ELECTRONIC ANALOG OF A FREE PISTON GAS GENERATOR

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UNITED STATES
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THESIS

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Submitted in partial fulfillment of
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ABSTRACT

The use of an electronic analog computer in solving the differential equation of motion of the free piston gas generator has been investigated. To accomplish this end, the free piston has been treated as a specialized mass-spring system, wherein the forces were derived from thermodynamic considerations rather than from elastic considerations. The first law of thermodynamics was solved on the computer three concurrent times in order to produce electronic analogs of an internal combustion engine, a reciprocating compressor, and a gas spring. Coulomb and viscous friction were also simulated. The results of these four analogs were then combined and integrated as indicated by Newton's second law. The computer yielded a solution which represented the piston motion. A total of 26 amplifiers, seven diode limiters, and six electronic multipliers were used to solve the problem.

The characteristics of an existing free piston machine were introduced into the computer as a final check of the analog. The accuracy of some of the results was less than desired due to errors in a two-quadrant electronic multiplication involved in the gas spring analog. The other three analogs performed very satisfactorily. For accurate and reproducible results, some other method, such as direct function generation or single quadrant multiplication, must be used in the simulation of the gas spring.

ACKNOWLEDGEMENT

The writer wishes to express his appreciation for the assistance and encouragement given in this investigation by Professor Robert E. Newton and Professor Paul F. Pucci of the U.S. Naval Postgraduate School; and to the Department of Mathematics and Mechanics of the U.S. Naval Postgraduate School for the use of the Boeing Electronic Analog Computer.

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Symbol	Meaning	Units
A	Cross-sectional area	ft^2
C	Coulomb friction force	lb_f
C	Electrical capacatance	μfarads
E	Internal energy	$ft\text{-}lb_f$
F	Force	lb_f
F_F	Frictional force	
HV	Heating value of fuel	$\frac{ft\text{-}lb_f}{lb_m}$
M	Mass of piston	lb_m
P	Pressure	lb_f/ft^2
P_t	Delivery pressure to turbine	
P_p	Engine pressure at start of compression	
P_s	Pressure in the scavenge space	
P_d	Discharge pressure of compressor	
P_i	Intake pressure of compressor	
P_o	Atmospheric Pressure	
P_b^o	Pressure in bounce at start-up	
Q	Energy as heat	$ft\text{-}lb_f$
Q_{rej}	Total energy leaving the system through cooling water	
Q_f	Energy of fuel	
R	Gas constant for air	$\frac{ft\ lb_f}{lb_m\ {}^oR}$
R	Electrical resistance	ohms
T	Temperature	^oR
T_o	Ambient temperature	
T_d	Temperature of compressor discharge	

TABLE OF SYMBOLS

Symbol	Meaning	Units
T_p	Temperature of engine charge	
V	Volume	ft^3
V_p	Volume of engine charge	
W	Energy as work	$ft\text{-}lb_f$
W_F	Frictional work	
Y_i	Length of compressor intake stroke	ft
a	Potentiometer constant	dimensionless
b	Bounce cylinder clearance when engine pistons are touching	ft
c	Compressor clearance when engine pistons are touching	ft
f	Frequency	sec^{-1}
h	Enthalpy	$\frac{ft\text{-}lb_f}{lb_m}$
h_o	Enthalpy of ambient air	
h_t	Enthalpy of exit gasses	
k	Ratio of specific heats	dimensionless
k	Mechanical spring constant	lb_f/ft
m	Mass	lb_m
m_d	Mass of air delivered by compressor	
m_e	Mass of engine charge	
m_f	Mass of fuel	
n	Polytropic exponent of engine compression	dimensionless
n'	Polytropic exponent of engine expansion	
n_c	Polytropic exponent of compressor compression and expansion	
s	Stroke	ft

TABLE OF SYMBOLS

Symbol	Meaning	Units
t	Time	sec
v	Viscous damping constant	lbf-sec/ft
x	Engine displacement from centerline	ft
$\dot{x},\dot{y},\dot{z}$	Piston velocity	ft/sec
$\ddot{x}$	Piston acceleration	ft/sec^2
$\frac{dQ}{dt}$	Rate of energy transfer as heat	ft-lbf/sec
α_i	Magnitude scaling factor	Units of i/volt
α_t	Time scaling factor	Computer time/ Real time
ξ	Auxiliary displacement variable	arbitrary

Subscripts:

E	Refers to engine cylinder	
c	Refers to compressor cylinder	
f	Refers to feedback on operational amplifier	
b	Refers to bounce cylinder	

Quantities with a bar represent computer voltages corresponding to the real physical quantities. Thus $P = \alpha_p \bar{P}$.

1. Introduction.

A great deal of effort has been put into the solution of the free piston engine problem. All of the methods currently in use are digital methods involving a point-by-point calculation of the variables associated with such a device. These methods are time-consuming and cumbersome due to both the trial and error element and the many geometric and operating parameters involved. In addition, the digital methods can be made tractable only by incorporating many numerical assumptions; some of which introduce considerable error.

London(1) has developed a "Thermodynamic-dynamic analysis" whereby the calculated parameters of a "standard" machine may be scaled by certain affinity relationships to establish the design of larger or smaller machines. This method eases somewhat the numerical burden of calculation, but the results proved to be quite sensitive to the assumptions made; and in order to predict the characteristics of an existing machine, the assumptions were relaxed in four successive groups before the desired accuracy was obtained. This procedure still presented a formidable numerical task.

In addition, a solution by digital methods becomes more complex in the investigation of part-load characteristics.

The purpose of this investigation is to determine the possibility of solving the differential equation of motion of the free piston on the Boeing Electronic Analog Computer (BEAC), which is described in Appendix VI. If this can be done, the computer will electronically simulate the free piston, presenting as a solution the piston motion, with most of the pertinent quantities also available. The advantages of this method include:

1. The feedback principle, typical of analog computation,

eliminates trial and error.

2. All of the parameters may be taken into consideration, and the effect of each may be observed.

3. The solution is immediate, thus permitting the study of many combinations of parameters in a short time.

4. Fewer simplifying assumptions are required, and these may be relaxed to any degree compatible with the size of the computer.

5. Part-load behavior can be studied as easily as full-load behavior.

6. Time may be scaled to permit slow-motion studies.

2. General Background of Free Piston Systems.

Figure 1 illustrates schematically two of the possible free piston configurations. Figure 1(a) illustrates the free piston air compressor, where the useful output is compressed air; and Fig. 1(b) illustrates the free piston gas generator, where the useful output is high pressure-temperature gas for use in a turbine.

This investigation will encompass only the free piston gas generator. A slightly larger computer is required to simulate the free piston compressor.

The free piston machine depicted in Fig. 1(b) consists of two opposed pistons, the inboard ends of which operate as a two stroke diesel engine. Another part of each piston acts as a reciprocating air compressor. As the pistons move outward, the expansion work of the engine and compressor is stored as energy in the bounce chambers. This energy is then used on the inward stroke for the compression and delivery of the air in the compressor and the compression of the engine charge.

The outstanding features of this free piston machine are:

1. Simplicity of construction due to absence of crank mechanisms.

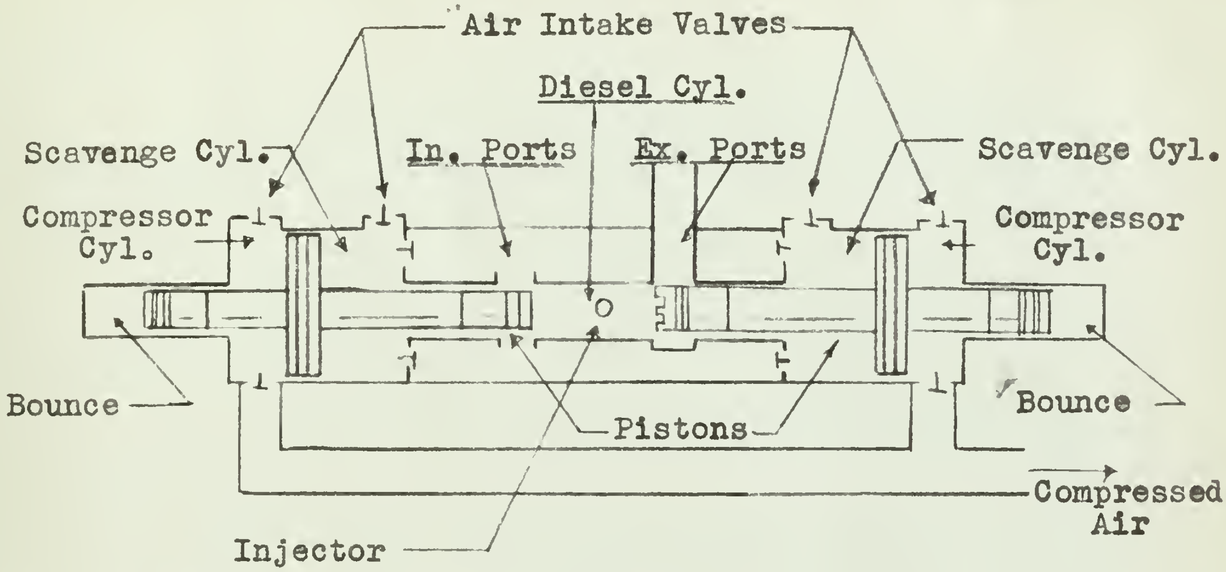
2. Opposed piston configuration eliminates vibration.

3. No piston side thrust arising from crank inertia forces.

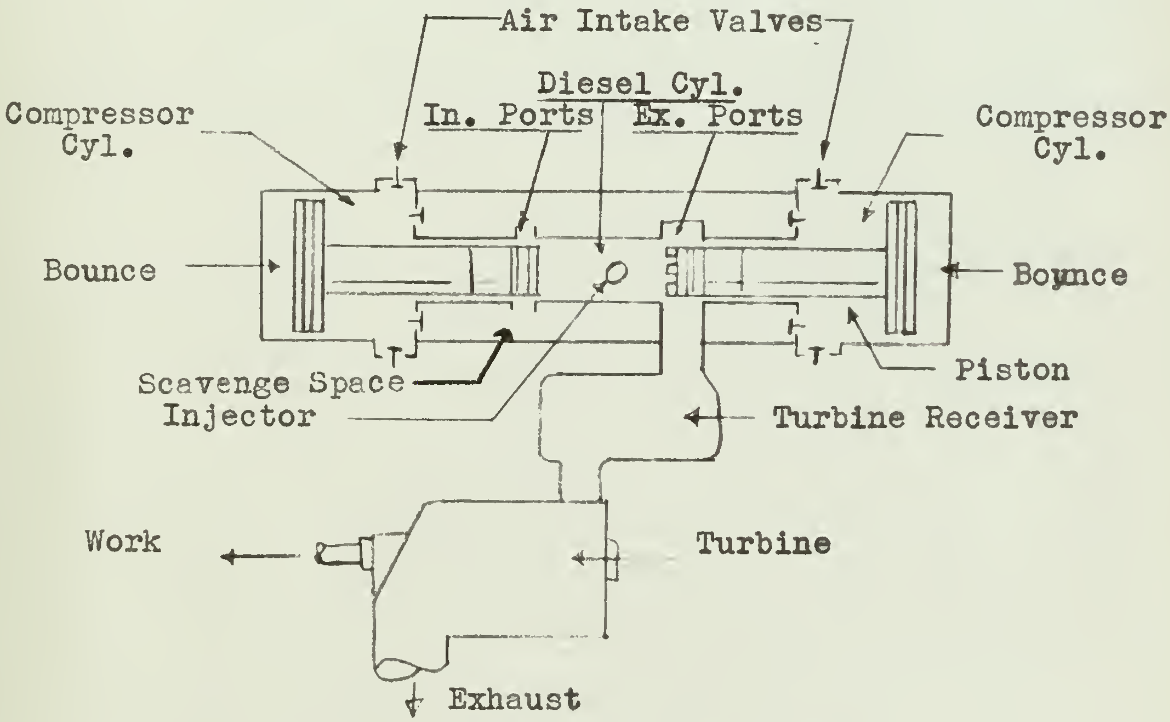
In the case of the gas generator, it would be well to review the features of a typical cycle.

Referring to Fig. 1(b), which illustrates, say, the pistons just starting the instroke. Beginning at this point, the following successive processes may be visualized:

1. High pressure in the bounce space forcing the pistons together; compressing and delivering the air in the compressor space, and



(a.) Free piston compressor system.



(b.) Free piston gas generator-turbine system.

Figure 1. Typical Free Piston Configurations.

compressing the engine charge when the scavenge ports are covered.

2. Reversal of piston motion occurring during injection and burning of fuel.

3. The expansion of the fluid in the engine cylinder and the intake stroke of the compressor; while the bounce cylinder is storing the expansion energy of the engine and compressor.

4. Opening of the engine ports during the latter portion of the outward stroke, delivering working fluid to turbine.

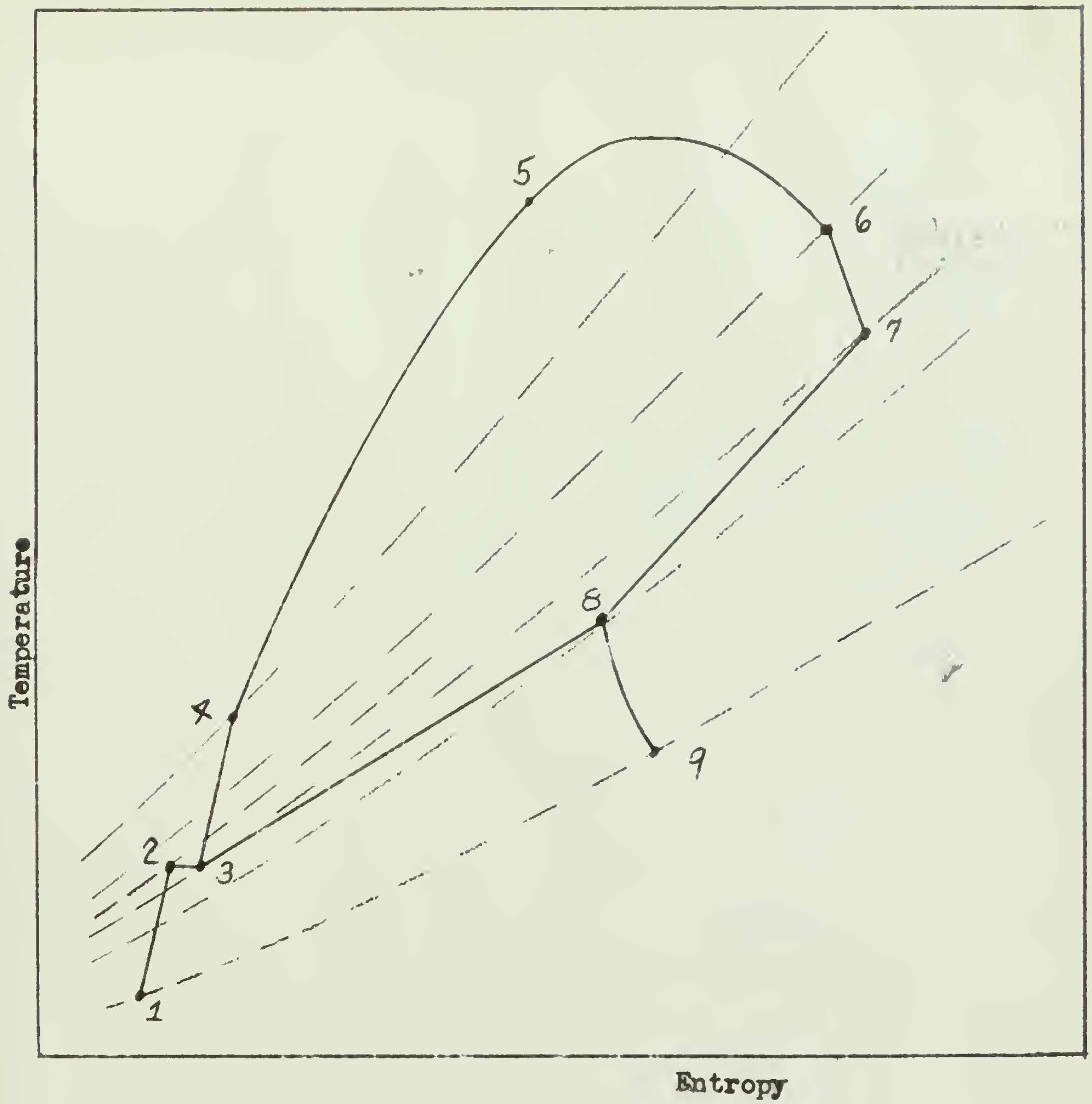
5. Pressure in bounce space causes reversal of piston motion, and cycle repeats.

The preceding cycle is represented on Temperature-Entropy coordinates in Fig. 2. It is to be noted that the processes involve different masses, and can be represented only on a "per pound" basis. This diagram is similar to that in common use in free piston literature. It should be pointed out that the following two assumptions are made in producing this diagram:

1. The scavenge air is furnished at constant pressure and temperature, related to the compressor discharge state.

2. Delivery to the turbine occurs at constant pressure and temperature.

These two assumptions are intimately connected with the mechanical design of the engine ports and the physical size of the turbine receiver. It is beyond the scope of this investigation to consider such problems. The foregoing, therefore, constitute the first two assumptions in this investigation. It must be pointed out, however, that these assumptions must be relaxed if it is desired to investigate transient behavior, inasmuch as these particular parameters are the primary transients. Before



Process 1-2 - Compression in Compressor.
Process 2-3 - Pressure Drop into Engine.
Process 3-4 - Compression in Engine.
Process 4-5 - Burning of Fuel.
Process 5-6 - Expansion in Engine.
Process 6-7 - Blowdown as Exhaust Ports Open.
Process 3,7-8 - Mixing and Scavenging.
Process 8-9 - Expansion in Turbine.

Figure 2. Temperature - Entropy Representation of Free Piston Generator-Turbine System.

proceeding further, the following are some additional assumptions*
covering the device as a whole:

1. That the compressor intake is at $70^{o}F.$ and 14.7 psia.

2. That the operating fluid obeys the ideal gas laws.

3. A nominal pressure drop of five per cent across each valve and engine port. This is arbitrary, and may be refined to any desired value.

4. That the geometry is symmetrical with respect to a plane passing through the center of the machine perpendicular to the horizontal axis. The end projection of this plane will be referred to as the "center line."

*Refer to Appendix I for all of the assumptions used.

3. The General Analog of the Free Piston.

From a dynamic point of view, the piston in a free piston machine may be considered as a rather special sort of mass-spring arrangement, wherein the "springs" yield forces derived from thermodynamic considerations rather than from elastic considerations.

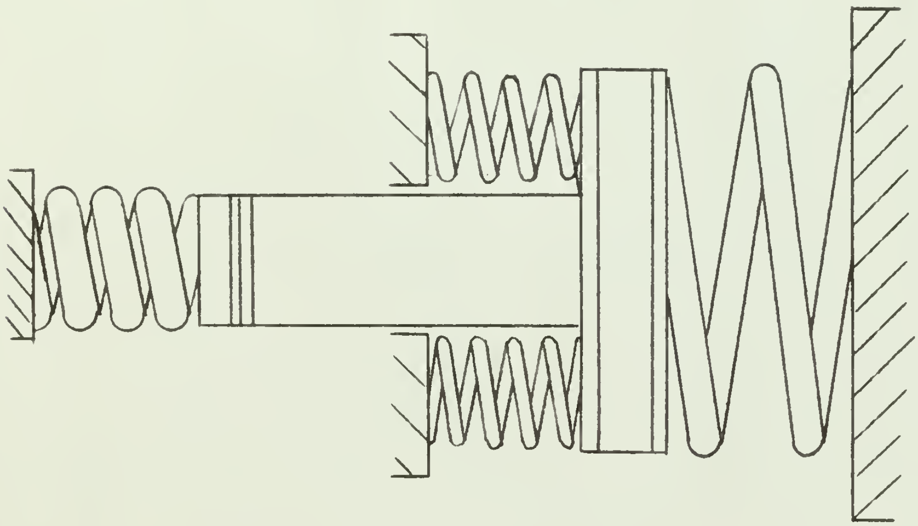


Figure 3. Spring Analogy of the Free Piston.

The system may be visualized as similar to that depicted in Fig. 3 above, where the spring forces are proportional to the conventional Pressure-Volume representation of the appropriate space. Not shown, but also present, is the force of static and viscous friction. Qualitative representations of these forces appear in Fig. 4.

