6	3	3¾	3	7	3	9½	3	7	6	0	5	11
,, ,, along outer surface,	2	7½	3	5½	3	10	4	1	3	10	6	5¼	6	4½
Height of mandible at condyle,	0	3¼	0	3½	0	4¾	0	4½	0	4½	0	7½	0	6½
,, ,, coronoid,	0	4¼	0	5	0	5½	0	5¾	0	5	0	9¼	0	9
,, ,, symphysis,	0	1¾	0	2¼	0	2¼	0	2	0	2½	0	4	0	3¾

* The tips of the premaxillæ had been broken off in the Elie, Burntisland, and Dunbar crania, so that the actual length of each was probably from 1 to 3 inches greater than is expressed in the Table.

the lower jaw with the articular surface of the temporal bone was 8 inches in length and 10 inches in transverse diameter.

The hyoid apparatus was present both in the Granton and Dunbar specimens. In the Granton specimen the transverse diameter between the tips of the great horns was $20\frac{1}{4}$ inches; in the Dunbar specimen it was 20 inches. In the Granton specimen the diameter from the tip of the anterior horn to the posterior border was $7\frac{1}{8}$ inches; in the Dunbar specimen it was $7\frac{1}{2}$ inches. The length of the stylo-hyoid in the Granton specimen was $11\frac{1}{2}$ inches; in the Dunbar specimen it was $11\frac{1}{4}$ inches. Compared with the hyoid of *Balænoptera borealis*, a description of which I have given in my account of that whale,* it will be seen that in the latter animal the hyoid was a somewhat larger bone; it differed also in the shape of the body, which in *B. rostrata* was convex from side to side and from before backwards on the inferior surface, whilst the great horn was fusiform in its long axis and not so flattened on its inferior surface as in *B. borealis*. The general shape of the hyoid in *B. rostrata* approximated, indeed, more to that of *B. sibbaldii*, though on a much smaller scale.

The length of the *tympanic* bone in the Granton specimen was $3\frac{1}{4}$ inches, the breadth was $1\frac{7}{8}$ inches.

Anterior Extremity.—The integument, the subjacent muscles, and other soft parts, together with the periosteum, were carefully removed from the bones of the limb, so that the carpal bones and joints and the digits might be seen and described *in situ*. The *Scapula* had the customary shape of the species, and possessed both coracoid and acromion processes. Its extreme antero-posterior diameter was 30 inches, whilst its breadth from the vertebral border to the middle of the inner border of the glenoid fossa was 16 inches. In Knox's young specimen the coracoid process, and about one-half of the acromion, consisted of unossified cartilage; the vertebral border was also cartilaginous, and at both the anterior and posterior angles, especially the latter, considerable plates of cartilage were still present. The *Humerus* also had the form of the species. It was $11\frac{1}{2}$ inches long and nearly 6 inches broad in the middle of the shaft. The head was elongated and encrusted with articular cartilage. The opposite end had broad articular areas for radius and ulna. The

* *Proc. Roy. Soc. Edin.*, Feb. 1882, and *Jour. of Anat. and Phys.*, April 1882.

Radius was $18\frac{1}{2}$ inches long and simple in form. The *Ulna* was nearly 18 inches long, and had a remarkably large hook-shaped olecranon process, of which 6 inches at and near the tip consisted of unossified cartilage. The carpal end of each of the two bones of the forearm was covered by a cartilaginous plate, which was separated by an imperfect joint cavity from the cartilage which covered the adjoining surface of the proximal row of carpal bones. The bones of the forearm had no separate bony epiphyses.

The *Manus* was tetradactylous. The *Carpus* consisted of five bones arranged in a proximal and a distal row, with in addition a pisiform bone. The five bones of the two rows were flattened on their dorsal and palmar surfaces where they were covered by periosteum and ligamentous bands, whilst the other surfaces (except the outer surface of the radiale) were encrusted by cartilage, and were articular. The original cartilaginous carpus was so well ossified that, except in the pisiform element, only narrow bands of cartilage separated the bones from each other. Depressions on the surface indicated the line of demarcation between the cartilages which covered adjoining carpal bones and the carpal ends of the metacarpal bones.

The proximal row, *procarpus*, consisted of radiale, intermedium, and ulnare. Their size may be gathered from the transverse diameter of their osseous parts : that of the radiale was $2\frac{1}{4}$ inches; of the intermedium the same; of the ulnare, $2\frac{1}{8}$ inches. The radiale articulated, by its proximal surface with the radius; by its distal with the metacarpal segment of digit *a*; by its ulnar with the intermedium and the more radial ($C\alpha$) of the two distal carpals; whilst its outer surface was non-articular. The intermedium articulated, proximally with both radius and ulna, especially the former; distally with the two carpals of the distal row; radially with the radiale; by its ulnar surface with the ulnare. The ulnare articulated, proximally with the ulna; distally with the more ulnar ($C\beta$) of the two distal carpals, and with the metacarpal segment of digit *d*; radially with the intermedium; by its ulnar surface with the pisiform. The pisiform element, like the olecranon process of the ulna, contained a considerable proportion of cartilage, which had a recurrent direction close to the border of the ulna; the pisiform articulated with the cartilage at the lower end of the ulna, with the ulnare, and slightly with the base of the metacarpal segment of digit *d*.

The distal row of carpalia, *mesocarpus*, consisted of only two bones, which were cut off from the two lateral margins of the carpus by the metacarpal segments of digits *a* and *d*, which passed to articulate the one with the radiale, the other with the ulnare. As there may be some difficulty in arriving at a conclusion, which of the five carpalia of the distal row of the type cetacean carpus are represented by these bones, I shall not designate them numerically, but distinguish them as Cα, Cβ. The transverse diameter of the bony part of each was about 2 inches. Cα articulated, proximally with the radiale and intermedium; distally with the metacarpal of digit *b*; radially with the metacarpal of digit *a*; by its ulnar surface with Cβ. Cβ articulated, proximally with the intermedium and ulnare; distally with the metacarpal of digit *c*; radially with Cα; on the ulnar side with the metacarpal of digit *d*.

The metacarpal segments of the four digits articulated as follows:—*a* with radiale and Cα; *b* with Cα; *c* with Cβ; *d* with Cβ, ulnare and pisiform. In addition to the metacarpal segments the digits were furnished with phalanges as follows:—*a* with three; *b* with seven; *c* with six; *d* with three. Each metacarpal bone and osseous phalanx had cartilage both at the distal and proximal ends, whilst the cartilage at the metacarpo-phalangeal joints and the larger inter-phalangeal joints was divided into two epiphysial plates, one for each bone; that between the 4th and 5th and 5th and 6th phalanges of digits *b* and *c* consisted of a single bar, not transversely segmented. Each terminal osseous phalanx had at its distal end a flattened plate of cartilage. Digit *b* was the longest, its terminal phalanx, in proximity to the tip of the flipper, was so small a plate of bone that unless care had been taken in dissecting the part it might easily have been missed. Digits *a*, *b*, and *c* were parallel and close to each other. Digit *d* diverged somewhat from *c*, so that the interval between them was greater than that between the other digits. The terminal phalanx of digit *d* reached the ulnar border of the flipper.

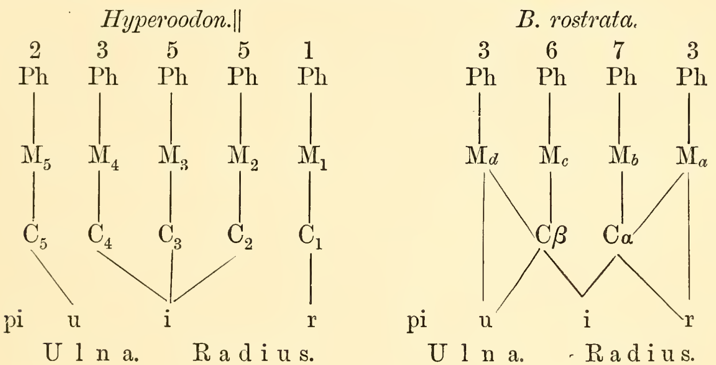
The dried manus of Knox's young specimen was examined, when three osseous nodules were seen to represent the radiale, intermedium, and ulnare; two nodules represented the bones of the distal row, and an unossified pisiform cartilage was also present. As regards the digits, four were present. Three of these, corresponding to *a*, *b*, and *c* in the description of the adult, were placed close together and parallel to each other; but the fourth, or most ulnar digit *d*, was set at an angle to digit *c*, so that an interval $1\frac{1}{4}$ inches wide occurred between its terminal phalanx and the phalanx of digit *c*, to which it was opposite. On the other hand, the intervals between digits *a* and *b*, and *b* and *c*, did not exceed a quarter of an inch. The existence of the well-marked interval between *c* and *d*, and the divergence of *c*, made me think that a rudimentary digit might be found in the intermediate structure. Accordingly, I dissected off the dried integument with care, but failed to see any osseous or cartilaginous structure in the interval.

As regards the general morphology of the carpus in *B. rostrata*, the proximal row presents no difficulty, for it possesses the three bones customary to the Cetacea, with in addition a pisiform. The distal row, as is generally admitted where only two distal carpalia exist, presents more difficulty. So long as it was thought that the missing phalanx in the cetacean tetradactylous hand was the pollex, it was natural to consider that the trapezium C_1 was also absent, or was fused with the radiale, and that the two distal carpalia represented, the one $C_2 + C_3$, the other $C_4 + C_5$. The recent discovery by W. Kükenthal* in an embryo Balænoptera of a rudimentary digit lying in the interval between the phalanges of digits *b* and *c*, raises the question whether it is not the middle digit, medius, rather than the pollex, that is absent in the manus of the Rorquals. Should this be so, the four digits would then represent the pollex, index, annularis, and minimus. It is, therefore, possible that the os magnum, carpale 3, as the distal carpal belonging to the medius, may, like its digit, be suppressed; that carpale $C\alpha$ may represent both trapezium and trapezoid $C_1 + C_2$, or possibly only the trapezoid C_2, C_1 being fused with the radiale; whilst $C\beta$ may represent the unciform, carpalia 4 and 5. $C_4 + C_5$, owing to the suppression of the os magnum, lies more to the radial side of the carpus than is customary, so as both to articulate with carpale 2, and to allow a portion of the metacarpal bone of the most ulnar digit to articulate with the ulnare of the proximal row. This is, however, merely a

* *Anatomischer Anzeiger*, December 27, 1890.

hypothesis, and would need more embryological evidence in its favour before it could be accepted.

In order to make the relations of the carpal bones and digits in *B. rostrata* more clear, I have constructed a graphic formula after the plan pursued by H. Leboucq,* Max Weber,† and W. Kükenthal ‡ in their several memoirs on the hand in the Cetacea. Alongside of it I have placed a similar scheme of the hand of an adult Hyperoodon, in which several years ago I demonstrated for the first time in the cetacean carpus § the presence of five separate osseous carpalia in the distal row, one for each digit. In the formula of *B. rostrata* I have used the descriptive terms employed in the text, which have been selected because they do not commit one to any hypothesis as to the homology either of the distal carpalia or the digits.



The *pelvic bones* were situated at the side of and extending forwards in front of the anal opening. They were fully ossified; one was 8 inches long, the other 8½ inches. They were somewhat curved, and each possessed two surfaces and two borders; the inner border was concave, the outer border for the most part convex. Bearing in mind Mr Perrin's description that "at the summit of the

* *Anatomischer Anzeiger*, March 1887.

† *Morphologisches Jahrbuch*, May 1888.

‡ "Die Hand der Cetaceen," *Denk. med. wissensch. Ges., zu Jena*, vol. iii., 1889.

§ *Jour. of Anat. and Phys.*, Oct. 1885, p. 184.

|| In the hand of an immature male, 20½ feet long, caught at Dunbar in November 1885, and the skeleton of which I have placed in the University Museum, the phalanx of digit 1 was still cartilaginous, as also was the terminal phalanx of digit 5.

outer convexity there was a small rough fibrous mass, about the size of a pea, in which were imbedded a number of very small cartilaginous plates," which he thought evidently represented the femur, I examined with care the corresponding part of the bone in this animal. I found several minute nodules about the size of small shot, which had been cut across in the process of removing the pelvic bone. They were attached to the convex border by fibrous bands. To the naked eye they had a cartilaginous appearance, but when examined microscopically they were seen to consist of fibrous tissue. In all probability the apparent nodules were the ends of small tendons, which had been cut across transversely close to their attachment to the pelvic bone. I may mention that many of the tendons in this animal have a dense homogeneous character, and in section look to the naked eye somewhat like cartilage.

In the following table the dates, localities, and other particulars of the properly authenticated Scottish specimens of *Balœnoptera rostrata* are summarised.

TABLE III.

Date.	Locality.	Length.	Sex.	Museum.	Authority.
		ft. in.			
Nov. 1808	Scalpa Bay, Orkney, . . .	17½ 0	...	... ...	Scoresby.
May 1832	Largo, Firth of Forth, . . .	14 0	...	... ...	...
Feb. 1834	Queensferry, Firth of Forth,. .	9 11	♀	University, Edinburgh.	Knox.
Sept. 1857	Bell Rock, ...	14 5	♂	... ...	M'Bain.
1858	Firth of Forth, ...	...	...	... ...	Alston.
1866	Islay, ...	...	...	University, Cambridge.	Clark.
July 1869	Arbroath, ...	13 0	...	... ...	Walker and Evans.
,, 1870	Aberdeen, ...	14½ 0	♀	University, Aberdeen.	J. Struthers.
1870	Hillswick, Shetland, . . .	...	...	University, Edinburgh.	W. Turner.
Sept. 1870	Burntisland, Firth of Forth, . .	18 0	♀	,, ,,	W. Turner.
,, 1871	Dunbar, ,, ,, . .	30? 0	♂	,, ,,	W. Turner.
April 1872	Anstruther, ,, ,, . .	15½ 0	♀	Science and Art, Edinburgh.	W. Turner.
Oct. 1872	Stornoway, ...	25 0	♂	University, Edinburgh.	W. Turner.
April 1877	Bervie, ...	16 0	♀	University, Aberdeen.	J. Struthers.
July 1879	Elie, Firth of Forth, . . .	18 0	♂	University, Edinburgh.	W. Turner.
Nov. 1879	Stromness, Orkney, . . .	16 0	...	... ...	W. Turner.
Jan. 1888	Granton, Firth of Forth, . .	28 4	♀	University, Edinburgh.	W. Turner.
Oct. 1888	Alloa, ,, ,, . .	...	...	,, ,,	W. Turner.

The Tactics adopted by certain Birds when Flying in the Wind. By Mr R. W. Western. (With a Plate.)

(Read January 18, 1892.)

All who watch the flight of hawks or kites are struck by the extraordinary power they have of sustaining themselves for a long time, sailing about in the air without flapping their wings or making any conspicuous effort whatever.

Most birds of prey, as distinguished from game, are able to do this, and thus hold commanding heights from which to descry food. The expression "wind-sucker," as used by Ben Jonson, refers to the kestrel, a kind of hawk unsuited for falconry ; the astonishing way in which it keeps itself poised in the air, facing the wind with beak outstretched, led to the name, which implies the only explanation of the phenomenon that the science of those days could furnish. Sea-birds are also very proficient.

The author's own observations were upon *Cheel*, a kind of kite, which after the parrot and crow, is the commonest bird in India (and excluding the pariah-dog, perhaps the most useful scavenger). The author once saw one of these get off a bough, wheel about in the air for two and a half minutes by the clock, and finally end up on the self-same bough, without having made a single flap of its wings during the whole time.

Phenomena of this kind are quite beyond the reach of any parachute theory which assumes that the birds present so large an expansion of wing area in proportion to their weight, that the falling which necessarily takes place may be sufficiently small to escape detection. It will, however, be advisable to find an expression for the amount of this falling, so as to determine it precisely.

We must make the conventional assumption about the relation of the velocity of air to the pressure produced by it on a flat surface, viz., that a flat surface destroys the motion of all the air that blows against it in a direction at right angles. Of course, the relation is the same whether the air blows against the surface, or the surface is carried through the air, as in the present instance.

This relation is not exact. When a ship lies at anchor, and swings to the tide, the pressure produced by the stream dividing at the bows is balanced by the waters closing again at the stern (provided her lines are sufficiently fine), so that the only force straining the cable is due to skin friction. If this is true for a dense fluid like water, how much more must it hold in thin air whose mobility enables it to get round the sharp corner of the most angular bodies with very little eddying? Still, by treating the area which a bird presents, as a flat surface, we introduce an error in the opposite direction, for the form tends to reproduce the velocity of impinging air in an opposite direction, thereby increasing the pressure sustained.

In the absence of better information on this point, we must suppose the two errors to cancel one another more or less, awaiting the time when the results of a number of delicate experiments will enable us to treat the matter with greater exactitude.

It may be noted that the distribution of feathers at the edge of the wing is such as to promote the formation of eddies.

To proceed, then, on the hypothesis explained above, the resistance of the air to the bird, when falling with outstretched wings at v feet per second, will be $\frac{\mathrm{GA}v}{g}v$, where A square feet is the area presented by the bird, and G lbs. is the weight of a cubic foot of air; $g = 33{\cdot}2$. It is obvious that this quantity cannot be greater than the weight of the bird. Thus from the equation $\mathrm{W} = \frac{\mathrm{GA}}{g}v^2$ we obtain $v = \sqrt{\frac{\mathrm{W}g}{\mathrm{GA}}}$ as the greatest velocity which the falling bird can attain.

From a number of measurements of the particular birds to which the observations of the author relate, W may be taken at $\frac{21}{16}$ lbs. and A at $2\frac{1}{2}$ square feet. So that the greatest rate of falling will be

$$\sqrt{\frac{\frac{21}{16} \times 32{\cdot}2}{{\cdot}0807 \times 2{\cdot}5}} = 14{\cdot}5 \text{ feet per second.}$$

The falling bird will never exactly attain this velocity, but it very quickly approaches it.

To find the velocity attained after falling any number of seconds t; we must equate the increase of motion, to the excess that the weight of the body has over the resistance of the air, after any interval of falling. Thus the downward acceleration

$$=\frac{W-\frac{GA}{g}v^2}{\frac{W}{g}}=g-\frac{GA}{W}v^2$$

or

$$\frac{dv}{dt}=g-\frac{GA}{W}v^2 \text{ and } dt=\frac{dv}{g-\frac{GA}{W}v^2}$$

As v^2 can never actually become equal to $\frac{Wg}{GA}$, the expression $g-\frac{GA}{W}v^2$ will never become zero; thus the left-hand side of the last equation is continuous, and can be integrated in all cases.

$$\int_0^t dt=\int_0^v \frac{dv}{g-\frac{GA}{W}v^2}$$

or

$$t=\frac{1}{2\sqrt{\frac{GA}{W}g}}\cdot\log\frac{\sqrt{g}+\sqrt{\frac{GA}{W}}v}{\sqrt{g}-\sqrt{\frac{GA}{W}}v}$$

$$=\frac{1}{2\sqrt{kg}}\log\frac{\sqrt{g}+v\sqrt{k}}{\sqrt{g}-v\sqrt{k}};\text{ where } k=\frac{GA}{W}$$

whence

$$\log^{-1}2t\sqrt{kg}=\frac{\sqrt{g}+v\sqrt{k}}{\sqrt{g}-v\sqrt{k}}$$

and

$$\frac{\log^{-1}2t\sqrt{kg}-1}{\log^{-1}2t\sqrt{kg}+1}=v\sqrt{\frac{k}{g}}$$

therefore

$$v=\sqrt{\frac{g}{k}}\left\{1-\frac{2}{e^{2t\sqrt{kg}}+1}\right\}$$

Whence it will be seen after the first second, the falling body will have attained a velocity of only $\frac{1}{\log^{-1}\sqrt{\frac{GA}{W}}g+1}$ short of its absolute maximum. Substituting in this the values for W and A already given, we obtain ·0237. So that the falling bird at the end of the first second will have attained a velocity only $2\frac{1}{4}$ per cent. less than its greatest possible value.

It is, however, also necessary to find how *far* the bird will fall in a given time. We must, therefore, evaluate

$$\int_0^t vdt$$

$$= \sqrt{\frac{g}{k}} \int_0^v \left\{ 1 - \frac{2}{e^{2t\sqrt{kg}}+1} \right\} dt$$

$$= \sqrt{\frac{g}{k}} \left\{ t - 2\log_e \frac{e^{2t\sqrt{kg}}}{e^{2t\sqrt{kg}}+1} + 2 \log \tfrac{1}{2} \right\}$$

$$= \sqrt{\frac{g}{k}} \left(t - 2\log_e \frac{\log^{-1} 2t\sqrt{\frac{GA}{W}g}}{\log^{-1} 2t\sqrt{\frac{GA}{W}g}+1} - 1{\cdot}386 \right)$$

Putting $t = 1$ in order to find the distance fallen during the first second, we see that the middle term becomes so small that it may be neglected. Then, substituting the proper value for W and A, we have

$$14{\cdot}5 \times -{\cdot}386 = -5{\cdot}6$$

or about 5 feet 7 inches in a downward direction.

For the next and succeeding seconds the drop, as we have seen, will exceed 14 feet.

Actual experiments on dead birds, with their wings extended with wire (correction being made for the increase of weight), more than confirm these results, and entirely dispose of the parachute theory as applied to these birds. We must, therefore, look to another principle to explain the support they derive from the air they fly in.

Such a principle is furnished in the example of a common paper kite. As everyone knows, the two essential conditions for making the kite to rise are, wind and string, of which a bird must necessarily lack the string. For this reason the bird can make no great use of this principle in a wind of constant velocity. When, however, the rate of the wind continually varies (as is usually the case), it may substitute for the string the resistance due to its own inertia.

We need only assume for the bird, a power of balancing itself at any inclination, so as to be able to present the area of its wings to the wind in the most advantageous position.

Suppose the wind blowing from right to left in fig. 1.

Let EF represent the trace of the surface which the bird creates; area A square feet. Then, if EF is inclined to the horizontal at the angle α, the area obstructive to the wind is A sin α, and the mass of air obstructed per second

$$=\frac{\mathrm{GA}\sin\alpha}{g}u$$

u being the relative velocity of the wind.

This produces a pressure at right angles to EF proportional to the resolved velocity in that direction

$$=\frac{\mathrm{GA}\sin\alpha}{g}u(u\sin\alpha) \quad \text{or} \quad \frac{\mathrm{GA}\sin^2\alpha}{g}u^2$$

Resolving this pressure vertically and horizontally, we obtain a vertical force $\frac{\mathrm{GA}\sin^2\alpha}{g}u^2\cos\alpha$ lbs. to support the weight of bird, and a horizontal force $\frac{\mathrm{GA}}{g}u\sin^3\alpha$ producing acceleration in the direction of the wind.

Let us suppose the bird to be just supported. Then equating the former force to W, the weight of the bird, we have

$$\frac{\mathrm{GA}}{g}u^2\sin^2\alpha\cos\alpha=\mathrm{W}$$

from which

$$u^2=\frac{\mathrm{W}g}{\mathrm{GA}}\cdot\frac{1}{\sin^2\alpha\cos\alpha}$$

As the bird is supposed to be able to poise itself in the most advantageous way, we must determine what value of α correlates with the smallest value of u.

Differentiating the expression for u^2, and equating the derived coefficient to zero, we get

$$2\sin\alpha\cos^2\alpha=\sin^3\alpha$$

or

$$\tan\alpha=\sqrt{2}$$

so that the most advantageous inclination is $\tan^{-1}\sqrt{2}$ or 54° 44′.

Substituting this value for α in the equation

$$u=\sqrt{\frac{\mathrm{W}g\sec\alpha}{\mathrm{GA}\sin^2\alpha}}$$

the least value of u is shown to be $14\cdot5\times\frac{1\cdot316}{\cdot8165}=23\cdot3$ feet per second; $\sqrt{\frac{\mathrm{W}g}{\mathrm{GA}}}$ being 14·5 in the present instance, as found before.

Hence our bird would remain supported at an inclination of 54° 44′ when a wind of 23·3 feet per second was blowing against it. This only constitutes a light breeze, No. 2 in Beaufort's scale, in which a well-conditioned man-of-war, with all sail set, and clean hull, would go about three knots an hour in smooth water.

There is seldom less wind than this in the air, and when this velocity is exceeded, an upward force is developed which is greater than the weight of the bird; the latter being thus raised to a higher level.

Let us now consider the effect of the horizontal force $\frac{\mathrm{GA}}{g}u^2\sin^3\alpha$, which is hurrying the bird along in the direction of the wind.

If the velocity of the wind is γ feet per second, as has been shown, our bird will no longer be supported after its own velocity in the same direction becomes $\gamma-23\cdot3$ feet per second.

Let us suppose the gust touches the rate of 50 feet per second, which is not much for its momentary highest velocity. Then our bird, when the support fails, will be travelling at $(50-23\cdot3)$ feet per second, or 26·7 feet per second. If the gust then comes to an end, and the wind falls, the bird can avail itself of this energy to rise in the air. 26·7 feet per second is equivalent to an increase of level of 11·7 feet, according to the relation $\mathrm{H}=\frac{v^2}{2g}$.

It will be necessary, of course, to discount this by loss due to skin friction or resistance produced by the air in passing over the feathers, also by loss of energy involved in changing the direction, neither of which sources of discrepancy are easily estimated.

It was only desired to demonstrate one case in which the bird may support itself without direct expenditure of energy for the purpose.

Sufficient has been proved to show the advantage which must accrue to one that skilfully negotiates a gust. But the resources of a sagacious bird are not exhausted.

When there are no gusts, and the wind is steady, or in the case under consideration, when the variations of its velocity do not exceed 23·3 feet per second, if the bird can, by utilising part of its acquired velocity, by swooping down, or making a flap or two, get into another current, or anywhere where the air is comparatively still, it will be in the same position as occurred at the lull, which we had supposed in the previous instance to follow the gust.

Actual observation, in the light of this theory, makes it quite evident they do make use of such tactics, and that it aids them considerably in sustaining their flight.

The author has observed *Cheel* ascend the spur of a mountain some hundreds of feet by following manœuvres which plainly illustrate the working of the above principles.

Take AB, fig. 2, for the axis of the spur. The curved line shows the course of the bird. Flying towards the spur, it ceases to use its wings at *a* and rises by virtue of its attained velocity to *b*, where it meets the wind coming over the edge of the hill, and is thus driven quickly to *c*; here it finds the air stagnant. Sheltered by the rising slope of the spur, it drops a little to reach still quieter air, and utilises its thereby further-increased velocity to bring it back again towards the hill, the operations being continued indefinitely.

In the instance observed by the author, the bird found it necessary to make from four to six flaps of its wings at points *c*, *e*, &c.

It may be also remarked that most birds in a high wind, when they pass such an object as a church steeple, for instance, will make use of the changed velocity of wind on its lee side to raise themselves perceptibly in their flight.

In such ways the wind, to us a hitherto insuperable obstacle to practical aeronautics, is turned to account by unintelligent members of the animal creation. They trade, as it were, between two regions of air, bringing energy acquired in one to make use of in another.

The difficulty of adopting the same practice with flying machines is the extreme rapidity with which movements must be made. In order to demonstrate this, let us consider the rate at which an object of weight, W lbs., presenting to the wind an effective area of A square feet, will acquire velocity.

Take the velocity of the wind γ feet per second, and its relative velocity to the object $(\gamma - v)$ feet per second. Then the pressure acting on the object will be

$$\frac{\mathrm{GA}(\gamma - v)}{g}(\gamma - v)$$

and dividing this by the mass of the body, we get its acceleration

$$\frac{\frac{\mathrm{GA}}{g}(\gamma - v)^2}{\frac{\mathrm{W}}{g}}$$

whence the equation

$$\frac{dv}{dt} = \frac{\mathrm{GA}}{\mathrm{W}}(\gamma - v)^2$$

inverting

$$dt = \frac{\mathrm{W}}{\mathrm{GA}} \frac{dv}{(\gamma - v)^2}$$

Then as v, the velocity of the body, can never reach γ, the rate of the wind, we may integrate both sides

$$t = \frac{\mathrm{W}}{\mathrm{GA}} \int_0^v (\gamma - v)^{-2} dv$$

i.e.,

$$t = \frac{\mathrm{W}}{\mathrm{GA}} \left\{ \frac{1}{\gamma - v} - \frac{1}{\gamma} \right\}$$

$$= \frac{\mathrm{W}}{\mathrm{GA}} \cdot \frac{v}{\gamma^2 - \gamma v}$$

or

$$t\gamma^2 - t\gamma v = \frac{\mathrm{W}}{\mathrm{GA}} v$$

or

$$t\gamma^2 = v\left(t\gamma + \frac{\mathrm{W}}{\mathrm{GA}}\right)$$

whence

$$v = \frac{t\gamma^2}{t\gamma + \frac{\mathrm{W}}{\mathrm{GA}}}$$

Since $\frac{W}{GA}$ must in all practical cases be a small quantity compared to γ, the velocity of the object will be almost equal to that of the wind in a few seconds.

In the example of the bird treated as an inclined plane, and just supported by a constant relative velocity of wind, the horizontal acceleration is $g \tan \alpha$, where α, as before, is the inclination of the plane to the horizontal. For the ratio of the horizontal to the vertical pressures, due to the wind, is

$$\frac{\frac{GA}{g} u^2 \sin^2 \alpha \cos \alpha}{\frac{GA}{g} u^2 \sin^3 \alpha} = \tan \alpha$$

Therefore since the latter force is by hypothesis equal to the weight of the bird, and its acceleration under that would be g, its acceleration under the former will be $g \tan \alpha$.

When α is given, the value 54° 44′ required to bring about the least value for the relative velocity u—

$$g \tan \alpha = 45 \text{ feet per second.}$$

There is no way out of this difficulty.

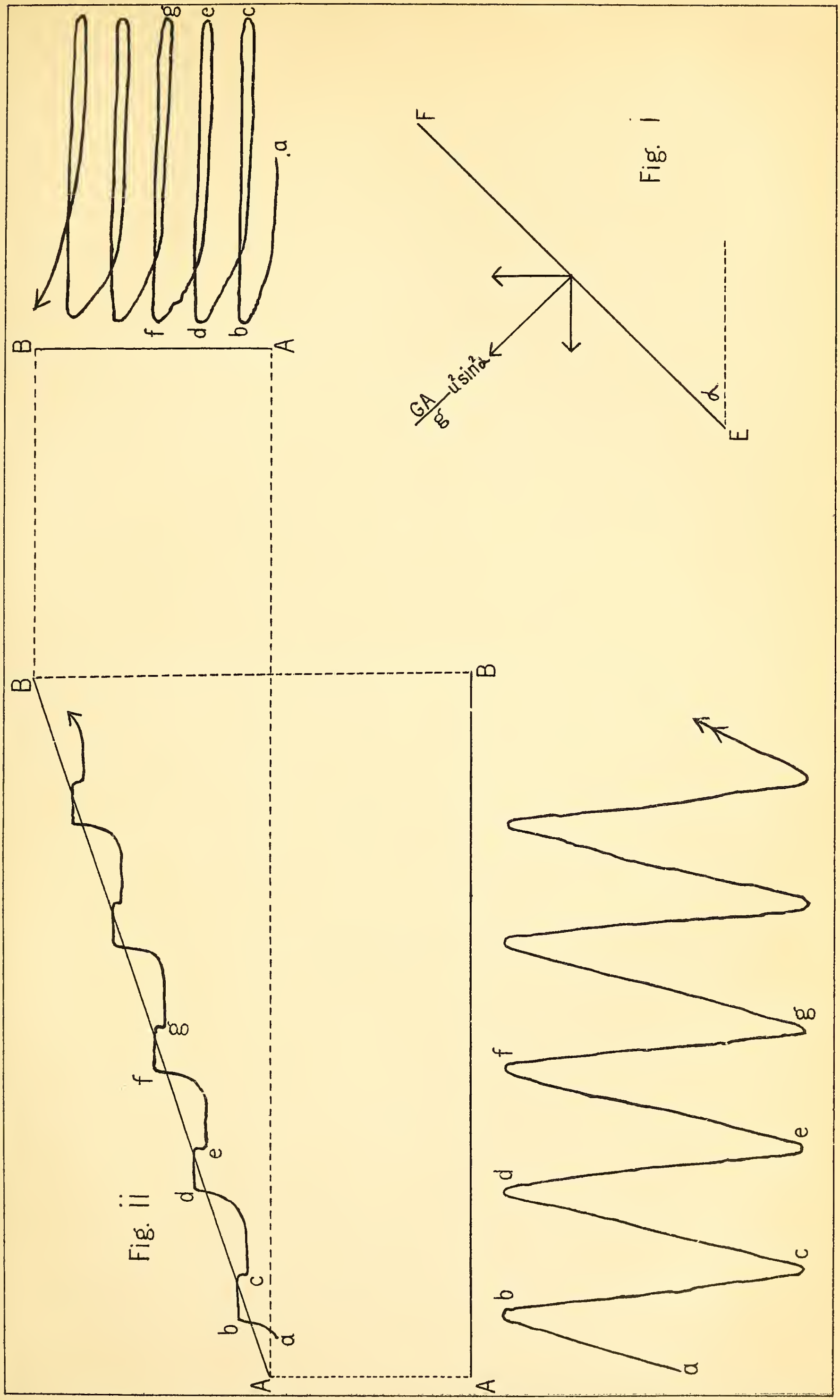
Plenty of energy is in the air, enough to support many flying machines, but one must be very quick to seize it.

Such things as upward currents in the air no doubt exist, and birds continually make use of them; to currents like this must be attributed the more inexplicable feats of sailing, which *Cheel* sometimes perform, round the tops of hills.

The vertical velocity of these currents in order to support the bird should be $\sqrt{\frac{Wg}{GA}}$; 17 feet per second for these, at least; being a velocity equal to their ultimate rate of falling. Such currents cannot often occur.

In cyclonic weather there is supposed to be a general upward current of the air, but it probably does not exceed 6 inches per second, and birds notoriously fly best in anticyclonic weather, when the very opposite conditions prevail.

WESTERN ON TACTICS OF BIRDS FLYING IN THE WIND.



Note on Certain Remarkable Volume Effects of Magnetisation. By Professor Cargill G. Knott, D.Sc., and A. Shand, Esq.

(Read May 16, 1892.)

In June of last year one of us gave before the Society a short account of certain preliminary experiments on the volume effects of magnetisation.* These experiments had been made in Japan; and the results obtained evidently called for a more thorough investigation. The facilities of the Physical Laboratory of Edinburgh University were kindly placed at our disposal by Professor Tait. Five tubes of Swedish iron, all cut from the same original bar but made with different bores, were prepared for study in much the same manner as formerly. Many experimental difficulties encountered us; and only now have these been satisfactorily overcome. For it must be remembered that the effect to be observed is very minute, and can be measured only in one way, viz., by the motion of the meniscus along the narrow capillary tube which is in connection with the iron tube, the whole being filled with fluid. But such an arrangement is obviously a very delicate thermometer; and, as a matter of fact, we have seldom got the meniscus really steady. Generally it is moving at a rate which, though as inappreciable to the naked eye as the motion of an hour hand, is only too evident under the microscopic power necessary for the observation of the magnetic effect.

To diminish the temperature effect upon the liquid contained in the tube, we used Professor Tait's device of filling up most of the volume by a hollow glass bulb nearly the same length and width as the bore. But there still remained the effect of temperature upon the iron walls. At first it might be supposed that with the three substances, iron, glass, and liquid, it would be a comparatively easy matter by calculation to so adjust the size of the glass bulb to the

* "On the Effect of Longitudinal Magnetisation on the Interior Volume of Iron and Nickel Tubes," by Professor Knott, D.Sc., F.R.S.E., *Proc. Roy. Soc. Edin.*, 1891.

volume of the iron tube as to compensate accurately for the degree or two of temperature variation that might occur in the course of a single experiment. But it is obvious that the iron tube will be first affected by a change of temperature coming in from without, and the glass last of all. Consequently, it seemed altogether a vain quest to seek for anything like a perfect compensation.

The liquid finally chosen to fill the volume of the tube was water. Mercury and coloured alcohol were both tried, simply to be discarded; the former because of its tendency to go by fits and starts in the capillary tube, the latter because of its high expansibility.

Only one iron tube, No. I., has so far been satisfactorily experimented with. Rough trials with Tubes II. and III. show that similar effects are produced with them. The present note, however, refers only to No. I., which has the widest bore or thinnest wall of the five. The very interesting results obtained with it call for a brief description.

On reference to the paper already cited, it will be seen that up to the magnetic fields used the internal capacities of the iron tubes all diminished. It was pointed out, however, that a maximum change seemed to have been about to be reached; and, reasoning from the remarkable behaviour of the nickel tube, we might expect not only the existence of a maximum contraction for a moderate field, but even a change from contraction to dilatation at higher fields. Such a result would not be surprising in the light of Mr Bidwell's experiments on the changes of *length* of magnetised rods.

But the extraordinary fact to be noted is that, with Tube I., we get not only a single maximum and a change of sign in the dilatation, but *three* maxima (or two maxima and one minimum) and *two* changes of sign. It will suffice to give one set of measurements. Very similar results were obtained in all the really good experiments.

The bore-space filled by the water during the experiment had a volume of 343·75 cub. cm., with a cross-section of approximately 8 sq. cm. and a length of nearly 42·7. The thickness of wall was ·33 cm., the external and internal diameters being respectively 3·84 and 3·19 cm.

The tube when being magnetised was well included in the magnetising coil.

The magnetising fields are given in magnetic units, and the changes of volume in the experimental scale unit. To reduce to cubic centimetres, multiply by $2{\cdot}23 \times 10^{-6}$. To reduce to dilatations, multiply by $6{\cdot}5 \times 10^{-9}$.

Field.	Volume Increment.	Field.	Volume Increment.	Field.	Volume Increment.
6·7	+ 3·3	81·5	− 52·8	523	+ 59·8
12·3	+ 1·7	100	− 24·9	616	+ 57
20·7	− 0·3	123	+ 3·9	701	+ 54·5
27·4	− 8·1	127	+ 8	614	+ 56
30·2	− 9·5	134	+ 12·5	500	+ 62
37·5	− 22·3	176	+ 35·3	361	+ 61·5
43·3	− 33·9	181	+ 42	377	+ 59
53·6	− 50·2	184	+ 37	296	+ 55·5
54·1	− 53·5	191	+ 38·5	180	+ 30·1
51·2	− 57	301	+ 60·3	143	+ 11·3
64	− 66	398	+ 61·8	85	− 49
67·2	− 63				

Here, as will be seen at a glance, the dilatation changes sign twice, first about field 20, and then about field 120. It attains a positive maximum about field 10, and again about field 400. The negative maximum is particularly sharp, and occurs about field 64. Comparing these volume dilatations with Mr Bidwell's results for linear dilatations in the direction of magnetisation, we may provisionally draw these conclusions. The linear dilatation (μ) at right angles to the direction of magnetisation is generally of opposite sign to the linear dilatation (λ) along the direction of magnetisation. At low fields the lengthwise elongation is numerically greater than twice the transverse contraction, so that the cubical dilatation ($\lambda + 2\mu$) is positive. Soon, however, as the field is taken stronger, the transverse contraction becomes numerically the greater, and the cubical dilatation changes sign. So long as the cubical dilatation remains negative, the transverse contraction has the advantage. It, however, seems to pass through a maximum and then diminish, so that the lengthwise elongation again recovers its superiority and the cubical dilatation becomes positive. Finally, since the cubical dilatation remains positive up to fields much higher than that at which the change in length becomes contraction instead of elongation, it follows that the transverse contraction must also change sign and become a transverse dilatation. In a general way we may say that the linear dilatations along and perpendicular to the magnetic field tend to be of opposite sign. Where the one is positive the other is negative. Each has a comparatively early maximum, and each changes sign in moderate fields. Their manners of variation differ, however, in detail; and hence arises the undulatory variation which has just been described as characteristic of the volume changes of a particular iron tube under the influence of increasing magnetising forces.

On the Eliminant of the Equations of the Ellipse-Glissette. By the Hon. Lord M'Laren.

(Read June 6, 1892.)

The problem consists in the elimination of θ from two equations which (after expressing $\cos^2\theta$ in terms of $\sin^2\theta$) may be written

$$-\mathrm{C}\sin\theta\cos\theta - \mathrm{B}\sin^2\theta + \alpha_1\cos\theta + \beta_1\sin\theta + \gamma_1 = 0 : \quad . \quad . \quad . \quad (1)$$
$$\mathrm{C}\sin\theta\cos\theta + \mathrm{B}\sin^2\theta + \alpha_2\cos\theta + \beta_2\sin\theta + \gamma_2 = 0 : \quad . \quad . \quad . \quad (2)$$

whence,

$$(\alpha_1 + \alpha_2)\cos\theta + (\beta_1 + \beta_2)\sin\theta + \gamma_1 + \gamma_2 = 0 :$$

or

$$\lambda\cos\theta + \mu\sin\theta + \nu = 0 : \quad . \quad . \quad . \quad (3)$$

As only two of the equations can be used, the elimination will be effected by means of (2) and (3), using the relation $\cos^2\theta + \sin^2\theta = 1$, where necessary, to suppress the term of $\cos^2$.

The original equations include five terms, and it is proposed to find the determinant of the 5th order; the first step to which is the formation of a determinant of the 8th order.

Putting X for $\cos\theta$, and Y for $\sin\theta$, we have

	Y^3	X^2Y	XY^2	XY	Y^2	X	Y	1	
	.	.	.	C	B	α	β	γ	(2)
	.	.	.	.	.	λ	μ	ν	(3)
	.	.	.	μ	$-\lambda$	ν	.	λ	(4)
$0=$	.	.	.	λ	μ	.	ν	.	(5)
	B	.	C	α	β	.	γ	.	(*b*)
	$-\lambda$	.	μ	ν	.	.	λ	.	(*c*)
	.	C	B	β	$-\alpha$	γ	.	α	(*d*)
	.	μ	$-\lambda$	.	$-\nu$	λ	.	ν	(*e*)
			$B\mu+\lambda C$	$B\nu+\lambda\alpha$	$\lambda\beta$	.	$\lambda(B+\gamma)$	.	(*f*)
			$\mu B+C\lambda$	$\mu\beta$	$C\nu-\mu\alpha$	$\mu\gamma-C\lambda$	.	$\mu\alpha-C\nu$	(*g*)
				$\alpha\lambda-\beta\mu+B\nu$	$\beta\lambda+\alpha\mu-C\nu$	$C\lambda-\gamma\mu$	$(B+\gamma)\lambda$	$C\nu-\alpha\mu$	(6)

The determinant consisting of the first eight rows is a solution, but is too high, being, as will presently be seen, of the 9th degree in the cöordinates of the curve. The determinant of the 5th order consists of the first four rows and the last row, or (2), (3), (4), (5), and (6). In applying the dialytic method, as has been done to the solution of the present case, I had the choice of *nine* equations of the 3rd degree; and the selection from these was determined by two considerations:—

1*st.* In order that the terms of the 3rd degree might be made to disappear without unduly raising the degree of the eliminant, it was found that the second group of equations should be formed in pairs by multiplying two equations of the first group by X and Y respectively. This is a condition which seems to be generally applicable to problems of elimination.

2*nd.* In order that the eliminant should have an absolute term (as required by the conditions of the case), it was seen to be necessary that the constant part of its terms of XY and Y^2 should not be the same for equation (2) and the resulting equation (6); because if the constant terms be equal with like signs, the whole function will vanish identically.

These conditions determine that the second group of equations should be formed from (2) and (4) by multiplying these functions by X and Y successively. [If we were using (1) or (2) with $\cos^2\theta$ instead of $\sin^2\theta$, the second group would be formed from (5), which would then have the sign of X^2 negative.]

In the preceding table, (f) is formed from (b) and (c), (g) is formed from (d) and (e), and (6) is $(f)-(g)$: the resulting equation (6) being only of the 2nd degree. The equations (2), (3), (4), (5), and (6) constitute the required determinant of the 5th order.

An equation conjugate to (6) may be formed by suppressing $\sin^2\theta$ in the original equations, and proceeding as above.

A partial expansion of the quintic determinant gives the following eliminant:— . . . . (A)

$$
\begin{aligned}
&(\alpha\lambda-\beta\mu+\mathrm{B}\nu)\{\mathrm{B}\nu^3+\alpha\lambda\nu^2+\beta\lambda^2\mu+\gamma\mu^2\nu-\mathrm{B}\lambda^2\nu-\alpha\lambda\mu^2-\beta\mu\nu^2-\gamma\lambda^2\nu\}\\
&+(\beta\lambda+\alpha\mu-\mathrm{C}\nu)\{\mathrm{C}\lambda^2\nu+\alpha\mu\nu^2+\alpha\lambda^2\mu+\beta\lambda\nu^2-\mathrm{C}\nu^3-\beta\lambda^3-2\gamma\lambda\mu\nu\}\\
&+(\mathrm{C}\lambda-\gamma\mu)\{\mathrm{C}\lambda\mu^2+\beta\lambda^2\nu+\beta\mu^2\nu-\mathrm{B}\lambda^2\mu-\mathrm{B}\mu\nu^2-\mathrm{C}\lambda\nu^2-\gamma\lambda^2\mu-\gamma\mu^3\}\\
&+(\mathrm{B}+\gamma)\lambda\ .\ \{\mathrm{B}\lambda^3+\mathrm{C}\mu\nu^2+\gamma\lambda^3+\gamma\lambda\mu^2-\mathrm{B}\lambda\nu^2-\mathrm{C}\lambda^2\mu-\alpha\lambda^2\nu-\alpha\mu^2\nu\}\\
&+\mathrm{C}\nu-\alpha\mu)\{2\mathrm{B}\lambda\mu\nu+\mathrm{C}\lambda^2\nu+\alpha\lambda^2\mu+\alpha\mu^3-\mathrm{C}\mu^2\nu-\beta\lambda^3-\beta\lambda\mu^2\}=0\,.
\end{aligned}
$$

This equation has been developed in two different combinations, and the results compared and found to agree.

Instead of forming the determinant of the 5th order from that of the 8th order, as above given, we may form the 5th row directly from the rows marked (2) and (4), as follows :—

By eliminating successively the 1st and 2nd terms between these rows, we form the two equations

$$(B\mu + C\lambda)XY \quad . \quad + (B\nu + \alpha\lambda)X + \lambda\beta Y + (\beta + \gamma)\lambda = 0$$
$$. \quad (B\mu + C\lambda)Y^2 + (\alpha\mu - C\nu)X + \beta\mu Y + (\gamma\mu - C\lambda) = 0\,.$$

By subtraction,—after multiplying the first equation by Y and the second by X,—we obtain the 5th row, being (6) of the tabular series. The operation is essentially the same as that already performed.

The determinant may be reduced to the 4th order by eliminating one of the higher terms between the 1st row and each of the others successively. But this is really a retrograde step; because in the development, the number of terms is greater than in the preceding. Apparently, to obtain the best results, the order of the determinant ought *not to be less* than the number corresponding to the number of terms of the original equation.

The equations of the glissette, as given by Professor Tait (*Proc. Roy. Soc. Edin.*, vol. xvii. p. 2), are

$$x - r\cos(\theta - \alpha) = (a^2\cos^2\theta + b^2\sin^2\theta)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (a)$$
$$y - r\sin(\theta - \alpha) = (b^2\cos^2\theta + a^2\sin^2\theta)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (b)$$

By expansion, and putting $r\cos\alpha = p$, $r\sin\alpha = (-q)$, and expressing $\cos^2\theta$ in terms of $\sin^2\theta$, these equations become

$$\left.\begin{aligned} -(a^2 - b^2 - p^2 + q^2)\sin^2\theta + 2pq\cos\theta\sin\theta + 2px\cos\theta - 2qx\sin\theta + a^2 - p^2 - x^2 = 0 \\ (a^2 - b^2 - p^2 + q^2)\sin^2\theta - 2pq\cos\theta\sin\theta + 2qy\cos\theta + 2py\sin\theta + b^2 - q^2 - y^2 = 0 \end{aligned}\right\}$$

Also

$$2(px + qy)\cos\theta + 2(py - qx)\sin\theta + a^2 + b^2 - p^2 - q^2 - x^2 - y^2 = 0 :$$

Comparing these questions with the generalised expressions (1), (2), and (3), we have

$$B = a^2 - b^2 - p^2 + q^2\,; \qquad C = -2pq\,;$$
$$\alpha_2 = 2qy\,; \qquad \beta_2 = 2py\,;$$
$$\lambda = 2(px + qy)\,; \qquad \mu = 2(py - qx)\,;$$
$$\gamma_2 = b^2 - q^2 - y^2\,; \qquad \nu = a^2 + b^2 - p^2 - q^2 - x^2 - y^2.$$

λ and ν are of the 2nd degree in the x-and-y cöordinates of the curve. α, β, λ, and μ are of the 1st degree in these cöordinates. B, C are functions of constants only.

These values being compared with the determinant, it is seen that the eliminant is of the 8th degree in x and y. The subjoined tables give respectively the highest and lowest indices of the cöordinates, x and y, for each term of the determinant :—

$$\left.\begin{array}{ccccc} 0 & 0 & 1 & 1 & 2 \\ . & . & 1 & 1 & 2 \\ 1 & 1 & 2 & . & 1 \\ 1 & 1 & . & 2 & 1 \\ 2 & 2 & 3 & 3 & 2 \end{array}\right\} = 0 : \qquad 0 \left\{\begin{array}{ccccc} 0 & 0 & 1 & 1 & 0 \\ . & . & 1 & 1 & 0 \\ 1 & 1 & 0 & . & 1 \\ 1 & 1 & . & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right.$$

The first table shows the eliminant to be of the 8th degree. The absolute term is given by the zero indices of the second table. It is included in the term $(B^2 + C^2)\nu^4$, of which the absolute part is

$$\{(b^2 - a^2 - p^2 + q^2)^2 + 4p^2q^2\}\{a^2 + b^2 - p^2 - q^2\}^4 .$$

This quantity may vanish by either factor becoming $=0$, when the curve passes through the point of intersection of the guides. For the case of $a^2 + b^2 = p^2 + q^2$, the curve has its principal axis in the 1st and 3rd quadrants. The other case apparently represents a curve whose axis of symmetry is in the 2nd and 4th quadrants.

A solution has been given by Dr Muir in the current volume of the *Proceedings*, vol. xix. p. 25, which was obtained by a different process. His eliminant as first obtained is of the 10th degree; and the extraneous factor being found, the equation is then reduced to the 8th degree, and expanded in terms of x and y.

It is proposed to verify the two solutions by comparing the constituent terms of the 8th degree in my solution with the corresponding terms as given by Dr Muir. The first step is to multiply out the factors of the eliminant above found, (A), p. 90, arranging the terms according to powers of the non-homogeneous elements γ and ν. The resulting expression is

$$
\begin{aligned}
&(B^2+C^2)\nu^4+(4C\lambda\mu-2B\lambda^2+2B\mu^2)\gamma\nu^2\\
&+2(B\alpha\lambda-C\beta\lambda+C\alpha\mu-B\beta\mu)\nu^3\\
&+(2\lambda^2\mu^2+\mu^4+\lambda^4)\gamma^2-2(\alpha\lambda^3+\alpha\lambda\mu^2+\beta\lambda^2\mu+B\mu^3)\gamma\nu\\
&+(\alpha^2\lambda^2+\beta^2\mu^2+\beta^2\lambda^2+\alpha^2\mu^2)\nu^2\\
&+(2BC\lambda\mu-2B^2\lambda^2-C^2\lambda^2-C^2\mu^2)\nu^2\\
&+2(B\beta\lambda^2\mu-B\alpha\lambda^3+C\beta\lambda^3-2B\alpha\lambda\mu^2+C\alpha\mu^3)\nu+2(B\lambda^4-C\mu\lambda^3\\
&\qquad+B\lambda^2\mu^2-C\lambda\mu^3)\gamma\\
&+C^2\lambda^2\mu^2-2BC\lambda^3\mu+B^2\lambda^4+2\alpha\beta\lambda^3\mu-\beta^2\lambda^2\mu^2-\alpha^2\lambda^2\mu^2-2\alpha\beta\lambda\mu^3\\
&\qquad-\alpha^2\lambda^4-\beta^2\lambda^4=0\,.
\end{aligned}
$$

The terms of the 8th degree in x and y are all contained in the first four lines of the preceding formula, and these only need to be expressed in terms of x and y. Substituting for α, β, λ, μ, in the above factors, and retaining only the highest terms of the expressions which are represented by ν^3, ν^4, $\gamma\nu^2$, we have

$$
\begin{aligned}
&(B^2+C^2)(x^8+4x^6y^2+6x^4y^4+4x^2y^2+y^8+\ \&c.)\\
&-\Big\{8\{B(q^2-p^2)-2Cpq\}x^2+16\{C(p^2-q^2)-2Bpq\}xy+8\{B(p^2-q^2)\\
&\qquad+2Cpq\}y^2\Big\}\times(x^4y^2+2x^2y^4+y^6+\ \&c.)\\
&-\Big\{8\{C(q^2-p^2)+2Bpq\}xy+8\{B(q^2-p^2)-2Cpq\}y^2\Big\}\times(x^6+3x^4y^2\\
&\qquad+3x^2y^4+y^6)\\
&\qquad+16(p^2+q^2)^2(x^4+2x^2y^2+y^4)\times(y^4+\ \&c.)\\
&\qquad-32(p^2+q^2)^2(x^2y^2+y^4)\times(x^2y^2+y^4+\ \&c.)\\
&\qquad+16(p^2+q^2)^2(x^2y^2+y^4)\times(x^4+2x^2y^2+y^4+\text{etc.})+f_6(x_1y)\\
&\qquad\quad+f'_4(x_1y)+f''_2(x_1y)=0\,.
\end{aligned}
$$

Observing that $C=-2pq$, $B=a^2-b^2-p^2+q^2$, the coefficients of the powers are seen to be

For

$$
\begin{aligned}
&x^8, && (B^2+C^2)=(a^2-b^2-p^2+q^2)^2+(2pq)^2\ \ldots\ldots =\Omega\\
&x^7y, && 8(C(p^2-q^2)-2Bpq)=-16pq(a^2-b^2)\\
&x^6y^2, && 4(B^2+C^2+4B(p^2-q^2)+4(p^2+q^2)+8Cpq)\\
& && \left.\begin{aligned}&=4(\Omega+4(p^2-q^2)(a^2-b^2)\\&=4(a^2-b^2+p^2-q^2)^2+(4pq)^2\end{aligned}\right\}=4\Gamma\\
&x^5y^3, && -16pq(a^2-b^2)\\
&x^4y^4, && 6(B^2+C^2)+32B(p^2-q^2)+32(p^2+q^2)^2+64Cpq=8\Gamma-2\Omega\\
&x^3y^5, && +16pq(a^2-b^2)\\
&x^2y^6, && 4\Gamma\ \ldots\ldots\ldots\ldots =4\Gamma\\
&xy^7, && +16pq(a^2-b^2)\\
&y^8, && (a^2-b^2-p^2+q^2)^2+(2pq)^2\ \ldots\ldots =\Omega
\end{aligned}
$$

These are the same values which are found in Dr Muir's paper.*

* There are two errata in the page referred to (p. 27, below the Table). Line 6 should be $(a^2-b^2+p^2-q^2)^2+(2pq)^2$; and the expression in the 10th line is meant to be equited to 4Γ.

Note on the Elimination of θ between two Equations of the 2nd Degree.

In the case of the glissette problem one of the equations is of the 1st degree. With two general equations of the 2nd degree $(a)=0$, $(b)=0$, and the relation $\cos^2\theta+\sin^2\theta=1$, we may form *two* new conjugate equations in the form (6) of the glissette system. To form the first of these, $\cos^2\theta$ is transformed to $\sin^2\theta$ before raising the equations to the 3rd degree. The terms of the 3rd degree are then eliminated between four equations of the forms, (b), (c), (d), (e) of that system. To form the conjugate equation, $\sin^2\theta$ is transformed before operating. A fifth equation is wanted, because we cannot use $\cos^2\theta+\sin^2\theta=1$ without introducing an additional term. This equation is got by eliminating $\cos\theta$ and the term independent of θ between the cubic equations, and then dividing by $\sin\theta$, or *vice versa.*

Supposing the term of $\cos^2\theta$ transformed, and the equations divided by their absolute terms, the last-mentioned equation will be formed as under, where X is put for $\cos\theta$, and Y for $\sin\theta$:—

	X^2Y	XY^2	X^2	XY	Y^2	X	Y	1	
(1) (a)	·	·	·	a_1	b_1	c_1	d_1	1	
(2) (b)	·	·	·	a_2	b_2	c_2	d_2	1	
(3)	a_1	b_1	·	d_1	$-c_1$	1	·	c_1	$=0$
(4)	a_2	b_2	·	d_2	$-d_2$	1	·	c_2	
(1, and 2)	·	·	·	(a_1-a_2)	(b_1-b_2)	(c_1-c_2)	(d_1-d_2)	·	(5)
(3, and 4)	$(c_1a_2-c_2a_1)$	$(c_1b_2-c_2b_1)$	·	$(c_1d_2-c_2d_1)$	·	(c_1-c_2)	·	·	(6)

Subtracting (5) from (6) and dividing by Y, we form

$$(c_1a_2-c_2a_1)X^2+(c_1b_2-c_2b_1)XY+(c_1d_2-c_2d_1+a_2-a_1)X+(b_2-b_1)Y+(d_2-d_1)=0$$

or

$$(c_1b_2-c_2b_1)XY+(c_2a_1-c_1a_2)Y^2+\left\{\begin{matrix}c_1d_2-c_2d_1\\+a_2-a_1\end{matrix}\right\}X+(b_2-b_1)Y+\left\{\begin{matrix}c_1a_2-c_2a_1\\+d_2-\ d_1\end{matrix}\right\}=0 \quad (c)$$

The other equations (formed as above directed) are

$$\left\{\begin{matrix}b_1c_2-b_2c_1\\+a_1d_2-a_2d_1\end{matrix}\right\}XY+\left\{\begin{matrix}b_1d_2-b_2d_1\\+a_2c_1-a_1c2\end{matrix}\right\}Y^2+(a_2-a_3)X+(b_1-b_2)Y+\left\{\begin{matrix}a_1c_2\\-a_2c_1\end{matrix}\right\}=0 \quad (d)$$

$$\left\{\begin{matrix}a_1c_2-a_2c_1\\+b_1d_2-b_2d_1\end{matrix}\right\}XY+\left\{\begin{matrix}a_1d_2-a_2d_1\\+b_2c_1-b_1c_2\end{matrix}\right\}Y^2+(b_1-b_2)X+\left\{\begin{matrix}a_1b_2-a_2b_1\\+a_1-a_2\end{matrix}\right\}Y+\left\{\begin{matrix}b_1c_2\\-b_2c_1\end{matrix}\right\}=0 \quad (e)$$

The coefficients in (a) (b) (c) (d) (e) constitute the required determinant of the 5th order, and the eliminant is of the 8th degree.

The eliminant of three general equations of the 2nd degree in x, y, z, may be formed in like manner: only in this case the coefficients are all of the 3rd degree, and are of the determinant form

$$\begin{vmatrix} a_1, & b_1, & c_1 \\ a_2, & b_2, & c_2 \\ a_3, & b_3, & c_3 \end{vmatrix} \quad \begin{vmatrix} a_1, & b_1, & d_1 \\ a_2, & b_2, & d_2 \\ a_3, & b_3, & d_3 \end{vmatrix} \quad \&c.$$

Two equations may be formed in the form (6) of the glissette problem, and two others by eliminating x and dividing by y, and conversely. Any three of these together with the three original equations constitute a determinant of the 6th order; and the eliminant is of the 12th degree. I have worked out the general solution, but it is not necessary to give it here as the method has been fully explained.

As an example of the application of this method, I take the case of the locus of the centre of a generating ellipse moving in contact with guides inclined at any angle, α. The equations (original and reduced) are

$$\left.\begin{aligned} a^2\cos^2(\phi-\alpha)+b^2\sin^2(\phi-\alpha)&=x^2 \qquad (1)\\ a^2\cos^2(\phi+\alpha)+b^2\sin^2(\phi+\alpha)&=y^2 \qquad (2)\end{aligned}\right\}$$

$$\left.\begin{aligned} (a^2-b^2)(\cos 2\alpha.\cos^2\phi+\sin 2\alpha.\sin\phi\cos\phi)+(a^2\sin^2\alpha+b^2\cos^2\alpha)-x^2&=0 \qquad (1)\\ (a^2-b^2)(\cos 2\alpha.\cos^2\phi-\sin 2\alpha.\sin\phi\cos\phi)+(a^2\sin^2\alpha+b^2\cos^2\alpha)-y^2&=0 \qquad (2)\end{aligned}\right\}$$

Putting A and B for the coefficients of $\cos^2\phi$ and $\sin\phi\cos\phi$ respectively, X for $\cos\phi$, and Y for $\sin\phi$, and multiplying by X^2 and XY successively, we have the following scheme, which is of the 7th order, but is reducible to the 3rd:—

	X^4	X^3Y	X^2Y^2	XY^3	X^2	XY	1		
	.	.	.	.	A	B	$c-x^2$		(1)
	.	.	.	.	A	$-$B	$c-y^2$		(2)
	A	B	.	.	$c-x^2$	.	.		(3)
	A	$-$B	.	.	$c-y^2$	.	.	$=0:$	(4)
	1	.	1	.	-1	.	.		(5)
	.	A	B	.	.	$c-x^2$	.		(6)
	.	A	$-$B	.	.	$c-y^2$	.		(7)
	.	1	.	1	.	-1	.		(8)
From (3, 4, 5)			-2A	.	$2(A+C)-x^2-y^2$	.	.		
From (6, 7, 8)			B	.	.	y^2-x^2	.		
Whence,			.	.	$2B(A+C)-B(x^2-y^2)$	$A(y^2-x^2)$	.		(9)

This gives a determinant of the 3rd order with (1), (2), and (9), and the reduced eliminant takes the form

$$(B^2 + A^2)(x^4 + y^4) + 2(B^2 - A^2)x^2y^2 - B^2(2A + 4C)(x^2 + y^2) + 4B^2C(A + C) = 0 .$$

The curve consists of four ovals. When the guides are rectangular we have

$$a = \frac{\varpi}{4} ; \quad A = 0 ; \quad B = (a^2 - b^2) ; \quad C = \tfrac{1}{2}(a^2 + b^2) ;$$

and the equation reduces to

$$(a^2 - b^2)^2(x^4 + y^4 + 2x^2y^2) - 2(a^2 - b^2)^2(a^2 + b^2)(x^2 + y^2) + (a^2 - b^2)^2(a^2 + b^2)^2 = 0 ;$$

or,

$$\{(x^2 + y^2) - (a^2 + b^2)\}^2 = 0 .$$

The ovals are then attenuated to two coincident circular arcs, which is the known form of the locus of the centre of an ellipse moving between rectangular guides.

