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The Great Telescope of the Lick Observatory. Aperture, 36 inches; Length, 57 feet.

## THE ELEMENTS OF ASTRONOMY

A TEXTBOOK

## BY

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REVISED EDITION<br>WITH ADDITIONS AND CORRECTIONS

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## PREFACE.

The present volume is a new work, and not a mere abridgment of the author's "General Astronomy." Much of the material of the larger book has naturally been incorporated into this, and many of its illustrations are used ; but everything has been worked over with reference to the wants of institutions which demand a more elementary and less extended course than that presented in the "General Astronomy."

It has not always been easy to decide just how far to go in cutting down and simplifying. On the one hand the students who are expected to use the present book are not children, but have presumably mastered the elementary subjects which properly precede the study of Astronomy, and it is an important part of their remaining education to make them familiar with astronomical terms and methods; on the other hand it is very easy to assume too much, and to make the book difficult and incomprehensible by the use of too many unfamiliar terms, and the unprepared presentation of new ideas and demonstrations - and the danger is greater in a brief course than in a longer one.

While therefore the writer has tried to treat every subject simply and clearly, he has not discarded the use of technical terms in proper places, and he has always sought
to stimulate thought, to discourage one-sided and narrow ways of looking at things, and to awaken the desire for further acquisition.

The book presupposes students anxious to learn, and an instructor who understands the subject in hand, and the art of teaching.

Special attention has been paid to making all statements correct and accurate as far as they go. Many of them are necessarily incomplete, on account of the elementary character of the work; but it is hoped that this incompleteness has never been allowed to degenerate into untruth, and that the pupil will not afterwards have to unlearn anything that the book has taught him.

In the text no mathematics higher than elementary algebra and geometry is introduced; in the foot notes and in the appendix an occasional trigonometric formula appears.

Certain subjects, which, while they certainly ought to be found within the covers of every text-book of Astronomy, are perhaps not essential to an elementary course, have been relegated to an appendix. Where time allows, the instructor will find it advisable to include some of them at least in the student's work.

A brief Uranography is also presented, covering the constellations visible in the United States, with maps on a scale sufficient for the easy identification of all the principal stars. It includes also a list of such telescopic objects in each constellation as are easily found and lie within the power of a small telescope.

The author is under special obligations to Messrs. Kelley of Haverhill, Lambert of Fall River, and Par-
menter of Cambridgeport, for valuable suggestions and assistance in preparing the work, and to his assistant, Mr. Reed, for help in the proof-reading: also to Warner \& Swasey for the cut of the Lick telescope which forms the frontispiece.

## PREFACE TO THE REVISED EDITION OF 1897.

The progress of Astronomy since the first publication of this work has been such as to require a thorough revision and partial rewriting of the book in order to make it fairly representative of the existing state of the science. Numerous changes and corrections have been made, with some considerable additions; but the necessary alterations have been so managed that it is believed that no serious inconvenience will arise in using the old and new editions together.

A "Synopsis for Review and Examination" has been added, which, it is hoped, will be found useful by both teachers and pupils.
C. A. Young.

## PREFACE TO ISSUE OF 1918

While the greater part of the text remains as it was written by its author, such changes have been made in this issue as are necessary to bring it down to date.

Mount Holyoke College
Anne Sewell Young
September, 1918
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## INTRODUCTION.

1. The earth is a huge ball about 8000 miles in diameter, composed of rock and water, and covered with a thin envelope of air and cloud. Whirling as it flies, it rushes through empty space, moving with a speed fully fifty times as great as that of the swiftest cannon-ball. On its surface we are wholly unconscious of the motion, because it is perfectly steady and without jar.

As we look off at night we see in all directions the countless stars, and conspicuous among them, and looking like stars, though very different in their real nature, are scattered a few planets. Here and there appear faintly shining clouds of light, like the so-called Milky Way and the nebulæ, and perhaps, now and then, a comet. Most'striking of all, if she happens to be in the heavens at the time, though really the most insignificant of all, is the moon. By day the sun alone is visible, flooding the air with its light and hiding the other heavenly bodies from the unaided eye, but not all of them from the telescope.
2. The Heavenly Bodies. - The bodies thus seen from the earth are known as the "heavenly bodies." For the most part they are globes like the earth, whirling on their axes, and moving swiftly through space, though at such distances from us that their motions can be detected only by careful observation.

They may be classified as follows: First, the solar system proper, composed of the sun, the planets which revolve around.
it, and the satellites, which attend the planets in their motion around the sun. The moon thus accompanies the earth, which herself belongs among the planets. The distances between these bodies are enormous as compared with the size of the earth, and the sun which rules them all is a body of almost inconceivable magnitude.

Next, we have the comets and the meteors which, while they acknowledge the sun's dominion, move in orbits of a different shape from those of the planets, and are bodies of a very different character. Finally, we have the stars and nebuloe, at distances from us immensely greater than even those which separate the planets. The stars are suns, bodies comparable with our own sun in size and nature, and, like it, self-luminous, while the planets and their satellites shine only by reflected sunlight. Of the nebulæ we know very little, except that they are cloud-like masses of self-luminous matter, and belong to the region of the stars.

## 3. Subject-Matter of Astronomy. - Astronomy is the science

 which treats of the heavenly bodies. It investigates (a) their motions and the laws which govern them; (b) their nature, dimensions, and characteristics ; (c) the influence they exert upon each other either by their attraction, their radiation, or in any other way.Astronomy is the oldest of the natural sciences: nearly the earliest records that we find in the annals of China and upon the inscribed "library bricks" of Assyria and Babylon relate to astronomical subjects, such as eclipses and the positions of the planets. Obviously in the infancy of the race the rising and setting of the sun, the progress of the seasons, and the phases of the moon must have compelled the attention of even the most unobservant.

As Astronomy is the oldest of the sciences, so also it is one of the most perfect, and in certain aspects the noblest, as being the most "unselfish," of them all.
4. Utility. - Although not bearing so directly upon the material interests of life as the more modern sciences of Physics and Chemistry, it is really of high utility. It is by means of Astronomy that the latitudes and longitudes of places upon the earth's surface are determined, and by such determinations alone is it possibee to conduct vessels upon the sea. If we can imagine that some morning men should awake with Astronomy forgotten, all almanacs and astronomical tables destroyed, and sextants and chronometers demolished, commerce would practically cease, and so far as intercourse by navigation is concerned, the world would be set back to the days before Columbus. Moreover, all the operations of surveying upon a large scale, such as the determination of the boundaries of countries, depend more or less upon astronomical observations. The same is true of all operations which, like the railway service, require an accurate knowledge and observance of the time; for the fundamental time-keeper is the diurnal revolution of the heavens, as determined by the astronomer's transit-instrument.

In ancient times the science was supposed to have a still higher utility. It was believed that human affairs of every kind, the welfare of nations, and the life history of individuals alike, were controlled, or at least prefigured, by the motions of the stars and planets; so that from the study of the heavens it ought to be possible to predict futurity. The pseudo-science of Astrology based upon this belief really supplied the motives that led to most of the astronomical observations of the ancients. Just as modern Chemistry had its origin in Alchemy, so Astrology was the progenitor of Astronomy.
5. Place in Education. - Apart from the utility of Astronomy in the ordinary sense of the word, the study of the science is of the highest value as an intellectual training. No other science so operates to give us on the one hand just views of our real insignificance in the universe of space, matter, and time, or to teach us on the other hand the dignity of the human intellect as the offspring, and measurably the
counterpart, of the Divine ; able in a sense to "comprehend" the universe, and know its plan and meaning. The study of the science cultivates nearly every faculty of the mind; the memory, the reasoning power, and the imagination all receive from it special exercise and development. By the precise and mathematical character of many of its discussions it enforces exactness of thought and expression, and corrects that vague indefiniteness which is apt to be the result of pure literary training. On the other hand, by the beauty and grandeur of the subjects it presents, it stimulates the imagination and gratifies the poetic sense. In every way it well deserves the place which has long been assigned to it in education.
6. The present volume does not aim to make finished astronomers of high-school pupils. That would require years of application, based upon a thorough mathematical training as a preliminary. Our little book aims only to present such a view of the elements of the science as will give the pupils of our high schools an intelligent understanding of its leading facts, - not a mere parrot-like knowledge of them, but an understanding both of the facts themselves and of the general methods by which we ascertain them. These are easily mastered by a little attention, and that without any greater degree of mathematical knowledge than may confidently be expected of pupils in the latter years of a high-school course. Nothing but the simplest Arithmetic, Algebra, and Geometry will be required to enable one to deal with anything in the book, except that now and then a trigonometric equation may be given in a note or in the Appendix for the benefit of those who understand that branch of mathematics.

The occasional references to "Physics" refer to Gage's " Principles of Physics"' (Goodspeed's revision), and those to " Gen. Ast." relate to the author's " General Astronomy," or college text-book.

## CHAPTER I.

## FUNDAMENTAL NOTIONS AND DEFINITIONS, AND THE DOCTRINE OF THE SPHERE.

7. The Celestial Sphere. - The sky appears as a hollow vault, to which the stars seem to be attached, like specks of gilding upon the inner surface of a dome. We cannot judge of the distance of the concavity from the eye, further than to perceive that it must be very far away, because it lies beyond even the remotest terrestrial objects. It is therefore natural, and it is extremely convenient from a mathematical point of view, to regard the distance of the heavens as unlimited. The celestial sphere, as it is called, is conceived of as so enormous that the whole material universe lies in its centre like a few grains of sand in the middle of the dome of the Capitol. The imaginary radius of the celestial sphere is assumed to be immeasurably greater than any actual distance known, and greater than any quantity asssignable, - in technical language, mathematically infinite.

There are other ways of regarding the celestial sphere, which are equally correct and lead to the same general results without requiring the assumption of an infinite radius, but on the whole they are more complicated and less convenient than the one above indicated, which is that usually accepted among astronomers.
8. Vanishing Point. - Since the radius of the celestial sphere is thus infinite, any set of lines which are parallel to each other, if extended indefinitely, will appear to pierce it at a single point. The real distances of the parallel lines from
each other remain, of course, unchanged however far they may be produced; so that whatever may be the radius of the sphere, they actually pierce the surface in a group of separate points. But since the radius of the sphere is "infinite," the apparent size of the group of points, as seen from the earth, will be less than any assignable quantity. In other words, to the eye the area occupied by the group on the surface of the sphere will shrink to a mere point, - the "vanishing point" of perspective. Thus the axis of the earth, and all lines parallel to it, pierce the heavens at the celestial pole; and the plane of the earth's equator, which keeps parallel to itself during her annual circuit around the sun, marks out only one celestial equator in the sky.
9. Place of a Heavenly Body. - This is simply the point where a line, drawn from the observer through the body in question and continued onward, pierces the sphere. It de-


Fig. 1. pends solely upon the direction of the body, and is obviously in no way affected by its distance. Thus in Fig. 1, $A, B, C$, etc., are the apparent places of $a, b, c$, the observer being at $O$. Objects that are nearly in line with each other, as $h, i, k$, will appear close together in the sky, however great their real distance from each other may be. The moon, for instance, ofter looks to us "very near" a star, which is really of course at an immeasurable distance beyond her.
10. Angular Measurement. - It is clear that we cannot properly measure the apparent distance of two heavenly bodies from each other in the sky by feet or inches. To say that two
stars are about five feet apart, for instance (and it is not very uncommon to hear such an expression), means nothing unless you tell how far from the eye the five-foot measure is to be held. If 20 feet away, it means one thing, and corresponds, nearly, to the apparent length of the "Dipper-handle" in the sky (see Art. 23) ; if 100 feet away, it corresponds to an apparent distance only about one-fifth as great, or to one of the shorter sides of the "Dipper-bowl" (see Art. 23); but if the five-foot measure were a mile away, its length would correspond to an apparent distance about one-tenth the apparent diameter of the moon. The proper units for expressing apparent distances in the sky are those of angle, viz. : radians, or else degrees $\left({ }^{\circ}\right)$, minutes ( ${ }^{\prime}$ ), and seconds ( ${ }^{\prime \prime}$ ). The Great Bear's tail or Dipper-handle is about 16 degrees long, the long side of the Dipper-bowl is about 10 degrees, the shorter sides are $4^{\circ}$ or $5^{\circ}$; the moon is about half a degree, or $30^{\prime}$, in diameter.
11. The student will remember that a degree is one three-hundred-and-sixtieth of the circumference of a circle, so that a quarter of the circumference, or the distance from the point overhead to the horizon, is $90^{\circ}$; also, that a minute is the sixtieth part of a degree, and a second the sixtieth part of a minute. The radian is the angle measured by an arc of the circumference equal to its radius. It is $\frac{360^{\circ}}{2 \pi}$, or (approximately) $57^{\circ} .3,3437^{\prime} .7$, or $206264^{\prime \prime} .8$.
It is very important, also, that the student in Astronomy as soon as possible should accustom himself to estimate celestial measures in these angular units. A little practice soon makes it easy, though at first one is apt to be embarrassed by the fact that the sky looks to the eye not like a true hemisphere, but a flattened vault, so that all estimates of angular distances for objects near the horizon are apt to be exaggerated. The moon when rising or setting looks to most persons much larger ${ }^{1}$ than when overhead, and the "Dipper-bowl"

1 This is due to the fact that when a heavenly body is overhead there are no intervening objects by which we can estimate its distance from us, while at the horizon we have the whole landscape between us and it. This
when underneath the pole seems to cover a much larger area than when above it.

## 12. Relation between the Distance and Apparent Size of an

 Object. - Suppose a globe of the radius $B C$, Fig. 2, equal to $r$. As seen from the point $A$ its "apparent" (that is, angular:) semi-diameter will be the angle $B A C$, or $s$, its distance being

Fig. 2.
$A C$, or $R$. Evidently the nearer $A$ is to $C$, i.e., the smaller $R$ may be, the greater will be the angle $s$. If the angle $s$ is $1^{\circ}$, $R$ will be $57.3+$ times as great as $r$; and if it were only $1^{\prime \prime}, R$ would then be 206,265 times $r$. As long as the angle $s$ does not exceed $1^{\circ}$ or $2^{\circ}$, we may without sensible error take

$$
r=R \times \frac{s^{\circ}}{57.3}, \text { or } R\left(\frac{s^{\prime \prime}}{206265}\right)
$$

The distances $R$ and $r$ are of course measured in units of length, or "linear" units, such as miles, kilometers, or feet. In
makes it seem remoter when low down, and as the angular diameter is unchanged, it makes it seem to us larger (Art. 12). It may be mentioned as a rather curious fact that people unaccustomed to angular measurements say, on the average, that the moon (when high in the sky) appears about a foot in diameter. This implies that the surface of the sky appears to them only about 110 feet away. Probably this is connected with the physiological fact that by the muscular sense, by means of the convergence of the eyes, we can directly estimate distances up to about 100 feet with more or less accuracy. Beyond that we depend for our estimate mainly on intervening objects.
general, therefore, for $r$, the radius ${ }^{1}$ (in linear units) of a globe whose angular semi-diameter is $s^{\prime \prime}$, we have

$$
r=R \frac{s^{\prime \prime}}{206265} \quad \text { Also } s^{\prime \prime}=206265 \frac{r}{R}
$$

We see therefore that the apparent diameter of the object varies directly as the linear diameter, and inversely as the distance. In the case of the moon, $R$ equals about 239,000 miles, and $r, 1081$ miles; whence, from the formula just given, her semi-diameter,

$$
s^{\prime \prime}=206265 \times \frac{1081}{239000},
$$

which equals $933^{\prime \prime}$ - a little more than $\frac{1^{\circ}}{4}$.

## CIRCLES OF THE SPHERE.

13. In order to be able to describe intelligibly the position of a heavenly body in the sky, it is convenient to suppose the inner surface of the celestial sphere to be marked off by circles traced upon it, - imaginary circles, of course, like the meridians and parallels of latitude upon the surface of the earth. Three distinct systems of such circles are made use of, each of which has its own special adaptation for its special purposes.

SYSTEM Which depends UPON the direction of the force of gravity at the point where the obSERVER STANDS.
14. The Zenith and Nadir. - If we suspend a plumb-line (consisting simply of a slender thread with a heavy ball attached to it), the thread will take a position depending upon

[^0]the direction of the force of gravity. If we imagine the line of this thread to be extended upward to the sky, it will pierce the celestial sphere at a point directly overhead, known as the astronomical zenith, ${ }^{1}$ or "the zenith" simply, unless some other qualifier is annexed.

As will be seen later (Art. 82), the plumb-line does not point exactly to the centre of the earth, because the earth rotates on its axis and is not strictly spherical. If an imaginary line be drawn from the centre of the earth upward through the observer, and produced to the celestial sphere, it marks a point known as the " geocentric zenith," which is never very far from the astronomical zenith, but is not identical, and must not be confounded, with it. For most purposes the astronomical zenith is the better practical point of reference, because its position can be determined directly by observation, which is not the case with the geocentric zenith.

The nadir (also an Arabic term) is the point opposite to the zenith in the invisible part of the celestial sphere directly underneath.
15. The Horizon. - If now we imagine a great circle drawn completely around the celestial sphere half way between the zenith and nadir (and therefore $90^{\circ}$ from each of them), it will be the horizon. ${ }^{2}$ Since the surface of still water, according to hydrostatic principles, is always perpendicular to the direction of gravity, we may also define the horizon as the great circle in which a plane, tangent to a surface of still water at the place of observation, cuts the celestial sphere; or, in slightly different words, the great circle where a plane passing through the observer's eye, and perpendicular to the plumb line, cuts the sphere.

[^1]Sensible and Rational Horizon. - Many writers distinguish between the "sensible" and "rational" horizons, the former being defined by a horizontal plane drawn thraugh the observer's. eye, while the latter is defined by a plane, parallel to this, but drawn through the centre of the earth. Since, however, the celestial sphere is infinite in diameter, the two lines traced upon it by these planes, though 4000 miles apart, confound themselves to the observer's eye into a single great circle, $90^{\circ}$ from both zenith and nadir, agreeing with the first definition given above. The distinction is not necessary.
16. Visible Horizon. - The word "horizon" means literally "the boundary," that is, the limit of landscape, where sky meets earth or sea; and this boundary line is known as the Visible Horizon. On land it is of no astronomical importance, being usually an irregular line broken by hills and trees and other objects; but at sea it is practically a true circle, nearly, though not quite, coinciding with the horizon as above defined. If the observer's eye were at the water-level, the coincidence would be exact; but if he is at an elevation above the surface, the line of sight drawn from. his eye tangent to the water inclines or "dips" downward by a small angle, on account of the curvature of the earth. This is illustrated


Fig. 3.- Dip of the Horizon. by Fig. 3, where $O H$ is the line of the true level from the observer's eye at $O$, situated at an elevation, $h$, while $O B$ is the line drawn to the visible horizon.

The visible horizon, therefore, is not a great circle of the celestial sphere, but technically a small circle, parallel to the true horizon and depressed below it by an amount measured by the angle $H O B$, which is called the Dip of the Horizon. ${ }^{1}$. In marine astronomy this visible horizon is of great importance, because it is the circle from which the observer measures with his

[^2]sextant the height of the sun or other heavenly body, in the operations by which he determines the place of his ship.

## 17. Vertical Circles; the Meridian and the Prime Vertical. -

 Vertical Circles are great circles drawn from the zenith at right angles to the horizon. Their number is indefinite: each star has at any moment its own vertical circle. That particular vertical circle which passes north and south is known as the Celestial Meridian, and is evidently the circle which would be obtained by continuing to the sky the plane of the terrestrial meridian upon which the observer is located. The vertical circle at right angles to the meridian is the Prime Vertical.

Fig. 4. - The Horizon and Vertical Circles.
$O$, the place of the Observer. OZ, the Observer's Vertical. $Z$, the Zenith; $P$, the Pole. SENW, the Horizon. $S Z P N$, the Meridian. $E Z W$, the Prime Vertical.
$M$, some Star.
ZMH, arc of the Star's Vertical Circle. TMR, the Star's Almucantar. Angle TZM, or arc SH, Star's Azimuth. Arc HM, Star's Altitude. Arc ZM, Star's Zenith Distance.
18. Parallels of Altitude, or Almucantars. - These are small circles of the celestial sphere drawn parallel to the horizon just as the parallels of latitude on the earth's surface are drawn. parallel to the equator. The term Almucantar (Arabic) is seldom used.
19. We are now prepared to designate the place of a body in the sky by telling how many degrees it is above the horizon, and how it "bears" from the observer.

Altitude and Zenith Distance. - The Altitude of a celestial body is its angular elevation above the horizon ; i.e., the number of degrees between it and the horizon, measured on a vertical circle passing through the object. In Fig. 4 the vertical circle $Z M H$ passes through the body $M$. The arc $M H$ measured in degrees is the Altitude of $M$, and the arc $Z M$ (the "complement" of $M H$ ) is called its Zenith Distance.
20. Azimuth and Amplitude. - The Azimuth (an Arabic word) of a heavenly body is the same as its "bearing" in surveying; measured, however, from the true meridian and not from the magnetic. ${ }^{1}$

It may be defined as the angle formed at the zenith between the meridian and the vertical circle which passes through the object; or, what comes to the same thing, it is the arc of the horizon intersected between the south point and the foot of this circle. In Fig. $4 S Z N$ is the meridian, and the angle $S Z M$ is the azimuth of $M$, as also is the arc $S H$, which measures the angle at $Z$. The distance of $H$ from the east or west point of the horizon is called the Amplitude of the body; $H E$ in the figure is the amplitude of $M$.
21. There are various ways of reckoning azimuth. Many writers express it in the same way as the bearing is expressed in Surveying ; i.e., so many degrees east or west of north or south. In the figure, the azimuth of $M$ thus expressed is about $\mathrm{S} .50^{\circ} \mathrm{E}$. The more usual way at present, however, is to reckon it from the south point clear around through the west to the point of beginning. Thus an object exactly in the southwest would have an azimuth of $45^{\circ}$, while that of $M$ would be the arc $S W N E H$, about $310^{\circ}$.

[^3]For example, to find a star whose azimuth is $260^{\circ}$ and altitude $60^{\circ}$, we must face N. $80^{\circ}$ E., and then look up two-thirds of the way to the zenith, the zenith distance being $30^{\circ}$.
22. Altitude and azimuth, however, are for many purposes inconvenient, because for a celestial object they continually change.

When the sun is rising, for instance, its altitude is zero. Half an hour later it is increased by several degrees, and the azimuth also is altered; for the sun does not (except to an observer at the earth's equator) rise vertically in the sky, but slopes upward, moving from the left towards the right; so also when it is setting: and the same is true, in a general way, of every heavenly body.

It is desirable, therefore, to use a different way of defining the place of a body in the heavens which shall be free from this objection, and this we can do by taking as the "fundamental line" of our system, not the direction of gravity as shown by the plumb-line (which is not the same at any two different points on the earth's surface, and is continually changing as the earth turns around), but the direction of the earth's axis.

## SYSTEM OF CIRCLES DEPENDING ON THE DIRECTION OF THE EARTH'S AXIS OF ROTATION.

23. The Apparent Diurnal Rotation of the Heavens. - If we go ${ }^{1}$ out on some clear evening in early autumn and face the north, we shall find the aspect of that part of the heavens directly before us substantially as shown in Fig. 5. In the

[^4]northwest is the constellation of the Great Bear (Ursa Major), characterized by the conspicuous group of seven bright stars, familiar to all our readers as the "Great Dipper." It now


Fig. 5. - The Northern Circumpolar Constellations.
lies with its handle sloping upward toward the west. The two easternmost stars of the four which form its bowl are called the "Pointers" because they point to the Pole-star, which is a solitary star not quite half-way from the horizon
to the zenith (in the latitude of New York). It is about as bright as the brighter of the two Pointers, and a curved line of small stars extending upward and westward joins it to the bowl of the "Little Dipper," the Pole-star being at the extremity of the handle. The two brightish stars, which correspond in position to the Pointers in the Great Dipper, are known as the "Guards" (of the pole).

High up on the opposite side of the Pole-star from the Great Dipper, and at nearly the same distance, is an irregular zigzag of five stars, about as bright as the Pole-star itself. This is the constellation of Cassiopeia.

Below Cassiopeia lies Perseus; and still lower, near the northeastern horizon, is Auriga (the Charioteer), with the bright star Capella, the only really first-magnitude star in all the region of the sky with which we are now dealing. Directly below the Pole-star the vacant space is occupied by the large but insignificant constellation of the Camelopard. Cepheus, also containing but few bright stars, is directly above Cassiopeia. Above the Pole-star, between it and the zenith, lies the head and neck of the Dragon (Draco), but its tail extends westward nearly half-way around the pole, and is marked by an irregular line of stars lying between the Great and Little Dippers.
(The above description, and the figure given, apply strictly to the appearance of the heavens on Sept. 22, at 8 P.m., as seen by an observer in latitude $40^{\circ}$.)
24. If now we watch these stars for only a few hours, we shall find that while all their configurations remain unaltered, their places in the sky are slowly changing. The Great Dipper slides downward towards the north, so that by eleven o'clock (on Sept. 22) the Pointers are directly under the Polestar. Cassiopeia still keeps opposite however, rising towards the zenith; and if we continue the watch long enough, we shall find that all the stars appear to be moving in concentric circles around a point near the Pole-star, revolving counter-clockwise (as we look towards the north) with a steady uniform motion, which takes them completely around once a day, or, to be more
exact, once in $23^{\mathrm{h}} 56^{\mathrm{m}} 4.1^{\mathrm{s}}$ of ordinary time. They behave just as if they were attached to the inner surface of a huge revolving dome.

At midnight (of Sept. 22) the position of the stars will be as indicated by the figure, if we hold it so that the XII in the margin is at the bottom; at 4 A.m. they will have come to the position indicated by bringing XVI to the bottom; and so on. On the next night at 8 o'clock we shall find things (very nearly) in their original position.

If instead of looking towards the north we now look southward, we shall find that there also the stars appear to move in the same kind of way. The stars which are not too near the Pole-star all rise somewhere in the eastern horizon, ascend obliquely to the meridian, and descend to set at points on the western horizon. The next day they rise and set again at precisely the same points, and the motion is always in an arc of a circle, called the star's diurnal circle, the size of which depends upon its distance from the pole. Moreover, all these ares are strictly parallel to each other.
25. The ancients accounted for these fundamental and obvious facts by supposing that the stars are really attached to the celestial sphere, and that this sphere really turns daily in the manner indicated. According to this view there must evidently be upon the sphere two opposite points which remain at rest, and these are the Poles.
26. Definition of the Poles. - The Poles, therefore, may be defined as those two points in the sky where a star would have no diurnal motion. Its exact position may be determined with proper instruments by finding the centre of the small diurnal circle described by some star near the pole, as for instance the Pole-star. ${ }^{1}$

[^5]This definition of the pole is that which would have been given by any ancient astronomer ignorant of the earth's rotation, and it is still perfectly correct. But knowing, as we now do, that this apparent revolution of the celestial sphere is due to the real rotation of the earth on its axis, we may also define the poles as the points where the earth's axis of rotation, produced indefinitely, would pierce the celestial sphere.

Since the two poles are dia-


Fig. 6. - The Plane of the Earth's Equator produced to cut the Celestial Sphere. metrically opposite in the sky, only one of them is usually visible from any given place. Observers north of the earth's equator see only the north pole, and vice versa for observers in the southern hemisphere.
27. The Celestial Equator, or Equinoctial.-This is a great circle of the celestial sphere, drawn half-way between the poles (therefore everywhere $90^{\circ}$ from each of them), and is the great circle in which the plane of the earth's equator cuts the celestial sphere. It is often called the "Equinoctial." Fig. 6 shows how the plane of the equator produced far enough would mark out such a circle in the heavens.

The equator cuts the horizon at the east and west points, but it does not cut it perpendicularly nor pass through the zenith unless the

Pole-star (at present) is about $1 \frac{1}{4}^{\circ}$ distant from the pole. If we draw an imaginary line from the Pole-star to the star Mizar (Zeta Ursæ Majoris, the one at the bend of the Dipper handle), it will pass almost exactly through the pole itself; the distance of the pole from the Pole-star being very nearly one-quarter of the distance between the two "Pointers."
observer is at the earth's equator ; at its highest point it is just as far below the zenith as the pole is above the horizon.
28. Parallels of Declination. - Small circles drawn parallel to the equinoctial, like the parallels of latitude on the earth, are known as Parallels of Declination. For any star situated on one of these parallels, the parallel is obviously identical with the star's diurnal circle. (The reason why these circles in the heavens are not called parallels of latitude will appear later.)
29. Hour-Circles. - The great circles of the celestial sphere which pass through the poles, like the meridians on the earth, and are therefore perpendicular to the celestial equator ("secondaries" to it), are called Hour-Circles. Some writers call them celestial "meridians," but the term is objectionable, since it is sometimes used to designate an entirely different set of circles (the secondaries to the ecliptic-Art. 38). That particular hour-circle which passes through the zenith at any moment is of course coincident with the Celestial Meridian, defined in Art. 17.
30. The Celestial Meridian and the Cardinal Points. - The best form for the definition of the Celestial Meridian is, the great circle which passes through the zenith and the poles. The points where this meridian cuts the horizon are the north and south points, and the east and west points of the horizon lie half-way between them; the four being known as the Cardinal Points. The student is especially cautioned against confounding the North Point with the North Pole; the former being on the horizon, the latter high up in the sky.

In Fig. $7 P$ is the north celestial pole, $Z$ is the zenith, and $S Q Z P N$ is the celestial meridian. $P m P^{\prime}$ is the hour-circle of the object $m$, and $a m R b V$ is its parallel of declination or diurnal circle. NESW is the horizon, and the points indicated by these letters are the four Cardinal Points.

By means of the hour-circles and the celestial equator we now have a second method of designating the position of an object in the heavens: for Altitude and Azimuth we can sub. stitute Declination and Hour-Angle.
31. Declination and Polar Distance. - The Declination of a star is its angular distance north or south of the celestial equator; + if north, - if south. It corresponds precisely with the Latitude of a place on the earth's surface ; it cannot, however,


Fig. 7. - Hour-Circles, etc.
$O$, place of the Observer; $Z$, his Zenith. SENW, the Horizon.
$P O P^{\prime}$, line parallel to the Axis of the Earth. $P$ and $P^{\prime}$, the two Poles of the Heavens. EQWT, the Celestial Equator, or Equinoctial.
$X$, the Vernal Equinox, or "First of Aries."
$P X P^{\prime}$, the Equinoctial Colure, or Zero Hour-Circle.
$m$, some Star.
Ym, the Star's Declination: Pm, its NorthPolar Distance.
Angle $m P R=\operatorname{arc} Q Y$, the Star's (eastern) Hour-Angle; $=24^{\mathrm{b}}$ minus Star's (western) Hour-Angle.
Angle $X P m=\operatorname{arc} X Y$, Star's Right Ascen. sion. Sidereal time at the moment $=24^{\mathrm{b}}$ minus $X P Q$.
be called celestial "Latitude," because that term has been preoccupied by an entirely different quantity (Art. 38).

In Fig. $7, m Y$ is the declination of $m$, and $m P$ is its North polar distance.
32. Hour-Angle. - The Hour-Angle of a star at any moment is the angle at the pole between the celestial meridian and the hour-circle of the star, which angle is measured by the arc of the celestial equator intercepted between the hour-circle of the star and the meridian. In Fig. 7, for the body $m$, it is the angle $m P Z$, or the arc $Q Y$. This angle (or arc) may of course be measured like any other, in degrees; but since it depends upon the time which has elapsed since the body was last on the meridian, it is more usual to measure it in hours, minutes, and seconds of time. The "hour" is then equivalent to $\frac{1}{24}$ of a circumference, or $15^{\circ}$, and the minute and second of time to 15 minutes and 15 seconds of arc respectively. Thus an hourangle of $4^{\mathrm{h}} 2^{\mathrm{m}} 3^{\mathrm{s}}$ equals $60^{\circ} 30^{\prime} 45^{\prime \prime}$.
33. The position of the body $m$ (Fig. 7) is, then, perfectly defined by saying that its declination is $+25^{\circ}$, and its hour-angle $40^{\circ}$ east (or simply $320^{\circ}$ if we choose to reckon completely around in the direction of the diurnal motion). Instead of 40 degrees, we might say $2^{\text {h }} 40^{\text {m }}$ east, or $21^{\mathrm{h}} 20^{\mathrm{m}}$ to correspond to the $320^{\circ}$.

The declination of a star (omitting certain minutiæ for the present) remains practically unaltered even for years, but the hour-angle changes continually and uniformly at the rate of $15^{\circ}$ for every (sidereal) hour.
34. The Vernal Equinox, or First of Aries. - The sun and the planets do not behave as if they were firmly fixed upon the celestial sphere like the stars; but rather as if they were glow-worms crawling slowly about upon its surface while it carries them in its diurnal rotation. As every one knows, the sun in the winter is far to the south of the equator, and in the summer far to the north; it crosses the equator, therefore, twice a year, passing from the south to the north about March 20th, and always at the same point (neglecting for the present the effect of what is known as precession, Art. 122). This point is known as the Vernal Equinox, or the First of

Aries, and is made the starting-point for certain important systems of celestial measurement. It is the "Greenwich" of the celestial sphere.

Unfortunately it is not marked by any conspicuous star; but a line drawn from the Pole-star through Beta Cassiopeiæ (the westernmost or "preceding" star in the zigzag) (see Fig. 5, Art. 23) and continued $90^{\circ}$ from the pole, strikes very near it.
35. Sidereal Time. - A sidereal clock is one that is set and rated so that it marks noon every day, not at the moment when the sun is crossing the meridian, but when the vernal equinox does so. When the clock is correct (i.e., neither too fast or slow), its face indicates the hour-angle of the vernal equinox; and we may therefore define the sidereal time at any moment as the hour-angle of the vernal equinox at that moment.

It is called " sidereal" time because the length of its day is the time that elapses between two successive passages of the same star across the meridian. It is not convenient for the purposes of ordinary life; but for many astronomical purposes it is not only convenient, but practically indispensable. It is usual to divide the face of the sidereal clock into 24 hours, and to reckon the time completely around, instead of counting it in two half-days of 12 hours each; moreover, its day, for reasons which will be explained later (Art. 128), is about four minutes shorter than the ordinary solar day.
36. Right Ascension. - The "Right Ascension" of a star may now be defined as the angle made at the celestial pole between the hour-circle of the star and the hour-circle which passes through the vernal equinox, and is known as the Equinoctial Colure. This angle is measured by the arc of the celestial equator intercepted between the vernal equinox and the point where the star's hour-circle cuts the equator, and is reckoned always eastward from the equinox completely around the circle, and may be expressed either in degrees or in hours. A star one degree west of the
equinox has a right ascension of $359^{\circ}$, or 23 hours and 56 minutes.

Evidently the diurnal motion does not affect the right ascension of a star, but, like the declination, it remains practically unchanged for years. In Fig. 7 , if $X$ be the vernal equinox, the right ascension of $m$ is the angle $X P m$, or the arc $X Y$ measured from $X$ eastward.
37. Observatory Definition of Right Ascension. - The right ascension of a star may also be correctly, and for many purposes most conveniently, defined, as the sidereal time at the moment when the star is crossing the meridian.

Since the sidereal clock is made to show zero hours, minutes, and seconds at the moment when the vernal equinox is on the observer's meridian, its face at any other time shows the hour-angle of the equinox; and this is just what was defined in the preceding section as the right ascension of any star which may then happen to be on the meridian.

Obviously the positions of the heavenly bodies with reference to each other may be indicated by their declinations and right ascensions, just as the positions of places on the earth's surface are indicated by their latitudes and longitudes. The declination of a star corresponds exactly to the latitude of a city, and the star's right ascension to the city's longitude; the vernal equinox taking, in the sky, the place of Greenwich on the earth.
38. Celestial Latitude and Longitude. - A different way of designating the positions of heavenly bodies in the sky has come down to us from very ancient times. Instead of the equator, it makes use of another circle of reference in the sky known as the Ecliptic. This is simply the apparent path described by the sun in its annual motion among the stars, and may be defined as the intersection of the plane of the earth's orbit with the celestial sphere, the "vernal equinox" being one of the
two places in the sky where the celestial equator crosses it. Before the days of clocks, the ecliptic was in many. respects a more convenient circle of reference than the equator, and was almost universally used as such by the old astronomers. Celestial longitude and latitude are measured with reference to the ecliptic in the same way that right ascension and declination are measured with respect to the equator. Too much care cannot be taken to avoid confusion between terrestrial latitude and longitude and the celestial quantities that bear the same name (Appendix, Art. 491).
39. Recapitulation. - The direction of gravity at the point where the observer happens to stand determines the zenith and nadir, the horizon and the almucantars (parallel to the horizon), and all the vertical circles. One of the verticals, the meridian, is singled out from the rest by the circumstance that it passes through the pole, thus marking the north and south points where it cuts the horizon. Altitude and azimuth (or their complements, zenith distance and amplitude) are the "coordinates " which designate the position of a body by reference to the zenith and meridian.

Evidently this set of points and circles shifts its position with every change in the place of the observer. Each place has its own zenith, its own horizon, and its own meridian.

In a similar way, the direction of the earth's axis (which is independent of the observer's place on the earth) determines the poles (Art. 26), the equator, the parallels of declination, and the hour-circles. Two of these hour-circles are singled out as reference lines: one of them is the meridian which passes through the zenith, and is a purely local reference line; the other, the equinoctial colure, which passes through the vernal equinox, a point chosen from its relation to the sun's annual motion.

Declination and hour-angle define the place of a star with reference to the pole and the meridian, while declination and
right ascension refer it to the pole and vernal equinox. The latter are the co-ordinates ordinarily given in star-catalogues and almanacs for the purpose of defining the position of stars and planets, and they correspond exactly to latitude and longitude on the earth, by means of which geographical positions are designated.

Finally, the earth's orbital motion gives us the great circle of the sky known as the ecliptic, and celestial latitude and longitude are quantities which define the position of a star with reference to the ecliptic and the vernal equinox. For most purposes this pair of co-ordinates is practically less convenient than right ascension and declination; but, as has been said, it came into use much earlier, and is not without its advantages in dealing with the planets and the moon.
40. Relation of the Place of the Celestial Pole to the Observer's Latitude. - If an observer were at the north pole of the earth; it is clear that the Pole-star would be very near his zenith, while it would be at the horizon if he were at the equator. The place of the pole in the sky, therefore, depends evidently on the observer's latitude, and in this very simple way - The Alititude of the Pole (its height in degrees above the horizon) is ALWAYS EQUAL TO THE Latitude of the Observer. This relation will be clear from Fig. 8. The latitude (astronomical) of a place may be defined as the angle between the direction of gravity at that place and the plane of the earth's equator, -the angle $O N Q$


Fig. 8. - Relation of Latitude to the Elevation of the Pole. in the figure. If, now, at $O$ we draw $H H^{\prime}$ perpendicular to $O . N$, it will be a "level"
line, and will lie in the plane of the horizon. From $O$ also draw $O P^{\prime \prime}$ parallel to $C P^{\prime}$, the earth's axis. Both $O P^{\prime \prime}$ and $C P^{\prime}$ being parallel, will be directed to the same "vanishing-point" in the celestial sphere, (Art. 8), and this point is the celestial pole (Art. 26).

The angle $H^{\prime} O P^{\prime \prime}$ is therefore the altitude of the pole as seen at $O$; and it obviously equals $O N Q$, since $O H^{\prime}$ is perpendicular to $O N$, and both $C P^{\prime}$ and $O P^{\prime \prime}$ are perpendicular to $Q Q^{\prime}$.
This fundamental relation, that the altitude of the pole is identical with the observer's latitude, cannot be too strongly impressed on the mind.
41. The Right Sphere. - If the observer is situated at the earth's equator (that is, in latitude zero), the pole will be in his horizon, and the celestial equator will be a vertical circle, coinciding with the "prime vertical" (Art. 17). All heavenly bodies will rise and set vertically instead of obliquely, as in our own latitudes; and their diurnal circles will all be bisected by the horizon, so that they will be 12 hours above and 12 hours below it, and the length of the night will always equal that of the day. This aspect of the heavens is called the Right Sphere.
42. The Parallel Sphere. - If the observer is at the pole of the earth, where his latitude equals $90^{\circ}$, then the celestial pole will be at the zenith, and the equator will coincide with the horizon. If he is at the north pole, all the stars north of the celestial equator will remain permanently above the horizon, never rising nor setting, but sailing around the sky on parallels of altitude. The stars in the southern hemisphere, on the other hand, will never rise to view.

Since the sun and moon move among the stars in such a way that during half the time they are north of the equator and half the time south of it, they will be half the time above the horizon and half the time below it (at least approximately, since this statement needs to be somewhat modified to allow for the effect of "refraction"-Art. 50). The moon will be visible for about a fortnight at a time, and the sun for about six months.

It is worth noting that for an observer exactly at the north pole the definitions of meridian and azimuth break down, since there the zenith coincides with the pole. Face in what direction he will, he is looking due south. If he changes his place a few steps, however, everything will come right.
43. The Oblique Sphere. - At any station between the pole and the equator the pole will be at an elevation above the horizon, and the stars will rise and set in oblique circles, as shown in Fig. 9. Those whose distance from the elevated pole is less than $P N$, the


Fig. 9. - The Oblique Sphere. latitude of the observer, will of course never set, remaining perpetually visible. The radius of this "circle of perpetual apparition," as it is called (the shaded cap around $P$ in the figure), is obviously just equal to the height of the pole, becoming larger as the latitude increases. On the other hand, stars within the same distance of the depressed pole will lie within the "circle of perpetual occultation," and will never rise above the horizon.

A star exactly on the celestial equator will have its diurnal circle, $E Q W Q^{\prime}$, bisected by the horizon, and will be above the horizon just as long as below it. A star north of the equator (if the north pole be the elevated one) will have more than half of its diurnal circle above the horizon, and will be visible for more than twelve hours of each day; as, for instance, a star at $A$, and of course the reverse will be true of the stars on the other side of the equator. Whenever the sun is north of the celestial equator, the day will therefore be longer than the night for all stations in northern latitude; how much
longer will depend both on the latitude of the place and the sun's distance from the equator (its declination).
44. Whenever the sun is north of the equator, it will, in northern latitudes, rise at a point north of east, as $B$ in the figure, and will continue to shine upon every vertical surface that faces the north, until, as it ascends, it crosses the prime-vertical $E Z W$ at some point $V$. In the latitude of New York the sun on the longest days of summer is south of the prime-vertical only about eight hours of the whole fifteen during which it is above the horizon. During seven hours of the day it shines into north windows.

If the latitude of the observer is such that $P N$ in the figure is greater than the sun's polar distance at the time when it is farthest north, the sun will make a complete circuit of the heavens without setting, as is the case at the North Cape and at all stations within the Arctic Circle. (See Art. 130.).
45. A celestial globe will be of great assistance in studying these diurnal phenomena. The north pole of the globe must be elevated to an angle equal to the latitude of the observer, which can be done by means of the degrees marked on the metal meridian ring. It will then be seen at once what stars never set, which ones never rise, and during what part of the 24 hours any heavenly body at a known distance from the equator is above or below the horizon.
(For a description of the Celestial Globe, see Appendix, Art. 524.)

## CHAPTER II.

FUNDAMENTAL PROBLEMS OF PRACTICAL ASTRONOMY. THE DETERMINATION OF LATITUDE; OF TIME; OF LONGITUDE; OF THE PLACE OF A SHIP; OF THE POSITION OF A HEAVENLY BODY.
46. There are certain problems of Practical Astronomy ${ }^{1}$ which are encountered at the very threshold of all investigations respecting the dimensions and motions of the heavenly bodies, the earth included. An observer must know how to determine his position on the surface of the earth, how to ascertain the exact time at which an observation is made, and how to observe the precise position of a heavenly body, and fix its right ascension and declination.

The first of these practical problems which we are to consider is
47. The Determination of the Observer's Latitude. - In Geography the latitude of a place is usually defined simply as its distance north or south of the earth's equator, measured in degrees. This is not explicit enough, unless it is stated how the degrees themselves are to be measured. If the earth were a perfect sphere, there would be no difficulty. But since the earth is quite sensibly flattened at its poles, the degrees (geographical) have somewhat different lengths in different parts of the earth.

[^6]An exact definition of the astronomical latitude of a place has already been given (Art. 40). It is (1) the angle between the direction of gravity and the plane of the equator, which is the same as the altitude of the pole. (2) It may also be defined


Fig. 10.-Determination of Latitude. as the declination of the zenith, as is clear from Fig. 10, where $P B$, the altitude of the pole, equals $Q Z$ (since $P Q$ and $Z B$ are each $90^{\circ}$ ), and $Q Z$ by the very definition of declination (Art. 31) is the declination of the zenith. The problem, then, is to determine, by observing some of the heavenly bodies, either the angle of elevation of the celestial pole, or the distance in degrees between the zenith and the celestial equator.
48. First Method. - By circumpolars. The most obvious method is by observing with a suitable instrument the altitude of some star near the pole (a "circumpolar" star) at the moment when it is crossing the meridian above the pole, and again 12 (sidereal) hours later when it is once more on the meridian below the pole. In the first case its elevation is the greatest possible ; in the second, the least possible. The mean of the two altitudes (each corrected for atmospheric refraction, which will shortly be considered) is the latitude of the observer.

The method has the great advantage that it is an "independent" one; i.e., the observer is not obliged to make use of any data that have been determined by his predecessors. But the method fails for stations very near the equator of the earth.
49. The Meridian Circle. - The instrument with which such observations are usually made in a fixed observatory is called the meridian circle. The principle of it is exhibited in Fig. 11. It consists of a telescope firmly attached to a stiff axis, which turns in bearings attached to two solid piers. These bearings are so adjusted that
the axis is exactly level and exactly east and west. Near $E$, at the eye end of the instrument, a "reticle" of spider-lines (see Appendix, Art.544) is so placed in the tube that on looking into the eye-piece it can be distinctly seen at the same time with the star which is to be observed - the field of view being slightly illuminated so that the spider-threads appear like dark lines drawn in the sky.

The telescope as it is rotated up or down upon the pivots is obviously always directed to the meridian; and by elevating or depressing it, it can be so set that any given star will be " bisected " by the horizontal spider-line of the reticle at the moment when it crosses the meridian. The instrument carries a large and carefully graduated circle so attached to the axis
 as to turn with the telescope.
Fig. 11. - The Meridian Circle (Schematic) scopes" fixed to the pier "read off" the graduation of this circle, and so determine the altitude of the object at which the telescope is pointed. (A fuller account of the instrument and its appendages is given in the Appendix, Art. 548.)
50. Refraction. - It was said that the observed altitudes must be corrected for atmospheric refraction. As the rays of light enter the earth's atmosphere from a distant object they are bent downward by refraction (Physics, page 295), except only such as strike the surface of the atmosphere perpendicularly. Fig. 12 illustrates this effect. Since the observer sees the object in the direction in which the rays enter the eye, without any reference to its real position, this bending down of the rays causes every object seen through the air to look higher up in the sky than it would if the air were absent.

Under average conditions the refraction elevates a body at the horizon about $35^{\prime \prime}$, so that the sun and moon in rising both appear
clear of the horizon while still wholly below it. At an altitude of only $5^{\circ}$ the refraction falls off to $10^{\prime}$; at $44^{\circ}$, it is $1^{\prime}$; and at


Fig. 12. the zenith, zero. Its amount at any given altitude varies quite sensibly however, with the temperature and barometric pressure, increasing as the thermometer falls or as the barometer rises; so that whenever great accuracy is required in measures of altitude of a heavenly body, we must have observations both of the thermometer and barometer to go with the readings of the "circle." In works on Practical Astronomy tables are given by which the refraction can be computed for an object at any altitude and in any state of the weather.

It is hardly necessary to say that this indispensable " refractioncorrection" of nearly all astronomical observations makes a great deal of trouble and involves more or less error and uncertainty.
51. Second Method of Determining the Latitude. - By the meridian altitude or zenith distance of a body whose declination is accurately known.

In Fig. 13 the circle $A Q P B$ is the meridian, $Q$ and $P$ being respectively the equator and the pole, and $Z$ the zenith. $Q Z$ is obviously the declination of the zenith, or the latitude of the observer (Art. 47). Suppose now that we observe $Z s$, the zenith distance of a star, $s$, south of the


Fig. 13. zenith as it crosses the meridian, and that $Q s$, the declination of the star, is known. Then, evidently, $Q Z$ equals $Q s+s Z$; i.e., the latitude equals the declination of the star plus its zenith distance.

If a star were at $s^{\prime}$, south of the equator, the same equation would still hold algebraically, because the declination $Q s^{\prime}$ is a minus quantity.

If the star were at $n$, between the zenith and pole, we should have, latitude equals the declination minus the zenith distance.

If we use the meridian circle in making our observations, we can always select stars that pass near the zenith where the refraction is small, which is in itself an advantage. Moreover, we can select the stars in such a way that some will be as much north of the zenith as others are south, and this will "eliminate" the refraction errors. On the other hand, in using this method we have to obtain our star declinations from the catalogues made by previous observers, and so the method is not an "independent" one.

There are many other methods in use, some of which are practically more convenient and accurate than either of the two described, but their explanation would take us too far.

See Art. 71* for a note upon Variation of latitude.

## DIFFERENT KINDS OF TIME.

52. Time is usually defined as " measured duration." From the earliest history the apparent diurnal rotation of the heavens has been accepted as the standard, and to it we refer all artificial measures of time, such as clocks and watches. In practice the accurate "determination of time" therefore consists in finding the hour-angle of the object which has been selected to mark the beginning of the day by its "transit" across the meridian.

In Astronomy, three kinds of time are now recognized, Sidereal Time, Apparent Solar Time, and Mean Solar Time, the last being the time of civil life and ordinary business, while the first is most used for astronomical purposes. Apparent solar time has now practically fallen out of use, except in half-civilized countries where watches and clocks are scarce and sun-dials are still the principal time-keepers.
53. Sidereal Time. - The celestial object which determines sidereal time by its position in the sky at any moment is, it
will be remembered, the "vernal equinox" or " first of Aries"; i.e., the point where the sun crosses the celestial equator about March 20th every year (Art. 34). As has already been explained (Art. 35), the sidereal time at any moment is, therefore, the hour-angle of the vernal equinox at that moment; or, what comes to the same thing, it is the time marked by a clock which is so set and adjusted as to show (sidereal) noon $\left(0^{\mathrm{h}}, 0^{\mathrm{m}}, 0^{\mathrm{s}}\right)$ at each transit of the "first of Aries." The sidereal " $d a y$ " is the interval between successive transits of this point, and within less than $\frac{1}{100}$ of a second, is equal to the interval between successive transits of any given star. The equinoctial point is, it is true, invisible; but its position among the stars is always known, so that its hour-angle can be determined by observing them.
54. Apparent Solar Time. - Just as sidereal time is the hour-angle of the vernal equinox, so apparent solar time at any moment is the hour-angle of the sun. It is the time shown by the sun-dial, and its "noon" occurs at the moment when the sun crosses the meridian. On account of the annual eastward motion of the sun among the stars, due to the earth's orbital motion (more fully explained farther on - Art. 128), the day of solar time is about four minutes longer than the sidereal day. Moreover, because the sun's motion in the sky is not uniform, the days of apparent solar time are not all of the same length. December 23d, for instance, is 51 seconds longer from noon to noon, reckoned by the sun, than Sept. 16th. For this reason, apparent solar time is unsatisfactory for scientific use, and it has been discarded in favor of mean solar time.
55. Mean Solar Time. - A fictitious sun is therefore imagined, which moves around the sky uniformly, and in the celestial equator, completing its annual course in exactly the same time
as that in which the actual sun makes the circuit of the ecliptic, that is, in one year; and this "fictitious sun" is made the time-keeper for mean solar time. The mean solar days are therefore all exactly of the same length, and equal in length to the average "apparent solar" day. It is mean noon when this "fictitious sun" crosses the meridian, and at any moment the hour-angle of the "fictitious sun" is the mean time for that moment.
56. Sidereal time will not answer for business purposes, because its noon (the transit of the vernal equinox) occurs at all hours of the day in different seasons of the year. On the 22d of September, for instance, it comes at midnight. Apparent solar time is unsatisfactory from the scientific point of view, because of the variation in the length of its days and hours. And yet we have to live by the sun : its rising and setting, daylight and night, control our actions. In mean solar time we find a satisfactory compromise - a time-unit which is invariable, and still in agreement with sun-dial time nearly enough for convenience. It is the time now used for all purposes except in certain astronomical work. The difference between apparent time and mean time, (never amounting to more than about a quarter of an hour, ) is called the "equation of time," and will be discussed hereafter in connection with the earth's orbital motion (Art. 128). The Nautical Almanac also furnishes data by means of which the sidereal time may be accurately deduced from the corresponding solar time, or vice versa, by a very brief ${ }^{1}$ calculation.
57. The Civil Day and the Astronomical Day. - The astronomical day begins at "mean noon"; the civil day, 12 hours earlier at midnight. Astronomical mean time is reckoned around through the whole 24 hours instead of being counted in two series of 12 hours each: thus, 10 A.m. of Wednesday,
${ }^{1}$ The approximate relation between sidereal and mean solar time is very simple. On March 20th, the two times agree, and after that the sidereal time gains two hours a month. On April 5th, therefore, the sidereal clock is one hour in advance, on April 20th, two hours, and so on.

Feb. 27th, civil reckoning, is Tuesday, Feb. 26th, 22 hours, by astronomical reckoning. Beginners need to bear this in mind in referring to the almanac.

## DETERMINATION OF TIME.

58. In practice the problem of determining time always takes the form of ascertaining the "error" or "correction" of a time-piece; that is, finding the amount by which a watch or clock is faster or slower than the time it ought to indicate. The method ordinarily em-


Fig. 14. - The Transit Instrument. ployed by astronomers is by means of the Transit Instrument, which is an instrument precisely like the meridian circle (Art. 49) without the circle and its reading microscopes. As the instrument (Fig. 14) is turned upon its axis, the vertical wire in the centre of the "reticle" exactly follows the meridian, when the instrument is in perfect adjustment. If, then, we know the instant shown by the clock when a known star is crossing this wire, we have at once the means of determining the error of the clock, because the sidereal time at that moment is equal to the star's right ascension (Art. 37). The difference between the right ascension of the star as given in the almanac and the time shown by the face of the clock at the moment of transit gives directly the "error" of the sidereal clock.

The observation of only a single star would give the error of the clock pretty closely, but it is much better and usual to observe a num.
ber of stars (from 8 to 10), reversing the instrument upon its pivots once at least during the operation. With a good instrument a skilled observer can thus determine the clock error within about a thirtieth of a second of time, provided proper means are taken to allow for his "personal equation."

If instead of observing a star we observe the sun with this instrument, the time shown by the (solar) clock ought to be noon plus or minus the equation of time for the day as given in the almanac. But for various reasons transit observations of the sun are less accurate than those of the stars, and it is better to deduce the mean solar time, when needed, from the sidereal by means of the almanac data. (For a fuller description of the transit instrument and its adjustments see Appendix, Art. 544.)
59. Personal Equation. - It is found that every observer has his own peculiarities of time-observation, and the so-called "personal equation" of an observer is the amount that must be added (algebraically) to the time observed by him in order to get the actual moment of transit. This personal equation differs for different observers, but is reasonably constant (though never strictly so) for one who has had sufficient practice. In the case of observations with the chronograph (see Appendix, Art. 547) it is usually less than 0.1 of a second.

The best device for eliminating it is one in which the observer simply keeps the star accurately bisected by a micrometer wire, the record being made automatically.
60. Other Methods of Determining Time. - While the method by the transit instrument is most used, and is on the whole the most convenient and accurate, several other methods are available. At sea, and ly travellers on scientific expeditions, the time is usually determined by observing the altitude of the sun some hours before or after noon (see Appendix, Art. 493). It can also be done roughly by means of a noon-mark, which is simply a true north and south line running from the bottom of some'vertical line, - a line drawn on the floor from the edge of the door-jamb, for instance. The moment when the shadow falls on this line is apparent noon, and must be corrected for the equation of time to get mean noon. As the shadow is somewhat indistinct, a determination made in this way is liable to an error of half a minute or so.

## LONGITUDE.

61. Having now the means of finding the true local time, we can approach the problem of the longitude, in many respects the most important of what may be called the "economic" problems of Astronomy. The great observatories of Greenwich and Paris were founded expressly to furnish the necessary data for determining the longitude of ships at sea, and the English government has given large prizes for the construction of clocks and chronometers to be used in such determinations.

The longitude of a place on the earth may be defined as the angle (at the pole of the earth) between the standard ${ }^{1}$ meridian and the meridian passing through the place; and this of course is equal to the arc of the equator intercepted between the two meridians.

Since the earth turns uniformly on its axis, this angle is strictly proportional to, and is measured by the time intervening between the transits of the mean sun or of any given star across the two meridians. It may therefore be defined as the difference of local times between the standard meridian and the place in question, or the amount by which noon at Greenwich is earlier or later than noon at the station of the observer. Accordingly, terrestrial longitude is now usually reckoned in hours, minutes, and seconds rather than in degrees.

Since an observer can easily find his own local time by the methods given above, the knot of the problem is simply this: To find Greenwich local time at any moment without going there.
62. First Method. - By Telegraph. Incomparably the best method, whenever it is available, is to make a direct telegraphic

[^7]comparison between the clock of the observer and that of some station the longitude of which is known. The difference between the two clocks will be the true difference of longitude of the places after the proper corrections for their errors and "personal equation" (Art. 59) have been applied.

Wireless telegraphy is now being used for this purpose. Anyone on the Atlantic seaboard who has a wireless receiving set can get accurate Washington time from the signals sent out daily from Arlington, Va.
63. Second Method. - By the Chronometer. The chronometer is merely a very accurate watch. It is set to Greenwich time at some place whose longitude is known, and thereafter keeps that time wherever carried. The observer has only to find the apparent "error" of such a chronometer with respect to his local time, and this apparent error is his longitude.

Practically, of course, no chronometer goes absolutely without gaining or losing; hence it is always necessary to know and allow for its gain or loss since the time it was set. Three or more should be used if possible, since any irregular going of either of them will then be pretty surely indicated by its disagreement with the others.
64. Other Methods. - Before the days of telegraphs and reliable chronometers, astronomers were generally obliged to get their Greenwich time from the moon, which may be regarded as a clock-hand with stars for dial figures. Since the laws of the moon's motion are now well known, so that the place which the moon will occupy is predicted in the Nautical Almanac for every hour of every Greenwich day, it is possible to deduce the Greenwich time at any moment when the moon is visible, by some observation which will determine her place among the stars. The almanac place, however, is the place at which the moon would be seen by an observer situated at the centre of the earth, and consequently the actual observations in most cases require rather complicated and disagreeable reductions before they can be made available.

Various kinds of observations of the moon are made use of for the purpose, of which the most satisfactory are those obtained on occasions when the moon "occults" a star or eclipses the sun. For fuller explanations, see " General Astronomy."
65. Local and Standard Time. - Until recently it has been always customary to use local time, each station determining its own time by its own observations. Before the days of the telegraph and while travelling was comparatively slow and infrequent, this was best. At present, for many reasons, it is better to give up the old system of local times in favor of a system of standard time. The change facilitates all railway and telegraphic business in a remarkable degree, and makes it practically easy for every one to keep accurate time, since it can be daily wired from some observatory to every telegraph office. According to the system now established in North America, there are five such standard times in use, - the colonial, the eastern, the central, the mountain, and the Pacific, - which differ from Greenwich time by exactly $4,5,6,7$, and 8 hours respectively, the minutes and seconds being everywhere identical.

At most places only one of these standard times is employed; but in cities where different systems join each other, as for instance at Atlanta and Pittsburgh, there are two standard times in use, differing from each other by exactly one hour, and from the local time by about half an hour. In some such places the local time also maintains itself.

In order to determine the standard time by observation, it is only necessary to determine the local time by one of the methods given, and correct it according to the observer's longitude from Greenwich.
66. Where the Day begins. - It is evident that if a traveller were to start from Greenwich on Monday noon and travel westward as fast as the earth turns to the east beneath his feet, he would keep the sun exactly upon the meridian all day long and have continual noon. But what noon? It was

Monday when he started, and when he gets back to London 24 hours later it will be Tuesday noon there; and yet he has had no intervening night. When did Monday noon become Tuesday?

It is agreed among mariners to make the change of date at the 180th meridian from Greenwich. Ships crossing this line from the east skip one day in so doing. If it is Monday afternoon when a ship reaches the line, it becomes Tuesday afternoon the moment she passes it, the intervening 24 hours being dropped from the reckoning on the log-book. Vice versa, when a vessel crosses the line from the western side, it counts the same day twice, passing from Tuesday back to Monday and having to do Tuesday over again.

This 180th meridian passes mainly over the ocean, hardly touching land anywhere. There is some irregularity in the date actually used on the different islands in the Pacific. Those which received their earliest European inhabitants via the Cape of Good Hope, have, for the most part, adopted the Asiatic date, even if they really lie east of the 180th meridian, while those that were first approached via Cape Horn have the American date. When Alaska was transferred from Russia to the United States, it was necessary to drop one day of the week from the official dates.

## PLACE OF A SHIP AT SEA.

67. The determination of the place of a ship at sea is the problem to which Astronomy mainly owes its economic importance. As was said a few pages back, national observatories and nautical almanacs were established in order to supply the mariner with the data needed to make this determination accurately and promptly. The methods employed are necessarily such that the required observations can be made with the sextant and chronometer. Fixed instruments (like the transit instrument and meridian circle) are obviously out of the question on board of a vessel.
68. Latitude at Sea. - This is obtained by observing with the sextant the sun's maximum altitude, which, of course, is reached when the sun is crossing the meridian. Since at sea, the sailor seldom knows beforehand the precise chronometer time of local noon, the observer takes care not to be too late and begins to measure the sun's altitude a little before noon, repeating his observations every minute or two. At first the altitude will keep increasing, but when noon is reached the altitude will become stationary, and then begin to decrease. The observer uses, therefore, the maximum altitude obtained, which, duly corrected for "semi-diameter," "dip of the horizon," "refraction," and " parallax," (see Appendix, Art. 492), gives him the true altitude of the sun's centre, and taking this from $90^{\circ}$ we get its zenith distance. Looking in the almanac, we find there the sun's declination given for Greenwich (or Washington) noon of every day, with the hourly change, so that we can easily deduce the exact declination at the moment of the observation. Then the observer's latitude comes out at once, because (Art. 51) the latitude of the observer equals the sun's zenith distance plus the sun's declination. It is easy in this way, with a good sextant, to get the latitude within about half a minute of are (or, roughly, half a mile).

## 69. Determination of the Local Time and Longitude at Sea.

- The usual method now employed for the longitude depends upon the chronometer. This is carefully "rated" in port; that is, its error and its daily gain or loss are determined by comparisons with an accurate clock for a week or two, the clock itself being kept correct by transit observations. By merely allowing for its gain or loss since leaving port and adding this gain or loss to the error which the chronometer had when brought on board, the seaman at once obtains the error of the chronometer on Greenwich time at any moment; and allowing for this error, he has the Greenwich time itself with an accuravy which depends only on the constancy of the chronometer's
rate: it makes no difference whether it is gaining much or little, provided its daily rate is always the same.

He must also determine his own local time, and it must be done with the sextant, since, as was said before, an instrument like the transit cannot be used at sea. He does it by measuring the altitude of the sun, not at or near noon, as generally supposed, but when the sun is as near east or west as the circumstances permit. From such an observation the sun's hourangle, i.e., the apparent time (and from this the mean time), is easily found, by means of the so-called $P Z S$ triangle, provided the ship's latitude is known (see Appendix, Art. 493). The longitude follows at once, being simply the difference between the Greenwich time and the local time (Art. 63).

For "Sumner's method" of determining a ship's place, see " The Manual of Astronomy " or "General Astronomy."

## DETERMINATION OF THE POSITION OF A HEAVENLY BODY.

As the basis of all our investigations in regard to the motions of the heavenly bodies, we require a knowledge of their "places" in the sky at known times. The "place," so-called, is defined by the body's right ascension and declination.
70. 1. By the Meridian Circle. - If a body is bright enough to be seen by the telescope of the meridian circle and comes to the meridian in the night time, its right ascension and declination are best determined by that instrument. If the meridian circle is in exact adjustment, the right ascension of the object is simply the sidereal time when it crosses the middle wire of the reticle of the instrument.

The "circle-reading," on the other hand, corrected for refraction and parallax, gives the polar distance of the object (the complement of its declination) if the "polar point" of the circle has been determined (see Appendix, Art. 549). A single complete observation with the meridian circle deter-
mines therefore both the right ascension and the declination of the object.
71. 2. By the Equatorial. -If the body, a comet for instance, is too faint to be observed by the telescope of the meridian circle, which is seldom very -powerful, or if it comes to the meridian only in the daytime, we usually accomplish our object by using the equatorial, and determine the position of the body by measuring the difference of right ascension and declination between it and some neighboring star, the place of which is given in a star-catalogue.

In measuring this difference of right ascension and declination, we usually employ a " micrometer" (Art. 542), attached to the telescope. The difference of right ascension between the star and the object to be determined is measured by simply observing with the chronograph the transits of the two objects across wires that are placed north and south ; the difference of declination, by bisecting each object with one of the micrometer wires as it crosses the middle of the field of view. The observed differences must be corrected for refraction, and also for the motion of the body, if it is appreciable.

In some cases "altitude and azimuth instruments," so-called, are used for such "extra-meridian" observations, especially in observing the moon.

Art. 71*. Variation of Latitude. It has been discovered since 1889 that latitudes on the earth's surface certainly undergo slight periodical changes, amounting sometimes to $0 .{ }^{\prime \prime} 6$ within a year. In other words, the earth's poles are not absolutely fixed, but wander about to the extent of 50 or 60 feet, never going more than 30 feet or so from their mean position. The motion is found to be compounded of two : one circular with a period of 14 months, the other in an elongated oval with a period of exactly a year. It is supposed to be due chiefly to causes depending upon the seasons, but the explanation is not yet complete.

There is so far no evidence that any considerable changes have ever occurred in the position of the poles.

## CHAPTER III.

THE EARTH: ITS FORM, ROTATION, AND DIMENSIONS. MASS, WEIGHT, AND GRAVITATION. - THE EARTH'S MASS AND DENSITY.
72. In a science which deals with the heavenly bodies it might seem at first that the earth has no place; but certain facts relating to it are just such as we have to investigate with respect to her sister planets, are ascertained by astronomical methods, and a knowledge of them is essential as a base of operations. In fact, Astronomy, like charity, "begins at home," and it is impossible to go far in the study of the bodies which are strictly "heavenly" until one has first acquired some accurate knowledge of the dimensions and motions of the earth itself.
73. The astronomical facts relating to the earth are broadly these:

1. The earth is a great ball about 7920 miles in diameter.
2. It rotates on its axis once in twenty-four sidereal hours.
3. It is not exactly spherical, but is flattened at the poles, the polar diameter being nearly twenty-seven miles, or about one two hundred and ninety-fifth part, less than the equatorial.
4. It has a mean density between 5.5 and 5.6 as great as that of water, and a mass represented in tons by six with twentyone ciphers following (six thousand millions of millions of millions of tons).
5. It is flying through space in its orbital motion around the sun with a velocity of about eighteen and a half miles a second; i.e., about seventy-five times as swiftly as an ordinary cannon. ball.

## I.

74. The Earth's Approximate Form and Size. - It is not necessary to dwell on the ordinary proofs of the earth's globularity. We will simply mention them.
75. It can be circumnavigated.
76. The appearance of vessels coming in from sea indicates that the surface is everywhere convex.
77. The fact that the dip of the sea horizon (Art. 16), as seen from a given elevation, is (sensibly) the same in all directions, and at all parts of the earth, shows that the surface is approximately spherical.
78. The fact that as one goes from the equator towards the north the elevation of the pole increases in proportion to the distance from the equator proves the same thing.
79. The outline of the earth's shadow, seen upon the moon during lunar eclipses, is such as only a sphere could cast.

We may add, as to the smoothness and roundness of the earth, that if the earth be represented by an 18 -inch globe, the difference between its greatest and least diameters would be only about one-sixteenth of an inch; the highest mountains would project only about one-eightieth of an inch, and the average elevation of continents and depths of the ocean would be hardly greater on that scale than the thickness of a film of varnish. Relatively, the earth is really much smoother and rounder than most of the balls in a bowling alley.

## 75. The Approximate Measure of the Earth's Diameter. -

 There are various ways of determining the diameter of the earth. The best, in fact the only accurate one, is by measuring arcs of meridian, so as to ascertain the number of miles or kilometres in a degree of the earth's circumference. This will be more fully discussed in Articles 86-89.There are various approximate methods, one of the simplest of which is the following :

Erect upon a reasonably level plane three rods in line, a mile
apart from each other, and cut off their tops at the same level, carefully determined with a surveyor's levelling instrument. It will then be found, on sighting across from $A$ to $C$ (Fig. 15), that the line, after allowing for refraction, passes about eight inches below $B$, the top of the middle rod.

Suppose the circle $A B C$ completed, and that $E$ is the point of the circumference opposite $B$, so that $B E$ equals the diameter of the earth (i.e., $B E=2 R$ ). By geometry,

$$
B D: B A:: B A: B E ; \text { whence } B E=\frac{B A^{2}}{B D}
$$



Fig. 15. - Curvature of the Earth's Surface.
Now $B A$ is one mile, and $B D$ equals two-thirds of a foot, or $\frac{1}{7920}$ of a mile; hence $B E$ equals

$$
\frac{1^{2}}{\frac{1}{7920}}=7920 \text { miles. }
$$

On account of refraction, however, the result cannot be made exact. The necessary correction is large, and varies with the thermometer and barometer ; so that the actual observed length of $B D$, instead of being 8 inches, ranges from 5 to 7 according to circumstances.

## 1.

76. The Rotation of the Earth. - At the time of Copernicus the only argument he could adduce in favor of the earth's rotation ${ }^{1}$ was that the hypotıesis is much more probable than the older one, that the heavens themselves revolve.
[^8]All the phenomena then known would be sensibly the same or either supposition. The apparent diurnal motion of the heavenly bodies can be fully accounted for (within the limits of the observations then possible) either by supposing that the stars are actually attached to a celestial sphere which turns daily, or that the earth itself rotates upon an axis; and for a long time the latter hypothesis did not seem to most people so probable as the older and more obvious one.

A little later, after the telescope had been invented, analogy could be adduced; for with the telescope we can see that the sun, moon, and many of the planets are rotating globes. Within the present century it has become possible to adduce experimental proofs which go still further, and absolutely demonstrate the earth's rotation: some of them even make it visible.

## 77. Foucault's Pendulum

 Experiment. - Among these experimental proofs the most impressive is the pendulum experiment devised and first executed by Foucault in 1851. From the dome of the Pantheon in Paris, he hung a heavy iron ball about a foot in diameter by a wire more than 200 feet long (Fig. 16). A circular rail some twelve feet across, with a little ridge of sand built upon it, was placed in such a way that a pin attached to the swinging ball would just scrape the sand and leave a mark at each vibration. To put the ball in motion it was drawn aside by a cotton cord andleft to come absolutely to rest; then the cord was burned off, and the pendulum started to swing in a true plane. But this plane seemed to deviate slowly towards the right, so that the pin on the pendulum-ball cut the sand-ridge in a new place at each swing, shifting at a rate which would carry the line completely around in about 32 hours, if the pendulum did not first come to rest. In fact the floor of the Pantheon was actually and visibly turning under the plane of the pendulum vibration. The experiment created great enthusiasm at the time, and has since been frequently performed.

For fuller discussion, see Appendix, Art. 494.
78. We merely mention (without discussion) a number of other demonstrations of the earth's rotation.
(a) By the gyroscope, an experiment also due to Foucault.
(b) The slight eastward deviation of bodies in falling from a great height. The idea of the experiment was first suggested by Newton, but its actual execution has only been carried out during the present century - by several different observers.
(c) The deviation of projectiles.
(d) Various phenomena connected with Meteorology and Physical Geography, such as the direction of the trade winds and the great currents of the ocean, which are determined by the earth's rotation; so also the direction of the revolution of cyclones, which in the northern hemisphere move contrary to the hands of a watch, while in the southern their motion is in the opposite direction (see "General Astronomy ").

It might seem at first that the rotation of the earth in 24 hours is not a very rapid motion. A point on the equator moves, however, nearly 1000 miles an hour, which is about 1500 feet per second, - very nearly the speed of a cannon-ball.
79. Invariability of the Earth's Rotation. - It is a question of great importance whether the day ever changes its length. Theoretically, it must almost necessarily do so. The friction of the tides and the deposit of meteoric matter upon the earth
both tend to retard the earth's rotation; while, on the other hand, the earth's loss of heat by radiation and the consequent shrinkage must tend to accelerate it and to shorten the day. Then geological causes act some one way and some the other. At present we can only say that the change, if any change has occurred since Astronomy became accurate, has been too small to be detected.

The day is certainly not longer or shorter by the $\frac{1}{100}$ part of a second than it was in the days of Ptolemy ; probably it has not changed by the $\frac{1}{1000}$ part. The criterion is found in comparing the times at which celestial phenomena, such as eclipses, transits of Mercury, etc., have occurred during the range of astronomical history. Professor Newcomb's investigations in this line make it highly probable, however, that the length of the day has not been absolutely constant during the last 150 years.
80. Effects of the Earth's Rotation upon Gravity on the Earth's Surface. - As the earth rotates, every particle of its matter is subjected to a so-called "centrifugal force" directed away from the axis of the earth (Physics, page 63), and this force depends upon the radius of the circle upon which the particle moves, and the velocity with which it moves.

The formula is $C=\frac{V^{2}}{R}$, in which $V$ is the velocity of the moving particle, $R$ the radius of the circle, and $C$ is the centrifugal force, expressed as an " acceleration," in the same way that gravity is expressed by $g$, - the velocity of $32 \frac{1}{6}$ feet, which a falling body acquires in the first second of its fall.

As stated in the Physics in the passage referred to, a body at the equator of the earth has its weight diminished by $\frac{1}{289}$ part, in consequence of this force. (But see Art. 91.)

## 81. Effect of Centrifugal Force in diminishing Gravity. -

 Between the equator and the poles the centrifugal force is less than at the equator, because the circle described each day by a body at the earth's surface is smaller, its distance fromthe axis being less. Moreover, as shown in Fig. 17, the centrifugal force $M T$ at $M$, since it acts at right angles to the earth's axis, $O P$, is not directly opposed to the earth's attraction, which acts (nearly) on the line $M O$; it is not, therefore, wholly effective in diminishing the weight of the body. To ascertain the effect produced upon the weight, $M T$ must be "resolved" (Physics, p. 52) into the two component forces $M R$ and $M S$. The first of these alone acts to les-


Fig. 17.
The Earth's Centrifugal Force. sen the weight.
82. Effect of the Horizontal Component of the Centrifugal Force. - The horizontal component $M S$ tends to make the plumb-line deviate from the line $M O$ (drawn to the earth's centre) towards the equator, so as to make a smaller angle with the earth's axis than it otherwise would.

In latitude $45^{\circ}$, this horizontal component of the centrifugal force has a maximum equal to about $\frac{1}{578}$ of the whole force of gravity, and causes the plumb-line to deviate about $6^{\prime}$ from the direction it would otherwise take. If the earth's surface were strictly spherical, this horizontal force would make every loose particle tend to slide towards the equator, and the water of the ocean would so move. But the earth bulges at the equator just enough to counteract this sliding tendency, and in consequence the deviation of the plumb-line from a line drawn to the earth's centre is nearly doubled. (See Art. 90.)
83. Gravity. - What is technically called "gravity" is not simply the attraction of the earth for a body upon its surface, but the resultant of the attraction combined with this centrifugal force. It is only at the equator and at the pole that "gravity" is directed strictly towards the earth's centre. Lines of level are always perpendicular to "gravity," and they are, therefore, not true circles around the earth's centre. If the earth's rota-
tion were to cease, the Mississippi River would at once have its course reversed, since the mouth of the river is several thousand feet further from the centre of the earth than are its sources.
84. Accurate Determination of the Earth's Dimensions. The form of the earth, instead of being spherical, is much more nearly that of an "oblate spheroid of revolution" (an orange-shaped solid) quite sensibly flattened at the poles; the polar diameter being shorter than the equatorial by about $\frac{1}{295}$ part. According to "Clarke's ${ }^{1}$ Spheroid of 1866 " (which is adopted by our Coast and Geodetic Survey as the basis of all calculations) the dimensions of the earth are as follows:-

| Equatorial radius, (a) $6,378,206.4$ metres $=3963.307$ miles. |  |
| :--- | :--- |
| Polar radius, | (b) $6,356,583.8$ metres $=3949.871$ |
| Difference, | " |
| "1,622.6 metres $=\frac{13.436}{}$ |  |

These numbers are likely to be in error as much, perhaps, as 100 metres, and possibly somewhat more ; they can hardly be 300 metres wrong.

This deviation of the earth's form from a true sphere is due simply to its rotation, and might have been cited as proving it. The centrifugal force caused by the rotation modifies the direction of gravity everywhere except at the equator and the poles (Art. 82) ; and so the surface necessarily takes the spheroidal form.
85. Methods of Determining the Earth's Form. - There are several ways of doing this : one by measurement of distances upon its surface in connection with the latitudes and longitudes of the points of observation. This gives not only the form, but the dimensions also; i.e., the size in miles or metres. An-

[^9]other method is by the observation of the force of gravity at various points - observations which are made by means of a pendulum apparatus of some kind, and determine only the form of the earth, but not its size.
86. The simplest form of the method by actual measurement is that in which we determine the length of degrees of latitude, some near the equator, and others near the poles, and still others intermediate.

If the earth were exactly spherical, the length of a degree would, of course, be everywhere the same. Since it is not, the length of a degree will be greatest where the earth is most nearly flat; i.e., near the poles; in other words, the distance between two points on the same meridian having their plumblines inclined to each other at an angle of one degree will be greatest where the surface is least curved.

The measurement of an "arc" involves two distinct sets of operations, one purely astronomical, the other geodetic. Having selected two terminal stations several hundred miles apart, and one of them as nearly as possible due north of the other, we must determine first the distance between them in feet or metres, and second (by astronomical observations), their difference of latitude in degrees, with the exact azimuth or bearing of the line that connects them.
87. Geodetic Operations. - The determination of the distance in feet or metres. It is not practicable to measure this with sufficient accuracy directly, as by simple "chaining," but we must have recourse to the process known as "triangulation."

Between the two terminal stations ( $A$ and $H$, Fig. 18) others are selected such that the lines joining them form a complete chain of triangles, each station being visible from at least two others. The angles at each station are carefully measured; and the length of one of the sides, called the "base," is also measured with all possible precision. It can be done with an error not exceeding an inch in ten
miles. ( $B U$ is the base in the figure.) Having the length of the base, and all the angles, it is then possible to calculate every other line in the chain of triangles. An error of more than three


Fig. 18.
feet in a hundred miles would be uupardonable.
88. Astronomical Operations. - By astronomical observations we must determine (a) the true bearing or azimuth of the lines of the triangulation, and also (b) the difference of latitude in degrees between $A$ and $H$.

To effect the first object, it will be sufficient to determine the azimuth of any one of the sides of the system of triangles by the method given in the Appendix, Art. 495. This being known, the azimuth of every other line is easily got from the measured angles, and we can then compute how many feet or metres one terminal station is north of the other, - the line $A h$ in the figure.
(b) The difference of latitude. This is obtained simply by determining the latitudes of $A$ and $H$ by one of the methods of Art. 47, or by any other method that will determine astronomical latitudes with precision. It is well, also, to measure the latitudes of a number of the intermediate stations, and to determine the azimuths of a number of lines of the triangulation instead of a single one (in order to lessen the effect of errors of observation).
89. The Length of a Degree of Latitude. - The geodetic and astronomical observations thus give the length of the line $A h$, both in feet and in degrees, so that we immediately find the number of feet in that degree of latitude which has its middle point half-way between $A$ and $h$. If the earth were spherical, the length of a degree would be everywhere the same, and the
earth's diameter would be found simply by multiplying the length of one degree by 360 and dividing the product by $\pi$, that is, 3.1415926 .
More than twenty such arcs have been measured in different parts of the world, varying in length from $48^{\circ}$ to $2^{\circ}$, and it appears clearly that the length of the degrees, instead of being everywhere the same, increases towards the pole.

| At the equat |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| At lat. $20^{\circ}$ | " | " | $=68.786$ | " |
| " $40^{\circ}$ | " | " | $=68.993$ | " |
| " " $60{ }^{\circ}$ | " | " | $=69.230$ |  |
| " " $80^{\circ}$ | " | " | $=69.386$ | , |
| At the pole, | " | " | $=69.407$ |  |

The difference between the equatorial and polar degree of latitude is more than seven-tenths of a mile, or over $3 \check{5} 00$ feet; while the probable error of measurement cannot exceed a foot or two to the degree.
90.. The Ellipticity or Oblateness of the Earth. - The calculations by which the precise form of the earth is deduced from such a series of measurements of arcs lie beyond our scope, but the net result is as stated in Ait. 84.

The fraction obtained by dividing the difference between the equatorial and polar radii, by the equatorial (i.e., the fraction $\frac{a-b}{a}$ ), is called by various names, such as the "Polar Compression," the "Ellipticity," ${ }^{1}$ or the Oblateness, of the earth ; the last term being most used.

Owing to the obvious irregularities in the form of the earth, the results obtained by combining the ares in different ways

[^10]are not exactly accordant, so that a very considerable variation is found in the ellipticity as deduced by different authorities.

## 91. Determination of the Earth's Form by Pendulum Experi-

 ments. -For details of experiments of this kind we must refer the reader to the "General Astronomy." It is sufficient for our purpose simply to say that such observations show that the force of gravity is greater at the pole than at the equator by about $\frac{1}{190}$ part; i.e., weighed in a spring balance, a man who weighs 190 pounds at the equator would weigh 191 at the pole. The centrifugal force of the earth's rotation accounts for about one pound in 289 of the difference. The remainder (about one pound in 555 ) has to be accounted for by the difference between the distances of the centre of the earth from the pole and from the equator. At the pole a body is nearer the centre of the earth than anywhere else on the earth's surface, and as a consequence the earth's attraction upon it is greater.The result of the pendulum observations thus far made indicates for the earth an ellipticity of about $\frac{1}{2} \frac{1}{3}$, - in practical (though not absolute) agreement with the result derived by measurement of arcs.

There are other purely astronomical methods of determining the form of the earth, depending upon certain irregularities in the moon's motion which are due to the "bulge" at the earth's equator; and upon the moon's effect in producing "precession" (Art. 124). They indicate a slightly smaller "oblateness" of about $\frac{1}{300}$.
92. Station Errors. - If the latitudes of all the stations in a triangulation, as determined by astronomical observations, are compared with their differences of latitude as deduced from trigonometrical operations, we find the discrepancies by no means insensible. They are in the main due not to errors of observation, but to irregularities in the direction of gravity, and depend upon the variations in the density of the crust and the irregularities of the earth's surface. Such irregularities are called station errors. According to the Coast Survey, in the
eastern part of the United States these station errors average about $1 \frac{1}{2}$ seconds of arc, affecting both the longitudes and latitudes of the stations, as well as the astronomical azimuths of the lines that join them. Station errors of from $4^{\prime \prime}$ to $6^{\prime \prime}$ are not very uncommon, and in mountainous countries these deviations occasionally amount to $30^{\prime \prime}$ or $40^{\prime \prime}$.

## 93. Distinction between Astronomical and Geographical Lat-

 itude. - The astronomical latitude of the station is that actually determined by astronomical observations. The geographical latitude is the astronomical latitude corrected for "station error." It may be defined as the angle formed with the plane of the equator by a line drawn from the place perpendicular to the surface of the "standard spheroid" at that station. Its determination involves the adjustment and even-ing-off of the discrepancies between the geodetic and astronomical results over extensive regions of country. The geographical latitudes are those used in constructing a map.For most purposes, however, the distinction may be neglected, since on the scale of an ordinary map the "station errors" would be insensible.
94. Geocentric Latitude. - The astronomical latitude at any place (Art. 47) is, it will be remembered, the angle between the plane of the equator and the direction of gravity at that place. The geocentric latitude, on the other hand, is the angle made at the centre of the earth, as the word implies, between the plane of the equator and a line drawn from the observer to the centre of the earth; which line, evidently does not coincide with the direction of gravity, since the earth is not spherical.

In Fig. 19, the angle $M N Q$ is the astronomical latitude of the point M. (It is also the geographical latitude, provided the "station error" at that point is insensible.) The angle $M O Q$ is the geocentric latitude. The angle $Z M Z^{\prime}$, the difference of the two latitudes, is called the angle of the vertical, and reaches its maximum, $11^{\prime} 31^{\prime \prime}$, at latitude $45^{\circ}$.


Fig. 19. - Astronomical and Geocentric Latitude.

The geocentric degrees are longer near the equator than near the poles, and it is worth noticing that if we form a table like that of Art. 89, giving the length of each degree of geographical latitude from the equator to the pole, the same table, read backwards, gives the length of the geocentric degrees; i.e., at the pole a degree of geocentric latitude is 68.704 miles, and at the equator 69.407 miles.

Geocentric latitude is seldom employed except in certain astronomical calculations in which it is necessary to "reduce the observations to the centre of the earth."
95. Surface and Volume of the Earth. - The earth is so nearly spherical that we can compute its surface and volume with sufficient accuracy by the formulæ for a perfect sphere, provided we put the earth's mean semi-diameter for $r$ in the formulæ. This mean semidiameter of an oblate spheroid is not $\frac{a+b}{2}$, but $\frac{2 a+b}{3}$, because if we draw through the earth's centre three axes of symmetry at right angles to each other, one will be the axis of rotation and both the others will be equatorial diameters. The mean semi-diameter $r$ of the earth thus computed is 3958.83 miles; its surface $\left(4 \pi r^{2}\right)$ is $196,944,00 \mathrm{C}$ square miles, and its volume $\left(\frac{4}{3} \pi r^{3}\right), 260,000$ million cubic miles, in round numbers.

## III.

## THE EARTH'S MASS AND DENSITY.

96. Definition of Mass. - The mass of a body is the "quantity of matter" in it; i.e., the number of "pounds," "kilograms," or "tons" of material it contains. It must not be confounded with the " volume" of a body, which is simply its bulk; i.e., the number of cubic feet or cubic miles in it; nor is it identical with its "weight," which is simply the force with which the body tends to move towards the earth. It is true that under ordinary circumstances the mass of a body and its
weight are proportional, and numerically equal; a mass of ten pounds "weighs" very nearly ten pounds under ordinary circumstances; but the word "pound" in the two halves of the sentence means two entirely different things; the pound of "mass" is one thing, the pound of "force" a very different one.
97. Mass and Force distinguished. - This identity of names for the units of mass and force leads to perpetual ambiguity, and is very unfortunate, though the reason for it is perfectly obvious, because in most cases we measure masses by weighing. The unit of mass is a certain piece of platinum, or some such unalterable substance which is kept at the national capital, and called the standard "pound," or " kilogram." The pound or kilogram of weight or force, on the other hand, is not the piece of metal at all, but the attraction between it and the earth at some given place, as, for instance, Paris. It is a pull or a stress.

The mass of a given body, - the number of mass-pounds in it - is invariable; its weight, on the other hand, - the number of force-pounds which measures its tendency to fall, - depends on where the body is. At the equator it is less than at the pole; at the centre of the earth it would be zero, and on the surface of the moon only about one-sixth of what it is on the earth's surface.
The student must always be on his guard whenever he comes to the word "pound," or any of its congeners, and consider whether he is dealing with a pound of mass or force.

Many high authorities now advocate the entire abandonment of these old force-units which bear the same names as the mass-units, and the substitution in all scientific work of the dyne (Physics, p. 33) and its derivative, the megadyne. The change would certainly conduce to clearness, but would, for a time at least, involve much inconvenience. The dyne equals $\frac{10000}{9805}$, or 1.0199 , times the weight of a milligramme at Paris; and the megadyne, 1.0199 times the weight of a kilogramme at the same place.
98. The Measurement of Mass. - This is usually effected by a process of "weighing" with some kind of balance, by means of which we ascertain directly that the "weight" of the body is the same as the weight of a certain number of the standard units in the same place, and thence infer that its mass is the same.

It may be done also, though in practice not very conveniently, by ascertaining what velocity is imparted to a body by the expenditure of a known amount of energy (see Appendix, Art. 496).

But it is obvious that neither of these methods could be used to measure the enormous mass of the earth, and we must look for some different process by which to ascertain the number of tons of matter it contains.

The end is accomplished by comparing the attraction which the earth exerts upon some body at its surface, with the attraction exerted upon the same body by a known mass at a known distance.
99. Gravitation. The Cause of "Weight." - Science cannot yet explain why bodies tend to fall towards the earth, and push or pull towards it when held from moving. But Newton discovered that the phenomenon is only a special case of the much more general fact which he inferred from the motions of the heavenly bodies, and formulated as "the law of gravitation," under the statement that any two particles of matter " attract" each other with a force which is proportional to their masses and inversely proportional to the square of the distance between them.

If instead of particles we have bodies composed of many particles, the total force between the bodies is the sum of the attractions of the different particles, each particle attracting every particle in the other body.

[^11]100. We must not imagine the word "attract" to mean too much. It merely states as a fact that there is a tendency for bodies to move toward each other, without including or implying any explanation of the fact.

Thus far no explanation has appeared which is less difficult to comprehend than the fact itself. Whether bodies are drawn together by some outside action, or pushed together, or whether they themselves can "act" across space with mathematical intelligence, - in what way it is that "attraction" comes about, is still unknown, and apparently as inscrutable as the very nature and constitution of an atom of matter itself. It is at present simply a fundamental fact, though it is not impossible that ultimately we may be able to show that it is a necessary consequence of the relation between particles of ordinary matter and the all-pervading "ether" to which we refer the phenomena of light, radiant heat, electricity, and magnetism (Physics, p. 267).
101. The Attraction of Spheres. - If the two attracting bodies are spheres, either homogeneous, or made up of concentric shells which are of equal density throughout, Newton showed that the action is precisely the same as if all the matter of each sphere were collected at its centre; and if the distance between the bodies is very great compared with their size, then, whatever their form, the same thing is very nearly true.

If the bodies are prevented from moving, the effect of attraction will be a stress or pull, to be measured in dynes or force-units (not mass-units), and is given by the equation

$$
F(\text { dynes })=G \times M_{1} \times M_{2} \div d^{2}
$$

where $M_{1}$ and $M_{2}$ are the masses of the two bodies expressed in mass-units (grammes), $d$ is the distance between their centres (in centimetres), and $G$ is a factor known as "The Constant of Gravitation," which equals 0.0000000666 according to the latest determination by Boys in 1894. $G$ will of course have a different numerical value if other units of mass and distance are used. ${ }^{1}$
${ }^{1}$ It will not do to write the formula $F=\frac{M_{1} \times M_{2}}{d^{2}}$ (omitting the $G$ ), unless the units are so chosen that the unit of force shall be equal to the
102. Acceleration by Gravitation. - If $M_{1}$ and $M_{2}$ are set free while under each other's attraction, they will at once begin to approach each other, and will finally meet at their common centre of gravity, having moved all the time with equal "momenta" (Physics, p. 29), but with velocities inversely proportional to their masses. At the end of the first second $M_{1}$ will have acquired a velocity of

$$
G^{1} \times \frac{M_{2}}{d^{2}}
$$

which, the student will observe, is entirely independent of $M_{1}$ itself: a grain of sand and a heavy rock fall at the same rate in free space under the attraction of the same body, at the same distance from it. (In the C. G. S. system $G^{1}$ is identical with $G$. In other systems of units it is usually numerically different.) Similarly $M_{2}$ will have acquired a velocity

$$
G^{1} \times \frac{M_{1}}{d^{2}} .
$$

The velocities with which the two bodies are approaching each other will be the sum of these velocities; and if we denote this "acceleration" (or the velocity of approach acquired in one second) by $f$, just as $g$ is used to denote the acceleration due to gravity in a second, we shall have

$$
f=G^{1}\left(\frac{M_{1}+M_{2}}{d^{2}}\right)
$$

This is the form of the law of gravitation which is most used in dealing with the motions of the heavenly bodies. The reader will notice that while the expression for $F$ (the force in dynes) has the product of the masses in its numerator, that for $f$ (the acceleration) has their sum.
103. We are now prepared to discuss the methods of measuring the earth's mass. It is only necessary, as has been
attraction between two masses each of one unit at a distance of one unit. It is not true that the attraction between two particles, each having a mass of one pound, at a distance of one foot, is equal to a stress of either one pound or one dyne.
already said (Art. 98), to compare the attraction which the earth exerts on a body, $X$, at its surface (at a distance, therefore, of 3959 miles from its centre) with the attraction exerted upon $X$ by some other body of a known mass at a known distance. The practical difficulty is that the attraction of any manageable body is so very small, compared with that of the earth, that the experiments are extremely delicate, and unless the mass is one of several tons, its attraction will be only a minute fraction of a grain of force, hard to detect and worse to measure.