To illustrate the analog computer approach to this type of problem, consider the simple mass-spring arrangement illustrated in Fig. 5. The differential equation of this motion may be immediately written down as

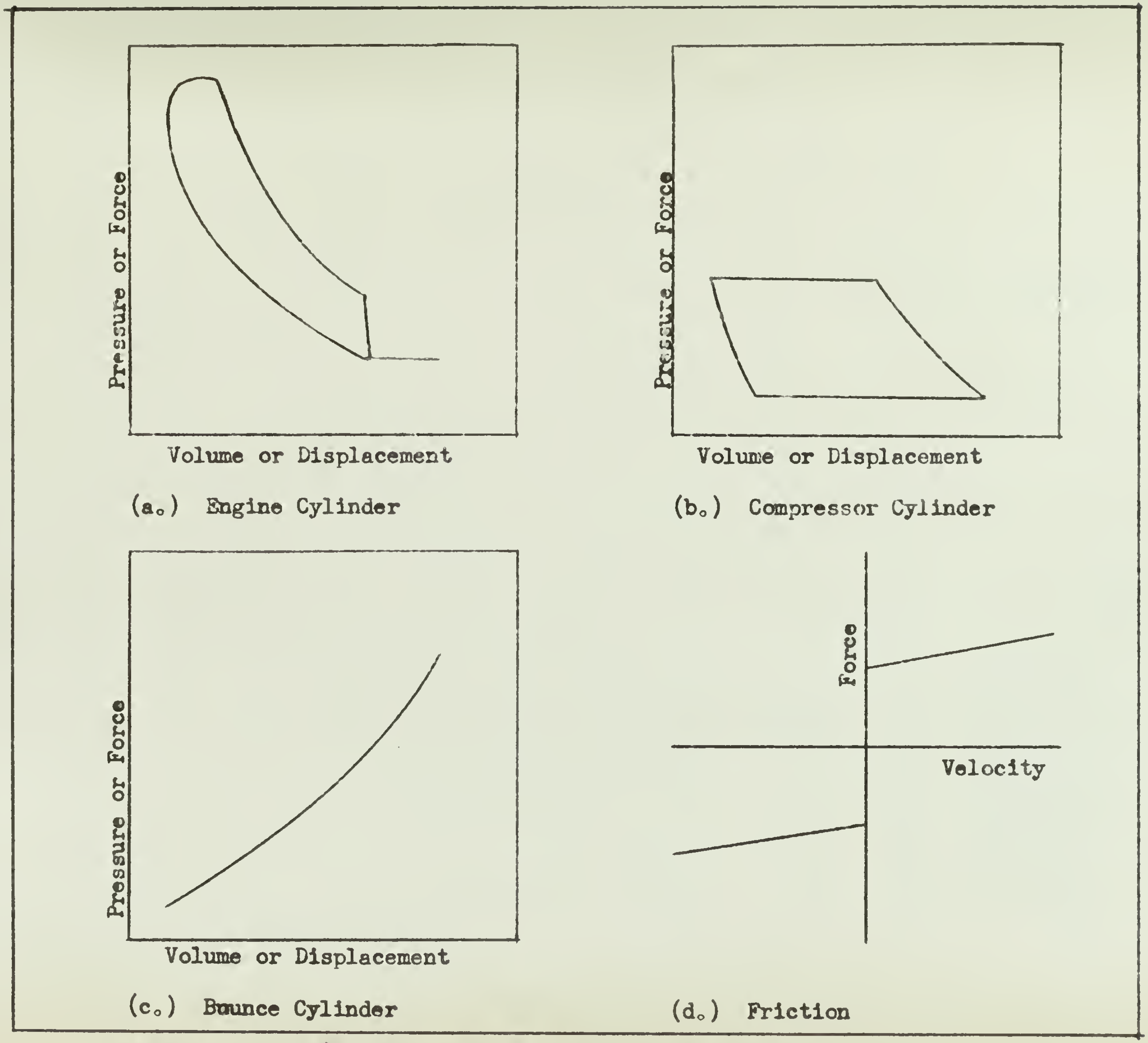


Figure 4. Typical Forces Acting on the Free Piston.

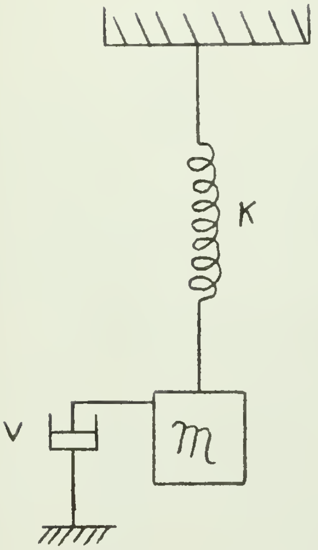


Figure 5. Simple Spring-Mass System.

$$m\ddot{\xi} + K\xi + V\dot{\xi} = 0,$$

where m is the mass
v is the damping constant
k is the spring constant
$\ddot{\xi}$ is the acceleration
$\dot{\xi}$ is the velocity
ξ is the displacement

By solving for the highest derivative,

$$\ddot{\xi} = -\left(\frac{K}{m}\xi + \frac{V}{m}\dot{\xi}\right).$$

Recalling that the operational amplifier can add and/or integrate with respect to time, a computer schematic may be immediately devised to solve this problem. This is illustrated below in Fig. 6.*

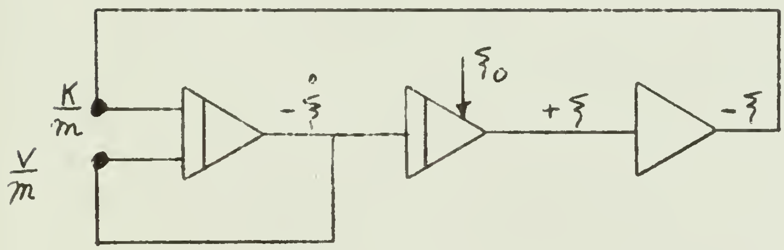


Figure 6. Electronic Analog of the Simple Spring-Mass System.

*Refer to Appendix II for analog computer table of symbols.

This simple problem points out the utility of analog feedback. That is to say, the immediate stabilization to the correct solution by using this solution to generate the derivatives, which are then integrated to form the solution again.

Turning now to the free piston problem, and referring to Fig. 3; the equation for this motion may be formulated in accordance with Newton's second law as

$$M\ddot{x} = F_E(x,\dot{x}) + F_c(x,\dot{x}) - F_b(x,\dot{x}) - F_F(\dot{x}), \qquad 3.1$$

where M is the piston mass
$F_E(x,\dot{x})$ is the force on the engine piston
$F_c(x,\dot{x})$ is the force on the compressor piston
$F_b(x,\dot{x})$ is the force on the bounce piston
$F_F(\dot{x})$ is the force due to friction
x is the piston displacement
$\dot{x}$ is the piston velocity
$\ddot{x}$ is the piston acceleration

In a manner similar to the simple mass-spring problem, a computing arrangement can be immediately proposed. Such an arrangement is outlined in Fig. 7. In this figure the block representing the engine, compressor, bounce, and friction, are rather complex computing networks with many more inputs than shown. The overall principle, however, is basically the same as that outlined in Fig. 6.

This fact has two very important consequences in that :

1. A completely compatible set of parameters must exist in order for the computer to produce a sustained oscillation. If this condition is not met, a situation exists which is analagous to overdamping or negative damping of the simple system.

2. Since equation 3.1 also expresses the work-energy equality, a solution of this equation will automatically satisfy the condition that the engine work appears as the sum of compressor and friction work.

The computer arrangement as shown in Fig. 7 will produce displace-

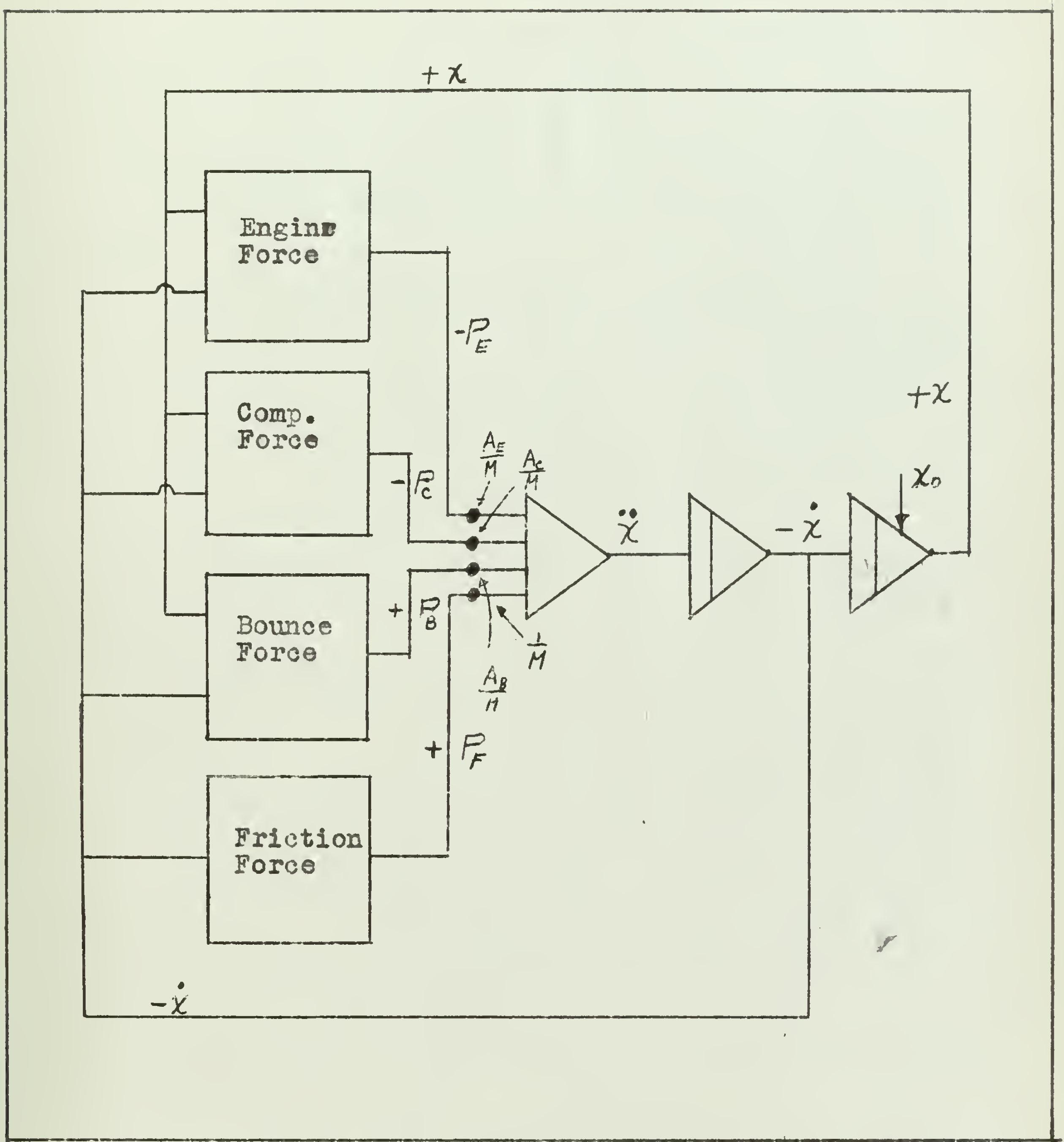


Figure 7. Basic Computer Arrangement for the Free Piston Problem.

ment as a function of time. Also available will be the various forces or pressures contributing to the motion. Related items such as air-fuel ratio, scavenge ratio, and power output will not be available directly from the computer but can be easily calculated.

The inputs to the system will be :

1. Geometric parameters, which will be fixed for a given machine.

2. Initial pressure conditions, which are:

   a. Intake and delivery pressure of the compressor.

   b. Pressure at the start of engine compression.

   c. An initial pressure in the bounce space.

3. Amount and heating value of fuel.

4. Thermodynamic parameters such as polytropic exponents, and ratio of specific heats.

All of the above parameters may be varied to suit almost any condition compatible with the assumed geometry. For instance, part load behavior may be simulated by merely changing the appropriate pressure initial conditions, and then determining the range of fuel energies for which the solution is stable. Of course, a different amount of fuel applied to the same pressure conditions will lead to a different displacement-frequency solution.

The usefulness of the analog computer approach to this problem is that:

1. The parameters are introduced as various constants and initial conditions and are not manipulated at all.

2. The computer solves the differential equation of motion.

3. Other quantities may be determined by a minimum amount of calculation.

It is to be re-emphasized that the analog computer eliminates the trial-and-error elements from this problem if the geometry is used as a starting point.

4. Geometric Parameters.

There are eight primary geometric parameters and variables associated with the analog approach to the free piston problem. These are:

1. Area of engine piston, A_E.
2. Area of compressor piston, A_c.
3. Area of bounce piston, A_b.
4. Engine piston displacement from centerline, x.
5. Compressor piston displacement from head, y.
6. Bounce piston displacement from head, z.
7. Engine piston position at port closure, x_p.
8. Piston mass, M.

Referring to Fig. 1(b), it may be readily seen that

$$A_b = A_E + A_c, \qquad 4.1$$

$$y = x + c, \qquad 4.2$$

and

$$z = b - x, \qquad 4.3$$

where c and b are the respective clearances of the compressor and bounce cylinders when the engine pistons are touching. Since the piston is in one piece, it is obvious that

$$\frac{dx}{dt} = \frac{dy}{dt} = -\frac{dz}{dt}. \qquad 4.4$$

These relationships are necessary in that x, y, and z and their time derivatives are used in the generation of the appropriate forces.

The mass, M, may be taken as any reasonable value compatible with piston size. A computer solution indicating a piston density greater than that of steel must be rejected.

The treatment of the free piston generator will encompass one-half of the system, which will be the right hand piston as illustrated in Fig. 3. Positive forces will be considered to be acting to the right. Since one of the computer processes will be the summation of these forces, the associated sign inversion requires that the negatives of the engine and compressor forces be generated. Since one-half of the system is being treated, all extensive quantities must be multiplied by two.

5. Thermodynamic Parameters.

Inasmuch as the computer is solving a mechanical problem, all thermodynamic quantities will be assumed to be given in compatable mechanical units. The use of Joule's constant will always be implied whenever necessary. Energy leaving a system as heat will be considered negative.

5.1 Pressures.

The pressure taken as the fundamental variable will be the turbine receiver pressure, P_t. This will be assumed to remain cyclically constant. Using this as a starting point, it is possible to work backwards through the system to establish all of the other pressures. These are

$$P_p = \frac{P_t}{r_e}, \qquad 5.1.1$$

$$P_s = \frac{P_p}{r_s} = \frac{P_t}{r_e r_s}, \qquad 5.1.2$$

$$P_d = \frac{P_s}{r_d} = \frac{P_t}{r_e r_s r_d}, \qquad 5.1.3$$

and

$$P_i = P_o r_i ; \qquad 5.1.4$$

where P_p is the engine pressure during scavenging,
P_s is the pressure in the scavenge cylinder,
P_d is the compressor delivery pressure,
P_i is the compressor intake pressure,
P_o is the ambient pressure, and
r is the ratio of downstream to upstream pressure for

"e" - Engine exhaust ports,
"s" - Engine scavenge ports,
"d" - Compressor discharge valves, and
"i" - Compressor intake valves.

5.2 Volumes.

In all cylinders, the volume is the product of total displacement and area.

5.3 Temperatures.

Temperature will not be used as such, but rather indirectly in the determination of the mass of air charged to the engine and the power output of the machine.

5.4 Mass of Engine Charge.

The mass of air charged to the engine is not at all a necessary parameter insofar as the computer is concerned, but must be known in order to determine the air-fuel ratio.

Using the assumption that there is no heat gained or lost in the scavenging space, it can be postulated that, upon completion of the scavenging process, the engine charge temperature, T_p, will be essentially that temperature of the compressor discharge. The mass of engine charge, m_e, will then be

$$m_e = P_p V_p / RT_p, \qquad 5.4.1$$

where V_p is the engine volume at the point of port closure.
Applying the compression ratio of the compressor to the above equation yields

$$m_e = \frac{P_p V_p}{R T_o \left(\frac{P_d}{P_i}\right)^{n_c - 1}}, \qquad 5.4.2$$

where T_o is the ambient temperature
n_c is the polytropic exponent of compression in the compressor.

This result may be expressed in terms of P_t by applying relationships 5.1.1 and 5.1.5:

$$m_e = \frac{V_p P_t^{(2-n_c)} \left(P_o r_e r_s r_d r_i\right)^{(n_c - 1)}}{R T_o r_e}. \qquad 5.4.3$$

If all r's are assumed the same, the above equation reduces to

$$m_e = \frac{V_p}{R T_o}\left[P_t^{(2-n_c)} P_o^{(n_c-1)} r^{(4n_c-5)}\right]. \qquad 5.4.4$$

Assuming that r is 0.95, and further postulating that n_c is 1.30, the above relationship becomes

$$m_e / V_p = 0.0113 \, P_t^{0.7}, \qquad 5.4.5$$

where P_t is expressed in pounds per square inch. For convenience, equation 5.45 is presented graphically in Fig. 8.

5.5 Power Output of the System.

To determine the power delivered to the turbine, an overall energy balance is necessary:

$$m_d h_o + m_f HV = (m_f + m_d)h_t + Q_{rej}, \qquad 5.5.1$$

where m_d is the mass of air delivered by the compressor per cycle.
m_f is the mass of fuel injected per cycle.
h_o is the enthalpy of entering air.
ht is the enthalpy of exit gases.
HV is the lower heating value of fuel
Qrej is the energy as heat leaving the system via the cooling water per cycle.

Equation 5.5.1 makes the customary assumption that the sensible enthalpy of the fuel may be neglected.

The quantity m_d may be readily determined by noting from the computer solution the length of the intake portion of the compressor stroke, and applying the ideal gas law

$$m_d = P_i A_c Y_i / RT_o, \qquad 5.5.2$$

where Y_i is the length of the intake portion of the compressor stroke.

The quantity m_f HV is a known operating variable and h_o is tabulated.

The quantity Q_{rej} is composed of:

1. Energy as heat leaving the engine.

2. Energy as heat leaving the compressor.

3. Energy lost through frictional heating. It will be assumed that this will be totally lost to the coolant.

The determination of Q_{rej} will be deferred until more specific differential equations are formulated.

Assuming for the moment that Q_{rej} is known, the only remaining unknown in equation 5.5.1 is h_t, the enthalpy of the exit gasses. From

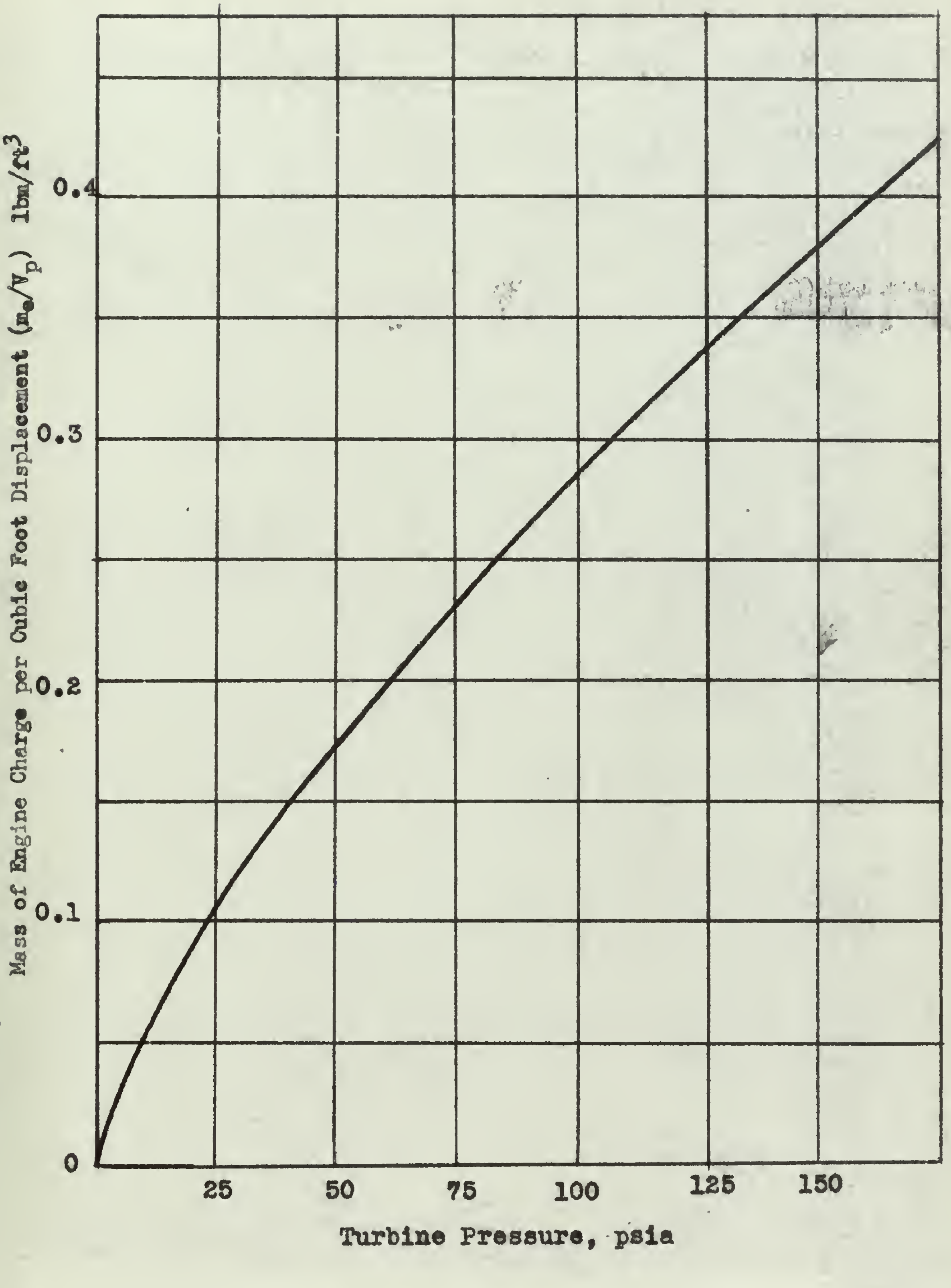


Figure 8. Engine Charge Mass per Cubic Foot Piston Displacement.

this quantity, the temperature of the exit gas may be determined, and also the ideal isentropic work available per pound of fluid. The product of this work, the total mass delivered per cycle, and the frequency will then give the air horsepower of the system. It is to be noted again that this treatment involves one-half of the machine, so that the "total" mass mentioned above is twice that given in equation 5.5.2.