Since this paper was in type, I have found that an equation equivalent to the equation (6) of the determinant given on p. 89 may be formed by a method identical in principle with Bezont's system of elimination for homogeneous equations. The original generalised equations may be stated thus—

$$\left.\begin{aligned} \{(C - A)X + BY + a_2\} . X + \{\beta_2 Y + \gamma_2 + A\} &= 0 \\ \{-\mu X + \lambda Y\} . X + \{\nu Y + \mu\} &= 0 \end{aligned}\right\} \quad \begin{aligned} &. \ . \ . \ (2) \\ &. \ . \ . \ (3) \cdot Y \end{aligned}$$

$$\therefore \{B\nu - \beta_2\lambda\}Y^2 + \{(C - A)\nu + \beta_2\mu\}XY + \{\gamma_2 + C\}\mu X + \{a_2\nu + B\mu\}Y + a_2\mu - \lambda(\gamma_2 + A) = 0 \quad (d)$$

The determinant may be arranged as under, where E is put for A − C :—

	Y^2	XY	X	Y	(1)		
(c)	.	.	λ	μ	ν		(1)
(a)	E	B	a_2	β_2	$\gamma_2 + C$		(2)
$(c)\cdot Y$	μ	λ	.	ν	.	$=0$	(3)
$(c)x$	$(-\lambda)$	μ	ν	.	λ		(4)
(d)	$B\nu - \beta_2\lambda$,	$\beta_2\mu - E\nu$,	$(\gamma_2 + C)\mu$,	$a\nu + B\mu$,	$a_2\mu - \lambda(\gamma_2 + A)$		(5)

This determinant may be reduced to the 4th order by combining (2) successively with (3) (4) and (5), so as to cause the 1st column to vanish; and as the coefficient of Y^2 in (2) only contains the constant E (or A − C), this can be done without raising the degree of the eliminant. The eliminant contains eighteen compound terms, and when developed is, of course, identical with that previously found, as I have verified by a partial expansion.

Ptomaïnes extracted from Urine in certain Infectious Diseases. By Dr A. B. Griffiths, F.R.S.E., F.C.S., &c.

(Read January 18, 1892.)

The author has extracted ptomaïnes from the urine of patients suffering from certain infectious diseases by the following method :—

(*a*) A considerable quantity of urine was made alkaline by the addition of a solution of sodium carbonate, and then agitated with half its volume of ether. (*b*) The ethereal solution (after standing) was filtered, and agitated with a solution of tartaric acid. The tartaric acid combines with any ptomaïnes present, forming soluble tartrates; and the solution of tartrates forms the lower layer of the liquid mass. (*c*) The tartaric acid solution (after being separated from the ether) was also made alkaline by the addition of sodium carbonate, and was once more agitated with half its volume of ether. (*d*) The ethereal solution (after standing) was separated, and the ether allowed to evaporate spontaneously. (*e*) The residue (after drying over sulphuric acid) was treated with water, an excess of pure calcium hydroxide added, and the mixture evaporated on a water-bath. The residue so obtained was treated with chloroform and filtered. The filtrate (after evaporation) yielded the ptomaïne in an isolated and a crystalline condition.

By this method the three following ptomaïnes were isolated :—

(1) From Scarlatina.

The ptomaïne is a white crystalline body, which is soluble in water. It has a slight alkaline reaction; and it forms a white hydrochloride, a yellow aurochloride, a yellowish white precipitate with phosphomolybdic acid, a white precipitate with phosphotungstic acid, a yellow precipitate with picric acid; and it is also precipitated by Nessler's solution.

Analyses of this ptomaïne gave the following results :—

	Found.			Calculated for
	I.	II.	III.	$C_5H_{12}NO_4$.
Carbon, . . .	39·91	40·04	—	40·00
Hydrogen, . .	8·20	8·16	—	8·00
Nitrogen, . .	—	—	9·30	9·33
Oxygen, . . .	—	—	—	42·66

The above figures correspond with the formula $C_5H_{12}NO_4$.

A ptomaïne with exactly the same composition and properties was extracted from pure cultivations of *Micrococcus scarlatinæ* in peptonized gelatine* by the following method :—

The contents of twenty tubes were boiled with water, filtered, and the filtrate precipitated with subacetate of lead. This precipitate was filtered off, a current of sulphuretted hydrogen passed through the filtrate, and the plumbic sulphide separated by filtration. The filtrate was concentrated by evaporation, and then extracted with amylic alcohol. The amylic solution was repeatedly treated with water, then concentrated, acidulated with sulphuric acid and repeatedly shaken with ether, which removes the oxy-aromatic acids. Freed from ether it was evaporated to a quarter of its volume, and thus volatile fatty acids were driven off. The sulphuric acid was precipitated by baryta, and the precipitate removed by filtration. The excess of baryta was precipitated by a current of carbon dioxide, and this was also removed by filtration. The fluid was heated for some time on a water-bath, cooled, and precipitated with mercuric chloride. The precipitate was washed and decomposed by sulphuretted hydrogen ; the mercuric sulphide was filtered off, and the filtrate concentrated. The hydrochloride of the ptomaïne was subsequently deposited in the crystalline condition. It was dissolved in water, and then treated with pure calcium hydroxide, which liberated the base. The ptomaïne was separated by chloroform, in which it is soluble ; and it was finally purified by washing with alcohol and water.

* These subcultures were ten weeks old.

(2) From Diphtheria.

The ptomaïne is also a white crystalline substance. It forms a white hydrochloride, a pale yellow aurochloride, a yellow precipitate with tannic acid, a white precipitate with phosphomolybdic acid, a yellow precipitate with picric acid, and a brown precipitate with Nessler's solution.

Analyses of this ptomaïne gave the following results :—

	Found.			Calculated for
	I.	II.	III.	$C_{14}H_{17}N_2O_6$.
Carbon, . . .	54·21	—	54·30	54·36
Hydrogen, . .	5·56	—	5·49	5·50
Nitrogen, . .	—	9·00	—	9·06
Oxygen, . . .	—	—	—	31·06

The above figures correspond with the formula $C_{14}H_{17}N_2O_6$.

The same ptomaïne was extracted from fifteen tubes containing pure cultivations of *Bacillus diphtheriæ* * on nutrient agar-agar, which had been kept at 36°–37° C. for twenty-one days. The method used for extracting and purifying this ptomaïne was the same as the one which has been already described.

(3) From Parotitis (Mumps).

This ptomaïne was extracted from urine in a case where the kidneys were involved, and the parotid and sub-maxillary glands were both affected. It crystallizes in white prismatic needles, which are soluble in water, ether, and chloroform. It has a neutral reaction and a slightly bitter taste. This base forms a yellow crystalline platinochloride, a pale yellow aurochloride, and a white crystalline hydrochloride. It combines with phosphomolybdic acid, forming a golden yellow precipitate. This ptomaïne produces a white precipitate with phosphotungstic acid, a slight yellow precipitate with mercuric-potassic iodide, a brown preci-

* Bacillus No. 2 of Klebs and Löffler, which is the same as Klein's *Bacillus diphtheriæ.*

pitate with iodine solution, and a flocculent precipitate with picric acid.

Analyses of this ptomaïne gave the following results :—

	Found.			Calculated for
	I.	II.	III.	$C_6H_{13}N_3O_2$.
Carbon, . . .	45·34	—	45·29	45·28
Hydrogen, . .	8·22	—	8·20	8·17
Nitrogen, . .	26·39	26·42	—	26·41
Oxygen, . .	—	—	—	20·12

The above figures correspond with the formula $C_6H_{13}N_3O_2$.

(*a*) When this ptomaïne was boiled for a few minutes with mercuric oxide, it yielded creatine (methylglycocyamine) according to the following equation :—

$$C_6H_{13}N_3O_2 + 3O_2 = C_4H_9N_3O_2 + 2CO_2 + 2H_2O.$$

On cooling, the creatine crystallized out in colourless rhombic prisms. These crystals were soluble in water, very slightly soluble in alcohol, and insoluble in ether. When heated alone they gave rise to ammoniacal products. An aqueous solution of the crystals was neutral to test papers, and on the addition of zinc chloride a crystalline precipitate was formed. These reactions are characteristic of creatine.

(*b*) When this ptomaïne was boiled for a long time with mercuric oxide, it yielded methylguanidine and oxalic acid, *i.e.*, it was first converted into creatine (as already stated), and finally the creatine yielded methylguanidine (methyluramine) and oxalic acid :—

$$C_4H_9N_3O_2 + O_2 = CH_4(CH_3)N_3 + C_2H_2O_4.$$

By the addition of pure milk of lime to an aqueous solution of the crystalline mass so formed, calcium oxalate was precipitated; and after standing and filtration, the filtrate contained a substance which was very deliquescent. Solutions of this substance were precipitated by salts of copper, lead, and iron (ferric); and when heated with baryta there was a distinct ammoniacal odour, no doubt due to the formation of ammonia and methylamine. From these reactions, the author concludes that this decomposition product was methylguanidine.

This ptomaïne is therefore related to creatine as well as guanidine, and is most likely propylglycocyamine :—

$$HN = C \begin{matrix} \diagup NH_2 \\ \diagdown N(C_3H_7) - CH_2 - CO_2H. \end{matrix}$$

This ptomaïne is poisonous. When administered to a cat it produced nervous excitement, cessation of the salivary flow, convulsions, and death.

The three ptomaïnes described in this paper do not occur in normal urines ; they are, therefore, produced within the body during the courses of the diseases.

On a Crystalline Globulin occurring in Human Urine. By D. Noël Paton.

(From the Laboratory of the Royal College of Physicians of Edinburgh.)

(Read March 7, 1892.)

In October 1890, Dr Bramwell sent to the Laboratory a small quantity of urine, with a request that I should give him a report upon it. To him I am indebted for the opportunity of making the series of observations here recorded.

On examination, I found that the amount of proteid was enormous, and that it was made up almost entirely of a globulin :—

Total proteids, ...	2·00
Albumin, ...	0·08
Globulin, ...	1·92

This condition of the urine persisted, with only slight changes, until the death of the patient.

The results of the analyses made at different dates are given below.

November 24, 25, *and* 26, 1890.

	24.11 11 P.M.	25.11 12 NOON.	25.11 6 P.M.	25.11 11.30 P.M.	26.11 7 A.M.
Total proteids,	4·80	2·000	1·500	2·500	3·250
Albumin, . .	...	0·087	0·087	0·160	0·130
Globulin, . .	...	1·913	1·413	2·340	3·120
$\frac{\text{Albumin,}}{\text{Globulin,}}$ . .	...	0·045	0·061	0·068	0·044
	...	...	...	...	...

The slight rise in the quotient towards the end of the day corresponds to what has been found in regard to the influence of food on the proteids in ordinary Bright's disease (Lecorche and Telamon and Noël Paton, *Brit. Med. Jour.*, vol. ii. 1890).

June 1891.

	13.6.91	17.6.91	22.6.91
Total proteids,	0·448	0·8	4·40
Albumin,	0·022	...	0·02
Globulin,	0·426	...	4·38
Albumin,	0·05	...	0·004
Globulin,	...	...	...

The patient was recovering from a prolonged attack of diarrhœa, and in the beginning of June was extremely ill, taking little nourishment, and presumably absorbing less. Under treatment, his condition improved as the month advanced; and with the improvement, the globulin in the urine increased.

September 30 (2.30 P.M.), 1891.

Total proteids,	5·16
Albumin,	0·24
Globulin,	4·92
Albumin,	0·043
Globulin,	...

The specific gravity of this urine was 1035, and, after the separation of the proteids, only 1007·5.

The patient was in a good state of health, and taking a fairly generous diet.

December 2, 3, 4, *and* 5, 1891.

	2.12	3.12	4.12	5.12
Total proteids,	3·66	3·86	4·02	3·82
Albumin,	0·03	0·07	0·064	0·088
Globulin,	3·63	3·79	3·956	3·732
Albumin,	0·008	0·018	0·01	0·02
Globulin,	...	...	...	...

A full account of the clinical history and progress of the case, and of the pathological conditions observed, *post-mortem*, has been published by Dr Bramwell in his *Atlas of Clinical Medicine*, vol. i. p. 170. It is sufficient to state here that the patient had no symptom of renal disease, and that after death the kidneys were normal except for a small degree of cirrhosis. The liver was slightly cirrhotic and intensely fatty.

My observations are confined to the determination of the nature of the globulin and to the investigation of its source in the body.

(*a*) Nature of the Globulin.

Method of Preparation.—To determine the nature of the globulin, it was precipitated from the urine with sulphate of ammonia. The precipitate was thrown on a filter-paper, and washed with half-saturated sulphate of ammonia solution. It was then placed in a sausage-paper of vegetable parchment, a few pieces of thymol being added, and dialysed for four or five days in running tap water. Finally, the dialysis was carried on for forty-eight hours into distilled water, which was frequently changed. The globulin precipitated inside the paper was removed and repeatedly washed with distilled water by decantation, until no reaction of proteids or sulphate could be obtained from the wash-water.

The precipitate thus obtained was white and crystalline.

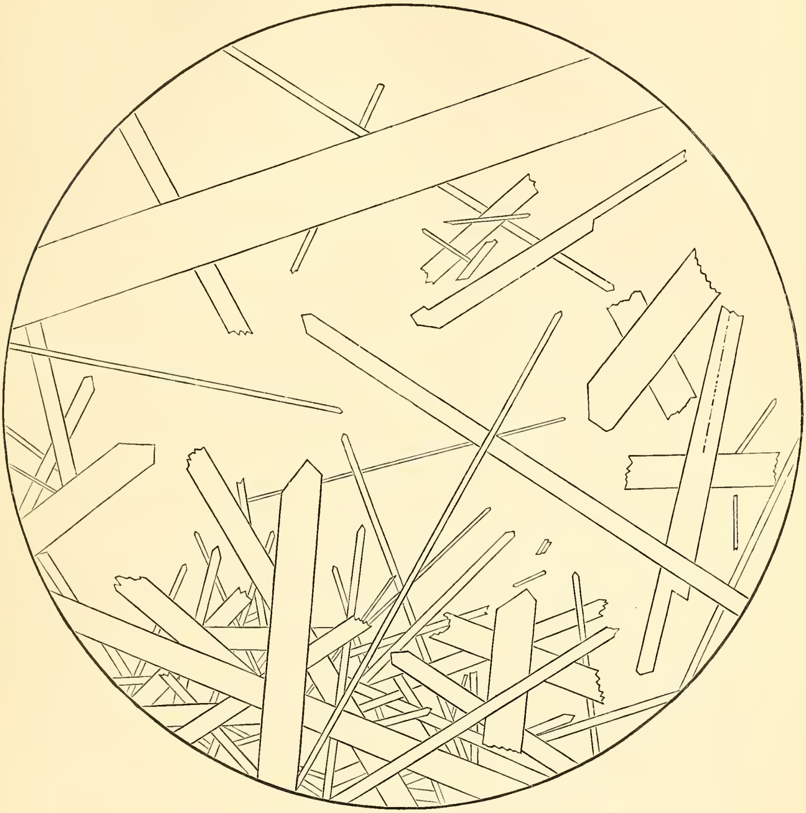
Spontaneous Crystallisation of Globulin.—But a most unexpected opportunity occurred of studying this globulin. Dr Bramwell found that if the urine was kept, sometimes after a day or two, sometimes after weeks or months, a copious white precipitate formed, which when mixed with the supernatant urine, gave it the appearance of watered silk.

On microscopic examination this deposit was found to be composed of elongated rhombic crystals, lying either singly or in rosettes.

These crystals are much larger than tyrosin crystals, and, instead of being acicular, terminate in a characteristic angulated extremity.

That these crystals are the globulin substance is shown, not only by their being identical in all their characters with the crystalline globulins prepared by dialyses, but also by the following observations.

The urine of September 30 was examined on October 14. A small deposit of crystals had already formed. These were filtered off and weighed, and the proteids in the filtrate estimated. Some of the filtrate was set aside till the 27th, when a further crop of crystals had separated. These were filtered off and weighed, and the proteids of the filtrate again determined. Again, some of the filtrate was set aside, when a further deposit of crystals occurred;



and on the 30th the crystals and proteids were again estimated. The results may be presented in a tabular form.

	Oct. 14.	Oct. 27.	Oct. 30.
Total proteids, ...	5·16	4·50	1·00
Crystals, ...	0·4	1·04	3·70
Nitrogen in filtrate by Kjeldahl,	0·78	0·80	...

This table shows most clearly the development of the crystals from the proteid in solution in the urine.

Chemical Characters of the Crystals.—A large quantity of the crystals were prepared by decanting the urine, and dialysing the residue for several days in sausage-papers with running water, to remove urea, salts, &c. The crystals were then taken from the dialyser, and repeatedly washed with distilled water by decantation to remove all albumin and the remains of the salts. They were finally washed with absolute alcohol and ether, and dried over sulphuric acid.

Ash.—The crystals, thus prepared, are practically ash-free; the combustion, at as low a temperature as possible, of 0·965 grm. of crystals dried at 110° C. yielded no weighable amount of ash.

Water of Crystallisation.—To determine the water of crystallisation, the crystals were washed with alcohol and ether, and were then dried over sulphuric acid at the ordinary atmospheric pressure, until the weight was constant. They were then dried at 110° C. until the weight again became constant. The percentage loss was, in two specimens, 6·01 and 6·6 per cent.

Grübler found that air-dried crystals of vegetable globulin lost 10 per cent. at 110° C.

General Reactions.—The crystals are insoluble in distilled water; soluble in dilute solutions of chloride of sodium and sulphate of ammonia; soluble in hydrochloric, in sulphuric, and in acetic acids; soluble in potassic hydrate and ammonia. As the ammonia evaporates off, the crystals are sometimes reprecipitated. They are insoluble in hot water and in absolute alcohol. They give the xanthoproteic reaction. When heated with strong sulphuric acid, they give a reddish colour. They give Liebermann's reaction with hydrochloric acid. They burn with the odour of burned feathers, and leave no ash.

Solubility.—The solubility of the crystals was tested in sulphate of magnesia, chloride of sodium and sulphate of ammonia.

With chloride of sodium a saturated solution was required to prevent solution. In all dilutions the crystals were dissolved.

In sulphate of magnesia solutions, the crystals were soluble in a 93 per cent. solution, but insoluble in 100 per cent. solution.

In sulphate of ammonia the crystals were partially dissolved in a 16 per cent. solution, and were freely soluble in greater dilutions.

Temperature of Coagulation.—In solution in the urine, the globulin commenced to be precipitated at between 59° and 60° C., and was completely separated out at 62° C.

When the pure crystals were dissolved in dilute solutions of neutral salts, the coagulation point varied somewhat with the nature of the salt employed and with the strength of the solution.

Sulphate of ammonia, ...	56° to 59° C.
Sulphate of magnesium, ...	58° ,, 59° C.
Chloride of sodium, ...	58° ,, 59° C.

In all, however, whatever dilution was used, the coagulation point was under 60° C.

The precipitate thrown down on boiling the urine is very characteristic. At first the usual milkiness appears, but as the temperature rises the precipitate suddenly separates out in a stringy fibrin-like mass, which adheres to the sides of the vessel or to the rod with which the fluid is stirred. When collected, this fibrin-like mass is found to possess a considerable degree of extensibility and elasticity.

The crystals are not precipitated from solution in dilute sulphate of ammonia by the addition of acetic acid.

Composition.—The crystals were dried at 110° C. before they were analysed.

For the determination of the carbon, hydrogen, and nitrogen, I have to thank Mr Murray, B.Sc., who was good enough to perform the combustions for me in the Chemical Laboratory of the University. The sulphur was determined by oxidising with caustic potash and nitrate of potash, and precipitating the sulphates with baric chloride.

	I.	II.	III.	IV.	V.	VI.	VII.	Average.
C	51·82	51·97	...	..	...	...	...	51·89
H	6·83	6·93	...	...	...	...	...	6·88
N	...	...	16·1	16·02	...	...	...	16·06
S	...	...	...	...	1·197	1·314	1·23	1·24
O	...	...	...	...	...	...	...	23·93
1	...	...	...	...	...	...	...	100

I believe that this is the first occasion on which a pure crystalline globulin from the animal body has been procured and examined.

A comparison of its composition with the globulins derived from plants, which have been so carefully studied by Chittenden and Hartwell, by Grübler, Ritthausen, and Barbieri, shows a smaller amount of nitrogen and a larger amount of sulphur in the globulin than occurs in vegetable globulins. (See *Journal of Physiology*, 1890, p. 440.)

A comparison of the composition of this pure crystalline ash-free globulin, with Hofmeister's analysis of crystalline ash-free albumin (*Zeitsch. f. phys. Chem.*, Bd. xvi. p. 189), is shown in the subjoined table :—

	Albumin.	Globulin.
C, ...	53·28	51·89
H, ...	7·26	6·88
N, ...	15·00	16·06
S, ...	1·09	1·24
O, ...	(23·37)	(23·93)

The globulin is thus poorer in carbon, but richer in nitrogen and sulphur.

When compared with Harnack's amorphous ash-free albumin (*Ber. d. deutsche chem. Gesell.*, Bd. xxiii. pp. 40–43), the difference in the amount of sulphur is rather in the opposite direction, Harnack finding a mean of 1·91 per cent. S as the result of his analysis.

Myosin, according to Kühne and Chittenden (*Zeitsch. f. Biol.*, Bd. xxv. p. 358), has the following composition :—

C, ...	52·79
H, ...	7·12
N, ...	16·86
S, ...	1·26
O, ...	22·97

Fibrin agrees fairly closely in composition with myosin (*Hammarsten Pflüger's Arch.*, Bd. xxii. p. 484) :—

C, ...	52·68
H, ...	6·83
N, ...	16·91
S, ...	1·1
O, ...	22·48

They appear to contain more carbon and nitrogen than this globulin from the urine.

It is unnecessary here to enter upon a discussion on the recent literature on the crystallisation of various proteids. An admirable résumé will be found in Bunges' *Physiological Chemistry* (translated by Wooldridge, p. 54).

Relationship to known Globulins of Animal Body.—It is difficult to identify this proteid with any of those known to occur in the animal body.

It undoubtedly belongs to the group of globulins of which Myosinogen, Fibrinogen, and Halliburton's Hepato-globulin are members, agreeing with them in its temperature of coagulation. It differs from Myosinogen in its solubility in more concentrated solutions of sulphate of magnesia and chloride of sodium, and in its not precipitating on the addition of acetic acid, and in the absence of any tendency to form Myosin.

From Fibrinogen it differs also in its greater solubility in neutral salts, and from the absence of any tendency to coagulate as fibrin.

Its distinction from the less known Hepato-globulin is less clearly defined, and at present I am engaged upon an investigation on this substance. The intense fatty degeneration of the liver, observed *post-mortem*, may point to some connection between the urinary globulin and that organ.

None of these proteids have been described as occurring in a crystalline condition.

That it is not connected with the Globin of the red-blood corpuscles seems to be indicated by its coagulation temperature. Hæmoglobin coagulates at 73° C.

That it is not Myoglobulin is shown by its lower coagulation point, and by the fact that it does not require saturation with sulphate of magnesium to precipitate it from its solutions.

Circumstances determining Crystallisation.—The precipitation of the crystals of globulin was usually delayed till some days after that on which the urine was passed—on many occasions it did not appear for several weeks.

Whether this deposition corresponds to a loss of acidity has not been definitely investigated; but certainly the urine was markedly acid, even after the deposition of the crystals had occurred, as is shown by the following examination of the urine of November 25th and 26th, after deposition of crystals :—

Acidity in per cent. Oxalic Acid.

November 25, 11 A.M.,	0·1512
,, ,, 6 P.M.,	0·1764
,, ,, 11.30 P.M.,	0·3969
,, 26, 7 A.M.	0·2961

Again, on the rare occasions upon which ammoniacal decomposition occurred, the crystals did not form.

The amount of globulin held in solution is enormous when compared with the amount of inorganic salts of the urine. On December 2nd these amounted to 1·475 per cent., and it is probably on this account that the globulin separates out, just as it does in the dialyser. A fall in temperature does not appear to be the factor bringing about crystallisation, because, while undoubtedly very copious precipitates of crystals were got in urine kept in ice, Dr Bramwell notes that a large precipitate appeared in the urine of June 10th on the day after it was passed.

Resistance to Ammoniacal Decomposition.—One point worthy of notice is the great resistance to the ordinary processes of decomposition offered by a urine so rich in proteids. Many specimens were kept standing for weeks on the Laboratory table without undergoing ammoniacal fermentation. The globulin undoubtedly exercises an antiseptic effect.

(*b*) Source of the Globulin.

Relationship to Blood Plasma.—Having studied the nature of the globulin, it next became necessary to investigate its mode of production.

Have we to do with a simple transudation of the proteids of the blood plasma?

Lecorche et Telamon (*Traité de l'Albuminurie*, Paris, 1888) and Estelle maintain that the proteids of the urine in albuminuria occur in the same proportion in which they exist in the blood plasma.

In another paper (*Brit. Med. Journal*, vol. ii. p. 197, 1890) I have shown reason to doubt their results, and have argued against the existence of any proportion between the relative amounts of the proteids in the urine and in the blood plasma. The present case appeared to afford a good opportunity of testing this point. An attempt was made to procure a sufficient quantity of blood with the artificial leech to allow an analysis of the serum, but was unsuccessful.

The patient, who all along showed a deep interest in the scientific aspect of his case, accordingly blistered himself on November 15th, 1890, and on the 16th the blister was opened, and about 2 c.c. of serum were obtained.

An analysis of this showed :—

Total proteids,	5·5
Albumin,	4·0
Globulin,	1·5
Albumin,	2·5
Globulin,	...

Buchardt has already shown that in such effusions the proportion of proteids corresponds to that in the blood serum—the proportion of serum albumin being somewhat greater.

On the same day the urine gave these results :—

	Gravimetric.	Esbach.
Total proteids,	1·4	1·45
Albumin,	0·0	trace
Globulin,	1·4	1·45

In this case the proteids of the urine showed no relationship to the proteids of the blood serum. With the small amount of serum at my disposal it was, of course, impossible to investigate the nature of the globulin present in it.

Method.—To estimate the proteids gravimetrically, the total proteids were first determined by boiling, and adding a drop of acetic acid and a drop of saturated solution of acetate of soda. This brought down a firm ropy precipitate, which could readily be collected on a weighed filter-paper washed with distilled boiling water, absolute alcohol, ether, and weighed.

The globulin was precipitated by the addition, to a known volume of the urine, of an equal volume of saturated solution of sulphate of ammonia; the mixture was allowed to stand twelve hours, then thrown on a dried filter-paper. The proteid of the filtrate, *i.e.*, the albumin, was estimated by precipitation by boiling, the precipitate being washed frequently with boiling water to remove all the ammonia salt, then with alcohol and ether, and weighed. The globulin was determined by difference.

To separate the globulin by Esbach's method, sulphate of magnesia was used, since sulphate of ammonia causes a precipitate with picric acid.

The results by these two methods corresponded closely, as is shown by the preceding tables.

Since the above was written, a paper by Csatàry (*Deutsche Arch. f. klin. Med.*, Bd. xlviii. p. 358, 1891) has fully confirmed my previous conclusions, and the result arrived at in the present case. By an extended series of observations Csatàry clearly shows that the proportion of albumin and globulin in the urine bears no proportion to their relative amounts in the blood serum.

Metabolism.—A quantitative estimation of the proteids of the urine showed that the patient was passing, on many occasions, between 60 and 70 grammes of these substances per diem.

Since this is an amount of proteid which, according to the results of the older observers, would be almost sufficient for the daily wants of a healthy man, or, according to the more recent results of Hirschfeld and Klemperer, is far more than sufficient, it became a matter of importance to study more carefully the changes in the metabolism which accompanied this great loss.

The patient was directed to select for himself a diet to which he was to adhere for four days. Each article of diet was roughly weighed, and a specimen diet was sent to the Laboratory for examination. From this a calculation of the amount of nitrogen taken was made. The total urine of each day during the observations was sent to the Laboratory, and was examined for the amount of proteids and the amount of nitrogen after the separation of the proteids. This was done by Kjeldahl's method. From this the nitrogen in the proteids and the total nitrogen of the urine were calculated. The following table gives the results of this observation. The amount of nitrogen in the food was calculated from diet tables, and is approximate.

These tables show that the patient consumed daily during this experiment about twice the ordinary amount of proteid food—30 grms. of nitrogen corresponding to 187·5 grms. of albumin. It should be mentioned that during the experiment he was taking rather more than his usual diet.

This large amount of proteid diet was tolerably well absorbed—between the nitrogen of the ingesta and that of the urine there was

DIET.

	Amount in grms.	Per cent. N as Basis of Calculation.	Total N in grms.
Bread, ...	168	1·3	2·2
Fish, ...	530	2·7	12·3
Eggs, ...	121	1·9	2·3
Cheese, ...	21	4·5	1·1
Potatoes, ...	150	0·33	0·5
Sugar, ...	40	...	...
Butter, ...	30	0·6	0·2
Milk, ...	21·30	0·6	12·2
	...	...	30·8

Total nitrogen of ingesta about 30 grms.

EXCRETED.

Date.	Amount of Urine in ccs.	N of Urine not Proteid.		Proteids of Urine in grms.	N in Proteids calculated at 16 per cent.	Total N of Urine.
		Per cent.	In 24 hours in grms.			
2.12.91	1680	·84	14·11	61·48	10·552	24·662
3. ,, ,,	2000	·87	17·40	77·20	13·120	30·524
4. ,, ,,	1750	·88	15·40	70·35	11·959	27·350
5. ,, ,,	1740	·87	15·10	66·47	11·300	26·403
Average,	1792	·865	15·5	68·87	11·732	27·234

a difference of only between 3 and 4 grms., *i.e.*, between 10 and 13 per cent., very little more than the results of Rubner show for a healthy man. Of the 170 grms. of proteids absorbed, only 97 grms. were changed to urea, *i.e.*, about the amount which, according to Voit, is required for a normal diet. Corresponding to the remaining nitrogen, we find the excretion of 69 grms. of the globulin substance.

Was this 69 grms. of the proteid of the food retained to make good the loss, or was it, having undergone some change in the liver, excreted as a globulin with the characters above described?

That it is derived, somewhat directly, from the proteids of the food seems to be indicated by the small amount passed, when a small amount of proteids were being absorbed (see p. 102). But since only percentage amounts, and not the total amount of twenty-four hours, were obtainable, no definite conclusions can be based on these figures.

A careful series of experiments on the influence of different diets and of muscular work would probably have thrown valuable light on the nature of the condition; and such a series of experiments had been planned and arranged for when the patient's death supervened.

Analyses.

Percentage Composition of Globulin—

For the carbon, hydrogen, and nitrogen I am indebted to Mr Murray, B.Sc., of the Chemical Laboratory of the University.

Sulphur—

Exp. I. 0·332 grms. crystals dried at 110° C. fused with KHO and $NaNO_3$—

$BaSO_4$ = 0·029
S = 0·003987
= 1·197 per cent.

Exp. II. 0·373 grms. crystals dried at 110° C. fused with KHO and $NaNO_3$—

$BaSO_4$ = 0·043
S = 0·005912
= 1·314 per cent.

Exp. III. 1·752 grms. crystals dried at 110° C. fused with KHO and $NaNO_3$—

$BaSO_4$ = 0·158
S = 0·021725
= 1·23 per cent.

Nitrogen of Urine not Proteid—

Dec. 1. Proteids precipitated by boiling and 5 cc. of filtrate boiled with H_2SO_4 and Hg, and N. estimated according to Argutinsky's modification of Kjeldahl's method (*Pflüger's Arch.*, 1889)—

N = 0·42 grm
= ·84 per cent.

Dec. 2. 5 ccm. of filtrate—

N = 0·0435 grm.
= 0·87 per cent.

Dec. 3. 5 ccm. of filtrate—

N = 0·0441 grm.
= 0·882

Dec. 4. 5 ccm. of filtrate—

N = 0·0434 grm.
= ·868 per cent.

On the Blood of the Invertebrata. By A. B. Griffiths, Ph.D., F.R.S.E., F.C.S., &c.

(Read July 4, 1892.)

Introduction.

The blood of the *Invertebrata*, like that of the *Vertebrata*, is not homogeneous. It consists of a transparent or semi-transparent liquid, and a number of small, solid corpuscles which float in it.

In the higher animals the corpuscles are of two kinds—red and colourless; but in the *Invertebrata* there are, as a rule, only colourless corpuscles. The red blood of Annelids is different from the red blood of Vertebrates, inasmuch as the plasma is coloured, and the corpuscles are colourless in the former,* while in the latter the plasma is colourless, and there are present coloured and colourless corpuscles.

The corpuscles in the blood of the *Invertebrata* are of different sizes, and the size varies greatly in the same individual. Their form, however, is generally spherical, and their surface has a raspberry appearance.

In the higher *Invertebrata* the blood clots after a variable period of time. Haycraft and Carlier † have recently examined the coagulation of the blood in certain Invertebrates. According to their investigations "the clot is formed, at any rate for the greater part, by the welding together of blood-corpuscles. These throw out processes which interlace to form a solid mass."

Although the blood of the *Invertebrata* contains corpuscles, its composition greatly varies. For instance, the blood of the lower, and some of the higher, Invertebrates is a watery fluid containing only a small amount of proteids. This kind of blood has been termed hydrolymph. But in the majority of the higher Invertebrates the blood is less watery, and much richer in proteids. This variety of blood is called hæmolymph. The distinction between

* There are a few exceptions to this general statement.

† *Proc. Roy. Soc. Edin.*, vol. xv. p. 423.

these two varieties lies not only in their composition, but also in the physiological functions which they perform.

The hydrolymph carries nutriment to the tissues and organs, and removes waste products. As a rule it has no respiratory function;* for in those animals which possess blood of the nature of a hydrolymph the gaseous exchanges occur directly between the tissues and the surrounding medium.

The hæmolymph has not only a nutritive, but also a respiratory function, and it frequently contains respiratory pigments.

In the blood of the *Vertebrata* there is only one respiratory pigment—hæmoglobin. In the hæmolymph of the *Invertebrata* there is not only hæmoglobin, but several pigments of a respiratory nature; for instance—hæmocyanin,† hæmerythrin, chlorocruorin,‡ pinnaglobin,§ and probably other pigments or proteids, which may have a similar function.

The Blood of Echinoderms.

The blood of these animals is a true hydrolymph. It is a thin watery liquid, holding in solution mineral matter and a small quantity of proteids, and in it float numerous amœboid corpuscles. These corpuscles have been described by Geddes,|| and many of them contain variously coloured globules. Among these pigments is MacMunn's echinochrome,¶ which has a respiratory function; but the majority of these pigments are lipochromes, and consequently have no respiratory function.

Foëttinger** has found hæmoglobin in the blood of *Ophiactis virens* (one of the *Ophiuridea*), and the same respiratory pigment has been found by Howells†† in the blood of *Thyonella gemmata* (belonging to the *Holothuridea*). In the integument of many

* In some Echinoderms, MacMunn has proved the existence of a respiratory pigment in the corpuscles; but, as a rule, hydrolymph has no respiratory function.

† See Dr A. B. Griffiths in *Comptes Rendus de l'Académie des Sciences*, t. cxiv. p. 496.

‡ *Ibid.*, t. cxiv. (May 30, 1892), p. 1277.

§ *Ibid.*, t. cxiv. p. 840.

|| *Archives de Zoologie Expérimentale et Générale*, t. viii.

¶ *Quart. Jour. Micro. Sci.*, 1885.

** *Zool. Anzeiger*, 1883, p. 416.

†† *Studies from Biol. Lab. Johns Hopkins University*, vol. iii. p. 284.

Echinoderms, MacMunn* has found hæmatoporphyrin, which is well known to be a decomposition product of hæmoglobin; and there is no doubt that tissue-respiration plays an important part in the respiration of these animals. Although echinochrome and hæmoglobin have been found in the blood of some of these animals, the blood of others has no respiratory function.

We now consider the composition of the saline matter contained in the blood of the following Echinoderms:—*Spatangus*, *Echinus*, *Uraster*, and *Solaster*. The percentages of saline matter contained in the blood of these animals have been ascertained by the author, as follows:—

	I.	II.	III.	IV.	V.	Average.
Spatangus, . . .	1·820	1·834	1·862	1·841	1·836	1·832
Echinus, . . .	1·786	1·752	1·761	1·706	1·772	1·755
Uraster, . . .	1·924	1·936	1·922	1·941	1·932	1·931
Solaster, . . .	1·973	1·962	1·983	1·985	1·968	1·974

The author has also submitted to analysis the ashes of the blood of these animals. The ashes were obtained by incinerating the blood, partially covered in a platinum dish, at a very low temperature. By so doing the alkaline metals are not volatilised as they are when a high temperature is used.

The following results represent the averages of three analyses in each case:—

	Spatangus.	*Echinus.*	*Uraster.*	*Solaster.*
Iron oxide (Fe_2O_3), . .	trace.	trace.	trace.	trace.
Lime (CaO), . . .	3·62	3·78	3·02	3·16
Magnesia (MgO), . .	1·05	1·12	1·38	1·36
Potash (K_2O), . . .	4·81	4·76	4·52	4·63
Soda (Na_2O), . . .	43·78	43·82	44·22	44·03
Phosphoric acid (P_2O_5), .	4·62	4·53	4·24	4·32
Sulphuric acid (SO_3), .	2·31	2·36	2·22	2·23
Chlorine, ...	39·81	39·63	40·40	40·22
	100·00	100·00	100·00	100·00

* *Jour. Phys.*, vol. vii. p. 240.

The following table represents the complete analysis of the blood of these Echinoderms :—

	Spatangus.	*Echinus.*	*Uraster.*	*Solaster.*
Water, ...	95·769	95·907	95·576	95·667
Solids, ...	4·231	4·093	4·424	4·333
Fibrin, ...	0·046	0·043	0·042	0·049
Albumin, . . .	2·365	2·298	2·460	2·322
Salts, ...	1·820	1·752	1·922	1·962

The coagulation of the blood of Echinoderms has been investigated by Geddes,* Schäfer,† and Haycraft and Carlier.‡

Geddes states that the amœboid cells coalesce into irregular masses and shoot out processes which bind the cells together. But, according to Schäfer, the clot is not a mere plasmodum; there is also a fibrin-like substance which separates from the plasma; and the above analyses prove the presence of fibrin in the blood of Echinoderms.

According to Haycraft and Carlier, "the blood of the sea-urchin varies very much in the number of corpuscles present in the different specimens. In most cases, when allowed to coagulate, the clot is very small, and not easy to demonstrate in a few drops of blood."

Such is the present state of our knowledge concerning the blood of the *Echinodermata.*

The Blood of Annelids.

The blood of these animals is of the nature of hæmolymph. The author has investigated the blood of many of them.

The following is a list of the different Annelids investigated, arranged in their several classes along with the names of the respiratory pigments contained in the blood :—

* *Proc. Roy. Soc. Lond.*, 1880. † *Ibid.*, 1883, p. 370.

‡ *Proc. Roy. Soc. Edin.*, vol. xv. p. 423.

	Class.	Respiratory Pigment.	Pigment in the
Sipunculus, . .	Gephyrea.	Hæmerythrin.	Corpuscles.
Hirudo, . .	Hirudinea.	Hæmoglobin.	Plasma.
Hæmophsis, . .	,,	,,	,,
Lumbricus, . .	Oligochæta.	,,	,,
Sabella, . .	Polychæta.	Chlorocruorin.*	,,
Serpula, . .	,,	,,	,,
Arenicola, . .	,,	Hæmoglobin.	,,
Aphrodite, . .	,,	,,	,,
Glycera, . .	,,	,,	Corpuscles.
Terebella, . .	,,	,,	Plasma.
Nereis, . .	,,	,,	,,

The percentages of saline matter contained in the blood of these Annelids are given in the following table:—

	I.	II.	III.	IV.	Average.
Sipunculus, ...	3·22	3·20	3·25	3·27	3·27
Hirudo, ...	3·23	3·40	3·32	3·34	3·32
Hæmophsis, ...	3·64	3·59	3·62	3·61	3·61
Lumbricus, ...	3·89	3·91	3·83	3·82	3·86
Sabella, ...	3·51	3·56	3·61	3·55	3·55
Serpula, ...	3·26	3·19	3·24	3·24	3·23
Arenicola, ...	3·92	3·91	3·87	3·84	3·88
Aphrodite, ...	3·77	3·72	3·71	3·80	3·75
Glycera, ...	3·65	3·59	3·62	3·60	3·61
Terebella, ...	3·44	3·41	3·45	3·39	3·42
Nereis, ...	3·81	3·83	3·79	3·82	3·81

	Sipunculus.	*Hirudo.*	*Hæmophsis.*	*Lumbricus.*	*Sabella.*	*Serpula.*
Iron oxide (Fe_2O_3)	0·13	0·25	0·22	0·26	0·18	0·17
Lime (CaO), .	3·00	3·33	3·31	3·21	3·42	3·40
Magnesia (MgO),	1·65	1·52	1·60	1·54	1·22	1·26
Potash (K_2O), .	5·02	4·99	5·10	5·00	4·03	4·10
Soda (Na_2O), .	44·31	43·98	44·11	44·10	45·23	45·26
Phosphoric acid (P_2O_5), .	4·78	4·89	4·80	4·76	4·56	4·55
Sulphuric acid (SO_3), .	2·86	2·92	2·82	2·85	2·10	2·14
Chlorine, . .	38·25	38·12	38·04	38·28	39·26	39·12
	100·00	100·00	100·00	100·00	100·00	100·00

* See Dr Griffiths in *Comptes Rendus*, t. cxiv. (May 30, 1892).

The author has also submitted to analysis the ashes of the blood of these animals, and the preceding and following results represent the averages of two analyses in each case.

	Arenicola.	*Aphrodite.*	*Glycera.*	*Terebella.*	*Nereis.*
Iron oxide (Fe_2O_3),	0·20	0·24	0·22	0·22	0·23
Lime (CaO),	3·63	3·21	3·32	3·64	3·20
Magnesia (MgO),	1·25	1·52	1·55	1·20	1·50
Potash (K_2O),	4·05	5·06	5·01	4·08	5·04
Soda (Na_2O),	45·20	44·32	44·20	45·21	44·36
Phosphoric acid (P_2O_5),	4·50	4·80	4·79	4·56	4·82
Sulphuric acid (SO_3),	2·11	2·81	2·82	2·00	2·79
Chlorine,	39·06	38·04	38·09	39·09	38·06
	100·00	100·00	100·00	100·00	100·00

These animals possess blood which is much richer in solid constituents than Echinoderms; and the next table gives the composition of the blood of the eleven Annelids investigated :—

	Sipunculus.	*Hirudo.*	*Hæmophsis.*	*Lumbricus.*	*Sabella.*	*Serpula.*
Water,	91·65	90·75	90·40	90·12	90·82	91·22
Solids,	8·35	9·25	9·60	9·88	9·18	8·78
Fibrin,	0·13	0·18	0·16	0·19	0·15	0·14
Albumin,	5·02	5·73	5·82	5·80	5·48	5·40
Salts,	3·20	3·34	3·62	3·89	3·55	3·24

	Arenicola.	*Aphrodite.*	*Glycera.*	*Terebella.*	*Nereis.*
Water,	90·24	90·34	90·48	90·60	90·26
Solids,	9·76	9·66	9·52	9·40	9·74
Fibrin,	0·17	0·19	0·18	0·19	0·17
Albumin,	5·72	5·70	5·75	5·80	5·74
Salts,	3·87	3·77	3·59	3·41	3·83

Besides the fact that the blood of worms contains certain pigments, very little is known concerning it. There is no real coagulation, although it contains fibrin, but a heat-coagulum is formed at 65° C. When this coagulum is filtered off, *i.e.*, in the case of those worms whose blood contains hæmoglobin and chlorocruorin, no proteid remains in solution; but in the case of those whose blood contains hæmerythrin there is an additional proteid present, which coagulates at 70° C.

In *Sipunculus* the blood-corpuscles contain a coloured fluid between the external wall and the central nucleus. It may be stated that this is probably the first appearance of a true coloured corpuscle, but it differs essentially from the coloured corpuscle of the *Mammalia*, for in the latter the colouring matter is distributed throughout the corpuscle.

The hæmoglobin present in the blood of worms is identical in chemical composition with that present in the higher animals.*

In *Glycera*, and a few other Annelids, the hæmoglobin is present in special corpuscles, but in the majority of these animals it is dissolved in the plasma.

The gases of the blood of the eleven Annelids (previously mentioned) have been investigated by using the mercurial air-pump, *i.e.*, by the method which has been already described in the *Proceedings of the Royal Society of Edinburgh*, vol. xviii. p. 288. This method gave the following results:—

	Oxygen.	Carbonic Anhydride.	Nitrogen.
Sipunculus, ...	12·31	28·29	1·82
Hirudo, ...	12·93	29·62	1·91
Hæmophsis, ...	12·99	30·00	1·94
Lumbricus, ...	13·02	30·15	1·96
Sabella, ...	12·50	28·30	1·89
Serpula, ...	12·46	28·04	1·83
Arenicola, ...	12·89	30·12	1·90
Aphrodite, ...	11·99	28·20	1·87
Glycera, ...	12·87	29·24	1·89
Terebella, ...	12·90	29·55	1·76
Nereis, ...	12·82	29·02	1·84

The above figures represent volumes of the gases per 100 volumes of blood (the volumes being reduced to 0° C. and 760 mm.), and they also represent the averages of three determinations in each case.

The nitrogen is simply dissolved in the blood, but the oxygen

* Dr Griffiths in *Proc. Roy. Soc. Edin.*, vol. xviii. p. 294.

and carbonic anhydride are partly dissolved and partly in a state of loose chemical combination with certain constituents of the blood. The oxygen is united to the hæmoglobin, hæmerythrin, or chlorocruorin, as the case may be; and possibly the greater part of the carbonic anhydride is united to certain salts contained in the blood.

Invertebrate blood which contains hæmoglobin has a greater power of combining with and absorbing oxygen than blood which contains either hæmerythrin or chlorocruorin. This statement may be represented by the following figures, which are the averages of the results represented in the previous table :—

Blood containing	hæmerythrin	absorbs, &c.,	. 12·31	per cent.	of oxygen.
,,	chlorocruorin	,,	. 12·48	,,	,,
,,	hæmoglobin	,,	. 12·80	,,	,,

Although hæmerythrin and chlorocruorin are not so active as respiratory proteids as hæmoglobin, the former are as important as the latter. It is probable that by a process of physiological selection the respiratory proteids may have become more complex, and their molecular instability * therefore greater as the animal body became more elaborated, and a necessity arose for the setting apart of respiratory proteids for abstraction of oxygen from the air.

The Blood of Insects.

In a large number of insects the blood is colourless, although sometimes it is of a green, yellow, or red hue. This colour is not due to the amœboid corpuscles, but to the plasma in which they float.

The blood-plasma of *Musca domestica* (house-fly),† and that of the larva of *Chironomus* ‡ (both belonging to the *Diptera*), contains hæmoglobin.

Poulton § has made a number of observations on the blood of the *Lepidoptera*. The colour of the blood in these insects is principally green ; but it varies, to a certain extent, with the food. This pigment has no respiratory function ; and, in fact, no respiratory

* For instance, hæmoglobin is less stable than hæmocyanin. See Dr Griffiths in *Comptes Rendus*, t. cxiv. p. 496.

† MacMunn, *Proc. Birmingham Phil. Soc.*, vol. iii. p. 385.

‡ Lankester, *Jour. Anat. and Phys.*, vol. ii. p. 114.

§ *Proc. Roy. Soc. Lond.*, 1885, p. 270.

pigment appears to be present in the blood of the *Lepidoptera.* The existence of a respiratory pigment is doubtful, especially when one bears in mind the anatomical disposition of the respiratory apparatus in these animals, *i.e.*, the penetration of the air by the tracheæ through all the living tissues. Nevertheless, the blood of the *Lepidoptera* is rich in proteids, and it is capable of abstracting and retaining a considerable amount of oxygen. The author has ascertained the percentages of saline matter contained in the blood of the larvæ of certain Lepidopterous insects, &c. These percentages are as follows :—

		I.	II.	III.	Average.
Lepidoptera.	*Pontia brassicæ*,	3·62	3·71	3·68	3·67
	Noctua pronuba,	3·84	3·77	3·81	3·80
	Vanessa io,	3·91	3·98	4·00	3·96
	Smerinthus tiliæ,	3·92	3·99	3·87	3·92
Coleoptera.	*Lucanus cervus*,	3·51	3·49	3·55	3·51
	Dytiscus dimidiatus,	3·48	3·50	3·46	3·48

The blood was obtained from a large number of individuals in each case.

The ashes of the blood of these insects yielded the following results * :—

	Pontia brassicæ.	*Noctua pronuba.*	*Vanessa io.*	*Smerinthus tiliæ.*	*Lucanus cervus.*	*Dytiscus dimidiatus.*
Iron oxide (Fe_2O_3),	0·10	0·09	0·11	0·09	0·10	0·08
Lime (CaO),	2·02	2·96	2·06	2·43	2·25	2·21
Magnesia (MgO),	1·00	1·12	1·09	1·13	1·16	1·09
Potash (K_2O),	4·12	4·09	4·06	4·10	4·20	4·17
Soda (Na_2O),	43·48	44·03	43·62	43·29	44·00	43·62
Phosphoric acid (P_2O_5),	3·21	3·61	3·50	3·40	3·32	3·41
Sulphuric acid (SO_3),	2·91	2·80	2·87	2·92	2·83	2·80
Chlorine,	43·16	41·30	42·69	42·64	42·14	42·62
	100·00	100·00	100·00	100·00	100·00	100·00

* The averages of two analyses in each case.

The composition of the blood of these insects is represented in the next table :—

	Pontia brassicæ.	*Noctua pronuba.*	*Vanessa io.*	*Smerinthus tiliæ.*	*Lucanus cervus.*	*Dytiscus dimidiatus.*
Water, . . .	88·49	88·09	87·94	88·21	88·29	88·35
Solids, . . .	11·51	11·91	12·06	11·79	11·71	11·65
{ Proteids, &c., .	7·89	8·10	8·08	7·92	8·20	8·17
{ Salts, . .	3·62	3·81	3·98	3·87	3·51	3·48

Concerning the coagulation of the blood of insects, it may be stated that Poulton * has shown that the blood clots after a variable period of time ; and he has known samples of blood that have not clotted at all. Larval blood coagulates far more rapidly than pupal blood. The author entirely confirms these observations of Poulton.

The gases of the blood of the above-mentioned insects, investigated by the use of the mercurial air-pump, gave the following results :—

	Oxygen.	Carbonic Anhydride.	Nitrogen.
Pontia brassicæ, . .	16·34	34·21	1·92
Noctua pronuba, . .	17·24	33·19	1·72
Vanessa io, . . .	16·56	34·00	1·81
Smerinthus tiliæ, .	17·10	33·09	1·76
Lucanus cervus, . .	16·28	34·97	2·31
Dytiscus dimidiatus, .	16·22	34·86	2·35

The above figures (the averages of two determinations in each case) represent volumes of the gases per 100 volumes of blood (the volumes being reduced to 0° C. and 760 mm.). Although the blood of the *Lepidoptera* appears to be devoid of respiratory pigments, it has a respiratory function. In fact, the want of respiratory pigments in this case confirms the idea that the mere *colour* of hæmoglobin, for example, is of no use. It may be remarked, however, that myohæmatin † (which is connected with hæmoglobin and its derivatives) occurs in the muscles of insects ; and tissue-respira-

* *Proc. Roy. Soc. Lond.*, 1885, p. 294.

† MacMunn, *Phil. Trans.*, 1886, pt. i. p. 272.

tion by means of this pigment is well developed in this class of animals.

It appears that in the *Lepidoptera* the transport of oxygen from the surrounding medium to the living tissues is made, to a considerable extent, by means of colourless proteids of the blood. These substances form oxygenised combinations which are unstable, and which are carried by the blood across the tissues and are there dissociated, yielding the oxygen to the elements of those tissues which require it.

The Blood of Arachnids.

The author has only ascertained the composition of the ashes of the blood of a few of these Invertebrates. These ashes gave the following results (the averages of two analyses in each case) :—

	Epeira.	*Tegenaria.*	*Pholcus.*
Copper oxide (CuO),	0·20	0·23	0·26
Lime (CaO),	3·56	3·62	3·50
Magnesia (MgO),	1·94	2·03	1·98
Potash (K_2O),	5·00	4·91	4·89
Soda (Na_2O),	44·12	44·63	43·92
Phosphoric acid (P_2O_5),	4·83	4·92	4·85
Sulphuric acid (SO_3),	2·82	2·78	2·80
Chlorine,	37·53	36·88	37·80
	100·00	100·00	100·00

The blood of these animals contains hæmocyanin, which is the well-known respiratory pigment containing copper.

Lankester * has also shown that the blood of *Scorpio* becomes blue on exposure to air, and that it contains hæmocyanin.

Myohæmatin is present in the muscular tissues of the *Arachnida.*

Concerning the uses of these two pigments, it may be stated that the hæmocyanin receives oxygen from the air, and carries it to the tissues; and the myohæmatin receives the oxygen from the blood, and retains it until the tissue elements are in need of it.

* *Quart. Jour. of Micr. Sci.*, vol. xxiv. p. 151.

The Blood of Molluscs.

The blood of the majority of the *Lamellibranchiata* is more of the nature of a hydrolymph than that of a hæmolymph, although there are several exceptions. For instance, the blood of *Pinna*, *Mytilus*, and *Anodonta* is a true hæmolymph.

The percentages of saline matter contained in the blood of certain Lamellibranchs have been ascertained to be as follows :—

	I.	II.	III.	IV.	Average.
Mya, ...	0·921	0·963	1·021	0·987	0·973
Solen, ...	0·987	0·989	1·000	1·032	1·902
Pecten, . . .	1·003	0·998	0·989	1·006	0·999
Lima, . . .	0·986	0·989	0·992	0·995	0·990

The ashes of the blood of these animals yielded the following results on analysis (averages of two analyses in each case) :—

	Mya.	*Solen.*	*Pecten.*	*Lima.*
Copper oxide, . .	trace.	...	trace.	trace.
Iron oxide (Fe_2O_3), . .	...	0·20	...	trace.
Lime (CaO), . . .	3·59	3·46	3·70	3·58
Magnesia (MgO), . .	1·86	1·79	1·80	1·85
Potash (K_2O), . . .	4·89	4·90	4·87	4·90
Soda (Na_2O), . . .	44·20	44·03	44·11	44·08
Phosphoric acid (P_2O_5), .	4·89	4·80	4·76	4·83
Sulphuric acid (SO_3), .	2·75	2·73	2·80	2·77
Chlorine, . . .	37·82	38·09	37·96	37·99
	100·00	100·00	100·00	100·00

The next table represents the complete analysis of the blood of these Lamellibranchs :—

	Mya.	*Solen.*	*Pecten.*	*Lima.*
Water, ...	98·356	98·250	98·270	98·273
Solids, ...	1·644	1·750	1·730	1·727
{ Fibrin, ...	0·036	0·040	0·038	0·035
{ Albumin, ...	0·621	0·721	0·689	0·700
{ Salts, ...	0·987	0·989	1·003	0·992

The blood of *Solen* contains hæmoglobin; and this pigment is present in special corpuscles.*

MacMunn † has proved that the brown colour of *Solecurtus strigillatus* (belonging to the family Solenidæ) is due to the presence of hæmatoporphyrin, and it is very probable that the blood of this Mollusc also contains hæmoglobin.

The author ‡ has already given some details concerning the blood of certain representatives of the higher *Mollusca* and *Crustacea*, and consequently the following data may be considered as additional matter to that already published.

The orange-coloured blood or hæmolymph of *Chiton* (one of the *Polyplacophora*) and *Patella* § (one of the *Branchiogasteropoda*) contains no hæmocyanin, and its composition is represented in the following table:—

	Chiton.	*Patella.*
Water,	90·572	89·972
Solids,	9·428	10·028
{ Organic matter,	7·820	8·322
{ Salts,	1·608	1·706

The gases of the blood of these two Molluscs, investigated by means of the mercurial air-pump, gave the following results:—

	Chiton.		*Patella.*	
	I.	II.	I.	II.
Oxygen,	12·92	12·87	13·21	12·99
Carbonic anhydride,	30·46	31·29	31·52	31·02
Nitrogen,	1·36	1·86	1·76	1·81

* Ray Lankester, *Proc. Roy. Soc. Lond.*, vol. xxi. p. 71.

† *Jour. Phys.*, vol. viii. p. 384.

‡ *Proc. Roy. Soc. Edin.*, vol. xviii. pp. 288–294; and *Jour. Chem. Soc.*, vol. lxii. (1892), p. 648.

§ Concerning the Physiology of *Patella*, see Dr Griffiths' papers in *Proc. Roy. Soc. Lond.*, vol. xlii. p. 393; vol. xliv. p. 328; and his book, *The Physiology of the Invertebrata*, pp. 108, 284 (Reeve & Co., London).

The above figures represent volumes of the gases per 100 volumes of blood (the gaseous volumes being reduced to 0° C. and 760 mm.).

The blood of these Molluscs readily coagulates, forming a gelatinous coagulum; and this coagulum rapidly becomes fluid again. Similar observations have been made by Krukenberg on other Molluscs.

The blood of *Murex* (a Branchiogasteropod) and *Loligo* (a Cephalopod) has also been investigated by the author. In both of these animals it is a true hæmolymph containing the respiratory pigment—hæmocyanin; and the following empirical formula has been recently assigned to hæmocyanin:*—

$$C_{867}H_{1363}N_{223}CuS_4O_{258}.$$

The percentages of saline matter contained in the blood of *Murex* and *Loligo* are given in the following table:—

	I.	II.	III.	Average.
Murex, ...	1·826	1·820	1·818	1·821
Loligo, ...	3·216	3·201	3·208	3·208

Analyses of the ashes of the blood of these two Molluscs gave the following results (average of two analyses in each case):—

	Murex.	*Loligo.*
Copper oxide (CuO), ...	0·24	0·21
Lime (CaO), ...	3·71	2·32
Magnesia (MgO), ...	1·82	1·54
Potash (K_2O), ...	4·86	4·90
Soda (Na_2O), ...	44·26	45·23
Phosphoric acid (P_2O_5), ...	4·53	4·80
Sulphuric acid (SO_3), ...	2·66	2·84
Chlorine, ...	37·92	38·16
	100·00	100·00

The blood of these animals is rich in solid constituents, as the following table shows:—

* Dr Griffiths in *Comptes Rendus*, t. cxiv. p. 496.

	Murex.	*Loligo.*
Water,	88·557	86·154
Solids,	11·443	13·846
{ Organic matter,	9·623	10·645
{ Salts,	1·820	3·201

One hundred volumes of the blood of each of these Molluscs contained the following volumes of the three gases, the volumes being reduced to 0° C. and 760 mm. :—

	Murex.		*Loligo.*	
	I.	II.	I.	II.
Oxygen,	14·21	13·98	13·46	13·22
Carbonic anhydride,	30·16	29·76	30·02	32·31
Nitrogen,	1·82	1·65	1·46	1·23

The oxygen and carbonic anhydride in the blood of Invertebrates do not obey the law of Dalton in regard to the absorption of a mixture of gases by a simple fluid. In this respect there is a close analogy to the blood of the *Vertebrata.*

Such is the present state of our knowledge concerning the chemistry, &c., of the blood of the *Invertebrata.*

Note on Dr Muir's Solution of Sylvester's Elimination Problem. By Professor Tait.

(Read May 2, 1892.)

The following method of treating the question occurred to me while Dr Muir was reading his paper at the last meeting of the Society. It seems to throw some new and curious light on the intrinsic nature of the problem. I have confined myself to an exceedingly brief sketch, but it is clear that the proposed mode of treatment opens a wide field of interesting work.

Write the equations as

$$\frac{x^2}{A} - 2\frac{C'}{\sqrt{AB}} \cdot \frac{xy}{\sqrt{AB}} + \frac{y^2}{B} = 0, \text{ \&c.},$$

or

$$\xi^2 - 2e_3\xi\eta + \eta^2 = 0, \text{ \&c.}$$

The two values of ξ/η, &c., are evidently reciprocals of one another. In fact, if we were to put

$$\xi/\eta = \tan\theta_3, \text{ \&c.},$$

the equations might be written

$$1 - e_3 \sin 2\theta_3 = 0, \text{ \&c.}$$

Since we have

$$\frac{\xi}{\eta} \cdot \frac{\eta}{\zeta} \cdot \frac{\zeta}{\xi} = 1,$$

while the values of the factors on the left are, respectively, t_3 or $\frac{1}{t_3}$, t_1 or $\frac{1}{t_1}$, t_2 or $\frac{1}{t_2}$, it is obvious that the fourth equation required for the elimination is

$$\frac{1}{(t_1t_2t_3)^6}(t_1t_2t_3 - 1)^2(t_1t_2 - t_3)^2(t_2t_3 - t_1)^2(t_3t_1 - t_2)^2 = 0.$$

Put $T = t_1t_2t_3$, and this is

$$\frac{1}{T^8}(T-1)^2(T-t_1^2)^2(T-t_2^2)^2(T-t_3^2)^2 = 0.$$

Expanding and regrouping, the expression is easily transformed to

$$\frac{16}{T^4}\left(1-\left(\frac{1+t_1^2}{2t_1}\right)^2-\left(\frac{1+t_2^2}{2t_2}\right)^2-\left(\frac{1+t_3^2}{2t_3}\right)^2+2\frac{1+t_1^2}{2t_1}\cdot\frac{1+t_2^2}{2t_2}\cdot\frac{1+t_3^2}{2t_3}\right)=0$$

or

$$\frac{16}{t_1^4t_2^4t_3^4}(1-e_1^2-e_2^2-e_3^2+2e_1e_2e_3)^2=0.$$

The factor in brackets is the square of the determinant

$$\begin{vmatrix} 1 & e_3 & e_2 \\ e_3 & 1 & e_1 \\ e_2 & e_1 & 1 \end{vmatrix}$$

and thus Dr Muir's result is reproduced when we insert the values of e_1, e_2, e_3 in terms of A, B, C, A′, B′, C′.

