MEASUREMENT OF SMALL HIGH FREQUENCY VIBRATING BEAM AMPLITUDES BY INTERFEROMETER PRINCIPLES

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PREFACE

The measurement by conventional means of small amplitudes of vibration encountered in machine parts and in testing equipment becomes difficult and inaccurate as the frequency increases and as the amplitude of vibration decreases. It was therefore decided to study the feasibility of such measurement by optical interferometry with an electronic circuit as an interpreting device. The accurate measurement of amplitudes of vibration on the order of five ten-thousandths of an inch at frequencies on the order of one thousand cycles per second was selected as a reasonable capability of the equipment.

Work was done by the author on the optical and electronic systems for the measurement of these amplitudes of vibration during the period from December, 1950, through May, 1951, at the United States Naval Postgraduate School, Annapolis, Maryland.

The author is indebted to Professor E. K. Gatcombe for much helpful assistance in making the study; to Professor S. H. Kalmbach for valuable assistance in the field of optics and optical measurements; and to Professor C. E. Menneken for assistance with electronic equipment. Acknowledgements are also due to Mr. Jones of the Public Works Department, United States Naval Academy, for machine work; and to members of the departments of Physics; Electrical, Mechanical, and Electronics Engineering for providing equipment and instruments necessary for the study.

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TABLE OF SYMBOLS AND ABBREVIATIONS

Å	Angstrom units. 1 Å = 10^{-10} meters.
e	Charge on an electron in coulombs.
f	Frequency output of photocell.
f_1	Frequency of reed oscillation.
f_m	Frequency, maximum, from photocell.
f'	Focal length of lens.
m	Mass of an electron in kilograms.
r	Radius of circle.
s	Distance.
t	Time.
V	Voltage.
$\bar{v}$	Velocity vector.
y	Amplitude of vibration.
λ	Wave length.
ω	Angular velocity.

CHAPTER I

INTRODUCTION

1. Methods of measuring small displacements.

In engineering applications the measurement of displacements of small amplitude fall into three general categories: a direct measurement in units by the use of micrometers and micrometer scales; and indirect measurement involving electric circuit parameters as resistance, capacitance, and inductance, with subsequent conversion to displacement units; and a miscellaneous group utilizing penumatic, hydrodynamic, and acoustical theory. Each of the above has its particular field of application and its limitations. When it becomes necessary to measure small displacements at relatively high frequencies, the above methods fail for several reasons: calibration is difficult and inaccurate; force required to actuate the gage may become excessive; inertia of measuring components may limit the determinations to vibrations of relatively low frequency.

Another instrument that has been used to a limited degree in engineering for the measurement of small non-cyclic displacements is the optical interferometer, Vose (1), which is used in the conventional manner to obtain a high degree of accuracy in small measurements. The interferometer has several desirable features: the actuating force required is very small; and no calibration of the instrument is required since the wave lengths of light are very accurately

known. A disadvantage of the interferometer is that due to the extreme accuracy of the equipment it is highly susceptible to outside disturbance which make the counting of the interference bands difficult except under the most favorable conditions.

2. Objective of thesis.

The author was of the opinion that the inherent high accuracy of the interferometer along with its other desirable features would make it an excellent instrument for measuring small displacements at high frequencies, provided the results could be interpreted. The object of this thesis is to determine the practibility of measuring small amplitudes of vibration with an interferometer and interpreting the results by means of an electronic circuit.

3. Summary.

The test set-up constructed, figurel, schematic, provides for an extended source of monochomatic light, an electromagnet for vibrating the reed supporting one interferometer flat, a Fabry-Perot type interferometer, a condensing lens, a vacuum tube photoelectric cell, a tunable amplifier, and a set of ear phones.