6. Electronic Analog of an Internal Combustion Engine.

6.1 General.

The force exerted upon the engine piston may be most appropriately represented by referring to a typical indicator diagram such as depicted in Fig. 4(a) on page 9. Since the free piston device has no crank mechanism, the only fixed geometrical point is that of port closure. The thermodynamic state at this point will be known if it is assumed that successive cycles are identical. All other point of force as a function of displacement are determined by the dynamics of the system. In order to simulate this situation it is necessary to obtain a computer solution of the differential equation of the first law of thermodynamics.

In the following analysis, the approximate air-standard is used, which assumes:

1. Air is the working fluid, and
2. The mass of fluid within the cylinder remains constant.

The latter assumption is also derived from the following two practical considerations:

1. The mathematical complication involved in a changing mass system, while possible to solve on the computer, would require more components than are available.

2. The mass change is within the same order of magnitude as the accuracy of the computer, this there would be no observable gain from such a refinement.

6.2 The General Differential Equation for the Internal Combustion Engine.

The first law of thermodynamics may be written in differential form as

$$dQ = dW + dE, \tag{6.2.1}$$

where dQ is the net energy entering or leaving the system as heat.
dW is the work done by the system.
dE is the change in internal energy of the system.

By taking the time derivative of this equation, and by replacing dW and dE by their respective equivalents in terms of pressure and volume, the following equation is obtained.

$$\frac{dQ}{dt} = P\frac{dV}{dt} + \frac{1}{K-1}\left(P\frac{dV}{dt} + V\frac{dP}{dt}\right). \qquad 6.2.2$$

By solving for the highest derivative of P;

$$\frac{dP}{dt} = \frac{1}{V}\left[(K-1)\frac{dQ}{dt} - KP\frac{dV}{dt}\right]. \qquad 6.2.3$$

Now, the volume is given by

$$V = A_E x, \qquad 6.2.4$$

and

$$\frac{dV}{dt} = A_E\frac{dx}{dt} = A_E\dot{x}. \qquad 6.2.5$$

When equations 6.2.4 and 6.2.5 are substituted into equation 6.2.2, the final result is

$$\frac{dP_E}{dt} = \frac{1}{x}\left[\frac{K-1}{A_E}\frac{dQ}{dt} - KP_E\dot{x}\right], \qquad 6.2.6$$

where P_E is the engine pressure.

This equation may be solved on the BEAC in the following steps.

1. Integrate $\frac{dP_E}{dt}$ to form $-P_E$ with the appropriate initial condition, P_p.

2. Multiply $-P_E$ by $k\dot{x}$ to form $-kP_E\dot{x}$.

3. Add $\frac{k-1}{A_E}\frac{dQ}{dt}$ to form $\left[\frac{K-1}{A_E}\frac{dQ}{dt} - KP_E\dot{x}\right]$.

4. Divide this sum by x. This result is, by equation 6.2.6, $\frac{dP_E}{dt}$, which is then fed back to Step 1.

The above steps are represented schematically in Fig. 9. This diagram also shows considerable computer detail which is explained in the following notes.

1. The inputs to Amplifier 5 are to separate the net $\frac{dQ}{dt}$ into "fuel" and "non-fuel" terms in order that these quantities may be varied independently. In addition, the "fuel" function is non-zero for only a portion of the cycle. The output of this amplifier is $\frac{dQ}{dt}$ net.

2. The relay in the feedback of Amplifier 1 establishes the initial condition, P_p, whenever x is greater than x_p.

3. The combination of the electronic multiplier and Amplifier 2 is pre-set to give one-fiftieth of the product.

4. The multiplier in the feedback of Amplifier 4 has the effect of dividing by x, where-x must always be negative for stability. The output of this amplifier is pre-set to give ten times the quotient.

5. Amplifier 3 is set to give ten times the sum of $-\frac{1}{100}\left(K P_E \dot{x}\right)$ and $\frac{1}{100}\left(\frac{K-1}{A_E}\frac{dQ}{dt}\right)$ in order to compensate for the constants inherent in the multiplication and division.

To gain a better insight into the computer solution of equation 6.2.6 a few simple qualitative examples will be discussed in the following three sections.

6.3 Computer Simulation of a Reversible Adiabatic Process.

The reversible adiabatic process is characterized by $\frac{dQ}{dt}$ equal to zero, in which case equation 6.2.6 reduces to

$$\frac{dP_E}{dt} = - \frac{K P_E \dot{x}}{x} . \qquad 6.3.1$$

It may be easily verified that $P_E x^k =$ constant is the solution to the above equation, which defines the familiar isentropic process. The computer solution to equation 6.3.1 appears in Fig. 10 with various

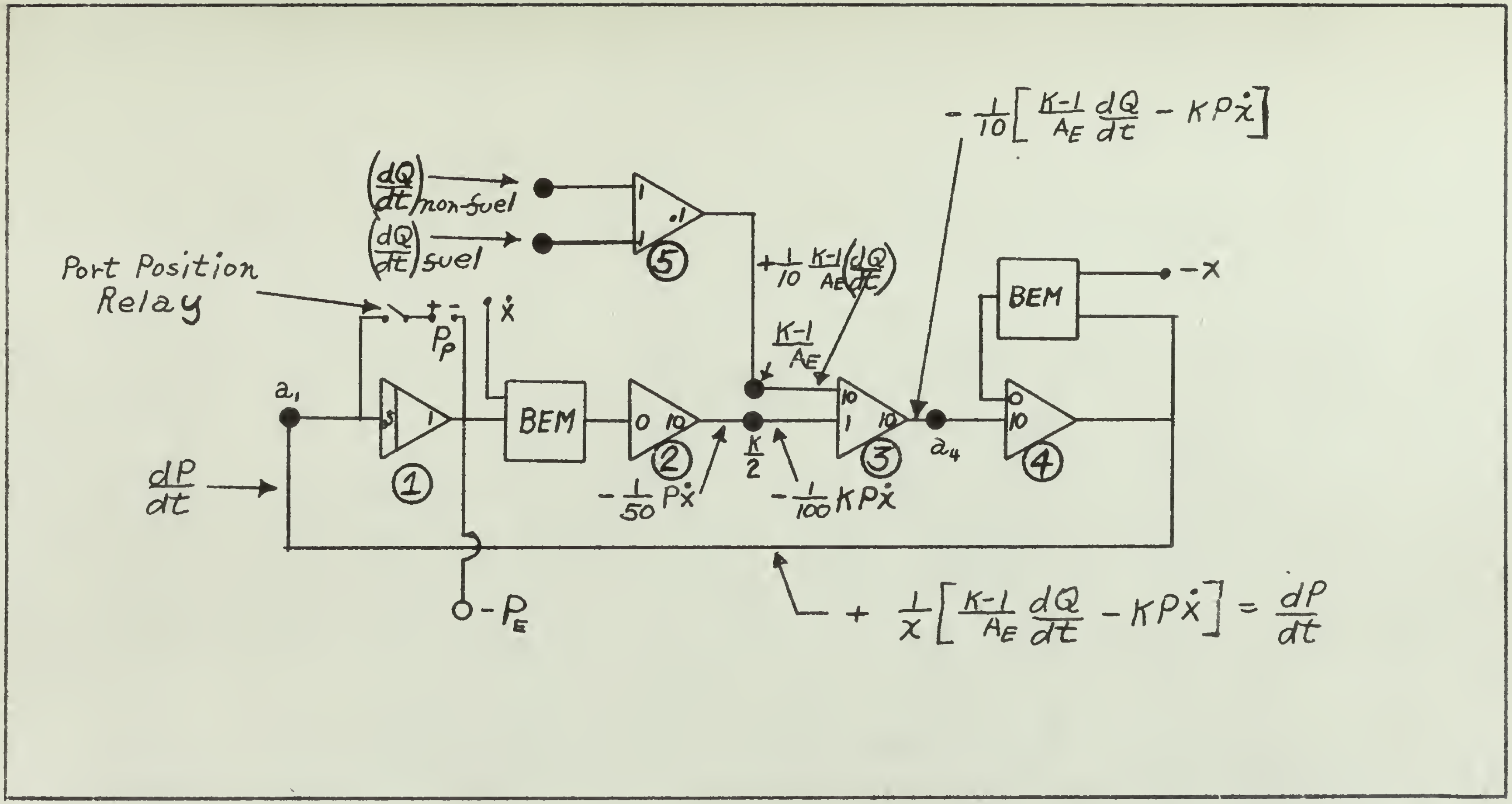


Figure 9. Electronic Analog of an Internal Combustion Engine.

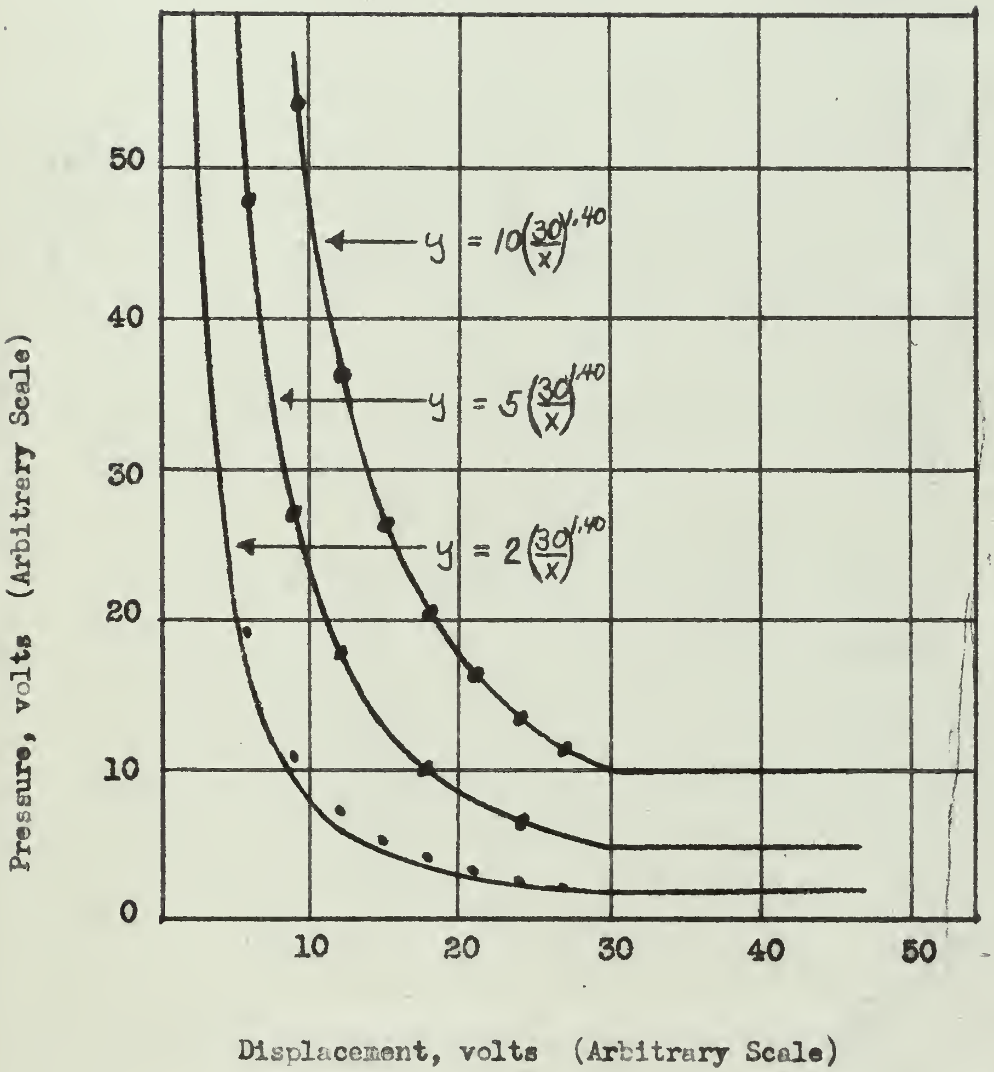


Figure 10. Computer Simulation of an Isentropic Process.

initial conditions. The input "x" was a displaced sinusoid simulating a piston motion at about one cycle per second. This was produced from the arrangement shown in Fig. 11, where ξ is an auxiliary variable solving the equation $\ddot{\xi} + \omega^2 \xi = 0$. Also shown are the "y" and "z" variables which will be used in the forthcoming discussions of the compressor and bounce cylinders. This is an artificial stroke input and will be by-passed when the entire free piston analog is assembled.

The solutions depicted in Fig. 10 can be seen to be excellent for all but the lowest values of initial condition. The plotted points are the numerical values of the indicated equations, and the solid lines are the computer solutions. In all cases, the integration was stopped whenever "x" was greater than 30 volts. The departure from the correct solution for the low value of initial condition is due to the fact that the inherent electronic errors are magnified whenever the computer is working with small voltages. This condition must be avoided if at all possible.

It is to be pointed out that the multiplication of $-P_E$ by $\dot{x}$ involves two quadrants. That is to say, $-P_E$ is always negative, while $\dot{x}$ is both positive and negative. Care must be taken to insure that this multiplication yields the same result in both quadrants. A slight maladjustment in this multiplier will not return the solution to the starting point since $\frac{dP_E}{dt}$ in one direction will be different from $\frac{dP_E}{dt}$ in the other.

6.4 Computer Simulation of a Reversible Polytropic Process.

The reversible polytropic process may be achieved by letting

$$\frac{dQ}{dt} = \frac{A_E}{K-1}(K-n)\, P_E \dot{x} \,. \qquad 6.4.1$$

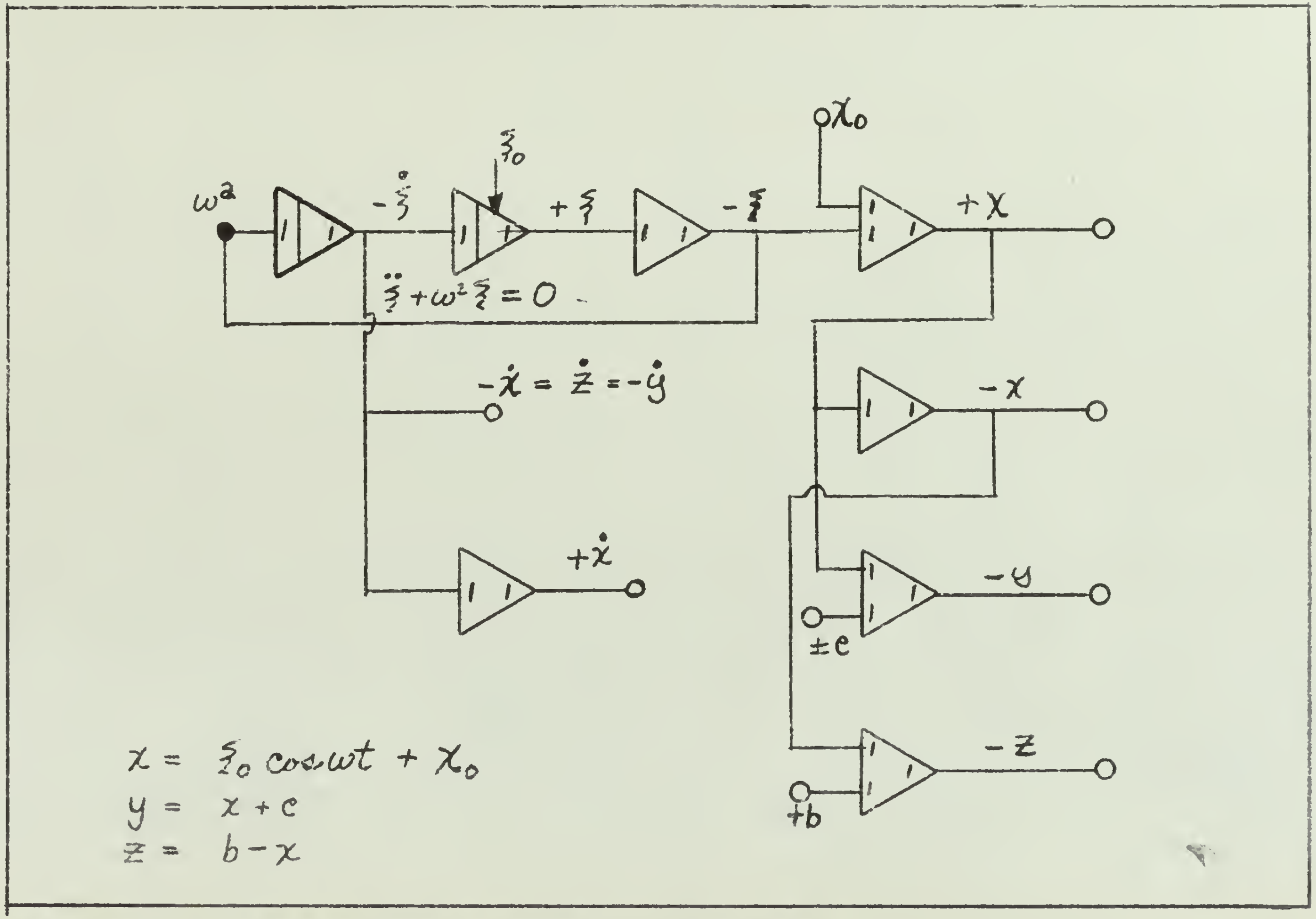


Figure 11. Generation of Artificial Stroke.

Upon substitution, equation 6.2.6 becomes

$$\frac{dP_E}{dt} = -\frac{n P_E \dot{x}}{x}, \qquad 6.4.2$$

It may be easily verified that $P_E x^n$ = constant is the solution to this equation. It should be pointed out that while mathematically $\frac{dQ}{dt}$ is as defined in equation 6.4.1, the same result may be achieved on the computer by letting $\frac{dQ}{dt}$ be zero, and applying the coefficient $\frac{n}{2}$ instead of $\frac{k}{2}$ after Amplifier 2, as suggested by equation 6.4.2.

Figure 12 illustrates the computer solution of a few samples of this case, again with excellent agreement. At this point, it is important to realize that there is no net loss of energy from the system over a cycle. The energy lost during compression is restored in the same manner during the expansion, or

$$\oint \left(\frac{dQ}{dt}\right) dt = 0 \qquad 6.4.3$$

This may be easily verified on the computer by integrating equation 6.4.1. Figure 13 shows displacement, pressure, $\frac{dQ}{dt}$, and $\int\left(\frac{dQ}{dt}\right)dt$ as functions of time. As expected, $\oint\left(\frac{dQ}{dt}\right)dt$ is zero at the end of a cycle.