One interesting point of the transformation seems to be the breaking up of this determinant into the four factors above specified; so that the equation

$$\begin{vmatrix} 1 & (\sin 2\theta)^{-1} & (\sin 2\alpha)^{-1} \\ (\sin 2\theta)^{-1} & 1 & (\sin 2\beta)^{-1} \\ (\sin 2\alpha)^{-1} & (\sin 2\beta)^{-1} & 1 \end{vmatrix}=0$$

has for roots, as values of $\tan\theta$,

$$\tan\alpha\tan\beta\,,\quad \frac{1}{\tan\alpha\tan\beta}\,,\quad \frac{\tan\alpha}{\tan\beta}\,,\quad\text{and}\quad \frac{\tan\beta}{\tan\alpha}.$$

But the novelty and value of the process seem to lie in the mode in which the elimination is effected by mere general reasoning.

Note on the Thermal Effect of Pressure on Water. By Professor Tait.

(Read July 18, 1892.)

I have just seen in the *Comptes Rendus* (June 27) an account of some experiments, on this subject, made by M. Galopin in the laboratory of Professor Pictet. As the effects obtained by him seem to be somewhat greater than my own experiments had led me to expect, I was induced to repeat my calculations with the view of trying to account for the difference. Unfortunately, M. Galopin's work is confined to 500 atmospheres, a pressure which lies a little beyond the range of my experiments; so that no very trustworthy comparison can be made. M. Galopin's results have one advantage over those of the *direct* experiments of the same kind which I made, inasmuch as he was able to use ordinary thermometers, while I employed thermo-electric junctions, in measuring the rise of temperature by compression. But they have a corresponding disadvantage, in the fact that mine were obtained instantaneously (by means of a dead-beat galvanometer) and required no correction; while his had to be corrected for the heat-equivalent of his apparatus to an amount not easy to estimate with accuracy.

I had assured myself of the general accuracy of my own work by showing that three altogether independent modes of estimating the effect of pressure on the maximum density point of water gave closely concordant results :—viz. a lowering of that point by about 1° C. for every 50 atmospheres. These investigations were described to the Society in 1881–4, and appear (in abstract) in vols. xi. and xii. of our *Proceedings;* more fully in the *Challenger Reports.* One mode of determination was *direct* (a modification of Hope's experiment); the others were theoretical deductions, from the compressibility of water at different temperatures, and from the rise of temperature produced by compression, respectively. M. Amagat subsequently obtained a result very closely agreeing with mine as given above.

His method differs from any of mine, for he seeks two temperatures, not very different, at which water has the same volume at the same pressure.

So far, I had been dealing with pressures of little more than 200 atmospheres. Higher pressures led to the result that the displacement of the maximum density point increases very much faster than does the pressure. For the terms in higher powers of the pressure begin to tell more and more; and another cause comes prominently into play, depending on the fact that water has a temperature of minimum compressibility (about 60° C. at ordinary pressures). This affects to a very much greater extent the lowering of the maximum density point by pressure than it affects the amount of heat developed by the compression. Both of these causes are indicated in my formulæ as contributing to such a result, but the small numerical factors of the terms which express them are not accurately known; and the calculation of the thermal effect of large pressures from data obtained by measuring compressibility at different temperatures is a very severe test of their accuracy. Besides, in giving a formula which exactly represented my determinations of the change of volume of water, under pressures from 150 to 450 atmospheres, and at temperatures 0° to 15° C., I expressly said that "it must not be extended, in application, much beyond" these limits. If, however, we venture to extend it to 500 atmospheres, it leads to the following expression, for the heating of water by the sudden application of that pressure,

$$\frac{t+3{\cdot}2}{26};$$

where t is the original temperature (C) of the water operated on. In obtaining this result it is assumed, in accordance with Kopp's data, that the expansibility of water at ordinary temperatures and at atmospheric pressure is approximately $(t-4)/72{,}000$. Other experimenters make it somewhat greater. [If the maximum density point were lowered 1° for *every* 50 atmospheres, the heating by 500 atmospheres would be about $(t+1)/22$ only. Comparing this with the result above, we see how considerably the causes, alluded to, affect the calculated amount of heating.]

Now I find that M. Galopin's results may be represented very closely (from 0° to 10° C., which are his temperature limits) by the analogous expression

$$\frac{t+5}{25}.$$

The difference between the denominators of these expressions is not serious, and may depend upon the uncertainty of the assumed expansibility of water, or upon an over-correction of his results by M. Galopin. [He increases his observed data by 52 per cent. in consequence of the thermal capacity of his apparatus.] But the difference between the numerators seems to show once more that M. Galopin's data have been over-corrected, or that it was scarcely warrantable to extend the application of my formula so far as 500 atmospheres.

Notes on the Wanyoro Tribe of Central Africa. By Robert W. Felkin, M.D., F.R.G.S., Fellow of the Anthropological Societies of London and Berlin, &c. (With a Plate.)

The paper which I have the honour of presenting to you to-night is similar in construction to those I have previously brought under your notice on the For, Madi, and Waganda tribes.

As in former papers, I have written from notes of my own observations made when in the country and from information supplied me by people of the tribe. This time, however, in order to complete the description of the people, I have had recourse to the works published by Sir Samuel Baker, Dr Emin Pasha, Capt. Casati, and the late Dr W. Junker, and I have to acknowledge my indebtedness to these authors for information gleaned from their pages.*

I have followed, as before, the order of the *Notes and Queries on Anthropology* published in 1874. I mention this because a new edition of this work has just been published, but I thought it better that my notes should coincide with my previous papers rather than with the new edition.

It might be thought unnecessary to publish an account of a tribe living in such near proximity to Uganda, but I think that a comparison between the two tribes will bring out marked differences, and illustrate the well-known fact that many diversities exist among African tribes which are in some respects closely akin.

The portrait is from a photograph (taken by Herr Richard Buchta) of Nickachuppi, a Wanyoro princess.

Unyoro is situated at an altitude of some 2600 to 3000 ft. above sea-level. It occupies a position towards the north, north-west, and west of Uganda, and is bounded on the north and east by the Nile, on the west by the Albert Nyanza, while on the south and south-west its boundaries are ill-defined. Its area is about 1500 square miles.

* See *The Albert Nyanza*, by Sir Samuel Baker; *Reisen in Africa*, by Dr Junker; *Emin Pasha in Central Africa*, by Emin Pasha; *Ten Years in Equatoria*, by Casati; and *Uganda and the Egyptian Soudan*, by Felkin and Wilson.

Physical Features of the Country.—It may be divided into two portions—a mountainous region, which extends from the Albert Nyanza to about 32° E. long.; and a plain, which slopes from thence to the Victoria Nile. The mountainous region consists of low mountain ranges and isolated peaks, many of the latter reaching a height of from 4000 to 5500 ft. above sea-level. Most of the hills are dome-shaped. From the mountain chains issue spurs, forming valleys through which streams flow. The country is extremely well watered, and practically drains to the west into the Albert Nyanza and to the north into the Nile; in the height of the rains a large quantity of water rushes down the deeply cut-out beds of the streams. Many of these are choked with papyrus, which also grows luxuriously around the Albert Nyanza. One river, the Kafur or Kafu, or River of Death, flows into the Nile near Mruli, and drains the eastern part of Unyoro. The country is fairly well wooded, but the forests are here and there separated by tracts of country overgrown with high grass; the underwood in the forest is very dense.

Geology.—The formation of the rocks in Unyoro is volcanic and metamorphic. The upper soil consists of grey fine-grained loamy detritus or thick dark-brown loam resting on red quartz. Sandy clay crops up at times, and a considerable quantity of ironstone is found, especially near the Albert Nyanza.

Climate.—Unyoro possesses a mild climate, and the temperature remains fairly constant throughout the year. The mean annual temperature may be taken to be about 78° F., and the extreme thermometric variations are not over 30° F. In February at lat. 2° 5′ N. Baker found the daily range to be 80° to 84° F., at night 56° to 58°. At Masindi, 4000 ft. above sea-level, the temperature was 62° at 6 a.m., 78° at noon, and Emin in October records the highest temperature at 79·7° F. The lowest temperature I observed was 69°, the highest 84° F., and the daily mean was 73°.

There is an abundant rainfall in Unyoro, the two periods of maximum rainfall being, as in Uganda, during April and May and October and November. Few observations have been made upon which one can base an estimate of the annual rainfall, but it is probably rather more than in Uganda, say 60 inches. Thunderstorms are very frequent, and the rainstorms in Unyoro occur with south and south-east winds. Emin notes that in October there was a fierce storm of hail and rain from the south-east, the hailstones being as large as horse beans, and continuing for half an hour, when they gave place to a perfect deluge of rain. Even during the driest season of the year the atmosphere is very moist, as is the soil; and Emin noticed the curious phenomenon of partial rains; he said they sometimes occurred in torrents, when at hardly ten minutes' distance there was no rain at all.

Population.—The population is about two and a half millions (1879). The people are Negroes, but they are not quite so fine a race as the Waganda, nor are they quite so tall. Their colour is, as a rule, a dark reddish brown (Broca's table, Nos. 42 and 43). Colour of the eyes, Broca's table No. 1. There are, however, a great many persons having a much lighter colour than that which is here indicated; in fact, they may be said to vary from reddish black to a dark reddish yellow, red being, however, the fundamental colour. The people are thin but well formed, save that the muscles of their legs are not well developed. They have good features and many of them are really good-looking; the lips are, however, very thick. Their teeth are good, but the four lower incisor teeth are extracted at puberty.

I regret that I have no detailed measurements of the Wanyoro.

Anatomy and Physiology.—The temperature taken in the axilla for five minutes, the subjects being at rest in the shade, at about 9 a.m., averaged 97·8° (98 observations taken). The number of respirations per minute under like conditions varied from 16 to 18; the pulse averaged 70. Like the Waganda, the Wanyoro do not bear cold well; they keep up fires in their huts during the night, and they do not like getting wet with the morning dew; they do not protect themselves from the sun. They are hardy, and bear privation fairly well, but are not quite so courageous as their neighbours. Their muscles are decidedly red in colour, and the fat has a yellow tinge. The mucous membrane of the mouth is highly stained with pigment. The skin is velvety, and they do not perspire much. The women have sufficient milk. They are delivered in a squatting position; a stake is often firmly driven into the ground, and the woman walks round about it in a circle until the commencement of each pain, when she squats down, supporting herself by the stake. Emin mentions that during delivery a woman will sometimes sit on her heels, her knees stretched apart, while one or two women support her back and arms, and the midwife sits in front of her ready to receive the child. (See *Reproduction.*)

Development and Decay.—I have no accurate information as to the average or extreme length of life among these people, but I saw a good many comparatively old men and women. Puberty is arrived at early, and the period of childbearing may be said to be from 14 to 25 years. Save that the mammary glands were remarkably well developed in the women, there was nothing remarkable about them. The Wanyoro's teeth are well preserved; I never saw any case of caries, nor do the teeth appear much worn. As in Uganda, it is considered unlucky if the children's teeth do not appear either at the right time or in the proper order.

Hair.—The Wanyoro have short, coarse, dull woolly hair. It is not plaited, and no care is spent on its cultivation. It is only shaved as a sign of mourning. Sometimes the women may be seen with flowers fastened in the hair, and, rarely, the men have a kind of bead head-dress, arranged something like an old-fashioned chignon, but they are not nearly so prone to such adornments as are the Longo, Shuli, or Shefalu. The musicians and medicine-men often wear long beards made from cows' tails or goats' beards. Comparatively few of the men have natural beards, and even those seen are not abundant. The hair from the rest of the body is generally removed by shaving or dipilation, but not so invariably as in Uganda.

Odour.—A distinctive odour is possessed by the Wanyoro, but it does not differ from that which obtains in Uganda.

Motions.—In expressing astonishment one hand is sometimes placed over the mouth; another method noticed by Emin is that of rapidly raising the closed fists to the crown of the head, from which they are drawn energetically to the forehead. When beckoning a person a movement as of grasping is made with the hand. When walking without loads the Wanyoro have a graceful carriage, and in standing, sitting, and lying their postures are graceful, although one does not notice the peculiarly dignified bearing seen in Uganda. In carrying loads their motions are somewhat ungainly; they then march with a short, quick step, the knees bent and the body inclined somewhat forwards. The arms are permitted to swing in walking. I did not observe that the feet were nearly so frequently employed for grasping articles as in Uganda, but all the joints are supple. They micturiate standing, the squatting position not as a rule being occupied when performing this function. In sleeping, the Wanyoro generally lie on the side or the back, and either an arm or a log of wood serves as a pillow. The hand is more commonly used than the fingers for pointing.

Physiognomy.—The Wanyoro have not such expressive faces as the Waganda; still the play of their features is fairly expressive. They gesticulate freely both in conversation, in palavers, or when telling stories.

Physical Powers.—The people are moderately strong, but their powers of endurance are not remarkable; they are not very good porters, for they appear distressed at a load greater than 50 or 60 lbs., and cannot march more than 15 or 16 miles a day when thus weighted. They require frequent rests.

Senses.—Their sight, hearing, and smell are abnormally acute, but I was unable to make any accurate observations in this connection.

Abnormalities.—Albinos are met with in the country; they are supposed to bring misfortune upon their relations. They do not result from the marriage of near kin. They often act the part of buffoons. I saw several dwarfs, and both Emin and Speke mention them; those I saw were evidently of Wanyoro origin, and had no connection whatever with the Akka or Tikki-tikki tribe. Deformities are, I believe, very uncommon, but hunchbacks may be occasionally met with, and I saw two children with harelips. I came across one case in which the toes were webbed and one in which there was a supernumerary thumb on both hands; the parents in both cases were normal. The practice of forming the "Hottentot apron" does not, so far as I am aware, obtain in Unyoro.

Crosses.—Practically the only crosses which are met with in Unyoro are those due to the marriage of the reigning family with the original Wanyoro, and, save for the fact that their skin is of a rather lighter shade, there is no difference to be noticed. Speke says that the lighter colour of the ruling class is attributed to the fact that "they do not do any work, but sit in the shade and drink abundance of milk."

Tattooing.—This is not practised in Unyoro, but I do not know of any laws which would forbid it. Emin says that he has seen scars and cuts used as ornaments, but only by women in the south-western districts, and I believe that this custom is not indigenous, but due to the imitation of their neighbours.

The Wanyoro tribal marks are two parallel burns on each temple. Some of the people wear earrings, but they are not very frequently seen. The four lower incisor teeth are extracted at puberty.

Dress.—The children in Unyoro are unclothed; the adults are variously clad; sometimes the women wear three or four folds of cloth made from the bark of the fig-tree suspended round their hips reaching to the knee; at other times they are clad in well-tanned goat-skins, or again a large bark-cloth is worn tightly fastened under the armpits. Some of the women wear in addition a cloak of the same material fastened round the neck and falling gracefully over the shoulders, much in the fashion of an Inverness cape. The men wear either tanned hide or bark-cloth; these are usually fastened over the right shoulder, the left being exposed. Head-dresses are rarely, if ever used.

Ornaments.—Some of the men wear rings round the ankles and wrists, and sometimes necklaces of beads or brass and copper rings. Sometimes, but usually only on festive occasions, they wear head-dresses manufactured out of the skins of wild animals, goats, antelopes, or leopards, and sometimes adorned with a variety of horns. The women are very fond of bead, iron, and copper ornaments; they often wear a series of anklets extending over two-thirds of the leg, and the arms may be covered with the same from the wrist to the elbow. Sometimes these rings are passed through three leather strips to keep them in position. The beadwork necklaces, cinctures, and bracelets are sometimes very tastefully made, and some of the ornaments constructed out of finely plaited grass, to which are suspended numerous bosses covered with fine beadwork, show considerable manipulative skill in their manufacture and taste in the arrangement of the colours. When on the warpath the men wear a simple loin-cloth and a strip of bark-cloth or banana leaf round the forehead. Some of the people disdain ornaments; at any rate I once sent a princess a present of some beadwork earrings and necklaces, and she refused to wear them, saying that she did not need to add to her charms.

Fat Wives.—A curious custom which obtains in Unyoro may here be mentioned. It seems that the king and some of his chiefs consider that obesity is a sign of beauty, and therefore some of their wives are compelled to subject themselves to a process of fattening to increase their charms. They are obliged to drink an immense quantity of milk, and so obese do they become that they can only crawl about on their hands and knees; it is impossible for them to walk. Personally I had no opportunity of seeing any of these beauties, but I quote an abbreviated description of one given by Casati:—"In the centre of a column was a palankin made of ox-hide fixed on two poles, upon which one of the royal wives sat wrapt in a bright-coloured mantle of peculiar pattern. . . . A cry of astonishment was uttered by everyone as a woman, almost a shapeless mass of flesh, with immense limbs and small eyes, sitting on a sort of sedan-chair, and supported by stout poles, was being carried across the royal threshold by four stalwart men. The accumulation of fat gradually proceeds so as to render the person unable to stand up. They are compelled to walk on their hands and knees, and even then move with great difficulty."—(Casati, *op. cit.*, vol. ii. p. 70.) These women are only permitted salt porridge made with broth twice a week, and sometimes a handful of salt.

Pathology.—My stay in the country was far too short to obtain anything but the briefest information respecting the diseases of the natives.

Malarial fevers are common enough on the banks of the Nile and on the shores of the Albert Nyanza, but I understand that they are far rarer in the higher parts of the country. At Kiroto, where I made special inquiries into the subject, I found that both the natives and the Egyptians rarely suffered from fever, but at Magungo Foweira, Kodj, and the adjacent districts malaria is very rife. Ague-cake was far from uncommon, and I saw many children suffering from moderately enlarged spleens, and a good many suffering from dropsy. Bronchitis and pneumonia I also met with, and these diseases must be fairly frequent, as they are treated with the actual cautery or cupping, and many persons bore the marks of these methods of treatment. In cupping, the cow's horn is used in place of a cupping-glass, a considerable quantity of blood is withdrawn, and the people are weak for several days subsequent to it.

Diarrhœa is common, and I saw several cases which I believed to be enteric fever. Headache is not very common; it is treated by cupping over the temples. Acute mania certainly occurs, but it appears to be of a transitory character. The individual afflicted with mental abberration often runs away and hides in the forest; if caught, he is compelled to drink large quantities of a decoction made from herbs and roots, which induces sleep and profuse sweating. I collected a number of specimens of these simples, but I had to leave them at Lado on account of difficulties of transit. Small-pox is much dreaded, as it causes a great mortality at times. Partial isolation of the patient is practised, the pustules are opened with a sharp thorn and the patient is washed, but the treatment is apparently not very successful. Epilepsy is as frequently seen as in Uganda, and occurs chiefly in girls. No treatment is known; girls suffering from this affection are difficult to get married, and anyone willing to take them off their fathers' hands can do so without giving a dowry. Skin diseases are very common in Unyoro, and this is rather surprising, as on the whole the people are cleanly, although their habitations are not well kept. Rheumatism, lumbago, and sciatica are frequently met with; the actual cautery is a favourite application. I never heard of a case of toothache. Leprosy is known in Unyoro, but does not appear to be very common; its hereditary nature is recognised, but this is not the case with regard to epilepsy. Venereal diseases are frequently seen; their introduction is said to date from Kamrasi's time, and they were brought into the country from east to west some forty years ago. Diseases of the eye and blindness are very rare.

Wounds heal rapidly and amputation is known; at anyrate the great chief Rionga showed me a boy whose arm he had amputated in the middle third on account of its having been shattered by a gunshot.

"Earth-eating" is practised in Unyoro, sometimes as a remedy for disease, but often to satisfy a simple craving. If long continued, it is said to cause discoloration of the skin and hair, emaciation and death. Hereditary cannibalism is known, and girls from such a family will rarely find a husband. Nightmare is known, and dreams thought to foretell coming events.

Medicine men and women often travel about the country, gaily dressed with bead ornaments and numbers of charms. They resemble the gipsies, having no fixed dwelling, but making themselves at home wherever they may be. They practise incantations to drive away the evil spirits which are supposed to have taken up their abode in sick people. In treating disease they employ many herbs, roots, and the bark of trees; they also apply the actual cautery, and cup their patients. Some of the medicine-men act as singers and dancers at festivities. They also prepare amulets and charms to ward off disease and death. They receive as payment for their services either beads, tanned skins, or cattle.

Casati says that the natives eat butterflies and insects to cure colds in the chest, and crocodile fat and dung is sometimes used as a cure for rheumatism.

*Reproduction.**—A lingering labour is hastened by rubbing the abdomen. If the head presents, it is considered to be a favourable sign. A footling forebodes a misfortune to the family. If a cross-birth occurs, turning is attempted by men, who receive payment for their services. If a woman dies before delivery, abdominal section is at once performed, and the child whether living or dead removed; the omission of this operation would be punished by the chief of the district with a heavy fine of cattle, goats, or even women, as it is believed that such a proceeding would entail ill-luck on the village. Post-partum hæmorrhage carries off many of the women; it is probably caused by attempts to remove a retained placenta. Sometimes, if the placenta does not come away readily, the abdomen is pressed or kneaded with a broad-ended pole. This pole is cut the convenient length, and placing one end on the ground, the woman presses the other against her abdomen; then swaying her body gently backwards and forwards or from side to side, she makes a rhythmical pressure on the fundus uteri. The umbilical cord is cut long and tied round the body of the child until it shrivels and falls off; to hasten separation it is rubbed with fat. Ligature of the umbilical cord is unknown. The placenta of a male child is buried in the house on the inner right-hand side of the door; that of a female child on the inner left-hand side. The child is washed in tepid water and then rubbed with a mixture of red clay and fat. It is not customary for the mother or child to leave the hut until the umbilical cord has fallen off, nor may the mother shave herself.

* For most of this information I am indebted to Emin Pasha.

About the 5th day after birth the mother sits on her threshold with her child, a sheep is killed and the child is named by the paternal grandfather, or should he be dead, by the maternal grandfather. It usually receives two names—one connected with some bodily peculiarity, the other chosen at pleasure. The skin of the sheep killed at the naming of the child is used to carry it in, the child being suspended at its mother's back in the skin so that its fore-legs are tied together round the mother's shoulders, its hind-legs round her waist. A woman suckles her child 18 months; it is then weaned and the breasts are rubbed with bitter juices. During suckling the women live apart from their husbands. The fecundity of women is from 12 to 25 years of age; many are barren, most of them have only two or three children. The women grow old quickly. Hermaphroditism is rather common. If women are sterile, they are sometimes returned to their parents, who return a proportion of the dowry paid for them. Twins are considered a great piece of good fortune, not only to the parents but also to the entire village. Universal festivities are held in the village, and rich gifts are brought to the mother from all sides. The firstborn twin, whether a boy or a girl, is called "Zingoma," the other "Kato." The placenta of living twins is placed in a large earthen vessel in a miniature hut hastily erected in the courtyard, where it remains four days. It is then carried in procession to another larger hut built in the jungle, and there it is left. Should the twins die, they, together with their placentæ, are left in an earthen vessel in the mother's hut until decomposition sets in. They are then removed to the miniature hut built in the courtyard and are there left for an arbitrary period, being watched by a man who scares away hyenas. During the time of this exposure the occupants of the house may not shave themselves, and they must keep aloof from everyone as a sign of mourning. When this period is accomplished, the men and women cut their hair and lay aside for a time all ornaments, and finally the house in which the birth took place is burnt, expiation thus being made.

Causes which limit Population.—Of the causes which limit population, polygamy is the most important. It is derogatory to the position of chiefs, even the most unimportant, to possess less than 10 or 15 wives. Poor men have three or four wives each.

As a chief has only four or five favourites, always chosen from among the youngest, perhaps exchanging them for still younger wives when they have lived with him for a few months, a large number of women are left fallow who would otherwise be capable of production. Early marriage also limits reproduction.

Habits and Customs.—The Wanyoro are an indolent, lazy people, and by no means possessed of great prowess. They are, however, fond enough of raiding and pillaging those who they think will be an easy prey. They are naturally intelligent, but do not possess nearly so much culture as do their neighbours the Waganda. The higher classes lounge about and smoke and drink all day, all work being left to their wives and slaves. Indeed it is only among the lower classes that the men work at all, and they will avoid manual labour as much as possible. The principal wives of the chiefs lead a luxurious and indolent life—sleeping, eating, and visiting being their occupations.

The Wanyoro are cleanly in their persons, frequent ablutions being practised, but, this notwithstanding, women frequently anoint their heads with a mixture of red ochre and fat. Many of the people anoint their bodies with oil, or sometimes with scent. One scent is a species of touchwood which smells like musk, the other is composed of very compact grey clay, which, according to Emin, is brought from the south and sold at a high price. Notwithstanding their cleanliness, the Wanyoro are infested by vermin which find shelter in their bark clothes. They get rid of them by fumigating the cloth every few days with the smoke of the stripped stalks of the dried papyrus ; this acts as a parasiticide, and gives a distinctive odour to the clothes. The Wanyoro are very neat, and all their handiwork is carefully performed. In their houses their stores of roots, &c. are neatly packed in either woven baskets or in packets made of banana leaves and tied with finely woven string. Their huts, however, are not well kept, and they are not very particular as to the removal of filth from their courtyards.

They cut their nails to a triangular point, the point being in the centre of the finger; the nail parings are carefully preserved, and from time to time taken and thrown into the jungle—why, I am unable to say.

There are not many ceremonials in Unyoro, but the people are courteous and friendly one to another. Both men and women salute each other at morning and night, saying Good-morning (Rairote; *ans.* Daabante); Good-evening (Geroba; *ans.* Gerōbera). When people meet in the road, if of equal rank, they say the word "Merembe," followed by each party giving vent to a series of curious nasal grunts of satisfaction. If a man of low rank meets one who is his superior in the street, he is expected either to kneel or to bend forward, supporting his body with his hands on his knees until his superior has passed. There are special forms of salutation for great chiefs, the customary address to the King being "Ngunzono Kali" (I greet the highest). The common people when expressing thanks use the word "Vebbala," the higher classes the word "Nkuebas." If a person has been to a drinking-party, when he leaves he salutes the host with the word "Nkuada" (thank you), and is answered by the salutation "Rainni." In all classes of society dried coffee-berries are offered when meeting or during conversation, and among the higher classes, after drinking beer, coffee-beans are used to remove the odour of the liquor. When strangers arrive at a hut the people sitting before its door always rise, and, after the customary salutations, beer, water, or food is produced, should the stranger require it. The women do not as a rule sit with the men, but form groups to themselves, and among the higher classes the sexes eat apart. The Wanyoro always wash their hands before eating; the viands are all covered with grass mats whilst being carried from the cooking-place, to prevent them from being bewitched by the evil eye. Kabrega may not eat poultry, and the chiefs are expected to refrain from this article of diet. Wooden spoons are used in eating, but the fingers serve the purpose of knives and forks. After meals, ablutions are customary.

Food.—The diet is principally vegetable, and consists of sweet potatoes, gourds, roasted bananas, durrah, eleusine, manioc, and various leguminous plants, coffee, sugar-cane, several species of beans, one of solanum, tullabone, sesamum, maize, cassava. When meat is scarce, blood boiled with butter and salt is eaten. Game and goats are very commonly partaken of, as well as fish and eggs. Beef can as a rule only be afforded by the rich. Elephant's flesh, and also crocodile and hippopotamus flesh, are not eaten, the latter being supposed to produce skin diseases. The Wanyoro are very fond of salt, and eat it in large quantities; but, unlike their neighbours, they avoid pepper, as they believe it produces sterility. Honey is greatly enjoyed and is usually taken with porridge, the latter being made of dried plantain or ground durrah. Milk is used as an article of diet, either fresh or curdled. In this connection I may mention that the milking of cows falls entirely to men; women are strictly forbidden to touch a cow's udder.

The great chiefs have separate kitchens in which the food is cooked respectively for men and women. The food for a great chief is cooked by a male cook with whom he has made blood-brotherhood. In the lower classes the women invariably prepare the food. The people usually partake of three meals during the day—the first soon after sunrise, the second at midday, and the third after sunset. The fireplaces made for cooking are constructed of stones, of which five are used; a long, flat, oblong stone is fixed in the ground lengthways, and on each side of it two smaller stones are fixed, so as to make six compartments. The pots used in cooking (see *Manufactures*) are globular in shape and of various sizes. Vegetables are boiled in them, and the people prefer their meat to be boiled too, though it is sometimes roasted or smoked. Fish is smoked likewise. Porridge and "Irish stew" are served in boat-shaped wooden dishes standing upon three or four feet, which raise them from the ground. Mats are used as table-cloths, and the food is only uncovered when the party are ready to dine. As in Uganda, strips of wet banana bark are used as napkins. Marrow bones are cooked in the ashes, then cracked and the marrow extracted; the people do not like raw marrow; white ants are often cooked in butter. When catching the ants for food large fires are lit near the ant-hill and the people either beat small drums or rattle sticks together, upon which the ants swarm.

Drinks.—Three varieties of beer are used in Unyoro. The first is Sandi, which is simply the juice of the ripe bananas expressed through porous mats and mixed with a little water. It is kept for a day or two before use and is only slightly fermented. It is a pleasant beverage and not intoxicating. Mwenge, the second drink, is prepared of artificially ripened bananas, to which is added water and roasted durrah. It is a sour beverage and very intoxicating. The Mervua beer is made from eleusine. The Wanyoro are great beerdrinkers, and the favourite method of drinking is to suck up the beer through three or four straws, or through a drinking-tube specially manufactured for the purpose.*

Fire.—Fire is produced by the friction of a sharpened piece of hard wood rotated between the hands in a notched piece of soft wood. No instruments are used for its production. The fire is caught by either bark-cloth tinder or sometimes tinder made of dried grass. The only fuel utilised is wood and charcoal. The natives sometimes carry fire with them on a journey, either in the form of a burning log or a slow match composed of a long coil of tightly twisted bark-cloth.

Habitations.—The Wanyoro are not very expert hut-builders, but they occupy a position between the neighbouring tribes in the north and the Waganda in the south-east. The huts are usually from 20 to 30 feet in diameter and 20 to 25 feet high, dome-shaped and circular or semi-circular. They are constructed of light wicker-work, which is supported by a central pole and some five or six other poles placed in almost a complete circle. They are thatched with grass, which reaches down to the ground. The entrance consists of a porch-like opening, 4 to 6 feet high, and closed at night with a strong basket-work door. Very usually the huts have a partition, sometimes formed of wicker-work and grass, at other times of bark-cloth curtains. Sometimes, but rarely, three or four huts will occupy a distinct compound, being surrounded by grass fences, but more commonly they form scattered groups without any inclosure. Smaller huts are sometimes built as storehouses, but more usually goods are stored in the common dwelling. The beds are either heaps of grass laid upon the floor and covered with hides, or sometimes they are raised shelves some two or three feet from the ground.

As before mentioned, the huts are very dirty. Goats and poultry live in them, and they are infested with mice, fleas, rats, and cock-roaches. No arrangements are made to carry away the rain which falls upon the huts, and consequently they are damp. The floor is simply composed of beaten earth.

* I have omitted a detailed description of brewing, as it does not differ from that which obtains in Uganda. See *Proceedings of the Royal Society*, vol. xiii. pp. 717 to 718. Each family brews its own liquor, but any which is in excess of their requirements is sold in the public markets.

Narcotics.—Both men and women in Unyoro smoke, although I do not think that the women are quite such inveterate smokers as their neighbours in Uganda. Two kinds of tobacco grow in the country—the Nicotiana Virginiana, and a smaller plant which is much stronger in flavour, has a smaller growth, and fawn-coloured flowers. According to Emin, this species is dying out. Tobacco is only used by smoking it in a pipe, and no substances are mixed with it. The tobacco-plants are carefully tended and the ground near them weeded; they are not allowed to seed; the leaves are dried in the sun and made into various-sized packages weighing from half a pound to about ten pounds. The only tobacco-pouches in vogue are either skin bags closed at the mouth by a string, or neatly made plaited grass baskets. The Wanyoro pipes are very various. Rionga showed me a very fine collection, one of which was capable of holding a pound and a half of tobacco. In Unyoro one can generally tell the rank of a man by the size of his pipe; the greater the man, the larger his pipe. The form of pipe is usually globular with a long stem; the bowl is made of clay, roughly glazed, and it is variously ornamented with circular dots, bands, chevrons, and sloping parallel lines. No mouth-pieces are used to the pipes. Sometimes a pipe may be made with two heads; I saw one with three. One peculiarity of the Wanyoro pipe is that, instead of the hole being carried from the bottom of the bowl through the stem, there is a ledge of clay extending half way across the bottom, and into this ledge the opening is made. As a rule, people use their own pipes, and carry them with them wherever they go. They are sometimes slung round the neck and hang down the back, but if a man does not happen to have a pipe with him when his friends are smoking they offer him their pipes occasionally. I never saw short pipes used, but on one occasion I saw a man smoking from a hole he had made in the ground.

Occupations.—In a country where the necessaries of life are easily procured the natives cannot be expected to be very industrious. The men build the houses, fight, if there is any fighting to be done, tend the cattle and milk them, and a few of them are employed in various manufactures (see later), but for the most part they avoid all work if they can. The women are more industrious; they cook, fetch wood and water, and do whatever agricultural work is necessary.

In all respects the people lead an easy-going life. They appear to be contented and satisfied, and I do not think they are either enterprising or ambitious enough to take readily to laborious employments. As far as I saw, they were lazy, unwilling, and unsatisfactory porters, but it must be admitted that their service was compulsory, their loads were heavy, and their hope of reward small. The Wanyoro living near the Nile and on the shores of the Albert Lake occupy themselves in fishing. After the evening meal the men collect together and carouse round the fires, occasionally singing and otherwise amusing themselves.

Amusements.—The people's amusements are not extensive, and consist chiefly in assemblies usually held at night at which dancing takes place, old and young alike participating with vigour. Sometimes men and women dance at the same time, but not together; at other times solos are indulged in. War-dances are also performed; and mimic warfare, in which great agility is exhibited, is a favourite pastime. Some of the dances are indecent in the extreme. The Wanyoro have some little theatrical ability, and sometimes their dances are arranged to illustrate some event, or to caricature some incident or person. The following scene which I witnessed at Kodj will illustrate what I mean :—A dance was arranged for my entertainment, and one of the dancers was called Abdul Aziz after Linant de Bellefonds. On one occasion Linant was riding and smoking, when he was suddenly attacked, so he immediately gave his pipe to his man in exchange for his gun. This was caricatured, two men dancing one after another, one carrying a pipe, the other a gun, the first one representing Linant. The steps resembled a horse galloping, and the dance was very amusing; every now and then the man, leaving off smoking, took his gun and fired into the crowd, and then resumed his pipe. Jesters and professional story-tellers are frequently met with; they go from village to village reciting legends or witty stories, which the people enjoy greatly. They usually wear fantastic head-dresses, and are not infrequently expert musicians of a kind. The occasion of births and marriages and the appearance of the new moon (see ceremonies connected with new moon) are gladly utilised by the people for carousal, dance and song.

Musical Instruments.—The Wanyoro possess various musical instruments, of which perhaps the drum is the chief. These are made of various sizes, hollowed out of the trunk of a tree, and gaily decorated with shells and beads and charms; they are usually beer-barrel in shape, and are generally played only on one end, either with the hands or with drumsticks. The drum may only be beaten as a signal for war and during its continuance, or on festive occasions. A fine follows disregard of these rules. Trumpets or horns are made out of antelope horns or bottle gourds. The flutes are constructed out of reeds. Some of the people also use a lyre made of five strings, which are arranged either projecting perpendicularly upwards from a gourd which serves as a sounding-board, or stretched over a board upon two bridges shaped like a harmonicon. As I am not musical, I cannot say much about the native tunes, but some of them seemed to be melodious; the choruses, accompanied by drums, horns, and flutes, are weird in the extreme. Baker considers that their music is a monotonous repetition of the three notes—*b*, *c*, *d*.

Hunting.—The only information I have respecting this subject is gained from Emin Pasha. He says that hunting-parties often take place. When they are arranged privately those that take part in them choose the leader from among themselves, but when they are set on foot by a district chief he appoints the leader. The man who throws the first spear at an animal receives the forefoot if it is killed. The division of the booty is effected by general agreement. If game is wounded and runs on to the ground belonging to another man and dies there, the owner of the land receives the right fore-foot. If a leopard or lion is killed near the king's dwelling the whole animal is carried to him, but if the distance is too great the skin alone is sent. One tusk of all elephants slain belongs by right to the king; the other may be kept by the hunter, but the king usually gives him a girl in exchange. The Wanyoro are not nearly such keen or brave hunters as the Waganda. Snares and pitfalls are sometimes employed to capture game; there is nothing distinctive in their construction. In fishing, the Wanyoro employ the hook and line, the spear, stationary nets and wicker-work basket traps; the latter are usually circular in character, and are of the shape of a hock bottle. The entrance to the trap is at the broad end, and is so arranged that fish swimming in cannot escape, for the bars forming the base will only open inwards.

Weapons.—The weapons employed by the Wanyoro are not numerous. The most primitive is the club, and various kinds of knobs are made—round, oval, and embossed. The spears are fairly well constructed; the heads are not large, only measuring from 6 to 8 inches, being 2 or 3 inches broad at the flange; they are usually ribbed. The shafts are from 5 to 6 feet in length. The same weapons are used in war and in chase. When hunting crocodiles lines are fastened to the spears, to which ambatch floats are attached. When going to war the sharpened spears are usually protected by leathern covers, neatly made and decorated with feathers or beads. Sometimes a peculiar spear is used for elephant hunting; it has a broad heavy head from 18 to 30 inches in length, 8 to 10 inches at its widest part. It is fastened to a short handle and heavily weighted with a mixture made of clay, straw, and cow's dung. It is dropped upon the animal by the hunter, who sits upon the branch of a tree. Sometimes these spears are hung over paths frequented by wild animals, and so arranged that they fall when the quarry passes beneath.

Agricultural Implements.—The people only possess one real agricultural implement, viz., the hoe. It is heart-shaped and slightly bent, forming with its neck a gentle curve. The handle is from 3 to 4 feet long, and a bent forked piece of wood is chosen for it, so that the whole when completed can be used like a pickaxe. The other implements used are simply curved knives or spear-heads, the latter being employed in planting and weeding, the former to prune or dress the plants.

Cultivation.—As before mentioned, the women till the fields, which are as a rule fairly well kept. Weeding is assiduously practised. Before sowing, the high grass is burnt, and after being allowed to lie upon the ground for two or three weeks, it is hoed and the various crops are planted. Owing to the richness of the soil, the people have little difficulty in raising sufficient produce for their necessities, and there is no doubt that with good cultivation the greater part of Unyoro would render surprisingly rich crops. It is unnecessary to detail here the various grains and vegetables grown in the country. (Tea, cinchona, and coffee would well repay introduction.)

Domestic Animals.—Here and there domesticated wild-cats are met with, and there are many dogs in Unyoro. They are long-bodied, lean, buff-coloured animals, lop-eared, and carrying their long short-haired tails erect. Many of them are very vicious. They are used in the chase, but from what I saw of them they are not well trained. Fowls are found everywhere; they are small and usually lean. Goats are very numerous; sheep more rarely seen, and they seem to be imported from the east of the Nile. The cattle mostly belong to the king or the chiefs; they are said to suffer from many diseases, for which copious bleeding from the neck seems to be the general remedy; the blood thus obtained is often used as food. Wahuma are the usual herdsmen, but in any case men or boys herd the cattle. The cattle's horns are always destroyed as soon as they commence to grow by cauterising them with a red-hot iron, in order to enable the animals to pass with greater ease through the jungle. In respect to cattle, Emin relates that those which supply milk for Kabrega's personal consumption are kept quite separate. They are milked in his presence every morning, and then go to pasture escorted by a man and a boy. The boy goes before them calling out loudly "the king's cattle," and everyone that happens to be near must withdraw as quickly as possible if he does not wish to be killed, as it is said that there are people who if they look upon cattle can turn their milk into blood.

Wild Animals.—In Unyoro there are very many animals—elephants, lions, leopards, jackals, hyena, buffalo, rhinoceros, wild-boars, various species of gazelles and antelopes; elan also are met with in considerable numbers, and sometimes white rhinoceros are seen. A very great number of hippopotami and crocodiles infest the Nile and also the larger streams, and here, as elsewhere, the crocodiles are very dangerous, many of the natives being carried off by them. I once shot 60 in a forenoon at Kodj. Boa-constrictors and other snakes are numerous. Many monkeys are found in the woods; as specially numerous may be mentioned the Cercopithecus griseo viridis, the Colobus guereza and the Colobus ruber, as well as the Paleoornis cudicularis, and some chimpanzees, which are however limited to the southern districts. The birds met with are very numerous; vultures are extremely common, and Emin mentions Sporothlastes fasciatus, Spermestes cucullatus, Trachyphonus margaritatus, Psittacus erythacus, Ortygospiza atricollis, and several Habropyga. Parrots are very numerous, as well as guinea-fowl and bats. Insect life in Unyoro is very profuse.

Manufactures.—Although the Wanyoro do not compare very favourably with the Waganda as artisans, yet a considerable amount of industry obtains in the country.

Tanning.—The Wanyoro are expert tanners, and the dressed skins are supple in the extreme. The skins of cattle, goats, antelopes, and monkeys are all employed; before being dried, they are pegged out upon the ground and carefully scraped. If the skins are to be employed for men's dress, the hair is not removed; if for women's garments, it is removed, with the exception of a fringe at the edge. As soon as the skins are moderately dry they are rubbed with butter; they are then frequently beaten between two flat stones. Should one skin not be sufficiently large to form a garment, several are sewn together with needles of home manufacture. These needles are rather rough, but are ingeniously made, the eye being formed by turning over one end of a piece of iron and beating its point into a small depression struck into the shaft of the needle.

String.—This is usually formed from banana fibre, but sometimes the fibre of creepers is used. It is very finely woven and serves innumerable purposes; sometimes it is plaited into mats and into bags, many kinds of the latter, from a heavy quality to a very fine and exquisitely woven variety, being manufactured. This twine, too, is used for the manufacture of bead ornaments, in which the Wanyoro exhibit considerable taste.

Bark-cloth.—This is manufactured as in Uganda.

Pottery.—The pottery in Unyoro is fairly good, though somewhat brittle. There are two varieties, a black and a red. The jars are mostly gourd-shaped, but oval and hemispherical bowls of various sizes are made. No flux is used. I have previously mentioned the pipes.

Blacksmiths.—The smithy work in Unyoro is rough. Either a stone or a mass of iron is used as an anvil, the hammer being either an oval stone or an oval piece of iron. I only once saw a hammer with a handle. The bellows are constructed of two earthen pots, over the mouths of which pliable skins are fastened, having a hollow stick tied in the middle. Clay tubes from the pots conduct the draught to the hearth. The smelting-ovens are oval and conical, usually three or three and a half feet high. The Wanyoro have an apparatus made of wood for drawing out wire.

Salt.—Except at Kibiro, the natives prepare salt by burning papyrus and then mixing the ashes with water. After allowing it to stand for 24 hours, it is filtered through a jar having a hole at the bottom and being half filled with tightly-pressed dried grass. It is also in times of need obtained from cow's urine. At Kibiro salt is manufactured on a larger scale, and Emin gives a very good description of it in a paper which was published by the Royal Scottish Geographical Society in 1887. He says—"The method of preparing the salt is tolerably simple. The earth from which the salt is to be extracted is placed in the evening under the end of a trough, whence a thin stream of water trickles over it all night long. In the morning the earth is put to dry for some hours; after this the women, with crescent-shaped pieces of iron, scratch off its superficial layers and with them fill other small troughs, out of which they shake it again into small heaps. The next day a certain quantity of this earth is mixed with water and then conveyed to the filtering apparatus. These consist of simple clay vessels, having holes at the bottom covered with a layer of fine hay; the vessels themselves stand upon an arrangement of three stones, and have beneath them smaller clay vessels into which the liquid drops. When the filtration is finished, if the saltmaker is not pressed for time, he allows the liquor to evaporate in the open air; it then leaves behind it a pure white salt. If, however, time cannot be given for this process of evaporation, it is accomplished by means of boiling within the huts, but the salt so obtained is darker in colour and less pure. The skill of the women consists in mixing the earth and water in the right proportions just before the filtering begins."

The chief of Kabiro pays an annual tribute in salt to the king.

Woodwork.—This is very primitive. Bowls are cut out of solid blocks of wood and fairly well finished; drums are made in the same way, but the paddles, walking-sticks, and the handles of hoes, &c., are not usually finished with any care.

Boatbuilding.—The Wanyoro canoes are neither large nor do they shew any advanced art in construction, being simply ordinary dug-outs. The boats, when finished, are clumsy and unwieldy. I saw, however, one interesting boat which was made of a curved tree and used for hunting hippopotami, the idea being that the hippo would rise in the curve, and therefore not upset the boat.

Basketwork.—Various grasses and canes are used for plaiting baskets, but the result is neither artistic nor are the forms graceful, and I rarely saw any distinctly pretty articles. Probably the best made wickerwork is the shield, upon which some care is spent.

Communications.—It may almost be said that the Wanyoro exercise no care in the establishment of roads or means of communication between one part of the country and another. As a rule, they content themselves by following the well-worn paths made by passengers between one village and another. It is true that sometimes, when a great chief travels, it is customary for him to order a path to be made, and this is done by sending forward a party of men who, following an ordinary footpath, trample down the grass on either side and remove the saplings; larger trees, however, are left standing. At the fords it is sometimes customary for a rope to be stretched across a stream. I have seen two tree-bridges made by simply felling a tree and causing it to fall across a stream, but I never heard of any greater attempt than this. In respect of ferries, I can only speak of those on the Nile itself; there did not seem to be any great regularity in their running, but certainly boats and men were stationed at various points near Kodj and Foweira. A small sum of beads was given to the ferryman by each passenger. In one or two swamps I noticed that logs of wood had been thrown in to facilitate passage.

Trade and Commerce.—The Wanyoro are keen traders. There is a lively interchange of produce in the country itself, but the exports are limited, consisting of ivory, salt, skins, and slaves. Although a few slaves may find their way into Uganda, the majority are bought by the Arab dealers and sent to the south *via* Karagwe. Salt is chiefly exported to Uganda. It is made up into cylindrical packets, wrapped in dried banana leaves, weighing about 25 lbs. and being sold for 400 cowries, equal in value perhaps to five and sixpence. As before mentioned, the one tusk of every elephant killed belongs to Kabrega the king, who usually purchases the other from the hunter; therefore nearly the whole of the ivory trade remains in his hands, and is carried on with the Arabs from Karagwe. The ivory varies very much in price, and a considerable quantity comes from districts outside Unyoro proper, but under vassalage to the king.

It is the custom for Arab traders when first arriving at the king's seriba to make him a considerable present of guns and ammunition, after which trading commences, and it is only when the Arabs have finished bartering with the king and the great chiefs that they are permitted to trade with the people. Markets are held once or twice a week in every group of villages; the people collect in large numbers, and beer, salt, coffee-berries, spears, knives, hoes, tobacco, beans, butter, bark-cloth, sheep, goats, and sometimes cattle are brought for barter. The Wanyoro are keen at a bargain, and considerable time is usually required before an exchange of goods is effected. Although the disputes as to value are sometimes very lively, the people keep their tempers, and real quarrels are rare. The cowrie, "simbi" as it is called, serves as the standard money, or rather a string of 100 cowries. It is difficult to give the exact values of produce, as they vary so much according to season and the quantity brought for sale, but about 2 lbs. of flour costs 10 cowries, a sheep or a goat from 1300 to 1500, an ox from 4000 to 5000. It should be mentioned, however, that there are not nearly so many cowries in Unyoro as there are in Uganda, and ten or twelve years ago, when I was there, produce was exchanged—butter for beer, meat for tobacco, flour for skins, and the like. Girls and small boys are usually made the objects of private barter, and are not exposed for sale in the markets. The price varies considerably, the Wahuma girls being the dearest (see *Marriage*). All the goods which the Wanyoro bring to market are neatly packed either in baskets or in banana-leaf parcels, and after the market is over the people form into groups and gossip and drink their beer.

Origin of the Wanyoro.—It seems to be undoubted that at present there are three fairly distinct classes of inhabitants in Unyoro—the old aboriginal inhabitants, the Wahuma or reigning families, and a mixture of the two. It is probable that the Wahuma came from the Galla country, crossed the Somerset Nile, and then either separated into two bands, the one going to Uganda, the other remaining in Unyoro and penetrating to Karagwe, or that Uganda was first conquered, then Unyoro, then Karagwe, and that it is only 50 or 60 years ago since, owing to revolutions, the three kingdoms became distinct. In 1878 Unyoro was divided into three distinct provinces, reigned over by three chiefs, each claiming the throne.

Kabrega ruled in the south-west, Amfina in the north near the Murchison Falls, and Rionga in the east, having his head-quarters at Kodj on the Nile. They were all descended from Issanca, and the following table will show their pedigree.

- Issanca.
  - Kwambe (Niantukera).
    - Sagārā.
      - RIONGA.
        - Kamiswa.
    - Kalehupe.
      - Furoka (dead).
      - AMFINA.
    - Mugene.
      - Kamrasi.
        - KABREGA.

Soon after I left the country, Rionga and Amfina were overthrown, and now Kabrega is the nominal ruler over the whole country. It is needless to repeat the history of the various events which took place when the Wahuma overran the country. Suffice it to say, that the Wahuma are distinctly herdsmen, and that the original inhabitants of Unyoro were tillers of the soil.

Legendary History.—On the origin of the Wanyoro dynasty, Casati relates as follows :—" The founder of the dynasty came from the countries beyond Lango, and established himself in Uganda. An expert huntsman, he taught the natives to kill the wild animals, and his fame spread as far as the queen, so that at last she was seized with a desire to see the brave stranger. He was a very handsome man and the lady fell in love with him, and did not hesitate for his sake to poison her husband. On his marriage with her he ascended the throne, and had two sons, to whom on his deathbed he gave as separate heritages the kingdoms of Uganda and Unyoro. And thus Kabrega asserted, on the strength of tradition, his Galla origin. That emigrations of this people had passed through the land of Unyoro is proved incontestably by the use (still common) of words of the Galla language, especially in the articles of food."

Government.—The government of Unyoro is moulded on the feudal system. The king is supreme, and he has more personal authority than the king of Uganda, and is not under the control of hereditary chiefs. The country is divided into large districts, ruled by great chiefs called Mkungus, and they possess supreme power, having under them a number of Mtongali who rule over smaller areas of land. Each district must be represented at court either by the Mkungo or by a person appointed by him. They receive orders from the king as to the amount of taxation and the levying of men either for purposes of war or raid. The Makungo are either hereditary chiefs or receive the appointment from the king as a special reward. When a Mkungo dies, one of his sons is appointed in his stead; it does not, however, follow that the eldest son succeeds to the coveted post, for if a younger son makes the king a larger present he may obtain the position. The Makungo are limited in number, and they used to be free from capital punishment. Casati, however, relates that Kabrega now gets out of this difficulty by first degrading a Mkungo and afterwards executing him. The collection of taxes is accompanied by extreme extortion, and, from the great Makungo down to the lowest Matongoli, each official tries to gain as much benefit for himself as possible. The king is maintained in power by a band of soldiers called Banasura, who are greatly feared and disliked by the people; they reside with the king, and follow him about during his journeys.

Kabrega is said to be the 16th king since the original conquest of Unyoro. It is the rule for a new king always to sleep two nights to the east of the Nile, and then march into the country along the road which his ancestors are said to have taken. Although I knew Amfina and Rionga personally, I never had the pleasure or otherwise of meeting Kabrega, and I must therefore refer you to the writings of Emin, Casati, Junker, and Baker for a description of him, merely saying that he is said to be the son of a Wahuma shepherdess, that he is about 5 ft. 10 inches in height, has an extremely light complexion, and that he is described as being of a cowardly and superstitious character. Amfina, on the contrary, was a thoroughly polished man, of fine soldierly bearing, with a remarkably intelligent face; he was very suave and friendly, although of a somewhat grasping disposition. He had adopted many Arabic customs and dressed in an Arabic kaftan. His divan was overspread with Turkish carpets, and the seats and bolsters were covered with spotless linen. When I visited him, coffee and cigarettes were gravely handed round by smartly dressed boys, and one forgot for the moment that one was not paying a visit of ceremony to some Arab Bey. Rionga was a fine old man, quiet and unassuming. He had not the same force of character as Amfina, but he was a man who loved peace, and one who remained up to the last firm in his allegiance to the Egyptians, whose ally he had been ever since he made blood-brotherhood with Sir Samuel Baker.

Land Laws.—In Unyoro the whole of the land belongs to the reigning powers. The people are treated as serfs; they are fixtures, and change masters with the land.

The king grants lands as fiefs, not only to his favourites from whom he receives a fixed tax and also a certain number of men in time of war, but also occasionally to his favourite wives. Should a wife possess one of these districts, she is permitted to visit it occasionally. During her residence at the court she appoints an agent who rules the district in her absence, sending her either produce or its equivalent. The king's sisters, who by the way are never permitted to marry, have also land granted to them for their support; and the king's brothers, who are practically imprisoned in order to prevent them raising rebellion, have an appointed district which serves for their support.

Crimes.—Justice is administered in the first instance by the Matongoli. There is an appeal from them to the Mkungo, and finally to the king. Theft is punished by confiscation of cattle or women for the benefit of the person robbed; after numerous offences, the thief may have a hand cut off. Adultery is punished by a fine of four cows; both parties may be beaten, and rarely the wife is divorced. A murderer may be summarily put to death; should he escape, the chief of the district receives 8 or 9 cows and three sheep or goats from the murdered man's relations; he then searches for the murderer, who, if found, is executed. On the whole, however, the death sentence is rarely carried out, for, as Emin relates, Kabrega expressed the opinion that a dead man pays no taxes. Executions are performed either by beating the culprit to death with knob-sticks or by the lance. Casati relates that culprits are often secured with ropes in a kneeling position and killed by three blows on the temple. He also says that if anyone seduces Kabrega's women their eyes are put out. There can be no doubt that Kabrega's jailors torture their prisoners with the object of extortion. Trial by augury obtains, fowls being used for the purpose; the medicine-men cut them up alive, and decide as to the guilt or otherwise of the accused by the colour of the fowls' intestines. Another method of determining guilt is as follows:—If one man charges another of a crime which he denies, both drink a decoction of herbs, and whoever becomes drowsy is considered the delinquent. (Emin says the decoction is given to two fowls.) The stocks are also used for punishment, the culprits having one leg secured in a heavy block of wood; sometimes they are left to die in the stocks, at others a large fine may release them. There is no real criminal class in Unyoro, and were it not that the people are so often incensed by the brutality and extortion of the Matongoli, the police, and the Bonasura, there would probably be very little crime indeed. The administration of justice has not reached the high standard to which it has attained in Uganda, and the trials in the former country are not conducted with the dignity and order which obtain in the latter.

Slavery.—Slavery only exists to a limited extent in Unyoro. Of course slaves are captured from the surrounding district, and, as before mentioned, a good number are exported from the country, but in Unyoro itself the king and the great chiefs are the only ones who possess slaves. On the other hand, it must be stated that practically the Wanyoro themselves are all slaves, as they are under the complete control of those set in authority over them.

Burial Customs.—Mourning in Unyoro is indicated by shaving the head and by laying aside all ornaments. Graves are dug three or four feet deep: for women, in the court of the house, on the right-hand side of the door; for men, to the left of the door. The corpse is placed on its right side in an attitude of ordinary sleep. Young children are buried in the gardens near the huts (Emin mentions that the Wanyoro living near the Albert Lake bury their dead, irrespective of sex, in the centre of the courtyard). Over the grave a miniature hut is erected, in which tobacco, pipes, bananas, and beer are placed. Speke gives an extraordinary account of a Wanyoro woman whose twins died. She kept two small jars in her hut as effigies of the children, and milked herself into these every evening, continuing to do so for five months, lest the spirits of the dead should persecute her. He says that the twins were not buried, as ordinary people, underground, but were placed in an earthen beer-jar, taken into the jungle and placed by a tree, with the pot turned mouth downwards.

After the death of a great chief one of his finger-bones and his hair are preserved; should he have been shaven, a piece of his bark-cloth dress is kept instead of his hair. When a king dies, the body is washed, anointed several times with fresh butter, and wrapt in a bark-cloth; a special seriba is constructed, in the centre of which a high scaffolding is erected, and on this the body is placed. A fire is lit underneath, and kept burning until the body is completely dried up and mummified. It is then wrapped in a great number of bark-cloths and taken into a new hut specially built for the purpose, and there it must remain until the new king ascends the throne. As the sons of the deceased monarch usually fight for the throne, the burial is delayed for an indefinite period. At the conclusion of the civil war the victor visits his father's body and sticks his spear into the ground near the right hand of the corpse. His first duty after ascending the throne is to bury his father. Speke describes the funeral somewhat as follows. A huge pit is dug, capable of containing several hundred people, and it is lined with new bark-cloths. The lower jaw of the late king having been cut out, it is preserved in a packet richly ornamented with beads. His favourite cow is killed, and its hide forms the only coffin.

Several of the deceased king's wives seat themselves in the centre of the grave and upon their knees the body is laid. During the succeeding night it is watched by the king's bodyguard, and the next morning the soldiers seize indiscriminately a great number of people as they issue from their huts in the morning. They are brought to the grave, their arms and legs are broken by clubs, and, their shrieks being drowned by tom-toms, the blowing of horns, whistles, and the yells of the assembled crowd, they are thrown into the pit and covered with earth. This is trampled into a compact mass, and a mound of earth is erected above them.

Persons who have been executed are not allowed to be buried, but are simply thrown into the high grass.

Relationships.—The only relationships recognised in Unyoro are those of grandparents, father, mother, brother, sister, brother-in-law, uncle (father's brother, mother's brother), son, daughter, son-in-law, daughter-in-law, and step-father.

Distribution of Property.—When a man dies, the eldest son inherits all that his father possessed, including his wives, with the exception of his own mother. The younger sons each receive two of the father's wives, but no other property unless the heir choose; as a rule they do receive a small portion of the estate. Wives and daughters have no inheritance. If a man die without an heir his eldest brother inherits the property. At a man's death his daughters are brought up by the principal heir: failing an heir, the chief of the district takes his place, usually incorporating the daughters in his harem and annexing the property.

Marital relations.—As under ordinary circumstances marriage in Unyoro is practically a matter of barter, the father of a family of healthy girls is a happy man. A marriage is arranged by the fathers, who as a preliminary measure visit one another frequently, and over their pipes and ale extol the virtues of their respective son and daughter. No secrecy obtains, but the discussion is carried on in the presence of friends or casual guests. When the fathers have reached a certain point in their negotiations it is customary for the bride's father to visit the bridegroom's father, and to offer his daughter as a friendly gift to the bridegroom. So far good; but then the question of price comes in, and often many days pass and many long discussions take place before it is finally settled. Rarely, cattle form the chief part of the dowry, but amongst the general population beads, bark-cloth, hoes, spears, sheep, and goats are more in vogue. The bargain struck, it is clinched by a feast, to which relations and friends are invited. After the price has been paid to the bride's father, a week may elapse, and then the bride, escorted by her friends and preceded by music, is led to her future husband's hut, where a great feast is held, both families contributing to it. The bridegroom's father kills an ox, the hind-quarters of which are given to the bride's father. Three days after this night of revelry the villagers again assemble, and, after decorating the marriage hut, they spend another night in dancing and singing. As among so many other African tribes, on the sixth day the bride returns alone to her father's house, where she remains for a week before finally and permanently returning to her husband. As far as I know, there is no marriage-contract. Polygamy is permitted, each wife as a rule, and certainly among the better classes, possessing her own establishment. There are no prohibited degrees of relationship as regards marriage, and a man may marry anyone, even his sister or daughter, with the exception of his own mother. Marriages of necessity sometimes take place, or, to put it more euphemistically, love-matches. It is considered quite proper in Unyoro for a marriageable girl to visit any lover she may choose during the night. A lover, however, may not visit his sweetheart; at least if he does, he runs the chance of a good thrashing and the fine of a cow. If, as a result of this free love, pregnancy ensues, the girl's father takes her to her lover's hut and he must maintain her until the child is born. After the birth of the child, the father claims her and the child, but the young man may purchase them back, the price, according to Emin, being 6 oxen and 4 sheep. If the child alone is bought, a boy costs 1 cow and 4 sheep, a girl 4 sheep or goats. Should an unmarried girl die in childbed, the lover is either killed or he ransoms himself by the payment of 6 to 9 cows, an almost prohibitive price except among the higher ranks. If a woman is sterile she may be returned to her father, who refunds part of the price paid for her, but this is uncommon and against popular sentiment. Emin says "if a man marries and his wife falls ill and dies during a visit to her father's house, the husband either demands a wife (a sister of the deceased) in compensation, or receives 2 cows. When a poor man is unable to procure the cattle required for his marriage at once, he may, by agreement with the bride's father, pay them by instalments. The children, however, born in the meantime belong to the wife's father, and each of them has to be redeemed with a cow." Divorce occasionally takes place in Unyoro. Either one spouse leaves the other or the wife is divorced for adultery. The procedure in such a case is described by Emin as follows:—"The injured husband cuts a piece of bark-cloth in two, keeping half of it himself and sending the other half with the wife to her father. When the cows formerly paid as the price of the bride are restored, this piece is returned to the husband, who then burns both pieces."