The different methods which have been actually used for determining the mass of the earth are enumerated and discussed in the " General Astronomy," to which the student is referred. We limit ourselves to the presentation of a single one, which is perhaps the best, and is not difficult to understand.
104. The Earth's Mass and Density determined by the Torsion Balance. - This is an apparatus invented by Michell, but first employed by Cavendish, in 1798. A light rod carrying two small balls at its extremities is suspended at its centre by a fine, metallic wire, so that it will hang horizontally. If it be allowed to come to rest, and then a very slight deflecting force be applied, the rod will be pulled out of position by an amount depending on the stiffness and length of the wire as well as the intensity of the force. When the deflecting force is removed, the rod will vibrate back and forth until brought to rest by the resistance of the air. The "torsional coefficient," as it is called, i.e., the stress which will produce a twist of one revolution, can be accurately determined by observing the time of vibration, when the dimensions and weight of the rod and balls are known. (See Physics (Anthony \& Brackett), p. 117.) This will enable us to determine what fraction of a grain (of force) or of a dyne is necessary to produce a twist of any number of degrees.
105. If, now, two large balls, $A$ and $B$, Fig. 20, are brought near the smaller ones, as shown in the figure, a deflection will be produced by their attraction, and the small balls will move from $a$ and $b$ to $a^{\prime}$ and $b^{\prime}$. By shifting the large balls to the other side at $A^{\prime}$ and $B^{\prime}$ (which can be done by turning the frame upon which these balls are supported) we get an equal


Fig. 20. - Plan of the Torsion Balance. deflection in the opposite direction; that is, from $a^{\prime}$ and $b^{\prime}$ to $a^{\prime \prime}$ and $b^{\prime \prime}$, and the difference between the two positions assumed by the two small balls, that is $a^{\prime} a^{\prime \prime}$ and $b^{\prime} b^{\prime \prime}$ will be twice the deflection.

It is not necessary, nor even best, to wait for the balls to come to rest. When vibrating slightly we note the extremities of their swing. The middle point of the swing gives the place of rest, while the time occupied by the swing is the period of vibration, which we need in determining the coefficient of torsion. We must also measure the distances $A a^{\prime}, A^{\prime} b^{\prime \prime}, B b^{\prime}$, and $B^{\prime} a^{\prime \prime}$ between the centre of each of the large balls and the point of rest of the small ball when deflected.
106. Calculation of the Earth's Mass from the Experiment. - The earth's attraction on each of the small balls evidently equals the ball's weight. The attractive force of the large ball on the small one near it is found from the observed deflection. If, for instance, this deflection is $1^{\circ}$, and the coefficient of torsion is such that it takes one grain (of force) to twist the wire one whole turn, then the deflecting force, which we will call $f$, will be $\frac{1}{36} \overline{0}$ of a grain. One-half of this deflecting force will be due to $A$ 's attraction of $a$; half, to $B$ 's attraction of $b$. Call
the mass of the large ball $B$ and that of the small ball $b$, and let $d$ be the measured distance $B b^{\prime}$ between their centres. We shall then have the equation

$$
\begin{equation*}
\frac{1}{2} f=G \frac{B \times b}{d^{2}}, \text { or } B=\frac{1}{2} \frac{f d^{2}}{G b} . \tag{1}
\end{equation*}
$$

Similarly calling $E$ the mass of the earth, and $R$ its radius, $w$ being the weight of the small ball (which weight measures the force of the earth's attraction upon it), we shall have

$$
\begin{equation*}
w=G\left(\frac{E \times b}{R^{2}}\right), \text { or } E=\frac{w R^{2}}{G b} ; \tag{2}
\end{equation*}
$$

whence (dividing the second equation by the first),

$$
\frac{E}{B}=2\left(\frac{w}{f}\right)\left(\frac{R^{2}}{d^{2}}\right),
$$

which gives the mass of the earth in terms of $B$.
107. Density of the Earth - Having the mass of the earth, it is easy to find its density. The volume or bulk of the earth in cubic miles has already been given (Art. 95), and can be found in cubic feet by simply multiplying that number by the cube of 5280 . Since a cubic foot of water contains $62 \frac{1}{2}$ masspounds (nearly), the mass the earth would have, if composed of water, follows. Comparing this with the mass actually obtained, we get its density. A combination of the results of all the different methods hitherto employed, taking into account their relative accuracy, gives about 5.53 as the most probable value of the density of the earth according to our present knowledge.
108. In the earlier experiments by this method, the small balls were of lead, about two inches in diameter, at the extremities of a light wooden rod five or six feet long enclosed in a case with glass ends; and their position and vibration was observed by a telescope looking
directly at them from a distance of several feet. The attracting masses, $A$ and $B$, were balls (also of lead) about one foot in diameter.

Boys in 1894 used a most refined apparatus in which the small balls of gold, $\frac{1}{4}$ of an inch in diameter, were hung at the end of a beam only half an inch long which was suspended by a delicate fibre of quartz. The deflections due to the attraction of two sets of lead balls, respectively $2 \frac{1}{4}$ and $4 \frac{1}{2}$ inches in diameter, were measured by observing with a telescope the reflection of a scale in a little mirror attached to the beam. He obtained 5.527 for the earth's density. Earlier determinations by Cornu and Wilsing have given 5.56 and 5.58 respectively. A still later important determination by a different method, completed by Richarz at Berlin in 1896, gives 5.505 .

From Equation (1), page 65, $G=\frac{1}{2} f d^{2} \div b \times B$; so that it is determinable directly from the observations, without any reference to the density of the earth.

## 109. Constitution of the Earth's Interior. - Since the average

 density of the earth's crust does not exceed three times that of water, while the mean density of the whole earth is about 5.55 , it is obvious that at the earth's centre the density must be very much greater than at the surface, - very likely as high as eight or ten times the density of water, and equal to that of the heavier metals. There is nothing surprising in this. If the earth were once fluid, it is natural to suppose that in the process of solidification the densest materials would settle towards the centre.Whether the centre of the earth is solid or fluid, it is difficult to say with certainty. Certain tidal phenomena, to be mentioned hereafter, have led Lord Kelvin to express the opinion that the earth as a whole is solid throughout, and "more rigid than glass," volcanic centres being mere "pustules," so to speak, in the general mass. To this most geologists demur, maintaining that at the depth of not many hundred miles the materials of the earth must be fluid or at least semi-fluid. This is inferred from the phenomena of volcanoes, and from the fact that the temperature continually increases with the depth so far as we have yet been able to penetrate.

## CHAPTER IV.

THE APPARENT MOTION OF THE SUN, AND THE ORBITAL MOTION OF THE EARTH. - PRECESSION AND NUTATION. -ABERRATION. - THE EQUATION OF TIME. -THE SEASONS AND THE CALENDAR.
110. The Sun's Apparent Motion among the Stars. - The sun has an apparent motion among the stars which makes it describe the circuit of the heavens once a year, and must have been among the earliest recognized of astronomical phenomena, as it is obviously one of the most important.

As seen by us in the United States, the sun, starting in the spring, mounts higher in the sky each day at noon for three months, appears to stand still for a few days at the summer solstice, and then descends towards the south, reaching in the autumn the same noon-day elevation which it had in the spring. It keeps on its southward course to the winter solstice in December, and then returns to its original height at the end of a year, marking and causing the seasons by its course.

Nor is this all. The sun's motion is not merely a north and south motion, but it also advances continually ecastward among the stars. In the spring the stars, which at sunset are rising in the eastern horizon, are different from those which are found there in summer or winter. In March the most conspicuous of the eastern constellations at sunset are Leo and Boötes. A little later Virgo appears, in the summer Ophiuchus and Libra; still later Scorpio, while in midwinter Orion and Taurus are ascending as the sun goes down.
111. So far as the obvious appearances are concerned, it is quite indifferent whether we suppose the earth to revolve around the sun, or vice versa. That the earth really moves, is absolutely demonstrated however by two phenomena too minute and delicate for observation without the telescope, but accessible to modern methods. One of them is the aberration of light, the other the annual parallax of the fixed stars. These can be explained only by the actual motion of the earth. We reserve their discussion for the present.
112. The Ecliptic, its Related Points and Circles. - By observing daily with the meridian circle the sun's declination, and the difference between its right ascension and that of some standard star, we obtain a series of positions of the sun's centre which can be plotted on a globe, and we can thus mark out the path of the sun among the stars. It turns out to be a great circle, as is shown by its cutting the celestial equator at two points just $180^{\circ}$ apart (the so-called "equinoctial points" or "equinoxes," Art. 34), where it makes an angle with the equator of approximately $23 \frac{1}{2}^{\circ} \cdot{ }^{1}$ This great circle is called the Ecliptic, because, as was early discovered, eclipses happen only when the moon is crossing it. It may be defined as the circle in which the plane of the earth's orbit cuts the celestial sphere, just as the celestial equator is the trace of the plane of the terrestrial equator.

The angle which the ecliptic makes with the equator at the equinoctial points is called the Obliquity of the Ecliptic. This obliquity is evidently equal to the sun's maximum declination, or its greatest distance from the equator, reached in June and December.
113. The two points in the ecliptic midway between the equinoxes are called the Solstices, because at these points the
sun "stands," i.e., ceases to move in declination. Two circles drawn through the solstices parallel to the equator are called the Tropics, or "turning lines," because there the sun turns from its northward motion to a southward, or vice versa.

The two points in the heavens $90^{\circ}$ distant from the ecliptic are called the Poles of the Ecliptic.

The northern one is in the constellation Draco, about midway between the stars Delta and Zeta Draconis, and on the Solstitial Colure (the hour-circle which runs through the two solstices), at a distance from the pole of the heavens equal to the obliquity of the ecliptic, or about $23 \frac{1}{2}$. Great circles drawn through the poles of the ecliptic, and therefore perpendicular, or "secondaries," to the ecliptic, are known as Circles of Latitude. It will be remembered (Arts. 38 and 39) that celestial latitude and longitude are measured with reference to the ecliptic and not to the equator.
114. The Zodiac and its Signs. - A belt $16^{\circ}$ wide ( $8^{\circ}$ on each side of the ecliptic) is called the Zodiac, or "Zone of Animals," the constellations in it, excepting Libra, being all figures of animals. It is taken of that particular width simply because the moon and the principal planets always keep within it. It is divided into the so-called Signs, each $30^{\circ}$ in length, having the following names and symbols:-

| Spring | $\left\{\begin{array}{l} \text { Aries } \quad \downarrow \\ \text { Taurus ४ } \\ \text { Gemini } \square \end{array}\right.$ | Autumn | $\left\{\begin{array}{lc} \text { Libra } & \bumpeq \\ \text { Scorpio } & \eta \\ \text { Sagittarius } & \uparrow \end{array}\right.$ |
| :---: | :---: | :---: | :---: |
| Summer | $\begin{cases}\text { Cancer } & \varpi \\ \text { Leo } & \Omega \\ \text { Virgo } & m\end{cases}$ | Winter | $\begin{cases}\text { Capricornus } & \text { vs } \\ \text { Aquarius } \\ \text { Pisces } & \neq x\end{cases}$ |

The symbols are for the most part conventionalized pictures of the objects. The symbol for Aquarius is the Egyptian character for water. The origin of the signs for Leo, Capricornus, and Virgo is not quite clear.

The zodiac is of extreme antiquity. In the zodiacs of the earliest history the Lion, Bull, Ram, and Scorpion appear precisely as now.
115. The Earth's Orbit. - The ecliptic is not the orbit of the earth, and must not be confounded with it. It is simply a great circle of the infinite celestial sphere, the trace made upon that sphere by the plane of the earth's orbit, as was stated in its definition. The fact that the ecliptic is a great circle gives us no information about the earth's orbit itself, except that it all lies in one plane passing through the sun. It tells us nothing as to the orbit's real form and size.
By reducing the observations of the sun's right ascension and declination through the year to longitude and latitude (the latitude would always be exactly zero except for some slight perturbations), and combining these data with observations of the sun's ap-


Fig. 21.
Determination of the Form of the Earth's Orbit. parent diameter, we can, however, ascertain the form of the earth's orbit and the law of its motion in this orbit. The size of the orbit-its scale of miles - cannot be fixed until we find the sun's distance.

## 116. To find the Form

 of the Orbit, we proceed thus: Take a point, $S$, for the sun, and draw through it a line, $S q{ }^{\rho}$ (Fig. 21), directed towards the vernal equinox, from which longitudes are measured. Lay off from $S$ lines indefinite in length, making angles with $S q$ equal to the earth's longitude as seen from the sun ${ }^{1}$ on each of the dayswhen observations were made. We shall thus get a sort of "spider," showing the direction of the earth as seen from the sun on each of those days.

Next as to the distances. While the apparent diameter of the sun does not tell us its absolute distance from the earth, unless we know this diameter in miles, yet the changes in the apparent diameter do inform us as to the relative distance at different times, the distance being inversely proportional to the sun's apparent diameter (Art. 12). If then on this "spider" we lay off distances equal to the quotient obtained by dividing some constant, say $10000^{\prime \prime}$, by the sun's apparent diameter at each date, these distances will be proportional to the true distance of the earth from the sun, and the curve joining the points thus obtained will be a true map of the earth's orbit, though without any scale of miles. When the operation is performed, we find that the orbit is an ellipse of small "eccentricity" (about $\frac{1}{60}$ ), with the sun not in the centre, but at one of the two foci.
117. Definitions relating to the Orbital Ellipse. - The ellipse is a curve such that the sum of the two distances from any point on its circumference to two points within, called the foci, is always constant and equal to the so-called major axis of the ellipse.

In Fig. 22, $S P+P F$ equals $A A^{\prime}, A A^{\prime}$ being the major axis. $A C$ is the semi-major axis, and is usually denoted by $A$ or $a$. $B C$ is the semi-minor axis, denoted by $B$ or $b$; the eccentricity, denoted by $e$, is the fraction or ratio $\frac{S C}{A C}$ or $\frac{c}{a}$, and is usually expressed as a decimal. ( $C S$ is $c$ ).

If a cone is cut across obliquely


Fig. 22. - The Ellipse. by a plane, the section is an ellipse, which is therefore called one of the "Conic Sections" (see Appendix, Art. 506).

The points where the earth is nearest to and most remote from the sun are called respectively the Perihelion and the Aphelion, the line joining them being the major-axis of the orbit. This line indefinitely produced in both directions is called the Line of Apsides, the major-axis being a limited piece of it. A line drawn from the sun to the earth or any other planet at any point in its orbit, as $S P$ in the figure, is called the planet's Radius vector, and the angle $A S P$, reckoned from the perihelion point, $A$, in the direction of the planet's motion towards $B$, is called its Anomaly.

The variations in the sun's diameter are too small to be detected without a telescope, so that the ancients failed to perceive them. Hipparchus, however, about 120 в.c., discovered that the earth is not in the centre of the circular ${ }^{1}$ orbit which he supposed the sun to describe with uniform velocity. Obviously the sun's apparent motion is not uniform, because it takes 186 days for the sun to pass from the vernal equinox to the autumnal, and only 179 days to return. Hipparchus explained this want of uniformity by the hypothesis that the earth is out of the centre of the circle.


Fig. 23. - Equable Description of Areas.
118. To find the Law of the Earth's Motion. - By comparing the measured apparent diameter with the differences of longitude from day to day we can deduce not only the form of the orbit but the "law" of the eartl's motion in it. On arranging the daily motions and apparent diameters in a table, we find that the daily motions vary directly as the squares of the diameters. From this

[^12]it can be shown to follow that the earth moves in such a way that its radius vector describes areas proportional to the times, a law which Kepler first brought to light in 1609 . That is to say, if $\alpha b, c d$, and ef, Fig. 23, be portions of the orbit described by the earth in different weeks, the areas of the elliptical sectors $a S b, c S d$, and $e S f$ are all equal. A planet near perihelion moves faster than at aphelion in just such proportion as to preserve this relation.
119. Kepler's Problem. - As Kepler left the matter, this is a mere fact of observation. Newton afterwards demonstrated that it is a necessary mechanical consequence of the fact that the earth moves under the action of a force always directed towards the sun (see Appendix, Art. 502). It is true in every case of elliptical motion, and enables us to find the position of the earth, or any planet, at any time when we once know the time of its orbital revolution (technically the "period") and the time when it was at perihelion. Thus, the angle $A S P$, Fig. 24, or the anomaly of the planet, must be such that the shaded area of the elliptical sector $A S P$ will be that portion


Fig. 2t. - Kepler's Problem. of the whole ellipse which is represented by the fraction $\frac{t}{T}, t$ being the number of days since the planet last passed perihelion, and $T$ the number of days in the whole period.

If, for instance, the earth last passed perihelion on Dec. 31st (which it did), its place on May 1st must be such that the sector $A S P$ will be 121 $\frac{121}{365_{4}^{1}}$ of the whole of the earth's orbit, since it is 121 days from Dec. 31st to May 1st. The solution of this problem, known as Kepler's Problem, leads to "transcendental" equations, and can be found in books on Physical Astronomy.
120. Changes in the Earth's Orbit. - The orbit of the earth changes slowly in form and position, though it is unchangeable (in the long run) as regards the length of its major axis and the duration of the year.
(1) Change in the Obliquity of the Ecliptic. The ecliptic slightly and very slowly shifts its position among the stars, thus altering their latitudes and the angle between the ecliptic and the equator. The obliquity is at present about $24^{\prime}$ less than it was 2000 years ago, and is still decreasing about $0^{\prime \prime} .5$ a year. It is computed that this diminution will continue for some 15,000 years, reducing the obliquity to about $22_{4}^{1 \circ}$, when it will begin to increase. The whole change can never exceed $1 \frac{1}{2}{ }^{\circ}$ on each side of the mean.
(2) Change of Eccentricity. At present the eccentricity of the earth's orbit (which is now 0.0168 ) is also slowly diminishing. According to Leverrier, it will continue to decrease for about 24,000 years until it becomes 0.003 and the orbit is almost circular. Then it will increase again for some 40,000 years until it becomes 0.02 . In this way the eccentricity will oscillate backwards and forwards, always, however, remaining between zero and 0.07 ; but the successive oscillations of both the eccentricity and obliquity are unequal in amount and in time, so that they cannot properly be compared to the "vibrations of a mighty pendulum," which is rather a favorite figure of speech in certain quarters.
(3) Revolution of the Apsides of the Earth's Orbit. The line of apsides of the orbit (which now stretches in both directions towards the opposite constellations of Sagittarius and Gemini) is also slowly and steadily moving eastward at a rate which would carry it around the circle in about 108,000 years. But the rate will vary.

These so-called "secular" changes are due to "perturbations" caused by the action of the other planets upon the earth. Were it not for their attraction, the earth would keep her orbit with reference to the sun strictly unaltered from age to age, except that possibly in the course of millions of years the effects of falling meteoric matter and of the attraction of the nearer fixed stars might become perceptible.
121. Besides these secular perturbations of the earth's Orbit, the earth itself is continually being slightly disturbed in its orbit. On account of its connection with the moon, it oscillates each month a
few hundred miles above and below the true plane of the ecliptic, and by the action of the other planets it is sometimes set forward or backward in its orbit to the extent of some thousands of miles. Of course every such displacement produces a corresponding slight change in the apparent position of the sun.
122. Precession of the Equinoxes. - The length of the year was found in two ways by the ancients: -

1st. By observing the time when the shadow cast at noon by a "gnomon" is longest or shortest: this determines the date of the solstice.

2d. By observing the position of the sun with reference to the constellations - their "heliacal" rising and setting, - i.e., the times when given constellations rise and set at sunset.

Comparing the results of observations made by these two methods at long intervals, Hipparchus in the second century B.C. found that they do not agree, the year reckoned from solstice to solstice or from equinox to equinox being about twenty minutes shorter than the year reckoned with reference to the constellations. The equinox moves westward on the ecliptic about $50^{\prime \prime} .2$ each year, as if advancing to meet the sun at each annual return. He therefore called this motion of the equinoxes "Precession."

On examining the latitudes of the stars, we find them to have changed but slightly in the last 2000 years. We know therefore that the ecliptic maintains its position sensibly unaltered. The right ascensions and declinations of the stars, on the other hand, are found to be both constantly changing, and this makes it certain that the celestial equator shifts its position. On account of the change in the place of the equinox, the longitudes of the stars grow uniformly larger, having increased nearly $30^{\circ}$ in the last 2000 years.
123. Motion of the Pole of the Heavens around the Pole of the Ecliptic. - The obliquity of the ecliptic, which equals the angular distance of the pole of the heavens from the pole of the ecliptic, is not sensibly affected by precession. That is to
say, - as the earth travels around its orbit in the plane of the ecliptic (just as if that plane were the level surface of a sheet of water in which the earth swims half immersed), its axis, $A C X$ (Fig. 25), always preserves the same constant angle of $23 \frac{1}{2}^{\circ}$ with the perpendicular $S C T$ which


Fig. 25. points to the pole of the ecliptic. But in consequence of precession, the axis while keeping its inclination unchanged, shifts conically around the line $S C T$ (like the axis of a spinning top before it becomes steady), taking up successively the positions $A^{\prime} C$, etc., thus carrying the equinox from $V$ to $V^{\prime}$, and so on.
In consequence of this shift of the axis, the pole of the heavens, i.e., that point in the sky to which the line $C A$ happens to be directed at any time, describes a circle around the pole of the ecliptic in a period of about 25,800 years $\left(360^{\circ} \div 50.2^{\prime \prime}\right)$. The pole of the ecliptic remains practically fixed among the stars, but the pole of the equator has moved many degrees since the earliest observations. At present the Pole-star (Alpha Ursæ Minoris) is about $11_{4}^{\circ}$ from the pole, while in the time of Hipparchus the distance was fully $12^{\circ}$; during the next century it will diminish to about $30^{\prime}$, and then it will increase again.

If upon a celestial globe we take the pole of the ecliptic as a centre and describe around it a circle with a radius of $23 \frac{1}{2}$, it will mark the
track of the celestial pole among the stars. It passes pretty near the star Vega (Alpha Lyræ), on the opposite side of the circle from the present Pole-star; so that, about 12,000 years hence, Vega will be the Pole-star, - a splendid one.

Reckoning backwards, we find that about 4000 years ago Alpha Draconis was the Pole-star, and about $3 \frac{1}{2}^{\circ}$ from the pole.

Another effect of precession is that the signs of the zodiac do not now agree with the constellations which bear the same name. The sign of Aries is now in the constellation of Pisces, and so on; each sign having "backed" bodily, so to speak, into the constellation west of it.
N. B. This precessional motion of the celestial pole must not be confounded with the motion of the terrestrial pole which causes variation of latitude (Art. 71*).
124. Physical Cause of Precession. - The physical cause of this slow conical rotation of the earth's axis around the pole of the ecliptic was first explained by Newton, and lies in the two facts that the earth is not exactly spherical, and that the sun and moon ${ }^{1}$ act upon the equatorial "ring" of matter which projects above the true sphere, tending to draw the plane of the equator into coincidence with the plane of the ecliptic by their greater attraction on the nearer portions of the "ring."

If it were not for the earth's rotation, this action of the sun and moon would bring the two planes of the ecliptic and the equator into coincidence; but since the earth is spinning on its axis, we get the same result that we do with the whirling wheel of a gyroscope, by hanging a weight at one end of the axis. We then have a combination of two rotations at right angles to each other, one the whirl

[^13]of the wheel, the other the "tip" which the weight tends to give the axis. Compared with the mass of the earth and its energy of rotation this disturbing force is very slight, and consequently the rate of precession extremely slow. Our space does not permit a discussion of the manner in which the forces operate to produce the peculiar result. For this, as also for an account of the so-called Equation of the Equinoxes and Nutation, the reader is referred to higher text-books.
125. Aberration. - The fact that light is not transmitted instantaneously, but with a finite velocity, causes the apparent displacement of an object viewed from any moving station, unless the motion is directly towards or from that object. If the motion of the observer is slow, this displacement or "aberration" is insensible; but the earth moves so swiftly ( $18 \frac{1}{2}$ miles per second) that it is easily observable in the case of the stars. Astronomical aberration may be defined, therefore, as the apparent displacement of a heavenly body due to the combina. tion of the orbital motion of the earth with that of light. The direction in which we have to point our telescope in observing a star is not the same as if we were at rest.


Fig. 26. - Aberration of a Raindrop.

We may illustrate this by considering what would happen in the case of falling raindrops observed by a person in motion. Suppose the observer standing with a tube in his hand while the drops are falling vertically: if he wishes to have the drops descend axially through the tube without touching the sides, he must obviously keep it vertical so long as he stands still; but if he advances in any direction the drops will strike his face and he must draw back the bottom of the tube (Fig. 26) by an amount which equals the advance he makes while a drop is falling through it; i.e., he must
incline the tube forward at an angle, ${ }^{1} a$, depending both upon the velocity of the rain-drop and the velocity of his own motion, so that when the drop, which entered the tube at $B$, reaches $A^{\prime}$, the bottom of the tube will be there also.

It is true that this illustration is not a demonstration, because light does not consist of particles coming towards us, but of waves transmitted through the ether of space. But it has been shown (though the proof is by no means elementary) that within very narrow limits, the apparent direction of a wave is affected in precisely the same way as that of a moving projectile.
126. The Effect of Aberration on the Place of a Star. - The velocity of light being 186,330 miles per second (according to the latest experiments of Newcomb and Michelson) while that of the earth in its orbit is 18.5 miles, we find that a star, situated on a line at right angles to the direction of the earth's motion, is apparently displaced by an angle which equals

$$
206,265^{\prime \prime} \times \frac{18.5}{186,330}, \text { or } 20^{\prime \prime} .5
$$

(The Astronomical Congress of 1896 adopted the value, $20^{\prime \prime} .47$.)
This is the so-called "Constant of Aberration."
If the star is in a different part of the sky its displacement will be less, the amount being easily calculated when the star's position is given.

A star at the pole of the ecliptic being permanently in a direction perpendicular to the earth's motion, will always be displaced by the same amount of $20^{\prime \prime} .5$, but in a direction continually changing. It must therefore appear to describe during the year a little circle, $41^{\prime \prime}$ in diameter.

A star on the ecliptic appears simply to oscillate back and forth in a straight line 41 " long. In general, the "aberrational orbit" is an ellipse, having its major axis parallel to the ecliptic and always $41^{\prime \prime}$ long, while its minor axis depends upon the star's latitude.
${ }^{1}$ Tang $\alpha=\frac{u}{V}$, or (when $\alpha$ is small) $\alpha=206265^{\prime \prime} \frac{u}{V}$, where $u$ is the veloc ity of the obseryer and $V$ that of the drop.
127. Determination of the Sun's Distance by means of the Aberration of Light. - Since (foot-note to Art. 125)

$$
a^{\prime \prime}=206265 \frac{u}{V}, \quad u=\frac{a^{\prime \prime}}{206265} V
$$

When, therefore, we have ascertained the value of $a^{\prime \prime}\left(20^{\prime \prime} .47\right)$ from observations of the stars, and of $V(186,330$ miles $)$ by physical experiments, we can immediately find $u$, the velocity of the earth in her orbit. The circumference of the earth's orbit is then found by multiplying this velocity, $u$, by the number of seconds in a sidereal year (Art. 133), and from this we get the radius of the orbit, or the earth's mean distance from the sun, by dividing the circumference by $2 \pi$. Using the values above given, the mean distance of the sun comes out $\mathbf{9 2 , 8 7 7}, 000$ miles. But the uncertainty of $a^{\prime \prime}$ is probably as much as $0^{\prime \prime} .03$, and this affects the distance proportionally, say one part in 600 , or 150,000 miles. Still the method is one of the very best of all that we possess for determining in miles the value of "the Astronomical Unit." See Appendix, Chap. XV.

## CONSEQUENCES OF THE EARTH'S ORBITAL MOTION.

128. Solar Time and the Equation of Time. - Since the sun makes the circuit of the heavens in a year, moving always towards the east, the solar day, or the interval between the two successive transits of the sun across any observer's meridian, is longer than that between two tiansits of a given star, or twenty-four sidereal hours. The difference must amount to exactly one day in a year ; i.e., while in a year there are $366 \frac{1}{4}$ (nearly) sidereal days, there are only $365 \frac{1}{4}$ solar days. The average daily difference is therefore a little less than $4^{\mathrm{m}}$.

Moreover, the sun's advance in night ascension between two successive noons varies materially, so that the apparent solar days are not all of the same length. Accordingly, as explained
in Arts. 54 and 55, mean time has been adopted, which is kept by a fictitious or mean sun, moving uniformly in the equator at the same average rate as that of the real sun in the ecliptic. The hour-angle of this mean sun is the local mean time, or clock time, while the hour-angle of the real sun is the apparent or sun-dial time.

The "equation of time" is the difference between these two times, reckoned as plus when the sun-dial is slower than the clock and minus when it is faster, i.e., it is the "correction" which must be added (algebraically) to apparent time in order to get mean time. ${ }^{1}$

The principal causes of this difference are two.

1. The variable motion of the sun in the ecliptic due to the eccentricity of the earth's orbit.
2. The obliquity of the ecliptic.

For an explanation of the manner in which these causes operate, see Appendix, Arts. 497-499.

The two causes mentioned are, however, only the principal ones. Every perturbation suffered by the earth comes in to modify the result; but all the other causes combined never affect the equation of time by more than a very few seconds.

The equation of time becomes zero four times a year, viz., about April 15th, June 14th, Sept. 1st, and Dec. 24th. The maxima are Feb. 11th, $+14^{\mathrm{m}} 32^{\mathrm{s}}$; May 14th, $-3^{\mathrm{m}} 55^{\mathrm{s}}$; July 26th, $+6^{\mathrm{m}} 12^{\mathrm{s}}$; and Nov. 2d, $-16^{\mathrm{m}} 18^{\mathrm{s}}$; but the dates and amounts vary slightly from year to year.
129. The Seasons. - The earth in its annual motion keeps its axis always nearly parallel to itself, for the mechanical reason that a spinning body maintains the direction of its axis

[^14]invariable, unless disturbed by extraneous force (very prettily illustrated by the gyroscope). On March 20th, the earth is so situated that the plane of its equator passes through the sun. At that time, therefore, the circle which bounds the illuminated portion of the earth passes through the two poles, and day and night are everywhere equal, as implied by the term "equinox." The same is again true on the 22d of September. About the 21st of June, the earth is so situated that its north pole is inclined towards the sun by about $23 \frac{1}{2}^{\circ}$. The south pole is then in the obscure half of the earth's globe, while the north pole receives sunlight all day long; and in all portions of the northern hemisphere the day is longer than the night, the difference depending upon the latitude of the place: in the southern hemisphere, on the other hand, the days are shorter than the nights. At the time of the winter solstice these conditions are, of course, reversed, and the southern pole has the perpetual sunshine.

At the equator of the earth the day and night are equal at all times of the year, and in that part of the earth there are no seasons in the proper sense of the word.

## 130. Diurnal Phenomena near the Pole. The Midnight Sun.

- At the north pole of the earth, where the celestial pole is in the zenith and the diurnal circles are parallel with the horizon (Art. 42), the sun will maintain the same elevation all day long, except for the slight change caused by its motion in declination during 24 hours. The sun will appear on the horizon at the date of the vernal equinox (in fact, about two days before it , on account of refraction), and will slowly wind upwards in the sky until it reaches its maximum elevation of $23 \frac{1}{2}$ degrees on June 21st. Then it will retrace its course until two or three days after the autumnal equinox, when it sinks out of sight.

At points between the north pole and the polar circle the sun will appear above the horizon earlier in the year than March 20th, and will rise and set daily until its declination becomes equal to the observer's distance from the pole. It will then make a complete circuit of the heavens daily, never setting again until it reaches the same decli-
nation in its southward course, after passing the solstice. From that time it will again rise and set daily until it reaches a southern declination just equal to the observer's polar distance. Then the long night begins, and continues until the sun, having passed the southern solstice, returns again to the same declination at which it made its appearance in the preceding spring.

At the polar circle itself, or, more strictly speaking, owing to refraction, about $\frac{30}{4}$ south of it, the "midnight sun" will be seen on just one day in the year - the day of the summer solstice.
131. Effects on Temperature. - The changes in the duration of "insolation" (exposure to sunshine) at any place involve changes of temperature and of other climatic conditions, thus producing the Seasons. Taking as a standard the amount of heat received in twenty-four hours on the day of the equinox, it is clear that the surface of the soil at any place in the northern hemisphere will receive daily from the sun more than this average amount of heat whenever he is north of the celestial equator; and for two reasons:-

1. Sunshine lasts more than half the day.
2. The mean altitude of the sun during the day is greater than at the time of the equinox, since he is higher at noon and in any case reaches the horizon at rising and setting. Now the more obliquely the rays strike, the less heat they bring to each square inch of the surface, as is obvious from Fig. 27. A beam of sunshine having a cross section, $A B C D$, when it strikes the surface at an angle, $h$, (equal to the sun's altitude)


Fig. 27.
Effect of Sun's Elevation on Amount of Heat imparted to the Soil. is spread over a much larger surface, $A c$, than when it strikes perpendicularly. This difference in favor of the more nearly vertical rays is exaggerated by the absorption of heat in the atmosphere, because rays that
are nearly horizontal have to traverse a much greater thickness of air before reaching the ground.

For these two reasons, therefore, the temperature rises rapidly at a place in the northern hemisphere as the sun comes north of the equator.
132. Time of Highest Temperature. - We, of course, receive the most heat in twenty-four hours at the time of the summer solstice; but this is not the hottest time of summer for the obvious reason that the weather is then getting hotter, and the maximum will not be reached until the increase ceases ; i.e., not until the amount of heat lost in twenty-four hours equals that received. The maximum is reached in our latitude about the 1st of August. For similar reasons the minimum temperature of winter occurs about Feb. 1st.

Since, however, the weather is not entirely "made on the spot where it is used," but is much influenced by winds and currents that come from great distances, the actual time of the maximum temperature at any particular place cannot be determined by mere astronomical considerations, but varies considerably from year to year.
133. The Three Kinds of Year. - Three different kinds of "year" are now recognized, - the Sidereal, the Tropical (or Equinoctial), and the Anomalistic.

The sidereal year, as its name implies, is the time occupied by the sun in apparently completing the circuit from a given star to the same star again. Its length is 365 days, 6 hours, 9 minutes, 9 seconds.

From the mechanical point of view, this is the true.year; i.e., it is the time occupied by the earth in completing its revolution around the sun from a given direction in space to the same direction again.

The tropical year is the time included between two successive passages of the vernal equinox by the sun. Since the equinox moves yearly, $50^{\prime \prime} .2$ towards the west (Art. 122), this
tropical year is shorter than the sidereal by about 20 minutes, its length being 365 days, 5 hours, 48 minutes, 46 seconds.

Since the seasons depend on the sun's place with respect to the equinox, the tropical year is the year of chronology and civil reckoning.

The third kind of year is the anomalistic year, the time between two successive passages of the perihelion by the earth. Since the line of apsides of the earth's orbit makes an eastward revolution once in about 108,000 years (Art. 120), this kind of year is nearly 5 minutes longer than the sidereal, its length being 365 days, 6 hours, 13 minutes, 48 seconds.

It is but little used, except in calculations relating to perturbations.
134. The Calendar. - The natural units of time are the day, the month, and the year. The day is too short for convenience in dealing with considerable periods, such as the life of a man, for instance, and the same is true even of the month, so that for all chronological purposes the tropical year (the year of the seasons) has always been employed. At the same time, so many religious ideas and observations have been connected with the change of the moon, that there has been a constant struggle to reconcile the month with the year. Since, however, the two are incommensurable, no really satisfactory solution is possible, and the modern calendar of civilized nations entirely disregards the lunar phases.

In the earliest times the calendar was in the hands of the priesthood and was predominantly lunar, the seasons being either disregarded or kept roughly in place by the occasional intercalation or the -dropping of a month. The Mohammedans still use a purely lunar calendar, having a "year" of 12 months, containing alternately 354 and 355 days. In their reckoning the seasons fall continually in dif. ferent months, and their calendar gains on ours about one year in thirty-three.
135. The Metonic Cycle and Golden Number. - Meton, a Greek astronomer, about 433 b.c., discovered that a period of 235 months is very nearly equal to 19 years of $365 \frac{1}{2}$ days each, the difference being hardly more than two hours. It follows that every 19th year the new moons recur on the same days of the month; so that, as far as the moon's phases are concerned, the almanacs of 1880 and 1899, for instance, would agree (but the way in which the intervening leap years come in may make a difference of one day).

The golden number of the year is its number in this Metonic cycle. It is found by adding 1 to the "date number" of the year and dividing by 19 : the remainder is the golden number, unless it comes out zero, in which case 19 itself is taken. Thus the golden number of 1890 is found by dividing 1891 by 19 ; the remainder, 10 , is the golden number of the year. This number is still employed in the ecclesiastical calendar for finding the date of Easter.
136. The Julian Calendar. - When Julius Cæsar came into power he found the Roman Calendar in a state of hopeless confusion. He, therefore, sought the advice of the astronomer Sosigenes, and in accordance with his suggestions established (в.c. 45) what is known as the Julian Calendar, which still, either untouched or with a trifling modification, continues in use among all civilized nations. Sosigenes discarded all consideration of the moon's phases, and adopting $365 \frac{1}{4}$ days as the true length of the year, he ordained that every fourth year should contain 366 days, the extra day being inserted by repeating the sixth day before the Calends of March, whence such a year is called "Bissextile." He also transferred the beginning of the year, which before Cæsar's time had been in March (as indicated by the names of several of the months, December, the tenth month, for instance) to January 1st.

Cæsar also took possession of the month Quintilis, naming it July. after himself. His successor, Augustus, in a similar manner appropriated the next month, Sextilis, calling it August, and to vindicate his dignity and make his month as long as his predecessor's, he added to it a day filched from February.

The Julian Calendar is still used unmodified in the Greek Church, and also in many astronomical reckonings.
137. The Gregorian Calendar. - The true length of the tropical year is not $365 \frac{1}{4}$ days, but 365 days, 5 hours, 48 minutes, 46 seconds, leaving a difference of 11 minutes and 14 seconds by which the Julian year is too long. This difference amounts to a little more than three days in 400 years. As a consequence, the date of the vernal equinox comes continually earlier and earlier in the Julian calendar, and in 1582 it had fallen back to the 11th of March instead of occurring on the 21 st , as it did at the time of the Council of Nice, A.d. 325. Pope Gregory, therefore, under the astronomical advice of Clavius, ordered that the calendar should be restored by adding ten days, so that the day following Oct. 4th, 1582, should be called the 15th instead of the 5th; further, to prevent any future displacement of the equinox, he decreed that thereafter only such "century years" should be leap years as are divisible by 400. (Thus $1700,1800,1900,2100$, and so forth, are not leap years, but 1600 and 2000 are.)
138. The change was immediately adopted by all Catholic countries, but the Greek Church and most Protestant nations refused to recognize the Pope's authority. The new calendar was, however, at last adopted in England by an act of Parliament passed in 1751. It provided that the year 1752 should begin on Jan. 1st (instead of March 25th, as had long been the rule in England), and that the day following Sept. $2 \mathrm{~d}, 1752$, should be reckoned as the 14th instead of the 3 d , thus dropping 11 days. At present (since 1800 and 1900 were leap years in the Julian calendar and not in the Gregorian) the difference between the two calendars is 13 days. Thus, in Russia the 23d of June is reckoned the 10 th; but in that country both dates are ordinarily used for scientific purposes, so that the date mentioned would be written June $\frac{10}{23}$. When Alaska was annexed to the United States, the official dates had to be changed by only eleven days, one day being provided for by the alteration from the Asiatic date to the American (Art. 66).

## CHAPTER V.

THE MOON. - HER ORBITAL MOTION AND THE MONTH. DISTANCE, DIMENSIONS, MASS, DENSITY, AND FORCE OF GRAVITY. - ROTATION AND LIBRATIONS. - PHASES. LIGHT AND HEAT. - PHYSICAL CONDITION. - TELESCOPIC ASPECT AND PECULIARITIES OF THE LUNAR SURFACE.
139. Next to the sun, the moon is the most conspicuous and to us the most important of the heavenly bodies: in fact, she is the only one except the sun, which exerts the slightest influence upon the interests of human life. If the stars and the planets were all extinguished, our eyes would miss them, and that is all. But if the moon were annihilated, the interests of commerce would be seriously affected by the practical cessation of the tides. She owes her conspicuousness and her importance, however, solely to her nearness, for she is really a very insignificant body as compared with stars and planets.
140. The Moon's Apparent Motion, Definition of Terms, etc. - One of the earliest observed of astronomical phenomena must have been the eastward motion of the moon with reference to the sun and stars, and the accompanying change of phase. If, for instance, we note the moon to-night as very near some conspicuous star, we shall find her to-morrow night at a point considerably farther east, and the next night farther yet; she changes her place about $13^{\circ}$ daily and makes a complete circuit of the heavens, from star to star again, in about $27 \frac{1}{3}$ days. In other words, she revolves around the earth in
that time, while she accompanies us in our annual journey around the sun.

Since the moon moves eastward among the stars so much faster than the sun (which takes a year in going once around), she overtakes and passes him at regular intervals; and as her phases depend upon her apparent position with reference to the sun, this interval from new moon to new moon is specially noticeable and is what we ordinarily understand as the "month."
The angular distance of the moon east or west of the sun at any time is called her "Elongation." ${ }^{1}$ At new moon it is zero, and the moon is said to be in "Conjunction." At full moon the elongation is $180^{\circ}$, and she is said to be in "Opposition." In either case the moon is in "Syzygy"; i.e., the sun, moon, and earth are arranged along a straight line. When the elongation is $90^{\circ}$ she is said to be in "Quadrature."
141. Sidereal and Synodic Months. -The sidereal month is the time it takes the moon to make her revolution from a given star to the same star again. It averages 27 days, 7 hours, 43 minutes, 11.55 seconds, or 27.32166 days, but varies some 3 hours on account of "perturbations." The mean daily motion is $360^{\circ} \div 27.32166$, or $13^{\circ} 11^{\prime}$. Mechanically considered, the sidereal month is the true one.

The synodic month is the time between two successive conjunctions or oppositions ; i.e., between successive new or full moons. Its average value is 29 days, 12 hours, 44 minutes, 2.864 seconds, but it varies $13^{\mathrm{h}}$, mainly on account of the eccentricity of the lunar orbit. As has been said already, this synodic month is what we ordinarily understand by the term "month."

[^15]If $M$ be the length of the moon's sidereal period, $E$ the length of the sidereal year, and $S$ that of the synodic month, the three quantities are connected by a simple relation which is easily demonstrated.
$\frac{1}{M}$ is the fraction of a circumference moved over by the moon in a day. Similarly $\frac{1}{E}$ is the apparent daily motion of the sun. The difference is the amount which the moon gains on the sun daily. Now it gains a whole revolution in one synodic month of $S$ days, and therefore must gain daily $\frac{1}{S}$ of a circumference. Hence we have the important equation

$$
\frac{1}{M}-\frac{1}{E}=\frac{1}{S}
$$

Another way of looking at the matter, leading, of course, to the same result is this:- In a sidereal year the number of sidereal months must be just one greater than the number of synodic months: the numbers are respectively $13.369+$ and $12.369+$.
142. The Moon's Path among the Stars. - By observing the moon's right ascension and declination daily with the meridian circle or other suitable instruments, we can map out its apparent path, just as in the case of the sun (Art. 112). This path turns out to be (very nearly) a great circle, inclined to the ecliptic at an angle of about $5^{\circ} 8^{\prime}$. The two points where it cuts the ecliptic are called the Nodes, the ascending node being the one where the moon passes from the south side to the north side of the ecliptic, while the opposite node is called the descending node.

The moon at the end of the month never comes back exactly to the point of beginning among the stars, on account of the socalled "perturbations," due mostly to the attraction of the sun. One of the most important of these perturbations is the "regression of the nodes." These slide westward on the ecliptic just as the vernal equinox does (precession), but much faster, completing their circuit in about 19 years instead of 26,000 .

When the ascending node of the moon's orbit coincides with the vernal equinox, the angle between the moon's path and the celestial
equator is $23^{\circ} 28^{\prime}+5^{\circ} 8^{\prime}$, or $28^{\circ} 36^{\prime}$; $9 \frac{1}{2}$ years later, when the descending node has come to the same point, the angle is only $23^{\circ} 28^{\prime}-5^{\circ} 8^{\prime}$, or $18^{\circ} 20^{\prime}$. In the first case the moon's declination will range during the month from $+28^{\circ} 36^{\prime}$ to $-28^{\circ} 36^{\prime}$, which makes a difference of more than $57^{\circ}$ in its meridian altitude. In the second case the whole range is reduced to $36^{\circ} 40^{\prime}$.
143. Interval between the Moon's Successive Transits; Daily Retardation of its Rising and Setting.-Owing to the eastward motion of the moon, it comes to the meridian later each day by about $51^{\mathrm{m}}$ on the average; but the retardation ranges all the way from 38 minutes to 66 minutes, on account of the variations in the rate of the moon's motion in right ascension. These variations are due to the oval form of its orbit and to its inclination to the celestial equator, and are precisely analogous to those of the sun's motion, which produce "the equation of time" (Art. 128) ; but they are many times greater.

The average retardation of the moon's daily rising and setting is also, of course, the same $51^{\mathrm{m}}$, but the actual retardation is still more variable than that of the transits, depending, as it does, to some extent on the latitude of the observer as well as on the variations in the moon's motion. At New York the range is from 23 minutes to 1 hour and 17 minutes. In higher latitudes it is still greater.

In latitudes above $61^{\circ} 30^{\prime}$ the moon, when it has its greatest possible declination of $28^{\circ} 36^{\prime}$ (Art. 142), will become circumpolar for a certain time each month, and will remain visible without setting at all (like the "midnight sun ") for a greater or less number of days, according to the latitude of the observer.
144. Harvest and Hunter's Moon. - The full moon that occurs nearest the autumnal equinox is known as the harvest moon, the one next following as the hunter's moon. At that time of the year the moon while nearly full rises for several consecutive nights nearly at the same hour, so that the moonlight evenings last for an unusually long time. The phenome-
non, however, is much more striking in Northern Europe than in the United States.

In September the full moon is near the vernal equinox, where its path is least inclined to the eastern horizon, so that the retardation of its rising is small. Fig. 28 represents a celestial globe looked at from the east. $H N$ is the horizon, $E$ the east point, $P$ the pole, and $E Q$ the equator. If, now, the first of Aries is rising at $E$, the line $J E J^{\prime}$ will be the ecliptic and will be inclined to the horizon at an


Fig. 28. - Explanation of the Harvest Moon.
angle less than QEH by $23 \frac{1}{2}^{\circ}$, which is the inclination of the equator. If, on the other hand, the first of Libra is rising, the ecliptic will be the line $D E D^{\prime}$. If the ascending node of the moon's orbit happens to coincide with the first of Aries, then, when this node is rising, the moon's path will lie still more nearly horizontal than $J J^{\prime}$, as shown by the dotted line $M E M^{\prime}$.
145. Form of the Moon's Orbit. - By observation of the moon's apparent diameter in connection with observations of her place in the sky, we can determine the form of her orbit around the earth in the same way that the form of the earth's orbit around the sun was worked out in Art. 116. The moon's apparent diameter ranges from $33^{\prime} 33^{\prime \prime}$ when as near as possible, to $29^{\prime} 24^{\prime \prime}$ when most remote. (Neison.)

The orbit turns out to be an ellipse like that of the earth around the sun, but one of much greater eccentricity, averag-
ing about $\frac{1}{18}$ (as against $\frac{1}{60}$ ). We say "averaging" because it varies from $\frac{1}{15}$ to $\frac{1}{2} \mathrm{r}$ on account of perturbations.

The point of the moon's orbit nearest the earth is called the perigee, that most remote, the apogee, and the indefinite line passing through these points, the line of apsides, while the major axis is that portion of this line which lies between the perigee and apogee. This line of apsides is in continual motion on account of perturbations (just as the line of nodes is - Art. 142); but it moves eastward instead of westward, completing its revolution in about nine years.

In her motion around the earth the moon also observes the same law of equal areas that the earth does in her orbit around the sun.

## THE MOON'S DISTANCE.

146. In the case of any heavenly body one of the first and most fundamental inquiries relates to its distance from us: until the distance has been somehow measured we can get no knowledge of the real dimensions of its orbit, nor of the size, mass, etc., of the body itself. The problem is usually solved by measuring the apparent "parallactic" displacement of the body due to a known change in the position of the observer. Before proceeding farther we must therefore briefly discuss the subject of parallax.
147. Parallax. - In gen r nal the word "parallax" means the difference between the $d^{\circ}$ ections of a heavenly body as seen by the observer and as seen from some standard point of reference. The "annual" or "heliocentric" parallax of a star is the difference of the star's direction as seen from the earth and from the sun. The "diurnal" or "geocentric" parallax of the sun, moon, or a planet, is the difference of its direction as seen from the centre of the earth and from the observer's station on the earth's surface, or what comes to the same thing, it is the angle at the body made by the two lines drawn
from it, one to the observer, the other to the centre of the earth. In Fig. 29 the parallax of the body, $P$, is the angle $O P C$, which equals $x O P$, and is the difference between $Z O P$ and $Z C P$. Obviously this parallax is zero for a body directly overhead at $Z$, and a maximum for a body just rising at $P_{h}$ Moreover, and this is to be specially noted, this parallax of a body at the horizon " the horizontal parallax" - is simply the angular semi-diameter of the earth as seen from the body. When we say that the moon's horizontal parallax is $57^{\prime}$, it is equivalent to saying that seen from the moon the earth appears to have a diameter of $114^{\prime}$.
148. Relation between Parallax and Distance. - When the horizontal parallax of any heavenly body is ascertained, its distance follows at once through our knowledge of the earth's dimensions. From Art. 12 we have the equation

$$
r=R\left(\frac{s^{\prime \prime}}{206265}\right)
$$

in which $r$ is the earth's radius, $R$ the distance of the body, and $s^{\prime \prime}$ the apparent semi-diameter of the earth (in seconds of arc) as seen from the body; i.e., $s^{\prime \prime}=$ the body's "horizontal parallax." If, as is usual, we write $p^{\prime \prime}$ instead of $s^{\prime \prime}$ for the horizontal parallax of the body, this gives

$$
R=r\left(\frac{206265}{p^{\prime \prime}}\right)
$$

This implies, of course, that a body whose horizontal parallax is $1^{\prime \prime}$ is at a distance 206,265 times the earth's radius; if the parallax is $10^{\prime \prime}$ it is only $\frac{1}{10}$ as far away, and so on.

Since the radius of the earth varies slightly in different latitudes, we take the equatorial radius as a standard, and the equatorial horizontal parallax is the earth's equatorial semi-diameter as seen from the body. It is this which is usually meant when we speak simply of "the parallax" of the moon, of the sun, or of a planet; (but never when we speak of the parallax of a star.)
149. Method of Determining the Moon's Parallax and Distance. - We limit ourselves to giving a single one, perhaps the simplest, of the different methods that are practically available. At each of two observatories, $B$ and $C$, Fig. 30, on, or very nearly on, the same meridian and very far apart (Santiago, and Cambridge, U.S., for instance), the moon's zenith distance, $Z B M$ and $Z^{\prime} C M$, is observed simultaneously with the meridian circle or some equivalent instrument. This gives in the quadrilateral $B O C M$ the two angles $O B M$ and $O C M$, each of which is the supplement of the moon's geocentric zenith distance at $B$ and $C$ respectively. The angle $B O C$, at the centre of the earth, is the difference of the geocentric latitudes of the two observatories (numeri-


Fig. 30. - Determination of the Moon's Parallax. cally, their sum).

Moreover, the sides $B O$ and $C O$ are known, being radii of the earth. The quadrilateral can, therefore, be solved by a simple trigonometrical process, ${ }^{1}$ and we can find the line $M O$. Knowing $M O$ and $O K$, the radius of the earth, the horizontal parallax, $O M K$, follows at once.

[^16]150. Parallax, Distance, and Velocity of the Moon. - The moon's equatorial horizontal parallax is found to average $3422^{\prime \prime} .0$ ( $57^{\prime} 2^{\prime \prime} .0$ ), according to Neison, but varies considerably on account of the eccentricity of the orbit. With this value of the parallax we find that the moon's average distance from the earth is about 60.3 times the earth's equatorial radius, or 238,840 miles, with an uncertainty of perhaps 20 miles.

The maximum and minimum values of the moon's distance are given by Neison as 252,972 and 221,614 . It will be noted that the average distance is not the mean of the two extreme distances.

Knowing the size and form of the moon's orbit, the velocity of her motion is easily computed. It averages 2287 miles an hour, or about 3350 feet per second. Her apparent angular velocity among the stars is about $33^{\prime}$ an hour on the average, which is just a little greater than the apparent diameter of the moon itself.


Fig. 31. - Moon's Path relative to the Suu.


Fig. 32.


Fig. 33.

Erroneous Representations of the Moon's Path.
151. Form of the Moon's Orbit with Reference to the Sun. While the moon moves in a small oval orbit around the earth, it also moves around the sun in company with the earth. This
common motion of the moon and earth, of course, does not affect their relative motion, but to an observer outside the system, the moon's motion around the earth would be only a very small component of the moon's whole motion as seen by him.
The distance of the moon from the earth is only about $\frac{1}{400}$ part of the distance of the sun. The speed of the earth in its orbit around the sun is also more than thirty times greater than that of the moon in its orbit around the earth; for the moon, therefore, the resulting path in space is one which is always concuve towards the sun, as shown in Fig. 31, and not like Figs. 32 and 33.

If we represent the orbit of the earth by a circle having a radius of 100 inches ( 8 feet, 4 inches), the moon would deviate from it by only one-quarter of an inch on each side, crossing it 25 times in one revolution, i.e., in a year.
152. Diameter, Area, and Bulk of the Moon. - The mean apparent diameter of the moon is $31^{\prime} 7^{\prime \prime}$. Knowing its distance, we easily compute from this by the formula of Art. 12 its real diameter, which comes out 2163 miles. This is 0.273 of the earth's diameter.

Since the surfaces of globes vary as the squares of their diameters, and their volumes as the cubes, this makes the surface area of the moon equal to about $\frac{1}{14}$ of the earth's, and the volume (or bulk) almost exactly $\frac{1}{49}$ of the earth's.

No other satellite is nearly as large as the moon in comparison with its primary planet. The earth and moon together, as seen from a distance, are really in many respects more like a double planet than like a planet and satellite of ordinary proportions. At a time, for instance, when Venus happens to be nearest the earth (at a distance of about twenty-five millions of miles) her inhabitants would see the earth about twice as brilliant as Venus herself at her best appears to us, and the moon would be about as bright as Sirius, oscillating backwards and forwards about half a degree each side of the earth.

## 153. Mass, Density, and Superficial Gravity of the Moon. -

 Her mass is about $\frac{1}{81.5}$ of the earth's mass, (0.0123).The accurate determination of the moon's mass is practically an extremely difficult problem. Though she is the nearest of all the heavenly bodies, it is far more difficult to "weigh" her than to weigh Neptune, the remotest of the planets. For the different methods of dealing with the problem, we must refer the reader to the "General Astronomy" (Art. 243), merely saying that one of the methods is by comparing the relative influences of the moon and of the sun in raising the tides.

Since the density of a body is equal to $\frac{\text { Mass }}{\text { Volume }}$, the density of the moon as compared with that of the earth is found to be 0.613 , or about 3.4 the density of water (the earth's density being 5.58). This is a little above the average density of the rocks which compose the crust of the earth. This small density of the moon is not surprising nor at all inconsistent with the belief that it once formed a part of the same mass with the earth, since if such were the case the moon was probably formed by the separation of the outer portions of that mass, which would be likely to have a smaller specific gravity than the rest.

The superficial gravity, or the attraction of the moon for bodies at its surface, is about one-sixth that at the surface of the earth. That is, a body which weighs six pounds on the earth's surface would at the surface of the moon weigh only one pound (by a spring balance). This is a fact that must be borne in mind in connection with the enormous scale of the surface structure of the moon. Volcanic forces on the moon would throw ejected materials to a vastly greater distance than on the earth.
154. Rotation of the Moon. - The moon rotates on its axis once a month, in exaclly the same time as that occupied by its revolution around the earth; its day and night are, therefore,
each about a fortnight in length, and in the long run it. keeps the same side always towards the earth: we see to-day precisely the same aspect of the moon as Galileo did when he first looked at it with his telescope, and the same will continue to be the case for thousands of years, if not forever.

It is difficult for some to see why a motion of this sort should be considered a rotation of the moon, since it is essentially like the motion of a ball carried on a revolving crank (Fig. 34). "Such a ball," they say, "revolves around the shaft, but does not rotate on its own axis." It does rotate, however : if we mark one side of the ball, we shall find the marked side presented successively to every point of the compass


Fig. 34. as the crank turns, so that the ball turns on its own axis as really as if it were whirling upon a pin fastened to the table.

By virtue of its connection with the crank, the ball has two distinct motions, (1) the motion of translation, which carries its centre in a circle around the axis of the shaft; (2) an additional motion of rotation ${ }^{1}$ around a line drawn through its centre of gravity parallel to the shaft. But the $\operatorname{pin} A$ (in the figure) and the hole in which the pin fits, both also turn at the same rate, so that the ball does not turn on the pin; nor the pin, in the hole.
155. Librations. - While in the "long run" the moon keeps the same face towards the earth, it is not so in the "short run": there is no crank-connection between them. With reference to the centre of the earth the moon is continually oscillating a little, and these oscillations constitute what are called librations, of which we distinguish three ; - viz., the

[^17]libration in latitude, the libration in longitude, and the diurnal libraiion.

The libration in latitude is due to the fact that the moon's equator does not coincide with the plane of its orbit, but makes with it an angle of about $6 \frac{1}{2}{ }^{\circ}$. This inclination of the moon's equator causes its north pole at one time in the month to be tipped a little towards the earth, while a fortnight later the south pole is similarly inclined towards us.

Moreover, since the moon's angular motion in its oval orbit is variable, while the motion of rotation is uniform like that of any other ball, the two motions do not keep pace exactly during the month, and we see alternately a few degrees around the eastern and western edges of the lunar globe. This is the libration in longitude, and amounts to about $7 \frac{3}{4}{ }^{\circ}$.

Then again when the moon is rising we look over its upper, which is then its western edge, seeing a little more of that part of the moon than if we were observing it from the centre of the earth. When it is setting we overlook in the same way its eastern edge. This constitutes the so-called diurnal libration, and amounts to about $1^{\circ}$. Strictly speaking, this diurnal libration is not a libration of the moon at all, but of the observer. The effect is the same, however, as that of a true libration.

Altogether, owing to librations, we see considerably more than half the moon's surface at one time or another. About 41 per cent of it is always visible, 41 per cent never visible, and a belt at the edge of the moon covering about 18 per cent is rendered alternately visible and invisible by the librations.
156. The Phases of the Moon. - Since the moon is an opaque body shining merely by reflected light, we can see only that hemisphere of her surface which happens to be illuminated, and of this hemisphere only that portion which happens to be turned towards the earth. When the moon is between the earth and the sun (at new moon) the dark side is then presented directly towards us, and the moon is entirely invisible. A week later, at the end of the first quarter, half of the
illuminated hemisphere is visible, and we have the half moon just as we do a week after the full. Between the new moon and the half moon, during the first and last quarters of the lunation, we see less than half of the illuminated portion, and then have the "crescent" phase. Between half moon and the full moon, during the second and third quarters of the lunation, we see more than half of the moon's illuminated side, and have then what is called the "gibbous" phase.


Fig. 35. - The Moon's Phases.

Fig. 35 (in which the light is supposed to come from a point far above the circle which represents the moon's orbit) shows the way in which the phases are distributed through the month.
157. The line which separates the dark portion of the disc from the bright is called the "terminator," and is always a semi-ellipse, since it is a semi-circle viewed obliquely. The illuminated portion of the moon's disc is, therefore, always a figure which is made up of a semicircle plus or minus a semi-ellipse, ${ }^{1}$ as shown in Fig. 36 A. It is sometimes incorrectly attempted to represent the crescent form by a construction like $36 B$, in which a smaller circle has a portion cut out of it by an arc of a larger one.

It is to be noticed also that $a b$, the line


Fig. 36. which joins the "cusps," or points of the crescent, is always perpendicular to a line drawn from the moon to the sun, so that the horns are always turned away from the sun. The precise position, therefore, in which they will stand at any time is perfectly predictable, and has nothing whatever to do with the weather. Artists are sometimes careless in the manner in which they introduce the moon into landscapes. One occasionally sees the moon near the horizon with the horns turned downwards, a piece of perspective fit to go with Hogarth's barrel, which showed both its heads at once.
158. Earth-Shine on the Moon. - Near the time of new moon the whole disc is easily visible, the portion on which sunlight does not fall being illuminated by a pale reddish light. This light is earth-shine, the earth as seen from the moon being then nearly full.

Seen from the moon, the earth would show all the phases that the moon does, the earth's phase being in every case exactly supplementary to that of the moon as seen by us at the time. Taking everything into account, the earth-shine by which the moon is illuminated near new moon is probably from 15 to 20 times as strong as the light of the full moon. The ruddy color is due to the fact that the light sent to the moon from the earth has passed twice through our atmosphere, and so has acquired the sunset tinge.

[^18]
## PHYSICAL CHARACTERISTICS OF THE MOON.

159. The Moon's Atmosphere. - The moon's atmosphere, if any exists, is extremely rare, probably not producing at the moon's surface a barometric pressure to exceed $\frac{1}{25}$ of an inch of mercury, or $\frac{1}{750}$ of the atmospheric pressure at the earth's surface. The evidence on this point is twofold: First, the telescopic appearance.

The parts of the moon near the edge of the disc or "limb" which, if there were any atmosphere, would be seen through its greatest possible depth, are visible without the least distortion. There is no haze, and all the shadows are perfectly black ; there is no sensible twilight at the cusps of the moon, and no evidence of clouds or storms, or of anything like the ordinary phenomena of the terrestrial atmosphere.

Second, the absence of refraction, when the moon intervenes between us and any more distant object.

At an eclipse of the sun there is no distortion of the sun's limb where the moon cuts it. When the moon "occults" a star, there is no distortion or discoloration of the star disc, but both the disappearance and reappearance are practically instantaneous. Moreover, an atmosphere of even slight density, quite insufficient to produce any sensible distortion of the image, would notably diminish the time during which the star would be concealed behind the moon, since the refraction would bend the rays from the star around the edge of the moon so as to render it visible both after it had really passed behind the limb and before it emerged from it.
160. Water on the Moon's Surface. - Of course, if there is no atmosphere there can be no liquid water, since the water would immediately evaporate and form an atmosphere of vapor if no air were present. It is not impossible, however, nor perhaps improbable that solid water, i.e., ice and snow, may exist on the moon's surface at a temperature too low to liberate vapòr in quantity sufficient to make an atmosphere dense enough to be observable from the earth.

## 161. What has become of the Moon's Air and Water? -

If the moon ever formed a part of the same mass as the earth, she must once have had both air and water. There are a number of possible, and more or less probable, hypotheses to account for their disappearance. (1) The air and water may have struck in, - partly absorbed by porous rocks, and partly disposed of in cavities left by volcanic action; partly also, perhaps, by chemical combinations and occlusion when the internal temperature became low enough. (2) The atmosphere may have flown away; - and this is perhaps the most probable hypothesis. If the "kinetic" theory of gases is true, no body of small mass, not extremely cold, can permanently retain any considerable atmosphere. A particle leaving the moon with a speed exceeding a mile and a half a second would never return. If she was ever warm, the molecules of her atmosphere must have been continually acquiring velocities greater than this, and deserting her one by one. See Physics, pages 231, 232.

In whatever way, however, it came about, it is quite certain that at present no substances that are gaseous or vaporous at low temperatures exist in any considerable quantity on the moon's surface, - at least not on our side of it.
162. The Moon's Light. - As to quality, it is simply sunlight, showing a spectrum identical in every detail with that of light coming directly from the sun itself; and this may be noted incidentally as an evidence of the absence of a lunar atmosphere, which, if it existed, would produce peculiar lines of its own in the spectrum.

The brightness of full moonlight as compared with sunlight is about $\frac{1}{600000}$ : according to this, if the whole visible hemisphere were packed with full moons, we should receive from it about one-eighth part of the light of the sun.

Moonlight is not easy to measure, and different experimenters have found results for the ratio between the light of the full moon and
 laston). The value now generally accepted is that determined by Zöllner, viz., $\frac{{ }^{\frac{1}{1}}{ }^{8} 000}{}$.

The half moon does not give, even nearly, half as much light as the full moon: near the full the brightness suddenly and greatly increases, probably because at any time except at the full moon, the moon's visible surface is more or less darkened by shadows.
The average albedo or reflecting power of the moon's surface Zöllner states as 0.174 ; i.e., the moon's surface reflects a little more than $\frac{1}{6}$ part of the light that falls upon it.

This corresponds to the reflecting power of a rather light-colored sandstone, and agrees well with the estimate of Sir John Herschel, who found the moon to be very exactly of the same brightness as the rock of Table Mountain when she was setting behind it. There are, however, great differences in the brightness of the different portions of the moon's surface. Some spots are nearly as white as snow or salt, and others as dark as slate.
163. Heat of the Moon. - For a long time it was impossible to detect the moon's heat by observation. Even when concentrated by a large lens, it is too feeble to be shown by the most delicate thermometer. The first sensible evidence of it was obtained by Melloni in 1846, with the newly invented "thermopile," by a series of observations from the summit of Vesuvius.

With modern apparatus it is easy enough to perceive the heat of lunar radiation, but the measurements are extremely difficult. A considerable percentage of the lunar heat seems to be heat simply reflected like light, while the rest, perhaps three-quarters of the whole, is "obscure heat"; i.e., heat which has first been absorbed by the moon's surface and then radiated, like the heat from a brick surface that has been warmed by sunshine. This is shown by the fact that a comparatively thin plate of glass cuts off some 86 per cent of the moon's heat.

The total amount of heat radiated by the full moon to the earth is estimated by Lord Rosse at about one eighty thousandth part of that sent us by the sun; but this estimate is probably too high : Prof. C. C. Hutchins in 1888 found it $\frac{1}{185000}$.
164. Temperature of the Moon's Surface. - As to the temperature of the moon's surface, it is difficult to affirm much with certainty. On the one hand the lunar rocks are exposed to the sun's rays in a cloudless sky for 14 days at a time, so that if they were protected by air like the rocks upon the earth they would certainly become intensely heated. During the long lunar night of 14 days, the temperature must inevitably fall appallingly low, perhaps $200^{\circ}$ below zero.

Lord Rosse's observations, some forty years ago, appeared to show that soon after the full the moon's surface reached a temperature above that of boiling water. This is confirmed by the conclusive investigations of Very (1899) so far as regards the moon's equator at points where the sun is nearly overhead. At the poles, on the other hand, and at the terminator, the temperature is very low; indeed, the average temperature of the moon's earthward surface is probably far below the freezing point of water, as indicated by Langley's older observations.

Lord Rosse has also found that during a total eclipse of the moon her heat-radiation practically vanishes, and does not regain its normal value until some hours after she has left the earth's shadow. This seems to indicate that she loses heat nearly as fast as it is received, and so can never get very warm.
165. Lunar Influences on the Earth. - The moon's attraction co-operates with that of the sun in producing the tides, to be considered later.

There are also certain distinctly ascertained disturbances of terrestrial magnetism connected with the approach and recession of the moon at perigee and apogee; and this ends the chapter of ascertained lunar influences.

The multitude of current beliefs as to the controlling influence of the moon's phases and changes upon the weather and the various conditions of life are mostly unfounded.

It is quite certain that if the moon has any influence at all of the sort imagined, it is extremely slight; so slight that it has not yet been demonstrated, though numerous investigations have been made ex-
pressly for the purpose of detecting it. We have never been able to ascertain with certainty, for instance, whether it is warmer or not, or less cloudy or not, at the time of full moon. Different investigations lead to contradictory results.
166. The Moon's Telescopic Appearance and Surface. - Even to the naked eye the moon is a beautiful object, diversified with markings which are associated with numerous popular superstitions. To a powerful telescope these markings mostly vanish, and are replaced by a countless multitude of smaller details which make the moon, on the whole, the finest of all telescopic objects, - especially so to instruments of a moderate size (say from six to ten inches in diameter) which generally give a more pleasing view of our satellite than instruments either much larger or much smaller.

An instrument of this size, with magnifying powers between 250 and 500 , virtually brings the moon within a distance ranging from 1000 to 500 miles, and since an object half a mile in diameter on the moon subtends an angle of about $0^{\prime \prime} .43$, it would be distinctly visible. A long line or streak even less than a quarter of a mile across can probably be seen. With larger telescopes the power can now and then be carried at least twice as high, and correspondingly smaller details made out, when the air is at its best.

For most purposes the best time to look at the moon is when it is between six and ten days old. At the time of full moon few objects on the surface are well seen.

It is evident that while with the telescope we should be able to see such objects as lakes, rivers, forests, and great cities, if they existed on the moon, it would be hopeless to expect to distinguish any of the minor indications of life, such as buildings or roads.
167. The Moon's Surface Structure. - The moon's surface for the most part is extremely broken. With us the mountains are mostly in long ranges, like the Andes and Himalayas. On the moon, the ranges are few in number; but, on the other hand, the surface is pitted all over with great "craters," which
resemble very closely the volcanic craters on the earth's surface, though on an immensely greater scale. The largest terrestrial craters do not exceed six or seven miles in diameter; many of those on the moon are fifty or sixty miles across, and some have a diameter of more than 100 miles, while smaller ones from five to twenty miles in diameter are counted by the hundred.

The normal lunar crater (Fig. 37) is nearly circular, surrounded by a ring of mountains which rise anywhere from a thousand to twenty thousand feet above the surrounding coun-


Fig. 37. - A Normal Lunar Crater (Nasmyth). try. The floor within the ring may be either above or below the outside level; some craters are deep, and some are filled nearly to the brim. In a few cases the surrounding mountain ring is entirely absent, and the crater is a mere hole in the plain. Frequently in the centre of the crater there rises a group of peaks, which attain about the same elevation as the encircling ring, and these central peaks often show holes or minute craters in their summits.

On some portions of the moon these craters stand very thickly; older craters have been encroached upon or more or less completely obliterated by the newer, so that the whole surface is a chaos of which the counterpart is hardly to be found on the earth, even in the roughest portions of the Alps. This is especially the case near the moon's south pole. It is noticeable, also, that, as on the earth the youngest mountains are generally the highest, so on the moon the more newly formed craters are generally deeper and more precipitous than the older.

The height of a lunar mountain or depth of a crater can be measured with considerable accuracy by means of its shadow; or in the case of a mountain, by the measured distance between its summit and the "terminator" (Art. 157), at the time when the top first catches the light and looks like a star quite detached from the bright part of the moon, as seen in Fig. 38.
168. The striking resemblance of these formations to terrestrial volcanic structures, like those exemplified by Vesuvius and others, makes it natural to assume that they had a similar origin. This, however, is not absolutely certain, for there are considerable difficul-


Fig. 38. - Photograph by Ritchey at Yerkes Observatory, 1900. ties in the way, especially in the case of what are called the great "Bulwark Plains." These are so extensive that a person standing in the centre could not even see the summit of the surrounding ring at any point; and yet there is no line of demarcation between them and the smaller craters, - the series is continuous. Moreover, on the earth, volcanoes necessarily require the action of air and water, which do not at present exist on the moon. It is obvious, therefore, that if these lunar craters are the result of volcanic eruptions, they must be, so to speak, "fossil" formations, for it is quite certain that there are no obvious appearances of present volcanic activity, though perhaps some suspicious indications.
169. Other Lunar Formations. - The craters and mountains are not the only interesting formations on the moon's surface.

There are many deep, narrow, crooked valleys that go by the name of "rills," some of which may once have been watercourses. Fig. 39 shows several of them. Then there are numerous straight "clefts," half a mile or so wide and of unknown depth, running in some cases several hundred miles, straight through mountain and valley, without any apparent regard for the accidents of the surface; they seem to be deep cracks in the crust of our satellite. Most curious of all are the light-colored streaks or " rays," which radiate from certain of the craters, extending in some cases a distance of many hundred miles. These are usually from five to ten miles wide, and neither elevated or depressed to any considerable extent with reference to the general surface. Like the clefts, they pass across valley and mountain, and sometimes through craters, without any change in width or color. No thoroughly satisfactory explanation has ever been given, though they have been ascribed to a staining of the surface by vapors ascending from rifts too narrow to be visible.

The most remarkable of these "ray systems" is the one connected with the great crater Tycho, not very far from the moon's south pole. The rays are not very conspicuous until within a few days of full moon, but at that time they and the crater from which they diverge constitute by far the most striking feature of the whole lunar surface.
170. Lunar Maps. - A number of maps of the moon have been constructed by different observers. The most recent and extensive is that by Schmidt of Athens, on a scale 7 feet in diameter: it was published by the Prussian government in 1878. Of the smaller maps available for ordinary lunar observation, perhaps the best is that given in Webb's "Celestial Objects for Common Telescopes." Two new photographic, large-scale, lunar maps have lately been published, from negatives made at the Lick and Paris observatories.
171. Lunar Nomenclature. - The great plains upon the moon's surface were called by Galileo "oceans" or "seas" (Maria), for he supposed that these grayish surfaces, which are visible to the naked eye and conspicuous in a small telescope, though not with a large one, were covered with water.

The ten mountain ranges on the moon are mostly named after terrestrial mountains, as Caucasus, Alps, Apennines, though two or three bear the names of astronomers, like Leibnitz, Doerfel, etc.

The conspicuous craters bear the names of eminent ancient and mediæval astronomers and philosophers, as Plato, Archimedes, Tycho, Copernicus, Kepler, and Gassendi ; while hundreds of smaller and less conspicuous formations bear the names of more modern astronomers.

This system of nomenclature seems to have originated with Riccioli, who made a map of the moon in 1651.
172. Changes on the Moon. - It is certain that there are no conspicuous changes, - there are no such transformations as would be presented by the earth viewed telescopically, - no clouds, no storms, no snow of winter, and no spread of vegetation in the spring. At the same time, it is confidently maintained by some observers that here and there alterations do take place in the details of the lunar surface, while others as stoutly dispute it.

The difficulty in settling the question arises from the great changes in the appearance of a lunar object under varying illumination. To insure certainty in such delicate observations, comparisons must be made between the appearance of the object in question, as seen at precisely the same phase of the moon, with telescopes (and eyes too) of equal power, and under substantially the same conditions in other
respects, such as the height of the moon above the horizon, and the clearness and steadiness of the air. It is, of course, very difficult to secure such identity of conditions. (For an account of certain supposed changes, see Webb's "Celestial Objects.")
173. Fig. 40 is reduced from a skeleton map of the moon by Neison, and though not large enough to exhibit much detail, will enable a student with a small telescope to identify the principal objects by the help of the key.

KEY TO THE PRINCIPAL OBJECTS INDICATED IN FIG. 40.

| A. Mare Humorum. | K. Mare Nubium. |
| :--- | :--- |
| B. Mare Nectaris. | L. Mare Frigoris. |
| C. Oeeanus Procellarum. | T. Leibnitz Mountains. |
| D. Mare Fecunditatis. | U. Doerfel Mountains. |
| E. Mare Tranquilitatis. | V. Rook Mountains. |
| F. Mare Crisium. | W. D'Alembert Mountains. |
| G. Mare Serenitatis. | X. Apennines. |
| H. Mare Imbrium. | Y. Caucasus. |
| I. Sinus Iridum. | Z. Alps. |

1. Clavius.
2. Schiller.
3. Maginus.
4. Schickard.
5. Tycho.
6. Walther.
7. Purbach.
8. Petavius.
9. "The Railway."
10. Arzachel.
11. Gassendi.
12. Catherina.
13. Cyrillus.
14. Alphonsus.
15. Theophilus.
16. Ptolemy.
17. Langrenus.
18. Hipparchus.
19. Grimaldi.
20. Flamsteed.
21. Messier.
22. Maskelyne.
23. Triesnecker.
24. Kepler.
25. Copernicus.
26. Stadius.
27. Eratosthenes.
28. Proclus.

28'. Pliny.
29. Aristarchus.
30. Herodotus.
31. Archimedes.
32. Cleomedes.
33. Aristillus.
34. Eudoxus.
35. Plato.
36. Aristotle.
37. Endymion.
174. Lunar Photography. - It is probable that the question of changes upon the moon's surface will soon be authoritatively decided by means of photography. The earliest success in lunar photography was that of Bond of Cambriadge (U.S.), in 1850, using the old

Daguerreotype process. This was followed by the work of De la Rue in England, and by Dr. Henry Draper and Mr. Rutherford in this country. Until very recently, Mr. Rutherfurd's pictures have remained absolutely unrivalled; but within the last few years, plates which


Fig. 40. - Map of the Moon, reduced from Neison.
have been taken at Cambridge, U. S., and at the Lick and Yerkes Observatories, as well as by Mr. Common in England and the Henry brothers at Paris, are far in advance of even the best of Rutherfurd's.

## CHAPTER VI.

THE SUN. - ITS DISTANCE, DIMENSIONS, MASS, AND DENSITY. - ITS ROTATION AND EQUATORIAL ACCELERATION. - METHODS OF STUDYING ITS SURFACE. SUN SPOTS. - THEIR NATURE, DIMENSIONS, DEVELOPMENT, AND MOTIONS. - THEIR DISTRIBUTION AND PERIODICITY. - SUN-SPOT THEORIES.

The sun is the nearest of the stars; a hot, self-luminous globe, enormous as compared with the earth and moon, though probably only of medium size among its peers; but to the earth and the other planets which circle around it, it is the grandest and most important of all the heavenly bodies. Its attraction controls their motions, and its rays supply the energy which maintains every form of activity upon their surfaces.
175. The Sun's Distance. - Its distance may be determined by finding its horizontal parallax (Art. 147); i.e., the semidiameter of the earth as seen from the sun. The mean value of this parallax is very near $8^{\prime \prime} .80$, with a probable error certainly less than $\pm 0^{\prime \prime} .01$. The distance may also be ascertained by measuring experimentally the velocity of light, and combining this with the so-called "Constant of Aberration" (Art. 127), or with the time required by light to travel from the sun to the earth, as deduced from the observation of the eclipses of Jupiter's satellites (Art. 355).

We reserve for the Appendix the discussion of the principal methods by which the parallax has been determined.

Taking the horizontal parallax at $8^{\prime \prime} .8$, the mean distance of the sun ( $a$ being the earth's equatorial radius) equals

$$
a \times \frac{206265}{8^{\prime \prime} .8}=23439 \times a . \quad \text { (See Art. 148.) }
$$

With Clark's value of $a$ (Art. 84), this gives 149,500,000 kilometres, or $92,897,000$ miles, which, however, is uncertain by at least 50,000 miles. The distance is variable, also, to the extent of about $3,000,000$ miles, on account of the eccentricity of the earth's orbit, the earth being nearer the sun in December than in June.

Knowing the distance of the sun, the orbital velocity of the earth is easily found by dividing the circumference of the orbit by the number of seconds in a sidereal year. It comes out 18.495 miles per second. (Compare this with the velocity of a cannon-ball - seldom exceeding 2000 feet per second.)

This distance is so much greater than any with which we have to do on the earth, that it is impossible to reach a conception of it except by illustrations. Perhaps the simplest is that drawn from the motion of a railway train, which, going a thousand miles a day (nearly 42 miles an hour without stops), would take $254 \frac{1}{3}$ years to make the journey. If sound were transmitted through interplanetary space, and at the same rate as through our own atmosphere, it would make the passage in about 14 years ; i.e., an explosion on the sun would be heard by us 14 years after it actually occurred. Light traverses the distance in 499 seconds.
176. Dimensions of the Sun. - The sun's mean apparent diameter is $32^{\prime} 4^{\prime \prime} \pm 2^{\prime \prime}$. Since at the distance of the sun, one second equals 450.36 miles, its diameter ${ }^{1}$ is 866,500 miles, or $109 \frac{1}{2}$ times that of the earth.

If we suppose the sun to be hollowed out, and the earth placed at the centre, the sun's surface would be 433,000 miles

[^19]away. Now, since the distance of the moon is about 239,000 miles, she would be only a little more than half-way out from the earth to the inner surface of the hollow globe, which would thus form a very good background for the study of the lunar motions. Fig. 41 illustrates the size of the sun and of such


Fig. 41. - Dimensions of the Sun compared with the Moon's Orbit.
objects upon it as the sun spots and prominences, as compared with the size of the earth and of the moon's orbit.

If we represent the sun by a globe two feet in diameter, the earth on that scale would be 0.22 of an inch in diameter, the size of a very small pea. Its distance from the sun would be just about 220 feet, and the nearest star, still on the same scale, would be 8000 miles away at the antipodes.

As a help to the memory, it is worth noticing that the sun's diameter exceeds the earth's just as many times as it is itself exceeded by the radius of the earth's orbit. Its diameter is nearly 110 times that of the earth, and it is also, roughly, the 110th part of its distance from us.

Since the surfaces of globes are proportional to the squares of their radii, the surface of the sun exceeds that of the earth in the ratio of $109.5^{2}: 1$; i.e., the area of its surface is about 12,000 times the surface of the earth.

The volumes of spheres are proportional to the cubes of their radii. Hence, the sun's volume or bulk is $109.5^{3}$, or 1,300000 times that of the earth.
177. The Sun's Mass. - The mass of the sun is very nearly 332,000 times that of the earth. There are various ways of getting at this result. Perhaps for our purpose the most convenient is by comparing the earth's attraction for bodies at her surface (i.e., the value of $g$ as determined by pendulum experiments, Physics, p. 68) with the attraction of the sun for the earth, or the central force which keeps her in her orbit. Put $f$ for this force (measured like gravity by the velocity it generates in one second), $g$ for the force of gravity ( 32 feet, 2 inches per second), $r$ the earth's radius, $R$ the sun's distance, and let $E$ and $S$ be the masses of the earth and sun respectively. Then the law of gravitation gives us the proportion
whence,

$$
\begin{gathered}
f: g:: \frac{S}{R^{2}}: \frac{E}{r^{2}} \\
S=E \times\left(\frac{f}{g}\right) \times\left(\frac{R}{r}\right)^{2} .
\end{gathered}
$$

From the size of the earth's orbit (considered as a circle), and the length of the year, $f$ is found ${ }^{1}$ to be 0.2333 inches.
. Therefore, $\frac{f}{g}=0.0006044=\frac{1}{1654}$ nearly. $\quad$ But $\frac{R}{r}=23,439$, the square of which is $549,387,000$, nearly ; whence,

$$
S=E \times \frac{1}{1654} \times 549,387,000=332,000 E
$$

${ }^{1}$ The formula is $f=\frac{V^{2}}{R}, V$ being the velocity of the earth in its orbit, 18.495 miles per second. We may also use the equivalent formula, $f=\frac{4 \pi^{2} R}{T^{2}}, T$ being the length of the year in seconds.

We note in passing that $\frac{1}{2} f$ expresses the distance which the earth falls towards the sun every second; just as $\frac{1}{2} g$ ( 16 feet) is the distance a body at the earth's surface falls in the first second. This quantity, $\frac{1}{2} f$ or 0.116 inches, is the amount by which the earth's orbit deviates from a straight line in a second. In travelling $18 \frac{1}{2}$ miles the deflection is only about one-ninth of an inch.
178. The Sun's Density. - Its density as compared with that of the earth may be found by simply dividing its mass by its volume (both as compared with the earth); i.e., the sun's density equals $\frac{332000}{1300000}=0.255$, a little more than a quarter of the earth's density.

To get its specific gravity (i.e., its density compared with water) we must multiply this by 5.53 , the earth's mean specitic gravity. This gives 1.41. That is, the sun's mean density is less than $1 \frac{1}{2}$ times that of water, - a very significant result as bearing on its physical condition, especially when we know that a considerable portion of its mass is composed of metals.

## 179. Superficial Gravity, or Gravity at the Sun's Surface. -

 This is found by dividing the sun's mass by the square of its radius, which gives 27.6 ; i.e., a body weighing one pound on the earth's surface would there weigh 27.6 pounds, and a person who weighs 150 pounds here, would there weigh nearly two tons. A body would fall 444 feet in a second instead of 16 feet as here, and a pendulum which vibrates seconds on the earth would vibrate in less than one-fifth of a second there.180. The Sun's Rotation. - Dark spots are often visible upon the sun's surface, which pass across the disc from east to west, and indicate an axial rotation. The average time occupied by a spot in passing around the sun and returning to the same apparent position as seen from the earth is on the average 27.25 days. This interval, however, is not the true or sidereal time of the sun's rotation, but the synodic, as is evi-
dent from Fig. 42. Suppose an observer on the earth at $E$ sees a spot on the centre of the sun's dise at $S$; while the sun rotates $E$ will also move forward in its orbit; and the observer, the next time he sees the spot on the centre of the disc, will be at $E^{\prime \prime}$, the spot having gone around the whole circumference plus the are $S S^{\prime}$.

The equation by which the true period is deduced from the synodic is the same as in the case of the moon (Art. 141); viz.,

$$
\frac{1}{T}-\frac{1}{E}=\frac{1}{S}
$$



Fig. 42.
Synodic and Sidereal Revolution of the Sun.
$T$ being the true period of the sun's rotation, $E$ the length of the year, and $S$ the observed synodic rotation. This gives $T=25.35$.

Different observers, however, get slightly different results. Carrington finds 25.38; Spoerer, 25.23.

The paths of the spots across the sun's disc are usually more or less oval, showing that the sun's axis is inclined to the ecliptic, and so inclined that the north pole is tipped about $7 \frac{1}{4}^{\circ}$ towards the position that the earth occupies near the


Fig. 43. - Path of Sun Spots across the Sun's Disc.
first of September. Twice a year the paths become straight, when the earth is in the plane of the sun's equator, - on June 3 and December 5th. Fig. 43 illustrates this.
181. Peculiar Law of the Sun's Rotation. - It was noticed quite early that different spots give different results for the period of rotation, but the researches of Carrington about 50 years ago first brought out the fact that the differences are systematic, so that at the solar equator the time of rotation is less than on either side of it. For spots near the sun's equator it is about 25 days; in solar latitude $30^{\circ}, 26.5$; and in solar latitude $40^{\circ}, 27$ days. Spots are almost never seen beyond the parallels of $45^{\circ}$, but fortunately by the use of the spectroscope we may find the rotation period in high as well as low latitudes; in this way it has been found that in latitude $80^{\circ}$ the time of rotation is 35 days, fully ten days longer than at the equator.

Clearly the sun's visible surface is not solid, but permits motions and currents like those of our air and oceans. It might be argued that the spots misrepresent the sun's real rotation, not being fixed upon its surface ; but the "faculæ" (Art. 184) give the same result, and so do spectroscopic observations (Art. 200).

Possibly this equatorial acceleration may be, in some way not jet explained, an effect of the tremendous outpour of heat from the solar surface ; but more likely, according to the most recent investigations, it is a long persisting "survival" from the sun's past history, and not attributable to causes now acting. If so, it will gradually die out, but it may be thousands or millions of years before it entirely disappears.
182. Arrangements for the Study of the Sun's Surface. The heat and light of the sun are so intense that we cannot look directly at it with a telescope as we do at the moon.

A very convenient method of exhibiting the sun to a number of persons at once is simply to attach to a small telescope a frame carrying a screen of white paper at a distance of a foot or more from the eye-piece, as shown in Fig. 44. With a proper adjustment of the focus, a distinct image is formed on the screen, which shows the main features very fairly; indeed with proper precautions, almost as well as the most elaborate
apparatus. Still, it is generally more satisfactory to look at the sun directly with a suitable eye-piece. With a small telescope, not more than $2 \frac{1}{2}$ or 3 inches in diameter, it is usual to introduce a simple shade-glass between the eye-piece and the eye, but the dark glass soon becomes very hot and is apt to crack. With larger instruments it is necessary to use eye-pieces specially designed for the purpose, and known as solar


Fig. 44. - Telescope and Screen. eye-pieces, or "helioscopes."


Fig. 45. - Herschel Eye-Piece.

The simplest, and a very good one, is known as Herschel's, in which the sun's rays are reflected at right angles by a plane of unsilvered glass (Fig. 45). With this apparatus, although the reflected light is still too intense for the unprotected eye, only a thin shade glass is required, and it does not become much heated. It is not a good plan to "cap" the object-glass in order to cut off part of the light. The smaller the object-lens of the telescope, the larger the image it makes of a luminous point, or the wider its image of a sharp line. To cut down the aperture is, therefore, to sacrifice the definition of delicate details.
183. Photography. - In the study of the sun's surface, photography is for some purposes very advantageous and much used. The instrument must, however, have lenses specially constructed for photographic operations, since an object-glass which would give admirable results for visual purposes would be worthless photographically, and vice versa .(see Appendix,

Art. 534). The dise of the sun on the negatives is usually from two to ten inches in diameter, but photographs of small portions of the solar surface are often on a very much larger scale, as in the remarkable pictures made by Janssen at Meudon.

Photographs have the great advantage of freedom from prepossession on the part of the observer, and in an instant of time they secure a picture of the whole surface of the sun such as would require a skilful draughtsman hours to copy. But, on the other hand, they take no advantage of the instants of fine seeing; they represent the solar surface as it happened to appear at the moment when the plate was uncovered, affected by all the momentary distortions due to atmospheric disturbances.
184. The Photosphere. - The sun's visible surface is called the "Photosphere," i.e., the "light sphere," and when studied under favorable conditions with rather a low magnifying power, it appears as a disc considerably darker at the edge than in the centre, and not smoothly bright but mottled, looking much like rough drawing paper. With a powerful instrument, and the best atmospheric conditions, the surface is seen to be made up, as shown in Fig. 46, of a comparatively darkish background, sprinkled over with grains or "nodules," as Herschel calls them, of something much more brilliant, - "like snow-flakes on gray cloth," according to Langley. These nodules or "rice grains" are from 400 to 600 miles across, and in the finest seeing, themselves break up into more minute "granules." For the most part, the nodules are about as broad as they are long, though of irregular form; but here and there, especially in the neighborhood of the spots, they are drawn out into long streaks, known as "filaments," "willow leaves," or "thatch straws."

Certain bright streaks called "faculoe" are also usually visible here and there upon the sun's surface, and though not very obvious near the centre of the disc, they become conspicuous
near the "limb," especially in the neighborhood of the spots. Very probably they are of the same material as the rest of


Fig. 46. -The Great Sun Spot of September, 1870, and the Structure of the Photosphere. From a Drawing by Professor Langley. From the "New Astronomy," by permission of the Publishers.
the photosphere, but elevated above the general level, and intensified in brightness.

Fig. 47 shows faculæ around a spot near the sun's limb. The photosphere is probably a sheet of clouds floating in a less luminous atmosphere, just as a cloud formed by the condensation of water-vapor floats in the air. It is intensely brilliant, for the same reason that the "mantle" of a Welsbach burner outshines the gas-flame which heats it: the radiating power of the solid and liquid particles which compose the clouds is extremely high.


Fig. 47. - Faculæ at Edge of the Sun. (De La Rue.)
185. Sun Spots. - Sun spots, whenever visible, are the most conspicuous and interesting objects upon the solar surface. The appearance of a normal sun spot, Fig. 48, fully formed, and not yet beginning to break up, is that of a dark central "umbra," more or less nearly circular, with a fringing "penumbra," composed of converging filaments. The umbra itself is not uniformly dark throughout, but is overlaid with filmy clouds, which usually require a good telescope and helioscope to make them visible, but sometimes, though rather infre-
quently, are conspicuous, - as in the figure. Usually, also, within the umbra there are a number of round and very black spots, sometimes called "nucleoli," but often referred to as "Dawes' holes" after the name of their first discoverer.
The darkest portions of the umbra, however, are dark only by contrast. Photometric observations show that even the


Fig. 48. - A Normal Sun Spot. (Secchi; modified.)
nucleus gives about one per cent as much light as a corresponding area of the photosphere: the blackest portion of a sun spot is really more brilliant than a calcium light.

Very few spots are strictly normal. They are often gathered in groups with a common penumbra, which is partly covered with brilliant "bridges" extending across from the outside photosphere. Frequentiy the umbra is out of the centre of the penumbra, or has a penumbra on one side only, and the penumbral filaments, instead of converging regularly towards the nucleus, are often distorted in every conceivable way.
186. Nature of Sun Spots. - Until very recently sun spots have been believed to be cavities in the photosphere, filled with gases and vapors cooler and therefore darker than the surrounding region. This theory is founded on the fact that


Fig. 49. - Hydrogen Flocculi around a Sun Spot.
many spots as they cross the sun's disc look just as if they were saucer-shaped hollows, with sloping sides colored gray and the bottom black.