6.5 Computer Simulation of a Non-polytropic Process.

Consider a process which is characterized by a net loss (or gain) of energy. This may be accomplished by letting $\frac{dQ}{dt}$ be some function other than that defined by equation 6.4.1. Suppose, for example, that $\frac{dQ}{dt}$ were a small negative constant. The resulting differential equation is not easy to solve. However, it can be deduced that a cyclic change in volume will not return the pressure to the starting point, since $\frac{dP_E}{dt}$ is different for each direction of motion. A few computer solutions of processes of this type are shown in Fig. 14. With the exception of the

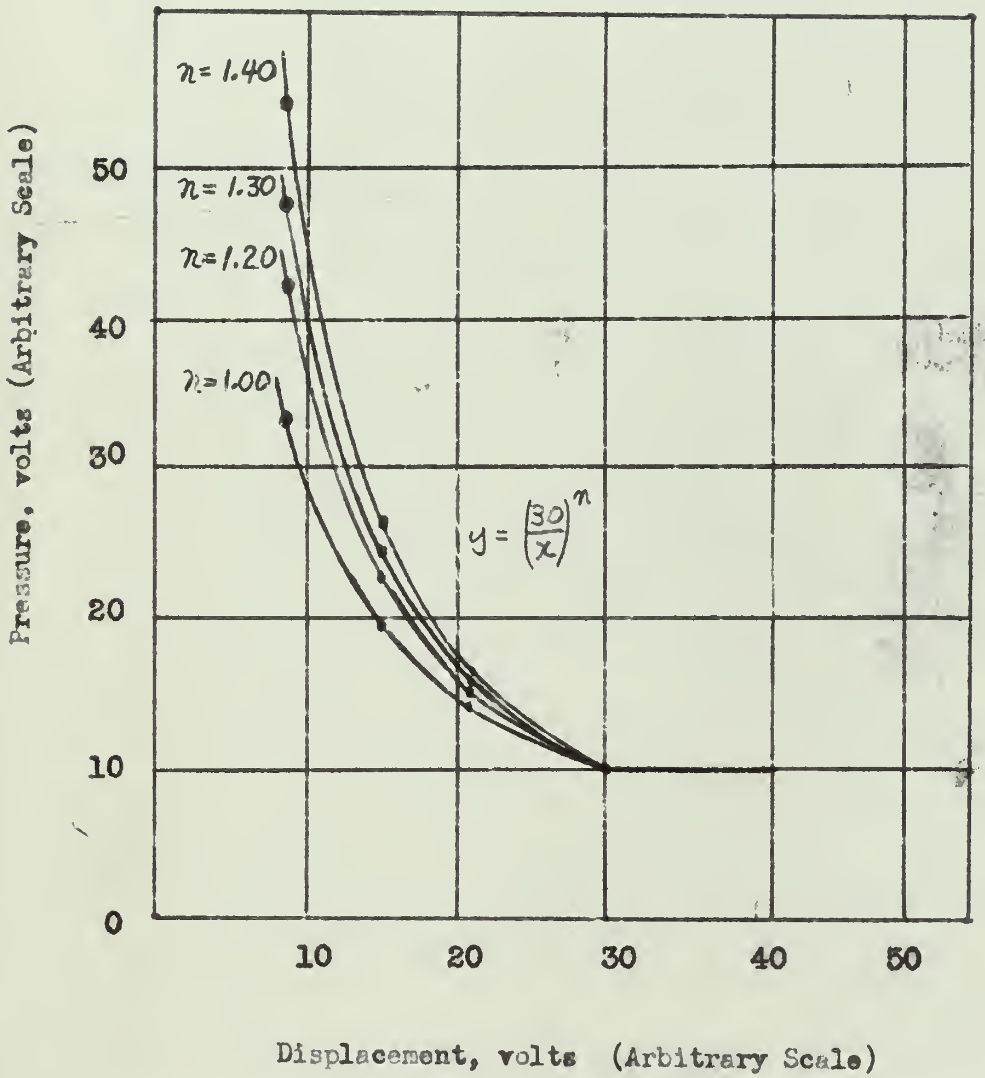


Figure 12. Computer Simulation of the Reversible Polytropic Process.

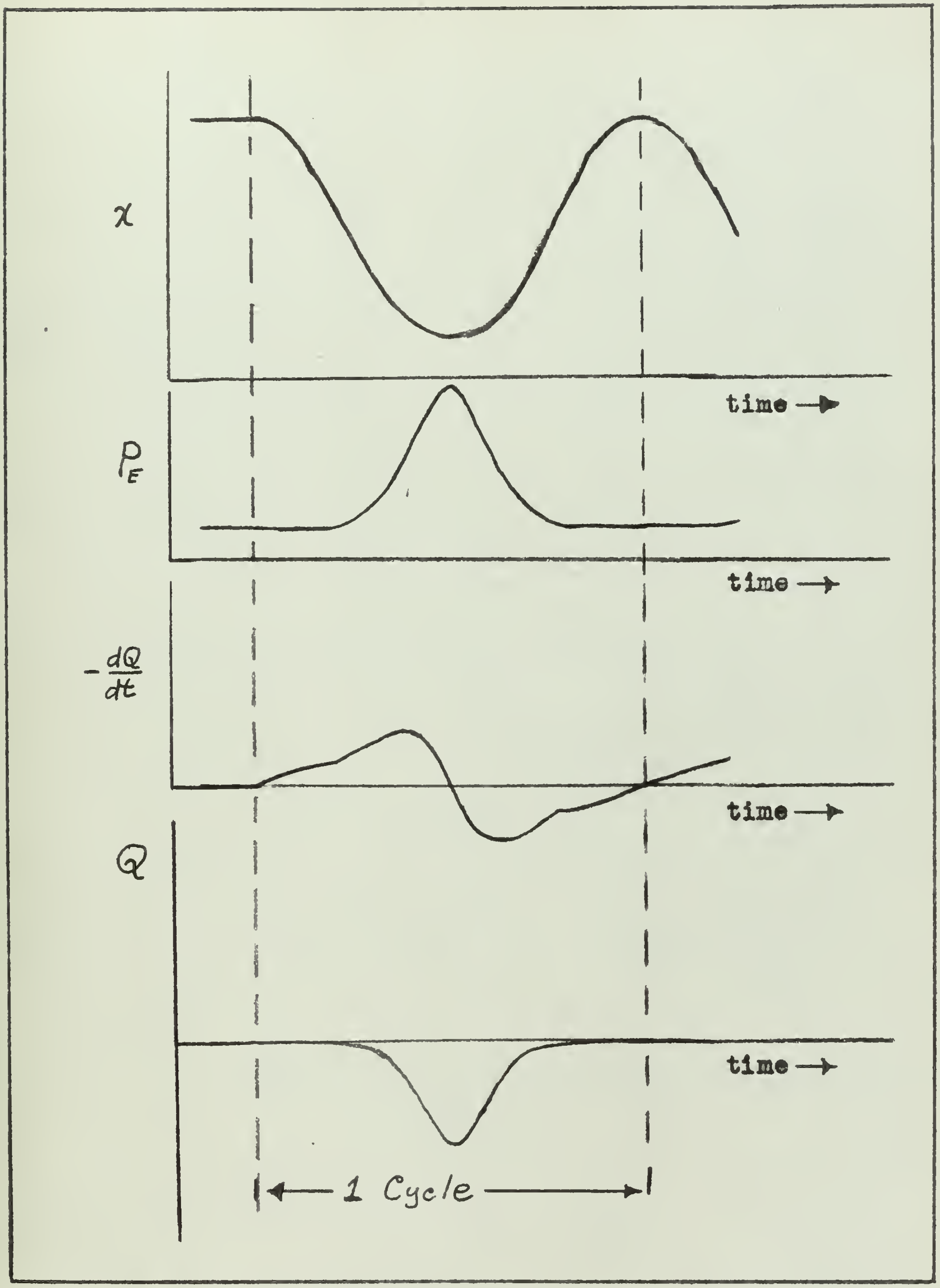


Figure 13. Reversible Polytropic Process as a Function of Time.

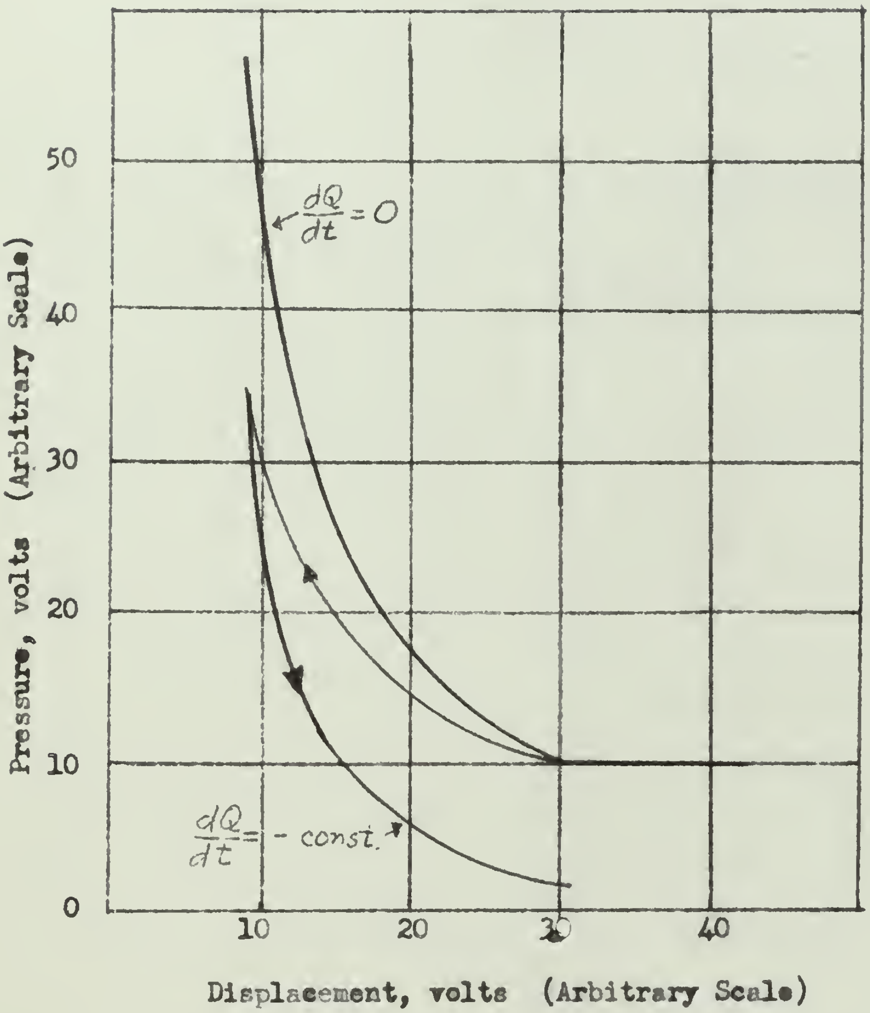


Figure 14. Computer Simulation of the Non-Polytropic Process.

isentropic case, there is net negative work done. This is, of course, the work equivalent of the energy lost by the system as heat. Figure 15 illustrates the same solutions as a function of time. Here it is noted that the net energy, Q, appears as a ramp function. The discontinuity in the pressure curve is due to the operation of the initial condition relay when the displacement exceeded 30 volts.

This and the preceding examples serve no purpose in the overall aim of this investigation, but are presented in order that the reader may become familiar with the rather peculiar sort of mathematical manipulations used in setting up the analog computer. In addition, they may serve as useful check points in the event that this problem is set up on a different computer.

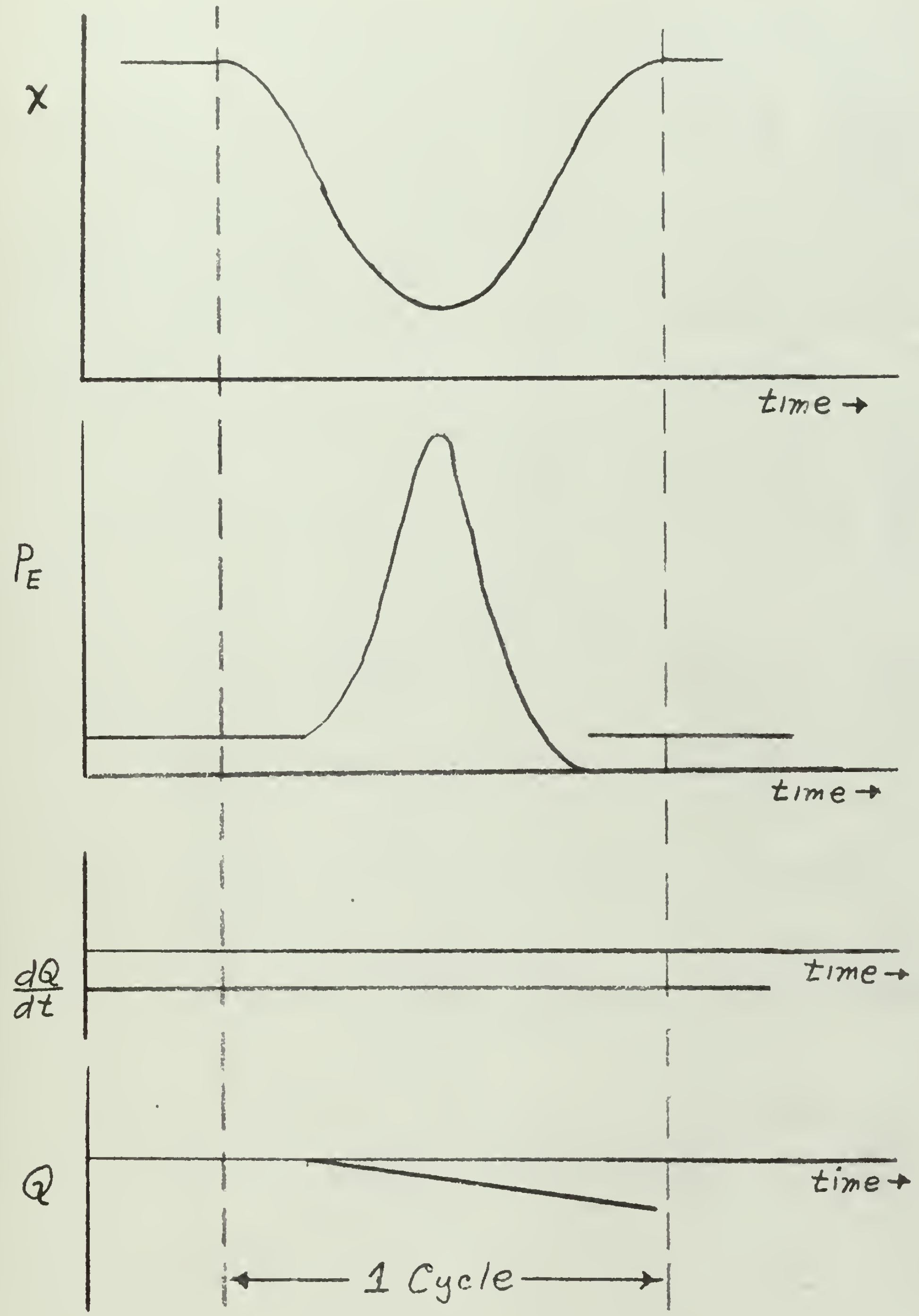


Figure 15. Non-Polytropic Process as a Function of Time.

7. Electronic Analog of the Standard Diesel Cycle.

7.1 General

Equation 6.2.6, and the associated computer arrangement illustrated in Fig. 9 represents the most general case of an engine cycle, wherein any form of heat removal or addition may be simulated by the appropriate selection of functions for the $\frac{dQ}{dt}$ terms. The investigation of solutions of this type, encompassing various methods of heat removal and fuel injection, is a very broad subject, and should be treated as a separate investigation. It will not be practical to do this within the wider scope of the free piston problem. In order that the results from this investigation may be more readily analysed, a standard diesel cycle will be employed for the engine. This has the additional practical advantage that excessive computer voltages will be avoided.

7.2 The Standard Diesel Cycle.

The standard diesel cycle consists of the following processes:

1. Polytropic compression, at exponent n, of the engine charge.
2. Constant pressure addition of energy over a portion of the outstroke.
3. Polytropic expansion of the engine charge and combustion products, at exponent n'.
4. Constant volume heat rejection to exhaust.
5. Constant pressure scavenging process.

Referring to equations 6.2.6 and 6.4.1, these processes may be described mathematically as follows:

1. Compression process.

$$\left(\frac{dP_E}{dt}\right)_n = -\frac{n P_E \dot{x}}{x} , \qquad 7.2.1$$

and

$$\left(\frac{dQ}{dt}\right)_n = A_E\left(\frac{K-n}{K-1}\right)P_E\dot{x}. \qquad 7.2.2$$

2. Combustion process.

$$\left(\frac{dP_E}{dt}\right)_p = 0, \qquad 7.2.3$$

and

$$\left(\frac{dQ}{dt}\right)_p = \frac{A_E K}{K-1}P\dot{x}. \qquad 7.2.4$$

3. Expansion process.

$$\left(\frac{dP_E}{dt}\right)_{n'} = -\frac{n' P_E \dot{x}}{x}, \qquad 7.2.5$$

and

$$\left(\frac{dQ}{dt}\right)_{n'} = A_E\left(\frac{K-n'}{K-1}\right)P_E\dot{x}. \qquad 7.2.6$$

The above equations may be readily solved on the computer by applying the following modifications to Fig. 9, on page 25.

1. Elimination of Amplifier 5.
2. Replacement of $\frac{1}{2}k$ by $\frac{1}{2}n$ when the velocity is negative; and by $\frac{1}{2}n'$ when the velocity is positive.
3. Disconnection of $\frac{dP_E}{dt}$ from Amplifier 1 during the prescribed interval of combusion, thus making $\frac{dP_E}{dt}$ zero.

Items 2 and 3 above may be accomplished through the use of relays. It should be noted, however, that since the free piston engine clearance is variable, the constant pressure combustion process must be initiated by a change in sign of velocity and terminated when a prescribed amount of energy has been added rather than at particular displacements. The reason for this is that a change in clearance should not affect the

amount of fuel energy added.

Figure 16 illustrates the computer arrangement to simulate the standard diesel cycle. The functions of the additional amplifiers and relays are as follows:

1. Relay 1, actuated by a change in sign of velocity, applies the coefficient $\frac{1}{2}n$ when velocity is negative, and the coefficient $\frac{1}{2}n'$ when the velocity is positive.

2. Relay 2, also actuated by a change in sign of velocity, disconnects $\frac{dP_E}{dt}$ from Amplifier 1 when the velocity goes positive; and connects the circuit so that $\frac{dP_E}{dt}$ may be reconnected by Relay 3 at the proper time.

3. Amplifier 5 performs the integration of $\frac{1}{10} n' P_E \dot{x}$, whenever $\dot{x}$ is positive. The quantity $\frac{1}{10} n' P_E \dot{x}$ may be rewritten as

$$\frac{1}{10} n' P_E \dot{x} = \frac{1}{10} n' \left(\frac{K-1}{K}\right)\left[\frac{K}{K-1} P_E \dot{x}\right] = \frac{1}{10} n' \frac{K-1}{K}\left(\frac{dQ}{dt}\right)_p \quad 7.2.7$$

This integration, therefore, yields a result proportional to the total instantaneous fuel energy. This is compared with the desired value of Q_f in Amplifier 6 which actuates Relay 3 when $\int\left(\frac{dQ}{dt}\right)_p dt$ equals $m_f HV$.

Figure 17 illustrates a typical solution using this arrangement. It should be pointed out that the operation of this analog does not depend upon a pre-set stroke or clearance.

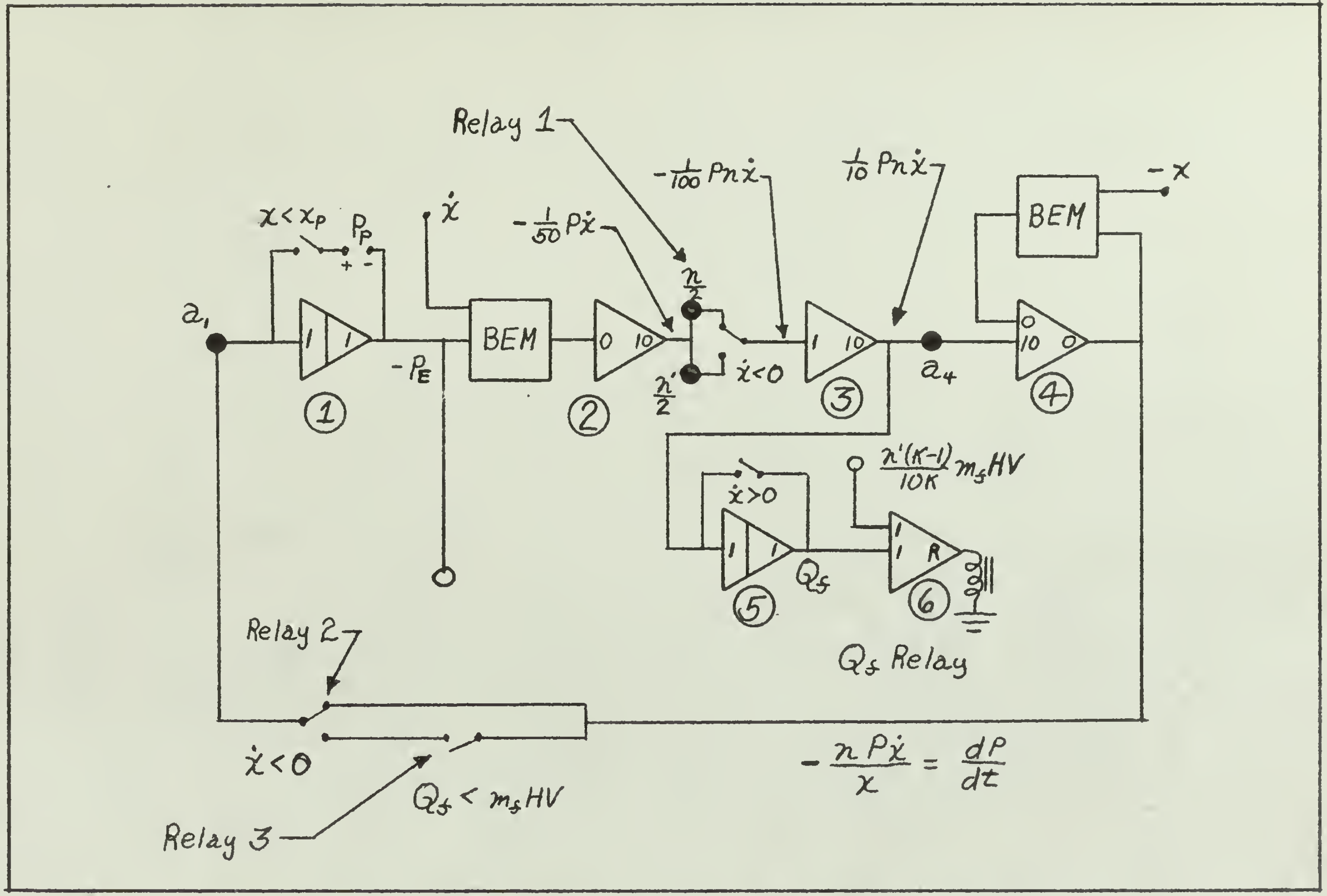


Figure 16. Electronic Analog of a Standard Diesel Cycle.