Unyoro is the only Negro State in which prostitutes are officially sanctioned. Emin thus describes their regulation:—"In Kabrega's establishment a great number of girls live as servants to his wives. They are usually good dancers, or are distinguished by corporeal advantages, and enjoy unlimited freedom at night. They are called *Vranga.* As soon as their day's work is finished, they go out, and if they are addressed by a man, they go with him and remain at his house for four or five days, according to his wishes. It often happens that they follow a man who pleases them of their own accord and stay with him. He is bound to comply with their wishes, and to provide them with food, &c. Their reward consists of cowries, bark-cloths, dressed hides, and even slaves, according to the circumstances of the man they fall in love with. Should the reward fall below their expectations, they always appeal to Kabrega, who in most cases decides in their favour, although he derives no benefit whatever from them. All that they earn belongs to them, and should one of them amass a fortune she sets up a seriba of her own, and perhaps marries one of the king's slaves. Should one of them bear a child it belongs to the king as a slave. If it be a boy, he is placed later on among the pages (*Vagaraggara*), and when grown-up is enrolled in the bodyguard, always as a slave, but no reproach clings to him because of his illegitimate birth. If it be a girl, she is brought up to her mother's profession, also remaining of course a slave of Kabrega, who however comes into no personal contact with these women. The institution seems to be very old, and Kabrega told me that the first of such women were not Wanyoro." When I was told of this custom I was informed that Kabrega exercised the right of *jus primae noctis* with these women. They are sometimes permitted to go about the country in couples; they wear a distinctive dress, which I saw. If they quarter themselves upon any individual, they take precedence over his wives, and expect a good present. Apart from this institution, the Wanyoro may be said to be fairly moral.

Language.—In all probability, the original language of the Wanyoro belonged to the Bantu group. It differs, however, distinctly from Luganda. I was not in Unyoro long enough to learn much about it, but I know that it is very much corrupted, in the northern districts the language being very nearly akin to that spoken by the Wachopi, while in the south-eastern districts there is a certain mixture of Luganda. As Emin mentioned, it is very curious that the market names for articles are different from those in everyday use. (See Appendix.)

Grant remarks that the language spoken in Unyoro differs but slightly from that spoken at Karagwe. It was not, however, understood by his followers until they had been some time in the country, and Emin mentions that he found that some of the Negro Shuli living to the east of the Nile, north-east of Unyoro, understood the language with comparatively little difficulty. Emin and Stanley have published vocabularies of the Wanyoro language, and I give in the Appendix a specimen. It must be remembered, however, that these observers used a different system in writing down the sounds.

Magicians and Divinations.—In Unyoro there are many magicians or wizards, from the queen-mother, who is very celebrated, down to a kind of hermit, either male or female, who lives apart from human habitations, in some secluded forest glade. In nearly all cases the magicians wear a distinctive dress, variously contrived head-dresses of beadwork or feathers, and a profusion of charms, crocodiles' teeth, lions' claws, &c. Speke describes a magician who was supposed to find stolen or missing goods as follows :—" A rain-gauge had been lost, and this magician was brought to find it. The necessary adept was an old man, nearly blind, dressed in strips of old leather fastened to the waist, and carrying in one hand a cow's horn primed with magic powder, carefully covered on the mouth with leather, from which dangled an iron belt. The old creature jingled the bell, entered our hut, squatted on his hands, looked first at one, then at the other, inquired what the missing things were like, grunted, moved his skinny arms round his head as if desirous of catching air from all four sides of the hut, then dashed the accumulated air on the head of his horn, smelt it to see if all was going right, jingled the bell again close to his ear and grunted with satisfaction: the missing articles must be found. To carry out the incantation more effectually, however, all my men were sent for to sit in the open before the hut, when the old doctor rose, shaking the horn and tinkling the bell close to his ear. He then, confronting one of the men, dashed the horn forward as if intending to strike him on the face, then smelt the head, then dashed at another, and so on till he became satisfied that my men were not the thieves. He then walked into Grant's hut, inspected that, and finally went to the place where the bottle had been kept. There he walked about the grass with his arm up and jingling the bell to his ear, first on one side, then on the other, till the track of hyena gave him the clue, and in three or more steps he found it. Hyena had carried it into the grass and dropped it there. Bravo for the infallible horn ! "

Amulets and talismans are greatly in repute, and the Wanyoro believe that charms and incantations can render people ill, or even kill them. Speke mentions a magic powder which was supposed to give the owner the power of speaking with tongues.

Divination by the appearance of the entrails of fowls is greatly in vogue: sometimes the fowl's stomach is slit open, and according to the state in which it is found a prognostication is given; at other times the intestines are cleansed from blood and floated out in warm water. The magicians are consulted before a man may take a journey or before war, and, as before mentioned, to discover a criminal. Baker wrote—"When my wife was expected to die, the guide procured a witch, who had killed a fowl to question it whether she would recover and reach the lake. The fowl in its dying struggle protruded its tongue, which sign is considered affirmative."

Religious beliefs and Superstitions.—It is a matter of extreme difficulty, if not impossible, to give any idea as to the religious beliefs of the Wanyoro. Personally, I was not able to ascertain anything directly on the subject. Baker writes of the Wanyoro that the people had no idea of a Supreme Being nor of any object of worship, their faith resting upon a simple belief in magic; and again, Emin says "no trace is to be found of the idea of a future life." Yet, in the light of their legends, their superstitions and their stories, it is almost impossible to agree with these statements. Else why the votive huts, or why the belief that Kamrasi, the late king, has an influence upon the destiny of the country. It seems to me that these things point to the belief in a future life, and the existence of a Supreme Being is evidently not altogether a strange idea to them, as is proved by their legends of the Great Magician. The human sacrifices also to the Great Father and the distinct ideas of spiritual beings who overrule personal destiny all indicate that further research will elucidate some primitive system of religion. No one, I think, has at present investigated this matter sufficiently, but what follows, and for which I have chiefly to thank the writings of Emin and Casati, as well as some legends I collected myself, will at any rate serve to fix a foundation upon which future observers will be able to build.

In Unyoro there is a singular belief that certain men leave their huts at night and kill travellers in order to eat their flesh or employ it in various magic arts. They retain the human form, but, owing to magic power, cannot be caught, as spears and bullets do not wound them. Curiously enough, it is said, however, that long sticks affect them, and with such a stick it is possible for a man to drive them about until dawn, when they can be seen and recognised. In some parts of Unyoro it is thought that both men and women can at night take at will the form of a lion, a leopard, or a hyena, travel long distances, and also may kill and devour lonely wanderers. This reminds one strongly of a belief which obtains in Darfour, and to which I referred in my paper on that country. It would seem also to point to the fact that cannibalism may have been once practised in Unyoro. The following stories, one of which is very like one told in Uganda, illustrate this point :—

One night a man was coming home from hunting ; he had been a long, long way, and was belated ; losing his path in the dark forest, he was greatly distressed, and called out loudly, hoping to attract attention. A lion, which was in reality a man, heard him, and coming up, offered to show him the way home. Instead of doing this, however, the lion led his victim deeper and deeper into the forest. After a time, seeing no lights, and not coming to any habitation, he got more frightened, and asked the lion where he was leading him. "To my cooking-place," said the lion ; at which the man tried to run away, but the lion killed, boiled, and ate him.

Many years ago, two men rose up before it was light one morning, as they wished to go a long journey. They had to pass by the hut of a noted medicine-woman, and were rather shy of doing so. Just as they reached the glade in which the witch lived they saw a hyena run into the hut. At that moment the sun rose, and the beast changed into the witch. The men rushed forward and caught her, and when they had told their neighbours what had happened, she was condemned to death. She said before she died that she had eaten many folk, and she threatened the people with much ill. Her threats availed nothing, however, and they clubbed her to death.

Casati says it is believed that any warlike enterprise is destined by supernatural power to have a glorious end if a human victim be offered as a holocaust on the frontier of the enemy, and is neither insulted nor taken away from the spot by them. "A child of unknown parents, stolen in a distant country, who from his youth is supposed to be pure and uncontaminated, is buried up to his shoulders on the road which the army must follow. Along the route the scouts cautiously proceed, silent and watchful, always in fear of the danger of ambuscades. The column follows at a distance; they discover the little head of the innocent victim from afar; it leans on its shoulder; it is the head of a corpse, or, agonising, it still moves, uttering its last wailing. The child being still there, according to their belief, was a sure sign of victory, and this was confirmed in their minds by the fact that the enemy had not even put in an appearance on the road or expected an assault, whence they inferred that there was nothing to fear, and that ultimate victory was certain. . . . . . No one dares to succour the wretched child of propitiation, or even address a word of consolation, for fear of breaking the charm."

Casati also gives a description of the ceremony of the *mpango* which I must quote :—" To appease the favour of the defunct Kamrasi, resource was had to the ceremony of *mpango* or the axe. The articles for this rite are a drum, round which is wound a thick brass wire (it is also ornamented with talismans consisting of pieces of wood, to which are attributed special magical virtues), a wooden stool, covered with lion and leopard skins, an iron lance of about 5 feet in length, the handle surrounded with brass wire, and lastly an axe (mpango), with the haft of wood covered by leopard skin, and with brass wire on the upper portion. The day passed. The sun hastened towards the west—a blow was struck on the grand drum, deep and solemn. In a moment songs ceased, all sounds were hushed, the market was empty, everyone went to his own habitation, the roads were deserted, and for three long days silence and sadness reigned around. Only the slow dismal rings of blows struck at intervals on the great drum told that they were fulfilling the mysterious rites of the *mpango*, which caused the miserable inhabitants to shudder with fear. It is the popular belief that the *nuggare* (drum) sounds without being beaten whenever the angry spirit of Kamrasi wishes to be appeased by human victims. The period of the mysterious rites was passed, the sun approached the end of his journey, the great *nuggare* gave forth its deepest sounds, cries of terror mixed with reverence echoed everywhere, the miserable passers-by, the peaceful husbandmen, were seized, bound with cords, and their throats cut as a holocaust to the Great Father. In Juaya ten unhappy creatures were killed. But the sacrifice was not to be completed till the next day, when the king stood erect in the hut of the *mpango*, on the threshold of the large door of ingress, dressed in the traditional habit (a great mantle of stuff made from the bark of trees), surmounted by a leopard-skin hanging at his back and round his neck, his head crowned with talismans, his wrists, neck, and ankles ornamented with large glass beads, and holding his lance in his right hand. The members of the condo and all the nobles were arranged in a semi-circle in the great court sitting on their little benches. The guardian of the *mpango* stood at the right hand of the king holding high the fatal axe. *Nuggare* and a small chair belonging to the grand rite were placed in the front; a large cup was on the ground a little way off. Terror and silence rested on the assembly. The king made a sign with his head, the great men rose, and bowing, as a sign of reverence, approached him. He touched one of them with the point of his spear on the shoulder; the chief advanced and extended his neck, the horrid axe descended, the blood was caught in the cup, the king with his fingers sprinkled some on his own forehead and cheeks, then on those of the great men; grasping the vase, he poured the remaining blood on the drum and on the seat. The sacrifice was complete. Nuggare, seat, lance, and cup were raised and carried to the queen-mother. . . . The drums and fife sounded for a feast. They killed oxen, opened jars of beer, and the drunken people danced and enjoyed themselves upon the ground just bathed with the blood of the victims." The rite is sometimes prolonged to the fifth day.

In Unyoro, as among most African tribes, the appearance of a new moon is the signal for great rejoicing. As soon as she becomes visible, guns are fired, music commences to play, and a dance and carousal, which extend throughout most of the night, take place. When the new moon appears it is the custom for the magicians, from Kabrega downwards, to prepare their powders, their amulets and talismans, and during the first few days after the arrival of the new moon divination is practised. With the exception of Casati, no one seems to have known or heard of human sacrifices on these occasions, but he writes as follows:—"At every new moon human beings are sacrificed to propitiate the favour of the invisible powers and to ensure happiness. These immolations are not, however, distinguished by any particular display. Ordinary business is suspended for three days, and the rites of the new moon are performed inside the palace by the sacrificing of some victim of propitiation, and by killing out of doors, in the direction in which they wish to frighten away the evil eye, a variable number of individuals. They kill bulls on the king's tomb monthly as a sacrifice, and human victims are often added to them."

Should the king or any member of the royal family be ill, human sacrifices, according to Casati, play an important part in their treatment.

It is a mark of high distinction and of great trust to be admitted to the ceremony of the milk. The members of the royal family and the great chiefs do not enjoy such an honour. The performance of heroic deeds in war, unalterable fidelity to the king, and, still more, being in sympathy to him, are reasons which may admit men to this highest of all distinctions in the kingdom. Night having fallen and the king's tables being set, those invited to the ceremony enter the grand hall of the royal mansion, the drums beat, the fifes whistle the royal march, the king takes a vase full of milk, and then passes it to those present, who in turn drink also. When the ceremony is finished, the doors are opened and the friends of the great men are admitted to the daily entertainment of getting intoxicated on copious libations, the king setting the example.

Even in Kamrasi's time, as Baker mentions, the king's throne, composed partly of copper, partly of wood, was said to have been handed down for many generations, and was considered to be a talisman, as also was an ancient drum which was greatly reverenced. These two articles were jealously guarded by special soldiers and seldom used. It was said that should the throne be lost or stolen the authority of the king would wane.

In Unyoro the king is the great rain-maker, but he compels his subjects to make him large presents in seasons of drought.

The natives of Unyoro have a great dread of the snowclad Ruwenzori. The people believe that its mysteries are guarded by demons who would prevent travellers from penetrating its recesses or ascending its higher peaks.

The custom of making blood-brotherhood obtains in Unyoro. It is accomplished by the contracting parties making a few small incisions upon the arm, or sometimes over the fifth right rib, and then respectively licking off the blood.

Although suicide is not very common in Unyoro, still it does obtain, usually owing to a man thinking that his honour is forfeited.

On starting for a voyage on Lake Albert a small present is usually thrown into the water, either beads or food. A similar custom obtains in Uganda, but, so far as I am aware, the Wanyoro have no distinct god of the lake.

A Wanyoro will not, if he can help it, return home by the road he has previously traversed. It is unlucky for a jackal or a hare to cross the path.

The parings of finger and toe nails must be carefully preserved and thrown into the high grass.

The king is very superstitious in regard to his food. He does not eat poultry, and confines himself to veal boiled with bananas, telabone porridge, and banana beer fermented with germinating corn, which is called Muenga. His food is cooked by persons whose fidelity has been tested, and is closely covered to protect it from the evil eye. The chiefs have to conform to the king's restrictions in their food.

Women have the monopoly of a certain power of charming which consists in bewitching vegetable or animal food with their eyes and then giving it for food; the eater is immediately seized with violent pains in the stomach, which do not pass off until the charmer is brought and spits three times on the body of the sufferer.

The belief in the evil eye of both men and women is universal, and no means of protection against it exist.

The cutting of children's upper incisors before the lower is feared as bringing misfortune; when it occurs, the magician is summoned to perform dances for the protection of the child, and is rewarded by a goat.

If an owl screeches near the house, his master dies. If a hyena or a jackal repeatedly approaches the house, misfortune is at hand; when the rhinoceros-bird croaks, rain may be looked for.

If a wagtail sings on the threshold, guests or presents arrive; if a man kills wagtails in the house fire breaks out in it. If a wagtail forsakes its nest made in the house, misfortune is near. Vultures and ravens are chief among the birds, and their slaughter causes illness. If vultures alight on the top of a poor man's house, he will receive rich gifts and presents.

A piece of the hide of the white rhinoceros worn on the body makes a man invulnerable. Skin of the otter worn on the body acts as an aphrodisiac.

An eclipse of the sun announces the death of a ruler. The Wanyoro spit three times whenever they see a shooting star.

To Emin and Casati I am indebted for the following stories and traditions:—"In primeval times, say the Wanyoro, people were numerous on the earth. They never died, but lived for ever. But as they became presumptuous and offered no gifts to the 'Great Magician' who rules the destinies of men, he grew angry, and, throwing the whole vault of heaven down upon the earth, killed them all. But in order not to leave the earth desolate, the 'Great Magician' sent down a man and a woman 'from above,' both of whom had tails. They produced a son and two daughters who married. One daughter bore a loathsome beast, the chameleon, the other a giant, the moon. Both children grew up, but soon disputes arose between them, for the chameleon was wicked and spiteful, and at last the 'Great Magician' took the moon up to the place from whence it still looks down upon the earth. But to keep in remembrance its earthly origin it becomes large and brilliant, and then decreases as though about to die, yet does not die, but in two days passes through the horizon from east to west, and appears again, tired from its journey, and therefore small in the western sky. But the sun was angry with the new rival and burnt it, so that the marks are still visible on its face. The chameleon and its progeny peopled the earth, the tails were lost, and the originally pale colour of the skin soon became dark under the glowing sun. At the present time the heavenly spheres are inhabited by people with tails, who have many herds. The stars are watchmen which the 'Great Magician' posts during the night; the sun is inhabited by giants."

According to Unyoro tradition, elephants and chimpanzees were once men, and the dog, too, was gifted with speech, but spoke only to his master.

"In ancient times a man had an honest son, but he himself was violent, and had taken many cattle from his neighbours. Once upon a time he ordered his son to go and occupy a neighbour's house, and if he did not do so he threatened to kill him. The son went and slept in that house, but found in the early morning that the inhabitants had fled. He dared not return home, whilst by himself he would have starved; so he prayed the 'Great Magician' to rescue him, and was thereupon, together with the house, turned into an elephant."

"An honest man had an only daughter, and she was wooed by a neighbour for his son, who had turned out badly. The young couple lived happily for a short time, but when the young wife absented herself occasionally from the house to visit her parents, her husband reproached her with availing herself of this excuse to go after other men. Each day he treated her worse, so she fled and returned to her father, to whom she related her misfortune; and he, angry at the stain that had fallen on his own and his daughter's honour, killed himself. At this moment the son-in-law arrived, and was transformed by the 'Great Magician' into a chimpanzee. But the wife, who would not desert him in spite of all that had happened, followed him, and from them are sprung the chimpanzees, who still talk among themselves like men, and have a fondness for women."

"'I am better off than thou,' said the francolin to the tortoise; 'I can both walk quickly and fly.' 'I congratulate you,' replied the tortoise. 'I draw myself along, and do my business in the best way I can.' Now it happened that some men out hunting set fire to the grass of the plain. The increasing conflagration drew closer, and made the circle smaller and smaller. The peril of the two animals was imminent and certain. The tortoise drew himself into a deep wet hole made by the foot of an elephant and was saved. The francolin tried to fly, but the smoke and fire overpowered him, and he fell down and died. He who boasts, fails when tested."

"The leopard entrusted her three little cubs to the custody of the dog, assuring him that he should have as a recompense for his services plenty of meat, on condition, however, that he never gnawed the bones. The arrangement went on very well for some time, but one day the dog, yielding to temptation, gnawed a bone, a splinter flew from it, and striking a cub on the head, killed it. He found it easy to deceive the mother on her return by bringing the two survivors up to her to be fed one after the other, the first twice. Soon the same fate befell the second cub; then the dog, seeing that his fault must be found out, fled and sought shelter with a man, who promised to protect and defend him on condition that he did not leave the house. The dog promised, but a few days afterwards, seeing a heap of bones at a little distance, he broke his word and went out. The leopard, who had been in search of him for some time to avenge the death of her cubs, sprang upon him, killed and devoured him, and from that day the leopard has not ceased waging war upon dogs and eating their flesh."

APPENDIX.

[As my vocabulary of the Wanyoro language was very small, and as it was important in the face of present events to publish a vocabulary, I have obtained permission of the Directors of the "Berliner Gesellchaft für Anthropologie, Ethnologie und Urgeschichte" to publish a very full and interesting vocabulary compiled by Emin Pasha, and published by them. It has never been published before in England. I have to sincerely thank them for their courtesy.]

Abrus precatorius . . .	Burúnga.
Air	Vikoi.
Albino	Nàmagsj.
Amomum (divers species) .	Matúnguru.
Animal	Kissólo (pl. bissóli).
Answer	Kabáka.
Ant	Ukángo, ussóssisi.
White ant	Uisvá.
Ant-hill	kisvá.
Arachis hypogæa . . .	Mpándi.
Arm	Mukóno.
Left arm, l. hand . . .	Mukónno gu mossó.
Right arm, r. hand . . .	Mukónno gu bulió.
Arrow	Mfíndu (ngóbbe. Wahuma).
Ashes	Nkókke.
To ask (also to describe the praying of Arabs) . . .	Kulégga.
To ask	Kussébba.
I ask	Nssébba.
Thou askest	Ussébba.
He asks	Assébba.
We ask	Tussébba.
You ask	Ussábba.
They ask	Bassébba.
Ass	Nkäïna.
Aulacodus Swinderianus . .	Mssuéh.
To awake	Kuimúka.
To wake	Kuiméra.
Axe	Mpángo.
Back	Mabégga.
Banana plantation, plantation .	Lussúkku.
The banana is ripe . . .	Kitóke kíri.
The banana is unripe . .	Kitóke kibíssi.

The banana is rotten . .	Kitóke kíssi káïri.
Banana beer (fermented) . .	Muënge
Banana beer (unfermented palm wine	Ssándi.
Sorghum beer (unknown in Uganda and made in Unyoro out of Eleusine) . . .	Mérua.
Banana fruit	Kitóke (pl. bitóke).
Banana-tree	Kitóke (pl. bitóke).
Bark-cloth (with a black pattern)	Mtoné.
Basin, lake, river (seldom) .	Niándja.
Basket	Kíbo.
Bat	Mbúggu-mbúggu.
Batalas edulis (sweet potatoes)	Biáta.
To bathe	Kunába.
To bathe oneself . . .	Kuénaba.
Glass beads	Kuánsi.
Red glass beads . . .	Ssággama.
White glass beads . . .	Moëro.
Green glass beads . . .	Bkónge.
Sky-blue glass beads . .	Gállama.
Beard	Mlídju.
Beard	Mlísu.
Beautiful, good . . .	Burúngi.
Because	Alláï.
Bed (stretcher) . . .	Ntábbu.
Bee	Udsóki.
Where hast thou just been ? .	Uá noláï.
Behind	Nío.
Bellows	Nsi.
Belly	Ndá.
Big, high	Mkúrru.
To bind	Kubbuá.
Bird	Njúnni.
The bird sings	Njúnni aksiolía.
Bitter, salty	Kissára.
Black	Merágusu.
Blatta orientalis . . .	Njénge.
Blind	Pimpíte.
Blood	Ssággama.
Blood-brother	Mfúmbiro.
Boat, ship	Mváto.
Body	Mobirri.
Bone	Kúgulú (pl. mágulu).
Bones	Gúmba (pl. magúmba).
Borassus Æthiopum . .	Betúgu.
Where were you born ? . .	Uá uéraï.
Bosom	Mawéri.
Bow	Kittá, buttá.
Box	Terekere.
Boy	Módjo.
Brass	Muhéri.

Bracelet	Nsóïre mukónno.
To break	Kuhénda.
Breakfast	Tuikuíre múnkoko.
Breast	Kissúbba.
I breathe	Kaokéra.
To bring, to give . . .	Kudéta.
Bring ! give!	Ndéta.
Brook	Kággera.
Brother	Mugánda.
My brother	Mugandawánga.
Thy brother	Mugandaáue.
His brother	Mugandawé.
Buceros (a species of) . .	Waholú.
Buffalo	Mbógo.
Bufo pantherinus . . .	Kikkerä.
Butter, oil	Mégitta.
To buy	Kúguea.
Calf	Njáuna.
I came	Ndsidsé.
Thou camest	Udsé.
He came	Adsé.
We came.	Tudsé.
You came	Udsé.
They came	Vadsé.
Canǎbal	Valiabántu, mlúggu.
Canavalia sp.	Nssórro.
The capital (town) . . .	Miboga.
Cat	Líssusi.
Cercopithecus (a species of) .	Nkúnda.
Cercopithecus griseo-viridis .	Nkénde.
Chameleon	Waisselokótto.
Cheek	Itámma (pl. matámma).
Chicken	Mussénje.
Chief, district governor, leader .	Makóngo.
Subordinate chief . . .	Matóngali.
Child	Muánna (pl. bavánna).
This woman has five children .	Mukkálli ávi äïre bavánna bĕtana. (wife thine has five children).
Chin	Kirésu.
Clay water-pot . . .	Nssúa.
Clean, neat	Mséro (lit. white, clean).
Clever	Magési.
Cloth	Mienda.
Clothes (stuff) (European) .	Dúbbugo wusúngu (clothes of a European).
The clothes are torn . .	Lúggoï témkire.
Cloud	Bitjú.
Coal (glowing and cold), wood-coal	Makkalá.

Cock	Mpánga.
The cock crows . . .	Nkóko kárraï.
Coffee	Muánni.
Coffee-tree	Ujamuánni.
Cold	Mpéo.
The cold, cold, it is cold . .	Mpéa, malómbe.
I am cold	Ndenempéo.
Collar	Ngíssa.
Colobus Guerza . . .	Ngéje.
Colocassia Antiquorum . .	(Only in South Unyoro) djúni.
Come, return	Idjá.
To come, to return . . .	Kídja.
I come	Nssidja.
Thou comest	Údja.
He comes	Adja.
We come	Túdja.
You come	Údja.
They come	Bádja.
Come later	Mbéuidjá.
I will come	Nádja.
Thou wilt come . . .	Unádja.
He will come	Anádja.
I will not come . . .	Tendíra.
I am content	Nessimere.
Cook	Mfúmbiro.
To cook	Kussúmba.
Copper	Kikómo.
To cut ripe corn . . .	Kussára.
Corvus scapulatus . . .	Tjikóna.
Cosmetornis Spekii . . .	Waibottó.
Cotton	Ngojé.
To cough	Kukuóhla.
To count	Kubálla.
Country	Ensi.
Cow	Nté.
Crocodile	Ssámbi.
To cry	Kotschúrra.
To cry	Kutschúrra.
To cut	Kuttäna.
To cut (with a knife) . .	Vuóggi.
Cypræa moneta . . .	Ssímbi.
Damp	Kisóberi.
To dance	Kubílla.
It is dark	Muirímma.
Darkness	Liala ula.
Daughter, girl	Mriánna.
Day after to-morrow . .	Luábirri.
Day (space of time) . . .	Runáku.
Day (as opposed to night) .	Mssánna.

Day before yesterday . .	Isaéri.
Dead	Iwafwa.
He is dead	Affíri.
Deaf	Kígara.
Dear (expensive) . . .	Agúmma.
To pay a debt	Kussassúra.
Delicate, weakly . . .	Muanúki.
Dew	Duménj.
To die	Kuffá.
Dioscoræa alata (Kisuahviasiku)	Biráï.
Dioscoræa (Helmia) bubbifera .	Makíngo.
Dirty	Kuirágura.
Dog	Mbuéne.
The dog barks . . .	Mbuéne Koigólla.
The dog bites	Mbuéne kumenna.
What are you doing? . .	Ukóla kí.
My dollar is lost . . .	Rialikáuge kanguíre.
To dress oneself . . .	Kusuála.
Dream	Ndosireh.
To dream	Kuróta.
Dressed goat skin . . .	Tajúmba
,, ,, ,, (from Uganda)	Buéra.
Give me a drink . . .	Tué tunjuë.
To drink	Kúnnua.
I drink	Únua.
Thou drinkest	Únnua.
He drinks	Annua.
We drink	Túnnua.
You drink	Únnua.
They drink	Bánnua.
Drink	Núa.
Drinking-tube	Dussaká.
Long drum	Mugédja.
Short drum	Ngómma.
He is drunk	Attamíre.
To dry	Kuúma.
Dry	Kiómere.
Dumb	Kibóbo.
Ear	Kutuí (pl. mattuí).
To eat	Kulía.
I eat	Ndía.
Thou eatest	Ulía.
He eats	Alía.
We eat	Tulía.
You eat	Ulía.
They eat	Balía.
Eave (over the door) . .	Kissákki.
Egg	Máŭle.
Elbow	Lukkogola.

Elephant	Ngédju.
Eleusine coracana	Buí'ta.
Empty	Udomukántu.
To end, ended	Kumára.
Enemy	Ujajána.
I have enough	Biná mari.
Entada sudanica	Mojóra.
Epilepsy	Nssimbo.
European, white man	Mŭsúngu (pl. wasúngu).
Eye	Lissó (pl. massó).
Evening meal	Okulía igóllo.
Face	Maïsso.
To fall	Kugguá.
Family	Lugánda.
Farm	Hanimiro.
Fat, thick, big	Mukóto.
Father	Oité, táta.
Father's brother (uncle)	Mugánda uá táta.
Is your father alive?	Viteo mómi (thy father living?).
Ficus sp. diff. for manufacture of cloth	Mtóhma.
To find	Kubbaniá.
Finger	Luála (pl. biála).
Finger-ring	Kátam (arabic).
Fire	Muró.
To make fire, to light a fire	Kuákja muró.
To put out the fire	Kukóllesa muró.
The fire burns	Muró guáka.
The fire has burnt me	Muró gúndjo kérrije.
Fish	Inquí.
Fisher or boatman	Balímba.
Fishing-hook	Ndóbbo.
Fist	Intómi.
Flea	Bunira.
Flea	Nssuéhra.
All my people fled	Bantuwánge bona vabúr (people mine all fled).
Flour	Buro.
Flute, whistle	Njámberi.
Fog	Kjohó.
Food	Kulía (lit. to eat).
Foot	Kiggerä (pl. biggerä).
Forehead	Bussió.
Friend	Munjuáni (pl. banjuáni).
Fruit	Bía mtíh.
Frumenty (Korn-brei)	Mkáti.
Full	Kisuíre.
Garden	Mssírri.
Giraffe	Ntwíga.

Give!	Mpá.
To give	Kuería.
I give	Nagábba.
Thou givest	Uagábba.
He gives	Jabágga.
We give	Tuagábba.
You give	Ugábba.
They give	Vagábba.
Go!	Génda.
We go	Tugénda.
You go	Ugénda.
They go	Bagénda.
To go	Kugénda.
I go	Ngénda.
Thou goest	Ugénda.
He goes	Agénda.
To go, to creep . . .	Kutámbola.
Goat	Mbulí.
Gold	(Unknown).
Good, bad	Murúngi-múbbi.
Good, well (adverb) . . .	Otió.
Good-day	Mssánna. answer-mssánna.
Good-evening	Geróba. answer-geróberi.
Good-morning	Raïróte. answer-daabánte.
Good people	Bántu varúngi.
Good things	Bíntu birúngi.
A good thing	Kíntu kurúngi.
Gourd	Múngo
Gourd-jar	Kissessí.
Grandfather	Djádja.
Grandmother	Djeidja.
Grass-hopper	Nzíge.
Grave	Kusíka.
Great lord, title of ruler . .	Mukámma.
I greet the highest . . .	Ngúnsono káli (to Kabrega).
,, ,, . . .	Ngúnsono díri (to Rionga).
,, ,, . . .	Ngúnsono bóki (to Amfina).
To greet	Kulamíkje.
Greeting after a visit . .	Nkuába. answer-rámmi.
Greeting (in the street) . .	Merémbe.
The ground	Btákka.
Guinea-fowl	Nssóllomi.
Gun	Bendúki.
Hail	Mebáli.
Hair	Issokí.
His hair is grey . . .	Issokí rimu émbui (Hair become grey).
Hammer	Nssámmu.

Hammer	Njóndo.
Hand	Biála.
Hard	Kigúmmu.
Harp, guitar	Ntóngoli.
Harp-string	Ugojé.
Hay	Ssúbbi.
Head	Mtué.
Back of the head	Nkóla.
Headache	Mtué vukúnduma.
Healthy, lively, strong	Muómi.
To hear	Kuúra.
Heavy	Kikuremérrá.
He-goat	Mpánja.
Hen (also used for cock)	Nkoko.
Hide	Haru.
High, long	Meláï.
Hill	Akasosyi.
Hippopotamus	Mbírsi.
Hoe	Mssikáh.
To hoe	Kullíma.
Hollow, hole	Kjúlu.
Honey	Djúru.
Horn	Dochombe (pl. madjémbe).
Horn, magic, amulet	Dschémbe.
Horn, trumpet	Kigguára.
Hot	Okókja.
House	Njúmba.
House door	Mliángo.
Hump-back	Bángo.
Hunch-backed	Vibángo.
I am hungry	Dinansálla.
To hunt	Kuígga.
Hunter	Muíggi.
Hyæna (H. crocuta)	Mfittíh.
Hystrix cristata	Njamanónto.
Ill	Mluáïri.
Iron	Tjúma.
Island	Kisínga, uyamési.
Jackal	Buá.
Ja-nein (not, never)	Jé-ngáine.
Jar	Tuágga.
Journey	Lugéndo.
To jump	Kugulka.
Kitchen	Fúmbiro.
To kill	Kuíta.
I kill	Natta.
Thou killest	Unátta.
He kills	Anátta.
We kill	Tunátta.

You kill	Unátta.
They kill	Vanátta.
To kill a goat	Kussálla mbúsi.
King	Kabákka.
The king's life-guard . .	Banassúra.
A king's son	Mulángere.
Knee	Kadjiví.
To kneel	Kukubamádjua.
Knife	Mujóh.
Knowing, intelligent, capable .	Namánj.
Lamb	Kanakatámma.
Lame	Mulémma muugaré.
Land	Kjálo.
Later	Manolleki.
To laugh	Kuendía.
Lay thee, lie down . . .	Kukubássia.
Lead	Lissáhssi.
Leaf	Dibbábi.
lean, thin	Aïanúkkerä.
Leg	Ntúmbuë.
Leopard	Ngóï.
A liar	Mugobbia.
To lie	Agóbbia.
Lion	Ntalí.
Lip	Munuá (pl. minuá).
Little, few	Kadóli.
Little girl	Kaála.
Liver	Iníh.
He still lives	Atjáli mómi (still living).
Lizard	Kigárra-gárra.
To lose	Kublía.
Louse	Maddá.
To love, to wish, to want .	Kuendía.
Low, short	Múmpi.
Lutra sp.	Ngónge.
Madness	Ilalú.
Mallet (for making bark cloth)	Nssámmu.
Man	Mssédja.
The fine man	Múntu murúngi.
An old man	Mumbué.
Mane	Nssínga.
Manihot utilissima (Kisuahili mahogo)	Lumónge mkállu.
Manis sp.	Ndúmmi.
Many	Bíngi.
Market	Kátali.
To marry	Kubáudoba.
Unmarried, widower, poor, forsaken, orphan, orphaned .	Munáku.
Plaited mat (from Uganda) .	Mkéka.

Tokplait mats	Kulúka mkéka.
Meal	Bussiáni.
Meat	Njámma.
Medicine	Móbbasi.
Men, people	Múntubántu.
Merchant	Mtúnsi.
To send a messenger	Kutúmma mkuénda.
Mid-day	Mujángue.
Mid-day meal	Tulíre mssánna.
Midnight	Muttúmbi.
Milk	Mattáï.
Mist	Kjohó.
Moon, month	Kuéhsi.
New moon (lit. moon newer)	Kuéhsi kudjá.
Morning (time)	Múnkoko.
Mosquito	Mébbu, nssúna.
Motacilla vidua	Njamakóngo.
Mother	Máma.
Mountain	Lussósi, rossósi.
Mouse	Mbéaba.
Mouth	Mummá.
Mud	Ssábu.
Murder	Mutémmu, muíti.
Musa Ensete	Kitémbe.
Mushroom	Búttosi, bkolío.
Nail	Nónno.
Nail (wood or iron)	Duíndu.
Naked	Buäre.
What is his name?	Ibara nue náni. (name his is what?)
Navel	Nkúndi.
Near	Maámpi.
Neck	Vikjá.
Needle	Nkínso.
Neighbour	Kullirána (pl. ballirána).
Net (for wild animals)	Kitímba.
New	Mojá.
Nicotiana rustica	Irkábuë (old word).
Nicotiana virginiana	Tába.
Night	Múkero.
Nightmare	Guppe.
No	N'ga.
Nose	Jendó.
Now, at once, soon	Átti.
Numbers.	
1	Tímmoi.
2	Bíbiri.
3	Bssáttu.
4	Bináj.
5	Btána.
6	Mukága.

Numbers—*continued.*

7	Mssánjo.
8	Muána.
9	Muénda.
10	Íkomi.
11	Íkomi na tímmoi.
12	,, ,, bíbiri.
13	,, ,, bsáttu.
20	Mákomi äbiri.
21	,, ,, na tímmoi.
22	,, ,, ,, bíri.
23	,, ,, ,, bsáttu.
24	,, ,, ,, binàj.
25	,, ,, ,, btána.
30	Mákomi assáttu.
40	Mákomi ánaj.
50	Mákomi attáhn.
60	Nkága.
70	Nssánjo.
80	Kenána.
90	Kijénda.
100	Igánna.
101	,, mu tímmoi.
102	,, ,, bibíri.
110	,, ,, miíkomi.
111	,, ,, ,, na tímmoi.
112	Igánna miíkomi na bíri.
120	,, muábiri.
130	,, muássatu.
140	,, muánaj.
150	,, muattáno.
160	,, munkága.
170	,, mussánjo.
180	,, mukinána.
190	,, mukijénda.
200	Magánábiri.
201	Magán ábiri natímmoi.
300	Magán bssáttu.
400	,, anáj.
500	,, atáhn.
600	,, mkága.
700	,, kssánjo.
800	,, kinána.
900	,, kijénda.
1000	Kássirísa.

To obey	Kuikíria.
Oh, there	Ué-táta.
	Thou father.
Old	Mukáïse.
He is old; old man . . .	Mukaïri.

To open	Kundúbula.
Oracle	Madúdu.
Ostrich	Údu.
Owl	Nssíndissi.
Owl	Nssíndissi.
Ox	Nommi.
Paddle	N'kassi.
Pain	Bruaïre.
Palace	Kjikáli.
Pavian (large, reddish) . .	N'kobbe.
Phacochœrus Æliani . .	Mberége.
Phaseolus Mungo . . .	Utógo.
Phaseolus sp. diff. . . .	Unverángo (large), nkóli (small).
Pipe-bowl	Njúngu.
Pipe-stem	Dussäkä
Plaited	Mabalá.
Plain	Kjéa.
To play	Kújánja.
Pool	Viasero.
Poor	Munáku.
Powder	Bugánga.
Prostitute	Oránga or vránga.
Psittacus erythacus . . .	Túkku.
Python Africanus . . .	Nsrjámr je.
To question	Kubulía.
Quick	Mángu.
Be quick	Jángua.
Rain	Indjurá.
It rains	Indjurá guire.
The rain has ceased . . .	Indjurá káïre.
Ram	Íhrmi.
Ready	Muéndi.
Red	Mutukuli.
Red pepper (capsicum conicum)	Kamráli.
Rhabdogale	Kovissímba.
Rhinoceros	Pióko.
Rib	Mbádju.
Rich	Mutúngi.
Ricinus communis . . .	Kissóga.
Right, true	Masímma.
Rind, shell (egg) . . .	Kjáï.
River	Muígga, kijámbo.
Road, path	Muhánda
Rock	Massádju.
Roof	Kusseráka.
Pointed roof	Kitíkro.
Root	Mssï.
Rope, string . .	Mugguá

To rub oneself with fat	Kuésiga mégitta.
Rudder	Ngáï.
Run	Eyeruka.
To run	Kuirúkka.
Saccharum officinarum	Bikáïdju.
He has said	Avádse.
I say	Uvása.
Thou sayest	Uvása.
He says	Avása.
We say	Tuvása.
You say	Uvása.
They say	Vavása.
What sayest thou?	Ugámba ki?
What hast thou said?	Ugámbje ki
Saliva	Malússu.
Salt	Múnju.
Sand	Mssénje.
I am satisfied	Nikutré.
Scales (fish)	Bigámba.
Rainy season	Wandjúrra.
To see	Kuvánna.
I see	Nvánna.
Thou seest	Uvánna.
He sees	Avánna.
We see	Tuvánna.
You see	Uvánna.
They see	Vavánna.
To seek	Kumónja.
To seek	Kumónja.
To sell	Kutúnda.
Sesamum orientale	Makjándi.
To sew	Kuvassíra.
Sharp	Bógi.
To shave	Kugérnba.
Sheep	Ntámma.
Shield	Ngábbu.
Shoes (red arabic)	Nkáïto.
Shooting-star	Kjivanémue.
Shoulders	Kibegga-begga (sing. ibegga).
To be silent	Kussílika.
Silver	(Unknown).
To sing	Kubínna.
Sister	Njakáïtu.
To sit, to stay	Kukára.
Sit, stay!	Ikkára (kutúdda. Wahuma).
Sit down	Ikkára vánssi.
Skin	Udíbba.
Skins worn as clothing	Kssáttu.
Woman's skirt made of bark-cloth	Mabúgo.
Skull	Kjiáuga.
Sky	Íguru.

The sky is clear . . .	Iguru tukvíri.
Slave	Muíro.
To call a slave	Kuíta múddu.
Sleep	Bássia.
To sleep	Kuebássia.
Slowly	Mpólan.
Small	Kadóli.
Small, narrow	Uaffúnda.
Smallpox	Blúndu, kulúndu.
Smith	Múehssi.
Smoke	Muíka.
Snail, mussel	Nssónko.
Snake	Udjoká.
The snake creeps . . .	Udjóka akuáwa.
To sneeze	Kuffulia.
Snout (lit. hand) . . .	Mukónno.
Soap	Ssabún.
Soft	Kikórobá.
Son	Mtabánu.
Sorghum sacchartum . .	Maïsia.
Sorghum vulgare . . .	Mógussa.
Sour	Nuégera.
To sow	Kussígga.
To speak / To talk	Kugámba.
I talk	Ngámba.
Thou talkest	Ugámba.
He talks	Agámba.
We talk	Tugámba.
You talk	Ugámba.
They talk	Bagámba.
To speak, to say . . .	Kuvása.
Spear, lance	Issomú.
Spear (elephant) . . .	Kidikía.
Spider	Uabúhbi.
Spoon	Ngámba.
Spring, natural reservoir . .	Kíssiba.
Stars	Njii yéhsi.
Stay with me	Túla nánge.
To steal	Kuíba.
Stepfather and brother-in-law .	Mukkóï.
Stick, staff	Míggo.
Stocks	Todu.
Stone	Ivari.
Stone	Kabáli.
Stool	Kitébbe.
Take the stool, take the stool away	Tuála ktébe.
To tell stories	Tukobádsa.
Stranger	Mŭgénj.
To strike	Kúkuba.
Stupid	Mssírru.
Suckling (babe) . . .	Káhua.

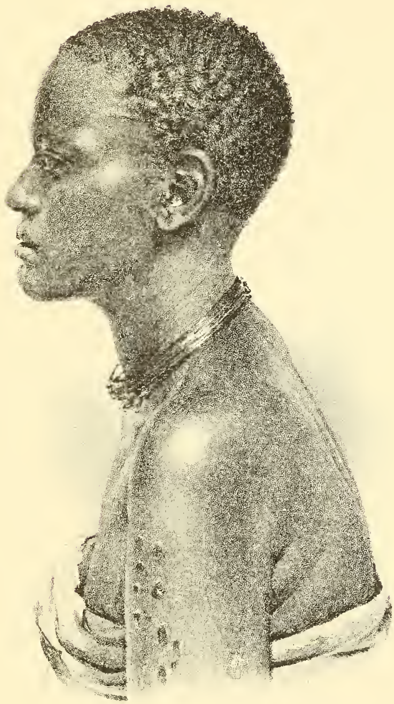
Sun	Issánna.
The sun is risen . . .	Issánna déulkré.
The sun is set	Issánna duguíre.
The noon (sun) is hot . .	Mujángue (issánna) riángri.
Time before sunset . . .	Waigóllo.
Time just after sunset . .	Gorúnmai.
Swallow	Utáï.
Sweat	Ntújo.
Sweet	Kussáï.
Sweet food	Kulía kussáï.
Syphilis	Kabrévendjú.
Tail	Mukkíra.
To take	Kutuála.
Tamarind tree . . .	Mkógi.
Tear	Malígga (pl. balígga).
Title for the first chief . .	Ngúnsono káli.
I thank (you)	Génda kubássia.
Thanks	Vébbali or ukuebassa.
Thief	Muíbi.
Thigh	Kibäro.
Thin	Budóli.
I am thirsty	Dinmáïro.
Thorn	Éïva (pl. mavvá).
Throat	Kadangíddo.
Thunder	Inkubbá.
To smoke tobacco . . .	Kúnnua tába.
To-day	Eréro.
To-morrow	Mkjá.
Will you come to-morrow? .	Olirá isó.
Tongue	Lulumí.
Tooth	Líhno (pl. máïno).
Toothache	Linío vukúnduma.
Tragelaphus scriptus . .	Ngábbi.
Tree	Mssáli (pl. kissáli).
Fallen trees	Kutáma kissáli.
Troglodytes spec (chimpanz) .	Kinjabántu (lit. manlike).
Turtle	Inkudú.
Tusk, ivory	Ssánga (pl. massánga).
Twins	Balóngo.
The firstborn (twin) . .	Nssínyoma.
The secondborn (twin) . .	Káto.
Ugly, bad	Múbbi.
Mother's brother (uncle) . .	Mugánda uá máma.
To urinate	Kunjára.
Venus	Njánsi jakué hsi (the moon's love).
Village	Kjika (múkka. Wahuma).
Voandzeia subterranea . .	Mpándi.
Vulture	Nségga.
Walk	Kuunga.
It is warm	Attagáttiri.
I am warm	Utomíre.
To wash	Kunawía.

To wash oneself . . .	Kuënawia.
Water	Maési.
Give me a little water . .	Ndéta maési kadóli (Give me water little).
High water	Maési gángi.
Low water	Maési kadóli.
The water boils . . .	Maési gattgaáttere.
The water is cold . . .	Maësi gakfuká.
The water is warm . . .	Maësi gákua kji.
Wilt thou shew me the way .	Ononjoléko kúbbu.
Week	Mssánjo.
Weir-basket for trapping fish .	Kuvénde, kijibénde.
He is well	Mussáï.
He went	Génsere.
West (lit. set of sun) . .	Bugguá.
Wind	Mujágga.
To wind	Kúmira.
Witch, wizard	Mbándua.
Whirlwind	Kísimu.
White	Moëro.
White pearl	Njáduë.
Woman	Mukkási (pl. bakkási).
The good woman . . .	Mukkási murúngu.
Many women	Bakkási, bangi.
Wood	Lukkuí (pl. nkuí).
Split wood	Kuattiá lukkuí.
Woodpecker (Picus badius) .	Nkónkona.
Wool, feathers	Vója.
Word	Kigámbo (pl. bigámbo).
To work	Kukóla.
Worm	Kiniángoro.
To write	Kukóla (to work).
Yard	Rúbuga.
Year (of 5 months) . . .	Muáka (pl. miáka).
Yes	Niho.
Yesterday	Isó.
He is still young . . .	Atjáli.
Zea Mais	Bitjóli.
Zebra	Ntlégge.

Emin found that at the market at Kabrega's capital, certain words were used which were not those usually employed in every-day language. He gives the following examples :—

	Usual Word.	Market Expression.
Banana	Bitoke .	Kohenda.
Banana beer	Muenge .	Viakouga.
Bark-cloth	Mbugu .	Kisseko.
Butter, oil	Megitta .	Djuru.
Coffee	Muanni .	Ndivua.
Meal	Bussiani .	Kabumba.
Meat	Ujamma .	Kjanjoa.
Salt	Munju .	Rengua.
Spear	Issomu .	Karamanjaso.
Sweet potatoes	Bjata .	Rumomoro.

DR. FELKIN ON THE WANYORO TRIBE.



SCOTT & FERGUSON EDINR

Note on the Division of Space into Infinitesimal Cubes. By Professor Tait.

(Read December 5, 1892.)

The proposition that "the only series of surfaces which, together, divide space into cubes are planes and their electric images" presented itself to me twenty years ago, in the course of a quaternion investigation of a class of *Orthogonal Isothermal Surfaces* (*Trans. R.S.E.,* Jan. 1872). I gave a second version of my investigation in vol. ix., p. 527, of our *Proceedings.* Prof. Cayley has since referred me to Note vi., appended by Liouville to his edition of Monge's *Application de l'Analyse à la Géométrie* (1850), in which the proposition occurs, probably for the first time. The proof which is there given is very circuitous; occupying some eight quarto pages of small type, although the reader is referred to a Memoir by Lamé for the justification of some of the steps. But Liouville concludes by saying:—"l'analyse précédente qui établit ce fait important n'est pas indigne, ce me semble, de l'attention des géométres." He had previously stated that he had obtained the result "en profitant d'une sorte de hasard." As Liouville attached so much importance to the theorem, and specially to his proof of it, it may not be uninteresting if I give other modes of investigation. The first of them is merely an improved form of what I have already given in our *Proceedings;* the second (which is the real object of this note) seems to have secured nearly all the advantages which Quaternions can afford, in respect alike of directness, clearness, and conciseness. It is very curious to notice that much of this gain in brevity is due simply to the fact that the *Conjugate* of a certain quaternion is employed along with the quaternion itself in my later work; while I had formerly dealt with the reciprocal, and had, in consequence, to introduce from the first the tensor explicitly. The investigation should present no difficulties to anyone who has taken the sort of trouble to remember elementary quaternion formulæ which every tyro in integration has to take to fix in his memory the values of $d \tan x$, $d \log. \frac{x+\sqrt{x^2-a^2}}{a}$, or $d \tan^{-1}x$, &c.

The only peculiarities of the question seem to be due to the contrast between the (apparently) great generality of the initial equation and the extremely restricted character of the sole solution. This will be abundantly evident from the discussions which follow, since it would almost appear as if the conditions arrived at were too numerous to be simultaneously satisfied. I find it very convenient to use a symbol ∂ $\left(\text{in the sense of } \frac{d}{dx}\right)$ to express rate of increase per unit of length. Thus

$$\nabla = i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz},$$

may be written

$$\nabla = \alpha\partial_1 + \beta\partial_2 + \beta\partial_3,$$

where α, β, γ are any rectangular unit-system.

The equation

$$d\sigma = uq^{-1}d\rho q, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

(where u is a scalar, and q a versor, function of ρ) ensures that an element of space at σ corresponds to a *similar* element at ρ; so that the transformation from ρ to σ, or *vice versâ*, is from one mode of dividing space into infinitesimal cubes to another. [From the purely analytical quaternion point of view the question may be regarded as simply that of finding u and q as functions of ρ, so that the right-hand member may be a complete differential.] We have at once

$$S\alpha d\sigma = -Sd\rho\nabla . S\alpha\sigma = uS.d\rho q\alpha q^{-1}$$

whatever constant unit vector α may be. Thus

$$-\nabla S\alpha\sigma = uq\alpha q^{-1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

A part, only, of the information given by this is contained in

$$V\nabla . uq\alpha q^{-1} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or

$$\begin{aligned} \mathrm{V}.qaq^{-1}\frac{\nabla u}{u} &= \mathrm{V}\nabla.qaq^{-1} \\ &= \mathrm{V}.(\nabla qq^{-1}qaq^{-1} + qaq^{-1}\nabla qq^{-1}) - 2\mathrm{S}.qaq^{-1}\nabla_1.\mathrm{V}q_1q^{-1} \\ &= 2qaq^{-1}\mathrm{S}.\nabla qq^{-1} - 2\mathrm{S}.qaq^{-1}\nabla_1.\mathrm{V}q_1q^{-1}. \end{aligned}$$

From the sum of the three equations of this form (each multiplied by its qaq^{-1}) it appears at once that

$$\mathrm{S}.\nabla q\, q^{-1} = 0\,; \qquad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

so that, as qaq^{-1} may be *any* unit vector,

$$\begin{aligned} \mathrm{V}.\alpha\frac{\nabla u}{u} &= -2\mathrm{S}\alpha\nabla_1.q_1q^{-1} \\ &= 2\partial_1 q\, q^{-1}. \qquad . \quad . \quad . \quad . \quad . \quad . \quad (5) \end{aligned}$$

From two of the three equations of this form we have

$$\partial_2\left(\mathrm{V}.\alpha\frac{\nabla u}{u}\cdot q\right) = \partial_1\left(\mathrm{V}.\beta\frac{\nabla u}{u}\cdot q\right);$$

and this, by means of (5), gives

$$-\mathrm{V}.\gamma(\alpha\partial_1 + \beta\partial_2)\frac{\nabla u}{u} = \frac{\nabla u}{u}\mathrm{S}\gamma\frac{\nabla u}{u},$$

or

$$\gamma\nabla\frac{\nabla u}{u} - \partial_3\frac{\nabla u}{u} = \frac{\partial_3 u}{u}\,\frac{\nabla u}{u}. \qquad . \quad . \quad . \quad (6)$$

There are, of course, three equations of this form, and they give by inspection

$$\frac{1}{u}\nabla\frac{\nabla u}{u} = \alpha\partial_1\frac{\nabla u}{u^2} = \beta\partial_2\frac{\nabla u}{u^2} = \gamma\partial_3\frac{\nabla u}{u^2} = \frac{1}{3}\nabla\frac{\nabla u}{u^2} \quad . \qquad (7)$$

The first and last of these equals give

$$\nabla^2(u^{\frac{1}{2}}) = 0\,,$$

whose general solution is known to be

$$u^{\frac{1}{2}} = \Sigma\frac{m}{\mathrm{T}(\rho - \epsilon)},$$

where m and ϵ are constants. The other members of (7) show that *one* term only of this Σ is admissible; so that, as no origin was fixed,

$$u^{\frac{1}{2}} = \frac{m}{T\rho} \cdot$$

From the three equations (5) we get also

$$\nabla (uq) = 0 \,,$$

so that

$$q = U\rho . a,$$

a being any constant versor. Thus we have the complete solution. It gives by (1)

$$d\sigma = m^2 a^{-1} \rho^{-1} d\rho \rho^{-1} a$$
$$= - m^2 a^{-1} d\rho^{-1} a,$$

so that σ is merely $-\rho^{-1}$, multiplied by a constant and subjected to a definite rotation.

But the following process is very much simpler. For we may get rid of the factor u, and so greatly simplify the investigation, by writing the equation of condition in the form

$$d\sigma = Kqd\rho q \quad . \quad . \quad . \quad . \quad (1)$$

It gives at once

$$\partial_2 \partial_1 \sigma = \partial_2 (Kq\alpha q) = \partial_1 (Kq\beta q)$$

or

$$V.\gamma(\nabla q - \gamma \partial_3 q) q^{-1} = 0 \,. \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Multiplying by γ, and adding the three equations of this form, we have

$$- \nabla q \, q^{-1} = \frac{\nabla Tq}{Tq} \,, \qquad \text{or} \qquad \nabla . q Tq = 0 \,.$$

By the help of this we may write (a) as

$$\frac{\nabla Tq}{Tq} = \gamma \left(\frac{\partial_3 Tq}{Tq} - \frac{\partial_3 Uq}{Uq} \right),$$

so that

$$\nabla \frac{1}{Tq} \cdot Uq = \gamma \partial_3 \frac{Uq}{Tq} = \beta \partial_2 \frac{Uq}{Tq} = \alpha \partial_1 \frac{Uq}{Tq} = \frac{1}{3} \nabla \frac{Uq}{Tq} \cdot \quad . \quad . \quad . \quad (b)$$

Thus, as the form of the three middle terms shows that their common value must be some *constant* quaternion,

$$d\frac{Uq}{Tq} = \Sigma \, \partial_1 \frac{Uq}{Tq} dx = d\rho a \, ,$$

or

$$\frac{Uq}{Tq} = \rho a \, ,$$

for we need not add a constant *vector* to ρ, and the form of the first of the five equal quantities above shows that no *quaternion* constant (except, of course, one of the form ϵa already referred to) can be added to the right-hand side.

Thus, finally, as before

$$d\sigma = -a\rho^{-1} d\rho \rho^{-1} \mathrm{K} a \, .$$

Though the methods employed in these two investigations are, at least at first sight, entirely different, it will be easily seen that the equations (7) and (b) to which they respectively lead are identical in meaning with one another, term by term. Yet the former shows two differentiations in every term, while the second appears to involve one only. Thus also, two distinct integrations were required in the first solution, while one sufficed for the second. But in the first, the tensor and versor of the quaternion were all along separated ; in the second the quaternion itself was directly sought.

On the Olfactory Organs of Helix. By Dr A. B. Griffiths, F.R.S. (Edin.), F.C.S., &c.

(Read May 2, 1892.)

Sochaczewer* has examined the olfactory organs in the *Pulmogasteropoda.* In these animals the organ of Semper, the pedal gland, and the tentacula have each been considered to have the function of an olfactory organ. Sochaczewer states that: (*a*) The organ of Semper is small in *Helix*, *Arion*, and *Limnæus;* but is well developed in *Limax.* In the last mentioned animal it has the form of four or five glandular lobate processes, which are set at the sides of the mouth. This organ is supplied with four nerve-fibres. The two median are muscular in character, while the lateral branches are the proper labiales, which give off, one on either side, a fine nerve-branch to the glandular branches of the organ of Semper. The cells of the constituent lobes resemble the glandular cells of the salivary glands; in other words, this organ has not an olfactory function. (*b*) The pedal gland is an olfactory organ. It is well supplied with nerves; but experiments are difficult to try in such an organ. (*c*) The tentacula of *Helix pomatia* are not the seat of the olfactory organs. After having cut off the tentacula, and allowed the wounds to heal, he then placed the snails on a flat plate, the edge of which was smeared with turpentine. He says that both the mutilated and unmutilated specimens turned away from the edges, and he therefore concludes that this proves that the tentacula are not the seat of the olfactory organ.

But Sochaczewer has overlooked the fact that the mutilated or excised individuals receded from the turpentine because this fluid gave off an irritating vapour which acted on the sensitive tissues generally; †

* *Zeitschrift für Wissenschaftliche Zoolgie*, vol. 35, p. 30.

† It should be borne in mind that the whole body is extremely sensitive to the action of irritating vapours; and in the tentacula this sensitiveness is much more delicate than over the general surface.

and it was this irritation or sense of pain which caused both the excised and non-excised animals to retract their steps. The vapours of bromine, ammonia, and other irritating liquids act in a similar manner; consequently, we are justified in rejecting Sochaczewer's suggestion that the pedal gland of *Helix* performs among other functions that of an olfactory organ. As the pedal gland is a highly cellular organ which secretes mucus, it may be stated that the vapours of irritating liquids cause a much more rapid secretion of this fluid, which is due to the stimulation or excitation of the secreting cells. For it is known that (*a*) every action that modifies the normal condition of a cell is an irritant of that cell; (*b*) every external force, provided it has a certain intensity, is capable of inducing cellular irritability; and there is no doubt the action of the vapours of turpentine, bromine, and ammonia on snails (either with or without their tentacula) cause them to retract their steps from such irritating vapours.

Now, as a matter of fact, the tentacula are the seat of the olfactory organs in *Helix pomatia.* Near the end of each tentaculum there is a ganglion from which nerve-fibres pass to the epithelium in which are sensory bulbs. To prove that the tentacula are the seat of the olfactory organs in *Helix pomatia* and *Helix aspersa*, the author placed a number of these animals on flat slabs, the edges of which were smeared with eau de Cologne, methyl alcohol, ether, and ethyl acetate. The vapours of these liquids do not act as irritants, for the secretion of mucus appeared to be perfectly normal. Those animals whose tentacula had been removed, gradually approached to the edges of the slabs; but those animals whose tentacula had been left intact did not approach, but turned away from the edges. This proves that the tentacula are the seat of the olfactory organs in *Helix;* because in these experiments excitation, due to irritating vapours, was reduced to nil. It will also be seen that non-irritating substances alone in the state of vapours, or of fine particles suspended in the atmosphere, can provoke olfactory sensations, not only in the higher animals, but also in the *Pulmogasteropoda*, and other Invertebrates.

On the Renal Organs of the Asteridea. By Dr A. B. Griffiths, F.R.S. (Edin.), F.C.S., &c.

(Read May 2, 1892.)

I have already shown * that the secretion of the five large cardiac sacs of the stomach of *Uraster rubens* contains uric acid; and consequently these sacs are considered to be the necessary apparatus for eliminating the nitrogenous products of the waste of the tissues, &c., from the blood or hydrolymph. It may be stated that Mr H. E. Durham † has raised the question that there is a possible source of error in concluding that these sacs have a renal function. He says that "it is possible that the uric acid arrived into the starfish's stomach in the interior of small mussels, &c., whose excretory organs contain that body, and of which as food the starfish appears to be very fond: anyhow, such a source of the uric acid must be eliminated before Griffiths' conclusion can be accepted; for obviously, if the presence of urates were demonstrated in the gastric contents of an individual who had recently supped off oysters, it would by no means follow that the stomach was the organ whereby the individual excreted his urates." Certainly not; but Mr Durham has overlooked the fact that if the uric acid had been introduced along with the food, I should also have found urea in the contents of the starfish's stomach; but I conclusively proved that urea was absent.

Now, as a matter of fact, Mr Follows and myself ‡ have shown that urea (as well as uric acid) is present in the secretion of the organs of Bojanus in the *Lamellibranchiata;* and, therefore, if the starfishes had supped off oysters or mussels, I should have found urea, in addition to uric acid, in the contents of the stomach. But

* *Proceedings of Royal Society of London*, vol. 44 (1888), p. 325.

† *Quarterly Journal of Microscopical Science*, vol. 33 (1891).

‡ See the *Chemical News*, vol. 51, p. 241; *Proceedings of Royal Society of Edinburgh*, vol. 14, p. 230; and the author's book, *The Physiology of the Invertebrata* (Reeve & Co., London).

this was not so, for urea is always absent in the true secretion of the cardiac sacs of the starfish's stomach.*

Then again, the animals used in the investigation had been kept in a tank for very many days without food; and to avoid any possible source of error, the alimentary canal had been previously washed out by injecting water through the oral aperture of each individual. This being done, each animal was then placed in a tank of water and kept as already stated for very many days before being used in the investigation.