Operation of the test set-up is as follows. The surfaces of the Fabry-Perot interferometer are brought into parallelism by means of adjusting screws, at which time there will appear on the surface of the optical flats a series of concentric rings known as Haidinger fringes. As the flats are brought closer together the rings disappear to-

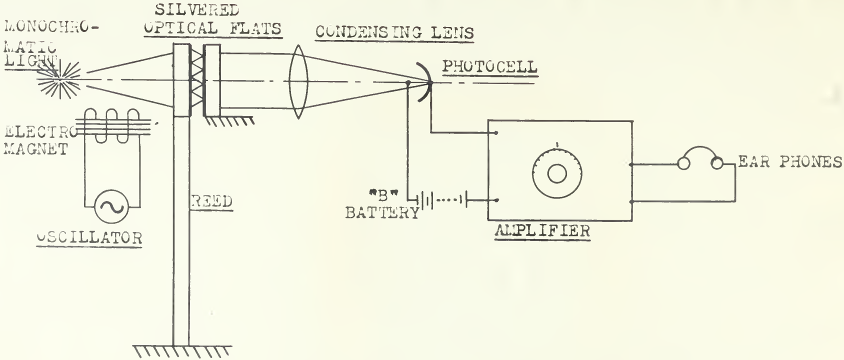


Figure 1, schematic.

wards the center, the central spot changing alternately from bright to dark. Similarly, as the flats are moved apart the concentric rings grow from the center. In either case, a change of the central spot from bright to dark to bright again indicates a change in the separation of the plates equal to one-half the wave length of the light being used. The central spot of the fringe system is focused on a photoelectric cell by means of the condensing lens. One optical flat is attached to a reed and is vibrated by the electromagnet. As this occurs there will be an electric impulse sent to the amplifier from the photocell each time the separation of the plates changes by one-half the wave length of light. The amplifier is tuned to the maximum frequency being generated as determined by audio signals in the ear phones. This maximum frequency will, in turn, be used to indicate the amplitude of the vibration of the reed carrying one of the interferometer plates.

CHAPTER II

DESIGN

1. Considerations.

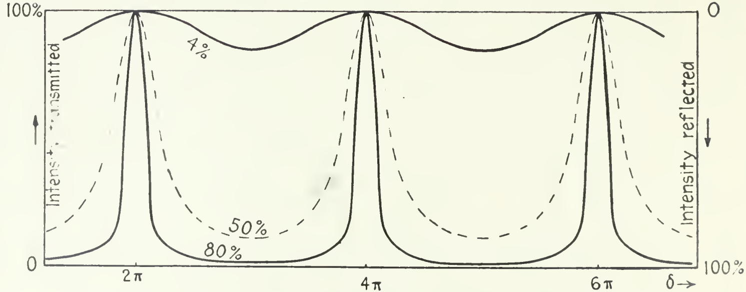
In the design of test equipment that would determine the feasibility and practicability of the interferometer and electronic circuit in measuring small amplitudes of vibration it was decided to use the simplest set-up with t the most favorable conditions obtainable. Each of the components will now be discussed with an explanation for the particular choice.

The monochromatic light source was chosen as the green line of a mercury arc with a wave length of 5461 Å. This selection was made for several reasons: availability of equipment, and good intensity. A green line filter is used to filter out unwanted light. Little difficulty is encountered with hyperfine-structure of the green due to the close proximity of the plates of the interferometer. The only effect of the hyperfine-structure is a partial illumination of the dark portion of the fringe system.

The Fabry-Perot type interferometer using transmitted light was chosen due to the sharpness of the fringes obtained with a maximum change in intensity of the light in going from a bright to a dark fringe. Figure 2 shows the sharpness of the fringe system. The inner faces of the optical flats making up the Fabry-Perot interferometer were silvered by vaporizing the silver in a vacuum jar

Figure 2

and permitting a light silver coating to be deposited on a clean glass surface. The plates were silvered to obtain a reflecting power of about 70 to 80%. Figure 3 shows the intensity contours for Haidinger fringes, the type used in this study.



—Intensity contours for Haidinger fringes, showing how their sharpness depends upon the reflecting power.

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Figure 3.

It is seen that the bright fringes suffer no loss in intensity, other than obsorption, while the dark portions have practically no transmitted intensity with the 80% reflective power coating. This fact is of great importance when considering the action of the photoelectric cell later on.

The interferometer plates must be brought into close proximity in order to obtain a central spot of sufficient size and of sufficient intensity. See appendix I for computation regarding plate separation considerations.

One interferometer plate was mounted in a bracket on the top of a steel cantilever beam. The beam or reed can be vibrated by means of an electromagnet supplied with an alternating current. The reed was designed for a high natural frequency in order that all tests could be conducted below the natural frequency of the reed, Den Hartog (4). This was desirable for several reasons: the end of the reed must move with simple harmonic motion, and passing through a natural frequency might cause the interferometer plates to strike against each other. The reed is supported in a clamp which is secured to the base of the test device.