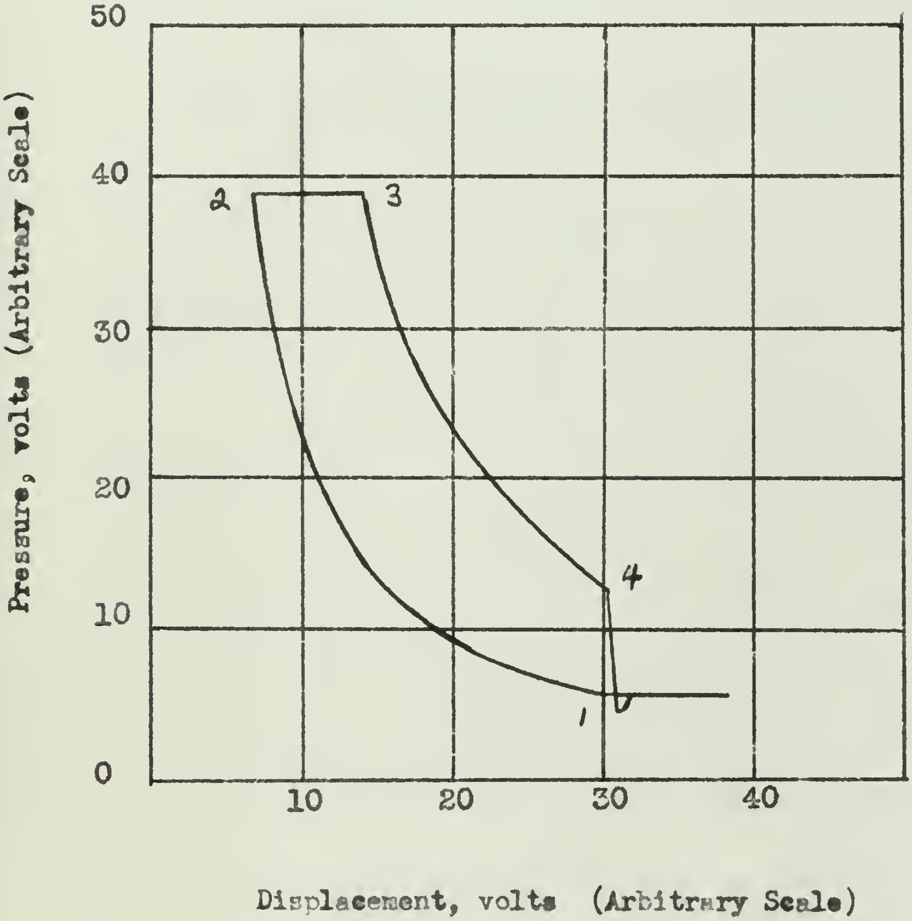


Figure 17. Computer Simulation of a Diesel Cycle.

8. Energy Relationships in the Engine.

In the operation of the analog of the standard diesel engine, it will be necessary to know the net energy lost as heat from the system; and the net energy added during the combustion process. These quantities may be determined by the integration of equations 7.2.2, 7.2.4, and 7.2.6 over the appropriate displacements. These are:

1. Energy as heat transferred during compression

$$Q_n = -\frac{K-n}{(K-1)(n-1)} P_1 V_1 \left[\left(\frac{V_1}{V_2} \right)^{n-1} - 1 \right]. \qquad 8.1$$

2. Net energy as heat transferred during combustion

$$Q_p = \frac{K}{K-1} P_2 \left(V_3 - V_2 \right). \qquad 8.2$$

3. Energy as heat transferred during expansion

$$Q_{n'} = -\frac{K-n'}{(K-1)(n'-1)} P_3 V_3 \left[\left(\frac{V_3}{V_4} \right)^{n'-1} - 1 \right]. \qquad 8.3$$

where the subscripts 1,2,3, and 4 refer to the points illustrated in Fig. 17. It can be seen that with the computer furnishing a picture similar to Fig. 17, all of the required pressures and volumes will be known, thus all of these Q's may be determined. The sum of Q_n and $Q_{n'}$ will be that portion of Q_{rej} chargeable to the engine; and Q_p is an idealized $m_f HV$ product which does not include heat lost during combustion, nor energy lost through incomplete combustion.

9. Electronic Analog of a Reciprocating Compressor.

The force exerted upon the compressor cylinder may be most conveniently represented by referring to a typical indicator diagram for a compressor, such as appears in Fig. 4(b), page 9. In the case of the free piston device, there is no fixed geometrical point. The only known quantities are the pressure limits.

In a manner similar to that used in the engine equations, the differential equation of the first law for the compressor may be written as

$$\frac{dP_c}{dt} = \frac{1}{y}\left[\frac{K-1}{A_c}\frac{dQ_c}{dt} - KP_c\dot{y}\right], \qquad 9.1$$

where A_c is the compressor piston area.
P_c is the compressor pressure.
y is the compressor displacement from the head.
$\frac{dQ_c}{dt}$ is the rate of heat transfer.

This equation, however, is valid only where the mass is constant, not during the delivery or intake processes. For these processes, it can be said that

$P_c = P_d$, for delivery, and

$P_c = P_i$, for intake. 9.2

The problem of the changing mass system of the compressor may be readily solved on the computer with essentially the same arrangement as that used for the engine, but with the use of diode limiters on the amplifier generating P_c. Such an arrangement is shown in Fig. 18, which solves the polytropic equation

$$\frac{dP_c}{dt} = -\frac{n_c P_c \dot{y}}{y}. \qquad 9.3$$

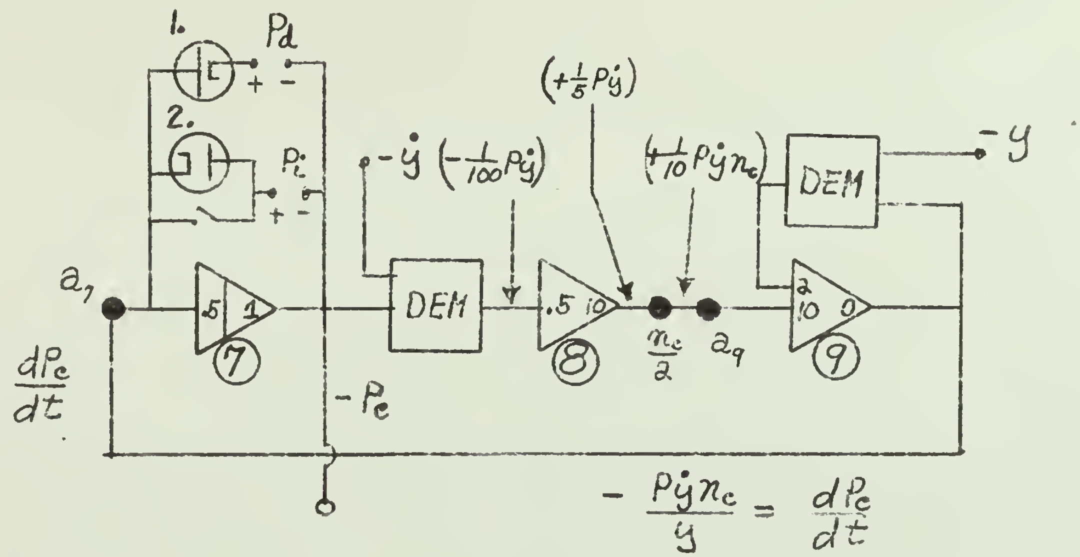


Figure 18. Electronic Analog of a Reciprocating Compressor.

It can be seen from Fig. 18 that $-P_c$ is always negative. With this being the case, the diodes in the feedback of Amplifier 7 will function as follows:

1. With $\dot{y}$ negative (instroke), $\frac{dP}{dt}$ will be positive, and $-P_c$, the output of Amplifier 1 , will tend to go more negative. This will continue until $-P_c$ overrides the bias on Diode 1 at which time this tube conducts, thus stopping the integration with $-P_c$ held constant at $-P_d$. This situation will continue as long as $\dot{y}$ is negative.

2. When $\dot{y}$ changes sign, $\frac{dP}{dt}$ will become negative and the output of this amplifier will become less negative. When this happens, the initial condition will be $-P_d$, which has been imposed by Diode 1.

3. The absolute value of P_c will continue to decrease until the bias on Diode 2 overrides $-P_c$. This tube will then conduct, thus holding the output at $-P_i$. This will continue until $\dot{y}$ changes sign and the integration is started at the initial condition $-P_i$. From this point on the cycle will repeat.

Thus it can be seen that by the use of two diode limiters, equation 7.1 may be solved in the case of a compressor without taking into consideration the changing mass problem. Furthermore, this arrangement will simulate a reciprocating compressor that has a variable stroke, which is precisely the situation in the free piston machine. This is because the integrations covering the compression and expansion strokes are initiated by a change in sign of the velocity, not at a particular position.

The analog illustrated in Fig. 18 was constructed using the assumption that the compression and expansion proceed at the same polytropic exponent.

Figure 19 illustrates two solutions of the compressor problem, corresponding to two different stroke lengths. Here it is again noted that a pre-set stroke and clearance is not necessary.

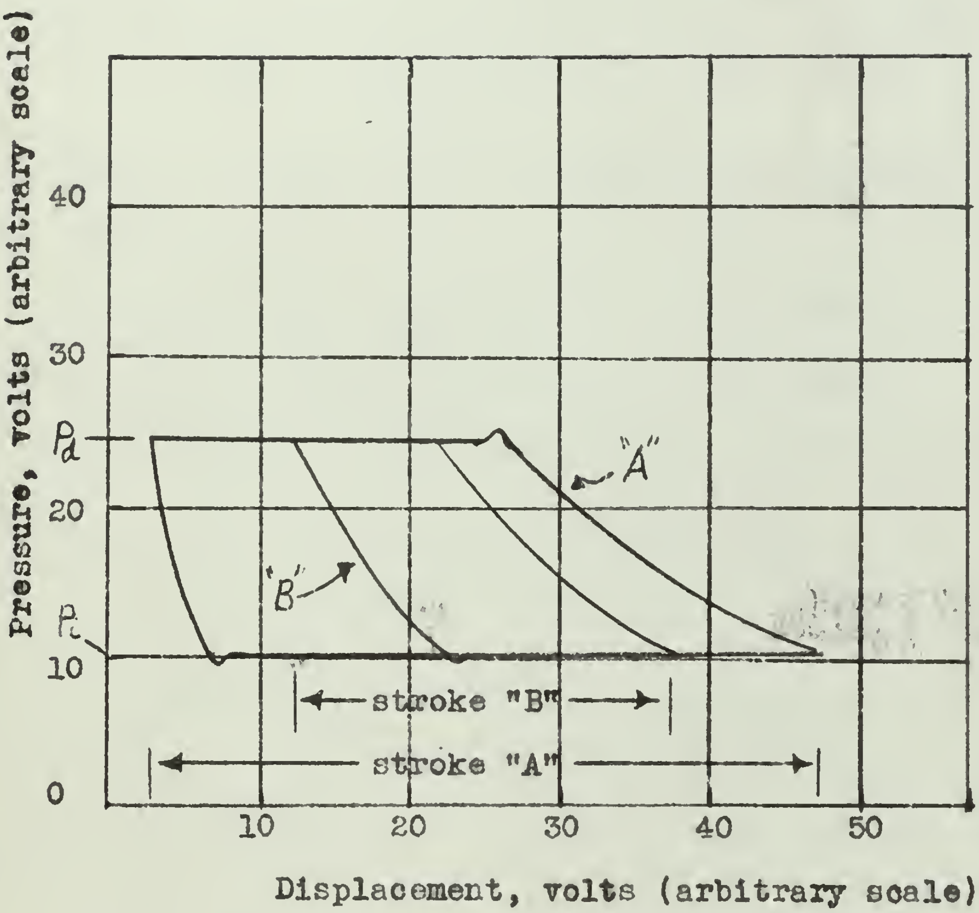


Figure 19. Computer Simulation of a Reciprocating Compressor.

10. Energy Relationships in the Compressor.

In order to solve equation 5.5.1 it is necessary to know the amount of energy removed as heat from the compressor per cycle. Recall that applying the coefficient n_c instead of k is equivalent to letting

$$\frac{dQ_c}{dt} = A_c\left(\frac{K-n_c}{K-1}\right)P_c\dot{y} , \qquad 10.1$$

which produces a reversible polytropic process wherein there is no net loss or gain of energy over a cycle. This will be true only in the case of a constant mass system. The compressor, however, compresses one mass and expands a different mass. By integrating equation 8.1 over the compression and expansion strokes, the net energy transferred as heat per cycle is

$$Q_c = -\frac{\dot{m}_d RT_o(K-n_c)}{(K-1)(n_c-1)}\left[\left(\frac{P_d}{P_i}\right)^{\frac{n_c-1}{n_c}}-1\right], \qquad 10.2$$

where m_d is the mass of air delivered by the compressor per cycle. This equation neglects the energy transferred during the delivery and intake processes. It can be seen that if m_d is zero, or if $k = n_c$, there will be no heat loss.

*See Appendix III for derivation.

11. Electronic Analog of a Gas Spring

The force exerted upon the bounce cylinder is that of a gas spring, i.e., Pz^k = constant. In this case, equation 6.3.1 may be rewritten as

$$\frac{dP_b}{dt} = -\frac{KP_b\dot{z}}{z}, \qquad 11.1$$

where P_b is the pressure on the bounce piston
z is the bounce piston displacement.

On the basis of the preceding analyses, the computer arrangement may be immediately depicted as that shown in Fig. 20. The diode in the feedback of Amplifier 10 performs the function of an atmospheric relief by not permitting P_b to fall below a voltage corresponding to one atmosphere, thus simulating the introduction of additional air into the bounce space. A typical solution from this arrangement is illustrated in Fig. 21, which also illustrates the functioning of the atmospheric relief.

In the case of the bounce cylinder analog, extreme caution must be used in the adjustment of the electronic multiplier forming the $P\dot{z}$ product, for the reason outlined in section 6.3. This situation is aggravated in this case since there may not be an initial condition imposed every cycle. That is to say, the solution will be "floating." A very slight maladjustment in this multiplier will cause the solution to drift rapidly away from the correct value. This malfunctioning is illustrated in Fig. 22. This situation must be avoided.

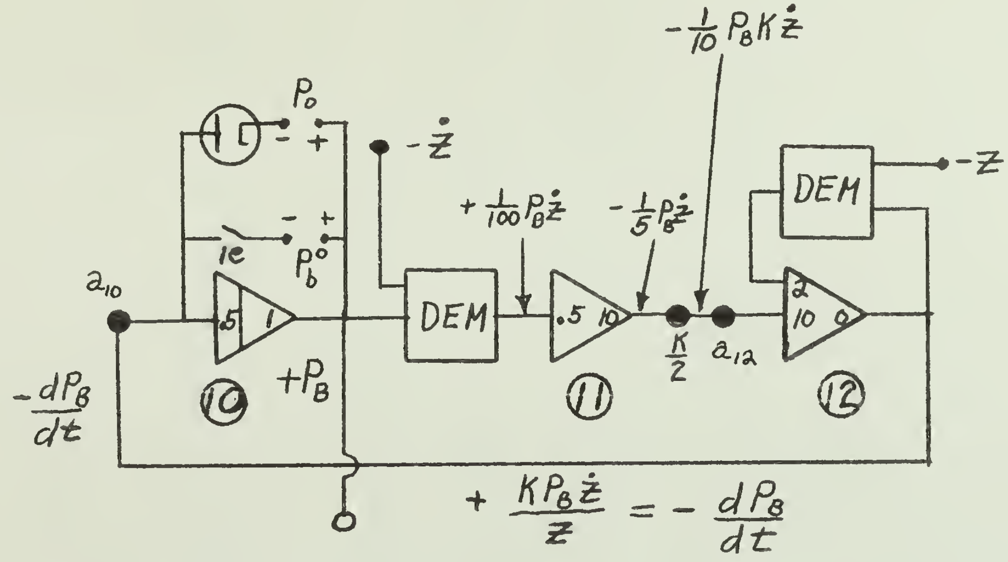


Figure 20. Electronic Analog of a Gas Spring.

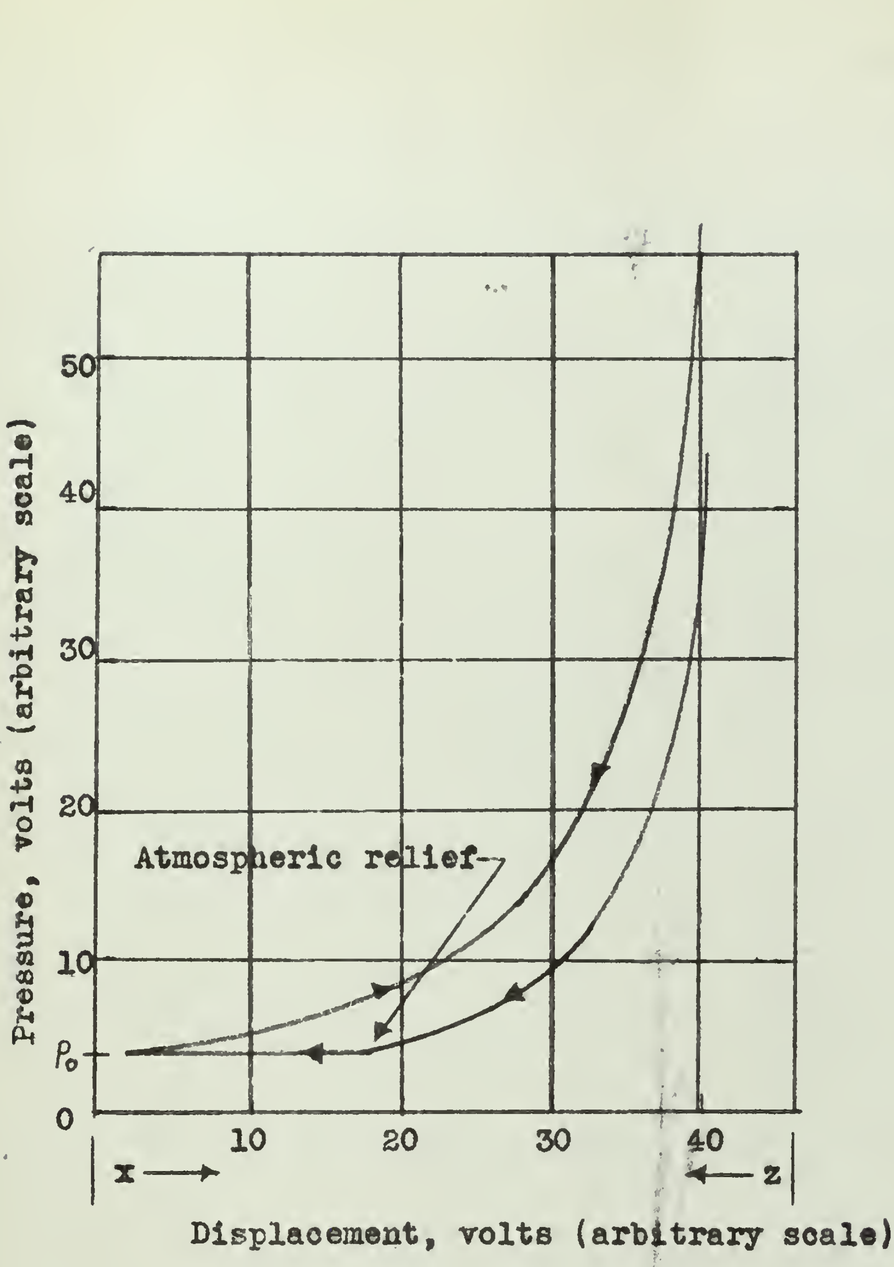


Figure 21. Computer Simulation of a Gas Spring.