Further, it may be stated that the sacculated walls of the cardiac sacs of the stomach of *Uraster rubens* take up indigo-carmine, and as A. Kowalewsky † has shown that this is characteristic of an excretory organ, we have additional evidence that these sacs in *Uraster* perform the function of a renal organ.

I am far from adversely criticising Mr Durham's important work on the excretory nature of wandering cells in Echinoderms and other Invertebrates, but it should be distinctly understood that in *Uraster* the cardiac sacs of the stomach form the chief apparatus by which uric acid is eliminated from the system. It is possible, nay probable, that wandering cells (carrying effete matter) may find their way to the cardiac sacs of the stomach of *Uraster* and other *Asteridea.* In fact, it may be stated that Shipley ‡ considers that in the *Hirudinea* wandering cells may collect and carry effete products to the nephridial sacs, and after undergoing degeneration, these products are excreted through the nephridium or segmental organ. The secretion, or rather excretion, of the nephridia of *Hirudo* contains uric acid.§

Since my paper (*loc. cit.*) was published (in 1888), I have proved, by the same methods of investigation, that the stomach of *Solaster* and *Astropecten* also performs the function of a renal organ, for uric acid is readily extracted from the secretion of that organ. It

* This statement may be readily confirmed when the proper tests are skilfully applied. I say "skilfully applied" because certain nitrogenous and other compounds have been said to be present in the secretion of the renal organs of many Invertebrates, which are not present at all.

† *Biologisches Centralblatt*, Bd. ix. (1889–90), pp. 33, 65, and 127.

‡ *Studies from Morphol. Labor., Cambridge*, vol. 5 (1890). See also Vejdovsky's *System der Oligochæten*, v., pp. 111, 112, and 127.

§ The author, in *Proceedings of Royal Society of Edinburgh*, vol. 14, p. 346.

may be, therefore, stated that the stomach or pyloric sac of the *Asteridea* is a digestive cavity and a renal organ; *i.e.*, it has a dual function. This is no exception among Invertebrates, for, as Darwin states in *The Origin of Species* (chap. vi.), "numerous cases could be given among the lower animals of the same organ performing at the same time wholly distinct functions."

In conclusion, it would be interesting to know whether the out-wandering cells in Echinoderms contain uric acid. If so, it would be proved that these cells form an additional means of eliminating this nitrogenous substance from the system—just as the skin in the higher animals supplements the kidneys in eliminating excretory products.

Note on Uniform Convergence. By Professor Cayley.

(Read December 5, 1892.)

It appears to me that the form in which the definition or condition of uniform convergence is usually stated, is (to say the least) not easily intelligible. I call to mind the general notion : We may have a series, to fix the ideas, say of positive terms

$$(0)_x + (1)_x + (2)_x, \quad . \quad . + (n)_x, \quad . \quad . \quad .$$

the successive terms whereof are continuous functions of x, for all values of x from some value less than a up to and inclusive of a (or from some value greater than a down to and inclusive of a) : and the series may be convergent for all such values of x, the sum of the series ϕx is thus a determinate function ϕx of x; but ϕx is not of necessity a continuous function; if it be so, then the series is said to be uniformly convergent; if not, and there is for the value $x = a$ a breach of continuity in the function ϕx, then there is for this value $x = a$ a breach of uniform convergence in the series.

Thus if the limits are say from 0 up to the critical value 1, then in the geometrical series $1 + x + x^2 + \quad . \quad . \quad .$, the successive terms are each of them continuous up to and inclusive of the limit 1, but the series is only convergent up to and exclusive of this limit, viz. for $x = 1$ we have the divergent series $1 + 1 + 1 + \quad . \quad . \quad .$, and this is *not* an instance; but taking, instead, the geometrical series $(1 - x) + (1 - x)\,x + (1 - x)\,x^2 + \quad . \quad . \quad .$, here the terms are each of them continuous up to and inclusive of the limit 1, and the series is also convergent up to and inclusive of this limit; in fact, at the limit the series is $0 + 0 + 0 + \quad . \quad . \quad .$ We have here an instance, and there is in fact a discontinuity in the sum, viz. $x < 1$ the sum is

$$(1 - x)(1 + x + x^2 + \quad . \quad . \quad .), = (1 - x).\frac{1}{1 - x}, = 1;$$

whereas for the limiting value 1, the sum is $0 + 0 + 0 + \quad . \quad . \quad ., = 0$. The series is thus uniformly convergent up to and exclusive of the value $x = 1$, but for this value there is a breach of uniform convergence.

I remark that Du Bois-Reymond in his paper, "Notiz über einen Cauchy'schen Satz, die Stetigkeit von Summen unendlicher Reihen betreffend," *Math. Ann.*, t. iv. (1871), pp. 135–137, shows that when certain conditions are satisfied, the sum ϕx is a continuous function of x, but he does not use the term "uniform convergence," nor give any actual definition thereof.

M. Jordan, in his "Cours d'Analyse de l'Ecole Polytechnique," t. i. (Paris 1882), considers p. 116 the series $s = u_1 + u_2 + u_3 + \ldots$, the terms of which are functions of a variable z, and after remarking that such a series is convergent for the values of z included within a certain interval, if for each of these values and for every value of the infinitely small quantity ϵ we can assign a value of n such that for every value of p,

$$\text{Mod}\,(u_{n+1} + u_{n+2} + \quad \ldots \quad + u_{n+p}) < \text{Mod}\,\epsilon,$$

ϵ being as small as we please, proceeds—

"Le nombre des termes qu'il est necessaire de prendre dans la série pour arriver à ce résultat sera en général une fonction de z et de ϵ. Néanmoins on pourra trés habituellement déterminer un nombre n fonction de ϵ seulement telle que la condition soit satisfaite pour toute valeur de z comprise dans l'intervalle considéré. On dira dans ce cas que la série s est *uniformement convergente* dans cette intervalle."

And similarly Professor Chrystal in his Algebra, Part II. (Edinburgh, 1889), after considering, p. 130, the series

$$\frac{x}{x+1.2x+1} + \frac{x}{2x+1.3x+1} \quad \ldots \quad + \frac{x}{(n-1)x+1.nx+1} + \quad \ldots$$

for which the critical value is $x = 0$, and in which when $x = 0$ the residue R_n of the series or sum of the $(n+1)$th and following terms is $= \dfrac{1}{nx+1}$ proceeds as follows:—Now when x has any given value, we can by making n large enough make $\dfrac{1}{nx+1}$ smaller than any given positive quantity α. But on the other hand, the smaller x is the larger must we take n in order that $\dfrac{1}{nx+1}$ many fall under α; and in general when x is variable there is no finite upper limit for

n independent of x, say v, such that if $n>v$ then $R_n<\alpha$. When the residue has this peculiarity the series is said to be *non-uniformly convergent;* and if for a particular value of x, such as $x=0$ in the present example, the number of terms required to secure a given degree of approximation to the limit is infinite, the series is said to *converge infinitely slowly.*

And he thereupon gives the formal definition: *If for values of x within a given region in Argand's diagram we can for every value of α, however small Mod. α, assign for n an upper limit v* INDEPENDENT OF x, such that, when $n>v$, Mod. $R_n<$Mod. α, then the series $\Sigma f(n,x)$ is said to be UNIFORMLY CONVERGENT within the region in question.

The two forms of definition (Jordan and Chrystal) appear to me equivalent, and it seems to me that construing the definition *strictly*, and applying it to the above instance $(1-x)+(1-x)x+(1-x)x^2+\ \ .\ \ .\ \ .$, the definition does not in either case indicate a breach of uniform convergency at $x=1$, viz. the definition shows uniform convergency from $x=0$ to $x=1-\epsilon$, ϵ being a positive quantity however small, or (as I have before expressed this) uniform convergency up to and exclusive of the limit 1; and further, it shows uniform convergency at the limit 1. For at this limit, the series of terms is $0+0+0+\ \ .\ \ .\ \ .$, the residue or sum of the $(n+1)$th and subsequent terms is thus also $0+0+0+\ \ .\ \ .\ \ .$, and we get the value of this residue, not approximately, but exactly, by taking a single term of the series. Jordan and Chrystal calculate, each of them, the residue from the general expression thereof by writing therein for x or z the critical value, and then comparing the value thus obtained with the values obtained for the $(n+1)$th and subsequent terms of the series on substituting therein for x or z the critical value, seem to argue that the discrepancy between these two values indicates the breach of uniform convergency.

It may be said that the objection is a verbal one. But it seems to me that the whole notion of the residue (although very important as regards the general theory of convergence) is irrelevant to the present question of uniform convergency, and that a better method of treating the question is as follows:

Considering as before the series

$$(0)_x+(1)_x+(2)_x\ \ .\ \ .\ \ .\ \ +(n)_x\ \ .\ \ .\ \ .$$

where the functions $(0)_x$, $(1)_x$, $(2)_x$, . . . are each of them continuous up to and inclusive of the limit $x=a$, and the series has thus a definite sum ϕx, this sum is *primâ facie* a continuous function of x, and what we have to explain is the manner in which it may come to be discontinuous. Suppose that it is continuous up to and exclusive of the limit $x=a$, but that there is a breach of continuity at this limit: write $x=a-\epsilon$, where ϵ is a positive quantity as small as we please, and consider the two equations

$$\phi x=(0)_x+(1)_x+(2)_x+ \quad . \quad . \quad .$$
$$\phi a=(0)_a+(1)_a+(2)_a+ \quad . \quad . \quad .$$

then we have

$$\phi a-\phi x=\epsilon\left\{\frac{(0)_a-(0)_x}{a-x}+\frac{(1)_a-(1)_x}{a-x}+ \quad . \quad . \quad . \right\}.$$

Hence if the sum of the series in { } is a finite magnitude M, not indefinitely large for an indefinitely small value of ϵ, we have $\phi \mathrm{a}-\phi x=\epsilon\mathrm{M}$, which is indefinitely small for ϵ indefinitely small, and there is no breach of continuity; the only way in which a breach of continuity can arise is by the series in { } having a value indefinitely large for ϵ indefinitely small, viz. if such a value is $\frac{\mathrm{N}}{\epsilon}$, then $\phi a-\phi x=\epsilon.\frac{\mathrm{N}}{\epsilon}, =\mathrm{N}$, and as x changes from $a-\epsilon$ to a, the sum changes abruptly from $\phi(a-\epsilon)$ to $\phi(a-\epsilon)+\mathrm{N}$.

The condition for a breach of uniform convergency for the value $x=a$, thus is, that writing $x=a-\epsilon$, ϵ a positive magnitude however small, the series

$$\frac{(0)_a-(0)_x}{a-x}+\frac{(1)_a-(1)_x}{(a-x)}+, \quad . \quad . \quad .$$

shall have a sum indefinitely large for ϵ indefinitely small, or say as before a sum $=\frac{\mathrm{N}}{\epsilon}$.

For the foregoing example, where the series is

$$(1-x)+x(1-x)+x^2(1-x)+ \quad . \quad . \quad .$$

the critical value is $a=1$: we have here $(n)_x=x^n(1-x)$, and consequently

$$\frac{(n)_a-(n)_x}{a-x}=\frac{a^n-x^n}{a-x}-\frac{a^{n+1}-x^{n+1}}{(a-x)}$$
$$=(a^{n-1}+a^{n-2}x \quad . \quad . \quad . \quad +x^{n-1})-(a^n+a^{n-1}x \quad . \quad . \quad . \quad +x^n)$$
$$=\ -x^n \text{ for } a=1.$$

The series

$$\frac{(0)_a-(0)_x}{a-x}+\frac{(1)_a-(1)_x}{a-x}+\quad . \quad . \quad .$$

thus is

$$-(1+x+x^2+\quad . \quad . \quad .)$$
$$=-\frac{1}{1-x},=\frac{-1}{\epsilon}\text{ for }x=1-\epsilon,$$

viz. we have

$$\phi 1-\phi(1-\epsilon)=\epsilon.\frac{-1}{\epsilon},=-1\ ;$$

which is right since by what precedes $\phi(1-\epsilon)=1$, $\phi 1=0$.

On a Certain Locus. By Professor P. H. Schoute,

University of Groningen, Holland.

(Read 5th December 1892.)

1. If the two ends of a strip of paper are pasted on one another after having twisted one of them through 180°, we obtain a surface with only one side. On the last meeting of the British Association Professor A. Crum Brown showed a model of a mathematical surface closely connected with it. This surface, with the common name of "marrow-bone," is the locus of the line AB (fig. 1), cutting a given circle C orthogonally and moving under the condition $\angle$ AOD $= 2 \angle$ BAE.

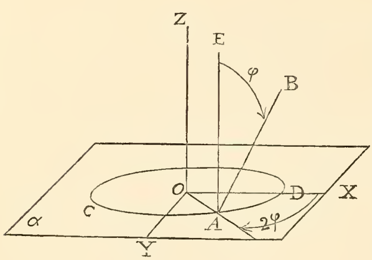


Fig. 1.

In the following lines I wish to determine the order of the locus and one of its peculiarities in the more general case, when the ratio of the two angular velocities instead of $2:1$ is $m:n$ (m and n integer and prime to one another).

2. Let us seek the section of the locus by the plane α of the circle. This section consists of the circle counted a certain number of times, and of a certain number of generators. As these two numbers are different for m odd and for m even, these two cases are to be treated separately.

First Case: m odd (surface with two sides).

The surface contains the different positions of the generator corresponding to the values of ϕ between 0 and 2π.

When $m\phi$ increases by 2π, the same point A is obtained. This proves that A lies on m different generators. For the angle $\dfrac{\angle \text{AOD} + 2k\pi}{m}$ admits m different values between 0 and 2π. In other words, the circle is an m-fold curve of the locus, and is to be counted m-times as part of the section.

On the other hand, when $n\phi$ is equal to $\frac{2k+1}{2}\pi$, the generator lies in α. This proves that α contains $2n$ generators. For the expression $\frac{2k+1}{2n}\pi$ admits $2n$ different values between 0 and 2π.

The section contains m-times the circle C and $2n$ generators. Therefore the locus is of the order $2(m+n)$.

Second Case: m even $=2m'$ (surface with only one side).

The surface only contains the different positions of the generator corresponding to the values of ϕ between 0 and π.

In the same manner is proved that the section contains m'-times the circle and n generators. Therefore the locus is of the order $2m'+n$ or $m+n$.

In the case of the "marrow-bone" the order is 3.*

* If we proceed analytically, the equation of the surface is obtained by eliminating p and ϕ among the three equations

$$\left.\begin{aligned} x &= (a+p\sin\phi)\cos 2\phi \\ y &= (a+p\sin\phi)\sin 2\phi \\ z &= p\cos\phi \end{aligned}\right\},$$

or of ϕ between

$$\left.\begin{aligned} x &= (a+z\tan\phi)\cos 2\phi \\ y &= (a+z\tan\phi)\sin 2\phi \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1).$$

Here two pit-falls might be indicated. Firstly, if we deduce the two equations

$$\frac{y}{x}=\tan 2\phi\,, \qquad x^2+y^2=(a+z\tan\phi)^2$$

the elimination of ϕ leads to an equation of the sixth order, containing also the result of the elimination of ϕ among

$$\left.\begin{aligned} x &= -(a+z\tan\phi)\cos 2\phi \\ y &= -(a+z\tan\phi)\sin 2\phi \end{aligned}\right\}.$$

Secondly, if we go on more cautiously and put $\tan\phi=t$, we find

$$\left.\begin{aligned} x &= (a+zt)\frac{1-t^2}{1+t} \\ y &= (a+zt)\frac{2t}{1+t^2} \end{aligned}\right\}$$

and by elimination of t

$$(x^2+y^2)(y-z)^2=(ay-xz)^2\,,$$

an equation of the fourth degree. Here however is it evident, that the terms x^2z^2 annihilate one another, and the rest can be divided by y, &c.

Synthetically the order of the general surface can also be found in seeking the section by a plane through OZ.

3. As the projections of the generators on α pass through the centre O of the circle, two generators can meet only in two different ways. If they lie in different planes through OZ, they can only meet in OZ; if their projections on α coincide, they can meet elsewhere. This remark will enable us to determine the order of the double curve on the surface.

The intersection of the generator ϕ with the axis OZ is given by the relation $z = -a \cot n\phi$. As $\phi + \frac{\angle \mathrm{BAE} + k\pi}{n}$ admits $2n$ different values between 0 and 2π, n different values between 0 and π, the line OZ is an $2n$-fold line for m odd and an n-fold line for m even.

The two sets of m (m') generators passing through two diametrically opposite points of the circle procure m^2 (m'^2) points of the double curve. As the expressions

$$\frac{n}{m}(m\phi + 2k\pi) \text{ and } \frac{n}{m}(m\phi + \pi + 2k\pi)$$

never are equal, it never happens that one of the m^2 (m'^2) points lies on the line OZ. Therefore the locus of the m^2 (m'^2) points is a double curve of the order m^2 (m'^2).

For m odd the double curve of the surface consists of a circle counted m-times, a right line counted $2n$-times, and a curve of the order m^2. This corresponds to a single double curve of the order

$$\frac{m(m-1)}{2}2 + \frac{2n(2n-1)}{2} + m^2, \text{ or } 2(m^2+n^2) - (m+n).$$

For m even the corresponding numbers are successively $\frac{m}{2}$, n, $\frac{m^2}{4}$, and $\frac{m^2+n^2}{2} - \frac{m+n}{2}$.

In the case of the marrow-bone is duly found the single double line, the line $\left.\begin{matrix} x = a \\ y = z \end{matrix}\right\}$.*

* Two generators with coincident projections on the plane α are represented by the equations

$$\left.\begin{aligned} x &= (a + p\sin\phi)\cos 2\phi \\ y &= (a + p\sin\phi)\sin 2\phi \\ z &= p\cos\phi \end{aligned}\right\}, \quad \left.\begin{aligned} x &= -(a + q\cos\phi)\cos 2\phi \\ y &= -(a + q\cos\phi)\sin 2\phi \\ z &= -q\sin\phi \end{aligned}\right\}$$

4. When the two angular velocities are incommensurable (m and n infinite), the surface is transcendental.

For the common point we find

$$\left.\begin{array}{l} p\cos\phi+q\sin\phi=0 \\ p\sin\phi+q\cos\phi=-2a \end{array}\right\},$$

or

$$p=\frac{2a\sin\phi}{\cos 2\phi}, \qquad q=-\frac{2a\cos\phi}{\cos 2\phi},$$

and therefore

$$x=a, \qquad y=a\tan 2\phi, \qquad z=a\tan 2\phi.$$

This proves that the locus of this point is the line $\left.\begin{array}{l} x=a \\ y=z \end{array}\right\}$. Really the equation $(x^2+y^2)(y-z)^2=(ay-xz)^2$ shows that this line is a double line of the surface.

Now the surface can easily be constructed by means of the circle and the double line. To that end we only have to join every point P of the double line with each of the two points common to the circle and the plane OPZ.

Recent Innovations in Vector Theory. By Professor C. G. Knott, D.Sc., F.R.S.E.

(Read December 19, 1892.)

(1.) Of late years there has arisen a clique of vector analysts* who refuse to admit the quaternion to the glorious company of vectors. Their high-priest is Professor Willard Gibbs. His reasons against the quaternion are given with tolerable fulness in *Nature*, April 2, 1891. His own vector analysis is presented in a pamphlet, "Elements of Vector Analysis, Arranged for the Use of Students in Physics—not Published" (1881–4). Mr Oliver Heaviside, in a series of papers published recently in the *Electrician*, and in an elaborate memoir in the *Philosophical Transactions* (1892), supports some of Gibbs's contentions, and cannot say hard enough things about the quaternion as a quantity which no physicist wants. Professor Macfarlane of Texas University sides with Heaviside in taking umbrage at a most fundamental principle of quaternions, and develops a pseudo-quaternionic system of vector algebra non-associative in its products.

Before proceeding to an examination of the positions taken by these writers, I wish to draw attention to some remarkable papers published between the years 1846–52, just at the time when Hamilton was developing the quaternion calculus. These papers were written by the Rev. M. O'Brien, Professor of Natural Philosophy and Astronomy in King's College, London. They give a vector analysis independent of the conception of the quaternion. It may be safely said that the anti-quaternionic vector analysts of to-day have barely advanced beyond the stage reached by O'Brien in his third and last paper "On Symbolic Forms derived from the Conception of the Translation of a directed Magnitude" (*Phil. Trans.*, 1852).

In this paper O'Brien defines two distributive products of two

* This paper is written wholly from the point of view of mathematical physics, for which a vector algebra is generally admitted to be of supreme importance. The purely analytical aspect of quaternions is not contemplated.

vectors, which he symbolises by the notations $\alpha.\beta$ and $\alpha\times\beta$. The product $\alpha\times\beta$ he calls the longitudinal translation of α along β; and the product $\alpha.\beta$ the lateral translation. The geometric meanings of these are sufficiently obvious. Thus the translation of $\overline{\mathrm{OA}}$ or α along $\overline{\mathrm{OB}}$ or β has, so to speak, a longitudinal part and a lateral part. The longitudinal translation $(\alpha\times\beta)$ is represented by the translation of OM (the resolved part of α along β) from O to B. It is measured by the product OM.OB or $ab\cos\mathrm{AOB}$, where a b are the lengths of the vectors α β. Similarly, the lateral translation $(\alpha.\beta)$ is represented by the translation of ON (the component of α perpendicular to β) from O to B. It is measured by the area of the parallelogram contained by α and β. Of course, these are simply Grassmann's "inner" and "outer" products. It must be noted, however, that O'Brien's $\alpha.\beta$ is not the same thing as Hamilton's $\mathrm{V}\alpha\beta$. To represent this conception he introduces the Directrix, and uses for it the symbol D. Thus D $(\alpha.\beta)$ is a directed line drawn perpendicular to the plane containing $\alpha\beta$, and of a length numerically equal to the area of the parallelogram $\alpha.\beta$. He then uses $\mathrm{D}\alpha$, $\mathrm{D}\beta$, $\mathrm{D}\gamma$, somewhat in the sense of Hamilton's i, j, k. In an interesting footnote he points out very precisely the difference between his system and Hamilton's, noting, for example, that the latter identifies α and $\mathrm{D}\alpha$, and finds in consequence that the square of any unit vector is equal to negative unity. He sees very clearly that unit vectors must have their squares all equal, but admits that his own value of positive unity is only an assumption. He concludes with these words: "If in any way I could show that -1 was the proper value for a unit of longitudinal translation, I should have $\alpha\alpha=\alpha\times\alpha+\alpha.\alpha=-1$." O'Brien applies his methods to general dynamics, and gives, besides, some applications of the linear and vector function in one of its standard forms, and also of the operator $\alpha\partial_1+\beta\partial_2+\gamma\partial_3$. In one of his early papers Hamilton refers to O'Brien's work in terms of high praise. It will be seen that O'Brien's methods of establishing his calculus are identical in principle with the long subsequent ones of Gibbs and Heaviside.

(2.) Professor Gibbs's position is best described in his own words (*Nature*, vol. xliii. pp. 511–12):—

"The question arises, whether the quaternionic product can claim a prominent and fundamental place in a system of vector analysis.

It certainly does not hold any such place among the fundamental geometrical conceptions as the geometrical sum, the scalar product, or the vector product. The geometrical sum $\alpha+\beta$ represents the third side of a triangle as determined by the sides α and β. $V\alpha\beta$ represents in magnitude the area of the parallelogram determined by the sides α and β, and in direction the normal to the plane of the parallelogram. $S\gamma V\alpha\beta$ represents the volume of the parallel*o*piped (*sic*) determined by the edges α, β, and γ. These conceptions are the very foundations of geometry.

"We may arrive at the same conclusion from a somewhat narrower but very practical point of view. It will hardly be denied that sines and cosines play the leading parts in trigonometry. Now the notations $V\alpha\beta$ and $S\alpha\beta$ represent the sine and cosine of the angle included between α and β, combined in each case with certain other simple notions. But the sine and cosine combined with these auxiliary notions are incomparably more amenable to analytical transformation than the simple sine and cosine of trigonometry, exactly as numerical quantities combined (as in algebra) with the notion of positive or negative quality are incomparably more amenable to analytical transformation than the simple numerical quantities of arithmetic.

"I do not know of anything which can be urged in favour of the quaternionic product of two vectors as a *fundamental* notion in vector analysis, which does not appear trivial or artificial in comparison with the above considerations. The same is true of the quaternionic quotient, and of the quaternion in general."

Now, what does the argument of the second paragraph quoted amount to? Certainly no quaternionist ever denied the importance of the sine and cosine in trigonometry; and Hamilton was unquestionably the first to show forth the analytical power of the functions $S\alpha\beta$ and $V\alpha\beta$. But, because these functions are so incomparably more amenable to analytical transformation than are their trigonometrical ghosts, are we to infer that they are necessarily more fundamental than *anything else?* And why should the angle itself be so unceremoniously left out? On the principle of answering a wise man according to his wisdom, might we not continue: It will hardly be denied that angles and their functions play the leading part in trigonometry. Now, the notation $\alpha\beta^{-1}$ represents the angle included between α and

β, combined with certain other simple notions. But the angle combined with these auxiliary notions is incomparably more amenable to analytical transformation than the simple angle of trigonometry, and so on—proving just as much and just as little as the great original itself. Heaviside, evidently with the above passage in his mind, says that "The justification for the treatment of scalar and vector products as fundamental ideas in vector algebra is the distributive property they possess." So be it; and is not the quaternion product as grandly distributive as any?

(3.) To appreciate the real character of the broad geometrical argument advanced by Gibbs, we must consider the meaning and purpose of a vector analysis. We are all agreed that vectors are of real importance in physics. Having then formed the conception of a vector, we have next to find what relations exist between any two vectors. We have to compare one with another, and this we may do by taking either their difference or their ratio. The geometry of displacements and velocities suggests the well-known addition theorem $\alpha + \delta = \beta$, recognised by Gibbs as essentially fundamental.

But this method, about which there is no dispute, does not seem to me to be more fundamental geometrically than the other method which gives us the quaternion. When we wish to compare fully two lengths a and b, we do not take their difference, but divide the one by the other. We form the quotient a/b, and this quotient is defined as the factor which changes b into a. Now a vector is a directed length. By an obvious generalisation, therefore, we compare two vectors by taking their quotient (α/β), and by defining this quotient as the factor which changes the vector β into the vector α. This is the germ out of which the whole of vector analysis naturally grows. A more fundamental conception it is impossible to make. Yet Gibbs says "it certainly does not" take rank as a fundamental conception in geometry with the conceptions of a vector-bounded area and of a vector-bounded volume, whose bounding vectors may have an infinity of values.

Again, a vector is an embodiment of direction; and to know how to change a direction is surely demanded of a vector analyst from the very beginning. But a change of direction is an angular displacement, that is, a versor or quaternion with unit tensor. Or, take the case of a body strained homogeneously. The relative vector of any two of its points is changed into its new value by a process which is a combination of turning and stretching. A simpler description cannot be imagined. It is completely symbolised by the quaternion with its tensor and versor factors. And *this*, we are taught, is trivial and artificial—as trivial, say, as the versor operation which every one performs when estimating the time that must be allowed to catch a train.

Professor Gibbs would have us base the whole of vector analysis on the two geometrical ideas embodied in the formulæ $V\alpha\beta$ and $-S\gamma V\alpha\beta$. But any thoughtful student, approaching the subject in this way, will almost certainly be struck with the arbitrariness of the definitions made at the outset. There is no very apparent reason, *at first*, why an area should be represented by a vector line drawn perpendicular to it; while the transition from $S\gamma V\alpha\beta$ to $S\alpha\beta$ can hardly fail to appear somewhat mysterious. This is the method adopted by Clifford in his Dynamic; but I believe this development to be logically faulty, and quite unsuited to a student otherwise ignorant of the properties of vector products. In quaternions, however, the quotient and product of two vectors being clearly defined —for $\alpha\beta$ is the operator which changes β^{-1} into α—it soon appears that to every quaternion q there is a conjugate q', so that their sum is a scalar and their difference a vector. From that the geometrical meanings of $V\alpha\beta$ and $S\alpha\beta$ are at once obtained, and the whole system is established firm and sure.*

At a recent meeting of the Physical Society of London, Professor Henrici is reported to have said, "Vectors must be treated vectorially;" and Mr Heaviside echoes the strain in § 175 of his "Electromagnetic Theory," as published in the *Electrician*. On the same sapient principle, I suppose, scalars must be treated scalarially, rotors rotorially, algebra algebraically, and geometry geometrically. That is to say, the remark is a very loose statement of a truism, or it is profound nonsense. Strictly speaking, to treat vectorially is to treat after the manner of vectors, or to treat *as vectors do*. Now, what does a vector do? Professor Gibbs, the prince of vector purists, says, on page 6 of his pamphlet, that "the effect of the skew [or vector] multiplication by α [any unit vector] upon vectors in a plane perpendicular to α is simply to rotate them all 90° in that

* See article "Quaternions" in Chambers's *Encyclopædia* (New Edit., 1892).

plane." Hence a vector acts versorially. To which Mr Heaviside, in fierce denunciation : "In a given equation [in *quaternion* vector analysis, that is] one vector may be a vector and another a quaternion. Or the same vector in one and the same equation may be a vector in one place and a quaternion (versor or turner) in another. This amalgamation of the vectorial and quaternionic functions is very puzzling. You never know how things may turn out." Puzzling? Then must Heaviside find his own system as puzzling as any. For, when he writes the vector product $ij=k$, he is simply acting on j by i or on i by j, and turning it through a right angle. It is impossible to get rid of this versorial effect of a vector.* But, if to treat vectors vectorially means that we must treat them versorially, on what principle of straw-dividing are we to debar their quaternionic treatment?

(4.) Before passing on to consider the systems of the innovators, I shall give three other short quotations from Gibbs's letter to *Nature*, as follows :—

"How much more deeply rooted in the nature of things are the functions $S\alpha\beta$ and $V\alpha\beta$ than any which depend on the definition of a quaternion will appear in a strong light, if we try to extend our formulæ to space of four or more dimensions. . . ."

To elucidate the "nature of *things*" by an appeal to the fourth dimension—to solve the Irish question by a discussion of social life in Mars—is a grand conception, worthy of the scorner of the trivial and artificial quaternion of three dimensions. But is it not the glory of quaternions that it is so pre-eminently a tridimensional calculus? Again, further on we read : "Vectors exist in such a [four dimensional] space, and there must be a vector analysis for such a space"—true ; and must there not be operators for changing one vector into another, geometrically analogous to the quaternion of three dimensions? Or, more generally, must there not be in n dimensional space the M-in-one corresponding to the 4-in-one of 3-dimensional space?

Again, we read that "nothing is more simple than the definition of a linear vector function, while the definition of a quaternion

* O'Brien alone of vector analysts uses a non-scalar product of two vectors, which does not involve directly this versorial characteristic ; but then he has to introduce his Directrix, so as to get out into space, as it were.

is far from simple." I fancy Gibbs has in mind the definition of a quaternion as the sum of a scalar and vector, although he nowhere tells us explicitly what he imagines a quaternion *fundamentally* to be. In like fashion Heaviside writes: "The quaternion is regarded as a complex of scalar and vector." The pure analyst may so regard it; but to the physicist it is made up of tensor and versor. Its *property* of being decomposable into scalar and vector parts, each with a geometric meaning at first sight distinct from its own fundamental characteristic, is an absolutely invaluable one. The quaternion comprehends within itself the conceptions of a rotation, a stretching, a vector area, and a projection. You may extract whichever part or parts may serve your purpose for the moment—they are all there uniquely determined when the quaternion is given. Here, truly, is a king of quantities. "Upon earth there is not his like."

(5.) In considering the claims of the rival systems of vector analysis, we will first glance at the notations suggested.

A good notation for a new calculus is half the battle won; and a notation must, of course, be in harmony with the principles of the calculus. Having to his own satisfaction abolished the quaternion, Professor Gibbs proceeds to argue that "we obtain the *ne plus ultra* of simplicity and convenience if we express the two functions [$V\alpha\beta$ and $-S\alpha\beta$] by uniting the vectors in each case with a sign suggestive of multiplication." Consequently he represents these functions in the forms $\alpha \times \beta$ and $\alpha.\beta$. These are the forms used long ago by O'Brien, only he used them the other way about.

Now there is a serious objection at the very outset to such a form as $\alpha \times \beta$ for the vector product of α and β. There corresponds to it no quotient amenable to symbolic treatment. The reason, of course, is that $\alpha \times \beta$ is not a complete product. It is only a *part* of the complete product, which Hamilton writes $\alpha\beta$. Given the quaternion equation $\alpha\beta = q$, any one quantity is uniquely determined if the other two are given. The quotient $\alpha = q/\beta$ has an intelligible meaning. But it is impossible, in spite of the suggestiveness of the form, to throw Gibbs's $\alpha \times \beta = \gamma$ into any such shape as $\alpha = \gamma \div \beta$. Now, in the quaternion notation, we have $V\alpha\beta = \gamma$, $S\alpha\beta = a$, where the *selective* symbols V and S pick out two important parts of the complete quaternion product. Since the essence of a selective symbol is the partialising of the conception, it is evidently out of the question to write $V\alpha = \gamma\beta^{-1}$ or $S\alpha = a\beta^{-1}$. Such transformations are meaningless.* The point I wish to emphasise is that Hamilton's notation does not even suggest the possibility of such a transformation. On the other hand, Grassmann's, O'Brien's, Gibbs's, and to a certain extent Heaviside's, notations do suggest such a possibility. It is certainly inexpedient, to say the least, to use a notation strongly resembling that for the multiplication of ordinary algebraic quantities, but having no corresponding process by which either factor can be carried over as a generalised divisor to the other side of an equation.

Heaviside uses $\alpha\beta$ in the sense of $-S\alpha\beta$, and to this notation exactly the same objection applies.

It is possible that Hamilton's notation might be improved, but certainly not in the way advocated by Gibbs. So long as we deal with two vectors only, there is no doubt a slight saving in symbols by use of forms like $\alpha\beta$ or $\alpha.\beta$ instead of $-S\alpha\beta$. But $\alpha \times \beta$ is no more easily written than is $V\alpha\beta$. Hamilton's form $S\alpha\beta\gamma$ is as easily written as Heaviside's $\alpha V\beta\gamma$, besides being more symmetrical and expressive; while $\alpha.\beta \times \gamma$ is clearly not in the running for compactness, perspicuity, or symmetry. Again, such a form as $V\alpha\beta S\gamma\delta$ becomes with Gibbs $\alpha \times \beta(\gamma.\delta)$, while Heaviside would probably write it $V\alpha\beta.\gamma\delta$. The quantity $\alpha S\beta\gamma\delta$ would take the forms $\alpha(\beta.\gamma \times \delta)$ and $\alpha.\beta V\gamma\delta$. Gibbs cannot write $V\alpha\beta\gamma$, which must be put into the form $-\alpha(\beta.\gamma) + \alpha \times (\beta \times \gamma)$, a hideous parody of $\alpha S\beta\gamma + V\alpha V\beta\gamma$.

The peculiar perspicuity of Hamilton's notation arises from the fact that the S and V are thrown out in such relief from amongst the Greek letters used for vectors and the small Roman letters used for quaternions and scalars. A glance suffices to tell whether a quantity is scalar or vector. We know at once what kind of quantity we have to deal with before we are called upon to inquire into its composition. Herein lies one great merit of the prefix method in such a calculus. Heaviside, to a large extent, destroys the contrast between the quantities and the selective symbols by using capital letters for vectors. In print the vectors are made heavy, and of course stand out prominently enough. But a vector analysis is a thing *to be used*; and it is *hopeless* with pencil or pen or chalk on a blackboard to try to prevent confusion between **A** and A. Heaviside virtually condemns his whole system by suggesting a

* Thus we cannot transform $\text{Sin}\, 3\theta = a$ into $\text{Sin}\,\theta = \frac{1}{3}a$.

suffix notation for manuscript; the result being that vectors and scalars are distinguishable only on close inspection. The conditions for a good notation are:—(1) an unmistakable difference between easily written symbols for scalar and vector quantities; (2) the scalar and vector parts of products and quotients thrown out in clear relief. This second is quite as important as the first condition. It is evident that, so far, Hamilton's notation easily holds its own.

It is easy to see that Professor Gibbs is compelled, for mere consistency's sake, to object to the selective principle of notation. He refuses to recognise that the scalar and vector products are parts of a complete product. The one he calls the *direct* product—an atrocious misnomer—and the other the *skew* product; the idea being, I suppose, that this product exists only when the vectors are inclined or skew towards one another. Presumably the recognised term vector product smacked too much of Hamilton and his ways, although there is no doubt it is infinitely more appropriate, even from Professor Gibbs's limited point of view.

(6.) One of the most important simplifications in quaternions is the identification of vectors with quadrantal quaternions, or of unit vectors with quadrantal versors.

Beginning with the quaternion quotient $q = \alpha/\beta$, we are quickly led by space considerations to study those quaternions which rotate a given vector through a right angle. Now, suppose we have two quadrantal quaternions $I\ I'$, and that we operate on the vector α which is perpendicular to the axes of both, then it is easy to show that

$$I\alpha + I'\alpha = (I + I')\alpha$$

gives a quadrantal quaternion $(I + I')$ bearing to I and I' exactly the same relation which would exist were I and I' vectors. That is, quadrantal quaternions are added and subtracted according to the recognised rules for vector addition and subtraction, which so far, be it noted, are all we know about vectors. Is there any *à priori* reason why a vector, acting on another vector at right angles, should not be regarded as a versor? Not only is there no reason against such a conception, but every vector analyst, excepting, perhaps, O'Brien, has made, and does make, that very assumption. As we said above (§ 3), Gibbs states it explicitly. That much being admitted, and the associative law being assumed to hold in products, it follows that consistency demands that $\alpha\alpha$ or α^2 should be put equal to *negative unity.* There seems to me to be absolutely no way out of this conclusion in any vector analysis in which the product of two perpendicular unit vectors is taken to be a third unit vector perpendicular to their plane, unless it be assumed that a vector operating a second time reverses its action, and *undoes* the effect of the first operation—in short, is a *reciprocating versor.*

Nevertheless Macfarlane, in his *Principles of the Algebra of Physics*, insists that Hamilton had no rational ground for putting the square of a unit vector equal to negative unity. This was exactly O'Brien's difficulty forty years ago. Heaviside takes the same view. Professor Gibbs *appears* to side with them by arguing that $-S\alpha\beta$ and not $S\alpha\beta$ is the quantity we should fix our attention on. He, however, does not admit into his calculus the complete product at all, even in the cases in which one of the parts vanishes. For example, with $i\,j\,k$, in their usual significance, he defines

$$i.i\,(=-Sii)=1,\ \ i\times j(=Vij)=k,\ \&c.\ \&c.,$$

but the product ij has no explicitly recognised place in his system.

On the other hand, both Heaviside and Macfarlane boldly write

$$ij=k,\ \ i^2=+1,\ \&c.,\ \&c.$$

Now let us take what is common to quaternions and to these other vector systems, namely, the well-known set of equations connecting mutually rectangular unit vectors:—

$$ij=k=-ji$$
$$jk=i=-kj$$
$$ki=j=-ik$$

and let us form the product of the three vectors i, $i+j$, j. By one mode of association

$$i(i+j)j=(i^2+ij)j=i^2j+kj=+i^2j-i.$$

By another mode of association

$$i(i+j)j=i(ij+j^2)=ik+ij^2=-j+ij^2.$$

Now it is evident at a glance that these cannot be the same unless $i^2=j^2=-1$. In other words, for the associative law to hold, we must of necessity take the squares of $i\,j\,k$ to be negative unity.* If

* See also Kelland and Tait's *Introduction to Quaternions* (chapter iii.) for an even simpler proof.

we follow O'Brien, Heaviside, and Macfarlane, and make $i^2 j^2 k^2$ equal each to $+1$, we get an algebra with non-associative products, an algebra which certainly is *not* quaternions.* Heaviside is apparently unaware of the non-associative beauties of his system, which he believes "to represent what the physicist wants;" for he says, much to the credit of the *Philosophical Transactions*, that his system is "simply the elements of Quaternions without the quaternions, with the notation simplified to the uttermost, and with the very inconvenient *minus* sign before scalar products done away with." (*Phil. Trans.*, vol. clxxxiii., 1892, p. 428.)

(7.) An important operator in quaternions is ∇ or $\alpha\partial_1 + \beta\partial_2 + \gamma\partial_3$, where $\partial_1\partial_2\partial_3$ are space-differentiations along the mutually rectangular directions of the unit vectors $\alpha\ \beta\ \gamma$. This operator has been called Nabla† by Robertson Smith, and Maxwell and Tait have adopted

* Macfarlane fully recognises this, and discusses at considerable length his two products of one arrangement of three vectors, and his five products of one arrangement of four vectors. If we represent the vectors of the new system by abcd . . . and the Hamiltonian vectors by $\alpha\beta\gamma\delta$ . . ., we see at once that $\text{ab} = -\text{K}(\alpha\beta) = -\beta\alpha$. Hence

$$(\text{ab})\text{c} = -\gamma\alpha\beta \text{ and } \text{a}(\text{bc}) = -\beta\gamma\alpha,$$

and the reason why (ab)c and a(bc) have different values is evident. But, by a similar process,

$$\alpha\beta\gamma = (\alpha\beta)\gamma = -\text{c}(\text{ab})$$
$$\text{and } \alpha\beta\gamma = \alpha(\beta\gamma) = -(\text{bc})\text{a}$$

so that, in this non-associative system, each association of one arrangement is equal to a particular association of one other arrangement. In quaternions the different values of the product of three vectors are got by permutation and by permutation only ; in this system they are got partly by different associations, and partly by permutation. But each system gives precisely the same number of different products.

Similarly, although there are *five* different products—((ab)c)d, (a(bc))d, (ab)(cd), a((bc)d), a(b(cd))—got from the one arrangement, any one of these has its four equivalents which are particular associations of other arrangements. Thus it may be shown that (ab)(cd) = ((ad)b)c = (d(ba))c = b((dc)a) = b(c(ad)). We commend this jungle, which the forsaker of the Hamiltonian track cannot escape if he only go far enough, to the careful consideration of Heaviside. It is small wonder that Grassmann (whose *Ausdehnungslehre* of 1862 does not hint at the possibility of putting $i^2 = -1$) never found leisure to apply his own system to angles in space. He never formed the geometric conception of a quaternion, his vector quotients being quite other things.

† Heaviside regards this name as "ludicrously inefficient," whatever that may mean. Some name is sorely needed. To invert Delta gives an awkward

the suggestion. Its square is *minus* the well-known Laplacean operator

$$\partial_1^2 + \partial_2^2 + \partial_3^2 \qquad \text{or} \left(\frac{d}{dx}\right)^2 + \left(\frac{d}{dy}\right)^2 + \left(\frac{d}{dz}\right)^2.$$

This *minus* sign is, of course, objected to by Heaviside and Macfarlane, who get rid of it (and a good deal else beside) by their assumption that the square of a unit vector is *plus* one.

For convenience of reference I shall give a table comparing the ∇ notations of the innovators with the real quaternion quantities. u is any scalar quantity, ω any vector quantity.

Hamilton-Tait.	Gibbs.	Heaviside.	Macfarlane.
∇u	∇u	∇u	∇u
$\nabla \omega$	non-existent in any.		
$V \nabla \omega$	$\nabla \times \omega$	$V \nabla \omega$	$\text{Sin} \nabla \omega$
$S \nabla \omega$	$- \nabla . \omega$	$- \nabla \omega$	$- \cos \nabla \omega$
$\nabla^2 u$	$- \nabla . \nabla u$	$- \nabla^2 u$	$- \nabla^2 u$
$\nabla^2 \omega$	$- \nabla . \nabla \omega$	$- \nabla^2 \omega$	$- \nabla^2 \omega$
$\nabla S \nabla \omega$	$- \nabla \nabla . \omega$	$- \nabla (\nabla \omega)$	$- \text{Sin} \nabla (\cos \nabla \omega)$
$\nabla V \nabla \omega$	$\nabla \times \nabla \times \omega$	$V \nabla V \nabla \omega$	$\text{Sin} \nabla (\text{Sin} \nabla \omega)$
$\nabla^{-1} \omega$	non-existent in any.		

and so on indefinitely.

Macfarlane has (and Heaviside should have) a quaternion-like quantity $\nabla\omega$, which is expressible in the quaternion form $-K\nabla\omega$. Again, in this system, there is (or should be) a quantity $\nabla(\nabla\omega)$ which is not the same as $\nabla^2\omega$—the result, of course, of the non-associative property of their vector products. Its value in quaternion symbolism is $-\nabla K \nabla \omega$ or $\nabla^2\omega - 2\nabla S\nabla\omega$, which Mr M'Aulay might express in the form $-\nabla \omega_1 \nabla_1$.

Consistent (so far) in his rejection of anything that suggests a quaternion product, Gibbs contents himself by picking out those

word, which does not euphoniously combine with other terms ; to use Delta is, of course, out of the question. *Nabla* is certainly euphonious, as any who have used it in lecturing can testify. Being quite innocent of any previous scientific significance, it has just the meaning that is put into it. Nabla occurs *only* in quaternion analysis; for the operator is not used by Gibbs in its full sense, and that which Heaviside and Macfarlane represent by ∇ is not Nabla.

bits which are either scalar or vector, and defining them independently of one another.* Thus $\nabla . \nabla \omega$ is *defined* to have the meaning

$$\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)\omega,$$

and a parenthetic note is added to the effect "that no meaning has been attributed to ∇ before a vector." One would think that the most natural inquiry for a vector analyst to make after he had formed the conception of the quantity ∇u would be—"What will be the effect of ∇ on a vector?" But no, says Gibbs in effect, be not so hasty, or you may get that "trivial" thing, a quaternion. So, in *beginning* the study of these functions, he makes no less than *four* definitions, the definitions, namely, of ∇u, $-S\nabla\omega$, $V\nabla\omega$, and $-\nabla^2\omega$,—the meanings of all of which follow in quaternions from *one* definition. The whole principle of Gibbs's and Heaviside's methods is, in fact, to represent more or less concisely certain quantities which occur frequently in mathematical physics; and, in doing so, they simply adopt from quaternions as much as they think they need. Unfortunately for themselves they take the shell and throw away the kernel.

For even with these four definitions—and others are introduced later as the subject is developed—Gibbs finds his system lacking in flexibility and true vitality. It has no power of self-growth, but has to be fitted with here an arm and there a leg, like a mechanical puppet. Then he has, so to speak, to lubricate its joints by pouring in the definitions of four other functions, with as many new symbols. One of these is the Potential—the others are called the Newtonian, Laplacian, and Maxwellian. They are symbolised thus—*Pot*, *New*, *Lap*, *Max*.

The meanings of these functions will be at once evident when they are exhibited in quaternion form. Thus, as is well known,

$$\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)\text{Pot}\, u = -4\pi u$$

from which at once

* The query suggests itself, could these bits have been discovered without quaternions as a guide?

$$\nabla^2 \text{Pot } u = +4\pi u \text{*}$$
$$\text{or} \qquad \text{Pot } u = 4\pi \nabla^{-2} u$$

Similarly, if ω be a vector quantity,

$$\text{Pot } \omega = 4\pi \nabla^{-2}\omega$$

Then we have

$$\text{New } u = \nabla \text{ Pot } u = 4\pi \nabla^{-1} u$$
$$\text{Lap } \omega = \text{V} \nabla \text{ Pot } \omega = 4\pi \text{V} \nabla^{-1}\omega$$
$$-\text{Max } \omega = \text{S} \nabla \text{ Pot } \omega = 4\pi \text{S} \nabla^{-1}\omega$$

Thus (Lap − Max) ω is $4\pi \nabla^{-1}\omega$, which probably Gibbs would write Tai ω—only, alas! he cannot use it.†

Now, Gibbs gives a good many equations—theorems, I suppose, they ape at being—which connect those functions and their various derivatives. All these equations are, in quaternions, *identities*, involving the *very simplest* transformations. But there is no such flexibility and simplicity in Gibbs's analysis. For example, he takes eight distinct steps to prove that

$$\nabla^{-2}\nabla^2\omega = \omega,$$

and even then he does not get it in this perfectly general form. He has to prove the special forms which this identity assumes, according as ω is solenoidal or irrotational. He finds that " $\frac{1}{4\pi}$Pot and $\nabla \times \nabla$ are inverse operators" for solenoidal functions, and that " $\frac{1}{4\pi}$Pot and $-\nabla\nabla.$ are inverse operators" for irrotational functions—all of which is included in the above identity. For if $\omega = \omega_1 + \omega_2$, so that $\text{S}\nabla\omega_1 = 0$ and $\text{V}\nabla\omega_2 = 0$, we get

$$\omega_1 + \omega_2 = \omega = \nabla^{-2}\nabla(\text{V}\nabla\omega_1 + \text{S}\nabla\omega_2).$$

Another of the theorems given, namely,

$$4\pi \text{ Pot } \omega = \text{Lap Lap } \omega - \text{New Max } \omega$$

* Tait proves this well-known equation of Poisson by purely quaternionic methods. It is an interesting commentary on the "simplicity" of the positive sign—the fetish alike of Heaviside and Macfarlane—that the *quaternion* ∇^2 gives 4π positive on the right-hand side!

† Taking his cue from Gibbs, Heaviside might possibly find *Ham* a less "ludicrously inefficient" name than Nabla for the operator ∇.

is simply the quaternion identity

$$4\pi \nabla^{-2}\omega = 4\pi \nabla^{-1} \nabla^{-1}\omega$$
$$= 4\pi \nabla^{-1} V \nabla^{-1}\omega + 4\pi \nabla^{-1} S \nabla^{-1}\omega.$$

Similarly the equation

$$4\pi \, \text{Pot}\, u = -\text{Max New}\, u$$

is a travesty of

$$4\pi \nabla^{-2} u = 4\pi \nabla^{-1} \nabla^{-1} u;$$

and the transformation

$$\omega = \nabla \nabla^{-1}\omega$$
$$= \nabla V \nabla^{-1}\omega + \nabla S \nabla^{-1}\omega$$

is hidden from sight in the symbolic mask

$$4\pi\omega = \nabla \times \text{Lap}\, \omega - \nabla \text{Max}\, \omega.$$

If these and other like equations are to be of any service, what brain is to be expected to carry the memory of them in their Max-Lap-New-Pot forms, which vary according as ω is solenoidal or irrotational? To the *worker* with the quaternion ∇, on the other hand, there is absolutely no difficulty. The functions will come in as they are needed, and transform into useful shapes by the simplest laws of their being. And all this exquisite potentiality of transformation Gibbs wilfully throws to the wind, because a quaternion is not a vector.

There seems little doubt that, in leaving the Hamilton causeway, Professor Gibbs has here fallen into a veritable slough, through which he has himself presumably passed laboriously enough, but wherein the hapless student who would follow will quickly reach the depths of despondency. No finer argument in favour of the real quaternion vector analysis can be found than in the tangle and the jangle of Sections 91 to 104 in the "The Elements of Vector Analysis."

It is little short of marvellous that, with such an appropriate and expressive symbolism and *calculus* as Hamilton has put to our hand, certain writers should think that anything is gained either in lucidity or completeness by notations like "Curl Pot ω," "Div Pot ω." To the uninitiated *Curl* and *Div* are as unintelligible as $V\nabla$ and $S\nabla$; while to the initiated the latter forms are infinitely to be preferred, simply because they belong to a complete and consistent system of analysis, have exactly the properties that belong to other similar operators, and consequently are amenable to analytical transformations of a general kind. The operator ∇, with its wondrous potentiality of transformation and of meaning, was regarded by Hamilton himself as one of his most remarkable creations. We have as yet only a glimmering of its full significance. As if to retard the fuller revelation, we are now asked to hack it into pieces small—into Curl, Div, Lap, Max, New, and Pot.

In the table of comparisons given above it was noted that $\nabla\omega$ had no place in the vector analysis of Professor Gibbs. Later on, however, he introduces a function resembling $\nabla\omega$ in appearance. He calls it the "dyadic," which is a vector operator (see below) but in this case "represents the nine differential coefficients of the three components of [the vector ω] with respect to x, y, and z, just as the vector ∇u represents the three differential coefficients of the scalar u with respect to x, y, and z."

That an operator can represent nine quantities *just as* a vector represents three is a novel mathematical truth; but it is difficult to see what is gained by this dyadic definition of what ought to be a quaternion. On inspection we find that Gibbs's functional operator $\nabla\omega$ corresponds to the quantity $S\sigma\nabla.\omega$ with the σ left out. In other words, it is what might be regarded as the operator on σ if the expression $S(\quad)\nabla.\omega$ could be contemplated from that point of view.

(8.) It seems expedient to look somewhat closely into Gibbs's system of dyadics, which Heaviside regards with such high admiration. For present purposes it will be necessary, occasionally, to use Gibbs's notation; but I shall represent his quantities and operators also in the quaternion symbolism. As usual, he starts off with some four or five "definitions," which of itself suffices to show how lacking in the elements of self-growth his system is.

If we form the quantity $\alpha S\lambda\rho$, we may consider it as an operator $\alpha S\lambda.$ acting on ρ. Professor Gibbs's notation for the quantity is $-\alpha\lambda.\rho$; and then the expression $\alpha\lambda$ he takes as his symbol of operation. That is, he uses the recognised symbolism for a product to represent an operation which by itself has no existence, and which possesses in its distributive quality only one of the ordinary attributes of a product. This operator he calls a *dyad*. The operator,

$$\alpha\lambda + \beta\mu + \gamma\nu,$$

where each term is a dyad, is called a *dyadic trinomial*, or simply a *dyadic*. Any linear and vector function can be expressed as a dyadic.

In his letter to *Nature*, Professor Gibbs argues strongly in favour of the dyadic as a functional symbol. To Hamilton's $S\sigma\phi\rho$ there corresponds $\sigma.(\alpha\lambda+\beta\mu+\gamma\nu).\rho$, in which the dyadic may be considered to act either on ρ or σ. If on σ, then $\sigma.(\alpha\lambda+\beta\mu+\gamma\nu)$ corresponds to Hamilton's $\phi'\sigma$ where ϕ' is the conjugate of ϕ. This conjugate may also be written $(\lambda\alpha+\mu\beta+\nu\gamma).\sigma$. In this double representation of the conjugate function, Professor Gibbs believes that his notation is much more flexible in analysis than Hamilton's, and admits of a freer development than the notation $\alpha S\lambda.+\beta S\mu.+\gamma S\nu$. That may, or may not, be so; but the question is not between the merits of an essentially artificial notation like $\alpha\lambda+\beta\mu+\gamma\nu$ and those of an expanded semi-cartesian form like $\alpha S\lambda\rho+\beta S\mu\rho+\gamma S\nu\rho$. The question is, whether the dyadic can do what the quaternion operator ϕ cannot do.

As an example of the artificiality of the dyadic, take the *definition* of the *direct product** of two dyads (indicated by a dot). Here, by definition

$$\{\alpha\beta\}\,.\,\{\gamma\delta\}=\beta.\gamma\alpha\delta.$$

Quaternions gives at once

$$\phi\psi\rho=\alpha S\beta(\gamma S\delta\rho)+\&c.=\alpha S\delta\rho S\beta\gamma+\&c.$$

There then follow the definitions of the *skew* products of ϕ and ρ, thus

$$\phi\times\rho=\alpha\lambda\times\rho+\beta\mu\times\rho+\&c.$$
$$\rho\times\phi=\rho\times\alpha\lambda+\rho\times\mu\beta+\&c.$$

These are operators or dyadics. To see what they mean, let them operate on some vector σ. Then we find

$$\phi\times\rho.\sigma=\alpha S\lambda\rho\sigma+\beta S\mu\sigma+\&c.=\phi V\rho\sigma$$
$$\rho\times\phi.\sigma=V\rho\alpha S\lambda\sigma+V\rho\beta S\mu\sigma+\&c.=V\rho\phi\sigma.$$

* Gibbs calls the quantity $\phi.\sigma$ (which is simply Hamilton's $\phi\sigma$) the *direct* product of the dyadic ϕ and the vector σ. The *direct* product of two vectors is $\alpha.\beta(=-S\alpha\beta)$, and this Heaviside calls the scalar product. Similarly translating the Gibbsian dialect, he speaks of $\phi\sigma$ as being the "scalar product of the dyadic and the vector"—and gets a scalar product equal to a vector! This "is most tolerable and not to be endured." Gibbs's own use of *direct* and of its symbolic "dot" in two quite different senses is itself open to criticism.

The first is simply $\phi\omega$, the old thing, and the second, $V\rho\phi\sigma$, is a very important quantity in the theory of the linear and vector function. Professor Gibbs continues :—" It is evident that

$$\{\rho \times \phi\} \, . \, \psi = \rho \times \{\phi \, . \, \psi\}$$
$$\{\rho \times \phi\} \, . \, a = \rho \times [\phi \, . \, a] \text{ \&c.,"}$$

with other three similar equations, all of which are meant to establish that the associative principle holds. In quaternion notation there is no difficulty, the two sides of the identity in nearly every instance having exactly the same form. Thus, the two given above are $V\rho\phi\psi\sigma$ and $V\rho\phi a$, the σ being introduced to make the significance of the former at once apparent. The comment on the dyadic equation

$$\psi \, . \, \{\rho \times \phi\} = \{\psi \times \rho\} \, . \, \phi$$

is that "the braces cannot be omitted without ambiguity." The quaternion expression is $\psi V\rho\phi\sigma$, where there is no chance of ambiguity, where everything is perfectly straightforward, and where there is much greater compactness in form. It seems to me that this last equation given by Gibbs condemns his whole principle of notation. It shows that one grand use of connecting symbols is to obscure the significance of a transformation ! It is interesting to note—as bearing upon the *intelligibility* of the notation—that Heaviside, who dotes so on the dyadic, writes $\phi \times \rho$ in the form $V\phi\rho$, so that he makes

$$\phi V\rho\sigma = -V\sigma\phi\rho \text{ ! !}$$

In the course of the development of the theory of the dyadic, Gibbs, with his usual proneness to lexicon products, invents a few names (or new meanings to old ones), such as Idemfactor, Right Tensor, Tonic, Cyclotonic, Shearer, and so on. These are all special forms of the linear and vector function ; and, excepting possibly the names, Professor Gibbs does not seem to have contributed anything of value to Hamilton's beautiful theory. In no case, so far as I have been able to see, do his methods compare, for conciseness and clearness, at all favourably with Hamilton's and Tait's. Take by way of further illustration the expression of $V\lambda'\mu'$ in terms of $V\lambda\mu$, where $\lambda' = \phi\lambda$ and $\mu' = \phi\mu$. At the very outset of the quaternion investigation we find

$$V\lambda'\mu' = m\phi'^{-1}V\lambda\mu$$

where m represents what unit volume becomes under the influence of the linear operator. The dyadic form of $m\phi'^{-1}$ is

$$\{\beta \times \gamma\mu \times \nu + \gamma \times \alpha\nu \times \lambda + \alpha \times \beta\lambda \times \mu\},$$

which Professor Gibbs further proposes (in *Nature*) to symbolise by $\phi{}^{\times}_{\times}\phi$. Thus, burden after burden, in the form of new notation, is introduced apparently for the sole purpose of exercising the faculty of memory. Further on in the pamphlet the quantity m is introduced in the form $[\phi]$, and is called the *determinant* of the dyadic; and then we are told that the relation of the surfaces $V\lambda'\mu'$ and $V\lambda\mu$ "may be expressed by the equation"

$$V\lambda'\mu' = [\phi]\phi_c^{-1}V\lambda\mu.$$

May be expressed!—as if this concise quaternion representation were a mere notation, overshadowed by the effulgence of the beautiful line-long bracket of dots and crosses given above.

(9.) On page 42 of Gibbs's pamphlet we have a beautiful example of giving back with the left hand what has been sternly removed with the right. We read:—"On this account we may regard the dyad as the most general form of product of two vectors. We shall call it the indeterminate product." And then he shows how to obtain a vector and a scalar "from a dyadic by insertion of the sign of skew or direct multiplication."

This is exquisite. From the *operator* $\alpha\lambda + \beta\mu + \gamma\nu$ he forms—heedless of his high-toned scorn for anything like a quaternion product—the conception of the sum of three such products, but quiets his conscience by calling them *indeterminate!* This sum of products then becomes, by simple insertion of dots and crosses, the vector

$$\phi_\times = \alpha \times \lambda + \beta \times \mu + \gamma \times \nu$$

and the scalar

$$\phi_s = \alpha.\lambda + \beta.\mu + \gamma.\nu$$

Language is impotent to characterise aright this remarkable feat of jugglery. "The quaternion product," we are told in effect, "is an abomination. But here is a nice thing we call the dyad, a vector operator of the purest blood, which (if occasion serve) may nevertheless be regarded as the most general [and therefore non-vectorial] form of product of two vectors. Being called indeterminate, it is not really of any account, except in so far as it yields two very useful quantities, the one a vector and the other a scalar. No connection, of course, with the quaternion."

The quaternion, indeed, must come in, though it be but fitfully; and Professor Gibbs is virtually obliged to introduce it in his treatment of the *versor*, which he regards as a special form of dyadic. To educe its properties he uses those quantities $\phi_\times$ ϕ_s formed by a so-called indeterminate process from the operator ϕ. He discovers, as any quaternionist could have told by intuition, that "$-\phi_\times$ and ϕ_s determine the versor without ambiguity." Observe the *negative* sign before the vector; and note the obstinacy of the writer in refusing to recognise explicitly the essentially quaternion character of the versor.