The penumbra is usually composed of "thatch straws," or long-drawn-out filaments of photospheric cloud, and these, as has been said, converge in a general way towards the centre of the spot.

At its inner edge the penumbra, from the convergence of these filaments, is usually brighter than at the outer. The inner ends of the filaments are ordinarily club-formed, but sometimes they are drawn out into fine points, which seem to curve downward into the umbra, like the rushes over a pool of water. The outer edge of the penum-
bra is usually pretty sharply bounded, and there the penumbra is darkest. In the neighborhood of the spot the surrounding photosphere is usually much disturbed and elevated into faculæ, as shown in Fig. 47.

Spectroheliograph pictures (Art. 203) show that there is a whirling motion of the hydrogen and calcium vapors that lie above and around the spots. Fig. 49, copied from such a photograph made at the Mount Wilson Solar Observatory, shows the curvature of the hydrogen filaments in the region surrounding a group of spots.
187. Dimensions of Sun Spots. - The diameter of the umbra of a sun spot varies all the way from 500 miles in the case of a very small one, to 40,000 or 50,000 miles, in the case of the largest. The penumbra surrounding a group of spots is sometimes 150,000 miles across, though that is an exceptional size. Not infrequently, sun spots are large enough to be visible with the naked eye, and can actually be thus seen at sunset or through a fog, or by the help of a colored glass.

The Chinese have many records of such objects, but the real discovery of sun spots dates from 1610, as an immediate consequence of Galileo's invention of the telescope. Fabricius and Scheiner, however, share the honor with him as being independent observers.
188. Duration, Development, and Changes of Spots. - The duration of sun spots is very various ; but they are always short-lived phenomena, astronomically speaking, sometimes lasting only for a few days, though more frequently for a month or two. In a single instance, the life of a spot group reached nearly eighteen months.

According to Secchi, the formation of a spot is usually announced some days in advance by a considerable disturbance of the surface of the photosphere, and by the formation of faculæ with groups of "pores," or minute dark points among them. These pores grow larger and coalesce, and at the same time the surrounding region takes on the filamentary structure of the penumbra, and the umbra finally
appears in the centre. The process ordinarily requires several days, but sometimes a few hours are sufficient. If the originating disturbance is particularly violent, it usually results not in a single normal spot, but in a group of irregular nuclei scattered within a common penumbra, and then the spots themselves usually break up into fragments, and these again into others which separate from each other with considerable velocity. Moreover, at each new disturbance, the forward portions of the group show a tendency to wade forward toward the east through the photosphere, leaving behind them a trail of smaller spots.

Occasionally a spot shows a distinct cyclonic motion, the filaments being drawn spirally inward; and in different members of the same group of spots, these cyclonic motions are not seldom in opposite directions, as of wheels gearing into each other.

When a spot vanishes, it is usualiy by the rapid encroachment of the photospheric matter, which, as Secchi expresses it, appears to "fall pell-mell into the cavity," completely burying it and leaving its place covered by a group of faculæ.
189. Motions of the Spots. - Spots within $15^{\circ}$ or $20^{\circ}$ of the sun's equator usually drift slowly towards it, while those in the higher latitudes drift away from it; but the motion is slight and exceptions are frequent. Spot groups in which the disturbance is violent, as intimated in the preceding section, seem to move towards the east on the sun's surface more rapidly than the quiet ones in the same latitude. Within and around the spot itself, the motion so far as can be observed, is usually inward towards the centre, and there downward. Not infrequently fragments at the inner edge of the penumbral filaments break off, move towards the centre of the spot, and there disappear as if swallowed up by a vortex.

Occasionally, though seldom, the downward motion at the centre of a spot is vigorous enough to be detected by the displacement of lines in the spectrum, while around the outer edges of the penumbra the same instrument in such cases usually shows a violent up-boiling from beneath (see Art. 200).

## 190. Distribution of Spots and Periodicity of Sun Spots. -

 It is a significant fact that the spots are confined mostly to two zones of the sun's surface between $5^{\circ}$ and $40^{\circ}$ of north and south latitude. A few are found near the equator about the time of the spot maximum, and practically none beyond the latitude of $45^{\circ}$. Fig. 50 shows the distribution of several thousand spots as observed by Spörer and Carrington.In 1843, Schwabe of Dessau, by the comparison of an exten-


Fig. 50. - Distribution of Sun Spots in Latitude.
sive series of observations then covering nearly twenty years, showed that the sun spots are probably periodic, being at sometimes much more numerous than at others, with a roughly regular recurrence every ten or eleven years. A few years later he fully established this remarkable result.

Wolf of Zurich has collected all the observations discoverable, and has obtained a pretty complete record back to 1610 , from which he has constructed the annexed diagram (Fig. 51) in which the ordinates represent what he calls his "relative numbers," which may be taken as the index of the sun's spottedness.

The average period is 11.1 years, but the maxima are somewhat irregular, both in time and as to the extent of spottedness. The two
last maxima occurred in 1906 and 1917. During the maximum, the surface of the sun is never free from spots, from 25 to 50 being frequently visible at once. During the minimum, on the contrary, weeks and even months pass without the appearance of a single one. The cause of this periodicity is not known, but it is probably due to causes within the sun itself rather than to anything external.


Fig. 51. - Wolf's Sun-Spot Numbers (see also p. 132).

190*. Another curious and important fact has been brought out by Spoerer, though not yet explained. Speaking broadly, the disturbance which produces the spots of a given period first manifests itself in two belts, about $30^{\circ}$ north and south of the sun's equator. These belts then draw in towards the equator, and the spot-maximum occurs when their latitude is about $16^{\circ}$; while the disturbance finally dies out at a latitude of from $5^{\circ}$ to $10^{\circ}$, about twelve or fourteen years after its first outbreak. Two or three years before this disappearance, however, two new zones of disturbance show themselves. Thus at the spot-minimum there are usually four well-marked spot-belts : two near the sun's equator, due to the expiring disturbance, and two in high latitudes, due to the newly beginning outbreak.
191. The Cause of Sun Spots. - As to this, very little can be said to be really known. Numerous theories more or less satisfactory have been proposed.

Professor Young, the author of our text, believed them to be the effect of eruptions breaking through the photosphere. He did not, however, consider them to be the holes, or craters, through which the eruptions break out, as Secchi at one time thought and as Mr. Proctor maintained to the very last, but rather thought, in accordance with Secchi's later views, that when an eruption takes place, a hollow, or sink, results in the neighboring cloud-surface, and in this hollow the cooler gases and vapors collect. It has been generally supposed that in some way they are due to matter descending from above upon the photosphere, although recent investigations make it seem possible that they are due to ascending currents - material flowing outward from the sun's interior, becoming cooler at the higher level, and therefore appearing dark against the brighter photosphere.

Faye considered them to be solar cyclones somewhat analogous to terrestrial storms, and interest in this theory has been revived by the discovery of a vortical structure of spots shown on photographs made at the Carnegie Solar Observatory at Mount Wilson, using the spectroheliograph (Art. 203) and the C line of hydrogen.

Several well-equipped solar observatories are applying new and ingenious methods of research to the study of sun spots, and we may confidently hope that in a few years our knowledge of their nature and cause will be greatly extended.
192. Terrestrial Influence of Sun Spots. - One influence of sun spots upon the earth is perfectly demonstrated. When the spots are numerous, magnetic disturbances (magnetic storms) are most numerous and most violent upon the earth, - a fact not to be wondered at, since notable disturbances upon the sun's surface have been in many cases immediately followed by magnetic storms with brilliant exhibitions of the Aurora Borealis, as in 1859 and 1883. The nature and mechanism of the connection is as yet unknown, but the fact is beyond doubt. The dotted lines in the figure of the sun-spot periodicity represent the magnetic storminess of the earth at the indicated dates (Fig. 51); and the correspondence between these curves
and the curves of the spottedness makes it impossible to question their relation to some common cause.

It has been attempted, also, to show that the periodical disturbance of the sun's surface is accompanied by effects upon the earth's meteorology, - upon its temperature, barometric pressure, storminess, and the amount of rainfall. On the whole, it can only be said that while it is entirely possible that real effects of the sort are produced, they must be slight and almost entirely masked by the effect of purely terrestrial disturbances. The results obtained by different investigations in attempting to co-ordinate sun-spot phenomena with meteorological phenomena are thus far unsatisfactory and even contradictory. We may add that the spots cannot produce any sensible effects by their direct action in diminishing the light and heat of the sun. They do not directly alter the amount of the solar radiation at any time by as much as one part in a thousand.


Fig. 51.*- Continuation of Fig. 51.

## CHAPTER VII.

THE SPECTROSCOPE, THE SOLAR SPECTRUM, AND THE CHEMICAL CONSTITUTION OF THE SUN. - THE CHROMOSPHERE AND PROMINENCES. - THE CORONA. - THE SUN'S LIGHT. - MEASUREMENT OF THE INTENSITY OF THE SUN'S HEAT. - THEORY OF ITS MAINTENANCE AND SPECULATIONS REGARDING THE AGE OF THE SUN.

About 1860 the spectroscope appeared in the field as a new and powerful instrument for astronomical research, resolving at a glance many problems which before seemed to be inaccessible even to investigation. It is not extravagant to say that its invention has done almost as much for the advancement of astronomy as that of the telescope.

It enables us to study the light that comes from distant objects, to read therein a record, more or less complete, of their chemical composition and physical conditions, to measure the speed with which they are moving towards or from us; and sometimes, as in the case of the solar prominences, to see, and observe at any time, objects otherwise visible only on rare occasions.
193. The Spectroscope. - The essential part of the instrument is either a prism or train of prisms, or else a diffraction "grating," ${ }^{1}$ which is capable of performing the same office of "dispersing" the rays of different color.

If with such a "dispersion piece," as it may be called

[^20](either prism or grating), one looks at a distant point of light, he will see instead of a point a long streak of light, red at one end and violet at the other. If the object looked at is a line of light, parallel to the edge of the prism or to the lines of the grating, then instead of a mere colored streak without width, one gets a "spectrum," - a colored band or ribbon of light, which may show markings that will give the observer most valuable information (Physics, p. 323). It is usual to form this line of light by admitting the light through a narrow "slit" placed at one end of a tube, which carries at the other end an achromatic object-glass having the slit in its principal focus (Physics, p. 318). This tube with slit and lens constitutes the "collimator"; so named because it is precisely the same as an instrument used in connection with the transit instrument to adjust its "line of collimation."

Instead of looking at the spectrum with the naked eye, it is better in most cases to use a small "view telescope" (so called to distinguish it from the large telescope to which the spectroscope is often attached).
194. Construction of the Spectroscope. - The instrument, therefore, as usually constructed, and shown in Fig. 52* consists of three parts, - collimator, dispersion piece, and view telescope, - although in the "direct-vision" spectroscope, shown in the figure, the view telescope is omitted.

Fig. 52, from "The Sun" by permission of Appleton \& Co., represents a large "tele-spectroscope," as the combination of telescope and spectroscope is called, arranged for photographic work.

If the slit, $S$, be illuminated by strictly "homogeneous light," say yellow, a yellow image of the slit will appear at $Y$. If, at the same time, light of a different wave length, red for instance, be also admitted, a red image will be formed at $R$, and the observer will then see a spectrum with two bright lines, the lines being really nothing more than images of the slit. If violet light be admitted, a violet image will be formed at $V$,


Frg. 52. - Telespectroscope, fitted for Photography.

and there will be three bright lines. If the light comes from a luminous solid, like the lime cylinder of a calcium light, or the filament of an incandescent lamp, or from an ordinary gas or candle flame (Physics, p. 318), there will be an infinite number of these slit images close together, without interval or break, and we then get what is called a continuous spectrum. If it comes, however, from an electric spark or a so-called Geissler tube, or from a Bunsen burner flame charged with the


Fig. 52.*-Different Forms of Spectroscope.
vapor of some volatile metal, the spectrum will consist of a series of bright lines, or bands.

194*. The Solar Spectrum. - If we look at sunlight, either direct or reflected (as from the moon), we get a spectrum, continuous in the main, but crossed by a multitude of dark lines, or missing slit-images. These dark lines are known as the "Fraunhofer lines," because Fraunhofer was the first to map them (in 1814). To some of the more conspicuous
ones he assigned letters of the alphabet which are still retained as designations: thus $A$ is a strong line at the extreme red end of the spectrum ; $C$, one in the scarlet ; $D$, in the yellow; $F$, in the blue ; and $H$ and $K$ are a pair at its violet extremity. Rowland's great photographic map of the spectrum contains several thousands, each as permanent a feature of the spectrum as rivers and cities are of a geographical chart. Their explanation remained an unsolved problem for nearly fifty years.
195. Principles upon which Spectrum Analysis depends. These, substantially as announced by Kirchhoff in 1858, are the three following : -

1st. A continuous spectrum is given by luminous bodies, which are so dense that the molecules interfere with each other in such a way as to prevent their free, independent, luminous vibration; i.e., by bodies which are either solid or liquid, or if gaseous are under high pressure.

2 d . The spectrum of a luminous gas under low pressure is discontinuous, and is made up of bright lines or bands: and these lines are characteristic ; i.e., the same substance under similar conditions always gives the same set of lines, and generally does so even under conditions differing quite widely ; but it may give two or more different spectra when the circumstances differ too widely.

3d. A gaseous substance absorbs from a beam of white light passing through it precisely those rays of which its own spectrum consists. The spectrum of white light which has been transmitted through it then exhibits a "reversed" spectrum of the gas ; i.e., a spectrum which shows dark lines in place of the characteristic bright lines.

This principle of reversal is illustrated by Fig. 53. Suppose that in front of the slit of the spectroscope we place a spirit lamp with a little carbonate of soda and some salt of thallium upon the wick. We shall then get a spectrum showing the two yellow lines of sodium and the green line of thallium, all bright. If now the lime light be started behind the flame, we
shall at once have the effect shown in the lower figure, - a continuous spectrum crossed by black ${ }^{1}$ lines which exactly replace the bright lines. Insert a screen between the lamp flame and the lime, and the dark lines instantly turn bright again. The explanation of the Fraunhofer lines, therefore, is that they are mainly due to the action of the gases and vapors of the solar atmosphere upon the light that comes from the liquid or solid particles composing the photospheric clouds. Some of them, how-


Fig. 53. - Reversal of the Spectrum.
ever, known as "Telluric lines," are due to the gases and vapors of the earth's atmosphere, - to water-vapor and oxygen especially.
196. Chemical Constituents of the Sun. - Numerous lines of the solar spectrum can be identified as due to the pres-
${ }^{1}$ Their darkening, however, when the light from the lime is transmitted through the flame, is only relative and apparent, not real. Their brightness is actually a little increased; but that of the background is increased immensely, making it so much brighter than the lines that, contrasted with it, they look black.
ence in the sun's atmosphere of known terrestrial elements in the state of vapor. To effect the comparison necessary for this purpose, the spectroscope must be so arranged that the observer can confront the


Fig. 54. - The Comparison Prism. spectrum of sunlight with that of the substance to be tested. In order to do this, half of the slit is covered by a little" comparison prism" (Fig. 54), which reflects into it the light from the sun, while the other half of the slit receives directly the light of some flame or electric spark. On looking into the eye-piece of the spectroscope, the observer will then see a spectrum, the lower half of which, for instance, is made by sunlight, while the upper half is made by light coming from an electric arc containing the vapor, say of iron.

In such comparisons photography may be most effectively used instead of the eye. Fig. 55 is a reproduction of a negative made in investigating the presence of iron in the sun. The lower half is the violet portion of the sun's spectrum and the upper half that of an


Fig. 55. - Comparison of the Spectrum of Iron with the Solar Spectrum. From a Negative by Professor Trowbridge.
electric arc charged with the vapor of iron. In the original every line of the iron spectrum coincides exactly with its correlative in the solar spectrum, though in the engraving many of the coincidences fail to be obvious. There are, of course, on the other hand, certain lines in the solar spectrum which do not find any correlative in that of iron, being due to other elements.
197. Elements known to exist in the Sun. - As the result of such comparisons, first made by Kirchhoff, but since repeated and greatly extended by late investigators, a large number of the chemical elements have been ascertained to exist in the solar atmosphere in the form of vapor.

Professor Rowland in 1890 gave the following preliminary list of thirty-six whose presence he regarded as certainly established, and it is probable that the completion of his research will add a number of others. ${ }^{1}$ The elements are arranged in the list according to the intensity of the dark lines by which they are represented in the solar spectrum: the appended figures denote the rank which each element would hold if the arrangement had been based on the number instead of the intensity of the lines. In the case of iron the number exceeds 2000.

| * Calcium, ır. | * Strontium, 23. | Copper, 30. |
| :---: | :---: | :---: |
| *Iron, r. | Vanadium, 8. | Zinc, 29. |
| * Hydrogen, 22. | * Barium, 24. | Cadmium, 26. |
| Sodium, 20. | Carbon, 7 . | *Cerium, ro. |
| * Nickel, 2. | Scandium, 12. | Glucinum, 33. |
| * Magnesium, 19. | Yttrium, $\mathrm{I}_{5}$. | Germanium, 3 |
| * Cobalt, 6. | Zirconium, 9. | Rhodium, 27. |
| Silicon, 21. | Molybdenum, 17. | Silver, ${ }^{\text {r }}$ - |
| Aluminium, 25 . | Lanthanum, i4. | Tin, 34. |
| *Titanium, 3 . | Niobium, 16. | Lead, 35 . |
| hromium, | Palladium, 8 8. | Erbium, |
| Langanese, | Neodymium, $\mathrm{r}_{3}$ | Potassium, |

An asterisk denotes that the lines of the element indicated appear often or always as bright lines in the spectrum of the chromosphere.

It will be noticed that all the bodies named in the list, carbon and hydrogen alone excepted, are metals (chemically hydrogen

[^21]acts much like a metal), and that many important terrestrial elements fail to appear; selenium, nitrogen, chlorine, bromine, iodine, sulphur, phosphorus, and boron are all missing.

We must be cautious, however, in drawing negative conclusions. It is quite conceivable that the spectra of these bodies under solar conditions may be so different from their spectra as presented in our laboratories that we cannot recognize them; for it is now unquestionable that many substances under different conditions give two or more widely different spectra, - nitrogen, for instance.

Mr. Lockyer thinks it nore probable that the missing substances are not truly " elementary," but are decomposed or "dissociated" by the intense heat, and so cannot exist on the sun, but are replaced by their components. He maintains, in fact, that none of our so-called "elements" are really elementary, but that all are decomposable, and are to some extent actually decomposed in the sun and stars, and some of them by the electric spark in our own laboratories. Granting this, many interesting and remarkable spectroscopic facts find easy explanation. At the same time the hypothesis is encumbered with serious difficulties, and has not yet been finally accepted by physicists and chemists.
198. The Reversing Layer.-According to Kirchhoff's theory the dark lines are formed by the passing of light emitted by the minute solid or liquid particles of which the photospheric clouds are supposed to be formed, through somewhat cooler vapors containing the substances which we recognize in the solar spectrum. If this be so, the spectrum of the gaseous envelope, which by its absorption forms the dark lines, should by itself show a spectrum of corresponding bright lines.

The opportunities are rare when it is possible to obtain the spectrum of this gas stratum separate from that of the photosphere; but at the time of a total eclipse, at the moment when the sun's disc has just been obscured by the moon, and the sun's atmosphere is still visible beyond the moon's limb, the observer ought to see this bright-line spectrum, if the slit of the spectroscope be carefully directed to the proper point.

The author succeeded in making this very observation at the Spanish eclipse of 1870 .

The lines of the solar spectrum, which up to the time of the final obscuration of the sun had remained dark as usual (with the exception of a few belonging to the spectrum of the chromosphere) were suddenly "reversed," and the whole length of the spectrum was filled with brilliant colored lines, which flashed out quickly and then gradually faded away, disappearing in about two seconds, - a most beautiful thing to see.

The natural interpretation of this phenomenon is that the formation of the dark lines in the solar spectrum is, mainly at least, produced by a very thin stratum close down upon the photosphere, since the moon's motion in two seconds would cover a thickness of only about 500 miles. It was not possible, however, to be certain that all the dark lines of the solar spectrum were reversed, and in this uncertainty lies the possibility of a different interpretation.

Several partial confirmations have been obtained by various eclipse observers since 1870, but at the eclipse of August, $1896,{ }^{1}$ Mr. Shackleton succeeded in photographing the phenomenon with a so-called "prismatic camera," or "objective prism spectroscope" (Arts. 458-9). He caught the critical moment exactly, with an exposure of only half a second, and his negative is apparently conclusive in favor of the inference stated above. A photograph taken only five seconds later shows merely some 20 bright lines belonging to the spectrum of the chromosphere (Art. 201), in place of the hundreds shown upon the earlier one.

Mr. Lockyer, however, still continues to dispute the existence of any such thin stratum, or "reversing layer." According to his view, the solar atmosphere is very extensive, and those lines of the iron spectrum, which, as he holds, correspond to the more complex combinations of its constituents, are formed only in the regions of lower temperature, high up in the sun's atmosphere.
199. Sun-Spot Spectrum. - The spectrum of a sun spot differs from the general solar spectrum not only in its diminished brilliancy, but in the great widening of certain lines, and the thinning and even "reversing" of others, especially those of hydrogen. The majority of the Fraunhofer lines, however, are not sensibly affected either way, a fact which Mr. Lockyer quotes as evidence that they originate high up in the
solar atmosphere rather than in the region of the reversing layer close to the photosphere.

The general darkness of the spot spectrum appears to be due to the presence of myriads of extremely fine dark lines, so closely packed as to be resolvable only by spectroscopes of great power. This indicates that the darkness is caused by the absorption of light by transmission through vapors, and is not simply due to the diminished brightness of the surface from which the light is emitted.
200. Distortion of Lines; Doppler's Principle. - Sometimes certain lines of the spectrum are bent and broken, as shown in Fig. 56. These distortions are explained by the swift motion towards or from the observer of the gaseous matter, which by


Fig. 56. - The C line in the Spectrum of a Sun Spot, Sept. 22, 1870. its absorption produces the line in question. In the case illustrated, hydrogen was the substance, and its motion was towards the observer, at one point at the rate of nearly 300 miles a second.

The principle upon which the explanation of this displacement and distortion of lines depends was first enunciated by Doppler in 1842. It is this: When the distance between us and a body which is emitting regular vibrations (either of sound or of light) is decreasing, then the number of vibrations received by us in each second is increased, and their wave-length, real or virtual, is correspondingly diminished.

Thus the pitch of a musical tone rises in the case supposed, and in the same way the refrangibility of a light wave, which depends upon its wave-length (Physics, p. 326) is increased, so that it will fall nearer the violet end of the spectrum (see Appendix, Art. 500).
201. The Chromosphere. - Outside the photosphere lies the chromosphere, of which the lower atmosphere, or "reversing layer," is only the densest and hottest portion. This chromosphere,
or "color sphere," is so called because it is brilliantly scarlet, owing the color to hydrogen, which is its main, or at least its most conspicuous, constituent. In structure it is like a sheet of flame overlying the surface of the photosphere to a depth of from 5000 to 10,000 miles, and as seen through the telescope at a total eclipse has been aptly described as like "a prairie on fire."

There is, however, no real "burning" in the case; i.e., no chemical combination going on between the hydrogen and some other element like oxygen. The hydrogen is too hot to burn in this sense, the temperature of the solar surface being above that of "dissociation"; so high that any compound containing hydrogen would there be decomposed.
202. The Prominences. - At a solar eclipse, after the sun is fairly hidden by the moon, a number of scarlet, star-like objects are usually seen blazing like rubies upon the contour of the moon's disc. In the telescope they look like fiery clouds of varying form and size, and as we now know, they are only projections from the chromosphere, or isolated clouds of the same material. They were called Prominences and Protuberances, as a sort of non-committal name, while it was still uncertain whether they were appendages of the sun or of the moon.

They were first proved to be solar during the eclipse of 1860, by means of photographs which showed that the moon's dise moved over them as it passed across the sun. Their real nature as clouds of incandescent gas was first revealed by the spectroscope in 1868, during the Indian eclipse of that year. On that occasion numerous observers recognized in their spectrum the bright lines of hydrogen along with another conspicuous yellow line, at first wrongly attributed to sodium, but afterwards, to a hypothetical element unknown in our laboratories and provisionally named "Helium," its yellow line being known as $D_{3}$ ( $D_{1}$ and $D_{2}$ being the sodium lines).

202*. Helium was discovered as a terrestrial element (or perhaps a mixture of two or more elements) in April, 1895, by Dr. Ramsay, of London, one of the discoverers of Argon. In examining the spec-
trum of the gas extracted from a specimen of cleveite, a species of pitch-blende, he found the characteristic $D_{3}$ line along with certain other unidentified lines which appear in the spectrum of the chromosphere and prominences. The same gas has since been found in a number of other minerals and in meteoric iron. Its density turns out to be about double that of hydrogen, but less than that of any other known element. Chemically it is extremely inert, refusing to enter into combination with other elements (as hydrogen does so freely), and therefore exists on the earth only in minute quantities. It seems, however, to be abundant in certain stars and nebulæ, where


Fig. 57. - Quiescent Prominences.
its lines are conspicuous along with those of hydrogen. The $D_{3}$ line is not the only helium line, but the chromosphere spectrum contains at least three others that are always observable, besides several that occasionally make their appearance. The $H$ and $K$ lines of calcium are also, like those of hydrogen and helium, always present as bright lines in the chromosphere; and several hundred lines of the spectra of iron, strontium, magnesium, sodium, \&c., have been observed in it now and then.
203. The Prominences and Chromosphere observable with the Spectroscope. - Janssen was so struck with the brightness of the hydrogen lines that he believed it possible to observe them in full daylight, and the next day he found it to be so; and not only this, but he found also that by proper management of his spectroscope he could study the forms and structure of the prominences nearly as well as during the eclipse.

Lockyer in England, a few days later, and quite independently, made the same discovery, and his name is always justly associated with Janssen's. See Appendix, Art. 501.

It is possible to photograph prominences by means of the spectroscope, using the $H$ and $K$ lines of calcium, but the best results are secured with the spectroheliograph. This is an instrument devised to allow light of one wave length only to fall upon the plate, so that the photograph is taken with light from a single element. A spectroscope is adjusted as described in Art. 501 , but the spectrum falls upon an opaque screen which has a narrow slit so placed that the single line of light to be used in the photograph may pass through to the sensitive plate. If this plate is moved at the same rate as the image drifts across the slit of the spectroscope (or if the two slits are moved together), the entire image will be photographed upon the plate. In this way the chromosphere and prominences around the whole circumference of the sun may be photographed at a single exposure.

Photographs of the sun's dise may be taken with either the $H$ or $K$ lines of calcium and show that there are bright clouds of calcium vapor known as calcium flocculi; hydrogen lines may be used in the same way, and Fig. 49 shows hydrogen floceuli.
204. Different Kinds of Prominences. - The prominences may be broadly divided into two classes, - the "quiescent" or "diffuse," and the "eruptive," or, as Secchi calls them, " the metallic," because they show in their spectrum the lines of many of the metals in addition to the lines of hydrogen.

The prominences of the former class, illustrated by Fig. 57, are immense clouds, often 50,000 or 60,000 miles in height and of corresponding horizontal dimensions. Ordinarily they either rest directly upon the chromosphere as a base or are connected with it by stems and columns, though in some cases they appear to be entirely detached from it. They are not very brilliant, and ordinarily show no lines in their spec-
trum except those of hydrogen and helium; nor are their changes usually rapid, but they continue sensibly unaltered, sometimes for days together ; i.e., as long as they remain in sight in passing around the limb of the sun. All their forms and behavior indicate that, like the clouds in our own atmosphere, they exist and float, not in a vacuum, but in a medium which must have a density comparable with their own, though for some reason not visible in the spectroscope. They are found on all portions of the sun's disc, not being confined to the sun-spot zones.


Prominences Sept. 7, 1871, 12.3 P.м. (a)


Same at 1.15 P.M. (b)

Fig. 58.
The eruptive prominences, on the other hand, appear only in the spot zones, and as a rule in connection with active spots. They usually seem to originate not within the spots themselves but in the surrounding region of disturbed faculæ. Ordinarily they are not so large as the quiescent prominences, but at times they become enormous, reaching elevations of several hundred thousand miles. They commonly take the form of "spikes," "flames," or "jets," and sometimes they are bright enough to be visible with the spectroscope on the disc of the sun itself.

They are most fascinating objects to watch, on account of the rapidity of their changes. Sometimes their actual motion can be perceived directly, like that of the minute hand of a clock, and this implies a velocity of at least 250 miles a second in the moving mass. In such cases the lines of the spectrum are also, of course, greatly displaced and distorted.

Fig. 58 represents a prominence of this sort at two times, separated by an interval of three quarters of an hour. The large quiescent prominence of Fig. (a) was completely blown to pieces by the "explosion," as it may be fairly called, which occurred beneath it. Such occurrences, of course, are not every-day affairs, but are by no means very uncommon.

The number of prominences of both kinds visible at one time on the circumference of the sun's disc ranges from one or two to twentyfive or thirty; the eruptive prominences being numerous only near the times of sun-spot maximum.
205. The Corona. - The corona is a halo or glory of light which surrounds the sun at the time of a total eclipse, and has been known from remote antiquity as one of the most beautiful and impressive of all natural phenomena. The portion of the corona near the sun is dazzlingly bright and of a pearly tinge, which contrasts finely with the scarlet prominences. It is made up of filaments and rays which, roughly speaking, diverge radially, but are strangely curved and intertwined. At a little distance from the edge of the sun the light becomes more diffuse, and the outer boundary of the corona is not very well defined, though certain dark rifts extend through it clear to the sun's surface. Often the filaments are longest in the sun-spot zones, giving the corona a roughly quadrangular form. This seems to be specially the case in eclipses which occur near the time of a sun-spot maximum. In eclipses which occur near the sun-spot minimum, on the other hand, the equatorial rays predominate - diffuse and rather faint, but of great extent. Near the poles of the sun there are often tufts of sharply defined threads of light, which curve both ways from the pole.

The corona varies greatly in brightness at different eclipses, according to the apparent diameter of the moon at the time. The total light of the corona is certainly always at least two or three times as great as that of the full moon.
206. Drawings and Photographs of the Corona. - There is very great difficulty in getting accurate representations of this phenomenon. The two or three minutes during which only it is visible at any given eclipse, do not allow time for trustworthy handwork; at any rate, drawings of the same corona made even by good artists, sitting side by side differ very much, sometimes ridiculously. Photographs are better, so far as they go, but hitherto they have not succeeded in bringing out many details of the phenomenon which are easily visible to the eye; nor do the pictures which show well the outer portions of the corona generally bring out the details near the sun's limb. The best results are obtained by making a sort of composite picture after the eclipse, combining in one representation all the features which appear with certainty in any of a series of photographs made with varying exposures. Fig. 59 is such a picture, engraved from a drawing by Wesley, combining sketches and photographs of the eclipse of 1900 . It shows a corona of the minimum type.
207. Spectrum of the Corona. - The characteristic feature of the visual spectrum is a bright line in the green, generally known as the " 1474 " line, ${ }^{1}$ because it coincides with a dark line which on Kirchhoff's map of the solar spectrum, is found at that point on his scale. This dark line had been previously identified by Ångström, as due to iron, and the coincidence was for a long time puzzling (since the vapor of iron is a very improbable substance to be found at such an elevation above the photosphere) until it was discovered that the line is really a close double, one of its two components being due to iron, while the other is due to some unknown gaseous element, which has been called "Coronium" after the analogy of Helium.

Besides this conspicuous green line there are several others in the violet (faintly shown in spectroscopic photographs) which are probably due to the same substance. The hydrogen and helium lines,

[^22]and $H$ and $K$ of calcium, have also been photographed as bright lines in the corona spectrum ; but observations made in 1893 and 1896 prove that they are caused by reflection (in our atmosphere) of light from the chromosphere and prominences and are not truly coronal.

The light of the corona is distinctly polarized; on one or two occasions observers have also reported in its visual spectrum the


Fig. 59. - Corona of Eclipse of 1900, Wadesboro, N.C.
presence of dark Fraunhofer lines, and these have now been photographed. The corona therefore unquestionably contains some matter which reflects sunlight. The faint continuous spectrum which forms the background for the bright and dark lines suggests the presence of incandescent solid or liquid particles.
208. Nature of the Corona. - The corona is proved to be a true solar appendage and not a mere optical phenomenon, nor due to either the atmosphere of the earth or moon, by two unquestionable facts: first, its spectrum as described above is not that of mere reflected sunlight, but of a glowing gas; and second, photographs of the corona made at widely differ-
ent stations on the track of an eclipse show identical details, ${ }^{1}$ and exhibit the motion of the moon across the coronal filaments. In a sense, then, the corona is a phenomenon of the sun's atmosphere, though the solar "atmosphere" is very far from bearing to the sun the same relations that are borne towards the earth by the air. The corona is not a simple spherical envelope of gas comparatively at rest, and held in equilibrium by gravity, but other forces than gravity are prevalent in it, and matter that is not gaseous probably abounds. The phenomena of the corona are not yet satisfactorily explained, and remind us far more of auroral streamers and comets' tails than of anything that occurs in the lower regions of our terrestrial atmosphere.

That the corona is composed of matter excessively rarified is shown by the fact that in a number of cases comets are known to have passed directly through it (as for instance in 1882) without the slightest perceptible disturbance or retardation of their motion. Its density must, therefore, be almost inconceivably less than that of the best vacuum we are able to make by artificial means.

## THE SUN'S LIGHT AND HEAT.

209. The Sun's Light. - By photometric methods, which we will not stop to explain (see "General Astronomy") it is found that the sun gives us about 1575 billions of billions ( 1575 followed by 24 ciphers) times as much light as a standard candle ${ }^{2}$ would do at that distance.
[^23]The light received from the sun is about 600,000 times that of the moon (Art. 162), about $7,000,000000$ times that of Sirius, the brightest of the fixed stars, about $50,000,000000$ times that of Vega or Arcturus, and fully 200,000,000000 times that of the Pole-star.

The "intensity" of sunlight, or the "intrinsic brightness" of the sun's surface, is quite a different matter from the total quantity of its light expressed in candle power. By intensity we mean the amount of light per square unit of luminous surface. From the best data we can get (only roughish approximations being possible) we find that the sun's surface is about 190,000 times as bright as that of a candle flame; and about 150 times as bright as the lime of a calcium light; even the darkest part of a sun spot outshines the lime light.

The brightest part of an electric arc comes nearer sunlight in intensity than anything else we know of, being from onehalf to one-quarter as bright as the solar surface itself.
210. Comparative Brightness of Different Portions of the Sun's Disc. - The sun's disc is brightest near the centre, but the variation is slight until we get pretty near the edge; there the light falls off rapidly, so that just at the sun's limb the intensity is not much more than one-third as great as at the centre. The color is modified also, becoming a sort of orange red, the blue and violet rays having lost much more of their brightness than the red and yellow.

This darkening is unquestionably due to the general absorption of light by the lower parts of the sun's atmosphere. Just how much the sun's brightness is diminished for us by this absorption, it is difficult to say. According to Langley, if the sun's atmosphere were suddenly removed, the surface would shine out somewhere from two to five times as brightly, and its tint would become strongly blue like the color of an electric arc.

## 211. The Quantity of Solar Heat: The Solar Constant. -

 The Solar Constant is the number of heat units which a square unit of the earth's surface, unprotected by any atmosphere, andexposed perpendicularly to the sun's rays would receive from the sun in a unit of time. The heat unit most used at present is the "Calory," ${ }^{1}$ which is the quantity of heat required to raise the temperature of one kilogram of water $1^{\circ} \mathrm{C}$.; and as the result of the best observations thus far made, it appears that the solar constant ${ }^{2}$ is between 19 and 22 of these "calories" to a square metre in a minute, under a vertical sun, and after allowing for the absorption of a large percentage of heat by the air. At the earth's surface a square metre would seldom actually receive more than from 10 to 15 calories in a minute.
212. Method of Determining the Solar Constant. - The principle is simple, though the practical difficulties are serious, and so far have made it impossible to obtain the accuracy desirable. The determination is made by allowing a beam of sunlight of known cross section to fall upon a known quantity of water for a known time and measuring the rise of temperature.

The difficulty lies partly in measuring and allowing for the heat received by the water from other sources than the sun, and for its own loss of heat by radiation; but especially and mainly in determining the proper allowance to be made for the absorption of the sun's heat in passing through the air. This atmospheric absorption changes continually with every change in the transparency of the air or of the sun's altitude. (For a description of the Pyrheliometer, seo Appendix, Art. 556.)

[^24]213. The Solar Heat at the Earth's Surface expressed in Terms of Melting Ice. - Since it requires $79 \frac{1}{4}$ calories of heat to melt a kilogram of ice with a specific gravity of 0.92 , it follows that, taking the solar constant at 21 , the heat received from a vertical sun would melt in an hour a sheet of ice 17.3 millimeters, or seven-tenths of an inch thick. From this it is easily computed that the amount of heat received by the earth from the sun in a year is sufficient to melt a shell of ice 124.2 feet thick all over the earth's surface.
214. Solar Heat expressed as Energy. - Since, according to the known value of the "mechanical equivalent of heat" (Physics, p. 260), a horse-power ( 33,000 foot pounds per minute) can easily be shown to be equivalent to about 10.7 calories per minute, it follows that each square meter of the earth's surface perpendicular to the sun's rays ought to receive about 2.0 horse-power continuously. Atmospheric absorption cuts this down to about $1 \frac{1}{4}$ horse-power, of which about $\frac{1}{8}$ can be realized by a suitable machine, such as Ericsson's solar engine.

The energy annually received from the sun by the whole of the earth's surface aggregates nearly 70 mile-tons to each square foot. That is, the average amount of heat annually received by each square foot of the earth's surface, if utilized in a theoretically perfect heat engine, would hoist nearly 70 tons to the height of a mile.
215. Solar Radiation at the Sun's Surface. - If now we estimate the amount of radiation at the sun's surface itself, we come to results which are simply amazing. We must multiply the solar constant observed at the earth by the square of the ratio between $93,000,000$ miles (the earth's distance from the sun) and 433,250 (the radius of the sun). This square is about 46,000 . In other words, the amount of heat emitted in a minute by a square meter of the sun's surface is about 46,000 times as great as that received by a square meter at the earth. Carrying out the figures, we find that this heat radiation at the sun's surface amounts to $1,000,000$ calories per square meter
per minute; that if the sun were frozen over completely, to a depth of 45 feet, the heat emitted would melt the shell in one minute; that if a bridge of ice could be formed from the earth to the sun by a column of ice 2.1 miles square and $93,000,000$ long, and if in some way the entire solar radiation could be concentrated upon it, it would be melted in one second, and in seven more would be dissipated in vapor.

Expressing it as energy, we find that the solar radiation is nearly 100,000 horse-power continuously for each square meter of the sun's surface.

These figures are based, of course, on the assumption that the sun radiates heat in all directions alike, and there is no reason known to science why it should not.

So far as we can see, only a minute fraction of the whole radiation ever reaches a resting place. The earth intercepts about $\frac{1}{2200000000}$, and the other planets of the solar system receive in all perhaps from ten to twenty times as much. Something like $\frac{10 \overline{10} \frac{1}{00000}}{}$ seems to be utilized within the limits of the solar system. As for the rest, science cannot yet give any certain account of it.
216. The Sun's Temperature. - As to the temperature of the sun's surface, we have no sure knowledge, except that it must be higher than that of any artificial heat. While we can measure with some accuracy the quantity of heat which the sun sends us, our laboratory experiments do not yet furnish the necessary data from which we can determine with certainty what must be the temperature of the sun's surface in order to enable it to send out heat at the observed rate.

The estimates of the temperature of the photosphere run all the way from the very low ones of some of the French physicists (who set it about $2500^{\circ} \mathrm{C}$. - higher, but not vastly higher than that of an electric arc) to those of Secchi and Ericsson, who put the figure among the millions. The prevailing opinion sets it between $5,000^{\circ}$ and $10,000^{\circ} \mathrm{C}$.; i.e., from $9,000^{\circ}$ to $20,000^{\circ}$ F. Scheiner (1899) adopts $7,000^{\circ}$ C. (about $12,000^{\circ}$ F.) as the most probable value.
217. The Burning Lens. - A most impressive demonstration of the intensity of the sun's heat lies in the fact that in the focus of a powerful burning lens all known substances melt and vaporize. Now at the focus of a lens the temperature can never more than equal that of the source from which the heat comes. Theoretically, the limit of temperature is that which would be produced by the sun's direct radiation at a distance such that the sun's apparent diameter would just equal that of the lens viewed from its focus.


Fig. 60.

The temperature produced at $F$, Fig. 60 , would, if there were no losses, be just the same as that of a body placed so near the sun that the sun's angular diameter equals $L F L^{\prime}$. Now, in the case of the most powerful lenses thus far made, a body at the focus was thus virtually carried to within about 240,000 miles from the sun's surface (about the same distance as that of the moon from the earth), and here, as has been said, the most refractory substances succumb immediately.
218. Constancy of the Sun's Heat. - By means of observations made at the same time in California, Washington, and Algiers, it has been proved beyond question that the quantity of heat radiated by the sun is not absolutely constant but may vary within a few days by as much as ten per cent. Such fluctuations would have no evident effect on climate.

As to any steady, progressive increase or decrease of solar heat, it is quite certain that no considerable change of that kind has been going on for the past 2000 years, because the distribution of plants and animals on the earth's surface is practically the same as in the days of Pliny; it is, however, rather probable than otherwise that the general climatic changes which Geology indicates as having formerly taken
place on the earth, - the glacial and carboniferous epochs, for instance, - may ultimately be traced to changes in the sun's condition.
219. Maintenance of Solar Heat. - One of the most interesting and important problems of modern science relates to the explanation of the method by which the sun's heat is maintained. We cannot here discuss the subject fully, but must content ourselves with saying first, negatively, that the phenomenon cannot be accounted for on the supposition that the sun is a hot, solid or liquid body simply cooling; nor by combustion, nor (adequately) by the fall of meteoric matter on the sun's surface, though this cause undoubtedly operates to a limited extent: second, positively, the solar radiation can be accounted for on the hypothesis first proposed by Helmholtz, that the sun is shrinking slouly but continuously. It is a matter of demonstration that an annual shrinkage of about 250 feet in the sun's diameter would liberate heat sufficient to keep up its radiation without any fall in its temperature. If the shrinkage were more than 250 feet, the sun would be hotter at the end of a year than it was at the beginning.

It is not possible to exhibit this hypothetical shrinkage as a fact of observation, since this diminution of the sun's diameter would not amount to a mile in 20 years, and nearly 10,000 years would be spent in reducing it by a single second of arc. No change much smaller than $1^{\prime \prime}$ could be certainly detected even by our most modern instruments.

We can only say that while no other theory yet proposed meets the conditions of the problem, this appears to do so perfectly, and therefore has high probability in its favor.
220. Age and Duration of the Sun. - If Helmholtz's theory is correct, it follows that in time the sun's heat must come to an end, and, looking backward, that it must have had a beginning. We have not the data for an accurate calculation of the sun's future duration, but if it keeps up its present rate of
radiation it must, on this hypothesis, shrink to about half its diameter in some $5,000,000$ years at longest. Since its mean density will then be eight times as great as now, it can hardly continue to be mainly gaseous (as it probably is at present), and its temperature must begin to fall quite sensibly. It is not, therefore, likely that the sun will continue to give heat enough to support such life on the earth as we are now familiar with, for much more than $10,000,000$ years, if it does it so long.
As to the past, we can be a little more definite. No conclusion of Geometry is more certain than this, - that the shrinkage of the sun to its present dimensions from a diameter larger than that of the orbit of Neptune, the remotest of the planets, would generate about $18,000,000$ times as much heat as the sun now radiates in a year. Hence, if the sun's heat has been and still is wholly due to the contraction of its mass, it cannot have been radiating heat at the present rate, on the shrinkage hypothesis, for more than $18,000,000$ years ; and on that hypothesis the solar system in anything like its present condition cannot be much more than as old as that.

But notice the "if." It is quite conceivable that the solar system may have received in the past other supplies of heat than that due to contraction. Radium and its congeners may have played a part.
221. Constitution of the Sun. - The received opinion as to the constitution of the sun is substantially as follows :-
(a) As to the condition of the central mass or nucleus of the sun, we cannot be said to have definite knowledge. It is probably gaseous, this being indicated by the sun's low mean density and high temperature ; but at the same time this gaseous matter must be in a very different condition from gases as we know them in our laboratories, on account of the intense heat and the enormous pressure due to the force of solar gravity. The central mass, while still possessing the characteristic properties of gas (that is, characteristic from
the scientific point of view), must be denser than water, and viscous, with a consistency something like pitch or tar.

While this doctrine as to the gaseous nature of the solar nucleus is generally assented to, there are, however, some authorities who still maintain that it is liquid.
222. (b) The Photosphere is in all probability a sheet of luminous clouds, constituted mechanically like terrestrial clouds, i.e., of minute solid or liquid particles floating in gas.

The photospheric clouds of the sun are supposed to be formed (just as snow and rain clouds are in our own atmosphere) by the cooling and condensation of vapors, and they float in the permanent gases of the solar atmosphere in the same way that our own clouds do in our own atmosphere. We do not know just what materials constitute these solar clouds, but naturally suppose them to be those indicated by the Fraunhofer lines, i.e., chiefly the metals, with carbon and its chemical congeners.
223. (c) The Reversing Layer. - The photospheric clouds float in an atmosphere containing a considerable quantity of the same vapors out of which they themselves have been formed, just as in our own atmosphere the air immediately surrounding a cloud is saturated with vapor of water. This vapor-laden atmosphere, probably comparatively shallow, constitutes the reversing layer, and by its selective absorption produces the dark lines of the solar spectrum, while by its general absorption it produces the peculiar darkening at the limb of the sun.

It will be remembered that Mr. Lockyer and others have been disposed to question the existence of any such shallow absorbing stratum, considering that the absorption takes place in all regions of the solar atmosphere up to a great elevation, but that the photographs made at the eclipse of 1896 (Art. 198) seem to establish its reality.
224. (d) The Chromosphere and Prominences are composed of permanent gases, mainly hydrogen, helium, and calcium, which
are mingled with the vapors of the reversing stratum in the region of the photosphere, but rise to far greater elevations than do the vapors. The appearances are for the most part as if the chromosphere were formed of jets of heated gases, ascending through the interspaces between the photospheric clouds, like flames playing over a coal fire.
225. (e) The Corona also rests on the photosphere, and the characteristic green line of its spectrum (Art. 207) is brightest just at the surface of the photosphere in the reversing stratum and in the chromosphere itself; but the corona extends to a far greater elevation than even the prominences ever reach, and it seems to be not entirely gaseous, but to contain, in addition the mysterious "coronium," dust and fog of some sort, perhaps meteoric. Many of the phenomena of the corona are still unexplained, and since thus far it has been observed only during the brief moments of total solar eclipses, progress in its study has been necessarily slow.

225*. Numerous attempts have been made to discover some method of observing the corona at other times than during an eclipse, but thus far without success. The corona line is not bright enough to render feasible the spectroscopic method, which succeeds with the prominences; and if it were, the fact that the streamers of the corona are probably in the main, not gaseous, but of dust-like constitution (giving therefore only the spectrum of reflected sunlight), would make the spectroscopic image, even if it could be obtained, extremely incomplete : the streamers would not appear in it at all.

Dr. Huggins, and others under his direction or following his suggestion, worked very hard for several years, beginning with 1883, in the endeavor to photograph the corona without an eclipse. Some of the plates showed, around the image of the sun, halo-forms which certainly looked very coronal. But it was soon found that plates exposed in rapid succession did not agree as to details, and there can be no doubt that the apparent "coronas" were not images of the real one. The illumination of our own atmosphere near the disc of the sun is far too brilliant to allow us, working through it, either to see, or to photograph, the much fainter corona behind it.

Various other methods have been vainly tried, and others have been planned and will be executed. The problem is by no means even yet given up as absolutely hopeless, though the prospect of success is not very promising.

As has been already said, the real nature of the corona is still problematical. In many ways it strongly resembles our terrestrial Aurora Borealis and the phenomena which, under certain circumstances, accompany electrical discharges in a partial vacuum. By many, therefore, it is regarded as something of the same sort on the solar scale of magnitude. Professor Schäberle, of the Lick Observatory, on the other hand, urges a purely "mechanical theory" which regards the corona as formed by streams or jets of some rare material ejected mainly from the sun-spot zones, repelled to planetary distances, and then falling back upon the solar surface. Many of the peculiar features of the visible corona are then easily explained as mere perspective effects due to the apparent superposition and interlacing of these ascending and descending streams.

Note to Art. 198. - At the eclipse of Jan. 22d, 1898, the results of Mr. Shackleton were fully confirmed, and even surpassed. The "flash spectrum," as it is called, was photographed by several different observers, with more powerful apparatus than his, and with complete success. It should be noted, however, that this bright line spectrum is not, and ought not to be, a mere negative of the dark line Fraunhofer spectrum. Its brightest lines do not always correspond to the darkest lines of the other, but there are great and significant differences of relative intensity: - as might be expected, since, for instance, the brilliant lines of Helium do not show at all as dark lines in the Fraunhofer spectrum, except under unusual conditions. A similar result was again obtained during the eclipses of May 28th, 1900, May 17th, 1901, and Aug. 31st, 1905.

Note to Art. 207. - It was discovered in 1898 from the photographs of the corona spectrum that the identification of the green line with Kirchhoff's " 1474 " is erroneous, and the later eclipses confirm the correction. The corona line is slightly more refrangible than "1474" ( $\lambda 5316$ ), having a wave-length of 5303 , and coincides with no dark line of the solar spectrum. The photographs also show that the spectrum of coronium, in addition to this green line, has at least six or eight others in the violet and ultra-violet region.

## CHAPTER VIII.

ECLIPSES. - FORM AND DIMENSIONS OF SHADOWS. ECLIPSES OF THE MOON. - SOLAR ECLIPSES. - TOTAL, ANNULAR, AND PARTIAL. - ECLIPTIC LIMITS AND NUMBER OF ECLIPSES IN A YEAR. - RECURRENCE OF ECLIPSES AND THE SAROS. - OCCULTATIONS.
226. The word "Eclipse" (literally a "faint" or "swoon") is a term applied to the darkening of a heavenly body, especially of the sun or moon, though some of the satellites of other planets besides the earth are also "eclipsed." An eclipse of the moon is caused by its passage through the shadow of the earth; eclipses of the sun, by the interposition of the moon between the sun and the observer, or, what comes to the same thing, by the passage of the moon's shadow over the observer.

The shadow (Physics, p. 274) is the space from which sunlight is excluded by an intervening body : geometrically speaking it is a solid, not a surface. If we regard the sun and the other heavenly bodies as spherical, these shadows are cones with their axes in the line joining the centres of the sun and the shadow-casting body, the point being always directed away from the sun.
227. Dimensions of the Earth's Shadow. - The length of the earth's shadow is easily found. In Fig. 61 we have, from the similar triangles $O E D$ and $E C a$,

$$
O D: E a:: O E: E C, \text { or } L .
$$

$O D$ is the difference between the radii of the sun and the earth, $=R-r . \quad E a=r$, and $O E$ is the distance of the earth from the sun $=D$. Hence

$$
L=D\left(\frac{r}{R-r}\right)=\frac{1}{108.5} D .
$$

(The fraction 108.5 is found by simply substituting for $R$ and $r$ their values.) This gives 857,000 miles for the length of the earth's shadow when $D$ has its mean value of 93,000000 miles. The length varies about 14,000 miles on each side of the mean, in consequence of the variation of the earth's distance from the sun at different times of the year.


Fig. 61. - The Earth's Shadow.
From the cone $a C b$ all sunlight is excluded, or would be were it not for the fact that the atmosphere of the earth by its refraction bends some of the rays into this shadow. The effect of this atmospheric refraction is to increase the diameter of the shadow about two per cent, but to make it less perfectly dark.
228. Penumbra. - If we draw the lines $B a$ and $A b$ (Fig. 61 ), crossing at $P$, between the earth and the sun, they will bound the "penumbra" within which a part, but not the whole, of the sunlight is cut off: an observer outside of the shadow but within this cone-frustum, which tapers towards the sun, would see the earth as a black body encroaching on the sun's disc.

While the boundaries of the shadow and penumbra are perfectly definite geometrically, they are not so optically. If a screen were placed at $M$, perpendicular to the axis of the shadow, no sharply defined lines would mark the boundaries of either shadow or penumbra. Near the edge of the shadow the penumbra would be very nearly as dark as the shadow itself, only a mere speck of the sun being there visible; and at the outer edge of the penumbra the shading would be still more gradual.
229. Eclipses of the Moon. - The axis or central line of the earth's shadow is always directed to a point directly opposite the sun. If, then, at the time of the full moon, the moon happens to be near the ecliptic (i.e., not far from one of the nodes of her orbit), she will pass through the shadow and be eclipsed. Since, however, the moon's orbit is inclined to the ecliptic, lunar eclipses do not happen very frequently,-seldom more than twice a year. Ordinarily the full moon passes north or south of the shadow without touching it.
Lunar eclipses are of two kinds, - partial and total: total when she passes completely into the shadow, partial when she only partly enters it, going so far to the north or south of the centre of the shadow that only a portion of her disc is obscured.
230. Size of the Earth's Shadow at the Point where the Moon crosses it. - Since $E C$, in Fig. 61, is 857,000 miles, and the distance of the moon from the earth is on the average about 239,000 miles, $C M$ must average 618,000 miles, so that $M N$, the semi-diameter of the shadow at this point, will be $\frac{618}{857}$ the earth's radius. This gives $M N=2854$ miles, and makes the whole diameter of the shadow a little over 5700 miles, about two and two-thirds times the diameter of the moon. But this quantity varies considerably; the shadow where it is crossed by the moon is sometimes more than three times her diameter, sometimes hardly more than twice.
An eclipse of the moon, when central, i.e., when the moon crosses the centre of the shadow, may continue total for about
two hours, the interval from the first contact to the last being about two hours more. This depends upon the fact that the moon's hourly motion is nearly equal to its own diameter.

The duration of a non-central eclipse varies, of course, according to the part of the shadow traversed by the moon.
231. Lunar Ecliptic Limit. - The lunar ecliptic limit is the greatest distance from the node of the moon's orbit at which the sun can be at the time of a lunar eclipse. This limit depends upon the inclination of the moon's orbit, which is somewhat variable, and also upon the distance of the moon from the earth at the time of the eclipse, which is still more variable. Hence we recognize two limits, the major and minor. If the distance of the sun from the node at the time of full moon exceeds the major limit, an eclipse is impossible; if it is less than the minor, an eclipse is inevitable. The major limit is found to be $12^{\circ} 15^{\prime}$; the minor, $9^{\circ} 30^{\prime}$. Since the sun, in its annual motion along the ecliptic, travels $12^{\circ} .15^{\prime}$ in less than 13 days, it follows that every eclipse of the moon must take place within 13 days from the time when the sun crosses the node.
232. Phenomena of a Total Lunar Eclipse. - Half an hour or so before the moon reaches the shadow, its limb begins to be sensibly darkened by the penumbra, and the edge of the shadow itself when it is first reached appears nearly black by contrast with the bright parts of the moon's surface. To the naked eye the outline of the shadow looks reasonably sharp; but even with a small telescope it is found to be indefinite, and with a large telescope and high magnifying power it becomes entirely indistinguishable, so that it is impossible to determine within half a minute or so the time when the boundary of the shadow reaches any particular point on the moon. After the moon has wholly entered the shadow, her dise is usually distinctly visible, illuminated with' a dull, cop-
per-colored light, which is sunlight, deflected around the earth into the shadow by the refraction of our atmosphere, as illustrated by Fig. 62.

Even when the moon is exactly central in the largest possible shadow, an observer on the moon would see the disc of the earth surrounded by a narrow ring of brilliant light, colored with sunset hues by the same vapors which tinge terrestrial sunsets, but acting with double power because the light has traversed a double thickness of our air. If the weather happens to be clear at this portion of the earth (upon its rim, as seen from the moon), the quantity of light transmitted through our atmosphere is very considerable, and the


Fig. 62. - Light bent into Earth's Shadow by Refraction.
moon is strongly illuminated. If, on the other hand, the weather happens to be stormy in this region of the earth, the clouds cut off nearly all the light. In the lunar eclipse of 1884 , the moon was absolutely invisible for a time to the naked eye, - a very unusual circumstance on such an occasion. During the eclipse of Jan. 28th, 1888, although the moon was pretty bright to the eye, Pickering found that its photographic power, when centrally eclipsed, was only about $\frac{1}{1400000}$ of what it was when uneclipsed.
233. Computation of a Lunar Eclipse. - Since all the phases of a lunar eclipse are seen everywhere at the same absolute instant wherever the moon is above the horizon, it follows that a single computation giving the Greenwich times of the different phenomena is all that is needed. Such computations are made and published in the Nautical Almanac. Each observer has only to correct the predicted time by simply adding or subtracting his longitude from Greenwich, in order to get the true local time. The computation of a lunar eclipse is not at all complicated, but lies rather beyond the scope of this work.

## ECLIPSES OF THE SUN.

234. Dimensions of the Moon's Shadow. - By the same method as that used for the shadow of the earth (Art. 227) we find that the length of the moon's shadow at any time is very nearly $\frac{1}{400}$ of its distance from the sun, and averages 232,150 miles. It varies not quite 4000 miles each way, ranging from 236,050 to 228,300 miles.

Since the mean length of the shadow is less than the mean distance of the moon from the earth ( 238,800 miles), it is evident that on the average the shadow will fall short of the earth. On account of the eccentricity of the moon's orbit, however, she is much of the time considerably nearer than at others, and may come within 221,600 miles from the earth's centre, or about 217,650 miles from its surface. If at the


Fig. 63. - The Moon's Shadow on the Earth.
same time the shadow happens to have its greatest possible length, its point may reach nearly 18,400 miles beyond the earth's surface. In this case the cross-section of the shadow where the earth's surface cuts it (at $O$ in Fig. 63) will be about 168 miles in diameter, which is the largest value possible. If, however, the shadow strikes the earth's surface obliquely, the shadow spot will be oval instead of circular, and the extreme length of the oval may much exceed the 168 miles.

Since the distance of the moon may be as great as 252,970 miles from the earth's centre, or nearly 249,000 miles from its surface, while the shadow may be as short as 228,300 miles, we may have the state of things indicated by placing the earth at $B$, in Fig. 63. The vertex of the shadow, $V$, will then fall

20,700 miles short of the surface, and the cross-section of the "shadow produced" will have a diameter of 196 miles at $O^{\prime}$, where the earth's surface cuts it. When the shadow falls near the edge of the earth, this cross-section may, however, be as great as 230 miles.
235. Total and Annular Eclipses. - To an observer within the true shadow cone (i.e., between $V$ and the moon, in Fig. 63) the sun will be totally eclipsed. An observer in the "produced" cone beyond $V$ will see the moon smaller than the sun, leaving an uneclipsed ring around it, and will have what is called an annular eclipse. These annular eclipses are considerably more frequent than the total, and now and then an eclipse is annular in part of its course across the earth and total in part. (The point of the moon's shadow extends in this case beyond the surface of the earth, but does not reach as far as its centre.)
236. The Penumbra and Partial Eclipses. - The penumbra can easily be shown to have a diameter on the line $C D$ (Fig. 63) of a trifle more than twice the moon's diameter. An observer situated within the penumbra has a partial eclipse. If he is near the cone of the shadow, the sun will be mostly covered by the moon, but if near the outer edge of the penumbra, the moon will only slightly encroach on the sun's disc. While, therefore, total and annular eclipses are visible as such only by an observer within the narrow path traversed by the shadow spot, the same eclipse will be visible as a partial one everywhere within 2,000 miles on each side of the path; and the 2,000 miles is to be reckoned perpendicularly to the axis of the shadow, and may correspond to a much greater distance on the spherical surface of the earth.

## 237. Velocity of the Shadow and Duration of Eclipses. -

 Were it not for the earth's rotation, the moon's shadow would pass an observer at the rate of nearly 2100 miles an hour.The earth, however, is rotating towards the east in the same general direction as that in which the shadow moves, and at the equator its surface moves at the rate of about 1040 miles an hour. An observer, therefore, on the earth's equator with the moon near the zenith would be passed by the shadow with a speed of about 1060 miles an hour ( $2100-1040$ ), and this is the shadow's lowest velocity, - about equal to that of a can-non-ball. In higher latitudes, where the surface velocity due to the earth's rotation is less, the relative speed of the shadow is higher ; and where the shadow falls very obliquely, as it does when an eclipse occurs near sunrise or sunset, the advance of the shadow on the earth's surface may become very swift, as great as 4000 or 5000 miles an hour.

A total eclipse of the sun observed at a station near the equator, under the most favorable conditions possible, may continue total for $7^{\mathrm{m}} 58^{\mathrm{s}}$. In latitude $40^{\circ}$, the duration can barely equal $6 \frac{1}{4}^{\mathrm{m}}$. The greatest possible excess of the radius of the moon over that of the sun is only $1^{\prime} 19^{\prime \prime}$.

At the equator an annular eclipse may last for $12^{\mathrm{m}} 24^{\mathrm{s}}$, the maximum width of the ring of the sun visible around the moon being $1^{\prime} 37^{\prime \prime}$.

In the observation of an eclipse four contacts are recognized: the first when the edge of the moon first touches the edge of the sun, the second when the eclipse becomes total or annular, the third at the cessation of the total or annular phase, and the fourth when the moon finally leaves the solar disc. From the first contact to the fourth the time may be a little over four hours.
238. The Solar Ecliptic Limits. - It is necessary, in order to have an eclipse of the sun, that the moon should encroach on the cone $A C B D$ (Fig. 64), which envelopes the earth and sun. In this case the true angular distance between the centres of the sun and moon, i.e., their distance as seen from the centre of the earth, would be the angle MES. This angle may range from $1^{\circ} 34^{\prime} 13^{\prime \prime}$ to $1^{\circ} 24^{\prime} 19^{\prime \prime}$, according to the changing distance of the sun and moon from the earth. The corresponding distances of the sun from the node, taking
into account the variations in the inclination of the moon's orbit, give $18^{\circ} 31^{\prime}$ and $15^{\circ} 21^{\prime}$ for the major and minor ecliptic limits.

In order that an eclipse may be central (total or annular) at any part of the earth, it is necessary that the moon should lie wholly inside the cone $A C B D$, as $M^{\prime}$, and the corresponding major and minor central ecliptic limits come out $11^{\circ} 50^{\prime}$ and $9^{\circ} 55^{\prime}$.


Fig. 64. - Solar Ecliptic Limits.
239. Phenomena of a Solar Eclipse. - There is nothing of special interest until the sun is mostly covered, though before that time the shadows cast by the foliage begin to be peculiar.

The light shining through every small interstice among the leaves, instead of forming as usual a circle on the ground, makes a little crescent - an image of the partly covered sun.

About ten minutes before totality the darkness begins to be felt, and the remaining light, coming, as it does, from the edge of the sun alone, is much altered in quality, producing an effect very like that of a calcium light, rather than sunshine. Animals are perplexed and birds go to roost. The temperature falls, and sometimes dew appears. In a few moments, if the observer is so situated that his view commands the distant horizon, the moon's shadow is seen coming, much like a heavy thunder-storm, and advancing with almost terrifying swiftness. As soon as the shadow arrives, and sometimes a little before, the corona and prominences become visible, while the brighter planets and the stars of the first three magnitudes
make their appearance. The suddenness with which the darkness pounces upon the observer is startling. The sun is so brilliant that even the small portion which remains visible up to within a very few seconds of the total obscuration so dazzles the eye that it is unprepared for the sudden transition. In a few moments, however, vision adjusts itself, and it is then found that the darkness is not really very intense.

If the totality is of short duration (that is, if the diameter of the moon exceeds that of the sun by less than a minute of arc), the corona and chromosphere, the lower parts of which are very brilliant, give a light at least three or four times that of the full moon. Since, moreover, in such a case the shadow is of small diameter, a large quantity of light is also sent in from the surrounding air, where, 30 or 40 miles away, the sun is still shining; and what may seem remarkable, this intrusion of outside light is greatest under a cloudy sky. In such an eclipse there is not much difficulty in reading an ordinary watch-face. In an eclipse of long duration, say five or six minutes, it is much darker, and lanterns become necessary.
240. Observation of an Eclipse. - A total solar eclipse offers opportunities for numerous observations of great importance which are possible at no other time, besides certain others which can also be made during a partial eclipse. We mention
(a) Times of the four contacts, and direction of the line joining the "cusps" of the partially eclipsed sun. These observations determine with extreme accuracy the relative positions of the sun and moon at the moment. (b) The search for intra-Mercurial planets. (c) Observations of certain peculiar dark fringes which appear upon the surface of the earth at the moment of totality. (d) Photometric measurement of the intensity of light at different stages of the eclipse. (e) Telescopic observations of the details of the prominences and of the corona. ( $f$ ) Spectroscopic observations (both visual and photographic) upon the spectra of the lower atmosphere of the sun, of the prominences, and of the corona. ( $g$ ) Observations with the polariscope upon the polarization of the light of the corona. ( $h$ ) Drawings and photographic pictures of the corona and prominences.
241. Calculation of a Solar Eclipse. - The calculation of a solar eclipse cannot be dealt with in any such summary way as that of a lunar eclipse, because the times of contact and other phenomena are different at every different station. Moreover, since the phenomena of a solar eclipse admit of extremely accurate observation, it is necessary to take account of numerous little details which are of no importance in lunar eclipses. The Nautical Almanacs give, three years in advance, a chart of the track of every solar eclipse, and with it data for the accurate calculation of the phenomena at any given place.

Oppolzer, a Viennese astronomer, lately deceased, published a few years ago a remarkable book, entitled " The Canon of Eclipses," containing the elements of all eclipses ( 8,000 solar and 5,200 lunar) occurring between the year 1207 в.c. and 2162 A.D., with maps showing the approximate track of the moon's shadow on the earth. It indicates total eclipses visible in the United States in 1900, 1918, 1923, 1925, 1945, 1979, 1984, and 1994 (Philippines not included).
242. Number of Eclipses in a Year. - The least possible number is two, both of the sun; the largest, seven, five solar and two lunar, or four solar and three lunar.
The eclipses of a given year always take place at two opposite seasons (which may be called the "eclipse months" of the year), near the times when the sun crosses the nodes of the moon's orbit. Since the nodes move westward around the ecliptic once in about 19 years (Art. 142), the time occupied by the sun in passing from a node to the same node again is only 346.62 days, which is sometimes called the "eclipse year."

In an eclipse year there can be but two lunar eclipses, since twice the maximum lunar ecliptic limit $\left(2 \times 12^{\circ} 15^{\prime}\right)$ is less than the distance the sun moves along the ecliptic in a synodic month $\left(29^{\circ} 6^{\prime}\right)$; the sun therefore cannot possibly be near enough the node at both of two successive full moons ; on the other hand, it is possible for a year to pass without any lunar eclipse, the sun being too far from the node at all four of the full moons which occur nearest to the time of its node-passage.

In a calendar year (of $365 \frac{1}{4}$ days) it is, however, possible to have three lunar eclipses. If one of the moon's nodes is passed by the sun in January, it will be reached again in December, the other node having been passed in the latter part of June, and there may be a lunar eclipse at or near each of these three node-passages. This actually occurred in 1898 and 1917.

As to solar eclipses, the solar ecliptic'limits are so much larger than the lunar, that there must be at least one solar eclipse at each node-passage of the year; and there may be two, thus making four in the eclipse year. (When there are two solar eclipses at the same node, there will always be a lunar eclipse at the full moon between them.) In the calendar year, a fifth solar eclipse may come in if the first eclipse month falls in January. Since a year with five solar eclipses in it is sure to have two lunar eclipses in addition, they will make up seven in the calendar year. This will happen next in 1935. In 1917 there were seven eclipses, four solar and three lunar.
243. Frequency of Solar and Lunar Eclipses. - Taking the whole earth into account, the solar eclipses are the more numerous, nearly in the ratio of three to two. It is not so, however, with those which are visible at a given place. A solar eclipse can be seen only from a limited portion of the globe, while a lunar eclipse is visible over considerably more than half the earth, - either at the beginning or the end, if not throughout its whole duration; and this more than reverses the proportion between lunar and solar eclipses for any given station.

Solar eclipses that are total somewhere or other on the earth's surface are not very rare, averaging one for about every year and a half. But at any given place the case is very different: since the track of a solar eclipse is a very narrow path over the earth's surface, averaging only 60 or 70 miles in width, we find that in the long run a total eclipse happens at any given station only once in about 360 years.

During the 19th century seven shadow tracks have traversed the United States, the last on May 28th, 1900. A total eclipse was seen in the West in 1918, and there will be one in New England in 1925.
244. Recurrence of Eclipses; the Saros. - It was known to the Chaldeans, even in prehistoric times, that eclipses occur at a regular interval of 18 years and $11 \frac{1}{3}$ days ( $10 \frac{1}{3}$ days if there happen to be five leap years in the interval). They named this period the "Saros." It consists of 223 synodic months, containing 6585.32 days, while 19 "eclipse years" contain 6585.78 . The difference is only about 11 hours, in which time the sun moves on the ecliptic about $28^{\prime}$. If, therefore, a solar eclipse should occur to-day with the sun exactly at one of the moon's nodes, at the end of 223 months the new moon will find the sun again close to the node ( $28^{\prime}$ west of it), and a very similar eclipse will occur again; but the track of this new eclipse will lie about 8 hours of longitude further west on the earth, because the 223 months exceed the even 6585 days by $\frac{32}{100}$ of a day, or 7 hours, 42 minutes. The usual number of eclipses in a Saros is about 71, varying two or three one way or the other.

In the Saros closing Dec. 22d, 1889, the total number was $72,-29$ lunar and 43 solar. Of the latter, 29 were central ( 13 total, 16 annular), and 14 were only partial. The following may be given as an example of the recurrence of eclipses at the end of a Saros: The four eclipses of 1878 occurred (1) on Feb. 2d, solar, annular ; (2) Feb. 17 th, lunar, partial ; (3) July 29th, solar, total ; (4) Aug. 12th, lunar, partial. In 1896 the corresponding eclipses were Feb. 13th, solar, annular ; Feb. 28th, lunar, partial ; Aug. 9th, solar, total ; Aug. 23d, lunar, partial. It is usual to speak of the eclipse of Aug. 9th, 1896, for instance, as a recurrence of the eclipse of July 29, 1878, one Saros period earlier.
245. Occultations of the Stars. - In theory and computation, the occultation of a star is identical with an eclipse, except that the shadow of the moon cast by the star is sensibly $a$ cylinder, instead of a cone, and has no penumbra. Since the moon always moves eastward, the star disappears at the moon's eastern limb, and reappears on the western. Under all ordinary circumstances, both disappearance and reappearance are
instantaneous, indicating not only that the moon has no sensible atmosphere, but also that the (angular) diameter of even a very bright star is less than $0^{\prime \prime} .02$. Observations of occultations determine the place of the moon in the sky with great accuracy, and when made at a number of widely separated stations they furnish a very precise determination of the moon's parallax and also of the difference of longitude between the stations.
246. Anomalous Phenomena. - Occasionally, the star, instead of disappearing suddenly when struck by the moon's limb (faintly visible by "earth-shine"), appears to cling to the limb for a second or two before vanishing. In a few instances it has been reported as having reappeared and disappeared a second time, as if it had been for a moment visible through a rift in the moon's crust. In some cases the anomalous phenomena have been explained by the subsequent discovery that the star was double, but many of them still remain mysterious, though it is quite likely that they were often illusions due to physiological causes in the observer.

## CHAPTER IX.

## CELESTIAL MECHANICS.

THE LAWS OF CENTRAL FORCE. - CIRCULAR MOTION. KEPLER'S LAWS, AND NEWTON'S VERIFICATION OF THE THEORY OF GRAVITATION. - THE CONIC SECTIONS. - THE PROBLEM OF TWO BODIES. - THE PROBLEM OF THREE BODIES AND PERTURBATIONS. - THE TIDES.

It is, of course, out of the question to attempt in the present work any extended treatment of the theory of the motion of the heavenly bodies; but quite within the reach of those for whom this volume is designed there are certain fundamental facts and principles, so important, and in fact, essential to an intelligent understanding of the mechanism of the solar system, that we cannot pass them in silence.
247. Motion of a Body Free from the Action of any Force. According to the first law of motion (Physics, p. 55) a moving body left to itself describes a straight line with a uniform velocity. If we find a body so moving, we may infer, therefore, that itis either acted on by no force whatever, or, if forces are acting upon it, that they exactly balance each other.

It is usual with some writers to speak of a body thus moving uniformly in a straight line as actuated by a "projectile force"; a most unfortunate expression, against which we wish to protest vigorously. It is a survival of the old Aristotelian idea that rest is more " natural" to matter than motion, and that when a body moves, force must operate to keep it moving. This is not true. Not motion, but the change of motion, either in speed or in direction, - this alone implies the action of "force."
248. Motion under the Action of a Force. - If the motion of a body is in a straight line but with varying speed, we infer a force acting exactly in the line of motion. If the body moves in a curve, we know that some force is acting across the line


Fig. 65. - Curvature of an Orbit. of motion: if the velocity keeps increasing, we know that the direction of the force that acts is forward, like $a b$ in Fig. 65, making an angle of less than $90^{\circ}$ with the "line of motion" ${ }^{1} a t$; and vice versa, if the motion of the body is retarded. If the speed is constant, we know that the force must continually act on the line $a c$ exactly perpendicular to the line of motion.

- Here, also, we find many writers, the older ones especially, bringing in the "projectile force," and saying that when a body moves in a curve, it does so "under the action of two forces; one the force that draws it sideways, the other the 'projectile force' directed along its path." We repeat, this "projectile force" has no present existence nor meaning in the problem of a body's motion. Such a force may have put the body in motion long ago, but its function has ceased, and now we have only to do with the action of one single force, - the deflecting force, which alters the direction and velocity of the body's motion. From a curved orbit we can only infer the necessary existence of one force. We do not mean to say that this force may not be the "resultant" of several ; it often is; but a single force is all that is necessary to produce motion in a curve.
> 249. Law of Equal Areas in the Case of a Body moving under the Action of a Force directed towards a Fixed Point. - In this case it is easy to prove that the path of the body will be a curve (not necessarily a circle), "concave" towards the centre of force; that it will all lie in one plane, and that the

[^25]body will move in such a way that its "radius vector" (the line that joins the body to the centre of force) will describe areas proportional to the time.

Thus in Fig. 66, the areas $a S b, c S d$, and $e S f$, are all equal; the arcs $a b, c d$, and $e f$, being each described in a unit of time, under the action of a "central" force, always directed towards $S$.

We have already seen that as a matter of fact the earth obeys this law in moving around the sun (Art. 118), and the moon in her orbit around the earth. Newton showed that if the force which keeps them in their orbits acts always along the radius vector, they must do so of necessity. See Appendix, Art. 502.
250. Circular Motion. - In the special case when the path of a body is a circle described under the action of a force directed to its centre, its velocity is constant. The force, moreover, is constant, and in works upon Mechanics (see Physics, p. 63; but in the Physics the letter $a$ replaces our $f$ ) is easily proved to be given by the formula

$$
\begin{equation*}
f=\frac{V^{2}}{r} \tag{a}
\end{equation*}
$$

In this formula, $V$ is the velocity in feet per second, while $r$ is the radius of the circle, and $f$ is the central force measured by the "acceleration" of the body towards the centre, ${ }^{1}$ - just as the force of gravity is expressed by the quantity $g$ ( $32 \frac{1}{6}$ feet, or 386 inches).

For many purposes it is desirable to have a formula which shall substitute for $V$ (which is not given directly by observation) the time
${ }^{1}$ If we want the central force in pounds, the formula becomes

$$
f \mathrm{lbs} .=W \times \frac{V^{2}}{g r}, W \text { being the weight of the body in pounds. }
$$

of revolution, $t$, which is so given. Since $V$ equals the circumference of the circle divided by $t$, or $\frac{2 \pi r}{t}$, we have at once, by substituting this value for $V$ in equation ( $a$ ),

$$
\begin{equation*}
f=4 \pi^{2} \times \frac{r}{t^{2}}, \tag{b}
\end{equation*}
$$

an expression to which we shall frequently refer.

## KEPLER'S LAWS.

251. Early in the 17 th century Kepler discovered, as unexplained facts, three laws which govern the motions of the planets, - laws which still bear his name. He worked them out from a discussion of the observations which Tycho Brahe had made through many preceding years. They are as follows :-
I. The orbit of each planet is an ellipse with the sun in one of its foci. See Art. 117.
II. The radius vector of each planet describes equal areas in equal times.
III. The squares of the periods of the planets are proportional to the cubes of their mean distances from the sun; i.e., $\dot{t}_{1}{ }^{2}: t_{2}{ }^{2}:: \alpha_{1}{ }^{3}: \alpha_{2}{ }^{3}$. This is the so-called "Harmonic law."
252. To make sure that the student apprehends the meaning and scope of this third law, we append a few simple examples of its application.
253. What would be the period of a planet having a mean distance from the sun of one hundred astronomical units; i.e., a distance a hundred times that of the earth?

$$
\begin{gathered}
1^{3}: 100^{3}=1^{2}(\text { year }): X^{2} \\
\text { whence, } X(\text { in years })=\sqrt{100^{3}}=1000 \text { years. }
\end{gathered}
$$

2. What would be the distance from the sun of a planet having a period of 125 years?
$1^{2}$ (year) : $125^{2}=1^{3}: X^{3} ;$ whence $X=\sqrt[3]{125^{2}}=25$ astron. units.
3. What would be the period of a satellite revolving close to the earth's surface?
$(\text { Moon's Dist. })^{3}:(\text { Dist. of Satellite })^{3}=(27.3 \text { days })^{2}: X^{2}$,

$$
\begin{gathered}
\text { or, } 60^{3}: 1^{3}=27.3^{2}: X^{2} ; \\
\text { whence, } X=\frac{27.3 \text { days }}{\sqrt{60^{3}}}=1^{\mathrm{h}} 24^{\mathrm{m}} .
\end{gathered}
$$

253. Many surmises were early made as to the physical meaning of these laws. More than one person guessed that a force directed towards the sun might be the explanation. Newton proved it. He demonstrated the law of equal areas and its converse as necessary consequences of the laws of motion under a central force. He also demonstrated that if a body moves in an ellipse, having the centre of force at its focus, then the force at different points in the orbit must vary inversely as the square of the radius vector at those points : and finally, he proved that, granting the "harmonic law," the force from planet to planet must also vary according to the same law of inverse squares. See Appendix, Art. 503.
254. Inferences from Kepler's Laws. - From Kepler's laws we are therefore entitled to infer, as Newton proved, First (from the second law), that the force which retains the planets in their orbits is directed towards the sun.

Second (from the first law), that the force which acts on any given planet varies inversely as the square of its distance from the sun.

Third (from the "harmonic law"), that the force is the same for one planet as it would be for another in the same place; or, in other words, the attracting force depends only on the mass and distance of the bodies concerned, and is sensibly independent of their physical conditions (such as temperature, chemical constitution, etc.). It makes no sensible difference in the motion of a planet around the sun whether it is large or
small, hot or cold, made of hydrogen or iron ; so far at least as we are yet able to detect.

## 255. Verification of the Theory of Gravitation by the Moon's

 Motion. - When Newton first conceived the idea of "universal gravitation" (Art. 99), he at once saw that the moon's motion around the earth ought to furnish a test of the theory. Since the moon's distance (as was even then well known) is about sixty times the earth's radius, the force with which it is "attracted" by the earth, on the hypothesis of gravitation, ought to be $\frac{1}{60^{2}}$, or $\frac{1}{3600}$, of what it would be if the moon were at the surface of the earth. Now, at the earth's surface, a body falls 193 inches in the first second. The moon, then, ought to fall towards the earth $\frac{1}{3600}$ of 193 inches, or 0.0535 inches per second. Calculation shows that it really does ${ }^{1}$ (see Appendix, Art. 505).The reader will notice, however, that the agreement between the moon's "fall per second" and the $\frac{193}{3600}$ of an inch does not establish the idea of gravitation; it only makes it probable. It is quite conceivable that the coincidence in this one case might be accidental. On the other hand, discordance would be absolute disproof. The complete demonstration of the law of gravitation lies in its entire accordance, not with one fact, but with a countless multitude of them, and in its freedom from a single contradiction shown by the most refined observations.

[^26]256. Newton did not rest with merely showing that the motion of the planets and of the moon could be explained by the law of gravitation, but he also solved the more general inverse problem, and determined what kind of motion is required by that law. He found that the orbit of a body moving around a central mass under the law of gravitation, is not, of necessity a circle, nor even an ellipse of slight eccentricity like the planetary orbits. But it must be a "conic." Whether it will be circle, ellipse, parabola, or hyperbola, depends on circumstances.

For a brief description of these "conics," or " conic sections" and the way in which they are formed by the cutting of a cone, see Appendix, Art. 506.
257. The Ellipse, Parabola, and Hyperbola. - Fig. 67 shows the appearance and relation of these curves as drawn upon a plane. The ellipse is a "closed curve" returning into itself,


Fig. 67. - The Relation of the Conics to each other.
and in it the sum of the distances of any point, $N$, from the two foci equals the major axis ; i.e., $F N+F^{\prime \prime} N=P A$. The hyperbola does not return into itself, but the two branches $P N^{\prime}$
and $P n^{\prime \prime}$ go off into infinity, becoming nearly straight and diverging from each other at a definite angle. In the hyperbola it is the difference of two lines drawn from any point on the curve to the two foci which equals the major axis: i.e., $F^{\prime \prime} N^{\prime}-F N^{\prime}=P A^{\prime}$.

The parabola, like the hyperbola, fails to return into itself, but its two branches, instead of diverging, become more and more nearly parallel. It has but one focus and no major axis; or rather, to speak more mathematically, it may be regarded either as an ellipse with its second focus $F^{\prime}$ removed to an infinite distance, and therefore, having an infinite major axis; or with equal truth it may be considered as an hyperbola, of which the second focus $F^{\prime \prime}$ is pushed indefinitely far in the opposite direction, so that it has an infinite negative major axis.
In the ellipse, the eccentricity $\left(\frac{F C}{P C}\right)$ is less than unity. In the hyperbola it is greater than unity $\left(\frac{F C^{\prime \prime}}{P C^{\prime}}\right)$. In the parabola it is exactly unity; in the circle, zero.

The eccentricity of a conic determines its form. All circles therefore have the same shape, and so do all parabolas : parabolas (when complete) differ from each other only in size.
258. The Problem of Two Bodies. - This problem, proposed and completely solved by Newton, may be thus stated: -

Given the masses of two spheres, and their positions and motions at any moment ; given also the law of gravitation: required the motion of the bodies ever afterwards, and the data necessary to compute their place at any future time.

The mathematical methods by which the problem is solved require the use of the Calculus, and must be sought in works on analytical mechanics and theoretical astronomy, but the general results are easily understood and entirely within the grasp of our readers.

In the first place the motion of the centre of gravity of the two bodies is not in the least affected by their mutual attraction.

In the next place, the two bodies will describe as orbits around their common centre of gravity two curves precisely similar in form, but of size inversely proportional to their masses, the form and dimensions of the two orbits being determined by the masses and velocities of the two bodies.

If, as is generally the case, the two bodies differ greatly in mass, it is convenient to ignore the centre of gravity entirely, and to consider simply the relative motion of the smaller one around the centre of the other. It will move with reference to that point precisely as if its own mass, $m$, had been added to the principal mass, $M$, while it had become itself a mere particle. This relative orbit will be precisely like the orbit which $m$ actually describes around the centre of gravity, except that it will be magnified in the ratio of $(M+m)$ to $M$; i.e., if the mass of the smaller body is $\frac{1}{100}$ of the larger oue, its relative orbit around $M$ will be just one per cent larger than its actual orbit around the common centre of gravity of the two.
259. Finally, the orbit will always be a "conic," i.e., an ellipse or an hyperbola; but which of the two it will be, depends on three things, viz., the united mass of the two bodies ( $M+m$ ), the distance $r$ between $m$ and $M$ at the initial moment, and the velocity, $V$, of $m$ relative to $M$. If this velocity, $V$, be less than a certain critical value (which depends only on $(M+m)$ and $r$ ), the orbit will be an ellipse; if greater, ${ }^{1}$ it will be an hyperbola. (See Appendix, Art. 507*.)
The direction of the motion of $m$ with respect to $M$, while it has influence upon the form and position of the orbit (its "eccentricity") has nothing to do with determining its species; and semi-major axis; nor its period in case the orbit is elliptic: these are all independent of the direction of $m$ 's motion.

[^27]The problem is completely solved. From the necessary initial data corresponding to a given moment we can determine the position of the two bodies for any instant in the eternal past or future, provided only that no force except their mutual attraction acts upon them in the time covered by the calculation.
260. Intensity of Solar Attraction. - The attraction between the sun and the earth from some points of view appears like a very feeble action. It is only able to' deflect the earth from a rectilinear course about $\frac{1}{9}$ of an inch in a second, during which time she travels more than 18 miles: and yet if it were attempted to replace by bands of steel the invisible gravitation which draws or pushes the earth towards the sun, it would be necessary to cover the whole surface of the earth with steel wires as large as telegraph wires, and only about half an inch apart from each other, in order to get a connection that could stand the strain. Such a ligament of wires would be stretched almost to the breaking point. The attraction between the sun and the earth, expressed as tons of force (not tons of mass, of course) is $3,600,000$ millions of millions of tons ( 36 with 17 ciphers). Similar stresses are acting through apparently empty space in all directions.

We renew, also, the caution that the student must not think that the word "attraction" implies any explanation whatever, or any understanding of the "force" that tends to make two masses approach each other (see Art. 100). But this does not at all affect the proof and certainty of its existence.

THE PROBLEM OF THREE BODIES: PERTURBATIONS AND THE TIIDES.
261. As has been said, the problem of two bodies is completely solved; but if instead of two spheres attracting each other we have three or more, the general problem of determining their motions and predicting their positions transcends the present power of human mathematics.
"The problem of three bodies" is in itself as determinate and capable of solution as that of two. Given the initial data, i.e., the masses, positions, and motions of the three bodies at a given instant; then, assuming the law of gravitation, their motions for all the future, and the positions they will occupy at any given date are absolutely predetermined. Our present resources of calculation are, however, inadequate.

But while the general problem of three bodies is intractable, all the particular cases of it which arise in the consideration of the motions of the moon and of the planets, have already been practically solved by special devices, Newton himself leading the way; and the strongest proof of the truth of his theory of gravitation lies in the fact that it not only accounts for the regular elliptic motions of the heavenly bodies, but also for their apparent irregularities.
262. It is quite beyond the scope of this work to discuss the methods by which we can determine the so-called "disturbing" forces and the effects they produce upon the otherwise elliptical motion of the moon or of a planet. We wish here to make only two or three remarks.

First, that the "disturbing force" of a third body upon two which are revolving around their common centre of gravity is not the whole attraction of the third body upon either of the two; but is generally only a small component of that attraction. It depends upon the difference of the two attractions exerted by the third body upon each of the pair whose relative motions it disturbs, - a difference either in intensity, or in direction, or in both. If, for instance, the sun attracted the moon and earth alike and in parallel lines, it would not disturb the moon's motion around the earth in the slightest degree, however powerful its attraction might be. The sun always attracts the moon more than twice as powerfully as the earth does; but the sun's disturbing force upon the moon when at its very maximum is only $\frac{1}{90}$ of the earth's attraction.

The tyro is apt to be puzzled by thinking of the earth as fixed while the moon revolves around it; he reasons, therefore, that at the time of new moon, when the moon is between the earth and sun, the sun would necessarily pull her away from us if its attraction were really double that of the earth : and it would do so if the earth were fixed. We must think of the earth and moon as both free to move, like chips floating on water, and of the sun as attracting them both with nearly equal power, - the nearer of the two a little more strongly, of course.
263. Second, it is only by a mathematical fiction that the "disturbed body" is spoken of as "moving in an ellipse": it does not really do so. The path of the moon, for instance, never returns into itself. But it is a great convenience for the purposes of computation to treat the subject as if the orbit were a material wire always of truly elliptical form, having the moving body strung upon it like a bead, this orbit being continually pulled about and changed in form and size by the action of the disturbing forces, taking the body with it of course in all these changes. This imaginary orbit at any moment is a true ellipse of determinable form and position, but is constantly changing. It is in this sense that we speak of the eccentricity of the moon's orbit as continually varying, and its lines of apsides and nodes as revolving.

The student must be careful, however, not to let this wire theory of orbits get so strong a hold upon the imagination that he begins to think of the orbits as material things, liable to collision and destruction. An orbit is simply, of course, the path of a body, like the track of a ship upon the ocean.
264. Third, the " disturbances" and " perturbations" are such only in a technical sense. Elliptical motion is no more natural or proper to the moon or to a planet than its actual motion is; nor in a philosophical sense is the pure elliptical motion any more regular (i.e., "rule-following") than the socalled disturbed motion.

We make the remark because we frequently meet the notion that the so-called "perturbations" of the heavenly bodies are imperfections and blemishes in the system. One good old theologian of our acquaintance used to maintain that they were a consequence of the fall of Adam.
265. Lunar Perturbations. - The sun is the only body which sensibly disturbs the motion of the moon. The disturbing force due to the solar attraction is continually changing its direction and intensity, and to attempt to trace out its effects would take us far beyond our purpose. We may say, however, that on the whole the sun's action upon the moon slightly diminishes the effect of the earth's attraction (by about $\frac{1}{3} \frac{1}{0}$ ) and so makes the month very nearly an hour longer than it otherwise would be.

Moreover, the continual variations in the disturbing force are answered by a correspondingly continual writhing and squirming of the lunar orbit, which introduces into the lunar motions an almost countless number of so-called "inequalities."

One or two of the largest of them, those especially which affect the time of eclipses, were discovered before the time of Newton; but it is only within the last hundred years that the "lunar theory" has been brought to anything like perfection, and it is by no means "finished" yet: the actual place of the moon still sometimes differs from the almanac place by as much as $5^{\prime \prime}$, or say about five miles. In the calculation of this almanac place, over a hundred separate "inequalities " are now taken account of.

## THE TIDES.

ת66. Just as the disturbing force of the sun modifies the intensity and direction of the earth's attraction on the moon, so the disturbing forces due to the attractions of the sun and moon act $\dot{j}$-non the liquid portions of the earth to modify the intensity $\dot{\sim} \underset{\mathfrak{\tau}}{\star t}$ direction of gravity and generate the Tides.

These consist in a regular rise and fall of the ocean surface, generally twice a day, the average interval between corresponding high waters on successive days at any given place being 24 hours, 51 minutes. This is precisely the same as the average interval between two successive passages of the moon across the meridian, and the coincidence, maintained indefinitely, of itself makes it certain that there must be some causal connection between the moon and the tides: as some one has said, the odd 51 minutes is "the moon's ear-mark."

That the moon is largely responsible for the tides is also shown by the fact that when the moon is in perigee, i.e., at the nearest point to the earth, they are nearly 20 per cent higher than when she is in apogee. The highest tides of all happen when the new or full moon occurs at the time when the moon is in perigee, especially if this perigeal new or full moon occurs about the first of January, when the earth is also nearest to the sun.
267. Definitions. - While the water is rising it is "flood" tide; when falling, it is "ebb." It is "high water" at the moment when the water level is highest, and "low water" when it is lowest. The "spring tides" are the largest tides of the month, which occur near the times of new and full moon, while the "neap tides" are the smallest, and occur at half moon, the relative heights of spring and neap tides being about as 8 to 3 . At the time of the spring tides, the interval between the corresponding tides of successive days is less than the average, being only about 24 hours, 38 minutes (instead of 24 hours, 51 minutes) and then the tides are said to "prime." At the neap tides, the interval is greater than the mean - about 25 hours, 6 minutes, and the tide "lags." The "establishment" of a port is the mean interval between the time of high water at that port and the next preceding passage of the moon across the meridian. The "establishnent" of New York, for instance, is 8 hours, 13 minutes; but the actual
interval between the moon's transit and high water varies nearly half an hour on each side of this mean value at different times of the month, and under varying conditions of the weather.
268. The Tide-Raising Force. - If we consider the moon alone, it appears that the effect of her attraction upon the earth, regarded as a liquid globe, is to distort the sphere into a slightly lemon-shaped form, with its long diameter pointing to the moon, raising the level of the water about two feet, both directly under the moon and on the opposite side of the earth (at $A$ and $B$, Fig. 68), and very slightly depressing it on the whole great circle which lies half way between $A$ and $B . \quad D$ and $E$ are two points on this circle of depression.