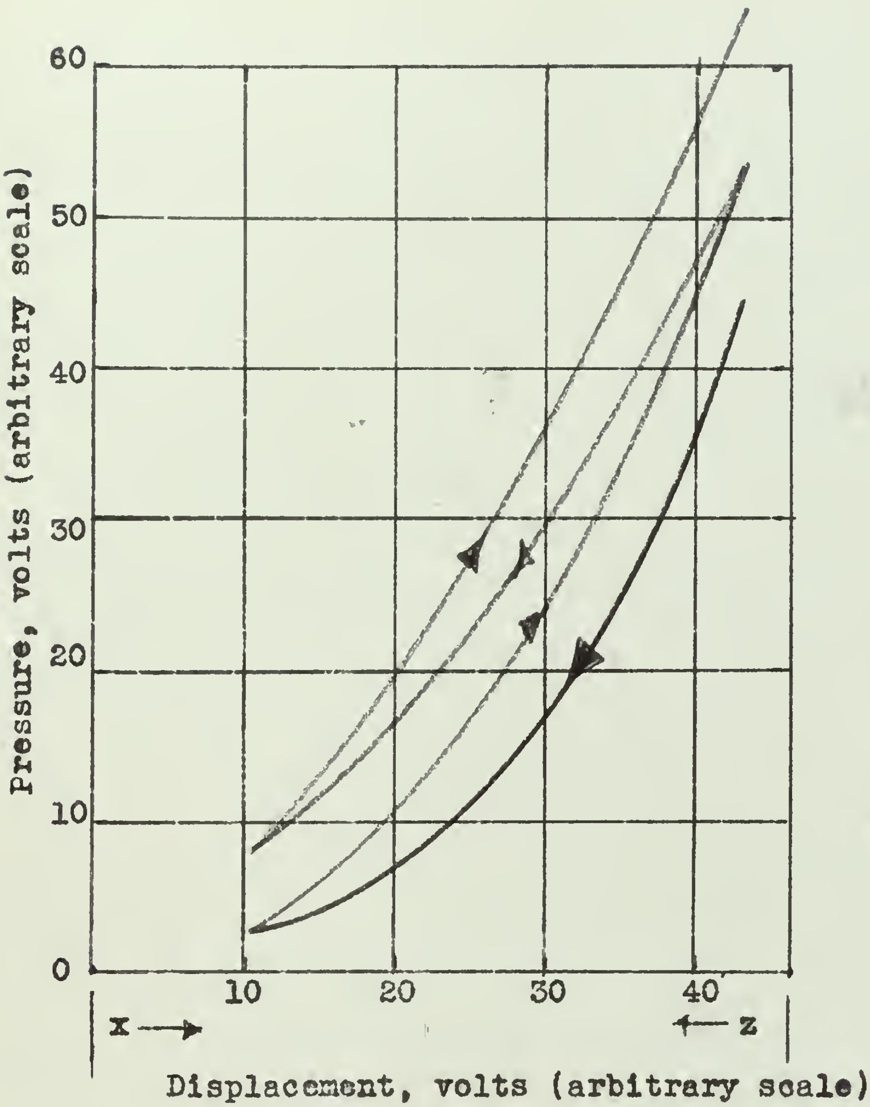


Figure 22. Result of Malfunctioning of the Gas Spring Analog.

12. Coulomb and Viscous Friction.

12.1 The Electronic Analog.

The force of friction is assumed to be composed of the two familiar types: Static friction, a constant force always opposing the motion, and viscous friction, a force proportional to the velocity, again opposing the motion. This total force is illustrated graphically in Fig. 4(d). This may also be expressed mathematically as

$$F_F = C - v\dot{x} \quad (\dot{x} < 0)$$
$$F_F = -C - v\dot{x} \quad (\dot{x} > 0), \qquad 12.1$$

where C is the Coulomb friction force

v is the viscous friction coefficient.

The analog to produce a voltage corresponding to this force is illustrated below in Fig. 23.

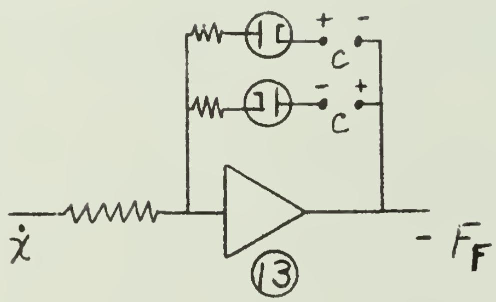


Figure 23. Electronic Analog of Coulomb and Viscous Friction

The amplifier has no feedback except for the two diodes and their series resistances. Whenever the input is different from zero, even by the smallest amount, the amplifier output will tend immediately towards saturation. This tendency is checked at the voltage "C" by one or the other of the diodes, which when conducting, places the feedback resistance in the circuit. Figure 24 illustrates the computer analog of the force of friction both as a function of velocity and of displacement.

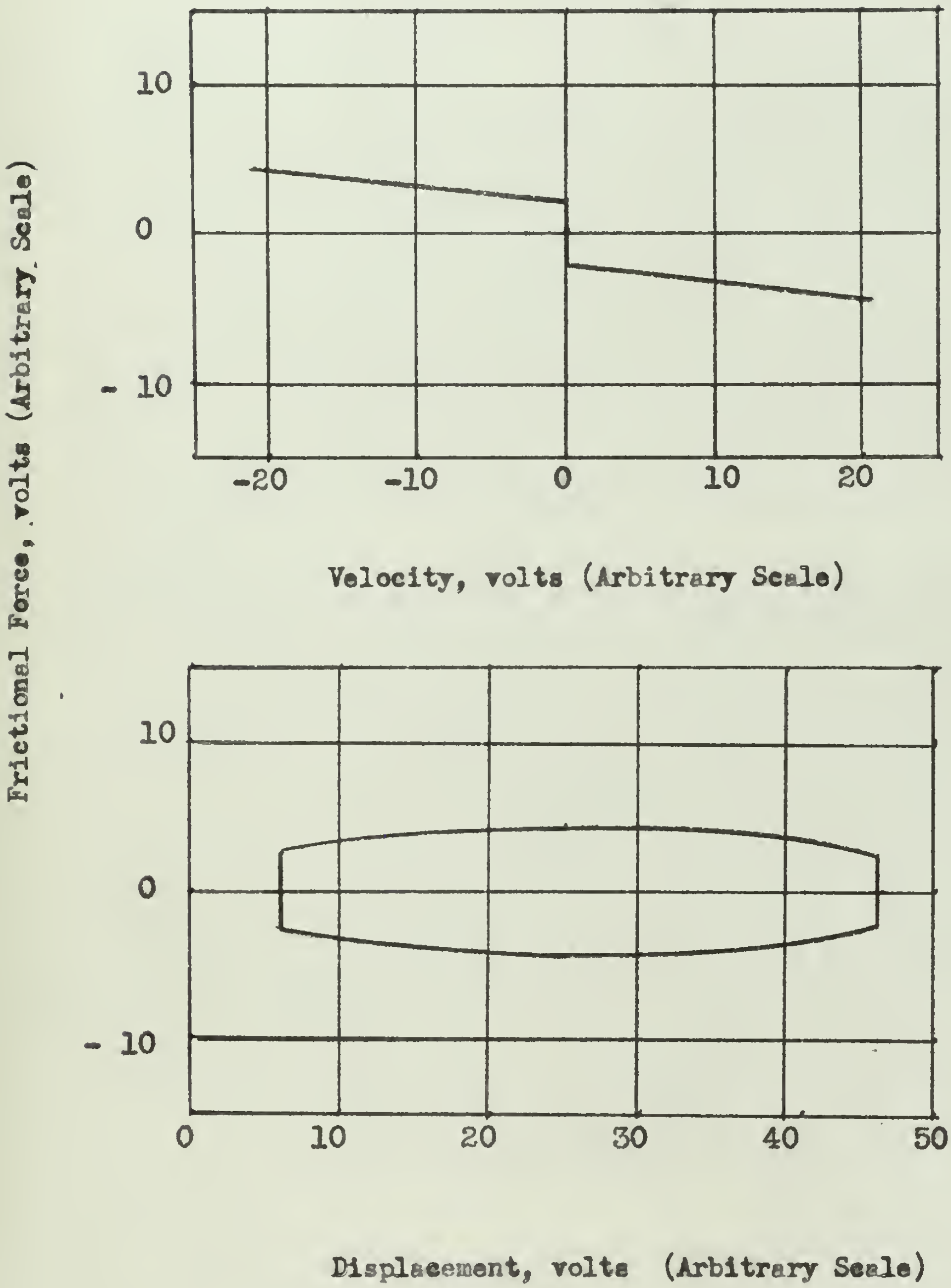


Figure 24. Computer Simulation of Coulomb and Viscous Friction.

12.2 Energy Dissipated by Friction.

The last remaining item as yet unknown in equation 5.5.1 is that portion of Qrej chargeable to friction.

The force of Friction is given by

$$F_F = C - v\dot{x}, \qquad \text{when } \dot{x} \text{ is negative,}$$

and 12.2.1

$$F_F = -C - v\dot{x}, \qquad \text{when } \dot{x} \text{ is positive.}$$

In order that this force may be integrated directly, it shall be assumed that the free piston has the approximate motion

$$x = c_e + \frac{s}{2}\cos 2\pi f t, \qquad 12.2.2$$

where c_e is the engine clearance

f is the frequency

s is the stroke

The work done by friction is

$$W_F = \oint F_f\,dx, \qquad 12.2.3$$

or

$$W_F = \int_0^{\frac{1}{f}} F_f\left(\frac{dx}{dt}\right)dt. \qquad 12.2.4$$

Upon substituting and integrating over a cycle, the net energy lost through friction work is given by

$$W_F = 2Cs + \frac{2\pi^2 f s^2}{4}. \qquad 12.2.5$$

13. Computer Operated Relays.

As outlined in Fig. 16, the use of five relays is required in the simulation of a diesel engine. For convenience, these are:

1. A relay to start and stop integration whenever x is greater than x_p, the port position. This has the effect of putting the "tail" on the engine picture, thus simulating the scavenging process.

2. Three relays necessary to make $\frac{dP_E}{dt}$ zero for a portion of the outstroke, thus simulating the constant pressure addition of fuel.

3. One relay to change exponent from compression to expansion.

The above relays, of course, must be actuated by quantities available in the computer. In all cases the relays were driven by the output of an operational amplifier by use of the scheme illustrated below in Fig. 25.

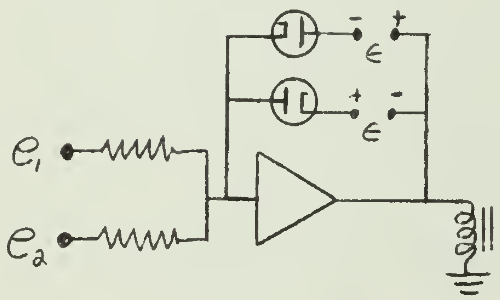


Figure 25. Use of Operational Amplifier to Actuate Relays.

As can be seen above, the amplifier has no feedback except diodes to limit the output to a voltage, ϵ, sufficient to actuate the relay. The action of the above arrangement is to yield an immediate step voltage to the relay whenever the algebraic sum of the inputs passes through zero. Figure 26 illustrates the output of such an amplifier actuated by a sine wave. It can be seen that this output is very nearly a square wave.

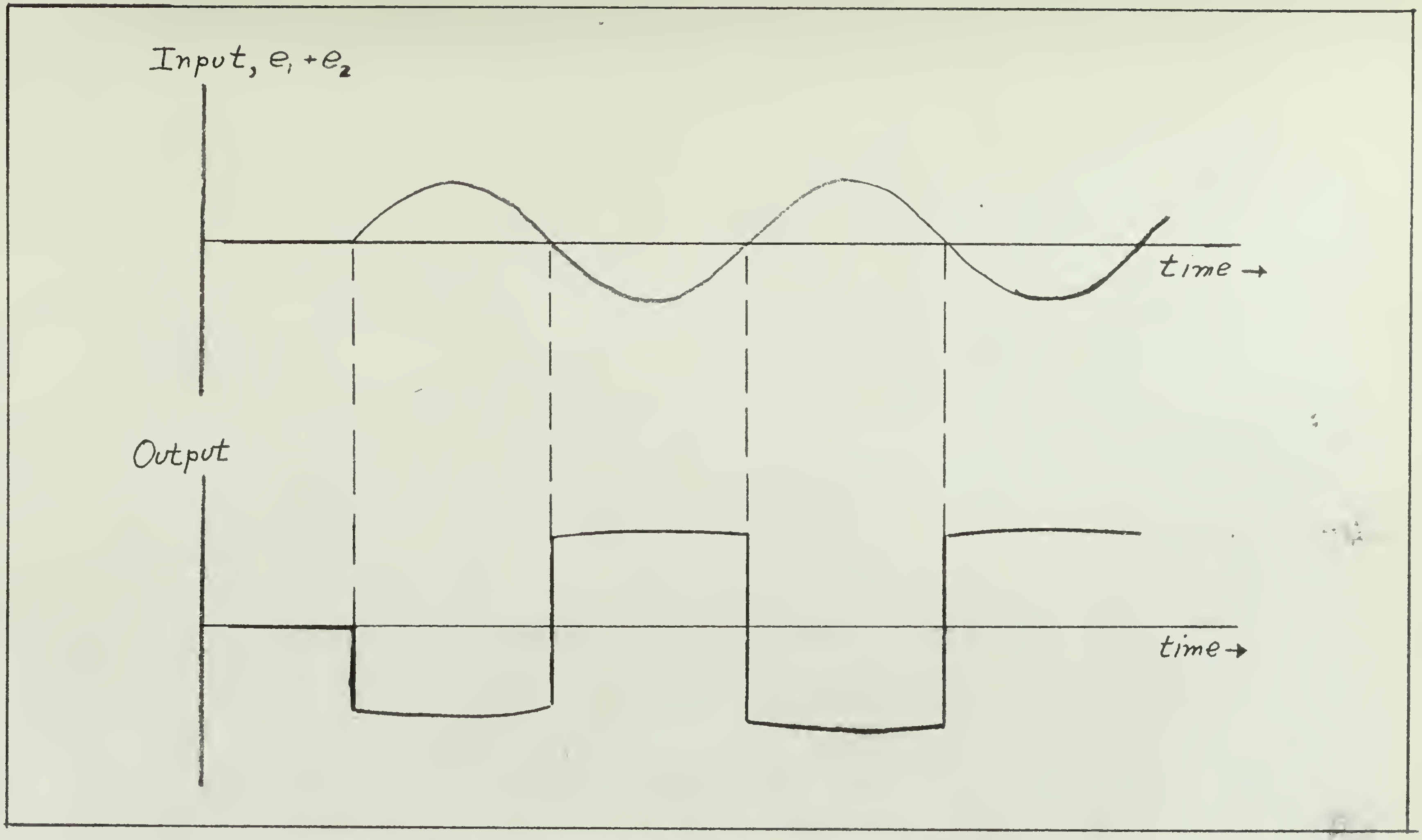


Figure 26. Operation of Computer Operated Relays.

14. Electronic Analog of the Free Piston Generator.

The preceding sections may be summarized by referring to an overall computer arrangement for the solution of the piston motion. This is illustrated in Fig. 27. This diagram is basically the same as Fig. 7, except that the details of Figs. 16, 18, 20, 23, and 25 are included. The numbers on the amplifiers correspond to those used in the above mentioned figures.

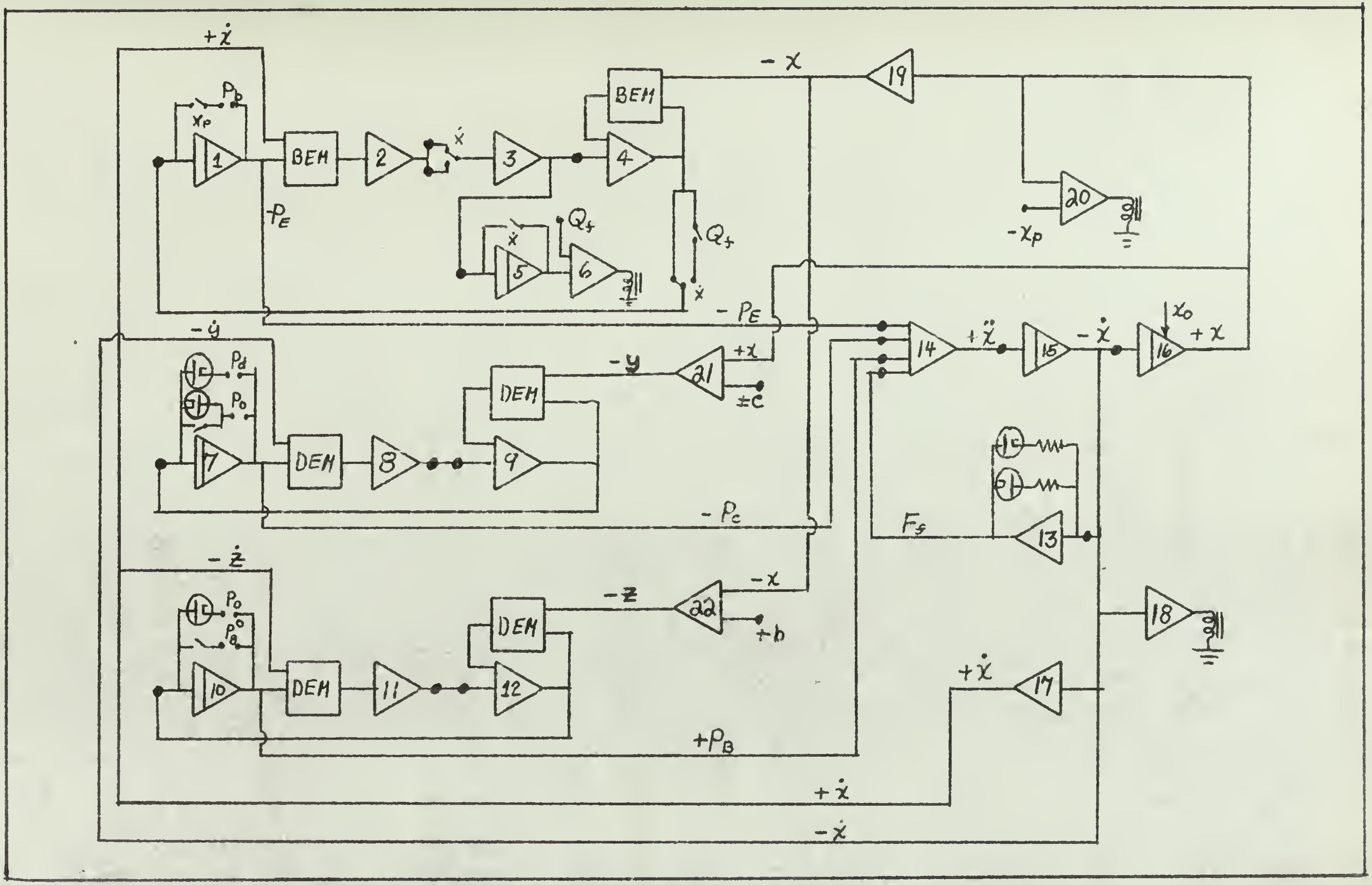


Figure 27. Electronic Analog of the Free Piston Gas Generator.

15. Time and Magnitude Scaling.

15.1 General.

All of the preceding sections have been rather qualitative in nature in order that the principles of the analog approach to the problem may be presented more clearly. In order to obtain a numerical result, however, it is necessary to consider magnitude and time scaling within the computer.

15.2 Scaling the Pressure Analogs.

As outlined in section 6.3, the various pressure analogs were constructed by using as a driving force a separately generated displaced sine wave, operating at a fixed frequency, based on real time. Such will not be the case when each of these analogs becomes a part of the overall hook-up as illustrated in Fig. 27. In this case, the integrations forming displacement and velocity will seek some natural frequency, based on computer time, not real time. To this end, a time scaling factor of 100 will be used. Thus, a real machine operating at 600 cycles per minute will appear on the computer as operating at 6 cycles per minute. The only criterion necessary in making the pressure integrations compatible time-wise with the displacement integrations is that a common time scale be used. When this is done, all integrations will proceed at the same rate.

The magnitude scaling is entirely arbitrary, depending upon the maximum value of the variable, and its relationship to the 50 volt limitation of the BEAC amplifiers. The only criterion here is that the same scale factor must be applied to the initial conditions.

There is a definite relationship between the various scale factors involved within each computing loop simulating the engine, compressor, and bounce pressures. The compressor, for example, solves the equation

$$\dot{P}_c = \frac{dP_c}{dt} = -\frac{n_c P_c \dot{y}}{y}, \qquad 15.2.1$$

or, in terms of computer voltages

$$\alpha_{\dot{P}_c} \overline{\dot{P}_c} = -\left(\frac{\alpha_{P_c} \alpha_{\dot{y}}}{\alpha_y}\right) n_c \frac{\overline{P_c}\,\overline{\dot{y}}}{\overline{y}}, \qquad 15.2.2$$

where α_i is the magnitude scale, units of "i" per volt, and the barred quantities represent the computer voltages corresponding to the real quantity. By comparing the above equations, it can be seen that $\alpha_{\dot{P}_c}$ may not be arbitrarily assigned, but is given by

$$\alpha_{\dot{P}_c} = \frac{\alpha_{P_c} \alpha_{\dot{y}}}{\alpha_y}. \qquad 15.2.3$$

Similar expressions exist in the engine and bounce analogs. These are

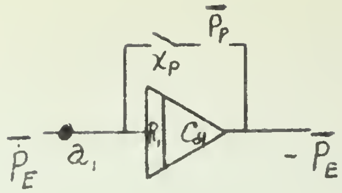
$$\alpha_{\dot{P}_E} = \frac{\alpha_{P_E} \alpha_{\dot{x}}}{\alpha_x}, \qquad 15.2.4$$

and

$$\alpha_{\dot{P}_B} = \frac{\alpha_{P_b} \alpha_{\dot{z}}}{\alpha_z}. \qquad 15.2.5$$

As outlined in section 6.2, there is no liberty selecting the magnitude scales involved in the multiplications and divisions. This scaling has already been performed for all amplifiers except those integrating to form the various pressures.