The expression

$$\{2\beta\beta - I\}\cdot\{2\alpha\alpha - I\}$$

represents in Gibbs's notation "a versor of which the axis is perpendicular to α and β, and the amount of rotation twice that which would carry α to β. It is evident that any versor may be thus expressed, and that either α or β may be given any direction perpendicular to the axis of rotation." Here we have the quaternion idea (in one simple instance of it) expressed as clearly as language can express it. The formula just given (in which "I" represents an *idemfactor*,—that is, unity) transforms at once into the quaternion form $\beta\alpha(\quad)\alpha\beta$, or more simply $q(\quad)q^{-1}$, where q is the quaternion that turns any vector (α) perpendicular to its axis through a definite angle. In this incomparable form we find for successive rotations the equation

$$rq(\quad)q^{-1}r^{-1} = rq(\quad)(rq)^{-1}$$

Taking $\theta_1 Sq = Vq$, $\theta_2 Sr = Vr$,

we have

$$\theta_3 = \frac{V(rq)}{S(rq)} = \frac{V.VrVq + VrSq + VqSr}{SVrVq + SrSq}$$

$$= \frac{\theta_1 + \theta_2 + V\theta_2\theta_1}{1 + S\theta_2\theta_1}$$

a perfectly general formula which Gibbs takes half a page to demonstrate, and even then his demonstration applies to versors only.

As bearing upon this subject let us inquire how Professor Gibbs would write the quaternion equation

$$\nabla . q\alpha(S\alpha\theta)q^{-1} = 0$$

where α may be any constant vector, and where the object is to find q and θ. This is one of the latest forms in which Professor Tait has expressed the historic problem of finding series of orthogonal isothermal surfaces. To represent it in Gibbs's notation we must first take the scalar and vector parts separately, for ∇ has no place in his system. Then to express $q(\quad)q^{-1}$ we must take some two vectors δ, ϵ, in terms of which the dyadics are to be expressed. One of these may be *any* vector in the plane perpendicular to the axis of q; so that in so expressing the versor we are really giving more than ought to be required. We then find for the compact and most expressive equation just given the following two

$$\nabla . \{\delta\delta - I\} . \{\epsilon\epsilon - I\} . \alpha\alpha . \theta = 0$$
$$\nabla \times \{\delta\delta - I\} . \{\epsilon\epsilon - I\} . \alpha\alpha . \theta = 0$$

in which $\nabla .$ and $\nabla \times$ act on both θ and ϵ, and where α may be any constant vector, say one of a rectangular system. The *expanded* quaternionic forms

$$S\nabla . \delta\epsilon(\alpha S\alpha\theta)\epsilon\delta = 0$$
$$V\nabla . \delta\epsilon(\alpha S\alpha\theta)\epsilon\delta = 0$$

are even more compact than these. We willingly leave to Professor Gibbs himself the task of translating the expression

$$S . \nabla q q^{-1} \nabla_1 . q_1 q^{-1}$$

where the suffix means that the second ∇ is to act only on the q which immediately follows it.

It has been already noted that Professor Gibbs refuses* to have anything to do with the quantity $\nabla\omega$ where ω is any vector. He uses the expression in the sense of a dyadic, that is, as an operator. Thus

$$d\rho . \nabla\omega = d\rho . \left(i\frac{d\omega}{dx} + j\frac{d\omega}{dy} + k\frac{d\omega}{dz}\right)$$
$$= d\omega$$

* If we except one extraordinary case to be noticed below.

In quaternions

$$-Sd\rho\nabla.\omega=d\omega$$

Yet he recognises that $\nabla\omega$ has a possible significance other than an operator when he writes $\{\nabla\omega\}_s$ and $\{\nabla\omega\}_\times$ as equivalent expressions for $\nabla.\omega$ and $\nabla\times\omega$. These suffixes are simply another way of *selecting* convenient bits of a thing complete in itself. They are more troublesome to write, distinctly more cumbrous in appearance, and less amenable to analytical transformations than are Hamilton's time-honoured forms. Besides, in quaternions, the treatment is uniform throughout.

(10.) There is something almost naïve in the way in which Heaviside, in § 192 of "Electromagnetic Theory" (*The Electrician*, November 18, 1892), introduces the dyadic as if nothing like it was to be found in either Hamilton or Tait. The truth is, it is all there. The "dyadic" of Heaviside's equations (137) and (138) is stated to be "of a very peculiar kind, inasmuch as its resultant effect on any vector is to reproduce that vector." It is what Gibbs calls an *idemfactor*. But it amounts to nothing more than equating a vector to the sum of its components along three given directions. The equations are simply Hamilton's ancient

$$\rho S\alpha\beta\gamma=\alpha S\beta\gamma\rho+\beta S\gamma\alpha\rho+\gamma S\alpha\beta,$$

expressed in the abridged form

$$\rho=\alpha S\alpha_1\rho+\beta S\beta_1\rho+\gamma S\gamma_1\rho$$

This is an almost fundamental formula, and yet it is characterised as being "very peculiar." As Hamilton showed long ago, if

$$\phi\rho=\alpha S\lambda\rho+\beta S\mu\rho+\gamma S\nu\rho$$

then

$$\phi^{-1}\rho=\lambda_1 S\alpha_1\rho+\mu_1 S\beta_1\rho+\nu_1 S\gamma_1\rho,$$

where $\alpha_1 S\alpha\beta\gamma=V\beta\gamma$, &c., &c.,

and $\lambda_1 S\lambda\mu\nu=V\mu\nu$, &c., &c.,

and then that "very peculiar" dyadic is seen to be $\phi\phi^{-1}=1$. Now Heaviside fusses greatly over this method of inverting ϕ; and any reader of § 172 would infer that the inventor of the name dyadic was the first to give this demonstration which Hamilton and Tait had somehow missed in their development of "the very clumsy way" of expressing $\phi^{-1}\rho$ in terms of ρ, $\phi\rho$, and $\phi^2\rho$. But the whole thing is given in Hamilton's *Elements* (p. 438, Equation XXVII.), and in Tait's *Quaternions* (p. 89, 2nd edition; p. 123, 3rd edition). Of course, this particular way of inverting ϕ depends on the particular semi-cartesian form into which it is thrown. The rare beauty of Hamilton's method, however, lies in its generality. For this Mr Heaviside has apparently only scoffings.

In § 171 of the same series of articles, Heaviside criticises some of Professor Tait's methods of developing the quaternion calculus. His criticisms are equally valid against Hamilton's own methods. Regarding these he says :—

"The reader is led to think that the object of the investigation is to invert a linear operator—that is given $[\rho = \phi\sigma]$ to find $[\sigma = \phi^{-1}\rho]$. But if this were all it would be a remarkable example of how not to do it. For the inversion of a linear operator can be easily effected by other far simpler and more natural means. The mere inversion is nothing. It is the cubic equation itself that is the real goal. The process of reaching it is simplified by the omission of inverse operations."

But what says Tait in § 174 of the 3rd edition (§ 162 of the 2nd) :— "It is evident from these examples that for special cases we can usually find modes of solution of the linear and vector equation which are simpler in application than the general process of § 160. The real value of that process, however, consists partly in its enabling us to express inverse functions of ϕ, such as $(\phi - g)^{-1}$ for instance, in terms of direct operations, a property which will be of use to us later; partly in its leading us to the fundamental cubic . . . . . whose interpretation is of the utmost importance with reference to the axes of surfaces of the second order, principal axes of inertia, the analysis of strains in a distorted solid, and various similar enquiries."

Could words be plainer or more emphatic? The most lenient hypothesis is, that our self-appointed critic has not really read Tait's *Quaternions*.

Then, as to the much vaunted "simplified" process of reaching the cubic, what does it amount to? It consists in the highly original trick of writing

$$m\mathrm{V}\mu\nu = \phi'\mathrm{V}\phi\mu\phi\nu$$

instead of

$$m\phi'^{-1}\mathrm{V}\mu\nu = \mathrm{V}\phi\mu\phi\nu \ !$$

(11.) Throughout his pamphlet, for reasons given in his letter to *Nature*, Gibbs refuses to admit that the complete product of two vectors has any claim upon his attention. He, nevertheless, smuggles it in, as we saw in § 9, so as to facilitate his treatment of the "dyadic" versor. If it is not really needed, then we must look upon it as a kind of catalytic agent. In general, however, the expression $\alpha\beta$ is taken to mean a "dyad" or operator of the form $\alpha S\beta$ or $\beta S\alpha$, according as the operand is affixed or prefixed.

What, then, are we to understand by the following equations :—

$$\iint d\sigma\omega = \iiint dv\nabla\omega \qquad (2) \text{ of } \S\ 164$$

and

$$\int d\rho\omega = \iint d\sigma \times \nabla\omega \qquad (2) \text{ of } \S\ 165$$
$$= \iint \nabla d\sigma\nabla . \omega$$

in quaternion symbolism, where by $d\sigma$ is meant νds, ν being unit vector drawn perpendicular to the surface element ds. Now, these equations are perfectly true in their ordinary quaternionic interpretation; but if we are to credit Gibbs with consistency, we cannot believe that he means us to regard $d\sigma\omega$, $d\rho\omega$, and $\nabla\omega$ as quantities (*i.e.*, quaternions). They ought to be dyads. If we take them as *prefactors*, we simply reproduce No. 1 of the respective sets of equations, namely,

$$\iint d\sigma u = \iiint dv\nabla u \text{ and } \int d\rho u = \iint d\sigma \times \nabla u$$

But if we take them as *postfactors*, we get for the first case,

$$\iint \omega S\nu\tau ds = \iiint S\tau\nabla . \omega dv,$$

an equation which is interpretable and true only if $S\nabla\tau = 0$,—that is, for an incompressible fluid. Similarly, the second equation becomes

$$\int \omega S\tau d\rho = \iint S\tau\nu\nabla . \omega ds,$$

an equation which is interpretable and true only if $S\nu\nabla\tau = 0$. But, since equations (2) are meant to be perfectly general, we must conclude that Gibbs, after all, does use the expressions $d\sigma\omega$, $d\rho\omega$, and $\nabla\omega$ in the sense of quantities. To what end, then, was his *Nature* letter written, and why does his pamphlet not contain the explicit definitions of the meanings of such quaternions?

A favourite argument used alike by Gibbs and Heaviside is, that even avowed quaternionists work mostly with vectors and scalars and rarely with quaternions. But this surely betokens a total misapprehension of the whole significance of the calculus. In trigonometrical analysis we work mostly with sines, cosines, tangents, secants, and so on. But, because sin θ occurs a myriad times for one occurrence of θ itself, do we not then deal with *angles* or *arcs* in trigonometry? By his notation Gibbs tries—with what success we have seen—to sink the quaternion quite out of sight. But Heaviside uses Hamilton's vector notation; and this notation emphasises the truth, that a quaternion is being dealt with. The selective symbols V and S are as quaternionic as T, U and K. The equation

$$V\rho\phi\sigma = 0$$

is as fundamentally a quaternion equation as is

$$\nabla . q\theta q^{-1} = 0$$

And, although Gibbs gets over a good deal of ground without the explicit recognition of the complete product, which is the difference of his "skew" and "direct" products, yet even he recognises in plain language the versorial character of a vector, brings in the quaternion whose vector is the difference of a linear vector function and its conjugate, and does not hesitate to use the accursed thing itself in certain line, surface, and volume integrals.

The principal arguments of the present paper may be conveniently summarised thus:—

(1.) The quaternion is as fundamental a geometrical conception as any that Professor Gibbs has named.

(2.) In every vector analysis so far developed, the versorial character of vectors in product combinations cannot be got rid of.

(3.) This being so, it follows as a natural consequence that the square of a unit vector is equal to negative unity.

(4.) The *assumption* that the square of a unit vector is positive unity leads to an algebra whose characteristic quantities are non-associative, but in no sense more general than the corresponding but associative quaternion quantities, and whose ∇ is not the real efficient *Nabla* of quaternions.

(5.) The invention of new names and new notations has added nothing of importance to what we have already learned from quaternions.

The true entrance into vector analysis is by quaternion gate. Some there be who have climbed over definition fences, or meandered in along "pleasant green" lanes. But these are not the true pilgrim; and their character will declare itself when a real and novel difficulty has to be surmounted. One indispensable accoutrement is the pure ∇, and that is out and out quaternionic.

But from the attacks of these innovators, Hamilton, one of the greatest analysts of any age, scarce needs defender. He is "the strong man fully armed." "When a stronger than he, coming upon him, shall overcome him" then—but when shall that be? There is, as yet, no sign of *him!*

On some Modifications of the Water-Bottle and Thermometer for Deep-Sea Research. By J. Y. Buchanan.

(Read January 10, 1893.)

The instruments exhibited were especially fitted for working with the "messenger," a perforated weight which is allowed to glide down the line or wire at a suitable interval behind the apparatus which it is to act on. The apparatus immediately acted on in this case is a *trigger*, or disengaging hook, of very simple construction, which is attached independently to the line above the principal apparatus. Separating the arrangement for freeing the apparatus (whether thermometer or water-bottle or other instrument) has many advantages. Being very light, it can be attached to the line by twine, or to the wire by a small clamp; and the principal apparatus can be attached in any convenient way, without it being necessary to pay minute attention to the knots or hitches that may be required. Further, in almost all cases, the principal apparatus and the separate trigger are much cheaper than the two combined. The separate trigger was exhibited attached to two forms of water-bottle for collecting water at intermediate depths. In one, which is a modification of a bottle for collecting bottom water, and is figured and described in the *Challenger Reports, Narrative,* vol. i. p. 117, the weights, which are provided to fall down and rest on the india-rubber valves *K* and *H* in the figure, are suspended by a line or wire to the trigger, and when the messenger falls on the trigger the line is slipped and the weights fall on the valves, which are then fixed, and enclose the sample. The pressure on the valves may be equally well produced by springs.

The other water-bottle was one of the stop-cock type, with which all the intermediate waters of the Challenger Expedition were collected. It is figured and described in *Challenger Narrative*, p. 113, and consists essentially of a straight tube with a stop-cock at each end. The stop-cocks are worked by levers, which are connected by a rod, so as always to be equally open or equally shut. When the messenger is not used, the connecting-rod carries a metal flap, which exposes its edge to the passing water during descent, but falls out at right angles to the axis of the instrument when the motion is reversed. The resistance which it thus offers to the passage of water produces a downward pressure sufficient to shut the stop-cocks and enclose the sample. This arrangement acts perfectly well, the only objection to it being that it requires a certain amount of practice and experience. For this reason the collection of intermediate waters on board the Challenger was never delegated, but was always superintended by myself.

Where the "messenger" is used, no skill and no experience are required. The stop-cocks are supported by a line or wire from the upper end of the connecting-rod to the trigger. When the messenger falls on the trigger, the stop-cocks close by the weight of the connecting-rod, and, if necessary, attached weights. When used without the messenger, the point which requires experience and practice is to regulate the stiffness of the stop-cock, so that there shall be no danger of its closing by the weight of the connecting-rod and flap during descent, but shall certainly close when exposed to the entire pressure produced by the passage of the extended flap through the water. When the stop-cocks are fixed open until the messenger strikes the trigger, no such adjustment is necessary; and the connecting-rod can be made as heavy and the stop-cocks as free as may be desired, so that there may be no question about the stop-cocks closing immediately they are released. This arrangement was used on board the yacht "Princess Alice," owned and commanded by H.S.H. Prince Albert of Monaco, during a cruise in the Mediterranean in September 1892.

The "messenger" is by no means a recent invention; it has been used by many early investigators, even in water of some depth. It was notably used by Aimé in his very remarkable investigations in the Mediterranean in the years 1846–48. Its great merit is certainty. It is often objected that it is too wasteful of time to be used in any considerable depth. But this is not so. The messenger, like any other body falling through water, attains a maximum and constant velocity before it has fallen more than a few fathoms. Its rate of descent can be perfectly accurately ascertained to a second by determining the time taken to reach the trigger at (say) 50 and 150 fathoms. The difference between the two times gives the velocity in seconds per hundred fathoms. I have made many experiments of this kind, both on sounding line and on wire, and I have invariably found the greatest uniformity in the rate for the same "messenger" and the same line. On board the "Princess Alice" the messenger used was very heavy; it consisted of a ring of lead, weighing rather over 2 kilogrammes. Its impact on the trigger, at a depth of 1000 metres, was felt distinctly at the surface, wire being used. It was found, by experiment, to descend

	200 metres in	150 seconds.
	500 ,, ...	380 ,,
	1000 ,, ...	752 ,,
Average	100 ,, ...	75 ,,

This rate is very uniform, but slow. In the "Buccaneer," where hemp line was used with the water-bottle, a messenger of sheet lead, rolled into a cylinder fitting loosely over the line, descended at the rate of 50 fathoms in 26 seconds, or 115 fathoms per minute. The slower rate on the wire is due to its cutting deeply into the lead. Whatever the rate may be, I have always found it very constant, as, in fact, it ought to be; because, after the maximum velocity has been reached, there is no reason why it should suffer alteration.

The system of sounding line and weights and instruments, after the maximum velocity has been reached, descends at a constantly decreasing rate, owing to the increased resistance of the continually increasing length of line. When wire is used, the speed is more uniform. In all cases, however, experience shows that it is possible to calculate to a minute or so when the instruments will reach the required depth, and how long the messenger will take to reach the same depth. With all thermometers it is necessary to allow them to remain a certain time in order to take the temperature of the medium. In the "Princess Alice" the thermometers were allowed to remain 10 minutes down, and there was no difficulty in despatching the messenger so that it would reach the required depth about 10 or 11 minutes after the thermometer. In this way we have all the certainty of the method without any appreciable loss of time.

An objection is sometimes advanced that, when the line "leads" out at a great angle, the messenger will not run down and close the instruments. This, however, must be looked on as rather an advantage than otherwise, because observations made under these circumstances should be rejected, and are therefore much better left unrecorded.

A very important operation in oceanographic research is taking the temperature of the water at a series of depths, and collecting samples of it for examination. The more easily and expeditiously this can be done, the more likely is it to be done, and the greater will be the number of samples and temperatures appearing in each series. With the thermometer and water-bottle, each attached independently to the line, a certain interval must necessarily be between them. This necessitates stopping the line when the thermometer comes up, reading it, hoisting the water-bottle up so as to be in position for tapping off the water, resetting the water-bottle, lowering again a little, resetting the thermometer, and then letting go. Recent experience has shown me that it would be a very great convenience if an arrangement could be made so that the water-bottle and thermometer came up at once and at the same level, suitable for reading, tapping, resetting, and then letting go. The differential motion of the connecting-rod, with regard to the body of the water-bottle, seemed to offer the means. The brass tube, meant to hold the reversing thermometer, is pivoted at its lower end to a socket which slides over the connecting-rod, and can be pinched to it by a screw. The cover of the brass tube carries a groove or collar, into which a brass hook engages when the instrument is upright and the stop-cocks open. The brass hook gears into the projecting extremity of the upper stop-cock lever, and consequently has a differential and downward motion with regard to the connecting-rod when it is falling and the stop-cocks are closing. This downward motion disengages it from the collar on the upper part of the thermometer tube, which springs outwards and turns through 180°, and engages with the lower part of the connecting-rod, being now in the reversed position. The method of fixing the thermometer in its reversed position is very simple. A hook is screwed into the heel of the tube, and in the place of its rotation. When upright, this hook is on the inside, and a large brass ring which encircles the connecting-rod engages in it. When the thermometer falls outward, the hook pulls the ring outwards, and it presently drops down over the thermometer, encircling both it and the connecting-rod, and necessarily keeping the thermometer in its place. The instruments exhibited did not have the ring, which was fitted the next day.

An arrangement was also exhibited by which tubes, such as those used by Professor Pettersson in the collection of samples of water for the determination of their gaseous contents, can be attached to the connecting-rod, and the finely drawn cut end be broken off by the motion of the lever in shutting the stop-cocks.

The differential motion of the connecting-rod and the water-bottle (which may be replaced by another rod) affords the means of easily performing a number of simple laboratory manipulations in the deep sea; for instance, collecting samples of water over mercury. In water of any considerable depth, exhausted tubes, such as Pettersson used in Scandinavian waters, would be almost certain to collapse, if not by the pressure alone, by the pressure assisted by the momentary disturbance of structural equilibrium produced by the sudden breaking of the point. In order to collect water in glass tubes at great depths, recourse must be had to the method by displacement of mercury, which is easily effected by means of the differential motion deep-sea frame. The parallel motion gas bracket, manufactured by Messrs Milne, and much used in drawing offices, supplies a frame of the kind almost ready for attachment to the sounding line.

Synthesis by Means of Electrolysis. By Professor Crum Brown and Dr James Walker. Part VI.

(Read January 10, 1893.)

Abstract.

Synthesis of Tetramethylsuccinic Acid.—Ethyl-potassium dimethylmalonate cannot be prepared from dimethylmalonic ether by following precisely the method which succeeded in all previous cases. When the ether is mixed, either at once or slowly, with the calculated quantity of caustic potash in alcoholic solution, half of the ether is completely saponified, while the other half is unacted on, only an insignificant fraction being half-saponified and converted into the ethyl-potassium salt. A good yield of the latter was, however, obtained by a modification of the process, the special points of which were great dilution with alcohol and low temperature.

By electrolysis of the ethyl-potassium salt in the manner described in former papers, an ethereal product was obtained, of which a large part boiled between 240° and 250°. This portion was mixed with its own volume of fuming hydrobromic acid and heated for twenty hours to 110°. The product was neutralised with caustic soda and distilled with steam, when ethyl bromide and some undecomposed ether distilled over. What remained in the distilling flask was then acidified and again distilled with steam. Part of the distillate solidified in the condenser in white crystals. What remained in the flask was dissolved in potash, evaporated to a small bulk, and decomposed with acid. The acid thus obtained was filtered off and purified by recrystallisation from alcohol. It was very slightly soluble in cold, considerably more in hot water: easily soluble in alcohol and benzene, less so in ether. Combustion gave the following results :—

0·1323 gramme substance gave 0·2669 gramme CO_2 and 0·0947 gramme H_2O.

	Calculated for $C_8H_{14}O_4$.	Found.
C . .	55·17	55·02
H . .	8·05	7·96

On quickly heating the acid fused (with decomposition) at 195°, a small quantity was converted into the anhydride, which, after purification by recrystallisation from hot ligroin, formed fine needles fusing at 147°–148°.

The electrolytic conductivity of the acid was measured, with the result K = 0·0311. Bethmann obtained 0·0314 for the acid prepared by Auwers and Meyer.

Electrolysis of Ethyl-potassium Diethylmalonate.—Ethyl-potassium diethylmalonate was prepared from diethylmalonic ether exactly as described above for the preparation of the corresponding dimethyl compound, and was electrolysed in the usual way. The ethereal product gave a large fraction boiling below 230°. The residue boiling above 230° might be presumed to contain tetraethylsuccinic ether, but no acid could be obtained from it by saponification. By distilling at reduced pressure a somewhat viscid, colourless oil was obtained, boiling constant at 170° under pressure of 12 mm. of mercury. It was perfectly neutral, insoluble in water, miscible with alcohol, and had a specific gravity of 1·0082 at 13°·5 compared with water at 4°. Combustion gave the following results:—

I. 0·1194 gr. gave 0·2845 gr. CO_2 and 0·1076 gr. H_2O
II. 0·2193 „ 0·5230 „ 0·1965 „

	Calculated for $C_{14}H_{26}O_4$.	Found. I.	Found. II.
C . . .	65·12	64·98	65·03
H . . .	10·08	10·01	9·97

The substance is therefore not tetraethylsuccinic ether, but contains in the molecule C_2H_4 less than this.

When heated with fuming hydrobromic acid it yielded ethyl bromide and a new neutral substance, insoluble in water, but soluble in alcohol and in benzene. It was purified by recrystallisation from benzene and from ligroin. In the cold it was odourless, but when warmed it had a smell exactly like that of camphor. It fused at 84°·5. Analysis gave the following results:—

I. 0·1342 gr. gave 0·3341 gr. CO_2 and 0·1146 gr. H_2O
II. 0·1683 gr. gave 0·4180 gr. CO_2 and 0·1436 gr. H_2O
III. 0·1528 gr. gave 0·3796 gr. CO_2 and 0·1307 gr. H_2O

	Calculated for $C_{12}H_{20}O_3$.	Found. I.	II.	III.
C . .	67·92	67·90	67·75	67·76
H . .	9·43	9·49	9·48	9·51

0·542 gramme dissolved in 7·90 gramme alcohol raised the boiling point of the latter by 0°·360, corresponding to molecular weight 219. 0·540 gramme dissolved in 18·16 grammes of glacial acetic acid lowered the freezing point of the latter by 0·530 corresponding to molecular weight 214. $C_{12}H_{20}O_3$ has molecular weight 212.

The substance obstinately resisted the action of acids and bases. It is obvious that it is not tetraethylsuccinic anhydride, but an isomeric substance, with perhaps such a formula as

$$\begin{array}{l} \quad C_2H_5—C———C—C_2H_5 \\ \qquad\quad\;\; \| \qquad\;\; \| \\ C_2H_5—O—C—O—C—O—C_2H_5 \end{array}$$

or furfuran to which the replacement of all the hydrogen may have given extra stability.

Dr Hugh Marshall examined the substance crystallographically. His report is given in full in the paper.

Secondary Reactions.

Besides the ethers $C_2H_5OOC{\cdot}R''{\cdot}R''{\cdot}COOC_2H_5$ there are produced by the electrolysis of the salts $C_2H_5OOC{\cdot}R''{\cdot}COOK$ other substances which may be called secondary products. There is always some oxidation, which does not, however, lead to any complication, but only to loss, as the products are practically carbonic acid and water. In almost all cases unsaturated ethers are formed in accordance with equations such as—

$$2C_2H_5OOC{\cdot}R''{\cdot}CH_2{\cdot}CH_2{\cdot}COO- = C_2H_5OOC{\cdot}R''{\cdot}CHCH_2$$
$$+ C_2H_5OOC{\cdot}R''{\cdot}CH_2{\cdot}CH_2{\cdot}COOH + CO_2.$$

$$\begin{array}{l} 2C_2H_5OOC{\cdot}C(CH_3){\cdot}CH_2 = C_2H_5OOC{\cdot}C(CH_3){:}CH_2 + C_2H_5OOC{\cdot}C(CH_3){\cdot}CH_2{\cdot} \\ \qquad\qquad\quad | \qquad\quad | \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad | \qquad\quad | \\ \qquad\qquad COO- \quad H \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad COOH \;\; H \end{array}$$

These unsaturated ethers have a much lower boiling point than the synthetic ethers formed at the same time, and can therefore be easily separated from them. They are formed in considerable quantity from the higher terms of the normal series, and seem to be produced more readily from the acids with side chains.

Methylacrylic Acid.—From the ethereal product of the electrolysis of ethyl-potassium dimethylmalonate a fraction boiling between 115° and 125° was obtained. From this, by saponification, and decomposition of the potassium salt with acid, the acid was prepared. It was purified by recrystallising its calcium salt. The purified acid had, when warmed, a very penetrating smell, and decolorised bromine in solution in bisulphide of carbon. When heated to about 130° it was suddenly transformed into a white solid mass, which agreed in all respects with the description given by Fittig and Engelhorn * of the polymer of methylacrylic acid. Dried at 130–135 it gave the following results on combustion :

0·1396 gr. gave 0·2846 gr. CO_2 and 0·0881 gr. H_2O.

	Calculated for $(C_4H_6O_2)_n$.	Found.
C . . .	55·81	55·60
H . . .	6·97	7·01

The potassium in the potassium salt of the original acid was determined.

0·2895 gramme gave 0·2024 gramme K_2SO_4, *i.e.* 31·4 per cent. potassium, as compared with 31·5 calculated for $KC_4H_5O_2$. When cooled with ice the acid solidified, and fused again at 14°. Methylacrylic acid fuses at 16°.

Ethylcrotonic Acid.—As already mentioned, a considerable part of the ethereal product from the electrolysis of ethyl-potassium diethylmalonate boiled at a comparatively low temperature. From a fraction boiling between 150° and 170°, ethylcrotonic acid was prepared by the method described for methylacrylic acid. The purified acid fused at 41°–42°. Ethylcrotonic acid fuses at 41°.†

Combustion gave the following results :—0·1285 gramme gave 0·2969 gramme CO_2 and 0·1027 gramme H_2O.

	Calculated for $C_6H_{10}O_2$.	Found.
C . . .	63·16	63·02
H . . .	8·77	8·88

The dibromide was prepared from the acid. It fused at 80°. Fittig and Howe gave 80°·5 as the fusing point.‡

* Fittig and Engelhorn, *Liebig's Annalen* 200, 70.

† Fittig and Howe, *Liebig's Annalen* 200, 23.

‡ *Liebig's Annalen* 200, 35.

The quantity of ethereal product boiling at a comparatively low temperature was much smaller in the case of the monoalkyl malonic acids than with the dialkyl acids. From methylmalonic acid, acrylic ether, and from ethylmalonic acid a crotonic ether might be expected. Small quantities of ethers probably containing acrylic ether and a crotonic ether respectively were obtained, but not identified with certainty. In the case of the crotonic ether it seemed to be accompanied by the corresponding saturated ether (butyric ether), a circumstance not theoretically unlikely.

Products from Sebacic Acid.—By careful working it was found possible to raise the yield of *n*-dicarbodecahexanic ether from 20 to about 40 per cent. of the theoretical. The solid product of the reaction was spread on porous plates, when about two-thirds were absorbed. What remained was nearly pure *n*-dicarbodecahexanic ether. The porous plates were broken up and extracted with ether in a fat-extracting apparatus. The ethereal extract was dried and distilled at atmospheric pressure. The chief fractions were, one between 240° and 270°, and one between 280° and 310°. The residue was mainly *n*-dicarbodecahexanic ether. The fraction 240°–270° consisted essentially of sebacic ether.

The oily acid from the fraction 240°–270° was purified as far as possible, but as the quantity was small the acid was not obtained in a perfectly pure state. Analysis indicated the formula $C_9H_{16}O_2$. 0·1881 gramme gave 0·4755 gramme CO_2 and 0·1785 gramme H_2O.

	Calculated for $C_9H_{16}O_2$.	Found.
C . . .	69·23	68·94
H . . .	10·26	10·54

The acid united slowly with bromine at ordinary temperature.

It was obviously an unsaturated acid and a normal product of the electrolysis. The barium was determined in its barium salt: 0·2106 gramme of the salt dried at 130° gave 0·1085 gramme $BaSO_4$.

	Calculated for $Ba(C_9H_{15}O_2)_2$.	Found.
Ba . . .	30·6	30·3

Limits of Applicability of the method of Electrolytic Synthesis of Dibasic Acids.—Experiments made with ethyl-potassium fumarate, ethyl-potassium maleate, ethyl-potassium citraconate showed that the method was not directly applicable to unsaturated acids of the fatty series. Similarly, ethyl-potassium phthalate gave no ethereal products on electrolysis.

The same was the case with ethyl-potassium tartrate and with ethyl-potassium dibromosuccinate, which gave large quantities of free bromine at the anode. So far, therefore, the method seems to be restricted to saturated fatty dibasic acids. The authors are not without hopes of discovering means of making it applicable to hydroxy-acids.

Behaviour of Oxalic Acid.—Ethyl-potassium oxalate, prepared by Claisen's method,* which gave most satisfactory results, was submitted to electrolysis, in order to see if the general equation applies in this case, where $R'' = 0$. If it were so, oxalic ether would be formed :

$$2C_2H_5OOC{\cdot}COO- \quad = \quad C_2H_5OOC{\cdot}COOC_2H_5 + 2CO_2.$$

The result showed that the reaction does not take this course, but that the anion is oxidised to carbonic acid, water and ethylene.

As Claisen has suggested that perhaps his method of preparing ethyl-potassium oxalate might be applicable to other dibasic acids, experiments were made with succinic ether and with some of the alkylmalonic ethers. The result with these ethers was that, where the ether was attacked at all, the dipotassium salt was formed.

Behaviour of Camphoric Acid.—Camphoric acid was found to behave exactly like a saturated dibasic fatty acid, ethyl-potassium camphorate giving on electrolysis large quantities of ethereal products. These are now being examined.

* *Berichte der deutschen chemischen Gessellschaft* 24, 127.

On the Volume Effects of Magnetisation. By Prof. Cargill G. Knott, D.Sc., and A. Shand, Esq.

(Read July 4, 1892.)

(*Second Note.*)

In a previous communication (see page 85) the remarkable changes of capacity of an iron tube when magnetised were described. In the present note it is proposed to record a few of the more striking results since obtained.

The main object was to compare the volume changes in the interior of five tubes of Swedish iron, similar in every respect, except in size of bore, and subjected to the same magnetising forces. The tubes were distinguished by number. They were cut from the same original bar, and were as nearly as may be of the same length and external diameter. When prepared for experiment, each tube was closed at the originally open end by a screw-stopper, through which projected a fine capillary glass tube which was in connection with the interior volume. The volumes and internal radii of the various tubes were as follows:—

Tube.	Volume of Bore.	Radius of Bore.
I.	344·83 cub. cm.	1·60 cm.
II.	223·40 „	1·28 „
III.	127·41 „	·965 „
IV.	67·70 „	·70 „
V.	18·81 „	·35 „

The external diameter of each tube was 3·84 cm.

In every case the dilatation due to magnetisation was obtained by dividing the measured change of volume by the corresponding total volume of the bore.

One peculiar feature which was dwelt on in the previous note was the manner in which Tube I. behaved as the field was taken larger and larger. In very small fields there was *positive* dilatation which, however, soon changed to *negative*, and continued so till about field 150, when it became positive again. This phenomenon of the initial positive dilatation was found to exist in the other tubes as well. But a careful study of all the circumstances showed that its existence depended on the previous magnetic history of the iron. In every case, Tube V. excepted, the phenomenon did not declare itself when the tube was treated for the first time. Under this condition, the dilatation began negative in the lowest fields. It was only after the tube had been left residually magnetised by the action of a larger field that the application of a very small field produced this positive dilatation. The result is, in fact, a particular case of a more general phenomenon. It is a case of magnetic after-effect.

The phenomenon, as it is shown with Tubes I., II., III., and IV., may be briefly described as follows :—

When a smaller field is applied after a larger field has been applied and removed, the contraction of volume is less when the successively applied fields are in the same direction than when they are in opposite directions. When the second field is small enough the contraction changes sign, becoming a positive dilatation when, and only when, this field has the *same direction* as the previously applied stronger field. For, when the small field is applied in the *reverse direction* to the previous strong field, this change of sign never takes place. It is obvious, indeed, that in magnetically unbiassed iron the change of volume should be exactly the same in whatever direction the field is applied. Now whenever, in the case of the tubes specified, this positive dilatation for low fields showed itself, the mere reversal of field produced a negative dilatation. Such a change of sign with the field demonstrated a magnetic bias. And even when the field was not low enough to produce this positive dilatation, an exactly similar magnetic bias was indicated by the different contraction produced when the field was simply reversed. To get rid of this bias, due to the after-effect of a previously applied stronger field, it was sufficient to subject the iron to a succession of gradually diminishing fields, alternating in direction. This method of *neutralising** by reversals left the iron in an approximately unbiassed condition, and quite destroyed the tendency to positive dilatation in low fields.

With Tube V., however, positive dilatation always showed itself in certain low fields. That this was not due to magnetic bias, was proved by the fact that the reversal of the field failed to change the sign of the dilatation.

* "Demagnetising" by reversals, it is usually but unfortunately called.

In all the tubes, and especially in Tube V. of narrowest bore, another interesting illustration of after-effect was observed. After a given field had been applied and removed several times in succession, the meniscus in the capillary tube, which by its motion measured the change of volume, first moved slightly backward, and then turned and completed its total forward motion. On the first application of the same field in the reverse direction this backward "flash" was not observed. But if we continued to remove and apply the field in its new direction, this initial spring-back unmistakably declared itself at each application of field. The phenomenon is, in fact, essentially the same as that already described, and depends upon the current and its associated field not attaining their maximum values at once. With the powerful electro-magnet, which of necessity formed part of the circuit, the current, and therefore the field, rose in value sufficiently slowly to permit of the small initial values having their peculiar effect upon the biassed iron.

In the final series of experiments on all the tubes, magnetic bias was removed as far as possible, and every field was applied in both directions. The very close agreement between the dilatations produced by a given field, taken first in one and then in the other direction, was a sufficient demonstration of the absence of any strong bias.

The broad results are embodied in the following table, which gives the dilatations in all the tubes for certain definite fields. Any one column, excepting the first, relates to one tube. Each row gives the dilatations due to one particular field, whose value is entered to the left in the first column :—

Table of Cubical Dilatations $\times 10^7$.

Tube. / Field.	I.	II.	III.	IV.	V.
50	−3·6	−1·5	−2	−1	+1·3
100	−3	−6·5	−9·5	−4	+0·2
150	+0·4	−7·2	−18·6	−12	−8·0
200	+2	−5·7	−17	−20	−12
300	+3	−4·4	−12·6	−24	−19
500	+3·5	−3·7	−14	−31	−34
700	+3·2	−3	−15·8	−40·5	−51·5
1000	+3·8	−3·3	−17·8	−46	−84
1400	+3·8	−3	−20	−53	−129

The full significance of these numbers cannot be judged of till we have measured the linear dilatations of the tubes when subjected to the same magnetising forces. There are, however, certain very interesting conclusions which may at once be drawn.

1. Positive dilatations in higher fields are shown only in the tube of widest bore (Tube I.); and positive dilatations in low fields occur only in the tube of narrowest bore (Tube V.).
2. A maximum negative dilatation is found only in the first three tubes, and the field in which the maximum occurs is higher for the narrower bore or thicker wall. The graphs for Tubes IV. and V., however, show an undulation suggestive of a maximum.
3. By running the eye along the successive rows, or better, by study of the graphs, we have evidence of the gradual penetration of the magnetisation in through the walls, which become thicker in the order I., II., III., IV., V.
4. The large values of the dilatations in the tubes of narrowest bore are particularly worthy of note. In Tubes IV. and V. there is no evidence of a limit being approached; the inner surfaces seem far from a condition of saturation.
5. A comparison of the results for Tubes I. and II. suggests the existence of a tube of intermediate bore, which would show no change of internal volume in moderate and high fields; and this again suggests the existence, within the substance of a magnetised bar, of a surface enclosing a volume of zero dilatation. This surface, beginning at or near the external surface, advances inwards towards a limiting position as the field is increased.

Note on the Law of Transformation of Energy and its Applications. By W. Peddie, D.Sc.

(Read May 16, 1892.)

The investigation of the relations which subsist amongst various physical quantities, and which are exemplified in the different transformations of energy, is one of the most interesting investigations in the whole range of physics. On the other hand, the mathematical methods which are usually employed for this purpose are of such a nature as to be unavailable to any who are unacquainted with higher mathematical methods. It is one of the objects of the present note to point out how these relations may be worked out by means of the most elementary methods. For the notation of the calculus, though used below as a matter of convenience, is quite unnecessary, since we might use, *e.g.*, small and capital type of the same letter to denote quantities of the same kind of different magnitudes.

The first relations of this kind which were investigated were those exemplified in the transformation of heat into mechanical work. In any such transformation, the dynamical principle of conservation of energy holds good; but it was found necessary to introduce another principle—the Second Law of Thermodynamics—before the problem could be fully solved; and this principle is not explicitly dynamical, but is based upon an axiom in agreement with experience. Further, when we pass from thermo-mechanical transformations to transformations between two forms of energy of which heat is not one, no law corresponding to the Second Law of Thermodynamics has been formulated. On the contrary, it is generally stated that, in these cases, this method reduces to the use of the principle of conservation alone—a principle which is often not sufficient for the purpose. A second object of this note is to show that the application of the principle of conservation to a certain cycle of reversible transformations directly expresses the principle of degradation of energy and leads to the generalised analogue of the Second Law.

Let us denote the quantities which fix the physical condition of the system considered by the letters a, b, c, &c. The change of energy in any process is

$$dE = \frac{\partial E}{\partial a}da + \frac{\partial E}{\partial b}db + \frac{\partial E}{\partial c}dc + \ldots\ldots$$

or

$$dE = Ada + Bdb + Cdc + \ldots\ldots$$

where A, B, C, &c., are, in the ordinary language of dynamics, *forces* of the types which cause the changes da, db, dc, &c. They are the quantities upon the magnitude of which the tendency to transference, or transformation, of energy depends. [In the dynamical view, transformation is merely transference to a dissimilar system.] When the system returns, at the end of the process, to its original condition, we get

$$Ada + Bdb + Cdc + \ldots = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This equation is the expression of the principle of conservation, for it asserts that any amount of energy of the type (A, a), which enters the system, passes out of it again either in the form (A, a) or in some other form into which it is changed by the medium of the system, and we get finally $da = 0$, $db = 0$, &c.

If now we apply the principle to a closed reversible cycle of operations, in which equilibrium is only disturbed to an infinitesimal extent at any stage, and in which, for example—1st, A increases by dA, while a is constant; 2nd, a increases by da, while A is constant; 3rd, A decreases by dA, while a is constant; 4th, a decreases by da, while A is constant, we get

$$d.dE = dA.da + dB.db + dC.dc + \ldots = 0 \quad . \quad . \quad (2)$$

In general, if the above cycle is strictly observed with regard to the energy (A, a), it cannot be simultaneously carried out for any other type. But the actual cycle for any other type may be supposed to be built up of an infinite series of such cycles on an infinitely smaller scale, so that the final effect is the same as if the cycle were carried out.

Suppose that we are dealing with two types of energy only. In this case (2) becomes

$$dAda - dBdb = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

[Here the sign has been changed for convenience. The question is discussed fully later.] It is easy to see that equation (3) is in reality no less general than equation (2); for, before we could apply (2) to any particular case in which (say) energy of the type (A, a) is transformed into other types, we should have to break up the total quantity of energy of type (A, a) into its several parts, which are transformed, as far as possible, into the several types; and we should then have to consider each part separately. That is to say, we should have to resolve equation (2) into a series of equations of the form (3).

We may write (3) in the form

$$dAda \quad = d\,.\,d\beta\,,$$

or

$$\frac{dA}{A}.Ada = d\,.\,d\beta\,,$$

i.e.,

$$\frac{dA}{A}da \quad = d\,.\,d\beta\,, \qquad \qquad (4)$$

where da is the amount of energy of the type (A, a) which enters the system in the first operation of the cycle described above, and $d\beta$ is the amount of the type (B, b) which is developed by transformation. The equation (4) expresses the principle of degradation, for it asserts that only the fraction dA/A of the whole amount of energy of the type (A, a) which is supplied is changed into energy of the type (B, b). If a_1 be the whole amount of the type (A, a) which is supplied, while a_2 is the amount of that type which remains untransformed, we may write (4) in the form

$$\frac{A_1 - A_2}{A_1}a_1 = a_1 - a_2\,.$$

This equation gives a definition of A on an absolute scale (which includes Kelvin's definition of absolute temperature as a special case), for a generalisation of Carnot's reasoning regarding the efficiency of a reversible heat-engine shows that the working of our more general reversible system is independent of the nature of the working-substance. It may be put in the form

$$\Sigma\left(\frac{a}{A}\right) = 0\,,$$

which applies to a completed reversible cycle, and includes, as a particular case, the usual analytical expression of the Second Law of Thermodynamics. It is preferable, however, to work with the equivalent equation (3).

To find whether the positive or negative sign must be used in that equation, consider A and B as external "forces." If A tends to cause increase (or decrease) of a while B tends to cause increase (or decrease) of b, the negative sign is to be used; if otherwise, the positive sign must be used. Again, if we wrote (3) thus, $dAda \pm dbdB = 0$, we must take the opposite sign to that which we take when we write the second term in the order $dBdb$. For the cycle represented by $dbdB$ is performed in the opposite order to that represented by $dBdb$.

Collecting the results we get, when the $\mp$ sign is used in (3),

$$\left(\frac{dA}{db}\right)_{a \text{ const.}} = \pm\left(\frac{dB}{da}\right)_{b \text{ const.}} ; \quad \left(\frac{dA}{dB}\right)_{a \text{ const.}} = \mp\left(\frac{db}{da}\right)_{B \text{ const.}} \quad . \quad (5)$$

$$da = \pm A\frac{dB}{dA}db = \mp AdB\frac{db}{dA} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

A few special examples are appended.

I. *Heat and Mechanical Work.*—Let θ, ϕ, p, v represent respectively temperature, entropy, pressure, and volume. The quantity $d\theta d\phi$ represents energy supplied to the system, while $dpdv$ represents work done by it. Hence

$$d\theta d\phi + dpdv = 0 ,$$

giving $\frac{d\theta}{dv} = -\frac{dp}{d\phi}$, $\frac{d\theta}{dp} = \frac{dv}{d\phi}$, $\frac{d\phi}{dp} = \frac{dv}{d\theta}$, $\frac{d\phi}{dv} = -\frac{dp}{d\theta}$, the four well-known thermodynamical relations.

II. *Latent Heat and Work during Evaporation.*—Let l be the amount of liquid, and let ρ be its density, while σ is the density of the vapour, and λ is the latent heat. From $d\theta d\phi + dpdv = 0$ we get by (6)

$$d\theta = dp\frac{dv}{d\phi}$$

Now
$$dv = \left(\frac{1}{\sigma} - \frac{1}{\rho}\right)dl$$

whence
$$d\theta = dp\frac{\rho - \sigma}{\sigma\rho}\frac{dl}{d\phi} .$$

But $$\lambda dl = \theta d\phi,$$

so $$d\theta = dp\frac{\theta}{\lambda}\,\frac{\rho - \sigma}{\rho\sigma},$$

which gives the change of the boiling-point dependent on the change dp of pressure. The result applies also to the change in the melting-point, ρ being the density of the solid.

III. *Heat and Electric Energy.*—Let I and i represent respectively electromotive force and quantity of electricity. Then $d\mathrm{I}di$ and $d\theta d\phi$ both represent energy supplied to the system, so that $d\mathrm{I}di - d\theta d\phi = 0$, whence

$$\frac{\theta d\phi}{di} = -\theta\frac{d\mathrm{I}}{d\theta}$$

represents the amount of heat which must be withdrawn from the system (say a voltaic cell), during the passage of unit quantity of electricity, in order that the temperature may remain constant. This is Helmholtz's well-known expression.

If I represents the (internal) electromotive force in a thermo-electric circuit we get

$$\frac{d\mathrm{I}}{d\theta} = \frac{1}{\theta}\cdot\frac{\theta d\phi}{di} = \frac{\pi}{\theta}$$

where π is the heat absorbed at the junction when unit quantity of electricity passes across it at temperature θ.

IV. *Electrical Energy and Work.*—Consider a sphere of capacity C charged to potential V. The electrical energy is $\frac{1}{2}\mathrm{CV}^2$. Hence $d\mathrm{V}d(\mathrm{CV}) + dpdv = 0$, and

$$dp = d\mathrm{V}\frac{d(\mathrm{CV})}{dv} = \mathrm{V}d\mathrm{V}\frac{d\mathrm{C}}{dv}$$

since we have to take the value of $d(\mathrm{CV})/dv$ when V is constant. Now $v = \frac{4}{3}\pi\mathrm{C}^3$, whence

$$dp = \mathrm{V}d\mathrm{V}\frac{1}{4\pi\mathrm{C}^2}\cdot$$

If Q be the total charge of electricity

$$dp = \frac{\mathrm{Q}d\mathrm{Q}}{4\pi\mathrm{C}^4}\cdot$$

Hence we get the known expression for p, the pressure due to the total charge Q,

$$p = \frac{1}{8\pi}\frac{Q^2}{C^4} = \frac{R^2}{8\pi}$$

where R is the resultant electric force at the surface of the sphere. If we suppose electric energy to be stored in the medium, we see that $Q^2/8\pi C^4$ or $R^2/8\pi$ is also the expression for the energy per unit of volume at the surface.

V. *Electrification and Vapour-Pressure.*—Let E be the electrical energy contained in a liquid sphere of radius C and density σ. We have

$$\frac{dE}{dC} = -\frac{1}{2}\frac{Q^2}{C^2}.$$

If m represents the mass of the vapour we have

$$dm = -4\pi C^2 \sigma dC.$$

Also we have

$$\frac{dE}{dm}\cdot dm = \frac{dE}{dC}\frac{dC}{dm}dm = \frac{Q^2}{8\pi C^4 \sigma}dm \equiv dp dv.$$

But $dv = -\left(\frac{1}{\rho} - \frac{1}{\sigma}\right)dm$, ρ being the vapour density. Hence

$$dp = -\frac{1}{8\pi}\frac{\rho}{\sigma - \rho}\frac{Q^2}{C^4}$$

which agrees with J. J. Thomson's result.

VI. *Surface-Tension and Vapour-Pressure.*—Let T and S represent respectively the surface-tension and the surface of a drop of liquid, and let the other quantities be indicated as in the previous case. We get $dTdS - dpdv = 0$. Now $dS = 8\pi C dC = -\frac{2dm}{C\sigma}$ $= \frac{1}{C}\frac{2\rho dv}{\sigma - \rho}$. Hence

$$dp = 2\frac{dT}{C}\frac{\rho}{\sigma - \rho},$$

or, summing from $T = 0$ to $T = T$,

$$p - p_0 = 2\frac{T}{C}\frac{\rho}{\sigma - \rho}$$

where p and p_0 represent respectively the equilibrium-pressure of the vapour in contact with a plane surface and a spherical surface of radius C. This result was first given by Lord Kelvin.

VII. *Induced Electromotive Forces.*—Consider two circuits in which electric currents of intensities i and j respectively circulate similarly. Let the coefficient of mutual induction be M, and let us take as an experimental result the expression

$$f = i\frac{d}{dx}(j\mathrm{M})$$

for the force f which tends to produce a displacement dx. If I and J are the electromotive forces acting in the circuits, we have

$$fdx = id(j\mathrm{M}) = d\mathrm{I} \, . \, idt$$

$$d\mathrm{I} = \frac{d}{dt}(j\mathrm{M}) \, .$$

Similarly,

$$d\mathrm{J} = \frac{d}{dt}(i\mathrm{M}) \, .$$

On the Particles in Fogs and Clouds. By John Aitken, Esq.

(Read February 6, 1893.)

(*Abstract.*)

Cloud Particles.

At the beginning of the paper some observations made on the water particles in clouds on the Rigi on the 21st of May last are described. Previous observations with the fog-particle counter had shown that there is a relation between the density of a cloud and the number of water particles observed. On the occasion above referred to the number was very much greater than corresponded with the density. It is pointed out that the number of dust particles in the air which become centres of condensation depends on the rate at which the condensation is taking place, quick condensation causing a large number of particles to become active, slow condensation causing a small number; and that after the condensation has ceased a process of differentiation takes place, the larger particles robbing the smaller ones of their water, owing to the vapour-pressure at the surface of drops of large curvature being less than at the surface of drops of smaller curvature. The particles in a cloud are by this process reduced in number, those remaining becoming larger and falling quicker, the cloud thus tending to become thinner by the reduction of the number of particles and by the falling of some of them. It is shown that the exceptional readings above referred to, obtained on the Rigi, were owing to the observations then made being taken in a new and rapidly-formed cloud, due to the strong wind blowing at the time causing a quick ascent and rapid cooling and condensation, the result being the formation of a large number of very small water particles. Though the number was very great, the particles were so small they were only just visible with great care with the magnifying power used in the instrument. Previous observations on cloud particles had been made in slowly-formed or in old clouds after the process of differentiation had been in play for some time, and after the drops had been reduced in number and increased in size.

Fog Particles.

Fog particles, when formed in pure air, conduct themselves like cloud particles, and they have much the same appearance and size when seen on the micrometer of the fog-particle counter. Differentiation takes place in the particles of fogs also, and after the fog has been formed some time the particles are large and fall rapidly. Some fogs seem to clear away in this manner by raining themselves out of existence. But in town fogs the conditions are much more complicated, owing to the impurities in the air caused by the formation, during the combustion of coal and other causes, of a great number of nuclei which have an affinity for water vapour. This affinity of the nucleus for vapour not only determines condensation and thickening of the atmosphere before it is saturated, but after condensation has taken place this affinity resists the differentiating process. The affinity of the nucleus for vapour resists the tendency which the smaller particles have to evaporate, and the differentiating process is stopped when the particles are reduced to a certain size. The number of particles in a town fog thus tends to be great, and to remain great, while in a country fog they tend to be few, and to fall. There are thus *persisting fog particles* and *particles which tend to vanish.* The one clings the stronger to water the smaller it is, the other parts with it the freer the smaller it is. The affinity of the nucleus thus not only increases the density of the fog, but it prevents the natural decay.

These points were illustrated by filling two large glass receivers with air containing different kinds of nuclei. One receiver was filled with the products of combustion from a paraffin-lamp, the other with the products from a gas-flame, in which was burned a very little sulphur. The two receivers were connected by means of tubes, so as to be under the same conditions as regards cooling when the air in them was expanded. When the pressure was slowly lowered, the chilling produced a condensation in both receivers, but the one containing the sulphur in the products was much the denser. To show that the air in the receiver containing the lamp products had plenty of nuclei to give a dense form of fogging, this receiver was disconnected from the other one, and the air in it expanded a second time, but the second time the expansion was made very rapidly. The result was a condensation as dense as that given by the products containing the sulphur impurities. But it was only at the moment of complete quick expansion that the products from the lamp gave as dense a fog as the other; in a few seconds it began to clear, owing to differentiation setting in and rapidly reducing the number of drops. The upper limit of the fog could also be seen to be descending, the air in the upper part of the receiver being quite clear of fog, and in a few minutes all the fog had gone, having rained itself to the bottom of the receiver. While these changes were taking place in the one receiver, the fog formed in the impure air of the other had cleared but little, and it showed no tendency to fall, the fogging extending to the upper part of the receiver, quite filling it. This fog remained thick for an hour, and could be detected hours afterwards. The fog in this receiver illustrated the characteristics of a town fog, and that in the other the characteristics of a country fog.

It is concluded that it is not so much the number as the composition of the particles produced by combustion which causes the density and persistence of town fogs. A large number of particles without affinity for water vapour give a dense form of fogging only when the condensation is rapid, more rapid than it generally is in nature; and even when they do form a dense condensation it rapidly becomes clearer by differentiation. It would thus appear that if we wish to reduce the density of town fogs, it is not so much a question of perfection of combustion, as of changing the nature of the products of combustion. Perfect combustion will reduce the amount of smoke, but it will not reduce the density of the fogging proper. To effect this we must change the nature of the sulphur and other products, and convert them into products which have no affinity for water vapour. It is hoped that some substance may be found which will admit of being burned along with the coal, or of being added to the air, which will change the water-attracting particles of town air into particles indifferent to vapour, so as to form fog particles having a tendency to clear.

The influence of the products of combustion of a few substances in producing fogs is referred to. Alcohol burned at a plantinum burner, or in an open vessel without a wick, scarcely increases the number of condensing nuclei; but if burned at a wick, and the flame shows the sodium yellow colour, the increase in nuclei is very great. A paraffin-lamp increases the number greatly, a candle gives but little increase, while a wax vesta or wooden match greatly increases the number. But in all these cases the particles have but little affinity for water vapour, and do not cause fogging unless the air tends to be supersaturated. The products of the combustion of coal from a perfectly clear fire, and from a stove burning char and glowing red to the top, both gave a large increase in the number of particles, and many of the particles had an affinity for water, causing fogging in moist air without supersaturation; and if a little ammonia was present in the air, the fogging was very dense, and in both cases *persistent.*

The condition of fog particles when the temperature is below the freezing-point is shortly referred to. The lowest temperature at which observations have as yet been made with the fog-particle counter is 27° Fahr. after a night minimum of 24°. In this as well as in all the other cases the particles were unfrozen. The effect on certain meteorological phenomena of the particles being in a liquid condition at temperatures below the freezing-point is referred to.

A new Solution of Sylvester's Problem of the Three Ternary Equations. By the Hon. Lord M'Laren.

(Read January 16, 1893.)

The solution is effected by the method of Symmetric Functions.

By a preliminary transformation* the equations of the original system are put into the form

$$\left.\begin{array}{l} AX^2-2C'XY+BY^2=0 \\ BY^2-2A'YZ+CZ^2=0 \\ CZ^2-2B'ZX+AX^2=0 \end{array}\right\}.$$

The system is to be again transformed by dividing each equation by BY^2, and putting $\xi=\frac{x}{y}\cdot\frac{\sqrt{A}}{\sqrt{B}}$; $\zeta=\frac{z}{y}\cdot\frac{\sqrt{C}}{\sqrt{B}}$; giving

$$\left.\begin{array}{l} \xi^2-\frac{2C'}{\sqrt{AB}}\cdot\xi+1=0 \\ 1-\frac{2A'}{\sqrt{BC}}\cdot\zeta+\zeta^2=0 \\ \zeta^2-\frac{2B'}{\sqrt{CA}}\cdot\zeta\xi+\xi^2=0 \end{array}\right\} \textit{i.e.,} \left\{\begin{array}{ll} \xi^2-\epsilon_1\xi+1=0 & \quad \ldots\ldots(1) \\ \zeta^2-\epsilon_2\zeta+1=0 & \quad \ldots\ldots(2) \\ \zeta^2-\epsilon_3\xi\zeta+\xi^2=0 & \quad \ldots\ldots(3) \end{array}\right.$$

The roots of (1) and (2) may be denoted respectively by a and $\frac{1}{a}$; b and $\frac{1}{b}$. Now, as (3) has a root of ξ in common with (1), and has also a root of ζ in common with (2), if we substitute in (3) successively the four systems of values of ξ and ζ, and multiply together the four resulting expressions, *one* of the factors must satisfy (3), and therefore their product will vanish.

The four factors are—

$$\left(a^2-\epsilon_3ab+b^2\right),\ \left(\frac{1}{a^2}-\epsilon_3\frac{1}{ab}+\frac{1}{b^2}\right),\ \left(a^2-\epsilon_3\frac{a}{b}+\frac{1}{b^2}\right),\ \text{and}\ \left(\frac{1}{a^2}-\epsilon_3\frac{b}{a}+b^2\right).$$

But since the 1st and 2nd factors (when cleared of fractions) are

* The transformation is effected by dividing the original equations respectively by x^2y^2, y^2z^2, z^2x^2, and then putting X, Y, Z for $\frac{1}{x}$, $\frac{1}{y}$, $\frac{1}{z}$. The transformation used by Professor Tait in his paper (which I have seen in proof) would equally answer the purpose of this solution.

identical, and the 3rd and 4th factors are also identical, we need only multiply together the 1st and 3rd factors, and write

$$\left(a^2 - \epsilon_3 ab + b^2\right)\left(a^2 - \epsilon_3 \frac{a}{b} + \frac{1}{b^2}\right) = 0 \quad . \quad . \quad . \quad . \quad (4)$$

On multiplying out, dividing by a^2, and arranging the terms in symmetric functions, we have

$$a^2 + \frac{1}{a^2} + b^2 + \frac{1}{b^2} - \epsilon_3\left(a + \frac{1}{a}\right)\left(b + \frac{1}{b}\right) + \epsilon_3{}^2 = 0 \quad . \quad . \quad (5)$$

In order to express the symmetric functions in terms of the coefficients of (1) and (2), we observe that

$$a + \frac{1}{a} = \epsilon_1 \,;\; a^2 + \frac{1}{a^2} = \epsilon_1{}^2 - 2 \,;\; b + \frac{1}{b} = \epsilon^2 \,;\; b^2 + \frac{1}{b^2} = \epsilon_2{}^2 - 2 \,;$$

whence, substituting in (5),

$$\epsilon_1{}^2 + \epsilon_2{}^2 + \epsilon_3{}^2 - \epsilon_1\epsilon_2\epsilon_3 - 4 = 0 \,, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

i.e.,

$$4\left\{\frac{C'^2}{AB} + \frac{A'^2}{BC} + \frac{B'^2}{CA} - 2\frac{A'B'C'}{ABC} - 1\right\} = 0$$

or

$$CC'^2 + AA'^2 + BB'^2 - 2A'B'C' - ABC = 0 \quad . \quad . \quad . \quad (7)$$

The reason why, in this solution, the eliminant is obtained in its true dimension (not squared), is because in the operation (4) one of each of the pairs of identical factors is thrown out.

By making (1), (2), and (3) homogeneous we see that the solution consists in reducing the given equations to the form,

$$\left.\begin{array}{l} x^2 - \epsilon_1 xy + y^2 = 0 \\ y^2 - \epsilon_2 yz + z^2 = 0 \\ z^2 - \epsilon_3 zx + x^2 = 0 \end{array}\right\}$$

the eliminant of which is the quadratic,

$$\epsilon_1{}^2 + \epsilon_2{}^2 + \epsilon_3{}^2 - \epsilon_1\epsilon_2\epsilon_3 = 4 \,.$$

Meetings of the Royal Society—Session 1891–92.

Monday, 23rd November 1891.

General Statutory Meeting. Election of Office-Bearers. *P.* xix. 1.

Monday, 7th December 1891.

Sir Douglas Maclagan, M.D., President, in the Chair.

1. The President gave an Opening Address. *P.* xix. 2.

The following Communications were read :—

1. The Chemistry of Strophanthidin, a Decomposition Product of Strophanthine. By Professor T. R. Fraser, M.D., F.R.S., and Leonard Dobbin, Ph.D. *T.* xxxvii. 1.

2. On Glissette Surfaces. By the Hon. Lord M'Laren.

The following Candidates for Fellowships were balloted for, and declared duly elected Fellows of the Society :—

George Robert Milne Murray, Natural History Department, Brit. Museum.
Robert C. Mossman, F.R. Met. Soc.

Monday, 21st December 1891.

Dr William Craig in the Chair.

The following Communications were read :—

1. Obituary Notice of the late Andrew Young, F.G.S. By the Rev. Dr Flint, V.P. *P.* xix. p. lxiii.

2. A Preliminary Communication on the Electrical Resistance of Various Urines. By Dr Dawson Turner, F.R.C.P.E. Communicated by Dr Crum Brown, F.R.S. *P.* xix. 20.

3. On some Eurypterid Remains from the Upper Silurian Rocks of the Pentland Hills. By Malcolm Laurie, B.Sc., F.L.S. Communicated by R. H. Traquair, M.D., F.R.S. *T.* xxxvii. 151.

4. On the Lateral Sense-Organs of Elasmobranchs. II. The Sensory Canals in *Raia batis.* By Professor J. C. Ewart, M.D., and J. C. Mitchell, B.Sc., University of Edinburgh. *T.* xxxvii. 87.

Monday, 4th January 1892.

Sir William Turner, M.B., Vice-President, in the Chair.