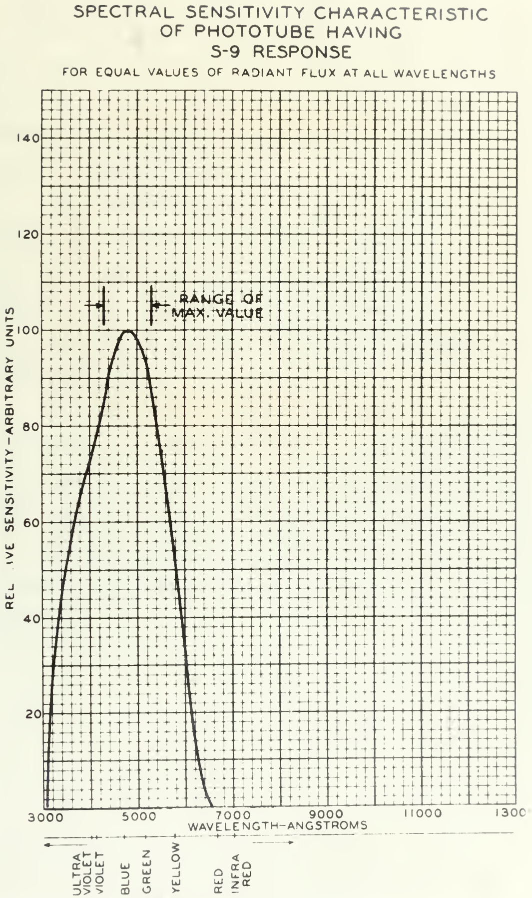
The second interferometer plate is supported in a bracket on the end of the telescope tube. The forward end of the telescope tube is mounted in a gimbal ring while the after end of the tube is mounted between three centering screws. The gimbal ring and centering ring are in turn sweated to a supporting block which is bolted to the steel base. All of the metal parts of the telescope tube, gimbal rings, centering ring, and supporting block are brass in the interest

of having them non-magnetic so they will be unaffected by the electromagnet which vibrates the reed.

A double convex lens with a focal length of 136 millimeters is located at the after end of the telescope tube. Since the Haidinger fringes appear to come from infinity the lens may be located at any convenient distance from the interferometer. A long focal length was chosen to give a reasonably large magnification to the image of the central spot.

The photoelectric cell is located 136 millimeters from the condensing lens at the focal point of the lens. The central spot of the fringe pattern at this point was made about one-quarter of an inch in diameter by suitable adjustment of the interferometer plate spacing. The central spot size was partially chosen from the subsequent consideration of the physical dimensions of the photocell.

The selection of the particular phototube to be used was dictated by several factors: (a) the cell must have a reasonably high response at 5461 Å; (b) the cell must be capable of handling light impulses at the rate of 300,000 per second; (c) the noise to signal ratio must be low. In view of these considerations a type 1P42 single element, vacuum tube photocell was chosen as being a suitable cell for this particular application. RCA Tube Handbook (3). The type 1P42 tube has a reasonably good response at 5461 Å, figure 4, the best of the commercially available tubes. The high frequency requirement rules out the possibility of using gas phototubes as the ionization time becomes appreciable and their use is



Courtesy RCA Tube Department.

Figure 4.

restricted to frequencies in the audio range. In the single element vacuum phototube the transit time for an electron is sufficiently small as to permit operation at 300 kilocycles and greater. The multiplier gas tube was not used in the interest of simplicity of the electronic circuit. The time required for an electron to acquire a charge is in the order of 10^{-9} seconds, Ryder (5).

The transit time for an electron in the photocell is a factor in determining the maximum frequency for which cell can be used. The transit time is a function of the anode voltage. Therefore the maximum frequency for which the cell can be used is a function of the anode voltage. See appendix II for these considerations.

The type 1P42 photocell is an end-on type with an aperature size of approximately one-quarter inch diameter. The aperature size was a factor in determining the plate separation used and the focal length of the condensing lens used.

The entire assembly was mounted on a steel plate eleven inches in diameter and one inch thick to provide a rigid mounting.