In the case of the engine, for example, the integrating amplifier may be treated as follows:



In this amplifier,

$$\alpha_{P_E}\overline{P}_E = \int \frac{\alpha_{\dot{P}_E}\overline{\dot{P}}_E}{\alpha_t}\,dt + \overline{P}_P , \qquad 15.2.6$$

where α_t is the time scaling factor, $\frac{\text{computer time}}{\text{real time}}$.

When the above constants are associated with the circuit parameters as outlined in Appendix II, the following equation is formed

$$\frac{\alpha_{\dot{P}_E}}{\alpha_{P_E}\alpha_t} = \frac{a_1}{R_1 C_{f1}} . \qquad 15.2.7$$

When equation 15.2.3 is substituted,

$$\frac{a_1}{R_1 C_{f1}} = \frac{\alpha_{\dot{x}}}{\alpha_x \alpha_t} . \qquad 15.2.8$$

By similar arguments, the circuit parameters involved in the compressor and bounce pressure integrations are given by:

$$\frac{a_7}{R_7 C_{f7}} = \frac{\alpha_{\dot{y}}}{\alpha_y \alpha_t} , \qquad 15.2.9$$

and

$$\frac{a_{10}}{R_{10} C_{f10}} = \frac{\alpha_{\dot{z}}}{\alpha_z \alpha_t} . \qquad 15.2.10$$

The subscripts 1, 7, and 10 used in the above equations refer to the number of the amplifier as given in Fig. 27.

Since x, y, and z refer to the same motion, it must be true that

$$\alpha_x = \alpha_y = \alpha_z , \qquad 15.2.11$$

and

$$\alpha_{\dot{x}} = \alpha_{\dot{y}} = \alpha_{\dot{z}} . \qquad 15.2.12$$

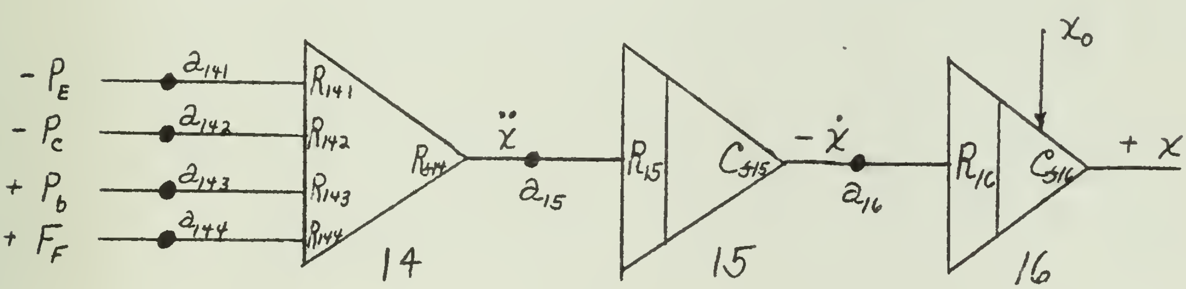
When this substitution is made, there finally results

$$\frac{a_1}{R_1 C_{f1}} = \frac{a_7}{R_7 C_{f7}} = \frac{a_{10}}{R_{10} C_{f10}} = \frac{\alpha_{\dot{x}}}{\alpha_x \alpha_t} . \qquad 15.2.13$$

This relationship specifies the definite time-magnitude relationship between the pressure integrations and the displacement integration.

15.3 Scaling the Displacement Integrations.

The displacement integrations may best be analysed by redrawing a portion of Fig. 27:



The following may be written for Amplifier 14.

$$\alpha_{\ddot{x}} \overline{\ddot{x}} = -\frac{1}{M}\left[-A_E \alpha_{P_E} \overline{P_E} - A_c \alpha_{P_c} \overline{P_c} + A_b \alpha_{P_b} \overline{P_b} + \alpha_{F_F} \overline{F_F} \right] . \qquad 15.3.1$$

From which

$$\frac{a_{141} R_{f141}}{R_{141}} = \frac{A_E \alpha_{P_E}}{M \alpha_{\ddot{x}}} , \qquad 15.3.2$$

$$\frac{a_{142} R_{f142}}{R_{142}} = \frac{A_c \alpha_{P_c}}{M \alpha_{\ddot{x}}} , \qquad 15.3.3$$

$$\frac{a_{143} R_{f143}}{R_{143}} = \frac{A_b \alpha_{P_B}}{M \alpha_{\ddot{x}}} , \qquad 15.3.4$$

and

$$\frac{a_{144} R_{f144}}{R_{144}} = \frac{\alpha_{F_F}}{M \alpha_{\ddot{x}}} . \qquad 15.3.5$$

The following may be written for Amplifier 15.

$$\alpha_{\dot{x}} \bar{\dot{x}} = - \int \frac{\alpha_{\ddot{x}} \bar{\ddot{x}}}{\alpha_t} dt , \qquad 15.3.6$$

or

$$\frac{a_{15}}{R_{15} C_{f15}} = \frac{\alpha_{\ddot{x}}}{\alpha_{\dot{x}} \alpha_t} . \qquad 15.3.7$$

And for Amplifier 16;

$$\alpha_x \bar{x} = - \int \frac{\alpha_{\dot{x}} \bar{\dot{x}}}{\alpha_t} dt , \qquad 15.3.8$$

or

$$\frac{a_{16}}{R_{16} C_{f16}} = \frac{\alpha_{\dot{x}}}{\alpha_x \alpha_t} . \qquad 15.3.9$$

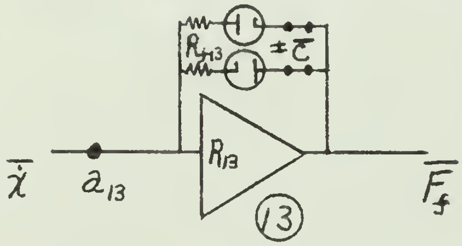
By comparing this equation with equation 15.2.13, it can be seen that

$$\frac{\alpha_{\dot{x}}}{\alpha_x \alpha_t} = \frac{a_1}{R_1 C_{f1}} = \frac{a_7}{R_7 C_{f7}} = \frac{a_{10}}{R_{10} C_{f10}} = \frac{a_{16}}{R_{16} C_{f16}} \,. \qquad 15.3.10$$

This equation means that the RC time constants in the x, P_E, P_c , and P_b integrations must be the same.

15.4 Scaling the Friction Analog.

Again referring to a portion of Fig. 27:



The equation for this amplifier is

$$\alpha_{F_f} \overline{F_f} = \pm \overline{C} - V \alpha_{\dot{x}} \bar{\dot{x}} \,, \qquad 15.4.1$$

from which

$$\frac{a_{13} R_{f13}}{R_{13}} = \frac{\alpha_{\dot{x}} V}{\alpha_{F_f}} \,. \qquad 15.4.2$$

No further scaling is required in the other amplifiers of Fig. 27.

16. The Pescara Machine on the Analog Computer.

16.1 Results.

Reference 1 contains the data and a digital analysis of a free piston gas generator manufactured by the Pescara organization in France. Appendix IV contains the applicable original data and some of the results given by London. It is interesting to note that the analog approach to the problem is nearly the reverse from the digital approach. The digital approach uses as a starting point an assumed stroke and clearances, and then proceeds to calculate some of the geometry, frequency, power output, and other quantities of interest. The analog computer approach uses as a starting point a particular geometry, and yields as a solution stroke, clearance, and frequency. Unfortunately, the data required by the computer were not entirely complete in the original Pescara paper. Conspicuously absent were:

1. The piston mass, M.
2. The absolute bounce clearance, b.
3. The initial bounce pressure, P_b^o.

All other items were either given or were determined by simple calculation.

Of course, the computer cannot produce a solution in the absence of the above items. London, however, had calculated these items in his analysis, and his results being the best available data, were used for these three parameters. It should be pointed out, however, that this digital analysis is not quite compatible with the computer solution in that:

1. The digital analysis used a modified Otto engine cycle, while the computer uses a diesel cycle.

2. The digital method charges friction individually to the

engine and compressor in the form of mechanical efficiencies, while such is charged to the machine as a whole in the computer approach.

3. The digital method uses "lumped" quantities such as net engine work, net compressor work, and net friction work. Such is not possible on the computer since the computer must consider differential items in solving the equation of motion.

With these exceptions noted, the following compares the computer solution* of the Pescara machine with the combined original data and London's analysis:#

1. Geometrical Results

Item	Analog	Pescara
Frequency, cycles/min.	600	613
Stroke, inches	17.5	17.5
Engine effective stroke, inches	9.74	9.70
Engine clearance, inches	1.10	1.14
Compressor clearance, inches	2.33	2.38
Bounce clearance	2.54	2.50

2. Mass Results

Item	Analog	Pescara
Compressor air delivered lbs/side/cycle	0.500	0.380
Ratio, engine/comp. air	0.370	0.516
Air-fuel ratio	22.7	32.0
Gas flow rate, lbs./hr.	36,600	29,000
Fuel flow rate, lbs./hr.	589	460

*Refer to Appendix IV for scaling of Pescara Machine

#Results and Calculations appear in Appendix V.

3. Thermodynamic Results.

Item	Analog	Pescara
Delivery Temperature	1550°R	1405°R
Air Horsepower	1200	1138

The net energy balance was also calculated. This showed a discrepancy of 6.6% between the engine work and the compressor plus frictional work.

Figures 28, 29, and 30 are the computer solutions of the engine pressure, compressor pressure, and bounce pressure, respectively, as functions of their respective displacements.

16.2 Discussion of Results.

As shown in the preceding section, the geometrical results were in excellent agreement with the actual machine data. The rest of the results, however, were close, but not as good as might be expected. This is attributable to two causes:

1. The analog computer was actually solving a different kind of problem than the Pescara machine, even though the geometry was essentially the same. This is because:

   a. A diesel cycle was used instead of a real engine cycle.

   b. Different thermodynamic constants were used.

   c. The frictional constants were assumed at much too low a value.

2. Computer error. In this regard it is to be noted that extreme difficulty was encountered in making the bounce analog stable. This phenomonon resulted from the error in forming the $P\dot{z}$ product by an electronic device. Since $\dot{z}$ changes sign it is imperative that $P\dot{z} = -P(-\dot{z})$, so that the solution will reproduce itself regardless of direction of motion. The electronic multiplier is not capable of the

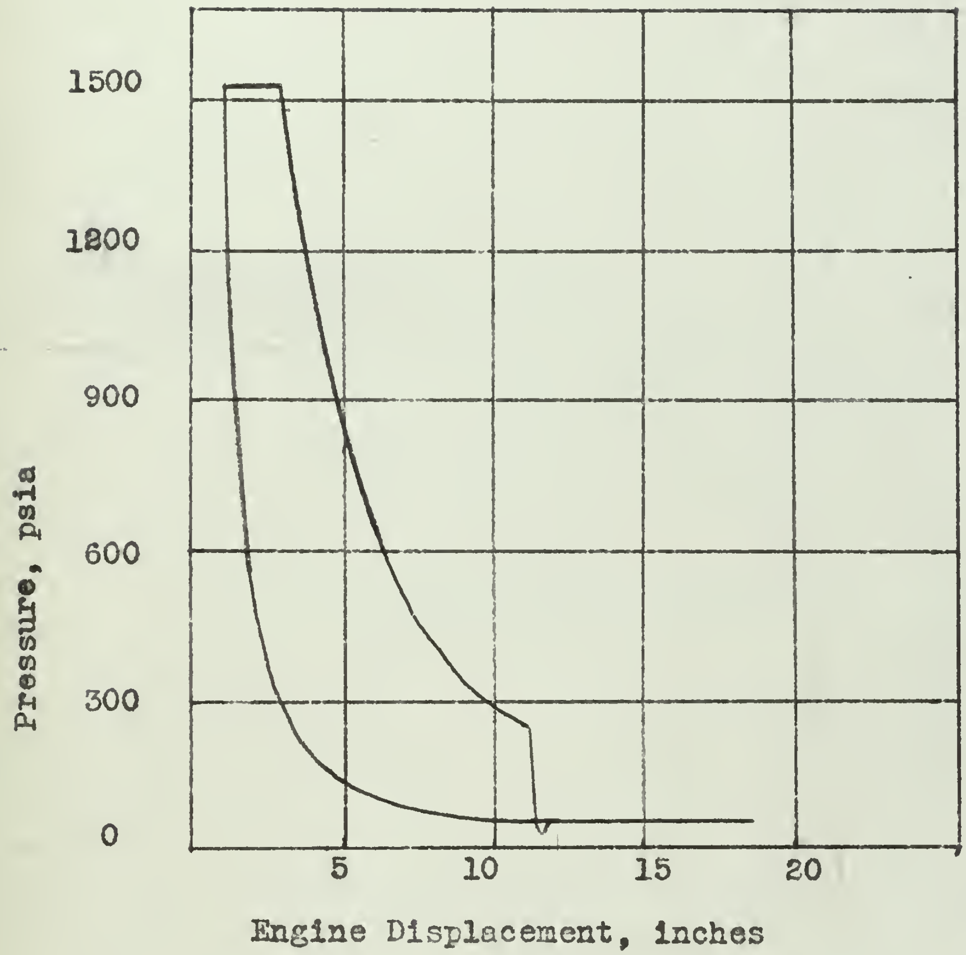


Figure 28. Computer Simulation of the Engine of the Pescara Machine.

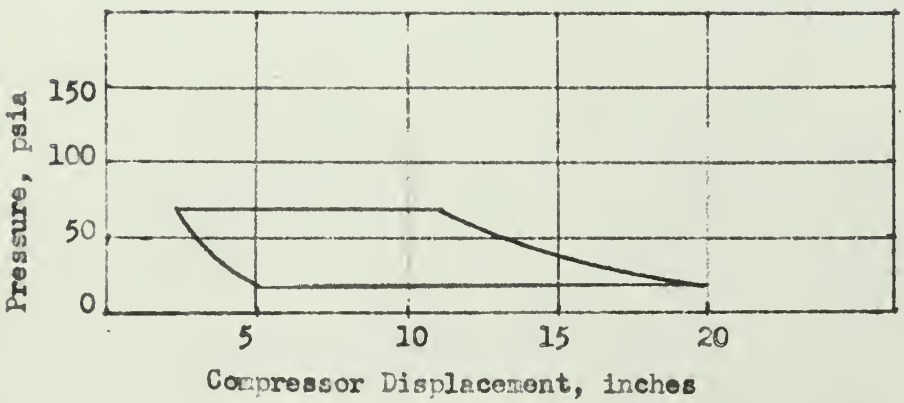


Figure 29. Computer Simulation of the Compressor of the Pescara Machine.

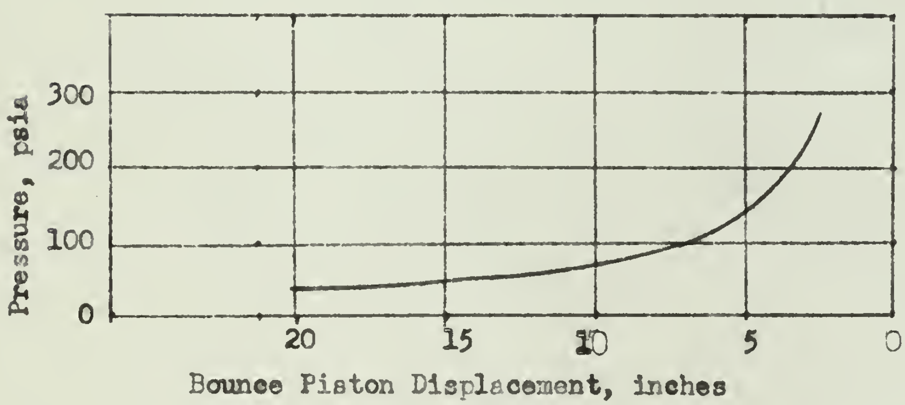


Figure 30. Computer Simulation of the Bounce of the Pescara Machine.

accuracy that is required in the case of the bounce piston. The engine and compressor analogs are not sensitive to this effect in that an initial condition is imposed each cycle which prohibits the solution from drifting away. Thus, the analog of the bounce piston exhibited a phenomenon similar to, but not as pronounced as, that shown in Fig. 23, page 51. The maximum drift was of the order of 3 volts. The scaling, however was such that a 3 volt departure represented a force error of about 59,000 pounds, which is sufficient to disrupt the entire analog, causing the solution to become unstable after about three cycles. Thus the solution as outlined in Appendix V approaches, but may not be, the steady state. This effect may account for most of the discrepancies.

17. Conclusions

The free piston engine problem may be solved on an electronic analog computer, if such a computer is capable of generating a stable solution of the gas spring. The computer can produce a solution to the equation of the first law of thermodynamics in the simulation of an internal combustion engine and of a reciprocating compressor. An analog computer is not capable of solving the gas spring problem to the required accuracy in this manner due to small inaccuracies in electronic multiplication.

Two alternative ways are suggested to produce an accurate analog of the gas spring. These are:

1. Use of a function generator to yield a function which may be used to give Pz^k = constant. Such a device would keep positive control of the bounce force at all times.

2. Use of one-quadrant multiplication. It has been pointed out that the primary difficulty was that electronic multiplication could not give exactly $P\dot{z} = -P(-\dot{z})$. This difficulty may be avoided by using $\dot{z}$ having always the same sign. This may be accomplished through the use of velocity relays and a sign changer to switch both the input and output of the multiplication such that the multiplier operates only in one quadrant.

It should also be pointed out that the solution of this problem required a good sized computer, in that a total of 26 amplifiers, eight diode limiters, eight voltage sources, and six electronic multipliers were required, in addition to the necessary read-out equipment. This is considered the minimum computer size to handle such a problem. Further refinements, or the addition of another compressor analog for

the solution of the free piston compressor would, of course, increase the computer requirements.

18. Bibliography.

1. A.K. Oppenheim and A.L. London — "The Free Piston Gas Generator - A Thermodynamic-Dynamic Analysis."

   Stanford University Technical Report FP-1

   Department of Mechanical Engineering, Stanford University; February 15, 1950.

2. G.A. Korn and T.M. Korn — "Electronic Analog Computers"

   McGraw Hill, 1956.

3. Boeing Airplane Company — "The Operation of the Boeing Electronic Analog Computer."

   Boeing Airplane Company Document D-12209, September 15, 1951.

APPENDIX I

LIST OF ASSUMPTIONS

1. The scavenge air is furnished at constant pressure and temperature.

2. The delivery to the turbine occurs at constant pressure and temperature.

3. The compressor intake is 70°F; one atmosphere.

4. The operating fluid obeys the ideal gas laws.

5. The pressure drop across each valve and engine port is five per cent.

6. The geometry is symmetrical with respect the centerline.

7. The compressor compresses and expands in accordance with a polytropic exponent of 1.30.

8. The sensible enthalpy of the fuel may be neglected.

9. The engine operates in accordance with an air standard diesel cycle.

10. The energy transferred as heat during the compressor intake and delivery is neglected.

11. The frictional force consists of coulomb and viscous friction.

12. The friction work is determined in advance by assuming that the stroke is sinesoidal.

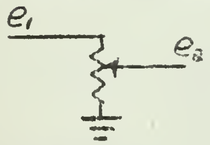
Other numerical assumptions regarding the specific problem of the Pescara machine are found in Appendix IV.