The following Communications were read :—

1. Note on a Theorem regarding a Series of Convergents to the Roots of a number. By T. MUIR, Esq., LL.D. *P.* xix. 15.

2. On the Development of the Lung-books of Scorpio, and the Relation of the Lung-books to the Gills of Aquatic forms. By MALCOLM LAURIE, B.Sc., F.L.S. Communicated by Professor COSSAR EWART.

3. On the Number of Dust Particles in the Atmosphere of certain Places in Great Britain and on the Continent, with remarks on the relation between the Amount of Dust and Meteorological Phenomena. Part II. By JOHN AITKEN, Esq., F.R.S. *T.* xxxvii. 17.

4. The action of the Valves of the Mammalian Heart. (With Lime-light Illustrations.) By D. NOËL PATON, Esq., M.D. *T.* xxxvii. 179.

The following Candidate for Fellowship was balloted for, and declared to be a duly elected Fellow of the Society :—

J. W. BALLANTYNE, M.D., F.R.C.P.E.

Monday, 18th January 1892.

Professor Chrystal, LL.D., Vice-President, in the Chair.

The following Communications were read :—

1. On Circular Magnetization accompanying Axial and Sectional Currents along Iron Tubes. By Professor C. G. KNOTT. *T.* xxxvii. 7.

2. On Tactics adopted by certain Birds when flying in the Wind. By R. W. WESTERN, Esq. Communicated by A. B. BROWN, Esq. *P.* xix. 76.

3. Ptomaines extracted from Urine in certain Infectious Diseases. By Dr A. B. GRIFFITHS. *P.* xix. 97.

4. On Impact, II. By Professor TAIT. T. xxxvii. 381.

5. On the Critical Isothermal of Carbonic Acid, as given by Amagat's Experiments. By Professor TAIT. *P.* xix. 32.

Monday, 1st February 1892.

Rev. Professor Flint, D.D., Vice-President, in the Chair.

The following Communications were read :—

1. The latest Physical Geography from Greenland. By C. PIAZZI SMYTH, LL.D., F.R.S.E., and late Astronomer-Royal for Scotland.

2. On the Equilibrium and Pressure of Arches, with a Practical Method of Ascertaining their true shape. Illustrated by examples. By R. Brodie, Esq.

3. Note on the Isothermals of Mixtures of Gases. By Professor Tait.

4. By permission of the Society, Dr Buchan communicated to the Meeting Observations of Temperature and Pressure in Edinburgh and Ben Nevis on the morning of February 1st, 1892, showing a sudden rise of Pressure and fall of Temperature.

The following Candidates for Fellowships were balloted for, and declared duly elected Fellows of the Society :—

Thomas Heath, B.A., Assistant Astronomer, Royal Observatory, Edinburgh.
J. H. Meining Beck, M.D.
H. J. Gifford.
Thomas Parker, Memb. Inst. C.E.
W. J. Brock, M.B., D.Sc.

Monday, 15th February 1892.

Sir William Turner, F.R.S., Vice-President, in the Chair.

The following Communications were read :—

1. Obituary Notice of Thomas Miller, LL.D. By Dr J. S. Mackay. *P.* xix. p. i.

2. On the New Star in the Constellation *Auriga.* By The Astronomer-Royal for Scotland. *T.* xxxvii. 51.

3. The lesser Rorqual (*Balænoptera rostrata*) in the Scottish Seas, with Observations on its Stomach. By Sir William Turner, F.R.S. *P.* xix. 36.

4. On the Relation between Kinetic Energy and Temperature in Liquids. By Professor Tait. *P.* xix. 32.

Monday, 29th February 1892.

Sir William Turner, F.R.S., Vice-President, in the Chair.

The following Communication was read :—

The Cranial Nerves of Man and Selachians. By Professor Ewart, M.D.

Monday, 7th March 1892.

The Rev. Professor Flint, D.D., Vice-President, in the Chair.

A copy of the "Rudolf Virchow Medal," presented by the Committee, was laid before the Society, and it was agreed that the Society express its thanks officially.

The following Communications were read :—

1. On the Changes in the Chemical Composition of Sea Water associated with Marine Blue Muds. By Dr JOHN MURRAY and ROBERT IRVINE.
2. On the Manganese Nodules in the Marine Deposits of the Clyde Sea Area. By Dr JOHN MURRAY and ROBERT IRVINE.
3. Note on Specimens of Chalk from Christmas Island in the Indian Ocean. By Dr JOHN MURRAY.
4. On a Crystalline Globulin occurring in Human Urine. By Dr D. NOËL PATON. *P.* xix. 102.
5. Additional Note on the Isothermals of CO_2 at volumes less than the critical volume. By Professor TAIT.

The following Candidates for Fellowships were balloted for, and declared duly elected Fellows of the Society :—

Professor J. T. MORRISON, M.A., B.Sc.
Rev. JOHN KERR, M.A.

Monday, 21st March 1892.

The Hon. Lord M'Laren in the Chair.

PRIZES.

The Keith Prize for 1889-91 was presented to R. T. OMOND, Esq., for his Contributions to Meteorological Science, many of which are contained in Vol. xxxiv. of the Society's *Transactions.*

LORD M'LAREN, on presenting the Prize, said :—

Mr R. T. Omond was appointed Superintendent of the Ben Nevis Observatory in the summer of 1883, and during these years he has carried on the work of the Observatory with conspicuous success, and it is to be added that particularly during the first winter this was done in the face of formidable difficulties. At the same time, he has undertaken and completed several inquiries of great importance in Meteorology, most of which have appeared in the Society's publications.

His first inquiry resulted in determining, by Robinson's anemometer, the exact numerical value at the rate of miles per hour of the estimations of wind-force made by the Observatory staff. An important paper followed on the formation of snow crystals from fog, which is one of the most striking phenomena of the meteorology of Ben Nevis. The conditions of growth, and rate of growth, of these crystals were detailed, as well as their intimate bearing on the methods to be adopted in carrying on any satisfactory system of the observation of air temperatures at High Level Observatories.

He has calculated the mean temperature of every day of the year, both for the Observatory and Fort William, and investigated their relations to each other, and the striking differences as regards their amount and the varying times of occurrence of their means, maxima, minima, and other noteworthy variations in the rise and fall of the annual curves. The question of the variation of the temperature at different heights above the ground has also been investigated. He has paid particular attention to coronas, halos, glories, and the other optical phenomena exhibited in sunshine by mountain mists. These have been examined by a very extensive series of measurements and accurate records of the different arrangements of colours of these phenomena, which are justly regarded as an important, and to a large extent novel, contribution to this branch of physics.

The relative frequency of the different winds has been ascertained, together with the percentage of the whole rainfall which falls with each wind. The relation of the results to cyclonic and non-cyclonic periods has been shown, from which it follows that four-fifths of the whole rainfall is precipitated during the passage of cyclones across that part of Europe. In non-cyclonic periods, almost the whole falls with N., N.W., and W. winds. The marked types of weather that accompany anticyclones according to the direction in which they lie to the Observatory at the time, and the equally marked wetness or dryness of particular winds according as they blow out of an anticyclone, or inwards upon a cyclone, have been clearly worked out. The diurnal variation in the direction of the summer winds has also been ascertained.

He has recently completed an exhaustive examination of the winds of Ben Nevis, in conjunction with Mr Rankin. In this paper it is shown that while the winds at the High Level Observatories on the

Continent exhibit the closest agreement with the winds at low levels in the same regions, the winds at the top of Ben Nevis, on the contrary, differ widely from the mean direction of the winds near sea level in that part of Great Britain. As regards Ben Nevis, this observed difference is a peculiarity of cyclonic regions, and thus marks out the Ben Nevis winds not merely different to, but as being wholly different in kind from, the winds of High Level Continental Observatories. But perhaps the most valuable result of the inquiry is the proof given that the veering of the wind at great heights, as required by the current theory that a cyclonic storm is a whirling column drawing the air in spirally below and pouring it out spirally above, is not confirmed by the Ben Nevis Observatories—a result making it plain that further observation and research are needed in order to place this vital question of atmospheric movements on a sound basis.

The Makdougall-Brisbane Prize for 1888–90 was presented to Dr LUDWIG BECKER for his Paper on "The Solar Spectrum at Medium and Low Altitudes," printed in Vol. xxxvi., Part 1, of the Society's *Transactions*:—

LORD M'LAREN, on presenting the Prize, said:—

I am now about to present to Dr Becker the Makdougall-Brisbane Medal for his paper on "The Solar Spectrum of Medium and Low Altitudes," and I can hardly doubt that the Society will most fully approve the award of the Council, when I say that in a field of research which has engaged the attention of many eminent astronomers and physicists, the results obtained by Dr Becker represent a very remarkable advance on the best observations hitherto published. Since the earliest days of Spectroscopy, men of Science have directed their attention to the bands produced by the absorption of the atmosphere. The object proposed is the discrimination of the lines of the solar spectrum due to air and aqueous vapour from those which represent absorption by the solar atmosphere, or other extra-mundane causes, and the method employed is that of high and low sun observations, and the comparison of the spectra thus obtained.

The possibility of accomplishing such work depends on the rapidity and accuracy with which the lines can be recorded while the sun is at a low altitude, and Dr Becker's first work was the invention of a recording apparatus. The idea worked out was that of the relative angular motion of collimator and grating magnified by wheels and screws; and so successfully was this principle carried out, that in the final apparatus the angular motion registered on the recording paper was 16,800 as large as the motion of the grating by which successive lines of the spectrum were brought into view, while the computed probable error in the registration was only about three-fourths of a second of arc.

The observations were made at a station set up by Lord Crawford on the Barmekin Hill on his property 850 feet in height. On this hill Dr Becker camped out during the summer months of 1887, 1888, 1889. The observations include 32 sunrises, 47 sunsets, and 18,000 observations of lines, which were made in 45 hours, and 8000 high sun observations.

The reductions of these observations, in the opinion of competent judges, have been made with the care and skill which we should expect from so proficient an observer. The theoretical reductions have been made by tables computed for the purpose, and the irregularities noted by the graphical process. In these reductions Rowland's standard lines were used.

It would be out of place on this occasion to enter into further detail; but I may mention that within the region 6024–4861, 3637 lines were recorded, whereof 928 were shown to be of telluric origin.

Work of the same kind was done by Thollon at the same time that Dr Becker was carrying on his observations, and I understand that his published results give only about two-thirds of the number of lines which Dr Becker has been able to record; and there is every reason to believe that the work of both is of the highest attainable degree of accuracy.

It seems unlikely that further detail can be found unless the Rowland grating be superseded by more perfect apparatus, though that seems hardly possible. What is most to be desired is, that the spectra of the elements believed to exist in the sun should be examined with the like powerful instruments and equal skill, in order that all the lines of the visible spectrum may eventually be referred to their proper constituent elements.

The following Communications were read:—

1. Further Remarks on *Nova Aurigæ*. By the ASTRONOMER-ROYAL FOR SCOTLAND.

2. List of the Fossil *Selachii* of Fife and the Lothians. By Dr R. H. TRAQUAIR, F.R.S.

Monday, 4th April 1892.

Sir Arthur Mitchell, K.C.B., Vice-President, in the Chair.

The Chairman, in accordance with the Laws, announced (as follows), the names of proposed new Foreign and British Honorary Fellows, to be submitted for Ballot at the Second Meeting in May :—

Foreign Honorary Fellows :—

GUSTAV WIEDEMANN, Leipzig.
WILLIAM DWIGHT WHITNEY, Yale College.
ÉMILE DUBOIS-REYMOND, Berlin.
ARMAND HIPPOLYTE LOUIS FIZEAU, Paris.

British Honorary Fellows—

DAVID GILL, LL.D., F.R.S., Cape of Good Hope.
Colonel ALEXANDER ROSS CLARKE, C.B., F.R.S.
Sir JAMES PAGET, Bart., LL.D.
The Right Rev. W. STUBBS, D.D., LL.D., Lord Bishop of Oxford.

The following Communications were read :—

1. A Problem of Sylvester's in Elimination. By THOMAS MUIR, LL.D.
2. Note on Professor Cayley's Proof that a Triangle and its Reciprocal are in Perspective. By the Same.
3. On the most recent Phases of Greek Literary Style. By Emeritus Professor BLACKIE. *T.* xxxvii. 107.
4. A Contribution to the Anatomy of Sutroa. By F. E. BEDDARD, M.A. *T.* xxxvii. 195.

The following Candidates for Fellowships were balloted for, and declared duly elected Fellows of the Society :—

PATRICK NEILL FRASER.
DAVID PAULIN.

Monday, 2nd May 1892.

Sir Douglas Maclagan, M.D., President, in the Chair.

The following Communications were read :—

1. Note on Dr Muir's Solution of Sylvester's Elimination Problem. By Professor TAIT. *P.* xix. 131.
2. On Borolanite — an Igneous Rock intrusive in the Cambrian Limestone of Assynt, Sutherlandshire, and the Torridon Sandstone of Ross-shire. By J. HORNE and J. J. H. TEALL, F.R.S., of the Geological Survey. *T.* xxxvii. 163.

3. The Lower Carboniferous Volcanic Rocks of East Lothian (Garlton Hills). By F. H. HATCH, Ph.D., F.G.S., of the Geological Survey. Communicated by Sir ARCHIBALD GEIKIE. *T.* xxxvii. 115.

4. On the Olfactory Organs of Helix. By Dr A. B. GRIFFITHS, F.C.S. *P.* xix. 198.

5. On the Renal Organs of the Asteridea. By the Same. *P.* xix. 205.

Monday, 16th May 1892.

Sir Douglas Maclagan, M.D., President, in the Chair.

The ASTRONOMER-ROYAL FOR SCOTLAND exhibited a Stellar Photograph by Dr GILL of the Cape Observatory.

The following Communications were read :—

1. Note on the Law of Transformation of Energy and its Applications. By W. PEDDIE, D.Sc. *P.* xix. 253.

2. Note on certain remarkable Volume effects of Magnetization. By Professor C. G. KNOTT, D.Sc., and A. SHAND, Esq. *P.* xix. 85.

3. On the Ventilation of Schools and Public Buildings. By C. HUNTER STEWART, M.B., C.M., B.Sc.

4. On the Glacial Succession in Europe. By Professor J. GEIKIE, F.R.S. *T.* xxxvii. 127.

The following were balloted for, and declared duly elected Foreign Honorary Fellows :—

GUSTAV WIEDEMANN, Professor of Physics in the University of Leipzig.

WILLIAM DWIGHT WHITNEY, Professor of Sanskrit and Comparative Philology in Yale College, United States.

ÉMILE DUBOIS-REYMOND, Professor of Physiology in the University of Berlin.

ARMAND HIPPOLYTE LOUIS FIZEAU, Mem. Inst. of France.

The following were balloted for, and declared duly elected British Honorary Fellows :—

DAVID GILL, LL.D., F.R.S., Her Majesty's Astronomer at the Cape of Good Hope.

Colonel ALEXANDER ROSS CLARKE, C.B., R.E., F.R.S.

Sir JAMES PAGET, Bart., LL.D., D.C.L., F.R.S., Cor. Mem. Inst. of France.

The Right Rev. W. STUBBS, D.D., LL.D., Bishop of Oxford.

The following Candidates for Fellowships were balloted for, and declared duly elected Fellows of the Society:—

George Young, Ph.D.
Joan Simpson Ford, F.C.S.
Henry Coates.
W. A. Taylor, M.A. Camb.
Arthur J. Pressland, M.A.
Patrick Doyle, C.E., M.R.I.A.
M. J. R. Dunstan, B.A., F.C.S.

Monday, 6th June 1892.

Sir William Turner, F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. Obituary Notice of David Davidson, Esq. By George Barclay, Esq. *P.* xix. p. ix.
2. On the Eliminant of the Equations of the Ellipse-Glissette. By the Hon. Lord M'Laren. *P.* xix. 89.
3. On the Composition of *Pinnaglobin*, a New Globulin. By Dr A. B. Griffiths, F.C.S.
4. On the Reproductive Organs of *Noctua pronuba.* By the Same.
5. On the Steam-line of CO_2. By Professor Tait.

The following Candidates for Fellowships were balloted for, and declared duly elected Fellows of the Society:—

J. B. Thackwell, M.B., C.M.
Peter Fyfe, Esq.

Monday, 20th June 1892.

Sir Douglas Maclagan, M.D., President, in the Chair.

Dr Ramsay Traquair, F.R.S., exhibited Remains of Animals occurring in Volcanic Tuff from Teneriffe.

The following Communications were read:—

1. On the Variations in the amount of CO_2 in the Ground-Air (*Grund-luft* of Pettenkofer). From the Public Health Laboratory of the University of Edinburgh. By C. Hunter Stewart, M.B., C.M.
2. On the Diurnal Oscillations of the Barometer in the Polar Regions during Summer. By Alexander Buchan, LL.D.

Monday, 4th July 1892.

The Hon. Lord M'Laren in the Chair.

The following Communications were read :—

1. The Rotatory Movements of the Human Vertebral Column. By A. W. Hughes, M.B., F.R.C.S. Communicated by Sir William Turner, F.R.S.
2. On the Genus *Lepidophloios*, Sternb. By Robert Kidston, Esq., F.G.S.
3. Further Notes on the Volume Effects of Magnetization. By Professor Cargill G. Knott and A. Shand, Esq. *P.* xix. 249.
4. On the Blood of the Invertebrata. By Dr A. B. Griffiths, F.C.S. *P.* xix. 116.
5. On the Laws of Motion. Part II. By Professor Tait.

The following Candidate for Fellowship was balloted for, and declared duly elected a Fellow of the Society :—

Francis John Martin, W.S.

Monday, 18th July 1892.

Sir Douglas Maclagan, M.D., President, in the Chair.

The Chairman made some Remarks on the Work of the Session.

The following Communications were read :—

1. Obituary Notice of Lord President Inglis. By Æneas J. G. Mackay, Esq., Advocate. *P.* xix. p. xiii.
2. The Wanyoro : a Central African Tribe. By Dr Felkin. *P.* xix. 136.
3. On the Fossil Flora of the South Wales Coal-Field. By R. Kidston, Esq., F.G.S.
4. On the Lateral Sense Organs of Elasmobranchs, III. The Ampullary Canals of Raia and Laemargus. By Professor Cossar Ewart, M.D.
5. The Cranial Nerves of the Skate. By Professor Cossar Ewart, M.D.
6. On the Thermal Effect of Pressure on Water. By Professor Tait. *P.* xix. 133.

Donations to the Library of the Royal Society from 1891 to 1892.

I. Transactions and Proceedings of Learned Societies, Academies, &c.

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OBITUARY NOTICES.

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Sir George Biddell Airy. By Professor Copeland.

(Read January 30, 1893.)

George Biddell Airy was born at Alnwick on the 27th of July 1801. He received his earlier education partly at Hereford and elsewhere, but chiefly at Colchester Grammar School. In the autumn of 1819 he was sent to Cambridge, where, at first, he studied at the expense of his maternal uncle, Arthur Biddell of Playford, near Ipswich. He entered Trinity College as a sizar, and although partly supporting himself by teaching, he found time while yet an undergraduate to contribute a paper on glass reflectors silvered at the back to the *Cambridge Philosophical Transactions*, then recently started. In 1823 he graduated as senior wrangler, and was elected a Fellow of his College in the following year. About this time he wrote a number of important papers on the figure of the earth and on other subjects, not the least valuable of these being one on astigmatism of the eyes and the simple modification of the spectacles, by the introduction of the single cylindrical surface necessary for its correction.

At the end of 1826 he was appointed Lucasian Professor, and early in 1828 Plumian Professor of Astronomy and Natural Philosophy, as well as Director of the new observatory. He at once threw himself heart and soul into both functions of his office, giving experimental lectures, and also systematically observing the sun, moon, planets, and stars with the Dollond transit instrument which, with a couple of clocks, formed the whole equipment of the observatory. The completely reduced observations were published year by year in worthy emulation of the example set on the Continent by Bessel and Struve some years previously. No trouble, we are told, was spared to secure accuracy in the results, the errors of the instrument being determined, and proper numerical corrections applied to the observations. Although fully ten weeks of the summer of 1828 were devoted to visiting the observatories of France and the north of Italy, and although for the first year Airy had no assistant, the volume containing the earlier Cambridge observations appeared in April of the year after they were made. An assistant was then appointed, without whose aid in the routine work of the observatory it would have been physically impossible for Airy to devote due attention to the investigation of the motion of Venus, which now engaged his attention. In 1831 he presented to the Royal Society his celebrated memoir "On an Inequality of Long Period in the Motions of the Earth and Venus," which, as Airy himself said, "contains the first specific improvement in the solar tables made in this country since the establishment of the theory of gravitation." About this time also he deduced the mass of Jupiter from elongations of the fourth satellite observed with a small equatorial telescope. These important investigations failed, however, to divert Airy altogether from the study of the Undulatory Theory of Light, which he materially helped to develop. Perhaps the most generally known of his discoveries in this field is that of the exquisitely beautiful phenomenon caused by the passage of polarised light through two thick plates of quartz cut from right and left-handed crystals, which will ever bear the name of Airy's Spirals.

For the first meeting of the British Association at York in 1831, Airy wrote a "Report on the Progress of Astronomy during the Present Century." It is in fact a masterly history of the science from the beginning of this century to the date of the meeting, and, as such, can never lose its value. Towards the close of 1832 the mural circle, 8 feet in diameter, was erected, and at the same time an additional assistant was appointed in the person of James Glaisher, afterwards so well known in the scientific world. About 1833 the Duke of Northumberland presented to the Cambridge Observatory a refracting telescope with an object-glass of the then unrivalled aperture of nearly 12 inches. The mounting for this instrument was designed by Professor Airy, who adopted the English arrangement of a long polar-axis, which when built in the solid style characteristic of all the apparatus constructed by him affords perfect facility for observation near the celestial equator, but is inconvenient to a degree when the telescope is directed to the immediate neighbourhood of the pole of the heavens.

In 1835 Pond, the sixth Astronomer-Royal, resigned, and Airy, the distinguished young Cambridge professor, was naturally appointed to the vacant post by Lord Auckland, then head of the Admiralty. Airy forthwith reorganised the national observatory, introducing the methods and arrangements that had proved so successful at Cambridge. From that time forth the annual volumes of Greenwich observations, in their familiar drab bindings, appeared with unfailing regularity. The field of work, too, was soon extended. For some years the study of terrestrial magnetism had been making rapid strides, mainly under the auspices of the *Magnetische Verein*, of which Gauss and Weber were the leading spirits. Desirous that this country should take a fitting share in investigations so closely bearing on its varied maritime interests, Airy in 1840 established the magnetical and meteorological services at Greenwich. The memorable Antarctic voyage of Captain (afterwards Sir) James Clark Ross at that time doubtless influenced the authorities in sanctioning this extension of the work carried on at Greenwich. The laborious eye observations in this department were replaced by photographic records as early as 1848. The interdependence of solar and magnetic phenomena, which at first escaped Airy, was established from the Greenwich observations many years afterwards by the indefatigable researches of Mr Ellis.

Not content with publishing and discussing his own work, Airy undertook the gigantic task of reducing all the lunar and planetary observations made by Bradley, Bliss, Maskelyne, and Pond from 1750 to 1830. Bradley's matchless observations of the sun and fixed stars for the first twelve years of that period had already been utilised by Bessel in his *Fundamenta Astronomiæ*. For the planetary reductions, Airy in 1847 received the gold medal of the Royal Astronomical Society for the second time, and in the following year the lunar reductions gained him the equivalent testimonial of the same society.

In 1847 was completed the Greenwich Altazimuth, designed by Professor Airy for the purpose of observing the moon near conjunction. In one respect the altazimuth marked a new departure in the construction of astronomical instruments. Instead of being built up of many parts bound together by numerous screws, it was made in as few pieces as possible, and the greatest care was taken that the critical parts, such as the micrometer screws, bore immediately against solid portions of the mounting. With this instrument the moon was closely followed through its smaller phases for many years, and if the resulting places have not fulfilled the expectations raised by them, it is chiefly because of the difficulty of obtaining a very exact record from any instrument free to move in two co-ordinates, and partly also from the somewhat insufficient optical power of the telescope employed. The altazimuth was, however, a bold experiment, and cleared the way for a new instrument about to be introduced at Greenwich, viz., the Universal Transit-Circle, which seems destined to accomplish in the most satisfactory manner the task originally proposed by Airy. In 1848 the late Astronomer Royal replaced the old zenith-sector by the reflex zenith-tube, with which γ Draconis, the star which passes almost exactly overhead at Greenwich, is observed from time to time. Greenwich is indeed favoured by the position of this star, so near at once to the zenith, and to the solstitial colure, that for many centuries it will continue to culminate within a few minutes of the zenith, just as it did in the days of Hooke, by whom it was first observed with extreme accuracy.

In 1850 the large and massive transit-circle built, like almost all the Greenwich instruments, by Troughton and Simms, superseded the transit instrument and mural circles till then in use. It is still a magnificent instrument, and one of the most stable of its class. Like the altazimuth it is almost altogether made of cast-iron, but unlike the greater number of transit-circles it cannot be reversed; this disadvantage is, however, very possibly outweighed in a national observatory by the unbroken nature of the records obtained. The chronograph was introduced at Greenwich some four years after the transit-circle.

In 1859 the Greenwich plant was augmented by a large equatorial of $12\frac{3}{4}$ inches aperture. The object-glass was made by Merz, but the mounting, which resembles that of the Northumberland telescope at Cambridge, was designed by Airy. So satisfactory and steady has this mounting proved that it is now being provided with a telescope of no less than 28 inches aperture made by Sir Howard Grubb. Some years later Airy devised the "Orbit-Sweeper"—a new form of equatorial, with a third axis in the direction of what would be the tube in an ordinary equatorial. When properly set and driven by clockwork the telescope continues to sweep along any given great circle of the heavens. The University Observatory at Strassburg is provided with a beautiful instrument of this kind.

The last considerable extension of the scope of Greenwich Observatory made by the late Astronomer-Royal was the introduction in 1873 of spectroscopy and the regular photography of solar phenomena.

Of Sir George Airy's many labours in fields outside the regular work of the observatory only the briefest mention can here be made. His eminently practical method of correcting the deviation of compasses in iron vessels, the outcome of numerous experiments on board ship, came into general use. The determination of the density of the earth by experiments in Harton Colliery will long be remembered in the North Country.

Of international as well as national importance was the share he took in the restoration of the standard weights and measures lost in the conflagration which destroyed the old Houses of Parliament. He took an active part too in the "battle of the gauges," which decided the width of railways in this country. Indeed, he may be said to have been general adviser on all matters connected with science not only to the government but to the whole country. He observed the total solar eclipse of 1842 in the north of Italy, and that of 1851 in Norway. To him astronomy is mainly indebted for the famous Eclipse Expedition to Spain in 1860, on which occasion the troopship "Himalaya" carried out a large party of observers, who, scattered along the line of total obscuration, shared in reaping perhaps the richest harvest of results ever secured on one of these rare occasions.

The best-known books by Sir George Airy are the *Mathematical Tracts*, *Gravitation*, and *Six Lectures on Astronomy*. For the *Encyclopædia Metropolitana* he wrote a number of articles, notably those on "The Figure of the Earth" and on "Tides and Waves." His contributions to the publications of learned societies were very numerous, no less than 242 papers standing under his name in the

Royal Society's catalogue, besides four papers of which he was joint author.

In addition to the well-earned honours bestowed on him by the Royal Astronomical Society, of which he was repeatedly President, he was elected President of the Royal Society, and received both a Copley and a Royal Medal. He was made a C.B. in 1871, and a K.C.B. in the following year. In 1875 he received the Freedom of the City of London, and also that of the Company of Spectacle Makers, the latter honour in acknowledgment of the signal manner in which he had given a refined development to the Company's craft. Oxford, Cambridge, and Edinburgh conferred on him their honorary degrees. Of the Royal Society he was President from 1871 to 1877, and Fellow for nearly sixty years. At the time of his death he was by many years the senior Honorary Fellow of this Society, having been elected in 1835. Numerous honours came to him from abroad, amongst them the Lalande Medal. He was one of the eight Foreign Associates of the French Academy.

In 1830 he married Richarda, daughter of the Rev. R. Smith of Edensor, by whom he had nine children; of these, three sons and three daughters survived him. Lady Airy died in 1875, almost exactly six years before he resigned the post of Astronomer-Royal, which he did on August 15, 1881, shortly after the completion of his eightieth year. The closing years of his life were spent not far from the great observatory he had directed for more than forty-six years. He died on January 2, 1892, from an internal complication, the result of a fall.

David Davidson. By George Barclay, M.A.

(Read June 6, 1892.)

The late Mr David Davidson was an Edinburgh man on both sides of the house,—his father being the Rev. Dr Davidson of Muirhouse, minister of the Tron Church, and his mother a sister of Lord Cockburn. He was born at No. 8 Heriot Row, 20th May 1808, and received his early education at the High School, completing it afterwards in England. Having chosen a business life as his career, he was at the age of 17 "apprenticed" to one of the great Leith firms of those days, whose offices it was a pretty costly favour to be admitted to (the "apprentice fee" running as high as £200 to £300), with the result, in Mr Davidson's case, that the "fee" had not been paid a couple of months when the great crash of 1825 occurred, which involved his employers, as well as many other Leith firms, in hopeless financial troubles, and eventual ruin. It was a hard school for the young apprentice, but it probably helped to give him the quiet self-possession and courage in face of difficulties which characterised him through life. And there were still harder times to follow. While yet in his teens, Mr Davidson started on an independent business career; but the shadow of the great collapse of 1825 lay heavy over Leith for many years; his undertakings did not prove fortunate, and after struggling on throughout the thirties at home, Mr Davidson was led in 1842 to seek a new field in Canada. He had married in 1834 Frances, daughter of Mr James Pillans, a well-known Leith merchant; with her and three children he now sailed, July 1842, for Montreal, where he assumed the managership of the Bank of British North America, a post which he afterwards exchanged for that of the Bank of Montreal, remaining at the head of that important institution until he was recalled to Edinburgh to take the management of the Bank of Scotland. Mr Davidson was fortunate in the time of his arrival in Canada, and his career there was a pleasant and successful one throughout. Montreal fifty years ago was a very different place from the great city of to-day. It was a comparatively small community and primitive life which Mr Davidson found there in

1842;—giving readier opportunity for close personal relations, and greater prominence to individual exertions for the commonweal, while the few were still acting as pioneers to the many that were to follow. In this society Mr Davidson soon began to take an active part. In public he interested himself in many philanthropic, and very specially in educational, institutions. With the assistance of his uncle, Lord Cockburn, he was the means of securing for the High School of Montreal such a staff of teachers from Scotland as soon raised it to the eminent position as a school which it to this day maintains; and he took a keen interest in the advancement of the M'Gill University, now a very important seat of learning in the colony. The "Davidson Gold Medal" has for many years been the highest prize in the High School of Montreal. And in private, Mr Davidson found abundant opportunities for the gratification of his constant desire to be helpful in alleviating the troubles and promoting the interests of those around him. It is pleasant to record that when, after twenty-seven years' absence, Mr Davidson's death was announced in Montreal, his son there received the most touching proofs how his name and memory were, both on public and private grounds, still "freshly remembered." There were many who could tell of acts of kindness which had been the saving or making of them and theirs; and there were still some old friends who remembered pleasant days in the little summer retreat near the city, which Mr Davidson had reclaimed from the waste, and with the aid of his wife's taste and skill had gradually converted into a little paradise of lawn and trees and flowers. No wonder, then, that Mr Davidson too always looked back on his twenty years' life in Canada with affectionate interest and satisfaction. It came to a close in 1862, when, as already mentioned, he was recalled to Edinburgh to take the management, as "Treasurer," of the Bank of Scotland. Thenceforward, during nearly twenty years, Mr Davidson's was a well-known personality in Edinburgh and Edinburgh life. Already past middle age, of large frame, strongly-marked features (very noticeably resembling Wordsworth's), and leisurely gait, his whole bearing was one of quiet dignity, to which his gravely simple manner and deliberate low-voiced speech gave additional relief. As head of the premier Scotch Bank, and Chairman of the Associated Banks, Mr Davidson now devoted himself not only to the successful management of the Bank of Scotland, but also to forwarding and safeguarding the common interests of all the Scotch banks in the many questions which arose in connection with the great extension of their business, including their settlement in London, during the period of his tenure of office. And when the great crisis caused by the failure of the City of Glasgow Bank occurred in 1878, Mr Davidson was able to render such service to the banks, and to the *people* of Scotland, as should always be gratefully remembered. Not altogether taken by surprise, Mr Davidson and his colleagues had made some preparation for the impending crash; but when it did come, in more hideous form than any had deemed possible, if a panic with more widespread disaster was averted, they who best know the details of those anxious days will probably be the readiest to admit, how much it was to the ability, calmness, and courage in council of Mr Davidson, that the result was due. The strain, however, proved too great for a man of 70. Mr Davidson's health soon after gave way; and though a holiday of some months partially restored it, a return to the hard work, and anxiety involved in bank management, was deemed unadvisable, and in 1879 Mr Davidson resigned his position as Treasurer of the Bank of Scotland. During his stay in Edinburgh, Mr Davidson took little or no part in public matters *proper;* interesting himself, however, in various concerns of public usefulness, specially in "Donaldson's" and the "Longmore" Hospitals, and in the "Walker Trust," in the development of which he did yeoman's service. Outside the bank his only business interest was in the "North British and Mercantile Insurance Company." Of this institution, Mr Davidson had been a Director for many years in Edinburgh, and when he now left Scotland and settled in London, he was in 1880 elected "Chairman of the General Court of Directors," a position which he filled with much acceptance and warm recognition of his services to the Company, till his death. It gave him sufficient occupation without overwork, and had the advantage of bringing him down once a year at least to Edinburgh. At Somerset Lodge, Wimbledon, Mr and Mrs Davidson passed the remainder of their days. Mrs Davidson though for years an invalid, under the unwearied and utterly self-forgetting tendance of her husband, survived till 23rd February 1891; and it was only after fifty-seven years of married life, that death for a little time severed a union of the most singular perfectness and mutual devotion. Mr Davidson bore the blow after the manner of his nature; seemed as months went by to be even somewhat regaining strength and cheerfulness; but the *hold* of life was gone, and a slight chill sufficed to carry him off on the 30th October of the same year (1891), with mental powers quite unimpaired, and a physical constitution still vigorous enough to have prolonged his life under other circumstances for years to come. Mr Davidson was 84 when he died in 1891; and as his father was born in 1745, their joint lives covered the somewhat remarkable space of 146 years. Besides his professional acquirements in matters of banking and finance, Mr Davidson had a widespread knowledge of men and things, derived from much reading as well as personal experience, and there were few subjects on which he could not either converse, or, at least, intelligently enjoy conversation. He was never much of a sportsman, though in early days he might be seen among those who some sixty years ago followed Mr Hepburn's hounds at Karswell, and in Canada he indulged in occasional fishing excursions, when his friend Dr Campbell, the well-known physician of Montreal, was his usual companion. But after his return to this country, his interest in sport consisted almost entirely in seeing it enjoyed in summer quarters by his sons. Mr Davidson's pleasures throughout life were mainly those of his home—a *country* home wherever possible. His family affections were of the deepest and most unselfish; and more and more as life advanced, he found his happiness in ministering to the happiness of his wife and children. Three sons and one daughter survive him. Mr Davidson became a member of the Royal Society of Edinburgh in 1867.

John Inglis, President of the Court of Session and Justice-General of the Court of Justiciary. By **Æneas J. G. Mackay**, Esq., Sheriff of Fife and Kinross.

(Read July 18, 1892.)

John Inglis was born in George Square, Edinburgh, on 21st August 1810, and died at Loganbank, in the parish of Glencorse, Mid-lothian, on 20th August 1891, a day before the completion of his eighty-first year. He was the youngest of four sons of the Reverend Dr John Inglis, the successor of Principal Robertson, the historian, as minister of Old Greyfriars' Church. The eldest was Harry Inglis, Writer to the Signet, between whom and John the fraternal bond was strengthened by mutual good offices and a closer intimacy, both in earlier and later life, than often falls to the lot of brothers. Their mother was Maria Moxham, daughter of Abraham Passmore of Rolle Farm, Devon, who again brought English blood into a family, the name of which indicates a remote English, probably Border, origin. His paternal grandfather was Harry Inglis, minister of Forteviot, in Perthshire—a county in which his father had been minister of Tibbermore before his appointment to Old Greyfriars'. While certain traits in his character may be traced to the English connection on the mother's side, and his education at Oxford, he continued through life a patriotic Scotchman, and a devoted member of the Scottish Presbyterian Church by law established. In its Assemblies his father was the chief leader of the Moderate Party, and he was reckoned one of the three best preachers of his time in Scotland, and it was the time of Chalmers. Allowing for the difference between the eloquence of the Assembly and Pulpit, and that of the Bar and Bench, the style of the son had a strong family likeness to that of his father. Both were distinguished by cogent reasoning, and facile, apt, and forcible expression; and as these were the product of Nature rather than Art, it may be inferred that the style indicated a similarity of character which might also be traced in the massive features he inherited from his father. Both were felt to be, as soon as seen, to use a phrase of Dr Chalmers, "men of weight."

He was educated at the High School of Edinburgh during the rectorship of Dr Carson, a learned scholar, and in the class of Mr Benjamin Mackay, who is described by his pupils as a practical Scottish schoolmaster of the best kind—intelligent, accurate, and thorough. Several of his class-fellows still happily survive, two of whom—Lord Moncreiff, the competitor of Inglis at the Bar and his friend on the Bench, and Sir Douglas Maclagan—held the office of President of the Royal Society, which was offered to, and declined by, Inglis for reasons afterwards noticed. From the High School he went to the University of Glasgow, where the lectures of Sir Daniel Sandford gave him a taste for literature in its best models—the classical authors of Greece. "I can never forget," he said many years after, when addressing the students of that University as their Rector, "with what delight I listened to the prelections of Sandford, whose reading of Greek poetry conveyed to the hearer the highest intellectual pleasure." In him, as in others, the living voice of the teacher wakened the intellect, and by the pleasure it gave evoked the taste for study and the study of good taste.

Having been elected to an exhibition on the foundation of Mr Snell, he matriculated at Balliol College, Oxford, in 1830, graduated as B.A. in 1834, and as M.A. in 1836. He did not obtain high academic honours, and was placed in the third class in the Classical School two years after Mr Gladstone and the same year in which Lord Selborne took degrees which more certainly indicated their future eminence.

He had entered probably with more zest into the social than the learned life of Oxford, but his ability, especially in argument, was recognised by his contemporaries. That he had not been a careless spectator of the University in all its bearings, was shown by his grasp of the principles of University education and accurate knowledge of the difference between the English and Scottish systems, with their respective merits and deficiencies, when called on to legislate for the Scottish Universities, and act as Chancellor of that of Edinburgh. But it was the profession he chose which first gave scope to his mental powers.

In 1835 he was admitted a member of the Faculty of Advocates.

For a few years his success in the profession, which at that time had perhaps as many good lawyers as at any period before or since, was not rapid; but soon after the year 1842, when he distinguished himself as junior counsel for Mr Nelson in the Hot Blast Patent Case, his advance to the highest position at the Scottish Bar was assured. The successful defence of Madeleine Smith in 1857, towards the end of his career at the Bar, did not create his reputation as an advocate, but extended it from the select circle interested in civil law-suits to the larger public which watches a *cause célèbre* in the criminal courts. In 1841 he married a daughter of Lord Wood, a judge of the Court of Session, who predeceased him, leaving two sons, Mr Alexander Wood Inglis, Secretary of the Board of Manufactures, and Mr Herbert Maxwell Inglis, W.S. In 1844 he became Advocate-Depute, in February 1852 Solicitor-General, and three months later Lord Advocate in the administration of Lord Derby. This early promotion was due to his position at the Bar as much as or more than to his attachment to the Conservative party. By mental constitution, as well as the time and circumstances of his birth, education, and profession, a Tory or Conservative of a type now almost extinct, he was a lawyer first and a politician afterwards.

Outside of the sphere of party, his opinions on political and social, as well as other subjects, might perhaps be best described as Liberal-Conservative, a term then current, though now rarely used. In November 1852 he ceased to be Lord Advocate on the resignation of Lord Derby. He was soon after elected Dean of the Faculty of Advocates, an office he held till 1858, and prized as the free and unanimous choice of his professional brethren. He described its duties in his farewell letter in the appropriate language which marked his speech and writing:—"My constant desire and earnest endeavour has been to render the office practically available for the purposes which it is intended to serve; to induce unity of sentiment and action within the Faculty: to maintain its privileges and independence; to secure a scrupulous observance of the rules of professional propriety; to promote that social harmony for which the Scottish Bar has been distinguished; to advance the reputation of the Faculty as a learned Society and a national institution; to encourage by all legitimate means the cultivation of learning and scholar-like accomplishments.

"To the efficient administration of Justice, not even the purity of the Bench is more requisite than the independence and integrity of the Bar."

This is not the place to enlarge on his character as an advocate; but it may be proper to mention, as the law occupied the largest share of his life, that his knowledge and practice embraced every department—Civil, Criminal, and Ecclesiastical; the conduct of Jury Trials; the examination of witnesses, which was strict but never rude, and knew when to leave off as well as how to begin; the composition of written pleadings (in his time still in frequent use), and the delivery of oral arguments, in which he was equally able in the statement of the case and in reply. His distinctive excellence as a pleader was concise, clear, logical, and dispassionate reasoning. It is almost impertinent to say that he never used the artifices which the satirist and the playwright sometimes associate with successful advocacy. But he scrupulously abstained even from the arts of the rhetorician, which few speakers in public altogether avoid. Sound sense, grasp of principle, accuracy in detail, ready memory, and practical as distinguished from scholastic logic, orderly arrangement, and an apt choice of words, were the qualities he cultivated and matured by constant industry and a wide experience of men and business.

In 1858 he became Lord Advocate for the second time. On the death of Lord Justice-Clerk Hope in the summer of that year, he was promoted to the Presidency of the Second Division of the Court of Session, and on Lord Colonsay's appointment as the Scottish legal member of the House of Lords in 1867, he became President of that Court, and Justice-General of the Court of Justiciary, offices which he held till his death. He had thus the long judicial experience of thirty-three years, during which he gained and held the complete confidence of the whole legal profession, of the public, and, so far as is possible, of the parties to litigation, by assiduous attention to the duties of his office, and the ability and fairness of his judgments.

From what has been said of his character as an advocate, it almost follows that he was still more eminent as a judge. Though not free on certain points from strong convictions, which appeared to those who did not share them strong prejudices, the chief characteristic of his mind was impartiality, constantly directed to the exact ascertainment of facts, and the application of the principles of justice to the facts ascertained. For his judicial merit it is sufficient to refer to the estimate contributed by a lawyer who had long practised before him, and became his colleague, Lord M'Laren, to the *Juridical Review*, a publication in whose success the late Lord President showed the interest he felt in every effort to maintain and extend the reputation of Scottish Jurisprudence.

Second only to his services to the Law, and possibly more interesting to the learned Society at whose request this notice is written, were those he rendered to University Education in Scotland. The accidents of party politics gave him a short experience of Parliamentary and London life, and preserved unimpaired his genuine Scottish patriotism. He sat for the burgh of Stamford only from 2nd March to 13th July 1858. He did not travel much, and his life was practically spent in Edinburgh and his country-seat of Glencorse in its immediate neighbourhood, with occasional visits to other parts of Scotland for sport or golf, his favourite amusements.

But it was his and their good fortune that he carried during his brief Parliamentary career the Act for the Reform of the Scottish Universities. While largely indebted to the preparatory labours of other University Reformers, chiefly members of his own profession, amongst whom may be named Lord Moncreiff, Mr Edward Maitland (Lord Barcaple), Professor Lorimer, and Mr Francis Russell, in its final form this Act was the fruit of his practical sagacity and prudence.

It was a further favour of Providence that the author of the Act presided over the Commission, and brought it into operation by a series of Ordinances, which dealt with almost every branch of University Education and Administration.

His hand may be specially traced in those relating to Finance, where he had to solve the problems of making the best of too slender endowments, and to the development of the Faculty of Law, in which he had to make the most of a too scanty professoriate, and to attach by graduation a class of students to the University where they had hitherto been often little more than casual visitors.

While Dean of Faculty, aided by several of his brethren, and in particular by Mr Patrick Fraser, afterwards a judge, he did much to raise the standard of the education of the Bar, and to preserve the high reputation of the Advocates' Library, a professional, which had almost become a national, institution without extraneous aid, and in spite of the discouragement of more than one ministry. As a University Reformer he kept in view all the Faculties, their mutual relations, their scientific not less than practical value, and controlled the natural tendency of professors to suppose their own subjects paramount. His mind sought as if by natural instinct the due proportion of things, and an unexaggerated expression of thoughts. He laid great stress on the vital importance to the Scottish University of the choice of the best professors, by placing the patronage in honest, firm, and discriminating hands, preferring for this purpose a Board of Curators to either Municipal or Government Patronage, and to leave to professors so chosen a wide liberty in the conduct of their special studies. But he insisted not less on what he called the great principle, that the "professors were made for the students, and not the students for the professors." He recognised the utility of extra-professorial, or, as it was commonly called, extra-mural teaching, which had contributed much to the credit of the Medical School in Edinburgh, but he doubted whether it was applicable to the smaller Universities or to the Faculties of Theology and Law, or even to Arts, until the professoriate was better endowed. In this, as in other points, he preferred slow and sure to rapid or experimental changes, and proved that the equal balance of his mind was not confined to the administration of justice. He was elected to the Rectorship of King's College, Aberdeen, in 1857, of Glasgow University in 1865, and to the Chancellorship of Edinburgh in 1869, an office which he held till his death, and in which he represented the University, at the celebration of its Tercentenary, to the learned world with the same dignity which marked his performance of the annual duty of conferring degrees.

These appointments proved the recognition of his services by the students as well as the graduates of the Scottish Universities. In 1858 he was created Doctor of Laws of the University of Edinburgh, and in the following year D.C.L. of the University of Oxford, and a member of the Privy Council. It is believed that he declined a Peerage, and it is certain that he might, had he wished, have become a legal member of the House of Lords. But, " content to live where life begun," he felt that Scotland was his home, and the Presidency of its Supreme Courts the sphere in which he could best serve his country. Before the centripetal force of London and other large cities was generally acknowledged to be a national risk, and before decentralisation had become a popular opinion, he proved by a practical example the value, not merely to the locality but to the nation, of the local application of talents in their kind of the highest order. In 1876 he was appointed Chairman of the Commission of Inquiry with regard to the Scottish Universities, which collected much useful information, though its recommendations, too largely influenced by the members who represented Physical Science and underestimated the value of Mental Philosophy, Language, and the Arts, did not meet with the general approval of Scottish Educational Reformers, and do not bear the stamp of the mind of the Chairman like the practical measures of the former executive Commission. His capacity lay rather in sifting and carrying out than in originating or advocating reforms. As Chairman of the Association for the Better Endowment of the University of Edinburgh, his position enabled him to direct public attention to a source of weakness in the Scottish Universities, which possess neither the ancient foundations of the English, nor the liberal support of Government enjoyed by Continental, American, and Colonial Universities, to enable them to fulfil their functions as National Institutions for the benefit of all classes in Scotland, as well as of English, Indian, and Colonial Students, who resort to them, attracted by the reputation of their professors and the practical and cosmopolitan character of their methods of study. He was a regular attendant of the meetings of the Board of Manufactures, and was able to show that something might be done, even with the scanty funds grudgingly allowed to Scotland, for the promotion of the Fine Arts, in which he took the interest of an intelligent amateur. The history of Scotland specially engaged his attention ; and the Scottish Text Society, originated by Dr Gregor of Pitsligo, for the preservation and publication of its early and characteristic language and literature,—strangely neglected by Scotchmen in spite of the efforts of Pinkerton and Chalmers, Sir Walter Scott, Irving, and Laing, until its linguistic value was pointed out by indefatigable German students,—found in him not merely an ornamental head but an active supporter. His own contributions to literature were few, and apart from occasional speeches in the academic offices he held, his published works were limited to two essays in *Blackwood's Magazine*, one on "The Present Position of the Church of Scotland," the other on "Montrose and the Covenanters of 1638;" an address to the Juridical Society of Edinburgh on "The Scottish Lawyers of the 17th Century;" and an Antiquarian Note on the name of the parish of Glencorse, when official ignorance proposed to alter it to Glencross. It was the opinion of qualified judges that he might have distinguished himself as an historical writer, and in particular that he might have written better than any lawyer of his time a History of the Law of Scotland, a task to which he incited, as yet without result, the members of the Juridical Society. When written, many elucidations of it will be derived from his judgments. But he knew best where his strength lay: in practical action rather than philosophic speculation, in judicial rather than literary composition. Lord M'Laren has noted, in the paper already referred to, that he set the example of a new and better style of judicial expression than had been common in the judgments of the Scottish Bench.

Some other directions of his activity, more of a private or semi-private than a public kind, must be omitted from a notice too short to convey an adequate conception of his character, but longer than the purpose for which it has been written perhaps justifies; yet it cannot be concluded without referring to the circumstances of his connection with the Royal Society of Edinburgh, which were those of a loyal supporter and steadfast friend, but neither of a contributor to nor an auditor of its proceedings.

On 5th February 1855, when Dean of Faculty, he was elected a Fellow of the Society. After the death of Professor Kelland, its President, on 7th May 1879, a meeting of the Council was held on 31st October to designate a successor, when it was stated that there was "a strong feeling among many of the Fellows that the next President should be a man of letters—the Society having been instituted for the promotion of literature as well as science." The name of the Lord Justice-General was unanimously adopted, and Sir Robert Christison and Professor Douglas Maclagan were deputed to obtain his consent. In a letter, dated 3rd November 1879, declining the nomination, he wrote:—"I need hardly say that I regard the appointment of President of that Society (the Royal Society of Edinburgh) as one of the greatest honours that can be bestowed on any Scotchman in Scotland, and I therefore appreciate the kindness of the Society in proposing to place me in a position of such importance. But I cannot help feeling, and the feeling grows stronger the longer I consider the matter, that it is a position for which I am in no way adequately qualified. Though a Fellow of the Royal Society, I have never hitherto taken any part in its proceedings, and at my time of life, with my judicial work, I cannot look forward to do so in the future. This might be of less importance if I were personally distinguished either by scientific acquirements or literary work. But you know as well as I do that of physical science I know next to nothing, and that a laborious professional life has left me no time for courting the Muses."

In a similar modest spirit, and with the knowledge of himself which has been deemed not the least difficult attainment of a philosopher, when solicited by a literary society to deliver a lecture he replied to its representatives:—"Do you know, gentlemen, that I have actually ventured to write my own epitaph, and that it runs, 'Here lies a man who has never given a lecture.'"

Although he did not take part in the proceedings of the Royal Society, the letter quoted shows his sense of its importance to the intellectual life of Scotland, and as an instrument for the advance of knowledge. He gave the Society the benefit of his influence in procuring an extension of its rooms, and a few weeks before his death visited its premises to inform himself how the much-needed space could be secured for its rapidly-increasing scientific library. Without being the least of a bookish man, he had a keen interest in libraries, and in rare and good books. So, without being either a scientific or a literary man, he appreciated and, on proper occasions, expressed his appreciation of the inventions and discoveries which during his lifetime enlarged the bounds of the physical or material sciences, as well as of the contributions to literature and philosophy, which have combined to make the Victorian as marked an era as the Elizabethan in the annals of thought. At a time when there was a risk that the absorbing cares of professional and mercantile pursuits and the rapid acquisition of wealth might lower the reputation of Scotland as a country, knowing the value of science and literature, such an example in the head of a profession, sometimes tempted to sink its character as a learned Corporation created for the administration of justice in that of an interesting and lucrative business, may perhaps be deemed an unwritten contribution to one of the objects for which the Royal Society exists.

Alexander Forbes Irvine of Drum. By Sheriff Æneas Mackay.

(Read January 30, 1893.)

Alexander Forbes Irvine of Drum, Advocate, twice a Vice-President of the Royal Society, was born on 18th February 1818 at Schivas, and died at Drum Castle in Aberdeenshire on 4th April 1892. As an example of a country gentleman and lawyer, who had a genuine interest in science and literature, it has been thought that his life merits a fuller notice than the President could afford to give in his account of the many losses the Society suffered during the past year. But I have accepted the invitation of the Council to write it with diffidence, knowing how difficult it is to express in words characteristics which impressed all with whom Mr Irvine came in contact, yet were as delicate as they were rare during the period in which he lived. It might be hard to find in the long list of present Fellows of the Society one who, in the same position, possessed the same qualities. It would be easier to discover a parallel in the seventeenth or eighteenth century amongst the members of the circles of Scott of Scottstarvet, or Gordon of Straloch, of Clerk of Penicuik, Lord Monboddo, or Lord Hailes, than in a time during which, in Scotland, intellectual, other than agrestic or forensic, tastes have too seldom been combined with the ownership of land or the profession of law.

Mr Irvine was the eldest son of Alexander Irvine of Drum, who died in 1861, and of Margaret, daughter of James Hamilton, a descendant of the Hamiltons of Little Earnick. The family he represented originally sprang from the strong Border stock of the Irvines of Bonshaw in Annandale. William de Irvine migrated to Aberdeenshire in the reign of Robert the Bruce. In that county the same family has held for more than five centuries and a half the estate of Drum, which derives its name from the ridge or rising ground on the north side of the Dee, about eleven miles from Aberdeen. The founder of the Aberdeenshire branch, William de Irvine, was, according to a tradition both of Annandale and Deeside, the son of a vassal of the Lords of Annandale, who served as armour-bearer and secretary of Robert the Bruce. He may have been the clerk who, in Barbour's poem, rode along with his master from the English Court in London to Lochmaben, which they reached on the fifth day. According to another cherished tradition of the family, he guarded Bruce during one of the perilous passages of his life, when he slept under a hollybush in the Forest of Drum. These and other services were rewarded by the charter in 1523 of the estate of Drum in free forestry, followed next year by a Charter converting the Estate into a barony, both of which still exist amongst the muniments of Drum. The arms of the family originally, or at least in the oldest form known, as given in Sir David Lyndsay's Register, three holly leaves vert on a field argent, with holly leaves for the crest, and the motto, "Sub sole sub umbra virens," give some corroboration to the latter tradition. The Chamberlain Rolls of 1329 prove that William de Irvine was Clerk of the Rolls in that year—a man of law and letters as well as of arms.

The subject of this notice was the twentieth in descent from the comrade of Bruce, and is said to have been also the twentieth head of the family who bore the name of Alexander.

Another ancestor, "the strong undoubted Laird of Drum," died fighting with Maclean of Duart for the king against the Highland host at the Battle of the Harlaw :—

"Gude Sir Alexander Irvine,
The much renowned Laird of Drum,
None in his days was better seen
When they were semblit all and sum.

"To praise him we suld not be dumm
For valour, witt, and worthiness,
To end his days he there did come,
Quhois ransom is remeidyless."
—*Ballad of the Battle of Harlaw*, Stanzas xxxix., xl.

His son received a grant about the year 1420 from the Abbot of Arbroath of the lands of Forglinn and the custody of the Breach Bannoch, which entailed the duty of leading the vassals of the Abbey when summoned to the royal host. This venerable relic was a casket or reliquary—perhaps that still preserved at Monymusk by a family whose predecessors at one period held the office of its custodier. It was believed to contain a fragment of the last earthly garment worn by St Columba, which was thought, long after his death, to be the omen and pledge of victory, by preserving some of the saint's miraculous virtue. [Bishop Reeves gives a notice of the Breach Bannoch in his edition of Bishop Adamnan's *Life of St Columba*, and it is figured in Dr Joseph Anderson's *Scotland in Early Christian Times.*] This Sir Alexander Irvine was one of the hostages for the ransom of James I. It was he who built Drum's Aisle or Chantry in the church of St Nicholas, Aberdeen, where the brass effigies and brief epitaphs of "Honorabilis et famosus miles dominus Alexander de Irwin," and his wife, "Nobilis domina Elizabeth de Keth militis marescalli Scotiæ filia," still remain, though the "Hic sub ista sepultura jacet" has been falsified by their removal from the site of the tomb to the wall of the church.

Another Sir Alexander, in the reign of James IV. and James V., held the office of Sheriff of Aberdeenshire, and appears in the Exchequer Rolls as Crown Receiver for Kintore, Coul, and O'Neil. His son took part in the defence of Scotland against Henry VIII., and lost his eldest son at the Battle of Pinkie.

Three other Lairds of Drum deserve a passing notice even in the brief genealogical reference which such a notice as the present permits. Sir Alexander Irvine of the time of James I.'s English reign and the commencement of that of Charles I. is described by Sir Samuel Forbes, writing in 1715, as "that Laird of Drum, who lived in our grandfathers' times, for his benignity and ample bounty to the poor, deserves to be remembered and praised. He lived decently, was a plain man, nicknamed Little Breeches, increased in wealth, bequeathed lands for maintenance of poor widows, poor maids, and for the education of several children at school, and of young men to be taught philosophy and theology." Neither the pious founder nor his panegyrist probably contemplated how ample this bounty was to become. Let us hope the bursars of Aberdeen bless the memory of "Little Breeches."

His son and successor was appointed Sheriff of Aberdeenshire by Charles I. and suffered much in the troubles of the civil war and ecclesiastical revolution. Drum Castle was taken in his absence by General Monro, who granted to the Lady Drum, its defender, that the garrison might retire with the honours of war. The forces of the Covenant for a considerable time garrisoned Drum, lived on the estate, and plundered the castle. The laird himself was more than once fined and imprisoned on account of his loyalty and refusal to swear to the Covenant. For his contumacy and his appeal to the civil power in the person of Overton, one of Cromwell's colonels, he was excommunicated by the Presbytery of Aberdeen. In his protest against their sentence, he declared "that he separated himself "from the discipline of Presbytery—in particular that of Aberdeen— "as a human invention, destructive to the civil peace of Christians, "and that he intended, by the aid of God, to walk and live in such a "Christian way as is conform to the Divine will in the sacred Word." Whitelock in his Memorials reports "that letters had come from "Scotland that the ministers of Scotland inflame the people against "England, and damn all their brethren and people who are not of "their opinion, and that the Laird of Drum had bid them defiance."

The first wife of this laird was Lady Mary Gordon, daughter of Lord Huntly; his second was Mary Coutts, the "shepherd's daughter" of the ballad of "The Laird of Drum," who, when his kinsmen would not bid her welcome, and the laird gave her the somewhat halting consolation—

> "Ye shall be cook in my kitchen,
> Butler in my ha', O,
> Ye shall be lady to my command
> When I ride far away, O."

replied with the spirit and plain speech of a Scottish wife—

> "But I told ye afore we were wed
> I was ower low for thee, O."
>
> "But . . . . .
>
> . . . . . .
>
> "An I were deid and ye were deid,
> And baith in a grave laid, O,
> And ye and I were tane up again,
> Wha could ken your moulds fra mine, O."

Something in the name of Irvine, as in that of Yarrow, seems to have attracted the ballad writer, and the heroine of the far finer song of "Helen of Kirkconnel Lea" was of the race of the Irvines of the Border. It was, indeed, a name which, in one or other of its forms of Irvine, Irwin, and Irving, spread wide as well as took deep root; and this may have helped to its welcome by the popular voice. Besides the original stem of Bonshaw and the branch of Drum, the name is found in Dumbartonshire, Forfarshire, and other Scottish counties. The Irwins of Roscommon, Cromwellian settlers, claim descent from the family of Drum; and the first great writer of the United States, proud that he united the name of Washington with that of Irvine, sent an engraving of his portrait to the Laird of Drum, in token of his belief that his Scottish blood was derived from the same source.

The son of Charles I.'s sheriff, as faithful a royalist and strenuous an anti-Covenanter as his father, was sentenced to death for his principles, but escaped execution by the victory of Montrose at Kilsyth. After the Restoration he declined a patent of nobility as Earl of Aberdeen, which had been offered to his father by Charles I., because the patent could not be made at the date of the offer.

The memories of such a name and ancestry clustering round the old tower, the ruined chapel, and the ancient Forest of Drum, could scarcely fail to transmit a taste for Scottish history, music, and song, and an attachment to Conservative principles and Episcopal and anti-Covenanting tenets. Nor will other traits in the character of his forebears, which with this view have been glanced at in the foregoing sketch, be found wanting in the subject of the present notice, as in a gallery of family pictures we are sometimes struck by the recurrence of the same features in a feudal baron, a cavalier, and a modern gentleman.

Mr Irvine was educated at home under a tutor, and afterwards at Marischal College, Aberdeen. During or shortly after completing his collegiate course, he came into contact with a group of men almost contemporaries, but most of them a little older than himself, who were destined to extend the credit of the town and county of Barbour, Boece, and Spalding, as the most fertile seed-plot in Scotland for historical talent; Hill Burton, the critical, yet vigorous, historian of Scotland; Dr Grub, the exact annalist of the Episcopal Church; John Stuart and Joseph Robertson, the two most thorough students of Scottish records in their time. With three other Aberdonians, who acquired honourable fame in the same field, he had early opportunities of intercourse—Mr Cosmo Innes, son of the neighbouring Laird of Durris, who became the Professor of History in the University of Edinburgh, and editor of the Acts of Parliament, commenced by Thomas Thomson, and of more chartularies than any man of his time; Mr William Forbes Skene, the historian of Celtic Scotland, son of the Laird of Rubbislaw, near Aberdeen; and Mr George Burnet, a younger son of the Laird of Kemnay, one of the worthiest successors of Sir David Lindsay in the office of Lyon. Though he did not become, like these scholars, an historical author, Mr Irvine shared their taste for antiquities, especially ecclesiastical and architectural, for books, especially those on history and law, and for literature, as one of the pleasures and ornaments of life. With Stuart, his connection by marriage, and Robertson, his associate by affinity of opinions and tastes, he was on terms of intimate friendship. When a happy selection transferred Robertson from the editor's desk to the table of the Historical Curator in the Register House in Edinburgh, his friendship with Mr Irvine was cemented by more frequent intercourse. For many years, until shortly before Robertson's death, they were companions in a Sunday walk to the chapel of Roslin or of Dalkeith, refreshing their bodies by exercise after the week's work, their minds by congenial conversation, and their spirits by common worship. To the end of his life Mr Irvine was a great walker, and, following a Scottish habit, took when alone a book as his companion, generally a volume of the classics. Although not an editor, through innate modesty, he was an early member of the Spalding Club of Aberdeen, which, unlike the Bannatyne, Maitland, and Abbotsford Clubs, did not merely print rare and valuable materials for history, but advanced that branch of knowledge by such works as the prefaces to Stewart's edition of the *Book of Deer* and *The Sculptured Stones of Scotland*, Robertson's admirably arranged *Collection of the Records of the County of Aberdeen*, Innes's *Memorials of King's College and Register of the Bishops of Aberdeen*, and Grub's edition of the *Early Ecclesiastical History of Father Innes.* He took part at a later period in the foundation of the New Spalding Club, which is now diligently gathering the fragments left by an older and more historical generation.