CHAPTER III

INTERPRETATION OF RESULTS

1. Limitations of mechanical recorders.

Mechanical recorders of dynamic strain measurements have a frequency limit due to inertia of the parts which limit these recorders to frequencies on the order of 100 cycles per second. String type oscillographs have a much higher limit yet the inertia and response of the instrument may introduce considerable error as its upper limit is reached. In any case, the calibration of such instruments is a lengthy and inaccurate process. The measurement of an output frequency is, on the other hand, accomplished with relative ease and is capable of a very high degree of accuracy. It is for this reason that an electronic circuit was selected as an interpreting device for this project.

2. Consideration of problem.

As has been stated previously, a change in the separation of the interferometer plates of one-half wave length of light will cause the central spot to go through a complete cycle of light intensity. One-half wave length of the green line used is 0.00001071 inches. Thus for a change of separation of 0.0005 inches there would be 46.68 cycles of light intensity in the central spot of the interferometer.

If we are measuring the amplitude of vibration of a reed by this method, let us assume that the end of the reed moves through an amplitude of 0.0005 inches at a frequency of 1000

cycles per second, figure 5. The number of light cycles appearing in the interferometer during one second would then be:

4 x 1000 x 46.68 or 186,720 per second.

To attempt to count such a number appears impractical.

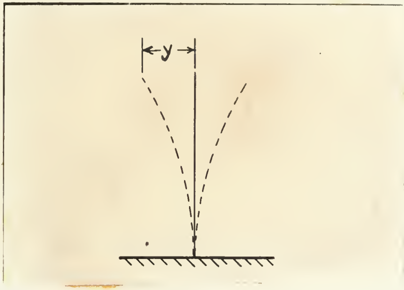


Figure 5

There is, however, a means of interpreting the results if one valid basic assumption is made: that the end of the reed moves with simple harmonic motion. The impulses from the photocell will then be frequency modulated such that in going from the maximum deflection in one direction to the maximum deflection in the other the frequency varies sinusoidally from zero to a maximum at the mid-point and back to zero again. Figure 6 and figure 7.

Taking the above problem where the amplitude was 0.0005 inches and the frequency of oscillation was 1000 cycles per second, the maximum frequency to be obtained is as follows:

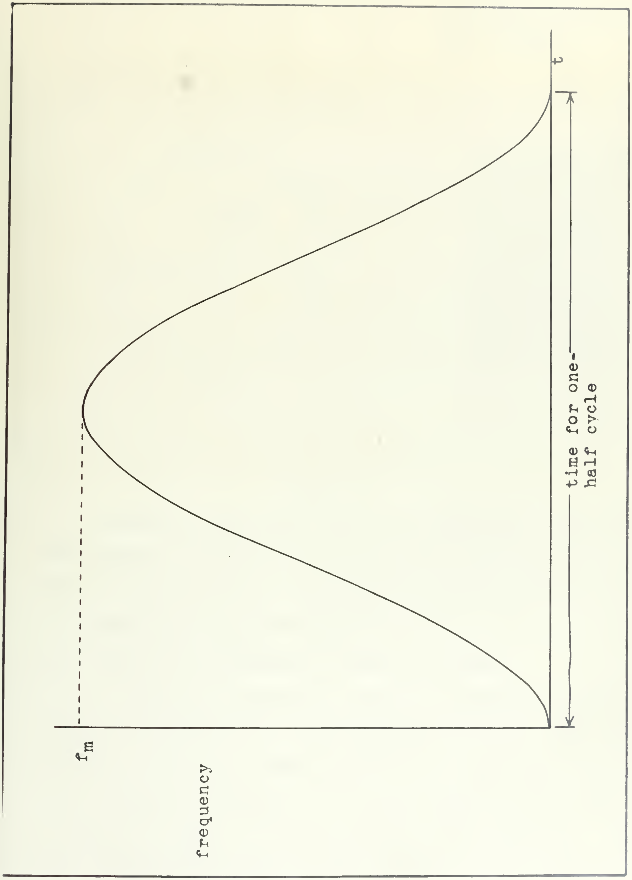


Figure 6.

Radius of circle 46.68 lines
Maximum rate of crossing lines $r \times \omega \times \sin \theta$

$46.68 \times 2\pi \times 1000 \times 1$

293,299 cycles per second.