Students seldom find any difficulty in seeing that the moon's attraction ought to raise the level at $A$; but they


Fig. 68.-The Tides. often do find it very hard to understand why the level should also be raised at $B$. It seems to them that it ought to be more depressed just there than anywhere else. The mystery to them is how the moon, when directly under foot, can exert a lifting force such as would diminish one's weight. The trouble is that the student thinks of the solid part of the earth as fixed with reference to the moon, and the water alone as free to move. If this were the case, he would be entirely right in supposing that at $B$ gravity would be increased by the earth's attraction, instead of diminished; the earth, however, is not fixed. but perfectly free to move.
269. Explanation and Calculation of the Diminution of Gravity at the Point opposite the Moon. - Consider three par-
ticles, Fig. 69, at $B, C$ and $A$, moving with equal velocities, $A a, B b$, and $C c$, but under the action of the moon, which attracts $A$ more powerfully than $C$, and $B$ less so. Then if the particles have no bond of connection, at the end of a unit of time they will be at $B^{\prime}, C^{\prime \prime}$, and $A^{\prime}$, having followed the curved paths indicated. But since $A$ is nearest the moon, its path will be the most curved of the three, and that of $B$ the least curved. It is obvious, therefore, that the distances of both $B$ and $A$ from $C$ will have been increased; and if they were connected to $C$ by an elastic cord, the cord would be stretched, both $A$ and $B$ being relatively pulled away from $C$, by practically the same amount. We say relatively, because $C$ is really pulled away from $B$, rather than $B$ from $C,-C$ being more attracted by the moon than $B$ is; but the two are separated all the same, and that is the point.
270. The Amount of the Moon's Tide-Raising Force. - When the moon is either in the zenith or nadir, the weight of a body at the earth's surface is diminished by about one part in eight and a half millions, or one pound in four thousand tons.

At a point which has the moon on its horizon, it can be shown that gravity is increased by just half as much, or about one seventeen-millionth.

The computation of the moon's lifting force at $A$ and $B$ (Fig. 68) is as follows : The distance of the moon from the earth's centre is 60 earth radii, so that the distances from $A$ and $B$ are 59 and 61 respectively. The moon's mass is about $\frac{1}{80}$ of the earth's. Taking $g$ for the force of gravity at the surface of the earth, we have, therefore, attraction of moon on $A=\frac{g}{80 \times 59^{2}}$, attraction on $C=\frac{g}{80 \times 60^{2}}$, and attraction on $B=\frac{g}{80 \times 61^{2}}$. From this we find,

$$
(A-C)=\frac{g}{8424000}, \text { and }(C-B)=\frac{g}{8835000} .
$$

Several attempts have been made within the last twenty years to detect this variation of weight by direct experiment, but so far unsuccessfully. The variations are too small.

The moon's attraction also produces everywhere except at $A, B, D$, and $E$ (Fig. 68) a tangential force which urges the particles along the surface towards the line $A B$, and this force actually preponderates in the tide-making.
271. The Sun's Tide-Producing Force. - The sun acts precisely as the moon does, but being nearly four hundred times as far away, ${ }^{1}$ its tidal action, notwithstanding its enormous mass, is less than that of the moon in the proportion (nearly) of 5 to 11 . At new and full moon, the tidal forces of the sun and moon conspire, and we then have the spring tides; while at quadrature they are opposed and we get the neap tides. The relative height of the spring and neap tides has already been stated as about 8 to 3 (i.e., $11+5: 11-5$ ).
272. The Motion of the Tides. - If the earth were wholly composed of water, and if it kept always the same face towards the moon (as the moon does towards the earth) so that every particle on the earth's surface were always subjected to the same disturbing force from the moon; then, leaving out of account the sun's action for the present, a permanent tide would be raised upon the earth as indicated in Fig. 68. The difference between the level at $A$ and $D$ would in this case be a little less than two feet.

[^28]Suppose, now, the earth to be put in rotation. It is easy to see that the two tidal waves $A$ and $B$ would move over the earth's surface, following the moon at a certain angle dependent on the inertia of the water, and tending to move with a westward velocity precisely equal to that of the earth's eastward rotation, - about 1000 miles an hour at the equator. The sun's action would produce similar tides superposed upon the lunar tides, and about five-elevenths as large, and at different times of the month these two pairs of tides would be differently related, as has already been explained, sometimes conspiring, and sometimes opposed.
If the earth were entirely covered with water, the tide waves would run around the earth regularly; and if the water were more than 14 miles deep, the two wave crests would still lie one under the moon, the other at its antipodes.

If the depth of the water were (as it really is) much less than 14 miles, the crests of the tide waves would, in the equatorial regions, follow the moon at an angle of $90^{\circ}$, while in the polar regions they would still move as in the case of the deeper ocean. In some intermediate latitude there would be a belt of eddying currents without rise or fall. We do not, however, undertake to prove these results.

These results are, of course, enormously complicated by the existence of the great continental barriers, running more or less north and south. Consider also the varying depth of the water of the different oceans and the irregular contours of the shores, and it is evident that the whole combination of circumstances makes it quite impossible to determine by theory what the course and character of the tide waves must be. We are obliged to depend upon observations, and observations are more or less inadequate because, with the exception of a few islands, our only possible tide-stations are on the shores of continents where local circumstances largely control the phenomena.
273. Free and Forced Oscillations. - If the water of the ocean is suddenly disturbed, as for instance by an earthquake, and then left to itself, a "free wave" is formed, which, if the horizontal dimensions of the wave are large as compared with the depth of the water, will travel at a rate depending solely on the depth.

Its velocity is equal, as can be proved, to the velocity acquired by a body in falling through half the depth of the ocean;
i.e., $\quad v=\sqrt{g h}$, where $h$ is the depth of the water.

Observations upon waves caused by certain earthquakes in South America and Japan have thus informed us, that between the coasts of those countries the Pacific averages between $2 \frac{1}{2}$ and 3 miles in depth.

Now, as the moon in its apparent diurnal motion passes across the American continent each day, and comes over the Pacific Ocean, it starts such a "parent" wave in the Pacific, and a second one twelve hours later. These waves, once started, move on nearly (but not exactly) like a free earthquake wave : not exactly, because the velocity of the earth's rotation being about 1050 miles an hour at the equator, the moon moves (relatively) westward faster than the wave can naturally follow it; and so for a while the moon slightly accelerates the wave. The tidal wave is thus, in its origin, a "forced oscillation": in its subsequent travel it is very uearly, but not entirely, " free."
274. Co-Tidal Lines. - Co-tidal lines are lines drawn upon the surface of the ocean connecting points which have their high water at the same moment of Greenwich time. They mark the crest of the tide wave for every hour, and if we could map them with certainty, we should have all necessary information as to the actual motion of the tide wave. Unfortunately we can get no direct knowledge as to the position of these lines in mid-ocean: we can only determine a few points here and
there on the coasts and on the islands, so that much necessarily is left to conjecture. Fig. 70 is a reduced copy of a co-tidal map, borrowed by permission, with some modifications, from Guyot's "Physical Geography."
275. Course of Travel of the Tidal Wave. - In studying this map, we find that the main or "parent" wave starts twice a day in the Pacific, off Callao, on the coast of South America. This is shown on the chart by a sort of oval "eye" in the co-tidal lines, just as on a topographical chart the summit of a mountain is indicated by an " eye" in the contour lines. From this point the wave travels northwest through the deep water of the Pacific, at the rate of about 850 miles an hour, reaching Kamtchatka in ten hours. Through the shallower water to the west and southwest, the velocity is only from 400 to 600 miles an hour, so that the wave arrives at New Zealand about 12 hours old. Passing on by Australia, and combining with the small wave which the moon raises directly in the Indian Ocean, the resultant tide crest reaches the Cape of Good Hope in about 29 hours, and enters the Atlantic. Here it combines with a smaller tide wave, 12 hours younger, which has "backed" into the Atlantic around Cape Horn, and it is also modified by the direct tide produced by the moon's action upon the Atlantic. The tide resulting from the combination of these three then travels northward through the Atlantic at the rate of nearly 700 miles an hour. It is about forty hours old when it first reaches the coast of the United States in Florida; and our coast is so situated that it arrives at all the principal ports within two or three hours of that time. It is 41 or 42 hours old when it reaches New York and Boston. To reach London, it has to travel around the northern end of Scotland and through the North Sea, and is nearly 60 hours old when it arrives at that port, and at the ports of the German Ocean.

In the great oceans, there are thus three or four tide crests travelling simultaneously, following each other nearly in the same track, but with continual minor changes. If we take into account the tides in rivers and sounds, the number of simultaneous tide crests must be at least six or seven ; i.e., the tidal wave at the extremity of its travel (up the Amazon River for instance) must be at least three or four days old, reckoned from its birth in the Pacific.


Fig. 70. - Map of Cotidal Lines.
276. Tides in Rivers. - The tide wave ascends a river at a rate which depends upon the depth of the water, the amount of friction, and the swiftness of the stream. It may, and generally does, ascend until it comes to a rapid where the velocity of the current is greater than that of the wave. In shallow streams, however, it dies out earlier. Contrary to what is usually supposed, it often ascends to an elevation far above that of the highest crest of the tide wave at the river's mouth. In the La Plata and Amazon, it goes up to an elevation of at least 100 feet above the sea level. The velocity of the tide wave in a river seldom exceeds 10 or 20 miles an hour, and is usually much less.
277. Height of Tides. - In mid-ocean, the difference between high and low water is usually between two and three feet, as obseved on isolated islands in deep water; but on continental shores the height is ordinarily much greater. As soon as the tide wave "touches bottom," so to speak, the ve-


Fig. 71. - Increase in Height of Tide on approaching the Shore.
locity is diminished, the tide crests are crowded more closely together, and the height of tne wave is increased somewhat as indicated in Fig. 71. Theoretically, it varies inversely as the fourth root of the depth; i.e., where the water is 100 feet deep, the tide wave should be twice as high as at the depth of 1600 feet.

Where the configuration of the shore forces the tide into a corner, it sometimes rises very high. At the head of the Bay of Fundy tides of 70 feet are said to be not uncommon, and some of nearly 100 feet have been reported.

## 278. Effect of the Wind, and Changes in Barometric Pressure.

- When the wind blows into the mouth of a harbor, it drives in the water by its surface friction, and may raise the level several feet. In such cases the time of high water, contrary to what might at first be
supposed, is delayed, sometimes as much as 15 or 20 minutes. This depends upon the fact that the water runs into the harbor for a longer time than it would do if the wind were not blowing.

When the wind blows out of the harbor, of course there is a corresponding effect in the opposite direction.

When the barometer at a given port is lower than usual, the level of the water is usually higher than it otherwise would be, at the rate of about one foot for every inch of difference between the average and actual heights of the barometer.
279. Tides in Lakes and Inland Seas. - These are small and difficult to detect. Theoretically, the range between high and low water in a land-locked sea should bear about the same ratio to the rise and fall of tide in mid-ocean that the length of the sea does to the diameter of the earth. On the coasts of the Mediterranean the tide averages less than 18 inches, but it reaches the height of three or four feet at the head of some of the gulfs. In Lake Michigan, at Chicago, a tide of about $1 \frac{3}{4}$ inches has been detected, the " establishment" of Chicago being about 30 minutes.
280. Effects of the Tides on the Rotation of the Earth. -

If the tidal motion consisted merely in the rising and falling of the particles of the ocean to the extent of some two feet twice daily, it would involve a very trifling expenditure of energy; and this is the case with the mid-ocean tide. But near the land this slight oscillatory motion is transformed into the bodily travelling of imınense masses of water, which flow in upon the shallows and then out again to sea with a great amount of fluid friction ; and this involves the expenditure of a very considerable amount of energy. From what source does this energy come?

The answer is that it must be derived mainly from the earth's energy of rotation, and the necessary effect is to lessen the speed of rotation, and to lengthen the day. Compared with the earth's whole stock of rotational energy, however, the loss by tidal friction even in a century is very small, and the theoretical effect on the length of the day extremely slight. Moreover, while it is certain that the tidal friction, by itself considered, lengthens the day, it does not follow that the day grows longer. There are counteracting causes,-for instance, the earth's radiation of heat into space and the consequent
shrinkage of her volume. At present we do not know as a fact whether the day is really longer or shorter than it was a thousand years ago. The change, if real, cannot well be as great as $\frac{1}{1000}$ of a second.
281. Effect of the Tide on the Moon's Motion. - Not only does the tide diminish the earth's energy of rotation directly by the tidal friction, but theoretically it also communicates a minute portion


Fig. 72.
Effect of the Tide on the Moon's Motion. of that energy to the moon. It will be seen that a tidal wave situated as in Fig. 72 would slightly accelerate the moon's motion, the attraction of the moon by the tidal protuberance, $F$, being slightly greater than that of the opposite wave at $F^{\prime}$. This difference would tend to draw it along in its orbit, thus slightly increasing its velocity, and so indirectly increasing the major axis of the moon's orbit, as well as its period. The tendency is, therefore, to make the moon recede from the earth and to lengthen the month.

Upon this interaction between the tides and the motions of the earth and moon, Prof. George Darwin has founded his theory of "tidal evolution," viz., that the satellites of a planet, having separated from it millions of years ago, have been made to recede to their present distances by just such an action. An excellent popular statement of this theory will be found in the closing chapter of Sir Robert Ball's "Story of the Heavens," and a fuller one in Darwin's book, "The Tides," Chapters XVI and XVII.

## CHAPTER X.

## THE PLANETS IN GENERAL.

BODE'S LAW. - THE APPARENT MOTIONS OF THE PLANETS. - THE ELEMENTS OF THEIR ORBITS. - DETERMINATION OF PERIODS AND DISTANCES. - STABILITY OF THE SYSTEM. - DETERMINATION OF THE DATA RELATING TO THE PLANETS THEMSELVES. - DIAMETER, MASS, ROTATION, SURFACE-CHARACTER, ATMOSPHERE, ETC. HERSCHEL'S ILLUSTRATION OF THE SCALE OF THE SYSTEM.
282. The stars preserve their relative configurations, however much they alter their positions in the sky from hour to hour. The Dipper remains always a "Dipper" in every part of its diurnal circuit. But certain of the heavenly bodies, and among them the most conspicuous, behave differently. The sun and moon continually change their places, moving always eastward among the constellations; and a few others, which to the eye appear as very brilliant stars (really not stars at all), also move, ${ }^{1}$ though not in quite so simple a way.

These moving bodies were called by the Greeks "planets."; i.e., "wanderers." The ancient astronomers counted seven of them, - the sun and the moon, and in addition Mercury, Venus, Mars, Jupiter, and Saturn. At present, the sun and

[^29]moon are not reckoned as planets, but the roll includes, in addition to the five other bodies known to the ancients, the earth itself, which Copernicus showed should be counted among them, and also two new bodies (Uranus and Neptune) of great magnitude, though inconspicuous because of their distance. Then there is, in addition, the host of the so-called asteroids.
283. The list of the planets in the order of their distance from the sun stands thus at present, - Mercury, Venus, the Earth, Mars,Jupiter, Saturn, Uranus, and Neptune; and between Mars and Jupiter, where another planet would naturally be expected, there have already been found more than eight hundred little planets, or "asteroids," which probably represent a single planet somehow either "spoiled in the making," so to speak, or else subsequently burst into fragments. These planets are all (probably) dark bodies, shining only by reflected light, globes which, like the earth, revolve around the sun in orbits nearly circular, moving all of them in the same direction, and (with some exceptions among the asteroids) nearly in the common plane of the ecliptic. All but the inner two and the asteroids are attended by "satellites." Of these the earth has one (the moon), Mars two, Jupiter nine, Saturn nine, Uranus four, and Neptune one.

## 284. Relative Distances of the Planets from the Sun ; Bode's

 Law. - There is a curious approximate relation between the distances of the planets from the sun, usually known as "Bode's Law."It is this: Write a series of 4's. To the second 4 add 3, to the third, add $3 \times 2$, or 6 ; to the fourth, $4 \times 3$, or 12 ; and so on, doubling the added number each time, as in the following scheme.

| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3 | 6 | 12 | 24 | 48 | 96 | 192 | 384 |
| 4 | $\frac{7}{7}$ | 10 | 16 | [28] | 52 | 100 | 196 | 388 |
| ¢ | ¢ | $\oplus$ | б | (i) | 4 | h | W | $\Psi$ |

The resulting numbers (divided by 10) are approximately equal to the true mean distances of the planets from the sun, expressed in radii of the earth's orbit (astronomical units); - excepting Neptune however; in his case the law breaks down utterly. For the present, at least, it is to be regarded as a mere coincidence, rather than a real "law." No satisfactory explanation of it has yet been found.

## 285. Table of Names, Distances, and Periods.

| Name. | Stmbol. | Distance. | Bode. | Diff. | Sid. Period. | Srin. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mercury | ¢ | 0.387 | 0.4 | -0.013 | $88^{\mathrm{d}}$ or $3^{\text {mo. }}$ | $116{ }^{\text {d }}$ |
| Venus | 9 | 0.723 | 0.7 | +0.023 | $224.7^{\text {d }}$ or $7 \frac{1}{2} \mathrm{mo}$. | $584{ }^{\text {d }}$ |
| Earth | $\oplus$ | 1.000 | 1.0 | 0.000 | $36511^{\text {d }}$ or $1^{\text {y }}$ |  |
| Mars | ¢ | 1.523 | 1.6 | $-0.077$ | $687^{\text {d }}$ or $1^{\text {y }} 10^{\text {mo. }}$ | $780^{\text {d }}$ |
| Mean Asteroid |  | 2.650 | 2.8 | -0.150 | $3^{5} .1$ to $8^{\text {r }} .0$ | various |
| Jupiter | 4 | 5.202 | 5.2 | +0.002 | 11 F .9 | $399{ }^{\text {d }}$ |
| Saturn. | h | 9.539 | 10.0 | -0.461 | 29 y .5 | $378{ }^{\text {d }}$ |
| Uranus | ¢ \& H | 19.183 | -19.6 | -0.417 | $84^{\text {r }} .0$ | $370^{\text {d }}$ |
| Neptune | $\Psi$ | 30.054 | 38.8 | -8.746 ! | 164.8 | $367{ }^{1}{ }^{\text {d }}$ |

The column headed "Bode" gives the distance according to Bode's law ; the column headed "Diff.," the difference between the true distance and that given by Bode's law.

Fig. 73 shows the smaller orbits of the system (including the orbit of Jupiter), drawn to scale, the radius of the earth's orbit being taken as one centimetre. On this scale the diameter of Saturn's orbit would be $19^{\mathrm{cm}} .08$, that of Uranus would be $38^{\mathrm{cm}} .36$, and that of Neptune $60^{\mathrm{cm}} .11$, or about 2 feet. The nearest fixed star, on the same scale, would be about a mile and a quarter away. It will be seen that the orbits of Mercury, Mars, Jupiter, and several of the asteroids are quite distinctly "out of centre" with respect to the sun.
286. Periods. - The Sidereal Period of a planet is the time of its revolution around the sun, from a star to the same star again, as seen from the sun. The Synodic Period is the time between two successive conjunctions of the planet with the sun, as seen from the earth.


Fig. 73. - Plan of the Smaller Planetary Orbits.
The sidereal and synodic periods are connected by the same relation as the sidereal and synodic months (Art. 141), namely,

$$
\frac{1}{S}=\frac{1}{P}-\frac{1}{E}
$$

in which $E, P$, and $S$ are respectively the periods of the earth and of the planet, and the planet's synodic period, and the numerical difference between $\frac{1}{P}$ and $\frac{1}{E}$ is to be taken without regard to sign; i.e., for an inferior planet, $\frac{1}{S}=\frac{1}{P}-\frac{1}{E}$; for a superior one, $\frac{1}{S}=\frac{1}{E}-\frac{1}{P}$. The two last columns of the table of Art. 285 give the approximate periods, both sidereal and synodic, for the different planets.
287. Apparent Motions. - If we imagine a line drawn through the sun perpendicular to the plane of the ecliptic, the planets, from a distant point on this line, would be seen to travel in their nearly circular orbits with a steady, forward motion; but viewed from the earth, the apparent motion is complicated, being made up of their own real motion around the sun, combined with an apparent motion due to the earth's own movement.
The apparent motion of a body relative to the earth can be very simply stated. Every body which is really at rest will appear, as seen from the earth, to move in an orbit identically like the earth's orbit, and parallel to it, but keeping in such a part of this apparent orbit as always to have its motion precisely equal and opposite the earth's own real motion at the moment. If a body is really moving, its apparent motion with respect to the earth will be found by combining its motion with another motion equal to that of the earth, but reversed. It is not difficult to prove this, but our space will not permit.
288. Effect of the Combination of the Earth's Motion with that of a Planet. - The apparent or "geocentric," motion of a planet is therefore made up of two motions, and appears to be that of a body moving once a year around the circumference of a circle equal to the earth's orbit, while the centre of this circle itself goes around the sun upon the real orbit of the planet, and with a periodic time equal to that of the planet.

Jupiter, for instance, appears to move as in Fig. 74, making 11 loops in each revolution, the smaller circle having a diameter of about one-fifth of the larger one, upon


Fig. 74.
Apparent Geocentric Motion of Jupiter. which its centre moves, since the diameter of Jupiter's orbit is about five times that of the earth.

As a consequence, we have an apparent back-andforth movement of the planets among the stars. They move eastward (technically "advance") part of the time, and part of the time they move westward (technically "retrograde"), the arc of retrogression being, however, always less than that of advance.
289. Explanation of Terms. - Fig. 75 illustrates the meaning of a number of terms which are used in describing a planet's position with reference to the sun, viz., Opposition, Quadrature, Inferior and Superior Conjunction, and Greatest Elongation. $E$ is the position of the earth, the inner circle being the orbit of an "inferior" planet, while the outer circle is the orbit of a "superior" planet. In general, the angle $P E S$ (the angle at the earth between lines drawn from the earth to the planet and the sun) is the planet's elongation. For a superior planet, it can have any value from zero to $180^{\circ}$; for an inferior, it has a maximum value that the planet cannot exceed, depending upon the diameter of its orbit.

## 290. Motion of a Planet in Right Ascension and Longitude.

 - Starting from the line of superior conjunction, the planet whether superior or inferior, moves eastward or "direct" for a time, but at arate continually slackening until the planet becomes "stationary"; then it reverses its course and moves westward, the middle of the arc of retrogression nearly coinciding with the point where it comes to opposition or inferior conjunction. After a time it becomes once more stationary, and then resumes its eastward motion until it again arrives at superior conjunction, having completed a synodic period. In time, as well as in degrees, the "advance" always exceeds the "retrogression."

As observed with a sidereal clock, all the planets come later to the meridian each night when moving direct, since their right ascension is then increasing ; vice versa, of course, when they are retrograding.


Fig. 75. - Planetary Configurations and Aspects.
291. Motion of the Planets with respect to the Sun's Place in the Sky; Change of Elongation. - The visibility of a planet depends mainly on its "elongation" (i.e., its angular distance from the sun), because when near the sun, the planet will be above the horizon only by day. As regards their motion, con-
sidered from this point of view, there is a marked difference between the inferior planets and the superior.
I. The superior planets drop always steadily westward with respect to the sun's place in the heavens, continually increasing their western elongation or decreasing their eastern. As observed by an ordinary time-piece (keeping solar time), they therefore invariably come earlier to the meridian every successive night, never moving eastward among the stars as rapidly as the sun does.

Beginning at conjunction, the planet is then behind the sun, at its greatest distance from the earth, and invisible. It soon, however, reappears in the morning, rising before the sun, and passes on to western quadrature, when it rises at midnight. Thence it moves on to opposition, when it is nearest and brightest, and rises at sunset. Still dropping westward and receding, it by and by reaches eastern quadrature and is on the meridian at sunset. Thence it still crawls sluggishly westward until it is lost in the evening twilight and completes its synodic period by again reaching conjunction.
292. II. The inferior planets, on the other hand, apparently oscillate across the sun, moving out equal distances on each side of it, but making the westward swing much more quickly than the eastward.

At superior conjunction an inferior planet is moving eastward faster than the sun. Accordingly it creeps out into the twilight as an evening star, and continues to increase its apparent distance from the sun until it reaches its "greatest eastern elongation" ( $47^{\circ}$ for Venus; for Mercury, from $18^{\circ}$ to $28^{\circ}$ ). Then the sun begins to gain upon it, and as the planet itself soon begins to retrograde, the elongation diminishes rapidly, and the planet rushes back towards inferior conjunction, passes it, and, as a morning star, moves swiftly out to its western elongatior. Then it turns and climbs slowly back to superior conjunction again.
293. Motions in Latitude. - If the planets' orbits lay precisely in the same plane with the earth's orbit, they would always keep exactly in the ecliptic; but while the orbits of the larger planets never go far from that circle, they do, in fact, deviate a few degrees, so
that their paths in the heavens form more or less complicated loops and kinks. Fig. 75* shows the loops made by Saturn and Uranus in 1897.


Fig. 75*. - Motion of Saturn and Uranus in 1897.
294. Ptolemaic System. - Assuming the fixity and central position of the earth and the actual revolution of the heavens, Ptolemy, who flourished at Alexandria about 140 A.d., worked out the system which bears his name. In his great work, the Almagest, which for 14 centuries was the authoritative "Scripture of Astronomy," he showed that all the apparent motions of the planets, so far as then observed, could be accounted for by supposing each planet to move around the circumference of a circle called the "epicycle," while the centre of this circle, sometimes called the "fictitious planet," itself moved around the earth on the circumference of another and larger circle called the "deferent." To account for some of the irregularities of the planets, however, it was necessary to suppose that both the deferent and epicycle, though circular, are eccentric.
295. The Copernican System. - Copernicus (1473-1543) asserted the diurnal rotation of the earth on its axis, and showed that this would fully account for the apparent diurnal revolution of the heavens. He also showed that nearly all the
known motions of the planets could be accounted for by supposing them to revolve around the sun (with the earth as one of them) in orbits circular, but slightly out of centre. His system, as he left it, was very nearly that which is accepted to-day, and Fig. 73 may be taken as representing it. He was obliged, however, to retain a few small epicycles to account for certain of the irregularities.

So far, no one had dared to doubt the exact circularity of the celestial orbits. It was considered metaphysically improper that heavenly bodies should move in any but perfect curves, and no curve but the circle was recognized as such. It was left for Kepler, some 65 years later than Copernicus, to show that the planetary orbits are elliptical, and to bring the system substantially into the form in which we know it now.
296. The Elements of a Planet's Orbit are a set of numerical quantities, seven in number, which embody a complete description of the orbit, and supply the data for the prediction (perturbations excepted) of the planet's exact place at any time in the past or future.

An explanation of them will be found in the Appendix, Art. 507.

There is a general method, the discussion of which lies quite beyond our reach, by which all the seven elements of a planet's orbit can be deduced from any three perfectly accurate observations of the right ascension and declination of the body, separated by a few weeks' interval (excepting, however, one or two special cases where the observed places are so peculiarly situated that a fourth observation becomes necessary). Of course, if the observations are not perfect, and they never are, the orbit deduced will be only approximate; but in ordinary cases three observations such as are now usually made at our standard observatories, with an interval of a month or so between the extremes, will give a very fair approximation to the orbit, which can then be corrected by further observation. This general method of computing the orbit from three observations was invented in 1801, by Gauss, then a young man of 23 , in connection with the discovery of Ceres,
the first of the asteroids, which, after its discovery by Piazzi, was soon lost to observation in the rays of the sun.

Since, however, the planetary orbits are for the most part approximately circular, and nearly in the plane of the ecliptic, they are described with sufficient accuracy for many purposes by giving simply the planet's mean distance from the sun with the corresponding period.
297. Determination of a Planet's Sidereal Period. - This may be effected by determining the mean synodic period of the planet from a comparison of the dates of two conjunctions or oppositions, widely separated in time. The exact instant of opposition is found from a series of right ascensions and declinations observed about the proper date; and by comparing the deduced longitudes with the corresponding longitudes of the sun, we easily find the precise moment when the difference was $180^{\circ}$. When the synodic period is found, the sidereal is given by the equations of Art. 286, viz., $\frac{1}{S}=\frac{1}{P}-\frac{1}{E}$ for an inferior planet and $\frac{1}{S}=\frac{1}{E}-\frac{1}{P}$ for a superior one. In the first case, $P=\frac{S E}{S+E}$; in the second, $\frac{S E}{S-E}$. It will not answer for this purpose to deduce the synodic period from two successive oppositions, because, on account of the eccentricity of the orbits, both of the planet and of the earth, the synodic periods are somewhat variable. The observations must be sufficiently separated in time to give a good determination of the mean synodic period.
298. Geometrical Method of Determining a Planet's Distance in Astronomical Units. - When we have found the planet's sidereal period, we can easily ascertain its distance from the sun in astronomical units by means of two observations of its elongation from the sun, made at two dates separated by an interval of exactly one of its periods.

The "elongation," it will be remembered, is the difference between the longitude of the planet and that of the sun as seen from the earth, and is determined for any given instant by means of a series of meridian-circle observations of both sun and planet covering the desired date.
299. To find the distance of Mars, for instance, we must have two elongations observed at an interval of 687 (686.95) days, so that the planet at the second observation will be at the same point, M, (Fig. 76) which it occupied at first. If the


Fig. 76. - Determination of the Distance of a Planet from the Sun.
earth was at $A$ at the first observation, then at the second she will be at a point, $C$, so situated that the angle $A S C$ will be • that which the earth will describe in the next $43 \frac{1}{2}$ days (the difference between 687 days and two complete years) and is
therefore known. The two angles $S A M$ and $S C M$ are the two elongations of the planet, given directly by the observations. The two sides $S A$ and $S C$ are also known, being the earth's distance from the sun at the two times of observation. Hence, knowing two adjacent sides of the quadrilateral and its angles, we can easily solve it (as in Art. 149), and find $S M$, and also the angle $A S M$ or $C S M$, which determines the direction of $M$ from $S$ at that point in its orbit.

The student can follow out for himself the process by which, from two elongations of Venus, $S A V$ and $S B V$, observed at an interval of 225 days, $S V$ can be determined.
300. From a sufficient number of such pairs of observations distributed around the planet's orbit, it will evidently be possible to work out completely the magnitude and form of the orbit, and it was actually in just this way that Kepler, from the observations of Tycho, showed that the planetary orbits are ellipses, and deduced their relative distances from the sun as compared with that of the earth. His harmonic law was then discovered by simply comparing the periods with the distances. Now that we have the "harmonic law," a planet's approximate mean distance can, of course, be much more easily found by applying the law (the period being given) than by the geometrical method.

## 301. Simple Method of find-

 ing the Distance of an Inferior Planet. - In the case of Venus, which has a practically circular orbit, the method illustrated by Fig. 77 may be used. When the planet is at its greatest elongation, the angle $S V E$ is sensibly a right angle, so that we needonly to know $S E$ and the angle of greatest elongation, $E S V$, in order to compute $S V$. Mercury's orbit is so eccentric that the metbod in his case will give only rough approximations.
302. Planetary Perturbations. - The attractions of the planets for each other disturb their otherwise elliptical motion around the sun, somewhat as the sun disturbs the motion of the moon around the earth; but the disturbing forces are in nearly all cases small, and the resulting perturbations, as a rule, are much less than in the case of the moon. They are divided into two great classes, - the periodic and the secular.
303. Periodic Perturbations. - The periodic are such as depend upon the relative positions of the planets in their orbits, and generally run through their course in less than a century, though there are some with periods exceeding a thousand years. For the most part they are trifling in amount.

Those of Mercury never exceed $15^{\prime \prime}$, as seen from the sun. Those of Venus may reach $30^{\prime \prime}$; those of the earth about $1^{\prime}$; and those of Mars a little over $2^{\prime}$. The mutual disturbances of Jupiter and Saturn are much larger, reaching $28^{\prime}$ and $48^{\prime}$ respectively. Those of Uranus never exceed $3^{\prime}$, and those of Neptune are smaller yet. In the case of asteroids, however, disturbed by Jupiter, the displacement is sometimes enormous, as much as $8^{\circ}$ or $10^{\circ}$.
304. Secular Perturbations. - These are such as depend not on the positions of the planets in their orbits, but on the relative positions of the orbits themselves. Since these positions change very slowly, these perturbations, though in the strict sense periodic also, are extremely slow in their development, running along, as the name implies, "from age to age," in periods to be reckoned only by myriads and millions of years.

La Place and La Grange a century ago supposed that they had proved that the major axes and periods of the orbits will
never be changed by these secular perturbations, but will remain, in the long run, absolutely constant. Poincaré has recently shown, however, that their proof is not conclusive. "Never" and "absolutely" are words too strong.

The nodes ${ }^{1}$ and perihelia, on the other hand, move around continuously. All the nodes regress, and all the lines of apsides advance (that of Venus alone excepted). The shortest of their periods of revolution is 37,000 years, the longest over half a million.

The inclinations of the orbits to the plane of the ecliptic oscillate back and forth, in periods (not regular, however) of many thousand years, but the oscillations are confined within a very few degrees.

The eccentricities also oscillate back and forth in a similar way, alternately increasing and decreasing, but only within narrow limits.
305. Stability of the System. - The solar system, therefore, will not for millions of years, perhaps never, suffer serious derangement from perturbations due to the mutual attraction of the planets. The mean distances and periods of the orbits will long remain practically unchanged; the changes in the position of node and perihelion are of no consequence, and the alterations in the inclination and eccentricity, which would be serious if they were extensive, are confined within narrow limits.

It does not follow, however, that because the mutual attractions of its members cannot seriously derange the system, there may not be other causes which can do so. There are many conceivable actions which would necessarily terminate in its destruction, such as the retardation of planetary motions which would be caused by a resisting medium, or by the encounter with a sufficiently dense swarm of meteoric matter. We add also that the asteroids have not the same guarantees of safety as the larger planets. The changes of their inclinations and eccentricities are not narrowly limited.'

[^30]
## THE PLANETS THEMSELVES.

306. In studying the planetary system, we meet a number of subjects of inquiry which refer to the individual planet, and not at all to its orbit, - such, for instance, as its magnitude ; its mass, density, and surface gravity; its axial rotation and ellipticity ; its brightness, phases, and reflecting power or "albedo," and the spectroscopic qualities of its light; its atmosphere; its surface-markings and topography; and, finally, its satellite system.
307. Magnitude. - The size of a planet is found by measuring its angular diameter with some form of micrometer (see Appendix, Art. 542). Since we can find the distance of a planet from the earth at any moment when we know the elements of its orbit, this will give us the means of finding at once the planet's linear diameter. It will come out in astronomical units (i.e., in terms of the earth's mean distance from the sun) if the planet's distance is expressed in such units : if we know the value of that unit (about 93,000000 miles), which depends upon our knowledge of the solar parallax, then we can also find the planet's diameter in miles.

The equation is simply

$$
\text { Linear diameter }=D \times \frac{d^{\prime \prime}}{206265},
$$

in which $D$ is the distance of the planet from the earth, and $d^{\prime \prime}$ the number of seconds of arc in its measured diameter.
308. It is customary to divide the real semi-diameter or radius of the planet by that of the earth, and call the quotient $r$ (i.e., $r$ is the number of times the planet's semi-diameter exceeds that of the earth). The area of the planet's surface (compared with that of the earth) then equals $r^{2}$, and its volume or bulk equals $r^{3}$.

If, as is nearly the case with Jupiter, the diameter is eleven times that of the earth, $r=11$; the surface of Jupiter $=r^{2}=121$, and the volume $=r^{3}=1331$ times that of the earth.

The nearer the planet, other things being equal, the more accurately $r$ and the quantities derived from it can be determined. An error of $0^{\prime \prime} .1$ in measuring the apparent diameter of Venus when nearest counts for less than 13 miles, while in Neptune's case it would correspond to more than 1300.
309. Mass, Density, and Surface Gravity. - If the planet has a satellite, its mass is very easily and accurately found from the proportion ${ }^{1}$

$$
\text { Mass of sun : mass of planet : : } \frac{A^{3}}{T^{2}}: \frac{a^{3}}{t^{2}}
$$

in which $A$ is the mean distance of the planet from the sun and $T$ its sidereal period of revolution; while $a$ is the distance of the satellite from the planet, and $t$ its sidereal period.

Substantially the same proportion may be used to compare the planet with the earth, viz.,

$$
\text { Earth }+ \text { moon }: \text { planet }+ \text { satellite }:: \frac{a_{1}^{3}}{t_{1}^{2}}: \frac{\alpha_{2}^{3}}{t_{2}^{2}},
$$

$a$ and $t$ being here the distance and period of the moon, and $\alpha_{2}$ and $t_{2}$ those of the planet's satellite.

For a demonstration of these proportions, see Appendix, Art. 508. When a planet has no satellite its mass can be determined only by means of the perturbations which it produces in the motion of othes planets, or of comets.

[^31]Having the planet's mass compared with the earth, we get its density by dividing the mass by the volume; i.e.,

$$
\text { Density }=\frac{m}{r^{3}}
$$

The superficial gravity, i.e., the force of gravity on the planet's surface compared with that of the earth (neglecting the centrifugal force due to its rotation) is simply $\frac{m}{r^{2}}$.
310. The Rotation Period and Data connected with it. The length of the planet's day, when it can be determined at all, is ascertained by observing some spot upon the planet's disc, and noting the interval between its returns to the same apparent position. The spectroscope may also be used.

In reducing the observations, account has to be taken of the continual change in the direction of the planet from the earth, and also of the variations of its distance, which alter the time taken by light in reaching us.

The inclination of the planet's equator to the plane of its orbit, and the position of its equinoxes, are deduced from the same observations that give the planet's rotation period; we have to observe the path pursued by a spot in its motion across the disc. Only Mars, Jupiter, and Saturn permit us to find these elements of their rotation with any considerable accuracy.

The "ellipticity," " oblateness," (Art. 90) or "polar compression" of the planet, due to its rotation, is found by micrometric measures of its polar and equatorial diameters.
311. Data relating to the Planet's Light. - The planet's brightness, and its reflecting power or "albedo," are determined by photometric observations; and the spectroscopic peculiarities of its light are, of course, studied with the spectroscope. The question of its atmosphere is investigated also
by means of various effects upon the planet's appearance and light. The planet's surface-markings and topography are studied directly with the telescope, by making careful drawings of the appearances noted at different times. If the planet has any well marked and characteristic spots upon it, by which the time of rotation can be found, then it soon becomes easy to identify such as are really permanent, and after a time to chart them more or less perfectly ; but we add immediately, that Mars is the only planet of which, so far, we have been able to make anything which can be called a map.
312. Satellite System. - The principal data to be ascertained are the distances and the periods of the satellites. These are obtained by micrometric measures of the apparent distance and direction of each satellite from the planet, but the reduction of the observations is rather complicated on account of the continual change in the planet's distance and direction from the earth.

In a few cases, also, it is possible to make observations by which we can determine the diameter of the satellites; and where there are a number together, their masses may sometimes be ascertained from their mutual perturbations.

With the exception of our moon and the outer satellites of Jupiter and Saturn, all the satellites of the solar system move almost exactly in the plane of the equator of the planet to which they belong ; at least, so far as known, for we do not know with certainty the position of the equatorial planes of Uranus and Neptune. Moreover, all the satellites but the moon, Hyperion, and those recently discovered, move in orbits that are practically circles.
313. Classification of Planets. - Humboldt has classified the planets in two groups, - the "terrestrial planets," as he calls them, and the "major planets." The terrestrial group
contains the four planets nearest the sun, - Mercury, Venus, the Earth, and Mars. They are all bodies of similar magnitude, ranging from 3000 to 8000 miles in diameter; not very different in density and probably roughly alike in physical constitution, though probably also differing very much in the extent, density, and character of their atmospheres.

The four major planets, Jupiter, Saturn, Uranus, and Neptune are much larger bodies, ranging from 32,000 to 90,000 miles in diameter; are much less dense ; and, so far as we can


Fig. 78. - Relative Sizes of the Planets.
make out, present only cloud-covered surfaces to our inspection. There are strong reasons for supposing that they are at a high temperature, and that Jupiter especially is a sort of "semi-sun"; but this is not certain.

As to the asteroids, the probability is that they represent a fifth planet of the terrestrial group, which, as has been already intimated, failed somehow in its evolution, or else has been broken to pieces.

Fig. 78 gives an idea of the relative sizes of the planets. The sun on the scale of the figure would be about a foot in diameter.
314. Tables of Planetary Data. - In the Appeudix we present tables of the different numerical data of the solar system, derives from the best authorities and calculated for a solar parallax of $8^{\prime \prime} .80^{\circ}$ : the sun's mean distance being, therefore, taken as $92,897,000$ miles. These tabulated numbers, however, differ widely in accuracy. The periods of the planets and their distances in astronomical units are very precisely known : probably the last decimal place in the table may be trusted. Next in certainty come the masses of such planets as have satellites, expressed in terms of the sun's mass. The masses of Venus and Mercury, however, are much more uncertain. The distances of the planets in miles, their masses in terms of the earth's mass, and their diameters in miles, all involve the solar parallax, and are affected by the slight uncertainty in its amount. For the remoter planets, moreover, diameters, volumes, and densities are subject to a very considerable percentage of error, as explained above. The student need not be surprised, therefore, at finding serious discrepancies between the values given in these tables and those given by others, amounting in some cases to 10 per cent or 20 per cent, or even more. Such differences merely indicate the actual uncertainties of our knowledge.

## 315. Sir John Herschel's Illustration of the Dimensions of

 the Solar System. - In his "Outlines of Astronomy," Herschel gives the following illustration of the relative magnitudes and distances of the members of our system : -"Choose any well-levelled field. On it place a globe two feet in diameter. This will represent the sun. Mercury will be represented by a grain of mustard seed on the circumference of a circle 164 feet in diameter for its orbit; Venus, a pea, on a circle of 284 feet in diameter; the Earth, also a pea, on a circle of 430 feet; Mars, a rather large pin's head, on a circle of 654 feet; the asteroids, grains of sand, on orbits having a diameter of 1000 to 1200 feet; Jupiter, a moderate-sized orange, on a circle nearly half a mile across; Saturn, a small orange, on a circle of four-fifths of a mile; Uranus, a full-sized cherry or small plum, upon a circumference of a circle more than a mile in diameter ; and, finally, Neptune, a good-sized plum, on a circle about $2 \frac{1}{2}$ miles in diameter."

We may add that on this scale, the nearest star would be on the opposite side of the earth, 8000 miles away.

## CHAPTER XI.

## THE TERRESTRIAL AND MINOR PLANETS.

316. Mercury has been known from the remotest antiquity. At first, astronomers failed to recognize it as the same body on the eastern and western side of the sun, and among the Greeks it had for a time two names, - Apollo, when it was morning star, and Mercury, when it was evening star. It is so near the sun that it is comparatively seldom seen with the naked eye, but when near its greatest elongation it is easily enough visible as a brilliant star of the first magnitude, low down in the twilight. It is best seen in the evening at such eastern elongations as occur in March and April. When it is morning star, it is best seen in September and October.

It is exceptional in the solar system in various ways. It is the nearest planet to the sun, receives the most light and heat, is the swiftest in its movement, and (excepting some of the asteroids) has the most eccentric orbit, with the greatest inclination to the ecliptic. It is also the smallest in diameter (again excepting the asteroids), and has the least mass of all the planets.
317. Its Orbit. - Its mean distance from the sun is $36,000-$ 000 miles, but the eccentricity of its orbit is so great ( 0.205 ) that the sun is 7,500000 miles out of the centre, and the radius vector ranges all the way from $28 \frac{1}{2}$ millions to $43 \frac{1}{2}$, while the velocity in its orbit varies from 36 miles a second at perihelion to only 23 at aphelion. A given area upon its surface receives on the average nearly 7 times as much light and heat as it would on the earth; but the heat received at perihelion
is greater than that at aphelion in the ratio of $9: 4$. For this reason there must be at least two seasons in its year due to the changing distance, even if the equator of the planet should be parallel to the plane of its orbit; and if the planet's equator is inclined nearly at the same angle as the Earth's, the seasons must be extremely complicated.
The sidereal period is 88 days, and the synodic period, (or the time from conjunction to conjunction,) 116. The greatest elongation ranges from $18^{\circ}$ to $28^{\circ}$, and occurs about 22 days before and after the inferior conjunction. The inclination of the orbit to the ecliptic is about $7^{\circ}$.
318. The Planet's Magnitude, Mass, Etc. - The apparent diameter of Mercury ranges from $5^{\prime \prime}$ to about $13^{\prime \prime}$, according to its distance from us ; and the real diameter is very near 3000 miles. This makes its surface about a seventh that of the earth, and its bulk or volume, one-eighteenth. The planet's mass is not accurately known ; it is very difficult to determine, since it has no satellite, and the values obtained from perturbations range very widely : it is probably between $\frac{1}{20}$ and $\frac{1}{25}$ of the mass of the earth. Its mass is, however, unquestionably smaller than that of any other planet, asteroids excepted. Our uncertainty as to its mass of course prevents us from assigning any certain values to its density, though probably it is not quite so dense as the earth.
319. Telescopic Appearances, Phases, Etc. - As seen through the telescope, the planet looks like a little moon, showing phases precisely similar to those of our satellite. At inferior conjunction the dark side is towards us; at superior conjunction, the illuminated surface. At greatest elongation the planet appears as a half moon. It is gibbous between superior conjunction and greatest elongation, while between inferior conjunction and elongation it shows the crescent phase.

Fig. 79 illustrates the phases of Mercury (and equally of Venus).

The atmosphere of the planet cannot be as dense as that of Venus, because at a "transit" it shows no encircling ring of light, as Venus does (Art. 324) ; both Huggins and Vogel, however, report spectroscopic ${ }^{1}$ observations which imply the presence of an atmosphere containing the vapor of water.

Generally, the planet is so near the sun that it can be observed only by day, but when the proper precautions are


Fig. 79. - Phases of Mercury and Venus.
taken to screen the object-glass of the telescope from direct sunlight, the observation is not specially difficult. The surface presents very little of interest. It is brighter at the edge than at the centre, but until recently no markings have been observed well enough defined to give us any trustworthy information as to its geography or even its rotation.

Schröter, a German astronomer, the contemporary of the elder Herschel, and, to speak mildly, an imaginative man, early in the century reported certain observations which would seem to indicate the existence of high mountains upon the planet, and he deduced from his observations a rotation period of 24 hours, 5 minutes. Later observers, with instruments certainly far more perfect, have not been able to verify his results, and they are now considered as of little weight.

[^32]In 1889 the Italian astronomer, Schiaparelli, announced that he had discovered certain permanent markings upon the surface of Mercury, and that from them he had ascertained that the planet rotates on its axis only once during its orbital period of 88 days; i.e., it keeps the same face always turned towards the sun, behaving in this respect just as the moon does towards the earth. Owing, however, to the great eccentricity of its orbit, the planet has a large "libration" (Art. 155), amounting to nearly $23 \frac{1}{2}^{\circ}$ on each side of the mean position ; i.e., seen from a favorable position on the planet's surface, the sun, instead of rising and setting daily as with us, would appear to oscillate back and forth in the sky to the extent of $47^{\circ}$ every 88 days.

This important discovery waited long for verification, the necessary observations being extremely difficult, but in 1896 Mr. Lowell reported that his observations at Flagstaff furnish a complete confirmation of Schiaparelli's result.
The "albedo," or reflecting power, of the planet is very low, only 0.13 ; somewhat inferior to that of the moon, and very much below that of any other of the planets. In the proportion of light given out at its different phases, it behaves like the moon, flashing out strongly near the full.
No satellite is known, and there is no reason to suppose that it has any.
320. Transits of Mercury. - At the time of inferior conjunction, the planet usually passes north or south of the sun, the inclination of its orbit being $7^{\circ}$; but if the conjunction occurs when the planet is very near its node, it crosses the disc of the sun, and becomes visible upon it as a small, black spot, - not, however, large enough to be seen without a telescope, like Venus under similar circumstances. Since the earth passes the planet's node on May 7th and Nov. 9th, transits can occur only near those dates.

If the planet's orbit were truly circular, the conditions of transit would be the same at each node; but at the May transits, the planet
is near its aphelion, and, as a consequence, they are only about half as numerous as the others. For the November transits, the interval is usually 7 or 13 years ; for the May transits, 13 or 46 . 22 synodic periods of Mercury are pretty nearly equal to 7 years; 41 still more nearly equal to 13 years; and 145 are almost exactly equal to 46 years. Hence, 46 years after a given transit another one at the same node is almost certain. During the first half of the twentieth century transits will occur as follows : -

May 7th, 1924, and May 10th, 1937 ; Nov. 14 th, 1907*; Nov. 7th, 1914*; Nov. 8th, 1927; and Nov. 12th, 1940. Only the two marked with an asterisk were (partially) visible in the United States.

Traṇsits of Mercury are of no particular astronomical importance, except as furnishing accurate determinations of the planet's place.

## VENUS.

321. The next planet in the order from the sun is Venus, the brightest and most conspicuous of all. It is so brilliant that at times it casts a distinct shadow, and is easily seen by the naked eye in the daytime. Like Mercury, the Greeks had two names for it, - Phosphorus as morning star, and Hesperus as evening star.

Its mean distance from the sun is 67,200000 miles, and its distance from the earth ranges from 26,000000 miles ( $93-67$ ) to $160,000000(93+67)$. No other body ever comes so near the earth, except Eros, the moon, and occasionally a comet. The eccentricity of its orbit is the smallest in the solar system (only 0.007 ), so that the greatest and least distances of the planet from the sun differ from the mean only 470,000 miles. Its orbital velocity is about 22 miles per second. Its sidereal period is 225 days, or $7 \frac{1}{2}$ months; and its synodic period, 584 days, - a year and 7 months. From superior conjunction to elongation on either side is 220 days, while from inferior conjunction to elongation is only 71 or 72 days. The greatest elongation is $47^{\circ}$ or $48^{\circ}$. The inclination of its orbit is about $3 \frac{1}{2}^{\circ}$.
322. Magnitude, Mass, Density, Etc. - The apparent diameter of the planet ranges from $67^{\prime \prime}$, at the time of inferior
conjunction, to only $11^{\prime \prime}$ at superior, the great difference depending upon the enormous variation in the distance of the planet from the earth. The real diameter of the planet in miles is about 7700 . Its surface, compared with that of the earth, is $\frac{95}{100}$; its volume, $\frac{92}{100}$. By means of the perturbations she produces upon the earth, the mass of Venus is found to be a little less than four-fifths of the earth's mass. Hence her density is about 86 per cent, and her superficial gravity 83 per cent of the earth's. A man who weighs 160 pounds here would weigh only about 133 pounds on Venus.


Fig. 80. - Telescopic Appearances of Venus.
323. General Telescopic Appearance, Phases, Etc. - The telescopic appearance of Venus is striking on account of her great brilliance, but exceedingly unsatisfactory because nothing is distinctly outlined upon the disc. When about midway between greatest elongation and inferior conjunction the planet nas an apparent diameter of $40^{\prime \prime}$, so that with a magnifying
power of only 45 she looks exactly like the moon four days old, and of the same apparent size. (Very few persons, however, would think so on the first view through the telescope: the novice always underrates the apparent size of a telescopic object.)

The phases of Venus were first discovered by Galileo in 1610, and afforded important evidence as to the truth of the Copernican System as against the Ptolemaic.

Fig. 80 represents the planet's disc as seen at five points in its orbit. 1,3 , and 5 are taken at superior conjunction, greatest elongation, and near inferior conjunction, respectively; while 2 and 4 are at intermediate points. (No. 2 is badly engraved, however; the sharp corners are impossible.)

The planet attains its maximum brightness when its apparent area is at a maximum, about 36 days before and after inferior conjunction.

According to Zöllner, the albedo of the planet is 0.50 ; i.e., about three times that of the moon, and almost four times that of Mercury. It is, however, slightly exceeded by the reflecting power of Uranus and Jupiter, while that of Saturn is about the same. This high reflecting power has generally been considered to indicate a surface mostly covered with clouds (though Lowell dissented from this, see Art. 325). The disc of Venus is brightest at the edge, as is also the case with Mercury, Mars, and the moon.
324. Atmosphere of the Planet. - Venus has an atmosphere of some density. When crossing the limb of the sun at a transit it is encircled with a thin bright ring due to atmospheric refraction, reflection, and diffusion of light. If due solely to refraction, which is more than doubtful, it would follow that the atmosphere must be about 55 miles deep, and much denser than our own. But at certain times when the planet, near inferior conjunction, is some minutes of arc from the sun the ring is still visible, with proper precautions.
and, as Russell pointed out in 1898, it is brightest on the edge nearest the sun, - indicating that it is caused mainly by reflection and diffusion, and only in part by refraction; so that the atmosphere may be, and probably is, less dense than that of the earth. There are doubtful spectroscopic indications of the presence of water-vapor.

Several observers have reported faint lights seen at times on the dark portions of the planet's disc. They cannot be explained by any reflection of foreign light from the planet's surface, but, if real, must originate there. They recall the Aurora Borealis and other electrical manifestations on the earth.
325. Surface-Markings, Rotation, Etc. - As has been said, Venus is a very unsatisfactory telescopic object. She presents no obvious surface-markings, - nothing to most observers but occasional indefinite shadings: sometimes, however, when in the crescent phase, intensely bright spots have been reported near the "cusps," or points, of the crescent. These may perhaps be "ice-caps," like those which are seen on Mars. The darkish shadings may possibly be continents and oceans, dimly visible, or they may be atmospheric objects ; observations do not yet decide. From certain irregularities occasionally observed upon the "terminator," some have maintained that there are high mountains on the surface, but the evidence is by no means satisfactory.

Lowell, in opposition to all previous observers, reported the discovery, at Flagstaff in 1896, of a system of permanent markings consisting of rather narrow, nearly straight, dark streaks, radiating like spokes from a sort of central "hub." He describes them as fairly definite in outline, but $\operatorname{dim}$, as if seen through a luminous though unclouded atmosphere of considerable depth ; and he goes so far as to give a map of the planet, with names attached to some of the leading features. His observations have been confirmed by other observers at Flagstaff.

No satellite has ever been discovered; not, however, for want of earnest searching. Venus probably has none.

325*. Rotation of the Planet. -Schröter, early in the century, assigned a rotation period of 23 h .21 m ., and the result was partially confirmed by some later observers, and generally accepted until recently, though not without misgivings. The planet's dise shows no sensible oblateness, as it ought to do if his figures were correct. The observations of Schiaparelli, on the other hand, while he did not consider them absolutely conclusive, indicate a very slow rotation, probably of 225 days, identical with the planet's orbital period, as in the case of Mercury and the moon. Mr. Lowell considered that his observations absolutely prove the correctness of this conclusion, and also that the equator of Venus is only very slightly inclined to her orbit.
326. Transits. - Occasionally Venus passes between the earth and the sun at inferior conjunction, giving us a so-called "transit." She is then visible even to the naked eye as a black spot on the disc, crossing it from east to west. When the transit is central, it occupies about eight hours, but when the track lies near the edge of the disc, the duration is of course correspondingly shortened. Since the sun passes the nodes of the orbit on June 5th and December 7th, all transits must occur near these dates, but they are very rare phenomena.

Their special interest consists in their availability for the purpose of finding the sun's parallax (see Appendix, Arts. 516519). The first observed transit (in 1639) was seen by only two persons, -Horrox and Crabtree, in England, but the four which have occurred since then have been extensively observed in all parts of the world where they were visible, by scientific expeditions sent out for the purpose by the different governments. The transits of 1769 and 1882 were visible in the United States.
327. Recurrence and Dates of Transits. - Five synodic, or thirteen sidereal revolutions of Venus are very nearly equal to eight years, the difference being little more than one day; and still more nearly-


Fig. 81.
Transit of Venus Tracks.
in fact, almost exactly - 243 years are equal to 152 synodic, or 395 sidereal revolutions. If, then, we have a transit at any time, we may have another at the same node eight years earlier or later. Sixteen years before or after, it will be impossible, and no other transit can occur at the same node until after the lapse of 235 or 243 years, although a transit or pair of transits may occur at the other node in about half that time. Transits of Venus have occurred, or will occur at the following dates:-

| $\begin{cases}\text { Dec. 7th, 1631. } \\ \text { Dec. 4th, 1639. }\end{cases}$ | $\left\{\begin{array}{l}\text { June 5th, } 1761 . \\ \text { June 3d, } 1769 .\end{array}\right.$ |
| :--- | :--- |
| $\left\{\begin{array}{l}\text { Dec. 9th, 1874. } \\ \text { Dec. 6th, 1882. }\end{array}\right.$ | $\left\{\begin{array}{l}\text { June 8th, 2004. } \\ \text { June 6th, 2012. }\end{array}\right.$ |

Fig. 81 shows the tracks of Venus across the sun's disc in the transits of 1874 and 1882.

## MARS.

328. This planet is also prehistoric as to its discovery. It is so conspicuous in color and brightness, and in the extent and apparent capriciousness of its movement among the stars, that it could not have escaped the notice of the very earliest observers.

Its mean distance from the sun is a little more than one and a half times that of the earth ( 141,500000 miles), and the eccentricity of its orbit is so considerable (0.093) that its radius vector varies more than 26,000000 miles. At opposition the planet's average distance from the earth is 48,600000 miles. When opposition occurs near the planet's perihelion, this distance is reduced to 35,500000 miles, while near aphelion it is over 61,000000. At superior conjunction, the average distance from the earth is 234,000000 .

The apparent diameter and brilliancy of the planet of course vary enormously with those great changes of distance. At a "favorable" opposition (when the distance is at its minimum),
the planet is more than fifty times as bright as at superior conjunction, and fairly rivals Jupiter; when most remote, it is hardly as bright as the Pole-star.

The favorable oppositions occur always in the latter part of August (at which time the earth as seen from the sun passes the perihelion of the planet), and at intervals of 15 or 17 years. The last such opposition was in 1909.

The inclination of the orbit is small, $1^{\circ} 51^{\prime}$.
The planet's sidereal period is 687 days, or 1 year $10 \frac{1}{2}$ months; its synodic period is much the longest in the planetary system, being 780 days, or nearly 2 years and 2 months. During 710 of the 780 days it moves eastward, and retrogrades during 70 .
329. Magnitude, Mass, Etc. - The apparent diameter of the planet ranges from $3^{\prime \prime} .6$ at conjunction to $24^{\prime \prime} .5$ at a favorable opposition. Its real diameter is approximately 4300 miles, with an error of perhaps 50 miles one way or the other. This makes its surface about two-sevenths, and its volume one-seventh of the earth's.

Its mass is a little less than $\frac{1}{9}$ of the earth's mass. This makes its density 0.73 and superficial gravity 0.38 ; a body which here weighs 100 pounds would have a weight of only 38 pounds on the surface of Mars.
330. General Telescopic Aspect, Phases, Albedo, Atmosphere, Etc. - When the planet is nearest the earth, it is more favorably situated ${ }^{1}$ for telescopic observation than any other heavenly body, - the moon alone excepted. It then shows a ruddy dise which, with a power of 75 , is as large as the moon.

[^33]Since its orbit is outside the earth's, it never exhibits the crescent phases like Mercury and Venus; but at quadrature it


Fig. 82.
Mars at Quadrature. appears distinctly gibbous, as in Fig. 82, about like the moon three days from the full. Like Mercury, Venus, and the moon, its dise is brighter at the limb (i.e., at the circular edge) than at the centre ; but at the "terminator," or boundary between day and night upon the planet's surface, there is a slight shading which, taken in connection with certain other phenomena, indicates the presence of an atmosphere. This atmosphere, however, contrary to opinions formerly held, is probably much less dense than that of the earth, the low density being indicated by the infrequency of clouds and of other atmospheric phenomena familiar to us upon the earth, to say nothing of the fact that since the planet's superficial gravity is less than $\frac{2}{5}$ the force of gravity on the earth, a dense atmosphere would be impossible.

More than forty years ago Huggins, Janssen, and Vogel all reported the lines of water-vapor in the spectrum of the planet's atmosphere ; but the observations of Campbell, at the Lick Observatory in 1894, throw great doubt on their result, and show that the water-vapor, if present at all, is too small in amount to give decisive evidence of its presence. ${ }^{1}$

Zöllner gives the albedo of Mars as 0.26 , just double that of Mercury, and much higher than that of the moon, but only about half that of Venus and the major planets. Near opposition, the brightness of the planet suddenly increases, in the same way as that of the moon near the full (Art. 162).
331. Rotation, Etc. - The spots upon the planet's disc enable us to determine its period of rotation with great precision.

[^34]Its sidereal day is found to be 24 hours, 37 minutes, 22.67 seconds, with a probable error not to exceed one-fiftieth of a second. This very exact determination is effected by comparing drawings of the planet made by Huyghens and Hooke more than 200 years ago with others made recently.
The inclination of the planet's equator to the plane of its orbit is very nearly $24^{\circ} 50^{\prime}\left(26^{\circ} 21^{\prime}\right.$ to the ecliptic). So far, therefore, as depends upon that circumstance, Mars should have seasons substantially the same as our own, and certain phenomena of the planet's surface, soon to be described, make it evident that such is the case.
The planet's rotation causes a slight but sensible flattening at the poles, -about $\frac{1}{20 \pi}$, according to the latest determinations.
(Much larger values, now known to be certainly erroneous, are found in the older text-books.)
332. Surface and Topography. - With even a small telescope, not more than three or four inches in diameter, the planet is a very beautiful object, showing a surface diversified with markings dark and light, which for the most part are found to be permanent objects. Occasionally, however, for a few hours at a time, we see others of a temporary character, supposed to be clouds; but these are surprisingly rare as compared with clouds upon the earth. The permanent markings on the planet are broadly divisible into three classes, -

First, The white patches, two of which are specially conspicuous near the planet's poles, and are by many supposed to be masses of snow or ice, since they behave just as would be expected if such were the case. The northern one dwindles away during the northern summer, when the North Pole is turned towards the sun, while the southern one grows rapidly larger; and vice versâ during the southern summer. But the probable low temperature of the planet (Art. 335) makes it at least doubtful whether the apparent "snow and ice" is really congealed water, or some quite different substance.

Second, Patches of bluish-gray or greenish shade, covering about $\frac{3}{8}$ of the planet's surface, until recently generally supposed to be bodies of water, and therefore called "seas" and "oceans." But more recent observations show a great variety of details within these areas, and such changes of appearance following the seasons of the planet, that this theory is no longer tenable, and they seem more likely to be regions covered with something like vegetation.

Third, Extensive regions of various shades of orange and yellow, covering nearly five-eighths of the surface, and interpreted as land. These markings are, of course, best seen when near the centre of the planet's disc; near the limb they are lost in the brilliant light which there prevails, and at the terminator they fade out in the shade.


Fig. 83. - Telescopic Views of Mars.
Fig. 83 gives an idea of the planet's general appearance, though without pretending to minute accuracy.
333. Recent Discoveries. The Canals and their Gemination.In addition to these three classes of markings, the Italian astronomer Schiaparelli in 1877 and 1879 reported the discovery of a great number of fine straight lines, or "canals" as he called them, crossing the ruddy portions of the planet's disc in all directions ; and in 1881 he announced that some of them become double at times. These new markings are faint, and very difficult to see, and for several years there was a strong suspicion that he was misled by some illusion; more recently, however, his results have been abundantly confirmed,
both in Europe and in the United States. It appears that in the observation of these objects the power of the telescope is less important than steadiness of the air and keenness of the observer's vision. Nor are they usually best seen when Mars is nearest, but their visibility depends largely upon the season on the planet; and this is especially the case with their "gem-


Fig. 83*.
ination." Fig. 83* from one of Mr. Lowell's drawings in 1894 , gives an idea of the extent and complexity of the canalsystem ; but the reader must not suppose that in the telescope it stands out with any such conspicuousness. The figure shows also how some of the canals cross the so-called "seas," and disprove the propriety of the name.

333*. As to the real nature and office of the "canals" there is a wide difference of opinion, and it is very doubtful if their true explanation has yet been reached. Indeed it is still quite possible that some of the peculiar phenomena reported are illusions, based on what the observers think they ought to see: it is easy to be deceived in attempting to interpret intelligibly what is barely visible. According to Flammarion, Lowell, and other zealous observers of the planet, the polar caps are really snow-sheets, which melt in the (Martian) spring, and send the water towards the planet's equator
over its nearly level plains (for no high mountains have yet been discovered there), obscuring for several weeks the well-known markings which are visible at other times. In Lowell's view the dark regions on the planet's surface are areas covered with some sort of vegetation, while the ruddy portions are barren deserts, intersected by the canals, which he believes to be really irrigating water courses; and on account of their straightness, and some other characteristics, he is disposed to regard them as artificial. When the water reaches these "canals" vegetation springs up along their banks, and these belts of verdure are what we see with our telescopes, - not the narrow water-channels themselves. Where the canals cross each other and the water supply is more abundant, there are dark, round "lakes," as they have been called, which he interprets as "oases." All of this theoretical explanation rests, however, upon the assumption that the planet's temperature is high enough to permit the existence of waterin the liquid state ; to say nothing of other difficulties. But whatever may be the explanation, there is no longer any doubt as to the existence of the "canals," nor that they (and other features of the surface) undergo real changes with the progress of the planet's seasons. Their "gemination," however, still remains a mystery, and in the report of the Harvard College Observatory for 1896 it is stated that some experiments recently made there throw a good deal of doubt on the "objective reality" of the doubling.
334. Maps of the Planet. - A number of maps of Mars have been constructed by different observers since the first was made by Maedler in 1830. Fig. 84 is reduced from one published in 1888 by Schiaparelli, and shows most of his "canals" and their " gemination." While there may be some doubt as to the accuracy of the minor details, there can be no question that the main features of the planet's surface are substantially correct. The nomenclature, however, is in a very unsettled condition. Schiaparelli has taken his names mostly from ancient geography, while the English areographers, ${ }^{1}$ following the analogy of the lunar maps, have mainly used the names of astronomers who have contributed to our knowledge of the planet's surface.

[^35]
335. Temperature. - As to the temperature of Mars we have no certain knowledge at present. Unless the planet has some unexplained sources of heat it ought to be very cold. Its distance from the sun reduces the intensity of solar radiation upon its surface to less than half its value upon the earth, and its atmosphere cannot well be as dense as at the tops of our loftiest mountains. On the other hand things look very much as if liquid water and vegetable life were present there. It is earnestly to be hoped that before long we may come into possession of some heat-measuring apparatus sufficiently delicate to decide whether the planet's surface is really intensely cold or reasonably warm, -for of course there are various conceivable hypotheses which might account for a high temperature at the surface of Mars.
336. Satellites. - The planet has two satellites, discovered by Hall, at Washington, in 1877. They are extremely small, and observable only with very large telescopes. The outer one, Deimos, is at a distance of 14,600 miles from the planet's centre, and has a sidereal period of 30 hours, 18 minutes; while the inner one, Phobos, is at a distance of only 5800 miles, and its period is only 7 hours, 39 minutes, - less than one-third of the planet's day. (This is the only case known of a satellite with a period shorter than the revolution of its primary.) Owing to this fact, it rises in the west, as seen from the planet's surface, and sets in the east, completing its strange backward diurnal revolution in about 11 hours. Deimos, on the other hand, rises in the east, but takes nearly 132 hours in its diurnal circuit, which is more than four of its months. Both the orbits are sensibly circular, and lie very closely in the plane of the planet's equator.

Micrometric measures of the diameter of such small objects are impossible, but from photometric observations, Prof. E. C. Pickering, assuming that they have the same reflecting power as that of Mars itself, estimates the diameter of Phobos as about seven miles, and
that of Deimos as five or six. Mr. Lowell, however, from his observations of 1894, deduced considerably larger values, viz. 10 miles for Deimos, and 36 for Phobos. If this is correct, Phobos, seen in the zenith from the point on the planet's surface directly beneath him, would appear somewhat larger than the moon but only about half as bright. Deimos would be no brighter than Venus.
337. Habitability of Mars. - As to this question we can only say that, different as must be the conditions on Mars from those prevailing on the earth, they differ less from ours than those on any other heavenly body observable with our present telescopes; and if life, such as we know it upon the earth, can exist on any of the planets, Mars is the one. If we could waive the question of temperature, and assume, with Flammarion and others, that the polar caps consist of frozen water, then it would become extremely probable that the growth of vegetation is the explanation of many of the phenomena actually observed.

Mr. Lowell went further and argued the presence of intelligent beings, possessed of high engineering skill, from the apparent "accuracy" with which the "canals" seem to be laid out, in a well planned system of irrigation. But at present, and until the temperature problem is solved, such speculations appear rather premature, to say the least.

## THE ASTEROIDS, OR MINOR PLANETS.

338. The asteroids ${ }^{1}$ are a multitude of small planets circling around the sun in the space between Mars an l Jupiter. It was early noticed that between Mars and Jupiter there is a gap in the series of planetary distances, and when Bode's Law (Art. 284) was published in 1772, the impression became very strong that there must be a missing planet in the space, - an impression greatly strengthened when Uranus was discovered in 1781, at a distance precisely corresponding to that law. An association was formed to search for the missing planet, but rather strangely the first discovery was made, not by a mem-
${ }^{1}$ They were first called "asteroids" (i.e., "star-like" bodies) by Sir William Herschel early in the century, because, though really planets, an ordinary telescope shows them only as stars, without a disc.
ber of the organization, but by the Sicilian astronomer, Piazzi, who on the very first night of the last century (Jan. 1st, 1801) discovered a planet which he named Ceres, after the tutelary divinity of Sicily. The next year Pallas was discovered by Olbers. Juno was found in 1804 by Harding, and in 1807 Olbers, who had broached the theory of an exploded planet, discovered the fourth, Vesta, the only one which is bright enough ever to be easily seen by the naked eye. The search was kept up for some years longer, but without success, because the searchers did not look for small enough objects. The fifth asteroid (Astræa) was found in 1845 by Hencke, an amateur who had resumed the subject afresh by studying the smaller stars. In 1847 three more were discovered, and every year since then has added from one to a hundred.

Since 1891, when Wolf of Heidelberg introduced photography as a means of discovery, the number has grown very rapidly. The list then included 321; now the number exceeds 800 . Most of the new discoveries are due to Wolf and his assistants, though others have made important contributions.

All the asteroids whose orbits have been determined are designated by numbers, mostly in the order of discovery ; all but the newer ones have names also.

Thus Ceres is (1), Thule is ( $\overline{279}$ ), etc.
339. Their Orbits. - The mean distances of the different asteroids from the sun differ considerably, and the periods, of course, correspond. Hungaria, ( $\overline{434}$ ), is nearest to the sun (excepting Eros; see p. 243), its distance from the sun being about 180,800000 miles, and its period about 2 years and 260 days. Hector, ${ }^{1}$ (624), one of the Jupiter group, is the most remote, with a mean distance of $5.2 \overline{(490,000,000 ~ m i l e s) . ~}$

The inclinations of the orbits to the ecliptic arerage nearly $8^{\circ}$. The orbit of Pallas, (2), is inclined at an angle of $35^{\circ}$, and several others exceed $25^{\circ}$. The eccentricity of the orbits is very

[^36]large in many cases. Ocllo, ( $\overline{4 \pi 5})$, has the almost cometary eccentricity of 0.38 and more than a dozen others have an eccentricity exceeding 0.30 .
340. The Bodies Themselves. - The four first discovered, and one or two others, when examined with a powerful telescope, show a perceptible dise, not large enough, however, for satisfactory measurement. Vesta is much the brightest of them, and until very recently was therefore supposed to be the largest. In 1894-5 Mr. Barnard, however, at the Lick Observatory, succeeded in making a set of micrometric measures which gave the following rather surprising results. He finds the diameter of Ceres to be 485 miles; of Pallas, 304 ; of Vesta, 243 ; and of Juno only 118. But the "probable error" must be considerable. The albedo of Vesta must much exceed that of the others. None of these bodies except the first four can be more than about 50 or 60 miles in diameter; and the newly discovered ones, barely visible in a 12 -inch telescope, cannot be larger than the moons of Mars, perhaps 10 or 20 miles in diameter.

As to the individual masses and densities, we have no certain knowledge.

If the deusity of Ceres is about the same as that of the rocks which compose the earth's crust, her mass may be as great as $\frac{1}{7000}$ that of the earth. If so, gravity on her surface would be about $\frac{1}{23}$ of gravity here, so that a body would fall about eight inches in the first second. Of course on the smaller asteroids it would be much less. If the hypothetical inhabitant and owner of one of these little planets should throw a stone, it would become independent, circling around the sun in an orbit of its own, and never returning to the planet.

It is, however, possible from the perturbations they produce on Mars to estimate a limit for the aggregate mass of the whole swarm. According to Leverrier, it cannot exceed onefourth the mass of the earth, but may be very much less. A still more recent computation by Ravené in 1896 indicates a total mass about $\frac{1}{115}$ that of the earth.

The united mass of those at present known would make only a small fraction of such a body, - hardly a thousandth of it; presumably, therefore, the number still undiscovered is to be counted by thousands, and they must be, for the most part, very much smaller than those already known. How long it will be considered worth while to hunt for new ones is a question now forcing itself on the attention of astronomers.
341. Origin. - As to this we can only speculate. It is hardly possible to doubt, however, that this swarm of little rocks in some way represents a single planet of the terrestrial group. A commonly accepted view is that the material, which, according to the nebular hypothesis, once formed a ring (like one of the rings of Saturn), and ought to have collected to make a single planet, has failed to be so united; and the failure is ascribed to the perturbations produced by the giant Jupiter, whose powerful attraction is supposed to have torn the ring to pieces, and thus prevented the normal development of a planet. Another view is that the asteroids may be fragments of an exploded planet. If so, there must have been not one, but many, explosions; first of the original body, and then of the separate pieces, for it is demonstrable that no single explosion could account for the present tangle of orbits.
342. Intra-Mercurial Planets. - It is not improbable that there is a considerable quantity of matter circulating around the sun inside the orbit of Mercury. This has been believed to be indicated by an otherwise unexplained advance of the perihelion of its orbit. It has been somewhat persistently supposed that this intra-Mercurial matter is concentrated into one, or possibly two, planets of considerable size, and such a planet has several times been reported as discovered, and has even been named "Vulcan." We can only say here that the supposed discoveries have never been confirmed, and the careful observations of total solar eclipses during the past ten years make it practically certain that there is no "Vulcan." Perhaps, however, there is an intra-Mercurial family of "asteroids." But they must be very minute or some of them would certainly have been found either during eclipses or crossing the sun's disc; a planet as much as 200 miles in diameter could hardly have escaped discovery.
343. The Zodiacal Light. - This is a faint beam of light extending from the sun both ways along the ecliptic. In the evening it is best seen in the months of February, March, and April, and in our latitudes then extends about $90^{\circ}$ eastward from the sun; in the tropics, it is said that it can be followed quite across the sky, forming a complete ring. At the point in the sky directly opposite to the sun, there is a patch of slightly greater luminosity, called the "Gegenschein" or "counter-glow." The region near the sun is fairly bright and even conspicuous, but the more distant portions are extremely faint and can be observed, like the fainter portions of the milky way, only in places where there is no illumination of the air by artificial lights. The spectrum is a simple, continuous spectrum, without markings of any kind.

We emphasize this, because it has often been mistakenly reported that the line which characterizes the spectrum of the Aurora Borealis appears in the spectrum of the zodiacal light.

The cause of the phenomenon is not certainly known. Some imagine that the zodiacal light is only an extension of the solar corona (whatever that may be), which is not perhaps unlikely; but on the whole the more prevalent opinion seems to be that it is due to sunlight, reflected from myriads of small meteoric bodies circling around the sun, nearly in the plane of the ecliptic, thus forming a thin, flat sheet (something like one of Saturn's rings), which extends far beyond the orbit of the earth. Near the sun, within the orbits of Mercury and Venus, they are supposed to be much more numerous than at a greater distance; thus accounting for the greater brightness of the zodiacal light in that part of the sky, notwithstanding the fact that all the meteoric particles within $90^{\circ}$ from the sun would present to us less than half their illuminated surface: if globular, they would show a crescent phase like the moon between new and half.

As for the " Gegenschein," this is explained by supposing that the particles opposite to the sun in the sky "flash out"
in the same way the moon does at the full. See Art. 162.
We have no direct evidence as to the size of the meteoric particles, though the analogy of shooting stars suggests that they are probably very small. See Art. 410.

The peculiar disturbance of Mercury, referred to in the preceding article, may be, perhaps, due to the denser portion of this ring near the sun, and this is the reason why we consider the subject here in connection with the planets, rather than later in connection with the subject of meteors. The theory has also been maintained that the zodiacal light is due to a meteoric ring surrounding the earth.

Note to Art. 339. - Eros, ( $\overline{433}$ ), is in many respects the most remarkable of the minor planets thus far known. It was discovered (photographically) by Witt at Berlin in August, 1898, and turns out to have a mean distance and period less than that of Mars. The semi-major axis of its orbit is $135,480,000$ miles, and its period 643.14 days. The eccentricity of its orbit is 0.223 , so that at aphelion it recedes to a distance of nearly 166,000000 miles from the sun, while at perihelion its distance is $105,290,000$ miles, which is onlýy 12,000000 miles greater than the earth's mean distance; but the two orbits are so situated that the planet's smallest possible distance from the earth is 13,500000 miles, and occurs only when Eros comes to opposition about January 21st. Observations of Eros made at such an opposition will furnish by far the most accurate determination of the solar parallax (see Art. 515). Unfortunately such oppositions are rare. One occurred in 1894 (before the planet was discovered) ; the next will be in 1931. In the winter of 1900-1901 the conditions, though not the best, - the least distance between the planet and the earth having been nearly 30,000000 miles, -were much better than usual, and an immense number of careful observations were made, the results of which have not yet been completely worked out. ${ }^{1}$ The inclination of the planet's orbit is nearly $11^{\circ}$, so that when nearest the earth, as in 1894, it goes far north, and at the time of opposition moves among the stars nearly from north to south instead of from east to west.

It is extremely small, probably not over 20 miles in diameter, and seldom visible except with the largest telescopes; but its changes of apparent brightness are extreme, and it is just possible that when nearest it may become visible to the naked eye. There is also a periodic variation which seems to indicate an axial rotation in about $5 \frac{1}{4}$ hours.

[^37]
## CHAPTER XII.

THE MAJOR PLANETS. - JUPITER: ITS SATELLITE SYS TEM ; THE EQUATION OF LIGHT, AND THE DISTANCE OF THE SUN. - SATURN: ITS RINGS AND SATELLITES.

- URANUS: ITS DISCOVERY, PECULIARITIES, AND SATELLITES. - NEPTUNE : ITS DISCOVERY, PECULIARITIES, AND SATELLITE.

344. Jupiter, the nearest of the major planets, stands next to Venus in the order of brilliance among the heavenly bodies, being fully five or six times as bright as Sirius, the most brilliant of the stars, and decidedly superior to Mars, even when Mars is nearest. It is not, like Venus, confined to the twilight sky, but at the time of opposition dominates the heavens all night long.

Its orbit presents no marked peculiarities. The mean distance of the planet from the sun is a little more than five astronomical units ( 483,000000 miles), and the eccentricity of the orbit is not quite $\frac{1}{20}$, so that the actual distance ranges about 21,000000 miles each side of the mean. At an average opposition, the planet's distance from the earth is about 390,000000 miles, while at conjunction it is distant about 580,000000 ; but it may come as near to us as 370,000000 , and may recede to a distance of nearly 600,000000 .

The inclination of its orbit to the ecliptic is only $1^{\circ} 19^{\prime}$. Its sidereal period is 11.86 years, and the synodic is 399 days (a figure easily remembered), a little more than a year and a month.
345. Dimensions, Mass, Density, Etc. - The planet's apparent diameter varies from $50^{\prime \prime}$ to $32^{\prime \prime}$, according to its distance from
the earth. The disc, however, is distinctly oval, so that while the equatorial diameter is 88,200 miles, ${ }^{1}$ the polar diameter is only 83,000 . The mean diameter, $\left(\frac{2 a+b}{3}\right)$ (see Art. 95), is 86,500 miles, or very nearly eleven times that of the earth.

Its surface, therefore, is 119 , and its volume or bulk 1300 times that of the earth. It is by far the largest of all the planets, - larger, in fact, than all the rest united.

Its mass is very accurately known, both by means of its satellites and from the perturbations it produces upon certain asteroids. It is $\frac{1}{1048}$ of the sun's mass, or about 316 times that of the earth.

Comparing this with its volume, we find its mean density to be 0.24 ; i.e., less than one-fourth the density of the earth, and almost precisely the same as that of the sun. Its surface gravity is about $2 \frac{2}{3}$ times that of the earth, but varies nearly 20 per cent between the equator and poles of the planet on account of its rapid rotation.
346. General Telescopic Aspect, Albedo, Etc.-In even a small telescope the planet is a fine object, for a magnifying power of only 60 makes its apparent diameter, even when remotest, equal to that of the moon. With a large instrument and a magnifying power of 200 or 300 , the disc is covered with an infinite variety of detail, interesting in outline and rich in color, changing continually as the planet turns on its axis. For the most part the markings are arranged in "belts" parallel to the planet's equator, as shown in Fig. 85.

The left-hand one of the two larger figures is from a drawing by Trouvelot (1870), and the other from one by Vogel (1880). The smaller figure below represents the planet's ordinary appearance in a three-inch telescope.

Near the limb the light is less brilliant than in the centre of the disc, and the belts there fade out. The planet shows no

[^38]perceptible phases, but the edge which is turned away from the sun is usually sensibly darker than the other. According to Zöllner, the mean albedo of the planet is 0.62 , which is extremely high, that of white paper being 0.78 . The question has been raised whether Jupiter is not to some extent self-luminous, but there is no proof and little probability that such is the case.


Fig. 85. - Telescopic Views of Jupiter.
347. Atmosphere and Spectrum. - The planet's atmosphere must be very extensive. The forms visible with the telescope are nearly all evidently atmospheric. In fact, the low mean density of the planet makes it very doubtful whether there is anything solid about it anywhere, - whether it is anything more than a ball of fluid overlaid by cloud and vapor.

The spectrum of the planet differs less from that of mere reflected sunlight than might have been expected, showing that the light is not obliged to penetrate the atmosphere to any
great depth before it encounters the reflecting envelope of clouds. There are, however, dark shadings in the red and orange parts of the spectrum that are probably due to the planet's atmosphere, and seem to be identical in position with certain bands which are intense in the spectra of Uranus and Neptune.
348. Rotation. - Jupiter rotates on its axis more swiftly than any other of the planets. Its sidereal day has a length of about 9 hours, 55 minutes. The time can be given only approximately, not because it is difficult to find and observe welldefined objects on the disc, but because different results are obtained from different spots, according to their nature and their distance from the equator, - the differences amounting to six or seven minutes. Speaking generally, spots near the equator indicate a shorter period of rotation than those near the poles, just as is the case with the sun.

In consequence of the swift rotation, the planet's "oblateness" or "polar compression" is quite noticeable, -about $\frac{1}{17}$. The plane of rotation nearly coincides with that of the orbit, the inclination being only $3^{\circ}$, so that there can be no wellmarked seasons on the planet due to the causes which produce our own seasons.
349. Physical Condition. - This is obviously very different from that of the earth or Mars. No permanent markings are found upon the disc, though occasionally some which may be called "sub-permanent" do appear, as, for instance, the " great red spot" shown in Fig. 85. This was first noticed in 1878, became extremely conspicuous for several years, and is still visible, although only a faded ghost of itself. Were it not that during the first 8 years of its visibility it changed the length of its apparent rotation by about six seconds (from 9 hours, 55 minutes, 34.9 seconds to 9 hours, 55 minutes, 40.2 seconds), we might suppose it permanently attached to the
planet's surface, and evidence of a coherent mass underneath. As it is, opinion is divided on this point.

Many things in the planet's appearance indicate a high temperature, as, for instance, the abundance of clouds, and the swiftness of their transformations; and since on Jupiter the solar light and heat are only $\frac{1}{27}$ as intense as here, we are forced to conclude that it gets very little of its heat from the sun, but is probably hot on its own account, and for the same reason that the sun is hot; viz., as the result of a process of condensation. In short, it appears very probable, as has been intimated before, that the planet is a sort of "semi-sun," hot, though not so hot as to be sensibly self-luminous.
350. Satellites. - Jupiter has nine satellites now known. Four of them are so large as to be easily seen with an opera glass, and were discovered by Galileo in 1610 with his newly invented telescope. They are usually known as the first, second, etc., in the order of distance from the planet. The distances range from 262,000 to $1,169,000$ miles, and their sidereal periods from 42 hours to $16 \frac{2}{3}$ days. Their orbits are sensibly circular, and very nearly in the plane of the planet's equator. The third is much the largest, - about 3600 miles in diameter; the others are between 2000 and 3000 . The fifth, discovered by Barnard in 1892 at the Lick Observatory, is extremely small and lies very close to the planet. The sixth and seventh, discovered by Perrine, also at the Lick Observatory, by photography, in $1905,{ }^{1}$ are still smaller, and at nearly equal distances of over $7,000,000$ miles, with periods of about $8 \frac{1}{2}$ and 9 months. The eighth was discovered by Melotte at Greenwich, also by photography, in 1908, and the ninth by Nicholson at the Lick Observatory in 1914.

The fourth satellite is very dark, so that on the planet's disc it is hardly distinguishable from its shadow. The others appear bright, invisible, or dark, according to the brightness of the background.

[^39]Spots are sometimes seen on their surfaces, and there are variations of brightness, conspicuous in the case of the fourth, which seem to indicate that, like the moon, they keep always the same face towards their primary.

Some observers have reported changes of form as if they were not solid but cloudlike.
351. Eclipses and Transits. - The orbits of the four large satellites are so nearly in the plane of the planet's orbit that excepting the fourth, which sometimes escapes, they are eclipsed at every revolution, and also cross the planet's disc at every conjunction. When the planet is either at opposition or conjunction, the shadow, of course, is directly behind it, and we cannot see the eclipse at all. At other times we ordinarily see only the beginning or the end; but when the planet is at or near quadrature the shadow projects so far to one side that the whole eclipse of every satellite, except the first, takes place clear of the disc. An eclipse is a gradual phenomenon, the satellite disappearing by becoming slowly fainter and fainter as it plunges into the shadow, and reappearing in the same leisurely way.

Two important uses have been made of these eclipses: they have been employed for the determination of longitude, and they furnish the means of ascertaining the time required by light to traverse the space between the earth and the sun.
352. The Equation of Light. - When we observe a celestial body we see it not as it is at the moment of observation, but as it was at the moment when the light which we see left it. If we know its distance in astronomical units, and know how long light takes to traverse that unit, we can at once correct our observation by simply dating it back to the time when the light started from the object. The necessary correction is called the "equation of light," and the time required by light to traverse the astronomical unit of distance is the "Constant
of the Light-equation" (not quite 500 seconds, as we shall see).

It was in 1675 that Roemer, the Danish astronomer, (the inventor of the transit instrument, meridian-circle, and prime-vertical instrument, - a man almost a century in advance of his day,) found that the eclipses of Jupiter's satellites show a peculiar variation in their times of occurrence, which he explained as due to the time taken by light to pass through space. His bold and original suggestion was neglected for more than 50 years, until long after his death, when Bradley's discovery of aberration proved the correctness of his views.
353. Eclipses of the satellites recur at intervals which are really almost exactly equal (the perturbations being very slight), and ihe interval can easily be determined and the times tabulated. But if we thus predict the times of the eclipses during a whole synodic period of


Fig. 86.
Deiermination of the Equation of Light. the planet, then, beginning at the time of opposition, it is found that as the planet recedes from the earth, the eclipses, as observed, fall constantly more and more behindhand, and by precisely the same amount for all four satellites. The difference between the predicted and observed time continues to increase until the planet is near conjunction, when the eclipses are almost 17 minutes later than the pre- . diction. After the conjunction they quicken their pace, and make up the loss, so that when opposition is reached once more they are again on time.

It is easy to see from Fig. 86 that at opposition the planet
is nearer the earth than at conjunction by just two astronomical units; i.e., $J B-J A=2 S A$, and light coming from $J$ to the earth when it is at $A$, will, therefore, make the journey quicker than when it is at $B$, by twice the time it takes light to pass from $S$ to $A$.

The whole apparent retardation of eclipses between opposition and conjunction must therefore be exactly twice the time ${ }^{1}$ required for light to come from the sun to the earth. In this way the "light-equation constant" is found to be very nearly 499 seconds, or 8 minutes, 19 seconds, with a probable error of perhaps two seconds.
354. Since these eclipses are gradual phenomena, the determination of the exact moment of a satellite's disappearance or reappearance is very difficult, and this renders the result somewhat uncertain. Prof. E. C. Pickering of Cambridge has proposed to utilize photometric observations for the purpose of making the determination more precise, and two series of observations of this sort and for this purpose are now nearly completed, one in Cambridge, and the other at Paris under the direction of Cornu, who has devised a similar plan. Pickering has also applied photography to the observation of these eclipses with encouraging success.
355. The Distance of the Sun determined by the "Lightequation." - Until 1849 our only knowledge of the velocity of light was obtained from such observations of Jupiter's satellites. By assuming as known the earth's distance from the sun, the velocity of light can be obtained when we know the time occupied by light in coming from the sun. At present, however, the case is reversed. We can determine the velocity of light by two independent experimental methods, and with a

[^40]surprising degree of accuracy. Then, knowing this velocity and the "light-equation constant," we can deduce the distance of the sun. According to the latest determinations the velocity of light is 186,330 miles per second. Multiplying this by 499 we get $92,979,000$ miles for the sun's distance (compare Art. 127).

## SATURN.

356. This is the most remote of the planets known to the ancients. It appears as a star of the first magnitude (outshining all of them, indeed, except Sirius), with a steady, yellowish light, not varying much in appearance from month to month, though in the course of 15 years it alternately gains and loses nearly 50 per cent of its brightness with the changing phases of its rings : for it is unique among the heavenly bodies, a great globe attended by nine satellites and surrounded by a system of rings, which has no counterpart elsewhere in the universe so far as known.

Its mean distance from the sun is about $9 \frac{1}{2}$ astronomical units, or 886,000000 miles ; but the distance varies nearly 100,000000 miles on account of the considerable eccentricity of the orbit ( 0.056 ). Its nearest opposition approach to the earth is about 774,000000 miles, while at the remotest conjunction it is 1028,000000 miles away. The inclination of the orbit to the ecliptic is about $2 \frac{1}{2}^{\circ}$. The sidereal period is about $29 \frac{1}{2}$ years, the synodic period being 378 days.
357. Dimensions, Mass, Etc. - The apparent mean diameter of the planet varies according to the distance from $14^{\prime \prime}$ to $20^{\prime \prime}$. The planet is more flattened at the poles than any other (nearly $\frac{1}{10}$ ), so that while the equatorial diameter is about 75,000 miles, the polar is only 68,000 : the mean diameter, therefore, $\left(\frac{2 a+b}{3}\right)$, is not quite 73,000 , - a little more than nine times the diameter of the earth. Its surface is about 84
times that of the earth, and its volume 770 times. Its mass is found (by means of its satellites) to be 95 times that of the earth, so that its mean density comes out only one-eighth that of the earth, actually less than that of water! It is by far the least dense of all the planetary family.

Its mean superficial gravity is about 1.2 times gravity upon the earth, varying, however, nearly 25 per cent between the equator and the pole. It rotates on its axis in about 10 hours, 14 minutes, as determined by Hall in 1876 from a white spot that for a few weeks appeared upon its surface. The observations of Stanley Williams in 1893, while generally confirming Hall's result, furnish evidence that spots in different latitudes have slightly different periods.

The equator of the planet is inclined about $27^{\circ}$ to the plane of its orbit.
358. Surface, Albedo, Spectrum. - The disc of the planet, like that of Jupiter, is shaded at the edge, and like Jupiter it shows a number of belts arranged parallel to the equator. The equatorial belt is very bright (not relatively quite so much so, however, as represented in Fig. 87), and is often of a delicate pinkish tinge. The belts in higher latitudes are comparatively faint and narrow, while just at the pole there is usually a cap of olive green. Occasionally there are slight irregularities in the edges of the belts.

Zöllner makes the mean albedo of the planet 0.52, about the same as that of Venus.

The planet's spectrum is substantially like that of Jupiter, but the dark bands are rather more pronounced. These bands, however, do not appear in the spectrum of the ring, which probably has very little atmosphere. As to the physical condition and constitution of the planet, it is probably much like Jupiter, though it does not seem to be "boiling" quite so vigorously.


Fig. 87. - Saturn and his Rings.
359. The Rings. - The most remarkable peculiarity of the planet is its ring system. The globe is surrounded by three thin, flat, concentric rings, like circular discs of paper pierced through the centre. They are generally referred to as $A, B$, and $C, A$ being the exterior one.