When Mr Irvine came to Edinburgh he devoted himself chiefly to Mathematics and Physics, and was urged by Professor James Forbes to continue them at the University of Cambridge. He had thought of the profession of Civil Engineering, but, yielding to his father's wish, entered on the study of Law, attended the lectures of that Faculty, and wrote in the chambers of a Writer to the Signet. He passed as an Advocate in 1843, and while he never enjoyed much practice at the bar, took a lively interest and possessed a sound knowledge in several departments of the Law. He was proud of and devoted to his profession, and supported by a modest estimate of his own talents, he did not, like so many, quit its ranks, but continued the study of the law in those branches which interested him most. He acquired what was then becoming rare, and has since become rarer, a considerable knowledge of Roman Law, and his ecclesiastical sympathies encouraged him to study its rival and successor, the Canon Law, whose remoteness from modern practice has led to its almost total neglect by Scottish lawyers, although some knowledge of it is necessary to an accurate acquaintance with the history and meaning of several parts of their own jurisprudence. The criminal law had always an attraction for him, and he edited several volumes of the Justiciary Reports, as well as a separate report of the trial of Madeline Smith ; but after the apprenticeship of circuit, he did not practise in the Criminal Courts.

Though not naturally inclined to authorship, he wrote a book on the Game Laws with an elegance not common in legal literature. He also wrote several reviews for Mr Joseph Robertson, when editor of the *Courant*, on Art, Painting, Music, and Italian Literature, a language he knew well and enjoyed. He was appointed to the principal clerkship of the Justiciary Court in 1867, and to the sheriffship of Argyle in 1871, and he discharged the duties of both offices with assiduity and conscientiousness, resigning the latter when the advance of age warned him that it was time to restrict his work.

As sheriff he was called upon to act in one of the varied (but fortunately rare) functions of that office, and led with discretion and success the naval and military force in the expedition to Tyree, where the law had been defied, and required to apply its ultimate sanction. In another ordinary, and more agreeable, portion of a sheriff's duties—the business of the Board of Northern Lights—he took great interest. He was constant in attendance at the meetings of that Board, as well as of the Board of Fisheries, and was rarely absent from the annual tour of the Committee for the Inspection of the Scottish Lighthouses. On the death of Mr James Crichton, Sheriff of the Lothians, he was unanimously chosen by his brethren Vice-Dean of the Faculty of Advocates. He never restricted himself jealously to his profession, as is now deemed by some lawyers indispensable to success in the profession of an advocate. He preferred the example of those who have thought that a profession calling itself liberal and learned, warrants, and even requires, the continuance in manhood of the liberal studies of youth. He was distinguished by the variety of his tastes, and the catholic spirit with which he cultivated them. If this variety prevented the attainment of excellence in any single subject, it at least saved him from the prejudices and bigotry sometimes associated with more concentrated intellects. An amateur in astronomy and physical science, he used to recall with pleasure that, through his acquaintance with Mrs Somerville, he had the good fortune to be present at the memorable dinner given to Sir John Herschell in 1836, when that philosopher returned after his splendid observations at the Cape of Good Hope. In later years he was a frequent visitor at the observatory at Dunecht, where his friends and neighbours, the late and present Earls of Crawford, aided by Professor Copeland and other astronomers, were advancing a science too much neglected in Scotland since the days when George Buchanan celebrated the triumphs, and James VI. visited the observatory of Tycho Brahé. He was one of the professional gentlemen of Edinburgh who invited Professor Tait to give lectures to them and their friends on the latest wonders of physical science. But he did not, like some students of physics, despise mental philosophy, or, like some lawyers, the philosophy of law, of which his friend, Professor Lorimer, was almost the solitary representative at this period in Great Britain.

Mr Irvine was by conviction, as well as by hereditary prepossession, a Scottish High Churchman, and he took a deep interest in the theological and liturgical studies of Bishop Forbes of Brechin, one of the most valued of an unusually large circle of friends. But this did not prevent him from sympathising with the valiant stand made by Mr Robertson Smith for freedom in linguistic and historical science as applied to biblical criticism, when that scholar was opposed by the conservative forces of all the churches, and deprived of the professorship of Hebrew at the Free Church College, Aberdeen, by the policy then followed, but since modified or abandoned by the leaders of the Free Church.

He had a warm appreciation of the Fine Arts, of which one of his grand-uncles had been an early connoisseur, and of classical as well as Scottish music, and also of the old and too often forgotten classics of the Scottish language. Along with the late Lord Justice-General Inglis, he was one of the few lawyers who promoted the Scottish Text Society, founded by Dr Gregor of Pitsligo, and supported by Professor Masson, both Aberdonians, and he served as a Vice-President of that Society. He had a curious memory for rare quotations, epigrams, and quaint and old stories, both of and outside of his profession. It was pleasant to see him preparing to tell one, and silently enjoying the smiles or laughter of the listener. He never lost the Aberdonian accent, but his voice had a mild intonation of its own; he had less of the ordinary Aberdonian temper and manner. By nature and habit he was reserved, and even shy, but always courteous, never combative or demonstrative, though firm in his convictions, and ready, when necessary, to maintain them. While his character was thoroughly Scotch, his culture was of a kind perhaps more common south than north of the Tweed. He was the only lawyer of his time who, following a fashion which lingers a little longer in the Houses of Parliament than in the Parliament House, appeared with a flower—by choice, violets—in his button-hole, bringing a breath of fresh air into the dusty purlieus of the law, and casting a gleam of sunshine over the musty books and keen visages of the daily labourers in the Courts. He was naturally fond of the garden, and its flowers were the only ones he cultivated, for he never indulged in rhetoric. He held the common opinion that most speeches are too long; and, what was less common, he acted on it, and when he spoke in public did so with grace and terseness.

He became a Fellow of the Royal Society of Edinburgh in 1874, and served as Vice-President from November 1883 to October 1886 and from November 1890 to April 1892. In 1887 he was made a Doctor of Laws by the University of Edinburgh, and he was one of the few who know something of both the laws of which he was an honorary graduate.

The ownership of a fine estate, bounded by one of the beautiful rivers of Scotland, and an old castle with historical associations, besides influencing his character, imposed on him the duties of a country gentleman, which have seldom been better fulfilled. The loss of a portion of his estate through an unfortunate law-suit he was bound, by honour as well as by interest, to defend, in which the House of Lords reversed the judgment of the Court of Session, and perhaps a share of the proverbial prudence of his fellow-countrymen, led him to practise economy in matters personal.

But his economy was the reverse of selfish. Its purpose was to enable him to be more hospitable and charitable on what he deemed the proper occasions for the exercise of these virtues. His estate was one of the best managed in the district where it lay, and best provided with houses for farmers and labourers. He left it free from debt, and it was estimated that he expended £40,000 on its improvement. He made a convenient and appropriate addition to the Castle of Drum, which will carry down its history in the nineteenth —as the strong Great Tower marks its origin in the days of the War of Independence and the larger addition by one of his ancestors records its history after the troubles of the seventeenth—century. During a period when many of the nobility and gentry were selling the books they inherited to pay the bets or stakes they had lost, he was extending his library, in which he had the interest of a reader, and not merely a collector. He restored with taste the decayed chapel, and practised in it a natural piety, after the manner of his forefathers and the rites of the branch of the Church Catholic of which he was a member. His unobtrusive talents and independent and honourable character received the recognition due to them from those amongst whom he lived, as well as in the profession to which he belonged. He acted for about thirty years as Chairman of the Committee of Publications of the Highland and Agricultural Society, and more than once as Director of the Philosophical Institution of Edinburgh. He served for several years after 1860 as Captain-Commandant of the 20th Aberdeenshire Rifle Corps. In 1862 he was chosen Convener of the Commissioners of Supply of the county of Aberdeen, and presided over their meetings with invariable courtesy and tact. On retiring from office, when the Local Government Act came into operation, he was presented with his portrait, painted by

Sir George Reid, *P*.R.S.A., whose brush has represented, with the fidelity of an Aberdonian and the insight of a master, his outward appearance—the sidelong glance of the shy and modest eyes which at times in later years seemed to look within, the gentle smile, and the gentlemanly bearing. He also acted as chancellor or legal adviser to his friend, Bishop Forbes of Brechin, and to his successor, the present Bishop and Primus of the Episcopal Church of Scotland.

He was not spared the trials which those who live long usually meet. He lost his first-born son, Alexander, when young; his next (also Alexander) in the prime of life, a scholar of New College, Oxford; and he bore these, as he did less serious misfortunes, with a brave spirit.

He was survived by two brothers, James Hamilton Irvine who early in his life went to Australia and settled in North Gippsland, and General Charles Irvine of the Indian army, and a sister, whose gift of Scottish song often charmed him and his friends. His wife, who shared his tastes and aided all his endeavours, also survived him. Her father, Colonel Forbes Leslie of Rothienorman, a learned archæologist, to whose manuscript history of the family of Drum this notice is indebted for several facts in its earlier annals, afforded another example of the pursuit of intellectual and artistic studies amongst the gentlemen of Aberdeenshire which, under fortunate conditions, may stimulate a generation and enlighten a neighbourhood. An only remaining son, Francis Irvine, has inherited an historic name, which Mr Irvine handed down, not dignified by any title or illustrated by any remarkable genius, but distinguished by a quality more useful than genius, which cannot be imitated and is rarely transmitted,—the faithful discharge of the duties of a scholar, a lawyer, and a gentleman in the station and offices which fell to his lot.

Alexander Keiller, M.D., LL.D., F.R.C.P.E., F.R.S.E.
By T. A. G. Balfour, M.D.

(Read January 16, 1893.)

Dr Alexander Keiller became a Fellow of this Society in 1866. He held a distinguished place as a physician, more especially in the gynæcological department of the profession, and was much and justly esteemed, respected, and loved by all his professional brethren.

His extensive experience, gained from many important public appointments and from a large private practice, and the numerous, varied, and valuable contributions with which, as the result of these advantages, he enriched the literature of his profession, placed him among the very foremost of those who have extended the confines of practical medicine, and furnished to his fellow-workers a tried and solid foundation upon which they may confidently build in prosecuting their further researches.

Dr Keiller was born at Arbroath on November 11th, 1811—a date easily remembered, if we adopt the mnemonics which he has humorously supplied: "I was born," he said, "at the 11th hour of the 11th day of the 11th month of the 11th year of this century."

His father, John Keiller (or Keelor, as it is written in the burgess ticket which he obtained at Aberbrothock in 1804, and which, through the kindness of a friend, is now before me), was a merchant in that town, but seems to have been originally from Dundee, as he is stated to have *returned* to Dundee from Arbroath in 1814.

Dr Keiller's early education was, I believe, at the grammar school at Dundee; and afterwards, with a view to prepare him more thoroughly for his preliminary medical studies, he was placed under the tuition of Mr James B. Lindsay, whose fame as an educationist was widely acknowledged over the northern counties of Scotland. At that time, on a house in Union Street, Dundee, you might have seen a signboard with the unpretentious inscription—James Lindsay, Teacher of Languages; and within that house there lived a man in many respects truly remarkable—a kind of prodigy of learning.

He was a profound mathematician, an able and accomplished scientist and experimentalist, and as a linguist it would be hardly too much to say that he was *facile princeps* of all the philologists who ever lived; for there was scarcely a language spoken on earth which he did not know and could not speak. It was to the training of such a man that Dr Keiller was now entrusted. His affections were drawn out towards his teacher, and almost the last of his public acts was to pay a loving tribute to the great merits of Lindsay in a paper which he read as lately as last August before the British Association, by showing that he was among the first, if not the very first, who invented the Electric Telegraph, and had actually worked one of his own construction between Dundee and the coast of Fife; and that he also successfully experimented on electricity as a source of lighting.

Contact with such a genius could hardly fail to inspire a like enthusiasm and perseverance, though these, in the case of this pupil, were directed into other channels, and must have done much to lay the foundation of his after success and eminence, both as a careful observer and skilful inventor. Some time after the death of his father, Mrs Keiller removed with her family to Edinburgh, and took up her abode for some years in Adam Square.

Before commencing his medical studies at Edinburgh, young Keiller had shown a strong bias to anatomical pursuits, and might have been found, when fifteen or sixteen years of age, at the *Earthen* Mound (as it was then called), sitting for hours together under a temporary shelter, and by means of an old book on Anatomy studying the various human bones which he had picked up from the loads of rubbish which from time to time, since 1782, had been laid down during its formation. With a proclivity so marked, we are prepared to learn that when, in 1830, he attended the lectures of Dr Knox, who was then at the zenith of his popularity, Keiller became a favourite pupil, and that that distinguished anatomist augured well regarding his success in that science.

After taking the diploma of the Royal College of Surgeons in 1833, his neatness of hand and proficiency in anatomy doubtless commended him to Dr A. Jardine Lizars, an able and successful teacher of that science in the Argyle Square Medical School, who afterwards became Professor of Anatomy at Aberdeen, and Dr

Keiller was appointed his Prosector and subsequently his Demonstrator; and while here he made, and superintended the drawings of, a large series of dissections for a work on Regional Anatomy.

After Dr Knox retired we find Dr Keiller, in conjunction with the late Dr Skae, lecturing on Anatomy on his own account. During his student days, however, a new subject had been brought under his notice, which was destined to form the special study of his future life. Dr John M'Intosh was then lecturing with much acceptance on Midwifery, and his pupil, and afterwards class assistant, Keiller became; and in this post, and at a Dispensary which Dr M'Intosh opened and left almost entirely to his assistant's care, he had an excellent opportunity of acquiring a knowledge of those subjects of which in later life he became the able exponent, and in the successful prosecution of which he gained his laurels.

In 1835 he took his degree of M.D. at St Andrews University, and afterwards became one of its examiners in Medicine; and in 1886, as a mark of their appreciation of his merits, the Senatus conferred on him the honorary title of LL.D.

After having lectured on Anatomy for two years in Edinburgh, he went to Dundee, and, following in the footsteps of Dr M'Intosh, he took part in establishing and conducting a Medical and Surgical Dispensary there, and gave special attention to that department of it which related to Midwifery and the Diseases of Women and Children.

Having laboured for seven years at Dundee, he was induced to return to Edinburgh by the urgent request of his friends, and notably by Professor (afterwards Sir James) Simpson, with whom he was on terms of the greatest intimacy. On his return, in addition to general practice, he commenced lecturing on Materia Medica, and subsequently took up the subject of Medical Jurisprudence; and ultimately in 1853, on the death of Dr Campbell, who had been Lecturer on Midwifery and the Diseases of Women and Children in connection with the Royal College of Surgeons, Dr Keiller was appointed to that honourable post, and to that special department he afterwards almost exclusively devoted himself.

The mere recital of these subjects on which Dr Keiller gave his prelections shows very clearly his extensive acquaintance with various branches of Medical Science, and the amount of collateral knowledge which he brought to bear on the subject to which at last his energies and talents were mainly directed.

In 1846 Dr Keiller was fortunate in securing as his partner in life the daughter of Major Roy, a lady in every way fitted to contribute to his happiness and comfort. He was elected a Fellow of the Royal College of Physicians in 1848, and was one of its examiners in Midwifery and Medical Jurisprudence; and after having for years been an active member of Council, he was elected President of that College in 1875. On the lamented death of Dr Seller, he was chosen a trustee, and very recently held the appointment of Morrison Lecturer; in connection with which he delivered two courses of lectures devoted to the consideration of the nervous diseases of women, in which the value of his early anatomical studies and work were apparent.

In 1851 he was appointed one of the Ordinary Physicians to the Royal Infirmary, and continued to discharge the duties of that office for fifteen years. While here he was the means of securing the setting apart of a special ward for the clinical study of the diseases of women. He afterwards became Consulting Physician to that institution; and at the same time was chosen one of the Ordinary Physicians to the Royal Hospital for Sick Children, which office he held for eight years, and during that time delivered clinical lectures on the diseases of children. He also held the office of Consulting Obstetric Physician to the Royal Public Dispensary, and was one of the physicians of the Royal Maternity Hospital.

He was appointed an examiner in Midwifery at the University of Edinburgh, and from time to time lectured to the class of Midwifery in that school of learning at the request of his attached friend, Sir J. Y. Simpson. When the deeply lamented death of that most distinguished physician occurred, Dr Keiller became a candidate for the vacant chair, and many were the testimonials which he received from all parts of the world, and several of them from those who had attained distinguished positions in the profession, as to his eminent fitness to become the successor even of one so pre-eminently renowned as its late occupant. I may add that Dr Keiller was an Honorary Fellow of the Obstetrical Society of London, and an Honorary Member of the Gynæcological Society of Boston, U.S.

His writings were very numerous, and were communicated to various Societies, both here and at London; but specially did our Obstetrical Society get the benefit of his ripe experience and soundness of judgment. For twenty or thirty years he was present at almost every meeting of that Society, and scarcely could a subject be introduced, or a specimen exhibited, on which he could not profitably dilate, as having in his own practice had cases of a similar kind which he had carefully studied.

To Dr Keiller we are also indebted for the invention and introduction of various useful instruments into obstetrical practice. In this connection it is interesting to know that a similar inventive faculty was possessed by a brother of Dr Keiller at Perth, who patented valuable machines, among which was a self-registering target, so as to avoid any risk to the marker at rifle competitions, and of this the Government, I understand, ultimately availed themselves. Another of his inventions, by his failing to renew the patent, was adopted by others, who thereby enriched themselves at his expense. As regards Dr Keiller, I may specify that it is to him that we are indebted for the introduction of caoutchouc bags into midwifery practice, he having brought them under the notice of the Obstetrical Society here at least a year before any competitor appeared on the field; and it is to be regretted that, in the minds of some, these instruments are even now associated with another name than that of Keiller, and thus he was subjected to the painful experience of the poet when he exclaimed:

> "Hos ego versiculos feci, tulit alter honores;
> Sic vos non vobis," &c., &c.—

an experience to which his brother also was not a stranger.

The retiring modesty of this estimable physician formed a marked feature of his character, and may, to some extent, have concealed his real merits, though, in the case of all who knew him well, it only added a fresh lustre to his other qualities, as in all discussions he was ever ready to acknowledge the merits of others, and there was a total absence of anything like self-assertion on his part. Sir J. Y. Simpson early observed this beautiful feature in his friend, for in 1843, in writing to him that he (Sir James) would be proud to acknowledge his zealous labours, he adds: "Do send something for our *Journals* yourself. You have in you the power of doing much more than you suppose."

While his talents and high professional standing secured for him well-merited honour and respect, his sincerity of heart, kindly and amiable disposition, and the vein of quiet humour which he possessed, greatly endeared him to all his professional brethren, and made him a most welcome guest at their social gatherings. His removal from the midst of us has left a great blank, and caused deep and sincere sorrow, and his memory will long be cherished with affectionate regard.

Dr Keiller's attention, however, was not limited by the things of time and sense, but extended to the higher and nobler objects beyond: as his minister, the Rev. Arthur Gordon, well said, "He had wisely cultivated the spiritual as well as the intellectual side of his nature." He availed himself of the means of grace in, and was a respected elder of, St Andrew's Parish Church. He held fast the old truths of the infallible Word of God, and had no sympathy with, but grieved over, that specious and pretentious, but really shallow rationalism and infidelity of the present day. Nor was he ashamed to own that he did so, as appears also in his remarks on Mr Lindsay, when, after speaking of his philological pursuits above referred to, Dr Keiller adds: "It may be mentioned, as imparting greater value to these researches, that Mr Lindsay's attention was originally riveted on philological studies, owing to doubts which he entertained as to the authenticity of Scripture history, more especially as regarded the origin of the human race from the primal pair; but that the more he studied the different languages, dead and extant, the more his doubts gave way, and the stronger did his conviction of the truth of the literal exactness of the Scripture statements on this subject become."

Dr Keiller died at North Berwick, where his country residence was. The cause of his death was apoplexy. The shock occurred on September 18, 1892, and proved fatal on the 26th day of the month. He had reached the ripe age of 81; but till within a few days of his death his corporeal and mental vigour seemed so little impaired that his friends were joyfully anticipating that his sojourn on earth would still be considerably prolonged. A large and attached group of mourners assembled at St Andrew's Church, and conveyed the remains of their beloved friend to their last resting-place in Warriston Cemetery.

George Husband Baird Macleod, Knt., M.D. Glas., LL.D. St And., F.R.S. and F.R.C.S. Ed., F.F.P.S.G., and F.R.C.S.I., Corresponding Member of the Société de Chirurgie de Paris and of the Académie de Médecine de Paris; Member of the Deutsche Gesellschaft für Chirurgie; Regius Professor of Surgery, Glasgow University; Senior Surgeon to the Glasgow Western Infirmary, and Surgeon in Ordinary to Her Majesty the Queen in Scotland. By the Rev. **W. H. Macleod**, B.A. Cantab., B.D.

(Read January 16, 1893.)

George Husband Baird Macleod was born in the Manse of Campsie on the 21st of September 1828, and died in Glasgow on the 31st of August 1892, after two days' illness. He was the son of Norman Macleod, D.D., Dean of the Chapel Royal, and one of Her Majesty's Chaplains, afterwards minister of St Columba's Church, Glasgow, famed as a Celtic scholar, his writings in the Gaelic language being unrivalled among modern authors. From this cause, added to his eloquence as a preacher, and his unwearied labours for the good of the Highlanders, his memory is yet fondly cherished wherever the Gaelic language is spoken. The grandfather of Sir George was Norman Macleod, minister of the parish of Morven, whose ministry, together with that of his son, who succeeded him (John Macleod, D.D., Dean of the Thistle, the well-known and much-honoured "High Priest of Morven"), extended to the remarkable term of 105 years. On his mother's side he was descended partly from a Lowland family, who achieved no little distinction in their day and generation, the last of whom, his grandfather, James Maxwell, being Commissioner to the Duke of Argyll over his estates in Mull, Morven, and Tiree. Through his mother he was also descended from several well-known Argyllshire families. It would be impossible here to do more than allude to the many influences which helped to mould his character, but no sketch would be complete if it did not make some mention of the atmosphere in which he had been brought up, or of the traditions which he unconsciously imbibed, inherited on both sides from a race of honoured ancestors. That he fully appreciated how much he had gained from the past, his own record of his early days gives abundant testimony; and to the dim but animating memorials of those Highland homes, where his parents passed their youth, he attributed much of the success in life which he, with other members of his father's family, gained—for they were memorials of families revered for their human sympathy, their unswerving rectitude, their kindly solicitude for the people around them, as well as their deep affection for one another. Speaking of the "Highland Parish," written by his well-known brother, Norman Macleod, D.D., minister of the Barony parish, Glasgow, where the picture is faithfully drawn of what those people were, and of the influences under which they lived and died, he writes: "It is impossible now, amidst the 'sturm und drang' of modern life, to define the nature and influence of that circumambient atmosphere which a Highland upbringing in the time of my parents produced, and which their children unconsciously found around them ever afterwards. A temperament moulded by much of poetry and legend and misty tales, and not free from a certain suspicion of superstition, gave a complexion to many of their views of life and of persons; and while good breeding and unswerving loyalty to old friends, with much gratitude for kindness received, were characteristic of them, yet prejudice and antipathies were not denied." But though this influence from the past must not be lost sight of, yet perhaps he owed more to the moulding and guiding example of his eldest brother Norman, apart from the wise and loving influence of his parents, than to anyone else. Concerning him he writes: "From earliest youth he was a personality in the family. We all looked up to him, and he did not fail to exercise over us the best and most enduring influence. His great talent, high aspirations, deep religious feeling, and broad, manly principles, full of all that was true and real and honest, and wholly free from cant and suspicion of hypocrisy, could not fail to have an abiding and continuous effect on all of us, who were his juniors, and all of whom were deeply attached to him. His example steadied us, his happy and affectionate nature welded us into a truly united family, and exercised over us an influence which augmented and confirmed the happy effects of our parents' more silent instruction."

Surrounded, therefore, by such influences, which all through his life continued to exercise their hold over him, and himself a son of the Manse, he was until the end loyal to the best traditions of such a home, and to the Church with which these abodes were connected.

His father removed to Glasgow in 1836; and after attending Mr William Munsie's well-known Academy in Nile Street, and being for some time at school in Arran, he joined the Junior Latin and Greek classes in the University at the beginning of the session 1843–44; but beyond taking the University Prize Essay, and gaining high honours in Philosophy, he did not specially distinguish himself in his Arts course, a circumstance which he chiefly attributed to the insufficient way he had been taught the rudiments of classics. The foundations of knowledge were never properly laid, and all through his life he felt the neglect in this respect to which he had been subjected. It was the Logic Class which first awakened in him the love of knowledge for its own sake, and gave him the first real impetus. "This class," he writes, "was the key to my brain, and emancipated me from what was to me the cramped and uninteresting field of classics, and set my feet on a firm and enduring rock. I owe everything to that class and its genial teacher" (Professor Robert Buchanan).

In 1848 he entered the Medical classes. It had been arranged that he should study for the Church; but Sir John Macleod of St Kilda, then a Director in the Old East India Company, having promised his father to nominate him to a medical post in that Company, it was thought that such a chance should not be lost, so he took up a subject for which at the time he had no special predilection, and of which he had no special knowledge. He, however, soon made his mark, taking a prize in Anatomy and Materia Medica, and a first in the Institutes of Medicine; but the extra strain which this inflicted upon him, and the almost incessant work which it implied —for more than once he sat up all night, and almost always far on into the morning—told upon his health, which was then far from satisfactory, and brought on spitting of blood and other dangerous symptoms, which necessitated almost complete cessation from study. It was therefore thought best that he should go to the south of England in the spring of 1851; but though greatly benefited by this change, it was deemed somewhat risky for him to remain in Glasgow during the winter, so he was sent abroad. Gibraltar was the place selected, and never did he cease to entertain the pleasantest recollections of his stay there. It also proved of the utmost benefit to him; for in the spring of 1852, after visiting Malta, he returned home, through Italy and Germany, perfectly restored to health. This was his first experience of foreign travel, and the love which he then imbibed for it never deserted him. Some part of every year was passed abroad; and as we glance through the many bulky volumes which contain the record of all his wanderings, it can easily be seen how much he enjoyed travelling. Many of the incidents which they contain are very amusing, and some of great interest. It may not be out of place to transcribe here an account which he gives of a scene which occurred on a Spanish lugger, dirty beyond all description, as Spanish luggers were, in which he crossed from Tangiers to Gibraltar, as it shows, in spite of the inconvenience and discomfort which he had to put up with, the keen sense of the ludicrous which he always possessed. "As the sun rose," he says, "all the cocks—and there were dozens—in our coops began to crow lustily, and those whose freedom enabled them clapped their wings with joy. It is curious, but probably an electric influence, which thus compels cocks to crow when they feel the sun. These birds all crew, and yet some of them were so uncomfortably situated that it defied me to understand what pleasure they could have in the act. Some were standing on their heads, or rather necks, with their long red eyelids winking on the deck; others on their backs formed the pedestals of innumerable feet, the bodies belonging to which were again the points of support of another living layer. Placed in every imaginable posture and ungraceful attitude, cramped and crushed to the utmost limits of endurance, and many in the centre totally excluded from a ray of light, these gallant trumpeters sounded their peal of joy as if they exulted in the thought that the time of their liberation was drawing near. At times the first note, which was delivered with emphasis, seemed as if it comprised the utmost exertion of which its author was capable; the succeeding prolongation, on which the whole effect depended, being wholly wanting, or dwindling down into an insignificant rattle. At other times a bravura was expelled in short, disjointed, but determined accents, as if the taste of the performer had led him to execute it in staccato, while every now and then some poor aspirant in the centre of the crowd, vainly endeavouring to balance himself on others during his debût, produced a dreadful tumult in the community by the living props giving way, and his putting the whole mass in commotion by his fall, while his own chivalric effort prematurely died away in choking accents, some violent neighbour having apparently made fierce attempts to garrotte him. Never, I think, was melody produced under more disadvantageous circumstances. Their chanticleeric endeavours, moreover, were not received with due encouragement by the crews of the ships around. At daylight our anchor had been raised to allow us to drift in with the tide; and as we passed through the shipping, innumerable redcowls from the bulwarks cursed our concert in every language under the sun."

He graduated in Medicine in the spring of 1853, and immediately afterwards went to Paris to continue his studies. That great Medical School was then presided over by men of European renown, such as Velpeau, Nélaton, Bouchardat, Jobert de Lamballe, Ricord, &c., and as his degree in medicine freed him from all fees for lectures or hospitals, he was able to make as much use as it was possible of his opportunities. As in Glasgow, so here, anatomy and surgery claimed his chief attention, and many were the hints and valuable the experience he gained. The French School ever exerted its influence over him, and it was to their works that he was in the habit of first turning in all his after preparation.

He returned from Paris in the autumn of 1853, spending some time on his way home in the London Hospitals, and settled down to practice in Glasgow, having by that time given up an idea of going to India, partly owing to family affairs, and partly because of better prospects presenting themselves at home. But it was not long before he was again away. Europe was beginning to echo with the call to arms, the Crimean War was on the tapis, and where better could a young surgeon gain that experience which was so necessary for him in his profession than on the field of battle? But how was he to get there? While on the alert to seize the first chance which presented itself to achieve this most desirable end, the opportunity was most unexpectedly put in his way. One evening, at a dance, his host asked him whether he would be willing to go with a friend of his on a yachting cruise to Constantinople. He jumped at the chance of thus getting near the seat of the future war, and on the

23rd April 1854, a few days after he had accepted the invitation, he started. This sudden resolve was the making of him, and was (though then very obscurely seen) the first rise of the tide which brought him long afterwards to the front. In the *Chance*, an 80-ton cutter, he visited many places in the Mediterranean, and it was when lying in Malta that the war broke out. Through the recommendations of his friends, he was strongly urged to go and try what chance there was of employment. Leaving, therefore, his yachting comrades, he pushed on with all speed to Constantinople, only to meet with disappointment. Nothing was to be done there; and though he made every effort to gain his end, every attempt met with failure, so that he was reluctantly compelled to return home. Yet, fruitless though the voyage appeared at the time, it was not really so, for in November of the same year (1854) he was again on the war-path. Colonel George (afterwards Sir George) Campbell of Garscube, who was in the 1st Royal Dragoons, had been severely wounded in the heavy cavalry charge at Balaclava, and his mother, Mrs Campbell, anxious for his safety, and desirous to find some one who would go out and bring him home, called one day and asked Macleod if he would undertake this duty. This request he gladly complied with, and he travelled night and day until he found his patient, at Scutari, very badly wounded, and much in need of some one to tend him. After nursing him for many weeks at Scutari, and afterwards at Constantinople, he brought him back in safety to London. But while waiting for his friend to gain sufficient strength for the journey home, his restless energy, and his determination to make himself as efficient in his profession as he could, did not allow him to pass the time in idleness, for he worked morning, noon, and night in the English and French Hospitals, and saw and did a great deal of surgery, and in the dead-house was able to practise all the operations frequently. This action of his was characteristic of him all his life long: never did he allow an opportunity escape of perfecting himself in that profession which had stirred his enthusiasm. By this time rumours of the unsatisfactory state of the hospitals in the East, and of the sufferings of the wounded, had reached England, and all were anxious to receive authentic information, none being more desirous of ascertaining the exact state of matters than the Government of the day. While passing through

London with Colonel Campbell, where he met Sir William Ferguson in consultation, he was asked by Lord Blantyre, who knew the Campbells, to go with him to the Minister of War, Mr Sidney Herbert, in order that he might give him all the information he could upon these important points. Macleod strongly recommended that wooden hospitals, like those used in Glasgow as temporary buildings during the fever epidemic, should be erected, and that the dirty and poisonous barracks then in use at Scutari should be abandoned. This recommendation of his was agreed to, and he was asked to obtain plans with as little delay as possible. On his return, therefore, to Glasgow a few days later, with the assistance of Professor Lawrie, and Mr James Smith, architect, plans were drawn up, modelled on those used in Glasgow, but with such alterations as seemed necessary for their new requirements, and Professor Lawrie returned with him to London to support the idea. They were afterwards adopted and set up on the Dardanelles. By this time, however, the Government found that, to make adequate provision for the proper treatment of the wounded, it would be necessary greatly to increase the medical staff, and they therefore determined to augment the regular army medical staff by a specially arranged staff of civil surgeons. Sir John Forbes and Mr (afterwards Sir William) Bowman had the organisation of it, and by the latter Macleod was offered an assistant-surgeonship. This he refused to accept, representing that, as he had been twice out, and knew more of the work there than any of those spoken of for the senior positions, it would not be worth his while to go unless he received a senior appointment. This he was given, and asked if he would go out at once to Smyrna with Major Storks, who had been appointed to organise the staff and hospital there. He received his appointment on Tuesday; and on Friday, 10th February 1854, three days later, he had started for Smyrna, by Paris and Marseilles, with Eddowes as his assistant-surgeon, and under the command of Major Storks. They arrived at Smyrna on the 25th, when, greatly to his surprise, he found himself made senior of the whole staff, and appointed interim superintendent. This piece of good luck came about partly through his having "so courageously" gone out at once when asked, and partly through the recommendation of the superintendent, who was home on leave, and Major Storks, who considered him best fitted for the post. Thus he found himself, at the age of twenty-six, in a position which many men, greatly his seniors, might have envied.

There was much need for reorganisation, but soon he had all things in good working order, associated as he was with a band of energetic men, nearly all of whom made their mark afterwards in the world—Spencer Wells; Ranke of Munich; Macdonnell, afterwards Professor of Surgery in Dublin; Rolleston of Oxford, and many others. Here at Smyrna he remained until the end of May 1855, when the work became lighter; and having the good excuse of an attack of Smyrna fever, he asked for leave, and started for "the front," being determined to see active warfare somehow. With letters of introduction from Colonel Storks to Dr (afterwards Sir John) Hall, principal medical officer in the Crimea, and to many others, he set out. Dr Hall received him most kindly, and to the weary and overburdened medical officers his help was most welcome. But he was not engaged long at such temporary work, for, a surgeon attached to the General Hospital having died from cholera, he was placed in orders by Dr Hall to succeed him. This was a most responsible position, as the General Hospital was of considerable size, and was "general," or for no special regiment or division. It was in the "lines" of the 3rd Division. He then received army rank (that of Major of comparative rank and first-class Staff Surgeon in the Medical Service), and remained "Senior Surgeon to the General Hospital before Sebastopol" from this time till the Crimea was evacuated in 1856. Of the hardships of that trying time,—and they were not easy to bear, as one can judge from his journals, kept most methodically during the whole time of his residence in the Crimea,—we cannot now speak. Several times under fire, he remained at his post until, as the result of all his surroundings combined—food, sleeping-quarters, bad water, fatigue, and ennui—he was struck down with erysipelas and camp fever. He and his tent companion, who afterwards succumbed, were seized at the same time; and although fried slices of salt pork and rum and water formed their chief staple of food, he finally rallied after having been sent down by sea to Therapia. Before he returned to duty he made a hasty visit to Smyrna to settle up his affairs there; and although at Constantinople, where he met the late Sir

William Aitken, he was seized with a sharp attack of jaundice, as soon as he could he was again at his post, remaining there through the winter of 1855–56, until the signing of Peace, in April 1856. For his services at the Battle of the Tchernaya he got the Sardinian medal, also the Turkish, and, we believe, was the only civil surgeon who received the English medal with clasp for Sebastopol, and on his return home he received a special gratuity from the Government for his services. He was also to have received the much coveted Legion of Honour, but, through some carelessness in making the return, he never got it. Leaving the Crimea in April 1856, he visited Palestine and Egypt, and before he reached England spent some time in Paris attending the Hospitals and renewing old friendships. On his return to Glasgow, in the autumn of 1856, being then but twenty-eight years old, he settled down to practice, and published, soon after his return, his *Notes on the Surgery of the Crimean War, with Remarks upon Gunshot Wounds*, a book which at once brought him into notice, attracting as it did a good deal of attention, and which even yet is recognised as one of the authorities upon the subject of which it treats. Besides the British edition, some 6000 copies were sold in America, and it was distributed by authority in both the Northern and Southern armies. But, though engaging in general practice, he was at heart a surgeon, and desired above all things to distinguish himself as a teacher. He therefore fitted up his dining-room as a lecture-room, and began a class of instruction in surgical apparatus—a subject which was not then taught even in the hospital. Encouraged by the success which attended this venture (for the first winter he had a class of thirty-two), he took a room the following winter in Cathedral Street, and announced a course of lectures in Systematic Surgery. Dr Robert Hunter then occupied the Chair of Surgery in Anderson's College, and though at first this rival class met with his opposition, he finally gave Macleod all the support he could. It was during this time that Macleod began to agitate, by pamphlets and otherwise, for certain reforms in the mode of clinical instruction, and of the appointment to office, then in vogue, in the hospital; and, though in later life he might not altogether have approved of his own recommendations, yet the controversy did good, though he suffered the penalty of a reformer by his being kept out of the Infirmary for some time. When, however, he was at length appointed one of the House Surgeons to the Royal Infirmary, the enthusiasm with which he threw himself into the work of clinical teaching, and the success which attended him, induced Professor Lawrie, to whom he was ever deeply attached, to appoint him to conduct his class when failing health prevented him from fulfilling his work; and, as an evidence of his popularity and success as a teacher, the students presented him at the close of the session with a handsome testimonial.

On the death of Dr Hunter in 1859, Macleod was appointed to succeed him in the Andersonian : this gave him the outlet he wished; and by gradually dropping certain departments of general practice, he was enabled to confine himself more entirely to his work as a teacher. In 1859 the Chair of Surgery in the University fell vacant, through the death of Professor Lawrie; and, though Sir Joseph (then Mr) Lister was appointed to the Chair, the ten years which he passed at the Andersonian were of incalculable value, for the experience which he gained there of teaching, in addition to the knowledge of his subject which he had previously received in Paris and in the Crimea, made his claims paramount when, in 1869, the Chair of Surgery in the University again fell vacant by the removal of Sir Joseph Lister to Edinburgh. From that time onward, having then dropped general practice altogether, his heart was completely bound up in the success of his classes at the University and the Western Infirmary. Only those who met him there can know the enthusiasm for his work which, even up to the day of his death, possessed him; and this enthusiasm he transferred to his students, who flocked to him in such numbers that every available corner of his large class-room was crowded, many having to content themselves with standing room, or to seek some insecure or uncomfortable resting-place upon a window sill or upon the floor. Yet with it all he never found any difficulty in maintaining the most absolute order, though often he expressed himself amazed at the attention he met with, and the earnestness and interest with which they followed his every word. It was his desire to help his students to be men of wide sympathy, and, raising them up above the mere drudgery and business of their profession, to make them feel something of the dignity of their calling, and cause them to hate and shun all that was mean and all that was base. His interest in them never failed; they might rely upon his ready help and warm sympathy though years might have passed since they sat under him at the University or followed him in his clinical teaching at the Infirmary. For it was ever a pleasure to him to welcome back old students, and hear from them how it had fared with them since they had left the shelter of the College walls.

Skilful though he was as an operator, and through firmness of nerve able to perform with success the most difficult operations, yet his interest in his patients did not then terminate. His care and watchfulness never relaxed until the cure was complete, and even then he was glad to see them again, and to hear of their welfare.

Those who followed him and heard him teach, and saw him operate, can testify to his power of attraction and to the care which he lavished upon every case; but it was only those who could follow him home who knew the strain and stress which it all involved, and how deeply he felt the suffering which he did all that in him lay to relieve. Many were the sleepless nights he spent, after the toils and anxieties of the day, imagining all the possible contingencies that might arise to frustrate his skill and care; and this highly strung nervous temperament, which in our estimate of him we should not forget, did much to wear out his otherwise robust constitution. It is, however, impossible to do justice to this latter portion of his life, for which, after all, the earlier portion was but the preparation; though, no doubt, as it has been most truly said, his best memorial lies in the hearts of the thousands of medical men scattered all over the globe, who owe to him mainly the groundwork of all their surgical knowledge. Perhaps we cannot do better than to insert here a passage from the *Glasgow University Magazine*, which, coming from such a source, may well be taken to indicate how much he was beloved by the students of his Alma Mater. Nothing would have cheered his heart so much than to have known that those, for whose welfare he had so earnestly and devotedly laboured, understood and reciprocated his feelings. After speaking of the loss which they felt they had sustained when the news of his sudden death reached them, and of how unexpected it was to those of his own class, who, at the close of the session, had listened to his usual hearty farewell till a welcome return to the winter's work, the article continues :—

"He lived for students, and died in their service, while his first aim in life—the impulse which anyone who knew him at the bedside or in the Operating Theatre at once saw to be the mainspring of his thorough method and perfected skill—was to cure the sick and ease the suffering, by the best and kindest methods the science he loved could teach him. It lends grace to our memory of him that the news of his death stayed the hands that were busy in the ward preparing for his visit. Every professor is a hero to his students; and could the boundless reminiscences with which Sir George freely entertained his followers, the stories unique in humour—at times in pathos—with which he enlivened his lectures, and all the acts of honest kindness ever ready for those who honoured him by doing his work—could all those be gathered from the hundreds who even to-day cannot realise that they shall enjoy such no more, they would form a volume limitless as rare. No professor could have been more ready to entertain every project where students asked his advice or sought his support. It was on account of a high ideal of home life that his figure was not oftener seen about College when the day's work was done and lighter work begun; but there are many who know that there was not a worthy movement but what he was anxious to support, and that liberally. His popularity among the students was unbounded; and long before our Queen recognised in him a fit knight, the citizens of our University had, with a significance deeper than stately figure and commanding presence, honoured him as their "Duke." And still we mourn him, as one whose place will never be filled by another, either in our memories or in our lives."

This allusion to his appearance will recall to all, who knew him, the remarkable height and splendid features of one whom no one could pass, however carelessly, without being impressed.

Though too busy to publish much, yet he made some valuable additions to the surgical literature of the day. Mention has already been made to his *Notes of the Surgery of the Crimean War*, which was published in 1858. In 1864 he issued his *Outlines of Surgical Diagnosis.* The edition published of this was sold out in three months, and a large edition was published and sold in America. Though repeated representations were made to him both from Great Britain and America for a new edition, and though for many years he collected materials for this end, he never found himself able to overtake it. We believe this was the first work of its kind published, although since then several have appeared.

In the second edition of *Cooper's Surgical Dictionary* he wrote several articles, and also for the *International Encyclopædia of Medicine and Surgery* he contributed an article upon the "Surgical Affections of the Neck." He wrote many articles for the leading medical periodicals of this country, besides printing separately numerous addresses on professional and general topics.

Besides being M.D. of Glasgow, F.R.C.S. Ed., F.F.P.S. Glas., he had a long list of honorary distinctions. He was a Fellow of this Society, and had conferred upon him the honorary Fellowship of the Royal College of Surgeons of Ireland, and as late as last spring the LL.D. of the University of St Andrews. He was also a corresponding member of the Société de Chirurgie de Paris and of the Académie de Médecine de Paris, Member of the Deutsche Gesellschaft für Chirurgie, Fellow of the College of Physicians of Philadelphia, and member of several other learned societies. He was Surgeon in Ordinary to the Queen for Scotland, which appointment he received on the removal of Sir Joseph Lister to London. In the year of Her Majesty's jubilee he received the honour of knighthood. He was, moreover, one of the Crown representatives in the General Council of Medical Education and Registration, and a D.L. and J.P. for Dumbartonshire.

So far we have spoken of him with special reference to his profession as a surgeon, yet one final word must be added. At heart a surgeon, he was by no means one-sided, as those with whom he came in contact soon discovered, for with his love of travel was combined a love for the history with which the places he visited was inseparably connected. When able to snatch a few minutes from the busy day, he took up and read and re-read some branch of historical study. After his profession, perhaps history had for him the greatest fascination, and never was he tired studying the checkered fortunes of a nation's life. Nothing he disliked more than to be considered a mere specialist, to whom the world and all things therein were of no interest, save as they served to provide subjects for the morning's lecture. In the many addresses which from time to time he delivered, subjects of historical interest were almost always his choice. His profession, instead of narrowing him, seemed to help to widen his sympathies and his tastes, and incline him to take a special interest in general literature.

Filled with that spirit of romance and warm-heartedness which his Highland upbringing did so much to sustain, he delighted to welcome to his house of Fiunary, on the Gareloch—called after that other Fiunary, on the Sound of Mull, so long the family's home—old fellow-students and old companions, and continue there the traditions for which that other home was ever so lovingly remembered. The soul of honour, a man beloved by those who have the best right to speak—his students and his friends—his loss will be deeply felt by a large circle who admired him as a man, and valued him as a friend.

The end came with startling suddenness, on the eve of his departing for his autumn holiday, before the work of the winter. He was seized on Monday night, the 29th August, with the severe pain which accompanies angina pectoris; and though on Tuesday able to make the arrangements necessary for a temporary absence from his work, on Wednesday morning, while those beside his bed were speaking to him and he to them, with no thought of immediate death, he suddenly passed painlessly and peacefully away.

He is survived by Lady Macleod, and a family of four sons and two daughters.

Thomas Miller, M.A., LL.D. By J. S. Mackay, LL.D.

(Read February 15, 1892.)

Thomas Miller was born at Greenbrae, in the parish of Ardoch, on the 16th October 1807. His father, William Miller, was a farmer, and intended the son to follow his own calling. The son, however, had a thirst for learning, and wished to enter the Church. He was sent—perhaps it would be more correct to say he was allowed to go—to St Andrews, for the expenses of his first session (1825–26) were defrayed from his own earnings as a teacher. He took the usual Arts course of four years, distinguishing himself in all his classes, and particularly in those of mathematics and natural philosophy, and graduated the first man of his year. He had come up to the University with what would now be considered rather a slender stock of knowledge, for he knew no algebra or geometry, his Greek was entirely self-acquired, and the instruction he had in Latin was such as could be obtained from one year's attendance at a country school. But if his preparation for college studies was inadequate, it was more than compensated for by his enthusiasm and his indomitable industry, two qualities which remained with him to the end of his life.

In accordance with his design of entering the Church, Mr Miller spent four years more at St Andrews in the study of theology. During this time he had not only many private pupils, but he was engaged by his friend, Professor Duncan, to superintend his competitions, to correct his class exercises, and to revise his *Elements of Solid Geometry*, which was then in manuscript. All this, with his own work for his divinity classes, kept him employed with little intermission from six in the morning till midnight.

At the close of his theological course the appointment of a mathematical master in the Madras College, St Andrews, fell to be made, and he was urged by Principal Haldane to become a candidate for the post. Diffident though he was of his qualifications (it is usually the best candidates who are diffident), he sent in his application, and was unanimously elected. This event changed the current of his life. He remained four years in the Madras College, teaching not only mathematics, but geography and elementary science with much efficiency, for the number of his pupils increased very greatly during his term of office.

On the death of Professor Jackson in 1837, Mr Miller became a candidate for the vacant chair of natural philosophy, but the appointment was given to Dr Adam Anderson, who was then Rector of the Perth Academy. Mr Miller became Dr Anderson's successor, and in October 1837 commenced his first session in Perth. The classes he had at first were not large, and in leaving St Andrews he knew he was making a heavy pecuniary sacrifice. The number of his pupils, however, increased considerably as time went on.

During his first sessions his leisure was occupied in compiling courses of mathematics and natural philosophy for his students (he always called his pupils "students," addressing them individually as "Mr" and collectively as "Gentlemen"), and making himself acquainted with the progress of scientific discovery. While he was interested in science mainly for its own sake, and for the sake of the benefits which its discoveries have conferred upon mankind, he was keenly alive to the educational importance of its historical development, and familiarised his pupils with the names and the achievements of the great masters from Euclid downwards. For many years he devoted special attention to the Differential and Integral Calculus, and was an assiduous student of the works of Biot, Poisson, Lagrange, and Laplace. In 1852 he was elected a fellow of the Royal Society of Edinburgh, and in 1853 he was offered the degree of LL.D., both by his own University and that of Aberdeen. He chose to accept the St Andrews doctorate. In 1854 appeared his *Treatise on the Differential Calculus, with its Application to Plane Curves, to Curve Surfaces, and to Curves of Double Curvature.* A corresponding volume which he drew up on the Integral Calculus was never published, the demand for books treating of the higher mathematics being extremely limited, and publication entailing a serious pecuniary loss. Not long after he came to Perth he helped to found a Mechanics' Institute, and for several years in succession delivered to the mechanics courses of lectures on natural philosophy, astronomy, chemistry, and geology. These lectures were given gratuitously, and he was much pleased with the only reward they brought him, beyond the enlightenment and the gratitude of his audiences,—a copy of Laplace's *Mécanique Céleste.*

For four and forty years Dr Miller discharged the duties of Rector of the Perth Academy, and (in the words of one of his distinguished pupils) "under his reign the Academy was less a school than a notable provincial college." It may be worth while, for purposes of pedagogic comparison, to give a general statement of the course of study through which he conducted his junior and senior classes. The course consisted of the theory of arithmetic, algebra, plane and solid geometry, geometrical conics, plane and spherical trigonometry, dynamics, elementary physics and astronomy, and inorganic chemistry. Occasionally he had pupils who gave a third or even a fourth year's attendance, for which he would accept no fee, and he took particular pleasure in initiating them into the mysteries of the calculus.

In 1881 he retired, and not long afterwards his friends and former pupils presented him with his portrait painted by J. M. Barclay, R.S.A. He had long been prominently connected with the charitable and philanthropic schemes of the city, and he continued to give them his support. He was a Justice of the Peace for the county, and at the time of his death had been for more than half a century an office-bearer in St Paul's Church. He died on the 9th September 1891.

Dr Miller possessed the qualifications which go to form a great schoolmaster. He was a man of high ability, he never ceased to be a student, he had genuine sympathy with youth, and while he was patient with the dullest, he could rouse the enthusiasm of all. No master was ever prouder of the successes of his pupils, or took a livelier interest in their after welfare. His culture was not that of science alone, for he was widely read in literature, and he could grapple with the philosophical and theological questions of the day. To the young men who were his assistants no head-master could be kinder or more considerate. With peculiar appropriateness one may say of him, in the words of a well-known writer, "he had the esteem of his fellow-citizens, and the love of his fireside; he bore good fortune meekly; he suffered evil with constancy; and through evil or good he maintained truth always."

Dr Miller was married in 1847 to Ann Buchanan, who now survives him. He has left also three sons, Surgeon-Major W. B. Miller, Rev. T. D. Miller, Mr R. H. Miller, LL.B., and a daughter, Mrs H. K. Davson.

Thomas Nelson, F.R.S.E. By W. Scott Dalgleish, M.A., LL.D.

(Read December 19, 1892.)

Thomas Nelson, the head of the publishing house of Thomas Nelson & Sons, died at his residence, St Leonard's, Edinburgh, on October 20th, 1892—within two months of completing his seventieth year. He was educated at the High School of Edinburgh; and when he was seventeen years of age he joined his father's business, which was that of a bookseller and publisher, carried on partly at the quaint old shop which stood till a few years ago at the head of the West Bow, and partly in the Gordon Mansion-house on the Castle Hill. His business faculty was rapidly developed, for in 1844, when he was barely twenty-two, he was entrusted with the organisation of the London branch. Two years later, the business was removed from the Castle Hill to new premises at Hope Park. Some years before that, Thomas Nelson, senior, the head of the firm, had been laid aside by illness; and the task of founding the extensive printing and publishing establishment at Hope Park was undertaken by his two sons, William and Thomas, who were comparatively young men. In this they succeeded so well that they brought to themselves fortune as well as fame.

When the Hope Park premises had been in existence for upwards of thirty years, and had been extended to the utmost capacity of the available ground, and when the business in its many and varied departments was in full career in Edinburgh, London, and New York, it received a sudden check from the occurrence of the disastrous fire of 1878. This seeming calamity, however, turned out to be a blessing in disguise; for it enabled the firm to make a fresh start at Parkside Works, in magnificent premises, conveniently arranged, and furnished with the latest and most approved forms of machinery. There, during the last fourteen years, the business has been carried on under the most favourable conditions, and with ever-increasing prosperity.

Enough has been said, for the present occasion, of the history of the firm with which Thomas Nelson was identified. Something may now be added as to his characteristics as a man of business, and as a man.

From his earliest years he showed a distinct and remarkable turn for mechanics. Both in bookbinding and in presswork, he devised many ingenious contrivances which are now generally adopted by printers and bookbinders. His greatest achievement in this department, however, was his invention, about the year 1850, of a rotary printing-press, with curved stereotype plates fixed on cylinders, and with a continuous web of paper. A working model of his machine, made by the engineers at Hope Park under his direction, was exhibited in the London International Exhibition of 1851, and attracted a great deal of attention; and the same model was again seen at work in the Edinburgh Exhibition of 1886.

The essential points of a rotary press are: (1) Stereotype plates cast in curved form; (2) a continuous web of paper; (3) a serrated knife to cut the paper into sheets as delivered from the machine. In these three particulars, the Nelson Press was unquestionably the original of all the rotary presses now in use for newspaper work.

Long before Mr Nelson took up the subject, the problem of rotary printing had engaged the attention of inventors, but it had not been solved. In 1790 William Nicholson of London patented a machine which anticipated many of the features of the modern press—in particular, the impression roller, and the distributing rollers on the ink-plate. But these features belong rather to the cylinder printing machine than to the rotary press. The only point in Nicholson's specifications bearing on the rotary press was that the "block, forme, plate, assemblage of types, or original," was to be placed on the face of one of his cylinders. Unfortunately, Nicholson never made a machine in conformity with his patent. That was done twenty-one years later by Friedrich Koenig, whose machine, patented in 1811, was the original of the impression-cylinder machines now universally used for book printing, but did not include the proposed type cylinder.

The latter idea was first realised in Applegath's machine of 1848, which was used for several years in *The Times* office. In this case the cylinder bearing the formes of type was vertical, and the paper in sheets was fed in by hand from eight platforms.

This was the position of the problem in 1850, when Mr Thomas Nelson invented his rotary press. Now, the points in which the Nelson Press went beyond all previous inventions were: (1) That its cylinders were covered not with types but with stereotype plates cast in a curved mould; (2) that the paper was in the form of a continuous web, and passed automatically through the machine; (3) that it had a serrated knife sunk into the face of one of the impression cylinders, for the purpose of cutting the paper into sheets. It was also a perfecting machine, printing both sides of the paper at one operation. These are what I have called the essentials of a rotary press, and they appeared in the Nelson Press for the first time.

The invention did not pass without notice. When the working model was exhibited in London in 1851, it was referred to in all the principal newspapers, and it was minutely described (with drawings) in Cassell's *Illustrated Exhibitor* (1852). Probably the reason why the plan of the Nelson Press was not at once adopted was, that it was suggested for the printing of books, for which it is admitted not to be well adapted. Its special applicability to newspaper work had not then been realised, though it was not improbably suggested by the exhibition of the machine. In the Hoe machine, with which *The Times* superseded the Applegath in 1858, the type formes were still affixed to the cylinder, only that cylinder was horizontal instead of being vertical. The paper also was fed-in in sheets from ten separate platforms. There was as yet no web.

The first machine made on the model of the Nelson Press was that of Marinoni of Paris, but even that was not in the first instance a web machine. The reel of paper, however, was soon added, and in its completed form, as patented in this country in 1872, the machine was an obvious copy of the Nelson Press, and indeed that was scarcely denied by the inventors. Since that time, the rotary press has been brought to a marvellous state of perfection in the "Walter Press," and in the "Hoe Double-web Press"; and while I, of course, admit that these machines contain many improvements and refinements that were not dreamed of in 1850, I think I am entitled to claim that the three essentials of these and of all rotary machines—namely, plates cast in the curve, a web of paper, and a serrated knife—were all found in the Nelson Press, as it was exhibited in 1851, and were there in combination for the first time.

Mr Nelson could never be induced to lay claim to the invention, and I believe he did not patent it. It was with difficulty that he was induced to allow the original model to be exhibited in the Edinburgh Exhibition of 1886. He did not care about the matter on personal grounds; but he was anxious that, as Edinburgh has the credit of introducing stereotyping in the British Islands, so Edinburgh should have the credit of having produced the first rotary printing-machine in the world.

Mr Nelson's fertile mind was always generating fresh ideas, many of which were turned to practical account in his business, and especially in his school-book work. Though not a professed man of letters, he had remarkable facility in writing for the young in a manner that arrested attention. His first school-books were edited by himself; and to the last he continued to write lessons and to project new books. Considering the thousands (I might say the millions) of school-books issued from his press during the last quarter of a century, it would be difficult to exaggerate the extent of his influence on the youth of the country. Though not a professed linguist, he had original views about the origin and development of language, and especially about grammar, which received the approval of distinguished scholars. Though not a professed scientist, he developed new ideas in connection especially with geography and map-making, which were endorsed by eminent specialists like Sir John Herschell.

He was a man of indefatigable energy, with a great capacity and an insatiable appetite for work. When a new idea had taken hold of him, he could not rest till he had carried it out. It possessed his whole mind, and in favour of it everything else had to be laid aside for the time. To this power of concentration much of his success in business was due. Much also was due, however, to his sound judgment, and to his possession in a remarkable degree of what he called the publisher's instinct. The projecting and the working out of new schemes gave him the keenest enjoyment. Difficulties only gave spur to his intent and zest to his labour; and if he was not too much elated by success, neither was he easily daunted by failure.

Perhaps the most striking instance of his resolution and his fearless mettle occurred on the occasion of the great fire already referred to. The total destruction of buildings so costly, so convenient, and so handsome, would have paralysed most men; but Thomas Nelson was not thus easily dismayed. On the day following the night of the fire, and while it was still smouldering, he took possession of the desk of the present writer in an adjoining house in Hope Park Terrace, and sent off thence telegrams and letters to England, France, and America, ordering new printing presses and other machines. At the same time, he arranged with architects and builders for the erection of a series of brick sheds on his own ground at St Leonard's, for the reception of these machines, which sheds formed the nucleus of the new Parkside Works. He also made arrangements with the leading printers in Edinburgh for the immediate production of new stock from the stereotype plates which had fortunately been saved. That the business, in spite of the fire, was carried on with scarcely an appreciable break, was due entirely to his enterprise and resource, which scarcely fell short of being heroic. So true is it that "In the reproof of chance lies the true proof of men."

Thomas Nelson was very little of a public man. He was a keen politician, but he seldom appeared on political platforms; and though a staunch Free Churchman, he avoided Church Courts. He knew that his strength lay in his business, and he wisely confined himself to that; but that did not prevent him from taking a deep and earnest interest in public affairs, or in religious, scientific, and educational movements. With such movements he frequently showed his sympathy in the form of handsome subscriptions, as in the case of the erection of the new Royal Infirmary, and of the new University buildings. He was a remarkable member of a remarkable family, which has done much to enhance the fame of Edinburgh in connection with its most characteristic industry, and to which Edinburgh, and indeed Scotland, is indebted for signal examples of personal worth of public spirit, and of patriotism.

Andrew Young, F.G.S. By Professor Flint, D.D.

(Read December 21, 1891.)

Andrew Young, the subject of this notice, was born at Edinburgh in 1807. His first instructor was his father, David Young, a successful teacher in the city during half a century. He early entered the University, and passed with distinction through the curricula of Arts and Theology. He had University Prizes awarded him for no less than five poems. These poems show a facility and skill in the metrical expression of his thoughts and feelings remarkable in a youth of from fifteen to seventeen years of age, and only to be explained by a naturally poetical disposition having prompted him almost from childhood to cultivate the art of versification. The poem on "The Scottish Highlands," warmly commended by Professor Wilson when declaring him Laureate of the Moral Philosophy Class, is not only the longest and most laboured of his compositions, but the one which gives the highest conception and fullest measure of his poetical talent and resources.

Having completed his course at the University he chose teaching as his profession in preference to the ministry. At the age of twenty-one he was appointed by the Town Council of Edinburgh to the head-mastership of Niddry Street School, a position which he held for eleven years. It was during this period that he composed that delightful Sabbath-school hymn, "The Happy Land," which has been so widely and richly productive of good, and which has endeared his name to multitudes in all parts of the world. It well deserves its success and influence, although it owes them not to any rare or remarkable felicities either of thought or expression, but to the admirable adjustment of the words to the melody, and of both to the minds and voices of the young. The secret of its charm and power reveals itself at once when heard sung by a fairly large number of children. As to the circumstances in which it was written I need only refer to Mr Young's own account of them in the preface to his "Poems."

In 1840 he was appointed head-master of the English Department in Madras College, St Andrews. After teaching there with acceptance for thirteen years, he retired into private life, and settled in Edinburgh. But he did not cease to manifest his love of teaching and his interest in the young. For many years he was the Superintendent of Greenside Parish Sunday School. No position could have been more congenial to him than was this.

He published in 1876 *The Scottish Highlands, and other Poems.* The volume was very favourably received by the press, and had a large circulation. If it nowhere shows us that its author was a great poet, it throughout shows us that he possessed a genuinely poetical nature, and a fervent and devout spirit. The feelings which inspire his verse are always pure and lovable, and the verse itself flows with ease and naturalness.

Mr Young was elected a Fellow of this Society on the 5th of December 1881. He was also a member of the Geological Society.

He died on the 30th of November 1889, and was buried on the 4th of December in Rosebank Cemetery. He was twice married, and is survived by a widow and daughter.

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