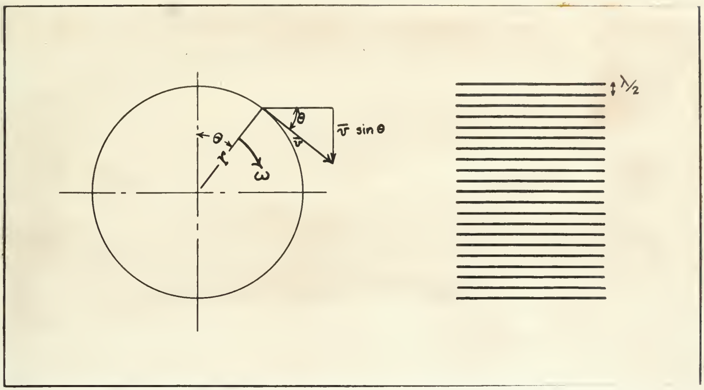


Figure 7

Therefore, any method that will enable one to determine the maximum frequency will indicate the amplitude of oscillation. If the oscillator frequency is held constant, then:

Amplitude is proportional to f_m.

3. Method of determining maximum frequency.

A variety of electronic circuits are available for the accurate determination of the maximum frequency. The one chosen by the author makes use of six stage tunable amplifier. The amplifier is tuned down from the high frequency range until a signal is heard in the ear phones of the amplifier. The highest frequency at which a signal is heard is the maximum frequency being produced. This maximum frequency can then be converted directly into an amplitude measurement of the vibrating reed.

Although the above method is perhaps not as accurate as several other of the other electronic methods available, it was selected on account of its simplicity and the availability of the equipment. Results sufficiently accurate for most purposes can be obtained with a calibrated amplifier using "sharp" tuning.

CHAPTER IV

RESULTS

1. Sensitivity of tuning.

With the equipment in operation it was found that tuning to the maximum frequency could be accomplished with a rather high degree of accuracy. Results to the nearest tenth of a kilocycle were obtained, provided spurious pickup from external sources was not present. In the lower frequencies, that is, below 100 kilocycles, it was very difficult to shield out spurious signals. However, by interrupting the light beam intermittently the audio signal resulting from the photocell pickup could be identified by ear and the proper tuning could be accomplished.

2. Capabilities of equipment.

The product of the amplitude of vibration, the frequency of vibration, and a constant, is equal to the maximum output frequency of the photocell. The constant depends upon the wave length of light used and the units of measurement. If amplitude is expressed in inches, wave length in inches, and the frequency of vibration in cycles per second, the constant is equal to $4\pi/\lambda$. It is seen that for a certain wave length of light that large amplitudes at low frequencies or small amplitudes at high frequencies can be measured. In the illustration above

$$f_m = \frac{4\pi}{\lambda} \cdot f_1 \cdot y$$

The maximum frequency which the type 1P42 cell is capable of handling is 1000 megacycles per second.* Therefore, by this method, with a given frequency of vibration the maximum amplitude which can be measured is easily calculated. One consideration which must be borne in mind, however, is the interferometer plate separation. The amplitude of vibration must not exceed the separation of the plates or damage to the optical flats will result.

3. Practical considerations in the construction of the interferometer.

While figure 3 suggests that the optimum reflecting power of the interferometer plates should be between 90% and 100%, the absorption of light by the coating becomes an appreciable factor. The author found by trial and error that a 70% to 80% reflecting power gave the best results, i.e., sharp contrast between bright and dark sections yet sufficient transmitted intensity to operate the photocell.

Silver coating rather than aluminum was used in order to minimize the absorption of light. One difficulty encountered in using silver coated flats was the tarnishing of the surface. It was noted, however, that considerable tarnishing of the surface could be tolerated with no undue adverse effect on the operation of the interferometer. The author experimented briefly with removal of the silver tarnish by placing a piece of aluminum foil in contact with the silvered surface and immersing in an electrolyte. This will remove the sulfide ion but it was found that the redeposited silver would not adhere to the

* Private communication between author and R.C.A. Tube Department, Radio Corporation of America, Harrison, New Jersey.

glass surface. The author is of the opinion that it is not practical to remove the tarnish from lightly silvered glass surfaces and suggests resilvering when the tarnish becomes excessive.