Galileo half discovered them in 1610 ; that is, he saw with his little telescope two appendages on each side of the planet; but he could make nothing of them, and after a while he lost them. The problem remained unsolved for nearly 50 years, until Huyghens explained the mystery in 1655 . Twenty years later D. Cassini discovered that the ring is double; i.e., composed of two concentric rings, with a dark line of separation between them; and in 1850 Bond of Cambridge, U.S., discovered a third "dusky" or "gauze" ring between the principal ring and the planet. (It was discovered a fortnight later, but independently, by Dawes in England.)

The outer ring, $A$, has a diameter of about 168,000 miles, ${ }^{1}$ and a width of about 10,000 . Cassini's division is about 1000 miles wide ; the ring, $B$, which is much the broadest of the three, is about 17,000 . The semi-transparent ring, $C$, has a width of about 9000 miles, leaving a clear space of from 9000 to 10,000 miles in width between the planet's equator and its inner edge. Their plane coincides with that of the planet's equator. The thickness of the rings is extremely small, -probably not over 100 miles, as proved by the appearance presented when once in 15 years we view them edgewise.
360. Phases of the Rings. - The rings are inclined about $28^{\circ}$ to the ecliptic, and, of course, maintain their plane parallel to itself at all times. Twice in a revolution of the planet, therefore, this plane sweeps across the orbit of the earth (too small to be shown in the figure - Fig. 88), occupying nearly a year in so doing; and whenever the plane passes between the earth and the sun the dark side of the ring is towards us, and the edge alone is visible.

[^41]While the plane of the ring is sweeping over the earth's orbit she crosses the plane once or three times according to circumstances; usually three times, causing iwo periods of disappearance. When the edge is exactly towards us only the largest telescopes can see the ring, like a fine needle of light piercing the planet's ball, as in the uppermost engraving of Fig. 87. The last disappearance occurred in 1907.


Fig. 88. - The Phases of Saturn's Rings.
361. Structure of the Rings. - It is now universally admitted that they are not continuous sheets, either solid or liquid, but mere swarms of separate particles, each pursuing its own independent orbit around the planet, though all moving nearly in a common plane.

The idea was first suggested by J. Cassini in 1715, but was lost sight of until again suggested by Bond in connection with his discovery of the semi-transparent or dusky ring; it has finally been established by the researches of Peirce, Maxwell, and others, that no continuous sheet of solid or liquid matter could stand the strain of rotation, while all the conditions of the problem are met by the meteoric hypothesis, and in 1895, Keeler, of the Allegheny Observatory, obtained a beautiful spectroscopic confirmation of the theory by showing, from a photograph of the spectrum of the planet and its rings, that the particles at the outer edge of the ring are moving more slowly than those at the inner edge, the velocities being respectively about 10.1 and 12.4 miles a second, - as they ought to be.

It is a question not yet settled whether the rings constitute a stable system, or are liable ultimately to be broken up.
362. Satellites. - Saturn has ten ${ }^{1}$ of these attendants, the largest of which was discovered by Huyghens in 1655. It looks like a star of the ninth magnitude, and is easily seen with a three-inch telescope.

Four others were discovered by D. Cassini, before 1700 , - two by Sir Williain Herschel, near the end of the last century, and one, Hyperion, by Bond of Cambridge, U.S., in September, 1848. The ninth and tenth ${ }^{1}$ were also discovered at Cambridge by W. H. Pickering, in 1898 and 1905, and named Phœbe and Themis.

Since the order of the discovery of the satellites does not agree with that of the order of the distance, it has been found necessary to designate them by the names assigned by Sir John Herschel, as follows, beginning with the most remote of the old satellites, viz. : Iapětus, (Hyperion), Titan, Rhea, Dione, Tethys; Encelădus, Mimas.
It will be noticed that these names, leaving out Hyperion, which was not discovered when the others were assigned, form a line and a half of a regular Latin pentameter.

The range of the system is enormous. ${ }^{1}$ Iapetus has a distance of $2,225,000$ miles, with a period of 79 days, nearly as long as that of Mercury. On the western side of the planet, this satellite is always much brighter than upon the eastern, showing that, like our own moon, it keeps the same face towards the planet at all times, - one-half of its surface having a higher reflecting power than the other.

Titan, as its name suggests, is by far the largest. Its distance is about 770,000 miles, and its period a little less than 16 days. It is probably 3000 or 4000 miles in diameter, and, according to Stone, its mass is $\frac{1}{6600}$ of Saturn's. The orbit of Iapetus is inclined about $10^{\circ}$ to the plane of the rings, but all of the other satellites move exactly in their plane, and all the five inner ones in orbits sensibly circular. It is not impossible, nor even improbable, that other minute satellites besides Hyperion and Themis may yet be discovered in the great gap between Titan and Iapetus.

[^42]
## URANUS.

363. Urănus was the first planet ever "discovered," and the discovery created great excitement and brought the highest honors to the astronomer. It was found accidentally by the elder Herschel on March 13, 1781, while "sweeping" the heavens for interesting objects with a seven-inch reflector of his own construction. He recognized it at once by its disc as something different from a star, but supposed it to be a peculiar sort of a comet, and its planetary character was not demonstrated until nearly a year had passed. It is easily visible to a good eye as a star of the sixth magnitude.

Its mean distance from the sun is about 19 times that of the earth, or about 1800,000000 miles, and the eccentricity of its orbit is about the same as that of Jupiter's, making the aphelion distance nearly 70,000000 miles greater than the distance at perihelion. The inclination of the orbit to the ecliptic is very slight - only $46^{\prime}$. The sidereal period is 84 years, and the synodic, $369 \frac{1}{3}$ days.

In the telescope it shows a greenish dise about $4^{\prime \prime}$ in diameter, which corresponds to a real diameter of about $32,000^{1}$ miles. This makes its bulk about 66 times that of the earth. The planet's mass is found by its satellites to be about 14.6 times that of the earth; so that its density and surface gravity are respectively 0.22 and 0.90 . The albedo of the planet, according to Zöllner, is very high, 0.64 , - even a little above that of Jupiter. The spectrum exhibits intense dark bands in the red, due to some unidentified substance in the planet's atmosphere, which is probably dense. These bands explain the marked greenish tint of the planet's light.

The disc is obviously oval, with an ellipticity of about $\frac{1}{14}$ according to the writer's observations, which are confirmed by those of Schiaparelli. There are no clear markings on the disc, but there seem to be faint traces of something like belts,

[^43]which, most singularly and inexplicably, seem to lie, not in the plane of the satellite orbits, but at an inclination of $20^{\circ}$ or so to that plane. The observations, however, are far from satisfactory or conclusive. No spots are visible from which to determine the planet's diurnal rotation.
364. Satellites. - The planet has four satellites, Ariel, Umbriel, 'Titania, and Oberon, Ariel being the nearest to the planet.

The two brightest, Oberon and Titania, were discovered by Sir William Herschel a few years after his discovery of the planet; Ariel and Umbriel, by Lassell in 1851.

They are telescopically among the smallest bodies in the solar system, and most difficult to see. In real size, they are, of course, much larger than the satellites of Mars, very likely measuring from 200 to $\check{0} 00$ miles in diameter.

Their orbits are sensibly circular, and all lie in one plane, which ought to be, and probably is, coincident with the plane of the planet's equator; but the belts raise questions.

They are very close packed also, Oberon having a distance of only 375,000 miles, and a period of 13 days, 11 hours, while Ariel has a period of 2 days, 12 hours, at a distance of 120,000 miles. Titania, the largest and brightest of them, has a distance of 280,000 miles, somewhat greater than that of the moon from the earth, with a period of 8 days, 17 hours.

The most remarkable thing about this system remains to be mentioned. The plane of their orbits is inclined $82^{\circ} .2$ to the plane of the ecliptic, and in that plane they revolve backwards; or we may say, what comes to the same thing, that their orbits are inclined to the ecliptic at an angle of $97^{\circ} .8$, in which case their revolution is considered as direct.

## NEPTUNE.

365. Discovery. - The discovery of this planet is reckoned the greatest triumph of mathematical astronomy. Uranus
failed to move in precisely the path computed for it, and was misguided by some unknown influence to an extent which could almost be seen with the naked eye. The difference between the actual and computed places in 1845 was the "intolerable quantity " of nearly two minutes of are.

This is a little more than half the distance between the two principal components of the double-double star, Epsilon Lyræ, the northern one of the two little stars which form the small equilateral triangle with Vega (Art. 468). A very sharp eye detects the duplicity of Epsilon without the aid of a telescope.

One might think that such a minute discrepancy between observation and theory was hardly worth minding, and that to consider it "intolerable" was putting the case very strongly. But just these minute discrepancies supplied the data which were found sufficient for calculating the position of a great world, until then unknown, and bringing it to light. As the result of a most skilful and laborious investigation, Leverrier wrote to Galle in substance:-
"Direct your telescope to a point on the ecliptic in the constellation of Aquarius, in longitude $326^{\circ}$, and you will find within a degree of that place a new planet, looking like a star of about the ninth magnitude, and having a perceptible disc."

The planet was found at Berlin on the night of Sept. 23, 1846, in exact accordance with this prediction, within half an hour after the astronomers began looking for it, and within $52^{\prime}$ of the precise point that Leverrier had indicated.

We cannot here take the space for a historical statement, further than to say that the English Adams fairly divides with Leverrier the credit for the mathematical discovery of the planet, having solved the problem and deduced the planet's approximate place even earlier than his competitor. The planet was being searched for in England at the time it was found in Germany. In fact, it had already been observed, and the discovery would necessarily have followed in a few weeks, upon the reduction of the observations.
366. Error of the Computed Orbit. - Both Adams and Leverrier, besides calculating the planet's position in the sky, had deduced elements of its orbit and a value for its mass, which turned out to be seriously wrong. The reason was that they assumed that the new planet's mean distance from the sun would follow Bode's Law, a supposition perfectly warranted by all the facts then known, but which, nevertheless, is not even roughly true. As a consequence their computed elements were erroneous, and that to an extent which has led high authorities to declare that the mathematically computed planet was not Neptune at all, and that the discovery of Neptune itself was simply a " happy accident." This is not so, however. While the data and methods employed were not by themselves sufficient to determine the planet's orbit with accuracy, they were adequate to ascertain the planet's direction from the earth. The computers informed the observers where to point their telescopes, and this was all that was necessary for finding the planet. In a similar case the same thing could be done again.
367. The Planet and its Orbit. - The planet's mean distance from the sun is a little more than 2800,000000 miles (instead of being over 3600,000000 , as it should be according to Bode's Law). The orbit is very nearly circular, its eccentricity being only 0.009 . Even this, however, makes a variation of over 50,000000 miles in the planet's distance from the sun. The inclination of the orbit is about $1 \frac{3}{4}^{\frac{3}{2}}$. The period of the planet is about 164 years (instead of 217 as it should have been according to Leverrier's computed orbit), and the orbital velocity is about $3 \frac{1}{3}$ miles per second.
Neptune appears in the telescope as a small star of between the eighth and ninth magnitudes, absolutely invisible to the naked eye, though easily seen with a good opera-glass. Like Uranus, it shows a greenish disc, having an apparent diameter of about $2^{\prime \prime} .6$. The real diameter of the planet is about $35,000^{1}$ miles ; but the probable error of this must be fully 500 miles. The volume is a little more than 90 times that of the earth.

[^44]Its mass, as determined by means of its satellite, is about 18 times that of the earth, and its density 0.20 .

The planet's albedo, according to Zöllner, is 0.46 , a trifle less than that of Saturn and Venus.

There are no visible markings upon its surface, and nothing certain is known as to its rotation.

The spectrum of the planet appears to be like that of Uranus, but of course is rather faint.

It will be noticed that Uranus and Neptune form a "pair of twins," very much as the earth and Venus do, being almost alike in magnitude, density, and many other characteristics.
368. Satellite. - Neptune has one satellite, discovered by Lassell within a month after the discovery of the planet itself. Its distance is about 223,000 miles, and its period $5^{\text {d }}, 21^{\text {h }}$. Its orbit is inclined to the ecliptic at an angle of $34^{\circ} 48^{\prime}$, and it moves backward in it from east to west, like the satellites of Uranus. It is a very small object, not quite as bright as Oberon, the outer satellite of Uranus. From its brightness, as compared with that of Neptune itself, we estimate its diameter as about the same as that of our own moon.
369. The Solar System as seen from Neptune. - At Neptune's distance the sun itself has an apparent diameter of only a little more than one minute of arc, - about the diameter of Venus when nearest us, and too small to be seen as a disc by the naked eye, if there are eyes on Neptune. Its light and heat are there only $\frac{1}{900}$ of what we get at the earth. Still, we must not imagine that the Neptunian sunlight is feeble as compared with starlight, or even moonlight. Even at the distance of Neptune the sun gives a light nearly equal to 700 full moons. This is about 80 times the light of a standard candle at one metre's distance, and is abundant for all visual purposes. In fact, as seen from Neptune, the sun would look very like a large electric arc lamp, at a distance of a few yards.
370. Ultra-Neptunian Planets. - Perhaps the breaking down of Bode's Law at Neptune may be regarded as an indication that the solar system terminates there and that there is no remoter planet, but of course it does not make it certain. If such a planet exists, however, it is sure to be found sooner or later, probably by means of the disturbances it produces in the motion of Uranus and Neptune. Prof. W. H. Pickering has recently determined the elements of such a hypothetical planet " O ," basing his determination upon the perturbations of Uranus. A photographic search for the planet has been made, but without success.

Note to Art. 350. - Jupiter's New Satellites. The orbits of the sixth and seventh are remarkable not only for their near equality and immense size but because they are both inclined to the planet's equator more than $30^{\circ}$. They face differently, however, so that they do not coincide, but interlock. The orbit of the sixth is quite eccentric. Both satellites are extremely small, the seventh much smaller than the other, neither of them being visible to the eye in any ordinary telescope.

The eighth and ninth are another pair of twins, twice as far from Jupiter as the sixth and seventh, and revolving in the opposite direction.

Note to Art. 362. - Saturn's New Satellites. Although the discovery of Saturn's ninth satellite, Phœebe, upon photographs made at Arequipa, was first announced in 1898, it was not until the spring of 1904 that it was finally confirmed. The satellite is more than eight million miles distant from the planet, about which it revolves once in 550 days. Its motion is backward, from east to west, like that of the eighth and ninth satellites of Jupiter.

The discovery of a tenth satellite having an orbit of nearly the same size and period as Hyperion was announced in 1905, but has not been fully confirmed.

## CHAPTER XIII.

## COMETS AND METEORS.

THE NUMBER, DESIGNATION, AND ORBITS OF COMETS. THEIR CONSTITUENT PARTS AND APPEARANCE.-THEIR SPECTRA AND PHYSICAL CONSTITUTION. - THEIR PROBABLE ORIGIN. - REMARKABLE COMETS. - AEROLITES, THEIR FALL AND CHARACTERISTICS. - SHOOTING STARS AND METEORIC SHOWERS. - CONNECTION BETWEEN COMETS AND METEORS.
371. From time to time bodies very different from the stars and planets appear in the heavens, remain visible for some weeks or months, pursue a longer or shorter path, and then vanish in the distance. These are the "comets," (from coma, i.e., "hair,") so called because when one of them is bright enough to be seen by the naked eye, it looks like a star surrounded by a luminous fog, and usually carries with it a streaming tail of hazy light. The large ones are magnificent objects, sometimes as bright as Venus and visible by day, with a head as large as the moon, having a train which extends from the horizon to the zenith, and is really long enough to reach from the earth to the sun. Such comets are rare, however. The majority are faint wisps of light, visible only with the telescope.

Fig. 89 is a representation of Donati's comet of 1858, which was one of the finest ever seen.

In ancient times comets were always regarded with terror, as of evil omen, if not personally malignant; and the notion


Fig. 89. - Naked-eye View of Donati's Comet, Oct. 4, 1858. (Bond.)
still survives in certain quarters, although the most careful research goes to prove that they really do not exert upon the earth the slightest perceptible influence of any kind.

Thus far, our lists contain nearly 800 , about 400 of which were observed before the invention of the telescope, and therefore must have been bright. Of those observed since then, only a small proportion have been conspicuous to the naked eye, perhaps one in five. The total number that visit the solar system must be enormous, for there is seldom a time when one at least is not in sight; and even with the telescope we see only such as come near the earth and are favorably situated for observation.
372. Designation of Comets. - A remarkable comet generally bears the name of its discoverer, or of some one who has acquired its "ownership," so to speak, by some important research coucerning it. Thus we have Halley's, Encke's, and Donati's comets. The common herd are designated only by the year of discovery, with a letter indicating the order of discovery in that year ; or, still again, by the year, with a Roman numeral denoting the order of perihelion passage. Thus, Donati's comet is "Comet 1858 -VI.," and also "Comet $f, 1858$. ." Comet $b$ is not, however, always Comet II., for Comet $c$ may beat it in reaching perihelion. In some cases, a comet bears a double name, as the Pons-Brooks Comet, which was discovered by Pons in 1812, and by Brooks on its recent return in 1883.
373. Duration of Visibility and Brightness. - The comet of 1811 was observed for 17 months, the great comet of 1861 for a year, and Comet 1889. I was followed at the Lick Observatory for nearly two years, - the longest period of visibility yet recorded. On the other hand the comet is sometimes visible only a week or two. The average is probably not far from three months.

As to brightness, comets differ widely. About one in five reaches the naked-eye limit, and a very few, say four or five in a century, are bright enough to be seen in the daytime. The great comet of 1882 and Comet $\alpha, 1910$, were so observed.
374. Their Orbits. - A large majority move in orbits that are sensibly parabolas. Of about 400 orbits thus far computed, more than 300 are of this kind. About 85 orbits are
more or less distinctly elliptical, and about half a dozen seem to be hyperbolas ; but hyperbolas differing so slightly from the parabola that the hyperbolic character is not certain in a single one of the cases. Comets which have elliptical orbits of course return at regular intervals; the others visit the sun only once, and never come back.

As in the case of a planet, three perfect observations of a comet's place are sufficient to determine its entire orbit. Practically, however, it is not possible to observe a comet with anything like the accuracy of a planet, nor usually with sufficient precision to determine certainly from three observations whether the orbit is or is not parabolic.


Fig. 90. - The Close Coincidence of Different Species of Cometary Orbits within the Earth's Orbit.
375. The plane of the orbit and its perihelion distance can in most cases be fairly settled from a few observations; but the eccentricity.
and the major axis (with its corresponding period), require a long series for their determination, and are seldom ascertained with much precision from observations made at a single appearance of the comet. In that part of the comet's path which can be observed from the earth, the three kinds of orbits diverge but little ; indeed, they may almost coincide (as shown in Fig. 90.)

It must be understood, moreover, that orbits which are sensibly parabolic are seldom strictly so; indeed, the chances are infinity to one against an exact parabola. If a comet were moving at any time exactly in such an orbit, then the slightest retardation due to the disturbing force of any planet, would change this parabola into an ellipse, and the slightest acceleration would make an hyperbola of it. (See Art. 259.)
376. The Elliptic Comets. - There are about a dozen of the elliptic orbits, to which computation assigns periods near or exceeding a thousand years. The real character of most of these orbits is still rather doubtful. About 75 comets, however, have orbits distinctly and certainly elliptical, and about 60 have periods of less than a hundred years. About 20 of these have been actually observed at two or more returns to perihelion ; as to the rest of them, some are now expected within a few years, and some have probably been lost to observation, either like Biela's comet, soon to be discussed, or by having their orbits transformed by perturbations.

The difficulty of determining whether a particular comet is or is not periodic is much increased by the fact that these bodies have not any characteristic "personal appearance" so to speak, by which a given individual can be recognized whenever seen - as Saturn could for instance. It is necessary to depend almost entirely upon the elements of its orbit for the identification of a returning comet, and this is not always satisfactory. (See Art. 377.)

The first comet ascertained to move in an elliptical orbit was that known as Halley's with a period of about 76 years, its periodicity having been discovered by Halley in 1681. It has since been observed in 1759 and 1835, and again in 1909 and 1910. The second of the periodic comets (in order of discovery) is Encke's, with the shortest period known, only $3 \frac{1}{2}$ years. Its periodicity was discovered in 1819.

Fig. 91 shows the orbits of a number of the short-period comets (it would cause confusion to insert more), and also that of Halley's Comet. These particular comets all (except Halley's) have periods ranging from $3 \frac{1}{2}$ to 8 years, and it will be noticed that they all pass very near the orbit of Jupiter.


Fig. 91. - Orbits of Short-period Comets.
Moreover, each comet's orbit crosses that of Jupiter near one of its nodes (the node is marked by a short cross line). The fact is extremely significant, showing that these comets at times come very near to Jupiter, and it points to an almost certain connection between that planet and these bodies.
377. Comet Groups. - There are several instances in which a number of comets, certainly distinct, chase each other along almost exactly the same path, at an interval, usually of a few months or years, though they sometimes appear simultaneously. The most remarkable of these "comet groups" is that composed of the great comets of $1668,1843,1880,1882$, and 1887 . It is
of course nearly certain that the comets of such a "group" have a common origin, perhaps from the disruption of a single comet by the attraction of the sun or a planet.
378. Perihelion Distance, Etc. - The perihelion distances of comets differ greatly. Eight of the 400 orbits approach the sun within less than 6,000000 miles, and four have a perihelion distance exceeding 200,000000 . A single comet, that of 1729 , had a perihelion distance of more than four astronomical units, or 375,000000 miles. It is one of the half dozen comets possibly hyperbolic, and must have been an enormous one to be visible under the circumstances. There may, of course, be any number of comets with still greater perihelion distances, because as a rule we are only able to see such as come reasonably near to the earth's orbit, - probably only a small percentage of the total number that visit the sun.

The inclinations of cometary orbits range all the way from zero to $90^{\circ}$. As regards the direction of motion, the six hyperbolic comets and all the elliptical comets having periods less than 100 years move direct, excepting only Halley's Comet and Tempel's Comet of 1866. The rest show no decided preponderance either way.
379. Comets are Visitors. - The fact that the orbits of most comets are sensibly parabolic and that their planes have no evident relation to the ecliptic, apparently indicates (though it does not absolutely demonstrate) that these bodies do not in any proper sense belong to the solar system. They are probably only visitors. They come to us precisely as if they simply dropped towards the sun from an enormous distance; and they leave the system with a velocity which, if no force but the sun's attraction acts upon them, will carry them away to an infinite distance, or until they encounter the attraction of some other sun. Their motions are just what might be expected of ponderable masses moving in empty space between the stars, under the law of gravitation.

A slightly different view is advocated by some high authorities. They think that our solar system in its journey through space (Art.
430) is accompanied by distant, outlying clouds of nebulous matter, and that they are the source and original "home" of the comets. It is argued that if the comets came from interstellar space the number of hyperbolic orbits would be much greater, because we should meet so many more comets than would overtake us.
380. Acceleration of Encke's Comet. - This little comet behaves in an exceptional manner. It shortens its period of $3 \frac{1}{2}$ years by about two hours at each revolution, having cut it down more than two days since its discovery in 1819. This effect is probably due to some resistance ${ }^{1}$ encountered by the comet, possibly by collision with swarms of meteors.

## PHYSICAL CONSTITUTION OF COMETS.

381. The Constituent Parts of a Comet. - (a) The essential part of a comet, that which is always present and gives the comet its name, is the Coma, or nebulosity, a hazy cloud of faintly luminous transparent matter.
(b) Next we have the Nucleus, which, however, is wanting in many comets, and makes its appearance only when the comet is near the sun. It is a bright, more or less star-like point near the centre of the comet, and is usually the object "observed on" in noting a comet's place. In some cases, the nucleus is double or even multiple.
(c) The Tail, or Train, is a stream of light which commonly accompanies a bright comet, and is sometimes present even with a telescopic one. As the comet approaches the sun, the tail follows it; but as the comet moves away from the sun, it precedes. It is usually, speaking broadly, directed away from the sun, though its precise form and position are determined

[^45]partly by the comet's motion. It is practically certain that it consists of extremely rarefied matter, which is thrown off by the comet and powerfully repelled by the sun. It certainly is not-like the smoke of a locomotive or the train of a meteor - matter simply left behind by the comet.
(d) Jets and Envelopes. The head of a brilliant comet is often veined by short jets of light, which appear to be spurted out of the nucleus; and sometimes the nucleus throws off a series of concentric envelopes, like hollow shells, one within the other. These phenomena, however, are seldom observed in telescopic comets.
382. Dimensions of Comets. - The volume or bulk of a comet is often enormous - almost beyond conception if the tail is included in the estimate. The head, as a rule, is from 40,000 to 150,000 miles in diameter : a comet less than 10,000 miles in diameter would stand little chance of discovery, and comets exceeding 150,000 are rather rare, though there are a considerable number on record.

The comet of 1811 at one time had a diameter of fully 1,200000 miles ( 40 per cent larger than that of the sun). The head of the comet of 1680 was 600,000 miles in diameter, that of Holmes's comet of 1892, 700,000, and that of Donati's comet about 250,000.

The diameter of the head keeps changing all the time; and what is singular is, that while the comet is approaching the sun, the head usually contracts, expanding again as it recedes.

The diameter of Encke's Comet contracts from about 300,000 miles (when it is 130,000000 miles from the sun) to a diameter not exceeding 12,000 or 14,000 , when it is at perihelion, at a distance of 33,000000 , the variation in bulk being more than 10,000 to 1 . No entirely satisfactory explanation is known, but Sir John Herschel has suggested that the change is merely optical, - that near the sun a part of the nebulous matter is evaporated by the solar heat and so becomes invisible, condensing and reappearing again when the comet gets to cooler regions.

The nucleus usually has a diameter ranging from 100 miles up to 5000 or 6000 , or even more. Like the comet's head, it also varies greatly in diameter, even from day to day; the changes, however, do not seem to depend in any regular way upon the comet's distance from the sun, but rather upon its activity in throwing off jets and envelopes.
The tail of a comet, as regards simple magnitude, is by far its most imposing feature. Its length is seldom less than 5,000000 or 10,000000 miles : it frequently attains 50,000000 , and there are several cases where it has exceeded 100,000000 .
383. Mass of Comets. - While the volume of comets is thus enormous, their masses are apparently insignificant, in no case at all comparable even with that of our little earth. The evidence on this point, however, is purely negative: it does not enable us in any case to say how great the mass really is, but only how great it is not; i.e., it only proves that the comet's mass is less than a certain very small fraction of the earth's mass. The evidence is derived from the fact that no sensible perturbations are produced in the motions of the planets when comets come even very near them; and yet in such a case the comet itself is fairly "sent kiting," showing that gravitation is fully operative between the comet and planet.

Lexell's Comet in 1770, and Biela's Comet on several occasions, came so near the earth that the length of the comet's period was changed by several weeks, while the year was not altered by so much as a single second. It would have been changed by many seconds if the comet's mass were as much as $\frac{10}{100000}$ that of the earth. At present this mass ( T odooo of the earth's mass) is very generally assumed as a probable "upper limit" for even a large comet. It is about ten times the mass of the earth's atmosphere, and is about equal to the mass of a ball of iron 150 miles in diameter.
384. Density of Comets. - This is, of course, extremely small, the mass being so minute and the volume so great. If
the head of a comet 50,000 miles in diameter has a mass $\frac{1}{100000}$ that of the earth, its mean density is about $\frac{1}{6000}$ of that of the air at the earth's surface. As for the tail, the density must be almost infinitely lower yet, far below that of the best vacuum we can make by any means.known to science: it is nearer to an "airy nothing" than anything else we know of. The extremely low density of comets is shown also by their transparency. Small stars can be seen through the head of a comet 100,000 miles in diameter, even very near its nucleus, and with hardly a perceptible diminution of lustre.
385. We must bear in mind, howerer, that the low mean density of a comet does not necessarily imply that the density of its constituent parts is small. A comet may be to a considerable extent composed of small heavy bodies, and still have a low mean density, provided they are widely separated. There is much reason, as we shall see, for supposing that such is really the case, - that the comet is largely composed of small meteoric stones, carrying with them a certain quantity of enveloping gas.

Another point should be referred to. Students often find it hard to conceive how such impalpable "dust clouds" can move in orbits like solid masses and with such enormous velocities. They forget that in a vacuum a feather falls as swiftly as a stone. Inter-planetary space is a vacuum, far more perfect than anything we can produce by artificial means, and in it the lightest bodies move as freely and swiftly as the densest, since there is nothing to resist their motion. I ${ }^{f}$ all the earth were suddenly annihilated except a single feather, the feather would keep on and pursue the same orbit, with the unchanged speed of nearly 19 miles a second.
386. The Light of Comets. - To some extent this may be mere reflected sunshine, but in the main it is light emitted by the comet itself, under the stimulus of solar action.

That the light depends in some way on the sun is shown by the fact that its intensity follows very closely the same law as the brightness of a planet; i.e., the comet's brightness is ordinarily proportional to the quantity $\frac{1}{R^{2} r^{2}}$, in which $R$ is the dis-
tance of the comet from the sun, and $r$ its distance from the earth.

But the brightness often varies rapidly and capriciously without any apparent reason; and that the comet is self-luminous when near the sun is proved by its spectrum, which is not that of sunlight at all, but is a spectrum of bright bands, three of which are usually seen and have been identified with the spectrum of gaseous hydrocarbons, or, probably, cyanogen. This spectrum is absolutely identical with that given by the blue base of a candle flame ; or, better, by a Bunsen


Fig. 92. - Comet Spectra.
(For convenience in engraving, the dark lines of the solar spectrum in the lowest strip of the figure are represented as bright.)
burner consuming ordinary coal gas. Occasionally a fourth band is seen in the violet, and when the comet approaches unusually near the sun, the bright lines of sodium, magnesium, (and probably iron) sometimes appear. There seem to be cases, also, in which different bands replace the ordinary
hydrocarbon bands, and the spectrum of Holmes's comet of 1892 was purely continuous. Mr. Lockyer maintains that as a regular thing the spectrum of a comet changes with its changing distance from the sun, but this is doubtful.

Fig. 92 represents the ordinary comet spectrum compared with the solar spectrum, and with that of a candle flame. Two anomalous spectra also are shown in the figure.


Fig. 93. - Head of Donati's Comet, Oct. 5, 1858. (Bond.)
The spectrum makes it almost certain that hydrocarbon gases are present in considerable quantity, and that these gases are somehow rendered luminous; not probably by any general heating, for there is no reason to think that the general temperature of a comet is high, but more likely by electric discharges between the solid particles, or by local heatings, due perhaps, as Mr. Lockyer maintains, to collisions between them. We are not to suppose, however, that the hydrocar-
bon gas, because it is so conspicuous in the spectrum, necessarily constitutes most of the comet's mass : more likely it is only a very small percentage of the whole.
387. Phenomena that accompany the Comet's Approach to the Sun. - When a comet is first discovered it is usually a mere round, hazy cloud of faint nebulosity, a little brighter near the middle. As it approaches the sun, it brightens rapidly, and the nucleus appears. Then on the sunward side the nucleus begins to emit luminous jets, or to throw off more or less symmetrical envelopes, which follow each other at intervals of some hours, expanding and growing fainter, until they are lost in the general nebulosity of the head. Fig. 93 shows the envelopes as they appeared in the head of


Fig. 94. - Tebbutt's Comet, 1881. (Common.) Donati's comet of 1858. At one time seven of them were visible together: very few comets, however, exhibit this phenomenon with such symmetry. More frequently the emissions from the nucleus take the form of mere jets and streamers, as shown in Fig. 94, which is a drawing of the head of Tebbutt's comet of 1881. This was an extremely active one, continually throwing out jets, breaking the nucleus into fragments, or exhibiting some other unexpected appearance.
388. Formation of the Tail. - The tail appears to be formed of material which is first projected from the nucleus of the comet towards the sun, and then afterwards repelled by the sun, as illus-
trated by Fig. 95. At least, this theory has the great advantage over all others which have been proposed, that it not only accounts for the phenomena in a general way, but admits of being worked out in detail and verified mathematically, by


Fig. 9j. - Formation of a Comet's Tail by Matter expelled from the Head. comparing the actual size and form of the comet's tail, at different points in the orbit, with that indicated by the theory; and the accordance is generally very satisfactory.

According to this theory, the tail is simply an assemblage of repelled particles, each moving in its own hyperbolic orbit around the sun, the separate particles having very little connection with or effect upon each other, and being almost entirely emancipated from the control of the comet's head. Since the force of the projection is seldom very great, all these orbits lie nearly in the plane of the comet's orbit, and the result is that the tail is usually a sort of a flat, hollow, curved, horn-shaped cone, open at the large end, as represented by Fig. 96.
389. Curvature of the Tails and Tails of Different Types. - The tail is curved, because the repelled particles after leaving the comet's head retain their original motion, so that they are arranged not along a straight line drawn from the sun to the comet, but on a curve convex to the direc-


Fig. 96.
A Comet's Tail as a Hollow Cone. tion of the comet's motion, as shown in Fig. 97: but the stronger the repulsion the less the curvature.

Bredichin (of Moscow) has found that in this respect the trains of comets may be classified under three different types, as indicated by Fig. 98.

First, the long, straight rays: they are composed of matter upon which the solar repulsion is from 12 to 15 times as great as the gravitational attraction, so that the particles leave the comet with a relative velocity of four or five miles a second, which is afterwards continually increased until it becomes enormous. The nearly straight rays shown in Fig. 89 belong to this type. For plausible reasons, connected with its low density, Bredichin considers them to be composed of hydrogen, possibly set free by the decomposition of hydrocarbons. They are rather uncommon, and in no case have been bright enough to allow a spectroscopic test of their nature.


Fig. 97. - A Comet's Tail at Different Points in ifs Orbit near Perihelion.
The second type is the curved plume-like train, like the principa! tail of Donati's comet. In trains of this type, supposed to be due to hydrocarbon vapors, the repulsive force varies from 2.2 times the gravitational attraction for particles on the convex edge of the train, to half that amount for those on the inner edge.

Third. A few comets show tails of still a third type, short, stubby brushes, violently curved, and due to matter upon which the repulsive force is feeble as compared with gravity. These are assigned to
metallic vapors of considerable density, iron perhaps, with an admixture of sodium, etc.

The nature of the force which repels the particles of a comet is


Fig. 98.
Bredichin's Three Types of Cometary Tails. still somewhat uncertain. Until very recently it has been generally assumed to be in some way electrical; but the recent experimental demorstrations of the direct impulsive action of light-waves, long recognized as a necessary consequence of the electromagnetic theory of light, make it quite possible that this light-repulsion may furnish the explanation. Nor is there any real antagonism between this and the electrical theory, now that the new theory of light is accepted.

There is no reason to suppose that the matter driven off to form the tail is ever recovered by the comet.
390. Unexplained and Anomalous Phenomena. - A curious phenomenon, not yet explained, is the dark stripe which, in a large comet nearing the sun, runs down the centre of the tail, looking very much as if it were a shadow of the comet's head. It is certainly not a shadow, however, because it usually makes more or less of an
angle with the sun's direction. It is well shown in Figs. 93 and 94 . When the comet is a greater distance from the sun, this central stripe is usually bright, as in Fig. 99.

Not infrequently, moreover, comets possess anomalous tails, -usually in addition to the normal tail, but sometimes substituted for it, - tails directed sometimes straight towards the sun, and sometimes at right angles to that direction. Then, sometimes, there are luminous "sheaths," which seem to envelop the head of the comet and project towards the sun; or little clouds of cometary matter which leave the main comet like puffs of smoke from a bursting bomb,


Fig. 99.
Bright-Centred Tail of Coggia's Comet. June, 1874. and travel off at an angle until they fade away (see Fig. 100). None of these appearances are contradictory to the theory above stated, although not yet clearly included in it.
391. The Nature of Comets. - All things considered, the most probable hypothesis as to the constitution of a comet is that its head is a swarm of small meteoric particles, widely separated (say pin-heads, many feet apart), each carrying with it a gaseous envelope, in which light is produced either by electric discharges or by some action due to the rays of the sun. As to the size of the constituent "particles," opinions differ widely. Some maintain that they are large rocks : Professor Newton calls a comet a " gravel-bank" : others say that it is a mere "dust-cloud." The unquestionable and close connection between meteors and comets, which we shall soon discuss, almost compels some " meteoric hypothesis."
392. Origin of Periodic Comets. - It is clear, as has been said, that the comets which move in parabolic orbits cannot
have originated in the solar system, but must be visitors rather than citizens : as to those which move in elliptical orbits, it is a question whether we are to regard them as native-born or only as naturalized.

It is evident that, in some way, many of them stand in peculiar relations to Jupiter and other planets (see Art. 376).

All the short period comets, i.e., those which have periods ranging from three to eight years, pass very close to the orbit of Jupiter, and are now recognized and spoken of as Jupiter's "comet-family." About thirty are known already, of which fifteen have already been observed twice or oftener. Similarly, Saturn is credited with two comets; Uranus with two (one of them is Tempel's comet, which.is closely connected with the November meteors, and was last due in 1900). Finally, Neptune has a family of six ; among them Halley's Comet and two others which have returned a second time to perihelion since 1880 .
393. The Capture Theory. - The generally accepted theory as to the origin of these "comet families" is one first suggested by La Place, - that the comets which compose them have been captured by the planet to which they stand related. A comet entering the system and passing near a planet will be disturbed, and either accelerated or retarded : if it is accelerated, the original parabolic orbit will be changed to an hyperbola, and the comet will never be seen again; but if it is retarded, the orbit becomes elliptical, and the comet will return at each revolution to the place where it was first disturbed. After a lapse of time the planet and the comet will come together a second time at or near this place. The result may $b e$ an acceleration which will send the comet out of the system again ; but it is an even chance, at least, that it may be a retardation, and that the orbit and period will thus be further diminished. And this may happen over and over again, until
the comet's orbit falls so far inside that of the planet that it suffers no further disturbance to speak of.

Given time enough and comets enough, and the ultimate result would necessarily be such a comet family as really exists. It is not permanent, however : sooner or later, if a captured comet is not first disintegrated, it will almost certainly encounter its planet under such conditions as to be thrown out of the system in an hyperbolic orbit.

The late R. A. Proctor declined to accept the above theory, and maintained with much vigor and ability the theory that comets and meteor swarms have been "ejected" from the great planets by eruptions of some sort. We cannot here stop to discuss the theory, but the objections to it are serious, and probably fatal.
394. Remarkable Comets. - Our space does not permit us to give full accounts of any considerable number. We limit ourselves to three, which, for various reasons, are of special interest.

Biela's Comet is (or rather was) a small comet some 40,000 miles in diameter, at times barely visible to the naked eye, and sometimes showing a short tail. It had a period of 6.6 years, and it was the second comet of short period known, having been discovered by Biela, an Austrian officer, in 1826; (the periodicity of Encke's Comet had been discovered seven year's earlier.) Its orbit comes within a few thousand miles of the earth's orbit, the distance varying somewhat, of course, on account of perturbations; but the approach is often so close that if the comet and the earth should happen to come along at the same time there would be a collision.

In 1832, some one started the report that such an encounter was to occur, and there was in consequence, a veritable panic in southern France, the first of the numerous " comet-scares." On this occasion, the comet passed the critical point nearly a month ahead of the earth, and was never at a distance less than 15,000000 miles.
395. At its return in 1846 it did a very strange thing, entirely unprecedented. It split into two. When first seen, on Nov. 28th, it was round and single. On Dec. 19th it was distinctly pear-shaped, and ten days later it had divided, the duplication being first noticed in this country (at New Haven and Washington) some weeks before it was observed in Europe. The twin comets travelled along for four months at an almost unchanging distance of about 165,000 miles, without any apparent effect upon each other's motions, but both very active from the physical point of view, showing remarkable variations of brightness, and also alternations, comet $A$ brightening up when $B$ was faint, and vice versa. In August, 1852, the twins were again observed, now at a distance of about 1,500000 miles; but it was impossible to tell which was which. Neither of them has ever been seen again, though they must have returned many times, if still existing as comets, and more than once in a favorable position.
396. There remains, however, another remarkable chapter in the story of this comet, though its proper place is under the head of meteors. In 1872, on Nov. 27th, just as the earth was crossing the track of the lost comet, but some millions of miles behind where the comet ought to be, she encountered a wonderful meteoric shower. As Miss Clerke expresses it, perhaps a little too positively, "it became evident that Biela's Comet was shedding over us the pulverized products of its disintegration." The same thing happened again in November, 1885.
It is not certain whether the meteor swarms thus encountered were really what was left of the comet itself, or whether they merely follow in its path. The comet must have been several millions of miles ahead of the place where these meteor swarms were met, unless it has been set back in its orbit since 1852 by some unexplained and improbable perturbations. But if the comet still exists and occupies the place it ought to, it
cannot be found; it must have somehow lost the power of shining.

The meteors connected with this comet are known both as " Bielids" and as "Andromedes," the latter name indicating that their so called "radiant" is in the constellation of Andromeda.
397. The Great Comet of 1882. - This will long be remembered, not only for its magnificent beauty, but for the great number of unusual phenomena it presented. It was first seen in the southern hemisphere about September 3rd, but not in the northern until the 17 th, the day on which it arrived at perihelion. On that day and. the next, it was independently discovered within two or three degrees of the sun near noon, by several observers who had not before heard of its existence. On the 17th, at the Cape of Good Hope, the observers followed it right up to the edge of the sun's disc, which it " transited" invisibly, showing neither as a light nor as a dark spot on the solar surface. It was visible to the naked eye in full sunshine for nearly a week after perihelion. It then became a splendid object in the morning sky, and it continued to be observed for six months. That portion of the orbit visible from the earth, coincides almost exactly with the orbits of four other comets, - those of $1668,1843,1880$, and 1887, with which it forms a "comet group," as already mentioned (Art. 377).

The striking peculiarity of the orbits of this "comet group" is the closeness of their approach to the sun, their perihelion distances all being less than 750,000 miles, so that they pass within 300,000 miles of the sun's surface ; i.e., right through the corona, and with a velocity exceeding 300 miles a second; and yet this passage through the corona does not disturb their motion perceptibly. The orbit of the comet, of 1882 , turns out to be a very elongated ellipse, with a period of abou: 800 years. The period of the comet of 1880 was computed as only 17 years, while the orbits of the other three appear to be sensibly parabolic.
398. Telescopic Features. - Early in October, the comet presented the ordinary features. The nucleus was round, a number of well-marked envelopes were visible in the head, and the dark stripe down the centre of the tail was sharply defined. Two weeks later, the nucleus had been broken up and transformed into a srooked
stream, some 50,000 miles in length, of five or six bright points ; the envelopes had vanished from the head, and the dark stripe was replaced by a bright central spine.

At the time of perihelion the comet's spectrum was filled with countless bright lines. Those of sodium were easily recognizable, and continued visible for several weeks; the other lines disappeared much more quickly, and were not certainly identified, although the general aspect of the spectrum indicated that iron, manganese, and calcium were probably present. By the middle of October it had become simply the normal comet spectrum, with the ordinary hydrocarbon bands.


Fig. 100. - The "Sheath," and the Attendants of the Comet of 1882.
399. The Tail. - The comet was so situated that the tail was directed nearly away from the earth, and so was not seen to good advantage, never having an apparent length exceeding $35^{\circ}$. The actual length, however, at one time was more than 100,000000 miles.

A unique, and so far unexplained, phenomenon was a faint, straightedged "sheath " of light that enveloped the portions of the comet near the head, and projected $3^{\circ}$ or $4^{\circ}$ in front of it, as shown in Fig. 100. Moreover, there were certain shreds of cometary matter accompanying the comet at a distance of $3^{\circ}$ or $4^{\circ}$ when first seen, but gradually receding and growing fainter. This also was something new in cometary history, though Brooks's Comet of 1889 (next to be mentioned) has since then done the same thing.

399*. Brooks's Comet 1889. V. (for a time known as the Lexell-Brooks Comet). Comet 1889. V was a small one discovered by Brooks in July, 1889, and was soon found to be moving in an elliptical orbit with a period of about 7 years. At the Lick Observatory it was observed to have three accompanying fragments. On investigating the orbit more carefully with the data then available, Mr. Chandler found that in 1886 it had passed very close to Jupiter, and that its orbit had been greatly changed from a much larger ellipse, with a period, according to his calculations, of about 27 years; and his calculations led him to conclude that it was very probably identical with the lost comet of Lexell, which was observed in 1770 (with a period which was then computed to be about $5 \frac{1}{2}$ years), but never returned. Laplace, and later, Leverrier, showed that it must have passed near to Jupiter in 1779, and been thrown into a much larger orbit. The data, however, were not sufficient to decide the dimensions exactly, nor at the time of Mr. Chandler's computation, to make his identification certain; but it seemed so probable, that the comet was for some time called the "Lexell-Brooks." It returned late in 1896, and from the observations in 1889 and 1896 Dr. Poor of Baltimore has shown that probably Chandler's identification cannot be maintained. At the same time the whole history of the investigation forms an admirable illustration of the "capture theory." In 1889 the comet passed certainly within 200,000 miles of Jupiter, and perhaps even closer. The motion of the satellites was not in the least affected (indicating the
minuteness of its mass), but it was probably at that time that the comet was pulled into four pieces, as observed by Barnard. In 1921, if it "lives" so long, i.e., if it does not become too much disintegrated to be still observable as a comet, it will again pass near to Jupiter ; and it remains to be seen what will happen to it then.

Holmes's comet of 1892-3 was in many ways remarkable. When first discovered it was already visible to the naked eye, and was apparently almost stationary, fast increasing in size as if swiftly approaching. For a time a popular impression prevailed that it was Biela's lost comet, and might strike the earth, which led to something like a "newspaper panic" in certain quarters. It was, however, really receding, and never came nearer than 150,000000 miles. It was never conspicuous, and had no nucleus or notable train ; but its bulk was enormous : at one time its diameter exceeded 700,000 miles. It experienced many capricious changes of apparent size and brightness, and its spectrum was purely continuous, - a thing unprecedented in comets. It moves in an orbit like that of an asteroid, with its perihelion just outside the orbit of Mars, and its aphelion close to that of Jupiter, its period being a few days less than seven years.

399**. Photography of Comets. - It is now possible to photograph comets, and the photographs bring out numerous peculiarities and details which are not visible to the eye even with telescopic aid. This is especially the case in the comet's tail. Fig. 100* is from Hussey's photograph of Rordame's comet of 1893, for which we are indebted to the kindness of Professor Holden, director of the Lick Observatory. As the camera was kept pointed at the head of the comet (which was moving pretty rapidly) the star-images during the hour's exposure are drawn out into parallel streaks, the little irregularities being due to faults of the driving-clock and vibrations of the telescope. The knots and streamers which characterize the comet's tail were none of them visible in the telescope, and differ trom those shown upon plates taken on the days preceding and


COMET RORDAME, JULY 13, 1893.
Photographed by W. J. Hussey, at the Lick Observatory.
following. Other plates, made the same evening a few hours earlier and later, indicate that the knots were swiftly receding from the comet's head at a rate exceeding 150,000 miles an hour.

Comets have several times lately been discovered by photography in the same way as asteroids, - first by Barnard in 1892.

## METEORS AND SHOOTING STARS.

400. Meteorites. - Occasionally bodies fall upon the earth out of the sky. Until they reach the air they are usually invisible, but as soon as they enter it they become conspicuous, and the pieces which fall are called "Meteorites," "Aerolites," " Uranoliths," or simply " meteoric stones."
If the fall occurs at night, a ball of fire is seen, which moves with an apparent velocity depending upon the distance of the meteor and the direction of its motion. The fire-ball is generally followed by a luminous train, which sometimes remains visible for many minutes after the meteor itself has disappeared. The motion is usually somewhat irregular, and here and there along its path the meteor throws off sparks and fragments, and changes its course more or less abruptly. Sometimes it vanishes by simply fading out in the distance, sometimes by bursting like a rocket. If the observer is near enough, the flight is accompanied by a heavy, continuous roar, emphasized now and then by violent detonations. The noise is frequently heard fifty miles away, especially the final explosion. ${ }^{1}$

If the fall occurs by day, the luminous appearances are mainly wanting, though sometimes a white cloud is seen, and the train may be visible. In a few cases, aerolites have fallen almost silently, and without warning.

[^46]401. The Aerolites Themselves. - The mass that falls is sometimes a single piece, but more usually there are many fragments, sometimes numbering thousands: as the old writers say, "It rains stones." The pieces weigh from 500 pounds to a few grains, the aggregate mass occasionally amounting to more than a ton.

By far the greater number of aerolites are stones, but a few, perhaps three or four per cent of the whole number, are masses of nearly pure iron more or less alloyed with nickel.

The total number of meteorites which have fallen and been gathered into our cabinets since 1800 is about $275,-11$ of which are iron masses. Nearly all, however, contain a large percentage of iron, either in the metallic form or as sulphide. Between 25 and 30 of the 275 fell within the United States, the most remarkable being those of Weston, Conn., in 1807; New Concord, Ohio, 1860; Amana, Jowa, 1875; Emmett County, Iowa, 1879 (mainly iron); and Johnson County, Ark., 1886 (iron).

Twenty-five ${ }^{1}$ of the chemical elements have been found in these bodies, but not one new element, though a large number of new minerals appear in them, and seem to be peculiar to and characteristic of aerolites. The most distinctive external feature of a meteorite is the thin, black, varnish-like crust that covers it. It is formed by the fusion of the surface during the meteor's swift flight through the air, and in some cases penetrates the mass in cracks and veins. The surface is generally somewhat uneven, having "thumb-marks" upon it, -hollows probably formed by the fusion of some of the softer minerals.

Fig. 101 is from a photograph given in Langley's "New Astronomy," where the body is designated - perhaps a little too positively - as "part of a comet."
402. Path and Motion. - When a meteor has been observed from a number of different stations, its path can be computed.

It usually first appears at an altitude of between 80 and 100 miles and disappears at an altitude of between 5 and 10 miles. The length of the path may be anywhere from 50 to 500 miles. The velocity ranges from 10 to 40 miles a second in the earlier part of its course, and this is reduced to one or two miles a second before the meteor disappears.

The average velocity with which these bodies enter the air seems to be very near the parabolic velocity of 26 miles a second, due to the sun's attraction at the earth's distance-just as should be the case, if, like the comets, they come to us from inter-stellar space; but more recent researches of Professor Newton seem to show such a decided


Fig. 101.
Fragment of one of the Amana Meteoric Stones. preponderance of direct motions and small inclinations to the ecliptic as would rather indicate on the other hand, that they are of planetary instead of stellar origin perhaps minute "outriders" of the asteroid group.
403. Observation of Meteors. - The object should be to obtain as accurate an estimate as possible of the altitude and azimuth of the meteor at moments which can be identified, and also of the time occupied in traversing definite portions of the path. By night, the stars furnish the best reference points from which to determine its position. By day, one must take advantage of natural objects and buildings to
define the meteor's place, the observer marking the precise spot where he stood, when the meteor disappeared behind a chimney for instance, or was seen to burst just over a certain twig in a tree. By taking a proper instrument to the place afterwards, it is then easy to translate such data into bearings and altitude. As to the time of fight, which is required in order to determine the meteor's velocity, it is usual for the observer to begin to repeat rapidly some familiar verse of doggerel when the meteor is first seen, reiterating it until the meteor disappears. Then by rehearsing the same before a clock, the nurnber of seconds can be pretty accurately determined.
104. The Light and Heat of Meteors. -These are due simply to the destruction of the meteor's velocity by the friction and resistance of the air. When a body moving with a high velocity is stopped by the resistance of the air, by far the greater part of its energy is transformed into heat. Sir William Thomson has shown that the thermal effect in the case of a body moving through the air with a velocity exceeding 10 miles a second is the same as if it were immersed in a flame having a temperature at least as high as that of the oxyhydrogen blow-pipe; and, moreover, this temperature is independent of the density of the air, - depending only on the velocity of the meteor. Where the air is dense, the total quantity of heat -i.e., the number of calories developed in a given time - is of course greater than where the air is rarefied; but the virtual temperature of the air where it rubs against the surface is the same in either case. During the meteor's flight its surface therefore is heated to lively incandescence and melted, and the liquefied portions are swept off by the rush of air, condensing as they cool to form the train. In some cases this train remains visible for many minutes, - a fact not easy to explain. It is hardly possible that such a smoke-like cloud should remain luminous by retaining its heat for so long a time, and it seems probable therefore that the material must be phosphorescent.
405. Origin of Meteors. - They cannot be, as some have maintained, the immediate products of eruption from volcanoes,
either terrestrial or lunar, since they reach our atmosphere with a velocity greater than $7 \frac{1}{2}$ miles a second, the "parabolic velocity" (see Art. 507*) due to the earth's attraction. This indicates that they come to us from the depths of space. There is no certain reason for assuming that they have originated in any way different from the larger heavenly bodies : at the same time many of them resemble each other so closely as almost to compel the surmise that these at least have a common source.

It is not, perhaps, impossible that such may be fragments which, ages ago, were shot out from now extinct lunar volcanoes, with a velocity which made planets of them for the time being. If so, they have since been travelling in independent orbits, until at last they encounter the earth at the point where her orbit crosses theirs. Nor is it impossible that some of them were thrown out by terrestrial eruptions when the earth was young, or from the planets, or from the stars.

## SHOOTING S'TARS.

406. Their Nature and Appearance. - These are the swiftly moving, evanescent, star-like points of light, which may be seen every few minutes on any clear, moonless night. They make no sound, nor (with perhaps one exception hereafter to be noted) has anything been known to reach the earth's surface from them, not even in the greatest "meteoric showers."

For this reason it is probably best to retain, provisionally at least, the old distinction between them and the great meteors from which aerolites fall. It is quite possible that the distinction has no real ground, - that shooting stars are just like other meteors except in size, being so small that they are entirely consumed in the air: but then, on the other hand, there are some things which favor the idea that the two classes of bodies differ about as asteroids do from comets.
407. Number of Shooting Stars. - Their number is enormous. A single observer averages from four to eight an hour ; but if
the observers are sufficiently numerous, and so organized as to be sure of noting all that are visible from a given station, about eight times as many are counted. From this it is estimated by Professor Newton that the total number which enter our atmosphere daily must be between 10,000000 and 20,000000, the average distance between them being about 200 miles. Besides those which are visible to the naked eye, there is a still larger number of meteors which are so small as to be observable only with the telescope.

The average hourly number about six o'clock in the morning is double the hourly number in the evening; the reason being that in the morning we are on the front of the earth, as regards its orbital motion, ${ }^{1}$ while in the evening we are in the rear. In the evening we see only such as overtake us. In the morning we see all that we either meet or overtake.
408. Elevation, Path, and Velocity. - By observations made at stations 30 or 40 miles apart, it is easy to determine these data with some accuracy. It is found that on the average the shooting stars appear at a height of about 74 miles, and disappear at an elevation of about 50 miles, after traversing a course of 40 or 50 miles, with a velocity of from 10 to 30 miles a second, - about 25 on the average. They do not begin to be visible at so great a height as the aerolitic meteors, and they are more quickly consumed, and therefore do not penetrate the atmosphere to so great a depth.
409. Brightness, Material, Etc. - Now and then a shooting star rivals Jupiter or even Venus in brightness. A considerable number are like first-magnitude stars, but the great majority are faint. The bright ones generally leave trains.

Occasionally it has been possible to get a "snap shot," so to

[^47]speak, at the spectrum of a meteor, and in it the bright lines of sodium and magnesium (probably) are fairly conspicuous among many others which cannot be identified by such a hasty glance.

Since these bodies are consumed in the air, all we can hope to get of their material is their "ashes." In most places its collection and identification is, of course, hopeless; but the Swedish naturalist, Nordenskiold, thought that it might be found in the polar snows. In Spitzbergen he therefore melted several tons of snow, and on filtering the water he actually detected in it a sediment containing minute globules of oxide and sulphide of iron. Similar globules have also been found in the products of deep-sea dredging. They may be meteoric, but what we now know of the distance to which smoke and fine volcanic dust is carried by the wind makes it not improbable that they may be of purely terrestrial origin.
410. Probable Mass of Shooting Stars. - We have no way of determining the exact mass of such a body; but from the light it emits, as seen from a known distance, an estimate can be formed which is not likely to be widely erroneous.

An efficient incandescent electric lamp consumes about 150 footpounds of energy per minute, for every candle power. Assuming for the moment, then, that the ratio of the light (or luminous energy) to the total energy is the same for a meteor as for the electric lamp, we can compute the total energy of a meteor which shines with known brightness for a given number of seconds; and we can then compute its mass ${ }^{1}$ from its known velocity.

If a meteor converted all its energy into light, wasting none in invisible rays, this calculation would give the mass several times too great. If, on the other hand, the meteor were only feebly luminous, the result would be too small.

[^48]It is likely on the whole that an ordinary meteor and a good incandescent lamp do not differ widely in their "luminous efficiency," and calculations on this basis indicate that the ordinary shooting stars weigh only a small fraction of an ounce, - from a grain or two up to 50 or 100 grains.
411. Effects produced by Meteors and Shooting Stars. We must content ourselves with merely alluding to certain effects which they must theoretically produce, adding that in no case have such effects been found sensible to observation.

1. In the first place, meteors add continually to the earth's mass - perhaps as much as 40,000 tons a year. If so, it would take about 1000 million years to accumulate a layer one inch thick on the earth's surface.
2. They diminish the length of the year: (a) by acting as a resisting medium, and so really shortening the major axis of the earth's orbit (like the orbit of Encke's comet); (b) by increasing the mass of the earth and sun, and so increasing the attraction between them; (c) by increasing the size of the earth, and thus slackening its rotation and lengthening the day.

Calculation shows, however, that the combined effect would hardly amount to more than $\frac{1}{1000}$ of a second in a million years.
3. Each meteor brings to the earth a certain amount of heat, developed in the destruction of its motion. According to the best estimates, however, all the meteors that fall upon the earth in a year supply no more heat than the sun does in about one-tenth of a second.
4. They must necessarily render inter-stellar space imperfectly transparent, if, as there is every reason to suppose, they pervade it throughout in any such numbers as in the domain of the solar system. But this effect is also so small as to defy calculation.
412. Meteoric Showers. - There are occasions when these bodies, instead of showing themselves here and there in the sky at intervals of several minutes, appear in "showers" of thousands; and at such times they do not move at random, but all their paths diverge or "radiate" from a single spot in the sky, known as the "radiant"; i.e., their paths produced backward all pass through it, though they do not usually start
there. Meteors which appear near the radiant are apparently stationary, or describe paths that are very short, while those in the more distant regions of the sky pursue long courses.

The "radiant" keeps its place among the stars sensibly unchanged during the whole continuance of the shower, for hours or days, it may be, and the shower is named according to the place of the radiant. Thus, we have the "Leonids," or meteors whose radiant is the constellation of Leo; the "Andromedes" (or Bielids) ; the "Perseids," the" Lyrids," etc.

Fig. 102 represents the tracks of a large number of the Leonids of 1866, showing the position of the radiant near Zeta Leonis.


Fig. 102. - The Meteoric Kadiant in Leo, Nov. 13, 1866.
The "radiant" is explained as a mere effect of perspective. The meteors are all moving in lines nearly parallel when encountered by the earth, and the radiant is simply the perspective "vanishing point" of this system of parallels: its
position depends entirely upon the direction of the motion of the meteor's relative to the earth. For various reasons, however, the paths of the meteors, after they enter the air, are not exactly parallel, and in consequence the radiant is not a mathematical point, but a "spot" in the sky, often covering an area of three or four degrees square.

Probably the most remarkable of all the meteoric showers that have ever occurred was that of the Leonids on Nov. 12th, 1833. The number at some stations was estimated as high as 100,000 an hour, for five or six hours. "The sky was as full of them as it ever is of snowflakes in a storm."
413. Dates of Meteoric Showers. - Meteoric showers are evidently caused by the earth's encounter with a swarm of meteors, and since this swarm pursues a regular orbit around the sun, the earth can meet it only when she is at the point where her orbit cuts the path of the meteors: this, of course, must always happen on or near the same day of the year, except as in the process of time the meteoric orbits shift their positions on account of perturbations. The Leonid showers, at present, appear on the 16 th of November, within a day or two; and the Andromedes about the 20th of the same month.

In some cases the meteors are distributed along their whole orbit, forming a sort of ring and rather widely scattered. In that case the shower recurs every year and may continue for several days, as is the case with the Perseids, or August meteors. On the other hand, the flock may be concentrated, and then the shower will occur only when the earth and the meteor-swarm both arrive at the orbit-crossing together. This is the case with both the Leonids and the Andromedes. The showers then occur, not every year, but only at intervals of several years, and always on or near the same day of the month. For the Leonids, the interval is about 33 years, and for the Bielids, usually 13.

The meteors which belong to the same group have certain family resemblances. The Perseids are yellow, and move with medium velocity. The Leonids are very swift (we meet them), and they are of a bluish green tint, with vivid trains. The Bielids are sluggish (they overtake the earth), are reddish, being less intensely heated than the others, and they usually have only feeble trains.
414. The Mazapil Meteorite. - As has been said, during these showers no sound is heard, no sensible heat perceived, nor do any masses reach the ground; with one exception, however, that on Nov. 27 th, 1885 , a piece of meteoric iron fell at Mazapil, in northern Mexico, during the shower of Andromedes which occurred that evening (Art. 396). Whether the coincidence is accidental or not, it is certainly interesting. Many high authorities speak confidently of this piece of iron as being a piece of Biela's comet itself; and this brings us to one of the most remarkable discoveries of nineteenth century astronomy.
415. The Connection between Comets and Meteors. - At the time of the great meteoric shower of 1833, Professors Olmsted and Twining, of New Haven, were the first to recognize the "radiant," and to point out its significance as indicating the existence of a swarm of meteors revolving around the sun in a permanent orbit. Olmsted even went so far as to call the body a "comet." Others soon showed that in some cases, at least, the meteors must be distributed in a complete ring around the sun, and Erman of Berlin developed a method of computing the meteoric orbit when its radiant is known. In 1864 Professor Newton of New Haven showed by an examination of the old records that there had been a number of great meteoric showers in November, at intervals of 33 or 34 years, and he predicted confidently a repetition of the shower on November 13 th or 14 th, 1866 . The shower occurred as predicted, and was observed in Europe; and it was followed by another in 1867 which was visible in America, the meteoric swarm being extended in so long a procession as to require more than two years to cross the earth's orbit. The swarm returned again in 1900 and 1901, but comparatively few meteors were seen.
416. Identification of Meteoric and Cometary Orbits. - Within a few weeks after the shower of 1866 it was shown by Leverrier and Oppolzer that the orbit of these meteors was identical with that of a faint comet known as Tempel's, observed a year before; and about the same time, in fact a few weeks earlier, Schiaparelli showed that the Perseids, or August meteors, move in an orbit identical with that of the bright comet of 1862, known as Tuttle's.

Now a single coincidence might be accidental, but hardly two. Five years later came the shower of Andromedes, following in the track of Biela's comet; and among the more than a hundred distinct meteor swarms, now recognized, Prof.


Fig. 103. - Orbits of Meteoric Swarms.
Alexander Herschel finds five others which are similarly related, each to its special comet. It is no longer possible to doubt that there is a real and close connection between these comets and their attendant meteors.

Fig. 103 represents four of these cometo-meteoric orbits.

41\%. Nature of the Connection. - This cannot be said to be ascertained. In the case of the Leonids and the Andromedes, the meteoric swarm follows the comet. But this does not seem to be so in the case of the Perseids, which scatter along more or less abundantly every year.

The prevailing belief seems to be at present that the comet itself is only the thickest part of the meteoric swarm, and that the clouds of meteors scattered along its path are the result of its disintegration.

It is easy to show that if a comet really is such a swarm, it must gradually break up more and more at each return to perihelion and disperse its constituent particles along its path, until the compact swarm has become a diffuse ring. The longer the comet has been


Fig. 104. - Origin of the Leonids.
moving around the sun, the more uniformly the particles will be distributed. The Perseids are supposed, therefore, to have been in the system for a long time, while the Leonids and Andromedes are believed to be comparatively newcomers. Leverrier, indeed, has gone so far as to indicate the year 126 A.d. as the time at which Uranus captured Tempel's comet, and brought it into the system (as illus-
trated by Fig. 104). But the theory that meteoric swarms are the product of cometary disintegration assumes the premise that comets enter the system as compact clouds, which, to say the least, is not yet certain.
418. Mr. Lockyer's Meteoric Hypothesis. - One of the English astronomers, Mr. Lockyer, has been enlarging greatly the astronomical importance of meteors. The probable meteoric constitution of the zodiacal light (Art. 343), as well as of Saturn's rings, and of the comets, has long been recognized; but he goes much farther and maintains that all the heavenly bodies are either meteoric swarms, more or less condensed, or the final products of such condensation; and upon this hypothesis he attempts to explain the evolution of the planetary system, the phenomena of variable and colored stars, the various classes of stellar spectra, and the forms and structure of the nebulæ, - indeed, pretty much everything in the heàvens from the Aurora Borealis to the sun. As a "working hypothesis" his theory is unquestionably suggestive and has attracted much attention, but it does not bear criticism in all its details.

## CHAPTER XIV.

## THE STARS.

THEIR NATURE, NUMBER, AND DESIGNATION. - STARCATALOGUES AND CHARTS. - PROPER MOTIONS AND THE MOTION OT THE SUN IN SPACE. - STELLAR PARALLAX. - STAR-MAGNITUDES. - VARIABLE STARS. STELLAR SPECTRA.
419. The solar system is surrounded by an immense void, peopled only by meteors. If there were any body a hundredth part as large as the sun within a distance of a thousand astronomical units, its presence would be indicated by considerable perturbations of Uranus and Neptune. The nearest star, as far as our present knowledge goes, is one whose distance is more than 200,000 units, - so remote that, seen from it, the sun would look about like the Pole-star, and no telescope yet constructed would be able to show a single one of all the planets of the solar system.

The spectra of the stars indicate that they are bodies like our sun, incandescent, shining each with its own peculiar light. Some are larger and hotter than the sun, others smaller and cooler; some, perhaps, hardly luminous at all. They differ enormously among themselves, not being, as was once thought, as much alike as individuals of the same race, but differing as widely as animalcules from elephants.
420. Number of the Stars. - Those that are visible to the eye. though numerous, are by no means countless. If we take
a limited region, as, for instance, the bowl of "The Dipper," we shall find that the number we can see within it is not very large, - hardly a dozen. In the whole celestial sphere, the number of stars bright enough to be distinctly seen by an average eye is only between 6000 and 7000 , - and that in a perfectly clear and moonless sky: a little hạze or moonlight will cut down the number full one-half. At any one time, not more than 2000 or 2500 are fairly visible, since near the horizon the smail stars (which are vastly the most numerous) disappear. The total number which could be seen by the ancient astronomers well enough to be observable with their instruments is not quite 1100 .

With even the smallest telescope the number is enormously increased. A common opera-glass brings out at least 100,000 , and with a $2 \frac{1}{2}$-inch telescope, Argelander made his "Durchmusterung" of the stars north of the equator, more than 300,000 in number. The Lick telescope, 36 inches in diameter, probably reaches about 100,000000 .
421. Constellations. - The stars are grouped in so-called "constellations," many of which are extremely ancient, all those of the Zodiac and all those near the northern pole being of pre-historic origin. Their names are for the most part drawn from the Greek and Roman mythology, many of them being connected in some way or other with the Argonautic expedition. In some cases the eye, with the help of a lively imagination, can trace in the arrangement of the stars a vague resemblance to the object which gives name to the constellation, but generally no reason is obvious for either its name or its boundaries.
Of the 67 constellations now generally recognized, 48 have come down from Ptolemy, the others having been formed by later astronomers to embrace stars not included in the old con. stellations, and especially to provide for the stars near the southern pole. Many other constellations have been proposed at one time or another, but since rejected as useless or imper-
tinent, though a few have obtained partial acceptance and at present find a place upon some maps.

A thorough knowledge of these artificial star groups and of the names and places of the stars that compose them, is not at all essential even to an accomplished astronomer ; but it is a matter of great convenience to be acquainted with the principal constellations, and to be able to recognize at a glance the brighter stars, -50 to 100 in number. This amount of knowledge is easily obtained in three or four evenings by studying the heavens in connection with a good celestial globe, or with star-maps - taking care, of course, to select the evenings in different seasons of the year, so that the whole sky may be covered. In the Uranography, which forms a supplement to this volume, we give a brief description of the various constellations, and directions for tracing them by the help of small star-maps, which are quite sufficient for this purpose, though not on a scale large enough to answer the needs of detailed study. For reference purposes ${ }^{1}$, the more elaborate atlases of Proctor, Heis, or Klein, are recommended.
422. Designation of the Stars. - There are various ways of designating particular stars.
(a) By Names. About sixty of the brighter stars have names of their own.

These names are partly of Latin and Greek origin (e.g., Capella, Sirius, Arcturus, Procyon, Regulus, etc.), and partly Arabic (e.g., Aldebaran, Vega, etc.).
(b) By the Star's Place in the Constellation. This was the usual method employed by Ptolemy for designating stars.

Thus, Spica is the star in the "spike of wheat" that Virgo carries; Cor Leonis (the lion's heart) is a synonym for Regulus; and Cynosure means "dog's tail"; since Ursa Minor was apparently a dog in the days of the early Greek astronomers, who gave that name to the star which is now the Pole-star.
(c) By Constellation and Letters. In 1603, Bayer, in publishing his star-map, adopted an excellent plan, ever since followed, of designating the stars in a constellation by the

[^49]letters of the Greek alphabet. The letters are ordinarily applied nearly in the order of brightness, Alpha (a) being the brightest star of the constellation, and Beta $(\beta)$ the next brightest, etc. But they are sometimes applied to the stars in their order of position rather than in that of brightness.

When the stars of a constellation are so numerous as to exhaust the letters of the Greek alphabet, the Roman letters are next used, and then, if necessary, we employ the numbers which Flamsteed assigned a century later. At present every naked-eye star can be referred to and identified by its letter or number in the constellation to which it belongs.
(d) By Catalogue Number. Of course the preceding methods all fail in the case of telescopic stars. To such we refer as number so-and-so of some one's catalogue : thus, $\mathrm{Ll}, 21,185$ is read "Lalande, 21,185 ," and means the star that is so numbered in Lalande's catalogue. At present, somewhere from 600,000 to 800,000 stars are contained in our catalogues, so that (except in the Milky Way) every star visible in a three-inch telescope can be found and identified in one or more of them.

Synonyms. Of course all the bright stars which have names have letters also, and are sure to be found in every catalogue which covers their part of the heavens. A conspicuous star, therefore, has usually many "aliases," and sometimes great care is necessary to avoid mistakes on this account.
423. Star-Catalogues. - These are lists of stars, giving their positions (i.e., their right ascensions and declinations, or latitudes and longitudes), and usually also indicating their socalled "magnitudes" or brightness.

The first of these star-catalogues was made about 125 b.c. by Hipparchus of Bithynia (the first of the world's great astronomers), giving the latitude and longitude of 1080 stars. This catalogue was republished by Ptolemy, 250 years later, the longitudes being corrected for precession ; and during the middle ages several others were made by the Arabic astronomers and those that followed them.

The modern catalogues are numerous. Some, like Argelander's "Durchmusterung," already referred to, give the place of a great number of stars rather roughly, merely as a means of ready identification; others are "catalogues of precision," like the Pulkowa and Greenwich catalogues, which give the places of only a few hundred so-called "fundamental stars," determined as accurately as possible, each star by itself. The immense catalogue of the German Astronomische Gesellschaft, now in process of publication, will contain accurate places of all stars above the 9th magnitude north of $15^{\circ}$ South Declination. The observations, by numerous co-operating observatories, have occupied nearly 20 years, but are at last finished.
424. Mean and Apparent Places of the Stars. - The modern star-catalogue contains the mean right ascension and declination of its stars at the beginning of some designated year ; i.e., the place the star would occupy if there were no equation of the equinoxes, nutation, or aberration. To get the actual (apparent) right ascension and declination of a star for some given date (which is what we always want in practice), the catalogue place must be "reduced" to that date; that is, it must be corrected for precession, aberration, etc. The operation is, however, a very easy one with modern tables and formulæ, involving perhaps from five to ten minutes' work.
425. Star-Charts and Stellar Photography. - For some purposes, accurate star-charts are even more useful than catalogues. The old-fashioned and laborious way of making such sharts was by "plotting" the results of zone observations, but at present it is being done by means of photography, vastly better and more rapidly. A co-operative campaign began in 1889 , the object of which is to secure a photographic chart of all the stars down to the 14 th magnitude. The work is now well advanced, the greater part of the necessary 23,000 negatives having been already made.

One of the most remarkable things about the photographic method is that with a good instrument there appears to be no limit to the faintness of the stars that can be photographed: by increasing the time of exposure, smaller and smaller stars


Fig. 105. - Photographic Telescope of the Paris Observatory.
are continually reached. With the ordinary plates and expo-sure-times not exceeding twenty minutes, it is now possible to get distinct impressions of stars that the eye cannot possibly see with the telescope employed.

Fig. 105 represents the photographic telescope ( 14 inches aperture and 11 feet focus) of the Paris Observatory. The others engaged in this star-chart campaign are all of identical optical power. Several other enormous photographic telescopes, ranging in diameter from 24 to 32 inches, and from 40 to 60 feet long, have recently been mounted in European observatories. The "Bruce telescope" (named for its donor, the late Miss Bruce of New York) has a four-lens objective of 24 inches diameter and 14 feet focal length. It is at present mounted at Arequipa, Peru.

## STAR MOTIONS.

426. The stars are ordinarily called "fixed," in distinction from the planets or "wanderers," because they keep their positions and configurations sensibly unchanged with respect to each other for long periods of time. Delicate observations, however, separated by sufficient intervals, have demonstrated that the fixity is not absolute, but that the stars are all really in motion; and by the spectroscope the rate of motion towards or from the earth can in some cases be approximately measured. In fact, it appears that their velocities are of the same order as those of the planets: they are flying through space incomparably more swiftly than cannon-balls, and it is only because of their inconceivable distance from us that they appear to change their places so slowly.
427. Proper Motions. - If we compare a star's position (́right ascension and declination), as determined to-day, with that observed a hundred years ago, it will always be found to have changed considerably. The difference is due in the main to precession, nutation, and aberration. Those, however, are none of them real motions of the stars, but are only apparent dis-
placements, and moreover are "common"; i.e., they are shared alike by all the stars in the same part of the sky. But after allowing for all these "common" and apparent motions of a star, it generally appears that within a century the star has really changed its place more or less with reference to others near it, and this real shifting of place is called its "proper" motion, - the word "proper". being in this case the antithesis of "common." Of two stars side by side in the same telescopic field of view, the proper motions may be directly opposite, while, of course, the common motions will be sensibly the same.

The maximum proper motion now known is that of an eleventh-magnitude star in Ophiuchus, found by Barnard to move $10^{\prime \prime} .3$ a year. An eighth-magnitude star known as No. 243, Gould's Cordoba Zones, Fifth Hour, drifts $8^{\prime \prime} .7$ annually, while the so-called "runaway star," 1830 Groombridge, changes its position 7 " each year.

While these stars of large proper motion are all faint, the proper motions of faint stars average less than those of the brighter, since on the average they are more distant. For the first-magnitude stars the average is about $\frac{1}{4}$ " annually, and for the sixth-magnitude stars - the faintest visible to the naked eye - it is about $\frac{1}{25}{ }^{\prime \prime}$. These motions are always sensibly rectilinear.

They were first detected in 1718 by Halley, who found that since the time of Hipparchus the star Arcturus had moved towards the south nearly a whole degree, and Sirius about half as much.

At present the best method of determining proper motion is to measure the positions of stars relative to each other on photographs, with at least ten years between them - the longer the interval the better.
428. Real Motions of the Stars. - The "proper motion" of a star gives us very little knowledge as to the real motion of that star in miles per second, unless we know its distance; nor even then unless we also know its rate of motion towards
or from us. The proper motion derived from the comparison of the catalogues of different dates is only the angular value of that part of the whole motion which is perpendicular to the line of vision. A star moving straight towards or from the earth has no "proper motion" at all in the technical sense ; i.e., no change of apparent place which can be detected by comparing observations of its position.

Fig. 106 illustrates the subject. If a star really moves in a year from $A$ to $B$, it will seem to an observer at the earth to have moved over the line $A b$, and the proper motion (in seconds of arc) will be $206,265 \times \frac{A b}{\text { Distance }}$. Since $A b$ cannot be greater than $A B$, we can in some cases fix a minor limit to the star's velocity. We know, for instance, that the


Fig. 106. - Components of a Star's Motion. distance of 1830 Gr . is probably not less than 2,000000 astronomical units; and, therefore, since $A b$ subtends an angle of $7^{\prime \prime}$ at the earth, its length must at least equal $\frac{7 \times 2,000000}{206,265}$ astronomical units. This represents the motion for a year and corresponds to a velocity of about 200 miles a second.

The real velocity along the line $A B$ is, of course, the hypothenuse of the right-angled triangle whose sides equal $A b$ and $A a$, and can be found only when both $A b$ and $A a$ are known.

In many cases a number of stars in the same region of the sky have a motion practically identical, making it almost certain that they are really neighbors and in some way connected, - probably by community of origin. In fact, it seems to be the rule rather than the exception that stars which are apparently near each other are really comrades. Five stars of the Great Dipper are apparently moving along parallel lines with equal velocities, and even some stars in other parts of the sky, notably Sirius, seem to belong to this same system. Many of the stars in the constellation of Taurus have a common motion, and another such group is found in Perseus.
429. Motion in Line of Sight. - The motion of a star directly towards or away from us (usually called radial motion) may be detected by means of the spectroscope. If a star is approaching us, the lines of its spectrum will be shifted towards the violet, according to Doppler's principle (Arts. 200 and 500), and vice versa if it is receding from us. The reality of these motions was first demonstrated by visual observations made by Huggins in 1868, but photographs are better adapted to the accurate measurements necessary.
The spectrum of titanium or iron is photographed on each side of the spectrum of the star. If the star is at rest relative to the earth, the corresponding lines of star and comparison spectrum will agree. If the star is approaching the earth, its lines will be shifted toward the violet end of the spectrum. In Fig. 107 the dark lines of the spectrum of $\eta$ Leonis are displaced toward the red by an amount which indicates that the star and earth were separating at


Fig. 107.
Spectrum of Eta Leonis (Yerkes) the rate of 18 miles a second.

When the lines of the spectrum are clear-cut and well defined, as in the illustration, very accurate results may be obtained, so that for such stars the radial velocity is known to within a small frac-
tion of a mile per second. Even when the lines are broad and hazy, we may know this component of a star's motion with greater accuracy than we do its cross motion, which involves a knowledge of the distance of the star.

For the most part these motions of the stars, so far as at present ascertained, seem to range between zero and 25 or 30 miles a second, with still higher speeds in a few exceptional cases.
430. The Sun's Way. - The proper motions of the stars are due partly to their own motions, but partly also to the motion of the sun, which, like the other stars, is travelling through space, taking with it the earth and the planets. Sir William

Herschel was the first to investigate and determine the direction of this motion a century ago.
The principle involved is this: The motion of a star relative to the solar system is made up of its own real motion combined with the sun's motion reversed. On the whole, therefore, the stars will apparently drift bodily in a direction opposite to the sun's real motion. Those in that quarter of the sky to which we are approaching will open out from each other, and those in the rear will close up behind us. Again, from the radial motion of the stars (spectroscopically measured) a result can be obtained. In the portion of the heavens toward which the sun is moving, the stars will on the whole seem to approach, and in the opposite quarter, to recede. The individual motions lie in all directions; but when we deal with them by the hundred the individual is lost in the general, and the prevailing drift appears.

About twenty different determinations of the point in the sky towards which the sun's motion is directed have been thus far made by various astronomers. There is a reasonable accordance of results, and they all show that the sun, with its attendant planets, is moving towards a point in the constellation of Hercules, having a right ascension of about $267^{\circ}(17$ hours, 48 minutes), and a declination of about $31^{\circ}$ north. This point is called the "Apex of the sun's way."

If there are any predominant drifts among the stars whose motions form the basis of this calculation, as recent investigations of Kapteyn and others would indicate, this computed position will be affected.

## THE PARALLAX AND DISTANCE OF THE STARS.

431. When we speak of the "parallax" of the sun, of the moon, or of a planet, we always mean the "diurnal" or "geocentric" parallax (Art. 147) ; i.e., the angular semi-diameter of the earth as seen from the body. In the case of a star, this kind
of parallax is hopelessly insensible, never reaching $\frac{1}{20^{\frac{1}{0.00}}}$ of a second of arc. The expression "parallax of a star" always refers, on the contrary, to its "annual" or "heliocentric" parallax; i.e., the angular semi-diameter, not of the earth, but of the earth's orbit as seen from the star. In Fig. 108 the angle at the star is its parallax.
Even this heliocentric parallax, in the case of all stars but a very few, is too minute to be fairly measured by our present


Fig. 108. - The Annual Parallax of a Star.
instruments, never reaching a single second. In the case of Alpha Centauri, which is our nearest neighbor so far as known at present, the parallax is about $0^{\prime \prime} .9$ according to the earlier observers, but only $0^{\prime \prime} .75$ according to the latest authorities. Only four or five other cases are now known in which the parallax exceeds $0^{\prime \prime} .3$.
432. In accordance with the principle of relative motion (Art. 287), every star really at rest must appear to move in the sky just as if it were travelling yearly around a little orbit 186,000000 miles in diameter, the precise counterpart of the earth's orbit, and with its plane parallel to the plane of the ecliptic. In this little orbit the star keeps always opposite to the earth, apparently moving in the opposite direction. If the star is near the pole of the ecliptic, its "parallactic orbit," as it is called, will be sensibly circular: if it is near the ecliptic, the orbit will be seen edge-wise as a straight line; while if a star is at an intermediate celestial latitude, the orbit will be an ellipse, which becomes more nearly circular as we approach the pole of the ecliptic.

If, now, we can measure the apparent size of this parallactic orbit in seconds of are, the star's distance immediately follows. It equals

$$
R \times \frac{206265}{p^{\prime \prime}}
$$

in which $R$ is the astronomical unit (the distance of the earth from the sun), and $p^{\prime \prime}$ is the star's parallax in seconds of are; i.e., the angular semi-major axis of its parallactic orbit.

For a discussion of methods, see Appendix, Arts. 521-523.
433. Unit of Stellar Distance; the Light Year. - The distances of the stars are so enormous that even the radius of the earth's orbit, the "astronomical unit" hitherto employed, is too small for a convenient measure. Astronomers use the parsec, a distance corresponding to a parallax of one second, as the unit of stellar distance. Another common unit is the light year, the distance light travels in a year, which is about $63,000^{1}$ times the distance of the earth from the sun.
A star with a.parallax of one second is at a distance of 3.262 light years ; and in general the distance in light years equals

$$
\frac{3.262}{p^{\prime \prime}}
$$

No star has as yet been found with a parallax greater than $0^{\prime \prime} .75$, which corresponds to a distance of four and three-tenths "light years" from the solar system; the naked-eye stars are probably for the most part within 200 or 300 years, and many of the remoter ones must be some thousands of years away.

For the parallaxes of a number of stars, see Table V. of the Appendix.

[^50]THE BRIGHTNESS AND LIGHT OF THE STARS.
434. Star Magnitudes. - Hipparchus and Ptolemy arbitrarily divided the stars into six classes, or so-called " magnitudes," according to their brightness, the stars of the sixth magnitude being those which are barely perceptible by an ordinary eye, while the first class comprised about twenty of the brightest. After the invention of the telescope, the same method was extended to the fainter stars, but without any settled system, so that the "magnitudes" assigned to telescopic stars by different observers are very discordant.
435. Of course, the stars grouped under one magnitude are not exactly alike in brightness, but shade from brighter to fainter, so that precision requires the use of fractional magnitudes, and we now ordinarily employ the decimal notation. Thus a star of magnitude 2.4 is a shade brighter than one whose magnitude is written 2.5.

Heis enumerates the stars clearly visible to the naked eye, north of the 35th parallel of south declination, as follows:-

| First Magnitude, 14. | Fourth Magnitude, 313. |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Second | " | 48. | Fifth | " | 854. |
| Third | $"$ | 152. | Sixth | " | 2010. |
|  |  | Total, 3391. |  |  |  |

It will be noticed how rapidly the numbers increase for the smaller magnitudes. Nearly the same holds good also for telescopic stars, though below the tenth magnitude the rate of increase falls off.
436. Light Ratio and Absolute Scale of Star Magnitude. The scale of magnitudes ought to be such that the "light ratio," or the number of times by which the brightness of any star exceeds that of a star just one magnitude smaller, should be the same through its whole extent. If $r$ represent this ratio, a first-magnitude star will be $r$ times as bright as a second, a second $r$ times as bright as a third, and so on. This would make a first-magnitude star $r^{5}$ times as bright as a sixth, and
as the light received from a first-magnitude star may be taken as 100 times that from a star of the sixth magnitude, the value of our light ratio is $\sqrt[5]{100}$, or 2.512 . The logarithm of this number is 0.4 .

An increase of 100 times in light means, therefore, a change of five magnitudes.

On this scale, Altair (Alpha Aquilæ) and Aldebaran (Alpha Tauri) may be taken as standard first-magnitude stars, while the Pole-star and the two Pointers are very nearly of the standard second magnitude.
437. Fractional and Negative Magnitudes and General Equation. - For stars which are brighter than the standard first magnitude we have to use fractional and even negative magnitudes. Thus, according to Pickering, the magnitude of Vega is 0.2 , of Arcturus 0.0 , and of Sirius -1.4 . This means that Sirius is $2.51^{2.4}$ (about 9.12) times as bright as Altair. The magnitude of the sun is about -26.3 .

If $b_{\mathrm{m}}$ is the brightness of a star of magnitude $m$, and $b_{\mathrm{n}}$ that of another of the $n$th magnitude, we have the equation:

$$
\frac{l_{\mathrm{m}}}{b_{\mathrm{n}}}=2.512^{(\mathrm{n}-\mathrm{m})} \text {, or, log. }\left(\frac{b_{\mathrm{m}}}{b_{\mathrm{n}}}\right)=\frac{4}{10}(n-m) .
$$

By way of illustration let us compare the brightness of two stars of magnitude 3.2 and 8.6 respectively. Using these numbers for $m$ and $n$, the logarithm of the ratio of brightness is 2.16 , corresponding to the number 145 . Therefore a 3.2 -magnitude star is 145 times as bright as one of magnitude 8.6.
438. Magnitudes and Telescopic Power. - If a good telescope just shows stars of a certain magnitude, then, since the light-gathering power of a telescope depends on the area of its object-glass (which varies as the square of its diameter). we must have a telescope with its aperture larger in the ratio of $\sqrt{2.512}: 1$, in order to show stars one magnitude smaller; $i e$. , the aperture must be increased in the ratio of 1.59 to 1 . A ten-fold increase in the diameter of an olject-glass theoretically carries the power of vision just five magnitudes lower.

It is usually estimated that the twelfth magnitude is the limit of vision for a 4 -inch glass. It would require, therefore, a 40 -inch glass to reach the seventeenth magnitude of the absolute scale; but on account of loss of light from the increased thickness of the lenses, and for other reasons, the powers of large glasses never quite reach the theoretical limit.
439. Measurement of the Brightness of Stars: Stellar Photometry. - Our space does not permit any extended discussion of this subject, which has of late attracted much attention.

When the magnitudes of a few stars have been determined, other stars may be arranged in order of brightness so as to form a "sequence" (including some of the known standards) and magnitudes assigned. This method was introduced by Herschel and may be used with or without a telescope. The light of all stars down to magnitude 6.5, and of many fainter stars in selected regions, has now been photometrically measured.

Magnitudes may also be found by comparing the size and blackness of star images on photographic plates. The method of sequences may be used, or, if the images are round and black, their diameters may be measured. In either case it is necessary to have on the plate a few standard stars of known magnitude. As the photographic plate is less sensitive than the eye to red light, a red star will appear comparatively faint upon the photograph, and we must distinguish between photographic and visual magnitudes. A red star may appear two or more magnitudes brighter to the eye than on the photograph.
440. Starlight compared with Sunlight.-Zöllner and others have endeavored to determine the amount of light received by us from certain stars, as compared with the light of the sun. A standard first-magnitude star, like Altair, gives us about $90 \overline{0} 0-\frac{1}{0} 0 \overline{0} 0 \overline{0}$ th as much light as the sun does, and it would take, therefore, about nine million million stars of the sixth magnitude to equal the light of the sun. These numbers, however, are very uncertain.
441. Assuming what is roughly but by no means strictly true, that Argelander's magnitudes agree with the absolute scale, it appears
that the 324,000 stars of his Durchmusterung, all of them north of the celestial equator, give a light about equivalent to 240 or 250 firstmagnitude stars. How much light is given by stars smaller than the $9 \frac{1}{2}$ magnitude (which was his limit) is not certain. It must vastly exceed that given by the larger stars. As a rough guess, we may, perhaps, estimate that the total starlight of both the northern and southern hemispheres is equivalent to about 3000 stars like Vega, or 1500 at any one time. According to this, the starlight on a clear night is about $\frac{1}{60}$ of the light of the full moon, or about $\frac{1}{33000000}$ of sunlight. More than 90 per cent of it comes from stars which are entirely invisible to the naked eye.
442. Heat from the Stars. - The stars send us heat as well as light, so little, however, as to defy detection by any ordinary thermoscopes. Until very recently all attempts at measurement had failed; but in 1898-1900 Professor E. F. Nichols, working at the Yerkes observatory with a radiometric apparatus of unprecedented delicacy, obtained distinct deflections from the rays of Vega, Arcturus, Jupiter, and Saturn. Arcturus appeared to give about 2.2 as much heat as Vega, Jupiter 4.7, and Saturn $\frac{3}{4}$.
443. Amount of Light emitted by Certain Stars. - When we know the parallax of a star (and therefore its distance in astronomical units) it is easy to compute its real light emission as compared with that of the sun. It is only necessary to multiply the light we now get from it (expressed as a fraction of sunlight) by the square of the star's distance. Thus, according to Gill, the distance of Sirius is about 550,000 units; and the light we receive from it is $\frac{1}{0000000} 0$ of sunlight. Applying the principle above stated, we find that Sirius is really emitting more than 40 times as much light as the sun. As for other stars whose distance and light have been measured, some turn out brighter and some darker than the sun; the range of variation is very wide, and the sun holds apparently a medium rank in brilliance among its kindred.
444. Why the Stars differ in Brightness. - The apparent brightness of a star, as seen from the earth, depends both on

[^51]its distance and on the quantity of light it emits, and the latter depends on the extent of its luminous surface and upon the brightness of that surface; as Bessel long ago suggested, "there may be as many dark stars as bright ones." Taken as a class, the bright stars undoubtedly average nearer to us than the fainter ones, and just as certainly they average larger in diameter, and also more intensely luminous. But when we compare a single bright star with a fainter one, we can seldom say to which of the three different causes it owes its superiority. We cannot assert that a particular faint star is smaller or darker or more distant than a particular bright star, unless we know something more than the simple fact that it is fainter.
445. Dimensions of the Stars. - We have very little absolute knowledge on this subject; in a few instances, as that of Algol (see Art. 454*), it has been possible to obtain an indirect measure, and we find that Algol is probably a little more than a million miles in diameter : considerably bulkier than the sun. The apparent, angular diameter of a star is probably in no case large enough to be directly measured by any of our present instruments. At the distance of Alpha Centauri the sun would have an angular diameter less than $0 . ״ 01$. We shall find that in the case of binary stars of which we happen to know the parallax, we can determine their masses; but diameters, volumes, and densities are at present quite beyond our reach except in cases like that of Algol.

## VARIABLE STARS.

446. Classes of Variables. - Many stars are found to change their brightness more or less, and are known as variable. They may be classified as follows : -
I. Stars that change their brightness slowly and continuously.
II. Those that fluctuate irregularly.
III. Temporary stars which blaze out suddenly and then disappear.
IV. Periodic stars of the type of "Omicron Ceti," usually with a period of several months.
V. Periodic stars having short periods, with a continual change of light.
VI. Periodic star's of the "Algol type," in which the variation is like what might be produced if the star were periodically eclipsed by some intervening object.
447. I. Gradual Changes. - The number of stars which are certainly known to be changing gradually in brightness is surprisingly small, considering that they are growing older all the time. On the whole, the stars present, not only in position, but in brightness also, sensibly the same relations as in the catalogues of Hipparchus and Ptolemy.