APPENDIX II

Analog Computer Symbols[2]

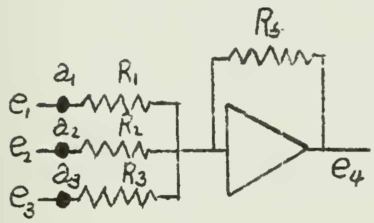
Circuit	Symbol	Function

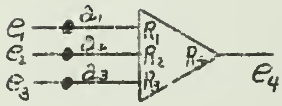
1. Potentiometer:



$$e_2 = a\,e_1$$

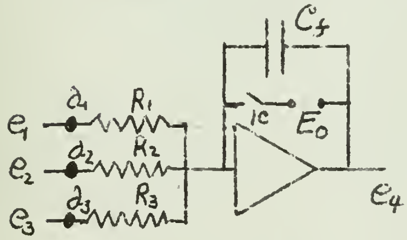
2. Summer:

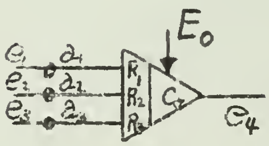




$$e_4 = -\left(\frac{a_1 R_f}{R_1} e_1 + \frac{a_2 R_f}{R_2} e_2 + \frac{a_3 R_f}{R_3} e_3\right)$$

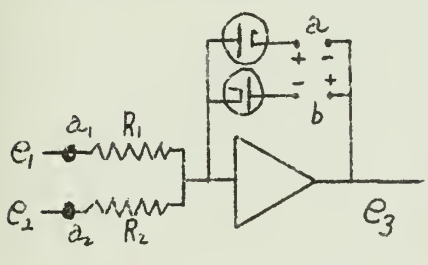
3. Summing Integrator:

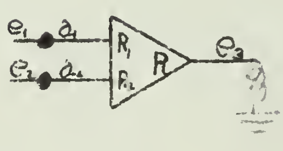




$$e_4 = -\int_0^t \left(\frac{a_1}{R_1 C_f} e_1 + \frac{a_2}{R_2 C_f} e_2 + \frac{a_3}{R_3 C_f} e_3\right) dt + E_0$$

4. Relay Driver:

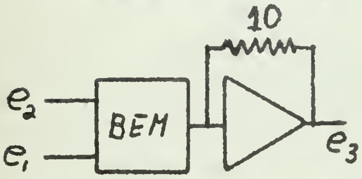


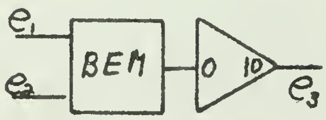


$$e_3 = \begin{cases} -a \text{ when } \dfrac{a_1 e_1}{R_1} + \dfrac{a_2 e_2}{R_2} > 0 \\ +b \text{ when } \dfrac{a_1 e_1}{R_1} + \dfrac{a_2 e_2}{R_2} < 0 \end{cases}$$

Circuit	Symbol	Meaning

5. Beeing Electronic Multiplier:

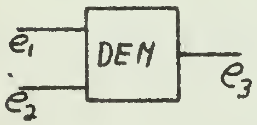




$$e_3 = +\frac{1}{50} e_1 e_2$$

Note: Direct multiplier output is approximately $-\dfrac{e_1 e_2}{5 \times 10^8}$

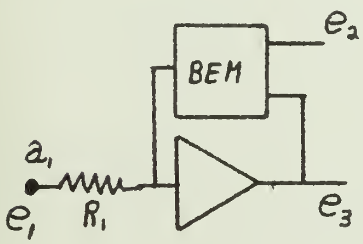
6. Donner Electronic Multiplier:

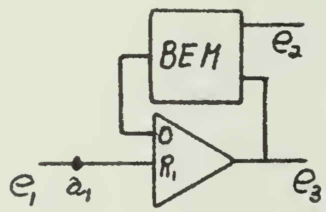


—

$$e_3 = -\frac{1}{100} e_1 e_2$$

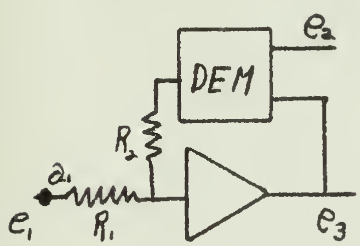
7. Division using Boeing Multiplier:

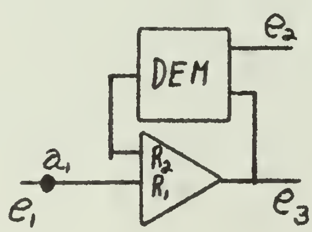




$$e_3 \approx -\frac{5 \times 10^8 a_1}{R_1} \frac{e_1}{e_2}$$

8. Division Using Donner Multiplier:





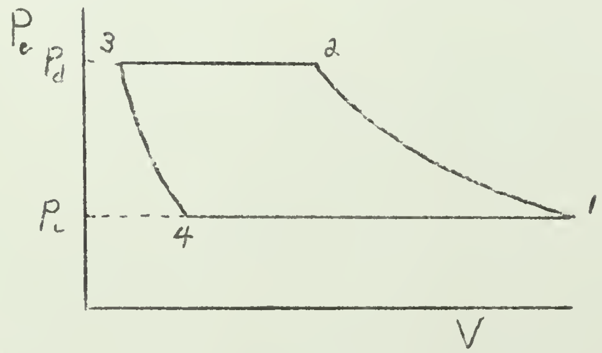
$$e_3 = -100 \frac{a_1 R_1}{R_2} \frac{e_1}{e_2}$$

Other Special Circuits are explained in text.

APPENDIX III

Heat Removal from the Compressor

The compressor cycle is given by the figure below:



For the processes 1-2 and 3-4, the rate of heat transfer is given by

$$\frac{dQ_c}{dt} = A_c\left(\frac{K-n_c}{K-1}\right)P_c\frac{dy}{dt}.$$

It shall be assumed that the delivery and intake processes involve no energy transfer as heat.

For process 1-2

$$P = P_1\left(\frac{y_1}{y}\right)^{n_c},$$

or

$$dQ = \left[A_c\left(\frac{K-n_c}{K-1}\right)P_1 y_1^{n_c}\right]\frac{dy}{y^{n_c}}.$$

Upon integrating between the volume limits 1 and 2,

$$Q_{12} = A_c\frac{K-n_c}{(K-1)(n_c-1)}P_1 y_1^{n_c}\left[\frac{1}{y_1^{n_c-1}} - \frac{1}{y_2^{n_c-1}}\right],$$

$$= A_c\frac{K-n_c}{(K-1)(n_c-1)}P_1 y_1\left[1 - \left(\frac{y_1}{y_2}\right)^{n_c-1}\right].$$

Now

$$A_c y_1 = V_1,$$

and

$$P_1 V_1 = m_1 R T_0.$$

Therefore

$$Q_{12}' = -\frac{K-n_c}{(K-1)(n_c-1)}Rm_1\left(T_2 - T_1\right).$$

By a similar process, Q_{34} is given by

$$Q_{34} = \frac{K - n_c}{(K-1)(n_c - 1)} R m_3 \left[T_3 - T_4 \right].$$

The net energy transfer is

$$Q_c = Q_{12} + Q_{34} = \frac{K - n_c}{(K-1)(n_c - 1)} (T_2' - T_1')(m_3 - m_1) R,$$

now

$$m_3 - m_1 = -m_d.$$

Therefore

$$Q_c = -\frac{K - n_c}{(K-1)(n_c - 1)} m_d R T_1' \left[\frac{T_2'}{T_1'} - 1 \right].$$

$$= -\frac{(K - n_c)}{(K-1)(n_c - 1)} m_d R T_0' \left[\left(\frac{P_d}{P_i} \right)^{\frac{n_c}{n_c - 1}} - 1 \right].$$

APPENDIX IV

PROGRAMMING THE ANALOG COMPUTER FOR THE PESCARA MACHINE

I. Data.

Listed below is the combined data from the original data and the results given by London:[1]

A. Geometry:

A_e	0.979 ft^2	(*)
A_c	5.73 ft^2	(*)
A_b	6.77 ft^2	(*)
M	2080 lb	(‡)
x_p	10.84"	(†)
c	1.24"	(†)
b	21.14"	(‡)

B. Operating Conditions:

P*	4.85	(*)
P_t	64.5psia	(*)
P_b^o	293 psia	(‡)

C. Assumed Quantities:

C	50 lb
v	0.1 lb sec/ft
n	1.30
n'	1.40
n_c	1.30

*Original Data.
†Calculated from original data.
‡From London's solution.

II. Assignment of Scale Factors.

A. Pressure Scaling.

1. The Engine.

$P_p = 64.5$ psia

The compression ratio is estimated as 10

Therefore Peak pressure $\approx$ 1500 psia

Let $\alpha_{p_E} = 30$ psia/volt = 4320psfa/volt

Then $\overline{P_p} = 2.15$ volts.

2. The Compressor.

$P_i = 14.0$ psia

$P_d = 14.0 \times 4.85 = 67.9$ psia

Let $\alpha_{p_c} = 10$ psia/volt = 1440 psfa/volt

$\overline{P_i} = 1.40$ volts

$\overline{P_d} = 6.79$ volts

3. The Bounce.

Let $\alpha_{p_b} = 20$ psia/volt = 2880 psfa/volt

$\overline{P_b^o} = 14.65$ volts

B. Displacement Scaling.

Let $\alpha_x = \frac{1}{2}''$ per volt = 1/24 ft per volt

Then: $\overline{x_p} = 21.68$ volts

$\overline{c} = 2.46$ volts

$\overline{b} = 42.28$ volts

C. Velocity Scaling.

It is known that

$x \approx 1 + 9 \cos 62.8t$ inches

$\dot{x}_{max} = 50$ ft/sec

Let $\alpha_{\dot{x}} = 2$ ft/sec/volt

D. Acceleration Scaling.

$\ddot{x}_{max} \approx 3000$ ft/sec^2

let $\alpha_{\ddot{x}} = 100$ ft/sec^2/volt

E. Time Scaling.

let $\alpha_t = 100$ seconds computer time/1 second real time

F. Friction Force Scaling.

let $\alpha_{F_F} = 20$ lbs/volt

then $\bar{C} = 2.5$ volts

G. Summary.

α_{P_E} = 4320 psfa/volt

α_{P_c} = 1440 psfa/volt

α_{P_b} = 2880 psfa/volt

α_x = 1/24 ft/volt

$\alpha_{\dot{x}}$ = 2 ft/sec/volt

$\alpha_{\ddot{x}}$ = 100 ft/sec^2/volt

α_{F_F} = 20 lb/volt

α_t = 100 sec/sec

III. Determination of Circuit Quantities

A. Pressure and Displacement Integrators (Amplifiers 1, 7, 10, and 16.)

From equation 15.3.10:

$$\frac{a_1}{R_1 C_{f1}} = \frac{a_7}{R_7 C_{f7}} = \frac{a_{10}}{R_{10} C_{f10}} = \frac{a_{16}}{R_{16} C_{f16}} = \frac{\alpha_{\dot{x}}}{\alpha_x \alpha_t} = 0.480.$$

Let R's = 1 meg

C's = 1 μf

a's = 0.480

B. The velocity Integration (Amplifier 15)

From equation 15.3.7

$$\frac{a_{15}}{R_{15} C_{f15}} = \frac{\alpha_{\ddot{x}}}{\alpha_{\dot{x}} \alpha_t} = 0.500$$

Let

$$C_{f15} = 1 \mu f$$
$$R_{15} = 1 \text{ meg}$$
$$a_{15} = 0.500$$

C. Summation of Forces (Amplifier 14)

From equations 15.3.2 to 15.3.5:

$$\frac{a_{141} R_{f14}}{R_{141}} = \frac{A_E \alpha_{P_E}}{M \alpha_{\ddot{x}}} = 0.654$$

$$\frac{a_{142} R_{f14}}{R_{142}} = \frac{A_C \alpha_{P_C}}{M \alpha_{\ddot{x}}} = 1.278$$

$$\frac{a_{143} R_{f14}}{R_{143}} = \frac{A_B \alpha_{P_B}}{M \alpha_{\ddot{x}}} = 3.02$$

$$\frac{a_{144} R_{f14}}{R_{144}} = \frac{\alpha_{F_F}}{M \alpha_{\ddot{x}}} = 0.0031$$

Let:

$R_{f14} = 1 \text{ meg}$

$R_{141} = 1 \text{ meg}$, then $a_{141} = 0.654$

$R_{142} = .5 \text{ meg}$, then $a_{142} = 0.639$

$R_{143} = .1 \text{ meg}$, then $a_{143} = 0.302$

$R_{144} = 10 \text{ meg}$, then $a_{144} = 0.0310.$

D. Formation of Frictional Force (Amplifier 13)

From equation 15.4.2

$$\frac{a_{13} R_{f13}}{R_{13}} = \frac{\alpha_{\dot{x}} V}{\alpha_{F_F}} = 0.01$$

Let:

$R_{f13} = 0.1$ meg

$R_{13} = 10$ meg, then $a_{13} = 1.00$

APPENDIX V

CALCULATION OF RESULTS FROM THE ANALOG COMPUTER SOLUTION

1. Frequency.

The measured frequency was six cycles per computer minute, corresponding to a real frequency of 600 cycles per minute.

2. Stroke and clearances.

Stroke from figure 28 - 17.5 inches.

Engine effective stroke - 9.74 inches.

Engine clearance - 1.10 inches.

Compressor clearance - 2.33 inches.

Bounce clearance - 2.54 inches.

3. Compressor delivery.

From equation 5.5.2: $m_d = P_i A_c Y_i / RT_o$.

$$\bar{Y}_i = 29.9 \text{ volts} = 14.95 \text{ in.}$$
$$P_i = 14.0 \text{ psia}$$
$$A_c = 5.73 \text{ ft}^2$$
$$T_o = 530^\circ \text{ R}$$

$$m_d = \frac{(14.0)(144)(5.73)(14.95)}{(53.3)(530)(12)} = 0.500 \text{ lb}$$

4. Engine charge mass.

From equation 5.4.5:

$$m_e/V_p = 0.0113 \, P_t^{0.7}$$
$$= 0.209 \text{ lb/ft}^3$$

$$V_p = \frac{(10.84)(0.979)}{12} = 0.884 \text{ ft}^3$$

4. (Continued)

$$m_e = (0.209)(0.884) = 0.185 \text{ lb.}$$

5. Ratio of compressor to engine air.

$$m_e/m_d = 0.185/0.500 = 0.370.$$

6. Mass of fuel.

From equation 8.2 : $m_f HV = \frac{k}{k-1} P_2 (V_3 - V_2)$

$$\bar{P}_2 = 51 \text{ volts} = 1530 \text{ psia}$$
$$\overline{\Delta V} = 3.6 \text{ volts} = 1.80 \text{ in.}$$

$$m_f HV = \frac{1.4}{0.4}(1530)(144)\left(\frac{1.8}{12}\right) = 115{,}500 \text{ ft-lb}$$

Assuming a LHV of 18,200 BTU/lb;

$$m_f = \frac{115{,}500}{(18{,}200)(778)} = 0.00818 \text{ lb.}$$

7. Air-fuel ratio.

a-f ratio = 0.185/0.00818 = 22.7

8. Fuel rate.

Fuel rate = (0.00818)(2)(600)(60) = 589 lb/hr

9. Gas rate.

Total mass per stroke per side = 0.508 lb.

Gas rate = (0.508)(2)(600)(60) = 36,600 lb/hr

10. Friction work.

From equation 12.2.5; $W_f = 2Cs + \frac{\pi^2 vfs^2}{2}$

$$W_f = (2)(50)\left(\frac{17.5}{12}\right) + \frac{(\pi^2)(0.1)}{2}\left(\frac{17.5}{12}\right)^2 = 153 \text{ ft-lb}$$

11. Heat lost by compressor.

From equation 10.2, $Q_c = \frac{m_d(K-n_c)RT_0}{(K-1)(n_c-1)}\left[(P^*)^{\frac{n_c}{n_c-1}} - 1\right]$

$$Q_c = \frac{(0.500)(53.3)(530)(0.1)}{(0.4)(0.3)}\left[(4.85)^{0.231} - 1\right]$$

$= 5170$ ft-lb.

12.Heat lost from engine.

From equation 8.1, $Q_{12} = \frac{(K-n)P_1V_1}{(K-1)(n-1)}\left[\left(\frac{V_2}{V_1}\right)^{n-1} - 1\right]$

$$Q_e = \frac{(0.1)(0.979)(64.5)(144)(10.84)}{(0.4)(0.3)(12)}\left[(8.5)^{0.3} - 1\right]$$

$=$ 8,000 ft lb

13. Temperature of exit gasses.

From equation 51 5.5.1:

$$m_d h_o + m_f HV = (m_f + m_d)h_t + Q_{rej}$$

$m_d h_o$ =	(0.500)(778)(126.7)	49,200 ft-lb
$m_f HV$ =		115,500 ft-lb
Q_{rej} =		13,300 ft-lb

from which

$$h_t = \frac{151,400}{(778)(0.508)} = 382 \text{ BTU/lb}$$

From the tables:

$T_t = 1550°R.$

14. Air Horsepower.

From the gas tables, an isentropic expansion from the state given in item 13 yields an ideal work output of 139 BTU/lb.

$$AHP = \frac{(139)(778)(0.508)(2)(600)(60)}{33,0000} = 1200$$

15. Work balance.

Area under the engine curve = 340 volts^2.

$W_e = (340)(4320)(0.979)(1/24) = 60,000$ ft-lbs

Area under the compressor curve = 160 volts^2.

$W_c = (160)(1440)(5.73)(1/24) = 55,200$ ft-lbs.

$W_f =$ 153 ft-lbs.

Discrepancy: 4,650 ft-lbs

APPENDIX VI

THE BOEING ELECTRONIC ANALOG COMPUTER

The Boeing Electronic Analog Computer (BEAC) consists of an equipment rack, in which is mounted a number of operational amplifiers, coefficient potentiometers, limiters, multipliers, and voltage sources. The rack also contains a common power supply for the various items. For this investigation, two such racks were used. A photograph of this equipment is shown in Fig. 31. Also illustrated is the x-y plotting device, and the time recorder. The hook-up to solve a particular problem may be made in two ways:

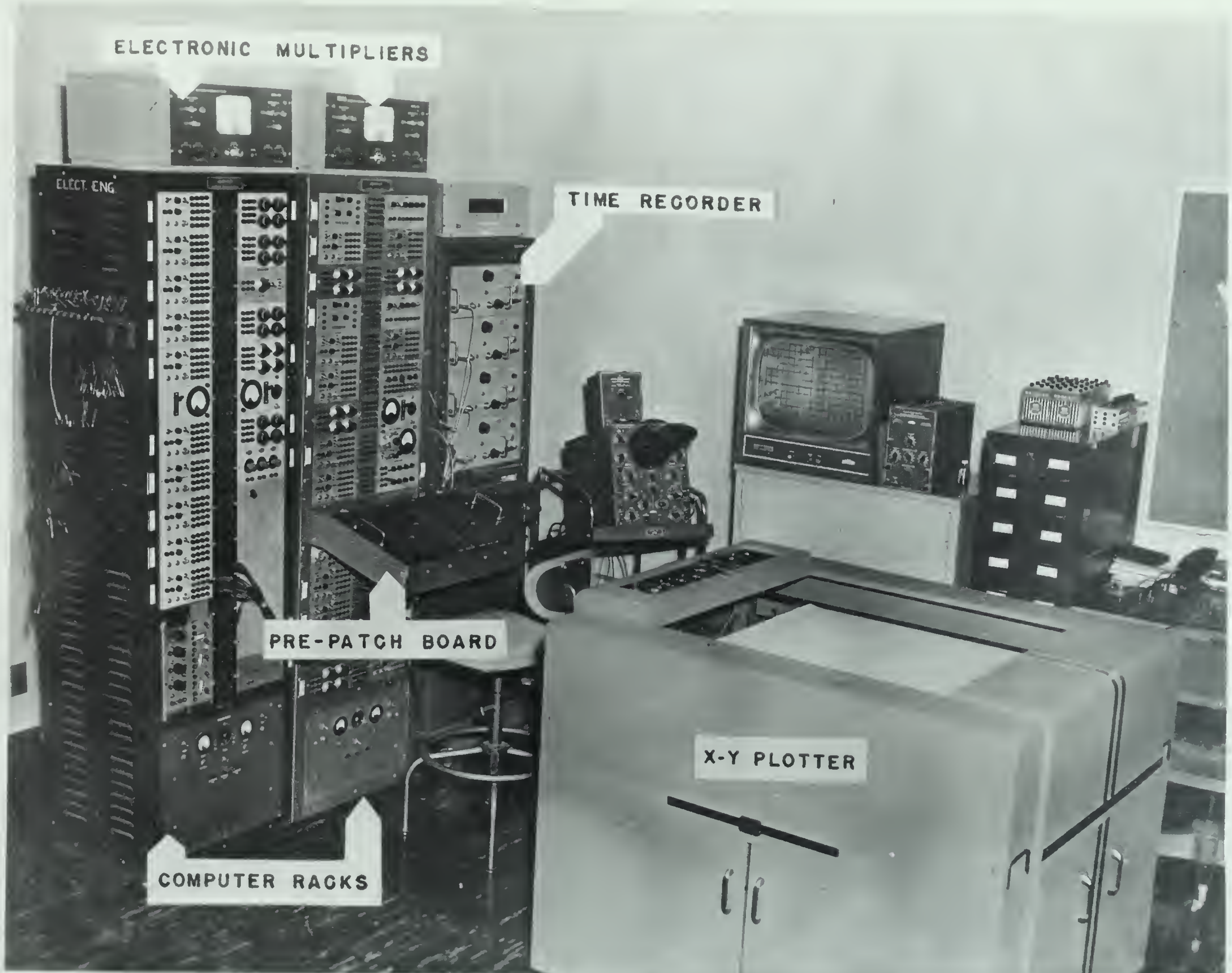
1. Directly on the front of the computer, or

2. On a pre-patch board, which is then mounted on the front of the machine. All the necessary connections are led in beneath the pre-patch panel.

The Boeing operational amplifier is a high-quality DC amplifier having an open loop gain of approximately 50,000. This amplifier will operate with stability with any feedback ratio up to unity. It may be used for summation, integration, differentiation or more complex operations. The amplifier remains linear as long as the output is less than 50 volts. A glow tube circuit shorts the output at about 51 volts, thus this value is an absolute operating limit.

The general operation of the computer is quite stable. This stability may be enhanced further through the use of an automatic balancing system, which restricts the amplifier drift to less than a few millivolts per month.

Detailed descriptions and operating procedures may be found in the manufacturer's data.[3]


ELECTRONIC MULTIPLIERS
ELECT. ENG.
TIME RECORDER
PRE-PATCH BOARD
X-Y PLOTTER
COMPUTER RACKS