CHAPTER V

CONCLUSION

1. Practicability of method.

The author is of the opinion that the measurement of small cyclic displacements by the method set forth in this thesis provides a degree of accuracy not usually attained by the ordinary methods. While the field of application of this principle is limited it is felt that one important application might be its use in the calibration of more rugged field test equipment. It was with this idea in mind that the author undertook the study of the problem set forth herein.

2. Suggested variations for further study.

As was stated earlier, there are a number of different electronic circuits available for interpreting results. The author confined his field of study to one in which the end of the reed moved with simple harmonic motion. Where simple harmonic motion does not exist, an alternate circuit is suggested consisting of the following components: a broad band amplifier, an electronic counting circuit, and a suitable recorder. The interference bands traversing the interferometer per cycle could be picked up by the photocell, amplified, and counted by the electronic circuit. The amplitude of vibration could then be obtained directly.

The author is of the opinion that a multiplier photocell could be used to advantage as its use would reduce the number of amplifier stages necessary. While the upper frequency which the multiplier photocell can accommodate is less than the single

element photocell, this is not considered to be a serious limitation.

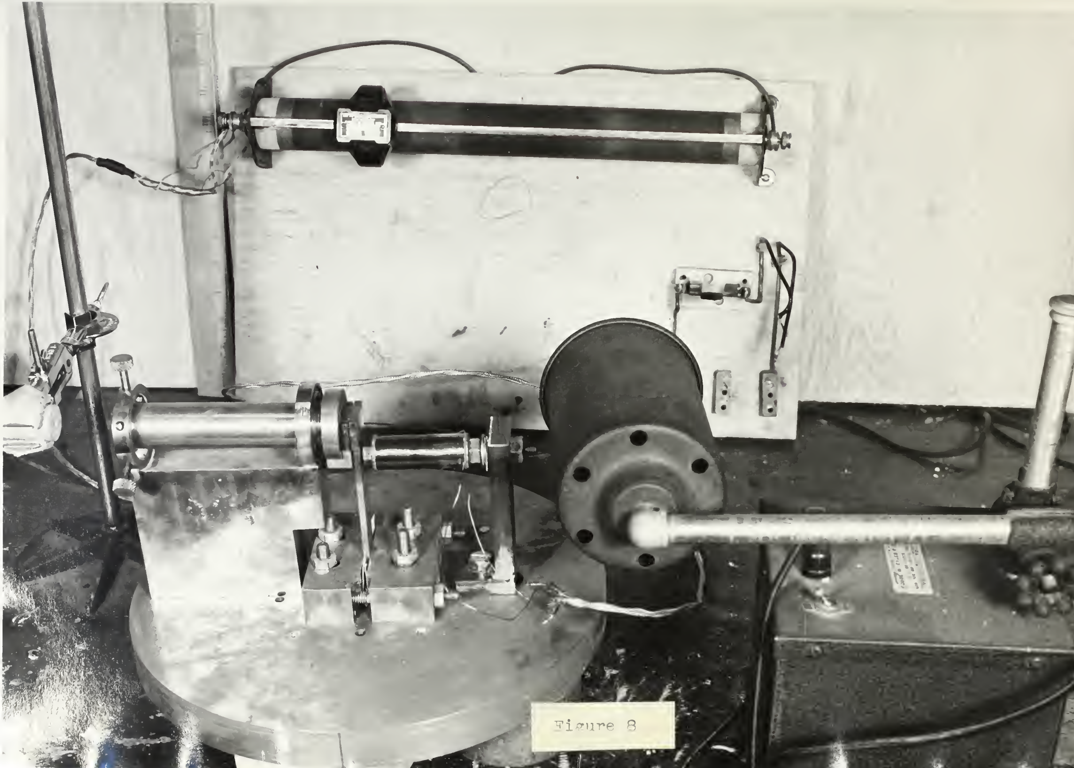


Figure 8

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5 Ryder, John D., Electronic fundamentals and applications. New York. Prentice Hall, 1950.