There are, however, instances in which it can hardly be doubted that considerable change has occurred even within the last two or three centuries. Thus Bayer in 1610 lettered Castor as Alpha Geminorum, while Pollux, which he called Beta Geminorum, is now considerably the brighter: there are about a dozen other similar cases known, and a much larger number suspected.
448. Missing and New Stars. - It is commonly believed that a considerable number of stars have disappeared since the first catalogues were made, and that some new ones have come into existence. While it is unsafe to deny absolutely that such things may have happened, we can say, on the other hand, that not a single case of the kind is certainly known. In numerous instances, stars recorded in the catalogues are now missing; but in nearly every case we can account for the fact either by a demonstrated error of observation or printing, or by the fact that the missing stars were planets. There is no case of a new star appearing and remaining permanently bright.
449. II. Irregular Fluctuations. - The most conspicuous variable star of the second class is Eta Argûs (not visible in the United States). It varies all the way from zero magnitude (in 1843 it stood next to Sirius in brightness) down to the seventh, which has been its status ever since 1865, although in 1888 it was reported as slightly
brightening up again. Alpha Orionis and Alpha Cassiopeiæ behave in a similar way, except that their range of brightness is small, not much exceeding half a magnitude.

450: III. Temporary Stars. - There are several well authenticated instances of stars which have blazed up suddenly, and then gradually faded away. The most remarkable of these was that known as Tycho's Star, which appeared in the constellation of Cassiopeia in November, 1572, was for some days as bright as Venus at her best, and then gradually faded away, until at the end of 16 months it became invisible; (there were no telescopes then.) It is not certain whether it still exists as a telescopic star: so far as we can judge, it may be either of half a dozen which are near the place determined by Tycho. There has been a curious and utterly unfounded notion that this star was the "Star of Bethlehem," and would reappear to herald the second advent of the Lord.

A temporary star, which appeared in the constellation of Corona Borealis in May, 1866, is interesting as having been spectroscopically examined by Huggins when near its brightest (second magnitude). It then showed the same bright lines of hydrogen which are conspicuous in the solar prominences. Before its outburst, it was an eighth-magnitude star of Argelander's catalogue, and within a few months it returned to its former low estate, which it still retains.

In August, 1885, a sixth-magnitude star suddenly appeared in the great nebula of Andromeda, very near the nucleus. It began to fade almost immediately, and in a few months entirely disappeared. Its spectrum was sensibly continuous.

In December 1891 a "Nova" appeared in the foot of Auriga. In February it was nearly of the 4th magnitude, and remained visible to the naked eye for about a month. Its spectrum was very interesting. The bright lines were numerous, those of hydrogen and helium with the $H$ and $K$ of calcium, being specially conspicuous; and each of them was accompanied by a dark line on the more refrangible side, as if two bodies were concerned: one of them giving bright lines
in its spectrum and receding from us, the other, with corresponding dark lines in its spectrum, but approaching. According to Vogel the relative velocity of the two masses must, if this is the true explanation, have exceeded 550 miles a second.

In April the star became invisible, but brightened up again in the autumn, and then showed an entirely different spectrum, closely resembling that of a nebula.

Since 1885 thirteen new stars have been found from a study of photographs at the Harvard College Observatory, being recognized first by their peculiar spectra and then found upon the chart plates. Ten of these were discovered by the late Mrs. Fleming. A new star was discovered by Turner at Oxford in 1903 upon plates taken for the Astrographic Catalogue.

Nova Persei, ${ }^{1}$ in 1901, and Nova Aquilæ, in 1918, were the most brilliant temporary stars of recent times, though Nova Geminorum No. 2 attained a magnitude of 3.5 ; their photographed spectra were similar to that of Nova Aurigæ. It is now clear that these objects are essentially alike in origin and character, and that they are not extremely rare. All thus far observed have been in, or near, the Milky Way.
451. IV. Variables of the "Omicron Ceti" Type. - These objects behave almost exactly like the temporary stars, in remaining most of the time faint, rapidly brightening, and then gradually fading away, - but they do it periodically. Omicron Ceti, or Mira (i.e., "the wonderful") is the type. It was discovered by Fabricius in 1596, and was the first variable known. During most of the time it is invisible to the naked eye, of about the 9 th magnitude at the minimum, but at intervals of about 11 months it runs up to the fourth or third, or even second, magnitude, and then back again; the rise is much more rapid than the fall. It remains at its maximum about a week or ten days. The maximum brightness varies very considerably, and its period, while always about 11 months, also varies to the extent of two or three weeks, and during the last
few years seems to have shortened materially. The spectrum of the star at its maximum is very beautiful, showing a large number of intensely bright lines, some of which are certainly due to hydrogen.

Its " light curve" is $A$ in Fig. 109.


Fig. 109. - Light Curves of Variable Stars.
A large proportion of the known variables belong to this class (nearly half of the whole), and a large proportion of them have periods which do not differ very widely from one year. None so far discovered exceed two years, and none are less than two months. Most of the periods, however, are more or less irregular. Some writers include the temporary stars in this class, maintaining that the only difference is in the length of period.
452. Class V. Short-Period Variables. - In these the periods range from about $3 \frac{1}{4}$ hours (that of XX Cygni, the shortest known at present) to three or four weeks, and the light of the star fluctuates continually. In many cases there are two or more maxima in a complete period, accompanied by complicated spectroscopic phenomena much like those observed in Nova Aurigæ. The light curves of Beta Lyræ and Eta Aquilæ, which are typical of this class, are given at B, Fig. 109.
453. Class VI. The Algol Type. - In this class the star remains bright for most of the time, but apparently suffers a periodical eclipse. The periods are mostly very short, ranging from ten hours to about five days.
Algol, or Beta Persei, is the type star. During most of the time it is of the second magnitude, and it loses about fivesixths of its light at the time of obscuration. The fall of brightness occupies about $4 \frac{1}{2}$ hours ; the minimum lasts about 20 minutes, and the recovery of light takes about $3 \frac{1}{2}$ hours. The period, a little less than three days, is known with great precision, - to a single second indeed, - and is given in connection with the light curve of the star in Fig. 109. Only about one hundred of this class are known at present.
454. Explanation of Variable Stars. - No single explanation will cover the whole ground. As to progressive changes, none need be looked for. The wonder rather is that as the stars grow old, such changes are not more notable. As for irregular changes, no sure account can yet be given. Where the range of variation is small (as it is in most cases) one thinks of spots on the surface of the star, more or less like sun spots ; and if we suppose these spots to be much more extensive and numerous than are sun spots, and also like them to have a regular period of frequency, and also that the star revolves upon its axis, we find in the combination a possible explanation of a large proportion of all the variable stars. For the temporary stars, we may imagine either great eruptions of glowing mat-
ter, like solar prominences on an enormous scale, or we may imagine that a dark star moving rapidly in space encounters a nebula in its path, and as a result of the friction its surface is suddenly raised to a very high temperature. Stars of the Mira type, according to Lockyer, owe their regular outbursts of brightness to the collisions due to the passage of a smaller swarm through the outer portions of a larger one, around which the smaller revolves in a long ellipse. But the great irregularity in the periods of variables belonging to this class is hard to reconcile with a true orbital revolution, which is usually an accurate time-keeper. Many of the spectroscopic phenomena of the temporary stars and of the periodic stars of Class IV resemble pretty closely those that appear in the solar chromosphere and prominences; suggesting in such cases a theory of explosion or eruption.

In the case of the short-period variables of Class V, the spectroscopic phenomena in some instances rather seem to indicate the mutual interaction of two or more bodies revolving close together around a common centre of gravity: this is the case with Beta Lyræ. Others admit of simpler explanation, as due merely to the axial rotation of a body with large spots upon its surface.

454*. Stellar Eclipses. - As to stars of the Algol type the most natural explanation, suggested by Goodricke more than a century ago, is that the obscuration is an eclipse produced by the periodical interposition of some opaque body between us and the star.

The truth of this theory was substantially demonstrated in 1889 by Vogel, who found by his spectroscopic observations (see Art. 429) that 17 hours before the minimum Algol is receding from us at the rate of nearly 27 miles a second, while 17 hours after the minimum it is coming toward us at practically the same rate. This is just what ought to happen if Algol had a large dark companion and the two were revolving around their common centre of gravity, in an almost
circular orbit, nearly edgewise towards the earth. Vogel's conclusions are that the distance of the dark star from Algol is about 3,250000 miles, that their diameters are about 840000 and 1,060000 miles respectively. Furthermore, their period being $2^{\mathrm{d}} 20^{\mathrm{h}} 48.9^{\mathrm{m}}$, it follows (see Art. 466) that their united mass is about two-thirds that of the sun, and their mean density only about one-fifth as great as his: less even than that of Saturn, and not much above the density of cork.

That the fainter star contributes a little to the light of the system is shown by a slight secondary minimum midway between principal minima, when the fainter star would be eclipsed by the brighter one. The loss of light is less than one tenth of a magnitude.
455. Number and Designation of Variables, and their Range of Variation. - Mr. Chandler's catalogue of known variables (published in 1896) includes 393 objects, and there are also a considerable number of suspected variables. About half of them are clearly periodic in their variation. Since then the list has grown with extreme rapidity, and at present includes more than 4000 , mostly stars which are never visible to the naked eye.

Such variable stars as had not familiar names of their own before the discovery of their variability, are generally indicated by the letters R, S, T, etc. ; i.e., R. Sagittarii is the first discovered variable in the constellation of Sagittarius; S. Sagittarii is the second, and so on.

In a considerable number of the earlier discovered variables, the range of brightness is from two to eight magnitudes; i.e., the maximum brightness exceeds the minimum from six to a thousand times. In the majority, however, the range is much less, often only a fraction of a magnitude.

It is worth noting that a large proportion of the variables, especially of Classes IV. and V., are reddish in their color. This is not true of the Algol type.

455*. Photography has lately come to the front as a most effective method of detecting variables. A very large proportion of all those discovered within the last ten years have been found by the com-
parison of photographic star charts made at the Harvard Observatory and at its Arequipa annex. Several foreign observatories have also contributed largely. Frequently attention has been attracted to a star by a peculiar "colonnaded" structure of its photographic spectrum, marking the star as suspicious, - a suspicion usually soon confirmed.

One of the most interesting discoveries is that of variable star-clusters, announced by E. C. Pickering in 1895, from the study of photographs made by Bailey at Arequipa. In several clusters, notably three known as Messier 3, Messier 5, and $\omega$ Centauri, there are in each more than a hundred variables. Photographs less than two hours apart show numerous conspicuous changes of brightness. The stars are of course all very small, - their periods generally less than two days. Most clusters show no variables.

Another discovery, dating from 1904, is that of certain limited areas several degrees in extent, within which variables specially abound. One of these fields is near $\gamma$ Aquilæ, another is the smaller Magellanic cloud, and a third is in Orion, including and surrounding the great Nebula.

In Table IV. of the Appendix, we give from Chandler's catalogue a list of the principal naked-eye variables which can be seen in the United States. The observation of variable stars is especially commended to the attention of amateurs, because with a very scanty instrumental equipment, work of scientific value can be done in this line. The observer should put hinuself in communication with the American Association of Variable Star Observers, whose secretary is Mr. W. T. Olcott of Norwich, Connecticut.

## STAR SPECTRA.

456. As early as 1824, Fraunhofer observed the spectra of a number of bright stars, by looking at them through a small telescope with a prism in front of the object-glass. In 1864, as soon as the spectroscope had taken its place as a recognized instrument of research, it was applied to the stars by Huggins and Secchi. The former studied comparatively few spectra, but very thoroughly, with reference to the identification of the chemical constituents of certain stars. He found with certainty in their spectra the lines of sodium, magnesium, calcium, iron, and hydrogen, and more or less doubtfully a number of
other metals. Secchi, on the other hand, examined great numbers of spectra, less in detail, but with reference to a classification of the stars from the spectroscopic point of view.
457. Secchi's Classes of Spectra. - He made four classes, as follows:-
I. Those which have a spectrum characterized by great intensity of the dark hydrogen lines, all other lines being comparatively feeble or absent. This class comprises more than half


Fig. 110.- Secchi's Types of Stellar Spectra (Keeler).
of all the stars examined, - nearly all the white or bluish stars. Sirius and Vega are its types.

This class includes also the "Orion" stars, in whose spectra helium lines are prominent.
II. Those which show a spectrum resembling that of the sun; i.e., with numerous fine dark lines in it. Capella (Alpha Aurigæ) and Pollux (Beta Geminorum) are conspicuous examples. The stars of this class are also numerous, the first and second classes together comprising fully seven-eighths of all the stars observed.

Certain stars like Procyon and Altair seem to be intermediate between the first and second classes.
III. Those which show spectra characterized by dark bands sharply defined at the upper or more refrangible edge and shading out towards the red. Most of the red stars and a large number of the variable stars belong to this class. Some of them show also bright lines in their spectra.
IV. This class comprises only a few small stars, which show, like the preceding, dark bands, but shading in the opposite direction; usually also they show a few bright lines. They are all of a deep-red color.

This classification is the basis of the Harvard system now very generally used for detailed study. Different types are indicated by letters, arranged so as to show a possible order of evolution based upon increasing complexity of spectrum. Secchi's Type I is subdivided into Harvard Types B and A, Type II into F, G and K; Type III corresponds to M, and Type IV to N.
458. Photography of Stellar Spectra. - The observation of these spectra by the eye is very tedious and difficult, and photography has of late been brought in most effectively. Huggins in England and Henry Draper in this country were the pioneers ; now the photography of stellar spectra is an important part of the program of nearly all the large observatories.


Fig. 111. - Arrangement of the Prisms in the Slitless Spectroscope.

Pickering uses an 11inch telescope, formerly belonging to Draper, with a battery of four enormous prisms placed in front of the objectglass, as shown in Fig. 111, forming thus a "slitless spectroscope." The edges of the prisms are placed east and west, and the clock-work on the telescope is made to run a trifle too fast or too slow, in order to give
width to the spectrum formed upon the sensitive plate, which is placed at the focus of the object-glass : if the clock-work followed the star exactly, the spectrum would be a mere narrow streak. With this apparatus and an exposure of 30 minutes, spectra are obtained which, before enlargement, are fully three inches long from the $F$ line to the ultra-violet extremity. They easily bear tenfold enlargement, and show hundreds of lines in the spectra of the stars belonging to Secchi's second class. Fig. 112 is from one of these photographs of the spectrum of Vega.

The great Bruce telescope (Art. 425) has also been provided with an object-glass prism, and with that instrument the spectra of very faint stars can now be reached.


Fig. 112. - Photographic Spectrum of Vega.
459. The slitless spectroscope has three great advantages, - first, that it utilizes all the light which comes from the star to the objectglass, much of which, in the usual form of instrument, is lost in the jaws of the slit; second, by taking advantage of the length of a large telescope, it produces a very high dispersion with even a single prism; third, and most important of all, it gives on the same plate and with a single exposure the spectra of all the many stars whose images fall upon the plate. Per contra, the giving up of the slit precludes all the usual methods of identifying the lines of the spectrum by actually confronting them with comparison spectra. This makes it very difficult to utilize the photographs for some purposes of scientific work. For instance, it has not yet been found possible to use the slitless spectroscope for determining the absolute motions of the stars in the line of sight, though Professor Pickering in 1896 devised an exceedingly ingenious method of using it to measure the relative motion of different stars photographed on the same plates, in such a way that any rapid motion of circulation among the stars of a single group (the Pleiades, for instance) might be detected. In the case of the Pleiades, however, the result was simply negative : no such relative motion was found. Vogel's apparatus, for this purpose (Art. 429), is of the ordinary form, with a slit upon which the image of the star is thrown.
460. Twinkling or Scintillation of the Stars. - Before closing the discussion of starlight, a word should be added upon this subject, though the phenomenon is purely physical and not in the least astronomical. It depends both upon the irregularities of refractive power in the air traversed by the light on its way to the eye, and also on the fact that the star is optically a luminous point without apparent size, a fact which gives rise to "interference" phenomena. Planets, which have dises measurable with a micrometer, do not sensibly twinkle. The scintillation is, of course, greatest near the horizon, and on a good night it practically disappears at the zenith. When the image of a scintillating star is examined with a spectroscope, dark interference bands are seen moving back and forth in the spectrum.

## Note supplementary to Art. 450.

Nova Persei. - This recent, and one of the most remarkable, of all the temporary stars was first seen on the 20th of February, 1901, as a star of the 2 d magnitude. On the 19 th it was below the 12th, not appearing on various photographs of its region made that evening. On the 24th it had become, next to Sirius, the brightest star in the heavens, having increased its brilliance more than 25,000 times within five days-the brightest "Nova" since Kepler's star of 1604. It soon faded, with curious oscillations of light, and by the end of the year had become telescopic, though still visible in large instruments.

On the 22d its spectrum as photographed by Pickering resembled that of the Orion stars, - mainly continuous with about 30 dark lines; but on the 24 th , when next photographed, it had become like that of Nova Aurigæ, and then gradually changed into that of a nebula, with the peculiarity that its bright lines were extremely broad and hazy.

Several observers have found that its proper motion and parallax are absolutely insensible, its distance being enormous, - probably many hundred light-years.

Before the star became telescopic, an extensive nebulosity appeared around it, and in November photographs (made with the reflectors of the Lick and Yerkes observatories) showed that certain knots and streaks in the nebula were receding swiftly from the central star, - at a rate of many thousand miles a second unless the star is really much nearer than the parallax observations indicate.

At present the explanation generally accepted is that first suggested by Kayser : - that the motion is only apparent, due to a progressive illumination of denser portions of the nebula as the light from the great central outburst advances at the rate of 186,000 miles a second. This would make the distance of the star about 300 light-years, a very probable value.

## CHAPTER XV.

DOUBLE AND MULTIPLE STARS; CLUSTERS AND NEBULA. - THE MILKY WAY AND THE DISTRIBUTION OF THE STARS IN SPACE. - THE STELLAR UNIVERSE. - COSMOGONY AND THE NEBULAR HYPOTHESIS.
461. Double Stars. - The telescope shows numerous cases in which two stars lie so near each other that they can be separated only by a high magnifying power. These are double stars, and at present about 16,000 such couples are known. There is also a considerable number of triple stars, and a few which are quadruple. Fig. 113 represents some of the bestknown objects of each class.

The apparent distances generally range from $30^{\prime \prime}$ downwards, very few telescopes being able to separate stars closer than onefourth of a second. In a large proportion of cases, perhaps onethird of all, the two components are very nearly equal; but in many they are very unequal; in that case (never when they are equal) they often present a contrast of color, and when they do, the smaller star, for some reason not yet known, always has a tint higher in the spectrum than that of the larger: if the larger is reddish or yellow, the small star will be green, blue, or purple. Gamma Andromedæ and Beta Cygni are fine examples for a small telescope.

The "distance" and "position angle" of a double star are usually measured with the filar micrometer (Appendix, Art. 542 ), the position angle being the angle made at the larger star between the hour-circle and the line which joins the stars. This angle is always reckoned from the north through the east, completely around the circle ; i.e., if the smaller star were northwest of the larger one, its position angle would be $315^{\circ}$.
462. Stars Optically and Physically Double. - Stars may be double in two ways, optically and physically. In the first case they are only nearly in line with each other, as seen from the earth. In the second case they are really near each other.

In the case of stars that are only optically double, it generally happens that we can, after some years, detect their mutual independence in the fact that their relative motion is in a straight line and uniform. This is a simple consequence of the combination of their independent "proper motions."


Fig. 113. - Double and Multiple Stars.
If they are physically connected, we find on the contrary that the relative motion is in a concave curve; i.e., taking one of them as a centre, the other moves around it. The doctrine of chances shows, what direct observation confirms, that the optical pairs must be comparatively rare, and that the great majority of double stars must be really physically connected,

- probably by the same attraction of gravitation which controls the solar system.

463. Binary Stars. - Stars thus physically connected are also known as binary stars. They revolve in elliptical orbits around their common centre of gravity in periods which range from 14 years to 1500 (so far as at present known), while the apparent major axis of the ovals ranges from $40^{\prime \prime}$ to $0^{\prime \prime} .5$. The elder Herschel, a little more than a century ago, first discovered this orbital motion of "binaries" in trying to ascertain the parallax of some of the few double stars known at his time. It was then supposed that they were simply "optical pairs," and he expected to detect an annual displacement of one of the stars with reference to the other. He failed in this, but found instead a true orbital motion.

At present the number of pairs in which this kind of motion has been certainly detected is about 200 , and is continually increasing as our study of the double stars goes on. Most of the double stars have been discovered too recently to show any sensible motion as yet, but about fifty pairs have progressed so far, either having completed an entire revolution or a large part of one, that it is possible to compute their orbits with some accuracy.
464. Orbits of Binaries. - In the case of a binary pair the apparent orbit of the smaller star with reference to the larger is always an ellipse ; but this apparent orbit is only the true orbit seen more or less obliquely. If we assume what is probable, ${ }^{1}$ though certainly not proved as yet, that the orbital motion of the pair is under the law of gravitation, we know that the

[^52]larger star must be in the focus of the true relative orbit described by the smaller ; and, moreover, that the latter must describe around it equal areas in equal times. By the help of these principles, we can deduce from the apparent oval the true orbital ellipse; but the calculation is troublesome and delicate.

The relative orbit is all that can be determined from micrometer observations of the distance and position angle measured between the two stars.


Fig. 114. - Orbits of Binary Stars.

Fig. 114 represents the orbits of four of the best determined doublestar systems.

In but a few cases, where we have sufficient meridian-circle observations, or where the two components of the pair have had their position and distance measured from a neighboring star not partaking of their motion, we can deduce the absolute motion of each of the two stars separately with respect to their common centre of gravity, and thus get data for determining their relative masses (Art. 466). The case of Sirius is in point. About 60 years ago it had been found
from meridian-circle observations to be moving for no assignable reason in a small oval orbit with a period of about 50 years. In 1862, Clark found near it a minute companion, which explained everything ; only we have to admit that this faint acolyte, which does not give ${ }_{12} \frac{1}{0} 00$ as much light as Sirius itself, has a mass more than a quarter part as great ; it seems to be one of Bessel's "dark stars."

Fig. 114* shows the absolute and relative orbits of the system of Sirius.


FIG. 114*. - Orbit of Sirius.

The dotted line represents the relative orbit of the companion with respect to Sirius, while the larger, full-line ellipse is its actual orbit around the common centre of gravity, $C$; the smaller oval being the orbit of Sirius itself around $C$, as given by meridian-circle observation.
465. Size of the Orbits. - The real dimensions of a doublestar orbit can be obtained only when we know its distance from us. Fortunately, a number of the stars whose parallaxes have been ascertained are also binary, and assuming the best available data as to parallax and orbit, we find the following results, - the semi-major axis in astronomical units being always
equal to the fraction $\frac{a^{\prime \prime}}{p^{\prime \prime}}$, in which $a^{\prime \prime}$ is the semi-major axis of the double-star orbit in seconds of arc, and $p^{\prime \prime}$ is its parallax.

| Name. |  | Assumed <br> Parallax. | Angular <br> Semi-axis. | Real <br> Serni-axis. | Period. | Mass. <br> $\odot=1$. |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| $\eta$ Cassiopeiæ . | . | $0^{\prime \prime} .35$ | $8^{\prime \prime} .21$ | 23.5 | $195 y .8$ | 0.33 |
| Sirius . . | . | 0.39 | 8.03 | 20.6 | 52.2 | 3.24 |
| a Centauri. | . | . | 0.75 | 17.70 | 23.6 | 81.1 |
| 70 Ophiuchi | . | 0.25 | ? | 4.55 | 18.2 | 88.4 |

These double-star orbits are evidently comparable in magnitude with the larger orbits of the planetary system, none of those given being smaller than the orbit of Uranus, and none twice as large as that of Neptune. The elements of the orbits are from the data of Dr. See. Observations since the reappearance of the companion in 1897 indicate that the true period and distance of Sirius are both a little less than here given, and its mass somewhat smaller.

465*. Spectroscopic Binaries. - One of the most interesting of recent astronomical results is the detection by the spectroscope of several pairs of double stars so close that no telescope can separate them. In 1889 the brighter component of the well-known double star Mizar (Zeta Ursæ Majoris, Fig. 113) was found by Pickering to show the dark lines double in the photographs of its spectrum, at regular intervals of about 52 days. The obvious explanation is that this star is composed of two, which revolve around their common centre of gravity, in an orbit whose plane is nearly edgewise towards us. When the stars are at right angles to the line along which we view them, one of them will be moving towards us, the other from us, and as a consequence, according to Doppler's principle (Arts. 200 and 500), the lines in their spectra will be shifted opposite ways. Now since the two stars are so close that their spectra overlie each other, the result will be simply to make the lines in the compound spectrum apparently double. From the distance between the two components of the lines thus doubled, the relative velocity of the two stars can be
found ; and from this (knowing the period) the size of the orbit and the united mass of the stars. Thus it appears that in the case of Mizar, the relative velocity of the two components is about 100 miles a second, the period 104 days, and the distance between the two stars about 140 millions of miles; from which it follows (Art. 466) that the united mass of the two is about 40 times that of the sun.

The lines in the spectrum of Beta Aurigæ exhibit the same peculiarity, but the doubling occurs once in four days. The relative velocity is about 150 miles a second, and the diameter of the orbit about 8,000000 miles, the united mass of the pair being about two and a half times that of the sun.

In $1896^{1}$ two other similar cases were announced by Professor Pickering, discovered on the South American spectrum photographs. The first is $\mathrm{Mu}_{1}$ Scorpii, with a period of $34^{\mathrm{h}} 42.5^{\mathrm{m}}$ : the size of the orbit not then determined. The second is 3105 Lacaille in Puppis, with a period of $74^{\mathrm{h}} 46 \mathrm{~m}$. In both cases the components are considerably unequal.

These observations were all made by photographing the spectrum with the slitless spectroscope (Art. 458), and are only possible in cases where the stars which compose the pair are both reasonably bright.

With his slit-spectroscope, Vogel, in 1889, as has already been stated in Art. 4554, detected about the same time a similar orbital motion in Algol, although the companion of the brighter star does not give light enough to form a spectrum of its own.

A year later he found another similar result in the case of the bright star Alpha Virginis (Spica). The star is really double, having a small companion like that of Algol, not bright enough to make a perceptible spectrum, but heavy enough to make its partner swing around in an orbit 6,000000 miles in diameter once in four days. The sun is not quite in the plane of the orbit, so that Spica is never, like Algol, eclipsed by its attendant.

Still more recently Belopolsky of Pulkowa, Lockyer, and others, especially Campbell of the Lick Observatory, have detected numerous other similar pairs, of which the most notable are the following : Castor, Delta Cephei (its period is identical with that of its lightvariation), Capella, and Polaris with the low radial-velocity range of only 4 miles a second, and a period less than 4 days. Beta Lyræ also falls into this class, but its spectrum lines double as well as shift. At the present time the number of known spectroscopic

[^53]binaries exceeds 300 , and is fast increasing. Campbell finds that about 1 in 4 of the stars above the fifth magnitude are of this character.
466. Masses of Binary Stars. - If we assume that the binary stars move under the law of gravitation, then when we know the semi-major axis of the orbit in astronomical units and the period of revolution, we can find the mass of the pair as compared with the sun by the proportion. (Art. 309)
$$
S+e: M+m:: 1: \frac{a^{3}}{t^{2}}
$$
in which $S+e$ is the united mass of the sun and earth ( $e$ is insignificant), $M+m$ is the united mass of the two stars, $a$ the semi-major axis of their orbit in astronomical units, and $t$ their period in years. This gives
$$
(M+m)=(S+e) \frac{a^{3}}{t^{2}} .
$$

The final column of the little table gives the masses of the starpairs resulting from the data which are presented; but the reader must bear in mind that they are not much to be relied on, because of the uncertainty of the parallaxes in question. A slight error in the parallax makes a vastly greater error in the resulting mass. The reader is also reminded of the fact that the mass of the pair gives no clue to the diameter or density of the stars.

466*. Evolution of Binary Systems. - As already remarked (Art. 462) the theory of probabilities indicates that the great majority of double stars must be physically connected, but our observations have not yet continued long enough to give us anything like an accurate knowledge of the orbits of more than a very few. Table VII (Appendix) presents a list of twenty, mostly computed by Dr. See, which may be regarded as fairly known. Two others of long period are added, not yet, however, to be accepted as reliable, the data being insufficient.

It will be noticed that the orbits are very eccentric as compared with those of the planets, the average eccentricity of the stellar orbits being nearly 0.50 . Dr. See has investigated the probable origin of these binary systems, and finds that
all the peculiarities of their orbits can be accounted for by the theory of "tidal evolution" (Art. 281). It is supposed that in such cases the primitive nebula as it whirls assumes the dumb-bell form known as the "apioid" : the two parts separate, and as they revolve around their common centre of gravity great tides are raised, which by their interaction push the spinning globes apart into eccentric orbits.
467. Planetary Systems attending Stars. - It is a natural question whether some, at least, of the stars have not planetary systems of their own, and whether some of the small "companions" that we see may not be the Jupiters of such systems. We can ouly say as to this that no telescope ever constructed could even come near to making visible a planet which bears to its primary approximately the relations of size, distance, and brightness which Jupiter bears to the sun. In the solar system, viewed from our nearest neighbor among the stars, Jupiter would be a star of about the twenty-first magnitude, not quite $5^{\prime \prime}$ distant from the sun, which itself would be a star of the second magnitude. To render a star of the twenty-first magnitude barely visible (apart from all the difficulties raised by the proximity of a larger star) would require a telescope of more than 20 feet aperture.
468. Multiple Stars. - There are a considerable number of cases where we find three or more stars connected in one system. Zeta Cancri consists of a close pair revolving in a nearly circular orbit, with a period of somewhat less than 60 years, while a third star revolves in the same direction around them, at a much greater distance, and with a period that must be at least 500 years. Moreover, the third star is subject to a peculiar irregularity in its motion, which seems to indicate that it has an invisible companion very near it, the system probably being really quadruple. In Epsilon Lyræ we have a most beautiful quadruple system composed of two pairs, each pair making its own slow revolution with a period of over 200 years; probably, moreover, since they have a common proper motion, the two pairs revolve around each other in a period only to be reckoned by milleniums. In Theta Orionis we have
a multiple star in which the six components are not organized in pairs, but are at not very unequal distances from each other (see Fig. 113).
469. Clusters. - There are in the sky numerous groups of stars, containing from a hundred to many thousand members. A few are resolvable by the naked eye, as, for instance, the


Fig. 115. - The Pleiades.
Pleiades (Fig. 115) ; some, like "Præsepe" (in Cancer), break up under the power of even an opera-glass; but most of them require a large telescope to show the separate components.

To the naked eye or small telescopes, if visible at all, they look merely like faint clouds of shining haze; but in a large telescope they are among the most magnificent objects the heavens afford. The cluster known as "13 Messier," not far from the "apex of the sun's way," is perhaps the finest.

The question at once arises whether the stars in such a cluster are comparable with our own sun in magnitude, and separated from each other by distances like that between the sun and Alpha Centauri, or whether they are really small and closely packed,-mere sparks of stellar matter, - whether the swarm is about the same distance from us as the stars, or far beyond them. The irregular ones, like the Pleiades and Præsepe clusters, undoubtedly belong to our own stellar system, but it may be otherwise with a few like the Hercules cluster ( 13 Messier), which are globular in form. While the evidence may not be conclusive, recent investigations seem to indicate that this cluster may be as distant as 100,000 light years. If this be so, individual stars must be more luminous than our sun, it may take 1000 years for light to travel from one side to the other, and the actual distances between members of the cluster must be enormous, though perhaps less than that which separates our sun from its nearest neighbor.

## NEBULE.

470. Besides the luminous clouds which under the telescope break up into separate stars, there are others which no telescopic power resolves, and among them some which are brighter than many of the clusters. These irresolvable objects, which now number something like 10,000 , are "the nebulæ." Two or three of them are visible to the naked eye, - one, the brightest of all, and the one in which the temporary star of 1885 appeared, is in the constellation of Andromeda, and is represented in Fig. 116 as seen in a good-sized telescope. Another most conspicuous and very beautiful nebula is that in the sword of Orion.

The larger and brighter nebulæ are mostly irregular in form, sending out sprays and streams in all directions, and containing dark openings and "lanes." Some of them are of enormous volume. The nebula of Orion (which includes within its boundary the multiple star Theta Orionis) covers several square degrees, and since we know with certainty that it is more remote than Alpha Centauri, its cross-section as seen


Fig. 116.
Telescopic View of the Great Nebula in Andromeda. from the earth must exceed the area of Neptune's orbit by many thousand times. The nebula of Andromeda is not quite so extensive, and it is rather more regular in its form. The smaller nebulæ are, for the most part, more or less nearly oval in form and brighter in the centre. In the so-called "nebulous stars," the central nucleus is like a star shining through a fog. The "planetary nebulæ" are nearly circular and of about uniform brightness throughout, and the rare "annular or ring nebulæ" are darker in the centre. Fig. 117 is a copy of a photograph of a nebula in the constellation of Canes Venatici and is a fine illustration of the spiral form. This is sometimes called the "Whirlpool Nebula." There are several double nebulæ and a few that are variable in brightness, though no periodicity has yet been ascertained in their variations.

The great majority of the thousands of nebulæ are ex-
tremely faint, but the few that are reasonably bright are very interesting objects.
471. Drawings and Photographs of Nebulæ. - Not very long ago the correct representation of a nebula was an extremely difficult task. A few more or less elaborate engravings exist of perhaps fifty of the most conspicuous of them ; but photography has recently taken possession of the field. The first success in this line was by Henry Draper of New York, in 1880, in photographing the nebula of Orion. Since his death in 1882, great progress has been made, both in Europe and this country, and at present the photographs are continually bringing out new and before unsuspected features. Fig. 118, for instance, is from a photograph of the nebula of Andromeda, taken by Mr. Roberts of Liverpool in December, 1888, and shows that the so-called "dark lanes," which hitherto had been seen only as straight and wholly inexplicable markings, as


Fig. 117. - A Spiral Nebula (Keeler). represented in Fig. 116, are really curved ovals, like the divisions in Saturn's rings. The photograph brings out clearly a distinct spiral structure pervading the whole nebula, though as yet not satisfactorily seen by the eye with any telescope.
The photographic work of Keeler has shown that great numbers of nebulæ are of a spiral form.

Thus, in the Pleiades, it has been found that nearly all the larger stars have wisps of nebulous matter attached to them, as indicated by the dotted outlines in Fig. 115: and in a small territory in and near


Fig. 118. - Mr. Roberts's Photograph of the Nebula of Andromeda.
the constellation of Orion, Pickering, with an eight-inch photographic telescope, found upon his star plates nearly as large a number of
new nebulæ as of those that were previously known within the same boundary.

The photographs of nebulæ require, generally, an exposure of from one hour to four or five, or even more. The images of all the brighter stars in the field are therefore enormously over-exposed, and seriously injure the picture from an artistic point of view.
472. Changes in Nebulæ. - It cannot perhaps be stated with certainty that sensible changes have occurred in any of the nebulæ, since they first began to be observed, - the early instruments were so inferior to the modern ones that the older drawings cannot be trusted very far; but some of the differences between the older and more recent representations make it extremely probable that real changes are going on. Probably after a reasonable interval of time, photography will settle the question.
473. Spectra of Nebulæ. - One of the most important of the early achievements of the spectroscope was the proof that the light of the nebulæ proceeds not from aggregations of stars, but from glowing gas in a condition of no great density. Huggins, in 1864, first made the decisive observation by finding bright lines in their spectra.

So far the spectra of all the nebulæ that show lines at all appear to be substantially the same. Four lines are usually easily observed, two of which are due to hydrogen; but the other two, which are brighter than the hydrogen lines, are not yet identified, and are almost certainly due to some element not yet detected on the earth or sun, and are apparently peculiar to the nebulæ. At one time the brightest of the four lines was thought to be due to nitrogen, and even yet the statement that this is the case is found in many books; but it is now certain that whatever it may be, nitrogen is not the substance.

Mr. Lockyer has ascribed this line to magnesium in connection with his "meteoric hypothesis"; but elaborate observations of Huggins and others show conclusively that this identification also is incorrect.

Fig. 119 shows the position of the principal lines so far observed : in the brighter nebulæ a number of others are also sometimes seen,
and over seventy lines have been photographed in the spectra of different nebulæ : the lines of helium are generally found to be present. One of Mr. Huggins's photographic spectra of the nebula of Orion shows, in addition to those that are visible to the eye, a considerable number of bright lines in the ultra-violet; and, what is interesting, these lines seem to pertain also to the spectrum of the stars in the so-called "Trapezium" (Theta Orionis), as if, which is very likely, the stars themselves were mere condensations of the nebulous matter.


Fig. 119. - Spectrum of the Gaseous Nebulæ.
Not all nebulæ show the bright-line spectrum. Those which do - probably only a few hundred - are of a greenish tint, at once recognizable in a large telescope. The white nebulæ, most of them spiral in form and greatly outnumbering the green, or gaseous ones, seem to present a continuous spectrum crossed by dark absorption lines. It would appear, therefore, that the quality of their light is the same as that of the stars, and the possibility of distant galaxies is suggested. But the telescopic evidence as to the non-stellar constitution of nebulæ is the same for all: no nebula resists all attempts at resolution more stubbornly than that of Andromeda, the largest one of this type.

Keeler at the Lick Observatory has very recently observed a number of the brighter nebulæ with a spectroscope of high dispersive powers, and has been able to detect and to measure the motion of several of them along the line of sight. The velocity of their motion appears to be of the same order as that of the stars, the nebulæ observed giving results ranging from zero up to about 40 miles a second - some approaching and others receding. The spiral nebulæ appear to have much greater velocities.

As to the real constitution of these bodies, we can only speculate. The fact that the matter which shines is mainly gaseous does not make it certain that they do not also contain dark matter, either liquid or solid. What proportion of it there may be we have at present no means of knowing.
474. Distance and Distribution of Nebulæ. - As to their distance, we can only say that the gaseous nebulæ seem to lie within the stellar universe, and not beyond its boundaries, as is clearly shown by the nebulous stars first pointed out and discussed by the older Herschel, and by such peculiar association of stars and nebulæ as we find in the Pleiades.

Moreover, in certain curious luminous masses known as the "Nubeculæ" (near the south pole), we have stars, star-clusters, and nebulæ intermingled promiscuously.
In the sky generally, however, the distribution of the nebulæ is in contrast with that of the stars. The stars crowd together near the Milky Way: the white nebulæ ${ }^{1}$ seem to be most numerous just where the stars are fewest, as if the stars had somehow consumed in their formation the substance of which the nebulæ are made; or as if, possibly, on the other hand, the nebulæ had been formed by the disintegration of stars, as a few astronomers have maintained, in opposition to the more common view.

## THE CONSTITUTION OF THE SIDEREAL HEAVENS.

475. The Galaxy or Milky Way. - This is a luminous belt of irregular width and outline, which surrounds the heavens nearly in a great circle. It is very different in brightness in different parts, and in several constellations it is marked by dark bars and patches which make the impression of overlying clouds : the most notable of them is the so-called "Coal-sack," near the southern pole. For about a third of its length (from Cygnus to Scorpio) it is divided into two roughly parallel streams. The telescope shows it to be made up almost wholly

[^54]of small stars, from the eighth magnitude down; it contains also numerous star-clusters, but very few white nebulæ.

The galaxy intersects the ecliptic at two opposite points not far from the Solstices, and at an angle of nearly $60^{\circ}$, its "northern pole" being, according to Herschel, in the constellation of Coma Berenices.

As Herschel remarks, "the 'galactic plane' is to the sidereal universe much what the plane of the ecliptic is to the solar system, - a plane of ultimate reference, and the ground plan of the stellar system."
476. Distribution of the Stars in the Heavens. - It is obvious that the distribution of the stars is not even approximately uniform : they gather everywhere in groups and streams. But besides this the examination of any of the great star-catalogues shows that the average number to a square degree increases rapidly and pretty regularly from the galactic pole to the galactic circle itself, where they are most thickly packed. This is best shown by the "star-gauges" of the elder Herschel, each of which consisted merely in an enumeration of the stars visible in a single field of view of his 20 -foot reflector, the field being $15^{\prime}$ in diameter.

He made 3400 of these "gauges," and his son followed up the work at the Cape of Good Hope with 2300 more in the south circumpolar regions. From the data of these star-gauges, Struve has deduced the following figures for the number of stars visible in one field of view :
Distance from Galactic Circle. Average No. of Stars in Field.

| $90^{\circ}$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | 4.15 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $75^{\circ}$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | 4.68 |
| $60^{\circ}$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | 6.52 |
| $45^{\circ}$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | 10.36 |
| $30^{\circ}$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | 17.68 |
| $15^{\circ}$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | 30.30 |
| 0 | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | 122.00 |

47\%. Structure of the Stellar Universe. - Herschel, starting
from the unsound assumption that the stars are all of about the same size and brightness, and separated by approximately equal distances, drew from his observations certain untenable conclusions as to the form and structure of the "galactic cluster," to which the sun was supposed to belong, - theories for a time widely accepted and even yet more or less current, though in many points certainly incorrect.

But although the apparent brightness of the stars does not thus depend mainly upon their distance, it is certain that, as a class, the faint stars are smaller, darker, and more remote than the brighter ones; we may, therefore, safely draw a few conclusions, which, so far as they go, substantially agree with those of Herschel.
478. I. The great majority of the stars we see are contained within a space having roughly the form of a rather thin, flat dise, with a diameter eight or ten times as great as its thickness, our sun being not very far from its centre.
II. Within this space the naked-eye stars are distributed rather uniformly, but with some tendency to cluster, as shown in the Pleiades. The smaller stars, on the other hand, are strongly " gregarious," and are largely gathered in groups and streams, which have comparatively vacant spaces between them.
III. At right angles to the "galactic plane" the stars are scattered more evenly and thinly than in it, and we find here on the sides of the dise the comparatively starless region of the nebulæ.
IV. As to the Milky Way itself, it is not certain whether the stars which compose it are ranged in a kind of ring, with a comparatively empty space in the middle where the sun is placed, or whether it is spiral in form.

As to the size of the disc-like space which contains most of the stars, very little can be said positively. Its diameter must be as great as 20,000 or 30,000 light years, -how much greater we cannot even guess; and as to "the beyond" we are still more ignorant. If, however, there are other stellar systems
of the same order as our own, it may be they are neither the nebulæ nor the clusters which the telescope reveals, but are far beyond the reach of any instrument at present existing.
479. Do the Stars form a System? - It is probable that gravitation ${ }^{1}$ operates between the stars (as indicated by the motions of the binaries), and they are certainly moving very swiftly in various directions. The question is whether these motions are governed by gravitation, and are "orbital" in the ordinary sense of the word.

There has been a very persistent belief that somewhere there is "a great central sun," around which the stars are all circling. As to this, there is no longer any question - the "central sun" speculation is certainly unfounded, though we have not space for the demonstration of its fallacy.

Another less improbable doctrine is that there is a general revolution of the mass of stars around the centre of gravity of the whole, a revolution nearly in the plane oĩ the Milky Way. Some years ago, Maedler, in his speculations, concluded that this centre of gravity of the stellar universe was not far from Alcyone, the brightest of the Pleiades, and that therefore this star was in a sense the "central sun." The evidence, however, is entirely inconclusive, nor is there yet proof of any such general revolution.
480. On the whole, the most probable view seems to be that the stars are moving much as bees do in a swarm, each star mainly under the control of the attraction of its nearest neighbors, though influenced more or less, of course, by that of the general mass. If so, the paths of the stars are not "orbits" in any periodic sense; i.e., they are not paths which return into themselves. The forces which at any moment act upon a given star are so nearly balanced that its motion must be sensibly rectilinear for thousands of years at a time.

[^55]From a study of both proper and radial motions it has been found that the stars are not moving at random, but there are apparently within what we call our universe two swarms, or streams of stars, moving in opposite directions. The stars in each swarm are flying about in every direction, but all share in the motion common to the swarm.
481. Cosmogony. - One of the most interesting and one of the most baffling topics of speculation relates to the process by which the present state of things has come about. In a forest, to use an old comparison of Herschel's, we see around us trees in all stages of their life history, from the sprouting seedlings to the prostrate and decaying trunks of the dead. Is the analogy applicable to the heavens, and if so which of the heavenly bodies are in their infancy, and which decrepit with age?

At present many of these questions seem to be absolutely beyond the reach of investigation even. Others, though at present unsolved, appear approachable, and a few we can already answer. In a general way we may say that the condensation of diffuse, cloud-like masses of matter under the force of gravitation, the conversion into heat of the energy of motion and of position (the "kinetic" and "potential" energy - Physics, p. 73) of the particles thus concentrated, the effect of this heat upon the mass itself, and the effect of its radiation upon surrounding bodies, - these principles cover nearly all the explanations that can thus far be given of the present condition of the heavenly bodies.
482. Genesis of the Planetary System. - Our planetary system is clearly no accidental aggregation of bodies. Masses of matter coming haphazard to the sun would move (as the comets actually do move) in orbits which, though always conic sections, would have every degree of eccentricity and inclination. In the planetary system this is not so. Numerous relations
exist for which the mind demands an explanation, and for which gravitation does not account.

We note the following as the principal:-

1. The orbits of the planets are all nearly circular.
2. They are all nearly in one plane (excepting those of some of the asteroids).
3. The revolution of all, without exception, is in the same direction.
4. There is a curious and regular progression of distances (expressed by Bode's Law ; which, however, breaks down with Neptune).

As regards the planets themselves :-
5. The plane of the planet's rotation nearly coincides with that of the orbit (probably excepting Uranus).
6. The direction of rotation is the same as that of the orbital revolution (excepting probably Uranus and Neptune).
7. The plane of the orbital revolution of the planet's satellites coincides nearly with that of the planet's rotation, wherever this can be ascertained.
8. The usual direction of the satellites' revolution is the same as that of the planet's rotation.
9. The largest planets rotate most swiftly.

Now this arrangement is certainly an admirable one for a planetary system, and therefore some have argued that the Deity constructed the system in that way, perfect from the first. But to one who considers the way in which other perfect works of nature usually attain to their perfection, - their processes of growth and development, - this explanation seems improbable. It appears far more likely that the planetary system was formed by growth than that it was built outright.
483. The Nebular Hypothesis. - The theory which in its main features is now generally accepted, as supplying an intelligible explanation of the facts, is that known as "the nebular hypothesis." In a more or less crude and unscientific form, it was first suggested by Swedenborg and Kant, and afterwards, about the beginning of the present century, was worked out in mathematical detail by La Place. He maintained -
(a) That at some time in the past ${ }^{1}$ the matter which is now gathered into the sun and planets was in the form of a nebula.
(b) This nebula, according to him, was a cloud of intensely heated gas. (As will be seen, this postulate is questionable.)
(c) Under the action of its own gravitation, the nebula assumed a form approximately globular, with a motion of rotation, the rotational motion depending upon accidental differences in the original velocities and densities of different parts of the nebula. As the contraction proceeded, the swiftness of the rotation would necessarily increase for mechanical reasons: since every shrinkage of a revolving mass implies a shortening of its rotation period.
(d) In consequence of the rotation, the globe would necessarily become flattened at the poles, and ultimately, as the contraction went on, the centrifugal force at the equator would become equal to gravity, and rings of nebulous matter, like the rings of Saturn, would be detached from the central mass. In fact, Saturn's rings suggested this feature of the theory.
(e) The ring thus formed would for a time revolve as a whole, but would ultimately break, and the material would collect into a globe revolving around the central nebula as a planet. ${ }^{2}$ La Place supposed that the ring would revolve as if solid, the particles at the outer edge moving more swiftly than those at the inner. If this were always so, the planet formed would necessarily rotate in the same direction as the ring had revolved.
$(f)$ The planet thus formed might throw off rings of its own, and so form for itself a system of satellites.

[^56]The theory obviously explains most of the facts of the solar system, which were enumerated in the preceding article, though some of the exceptional facts, such as the short periods of the satellites of Mars, and the retrograde motions of those of Uranus and Neptune, cannot be explained by it alone in its original form. Even they, however, do not contradict it, as is sometimes supposed.
Many things also make it questionable whether the outer planets are so much older than the inner ones, as the theory would indicate. It is not impossible that they may even be younger.
484. On the whole, we may say that while in its main outlines the theory may perhaps be true, it certainly needs serious modifications in details. It is rather more likely, for instance, that the original nebula was a cloud of ice-cold meteoric dust than an incandescent gas, or a "fire-mist," to use a favorite expression; and it is likely that planets and satellites were often separated from the mother-orb otherwise than in the form of rings. Nor is it possible that a thin, wide ring could revolve in the same way as a solid, coherent mass : the particles near the inner edge must make their revolution in periods much shorter than those upon the circumference.

A most serious difficulty arises also from the apparently irreconcilable conflict between the conclusions as to the age and duration of the system, which are based on the theory of heat (see Art. 489) and the length of time which would seem to be required by the nebular hypothesis for the evolution of our system.

Our limits do not permit us to enter into a discussion of Darwin's "tidal theory" of satellite formation, which may be regarded as in a sense supplementary to the nebular hypothesis; nor can we more than mention Faye's proposed modification of it. According to him, the inner planets are the oldest.
485. The Planetesimal, or Spiral Nebula, Hypothesis. According to a theory recently proposed and developed by

Chamberlin and Moulton, the solar system was at one time in the form of a spiral nebula. Such a nebula is supposed to have been made up of discrete particles ("planetesimals") revolving in elliptical orbits about the central nucleus and across the arms of the spiral rather than along them. The arms show simply the distribution of matter at a given time. It is supposed that spiral nebulæ may be developed by tidal disruption when two suns pass near each other.

According to this hypothesis the sun was formed from the central mass, the planets from the local condensations or nuclei in the coils of the spiral, their masses having been increased by the sweeping up of the scattered particles whose orbits they crossed. In a similar way satellites were formed from smaller nuclei.

While it is obviously impossible to prove absolutely a theory of so wide application, the Planetesimal Hypothesis offers some decided advantages over the Laplacian. It accounts satisfactorily for the facts in harmony with the older theory and also for the direct rotation of the planets, the large eccentricities of the orbits of Mercury, Mars, and the asteroids, and the present distribution of the moment of momentum of the solar system. The retrograde motions of the satellites of Uranus and Neptune offer no difficulty, neither do the rapid revolutions of Phobos and the inner portion of Saturn's ring.

The fact that photography has shown that the spiral is a common form of nebula, while none of the Laplacian type has been found, gives added weight to the hypothesis. We must remember, however, that the solar nebula was probably much smaller than any of those photographed.
486. Stars, Star-clusters, and Nebulæ. - It is obvious that the nebular hypothesis in all of its forms applies to the explanation of the relations of these different classes of bodies to each other. In fact, Herschel, appealing only to the "law of continuity," had concluded, before La Place formulated his theory, that the nebulæ develop sometimes into clusters, some-
times into double ${ }^{1}$ or multiple stars, and sometimes into single stars. He showed the existence in the sky of all the intermediate forms between the nebula and the finished star. For a time, half a century ago, while it was generally believed that all the nebulæ were nothing but star-clusters, only too remote to be resolved by existing telescopes, his views fell rather into abeyance; but they regained acceptance in their essential features when the spectroscope demonstrated the substantial difference between gaseous nebulæ and the star-clusters.
487. Conclusions from the Theory of Heat. - Kant and La Place, as Newcomb says, seem to have reached their results by reasoning forwards. Modern science comes to very similar conclusions by working backwards from the present state of things.

Many circumstances go to show that the earth was once much hotter than it now is. As we penetrate below the surface, the temperature rises nearly a degree (Fahrenheit) for every 60 feet, indicating a white heat at the depth of a few miles only; the earth at present, as Sir William Thomson says, "is in the condition of a stone that has been in the fire and has cooled at the surface."

The moon bears apparently on its surface the marks of the most intense igneous action, but seems now to be entirely chilled.

The planets, so far as we can make out with the telescope, exhibit nothing at variance with the view that they were once intensely heated, while many things go to establish it. Jupiter and Saturn, Uranus and Neptune, do not seem yet to have cooled off to anything like the earth's condition.
488. As to the sun, we have in it a body continuously pouring forth an absolutely inconceivable quantity of heat without any visible source of supply. As has been explained

[^57]already (Art. 219), the only rational explanation of the facts, ${ }^{1}$ thus far presented, is that which makes it a huge cloud-mantled ball of elastic substance, slowly shrinking under its own central gravity, and thus converting into the kinetic energy of heat ${ }^{2}$ the potential energy of its particles, as they gradually settle towards the centre. A shrinkage of 250 feet a year in the sun's diameter ( 125 feet in its radius) will account for the whole annual out-put of radiant heat and light.
489. Age and Duration of the System. - Looking backward, then, and trying to imagine the course of time and of events reversed, we see the sun growing larger and larger, until at last it has expanded to a huge globe that fills the largest orbit of our system. How long ago this may have been, we cannot state with certainty. If we could assume that the amount of heat yearly radiated by the solar surface had remained constantly the same through all those ages, and, moreover, that all the radiated heat came only from one single source, the slow contraction of the solar mass, apart from any considerable original capital in the form of a high initial temperature, and without any reinforcement of energy from outside sources, if we could assume these premises, it is easy to show that the sun's past history must cover about 15,000000 or 20,000000 years. But such assumptions are at least doubtful; and, if we discard them, all that can be said is that the sun's age must be greater, and probably many times greater, than the limit we have named.

Looking forward, on the other hand, from the present towards the future, it is easy to conclude with certainty that if the sun continues its present rate of radiation and contrac-

[^58]tion, and receives no subsidies of energy from without, it must within 5,000000 or 10,000000 years become so dense that its constitution will be radically changed. Its temperature will fall and its function as a sun will end. Life on the earth, as we know life, will be no longer possible when the sun has become a dark, rigid, frozen globe. At least this is the inevitable consequence of what now seems to be the true account of the sun's present activity, and the story of its life.
490. The Present System not Eternal. - One lesson seems to be clearly taught: That the present system of stars and worlds is not an eternal one. We have before us everywhere evidence of continuous, irreversible progress from a definite beginning towards a definite end. Scattered particles and masses are gathering together and condensing, so that the great grow continually larger by capturing and absorbing the smaller. At the same time the hot bodies are losing their heat and distributing it to the colder ones, so that there is an unremitting tendency towards a uniform, and therefore useless, temperature throughout our whole universe: for heat is avail. able as energy (i.e., it can do work) only when it can pass from a warmer body to a colder one. The continual warming up of cooler bodies at the expense of hotter ones always means a loss, therefore, not of energy, for that is indestructible, but of available energy. To use the ordinary technical term, energy is continually "dissipated" by the processes which constitute and maintain life on the universe. This dissipation of energy can have but one ultimate result, that of absolute stagnation when the temperature has become everywhere the same.

If we carry our imagination backwards, we reach "a beginning of things," which has no intelligible antecedent; if forwards, we come to an end of things in dead stagnation. That in some way this end of things will result in a "new heavens and a new earth" is, of course, very probable, but science as yet can present no explanation of the method.

Note (1) to Art. 465*. - The spectroscopic binaries $\mathrm{Mu}_{1}$ Scorpii and Lac. 3105 (A. G. C. 10534) are of the 4th and 5th magnitudes respectively, and turn out to be very remarkable for the velocity of their motion.

In the case of $\mathrm{Mu}_{1}$ Scorpii, the relative velocity is about 300 miles a second, and since the period is $34^{\mathrm{h}} 42.5^{\mathrm{m}}$, the diameter of the relative orbit is 6,050000 miles, and the mass of the system about 17.6 times that of the sun.

The other pair has a relative velocity still higher, 385 miles a second. Taking the period as $74^{\mathrm{h}} 46^{\mathrm{m}}$, we find that the diameter of the relative orbit is $16 \frac{1}{2}$ million miles, and the united mass of the pair is nearly 77 times that of the sun.

A few other spectroscopic binaries of the same class have been detected lately, Beta Lupi being the only conspicuous one. Details as to its period and velocity are not yet at hand.

Note on Eros, Page 243. - In 1904 Professor Hinks of Cambridge (England) published the results of a most careful measurement and discussion of 295 Eros plates made at nine different observatories during the week in which the planet was nearest the earth in 1900. He obtains for the solar parallax $8^{\prime \prime} .7966 \pm 0^{\prime \prime} .0047$, - practically identical with $8^{\prime \prime} .80$, the value generally accepted.

In 1909 he announced as the final result from the measures of photographs, $8^{\prime \prime} .806 \pm 0^{\prime \prime} .0027$; from micrometer measures, $8^{\prime \prime} .802 \pm 0^{\prime \prime} .0036$.

Note (3) то Аrt. 488. - The discovery of radium and other radioactive substances has greatly changed the aspect of affairs. If these bodies are really present in the sun in any considerable quantity, it is quite possible and not even unlikely that a large part of the solar radiation may be due to them. In that case all speculations on the age and duration of the sun and the solar system are simply futile.

Additional Notes on Asteroids and Minor Planets. - When first announced each new object is designated provisionally by letters (two at present) in a regular alphabetical arrangement following the order of discovery: thus Eros was for a time known as $D Q . Z Z$ was found in 1907, but the system has been continued, always, however, prefixing the year. As soon as sufficient observations have been obtained to determine the planet's orbit the director of the Berlin Recheninstitut assigns the-
"number" by which it is to be permanently known, and the discoverer may then, if he chooses, give the planet a name.

Among the recently discovered asteroids Achilles, discovered at Heidelberg in February, 1906, is the most interesting. Its mean distance from the sun, according to the still somewhat uncertain orbit based on the first observations, far exceeds even that of Thule, being a little greater than that of Jupiter, and its period is twelve years. Moreover, at the time of its discovery it was, as pointed out by Charlier, nearly at the vertex of an equilateral triangle formed by it with Jupiter and the sun. Lagrange had long ago shown that a body so placed would, under certain conditions, move so as to preserve permanently its configuration with the other two ; and now Charlier has found that, even if the necessary conditions are not exactly met, the result will be nearly the same, - that is, the body will never get far away from the equilateral vertex, but will travel around it in a small orbit.

The planet apparently presents, in the "problem of three bodies," a special case which admits of a rigorous solution, and is therefore of great mathematical interest.

Three other planets - Hector, Patroclus, and Nestor - have also been found whose orbits are similar to that of Achilles.

## A P P E N D I X.

## CHAPTER XVI.

## MISCELLANEOUS AND SUPPLEMENTARY.

CELESTIAL LATITUDE AND LONGITUDE. - CORRECTIONS TO AN ALTITUDE MEASURED AT SEA. - CALCULATION OF THE LOCAL TIME FROM A SINGLE ALTITUDE OF THE SUN. - DETERMINATION OF AZIMUTH. - THEORY OF THE FOUCAULT PENDULUM. - MEASUREMENT OF MASS INDEPENDENT OF GRAVITY. - THE EQUATION OF TIME. - HOW THE SPECTROSCOPE MAKES THE SOLAR PROMINENCES VISIBLE. - THE EQUATION OF DOPPLER'S PRINCIPLE. - AREAL, LINEAR, AND ANGULAR VELOCITIES. KEPLER'S HARMONIC LAW AND THE LAW OF GRAVITATION. - CORRECTION TO THE HARMONIC LAW. NEWTON'S VERIFICATION OF GRAVITATION BY MEANS OF THE MOON. - TEE CONIC SECTIONS. - FORMULA FOR THE MASS OF A PLANET. - ELEMENTS OF A PLANET'S ORBIT. - DANGER FROM COMETS. - TWILIGHT.
491. Relation of Celestial Longitude and Latitude to Right Ascension and Declination (supplementary to Art. 38). - In Fig. $120 E C$ represents the ecliptic, and $E Q$ the celestial equator, the point $E$ being the vernal equinox. $K$ is the pole of the ecliptic, and $P$ that of the equator, $K P C Q$ being an are of the solstitial colure, the circle which is perpendicular both
to the equator and the ecliptic. Let $S$ be any star. Through it draw $K L$ and $P R$ which will be "secondaries" respectively to the ecliptic and equator. Then the star's longitude is $E L$


Fig. 120. or $\lambda$, and its latitude is $S L$ or $\beta$. In the same way the right ascension is $E R$ or a, and the declination $S R$ or $\delta$.

When $a$ and $\delta$ are given, together with the obliquity of the ecliptic or the angle $C E Q$, it is a simple problem of spherical trigonometry to find $\lambda$ and $\beta$. In the triangle $K P S, K P$ is equal to the obliquity of the ecliptic; $P S=$ $90^{\circ}-\delta ; K S=90^{\circ}-\beta$; the angle $K P S$ is $\left(90^{\circ}+\alpha\right)$, because $Q P R=Q R=\left(90^{\circ}-\alpha\right) ; P K S=C K L=\left(90^{\circ}-\lambda\right) . \quad K P$ is always the same $23^{\circ} 28^{\prime}$, and so when any two of the other quantities are given the triangle can be solved.
492. Corrections to an Altitude measured at Sea (supplementary to Arts. 67-69). - (1) Correction for "Semi-diameter." Since the observer measures with his sextant the altitude, not of the centre, but of the lower edge of the sun's dise (technically its lower " limb"), it is necessary to add to the measured height the sun's angular semi-diameter as given in the almanac. This never differs more than $20^{\prime \prime}$ from $16^{\prime}$.
(2) Correction for "Dip." This correction results from the fact that the marine observer measures altitudes from the visible horizon (Art. 16). The dip is the angle $H O B$ in Fig. 3, p. 11, and depends upon the observer's height above the sealevel. Its value is given in a little table contained in every work on navigation, but may be approximately calculated by the simple formula, -

$$
\text { Dip }(\text { in minutes of arc })=\sqrt{\text { Height in feet. }}
$$

(For demonstration, see " General Astronomy," Art. 81, note.)

That is, if the eye is 20 feet above the water the dip by the formula is $\sqrt{20}$ minutes, or $4^{\prime} .47$. (The table makes it $4^{\prime} 20^{\prime \prime}$; i.e., $4^{\prime} .33$.) The dip is to be subtracted from the measured altitude, because the visible horizon is always below the true horizon.
(3) Refraction. This has already been explained in Art. 50. The amount of the correction for any observed altitude is found from a table given in all works on navigation or practical astronomy. Like the dip, the refraction correction must always be subtracted from the observed altitude.
(4) Parallax. The declination of the sun is given in the almanac as it would be if seen from the centre of the earth. Before we can apply the equation of Art. 51, we must therefore reduce the actually observed altitude to what it would be if observed from that point: the needed correction is what is called the geocentric (or diurnal) parallax. It may be defined as the difference of direction between two lines drawn to the body, one from the observer, the other from the earth's centre: or what comes to the same thing, it is the angle at the body made by these two lines. In Fig. 121, where $S$ is the sun, $C$ the earth's centre, and $O$ the observer, it is the angle OSC, which is the difference between the observed or "apparent" zenith distance


Fig. 121. $Z O S$, and the "true" zenith distance $Z C S$. Since $Z C S$ is always smaller than $Z O S$, the correction for parallax always has to be added to the observed altitude. In the case of the sun, this correction never exceeds $9^{\prime \prime}$.
493. Method of Calculating the Local Time from the Sun's Altitude (supplementary to Arts. 60 and 69). - In Fig. 122 the circle $N Z M$ is the meridian, $P$ being the pole, $Z$ the zenith, and
$E Q$ the equator. $S$ is the sun, whose altitude $S H$ has been measured. This altitude (properly corrected for semi-diameter, dip, refraction, and parallax), and subtracted from $90^{\circ}$, gives $Z S$, the sun's zenith


Fig. 122.
Determination of Time by the Sun's Altitude. distance, as one side of the triangle. The second side $Z P$ is the complement of $N P$, which is the observer's latitude. Finally, since $A S$ is the declination of the sun (given in the almanac) the third side $P S$ is the complement of the declination. We, therefore, know the three sides of the spherical triangle, ZPS, and can find either of its angles. The angle at $P$ is the one we want, - the sun's hour-angle (Art. 32 ) ; i.e., the apparent time.

The trigonometrical formula ordinarily used in computing it is

$$
\sin \frac{1}{2} P=\sqrt{\frac{\sin \frac{1}{2}[z+(\phi-\delta)] \sin \frac{1}{2}[z-(\phi-\delta)]}{\cos \phi \cos \delta}},
$$

in which $z$ is the sun's zenith distance, $\delta$ its declination, and $\phi$ the latitude of the observer. We may add that the angle $P Z S$ is the sun's azimuth at the time of the observation. The third angle, $Z S P$, is called the "parallactic angle" for reasons we cannot here stop to explain.

The observation should be made not near noon, but when the sun is as near to the prime vertical (Art. 17) as possible, because when the angle at $Z$ is nearly $90^{\circ}$ any uncertainty in the side $P Z$ (which depends on the ship's latitude) will produce the least possible error in computing the hour-angle.

The apparent time, corrected for the equation of time (which is given in the almanac), gives the local mean time, and the
difference between this local time and the Greenwich time (furnished by the chronometer) is the longitude.
494. Theory of the Foucault Pendulum (supplementary to Art. 77). - The approximate theory of the experiment is very simple. A pendulum suspended so as to be equally free to swing in any plane (unlike the common clock pendulum in this freedom), if set up at the pole of the earth would appear to shift around in 24 hours. Really in this case the plane of vubration remains fixed, while the earth turns under it.

This can be easily seen by setting up upon a table a similar apparatus, consisting of a ball hung from a frame by a thread and then, while the ball is swinging, turning the table around upon its castors with as little jar as possible. The plane of the swing will remain unchanged by the motion of the table.

It is easy to see, moreover, that at the equator there will be no such tendency to shift; while in any other latitude the effect will be intermediate, and the time required for the pendulum to complete the revolution of its plane will be longer


Fig. 123.
Explanation of the Foucault Pendulum Experiment. than at the pole. The northern edge of the floor of a room (in the northern hemisphere) is nearer to the axis of the earth than is its southern edge, and therefore is carried more slowly eastward by the earth's rotation. Hence the floor must "skew" around continually, like a post-age-stamp gummed upon a whirling globe anywhere except at the globe's equator. Every line drawn on the floor, therefore, continually shifts its direction, and a free pendulum set at first to swing on any such line must apparently deviate at the same rate in the opposite direction.

The total amount of this deviation in a day is easily estimated geometrically. Suppose a parallel of latitude drawn through the place where the experiment is made, and a series of tangents drawn at


Fig. 124. - Developed Cone. points close together on this parallel. All these tangents will meet at some point $V$ (Fig. 123) which is on the earth's axis produced, and taken together they form a cone with its point at $V$. Now if we suppose this cone cut down on one side and opened up (technically, "developed"), it would give us a sector of a circle, as in Fig. 124, and the angle of the sector - the unshaded angle $A V A^{\prime}$ of Fig. 124 - would be the sum total of the angles between all the tangent lines of which the cone is composed. It is easy to prove that $A B A^{\prime}=$ $360^{\circ} \times \sin$ lat. (see "General Astronomy ").

In the northern hemisphere the plane of vibration of the Foucault pendulum moves round with the hands of a watch; in the southern, the motion is reversed.
495. Determination of Azimuth (supplementary to Art. 88). An important problem of practical astronomy, especially in geodetic work, is that of finding the true bearing or azimuth of a line on the earth's surface. The process is this:-With a carefully adjusted theodolite the observer points alternately upon the Pole-star and upon a distant signal erected for the purpose, the signal being of course such that it can be observed at night, -usually it is a small hole in a screen with a lantern behind it, looking like a star as seen through the observer's telescope. The readings of the circle of the theodolite then give directly the angle between the signal and the Pole-star at the moment of each observation, and if the Pole-star were exactly at the pole, this angle would be the azimuth of the signal. In the actual state of the case it is necessary to note accurately the sidereal time of each observa-
tion of the star, and from this its azimuth at that moment can easily be calculated, by means of the $P Z S$ triangle, Fig. 125.

In this we know the side $P Z$, the complement of the observer's latitude; also the side $P S$, which is the complement of the star's declination; and finally, we know the hour-angle $S P Z$, which is simply the difference between the observed sidereal time at the moment of observation and the Pole-star's right ascension. Hence we can easily compute the angle at $Z$, which is the star's azimuth; and when the azimuth of the star is known, that of the nightsignal follows at once. The results are
 most accurate when the Pole-star hap- Fig. 125.-Determination of Azimuth. pens to be near one of its "elongations," as at $S^{\prime}$ or $S^{\prime \prime}$. Then errors of even several seconds in noting the time are practically harmless.

The azimuth of the night-signal being determined, the observer measures the next day the horizontal angle between it and the object whose azimuth is required.
496. Measurement of Mass Independent of Gravity (supplementary to Art. 98). - It is quite possible to measure masses without weighing. In Fig. $126 B$ is a receptacle carried at the end of a horizontal arm $A$, which is itself attached to an axis $M N$, exactly vertical and free to turn on pivots at top and bottom. A spiral spring $S$, like the hair spring of a watch, is connected with this axis so that if $A$ is disturbed it will oscillate back and forth at a rate which depends upon the stiffness of the spring and the total inertia of the apparatus. If we put into $B$ one standard "pound " (of mass), it will vibrate a certain number of times a minute; if two pounds, it will vibrate more slowly; if three, still more slowly; and so on : and this time of vibration can be determined and
tabulated. To determine now the mass of a body $X$, we have only to put it into the receptacle $B$, set the apparatus vibrat. ing, and count the number of swings in a minute. Referring to our table, we find what number of "pounds" in $B$ would


Fig. 126. have given the same rate of vibration. We know then that the "inertia" of $X$ is the same as that of this number of "pounds," and therefore its mass is the same.

This determination is independent of all considerations of weight : the apparatus would give the same results on the surface of the moon, or on that. of Jupiter, as on the earth. It is obvious, however, that an instrument of this sort could not compete in accuracy or convenience with a well-made balance, because of the friction of the pivots, the resistance of the air, etc. We introduce it simply to assist in separating in the pupil's mind the idea of mass from that of weight.

## discussion of the equation of time.

(Supplementary to Art. 128.)
497. Effect of the Eccentricity of the Orbit. - Near perihelion, which occurs about Dec. 31st, the sun's eastward motion on the ecliptic is most rapid. At this time, accordingly, the apparent solar days exceed the sidereal by more than the average amount, making the sun-dial days longer than the mean. The sun-dial will therefore lose time at this season, and will continue to do so until the motion of the sun falls to its average value, as it will at the end of about three months. Then the sun-dial will gain until aphelion; and at that time (if the clock and the sun-dial were started together at perihelion)
they will once more agree. During the remaining half of the year, the action will be reversed; i.e., for the first three months after aphelion the sun-dial will gain, and in the next three lose what it had gained. Thus, twice a year, so far as the eccentricity of the earth's orbit is concerned, the clock and the sun-dial will agree, - at the times of perihelion and aphelion, - while half-way between they will differ by about eight minutes. The equation of time (so far as due to this cause only) is about +8 minutes in the spring, and -8 in the autumn.

## 498. Effect of the Inclination of the Ecliptic to the Equator.

- Even if the sun's motion in longitude, i.e., along the ecliptic, were uniform, its motion in right ascension would be variable. If the true and fictitious suns started together at the vernal equinox, one moving uniformly in the ecliptic and the other in the equator, they would indeed be together (i.e., have the same right ascensions) at the two solstices and at the other equinox, because it is just $180^{\circ}$ from equinox to equinox, and the solstices are exactly half-way between them; but at any intermediate points their right ascensions would differ. This is easily seen by taking a celestial globe and marking on the ecliptic the point $m$, Fig. 127, ${ }^{1}$ half-way be-


Fig. 127. - Effect of Obliquity of Ecliptic in producing Equation of Time. tween the vernal equinox and the solstice, and also marking a point $n$ on the equator, $45^{\circ}$ from the equinox. It will be seen at once that the former point is west of $n$; so that $m$ in the apparent diurnal revolution of the sky will come first to the meridian. In other words, when the sun is half-way between the vernal equinox and the summer solstice, the sun-dial, so far as the obliquity of the ecliptic is concerned, is faster than the clock, and this component of the equation of time is minus. The difference, measured by the arc of the equator $m^{\prime} n$, amounts to nearly 10 minutes. Of course the same thing holds, mutatis mutandis, for the other quadrants.

[^59]If the ecliptic be divided into equal portions from $E$ to $C$, and hour-circles be drawn from $P$ through the points of division, it will at once be seen that near $E$ the portions of the ecliptic are longer than the corresponding portions of the equator, the arc of the ecliptic being the hypothenuse of a right-angled triangle which has the arc of the equator for its base. On the other hand, near the solstice $C$, the arc of the ecliptic is shorter than the corresponding arc of the equator, on account of the divergence of the hour-circles as they recede from the pole.
499. Combination of the Effects of the Two Causes. - We can represent the two components of the equation of time and the result of their combination by a graphical construction - as follows (Fig. 128) : -

The central horizontal line is a scale of dates one year long, the letters denoting the beginning of each month. The dotted curve


Fig. 128. - The Equation of Time.
shows that component of the equation of time which is due to the eccentricity of the earth's orbit (Art. 497). Starting at perihelion ou Dec. 31st, this component is zero, rising to a value of about +8 minutes on April 2d, falling to zero on June 30th, and reaching the second maximum of -8 minutes about October 1st. In the same way the broken-line curve denotes the effect of the obliquity of the ecliptic (Art. 498), which, alone considered, would produce an equation of time having four maxima of approximately ten minutes each,
on about the 6th of February, May, August, and November, and reducing to zero at the equinoxes and solstices. The full-lined curve represents their combined effect, and is constructed by making its "ordinate" at each point equal to the sum (algebraic) of the ordinates of the two other curves.
500. The Equation of Doppler's Principle (supplementary to Art. 200). If $V$ is the velocity of light ( 186,330 miles a second), and $r$ and $s$ are the velocities with which the observer and luminous object respectively are receding from each other ; then, if $L$ be the normal wave-length of a ray, and $L_{1}$ its observed wave-length as affected by the motions, Doppler's equation is $L_{1}=L\left(\frac{V+s}{V-r}\right)$, which holds good for all values of $r$ and $s$. When they are small compared with $V$, as is always practically the case, the equation becomes, very approximately, $L_{1}=L\left(\frac{V+(r+s)}{V}\right)$, or $\frac{L_{1}-L}{L}=\frac{r+s}{V}$. If the bodies are approaching, $r$ and $s$ become negative; i.e. $L_{1}$ is less than L. $A$ ray of wave-length L will therefore be found in the observed spectrum where a ray of wave-length $\mathrm{L}_{1}$ would fall were it not for the motion; in other words, the place of the ray will be shifted in the spectrum.

The rate at which the distance between the observer and the body is increasing is obviously $(r+s)$, for which we may put the single quantity $v$, since the observations do not decide what part of the whole change of distance is due to the motion of the observer. We then have, $v=V\left(\frac{L_{1}-L}{L}\right)$, which is the formula generally given.

In this way, with powerful spectroscopes, motions of approach or recession along the line of sight can be detected if they amount to more than one or two miles a second, but the exact measurement is very delicate and difficult, and is embarrassed by the recently discovered fact that the wave-lengths of the rays from a luminous gas are slightly increased by pressure.
501. How the Spectroscope enables us to see the Chromosphere and Prominences without an Eclipse (supplementary to Art. 203). - The reason why we cannot see the prominences and chromosphere at any time by simply screening off the sun's disc, is the brilliant illumination of our own atmosphere.

When we point the "telespectroscope" (Art. 194) so that the sun's image falls as shown in Fig. 129, with its limb just tangent to the edge of the slit, then if there is a prominence at that point we shall get two overlying spectra; one, the spectrum of the air light, the other, that of the prominence itself. The latter


Fig. 129. - Spectroscope Slit adjusted for Observation of the Prominences. is a spectruin composed of bright lines, or, if the slit be opened a little, of bright images of whatever part of the prominences may fall into the jaws of the slit; and the brightness of these lines or images is independent of the dispersive power of the spectroscope, since increase of dispersion merely sets the images farther apart without making them fainter. The spectrum of the aerial illumination, on the other hand, is simply that of sunlight, - a continuous spectrum showing the usual Fraunhofer lines, and this spectrum is made faint by its extension. Moreover, it presents dark lines or spaces just at the very places in the spectrum where the bright images of the prominences fall, so that they become easily visible.

A grating spectroscope of ordinary power, attached to a telescope of three or four inches aperture, gives a very satisfactory view of these beautiful and interesting objects. The red image, which corresponds to the $C$ line of hydrogen, is by far the best for such observations. When the instrument is properly adjusted, the slit open a little, and the image of the sun's limb brought exactly to its edge, the observer at the eye-piece of the spectroscope


Fig. 130. - The Chromosphere and Prominences seen in the Spectroscope. will see things about as we have attempted to represent them in Fig. 130, as if he were looking at the clouds in an evening sky from across the room through a slightly opened window blind.
502. Areal, Linear, and Angular Velocities (supplementary to Art. 249). - The number of units of area (acres or square miles) included in the sector of the orbit described in a unit of time is called the body's areal velocity. The linear velocity is simply the "speed" with which it is moving - the number of feet or miles per second - and is called "linear" because it is measured in "linear units." The angular velocity is the number of angular units (degrees, or "radians") swept over by the radius vector in a unit of time.

In Fig. 131 the area of the sector $A S B$ is the areal velocity; the length of the line $A B$ is the linear velocity; and the angle $A S B$ is the angular velocity ( $A$ and $B$ are supposed to be occupied by the body in two successive seconds).

Since the area described in a unit of time is the same all through the orbit, it can easily be proved, first, that the linear velocity (usually denoted by $V$ ) is always inversely proportional to $S b$, the


Fig. 131.
Linear and Angular Velocities. perpendicular drawn from $S$ upon $A B$, produced if necessary : secondly, that the angular velocity (ordinarily denoted by $\omega$ ) at any point of the orbit is inversely proportional to the square of $A S$, the radius vector.

In every case of motion under central force we may say, therefore :
I. The areal velocity (a cres per second) is constant.
II. The linear velocity (miles per second) varies inversely as the distance from the centre of force to the body's line of motion at the moment.
III. The angular velocity (degrees per second) varies inversely as the square of the radius vector.

These three statements are not independent laws, but only geometrical equivalents for each other. They hold good regardless of the nature of the force, requiring only that when it acts it act directly towards or from the centre, so as to be directed always along the line
of the radius vector. It makes no difference whether the force varies with the square or the logarithm of the distance; whether it is increasing or decreasing, attractive or repulsive, continuous or intermittent, provided only it be always "central."
503. Proof of the Law of Inverse Squares, from Kepler's Harmonic Law (supplementary to Art. 253). - For circular orbits the proof is very simple. From equation (b), Art. 250, we have for the first of two planets,

$$
f_{1}=4 \pi^{2} \frac{r_{1}}{t_{1}^{2}}
$$

in which $f_{1}$ is the central force (measured as an acceleration), and $r_{1}$ and $t_{1}$ are respectively the planet's distance from the sun and its periodic time.

For a second planet,

$$
f_{2}=4 \pi^{2} \frac{r_{2}}{t_{2}^{2}}
$$

Dividing the first equation by the second, we get

$$
\frac{f_{1}}{f_{2}}=\frac{r_{1}}{r_{2}} \times\left(\frac{t_{2}^{2}}{t_{1}^{2}}\right)
$$

But by Kepler's third law

$$
t_{1}^{2}: t_{2}^{2}=r_{1}^{3}: r_{2}^{3} ; \text { whence, } \frac{t_{2}^{2}}{t_{1}^{2}}=\frac{r_{2}^{3}}{r_{1}^{3}} ;
$$

substituting this value of $\frac{t_{2}^{2}}{t_{1}^{2}}$ in the preceding equation, we have

$$
\frac{f_{1}}{f_{2}}=\frac{r_{1}}{r_{2}} \times \frac{r_{2}^{3}}{r_{1}^{3}}=\frac{r_{2}^{2}}{r_{1}^{2}}
$$

i.e., $f_{1}: f_{2}=r_{2}^{2}: r_{1}^{2}$, - which is the law of inverse squares.

In the case of elliptical orbits the proposition is equally true if, for $r$, we substitute $a$, the semi-major axis of the orbit. but the demonstration is more complicated.
504. Correction to Kepler's Third Law (supplementary to Art. 253). - The Harmonic Law as it stands in Art. 251 is not strictly true: it would be so if the planets were mere particles, infinitesimal as compared with the sun; but this is not the case. The accurate statement is

$$
t_{1}^{2}\left(M+m_{1}\right): t_{2}^{2}\left(M+m_{2}\right)=r_{1}^{3}: r_{2}^{3},
$$

in which $M$ is the sun's mass, and $m_{1}$ and $m_{2}$ are the masses of the two planets compared.
505. Newton's Verification of the Idea of Gravitation by means of the Moon (supplementary to Art. 2505). - Regarding the moon's orbit as a circle, we can easily compute how much she falls toward the earth in a second.

In Fig. 132 let $A E$ be the distance the moon travels in a second, then $D E$, or its equal (sensibly) $A B$, is the virtual "fall" of the moon towards the earth in one second; i.e., the amount by which the earth's attraction deflects the moon away from the rectilinear path which it would otherwise pursue. By Geometry, since the triangle $A E F$ (being inscribed in a semi-circte) is right-angled


Fig. 132.
Verification of the Hypothesis of Gravitation by Means of the Motion of the Moon. at $E$, we have

$$
A B: A E:: A E: A F, \text { or } A B=\frac{(A E)^{2}}{2 R}
$$

$R$ being the radius of the orbit. Now $A E$ is found by dividing the circumference of the circle $2 \pi R$ by $T$, the number of seconds in a sidereal month;
whence $(A E)^{2}=\frac{4 \pi^{2} R^{2}}{T^{2}}$, and $A B$, which is found by dividing this by $2 R$, comes out $=\frac{2 \pi^{2} R}{T^{2}}$. If for $R$ we put its equivalent $60 \times r$ ( $r$ being the radius of the earth), we have, finally,

$$
A B=\frac{120 \pi^{2} r}{T^{2}}
$$

Working out this formula with the now known values of $r$ and $T$, we get $A B=0.0534$ inches - which is quite as near to $\frac{193}{3600}$ as could possibly be expected; i.e., the moon does fall towards the earth in a second just as much as a stone at that distance from the earth's centre ought to, if the hypothesis of gravitation is correct.
506. The Conic Sections (supplementary to Art. 256). - (a) If a cone of any angle (Fig. 133) be cut by a plane which makes with its axis, $V C$, an angle greater than $B V C$, the semiangle of the cone, the section is an ellipse (as EF). In this case, the plane of the section cuts completely across the cone. The ellipse formed will vary in shape and size according to the position of the plane, - the circle being simply a special case when the cutting plane is perpendicular to the axis.
(b) When the cutting plane makes with the axis an angle less than $B V C$ (the semi-angle of the cone), it plunges continually deeper into the cone and never comes out on the other side, the cone being supposed to be indefinitely prolonged. The section in this case is a hyperbola, $G H K$. If the plane of the section be produced upward, however, it encounters "the cone produced," cutting out from it a second hyperbola, $G^{\prime} H^{\prime} K^{\prime}$, exactly like the original one, but turned in the opposite direction. The axis of the hyperbola is always reckoned as negative, lying outside of the curve itself (in the figure it is the line $\boldsymbol{H} \boldsymbol{H}^{\prime}$ ). The centre of the hyperbola is the middle point of this axis, a point also outside the curve.


Fig. 133. - The Conics.
(c) When the angle made by the cutting plane with the axis is exactly equal to the cone's semi-angle, the plane will be parallel to a plane which is tangent to the conical surface, and we then get the special case of the parabola, which is, so to speak, the boundary or partition between the infinite variety of ellipses and hyperbolas which can be cut from a given cone. All parabolas are of the same shape, just as all circles are, differing only in size.

This fact is by no means self-evident, and we cannot stop to prove it; but it is true. It does not mean, of course, that an are of a parabola has the same shape as an arc of another parabola taken from a different part of the curve, but that the complete parabolas, cut from infinitely extended cones, are similar, whatever the angle of the cone.
507. Elements of a Planet's Orbit (supplementary to Art. 296). - In order to describe a planet's orbit intelligibly, and to supply the data required for the prediction of its position at any time, it is necessary to know certain quantities called the "elements" of its orbit. Those ordinarily employed are seven in number, as follows :-

1. The semi-major axis . . . a.
2. The eccentricity . . . e.
3. The inclination of the orbit to the plane of the ecliptic . . . i.
4. The longitude of the ascending node . . . $\Omega$.
5. The longitude of perihelion . . . p.
6. The epoch . . . E.
7. The sidereal period (or else the mean daily motion) . . . T, or else $\mu$.

The first five of these describe the orbit itself ; the two last furnish the means of finding the planet's place in the orbit. The semi-major axis determines the orbit's size; the eccentricity defines its shape; the inclination and longitude of the node, taken together, determine the position of the plane of the orbit; and, finally, the longitude of perihelion determines how the major axis (or line of apsides) of the orbit lies upon this plane.

To determine the place of the planet in the orbit we need two more data. One is the starting-point or "epoch," which is simply the longitude of the planet at some given date (usually Jan. 1st, 1850). The other is commonly the time of revolution; though, instead of it, we may use the mean daily motion.

If Kepler's Harmonic Law were strictly true, the period could at once be found from the major axis by the proportion
$(1 \text { year })^{2}: T^{2}::(\text { earth's distance from the sun })^{3}: a^{3}$
(Art. 251), which gives $T$ (in years) $=a^{\frac{2}{2}}, a$ being expressed in astronomical units. But as the law is only approximate (Art. 504), $a$ and $T$ must be treated as independent quantities where precision is needed.

Having these seven elements of a planet's orbit, it would be possible, were it not for perturbations, to compute exactly the precise place of the planet for any date whatever, either in the past or future.

507*. (Supplementary to Art. 259.) The Critical or Parabolic Velocity, and its Relation to the Major Axis of the Orbit. - If we let $U$ represent the "critical velocity" due to the attraction between two bodies, $M$ and $m$, at the distance $r ; V$; the velocity of $m$ at that point, relative to $M$ considered as fixed; and $a$, the semimajor axis of the conic described by $m$ relative to $M$; then it can be proved that $a=\frac{r}{2}\left(\frac{U^{2}}{U^{2}-V^{2}}\right)(1),-$ a relation of great importance.

If $V=U$, the denominator becomes zero, $a$ becomes infinite, and the corresponding conic is a parabola. For this reason $U$ is generally called the "parabolic velocity," corresponding to the distance $r$.
If $V$ exceeds $U$, the denominator becomes negative, making $a$ also negative, and indicating au hyperbolic orbit.

If, however, $V$ is less than $U$, the denominator will be positive and finite ; $a$ will be so also, and the orbit will be an ellipse, in which $m$ will revolve around $M$ with a regular period. Moreover, since only $r, V$, and $U$ appear in the equation which gives the value of $a$, it is clear that $a$, and therefore the period also according to Kepler's third law, are independent of everything else, -as, for instance, of the direction in which $m$ is moving when its distance from $M$ is equal to $r$.
Finally, if the orbit is circular, $a$ must equal $r$; which requires that $V^{2}=\frac{1}{2} U^{2}$, and $V=U \sqrt{\frac{1}{2}}=0.707 \times U$. In other words, the velocity of a planet moving in a circular orbit around the sun at a distance $r$, is .707 of the "parabolic velocity" due to the sun's attraction at that distance.

This "parabolic velocity" at the distance $r$ is sometimes also called the "velocity from infinity," because it is that which would be acquired by a particle $m$ in falling towards $M$ from an infinite distance until it reaches a distance equal to $r$; supposing, of course, that $M$ is fixed and that $m$ starts from rest and is not acted upon during its fall by any force except the mutual attraction between itself and $M$. It might be supposed that this "velocity from infinity" would itself be infinite, but it is not. Its value is given by the equation $U=k \sqrt{\frac{M}{r}}$
(2), in which $k$ is a constant factor depending upon the units in which velocity, distance, and time are measured. In the solar system, if we take the mass of the sun as the unit of mass and the radius of the earth's orbit as the unit of distance, then
for the velocity acquired by a particle falling freely towards the sun from an infinite distance to the distance $r$, we have

$$
U(\text { miles per second })=26.156\left(\frac{1}{\sqrt{r}}\right)
$$

If $r$ is unity, $U=26.156$ miles per second, so that if the earth's velocity were increased to this, it would fly off in a parabola. At a distance one-fourth that of the earth from the sun, $U=52.3$, and at the surface of the sun (where $r=\frac{1}{214.4}$ ) it is 383 ; while at Neptune ( $r=30.05$ ), $U=4.77$ miles a second.

Formula (2) enables us also to compute the parabolic velocity at the surface of a planet due to its own attraction. Thus for the earth we put $M=\frac{1}{331100}$, and $r=\frac{1}{23} \frac{1}{440}$ (Art. 178). $U$ then comes out 6.9 miles per second; and since this is the velocity which a body would attain in falling under her attraction from an infinite distance to her surface, it follows that a body projected from the earth with this or any higher velocity would never return, unless brought back by other forces than her attraction. At the surface of the moon the "parabolic velocity" due to the moon's attraction is only 1.48 miles, or less than 8000 feet ; and this probably explains (Art. 161) why she has lost her atmosphere.
508. Formula for the Mass of a Planet (supplementary to Art. 309). - The derivation of the formula for a planet's mass is very simple in the case of circular orbits. From the law of gravitation we have the accelerating force with which the planet and satellite attract each other (Art. 102), given by the equation,

$$
f=\frac{P_{1}+s_{1}}{R_{1}^{2}},
$$

in which $P_{1}$ and $s_{1}$ are the masses of the planet and satellite, and $R_{1}$ the radius of its orbit. Also from equation $b$ (Art. 250), for a body moving in a circle,

$$
f=\frac{4 \pi^{2} R_{1}}{t_{1}^{2}}
$$

Equating these values of $f$, we get

$$
\frac{4 \pi^{2} R_{1}}{t^{2}}=\frac{\left(P_{1}+s_{1}\right)}{R_{1}{ }^{2}} ; \text { whence }\left(P_{1}+s_{1}\right)=\frac{4 \pi^{2} R_{1}^{3}}{t_{1}^{2}} .
$$

For a second planet and satellite we should get similarly

$$
\left(P_{2}+s_{2}\right)=\frac{4 \pi^{2} \times R_{2}{ }^{3}}{t_{1}{ }^{2}} ;
$$

whence we have

$$
\left(P_{1}+s_{1}\right):\left(P_{2}+s_{2}\right):: \frac{R_{1}{ }^{3}}{t_{1}{ }^{2}}: \frac{R_{2}{ }^{3}}{t_{2}{ }^{2}}{ }^{2}
$$

This is equally true of elliptical orbits, provided we put $a_{1}$ and $a_{2}$ for $R_{1}$ and $R_{2}$; but the proof of that statement is beyond our reach here.
509. Danger from Comets. - It has been supposed that a comet might damage the earth in either of two ways, - by actually striking us, or by falling into the sun and so causing a sudden and violent increase of solar radiation.

There is no question that a comet may strike the earth, and it is very probable that one will do so at some time. Biela's Comet is not the only one whose orbit passes ours at a distance less than the comet's semi-diameter. Such encounters will be rare, however, - Babinet says once in about 15,000000 years in the long run.

As to the consequences of a comet's striking the earth, everything, so far as the earth is concerned (it will certainly be bad for the comet), depends upon the size of the "particles" of which it is composed. If they weigh tons, the bombardment will be serious; if only pounds, they will perhaps do some mischief. If only ounces or grains, they would burn in the air like shooting stars, and we should simply have a beautiful meteoric shower, - and this is decidedly the most probable, as well as the most comfortable, hypothesis.

As regards the fall of a comet into the sun, it is practically certain that it will do us no harm whatever. The total amount of energy due to the striking of the sun by a comet having a mass $\frac{10}{100000}$ that of the earth would be only about as much as the sun radiates in eight or nine hours; and the transformation of this energy into heat would take place almost entirely beneath, or at least within, the photosphere, and would appear not so much in a rise of temperature and increase of radiation as in an expansion of the sun's volume. Probably if a comet were to strike the sun, we should know nothing of it, unless some observer happened to be watching the sun at the moment. He might see a sudden increase of brilliance on a certain portion of the surface, lasting for a few minutes, and very likely our magnetometers would show a disturbance.

509*. Twilight. - This is caused by the reflection of sunlight from the upper portions of the earth's atmosphere. After the sun has set, its rays, passing over the observer's head, still continue to shine through the air above him, and twilight continues as long as any portion of this illuminated air remains visible from where he stands. It is considered to end when stars of the sixth magnitude become visible near the zenith, which does not occur until the sun is about $18^{\circ}$ below the horizon : but this is not strictly the same for all places.

The length of time required by the sun after setting to reach this depth of $18^{\circ}$ below the horizon, varies with the season and with the observer's latitude. In latitude $40^{\circ}$ it is about 90 minutes on March 1 st and Oct. 12th; but more than two hours at the summer solstice. In latitudes above $50^{\circ}$, when the days are longest, twilight never quite disappears, even at midnight. On the mountains of Peru, on the other hand, it is said never to last more than half an hour.

## CHAPTER XVII.

## ME'THODS OF DETERMINING THE PARALLAX AND DISTANCE OF THE SUN AND STARS.

Importance and difficulty of the Problem. - hisTORICAL. - CLASSIFICATION OF METHODS. - GEOMETRICAL METHODS. - OPPOSITIONS OF MARS AND TRANSITS OF VENUS. - GRAVITATIONAL METHODS. - DETERMI- . NATION OF STELLAR PARALLAX.
510. In some respects the problem of the sun's distance is the most fundamental of all that are encountered by the astronomer. It is true that many important astronomical facts can be ascertained before it is solved: for instance, by methods which have been given in Arts. 299 and 300, we can determine the relative distances of the planets and form a map of the solar system, correct in all its proportions, although the unit of measurement is still undetermined, - a map without any scale of miles. But to give the map its use and meaning, we must ascertain the scale, and until we do this we can have no true conception of the real dimensions, masses, and distances of the heavenly bodies. Any error in the assumed value of the astronomical unit propagates itself proportionally through the whole system, not only solar but stellar.

The difficulty of the problem is commensurate with its importance. It is no easy matter, confined as we are to our little earth, to reach out into space and stretch a tape-line to the sun. In Arts. 127 and 355 we have already given the two methods of determining the sun's distance, which depend on
our experimental knowledge of the velocity of light. They are satisfactory methods and sufficient for the purposes of the text. But methods of this kind have become available only since 1849. Previously astronomers were confined entirely to purely astronomical methods, depending either upon geometrical measurement of the distance of one of the nearer planets when favorably situated, or else upon certain gravitational relations which connect the distance of the sun with some of the irregular motions of the moon, or with the earth's power of disturbing her neighboring planets, Venus and Mars.
511. Historical. - Until nearly 1700 no even approximately accurate knowledge of the sun's distance had been obtained. Up to the time of Tycho it was assumed (on the authority of Ptolemy, who rested on the authority of Hipparchus, who in his turn depended upon an erroneous observation of Aristarchus) that the sun's horizontal parallax is $3^{\prime}$, a value more than 20 times too great. Kepler, from Tycho's observations of Mars, satisfied himself that the parallax certainly could not exceed $1^{\prime}$, and was probably much smaller; and at last, about 1670 , Cassini, also by means of observations of Mars made in France and South America for the purpose, showed that the solar parallax could not exceed $10^{\prime \prime}$. He set it at $9^{\prime \prime} .5$, -the first reasonable approximation to the true value, though still about eight per cent too large.
The transits of Venus in 1761 and 1769 furnished data that proved it to lie between $8^{\prime \prime}$ and $9^{\prime \prime}$, and the discussion of all the available observations, published by Encke about 1824, gave as a result for the parallax, $8^{\prime \prime} .5776$, corresponding to a distance of about 95,000000 miles. The accuracy of this determination was, however, by no means commensurate with the length of the decimal, and its error began to be obvious about 1860; since then it has been practically settled that the true value of the sun's parallax lies somewhere between $8^{\prime \prime} .75$ and $8^{\prime \prime} .85$, its distance being between $92,400,000$ and $93,500,000$
miles. Indeed it is now certain that the figure $8^{\prime \prime} .8$, adopted in the text, must be extremely near the truth.
512. The methods available for determining the distance of the sun may be classified under three heads,-geometrical, gravitational, and physical. The physical methods (by means of the velocity of light) have been already discussed (Arts. 127 and 355 ). We proceed to present briefly the principal methods that belong to the two other classes.

## GEOMETRICAL METHODS.

513. The direct geometrical method of getting the sun's parallax (by observing the sun itself at stations widely separated on the earth, in the same way that the parallax of the moon is measured - Art. 149) is practically worthless, the inevitable errors of observation being too large a fraction of the quantity sought. We may add that the sun, on account of the effect of its heat upon an instrument, is a very intractable subject for observation.

Since, however, we know at any time the distance of the planets in ästronomical units, our end will be just as perfectly obtained by measuring the distance (in miles) of one of them.
514. Observations of Mars. - In the case of Mars at the time of its nearest approach to the earth this "can be done satisfactorily. There are two ways of proceeding :-

1. By observations made from two or more stations widely separated in latitude.
2. By observations from a single station near the equator.

In the first case the observations may be ( $\alpha$ ) meridian circle observations of the planet's zenith distance, precisely such as are used for getting the moon's parallax in Art. 149, just cited ; or they may be (b) micrometer measurements of the difference of declination between the planet and the surrounding stars.

Since, however, different observers and different instruments are concerned in the observations, the results of both these processes seem to be less trustworthy than those obtained by the second method.


Fig. 134.
515. In the second case, a single observer, by measuring with a heliometer (Art. 543) the apparent distance between the planet and small stars nearly east and west of it, can determine its parallax, and hence its distance, with great accuracy. Fig. 134 exhibits the principle involved. When the observer is at $\boldsymbol{A}$ (a point on or near the earth's equator), the planet $M$ is just rising to him, and he sees it at $a$, a point in the sky which is east of $c$, the point where it would be seen from the centre of the earth, the angle CMA being its horizontal parallax. On the other hand, 12 hours later, when the rotation of the earth has taken the observer to $B$ and the planet is setting, its parallax will displace it to the west of $c$, and by the same amount. When the planet is rising, its parallax increases its right ascension; when setting, it diminishes

Suppose for the moment that the orbital motions of Mars and the earth are suspended, the planet being at opposition and as near as possible to us. If, then, when the planet is rising we measure carefully its distance $M_{\mathrm{e}} S$, Fig. 135, from a star which is west of it, and 12 hours later measure it again, the planet being now at $\boldsymbol{M}_{\mathrm{w}}$, the difference of the two measures will give the distance $M_{\mathrm{e}} M_{\mathrm{w}}$, which is twice the parallax of the planet. The earth's rotation will have performed for the observer the function of a long journey, by transporting him in 12 hours, free of expense and trouble, from one station to another 8000 miles away.

In practice the observations are not limited to the moment when the planet is exactly on the horizon, and measures are made not from one star alone, but from a number. Moreover, the orbital motions, both of the earth and the planet, during the interval between the observations, must be taken into account; but this presents no considerable difficulty.

On the whole, this method of determining the astronomical unit is about the most accurate of all the geometrical methods. Though long ago suggested by Flamsteed, it was first thoroughly carried out by Gillat Ascension Island, in 1877. His result fixed the solar parallax at $8^{\prime \prime} .783 \pm$ $0^{\prime \prime} .015$. The size of the planet's disc, however, interferes somewhat with the precision of the necessary measurements, and itis * found that even more accurate results can be obtained from some of the asteroids which at opposition come nearest to us. The observations of Iris, Sappho, and Victoria, made in 1889-91,


Fig. 135. - Micrometric Comparison of Mars with Neighboring Stars. by Gill at the Cape of Good Hope, in concert with other observers in Europe and America, give $8^{\prime \prime} .802 \pm 0^{\prime \prime} .005$. The observations of Eros for this purpose, made in 1900-1901, have already been mentioned on page 243.

The method cannot be used with Venus because when nearest she is visible only in the daytime.
516. Transits of Venus. - Now and then, however, Venus passes between us and the sun and "transits" the disc, as explained in Art. 326. Her distance from the earth is then only about 26,000000 miles, and her horizontal parallax is between three and four times as great as that of the sun. If viewed
by two observers at different stations on the earth, she will therefore be seen at different points on the sun's disc, and her apparent displacement on the disc will be the difference between her own parallactic displacement (corresponding to the difference in distance between the two stations) and that of the sun itself. This relative displacement is more than $2 \frac{1}{2}$ times the parallax of the sun, or, more exactly, $\frac{723}{277}$ as great.

In other words, if two observers


Fig. 136.
Contacts in a Transit of Venus. are situated so far apart that the distance between them would subtend an angle of $8^{\prime \prime}$, as seen from the sun, then the apparent displacement of Venus on the sun's disc, as seen from their two stations, would be 2.61 times $8^{\prime \prime}$, or nearly $21^{\prime \prime}$, a quantity quite measurable.

To determine the solar parallax then, by means of a transit of Venus, we must find the means of somehow measuring the angular distance between the two positions which Venus occupies on the sun's disc, as seen simultaneously from two widely distant stations of known latitude and longitude. The methods earliest proposed and executed depend upon observations of the times of contact between the planet and the edge of the sun's disc. There are four of these contacts, as indicated in Fig. 136, the first and fourth being "external," the second and third "internal."
517. Halley's Method, or the Method of Durations. - The method suggested by Halley, who first noticed, in 1679, the peculiar advantages that would be presented by a transit of Venus as a means of finding the sun's distance, consists in observing the duration of the transit at two stations differing greatly in latitude, and so chosen that the difference of dura-
tions will be as large as possible. It is not necessary to know the longitude of the stations very accurately, since absolute time does not come into the question. All that is necessary is to know the latitudes accurately (which were easily obtained even in Halley's time), and the clock-rates for the four or five hours between the beginning and end of the transit. Halley expected to depend mainly on the second and third contacts, which he supposed could be observed within a single second. If so, the sun's parallax could easily be determined within $\frac{1}{500}$ of its true value.

Having the durations of the transit at the two stations, and knowing the angular motion of Venus in an hour, we have at once very accurately the length of the two chords $a c$ and $d f$ (Fig. 137) described by Venus upon the sun, expressed in seconds of arc - more accurately than they could be measured by any micrometer. We also


Fig. 137. - Halley's Method.
know the sun's semi-diameter in seconds, and hence in the triangles $S a b$ and $S d e$, we can compute the length (in seconds still) of $S b$ and Se. Their difference, $b e$, is the displacement due to the distance between the stations on the earth. The virtual base line is of course not the direct distance between $B$ and $E$, because that line is not perpendicular to the line of sight from the earth to Venus, but the true value to be used is easily found by Trigonometry. Calling this true base line $m$, and putting $p^{\prime \prime}$ for the sun's horizontal parallax, we have

$$
p^{\prime \prime}=[b e]^{\prime \prime} \times\left(\frac{277}{723}\right) \times\left(\frac{r}{m}\right),
$$

$r$ being the radius of the earth. The rotation of the earth of course comes in to shift the places of $E$ and $B$ during the transit, but the shift is easily allowed for.

In order that the method may be practically successful, it is also necessary that the transit tracks should lie near the edge of the sun, for obvious reasons. If they crossed near the centre of the disc, it would be impossible to compute the distances, $S b$ and $S e$, with much accuracy.


Fig. 138. - Atmosphere of Venus as seen during a Transit. (Vogel, 1882.)

Halley died before the transits of 176169, but his method was thoroughly tried, and it was found that the observations of contact, instead of being liable to an error of a single second, are uncertain to fully 10 times that amount. This is due to the fact that at the time of internal contact the planet does not present the appearance of a round, black dise neatly touching the edge of the sun, but is slightly distorted by optical imperfections of the telescope and of the observer's eye; and, moreover, it is surrounded by an undefined, luminous ring, caused by the refraction of sunlight through its atmosphere, - a very beautiful phenomenon, but quite incompatible with accurate observation of "contacts." Fig. 138 illustrates the appearances seen by Vogel during the transit of 1882.
518. De l'Isle's Method. - Halley's method requires stations in high latitudes, uncomfortable and hard to reach, and so chosen that both the beginning and end shall be visible. More-
over, if the weather prevents the end from being visible after the beginning has been observed, the method fails.

De l'Isle's method, on the other hand, employs pairs of stations near the equator, and does not require that the observer should see both the beginning and end of the transit. Observations of either phase can be utilized, which is a great advantage. But it does require that the longitudes of the stations should be known with extreme precision, since it consists essentially in observing the absolute time of contact (i.e., Greenwich or Paris time) at both stations.

Suppose that an equatorial observer, E, Fig. 139, on one side of the earth notes the moment of internal contact in Greenwich time, the planet being then at $V_{1}$; when $W$ notes the contact (also in Greenwich time), the planet will be at $V_{2}$, and the angle $V_{1} D V_{2}$ is the earth's apparent diameter as seen from the sun; i.e., twice the sun's horizontal parallax. Now


Fig. 139. - De l'Isle's Method.
the angle at $D$ is at once determined by the time occupied by Venus in moving from $V_{1}$ to $V_{2}$. It is simply just the same fraction of $360^{\circ}$, that the time is of 584 days, the planet's synodic period. If, for example, the time were 12 minutes, we should find the angle at $D$ to be about $18^{\prime \prime}$.
519. Heliometric and Photographic Observations. - Instead of observing merely the four contacts and leaving the rest of the transit unutilized, we may either keep up a continued series of measurements of the planet's position upon the sun's disc with a heliometer, or we may take a series of photographs to be measured up at leisure. Such heliometer meas-
ures or photographs, taken in connection with the recorded Greenwich times at which they were made, furnish the means of determining just where the planet appeared to be on the sun's dise at any given moment, as seen from the observer's station. A comparison of these positions with those simultaneously occupied by the planet, as seen from another station, gives at once the means of deducing the parallax.

In 1874-82 several hundred heliometer measures were made, mostly by German parties, and several thousands of photographs were obtained at stations in all quarters of the earth where the transits could be seen. The final result of all these observations ${ }^{1}$ is given by Newcomb as $8^{\prime \prime} .857 \pm 0.23$, differing to an unexpected degree from the figures given by other methods, and rather discordant among themselves. It would almost seem that measurements of this sort must be vitiated by some constant source of error.
520. Gravitational Methods. - These hardly admit of elementary discussion. We merely mention them.

1. By the moon's parallactic inequality. This is an irregularity in the moon's motion, which depends simply on the ratio between the distance of the sun and the radius of the moon's orbit. If, what is practically very difficult, we could determine by observation exactly the amount of this inequality (which reaches about $2^{\prime}$ at its maximum), we could at once get the solar parallax.
2. The perturbations produced by the earth in the motions of Mars and Venus give the means of determining the ratio between the mass of the sun and that of the earth. Now from Art. 309,

$$
S: E=\frac{R^{3}}{T^{2}}: \frac{r^{3}}{t^{2}}
$$

[^60]in which $S$ and $E$ are respectively the masses of the sun and earth, $R$ and $r$ are the radii of the orbits of the earth and moon, $T$ is the length of the sidereal year, and $t$ that of the sidereal month, corrected for perturbations. Hence
$$
\frac{S}{E}(\text { or } M)=\frac{R^{3}}{r^{3}} \times \frac{t^{2}}{T^{2}}
$$

If, then, $M$ is known from planet observations, $R$ is at once deducible in terms of $r$ (moon's distance).

There are also other equations available which do not involve the moon at all, but substitute measurements of gravity by the pendulum.

Even at present this method approaches closely in value to the others. Ultimately, it must supersede them all, because as time goes on and the secular perturbations of our two neighboring planets accumulate, the precision with which $M$ is determined continually improves, and apparently without limit.

## METHODS OF MEASURING STELLAR PARALLAX.

521 (supplementary to Art. 432). The determination of stellar parallax had been attempted over and over again from the time of Tycho Brahe down, but without success, until in 1838 Bessel at last demonstrated and measured the parallax of 61 Cygni ; and the next year Henderson of the Cape of Good Hope, determined that of Alpha Centauri. The operation of measuring the parallax of a star is on the whole the most delicate in the whole range of practical astronomy. Two methods have been successfully employed so far, known as the absolute and the differential.
(a) The first method consists in making meridian observations of the star's right ascension and declination with the extremest possible accuracy, at different times of the year, applying rigidly all the known
corrections (for precession, nutation, proper motion, etc.) and then examining the deduced positions. If the star is without parallax, they will all agree. If it has sensible parallax, they will show, when plotted on a chart, an apparent annual orbital motion of the star and will determine the size of its "parallactic orbit" (Art. 432). Theoretically this method is perfect: practically it seldom gives satisfactory results, because the annual changes of temperature and moisture disturb the instrument in such a way that its errors intertwine themselves with the parallactic displacement of the star in a manner that defies disentanglement. No process of multiplying observations and taking averages helps the matter very much, because the instrumental errors involved are themselves annually periodic, just as is the parallax itself. Still, in a few cases, the method has proved successful, as in the case of Alpha Centauri, above cited.
522. (b) The Differential Method. - This consists in measuring the change of position of the star whose parallax we are seeking, with respect to other small stars, near it in apparent position (i.e., within a few minutes of arc), but presumably so far beyond as to liave no sensible parallax of their own. The great advantage of the method is that it avoids entirely the difficulties due to the uncertainty in respect to the precise amount of the currections for precession, aberration, nutation, etc., since these are almost the same for the principal star as for the comparison stars; to a considerable extent also, the method evades the effects of refraction and temperature disturbances. But per contra, it measures not the whole parallax of the star investigated, but only the difference between its parallax and that of the stars with which it is compared.

Suppose, for instance (see Fig. 140), that in the same telescopic field of view, we have the star $a$, which is near us, the stars $c, d$, and $e$, which are so remote that they have no sensible parallax at all, and the star $b$, which is about twice as far away as $a$. $a$ and $b$ will describe their parallactic orbits every year, just alike in form, but $a$ 's orbit twice as large as $b$ 's. If now, during the year, we continually measure the distance (in seconds of arc) and the direction from $c$ or $e$ to $a$ and $b$, the results will give us the true dimensions of their parallactic
orbits. If, however, we had taken $b$ as the starting point to measure $a$ 's motion, we should have found only half the true value. It follows that if the measurements are absolutely accurate, the parallax deduced by this method can never be too large, but may be too small: the distance of the star will certainly be more or less exaggerated.


Fig. 140. - Differential Method of determining Stellar Parallax.
The necessary measurements, if the comparison stars are within a minute or two of arc from the star under investigation, may be made with the wire micrometer; but if the distance exceeds a few minutes, we must resort to the "heliometer" (Appendix, Art. 543) with which Bessel first succeeded. The best results are obtained from measures of photographs taken six months apart, just when the parallactic displacements of the star are greatest and in opposite directions. If several comparison stars are used, any having a sensible parallax of its own may be detected.
523. Selection of Stars to be examined for Parallax. - It is obviously necessary to choose for observations of this sort stars that are presumably near. The most important indication of proximity is a large proper motion. Brightness also is of course confirmatory. Still, neither of these indications is certain. A star which happens to be moving directly towards or from us shows no proper motion at all, however near; and among the millions of faint stars it is quite likely that some few individuals, at least, are nearer than Alpha Centauri.

## CHAPTER XVIII.

## ASTRONOMICAL INSTRUMENTS.

THE CELESTIAL GLOBE. - THE TELESCOPE: SIMPLE, ACHROMATIC, AND REFLECTING. - THE EQUATORIAL. - THE FILAR MICROMETER. - THE HELIOMETER. - THE TRANSIT INSTRUMENT. - THE CLOCK. - THE CHRONOGRAPH.

- THE MERIDIAN CIRCLE. - THE SEXTANT. - THE PYRHELIOMETER.

524. The Celestial Globe. - The celestial globe is a ball, usually of papier-mâché, upon which are drawn the circles of the celestial sphere and a map of the stars. It is mounted in a framework which represents the horizon and the meridian, in the manner shown by Fig. 141.

The "Horizon," $H H^{\prime}$ in the figure, is usually a wooden ring three or four inches wide and perhaps three-quarters of an inch thick, directly supported by the pedestal. It carries upon its upper surface at the inner edge a circle marked with degrees for measuring the azimuth of any heavenly body, the graduation beginning at the south point where the horizon is intersected by the metal circle which represents the meridian. Next comes the zodiacal circle, containing in order the names of the 12 signs of the zodiac. Outside of this is a narrow circle marked with the degrees of celestial longitude, the zero of this graduation being made to correspond with the sign Aries in the zodiacal circle. Just outside of this circle is a second similar one, marked with the months of the year and the days of the month, the days being set against the longitude graduar
tion so that each day is opposite to the degree of longitude occupied by the sun on that day of the year. In the circle of the months is also given at different points the equation of time for corresponding days.
525. The Meridian Ring, $M M^{\prime}$, is a circular ring of metal which carries the bearings of the axis on which the globe revolves. Things are so arranged, or ought to be, that the mathematical axis of the globe is exactly in the same plane as


Fig. 141. - The Celestial Globe.
the graduated face of the ring, which is divided into degrees and fractions of a degree. The meridian ring fits into two notches in the horizon-circle, and is held underneath the globe
by a support with a clamp, which enables us to fix it securely in any desired position, the mathematical centre of the globe being precisely in the planes both of the meridian ring and the horizon.
526. The Surface of the Globe is marked first with the celestial equator (Art. 27), next with the ecliptic (Art. 38), crossing the equator at an angle of $23 \frac{1}{2}^{\circ}$ (at $V$ in the figure), and each of these circles is divided into degrees and fractions. The equinoctial and solstitial Colures (Art. 113), are also always represented. As to the other circles, usage differs. The ordinary way at present is to mark the globe with 24 hour-circles, 15 degrees apart (the Colures being two of them), and with parallels of declination 10 degrees apart.

On the surface of the globe are plotted the positions of the stars and the outlines of the constellations.
527. The Hour Index is a small metal circle three or four inches in diameter, which is fitted to the northern pole of the globe with stiffish friction, so that it can be set like the hands of a clock, but once set will turn with the globe without shifting its adjustment.
528. To Rectify a Globe, - that is, to set it so as to show the aspect of the heavens at any given time :-
(1) Elevate the North Pole of the globe to an angle equal to the observer's latitude by means of the graduation on the meridian ring, and clamp the ring securely.
(2) Look up the day of the month on the "horizon" of the globe, and opposite to the day find, on the longitude circle, the sun's longitude for that day.
(3) On the ecliptic (on the surface of the globe) find the degree of longitude thus indicated and bring it to the graduated face of the meridian ring.
The globe is then set to correspond to (apparent) noon of
the day in question. (It may be well to mark the place of the sun temporarily with a bit torn from the corner of a postagestamp, and gummed on at the proper place in the ecliptic: it can easily be wiped off with a damp cloth, after using.)
(4) Holding the globe fast, so as to keep the place of the sun on the meridian, turn the hour index until it shows at the edge of the meridian ring the mean time of apparent noon; i.e., $12^{\mathrm{h}} \pm$ the equation of time given for the day on the horizon ring. If standard time is used, the hour index must be set to the standard time of apparent noon.
(5) Finally, turn the globe until the hour for which it is to be set is brought to the meridian, as indicated on the hour index. The globe will then show the true aspect of the heavens.

The positions of the moon and planets are not given by this operation, since they have no fixed places in the sky and therefore cannot be put in by the globe-maker. If one wants them represented, he must look up their right ascensions and declinations for the day in some almanac, and mark the corresponding places on the globe with bits of wax or paper.

## TELESCOPES.

529. Telescopes are of two kinds, refracting and reflecting.

The refractor was first invented, early in the 17 th century, and is much more used; but the largest instruments ever made are reflectors. In both, the fundamental principle is the same. The large lens of the instrument (or else its concave mirror) forms a real image of the object looked at, and this image is then examined and magnified by the eye-piece, which in principle is only a magnifying glass.

In the form of instrument, however, which was originally devised by Galileo and is still used as the "opera-glass," the rays from the object-glass are intercepted, and brought to parallelism by the concave lens, which serves as an eye-glass, before they form the image. Tele-
scopes of this construction are never made of any considerable power, being very inconvenient on account of the smallness of the field of view.
530. The Simple Refracting Telescope. - This consists essentially, as shown in Fig. 142, of two convex lenses; one, the object-glass $A$, of large size and long focus; the other, the eye-glass $B$, of short focus, - the two being set at a distance nearly equal to the sum of their focal lengths. Recalling the optical principles relating to the formation of images by lenses (Physics, p. 310), we see that if the instrument is pointed towards the moon, for instance, all the rays that strike the object-glass from the top of the crescent will be collected to a focus at $a$, while those from the bottom will come to a focus at $b$; and similarly with rays from the other points on the surface


Fig. 142. - The Simple Refracting Telescope.
of the moon. We shall, therefore, get in the "focal plane" of the object-glass a small inverted "image" of the moon. The image is a real one; i.e., the rays really meet at the focal points, so that if we insert a photographic plate in the focal plane at $a b$ and properly expose it, we shall get a picture of the object. The size of the picture will depend upon the apparent angular diameter of the object and the distance from the object-glass to the image $a b$, and is determined by the condition that, as seen from the point $O$ (the "optical centre" of the object-glass), the object and its image subtend equal angles, since rays which pass through the point 0 suffer no sensible deviation.

If the focal length of the lens $A$ is 10 feet, then the image of the moon formed by it will appear, when viewed from a distance of 10 feet, just as large as the moon itself: viewed from a distance of one
foot, the image will, of course, appear 10 times as large. With such an object-glass, therefore, even without an eye-piece, one can see the mountains of the moon and the satellites of Jupiter by simply putting the eye in the line of the rays, at a distance of 10 or 12 inches back of the eye-piece hole (the eye-piece having been, of course, removed).
531. Magnifying Power. - If we use the naked eye, we cannot see the image distinctly from a distance much less than a foot, but if we use a magnifying lens of, say, one inch focus, we can view it from a distance of only an inch, and it will look correspondingly larger. Without stopping to prove the principle, we may say that the magnifying power is simply equal to the quotient obtained by dividing the focal length of the object-glass by that of the eye-lens; or, as a formula

$$
M=\frac{F}{f} ; \text { that is, } \frac{O d}{c d} \text { in the figure. }
$$

If, for example, the focal length of the object-glass be four feet and that of the eye-lens one-quarter of an inch, then

$$
M=\frac{48}{\frac{1}{4}}=192
$$

It is to be noted, however, that a magnifying power of unity is sometimes spoken of as no magnifying power at all, since the image appears of the same size as the object.

The magnifying power of a telescope is changed at pleasure by simply interchanging the eye-pieces, of which every telescope of any pretensions always has a considerable stock, giving various powers.
532. Brightness of the Image. - This depends not upon the focal length of the object-glass, but upon its diameter; or, more strictly, its area. If we estimate the diameter of the pupil of the eye at one-fifth of an inch, as it is usually reckoned, then (neglecting the loss from want of perfect transparency in the lenses) a telescope one inch in diameter collects into the image of a star 25 times as much light as the naked
eye receives; and the great Lick telescope of 36 inches in diameter, 32,400 times as much, or about 25,000 after allowing for the losses. The amount of light is proportional to the square of the diameter of the object-glass.

The apparent brightness of an object which, like the moon or a planet, shows a disc, is not, however, increased in any such ratio, because the light gathered by the object-glass is spread out by the magnifying power of the eye-piece. In fact, it can be demonstrated that no optical arrangement can show an extended surface brighter than it appears to the naked eye. But the total quantity of light in the image of the object greatly exceeds that which is available for vision with the naked eye, and objects which, like the stars, are mere luminous points, have their brightness immensely increased, so that with the telescope millions otherwise invisible are brought to light. With the telescope, also, the brighter stars are easily seen in the daytime.
533. The Achromatic Telescope. - A single lens cannot bring the rays which emanate from a single point in the object to any exact focus, since the rays of different color (wavelength) are differently refracted, the blue more than the green, and this more than the red (Physics, p. 315). In consequence of this so-called "chromatic aberration," the simple refracting telescope is a very poor ${ }^{1}$ instrument.

About 1760, it was discovered in England that by making the object-glass of two or more lenses of different kinds of glass, the chromatic aberration can be nearly corrected. Ob-ject-glasses so made - none others are now in common use are called achromatic, and they fulfil with reasonable approxi-

[^61]mation the "condition of distinctness"; viz., that the rays which emanate from any single point in the object should be collected to an absolute mathematical point in the image. In practice, only two lenses are ordinarily used in the construction of an astronomical glass, - a convex of crown glass, and a concave of fint glass, the curves of


Gauss


Clark
Fig. 143. - Different Forms of the Achromatic Object-glass. the two lenses and the distances between them being so chosen as to give the most perfect possible correction of the "spherical" aberration (Physics, p. 311) as well as of the chromatic. Many forms of objectglass are made: three of them are shown in Fig. 143.
534. Secondary Spectrum. - It is not possible with the kinds of glass hitherto available to obtain a perfect correction of color. Even the best achromatic telescopes show a purple halo around the image of a bright star, which, though usually regarded as "very beautiful" by tyros, seriously injures the definition : it is especially obnoxious in large instruments.

This imperfection of achromatism makes it impossible to get satisfactory photographs with an ordinary object-glass, corrected for vision. An instrument for photography must have an object-glass specially corrected for the purpose, since the rays which are most efficient in making the image upon the photographic plate are the blue and violet rays, which in the ordinary object-glass are left to wander very wildly.

Bright objects may, however, be photographed with a visual telescope if a color filter which absorbs these blue and violet rays is used with special plates made sensitive to the less refrangible light rays. The method cannot be used successfully for the nebulæ, because their light is deficient in the red, yellow, and green rays, which are the only ones that can pass through the filter.

When the photographic lens is used, red stars appear relatively faint; but with a certain combination of color filter and prepared plate, photographs may be taken upon which the relative magnitudes of the stars agree with what is seen with the eye.

534*. Diffraction and Spurious Dises. - Even if a lens were absolutely perfect as regards the correction of aberrations, both spherical and achromatic, it would still be unable to fulfil strictly "the condition of distinctness." Since light consists of waves of finite length, the image of a luminous point can never be also a point, but must of mathematical necessity according to the laws of diffraction be a disc of finite diameter surrounded by a series of interference rings; and the image of a line will be a streak and not a mathematical line. The diameter of the "spurious disc" of a star, as it is called, varies inversely with the diameter of the object-glass, and the larger the telescope the smaller the image of a star with a given magnifying power.

With a good telescope and a power of about 30 to the inch of aperture ( 120 for a 4 -inch telescope) the image of a small star, when the air is steady (a condition unfortunately seldom fulfilled), should be a clean, round disc with a bright ring around it, separated from the disc by a clear black space. According to Dawes, the image of a star with a $4 \frac{1}{2}$-inch telescope should be about $1^{\prime \prime}$ in diameter; with a 9 -inch instrument $0^{\prime \prime} .5$, and $\frac{1}{8}$ " for a 36 -inch glass.

If too deep an eye-piece be used, raising the power of the telescope too high (more than about 60 to the inch), the spurious disc of the star will become hazy at the edge, so that there is very little use with most objects in pushing the magnifying power any higher.

This effect of diffraction has much to do with the superiority of large instruments in showing minute details; no increase of magnifying power on a small telescope can exhibit the object as sharply as the same power on a large one, provided, of course, that the object-glasses are equally good in workmanship and that the atmospheric conditions are satisfactory. (But a given amount of atmospheric disturbance injures the performance of a large telescope much more than that of a small one.)
535. Eye-pieces. - For some purposes the simple convex lens is the best "eye-piece" possible; but it performs well only for a small object, like a close double star, exactly in the
centre of the field of view. As soon as the object is a little away from the centre, the image becomes hideous. Generally, therefore, we employ "eye-pieces" composed of two or more lenses, which give a larger field of view than a single lens and define satisfactorily over the whole extent of the field. They fall into two general classes, the positive and the negative.

The positive eye-pieces are much more generally useful. They act as simple magnifying glasses, and can be taken out of the telescope and used as hand-magnifiers if desired. The image of the object formed by the object-glass lies outside of this kind of eye-piece, between it and the object-glass.

In the negative eye-piece, on the other hand, the rays from the object-glass are intercepted by the so-called "field lens" before reaching the focus, and the image is formed between


Fig. 144. - Various Forms of Telescope Eye-piece.
the two lenses of the eye-piece. It cannot therefore be used as a hand-magnifier.

Fig. 144 shows the two most ordinary forms of eye-piece, and also the "solid eye-piece" constructed by Steinheil; but there are a multitude of other kinds.

The ordinary eye-pieces show the object in an inverted position, which is of no importance as regards astronomical observations.

The erecting eye-piece used in spy-glasses is constructed differently, having in it four lenses. It is essentially a compound microscope, and produces erect vision by inverting a second time the already inverted image formed by the object-glass.

It is evident that in an achromatic telescope, the object-glass is by far the most important and expensive member of the instrument. It costs, according to size, from $\$ 100$ or $\$ 200$ up to $\$ 50,000$, while the eye-pieces cost from $\$ 5$ to $\$ 25$ apiece.
536. Reticle. - When the telescope is used for pointing upon an object, as it is in most astronomical instruments, it must be provided with a "reticle" of some sort. The simplest form is a metallic frame with spider lines stretched across it, the intersection of the spider lines being the point of reference. This reticle is placed not at or near the object-glass, as is often supposed, but in its focal plane, as $a b$ in Fig. 142. Of course, positive eye-pieces only ${ }^{1}$ can be used in connection with such a reticle. Sometimes a glass plate with fine lines ruled upon it is used instead of spider lines. Some provision must be made for illuminating the lines, or "wires," as they are usually called, by reflecting into the instrument a faint light from a lamp suitably placed.
537. The Reflecting Telescope. - About 1670, when the chromatic aberration of refractors first came to be understood (in consequence of Newton's discovery of the " decomposition of light"), the reflecting telescope was invented. For nearly 150 years it held its place as the chief instrument for star-gazing, until about 1820, when large achromatics began to be made. There are several varieties of reflecting telescope. They differ in the way in which the image formed by the mirror is brought within reach of the magnifying eye-piece. Fig. 145 illustrates three of the most common forms. The Herschelian form is practicable only with very large instruments, since the head of the observer cuts off part of the light. The Newtonian is the one most used, but one or two large reflectors now in use are of the Cassegrainian form, which is exactly like the Grego-

[^62]rian shown in the figure, with the exception that the small mirror is convex instead of concave.

Until about 1870, the large mirror (technically "speculum ") was always made of speculum metal, a composition of copper and tin. It is now usually made of glass, silvered on the front by a chemical process. When new, these silvered films reflect, much more light than the old speculum metal: they tarnish rather easily, but fortunately they can be easily renewed.


Fig. 145. - Different Forms of Reflecting Telescope.

1. The Herschelian; 2. The Newtonian; 3. The Gregorian.
2. Large Telescopes. - The largest telescopes ever made have been reflectors. At the head stood the enormous instrument of Lord Rosse of Birr Castle, Ireland, six feet in diameter and 60 feet long, made in 1842, and still used. One still larger, 100 inches in diameter, was mounted in 1917 at the Carnegie Solar Observatory on Mt. Wilson, where a five-foot reflector was mounted in 1908. Another five-foot reflector, made by Mr. Common, in England, in 1889, was acquired by the Harvard College Observatory in 1905. The Crossley reflector at the Lick Observatory, with which Keeler made his wonderful photographs, has an aperture of three feet.

Of the refractors, the largest is that of the Yerkes Observatory, of the Chicago University. It has an aperture of 40 inches and a focal
length of 65 feet. Next follows that of the Lick Observatory (see Frontispiece), which has a diameter of 36 inches and a length of 57 feet. The next in size is the 32 -inch (visual) telescope at Meudon, which is followed by the Potsdam telescope, $31 \frac{1}{2}$ inches in diameter; and this is nearly equalled by the telescopes at Pulkowa, Nice, Allegheny and Paris ( $29 \frac{1}{2}$ to 30 inches). Then come the new Greenwich tele-


Fig. 146. - The Melbourne Reflector.
scope, 28 inches ; the Vienna telescope, 27 inches ; the two telescopes at Washington and the University of Virginia, $26 \frac{1}{4}$ inches ; and the Newall telescope (lately presented to the University of Cambridge, England), 25 inches. This list includes most of the refractors which have an aperture exceeding two feet, but a number of others are now
under construction. The two largest of these object-glasses, and those of the Pulkowa, Washington, and University of Virginia telescopes, were made by the Clarks, as well as the 24 -inch telescope of the Lowell Observatory and the 23 -inch instrument at Princeton.
539. Relative Advantages of Reflectors and Refractors. There is much earnest discussion on this point, each form of instrument having its earnest partisans. In favor of the reflectors we may mention

1. Ease of construction and cheapness: the speculum has but one surface to be worked, while the object-glass has four of them. Moreover the material of the speculum is much more easily obtained, since the light does not go through it as in the case of a lens; so that slight imperfections of internal structure and homogeneity are not very important.
2. Reflectors can be made larger than refractors.
3. Reflectors are perfectly achromatic: this is an immense advantage, especially in photographic and spectroscopic work.

On the whole, however, the balance of advantage is now generally considered to lie with the refractors.

1. The refractor gives a brighter image than a reflector of the same size. A heavy percentage of the light is lost by the two reflections, while in a refractor much less is lost in passing through the lenses.
2. Refractors generally define much better. Any error of form at a point in the surface of a lens, whether it be due to distortion by the weight of the lens or to the fault of the workmanship, affects the rays passing through that point only one-third as much as in the case of a speculum. Moreover, when a lens is slightly distorted by its weight, its two opposite surfaces are affected in a nearly compensatory manner. In a mirror there is no such compensation; the slightest distortion of a speculum is fatal to its performance.
3. A lens once made and fairly taken care of does not deteriorate with age. A speculum, on the other hand, must be re-silvered or re-polished every few years.
4. As a rule, refractors are much more convenient to use than reflectors, being lighter and less clumsy.
5. Mounting of a Telescope, - the Equatorial. - A telescope, however excellent optically, is not good for much unless firmly and conveniently mounted. ${ }^{1}$

At present some form of equatorial mounting is practically universal. Fig. 147 represents schematically the ordinary arrangement of the instrument. Its essential feature is that its "principal axis" (i.e., the one which turns in fixed bearings attached to the pier and is called the polar axis) is placed parallel to the earth's axis, pointing to the celestial pole, so that the circle $H$, attached to it, is parallel to the celestial equator. This circle is sometimes called the hour-circle, sometimes the right-ascension circle. At the extremity of the polar axis a "sleeve" is fastened, which carries the declination axis $D$, and to this declination axis is attached the telescope tube $T$, and also the declination circle $C$.
541. The advantages of this mounting are very great. In the first place, when the telescope is once pointed upon an object it is not necessary to move the declination axis at all in order to keep the object in the field, but only to turn the

[^63]polar axis with a perfectly uniform motion, which can be, and usually is, given by clock-work (not shown in the figure).

In the next place, it is very easy to find an object even if invisible to the eye (like a faint comet, or a star in the daytime), provided we know its right ascension and declination, and have the sidereal time, - a sidereal clock or chronometer being an indispensable accessory of the equatorial. We simply set the declination circle to the declination of the object, and then turn the polar axis until the hour-circle shows the proper hour-angle, which hour-angle is simply the difference between the right ascension of the object and the sidereal time at the moment. When the telescope has been so set, the object will be found in the field of view, provided a low-power eye-piece is used. On account of refraction, the setting does not direct the instrument precisely to the apparent place of the object, but only very near it: near enough for easy finding, however.

The equatorial ${ }^{1}$ does not give very accurate positions of heavenly bodies by means of the direct readings of its circles, but it can be used, as explained in Art. 71, to determine very precisely the difference between the position of a known star and that of a comet or planet.

The frontispiece shows the actual mounting of the Lick telescope. Fig. 105, Art. 425, represents another form of equatorial nounting, adopted for several of the instruments of the photographic campaign, and Fig. 146 is a view of the great Melbourne reflector. Lord Rosse's six-foot telescope is not equatorially mounted.
542. Micrometer. - Micrometers of various forms are employed in connection with the equatorial, the most common and generally useful micrometer being that known as the filar-position micrometer (shown in Fig. 148) - a small instrument which screws into the eye end of the telescope. It usually contains a set of fixed wires, two or three of them parallel to each other (only one, e, is shown in Fig. 149, which represents

[^64]the internal construction of the instrument), crossed at right angles by a single line or set of lines. Over the plate which carries the fixed threads lies a fork, moved by a carefully made


Fig. 148. - The Filar Position-Micrometer. screw with a graduated head, and this fork carries one or more wires parallel to the first set, so that the distance between the wires $e$ and d, Fig. 149, can be adjusted at pleasure and "read off" by means of the scale that is shown and the screw-head graduation. The box, $B$, Fig. 148, containing the wires and micrometer, is so arranged that it can itself be rotated around the optical axis of the telescope and set in any desired "position," - for example, so that the movable wire $d$ shall be parallel to the celestial equator. When so set that the


Fig. 149. - Construction of the Micrometer.
movable wire points from one star to another in the field of view, the "position angle" is then read off on the circle, $A$.

With such a micrometer we can measure at once the distance in seconds of are between any two stars which are near enough
to be seen in the same field of view, and can determine the posi tion angle of the line joining them. The available range in a small telescope may reach $30^{\prime}$. In large ones, which with the same eye-pieces give much higher magnifying powers, the range is correspondingly less - from $5^{\prime}$ to $10^{\prime}$. When the distance between the objects exceeds $1^{\prime}$ or $2^{\prime}$, however, the filarmicrometer becomes difficult to use and inaccurate, and we have to resort to instruments of a different kind.
543. The Heliometer. - This instrument, as its name implies, was originally designed to measure the apparent diameter of the sun, and is capable of measuring with extreme precision angular distances ranging all the way from a few seconds up to two or three degrees. It is a form of "double-image" micrometer, the measurement of the distance between two objects being made by superposing the image of one of them upon that of the other. In using the filar-micrometer we have to look "two ways at once" to be sure that each of the two wires accurately bisects the star upon which it is set: with a double-image micrometer, the observer's attention is concentrated upon a single


Fig. 150. - The Heliometer. point without distraction.
The heliometer is a complete telescope, equatorially mounted, and having its object-glass (usually from four to six inches in diameter) divided along its diameter, as shown in Fig. 150. The semi-lenses are so mounted that they can slide past each other for a distance of three or four inches, the distance being accurately measured by a delicate scale, which is read by a long microscope that comes down through the telescope-tube to the eye end. The tube is mounted in such a way that it can be turned around in its cradle, so as to make the line of division of the lenses lie at any desired position angle.

When the two halves of the object-glass are so placed that their optical centres, 0 and 1 or 0 and 2 , coincide, they act as a single lens, and form but a single image for each object in the field of view: but as soon as they are separated, each half-lens forms its own image. The distance between any two objects in the field of view is measured by making their images coincide, as indicated in the lower part of the figure, where $M_{0}$ and $S_{0}$ are the images of Mars and of a star, formed by the stationary half, $B$, of the object-glass, which has its centre at $0 . M_{1}$ and $S_{1}$ are the images formed by the other half. lens, $A$, when its centre is 1 , and $M_{2}$ and $S_{2}$ are the images when its centre is 2 . The distance between the images of $S$ and $M$ is therefore either 01 or 02 , read off on the sliding scale. The direction in which the line 102 has to be set to effect the coincidence, gives the direction, or position angle, from $M$ to $S$.
544. The Transit Instrument. - This instrument has already been mentioned, and figured in outline in Art. 58, Fig. 14. It consists of a telescope carrying at the eye end a reticle, and mounted on a stiff axis that turns in Y's which can have their position adjusted so as to make the axis exactly perpendicular to the meridian. A delicate spirit level, which can be placed upon the pivots of the axis to ascertain its horizontality, is an essential accessory, and it is practically necessary to have a graduated circle attached to the instrument in order to "set it" for a star, in readiness for the star's transit across the meridian. It is very desirable also that the instrument should have a "reversing apparatus," by which the axis may be easily and safely reversed in the Y's.

The reticle usually contains from five to fifteen vertical "wires," crossed by two horizontal ones. Fig. 151 shows the reticle of a small transit intended for observations by "eye and ear." When the chronograph is to be used, the wires are made more numerous and placed nearer together. In order to make the wires visible at night, one of the pivots of the in-
strument is pierced (sometimes both of them) so that the light from a lamp will shine through the axis upon a small reflector placed in the central cube of the instrument, where the axis and the tube are joined. This little reflector sends sufficient light towards the eye to illuminate the field, while it does not cut off any considerable portion of the rays from the object.

The instrument must be thoroughly stiff and rigid, without any loose joints or shakiness, especially in the mounting of the object-glass and reticle. Moreover, the two pivots must be accurately round, without taper, and precisely in line with each other, -in


Fig. 151. - Reticle of the Transit Instrument. other words, they must be portions of one and the same geometrical cylinder. To fulfil this condition, with errors nowhere exceeding $\frac{100^{1} 000}{}$ of an inch, taxes the highest skill of the mechanician. When accurately constructed and adjusted, the middle wire of the instrument always exactly coincides with the meridian, however the instrument may be turned on its axis; and the sidereal time when a star crosses that wire is therefore the star's right ascension (Art. 37).
545. The Adjustments of the Transit. - These are four in number :

1. The reticle must be exactly in the focal plane of the object-glass, and the middle wire truly vertical.
2. The line of collimation (i.e., the line which joins the optical centre of the object-glass to the niiddle wire) must be exactly perpendicular to the axis of rotalion. This may be tested by pointing the instrument on a distant object, and then reversing the instrument. If the adjustment is correct, the middle wire will still bisect the object after the reversal. If it does not, the reticle must be set right by adjusting screws provided for the purpose.
3. The axis must be level. This adjustment is made by the help of the spirit-level. One of the Y's has a screw by which it can be slightly raised or lowered, as may be necessary.
4. The azimuth of the axis must be exactly $90^{\circ}$; i.e., it must point exactly east and west. This adjustment is made by means of star observations with the help of the sidereal clock. Without going into detail, we may say that if the instrument is correctly adjusted, the time occupied by a star near the pole in passing from its transit across the middle wire, above the pole, to its next transit across the same wire, below the pole, must be exactly 12 sidereal hours. Moreover, if two stars are observed, one near the pole and another near the equator, the difference between their times of transit ought to be precisely equal to their difference of right ascension. By availing himself of these principles, the astronomer can determine the errors of adjustment in azimuth and correct them.

An observation with the instrument consists in noting the precise moment by the clock when the object observed crosses each wire. The mean is taken to give the time of transit across the middle wire. If the "error" of the clock is known and the instrument is in exact adjustment, the moment of crossing the middle wire will, as already said, give the right ascension of the object; or, if the right ascension of the object is already known (if, for instance, the object is an "almanac star"), the difference between this and the clockface indication will give the "clock error" (Art. 58).
546. Astronomical Clock. - Obviously a good clock or chronometer is an essential adjunct of the transit, and equally so of most other astronomical instruments. The invention of the pendulum clock by Huyghens was almost as important for the advancement of Astronomy as the invention of the telescope by Galileo; and the improvement of the clock and chronometer in the invention of temperature compensation by Harrison and Graham in the 18th century, is fully comparable with the improvement of the telescope by the invention of the achromatic object-glass.

The astronomical clock differs in no respect from any other clock, except that it is made with extreme care, and has a pendulum so compensated that its rate will not be sensibly affected by changes of temperature. The pendulum usually beats seconds, and the clock-face ordinarily has its second-hand, minute-hand, and the hour-hand, each moving on a separate centre, while the hours are numbered from 0 to 24 .

Excellence in an astronomical clock consists in its maintaining a constant rate; i.e., in gaining or losing precisely the same amount each day; and for convenience, the rate should be small. It is adjusted by raising or lowering the pendulum bob, and is generally made less than a second a day. The clock error or "correction" can of course be adjusted at any time by merely setting the hands.

The old-fashioned method of time observation consisted simply in noting by "eye and ear" the moment (in seconds and tenths of a second) when the phenomenon occurred; as, for instance, when a star passed the wire: the tenths, of course, were merely estimated, but the skilful observer seldom makes a mistake of a whole tenth in his estimation.
547. The Chronograph. - At present such observations are usually made by the help of electricity. The clock is so arranged that at every other beat of the pendulum an electric circuit is made or broken for an instant, and this causes a sudden sideways jerk in the armature of an electric magnet, like that of a telegraph sounder. This armature carries a pen, which writes upon a sheet of paper moving beneath it. The sheet is wrapped around a cylinder six or seven inches in diameter, and the cylinder itself turns uniformly once a minute: at the same time the pen-carriage is drawn slowly along, so that -the marks on the paper form a continuous spiral, graduated off into two-second spaces by the clock beats. When taken off the cylinder, the paper presents the appearance of an ordinary page crossed by parallel lines,
spaced off into two-second lengths, as shown in Fig. 152, which is a part of an actual record.


Fig. 152. - A Chronograph Record.


Fig. 153. - A Chronograph by Warner \& Swasey.
The observer at the moment of a star-transit merely presses a key. which he holds in his hand, and thus interpolates a mark of his own among the clock beats on the sheet; as, for instance,
at $X$ and $Y$ in the figure. Since the beginning of each minute is indicated on the sheet in some way by the mechanism of the clock beats, it is very easy to read the time of $X$ and $Y$ by applying a suitable scale, the beginning of the mark being the true moment of observation. In the figure, the initial minute, marked when the chronograph was started, happened to be 9 hours, 35 minutes, the zero in the case of this clock being indicated by a double beat. The signal at $X$, therefore, was made at $9^{\mathrm{h}} 35^{\mathrm{m}} 55^{\mathrm{s}} .45$, and that of $Y$ at $9^{\mathrm{h}} 36^{\mathrm{m}} 58^{\mathrm{s}} .63$. The "rattle" just preceding $X$ was the signal that a star was approaching the transit wire. Fig. 153 is a representation of a complete chronograph.
548. The Meridian Circle. - This has already been briefly described in Art. 49, but not in sufficient detail to give much real understanding of its construction and appearance. It is a transit instrument of large size and most careful construction, plus a large graduated circle (or circles), attached to the axis and turning with it. The utmost resources of mechanical art are expended in graduating this circle with accuracy. The divisions are now usually made either two minutes or five minutes, and the farther subdivision is effected by the socalled "reading microscopes," four or six of which are always used in the case of a large instrument. Since $1^{\prime \prime}$ on the circumference of a circle is $\frac{1}{206265}$ part of its radius, it follows that on a circle two feet in diameter $1^{\prime \prime}$ is only about $\frac{1}{17000}$ of an inch. An error of that amount is now very seldom made by reputable constructors in placing any graduation line. Fig. 154 represents a rather small instrument of this kind, having circles two feet in diameter, with a four and a half inch telescope.

Our limits do not permit a description of the reading microscopes seen at $A, B, C$, and $D$, by means of which the circle is read. For these, see works on Practical Astronomy.
549. Zero Points. - The instrument is used to measure the altitude or else the polar distance of a heavenly body at the time
when it is crossing the meridian. As a preliminary, we must determine a zero point upon the circle, - the nadir point if we wish to measure altitudes or zenith distances, the polar point if polar distances or declinations.


Fig. 154. - A Meridian Circle.
The polar point is determined by taking the circle reading for some star near the pole when it crosses the meridian above the pole, and then doing the same thing again twelve hours later when it crosses it below. The mean of the two readings, corrected for refraction, will be the reading the circle would
give when the telescope is pointed exactly to the pole ; technically, the "polar point."

The nadir point is the reading of the circle when the telescope is pointed vertically downward. It is determined by means of a basin of mercury underneath the instrument, the telescope being so set that the image of the horizontal wire of the reticle, as seen by reflection from the mercury, coincides with the wire itself. Since the reticle is exactly in the principal focus of the object-glass, the rays emitted from any point in the reticle will form a parallel beam after passing through the lens, and if this beam strikes perpendicularly upon a plane mirror, it will be returned as if from an object in the sky, and the lens will re-collect the rays to a focus in the focal plane. When, therefore, the image of the central wire of the reticle formed by reflection from the mercury coincides with the wire itself, we know that the line of collimation of the telescope is exactly perpendicular to the surface of the mercury; i.e., precisely vertical.
550. Collimating Eye-Piece. - To make this reflected image visible, it is necessary to illuminate the reticle by light thrown towardis the object-glass from behind the wires, for the ordinary illumination used during observations comes from the opposite direction. This peculiar illumination is effected by what is known as the Bohnenberger "collimating eye-piece," shown in Fig. 155. A thin glass plate inserted at an angle of $45^{\circ}$ between the lenses of a Ramsden eyepiece throws down sufficient light, and yet permits the observer to see ihe wires through the glass.


Fig. 155.
The Collimating Eye-piece.

Of course the zenith point is just $180^{\circ}$ from the nadir point, so that the zenith distance of any star is found by merely taking the difference between its circle reading (corrected for refraction) and the zenith reading.

Obviously the meridian circle can be used simply as a transit, if desired, so that with this instrument and a clock, the observer is in a position to determine both the right ascension and the declination of any heavenly body that can be seen when it crosses the meridian.
551. There are a number of other instruments which are more or less used in special observations. Our space barely permits their mention. The principal among them are the so-called "Universal Instrument," or Astronomical Theodolite; the "Prime Vertical" instrument (simply a transit faced east and west instead of moving in the meridian) ; the "Zenith Telescope"; and a new instrument by Chandler, known as the "Almucantar," which is used to observe the time when certain known stars reach a fixed altitude, usually that of the pole, an observation from which the time and the latitude of the place can be very accurately determined. It is simply a telescope carried on a "raft," so to speak, which floats on mercury, the telescope being pointed upwards at an angle approximately equal to the latitude and keeping automatically always precisely the same elevation.
552. The Sextant. - All the instruments so far mentioned, except the chronometer, require firmly fixed supports, and are therefore absolutely useless at sea. The sextant is the only one upon which the mariner can rely. By means of it he can measure the angular distance between two points (as, for instance, the sun and the visible horizon), not by pointing first on one and afterwards on the other, but by sighting them both simultaneously and in apparent coincidence, -a "double-image" measurement; in that respect the sextant is analogous to the heliometer. This measurement can be accurately made even when the observer has no stable footing.

Fig. 156 represents the instrument. Its graduated limb is usually about a sixth of a complete circle (as its name indicates) with a radius of from five to eight inches. It is graduated in half-degrees which are numbered as whole degrees, and so can measure any angle not much exceeding $120^{\circ}$. The index
arm, or "aiidade," $M N$ in the figure, is pivoted at the centre of the arc, and carries a "vernier," which slides along the limb and can be fixed at any point by a clamp with an attached tangent screw, T. The reading of this vernier gives the angle measured by the instrument ; the best instruments read to $10^{\prime \prime}$.

Just over the centre of the arc the "index-mirror," M, about


Fig. 156. - The Sextant.
two inches by one and one-half in size, is fastened securely to the index-arm, so as to move with it, keeping always perpendicular to the plane of the limb. At $H$, the "horizonglass," about an inch wide and about the same height as the index-glass, is secured to the frame of the instrument in such a position that when the vernier reads zero the index-mirror and horizon-glass will be parallel to each other. Only half of the horizon-glass is silvered, the upper half being left transparent. $E$ is a small telescope screwed to the frame and directed towards the horizon-glass.
553. If the vernier stands near zero (but not exactly at zero) an observer looking into the telescope will see together in the field of view two separate images of the object towards which the telescope is directed ; and if, while still looking, he slides the vernier, he will see that one of the images remains fixed while the other moves. The fixed image is formed by the rays which reach the object-glass directly through the unsilvered half of the horizon-glass; the movable image, on the other hand, is produced by the rays which have suffered two reflections, being reflected from the index-mirror to the hori-zon-glass and again reflected a second time at the lower half of the horizon-glass. When the two mirrors are parallel, the two images coincide, provided the object is at a considerable distance.

If the vernier does not stand at or near zero, an observer looking at an object direetly through the horizon-glass will see not only that object, but also, in the same field of view, whatever other object is so situated as to send its rays to the telescope by reflection from the mirrors; and the reading of the vernier will give the angle at the instrument between the two objects whose images thus coincide; the angles between the planes of the two mirrors being just half the angle between the two objects, and the half-degrees on the limb being numbered as whole ones.
554. Use of the Instrument. - The principal use of the instrument is in measuring the altitude of the sur. At sea, an observer, holding the instrument in his right hand and keeping the plane of the are vertical, looks directly towards the visible horizon through the horizon-glass (whence its name) at the point under the sun; then by moving the vernier with his left hand, he inclines the index-mirror upward until one edge of the reflected image of the sun is brought down to touch the horizon line. He also notes the exact time by the chronometer, if necessary. The reading of the vernier, after due
correction (Art. 492), gives the sun's true altitude at the moment.

On land we have recourse to an "artificial horizon." This is merely a shallow basin of mercury, coverel with a roof made of glass plates having their surfaces accurately plane and parallel. In this case, we measure the angle between the sun's image reflected in the mercury and the sun itself. The reading of the instrument corrected for index-error gives twice the sun's apparent altitude.

The skilful use of the sextant requires considerable dexterity, and from the small size of the telescope the angles measured are less precise than those determined by large fixed instruments. $O$ But the portability of the instrument and its applicability at sea render it absolutely invaluable. It was invented by Godfrey of Philadelphia in 1730.


Fig. 157. - Principle of the Sextant.
555. The principle that the angle between the objects whose images coincide in the sex. tant is twice the angle between the mirrors (or between their normals) is easily demonstrated as follows (Fig. 157) : -

The ray $S M$ coming from an object, after reflection first at $M$ (the index-mirror) and then at $H$ (the horizon-glass) is made to coincide with the ray $O H$, coming from the horizon.

From the law of reflection, we have the two angles SMP and $P M H$ equal to each other, each being $x$. In the same way, the two angles marked $y$ are equal. From the geometric principle that the angle $S M H$, exterior to the triangle $H M E$, is equal to the sum of the opposite interior angles at $H$ and $\dot{E}$, we get $E=2 x-2 y$. In the same way, $Q=x-y ;$ whence $E=2 Q=2 Q^{\prime}$.

## THE PYRHELIOMETER.

556. The pyrheliometer is an instrument devised by Pouillet for measuring the amount of heat received from the sun, and he made with it, in 1838, some of the earliest determinations of the "solar constant." In Fig. $158 a \alpha^{\prime}$ is a little snuff-box-like capsule, made of thin silver, and containing 100 grams of water. The bulb of a delicate thermometer is inserted in the water, and the temperature is read at a point $m$, near the middle of the stem. The disc, ss', enables us to point the instrument exactly towards the sun by making the shadow of $\alpha \alpha^{\prime}$ fall concentrically upon it. The upper surface of the box is just one decimeter in diameter, and is carefully coated with lampblack. The instrument is used by pointing it towards the sun, and first holding an umbrella over it until the temperature becomes stationary or nearly so, after which the umbrella is taken away, and the sun allowed to shine squarely upon the blackened surface for five minutes or so, the apparatus being occasionally turned on $d m$ as an axis, to stir up the water in the box. The rise of temperature in a minute would give the solar constant directly, were it not for the troublesome and uncertain corrections depending upon the continually varying absorption of the solar heat by our atmosphere.

For a description of Violles's actinometer (used for the same purpose), see "General Astronomy," Art. 341.
557. The Equatorial Coudé. - A great telescope mounted in the usual manner is very unwieldy, and various arrangements have been devised for relieving the difficulties. In the Equatorial Coudé, or "Elbowed Equatorial," now in use at Paris and in several other observatories, the inconveniences are overcome by the use of two mirrors, one outside of the objectglass, the other in the angle where the tube joins the polar axis, as shown in the figure. The observer sits in a fixed posi-


Fig. 159.-The Equatorial Coudé.
tion, under shelter, looking down through the polar axis, which is hollow and forms part of the telescope tube. The instrument is of course much more expensive than the usual form; but the costly revolving dome is dispensed with.

There is a considerable loss of light by the two reflections, and another, much simpler form with only one mirror is now in use at Cambridge (England). It commands, however, only a limited portion of the heavens, from $-30^{\circ}$ to $+75^{\circ}$ of declination.
558. The Siderostat. - In various lines of astronomical work, photographic and spectroscopic especially, there are great advantages in having the telescope and its accessories mounted solidly upon independent piers in a convenient position, the object under observation being reflected into it steadily by a plane mirror suitably moved by clock-work. The Siderostat of Foucault consists of a mirror with a cylindrical stem projecting from its back, and so mounted that it can be turned in any direction. The stem is engaged in a sliding sleeve at the end of an arm, which is attached to a polar axis and moved around by it once in 24 hours like a clock-hand. The angle which the arm makes with the stem is adjustable to correspond with the declination of the object observed. The mirror can be made to throw the reflected ray in any desired direction (horizontally towards the south in most cases), and means are provided by which the observer can adjust it from his distant position. This is perfectly satisfactory for all work which requires merely a fixed and convenient direction for the central line of the reflected beam, - as for the study of starspectra or the general spectrum of the sun. But if the instrument is used to photograph the sun or a group of stars, it has the serious disadvantage that the image revolves around its central point, and with a speed which varies continually in a manner which cannot be followed by any automatic arrangement.
559. The Cœlostat. - The Cœlostat differs in that the polar axis, to which the mirror is directly attached, turns once in 48 hours instead of 24 . The reflected image in this case does not revolve. But the direction in which it can be thrown is not arbitrary, as with the Siderostat, but is different for objects of different declination, and sometimes very-inconvenient. The difficulty can be overcome by using a second mirror which can be moved so as to receive the reflected ray and send it in the desired direction; but this of course adds materially to the expense, and involves loss of light.

## TABLE I. - ASTRONOMICAL CONSTANTS.

TIME CONSTANTS.
The sidereal day $=23^{h} 56^{\mathrm{m}} 4^{s} .090$ of mean solar time.
The mean solar day $=24^{\mathrm{h}} 3^{\mathrm{m}} \check{5} 6^{\mathrm{s}} .5 \check{5} 6$ of sidereal time.
To reduce a time interval expressed in units of mean solar time to units of sidereal time, multiply by 1.00273791; Log. of $0.00273791=[7.4374191]$.
To reduce a time interval expressed in units of sidereal time to units of mean solar time, multiply by $0.99726957=$ $(1-0.00273043) ;$ Log. $0.00273043=[7.4362316]$.
Tropical year (Leverrier, reduced to 1900), $365^{\text {d }} 5^{\mathrm{h}} 48^{\mathrm{m}} 45^{\mathrm{s}} .51$.
Sidereal year " " $\quad$ " 365 6 9 8.97.
Anomalistic year " " " $365 \quad 6 \quad 13$ 48.09.
Mean synodical month (new moon to new), $29^{\mathrm{d}} 12^{\mathrm{h}} 44^{\mathrm{m}} 2^{\mathrm{s}} .864$.
Sidereal month, . . . . . $27 \quad 74311.545$.
Tropical month (equinox to equinox), $\begin{array}{llll}27 & 7 & 43 & 4.68 .\end{array}$
Anomalistic month (perigee to perigee), $.271318 \quad 37.44$.
Nodical month (node to node), . $\begin{array}{lllll}27 & 5 & 5 & 35.81 .\end{array}$
Obliquity of the ecliptic (Newcomb), $23^{\circ} 27^{\prime} 08^{\prime \prime} .26-0^{\prime \prime} .4685(t-1900)$.
Constant of precession (Newcomb), $50^{\prime \prime} .248+0.000222(t-1900)$.
Constant of nutation (Astron. Conference, 1896), $9^{\prime \prime} .21$.
Constant of aberration (Astron. Conference, 1896), $20^{\prime \prime} .47$.
Equatorial semi-diameter of the earth (Clarke's spheroid of 1878), $-20926202^{\text {feet }}=6378190^{\text {metres }}=3963.296^{\text {miles }}$.

Polar semi-diameter, -
$20854895^{\text {feet }}=6356456^{\text {metres }}=3949.790^{\text {miles }}$.
Oblateness (Clarke), $\overline{29 \frac{1}{3} \cdot 45}$; (Harkness), $\frac{1}{3} \frac{1}{6}$.


TABLE III. - THE SATELLITES OF THE SOLAR SYSTEM.


## TABLE IV. - THE PRINCIPAL VARIABLE STARS.

A selection from S. C. Chandler's third catalogue of variables ("Astronomical Journal," July, 1896), containing such as, at the maximum, are easily visible to the naked eye, have a range of variation exceeding half a magnitude, and can be seen in the United States.


TABLE V.-STELLAR PARALLAXES AND PROPER MOTIONS.
(From Oudeman's Table, Ast. Nach., Aug., 1889.)

| No. | Name. | Mag. | Proper Motion. | Annual Parallax. | Distance Light Years. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\alpha$ Centauri | 0.7 | $3^{\prime \prime} .67$ | $0^{\prime \prime} .75$ | 4 |
| 2 | Ll. 21185. | 6.9 | 4.75 | 0.50 | 6.5 |
| 3 | 61 Cygni . | 5.1 | 5.16 | 0.40 | 8 |
| 4 | Sirius . | -1.4 | 1.31 | 0.39 | 8.3 |
| 5 | $\Sigma 2398$. | 8.2 | 2.40 | 0.35 | 9.3 |
| 6 | Ll. 9352 | 7.5 | 6.96 | 0.28 | 12 |
| 7 | Procyon . | 0.5 | 1.25 | 0.27 | 12.3 |
| 8 | Ll. 21258. | 8.5 | 4.40 | 0.26 | 12.5 |
| 9 | Altair | 1.0 | 0.65 | 0.20 | 16.3 |
| 10 | $\epsilon$ Indi | 5.2 | 4.60 | 0.20 | 16.3 |
| 11 | $o^{2}$ Eridani | 4.5 | 4.05 | 0.19 | 17 |
| 12 | Vega . | 0.2 | 0.36 | 0.16 | 20 |
| 13 | $\beta$ Cassiopeiæ, | 2.4 | 0.55 | 0.16 | 20 |
| 14 | 70 Ophiuchi . | 4.1 | 1.13 | 0.15 | 21 |
| 15 | $e$ Eridani . | 4.4 | 3.03 | 0.14 | 23 |
| 16 | Aldebaran | 1.0 | 0.19 | 0.12 | 27 |
| 17 | Capella | 0.2 | 0.43 | 0.11 | 29 |
| 18 | Regulus | 1.4 | 0.27 | 0.10 | 32 |
| 19 | Polaris | 2.1 | 0.05 | 0.07 | 47 |

-These are not all the stars upon Oudeman's list which are given as having parallaxes exceeding $0^{\prime \prime} .1$; but they are probably the best determined ones.
TABLE VI. - VELOCITY OF STARS IN THE LINE OF SIGHT. - Vogel.
The velocities are given in English miles per second. The sign + indicates recession.



| No. | Name. | R. A. (1900). | Dec. (1900). | Period. | a" | e | Mag. | Authority. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Ll. 9091 ( $\beta 883$ ) (A-B) | $4^{\text {h }} 45^{\mathrm{m}} .9$ | $+10^{\circ} 54^{\prime}$ | $5.5{ }^{\text {y }}$ 土 0.2 |  |  | 8-8 | See. |
| 2 | $\kappa$ Pegasi . . . . | $21 \quad 40$. 1 | $+2511$ | $11.42 \pm 0.4$ | $0^{\prime \prime} .42$ | 0.49 | 4.3-5 | 6 |
| 3 | $\delta$ Equulei (A-B) | $\begin{array}{lll}21 & 9 & .6\end{array}$ | + 937 | $11.45 \pm 0.2$ | 0.45 | 0.17 | 4.5-5 | 6 |
| 4 | $\zeta$ Sagittarii . | $18 \quad 56.3$ | $-301$ | $18.85 \pm 1.0$ | 0.69 | 0.28 | 3.9-4.4 | 6 |
| 5 | 42 Comæ Ber. | $13 \quad 5.1$ | + 184 | $25.56 \pm 0.1$ | 0.64 | 0.46 | 6-6 | 6 |
| 6 | $\zeta$ Herculis . | $\begin{array}{llll}16 & 37\end{array}$ | + 3147 | $35.00 \pm 0.3$ | 1.43 | 0.50 | 3-6 | 6 6 |
| 7 | $\eta$ Coronæ Bor. | $\begin{array}{ll}15 & 19\end{array}$ | + 3039 | $41.60 \pm 0.1$ | 0.91 | 0.27 | 5.5-6 | 6 |
| 8 | Sirius . | $6 \quad 40$. 4 | -16 34 | $51.80 \pm 0.2$ | 7.62 | 0.60 | -1.4-9 | Burnham. |
| 9 | $\gamma$ Andromedæ (B-C) | 1578 | $+4151$ | $54.0 \pm 1.0$ | 0.37 | 0.86 | 5.5-7 | See. |
| 10 | $\xi$ Ursæ Majoris . | $\begin{array}{llll}11 & 12 & .9\end{array}$ | $+326$ | $60.0 \pm 0.1$ | 2.50 | 0.40 | 4-5 | 6 |
| 11 | $\zeta$ Cancri ( $\mathrm{A}-\mathrm{B}$ ) . | $8 \quad 06.2$ | + 1758 | $60.0 \pm 0.5$ | 0.85 | 0.34 | 5.5-6.2 | 6 |
| 12 | $\gamma$ Coronæ Bor. | $\begin{array}{llll}15 & 38\end{array}$ | + 2636 | $73.0 \pm 2.0$ | 0.73 | 0.48 | 4-7 | ${ }^{6}$ |
| 13 | a Centauri . | $\begin{array}{ll}14 & 32\end{array} 6$ | $-6025$ | $81.1 \pm 0.3$ | 17.70 | 0.53 | 1-2 | 6 |
| 14 | 70 Ophiuchi | $18 \quad 0 \quad .4$ | + 233 | $88.4 \pm 1.0$ | 4.54 | 0.50 | 4.5-6 | 6 |
| 15 | $\phi$ Ursæ Majoris . | $\begin{array}{llll}9 & 45\end{array}$ | $+5433$ | $97.0 \pm 5.0$ | 0.34 | 0.44 | 5.5-5.5 | 6 |
| 16 | $\omega$ Leonis . . | $\begin{array}{lll}9 & 23\end{array}$ | + 930 | $116.2 \pm 1.0$ | 0.88 | 0.54 | 6.7 | 6 |
| 17 | $\boldsymbol{\xi}$ Boötis | $14 \quad 46 \quad .8$ | $+1931$ | 128.0 士 1.0 | 5.56 | 0.72 | 4.5-6.5 | 6 |
| 18 | $\gamma$ Virginis. | $\begin{array}{llll}12 & 36\end{array}$ | - 054 | $194 \pm 4$ | 4.00 | 0.90 | 3-3.2 | 6 6 |
| 19 | $\eta$ Cassiopeiæ . | 0 42 | + 5718 | $196 \pm 10$ | 8.21 | 0.51 | 4-7 | 66 |
| 20 | $\sigma$ Coronæ Bor. | $16 \quad 11.0$ | + 347 | $370 \pm 25$ | 3.82 | 0.54 | 6-7 | 6 |
| 21 | 36 Andromedæ | $0 \quad 49.6$ | +235 | 349 ? | 1.54 | 0.63 | 6-7 | Doberck. |
| 22 | Castor . . | $7 \quad 28.2$ | + 328 | 397? | 7.54 | 0.34 | 2.5-3 | Thiele. |

## THE GREEK ALPHABET.

| Letters. | Name. | Letters. | Name. | Letters. | Name. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A, $\alpha$, | Alpha. | I, ı, | Iota. | P, $\rho, \rho$, | Rho. |
| B, $\beta$, | Beta. | K, к, | Kappa. | $\Sigma, \sigma, s$, | Sigma. |
| 「, $\gamma$, | Gamma. | $\Lambda, \lambda$, | Lambda. | T, $\tau$, | Tau. |
| $\Delta$, $\delta$, | Delta. | $\mathbf{M}, \mu$, | Mu. | $\boldsymbol{\Upsilon}$, v, | Upsilon. |
| E, $\epsilon$, | Epsilon. | $\mathrm{N}, \nu$, | Nu . | $\Phi, \phi$, | Phi. |
| Z, $\zeta$, | Zeta. | E, $\xi$, | Xi. | $\mathbf{X}, \chi$, | Chi. |
| H, $\eta$, | Eta. | O, o, | Omicron. | $\Psi, \psi$, | Psi. |
| $\Theta, \theta, \vartheta$, | Theta. | $\Pi, \pi, \infty$ |  | $\boldsymbol{\Omega}, \omega$, | Omega. |

## MISCELLANEOUS SYMBOLS.

ठ, Conjunction.
$\square$, Quadrature.
8, Opposition.
$\Omega$, Ascending Node.
Ү, Descending Node.
$\omega$, Angle between line of nodes and line of apsides; also the obliquity of the ecliptic.

## SUGGESTIVE QUESTIONS

## FOR USE IN REVIEWS.

To many of these questions direct answers will not be found in the book; but the principles upon which the answers depend have been given, and the student will have to use his own thinking in order to make the proper application. They are inserted at the suggestion of an experienced teacher, who has found such exercises useful in her own classes.

1. What point in the celestial sphere has both its right ascension and declination zero?
2. What are the hour angle and azimuth of the zenith?
3. What angle does the (celestial) equator make with the horizon?
4. Name the (fourteen) principal points in the celestial sphere (zenith, etc.).
5. What important circles in the heavens have no correlatives on the surface of the earth?
6. If Vega comes to the meridian at 8 o'clock to-night, at what time (approximately) will it transit eight days hence?
7. What bright star can I observe on the meridian between 4 and 5 p.m., in the middle of August? (See star-maps.)
8. At what time of the year will Sirius be on the meridian at midnight?
9. The declination of Vega is $38^{\circ} 41^{\prime}$; does it pass the meridian north of your zenith, or south of it?
10. What are the right ascension and declination of the north pole of the ecliptic?
11. What are the longitude and latitude (celestial) of the north celestial pole (the one near the Pole-star)?
12. Can the sun ever be directly overhead where you live? If not, why not?
13. What is the zenith distance of the sun at noon on June 22d in New York City (lat. $40^{\circ} 42^{\prime}$ )?
14. What are the greatest and least angles made by the ecliptic with the horizon at New York? Why does the angle vary?
15. If the obliquity of the ecliptic were $30^{\circ}$, hov wide would the temperate zone be? How wide if the obliquity were $50^{\circ}$ ? What must the obliquity be to make the two temperate zones each as wide as the torrid zone?
16. Does the equinox always occur on the same days of March and September? If not, why not; and how much can the date vary?
17. Was the sun's declination at noon on March 10th, 1887, precisely the same as on the same date in 1889?
18. In what season of the year is New Year's Day in Chili?
19. When the sun is in the constellation Taurus, in what sign of the zodiac is he?
20. In what constellation is the sun when he is vertically over the tropic of Cancer? Near what star? (See star-map.)
21. When are day and night most unequal?
22. In what part of the earth are the days longest on March 20th? On June 20th? On Dec. 20th?
23. Why is it warmest in the United States when the earth is farthest from the sun?
24. What will be the Russian date corresponding to Feb. 28th, 1900, of our calendar? To May 1st?
25. Why are the intervals from sunrise to noon and from noon to sunset usually unequal as given in the almanac (For example, see Feb. 20th and Nov. 20th.)
26. At what rate does a star change its azimuth when rising or setting? (See Arts. 77 and 494, last paragraph.)
27. If the earth were to shrink to half its present diameter, what would be its mean density?
28. Is it absolutely necessary, as often stated, to find the diameter of the earth in order to find the distance of the sun from the earth? (See Arts. 127 and 3555.)
29. How will a projectile fired horizontally on the earth deviate from the line it would follow if the earth did not rotate on its axis?
30. If the earth were to contract in diameter, how would the weight of bodies on its surface be affected?
31. What keeps up the speed of the earth in its motion around the sun?
32. How many forces are necessary to keep the moon in its orbit?
33. Why is the sidereal month shorter than the synodic?
34. Does the moon rise every day of the month?
35. If the moon rises at 11.45 Tuesday night, when will it rise next?
36. How many times does the moon turn on its axis in a year?
37. What determines the direction of the horns of the moon?
38. Does the earth rise and set for an observer on the moon? If so, at what intervals?
39. How do we know that the moon is not self-luminous?
40. How do we know that there is no water on the moon?
41. How much information does the spectroscope give us about the moon?
42. What conditions must concur to produce a lunar eclipse?
43. Can an eclipse of the moon occur in the day-time?
44. Why can there not be an annular eclipse of the moon?
45. Which are most frequent at New York, solar eclipses or lunar?
46. Can an occultation of Venus by the moon occur during a lunar eclipse? Would an occultation of Jupiter be possible under the same circumstances?
47. How much difference would it make with the tides if the moon were one-fourth nearer? (Art. 271, note.)
48. Which of the heavenly bodies are not self-luminous?
49. When is a planet an evening star?
50. What planets have synodic periods longer than their sidereal periods?
51. When a planet is at its least distance from the earth, what is its apparent motion in right ascension?
52. A planet is seen $120^{\circ}$ distant from the sun; is it an inferior or a superior planet? $\cdot$
53. Can there be a transit of Mars across the sun's dise?
54. When Jupiter is visible in the evening, do the shadows of the satellites precede or follow the satellites themselves as they cross the planet's disc ?
55. Can the transits of Mercury be utilized to determine the distance of the sun, like the transits of Venus?
56. What would be the length of the month if the moon were four times as far away as now? (Apply Kepler's third law.)
57. What is the mass of a planet which has a satellite revolving in one-fifth of a lunar month, at a distance equal to that of the moon from the earth? (See Art. 309.)
58. What is the distance from the sun of an asteroid which has a period of eight years?
59. How much would the mass of the earth need to be increased to make the moon at its present distance revolve in one-fourth its present period? (See Arts. 309 and 508.)
60. Upon what circumstances does the apparent length of a comet's tail depend?
61. How can the distance of a meteor from the observer, and its height above the earth, be determined?
62. What heavenly bodies are not included in the solar system?
63. How do we know that stars are suns? How much is meant by the assertion that they are?
64. Suppose that in attempting to measure the parallax of a bright star by the differential method (Art. 522) it should turn out that the small star taken as the point to measure from, and supposed to be far beyond the bright one, should really prove to be nearer. How would the measures show the fact?
65. If $\alpha$ Centauri were to travel straight towards the sun with a uniform velocity equal to that of the earth in its orbit, how long would the journey take, on the assumption that the star's parallax is $0^{\prime \prime} .75$ ?
66. If Altair were ten times as distant from us, what would be its apparent "magnitude"? What, if it were a thousand times as remote? (See Arts. 436, 437; and remember that the apparent brightness varies inversely with the square of the distance.)

## SYNOPSIS FOR REVIEW AND EXAMINATION.

This synopsis is intended to facilitate the work of teacher and pupil in reviews and in preparation for examination.

A student who has been reasonably faithful in the original class work generally needs, in review, to look up only a comparatively small proportion of the topics he has studied; the difficulty is to know beforehand just what those topics are without going over the whole ground.

By an intelligent use of the synopsis he will be able at once to discriminate those with respect to which his memory and understanding are clear from those with respect to which he is consciously doubtful. He can thus avoid much waste of time and labor by confining his attention to the points that require it, and in this way will find it possible to deal as easily with a review lesson of fifty pages as he could with one of half the length without some such guide. The synopsis is made very full, and includes references to the appendix as well as to the body of the text. Of course it is not expected that pupils whose course has been limited to the text will look up these appendix articles unless time is specially allowed them for the purpose.

In a few instances, also, topics not mentioned in the book at all are introduced (as in article 17), with "teacher's notes" added; in hopes that the instructor will look up some of these subjects, and supplement the necessarily scanty information of the book. The interest and value of text-book work is greatly increased by the occasional introduction of something fresh from outside sources.

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## URANOGRAPHY

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A BRIEF DESCRIPTION OF

# THE CONSTELLATIONS VISIBLE IN THE UNITED STATES 

WITH

STAR-MAPS, AND LISTS OF OBJECTS OBSERVABLE WITH A SMALL TELESCOPE

## BY

## C. A. YOUNG, Ph.D., LL.D.

## Late Professor of Astronomy in Princeton University

A SUPPLEMENT TO THE AUTHOR'S "ELEMENTS OF ASTRONOMY FOR HIGH SCHOOLS AND ACADEMIES"

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## PREFACE.

This brief description of the constellations was prepared, at the suggestion of a number of teachers, as an integral part of the author's "Elements of Astronomy." It has been thought best, however, for various reasons, to put it into such a form that it can be issued separately, and used if desired in connection with the larger "General Astronomy," or with any other text-book. Since the Uranography also has to be used more or less in the open air at night, many will probably prefer to have it by itself, so that its use need not involve such an exposure of the rest of the text-book. All references marked Astr. are to the articles of the "Elements of Astronomy."

## ALPHABETICAL LIST OF THE CONSTELLATIONS DESCRIBED OR MENTIONED IN THE URANOGRAPHY.



## THE GREEK ALPHABET.

| Letters. | Name. | Letters. | Name. | Letters. | Name. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A, $\alpha$, | Alpha. | I, ı, | Iota. | P, $\rho, \rho$, | Rho. |
| B, $\beta$, | Beta. | K, к, | Kappa. | $\Sigma, \sigma, s$, | Sigma. |
| $\Gamma, \gamma$, | Gamma. | $\Lambda, \lambda$, | Lambda. | T, $\tau$, | Tau. |
| $\Delta, \delta$, | Delta. | $\mathbf{M}, \mu$, | Mu. | $\boldsymbol{r}$, v, | Upsilon. |
| E, $\epsilon$, | Epsilon. | $\mathrm{N}, ~ v$, | Nu. | $\Phi, \phi$, | Phi. |
| Z, $\zeta$, | Zeta. | $\Xi, \xi$, | Xi. | $\mathbf{X}, \chi$, | Chi. |
| H, $\eta$, | Eta. | O, о, | Omicron. | $\Psi, \psi$, | Psi. |
| $\Theta, \theta, \vartheta$ | Theta. | $\Pi, \pi$, | Pi. | $\Omega, \omega$, | Omega. |

## URANOGRAPHY,

## or

## A Description of the Constellations.

1. A general knowledge of the constellations sufficient to enable one to recognize readily the more conspicuous stars and their principal configurations, is a very desirable accomplishment, and not difficult to attain. It requires of course the actual study of the sky for a number of evenings in different parts of the year; and the study of the sky itself must be supplemented by continual reference to a celestial globe or star-map, in order to identify the stars observed and fix their designations. A well-made globe of sufficient size is the best possible help, because it represents things wholly without distortion, and is easily "rectified" (Astr. $528^{1}$ ) for any given hour, so that the stars will all be found in the proper quarter of the (artificial) heavens, and in their true relations. But a globe is clumsy, inconvenient out of doors, and liable to damage; and a good star-map properly used will be found but little inferior in efficiency, and much more manageable.
2. Star-Maps. - Such maps are made on various systems, each presenting its own advantages. None are without more or less distortion, especially near the margin, though they

[^65]differ greatly in this respect. In all of them the heavens are represented as seen from the inside, and not as on the globe, which represents the sky as if seen from the outside; i.e., the top of the map is north, and the east is at the left hand; so that if the observer faces the south and holds up the map before and above him, the constellations which are near the meridian will be pretty truly represented.
3. We give a series of four small maps which, though hardly on a large enough scale to answer as a satisfactory celestial atlas, are quite sufficient to enable the student to trace out the constellations and identify the principal stars.

In the map of the north circumpolar regions (Map I.), the pole is in the centre, and at the circumference the right-ascension hours are numbered in the same direction as the figures upon a watch face; but with 24 hours instead of 12 . The parallels of declination are represented by equidistant and concentric circles. On the three other rectangular maps, which show the equatorial belt of the heavens lying between $+50^{\circ}$ and $-50^{\circ}$ of declination, the parallels of declination are equidistant horizontal lines, while the hour-circles are vertical lines also equidistant, but spaced at a distance which is correct for declination $35^{\circ}$, and not at the equator. This keeps the distortion within reasonable bounds even near the margin of the map, and makes it very easy to lay off the place of any object for which the right ascension and declination are given.

The hours of right ascension are indicated on the central horizontal line, which is the equator, and at the top of the map are given the names of the months. The word September, for instance, means that the stars which are directly under it upon the map will be near the meridian about nine o'clock in the evening during that month.
4. The maps show all the stars down to the $4 \frac{1}{2}$ magnitude - all that are easily visible on a moonlight night. A few smaller stars are also inserted, where they mark some peculiar configuration or point out some interesting telescopic object. So far as practicable, i.e., north of $-30^{\circ}$ Declination, the magnitudes of Pickering's "Harvard Photometry" are used. The places of the stars are for 1900.

In the designation of clusters and nebulæ the letter M. stands for "Messier," who made the first catalogue of 103 such objects in 1784 ; e.g., 97 M. designates No. 97 on that list. A few objects from Herschel's catalogue are denoted by $\begin{aligned} \\ \text { with a number following. }\end{aligned}$

The student or teacher who possesses a telescope is strongly urged to get Webb's "Celestial Objects for Common Telescopes." It is an invaluable accessory. (Longmans, Green \& Co., N. Y.)

## THE CIRCUMPOLAR CONSTELLATIONS.

We begin our study of Uranography with the constellations which are circumpolar (i.e., within $40^{\circ}$ of the north pole), because these are always visible in the United States, and so can be depended on to furnish land (or rather sky) -marks to aid in identifying and tracing out the others.
5. Ursa Major, the Great Bear (Map I.). - Of these circumpolar constellations none is more easily recognizable than Ursa Major. Assuming the time of observation as about eight o'clock in the evening on Sept. 22d (i.e., $20^{\mathrm{h}}$ sidereal time), it will be found below the pole and to the west. Hold the map so that the VIII. is at the bottom, and it will be rightly placed for the time assumed.

The familiar Dipper is sloping downward in the northwest, composed of seven stars; all of about the second magnitude excepting $\delta$ (at the junction of the handle to the bowl), which is of the third. The stars $\alpha$ and $\beta$ are known as the "Pointers," because the line drawn from $\beta$ through $\alpha$, and produced about $30^{\circ}$, passes very near the Pole-star.

The dimensions of the Dipper furnish a convenient scale of angular measure. From $\alpha$ to $\beta$ is $5^{\circ} ; \alpha$ to $\delta$ is $10^{\circ} ; \beta$ to $\gamma, 8^{\circ}$; from $\alpha$ to $\eta$ at the extremity of the Dipper-handle (which is also the Bear's tail) is $26^{\circ}$.
6. The Dipper (known also in England as the "Plough," and as the "Wain," or wagon) comprises but a small part of
the whole constellation. The head of the Bear, indicated by a scattered group of small stars, is nearly on the line from $\boldsymbol{\delta}$ through $a$, carried on about $15^{\circ}$; at the time assumed ( $20^{\text {h }}$ sid. time), it is almost exactly below the pole. Three of the four paws are marked each by a pair of third or fourth magnitude stars $1 \frac{1}{2}^{\circ}$ or $2^{\circ}$ apart. The three pairs are nearly equidistant, about $20^{\circ}$ apart, and almost on a straight line parallel to the diagonal of the Dipper-bowl from $a$ to $\gamma$, but some $20^{\circ}$ south of it. Just now ( $20^{\mathrm{h}}$ sid. time) they are all three very near the horizon for an observer in latitude $40^{\circ}$, but during the spring and summer they can be easily made out.

## 7. Names ${ }^{1}$ of Principal Stars. -

$$
\begin{array}{ll}
\text { a. Dubie. } & \text { є. Alioth. } \\
\beta . \text { Merak. } & \zeta . \text { Mizar. The little star near it is } \\
\gamma . \text { Phecda. } & \text { Alcor, the "rider on his horse." } \\
\delta . \text { Megrez. } & \eta \text { Benetnasch or Alkaid. }
\end{array}
$$

Double Stars: (1) $\zeta$ (Mizar), Mags. 3 and 5; Pos. ${ }^{2} 149^{\circ}$; Dist. $14^{\prime \prime} .5$. In looking at this object the tyro will be apt to think that the small star shown by the telescope is identical with Alcor: a very low power eye-piece will correct the error. (Astr. Fig. 113.) The large star is itself a "spectroscopic binary" (see Art. 465*). (2) $\xi$, the southern one of the pair which marks the left hind paw. Binary: Mags. 4 and 5; Pos. (1910) (about) $120^{\circ}$, Dist. (about) $3^{3 \prime}$. Position and distance both change rapidly, the period being only 61 years. This was the first binary whose orbit was computed.

Clusters and Nebulce: (1) 81 and 82 M., A.R. $9^{\text {h }} 45^{\mathrm{m}}$, Dec. $69^{\circ} 44^{\prime}$. Two nebulx, one pretty bright, about half a degree apart. (2) 97 M ., A.R. $11^{\mathrm{h}} 07^{\mathrm{m}}$, Dec. $55^{\circ} 43^{\prime}-2^{\circ}$ south-following $\beta$. A planetary nebula.

[^66]8. Ursa Minor, the Lesser Bear (Map I.). - The line of the "Pointers" unmistakably marks out the Pole-star ("Polaris" or "Cynosura"), a star of the second magnitude standirig alone. It is at the end of the tail of Ursa Minor, or at the extremity of the handle of the "Little Dipper," for in Ursa Minor, also, the seven principal stars form a dipper, though with the handle bent in a different way from that of the other Dipper. Beginning at "Polaris" a curved line (concave towards Ursa Major) drawn through $\delta$ and $\epsilon$ brings us to $\zeta$, where the handle joins the bowl. Two bright stars (second and third magnitude), $\beta$ and $\gamma$, correspond to the pointers in the larger Dipper, and are known as the "Guardians of the


Fig. A. - Measurement of Distance and Position-Angle of a Double Star. Pole": $\beta$ is called "Kochab." The remaining corner of the bowl is marked by the faint star $\eta$ with another still smaller one near it.

The Pole lies about $1_{4} \frac{1}{4}^{\circ}$ from the Pole-star, on the line joining it to $\zeta$ Ursæ Majoris (at the bend in the handle of the large Dipper).

Telescopic Object. Polaris has a companion of the $9 \frac{1}{2}$ magnitude, distant $18^{\prime \prime} .6$, - visible with a two-inch telescope.
9. Cassiopeia (Map I.).-This constellation lies on the opposite side of the pole from the Dipper at about the same
distance as the "Pointers," and is easily recognized by the zigzag, "rail-fence" configuration of the five or six bright stars that mark it. With the help of the rather inconspicuous star $\kappa$, one can make out of them a pretty good chair with the feet turned away from the pole. But this is wrong. In the recognized figures of the constellation the lady sits with feet towards the pole, and the bright star $\alpha$ is in her bosom, while $\zeta$ and the other faint stars south of $a$, are in her head and uplifted arms: $\iota$, on the line from $\delta$ to $\epsilon$ produced, is in the foot. The order of the principal stars is easily remembered by the word Bagdei; i.e., $\beta, a, \gamma, \delta, \epsilon$, ८.

Names of Stars: $a$ (which is slightly variable) is known as Schedir; $\beta$ is called Caph.

Double Stars: (1) $\eta$, Mags. 4-7 $\frac{1}{2}$. Large star orange; small one purple. Pos. $170^{\circ} \pm$, Dist. $5^{\prime \prime} .5$. Binary, with a period of some 200 years. Easily recognized by its position about half-way between $\alpha$ and $\gamma$, a little off the line. (2) $\psi$, A.R. $1^{\mathrm{h}} 17^{\mathrm{m}}$, Dec. $67^{\circ} 21^{\prime}$; Triple; Mags. $4 \frac{1}{2}, 9$ and 9 ; Pos. A to (B+C) $106^{\circ}$, Dist. $29^{\prime \prime}$; Pos. B-C $257^{\circ}$, Dist. 2'.9. Found on a line from $\eta$ through $\gamma$ produced three times the distance $\eta-\gamma$ : rather difficult for a four-inch telescope.
10. The Sidereal Time determined by the Apparent Position of Cassiopeia. - The line from the Pole-star through Caph or $\beta$ Cassiopeix (which is the leader of all the bright stars of the constellation in their daily motion) is almost exactly parallel to the Equinoctial Colure. When, therefore, this star is vertically above the Pole-star it is sidereal noon; it is $6^{\mathrm{h}}$ when it is on the great circle (not the parallel of altitude) drawn from the Pole-star to the west point of the Horizon; $12^{\mathrm{h}}$ when vertically below it; and $18^{\mathrm{h}}$ when due east. A little practice will enable one to read the sidereal time from this celestial clock with an error not exceeding 15 or 20 minutes.
11. Cepheus (Map I.). - This constellation contains very few bright stars. At the assumed time ( $20^{\mathrm{h}}$ sidereal) it is above and west of Cassiopeia, not having quite reached the meridian above the pole. A line carried from a Cassiopeiæ through $\beta$, and produced $20^{\circ}$ (distance $a \ldots \beta=\check{b}^{\circ}$ nearly)
will pass very near to $\alpha$ Cephei, a star of the third magnitude, in the king's right shoulder. $\beta$ Cephei is about $8^{\circ}$ due north of $\alpha$, and $\gamma$ about $12^{\circ}$ from $\beta$, both also of third magnitude : $\gamma$ is so placed that it is at the obtuse angle of a rather flat isosceles triangle of which $\beta$ Cephei and the Pole-star form the two other corners. Cepheus is represented as sitting behind Cassiopeia (his wife) with his feet upon the tail of the Little Bear, $\gamma$ being in his left knee. His head is marked by a little triangle of fourth magnitude stars, $\delta, \epsilon$, and $\zeta$, of which $\delta$ is a remarkable variable with a period of $5 \frac{1}{3}$ days (see Astr. Table IV.). There are several other small variables in the same neighborhood, but none of them are shown on the map.