APPENDIX I

INTERFEROMETER PLATE SEPARATION

If a pair of parallel interferometer plates are separated by a distance s, there will be either a bright or dark spot in the center of the fringe pattern when $n\lambda = 2s$. The next corresponding bright or dark spot will occur when $(n-1)\lambda = 2s\cos\theta_1$, where θ_1 is the angle subtended by the second fringe. By the addition of the above equations we obtain:

$$\cos\theta_1 = 1 - \frac{\lambda}{2s}$$

The first order approximation of the cosine θ_1 is $1 - \frac{\theta_1^2}{2}$. Then:

$$1 - \frac{\theta_1^2}{2} = 1 - \frac{\lambda}{2s}$$

$$\theta_1 = \sqrt{\lambda/s}$$

The diameter of the image of the central spot when a lens of focal length f' is used is $f'\theta_1$, or diameter = $f'\sqrt{\lambda/s}$. The values of the various items in the above problem are as follows: $\lambda = 5461 \times 10^{-8}$ cm

f' = 13.6 cm.

diameter central spot = 0.63 cm.

Then:

$$0.63 = 13.6\sqrt{\frac{5461}{s} \times 10^{-8}}$$

$$s = 0.02185 \text{ cm., or } 0.0086 \text{ inches.}$$

APPENDIX II

TRANSIT TIME FOR AN ELECTRON IN AN ELECTRIC FIELD

An electron in traversing the distance between two charged plates does work as follows:

$$\text{work} = Ve = \frac{m(v^2 - v_0^2)}{2}$$

If the electron is emited from one surface the initial velocity v_o is zero, and

$$v^2 = 2V\frac{e}{m}, \quad \text{or} \quad v = \sqrt{2V\frac{e}{m}}$$

The transit time for a particle being uniformly accelerated in traversing a distance s is given by: $t = 2s/v$.

Therefore, $t = 2s \Big/ \sqrt{2V\frac{e}{m}}$.

Since s, e, and m are all constant, t is inversely proportional to the square root of the anode voltage. Ryder (5).

Considering the problem where the anode voltage is 90 volts, and the value of e/m is 1.76×10^{11} coulombs per kilogram, and s = .002 meters: (Note: this is the anode cathode separation in the 1P42 phototube)

$$t = \frac{2s}{\sqrt{2V\frac{e}{m}}} = \frac{2 \times 0.002}{\sqrt{2 \times 90 \times 1.76 \times 10^{11}}} = 0.711 \times 10^{-9} \text{ sec}$$

APPENDIX III

MAXIMUM THEORETICAL ERROR

To demonstrate the maximum theoretical error involved in using the author's method for the determination of amplitude, consider the case where the amplitude is 46.5 half-wave lengths of light. This corresponds to approximately 0.0005 inches. In this case an interference band does not appear at the midpoint of oscillation but rather one-quarter wave length on either side of the midpoint. Refer to Figure 7.

$$\theta = \cos^{-1} \frac{\text{one-half line}}{46.5 \text{ lines}}, \text{ and,}$$

$$\sin\theta = \frac{46.47}{46.50} = 0.99935.$$

The error involved, therefore, is the difference between 0.99935 and 1.0000 (sin of 90°), or 0.065%. Note also that the error is negative and cannot be positive. In inches this error is 0.325×10^{-6} inches.

The theoretical error is a function of the amplitude. For smaller amplitudes the error will be greater; for larger amplitudes it will be smaller.

PRACTICAL ERRORS

The actual error in a reading will, of course, be greater than the theoretical error computed above. The accuracy of the actual measurement will depend on the amplifier calibration, the sensitivity of the maximum frequency determination, and the accuracy of the determination of the frequency of the vibration of the beam or reed. If we assume that in the above

problem the amplifier calibration is good to within $\pm$ 1%, the sensitivity of the frequency determination is good to within $\pm$0.33%, and the determination of the frequency of vibration is accurate to within $\pm$1.0%, then the maximum possible error will be:

$$-0.065 \pm 1.00 \pm 1.00 \pm 0.33 = \begin{matrix} +2.330\ \% \\ -2.395\ \% \end{matrix}$$

The error involved in inches in an amplitude of 0.0005 inches would be about 12×10^{-6} inches.

Greater accuracy can be obtained by careful calibration of the frequency checking devices. Actually, frequencies can be checked to within negligible limits, should the need arise, by comparison with the various crystal oscillators which are available in electronics laboratories.