Names of Stars : $\alpha$ is Alderamin, and $\beta$ is Alphirk.
Double Stars : (1) $\beta$, Mags. 3 and 8; Pos. $251^{\circ}$; Dist. $14^{\prime \prime}$. (2) $\delta$, Mags. larger star 3.7 to 5 (variable), smaller one 7 ; Pos. $192^{\circ}$, Dist. $41^{\prime \prime}$; Colors, yellow and blue. (3) к, A.R. $20^{\mathrm{h}} 13^{\mathrm{m}}$, Dec. $77^{\circ} 19^{\prime}$; Mags. 4.5 and 8.5 ; Pos. $124^{\circ}$; Dist. 7. ${ }^{\prime \prime} 5$; Colors, yellow and blue.
12. Draco (Map I.). - The constellation of Draco is characterized by a long, sinuous line of stars, mostly small, extending half-way around the pole and separating the two Bears. A line from $\delta$ Cassiopeiæ drawn through $\beta$ Cephei and extended about as far again will fall upon the head of Draco, marked by an irregular quadrilateral of stars, two of which are of the $2 \frac{1}{2}$ and 3 d magnitude. These two bright stars about $4^{\circ}$ apart are $\beta$ and $\gamma$; the latter in its daily revolution passes almost exactly through the zenith of Greenwich, and it was by observations upon it that the aberration of light was discovered (Astr. 125). The nose of Draco is marked by a smaller star, $\mu$, some $5^{\circ}$ beyond $\beta$, nearly on the line drawn through it from $\gamma$. From $\gamma$ we trace the neck of Draco, eastward and downward ${ }^{1}$ towards the Pole-star until we come to $\delta$ and $\epsilon$ and some smaller stars near them. There the direction of the line is reversed,

[^67]so that the body of the monster lies between its own head and the bowl of the Little Dipper, and winds around this bowl until the tip of the creature's tail is reached at the middle of the line between the Pointers and the Pole-star. The constellation covers more than $12^{\mathrm{h}}$ of right ascension.
13. One star deserves special notice: $a$, a star of the $3 \frac{1}{2}$ magnitude which lies half-way between Mizar ( $\zeta$ Urs. Maj.) and the Guards ( $\beta$ and $\gamma$ Urs. Min.) ; 4700 years ago it was the Pole-star, within $10^{\prime}$ or $15^{\prime}$ of the pole, and much nearer than Polaris is at present, or ever will be. It is probable that its brightness has considerably diminished within the last 200 years; since among the ancient and mediæval astronomers it was always reckoned of the second magnitude.

Names of Stars : $\alpha$ is Thuban ; $\beta$, Alwaid; and $\gamma$, Etanin.
Double Stars: (1) $\mu$, Mags. 4 and $4 \frac{1}{2}$; Pos. $165^{\circ}$; Dist. $2^{\prime \prime} .5$. Binary, with a probable period of about 600 years. (2) $\epsilon$, Mags. 4, 8 ; Pos. $0^{\circ} .0$; Dist. $2^{\prime \prime} .9$; yellow and blue. Nebula, A.R. $17^{\mathrm{h}} 59^{\mathrm{m}}$; Dec. $66^{\circ} 38$. Planetary, like a star out of focus. This object is almost exactly at the pole of the ecliptic, about midway between $\delta$ and $\zeta$ Draconis, but a little nearer $\zeta$.
14. Camelopardalis (Map I.).-This is the only remaining one of the strictly circumpolar constellations - a modern asterism containing no stars above fourth magnitude, and constituted by Hevelius simply to cover the great empty space between Cassiopeia and Perseus on one side, and Ursa Major and Draco on the other. The animal stands on the head and shoulders of Auriga, and his head is between the Pole-star and the tip of the tail of Draco.

The two constellations of Perseus (which at $20^{\mathrm{h}}$ sidereal time is some $20^{\circ}$ below Cassiopeia) and of Auriga are partly circumpolar, but on the whole can be more conveniently treated in connection with the equatorial maps. Capella, the brightest star of Auriga, and next to Vega and Arcturus the brightest star in the northern hemisphere, at the time assumed (Sept. 22, 8 p.m.), is a few degrees above the horizon in the N.E. Between it and the nose of Ursa Major is part of the constellation of the Lyñx, - a modern asterism made, like Camelopardalis, merely to fill a gap.
15. The Milky Way in the Circumpolar Region. - The only circumpolar constellations traversed by it are Cassiopeia and Cepheus. It enters the circumpolar region from the constellation of Cygnus, which at $20^{\mathrm{h}}$ sidereal time is just in the zenith, sweeps down across the head and shoulders of Cepheus, and on through Cassiopeia and Perseus to the northeastern horizon in Auriga. There is one very bright patch a degree or two north of $\beta$ Cassiopeiæ; and half-way between Cassiopeia and Perseus there is another bright cloud in which is the famous cluster of the "Sword Handle of Perseus" - a beautiful object for even the smallest telescope.
16. Andromeda (Map II.). - Passing now to the equatocial maps and beginning with the northwestern corner of Map No. II., we come first to the constellation of Andromeda, which will be found exactly overhead in our latitudes about 10 o'clock in the middle of November, or at 8 o'clock a month later. Its characteristic configuration is the line of three second-magnitude stars, $\alpha, \beta$, and $\gamma$, extending east and north from $\alpha$, which itself forms the N.E. corner of the so-called " Great Square of Pegasus," and is sometimes lettered as $\delta$ Pegasi. This star may readily be found by extending an imaginary line from Polaris through $\beta$ Cassiopeiæ, and producing it about as far again: $\alpha$ is in the head of Andromeda, $\beta$ in her waist, and $\gamma$ in the left foot. About half-way from $\alpha$ to $\beta$, a little south of the line, is $\delta$ (of the third magnitude) with $\pi$ and $\epsilon$ of the fourth magnitude near it. A line drawn northwesterly from $\beta$ nearly at right angles to the line $\beta \gamma$, will pass through $\mu$ at a distance of about $5^{\circ}$, and produced another $5^{\circ}$ will strike the "great nebula" (Astr. 470), which forms a little obtuseangled triangle with $\nu$ and a sixth-magnitude star known as 32 Andromedæ.

Andromeda has her mother, Cassiopeia, close by on the north, and at her feet is Perseus, her deliverer, while her head rests upon the
shoulder of Pegasus, the winged horse which brought Perseus to her rescue. To the south, beyond the intervening constellations of Aries and Pisces, Cetus, the sea-monster, who was to have devoured her, stretches his ungainly bulk.

Names of Stars. $\quad \alpha$, Alpheratz ; $\beta$, Mirach; $\gamma$, Almaach.
Double Stars. (1) $\gamma$, Mags. 3, 5; Pos. $62^{\circ}$; Dist. $11^{\prime \prime}$; colors, orange and greenish blue - a beautiful object. The small star is itself double, but at present so close as to be beyond the reach of any but very large instruments (Astr. Fig. 113). (2) $\pi$ ( $2^{\circ}$ N. and a little west of $\delta$ ), Mags. 4, 9 ; Pos. $174^{\circ}$; Dist. $36^{\prime \prime}$; white and blue.

Nebulæ. M. 31; the great nebula; visible to naked eye. M. 32; small, round, and bright, is in the same low-power field with 31 ; south and east of it.
17. Pisces (Map II.). - Immediately south of Andromeda lies Pisces, the first of the zodiacal constellations, though now occupying (in consequence of precession) the sign of Aries. It has not a single conspicuous star, and is notable only as containing the vernal equinox, or first of Aries, which lies near the southern boundary of the constellation in a peculiarly starless region. A line from $\alpha$ Andromedæ through $\gamma$ Pegasi continued as far again strikes about $2^{\circ}$ east of the equinox.

The body of the southern fish lies about $15^{\circ}$ south of the middle of the southern side of the great square of Pegasus, and is marked by an irregular polygon of small stars, $5^{\circ}$ or $6^{\circ}$ in diameter. A long crooked "ribbon" of litcle stars runs eastward for more than $30^{\circ}$, terminating in a Piscium, called El Rischa, a star of the fourth magnitude $20^{\circ}$ south of the head of Aries. From there another line of stars leads up N.W. in the direction of $\delta$ Andromedæ to the northern fish, which lies in the vacant space south of $\beta$ Andromedæ.

Double Stars. (1) a, Mags. 4, 5.5; Pos. $324^{\circ}$; Dist. $3^{\prime \prime}$.
$\psi^{\prime}\left(2^{\circ}\right.$ S.E. of $\eta$ Andromedæ - see map), Mags. 4.9, 5; Pos. $160^{\circ}$; Dist. 31".
18. Triangulum (Map II.). - This little constellation, insignificant as it is, is one of Ptolemy's ancient 48. It lies half-way between $\gamma$ Andromedæ and the head of Aries, characterized by three stars of the third and fourth magnitudes.

Double Stars. (1) 1 or 6 ( $5^{\circ}$ nearly due south of $\beta$ Trianguli, and at the obtuse angle of an isosceles triangle of which $\alpha$ and $\gamma$ are the other two corners), Mags. 5, 6.5; Pos. $76^{\circ}$; Dist. $4^{\prime \prime}$; topaz-yellow and green.
19. Aries (Map II.). - This is the second of the zodiacal constellations (now occupying the sign of Taurus). It is bounded north by Triangulum and Perseus, west by Pisces, south by Cetus, and east by Taurus. The characteristic star-group is that composed of $a, \beta, \gamma$ (see map), about $20^{\circ}$ due south of $\gamma$ Andromedæ: $\alpha$, a star of the $2 \frac{1}{2}$ magnitude is fairly conspicuous, forming as it does a large isosceles triangle with $\beta$ and $\gamma$ Andromedæ.

Names of Stars. a, Hamal ; $\beta$, Sheratan; $\gamma$, Mesartim.
Double Stars. (1) $\gamma$, Mags. 4.5, 5; Pos. $0^{\circ}$; Dist. $8^{\prime \prime}$.8. (This is probably the earliest known double star; noticed by Hooke in 1664.) (2) $\epsilon$, Mags. 5, 6.5; Pos. $200^{\circ}$; Dist. $1^{\prime \prime} .2$. (About one-third of the way from $a$ Arietis towards Aldebaran, $\zeta$ is $4^{\circ}$ beyond it on the same line.) This is probably too difficult for any instrument less than 4 or $4 \frac{1}{2}$ inches' aperture. (3) $\pi$, Triple ; Mags. 5, 8.5, and 11; A-B, Pos. $122^{\circ}$; Dist. $3^{\prime \prime} .1$; A-C, Pos. $110^{\circ}$; Dist. $25^{\prime \prime}$. (At the southern corner of a nearly isosceles triangle formed with $\epsilon$ and $\tau, \epsilon$ being at the obtuse angle.)

The star 41 Arietis ( $3 \frac{1}{2}$ mag.), which forms a nearly equilateral triangle with $\alpha$ Arietis and $\gamma$ Trianguli, constitutes, with two or three other small stars near it, the constellation of Musca (Borealis), a constellation, however, not now generally recognized.
20. Cetus (Map II.). - South of Aries and Pisces lies the huge constellation of Cetus; which backs up into the sky from the southeastern horizon. The head lies some $20^{\circ}$ S.E. of $\alpha$ Arietis, marked by an irregular pentagon of stars, each side of which is $5^{\circ}$ or $6^{\circ}$ long. The southern edge of it is formed by the stars $\alpha$ ( $2 \frac{1}{2}$ mag.) and $\gamma\left(3 \frac{1}{2}\right.$ mag.) : $\delta$ lies nearly south of $\gamma$. $\beta$, the brightest star of the constellation (2d magnitude), stands alone nearly $40^{\circ}$ west and south of a. About half-way
from $\beta$ to $\gamma$ the line joining them passes through a characteristic quadrilateral (see map), the N.E. corner of which is composed of two fourth-magnitude stars, $\zeta$ and $\chi$. The remarkable variable o Ceti (Mira) lies almost exactly on the line joining $\gamma$ and $\zeta$, a little nearer to $\gamma$ than to $\zeta$. It is visible to the naked eye for about a month or six weeks every eleven months, when near its maximum.

Names of Slars. a, Menkar; $\beta$, Diphda or Deneb Kaitos; $\zeta$, Baten Kaitos; o, Mira.

Double Stars. (1) $\gamma$, Mags. 3.5, 7; Pos. $290^{\circ}$; Dist. $2^{\prime \prime} .5$; yellow and blue.
South of Cetus lies the constellation of Sculptoris Apparatus (usually known simply as Sculptor), which, however, contains nothing that requires notice here. South of Sculptor, and close to the horizon, even when on the meridian, is Phœenix. It has some bright stars, but none easily observable in the United States.
21. Perseus (Maps I. and II.). - Returning now to the northern limit of the map, we come to the constellation of Perseus. Its principal star is $a$, rather brighter than the standard second magnitude, situated very nearly on the prolongation of the line of the three chief stars of Andromeda. A very characteristic configuration is "the segment of Perseus" (Map I.), a curved line, formed by $\delta, a, \gamma$, and $\eta$, with some smaller stars, concave towards the northeast, and running along the line of the Milky Way towards Cassiopeia. The remarkable variable star $\beta$, or Algol (Astr. 453), is situated about $9^{\circ}$ south and a little west of $\alpha$, at the right angle of a right-angled triangle which it forms with $a$ (Persei) and $\gamma$ Andromedæ. Some $8^{\circ}$ south and slightly east of $\delta$ is $\epsilon$, and $8^{\circ}$ south of $\epsilon$ are $\zeta$ and $o$ of the fourth magnitude in the foot of the hero. Algol and'a few small stars near it form "Medusa's Head."

Names of Stars. $a$ is Marfak, or Algenib; $\beta$ is Algol.
Double Stars. (1) є, Mags. 3.5, 9; Pos. $10^{\circ}$; Dist. $8^{\prime \prime} .4$. (2) ¢́,

Quadruple; Mags. 3.5, 10, 11, 12 ; Pos. A-B, $207^{\circ}$; Dist. $13^{\prime \prime} .2,83^{\prime \prime}$, $121^{\prime \prime}$. (3) $\eta$, Mags. 5, 8.5 ; Pos. $300^{\circ}$; Dist. $28^{\prime \prime}$; orange and blue.

Clusters. (1) H VI. 33 and 34. Magnificent. Half-way between $\gamma$ Persei and $\delta$ Cassiopeiæ. (2) M. 34; A.R. $2^{\mathrm{h}} 34^{\mathrm{m}}$; Dec. $42^{\circ} 11^{\prime}$; coarse, with a pretty double star (eighth mag.) included.
22. Aurīga (Maps I. and II.). - Proceeding east from Perseus we come to Auriga, instantly recognized by the bright yellow star Capella (the Goat) and her attendant Hœdi (or Kids). Capella, a Aurigæ, according to Pickering, is precisely of the same brightness as Vega (Mag. $=0.2$ ), both of them being about $\frac{1}{5}$ of a magnitude fainter than Arcturus, but distinctly brighter than any other stars visible in our latitudes except Sirius itself. About $10^{\circ}$ east of Capella is $\beta$ Aurigæ of the second magnitude, and $8^{\circ}$ south of $\beta$ is $\theta$ of the third magnitude; $\delta$ Aurigæ is $10^{\circ}$ north of $\beta$ in the circumpolar region. $\epsilon, \zeta$, and $\eta, 4^{\circ}$ or $5^{\circ}$ S.W. of $\alpha$, are the "Kids."

Names of Stars. a, Capella; $\beta$, Menkalinan.
Double Stars. (1) $\omega$, Mags. 5, 9; Pos. $353^{\circ}$; Dist. $7^{\prime \prime}$; white, light blue. $\beta$ is a spectroscopic double (see Art. 465*).

Clusters. (1) M. 37; A.R. $5^{\mathrm{h}} 44^{\mathrm{m}}$; Dec. $32^{\circ} 31^{\prime}$ (on the line from $\theta$ Aurigæ to $\zeta$ Tauri, one-third of the way from $\theta$ ). Fine for small instrument. (2) M. 38; A.R. $5^{\mathrm{h}} 21^{\mathrm{m}}$; Dec. $35^{\circ} 47$ '. Nearly at the middle of the line from $\theta$ to $\omega$. (3) M. 36 ; A.R. $5^{\mathrm{h}} 28^{\mathrm{m}}$; Dec. $34^{\circ} 3^{\prime}$. One-third of the way from M. 38 to M. 37.
23. Taurus (Map II.). - This, the third of the zodiacal constellations, is bounded north by Perseus and Auriga, west by Aries, south by Eridanus and Orion, and east by Orion and Gemini. It is unmistakably characterized by the Pleiades, and by the V-shaped group of the Hyades which forms the face of the bull, with the red Aldebaran ( $\alpha$ Tauri) blazing in the creature's eye, as he charges down upon Orion. His horns reach out towards Gemini and Auriga, and are tipped with the second and third magnitude stars $\beta$ and $\zeta$. As in the case of Pegasus, only the head and shoulders appear in the constella-
tion. Six of the Pleiades are easily visible, and on a dark night a fairly good eye will count nine (see Astr. 469). With a 3 -inch telescope about 100 stars are visible in the cluster. In the Hyades the pretty naked-eye double $\theta_{1}, \theta_{2}$, is worth noting.

Names of Stars. a, Aldebaran ; $\beta, E l$ Nath; $\eta$ (the brightest of the Pleiades), Alcyone. For the names of the other Pleiades, see the figure in Art. 469 of the Astronomy.

Double Stars. (1) a has a small, distant companion, 12th magnitude; Pos. $36^{\circ}$; Dist. $1^{\prime} 48^{\prime \prime}$. It has also a second companion much nearer and more minute, but far beyond the reach of ordinary telescopes. (2) $\tau$, Mags. 5 and 8; Pos. $210^{\circ}$; Dist. $62^{\prime \prime}$; white and violet. Found by drawing a line from $\gamma$ (at the point of the V of the Hyades) through $\epsilon$, and producing it as far again.

Nebula. M. 1; A.R. $5^{\text {b }} 27^{\mathrm{m}}$; Dec. $21^{\circ} 56^{\prime}$, about $1^{\circ}$ west and a little north of $\zeta$. Often mistaken for a comet. The so-called "Crab Nebula."
24. Orīon (not O'rǐ-on) (Map II.). - On the whole this is the finest constellation in the heavens. As he stands facing the bull his shoulders are marked by the two bright stars, a and $\gamma$, the former of which in color and brightness closely matches Aldebaran. In his left hand he holds up the lion skin, indicated by the curved line of little stars between $\gamma$ and the Hyades. The top of the club, which he brandishes in his right hand, lies between $\zeta$ Tauri and $\mu$ and $\eta$ Geminorum. His head is marked by a little triangle of stars of which $\lambda$ is the chief. His belt consists of three stars of the second magnitude pointing obliquely downward towards Sirius. It is very nearly $3^{\circ}$ in length, with the stars in it equidistant like a measuring-rod, so that it is known in England as the "Ell and Yard." From the belt hangs the sword, composed of three smaller stars lying more nearly north and south: the middle one of them is the multiple $\theta$ in the great nebula. $\beta$ Orionis, or Rigel, a magnificent white star, is in the left foot, and $\kappa$ is in the right knee. Orion has no right foot, or if he has, it is
hidden behind Lepus. The quadrilateral $a, \gamma, \delta, \kappa$, with the diagonal belt $\delta, \epsilon, \zeta$, once learned can never be mistaken for anything else in the heavens.
25. Names of Stars. $\alpha$, Betelgeuse; $\beta$, Rigel; $\gamma$, Bellatrix; $\kappa$, Saiph; $\delta$, Mintaka; $\epsilon$, Alnilam; $\zeta$, Alnitak.

Double Stars. In these Orion is remarkably rich. (1) $\beta$ (Rigel), Mags. 1 and 9 ; Pos. $200^{\circ}$; Dist. $9^{\prime \prime} .5$; both white, - a beautiful and easy object. (2) $\delta$ (the westernmost star in the belt), Mags. 2.5 and 7 ; Pos. 0; Dist. $53^{\prime \prime}$. (3) $\zeta$, Triple; Mags. 2.5, 6.5, 10 ; A-B, Pos. $155^{\circ}$, Dist. $2^{\prime \prime} .4$; A-C, Pos. $9^{\circ}$, Dist. $59^{\prime \prime}$. (4) 九, Triple; Mags. 3.5, 8.5, 11 ; A-B, Pos. $142^{\circ}$, Dist. $11^{\prime \prime} .5$; A-C, Pos. $103^{\circ}$, Dist. $49^{\prime \prime}$. (This is the lowest star in the sword, just below the nebula.) (5) $\theta$, Multiple, the trapezium in the nebula. Four stars are easily seen by small telescopes (Astr. Fig. 113). (6) $\sigma$, Triple; Mags. 4, 8, 7; A-B, Pos. $84^{\circ}$, Dist. $12^{\prime \prime} .5$; A-C, Pos. $61^{\circ}$, Dist. $42^{\prime \prime}\left(1_{2}^{\circ} \mathrm{S}\right.$.W. of $\left.\zeta\right)$.

Nebula. M. 42 ; attached to the multiple star $\theta$. The nebula of all the heavens; by far the finest known, though in a small telescope wanting much of the beauty brought out by a larger one.
26. Eridănus (Map II.). - This constellation lies south of Taurus, in the space between Cetus and Orion, and extends far below the southern horizon. Its brightest star a (Achernar) is never visible in the United States.

Starting with $\beta$ of the third magnitude, about $3^{\circ}$ north and a little west of Rigel ( $\beta$ Orionis), one can follow a sinuous line of stars, some of them of the third and fourth magnitudes, westward about $30^{\circ}$ to the paws of Cetus, $10^{\circ}$ south of $a$ Ceti; there the stream turns at right angles southwards for $10^{\circ}$, then southeast for about $20^{\circ}$, and finally southwestward to the horizon. One could succeed in fully tracing it out, however, only by help of a map on a larger scale than the one we are able to present.

Names of Stars. $\beta$, Cursa; $\gamma$, Zaurack.
Double Stars. (1) 32 ; A.R. $3^{\text {h }} 48^{\mathrm{m}}$, Dec. S. $3^{\circ} 19^{\prime}$; Mags. 5, 7 ; Pos. $347^{\circ}$; Dist. $6^{\prime \prime} .6$; yellow, blue ; very fine. (2) $0^{2}$, Triple; A.R.
$4^{\mathrm{h}} 10^{\mathrm{m}} ;$ Dec. S. $7^{\circ} 50^{\prime}$; Mags. $5,10,10$; Pos. A, $\frac{(\mathrm{B}+\mathrm{C})}{2}, 108^{\circ}$; Dist. $83^{\prime \prime}$; Pos. B-C, $110^{\circ}$, Dist. $4^{\prime \prime}$; very pretty.
27. Lepus (Map II.). - This little constellation (one of the ancient 48) lies just south of Orion, occupying a space of some $15^{\circ}$ square. Its characteristic configuration is a quadrilateral of third and fourth magnitude stars, with sides from $3^{\circ}$ to $5^{\circ}$ long, $10^{\circ}$ south of $\kappa$ Orionis, and $15^{\circ}$ west and a little south of Sirius.

Double Stars. (1) $\gamma$ (the S.E. corner of the quadrilateral) is a coarse double. Mags. 4, 6.5; Pos. $350^{\circ}$; Dist. $93^{\prime \prime}$. (2) $\kappa\left(5 \frac{1}{2}^{\circ}\right.$ south of Rigel), Mags. 5 and 9 ; Pos. $0^{\circ}$; Dist. $3^{\prime \prime} . /$.
28. Columba (Noah's Dove) (Map II.). - This is next south of Lepus : too far south to be well seen in the Northern States. Its principal star, $\alpha$, or Phact, of the $2 \frac{1}{2}$ magnitude, is readily found by drawing a line from Procyon to Sirius, and prolonging it nearly the same distance. And in passing we may note that a similar line drawn from $\alpha$ Orionis through Sirius and produced, will strike near $\zeta$ Argus, or " Naos," a star about as bright as Phact, - the two lines which intersect at Sirius making the so-called "Egyptian X."
29. Lynx (Map I., II., and III.). - Returning now to the northern limit of the map, we find the modern constellation of the Lynx lying just east of Auriga and enveloping it on the north and in the circumpolar region. It contains no stars above the fourth magnitude, and is of no importance except as occupying an otherwise vacant space.

Double Stars. (1) 38, or $\rho$ Lyncis, A.R. $9^{\text {h }} 11^{\mathrm{m}}$; Dec. $37^{\circ} 21^{\prime}$; Mags. $4,7.5$; Pos. $240^{\circ}$; Dist. $2^{\prime \prime} .9$; white and lilac. (This is the northern one of a pair of stars which closely resembles the three pairs that mark the paws of Ursa Major. This pair makes nearly an isosceles triangle with the two pairs $\lambda \mu$ and $\iota \kappa$, Ursæ Majoris - see map.)
30. Gemini (Map II.). - This is the fourth of the zodiacal constellations (mostly in the sign of Cancer), containing the summer solstitial point about $2^{\circ}$ west and a little north of the star $\eta$. It lies northeast of Orion and southeast of Auriga, and is sufficiently characterized by the two stars $\alpha$ and $\beta$ (about $4 \frac{1}{2}^{\circ}$ apart), which mark the heads of the twins. The southern
one, $\beta$, or Pollux, is the brighter, but a (Castor) is much the more interesting, as being double. The feet are marked by the third-magnitude stars $\gamma$ and $\mu$, some $10^{\circ}$ east of $\zeta$ Tauri, and the map shows how the lines that join these to $\beta$ and $\alpha$ respectively mark the places of $\delta$ and $\epsilon . \quad \eta, 2^{\circ}$ west of $\mu$, is a variable, and is also double, though as such it is beyond the power of ordinary telescopes.

Names. a, Castor; $\beta$, Pollux ; $\gamma$, Alhena, $\mu$, Tejat (Post); $\eta$, Tejat (Prior) ; $\delta$, Wasat ; $\epsilon$, Meboula.

Double Stars. (1) a, Mags. 2.5, 3 ; Pos. $225^{\circ}$; Dist. $5^{\prime \prime}$.5. Binary; period undetermined, but certainly over 200 years. The larger of the close pair is also a spectroscopic binary, with period of about 3 days (see Art. 465*). There is also a companion of ninth mag., distant about $74^{\prime \prime}$; Pos. $164^{\circ}$. (2) $\delta$, Mags. 3, 8 ; Pos. $203^{\circ}$; Dist. $\boldsymbol{\gamma}^{\prime \prime}$. (3) $\mu$, Mags. 3, 11 ; Pos. $79^{\circ}$; Dist. $80^{\prime \prime}$.

Nebulce and Clusters. (1) M. 35; A.R. $6^{\mathrm{h}} 01^{\mathrm{n}}$; Dec. $24^{\circ} 21^{\prime}$; N.W. of $\eta$ at the same distance as that from $\mu$ to $\eta$, and on the line from $\gamma$ through $\eta$ produced. The map is not quite right in this respect. (2) 팡 IV. 45 ; A.R. $7^{\mathrm{h}} 22^{\mathrm{m}}$; N. $21^{\circ} 10^{\prime}$. A nebulous star in a small telescope ; in a large telescope, very peculiar - $2^{\circ}$ southeast of $\delta$.
31. Canis Minor (Map II.). - This constellation, just south of Gemini, is sufficiently characterized by the bright star Procyon, which is $25^{\circ}$ due south of the mid-point between Castor and Pollux. a, $\beta$, and $\gamma$ together form a configuration closely resembling that formed by $\alpha, \beta$, and $\gamma$ Arietis. Procyon, $\alpha$ Orionis, and Sirius form nearly an equilateral triangle with sides of about $25^{\circ}$.

Names. a, Procyon ; $\beta$, Gomelza.
Double Stars. (1) Procyon has a small companion, Dist. $40^{\prime \prime}$, Pos. $312^{\circ}$, - too small, however, for anything less than an 8 -inch telescope. In 1896 a still smaller companion, like that of Sirius, was found much nearer the large star. (See Art. 464.) (2) ( $\Sigma 1126$ ) (following Procyon 43 s, and $2^{\prime}$ south, - the brightest of the stars in that field), Mags. $7,7.5$; Pos. $145^{\circ}$; Dist. $1^{\prime \prime} .5$; a good test for a 4 -inch glass.
32. Monoceros (The Unicorn) (Map II.). - 'This is one of the new constellations organized by Hevelius to fill the gap between Gemini and Canis Minor on the north, and Argo Navis and Canis Major on the south. It lies just east of Orion. It has no conspicuous stars, but is traversed by a brilliant portion of the Milky Way. The a (fourth mag.) of the constellation lies about half-way between $\alpha$ Orionis and Sirius, a little west of the line joining them.

Double Stars. (1) 8, or $b$ ( $7 \frac{1}{2}^{\circ}$ east and $3^{\circ}$ south of $a$ Orionis), Mags. 5, 8 ; Pos. $24^{\circ}$; Dist. $12^{\prime \prime} .9$; colors, orange and lilac. A fine low-power field. (2) 11 Monocerōtis, a fine triple (see Fig. 113 of Astr.), A.R. $6^{\mathrm{h}} 24^{\mathrm{m}}$; Dec. south $6^{\circ} 57^{\prime}$; A to B-C, Pos. $130^{\circ}$, Dist. $8^{\prime \prime}$; B-C, Pos. $120^{\circ}$, Dist. $2^{\prime \prime} .5$. The star is very nearly pointed at by a line drawn from $\zeta$ Canis Majoris, north through $\beta$, and continued as far again.

Clusters. (1) H VII. 2; A.R. $6^{\mathrm{h}} 24^{\mathrm{m}}$; Dec. N. $5^{\circ} 2^{\prime}$ (visible to the naked eye about $1_{2}^{10} \mathrm{~N} . \mathrm{E}$. of 8 Monocerotis described above). A fine cluster for a low power. (2) M. 50 ; A.R. $6^{\mathrm{h}} 57^{\mathrm{m}}$; Dec. S. $8^{\circ} 9^{\prime}$. In the Milky Way, on the line from Sirius to Procyon, two-fifths of the distance.
33. Canis Major (Map II.). - This glorious constellation needs no description. Its $a$ is the Dog Star, Sirius, beyond all comparison the brightest in the heavens, and probably one of our nearer neighbors. It is nearly pointed at by the line drawn through the three stars of Orion's belt. $\beta$, at the extremity of the uplifted paw, is of the second magnitude, and so are several of those farther south in the rump and tail of the animal, who sits up watching his master Orion, but with an eye out for Lepus.

Names. a, Sirius; $\beta$, Mirzam; $\gamma$, Muliphen; $\delta$, Wesen; $\epsilon$, Adara. $y$ is said to have disappeared from 1670 to 1690 , but at present is not recognized as variable, though much fainter than would be expected from its being lettered as $\gamma$.

Double Stars. (1) Sirius itself has a small companion (see Art. 463). (2) $\mu$ ( $4^{\circ}$ N.E. of Sirius), Mags. $5,9.5$; Pos. $335^{\circ}$; Dist. $3^{\prime \prime} .5$.

Clusters. (1) M. 41 ( $4^{\circ}$ south of Sirius); a fine group with a red star near centre.
34. Argo Navis (The Ship) (Maps II. and III.). - This is one of the largest, most important, and oldest of the constellations, lying south and east of Canis Major. Many Uranographers now divide it into three, Puppis, Vela, and Carina. Its brightest star, a Argûs, Canopus, ranks next to Sirius, but is not visible anywhere north of the parallel of $38^{\circ}$. The constellation, huge as it is, is only a half one, like Pegasus and Taurus, - only the stern of a vessel, with mast, sail, and oars ; the stem being wanting. In the part of the constellation covered by our maps the most conspicuous stars lie east and southeast of Canis Major. We have already mentioned $\zeta$, or Naos (Art. 28), at the southeast extremity of the "Egyptian X"; and about $8^{\circ}$ south and a little east of it is $\gamma$, nearly of the second magnitude.

Clusters. One or two clusters are accessible in our latitudes. (1) (Hy VIII. 38), A.R. $7^{\mathrm{h}} 31^{\mathrm{m}}$; Dec. S. $14^{\circ} 12^{\prime}$. Pointed at by the line from $\beta$ Can. Maj. through Sirius, continued $2 \frac{1}{2}$ times as far. Visible to naked eye ; rather coarse. (2) M. 46 ; a little more than $1^{10}$ east and south of the preceding. (3) M. 93, A.R. $7^{\mathrm{h}} 39^{\mathrm{m}}$; Dec. S. $23^{\circ} 34^{\prime}$; about $2^{\circ}$ N.W. of $\zeta \operatorname{Argûs}$.
35. Cancer (Maps II. and III.). - This is the fifth of the zodiacal constellations, bounded north by Lynx and Leo Minor, south by the head of Hydra, west by Gemini and Canis Minor, and east by Leo. It does not contain a single conspicuous star, but is easily recognizable from its position, and in a dark night by the nebulous cloud known as "Præsepe," or the "Manger," with the two stars $\gamma$ and $\delta$ near it, - the so-called "Aselli," or "Donkeys." Præsepe (sometimes also called the "Beehive") is really a coarse cluster of seventh and eighth magnitude stars, resolvable by an opera-glass. The line from Castor through Pollux, produced about $12^{\circ}$, passes near enough to it to serve as a pointer. $a$, of the fourth magnitude, is on the line drawn from Præsepe through $\delta$ (the southern Asellus), produced about $7^{\circ} ; \beta$ may be recognized by drawing a line from $\gamma$ (the northern Asellus) through Præsepe, and continuing it about $12^{\circ}$.

Double Stars. (1) ८, Mags. 4, 6.5ँ; Pos. 308 ; Dist. 30"; orange and blue; nearly due north of $\gamma$, distance twice that between the Aselli.
(2) ち, Triple (see Astr. Fig. 113); A-B, Mags. 6 and 7, Pos. (1905) $346^{\circ}$, Dist. $1^{\prime \prime}$; in rapid motion ; period about 60 years. A-C, Pos. (1905) $110^{\circ}$, Dist. $5^{\prime \prime}$; also in motion, but period unknown and much longer. Easily found by a line from $\alpha$ Gem. through $\beta$, produced two and a half times as far.
36. Leo (Map III.). - East of Cancer lies the noble constellation of Leo, which adorns the evening sky in March and April; it is the sixth of the zodiacal constellations, now occupying the sign of Virgo. Its leading star Regulus, or "Cor Leonis," is of the first magnitude, and two others, $\beta$ and $\gamma$, are of the second. $\alpha, \gamma, \delta$, and $\beta$ form a conspicuous irregular quadrilateral (see map), the line from Regulus to Denebola being $26^{\circ}$ long. Another characteristic configuration is "The Sickle," of which $a, \eta$ is the handle, and the curved line $\eta, \gamma, \zeta$, $\mu$, and $\epsilon$ is the blade, the cutting edge being turned towards Cancer. The "radiant" of the November meteors lies between $\xi$ and $\epsilon$.

Names. $\alpha$, Regulus ; $\beta$, Deńebola ; $\gamma$, Algeiba; $\delta$, Zosma.
Double Stars. (1) $\gamma$, Mags. 2, 3.5 ; Pos. $116^{\circ}$; Dist. $3^{\prime \prime} .4$; binary; period about 400 years. (2) $\iota$, Mags. 4 and 7; Pos. $65^{\circ}$; Dist. $2^{\prime \prime} .5$; yellow and bluish; easily recognized by aid of the map. (3) 54 , Mags. 4.5, 7 ; Pos. $103^{\circ}$; Dist. $6^{\prime \prime}$.2. Found by producing the line from $\beta$ through $\delta$ half its length.
37. Leo Minor and Sextans (Map III.). - Leo Minor is an insignificant modern constellation composed of a few small stars north of Leo, between it and the hind feet of Ursa Major. It contains nothing deserving special notice. A similar remark holds as to Sextans even more emphatically.
38. Hydra (Map III.). - This constellation, with its riders Crater and Corvus, is a large and important one, though not very brilliant. The head is marked by a group of five or six fourth and fifth magnitude stars just $15^{\circ}$ south of Præsepe. A curving line of small stars leads down southeast to $\alpha$, "Cor. Hydroe," a small secund or bright third magnitude star stand.
ing very much alone. From there, as the map shows, an irregular line of fourth-magnitude stars running far south and then east, almost to the boundary of Scorpio, marks the creature's body and tail, the whole covering almost six hours of right ascension, and very nearly $90^{\circ}$ of the sky. About the middle of his length, and just below the hind feet of Leo ( $30^{\circ}$ due south from Denebola), we find the little constellation of Crater; and just east of it the still smaller but much more conspicuous one of Corvus, with two second-magnitude stars in it, and four of the third and fourth magnitudes. It is well marked by a characteristic quadrilateral (see map), with $\delta$ and $\eta$ together at its northeast corner. The order of the letters differs widely from that of brightness in this constellation, suggesting that changes may have occurred.

Names. a Hydræ, Alphard or Cor Hydra; a Crateris, Alkes; $\alpha$ Corvi, Alchiba; $\delta$ Corvi, Algores.

Double Stars. (1) $\in$ Hydræ (the northernmost one of the group that marks the head), Mags. 4, 8 ; Pos. $220^{\circ}$; Dist. $3^{\prime \prime} .5$; yellow and purple. (2) $\delta$ Corvi, Mags. 3, 8; Pos. $210^{\circ}$; Dist. 24" ; yellow and purple. (3) Nebula, 파 IV. 27, A.R. $10^{\mathrm{h}} 19^{\mathrm{m}}$; Dec. S. $18^{\circ} 2^{\prime}\left(3^{\circ}\right.$ S. and $\frac{1}{2}$ O $W$. of $\mu$-see map). Bright planetary nebula, about as large as Jupiter.
39. Virgo (Map III.). - East and south of Leo lies Virgo, the seventh zodiacal constellation, bounded on the north by Boötes and Coma Berenices, on the east by Boötes and Libra, and on the south by Corvus and Hydra. Its $a$, Spica Virginis, is of the $1 \frac{1}{2}$ magnitude and, standing rather alone $10^{\circ}$ south of the celestial equator, is easily recognized as the southern apex of a nearly equilateral triangle which it forms with Denebola ( $\beta$ Leonis) to the northwest, and Arcturus northeast of it. $\beta$ Virginis of the third magnitude is $14^{\circ}$ due south of Denebola. A line drawn eastward and a little south from $\beta$ (third magnitude) and then carried on, curving northward, passes successively (see map) through $\eta, \gamma, \delta$, and $\epsilon_{\text {, }}$
of the third magnitude (notice the word formed by the letters Bēgde, like Bagdei in Cassiopeia, Art. 9). $\theta$ lies nearly midway between $a$ and $\delta$. There are also a number of other fourthmagnitude stars.

Names. $\quad a$, Spica and Azimech; $\beta$, Zavijava; $\epsilon$, Vindemiatrix.
Double Stars. (1) $\gamma$, (Binary ; period 185 years; not quite half-way from Spica to Denebola, and a little west of the line), Mags. 3, 3; Pos. (1905) $328^{\circ}$; Dist. $6^{\prime \prime} .2$; very easy and fine (Astr. Fig. 113). (2) $\theta$ (two-fifths of the way from Spica towards $\delta$ ), Triple; Mags. A 4.5, B 9, C 10 ; Pos. A-B, $345^{\circ}$, Dist. $7^{\prime \prime}$; A-C, Pos. $295^{\circ}$, Dist. $65^{\prime \prime}$. (3) o (one-third of the way from Denebola towards $\gamma$ Virginis), Mags. 6 and 8; Pos. $228^{\circ}$; Dist. $3^{\prime \prime} .5$. Spica is a spectroscopic binary (Art. 465*).

Nebulce. (1) M. 49 ; A.R. $12^{\mathrm{h}} 24^{\mathrm{m}}$; Dec. $+8^{\circ} 40^{\prime}$. Forms an equilateral triangle with $\delta$ and $\epsilon$. It lies in the remarkable "nebulous" region of Virgo. But most of the nebulæ are faint, and observable only with large telescopes. (2) झ II. 74 and 75 ; A.R. $12^{\mathrm{h}} 47^{\mathrm{m}}$; Dec. $+11^{\circ} 53^{\prime}$; two in one field, $2^{\circ}$ west and a little north of $\epsilon$. (3) M. 86 (midway between Denebola and $\epsilon$ ) ; A.R. $12^{\mathrm{h}} 20^{\mathrm{m}}$; Dec. $+13^{\circ} 36$. A large telescope shows nearly a dozen nebulæ within $2^{\circ}$ of this place.
40. Coma Berenices (Map III.). - This little constellation, composed of a great number of fifth and sixth magnitude stars, lies $30^{\circ}$ north of $\gamma$ and $\eta$ Virginis, and about $15^{\circ}$ northeast of Denebola. It contains a number of interesting double stars, but they are not easily found without the help of an equatorial mounting and graduated circles.
41. Canes Venatici (The Hunting Dogs). - These are the dogs with which Boötes is pursuing the Great Bear around the pole: the northern of the two is Asterion, the southern Chara. Most of the stars are small, but $\alpha$ is of the $2 \frac{1}{2}$ magnitude, and is easily found by drawing from $\eta$ Ursæ Majoris (the star in the end of the Dipper-handle) a line to the southwest, perpendicular to the line from $\eta$ to $\zeta$ (Mizar) and about $15^{\circ}$ long: it is about one-third of the way from $\eta$ Ursæ Majoris to $\delta$ Leonis. With Arcturus and Denebola it forms a triangle much like that which they form with Spica.

Names. $\alpha$ is known as Cor Caroli (Charles II. of England).
Double Stars. (1) a, or 12 Canum, Mags. 3 and 5; Pos. $227^{\circ}$; Dist. $20^{\prime \prime}$. (2) 2 Canum (one-third of the way from $a$ towards $\delta$ Leonis), Mags. 6 and 8; Pos. $260^{\circ}$; Dist. $41^{\prime \prime} .3$; orange, smalt blue.

Nebulce. (1) M. 51 ; A.R. $13^{\mathrm{h}} 25^{\mathrm{m}}$; Dec. $47^{\circ} 49^{\prime}$ ( $3^{\circ}$ west and somewhat south of Benetnasch). A faint double nebula in small telescopes; in great ones, the wonderful "Whirlpool Nebula" of Lord Rosse. (2) M. 3 ; bright cluster (half a degree north of the line from $a$ Canum to Arcturus, and a little nearer the latter). It is one of the variable-star clusters discovered in 1895 (see Art. 455*).
42. Boötes (Maps III. and I.). - This fine constellation is bounded on the west by Ursa Major, Canes Venatici, Coma Berenices, and Virgo, and on the south by Virgo. It extends more than $60^{\circ}$ in declination, from near the equator quite to Draco, where the uplifted hand overlaps the tail of the Bear. Its principal star, Arcturus, is of a ruddy hue, and in brightness is excelled only by Sirius among the stars visible in our latitudes. Canopus and $a$ Centauri are reckoned brighter, but they are southern circumpolars. Arcturus is at once recognized by its forming with Spica and Denebola the great triangle already mentioned (Art. 39). Six degrees west and a little south of it is $\eta$, of the third magnitude, which forms with it, in connection with $v$, a configuration like that in the head of Aries. $\epsilon$ is about $10^{\circ}$ northeast of Arcturus, and in the same direction about $10^{\circ}$ farther lies $\delta$. A pentagon is formed by these two stars along with $\beta, \gamma$, and $\rho$. "Boötes" means "the shouter" (or, according to others, "the herdsman ").

Names. a, Arcturus; $\beta$, Nekkar; $\epsilon$, Izar; $\eta$, Muphrid, $\gamma$, Seginus.

Double Stars. (1) $\epsilon$, Mags. 3, 6 ; Pos. $325^{\circ}$; Dist. $3^{\prime \prime} .1$; orange and greenish blue; very fine. (2) $\zeta$ (about $9^{\circ}$ southeast from Arcturus, at right angles to the line $\alpha \epsilon$ ), Mags. $3.5,4$; Pos. $295^{\circ}$; Dist. $0^{\prime \prime} .8$; a good test for a 4 -inch glass. (3) $\pi\left(2_{2}^{10}\right.$ north of $\zeta$ ), Mags. 4.9, 6 ; Pos. $101^{\circ}$; Dist. $5^{\prime \prime}$.3. (4) $\xi\left(10^{\circ}\right.$ due east from Arcturus, $3^{\circ}$ N.E. from $\pi$ ), Mags. 4.7, 6.6 ; Pos. (1905) $176^{\circ}$; Dist. $2^{\prime \prime} .5$; yellow and purple. Binary; period 127 years.
43. Corona Borealis (Map III.). - This beautiful little constellation lies $20^{\circ}$ northeast of Arcturus, and is at once recognizable as an almost perfect semicircle composed of half a dozen stars, among which the brightest, $a$, is of the second magnitude. The extreme northern one is $\theta$; next comes $\beta$, and the rest follow in the $\beta a \gamma \delta \in \iota$ (Bagdei) order, just as in Cassiopeia.

## Names. a, Gemma, or Alphacca.

Double Stars. (1) $\zeta$ (nearly pointed at by $\epsilon-\delta$ Boötis ; $7^{\circ}$ from $\epsilon$ ), Mags. 5, 6 ; Pos. $301^{\circ}$; Dist. $6^{\prime \prime}$; white and greenish. (2) $\eta$, rapid binary, at certain times can be split by a 4 -inch glass. Mags. 6, 6.5; pointed at by the line from $\alpha$ through $\beta, 2^{\circ}$ beyond $\beta$. The temporary star of 1866 (Astr, 450) lies $1 \frac{1}{2}{ }^{\circ}$ S.E. of $\epsilon$ Coronæ.
44. Libra (Map III.). - This is the eighth of the zodiacal constellations, and lies east of Virgo, bounded on the south by Centaurus and Lupus, on the east by the upstretched claw of Scorpio, and on the north by Serpens and Virgo. It is inconspicuous, the most characteristic figure being the trapezoid formed by the lines joining the four stars $\alpha, \imath, \gamma, \beta$. $\beta$, which is the northernmost of the four, is the brightest ( $2 \frac{1}{2}$ magnitude), and is about $30^{\circ}$ nearly due east from Spica, while $\alpha$ is about $10^{\circ}$ southwest of $\beta$. The remarkable variable $\delta$ Libræ is $4^{\circ}$ west and a little north from $\beta$. Most of the time it is of the $4 \frac{1}{2}$ or fifth magnitude, but runs down nearly two magnitudes at the minimum.

Names. $\alpha$, Zuben el Genubi; $\beta$, Zuben el Chamali.
Cluster. M. 5; A.R. $15^{\mathrm{h}} 12^{\mathrm{m}}$; Dec. N. $2^{\circ} 32^{\prime}$. This is within the boundaries of Serpens, and just a little north and west of the fifthmagnitude star 5 Serpentis. It is a variable-star cluster (Art. 455*).
45. Antlia, Centaurus, and Lupus (Map III.). - These constellations lie south of Hydra and Libra. Antlia Pneumatica (the "AirPump") is a modern constellation of no importance and hardly recognizable by the eye, having only a single star as bright as the $4 \frac{1}{2}$ mag.
nitude. Centaurus, on the other hand, is an ancient and extensive asterism, containing in its (south) circumpolar portion two stars of the first magnitude : $a$ Centauri stands next after Sirius and Canopus in brightness, and, as far as present knowledge indicates, is our nearest neighbor among the stars. The part of the constellation which becomes visible in our latitudes is not specially brilliant, though it contains several stars of the $2 \frac{1}{2}$ and third magnitude in the region that lies south of Corvus and Spica Virginis. A line from $\epsilon$ Virginis through Spica, produced a little more than its own length, will strike very near $\theta$, a solitary star of the $2 \frac{1}{2}$ magnitude in the Centaur's left shoulder. $\iota$ (third mag.) lies $11^{\circ}$ west of $\theta$, and $\eta$ (third mag.) $9^{\circ}$ southeast; while $5^{\circ}$ or $6^{\circ}$ south of the line from $\theta$ to $\iota$ lies a tangle of third-magnitude stars, which, if they were at a higher elevation, would be conspicuous. Centaurus is best seen in May or early in June.

Lupus, also one of Ptolemy's constellations, lies due east of Centaurus and just south of Libra. It contains a considerable number of third and fourth magnitude stars ; but is too low for any satisfactory study in our own latitudes. It is best seen late in June. These constellations contain numerous objects interesting for a southern observer, but nothing available for our purpose.
46. Scorpio (or Scorpius) (Map III.). - This, the ninth of the zodiacal constellations, and the most brilliant of them, lies southeast of Libra, which in ancient times used to form its claws (Cheloe). It is bounded north by Ophiuchus, south by Lupus, Norma, and Ara, and east by Sagittarius. It is recognizable at once on a summer evening by the peculiar configuration, like a boy's kite, with a long streaming tail reaching far down to the southern horizon. Its principal star, Antares, is of the first magnitude and fiery red, like the planet Mars. From this it gets its name, which means "the rival of Ares" (Mars). $\beta$ (second magnitude) is in the arch of the kite bow, about $8^{\circ}$ or $9^{\circ}$ northwest of Antares, while the star which Bayer lettered as $\gamma$ Scorpii is well within Libra, $20^{\circ}$ west of Antares. (There is no little discordance and confusion among Uranographers as to the boundary between the
two constellations.) The other principal stars of the constellation are easily found on the map; $\delta$ is $3^{\circ}$ southwest of $\beta$, while $\epsilon, \zeta, \eta, \theta, \iota, \kappa$, and $\lambda$ follow along in order in the tail of the creature, except that between $\epsilon$ and $\zeta$ is interposed the double $\mu$. $\quad \epsilon, \theta$, and $\dot{\lambda}$ are all of the second magnitude, and the others of the third.

## 47. Names. $\alpha$, Antares; $\beta$, Akrab.

Double Stars. (1) a, Mags. 1 and 7; Pos. $270^{\circ}$; Dist. 3.15 ; fiery red and vivid green. A beautiful object when the state of the air allows it to be well seen. (2) $\beta$, Triple; Mags. A 2, B 4, C 10; A-B, Pos. $25^{\circ}$, Dist. $13^{\prime \prime}$; A-C, Pos. $89^{\circ}$, Dist. 04.9. (3) $v\left(2^{\circ}\right.$ due east of $\beta$ ), Quadruple; Mags. A 4, B 5, C 7, D 8; A-B, Pos. $7^{\circ}$, Dist. $0^{\prime \prime} .8$; A-C, Pos. $337^{\circ}$, Dist. $41^{\prime \prime}$; C-D, Pos. $47^{\circ}$, Dist. $2^{\prime \prime}$.4. A beautiful object. (4) $\xi$ Scorpii ( $88^{10}$ due north from $\beta$ ), Triple; Mags. A 5, B 5.2, C 7.5 ; A-B, (Binary) Pos. $200^{\circ}$, Dist. $1^{\prime \prime} .4$; $\frac{1}{2}(A+B)$ to C, Pos. $65^{\circ}$, Dist. $7^{\prime \prime}$.3. $\mu^{1}$ is a spectroscopic binary (Art. $465^{*}$ ).

Clusters. (1) M. 80, A.R. $16^{\mathrm{h}} 10^{\mathrm{m}}$; Dec. S. $22^{\circ} 42^{\prime}$; half-way between $\alpha$ and $\beta$; one of the finest clusters known. (2) M. 4, A.R. $16^{\mathrm{h}} 16^{\mathrm{m}}$; Dec. S. $26^{\circ} 14^{\prime} ; 1_{2}^{10}$ west of $\alpha$; not so fine as the preceding.

Norma lies west of Scorpio, between it and Lupus, while Ara lies due south of $\eta$ and $\theta$. Both are small and of little importance, at least to observers in our latitudes.
48. Ophiūchus (or Serpentarius) and Serpens (Map III.). Ophiuchus means the "serpent-holder." The giant is represented as standing with his feet on Scorpio, and grasping the "serpent," the head of which is just south of Corona Borealis, while the tail extends nearly to Aquila. The two constellations therefore are best treated together. The head of Serpens is marked by a group of small stars $20^{\circ}$ due east of Arcturus, and $10^{\circ}$ south of Corona. $\beta$ and $\gamma$ are the two brightest stars in the group, their magnitudes three and a half and four. $\delta$ lies $6^{\circ}$ southwest of $\beta$, and there the serpent's body bends southeast through $\alpha$ and $\epsilon$ Serpentis (see map) to $\delta$ and $\epsilon$ Ophiuchi in the giant's hand. The line of these five stars carried upwards passes nearly through є Boötis, and downwards
through $\zeta$ Ophiuchi. A line crossing this at right angles, nearly midway between $\epsilon$ Serpentis and $\delta$ Ophiuchi, passes through $\mu$ Serpentis on the southwest, and $\lambda$ Ophiuchi to the northeast. The lozenge-shaped figure formed by the lines drawn from $a$ Serpentis and $\zeta$ Ophiuchi to the two stars last mentioned forms one of the most characteristic configurations of the summer sky.
$\alpha$ Ophiuchi ( $2 \frac{1}{2}$ magnitude) is easily recognized in connection with $\alpha$ Herculis, since they stand rather isolated, about $6^{\circ}$ apart, on the line drawn from Arcturus through the head of Serpens, and produced as far again. $\alpha$ Ophiuchi is the eastern and the brighter of the two. It forms with Vega and Altair a nearly equilateral triangle. $\beta$ Ophiuchi lies about $9^{\circ}$ southeast of $\alpha$; and $5^{\circ}$ east and a little south of $\beta$ are five small stars in the Milky Way, forming a $V$ with the point to the south, much like the Hyades of T'aurus. They form the head of the now discredited constellation "Poniatowski's Bull" (Taurus Poniatovii), proposed in 1777.
49. Names. $\alpha$ Ophiuchi, Ras Alaghue ; $\beta$, Cebalrai ; $\delta$, Yed; $\lambda$, Marfic: a Serpentis, Unukalhai; $\theta$, Alya.

Double Stars. (1) $\lambda$ Ophiuchi, Binary ; period, 234 years; Mags. 4, 6 ; Pos. (1905) $60^{\circ}$; Dist. $1^{\prime \prime} .2$. (2) 70 Ophiuchi (the middle star in the eastern leg of the V of Poniatowski's Bull), Binary ; period, 93 years; Mags. 4.5, 7 ; Pos. (1905) $180^{\circ}$; Dist. $2^{\prime \prime}$. The position angle changes very rapidly just now, and the star is too close to be resolved by a small instrument. (3) $\delta$ Serpentis, Mags. 4, 5; Pos. $185^{\circ}$; Dist. $3^{\prime \prime} .6$; very pretty. (4) $\theta$ Serpentis, Mags. 4, 4.5; Pos. $104^{\circ}$; Dist. $21^{\prime \prime}$. (5) $\nu$ Serpentis ( $4^{\circ}$ N.E. of $\eta$ Ophiuchi), Mags. 4.5, 9 ; Pos. $31^{\circ}$; Dist. $51^{\prime \prime}$; sea-green and lilac.

Clusters. (1) M. 23, A.R. $17^{\mathrm{h}} 50^{\mathrm{m}}$; Dec. S. $19^{\circ} 0^{\prime}$. Fine lowpower field. (2) M. 12, A.R. $16^{\mathrm{h}} 41^{\mathrm{m}}$; Dec. S. $1^{\circ} 45^{\prime}$. On the line between $\beta$ and $\epsilon$ Ophiuchi, one-third of the way from $\epsilon$. (3) M. 10, A.R. $16^{\mathrm{h}} 51^{\mathrm{m}}$; Dec. S. $3^{\circ} 56^{\prime}$. On the line between $\beta$ and $\zeta$ Ophiuchi, two-fifths of the way from $\zeta$. (4) Hi VIII. 72, A.R. $18^{\text {h }} 22^{\mathrm{m}}$; Dec. N. $6^{\circ} 29^{\prime}$. Pointed at by the eastern leg of the Poniatowski V. $8^{\circ}$ from 70 Ophiuchi.
50. Hercules (Maps I. and III.). - This noble constellation lies next north of Ophiuchus, and is bounded on the west by Serpens, Corona, and Boötes, while to the east lie Aquila, Lyra, and Cygnus. On the north is Draco. The hero is represented as resting on one knee, with his foot on the head of Draco, while his head is close to that of Ophiuchus. The constellation contains no stars of the first or even of the second magnitude, but a number of the third. The most characteristic figure is the keystone-shaped quadrilateral formed by the stars $\epsilon, \zeta, \eta$, with $\pi$ and $\rho$ together at the northeast corner. It lies about midway on the line from Vega to Corona. The line $\pi \epsilon$, carried on $11^{\circ}$, brings us to $\beta$, the brightest star of the asterism; and $\gamma$ and $\kappa$ lie a few degrees farther along on the same line continued toward $\gamma$ Serpentis. The angle $\epsilon \beta \alpha$ is a right angle opening towards Lyra. $\alpha$ is irregularly variable, besides being double.
51. Names. a, Ras Algethi; $\beta$, Korneforos.

Double Stars. (1) $\alpha$, Mags. 3, 6 ; Pos. $119^{\circ}$; Dist. $4^{\prime \prime} .5$; orange and blue. A very beautiful object for a 4 -inch glass (Astr. Fig. 113). (2) $\zeta$ (the S.W. corner of the "Keystone"), Binary ; period, 34 y. (Astr. Fig. 113); Mags. 3, 6.5; Pos. (1905) $190^{\circ}$; Dist. 1".5. Rather difficult for a small instrument. (3) $\rho\left(21_{2}^{\circ}\right.$ east of $\pi$ at the N.W. corner of the "Keystone"), Mags. 4, 5; Pos. $312^{\circ}$; Dist. $4^{\prime \prime}$; white, emerald green. (4) $\delta$ (on the line from $\eta$ through $\epsilon$ produced nearly its own length), Mags. 3, 8; Pos. $184^{\circ}$; Dist. $18^{\prime \prime}$; white, light blue. Apparently an "optical pair"; the relative motion being rectilinear. (5) $\mu$ (nearly midway between Vega and $\alpha$ Herculis - see map), Triple; Mags. A 4, B $9.5, \mathrm{C} 10 ; \mathrm{A}, \frac{\mathrm{B}+\mathrm{C}}{2}$, Pos. $246^{\circ}$; Dist. $31^{\prime \prime}$. B-C, too faint and close for separation by any but large telescopes; Dist. about $1^{\prime \prime}$; position angle rapidly changing - about $20^{\circ}$ in 1890. (6) 95 Herculis (the N.W. corner of a little quadrilateral [sides $1^{\circ}$ to $2^{\circ}$ ] of fourth and fifth mag. stars, on line from $\rho$ through $\mu$, produced two-thirds its leugth), Mags. 5.5 and 6 ; Pos. $262^{\circ}$; Dist. $6^{\prime \prime}$; light green, cherry-red. Peculiar in showing contrast of color between nearly equal components. Clusters. (1) M. 13 , A.R. $16^{\mathrm{h}} 37^{\mathrm{m}}$; Dec. $36^{\circ} 41^{\prime}$. Exactly on the western boundary of the Keystone, one-third the way from $\eta$ towards
§. On the whole, the finest of all star clusters. (2) M. 92, A.R. $17^{\mathrm{h}} 13^{\mathrm{m}}$; Dec. $43^{\circ} 16^{\prime}\left(6^{\circ}\right.$ north and a little west of $\rho$ ). Fine, but not equal to the other.
52. Lyra (Map IV.). - The great white or blue star Vega sufficiently marks this constellation. It is attended on the east by two fourth-magnitude stars, $\epsilon$ and $\zeta$, which form with it a little equilateral triangle having sides about $2^{\circ}$ long. $\beta$ and $\gamma$ of the third magnitude ( $\beta$ is variable) lie about $8^{\circ}$ southeast from Vega, $2 \frac{1}{2}^{\circ}$ apart.

Double Stars. (1) Vega itself has a small companion, 11th mag.; Pos. $160^{\circ}$; Dist. $48^{\prime \prime}$. Only optically connected; the small star does not share the proper motion of the larger one, and has been used as a reference point in measuring Vega's parallax. (2) $\beta$, multiple; i.e., it has three small stars near it, forming a very pielly object with a low power. (3) $\epsilon_{1}$ and $\epsilon_{2}$, Quadruple (the northern of the two which form the little triangle with $\alpha$.) A sharp eye unaided by a telescope splits the star, and a small telescope divides both the components (see Astr. 468, and Fig. 113): $\epsilon_{1}$ (or 4 Lyræ), Mags. 6, 7 ; Pos. $12^{\circ}$; Dist. 3".2. $\epsilon_{2}$ (or 5 Lyræ), Mags. 5.5, 6 ; Pos. $132^{\circ}$; Dist. 2".5. $\epsilon_{1} \epsilon_{2}$, Pos. $173^{\circ}$; Dist. 207'I. On the whole, the finest object of the kind. (4) $\zeta$, Mags, 4,6 ; Pos. $150^{\circ}$; Dist. $44^{\prime \prime}$. (5) $\eta$ ( $10^{\circ}$ E. of Vega), Mags. 4.5, 8; Pos. $85^{\circ}$; Dist. $28^{\prime \prime}$; yellow, indigo. (6) $\delta$; fine field for low powers.

Nebula. M. 57, the Annular Nebula. A.R. $18^{\mathrm{h}} 49^{\mathrm{m}}$; Dec. $32^{\circ} 53^{\prime}$. Between $\beta$ and $\gamma$, one-third of the way from $\beta$. (Art. 470.)
53. Cygnus (Maps I. and IV.). - This lies due east from Lyra, and is easily recognized by the cross that marks it. The bright star $\alpha$ ( $1 \frac{1}{2}$ magnitude) is at the top, and $\beta$ (third magnitude) at the bottom, while $\gamma$ is where the cross-bar from $\delta$ to $\epsilon$ intersects the main piece, which lies along the Milky Way from the northeast to the southwest. $\zeta$ is (nearly) on the prolongation of the line from $\gamma$ through $\epsilon$, not quite so far from ₹ as $\in$ from $\gamma$.

Names. $\alpha$, Arided, or Deneb Cygni (there are other Denebs; e.g., Deneb Kaitos in Cetus) ; $\beta$, Allireo; $\gamma$, Sadr.

Double Stars. (1) $\beta$, Mags. 3.5, 7; Pos. $56^{\circ}$; Dist. 35"'; orange, smalt blue. This is the finest of the colored pairs for a small telescope. (2) $\mu$ (as far beyond $\zeta$ as $\zeta$ is east of $\epsilon$, at the tip of the eastern wing), Mags. 5 and 6 ; Pos. $118^{\circ}$; Dist. $3^{\prime \prime}$.8. (3) $\chi$ (one-third of the way from $\beta$ towards $\gamma$ ), Mags. 5 and 9 ; Pos. $73^{\circ}$; Dist. 26"; yellow and blue. (4) 61 Cygni (easily found by completing the parallelogram of which $\alpha, \gamma$, and $\epsilon$ are the other three corners. $\sigma$ and $\tau$ form a little triangle with 61, which is the faintest of the three), Mags. $5.5,6$; Pos. (1905) $127^{\circ}$; Dist. 22 $2^{\prime \prime}$. This is the star of which Bessel measured the parallax in 1838 (Astr. 521), - apparently our second nearest neighbor.
$\delta$ is also a fine double, but too difficult for an instrument of less than six inches' aperture.

Clusters. (1) M. 39 , A.R. $21^{\mathrm{h}} 28^{\mathrm{m}}$; Dec. $47^{\circ} 54^{\prime}$ (about $3^{\circ}$ north of $\rho$; $\rho$ itself (fourth mag.) being found by drawing a line from $\delta$ through a, and carrying it an equal distance beyond. (2) H VIII. 56 , A.R. $20^{\mathrm{h}} 19^{\mathrm{m}}$; Dec. $40^{\circ} 20^{\prime}$. Beautiful group, $\frac{10}{2}{ }^{\circ}$ north and a little east of $\gamma$. The bright spots in the Milky Way all through Cygnus afford beautiful fields for a low power.
54. Vulpecula et Anser (Map IV.).-This little constellation is one of those originated by Hevelius, and has obtained more general recognition among astronomers than most of his creations. It lies just south of Cygnus, and $\cdot$ is bounded to the south by Delphinus, Sagitta and Aquila.

It has no conspicuous stars, but contains one very interesting telescopic object, - the " Dumb-Bell Nebula," - M. 27, A.R. $19^{\mathrm{h}} 54^{\mathrm{m}}$; Dec. $22^{\circ} 23^{\prime}$. On a line from $\gamma$ Lyræ through $\beta$ Cygni, produced as far again, where this line intersects another drawn from a Aquilæ through $\gamma$ Sagittæ, $3 \frac{1}{2}{ }^{\circ}$ north and half a degree east of the latter star.
55. Sagitta (Map IV.). - This little asterism, though very inconspicuous, is one of the old 48. It lies south of Vulpecula, and the two stars $\alpha$ and $\beta$, which mark the feather of the arrow, lie nearly midway between $\beta$ Cygni and Altair, while its point is marked by $\gamma, 5^{\circ}$ farther east and north.

Double Stars. (1) $\zeta\left(\frac{3}{4} 0^{\circ}\right.$ N.W. of $\delta$, the middle star in the shaft of the arrow), Mags. 5.5, 9 ; Pos. $312^{\circ}$; Dist. $8^{\prime \prime} .6$ : the larger star is itself close double, distance about $\frac{1_{4}^{\prime \prime}}{\prime \prime}$, making an interesting triple system.
56. Aquila (Map IV.). - This constellation lies on the celestial equator, east of Ophiuchus and north of Sagittarius and Capricornus. It is bounded on the east by Aquarius and Delphinus, and on the north by Sagitta. Its characteristic configuration is that formed by Altarr (the standard first-magnitude star), with $\gamma$ to the north and $\beta$ to the south. It lies about $20^{\circ}$ south of $\beta$ Cygni, and forms a fine triangle with Vega and $\boldsymbol{a}$ Ophiuchi.

Double Star. (1) $\pi$ Aquilæ ( $1 \frac{1}{2}{ }^{\circ}$ N.E. of $\gamma$ ), Mags. 6 and 7; Pos. $120^{\circ}$; Dist. $1^{\prime \prime} .5$. Good test for $3 \frac{1}{2}$-inch glass.

Cluster. M. 11, A.R. $18^{\text {h }} 45^{\mathrm{m}}$; Dec. S. $6^{\circ} 24^{\prime}$. A fine fan-shaped group of stars in the Milky Way. A line carried from Altair through $\delta$ Aquilæ (see map), and prolonged once and a half as far again, will find it about $4^{\circ} \mathrm{S} . \mathrm{W}$. of $\lambda$.

The southern part of the region allotted to Aquila on our maps has been assigned to Antinoïs. This constellation was recognized by some even in Ptolemy's time; but he declined to adopt it. Hevelius appropriated the eastern portion of "Antinouis" for his constellation of "Scutum Sobieski," and M. 11 falls just within its limits.
57. Sagittarius (Map IV.). - This, the tenth of the zodiacal constellations, is bounded north by Aquila and Ophiuchus, west by Scorpio and Ophiuchus (though Bode and some other authorities crowd in a piece of "Telescopium" between it and Scorpio), south by Corona Australis, Telescopium, and Indus, and east by Microscopium and Capricornus. It contains no stars of the first magnitude, but a number of the $2 \frac{1}{2}$ and third.

The most characteristic configuration is the little inverted "milk dipper" formed by the five stars, $\lambda, \phi, \sigma, \tau$, and $\zeta$, of which the last four form the bowl, while $\lambda$ (in the Milky Way) is the handle. $\delta, \gamma$, and $\epsilon$, which form a triangle right-angled at $\delta$, lie south and a little west of $\lambda$, the whole eight together forming a very striking group. There is a curious disregard of any apparent principle in the lettering of the stars of this constellation; $\alpha$ and $\beta$ are stars not exceeding in brightness
the fourth magnitude, about $4^{\circ}$ apart on a north and south line and lying some $15^{\circ}$ south and $5^{\circ}$ east of $\zeta$ (see map). The Milky Way in Sagittarius is very bright, and complicated in structure, full of knots and streamers, and dark pockets.

Names. $\lambda$, Kaus Borealis; $\delta$, Kaus Media; $\epsilon$, Kaus Australis ; $\sigma$, Sádira. This star is strongly suspected of irregular variability.

Double Stars. (1) $\mu^{1}\left(7^{\circ}\right.$ N.W. of $\lambda$; on the line from $\zeta$ through $\phi$ produced), Triple; Mags. A 3.5, B 9.5, C 10 ; A-B, Pos. 315, Dist. $40^{\prime \prime}$; A-C, Pos. $114^{\circ}$, Dist. $45^{\prime \prime}$.

Clusters and Nebulce. (1) M. 22, A.R. $18^{\mathrm{h}} 29^{\mathrm{m}}$; Dec. S. $24^{\circ} 0^{\prime}\left(3^{\circ}\right.$ N.W. of $\lambda$, and midway between $\mu$ and $\sigma$ ). Capital object for a 4 -inch telescope. (2) M. 25, A.R. $18^{\mathrm{h}} 25^{\mathrm{m}}$; Dec. S. $19^{\circ} 10^{\prime}\left(7^{\circ}\right.$ north and $1^{\circ}$ east of $\lambda$; visible to naked eye). (3) M. 8; A.R. $17^{\mathrm{h}} 57^{\mathrm{m}}$; Dec. S. $24^{\circ} 21^{\prime}$ (a little south of the line $\phi \lambda$ produced, and as far from $\lambda$ as $\lambda$ from $\phi$; also visible to naked eye). (4) 핑 IV. 41, The Trifid Nebula, A.R. $17^{\mathrm{h}} 55^{\mathrm{m}}$; Dec. S. $23^{\circ} 2^{\prime \prime}\left(1_{1}^{10}\right.$ north of M. 8, and almost exactly on the line $\phi \lambda$ produced). A very beautiful and interesting object.
58. Capricornus (Map IV.). - This, the eleventh of the zodiacal constellations, follows Sagittarius on the east. It has Aquarius and Aquila (Antinoüs) on the north, Microscopium and Piscis Austrinus on the south, and Aquarius on the east. It has no bright stars, but the configuration formed by the two $\alpha$ 's ( $\alpha_{1}$ and $\alpha_{2}$ ) with each other and with $\beta, 3^{\circ}$ south, is characteristic and not easily mistaken for anything else. The two $\alpha$ 's, a pretty "double" to the naked eye, lie on the line from $\beta$ Cygni (at the foot of the cross) through Altair, produced about $25^{\circ}$. On the line $\alpha \beta$, about $3^{\circ}$ distant, lies $\rho$ (of the fourth magnitude), with two other small stars near it. From this a line $20^{\circ} \mathrm{long}$, carried due east through $\theta$ and $\iota$ (of the fourth magnitude), brings the eye to $\gamma$ and $\delta$ of the third, the latter marking the constellation's eastern limit.

Names. a, Algiedi ( prima and secunda) ; $\delta$, Deneb Algiedi.
Double Stars. (1) $\alpha_{1}$ and $\alpha_{2}$ (pretty with a very low power), Mags. 3 and 4 ; Dist. $6^{\prime \prime} 13^{\prime \prime} . a_{2}$ has also a very faint companion, invisible
with any telescope of less than 6 -inch aperture; Pos. $150^{\circ}$; Dist. $7^{\prime \prime} .5$. The companion is itself double; Dist. about $1^{\prime \prime}$; Pos. $240^{\circ}$. (2) $\beta$, Mags. 3.5, 7 ; Pos. $267^{\circ}$; Dist. $3^{\prime} 25^{\prime \prime}$. The companion is also a close and difficult double. (3) $\rho$ (the northern star in the little triangle it forms with $\pi$ and o), Mags. 5, 9 ; Pos. $177^{\circ}$; Dist. $3^{\prime \prime} .8$. (4) $\pi$ (the S.W. one in the same triangle), Mags. 5, 9 ; Pos. $146^{\circ}$; Disć. $3^{\prime \prime} .5$.

Nebula. M. 30, A.R. $21^{\mathrm{h}} 34^{\mathrm{m}}$; Dec. S. $23042^{\prime}$ (about $1^{\circ}$ west and a little north of 41 Capricorni, a fifth-magnitude star, $7^{\circ}$ south of $\gamma$ Capricorni).
59. Delphinus (Map IV.). - This little asterism is ancient, and unmistakably characterized by the rhombus of third-magnitude stars known as " Job's Coffin." It lies about $15^{\circ}$ northeast of Altair, bounded north by Vulpecula and west by Aquila. There are a few stars visible to the naked eye in addition to the four that form the rhombus. Epsilon, about $3^{\circ}$ southwest, is the only conspicuous one.

Names. $\alpha$, Svalocin; $\beta$, Rotanev. These were given in joke by Nicolaus Cacciatore, a Sicilian astronomer, about 1800. The letters of the two names reversed make Nicolavs Venator; Venator being the translation of the Italian "Cacciatore," which means "Hunter." The joke is good enough to keep.

Double Stars. (1) $\gamma$ (at the N.E. angle of the rhombus), Mags. 4, 7 ; Pos. $271^{\circ}$; Dist. $11^{\prime \prime} .3$. (2) $\beta$, a very close and rapid binary, beyond the reach of all but large telescopes. It has, however, two little companions, distant about $30^{\prime \prime}$.
60. Equuleus (Map IV.). - This little constellation is still smaller than the Dolphin, and contains no such characteristic star group. It lies about $20^{\circ}$ due east of Altair, and $10^{\circ}$ S.E. of Delphinus (see map).

Double Stars. (1) є, Mags. 5, 7.5; Pos. $73^{\circ}$; Dist. $11^{\prime \prime}$. The larger star is also close double ; Mags. 5.5. and 7 ; Pos. $290^{\circ}$; Dist. $0^{\prime \prime} .9$. Perhaps resolvable with a 4 -inch telescope.
61. Lacerta (Maps I. and IV.). - This is one of Hevelius's modern constellations, lying between Cygnus and Andromeda, with no stars above the $4 \frac{1}{2}$ magnitude. It contains a few telescopic objects, but nothing suited to our purpose.
62. Pégăsus (not Pegas'us) (Map IV.). - This covers an immense space which is bounded on the north by Andromeda and Lacerta, on the west by Cygnus, Vulpecula, Delphinus, and Equuleus, on the south by Aquarius and Pisces, and on the east by Pisces and Andromeda. Its most notable configuration is "the great square," formed by the second-magnitude stars $\alpha, \beta$, and $\gamma$ Pegasi, in connection with $\alpha$ Andromedæ (sometimes lettered $\delta$ Pegasi) at its northeast corner. The stars of the square lie in the body of the horse, which has no hindquarters. The line drawn from a Andromedæ through $\alpha$ Pegasi, and produced about an equal distance, passes through $\xi$ and $\zeta$ in the animal's neck, and reaches $\theta$ (third magnitude) in his ear. Epsilon, $8^{\circ}$ northwest of $\theta$, marks his nose. The forelegs are in the northwestern part of the constellation just east of Cygnus, and are marked, one of them by the stars $\eta$ and $\pi$, the other by $\iota$ and $\kappa$.

Names. $\alpha$, Markal ; $\beta$, Scheat ; $\gamma$, Algenib; $\epsilon$, Enif.
Double Star. к, Mags. 4, 11; Pos. $302^{\circ}$; Dist. $12^{\prime \prime}$. The large star is also itself an extremely close double; Dist. $0^{\prime \prime} .3$; (pointed at by the northern edge of the "square," at a distance one and a quarter times its length.)

Cluster. M. 15, A.R. $21^{\mathrm{h}} 24^{\mathrm{m}}$; Dec. $11^{\circ} 38^{\prime}$ (on the line from $\theta$ through $\epsilon$, produced half its length, and just west of a sth-magnitude star).
63. Aquarius (Map IV.). - This, the twelfth and last of the zodiacal constellations, extends more than $3 \frac{1}{2}$ hours in right ascension, covering a considerable region which by rights ought to belong to Capricornus. It is bounded north by Delphinus, Equuleus, Pegasus, and Pisces; west by Aquila and Capricornus; south by Capricornus and Piscis Austrinus, and east by Cetus. The most notable configuration is the little Y of third and fourth magnitude stars which marks the "water jar" from which Aquarius pours the stream that meanders down to the southeast and south for $30^{\circ}$, till it
reaches the Southern Fish. The middle of the Y is about $18^{\circ}$ south and west of $a$ Pegasi, and lies almost exactly on the celestial equator. A line drawn west and a little south from $\gamma$ (the westernmost star of the Y ) to $a$ Capricorni, passes through $\beta$ (third magnitude) at one-third of the way, and through $\mu$ and $\epsilon$ (fourth and $3 \frac{1}{2}$ ) two-thirds of the way. $a$ (third magnitude) lies $4^{\circ}$ west and a little north of $\gamma$. $\delta$ (third magnitude) lies about half-way between the Y and Fomalhaut in the Southern Fish, $3^{\circ}$ or $4^{\circ}$ east of the line that joins them.

Names. a, Saad el Melik; $\beta$, Saad el Suna, $\delta$, Skat.
Double Stars. (1) $\zeta$ (the central star of the Y), Mags. 4, 4.5; Pos. $332^{\circ}$; Dist. $3^{\prime \prime} .6$; pretty and easy. (2) 12 Aquarii ( $7^{\circ}$ due west of $\beta$, and the brightest star in the vicinity), Mags. 5.5, 8.5; Pos. $190^{\circ}$; Dist. $2^{\prime \prime} .8$; yellowish white and light blue.

Clusters and Nebulce. (1) M. 2 ; A.R. $21^{\mathrm{h}} 17^{\mathrm{m}}$; Dec. S. $1^{\circ} 22^{\prime}$ (on the line drawn from $\zeta$ through $\alpha$, produced one and a quarter times its own length). (2) H IV. 1, A.R. $20^{\mathrm{h}} 58^{\mathrm{m}}$; Dec. S. $11^{\circ} 50^{\prime}$ (nearly on the line from $\alpha$ through $\beta$, produced its own length, and $1 \frac{1}{3}$ west of $\nu$; fifth magnitude); planetary nebula, bright and vividly green.
64. Piscis Austrinus (or Australis) (Map IV.). - This small constellation, lying south of Aquarius and Capricornus, presents little of interest. It has one bright star, Fomalhaut (pronounced Fomalhawt), of the $1 \frac{1}{2}$ magnitude, which is easily recognized from its being nearly on the same hour-circle with the western edge of the great square of Pegasıs, $45^{\circ}$ to the south of $a$, and solitary, having no star exceeding the fourth magnitude within $15^{\circ}$ or $20^{\circ}$. It contains no telescopic objects available for our purpose.

South of it, barely rising above the southern horizon, lie the constellations of Microscopium and Grus. The former is of no account. The latter is a conspicuous constellation in the southern hemisphere, and its two brightest stars, $\alpha$ and $\beta$, of the second magnitude, rise high enough to be seen in latitudes south of Washington. They lie about $20^{\circ}$ south and west of Fomalhaut.





Magnitudes $0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$ Nebula Cluser Symbols 深 * * +000


MAP II.





Magnitudes $0 \quad 1 \quad 2 \quad 3 \quad 45$ Nebula Cluster





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3
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4


[^0]:    ${ }^{1}$ The exact trigonometric equation is $\sin s=\frac{r}{R}$, whence $r=R \sin s$. This equation is exact, even if $s$ is a large angle.

[^1]:    ${ }^{1}$ The word "zenith" is derived from the Arabic, as are many other astronomical terms. It is a reminiscence of the centuries when the Arabs were the chief cultivators of science.
    ${ }^{2}$ Pronounced ho-rì'zon; beware of the vulgar pronunciation hor ${ }^{\prime}-i-z o n$.

[^2]:    ${ }^{1}$ The Dip (in minutes) $=\sqrt{h \text { (in feet) }}$ nearly.

[^3]:    ${ }^{1}$ The reader will remember, of course, that the magnetic needle does not point exactly north. Its direction varies widely in different parts of the earth, and not only so, but it changes slightly from hour to hour during the day, as well as from year to year.

[^4]:    ${ }^{1}$ The teacher is earnestly recommended to arrange to give the class, as early in the course as possible, an evening or two in the open air. It is the best and quickest way to secure an intelligent comprehension of the fundamental points and circles of the celestial sphere; and the study of the constellations, though not of much account considered as astronomy, is always interesting to young people and awakens interest in the science. If the class can have access to a good celestial globe at the same time, it will make the exercise easier and more profitable.

[^5]:    ${ }^{1}$ The student must be careful not to confound the pole with the Polestar. The pole is an imaginary point; the Pole-star is only that one of the conspicuous stars which happens now to be nearest to that point. The

[^6]:    ${ }^{1}$ Practical Astronomy is that branch of Astronomy which treats of the methods of making astronomical observations, the instruments used, and the calculations by which the results are deduced.

[^7]:    ${ }^{1}$ As to the standard meridian, there is some variation of usage among different nations. The French reckon from Paris, but most other nations use the meridian of Greenwich.

[^8]:    ${ }^{1}$ The word "rotate" denotes a spinning motion like that of a wheel on its axis. The word "revolve" is more general, and may be applied either to describe such a spinning motion, or (and this is the more usual use in Astronomy), to describe the motion of a body travelling around another, as when we say that the earth "revolves" around the sun.

[^9]:    ${ }^{1}$ Col. A. R. Clarke, for many years at the head of the English Ordnance Survey but now retired.

[^10]:    ${ }^{1}$ This "ellipticity" of the earth's elliptical meridian must not be confounded with its "eccentricity," the formula for which is $\sqrt{\frac{a^{2}-b^{2}}{a^{2}}}$. The "ellipticity" of the earth's meridian is about $\frac{1}{295}$, its "eccentricitv" nearly $\frac{1}{12}$.

[^11]:    ${ }^{1}$ The word " gravitation" is used to denote the attraction of bodies for each other in general, while "gravity" (French "pesanteur") is limited to the force which makes bodies fall at the surface of the earth or other heavenly body.

[^12]:    ${ }^{1} \mathrm{He}$ (and every one else until the time of Kepler) assumed on metaphysical grounds that the sun's orbit must necessarily be a circle, and described with a uniform motion, because (they said) the circle is the only perfect curve, and uniform motion is the only perfect motion proper to heavenly bodies.

[^13]:    ${ }^{1}$ The planets exert a very slight influence upon the motion of the equinox, not, however, by producing a true precession, but by slightly disturbing the position of the plane of the earth's orbit (Art. 120 (1)). This effect is in the opposite direction from the true precession produced by the sun and moon, and is about $0^{\prime \prime} .16$ annually.

[^14]:    ${ }^{1}$ Since it is the difference between the hour-angles of the fictitious and real suns at any moment, it may also be defined as the difference between their right ascensions; or, as a formula, we may write $E=a_{t}-a_{m}$, in which $a_{m}$ is the right ascension of the mean sun, and $a_{6}$ that of the true sun.

[^15]:    ${ }^{1}$ There is a slight difference between elongation in right ascension and elongation in longitude, and a corresponding difference between conjunction and opposition in right ascension and longitude respectively. Conjunction in right ascension occurs when the difference of right ascension of the sun and moon is zero; conjunction in longitude when the difference of longitude (reckoned on the ecliptic, it will be remembered) is zero.

[^16]:    ${ }^{1}$ The solution is effected as follows: (1) In the triangle $B O C$, we have given $B O, O C$, and the included angle $B O C$. Hence we can find the side $B C$, and the two angles $O B C$ and $O C B$. (2) In the triangle $B C M, B C$ is now known, and the two angles $M B C$ and $M C B$ are got by simply subtracting $O B C$ from $O B M$, and $O C B$ from $O C M$ : hence we can find $B M$ and $C M$. (3) In the triangle $O B M$, we know $O B, B M$, and the included angle $O B M$, from which we can find $O M$, the moon's distance from the centre of the earth.

[^17]:    ${ }^{1}$ The motion known as " rotation" consists essentially in this: That a line.connecting any two points not in the axis of the rotating body, and produced to the sky, will sweep out a circle on the celestial sphere.

[^18]:    1 At new moon or full moon the semi-ellipse of course becomes a semi circle.

[^19]:    ${ }^{1}$ It is quite possible that the sun's diameter is variable to the extent of a few hundred miles, since the sun is not solid.

[^20]:    ${ }^{1}$ The "grating" is merely a piece of glass or speculum metal, ruled with many thousand straight, equidistant lines, from 5000 to 20,000 in the inch.

[^21]:    ${ }^{1}$ Helium (see Art. 202*) was added in 1895, oxygen more recently.

[^22]:    ${ }^{1}$ See second note on page 160.

[^23]:    ${ }^{1}$ There are, however, unquestionable changes in the corona in the course of the half-hour or so which may elapse between the pictures when the stations are as much as 1000 miles apart; but the changes are not of such a sort as to affect the argument.

    2 For definition of standard candle, see Physics, p. 277.

[^24]:    ${ }^{1}$ Many writers use a unit of heat a thousand times smaller, i.e., the quantity of heat which raises the temperature of one gram of water $1^{\circ} \mathrm{C}$.; and they give as the "solar constant" the number of these units received per square centimeter in a minute. Since there are 10,000 square centimeters in a square meter, this makes the number expressing the solar constant a number just one-tenth as large as that stated above. Thus Langley puts it at 2.1 "small calories."

    There would be some advantage in expressing this "constant" on the "c. g. s. system" (Physics, p. 3). To do this, we should have to divide the "number" last given by 60 to reduce the minutes to seconds, so that, according to Langley, we should have the solar constant (c. G. s.) 0.035 "small calories" per square centimeter per second.
    ${ }^{2}$ Recent observations indicate that the so-called "constant" is really subject to slight variations.

[^25]:    ${ }^{1}$ The "line of motion" of a body at any instant is the tangent drawn to the curve in which the body travels, at the point occupied by the body.

[^26]:    ${ }^{1}$ At the time when Newton first made the calculation, he assumed the length of a degree to be 60 miles, while as we now know, it is more than 69 ; this made his result 16 per cent too small - so far out of the way that he loyally abandoned his theory as contradicted by facts. Some years later when Picard's measurement of a degree in France gave a near approximation to its real length, Newton at once resumed his work where he had dropped it, and soon completed his theory.

[^27]:    ${ }^{1}$ If precisely equal, the path will be a parabola, which may be regarded as either an ellipse or an hyperbola of infinite major axis (Art. 257). It is the boundary line, so to speak, between ellipses and hyperbolas.

[^28]:    ${ }^{1}$ The "tide-producing force" of a heavenly body varies inversely as the cube of its distance, and directly as its mass.

[^29]:    ${ }^{1}$ When we speak of the "motion" of the planets, the reader will understand that the apparent diurnal motion is not meant. We refer to their motions among the stars; i.e., their change of right ascension and declination.

[^30]:    ${ }^{1}$ The nodes of a planet's orbit are the points where their orbits cut the ecliptic, like the nodes of the moon's orbit (Art. 142). The perihelia are the points nearest the sun.

[^31]:    ${ }^{1}$ The proportion given is not absolutely correct. Strictly the first ratio of the proportion should be

    $$
    \text { Mass of sun }+ \text { planet }: \text { mass of planet }+ \text { satellite } \text {; }
    $$

    and moreover, the $\boldsymbol{T}$ and $t$ used must be, not exactly the actual periods, but the periods cleared of perturbations; the difference in the result is, however, insignificant, except in cases involving the earth and moon.

[^32]:    ${ }^{1}$ The planet's spectrum, in addition to the ordinary dark lines belonging to the spectrum of reflected sunlight, shows certain bands known to be due to water vapor, but it is not yet quite certain whether the vapor is in the planet's atmosphere or in our own.

[^33]:    ${ }^{1}$ Venus at times comes nearer; but when nearest she is visible only by daylight, and the hemisphere presented to the earth is mostly dark.

[^34]:    ${ }^{-1}$ A similar result was obtained by Campbell in 1909, when the observations were repeated under favorable conditions.

[^35]:    ${ }^{1}$ The Greek name of Mars is Ares, hence " Areography " is the description of the surface of Mars.

[^36]:    ${ }^{1}$ See additional notes on page 360 b .

[^37]:    ${ }^{1}$ See note on page 360 a.

[^38]:    ${ }^{1}$ The recent measures of Barnard and See make the diameter nearly 2000 miles greater.

[^39]:    ${ }^{1}$ See note on page 263.

[^40]:    ${ }^{1}$ The student's attention is specially directed to the point that the observations of the eclipses of Jupiter's satellites give directly neither the velocity of light nor the distance of the sun : they give only the time required by light to make the journey from the sun. Many elementary text-books, especially the older ones, state the case carelessly.

[^41]:    ${ }^{1}$ The recent measures of See make the outside diameter about 173,000 miles, and differ somewhat in other respects.

[^42]:    ${ }^{1}$ For note on the new satellites, see page 263.

[^43]:    1 28,500, according to See.

[^44]:    ${ }^{1}$ More recent measures by Struve, Barnard, and See make it about 28,000 miles - a surprising change.

[^45]:    ${ }^{1}$ It seems at first a singular paradox that resistance should shorten a comet's period and make it go faster. It does so by diminishing the size of the orbit. Every diminution of the comet's velocity will decrease $a$, the semi-major axis of its orbit; but the time of revolution is proportional to $a^{\frac{3}{2}}$, and is hence also diminished.

[^46]:    ${ }^{1}$ The observer must not expect to hear the explosion at the moment when he sees it, since sound travels only about 12 miles a minute. Usually the sound is some minutes on the way.

[^47]:    ${ }^{1}$ The earth's orbital motion at any instant is (very nearly) directed towards that point on the ecliptic which is $90^{\circ}$ west of the sun.

[^48]:    ${ }^{1}$ If the energy is expressed in foot-pounds (pounds of force) and the mass is wanted in mass-pounds, the equation for the energy is

    $$
    E=\frac{M V^{2}}{2 g}=\frac{M V^{2}}{64}, \text { nearly ; whence } M=\frac{64 E}{V^{2}} .
    $$

[^49]:    ${ }^{1}$ An excellent star-atlas by Professor Upton has lately been published by Ginn \& Co.

[^50]:    ${ }^{1}$ This number, 63,000 , is found by dividing the number of seconds in a year by 499 , the number of seconds that it requires light to travel from the sun to the earth.

[^51]:    ${ }^{1}$ Newcomb's more recent estimate is 750 Vegas.

[^52]:    ${ }^{1}$ As has been often pointed out, the question can be decided by spectroscopic observations whenever we become able to observe separately the two spectra of the components of a binary and so can determine the radial velocity of each at several different points in the orbit. The difficulties are great, but probably not insurmountable.

[^53]:    ${ }^{1}$ See note (1) on page 360 a.

[^54]:    ${ }^{1}$ Most of the gaseous nebulæ are near the plane of the Milky Way.

[^55]:    ${ }^{1}$ It must be remembered, however, that Hall and others have shown that the motion of the binaries does not absolutely prove the operation of gravitation.

[^56]:    ${ }^{1}$ As to the origin of the nebula itself, he did not speculate. There was no assumption, as is often supposed, that matter was first created in the nebulous condition. It was only assumed that, as the egg may be taken as the starting-point for the life history of an animal, so the nebula is to be regarded as the starting-point of the life history of the planetary system.
    ${ }^{2}$ It has been suggested by Huggins and others that the small nebulæ near the great nebula of Andromeda (see Fig. 118) may be "planets" in process of formation.

[^57]:    ${ }^{1}$ Dr. See has recently worked out the theory of the development of a binary pair from a nebula, by a process of tidal evolution. (Art. 466*.)

[^58]:    ${ }^{1}$ See note (3) on page 360 a.
    ${ }^{2}$ So far we have no decisive evidence whether the sun has passed its maximum of temperature or not. Mr. Lockyer thinks its spectrum (resembling as it does that of Capella and the stars of the second class) proves that it is now on the downward grade and growing cooler; but others do not consider the evidence conclusive.

[^59]:    ${ }^{1}$ The figure represents a globe seen from the west side, with the vernal equinox at $E . \quad E C$ is the ecliptic, and $E Q$ the equator.

[^60]:    ${ }^{1}$ The more than 2000 photographs which were made during the two transits at the stations occupied by American parties give a solar parallax of $8^{\prime \prime} .84$.

[^61]:    ${ }^{1}$ By making it extremely long in proportion to its diameter, the indistinctness of the image is considerably diminished, and in the middle of the 17th century instruments more than 200 feet in length were used by Cassini and others. Saturn's rings and several of his satellites were discovered by Huyghens and Cassini with instruments of this kind.

[^62]:    ${ }^{1}$ In sextant telescopes a negative eye-piece is sometimes used, with the wires between the two lenses.

[^63]:    1 We may add that it must, of course, be mounted where it can be pointed directly at the stars, without any intervening window-glass between it and the object. We have known purchasers of telescopes to complain bitterly-because they could not see Saturn well through a closed window.

[^64]:    ${ }^{1}$ For the Equatorial Coudé, Siderostat, and Cœlostat see Arts. 557. and 558 ,

[^65]:    ${ }^{1}$ The references are to the articles in the Author's "Elements of Astronomy," to which this Uranography is a supplement.

[^66]:    ${ }^{1}$ Capitals denote names that are generally used ; the others are met with only rarely.
    ${ }^{2}$ The "position angle" of a double star is the angle which the line drawn from the larger star to the smaller one makes with the hour-circle. It is always reckoned from the north completely around through the east, as shown in Fig. A.

[^67]:    ${ }^{1}$ The description here applies strictly only at $20^{\mathrm{h}}$ sid. time.

